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THE AEROPLANE HANDBOOK

EDITED BY

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(LATE R.E.)

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INTRODUCTORY NOTE

In compiling this pocket-book the aim has been to provide a manual for the use of those who are concerned in the manufacture and use of aircraft.

Though it may contain a considerable amount of information of interest to the pilot, it is not primarily intended for his use, for there is not a great deal in his vocation that can be learnt from books.

Technical information, which has advanced by leaps and bounds during the war, was largely treated as confidential by the authorities, and was therefore unavailable until some time after the armistice. This fact, together with the continued advance of the science, will necessitate the frequent addition of new matter and the revision of articles which, though at the time of their being written may have been the *dernier mot*, become less up to date by the time that they have emerged from the present much congested channels of the printers' trade.

To the designer, the draughtsman, the manufacturer, and the conveyer of goods or passengers by air, the Editor hopes that the present volume may be of use and that information of service to the controllers and organizers of airways—information of a commercial nature, that is – may be patiently awaited with the advent of further editions, such information being at the present moment, if not totally lacking, wholly unreliable as a guide to the future, and even then jealously guarded.

As the railway or hydraulic engineer now turns to one of the established engineering pocket-books for complete information on his own subject, so in due course, we hope, will the air engineer find all the information that he wants in this and future issues of the "Aeroplane Handbook."

THE EDITOR.

London,
December, 1919.

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THE
AEROPLANE HANDBOOK

AEROPLANE AND AERONAUTICAL INSTRUMENTS

By Capt. A. F. C. POLLARD, R.A.F.

The standard aeronautical instruments briefly described in this article have been divided into two main categories—Flying Instruments and Engine Instruments.

Flying Instruments include those instruments the indications of which help the aeronaut to judge his position, and the rate at which he is changing his position, in space, as well as the direction in which he may be moving. Those instruments which are merely computers, or mechanical calculators, will also be included in this class, since they are used to replace the draughting board or logarithmic tables in making rapid and approximate calculations while in the air, with the variables given by the indicating instruments. Only the most successful and commonly used computer will be described, though a great many have been devised and tried.

Engine Instruments include those instruments the indications of which enable the pilot to watch the performance and behaviour of his engine or engines, and the correct functioning of the various parts of the engine installation, the smooth working of which is so necessary to uninterrupted flight. Three of the standard instruments will be briefly described.

FLYING INSTRUMENTS

I.—Instruments for measuring the Height of Aircraft above the Surface of the Earth.

The problem of measuring accurately the height of aircraft above the surface of the earth is a difficult one, and no instrument has yet been devised which solves this problem really satisfactorily.

The Altimeter.—The instrument used universally by aeronauts is the *altimeter,* which is merely an aneroid barometer measuring the pressure of the atmosphere. An approximation to the true height is then obtained from an assumed relation between atmospheric pressure and height. In the first place, the atmosphere is assumed to have a constant temperature of 10° C. throughout, and, in the second place, a ground pressure of 760 mms. of mercury, which is equivalent to 29·92 inches of mercury. English makers take the conventional figure of 29·90 inches, which is about 14·71 lb. weight per square inch.

The following table gives the relation between height and pressure in such an ideal atmosphere, and the height thus obtained from a pressure-measuring instrument is known as the *isothermal height* in distinction from the *true height*, to which it can only be a rough approximation.

Isothermal Height in Feet.	*Barometer (Inches).*	*Isothermal Height in Feet.*	*Barometer (Inches).*	*Isothermal Height in Feet.*	*Barometer (Inches).*
0	29·90	11,000	19·95	21,000	13·80
1,000	28·82	12,000	19·23	22,000	13·31
2,000	27·78	13,000	18·53	23,000	12·83
3,000	26·77	14,000	17·86	24,000	12·36
4,000	25·81	15,000	17·22	25,000	11·92
5,000	24·88	16,000	16·61	26,000	11·49
6,000	23·98	17,000	16·00	27,000	11·07
7,000	23·11	18,000	15·42	28,000	10·67
8,000	22·27	19,000	14·86	29,000	10·29
9,000	21·47	20,000	14·33	30,000	9·92
10,000	20·69				

The instrument consists of a flat, shallow, cylindrical box *B* (see Fig. 1) with corrugated ends, partially exhausted, so that an increase of atmospheric pressure tends to make the circular ends bulge inwards towards one another. The central part of the lower surface of the box is attached to the base plate, and a flat spring *S*, supported at one end on a brass bracket *C* attached to the

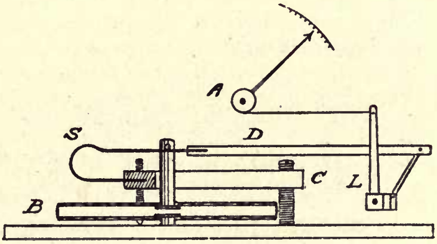

FIG. 1.

base plate by three screws, is linked at its free end to the central part of the upper surface of the box, tending to make the circular ends bulge outwards or away from one another.

In this manner the atmospheric pressure is balanced against the spring, and the movement of the upper surface of the box or the free end of the spring, which is linked to the upper surface of the box, indicates change of atmospheric pressure. As the range of this movement is extremely small, it has to be mechanically magnified by a series of levers. To the free end of the spring a rod *D* is attached, which in temperature compensated instruments is a compound strip of iron and brass. The end of this rod is linked to the short arm of a bell crank lever *L*, and the long arm of the lever is connected to a small drum *A* by a flexible chain. The pointer of the instrument is rigidly attached to the spindle of the drum. A hair spring is carried by the spindle, to which the pointer and drum are fixed in order that the backlash of the mechanism may be taken up. The short arm of the lever *L*, is so arranged that its length from the axis of rotation is adjustable by a set screw, in order that the openness of the scale may be altered. The scale is calibrated in feet, but it would be more useful to calibrate the instrument in inches of mercury and compute the height with a computer, after the temperature of the air has been ascertained from a strut thermometer, the ground pressure and temperature being known. The altimeter could then be used to give a more correct air speed, when used in conjunction with the air speed indicator, than is possible with the present calibration.

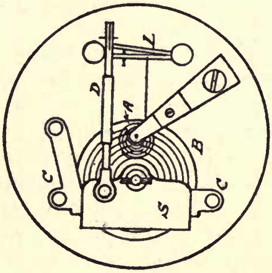

FIG. 2.

A top view of a practical form of the apparatus is shown in Fig. 2 with dial and pointer removed, the parts being lettered to correspond to the scheme of the working parts shown in Fig. 1.

The errors to which this instrument is subject are chiefly:

(i.) Errors due to the assumption of the ideal atmosphere already described and the constancy of temperature throughout this atmosphere.

The altimeter may be adjusted to read zero at the ground level whatever the barometric pressure may be, but, although this be done, an error of 250 feet may occur after a flight of 200 miles on account of the heterogeneity of the atmosphere. In the same way the pressure may change with the time at the same place, so that an instrument set for zero may be in error to as much as 100 feet in a few hours. The temporary changes of pressure due to eddies or gusts are too small to cause sensible error. Further, when the temperature of the atmosphere falls uniformly with increase of height, the true height can be obtained, to a better approximation, by multiplying the indicated height by the factor $273+t/283$, where t is the mean temperature, in degrees Centigrade, throughout the column of air, at the summit of which the instrument is supposed to be.

An indicated height of 20,000 feet may be in excess of the true height by 2,000 feet in winter on account of temperature error. This is an error of 10 per cent., and alone shows what a rough instrument the altimeter is.

(ii.) Instrumental errors. Of these the most important are due to—

(*a*) Imperfect temperature compensation.

Although the standard service instruments are compensated for ranges of temperature from −20° C. to 45° C. satisfactorily for ground pressures, this compensation is not found to be satisfactory for all pressures, and the corresponding error may be as much as 1,000 feet at high altitudes and low temperatures.

(*b*) Hysteresis or elastic fatigue of the metal of the aneroid box, which depends upon the rate of change of pressure.

This error shows itself during ascent or descent, and is usually greater during and after descending than after ascending or climbing. No instrument is accepted which shows an error of more than 550 feet during an ascent and descent at the rate of 1,000 feet per minute. Most accepted instruments then show an error of two-thirds of this maximum.

II.—Instruments for measuring the Position of Aircraft with Reference to the Latitude and Longitude, and Variations of these Co-ordinates.

By this heading it is not intended to convey the impression that the instruments classed under it actually indicate the latitude and longitude of aircraft, but indicate the position of the foot of the normal drawn from the aircraft to the earth's surface, and the speed and the direction in which it may be moving with reference to these co-ordinates *in plano.*

Three of the standard instruments and a useful protractor will be briefly described.

1. **The Compass.**—A very short description of the mechanical details of this well-known instrument will be given only.

The compass essentially consists of a card carrying a system of magnets, pivoted at its centre so that it may turn freely and be stable in any latitude. The card is enclosed in a liquid tight case, with a glass window so that the card can be seen, and the case is filled with a non-freezing liquid, such as alcohol. The card usually carries a float so that the pivot pressure may be relieved and the friction reduced. The case contains an expansion chamber to allow the liquid to alter its volume with change of temperature.

The card is divided into 360 degrees, with the N point marked zero, in clockwise direction through E, S, and W. The points of the compass are also marked.

When the compass is mounted in aircraft, usually some fixed mark on the casing or bowl, called the Lubber Line, is so placed that a vertical plane passing through it and the centre of the card is parallel to the fore and aft vertical plane of the machine.

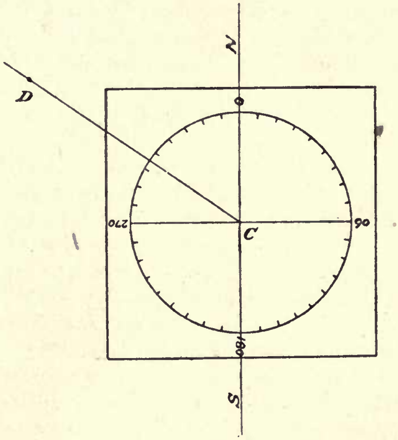

FIG. 3.

2. **The Creagh-Osborne Protractor.**—This protractor is a very simple and useful instrument for laying off bearings, especially for those unaccustomed to compass work.

It consists of a piece of transparent celluloid sheet (see Fig. 3) cut into a square with a circle described about the middle point, and divided into degrees from 0 to 360 similarly to the compass card. A silk thread is attached to the centre, and the protractor is laid upon the map with the centre *C* on the observation point. The thread is then stretched across the objective *D*, and if the north and south line of the protractor is parallel to the true or magnetic north and south line on the map, it is evident that the intersection of the silk thread with the graduated circle gives the bearing of the objective.

The top and bottom edges of the protractor are engraved with scales in metres and yards, which are generally used on continental maps. These scales have a representative fraction usually of $\frac{1}{80000}$ and $\frac{1}{100000}$.

3. **Aero Bearing Plate or Pelorus.**—The aero bearing plate for the use of observers is simply a compass card fitted with sights and a radius bar, in order that the amount of drift and the ground speed can be estimated.

It consists of three relatively rotatable rings, the inner one of which is graduated in the same manner as the compass card into degrees; the outer ring bears the fore and back sights and a radius bar. The middle ring serves to carry these two rings, and constitutes a base plate furnished with a securing bracket, which slips over a shoe fixed to the side of the fuselage of the machine. This ring is marked with two arrow-heads on a diameter forming the lubber mark, and this diameter is placed parallel to the fore and aft axis of the machine. The radius bar consists of two parallel strips between the two sights. The sight-bearing ring can then be rotated until objects on the ground appear to move parallel to the radius bar. After this adjustment is made the drift can be read off the graduated ring, when this has been set so that the lubber mark is opposite the same reading which is opposite the lubber line on the compass card.

In the Sperry Synchronised Drift Sight the graduated ring is automatically set to correspond with the compass card.

4. **Aero Course and Distance Calculator.**—This instrument solves all problems of the vector* triangle, the sides of which represent air speed, which is given by the air speed indicator in conjunction with the altimeter and compass, the velocity of the wind, and the ground speed on the velocity of the aircraft relative to the earth. It replaces the draughting board, and though of small dimensions gives results the accuracy of which is all that can be hoped for with the existing indicating instruments.

The Service 7″ Mark II. instrument consists of two celluloid discs pivoted at the centre and rotatable over one another. The lower disc of opaque white celluloid is 7″ in diameter, while the upper disc is of transparent celluloid $6\frac{1}{2}''$ in diameter. This upper disc has engraved upon it a network of squares, five to the linear inch, bounded by a circle of $2\frac{2}{5}''$ radius, and one diameter of this circle is marked as an arrow. The side of each square represents 10 units, and the radius is therefore 120 units. Two arms, also pivoted at the centre and held in position stiffly by a spring washer, carry on their upper surfaces scales similar to that on the upper disc. A small steel spring pointer slides along each arm.

Corresponding to the periphery of the squared circle on the upper disc, the compass card is engraved upon the lower white

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* See footnote on p. 10 for the term *vector*.

disc and marked every five degrees, as well as every two points of the compass.

The circular edge of the upper disc is engraved with a log scale from 10 to 270, and a corresponding scale is engraved on the lower disc, so that the outer edge of the calculator can be used in exactly the same way as a slide rule for arithmetical calculations. The upper disc bears three studs to enable it to be easily rotated over the lower disc with the gloved hand.

The following problems are worked out to illustrate one use of the calculator:

PROBLEM I.

To determine the magnitude and direction of the wind, when the air speed, compass course, the ground speed, and drift have been observed.

1. Set one arm and pointer to the drift and the ground speed —*i.e.*, to the ground speed vector.

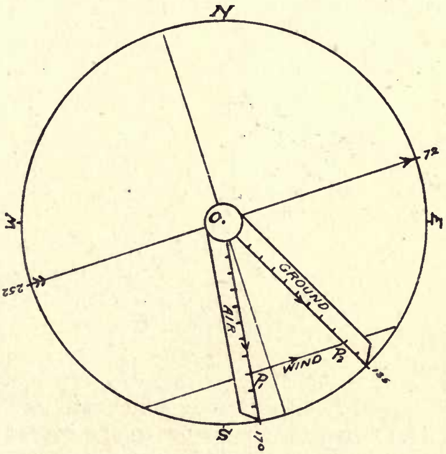

FIG. 4.

2. Set the other arm and pointer to the compass course and air speed—*i.e.*, to the air speed vector.

3. Set the upper disc so that the arrow is parallel to the line joining the two pointers.

Then the arrow gives the bearing of the wind and the distance between the two pointers the magnitude of it.

EXAMPLE (see Fig. 4).—Compass reads 170° and the air speed is 90 miles per hour. The drift is 135° and the ground speed 100 miles per hour.

Set the arms and pointers P_1 and P_2 in the manner indicated in the diagram. It is evident that the wind must blow from

P_1 to P_2 in order that the aircraft having travelled from O to P_1 in the air has only moved from O to P_2 relative to the ground. Turn the upper disc so that the arrow, or the nearest line in the squared network, is parallel to $P_1 P_2$. The wind is then seen to be blowing from 252°, and is 54 miles per hour in magnitude.

PROBLEM II.

The wind being known, to determine the course to steer in order to make good a given course.

EXAMPLE (see Fig. 5).—Assume the wind to be from 252 at 54 as already determined, the air speed 90, and the course to be made good 145°.

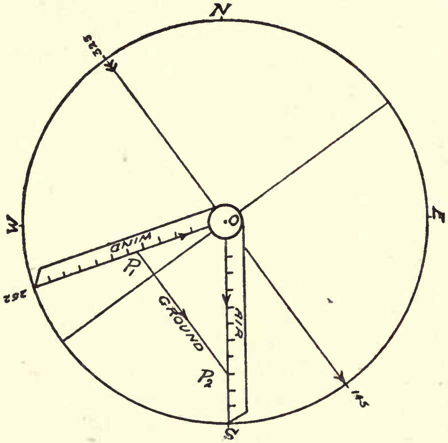

FIG. 5.

Set one arm parallel to the arrow, which was at 252° and the pointer P_1 at 54. Then set the arrow to the course to be made good. Turn the other arm with its pointer P_2 at 90 until P_1 and P_2 line on a line parallel to the arrow.

Then it is seen that the aircraft must now be steered due south, and the ground speed is 88 miles per hour.

Instead of miles per hour the nautical knot might be taken as the unit of velocity.

Since 1 nautical mile=185,325 cms.
1 English mile=160,934 cms.
1 knot=1 nautical mile per hour.
=1·152 miles per hour.
=51·47 cms. per second.
1 mile per hour=0·868 knots.
=44·7 cms. per second.

The use of the instrument for all problems involving the solution of a vectorial triangle must now be obvious.

III.—Instruments for indicating the Orientation of Aircraft.

No gravity controlled instrument of the nature of a simple pendulum can possibly indicate the true vertical in aircraft at all times. If the machine is flying level and accelerating or retarding its motion, the accelerating or retarding force compounded with the vertical acceleration due to gravity gives a resultant inclined to the latter fore and aft.

If the machine is performing a turn correctly banked, the resultant of the centrifugal force or the acceleration due to the rate of change of direction of the aircraft's path, directed along the radius of the turn and outwards, and the acceleration due to gravity, directed vertically downwards, is an acceleration plumb to the machine and the machine flies tilted. In consequence of this a pendulum would remain vertical with regard to the machine and would give no indication of the aircraft's orientation.

Gyroscopic motion, however, offers a complete solution to the problem of determining the true vertical in aircraft at all times, and the practical design of a suitable gyro-top is gradually being evolved.

1. **The Clinometer.**—This instrument is an "inverted" pendulum, and consists of a curved glass tube sealed at both ends. The tube contains a non-freezing liquid such as alcohol and a small bubble. The tube is mounted so that its highest point is in the middle, and a scale of degrees is marked on the metal mounting, 20° on either side of the zero or middle position.

The liquid in the tube represents the mass of a pendulum, and the bubble moves in the opposite direction to that in which a pendulum would move under the same circumstances. When the level is at rest, the bubble is at the highest point of the tube. If the clinometer be mounted across the axis of the machine, the bubble, during a turn, takes up such a position that the tangent to the tube at the bubble lies in a plane parallel to the correct banking plane, and when the machine turns without side slip the bubble will be at the highest point. During an "underbank" the bubble will be on that side of the zero position which is nearest the centre of the turn, and in an "overbank" the bubble will move to the opposite side.

Of the Service instruments, the Mark V. and Mark V.*a* have an electric lamp fixed to one end of the tube, so that the bubble can be seen at night.

This instrument is not of very great utility and, generally speaking, the pilot, feeling a draught of cold air on one side of the face during an improperly banked turn, will learn to fly by the feel of the machine, rather than entirely depend upon the bubble, by moving his stick in the same direction in which the bubble has gone.

IV.—Instruments which Function by Means of the Air and measure with Reference to the Air.

The correct measurement of the speed with which aircraft move through the air is important for navigational purposes. It is evident that if the machine is being carried in some direction by a wind, as well as moving forward by its own power in that wind, its motion relative to the earth is the resultant of these two velocities. Then in order that the aircraft may be correctly steered to make good a given course the magnitude and direction of the wind must be known; that is to say, the wind vector* must be specified.

This can be done by the course and distance calculator as soon as the ground and air speed vectors are known. The direction of the air speed vector or the course to steer can then be at once computed (see Problem II., p. 8).

Since the aircraft is constantly moving into unknown winds, the navigator frequently has to make these computations in order that the pilot may reach a given objective with moderate precision, and it is at once evident how necessary it is to determine quickly the ground speed and the air speed vectors.

The former is determined with the bearing plate already described, and the latter with the air speed indicator corrected for air density.

1. **The Air Speed Indicator and Pressure Heads.**—The air speed indicator is a delicate pressure gauge, consisting of an air-tight box divided into two compartments by means of an elastic diaphragm which separates the two compartments in an air-tight manner. The compartments are connected to the pressure heads, and when these are in the air stream, one compartment is maintained at a higher pressure than the other, and the diaphragm moves or bulges out from the side of higher pressure. This movement is transmitted by suitable mechanism to the indicating pointer.

There are three indicators in use—the Hollocombe Clift indicator, the Ogilvie indicator, and the R.A.F. speed indicator.

In the first two the air-tight box is separated into two compartments, by a thin leather or oiled silk diaphragm in the Clift indicator, and by a thin rubber diaphragm in the Ogilvie indicator, which is an old Admiralty type. This latter is not considered a good design, as the rubber tends to change its elastic properties in cold atmospheres and deteriorates with time. The R.A.F. speed indicator (see Fig. 6) is differently constructed, and the dia-

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* A *vector* quantity has direction as well as magnitude, and until these two *scalar* quantities are known the vector is not specified.

phragm is replaced by two thin metal aneroid boxes, *B*, connected to one another and the Pitot head by branched piping, whilst the air-tight case containing the aneroid boxes is connected to the static head of the pressure heads.

In all three instruments the movement of the central point of the diaphragms and the aneroid boxes is transmitted by suitable mechanism to the pointer, which moves over a circular scale representing air speed in miles per hour. In the Clift indicator the transmitting mechanism consists of a spindle attached to the centre of the diaphragm working in a guide. The end of the spindle presses against the free end of a thin, flat spring, and with increased pressure the diaphragm bulges and deflects the spring proportionately to the pressure. Resting against the other side of the spring is the arm of a bell crank lever, the other arm of which engages with a slot in the radial arm of a toothed and pivoted quadrant. The quadrant engages with a pinion on the spindle of the pointer, and as the lever oscillates the quadrant turns about its pivot and rotates the spindle of the pointer. A hair spring on the spindle takes up backlash.

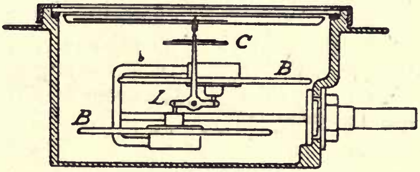


FIG. 6.

In the Ogilvie indicator the transmitting mechanism is more simple. A silk cord is attached to a small metal disc fixed to the centre of the rubber diaphragm, and passes over a small pulley to a drum on the spindle of the pointer. The spindle carries a hair spring which keeps the cord taut, and as the pressure increases the diaphragm bulges, pulls upon the cord, and turns the spindle.

The two aneroid boxes, *B*, of the R.A.F. speed indicator are held opposite one another by a rigid aluminium bracket (not shown in the diagram, Fig. 6) attached to the two outside faces of the boxes and the inside of the case of the instrument. The two inner faces of the boxes are thus free to bulge towards one another when an increase of pressure takes place within them. The central points of these two free faces are connected by pin joints to either side of a crank *L*, pivoted at its centre. The crank carries a lever arm which engages with a slot cut in the radial arm of a toothed quadrant *C*, but, unlike the Clift indicator, the slot is on the side of the pivot opposite to the quadrant. The toothed quadrant engages with a pinion on the spindle of the pointer, and a hair spring attached to the spindle takes up backlash.

The indicator is connected by two aluminium tubes, with rubber tubing jointing pieces, to the pressure heads, which are fixed to a strut as far out on the wing span as possible. These pressure heads project forwards into the air stream beyond the plane containing the front edges of the wings. The heads consist of two open tubes—one, the Pitot or *pressure head,* has its open end facing the air current; whilst the other, the *static head,* is closed at the end facing the air current, but is open to the atmosphere by a number of small holes piercing the curved surface of the tube some little distance behind the closed end. This closed end is stream-line shaped so that the air may slip past the small holes without eddy motion and produce no change of pressure inside the tube. The openings of both heads must be carefully finished so that eddy motion and consequently no erratic pressure be produced in the neighbouring air by burrs or irregularities. This is extremely important, and should be subject to special inspection.

When such pressure heads are placed in a current of air moving with velocity V, a pressure is set up in the Pitot tube, and no pressure change or approximately no pressure change is evoked in the static tube, provided these are connected to the indicator and no actual flow of air takes place during equilibrium with the elastic linkage of the mechanism.

If ρ is the density of the air and p is the difference of pressure in the two tubes, then it is found that

$$p = c\rho V^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where c is a constant, depending chiefly upon the inclination or the *angle of incidence* of the axis of the pressure heads to the air current.

The variation of c is negligible for an angle of incidence less than 10°, but becomes great for values greater than 10°.

It is evident from the above formula that to calibrate the instrument for a given density of the air is a straightforward matter, but the density of the air varies with temperature and altitude, and the correction for this variation of the density in order to determine the air speed V, when a machine is flying, is a matter which has caused a great deal of unnecessary dispute and some confusion.

If v is the volume and P the pressure of unit mass of gas at absolute temperature τ, then it is well known, for a gas far removed from its liquefying point like atmospheric air, that

$$Pv \text{ is proportional to } \tau;$$

$$\text{but the density } \rho \propto \frac{1}{v},$$

$$\text{hence } \rho \propto \frac{P}{\tau}.$$

For some other pressure P′ and temperature τ' the corresponding density $\rho' \propto P'/\tau','$

$$\text{hence } \frac{\rho}{\rho'} = \frac{P}{P'} \cdot \frac{\tau'}{\tau} \quad . \quad . \quad . \quad . \quad (2)$$

Now air speed indicators are calibrated in air at a pressure of 29·92 inches of mercury and temperature 16° C., which means that the density ρ' of the air is 1221 grammes per cubic metre.

Equation (2) then becomes

$$\frac{\rho}{1221} = \frac{P}{273 + t} \cdot \frac{273 + 16}{29 \cdot 92} \quad . \quad . \quad . \quad (3)$$

where ρ is the density of the air at pressure P inches of mercury and t is its temperature in degrees Centigrade.

If, now, V′ is the value of V in equation (1) for the standard conditions of calibration, then

$$\frac{V}{V'} = \sqrt{\frac{1221}{\rho}},$$

and substituting the value of the right-hand member of this equation by that given in (3)

$$V = V' \sqrt{\frac{29 \cdot 92}{273 + 16}} \sqrt{\frac{273 + t}{P}}$$

$$= V' \times \quad 0 \cdot 322 \quad \sqrt{\frac{273 + t}{P}}.$$

It is now a simple matter to correct the air speed reading given by the instrument in the air, provided that the barometric pressure, P, of the air near the aircraft and its temperature t can be determined. The pressure P can be read direct from the altimeter if this is calibrated in inches of mercury, instead of feet, and the temperature t can be read from a strut thermometer and the correction $\cdot 322 \sqrt{(273 + t)/P}$ applied in a few seconds to the reading V′ to give the true air speed V by means of a suitably designed computer.

This matter has been dealt with at some length because it is important for rapid and accurate present-day navigation that the three vector quantities in the vector triangle—the air speed, the wind, and the ground speed—be as accurately specified as possible.

It will now be evident to the reader how the five instruments, the compass, the altimeter, the strut thermometer, the bearing plate, and the air speed indicator, are intimately connected in the one object—*i.e.*, to reach a definite place on the map by the best route and in the least time.

ENGINE INSTRUMENTS

1. **The Revolution Indicator.**—This instrument indicates the rate at which the crank-shaft of the engine revolves, and is connected to it by means of a flexible drive. The flexible drive and the indicator rotate at one quarter of the engine speed, to prevent undue wear, and the reduction in speed is achieved by a special gear box connected to the engine shaft.

The most usual method for indicating the rate of revolution is based on the phenomenon of " centrifugal force."

If a spindle *R* (Fig. 7), to which are linked weights *A*, is rotated, a couple is set up by the centrifugal force *F*, tending to turn the arm carrying the weights into a position at right angles to the shaft *R*. This rotation of the arm *AA* is balanced by a spring *S*, and the position of the arm with regard to the shaft *R* is a measure of the rate of revolution, and is independent of the sense of rotation.

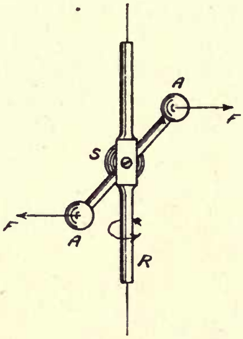


FIG. 7.

This device is used in the Elliott and Nicole Nielson revolution indicators. In the former instrument, one of the weights *A* is linked to a collar which slides upon the shaft *R*. The pin of a crank connected with a spindle engages with a groove in the collar. The other end of the spindle carries a toothed quadrant, which in turn engages with a pinion on the pointer spindle. When the shaft *R* rotates, the weights *A*, flying outwards, take up a definite position of equilibrium with the spring *S* for a given speed of revolution, and the movement is transmitted by the sliding collar to the pointer over the dial.

The Nicole Nielson instrument differs only in mechanical detail from the Elliott indicator.

Other types of indicator are made, such as the Elliott " electric generator type," in which the rate of revolution is measured by the current produced by the rotation of a small armature in a magnetic field.

2. **Pressure Gauges.**—These consist of oil and air pressure gauges, the former being used to indicate the pressure in the oil circulating system of aeroplane engines, and the latter to indicate the air pressure in pressure feed petrol tanks.

The majority of these gauges are constructed, after the Bourdon design, by Shaffer and Budenberg.

The moving element in this design is a hollow tube of elliptical cross section bent into the arc of a circle. The tube is closed at its free end and sweated to a brass box at the fixed end, the inside of the tube being connected with the pressure supply through the box. If the pressure inside the tube increases, it tends to straighten itself, and the amount of the movement of the free end depends upon the difference of pressure between the inside and outside of the tube.

If the variations of the atmospheric pressure outside the tube and inside the case of the instrument be of a sensible percentage of the pressure inside the tube, they will cause sensible error in the indications. In some of the standard gauges these variations are too small a percentage of the indications to cause sensible error.

The free end of the tube is in some instruments simply pinned eccentrically to the spindle of the pointer, and in others it is attached to the radial arm of a toothed quadrant which engages with a pinion on the pointer spindle.

Air pressure gauges are usually constructed with an eccentric scale to distinguish them from the oil pressure gauges, which have a concentric scale.

3. **Radiator Thermometers.**—Radiator thermometers are used to warn the pilot when the cooling water, in a water-cooled engine, is overheating, and the bulb of these instruments is inserted in the uptake pipe from the water jacket or the upper part of the radiator, near the water inflow.

The bulb is connected to the indicator on the instrument board by a copper pipe of 1 mm. bore. The bulb, which is cylindrical and about 4″ long and $\frac{7}{16}$″ outside diameter, is filled with ethyl ether and its vapour. The pressure of the vapour is measured by the indicator, which is similar to the Bourdon oil and air pressure gauges already described.

The principle on which the instrument depends is that the pressure of a vapour in contact with its own liquid depends only upon the temperature and not upon the volume it occupies.

It is extremely important, when fitting these instruments, to avoid contact of the transmitting tube with any hot parts of the engine, for if ether collects in the part so heated, the pressure of the vapour is produced hydrostatically throughout the whole system, and error is at once introduced into the readings of the indicator.

STRENGTH CALCULATIONS

By JOHN CASE M.A., A.F.R.Ae.S.

I.—Wings

1. **Normal Load** means the load taken by the wings when the machine is flying at a steady speed in a horizontal straight line.

2. **Load Factor.**—The method of stressing at present in use is to decide that a machine shall be able to bear n times normal load without the stresses exceeding a predescribed limit; the factor n is called the Load Factor.

3. **Greatest Loads on the Wings.**—These occur when the machine is made to have an angle of incidence approaching the stalling angle when the speed is much higher than that appropriate to such an angle. As a rule the greatest loads occur when the machine is flattened out from a vertical dive, or suddenly stalled when flying level at a high speed. The same effect is also produced by an upward gust meeting the machine flying horizontally. It would probably be possible to break any machine yet built by flattening out too suddenly when diving at the limiting velocity, when the wings might be called upon to carry as much as fifteen times normal load. The best course is to decide that a machine shall be unbreakable at some speed V, then the corresponding load factor is given by $n = V^2/V^2_s$, where V_s is the stalling speed.

If the machine be designed for these conditions it will probably be strong enough, in proportion, to meet all others. In all the cases mentioned the centre of pressure is in its most forward position when the greatest loads occur. We say, then, that *the wings of an aeroplane should be designed with the machine in the stalling position, and with the centre of pressure in its most forward position.* It can afterwards be checked for other conditions, particularly a vertical nose dive, and, in some cases, high speed flight at a small angle of incidence. In a vertical dive (*q.v.*), there is a down load on the front truss and an up load on the rear truss.

4. **Stalling: Strength Calculations with Centre of Pressure Forward.**—The stalling angle and centre of pressure coefficient may be taken from tests of the aerofoil used, though full-scale experiments show that the latter usually moves further forward than tests on a model monoplane would indicate; about 0·2 of the chord from the leading edge represents the likely limit.

In steady horizontal flight the total lift on the wings may be taken as equal to the total weight of the aeroplane (see § 4, General Theory).

As a rule correction for tail load is not necessary unless the engine is set at some considerable distance from the c.g., in which case the tail is usually placed in the slip-stream of the propeller and takes considerable load.

The normal load on the wings is thus W—weight of wings. All calculations are supposed made at n times this load.

5. **Distribution of Load.**—In the absence of model tests of the proposed wing combination, we take—

$$\text{For a biplane} \quad \frac{\text{load per ft.}^2 \text{ on top plane}}{\text{ditto on bottom plane}} = 1{\cdot}2.$$

For a triplane the loads per ft.² are in the following ratio:

$$\frac{\text{Top plane}}{1} = \frac{\text{Middle plane}}{0{\cdot}65} = \frac{\text{Bottom plane}}{0{\cdot}85}.$$

6. **Load Curves.**—Suitable curves for the distribution of load along the wings of a biplane are shown in Fig. 1, A and B, for angles of incidence approaching the stalling angle.*

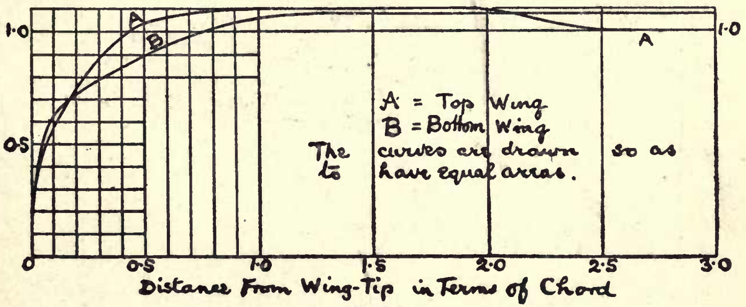


FIG. 1.

The load curves for the front and rear spars can be deduced from these:

Draw the load curve for the semi-span, and the plan form of the wing, to any convenient scales, as in Figs. 2 and 3, and on the latter draw the centre lines of the two spars. In the figure the curves are only shown as far as the outer struts.

The c.p. coefficient is assumed constant right up to the wing-tip; draw on the wing the locus of the c.p. for the chosen value of

---

* The present custom is to use curve B for both upper and lower wings, but the curves in the section on General Theory, § 20, suggest that curve A is more suitable for the upper wing.

the c.p. coefficient. Thus, if the c.p. coefficient be 0·28, the c.p. is everywhere 0·28 of the chord from the leading edge.

At each point along the span find the value of b/a (Fig. 2) and multiply it by the height of the load curve at the corresponding point. In the case shown, at 0·1 of the chord, $b/a=0·58$, the height of the load curve is 0·57; $0·57\times0·58=0·33$, which gives the load per unit length on the front spar at this point. By plotting the values found we obtain the load curve for the front spar; that for the rear spar can be found in the same way or by subtracting the front spar from the whole.

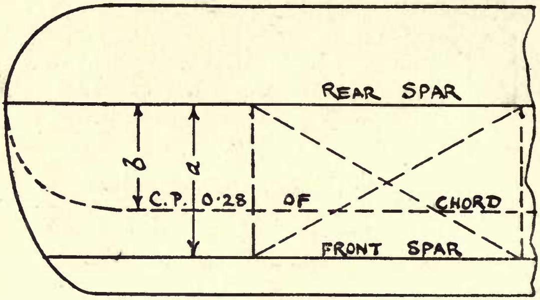
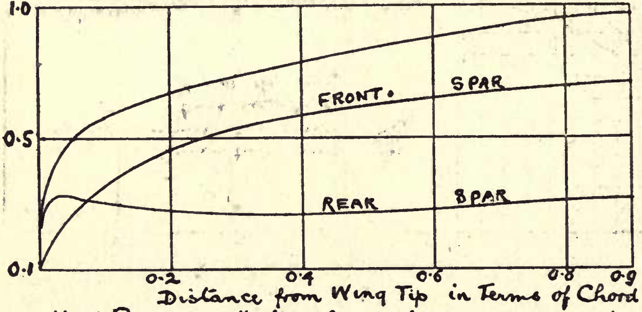


FIG 2



Half Span =159"; Total load on half-span = 304 lbs
Area of load curve for half span = 27·2 in²; length = 14·7"

FIG.3.

The scale of the load curves is found thus: If w = total load on half-span ÷ half-span, and h = area of load curve ÷ length of base, then the scale of the load curves is 1 unit $= w/h$ lb. per unit length. Thus in Fig. 3, $w=305/159=1·92$ lb./inch, $h=27·2/14·7=1·85$; the scale of the load curves 1 unit $=\frac{1·92}{1·85}=1·04$ lb./inch.

7. **Balanced Ailerons.**—The whole of the load outside the line *xy* (Fig. 4) must be taken on the rear spar.

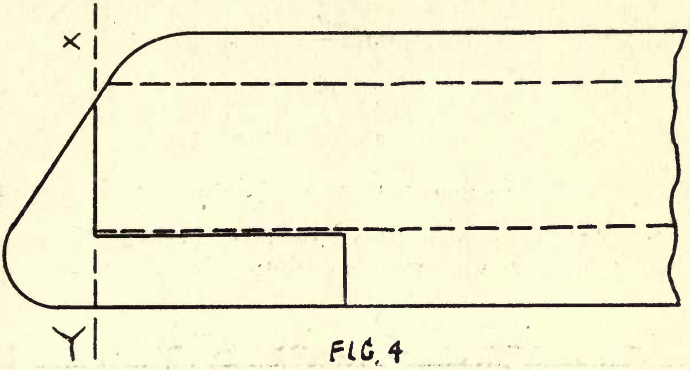


FIG. 4

The above process must be carried out on the top and bottom wings.

8. **Shearing Forces** on the overhang are given by integrating the load curve—*i.e.*, by plotting, at each point along the span up to the outer strut, the area of the load-curve from the end of the wing to that point. If the scales of the load curves are 1 inch = l inches; 1 inch equal w lb./inch, and the shear curve is drawn so that 1 inch $= s$ sq. inches of the load curve, the scale of the shear curve will be 1 inch $= wls$ lb.

9. **Bending Moments on the Overhang.**—Integrate the shearing force curve in the same way as the load curve was integrated. If we draw it so that 1 inch $= m$ sq. ins. of the shear curve the scale of bending moments is 1 inch $= mwl^2s$ lb./inches.

In this way find the B.M. at each of the outer strut points.

10. **First Approximation to the Bending Moments at all the Strut Points.**—These are found by the Theorem of Three Moments. Referring to Fig. 5:

$$\left.\begin{array}{l} M_A + 2(l_1 + l_2)M_B + l_2 M_c + \frac{1}{4}w_1 l^3 + \frac{1}{4}w_2 l_2{}^3 = 0 \\ M_B + 2(l_2 + l_3)M_c + l_3 M_D + \frac{1}{4}w_2 l_2{}^3 + \frac{1}{4}w_3 l_3{}^3 = 0 \end{array}\right\} \quad . \quad (1)$$

etc., etc.,

where w_1 is the mean load per inch on the span AB, and so on, M_A, M_B, etc., are the bending moments at A, B, etc., and are positive when they cause curvature in the same direction as w. We know M_A, and that $M_D = M_E$; these equations give M_B, M_C, etc. At a pin-joint M is zero.

11. **Reactions at A, B, C, . . .**—Referring to Fig. 5, S_{AL} is

the shearing-force to the left of A, S_{AR} that to the right of A, and so on.

S_{AL} is found from the load curves.

$$\left.\begin{aligned} S_{AR} &= \tfrac{1}{2}w_1 l_1 + \frac{M_B - M_A}{l_1} \\ S_{BL} &= \tfrac{1}{2}w_1 l_1 + \frac{M_A - M_B}{l_1} \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

etc., etc.

Then—

$$R_A = \text{reaction at } A = S_{AL} + S_{AR}, \text{ and so on} \quad . \quad . \quad (3)$$

Note that the equations (1) will usually make the M's negative; this sign must be kept when substituting in (2). Do this for all four spars, and so obtain the values of R_A, R_B . . ., . . . R'_B, R'_C . . .

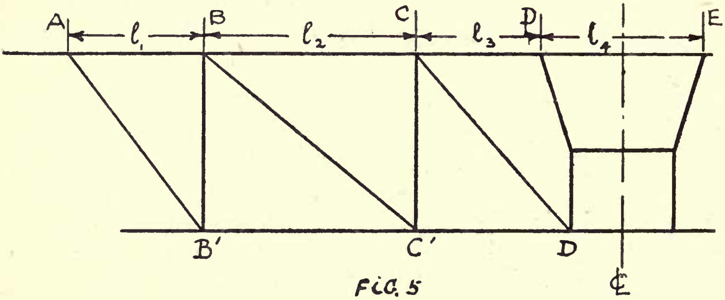


FIG. 5

12. **Force Diagram for Struts and Wires.**—A force diagram for each truss is now drawn by the usual methods of graphical statics,* to obtain the thrusts in AB, BC, etc., and the tensions in B'C', etc. Or, if P_1 P_2 . . . denote the thrusts in the top spars, and T_2, T_3 . . . the tensions in the bottom spars,

$$P_1 = \frac{l_1}{BB'} R_A = T_2.$$

$$P_2 = P_1 + \frac{l_2}{CC'}(R_A + R_B + R'_B) = T_3 \ . \ . \ . \text{ etc.}$$

13. **Second Approximation to Bending Moments.**—We now repeat the process of finding M_B, M_C, etc. It frequently happens that the line of the lift wire does not cut the centre line of the spar in the same point as the centre line of the strut. This introduces extra bending moments.

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* See Mathematics, Section II., § 6.

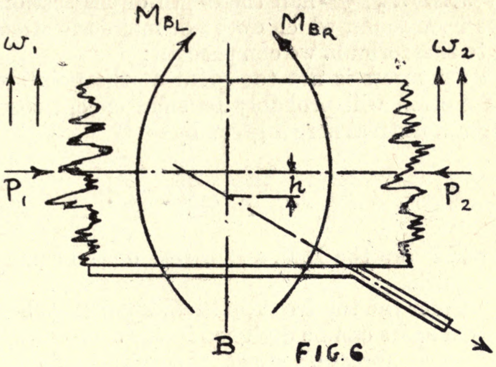


FIG. 6

$$\text{Here } M_{BL} - M_{BR} = (P_2 - P_1)\,h \quad . \quad . \quad . \quad (4)$$

where M_{BL} and M_{BR} = the B.M.'s to the left and right of B. The moment equations now become—

$$M_{AR} + 2_1 M_{BL} + 2_2 M_{BR} + {}_2M_{CL} + \tfrac{1}{4}w_1 l_1{}^3 + \tfrac{1}{4}w_2 l_2{}^3 = 0 \quad . \quad (5)$$

etc.

Hence find all the bending moments, and shearing forces, and reactions. We now have—

$$\left.\begin{aligned} S_{AR} &= \tfrac{1}{2}w_1 l_1 + \frac{M_{BL} - M_{AR}}{l_1} \\ S_{BL} &= \tfrac{1}{2}w_1 l_1 + \frac{M_{AR} - M_{BL}}{l_1} \end{aligned}\right\} \quad . \quad . \quad . \quad (6)$$

etc., etc.

14. **Section of Spar.**—The requisite section for the spar depends upon the maximum compression stress (assuming the spar is made of wood) allowed, and can now be roughed out. The maximum stress can be found approximately thus:

Taking the bay BC (Fig. 5), let λ_2 be the length of spar between the points of inflection (zero bending moment) then—

$$\lambda^2{}_2 = \frac{4}{w^2{}_2 l^2{}_2}\left[(M_{BR} - M_{CL})^2 + w_2 l^2{}_2\left(\frac{w_2 l^2{}_2}{4} + (M_{BR} + M_{CL})\right)\right] \quad (7)$$

A suitable section for the spar must now be guessed, and its moment of inertia, I, and area, A, found. Then the maximum stress is approximately—

$$f_c = \frac{w_2 \lambda^2{}_2}{8} \cdot \frac{Q_2 + 0{\cdot}3P_2}{Q_2 - P_2} \cdot \frac{y}{I} + \frac{P_2}{A} \quad . \quad . \quad . \quad (8)$$

where $Q_2 = \pi^2 EI/\lambda^2_2$; y = half the depth of the section. When a section has been found which does not make the stress too great according to this formula we can proceed:

See that the stress is not too great at the points of support, where the section will probably be solid except for bolt holes. The compression stress here is given by—

$$f_c = \frac{My}{I} + \frac{P}{A} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where M and P are the B.M. and thrust on the same side of the point of support.

In this manner the top front and rear spars may be designed.

The bottom spars can be designed in the same way except that the maximum compression stress in a bay should be estimated as—

$$f_c = \frac{1}{8} w\lambda^2 \frac{y}{I} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and the tensile stress as—

$$f_t = \frac{1}{8} w\lambda^2 \frac{y}{I} + \frac{T}{A} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The former of these makes no allowance for the reduction of compression stress by the end tensions; this leaves a margin for the end-load effect which occurs in nose-diving.

The four spars have now been roughed out, but their strength should be tested by more accurate methods.

15. **Checking the Spar Strengths.**—The first step is to find the end loads more accurately, and we must take into account drag forces and stagger forces.

Drag Forces.—If a curve of L/D for the aerofoil be available, use the value of L/D at the stalling angle to find the drag forces. If no curve is available take $L/D = 7$.

It is sufficiently accurate, in dealing with drag forces, to take the spars as supported at all the drag struts, and to find the reactions at these points on the assumption that there is a uniformly distributed load $= D/L$ times the lift load, and that the spars are pin-jointed at their fixing to the drag struts.

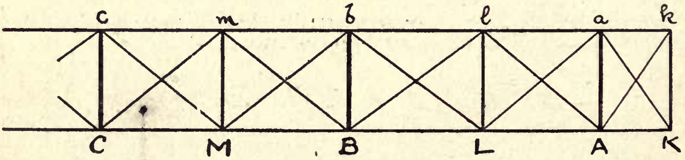


FIG 7

Fig. 7 represents part of the top or bottom wing. The drag forces are—

$$D_K = \frac{w}{2} \cdot AK, \quad D_A = \frac{w}{2} LK, \quad D_L = \frac{w}{2} BA, \text{ etc.,}$$

where D_K = drag force at K, etc., w is D/L times the mean lift load per unit length of wing.

The Stagger Forces arise from the fore-and-aft slope of the struts and lift wires. It is best to take them into account at the same time as the drag forces.

Fig. 8 shows the front and rear trusses, top and bottom trusses, and incidence panels of a two-bay machine.

The method is the same for any type.

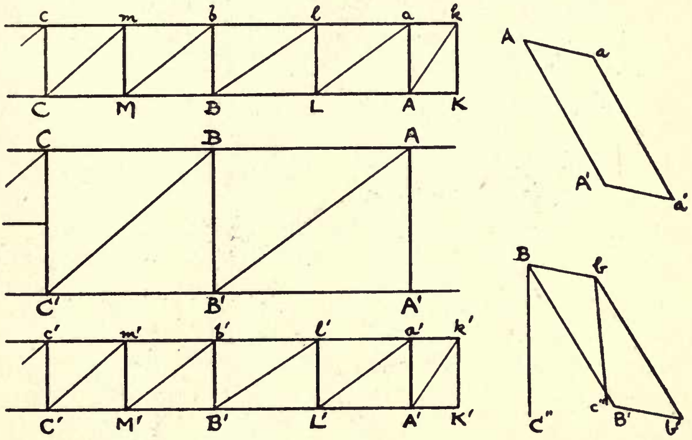


FIG 8.

Begin at bottom front outer strut joint, A'. The forces acting there are the lift R'_A, the drag D'_A, together with the drag D'_K, carried on by the wire $k'A'$; draw these as shown in Fig. 9 by 01′ and 1′2′, giving a resultant 02′, which can be resolved into 32′ parallel to $A'A$ and 03 parallel to $A'a'$; the former is carried to A by the strut and the latter to a' by the drag strut. Next take the point a': there is the lift force $R'a$ represented by 10 which combines with 03 to give a total force 13 at a', and this is resolved into 23, which goes up the strut $a'a$, and 12, which is carried on by the wire $a'L'$ to L' and thence to B'. Now go to A. The forces there are 32′ coming up the strut, R_A, D_A, together with D_K, as shown by 2′4′ and 4′5′; the resultant force at A is 35′, and this we resolve into 36 along the drag strut Aa, and 65′ parallel to $A'A$; this is carried on by the lift wire AB' to B'. Similarly, by adding R_a to 23 and 36 we obtain the resultant 46 at a, and this in turn gives 56 to be carried on by the lift wire *ab'*, and 45 which goes down the drag wires to *L* and thence to *B*.

We next do *B'*. The forces acting at *B'* are 65' carried on from *A* and R'_B; the force 12 carried from *a'* by the drag wires; D'_B, together with D'_L. Transfer 12 by parallels to 7'8' and add on $8'9' = D'_B + D'_L$. The resultant is 69', which gives 9', 10 for the strut *B'B*, and 6, 10 for the drag strut *B'b'*. The remainder of the forces are found in the same way, as shown in the diagram.

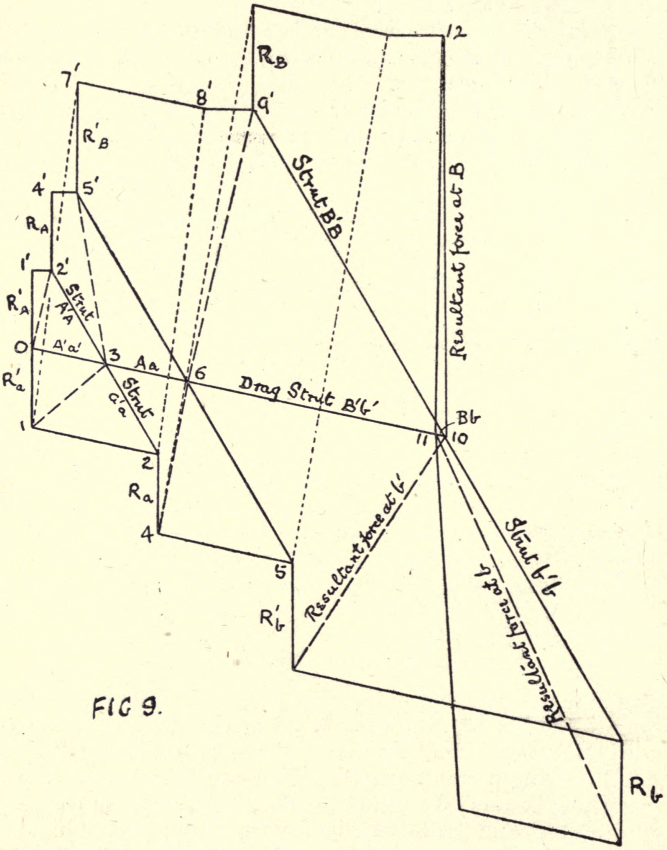


FIG 9.

The loads in the interplane struts are indicated on the diagram.

The load in drag strut Kk $= D_k$.
" " Aa = 3, 6.
" " Ll $= 36 + D_L$.
" " Bb = 10, 11 and so on.

The loads in the drag wires, and the tension and compressions produced by them in the spars, are then easily found.

Now make a table thus:

TOP PLANE.

Load due to—	*Front Spar.*					*Rear Spar.*				
	KA	*AL*	*LB*	*BM*	*MC*	*ka*	*al*	*lb*	*bm*	*mc*
Lift Wires										
Drag Wires										
Total										
Mean										

The first row is filled in from the values found in § 7; the second from those just found above; and the third by adding the first two. In the last row fill in the mean end load for the bays AL, LB and so on. In making this table take compressions plus and tensions minus. Make a similar table for the bottom plane.

16. Now find all the bending moments, taking into account end loads. A new form of the moment equation is required; many forms have been given but probably the most useful is that given by Webb and Thorne (*Aeronautics*, January 1, 1919):

$$\left.\begin{aligned}
&\frac{1}{(Q_1-P_1)l_1}\left(M_{AB}\left[1+0{\cdot}2\frac{P_1}{Q_1}\right]+2M_{BL}\left[1-0{\cdot}38\frac{P_1}{Q_1}\right]\right.\\
&\qquad\qquad\left.+\frac{w_1l_1^2}{4}\left[1-\frac{1}{70}\cdot\frac{P_1}{Q_1}\right]\right)\\
&+\frac{1}{(Q_2-P_2)l_2}\left(M_{CL}\left[1+0{\cdot}2\frac{P_2}{Q_2}\right]+2M_{BR}\left[1-0{\cdot}38\frac{P_1}{Q_1}\right]\right.\\
&\qquad\qquad\left.+\frac{w_2l_2^2}{4}\left[1-\frac{1}{70}\cdot\frac{P_2}{Q_2}\right]\right)
\end{aligned}\right\}\qquad(12)$$

where M_{AB}, etc., have the same meaning as above.

P_1 = mean end load in bay *AB*, found in table above.
Q_1 = Euler crippling load of $AB=\pi^2EI_1/l_1^2$.

P_2 and Q_2 have the same meanings for bay *BC*.

Write down these equations for the consecutive points, and obtain the values of M_{AB}, etc.

The maximum bending moment in AB is

$$\left.\begin{array}{l}\dfrac{Q}{Q-P}\left(\frac{1}{2}\Big[M_{AR}+M_{BL}\Big]\Big[1+0{\cdot}26\dfrac{P}{Q}\Big]+1{\cdot}02\dfrac{wl^2}{8}\right)\\ \qquad\qquad\qquad + \dfrac{(M_{BL}-M_{AR})^2}{2wl^2}\end{array}\right\} \quad . \quad (13)$$

at a distance from A given by

$$\tfrac{1}{2}l+\frac{M_{BL}-M_{AR}}{wl} \quad . \quad . \quad . \quad . \quad (14)$$

The points of inflection are at distances from A given by

$$\tfrac{1}{2}l+0{\cdot}9\frac{M_{BL}-M_{AR}}{wl}\pm 1{\cdot}2\sqrt{\frac{M_{max}}{w}} \quad . \quad . \quad (15)$$

The maximum compression stress within the bay is

$$f_c=M_{max}\cdot\frac{y}{I}+\frac{P}{A} \quad . \quad . \quad . \quad . \quad (16)$$

Note that (12) usually makes M_{AR}, etc., negative; this negative sign must be kept in substituting in (13), (14), etc.

As a rule it is not necessary to test the bottom spars in this way unless for some reason there are considerable end thrusts, except for nose-diving.

17. **The Spars must next be tested for Shear.**—The shearing forces on each side of the struts are found from (6) above, using the M's given by (12). The shearing force diagram is as shown in Fig. 10.

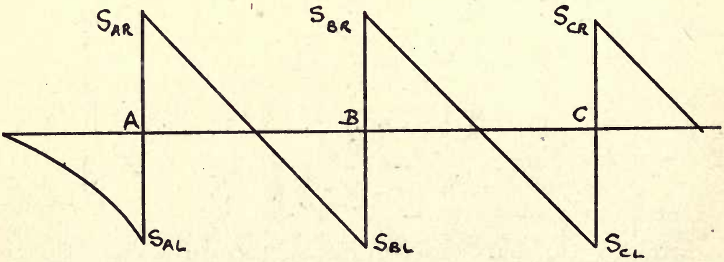


FIG. 10.

The part outside the outer strut is found from the load curve. At each point such as A, the total height S_{AR} to S_{AL} is R_A, and in going from A to B we drop a height equal to the total load on AB, and so on. The shearing force at any point is given by the height of the diagram above *ABC*. . . . At any point along the spar the maximum shear stress is given by

$$f_s=\frac{FZ}{It},$$

where F = shearing force; Z = moment of half the area of cross section about middle line; I = moment of inertia; t = smallest thickness of web.

This completes the stress calculations for the spars for this condition of flight.

The loads in all the wires have been found, and it is a simple matter to choose the right size.

For a high-speed machine, it will be desirable to repeat the process of checking the spar strengths when the c.p. is farther back, in the position corresponding with top speed. The factor then should be $\nless$70 per cent. of that taken with c.p. forward.

Nose-Diving.

18. For load curve use Fig. 1 B, for bottom and top wings. The following estimation of the loads in a limiting nose-dive may be taken to be on the safe side.

$$\left.\begin{array}{ll}\text{Total down load on front truss} & =\dfrac{Wcl_1}{lb}\\ \text{Total up load on rear truss} & =\dfrac{Wcl_2}{lb}\end{array}\right\} \quad . \quad . \quad (17)$$

where l_1 = distance of rear truss from c.p. of tail plane.
l_2 = ,, front ,, ,, ,,
l = ,, c.g. from c.p. of tail plane.
b = distance between spar centres.
c = the chord of the equivalent plane (*q.v.*).

It is sufficiently accurate to assume that the c.p. of the tail-plane is one-third of the tail plane chord from its leading edge. The total down-load and total up-load may each be divided in the ratio of the areas. The spar loadings, bending moments, and reactions must be worked out as above.

In addition to these loads there is a large drag on the wings, which may be taken as half the total weight of the machine, or, if the drag coefficients be known, it may be worked out.

In this condition of flight the incidence wires play an important part in determining the direct loads on the spars, struts, and wires. In dealing with this matter it is good enough to take the angle of incidence as zero. If the incidence wires are omitted from the calculations the errors may be enormous. The following is a summary of a method given by the author (*Aeronautics*, December, 1918). The formulæ look terrible, but there is nothing difficult in the work. Each bay is treated separately, beginning with the outer. The procedure is:

(i.) Assume that the anti-drag wires of the top plane, and the drag wires of the bottom plane, are acting, and can if necessary take a thrust. On this basis find all the loads omitting the incidence wire.

(ii.) Find the tension in the outer incidence wire by the formula below. The notation is for the most part explained by Fig. **11**.

(*a*) One drag bay, top and bottom.

$$\frac{Ti}{Li}=\left.\frac{\begin{gathered}K_Ft_F+K_ft_f-\mu K_Dt_D-\nu K'_Dt_D-\lambda\Big(KQ-\frac{a'}{a}kq\\ -\frac{a}{a'}K'Q'-\nu k'q'\Big)\end{gathered}}{\begin{gathered}K_FL_F+K_fL_f+K_iL_i+\mu^2K_DL_D+\nu^2K'_DL'_D\\ +\lambda\Big(\mu^2Ka+\frac{a'}{a}{}^2k+\frac{a^2}{a'}K'+\nu^2k'a'\Big)\end{gathered}}\right\}\quad . \qquad (18)$$

(*b*) Two drag bays, top and bottom.

$$\frac{Ti}{Li}=\left.\frac{\left[\begin{gathered}K_Ft_F+K_ft_f-\mu(K_{D1}t_{D1}+K_{D2}t_{D2})-\nu(K'_{D1}t'_{D1}+K'_{D2}t'_{D2})\\ -\lambda(\mu K_1Q_1+\mu B_2K_2Q_2-b_1k_1q_1-b_2k_2q_2)\\ +\lambda(B'_1K'_1Q'_1+B'_2K'_2Q'_2-\nu k'_1q'_1-\nu b'_2k'_2q'_2)\end{gathered}\right]}{\left[\begin{gathered}K_FL_F+K_fL_f+K_iL_i+\mu^2(K_{D1}L_{D1}+K_{D2}L_{D2})\\ +\nu^2(K'_{D1}L'_{D1}+K'_{D2}L'_{D2})+\lambda(\mu^2K_1a_1+\mu^2B_2{}^2K_2a_2\\ +b_1{}^2k_1a_1+b_2{}^2k_2a_2+B'_1{}^2K'_1a'_1+B'_2{}^2K'_2a'_2\\ +\nu^2k'_1a'_1+\nu^2b'_2{}^2k'_2a'_2)\end{gathered}\right]}\right\}(19)$$

In these formulæ—

$$\mu=1-\frac{G}{c'}\cot\phi-\frac{\gamma}{c}\cot\theta_1,\ v=\frac{c}{c'}-\frac{\gamma}{c'}(\cot\psi'-\cot\theta_1),$$

$$b_1=L'_2/a_1,\qquad b_2=\frac{L'_2+\nu a_1}{a_2},\qquad b_2'=L'_2/a'_2,$$

$$B_2=L_2/a_2,\qquad B'_1=L_2/a'_1,\qquad B'_2=\frac{L_2+\nu a'_1}{a'_2}.$$

K, *K′*, *k*, *k′*, etc. = (length)^2 ÷ cross section.

L = length of wires; the other dimensions are shown in Fig. **11**.

F, suffix, denotes front anti-lift wire.

f, ,, ,, rear lift wire.

i, ,, ,, incidence wire.

D, ,, ,, drag or anti-drag wire.

t denotes the values of the *T*'s in Fig. **11**, when the incidence wire is omitted; similarly, *Q* and *q* denote the values of *P* and *p* when the incidence wire is omitted.

$\lambda=E$ for steel ÷ E for the spars.

Each term is of the same type, and if the arithmetic be done in tabular form, these formulæ do not give any trouble.

(iii.) Resolve the tension of the incidence wire into two components, at each end, one along the lift axis, and one along the drag axis. Treat these as extra external forces, find the loads they cause on all the members, and add the results to those already found without the incidence wire.

(iv.) Proceed to the next bay, carrying on from the first the corrected loads in the wires, etc.

Having found all the end loads check the strength of the spars. It is these calculations which will probably decide the size of drag bracing required. In a machine intended for "stunting" the drag bracing should be strong enough to support the whole weight of the aeroplane divided between the top and bottom wings in the ratio of their areas.

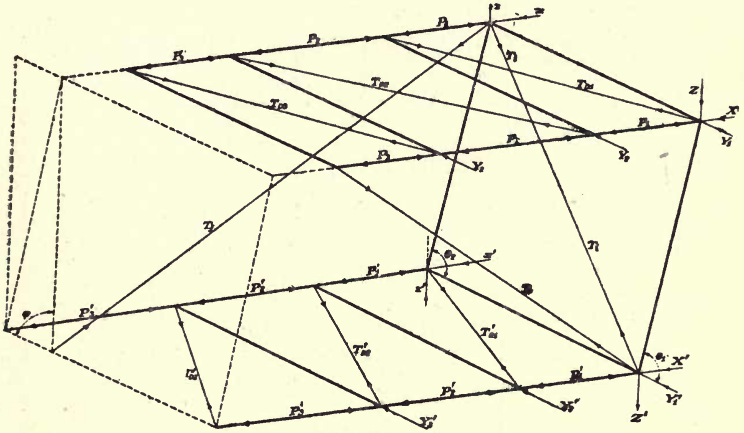


FIG 11.

The spars and wires are now all settled, and the interplane struts can be designed.

19. **Interplane Struts.**—(i.) *Parallel Struts,* if of wood, solid or hollow, can be designed from the formula—

$$I=\frac{PL^2}{\pi^2 E}.$$

P = end-load, L = length between pins, E = Young's modulus, I = smallest moment of inertia.

Steel tubular struts can be designed from the curve given in Fig. 12, provided care be taken that the crinkling stress is not exceeded. The crinkling stress may be taken as follows:

Annealed M.S. tubes, $\frac{P}{A} \ngtr 500{,}000\,\frac{t}{r}$ if $\frac{t}{r} < 0{\cdot}06,$

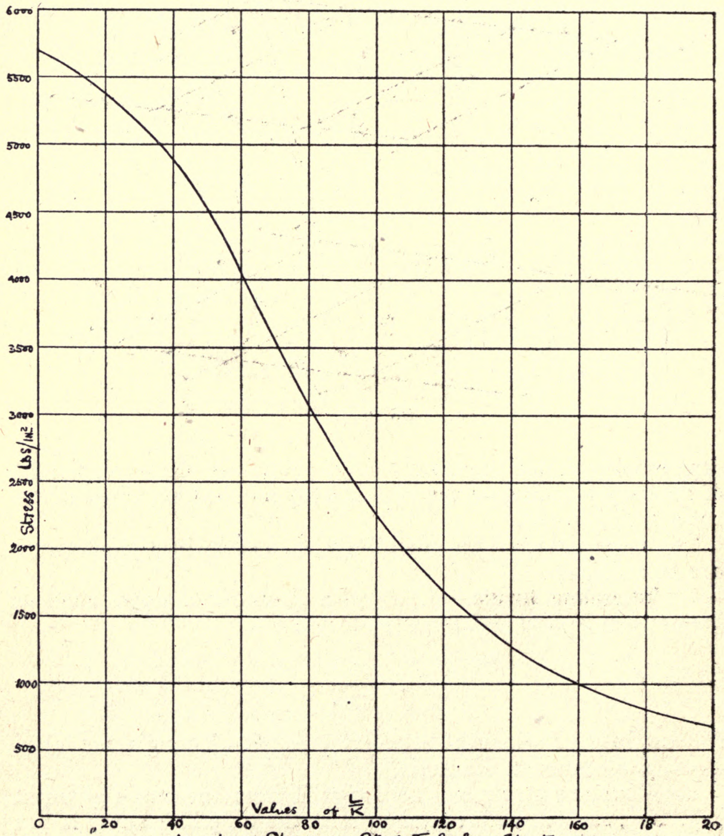
$\ngtr 30{,}000$ if $\frac{t}{r} > 0{\cdot}06.$

Annealed H.T.S. tubes, $\frac{P}{A} \ngtr 650{,}000\,\frac{t}{r}$ if $\frac{t}{r} < 0{\cdot}10,$

$\ngtr 65{,}000$ if $\frac{t}{r} > 0{\cdot}10.$

Hard H.T.S. tubes, $\frac{P}{A} = 110{,}000$ if $\frac{t}{r} > 0{\cdot}05,$

The units being pounds and inches.



Limiting Stress in Steel Tubular Struts

L = Length between pins (inches). K = radius of gyration of section = $\sqrt{I/A}$

Yield Point = 28 Tons/in² E = 13.600 Tons/in²

FIG. 12.

In connection with this reference should be made to a paper by Popplewell and Carrington (*Proc. I.C.E.*, October, 1918).

(ii.) *Solid Tapered Wooden Struts.*—The following method of design, due to Webb and Barling, gives the lightest strut to support a given thrust, neglecting any side load. Correct tapering saves 13 per cent. weight, and 8 per cent. resistance. For a given length, the proper taper depends on the end load. All cross-sections of the strut are supposed geometrically similar with their c.g.'s in a straight line.

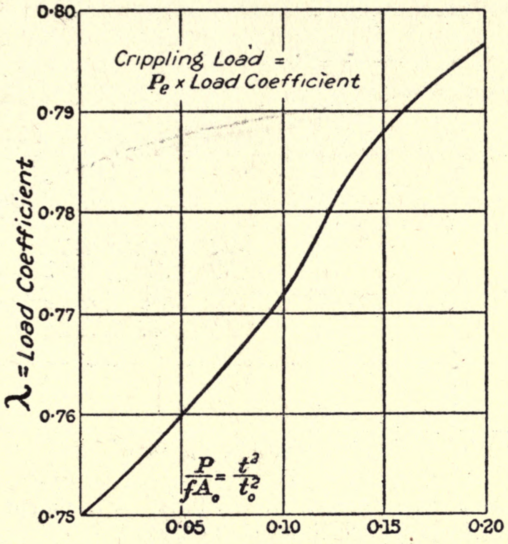


FIG. 13.

Proceed thus:

First Approximation.—Find I_0, the moment of inertia of largest (centre) section from

$$P = 0{\cdot}8\pi^2 EI_0/L^2.$$

Hence find t_0, the thickness at the centre, and A_0, the area at the centre.

Second Approximation.—Find the value of P/fA_0, where f is the maximum stress allowed.

From Fig. 13 find the "load coefficient." Then find I_0 from

$$P = \lambda\pi^2 EI_0/L^2.$$

Hence find t_0. The thickness t_1 at the ends is given by

$$t_1/t_0 = \sqrt{P/fA_0}.$$

Then take that curve in Fig. 14,* which gives t_1/t_0 nearest to this. The curve so chosen is the shape of the strut. The curves give t/t_0 plotted against x/l, t being the thickness at a distance x from the centre, and l being the semi-length. The value of t_1/t_0 is given by the height at which the curves cut the right-hand border of the diagram.

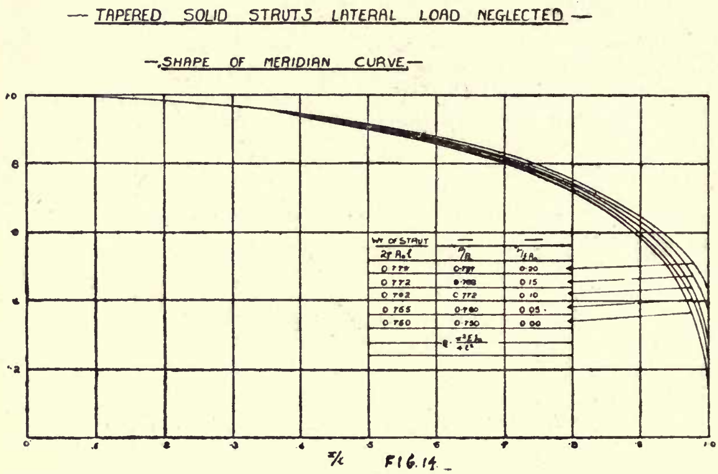


FIG. 14

(iii.) *Hollow Taper Struts,* the material of the strut being of uniform thickness d, such that for any section

$$A = atd,\ I = bt^3d,$$

where a and b are constants, and t is outside thickness of the section.

Proceed as above. For the first approximation take

$$P = \frac{16EI_0}{9L^2}.$$

The curves for λ and the shape of the strut are given in Figs. 13 and 15.

20. **Secondary Failure of Spars.**—The strength of the top and bottom planes to resist lateral buckling of the spars must be examined. Each drag bay should be taken for the case which makes the sum of the thrusts in the two spars greatest. The failing load is given by

$$P_1 + P_2 = \frac{\pi^2 E}{l^2}(I_1 + I_2) \quad . \quad . \quad . \quad (20)$$

where $P_1 + P_2$ = sum of thrusts in the two spars

$I_1 + I_2$ = sum of moments of inertia of the two spars about their vertical centre line.

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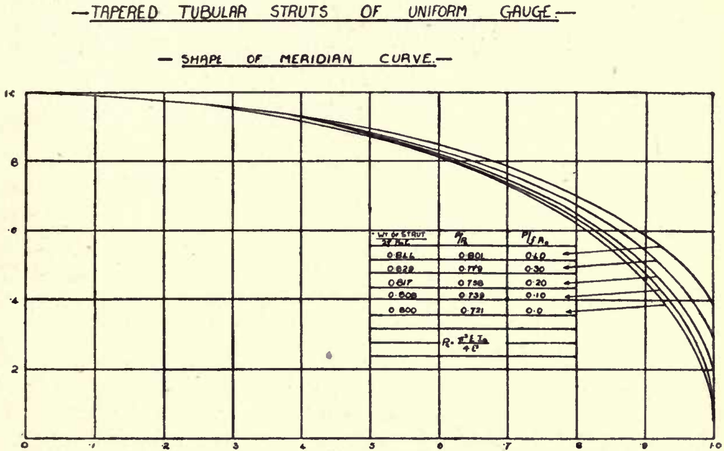
* By kind permission of the Advisory Committee for Aeronautics.

21. **Outrigger Aeroplanes.**—The tail load is transmitted through the outrigger booms to the wings. Allowance must be made for this. The tail load being known, the forces in the top and bottom booms are calculated, and these act as extra external forces on the wings. In a nose-dive the tail load can be taken as Wc/l (*cf.* Equations 17). In horizontal flight it $= M/l$ (*cf.* Equation I.).

22. **Multiple Engine Aeroplanes.**—When there are engines, or other heavy weights, set out on the wings, the dead-weight must be allowed for, and it must be multiplied by the factor of safety which is being used. The presence of these weights makes an enormous difference to the loads in the bays between the engines. The downward forces to be introduced on the front and rear spars are easily found if the c.g. of the engines is known. There is no new principle introduced into the calculations, but the loads in all the members between the engines should be very carefully obtained, with both wing engines running, and with only one. The balancing out of the drag forces requires some care, but it is not difficult if it be remembered that the "points of support" of the drag structure are the propeller thrusts.


—TAPERED TUBULAR STRUTS OF UNIFORM GAUGE.—

— SHAPE OF MERIDIAN CURVE.—

FIG. 15.

Reactions on the engine bearers, due to torque reaction, must be introduced. Their magnitude is 33,000 H.P.$/2\pi nd$, where $n =$ the r.p.m., and $d =$ the distance between the engine bearers in such directions as to produce a couple in opposite direction to that of the rotation of the engine.

23. **Landing.**—If the wings of an aeroplane be strong enough to stand the flight loads they will usually be strong enough to bear the landing stresses. If it be desired to check this the wings should

be stressed under a uniformly distributed down load equal to six times their own weight. The landing chassis should stand 4 or 5 times the weight of the machine. In a multiple engine machine the portion of the structure between the engines may have large loads thrown upon it.

II.—THE FUSELAGE.

24. The strength of the rear part of this must be worked out for two cases: stresses due to down load on the tail, and the stresses due to the load on the tail-skid in landing. The requisite sizes of the longerons, struts, and wires are found from these calculations. Afterwards the strength should be tested for side loads due to fin and rudder, and for torsion.

The greatest tail load which the machine has to stand occurs in suddenly stalling or in flattening out from a vertical dive, and is rather uncertain. One of two methods should be used: either (i.) guided by experience take some arbitrary loading per square foot on the tail and stress the fuselage for this, or (ii.) assume a value for the maximum pull the pilot can put on the control column, and, knowing the gearing of the controls, work out what total tail load this corresponds with. The rear part of the fuselage is treated as a cantilever from the point of attachment of the rear lift wire, *R* (Fig. 16), and half the total tail load is taken by each side. The loads in the struts, etc., can be found graphically by a force diagram, as in Fig. 16, or by calculation.

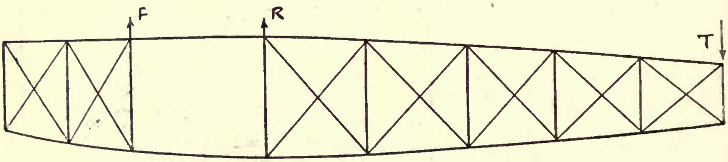


FIG 16

Any weights on the fuselage, or the weight of the structure itself, may be taken into account if necessary. In flattening out from a dive these weights will act along the fuselage towards the front of the machine; in suddenly stalling from horizontal flight they act downwards at right angles to the fuselage.

The load on the tail-skid when standing on the ground is easily found. Stress the fuselage for an up load equal to about six times this.

The fore part of the fuselage may be stressed in the same way, and particular care should be taken to see that there is adequate bracing to take the engine torque. The loads here are the deadweight of engines, tanks, or whatever may be situate in this part.

In making these calculations all weights should be multiplied by the factor of safety employed.

25. **The Strength of the Longerons.**—Each bay may be treated as a pin-jointed strut, but they are too short as a rule for Euler's formula. Their strength may be calculated from the Rankine formula—

$$P=\frac{f_cA}{1+a\left(\frac{l}{k}\right)^2},$$

where f_c = maximum compression stress allowed,
a = a constant,
A = area of cross section,
l = length,
k = smallest radius of gyration.

When the fuselage is covered with fabric we may take $a=1/5200$, if the fabric is attached in such a way as to support the longerons, otherwise $a=1/3000$.

26. **The Strength of Vertical and Horizontal Struts** may be calculated from the above formula, taking $a=1/3000$. For struts having a straight centre portion and tapering towards the ends, the most economical strut is obtained when the straight part has a length = 0·4 of the whole (Webb).

III.—Tail-Plane.

27. The stresses in the tail-plane are calculated in much the same way as the wings. There is an absence of data on pressure distribution on tails; failing other information the load curve of Fig. 1 may be taken, and the load should be taken as coming all on the front spar and all on the rear spar in turn. The elevators produce concentrated loads on the rear spar, acting at the hinges.

IV.—Fin, Rudder, Ailerons, etc.

28. Here also there is at present a dearth of data, and experience is the best guide. Having decided what total load must be allowed for, enough has been said above to enable stress calculations to be made, for, whether we are stressing wings or tails, fins or rudders, we are always dealing with beams under combined bending and end load, and struts. Care should be taken that the spars of ailerons and elevators, and rudder posts, are strong enough to take any twist they may be called upon to bear; for this reason square or circular spars should be used in preference to **H** section.

GENERAL THEORY

By JOHN CASE, M.A., A.F.R.Ae.S.

I.—The Dynamics of Steady Flight.

1. **Lift and Drag.**—Air moving with velocity V strikes a surface AB, inclined at an angle α (Fig. 1), to the direction of the wind. The resultant force R on the surface can be resolved into—(i.) L, $\perp^{R}$ to the direction of V=the *lift* on the surface; (ii.) D, in the direction of V=the *drag* on the surface; α is called the angle of incidence.

The same is true if the air is still and AB moves with velocity V. If AB be the wing of an aeroplane, L is the force which supports the weight, D is the force which has to be overcome by the propeller.

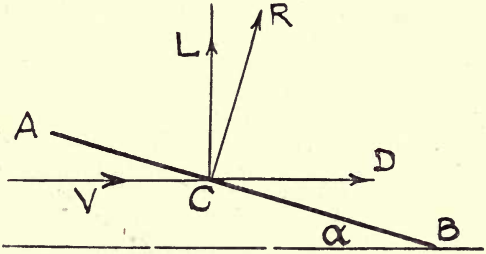


FIG. 1.

2. **Centre of Pressure.**—The point C, where R cuts AB, is called the centre of pressure; its position on AB depends on the angle α and the shape of AB.

3. **Lift and Drag Coefficients** are defined by the equations:

$$L=(\rho/g)SV^2k_L, \qquad \text{and} \qquad D=(\rho/g)SV^2k_D \quad . \quad . \quad (1)$$

where ρ=density of air; S=area of the surface AB; k_L=the absolute lift coefficient; k_D the absolute drag coefficient; L and D are the total forces in the directions stated. k_L and k_D are independent of the units used. For standard density, $\rho/g=0{\cdot}00237$, in ft. sec. units.

4. **Horizontal Straight Line Flight.**—Let W=total weight of aeroplane; S=total wing area; k_R=the resistance coefficient for

the whole machine minus the wings, such that the corresponding resistance $=(\rho/g)SV^2k_R$. Then the total resistance of the machine is—

$$D=\frac{\rho}{g}SV^2(k_D+k_R) \quad . \quad . \quad . \quad . \quad (2)$$

and the total lift is—

$$L=\frac{\rho}{g}SV^2k_L \quad . \quad . \quad . \quad . \quad . \quad (3)$$

There is also an upward or downward force on the tail, and some lift due to the body, but both these may usually be neglected, and we can take $W=L=(\rho/g)SV^2k_L$. From this—

$$V=\sqrt{\frac{W}{Sk_L\cdot\rho/g}} \quad . \quad . \quad . \quad . \quad (4)$$

For a given angle of incidence k_L is fixed, and therefore V is fixed. The minimum speed which will keep the machine in the air is obtained when k_L is a maximum. This will be the landing speed:

$$Vmin=\sqrt{\frac{W}{S(\rho/g)k_{L\ max}}} \quad . \quad . \quad . \quad . \quad (5)$$

The tractive force required is $T=D$, and from (2) and (3)

$$\frac{L}{D}=\frac{k_L}{k_D+k_R}$$

$$\therefore\ T=\frac{k_D+k_R}{k_L}=W \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{The power required}=TV=DV=\frac{\rho}{g}SV^3(k_D+k_R) \quad . \quad (7)$$

$$\text{But from (4)}\ V^3=\left(\frac{g}{\rho}\right)^{\frac{3}{2}}\left(\frac{W}{SK_L}\right)^{3/2}$$

$$\therefore P=\text{power required}=\frac{g}{\rho}\cdot\frac{W^{\frac{3}{2}}}{S^{\frac{1}{2}}}\cdot\frac{k_D+k_R}{k_L{}^{3/2}} \quad . \quad . \quad (8)$$

Hence the power required varies inversely as $\sqrt{\rho}$; this shows that the power required depends on the altitude for a given angle of incidence.

We can also write, from (4) and (6)—

$$P=WV\frac{k_D+k_R}{k_L}$$

$$\therefore\ V=\frac{P}{W}\cdot\frac{k_L}{k_D+k_R} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence for a given horse-power, the maximum speed occurs when $k_L/(k_D+k_R)$ is a maximum.

5. **Climbing.**—If the machine be climbing at an angle α to the horizon, the components of W along and perpendicular to the flight path are $W \sin\theta$ and $W \cos\theta$. Hence, in this case we must have—

$$T = W \sin\theta + \frac{\rho}{g} S V^2 (k_D + k_R) \quad . \quad . \quad . \quad (10)$$

and—

$$W \cos\theta = L = \frac{\rho}{g} S V^2 k_L \quad . \quad . \quad . \quad . \quad (11)$$

$$\text{From (11)} : V = \sqrt{\frac{W \cos\theta}{S k_L (\rho/g)}} \quad . \quad . \quad . \quad (12)$$

If V_o = speed flying horizontally, and V_c = speed climbing, then, for a given angle of incidence,

$$V_c / V_o = \sqrt{\cos\theta} \quad . \quad . \quad . \quad . \quad (13)$$

The rate of climb is $V_c \sin\theta = v$ say. Then $v = V_o \sin\theta \sqrt{\cos\theta}$. For a given angle of incidence V_o is fixed, and therefore v is a maximum when $\sin\theta\sqrt{\cos\theta}$ is a maximum—*i.e.*, when $\tan\theta = \sqrt{2}$, which gives $\theta = 54°$, and $v = 0{\cdot}62\ V_o$. This gives the maximum possible rate of climb, no matter how great the power available, but it requires a thrust equal to about $0{\cdot}85 W$, and is never realized.

$$P = TV = WV \sin\theta + \frac{\rho}{g} S V^3 (k_D + k_R) \quad . \quad . \quad (14)$$
$$= Wv + P_o \cos^{3/2}\theta,$$

where P_o = power taken for horizontal flight at the same angle of incidence. Therefore, for a given angle of climb, the maximum rate of climb is given by—

$$v = \frac{P - P_o \cos^{3/2}\theta}{W} \quad . \quad . \quad . \quad . \quad (15)$$

6. **Gliding in a Straight Line.**—Let the machine be gliding down at an angle θ to the horizontal. The speed will increase until the component of the weight along the flight path is equal to the total resistance of the aeroplane. The speed which is reached is called the limiting velocity or terminal velocity. At this stage we have—

$$W \sin\theta = \frac{\rho}{g} S V^2 (k_D + k_R) \quad . \quad . \quad . \quad (16)$$

$$W \cos\theta = \frac{\rho}{g} S V^2 k_L$$

Hence the angle of descent, for a given angle of incidence, is given by—

$$\tan\theta = \frac{k_D + k_R}{k_L} \quad . \quad . \quad . \quad . \quad (17)$$

and θ is least when $k_L/(k_D+k_R)$ is a maximum. From (16) we have, by squaring and adding—

$$\frac{\rho}{g}V^2=\frac{W}{S}[(k_D+k_R)^2+k^2{}_L]^{-\frac{1}{2}} \quad . \quad . \quad . \quad (18)$$

or—

$$V=\sqrt{\frac{W\sin\theta}{\rho/g\cdot S(k_D+k_R)}} \quad . \quad . \quad . \quad (19)$$

7. **In a Vertical Dive,** $\theta=90°$, and the limiting velocity is given by—

$$V=\sqrt{\frac{W}{\rho/g\cdot S(k_D+k_R)}} \quad . \quad . \quad . \quad (20)$$

The angle of incidence being now such that the total lift on the wings is *almost* zero; the lift is not quite zero for the following reasons: near the angle of no lift the air forces still produce a couple tending to rotate the machine about its c.g., and a load on the tail is necessary to balance this. The condition for a vertical dive is that the sum of the lifts on the wings and tail should be zero, consequently the lift on the wings is not zero, but equal and opposite to the load on the tail.

The velocities given by (19) and (20) are not reached instantly, but asymptotically; in practice the velocity attained in a few seconds is nearly equal to that which would be obtained after a very great space of time.

8. **A Horizontal Turn.**—A force has to be provided to cause the necessary acceleration towards the centre of the circle in which the machine is turning. This is obtained by banking the machine, and the lift forces then have a component towards the centre of the circle. Let ϕ be the angle of bank (Fig. 2). Then if there be no sideslip,

$$W=L\cos\phi=\frac{\rho}{g}SV^2k_L\cos\phi \quad . \quad . \quad (21)$$

and—

$$\frac{WV^2}{gr}=L\sin\phi=\frac{\rho}{g}SV^2k_L\,sin\,\phi \quad . \quad . \quad (22)$$

Hence,

$$\tan\phi=\frac{V^2}{gr} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and—

$$W^2\left(1+\frac{V^4}{g^2r^2}\right)=\left(\frac{\rho}{g}SV^2k_L\right)^2$$

$$\therefore\ V^2=\sqrt{\frac{W}{\left(\frac{\rho}{g}Sk_L\right)^2-\frac{W^2}{g^2r^2}}} \quad . \quad . \quad . \quad (24)$$

From (24) we see that the smaller r is the larger must V be for a given angle of incidence. Then (23) shows that ϕ, the angle of bank, increases rapidly as r is decreased. By (21), $L = W \sec \phi$, so that the total lift on the wings may be much greater than W.

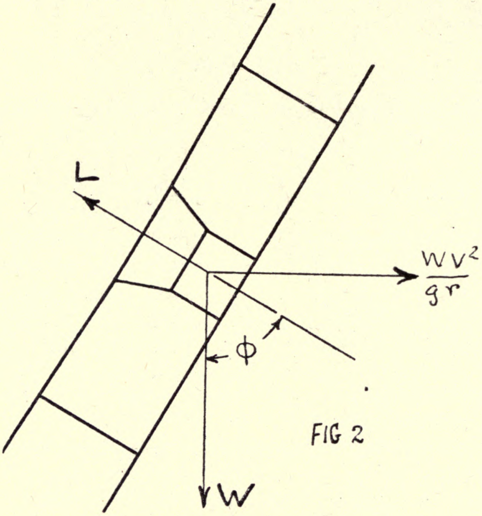


FIG 2

For a given angle of incidence, let the suffix 0 refer to horizontal straight-line flight. Then—

$$V = V_o\sqrt{\sec \phi}$$

From (23)—

$$\sec \phi = \sqrt{1 + \frac{V^4}{g^2r^2}}$$

$$\therefore\ V = \frac{V_o}{\sqrt[4]{1 - \frac{V_o^2}{g^2r^2}}} \quad . \quad . \quad . \quad . \quad (25)$$

The tractive force required is—

$$T = \frac{\rho}{g}S\,V^2(K_D + K_R) = \frac{\frac{\rho}{g}S\,V_o^2(K_D + K_R)}{\sqrt{1 - \frac{V_o^2}{g^2r^2}}}$$

or

$$T = \frac{T_o}{\sqrt{1 - \frac{V_o^2}{g^2r^2}}} \quad . \quad . \quad . \quad . \quad (26)$$

The power required is—

$$P=TV=\frac{P_o}{\left[1-\frac{V_o^2}{g^2r^2}\right]^{\frac{3}{4}}} \quad . \quad . \quad . \quad . \quad (27)$$

which increases rapidly as the radius r is decreased, and the power available sets a lower limit to the possible radius of turn.

In order to make the aeroplane turn in a circle without "yaw" two actions are necessary:

1. A force is necessary to give the whole machine an acceleration towards the centre of the circle, and we have seen how this is provided.
2. The aeroplane must be given a rotation about its c.g., otherwise it would move in the manner shown, becoming more and more "broadside-on" to its direction of motion—*i.e.*, "yawing." This is prevented by putting the rudder over in order to provide the necessary turning moment. After the machine is banked the elevators take the place of the rudder since the machine is more or less on its side.

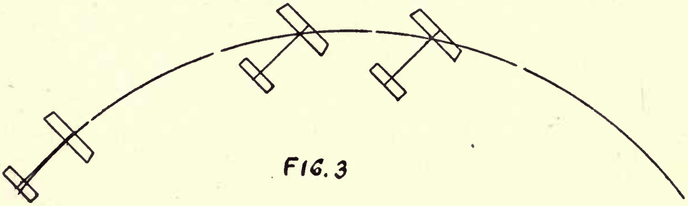


FIG. 3

9. **Spiral Glides and Spinning.**—The machine glides downwards in a spiral, or, more accurately, a helix. In Fig. 4, *A*, *B*, *C*, *D*, is the flight path—a helix described on a vertical circular cylinder, of which the axis is *a*, *b*, *c* . . . ; *aA*, *bB*, *cC* . . . being successive radii. Consider the aeroplane when it is at *C*: *YY'* represents the wings; *X C T* is the tangent to the flight path, making an angle θ with the horizontal; ϕ is the angle of bank; *CZ* is the lift axis of the machine, $\perp^{\text{R}}$ to *CY* and *CX*.

Suppose that the motion is steady, and that there is no sideslip. Then—

$$\frac{\rho}{g}SV^2(k_D+k_R)=W\sin\theta \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$0=W\cos\theta\sin\phi-\frac{WV^2}{gr}\cos\phi \quad . \quad (29)$$

$$\frac{\rho}{g}SV^2k_L=W\cos\theta\cos\phi+\frac{WV^2}{gr}\sin\phi \quad . \quad (30)$$

where r is the radius of turn. If ω be the angular velocity—

$$\omega = \frac{V}{r} \cos \theta \qquad . \quad . \quad . \quad . \quad (31)$$

From (29)—

$$r = \frac{V^2 \cot \phi}{g \cos \theta} \qquad . \quad . \quad . \quad . \quad (32)$$

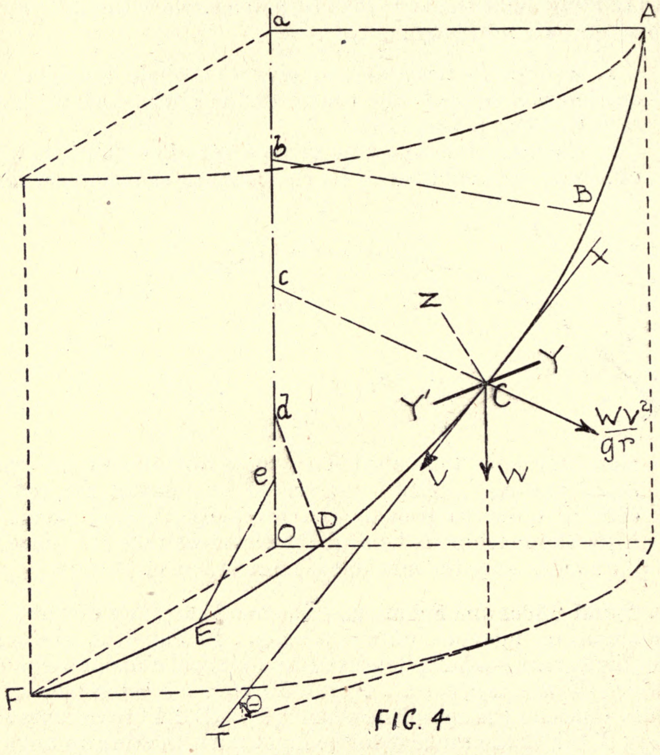


FIG. 4

From (29) and (30) we have—

$$\frac{\rho}{g} S V^2 k_L = W \cos \theta / \cos \phi \qquad . \quad . \quad . \quad (33)$$

∴ by (28)—

$$\sec \phi = \frac{k_L}{k_D + k_R} \tan \theta \qquad . \quad . \quad . \quad (34)$$

If V_o = the speed of horizontal straight-line flight for a given angle of incidence, then

$$V = \sqrt{\frac{W}{\rho/g \cdot S k_L} \cdot \frac{\cos \theta}{\cos \phi}} = V_o \sqrt{\frac{\cos \theta}{\cos \phi}} \qquad . \quad . \quad (35)$$

For a given angle of incidence, and a given angle of descent θ, the angle of bank is fixed by (34), and the speed is given by (28) or (35); ω and r are found from (31) and (32).

A spin is a spiral glide with a very large angle of incidence, far beyond the stalling angle (see § 13 below). The speed is then low, and the weight of the machine is supported mostly by the large drag. Owing to rotation the angle of incidence varies along the span, perhaps as much as 1° per foot run. In an observed spin of B.E.2 E., the radius of turn was 20 ft., the angle of descent 71°, the angle of bank 75°, the mean angle of incidence 32°, and the speed 80 ft./sec.

10. **Spiral Flight with the Engine on.**—Equation (28) becomes

$$T=\frac{\rho}{g}SV^2(k_D+k_R)\pm W\sin\theta \quad . \quad . \quad . \quad (36)$$

the plus sign being taken if the machine be climbing, and the negative sign if descending. Equations (29)-(33) still hold; also (35), but (34) is no longer true.

For a given radius the necessary angle of bank is given by (32) —*i.e.*, $\tan\phi = V^2/(gr\cos\theta)$, and the speed is then found from (35). The necessary airscrew thrust is given by (36) and the power taken is—

$$P=TV=\frac{\rho}{g}SV^3(k_D+k_R)\pm WV\sin\theta$$

$$=P_0\left(\frac{\cos\theta}{\cos\phi}\right)^{\frac{3}{2}}\pm Wv$$

where v = the speed of rising (+) or falling (−), and P_0 is the power taken for horizontal straight-line flight at the same angle of incidence and speed.

II.—PROPERTIES OF AEROFOILS, ETC.

11. **Flow of Air past Aerofoils, etc.**—When air flows past a solid obstacle such as A (Fig. 5), the air flows off on each side as shown, and in the space on the downstream side of the body the steady flow is replaced by an eddy motion, the air forming whirlpools or vortices. If, instead of the air flowing past the solid, the

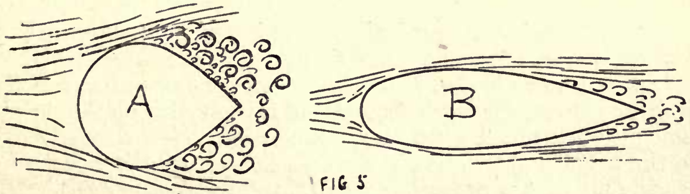


FIG 5

solid be moved through the air the same thing happens, and the formation of these eddies adds to the force required to push the solid through the air. If the tail of the body be stretched out as in *B*, the space, which would otherwise be filled by the eddies, is filled by the solid, the eddies have not so much chance of forming, and the resistance is reduced. The pressure on the extreme front of the body is always $P V^2/2g$ per unit area. It usually happens that, up to a certain speed, the eddies form and pass away from the solid in fairly uniform succession; but when this speed is reached the type of flow is entirely changed, and the regular eddy-formation is replaced by a state of violent turbulence. This velocity is the critical velocity for the solid. The same thing happens when the attitude of the body to the relative stream is changed: there is some critical position for which the *type* of flow changes almost suddenly. The flow past an aerofoil partakes of the nature shown in Fig. 6, and experiments made to measure the pressure all over the section show that it is positive on the surface facing the wind, and a little way round the nose—*i.e.*, over ACB; on the lee side, ADB, of the aerofoil, the pressure is negative—*i.e.*, the air exerts pressure on ACB and suction on ADB. The

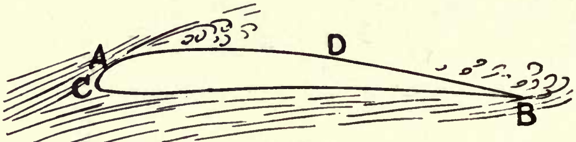
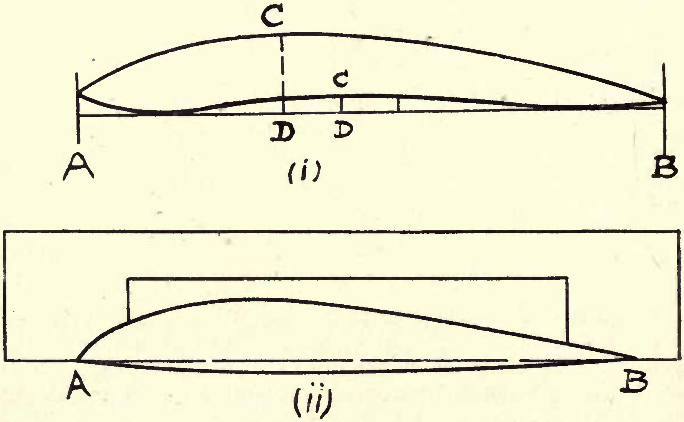


FIG. 6.



(i)

(ii)

FIG. 7.

pressure is greatest at the nose C, and the suction is greatest just in front of the maximum ordinate of the upper surface.

12. **Definitions of Terms relating to Aerofoils:**

(i.) The *chord* is defined in the manner shown in Fig. 7; if the lower surface be concave, (i.), the chord is defined by a straight edge; if convex, (ii.), by a templet. In both cases the length of the chord is *AB*.

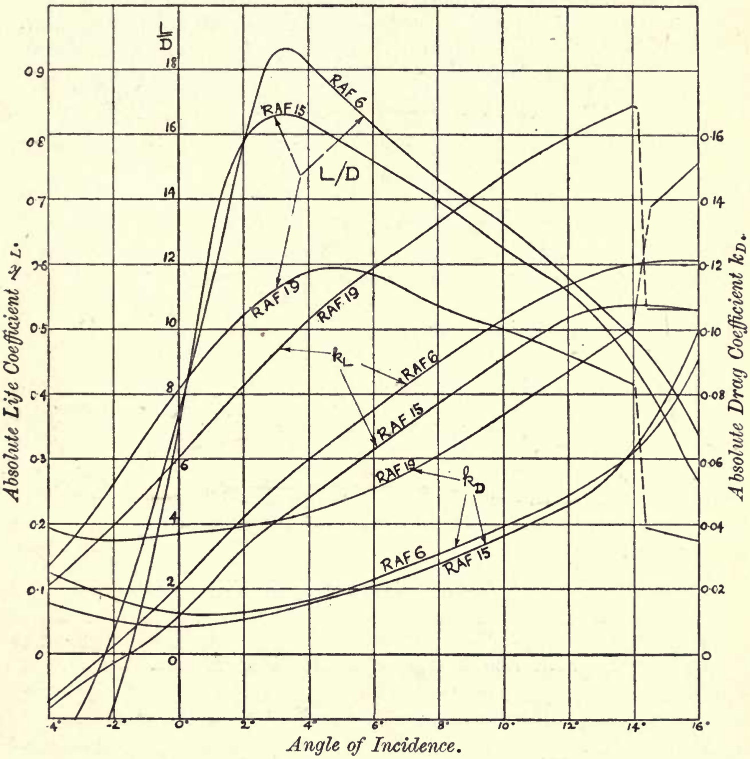


FIG. 8.—CHARACTERISTIC CURVES OF AEROFOILS, FROM MODELS 18′ × 3′. Wind Speed, 40 ft./sec.

(ii.) The *camber* of the surface is the ratio *CD/AB*, where *CD* is the maximum ordinate measured from the chord.

(iii.) *A* is the leading edge, *B* the trailing edge, the aeroplane moving in the direction *BA*.

(iv.) *The Angle of Incidence* is the angle between the chord and the direction of motion.

13. **The Air Forces on an Aerofoil.**—The total force exerted by the air on the aerofoil is measured by its two components, lift and

drag (see § 3 above). The lift is usually zero at a small negative angle of incidence—*i.e.*, when the chord is inclined slightly below the direction of the relative wind—and increases fairly uniformly up to the critical (stalling) angle, when it begins to decrease. The suction on the upper surface contributes more to the lift than the pressure on the lower surface. A few typical curves are shown in Fig. 8.

14. **Aerofoil Characteristics.**—In addition to k_L and k_D (§ 3), another quantity is necessary before we know all the properties of the aerofoil: the air forces exert a moment tending to rotate the aerofoil, and to calculate this we must know the position of the centre of pressure C (Fig. 9). The centre of pressure coefficient (μ) is defined by the ratio AC/AB. The moment coefficient is defined by the equation

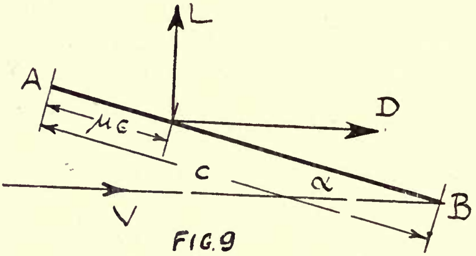


FIG. 9

$$M = \frac{\rho}{g} c k_m S V^2 = -\mu c (L \cos a + D \sin a)$$

or

$$k_m = -\mu(k_L \cos a + k_D \sin a),$$

where M = the moment of the air forces about A, k_m = the moment coefficient, c = the chord, a = the angle of incidence. M and k_m are positive when the forces tend to increase the angle of incidence.

15. **Influence of Shape of Aerofoil Section.**—The effect of modifying the various dimensions of an aerofoil on its most important properties will be seen by studying the accompanying table (pp. 48, 49), so far as available data allow. 1–7 in that table show the effects of progressive changes in certain quantities; 8 shows the effect of a particular modification of R.A.F. 14; 9–12 show the properties of certain sections.

16. **Effect of Plan Form.**—Tests on R.A.F. 6, with an aspect ratio of 6, with rectangular tips, and tips rounded in the form of an ellipse, show a small rise in k_L and fall in k_D, except for large positive angles, as the rounding off is increased. In a plan drawing of the wing the ellipse was drawn with its major axis along the centre of the wing. The maximum L/D is increased considerably

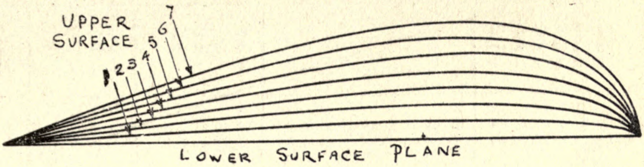


FIG. 10.

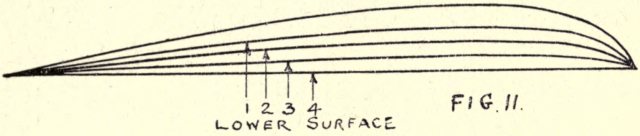


FIG. 11.

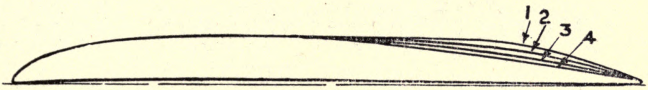


FIG. 12.

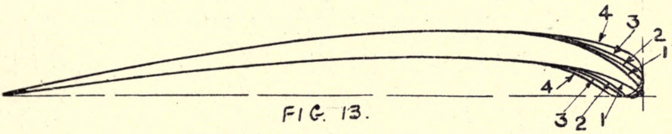


FIG. 13.

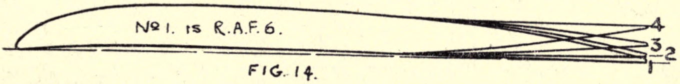


FIG. 14.

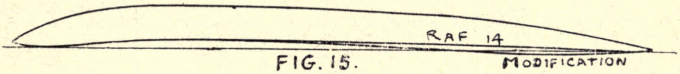


FIG. 15.

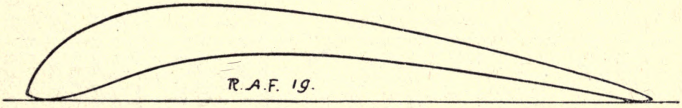


FIG. 16.

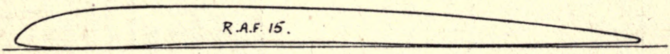


FIG. 17.

	Reference.	*Camber.**		*X/C*†		*Stalling ∠ Degrees.*	*Max. k_L.*	*Max. L/D.*	*k_L at Max. L/D.*	*L/D at $k_L=0·1$.*	*Angle of Zero Lift.*
		Top.	*Bottom.*	*Top.*	*Bottom.*						
Varying camber of top surface, plane under surface. (Fig. 10.) (1)	A.C.A., 1911-12	0·025	0	0·292	—	16·0	0·48	13·8	0·16	13·6	– 0·7
		0·050	0	0·292	—	14·5	0·57	14·9	0·22	7·8	– 1·6
		0·075	0	0·292	—	13·0	0·61	14·4	0·27	6·0	– 2·6
		0·100	0	0·292	—	11·4	0·59	12·7	0·32	4·5	– 3·4
		0·125	0	0·292	—	10·0	0·57	11·75	0·36	3·5	– 4·7
		0·150	0	0·292	—	8·5	0·53	11·1	0·39	3·0	– 6·0
		0·175	0	0·292	—	6·5	0·49	10·2	0·42	2·2	– 7·2
Shifting the max. ordinate of top surface; plane under surface. The 6th of the series is the 4th of series 1 above. (2)	A.C.A., 1912-13	0·10	0	0·5	—	17·0	0·62	11·3	0·47	4·0	
		0·10	0	0·38	—	17·0	0·62	13·3	0·34	4·6	
		0·10	0	0·355	—	17·0	0·67	13·6	0·34	4·6	
		0·10	0	0·332	—	16·0	0·71	13·9	0·32	4·6	
		0·10	0	0·310	—	13·0	0·61	13·9	0·33	4·7	
		0·10	0	0·292	—	11·0	0·57	13·3	0·33	4·7	
		0·10	0	0·252	—	10·5	0·53	12·7	0·304	4·8	
		0·10	0	1·220	—	9·0	0·45	12·0	0·301	4·6	
		0·10	0	0·164	—	8·5	0·41	11·0	0·285	4·4	
Varying under-camber, ordinate of bottom in constant ratio to those of top. The 1st is 4th of series 1 above. (Fig. 11.) (3)	A.C.A., 1911-12	0·10	0	0·292	0·292	11·4	0·59	12·7	0·32	4·5	– 3·4
		0·10	0·02	0·292	0·292	11·4	0·62	13·0	0·36	4·0	– 3·7
		0·10	0·04	0·292	0·292	11·6	0·65	12·5	0·38	3·6	– 4·0
		0·10	0·06	0·292	0·292	11·8	0·67	12·9	0·41	3·0	– 4·2
Convex under surface. Upper surface R.A.F. 6*a* for the 1st four; and section 4*a* (R. and M., 152) for the 2nd four. The ordinates of the bottom surface all proportional to those of top. (4)	R. and M., 195	0·0785	0	0·32	—	14·0	0·604	17·1	0·27	7·2	– 2·3
		0·0785	0·0262	0·32	0·32	14·0	0·565	16·4	0·25	9·3	– 1·7
		0·0785	0·0523	0·32	0·32	14·0	0·536	15·8	0·24	8·9	– 1·0
		0·0785	0·0785	0·32	0·32	14·0	0·476	13·4	0·28	7·0	0
	A.C.A.	0·080	0	0·30	—	14·0	0·617	17·9	0·25	8·9	– 2·5
		0·080	0·0267	0·30	0·30	14·0	0·526	16·8	0·24	10·7	– 1·2
		0·080	0·0534	0·30	0·30	14·0	0·482	15·9	0·26	10·0	– 0·5
		0·080	0·080	0·30	0·30	14·0	0·466	15·7	0·25	9·5	+ 2·0

Thickening towards trailing edge. The 3rd of the series is similar to R.A.F. 6. (Fig. 12.) (5)	A.C.A., 1912-13	0·076 0·076	0·008 0·008	0·30 0·30	0·30 0·30	14·5 14·5 14·0 13·0	0·62 0·62 0·63 0·59	13·3 13·4 14·3 14·6	0·34 0·33 0·30 0·29	5·3 5·0 5·8 7·0	
Thickening the nose (see Fig. 13). The 1st of the series is the last of series 3 above. (6)	A.C.A., 1912-13	0·100	0·06	0·292	0·292	11·8 12 0 11·0 9·0	0·67 0·73 0·63 0·54	12·9 11·5 11·1 10·4	0·41 0·42 0·41 0·33	3·0 3·0 3·0 3·0	– 4·2
Reversed curvature‡ of trailing edge (see Fig. 14). The first is similar to R.A.F. 6. (7)	A.C.A., 1912-13	0·076	0·008	0·30	0·300	15·0 16·0 16·0 18·0	0·63 0·575 0·55 0·48	15·6 15·0 14·2 13·0	0·30 0·24 0·25 0·30	6·5 7·0 8·2 7·8	– 2·2 – 1·3 – 0·5 + 2·5
Adding to the under surface of R.A.F. 14 (see Fig. 15). (8)	R. and M., 195	0·0706	0·0139	0·30	0·30	15·0 16·0	0·55 0·51	15·3 15·0	0·26 0·26	9·5 8·3	– 2·3 – 1·2
Two Admiralty sections. (9)	R. and M., 195	0·10 0·083	0·038 0·042	0·33 0·38	0·34 0·42	18·0 15·0	0·763§ 0·630	16·3 16·3	0·40 0·36	4·0 4·5	– 3·2 – 3·2
R.A.F. 19. (Fig. 16.) (10)	R. and M., 415	0·142	0·065	0·30	0·39	14·0	0·845	12·0	0·55	2·6	– 8·0
R.A.F. 15. (Fig. 17.) (11)	R. and M., 432	0·0669	0·0084	0·30	0·30	14·0	0·54	16·6	0·20	12·0	– 1·5
German Albatross. (12)	R. and M., 375	0·0964	0·0262	0·30	0·40	14·0	0·68	15·8	0·35	5·0	– 3·5

* Negative camber on the bottom surface=convex downwards.

† X=distance of max. ordinate from leading edge; C=chord.

‡ Increasing the upper camber gave a still larger (k_L) max., but reduced max. L/D.

§ The movement of the c.p. is greatly affected by the reversed curvature. In the third case, it moves from 0·33 of the chord for k^L=0·1 to 0·25 for k_L=0·5, whilst for the fourth the movement is reversed: 0·05 to 0·20 for the same range. Further experiments A. C.A., 1913-14) confirm these results.

by rounding off, and a general improvement is obtained up to the case when major axis of the ellipse is a quarter of the span.

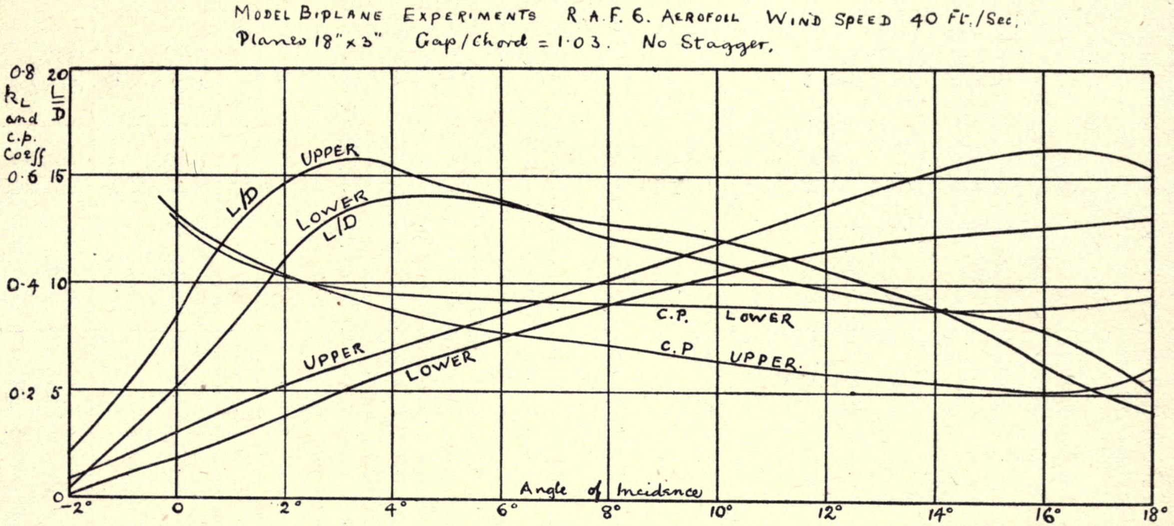
17. **Aspect Ratio.**—Experiments on an aerofoil of section similar to "Bleriot XI., *bis*" (a section having large top and bottom camber), show that as the aspect ratio increases (i.) the stalling angle is decreased, $(k_L)_{\max.}$ is increased slightly, and max. L/D is considerably increased (A. C. A., 1911-12). Similar experiments on R.A.F. 6*a* show the same thing (R. and M., 439). The following are the figures:

	Aspect Ratio.	*Stalling Angle.*	*Max.* k_L.	*Max.* L/D.	k_L *at Max.* L/D.	L/D *at* $k_L = 0{\cdot}1$.	*Angle of no Lift.*
"Bleriot XI., *bis*."	3	19°	0·67	10·1	0·39	4·7	– 2·4
	4	18°	0·68	11·5	0·32	4·4	– 2·4
	5	16°	0·685	12·9	0·316	4·7	– 2·8
	6	16°	0·686	14·0	0·313	5·0	– 2·9
	7	16°	0·69	15·1	0·310	4·7	– 3·2
	8	15°	0·686	15·5	0·305	4·4	– 3·6
R.A.F. 6*a*.	6	14°	0·59	17·1	0·25	8·0	– 2·3
	13	12°	0·605	23·0	0·29	8·0	– 2·3
	15·65	12°	0·615	25·0	0·33	8·0	– 1·9

The last example had rounded ends, and probably the aerodynamic superiority of this over the one with A.R. = 13 is due to this rather than to the increase of A.R. All the others had square ends. Other evidence goes to show that it is unlikely that L/D would be increased any more at still larger aspect ratios. At small values of k_L the A.R. does not appear to have much effect on L/D.

18. **Biplanes.**—Data obtained from tests of model monoplanes cannot be applied to biplanes or triplanes without modification. The lift coefficient is reduced, particularly on the lower plane, and consequently L/D is reduced as well. The effects are shown in Fig. 18 for R.A.F. 6.

There are two points upon which biplane effect chiefly depends: the ratio of the gap between the planes to the chord, and the "stagger"—*i.e.*, the relative fore-and-aft position of the planes. Stagger is measured by the angle θ (Fig. 19). It is easy to understand at once how the top plane interferes with the lower plane if the flow from the latter be considered (Fig. 19). The flow on the under surface of the top plane and the upper surface of the bottom plane are disturbed, consequently, since, in both cases, it is the upper surface which gives the greater part of the lift, we expect the lift of the lower wing to be reduced more than that of the upper; Fig. 19 indicates how the interference is influenced by



Model Biplane Experiments R.A.F. 6. Aerofoil Wind Speed 40 Ft./Sec.
Planes 18" x 3" Gap/Chord = 1·03. No Stagger.

FIG. 18.

gap and stagger. The effect of gap/chord ratio and stagger for R.A.F. 6 is shown below (R. and M., 196); the figures refer to the complete biplane.

Stagger.	*Gap/ Chord.*	*Stalling Angle.*	*Max. k_L.*	*Max. L/D.*	*k_L at Max. L/D.*	*L/D for k_L=0·1.*
0	0·67	17·5°	0·531	14·1	0·25	7·5
	1·00	—	0·562	15·3	0·24	7·5
	1·33	—	0·57	16·0	0·26	7·5
	1·67	—	0·6	16·1	0·25	7·5
	2·00	—	0·6	16·7	0·25	7·5
	2·33	16°	0·6	17·2	0·25	7·5
+30°*	0·9	16°	0·585	14·8	0·25	7·5
	0·9	16°	0·555	14·6	0·25	7·5
−20°	0·9	16°	0·525	14·9	0·25	7·5

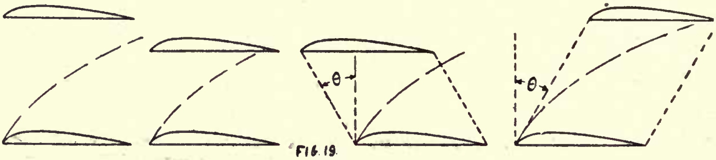


FIG. 19.

19. **Triplanes.**—The centre plane suffers most from interference in this case. The general effects of altering the gap/chord ratio and stagger are the same as for a biplane. Fig. 20 shows the properties of the several planes for R.A.F. 15 as a triplane, and the table below shows the effects of the gap/chord ratio and stagger for R.A.F. 6. In all cases the two gaps are equal, and the leading edges of the three sections are in a straight line. The figures refer to the complete triplane.

Further triplane experiments are given by Hunsaker in *Engineering*, July, 1916.

Stagger.	*Gap/ Chord.*	*Stalling Angle.*	*Max. k_L.*	*Max. L/D.*	*k_L at Max. L/D.*	*L/D for k_L=0·1.*
0	0·5	20°	0·439	10·0	0·224	6·0
	0·75	19°	0·493	10·8	0·200	—
	1·0	18°	0·532	11·6	0·216	6·5
	1·5	17°	0·558	12·4	0·242	—
	2·0	16°	0·575	13·7	0·254	6·7
−30°	1·0	15°	0·48	11·8	0·238	—
0	1·0	18°	0·532	11·6	0·216	6·5
+30°	1·0	18°	0·59	12·2	0·242	7·0
+60°	1·0	18°	0·621	12·1	0·25	—

---

* +Stagger means that the top plane is in front of the bottom.

20. **Pressure Distribution along the Span.**—The intensity of loading is not constant along the wing, but falls off towards the tip; the manner of distribution varies somewhat according to the angle of incidence, and is different for the top and bottom planes.

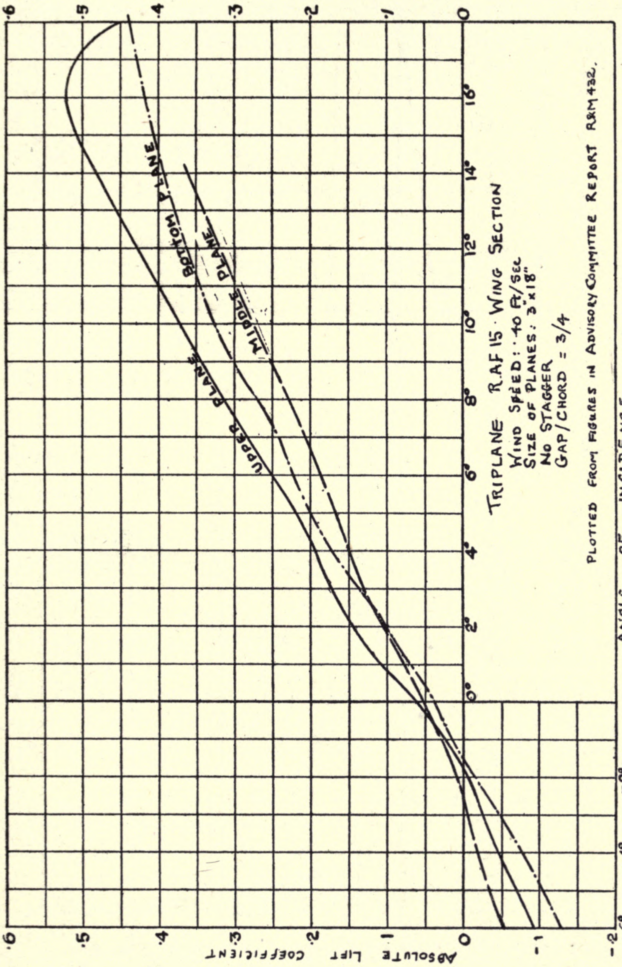


FIG. 20

Experiment indicates (1) that the load per inch run over the wingtip is independent of the chord, a decrease in chord being compensated by a rise of pressure intensity; (2) that the c.p. coefficient is practically constant all along the wing. The critical angle of the lower plane is much greater than that of the top plane for all points along the span, and the centre sections reach the critical angle before the outer. The latter effect is very pronounced on the top wing, and causes the load per inch run to rise over the outer sections at

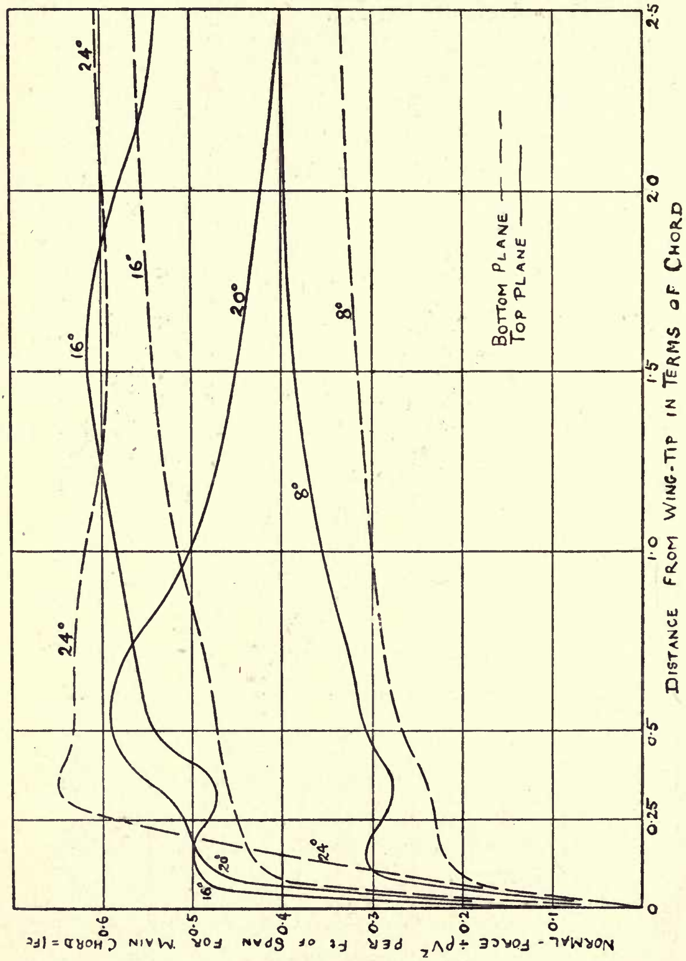


FIG. 21.

large angles of incidence. Since the top plane stalls before the bottom, the loading on the latter exceeds that on the former at large angles. Examples of load distribution are given in Fig. 21 (R. and M., 355).

21. **The "Equivalent Plane" of a Biplane** is defined in Fig. 22, a and b being given by—

$$\frac{a}{b}=\frac{S_2k_{L2}}{S_1k_{L1}}$$

S_1 = area of top plane AB, S_2 = area of bottom plane CD, k_{L1} and k_{L2} = their respective lift coefficients. For stress calculations it is usual to take $k_{L1}/k_{L2}=1{\cdot}2$.

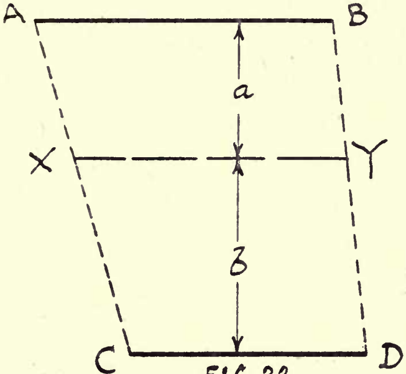


FIG. 22.

22. **Scale Effect.**—It can be shown that, if L and l denote corresponding dimensions of any part of an aeroplane and a model of that part, and if V and v are the velocities, the forces on the model and on the full-scale article will be the same provided $VL=vl$. It is not practicable to realize this condition with model experiments: if the model be 1/10 scale the air speed would have to be 10 times its full-scale value—*i.e.*, something of the order of 1,000 miles per hour. For this reason it is necessary to see how the force coefficients vary as vl is increased; these variations are called "scale-effect." If a model aerofoil has a chord = 3 inches and the wind speed = 60 ft./sec., $vl=15$; if an aeroplane has a chord of 5 feet and a speed of 150 ft./sec., $vl=750$, in ft./sec. units. Hence the range of vl to be covered is great. Various experiments have been made at the N.P.L. in this direction, and point to the following conclusions: For $vl>10$, the lift coefficient does not change appreciably over the range from max. L/D to max. lift, and neither does the critical angle; the drag coefficient decreases continuously up to vl = about 10; the max. L/D increases continuously to the end of the experimental range ($vl=12{\cdot}5$), is still increasing slightly, but it appears to become constant at vl = about 30; the angle at which the max. L/D occurs is reduced, and the angle of no lift is also reduced (*i.e.*, has a greater negative value), as vl increases. Scale-

effect corrections are given in the section dealing with performance. Since the range of *vl* is so great it is better to take log *vl* as a base when plotting curves; the errors in extrapolation will be reduced.

The above figures refer to tests on an R.A.F. 6 aerofoil (A.C.A., 1912–13 and 1913–14). Later experiments on a model biplane of the same aerofoil show similar results.

MATHEMATICS

By JOHN CASE, M.A., A.F.R.Ae.S.

Section I.—Pure Mathematics.

1. Algebra.

1. **Indices**:

$$x^m \cdot x^n = x^{m+n} \qquad x^m / x^n = x^{m-n}$$

$$1/x^m = x^{-m} \qquad x^0 = 1 \qquad \sqrt[m]{x} = x^{1/m}$$

2. **Logarithms**:

$$\log mn = \log m + \log n. \qquad \log x^n = n \log x.$$

$$\log \frac{m}{n} = \log m - \log n. \qquad \log 1 = 0.$$

3. **Use of Logarithm Tables.**—The logarithm of a number consists of a decimal part and a whole number. The decimal only is given in the tables, and is always +. The whole number depends only on the position of the decimal point; the rule is: for the logarithm of a number >1 the whole number is 1 *less* than the number of figures to the left of the decimal point, and is plus; for a number <1 it is 1 *more* than the number of 0's after the decimal point, and is minus. Thus:

From the tables, $\log 2{\cdot}0 = 0{\cdot}3010$.

2 is *one* figure to the left of the decimal, so the number to be added to 0·3010 is zero.

$\log 20 = 1 + 0{\cdot}3010 = 1{\cdot}3010$; $\log 200 = 2{\cdot}3010$.
$\log 0{\cdot}2 = -1 + 0{\cdot}3010$; written $\bar{1}{\cdot}3010$.
$\log 0{\cdot}02 = -2 + 0{\cdot}3010$; written $\bar{2}{\cdot}3010$.

In multiplying and dividing numbers like $\bar{1}{\cdot}3010$ by another number remember the decimal part is +. Thus $7(\bar{1}{\cdot}3010) = 7(-1 + 0{\cdot}3010) = -7 + 2{\cdot}1070 = \bar{5}{\cdot}1070$.

The logarithms given are $\log_{10}$. To find $\log_e$ multiply by 2·3026.

4. **Equations.**—The solutions of $x^2 + bx + c = 0$ are—

$$x = -\frac{b}{2} + \tfrac{1}{2}\sqrt{b^2 - 4c} \text{ and } -\frac{b}{2} - \tfrac{1}{2}\sqrt{b^2 - 4c}.$$

If $b^2 < 4c$ the roots are imaginary.

Equations of a higher degree than the second are best solved by successive guesses. The following rules may be helpful:

If we know the answer is *large*, a first idea of the root may be obtained by ignoring all powers of x except the highest; if we know the answer is *small*, neglect all powers of x except the lowest.

Try to spot where the function changes sign. Usually in such equations we only require one root, and we very likely have some idea of its size; probably the other roots are meaningless or silly. For instance, if we are trying to find the weight of something, and it is given by a cubic equation, we only want the real positive root of the equation: if it has two negative or imaginary roots as well we do not mind; of course, it *may* have two real positive roots, but the nature of the problem will usually help us.

Example of a cubic:

$$x^3 + 2x - 1000 = 0 \text{ and we know } x \text{ is large.}$$

As a first shot $x^3 = 1000$, $x = 10$. $x = 10$ makes the left side $= +20$; $x = 0$ makes the left side $= 1{,}000$; so clearly x is a little less than 10. Try $x = 9{\cdot}9$; this makes the left side $= -12$. So x is between 9·9 and 10, and nearer the former; 9·94 makes the left side $= 1{\cdot}99$. Probably this is near enough. Squared paper may be used as a help.

5. **Binomial Theorem.**—Definition of Factorial n, written n or $\underline{|n}$:

$$n! = \underline{|n} = n(n-1)\,(n-2)\;.\;.\;.\;3.2.1.$$

Expansions:

$$(x+a)^n = x^n + nx^{n-1}a + \frac{n(n-1)}{2!}x^{n-2}a^2 + \frac{n(n-1)\,(n-2)}{3!}x^{n-3}a^3 + \;.\;.\;.$$

for all values of n.

When x is small we can take—

$$(1+x)^n = 1 + nx. \qquad (1-x)^n = 1 - nx.$$

$$\frac{1}{(1+x)^n} = 1 - nx. \qquad \frac{1}{(1-x)^n} = 1 + nx.$$

2. **Trigonometry.**

6. **Defs.:**

$$\sin\theta = \frac{\mathrm{PM}}{\mathrm{OP}} \qquad \cos\theta = \frac{\mathrm{OM}}{\mathrm{OP}} \qquad \tan\theta = \frac{\mathrm{PM}}{\mathrm{OM}}$$

$$\operatorname{cosec}\theta = \frac{\mathrm{OP}}{\mathrm{PM}} \qquad \sec\theta = \frac{\mathrm{OP}}{\mathrm{OM}} \qquad \cot\theta = \frac{\mathrm{OM}}{\mathrm{PM}}$$

OP is +. OM is + to the right; − to the left.
PM is + above OA; − below OA.

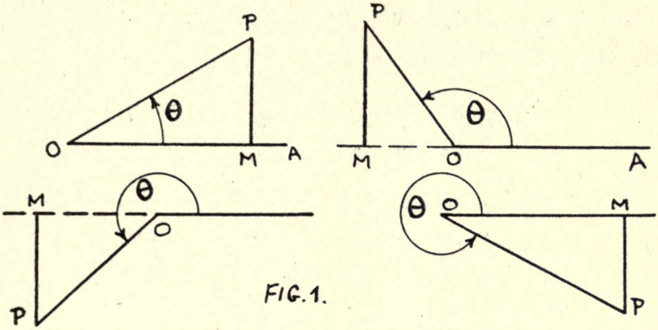


FIG. 1.

7. Relations between the Functions:

$$\sin^2\theta = 1 - \cos^2\theta. \qquad \cos^2\theta = 1 - \sin^2\theta.$$
$$\tan\theta = \sin\theta/\cos\theta. \qquad \cot\theta = \cos\theta/\sin\theta.$$
$$\sin\theta = \frac{\tan\theta}{1+\tan^2\theta}. \qquad \cos\theta = \frac{1}{1+\tan^2\theta}.$$

8. Functions of $(90^\circ \pm \theta)$ and $(180^\circ \pm \theta)$:

$$\sin(90^\circ \pm \theta) = \cos\theta. \qquad \cos(90^\circ \pm \theta) = \mp\sin\theta.$$
$$\sin(180^\circ \pm \theta) = \mp\sin\theta. \qquad \cos(180^\circ \mp \theta) = \cos\theta.$$

9. Solutions of Triangles:

In a right-angled triangle ABC—

$$c^2 = a^2 + b^2.$$
$$a = c\sin A = c\cos B = b\tan A.$$
$$b = c\cos A = c\sin B = a\tan B.$$

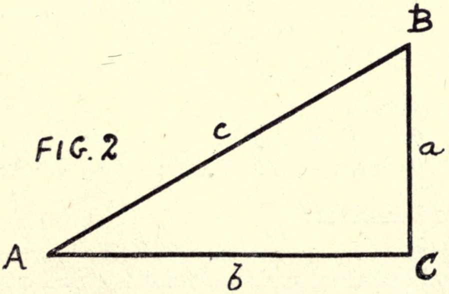


FIG. 2

In any triangle:

(i.) Given o, c, A, to find a, B, C.

$a^2 = b^2 + c^2 - 2bc \cos A$ gives a;

then $\sin B = \frac{a}{b} \sin A$ gives B, and $C = 180° - A - B$.

(ii.) Given b, c, B, to find a, A, C.

$\sin C = \frac{c}{b} \sin B$ gives C, and then $A = 180° - B - C$, and

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

(iii.) Given a, b, c, to find A, B, C.

$$\cos \mathrm{A} = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

give A and B, and then $C = 180° - A - B$.

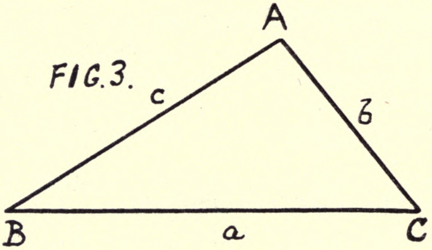


FIG. 3.

(iv.) Given A, B, c, to find a, b, C.

$$C = 180° - A - B.$$

$$a = c\frac{\sin A}{\sin C} \qquad b = c\frac{\sin B}{\sin C}.$$

10. **Sum of Difference of Two Angles:**

$$\sin(\theta \pm \phi) = \sin\theta \cos\phi \pm \cos\theta \sin\phi$$
$$\cos(\theta \pm \phi) = \cos\theta \cos\phi \mp \sin\theta \sin\phi$$
$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta \tan\phi}.$$

11. **Double Angles:**

$$\sin 2\theta = 2 \sin\theta \cos\theta.$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta.$$

12. **Inverse Functions:**

$\sin^{-1} x$ means the angle whose sine is x.
$\cos^{-1} x$ means the angle whose cosine is x, etc.

13. **Radian Measure of an Angle.**—The value of θ measured in radians is the arc $AB \div$ radius R. $\theta^\circ = \pi\theta/180$ radians.
θ radians $= 180\theta/\pi$ degrees.

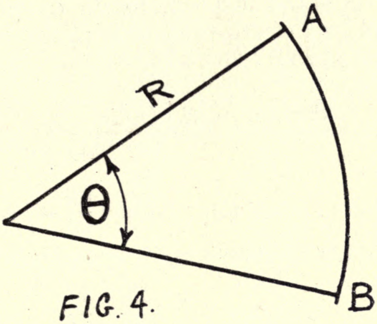


FIG. 4.

14. **Expansions:**

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \quad \ldots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \quad \ldots$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \ldots$$

where θ is measured in radians.

3. Differential Calculus.

15. In what follows, u, v, y are any functions of x—*i.e.*, any expressions involving x; other letters, a, b, c, n, m, are constants—*i.e.*, numbers.

Meaning of Differential Coefficient.—The differential coefficient of y with respect to x, written $\frac{dy}{dx}$, means the rate at which y is changing compared with x.

16. **List of Standard Differential Coefficients:**

y	$\frac{dy}{dx}$	y	$\frac{dy}{dx}$
any constant	0	$a \cot bx$	$-ab \operatorname{cosec}^2 bx$
ax^n	nax^{n-1}	$a \sec bx$	$ab \sec bx \tan bx$
ae^{nx}	nae^{nx}	$a \operatorname{cosec} bx$	$-ab \operatorname{cosec} bx \cot bx$
$a \log_e nx$	$\frac{an}{x}$	$a \sin^{-1} bx$	$\frac{ab}{\sqrt{1-b^2x^2}}$
$a \sin bx$	$ab \cos bx$	$a \tan^{-1} bx$	$\frac{ab}{1+b^2x^2}$
$a \cos bx$	$-ab \sin bx$		
$a \tan bx$	$ab \sec^2 bx$		

17. Methods of Differentiating:

(i.) Sum of several functions: differentiate each and add.

(ii.) Product of several functions; multiply the differential of each by all the others and add—*e.g.*, for three functions, 1, 2, 3, the differential coefficient of the product is (2nd × 3rd × diff. of 1st) + (1st × 3rd × diff. of 2nd) + (1st × 2nd × diff. of 3rd).

(iii.) A fraction:

$$\text{Diff. of Fraction} = \frac{\text{denom.} \times \text{diff. of num.} - \text{num.} \times \text{diff. of denom.}}{(\text{denominator})^2}$$

(iv.) A function of a function: To find $\frac{dy}{dx}$ where y is a function of u, and u in turn is a function of x:

$$\text{The rule is } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx};$$

e.g., $y = \log(\sin x) = \log u$, where $u = \sin x$;

$$\frac{dy}{du} = \frac{1}{u}; \qquad \frac{du}{dx} = \cos x;$$

$$\therefore \frac{dy}{dx} = \frac{\cos x}{u} = \frac{\cos x}{\sin x} = \cot x.$$

We proceed in the same way if the function process is carried to three or more operations:

$$y = \log\left[\sin(e^{x^2})\right]$$

$$\frac{dy}{dx} = \left(\frac{1}{\sin e^{x^2}}\right)(\cos e^{x2})\ ({e_x}^2)\ (2x).$$

The method can be put into words thus: any function of x given in the table above is obtained by doing something to x—we square it, or take the logarithm of it, or take the sine of it, or something. To each operation performed on x there is a corresponding operation to be done when we want the differential coefficient; this is what is given under the column dy/dx; *e.g.*, if we make a function by taking the logarithm of something, when we want the differential coefficient we must take 1 over this something—*i.e.*, turn it upside down. So, when we want to differentiate a complicated expression like the one given above, we see what has been done to x to produce the function; we take each of these operations, beginning as far away from x as possible, and perform the proper operation to obtain the differential coefficient, and multiply all the results. Thus, taking the function above, the *last* thing done was to take a logarithm, so the *first* thing to do to find the differential coefficient is to turn upside down what comes after the logarithm;

before the logarithm a sine was taken, so we take the cos of what comes after sin, and so on.

(v.) If x and y be given as functions of a third variable u,

$$\frac{dy}{dx}=\frac{dy}{du}\div\frac{dx}{du}.$$

(vi.) If x and y are given by $f(x, y)=0$: differentiate f with respect to x, treating y as a constant and call the result f_x; differentiate f with respect to y, treating x as a constant, and let the result be f_y. Then—

$$\frac{dy}{dx}=-\frac{f_x}{f_y}.$$

18. **Successive Differentiation.**—If we differentiate $\frac{dy}{dx}$ we obtain $\frac{d^2y}{dx^2}$, and so on.

19. **Maxima and Minima.**—Fig. 5 shows what we mean by max. and min. here: the points marked A are maxima, B are minima.

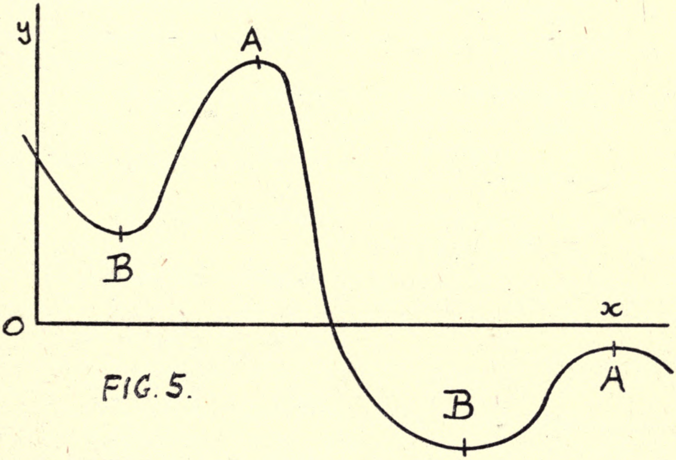


FIG. 5.

To see when y is a max. or min., find $\frac{dy}{dx}$, and put the result $=0$; solve the resulting equation for x. Then find $\frac{d^2y}{dx^2}$, and substitute in the result the values of x found from the equation $dy/dx=0$; then,

those values of x which make $\frac{d^2y}{dx}+$, make $y=\text{min}$.

those values of x which make $\frac{d^2y}{dx}$, make $y=\text{max}$.

20. **Properties of Curves.**—The slope of the tangent at (x, y) is $\frac{dy}{dx}$.

$$\rho = \text{the radius of curvature} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} \Big/ \frac{d^2y}{dx^2}.$$

If the curvature is slight we can take $\frac{1}{\rho}=\frac{d^2y}{dx^2}$.

4. Integral Calculus.

21. The Integral Calculus can be regarded as useful for two purposes:

(i.) Given $\frac{dy}{dx}$ to find y: if $\frac{dy}{dx}=u,\ y=\int u.dx$.

(ii.) Given a little bit of something, to find the whole; if we can find the value of a little bit of an area bounded by a certain curve we can find the whole, and so on.

22. **List of Useful Integrals:**

u	$\int u.dx$
ax^n	$\frac{ax^{n+1}}{n+1}$
$\frac{1}{x+a}$	$\log_e(x+a)$
e^{ax}	e^{ax}/a
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x^*}{a}$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\log_e\frac{x-a}{x+a}\ (x>a)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\log_e\frac{a-x}{a+x}(x<a)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x^*}{a}$
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\frac{x}{a}$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\frac{x}{a}(x>a>0)^*$
$\sqrt{a^2-x^2}$	$\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x^*}{a}$
$\sqrt{x^2-a^2}$	$\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\cosh^{-1}\frac{x}{a}$

u	$\int u.dx$
$\sin ax$	$-\frac{1}{a}\cos ax$
$\cos ax$	$\frac{1}{a}\sin ax$
$\tan ax$	$-\frac{1}{a}\log\cos ax$
$\operatorname{cosec} x$	$\log\tan\frac{x}{2}$
$\sec x$	$\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$
$\cot ax$	$\frac{1}{a}\log\sin ax$
$\sin^2 x$	$\frac{x^*}{2}-\frac{\sin 2x}{4}$
$\cos^2 x$	$\frac{x^*}{2}+\frac{\sin 2x}{4}$
$\tan^2 x$	$\tan x - x^*$
$\cot^2 x$	$-x^* - \cot x$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\cosh ax$	$\frac{1}{a}\sinh ax$
$\sinh ax$	$\frac{1}{a}\cosh ax.$

* The angle is to be expressed in radians.

u	$\int u.dx$
$\sqrt{x^2+a^2}$	$\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\sinh^{-1}\frac{x}{a}$
$x \cos nx$	$\frac{x \sin nx}{n}+\frac{\cos nx}{n^2}$
$x \sin nx$	$-\frac{x \cos nx}{n}+\frac{\sin nx}{n^2}$
$x^2 \cos nx$	$\frac{x^2 \sin nx}{n}+\frac{2x \cos nx}{n^2}-\frac{2 \sin nx}{n^3}$
$x^2 \sin nx$	$-\frac{x^2 \cos nx}{n}+\frac{2x \sin nx}{n^2}+\frac{2 \cos nx}{n^3}$
$e^{ax} \sin bx$	$e^{ax} \cdot \frac{a \sin bx - b \cos bx}{a^2+b^2}$
$e^{ax} \cos bx$	$e^{ax} \cdot \frac{b \sin bx + a \cos bx}{a^2+b^2}$

23. **Rules for Integration:**

(i.) To integrate the sum of several functions, integrate each separately and add.

In order to integrate a function which is not included in the above list, we must turn it into one of those forms or the sum of several such forms. The following methods are available:

(ii.) Try substituting x = some function of a new letter z, and $dx=\frac{dx}{dz}\cdot dz$. When the integration has been done substitute back for z in terms of x.

(iii.) If the function consist of the product of two functions of the form $\frac{du}{dx}\cdot f(u)\cdot dx$, we can integrate at once—*e.g.*,

$$\int(2ax+b)\sin(ax^2+bx+c)dx=-\cos(ax^2+bx+c).$$

(iv.) If the function consist of the product of two functions, one of which can be integrated by itself, and the other of which reduces to a simpler form when differentiated, use the formula:

$$\int uvdx=u\int vdx-\int\left[\int vdx\right]\frac{du}{dx}\cdot dx$$

$$\begin{aligned} e.g., \int x \cos x \cdot dx &= x\int \cos x\, dx-\int \sin x\cdot\frac{dx}{dx}dx \\ &= x \sin x-\int \sin dx\cdot dx \\ &= x\sin x+\cos x \end{aligned}$$

(v.) Rational Algebraic Fractions of the form $\frac{f(x)}{ax^2+bx+c}$.

If the numerator be higher than the first degree divide out until it is of first degree. This gives terms like x^n + a fraction. The x terms integrate, and the fraction is of the form—

$$\frac{px+q}{ax^2+bx+c}.$$

Arrange this so:

$$\frac{\frac{p}{2a}(2ax+b)+\left(q-\frac{pb}{2a}\right)}{ax^2+bx+c}.$$

The integral of this is—

$$\frac{p}{2a}\log_e(ax^2+bx+c)+\left(q-\frac{pb}{2a}\right)\frac{dx}{ax^2+bx+c}.$$

The latter integral

$$=\frac{1}{a}\frac{dx}{\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b}{4a^2}\right)}.$$

Put $x+\frac{b}{2a}=y$, and this is one of the forms given in the table.

(vi.) *Other Rational Algebraic Fractions.*—Try to break them up into fractions which can be integrated.

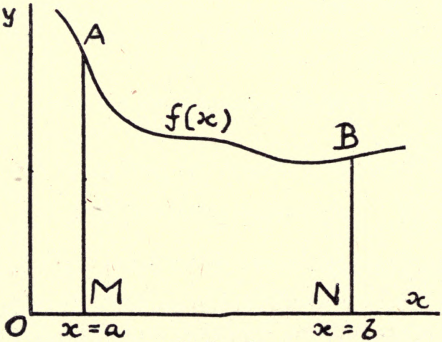


FIG. 6.

24. **Definite Integrals.**—If we want the value of an integral from $x=a$, to $x=b$, substitute $x=a$ and $x=b$ in the value of the integral, and subtract—*e.g.*:

$$\int_0^{\frac{\pi}{4}}\cos x\cdot dx=\sin\frac{\pi}{4}-\sin 0=\frac{1}{\sqrt{2}}.$$

If we want the value of a definite integral, and we cannot perform the integration, the value can be found by plotting the function to be integrated.

The value of $\int_a f(x)\,dx$ is the area shown in the diagram, and can be found by a planimeter.

25. **Lengths, Areas, etc.**—If we know the equation of a curve in the form $y=f(x)$, then (Fig. 6)—

(i.) The length of the curve $AB=\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$.

(ii.) The area $MABN=\int_a^b ydx$.

(iii.) The volume of the solid generated by AB revolving round Ox is $\pi\int_a^b y^2dx$.

SECTION II.—APPLIED MECHANICS.

1. The component of a force P along $OA=P\cos\theta$; the component perpendicular to $OA=P\sin\theta$.

2. The moment of P, about any point B, is P multiplied by BM.

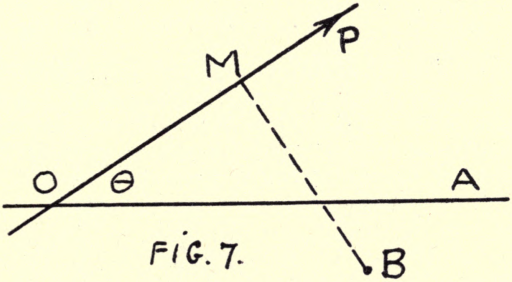


FIG. 7.

3. For any body which is in equilibrium under forces in one plane, equations for finding the unknown quantities are found by—

(i.) The sum of the components, in any direction, of all the forces must $=O$.

(ii.) The sum of the moments, about any point, of all the forces $=O$.

These two conditions will solve any problem on forces in one plane.

4. **Forces in Equilibrium.**—If lines are drawn parallel to the direction of the forces, consecutively, with lengths proportional to the forces, the lines so drawn must form a closed polygon, as in the diagram. Hence (i.) if we know the forces 1, 2, 3, 4, but nothing about 5, we find the magnitude and direction of 5 by drawing *abcde*, and then the line *ea*, required to complete the figure,

gives 5; alternatively *ae*, equal and opposite to 5, is the resultant of 1, 2, 3, 4. This gives the size and direction of 5; to find its position proceed thus:

In the figure *abcde* take any point *O* and join itto the corners; take any point 1 on the line of action of 1, and draw a line, parallel to *Ob*, to meet the line of action of 2 in 2; from 2 draw 23 parallel to *Oc*, and so on till 4 is reached. Then from 1 and 4 draw lines parallel to *Oe* and *Oa*; where they cut is a point on 5.

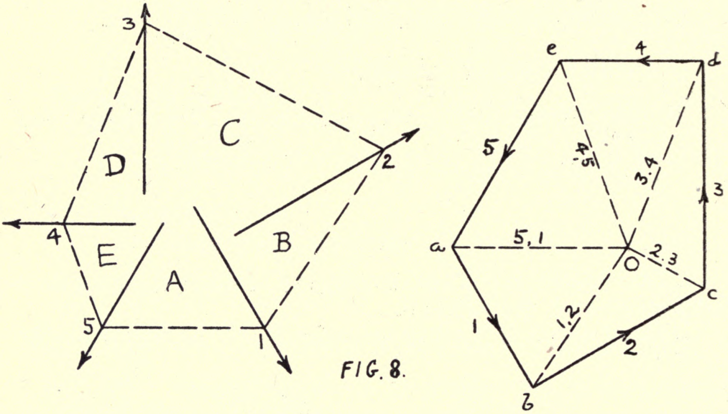


FIG. 8.

(ii.) If the forces meet at one point and we know all but two of them, and know the directions of these two, we can find their sizes; if we know 1, 2, 3, draw *abcd*, and then draw *de* and *ae* in the known directions of 4 and 5; where they cut is *e*.

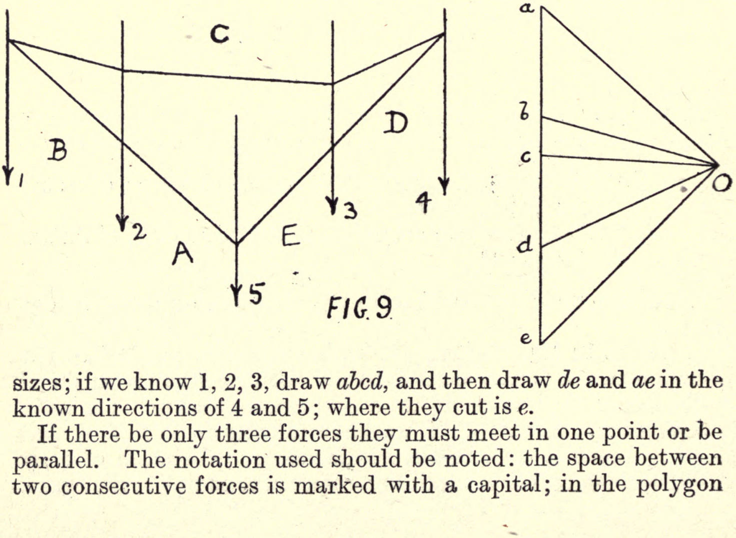


FIG. 9

If there be only three forces they must meet in one point or be parallel. The notation used should be noted: the space between two consecutive forces is marked with a capital; in the polygon the line representing a force is marked with the letters on each side of the force in the force diagram; the order of going round the two figures must be the same.

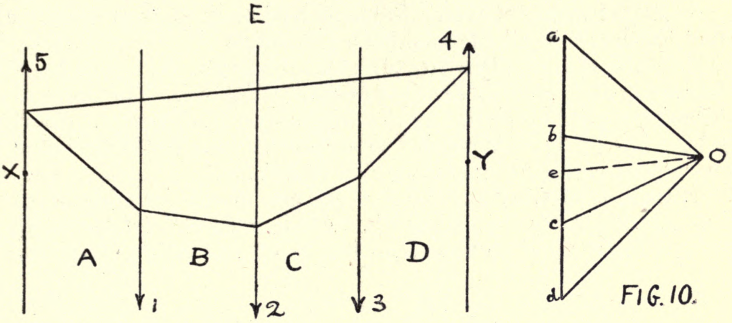


FIG. 10.

5. **Parallel Forces.**—The same method can be used for parallel forces. Fig. 9 shows the manner of finding the resultant (5) of 1, 2, 3, 4. Its size is the sum of 1, 2, 3, 4.

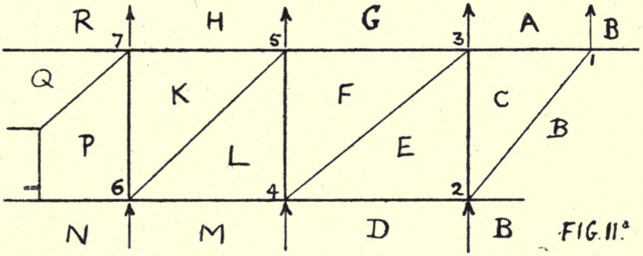


FIG. 11a.

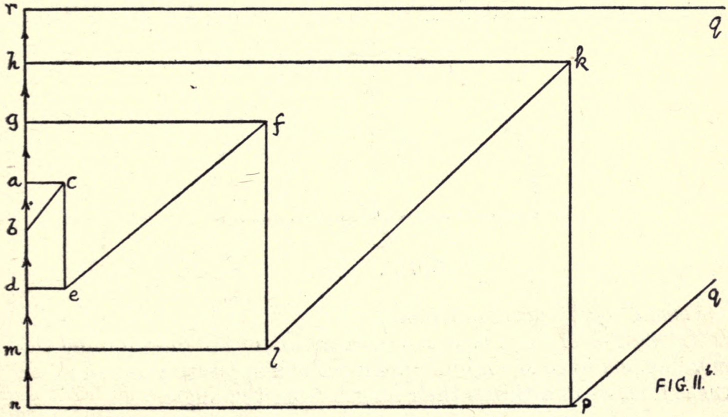


FIG. 11b.

Fig. 10 shows how to find two forces, 4 and 5, passing through two given points *X*, *Y*, which keep equilibrium with 1, 2, 3. Figs. 8, 9 and 10 are all lettered to correspond, so that the relation between them may be seen.

6. **Frameworks.**—Given a frame under known applied loads, to find the forces in all the members. Example:

The forces (*A*,*B*), (*D*,*B*), (*G*, *A*), etc., are known. The polygon of forces for each point is drawn as described above. The order of taking the points is shown by the numbers on the diagram. Each line in the top figure is parallel to the line between the same letters in the bottom diagram: *ef* is parallel to *EF*, etc.

7. **Centres of Gravity.**—To find the c.g. of any area: divide it into parallel strips of equal width. Along the middle of each strip take a force equal to the mean length of this strip. Find the position of the resultant of all these parallel forces: let it be the line *XX*. Repeat with strips in a perpendicular direction, and let the resultant act along *YY*: the c.g. is where *XX* and *YY* cut.

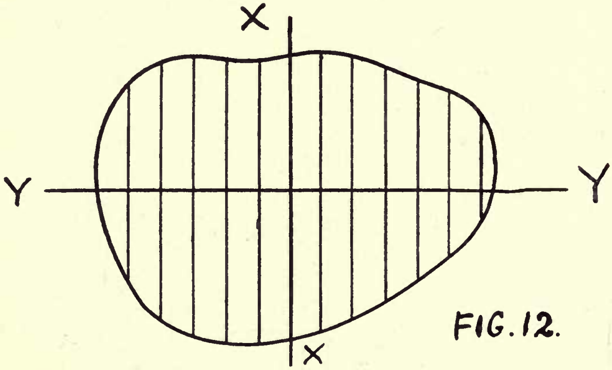


FIG. 12.

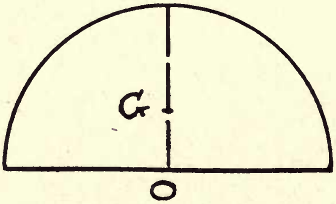


FIG. 13.

Some Useful Centres of Gravity:

Triangle.—The c.g. is at the intersection of the lines joining the angular points to the middle points of the opposite sides—*i.e.*, on these lines at two-thirds their length from the angle.

Parallelogram.—The c.g. is at the intersection of the diagonals.

For plate semicircular OG (Fig. 13) $=0{\cdot}4244\ R$.

For hemisphere $Og=0{\cdot}375\ R$.

$$\text{In Fig. 14: GN}=\frac{A_1y_1+A_2y_2}{A_1+A_2+A_3},$$

$$\text{GM}=\frac{A_1x_1+A_2x_2+A_3x_3}{A_1+A_2+A_3},$$

where $A_1=$ area of AB; $A_2=$ area of CD; $A_3=$ area of EF.

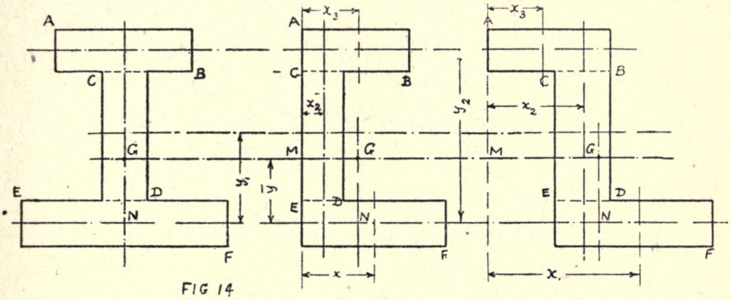


FIG 14

8. **Moments of Inertia.**—The M.I. of an area about a line XY is the sum of the products of the areas of all little strips like AB multiplied by x^2—*i.e.*, $\text{I}=\int x^2y\,dx$.

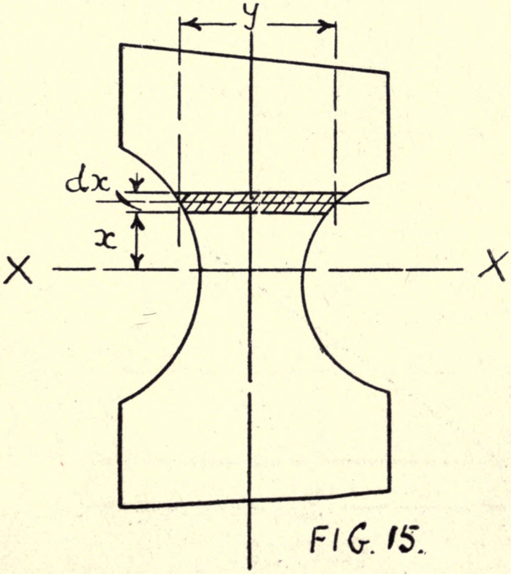


FIG. 15.

It is found easily by squared paper.

If I_x = the M.I. of an area A about XX, through the c.g., then the M.I. about YY, parallel to XX, is $I_y = I_x + Ah^2$.

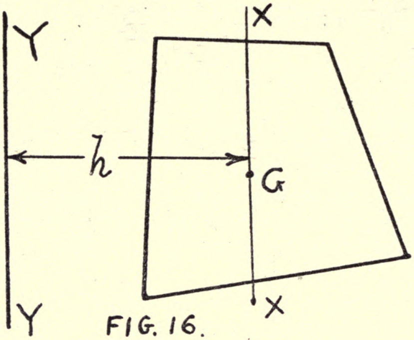


FIG. 16.

Table of Moments of Inertia of Areas.

$I^x = \frac{BD^3}{12}$. $\frac{BD^3 - bd^3}{12}$. $\frac{\pi(D^4 - d^4)}{64}$.

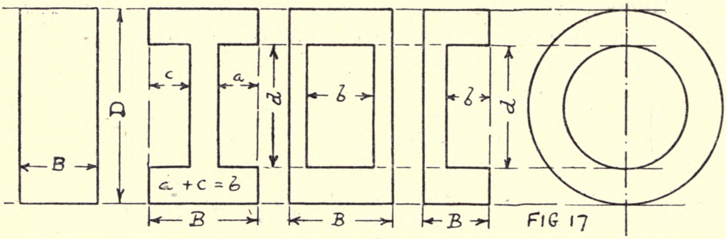


FIG 17

$\frac{BD^3}{63}$.

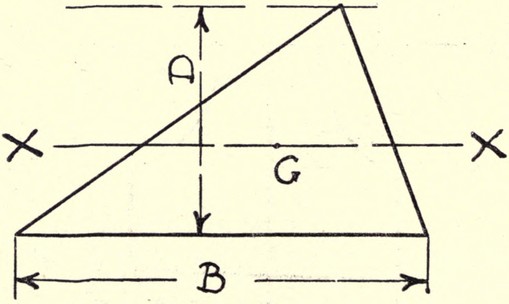


FIG. 18.

Ellipse.—

$$I_x = \frac{\pi(AB^3 - ab^3)}{64}.$$

$$I_y = \frac{\pi(A^3B - a^3b)}{64}.$$

If we write $I = Ak^2$, k is the radius of gyration.

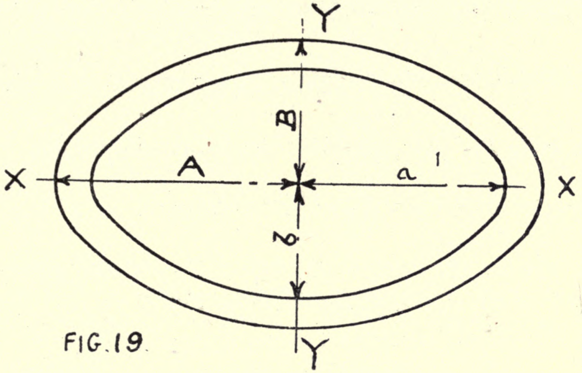


FIG. 19

9. **Mass.**—The mass of a body = its weight ÷ g; in the foot, pound, second system, $m = \text{mass} = \frac{W}{g}$ *slugs.*

10. **Mass Moments of Inertia** (I).—In the dynamics of rotating bodies we require the (mass) moment of inertia—*i.e.*, the sum of the products of the mass of each element of the body by the square of its distance from the axis of rotation. The units are slug - ft.^{2}.

Table.

(i.) *Uniform Rod*, length l,	
about axis through one end, perp. to length	$\frac{W}{g} \cdot \frac{l^2}{3}$
about axis through centre, perp. to length -	$\frac{W}{g} \cdot \frac{l^2}{12}$
(ii.) *Rectangular Plate* (sides a, b),	
about axis through centre parallel to side a	$\frac{W}{g} \cdot \frac{b^2}{12}$
about axis through centre $\perp^{R}$ to its plane	$\frac{W}{g} \cdot \frac{a^2 + b^2}{12}$

(iii.) *Rectangular Block* (sides a, b, c),

about axis through c.g., parallel to a - - $\frac{W}{g}\cdot\frac{b^2+c^2}{12}$

(iv.) *Circular Disc, or Cylinder* (radius r),

about its axis - - - - - - - $\frac{W}{g}\cdot\frac{r^2}{2}$

(v.) *Circular Ring*, about its axis - - - - $\frac{W}{g}\cdot r^2$

(vi.) *Solid Sphere*, about diameter - - - $\frac{W}{g}\cdot\frac{3r^2}{5}$

If we write $I=\frac{W}{g}k^2$, k is called the radius of gyration.

Analogous to (8) above $I_Y=\frac{W}{g}(k^2+h^2)$.

11. *Velocity, Acceleration, Force, etc.*

	Straight Line Motion.	*Rotation.*
With constant acceleration (*i.e.*, constant forces):		Note: radians/sec. $=\frac{2\pi}{60}\times$ r.p.m.
Initial speed	u ft./sec.	ω_1 radians/sec.
Speed after t secs. ..	v ft./sec.	ω_2 radians/sec.
Distance	s ft.	θ radians
Acceleration ..	f ft./sec.^{2}	α rad./sec.^{2}
	$v=u+ft.$	$\omega_2=\omega_1+\alpha t$
	$s=ut+\frac{1}{2}ft.^2$	$\theta=\omega_1 t+\frac{1}{2}\alpha t^2$
	$v^2=u^2+2ft.$	$\omega_2{}^2=\omega_1{}^2+2\alpha\theta$
	Force = P lb.	Couple = T lb.ft.
	W*t*. = W lb.	M.I. = I lb.ft.^{2}
Momentum	wv/g	$\mathrm{I}\omega/g$
Kinetic Energy ..	$wv^2/2g$	$\mathrm{I}\omega^2/2g$
	$\mathrm{P}=\mathrm{W}f/g$	$\mathrm{T}=\mathrm{I}\alpha/g$
Work done	$\mathrm{P}s$	$\mathrm{T}\theta$
Rate of working (h.p.)	$\mathrm{P}v/550$	$\mathrm{T}\omega/550$
If acceleration and forces not constant	$v=\frac{ds}{dt}$	$\omega=\frac{d\theta}{dt}$
	$f=\frac{dv}{dt}=\frac{d^2s}{dt^2}=v\frac{dv}{ds}$	$\alpha=\frac{d\omega}{dt}=\frac{d^2\theta}{dt^2}=\omega\frac{d\omega}{d\theta}$
	$\mathrm{P}=\frac{\mathrm{W}f}{g}$	$\mathrm{T}=\frac{\mathrm{I}\alpha}{g}$

SECTION III.—STRENGTH OF MATERIALS.

1. E = Young's modulus.
N = Modulus of rigidity.
p or f_t = Tensile stress.
p or f_c = Compressive stress.
q or f_s = Shear stress.

R = Pull or Thrust.
M = Bending moment.
T = Torque.
A = Area of section.
1 = Length.

F = Shearing force.
e = Elongation or contraction per unit of original length.
ϕ = Shear strain (radians).

2. **Relation between Stress and Strain:**

Shear $\phi = \frac{F}{NA}$ $q = \frac{F}{A}$.

Tension or compression $e = \frac{R}{EA}$ $p = \frac{R}{A}$.

3. **Principal Stresses.**—If a material be under two perpendicular stresses p_1 and p_2, and shear stress q, as shown, there are two planes for which the shear stress is zero. Across one the direct stress is a maximum, across the other a minimum. These are the principal stresses. They are given by—

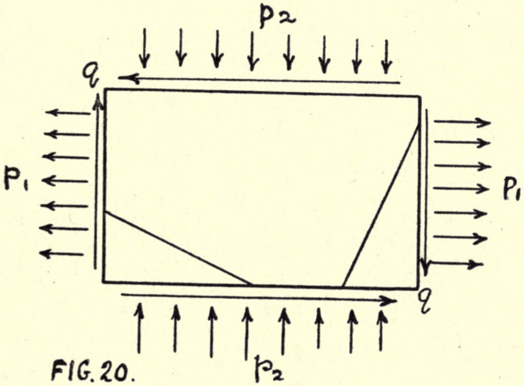


FIG. 20.

$$\tfrac{1}{2}(p_1 - p_2) \pm \tfrac{1}{2}\sqrt{(p_1 + p_2)^2 + 4q^2},$$

p_1 being a tensile stress and p_2 compressive, and the inclinations of the planes are given by—

$$\tan 2\theta = \frac{2q}{p_1 + p_2},$$

which gives two values for θ, differing by 90°. Either p_1 or p_2 may be negative or zero.

4. **Strain Energy.**—The work done in stretching (or compressing) a rod is $\frac{R^2}{2EA}$.

Loads suddenly applied produce double the stress they would produce if applied gradually.

5. **Bending of Beams.**—If a beam be loaded in any manner the bending moment at any point P is the algebraic sum of the moments about P of all the forces (including reactions) between P and either end of the beam. The shearing force at P is the algebraic sum of all the forces on one side of P.

	Max. B.M.	*Max. Shear.*	*Max. Deflection.*
Cantilever:			
W concentrated at end ..	Wl	W	$\frac{Wl^3}{3EI}$
w per unit length ..	$\frac{1}{2}wl^2$	w	$\frac{wl^4}{8EI}$
Freely supported beam:			
W concentrated at distance a ($<l/2$) from one end	$\frac{W a(l-a)}{l}$	$\frac{W(l-a)}{l}$	$\frac{W a^2(l-a)^2}{3EI}$
w per unit length ..	$\frac{1}{8}wl^2$	$\frac{1}{2}wl$	$\frac{5}{384}\frac{wl^4}{EI}$

In the above table, l = length, I = moment of inertia of cross-section (supposed constant).

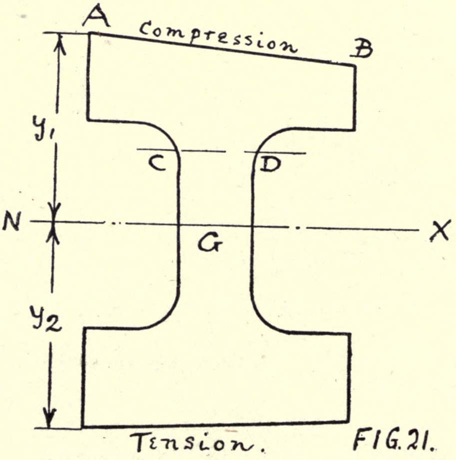


FIG. 21.

6. **Stresses due to Bending.**—In the diagram the curvature is supposed concave upwards. The part above NX is in compression; maximum stress is $f_c = \frac{My_1}{I}$.

The part below NX is in tension; maximum stress is $f_t = \frac{My_2}{I}$.

Shear stresses are in the plane of the section, and in longitudinal planes parallel to NX. The maximum shear stress occurs at the thinnest part of the web.* The shear stress at any point on CD is given by—

$$f_s = \frac{F \times \text{moment of CABD about } NX}{I \times \text{CD}}.$$

This gives the intensity of shear, either in the plane ABCD or in the longitudinal plane through CD.

7. **Torsion.**—A rod of length l is twisted by a torque T through an angle ϕ. Then—

$$T = NC\phi/l,$$

where C is the torsional stiffness of the section.

Section.	*C.*	*Max. Shear Stress.*
Solid circle (diam. $=D$) ..	$\frac{\pi D^4}{32}$	$\frac{16T}{\pi D^3}$ on surface
Hollow circle (outside diam. D; inside diam. d)	$\frac{\pi}{32}(D^4 - d^4)$	$\frac{16TD}{\pi(D^4 - d^4)}$ on surface
Square, side a	$\frac{a^4}{7 \cdot 11}$	$\frac{T}{0{\cdot}208a^3}$ at middle of side
Ellipse, and major axis, a; minor axis, b	$\frac{\pi a^3 b^3}{16(a^2 + b^2)}$	$\frac{16T}{\pi ab^2}$ at end of minor axis
Rectangle, $a \times b$, $a > b$ ..	—	$\frac{3a + 1{\cdot}8b}{a^2 b^2} T$ {at middle of long side

For empirical method of finding torsional stiffness of other sections see a paper by Griffiths and Taylor, *Proc. Inst. Mech. Eng.*, 1917. Remember that projections do not help torsional stiffness; thus a square section is only 6 per cent. stiffer than the inscribed circle; I sections are very weak in torsion.

SECTION IV.—MISCELLANEOUS.

1. **To Draw a Parabola, given Height and Base.**—(i.) Divide base and height equally, and the points on the curve are found as shown.

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* Unless the corners are very sharp.

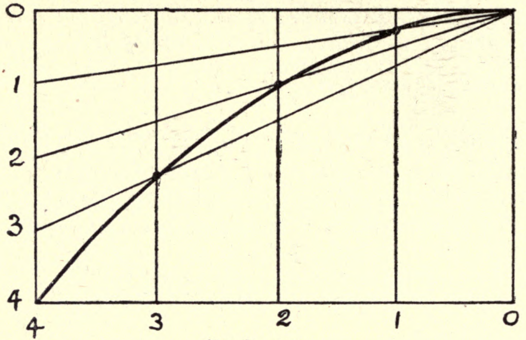


FIG 22

(ii.) Produce *DC* to *E* so that *EC*=*CD*. Divide *EA* and *EB* into equal parts; join as shown. The joining lines are tangents to the curve.

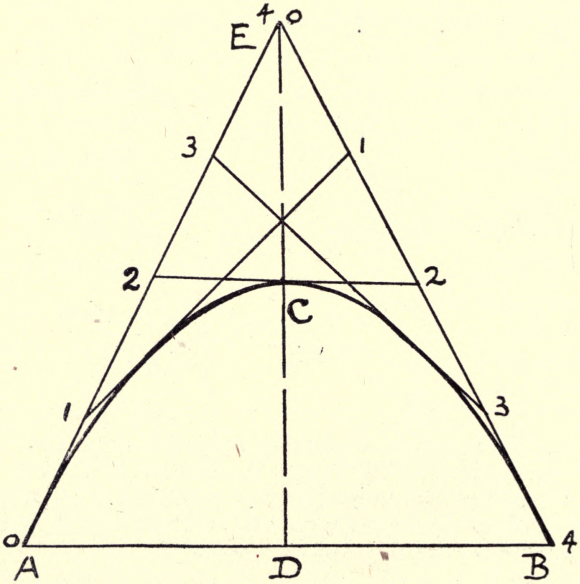


FIG. 23

2. **Formulæ connecting Angles in Three Dimensions** (useful in drawing wiring plates, etc.).—*Ox*, *Oy*, *Oz*, are three convenient reference lines mutually at right angles. *OP* is any line drawn through *O*.

α, β, γ are the inclinations of OP to the three reference lines Ox, Oy, Oz.

θ, ϕ, ψ are the "plan angles" on the planes yOz, zOx, and xOy respectively.

(Or we can take the planes in pairs—*e.g.*, xOy and yOz—and say

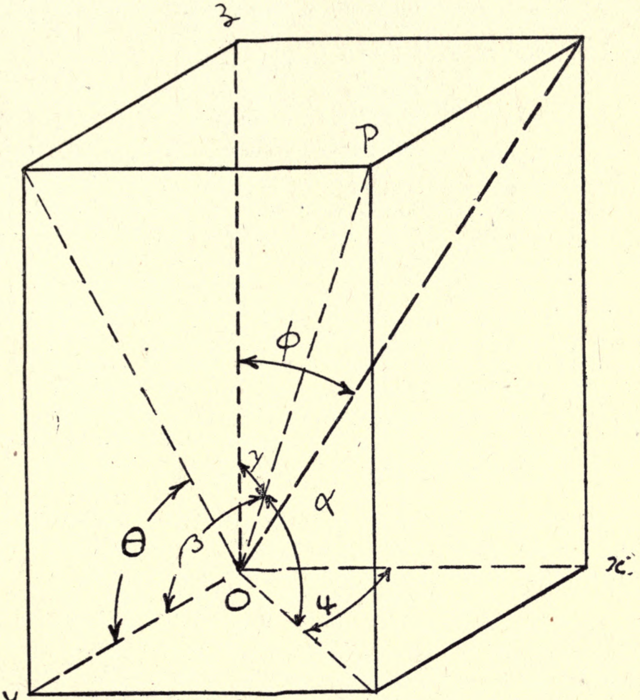


FIG. 24

that ψ is the plan angle on the plane xOy and θ is the elevation angle on the plane yOz, and so on.)

(i.) *Given two of the three angles* α, β, γ, *to find the third.*

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

(ii.) *Given two of the angles* α, β, γ, *to find the true angles between* OP *and each of the planes* xOy, yOz, zOx.

(*a*) Find the third angle out of α, β, γ, by (1), then—

(*b*) Inclination to plane
$$\left.\begin{array}{lll} \text{Inclination to plane} & & xOy = 90^\circ - \gamma \\ \quad ,, & ,, & yOz = 90^\circ - \alpha \\ \quad ,, & ,, & zOx = 90^\circ - \beta \end{array}\right\}$$

(iii.) *Given* α, β, γ, *to find the projected angles* θ, ϕ, ψ.

$$\cos\theta=\frac{\cos\beta}{\sin\alpha}\qquad\cos\phi=\frac{\cos\gamma}{\sin\beta}\qquad\cos\psi=\frac{\cos\alpha}{\sin\gamma}.$$

(iv.) *Given two of the angles* θ, ϕ, ψ, *to find the third.*

$$\tan\theta\ \tan\phi\ \tan\psi=1.$$

(v.) Given θ, ϕ, ψ, to *find* α, β, γ.

$$\begin{aligned}\cot\alpha&=\tan\phi\sin\theta.\\ \cot\beta&=\tan\psi\sin\phi.\\ \cot\gamma&=\tan\theta\sin\psi.\end{aligned}$$

HIGH TENSILE STEEL WIRE AND CABLES

By JOSEPH WILSON

Lieutenant R.A.F. (Air Construction Service)

THE place of wire in the construction of aircraft combines, in its great importance, many considerations which manufacturers and consumers alike have been prone to overlook in the hurry of development during the period covered by the Great War. It is therefore necessary to examine such considerations as are worth while at the moment, and which may be of value in the future.

Wire Drawing.—Undoubtedly one of the most ancient of handicrafts, and, like many such, hedgebound in this commercial age by subtleties of tradition wellnigh akin to the buying and selling of horses. Great is the manufacturer of wire who can boast in legitimate pride that he appreciates the true relativity of *torsion* to *tensility*, superior to the business principle. In this direction lies " the only way " to the production of a sound, usable, safe article.

Raw Material.—Starting with the wire rod, from which all denominations of size or gauge are drawn, it is discoverable that many rod qualities are available from which wire can be produced up to any degree of tensility required.

For instance, so-called high tensile wire can be manufactured from basic steel, which, under ordinary test (a straight pull) would give a high tensile strain. But in order to attain tha the wire has had to be very severely punished in the processes of drawing, and is extremely brittle, a condition which is not realized (unless the torsion demanded is relatively high and searching) until such wire is made up into rope form, when abject failure under ordinary shock stress at once results. This failure is the direct consequence of the wire having no ductility, or, say, quality.

Slightly better than basic is the acid process steel rod.

In this case the processes of drawing are not so drastic, but there is always the danger that the slight carbon content is subordinated by an element (sulphur-phosphorus) frequently encountered in all acid steels.

It follows, then, that, owing to the variant character of acid steel, proper relativity (tenacity and elasticity) cannot be relied upon in wire for high tensile duty.

The ideal material, from which are manufactured all wires required for aircraft purposes, is the rod produced from the so-called "Swedish" ores, and which can always be relied upon to be absolutely pure.

The Swedish ores contain, when mined, more than 65 per cent. of iron, either magnetic or hematite, or both, in intimate combination with quartz, apatite, chlorite, and calcareous spar.

To appreciate the wonderful purity of Swedish ore, which, it is of interest to know, is mostly mined within the confines of the Arctic Circle, the contrasting average percentage of iron found in rods for the rest of the ore-producing countries of the world is 38 per cent.

The case, therefore, for the Swedish rod, no matter under what other name it might be called ("plough," etc.), is unanswerable for the purpose of aircraft, whilst acid steels should only be used under the most careful conditions; and as for basic, its use should be made a penal offence.

It is rather a remarkable fact that the poorest of the Allies is, at the time of writing, using Swedish rods for the production of their aircraft wires, in spite of price and import difficulties.

Torsion *v.* **Hardness.**—Apart from official specified requirements, which in regard to torsion (ductility) are invariably low, the minimum number of torsions, or twists, for H.T. wire should be as follows:

Tons per Square Inch.		
120 to 130	..	34 twists per length of 100 diams.
130 to 145	..	32 ,, ,, ,,
145 to 160	..	30 ,, ,, ,,

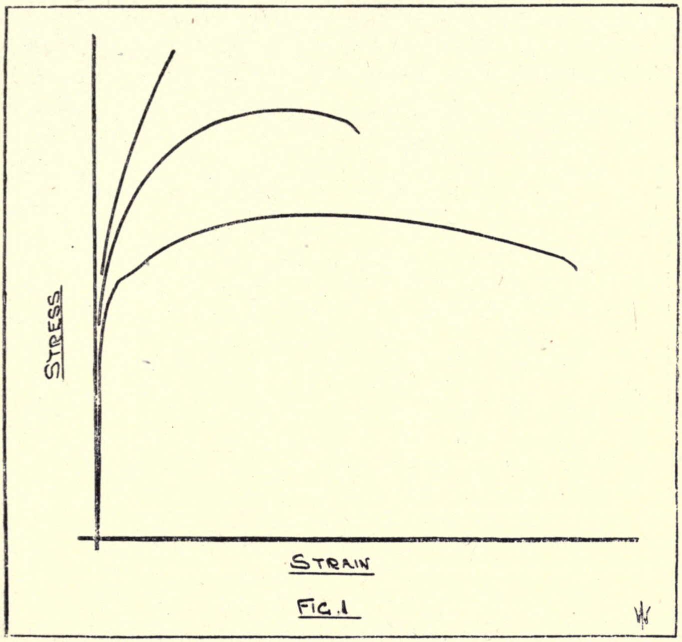
This table is based upon actual working experience, and, whilst not being correct to official requirements, will be found to assist greatly in searching wire of the stated tonnages for brittleness.

An important factor bearing upon this matter is the amount of hardness which the wire might have attained during the process of drawing. This point is illustrated graphically in Fig. 1, which shows how hardness can be accentuated, and the wire correspondingly punished, by being "pushed" through its drawing stages, using only a minimum number of holes.

This usage naturally affects ductility to such an extent as to practically eliminate any semblance to relative torsion—*i.e.*,in the upper curve especially, which has advanced from 80 tons per square inch tensility, through six holes or drawing stages, and reaches the greatly enhanced tensility of 160 tons per square inch.

The lower curve shows that by drawing the wire through successive easy stages—in this instance, twelve holes—the hardness has been increased by an amount corresponding to an increase in tenacity of barely 10 tons per square inch over the minimum strain required.

This latter is a range of hardness which is quite sufficient for any manufacturing purposes, and also provides for any qualitative variance in the rods.



The result, as shown in the above diagram, can be applied to any quality of rod, starting from, say, 80 tons per square inch.

Drawing wire from rods of this quality should, if the drawing operations be properly prepared, as to number of sizes, or holes, or speed of drawing finished at not more than 130 tons per square inch, and similarly from rods of any lesser quality, and so on.

Fig. 2 illustrates the foregoing in greater detail, and amply repays the keen student of wire for close attention.

If the stresses which are set up during drawing could be kept within the range *QA*, then the elastic limit *A* would not be passed, and hardness not be encountered; if, however, the stresses produce a condition represented by the point *B*, some distance beyond the elastic limit, the material will then have received a certain amount of permanent set—in other words, hardness has crept in, and the wire is said to be hard-drawn.

On removing the load the specimen assumes a condition represented by the point *H*—*i.e.*, it has received a permanent set equal to *GH*. Similarly, if the load is carried to points *C*, *D*, *E*, and *F*, the material will be taking on more and more permanent set, until finally the breaking-point, *G*, is reached.

A is the primitive elastic limit, whilst *B*, *C*, *D*, *E*, and *F*, are artificial elastic limits corresponding to different degrees of hardness.

The condition of the wire intended for high duty ropes should not lie beyond point *D*, and beyond this point the material becomes dangerously brittle.

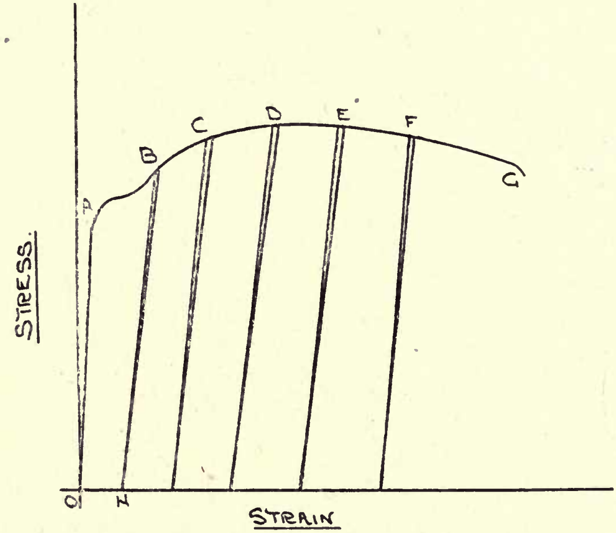


FIG 2

The dotted line shows the stress strain curve indicating the presence of over-hardness.

Sorting the Rods.—The careful manufacturer always sorts his rods when size has been reduced to, say, 12 or 13 S.W.G., as it is at this stage that rods can be detected for quality sufficient for drawing to fine sizes.

Poor rods are subsequently drawn down to nothing smaller than 0·036 diameter, whilst the good rods are set aside for sizes down to 0·076. Swedish rods will draw down to anything.

Whilst on the subject of rods, it might be well to suggest that, with a proper system of analysis, the temptation which is ever present with a certain type of manufacturer to push the whole of his rods through, irrespective of quality, hard or soft, would be removed.

Testing Wire.—The chief importance in testing wire of the larger diameters is undoubtedly the bending and torsion test.

The tensility can always be detected by the number of twists the wire resists—*i.e.*, few torsions, high tensility; many torsions, low tensility.

Whenever the torsion is low, greater watchfulness should be devoted to the bend test.

Where torsion nears the minimum number of twists specified, and tensile is near the maximum, suspect the wire for bend.

In the smaller diameters the bend test is, of course, not demanded, nor is it necessary, the specified tests being torsion only. An additional test can, however, be taken, especially on coils of fine wire which produce high tensility and low torsion.

Take, for instance, a coil of wire, say 0·010 inch diameter, the specified break being 21 to 24 lb.: the minimum number of twists required are 110 in a length of 4 inches.

Actual tests may have resulted in the maximum tensility of 24 lb., and the torsion 110 twists only.

When these conditions are encountered, a twelve-inch length of wire should be held firmly in the hands, and reversed in opposite directions, so as to form a loop, or bite, in the middle of the length, and then, by a gentle, steady pull with each hand, thus applying a combined bending and shearing stress, the wire, if brittle, will quickly fracture, whilst if sound the fracture comes slowly. With practice and a little observation brittleness can be at once "felt" where existent, and readily proven.

By this simple method the writer has seen tensility guessed to within half a pound of the actual break.

Table I., which follows, will be found useful in readily computing or breaking loads of all wire sizes now used in aircraft production.

To carefully calibrate, the tensile machine should be a *sine qua non*, particularly before proceeding to test the smaller denominations of wire, and, having done so, the elimination of slip in the box grips should have careful attention.

During testing operations, manipulation of the adjusting screw at the foot of the machine, which is sometimes necessary in taking up slip on elongation, should not be carried on after the jockey weight has traversed 50 per cent. of the load on the lever, as further adjustment at this stage tends to hasten the break. In this way much hard or brittle wire has been made to pass muster, by being dodged through the machine before inspectors with both eyes open and observation shut.

TABLE I.

Size of Wire.	*Breaking Load in Pounds.*	*Size of Wire.*	*Breaking Load in Pounds.*	*Size of Wire.*	*Breaking Load in Pounds.*
0·0070	0·0862	0·0091	0·1455	0·022	0·8500
0·0071	0·0886	0·0092	0·1491	0·023	0·9300
0·0072	0·0911	0·0093	0·1525	0·024	1·014
0·0073	0·0936	0·0094	0·1555	0·025	1·100
0·0074	0·0965	0·0095	0·1591	0·026	1·188
0·0075	0·0990	0·0096	0·1625	0·027	1·280
0·0076	0·1015	0·0097	0·1655	0·028	1·380
0·0077	0·1042	0·0098	0·1691	0·029	1·480
0·0078	0·1080	0·0099	0·1725	0·030	1·580
0·0079	0·1100	0·010	0·1760	0·031	1·700
0·0080	0·1126	0·011	0·2128	0·032	1·800
0·0081	0·1155	0·012	0·2530	0·033	1·930
0·0082	0·1183	0·013	0·2970	0·034	2·040
0·0083	0·1210	0·014	0·3448	0·035	2·160
0·0084	0·1240	0·015	0·3960	0·036	2·300
0·0085	0·1275	0·016	0·4500	0·037	2·400
0·0086	0·1295	0·017	0·5080	0·038	2·560
0·0087	0·1330	0·018	0·5700	0·039	2·700
0·0088	0·1360	0·019	0·6350	0·040	2·800
0·0089	0·1394	0·020	0·7000		
0·0090	0·1425	0·021	0·7750		

Directions for Use of Table.

(*a*) Required, the breaking stress of a wire of given section. First find the breaking load of the wire by means of a tensile test, let it be *w* lb. breaking load.

Assume that the wire is 0·020 diameter. Opposite 0·020 in the table is given 0·7000 lb. Hence—

$$\text{Breaking stress}=\frac{W}{0{\cdot}7000}\text{ lb. per sq. inch.}$$

Thus, if the breaking load given by the testing machine is 70 lb.,

$$\text{Breaking stress}=\frac{70}{0{\cdot}7000}=100\text{ tons per sq. inch.}$$

(*b*) Given the tensile stress required, to find at what load failure should occur.

Assume that a tensile stress of 100 tons per sq. inch is required.

Then, if the wire diameter is 0·008 inch., we find opposite this figure in the table the value 0·1126 lb.

Thus, the breaking load is 100×0·1126=11·26 lb.

Wire Ropes and Cables.—A glance at Table II. will show the wide range of wire ropes now being used for aircraft, the larger sizes being for various field purposes.

The construction adopted in each case is one giving flexibility and ease of splicing, but other constructions are now being adopted as wider knowledge of roping comes to the designer, and those concerned with the production of aircraft.

One of the chief requisites in a rope, whether it is intended for control or standing (bracing), purposes, is that the individual wires forming the strands shall be continuous, as far as possible; and where joints occur, such should be permanent in character, like, for instance, the Britannia joint used by electricians.

Where wire ends are simply tucked into the body or core of the strand, and there are more than, say, three such joints in a length of twelve feet of rope, the length should be cut out and destroyed.

Within the writer's experience much carelessness in the manufacture of important control ropes unfortunately exists, and is chiefly expressed in the form of a vagrant wire laid up into a rope like a separate strand. This condition is caused by a short single wire, having been tucked while stranding, coming adrift from the strand whilst still in the machine during closing, and so laid into the rope.

A vagrant wire being slack in lay might easily foul a control, either at the pulley or sheave, with disastrous results. The term "stranding" indicates the process by which the single wires are formed into strands, and these strands are then put into another machine and "closed" into the finished rope. Sufficient lay, or twist, is given to the wires when being stranded to keep them nicely bundled, whilst the lay given during the "closing" operation is designed to effect flexibility. A flexible rope has a short or quick lay, or many twists, whilst a non-flexible rope has a long lay.

Testing Wire Rope.—The most efficient rope is one which weighs the minimum amount per unit load, and yet possesses sufficient toughness to withstand the shocks met with in practice. Tests should, as far as possible, seek to discover whether brittleness exists in the rope, as this forms a disability which decides its future usefulness. It is therefore necessary, when cutting lengths from the coil for tensile tests, to take an extra piece, carefully unwind a few single wires, and feel for brittleness as formerly described, and if unsatisfactory, take torsion tests based upon the table given under "Wire." The bend test specified for ropes is also useful, and should never be omitted.

The preparation of a length of rope for tensile tests is best arranged with metalled ends, instead of spliced loops, because, if the splice is not carefully made, the break will occur in the splice. For rough testing, a length of rope may be cut from the coil, and gripped in the testing machine.

The proper preparation, however, is to have the ends metalled, and no test length should have a lesser breaking length than ten inches. Table II. gives a range of ropes which practically includes the whole of the flexible order now being used for aircraft, and it should be noted that diameters and weights are approximate only.

TABLE II.

Breaking Load.	Maximum Circumference of Rope.	Maximum Diameter of Rope.	Construction of Cable.			Weight per Length of 10 Feet.
			Number of Strands.	Wires per Strand.	Diameter of Wire.	
Cwt.	*Inches.*	*Inches.*			*Inches.*	*Ounces.*
5	0·213	0·068	7	7	0·0076	—
10	0·345	0·110	7	14	0·0084	—
15	0·445	0·142	7	19	0·0092	5
20	0·508	0·162	7	19	0·010	6
25	0·560	0·177	7	19	0·011	8
30	0·607	0·193	7	19	0·012	9
35	0·630	0·201	7	19	0·013	10
40	0·669	0·213	7	19	0·014	12½
45	0·728	0·232	7	19	0·015	14
50	0·756	0·241	7	19	0·016	16
						Pounds.
60	0·827	0·264	7	19	0·017	1·25
70	0·854	0·272	7	19	0·018	1·25
75	0·906	0·289	7	19	0·019	1·35
80	1·016	0·323	7	19	0·020	1·46
85	1·031	0·328	7	19	0·021	1·58
90	—	—	—	—	—	—
95	1·062	0·338	7	19	0·022	1·8
Tons.						
5	1·094	0·349	7	19	0·023	1·96
5½	1·156	0·368	7	19	0·024	2·13
6	1·1875	0·378	7	27	0·021	2·25
6½	—	—	—	—	—	—
7	1·219	0·388	7	27	0·022	2·53
7½	1·25	0·398	7	37	0·019	2·63
8	1·3125	0·418	7	37	0·020	2·89
9	1·375	1·437	7	37	0·021	3·08
10	1·437	0·457	7	37	0·022	3·49
11	1·500	0·478	7	37	0·023	3·81
12	1·594	0·508	7	37	0·024	4·15

General.—Coils of approved ropes should be carefully inspected for loose or vagrant wires, and for multiple joints; the crown, or outside wires, visualized for frictional wear, caused by the high speeds at which some makers close ropes.

Splicing of control ropes should be long, and "over-and-under" method of splice employed. The so-called "Liverpool" splice is more shapely, but less secure. A splice should never be served or covered before inspection, and there must always be a minimum of four and a half turns in a spliced length.

Thimbles should be tight in the loop, especially at the crown, and must be correct to width, suited to the diameter of the rope.

Steel shackles must engage the thimbles easily, and should roll comfortably when fitted.

The thick bottom ends of shackles should be nicely radiused, otherwise there is a tendency to cut the thimble and rope, and consequent fracture. Pulleys and sheaves should be as large as possible.

Never take a rope on trust, no matter by whom approved or manufactured.

AMERICAN WIRE AND CABLE STANDARDS*

THE following extracts from the report of the U.S. Advisory Committee for Aeroplane Material, Specifications, and Inspection were recommended for standardization at the Washington meeting of the Society of Automotive Engineers:

Aircraft Round Wire.

This specification covers solid high strength steel wire, round section, used in the construction of aircraft when flexibility is of minor importance. A tension test is to be made on each piece or coil of wire. When an individual coil of wire is to be divided into smaller coils to meet special requirements it is sufficient to make one test on the original coil and to cut and seal the smaller coils in the presence of the inspector. Samples

TABLE OF PHYSICAL PROPERTIES FOR AIRCRAFT ROUND WIRE.

Brown and Sharpe Gauge.	*Diameter (Inches).*	*Minimum Torsion for 6 Inch.*	*Weight in Pounds per 100 Feet.*	*Number of Bends through 90 Degrees.*	*Breaking Strength, Minimum Pounds.*
6	0·162	11	7·01	5	4,500
7	0·144	12	7·01	6	3,700
8	0·128	14	4·40	8	3,000
9	0·114	16	3·50	9	2,500
10	0·102	18	2·77	11	2,000
11	0·090	21	2·20	14	1,620
12	0·081	24	1·744	17	1,300
13	0·072	27	1·383	21	1,040
14	0·064	31	1·097	25	830
15	0·057	34	0·870	29	660
16	0·051	39	0·690	34	540
17	0·045	44	0·547	42	425
18	0·040	49	0·434	52	340
19	0·036	55	0·344	70	280
20	0·032	61	0·273	85	225
21	0·028	70	0·216	105	175

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* Reproduced from *The Aeroplane.*

for tension test shall be no less than 15 inches long, and free from bends and kinks.

In making tensile tests on aircraft wire the distance between jaws of testing machine with sample in place and before test shall be 10 inches. Wire for this purpose must break at no less than the amount specified in the attached table, which herewith becomes a part of this specification.

AIRCRAFT GALVANIZED SINGLE STRAND CABLE. (Non-flexible.)

Diameter of Cable (Inches).	*Number of Wires.*	*Breaking Strength of Cable (Pounds).*	*Approximate Weight (Pounds per 100 Feet).*
$\frac{5}{16}$	19	12,500	20·65
$\frac{1}{4}$	19	8,000	13·50
$\frac{7}{16}$	19	6,100	10·00
$\frac{3}{16}$	19	4,600	7·70
$\frac{5}{32}$	19	3,200	5·50
$\frac{1}{8}$	19	2,100	3·50
$\frac{7}{64}$	19	1,600	2·60
$\frac{3}{32}$	19	1,100	1·75
$\frac{5}{64}$	19	780	1·21
$\frac{1}{16}$	19	500	0·78
$\frac{1}{32}$	7	185	0·30

A torsion test is to be made on a sample cut from each piece or coil of wire as above. Samples for torsion tests shall be straight and no less than 10 inches long. The sample shall be gripped by two vices 6 inches apart; one vice shall be turned

AIRCRAFT GALVANIZED MULTIPLE CABLE 6×7* COTTON CENTRE.
(Flexible.)

Diameter of Cable (Inches).	*Breaking Strength of Cable (Pounds).*	*Approximate Weight (Pounds per 100 Feet).*
$\frac{5}{16}$	7,900	15·00
$\frac{1}{4}$	5,000	9·50
$\frac{7}{32}$	4,000	7·43
$\frac{3}{16}$	2,750	5·30
$\frac{5}{32}$	2,200	4·20
$\frac{1}{8}$	1,150	2·20
$\frac{7}{64}$	830	1·50
$\frac{3}{32}$	780	1·30
$\frac{5}{64}$	480	0·83
$\frac{1}{16}$	400	0·73

* Six strands of seven wires each.

uniformly at a speed not exceeding 60 r.p.m. (on the larger sizes of wire this speed shall be reduced sufficiently to avoid undue heating of the wire). One vice shall have free axial movement in either direction. All wire shall be required to stand the minimum number of complete turns shown in the first table.

Reels for Aircraft Strand and Cable.

(All Dimensions in Inches.)

Diameter of Strand or Cable.	*Diameter Head.*	*Traverse or Distance between Heads.*	*Diameter Bbl.*	*Diameter Arbor Hole.*	*Diameter Head.*	*Traverse or Distance between Heads.*	*Diameter Bbl.*	*Diameter Arbor Hole.*
	1,000 *Feet.*				3,000 *Feet.*			
1/32	12	4	8	1⅛	12	4	8	1⅛
1/16	12	4	8	1⅛	12	4	8	1⅛
5/64	12	4	8	1⅛	16	4	10	1⅛
3/32	12	4	8	1⅛	16	4	10	1⅛
7/64	16	4	10	1⅛	16	7	12	1⅛
1/8	16	4	10	1⅛	16	7	12	1⅛
9/64	16	7	12	1⅛	16	10	8	1⅛
5/32	16	7	12	1⅛	16	10	8	1⅛
3/16	18	7	12	2⅛	18	10	8	2⅛
7/32	18	7	12	2⅛	18	10	8	2⅛
1/4	18	10	10	2⅛	24	10	10	2⅛
5/16	18	10	10	2⅛	24	10	10	2⅛
11/32	18	10	8	2⅛	32	16	16	2⅛
3/8	18	10	8	2⅛	32	16	16	2⅛
	5,000 *Feet.*				10,000 *Feet.*			
1/32	12	4	8	1⅛	16	4	10	1⅛
1/16	16	4	10	1⅛	16	4	12	1⅛
5/64	16	7	12	1⅛	16	10	8	1⅛
3/32	16	7	12	1⅛	16	10	8	1⅛
7/64	16	10	8	1⅛	18	10	8	1⅛
1/8	16	10	8	1⅛	24	10	10	1⅛
9/64	24	10	10	1⅛	24	16	10	2⅛
5/32	24	10	10	1⅛	24	16	10	2⅛
3/16	24	10	10	2⅛	24	16	10	2⅛
7/32	24	10	10	2⅛	32	20	16	3⅛
1/4	32	18	16	2⅛	36	22	18	3⅛
5/16	32	18	16	2⅛	36	22	18	3⅛
11/32	32	20	16	3⅛	50	16	26	3⅜
3/8	32	20	16	3⅛	50	16	26	3⅜

In making racks for the above reels, allow 4 inches more width than the traverse specified above.

A bend test is to be made on a sample cut from each piece of wire as above. Sample for bend test shall be straight and no less than 10 inches long. One end of the sample shall be clamped between jaws having their upper edges rounded to $\frac{3}{16}$ inch radius. The free end of the wire shall be held loosely between two guides and bend 90° over one jaw; this is to be counted as one bend. On raising to a vertical position the count will be two bends. Wire shall then be bent to the other side, and so forth, alternating to fracture. The minimum number of bends required is stated in the table.

A wrapping test is to be made on at least 10 per cent. of the total number of coils offered for inspection at one time. The wire shall be tightly wrapped eight consecutive turns around its own diameter with a pitch of substantially the diameter of the wire and then unwrapped, maintaining the free end at approximately 90° with the mandril.

Aircraft Galvanized Multiple Cable 7×7* Wire Centre.
(Flexible.)

Diameter of Cable (Inches).	*Breaking Strength of Cable (Pounds).*	*Approximate Weight (Pounds per 100 Feet).*
$\frac{5}{16}$	9,200	16·70
$\frac{1}{4}$	5,800	10·50
$\frac{7}{32}$	4,600	8·30
$\frac{3}{16}$	3,200	5·80
$\frac{5}{32}$	2,600	4·67
$\frac{1}{8}$	1,350	2·45
$\frac{7}{64}$	970	1·75
$\frac{3}{32}$	920	1·45
$\frac{5}{64}$	550	0·93
$\frac{1}{16}$	480	0·81

It must stand this test without fracture. Because of the possibility of personal error in making this test, failure on one test is not considered conclusive, and, if requested to do so, the inspector shall make at least one but no more than two additional tests on the same piece of wire. If any of these tests are successful the material shall be passed as satisfactory in this respect.

All wire for this purpose shall be furnished in decimal sizes corresponding to the American Standard Wire Gauge (Brown and Sharpe Gauge). The wire shall be cylindrical and smooth and show no evidence of scapes, splints, cold shuts and rough tinning, not conforming with best commercial practice. A permissible variation of 0·002 inch above gauge on all sizes will be acceptable, but no wire will be accepted having a variation of more than 0·0005 inch below gauge.

* Seven strands of seven wires each.

Single Strand, Multiple Strand 6 × 7, 7 × 7, and 7 × 19.

A tensile test shall be made upon each individual reel of cable for shipment containing 10,000 feet or more. On shipments of reels of cable containing less than 10,000 feet, each test shall be made for each 5,000 feet of a size. The tension testing machine shall be of a type satisfactory to the representative of the U.S. Government, and shall be standardized with U.S. Standard Weights when considered necessary. Samples of cable for testing for tensile strength shall be no less than 24 inches long. In making tensile tests the distance between jaws of testing machine with sample in place and before test shall be no less than 10 inches. Samples for tensile test may be clamped in the jaws of the testing machine in the usual manner to facilitate speed of testing, but in case of failure or dispute on individual tests, and at the request of the manufacturer, check tests shall be made by socketing the samples with pure zinc. Cables for use in the construction of aircraft must break at no less than the amount specified in the attached table.

AIRCRAFT TINNED MULTIPLE STRAND CABLE 7 × 19* WIRE CENTRE.
(Extra Flexible.)

Diameter of Cable (Inches).	*Breaking Strength of Cable (Pounds).*	*Approximate Weight (Pounds per 100 Feet).*
$\frac{3}{8}$	14,400	26·45
$\frac{11}{32}$	12,500	22·53
$\frac{5}{16}$	9,800	17·71
$\frac{9}{32}$	8,000	14·56
$\frac{1}{4}$	7,000	12·00
$\frac{7}{32}$	5,600	9·50
$\frac{3}{16}$	4,200	6·47
$\frac{5}{32}$	2,800	4·44
$\frac{1}{8}$	2,000	2·88

At least one bending test is to be made on a sample cut from each 50,000 feet of cable of a given size. Each sample must be bent once around its own diameter and straightened again at least 20 times in succession in the same direction of bending without any of the wires breaking.

A torsion test is to be made on one wire from each sample of cable taken for tensile test. The wire is to be gripped by two vices 4 inches apart; one vice shall be turned uniformly at as high a rate of speed as possible without perceptibly heating the wire. One vice shall have free axial movement in either

* Seven strands of nineteen wires each.

direction. The number, N, of whole turns through which the wire must stand twisting is given by the formula—

$$N = \frac{1 \cdot 1}{d},$$

where d equals the diameter of the wire in inches. Failure of one piece of wire to show full number of turns specified in the above torsion test shall not be considered cause for rejection, but in each case two additional tests shall be made on two more wires from the same sample of cable, and if both samples meet the requirements of the specifications the strand shall be accepted in this respect.

No variation is permissible below nominal diameter. Cable having a diameter of $\frac{1}{32}$ to $\frac{5}{32}$ inch inclusive shall have a permissible variation of 10 per cent. above size, and cable having a diameter of $\frac{3}{16}$ to $\frac{3}{8}$ inch inclusive shall have a permissible variation of 7 per cent. above size.

Protective Coatings for Metals.

The Aeronautical Division of the Standards Committee of the S.A.E. is now engaged in a study of the various kinds of anti-rust treatment of metals. This subject is exceedingly important at present, on account of the need for protecting many of the bolts, nuts, and fittings used on aircraft from the action of the elements. It is planned to secure as many samples as possible of treatments, making use of chemical or heat methods, or a combination of the two, for protecting metals.

ALLOWANCES FOR BENDING SHEET METAL*

By RANDOLPH F. HALL

It is an important point in the design of fittings to make correct allowances for bends in sheet metal. The writer, together with B. D. Thomas, has carried out a number of measurements on bends round a 1 inch square bar, and the results are embodied in the chart of Fig. 1 on lengths measured along the neutral axis.

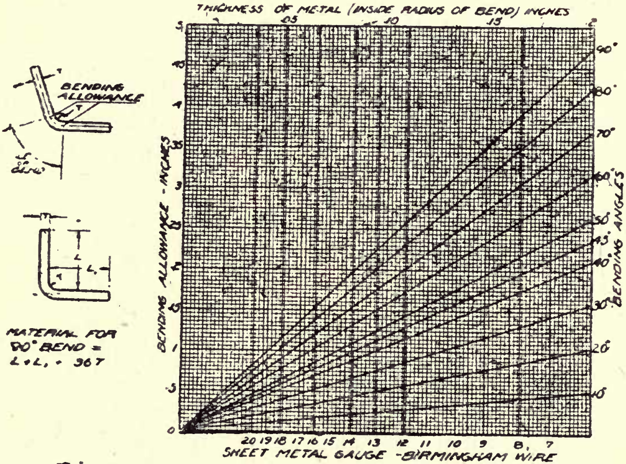

Fig 1.

BENDING ALLOWANCE CHART
THICKNESS OF METAL EQUALS INSIDE RADIUS OF BEND

A number of strips, 18, 16, 14, and 12 gauge sheet-steel, were bent over a 1 inch square steel bar, with corners rounded in each case with the radius equal to the thickness of the metal. Had no allowance been required for bending, the ends of the strip would have met in the centre of a face of the bar; but instead a gap resulted, which, divided by four, gave actual allowance for one bend. The allowances required per bend were between

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* Reproduced from *The Aeroplane.*

thirty-three hundredth and thirty-eight hundredth times the thickness of the metal, and the mean of ten different tests confirmed the allowances computed on the neutral axis. From these experiments the curves of the above diagram were deduced, allowances for angles less than 90° being made proportionately.

Results in the factory verify the chart. If the corners are hammered flat, less allowance is required, but the fitting is weakened.

THE VIBRATIONS METHOD OF TUNING-UP*

BY INGEGNERE CARLO MAURILIO LERICI

IN the workshops for aeronautical construction of the Società Italiana Aviazione (F.I.A.T. Co., Turin), as well as in several other aeroplane factories, a new method for regulating machines was first successfully introduced about a year ago.

This method utilizes the well-known phenomena of vibratory resonance for measuring the tensions of shrouds—cables and wires—and enables, therefore, the operator to " tune up " the machine with a precision and constancy which cannot be reached by the empirical proceedings still in use in most factories.

The consequences of a wrong regulation—namely, lack of symmetry and excessive bracing tensions—are too well known for being insisted upon.

It is sufficient to remember how the low safety factor of modern machines may be influenced by an inaccurate tuning. The deformations of wood elements, due to excessive initial compressive stresses as well as the wrong straining of bracing wires, are often responsible for a decrease of 15 to 20 per cent. of the safety factor. This is confirmed by the results of static tests carried on body-cellules of several up-to-date machines of various types.

The " vibrations method " makes use of a simple frequency-meter, by means of which the frequency of vibration of a straightened wire can be easily measured. This value is proportional to the tension according to the formula—

$$T = \frac{4l^2w}{g}n^2.$$

T = tension.
l = length of vibrating wire.
w = weight per unit of length.
g = acceleration of gravity.
n = frequency (vibrations per second).

Thus the tension is equal to Kn^2, K being a constant for any given cable or wire.

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* Reproduced from *The Aeroplane*.

The frequency-meter, which acts really as a vibrations tension-meter, is made up of a series of small steel plates of various lengths, which are clamped at one extremity. (See Fig. 1 (*d*) accompanying illustration.)

If one places the support of the meter in light contact with a wire which has been set in vibration, the plate, the natural period of which approaches that of the wire, will soon assume a distinct state of vibration.

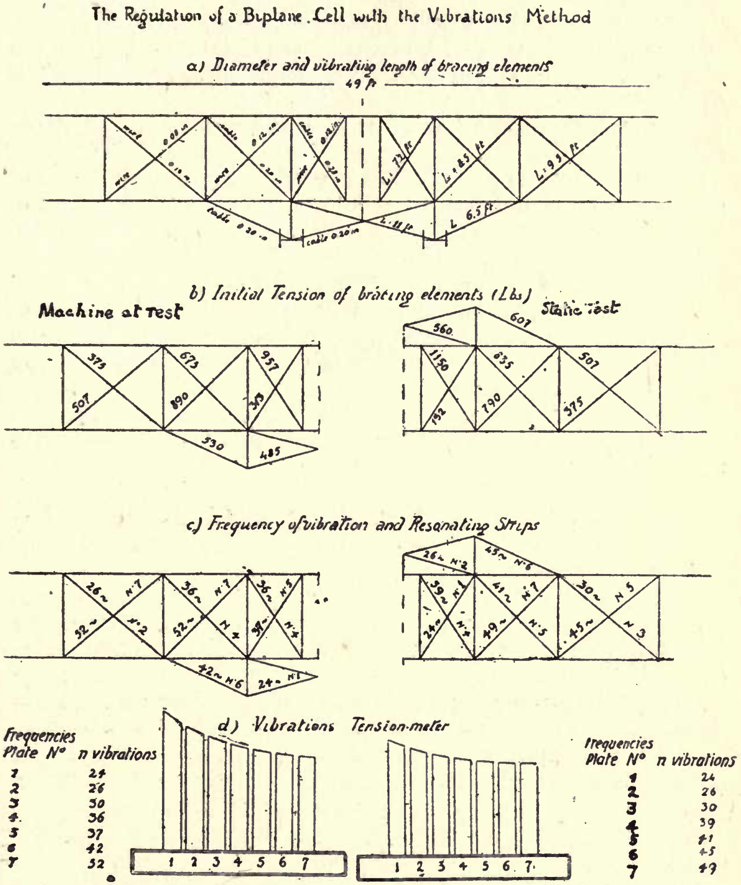

FIG. 1.

If the instrument includes plates of frequencies from n to n^1 vibrations, n^1 being greater than $2n$, resonance will occur simultaneously with two or three plates.

This corresponds to the fact that a loaded wire can set in vibration plates whose periods of oscillation are n, $2n$, $3n$, etc., according to the frequencies of the harmonics of the vibrating wires.

In order to measure the tension one must consider only the fundamental vibration of lesser frequency n.

It was found that for the regulation (tuning-up) of all modern biplanes, including recent types, a series of plates comprising the frequencies from 25 to 50 per second is sufficient. All uncertainty due to multiple resonance is thus eliminated.

The error due to the presence of turnbuckles is about 3 per cent. in excess.

It was found also possible, in the course of destructive tests of new types of aeroplanes, to follow quickly the distribution of tensions in all the wires, and to determine irregularities which calculations did not predict.

The vibrations tension-meter allows, furthermore, to recognize all the elements (struts, cables, and wires) which are capable of being influenced during flight by resonance with the motor, the fundamental frequency of which is of the order of 22–25 per second (1,300 to 1,500 revs. per minute).

A practical example will evidence better than any detailed description the proceeding for regulating a biplane cell with the new method.

The drawings show—

(*a*) The framework of a 250 h.p. pusher biplane; the diameters of cables and wire and their "vibrating length," as measured between terminals.

(*b*) The tensions required in both cases for a machine at rest and one suspended for static test. Such tensions were determined, of course, with the view of allowing the best distribution of stresses under load.

(*c*) The frequencies of vibration of all the tension elements determined either by means of the formula mentioned above, or employing a graphic diagram showing the relations existing between the variables T, n, l, w. The number of the meter plates whose frequencies are equal to that of the wires.

(*d*) The diagram of a seven-plate tension-meter for both cases.

To employ the vibrations tension-meter, the wire must be free to vibrate between its terminal attachments.

After having set up the vibration, the meter support is placed in slight contact with one extremity of the vibrating strand. If the "resonating" plate is that corresponding to the tension required, a further regulation is not needed.

Otherwise the resonance of a smaller plate (higher frequency) indicates excessive stress, while that of a greater one (lower frequency) indicates, on the contrary, lack of tension.

When the "tuning" of a strand is liable to influence the tension of another element the latter must be, if necessary, tuned a second time.

In the factories where this method is in use the tensionmeter is accompanied merely by a simple diagram analogous to that (*c*) of the drawing.

The time required for the complete timing of a 200–300 h.p. modern biplane ranges from 10 to 20 minutes, according to the size and to the approximation of the primary regulation.

AIRCRAFT TESTING*

AEROPLANE TESTING.

THE accurate testing of aeroplanes is one of the many branches of aeronautics which have been greatly developed during the war, and especially during the last year. For some months after the war began, a climb to 3,000 to 5,000 feet by aneroid and a run over a speed course was considered quite a sufficient test of a new aeroplane; now we all realize that for military reasons certainly, and probably for commercial reasons in the future, it is the performance of a machine at far greater heights with which we are mainly concerned.

In this paper I propose to give a short general account of some of the methods of testing now in use at the Testing Squadron of the Royal Air Force, and to indicate the way in which results of actual tests may be reduced, so as to represent as accurately as possible the performance of a machine independently of abnormal weather conditions, and of the time of the year.

So far as England is concerned, I believe that the general principles of what may be called the scientific testing of aeroplanes were first laid down at the Royal Aircraft Factory. Our methods of reduction were based on theirs to a considerable extent, with modifications that were agreed upon between us; they have been still further modified since, and recently a joint discussion of the points at issue has led to the naval and military tests being co-ordinated, so that all official tests are now reduced to the same standard.

It should be emphasized that, once the methods are thought out, scientific testing does not really demand any high degree of scientific knowledge; in the end, the accuracy of the results really depends upon the pilot, who must be prepared to exercise a care and patience unnecessary in ordinary flying. Get careful pilots, whose judgment and reliability you can trust, and your task is comparatively easy; get careless pilots, and it is impossible.

At the outset it may be useful to point out by an example the nature of the problems that arise in aeroplane testing. Suppose that it is desired to find out which of two wing sections is most

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* From a lecture by Captain H. T. Tizard; reproduced from *The Aeroplane.*

suitable for a given aeroplane. The aeroplane is tested with one set of wings, which are then replaced by the other set, and the tests repeated some days later. The results might be expressed thus:

	A Wings.	*B Wings.*
Speed at 10,000 feet ..	90 m.p.h.	93 m.p.h.
Rate of climb at 10,000 feet ..	250 ft. a minute.	300 ft. a minute.

Now, the intelligent designer knows that, firstly, an aneroid may indicate extremely misleading " heights," and, secondly, that even if the actual height above the ground is the same in the two tests the actual conditions of atmospheric pressure and temperature may have been very different on the two days.

He will, therefore, say, if he is an intelligent designer, as all British designers are now, What does that 10,000 mean ? Do you mean that your aneroid read 10,000 feet ? or do you mean 10,000 feet above the spot you started from, or 10,000 feet above sea-level ? If he proceeds to think a trifle further, he will say—What was the density of the atmosphere at your 10,000 feet ? Was it the same in the two tests ? If not, the results do not convey much.

There he will touch the keynote of the whole problem, for it is on the density of the atmosphere that the whole performance of an aeroplane depends; the power of the engine and the efficiency of the machine depend essentially on the density; the resistance to the motion of the machine through the air is proportional to the density, and so finally is the lift on the wings. None of these properties are proportional solely to the pressure of the atmosphere, but to the density—that is, the weight of air actually present in unit volume.

It follows that it is essential when comparing the performances of machines to compare them as far as possible under the same conditions of atmospheric density, not as is loosely done at the same height above the earth, since the density of the atmosphere at the same height above the earth may vary considerably on different days, and on the same day at different places.

At the same time, in expressing the final results, this principle may be carried too far. Thus, if the speed of a machine were expressed as 40 metres a second at a density of 0·8 kilogramme per cubic metre, the statement, though it may be strictly and scientifically accurate, will convey nothing to 99 per cent. of those directly concerned with the results of the test.

The result is rendered intelligible, and indeed useful, by the form " 90 m.p.h. at 10,000 feet," or whatever it is. With this form of statement, in order that all the statements or results may be consistent and comparative, we must be careful to mean by

"10,000 feet" a certain definite density, in fact, the average density of the atmosphere at a height of 10,000 feet above mean sea-level. This is what the problem of "reduction" of tests boils down to; what is the relation between atmospheric density and height above sea-level ? This knowledge is obtained from meteorological observations. We have collected all the available data —mostly unpublished—with results shown in the following table:

TABLE I.

MEAN ATMOSPHERIC PRESSURE, TEMPERATURE, AND DENSITY AT VARIOUS HEIGHTS ABOVE SEA-LEVEL.

Height in Kilometres.	*Height in Equivalent Feet.*	*Mean Pressure in Millibars.*	*Mean Temperature in Absolute Degrees Centigrade.*	*Mean Density in Kilogrammes per Cubic Metre.*
0	0	1,014	282	1·253
1	3,280	900	278	1·128
2	6,560	795	273	1·014
3	9,840	699	268	0·909
4	13,120	615	262	0·818
5	16,400	568	255	0·735
6	19,680	469	248	0·658
7	22,960	407	241	0·589

These are the mean results of a long series of actual observations made mainly by Dr. J. S. Dines. It is convenient to choose some density as standard, call it unity, and refer all other densities as fractions or percentages of this "standard density." We have taken, in conformity with the R.A.F., the density of dry air at 760 mm. pressure and 16 deg. of Centigrade as our standard density ; it is 1·221 kgm. per cubic metre. The reason this standard has been taken is that the air speed indicators in use are so constructed as to read correctly at this density, assuming the law—

$$p=\tfrac{1}{2}\rho V^2,$$

where V is the air speed, p the pressure obtained, ρ the standard density.

In some ways it would doubtless be more convenient to take the average density at sea-level as the standard density, but it does not really matter what you take so long as you make your units quite clear. Translated into feet, and fraction of the standard density, the figures in the above table become as in Table II.

Let us briefly consider what these figures mean. For example, we say that the density at 10,000 feet is 74 per cent. of our standard density, but it is not meant that at 10,000 feet above mean sea-level the atmospheric density will always be 74 per cent. of the standard density. Unfortunately for aeroplane tests this is far from true.

The atmospheric density at any particular height may vary considerably from season to season, from day to day, and even from hour to hour; what we do mean is that if the density at 10,000 feet could be measured every day, then the average of the results would be, as closely as we can tell at present, 74 per cent. of the standard density.

The above table may therefore be taken to represent the conditions prevailing in a "normal" or "standard" atmosphere, and we endeavour, in order to obtain a strict basis of comparison, to reduce all observed aeroplane performances to this standard atmosphere—*i.e.*, to express the final results as the performance which may be expected of the aeroplane on a day on which the atmospheric density at every point is equal to the average density at the point. Some days the aeroplane may put up a better performance, some days a worse, but on the average, if the engine power and other characteristics of the aeroplane remain the same, its performance will be that given.

TABLE II.

Height in Feet.	*Percentage of Standard Density.*	*Height in Feet.*	*Percentage of Standard Density.*
0	102·6	11,000	71·7
1,000	99·4	12,000	69·5
2,000	96·3	*13,000	67·3
3,000	93·2	14,000	65·2
4,000	90·3	15,000	63·0
5,000	87·4	16,000	61·1
6,000	84·6	*16,500	60·1
*6,500	83·3	17,000	59·1
7,000	81·9	18,000	57·1
8,000	79·2	19,000	55·2
9,000	76·5	20,000	53·3
*10,000	74·0		

It must be remembered that a standard atmosphere is a very abnormal occurrence; besides changes in density there may occur up and down air currents which exaggerate or diminish the performance of an aeroplane, and which must be taken carefully into account. They show themselves in an otherwise unaccountable increase or decrease in rate of climb or in full speed flying level at a particular height.

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* 6,500 feet is introduced as corresponding roughly to the French test height of 2,000 metres. 10,000 feet similarly corresponds roughly to comparing aeroplane test performances with the French standard of 3,000 metres, and similarly for 13,000 and 16,500 feet.

ACTUAL TESTS.

We now pass to the actual tests, beginning with a description of the observations which have to be made, and thereafter to the instruments necessary. The tests resolve themselves mainly into—

(*a*) A climbing test at the maximum rate of climb for the machine.

(*b*) Speed tests at various heights from the "ground," or some other agreed low level, upwards.

Experience agrees with theory in showing that the best climb is obtained by keeping that which is frequently called the air speed of an aeroplane—viz., the indications of the ordinary air speed indicator—nearly constant, whatever the height. (In other words, ρV^2 is kept constant.) We can look at this in this way:

There is a limiting height for every aeroplane, above which it cannot climb; at this limiting height, called the "ceiling" of the machine, there is only one speed at which the aeroplane will fly level; at any other air speed higher or lower it will descend.

Suppose this speed be 55 m.p.h. on the air speed indicator. Then the best rate of climb from the ground is obtained by keeping the speed of the machine to a steady indicated 55 m.p.h.

Fortunately, a variation in the speed does not make very much difference to the rate of climb; for instance, a B.E.2*c* with a maximum rate of climb at 53 m.p.h. climbs just as fast up, say, to 5,000 feet at about 58 m.p.h. This is fortunate, as it requires considerable concentration to keep climbing at a steady air speed, especially with a light scout machine; if the air is at all "bumpy," it is impossible.

At great heights the air is usually very steady, and it is much easier to keep to one steady air speed.

RATE OF CLIMB.

It is often difficult to judge the best climbing speed of a new machine; fliers differ very much on this point, as on most. The Testing Squadron, therefore, introduced some time ago a "rate of climb" indicator, intended to show the pilot when he is climbing at the maximum rate.

It consists of a thermos flask, communicating with the outer air through a thermometer tube leak. A liquid pressure gauge of small bore indicates the difference of pressure between the inside and outside of the vessel. Now, when climbing, the atmospheric pressure is diminishing steadily; the pressure inside the thermos flask tends, therefore, to become greater than the outside atmospheric pressure. It goes on increasing until air is being forced out through the thermometer tubing at such a rate that the rate of change of pressure inside the flask is equal to the rate of change of atmospheric pressure due to climbing. When climbing at a maximum rate, therefore, the pressure inside the thermos flask is a maximum. The pilot, therefore, varies his air speed until the liquid in the gauge is as high as possible, and this is the best climbing speed for the machine.

Reduction to Standard.

What observations during the test are necessary in order that the results may be reduced to the standard atmosphere ? Firstly, we want the time from the start read at intervals, and the height reached noted at the same time. Here we encounter a difficulty at once, for there is no instrument which records height with accuracy.

The aneroid is an old friend now of aeronauts as well as of mountaineers, but it is doubtful whether 1 per cent. of those who use it daily realize how extraordinarily rare it is that it ever does what it is supposed to do—that is, indicate the correct height above the ground, or starting-place.

The faults of the aeroplane aneroid are partly unavoidable and partly due to those who first laid down the conditions of its manufacture. An aneroid is an instrument which in the first place measures only the pressure of the surrounding air. Now, if p_1 and p_2 are the pressures at two points in the atmosphere, the difference of height between these points is given very closely by the relation,

$$h \propto \theta \log_e {}^{p_1}/_{p_2},$$

where θ is the average temperature, expressed in "absolute" degrees, of the air between the two points.

It is obvious that, if we wish to graduate an aneroid in feet, we must choose arbitrarily some value for θ. The temperature that was originally chosen for aeroplane aneroids was 50 degrees Fahrenheit or 10 degrees Centigrade.

The Defective Aneroid.

An aneroid, as now graduated, will therefore only read the correct height in feet if the atmosphere has a uniform temperature of 50 degrees Fahrenheit from the ground upwards, and it will be the more inaccurate the greater the average temperature between the ground and the height reached differs from 50 degrees Fahrenheit.

Unfortunately 50 degrees Fahrenheit is much too high an average temperature; to take an extreme example, it is only on the hottest days in summer, and even then very rarely, that the average temperature between the ground and 20,000 feet will be as high as 50 degrees Fahrenheit. On these very rare occasions an aneroid will read approximately correctly at high altitudes; otherwise it will always read too high. In winter it may read on cold days 2,000 feet too high at 16,000 feet—*i.e.*, it will indicate a height of 18,000 feet when the real height is only 16,000 feet. It is always necessary, therefore, to "correct" the aneroid readings for temperature. This is very disappointing to the pilot who thinks he has been up to 20,000 feet on a cold day and finds he has only been to 17,500 feet. The equation

$$H=\frac{273+t}{283}\cdot h$$

gives us the necessary correction. Here H is the true difference in height between any two points, t the average temperature in degrees Centigrade between the points, and h the difference in height indicated by aneroid. It is convenient to draw a curve showing the necessary correction factors at different temperatures, some of which are given in the following table:

TABLE III.

ANEROID CORRECTION FACTORS.

Temperature.	*Correction Factor.*
70° Fahr.	1·040
50° ,,	1·000
30° ,,	0·961
10° ,,	0·922
– 10° ,,	0·883

For example, if a climb is made through 1,000 feet by aneroid, and the average temperature is 10 degrees Fahrenheit, the actual distance is only 1,000 × 0·922 = 922 feet. The above equation is probably quite accurate enough for small differences of height—up to 1,000 feet, say—and approximately so for bigger differences.

The magnitude of the correction which may be necessary shows how important it is that observations of temperature should be made during every test. For this purpose a special thermometer is attached to a strut of the machine, well away from the fuselage, and so clear of any warm air which may come from the engine. The French, I believe, do not measure temperature, but note the ground temperature at the start of a test, and assume a uniform fall of temperature with height.

This, undoubtedly, may lead to serious errors. The change of temperature with height is usually very irregular, and only becomes fairly regular at heights well above 10,000 feet. It is not safe to assume a 3-degree drop in temperature per 1,000 feet, as some test stations do.

THE REAL TEST.

The aneroid being what it is, one soon comes to the conclusion that the only way to make use of it in aeroplane tests is to treat it purely as a pressure instrument. For this reason it is best to do away with the zero adjustment for all test purposes, and lock the instrument so that the zero point on the height scale corresponds to the standard atmospheric pressure of 29·9 inches, or 760 mm. of mercury. Every other height then corresponds to definite pressure; for instance the locked aneroid reads 5,000 feet when the atmospheric pressure is 24·88 inches, and 10,000 feet when it is 20·70 inches, and so on.

If the temperature is noted at the same time as the aneroid reading, we then know both the atmospheric pressure and temperature at the point, and hence the density can be calculated, or, more conveniently, read off curves drawn for the purpose. The observations necessary (after noting the gross aeroplane weight and net or useful weight carried) are, therefore, (i.) aneroid height every 1,000 feet; (ii.) time which has elapsed from the start of the climb; and (iii.) temperature. To these should be added also (iv.) the air speed, and (v.) engine revolutions at frequent intervals. Petrol consumption is also noted.

The observed times are then plotted on squared paper against the aneroid heights and a curve drawn through them. From this curve the rate of climb at any part (also in aneroid feet) can be obtained by measuring the tangent to the curve at the point. This is done for every 1,000 feet by aneroid.

The true rate of climb is then obtained by multiplying the aneroid rate by the correction factor corresponding to the observed temperature. These true rates are then plotted afresh against standard heights, and from this curve we can obtain the rates of climb corresponding to the standard heights, 1,000, 2,000, 3,000, etc. Knowing the change of rate of climb with height, the time to any required height is best obtained by graphical integration. Table IV. (p. **110**) gives the results of an actual test.

At least two climbing tests of every new machine are carried out up to 16,000 feet or over by aneroid. If time permits three or more tests are made. The final results given are the average of the tests, and represent as closely as possible the peformance on a standard day, with temperature effects, up and down currents, and other errors eliminated.

If we produce the rate of climb curve upwards it cuts the height axis at a point at which the rate of climb would be zero, and, therefore, the limit of climb reached. This is the "ceiling" of the machine.

TABLE IV.

Machine.......... Engine..........

Height in Aneroid Feet.	*Observed Temperature.*	*Percentage of Standard Density.*	*Observed Time.*	*Rate of Climb in Aneroid Feet.*	*Real Rate of Climb (corrected for Temperature).*	*Standard Height.*	*From Curve.*		
							Percentage of Standard Density.	*Time.*	*Rate of Climb.*
	Fahr.								
0	36°		0·0						
1,000	38°	101·0	1·0	835	814	1,000	99·40	1·20	775
2,000	38°	97·2	2·10	735	718	2,000	96·30	2·56	685
3,000	36°	94·0	3·70	640	622	3,000	93·26	4·11	610
4,000	36°	90·7	5·40	560	544	4,000	90·25	5·85	545
5,000	36°	87·4	7·25	510	495	5,000	87·35	7·80	490
6,000	33°	84·7	9·40	450	435	6,000	84·50	9·96	435
7,000	30°	82·1	11·90	405	389	7,000	81·80	12·40	385
8,000	26°	79·9	14·25	365	347	8,000	79·16	15·14	345
9,000	22°	77·6	17·00	330	312	9,000	76·55	18·20	310
10,000	23°	74·7	20·25	310	294	10,000	74·00	21·61	280
11,000	21°	72·2	23·60	280	264	11,000	71·70	25·41	245
12,000	20°	69·8	27·40	230	216	12,000	69·50	29·81	210
13,000	17°	67·7	31·90	195	182	13,000	67·32	35·13	170
14,000	12°	65·9	37·90	150	139	14,000	65·17	41·88	130
15,000	8°	64·1	45·25	110	101	14,500	64·11	46·23	105

SPEEDS.

His 16,000 feet, or whatever it is, reached, the flier's next duty is to measure the speed flying level by air speed indicator at regular intervals of height (generally every 2,000 feet) from the highest point downwards. To do this he requires a sensitive instrument which will tell him when he is flying level. The aneroid is quite useless for this purpose, and a "statoscope" is used. The principle of this instrument is really the same as that of a climbmeter.

It consists of a thermos flask connected to a small glass gauge, slightly curved, but placed about horizontally. In this gauge is a small drop of liquid, and at either end are two glass traps which prevent the liquid from escaping either into the outside air or into the thermos flask. As the machine ascends, and the atmospheric pressure being smaller and the pressure in the flask being higher than the external pressure, the liquid is pushed up to the right-hand trap, where it breaks, allowing the air to escape. On descending, the reverse happens; the liquid travels to the left, breaks, and air enters the flask. When flying truly level the drop remains stationary, moving neither up nor down. The instrument is made by the British Wright Co.

The flier or the observer notes the maximum speed by the air speed indicator—*i.e.*, the speed at full engine throttle. At one or more heights also he observes the speed at various positions of the throttle down to the minimum speed which will keep the machine flying at the height in question. The petrol consumption and the engine revolutions are noted at the same time, as well, of course, as the aneroid height and temperature.

Accurate observation of speeds needs very careful flying—in fact, much more so than in climbing tests. If the air is at all bumpy observations are necessarily subject to much greater error, since the machine is always accelerating and decelerating.

TRUE LEVEL FLYING.

The best way to carry out the test seems to be as follows: The machine is flown first just down hill and then just up hill, and the air speeds noted. This will give a small range between which the real level speed must lie.

The flier must then keep the speed as steadily as possible on a reading midway between these limits, and watch the statoscope with his other eye.

If it shows steady movement, one way or the other, the air speed must be altered accordingly by 1 m.p.h. In this way it is always possible at heights where the air is steady to obtain the reading correct, at any rate, to 1 m.p.h., even with light machines, provided always sufficient patience is exercised. The r.p.m. at this speed are then noted.

One difficulty, however, cannot be avoided. If at any height there is a steady up or down air current, then, though the air may appear calm—*i.e.*, there may be no “ bumps ”—the air speed indicator reading may be wrong, since to keep the machine level in an up current it is necessary to fly slightly down hill relatively to the air. Such unavoidable errors are, however, eliminated to a large extent by the method of taking speeds every 2,000 feet, and finally averaging the results.

AIR SPEED INDICATOR ERRORS.

We must now consider how the true speed of the aeroplane is deduced from the reading of the air speed indicator. It is well known that an air speed indicator reads too low at great heights—for example, if it reads 70 m.p.h. at 8,000 feet the real speed of the machine through the air is nearer 80 m.p.h.

The reason for this is that the indicator, like the aneroid, is only a pressure gauge—a sensitive pressure gauge, in fact, which registers the difference of pressure between the air in a tube with its open end pointing forward along the lines of flight of the machine, and the real pressure (the static pressure) of the external air.

This difference of pressure is as nearly as we can judge by experiment $=\frac{1}{2}p V^2$ (where p is the density of the air and V the speed of the machine), provided that the open end of the tube is well clear of wings, struts, fuselage, etc., and so is not affected by eddies and other disturbances.

Now, assuming this law, air speed indicators are graduated to read correctly, as I have said above, at a density of 1·221 kgms. per cubic metre, which we have taken as our standard density and called "unity." It corresponds on an average to a height of about 800 feet above sea-level.

Then suppose the real air speed of an aeroplane at a height of h feet is V m.p.h., and the indicated air speed is 70 m.p.h., this means that the excess pressure in the tube due to the speed is proportional to 1×70^2,

$$\text{or } \rho \times V^2 = 1 = 70^2 \cdot$$

where ρ is the density at the height in question, expressed as a fraction of the standard density.

To correct the observed speed, we therefore divide the reading by the square root of the density. Thus, observation of the maximum speed of an aeroplane at a height of 8,000 feet by the locked aneroid gave 80 m.p.h. on the indicator, the temperature being 31 degrees Fahrenheit. From the curve we find that the density corresponding to 8,000 feet and 31 degrees is 0·85 of standard density. The corrected air speed is therefore—

$$= \frac{80}{\sqrt{0{\cdot}85}} = 86{\cdot}7 \text{ m.p.h.}$$

This "corrected" air speed will only be true if the above law holds—that is to say, if there are no disturbances due to the pressure head, being in close proximity to struts or wings. It is always necessary to find out the magnitude of this possible error—that is, to calculate the air speed meter—and the only way to do this is to measure a real air speed at some reasonable altitude for easy observation of the aeroplane by actual timed observations from the ground, and from these timed results check those deduced from the air speed indicator readings.

This calibration is the most important and difficult test of all, since on the accuracy of the results depends the accuracy of all the other speed measurements. It can either be done by speed trials over a speed course close to the ground, or when the aeroplane is flying at a considerable height above the ground. In the Testing Squadron we have always attached much more importance to the latter method, mainly because the conditions approximate more to the conditions of the ordinary air speed measurements at different heights, and because the weather conditions are much steadier, and the flier can devote more attention to flying the machine at a constant air speed than he can when very close to the ground.

VISUAL SPEED TESTS.

One method is to use two camera obscuras, one of which points vertically upwards, and the other is set up sloping towards the vertical camera. At one important testing centre the cameras are a mile apart, and the angle of the sloping camera is 45 degrees. By this arrangement, if an aeroplane is directly over the vertical camera, it will be seen in the field of the sloping camera if its height is anywhere between 1,500 and 15,000 feet, although at very great heights it would be too indistinct for measurements except on a very clear day. The height the tests are usually carried out is 4,000 feet to 6,000 feet.

The aeroplane is flown as nearly as possible directly over the vertical camera, and in a direction approximately at right angles to the line joining the two cameras. The pilot flies in as straight a line and at as constant an air speed as he can. Observers in the two cameras dot in the position of the aeroplane every second. A line is drawn on the tables of each camera pointing directly towards the other camera, so that if the image of the aeroplane is seen to cross the lines in the one camera it crosses the line in the other simultaneously.

From these observations it is possible to calculate the height of the aeroplane with considerable accuracy; the error can be brought down to less than 1 part in a 1,000 with care. Knowing the height, we can then calculate the speed over the ground of the aeroplane by measuring the average distance on the paper passed over per second by the image in the vertical camera. If x inches is this distance, and f the focal length of the lens, the ground speed is $x \times h/f$ feet per second.

It is necessary to know also the speed and direction of the wind at the height of the test. For this purpose the pilot or his observer fires a smoke puff slightly upwards when over the cameras, and the observer in the vertical camera dots in its trail every second. The height of the smoke puff is assumed to be the same as that of the aeroplane—it probably does not differ from this enough to introduce any appreciable error in the results.

The tests are done in any direction relative to the wind, and generally at three air speeds, four runs being made at each air speed.

The advantages of this method are:

1. Being well above the earth the pilot can devote his whole attention to the test.

2. Within reasonable limits any height can be chosen, so that it is generally possible to find a height at which the wind is steady.
3. It does not matter if the pilot does not fly along a level path so long as he does so approximately. What is more important is that he should fly at a constant air speed.
4. It is not necessary that there should be any communication between the two cameras, although it is convenient. The two tracks are made quite independently, and synchronized afterwards from the knowledge that the image must have passed over the centre line simultaneously in the two cameras.

The main disadvantage is that somewhat elaborate apparatus is necessary, but this is of not much importance in a permanent testing station.

There are often periods, however, when an aeroplane has to be tested quickly, and low cloud layers and other causes prevent the camera test from being carried out. It is then necessary to rely on measurements of speeds near the ground for the calibration of the air speed indicator. In this method the aeroplane is flown about 10 feet off the ground, and is timed over a measured run.

There are two observers, one at each end of the course; when the aeroplane passes the starting-point the observer sends a signal and starts his stop-watch simultaneously; the second observer starts his stop-watch directly he hears the signal, and in his turn sends a signal and stops his watch when the aeroplane passes the finishing-point.

By this double timing errors due to the so-called "reaction time" of the observers are practically eliminated, for the observer at the end of the course tends to start his watch late, while the first observer stops his late. The mean of the two observations gives the real time. Four runs, two each up and down the course, are done at each air speed, the pilot or his observer noting carefully the average air speed during the run.

Observations of the atmospheric pressure and temperature from which the density can be obtained are also taken. The average strength and direction of the wind during each trial are noted from a small direct reading (or recording) anemometer, and the speed corrected in the same way as in the camera tests.

If there is a strong cross wind the aeroplane may have to be pointed at a considerable angle to the course, and this makes the test a very difficult one to carry out well. Generally speaking, it is only reliable when the wind is quite light, not more, at any rate, than 10 m.p.h. Even this is too strong if it is a cross wind.

A further difficulty is that at high speeds, over 100 m.p.h., an aeroplane may take quite a considerable time to accelerate up to a steady speed, and so it must fly level for a long distance each end before reaching the actual course. At the testing station previously alluded to the course is a mile long, and there is a clear half-mile or more at each end, but it is doubtful whether even this distance is enough for the machine to attain steady speed before the starting-point.

Finally, the flier of a single-seater is generally too busy watching the ground to do more than glance at his air speed indicator occasionally during the run. Doubtless it would be better in such a case to use some form of recording air speed instrument, although then other difficulties would arise.

Having got the true air speed from camera or speed course tests, and knowing the density at the height at which the test was carried out, we obtain what the air speed indicator should have read by multiplying the measured air speed by the square root of the density. By comparing this with the actual reading of the indicator we obtain the necessary correction. The whole procedure may be shown best by a table (V.) giving part of the results of a camera test.

TABLE V.

Calibration Test of Air Speed Indicator No. ....
On .... Machine.

Run No.	*Measured Ground Speed.*	*Measured Wind Speed and Direction.*	*Corrected True* $(=V)$ *Air Speed.*	*Observed Aneroid Height.*
1	59·1 m.p.h.	31·0 m.p.h. 161·5	89·2	5,100
2	123·4 ,,	28·6 ,, 5·5	93·7	5,100
3	62·0 ,,	32·3 ,, 168·5	93·8	5,050
4	124·7 ,,	32·3 ,, 21·0	95·6	5,000

Run No.	*Observed Temp.*	*Density* $(=\rho)$ *Referred to Standard Density.*	*Observed Air Speed.*	$V \times \sqrt{\rho}$.	*Correction Necessary.*
1	31	0·879	80·0	83·6	×3·6
2	31	0·879	85·0	87·8	×2·8
3	31	0·881	85·0	88·1	×3·1
4	31	0·882	88·0	88·8	×2·8
				Mean	×3·1

A summary of the complete speed tests may now be given. Firstly, the air speed and engine revolutions are noted flying level at full throttle every 2,000 feet, approximately, by aneroid. From the aneroid reading and temperature observation at each height the density is obtained. The reading of the air speed indicator is then first corrected for instrumental errors by adding or subtracting the correction found by calibration tests over the cameras or speed course.

This number is then again corrected for height by dividing by the square root of the density. The result should give the true air speed, subject, of course, to errors of observation.

The numbers so obtained are plotted against the "standard" heights—*i.e.*, the average height in feet corresponding to the density during the test. A smooth curve is then drawn through the points and the air speeds at standard heights of 3,000, 6,500, 10,000, 13,000, and 16,500 read off the curve. These heights are chosen because they correspond closely with 1, 2, 3, etc., kilometres.

The indicated engine revolutions are also plotted against the standard heights, because these observations form a check on the reliability of the results; also, the ratio of speed to engine revolutions at different heights may give valuable information with regard to the propeller.

Table VI. gives complete results of one of our tests of air speed at heights. The table refers to the same machine as Table V., which gives the results of calibration tests of the air speed indicator.

TABLE VI.

Air Speeds at Heights.

Aneroid Height.	*Observed Temperature.*	*Standard Density.*	*Corresponding Standard Height.*	*Observed Air Speed.*	*For Calibration Tests.*	*Corresponding for Density.*	*Observed R.P.M.*
	Fahr.						
3,000	39°	0·935	2,900	95 m.p.h.	98	101½	1,280
5,000	35°	0·875	4,900	93 ,,	96	102½	1,280
7,000	30°	0·821	6,900	88 ,,	91	100½	1,240
9,200	24°	0·767	9,000	81 ,,	84	96	1,220
10,800	19°	0·731	10,400	80 ,,	83	97	1,220
12,800	17°	0·682	12,600	72 ,,	76	92	1,200
13,800	12°	0·664	13,400	68 ,,	72	88½	1,180
15,200	8°	0·636	14,800	65 ,,	69	86½	1,160

Final Results from Curve.

Standard Height.	*Air Speed.*	*R.P.M.*
3,000	103·0 m.p.h.	1,290
6,500	100·5 ,,	1,250
10,000	96·5 ,,	1,215
13,000	94·5 ,,	1,180
15,000	86·0 ,,	1,160

WIRELESS TELEGRAPHY AS APPLIED TO AIRCRAFT

By R. D. BANGAY
(Marconi Wireless Telegraph Co., Ltd.)

THE uses to which wireless apparatus is put in connection with aircraft are:

1. Transmission of messages from aircraft to shore or aircraft to aircraft.
2. Reception of messages.
3. Navigation.

The medium through which "wireless" communication is effected is the ether, in which disturbances, known as electric waves, are set up at the transmitting station and radiated to the receiving station, where they are converted into sound.

TRANSMITTERS.

Types of Transmitters.—Apart from their power and range, there are two distinct types of transmitters—namely, (1) those that produce successive groups of damped waves, usually known as **Spark Transmitters,** owing to the employment of a spark gap in the primary circuit; (2) those that produce a continuous stream of undamped waves, usually known as Continuous Wave or **C.W. Transmitters.** This latter class embraces those used (*a*) for C.W. Telegraphy; (*b*) for Tonic Train Telegraphy; and (*c*) for Telephony.

Spark Transmitters.

Spark transmitters have three distinct circuits—namely, the charging or low frequency circuit, the primary oscillatory circuit, and the secondary oscillatory circuit, or aerial circuit. Fig. 2 represents a spark transmitter, in which an accumulator battery is the source of energy, and Fig. 3 represents a spark transmitter in which an alternator is the source of energy.

Action of Spark Transmitter.—Briefly, the action of a spark transmitter is as follows:

The primary oscillatory circuit, which consists essentially of a condenser and an inductive winding, has a definite electrical

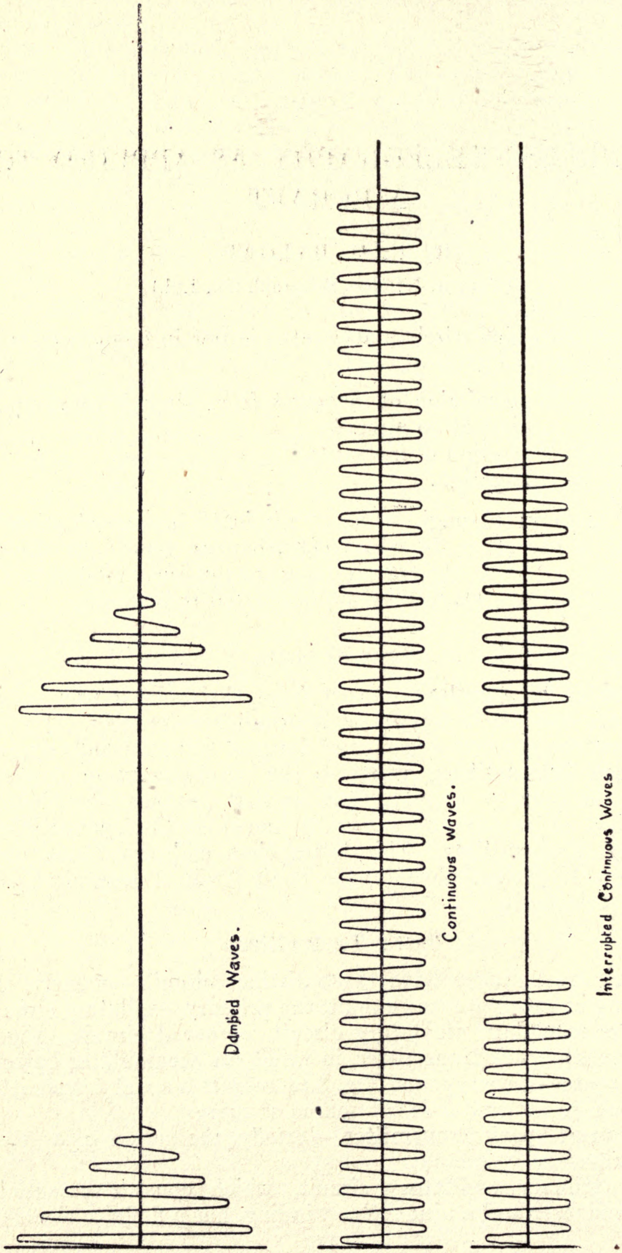

Damped Waves.
Continuous Waves.
Interrupted Continuous Waves

Fig. I.

time period, depending upon the capacity of the condenser and the inductance of the inductive winding (*vide* Formula I.). When such a circuit is energized, electricity will flow backwards

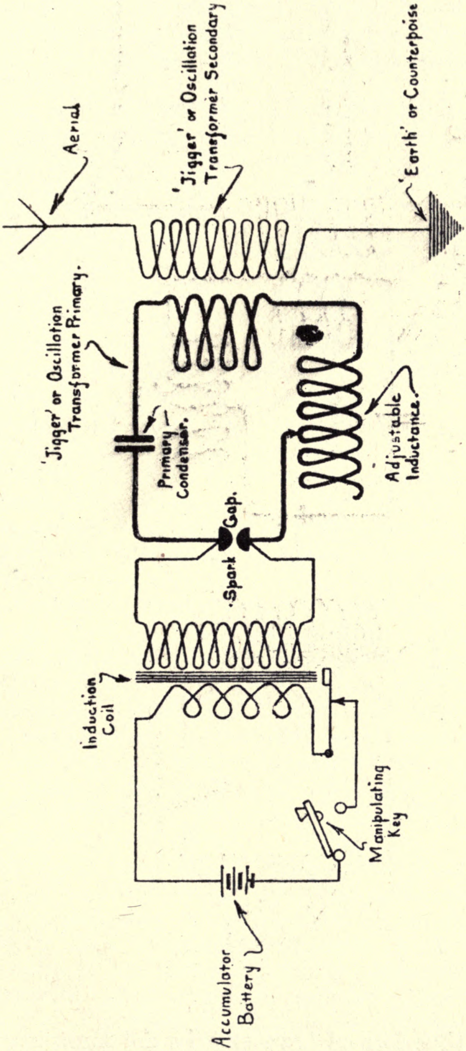


Fig. 2

and forwards through the inductive winding, first charging the condenser in one direction, and then in the other. The current will continue thus to oscillate until all the initial energy given to the circuit has been expended either in overcoming the resistance of the circuit or in creating waves in the ether which radiate the energy into space.

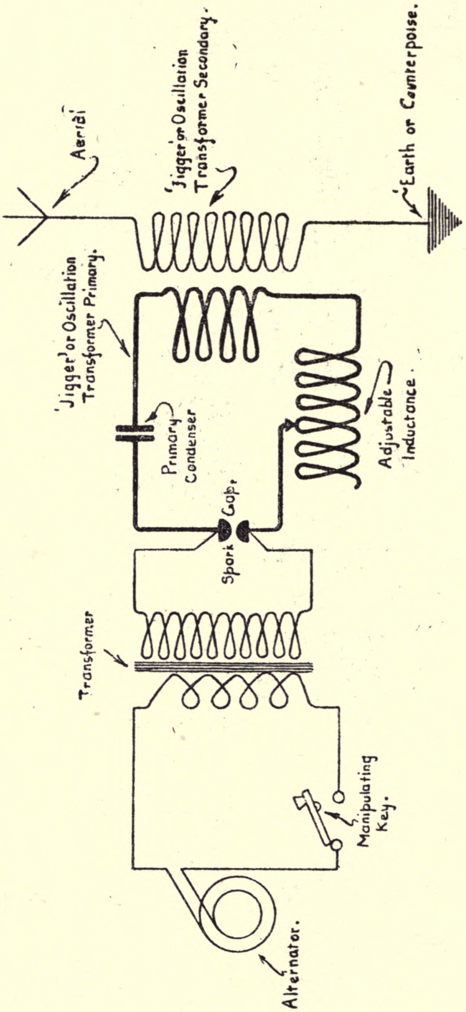


Fig 3

The waves thus formed have exactly the same frequency as the oscillatory currents producing them, and since the speed at which they travel through the ether is constant—namely, 3×10^8 metres per second—the length of the waves can be calculated, either from the frequency of the oscillations (*vide* Formula II.) or from the values of the capacity and inductance of the oscillatory circuit (*vide* Formula III.).

A closed oscillatory circuit, such as the primary circuit just described, is not a good radiator of electric waves, and for this reason it is inductively coupled to an aerial circuit consisting of an aerial wire and an "earth," or counterpoise, which is a better radiator.

When two such circuits are inductively coupled, any oscillations of current set up in the one create similar oscillations in the other; thus, if the primary circuit is energized, the energy is gradually transferred to the aerial circuit, from which it is radiated in the form of electric waves.

To enable this transfer of energy to take place freely the two circuits must be in tune—that is to say, they must both have exactly the same time period, or be tuned to the same wave-length.

The adjustment of the time period of the primary oscillatory circuit is usually accomplished by varying its inductance.

In some cases a separate winding, known as the "primary tuning inductance," is provided, so arranged that more or less of the winding can be included in the circuit. In the Marconi aircraft spark transmitters, the jigger primary is so arranged that part of the windings can be moved relatively to the rest, so that the two parts either oppose or assist one another, thus varying the inductance and consequently the wave-length of the circuit.

The Aerial.—The adjustment of the wave-length of the aerial circuit is accomplished either by varying the length of the aerial wire or by varying the amount of extra inductance included in the aerial circuit. Part of this extra inductance, known as the "aerial tuning inductance," is provided by the jigger secondary; but sometimes a separate winding is also provided, with tappings.

The aerial usually consists of a single wire, which, when not in use, is wound on an insulated reel within easy reach of the operator. The wire is arranged to break if the strain exceeds 50 pounds, in case a machine should inadvertently descend when the aerial wire is trailing. The length of the wire used depends upon the wave-length it is desired to transmit.

The fundamental wave-length of the aerial depends upon its length, and to a certain extent upon the self-capacity of the "earth," or counterpoise. If, as in most cases, the capacity of the counterpoise is large compared with that of the aerial wire, the wave-length of the aerial can be approximately calculated from Formula IV. Any inductance included in the aerial circuit will increase its wave-length, so that due allowance must be made for the inductance of the jigger secondary, and of any extra aerial tuning inductance used.

The "Earth."—The "earth," or counterpoise, is usually provided by the metal work (such as stay wires, engine, etc.) of the machine itself. This can be supplemented, if necessary, by stretching a few wires within the wing coverings.

In some cases an extra wire is trailed from the aircraft to act as the counterpoise. This wire should be so weighted that it trails at some distance from the aerial wire, otherwise radiation will be very considerably reduced, and also the aerial system becomes extremely directional.

Tuning the Transmitter.—Owing to the fact that the capacity of the primary oscillatory circuit is a fixed value for any particular instrument, the wave-length of that circuit is a function of the inductance included, and a calibration chart is usually provided with each instrument showing the wave-length obtained on the different tappings or adjustments of the primary tuning inductance. It is, therefore, a simple matter to set the primary circuit to give the desired wave-length.

The adjustment of the aerial wave-length is not so simple, for the reason that the capacity of the aerial will vary to a certain extent with different machines.

A tuning chart, showing the approximate length of the aerial and the approximate adjustment of the aerial tuning inductance for different wave-lengths, is useful for roughly setting the circuit, but the tuning should be finally adjusted by including a current indicator in series with the circuit. Then, after setting the primary to the desired wave-length, the length of the aerial wire can be varied until the maximum current reading is obtained on the indicator.

The current indicator can be either a hot wire ampèremeter or a small incandescent lamp. The former indicates the value of the current by the scale reading, and the latter by the brilliancy of the filament.

Excitation of Spark Transmitter.—The method of energizing the primary oscillatory circuit is to give the condenser an initial charge from the accumulator battery or dynamo, as the case may be, and to allow the condenser to discharge itself through the oscillatory circuit. To enable this initial charge to be given to the condenser, the oscillatory circuit, which would otherwise form a short circuit to the condenser, is broken by the spark gap, and the high tension terminals of the induction coil or transformer are connected across the condenser, or, what comes to the same thing, across the spark gap, as shown in Figs. 2 and 3.

When the condenser has been charged to a sufficiently high voltage (depending upon the length of the spark gap) the insulation of the latter is broken down, and the spark thus produced forms a temporary bridge, allowing the condenser to discharge through the oscillatory circuit. Thus, for every spark an oscillatory current of gradually diminishing strength flows in the primary circuit, producing in turn a group of damped waves.

The energy in each charge given to the condenser depends upon the capacity of the condenser and the voltage to which it is charged, and can be calculated from Formula V. The power given to the primary oscillatory circuit, therefore, is the quotient of the energy in each charge, and the number of times per second that the condenser is discharged through the circuit, and can be calculated from Formula VI.

Continuous Wave Transmitters.—There are several methods of generating continuous, or undamped, oscillations in a high-frequency oscillatory circuit for producing continuous waves.

For small power sets, such as are used on aircraft, the "oscillation valve" is practically the only method employed, on account of its simplicity and reliability.

The valve itself is shown diagrammatically in Fig. 4, and consists essentially of a hot electrode, termed the "filament," and a cold electrode, termed the "sheath," enclosed in a vacuum chamber similar to an electric-light bulb. Placed between the filament and the sheath is the controlling electrode, made of perforated metal or wire gauze, termed the "grid."

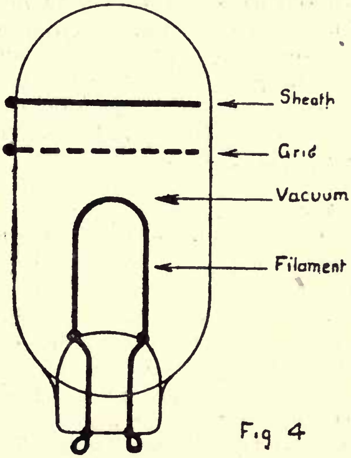


Fig 4

Action of the Valve.—The principle underlying its action is, briefly, as follows: When the filament is rendered incandescent by artificial means, electrons are liberated from the surface of the filament. The electrons being of negative polarity can be attracted to the sheath by charging the latter positively with respect to the filament; thus a current of electricity can flow through the valve, but only in the one direction.

The value of this current is limited by (1) the rate at which electrons are made available; (2) the resistance of the circuit, including the internal resistance of the valve and the resistance of the external circuit; and (3) the E.M.F. applied between the sheath and the filament.

The value of this current can also be controlled by the grid. Any negative charge applied to the grid tends to shield the filament from the effects of the positive charge applied to the sheath, resulting in a reduction of the flow of electrons from the filament to the sheath, and a consequent reduction in the current flowing through the valve.

In a suitably designed valve a comparatively low negative E.M.F. applied to the grid will neutralize a very high positive E.M.F. applied to the sheath.

High-frequency oscillatory currents can be generated by using the valve to impulse an oscillatory circuit.

If a pendulum be given a series of small impulses at intervals corresponding to the natural time period of the pendulum, it is well known that the latter will get up a swing backwards and forwards, even though the impulses are unidirectional. The swings will reach a greater and greater amplitude until the rate at which energy is lost by the pendulum in overcoming the resistance of the air, etc., is equal to the rate at which energy is supplied by the impulses.

In the same way, if a series of unidirectional current impulses be given to an inductance which forms part of an oscillatory circuit at intervals corresponding to the natural electrical time period of that circuit, current oscillations will be set up in the oscillatory circuit which will reach a greater and greater amplitude until the rate at which energy is lost by the oscillatory currents in overcoming the resistance of the circuit and in radiation, is equal to the rate at which energy is supplied by the impulses.

Fig. 5 shows diagrammatically how this can be accomplished. The sheath of the valve, S, is connected to one end of the inductance, L, which forms part of the oscillatory circuit, L, C. The other end of this inductance is connected through a high-tension battery, or D.C. Generator, B, to the filament of the valve, F. This battery must be connected so as to create a positive potential of, say, 1,000 volts on the sheath and a negative potential on the filament. The filament is rendered incandescent by a small four-volt accumulator battery, A.

Neglecting for the moment the action of the grid, it will be seen that under these conditions a unidirectional current will flow from the battery B through the inductance, and through the valve.

A coil, R, known as the reaction coil, is inductively coupled to the inductance, L, and connected between the filament of the valve and the grid electrode, and wound in such a direction that the current flowing in the inductance generates an E.M.F. in the reaction coil, which produces a negative charge on the grid of the valve, thus interrupting for the moment the flow of current from the battery, B. Owing to the inductance of the winding, L, however, the current which has already been started in it continues to flow in the oscillatory circuit, and charges up the condenser, C. As this condenser becomes charged the current gradually falls to zero, and then flows in the reverse direction as the condenser discharges itself. The result of this reversal of the direction of the current is to generate an opposite E.M.F. in the reaction coil, which reverses the potential on the grid, and allows another current impulse to flow from the battery B.

In a suitably designed circuit the difference in phase of the current flowing in the oscillatory circuit and that generated in the reaction coil is such that the potential of the grid is reversed and the valve consequently rendered conductive at the right moment—that is to say, at the moment when the current impulse from the battery assists and augments the current flowing in the oscillatory circuit, thus building up the oscillations with each successive impulse, until the loss in the oscillatory circuit, due to resistance and radiation, balances the rate at which energy is supplied by the battery.

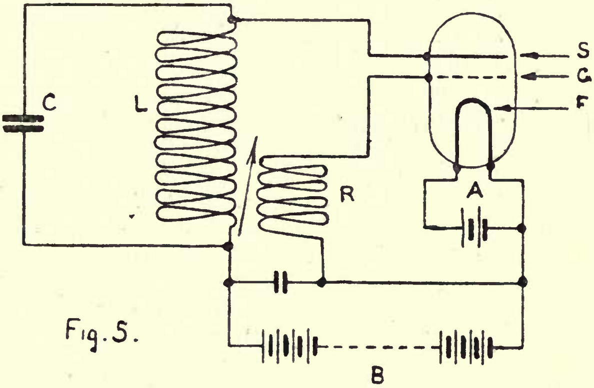


Fig. 5.

In the case of a valve transmitter, the aerial circuit is directly energized from the valve by including an inductive winding in series with the aerial, and connecting the valve across this winding, as shown in Fig. 6. The manipulating key can either be connected in the high-tension circuit, as shown by K1, but is more generally connected in the grid circuit, as shown by K2.

The same principle is employed in tonic train and telephone transmitters, these being provided in addition with suitable means of varying the amplitude of the oscillations generated in the aerial. This is accomplished by superimposing a low frequency current on the high frequency oscillations generated in the grid circuit. In the case of tonic train transmitters a low frequency current of uniform frequency and amplitude is superimposed on the high frequency oscillations acting on the grid of the transmitting valve. These low frequency impulses can be generated by means of a buzzer connected in series with the primary of a small transformer, the secondary of which is connected in the grid circuit of the valve in place of the manipulating key, K2, Fig. 6.

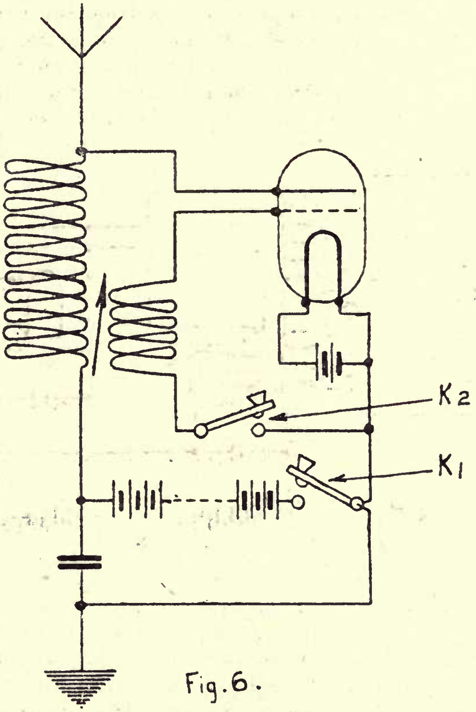


Fig. 6.

In the case of telephone transmitters a similar arrangement is used except that the buzzer is replaced by a microphone. The fluctuations in the microphone current due to speech are thus made to cause corresponding variations in the amplitude of the oscillations generated in the aerial.

The efficiency of the set as a telephone transmitter depends upon the degree to which the acoustic variations can be made to control the amplitude of the high frequency oscillations without distortion, and the method described above has certain limitations which make it unsuitable in some circumstances.

Fig. 7 shows an improved method of microphone control which has been adopted in the latest Marconi telephone transmitters.

The explanation of the action of this transmitter is briefly as follows: An oscillatory current is maintained in the aerial by the valve V_1. The high tension current from the source of power P is, however, fed through a highly inductive choking coil C. Across this " choke " is connected what is known as a " control valve " V^2, which deals only with the low frequency current variations created by the microphone M.

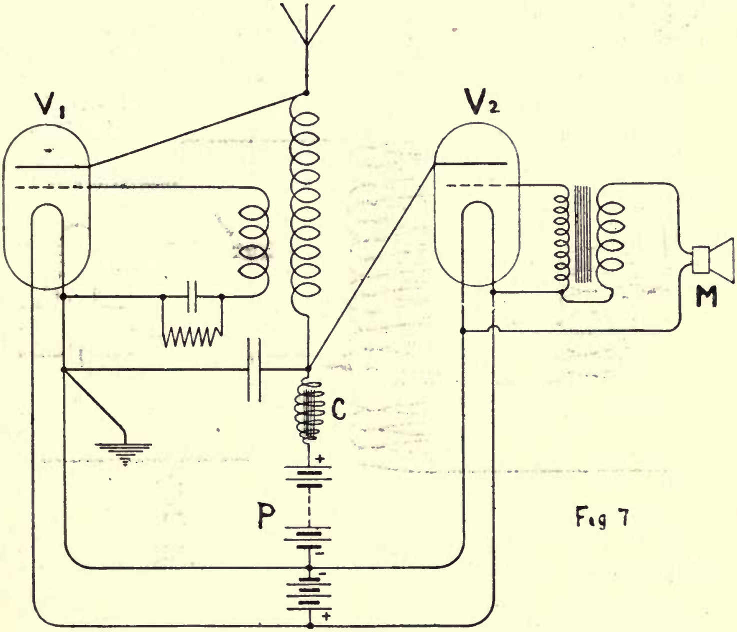


Fig 7

The microphone is connected to the grid circuit of the control valve so that the variations in the microphone current due to speech cause corresponding variations in the grid potential of the control valve. These in turn cause very much bigger variations in the sheath potential of the control valve, which, as already noted, is connected across the feed choke. The effect is equivalent to supplying the oscillation valve V_1 with current from the source of power P at a varying potential, thus varying the amplitude of the oscillations which it generates in the aerial.

RECEIVERS.

The function of the Receiver is to convert the electric waves which are radiated from the transmitter into something which is perceptible to the human senses. In nearly all modern wireless telegraph stations sound-reading is adopted as being the most accurate, as well as the most sensitive.

When a group of electric waves passes across a conductor which forms part of an oscillatory circuit, current oscillations are set up in that circuit. If the natural time period or wave-length of the oscillatory circuit is different to that of the incoming wave, the oscillatory current produced will be comparatively feeble. If, however, the wave-length of the circuit is the same as that of the incoming waves, each successive wave will assist the current oscillations, and thus produce a comparatively strong result.

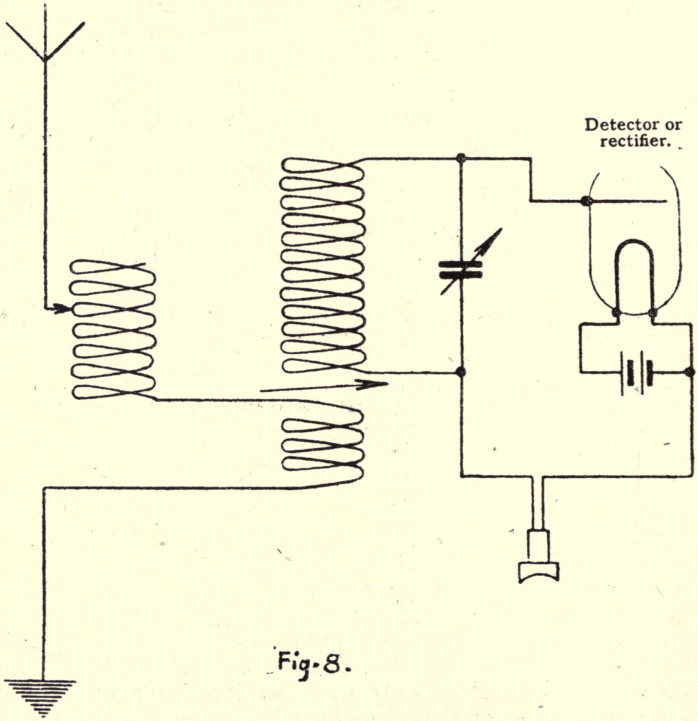


Fig. 8.

An oscillatory circuit which is a good radiator of electric waves is also a good receiver. Hence, an open oscillatory circuit consisting of an aerial wire connected at one end to the "earth," or counterpoise, is used at the receiving station for "picking up" the waves.

To enable the circuit to be tuned to the incoming wave, an adjustable inductance, called the "aerial tuning inductance," or an adjustable condenser, called the "aerial tuning condenser," or both, are connected in series with it.

The telephone receiver is the most sensitive ultimate detector of feeble electric currents, but it will not respond to high-frequency oscillatory currents.

To enable the high-frequency currents generated in the aerial by the incoming waves to be detected by the telephone it is necessary to rectify them—that is, to convert them into unidirectional currents.

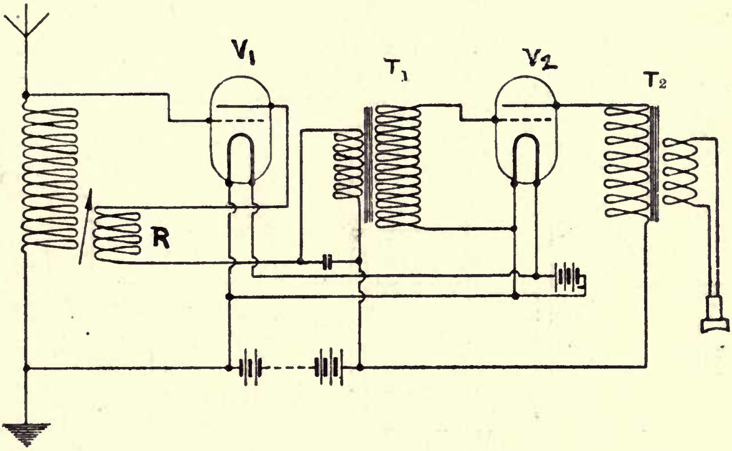


Fig. 9

One of the properties of the oscillation valve just described—namely, that it will only allow current to flow though it in one direction—can be used for this purpose.

For obvious reasons it cannot be efficiently employed directly in series with the aerial, as it would prevent oscillatory currents from flowing in that circuit, but it can be employed either across the aerial tuning inductance, or across the inductance of a secondary oscillatory circuit inductively coupled to the aerial, as shown in Fig. 8.

Magnifying Receivers.—To enable signals to be read above the noise of the engine on board an aircraft it is necessary that a very great volume of sound is produced in the telephone receiver. The sound produced in a simple receiver, as just described, is insufficient for this purpose. To increase the sound some form of relay or magnifier must be used.

The ordinary telephone relay is unsuitable, as it is sensitive to mechanical vibration. By far the best results are obtained by using the three-electrode oscillation valve, which can be used both for rectifying and magnifying the received oscillations.

Fig. 9 shows one type of valve receiver in which the first valve, V^1, rectifies the oscillatory currents induced in the aerial and communicates these rectified impulses through a transformer, T_1, to the grid of second valve, V_2. The variations thus produced in the grid potential of the second valve cause very much greater variations in the potential of its sheath, which, as shown in Fig. 9, is connected to the telephone transformer, T_2. A reaction coil, R, is included in the sheath circuit of the first valve, which, being inductively coupled to the aerial circuit, allows some of the energy liberated in the sheath circuit to be transferred to the aerial, thereby supplementing the oscillations created by the incoming signals and increasing the sensitiveness of the receiver.

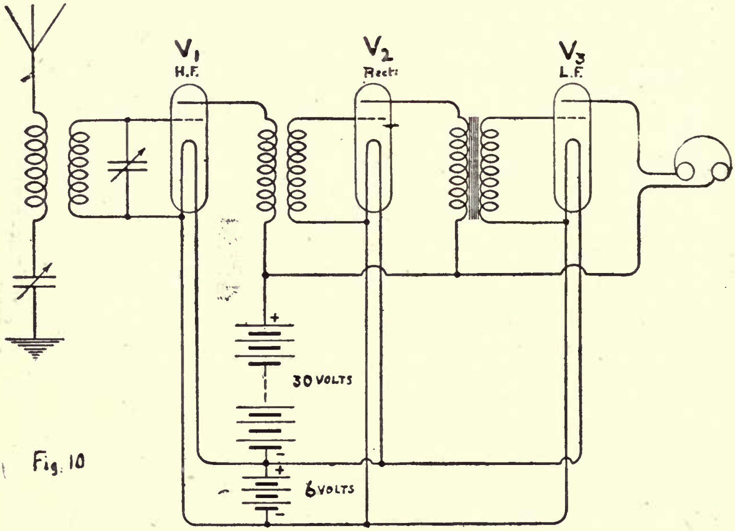


Fig. 10

If the coupling between the reaction coil and the aerial is increased, sufficient energy will be transferred to the aerial to maintain the oscillations indefinitely, and the receiver can then be used for the reception of continuous waves by slightly mistuning the aerial to the incoming signal.

In the receiver shown in Fig. 9 the high frequency currents are rectified by the valve V_1 before they are magnified by the valve

V_2. Fig. 10 shows another type of receiver in which the high frequency oscillations received in the aerial are first magnified by the valve V_1 before being rectified by the valve V_2. After rectification the signals are again magnified by the valve V_3.

A valve used for magnifying high frequency currents is known as a high frequency magnifier, and one used for magnifying the rectified currents is known as a low frequency or note magnifier. The advantage of high frequency magnification is to increase the sensitiveness of the receiver to extremely weak signals. On the other hand the advantage of note magnification is to increase the volume of sound produced in the telephones by signals of medium strength.

NAVIGATION BY WIRELESS.

Owing to the rapid variations in the direction and strength of air currents, the navigation of an airship during long-distance flights is extremely difficult, unless the pilot has some means of determining his exact position at frequent intervals. This means can be supplied by wireless telegraphy apparatus especially constructed for the purpose.

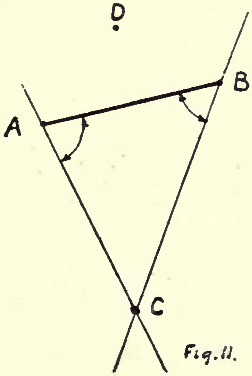


Fig. 11.

The principle is a simple one. The direction from which an electric wave emanates can be exactly determined with suitable apparatus. If such apparatus be erected at two points, A and B, Fig. 11, and simultaneous readings be taken at both of these points of the direction of a transmitting station, C, then by a simple trigonometrical calculation the exact position of C with relation to A and B can be determined.

For example, suppose that A and B are direction-finding stations erected at Hull and at Ipswich respectively, and suppose that C is an airship desirous of knowing its position: C would transmit a predetermined signal for a short period, which would be picked up by A and B. A would determine the direction by measuring the angle BAC, and B by measuring the angle ABC. Each station would then communicate the result to a transmitting station, D, situated at any convenient point. From the information thus obtained D would be able to calculate the exact position of C, and would transmit the result to him.

The principle on which the direction-finding apparatus is constructed is as follows: Any closed oscillatory circuit is highly directional whether used for transmission or reception. If a loop of wire forming part of a closed oscillatory circuit be suspended in the form of a rectangle, or triangle, as shown in Fig. 12, it will act as a fairly good receiver of incoming signals, and at the same time will be highly directional, inasmuch as it will not respond at all to signals coming from a direction at right angles to the plane in which the loop lies, and it responds best to signals when the plane of the loop corresponds with the direction of the transmitting station.

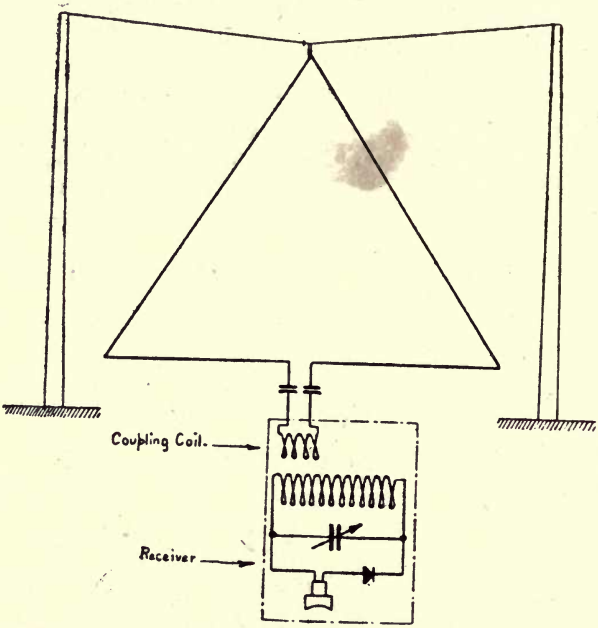


Fig. 12.

There are two methods of using this quality possessed by such an aerial for the purpose of finding the direction of any given transmitting station.

The first is to couple a receiver to the loop aerial in the ordinary way, and to arrange that the loop can be rotated through 180 degrees and its position indicated on a scale inside the operating-room. The scale reading thus indicates the direction of the transmitting station.

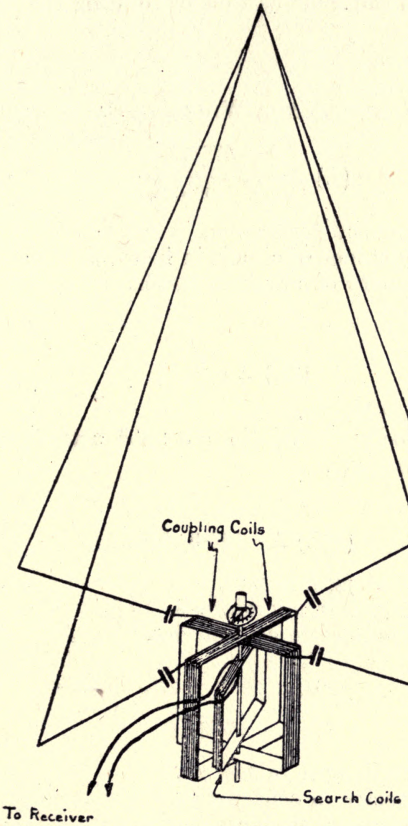


Fig. 13.

The second method is to erect two loops fixed permanently with their planes at right angles to one another, as shown in Fig. 13. Each aerial is then coupled to a common receiver through exactly symmetrical though independent inductive windings, these windings being fixed inside the receiver at right angles to one another. The coupling coil of the receiver, called the "search coil," is placed centrally to both the aerial coils, and is so arranged that it can be rotated with regard to them; thus, when its coupling is a maximum to one aerial it is also zero to the other, and *vice versa.*

It is evident that the effect of rotating the search coil in this second method is the same as that of rotating the aerial in the first method.

USEFUL FORMULÆ FOR WIRELESS CALCULATIONS.

$$\text{(I.)}\quad n=\sqrt{\frac{1}{4\pi^2LK}}.$$

n = frequency per second.
L = inductance of circuit in henries.
K = capacity of circuit in farads.

$$\text{(II.)}\quad \lambda=\frac{V}{n}.$$

λ = wave-length.
V = velocity of radiation $= 3 + 10^8$ metres per second.
n = frequency.

$$\text{(III.)}\quad \lambda=196{\cdot}8\sqrt{KL}.$$

λ = wave-length in feet.
K = capacity in microfarads.
L = inductance in centimetres.

or,

$$\lambda=2\sqrt{KL}.$$

λ = wave-length in metres.
K = capacity in centimetres.
L = inductance in microhenries.

N.B.—1 microhenri = 1,000 cms.
1 microfarad $= 9 \times 10^5$ cms.

$$\text{(IV.)}\quad \lambda=3{\cdot}4\,l \text{ approximately.}$$

λ = fundamental wave-length of single wire (aircraft aerial).
l = length of aerial wire.

(V.) $E = \frac{1}{2} K V^2$.

E = energy in joules.
K = capacity of condenser in farads.
V = voltage of condenser.

(VI.) $W = \frac{1}{2} K V^2 S$.

W = power of transmitter in watts.
K = capacity of condenser in farads.
V = voltage of condenser.
S = spark frequency per second.

SPRAY PAINTING, VARNISHING, AND DOPING

By C. L. BURDICK

The blowing on of paints, varnishes, and the like, with paint sprayers is an art which the war has helped to develop.

For a matter of twenty-five years machines of this nature have been on the market, but the present shortage of labour has brought them rapidly to the front for "speeding up."

Spray painting has been applied to such widely different classes of work as that of preparing the drawings of aero engines, painting and varnishing of shells, mines, hand-grenades, motor-cars, ammunition-boxes, gun-carriages, flying machines, observation balloons.

A certain amount of practice is required, but nothing like the degree of skill is called for that is necessary in painting with hog's hair.

All sorts of covering media are used—enamel paint, stoving paint, dope, ships' composition, tar and cement, varnish, polish, lacquers, bronzes, and, in fact, anything that can be applied with brushes; moreover, the work can be done on any surface which is suitable for painting.

One might suppose that a slender article like a bicycle frame could not be done with a spray; as a matter of fact, it can be done quickly, and with a saving of paint, with a paint-spraying machine.

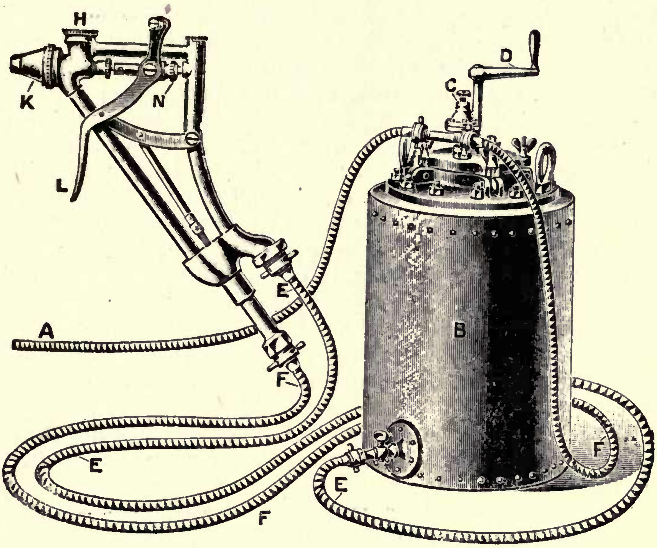
In the best-known types the apparatus consists of a hand-piece, in which an air valve and a valve for the liquid or paint are controlled by a single finger lever, while the paint is fed (under pressure) to the hand-piece through a flexible pipe; the air valve simply turns the compressed air full on when the lever is pressed, but the liquid or paint valve is finely adjusted, so that a very small spot of paint can be delivered, or, by opening wide, a large quantity can be delivered.

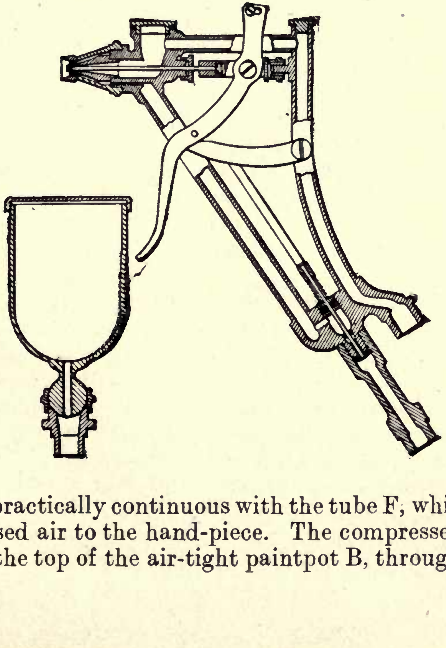
Sprays can be made large enough to cover the side of a building in a few seconds, but as a matter of practice it is found that a spray which under test conditions will deliver three square yards in a minute is as much as an operator can manage and obtain a level or uniform coating.

The illustration (on p. 137) shows the method of painting with a pressure paintpot.

The paintpot (B) is supplied in 1 or 4 gallon sizes or larger to order. Spray-painter nozzles are made for all classes of work.

The compressed air is introduced through the flexible tube A.





This tube is practically continuous with the tube F, which conducts the compressed air to the hand-piece. The compressed air is also admitted to the top of the air-tight paintpot B, through the reducing valve C; the pressure on the top of the paint will force the paint through the tube E to the hand-piece.

The advantage of this system is that by adjusting the valve C (by the nut at the top) the flow of the paint or liquid is under perfect control. If the operator is painting at a considerable height from the floor, screwing down the nut will send the paint up in sufficient quantity. The method is also convenient for paints or liquids of varying consistency, as the pressure can be easily and quickly regulated to suit the liquid. The flexible air and paint tubes are provided with union nuts and are easily assembled or taken apart. Usually asbestos-lined flexible tubing is provided for the paint tube (marked E on illustration), while rubber-lined metallic tubing is provided for the air (tubes A and F). A stirrer, D, is provided for agitating the paint. Hinged bolts with wing nuts enable the cover of the pot to be quickly removed.

When cleaning the hand-piece and paint tube, first allow the air to escape from the top of the paint by loosening the wing nuts, then, placing the hand with a rag over the nozzle of the instrument, turn the air back through the paint tube; this will blow the paint back into the pot; while the air is still turned back through the paint, turn the tap at the bottom of the pot. The paint tube may then be detached and hung up so that any remaining paint will run out.

If more thorough cleaning is required the detached end may be immersed in turps, and turning on the air at the hand-piece will suck the turps into the tube, which can then be blown out at the nozzle. The instrument can be set to give a fixed quantity by regulating the nut N. An air nozzle at K may be removed for cleaning, and an inner nozzle for paint may be removed by a spanner to get at the needle valve if necessary.

When coating small articles, a cup to hold a small quantity of liquid may be affixed at H (see illustration), and for many purposes the cup container is quite efficient. Where the painter is working in a fixed position an overhead paintpot with a fall of eight or ten feet will work for all paints which are fairly fluid; but to meet all conditions the pressure paint container is the best. The feed of paint can be nicely adjusted; also, the evaporation from such liquids as dope is prevented.

Paint sprayers are used in many departments of aeronautics; the big observation balloons are "doped" with this machine; the wings of aeroplanes are sometimes doped, and usually varnished with spray, and it is found that even the slender parts of the framework can be varnished much quicker and better with a sprayer than with the ordinary brush.

To paint with a spray requires a certain amount of practice, especially with a heavy liquid like dope. If the spray is held too far away from the surface of the wing the media evaporate and the dope goes on too dry; on the other hand, a level coating is difficult if held too close. A little practice will enable the operator to steer between Scylla and Charybdis.

Another advantage claimed for the spray method is that the same number of coats are not necessary. When a paint or liquid is laid on with an ordinary brush the tendency is to wipe away a portion as the paint is laid on. With a spray the surface may be "loaded," so that there is no "wiping away." The result is that three coats with a spray are equal to about five coats with an ordinary brush.

The pressure of the air for spray painting varies with the character of the liquid to be sprayed. A thin liquid-like lacquer can be handled with a pressure of 18 or 20 pounds to the square inch. The lower the pressure at which one can work the better, as it does not send so much of the liquid into the air of the room. Thirty pounds is sufficient for heavy paints or dope.

A room should be properly ventilated where spray painting is being done. When a medium like dope is being used the air should be warmed as it enters the room, and should be drawn out from near the floor, as the gases from the medium are heavier than the air. If this is done the air of the room can be kept quite clean and free from injurious fumes.

DOPE*

By A. J. A. WALLACE BARR

History of Dope.—At the present time all dopes used have a base either of cellulose acetate or cellulose nitrate, the former being the basis of non-flammable celluloid, and the latter, cellulose nitrate, which is another name for guncotton, being the basis of celluloid. It must not, however, be supposed that these materials have always been used for the purpose of proofing fabric coverings of aeroplanes, etc.

In the very early days, the wings, rudders, etc., were covered with unproofed cotton, which was stretched as tightly as possible, no attempt being made to make the fabric either petrol, water, air, or oil proof. In point of fact, it is very probable that the early pioneers did not realize the necessity of these points, for their machines were only used in the best possible weather, and they had no opportunity of learning the tremendous loss they suffered by using fabric which was not airproof. However, they soon began to realize the necessity of some form of proofing, and immediately started to use rubber fabric similar to that employed in the manufacture of balloons, although of a lighter grade.

This material was very advantageous for the purpose owing to the fact that it could be easily joined with rubber solution, and was perfectly waterproof. On the other hand, this fabric slackened off in wet weather, and the rubber became decomposed by the oil from the engine. This class of material was then abandoned in the majority of cases, although the late S. F. Cody utilized rubber-proofed material doped with cellulose acetate dope for the machine on which he won the Military Trials in 1912. This, however, was an isolated case, and the majority of constructors turned their attention to various varnishes with which they endeavoured to treat their fabric satisfactorily. Boiled linseed-oil was tried, but proved unsuccessful. Later, collodion solution containing castor-oil was tested, but it was found that although this material had advantages in weight, tautness, and other respects, the high inflammability of the film made its use somewhat dangerous.

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* This article was written just before the cessation of hostilities in November, 1918.

Some of the early school machines in this country were doped with a solution of sago, and this was found quite successful, although the slightest change in the humidity of the atmosphere affected the tautness. At a later date various varnishes were tried, also casein solutions; but all these materials had various properties which made them unsuitable for use. Finally, however, in 1910, dopes with a base of cellulose acetate came into fairly general use.

These dopes were easy to apply, quick drying, and the film was found to be thoroughly water, oil, petrol, and air proof, the film, in addition, tightening the fabric, and thereby increasing the speed and efficiency of the machine, and also reducing the inflammability. From that date onwards acetate dopes held a leading place in this country, and it was not until after the outbreak of war, when the shortage of supplies of acetate of cellulose became acute, that nitro-cellulose dopes were again considered. From then onwards various dope manufacturers have manufactured nitro-cellulose dopes, in some cases for use on all classes of land machines, but in the majority of cases for use on home service school machines only.

With regard to the early dope, when the use of tetrachlor-ethane as a softening agent and solvent was employed, this material was found extremely suitable so long as it was absolutely pure, and provided the other materials were the best that could possibly be obtained. However, with the shortage of materials after the outbreak of war, it was found that the tetrachlor-ethane set up decomposition, and the doped fabric rotted after a very short period of weathering.

The vapour of tetrachlor-ethane also had a toxic action on the workers, and in 1915 there were a considerable number of deaths through its use amongst the operatives employed. The Home Office laid down stringent regulations with regard to ventilation, but even then it was felt that the use of dope containing this ingredient entailed a certain amount of risk, and this type of dope was finally abolished in the year 1916.

It is, however, a very open point whether this type of dope was abolished owing to its poisonous nature or owing to the fact that it was impossible to obtain the necessary pure materials for its manufacture.

When the research work which was carried out regarding decomposition of tetrachlor-ethane dopes proved that the action was due to certain rays of sunlight, the Royal Aircraft Factory produced a dope covering containing certain pigments which entirely kept out these rays. This covering also served a twofold purpose by providing an excellent camouflage for machines employed in land operations. It was known as P.C. 10.

After the abolition of dopes containing tetrachlor-ethane their place was taken by non-poisonous dopes; but, notwithstanding their title, the Home Office still insist on the same amount of ventilation, and the use of pigmented covering is still continued, not only on account of its camouflage properties, but also on account of the very excellent results obtained by its use.

During this period the pigmented covering, or P.C. 10, was used only for the top surfaces of the planes and sides of fuselages, etc. —that is, those parts of the machines exposed to direct sunlight. The covering for the under surfaces of machines was a material essentially similar, in all respects, with the exception that the pigments were omitted. This dope covering was also a R.A.F. product, and known as V. 114.

From the middle of 1916 to the beginning of 1918, practically no change in the composition of dope and dope coverings was made, but in the beginning of 1918 the shortage of raw materials rendered it imperative to eliminate the use of the nitro-cellulose dope coverings, and it was decided to substitute oil varnish in their place. It was further decided that the under surfaces of the planes for land work need have no protective covering over acetate dopes, although it was decided to continue to coat the under surfaces of seaplanes with transparent oil varnish after doping, the top surfaces of both land machines and seaplanes being covered with pigmented oil varnish.

The varnish of this nature, although filling to a certain extent the function of the nitro-cellulose covering, dried slower, and therefore tended to reduce the output, and it is very doubtful whether it could be used successfully in the field for repair purposes. This point, however, remains to be seen.

Properties of Dope.—After the woodwork of the machine is completed the fabric is attached to various components which need covering, and is stretched as tightly as possible by the worker. No matter how tightly they are able to stretch it, it is necessary that it should be shrunk still further in order to obtain the maximum efficiency and safety for flying. In addition, it is necessary to proof the fabric against varying conditions which have to be met with on active service. The dope is used, therefore, to fulfil the following functions and conditions:

1. To tighten fabric.
2. To fill up the interstices between the warp and the weft of the fabric so as to prevent the penetration of wind and moisture, and to make the surface of the fabric smooth in order to reduce the skin friction.
3. To prevent contraction and expansion of the fabric during the varying weathers and varying temperatures.
4. To be petrol and oil proof, and to be capable of being washed and cleaned.

Application of Dope.—The dope must be applied in an approved dope shop, having a temperature of between 65° and 70° F. The air in this dope shop must comply with the Home Office regulations regarding ventilation.

It is essential that the fabric is thoroughly dry before the dope is applied, and it is also essential that the atmosphere of this dope shop contains the minimum humidity. This is necessary owing to the rapid evaporation of the dope solvents, causing a reduction in temperature, which causes moisture to condense on the dope film, thus turning it white.

The usual method for application of dope is by means of a brush, and the use of special covered dope cans is advised. These cans prevent evaporation, and so keep the dope at a uniform viscosity.

Application by Spray.—Attempts have been made to spray dope, but so far without much success—except in the case of nitro dopes and nitro dope coverings. Acetate dopes are not suitable for spraying, owing to the fact that the solvents evaporate before the dope reaches the fabric and the cellulose is precipitated. Nitro dopes, however, do not behave in this manner, owing to the fact that the solvents used are less volatile.

Dope Coverings.—Reference has already been made to dope coverings P.C. 10 and V. 114, and although these materials are essentially suitable for use both for home service and in Europe, it has been found that they are not suitable for use in tropical climates owing to the fact that the dark surface of P.C. 10 tends to absorb the heat, causing the temperature inside the plane to become too high, with the result that warping and twisting of the framework of the doped component is liable to take place. In order to obviate this, an aluminium, or in some cases white, covering is used in tropical climates.

This material is essentially the same as P.C. 10, with variation of pigments in order to produce different colours. For night flying also a different covering is used, and for this purpose a dark green colour is found to be most suitable.

German Dope.—The Germans in doping their aeroplanes appear to follow very closely the methods used in this country in so far as the raw materials are concerned; and although they used both acetate and nitro dopes before the war, they are now apparently confining their attention to acetate dope. This is probably accounted for by the fact that Germany was before the war the largest producer of cellulose acetate, and produced a very high quality material.

In addition, their resources for solvents, etc., are excellent. Having regard to these facts, it is somewhat surprising to notice to what extent the quality of the German dope has deteriorated during the progress of the war; but this deterioration is only a parallel instance of the very rough finish they give to their machines, and probably they only produce a dope which, in their opinion, will last as long as the average flying life of a machine on active service.

The Germans at the beginning of the war used acetate dope and covered it with transparent oil varnish; but at a later date, for camouflage purposes, and also for means of identification, they commenced to use oil and spirit paints, apparently paying very little regard to the high increase of weight thus caused. At the end of 1917, however, they commenced to use a camouflage fabric. This fabric was printed in various colourings, the chief of which were yellow, mauve, pink, blue, and green, and over this a very thin layer of dope was placed, the dope deposit being considerably lighter than the weight insisted upon by the authorities on this side, and, in addition, no covering of any sort was put over the dope. It has been found, however, from experiments made on fabric from captured machines, that the printed fabric faded rapidly when exposed to sunshine, and even this latest dope deteriorated very much faster than our English materials.

Doping for Commercial Machines.—The question of doping for commercial machines for after-the-war purposes is a somewhat difficult one to deal with, as it is impossible to give any indication as to what the probable price of the dope will be. It is almost certain, however, that the doping of machines will be treated in the same way as the painting of motor-cars, and it is hoped that they will not need to be redoped for a period of eighteen months or two years. Instead of the highly concentrated dopes now used owing to the shortage of solvents, in all probability about seven or eight coats of very thin dope will be given, and this will then be covered with a suitable pigmented covering in order to prevent the effect of ultra-violet rays on the dope and fabric, and on top of this pigmented covering another dope covering will be given, which will be made to any desired tint preferred by the user of the machine, or pigmented dopes may be used to reduce the number of coats.

One rather assumes that the mail-carrying machines will have a definite colour settled by the Government, and in all probability the public passenger-carrying machines will also have a special colour; but for the privately owned aeroplane, the owner will have as wide a range of colourings to choose from as he now has for his motor-car.

PATENTS AND PATENT LAW

By ARTHUR HUNT

Application.—The necessary forms for an application for a patent for the United Kingdom and the Isle of Man can be obtained on application to the Patent Office, 9, Southampton Buildings, Chancery Lane, London, or at the chief Post Office in most of the large towns in the United Kingdom. "Instructions to Applicants" are also obtainable at the Patent Office. Applications may be either left or sent by post, and must be made in the applicant's real names, and not in assumed or trade names. The applicant must be the true and first inventor of the invention, or he must apply with the true and first inventor, or he can apply provided he has received the invention from abroad; and the application must be accompanied by two copies of the specification. This may either be a "Provisional" or "Complete" specification.

It is customary only to file a Provisional Specification with the application. This must contain particulars of the nature of the invention, and gives provisional protection. It should give a fair description of the invention without too much detail. Drawings may be added if necessary, but not models. Pending the actual grant of the patent this enables the invention to be used and published. The fee payable on application is £1. The Complete Specification has eventually to be deposited, and this must be done within six months of the original application, when a further fee of £3 is payable.

All fees are payable on stamped forms, and not otherwise. The Complete Specification can be left with the original application if it is desired to obtain a grant quickly, but often the inventor wants to protect his invention before he has fully worked it out, and he is thus enabled to obtain provisional protection for his work, although he cannot take proceedings for infringement until after the patent is actually granted, and then only for infringements after the date of the filing of the Complete Specification, so that delay is dangerous. If later he discovers it does not come up to expectations he need not proceed farther, and can so save the full fees payable with a Complete Specification.

Title of Invention.—Every specification must commence with a descriptive title covering the subject-matter, which title becomes part of the patent. This must not be too wide nor too vague, and it is not intended that it should be the name by which the invention is to be generally known.

Examination of Specification.—The Complete Specification when filed is examined carefully by the authorities, and if it is found that it has been wholly or in part included in any patent granted during the previous fifty years the applicant is usually given an opportunity of amending his specification. The application may, however, be wholly refused, though there is a right of appeal.

Restriction of Application.—If the Complete Specification should include an invention not included in the Provisional Specification, the original application may be allowed to proceed so far as the invention included in both specifications is concerned, and the claim for the additional invention which is included in the Complete Specification may be treated as a separate application for that invention.

Inspection of Specifications.—When a Complete Specification is accepted the fact is advertised in the Official Journal of Patents, and the specification and drawings are open to the inspection of the public. Anyone wishing to be informed when a Complete Specification for any particular invention has been accepted can be furnished with the information on forwarding the prescribed form bearing a 5s. stamp. The Official Journal of Patents will be found at many Public Libraries throughout the kingdom. Within two months of this advertisement the grant of the patent may be opposed on various grounds by notice given at the Patent Office.

Improvements.—A " Patent of Addition " for any improvement in an invention can be obtained, and may be granted for the unexpired term of the original patent. A fee of £1 is payable on this application, but no fees are payable for its renewal.

Infringement.—Proceedings for the infringement of a patent cannot be instituted until the patent has been actually granted. Threats of legal proceedings in respect of any alleged infringement of patent may be stopped by injunction, and damages, if any are sustained, can be recovered if it can be shown that such alleged infringement was not, in fact, an infringement of any legal rights of the person making such threats.

Assignment.—A patentee may assign his patent for any place in or part of the United Kingdom or Isle of Man as effectually as if the patent were originally granted to extend to that place or part only.

Term of Patent.—Patents are granted for fourteen years subject to the payment of the prescribed fees, which become payable at intervals. It is possible in some cases by petitioning the Court to obtain a further extension for a further seven or fourteen years, and particularly if it can be shown that a patentee has not been adequately remunerated by his patents.

Lapse of Patents.—Patents lapse if the fees are not paid as they become due, but an application can be made for a lapsed patent to be restored on payment of the prescribed fees, and if good reason can be given for the omission of payment of the fees.

Register of Patents.—A Register of Patents is kept at the Patent Office, where all assignments of patents or grants of licences under patents should be registered.

Deceased Inventor.—Patents may be granted to the legal representative of any person claiming to be the inventor who has died before making his application.

Lost Patents.—Sealed duplicates of patents lost or destroyed may be obtained.

Record of Patents.—An illustrated journal of patented inventions, giving reports of patent cases and other information, is issued by the Patent Office, and copies of all Complete Specifications of patents in force, together with drawings relating thereto, are also obtainable by the public from this office for a small charge.

Secret Patents.—The inventor of an improvement in instruments or munitions of war may assign the invention to the Secretary of State for War or the Admiralty, in which case the Secretary of State or the Admiralty may before the publication of the Complete Specification certify that it is in the public interest that the particulars of the invention, and the manner in which it is to be performed, should be kept secret, in which case the documents are left in a sealed packet instead of being lodged in the ordinary way.

International Patents.—An International Convention exists between Great Britain and various Colonies and States. The application in each must be made within twelve months of the first application, but it is possible to get this time extended for three months, but not more. An applicant in any one of the States in the Convention may obtain priority in any other of the States during this twelve months.

Patent Office Library.—The Free Library of the Patent Office is open daily. The Colonial and Foreign patent laws may be consulted there, in addition to the patent specifications of various countries.

Patent Agents.—Applicants can always obtain the assistance of patent agents, and it is generally wiser to adopt this course in preparing the specifications. A register of names is kept at the Institute of Patent Agents.

THE HEAT TREATMENT OF STEEL*

By J. M. ROGERS, M.E.

The heat treatment of steel is of great importance in aeroplane construction. Aeroplane fittings and engine parts should be made as light as possible without impairing their strength. The steel from which they are made should therefore have the greatest possible strength and sufficient resilience to resist the shocks to which fittings or engine parts are subjected. For steel to attain the above properties it is necessary that it receive very careful heat treatment.

The three principal reasons for heat treating are to harden and toughen the steel, and refine its structure. There is always a great deal of uncertainty as to the physical condition of steel that has not been heat treated. Several portions of a bar or forging may have been finished at different temperatures or one bar or forging may have been finished at a higher or lower temperature than another. These conditions would cause a variation in the physical properties of the steel, which would not be at all desirable.

It will also be found that the structures of steel castings and drop forgings which have been finished at high temperatures are very coarse. All important steel castings and drop forgings should be heat treated for the purpose of refinement, as a coarse structure greatly impairs strength and ductility. It is evident from the above statements that it is necessary to heat treat steel when the best physical properties obtainable are desired.

There are five different structures found in steel: austenite, martensite, troostite, sorbite, and pearlite. When a piece of steel is heated to a temperature above the critical range—*i.e.*, temperature range over which the transformations of structure take place—the structure becomes austenitic, and this transformation is accompanied by a refinement of grain. As the piece is slowly cooled through its critical range, there is a transformation from an austenitic to a pearlitic structure, the steel passing through the transitional structures in the order given above. The physical properties of these structures for carbon steel are given in the table on p. 149.

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* Reproduced from *Aviation*, New York.

Structure.	*Strength in Thousands of Pounds per Square Inch.*	*Ductility.*	*Hardness.*	*Resistance to Shock.*	*Resistance to Fatigue.*
Austenite ..	150–200	30	1·5	very good	good
Martensite ..	?	0	4-5	very poor	very poor
Troostite ..	200	2–5	2	fair	very good
Sorbite ..	160	5–10	1·6	good	very good
Pearlite ..	140	10–15	1·5	good	poor
Ferrite (pure *a* iron).. ..	40	40	1	fair	fair

The completeness of this transformation depends upon the rate of cooling throughout the critical range. The transformation from austenite to martensite is very rapid, and austenite is never found in commercially heat-treated steel except in the special case of austenitic alloy steels.

A high carbon steel quenched from above the critical range will contain a large percentage of martensite, but as the carbon content decreases the structure becomes troostitic or sorbitic. After quenching, there are internal stresses in the steel resulting from the rapid cooling, and it is necessary to draw—*i.e.*, reheat the piece to a temperature below the upper limit of the critical range—in order to remove them. The higher the drawing temperature, the greater is the tendency for the steel to become pearlitic; and by properly choosing the drawing temperature any desired completeness of this transformation may be obtained.

The physical properties of steel may be greatly improved by the addition of certain elements. Nickel, chromium, and vanadium are used to a great extent. The steels to which these elements have been added are known as alloy steels. Nickel increases the hardness, toughness, and tensile strength, with only a slight decrease in ductility; chromium imparts great hardness; and vanadium gives a very good combination of strength and toughness. All of the above elements increase the elastic ratio. Alloy steels can be greatly improved by heat treating.

The determination of the proper quenching temperature is of great importance. The steel must be heated sufficiently above the critical range to refine the structure and give the maximum rate of cooling on quenching as the temperature passes through the critical range. The location of the critical range can be determined from a thermal curve, and usually can be obtained from the manufacturer of the steel.

The question as to how far above the critical range the quenching temperature should be depends upon a number of considerations. There is a tendency for the crystals to grow when the steel is heated above its critical range, and the rapidity of this growth increases with the temperature. Therefore it is not desirable to heat the steel any more above the critical range than necessary, or to hold it at that temperature any longer than necessary.

If the grain of a piece of steel is very coarse, it is necessary to heat it considerably above the critical range to refine the steel, but the resulting structure will not be as fine as it would have been had the structure been finer at the start. This difficulty may be overcome by heating the piece sufficiently to refine it, then cooling slowly or rapidly, and reheating slightly above the critical range, and finally quenching.

The size of the piece and the temperature drop between the furnace and the quenching bath are important considerations in determining the proper quenching temperature. In heating a piece of steel to its quenching temperature, the furnace temperature should be brought up slowly enough to give a uniform heat, and the piece should be permitted to soak at the quenching temperature for a few minutes only. The temperature of the quenching medium should not be permitted to vary sufficiently to appreciably affect its quenching speed. There are quenching oils which will give uniform quenching speeds over large temperature variations.

Steel is drawn after quenching in order to remove any strains resulting from the rapid cooling, and to toughen it if it has been made too brittle. The time of drawing depends upon the size of the piece, a longer time being required for larger pieces. A salt bath is very satisfactory for drawing at low temperatures.

The successful heat treating of steel depends a great deal upon the furnace man and the equipment with which he has to work. He must be able to regulate his furnaces accurately; use his judgment as to whether the steel is up to furnace temperature. and look after a number of small details, such as the arrangement of the steel in the furnace, the method of quenching, the location of the pyrometer fire iron in the furnace, etc.

The furnaces should be so constructed as to give uniform heats and absolute temperature regulation. An accurate pyrometer is absolutely essential. The potentiometer and high resistance types of pyrometers are most accurate, and it is quite important that they be compensating for cold end corrections.

It is quite important that there be a constant check kept upon the heat-treated products by making physical tests and microscopic examinations of the structure. In this way any irregularities in physical properties and structure are immediately detected. To obtain a uniform heat-treated product, it is very important that there should be no appreciable variations in the chemical composition of the steel, and frequent chemical analyses should be made to detect any such variations.

SPINNING*

By GEORGE H. BETTINSON

Most pilots, more especially pupils, are sure to experience some time during their career a " spin." These few notes, therefore, are written in the hopes that they will save some accidents caused by careless and inexperienced pilots, who are in nine cases out of ten directly responsible for their machines " spinning."

Spinning arises from " side-slipping." Side-slipping is of two kinds: (1) Outwards side-slip, and (2) inwards side-slip.

When a spin occurs a pilot must immediately ascertain which of these his side-slip is, and also whether his air speed is normal or abnormally low. The former is almost always known when it is possible for the flier to be aware that he is turning (*i.e.*, when he is not in a fog or a cloud). The air speed can only be known with any assurance from the air speed indicator.

In general, when a machine gets out of control, the first thing to do is to throttle the engine right down immediately. Then, if there is fair height, the machine should be dived. If there is not room to do this there will probably be a smash in any case, and it is accordingly better that the engine be switched right off so that all danger of fire is avoided.

The problem of spinning is best considered on the stable and unstable aeroplane separately.

1. If the aeroplane is a stable one, and the speed correct, side-slips in nine cases out of ten correct themselves. Suppose the side-slip is caused by the pilot doing too wide a turn with an excessively heavy bank, the aeroplane will, under gravity, side-slip towards the ground, and the resultant side-wind directed upwards from the earth will tend rapidly to re-align the planes, thus curing the side-slip, if there is space enough.

2. On the other hand, if the speed is correct but the aeroplane unstable, the only way a spin can occur is by the fin surface aft being inadequate or else wrongly disposed The flier's best course of action is to throttle down his engine and dive the machine for speed recovery, at the same time firmly holding his rudder appropriately—the straight position is generally best.

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* Reproduced from *The Aeroplane.*

We have now considered the causes of slipping on stable and unstable aeroplanes, given that the air speed in each case is correct (*i.e.*, normal or slightly abnormal). We will now consider the cases of the unstable and stable aeroplanes with the speed too slow.

3. Several conditions may occur in various unstable aeroplanes.

(*a*) The fin surface alone or the fin surface plus rudder surface may be inadequate.

(*b*) The rudder reaction may be too strong for the pilot to master it.

(*c*) The total fin surface aft (*i.e.*, fin plus rudder) may be excessive.

(*d*) The operation of the wing controls at extreme slow speeds may produce spinning tendencies.

(*e*) The fin surface may be unsymmetrical below and above the body of the aeroplane.

In (*a*) and (*b*) the pilot should hold the rudder firmly straight and dive for speed recovery, or the rudder may be slightly turned in opposition to the direction of the spin.

(*c*) This case is most unusual. The effect of too large a fin area aft would probably only be got by adding to the fin area the area of the rudder, which is supposed to be held rigid. The remedy is to let go the rudder for a moment and dive the machine whilst she re-aligns and becomes controllable again.

In (*d*) the only occasion when a spin can arise is when an underbanked turn is made, presumably because the machine is very near the ground. The mere fact that there is a spin tends to raise the outer wing, which is revolving at a greater speed.

Let us suppose now that the pilot for some reason or other is trying to avoid the bank. He accordingly pulls the inner wing flap down in order to increase the lift of this wing, but, much to his surprise, unless he is an experienced pilot, his action (contrary to the effect at normal flying speeds) diminishes the lift of this wing, thereby increasing the bank. This action certainly gives the aeroplane its proper bank, but at the same time it also causes a more rapid spin around the inner wing tip, and there will be a smash unless there is sufficient ground clearance.

The general instruction in (*d*), therefore, to the flier whose air speed is abnormally low is to turn the outer flap down, which will retard the spin. We learn by this that lateral control effects are reversed at stalling speeds.

To continue, the moment the aeroplane is straightened out by this inverted action the pilot should open the engine " full out " in the hopes that the machine will attain sufficient speed to avert a serious " pancake."

The moral, therefore, of the latter case is, never to attempt a turn with the speed of your machine near stalling point. Unfortunately, this particular form of error is one in which there are special excuses for falling into.

For example: If the aeroplane is over ground which cannot be alighted upon, and the engine suddenly "peters out," the flier will want to do two things: (1) Turn the machine back towards good landing-ground just quitted, and (2) maintain as much height as possible.

The writer himself has experienced this when flying a machine over the East Coast. In trying to avoid some telegraph-wires on the cliffs, the act of turning caused considerable loss of speed, and consequent loss of height, and the machine, which, by-the-by, was a stable one (a B.E.2c.), got into a spin, ending in a nose-dive into the compromising place he was trying to avoid—the sea.

Case (*e*) generally applies to small scouting aeroplanes having large engines in relation to their size. When flying at low speeds the swirl of the slip-stream affects the unsymmetrical fin surface, creating a bank at stalling speed.

4. Stable aeroplanes with speed too slow: First, considering outward side-slips under each of the different heads given in paragraph 3—

(*a*) Is non-existent.
(*b*) The rudder must be held firmly straight.
(*c*) Is non-existent.
(*d*) The remedy is the same as when the speed is normal.
(*e*) Ditto.

Inwards side-slips generally only occur with stunt flying, except in the case of flying in a cloud or in a fog, when the machine may unwittingly be held on a bank with the rudder in a straight position, giving the same results as an inward side-slip due to an over-banked turn. The remedy is to cut out the engine and hold the rudder over in the direction of the side-slip, then dive.

As soon as the machine is moving almost vertically downwards the rudder must be kicked straight and the elevator pulled gently backwards. (N.B.—This procedure has been confirmed experimentally in the air.)

It does not appear worth while to discuss cases (*a*) to (*f*) separately. The main point that the pilot must remember is never to fly his machine at an air speed which is practically stalling point, and this rule is very easily observed if the pilot will watch his indicator board closely.

Stalling and flying too slowly: Some pilots have a natural tendency to judge the speed of their machines by their ground speed, and seem to forget entirely that "ground speed" is an entirely different speed from "air speed."

A pilot should always be very careful when turning a machine from flying against the wind into flying with the wind. Having turned a machine down wind, she may appear to be flying quite fast, but in reality the air speed may be almost on stalling point.

Thus, for example, if the pilot is flying a 40 m.p.h. aeroplane (40 m.p.h. being the lowest speed at which she will not stall) against a 20-mile wind, an air speed of 40 m.p.h. is quite safe (20 m.p.h. being the ground speed). Now, when the pilot turns the machine down wind he is only just safe at a ground speed of 60 miles an hour, and in reality, although the machine in relation to the ground beneath him is travelling at an enormous speed, he is only just inside the limit of stalling speed.

AIR SPEED INDICATORS—THE PITOT TUBE*

By WINSLOW H. HERSCHEL

THERE are two dangers which an aviator must guard against by the use of a speed indicator. Too low a speed may lead to stalling—that is, the speed is so low that the weight of the aeroplane is not sustained—and, on the other hand, too high a speed may cause excessive stress and possible breakage. The speed indicator must be independent of the engine, because it is in gliding, with the engine still, that excessive speed is most likely to occur. On this account no form of engine tachometer can fulfil the purpose of the air speed meter.

Speed indicators, in general, consist of two parts—the head, which receives the impact of the relative wind, and the gauge, which must be placed within view of the pilot. It is important that the head should be placed sufficiently far from the body so that it will not be influenced by eddy currents, and a position near a wing-tip seems preferable. Even when so placed it must be remembered that the disturbance produced by a body passing through the air extends some distance in front of it. On account of the considerable distance between the head and he gauge, the connection between them should be easily changed in length without causing a change in the indications of the instrument, as otherwise, if the instrument were applied to another make of aeroplane, a new graduation of the gauge might become necessary.

While, strictly speaking, all types of speed indicators, with the exception of the rotary anemometer, have to be pointed directly into the wind in order to give correct readings, the angle between the relative wind and the direction of flight would be so small, in the case of a high-speed aeroplane, as not seriously to invalidate the readings of the indicator. It is therefore possible to fix the head permanently in the direction of flight, which avoids the undesirable complication of mounting it on a wind vane. It would be desirable, however, to mount the instrument so that it could be tilted when the angle of incidence was changed, as otherwise a greater error might be introduced than would be caused by the absence of a wind vane.

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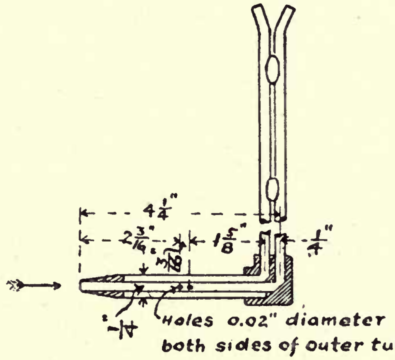
* Reproduced from *Aviation*, New York.

As an aeroplane does not always remain in an upright position, any instrument should be constructed so as to be independent of the effect of gravity, and, if so designed, it would also be independent of the effect of vertical acceleration.

In the great majority of cases the determination of true velocity is of secondary importance, so that it is practically no objection to a speed indicator if correction has to be made for air density when the true velocity is desired. On the other hand, it is an important advantage if no correction has to be made to the readings in order to find the relative sustaining power.

The principal types of speed indicator, which are to be considered with respect to their fitness for aeroplane conditions, are the rotary anemometer, pressure plate, Pitot tube, and Venturi tube. Speed indicators of the true anemometer type, such as the Robinson anemometer, are affected little, if at all, by changes in air density.



Example of a Pitot Tube Used by Rowse, Designed by the American Blower Company

A speed indicator adapted for aeroplanes made originally by Wilhelm Morell, of Leipzig, by combining a Robinson anemometer with a tachometer of the centrifugal pendulum type. Such an instrument should be very serviceable if used primarily to determine velocity rather than safety in flight.

The pressure plate is objectionable on account of the large head resistance, and the necessity of having a rigid connection between the head and the gauge. There is also considerable doubt as to the extent that the indications of such an instrument are affected by differences in air density. If pressure plates and rotary anemometers are omitted from further consideration, the Pitot tube and Venturi tube remain as the most important types. We shall now deal with the Pitot tube.

THE PITOT TUBE.

In the Pitot tube the head consists of a dynamic opening pointing directly into the current, and a static opening at right angles to it. If the two openings are connected to the two sides of a U-tube manometer containing a liquid of density d, while the density of the air in which the Pitot tube is placed is ρ, the velocity of the air may be obtained from the formula—

$$V=C\sqrt{\frac{2ghd}{\rho}} \quad . \quad . \quad . \quad . \quad (1)$$

where h is the head shown on the manometer, g is the acceleration of gravity, 32·17 feet-seconds squared (9·81 metres-seconds squared), and C is a coefficient to be determined by experiment. In the case of a Pitot tube used on an aeroplane, V would be the velocity of flight.

It has been shown by Rowse* that with proper care in the design and use of the Pitot tube C may, for all practical purposes, be taken equal to unity. To give correct readings, the static tube must be constructed with great care, though almost any form of dynamic opening will give good results. The dynamic tube must be pointed directly against the current, which is not possible when there is turbulence, and there must not be rapid variations in velocity, as the velocity calculated from the average head, indicated by the gauge, is not the true average velocity.

It has been claimed by some experimenters that a different value of C was obtained according to whether the tube was calibrated by moving it through still air or placing it in a current of moving air, but Fry and Tyndall† have shown that for velocities above 11 miles an hour (17·7 kilometres) both methods of standardization gave the same result. This shows that if the tube is of a form found by tests in moving air to have a coefficient of unity, then the coefficient C in Equation 1 may be taken equal to unity when calibrating a speed meter by moving it through still air, by means of an automobile or on the aeroplane itself.

If C is taken equal to unity, Equation 1 may be written—

$$V=K\sqrt{\frac{h}{\rho}}. \quad . \quad . \quad . \quad . \quad (2)$$

and values of K taken from Table I.

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* W. C. Rowse, *Transactions American Society of Mechanical Engineers.*

† J. D. Fry and A. M. Tyndall, *Philosophical Magazine.*

TABLE I.

Values of K for Equation 2.

h measured in:	ρ measured in:	V measured in:	K
Inches of water at 20° C. (68° F.)	$\frac{\text{Pounds}}{\text{Feet}^3}$	Feet per second	18·28
		Miles per hour	12·46
Minimum of water at 20° C. (68° F.)	$\frac{\text{Kilogrammes}}{\text{Metres}^3}$	Metres per second	4·426
		Kilometres per hour	15·93

To calculate the air density, ρ, the following quantities must be known or assumed:

B = the barometric pressure.
t = the temperature of the air.
P = pressure of saturated steam at t degrees, from the steam tables.
H = the relative humidity.

Then, in English units, if B and P are in inches of mercury, and t in degrees F.,

$$\rho = 1{\cdot}327 \frac{B - 0{\cdot}376PH}{460 + t} \text{pounds/feet}^3 \quad . \quad . \quad . \quad (3)$$

or, in metric units, if B and P are in millimetres of mercury, and t in degrees C.,

$$\rho = 1{\cdot}464 \frac{B - 0{\cdot}376PH}{273 + t} \text{kilogrammes/metres}^3 \quad . \quad (4)$$

The calculations required by Equation 3 may be avoided by the use of diagrams given by Rowse and by Taylor.* Hinz gives a diagram showing the gas constant of moist air which may be used in place of Equation 4.

The above equations and table give all the data necessary to calibrate a Pitot tube. If the gauge is correctly graduated and the apparatus is in good order, the head calculated from the observed velocity by means of Equation 2 should agree with the observed head, allowance being made for the difference between the air density during test and that assumed by the maker of the instrument.

In considering the error due to changes in air density, Darwin† makes the usual assumption that there is a fall of 1 degree F. for every 300 feet (0·61 degree C. per 100 metres) rise above the earth's surface. He gives Table 2, showing velocities indicated by a Pitot tube manometer, the actual speed being at all times 100 miles (161 kilometres) per hour.

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* D. W. Taylor, Society of Naval Architects and Marine Engineers.

† H. Darwin, *Aeronautical Journal.*

TABLE II.

PITOT TUBE READINGS FOR VARIOUS ALTITUDES, FOR $V = 100$ MILES PER HOUR.

Height.		*Speed Readings. Temperature Constant.*		*Speed Readings. Temperature Falling with Height.*	
Feet.	*Metres.*	*Miles.*	*Kilometres.*	*Miles.*	*Kilometres.*
0	0	100·0	160·9	100·0	160·9
1,000	305	98·3	158·0	98·6	158·7
2,000	610	96·5	155·3	97·1	156·1
3,000	914	94·7	152·3	95·7	154·0
4,000	1,219	93·0	149·8	94·3	151·8
5,000	1,524	91·3	147·0	92·9	149·7

While the errors shown by the table are considerable, it should be remembered that the safety against stalling is the same for a given manometer reading, whatever may be the density of the air and the consequent error in the readings as an indication of actual velocity.

Taking the weight of a cubic foot of air as 0·0715 pound (1·145 kilogrammes/metres^3), Equation 2 gives a head of 1·15 inches (29·2 millimetres for a speed of 50 miles, or 80 kilometres, per hour), which shows the desirability of magnifying the manometer reading. On account of the rocking of the aeroplane, this could not be done by using inclined columns of liquid such as are conveniently employed under other conditions. Some advantage may be derived from the use of a differential gauge, and it is claimed for a manometer of this type, especially designed for aeroplanes, that the error is only 1·5 per cent. when the manometer is 10 degrees out of the vertical.

It is thus apparent that the problem with the Pitot tube is to make a satisfactory gauge, without the use of liquids, which shall be sufficiently sensitive to give the required open scale readings at velocities approaching the stalling speed, and must also be durable enough not to suffer from vibrations and shocks experienced in landing.

METEOROLOGY

THE importance to aviation of the dissemination of trustworthy meteorological information was strongly emphasized by early experiences in the war, when it was realized that knowledge available for those moving by land or sea was of little value to those moving in the upper air. In a lecture on Commercial Aviation early in 1919, Major-General Sir Frederick Sykes pointed out that the weather was still the natural enemy of aviation, notwithstanding the progress made. At that time there were thirty-one Royal Air Force meteorological stations, and he urged the necessity for increasing the control of the meteorological service by flying men. In a paper on Meteorology, Colonel H. G. Lyons, R.E., Acting Director, Meteorological Office, illustrated the risks involved in ignorance of atmospheric conditions by the fate of the Zeppelins which raided the British Isles in February, 1915. A deep depression moving rapidly eastwards resulted in the loss of two of the raiders off the coast of Denmark.

The report of the Civil Aerial Transport Committee, issued in 1919 [Cd. 9218], contains among its appendices "A Report on Weather Service for Aerial Transport," signed by Lord Montagu of Beaulieu and Major G. I. Taylor, "A Memorandum on Research in Regard to Meteorology," by Colonel Lyons, and a commentary on that Memorandum by Sir Napier Shaw, who insists on the need of provision for studying the structure and properties of the atmosphere from the special point of view of aircraft. Lord Montagu and Major Taylor say that the chief meteorological requirements for the guidance of Aerial Transport may be divided into three groups: (1) Statistical information; (2) Forecasts; (3) Knowledge of the momentary meteorological conditions along aerial routes. Following are the chief passages from the Montagu-Taylor Report:

1. STATISTICAL INFORMATION.

This would be useful for such purposes as choosing routes and the sites for aerodromes and buildings. The statistical information at present obtainable differentiates between areas of 50 or 100 miles square; for instance, between the meteorological conditions which are met with on opposite sides of a mountain chain. But there is as yet little information relating to the local variations in meteorological conditions. Statistics already collected are available for determining such questions as whether it is better to leave Switzerland on the port or starboard hand in flying to Italy, or whether it would pay to go southwards by the Azores in flying to America. Statistical information available is not capable of discriminating between the average meteorological conditions of two alternative sites for aerodromes situated within a few miles, or even twenty or thirty miles of one another, except in so far as it shows which of the meteorological conditions are likely to be the same in two neighbouring sites. Statistics show, for instance, that the cloudiness and rainfall in two neighbouring sites may be the same unless special circumstances interfere. On the other hand, fogginess and windiness depend almost exclusively on such local conditions that the present statistics are of little use except for the actual station at which observations were made.

A small staff at the Meteorological Office would collect the available useful information, but such a staff would have to be directed by someone in touch with aeronautics, in order that the information might be collected and tabulated in the form most useful for flying. In dealing with wind statistics, for instance, it is more useful to know the number of hours during which certain winds blow with certain strengths, and from the various directions, than to know the mean wind velocity or direction. It is advisable that a system should be organized by which observations are taken and recorded regularly by aeroplanes travelling on defined routes. The work of collecting and comparing these statistics and the superintendence of instruments might be undertaken by the Meteorological Office

2. FORECASTING.

An extension of the present system in operation at the Meteorological Office should provide everything that can be expected in the present state of our meteorological knowledge.

1. *Extension of Range Covered.*—Weather telegrams are already obtained from a few distant places, as Iceland, Cairo, and the Azores, and from ships by wireless. The number of distant stations could be increased and the range extended so as to include Russia, the Balkans, North Africa, and America. Before the war, the observations sent by wireless from ships usually arrived too late to be used in the forecasts, but the growing importance of messages from the Atlantic in connection with aerial transport will make it worth while to organize these reports in such a way that the messages arrive in time. For this purpose, fuller co-operation with the Royal Navy and the Mercantile Marine should be sought. By increasing in this way the number of data on which forecasts are based, it will be possible to increase the size of the region for which the forecast is issued and to make it more accurate in all ways.

2. *Extension in the Number of Weather Conditions Predicted.*—Forecasts might be much more useful to flying men if they were made to include the heights at which clouds are to be expected, and the velocity and direction of the upper winds. Such forecasts would be facilitated by increasing the number of upper air observations and of those on mountains.

3. KNOWLEDGE OF THE MOMENTARY WEATHER.

For this purpose the present system is quite inadequate A continuous weather service would be necessary. All landing grounds would evidently be suitable weather observation stations, and continuous readings and reports could be arranged for, if necessary. In distributing information two systems could be used. Either the observations could be communicated at frequent intervals—say, every hour—to a Central Office, and could then be telegraphed or telephoned when required to aerodromes, or the observations could be taken only when required and telephoned direct to the termini.

The chief difficulty which is likely to be experienced in this branch of the weather service is that of communicating the observations sufficiently quickly to the termini. It seems probable that the direct method would be quicker than the Central Office method. Another point in favour of direct communication is that the observations taken at an intermediate landing-ground are only immediately interesting to the termini of the route on which it lies. These two points appear to us to outweigh the advantages of the Central Office method. If there is a direct telephone wire along the route or constant wireless communication, the whole problem is greatly simplified. If the ordinary telephone lines have to be used, some sort of priority will have to be arranged if they are to be of any real use.

The height of the clouds is one of the most important things for a pilot to know. It would not be difficult to fit a range-finder which would give the height of the clouds at a glance. The velocity and direction of the wind at various heights are also important, but it needs a skilled observer to make a pilot-balloon ascent, and even then the information is not available for about three-quarters of an hour after the balloon is sent off. On a clear day the upper wind at any particular height would be found in a few seconds by means of a smoke-shell fired vertically and timed so as to explode at the right height.

One of the chief functions of the observers at the landing-grounds would be to report the appearance and disappearance of fog on their own aerodrome. A corps of such observers should form an integral part of the military or civil air services, as, in fact, they already form a part of the Royal Naval Air Service.

GLUE*

BY DR. GEORGE F. LULL, U.S.A.

IT is not as easy as it would appear on the face of it to select a suitable glue joining two surfaces together.

There are many adhesives that are called "glue" that do stick articles together. We have a vegetable glue, made from starch, caustic soda, and water, that will hold wood together under certain conditions. Gum arabic and water glue papers together. Casein, borax, and water hold wood even under hard conditions, and sometimes come under the waterproof glue class.

A glue made from animal products, hide fleshings, sinews, bones, and parts, is the true animal glue. It has the special property that on cooling into its place in a joint it possesses a distinct film with the body, and thus makes a better holding medium than many of the other adhesives which, however thick they may appear when made up or applied, after drying out in the film or joint become extremely thin or fade away to almost nothing. This feature of the animal glue gives it its great superiority over other compositions. Animal glue is a colloid of about the formula $C_{76}H_{124}N_{24}O_{29}$. Much research work is being done to-day to discover more regarding it. The fact of its being an animal product is the great obstacle to the work.

PREPARATION OF GLUE FOR WORK.

A jacketed kettle is absolutely necessary for the proper dissolving and cooking of the dry glue. No glue should be mixed with water in a pan and melted—cooked—over a direct flame, nor should a jet of live steam be used directly on the glue in a dissolving kettle, because it will inevitably be burnt, and its full value of adhesiveness destroyed.

In general all animal glue should first be soaked up in cold water, first putting all, or the larger portion of, the full amount of water that is necessary for the solution of the glue into the kettle. Then the glue should be added slowly, and continuously stirred with awooden paddle so that it will be evenly liquefied

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* Reproduced from *The Aeroplane.*

and no lumps of dry glue balls appear. This stirring may be continued at intervals, if necessary, to keep it well mixed.

When properly soaked up, heat should be started, and the contents of the jacketed kettle gradually warmed up. This is one of the vital points of the preparation of the glue. With the stirring the mass of glue and water should be gradually raised to a temperature of 140 to 150 degrees F., and no higher, or the full strength of the glue will be lowered. It is not advisable to keep a batch of glue heated too long—say, two or three hours as the extreme limit. It is better to use up a smaller making and keep the glue fresher, and not allow it to skim over or make a hard crust on its surface. Also, it is better to make up only enough to use, and when done, empty out the kettle, clean it of all glue, and then there will be no trouble with bad-smelling glue.

WATERPROOF GLUE.

There are several chemicals that can be added to animal glue that will convert it on standing more or less into a waterproof glue. The resultant waterproof glue generally is a variable product, and must be carefully watched in order to get concordant results, as a light variation in the chemical or a change in the glue stock used may produce widely varying results. Formaldehyde formalin HCHO is largely used, and gives good results. This formaldehyde should be used very carefully—not over 5 per cent. of the liquid solution at the most, and as low as 1 per cent. with some glues has very good waterproofing effects. This formaldehyde must be added to the hot liquid solution of the glue just before using, making up only about enough for the work at hand, as left-over stock will be of no value, having set or hardened. The percentage should be computed as to the dry glue—*i.e.*, 100 pounds dry glue to start with to one-half per cent. of formaldehyde liquid.

The different bichromate salts, ammonium, aluminium, ammonia, chromate, chromate of lime, bichromate of potash, chrome alum, etc., all containing chromates, can be used to good advantage to convert animal glue to an insoluble waterproof glue. Greater care in handling these products must be taken, with but little exposure to light, and no overheating. The chromate must be dissolved in water and added to the glue mixture just before the removal from the kettle, or allowing only time for a thorough mixing at the point when ready to use.

One per cent. of bichromate of potash would be a large dose, about the limit—*i.e.*, 1 lb. chromate to 100 lb. glue. Smaller proportions can often be used, all depending upon the combination with the glue. This must be determined individually for each different sample of glue, care being exercised not to add such a quantity of the waterproofing element as to cause the glue to go "stringy" or "liver up." A small sample—say 100 grs. of glue 150 grs. water—can be dissolved up and when in good solution 1 gr. of bichromate of potash or chrome alum substance can be added. Results should be carefully watched.

Should the glue go stringy too much of the chemical has been added and the process will have to be repeated, using about one-half as much of the chemical to the regular amount of glue and water until a working formula has been established. I do not know of an absolutely waterproof glue made from animal glue that can be bought on the open market. Good results can be obtained by following out the above directions.

Some varnishes may be made sticky enough to hold wood together. Shellac in alcohol also may do it. Many formulas are found published for making waterproof glues, but they should be tried very carefully before being adopted as a true working proposition.

METRIC CONVERSION TABLES*

THE tables given on pp. 168 and 169 are designed to be used for both metres (m.) and millimetres (mm.) since an aeroplane is a big thing built up of a lot of little ones. The figures as they stand, with the point in the place shown, are for millimetres converted into inches. To convert metres into inches (which can readily be turned into feet, if necessary), the decimal point is to be taken as occurring three spaces later, as shown by the slight gap between the figures.

To use the table for millimetres, the column headed 0·0 contains the English equivalent in inches of the even number of millimetres shown down the left side. These run from 1 to 40, and all tenths of a millimetre from 0·0 up to this figure are shown in the columns, progressing by one-tenth at a time to the right. To convert a number of even mm. greater than 40, it is only necessary to adjust the decimal point as shown in the following examples.

For 6·0 mm. take the figure opposite 6 in col. headed 0·0 = 0·23622 in.

For 6·7 mm. take the figure opposite 6 in col. headed 0·7 = 0·26378 in.

For 67·0 mm. take this latter figure and, since 67·0 is 10 times greater than 6·7, move the point one to the right, which multiplies the result by 10 to agree = 2·6378 in.

For 278·0 mm. take figure opposite 27 in col. 0·8 and move point one to right = 10·9449 in.

For 2·78 mm. same figure, but point one place to the left (ten times less) = 0·10944 in.

For 275·0 mm. take 2·75 mm. and move the decimal point two places to right, since it is 100 times less than the figure required; or take 27·5 mm. and move the point one place to the right.

For the conversion of metric measures, other than those of length, which are covered by the table, the subjoined schedule of conversion factors will be found of use, and will cover any need likely to arise. It is not always necessary to work to the full

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* Reprinted from *The Aeroplane.*

number of decimal places given, but the accuracy desired must govern the factor used.

Multiply—	*By—*	*To Convert to—*
Miles	1·6093	Kilometres.
Kilometres	0·62138	Miles.
Square inches	6·4517	Square centimetres.
Square centimetres	0·155	Square inches.
Square feet	0·092903	Square metres.
Square metres	10·7639	Square feet.
Cubic inches	16·387	Cubic centimetres.
Cubic centimetres	0·061025	Cubic inches.
Cubic feet	0·028317	Cubic metres.
Cubic metres	35·314	Cubic feet.
Cubic feet	28·317	Litres.
Litres	0·035315	Cubic feet.
Lb.	0·45359	Kilogrammes.
Kilogrammes	2·2046	Lb.
Ounces (avoir.)	28·348	Grammes.
Grammes	0·035275	Ounces.
Lb. per square inch ..	0·070308	Kilogs. per square centimetre
Kilogs. per square centimetre	14·223	Lb. per square inch.
Lb. per square foot	4·8825	Kilogs. per square metre.
Kilogs. per square metre ..	0·20481	Lb. per square foot.

Fractions.				*Decimals.*			
Mm.	*Inches.*	*Mm.*	*Inches.*	*Mm.*	*Inches.*	*Mm.*	*Inches.*
$\frac{1}{16}$	0·002461	$\frac{9}{16}$	0·022145	0·005	0·000196	0·05	0·001968
$\frac{1}{8}$	0·004921	$\frac{5}{8}$	0·024606	0·01	0·000393	0·06	0·002362
$\frac{3}{16}$	0·007382	$\frac{11}{16}$	0·027067	0·02	0·000787	0·07	0·002755
$\frac{5}{16}$	0·012303	$\frac{13}{16}$	0·031988	0·03	0·001181	0·08	0·003149
$\frac{3}{8}$	0·014764	$\frac{7}{8}$	0·034448	0·04	0·001574	0·09	0·003543
$\frac{7}{16}$	0·017224	$\frac{15}{16}$	0·036909				

METRES AND MILLIMETRES INTO INCHES.

BASIS FACTOR: 1 metre equals 39·370113 inches. *Adopted by Order in Council*, May, 1898.

(Reproduced from *The Aeroplane.*)

M. or Mm.	0·0	0·1	0·2	0·25	0·3	0·4	0·5	0·6	0 7	0·75	0·8	0·9
0	—	0·003 94	0·007 87	0·009 84	0·011 81	0·015 75	0·019 69	0·023 62	0·027 56	0·029 53	0·031 50	0·035 43
1	0·039 37	0·043 31	0·047 24	0·049 21	0·051 18	0·055 12	0·059 06	0·062 99	0·066 93	0·468 90	0·070 87	0·074 80
2	0·078 74	0·082 68	0·086 61	0·088 58	0·090 55	0·094 49	0·098 43	0·102 36	0·106 30	0·108 27	0·110 24	0·114 17
3	0·118 11	0·122 05	0·125 98	0·127 95	0·129 92	0·133 86	0·137 80	0·141 73	0·145 67	0·147 64	0·149 61	0·153 54
4	0·157 48	0·161 42	0·165 35	0·167 32	0·169 29	0·173 23	0·177 17	0·181 10	0·185 04	0·187 01	0·188 98	0·192 91
5	0·196 85	0·200 79	0·204 72	0·206 69	0·208 66	0·212 60	0·216 54	0·220 47	0·224 41	0·226 38	0·228 35	0·232 28
6	0·236 22	0·240 16	0·244 09	0·246 06	0·248 03	0·251 97	0·255 91	0·259 84	0·263 78	0·265 75	0·267 72	0·271 65
7	0·275 59	0·279 53	0·283 46	0·285 43	0·287 40	0·291 34	0·295 28	0·299 21	0·303 15	0·305 12	0·307 09	0·311 02
8	0·314 96	0·318 90	0·322 83	0·324 80	0·326 77	0·330 71	0·334 65	0·338 58	0·342 52	0·344 49	0·346 46	0·350 39
9	0·354 33	0·358 27	0·362 21	0·364 17	0·366 14	0·370 08	0·374 02	0·377 95	0·381 89	0·383 86	0·385 83	0·389 76
10	0·393 70	0·397 64	0·401 58	0·403 54	0·405 51	0·409 45	0·413 39	0·417 32	0·421 26	0·423 23	0·425 20	0·429 13
11	0·433 07	0·437 01	0·440 95	0·442 91	0·444 88	0·448 82	0·452 76	0·456 69	0·460 63	0·462 60	0·464 57	0·468 50
12	0·472 44	0·476 38	0·480 32	0·482 28	0·484 25	0·488 19	0·492 13	0 496 06	0·500 00	0 501 96	0·503 94	0·507 87
13	0·511 81	0·515 75	0·519 69	0·521 65	0·523 62	0·527 56	0·531 50	0 535 43	0·539 37	0·541 34	0·543 31	0·547 24
14	0·551 18	0·555 12	0·559 06	0·561 02	0·562 99	0·566 93	0·570 87	0·574 80	0·578 74	0·580 71	0·582 68	0·586 61
15	0·590 55	0·594 49	0·598 43	0·600 39	0·602 36	0·606 30	0·610 24	0·614 17	0·618 11	0·620 08	0·622 05	0·625 98
16	0·629 92	0·633 86	0·637 80	0·639 76	0·641 73	0·645 67	0·649 61	0·653 54	0·657 48	0·659 45	0·661 42	0·665 35
17	0·669 29	0·673 23	0·677 17	0·679 13	0·681 10	0·685 04	0·688 98	0·692 91	0·696 85	0·698 82	0·700 79	0·704 73
18	0·708 66	0·712 60	0·716 54	0·718 50	0·720 47	0·724 41	0·728 35	0·732 28	0·736 22	0·738 19	0·740 16	0·744 10
19	0·748 03	0·751 97	0·755 91	0·757 87	0·759 84	0·763 78	0·767 72	0·771 65	0·775 59	0·777 56	0·779 53	0·783 47
20	0·787 40	0·791 34	0·795 28	0·797 24	0·799 21	0·803 15	0·807 09	0·811 02	0·814 96	0·816 93	0·818 90	0·822 84

METRES AND MILLIMETRES INTO INCHES—*Continued.*

M. or Mm.	0·0	0·1	0·2	0·25	0·3	0·4	0·5	0·6	0·7	0·75	0·8	0·9
21	0·826 77	0·830 71	0·834 65	0·836 61	0·838 58	0·842 52	0·846 46	0·850 39	0·854 33	0·856 30	0·858 27	0·362 21
22	0·866 14	0·870 08	0·874 02	0·875 98	0·877 95	0·881 89	0·885 83	0·889 76	0·893 70	0·895 67	0·897 64	0·901 58
23	0·905 51	0·909 45	0·913 39	0·915 35	0·917 32	0·921 26	0·925 20	0·929 13	0·933 07	0·935 04	0·937 01	0·940 95
24	0·944 88	0·948 82	0·952 76	0·954 72	0·956 69	0·960 63	0·964 57	0·968 50	0·972 44	0·974 41	0·976 38	0·980 32
25	0·984 25	0·988 19	0·992 13	0·994 09	0·996 06	1·000 00	1·003 94	1·007 87	1·011 81	1·013 78	1·015 75	1·019 69
26	1·023 62	1·027 56	1·031 50	1·033 46	1·035 43	1·039 37	1·043 31	1·047 25	1·051 18	1·053 15	1·055 12	1·059 06
27	1·062 99	1·066 93	1·070 87	1·072 83	1·074 80	1·078 74	1·082 68	1·086 62	1·090 55	1·092 52	1·094 49	1·098 43
28	1·102 36	1·106 30	1·110 24	1·112 20	1·114 17	1·118 11	1·122 05	1·125 99	1·129 92	1·131 89	1·133 86	1·137 80
29	1·141 73	1·145 67	1·149 61	1·151 57	1·153 54	1·157 48	1·161 42	1·165 36	1·169 29	1·171 26	1·173 23	1·177 17
30	1·181 10	1·185 04	1·188 98	1·190 94	1·192 91	1·196 85	1·200 79	1·204 73	1·208 66	1·210 63	1·212 60	1·216 54
31	1·220 47	1·224 41	1·228 35	1·230 31	1·232 28	1·236 22	1·240 16	1·244 10	1·248 03	1·250 00	1·251 97	1·255 91
32	1·259 84	1·263 78	1·267 72	1·269 68	1·271 65	1·275 59	1·279 53	1·283 47	1·287 40	1·289 37	1·291 34	1·295 28
33	1·299 21	1·303 15	1·307 09	1·309 05	1·311 02	1·314 96	1·318 90	1·322 84	1·326 77	1·328 74	1·330 71	1·334 65
34	1·338 58	1·342 52	1·346 46	1·348 42	1·350 39	1·354 33	1·358 27	1·362 21	1·366 14	1·368 11	1·370 08	1·374 02
35	1·377 95	1·381 89	1·385 83	1·387 79	1·389 76	1·393 70	1·397 64	1·401 58	1·405 51	1·407 48	1·409 45	1·413 39
36	1·417 32	1·421 26	1·425 20	1·427 16	1·429 14	1·433 07	1·437 01	1·440 95	1·444 88	1·446 85	1·448 82	1·452 76
37	1·456 69	1·460 63	1·464 57	1·466 53	1·468 51	1·472 44	1·476 38	1·480 32	1·484 25	1·486 22	1·488 19	1·492 13
38	1·496 06	1·500 00	1·503 94	1·505 90	1·507 88	1·511 81	1·515 75	1·519 69	1·523 62	1·525 59	1·527 56	1·531 50
39	1·535 43	1·539 37	1·543 31	1·545 27	1·547 25	1·551 18	1·555 12	1·559 06	1·562 99	1·564 96	1·566 93	1·570 87
40	1·574 80	0·1	0·2	0·25	0·3	0·4	0·5	0·6	0·7	0·75	0·8	0·9

MILLIMETRE EQUIVALENTS OF FRACTIONAL INCHES.

		1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″	12″
—	—	25·3995	50·7990	76·1986	101·598	126·998	152·397	177·797	203·196	228·596	253·995	279·394	304·794
$\frac{1}{32}''$	0·7937	26·1932	51·5928	76·9923	102·391	127·791	153·190	178·590	203·990	229·389	254·789	280·188	305·588
$\frac{1}{16}''$	1·5874	26·9870	52·3865	77·7860	103·185	128·585	153·984	179·384	204·783	230·183	255·582	260·982	306·381
$\frac{3}{32}''$	2·3812	27·7807	53·1802	78·5798	103·979	129·378	154·778	180·177	205·577	230·977	256·376	281·776	317·175
$\frac{1}{8}''$	3·1749	28·5744	53·9740	79·3735	104·773	130·172	155·572	180·971	206·370	231·770	257·170	282·569	307·969
$\frac{5}{32}''$	3·9686	29·3682	54·7677	80·1672	105·566	130·966	156·365	181·765	207·164	232·564	257·964	283·363	308·763
$\frac{3}{16}''$	4·7624	30·1619	55·5614	80·9610	106·360	131·760	157·159	182·559	207·958	233·358	258·757	284·157	309·556
$\frac{7}{32}''$	5·5561	30·9556	56·3552	81·7547	107·154	132·553	157·953	183·352	208·752	234·152	259·551	284·951	310·350
$\frac{1}{4}''$	6·3498	31·7494	57·1489	82·5485	107·948	133·347	158·747	184·146	209·546	234·945	260·345	285·744	311·144
$\frac{9}{32}''$	7·1436	32·5431	57·9426	83·3422	108·741	134·141	159·540	184·940	210·339	235·739	261·139	286·538	311·938
$\frac{5}{16}''$	7·9373	33·3368	58·7364	84·1359	109·535	134·935	160·334	185·734	211·133	236·532	261·932	287·332	312·731
$\frac{11}{32}''$	8·7310	34·1306	59·5301	84·9297	110·329	135·728	161·128	186·527	211·927	237·326	262·726	288·126	313·525
$\frac{3}{8}''$	9·5248	34·9243	60·3239	85·7234	111·122	136·522	161·922	187·321	212·721	238·120	263·520	288·919	314·319
$\frac{13}{32}''$	10·3185	35·7180	61·1176	86·5171	111·916	137·316	162·715	188·115	213·514	238·914	264·313	289·713	315·113
$\frac{7}{16}''$	11·1122	36·5118	61·9113	87·3109	112·710	138·109	163·509	188·909	214·308	239·708	265·107	290·507	315·906
$\frac{15}{32}''$	11·9060	37·3055	62·7051	88·1046	113·504	138·903	164·303	189·702	215·102	240·501	265·901	291·300	316·700
$\frac{1}{2}''$	12·6997	38·0993	63·4988	88·8983	114·297	139·697	165·097	190·496	215·896	241·295	266·695	292·094	317·494
$\frac{17}{32}''$	13·4934	38·8930	64·2925	89·6921	115·091	140·491	165·890	191·290	216·689	242·089	267·488	292·888	318·287
$\frac{9}{16}''$	14·2872	39·6867	65·0863	90·4858	115·885	141·284	166·684	192·084	217·483	242·883	268·282	293·682	319·081
$\frac{19}{32}''$	15·0809	40·4805	65·8800	91·2795	116·679	142·078	167·478	192·877	218·277	243·676	269·076	294·475	319·875
$\frac{5}{8}''$	15·8747	41·2742	66·6737	92·0733	117·472	142·872	168·271	193·671	219·071	244·470	269·870	295·269	320·669
$\frac{21}{32}''$	16·6684	42·0679	67·4675	92·8670	118·266	143·666	169·065	184·465	219·864	245·263	270·663	296·063	321·462
$\frac{11}{16}''$	17·4621	42·8617	68·2612	93·6608	119·060	144·459	169·859	195·258	220·658	246·058	271·457	296·857	322·256
$\frac{23}{32}''$	18·2559	43·6554	69·0549	94·4545	119·854	145·253	170·653	196·052	221·452	246·851	272·251	297·650	323·050
$\frac{3}{4}''$	19·0496	44·4491	69·8487	95·2482	120·647	146·047	171·446	196·846	222·245	247·645	273·045	298·444	323·844
$\frac{25}{32}''$	19·8433	45·2429	70·6424	96·0419	121·441	146·841	172·240	197·640	223·039	248·439	273·838	299·238	324·638
$\frac{13}{16}''$	20·6371	46·0366	71·4362	96·8357	122·235	147·634	173·034	198·433	223·883	249·232	274·632	300·032	325·431
$\frac{27}{32}''$	21·4308	46·8303	72·2299	97·6294	123·029	148·428	173·828	199·227	224·627	250·026	275·426	300·825	326·225
$\frac{7}{8}''$	22·2245	47·6241	73·0236	98·4232	123·822	149·222	174·621	200·021	225·420	250·820	276·220	301·619	327·019
$\frac{29}{32}''$	23·0183	48·4178	73·8173	99·2169	124·616	150·016	175·415	200·815	226·214	251·614	277·013	302·413	327·812
$\frac{15}{16}''$	23·8120	49·2116	74·6111	100·0107	125·410	150·809	176·209	201·608	227·008	252·407	277·807	303·207	328·606
$\frac{31}{32}''$	24·6057	50·0053	75·4048	100·8044	126·203	151·603	177·003	202·402	227·802	253·201	278·601	304·000	329·400

MILLIMETRES TO DECIMAL INCHES.

Table of Inch Equivalents.

1 to 100 MM.

MM.		10	20	30	40	50	60	70	80	90
1	0·03937	0·43307	0·82677	1·22047	1·61417	2·00787	2·40157	2·79527	3·18897	3·58267
2	0·07874	0·47244	0·86614	1·25984	1·65354	2·04724	2·44094	2·83464	3·22834	3·62204
3	0·11811	0·51181	0·90551	1·29921	1·69291	2·08661	2·48031	2·87401	3·26771	3·66141
4	0·15748	0·55118	0·94488	1·33858	1·73228	2·12598	2·51968	2·91338	3·30708	3·70078
5	0·19685	0·59055	0·98425	1·37795	1·77165	2·16535	2·55905	2·95275	3·34645	3·74015
6	0·23622	0·62992	1·02362	1·41732	1·81102	2·20472	2·59842	2·99212	3·38582	3·77952
7	0·27559	0·66929	1·06299	1·45669	1·85039	2·24409	2·63779	3·03149	3·42519	3·81889
8	0·31496	0·70866	1·10236	1·49606	1·88976	2·28346	2·67716	3·07086	3·46456	3·85826
9	0·35433	0·74803	1·14173	1·53543	1·92913	2·32283	2·71653	3·11023	3·50393	3·89763
10	0·39370	0·78740	1·18110	1·57480	1·96850	2·36220	2·75590	3·14960	3·54330	3·93700

101 to 200 MM.

MM.	100	110	120	130	140	150	160	170	180	190
1	3·97637	4·37007	4·76377	5·15747	5·55117	5·94487	6·33857	6·73227	7·12597	7·51967
2	4·01574	4·40944	4·80314	5·19684	5·59054	5·98424	6·37794	6·77164	7·16534	7·55904
3	4·05511	4·44881	4·84251	5·23621	5·62991	6·02361	6·41731	6·81101	7·20471	7·59841
4	4·09448	4·48818	4·88188	5·27558	5·66928	6·06298	6·45668	6·85038	7·24408	7·63778
5	4·13385	4·52755	4·92125	5·31495	5·70865	6·10235	6·49605	6·88975	7·28345	7·67715
6	4·17322	4·56692	4·96062	5·35432	5·74802	6·14172	6·53542	6·92912	7·32282	7·71652
7	4·21259	4·60629	4·99999	5·39369	5·78739	6·18109	6·57479	6·96849	7·36219	7·75589
8	4·25196	4·64566	5·03936	5·43306	5·82676	6·22046	6·61416	7·00786	7·40156	7·79526
9	4·29133	4·68503	5·07873	5·47243	5·86613	6·25983	6·65353	7·04723	7·44093	7·83463
10	4·33070	4·72440	5·11810	5·51180	5·90550	6·29920	6·69290	7·08660	7·48030	7·87400

TABLES AND DATA

CONVERSION OF DEGREES TO RADIANS.

(DIFFERENCE FOR 1′=0·00029.)

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
0	0·00000	0·00175	0·00349	0·00524	0·00698	0·00873	0·01047	0·01222	0·01396	0·01571
1	0·01745	0·01920	0·02094	0·02269	0·02443	0·02618	0·02793	0·02967	0·03142	0·03316
2	0·03491	0·03665	0·03840	0·04014	0·04189	0·04363	0·04538	0·04712	0·04887	0·05061
3	0·05236	0·05411	0·05585	0·05760	0·05934	0·06109	0·06283	0·06458	0·06632	0·06807
4	0·06981	0·07156	0·07330	0·07505	0·07679	0·07854	0·08029	0·08203	0·08378	0·08552
5	0·08727	0·08901	0·09076	0·09250	0·09425	0·09599	0·09774	0·09948	0·10123	0·10297
6	0·10472	0·10647	0·10821	0·10996	0·11170	0·11345	0·11519	0·11694	0·11868	0·12043
7	0·12217	0·12392	0·12566	0·12741	0·12915	0·13090	0·13264	0·13439	0·13614	0·13788
8	0·13963	0·14137	0·14312	0·14486	0·14661	0·14835	0·15010	0·15184	0·15359	0·15533
9	0·15708	0·15882	0·16057	0·16232	0·16406	0·16581	0·16755	0·16930	0·17104	0·17279
10	0·17453	0·17628	0·17802	0·17977	0·18151	0·18326	0·18500	0·18675	0·18850	0·19024
11	0·19199	0·19373	0·19548	0·19722	0·19897	0·20071	0·20246	0·20420	0·20595	0·20769
12	0·20944	0·21118	0·21293	0·21468	0·21642	0·21817	0·21991	0·22166	0·22340	0·22515
13	0·22689	0·22864	0·23038	0·23213	0·23387	0·23562	0·23736	0·23911	0·24086	0·24260
14	0·24435	0·24609	0·24784	0·24958	0·25133	0·25307	0·25482	0·25656	0·25831	0·26005
15	0·26180	0·26354	0·26529	0·26704	0·26878	0·27053	0·27227	0·27402	0·27576	0·27751
16	0·27925	0·28100	0·28274	0·28449	0·28623	0·28798	0·28972	0·29147	0·29322	0·29496
17	0·29671	0·29845	0·30020	0·30194	0·30369	0·30543	0·30718	0·30892	0·31067	0·31241
18	0·31416	0·31590	0·31765	0·31940	0·32114	0·32289	0·32463	0·32638	0·32812	0·32987
19	0·33161	0·33336	0·33510	0·33685	0·33859	0·34034	0·34208	0·34383	0·34558	0·34732
20	0·34907	0·35081	0·35256	0·35430	0·35605	0·35779	0·35954	0·36128	0·36303	0·36477
21	0·36652	0·36826	0·37001	0·37176	0·37350	0·37525	0·37699	0·37874	0·38048	0·38223

22	0·38397	0·38572	0·38746	0·38921	0·39095	0·39270	0·39444	0·39619	0·39794	0·39968
23	0·40143	0·40317	0·40492	0·40666	0·40841	0·41015	0·41190	0·41364	0·41539	0·41713
24	0·41888	0·42062	0·42237	0·42411	0·42586	0·42761	0·42935	0·43110	0·43284	0·43459
25	0·43633	0·43808	0·43982	0·44157	0·44331	0·44506	0·44680	0·44855	0·45029	0·45204
26	0·45379	0·45553	0·45728	0·45902	0·46077	0·46251	0·46426	0·46600	0·46775	0·46949
27	0·47124	0·47298	0·47473	0·47647	0·47822	0·47997	0·48171	0·48346	0·48520	0·48695
28	0·48869	0·49044	0·49218	0·49393	0·49567	0·49742	0·49916	0·50091	0·50265	0·50440
29	0·50615	0·50789	0·50964	0·51138	0·51313	0·51487	0·51662	0·51836	0·52011	0·52185
30	0·52360	0·52534	0·52709	0·52883	0·53058	0·53233	0·53407	0·53582	0·53756	0·53931
31	0·54105	0·54280	0·54454	0·54629	0·54803	0·54978	0·55152	0·55327	0·55501	0·55676
32	0·55851	0·56025	0·56200	0·56374	0·56549	0·56723	0·56898	0·57072	0·57247	0·57421
33	0·57596	0·57770	0·57945	0·58119	0·58294	0·58469	0·58643	0·58818	0·58992	0·59167
34	0·59341	0·59516	0·59690	0·59865	0·60039	0·60214	0·60388	0·60563	0·60737	0·60912
35	0·61087	0·61261	0·61436	0·61610	0·61785	0·61959	0·62134	0·62308	0·62483	0·62657
36	0·62832	0·63006	0·63181	0·63355	0·63530	0·63705	0·63879	0·64054	0·64228	0·64403
37	0·64577	0·64752	0·64926	0·65101	0·65275	0·65450	0·65624	0·65799	0·65973	0·66148
38	0·66323	0·66497	0·66672	0·66846	0·67021	0·67195	0·67370	0·67544	0·67719	0·67893
39	0·68068	0·68242	0·68417	0·68591	0·68766	0·68941	0·69115	0·69290	0·69464	0·69639
40	0·69813	0·69988	0·70162	0·70337	0·70511	0·70686	0·70860	0·71035	0·71209	0·71384
41	0·71558	0·71733	0·71908	0·72082	0·72257	0·72431	0·72606	0·72780	0·72955	0·73129
42	0·73304	0·73478	0·73653	0·73827	0·74002	0·74176	0·74351	0·74526	0·74700	0·74875
43	0·75049	0·75224	0·75398	0·75573	0·75747	0·75922	0·76096	0·76271	0·76445	0·76620
44	0·76794	0·76969	0·77144	0·77318	0·77493	0·77667	0·77842	0·78016	0·78191	0·78365
45	0·78540	0·78714	0·78889	0·79063	0·79238	0·79412	0·79587	0·79762	0·79936	0·80111

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CONVERSION OF DEGREES TO RADIANS—*Continued.*

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′
46	0·80285	0·80460	0·80634	0·80809	0·80983	0·81158	0·81332	0·81507	0·81681	0·81856
47	0·82030	0·82205	0·82380	0·82554	0·82729	0·82903	0·83078	0·83252	0·83427	0·83601
48	0·83776	0·83950	0·84125	0·84299	0·84474	0·84648	0·84823	0·84998	0·85172	0·85347
49	0·85521	0·85696	0·85870	0·86045	0·86219	0·86394	0·86568	0·86743	0·86917	0·87092
50	0·87266	0·87441	0·87616	0·87790	0·87965	0·88139	0·88314	0·88488	0·88663	0·88837
51	0·89012	0·89186	0·89361	0·89535	0·89710	0·89884	0·90059	0·90234	0·90408	0·90583
52	0·90757	0·90932	0·91106	0·91281	0·91455	0·91630	0·91804	0·91979	0·92153	0·92328
53	0·92502	0·92677	0·92852	0·93026	0·93201	0·93375	0·93550	0·93724	0·93899	0·94073
54	0·94248	0·94422	0·94597	0·94771	0·94946	0·95120	0·95295	0·95470	0·95644	0·95819
55	0·95993	0·96168	0·96342	0·96517	0·96691	0·96866	0·97040	0·97215	0·97389	0·97564
56	0·97738	0·97913	0·98088	0·98262	0·98437	0·98611	0·98786	0·98960	0·99135	0·99309
57	0·99484	0·99658	0·99833	1·00007	1·00182	1·00356	1·00531	1·00706	1·00880	1·01055
58	1·01229	1·01404	1·01578	1·01753	1·01927	1·02102	1·02276	1·02451	1·02625	1·02800
59	1·02974	1·03149	1·03323	1·03498	1·03673	1·03847	1·04022	1·04196	1·04371	1·04545
60	1·04720	1·04894	1·05069	1·05243	1·05418	1·05592	1·05767	1·05941	1·06116	1·06291
61	1·06465	1·06640	1·06814	1·06989	1·07163	1·07338	1·07512	1·07687	1·07861	1·08036
62	1·08210	1·08385	1·08559	1·08734	1·08909	1·09083	1·09258	1·09432	1·09607	1·09781
63	1·09956	1·10130	1·10305	1·10479	1·10654	1·10828	1·11003	1·11177	1·11352	1·11527
64	1·11701	1·11876	1·12050	1·12225	1·12399	1·12574	1·12748	1·12923	1·13097	1·13272
65	1·13446	1·13621	1·13795	1·13970	1·14145	1·14319	1·14494	1·14668	1·14843	1·15017
66	1·15192	1·15366	1·15541	1·15715	1·15890	1·16064	1·16239	1·16413	1·16588	1·16763

67	1·16937	1·17112	1·17286	1·17461	1·17635	1·17810	1·17984	1·18159	1·18333	1·18508
68	1·18682	1·18857	1·19031	1·19206	1·19381	1·19555	1·19730	1·19904	1·20079	1·20253
69	1·20428	1·20602	1·20777	1·20951	1·21126	1·21300	1·21475	1·21649	1·21824	1·21999
70	1·22173	1·22348	1·22522	1·22697	1·22871	1·23046	1·23220	1·23395	1·23569	1·23744
71	1·23918	1·24093	1·24267	1·24442	1·24617	1·24791	1·24966	1·25140	1·25315	1·25489
72	1·25664	1·25838	1·26013	1·26187	1·26362	1·26536	1·26711	1·26885	1·27060	1·27234
73	1·27409	1·27584	1·27758	1·27933	1·28107	1·28282	1·28456	1·28631	1·28805	1·28980
74	1·29154	1·29329	1·29503	1·29678	1·29852	1·30027	1·30202	1·30376	1·30551	1·30725
75	1·30900	1·31074	1·31249	1·31423	1·31598	1·31772	1·31947	1·32121	1·32296	1·32470
76	1·32645	1·32820	1·32994	1·33169	1·33343	1·33518	1·33692	1·33867	1·34041	1·34216
77	1·34390	1·34565	1·34739	1·34914	1·35088	1·35263	1·35438	1·35612	1·35787	1·35961
78	1·36136	1·36310	1·36485	1·36659	1·36834	1·37008	1·37183	1·37357	1·37532	1·37706
79	1·37881	1·38056	1·38230	1·38405	1·38579	1·38754	1·38928	1·39103	1·39277	1·39452
80	1·39626	1·39801	1·39975	1·40150	1·40324	1·40499	1·40674	1·40848	1·41023	1·41197
81	1·41372	1·41546	1·41721	1·41895	1·42070	1·42244	1·42419	1·42593	1·42768	1·42942
82	1·43117	1·43292	1·43466	1·43641	1·43815	1·43990	1·44164	1·44339	1·44513	1·44688
83	1·44862	1·45037	1·45211	1·45386	1·45560	1·45735	1·45910	1·46084	1·46259	1·46433
84	1·46608	1·46782	1·46957	1·47131	1·47306	1·47480	1·47655	1·47829	1·48004	1·48178
85	1·48353	1·48528	1·48702	1·48877	1·49051	1·49226	1·49400	1·49575	1·49749	1·49924
86	1·50098	1·50273	1·50447	1·50622	1·50796	1·50971	1·51146	1·51320	1·51495	1·51669
87	1·51844	1·52018	1·52193	1·52367	1·52542	1·52716	1·52891	1·53065	1·53240	1·53414
88	1·53589	1·53764	1·53938	1·54113	1·54287	1·54462	1·54636	1·54811	1·54985	1·55160
89	1·55334	1·55509	1·55683	1·55858	1·56032	1·56207	1·56381	1·56556	1·56731	1·56905

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FUNCTIONS OF ANGLES IN RADIANS.

Radians.	Degrees.	Sine.	Cosine.	Tangent.	Radians.	Degrees.	Sine.	Cosine.	Tangent.
0·01	0·573	0·00989	0·99995	0·00989	**0·70**	40·11	0·64435	0·76473	0·84258
0·02	1·146	0·02000	0·99980	0·02000	**0·72**	41·25	0·65935	0·75184	0·87698
0·03	1·719	0·02996	0·99955	0·02997	**0·74**	42·40	0·67430	0·73846	0·91313
0·04	2·292	0·03984	0·99921	0·03987	**0·76**	43·55	0·68899	0·72477	0·95062
0·05	2·865	0·05001	0·99875	0·05007	**0·78**	44·69	0·70339	0·71080	0·98958
0·06	3·438	0·05989	0·99821	0·05997	**0·80**	45·84	0·71732	0·69675	1·0295
0·07	4·011	0·07005	0·99754	0·07022	**0·82**	46·98	0·73116	0·68221	1·0717
0·08	4·584	0·07991	0·99680	0·08017	**0·84**	48·13	0·74470	0·66740	1·1158
0·09	5·157	0·09005	0·99594	0·09042	**0·86**	49·28	0·75794	0·65232	1·1619
0·10	5·730	0·09990	0·99500	0·10040	**0·88**	50·42	0·77070	0·63720	1·2095
0·11	6·302	0·10973	0·99396	0·11040	**0·90**	51·57	0·78333	0·62160	1·2602
0·12	6·876	0·11985	0·99279	0·12072	**0·92**	52·72	0·79565	0·60576	1·3135
0·13	7·447	0·12966	0·99156	0·13076	**0·94**	53·86	0·80765	0·58967	1·3697
0·14	8·022	0·13946	0·99023	0·14084	**0·96**	55·00	0·81915	0·57358	1·4281
0·15	8·594	0·14954	0·98876	0·15124	**0·98**	56·15	0·83050	0·55702	1·4910
0·16	9·167	0·15931	0·98723	0·16137	**1·00**	57·30	0·84151	0·54024	1·5577
0·17	9·733	0·16906	0·98561	0·17153	**1·02**	58·45	0·85218	0·52324	1·6287
0·18	10·31	0·17909	0·98383	0·18203	**1·04**	59·59	0·86251	0·50603	1·7045
0·19	10·88	0·18881	0·98201	0·19227	**0·06**	60·73	0·87235	0·48888	1·7844
0·20	11·46	0·19880	0·98004	0·20285	**1·08**	61·88	0·88199	0·47127	1·8715
0·22	12·61	0·21814	0·97592	0·22353	**1·10**	63·03	0·89127	0·45347	1·9654
0·24	13·75	0·23769	0·97134	0·24470	**1·12**	64·18	0·90019	0·43549	2·0671
0·26	14·90	0·25713	0·96638	0·26608	**1·14**	65·32	0·90863	0·41760	2·1758
0·28	16·04	0·27648	0·96102	0·28769	**1·16**	66·47	0·91683	0·39928	2·2962
0·30	17·19	0·29543	0·95536	0·30923	**1·13**	67·62	0·92466	0·38080	2·4282
0·32	18·34	0·31454	0·94924	0·33136	**1·20**	68·76	0·93211	0·36217	2·5737
0·34	19·48	0·33353	0·94274	0·35379	**1·22**	69·91	0·93909	0·34366	2·7326
0·36	20·63	0·35239	0·93585	0·37654	**1·24**	71·05	0·94580	0·32474	2·9125
0·38	21·77	0·37083	0·92870	0·39930	**1·26**	72·20	0·95213	0·30570	3·1146
0·40	22·92	0·38939	0·92107	0·42276	**1·28**	73·34	0·95807	0·28652	3·3438
0·42	24·07	0·40780	0·91307	0·44662	**1·30**	74·49	0·96355	0·26752	3·6018
0·44	25·21	0·42604	0·90470	0·47092	**1·32**	75·64	0·96873	0·24813	3·9042
0·46	26·36	0·44385	0·89610	0·49532	**1·34**	76·78	0·97351	0·22863	4·2580
0·48	27·50	0·46175	0·88701	0·52057	**1·36**	77·93	0·97790	0·20905	4·6779
0·50	28·65	0·47946	0·87756	0·54635	**1·38**	79·07	0·98185	0·18967	5·1767
0·52	29·80	0·49697	0·86777	0·57271	**1·40**	80·22	0·98546	0·16992	5·7994
0·54	30·94	0·51429	0·85762	0·59967	**1·42**	81·37	0·98867	0·15011	6·5863
0·56	32·09	0·53115	0·84728	0·62689	**1·44**	82·51	0·99148	0·13024	7·6129
0·58	33·23	0·54805	0·83645	0·65521	**1·46**	83·66	0·99386	0·11060	8·9860
0·60	34·38	0·56473	0·82528	0·68429	**1·48**	84·80	0·99588	0·09063	10·9881
0·62	35·53	0·58118	0·81378	0·71417	**1·50**	85·95	0·99750	0·07063	14·1235
0·64	36·67	0·59716	0·80212	0·74447	**1·52**	87·10	0·99872	0·05059	19·7403
0·66	37·82	0·61314	0·78998	0·77615	**1·54**	88·24	0·99952	0·03083	32·4213
0·68	38·96	0·62887	0·77751	0·80882	**1·56**	89·39	0·99994	0·01076	92·9085
					$\pi/2$	90			

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TABLE OF e^x AND e^{-x}.

x	e^x	e^{-x}	x	e^x	e^{-x}	x	e^x	e^{-x}
0·02	1·0202	0·9802	0·39	1·4770	0·6771	2·7	14·800	0·06721
0·04	1·0408	0·9608	0·40	1·4918	0·6703	2·8	16·445	0·06081
0·06	1·0618	0·9418	0·41	1·5068	0·6637	2·9	18·174	0·05502
0·08	1·0833	0·9231	0·42	1·5220	0·6571	3·0	20·086	0·04979
0·10	1·1052	0·9048	0·43	1·5373	0·6505	3·1	22·199	0·04505
0·11	1·1163	0·8958	0·44	1·5527	0·6440	3·2	24·533	0·04076
0·12	1·1275	0·8869	0·45	1·5683	0·6376	3·3	27·113	0·03688
0·13	1·1388	0·8781	0·46	1·5841	0·6313	3·4	29·964	0·03337
0·14	1·1503	0·8694	0·47	1·6000	0·6250	3·5	33·116	0·03020
0·15	1·1618	0·8607	0·48	1·6161	0·6188	3·6	36·598	0·02732
0·16	1·1735	0·8521	0·49	1·6323	0·6126	3·7	40·447	0·02472
0·17	1·1853	0·8437	0·5	1·6487	0·6065	3·8	44·701	0·02237
0·18	1·1972	0·8353	0·6	1·8221	0·5488	3·9	49·402	0·02024
0·19	1·2093	0·8270	0·7	2·0138	0·4966	4·0	54·598	0·01832
0·20	1·2214	0·8187	0·8	2·2255	0·4493	4·1	60·340	0·01657
0·21	1·2337	0·8106	0·9	2·4596	0·4066	4·2	66·686	0·01500
0·22	1·2461	0·8025	1·0	2·7183	0·3679	4·3	73·700	0·01357
0·23	1·2586	0·7945	1·1	3·0042	0·3329	4·4	81·451	0·01228
0·24	1·2713	0·7866	1·2	3·3201	0·3012	4·5	90·017	0·01111
0·25	1·2840	0·7788	1·3	3·6693	0·2725	4·6	99·484	0·01005
0·26	1·2969	0·7711	1·4	4·0552	0·2466	4·7	109·95	0·00910
0·27	1·3100	0·7634	1·5	4·4817	0·2231	4·8	121·51	0·00823
0·28	1·3231	0·7558	1·6	4·9530	0·2019	4·9	134·29	0·00745
0·29	1·3364	0·7483	1·7	5·4740	0·1827	5·0	148·41	0·00674
0·30	1·3499	0·7408	1·8	6·0497	0·1653	5·1	164·02	0·00610
0·31	1·3634	0·7335	1·9	6·6859	0·1496	5·2	181·27	0·00552
0·32	1·3771	0·7262	2·0	7·3891	0·1353	5·3	200·34	0·00499
0·33	1·3910	0·7190	2·1	8·1662	0·1225	5·4	221·41	0·00452
0·34	1·4050	0·7118	2·2	9·0250	0·1108	5·5	244·69	0·00409
0·35	1·4191	0·7047	2·3	9·9742	0·1003	5·6	270·43	0·00370
0·36	1·4333	0·6977	2·4	11·0232	0·09072	5·7	298·87	0·00335
0·37	1·4477	0·6907	2·5	12·183	0·08208	5·8	330·30	0·00303
0·38	1·4623	0·6839	2·6	13·464	0·07427	5·9	365·04	0·00274
						6·0	403·43	0·00248

NOTE.—Cosh $x=\frac{1}{2}(e^x+e^{-x})$.
sinh $x=\frac{1}{2}(e^x-e^{-x})$.
tanh $x=(e^x-e^{-x}/(e^x+e^{-x})$.

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SQUARES.

From 1 to 5·499.

	0	1	2	3	4	5	6	7	8	9	*Mean Differences.*								
											1	2	3	4	5	6	7	8	9
1·0	1·0000	1·0201	1·0404	1·0609	1·0816	1·1025	1·1236	1·1449	1·1664	1·1881	21	42	63	84	105	126	147	168	189
1·1	1·2100	1·2321	1·2544	1·2769	1·2996	1·3225	1·3456	1·3689	1·3924	1·4161	23	46	69	92	115	138	161	184	207
1·2	1·4400	1·4641	1·4884	1·5129	1·5376	1·5625	1·5876	1·6129	1·6384	1·6641	25	50	75	100	125	150	175	200	225
1·3	1·6900	1·7161	1·7424	1·7689	1·7956	1·8225	1·8496	1·8769	1·9044	1·9321	27	54	81	108	135	162	189	216	243
1·4	1·9600	1·9881	2·0164	2·0449	2·0736	2·1025	2·1316	2·1609	2·1904	2·2201	29	58	87	116	145	174	203	232	261
1·5	2·2500	2·2801	2·3104	2·3409	2·3716	2·4025	2·4336	2·4649	2·4964	2·5281	31	62	93	124	155	186	217	248	279
1·6	2·5600	2·5921	2·6244	2·6569	2·6896	2·7225	2·7556	2·7889	2·8224	2·8561	33	66	99	132	165	198	231	264	297
1·7	2·8900	2·9241	2·9584	2·9929	2·0276	2·0625	2·0976	3·1329	3·1684	3·2041	35	70	105	140	175	210	245	280	315
1·8	3·2400	3·2761	3·3124	3·3489	3·3856	3·4225	3·4596	3·4969	3·5344	3·5721	37	74	111	148	185	222	259	296	333
1·9	3·6100	3·6481	3·6864	3·7249	3·7636	3·8025	3·8416	3·8809	3·9204	3·9601	39	78	117	156	195	234	273	312	351
2·0	4·0000	4·0401	4·0804	4·1209	4·1616	4·2025	4·2436	4·2849	4·3264	4·3681	41	82	123	164	205	246	287	328	369
2·1	4·4100	4·4521	4·4944	4·5369	4·5796	4·6225	4·6656	4·7089	4·7524	4·7961	43	86	129	172	215	258	301	344	387
2·2	4·8400	4·8841	4·9284	4·9729	5·0176	5·0625	5·1076	5·1529	5·1984	5·2441	45	90	135	180	225	270	315	360	405
2·3	5·2900	5·3361	5·3824	5·4289	5·4756	5·5225	5·5696	5·6169	5·6644	5·7121	47	94	141	188	235	282	329	376	423
2·4	5·7600	5·8081	5·8564	5·9049	5·9536	6·0025	6·0516	6·1009	6·1504	6·2001	49	98	147	196	245	294	343	392	441
2·5	6·2500	6·3001	6·3504	6·4009	6·4516	6·5025	6·5536	6·6049	7·6564	6·7081	51	102	153	204	255	306	357	408	459
2·6	6·7600	6·8121	6·8644	6·9169	6·9696	7·0225	7·0756	7·1289	7·1824	7·2361	53	106	159	212	265	318	371	424	477
2·7	7·2900	7·3441	7·3984	7·4529	7·5076	7·5625	7·6176	7·6729	7·7284	7·7841	55	110	165	220	275	330	385	440	495
2·8	7·8400	7·8961	7·9524	8·0089	8·0656	8·1225	8·1796	8·2369	8·2944	8·3521	57	114	171	228	285	342	399	456	513
2·9	8·4100	8·4681	8·5264	8·5849	8·6436	8·7025	8·7616	8·8209	8·8804	8·9401	59	118	177	236	295	354	413	472	531
3·0	9·0000	9·0601	9·1204	9·1809	9·2416	9·3025	9·3636	9·4249	9·4864	9·5481	61	122	183	244	305	366	427	488	549

3·1	9·6100	9·6721	9·7344	9·7969	9·8596	9·9225	9·9856	10·049	10·112	10·176	64	126	189	252	315	378	441	504	567
3·2	10·240	10·304	10·368	10·433	10·498	10·563	10·628	10·693	10·758	10·824	7	13	20	26	33	39	46	52	59
3·3	10·890	10·956	11·022	11·089	11·156	11·223	11·290	11·357	11·424	11·492	7	13	20	27	34	40	47	54	60
3·4	11·560	11·628	11·696	11·765	11·834	11·903	11·972	12·041	12·110	12·180	7	14	21	28	35	41	48	55	62
3·5	12·250	12·320	12·390	12·461	12·532	12·603	12·674	12·745	12·816	12·888	7	15	21	28	36	43	50	57	64
3·6	12·960	13·032	13·104	13·177	13·350	13·323	13·396	13·469	13·542	13·616	7	15	22	29	37	44	51	58	66
3·7	13·690	13·764	13·838	13·913	13·988	14·063	14·138	14·213	14·288	14·364	8	15	23	30	38	45	53	60	68
3·8	14·440	14·516	14·592	14·669	14·746	14·823	14·900	14·977	15·054	15·132	8	15	23	31	39	46	54	62	69
3·9	15·210	15·288	15·366	15·445	15·524	15·603	15·682	15·761	15·840	15·920	8	16	24	32	40	47	55	63	71
4·0	16·000	16·080	16·160	16·241	16·322	16·403	16·484	16·565	16·646	16·728	8	16	24	32	41	49	57	65	73
4·1	16·810	16·892	16·974	17·057	17·140	17·223	17·306	17·389	17·472	17·556	8	17	25	33	42	50	58	66	75
4·2	17·640	17·724	17·808	17·893	17·978	18·063	18·148	18·233	18·318	18·404	9	17	26	34	34	51	60	68	77
4·3	18·490	18·576	18·662	18·749	18·836	18·923	19·010	19·097	19·184	19·272	9	17	26	35	44	52	61	70	78
4·4	19·360	19·448	19·536	19·625	19·714	19·803	19·892	19·981	20·070	20·160	9	18	27	36	45	53	62	71	80
4·5	20·250	20·340	20·430	20·521	20·612	20·703	20·794	20·885	20·976	21·068	9	18	27	36	46	55	64	73	82
4·6	21·160	21·252	21·344	21·437	21·530	21·623	21·716	21·809	21·902	21·996	9	19	28	37	47	56	65	74	84
4·7	22·090	22·184	22·278	22·373	22·468	22·563	22·658	22·753	22·848	22·944	10	19	29	38	48	57	67	76	86
4·8	23·040	23·136	23·232	23·329	23·426	23·523	23·620	23·717	23·814	23·912	10	19	29	39	49	58	68	78	87
4·9	24·010	24·108	24·206	24·305	24·404	24·503	24·602	24·701	24·800	24·900	10	20	30	40	50	59	69	79	89
5·0	25·000	25·100	25·200	25·301	25·402	25·503	25·604	25·705	25·806	25·908	10	20	30	40	51	61	71	81	91
5·1	26·010	26·112	26·214	26·317	26·420	26·523	26·626	26·729	26·832	26·936	10	21	31	41	52	62	72	82	93
5·2	27·040	27·144	27·248	27·353	27·458	27·563	27·668	27·773	27·878	27·984	11	21	32	42	53	63	74	84	95
5·3	23·090	28·196	28·302	28·409	28·516	28·623	28·730	28·837	28·944	29·052	11	21	32	43	54	64	75	86	96
5·4	29·160	29·268	29·376	29·485	29·594	29·703	29·812	29·921	30·030	30·140	11	22	23	44	55	65	76	87	98

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

SQUARES.

From 5·500 to 10.

	0	1	2	3	4	5	6	7	8	9	*Mean Differences.*								
											1	2	3	4	5	6	7	8	9
5·5	30·250	30·360	30·470	30·581	30·692	30·803	30·914	31·025	31·136	31·248	11	22	33	44	56	67	78	89	100
5·6	31·360	31·472	31·584	31·697	31·810	31·923	32·036	32·149	32·262	32·376	11	23	34	45	57	68	79	90	102
5·7	32·490	32·604	32·718	32·833	32·948	33·063	33·178	33·293	33·408	33·524	12	23	35	46	58	69	81	92	104
5·8	33·640	33·756	33·872	33·989	34·106	34·223	34·340	34·457	34·574	34·692	12	23	35	47	59	70	82	94	105
5·9	34·810	34·928	35·046	35·165	35·284	35·403	35·522	35·641	35·760	35·880	12	24	36	48	60	71	83	95	107
6·0	36·000	36·120	36·240	36·361	36·482	36·603	36·724	36·845	36·966	37·088	12	24	36	48	61	73	85	97	109
6·1	37·210	37·332	37·454	37·577	37·700	37·823	37·946	38·069	38·192	38·316	12	25	37	49	62	74	86	98	111
6·2	38·440	38·564	38·688	38·813	38·938	39·063	39·188	39·313	39·438	39·564	13	25	38	50	63	75	88	100	113
6·3	39·690	39·816	39·942	40·069	40·196	40·323	40·450	40·577	40·704	40·832	13	25	38	51	64	76	89	102	114
6·4	40·960	41·088	41·216	41·345	41·474	41·603	41·732	41·861	41·990	42·120	13	26	39	52	65	77	90	103	116
6·5	42·250	42·380	42·510	42·641	42·772	42·903	43·034	43·165	43·296	43·428	13	26	39	52	66	79	92	105	118
6·6	43·560	43·692	43·824	43·957	44·090	44·223	44·356	44·489	44·622	44·756	13	27	40	53	67	80	93	106	120
6·7	44·890	45·024	45·158	45·293	45·428	45·563	45·698	45·833	45·968	46·104	14	27	41	54	68	81	95	108	122
6·8	46·240	46·376	46·512	46·649	46·786	46·923	47·060	47·197	47·334	47·472	14	27	41	55	69	82	96	110	123
6·9	47·610	47·748	47·886	48·025	48·164	48·303	48·442	48·581	48·720	48·860	14	28	42	56	70	83	97	111	125
7·0	49·000	49·140	49·280	49·421	49·562	49·703	49·844	49·985	50·126	50·268	14	28	42	56	71	85	99	113	127
7·1	50·410	50·552	50·694	50·837	50·980	51·123	51·266	51·409	51·552	51·696	14	29	43	57	72	86	110	114	129
7·2	51·840	51·984	52·128	52·273	52·418	52·563	52·708	52·853	52 998	53·144	15	29	44	58	73	87	102	116	131
7·3	53·290	53·436	53·582	53·729	53·876	54·023	54·170	54·317	54·464	54·612	15	29	44	59	74	88	103	118	132
7·4	54·760	54·908	55·056	55·205	55·354	55·503	55·652	55·801	55·950	56·100	15	30	45	60	75	89	104	119	134
7·5	56·250	56·400	56·550	56·701	56·852	57·003	57·154	57·305	57·456	57·608	15	30	45	60	76	91	106	121	136

7·6	57·760	57·912	58·064	58·217	58·370	58·523	58·676	58·829	58·982	59·136	15	31	46	61	77	92	107	122	138
7·7	59·290	59·444	59·598	59·753	59·908	60·063	60·218	60·373	60·528	60·684	16	31	47	62	78	93	109	124	140
7·8	60·840	60·996	61·152	61·309	61·466	61·623	61·780	61·937	62·094	62·252	16	31	47	63	79	94	110	126	141
7·9	62·410	62·568	62·726	62·885	63·044	63·203	63·362	63·521	63·680	63·840	16	32	48	64	80	95	111	127	143
8·0	64·000	64·160	64·320	64·481	64·642	64·803	64·964	65·125	65·286	65·448	16	32	48	64	81	97	113	129	145
8·1	65·610	65·772	65·934	66·097	66·260	66·423	66·586	66·749	66·912	67·076	16	33	49	65	82	98	114	130	147
8·2	67·240	67·404	67·568	67·733	67·898	68·063	68·228	68·393	68·558	68·724	17	33	50	66	83	99	116	132	149
8·3	68·890	69·056	69·222	69·389	69·556	69·723	69·890	70·057	70·224	70·392	17	33	50	67	84	100	117	134	150
8·4	70·560	70·728	70·896	71·065	71·234	71·403	71·572	71·741	71·910	72·080	17	34	51	68	85	101	118	135	152
8·5	72·250	72·420	72·590	72·761	72·932	73·103	73·274	73·445	73·616	73·788	17	34	51	68	86	103	120	137	154
8·6	73·960	74·132	74·304	74·477	74·650	74·823	74·996	75·169	75·342	75·516	17	35	52	69	87	104	121	138	156
8·7	75·690	75·864	76·038	76·213	76·388	76·563	76·738	76·913	77·088	77·264	18	35	53	70	88	105	123	140	158
8·8	77·440	77·616	77·792	77·969	78·146	78·323	78·500	78·677	78·854	79·032	18	35	53	71	89	106	124	142	159
8·9	79·210	79·388	79·566	79·745	79·924	80·103	80·282	80·461	80·640	80·820	18	36	54	72	90	107	125	143	161
9·0	81·000	81·180	81·360	81·541	81·722	81·903	82·084	82·265	82·446	82·628	18	36	54	72	91	109	127	145	163
9·1	82·810	82·992	83·174	83·357	83·540	83·723	83·906	84·089	84·272	84·456	18	37	55	73	92	110	128	146	165
9·2	84·640	84·824	85·008	85·193	85·378	85·563	85·748	85·933	86·118	86·304	19	37	56	74	93	111	130	148	167
9·3	86·490	86·676	86·862	87·049	87·236	87·423	87·610	87·797	87·984	88·172	19	37	56	75	94	112	131	150	168
9·4	88·360	88·548	88·738	88·925	89·114	89·303	89·492	89·681	89·870	90·060	19	38	57	76	95	113	132	151	170
9·5	90·250	90·440	90·630	90·821	91·012	91·203	91·394	91·585	91·776	91·968	19	38	57	76	96	115	134	153	172
9·6	92·160	92·352	92·544	92·737	92·930	93·123	93·316	93·509	93·702	93·896	19	39	58	77	97	116	135	154	174
9·7	94·090	94·284	94·478	94·673	94·868	95·063	95·258	95·453	95·648	95·844	20	39	59	78	98	117	137	156	176
9·8	96·040	96·236	96·432	96·629	96·826	97·023	97·220	97·417	97·614	97·812	20	39	59	79	99	118	138	158	177
9·9	98·010	98·208	98·406	98·605	98·804	99·003	99·202	99·401	99·600	99·800	20	40	60	80	100	119	139	159	179

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SQUARE ROOTS.

From 1 to 5·499.

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
1·0	1·0000	1·0050	1·0100	1·0149	1·0198	1·0247	1·0296	1·0344	1·0392	1·0440	5	10	15	20	24	29	34	39	44
1·1	1·0488	1·0536	1·0583	1·0630	1·0677	1·0724	1·0770	1·0817	1·0863	1·0909	5	9	14	19	23	28	33	37	42
1·2	1·0954	1·1000	1·1045	1·1091	1·1136	1·1180	1·1225	1·1269	1·1314	1·1358	4	9	13	18	22	27	31	36	40
1·3	1·1402	1·1446	1·1489	1·1533	1·1576	1·1619	1·1662	1·1705	1·1747	1·1790	4	9	13	17	22	26	30	34	39
1·4	1·1832	1·1874	1·1916	1·1958	1·2000	1·2042	1·2083	1·2124	1·2166	1·2207	4	8	13	17	21	25	29	33	37
1·5	1·2247	1·2288	1·2329	1·2369	1·2410	1·2450	1·2490	1·2530	1·2570	1·2610	4	8	12	16	20	24	28	32	36
1·6	1·2649	1·2689	1·2728	1·2767	1·2806	1·2845	1·2884	1·2923	1·2961	1·3000	4	8	12	16	19	23	27	31	35
1·7	1·3038	1·3077	1·3115	1·3153	1·3191	1·3229	1·3266	1·3304	1·3342	1·3379	4	8	11	15	19	23	27	30	34
1·8	1·3416	1·3454	1·3491	1·3528	1·3565	1·3601	1·3638	1·3675	1·3711	1·3748	4	7	11	15	18	22	26	29	33
1·9	1·3784	1·3820	1·3856	1·3892	1·3928	1·3964	1·4000	1·4036	1·4071	1·4107	4	7	11	14	18	22	25	29	32
2·0	1·4142	1·4177	1·4213	1·4248	1·4283	1·4318	1·4353	1·4387	1·4422	1·4457	4	7	11	14	18	21	24	28	31
2·1	1·4491	1·4526	1·4560	1·4595	1·4629	1·4663	1·4697	1·4731	1·4765	1·4799	3	7	10	14	17	20	24	27	31
2·2	1·4832	1·4866	1·4900	1·4933	1·4966	1·5000	1·5033	1·5067	1·5100	1·5133	3	7	10	13	17	20	24	27	30
2·3	1·5166	1·5199	1·5232	1·5264	1·5297	1·5330	1·5362	1·5395	1·5427	1·5460	3	7	10	13	16	20	23	26	29
2·4	1·5492	1·5524	1·5556	1·5588	1·5620	1·5652	1·5684	1·5716	1·5748	1·5780	3	6	10	13	16	19	22	26	29
2·5	1·5811	1·5843	1·5875	1·5906	1·5937	1·5969	1·6000	1·6031	1·6062	1·6093	3	6	9	13	16	19	22	25	28
2·6	1·6125	1·6155	1·6186	1·6217	1·6248	1·6279	1·6310	1·6340	1·6371	1·6401	3	6	9	12	15	18	22	25	28
2·7	1·6432	1·6462	1·6492	1·6523	1·6553	1·6583	1·6613	1·6643	1·6673	1·6703	3	6	9	12	15	18	21	24	27
2·8	1·6733	1·6763	1·6793	1·6823	1·6852	1·6882	1·6912	1·6941	1·6971	1·7000	3	6	9	12	15	18	21	24	27
2·9	1·7029	1·7059	1·7088	1·7117	1·7146	1·7176	1·7205	1·7234	1·7263	1·7292	3	6	9	12	15	18	20	23	26
3·0	1·7321	1·7349	1·7378	1·7407	1·7436	1·7464	1·7493	1·7521	1·7550	1·7578	3	6	9	11	14	17	20	23	26

3·1	1·7607	1·7635	1·7664	1·7692	1·7720	1·7748	1·7776	1·7804	1·7833	1·7861	3	6	9	11	14	17	20	23	25
3·2	1·7889	1·7916	1·7944	1·7972	1·8000	1·8028	1·8055	1·8083	1·8111	1·8138	3	6	8	11	14	17	19	22	25
3·3	1·8166	1·8193	1·8221	1·8248	1·8276	1·8303	1·8330	1·8358	1·8385	1·8412	3	5	8	11	14	16	19	22	25
3·4	1·8439	1·8466	1·8493	1·8520	1·8547	1·8574	1·8601	1·8628	1·8655	1·8682	3	5	8	11	13	16	19	22	24
3·5	1·8708	1·8735	1·8762	1·8788	1·8815	1·8841	1·8868	1·8894	1·8921	1·8947	3	5	8	11	13	16	19	21	24
3·6	1·8974	1·9000	1·9026	1·9053	1·9079	1·9105	1·9131	1·9157	1·9183	1·9209	3	5	8	10	13	16	18	21	24
3·7	1·9235	1·9261	1·9287	1·9313	1·9339	1·9365	1·9391	1·9416	1·9442	1·9468	3	5	8	10	13	16	18	21	23
3·8	1·9494	1·9519	1·9545	1·9570	1·9596	1·9621	1·9647	1·9672	1·9698	1·9723	3	5	8	10	13	15	18	21	23
3·9	1·9748	1·9774	1·9799	1·9824	1·9849	1·9875	1·9900	1·9925	1·9950	1·9975	3	5	8	10	13	15	18	20	23
4·0	2·0000	2·0025	2·0050	2·0075	2·0100	2·0125	2·0149	2·0174	2·0199	2·0224	2	5	7	10	12	15	17	20	22
4·1	2·0248	2·0273	2·0298	2·0322	2·0347	2·0372	2·0396	2·0421	2·0445	2·0469	2	5	7	10	12	15	17	20	22
4·2	2·0494	2·0518	2·0543	2·0567	2·0591	2·0616	2·0640	2·0664	2·0688	2·0712	2	5	7	10	12	15	17	19	22
4·3	2·0736	2·0761	2·0785	2·0809	2·0833	2·0857	2·0881	2·0905	2·0928	2·0952	2	5	7	10	12	14	17	19	22
4·4	2·0976	2·1000	2·1024	2·1048	2·1071	2·1095	2·1119	2·1142	2·1166	2·1190	2	5	7	9	12	14	17	19	21
4·5	2·1213	2·1237	2·1260	2·1284	2·1307	2·1331	2·1354	2·1378	2·1401	2·1424	2	5	7	9	12	14	16	19	21
4·6	2·1448	2·1471	2·1494	2·1517	2·1541	2·1564	2·1587	2·1610	2·1633	2·1656	2	5	7	9	12	14	16	19	21
4·7	2·1679	2·1703	2·1726	2·1749	2·1772	2·1794	2·1817	2·1840	2·1863	2·1886	2	5	7	9	12	14	16	18	21
4·8	2·1909	2·1932	2·1954	2·1977	2·2000	2·2023	2·2045	2·2068	2·2091	2·2113	2	5	7	9	11	14	16	18	20
4·9	2·2136	2·2159	2·2181	2·2204	2·2226	2·2249	2·2271	2·2293	2·2316	2·2338	2	5	7	9	11	14	16	18	20
5·0	2·2361	2·2383	2·2405	2·2428	2·2450	2·2472	2·2494	2·2517	2·2539	2·2561	2	4	7	9	11	13	16	18	20
5·1	2·2583	2·2605	2·2627	2·2650	2·2672	2·2694	2·2716	2·2738	2·2760	2·2782	2	4	7	9	11	13	15	18	20
5·2	2·2804	2·2825	2·2847	2·2869	2·2891	2·2913	2·2935	2·2956	2·2978	2·3000	2	4	7	9	11	13	15	17	20
5·3	2·3022	2·3043	2·3065	2·3087	2·3108	2·3130	2·3152	2·3173	2·3195	2·3216	2	4	6	9	11	13	15	17	19
5·4	2·3238	2·3259	2·3281	2·3302	2·3324	2·3345	2·3367	2·3388	2·3409	2·3431	2	4	6	9	11	13	15	17	19

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

SQUARE ROOTS.

From 5·500 to 10.

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
5·5	2·3452	2·3473	2·3495	2·3516	2·3537	2·3558	2·3580	2·3601	2·3622	2·3643	2	4	6	8	11	13	15	17	19
5·6	2·3664	2·3685	2·3707	2·3728	2·3749	2·3770	2·3791	2·3812	2·3833	2·3854	2	4	6	8	11	13	15	17	19
5·7	2·3875	2·3896	2·3917	2·3937	2·3958	2·3979	2·4000	2·4021	2·4042	2·4062	2	4	6	8	10	12	15	17	19
5·8	2·4083	2·4104	2·4125	2·4145	2·4166	2·4187	2·4207	2·4228	2·4249	2·4269	2	4	6	8	10	12	14	16	19
5·9	2·4290	2·4310	2·4331	2·4352	2·4372	2·4393	2·4413	2·4434	2·4454	2·4474	2	4	6	8	10	12	14	16	18
6·0	2·4495	2·4515	2·4536	2·4556	2·4576	2·4597	2·4617	2·4637	2·4658	2·4678	2	4	6	8	10	12	14	16	18
6·1	2·4698	2·4718	2·4739	2·4759	2·4779	2·4799	2·4819	2·4839	2·4860	2·4880	2	4	6	8	10	12	14	16	18
6·2	2·4900	2·4920	2·4940	2·4960	2·4980	2·5000	2·5020	2·5040	2·5060	2·5080	2	4	6	8	10	12	14	16	18
6·3	2·5100	2·5120	2·5140	2·5159	2·5179	2·5199	2·5219	2·5239	2·5259	2·5278	2	4	6	8	10	12	14	16	18
6·4	2·5298	2·5318	2·5338	2·5357	2·5377	2·5397	2·5417	2·5436	2·5456	2·5475	2	4	6	8	10	12	14	16	18
6·5	2·5495	2·5515	2·5534	2·5554	2·5573	2·5593	2·5612	2·5632	2·5652	2·5671	2	4	6	8	10	12	14	16	18
6·6	2·5690	2·5710	2·5729	2·5749	2·5768	2·5788	2·5807	2·5826	2·5846	2·5865	2	4	6	8	10	12	14	16	17
6·7	2·5884	2·5904	2·5923	2·5942	2·5962	2·5981	2·6000	2·6019	2·6038	2·6058	2	4	6	8	10	12	14	15	17
6·8	2·6077	2·6096	2·6115	2·6134	2·6153	2·6173	2·6192	2·6211	2·6230	2·6249	2	4	6	8	10	11	13	15	17
6·9	2·6268	2·6287	2·6306	2·6325	2·6344	2·6363	2·6382	2·6401	2·6420	2·6439	2	4	6	8	10	11	13	15	17
7·0	2·6458	2·6476	2·6495	2·6514	2·6533	2·6552	2·6571	2·6589	2·6608	2·6627	2	4	6	8	9	11	13	15	17
7·1	2·6646	2·6665	2·6683	2·6702	2·6721	2·6739	2·6758	2·6777	2·6796	2·6814	2	4	6	7	9	11	13	15	17
7·2	2·6833	2·6851	2·6870	2·6889	2·6907	2·6926	2·6944	2·6963	2·6981	2·7000	2	4	6	7	9	11	13	15	17
7·3	2·7019	2·7037	2·7055	2·7074	2·7092	2·7111	2·7129	2·7148	2·7166	2·7185	2	4	6	7	9	11	13	15	17
7·4	2·7203	2·7221	2·7240	2·7258	2·7276	2·7295	2·7313	2·7331	2·7350	2·7368	2	4	5	7	9	11	13	15	16
7·5	2·7386	2·7404	2·7423	2·7441	2·7459	2·7477	2·7495	2·7514	2·7532	2·7550	2	4	5	7	9	11	13	15	16

7·6	2·7568	2·7586	2·7604	2·7622	2·7641	2·7659	2·7677	2·7695	2·7713	2·7731	2	4	5	7	9	11	13	14	16
7·7	2·7749	2·7767	2·7785	2·7803	2·7821	2·7839	2·7857	2·7875	2·7893	2·7911	2	4	5	7	9	11	13	14	16
7·3	2·7928	2·7946	2·7964	2·7982	2·8000	2·8018	2·8036	2·8054	2·8071	2·8089	2	4	5	7	9	11	13	14	16
7·9	2·8107	2·8125	2·8142	2·8160	2·8178	2·8196	2·8213	2·8231	2·8249	2·8267	2	4	5	7	9	11	12	14	16
8·0	2·8284	2·8302	2·8320	2·8337	2·8355	2·8373	2·8390	2·8408	2·8425	2·8443	2	4	5	7	9	11	12	14	16
8·1	2·8460	2·8478	2·8496	2·8513	2·8531	2·8548	2·8566	2·8583	2·8601	2·8618	2	4	5	7	9	11	12	14	16
8·2	2·8636	2·8653	2·8671	2·8688	2·8705	2·8723	2·8740	2·8758	2·8775	2·8792	2	3	5	7	9	10	12	14	16
8·3	2·8810	2·8827	2·8844	2·8862	2·8879	2·8896	2·8914	2·8931	2·8948	2·8965	2	3	5	7	9	10	12	14	16
8·4	2·8983	2·9000	2·9017	2·9034	2·9052	2·9069	2·9086	2·9103	2·9120	2·9138	2	3	5	7	9	10	12	14	15
8·5	2·9155	2·9172	2·9189	2·9206	2·9223	2·8240	2·9257	2·9275	2·9292	2·9309	2	3	5	7	9	10	12	14	15
8·6	2·9326	2·9343	2·9360	2·9377	2·9394	2·9411	2·9428	2·9445	2·9462	2·9479	2	3	5	7	9	10	12	14	15
8·7	2·9496	2·9513	2·9530	2·9547	2·9563	2·9580	2·9597	2·9614	2·9631	2·9648	2	3	5	7	9	10	12	14	15
8·3	2·9665	2·9682	2·9698	2·9715	2·9732	2·9749	2·9766	2·9783	2·9799	2·9816	2	3	5	7	8	10	12	13	15
8·9	2·9833	2·9850	2·9866	2·9883	2·9900	2·9917	2·9933	2·9950	2·9967	2·9983	2	3	5	7	8	10	12	13	15
9·0	3·0000	3·0017	3·0033	3·0050	3·0067	3·0083	3·0100	3·0116	3·0133	3·0150	2	3	5	7	8	10	12	13	15
9·1	3·0166	3·0183	3·0199	3·0216	3·0232	3·0249	3·0265	3·0282	3·0299	3·0315	2	3	5	7	8	10	12	13	15
9·2	3·0332	3·0348	3·0364	3·0381	3·0397	3·0414	3·0430	3·0447	3·0463	3·0480	2	3	5	7	8	10	11	13	15
9·3	3·0496	3·0512	3·0529	3·0545	3·0561	3·0578	3·0594	3·0610	3·0627	3·0643	2	3	5	7	8	10	11	13	15
9·4	3·0659	3·0676	3·0692	3·0708	3·0725	3·0741	3·0757	3·0773	3·0790	3·0806	2	3	5	7	8	10	11	13	15
9·5	3·0822	3·0838	3·0854	3·0871	3·0887	3·0903	3·0919	3·0935	3·0952	3·0968	2	3	5	6	8	10	11	13	15
9·6	3·0984	3·1000	3·1016	3·1032	3·1048	3·1064	3·1081	3·1097	3·1113	3·1129	2	3	5	6	8	10	11	13	14
9·7	3·1145	3·1161	3·1177	3·1193	3·1209	3·1225	3·1241	3·1257	3·1273	3·1289	2	3	5	6	8	10	11	13	14
9·8	3·1305	3·1321	3·1337	3·1353	3·1369	3·1385	3·1401	3·1417	3·1432	3·1448	2	3	5	6	8	10	11	13	14
9·9	3·1464	3·1480	3·1496	3·1512	3·1528	3·1544	3·1559	3·1575	3·1591	3·1607	2	3	5	6	8	9	11	13	14

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

SQUARE ROOTS.

From 10 to 54·99.

	0	1	2	3	4	5	6	7	8	9	*Mean Differences.*								
											1	2	3	4	5	6	7	8	9
10	3·1623	3·1780	3·1937	3·2094	3·2249	3·2404	3·2558	3·2711	3·2863	3·3015	15	31	46	62	77	92	108	123	139
11	3·3166	3·3317	3·3466	3·3615	3·3764	3·3912	3·4059	3·4205	3·4351	3·4496	15	30	44	59	74	89	104	118	133
12	3·4641	3·4785	3·4928	3·5071	3·5214	3·5355	3·5496	3·5637	3·5777	3·5917	14	28	43	57	71	85	99	114	128
13	3·6056	3·6194	3·6332	3·6469	3·6606	3·6742	3·6878	3·7014	3·7148	3·7283	14	27	41	54	68	82	95	109	122
14	3·7417	3·7550	3·7683	3·7815	3·7947	3·8079	3·8210	3·8341	3·8471	3·8601	13	26	39	52	66	79	92	105	118
15	3·8730	3·8859	3·8987	3·9115	3·9243	3·9370	3·9497	3·9623	3·9749	3·9875	13	25	38	51	64	76	89	102	114
16	4·0000	4·0125	4·0249	4·0373	4·0497	4·0620	4·0743	4·0866	4·0988	4·1110	12	25	37	49	62	74	86	98	111
17	4·1231	4·1352	4·1473	4·1593	4·1713	4·1833	4·1952	4·2071	4·2190	4·2308	12	24	36	48	60	72	84	96	108
18	4·2426	4·2544	4·2661	4·2778	4·2895	4·3012	4·3128	4·3243	4·3359	4·3474	12	23	35	46	58	70	81	93	104
19	4·3589	4·3704	4·3818	4·3932	4·4045	4·4159	4·4272	4·4385	4·4497	4·4609	11	23	34	45	57	68	79	90	102
20	4·4721	4·4833	4·4944	4·5056	4·5166	4·5277	4·5387	4·5497	4·5607	4·5717	11	22	33	44	56	67	78	89	100
21	4·5826	4·5935	4·6043	4·6152	4·6260	4·6368	4·6476	4·6583	4·6690	4·6797	11	22	32	43	54	65	76	86	97
22	4·6904	4·7011	4·7117	4·7223	4·7329	4·7434	4·7539	4·7645	4·7749	4·7854	11	21	32	42	53	63	74	84	95
23	4·7958	4·8062	4·8166	4·8270	4·8374	4·8477	4·8580	4·8683	4·8785	4·8888	10	21	31	41	52	62	72	82	93
24	4·8990	4·9092	4·9193	4·9295	4·9396	4·9497	4·9598	4·9699	4·9800	4·9900	10	20	30	40	51	61	71	81	91
25	5·0000	5·0100	5·0200	5·0299	5·0398	5·0498	5·0596	5·0695	5·0794	5·0892	10	20	30	40	50	59	69	79	89
26	5·0990	5·1088	5·1186	5·1284	5·1381	5·1478	5·1575	5·1672	5·1769	5·1865	10	19	29	39	49	58	68	78	87
27	5·1962	5·2058	5·2154	5·2249	5·2345	5·2440	5·2536	5·2631	5·2726	5·2820	10	19	29	38	48	57	67	76	86
28	5·2915	5·3009	5·3104	5·3198	5·3292	5·3385	5·3479	5·3572	5·3666	5·3759	9	19	28	38	47	56	66	75	85
29	5·3852	5·3944	5·4037	5·4129	5·4222	5·4314	5·4406	5·4498	5·4589	5·4681	9	18	28	37	46	55	64	74	83
30	5·4772	5·4863	5·4955	5·5045	5·5136	5·5227	5·5317	5·5408	5·5498	5·5588	9	18	27	36	46	55	64	73	82

31	5·5678	5·5767	5·5857	5·5946	5·6036	5·6125	5·6214	5·6303	5·6391	5·6480	9	18	27	36	45	53	62	71	80
32	5·6569	5·6657	5·6745	5·6833	5·6921	5·7009	5·7096	5·7184	5·7271	5·7359	9	18	26	35	44	53	62	70	79
33	5·7446	5·7533	5·7619	5·7706	5·7793	5·7879	5·7966	5·8052	5·8138	5·8224	9	17	26	34	43	52	60	69	77
34	5·8310	5·8395	5·8481	5·8566	5·8652	5·8738	5·8822	5·8907	5·8992	5·9076	9	17	26	34	43	51	60	68	77
35	5·9161	5·9245	5·9330	5·9414	5·9498	5·9582	5·9666	5·9749	5·9833	5·9917	8	17	25	34	42	50	59	67	76
36	6·0000	6·0083	6·0166	6·0249	6·0332	6·0415	6·0498	6·0581	6·0663	6·0745	8	17	25	33	42	50	58	66	75
37	6·0828	6·0910	6·0992	6·1074	6·1156	6·1237	6·1319	6·1400	6·1482	6·1563	8	16	25	33	41	49	57	66	74
38	6·1644	6·1725	6·1806	6·1887	6·1968	6·2048	6·2129	6·2209	6·2290	6·2370	8	16	24	32	41	49	57	65	73
39	6·2450	6·2530	6·2610	6·2690	6·2769	6·2849	6·2929	6·3008	6·3087	6·3166	8	16	24	32	40	48	56	64	72
40	6·3246	6·3325	6·3403	6·3482	6·3561	6·3640	6·3718	6·3797	6·3875	6·3953	8	16	24	32	40	47	55	63	71
41	6·4031	6·4109	6·4187	6·4265	6·4343	6·4420	6·4498	6·4576	6·4653	6·4730	8	16	23	31	39	47	55	62	70
42	6·4807	6·4885	6·4962	6·5038	6·5115	6·5192	6·5269	6·5345	6·5422	6·5498	8	15	23	31	39	46	54	62	69
43	6·5574	6·5651	6·5727	6·5803	6·5879	6·5955	6·6030	6·6106	6·6182	6·6257	8	15	23	30	38	46	53	61	68
44	6·6332	6·6408	6·6483	6·6558	6·6633	6·6708	6·6783	6·6858	6·6933	6·7007	8	15	23	30	38	45	53	60	68
45	6·7082	6·7157	6·7231	6·7305	6·7380	6·7454	6·7528	6·7602	6·7676	6·7750	7	15	22	30	37	44	52	59	67
46	6·7823	6·7897	6·7971	6·8044	6·8118	6·8191	6·8264	6·8337	6·8411	6·8484	7	15	22	29	37	44	51	58	66
47	6·8557	6·8629	6·8702	6·8775	6·8848	6·8920	6·8993	6·9065	6·9138	6·9210	7	15	22	29	37	44	51	58	66
48	6·9282	6·9354	6·9426	6·9498	6·9570	6·9642	6·9714	6·9785	6·9857	6·9929	7	14	22	29	36	43	50	58	65
49	7·0000	7·0071	7·0143	7·0214	7·0285	7·0356	7·0427	7·0498	7·0569	7·0640	7	14	21	28	36	43	50	57	63
50	7·0711	7·0781	7·0852	7·0922	7·0993	7·1063	7·1134	7·1204	7·1274	7·1344	7	14	21	28	35	42	49	56	63
51	7·1414	7·1484	7·1554	7·1624	7·1694	7·1764	7·1833	7·1903	7·1972	7·2042	7	14	21	28	35	42	49	56	63
52	7·2111	7·2180	7·2250	7·2319	7·2388	7·2457	7·2526	7·2595	7·2664	7·2732	7	14	21	28	35	41	48	55	62
53	7·2801	7·2870	7·2938	7·3007	7·3075	7·3144	7·3212	7·3280	7·3348	7·3417	7	14	20	27	34	41	48	54	61
54	7·3485	7·3553	7·3621	7·3689	7·3756	7·3824	7·3892	7·3959	7·4027	7·4095	7	14	20	27	34	41	48	54	61

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

SQUARE ROOTS.

From 55 to 100.

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
55	7·4162	7·4229	7·4297	7·4364	7·4431	7·4498	7·4565	7·4632	7·4699	7·4766	7	13	20	27	34	40	47	54	60
56	7·4833	7·4900	7·4967	7·5033	7·5100	7·5166	7·5233	7·5299	7·5366	7·5432	7	13	20	27	34	40	47	54	60
57	7·5498	7·5565	7·5631	7·5697	7·5763	7·5829	7·5895	7·5961	7·6026	7·6092	7	13	20	26	33	40	46	53	59
58	7·6158	7·6223	7·6289	7·6354	7·6420	7·6485	7·6551	7·6616	7·6681	7·6746	7	13	20	26	33	39	46	52	59
59	7·6811	7·6877	7·6942	7·7006	7·7071	7·7136	7·7201	7·7266	7·7330	7·7395	7	13	20	26	33	39	46	52	59
60	7·7460	7·7524	7·7589	7·7653	7·7717	7·7782	7·7846	7·7910	7·7974	7·8038	6	13	19	26	32	38	45	51	58
61	7·8102	7·8166	7·8230	7·8294	7·8358	7·8422	7·8486	7·8549	7·8613	7·8677	6	13	19	26	32	38	45	51	58
62	7·8740	7·8804	7·8867	7·8930	7·8994	7·9057	7·9120	7·9183	7·9246	7·9310	6	13	19	25	32	38	44	50	57
63	7·9373	7·9436	7·9498	7·9561	7·9624	7·9687	7·9750	7·9812	7·9875	7·9937	6	13	19	25	32	38	44	50	57
64	8·0000	8·0062	8·0125	8·0187	8·0250	8·0312	8·0374	8·0436	8·0498	8·0561	6	12	19	25	31	37	43	50	56
65	8·0623	8·0685	8·0747	8·0808	8·0870	8·0932	8·0994	8·1056	8·1117	8·1179	6	12	19	25	31	37	43	50	56
66	8·1240	8·1302	8·1363	8·1425	8·1486	8·1548	8·1609	8·1670	8·1731	8·1792	6	12	18	24	31	37	43	49	55
67	8·1854	8·1915	8·1976	8·2037	8·2098	8·2158	8·2219	8·2280	8·2341	8·2401	6	12	18	24	31	37	43	49	55
68	8·2462	8·2523	8·2583	8·2644	8·2704	8·2765	8·2825	8·2885	8·2746	8·3006	6	12	18	24	30	36	42	48	54
69	8·3066	8·3126	8·3187	8·3247	8·3307	8·3367	8·3427	8·3487	8·3546	8·3606	6	12	18	24	30	36	42	48	54
70	8·3666	8·3726	8·3785	8·3845	8·3905	8·3964	8·4024	8·4083	8·4143	8·4202	6	12	18	24	30	36	42	48	54
71	8·4261	8·4321	8·4380	8·4439	8·4499	8·4558	8·4617	8·4676	8·4735	8·4794	6	12	18	24	30	53	41	47	53
72	8·4853	8·4912	8·4971	8·5029	8·5088	8·5147	8·5206	8·5264	8·5323	8·5381	6	12	18	24	30	35	41	47	53
73	8·5440	8·5499	8·5557	8·5615	8·5674	8·5732	8·5790	8·5849	8·5907	8·5965	6	12	17	23	29	35	41	46	52
74	8·6023	8·6081	8·6139	8·6197	8·6255	8·6313	8·6371	8·6429	8·6487	8·6545	6	12	17	23	29	35	41	46	52
75	8·6603	8·6660	8·6718	8·6776	8·6833	8·6891	8·6948	8·7006	8·7063	8·7121	6	12	17	23	29	35	41	46	52

76	8·7178	8·7235	8·7293	8·7350	8·7407	8·7464	8·7521	8·7579	8·7636	8·7693	6	11	17	23	29	34	40	46	51
77	8·7750	8·7807	8·7864	8·7920	8·7977	8·8034	8·8091	8·8148	8·8204	8·8261	6	11	17	23	29	34	40	46	51
78	8·8318	8·8374	8·8431	8·8487	8·8544	8·8600	8·8657	8·8713	8·8769	8·8826	6	11	17	22	28	34	39	45	50
79	8·8882	8·8933	8·8994	8·9051	8·9107	8·9163	8·9219	8·9275	8·9331	8·9387	6	11	17	22	28	34	39	45	50
80	8·9443	8·9499	8·9554	8·9610	8·9666	8·9722	8·9778	8·9833	8·9889	8·9944	6	11	17	22	28	34	39	45	50
81	9·0000	9·0056	9·0111	9·0167	9·0222	9·0277	9·0333	9·0388	9·0443	9·0499	6	11	17	22	28	33	39	44	50
82	9·0554	9·0609	9·0664	9·0719	9·0774	9·0830	9·0885	9·0940	9·0995	9·1049	6	11	17	22	28	33	39	44	50
83	9·1104	9·1159	9·1214	9·1269	9·1324	9·1378	9·1433	9·1488	9·1542	9·1597	6	11	17	22	28	33	39	44	50
34	9·1652	9·1706	9·1761	9·1815	9·1869	9·1924	9·1978	9·2033	9·2087	9·2141	5	11	16	22	27	32	38	43	49
85	9·2195	9·2250	9·2304	9·2358	9·2412	9·2466	9·2520	9·2574	9·2628	9·2682	5	11	16	22	27	32	38	43	49
86	9·2736	9·2790	9·2844	9·2898	9·2952	9·3005	9·3059	9·3113	9·3167	9·3220	5	11	16	22	27	32	38	43	49
87	9·3274	9·3327	9·3381	9·3434	9·3488	9·3541	9·3595	9·3648	9·3702	9·3755	5	11	16	21	27	32	37	42	48
88	9·3808	9·3862	9·3915	9·3968	9·4021	9·4074	9·4128	9·4181	9·4234	9·4287	5	11	16	21	27	32	37	42	48
89	9·4340	9·4393	9·4446	9·4499	9·4552	9·4604	9·4657	9·4710	9·4763	9·4816	5	11	16	21	27	32	37	42	48
90	9·4868	9·4921	9·4974	9·5026	9·5079	9·5131	9·5184	9·5237	9·5289	9·5341	5	11	16	21	27	32	37	42	48
91	9·5394	9·5446	9·5499	9·5551	9·5603	9·5656	9·5708	9·5760	9·5812	9·5864	5	11	16	21	26	31	36	42	47
92	9·5917	9·5969	9·6021	9·6073	9·6125	9·6177	9·6229	9·6281	9·6333	9·6385	5	11	16	21	26	31	36	42	47
93	9·6437	9·6488	9·6540	9·6592	9·6644	9·6695	9·6747	9·6799	9·6850	9·6902	5	10	16	21	26	31	36	42	47
94	9·6954	9·7005	9·7057	9·7108	9·7160	9·7211	9·7263	9·7314	9·7365	9·7417	5	10	15	20	26	31	36	41	46
95	9·7463	9·7519	9·7570	9·7622	9·7673	9·7724	9·7775	9·7826	9·7877	9·7929	5	10	15	20	26	31	36	41	46
96	9·7980	9·8031	9·8082	9·8133	9·8184	9·8234	9·8285	9·8336	9·8387	9·8438	5	10	15	20	26	31	36	41	46
97	9·8489	9·8539	9·8590	9·8641	9·8691	9·8742	9·8793	9·8843	9·8894	9·8944	5	10	15	20	26	31	36	41	46
98	9·8995	9·9045	9·9096	9·9146	9·9197	9·9247	9·9298	9·9348	9·9398	9·9448	5	10	15	20	25	30	35	40	45
99	9·9499	9·9549	9·9599	9·9649	9·9700	9·9750	9·9800	9·9550	9·9000	9·9550	5	10	15	20	25	30	35	40	45

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

CUBES OF NUMBERS.

From 1 to 5·49.

	0	1	2	3	4	5	6	7	8	9
1·0	1·0000	1·0303	1·0612	1·0927	1·1249	1·1576	1·1910	1·2250	1·2597	1·2950
1·1	1·3310	1·3676	1·4049	1·4429	1·4815	1·5209	1·5609	1·6016	1·6430	1·6852
1·2	1·7280	1·7716	1·8158	1·8609	1·9066	1·9531	2·0004	2·0484	2·0972	2·1467
1·3	2·1970	2·2481	2·3000	2·3526	2·4061	2·4604	2·5155	2·5714	2·6281	2·6856
1·4	2·7440	2·8032	2·8633	2·9242	2·9860	3·0486	3·1121	3·1765	3·2418	3·3079
1·5	3·3750	3·4430	3·5118	3·5816	3·6523	3·7239	3·7964	3·8699	3·9443	4·0197
1·6	4·0960	4·1733	4·2515	4·3307	4·4109	4·4921	4·5743	4·6575	4·7416	4·8268
1·7	4·9130	5·0002	5·0884	5·1777	5·2680	5·3594	5·4518	5·5452	5·6398	5·7353
1·8	5·8320	5·9297	6·0286	6·1285	6·2295	6·3316	6·4349	6·5392	6·6447	6·7513
1·9	6·8590	6·9679	7·0779	7·1891	7·3014	7·4149	7·5295	7·6454	7·7624	7·8806
2·0	8·0000	8·1206	8·2424	8·3654	8·4897	8·6151	8·7418	8·8697	8·9989	9·1293
2·1	9·2610	9·3939	9·5281	9·6636	9·8003	9·9384	10·078	10·218	10·360	10·503
2·2	10·648	10·794	10·941	11·090	11·239	11·391	11·543	11·697	11·852	12·009
2·3	12·167	12·326	12·487	12·649	12·813	12·978	13·144	13·312	13·481	13·652
2·4	13·824	13·998	14·172	14·349	14·527	14·706	14·887	15·069	15·253	15·438
2·5	15·625	15·813	16·003	16·194	16·387	16·581	16·777	16·975	17·174	17·374
2·6	17·576	17·780	17·985	18·191	18·400	18·610	18·821	19·034	18·249	19·465
2·7	19·633	19·903	20·124	20·346	20·571	20·797	21·025	21·254	21·485	21·718
2·8	21·952	22·188	22·426	22·665	22·906	23·149	23·394	23·640	23·888	24·138
2·9	24·389	24·642	24·897	25·154	25·412	25·672	25·934	26·198	26·464	26·731
3·0	27·000	27·271	27·544	27·818	28·094	28·373	28·653	28·934	29·218	29·504

3·1	29·791	20·080	30·371	30·664	30·959	31·256	31·554	31·855	32·157	32·462
3·2	32·768	33·076	33·386	33·698	34·012	34·328	34·646	34·966	35·288	35·611
3·3	35·937	36·265	36·594	36·926	37·260	37·595	37·933	38·273	38·614	38·958
3·4	39·304	39·652	40·002	40·354	40·708	41·064	41·422	41·782	42·144	42·509
3·5	42·875	43·244	43·614	43·987	44·362	44·739	45·118	45·499	45·883	46·268
3·6	46·656	47·046	47·438	47·832	48·229	48·627	49·028	49·431	49·836	50·243
3·7	50·653	51·065	51·479	51·895	52·314	52·734	53·157	53·583	54·010	54·440
3·8	54·872	55·306	55·743	56·182	56·623	57·067	57·512	57·961	58·411	58·864
3·9	59·319	59·776	60·236	60·698	61·163	61·630	62·099	62·571	63·045	63·521
4·0	64·000	64·481	64·965	65·451	65·939	66·430	66·923	67·419	67·917	68·418
4·1	68·921	69·427	69·935	70·445	70·958	71·473	71·991	72·512	73·035	73·560
4·2	74·088	74·618	75·151	75·687	76·225	76·766	77·309	77·854	78·403	78·954
4·3	79·507	80·063	80·622	81·183	81·747	82·313	82·882	83·453	84·028	84·605
4·4	85·184	85·766	86·351	86·938	87·528	88·121	88·717	89·315	89·915	90·519
4·5	91·125	91·734	92·345	92·960	93·577	94·196	94·819	95·444	96·072	96·703
4·6	97·336	97·972	98·611	99·253	99·897	100·54	101·19	101·85	102·50	103·16
4·7	103·82	104·49	105·15	105·82	106·50	107·17	107·85	108·53	109·22	109·90
4·8	110·59	111·28	111·98	112·68	113·38	114·08	114·79	115·50	116·21	116·93
4·9	117·65	118·37	119·10	119·82	120·55	121·29	122·02	122·76	123·51	124·25
5·0	125·00	125·75	126·51	127·26	128·02	128·79	129·55	130·32	131·10	131·87
5·1	132·65	133·43	134·22	135·01	135·80	136·59	137·39	138·19	138·99	139·80
5·2	140·61	141·42	142·24	143·06	143·88	144·70	145·53	146·36	147·20	148·04
5·3	148·88	149·72	150·57	151·42	152·27	153·13	153·99	154·85	155·72	156·59
5·4	157·46	158·34	159·22	160·10	160·99	161·88	162·77	163·67	164·57	165·47

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

CUBES OF NUMBERS.

From 5·50 to 10.

	0	1	2	3	4	5	6	7	8	9
5·5	166·38	167·28	168·20	169·11	170·03	170·95	171·88	172·81	173·74	174·68
5·6	175·62	176·56	177·50	178·45	179·41	180·36	181·32	182·28	183·25	184·22
5·7	185·19	186·17	187·15	188·13	189·12	190·11	191·10	192·10	193·10	194·10
5·8	195·11	196·12	197·14	198·16	199·18	200·20	201·23	202·26	203·30	204·34
5·9	205·37	206·43	207·47	208·53	209·58	210·64	211·71	212·78	213·85	214·92
6·0	216·00	217·08	218·17	219·26	220·35	221·45	222·55	223·65	224·76	225·87
6·1	226·98	228·10	229·22	230·35	231·48	232·61	233·74	234·89	236·03	237·18
6·2	238·33	239·48	240·64	241·80	242·97	244·14	245·31	246·49	247·67	248·86
6·3	250·05	251·24	252·44	253·64	254·84	256·05	257·26	258·47	259·69	260·92
6·4	262·14	263·37	264·61	265·85	267·09	268·34	269·59	270·84	272·10	273·36
6·5	274·63	275·89	277·17	278·45	279·73	281·01	282·30	283·59	284·89	286·19
6·6	287·50	288·80	290·12	291·43	292·75	294·08	295·41	296·74	298·08	299·42
6·7	300·74	302·11	303·46	304·82	306·18	307·55	308·92	310·29	311·67	313·05
6·8	314·43	315·82	317·21	318·61	320·01	321·42	322·83	324·24	325·66	327·08
6·9	328·51	329·94	331·37	332·81	334·26	335·70	337·15	338·61	340·07	341·53
7·0	343·00	344·47	345·95	347·43	348·91	350·40	351·90	353·39	354·89	356·40
7·1	357·91	359·43	360·94	362·47	363·99	365·53	367·06	368·60	370·15	371·69
7·2	373·25	374·81	376·37	377·93	379·50	381·08	382·66	384·24	385·83	387·42
7·3	389·02	390·62	392·22	393·83	395·45	397·07	398·69	400·32	401·95	403·58
7·4	405·22	406·87	408·52	410·17	411·83	413·49	415·16	416·83	418·51	420·19
7·5	421·88	423·56	425·26	426·96	428·66	430·37	432·08	433·80	435·52	437·25

7·6	438·98	440·71	442·45	444·19	445·94	447·70	449·46	451·22	452·98	454·76
7·7	456·53	458·31	460·10	461·89	463·68	465·48	467·29,	469·10	470·91	472·73
7 8	474·55	476·38	478·21	480·05	481·89	483·74	485·59	487·44	489·30	491·17
7·9	493·04	494·91	496·79	498·68	500·57	502·46	504·36	506·26	508·17	510·08
8·0	512·00	513·92	515·85	517·78	519·72	521·66	523·61	525·56	527·51	529·48
8·1	531·44	533·41	535·39	537·37	539·35	541·34	543·34	545·34	547·34	549·35
8·2	551·37	553·39	555·41	557·44	559·48	561·52	563·56	565·61	567·66	569·72
8·3	571·79	573·86	575·93	578·01	580·09	582·18	584·28	586·38	588·48	590·59
8·4	592·70	594·82	596·95	599·08	601·21	603·35	605·50	607·65	609·80	611·96
8·5	614·12	616·30	618·47	620·65	622·84	625·03	627·22	629·42	631·63	633·84
8·6	636·06	638·28	640·50	642·74	644·97	647·21	649·46	651·71	653·97	656·23
8·7	658·50	660·78	663·05	665·34	667·63	669·92	672·22	674·53	676·84	679·15
8·8	681·47	683·80	686·13	688·47	690·81	693·15	695·51	697·86	700·23	702·60
8·9	704·97	707·35	709·73	712·12	714·52	716·92	719·32	721·73	724·15	726·57
9·0	729·00	731·43	733·87	736·31	738·76	741·22	743·68	746·14	748·61	751·09
9·1	753·57	756·06	758·55	761·05	763·55	766·06	768·58	771·10	773·62	776·15
9·2	778·69	781·23	783·78	786·33	788·89	791·45	794·02	796·60	799·18	801·77
9·3	804·36	806·95	809·56	812·17	814·78	817·40	820·03	822·66	825·29	827·94
9·4	830·53	833·24	835·90	838·56	841·23	843·91	846·59	849·28	851·97	854·67
9·5	857·38	860·09	862·80	865·52	868·25	870·98	873·72	876·47	879·22	881·97
9·6	884·74	887·50	890·28	893·06	895·84	898·63	901·43	904·23	907·04	909·85
9·7	912·67	915·50	918·33	921·17	924·01	926·86	929·71	932·57	935·44	938·31
9·8	941·19	944·08	946·97	949·86	952·76	955·67	958·59	961·50	964·43	967·36
9·9	970·30	973·24	976·19	979·15	982·11	985·09	988·05	991·03	994·01	997·00

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CUBE ROOTS.

From 1 to 5·499.

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
1·0	1·0000	1·0033	1·0066	1·0099	1·0132	1·0164	1·0196	1·0228	1·0260	1·0291	3	6	10	13	16	19	23	26	29
1·1	1·0323	1·0354	1·0385	1·0416	1·0446	1·0477	1·0507	1·0537	1·0567	1·0597	3	6	9	12	15	18	21	24	27
1·2	1·0627	1·0656	1·0685	1·0714	1·0743	1·0772	1·0801	1·0829	1·0858	1·0886	3	6	9	11	14	17	20	23	26
1·3	1·0914	1·0942	1·0970	1·0997	1·1025	1·1052	1·1079	1·1106	1·1133	1·1160	3	5	8	11	14	16	19	22	25
1·4	1·1187	1·1213	1·1240	1·1266	1·1292	1·1319	1·1344	1·1370	1·1396	1·1422	3	5	8	10	13	16	18	21	23
1 5	1·1447	1·1473	1·1498	1·1523	1·1548	1·1573	1·1598	1·1623	1·1647	1·1672	2	5	7	10	12	15	17	20	22
1·6	1·1696	1·1720	1·1745	1·1769	1·1793	1·1817	1·1840	1·1864	1·1888	1·1911	2	5	7	10	12	14	17	19	22
1·7	1·1935	1·1958	1·1981	1·2005	1·2028	1·2051	1·2074	1·2096	1·2119	1·2142	2	5	7	9	11	14	16	18	21
1·8	1·2164	1·2187	1·2209	1·2232	1·2254	1·2276	1·2298	1·2320	1·2342	1·2364	2	4	7	9	11	13	16	18	20
1·9	1·2386	1·2407	1·2429	1·2450	1·2472	1·2493	1·2515	1·2536	1·2557	1·2578	2	4	6	9	11	13	15	17	19
2·0	1·2599	1·2620	1·2641	1·2662	1·2683	1·2703	1·2724	1·2745	1·2765	1·2785	2	4	6	8	10	12	14	17	19
2·1	1·2806	1·2826	1·2846	1·2866	1·2887	1·2907	1·2927	1·2947	1·2966	1·2986	2	4	6	8	10	12	14	16	18
2·2	1·3006	1·3026	1·3045	1·3065	1·3084	1·3104	1·3123	1·3142	1·3162	1·3181	2	4	6	8	10	12	14	16	17
2·3	1·3200	1·3219	1·3238	1·3257	1·3276	1·3295	1·3314	1·3333	1·3351	1·3370	2	4	6	8	9	11	13	15	17
2·4	1·3389	1·3407	1·3426	1·3444	1·3463	1·3481	1·3499	1·3518	1·3536	1·3554	2	4	5	7	9	11	13	15	17
2·5	1·3572	1·3590	1·3608	1·3626	1·3644	1·3662	1·3680	1·3698	1·3715	1·3733	2	4	5	7	9	11	13	14	16
2·6	1·3751	1·3768	1·3786	1·3803	1·3821	1·3838	1·3856	1·3873	1·3890	1·3908	2	3	5	7	9	10	12	14	16
2·7	1·3925	1·3942	1·3959	1·3976	1·3993	1·4010	1·4027	1·4044	1·4061	1·4078	2	3	5	7	9	10	12	14	15
2·8	1·4095	1·4111	1·4128	1·4145	1·4161	1·4178	1·4195	1·4211	1·4228	1·4244	2	3	5	7	8	10	12	13	15
2·9	1·4260	1·4277	1·4293	1·4309	1·4326	1·4342	1·4358	1·4374	1·4390	1·4406	2	3	5	6	8	10	11	13	15
3·0	1·4422	1·4439	1·4454	1·4470	1·4486	1·4502	1·4518	1·4534	1·4550	1·4565	2	3	5	6	8	10	11	13	14

3·1	1·4581	1·4597	1·4612	1·4628	1·4643	1·4659	1·4674	1·4690	1·4705	1·4721	2	3	5	6	8	9	11	12	14
3·2	1·4736	1·4751	1·4767	1·4782	1·4797	1·4812	1·4828	1·4843	1·4858	1·4873	2	3	5	6	8	9	11	12	14
3·3	1·4888	1·4903	1·4918	1·4933	1·4948	1·4963	1·4978	1·4993	1·5007	1·5022	1	3	4	6	7	9	10	12	13
3·4	1·5037	1·5052	1·5066	1·5081	1·5096	1·5110	1·5125	1·5139	1·5154	1·5168	1	3	4	6	7	9	10	12	13
3·5	1·5183	1·5197	1·5212	1·5226	1·5241	1·5255	1·5269	1·5283	1·5298	1·5312	1	3	4	6	7	9	10	11	13
3·6	1·5326	1·5340	1·5355	1·5369	1·5383	1·5397	1·5411	1·5425	1·5439	1·5453	1	3	4	6	7	8	10	11	13
3·7	1·5467	1·5481	1·5495	1·5508	1·5522	1·5536	1·5550	1·5564	1·5577	1·5591	1	3	4	6	7	8	10	11	12
3·8	1·5605	1·5619	1·5632	1·5646	1·5659	1·5673	1·5687	1·5700	1·5714	1·5727	1	3	4	5	7	8	10	11	12
3·9	1·5741	1·5754	1·5767	1·5781	1·5794	1·5808	1·5821	1·5834	1·5848	1·5861	1	3	4	5	7	8	9	11	12
4·0	1·5874	1·5887	1·5900	1·5914	1·5927	1·5940	1·5953	1·5966	1·5979	1·5992	1	3	4	5	7	8	9	11	12
4·1	1·6005	1·6018	1·6031	1·6044	1·6057	1·6070	1·6083	1·6096	1·6109	1·6121	1	3	4	5	6	8	9	10	12
4·2	1·6134	1·6147	1·6160	1·6173	1·6185	1·6198	1·6211	1·6223	1·6236	1·6249	1	3	4	5	6	8	9	10	11
4·3	1·6261	1·6274	1·6287	1·6299	1·6312	1·6324	1·6337	1·6349	1·6362	1·6374	1	3	4	5	6	8	9	10	11
4·4	1·6386	1·6399	1·6411	1·6424	1·6436	1·6448	1·6461	1·6473	1·6485	1·6497	1	2	4	5	6	7	9	10	11
4·5	1·6510	1·6522	1·6534	1·6546	1·6558	1·6571	1·6583	1·6595	1·6607	1·6619	1	2	4	5	6	7	8	10	11
4·6	1·6631	1·6643	1·6655	1·6667	1·6679	1·6691	1·6703	1·6715	1·6727	1·6739	1	2	4	5	6	7	8	10	11
4·7	1·6751	1·6763	1·6774	1·6786	1·6798	1·6810	1·6822	1·6833	1·6845	1·6857	1	2	4	5	6	7	8	9	11
4·8	1·6869	1·6380	1·6892	1·6904	1·6915	1·6927	1·6939	1·6950	1·6962	1·6973	1	2	3	5	6	7	8	9	10
4·9	1·6985	1·6997	1·7008	1·7020	1·7031	1·7043	1·7054	1·7065	1·7077	1·7088	1	2	3	5	6	7	8	9	10
5·0	1·7100	1·7111	1·7123	1·7134	1·7145	1·7157	1·7168	1·7179	1·7190	1·7202	1	2	3	5	6	7	8	9	10
5·1	1·7213	1·7224	1·7235	1·7247	1·7258	1·7269	1·7280	1·7291	1·7303	1·7314	1	2	3	4	6	7	8	9	10
5·2	1·7325	1·7336	1·7347	1·7358	1·7369	1·7380	1·7391	1·7402	1·7413	1·7424	1	2	3	4	6	7	8	9	10
5·3	1·7435	1·7446	1·7457	1·7468	1·7479	1·7490	1·7501	1·7512	1·7522	1·7533	1	2	3	4	5	7	8	9	10
5·4	1·7544	1·7555	1·7566	1·7577	1·7587	1·7598	1·7609	1·7620	1·7630	1·7641	1	2	3	4	5	7	8	9	10

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

CUBE ROOTS.

From 5·500 to 10.

	0	1	2	3	4	5	6	7	8	9	*Mean Differences.*								
											1	2	3	4	5	6	7	8	9
5·5	1·7652	1·7662	1·7673	1·7684	1·7694	1·7705	1·7716	1·7726	1·7737	1·7748	1	2	3	4	5	6	7	8	10
5·6	1·7758	1·7769	1·7779	1·7790	1·7800	1·7811	1·7821	1·7832	1·7842	1·7853	1	2	3	4	5	6	7	8	9
5·7	1·7863	1·7874	1·7884	1·7894	1·7905	1·7915	1·7926	1·7936	1·7946	1·7957	1	2	3	4	5	6	7	8	9
5·8	1·7967	1·7977	1·7988	1·7998	1·8008	1·8018	1·8029	1·8039	1·8049	1·8059	1	2	3	4	5	6	7	8	9
5·9	1·8070	1·8080	1·8090	1·8100	1·8110	1·8121	1·8131	1·8141	1·8151	1·8161	1	2	3	4	5	6	7	8	9
6·0	1·8171	1·8181	1·8191	1·8201	1·8211	1·8222	1·8232	1·8242	1·8252	1·8262	1	2	3	4	5	6	7	8	9
6·1	1·8272	1·8282	1·8292	1·8302	1·8311	1·8321	1·8331	1·8341	1·8351	1·8361	1	2	3	4	5	6	7	8	9
6·2	1·8371	1·8381	1·8391	1·8400	1·8410	1·8420	1·8430	1·8440	1·8450	1·8459	1	2	3	4	5	6	7	8	9
6·3	1·8469	1·8479	1·8489	1·8498	1·8508	1·8518	1·8528	1·8537	1·8547	1·8557	1	2	3	4	5	6	7	8	9
6·4	1·8566	1·8576	1·8586	1·8595	1·8605	1·8615	1·8624	1·8634	1·8643	1·8653	1	2	3	4	5	6	7	8	9
6·5	1·8663	1·8672	1·8682	1·8691	1·8710	1·8710	1·8720	1·8729	1·8739	1·8748	1	2	3	4	5	6	7	8	9
6·6	1·8758	1·8767	1·8777	1·8786	1·8796	1·8805	1·8814	1·8824	1·8833	1·8843	1	2	3	4	5	6	7	8	8
6·7	1·8852	1·8861	1·8871	1·8880	1·8889	1·8899	1·8908	1·8917	1·8927	1·8936	1	2	3	4	5	6	7	8	8
6·8	1·8945	1·8955	1·8964	1·8973	1·8982	1·8992	1·9001	1·9010	1·9019	1·9029	1	2	3	4	5	6	7	8	8
6·9	1·9038	1·9047	1·9056	1·9065	1·9074	1·9084	1·9093	1·9102	1·9111	1·9120	1	2	3	4	5	6	7	8	8
7·0	1·9129	1·9133	1·9148	1·9157	1·9166	1·9175	1·9184	1·9193	1·9202	1·9211	1	2	3	4	5	5	6	7	8
7·1	1·9220	1·9229	1·9238	1·9247	1·9256	1·9265	1·9274	1·9283	1·9292	1·9301	1	2	3	4	5	5	6	7	8
7·2	1·9310	1·9319	1·9328	1·9337	1·9345	1·9354	1·9363	1·9372	1·9381	1·9390	1	2	3	4	4	5	6	7	8
7·3	1·9399	1·9408	1·9416	1·9425	1·9434	1·9443	1·9452	1·9461	1·9469	1·9478	1	2	3	4	4	5	6	7	8
7·4	1·9487	1·9496	1·9504	1·9513	1·9522	1·9531	1·9539	1·9548	1·9557	1·9566	1	2	3	3	4	5	6	7	8
7·5	1·9574	1·9583	1·9592	1·9600	1·9609	1·9618	1·9626	1·9635	1·9644	1·9652	1	2	3	3	4	5	6	7	8

7·6	1·9661	1·9670	1·9678	1·9687	1·9695	1·9704	1·9713	1·9721	1·9730	1·9738	1	2	3	3	4	5	6	7	8
7·7	1·9747	1·9755	1·9764	1·9772	1·9781	1·9789	1·9798	1·9806	1·9815	1·9823	1	2	3	3	4	5	6	7	8
7·8	1·9832	1·9840	1·9849	1·9857	1·9866	1·9874	1·9883	1·9891	1·9899	1·9908	1	2	3	3	4	5	6	7	8
7·9	1·9916	1·9925	1·9933	1·9941	1·9950	1·9958	1·9967	1·9975	1·9983	1·9992	1	2	3	3	4	5	6	7	8
8·0	2·0000	2·0008	2·0017	2·0025	2·0033	2·0042	2·0050	2·0058	2·0066	2·0075	1	2	2	3	4	5	6	7	7
8·1	2·0083	2·0091	2·0100	2·0108	2·0116	2·0124	2·0132	2·0141	2·0149	2·0157	1	2	2	3	4	5	6	7	7
8·2	2·0165	2·0173	2·0182	2·0190	2·0198	2·0206	2·0214	2·0223	2·0231	2·0239	1	2	2	3	4	5	6	7	7
8·3	2·0247	2·0255	2·0263	2·0271	2·0279	2·0288	2·0296	2·0304	2·0312	2·0320	1	2	2	3	4	5	6	7	7
8·4	2·0328	2·0336	2·0344	2·0352	2·0360	2·0368	2·0376	2·0384	2·0392	2·0400	1	2	2	3	4	5	6	6	7
8·5	2·0408	2·0416	2·0424	2·0432	2·0440	2·0448	2·0456	2·0464	2·0472	2·0480	1	2	2	3	4	5	6	6	7
8·6	2·0488	2·0496	2·0504	2·0512	2·0520	2·0528	2·0536	2·0543	2·0551	2·0559	1	2	2	3	4	5	6	6	7
8·7	2·0567	2·0575	2·0583	2·0591	2·0599	2·0606	2·0614	2·0622	2·0630	2·0638	1	2	2	3	4	5	6	6	7
8·8	2·0646	2·0653	2·0661	2·0669	2·0677	2·0685	2·0692	2·0700	2·0708	2·0716	1	2	2	3	4	5	5	6	7
8·9	2·0724	2·0731	2·0739	2·0747	2·0755	2·0762	2·0770	2·0778	2·0785	2·0793	1	2	2	3	4	5	5	6	7
9·0	2·0801	2·0809	2·0816	2·0824	2·0832	2·0839	2·0847	2·0855	2·0862	2·0870	1	2	2	3	4	5	5	6	7
9·1	2·0878	2·0885	2·0893	2·0901	2·0908	2·0916	2·0923	2·0931	2·0939	2·0946	1	2	2	3	4	5	5	6	7
9·2	2·0954	2·0961	2·0969	2·0977	2·0984	2·0992	2·0999	2·1007	2·1014	2·1022	1	2	2	3	4	5	5	6	7
9·3	2·1029	2·1037	2·1045	2·1052	2·1060	2·1067	2·1075	2·1082	2·1090	2·1097	1	2	2	3	4	5	5	6	7
9·4	2·1105	2·1112	2·1120	2·1127	2·1134	2·1142	2·1149	2·1157	2·1164	2·1172	1	1	2	3	4	4	5	6	7
9·5	2·1179	2·1187	2·1194	2·1201	2·1209	2·1216	2·1224	2·1231	2·1238	2·1246	1	1	2	3	4	4	5	6	7
9·6	2·1253	2·1261	2·1268	2·1275	2·1283	2·1290	2·1297	2·1305	2·1312	2·1319	1	1	2	3	4	4	5	6	7
9·7	2·1327	2·1334	2·1341	2·1349	2·1356	2·1363	2·1371	2·1378	2·1385	2·1392	1	1	2	3	4	4	5	6	7
9·8	2·1400	2·1407	2·1414	2·1422	2·1429	2·1436	2·1443	2·1451	2·1458	2·1465	1	1	2	3	4	4	5	6	6
9·9	2·1472	2·1480	2·1487	2·1494	2·1501	2·1508	2·1516	2·1523	2·1530	2·1537	1	1	2	3	4	4	5	6	6

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

CUBE ROOTS.

From 10 to 54·99.

	0	1	2	3	4	5	6	7	8	9	Mean Differences. 1	2	3	4	5	6	7	8	9
10	2·1544	2·1616	2·1687	2·1757	2·1828	2·1898	2·1967	2·2036	2·2104	2·2172	7	14	21	28	35	42	49	56	62
11	2·2240	2·2307	2·2374	2·2440	2·2506	2·2571	2·2637	2·2702	2·2766	2·2831	7	13	20	26	33	39	46	52	59
12	2·2894	2·2958	2·3021	2·3084	2·3146	2·3208	2·3270	2·3331	2·3391	2·3453	6	12	18	24	30	37	43	49	55
13	2·3513	2·3573	2·3633	2·3693	2·3752	2·3811	2·3870	2·3928	2·3986	2·4044	6	12	18	23	29	35	41	47	53
14	2·4101	2·4159	2·4216	2·4272	2·4329	2·4385	2·4441	2·4497	2·4552	2·4607	6	11	17	22	28	34	39	45	51
15	2·4662	2·4717	2·4771	2·4825	2·4879	2·4933	2·4987	2·5040	2·5093	2·5146	5	11	16	21	27	32	38	43	48
16	2·5198	2·5251	2·5303	2·5355	2·5407	2·5458	2·5510	2·5561	2·5612	2·5662	5	10	15	21	26	31	36	41	46
17	2·5713	2·5763	2·5813	2·5863	2·5913	2·5962	2·6012	2·6061	2·6110	2·6159	5	10	15	20	25	30	35	39	44
18	2·6207	2·6256	2·6304	2·6352	2·6400	2·6448	2·6495	2·6543	2·6590	2·6637	5	10	14	19	24	29	33	38	43
19	2·6684	2·6731	2·6777	2·6824	2·6870	2·6916	2·6962	2·7008	2·7053	2·7099	5	9	14	18	23	28	32	37	41
20	2·7144	2·7189	2·7234	2·7279	2·7324	2·7369	2·7413	2·7457	2·7501	2·7545	4	9	13	18	22	27	31	36	40
21	2·7589	2·7633	2·7677	2·7720	2·7763	2·7806	2·7849	2·7892	2·7935	2·7978	4	9	13	17	22	26	30	34	39
22	2·8020	2·8063	2·8105	2·8147	2·8189	2·8231	2·8273	2·8314	2·8356	2·8397	4	8	13	17	21	25	29	34	38
23	2·8439	2·8480	2·8521	2·8562	2·8603	2·8643	2·8684	2·8724	2·8765	2·9805	4	8	12	16	20	24	28	32	37
24	2·8845	2·8885	2·8925	2·8965	2·9004	2·9044	2·9083	2·9123	2·9162	2·9201	4	8	12	16	20	24	28	32	36
25	2·9240	2·9279	2·9318	2·9357	2·9395	2·9434	2·9472	2·9511	2·9549	2·9587	4	8	12	15	19	23	27	31	35
26	2·9625	2·9663	2·9701	2·9738	2·9776	2·9814	2·9851	2·9888	2·9926	2·9963	4	8	11	15	19	23	26	30	34
27	3·0000	3·0037	3·0074	3·0111	3·0147	3·0184	3·0221	3·0257	3·0293	3·0330	4	7	11	15	18	22	26	29	33
28	3·0366	3·0402	3·0438	3·0474	3·0510	3·0546	3·0581	3·0617	3·0652	3·0688	4	7	11	14	18	21	25	29	32
29	3·0723	3·0758	3·0794	3·0829	3·0864	3·0899	3·0934	3·0968	3·1003	3·1038	3	7	10	14	17	21	24	28	31
30	3·1072	3·1107	3·1141	3·1176	3·1210	3·1244	3·1278	3·1312	3·1346	3·1380	3	7	10	14	17	21	24	28	31

31	3·1414	3·1448	3·1481	3·1515	3·1548	3·1582	3·1615	3·1648	3·1682	3·1715	3	7	10	13	17	20	23	27	30
32	3·1748	3·1781	3·1814	3·1847	3·1880	3·1913	3·1945	3·1978	3·2010	3·2043	3	7	10	13	16	20	23	26	29
33	3·2075	3·2103	3·2140	3·2172	3·2204	3·2237	3·2269	3·2301	3·2333	3·2364	3	6	10	13	16	19	22	26	29
34	3·2396	3·2428	3·2460	3·2491	3·2522	3·2554	3·2586	3·2617	3·2648	3·2679	3	6	10	13	16	19	22	25	28
35	3·2711	3·2742	3·2773	3·2804	3·2835	3·2866	3·2897	3·2927	3·2958	3·2989	3	6	9	12	15	18	22	25	27
36	3·3019	3·3050	3·3080	3·3111	3·3141	3·3171	3·3202	3·3232	3·3262	3·3292	3	6	9	12	15	18	21	24	27
37	3·3322	3·3352	3·3382	3·3412	3·3442	3·3472	3·3501	3·3531	3·3561	3·3590	3	6	9	12	15	18	21	24	27
38	3·3620	3·3649	3·3679	3·3708	3·3737	3·3767	3·3796	3·3825	3·3854	3·3883	3	6	9	12	15	18	20	23	26
39	3·3912	3·3941	3·3970	3·3999	3·4028	3·4056	3·4085	3·4114	3·4142	3·4171	3	6	9	12	14	17	20	23	26
40	3·4200	3·4228	3·4256	3·4285	3·4313	3·4341	3·4370	3·4398	3·4426	3·4454	3	6	8	11	14	17	20	23	25
41	3·4482	3·4510	3·4538	3·4566	3·4594	3·4622	3·4650	3·4677	3·4705	3·4733	3	6	8	11	14	17	19	22	25
42	3·4760	3·4788	3·4815	3·4843	3·4870	3·4898	3·4925	3·4952	3·4980	3·5007	3	5	8	11	14	16	19	22	25
43	3·5034	3·5061	3·5088	3·5115	3·5142	3·5169	3·5196	3·5223	3·5250	3·5277	3	5	8	11	13	16	19	22	24
44	3·5303	3·5330	3·5357	3·5384	3·5410	3·5437	3·5463	3·5490	3·5516	3·5543	3	5	8	11	13	16	19	21	24
45	3·5569	3·5595	3·5622	3·5648	3·5674	3·5700	3·5726	3·5752	3·5778	3·5805	3	5	8	10	13	16	18	21	23
46	3·5830	3·5856	3·5882	3·5908	3·5934	3·5960	3·5986	3·6011	3·6037	3·6063	3	5	8	10	13	15	18	21	23
47	3·6088	3·6114	3·6139	3·6165	3·6190	3·6216	3·6241	3·6267	3·6292	3·6317	3	5	8	10	13	15	18	20	23
48	3·6342	3·6368	3·6393	3·6418	3·6443	3·6468	3·6493	3·6518	3·6543	3·6568	3	5	8	10	13	15	18	20	23
49	3·6593	3·6618	3·6643	3·6668	3·6692	3·6717	3·6742	3·6766	3·6791	3·6816	2	5	7	10	12	15	17	20	22
50	3·6840	3·6865	3·6889	3·6914	3·6938	3·6963	3·6987	3·7011	3·7036	3·7060	2	5	7	10	12	15	17	20	22
51	3·7084	3·7109	3·7133	3·7157	3·7181	3·7205	3·7229	3·7253	3·7277	3·7301	2	5	7	10	12	14	17	19	22
52	3·7325	3·7349	3·7373	3·7397	3·7421	3·7444	3·7468	3·7492	3·7516	3·7539	2	5	7	10	12	14	17	19	22
53	3·7563	3·7586	3·7610	3·7634	3·7657	3·7681	3·7704	3·7728	3·7751	3·7774	2	5	7	9	12	14	16	19	21
54	3·7798	3·7821	3·7844	5·7868	3·7891	3·7914	3·7937	3·7960	3·7983	3·8006	2	5	7	9	12	14	16	19	21

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

CUBE ROOTS.

FROM 55 TO 100.

	0	1	2	3	4	5	6	7	8	9	*Mean Differences.*								
											1	2	3	4	5	6	7	8	9
55	3·8030	3·8053	3·8076	3·8099	3·8121	3·8144	3·8167	3·8190	3·8213	3·8236	2	5	7	9	11	14	16	18	21
56	3·8259	3·8281	3·8304	3·8327	3·8350	3·8372	3·9395	3·8417	3·8440	3·8463	2	5	7	9	11	14	16	18	20
57	3·8485	3·8508	3·8530	3·8552	3·8575	3·8597	3·8620	3·8642	3·8664	3·8687	2	4	7	9	11	13	16	18	20
58	3·8709	3·8731	3·8753	3·8775	3·8798	3·8820	3·8842	3·8864	3·8886	3·8908	2	4	7	9	11	13	15	18	20
59	3·8930	3·8952	3·8974	3·8996	3·9018	3·9040	3·9061	3·9083	3·9105	3·9127	2	4	7	9	11	13	15	18	20
60	3·9149	3·9170	3·9192	3·9214	3·9235	3·9257	3·9279	3·9300	3·9322	3·9343	2	4	6	9	11	13	15	17	19
61	3·9365	3·9386	3·9408	3·9429	3·9451	3·9472	3·9494	3·9515	3·9536	3·9558	2	4	6	9	11	13	15	17	19
62	3·9579	3·9600	3·9621	3·9643	3·9664	3·9685	3·9706	3·9727	3·9748	3·9770	2	4	6	8	11	13	15	17	19
63	3·9791	3·9812	3·9833	3·9854	3·9875	3·9896	3·9916	3·9937	3·9958	3·9979	2	4	6	8	10	13	15	17	19
64	4·0000	4·0021	4·0042	4·0062	4·0083	4·0104	4·0125	4·0145	4·0166	4·0187	2	4	6	8	10	12	14	17	19
65	4·0207	4·0228	4·0248	4·0269	4·0290	4·0310	4·0331	4·0351	4·0372	4·0392	2	4	6	8	10	12	14	16	18
66	4·0412	4·0433	4·0453	4·0474	4·0494	4·0514	4·0535	4·0555	4·0575	4·0595	2	4	6	8	10	12	14	16	18
67	4·0615	4·0636	4·0656	4·0676	4·0696	4·0716	4·0736	4·0756	4·0777	4·0797	2	4	6	8	10	12	14	16	18
68	4·0817	4·0837	4·0857	4·0877	4·0896	4·0916	4·0936	4·0956	4·0976	4·0996	2	4	6	8	10	12	14	16	18
69	4·1016	4·1035	4·1055	4·1075	4·1095	4·1115	4·1134	4·1154	4·1174	4·1193	2	4	6	8	10	12	14	16	18
70	4·1213	4·1232	4·1252	4·1272	4·1291	4·1311	4·1330	4·1350	4·1370	4·1389	2	4	6	8	10	12	14	16	18
71	4·1408	4·1428	4·1447	4·1466	4·1486	4·1505	4·1524	4·1544	4·1563	4·1582	2	4	6	8	10	12	14	16	17
72	4·1602	4·1621	4·1640	4·1659	4·1679	4·1698	4·1717	4·1736	4·1755	4·1774	2	4	6	8	10	11	13	15	17
73	4·1793	4·1812	4·1832	4·1851	4·1870	4·1889	4·1908	4·1927	4·1946	4·1964	2	4	6	8	10	11	13	15	17
74	4·1983	4·2002	4·2021	4·2040	4·2059	4·2078	4·2097	4·2115	4·2134	4·2153	2	4	6	8	9	11	13	15	17
75	4·2172	4·2190	4·2209	4·2228	4·2246	4·2265	4·2284	4·2302	4·2321	4·2340	2	4	6	7	9	11	13	15	17

76	4·2358	4·2377	4·2395	4·2414	4·2432	4·2451	4·2469	4·2488	4·2506	4·2525	2	4	6	7	9	11	13	15	17
77	4·2543	4·2562	4·2580	4·2598	4·2617	4·2635	4·2653	4·2672	4·2690	4·2708	2	4	6	7	9	11	13	15	17
78	4·2727	4·2745	4·2763	4·2781	4·2799	4·2818	4·2836	4·2854	4·2872	4·2890	2	4	5	7	9	11	13	14	16
79	4·2908	4·2927	4·2945	4·2963	4·2981	4·2999	4·3017	4·3035	4·3053	4·3071	2	4	5	7	9	11	13	14	16
80	4·3089	4·3107	4·3125	4·3143	4·3160	4·3178	4·3196	4·3214	4·3232	4·3250	2	4	5	7	9	11	13	14	16
81	4·3267	4·3285	4·3303	4·3321	4·3339	4·3356	4·3374	4·3392	4·3409	4·3427	2	4	5	7	9	11	12	14	16
82	4·3445	4·3462	4·3480	4·3498	4·3515	4·3533	4·3551	4·3568	4·3586	4·3603	2	4	5	7	9	11	12	14	16
83	4·3621	4·3638	4·3656	4·3673	4·3691	4·3708	4·3726	4·3743	4·3760	4·3778	2	3	5	7	9	10	12	14	16
84	4·3795	4·3813	4·3830	4·3847	4·3865	4·3882	4·3899	4·3917	4·3934	4·3951	2	3	5	7	9	10	12	14	16
85	4·3968	4·3986	4·4003	4·4020	4·4037	4·4054	4·4072	4·4089	4·4106	4·4123	2	3	5	7	9	10	12	14	16
86	4·4140	4·4157	4·4174	4·4191	4·4208	4·4225	4·4242	4·4259	4·4276	4·4293	2	3	5	7	9	10	12	14	15
87	4·4310	4·4327	4·4344	4·4361	4·4378	4·4395	4·4412	4·4429	4·4446	4·4462	2	3	5	7	9	10	12	14	15
88	4·4480	4·4496	4·4513	4·4530	4·4547	4·4564	4·4580	4·4597	4·4614	4·4631	2	3	5	7	8	10	12	13	15
89	4·4647	4·4664	4·4681	4·4698	4·4714	4·4731	4·4748	4·4764	4·4781	4·4797	2	3	5	7	8	10	12	13	15
90	4·4814	4·4831	4·4847	4·4864	4·4880	4·4897	4·4913	4·4930	4·4946	4·4963	2	3	5	7	8	10	12	13	15
91	4·4979	4·4996	4·5012	4·5029	4·5045	4·5062	4·5078	4·5094	4·5111	4·5127	2	3	5	7	8	10	12	13	15
92	4·5144	4·5160	4·5176	4·5193	4·5209	4·5225	4·5242	4·5258	4·5274	4·5290	2	3	5	7	8	10	11	13	15
93	4·5307	4·5323	4·5339	4·5355	4·5371	4·5388	4·5404	4·5420	4·5436	4·5452	2	3	5	6	8	10	11	13	14
94	4·5468	4·5484	4·5501	4·5517	4·5533	4·5549	4·5565	4·5581	4·5597	4·5613	2	3	5	6	8	10	11	13	14
95	4·5629	4·5645	4·5661	4·5677	4·5693	4·5709	4·5725	4·5741	4·5757	4·5773	2	3	5	6	8	10	11	13	14
96	4·5789	4·5804	4·5820	4·5836	4·5852	4·5868	4·5884	4·5900	4·5915	4·5931	2	3	5	6	8	9	11	13	14
97	4·5947	4·5963	4·5979	4·5994	4·6010	4·6026	4·6042	4·6057	4·6073	4·6089	2	3	5	6	8	9	11	13	14
98	4·6104	4·6120	4·6136	4·6151	4·6167	4·6183	4·6198	4·6214	4·6229	4·6245	2	3	5	6	8	9	11	13	14
99	4·6261	4·6276	4·6292	4·6307	4·6323	4·6338	4·6354	4·6369	4·6385	4·6400	2	3	5	6	8	9	11	12	14

We are indebted to the courtesy of Messrs. Macmillan and Co., Ltd., for permission to quote these tables from "Five Figure Logarithmic and other Tables," by Frank Castle.

CUBE ROOTS.

From 100 to 549.

	0	1	2	3	4	5	6	7	8	9
100	4·6416	4·6570	4·6723	4·6875	4·7027	4·7177	4·7326	4·7475	4·7622	4·7769
110	4·7914	4·8059	4·8203	4·8346	4·8488	4·8629	4·8770	4·8910	4·9049	4·9187
120	4·9324	4·9461	4·9597	4·9732	4·9866	5·0000	5·0133	5·0265	5·0397	5·0528
130	5·0658	5·0788	5·0916	5·1045	5·1172	5·1299	5·1426	5·1551	5·1676	5·1801
140	5·1925	5·2048	5·2171	5·2293	5·2415	5·2536	5·2656	5·2776	5·2896	5·3015
150	5·3133	5·3251	5·3368	5·3485	5·3601	5·3717	5·3832	5·3947	5·4061	5·4175
160	5·4288	5·4401	5·4514	5·4626	5·4737	5·4848	5·4959	5·5069	5·5178	5·5288
170	5·5397	5·5505	5·5613	5·5721	5·5828	5·5934	5·6041	5·6147	5·6252	5·6357
180	5·6462	5·6567	5·6671	5·6774	5·6877	5·6980	5·7083	5·7185	5·7287	5·7388
190	5·7489	5·7590	5·7690	5·7790	5·7890	5·7989	5·8088	5·8186	5·8285	5·8383
200	5·8480	5·8578	5·8675	5·8771	5·8868	5·8964	5·9059	5·9155	5·9250	5·9345
210	5·9439	5·9533	5·9627	5·9721	5·9814	5·9907	6·0000	6·0092	6·0185	6·0277
220	6·0368	6·0459	6·0550	6·0641	6·0732	6·0822	6·0912	6·1002	6·1091	6·1180
230	6·1269	6·1358	6·1446	6·1534	6·1622	6·1710	6·1797	6·1885	6·1972	6·2058
240	6·2145	6·2231	6·2317	6·2403	6·2488	6·2573	6·2658	6·2743	6·2828	6·2912
250	6·2996	6·3080	6·3164	6·3247	6·3330	6·3413	6·3496	6·3579	6·3661	6·3743
260	6·3825	6·3907	6·3988	6·4070	6·4151	6·4232	6·4312	6·4393	6·4473	6·4553
270	6·4633	6·4713	6·4792	6·4872	6·4951	6·5030	6·5108	6·5187	6·5265	6·5343
280	6·5421	6·5499	6·5577	6·5654	6·5731	6·5808	6·5885	6·5962	6·6039	6·6115
290	6·6191	6·6267	6·6343	6·6419	6·6494	6·6569	6·6644	6·6719	6·6794	6·6869
300	6·6943	6·7018	6·7092	6·7166	6·7240	6·7313	6·7387	6·7460	6·7533	6·7606

310	6·7679	6·7752	6·7824	6·7897	6·7969	6·8041	6·8113	6·8185	6·8256	6·8328
320	6·8399	6·8470	6·8541	6·8612	6·8683	6·8753	6·8842	6·8894	6·8964	6·9034
330	6·9104	6·9174	6·9244	6·9313	6·9382	6·9451	6·9521	6·9589	6·9658	6·9727
340	6·9795	6·9864	6·9932	7·0000	7·0068	7·0136	7·0203	7·0271	7·0338	7·0406
350	7·0473	7·0540	7·0607	7·0674	7·0740	7·0807	7·0873	7·0940	7·1006	7·1072
360	7·1138	7·1204	7·1269	7·1335	7·1400	7·1466	7·1531	7·1596	7·1661	7·1726
370	7·1791	7·1855	7·1920	7·1984	7·2048	7·2112	7·2177	7·2240	7·2304	7·2368
380	7·2432	7·2495	7·2558	7·2622	7·2685	7·2748	7·2811	7·2874	7·2936	7·2999
390	7·3061	7·3124	7·3186	7·3248	7·3310	7·3372	7·3434	7·3496	7·3558	7·3619
400	7·3681	7·3742	7·3803	7·3864	7·3925	7·3986	7·4047	7·4108	7·4169	7·4229
410	7·4290	7·4350	7·4410	7·4470	7·4530	7·4590	7·4650	7·4710	7·4770	7·4829
420	7·4889	7·4948	7·5007	7·5067	7·5126	7·5185	7·5244	7·5302	7·5361	7·5420
430	7·5478	7·5537	7·5595	7·5654	7·5712	7·5770	7·5828	7·5886	7·5944	7·6001
440	7·6059	7·6117	7·6174	7·6232	7·6289	7·6346	7·6403	7·6460	7·6517	7·6574
450	7·6631	7·6688	7·6744	7·6801	7·6857	7·6914	7·6970	7·7026	7·7082	7·7138
460	7·7194	7·7250	7·7306	7·7362	7·7418	7·7473	7·7529	7·7584	7·7639	7·7695
470	7·7750	7·7805	7·7860	7·7915	7·7970	7·8025	7·8079	7·8134	7·8188	7·8243
480	7·8297	7·8352	7·8406	7·8460	7·8514	7·8568	7·8622	7·8676	7·8730	7·8784
490	7·8837	7·8891	7·8944	7·8998	7·9051	7·9105	7·9158	7·9211	7·9264	7·9317
500	7·9370	7·9423	7·9476	7·9528	7·9581	7·9634	7·9686	7·9739	7·9791	7·9843
510	7·9896	7·9948	8·0000	8·0052	8·0104	8·0156	8·0208	8·0260	8·0311	8·0363
520	8·0415	8·0466	8·0517	8·0569	8·0620	8·0671	8·0723	8·0774	8·0825	8·0876
530	8·0927	8·0978	8·1028	8·1079	8·1130	8·1180	8·1231	8·1281	8·1332	8·1382
540	8·1433	8·1483	8·1533	8·1583	8·1633	8·1683	8·1733	8·1783	8·1833	8·1882

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CUBE ROOTS.

From 550 to 1000.

	0	1	2	3	4	5	6	7	8	9
550	8·1932	8·1982	8·2031	8·2081	8·2130	8·2180	8·2229	8·2278	8·2327	8·2377
560	8·2426	8·2475	8·2524	8·2573	8·2621	8·2670	8·2719	8·2768	8·2816	8 2865
570	8·2913	8·2962	8·3010	8·3059	8·3107	8·3155	8·3203	8·3251	8·3300	8·3348
580	8·3396	8·3443	8·3491	8·3539	8·3587	8·3634	8·3682	8·3730	8·3777	8·3825
590	8·3872	8·3919	8·3967	8·4014	8·4061	8·4108	8·4155	8·4202	8·4249	8·4296
600	8·4343	8·4390	8·4437	8·4484	8·4530	8·4577	8·4623	8·4670	8·4716	8·4763
610	8·4809	8·4856	8·4902	8·4948	8·4994	8·5040	8·5086	8·5132	8·5178	8·5224
620	8·5270	8·5316	8·5362	8·5408	8·5453	8·5499	8·5544	8·5590	8·5635	8·5681
630	8·5726	8·5772	8·5817	8·5862	8·5907	8·5952	8·5997	8·6043	8·6088	8·6132
640	8·6177	8·6222	8·6267	8·6312	8·6357	8·6401	8·6446	8·6490	8·6535	8·6579
650	8·6624	8·6668	8·6713	8·6757	8·6801	8·6845	8·6890	8·6934	8·6978	8·7022
660	8·7066	8·7110	8·7154	8·7198	8·7241	8·7285	8·7329	8·7373	8·7416	8·7460
670	8·7503	8·7547	8·7590	8·7634	8·7677	8·7721	8·7764	8·7807	8·7850	8·7893
680	8·7937	8·7980	8·8023	8·8066	8·8109	8·8152	8·8194	8·8237	8·8280	8·8323
690	8·8366	8·8408	8·8451	8·8493	8·8536	8·8578	8·8621	8·8663	8·8706	8·8748
700	8·8790	8·8833	8·8875	8·8917	8·8959	8·9001	8·9043	8·9085	8·9127	8·9169
710	8·9211	8·9253	8·9295	8·9337	8·9378	8·9420	8·9462	8·9503	8·9545	8·9587
720	8·9628	8·9670	8·9711	8·9752	8·9794	8·9835	8·9876	8·9918	8·9959	9·0000
730	9·0041	9·0082	9·0123	9·0164	9·0205	9·0246	9·0287	9·0328	9·0369	9·0410
740	9·0450	9·0491	9·0532	9·0572	9·0613	9·0654	9·0694	9·0735	9·0775	9·0816
750	9·0856	9·0896	9·0937	9·0977	9·1017	9·1057	9·1098	9·1138	9·1178	9·1218

760	9·1258	9·1298	9·1338	9·1378	9·1418	9·1458	9·1498	9·1537	9·1577	9·1617
770	9·1657	9·1696	9·1736	9·1775	9·1815	9·1855	9·1894	9·1933	9·1973	9·2012
780	9·2052	9·2091	9·2130	9·2170	9·2209	9·2248	9·2287	9·2326	9·2365	9·2404
790	9·2443	9·2482	9·2521	9·2560	9·2599	9·2638	9·2677	9·2716	9·2754	9·2793
800	9·2832	9·2870	9·2909	9·2948	9·2986	9·3025	9·3063	9·3102	9·3140	9·3179
810	9·3217	9·3255	9·3294	9·3332	9·3370	9·3408	9·3447	9·3485	9·3523	9·3561
820	9·3599	9·3637	9·3675	9·3713	9·3751	9·3789	9·3827	9·3865	9·3902	9·3940
830	9·3978	9·4016	9·4053	9·4091	9·4129	9·4166	9·4204	9·4241	9·4279	9·4316
840	9·4354	9·4391	9·4429	9·4466	9·4503	9·4541	9·4578	9·4615	9·4652	9·4690
850	9·4727	9·4764	9·4801	9·4838	9·4875	9·4912	9·4949	9·4986	9·5023	9·5060
860	9·5097	9·5134	9·5171	9·5287	9·5244	9·5281	9·5317	9·5354	9·5391	9·5427
870	9·5464	9·5501	9·5537	9·5574	9·5610	9·5647	9·5683	9·5719	9·5756	9·5792
880	9·5828	9·5865	9·5901	9·5937	9·5973	9·6010	9·6046	9·6082	9·6118	9·6154
890	9·6190	9·6226	9·6262	9·6298	9·6334	9·6370	9·6406	9·6442	9·6477	9·6513
900	9·6549	9·6585	9·6620	9·6656	9·6692	9·6727	9·6763	9·6799	9·6834	9·6870
910	9·6905	9·6941	9·6976	9·7012	9·7047	9·7082	9·7118	9·7153	9·7188	9·7224
920	9·7259	9·7294	9·7329	9·7364	9·7400	9·7435	9·7470	9·7505	9·7540	9·7575
930	9·7610	9·7645	9·7680	9·7715	9·7750	9·7785	9·7819	9·7854	9·7889	9·7924
940	9·7959	9·7993	9·8028	9·8063	9·8097	9·8132	9·8167	9·8201	9·8236	9·8270
950	9·8305	9·8339	9·8374	9·8408	9·8443	9·8477	9·8511	9·8546	9·8580	9·8614
960	9·8648	9·8683	9·8717	9·8751	9·8785	9·8819	9·8854	9·8888	9·8922	9·8956
970	9·8990	9·9024	9·9058	9·9092	9·9126	9·9160	9·9194	9·9227	9·9261	9·9295
980	9·9329	9·9363	9·9396	9·9430	9·9464	9·9497	9·9531	9·9565	9·9598	9·9632
990	9·9666	9·9699	9·9733	9·9766	9·9800	9·9833	9·9866	9·9900	9·9933	9·9967

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AREAS OF CIRCLES.

Diameters advancing by Tenths.

For a movement by one place of the decimal point in the diameter, move it two places in the same direction in the area, and so on.

Diameter.	0	1	2	3	4	5	6	7	8	9
10	78·540	80·118	81·713	83·323	84·949	86·590	88·247	89·920	91·609	93·313
11	95·033	96·769	98·520	100·29	102·07	103·87	105·68	107·51	109·36	111·22
12	113·10	114·99	116·90	118·82	120·76	122·72	124·69	126·68	128·68	130·70
13	132·73	134·78	136·85	138·93	141·03	143·14	145·27	147·41	149·57	151·75
14	153·94	156·14	158·37	160·61	162·86	165·13	167·41	169·72	172·03	174·37
15	176·71	179·08	181·46	183·85	186·26	188·69	191·13	193·59	196·07	198·56
16	201·06	203·58	206·12	208·67	211·24	213·82	216·42	219·04	221·67	224·32
17	226·98	229·66	232·35	235·06	237·79	240·53	243·28	246·06	248·85	251·65
18	254·47	257·30	260·15	263·02	265·90	268·80	271·72	274·65	277·59	280·55
19	283·53	286·52	289·53	292·55	295·59	298·65	301·72	304·80	307·91	311·02
20	314·16	317·31	320·47	323·65	326·85	330·06	333·29	336·53	339·79	343·07
21	346·36	349·67	352·99	356·33	359·68	363·05	366·43	369·84	373·25	376·68
22	380·13	383·60	387·08	390·57	394·08	397·61	401·15	404·71	408·28	411·87
23	415·43	419·10	422·73	426·38	430·05	433·74	437·43	441·15	444·88	448·63
24	452·39	456·17	459·96	463·77	467·59	471·43	475·29	479·16	483·05	486·95
25	490·87	494·81	498·76	502·73	506·71	510·70	514·72	516·75	522·79	526·85
26	530·93	535·02	539·13	543·25	547·39	551·55	555·72	559·90	564·10	568·32
27	572·56	576·80	581·07	585·35	589·65	593·96	598·28	602·63	606·99	611·36
28	615·75	620·16	624·58	629·02	633·47	637·94	642·42	646·92	651·44	655·97
29	660·52	665·08	669·66	674·26	678·87	684·49	688·13	692·79	697·46	702·15
30	706·86	711·53	716·31	721·07	725·83	730·62	735·41	740·23	745·06	749·91

31	754·77	759·64	764·54	769·45	774·37	779·31	784·27	789·24	794·23	799·23
32	804·25	809·28	814·33	819·40	824·48	829·58	834·69	839·82	844·96	850·12
33	855·30	860·49	865·70	870·92	876·16	881·41	886·68	891·97	897·27	902·59
34	907·92	913·27	918·63	924·01	929·41	934·82	940·25	945·69	951·15	956·62
35	962·11	967·62	973·14	978·68	984·23	989·80	995·38	1000·98	1006·6	1012·2
36	1017·9	1023·5	1029·2	1034·9	1040·6	1046·3	1052·1	1057·8	1063·6	1069·4
37	1075·2	1081·0	1086·9	1092·7	1098·6	1104·5	1110·4	1116·3	1122·2	1128·1
38	1134·1	1140·1	1146·1	1152·1	1158·1	1164·2	1170·2	1176·3	1182·4	1188·5
39	1194·6	1200·7	1206·9	1213·0	1219·2	1225·4	1231·6	1237·9	1244·1	1250·4
40	1256·6	1262·9	1269·2	1275·6	1281·9	1288·2	1294·6	1301·0	1307·4	1313·8
41	1320·3	1326·7	1333·2	1339·6	1346·1	1352·6	1359·2	1365·7	1372·3	1378·8
42	1385·4	1392·0	1398·7	1405·3	1412·0	1418·6	1425·3	1432·0	1438·7	1445·4
43	1452·2	1459·0	1465·7	1472·5	1479·3	1486·2	1493·0	1499·9	1506·7	1513·6
44	1520·5	1527·4	1534·4	1541·3	1548·3	1555·3	1562·3	1569·3	1576·3	1583·4
45	1590·4	1597·5	1604·6	1611·7	1618·8	1626·0	1633·1	1640·3	1647·5	1654·7
46	1661·9	1669·1	1676·4	1683·6	1690·9	1698·2	1705·5	1712·9	1720·2	1727·6
47	1734·9	1742·3	1749·7	1757·2	1764·6	1772·0	1779·5	1787·0	1794·5	1802·0
48	1809·6	1817·1	1824·7	1832·2	1839·8	1847·4	1855·1	1862·7	1870·4	1878·0
49	1885·7	1893·4	1901·2	1908·9	1916·6	1924·4	1932·2	1940·0	1947·8	1955·6
50	1963·5	1971·4	1979·2	1987·1	1995·0	2003·0	2010·9	2018·9	2026·8	2034·8
51	2042·8	2050·8	2058·9	2066·9	2075·0	2083·1	2091·2	2099·3	2107·4	2115·6
52	2123·7	2131·9	2140·1	2148·3	2156·5	2164·7	2173·0	2181·3	2189·6	2197·9
53	2206·2	2214·5	2222·9	2231·2	2239·6	2248·0	2256·4	2264·8	2273·3	2281·7
54	2290·2	2298·7	2307·2	2315·7	2324·3	2332·8	2341·4	2350·0	2358·6	2367·2

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AREAS OF CIRCLES.

DIAMETERS ADVANCING BY TENTHS.

For a movement by one place of the decimal point in the diameter, move it two places in the same direction in the area, and so on.

Diameter.	0	1	2	3	4	5	6	7	8	9
55	2375·8	2384·5	2393·1	2401·8	2410·5	2419·2	2428·0	2436·7	2445·4	2454·2
56	2463·0	2471·8	2480·6	2489·5	2498·3	2507·2	2516·1	2525·0	2533·9	2542·8
57	2551·8	2560·7	2569·7	2578·7	2587·7	2596·7	2605·8	2614·8	2623·9	2633·0
58	2642·1	2651·2	2660·3	2669·5	2678·6	2687·8	2697·0	2706·2	2715·5	2724·7
59	2734·0	2743·2	2752·5	2761·8	2771·2	2780·5	2789·9	2799·2	2808·6	2818·0
60	2827·4	2836·9	2846·3	2855·8	2865·3	2874·7	2884·3	2893·8	2903·3	2912·9
61	2922·5	2932·1	2941·7	2951·3	2960·9	2970·6	2980·2	2989·9	2999·6	3009·3
62	3019·1	3028·8	3038·6	3048·4	3058·1	3068·0	3077·8	3087·6	3097·5	3107·4
63	3117·2	3127·1	3137·1	3147·0	3157·0	3166·9	3176·9	3186·9	3196·9	3206·9
64	3217·0	3227·1	3237·1	3247·2	3257·3	3267·4	3277·6	3287·7	3297·9	3308·1
65	3318·3	3328·5	3338·8	3349·0	3359·3	3369·6	3379·8	3390·2	3400·5	3410·8
66	3421·2	3431·6	3442·0	3452·4	3462·8	3473·2	3483·7	3494·1	3504·6	3515·1
67	3525·7	3536·2	3546·7	3557·3	3567·9	3578·5	3589·1	3599·7	3610·3	3621·0
68	3631·7	3642·4	3653·1	3663·8	3674·5	3685·3	3696·1	3706·8	3717·6	3728·4
69	3739·3	3750·1	3761·0	3771·9	3782·8	3793·7	3804·6	3815·5	3826·5	3837·5
70	3848·5	3859·4	3870·5	3881·5	3892·6	3903·6	3914·7	3925·8	3936·9	3948·0
71	3959·2	3970·3	3981·5	3992·7	4003·9	4015·1	4026·4	4037·6	4048·9	4060·2
72	4071·5	4082·8	4094·2	4105·5	4116·9	4128·2	4139·6	4151·1	4162·5	4173·9
73	4185·4	4196·9	4208·3	4219·9	4231·4	4242·9	4254·5	4266·0	4277·6	4289·2
74	4300·8	4312·5	4324·1	4335·8	4347·5	4359·2	4370·9	4382·6	4394·3	4406·1
75	4417·9	4429·6	4441·5	4453·3	4465·1	4477·0	4488·8	4500·7	4512·6	4524·5

76	4536·5	4548·4	4560·4	4572·3	4584·3	4596·3	4608·4	4620·4	4632·5	4644·5
77	4656·6	4668·7	4680·8	4693·0	4705·1	4717·3	4729·5	4741·7	4753·9	4766·1
78	4778·4	4790·6	4802·9	4815·2	4827·5	4839·8	4852·2	4864·5	4876·9	4889·3
79	4901·7	4914·1	4926·5	4939·0	4951·4	4963·9	4976·4	4988·9	5001·4	5014·0
80	5026·6	5039·1	5051·7	5064·3	5076·9	5089·6	5102·2	5114·9	5127·6	5140·3
81	5153·0	5165·7	5178·5	5191·2	5204·0	5216·8	5229·6	5242·4	5255·3	5268·1
82	5281·0	5293·9	5306·8	5319·7	5332·7	5345·6	5358·6	5371·6	5384·6	5397·6
83	5410·6	5423·6	5436·7	5449·8	5462·9	5476·0	5489·1	5502·3	5515·4	5528·6
84	5541·8	5555·0	5568·2	5581·4	5594·7	5607·9	5621·2	5634·5	5647·8	5661·2
85	5674·5	5687·9	5701·2	5714·6	5728·0	5741·5	5754·9	5768·3	5781·8	5795·3
86	5808·8	5822·3	5835·8	5849·4	5863·0	5876·6	5890·1	5903·7	5917·4	5931·0
87	5944·7	5958·3	5972·0	5985·7	5999·5	6013·2	6027·0	6040·7	6054·5	6068·3
88	6082·1	6096·0	6109·8	6123·7	6137·5	6151·4	6165·3	6179·3	6193·2	6207·2
89	6221·1	6235·1	6249·1	6263·1	6277·2	6291·2	6305·3	6319·4	6333·5	6347·6
90	6361·7	6375·9	6390·0	6404·2	6418·4	6432·6	6446·8	6461·1	6475·3	6489·6
91	6503·9	6518·2	6532·5	6546·8	6561·2	6575·6	6589·9	6604·3	6618·7	6633·2
92	6647·6	6662·1	6676·5	6691·0	6705·5	6720·1	6734·6	6749·1	6763·7	6778·3
93	6792·9	6807·5	6822·2	6836·8	6851·5	6866·1	6880·8	6895·6	6910·3	6925·0
94	6939·8	6954·6	6969·3	6984·1	6999·0	7013·8	7028·7	7043·5	7058·4	7073·3
95	7088·2	7103·1	7118·1	7133·1	7148·0	7163·0	7178·0	7193·1	7208·1	7223·2
96	7238·2	7253·3	7268·4	7283·5	7298·7	7313·8	7329·0	7344·2	7359·4	7374·6
97	7389·8	7405·1	7420·3	7435·6	7450·9	7466·2	7481·5	7496·9	7512·2	7527·6
98	7543·0	7558·4	7573·8	7589·2	7604·7	7620·1	7635·6	7651·1	7666·6	7682·1
99	7697·7	7713·2	7728·8	7744·4	7760·0	7775·6	7791·3	7806·9	7822·6	7838·3

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USEFUL DATA.

I. CONVERSION FACTORS FOR LENGTHS, ETC.

1. *Numerical.*

$\pi = 3{\cdot}1416$.
$\pi^2 = 9{\cdot}8696$.
$\sqrt{\pi} = 1{\cdot}7725$.

$e = 2{\cdot}7183$.
$\log_e N = 2{\cdot}3026 \log_{10} N$.
1 radian $= 57{\cdot}296°$.

2. *Length.*

1 nautical mile = 1·15 statute miles = 6,080 feet.
1 centimetre = 0·394 inch = 0·0328 foot.
1 metre = 3·281 feet.
1 kilometre = 0·6214 mile.
1 inch = 2·54 centimetres.
1 foot = 30·48 centimetres.
1 mile = 1·609 kilometres.

3. *Area.*

1 $\text{cm.}^2 = 0{\cdot}1550\ \text{in.}^2$.
1 $\text{in.}^2 = 6{\cdot}452\ \text{cm.}^2$.

4. *Volume.*

1 $\text{cm.}^3 = 0{\cdot}0609\ \text{in.}^3$.
1 $\text{in.}^3 = 16{\cdot}387\ \text{cm.}^3$.
1 imperial gallon $= 277{\cdot}3\ \text{in.}^3$.

5. *Mass.*

1 kilogramme = 2·205 pounds.
1 French ton = 1·016 tons.
1 cubic foot of water weighs 62·3 pounds.

6. *Velocity.*

60 miles per hour = 88 feet per second.
1 knot = 1·15 miles per hour = 1·7 feet per second.
1 mile per hour = 44·7 centimetres per second.

7. *Power.*

1 horse-power = 33,000 foot-pounds per minute.
" " = 550 foot-pounds per second.
" " = 746 watts.

II. WEIGHTS.

1. Weights of Materials, Pounds per Cubic Inch.

(i.) *Metals and Alloys.*

Aluminium, cast	0·092	Iron, cast ..	from 0·252 to 0·273
,, sheet	0·096	,, wrought	from 0·273 to 0·281
Brass	0·30	Lead	0·410
Copper, sheet	0·316	Mercury	0·491
,, wire	0·320	Steel	0·283
Duralumin	0·102	Tin	0·262
Gun-metal	0·300	Zinc	0·248

(ii.) *Liquids.*

Alcohol	0·028	Petrol	0·033
Lubricating oil	0·034	Waler, distilled	0·036

(iii.) *Woods.*

Ash	0·026	Lignum vitæ	0·040
Bamboo	0·013	Mahogany, Honduras ..	0·020
Beech	0·027	,, Spanish	0·031
Birch	0·026	Oak, African	0·035
Boxwood	0·046	,, American, red ..	0·030
Cedar, American	0·021	,, ,, white ..	0·028
,, West Indian ..	0·026	,, English from 0·028 to	0·034
Cork	0·0087	Pine, yellow	0·018
Ebony	0·043	Poplar	0·013
Elm, English	0·021	Spruce	0·017
,, Canadian	0·026	Teak	0·029
Fir	0·0203	Walnut	0·024
Hickory	0·029		

(iv.) *Miscellaneous.*

Celluloid	0·0442	Glass, plate	0·100
Cork	0·0087	Gutta-percha	0·035
Ebonite	0·050	India-rubber, pure	0·034
Felt	0·0061	,, for joints ..	0·09
Fibre	0·051	Ivory	0·066
Glass, crown	0·091	Leather	0·035
,, flint	0·111	Vulcanite	0·055

2. Weight of Metal Sheets.

Gauge S.W.G.	*Thickness in Inches.*	*Steel.*		*Aluminium.*		*Duralumin.*	
		Weight per Square Inch.	*Weight per Square Foot.*	*Weight per Square Inch.*	*Weight per Square Foot.*	*Weight per Square Inch.*	*Weight per Square Foot.*
		Pounds.	*Pounds.*	*Pounds.*	*Pounds.*	*Pounds.*	*Pounds.*
0	0·324	0·09170	13·204	0·0312	4·500	0·03338	4·806
1	0·300	0·08492	12·222	0·0289	4·170	0·03091	4·450
2	0·276	0·07811	11·246	0·0267	3·840	0·02843	4·094
3	0·252	0·07132	10·269	0·0244	3·510	0·02596	3·738
4	0·232	0·06566	9·452	0·0214	3·080	0·02390	3·441
5	0·212	0·06000	8·639	0·0198	2·810	0·02183	3·144
6	0·192	0·05434	7·822	0·0176	2·540	0·01978	2·848
7	0·176	0·04982	7·170	0·0162	2·340	0·01813	2·611
8	0·160	0·04528	6·519	0·0147	2·120	0·01648	2·373
9	0·144	0·04075	5·867	0·0133	1·910	0·01483	2·136
10	0·128	0·03624	5·215	0·0118	1·700	0·01319	1·899
11	0·116	0·03283	4·726	0·0107	1·540	0·01195	1·721
12	0·104	0·02943	4·238	0·0096	1·380	0·01071	1·542
13	0·092	0·02640	3·748	0·0089	1·283	0·00948	1·365
14	0·080	0·02267	3·260	0·0077	1·115	0·00824	1·187
15	0·072	0·02038	2·934	0·0069	1·004	0·00742	1·068
16	0·064	0·01812	2·608	0·0062	0·892	0·00659	0·949
17	0·056	0·01585	2·282	0·0054	0·781	0·00577	0·831
18	0·048	0·01358	1·956	0·0046	0·669	0·00494	0·712
19	0·040	0·01132	1·630	0·0039	0·558	0·00412	0·593
20	0·036	0·01019	1·467	0·0035	0·502	0·00371	0·534
21	0·032	0·00905	1·304	0·0031	0·446	0·00333	0·475
22	0·028	0·00792	1·141	0·0027	0·390	0·00288	0·415
23	0·024	0·00679	0·978	0·0023	0·335	0·00247	0·356
24	0·022	0·00623	0·896	0·00213	0·307	0·00226	0·326
25	0·020	0·00566	0·815	0·00193	0·278	0·00206	0·296
26	0·018	0·00509	0·733	0·00174	0·251	0·00185	0·267
27	0·016	0·00453	0·652	0·00155	0·223	0·00165	0·237
28	0·014	0·00396	0·574	0·00135	0·195	0·00144	0·208
29	0·013	0·00368	0·530	0·00126	0·181	0·00134	0·193
30	0·012	0·00340	0·489	0·00116	0·167	0·00124	0·178

These tables are compiled from the following data:

	Steel.	*Aluminium.*	*Duralumin.*
Specific Gravity	7·85	2·67	2·85
Weight in Pounds/Inches^3 ..	0·283	0·0968	0·103

III. ELASTIC PROPERTIES OF MATERIALS.

The units are pounds and inches. E = Young's Modulus; N = Modulus of Rigidity.

1. *Metals.*

Material.	*Tension.*			*Compression.*		*Shear.*	
	$E \div 10^6$.	*Elastic Limit.*	*Ultimate Strength.*	*Elastic Limit.*	*Ultimate Strength.*	$N \div 10^6$.	*Ultimate Strength.*
Aluminium, cast	12·5	7,000	18,000	—	—	—	—
,, rolled	—	12,500	27,000	—	—	—	13,000
,, wire	18·5	—	—	—	—	—	—
Copper, rolled	—	—	38,000	—	48,000	4·5 to 6·5	35,000
,, cast	9·1	14,000	30,000	—	37,000	—	28,000
,, wire	—	—	50,000	—	—	—	
Brass *from*	12·0	—	18,000	—	11,000	5·5	18,000
,, wire	—	—	80,000	—	—	—	—
Duralumin, rod	10·5	40,000	60,000	16,000 to 20,000	45,000 to 65,000	3·8	35,000
,, sheet	—	30,000	55,000	—	—	—	—
,, wire	—	40,000	60,000	—	—	—	—
Iron, cast *from*	14·0	—	13,400	—	80,000	5·5	18,000
,, ,, *to*	23·0	—	29,000	—	110,000	8·0	22,000
,, wrought *from*	27·0	23,000	36,000	21,000	36,000	11	37,000
,, ,, *to*	29·0	40,000	65,000	24,000	60,000	13·5	41,000
,, wire	25·0	—	90,000	—	—	—	—
Steel, mild *from*	29	40,000	60,000	40,000	60,000	12	47,000
,, ,, *to*	31	50,000	70,000	50,000	70,000	14·5	54,000

The strengths of special steels are too variable to be given. The particular specification must be referred to.

Gun-metal	—	17,000	31,000	—	—	—	—
Lead	—	—	3,300	—	—	—	—
Tin, cast	4·6	—	4,500	—	—	—	3,000
Zinc, ,,	—	—	7,400	—	—	—	9,000

2. *Timber.*

Material.	*Tension.*			*Compression.*		*Shear.*	
	$E \div 10^6$ Parallel to Grain.	*Elastic Limit.*	*Modulus of Rupture.*	*Strength Perpendicular to Grain.*	*Strength Parallel to Grain.*	*$N \div 10^6$.*	*Ultimate Strength Parallel to Grain.*
				Elastic Limit.	Max.		
Ash	1·4 to 1·65	7,700	12,000	1,300	6,000	0·14	1,750
Beech	1·5	7,400	12,000	—	6,000	—	1,700
Birch, American	1·8 to 1·95	8,400	13,000	1,000	6,600	0·17	1,620
Cedar from	1·0	4,200	6,000	400	4,000	—	800
,, to	1·7	6,000	10,000	700	5,300	—	1,100
Deal, Christiania	1·6	—	—	—	—	—	—
Elm, English	1·3	5,500	—	—	—	—	—
,, Canadian	1·4	6,700	12,500	1,200	5,800	—	1,650
Hickory	1·9	8,900	16,000	1,800	7,300	—	1,800
Mahogany, Honduras	1·3	7,000	10,000	1,000	5,500	0·1	1,300
,, Spanish	1·4	—	—	—	—	—	1,400
Oak, white commercial	1·4	6,700	12,000	1,300	6,000	—	1,760
Pine, white	1·2	5,000	7,400	530	4,500	—	850
,, Norway	1·7	8,000	11,000	720	6,000	—	1,100
Padouk, dark	2·4	10,500	—	—	—	0·2	2,000
Spruce	1·4 to 1·81	5,500	7,900	500	5,500	0·08	500
Teak	2·4	—	7,000	—	—	—	—
Walnut, Brazil	1·3 to 1·5	5,800 to 8,000	11,000	1,000	6,100	0·125	1,500

3. *Strength of Miscellaneous Materials.*

Cotton Fabric	85 pounds per inch.
Linen ,,	90 ,, ,,

4. *Strength of Streamline Wires.*

Size.	*Ultimate Strength in Pounds.*	*Size.*	*Ultimate Strength in Pounds.*
4 B.A.	1,050	$\frac{13}{32}$	10,250
2 ,,	1,900	$\frac{7}{16}$	11,800
$\frac{1}{4}$ B.S.F.	3,450	$\frac{15}{32}$	13,800
$\frac{9}{32}$,,	4,650	$\frac{1}{2}$	15,500
$\frac{5}{16}$,,	5,700	$\frac{9}{16}$	20,200
$\frac{11}{32}$,,	7,150	$\frac{5}{8}$	24,700
$\frac{3}{8}$,,	8,500		

PERFORMANCE

BY JOHN CASE, M.A., A.F.R.AE.S.

(Number references in brackets refer to the section dealing with General Theory.)

1. IN order to predict the performance of an aeroplane—*i.e.*, to foretell its speed, power consumption, rate of climb, etc.—we must know, as accurately as possible, the total resistance of the machine as a function of the angle of incidence, and the speed, the lift coefficient, the effective wing area, the total weight, and the power curves for the propeller; and the more accurately these are known, the more likely are we to predict accurately the performance of the machine. On account of the variation of the density of the air with altitude, the performance of a machine will be different at, say, 10,000 feet from what it is at ground level. Several of the fundamental equations have been given in the article on General Theory, and will be referred to here.

2. Before considering how to estimate the resistance, etc., of a machine let us see what are the properties of the aerofoil upon which depends the performance.

First, the minimum speed of horizontal flight is given by (5) above, and varies inversely as $k_{L\ max.}$ This is the landing speed of the machine, and for any other angle of incidence the speed is given by

$$V = V_L \sqrt{(k_{L\ max.})/k_L}.$$

From (9) the speed is a max. when L/D for the whole machine is a max., and so we can write:

$$\frac{V_{min.}}{V_{max.}} = \sqrt{\frac{k_L \text{ at } L/D_{max.}}{k_{L\ max.}}},$$

but this usually gives too small a speed range, and it becomes necessary to design so that the top speed occurs for some smaller value of k_L, usually about 0·1 for fast machines, thereby getting a smaller L/D, and, by (9), requiring more power. Evidently the best aerofoil for high-speed aeroplanes is one giving the greatest L/D for a lift coefficient given by—

$$k_L = \left(\frac{V_{min.}}{V_{max.}}\right)^2 (k_{L\ max.}).$$

If the rate of climbing be the chief consideration, then we want an aerofoil which gives the highest value of $(L/D)_{max.}$: (15) above shows that the greatest rate of climb is obtained when P_0 is as small as possible, and we have seen that this is so when L/D is as large as possible. A machine designed purely on this basis will have a small range of speed.

Now consider a machine designed for weight-carrying; and suppose the landing speed and top speed given. Then the area of the wings required is—

$$S = \frac{W}{\rho/g \cdot (k_L)_{max.}\ V^2_{min.}},$$

and the power required is, in horse-power—

$$P = \frac{WV}{550\ (L/D)},$$

where V is the top speed. We can write the total weight, W, thus:

$$W = aS + bP + w,$$

where a and b are constants, and w the weight of everything except the wings and power plant. Then the object is to make w/W as large as possible, for this

$$a\frac{S}{W} + b\frac{P}{W} = \frac{a}{\rho/g \cdot (k_L)_{max.} \cdot V^2_{min.}} + \frac{bv}{500\ (L/D)}$$

must be as small as possible—*i.e.*, we want a large $k_{L\ max.}$, and large L/D at the flying speed. The former is much the more important from our present point of view. For example: take $a = 1$, $V_{min.} = 60$, weight of power unit $= 4$ lb. per h.p., and propeller efficiency 70 per cent., giving $b = 0{\cdot}0104$; we have to consider changes in the quantity.

$$\frac{1}{8{\cdot}6\ k_L} + \frac{0{\cdot}624}{L/D}.$$

If $k_L = 0{\cdot}55$ and $L/D = 14$, this $= 0{\cdot}257$; now substitute an aerofoil having the same $k_{L\ max.}$; but $L/D = 16$, the quantity becomes 0·251, a reduction of 2 per cent.; instead of this, take an aerofoil whose $_{max.}\ k_L = 0{\cdot}7$, and $L/D = 14$, the quantity becomes 0·211, a reduction of 18 per cent.

These elementary considerations should give an idea of the points to consider in selecting an aerofoil for a given purpose.

3. **Details of Performance Calculations.**—Suppose the total weight (w) and landing speed (V_L) given, then the wing area is found from:

$$S = W/\frac{\rho}{g} V_L^2 \, k_{L\,max.}$$

When S has been found the speeds for all angles of incidence can be calculated. Thus, the first thing we require is a curve of k_L against α. If a model test of the complete machine, or of the complete wings, be available so much the better ; if not, the curves for the model test of the aerofoil must be corrected for biplane effect, gap/chord ratio, aspect ratio, stagger, and scale effect, as well as may be, the latter correction being applied last. As an aid to this the following information is given:

CORRECTION FACTOR FOR MAXIMUM k_L FOR ASPECT RATIO.

Aspect Ratio ..	6	7	8	9	10	11	12	13	14	15	16
Percentage increase of k_L maximum	0	0·5	1·0	1·6	2·3	3·4	4·6	5·6	6·4	7·0	7·4

TABLE OF PERCENTAGE INCREASE OF L/D FOR ASPECT RATIO.

		Aspect Ratio.										
		6	7	8	9	10	11	12	13	14	15	16
Values of k_L/k_L maximum.	0·3	0	1·0	1·8	2·8	4·5	6·5	9·3	11·8	13·7	15	15·8
	0·4	0	2·5	5·3	8·5	11·7	16·3	21·1	25·2	29·0	31·6	34·2
	0·5	0	4·5	9·4	14·5	19·6	25·3	32·3	38·0	42·5	45·8	48·0
	0·6	0	5·9	11·9	18·0	23·9	30·0	35·8	41·8	47·7	54·0	60·6
	0·7	0	6·5	13·0	19·5	26·0	32·7	39·4	45·8	53·0	60·6	70·0
	0·8	0	6·2	12·5	18·6	24·7	31·0	37·5	44·0	50·0	54	59
	0·9	0	5·5	11·0	17·4	23·6	31·0	39·5	46·5	51·5	54	58

GAP/CHORD CORRECTION FROM MONOPLANE.

$\frac{\text{Gap}}{\text{chord}}$	3	2	1·7	1·65	1·60	1·5	1·4	1·35	1·3	1·2	1·0	0·8	0·7
Coeff. for $k_{L\,max.}$	·993	·990	·990	·988	·983	·967	·950	·943	·940	·937	·928	·907	·880

CORRECTING FACTOR FOR L/D FOR GAP/CHORD RATIO.

		Gap/Chord.									
		3	2·5	2·0	1·8	1·6	1·4	1·2	1·0	0·8	0·7
$k_L/k_{L\ max.}$	0·2	1·085	1·08	1·033	1·00	0·975	0·96	0·946	0·92	0·883	0·85
	0·3	1·037	1·03	0·995	0·975	0·95	0·93	0·912	0·893	0·865	0·84
	0·4	1·006	1·00	0·98	0·960	0·935	0·92	0·902	0·883	0·855	0·825
	0·5	0·98	0·975	0·945	0·920	0·900	0·896	0·885	0·856	0·836	0·805
	0·6	0·95	0·93	0·885	0·86	0·85	0·845	0·836	0·80	0·79	0·775
	0·7	0·941	0·92	0·880	0·86	0·85	0·842	0·820	0·80	0·78	0·765
	0·8	0.928	0·90	0·858	0·84	0·83	0·819	0·805	0·777	0·77	0·76
	0·9	0·91	0·883	0·843	0·83	0·815	0·803	0·795	0·76	0·75	0·73

CORRECTIONS FOR SCALE EFFECT: MULTIPLY k_L MAX. BY 1·02, AND L/D BY THE FACTOR GIVEN BELOW.

$\frac{k_L}{k_{L\ max.}}$ ..	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
Factor ..	1	0·95	1·005	1·080	1·085	1·067	1·030	1·000	1·008	1·050

To use the above tables, when the criterion is $k_L/k_{L\ max.}$, proceed as follows: From the tests of the model monoplane plot a curve of L/D taking k_L as abscissæ, and from this deduce a curve of L/D with $k_L/k_{L\ max.}$ as abscissæ. Then multiply the ordinates of this curve, for abscissæ = 0·2, 0·3, etc., by the number given in the tables.

By this means probable full-scale curves for the wings are obtained. If sufficient data are available the lift curve should be corrected for the tail load:

$$k'_L = k_L - \frac{c}{l}\bar{k}_m.$$

k'_L = effective lift coefficient; k_L = lift coefficient of wings; c = equivalent chord; l = distance from c.g. to an assumed c.p. of tail; $\bar{k}_m$ = moment coefficient of wings about the c.g., which can easily be found when the c.p. coefficient for the wings and the position of the c.g. are known.

Next estimate the resistance of the whole machine. Strictly speaking, we ought to consider separately those parts of the machine which are in the propeller slip-stream, as they have a greater resistance, on account of the higher air velocity, but the experimental information available does not allow this correction to be made with any certainty.* The increase of resistance is partly compensated by an apparent increase in the efficiency of the propeller.

Having deduced the proper lift curve, the speed can be calculated for all angles of incidence, and from this the total resistance can be estimated for all angles, as shown below.

Having estimated the resistance of the body, etc., for all angles of incidence, and the *L/D* curve for the wings, a curve of drag for the complete machine can be plotted.

The calculations should be done as a table, thus:

Wing Incidence.	*Corrected k_L.*	*Speed Ft./Sec.*	*L/D for Wings.*	*Drag of Wings.*	*Resistance of Body, etc.*	*Total Drag.*	*H.P. Required.*

The seventh column is obtained by adding the fifth and sixth, and the h.p. required = Total Drag × Speed ÷ 550. The total drag and h.p. should now be plotted against *V* (Fig. 1).

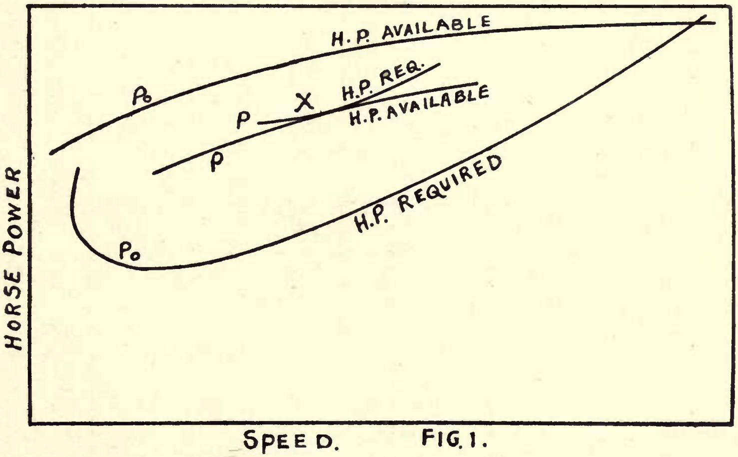
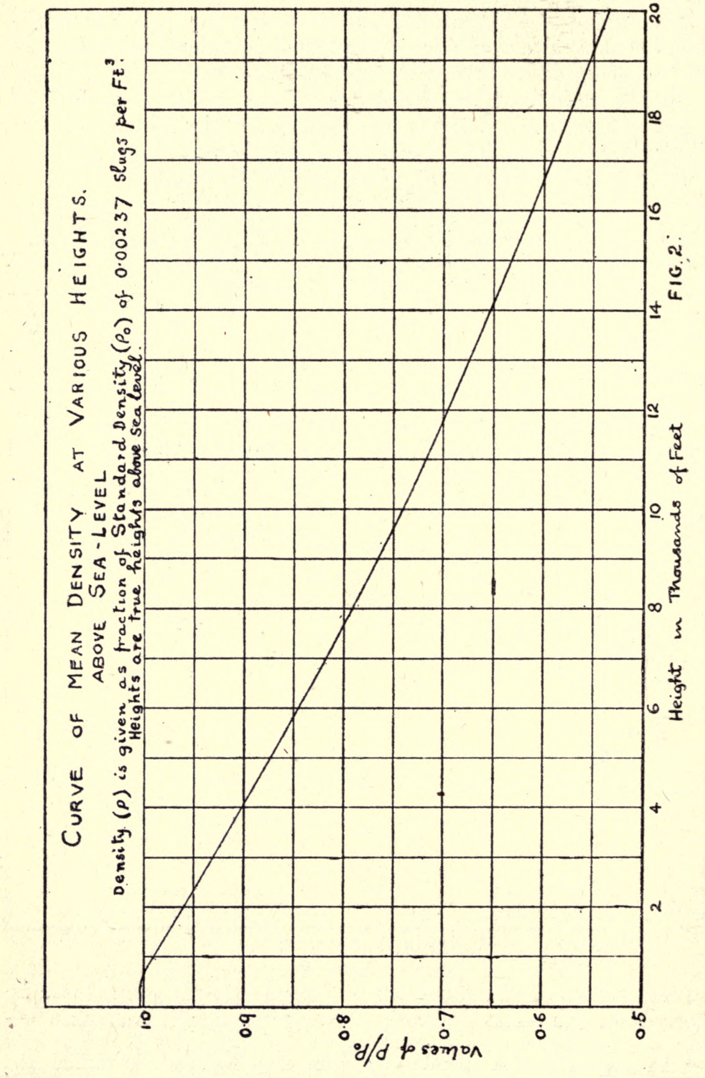


FIG. 1.

From the data of the propeller, supposed known, a curve can be plotted on the same diagram showing the "h.p. available"—

---

* An approximation to this correction may be made by taking the velocity of the air $= V + \frac{H\text{-}P \times 550}{\rho/g \cdot AV}$, where the *H-P* is the torque horsepower of the propeller, and *A* the cross-sectional area of the slip-stream, $= 0{\cdot}6$ times the disc area of the propeller. The body resistance should then be multiplied by 0·75.



CURVE OF MEAN DENSITY AT VARIOUS HEIGHTS. ABOVE SEA-LEVEL

Density (ρ) is given as fraction of Standard Density (ρ_0) of 0·00237 slugs per Ft^3. Heights are true heights above sea level.

FIG. 2.

i.e., the maximum thrust h.p., which can be obtained from the propeller at given forward speeds.

4. The above calculations apply to ground level, at standard density; at any other altitude the curves will be different. The resistance must be multiplied by ρ/ρ_0, ρ being the density at the height considered, and ρ_0 that at ground level; a curve of ρ is given in Fig. 2. The h.p. required must be multiplied by $\sqrt{\rho_0/\rho}$. The available thrust h.p. must also be corrected for density. This is done as follows: The b.h.p. of the engine, as a function of the r.p.m., is known, from test conditions, at ground level, and may be taken as proportional to ρ. The propeller torque h.p. is known, also as a function of the r.p.m. for various forward speeds. These curves are plotted as in Fig. 3, and the intersections of these curves give the relations between h.p., forward speed, and r.p.m. for any given density (or altitude). Thus, at a speed V, and altitude corresponding with ρ_1, we know that the torque h.p. is OM, and the r.p.m. = ON. Knowing the efficiency of the propeller (η), the available thrust h.p. at this height $= \eta \cdot$ OM. In this way h.p. curves at various altitudes can be drawn, as in Fig. 1.

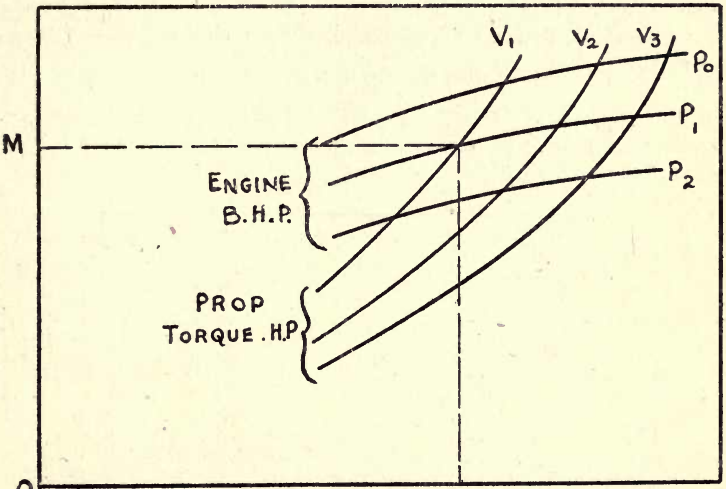


FIG. 3.

Climbing.—The difference between the ordinates of curves of equal altitude, at any given speed, gives the h.p. available for climbing at that speed and height. The rate of climb is

$$v = \frac{\text{h.p. available} - \text{h.p. req. for horizontal flight}}{550\ W.} \text{ft./sec.}$$

When the two corresponding h.p. curves *touch*, as at X, there is no power left for climbing, and this gives the ceiling for the machine.

The angle of climb $= \sin^{-1}(v/V)$.

5. **Circular Flight.**—Curves of h.p. required are easily deduced from that for horizontal straight line flight by means of equation (27) above. For a given value of r, P/P_0 is worked out for each value of V_0, and thus power curves for a series of values of r can be drawn, as in Fig. 4. The intersections, X, Y, Z, of these curves with the curve of power available give the maximum speed for a given radius. From (23) a curve of maximum angle of bank can also be drawn. The method can be extended to the case of spiral flight if desired.

6. **The Calculation of Resistance.**—The resistance of the wings $= \frac{\rho}{g} S k_D V^2$. The following figures all refer to 100 ft./sec.

The Resistance of the Body.—The following table gives the resistance, in pounds, of various bodies, and this can be used as a guide.

Body.	*Resistance.*	*Max. Cross Section. Area Ft.²*	*Lb./Ft.² of Max. Section.*	*Authority.*
1. Perfect stream-line, circular section ..	—	—	1·2–1·3	N.P.L.
2. Ditto, with cockpit, pilot screen, and shield	—	—	1·7	,,
3. Same as 1, sq. sec...	—	—	1·6	,,
4. Same as 2, sq. sec...	—	—	2·1	,,
5. F.E. 4	42	13·2	3·2	,,
6. R.E. 7. Beardmore engine	35	8·8	4·0	R.A.E.
7. R.E. 7, R.A.F. 1A engine	55	8·8	6·3	,,
8. R.E. 8	53	10·0	6·6	,,
9. F.E. 2D. Rolls-Royce engine	93	13·2	7·0	,,
10. S.E. 5, large wind-screen, and Lewis gun	39	6·6	5·9	,,
11. B.E. 2C.	47·5	6·5	7·3	R.A.E. N.P.L.
12. B.E. 2E:				
0° wing incidence..	58	—	—	N.P.L.
14° ,, ,,	68	—	—	

Resistance of Other Parts.—Circular cylinder, length/diam. = 1, 4 lb./ft.² of projected area, at 100 ft./sec.; $l/d = 10$; 10 lb./ft.², $l/d = \infty$, 12·7 lb./ft.²—Eiffel.

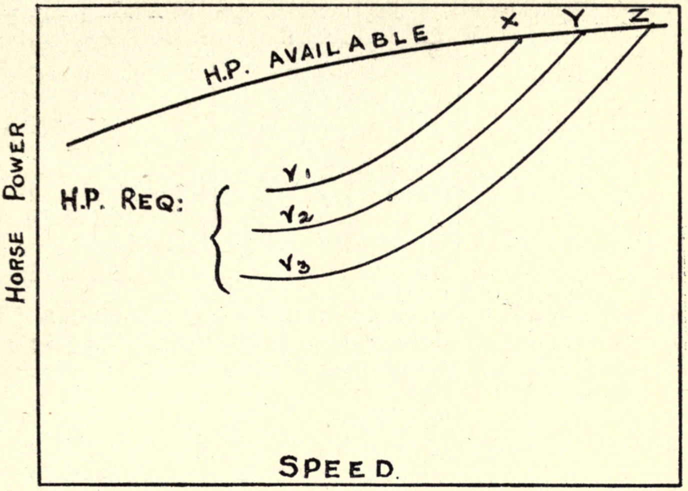


FIG. 4.

Round Wires.—Resistance in lb./ft. at 100 ft./sec. (N.P.L.).

Res. gauge ..	2	3	4	5	6	8	10	12	14	16
Res. piano wire	0·320	0·291	0·266	0·244	0·218	0·179	0·140	0·111	0·082	0·063
Res. cable ..	0·384	0·349	0·319	0·293	0·262	0·214	0·168	0·133	0·098	0·076
Breaking load, cwt.	70	50	45	40	30	20	—	—	—	5

Include end fittings and wiring plates in the length, and add 1 foot.

R.A.F. Wires:

Size.	*Resistance, Lb./Ft.*	*Add for End Fittings and Wiring Plates (Pounds).*
4 B.A.	0·25	0·210
2 B.A.	0·27	0·302
1/4 B.S.F.	0·34	0·441
9/32 ,,	0·37	0·529
5/16 ,,	0·41	0·675
11/32 ,,	0·49	0·763
3/8 ,,	0·52	0·895
13/32 ,,	0·60	1·059
7/16 ,,	0·64	0·152
15/32 ,,	0·67	0·323
1/2 ,,	0·71	1·806
	(N.P.L.)	(R.A.E.)

For inclined wires take projected length.

Struts (N.P.L.):

Fineness Ratio.	*Resistance Lb./Ft.*	
2·5	1·16	Add 0·2 lb. or each end fitting.
3·0	1·04	
3·5	1·00	
4·0	1·00	
4·5	1·03	

for good streamline sections. Others should be tested. For sloping struts, measure fineness ratio in a plane parallel to direction of relative wind, and take the projected length.

Radiators.—8·14 lb./ft.^{2}

Rudders and Fins.—0·10 lb./ft^2.

Wheels.—The resistance of one wheel with 800 mm. × 150 mm., Palmer tyre, at 100 ft./sec. has a resistance of 6·2 lb. if faired from hub to rim, and 10 lb. if not faired. For other sizes take the resistance as proportional to the size of tyre (N.P.L. and R.A.E.).

For odd fittings, etc., take 5 lb. per ft.^{2} of projected area. For speed, V, multiply figures given by $(V/100)^2$.

A NOTE ON STABILITY

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THE calculations affecting the stability of an aeroplane are of so deep and abstruse a nature that it is not possible in a work of this type to deal with them exhaustively. The object of this note is to deal briefly with certain fundamental points, avoiding the introduction of mathematics.

An aeroplane has six degrees of freedom, three being motions of translation, and three motions of rotation. It can move bodily forwards, bodily sideways, and bodily up and down, and it can, further, rotate about its longitudinal axis (*i.e.*, " roll "), rotate about its transverse axis (*i.e.*, " pitch "), or rotate about its vertical axis (*i.e.*, " spin "). Its movement can be combined of any two, three, four, five, or six of these free motions. Thus, in climbing on a turn there will be, first, the forward translational motion; second, the upward translational motion; third, the spinning motion of rotation about the vertical axis; fourth, the rolling, or banking, motion about the fore and aft axis; and, fifth, a probable " yaw," or sideways motion of translation, brought about by the centrifugal effect of the turn.

In considering the question of stability in an aeroplane it is clear that it will answer all requirements if it can be furnished with means whereby, without the assistance of the control normally exercised by the pilot, it can correct, of itself, any tendency either to alter its altitude, change its direction, " roll," or " yaw." The normal forward motion of translation is essential to its support, and as to the upward motion of translation, that is a matter which is concerned solely with engine power.

In a perfectly stable aeroplane flying in still air, no control beside the engine throttle would be necessary provided the machine were set to fly on a perfectly straight course, since to gain or lose altitude would simply be accomplished by allowing more or less gas to enter the engine. Still air is, however, manifestly an impossibility, and hence a form of direction control in the shape of a rudder has to be added in order to allow a course in the horizontal plane to be maintained. In a machine of inherent stability, of which many types are produced to-day, granting a reasonable altitude, no control is required beyond that of the throttle and the rudder to fly in any given direction.

The ability which an inherently stable aeroplane possesses demands a source of power which must come either from the engine or from gravity. In the following notes it will be considered that the whole of the power for stabilizing purposes comes from gravity, especially so as, for the sake of the argument, the aeroplane is supposed to be flying itself and to be deprived of any control whatever. Such being the case, it is obvious that any correction effect called for by the machine being forced off its course, or caused to pitch, and so forth, by some accidental cause, must involve a loss in altitude. In consequence of this fact, that it cannot correct itself unless it has sufficient height for the acquisition of the requisite gravitational power, a stable aeroplane cannot be handled as such when near the ground—as, for instance, when getting off and alighting—and must therefore be furnished, in addition to any features which it may require for stabilizing purposes, with adequate organs of control. Incidentally, it is pointed out, that whereas, in a controlled aeroplane, loss in speed can be substituted for loss in altitude, this cannot take place in an inherently stable aeroplane, but is dependent purely upon the alteration of the setting of the elevator.

It must here be remarked that, owing to looseness of terms, the word "stability" has been much abused in the past, and has not infrequently been associated with mechanisms such as gyroscopes, the object of which was to provide some form of automatic control. They were, however, commonly referred to as apparatus for automatic "stability." What we are now considering is not a form of mechanical control, but a form of stability founded upon aerodynamic principles, and inherent in the design of the lifting and subsidiary surfaces of the aeroplane.

A point of the utmost importance is that any such stability must be concerned, not only with maintaining the aeroplane travelling nose foremost, and preventing it from pitching and rolling, etc., but also with the maintenance of speed, since it is upon this factor that the support of the aeroplane depends. If a stable aeroplane is set with such a throttle opening that it just flies horizontally, it will neither gain nor lose altitude permanently, although the effect of gusts will be to make it temporarily gain or lose altitude, and hence there will also be alternations in its air speed, though its average air speed will remain constant.

It is also desirable to note that if a stable aeroplane is controlled so as to adopt a certain position, it should, if disturbed from that position, return to it providing the control is kept as it were fixed. Thus, if by means of his controls a pilot sets his machine to do a side-slip, it may be struck by a gust and thrown on to an even keel again; but if it is inherently stable, it should immediately return to its previous state of side-slipping.

The various motions of a machine will now be dealt with seriatim, and since longitudinal, or fore and aft, stability is generally regarded as of first importance, it will be dealt with first. This is the quality of stability that resists rotation about the transverse axis of aeroplane, so that if from any cause it pitches up or down it tends to return to its normal horizontal altitude.

A small ballasted aerofoil, consisting of a sheet of paper or mica furnished with a weight on its leading edge, evinces a high degree of fore and aft stability when allowed to glide. In doing so it will acquire a definite gliding angle and a suitable angle of incidence. The object of the weight, or ballast, on the leading edge is to bring the centre of gravity coincident with the centre of pressure in these circumstances. The normal altitude of the aerofoil will thus be as in Fig. 1, and it will be in equilibrium provided that the c.p. and the c.g. are coincident. If, however, through the agency of a gust it should be disturbed from this altitude so that it pitches upwards and increases its angle of incidence, the centre of pressure, owing to the plane being flat, will retire towards the trailing edge, and the lift and weight forces will, consequently, no longer meet in a point, but will form a couple, as indicated in Fig. 2, the obvious effect of which couple is to tend to restore the aerofoil back to its original attitude, as in Fig. 1. If, however, the effect of the disturbance is to cause the aerofoil to decrease its angle of incidence, an exactly converse state of affairs is brought into existence, the c.p. moves forward, and once more a couple is formed, but this time, as set out in Fig. 3, it acts in the opposite direction, and tends to increase the angle of incidence until it is again normal.

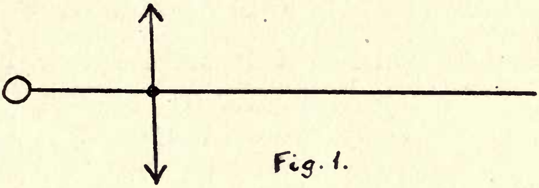

Fig. 1.

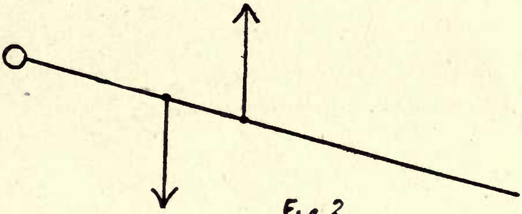

Fig. 2.

This phenomenon of complete fore and aft stability in the ballasted aerofoil is a result of the fact that with a flat plane the movement of the centre of pressure with variations in angle of incidence is a regular one—that is to say, as the angle is increased the locus of the c.p. tends to move steadily towards the centre of symmetry of the aerofoil. As lifting surfaces flat planes are highly inefficient, and give place to planes of curved and cambered section, in which the c.p. movement is not of the same type but is highly irregular. Thus, at very small angles of incidence the c.p. on a curved plane may be at or about the centre of the chord. As the angle is increased up to 10 or 12 degrees the c.p. will move forward, but when the angle is further increased it moves backwards again, until when normal to the air flow the c.p. is again in a more or less central position. It is evident, therefore, that a curved aerofoil will not be stable when merely ballasted and used by itself, as no restoring couple will be formed by the weight and lift forces; but, on the other hand, as shown in Fig. 4, the couple produced will have an exactly contrary effect.

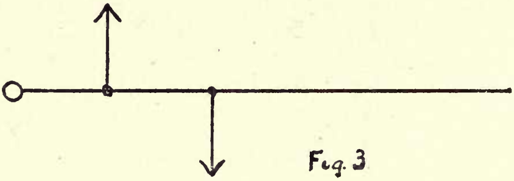

Fig. 3

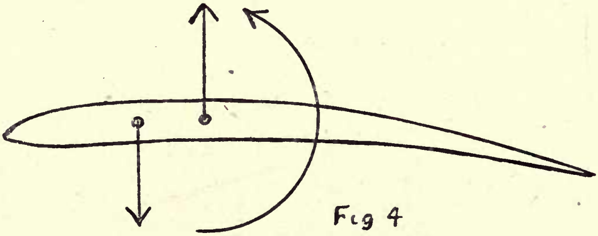

Fig 4

A ballasted aerofoil with a curved section will, however, show a considerable degree of stability when flown upside down—*i.e.*, with the concave surface uppermost; in fact, in this position it exhibits greater longitudinal recovering power than a purely flat surface. The reason for this will be made clear a little later.

The only way in which the irregular movement of the c.p. in a curved plane can be regulated is to furnish a secondary surface, the movement of the c.p. on which will act in opposition to that of the curved plane, and not only oppose it, but overcome it, to such extent that with any increase in angle of incidence the c.p. of the whole system will retire, and conversely with a decrease in angle below normal the resultant c.p. will move forward, so that the couple formed will always be in the direction of restoration to normal altitude.

This subsidiary plane can be placed either in front of the main plane or behind it, and it will tend to have the desired effect always provided that, however it be placed, the angle of incidence of the leading plane is greater than that of the trailing plane. When one has two planes in tandem, as suggested in Fig. 5, the resultant position of the c.p. of the whole system depends upon the positions of the c.p. on each plane individually, and also upon the relative amount of lift force exerted through each c.p. If in any positive change of angle the relative lift of the trailing plane is increased to a greater extent than that of the leading plane the resultant c.p. will move backwards, and in order that such may be the case it is necessary to have the trailing pane at a smaller angle of incidence than that of the forward surface. If, considering the lift as proportional to the angle of incidence (which at small angles is not very wide of the mark), suppose we have a leading plane set at an angle of 5 degrees and a trailing plane at an angle of 1 degree. Now, let it be supposed that the machine alters its attitude through 2 degrees, so that the angles of each plane are increased by this amount. This means that the lift on the front plane is enhanced by $\frac{2}{5}=40$ per cent., whilst that on the rear plane is increased by $\frac{2}{1}=200$ per cent. The relative lift on the latter is thus increased to a much greater extent than the former, with the result that the total c.p. tends to move in the desired direction. This is, of course, purely a qualitative theorem. Considering it quantitatively it is clear that what has to be dealt with is the moment of the c.p. of the trailing plane about the c.g. of the machine, relative to the moment of the c.p. of the leading plane about the same point; hence the area of the controlling plane and its distance from the c.g. are matters of great importance. Obviously, if the frame is very long the tail plane can be made very small, and *vice versa*, always providing that the moment provided by the tail is greater than the moment provided by the shift of the c.p. in the leading plane.

Fig. 5.

It is a matter of no consequence whether the actual organ of lift is in front or behind. If in the latter position, as suggested in Fig. 6, exactly the same state of affairs exists as in the conditions set forth in Fig. 5, at all events, so far as longitudinal stability is concerned. One may, in fact, regard the leading plane as a very small main plane, and the trailing plane as a very large tail, irrespective of which bears the principal burden of providing the lift.

Returning for a moment to the ballasted aerofoil, one perceives that if the plane is of curved section with the concave side uppermost it will naturally tend to be stable, since, regarding it as made up of two parts separated only by an imaginary line, the front part is set at a big angle of incidence and the rear part at a small angle, which conforms to the conditions laid out in Figs. 5 and 6. On the other hand, if the aerofoil is flown concave side downwards the conditions are exactly reversed, and the centre of pressure must always move, during small angles of incidence, in the direction opposite to that required for the formation of a recuperative couple.

Fig. 6.

It is important to realize, in connection with the state of affairs shown in Fig. 5, that although at big angles the c.p.'s of the main and subsidiary planes move in the same direction, at small angles they move in directions opposite to one another. This will be made clear if we take a simple case and assign reasonable numerical values to the varying quantities. In Fig. 7 the main plane is supposed to have an incidence angle of 5 degrees, and the tail one of 4 degrees. This is the normal attitude of the machine; as it is presumed to be in equilibrium it follows that the lift at $M \times MG =$ lift at $T \times GT$. Let us call MG 12 inches, and GT 60 inches, and the lift at M 200 pounds, whilst that at T is 40 pounds. If the machine now tips its nose up through 2 degrees, the angle of the main plane becomes 7 degrees, and the lift being roughly proportional to the angle (supposing the speed to remain constant) the lift at M becomes 280 pounds. The subsidiary plane, T, now flies at an angle of 6 degrees,

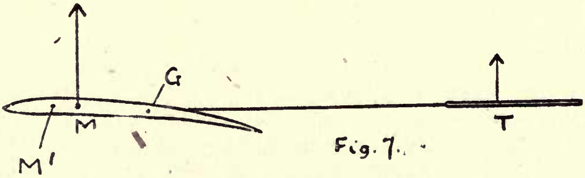

Fig. 7.

and its lift is increased to approximately 60 pounds. The movement of the tail is thus proportional to 60 in. × 60 lb. = 3,600 in.-lb. It will actually be a little more owing to the rearward movement of the c.p. on the tail. As the chord of the latter is small the difference will, however, not be noticeable. The c.p. of the main plane now moves, not backwards, but forwards to *M*¹, the distance *M*¹ *G* being, say, 15 inches. The moment is thus 280 lb. × 15 in. = 4,200 in.-lb. It is clear that the tendency for this machine will be for a slight upward pitch to increase itself as the resulting couple will be in the direction indicated in the figure. To remedy this state of affairs, either the tail must be made of larger area, or the length of the tail booms must be increased, or, what is more practically valuable than either of these alternatives, the normal angle of the tail plane must be decreased, so that when the aeroplane is flying at its normal attitude this plane carries a smaller fraction of the load.

The possession of fore and aft stability implies that the aeroplane should be able to recover its attitude in the vertical by a rotation about the transverse axis, and this rotation is not necessarily limited to small angles. Thus the aeroplane might be forced into a completely upside-down position. In this case, if left to itself, it would normally adopt a course of the type illustrated in Fig. 8, the main plane having a strong downward "lift," whilst the tail has a very much less one. This results in the formation of a couple in a clockwise direction, which with the forward motion of the aeroplane, results in a circular path being followed until at the point A the conditions of equilibrium are again complied with.

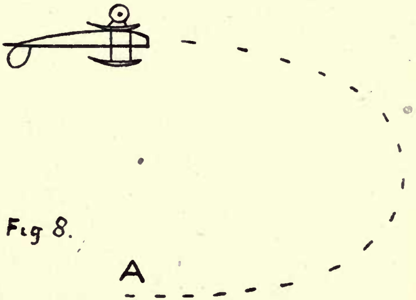

Fig 8.

Again, suppose the machine to lose all forward motion whatsoever, so that it stops momentarily horizontal, and then commences to fall vertically. In this case the first tendency, as indicated in Fig. 9, is for the machine to move backwards, due to the positive angle of the main plane giving rise to a horizontal force, *P*, as well as a vertical one, *R*, the former of which acts in the direction of the tail. Now, the slightest backward travel means that the c.p.'s of both the tail plane and the main plane tend to move towards their trailing edges, with the result that a counter-clockwise couple is formed, tending to depress the nose of the

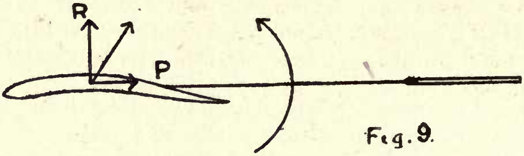

Fig. 9.

aeroplane, which is followed by a recovering dive in its normal direction.

If the aeroplane should be forced into such a position that it is, as it were, "standing on its tail," it will recover a normal

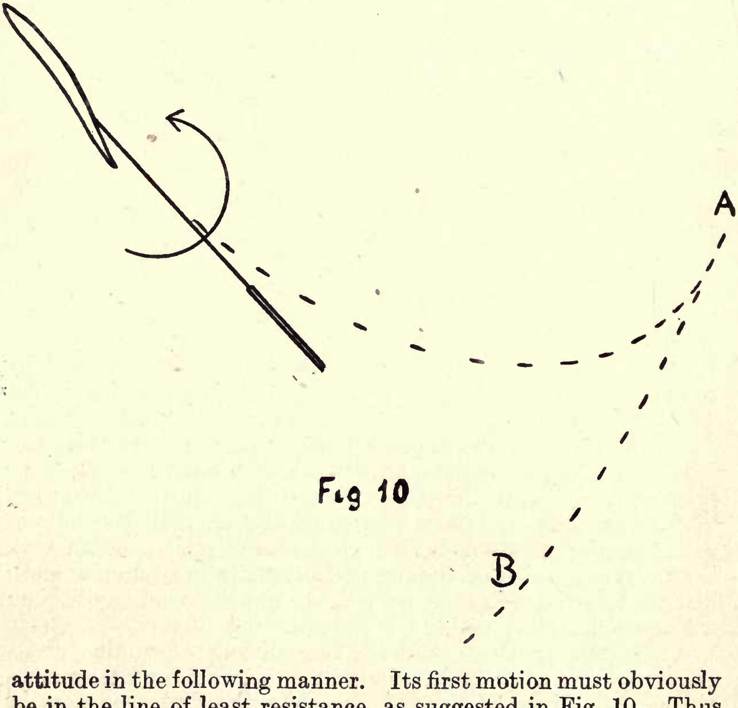

Fig 10

attitude in the following manner. Its first motion must obviously be in the line of least resistance, as suggested in Fig. 10. Thus momentarily it will be flying tail first. Now the angle of the tail being now greater (since the direction is reversed) than that of the main plane, it follows that a counter-clockwise couple will be formed, which, in conjunction with the backward travel of the aeroplane as a whole, will cause it to trace a circular path, as suggested in dotted lines, and it will continue to do this until it comes to the point *A*, at which all the kinetic energy attained during the tail dive is now absorbed is potential, and at the same time the machine is brought to rest with its nose pointing downwards, this position being exactly suitable for a nose-dive which enables it to recover its normal speed and attitude, as shown in the dotted lines, *AB*.

This nose-dive is shown in more detail in Fig. 11, in which the aeroplane is supposed to be dropping nose foremost. Here the lift on the main plane, the angle of which is greater than that of the tail, causes the formation of a clockwise couple which returns the aeroplane to an even keel.

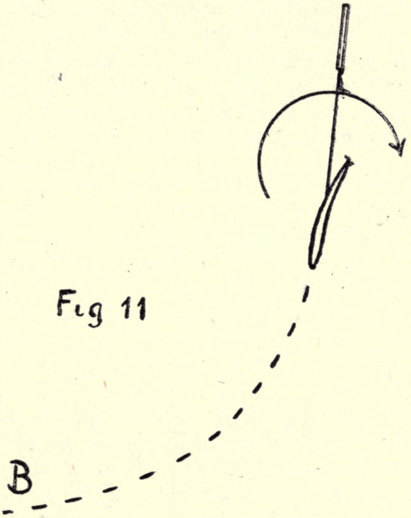
Fig 11

It will be realized that not only must the fore and aft stability of the aeroplane look after its normal attitude, and tend to maintain it constant, but it must also do the same in regard to speed, which is equally important for its support. If speed should tend to fall off, the loss of lift in the machine as a whole, whilst not immediately altering its attitude relative to the earth, alters it relative to its flight path, or, what amounts to the same thing, to the relative wind. The normal path of the aeroplane in Fig. 12 is along the dotted line. With the falling off in lift the effect of gravity is to make it follow the solid arrows. As a result, its real angle of incidence is increased, being now the angle *ABC* instead of the angle *ABD*. In consequence of this the c.p. moves towards the tail, a counter-clockwise couple is formed, and the aeroplane tips its nose down, thus reducing its angle of incidence and resistance, and increasing its speed.

If, on the other hand, by any means the relative speed of the aeroplane should be increased above its normal it would tend to climb rapidly, and thus increasing the angle of incidence between the wings and the flight path, it would also tend to pitch its nose up, with the result that the excess of kinetic energy would be converted into potential, and it would flatten out at a greater height after returning to its normal speed.

In Fig. 11 the ideal case of a recovering nose-dive is shown, which is inclined to neglect the fact that the aeroplane, being a solid structure, has both momentum and moments of inertia. The effect of the latter is to tend to make the aeroplane continue rotating when once rotation has started, exactly analogously to the case of a flywheel. Now in Fig. 11 the machine is supposed to rotate through practically 90 degrees in the vertical plane. When, therefore, it comes to the point, *B*, at which its attitude is the normal one and its speed also normal, it will not continue straightforward on its natural flight path, or glide if the engine be cut off, but the inertia effects will now come into play and cause it to pitch its nose up a little. As a result of this upward pitching there will be an increase in the angle of incidence and a loss of speed, followed, as shown above, by a tendency to dive and recover speed. During the dive the same rotation will take place and the same inertia effects be felt; hence, even if the latter are quite small, the flight path will cease to be the ideal one, and will degenerate into a series of swoops, as shown in Fig. 13, line *A*. Providing the longitudinal stability is sufficiently marked to overwhelm this swooping tendency, the swoops will gradually die out and become a simple glide; but if the setting of the planes does not conduce to such great stability the swoops, instead of decreasing in amplitude, as at *A*, may actually increase, as shown at *B*; or a third state of affairs may exist, in which the swoops tend neither to increase nor decrease. It will thus be observed that the disposition of the principal weights in the aeroplane have a considerable influence upon its stability,

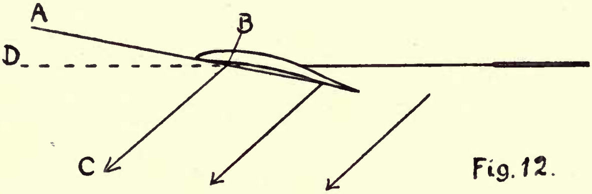

Fig. 12.

which cannot be regarded as entirely guaranteed by the setting of the supporting and subsidiary surfaces.

The above are primary effects, but there are secondary ones associated with the influence of the leading plane upon the air which traverses the trailing surface, and also with the influence of the screw thrust. These may be conveniently touched upon here rather than later.

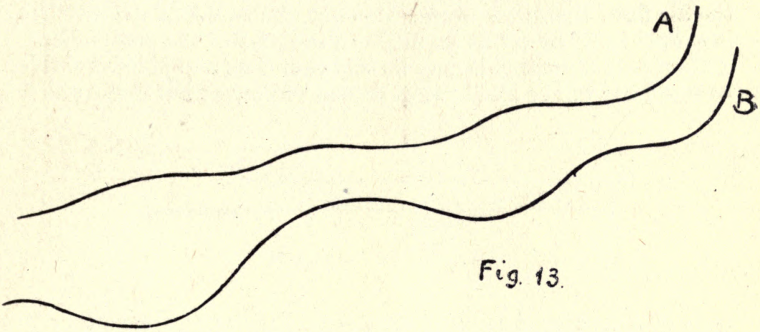

Fig. 13.

First of all, in regard to the relative positions of the surfaces, upon the disposition and area of which stability depends; it is evident that, since they are no very great distance apart, the leading plane must have some sort of effect upon the trailing one, inasmuch as the latter is dealing with a stream of air which has already been deflected and accelerated downwards. This is made clear in Fig. 14, which illustrates the fact that there is a considerable difference between the real angle of incidence of the tail *ABC*, and its apparent angle in relation to the flight path of the machine as a whole—namely, *CBD*. It is thus easy to perceive that the tail plane might actually have a greater angle of incidence than the main plane when the aeroplane was

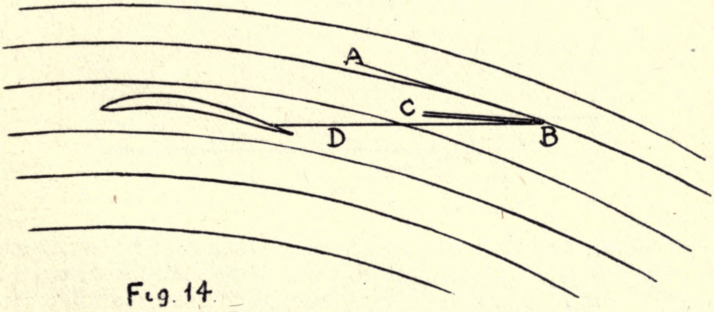

Fig. 14

at rest and yet satisfy the conditions of stability when the screw was working. It is, consequently, not possible, by merely glancing at the setting of the subsidiary surfaces of a design, to say that it is longitudinally stable or not. In a pusher machine this deflection effect does not exist to so marked an extent, as, although the leading plane imparts a downward component to the air flow, the screw washes this out almost completely, unless it should itself be set at an angle to the axis of the aeroplane.

Generally speaking, it may be said that if an aeroplane is stable as a glider it will be stable when driven by power; but in the above case of a tractor it is clear that some difference must occur in the lift of the tail according to whether the engine is running or no, and irrespective of whether the tail plane is designed to have a positive or negative lift. The latter is rarely employed, it being more common practice to dispose this surface so that it is neutral, and virtually carries no load at all. On the other hand, if it is set at a negative angle to the slip stream, it will have, when the engine is opened out, a tendency to pitch the nose of the machine up. This is by no means undesirable, as it means that when the engine is switched off the tendency is for the nose to pitch down and encourage the machine immediately to assume its natural gliding path. If the tail is normally designed to carry a certain amount of load, opening up the engine will cause the aeroplane to pitch its nose down and gain speed, and though this in itself may be not undesirable, it must also be remembered that, conversely, switching the engine off encourages an upward pitch, so that the aeroplane may stall or partially stall before it gets into gliding flight. In general it is preferable that the former conditions should be observed rather than the latter.

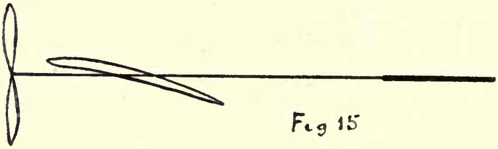

Fig 15

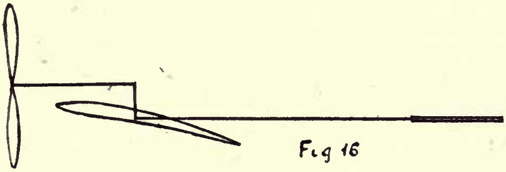

Fig 16

The position of the engine, in so far as it affects the application of thrust, is also a matter of some importance in a consideration of fore and aft stability. If the line of thrust, as indicated in Fig. 15, is symmetrical with the c.p. of the machine as a whole, so that thrust and resistance are directly opposed, it is clear that the extent to which the engine is opened up will have no immediate bearing upon the trim of the machine, neglecting the effect which has already been touched on above, and also neglecting the fact that the c.p. may, with changes of angle of the main planes, suffer a slight vertical displacement as well as a horizontal one. As will be shown later, there are certain considerations which favour a low position of the weights, which of course implies a low situation of the engine, and a corresponding low line of thrust. The latter is exemplified in Fig. 17. Assuming that the lift and weight forces are in equilibrium, it is evident that the thrust and head resistance forces are not so, and hence opening up of the engine will promote a couple tending to pitch the nose of the machine up so that it climbs rapidly. On the other hand, it will tend to dive when the thrust is removed. This, as mentioned above, is quite desirable. If the thrust is high the aeroplane tends to flatten out and increase speed when the engine is opened, and pitch upwards when it is throttled—that is, of course, assuming that no controlling organs are brought into play to resist this influence. It will be perceived that in the former case the machine, left to itself, except in so far as engine power and thrust are concerned, tends to maintain a constant speed, so that more engine power available simply means a greater attainment of altitude. In the latter case the effect may be said to be that a high thrust tends to keep the machine at a more or less constant altitude, so that its forward speed varies with the engine power. It is difficult to imagine any conditions of flying in which this state of affairs is particularly called for, and therefore, on all counts, the reasonably low line of

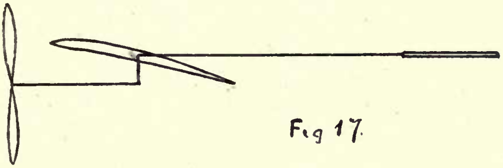

Fig 17.

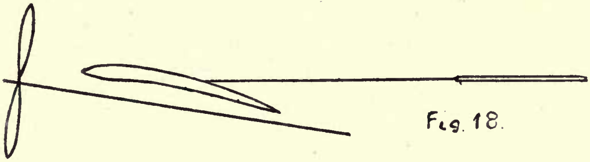

Fig. 18.

thrust is preferable; but it can readily be imagined that this would have an evil influence if carried to excess.

In some designs, notably the R.E.8, it has been sought to enhance the climb performance by setting the axis of the screw at an angle to the axis of the aeroplane, as suggested in Fig. 18. This has an effect on fore and aft stability similar to that of a low line of thrust, though the engine, so far as weight is concerned, is kept in a symmetrical position. The influence of the gyroscopic force of a rotary engine will be discussed later.

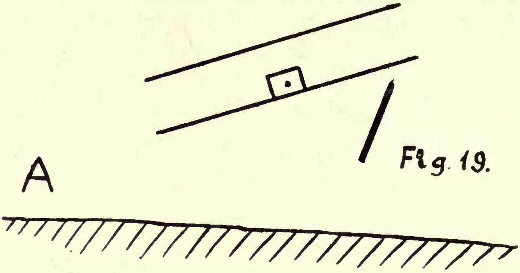

Fig. 19.

Lateral Stability.—The second undesirable motion which an aeroplane can undergo is a motion of rotation about the longitudinal axis. This is commonly called "rolling," and its tendency to resist this influence is the "lateral stability" of the machine.

When a machine rolls so that its surfaces are not symmetrically placed with regard to the ground, as shown in Fig. 19, it loses lift, inasmuch as, although, if the speed remains the same, the same amount of air in unit time is being dealt with by the planes, this air is not being accelerated in the same direction as gravity. In consequence of the reduced lift the machine tends to lose height, and as in doing this it will tend to follow the line of least resistance, it will accordingly move towards *A*—*i.e.*, in the direction pointed by the lower wing-tip. This motion is, of course, compounded with the forward travel of the aeroplane as a

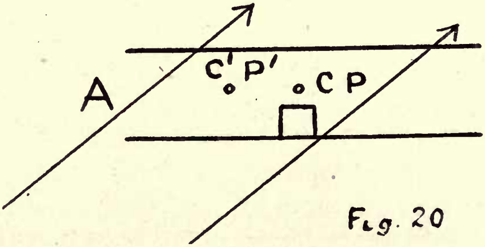

Fig. 20

whole, the result being a diagonal motion, so that the air passes over the planes in the manner indicated in Fig. 20. When the air passes normally over the plane surface the c.p. is on the centre of the wing span, as marked in the sketch; but, since the c.p. always tends to move towards the entering edge of a plane, it follows that as the air adopts a diagonal flow the c.p. takes up a fresh position somewhere nearer the wing-tip, *A*, than formerly.

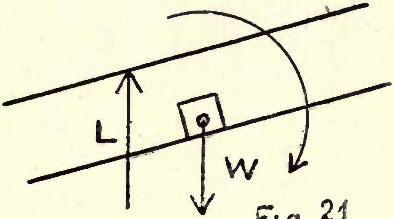

Fig. 21

Looked at from the front, then, the aeroplane is subject to a new arrangement of forces. We have the weight force, *W*, acting through the c.g., *L* the lift force acting through its new c.p., which is no longer coincident with the c.g. A couple is thereby formed, the tendency of which is to rotate the aeroplane in the direction of the arrow—*i.e.*, back on to an even keel.

In order to promote their lateral stability most designs incorporate a dihedral setting of the main planes, Fig. 22. In this case, when the machine starts to move sideways as a result of its initial roll, the wing *AB* presents a larger angle of incidence to the air than the wing *BC*. The effect of this is to give *AB* a relatively greater lift than *BC*, so that the c.p. through which the resultant lift reacts moves rapidly in the direction of *A*, and quickly restores the aeroplane by enhancing the righting couple.

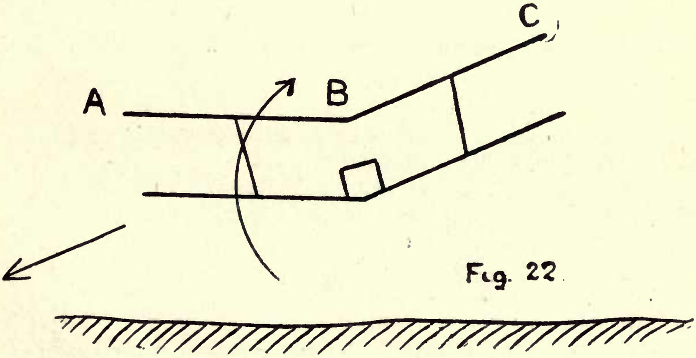

Fig. 22

It is particularly important to note that the lateral restoration of the aeroplane depends directly upon the fact that when it rolls it immediately side-slips as well. Unless this side-slip takes place the recovering action is absent. Thus in the above the wing *AB* only has a greater lift than *BC* when the machine as a whole has, to however slight an extent, some component of motion in the direction of *A*. So long as the course of the machine is straight forward wholly, the wings, being symmetrical, have equal lift. One often sees the explanation given that the recovering couple is provided by the fact that *AB* has a bigger lift effect than *BC* simply because it is "lying flatter," and the length it projects on to a horizontal line is greater. This is sheer nonsense. It is the side-slip which is primarily responsible for the stabilizing action, and a flat plane, though less so than one with a dihedral setting, is stable. If the above-mentioned fallacy were true it would have no stability at all.

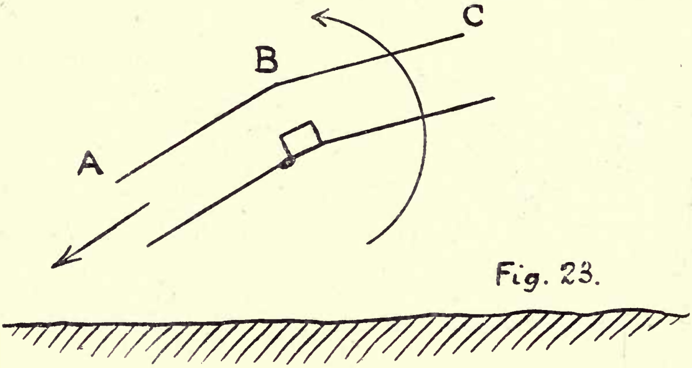

Fig. 23.

If the wings are set with an anhedral (or cathedral) angle, with the apex pointing upwards, the conditions promote definite instability in the lateral sense. In this case any tendency of the c.p. to move in the direction of *A* by reason of the side-slip which follows the roll is counterbalanced and overwhelmed by the fact that *BC*'s angle of incidence causes more lift reaction on that wing than on the other, hence the c.p. will actually move in the direction of *C*, providing, with the weight force, a couple of anti-clockwise direction which serves to accelerate the roll instead of checking it, as the requirements of stability demand.

It is not necessary for the dihedral angle to be in the vertical plane, for it has an exactly similar restoring action if arranged in the horizontal plane. In Fig. 24 a machine with swept-back wings is seen from above in the act of side-slipping after a roll. *AB* is the wing nearer the ground, and as a result of the side-slip the air is moving across the planes more or less diagonally. It will now be seen that the wing *AB* is disposed to have a greater lifting effect than *BC*, since the former is travelling with its entering edge relatively square with the air-flow, whilst that of *BC* is sharply inclined thereto. The result of this difference in lift effect is that the c.p. once more moves towards the tip *A*, and the necessary restoring couple is set in motion.

It is to be observed that a dihedral setting, either in the vertical or the horizontal plane, slightly reduces the efficiency of the planes as lifting surfaces; for stabilizing purposes the dihedral angle, however, need only be quite small, as nothing is gained by making it exceed a maximum of 5 degrees, and at or below this amount the loss in lift is quite negligible.

In some aeroplanes—the Gotha bomber is a good instance of this—the planes have a horizontal dihedral as well as a vertical one. Each arrangement has its own particular advantages and disadvantages; for instance, the effect of the vertical dihedral is relatively to lower the c.g. of the aeroplane, which may or may not be desirable, according to how it is designed. With this type of dihedral angle the edge-resistance of the main planes—that is to say, their resistance to a sideways movement—is greater than if they are flat; hence, as we shall see, a bigger fin is required on the tail, or conversely the fuselage must be made larger.

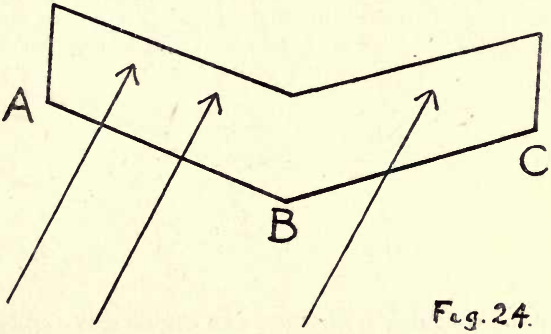

Fig. 24.

The arrangement of the dihedral setting of the planes and tail plane, as seen from the side of the machine, in the interests of fore and aft stability, is exactly analogous to the dihedral setting of the wings as seen from in front, or from above, in the interests of lateral stability. Just as, in the former case, there can be no stabilizing effect unless the aeroplane is moving either forwards or backwards, so in the lateral sense there can be no restoring action unless the machine side-slips in one direction or the other.

Secondary Effects.—When an engine is driving a screw propeller, and the crank chamber of the former is rigidly attached to the body of the aeroplane, there is a tendency, called the torque reaction of the screw, for the screw to remain stationary, and for the aeroplane to turn round axially. This torque reaction is dependent, in its effects, upon the relative areas of the wings and the screw blades. The former are, of course, very much greater in area than the latter; but still the tendency exists, and if a single engine and screw is used, it follows that the effect will be that one wing, as shown in Fig. 25, has its load increased by the torque reaction, and the other has its load decreased. When controlling the machine the pilot has in the ailerons an easy means of trimming the aeroplane back on to an even keel, according to whether his engine is fully opened out or partially throttled. But if we imagine that the aeroplane is entirely uncontrolled, and depending upon its inherent stability, it will be perceived that the effect of the torque reaction is to make the machine maintain a certain angle of roll, which in turn will encourage a slight but continuous side-slip. This tendency can be overcome by employing either two screws driven in opposite directions by a single engine, or two screws each driven by its own engine, and each rotating in a different direction.

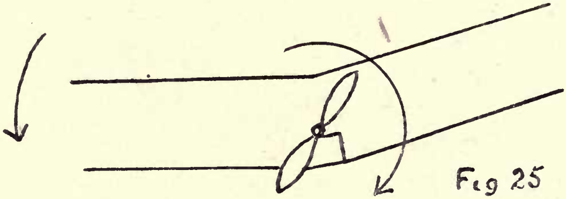

Fig 25

Directional Stability.—If the aeroplane is to retain its lift it is necessary that the relative air shall move across the surfaces from leading edge to trailing edge, and not, either wholly or partly, from wing-tip to wing-tip. It is, consequently, essential that the machine should "follow its nose," and that its only motion of translation should be along its fore and aft axis. In order to furnish an aeroplane with a directional sense, and give it the ability to resist rotation about its vertical axis, the principle of the arrow is adopted, and a tail fin provided whereby the effective side area of the machine in the rear of the c.g. is greater than that in front of the c.g.—*i.e.*, whereby the moment of the fin area behind is greater than the moment of the fin area in front. In Fig. 26 an aeroplane is shown undergoing a "spin"—that is to say, it is not following its nose, but its axis is inclined to its line of flight, which latter is represented by the dotted line. The relative wind is suggested by the arrows. If the moment of the fin area behind the c.g. is sufficiently great a couple will be immediately formed in the clockwise direction indicated by the arrow, and the effect of this couple will be to restore the aeroplane to its proper attitude to the flight path. In this connection it is important not to confuse a "spin" with the idea of the aeroplane altering its course, inasmuch as the path of the c.g. of the machine is not necessarily affected by a spin.

If the fin is of sufficient size to balance the side area in front of the c.g. (all side area has a fin effect, and hence the rearward extension of the fuselage, the tail skid, the edges of the tail plane, all act as fins) the aeroplane acts as a weathercock, and, as is obvious from the conditions shown in the last figure, tends when struck by a gust to turn its nose into the gust. This involves the aeroplane in altering its course from time to time if left to itself, but this, in any case, in the absence of reasoning control, is entirely unavoidable. It is obviously better to have the machine possess weathercock stability than its converse, in which the effective fin area in front of the c.g. is greater than that behind. In this event, the moment a spin begins it is, instead of being checked, immediately encouraged, and would continue until the machine lost all sense of direction and simply fell to the ground.

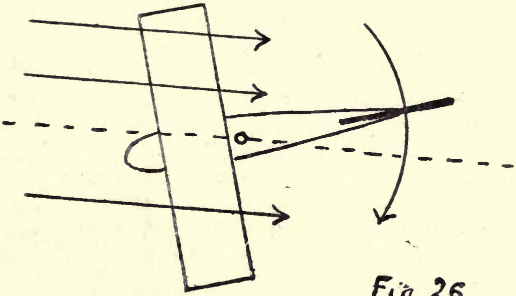

Fig. 26.

At the same time it is possible to have too large a fin. It will be understood that in dealing with the action of an aeroplane rolling and side-slipping, in the last section, only the effect of the main planes was considered, but that there is also an important effect upon the subsidiary surfaces, especially the fin. Fig. 27 shows a machine side-slipping—such a side-slip may be produced by a roll, or by under- or over-banking on a turn, or be caused deliberately by working the ailerons in opposition to the rudder. If the fin is very large, the moment the side-slip occurs, the machine tends to spin, as shown by the arrow, and as this will involve it in a change of direction it is not necessarily desirable. If the fin is made of reasonable size, neither too large nor too small, it will give the aeroplane a general weathercock tendency, which is desirable; but, at the same time, it will not allow the machine to be so sensitive as to immediately change direction at the least sign of side-slip.

Secondary Effects.—When an aeroplane spins on its vertical axis, from whatever cause, the relative speed through the air of one wing is increased and that of the other decreased. In the above figure the speed of the wing *B* is increased by the clockwise spin, and hence its lift is increased. As a consequence the spin is accompanied by a roll, as shown in the accompanying figure (Fig. 28). The original side-slip tends, as shown in the consideration of lateral stability in a previous section, to cause the machine to roll with the wing *A* uppermost. The spin tends to give more lift to *B*, and so correct this discrepancy, and if rapid may actually overwhelm it, so as to promote the anti-clockwise couple shown in the figure. The subsequent roll will be again accompanied by a side-slip, and the side-slip accompanied by a spin. It will, of course, be realized that these motions may only take place to a very small extent, but that they exist none the less, and must be taken into consideration. It will also

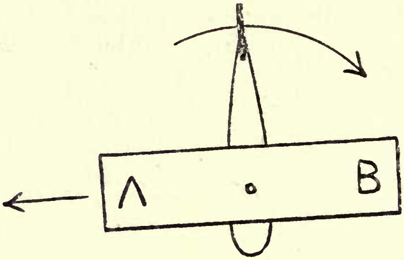

Fig. 27.

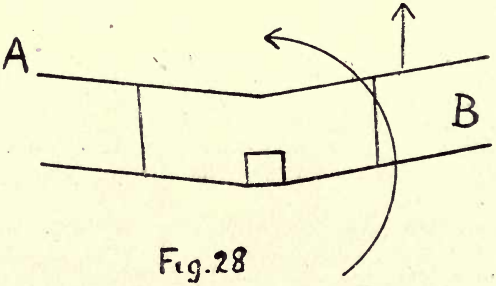

Fig. 28

be perceived that there is a most intimate connection between spinning, rolling, and side-slipping, and that each can reproduce the other.

It is sometimes supposed by the lay mind, and suggested by the fallacy-monger, that if an aeroplane be truly stable in a directional sense, it can, without help from the pilot, follow any given course entirely irrespective of any gusts that strike it. This is, of course, absurd. It is of no consequence to the aeroplane in what direction the air is moving relative to the ground, the only thing that it takes note of and obeys are changes in wind direction; and as these are entirely unsystematic and fortuitous, it follows that, under actual flying conditions—the air, however apparently calm it may be, is never really in a state of rest—a stable machine left to itself will follow a most erratic course. This is well seen by the behaviour of free models.

If an aeroplane is laterally stable it may be controlled directionally by means of the rudder alone, and aileron control is only called for for purposes of correction near the ground, and also for the execution of very rapid turns. When the rudder is thrown over to the right, as shown in Fig. 29, a force is produced upon it which has two effects. Firstly, it tends to turn the machine in the direction of the arrow (dotted); and, secondly, it causes the whole machine to yaw or side-slip in the direction of the wing *A*. This side-slip is, as explained before, necessarily accompanied by a roll, and if the aeroplane was previously flying level it will now turn and change direction, and at the same time it will bank itself in the desired manner as shown in Fig. 30. Such banking will be automatically correct, and will go on increasing until the tendency to side-slip outwards, produced by the centrifugal force of the turn, is equal and opposite to the tendency to side-slip inwards, due to the banking effect.

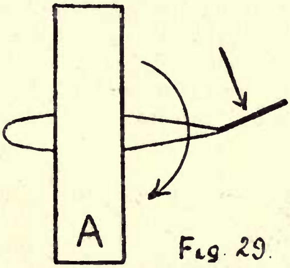

Fig. 29.

It is in this connection that the importance of the fin size becomes manifest. If the rear fin moment is very big the tendency will be for the aeroplane to resist the yaw, which gives rise to the desired banking effect, whereas if the fin be too small the tendency will be for the turn to be too quick for the amount of bank that is available. Hence, to produce an aeroplane that shall be thoroughly stable and capable of looking after itself on turns, a nice adjustment of fin surfaces is required.

It may be noted, in passing, that in a tractor aeroplane the screw, placed as it is some distance from the centre of gravity, has a by no means negligible fin effect. Let us suppose that we are looking at a counter-clockwise screw from the front, as shown in Fig. 31, and that the screw is struck by a gust coming from the left, as suggested by the arrows. In this event the relative speed between the blade and the air is greater when it is at the top than at the bottom, and this unsymmetrical disposition of resistance is exactly the equivalent of a fin placed in the same place as the screw. The same thing applies to a pusher screw; but as this is invariably very close to the c.g. of its machine the moment of its finnage is negligible.

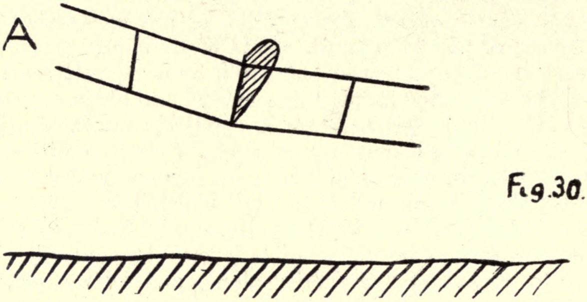

Fig.30.

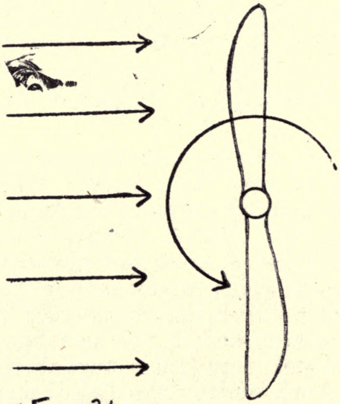

Fig. 31

It is commonly supposed that a stable machine is *ipso facto* difficult to control. To the extent that a stable machine has, so to speak, a "will of its own," this is true; or, at all events, it is true that the stable machine is quite naturally a little slower in response. This, however, is only disadvantageous in such machines as those in which intense manœuvrability is demanded for fighting or stunting purposes. Providing a pilot has sufficient elevator control to "stall" the aeroplane when required, and sufficient aileron to overwhelm its natural lateral stability, and enough rudder to resist its weathercock tendency, there is no reason why stability should interfere with controllability. Owing to its tendency to trim itself to suit every gust a stable machine, if left to itself, is likely to be more bumpy and fractious when flying in bad weather than an unstable one; but if the same control is exercised in both cases this difference is reduced to a negligible amount. One is quite safe in prophesying that for peace-time purposes, as opposed to the requirements of offensive warfare, the stable machine is desirable on all accounts.

PROPELLERS

BY E. P. KING, B.SC. (ENG.), LOND. A.M.I.MECH.E.

§ 1. **Terminology.**—According to the strict meaning of the word, any device or instrument used for propulsion may be termed a *propeller*, whether it be of the screw, jet, paddle, or any other type. The *screw propeller* being, however, the most efficient and only really successful form of propelling device both for marine and air craft, the significance of the term *propeller* has been narrowed to refer only to the screw actuator. Some aeronautical engineers, indeed, have endeavoured to restrict the use of the term to airscrews located at the *rear* of the body, or behind the engines of an aircraft, in a position which, more or less, corresponds to that of the marine propeller. According to general usage, however, the term *propeller* may be employed synonymously with *airscrew*, regardless of the position which this may occupy on the aircraft.

§ 2. **The Newtonian Method.**—In order that simple hydrostatic and aerodynamic phenomena may be dealt with mathematically according to the fundamental laws of force and motion, Newton makes use of a hypothetical medium, which is defined as consisting of a large number of material particles equally distributed in space, possessing mass, but of no sensible magnitude. Each of these particles is supposed to move independently, and is quite unconnected, and without influence on any of the neighbouring particles. In such a hypothetical medium the phenomenon of "action at a distance" does not exist, and the only way that momentum can be communicated is by actual contact with the impeller. Bodies traversing such a medium experience a drag, which, by the second Law of Motion,* is proportional to the momentum communicated per second in the direction of the motion of the body. Similarly, the thrust obtained from an actuator operating in this medium will be proportional to the momentum imparted per second to the rearward wake stream. In his classic paper of 1865 before the Institution of Naval Architects,† Dr. Rankine says: "The reaction of the stream

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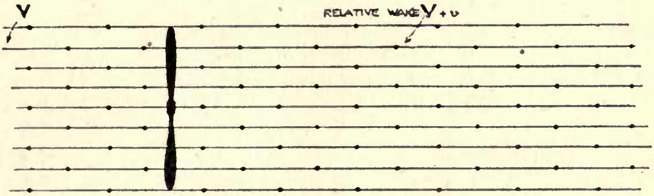
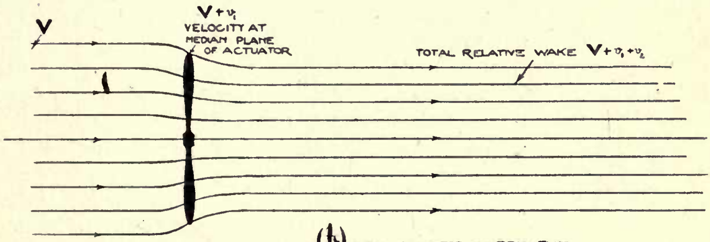
* "Rate of change of momentum is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts."

† "On the Mechanical Principles of the Action of Propellers," by Dr. W. J. Rankine, *Trans. I.N.A.*, 1865, vol. vi., p. 13.

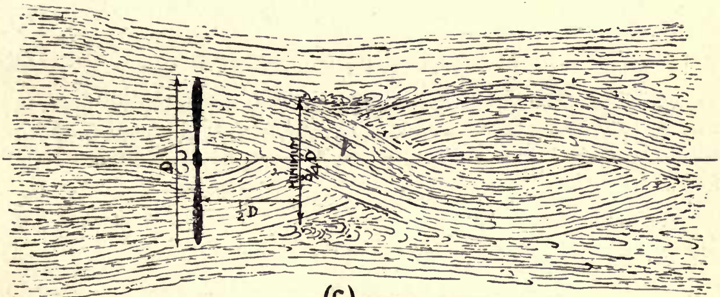
of fluid acted upon by any propelling instrument is the product of three factors—the mass of a cubic foot of fluid, the number of cubic feet acted-on per second, and the velocity impressed on the fluid." In Fig. 1 (*a*) an ideal disc actuator is shown operating in a hypothetical medium such as we have been considering.



(a) ACTUATOR OPERATING IN NEWTON'S HYPOTHETICAL MEDIUM



(b) IDEAL ACTUATOR IN REAL FLUID



(c) AIRFLOW IN ACTUAL CASE

Fig. 1.

Since the medium is discontinuous, and there is no "action at a distance," the rearward momentum must be communicated instantaneously at the moment the particles of the medium pass through the actuator disc. If ρ be the mass-density of the medium,* A the area of the actuator disc, V the velocity of this

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* $\rho=\frac{\delta}{g}=\frac{0{\cdot}076}{32}=0{\cdot}00238$ slugs/foot^3 for air at ground level. See § 21.

latter relatively to the oncoming stream, and v the added wake velocity, the thrust, T, may be expressed thus—

$$T = \rho A V v \quad . \quad . \quad . \quad . \quad (1)$$

Now the efficiency of working will be given by the ratio—

$$\text{Efficiency } \eta = \frac{\text{useful work/sec.}}{\text{total work expended/sec.}}$$

$$= \frac{\text{useful work/sec.}}{\text{useful work/sec.} + \text{energy lost to wake/sec.}}$$

$$\text{or, } \eta = \frac{TV}{TV + \frac{1}{2}Mv^2} = \frac{MVv}{MVv + \frac{1}{2}Mv^2} = \frac{V}{V + \frac{v}{2}} \quad . \quad . \quad (2)$$

§ 3. **R. E. Froude's Result.**—Let us now consider the case of the ideal disc actuator operating in a real fluid such as air. The continuity of such a medium would naturally cause the oncoming air to start accelerating before reaching the actuator disc, so as to give a smooth velocity and pressure gradient through the actuator. The problem is to find the magnitude of this inflow, or intake, velocity in terms of the total added velocity. If T be the thrust, M the mass of air dealt with per second, V the actuator velocity, v_1 the inflow velocity, and v_2 that part of the total added velocity, v, which is communicated to the air at the rear of the median plane of actuator, we have the following relations. Note that work per second done on air is equivalent to the product of thrust and air velocity at moment of passing through actuator. Thus we may write—

$$T(V + v_1) = M(v_1 + v_2)\,(V + v_1) \quad . \quad . \quad . \quad (3)$$

But this must be equivalent to the difference in kinetic energy of stream (per second) before and after passing through the actuator disc. The gain in kinetic energy per second will obviously be given by—

$$\tfrac{1}{2}M[(V + v_1 + v_2)^2 - V^2] \quad . \quad . \quad . \quad (4)$$

By equating expressions (3) and (4) we have—

$$(v_1 + v_2)\,(V + v_1) = \tfrac{1}{2}[(V + v_1 + v_2)^2 - V^2] \quad . \quad . \quad (5)$$

From this equation we find $v_1 = v_2$; but $v = v_1 + v_2$, so that $v_1 = v_2 = \frac{v}{2}$. We conclude that one-half of the added slip velocity is *antecedent*, and the other half *posterior*, to the median plane of the ideal actuator. Under such conditions of operation the air velocity at the moment of passing the actuator disc will be given by $V + v_1$ or $V + \frac{v}{2}$, and the mass of air dealt with per second will therefore be given by $\rho A(V+v_1)$ or $\rho A\left(V+\frac{v}{2}\right)$. The thrust of the actuator may accordingly be expressed thus—

$$T=\rho A(V+v_1)v=\rho A\left(V+\frac{v}{2}\right)v \quad . \quad . \quad . \quad (6)$$

§ 4. **Inflow Results applicable to an Airscrew.**—It is a remarkable fact that the ratio of inflow to total slip velocity, obtained as above by the simple application of first principles to the hypothetical actuator, agrees with the ratio which, used in conjunction with the orthodox method of airscrew analysis and design, gives results in close agreement with those obtained in actual practice. This result has also been confirmed by an important investigation recently made at the National Physical Laboratory at Teddington.* It has been shown that the calculated performance of an airscrew by the momentum method and by the blade element method may be made to agree with each other, and also with the experimental results, when the ratio of the total added wake velocity to the inflow velocity has a mean value of 2·1. The inflow ratio $\frac{v_1}{V}$ is, in the report referred to, represented by the symbol a, so that v_1, the intake velocity, will be given by aV, and the total added wake velocity, v, will be given by the expression $2{\cdot}1aV$. The air velocity at the propeller disc thus becomes $V+v_1=V\left(1+\frac{v_1}{V}\right)=V(1+a)$, the mass dealt with per second being accordingly given by $\rho AV(1+a)$. The thrust, T, may therefore be expressed in the following form—

$$T=\rho AV^2(1+a)2{\cdot}1a \quad . \quad . \quad . \quad . \quad (7)$$

If now we substitute $v=2v_1=2aV$ in the Froude equation (6) we have the practically identical result—

$$T=\rho AV^2(1+a)2a \quad . \quad . \quad . \quad . \quad (8)$$

which may also be written in the following way by substituting $A=\frac{\pi D^2}{4}$—

$$T=\rho D^2V^2(1+a)\frac{\pi}{2}a \quad . \quad . \quad . \quad . \quad (9)$$

It will be seen that the value of the inflow ratio, a, depends only on that of the "absolute thrust coefficient" $T'_c=\frac{T}{\rho V^2D^2}$.†

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* *Vide* Report of the National Advisory Committee for Aeronautics, No. 328, May, 1917.

† *Vide* § 17.

The variation of a with T'_c may be conveniently represented by means of a curve such as shown in Fig. 2. It should be noted that

$$T=\frac{THP\times 550}{V}=\frac{BHP\times\eta\times 550}{V} \quad . \quad . \quad . \; (10)$$

so that we have—

$$T'_c=\frac{T}{\rho V^2D^2}=\frac{BHP\times\eta\times 550}{\rho V^3D^2} \quad . \quad . \quad . \; (11)$$

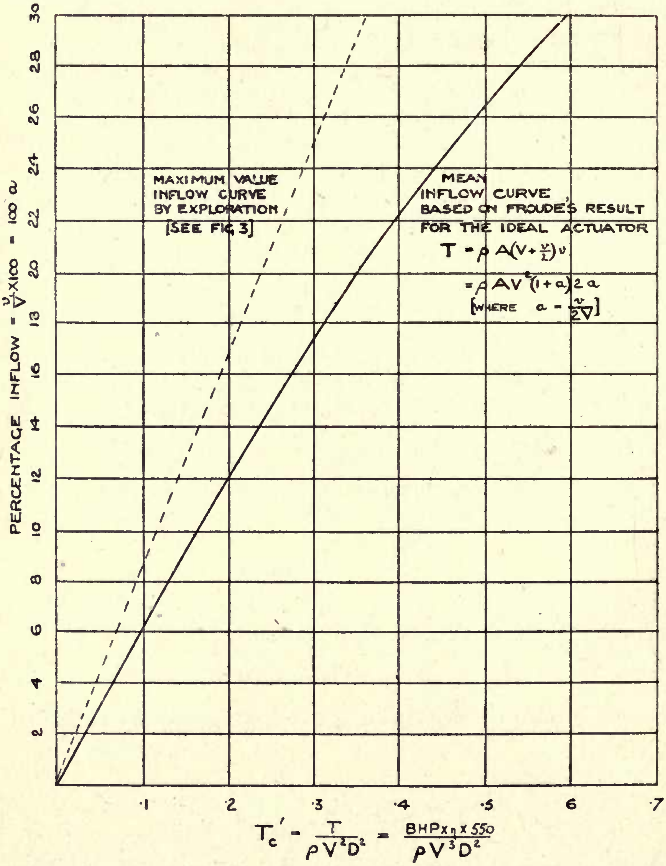

FIG. 2.

§ 5. **Configuration of Air-flow past an Actual Airscrew.**—Under ideal conditions of operation the air passing through an actuator is assumed to enter and leave in a cylindrical column such as shown in Fig. 1 (*b*). The state of the air which actually exists near and around a screw propeller when in operation is, however, very different, having a motion somewhat as shown in Fig. 1 (*c*). It is there seen that the wake stream takes up a configuration similar to that of a coarsely stranded rope, the number of air strands being determined by the number of blades to the propeller. From actual exploration it has been found* that the wake stream of any normal propeller contracts as it passes rearwards, and that the minimum diameter occurs at an axial distance about equal to half the diameter of the propeller measured rearwards from the median plane of operation. At this point the "waist" diameter is about three-quarters that of the propeller itself. It is found that the slip stream has a sharply defined boundary consisting of a narrow zone of air moving with a small negative velocity. As the wake stream expands this boundary phenomenon gradually disappears, so that at eight to ten diameters rearwards only a small eddy motion remains. The maximum velocity communicated to the air occurs at about two-thirds the propeller radius from boss centre, this result being in good agreement with that obtained by Eiffel for a small screw one metre in diameter.† From the tests made at the National Physical Laboratory,‡ it appears that the effects of inflow commence to appear at an axial distance equal to the diameter of the screw when operating at high values of the thrust coefficient T'_c, but that at small values of T'_c, no manifestation of inflow occurs till within one-half the propeller diameter forward of the screw plane. The rotational spin does not appear till within one-eighth the diameter measured forward of the propeller, but even at eight diameters rearwards this air-spin is still noticeable. In Fig. 3 the velocity and spin gradients for the N.P.L. propeller (16 inches diameter) are shown for various values of T'_c. It will be seen that the inflow ratios are always greater than those given for the same T'_c values in Fig. 2. This is explained by the fact that only *maximum* values are plotted in Fig. 3, whereas Fig. 2 is based on *uniform distribution* of air velocity over the entire cross section of slip-stream.

§ 6. **Classification of Airscrews.**—Airscrews are usually classed according to their direction of rotation when viewed from the *extreme rear of machine*, and also according to their position *in relation to the main planes* of an aeroplane. An airscrew located in front of its power unit, or in front of the body or main planes of an aeroplane, is termed a *tractor screw*, while one located at the rear of its power unit, or behind the main planes, is termed a *pusher screw*. A right-hand tractor screw is one which, being situated in front of the body or main planes of an aeroplane,

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* *Vide* Report of the National Advisory Committee for Aeronautics' No. 371, December, 1917.

† *Vide* "Résistance de l'air et l'aviation," by G. Eiffel, 1914, p. 323.

‡ Already referred to above. Report No. 371.

turns in a clockwise direction when viewed from the tail of the machine. Similarly, a right-hand pusher screw is one located to the rear of the main planes, but also turning in a clockwise direction when viewed from the rear of the machine. Left-hand tractors and pushers rotate in the opposite direction—that is to say, anti-clockwise when viewed from the tail of an aeroplane.

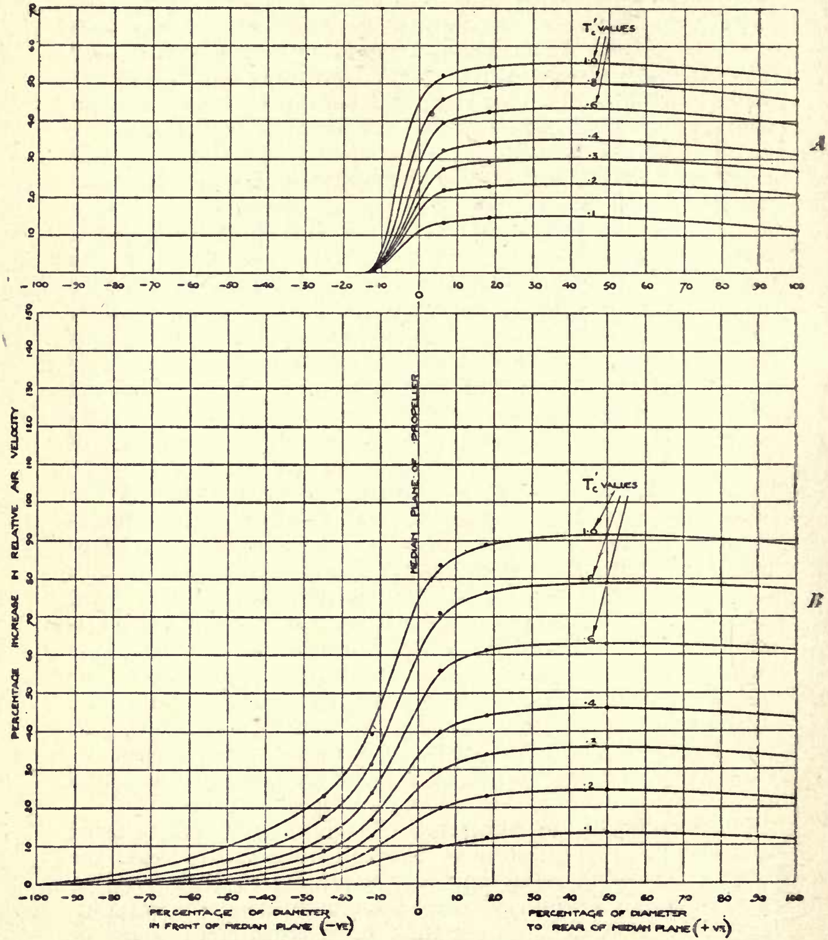

FIG. 3.

A. Curves showing Maximum Rotatunal Velocity increase in terms of translational speed.

B. Curves showing Maximum Axial Velocity increase in terms of translational speed.

§ 7. **Types of Airscrew.**—The aerodynamic *type* and performance of a propeller depends chiefly on its plan form, number of blades, and their shape in cross-section. It also depends on the distribution of the angle of attack over the working part of the blade. Each type of propeller may be used as a model for any number of geometrically similar propellers of different diameters, and it may also become the progenitor of a whole family of propellers having the same plan form, blade section, and attack distribution, but of different diameter and pitch values.

§ 8. **Aspect Ratio and Shape of Propeller Blades.**—The aspect ratio of a propeller is usually defined as the quotient of full diameter by maximum blade width. Experiments made by Dorand* have shown that the best aspect ratio is about 10, and that only very little, if any, advantage is gained at higher values. This result is confirmed by experiments on four-bladed screws made at the National Physical Laboratory,† practically no increase in efficiency being obtained by increasing the aspect ratio from 10 to 15. A report of forty-eight propeller tests is given by Dr. Durand,‡ the conclusion being that fairly narrow blades give the highest maximum efficiency, but that, generally speaking, wide blades are better when a propeller is operating at high slip values or under climbing conditions. In the standard blade form curves shown in Fig. 4, the aspect ratio is seen to be 9. The precise *shape* to be given to a screw propeller blade in order to secure the best possible maximum efficiency is another difficult and complex matter to decide. Certain it is, however, that all abnormal blade shapes, such as those of the snail-shell and turbine types, give a very low maximum efficiency. Very wide or very narrow blade tips should be avoided; but, as a general rule, the blades of slow-running propellers may be wider or fuller at the tips than those for high-"revving" screws. Bejeuhr, who has conducted many experiments on propellers with different numbers and types of blade,§ found that the best results were obtained with propellers having a few narrow blades evenly tapered and faired off to the tips.

§ 9. **Number of Blades.**—Practically all successful types of airscrew are of the two-bladed or else of the four-bladed variety, although, of course, there is no real difficulty in constructing a propeller with many more blades. From first principles it appears

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* "Étude experimentale des hélices propulsives au Laboratoire d'Aéronautique Militaire de Chalais Meudon," *La Technique Aéronautique*. June 1, 1913.

† *Vide* Report of the National Advisory Committee for Aeronautics, No. 316, February, 1917.

‡ *Vide* Third Annual Report of the (American) Advisory Committee for Aeronautics. Report No. 14, Part II., by Dr. W. F. Durand.

§ These experiments were carried out on a special track laid down in conjunction with the Frankfort International Exhibition of 1909.

that the larger the number of blades the more nearly does the stranded slip-stream approach to a cylindrical column of air moving with uniform velocity, and consequently the smaller the $\frac{1}{2}mv_s^2$ loss. This, however, does not take the *instrumental* losses into account which are incidental to the setting up of the more uniform wake-stream. Blade interference becomes very serious with low-pitched multibladed screws, and as a result of tests made with propellers having 8, 16, and 24 blades, Constanzi* confirmed the conclusions of Bejeuhr that the best results are obtained with but few blades. Actually, the designer has little choice in the matter, as only two- and four-bladed screws are now permissible, the number of blades being fixed chiefly by the rotational speed of the propeller shaft. As a rough guide, it may be stated that high-"revving" propellers required to operate above 1,400 r.p.m. give the better performance when constructed as *two*-bladers, while low-"revving" screws below 1,400 r.p.m. are more satisfactory as *four*-bladers. This result, arrived at from actual performance data, is shown in Fig. 5, where it is seen that the "first approximation" efficiency curves intersect at a tip speed of about 700 ft./sec.—corresponding to 1,400 r.p.m.

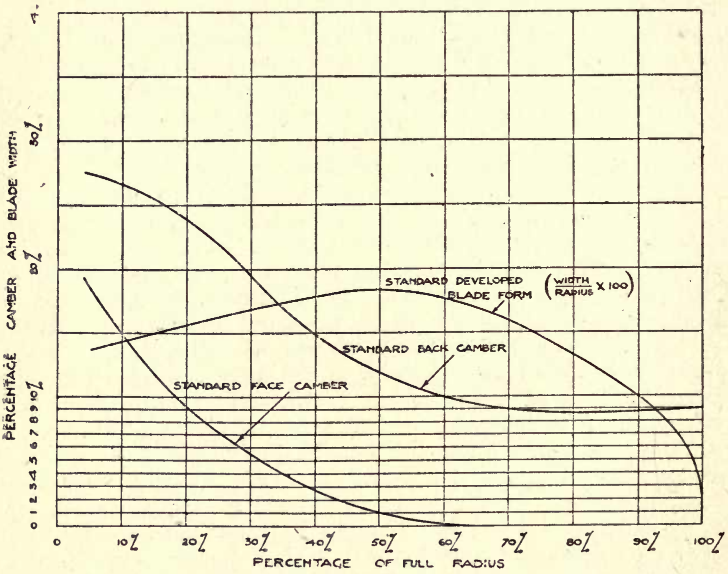

Fig. 4.

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* *Vide* "Nota sulla resistenza delle eliche autorotanti," 1913.

with a propeller 9 feet 6 inches in diameter. Propellers driven directly off the engine shaft are generally more efficient as two-bladers, while propellers used in conjunction with geared-down engines are usually more satisfactory as four-bladers. Some important experiments on the relative merits of two- and four-bladed screws have been carried out at the National Physical Laboratory,* the blades themselves being identical, and the screws, therefore, having equal face-pitch values in each case. The resulting efficiency curves for type A airscrews, having an aspect ratio 10, are shown in Fig. 6. It is seen there that (*a*) the maximum efficiency of 73½ per cent. is attained with the two-bladed screw; (*b*) a maximum efficiency of 70½ per cent. is attained with the four-bladed screw, in which all the blades operate in the same plane of rotation; and that (*c*) a maximum efficiency

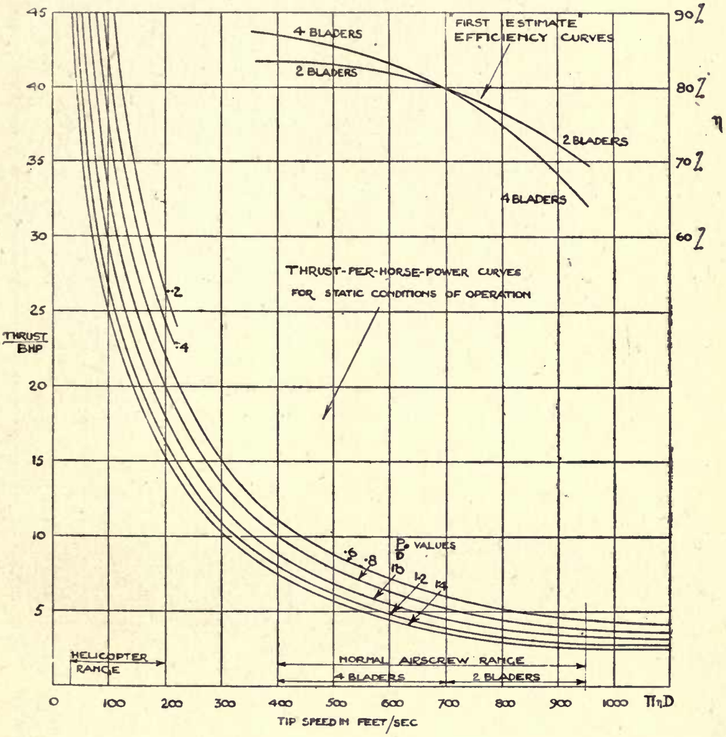

FIG. 5.

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* Report of the National Advisory Committee for Aeronautics, No. 316, February, 1917.

of only 68 per cent. is attained with a four-blader formed by superimposing a pair of two-bladers at right angles to each other. At the same forward and rotational speed values it was found that the torque, and consequently the brake power, required to operate the four-bladed screw was about 1·80 that required by the two-blader; while the thrust was only 1·72 times as great in the case of the single piece propeller, and 1·70 times as great in the case of the four-blader in two sections.

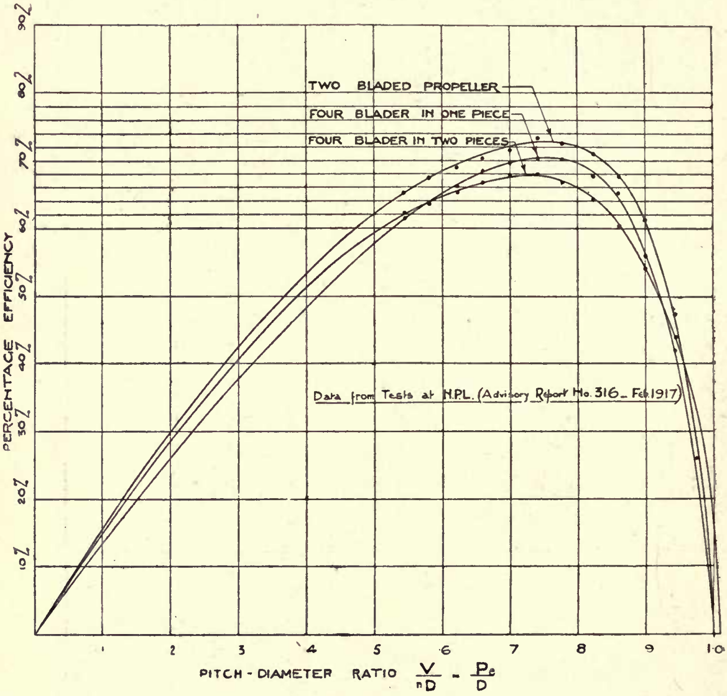

Fig. 6.

§ 10. **Distribution of Angle of Attack.**—The manner in which the attack of a propeller blade varies from root to tip has an important bearing on the shape of the efficiency curve, and consequently on the performance of a propeller. According to the definition of M. Soreau,* a "rational" propeller is one designed to operate at a constant angle of attack over the whole blade. This is, indeed, a satisfactory method of design where it is more

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* "L'hélice propulsive," Mémoires de la Société des Ingenieurs Civils de France, September, 1911.

important to secure a well-" rounded " top to the efficiency curve and a good " all-round " performance rather than one which is especially good when the propeller is operating under a particular set of aerodynamic conditions. If, for example, the very highest performance be required when operating " all-out level," the angle of attack of each blade element should be so increased, with increase of camber towards the propeller boss, that each section may function at its maximum lift-drag ratio everywhere over the blade. A very " peaky " propeller characteristic usually results from such a distribution of attack, a very high maximum efficiency being, however, obtainable.

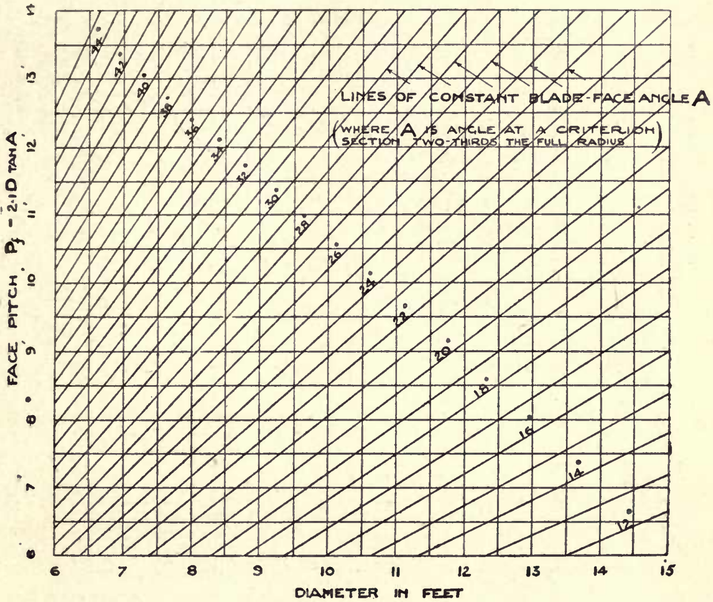


FIG. 7.

§ 11. **Definition of Pitch.**—Much ambiguity has arisen as to the meaning and use of the term *pitch* in connection with an airscrew. It is important, therefore, to clearly discriminate between the *face* pitch, P_f, the *effective* pitch, P_e, and the *mean experimental* pitch, P_o. The *face* pitch, P_f, is obtained according to the following geometric rule, where A is the blade-face angle at a criterion section two-thirds of the full propeller radius measured from the boss centre. Thus we have—

$$P_f = \frac{2}{3}\pi D \tan A = 2{\cdot}1 D \tan A \quad . \quad . \quad . \quad (12)$$

A chart for quickly finding P_f for all propellers from 6 to 15 feet diameter is given in Fig. 7. The face pitch is useful as a rough guide in judging the suitability of a propeller for any particular engine and aeroplane combination. It is the figure ordinarily referred to in making use of the term *pitch* without qualification, and it is that which is stamped on the propeller boss. The *mean effective* pitch is the advance per revolution which an airscrew makes in an axial direction when operating under some definite condition of speed or climb. It is a quantity which varies with each speed and manœuvre of an aircraft, being zero when the propeller is simply turning under static conditions, and a maximum when flying "all-out level." If P_e is in feet, V in ft./sec., n in revs./sec., we have—

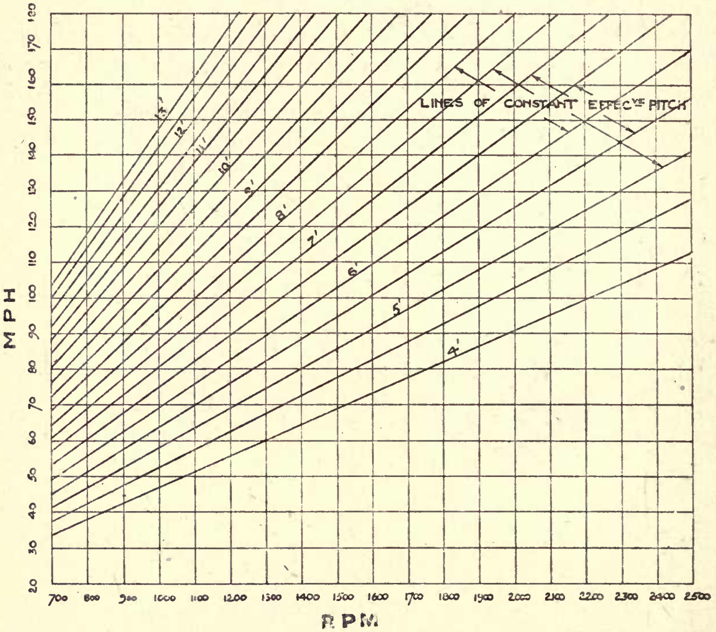


FIG. 8.

$$P_e = \frac{V}{n} = 88 \frac{MPH}{RPM} \quad . \quad . \quad . \quad . \quad (13)$$

A chart for quickly finding P_e for all propellers operating at a rotational speed between 700 r.p.m. and 2,500 r.p.m. is given in Fig. 8. The *mean experimental* pitch, P_o, of a propeller is the axial advance per revolution which must be "artificially" imposed on the screw in order that the thrust may be reduced to zero. Under such conditions of operation the slip also vanishes, and the propeller simply "screws its way" into the air without leaving any axial wake. It is, of course, impossible to reproduce these conditions in *normal* flight, although in the case of a steep nose-dive a propeller may change its function into that of a windmill, the *effective-pitch* value, P_e, then being even greater than the *mean experimental* pitch, P_0. As we shall see presently,

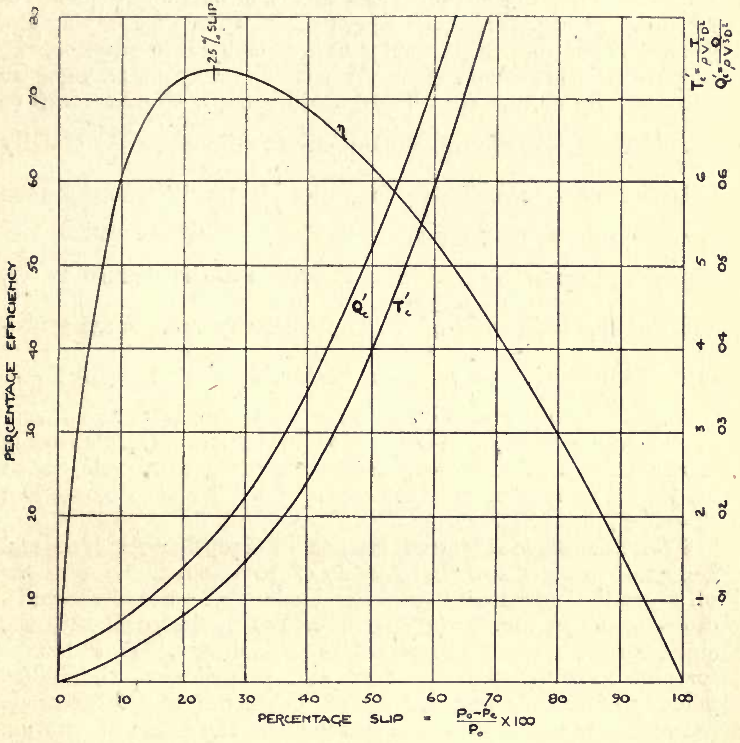


FIG. 9.

the best efficiency is usually attained when the *mean effective* pitch of a propeller is three-quarters of its *mean experimental* pitch. If it is desired to obtain a theoretical estimate of the *zero-thrust* pitch of a propeller, the rule already given for finding the *face* pitch may be used, the blade-face angle, A, now being increased by the inclination α_0 between the no-lift line and face line of the criterion propeller section. Thus we have approximately—

$$P_0 = 2{\cdot}1 D \tan (A + \alpha_0) \quad . \quad . \quad . \quad (14)$$

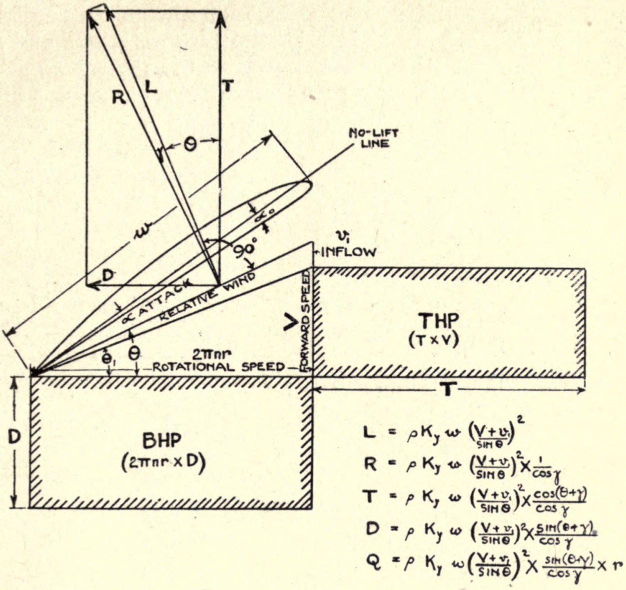
§ 12. **Definition of Pitch-diameter, Velocity and Slip Ratios.**—There is abundant evidence to show that the maximum efficiency of any member of a family of propellers, conforming to the same type of blade, chiefly depends on the *zero-thrust* pitch-diameter ratio $\frac{P_o}{D}$. Of course, the actual efficiency attainable in any particular case also depends on the blade shape, section, and distribution of attack; but this efficiency may be closely estimated by making use of such a family of characteristic curves as shown in Fig. 14. Even where the wisdom of applying the general result to any particular case is called into question, the *shape* of the maximum efficiency envelope may be assumed as substantially correct for all types of airscrew, whether two- or four-bladed. The *effective* pitch-diameter ratio, $\frac{V}{nD} = \frac{P_e}{D}$, normally falls short of the *zero-thrust* pitch ratio, $\frac{P_o}{D}$, the difference between the two values being representative of the angle of attack and slip of the screw blades. The velocity ratio is defined as $\frac{P_e}{P_o}$, and the slip ratio as $\frac{P_o - P_e}{P_o}$. The efficiency curve for the two-bladed screw of type A shown in Fig. 6, plotted against $\frac{V}{nD}$, has been replotted in Fig. 9, on a slip-ratio base, the thrust coefficient curve, T'_c, and torque coefficient curve, Q'_c, also being shown in Fig. 9. In accordance with the general rule stated in § 11, the maximum efficiency of propeller *A* occurs when $P_e = \cdot 75\, P_o$, or at 25 per cent. slip.

§ 13. **The Blade Element Method of Propeller Analysis and Design.**—Consider now the intensity of the aerodynamic reactions on a small element of a propeller blade such as that shown in cross-section in Fig. 10 (*a*) and (*b*). Let us first deal with the simpler case, neglecting forward rotational spin. If V be the forward velocity of the aircraft and propeller, v_1 the inflow velocity of the air just in front of the blades, and θ the angle between the resultant wind direction and the plane of rotation of the propeller, the magnitude of the oncoming wind velocity relatively to the moving blade element will be given by $\frac{V + v_1}{\sin \theta}$, and the "absolute" angle of attack will be given by the relation—

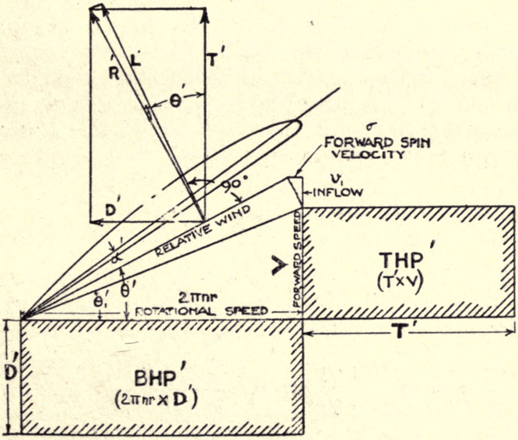
$$a = A + a_o - \theta \quad . \quad . \quad . \quad . \quad (15)$$

In accordance with the usual rule for finding the normal reaction on an aerofoil, we have—

$$L = \rho K_y w \left(\frac{V + v_1}{\sin \theta}\right)^2 \quad . \quad . \quad . \quad . \quad (16)$$



(a) REACTIONS ON BLADE ELEMENT NEGLECTING FORWARD SPIN



(b) REACTIONS ON BLADE ELEMENT ALLOWING FOR FRONT SPIN.

FIG. 10.

where L is the *intensity* of the reaction normal to the relative wind direction, w is the width of the blade element and ρ, K_y, etc., have their usual significance. Now the gliding angle, γ, of the propeller element may be obtained from the simple relation $\gamma = \tan^{-1}\left(\frac{D}{L}\right)$, so that the intensity of total reaction, R, may be expressed—

$$R = \rho K_y w \left(\frac{V + v_1}{\sin\theta}\right)^2 \times \frac{1}{\cos\gamma} \quad . \quad . \quad . \quad (17)$$

Further, the intensity of reaction, R, may be split up into two mutually normal components—a thrust component, T, acting axially in the direction of translation, and a drag component, D, acting tangentially to the circular path of the blade element, and in its plane of rotation. From the geometry of the figure it will be seen that—

$$T = \rho K_y w \left(\frac{V + v_1}{\sin\theta}\right)^2 \frac{\cos(\theta + \gamma)}{\cos\gamma} \quad . \quad . \quad (18)$$

$$D = \rho K_y w \left(\frac{V + v_1}{\sin\theta}\right)^2 \frac{\sin(\theta + \gamma)}{\cos\gamma} \quad . \quad . \quad (19)$$

Since r is the radius of rotation of the propeller element the torque, Q, may be expressed—

$$Q = \rho K_y w \left(\frac{V + v_1}{\sin\theta}\right)^2 \frac{\sin(\theta + \gamma)}{\cos\gamma} \times r \quad . \quad . \quad (20)$$

Now the efficiency of the blade element will be given by the ratio of useful power to power expended. The useful thrust-power is obtained by taking the product of thrust and axial speed thus, $THP = T \times V \div 550$. The brake power expended is given by the product of drag and rotational speed thus, $BHP = D \times 2\pi rn \div 550$. The efficiency, η, may therefore be written—

$$\eta = \frac{T}{D} \times \frac{V}{2\pi rn} = \frac{T}{D} \tan\theta_1 \quad . \quad . \quad . \quad (21)$$

Substituting for T and D the values obtained in equations (18) and (19), we have the following important result for the efficiency of a propeller blade element—

$$\eta = \frac{\cos(\theta + \gamma)}{\sin(\theta + \gamma)} \tan\theta_1 = \frac{\tan\theta_1}{\tan(\theta + \gamma)} \quad . \quad . \quad (22)$$

In Fig. 10 the thrust and brake powers are represented by rectangles constructed on the velocity vectors, V and $2\pi nr$, respectively. The efficiency of operation will then be given by the ratio of the areas enclosed within these rectangles.

§ 14. **Effect of Forward Air-spin.**—If we take the front air-spin into account it will be apparent that the blade element is virtually operating at a smaller relative air speed than previously assumed; for the air just in front of the blades has a velocity component, σ, in the same plane and direction of rotation as the blades themselves. The relative air speeds in an axial and rotational direction are thus given by the expressions $(V+v_1)$ and $(2\pi nr-\sigma)$ respectively, these component velocities combining to give the relative oncoming velocity $\sqrt{(V+v_1)^2+(2\pi nr-\sigma)^2}$.

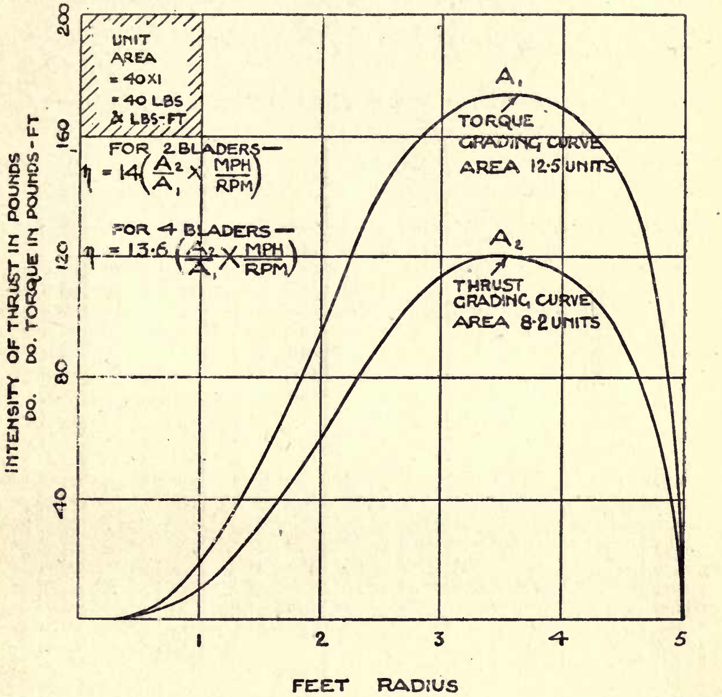


FIG. 11.

Obviously this result is less than that obtained by neglecting front spin. It will also be clear from a comparison of diagrams (*a*) and (*b*) in Fig. 10 that the calculated angle of attack of any element will, if forward spin be allowed for, be less than previously determined. The analytical thrust and torque of the propeller will accordingly be smaller, both on account of the reduced relative air speed and also on account of the decreased angle of attack. The effect of forward spin in decreasing the efficiency of a blade element will be apparent from an examination of equation (22). It will be seen that at the same forward and rotational speeds as before, and at the same gliding angle, the efficiency of working becomes less as the slope of the relative oncoming wind becomes greater. That is, for constant θ_1 and γ values the efficiency η becomes less as θ becomes greater. Actually, the value of γ will change with the angle of attack, and if the original analysis showed the blade to be operating at its smallest γ value, the reduction in attack, due to the allowance for forward spin, will cause γ to increase, and the efficiency to suffer accordingly. A useful table for finding the gliding angle, γ, corresponding to all values of $\frac{L}{D}$ from 5 to 23 is given herewith:

Table of Gliding Angles for L/D Ratios from 5 to 23.

L/D	γ	L/D	γ	L/D	γ	L/D	γ	L/D	γ	L/D	γ
5·0	11° 19′	8·0	7° 08′	11·0	5° 12′	14·0	4° 05′	17·0	3° 23′	20·0	2° 52′
5·1	11° 06′	8·1	7° 04′	11·1	5° 09′	14·1	4° 04′	17·1	3° 22′	20·1	2° 52′
5·2	10° 52′	8·2	6° 58′	11·2	5° 05′	14·2	4° 02′	17·2	3° 20′	20·2	2° 51′
5·3	10° 42′	8·3	6° 54′	11·3	5° 02′	14·3	4° 00′	17·3	3° 19′	20·3	2° 50′
5·4	10° 29′	8·4	6° 47′	11·4	5° 01′	14·4	3° 57′	17·4	3° 19′	20·4	2° 49′
5·5	10° 19′	8·5	6° 44′	11·5	4° 58′	14·5	3° 56′	17·5	3° 18′	20·5	2° 48′
5·6	10° 09′	8·6	6° 37′	11·6	4° 55′	14·6	3° 54′	17·6	3° 16′	20·6	2° 48′
5·7	9° 56′	8·7	6° 34′	11·7	4° 52′	14·7	3° 52′	17·7	3° 16′	20·7	2° 47′
5·8	9° 46′	8·8	6° 30′	11·8	4° 50′	14·8	3° 52′	17·8	3° 14′	20·8	2° 46′
5·9	9° 54′	8·9	6° 24′	11·9	4° 48′	14·9	3° 50′	17·9	3° 13′	20·9	2° 45′
6·0	9° 29′	9·0	6° 20′	12·0	4° 45′	15·0	3° 49′	18·0	3° 12′	21·0	2° 44′
6·1	9° 19′	9·1	6° 17′	12·1	4° 43′	15·1	3° 48′	18·1	3° 10′	21·1	2° 43′
6·2	9° 09′	9·2	6° 13′	12·2	4° 41′	15·2	3° 47′	18·2	3° 09′	21·2	2° 42′
6·3	9° 02′	9·3	6° 10′	12·3	4° 38′	15·3	3° 47′	18·3	3° 09′	21·3	2° 42′
6·4	8° 52′	9·4	6° 03′	12·4	4° 36′	15·4	3° 43′	18·4	3° 08′	21·4	2° 41′
6·5	8° 45′	9·5	6° 00′	12·5	4° 35′	15·5	3° 42′	18·5	3° 07′	21·5	2° 40′
6·6	8° 39′	9·6	5° 56′	12·6	4° 31′	15·6	3° 40′	18·6	3° 06′	21·6	2° 39′
6·7	8° 29′	9·7	5° 53′	12·7	4° 30′	15·7	3° 38′	18·7	3° 06′	21·7	2° 38′
6·8	8° 22′	9·8	5° 50′	12·8	4° 28′	15·8	3° 36′	18·8	3° 04′	21·8	2° 38′
6·9	8° 15′	9·9	5° 46′	12·9	4° 26′	15·9	3° 36′	18·9	3° 03′	21·9	2° 37′
7·0	8° 08′	10·0	5° 43′	13·0	4° 24′	16·0	3° 36′	19·0	3° 02′	22·0	2° 36′
7·1	8° 02′	10·1	5° 39′	13·1	4° 21′	16·1	3° 34′	19·1	3° 00′	22·1	2° 35′
7·2	7° 55′	10·2	5° 36′	13·2	4° 20′	16·2	3° 33′	19·2	2° 59′	22·2	2° 35′
7·3	7° 48′	10·3	5° 33′	13·3	4° 18′	16·3	3° 30′	19·3	2° 59′	22·3	2° 34′
7·4	7° 41′	10·4	5° 29′	13·4	4° 16′	16·4	3° 30′	19·4	2° 58′	22·4	2° 34′
7·5	7° 35′	10·5	5° 26′	13·5	4° 14′	16·5	3° 30′	19·5	2° 57′	22·5	2° 32′
7·6	7° 31′	10·6	5° 22′	13·6	4° 13′	16·6	3° 28′	19·6	2° 56′	22·6	2° 31′
7·7	7° 25′	10·7	5° 19′	13·7	4° 11′	16·7	3° 27′	19·7	2° 55′	22·7	2° 31′
7·8	7° 18′	10·8	5° 18′	13·8	4° 08′	16·8	3° 26′	19·8	2° 54′	22·8	2° 30′
7·9	7° 14′	10·9	5° 16′	13·9	4° 06′	16·9	3° 24′	19·9	2° 53′	22·9	2° 30′

§ 15. **Thrust and Torque Grading Curves. Calculation of H.P. and η.**—The thrust and torque grading curves of a propeller are obtained by plotting the thrust and torque intensities against radius of rotation. The curves shown in Fig. 11 apply to a particular four-bladed propeller 10 feet in diameter, the rotational speed being 1,200 r.p.m. (20 revs./sec.), and the forward speed 104 m.p.h. (153 ft./sec.). The area of the torque diagram is seen to be 12·3 units, and that of the thrust diagram 8·2 units; but, since each unit corresponds to 40 lb.-ft. and 40 lb. respectively, the total torque per blade is obviously $12{\cdot}3 \times 40 = 492$ lb.-ft., and the total thrust $8{\cdot}2 \times 40 = 328$ lb. Now we have seen that the total torque of a four-bladed screw is $1{\cdot}8 \times 2 = 3{\cdot}6$ times that of a single blade, while the total thrust is only $1{\cdot}72 \times 2 = 3{\cdot}44$ times that of the single blade, so that we have—

$$B.H.P. = \frac{\text{total torque in lb.-ft.} \times 2\pi \times \text{revs./sec.}}{550}$$

$$= \frac{492 \times 3{\cdot}6 \times 2\pi \times 20}{550} = 405 \; B.H.P.$$

$$T.H.P. = \frac{\text{total thrust in lb.} \times \text{axial speed in ft./sec.}}{550}$$

$$= \frac{328 \times 3{\cdot}44 \times 153}{550} = 314 \; T.H.P.$$

$$\text{Hence, } \eta = \frac{T.H.P.}{B.H.P.} = \frac{314}{405} = 77{\cdot}5 \text{ per cent.}$$

§ 16. **Aerodynamic Data on Airscrew Sections.**—The geometrical contours and aerodynamic characteristics of six sections suitable for airscrew design are given in Fig. 12. The shape and proportions of the various sections were taken from an actual airscrew, the models tested being made to a uniform size of 6 inches × 36 inches. The air speed in the test channel was kept as nearly as possible constant at 80 ft./sec., so that we have $vl = 40$ ft.^{2}/sec. units. As the models were tested at this high value of vl, it may be assumed that the full scale characteristics will be practically identical with those shown in the curves. This assumption is justified by the close agreement usually obtained between the calculated and actual performance of an airscrew when high-speed aerofoil data is employed.

§ 17. **The Characteristic Coefficients of an Airscrew.**—In applying the fundamental force equation to the case of the ideal actuator we have already made use of the symbol T'_c to represent the absolute thrust coefficient. It has been shown that the inflow ratio a and efficiency η are entirely dependent on T'_c. Let us now consider how the thrust, torque, and power of corresponding blade elements of geometrically similar propellers varies with V, n, and D, when operating under similar aerodynamic conditions —that is to say, at the same angle of attack, α, and therefore at the same values of K_y, γ, θ, a, and $\frac{V}{nD}$. From the thrust and torque equations (18) and (20) and the corresponding power equation it will then be clear that—

$$T \propto \rho w V^2(1+a)^2 \quad . \quad . \quad . \quad . \quad (23)$$
$$Q \propto \rho w V^2(1+a)^2 D \quad . \quad . \quad . \quad . \quad (24)$$
$$P \propto \rho w V^3(1+a)^2 \quad . \quad . \quad . \quad . \quad (25)$$

Now the area of corresponding blade elements of similar screws varies as wD or as D^2. Also, for similar conditions of operation,

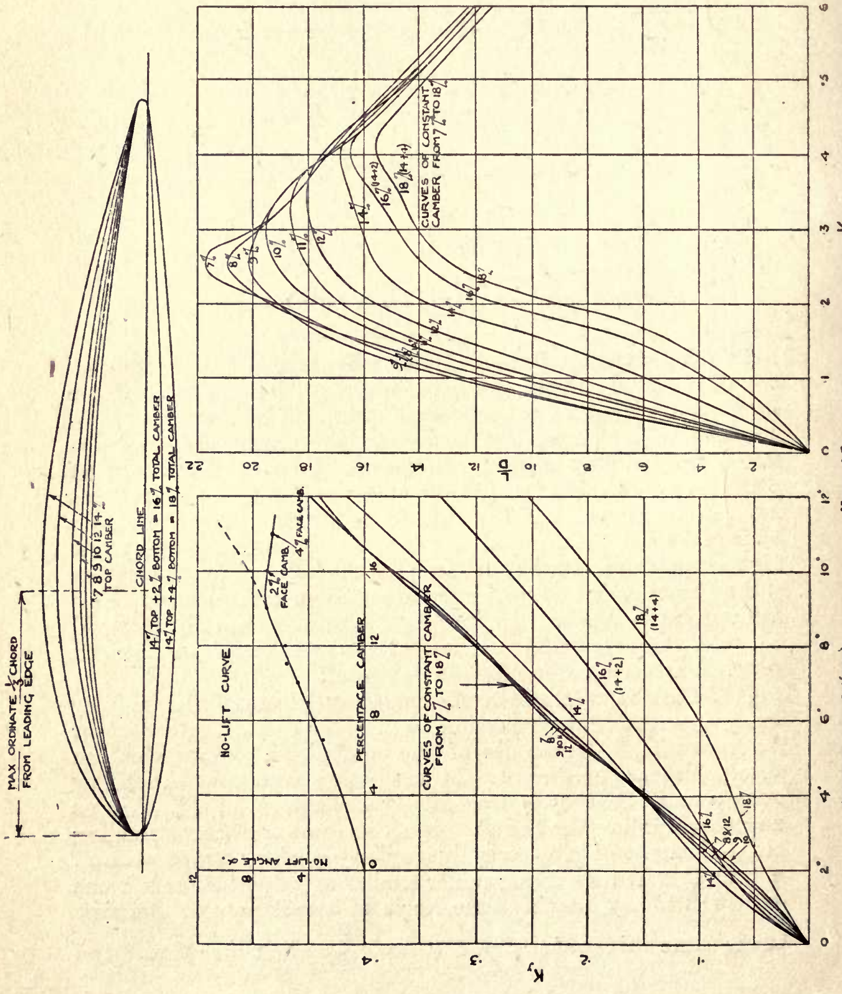


FIG. 12.

the inflow ratio a is constant and $V \propto nD$, so that we may rewrite the above equations in the following alternative forms—

$$T \propto \rho V^2 D^2 \qquad \propto \rho n^2 D^4 \qquad \propto \rho \frac{V^4}{n^2} \quad . \quad . \quad (26)$$

$$Q \propto \rho V^2 D^3 \qquad \propto \rho n^2 D^5 \qquad \propto \rho \frac{V^5}{n^3} \quad . \quad . \quad (27)$$

$$P \propto \rho V^3 D^2 \qquad \propto \rho n^3 D^5 \qquad \propto \rho \frac{V^5}{n^2} \quad . \quad . \quad (28)$$

By introducing appropriate coefficients and transposing terms, we have the following important results—

$$T_c' = \frac{T}{\rho V^2 D^2} \qquad T_c'' = \frac{T}{\rho n^2 D^4} \qquad T_c''' = \frac{Tn^2}{\rho V^4} \quad . \quad (29)$$

$$Q_c' = \frac{Q}{\rho V^2 D^3} \qquad Q_c'' = \frac{Q}{\rho n^2 D^5} \qquad Q_c''' = \frac{Qn^3}{\rho V^5} \quad . \quad (30)$$

$$P_c' = \frac{P}{\rho V^3 D^2} \qquad P_c'' = \frac{P}{\rho n^3 D^5} \qquad P_c''' = \frac{Pn^2}{\rho V^5} \quad . \quad (31)$$

In Fig. 9 the values of the absolute coefficients T_c' and Q_c' are shown plotted against slip ratio $\frac{P_o - P_e}{P_o}$. The rapid rise of the curves at large slip values should be noticed. Since V is involved in the denominator of T_c' and Q_c', it will be obvious that their value becomes infinitely great when V is reduced to zero—that is to say, the "absolute" thrust and torque coefficients have no useful significance under static conditions of operation. Under these conditions, the coefficients T_c'', Q_c'' and P_c'' must be employed.*

§ 18. **Static Thrust of Propellers and Helicopters.**—The quality of an airscrew as regards static thrust is generally expressed in terms of thrust per horse-power. Now for similar, and similarly operating, airscrews we have $\frac{T}{P} \propto \frac{\rho n^2 D^4}{\rho n^3 D^5} \propto \frac{1}{nD}$, so that the thrust per horse-power under such conditions will vary inversely as the tip speed. This result is seen in the mean experimental curves plotted in Fig. 5, where it is also apparent that the thrust per horse-power also depends on the pitch-ratio of the screwemployed, being greater as the pitch-ratio becomes smaller at the same tip speed. In order to secure the best results from the large direct lifting screw or helicopter, very small tip speeds should be em-

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* It should be noticed that various authorities differ in the choice of symbols representative of the various characteristic coefficients of a propeller. Thus the N.P.L. makes use of a_o, b_o, for T_c'', Q_c'', under static conditions of operation.

ployed—not greater than about 200 ft./sec., as shown in Fig. 5. The pitch-diameter ratio $\frac{P_o}{D}$ should also be very small, which means that, normally, very large diameters are necessary in order that the requisite horse-power may be absorbed to the best advantage. A convenient chart for finding the tip speeds of all propellers from 4 feet to 15 feet in diameter, operating at all rotational speeds from 700 r.p.m. to 2,500 r.p.m., is given in Fig. 13.

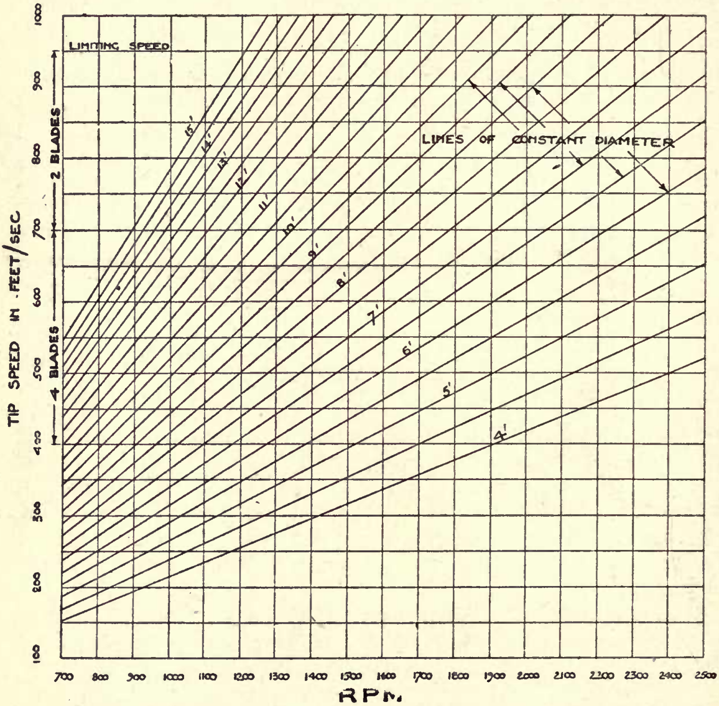

Fig. 13.

§ 19. **The Eiffel Diagram.**—A family of efficiency curves applicable to a particular type of two-bladed propeller is given in Fig. 14, and in Fig. 15 the corresponding power curves are plotted on a logarithmic diagram in accordance with the method evolved by M. Eiffel.* It should be particularly noted that the horse-power and velocity co-ordinates only apply *directly* to the

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* *Vide* "Résistance de l'air et l'aviation," chap. x.

characteristic curves when the propellers are 10 feet in diameter, and are operating at the datum rotational speed of 1,000 r.p.m. (16·7 revs./sec.). At all other diameters and rotational speeds the corresponding power and velocity values are obtained by marking-off two segments of appropriate length and slope determined by the *D* and r.p.m. scales at the right-hand side of the diagram. It is sometimes found useful to consider the iso-efficiency lines as analogous to the contour lines of a map, the enveloping efficiency curve of Fig. 14 (which is, for all practical purposes, coincident with the locus of maximum efficiency) then becoming the line of least slope in Fig. 15. Having given the engine power and estimated speed of an aircraft, it is frequently required to find the best propeller diameter to employ at a stated rotational speed, or the optimum diameter and best gear ratio to employ in order to attain the efficiency maximum maximorum. Such problems as these may be readily solved by means of the Eiffel diagram if it be borne in mind that the oblique scales should be manipulated so as to reach as nearly as possible to the centre of the diagram along the line of maximum efficiency. The method of using the diagram may be best illustrated by means of an example. Suppose it be required to find the optimum diameter of an airscrew and the best gear reduction to be fitted

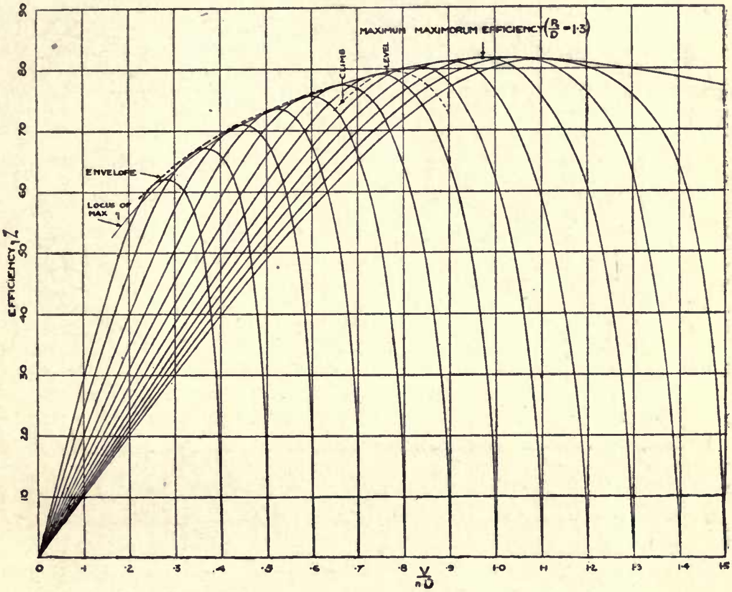

Fig. 14.

to an engine developing 575 b.h.p. at a nominal rotational speed of 1,750 r.p.m. We shall suppose that at maximum maximorum efficiency the aircraft is estimated to attain a speed of 110 m.p.h. near ground level. Now, obviously the propeller will require to be a four-blader, so that before we can make use of the two-blader characteristics shown in Fig. 15, it will be necessary to reduce the given horse-power according to the result given in § 9.

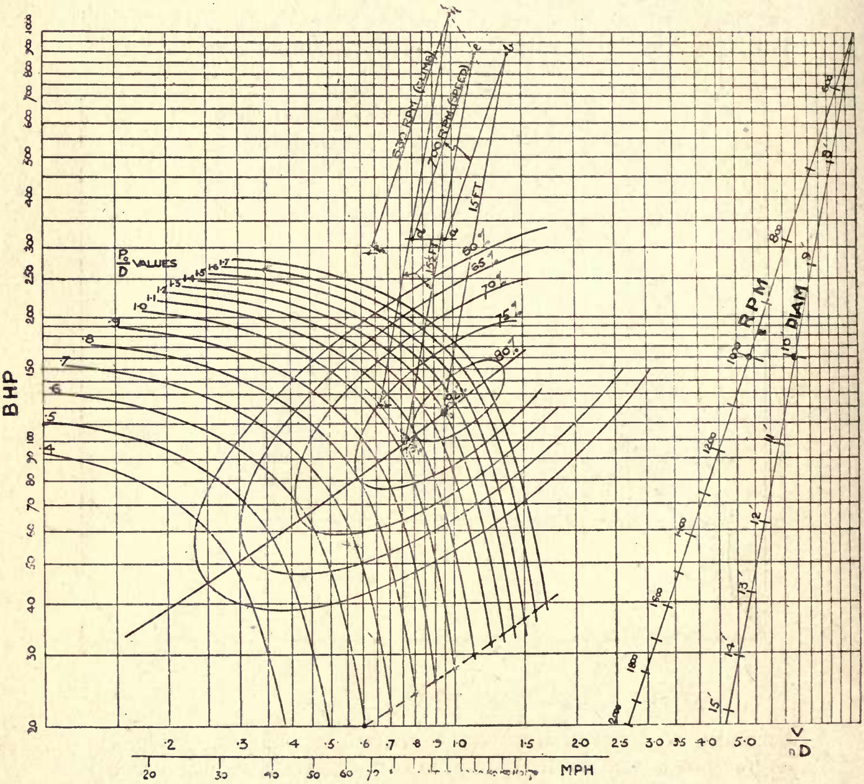

Fig. 15.

Thus the horse-power absorbed by a two-bladed screw having identical blades, and operating at the same forward and rotational speeds as a four-blader absorbing 575 b.h.p., will be given by $\frac{575}{1\cdot8} = 320$ b.h.p. In Fig. 15 we may now find the point *a* corresponding to 320 b.h.p. and 110 m.p.h. The points *a* and *o* being fixed, it only requires a simple manipulation of the oblique scales to effect the solution required. It is seen that the optimum diameter represented by the segment *ob* is 15 feet, and the rotational speed represented by the segment *ab* is 700 r.p.m. The best gear ratio is thus $\frac{700}{1750}=0{\cdot}4$. If we assume a forward speed of 90 m.p.h. and 0·4 gear reduction, it will be seen from Fig. 15 that the diameter should be increased to 15½ feet, the efficiency of operation thereby falling from 82 per cent. to 79 per cent.—a loss of 3 per cent., due entirely to the reduction of forward speed. Suppose now it be required to find the drop in rotational speed and efficiency with the 15½ feet screw climbing at an air speed of 70 m.p.h., we proceed by the method of "trial and error" thus: Having assumed a horse-power value (about 90 per cent. of that when operating "all-out level"), we mark the appropriate point, *g*, and set up *gh* representative of the propeller revolutions at the assumed horse-power. The horse-power and rotational speed must agree with a point on the engine curve, and the diameter represented by the closing segment *hk* must be 15½ feet, since the diameter has not changed during the climbing manœuvre. If the latter condition be not satisfied at the first attempt, other horse-power and rotational speed values must be taken from the engine curve, and the position of *g* and the length of *gh* adjusted till $ef=hk$.

It will be seen from Fig. 15 that the rotational speed when climbing at 70 m.p.h. has fallen to 630 r.p.m., the efficiency at the same time having decreased to 75 per cent.—or 4 per cent. below the value in level flight. The points of operation corresponding to the above results are shown in the appropriate efficiency curve in Fig. 14. Of course the accuracy of the results obtained in any case depends on the accuracy with which the family characteristics can be determined. We have already seen that their *shape* depends on the *type* of propeller, as also does the pitch ratio corresponding to the efficiency maximum maximorum. In Figs. 14 and 15 it is seen that the max. max. efficiency of 82 per cent. is attained at a $\frac{P_e}{D}$ value of 1·3, corresponding to $\frac{P_e}{D}$ value of 0·96. This agrees with the result obtained by Dorand,* a max. max. efficiency of 82 per cent. being attained at a best pitch diameter ratio of 1·29. It is interesting to note from the discussion of R. E. Froude's paper of 1886† that an efficiency of 70 per cent. can be attained with *marine* propellers when operating at a pitch-diameter ratio of 1·15.

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* "Étude expérimentale des hélices propulsives au Laboratoire Militaire de Chalais Meudon," *La Technique Aéronautique*, June 13, 1913.

† "On the Determination of the most suitable Dimensions of Screw Propellers," by Dr. R. E. Froude, *Proceedings of the Institution of Naval Architects*, vol. xxvii., 1886.

§ 20. **The Strength of Airscrew Blades.**—In seeking to make an accurate estimate of the stresses in, and strength of, a propeller blade, the following aerodynamic and mechanical phenomena must be properly taken into account—

(*a*) Bending moment due to axial thrust.
(*b*) Bending moment due to any gyroscopic action.
(*c*) Direct centrifugal force.
(*d*) Bending moment due to the latter.
(*e*) Twist of blades due to movement of centre of pressure and centrifugal force.

In the calculation of (*a*), the stresses due to blade thrust, it is usual to first derive the bending moment diagram from the thrust grading curve, and, having obtained the moment of inertia of a number of blade sections about their least radii of gyration, to apply the well-known rule $f = \frac{M}{Z}$. The torque due to (*b*), gyroscopic action, only occurs when the propeller axis deviates from its straight course on account of some manœuvre of the aircraft. The torque at the root of each blade may then be calculated from the rule $M = I\omega_1\omega_2$, where M is the total torque in lb.-ft. tending to bend the engine shaft, I, the moment of inertia of the revolving propeller in lb.-ft.^{2} units, ω_1 the angular velocity of rotation in radians/sec. and ω_2 the maximum angular velocity which can be attained in any manœuvre of the aircraft. It is usual to assume that a maximum angular velocity of one radian/sec. may be instantaneously attained by an aeroplane in "flattening out" after a steep nose-dive. In determining (*c*) the centrifugal force on the propeller blade at any section, we apply the rule c.f. $= \frac{W}{g}\omega_1{}^2r$, where W is the weight of that portion of the blade beyond the section considered and r is the radius of rotation of its centre of gravity. The calculation of (*d*), the bending moment due to centrifugal force, is not so simple.

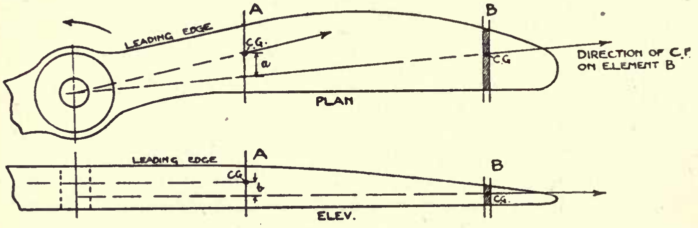
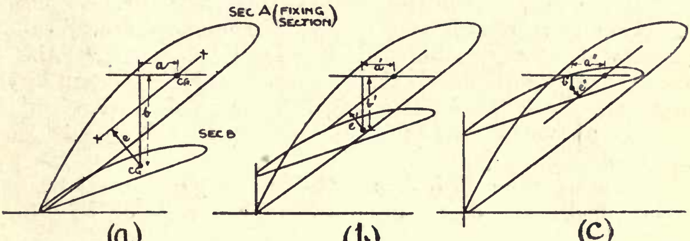


FIG. 16.

The object of the designer is to so proportion and dispose the material of the propeller blade that a negative bending moment shall be obtained sufficient to cancel (*a*), the bending moment due to direct aerodynamic thrust. The usual method of effecting this end by swinging the propeller tip forward in an axial direction is illustrated in Fig. 16. It will be seen that the direction of the centrifugal force on a small blade element, *B*, intersects the "fixing section," *A*, at a *plan* distance, *a*, from its c.g. nearer to the trailing edge, and an *elevation* distance, *b*, also nearer the trailing edge of the propeller. These distances are set out in the enlarged end view at Fig. 17 (*a*), the "effective eccentricity," or lever arm, at which the elemental centrifugal force acts being shown at *e*. By swinging the tip forward, as in diagram (*b*), the eccentricity is reduced to *e*′, while in diagram (*c*) the eccentricity, *e*″, is negative, and a negative bending is produced in the blade tending to "wash-out" the thrust bending.



(a) (b) (c)

FIG. 17.

The estimation of (*e*), the twist of the blades due to the movement of the centre of pressure, is another difficult problem. Dorand has conducted many experiments on this point, and he concludes that the aerodynamic reactions, normally falling ahead of the centre of gravity of the various sections, tends to *increase* the pitch of a propeller, while the centrifugal force has the opposite effect.*

§ 21. **Useful Miscellaneous Data.**—(*a*) The density of standard air is defined as 0·0762 lb./ft.^{3}, or 1·221 kilog./m.^{3} at a pressure of 29·9 inches, or 759·5 mm. of mercury and at a temperature of 16° C. or 60·6° F. The normal rate of fall-off of air density and engine-power with height is given in the following table in terms of the standard air density, which normally occurs at about 800 feet altitude.

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* *Vide* "La déformation des hélices et leurs coefficients de sécurité," *La Technique Aéronautique*, May 1, 1912.

TABLE SHOWING FALL-OFF OF AIR DENSITY AND ENGINE-POWER WITH HEIGHT.

Height in Feet.	*Percentage of Standard Density.*	*Percentage of Normal Power.*
0	102·6	103·0
5,000	87·5	85·5
10,000	74·0	69·5
15,000	63·1	56·5
20,000	53·3	45·0

(*b*) The average density of mahogany is 35 lb./ft.^{3} (·02 lb./in.^{3}).

(*c*) If c and t be the chord length and thickness respectively of a well-proportioned propeller section, the area will be approximately given by the rule $A = \cdot 7\ ct$. and the moment of inertia about its axis of least gyration will be given by $I = \cdot 04\ ct.^3$ Also, if the maximum ordinate be situated at one-third the chord length measured from the leading edge, the c.g. of section will be situated at a point 0·37 c from the leading edge, and 0·42 t above the chord line.

(*d*) 100 m.p.h. = 147 ft./sec. = 44·7 metres/sec. = 87 knots.

(*e*) 1 inch = 2·54 cms. 1 foot = 30·5 cms. 1 kilog. = 2·2 lb.

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