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Climatology of Ground-Based Radio Ducts*

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An atmospheric duct is defined as occurring when geometrical optics indicate that a radio ray leaving the transmitter and passing upwards through the atmosphere is sufficiently refracted that it is traveling parallel to the earth's surface. Maximum observed incidence of ducts was determined as 13 percent in the tropics, 10 percent in the arctic and 5 percent in the temperate zone by analysis of 3 to 5 years of radiosonde data for a tropical, temperate, and arctic location. Annual maximums are observed in the winter for the arctic and summer for the tropics. The arctic ducts arise from ground-based temperature inversions with the ground temperature less than −25° C while the tropical ducts are observed to occur with slight temperature and humidity lapse when the surface temperature is 30° C and greater.

1. Introduction

The author has recently had cause to investigate the limitations placed upon ray tracing of vhf-uhf radio waves by the occurrence of atmospheric ducts [1].[1] Ducting is defined as occurring when a radio ray originating at the earth's surface is sufficiently refracted during its upward passage through the atmosphere so that it either is bent back towards the earth's surface or travels in a path parallel to the earth's surface. Although the proper treatment of ducting involves consideration of the wave equation solution [2] rather than a simple ray treatment, the present study will be based upon a geometrical optics definition of the limiting case in which ray tracing techniques may be used. This simple criterion is then applied to several years of radiosonde observations from stations typical of arctic, temperate and tropical climates to derive estimates of the variation of the occurrence of radio ducts with climatic conditions.

2. Background

The property of the atmosphere basic to radio-ray tracing is the gradient of the radio refractive index of the atmosphere, n. For standard conditions near the surface of the earth n is a number of the order 1.0003 and its gradient is about 40×10^{-6} per kilometer. It is instructive to consider the order of magnitude of refractive index gradient needed for trapping for several commonly observed refractive index profiles. Snell's law may be written, for cylindrical coordinates,

$$n_t r_t \cos\theta_t = n_d r_d \cos\theta_d, \tag{1}$$

where θ is the elevation angle made by the ray at the point under consideration. The subscripts t and d refer to the values of the variables at the transmitter height and the top of the trapping layer respectively. Thus trapping occurs when the ray is traveling parallel to the earth, i.e., $\cos\theta_d=1$ and

$$\frac{n_t r_t}{n_d r_d} \geq 1. \tag{2}$$

The angle of penetration at the transmitter, θ_p, found by setting

$$\frac{n_t r_t}{n_d r_d}\cos\theta_p = 1, \tag{3}$$

divides the ray family into two groups since all rays of $\theta_0 \leq \theta_p$ are trapped within the duct and those rays of $\theta_0 > \theta_p$ are not. The n gradient for a given value of θ_p is then given by

$$\frac{\Delta n}{\Delta r} = -\frac{n_t - n_d}{r_d - r_t}, \tag{4}$$

where, for the ducting case, n_d must satisfy (3), i.e.,

$$n_d = \frac{n_t r_t}{r_d}\cos\theta_p. \tag{5}$$

By designating $r_d = r_t + \Delta h$ then (4) may be written

$$\frac{\Delta n}{\Delta r} = -\frac{n_t}{\Delta h}\left[1 - \frac{r_t}{r_t+\Delta h}\cos\theta_p\right]. \tag{6}$$

By rewriting (6),

$$\frac{\Delta n}{\Delta r} = -\frac{n_t}{\Delta h}\left[1 - \frac{1}{1+\frac{\Delta h}{r_t}}\cos\theta_p\right], \tag{7}$$

and expanding $\left(1+\frac{\Delta h}{r_t}\right)^{-1}$ and $\cos\theta_p$ one obtains the expression

$$\frac{\Delta n}{\Delta r} \cong -n_t\left[\frac{1}{r_t} + \frac{\theta_p^2}{2\Delta h}\right] \tag{8}$$

by neglecting terms of the order $\frac{\theta_p^4}{4!}$ and $\left(\frac{1}{r_t}\right)^2$.

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[1] Figures in brackets indicate the literature references at the end of this paper.

For the case of $\theta_p=0$ and the transmitter at sea level, (8) reduces to

$$-\frac{\Delta n}{\Delta r}=\frac{n}{a}\cong\frac{1}{a}\cong 157\, N \text{ units/km.} \qquad (9)$$

using $a=6{,}373$ km and $N=(n-1)\,10^6$. Note that the n gradient is referred to here and hereafter as parts per million, or in popular parlance, N units.

It is seen from (8) that the n gradient necessary to trap a radio ray at a given value of θ_p is practically independent of transmitting antenna height above the earth. For example, a $\theta_p=0$ ray will be trapped by an n gradient of -157.0 N units/km at sea level when $n_t=1.0003$ while the necessary n gradient at 3 km above sea level will be -156.9 N units/km for an $n_t=1.0002$, thus indicating, for all practical applications, that the necessary n gradient for trapping is independent of altitude. Further, by considering the temperature and humidity gradients encountered in the troposphere one is led to the conclusion that ducting gradients would not be expected to occur at heights greater than 3 km above the earth's surface. In fact, Cowan's [3] investigation indicates that trapping gradients are nearly always confined to the first kilometer above the surface.

A consideration of (8) indicates that the magnitude of the negative gradient necessary for ducting is $1/a$ for $\theta_o=0$ but is increased by the amount $n_t\theta_p^2/2\Delta h$ for other values of θ_p. The gradients necessary for atmospheric ducts as a function of θ_o are given for several different but typical n profiles in figure 1. An analysis of radiosonde data indicates that gradients in excess of 500 N units/km

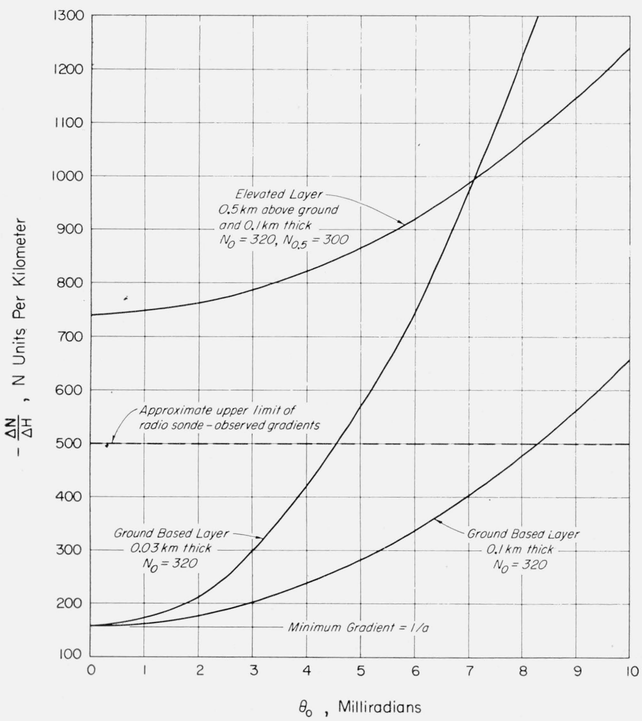

FIGURE 1. *Refractivity gradients needed for radio ducts.*

are seldom indicated within atmospheric layers. Note how rapidly the necessary gradients increase to the approximate upper limit of gradients derived from radiosonde observations; a ground-based layer 100-m thick attains this gradient at 8.3 milliradians while the maximum observed gradient is intercepted by the 30-m layer curve at 4.5 milliradians of θ_o. A third example was calculated for an elevated layer 0.5 km above the ground and 100-m thick by assuming normal refraction between the ground and the base of the layer and solving for the necessary ducting gradient within the layer. The large values of the n gradient necessary for this case indicate that elevated ducts would rarely be observed. Although the preceding examples were calculated for a ground transmitter, the combinations of θ_p, $\Delta n/\Delta h$, and Δh are very nearly the same as would be obtained for any other transmitter height within the first 3 km above the surface.

3. Description of Observed Ground-Based Atmospheric Ducts

Radiosonde data were examined for the occurrence of ducts. Three consecutive years of data were analyzed for the months of February, May, August, and November at each of three Weather Bureau stations. The three stations were chosen to represent a range of climates: Fairbanks, Alaska for an arctic climate, Washington, D. C., for a temperate climate and Swan Island, West Indies, as an example of a tropical climate. The procedure used to determine the occurrence of a radio duct was to:

(a) Determine the value of N from the expression [4]

$$N=(n-1)10^6=\frac{77.6}{T}\left(P+\frac{4810\,e_s RH}{T}\right), \qquad (10)$$

where P is the station pressure in millibars, RH is the percent of the saturation vapor pressure, e_s, in millibars at the absolute temperature, T, in degrees Kelvin;

(b) note all instances when the N gradient equaled or exceeded the minimum ducting gradient indicated by (9), i.e.,

$$-\frac{\Delta n}{\Delta r}=\frac{n}{a}\sim\frac{1}{a}=157\, N \text{ units/km;} \qquad (11)$$

(c) if, for the instances selected by (b),

$$\frac{n_t r_t}{n_d r_d}\geq 1, \qquad (2)$$

where $r_t=a$, then the duct was said to trap rays from an antenna resting on the surface of the earth. Further, under this condition the particular duct would trap all rays from $\theta_o=0$ up to the angle of penetration,

$$\cos\theta_p=\frac{n_d r_d}{n_t a}. \qquad (12)$$

The ducts selected by procedure (a) through (c) are defined as ground-based ducts. Statistics of ground-based ducts are given below.

The percentage occurrence of ducts is shown on figure 2. The maximum occurrences of 13.8 percent for August at Swan Island and 9.2 percent for Fairbanks in February are significantly greater than the values observed at other times of the year. The Washington data display a summertime maximum of 4.6 percent indicating the temperate zone maximum incidence is about one-half the wintertime maximum incidence in the arctic, and about one-third of the summertime tropical maximum.

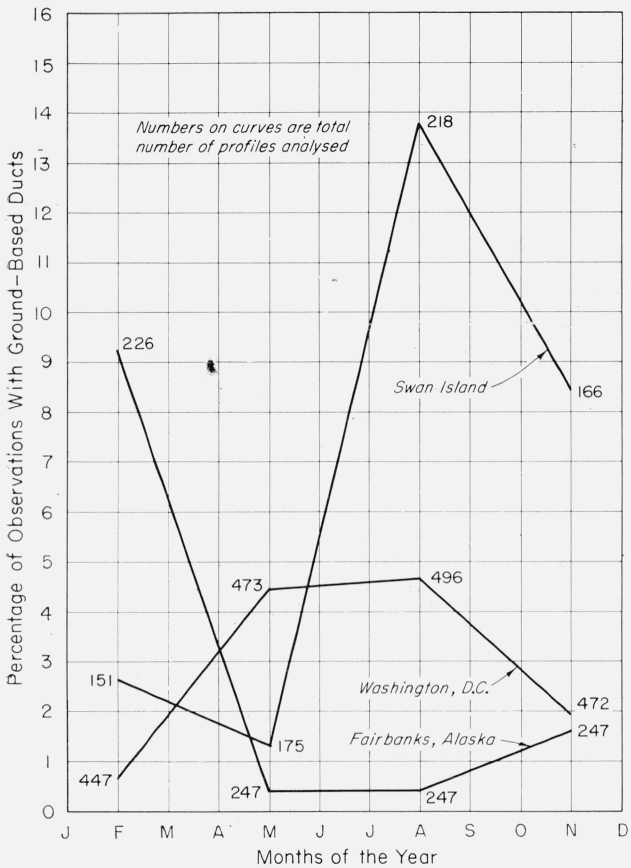

FIGURE 2. *Frequency of occurrence of ground-based ducts.*

The range of values of θ_p observed are shown in figure 3. The mean value calculated for each month as well as the maximum and mean values of θ_p observed for the limiting cases are given for each month and location. The mean value of the angle of penetration under these conditions is between 2 and 3 milliradians and appears to be independent of climate. The maximum value of θ_p observed during ducting is 5.8 milliradians.

The refractivity gradients observed during ducting are given on figure 4. The maximum gradient of 420 N units/km was observed during February at Fairbanks, Alaska. The mean values of N gradient appear to follow a slight climatic trend from a high

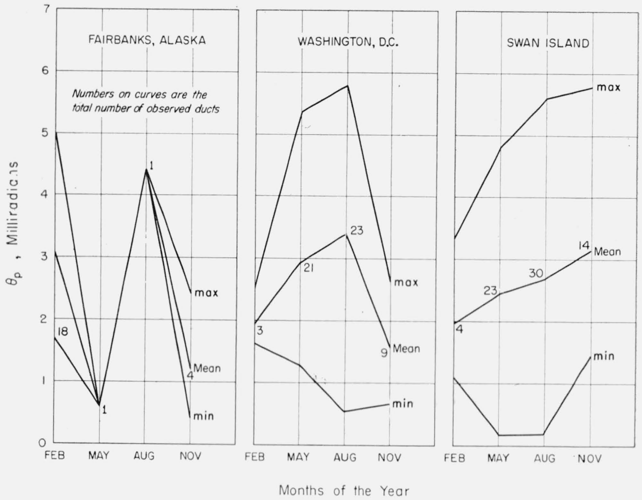

FIGURE 3. *Angle of penetration of ground-based ducts.*

value of 230 N units/km at Fairbanks to a value of 190 N units/km at Swan Island.

Another property of radio ducts is their thickness which is given in figure 5. Again there is observed a slight climatic trend as the median thickness increases from 66 m at Fairbanks to 106 m at Swan Island. These values of thickness correspond to the gradients given in figure 4. The steepness of the distribution curve for Washington appears to be due to a mixture of N profile types that give rise to ground-based ducts. The thickest ducts at Washington are actually elevated layers accompanied by a relatively small N gradient between the ground and the base of the layer. This is in contrast to the Fairbanks and Swan Island profiles which tend to be composed of a single ducting gradient from the surface to the top of the duct.

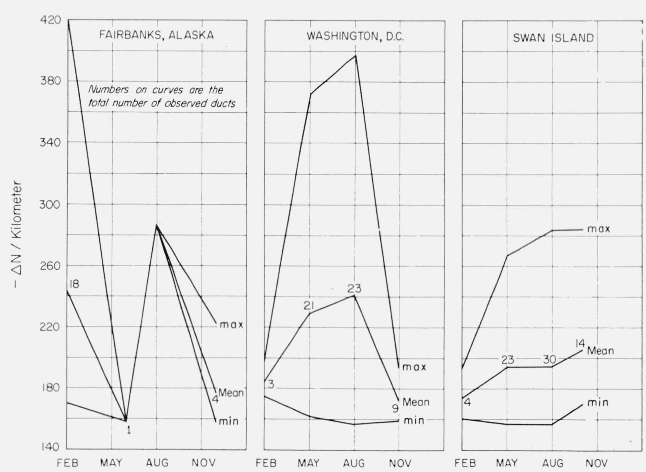

FIGURE 4. *Refractivity gradients of ground-based ducts.*

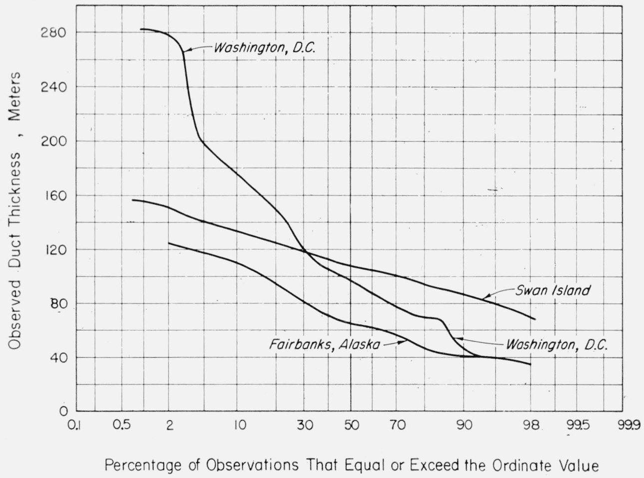

FIGURE 5. *Observed ground-based duct thickness.*

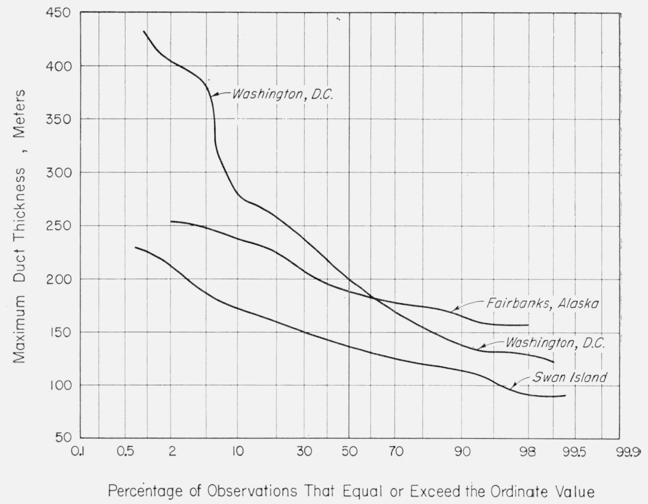

FIGURE 6. *Maximum ground-based duct thickness.*

One may obtain yet another thickness by linearly extrapolating to obtain the height at which the gradient is equal to $-1/a$, that is, the height corresponding to the gradient just sufficient to trap the ray at $\theta_o=0$. These values, shown in figure 6, display an increase in the median thickness of about 25 percent for Swan Island, 100 percent for Washington, and 200 percent for Fairbanks, which results in a reversal of the climatic trend of the observed thickness between Fairbanks and Swan Island. This increase in height emphasizes the information of the preceding figures, namely, Fairbanks is characterized by shallow layers with relatively intense gradients.

These maximum duct widths may be used to estimate the minimum frequencies that are trapped by reference to a ducting theory that assumes a linear decay of refractive index within the duct such as that given by Kerr [5] where the maximum wavelength, λ, trapped by a given thickness, d, is given by

$$\lambda_{\max}=c\gamma^{1/2}d^{3/2}, \tag{13}$$

where c is a constant and γ is a function of the n gradient excess over the minimum value of $\Delta N/\Delta H=1/\underline{a}$. If $\lambda_{\max}$ is to be expressed in centimeters, d in meters and

$$\gamma=\left(\frac{N_t-N_d}{d}-15.7\right)10^{-8}, \tag{14}$$

then

$$c=2.514\times 10^2.$$

By the use of (13) the minimum frequencies trapped during ducting conditions were estimated for the maximum duct thicknesses of figure 6. These values, given in table 1, were determined for the month with the maximum occurrence of ducts, thus allowing an estimate of the radio frequencies likely to be effected by ducting conditions. Note, for example, that the data of table 1 indicate that 500-Mc rays will be trapped by 50 percent of the ducts regardless of location.

The reader is cautioned that an atmospheric duct does not have the sharp boundaries of a metallic waveguide and thus the minimum frequencies given by table 1 do not represent cutoff frequencies but, as Kerr is so careful to emphasize, merely yields a suggested lower limit of the frequencies strongly affected by the duct under the assumptions of this rudimentary theory.

TABLE 1. *Estimated minimum frequency trapped at* $\theta_o=0$

Station	Minimum frequency in megacycles trapped by the indicated percentage of ducts						
	95%	*90%*	*75%*	*50%*	*25%*	*10%*	*5%*
Fairbanks, Alaska (February)	1, 500	1, 300	1, 200	890	690	490	435
Washington, D.C. (August)	4, 300	3, 000	1, 100	600	270	180	150
Swan Island, West Indies (August)	2, 500	1, 300	725	500	365	325	225

4. Temperature and Humidity Distributions Associated with Ground-Based Ducts

The temperature and humidity structure within ground-based ducts appears to be somewhat different from that normally encountered in the atmosphere. This departure from the normal structure is an aid in distinguishing the different atmospheric mechanisms that give rise to ducts as well as helping the meteorologist forecast ducting from his experience with the normal meteorological variables.

The temperature and humidity structure of the atmosphere during ducting conditions may be evaluated by noting that N is composed of a term proportional to the air density, D, plus a term involving the partial pressure of water vapor, W.

These components are given by

$$D=\frac{77.6}{T}P, \tag{15}$$

and

$$W=\frac{3.73\times10^5 e_s RH}{T^2}. \tag{16}$$

The gradient, ΔN, with respect to height may then be expressed:

$$\Delta N=\Delta D+\Delta W. \tag{17}$$

Average values of ΔN, ΔD, and ΔW are given for two increments between the earth's surface and 1 km above sea level for Fairbanks, Alaska, Washington, D.C., and Swan Island, W.I., in table 2. These data were determined from the Weather Bureau publication on long term mean upper air data [6].

TABLE 2. *Gradient of* N, D, *and* W (N *units/km*)

Station	Height increment	February			August		
		$-\Delta N$	$-\Delta D$	$-\Delta W$	$-\Delta N$	$-\Delta D$	$-\Delta W$
Fairbanks, Alaska	surface to 0.5 km	37	41	−4	31	27	4
	0.5 km to 1.0 km	35	35	0	36	24	12
Washington, D.C.	surface to 0.5 km	41	34	7	60	28	32
	0.5 km to 1.0 km	30	26	4	46	24	22
Swan Island, West Indies	surface to 0.5 km	39	24	15	47	26	21
	0.5 km to 1.0 km	58	24	34	66	24	42

Several general observations may be made of the data of table 2. The gradient of the dry term is relatively less variable than that of the wet term when considered as a function of season or height; the increase of ΔN from winter to summer at a particular location or from arctic to tropical climate at a given time is most strongly reflected in ΔW rather than in ΔD. The marked increase of gradient with height for Swan Island reflects the drop of refractivity across the interface of the trade wind inversion where dry subsiding air overlies the moist oceanic surface layer. Note, however, that in all cases the average N gradient is significantly less than the -157 N units/km needed for ducting.

Examples of the temperature and humidity structures within ducts typical of each of our three climates are shown in figure 7. The Fairbanks duct is accompanied by a surface temperature of −25° C with a strong temperature inversion and a slight humidity lapse indicating temperature inversions associated with wintertime cooling of the air next to the ground. The Washington example appears to be typical of the temperate-zone temperature inversion. The Swan Island profile, however, shows a moderate lapse of both temperature and humidity. This apparent contradiction is explained by the strong lapse in vapor pressure associated with the moderate lapse in temperature when the initial temperature is near 30° C. The strong vapor pressure lapse presumably arises from evaporation off the sea surface. This effect was further examined by studying the percentage of the total N gradient of each duct that was contributed by the gradient of the dry and wet terms. The median contribution of the dry term gradient, summarized in table 3, displays strong seasonal and geographic differences. The dry term contribution decreases from summer to winter and from arctic to tropical climates. The Swan Island ducting gradients are at least 90 percent due to humidity lapse, while the Fairbanks wintertime maximum is due to the strong temperature inversion associated with very low surface temperatures. In fact, under these conditions at Fairbanks the vapor pressure actually increases with height with the result that the dry term contributes more than 100 percent of the ducting gradient.

TABLE 3. *Median contribution of* $\Delta D/\Delta H$ *to* $\Delta N/\Delta H$ *for ducting conditions*

	Fairbanks	Washington, D.C.	Swan Island
	%	%	%
February	103.0	73.0	9.5
May	40.5	33.5	2.0
August	37.0	26.5	4.5
November	62.0	55.0	6.0

The data for Washington, D. C., however, appears to indicate that temperate zone ducting arises from a mixture of the arctic and tropic mechanisms dependent upon the season. It appears that the wintertime ducts in the temperate zone are of the dry-term arctic type while the summertime ducts are of the tropical humidity-lapse variety.

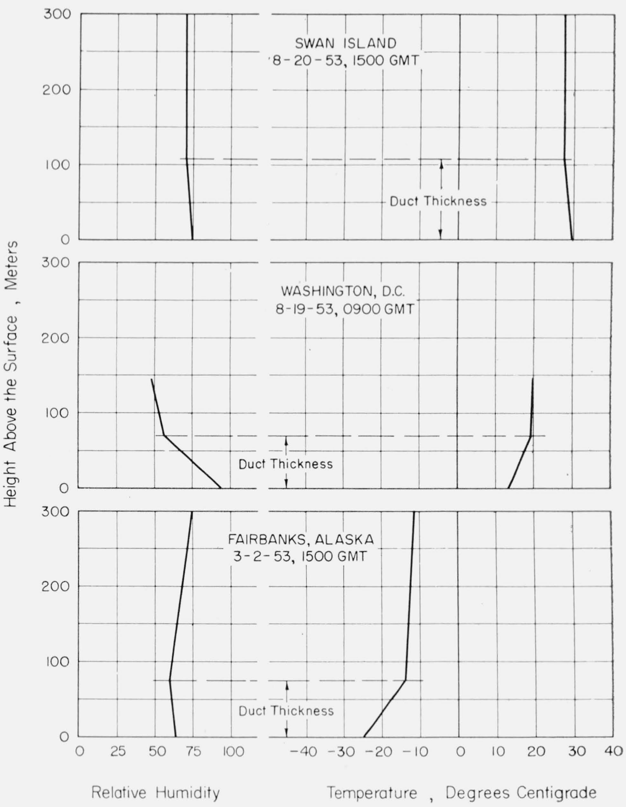

FIGURE 7. *Temperature and humidity profiles for typical ground-based ducts.*

5. Conclusions

The results given above were derived from a consideration of radiosonde data. Although the radiosonde is not an extremely sensitive instrument, it is readily available and forms the only source of climatic information involving the temperature and humidity structure of the atmosphere. It is believed that the radiosonde data will at least yield information as to the climatic trend of radio ducts as well as their probable temperature and humidity distributions. Further, it is evident that the choice of stations may affect the actual percentage of ducts observed. For example, it is certain that a greater percentage of ducts would be observed over water in the subtropics than over Swan Island. Indeed, the kite soundings of the Meteor Fxpedition [7] indicate about a 50-percent incidence of *ducting gradients* (but not necessarily ground-based ducts) between 6° and 20° N latitude and between 9° and 30° S latitude.

With these reservations in mind, the present study has shown:

(a) Limiting layers occur no more than 15 percent of the time.

(b) The annual cycle of the incidence of limiting layers is reversed for the arctic and tropical stations studied. The arctic station has a wintertime maximum and the tropic station a summertime maximum. The temperate station has a summertime maximum incidence of less than 5 percent.

(c) The maximum angle of penetration observed is 5.8 milliradians with a mean value of about 3 milliradians.

(d) The maximum observed lapse rate of N is -420 N units/km.

(e) The maximum thickness of observed limiting layers is such as to trap radio waves with frequencies greater than about 500 Mc at all locations for at least 50 percent of the observed ducts.

(f) The limiting layers in the arctic appear to be associated with temperature inversions at ground temperatures of $-25°$ C or less; temperate zone, with the common radiation inversion and accompanying humidity lapse; tropics, with a moderate temperature and humidity lapse for temperatures of about 30° C.

One may wonder how much the above conclusions are affected by the choice of the antenna height $h_t=0$. This question may be partially answered by a consideration of the climatic occurrence of ducting gradients with respect to height. Cowan's analysis of ducting gradients has shown that such gradients may be found in as many as 40 percent of the observations during some months at some locations. This same study indicates that the majority of ducting gradients are observed within the first kilometer above the earth and that the incidence of ducting gradients decreases rapidly with increasing height becoming negligible for heights in excess of 3 km. The inference of these data is that the ground-based duct statistics are representative of maximum conditions and that the effects of ducts will be negligible above 3 km.

If, on the other hand, a climate is such that elevated ducts consistently form at some altitude, then the choice of antenna height would definitely bias the ducting statistics. For example, during the summer at San Diego and Oakland, Calif., an elevated duct is observed for 37 to 54 percent of the observations [8] with an average base 0.55 km above the surface, an average thickness of 0.275 km and an average gradient of 195 N units/km. This elevated duct would not trap any rays if $h_t=0$. If, however, the antennas were located at the base of the duct ($h_t=$ 0.55 km) then all rays of $\theta_0 \leq 12$ milliradians would be trapped. The angle of penetration would become less with increasing distance of the antenna below the base of the duct becoming zero when h_t is 375 m below the duct base.

In any case encountered in practice, however, one should either determine the duct incidence for any particular combination of antenna height and climate or consider the above statistics to be indicative of the relative climatic distribution of radio ducts.

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