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THE CONSTRUCTION

OF

GRAPHICAL CHARTS

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THE CONSTRUCTION OF
GRAPHICAL CHARTS

BY

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PREFACE

Much of the work of calculation done by the engineer or designer is in the repeated application of a limited number of formulas to a variety of different conditions, which involves merely the substitution of different variables in identical equations.

Any mechanical means for performing this operation expeditiously will not only lead to a saving of time and mental wear and tear, but will also minimize the chances for error.

Such a device is the calculating chart, or nomogram, and the increasing frequency with which it is employed in the more recent technical publications is a good evidence of the growing recognition of its value.

Many excellent examples of these charts have appeared of late years and are available for use, but it is evident that to realize their full value as useful instruments the engineer should have a sufficient acquaintance with their underlying principles to construct charts suited to his individual needs.

Some of the chart forms employed to-day have been known and used for many years, but it is only within recent times that any systematic study has been made of the subject as a whole or any attempt to properly classify and correlate the different types.

In this work the French have been pioneers, and it is to one of them, Maurice d'Ocagne, that we owe what is probably the most thorough and comprehensive text on the subject, his "Traité de Nomographie."

Although books on nomography have been published in many foreign languages, there does not appear to have been anything written on the subject in English outside of a few scattered magazine articles which have covered only restricted portions of the field. Books in English on graphical calculus and computation are by no means uncommon, but this is generally looked upon as something different from nomography, although a strict line of demarcation between the two subjects would be somewhat difficult to trace.

It was with the idea of supplying an elementary English text in this neglected field that the following chapters (originally contributed in serial form to the *American Machinist*) were written.

Believing that the subject should be particularly useful to the practising engineer, who is often a trifle rusty in some parts of his mathematics, an effort has been made to simplify the mathematical treatment. A series of illustrative problems has also been worked out in detail for nearly all the chart forms which are here described, as it was thought that a study of these would afford a clearer insight into the methods and a better understanding of the difficulties likely to be encountered than would be possible from a purely theoretical analysis.

The desire for simplicity in mathematical treatment has made it necessary to restrict the application of the charts to the simpler forms of equation. Equations of the more complex types may be and have been charted, but the mathematical difficulties are such as to make a discussion of the methods used out of place in the present volume.

The processes described here, if thoroughly understood, should be sufficient to cover a large proportion of the formulas in common use. Those of my readers who wish to pursue the subject further are referred to the more ambitious works of d'Ocagne, Soreau, and others.

JOHN B. PEDDLE.

August, 1910.

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CONSTRUCTION OF GRAPHICAL CHARTS

CHAPTER I.

Charts Plotted on Rectangular Co-ordinates.

The Simplest Form of Chart.

The simplest form of graphical chart is that which is frequently used to compare different systems of units of the same character with each other. It is often used, for instance, to show the relative values of temperatures as measured on the Centigrade and Fahrenheit scales.

It is exceedingly simple to construct and to use.

If an equation containing but one variable and its function is to be represented, one side of a straight line is graduated to represent one of the variables, and the equation solved to give as many corresponding values of the other variable as are needed. These are laid off on the other side of the line, and in order to read the chart we have merely to run across the line from one scale to the other to get corresponding values of the variables. It may be used for a variety of equations, such as $y = ax + b$, $v = \sqrt{2\,g\,h}$, $s = 1/2\;a\,t^2$, $a = \frac{\pi}{4} d^2$, $y = log.\,x$, $y = sin.\,x$, etc.

For purposes of illustration I have plotted the two charts shown in Fig. 1 to represent the corresponding values of the diameter and area of the circle. Such a chart is of very little practical value, since a table of circular areas will give the desired results with much greater accuracy and convenience. I have introduced it here partly to illustrate the type of chart, but mainly for the purpose of discussing the relative merits of the two systems of graduation which are shown.

It will be noted that in Chart *A* the diameters are expressed in equal scale divisions, and the areas by divisions which diminish in size as the areas increase. In *B* the areas are represented by equal divisions and the diameters by divisions which increase in size as the diameters increase.

The accuracy with which we can read such charts will evidently depend upon the size of the divisions. In general, the conditions represented in *A* are preferable for, although the absolute error in reading the upper part of the unequal scale will be greater than in the lower part, the *percentage* of error throughout the scale will be more nearly equal with *A* than with *B*.

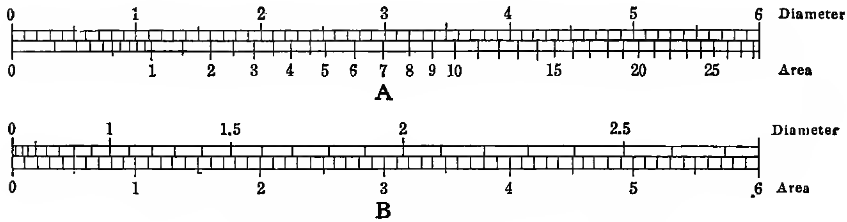

FIG. 1.—Plotted scales of the diameter and area of circles.

On the other hand, if most of our readings are to be about the upper part of the scale, it may pay us to use the *B* arrangement in order to take advantage of its larger divisions.

CHARTS PLOTTED ON RECTANGULAR CO-ORDINATES.

Let us take an equation of the form

$$y=b\pm a\,x. \tag{1}$$

This equation, when plotted on rectangular coördinates, gives us a straight line. That is, if we lay off values of y on the vertical or Y-axis and of x on the horizontal or X-axis and erect perpendiculars to these axes at corresponding values of x and y, these perpendiculars will intersect at points which lie on the same straight line. Thus in Fig. 2, line 7 corresponds to the equation

$$y=10+1/2\,x.$$

If we erect a perpendicular to any point on the X-axis, say 40, find its intersection with line 7, and then run horizontally to the Y-axis, we will get the corresponding value of y as 30.

If we give b different values, say 15, 20, and 25, leaving a the same, we get the parallel lines 6, 5, and 4, which intersect the Y-axis in the new values of b. If we change a, we change the slope of the line; if we make it negative, we get the downward sloping lines 3, 2, and 1.

Suppose we make a in the equation equal to 1. Our sloping lines will now run at an angle of 45 degrees. Taking a new chart to avoid confusion we will have something like Fig. 3. Two sets of diagonals are shown: one sloping up as we move to the right and the other sloping down. The first corresponds to

$$y=b+x \tag{2}$$

and the other to

$$y=b-x \tag{3}$$

According to the first equation, we have y as the sum of b and x. If, therefore, we enter on the X-axis at, say 24, run up as indicated by the heavy line to diagonal 15, and thence to the Y-axis, we will read the sum, or 39. Subtraction would be performed by going in the opposite direction or by using the b lines designated by negative values.

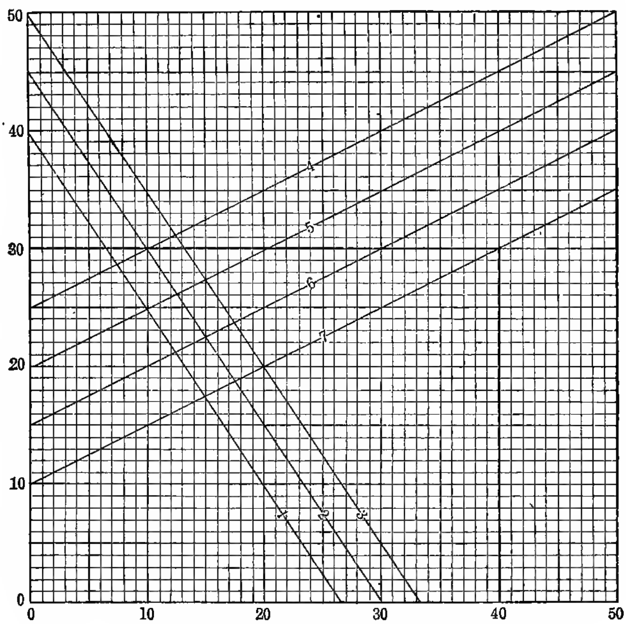

Fig. 2.—Lines plotted from the general equation $y=b \pm ax$.

Right here it might be well to suggest that the quantities represented by the diagonal lines in this or any other chart should be such as are not likely to vary much, and are capable of being expressed in round numbers. Fractional values can be much more easily picked off of the scales on the axes. A large number of diagonals on the drawing is very likely to cause confusion in reading, and will certainly entail additional labor to construct.

Let us now consider the other set of diagonals, corresponding to

$$y=b-x.$$

This may also be written

$$b=x+y. \tag{4}$$

It indicates that if we enter the X- and Y-axes with two numbers to be added and run the perpendiculars out to their intersection, this intersection will be found on the diagonal numbered with the sum. Thus entering the X- and Y-axes at 26 and 44, and running as indicated by the heavy lines, we find the intersection on diagonal 70. Next suppose $b=0$, and give a different values. The diagonals will now be a series of radiating lines from the intersection of the X- and Y-axes. This is shown in Fig. 4.

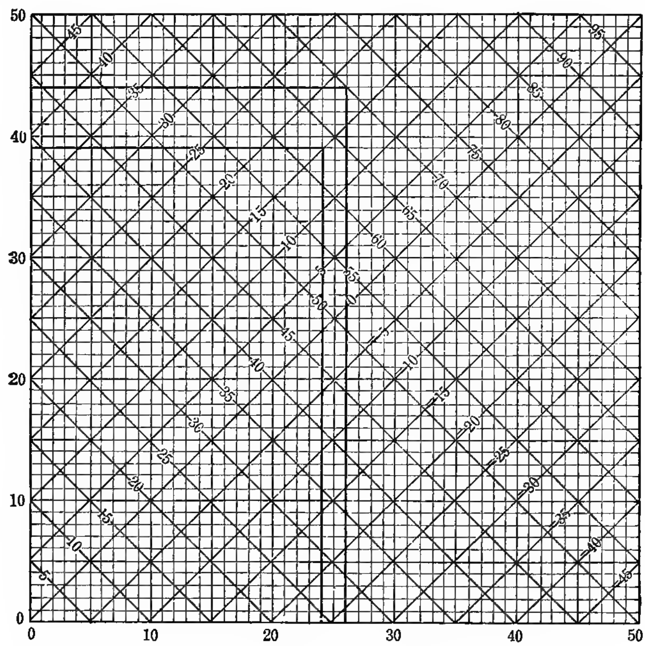

FIG. 3.—Lines plotted from the general equation $y=b\pm x$.

Here, as our equation informs us, the chart may be used for multiplication. Entering the X-axis at 2, running up to the diagonal 3, and from there to the Y-axis, we read the product, 6. Division is, of course, performed by going through the chart in the opposite direction.

This chart, while simple in appearance, is not very practical where the multipliers differ greatly in value. It is easily seen that if we wish to multiply any number on the X-axis by 10, it will be necessary to have the chart 10 times as high as it is wide. Moreover, the intersection of the vertical lines with the diagonals near the 10-line is very acute and necessarily difficult to read accurately. The best position for the diagonal for this purpose is on or near the 45-degree angle.

These difficulties may be partly overcome by changing the scale values. If we renumber the diagonals from 0.1 to 1 making their values 10 times as great, as shown in the parentheses, and also give the graduations on the Y-axis a double set of numbers, we may be able to keep the dimensions of the chart within reasonable limits and also use diagonals which are more favorably disposed for accurate reading. In any case, however, there will be an unavoidable crowding together of the diagonals near the

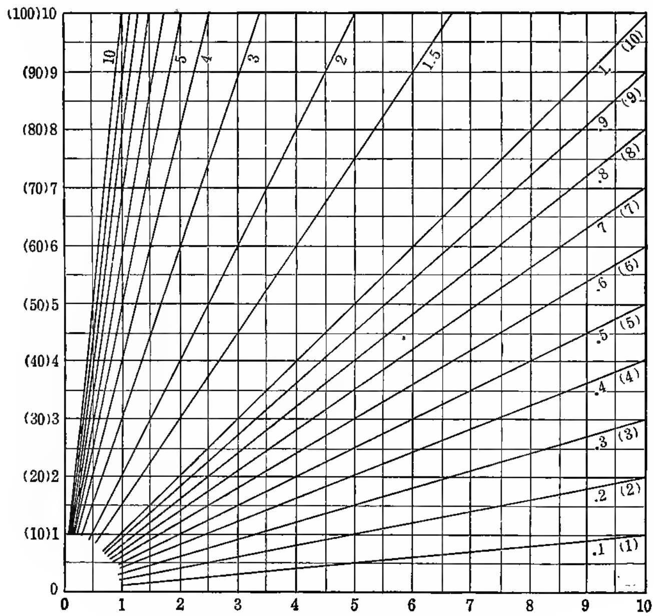

FIG. 4.—Lines plotted from the equation $y = ax$.

origin which will make the readings about the low numbers difficult, if not impossible.

There was no real need to suppose that b in the equation was zero. It was done merely for convenience in illustrating the point I wished to explain. If b had had any value, positive or negative, we should have had the same set of radiating lines, but their point of intersection would have been shoved up or down the Y-axis by the value which we give to b.

Let us now investigate another form of chart for multiplying, writing our equation

$$a = x\,y \tag{5}$$

If we give a a definite value and find corresponding values of x and y, it will be found that perpendiculars erected at these corresponding points will intersect on a curve called the equilateral hyperbola. For each different value of *a* we will have a different curve.

A chart constructed with them, like Fig. 5, could therefore be used for multiplication and, of course, for its converse, division. We have only to pick out the numbers to be multiplied on the two axes, follow up their perpendiculars to their point of intersection, which will be found on the curve numbered with the product. Should this point fall between two curves, instead of on one of them, the product must be interpolated

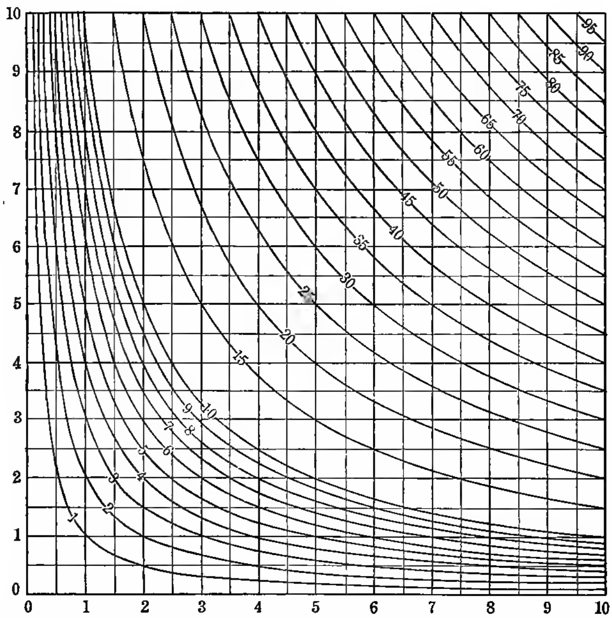

FIG. 5.—Chart for multiplication and division, plotted from the equation $a = xy$.

by eye. It will readily be seen that this method is not at all suited to any case in which the desired number of products is large, since the labor of drawing in the curves would be prohibitory.

Note that curves 1 or 10 might be used as tables of reciprocals.

Next, let us consider a case in which some power of one of the quantities is involved. We will select a case involving several multiplications in order to show how some of the principles already discussed are applied. This will be done in some detail in order to clearly show the process of attacking a simple problem.

Chart for Proportions of Band Brakes.

Take the formula for the band brake

$$P = AT(1 - 10^{-0.00758 f a}),$$

in which P represents the resultant tangential pull on the brake, A the area of the cross section of the brake band in square inches, T the tension in the tight side of the band in pounds per square inch, f the coefficient of friction (0.18 in the case of iron on iron) and a the arc of contact of the band in degrees.

Inside of the parenthesis in our equation there is only one quantity which need be considered as a variable, a the arc of contact; f will be constant for any given materials for band and drum and, as indicated above, will be taken as 0.18. Under these circumstances, instead of drawing a separate line or set of lines for each quantity inside the parenthesis, we need only draw one line for the parenthesis as a whole, getting the different values for plotting this line by letting a vary. We will have to assume the limits within which this variation is to take place. Suppose we take these as 200 and 300 degrees. Then solve the parenthesis for every 10 degrees between these limits.

In Fig. 6 the results of these calculations are shown plotted as ordinates on the chart, while the corresponding arcs of contact are taken as abscissas. In laying off the latter, one small scale division on the horizontal scale is used to represent two degrees of arc. The vertical scale will need to be large as the values of the parenthesis only vary from 0.4666 to 0.6103, and this, if plotted to a small scale, would make a very flat and therefore undesirable curve.

Suppose we make one small scale division on the vertical scale equal to 0.01. This has been done on the chart, and the curve drawn through the points thus found. These values must now be multiplied by the assumed values of T, the tension per square inch in the band. According to one authority, the safe values for T will range from 4500 to 6500 for wrought iron, and from 8500 to 11,500 for steel. We have therefore to provide for a total range of 7000 pounds and we will cover this by steps of 500 pounds. We will adopt the multiplying method shown in Fig. 4, making the radiating lines stand for the different tensions. They must converge to a point somewhere on the zero line of the curve just drawn, and this point may be chosen at will. In reading the chart we must run up or down a vertical line until we strike the curve, and then go horizontally until we reach the desired T-line. It is evident that all the T-lines must be in such a position that they may be intersected by any horizontal drawn from the curve.

They must be so drawn that the tangents of the angles they make with the vertical will be proportional to the tensions they represent. Let us run up ten of the large divisions from the zero line and then horizontally 4 1/2, 5, 5 1/2, 6, etc., of the large divisions, corresponding to tensile stresses of

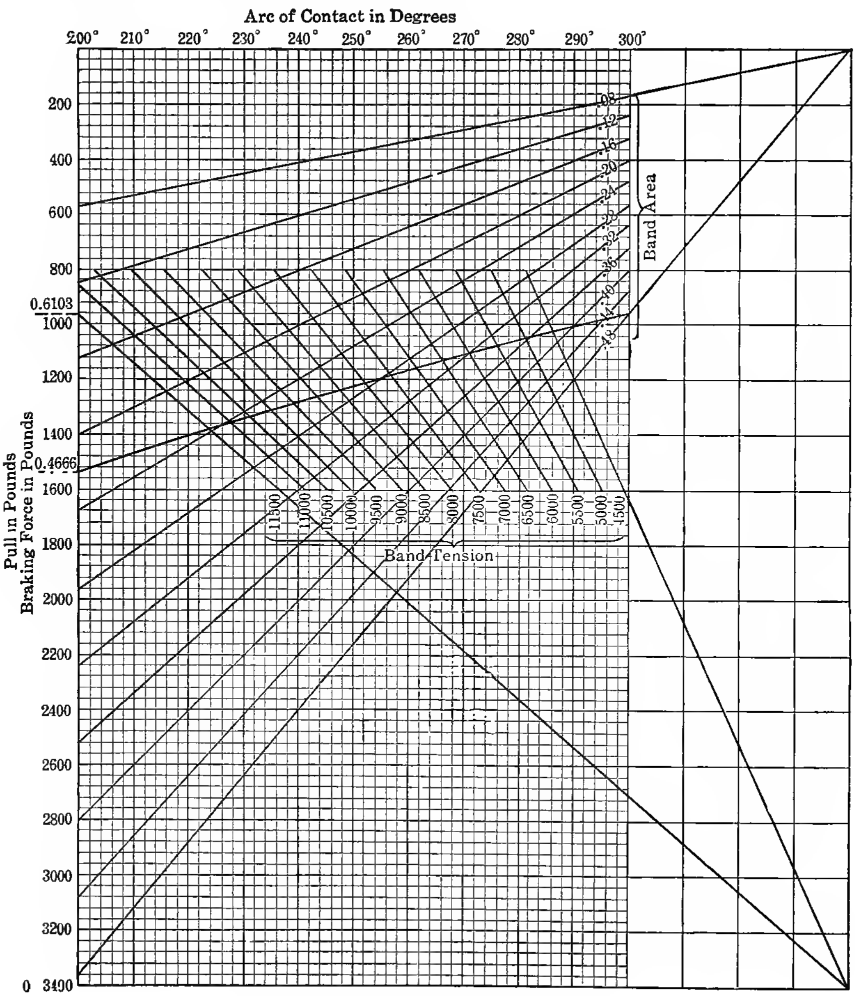

FIG. 6.—Proportions of band brakes.

4500, 5000, 5500, 6000, etc., so as to get the lines well spread out. If we take the point of convergence at 14 large divisions to the right of the left-hand edge of the chart, the conditions we have imposed above will be fulfilled, and this has accordingly been done. The results of this multiplication will be read on some horizontal axis, and they must next be multiplied by the assumed values of A, the area of the cross section of the band.

We could use the same point of convergence for the A-lines as for the T-, but inasmuch as this would cause some confusion in reading the diagram, it will be better to use some other center, which, however, must be located on the vertical line passing through the T center. According to the authority quoted above, the thickness of the band for ordinary cases should vary between 0.08 inch and 0.16 inch, corresponding, roughly, to No. 12 and No. 6 Brown & Sharpe gage. If our bands are not to be less than 1 inch nor more than 3 inches in width, the maximum variation in area will be between 0.08 square inch and 0.48 square inch. For convenience let the areas vary by steps of 0.04 square inch, although any other size of step might have been chosen. This will give us 11 lines which must be so drawn that the tangents of the angles they make with the horizontal will be proportional to 0.08, 0.12, 0.16, etc.

We must now determine the limits within which our results, the desired values of P, must fall. For the least area, 0.08 square inch; the least tension, 4500; and the smallest contact angle, 200 degrees, we have $P=168$. For the largest values of the same quantities we have $P=3369$. These values of P will be read on a vertical scale. It will be found that if we allow 1 large division on the vertical scale to represent 200 pounds it will give us a convenient scale length and readings may be made with an accuracy which is sufficient for all practical purposes. The length of the vertical scale will thus be about 17 of the large divisions.

Therefore, going up 17 large divisions from the zero of the curve, we locate the center for the radiating A-lines on the vertical line which passes through the center of the T-lines. From this center we go 10 large divisions to the left and, going down 2, 3, 4, 5, etc., large divisions (proportional to 0.08, 0.12, 0.16, 0.20, etc)., we locate the points through which the A-lines must be drawn from their center. By this arrangement they will cover the desired length on the scale of P. Our chart is now complete except for lettering the lines and scales. The left-hand scale must of course be lettered so as to make each large division represent 200 pounds' pull.

To read the chart, enter at the bottom or top at the assumed arc of contact and run up or down to the curve, from there go horizontally to the desired tension in the band, then vertically to the area line, and then horizontally to the vertical scale representing the tangential pull. Or, if the pull, arc of contact and tension are known, enter as before at the arc of contact, run vertically to the curve, thence to the tension line, and the intersection of the vertical through this point with the horizontal drawn from the desired pull will be on or near one of the area lines, thus giving the necessary size of the band.

It is obvious that for all practical purposes our chart might have been trimmed off on the right-hand side at the end of the curve so as to omit all of the diagram not sectioned with the small divisions, also that there is no need of continuing the *T*-lines above or below the curve.

Charts with Irregular Scales.

There is no necessity in these charts for having the scale divisions equal, as has been the case in all the charts except the first. If we admit this, there is a distinct advantage in many cases in having them irregular.

Chart for the Focal Distance of a Lens.

For instance, take the formula connecting the two foci of a lens with its principal focus

$$\frac{1}{f}+\frac{1}{f'}=\frac{1}{p}$$

where f and f' are conjugate focal distances and p the principal focal distance.

Make

$$x=\frac{1}{f},\ y=\frac{1}{f'},\ \text{and}\ b=\frac{1}{p}.$$

The above equation becomes $x+y=b$ which is identical with equation (4) above.

We have, in this case, to lay out on the X- and Y-axes the reciprocals of f and f' and draw in the diagonals as shown in Fig. 7, just as we did in Fig. 3. Knowing the principal focal distance of our lens, we select the diagonal corresponding to it, enter the X-axis, say, at the distance of the object from the lens, run up to the diagonal, from there to the Y-axis, and read off the distance at which the object will be in focus.

Logarithmic Charts.

A more important case is where the divisions are laid off to a logarithmic scale. Paper ready ruled in this way may now be had from dealers in mathematical instruments and is valuable for many purposes. On it many problems which would have to be solved by tediously drawn curves, may be worked with ease by straight lines.

Let us return to equation (5) $a = xy$. This may also be written *log.* $a =$ *log.* $x +$ *log.* y which is identical with (4), the equation for a straight line. The paper in question is graduated on its horizontal and vertical axes so that the lengths from the origin are equal to the logarithms of the numbers placed opposite the graduation marks.

If in Fig. 8 we connect 2 on the vertical axis with 2 on the horizontal axis, 3 with 3, and so on, we get a chart similar to Fig. 3, which was used for addition, but in this case is for multiplication. It also bears some resemblance to Fig. 5, the equilateral hyperbolas used there being replaced by straight lines. To use the chart enter at the X- and Y-axes with the numbers to be multiplied and follow out the perpendiculars at these points to their point of intersection, which will be found at the diagonal numbered with the product.

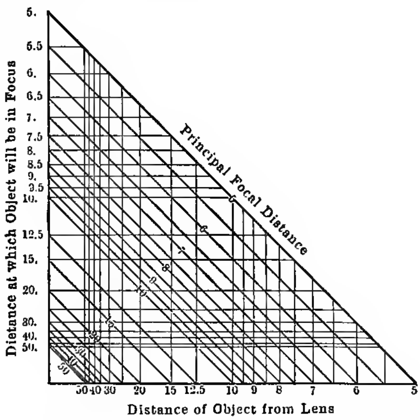

FIG. 7.—Chart for focal distances of a lens.

We might also draw the diagonals so as to slope upward from left to right instead of downward, as shown on the same chart. This is identical with the second form of addition chart of Fig. 3, and may also be used for multiplication. Thus, entering on the X-axis with the multiplicand we run up till we strike the diagonal numbered with the multiplier, and thence over to the product on the Y-axis. Such paper is also very convenient for handling equations containing powers and roots of the variables, and especially where these powers and roots are fractional.

For instance, $y = x^2$ may be written *log.* $y = 2$ *log.* x.

This indicates that a line drawn so that its tangent with the horizontal is 2 could be used for squaring numbers on the X-axis, or conversely for extracting the square roots of numbers on the Y-axis. This is shown in Fig. 9. The top of the diagram is bisected, and a line drawn to this point from the origin, enabling us to find any square not exceeding 10. Entering at 2 on the X-axis and running up till we strike this line and from there to the Y-axis, we read 2^2, or 4.

To get squares greater than 10 we should have to extend our chart above the 10-line. It would be exactly similar, however, to the part below, and it is therefore only necessary to lower our squaring line so as to

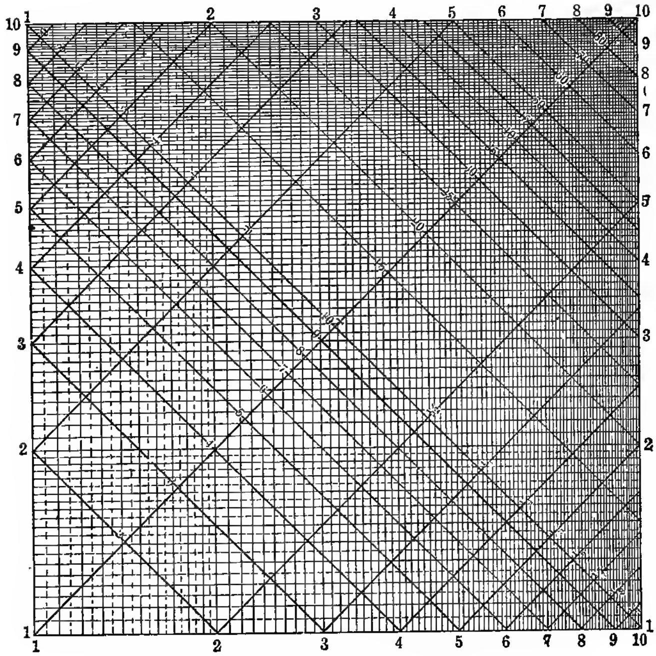

FIG. 8.—Logarithmic chart for multiplication.

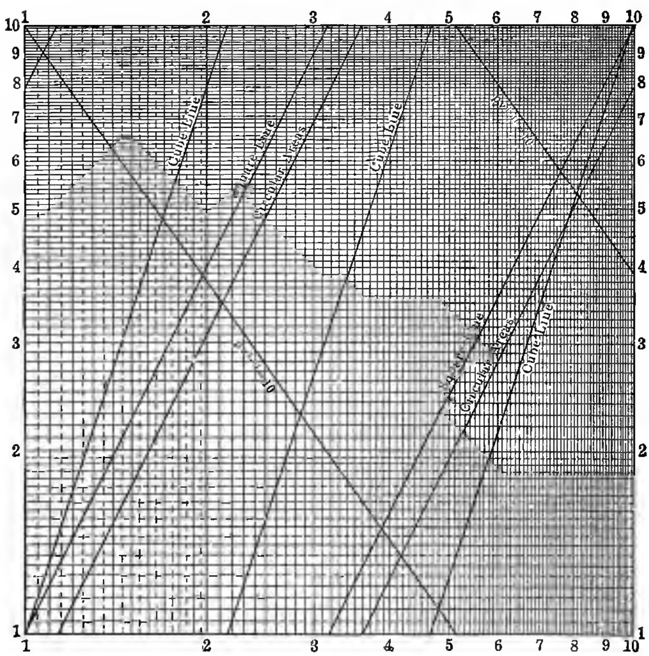

FIG. 9.—Lines of powers, roots, etc., on logarithmic paper.

cut the base of the chart in the middle, and make it pass through the upper right-hand corner. We thus get a chart which may be used for getting the square or square root of any number, the only thing to be noted in the latter operation is that we must use one or the other section of the line according to the position of the decimal point. If the number whose square root is desired has one, or three, or five places (any odd number)

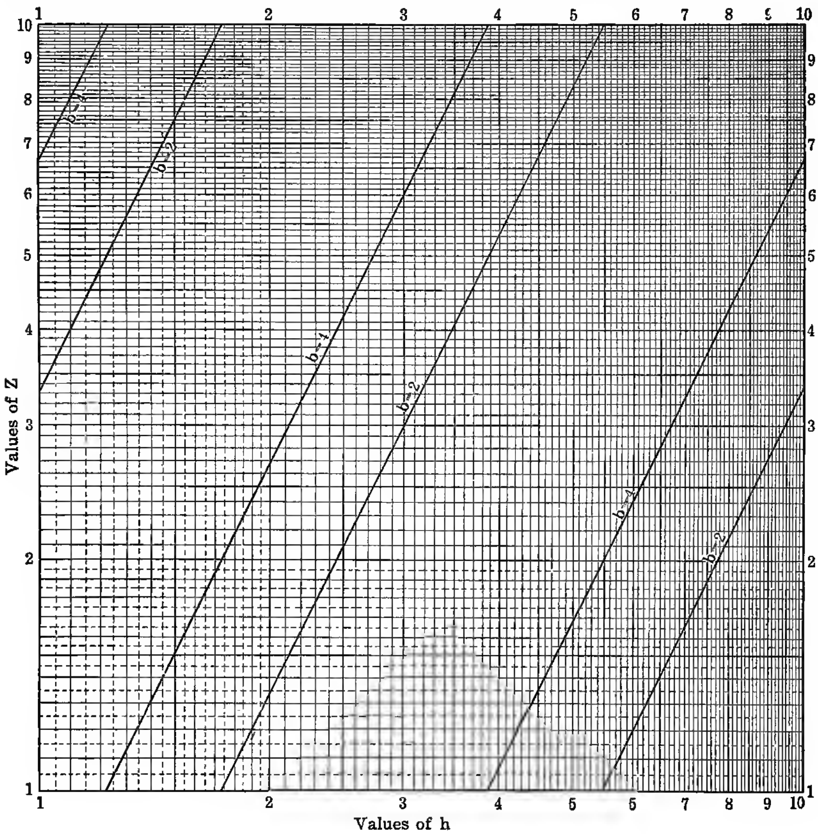

Fig. 10.—Logarithmic charts plotted from the equation $Z=\frac{bh^2}{6}$

before the decimal point, use the first section of the line; if it has two, or four, or six places (any even number), before the decimal point, use the second section.

From what has been said it is plain that the cube line should be drawn by dividing the upper and lower edges of the diagram into three parts so as to make the tangent of the angle of slope 3. Here there will be three lines crossing the diagram. For getting cube roots the first section should be used where the number of places before the decimal point is 1, 4, or 7, etc., the second section where the number of places is 2, 5, or 8, etc., while the third section is used where the number of places is 3, 6, or 9, etc.

For getting fractional powers or roots the tangent of the angle of the slope must, of course, be equal to this fractional exponent. Equations such as $pv^n=c$ are easily solved. In Fig. 9 the line representing $pv^{1.41}=10$ has been drawn for purposes of illustration, v being read on the horizontal, and p on the vertical axis. On the same chart a line has been drawn for getting circular areas, showing the extreme simplicity of the method. Diameters are read on the horizontal and areas on the vertical axis.

In Fig. 10 is shown the application of this paper to the formula for the section modulus of a beam of rectangular section

$$Z=\frac{b\,h^2}{6}.$$

In this chart, values of h are read on the base line, b on the diagonals, and Z on the vertical or Y-axis. For the sake of clearness only two of the diagonals representing b have been drawn. They are for $b=2$ and $b=4$. The intersections with the vertical or Z-axis are found by letting $h=1$. The tangent of the angle of slope is 2.

In reading any of the logarithmic charts here given, significant figures only will be found. No definite rules need be given for finding the position of the decimal point. As with the slide rule, it needs only the application of a little common sense.

CHAPTER II.

THE ALINEMENT CHART.

A type of chart which has received considerable attention of late years and which differs radically from those already described is that known as the alinement chart. In the charts hitherto examined the necessary lines were plotted on what are known as rectangular coördinates; that is, the axes on which the values of x and y were plotted met at a right angle. This is by no means a necessary condition. The axes may be parallel, and, in fact, I have a little book in which the author has developed a system of coördinate geometry based on parallel, instead of rectangular coördinates.

To aid us in understanding this form of chart, let us take an equation of the form

$$a\,u+b\,v=c \tag{6}$$

where u and v are variables and a, b, and c are constants. This is the equation of a straight line where rectangular coördinates are used. To illustrate, let us assume $a=4$, $b=6$, and $c=60$, and draw the line represented by the equation as shown in Fig. 11.

Since u and v may have any values, let $u=0$, then $6\,v=60$, and $v=10$. Again, letting $v=0$, $4\,u=60$, and $u=15$. This gives us the coördinates of two points on the line, one on each axis. Lay off $u=15$ on the Y-axis and $v=10$ on the X-axis and join them. Then for $v=4$, $u=9$, as shown by the heavy line on the chart.

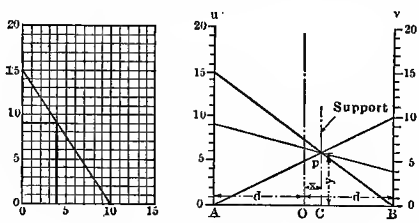

FIG. 11.—Comparison of a rectangular and alinement chart.

Now let us lay off the same quantities on the parallel axes in the second chart of Fig. 11. On the axis marked $A\,u$ lay off 15 and join it to $v=0$ on the $B\,v$ axis. Lay off $v=10$ on the $B\,v$ axis and join it to $u=0$ on the $A\,u$ axis. These two lines meet at the point marked p. It will be found that all lines joining corresponding values of u and v, as found from the equation, will pass through the same point. Or, if we take 4 on the $B\ v$ axis and join it with p, this line prolonged will cut $A\ u$ at 9, giving us the same result that we got by the other chart.

Thus, what was the equation of a line with the rectangular system becomes now the equation of a point. Keeping a and b constant and changing c gives us, with the rectangular system, a series of parallel lines. With the parallel coördinates this merely moves the point p up or down on the line $C\ p$. This line is called the "support" for the points of intersection p. On the other hand, changes of a or b will shift p to the right or left of the support $C\ p$. To establish the point p it will generally be sufficient to solve the equation for a few easily determined values of u and v, lay them off on their axes, and join corresponding points, as has just been done. Or it may be worked out analytically as follows:

Let the position of the point be supposed to be located by reference to rectangular coördinates, of which the line $A\ B$ represents the X-axis, and the line through O midway between $A\ u$ and $B\ v$ and parallel to them the Y-axis. Draw a horizontal line through p. It will intersect the lines $A\ u$ and $B\ v$ at the same height above $A\ B$ as p. Call this distance y.

Equation (6) may now be written

$$a\,u+b\,v=a\,y+b\,y=c,$$

or

$$y=\frac{c}{a+b}. \qquad (7)$$

This gives the distance of the point p above $A\ B$.

If x', y', and x'' y'' are the rectangular coördinates of two known points on a line, analytic geometry teaches us that

$$\frac{x-x'}{x'-x''}=\frac{y-y'}{y'-y''}.$$

Let us apply this to the two lines originally drawn to locate p, and call the distances $O\ A$ and $O\ B$, $-\ d$ and $+\ d$, respectively. The coördinates of the points at the two ends of one of the lines are:

$$x'=-d,\ \ y'=\frac{c}{a},\ \text{and}\ \ x''=d,\ \ y''=0.$$

Therefore

$$\frac{x+d}{-d-d}=\frac{y-\dfrac{c}{a}}{\dfrac{c}{a}-0}.$$

For the other line $x'=-d$, $y'=0$, and $x''=d$, $y''=\dfrac{c}{b}$ are the co-ordinates of the ends, and its equation will be

$$\frac{x+d}{-d-d} = \frac{y-0}{0-\frac{c}{b}}.$$

Combining these equations so as to eliminate y we get

$$x = \frac{d(b-a)}{b+a}. \tag{8}$$

thus giving us the distance of the point from the vertical line through O. It shows, also, that this distance is independent of c, and that, therefore, however c varies (if a and b are constant), p will always lie on the line $C\,p$.

The actual location of the various points on this line may be found by solving the equation for y for different values for a and b, or, as said before, by joining up corresponding points on the $A\,u$ and $B\,v$ axes by lines whose intersection with the locus, or support, of p will give us the desired points.

A practical example worked through will, perhaps, give us a better idea of the methods used than we should gain by a purely abstract discussion.

Chart for Areas.

A man engaged in making blueprints asked me to make him a chart for calculating the areas of his prints in order to aid him in fixing his charges. As it makes a very simple problem for this purpose I will use it in this demonstration, merely remarking that the diagram furnished him differs in some particulars from the one used for illustration here, in order to better adapt it to his needs.

The formula used was

$$A = \frac{WL}{144};$$

in which A is the area in square feet, W the width in inches and L the length in inches.

Write this in its logarithmic form

$$\log. W + \log. L = \log. A + \log. 144.$$

Let us plot *log.* W on the $A\,u$ axis, *log.* L on the $B\,v$ axis and *log.* A on the intermediate support.

We must first decide upon the scales by which these lengths are to be measured on their axes. For instance, u, the measured length of *log.* W on the $A\,u$ axis, is obtained by multiplying *log.* W by some "modulus" or coefficient to get the desired length in inches. Let us call this modulus l_1 for the $A\ u$ axis, l_2 for the $B\ v$ axis, and l_3 for the intermediate support. Then

$$u = l_1 \log. W,$$
$$v = l_2 \log. L,$$

and

$$\log. W = \frac{u}{l_1},$$
$$\log. L = \frac{v}{l_2}.$$

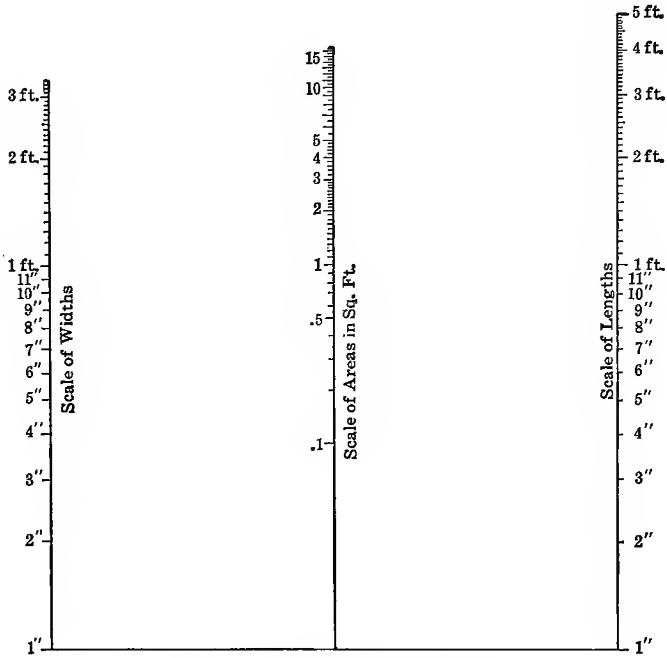

FIG. 12.—Alinement chart for areas.

Calling $\log. A + \log. 144 = c$, we have

$$\frac{u}{l_1} + \frac{v}{l_2} = c,$$

or

$$l_2 u + l_1 v = l_1 l_2 c.$$

From equation (8) we have

$$x = d\,\frac{b-a}{b+a} = d\,\frac{l_1 - l_2}{l_1 + l_2},$$

from which

$$\frac{l_1}{l_2} = \frac{d+x}{d-x} = \frac{C\,A}{C\,B}, \tag{9}$$

thus locating the support for the product.

From the equation (7) we see that

$$y = \frac{l_1\,l_2\,c}{l_2 + l_1},$$

c must therefore be multiplied by

$$\frac{l_1\,l_2}{l_2 + l_1}$$

in order to give the measured lengths along the third axis, or support, and this quantity must be its modulus, or

$$l_3 = \frac{l_1\,l_2}{l_1 + l_2}. \tag{10}$$

The graduated lengths along the different axes may be anything we choose to make them. In general, they should be about equal and as long as possible while keeping the size of the chart within reasonable limits. The largest size of print which was called for in this problem was 40 x 60 inches. The logarithm of 40 is 1.602, and of 60 is 1.778. These numbers are so nearly equal that it will not pay us to use different scales in laying them out in order to represent them by exactly equal lengths on the axes, and l_1 will accordingly be made equal to l_2.

The best scale to use for a practical problem would probably be 1 inch for a logarithmic value of 0.1, thus making the length of the longest axis about 18 inches. In the drawing made for this article the twentieth scale was used, giving a chart half this size. This does not refer to the cut which, of course, has been reduced.

Since l_1 and l_2 are to be equal, equation (9) shows that the distances $C\,A$ and $C\,B$ are equal, or the support for the areas must be midway between the outside axes. For the modulus on the third axis we have from equation (10)

$$l_3 = \frac{l_1\,l_2}{l_1 + l_2} = \frac{l_1}{2}$$

or we must use a scale of fortieths on the middle axis, on which the areas are plotted, if we use twentieths on the outside axes. The distance between the outside axes may be anything we wish. If the axes are too close we get a compact chart, but the intersection of the index line with the axes may, in some positions, be so acute as to make accurate reading difficult. The farther the axes are apart the better this condition will be, but we must not make the distance so great as to get a chart which will be awkward to handle.

Perhaps the best arrangement for average conditions will be to have the chart about square, in which case the index line will never make a smaller angle with the axes than 45 degrees; this is not objectionable.

The two outside axes are now to be graduated so as to represent the logarithms of the desired lengths and widths expressed in inches. Start with 1 inch (whose logarithm is 0) on the *A B* line.

On the middle axis instead of putting 1 on the *A B* line we must remember that logarithm *A* is to be added to logarithm 144 (which is 2.158), and we therefore run up 21.58 measured with the fortieth scale before beginning to graduate. Calling this point 1, we lay off from it the logarithms of 2, 3, 4, etc., and such subdivisions of them as may be necessary, till we reach 17, a trifle beyond the limits of our other scales. The chart is now complete with the exception of the lettering.

To read it, lay a straight-edge or draw a fine thread tightly across the chart so as to join the points representing the length and width of the print, and the intersection of the line with the middle axis will give the area in square feet. Better, perhaps, than either the straight-edge or thread is a piece of glass or thin celluloid with a straight line scratched on its under surface.

Such charts as this will ordinarily show a very marked advantage over those previously described. They are usually much simpler to construct, and they avoid the confusing tangle of lines so often found with the rectangular type. Moreover, since we do not have to draw a separate line for each value of the variable, as is sometimes necessary with the other form, it will be easier to get close readings by interpolation.

The scope of this chart might have been somewhat enlarged, without much trouble, had it been thought desirable. The prices corresponding to the different areas might have been marked on the other side of the area line in something the same manner as was done in Fig. 1. Also the chart might have been extended to give the total area or price of a number of prints of given size. To do this we should merely have to consider the area line as the outside axis of a new diagram, the other outside axis being graduated to represent any desired number of prints, and the product would be read off on a new intermediate axis. The *A B*, or base, line need not have been left on the chart, as it is of no use after the construction is once made, and it will generally be omitted.

Chart for Collapsing Pressure of Tubing.

Another formula charted on this plan is shown in Fig. 13. It is

$$P = 50{,}210{,}000 \left(\frac{t}{D}\right)^3,$$

and will be recognized as Stewart's formula for the collapsing pressure for bessemer-steel tubing, to be applied to pressures not exceeding 581 pounds or to values of $\frac{t}{D}$ not exceeding 0.023.

In it P is the external pressure in pounds per square inch, t the thickness of the tube in inches, and D the external diameter of the tube, also in

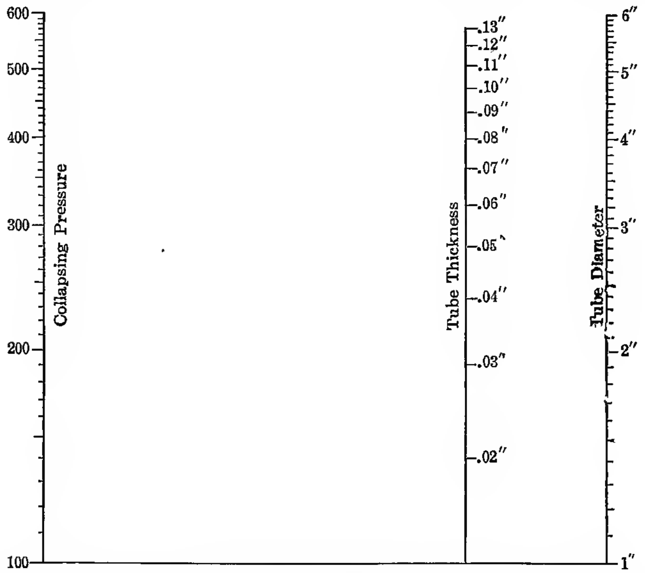

Fig. 13.—Alinement chart for Stewart's formula for collapsing pressures of Bessemer tubing.

inches. It is very similar to the case we have just worked out, but there are one or two practical points in which they differ which will make it worth our while to hastily run through the construction.

The formula may also be written

$$P\,D^3 = 50{,}210{,}000t^3$$

or

$$log.\ P + 3\ log.\ D = log.\ 50{,}210{,}000 + 3\ log.\ t.$$

Its essential similarity with our fundamental equation will be readily seen.

Suppose we take the range in tube diameters from 1 inch to 6 inches, and let our pressures vary from 100 pounds to 600 pounds, the latter a trifle above the 581 pounds for which the formula is supposed correct. *Log.* 1 is 0 and *log.* 6 is 0.778; *log.* 100 is 2 and *log.* 600 is 2.778. This gives us a range of 0.778 in the value of the logarithm in each case. Let us make the length of line corresponding to this range the same on the two outside axes, say 7.78 divisions on whatever scale may be convenient.

On account of this equality we may write for the maximum values of P and D

$$l_1\,(log.\,P - 2) = 3\,l_2\,log.\,D$$

or

$$l_1 = 3\,l_2 \text{ and } l_2 = \frac{l_1}{3}.$$

Then $l_1\,(log.\,\mathrm{P}-2) = l_1\,log.\,D$, showing that the two scales are identical so far as graduation is concerned.

The logarithms of the values of D will be laid off from the horizontal base line; the logarithms of P, *above* 100, from the same line. But it must be remembered that the real zero for the P-line is 20 divisions (on the scale we have chosen) below the base line, and that consequently the line corresponding to $A\,B$ of Fig. 11 will slope up from this point to the point marked 1 inch on the D-line. There is no need to draw it, however.

The location of the t-line, as given by equation (9), is

$$\frac{l_1}{l_2} = \frac{CA}{CB} = \frac{3}{1}.$$

Next let us determine the modulus l_3 for the support, or axis, for t. According to equation (10)

$$l_3 = \frac{l_1\,l_2}{l_1 + l_2} = \frac{l_1}{4}.$$

Inasmuch, however, as the *log.* t is multiplied by 3, it will be convenient to consider its modulus as $\frac{3}{4}\,l_1$, and graduate *log.* t directly with this scale instead of using a modulus of $\frac{1}{4}$ and laying off the values of 3 *log.* t. The other quantity *log.* 50,210,000 laid off on this axis will, however, only be affected by the modulus $\frac{1}{4}\,l$, since the coefficient of this logarithm is 1 instead of 3.

In graduating the t-line note that we must add *log.* 50,210,000 or 7.7007 to *log.* t, and that the zero from which the graduations are measured must be on the sloping $A\,B$-line referred to above. If the left-hand end of the line, corresponding to point A, is 20 divisions below the horizontal base line, the zero for the t-line will be 5 of these same divisions below. Now, using a scale one-fourth the size of that used on the outside axes (since $l_3 = \frac{1}{4} l_1$), lay up 77.007 divisions. This will give us the point corresponding to 1 inch on the t-line. Our values for t, being less than 1, will all fall below this.

For example, take $t = 0.1$ inch; $\log. t = -1$. This will be measured down from point 1 on the t-axis, the length being 30 divisions on the one-fourth scale or 10 divisions on the three-fourths scale; or, what is the same thing, we may go up 47.007 divisions from the zero. The other points on this axis may be located in the same way or by joining up suitable points on the outside axes. The chart now needs only to be lettered to be complete.

A simple modification of the alinement chart as already described is sometimes of value.

Let our general equation have the form $au + bv = 0$.

In this equation c has been made zero, and, since this is so, y in equation (7) is also zero. This shows that the support for the points of intersection is now the line $A\ B$. In order to have the points of intersection lie between the points A and B it will be necessary that Au and Bv axes lie on opposite sides of the line $A\ B$. As indicated in the last example, there is no necessity that $A\ B$ should lie perpendicular to the axes, and it will evidently be to our advantage to make it sloping, since in this way the chart can be made to occupy less room.

Chart for Twisting Moment of a Shaft.

The methods followed in constructing this diagram will be shown by working out another practical example. For this purpose let us take the equation for the twisting moment in a cylindrical shaft

$$M = 0.196\, D^3 f,$$

or

$$-0.196\, D^3 f + M = 0,$$

where M is the twisting moment, D the diameter of the shaft in inches, and f the fiber stress in pounds per square inch.

Let

$$u = -\, l_1 f \quad \text{and} \quad v = l_2 M,$$

then

$$f = \frac{u}{-l_1} \quad \text{and} \quad M = \frac{v}{l_2},$$

and

$$\frac{0.196\, D^3 u}{l_1} + \frac{v}{l_2} = 0$$

or

$$0.196\, D^3 l_2 u + l_1 v = 0$$

Now from equation (8)

$$x = d\frac{b-a}{b+a} = d\frac{l_1 - 0.196\, D^3 l_2}{l_1 + 0.196\, D^3 l_2} \qquad (11)$$

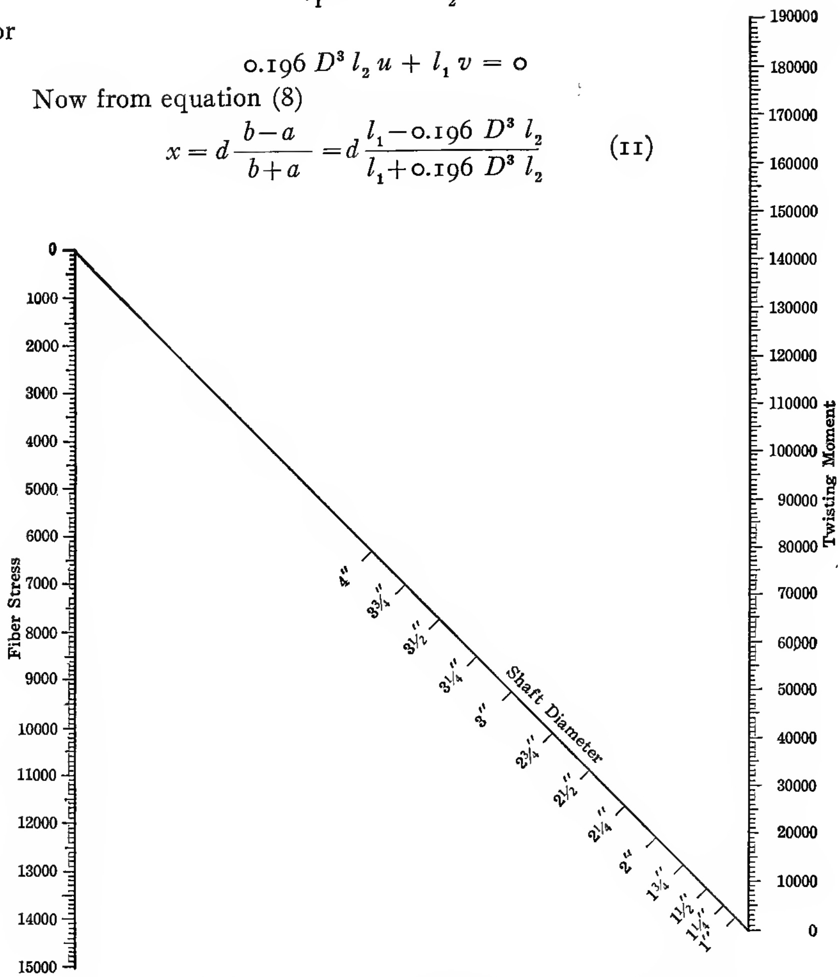

FIG. 14.—Alinement chart for the twisting moment in cylindrical shafts.

This is the equation for graduating the support for D. The two axes must be graduated according to the equations $u = -l_1 f$, and $v = l_2 M$, which show that the divisions on each axis are to be equal among themselves, or that the graduation is regular. Let us assume that the greatest fiber stress we shall need is 15,000 pounds and that our largest shaft will be 4 inches in diameter. Our maximum moment will then be about 188,200. If we make 1 inch equal to 1000 pounds on the f-axis, this axis will have to be 15 inches long. Making 1 inch equal to 10,000 pounds on the moment axis will give us a length of about 19 inches; l_1 will, therefore, equal 10 l_2. Suppose we say that 20 inches will be a convenient length for the diagonal, then d will equal 10 inches.

Now graduate the outside axes into inches and tenths, taking as the zero point on each the intersection of the axis and the diagonal. The graduations for the D-axis or diagonal will be determined by solving our equation for x. Let us find the point corresponding to the 4-inch diameter. From equation (11)

$$x = 10\frac{10-0.196\times 64\times 1}{10+0.196\times 64\times 1} = -1.11.$$

The division mark for the 4-inch diameter will, therefore, be placed 1.11 inches to the left of the middle of the diagonal. As many other points as may be considered necessary are found and laid off in the same manner. In Fig. 14 this has been done for every quarter inch from 1 inch to 4 inches. To save work, the graduations on the fiber stress line need not have been extended below, say, 8000. The line on which the diameters are laid off need not extend beyond the 4-inch graduation, but for the sake of clearness it has been retained here.

Doubled or Folded Scales.

When an alinement chart is intended to cover a considerable range of values we are confronted with the difficulty that it must be large, and therefore awkward to handle, or we must have scale divisions which are too small for accurate reading. These difficulties may be overcome with but little additional trouble by a system of double graduation of the axes.

In Fig. 15 let A and C be the outside axes of an alinement chart, and B the support on which the results are to be read. Say we wish to graduate the A-axis for a length equal to a-c, and that this length is too great for our chart if we use a desirable scale unit. Take a length a-b, equal to about half of a-c and lay this off on the left-hand side of A and graduate it. On the right-hand side of A lay off the rest of the length, or b-c. Call

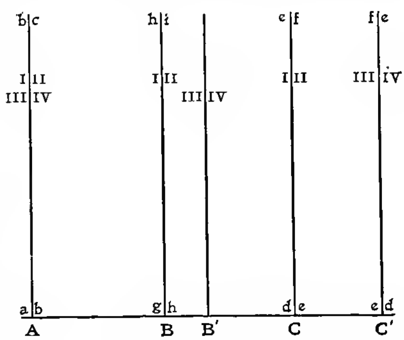

FIG. 15.—Diagram of an alinement chart with doubled scales.

the first scale I and the second II. On the C-axis do the same, graduating the first half of the desired length (which we will call d-e) up the left-hand side of the axis, and the second half, or e-f, on the other side. Mark them I and II to correspond with A. The location of the central support and its scale unit, or modulus, is determined as previously

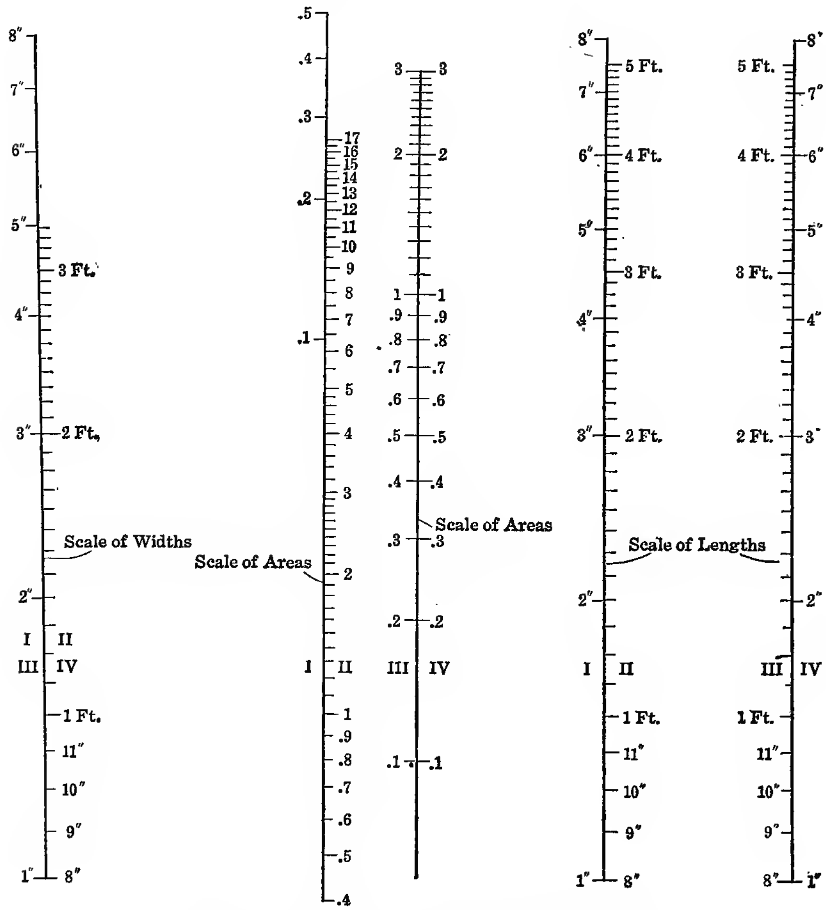

FIG. 16.—An alinement area chart with doubled scales.

explained for the simple alinement chart. The left-hand side will be graduated with values, say from g to h, corresponding to I and I on A and C, and marked I, while the right-hand side will be graduated from h to i, corresponding to II and II on A and C, and marked II.

So long as we wish to get values on B corresponding to I and I, or to II and II on A- and C-axes, we evidently have no trouble, but if we attempt to combine I on A with II on C we find no place on B where the result can be read. We are, therefore, compelled to use two new axes, one for values of B and the other for C. Call these new axes B' and C'. On C' graduate the left-hand side exactly the same as the right-hand side of C, or from e to f, and the other side like the opposite side of C, or from d to e. Mark these scales III and IV, respectively, and since the III side of C' is to be combined with the a-b length on A, the latter had better also be marked III. For the same reason mark the right-hand side of A, IV.

The central axis, B', must be located in the same relation to A and C' as was B to A and C, and will be graduated on the left to correspond with the combination III-III on A and C', and on the other side to correspond with the combination IV-IV on the same axes.

At first sight this diagram is a little confusing and there is always a chance for mistakes in connecting up wrong pairs of axes. If a little care is taken, however, to see that the readings are made on axes bearing the same Roman numeral, the seeming confusion will disappear and the liability to error will be small.

The process of constructing this chart is so simple that further explanation seems unnecessary. For purposes of illustration the area chart shown in Fig. 12 is reproduced in Fig. 16 by this method. A comparison with Fig. 12 will show that while the new chart is somewhat more complex in appearance, it permits the use of divisions which are so much larger that they compensate in a large measure for the additional confusion.

Alinement Chart with Curved Support.

All of the alinement charts dealt with so far have had straight-line axes or supports for the different scales. This is by no means necessary since any one or all of them may be curved.

A case in which the intermediate support is curved will next be considered. Let the equation take the form

$$S = Vt + \frac{gt^2}{2}$$

This is the equation for the space passed over by a body falling under the influence of gravity and starting with an initial velocity. In it S = the space moved over in the time t, V = the inital velocity, and g = the acceleration of gravity, which we will call 32.

The formula is chosen not so much for its practical value as because

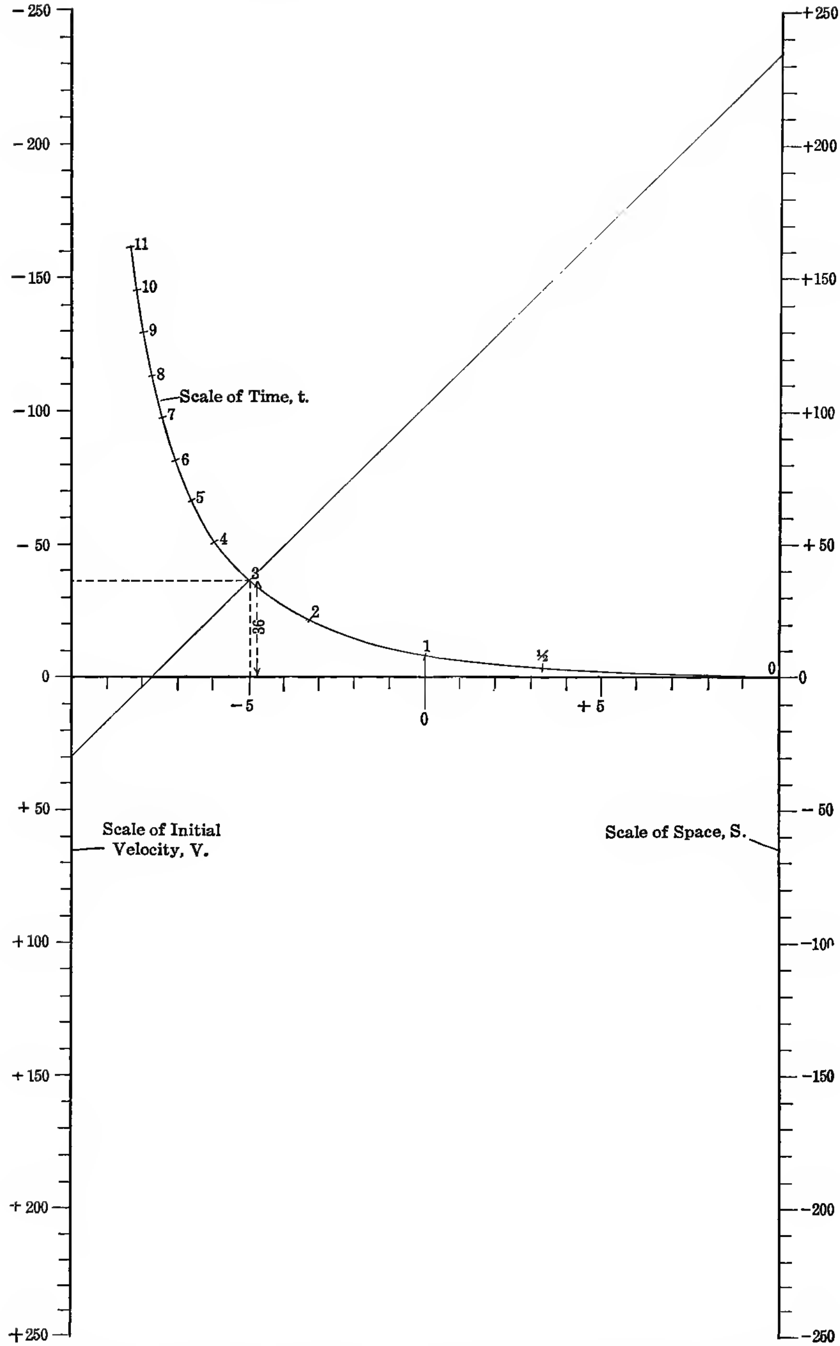

FIG. 17.—An alinement chart with a curved support, solving the equation $S = Vt + \frac{gt^2}{2}$

its form is a good one for the purpose of illustrating this type of chart. Let us write it

$$S = V\,t + 16\,t^2,$$

or

$$-V\,t + S = 16\,t^2.$$

Make

$$u = -l_1 V \text{ and } v = l_2 S,$$

then

$$V = \frac{u}{-l_1} \text{ and } S = \frac{v}{l_2},$$

Substituting above we have

$$\frac{u\,t}{l_1} + \frac{v}{l_2} = 16\,t^2.$$

or

$$l_2\,t\,u + l_1 v = 16\,l_1 l_2 t^2.$$

This is evidently identical with the fundamental equation for the alinement chart. From equations (7) and (8) for y and x we have

$$y = \frac{16\,l_1\,l_2\,t^2}{l_1 + l_2\,t},$$

and

$$x = d\,\frac{l_1 - l_2\,t}{l_1 + l_2\,t}.$$

These are the equations of the points constituting the support for t.

The choice of the scale units is of little or no importance in this case, since we are not obliged to work between any definite limits. For simplicity in calculation, then, let us take $l_1 = l_2$.

Then

$$y = 16\,l_1 \frac{t^2}{1 + t}$$

and

$$x = d\frac{1 - t}{1 + t}.$$

Here our formula does not have a logarithmic form, and we can, therefore, graduate our scales in lengths proportional to the numerical values of the quantities involved and not of their logarithms. This has been done on the scales for V and S. It should be observed that since the modulus for V, or l_1, is negative, the positive values of that quantity are measured down from the base line. The distance between the axes may be anything we like, but to simplify our calculations we will make it 20 of some unit in order that the half distance, or d, may be 10.

Solve the equation

$$x = 10\frac{1-t}{1+t}$$

for as many values of t as are wanted. In the chart shown in Fig. 17 the values taken for t were 0, 1/2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11. The corresponding values of x are 10, 3.33, 0, −3.33, −5, −6, −7.15, −7.5, −7.78, −8, −8.18, −8.33. For the same values of t we have for y: 0, 2.6, 8, 21.3, 36, 51.1, 66.6, 82.2, 98, 113.8, 129.5, 145.2, 161.4.

Plot the curve for these values of x and y, and letter it to correspond with t. The construction for the point $t=3$ has been indicated by dotted lines.

The horizontal axis on which x is plotted is only used for the construction of the curve and may be omitted in the completed chart. It is retained in Fig. 17 in order that the process may be clearly indicated.

If we connect two points on the outside axes by a straight line, the intersection of this line with the curved support will give t, or by connecting the initial velocity V with the time on the curved support we read on the S-line the distance passed over. This has been done in the figure for $V=30$ and $t=3$, giving the value for S as 234. By making the index line pass through $V=0$ and the given time we get a case corresponding to the simple law of falling bodies. If V be taken negative we may get two intersections with the t-line, and either of the times thus found will satisfy the equation.

CHAPTER III.

Alinement Charts for More than Three Variables

Chart for Helical Compression Spring.

So far the alinement charts as described have only taken account of three variables. This is not a necessary limitation and we will next consider a case in which the number of variables is four. For illustration we will use the formula for the load supported by a helical compression spring

$$P=0.196\frac{d^3}{r}f,$$

where P is the load, d the diameter of the wire, r the mean radius of the coil, and f the fiber stress. Say we wish to have our chart cover wire from No. 10 to No. 0000 B. S. gage, or from 0.102 to 0.46 inch diameter. Let us assume that the mean radius of the smallest spring will be 1/2 inch and of the largest 2 inches, and that f may vary between 30,000 and 80,000 pounds. Put the equation into its logarithmic form

$$log.\ P = log.\ 0.196 + 3\ log.\ d + log.\ f - log.\ r.$$

We will have to make two steps in getting our solution, and in each step but three variables must appear. Therefore let us say

$$log.\ 0.196 + 3\ log.\ d - log.\ r = log.\ q$$

and

$$log.\ P = log.\ q + log.\ f.$$

These two equations are evidently of the same form as those previously treated by the alinement chart, and will be charted by exactly the same methods. The quantities d and r, or rather their logarithms, we will plot on the outside axes and read q on the intermediate support. See Fig. 18. Since *log. d* and *log. r* are affected by opposite signs, the positive values of these quantities will be laid off in opposite directions from the base line. As previously explained, the base line may be made sloping, and for convenience we will suppose that this has been done here. Our former constructions depended upon a knowledge of the position of this line, but once the matter is understood there is no real necessity for actually locating it, and in the present instance it will be disregarded.

We have assumed that the values of r are to lie between 0.5 inch and 2 inches. The logarithm of 0.5 is −0.301 and of 2 is +0.301, making a total range of 0.602. Choosing a suitable scale unit, this length is laid off on a vertical line at the right of the paper. The middle point will

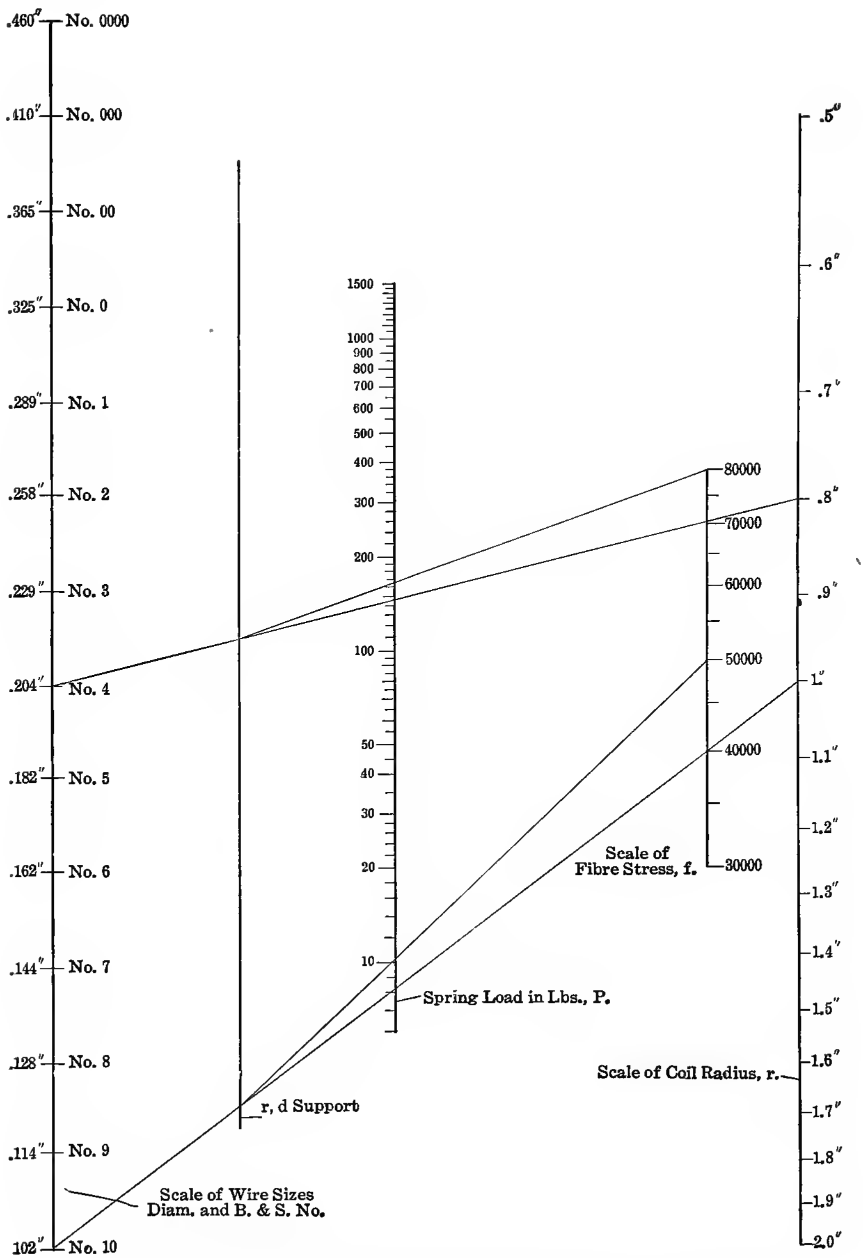

FIG. 18.—Alinement chart for determining load supported by a helical spring.

evidently be lettered 1, and will be the point at which this axis is intersected by the base line—a matter of no importance, however, in the present instance. If we call directions upward positive and downward negative, and remember that *log. r* has a minus sign, we will see that the point corresponding to 2 will be at the lower end of the line and that corresponding to 0.5 at the upper end. Graduate the intermediate portions for as many values as are desired, of course, in their logarithms. The other outside axis, on which *d* is to be laid off, is drawn to the left of the axis just constructed and may be placed in any convenient position. The values of *d* called for lie between 0.102 and 0.46 inch for which the logarithms are $\bar{1}.0086$ (or -0.9914) and $\bar{1}.6628$ (or -0.3372). *Log. d* is to be multiplied by 3, however, and therefore these values become -2.9742 and -1.0116. Their difference is 1.9626 which, after multiplication with the scale unit, gives the graduated length of the *d*-axis. If we use a scale unit of 1/3 the size of that used on the *r*-line we will get substantially the same length for the two axes. It will be convenient, in graduating this line, to take the *r*-line unit and graduate the logarithms of *d* directly from it rather than use the 1/3 scale and then multiply by three, since *log. d* is to be multiplied by 3.

The position of the zero on this line (corresponding to 1) will evidently be beyond the upper end, since all the logarithmic values are negative. The point marked 0.46, being nearer 1 than 0.102, will be at the upper end and the other at the lower. Having chosen the positions for the limits of this line, we proceed to graduate it.

The logarithm of 0.102 is $\bar{1}.0086$. Lay your engineer's scale on the line so that the point chosen to represent 0.102 is opposite 0.86 on the scale. Then with the aid of a table of logarithms pick off the intermediate points up to 66.28. Our formula shows that *log.* 0.196 should be added to 3 *log. d*. The method of making this addition was explained in the previous problems where we worked from the base line. In the present case where we are ignoring the exact position of the base line we disregard the *log.* 0.196 since its only effect is to change the distance of our indefinite base line from what we must look upon as the fixed position of the *d*-line graduations.

While speaking of the *d*-line I should like to call attention to the remarkably regular appearance of the graduations. The nearly equal spacing means that the diameters of the wire increase by approximately a geometrical progression.

We must next locate the position of the *r-d* or *q* support and determine the value of its scale unit. The scale units on the *r*- and *d*- lines are in the ratio of 1 to 1/3. If we take the unit for r as the standard of reference, we find from equation (10) that the unit for the r-d support will be

$$\frac{1\times\frac{1}{3}}{1+\frac{1}{3}}=\frac{1}{4},$$

and from the ratio of the unit lengths on the outside axes we find that the intermediate support should divide the distance between them in the proportion of 1/4 to 3/4. Equation (9.) This line may now be drawn and might be graduated in the unit we have determined if there were any need to have the numerical result of the r-d operation. As this will not usually be wanted, we will save ourselves the trouble.

Take now the second of the equations started with,

$$log.\ P = log.\ q + log.\ f$$

which shows that P is the product of the multiplication of q and f. Their scales will be the outside axes of a new chart and P will be graduated on an axis between them. We have assumed a variation of f from 30,000 to 80,000. The logarithm of 30,000 is 4.4771 and of 80,000 is 4.9031. The difference, 0.4260, multiplied by the scale unit chosen, gives the length of the axis. In the chart made for this article the scale unit selected for the f-axis is 1/2 that of the reference standard used on r. It would have been better on some accounts if the unit had been made larger in order to get greater scale lengths on the different axes. I found, however, that any larger scale unit that I could use would give a unit for graduating the P-axis which would be utterly impracticable with the ordinary engineer's scale. The graduation of the P-scale might, of course, be made by a series of projections from the other axes had there been any pressing need to have the f-scale long, but this is a tedious operation. In the problem we are considering the values of f will generally be expressed in round numbers, and there will be no need of minute subdivision—the chief advantage of a long scale. Accordingly, the unit value of 1/2 was chosen for f.

In locating the f-line it was simply placed as far to the right as it would conveniently go without interfering with the r-line, and, as with the other axes, the graduations are located on it in any position we please. Beginning at the lower end, which we mark 30,000, we graduate up with the logarithms of the desired fiber stresses until we reach 80,000.

Lastly, we must locate and graduate the P-axis. The scale unit for the r-d support has been found to be 1/4; that of the f-line is 1/2. Therefore, substituting in formula (10), we get for the scale unit for P

$$l_3=\frac{\frac{1}{2}\times\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}}=\frac{\frac{1}{8}}{\frac{3}{4}}=\frac{1}{6}$$

The P-axis will divide the distance between the f- and q-lines into parts which have a ratio of 2/3 to 1/3, since the units on the side lines are 1/2 and 1/4. Equation (9). The range over which we must suppose the values of P to vary is a trifle indefinite. It will not do to substitute the values of the variables already settled upon, which give the minimum and maximum values of P, for this would lead to absurd combinations. It is not probable, for instance, that a spring would be made of No. 10 wire and a 2-inch coil radius, and it is still less likely that wire 0.46 inch in diameter would be used in a spring whose coil radius was 1/2 inch. Taking average conditions, I find that the range for P should be somewhere in the neighborhood of from 10 to 1000 pounds. To make sure of being on the safe side, I have extended the limits a little beyond each of these values.

Now, when it comes to starting the graduations on P we ought, properly speaking, to know the location of the base line; but we have completely lost track of this, and it cannot, therefore, serve us. We may easily locate one point on P, however, if we run through a trial calculation. Let $d = 0.102$ inch, $r = 1$ inch, and $f = 50{,}000$ pounds. Then

$$P = 0.196 \frac{0.102^3}{1} 50{,}000 = 10.4.$$

On the chart join up $d = 0.102$ with $r = 1$, and find the intersection with the r-d support. From this point draw a line to 50,000 on the f-line and find its intersection with the line chosen for P. This must be the point corresponding to 10.4, whose logarithm is 1.017. We have thus found a starting point for our graduations, and the other marks may easily be located with the proper scale, 1/6 that of r. The chart is now complete except for lettering. For convenience in reading I have given the d-line a double set of numbers, one for the diameters and the other for the corresponding gage numbers.

To read the chart draw a line between the selected values of r and d (say 0.8 and 0.204) and get the intersection with the r-d support. Connect this point with the chosen fiber stress (say 80,000). The intersection with P, which is at 166, gives the load the spring will carry.

Chart for Strength of Gear Teeth.

Next, let us take a formula containing five variables instead of four. The principles involved are precisely the same as those already discussed: we merely carry the process one step further. For the sake of variety a slight change will be made in the disposition of the axes. The formula chosen for charting is the well-known one by Lewis for the strength of gear teeth

$$W = s\,p\,f\,y$$

where W is the pitch-line load, s the fiber stress, p the circular pitch, f the face width, and y a constant corresponding to the number of teeth.

Let us separate the right-hand side of the equation into two parts and construct a separate chart for each, one giving the product of s and y, and the other the product of p and f. Then, if we take the resulting product lines as the outside lines of a new chart, we will find the value of W (their product) on their intermediate support. See Fig. 19.

We must impose the customary limits on the variables in order to determine the size of the chart. Suppose we let p vary between 1/2 and 2 inches and f between 1 and 6 inches. According to the tables which usually accompany the formula, s may vary between 1700 and 20,000 and y from 0.067 for a 12-tooth pinion to 0.124 for the rack. For the sake of simplicity we will suppose the application of the chart to be limited to the 15-degree involute teeth.

Take first the values of y. The logarithm of 0.067 is $\bar{2}.8261$, and of 0.124 it is $\bar{1}.0934$, giving a difference between the extremes of 0.2673; this multiplied by the scale unit chosen gives the graduated length of the axis. Pick out the values of y from the table, find their logarithms, and lay down the latter on the axis, making the lower end of the line the logarithm of 0.067. Lewis gives a formula

$$y = 0.124 - \frac{0.684}{n}$$

for calculating the value of y from the number of teeth. I have made these calculations and laid off the results on the other side of the line for purposes of comparison. It will be noted that the tabular values are spaced somewhat irregularly as compared with the calculated. This is a matter of passing interest, but the chief point to which I wish to direct attention is the ease with which empirical constants, which are connected by no known law, may be handled by these diagrams. There is no need of trying to force them to fit some arbitrary equation for they may be inserted in the chart exactly as they were obtained from experiment. In lettering this line we place opposite the graduations the numbers of teeth corresponding to the different values of y, which we have plotted, instead of the y-number themselves. The former we know from our given gear, while the latter is of no special interest. This line from now on will be called the n- instead of the y-axis.

Opposite and parallel to this line we draw the axis for s, the fiber stress.

The logarithm for its lowest value, 1700, is 3.2305, and for the highest, 20,000, is 4.3010, giving a difference of 1.0705. If we take a scale-unit value of one-fourth that used on the n-axis the two lines will be approxi-

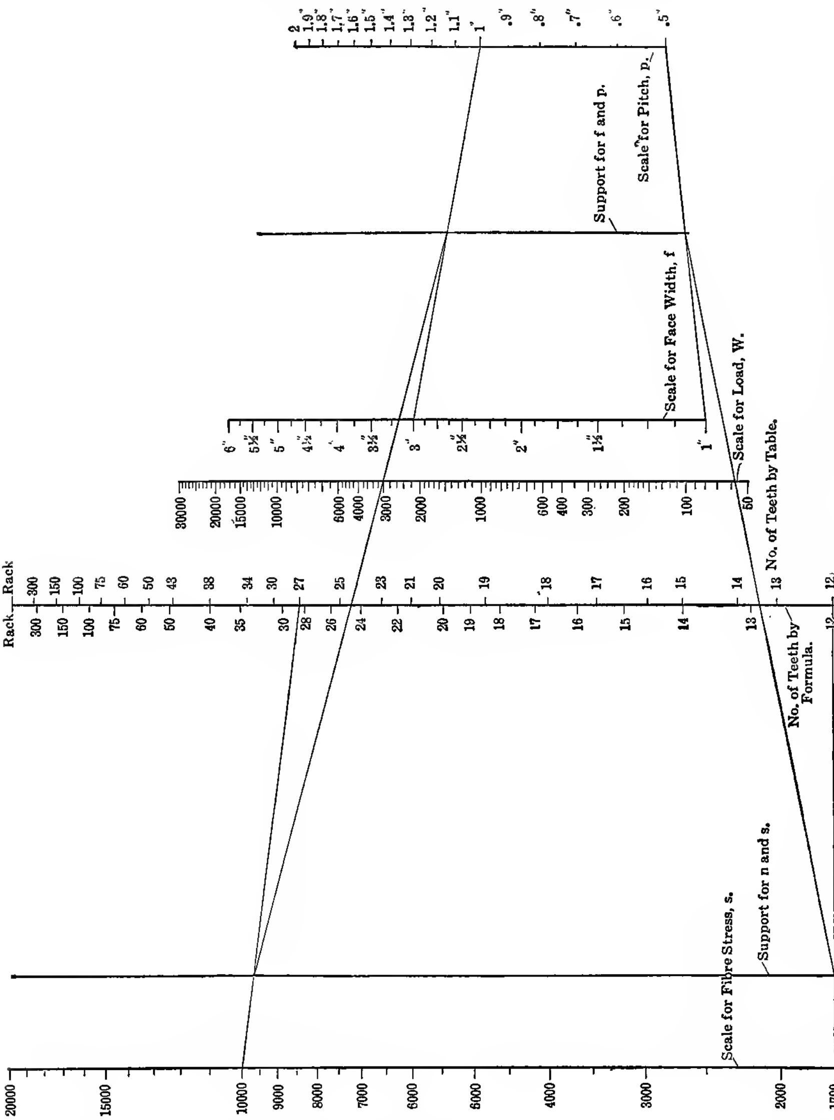

FIG. 19.—An alinement chart for the strength of gear teeth, based on Lewis' formula.

mately equal. Taking the lower end of the line at any convenient point, mark it 1700 and graduate up to the top in the logarithms of the desired values of s. In lettering this line it might be well, in case the gears for which the chart is to be used are all to be of the same material, to place opposite the fiber stresses the appropriate speeds as shown by the table, thus making the chart entirely self-contained. Where several different materials are to be used this would probably cause a considerable amount of confusion, and it has therefore been omitted here.

The lengths of the scale units on the outside axes being 1 and 1/4, we find the unit length for use on the intermediate support to be

$$\frac{1 \times \frac{1}{4}}{1 + \frac{1}{4}} = \frac{1}{5},$$

and the support will divide the distance between the outside axes into intervals whose lengths are 1/5 and 4/5 of this distance. We do not graduate the intermediate support, since the numerical results of the multiplication are not wanted.

Next take the values of p and f; p varies from 1/2 inch to 2 inches. The corresponding logarithms are —0.301 and +0.301, making a total range of 0.602. The lowest value of f is 1 inch (*log.* = 0) and the highest 6 inches (*log.* = 0.778), making a total range of 0.778. Since these two lengths are so nearly equal we might as well use the same scale unit for each, and it will be found convenient to make it 1/5 that used on the n-axis. The support for the product will have a scale unit 1/2 the size of that used on the outside axes, or will be equal to 1/10 the length of that which was used on the n-line. This support and the one previously located are to be used as the outside axes for the last multiplication, whose product is W. The size of the scale unit on the W-line, since those on its outside axes are 1/10 and 1/5, is

$$\frac{\frac{1}{10} \times \frac{1}{5}}{\frac{1}{10} + \frac{1}{5}} = \frac{1}{15}$$

and the line itself will divide the distance between these axes in the ratio of 1/3 to 2/3.

It will be convenient to have the W-line fall between the diagrams used for the preliminary multiplications in order to avoid confusion. Therefore, locate it somewhat to the right of the n-axis and then draw a vertical for the support for the p-f product so that its distance from the W-line is 1/2 the distance of the latter from the n-s support. At convenient equal distances from the p-f support draw the p and f-axes, and graduate them with the logarithms of p and f, using a scale unit 1/5 the size of that we used for the n-graduation. As before, we may locate the graduated parts of these lines anywhere we please on them.

The W-axis is now to be graduated, and its graduations, unlike the others, must start at some definite point. Solve the equation for any values within the prescribed limits. Take, for instance, $n=12$, $y=0.067$, $s=1700$, $f=1$, and $p=1/2$. Then

$$W = 1700 \times 1/2 \times 1 \times 0.067 = 56.95.$$

On the chart join 1700 on the s-line with 12 on n, and get the intersection with the intermediate axis, which will be at the product (unknown) of the two. Join 1 on the f-line with 0.5 on the p-line, and get the intersection with their intermediate axis, giving again the product (unknown). Join the the product of n and s with that of p and f, and the intersection with the W-line must be the point corresponding to 56.95. Its logarithm is 1.7555. Lay an engineer's scale with the proper-sized graduations (1/15 that used on the n-line) on the W-line so that 1.7555 on it is at the point we have located, and graduate the rest of the line from a table of logarithms. The method of using the chart should be obvious from what has preceded, but may be briefly recapitulated. Join the desired values on n and s, say, 27 and 10,000, by a straight line and mark its intersection with the support. Join the desired values of p and f, say, 1 and 3, by a straight line and get its intersection with their support. Join these two points by a third line, and its intersection with the W-line gives the load, 3000 pounds, which the gear will carry safely.

I believe that a comparison of this diagram with others which have been published for the solution of this equation will show that it has some very marked advantages over them in point of clearness of reading and simplicity of construction. The only point which gave any trouble in construction was the selection of scale values for the different lines so that they might all be read from an ordinary engineer's scale. Several trials were necessary before they were finally settled.

Enough has been said, I think, to indicate the general method to be followed in cases where the equation to be charted contains more than three variables, and there should be no difficulty in extending the method to any case where more than five—the largest number treated here—are involved. Before leaving this part of the subject, however, I wish to take up briefly another case, differing slightly from those which have gone before, and which is occasionally serviceable in special problems.

Chart for Strength of a Rectangular Beam.

Suppose we have an equation of the form

$$\frac{WL}{8} = \frac{bh^2}{6} f.$$

This is the equation for a rectangular beam, supported at the ends and uniformly loaded. In it W is the total load, L the length of the beam in inches, b the breadth, and h the height of the rectangular, cross-section of the beam, both in inches, and f the fiber stress.

Let us suppose for convenience that the beam is of white oak or long-leaf yellow pine for which the "Cambria" pocket book gives a safe fiber stress of 1200. Our formula may then be simplified to read

$$WL = 1600\, b\, h^2.$$

For our limits let us say that L varies between 10 and 24 feet, or 120 and 288 inches, b from 2 to 10 inches, and h from 4 to 14 inches. Then W will vary from about 178 to 26,100. Suppose, now, we construct two charts, one for multiplying W and L and the other for 1600 b times h^2, Fig. 20. The two products are to be equal. We may, therefore, use the same line as the support for the product for each chart if the scale units on the two supports have the same value. The base lines for the two charts may or may not coincide, but it is essential that they intersect the intermediate support at the same point if we expect the two index lines to cut it at a common point. This must be the case if the products of the two multiplications are to be equal as we have supposed. As in the previous illustrations, there is no necessity for actually drawing the base line. The general method of procedure in constructing this diagram is so similar to what has gone before that it will not be described in detail.

After finding the range of values required for the L-line we choose a convenient unit length and graduate the line in the logarithms of the desired values. The W-line is placed opposite it at any convenient distance and graduated with a scale unit whose length is one-quarter that used on L. The support for the product of these quantities must, therefore, divide the distance between them in the ratio of 1/5 to 4/5, and its scale unit will be

$$\frac{1 \times \frac{1}{4}}{1 + \frac{1}{4}} = \frac{1}{5}.$$

For the b- and h-lines it will be found that a scale unit of the same size as the standard used on L may be taken for b, and one of one-quarter the standard for h. These will give convenient lengths for the two axes, and the intermediate axis will also have a scale unit of 1/5, since again

$$\frac{1 \times \frac{1}{4}}{1 + \frac{1}{4}} = \frac{1}{5}.$$

This is essential if, as remarked above, the products of the two multiplications are to be represented by equal lengths on the common support. Remember that when h^2 is plotted the lengths of the logarithms of h must be multiplied by 2. The scale units chosen for the b- and h-lines being 1 and 1/4, the support must be distant from these lines in the ratio of 4/5 to 1/5. Lay off the *b*- and *h*-lines at any convenient distances from the *W-L* support which will satisfy this ratio.

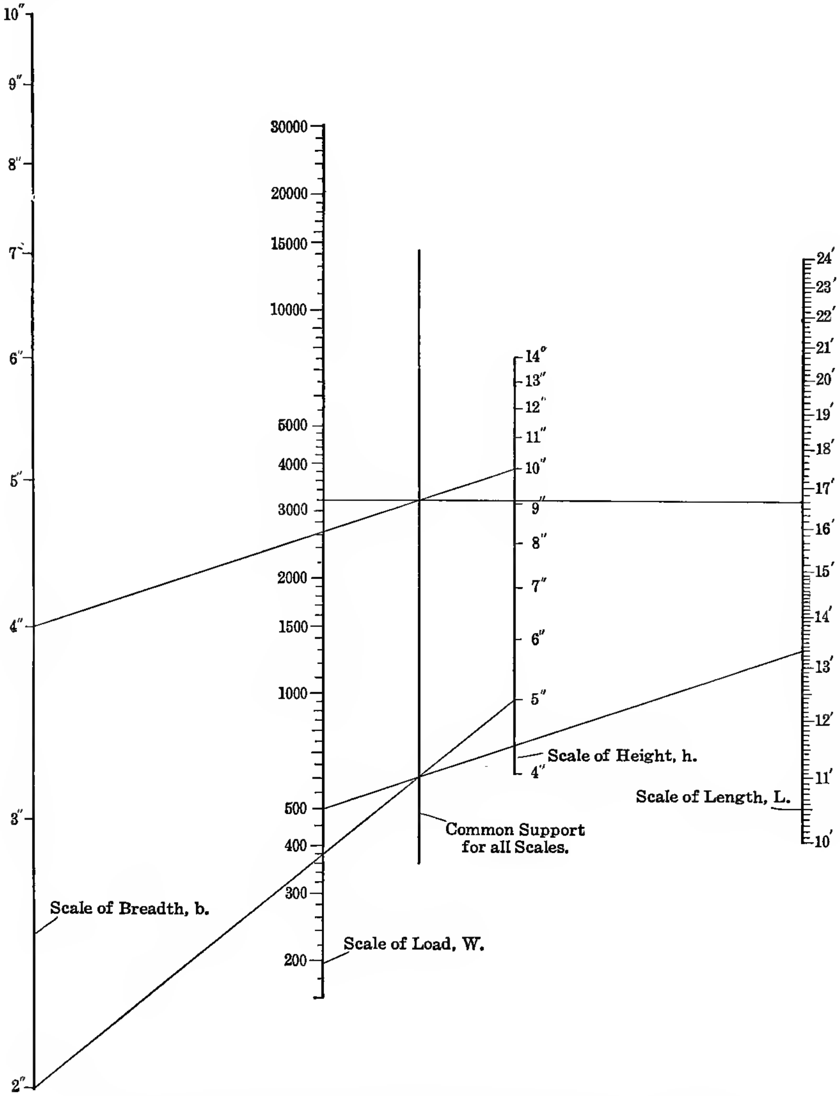

FIG. 20.—An alinement chart plotted from the equation $WL = 1600\,bh^2$.

Graduate the *b*-line in the logarithms from 2 to 10 with a scale unit of 1. The *h*-line is to be graduated in twice the logarithms of the numbers between 4 and 14. The position of the graduations on *b* is chosen arbitrarily, but for h must be found by a simple trial calculation, since the location of the base line is unknown.

Assume $b=2$ inches, $h=5$ inches, and $L=160$ inches (13 feet 4 inches). Then

$$W=1600\frac{2\times 25}{160}=500.$$

Join 500 on the W-line with 160 inches (13 feet 4 inches) on the L-line and mark the intersection with the intermediate axis. Through this point of intersection draw another line so as to pass through 2 on the b-line. Where this line intersects the h-line must be the point numbered 5. Its logarithm is 0.699. Knowing this and the proper scale length, we may easily find the other points on this line. To read the chart draw a line between the chosen values of W and L and mark its intersection with the support for the product. Any line drawn through this point to the b- and h-axes will intersect them in values which will give the necessary strength to the beam. Thus, on the chart, the solution has been found for the case where $W=3200$ and $L=200$ inches (16 feet 8 inches). It is found that a beam 4 x 10 inches will satisfy the conditions as to strength.

CHAPTER IV.

THE HEXAGONAL INDEX CHART.

A type of chart of quite a different character from any of those previously described will now be considered. Suppose we have a diagram like Fig. 21, where $A\ O\ C$ is any angle whatever, and $O\ B$ its bisector. Measure equal distances Oa and Oc on the OA- and OC-axes and erect perpendiculars ab and cb. They meet, of course, on $O\ B$. The length $Oa = Oc = Ob \cos.\ A\ OB$, or $Oa + Oc = 2\ Ob \cos.\ A\ O\ B$.

Now suppose b moved out to b' on the perpendicular $b\ b'$. Project b' to a' and c'. Then since $b\ b^1$ makes the same angles with $O\ A$ and $O\ C$, its projections on these two axes will be of equal length, or $a\ a'$ will equal $c\ c'$. Therefore,

$$Oa + Oc = Oa' + Oc' = 2\ O\ b \cos.\ A\ O\ B.$$

We have here, evidently, a new form of addition chart. If the scale values on $O\ A$ and $O\ C$ are equal, and that on $O\ B$ is this unit times $\frac{1}{2 \cos A\ O\ B}$, the length $O\ b$ measured to this unit is equal to the sum of Oa' and Oc'. If $A\ O\ C$ is 90 degrees the unit length for the $O\ B$-axis is that used on $O\ A$ multiplied by $\frac{1}{\sqrt{2}}$, and if $AO\ C$ is 120 degrees the unit lengths on all three axes are the same.

If we were to graduate the three axes with their proper units and then erect perpendiculars to the axes at the division points we could find the sum of Oa' and Oc' by finding the perpendicular from $O\ B$ which passes through the point of intersection of the perpendiculars from a' and c'. It will readily be seen, however, that this would entail a very confusing network of lines, and it is, therefore, customary with this form of chart to use what is known as a transparent index. It consists of a transparent sheet, preferably of thin celluloid, on the lower side of which are ruled three lines meeting at a point; each line is perpendicular to one of the axes. The axes having been properly graduated, the index is laid on the chart (care being taken that the index lines are perpendicular to their respective axes) and is so adjusted that one perpendicular passes through the selected value on $O\ A$ and the second through that on $O\ C$. The third perpendicular will then intersect $O\,B$ at the sum of the two quantities. The angle $A\,O\,C$ may be anything we like, but since we get equal scale units on the three axes with an angle of 120 degrees, it is advantageous, in general, to use that value. Where this is done the arrangement is known as the "hexagonal" type. The whole thing is so simple and self-

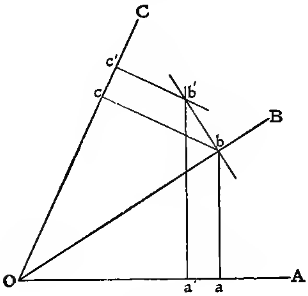

FIG. 21.

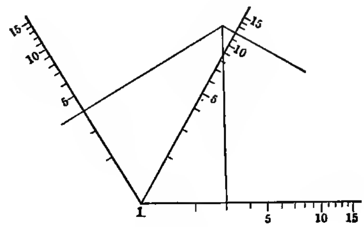

FIG. 22.

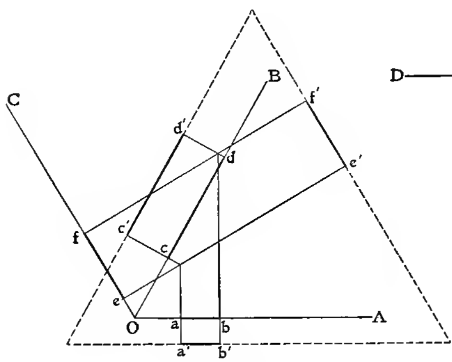

FIG. 23.

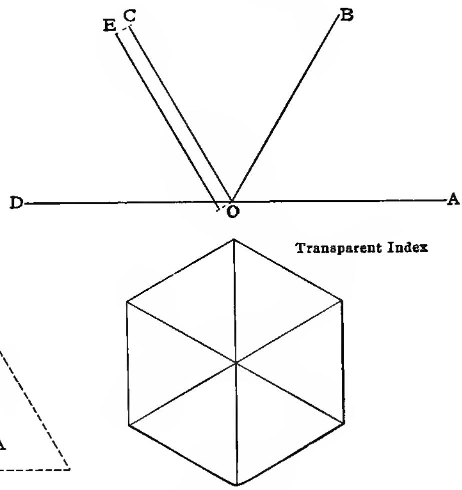

FIG. 24.

Diagrams illustrating the hexagonal index chart.

evident that it scarcely seems to call for an illustrative example, and I will, therefore, not attempt to do more than refer to some of its more important peculiarities.

Like the other forms of addition chart already examined, it may be turned into a chart for multiplication by graduating the axes in the logarithms of the numbers instead of the number themselves, Fig. 22. When the graduation of the middle or $O\,B$-axis is identical in general form (not necessarily in length) with those on the side axes, it may be projected from them by parallel lines whose angle with OA or OC is the supplement of the angle which OB makes with them. This is a convenience in case the angle OAC is, say, 90 degrees, as it does away with the necessity for a scale whose unit length is $\frac{1}{\sqrt{2}}$ times that used on OA and OC. It will also be noted by reference to Fig. 23 that the graduated lengths on the three axes may be moved as far as we please in a direction *perpendicular* to these axes without changing the points at which the index line cuts them. This is sometimes an advantage in that it allows us to shift our scales so as to get a more compact and convenient arrangement of the chart than is always possible if the axes are to meet at O. For instance, suppose we wished to arrange the three scales on the sides of an equilateral triangle, shown dotted in Fig. 23. It is plain that we get precisely the same results with the lines $a'b'$, $c'd'$, and $e'f'$ that we do with the lines ab, cd, and ef; *i.e.*, if $ab+ef=cd$ it is likewise true that $a'b'+e'f'=c'd'$. It is also advantageous in case any of the quantities is affected by a number of different coefficients. In this case it is only necessary to draw a separate parallel line for each value of the coefficient multiplied into the variable, and graduate it with the product of the two. Then, taking the line affected by the desired coefficient, pick out the required point on it and pass the index line through this point.

This form of chart may be arranged easily to take care of a larger number of variables than three. On Fig. 24 the product of OA and OC will be found on OB. If we draw a new axis OD, making an angle of 120 degrees with OB, we have a new diagram on which we may obtain the product of OB and OD. The product will be read on OC, or any line, as OE, parallel to it. This operation may be repeated an indefinite number of times, and it is here that the advantage of being able to move the scales in a direction perpendicular to themselves becomes most apparent. It enables us to handle a large number of variables and have a separate scale for each one of them.

In problems of this sort it is an advantage to have a transparent index made in the shape shown in the same figure. It is a hexagon with the sides parallel to the index lines. This chart takes its name from the shape of this index sheet. After setting the index to get the product on OB, place a straight-edge against the side parallel to the OB index line, and it is easy to slide it into position for the next reading without losing its orientation, and at the same time always keep the index through the point last found on OB.

A Modification of the Preceding Type.

Personally, I must confess, the method of the transparent index does not appeal to me very strongly. It has the disadvantage of not being self-contained, and unless we provide a special index for each chart the two are not likely to be found together when they are wanted. In the second place, it is easier to "fudge," or force the index to give the desired results than with most of the other types. Still it must be admitted that it has its advantages in certain cases, and I have had one or two problems to chart which it seemed impossible to handle with any approach to simplicity by any other method.

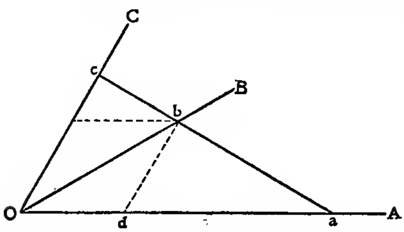

Fig. 25.—Diagram illustrating a modification of the preceding type.

A form of chart which is related to both the hexagonal and alinement types is shown in Fig. 25 In it the axes OA and OC make any angle, and OB bisects it. Draw any line ac. Then from similar triangles we have

$$\frac{Od}{Oa} = \frac{Oc - bd}{Oc},$$

or

$$\frac{Od}{Oa} = 1 - \frac{bd}{Oc}$$

and

$$\frac{Od}{Oa} + \frac{bd}{Oc} = 1.$$

Now

$$Od = bd = \frac{Ob}{2 \cos. AOB}$$

$$\therefore \frac{1}{Oa} + \frac{1}{Oc} = \frac{2 \cos. AOB}{Ob}.$$

The simplest case is where the angle AOB is 60 degrees; then $\cos. AOB = 1/2$ and

$$\frac{1}{Oa} + \frac{1}{Oc} = \frac{1}{Ob}.$$

This is in reality the "reciprocal" form of the type just described.

The equation we have derived is of the same form as that which was used in plotting the chart shown in Fig. 7,

$$\frac{1}{f} + \frac{1}{f'} = \frac{1}{p}.$$

As a matter of interest, this formula has been recharted by the new method. In Fig. 26 f and f' are graduated on the outside axes and p on the middle. To read the chart join up points on two of the axes which are known and get the intersection of the line with the third. This will be the value necessary to satisfy the equation.

The 60-degree arrangement of the axes is not quite so satisfactory in this type of chart as in the last described, since when we are working out toward the limits, the index line is likely to cut some one of the axes at an angle which is disagreeably acute. For this reason it is generally considered that the advantage lies with a smaller angle even if the work of graduating is somewhat more difficult.

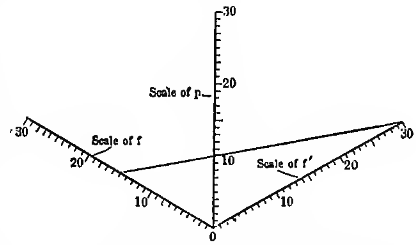

FIG. 26.—Chart plotted by method illustrated by Fig. 11.

Where the two outside axes are graduated alike, the central axis may be marked off without much difficulty by simply joining like points on the outsides. The marks thus found on the middle axis will have numbers whose values are one-half those on the outside lines. This form of chart might be used for multiplication by plotting the reciprocals of the logarithms of the numbers to be multiplied on the outside lines and of their products on the middle. The advantages of such an arrangement are not very apparent, however, and it has but little practical interest.

CHAPTER V.

PROPORTIONAL CHARTS.

A family of chart-forms of great structural simplicity is that which is known under the general name of the "proportional" or "parallel alinement" type. The ease with which they may be laid out and the fact that they may be used with certain forms of equations which cannot be handled so conveniently by those types previously described are strong recommendations for their use in these cases.

Take any two lines meeting at any angle and lay off the distance a and b, as shown in Fig. 27. Connect the points at the ends of these lengths by a straight line and draw a parallel to it. This parallel intersects the axes at the lengths c and d. From similar triangles we have

$$\frac{a}{b}=\frac{c}{d}.$$

If, therefore, we lay off on one side of the vertical axis a scale for the values of a, and on the other side for c, and similarly, on the horizontal axis, the scales for b and d, we have a chart which takes account of four variables. Knowing a, b, and c, for instance, we join a and b by a straight line and draw a parallel to it through c. This line intersects the d axis at the required value of that variable.

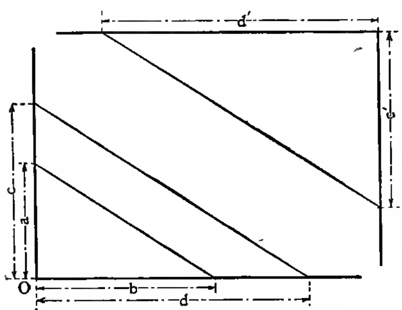

FIG. 27.—Diagram of proportional chart.

It may be advantageous, in certain cases, to have the scales graduated on separate lines instead of doubling up as was done with a and c or b and d. This is also shown in Fig. 27 where two lines parallel to the original axes have been drawn. The solution d' is found by drawing through c' a parallel to the original $a\ b$ line.

CHART FOR STRENGTH OF THICK HOLLOW CYLINDERS.

As an illustration of this type of chart take the Lamé formula for the strength of thick hollow cylinders subjected to internal pressure

$$\frac{D}{d}=\sqrt{\frac{f+p}{f-p}},$$

where D is the outside diameter of the cylinder, d the inside diameter (both in inches), f the fiber stress in the material, and p the internal pressure (both in pounds per square inch).

Squaring both sides of the equation we have

$$\frac{D^2}{d^2} = \frac{f+p}{f-p}.$$

This has the same form as the fundamental equation. Plot on the horizontal and vertical axes the desired values of D^2 and d^2. On the same axes plot as many values of $f+p$ and $f-p$ as may be deemed neces-

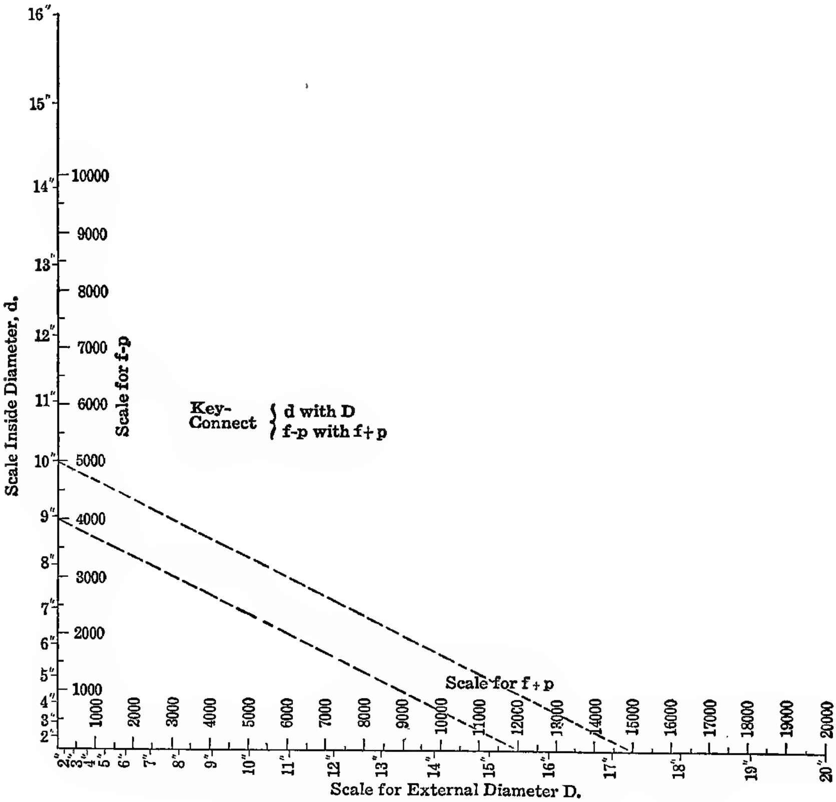

FIG. 28.—Proportional chart for the strength of thick hollow cylinders.

sary. The scale units used for corresponding quantities on the two axes may be equal or not, as we please. In this case if we use equal scale units the horizontal axis will be considerably longer than the other, and the index lines are likely to cut it at a disagreeably acute angle. Accordingly the values of D and $f+p$ are laid off with a scale unit whose length is $2/3$ that used for d and $f-p$. On the chart the solution is shown for $f=8000$, $p=4000$ and $d=10$ inches, giving $D=17.3$ inches.

The only objection which might be raised against the chart just shown is the fact that a preliminary calculation—the addition and subtraction of the quantities f and p—is necessary before the chart is used. This, however, is not the fault of the chart but of the equation which was purposely chosen to bring up this point. A makeshift of this sort should, of course, be avoided where possible, but is often not objectionable. In this case where the values of f and p will usually be given in round numbers the necessary computations of $f+p$ and $f-p$ are easily made mentally and no serious difficulty will result. I have, however, seen this scheme used on some charts where it involved quite a little calculation or consultation of tables and where, on account of the complexity of the equation, it was evidently the only method which permitted it to be charted at all.

The Rotated Proportional Chart.

This type of chart is susceptible of a slightly different arrangement which is sometimes considered advantageous. Suppose the lines carrying the quantities c and d, Fig. 27, to have been rotated about the origin, O, through 90 degrees. We will have a diagram like Fig. 29. In making this rotation the line joining the points c and d will likewise turn through 90 degrees, and will be at right angles instead of parallel to that joining a and b. In reading such a chart it is generally customary to have a transparent index consisting of a sheet of thin celluloid with two lines, at right angles to each other, scratched on its lower surface. This is laid on the chart in such a way as to have one of the lines pass through a and b and the other through c. The intersection of the latter with the d scale then gives the required value of that quantity. The same result may be obtained, of course, by a pair of draftsman's triangles laid against each other.

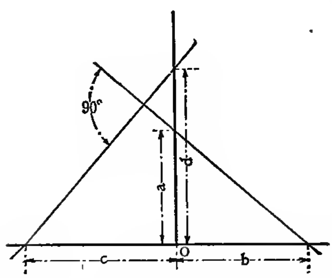

FIG. 29.—Diagram of rotated proportional chart.

With this chart, as with the first one described, there is no need that the axes carrying a and d, or b and c should coincide. Every condition will be satisfied if the lines are separate but parallel. The advantage of this arrangement of chart over the other is not very marked, and I do not incline much toward its use. Some authorities, however, seem to look upon it with considerable favor and that is my main reason for referring to it at all.

Chart for Resistance of Earth to Compression.

One formula will be worked out showing its application. For this purpose let us take the formula for the resistance of earth to compression, used in calculations for foundations. It is:

$$P = w\,h\left(\frac{1 + sin.\ \phi}{1 - sin.\ \phi}\right)^2,$$

where P is the ultimate load on the earth in pounds per square foot, w is the weight of the earth in pounds per cubic foot, h is the depth in feet and ϕ the angle of repose of the earth.

The expression

$$\left(\frac{1+sin.\ \phi}{1-sin.\ \phi}\right)^2$$

may be treated as a single variable and the equation arranged

$$\frac{P}{h}=\frac{w}{\left(\frac{1-sin.\ \phi}{1+sin.\ \phi}\right)^2}.$$

This gives us the simple proportion we need for this type of chart. The limits were determined as follows: The friction angles given by Rankine for different conditions lie, roughly, between 15 and 45 degrees, though they exceed this in a few cases. To cover them all the graduations on the ϕ scale will be run up to 60 degrees, though it is probable that most of the values wanted will lie below 40 degrees. The extreme value of h was arbitrarily taken as 15 feet. The values of w given in the pocket-books range from about 70 to 130 pounds. Taking h as 15 feet, w as 130 pounds, and ϕ as 40 degrees, we find P to be about 40,000.

Next let us choose our scale units. If we take the scale unit for h (which we will call l_1) as 1/4, then $15\times 1/4=3\ 3/4$ inches, which is about the length wanted in the original drawing. For w let the scale unit (l_4) be taken as $\frac{1}{40}$. Then $130\times 1/40=3\ 1/4$ inches, again a convenient length. For the ϕ-axis let the unit length (l_3) be $\frac{1}{0.04}$ The maximum value of the parenthesis containing ϕ is 0.347 when $\phi=15$ degrees. Then $0.347\times \frac{1}{0.04}=8.675$ inches, which will be about right.

Now the scale units should be in the same ratio as the quantities they affect. Hence, calling the scale unit for P l_2 we have

$$\frac{l_2}{l_1}=\frac{l_4}{l_3},\ \text{or}\ \frac{l_2}{\frac{1}{4}}=\frac{\frac{1}{40}}{\frac{1}{0.04}}$$

Then

$$l_2 = \frac{\frac{1}{4} \times \frac{1}{40}}{\frac{1}{0.04}} = \frac{0.04}{160} = \frac{1}{4000}$$

Multiplying the maximum value of P by this unit we get,

$$40{,}000 \times \frac{1}{4000} = 10 \text{ inches}$$

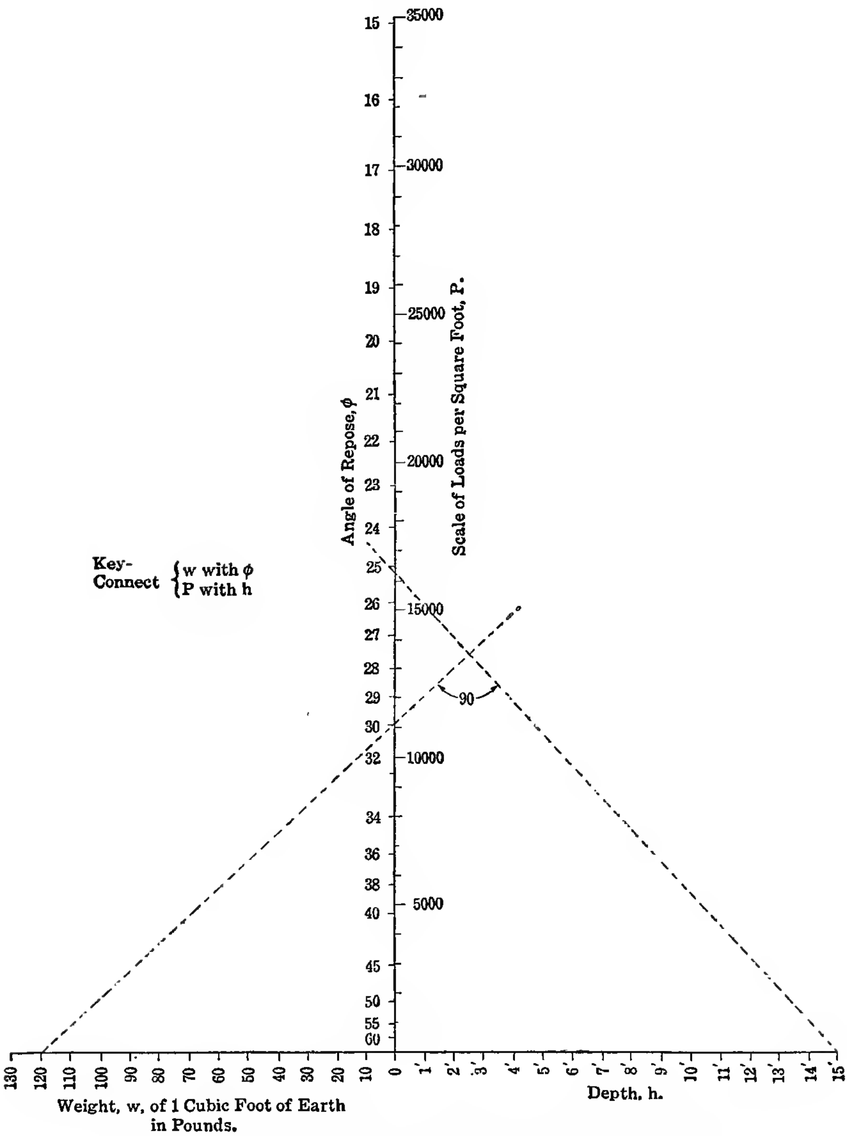

FIG. 30.—Proportional chart for earth resistance in compression.

as the length of the P axis. This was a trifle greater than I wanted for the limits I had placed on the size of the chart and I arbitrarily reduced it to about the same length as the ϕ-line, making the maximum value for P, 35,000. This would correspond to an angle ϕ of about 38 degrees with h and w at their maximum, and would probably cover most cases. It was, however, entirely a matter of convenience and there is no reason, in a practical case, why the scale should not extend as much further as the conditions in the problems likely to be encountered would seem to require. The graduation of the different scales is now an easy matter and the completed diagram is shown in Fig. 30.

The broken lines show the position of the index for $w = 120$, $\phi = 30$ degrees, and $h = 15$ feet, giving the load $P = 16{,}200$ This is, as stated above, the ultimate strength of the soil. If a fixed factor of safety may be used for all cases likely to be met with, it might easily be introduced when P is plotted; that is, the numbers placed opposite the graduation marks on this scale would be divided by whatever factor we chose. Then the diagram would give us safe, instead of ultimate loads.

Charts with Parallel Axes for Sums or Differences.

Next let us take a case like that shown in Fig. 31. Here the quantities are laid off from an arbitrary zero line on two axes which are parallel. Draw a transversal between the ends of the lengths a and b, and another parallel to it cutting the axes in the lengths c and d. An inspection of the diagram shows that

$$a - b = c - d,$$

or the difference between the lengths on the two axes cut by any system of parallel lines is constant. If one pair of corresponding quantities had been laid off below the zero and the other above it we should have had a constant sum instead of a constant difference. We may even get a case corresponding to

$$a - b = c + d,$$

if we lay off the quantity d below the zero and the others above. This will be referred to later. As in the previous type of chart, there is no need to have the values a and c, or b and d laid off on the same axes. They may be laid off on parallel axes if the distance between each pair of axes is the same. This distance might be varied if the need for it arose, but it would require an alteration in the scale units to correspond.

To save referring to it again, it might as well be noted here, once for all, that this chart, and all of those yet to be described involving four variables, has the same rotational property as was indicated in Fig. 29 for the first type. This is shown in Fig. 32, where the *c* and *d* axes have been turned through 90 degrees without altering their relative positions. The position of the index is shown by the fine lines, and the construction is sufficiently clear, I think, to render any further explanation unnecessary.

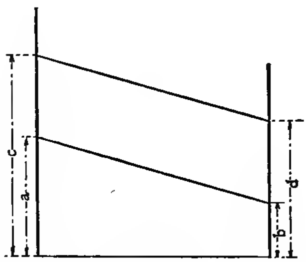

FIG. 31.

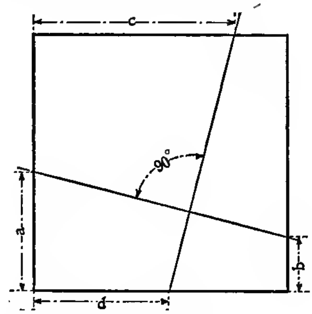

FIG. 32.

Diagrams of proportional charts.

A chart of the kind we have been examining is of little importance if we are only to use it for addition and subtraction; but it acquires an added value if, instead of plotting the numbers themselves on the axes, we plot their logarithms. This transforms the chart into one for multiplication and division.

CHART FOR CENTRIFUGAL FORCE.

An example of the use of it is given in Fig. 33. The formula used is that for centrifugal force,

$$C = \frac{w\,v^2}{g\,r},$$

where C is the centrifugal force in pounds, w is the weight in pounds, g is the acceleration of gravity, v is the velocity in feet per second, and r is the radius in feet of the path of the weight. Rewrite the equation

$$\frac{C}{w} = \frac{v^2}{gr}$$

Then

$$log.\, C - log.\, w = 2\, log.\, v - log.\, gr$$

which is identical with the fundamental equation given above. The limits between which we are to work are of no special importance here, since the chart is not supposed to be applied to any particular problem. We will have to fix some conditions, however, so let us say that w varies from 1 pound to 100 pounds, v from 1 foot to 50 feet, and r from 0.1 foot to 10 feet. The maximum value of C will be 776.4, and of its logarithm 2.89. The maximum value of *log.* v^2 is 3.398, of *log.* w is 2, and of *log.* 32.2 r is 2.508.

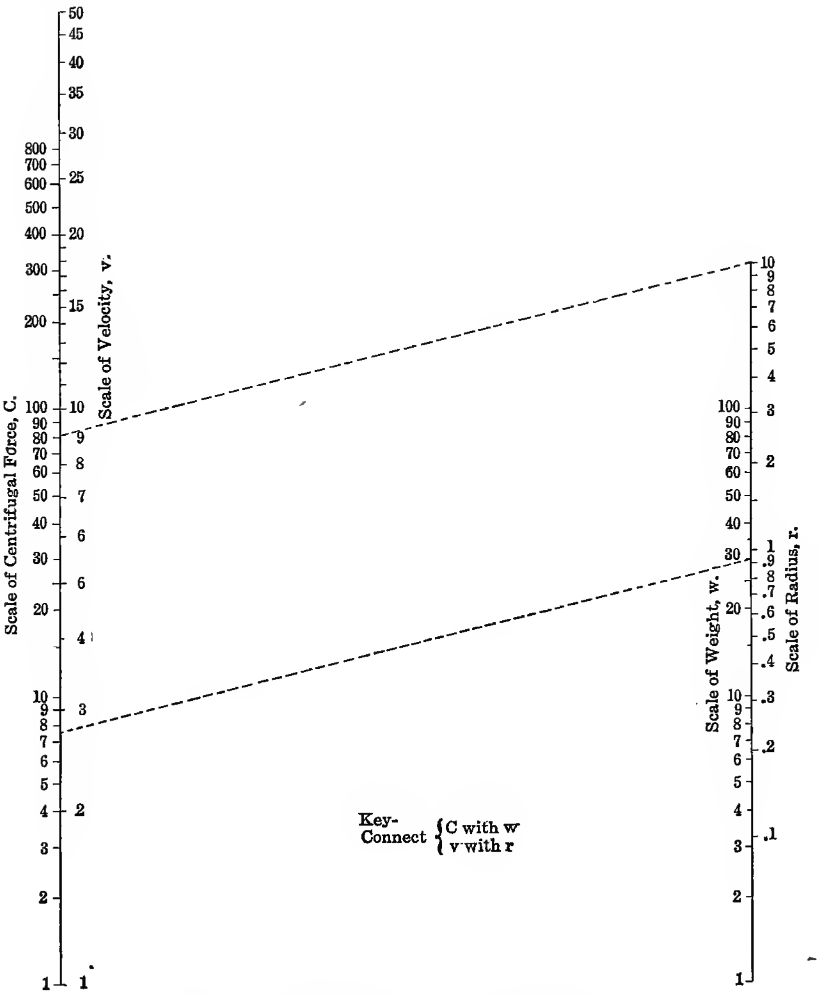

FIG. 33.—Proportional chart for centrifugal force.

In graduating the axes the same scale unit must be used throughout. All except the r-scale commence with mark 1 at the zero point, and are laid off in any convenient sized divisions from a table of logarithms. The C-scale was extended to 800 instead of stopping at its exact upper limit, 776.4. In the case of the *r*-scale we must place mark 1 at a distance of 1.508 (*log.* 32.2) above the zero, and graduate above and below this as desired. Fig. 33 shows the completed diagram with transversals drawn to indicate a solution for $w = 30$, $v = 9$, and $r = 10$; *C* should then equal 7.55.

Chart for Piston-rod Diameter.

With this type of chart it is not necessary that the equation be in the form of a simple proportion, though it should be capable of being placed in that form by a little manipulation. For instance, take the formula given by Kent for the diameter of the piston rod of a steam engine,

$$d = 0.013 \sqrt{D\ l\ p},$$

where *d* is the diameter of the rod, *D* is the piston diameter, and *l* the length of the stroke, all in inches, and *p* the maximum steam pressure in pounds per square inch. Squaring, this becomes

$$d^2 = 0.000169\ D\ l\ p = \frac{D\ l\ p}{5917}.$$

In its proportional form it is:

$$\frac{d^2}{D} = \frac{l}{\frac{5917}{p}},$$

or in logarithms,

$$2\ log.\ d - log.\ D = log.\ l - (log.\ 5917 - log.\ p),$$

which agrees with the fundamental equation for this type of chart.

Here we plot the logarithms of d^2, *D* and *l* as usual. In the case of *p*, however, we first lay off the *log.* of 5917 (=3.772) from the zero and from that point plot the logarithms of *p downward*, since we use the reciprocal of *p* and not *p* itself in the last member of the proportion. For the sake of compactness it is well to have all four scales on about the same horizontal zone, and since those of *l* and *p* are much higher than the others we drop their zeros by equal distances below those of *d* and *D*. The zeros are not shown in the chart, Fig. 34, since none of the graduations go down that far, and only the working parts of the scales are needed. No error is introduced by this shifting of the scales since the slopes of the lines joining them are the same before and after the transfer. The units used in graduation must be the same for all scales unless a different one is indicated by the exponent of the quantity. Therefore, *D*, *l* and *p* are plotted with one unit, and *d* (since it is squared) with one twice as large. The resulting chart is shown in Fig. 34, and the broken parallel lines give the solution for the case where $D = 20$ inches, $l = 30$ inches, and $p = 100$ pounds. Then $d = 3.18$ inches.

It may be mentioned here that this type of chart may be applied to equations containing but three variables. If, for instance, in our equation

$$\frac{a}{b} = \frac{c}{d},$$

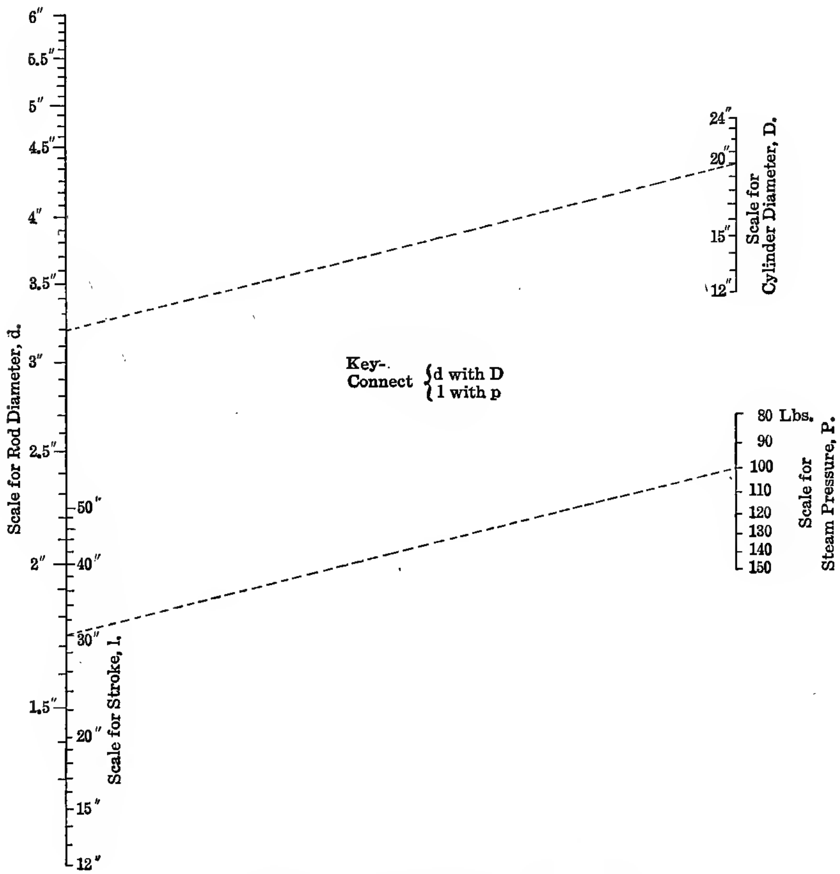

FIG. 34.—Proportional chart to determine piston-rod diameter.

a, b and d are variables and c a constant, the c graduation is reduced to a single point through which all lines referring to c must pass. The method of using such a chart is precisely the same as for those just described, and it is hardly of sufficient importance to merit more than a passing notice.

THE Z-CHART.

The examples which have been given will illustrate sufficiently well, I think, the general methods to be followed in cases involving a simple proportion, and we will now proceed to examine a new type which, while it bears a family resemblance to some of those previously described, differs from them in several important particulars.

In Fig. 35 we have three axes arranged in the form of a letter Z. Draw a transversal across them. From similar triangles we have

$$\frac{a}{d} = \frac{b}{c}, \text{ or } a = \frac{b\,d}{c}.$$

Add d to each side of the equation and we have

$$a + d = \frac{b\,d}{c} + d = \frac{d}{c}(b + c).$$

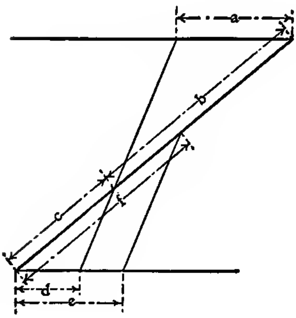

FIG. 35.

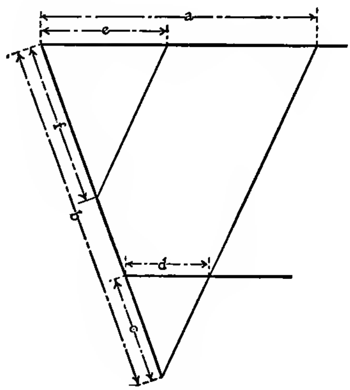

FIG. 36.

Diagrams illustrating Z-charts.

Now $(b + c)$, the length of the diagonal of the Z, is a constant which we may call k.

$$\therefore\ a + d = \frac{d}{c}k. \tag{12}$$

Draw a second line parallel to the first transversal. Then

$$\frac{d}{c} = \frac{e}{f},$$

and the original equation becomes

$$a + d = \frac{e}{f}k. \tag{13}$$

If then the equation which we are to chart has the form

$$u + v = \frac{x}{y},$$

we lay off u on the upper horizontal, v on the lower, x also on the lower and y on the diagonal. Joining the values of u and v corresponding to a and d by a transversal, and drawing a parallel to it through the value of x corresponding to e we get the resulting value of y, or f, on the diagonal.

Similarly, we may get the solution of a problem where the difference of two quantities is used instead of their sum. Fig. 36 shows the arrangement. Here, as before,

$$\frac{a}{d}=\frac{b}{c} \text{ or } a=\frac{b\,d}{c},$$

and

$$a-d=\frac{b\,d}{c}-d=\frac{d}{c}(b-c)=\frac{d}{c}k$$

$$\therefore a-d=\frac{e}{f}k.$$

The selection of the scale units is of some importance with this chart and a brief discussion of their mutual relation is necessary. It is understood, of course, that the numerical values of the quantities u, v, x and y are to be multiplied by certain scale units in order to get their measured lengths, a, d, e and f on the axes. Let these lengths be l_1, l_2, l_3 and l_4 for u, v, x and y, respectively. For u and v the scale unit must be the same $(l_1=l_2)$, since otherwise parallel lines joining their scales would not indicate a constant sum, but l_3 and l_4 may be chosen at will. Now since $a=l_1 u$, $d=l_1 v$, $e=l_3 x$ and $f=l_4 y$ we have by substitution in equation (13)

$$l_1(u+v)=\frac{l_3\,x}{l_4\,y}k$$

or since

$$u+v=\frac{x}{y},$$

$$l_1=\frac{l_3}{l_4}k$$

and

$$k=(b+c)=\frac{l_1 l_4}{l_3}, \tag{14}$$

which gives the necessary length of the diagonal of the Z.

For the subtraction formula this becomes

$$(b-c)=\frac{l_1 l_4}{l_3}. \tag{15}$$

Sometimes we wish to make $l_3 = l_1$. In this case

$$(b+c) = l_4, \qquad (16)$$

or the diagonal is the same length as the scale unit used in graduating it.

It should be noted here that the *a* and *d* scales may be shifted along their axes the *same* amount and in the *same* direction as far as we please, without changing the direction of the transversal joining them and that, therefore, no error will be introduced. This sometimes permits us to make a more convenient arrangement of the scales as will be shown later in connection with the chart for chimney draft.

Chart for Polar Moment of Inertia.

As the first illustration for the construction of the *Z*-chart I have chosen the formula for the polar moment of inertia of a flat rectangular plate about an axis perpendicular to its plane and passing through the center. It is sometimes used in the power calculations for the draw spans of bridges, the assumption being that the span may be taken as having approximately the same polar moment of inertia as the flat plate. The formula is:

$$I = \frac{W}{12}(B^2 + L^2),$$

where I is the polar moment of inertia, W the weight of the plate, and B and L its breadth and length. The weight will be expressed in pounds and B and L in feet. The engineer who wishes to have the forces in his final results in pounds instead of poundals, will usually prefer to divide at once by g, instead of doing this at the end of his calculations; in which case the formula becomes

$$I' = \frac{W}{386.4}(B^2 + L^2).$$

This may be written

$$\frac{I'}{\dfrac{W}{386.4}} = (B^2 + L^2),$$

and we evidently have an equation suited to the *Z*-type of chart.

When planning this chart my intention was to give it something like a practical form by taking the maximum values of B and L as about 10 and 80. However, since these quantities are to be squared and $L^2 =$ 6400, while B^2 is only 100, it is evident that if we lay them off to the same scale and use any practicable length for 6400, 100 would be so small (after the necessary reduction by the engraver) that its subdivisions for

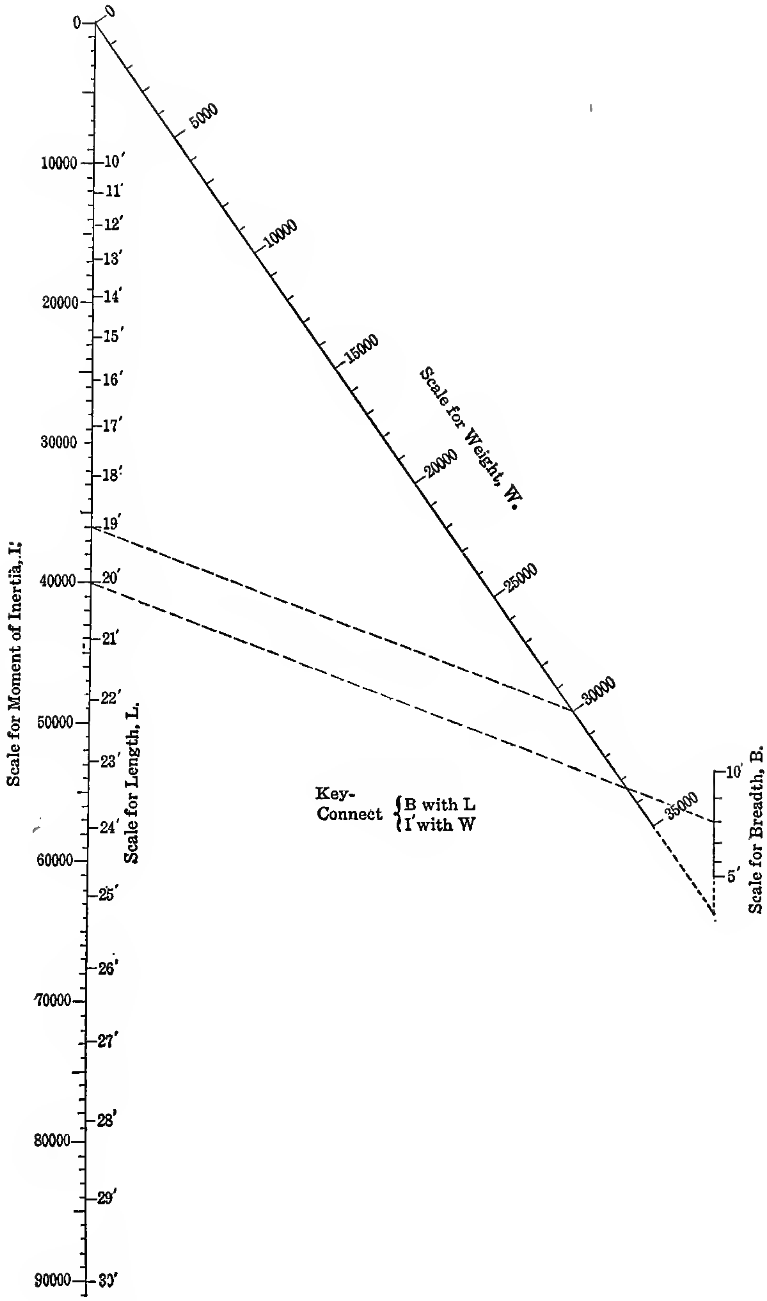

FIG. 37.—Z-chart plotted from formula for polar moment of inertia.

smaller values of B would be illegible in the cut. In view of the fact, however, that the charts which illustrate this book are intended primarily as examples of methods of construction and application, I have not hesitated in many cases to sacrifice a practical chart for the sake of getting one which showed a process clearly, and this I shall do in the present instance. The conditions are assumed to give clear reading scales in the cut, but the chart in its present form will have little practical value for the bridge designer.

Let us say then that the maximum value for W is to be 35,000, that the maximum value for B is 10 feet, and for L, 30 feet. Then the maximum value for I' is very nearly 90,000. The scale units and scale lengths must next be fixed. I wished to keep the original drawing inside of a length of 10 inches. By making the scale unit for I' $\frac{1}{10000}$ I get a graduated length of

$$90{,}000 \times \frac{1}{10000} = 9 \text{ inches.}$$

This is the scale unit we called l_3 in the preliminary explanation. For the W-line it will be convenient to make the scale unit, l_4, equal to $\frac{386.4}{5000}$.

Then

$$\frac{35{,}000}{386.4} \times \frac{386.4}{5000} = 7 \text{ inches}$$

is the graduated length of this axis. This unit makes it possible to plot W directly from the 50 side to an engineer's scale without bothering about the coefficient $\frac{1}{386.4}$. For the L and B scales let us take $l_1 = 1/100$. Then for L^2 we have a graduated length of $900 \times 1/100 = 9$ inches, the same as for I', and for B^2 $100 \times 1/100 = 1$ inch. Substituting the scale units thus found in equation (14) we get for the length of the diagonal of the Z,

$$(b + c) = \frac{l_1\, l_4}{l_3} = \frac{\frac{1}{100} \times \frac{386.4}{5000}}{\frac{1}{10000}} = \frac{3{,}864{,}000}{500{,}000} = 7.728 \text{ inches.}$$

Having drawn our axes (the diagonal making any convenient angle with the parallels) we have only to graduate them, and this is a simple matter, B and L being plotted in the squares of the desired values with a scale unit of $1/100$, while the W- and I'-lines are plotted directly from the $\frac{1}{5000}$ and $\frac{1}{10000}$ scales. The parallel broken lines show how the chart is read for the case where $B = 8, L = 20$, $W = 30{,}000$, which gives for I' 36,000.

Chart for Intensity of Chimney Draft.

The next formula which I have charted is one for the intensity of chimney draft

$$f = h\left(\frac{7.64}{T_2} - \frac{7.95}{T_1}\right),$$

where f is the draft expressed in inches of water, h the height of the chimney in feet, T_1 the absolute temperature of the chimney gases, and T_2 the absolute temperature of the external air.

The formula will be seen at once to belong to the second type of Z-chart where we have a difference instead of a sum of two variables. The general method of procedure is identical with that just described, but there are a few differences of minor detail which require a brief description. Thus the variables T_1 and T_2 appear in the denominators of the fractions instead of the numerators, which indicates that the plotted values are proportional to the reciprocals of these quantities and not to the quantities themselves. For our limits let us take h as varying between 50 and 150 feet, T_1 from 761 to 1161 degrees absolute (or from 300 degrees Fahrenheit to 700 degrees Fahrenheit), and T_2 from 461 to 561 degrees absolute (or from 0 degrees Fahrenheit to 100 degrees Fahrenheit). Then f will have a maximum value of 1.46 inches and we will graduate it from 0 to 1.5 inches.

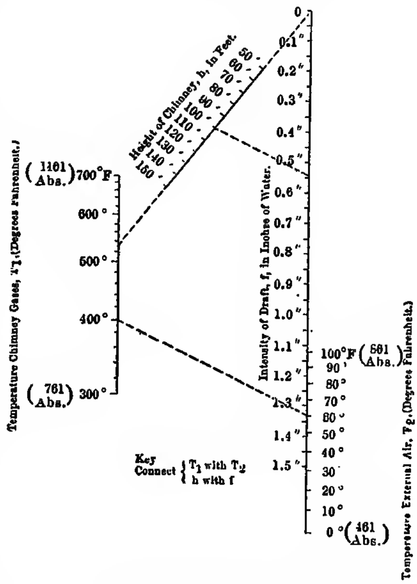

Fig. 38.—Z-chart to determine intensity of chimney draft.

The scale unit on the f-line (l_3) I took as $\frac{1}{0.2}$ which gives a length of $1.5 \times \frac{1}{0.2} = 7.5$ inches for its graduations; l_4, the h-scale unit was taken as $1/40$. This gave a graduated length from zero of $150 \times 1/40 = 3.75$ inches; l_1, the unit used for T_1 and T_2 was made $1/0.001$. The extreme length of the T_2-line from its zero will then be

$$\frac{7.64}{461} \times \frac{1}{0.001} = 16.6 \text{ inches},$$

too great for the size of chart planned which I wished to keep within a length of 10 inches. The lower limit of the graduations on this line is

$$\frac{7.64}{561} \times \frac{1}{0.001} = 13.62 \text{ inches}$$

from the zero. This is an empty space which is of no advantage, and the chart will be improved in appearance and compactness if we slip the graduations along the axis toward the zero point or the point where the diagonal intersects this axis. In my drawing the graduations were shifted a distance of 8 inches, which brought them within the prescribed limits. The T_1-graduations were shifted the same amount in the same direction, and thus no change was made in the direction of the transversals joining them and no error introduced.

The length of the diagonal, from equation (15) is

$$(b-c) = \frac{l_1 l_4}{l_3} = \frac{\frac{1}{0.001} \times \frac{1}{40}}{\frac{1}{0.2}} = \frac{0.2}{0.04} = 5 \text{ inches.}$$

As many values of $\frac{7.64}{T_2}$ and $\frac{7.95}{T_1}$ as are wanted are now calculated and plotted, remembering that their zeros are 8 inches beyond the points where the diagonal intersects their axes, h is plotted on the diagonal and f on the same axis with T_2.

In lettering the T_1- and T_2-lines it will be a convenience for the person who uses the chart to have the temperatures marked in the Fahrenheit scale instead of from the absolute zero. This has been done on the chart, Fig. 38, but the absolute temperatures have been retained at each end of the scales as an aid to a clearer understanding of the construction. The position of the parallel index lines shows the application of the chart to the case where the chimney temperature is 400 degrees Fahrenheit, the temperature of the outside air 60 degrees Fahrenheit, and the height of the chimney 100 feet. The draft gage reading should then be a trifle over 0.54 inch.

Chart for Safe Load on Hollow Cast-iron Columns.

An interesting application of the Z-type of chart is to certain equations where a variable appears twice. Since each time it appears it occupies a scale, the number of variables we can handle is reduced from four to three. Suppose the fundamental equation to be of the form

$$u + v = \frac{v}{y}.$$

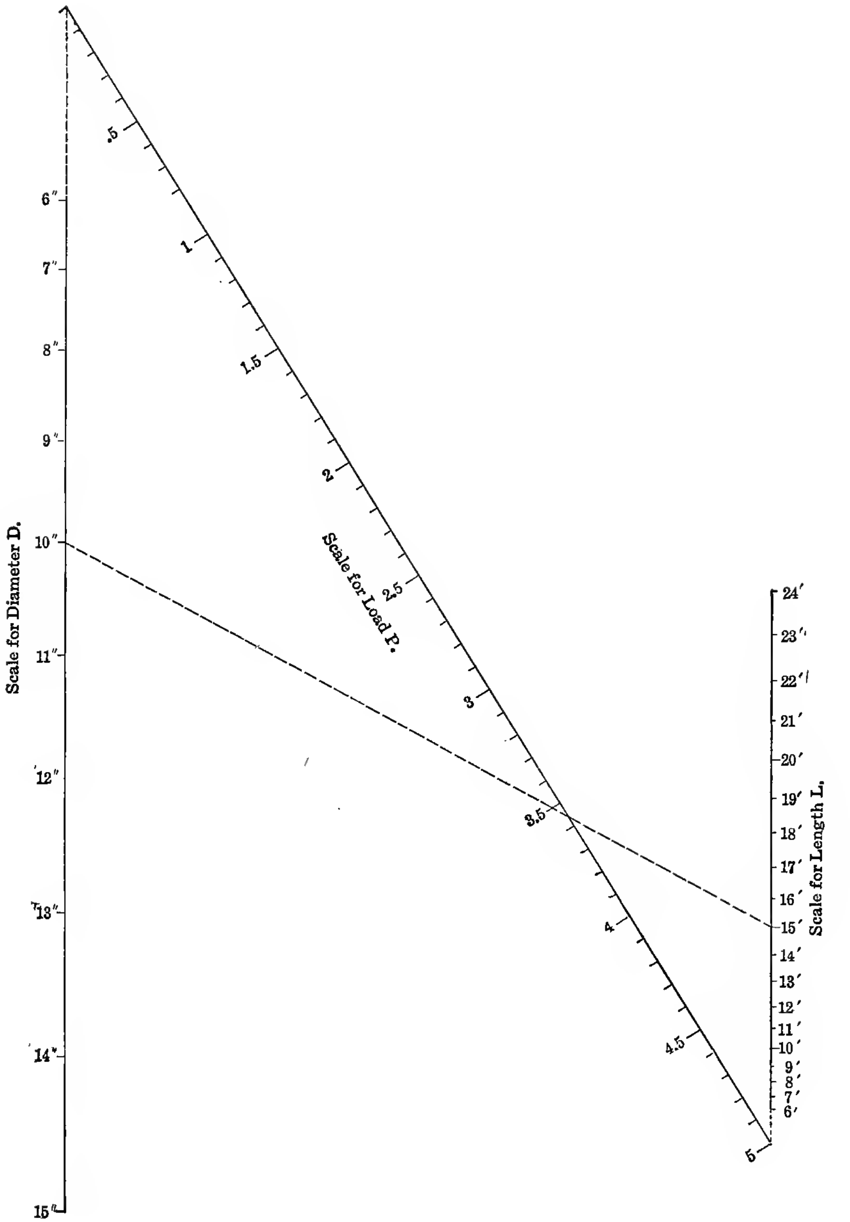

FIG. 39.—Z-chart to determine safe load of hollow cast-iron columns.

This evidently refers to equation (12) used in demonstrating the Z-chart. Here one index line instead of two is used in making a reading.

This gives a particularly useful chart since equations of this type are by no means uncommon and are awkward things to handle by any of the methods hitherto described. A formula which will serve as an excellent illustration is the one given below which is taken from the "Cambria" pocket book:

$$P = \frac{5}{1 + \frac{L^2}{800\ D^2}}.$$

It is the formula for the safe load on hollow round cast-iron columns with flat ends. In it P = the safe load in tons (of 2000 pounds) per square inch of column section, D is the outside diameter of the column in inches and L the length of the column also in inches. The successive steps required to put the formula into working shape are indicated below

$$1 + \frac{L^2}{800\ D^2} = \frac{5}{P}$$

and

$$800\ D^2 + L^2 = \frac{800\ D^2}{0.2\ P}$$

Let us take the limits for D as 6 and 15 inches, for L as 72 inches (=6 feet), and 288 inches (=24 feet). Then P will vary between 1.3 tons and 4.86 tons. The maximum value to be laid off on the L-line will be $\overline{288}^2$ or 82,944, on the D-line $800 \times \overline{15}^2 = 180{,}000$, and on P, $0.2 \times 4.86 = 0.972$. The scale units for L and D being the same, it is evident that the value 180,000 will control the choice of the scale unit if we are planning a chart of a certain size. Suppose the scale unit l_1 is made 1/20000. Then

$$180{,}000 \times 1/20000 = 9 \text{ inches},$$

which is about right. For L we have

$$82{,}944 \times 1/20000 = 4.147 \text{ inches}.$$

The scale unit l_4 used in graduating P on the diagonal will be taken as 1/0.1 and the graduated length will, therefore, be

$$0.972 \times 1/0.1 = 9.72 \text{ inches}.$$

Since $l_3 = l_1$ the length of the diagonal intercepted between the parallel axes is, according to equation (16), l_4 or 1/0.1 = 10 inches. On the chart, Fig. 39, D^2 has been graduated for every inch between 6 and 15 inches, and from its zero the diagonal, 10 inches long, has been drawn in any convenient direction. On it we may consider that we are graduating 0.2 P with a scale unit of 1/0.1 or P with a scale unit of 1/0.5. Lastly, the second parallel is drawn from the end of the diagonal and graduated for L^2. This has been done for every 12 inches and the points marked with the corresponding values in feet.

As noted above, but a single transversal or index line is required for reading this chart.

By joining 15 feet on the L-line with 10 inches on the D-line we find the safe load per square inch on the column is 3.56 tons.

Before leaving this subject it might be well to call attention to the fact that another way of treating three variables by the Z-chart is to imagine that one of the four which normally belong to it is replaced by a constant. The scale belonging to it then reduces to a point through which all of the lines pertaining to it must pass.

CHAPTER VI.

EMPIRICAL EQUATIONS.

In the previous chapters I have discussed some of the methods used in plotting curves and charts from given equations. The present one will be devoted to the reverse process, namely, the derivation of equations to fit a given set of empirical data when these data are plotted in the form of a curve or chart.

The subject is one which is full of difficulties, and, so far as I know, no systematic general method has ever been devised which will give the correct form of equation to be used. The discovery of the equation's form is to a large extent a matter of intuition which can only be acquired by long experience. Some persons seem to be peculiarly gifted in the ability to pick out the proper kind of equation for use in compensating a particular set of observations, but for the rank and file of the men engaged on experimental work this is, and probably always must be, a matter of pure guesswork, which must be verified by cut-and-try methods.

In getting an algebraic expression to show the relations between the components of a given set of data there may be two entirely distinct objects in view, one being to determine the physical law controlling the results and the other to get a mathematical expression, which may or may not have a physical basis, but which will enable us to calculate in a more or less accurate manner other results of a nature similar to those of the observations.

To attain the first result it will generally be necessary to have as a starter some sort of hypothesis as to the physical relations of the data in question, although in a few isolated cases it has been possible to arrive at hitherto unknown laws by a fortuitous treatment of the observations. In such a case as this, questions as to the intricacy or convenience of the formula in use are considered subordinate to correctness of form.

In the second case, where we want an expression which will enable us to calculate results of the same general character as the observations, form will generally be sacrificed to convenience of handling and no pretense will be made that the derived formula conforms to any physical law. This condition is one very commonly met with in engineering practice, and will be the one with which this chapter is chiefly concerned.

It has been a common matter of complaint among the so-called "practical" men that the "theorists" who are responsible for the formulas are very prone to unnecessary complication, and that the formulas they offer are in many cases no more exact than others of a much simpler type. It cannot be denied that there is some justification for these charges, due, perhaps, to a popular impression that a complicated formula presupposes brain work of a high order for its production.

That this is not necessarily true needs no special proof, but, on the other hand, we should be carefully on our guard lest we be led by a desire for simplicity into devising mere rules of thumb, applicable, perhaps, to the very special conditions in which they originated, but nowhere else. As an example of this, take the numerous formulas which have been proposed in the past for the strength of gear teeth; formulas giving results which in some instances differ from each other by several hundred per cent.

A few words of caution may be necessary at the start to prevent the reader from expecting too much of the processes described. Except in some of the simplest cases where the line connecting the plotted data is straight, it will generally be possible to fit a number of very different forms of equation to the same curve, none of them exactly, but all agreeing with the original about equally well. Interpolation on any of these curves will usually give results within the desired degree of accuracy. The greatest caution, however, should be observed in *exterpolation*, or the use of the equation *outside* of the limits of the observations.

If the form of the equation is known at the start to be correct and the observations are merely used to determine the constants, exterpolation will generally be safe. If, on the contrary, the form of the equation has been guessed at, exterpolation is hazardous in the extreme, and, if an attempt is made to use the formula much outside of the range of the observations on which it is based, serious errors may be committed.

The whole subject is full of pitfalls against which one must constantly be on guard.

About the only process for getting empirical equations which is discussed in the text-books is that known as the method of least squares. It will yield satisfactory results where a good equation has been chosen at the start, but it is tedious and laborious in the extreme even under the most favorable circumstances, while for certain forms of equation the difficulties of the method are so great that it can hardly be considered as practicable. On this account, and because it can be found fully described in the ordinary text-books, I shall not touch upon it here, but confine myself to a number of graphical or semigraphical methods with which I am acquainted. Some of these at least are but little known. Nevertheless, there are some very decided advantages in their use, as I hope to show later.

Finding the Equation for a Straight Line.

To begin with, let us take a very simple case where the relation between the variables in the equation is linear; that is, where the plotted results fall upon a straight line. The literature of engineering contains numerous examples of this type, and I have chosen as illustrations two charts taken from Bulletin No. 252 of the University of Wisconsin, entitled, "Current Practice in Steam Engine Design," by O. N. Trooien. Fig. 36 of this bulletin is reproduced here in Fig. 40, and is intended to show the relation between the indicated horsepower and square feet of belt surface per minute for Corliss engines.

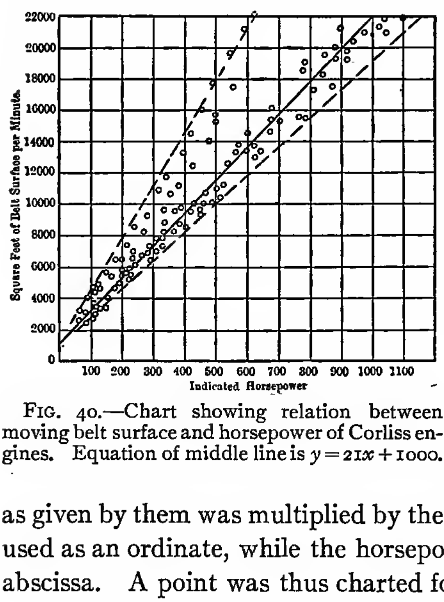

Fig. 40.—Chart showing relation between moving belt surface and horsepower of Corliss engines. Equation of middle line is $y = 21x + 1000$.

The necessary data for plotting this diagram were obtained from a number of engine builders. The rim speed of the belt pulley as given by them was multiplied by the width of the belt and the result was used as an ordinate, while the horsepower of the engine was taken as the abscissa. A point was thus charted for each engine, as shown in Fig. 40.

In this case, as in many others which occur in practice, the chart looks as if a charge of bird shot had been fired at it, and it is manifestly impossible to find a line which shall even approximately pass through all of the points.

If we have any reason to suppose that a rational formula connecting the belt surface and horsepower would be of a simple linear type, all we have to do is to draw a straight line which will coincide as nearly as possible with the "axis" of the group of points and take its equation as the best representation we can get for the data. In the case of horsepower and belt surface it is generally assumed that there is a rough proportionality between them; hence a straight line is used here. The points in Fig. 40 show two fairly distinct groups of points, and judging by eye, Mr. Trooien appears to have favored the lower group in drawing his line of average values.

This proceeding is in many instances not only justifiable, but imperative, if we wish to have our line represent the best probable values. It often happens that certain observations are known to be more accurately made than others, and hence should be given greater weight in determining the final result. In the least-square method the better observations are affected by coefficients corresponding to their greater accuracy and in the graphical method the same end is attained by causing our line to pass closer to the points representing the better observations. Just what reason Mr. Trooien had for giving greater weight to one group than to the other is not stated. It may be that the builders had a better reputation or the results may have been more in conformity with theoretical considerations.

Our average line being located (and it will generally be found advantageous to use a fine thread stretched through the points for this purpose) its equation is easily determined. The general form will, of course, be,

$$y = a\,x + b,$$

where b is the height of the intercept on the Y-axis (in this case at 1000) and a is the tangent of the angle made by the line with the horizontal. The Y-axis, as just noted, is cut at 1000, and the ordinate, through 1000 horse power is cut at 22,000. The difference is 21,000. Dividing this by 1000, the horizontal distance, gives 21 as the value of a.

Our formula then reads,

$$y = 21\,x + 1000,$$

or, as y represents belt speed and x the horsepower,

$$S = 21\,H.P. + 1000.$$

Where the points representing the observations scatter as badly as is the case here, the formula must be looked upon as a very rough approximation, and considerable deviation from it may be allowed in practice when for any reason this seems desirable. To indicate the limits within which this deviation may be made without departing from common practice, Mr. Trooien draws two lines to include the extreme cases and derives the constants for them in the same manner as before. Since all the lines meet at the same point on the axis, the value for b is 1000 in each case, while a varies from a maximum of 35 to a minimum of 18.2.

The quantity laid out on the X-axis does not have to be of the first power, as in the case just discussed, and may even be itself a product of several variables. In such a case we must lay off not x itself, but x^2, x^3, $\sqrt{x}$, etc., as the case may be, or $x\,z$ if it is a product.

Another Illustration of Finding the Equation for a Straight Line.

This may be illustrated by the chart for the connecting rods of Corliss engines shown in Fig. 16 of the same bulletin and reproduced here in Fig. 41.

If the Euler formula for struts be taken as correct for the connecting rod, it may be reduced to the expression.

$$d = C\sqrt{DL}$$

where d is the diameter of the rod, C a constant whose value is to be determined, D the diameter of the piston (supposed to be acted upon by a standard steam pressure), and L the length of the rod.

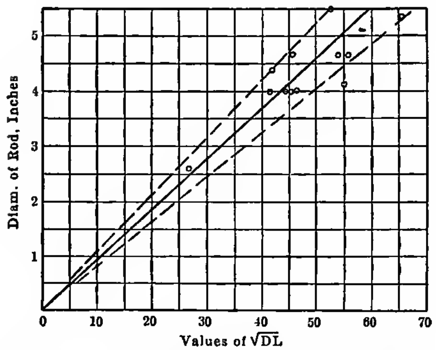

Fig. 41.—Chart showing relation between diameter of connecting rod and square root of piston diameter times the length of rod for Corliss engines. Equation of middle line is $d = 0.092\sqrt{DL}$.

From the data furnished by the engine builders the points in the chart shown in Fig. 41 were plotted, using the values of d as ordinates and of $\sqrt{DL}$ as abscissas. The resulting line should be straight and pass through the origin, and the angle with the horizontal gives the desired value of C as $5.5/60$, or 0.092, for the mean and 0.104 and 0.081 as the maximum and minimum values.

Finding the Equation of a Curve.

Next let us consider the case where the line connecting the observations is curved. Here we have no ready-made equation as with the straight line, requiring merely the discovery of a couple of constants. The general form of the equation must be guessed at if the physical law is unknown, and here we encounter one of the greatest difficulties connected with the subject and one for which it is practically impossible to offer much real help.

The appearance of the curve may or may not afford a clue, and in this connection it is suggested that a book like Frost's "Curve Tracing," may be useful for reference. It contains a large number of curves plotted from various equations and their shapes will sometimes suggest a good form of equation if we are at fault.

It has sometimes been suggested as a solution of this difficulty that we plot a considerable number of functions, such $y=x^2$, $y=x^3$, $y=log.\ x$, $y=1/x$, etc., on a straight line and then from any pole draw a series of radiating lines through the points thus found as shown in Fig. 42 where $y=x^2$ has been used. The observed results are plotted on a similar straight line for equally spaced values of the variable. This graduated line is then laid on the radiating lines and shifted around until we get the plotted points falling on them. Such an agreement would indicate at once the proper function to use, and a measurement of its distance from the pole would indicate the coefficient.

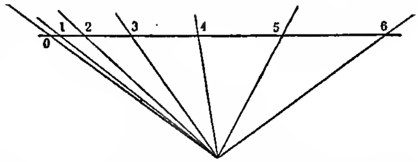

FIG. 42.—Trial diagram of a known function. In this case $y=x^2$.

While this looks promising, my own experience leads me to accord it but little practical value. The observation points can hardly ever be made to agree even approximately with the trial function.

Many experimenters assume that an equation of the parabolic form,

$$y=a+b\,x+c\,x^2. \ . \ . \ . \ etc.$$

may be used for almost any class of observations with good results, and it is surprising sometimes how closely it may be made to fit unpromising conditions.

It should not, however, be blindly used for all cases, for while, on the one hand, it may be forced, with a sufficient number of terms, into the semblance of an agreement with almost any set of data, on the other hand, a large number of terms is detrimental to its subsequent use in calculation and in many cases a far simpler equation may be discovered which will not only be easier to handle, but may even give more accurate results. For instance, the crest of a sine curve may be made to agree quite closely with a parabola, but the longer this arc is the greater is our difficulty in getting a fit.

In fitting an equation to a given set of observations the first step is to draw through the plotted points a smooth curve. If the experimental work has been carefully and accurately done the curve may be made to pass through, or close to, almost all the points. If not, the curve must be drawn in such a way as to represent a good probable average; that is, so as to leave about an equal number of points at about equal distances on either side of it, these distances, of course, being kept as small as possible. Such a curve is assumed to represent the most probable values of the observations, and we then attempt to get its equation.

It may be stated that it is always possible to get an equation which will agree exactly with a given curve at any desired number of points, providing we use an equal number of constants in our equation.

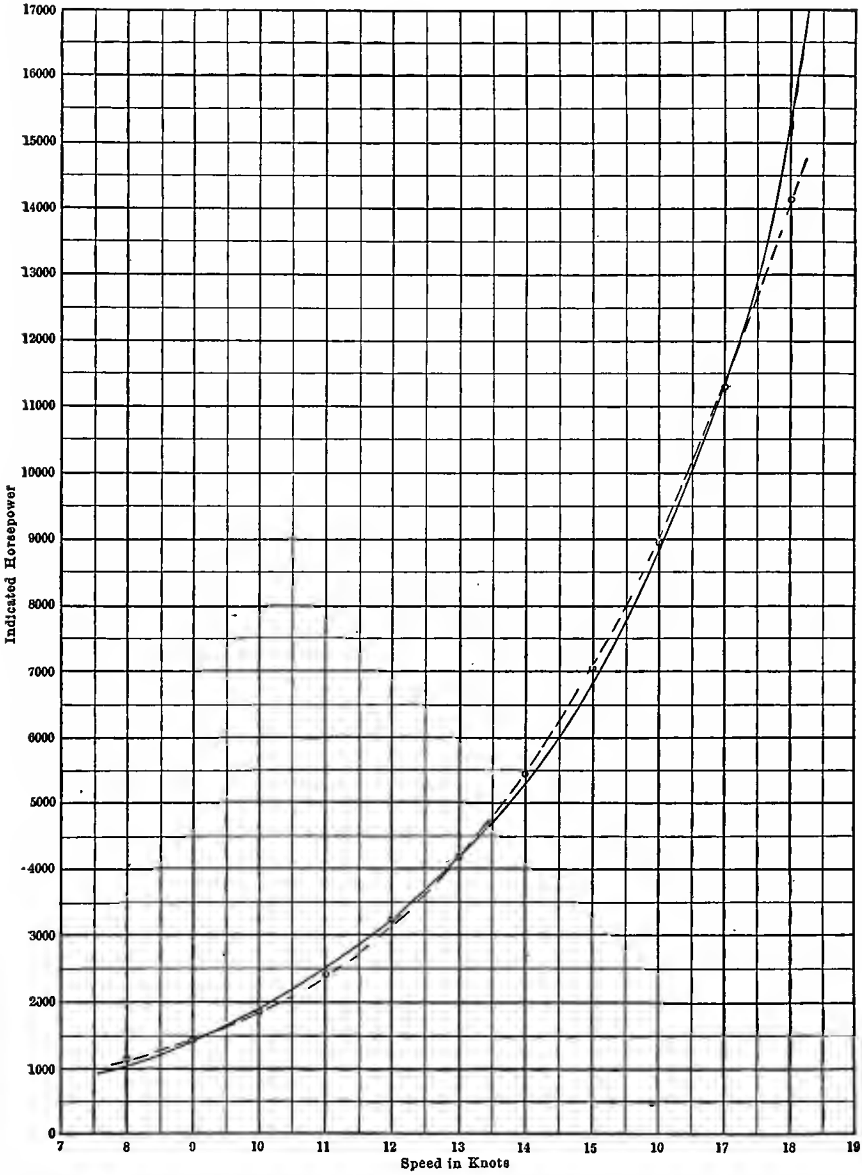

FIG. 43.—Chart showing relation between indicated horsepower and speed in knots for the battleship "Maine." Equation of the dashed curve is $y=440.5x-82.32x^2+5.65x^3$.

Method of Selected Points.

This is called the method of selected points and will be described first as it is the simplest and quickest method and, if a good equation has been chosen at the start, we may get results of a very satisfactory character.

For purposes of illustration I have chosen a curve given in the *Journal* of American Society of Naval Engineers for November, 1902. It shows the relation between the speed in knots and the indicated horsepower for the battleship "Maine" and is reproduced by the solid line in Fig. 43.

The data from which the curve was plotted are not given and there is no means of knowing how accurately it represents the results of the test. It will, therefore, be taken as it stands and an attempt made to find the compensating equation. As to the form of the equation, we will disregard all theoretical considerations and assume it to be parabolic since it has most of the ear-marks of this type.

The curve stops at about eight knots and we have nothing to guide us as to its shape below this point. The assumption will be made, however, that the horsepower and speed became zero together; that is, that the curve passes through the origin. If this is so the first constant in the general parabolic equation (the one unattached to a variable) vanishes.

Let us assume that the equation contains the first three powers of x, or that

$$y=a\,x+b\,x^2+c\,x^3$$

where y represents the indicated horsepower and x the speed in knots. We have here three constants whose values must be determined. To do this take three points on the curve, one at about the middle and the others at or near the ends, and form three equations, inserting in them the values of y and x for these points taken from the curve.

In the case in question I have selected the points at 9, 13, and 17 knots. The corresponding values for y (the horsepower) are 1400, 4180, and 11,350.

Inserting these in the chosen equation we have:

$$1{,}400=9\,a+81\,b+729\,c,$$
$$4{,}180=13\,a+169\,b+2{,}197\,c,$$
$$11{,}350=17\,a+289\,b+4{,}913\,c.$$

These equations are solved by the customary methods for a, b, and c, giving us 440.5 for a, -82.32 for b, and 5.628 for c.

The equation then reads:

$$y = 440.5\,x - 82.32\,x^2 + 5.628\,x^3,$$

or

$$H.P. = 440.5\,S - 82.32\,S^2 + 5.628\,S^3.$$

The curve for this equation has been drawn as a broken line on the same chart as the original curve, and is seen to pass through the chosen points exactly and to give a very fair agreement at nearly every other point.

At the upper end, however, although the two curves are not much separated, there is a considerable difference in the horsepower as read from the two curves, and the indications are that this will become worse as we overstep the limits of the chart. Up to about 17 1/2 knots, however, the equation would usually be considered a passable fit. The rapid rise in the horsepower as the speed increases at the upper end of the curve would indicate that better results might have been reached by the use of a higher power of x in the equation.

Another Illustration of the Method of Selected Points.

The above method will answer every requirement in many cases, but too much reliance should not be placed in it without an actual test of the results. As an example of the danger of this I have applied the method to a series of experiments showing the variation of the coefficient of friction of straw-fiber friction drives with the slip.

The experiments were made by Professor Goss, who describes them in the *Transactions* of the American Society of Mechanical Engineers for 1907, page 1099.

To avoid a confusion of notation, I have replotted the curve from the original paper in Fig. 44, with the ordinates and abscissas interchanged. The small circles represent the observations and the solid curve is Professor Goss' idea of the best representation of their average value. We will attempt to compensate this curve by a suitable equation.

At first glance the curve seems to have some of the characteristics of the parabolic type, enough at any rate to make it amenable to treatment by that form of equation. It straightens out suspiciously, however, in each direction, as it leaves the region of greatest curvature near the ordinate erected at 0.4, and this would suggest the hyperbolic rather than the parabolic type. As an experiment, however, we will run it out on the assumption of its being a parabola and will try compensating by the equation

$$y = a + b\,x + c\,x^2 + d\,x^3.$$

The four constants will make it possible to get four points of exact agreement instead of three, as in the previous example, and we should naturally expect that the general agreement would be better on account of this larger number of points.

Let these points be $y=0.55$, $x=0.15$; $y=0.825$, $x=0.3$; $y=1.42$, $x=0.4$; $y=2.7$, $x=0.45$. The four equations then become:

$$0.55 = a+0.15\,b+0.0225\,c+0.003375\,d,$$
$$0.825 = a+0.3\,b+0.09\,c+0.027\,d,$$
$$1.42 = a+0.4\,b+0.16\,c+0.064\,d,$$
$$2.7 = a+0.45\,b+0.2025\,c+0.091125\,d.$$

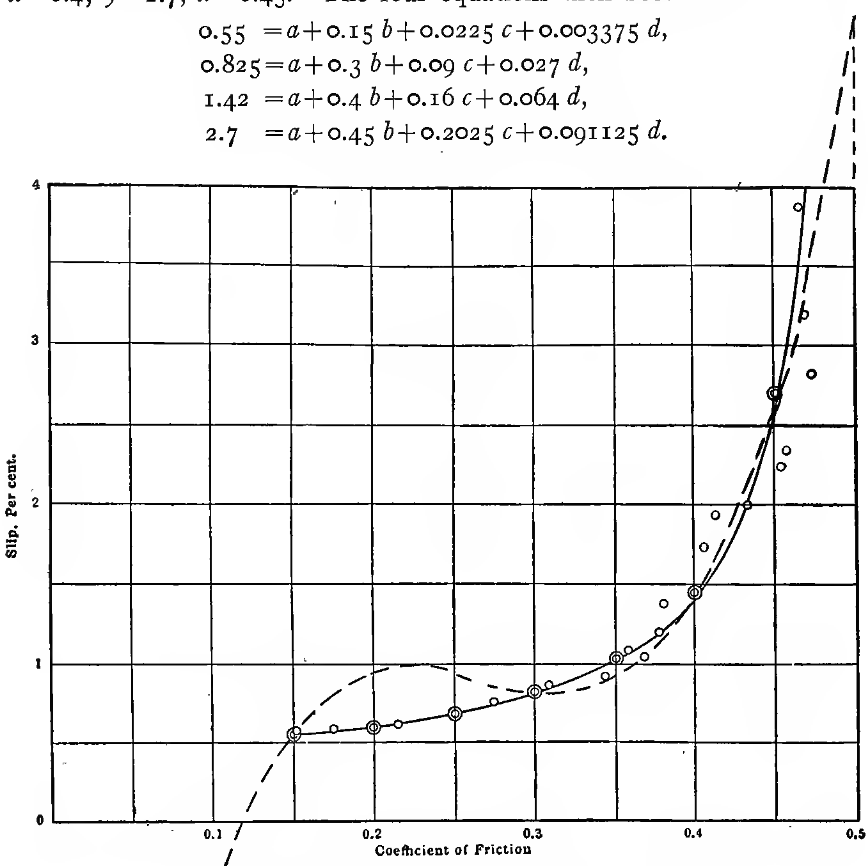

FIG. 44.—Chart showing relation between coefficient of friction and slip for straw-fiber frictions. Equation is $y=\dfrac{-0.13}{x-0.502}+0.181$.

The solution of these equations gives us $a=-5.89$, $b=80.5$, $c=-308$, $d=381.8$,

or,

$$y=-5.89+80.5\,x-308\,x^2+381.8\,x^3.$$

The values of y were now calculated for every 0.05 of x from 0.1 to 0.5, and the result is shown by the broken line on the same diagram. It hits the selected points with practical exactness, but it would require a vivid imagination to say that the fit elsewhere was even fairly good. A larger number of selected points and constants would undoubtedly have helped materially in improving this state of affairs, but the most cursory inspection of the diagram will show that the trouble is not due to the small number of points, but rather to the choice of an improper form of equation.

Returning now to the suggestion made above as to its hyperbolic form, let us see what can be done on that supposition. We will assume that the curve is a rectangular hyperbola of which we do not know the asymptotes.

Let us try an equation of the form

$$(y+a)(x+b)=c.$$

The three constants will demand three equations, and we will select for our points $y=0.55$, $x=0.15$; $y=0.825$, $x=0.3$; $y=2.7$, $x=0.45$.

Substituting these values in the equation above we have,

$$(0.55+a)(0.15+b)=c,$$
$$(0.825+a)(0.3+b)=c,$$
$$(2.7+a)(0.45+b)=c.$$

The solution of these equations for a, b, and c, gives us

$$a=-0.1806,\ b=-0.5015,\ \text{and}\ c=-0.1298.$$

These values, in round numbers, substituted in the original equation give us

$$(y-0.181)(x-0.502)=-0.13,$$

or,

$$y=\frac{-0.13}{x-0.502}+0.181.$$

If, now, we substitute values of x for every 0.05 from 0.15 to 0.45, we get the points represented by the double circles in the chart. They agree so closely with the original curve as to be practically identical with it. Thus, with a less number of points we have obtained an extremely satisfactory fit, and have given a practical illustration of the statement made above as to the desirability of starting with a good equation rather than trying to force a fit by the use of an unsuitable equation and a large number of constants.

Value of Logarithmic Cross-section Paper in Determining Form and Constants of an Equation.

This may be a good place to say that the logarithmic paper described in a previous chapter is often of great service in determining the form and constants of an equation.

If the equation involves only a simple product or quotient with no addition or subtraction, its trace on logarithmic paper will be a straight line. The tangent of the angle made by this line with the horizontal (and this may be positive or negative) will give the exponent of the variable, while the intercept on the Y-axis will give the constant by which the variable is multiplied.

It is much to be regretted that the ordinary commercial logarithmic paper is only laid off from 1 to 10 on the axes, for my experience is that almost invariably the line will extend beyond these limits, and it then becomes difficult to see clearly if it is rectilinear, since it must be broken and appear in two or more places on the sheet. If such paper were printed with graduations on each axis from 1 to 100 instead of from 1 to 10, it would greatly facilitate many of these operations. Any curve having the aspects of the hyperbolic or parabolic type should always be so plotted, since, if it does appear as a straight line, it saves a large amount of labor in determining its equation.

One special case may be mentioned here, which is sometimes useful in gas-engine work; namely, the determination of the exponent of the v in the equation for the expansion curve. If we have an indicator diagram we take the ordinates representing the pressures (absolute) and lay them out on the logarithmic paper from points on the X-axis representing the volumes (which must include the clearance). The points thus found should fall upon a line which is sensibly straight if the exponent is constant for all parts of the curve. Otherwise the exponent must be determined for any particular point by drawing the tangent to the curve there.

As an illustration, I have reproduced the expansion line from the indicator diagram of an old Clerk gas engine. The volumes are measured from the clearance line in any convenient unit. The length of the diagram made it convenient to call the clearance volume 9. From there on, the indicator diagram was divided, as shown in Fig. 45 (*a*), and the logarithms corresponding to the numbers on the X-axis were laid off on the X-axis of the lower logarithmic diagram, (*b*) of Fig. 45.

The pressures from the absolute zero were then measured from the indicator card and their logarithms laid off from the corresponding points of the X-axis of (*b*).

A straight line was now drawn to indicate the general direction of the middle set of points and then a parallel to it through 10 on the X-axis. Its intercept on the Y-axis measured in linear (not logarithmic) units gives the tangent of the angle of slope. In this case it is 1.32 which, divided by 1 (the distance to 10 measured on X), gives 1.32 as the value of the exponent.

The method of selected points, while accurate enough for many purposes, especially where the form of the equation is definitely known at the

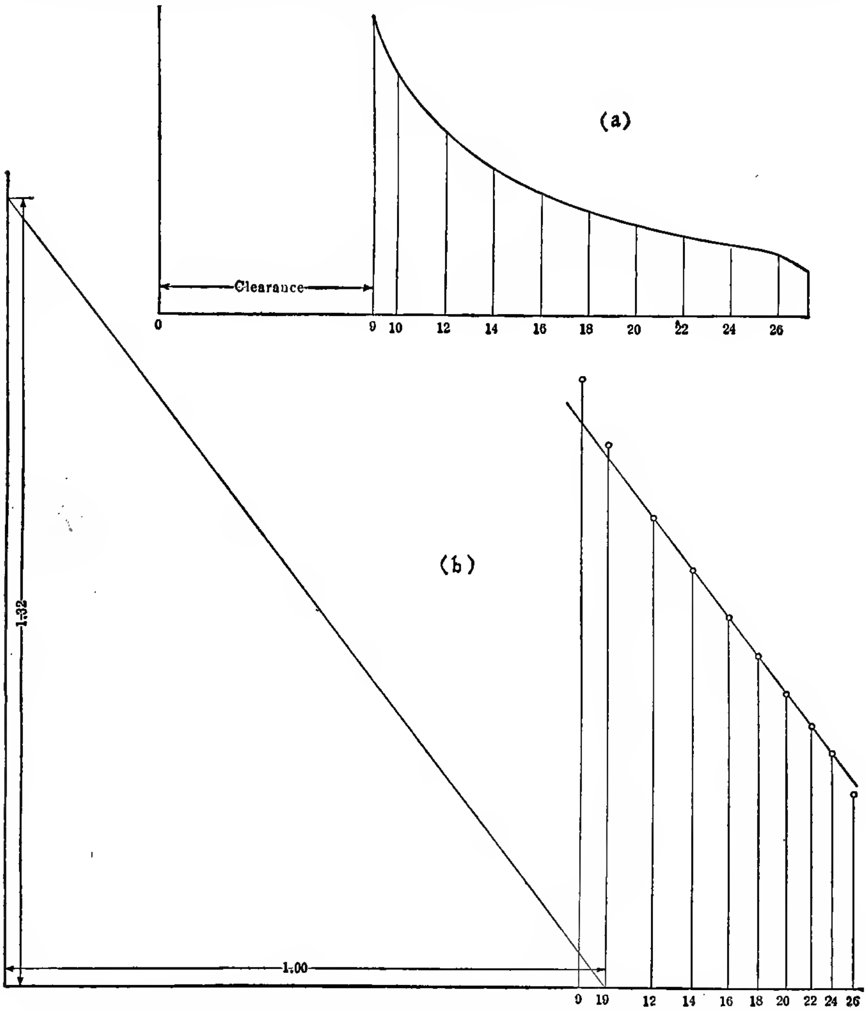

FIG. 45.—Expansion line of a gas-engine indicator card and logarithmic determination of value of exponents in the equation of the expansion curve.

start, is not so satisfactory when we wish for greater refinement, and especially when we are in the dark as to the proper form of equation. The number of points which can influence the result is no more than the number of constants employed, and if we wish to use a small number of constants we cannot expect any high degree of accuracy in the fit. Some method by which a larger number of points on our curve may enter into the result without burdening the equation with constants is, therefore, much to be desired.

Method of Equating the Area and Moments Obtained from Measuring the Area under a Curve with the Integration of the Assumed Equation of the Curve.

Suppose that, an observation curve being drawn, we obtain its area by any planimetric method. If, now, we find the area of the curve of the assumed equation by integration and equate it to the area just found of the observation curve, we evidently have a condition in which we can take account of as large a number of points as we please without necessarily using a large number of constants. In fact, this one equation takes care of one and only one constant. It would, of course, be possible to have two curves of equal area and quite different shape if the assumed formula were not well chosen.

Suppose, however, that we get the moment of the area of the original curve by dividing it up into a number of vertical slices, taking the area of each slice above the X-axis and multiplying it by the distance of its center from any arbitrary vertical axis, generally Y, and then adding the moments thus found; we shall in this way obtain the moment about the assumed axis of the entire area between the curve and X. Its value will evidently depend upon the form as well as the area of the curve. The moment of the assumed curve may likewise be determined by integration and can be placed equal to the measured moment. This accounts for another constant. Similarly we may obtain second and third moments, etc., by multiplying the areas of the slices by the square and cube of the distances from the assumed axis and, from each of these, form equations with the same moments of the theoretical curve. We must, of course, have as many of these equations as we have constants to determine.

Any of the well-known methods for getting the areas and moments may be used, but as it will make the explanation simpler I shall get my areas and moments in what follows by taking the mean ordinates of a series of vertical slices in the same way that we do when averaging an indicator diagram, and assume that all necessary accuracy can be secured by making the strips narrow and of considerable number. As an illustration of the application of the method it will be interesting for purposes of comparison to take again the curve for the speed and horsepower of the "Maine."

The same formula will be assumed as before, having three constants to be determined and, therefore, demanding three equations. The curve extends practically from 8 to 18 knots and these will be taken as the limits within which to work.

For convenience we will divide this space into 10 vertical slices. A greater number would lead to greater accuracy, but the work of calculation is laborious at the best and for illustrative purposes this will be amply sufficient. The height of the curve at the middle of each of these spaces is then measured and tabulated in the column headed y alongside of the corresponding value of x. Next to the x column is one of x^2. Then follow columns for $x\,y$ (the first moment) and $x^2\,y$ (the second moment).

y	x	x^2	xy	x^2y
1,260	8.5	72.25	10,710	91,077
1,660	9.5	90.25	15,770	149,810
2,150	10.5	110.25	22,575	237,040
2,850	11.5	132.25	32,775	376,920
3,670	12.5	156.25	45,875	573,440
4,730	13.5	182.25	63,855	862,040
6,050	14.5	210.25	87,725	1,272,000
7,750	15.5	240.25	120,130	1,862,000
10,000	16.5	272.25	165,000	2,722,500
13,050	17.5	306.25	228,370	3,996,500
53,170 = Area			792,785 = M_1	12,143,327 = M_2

Since the width of the slices is 1, the height of the middle ordinate gives us its area at once, and the sum of the ordinates will be the area of the space under the entire curve.

The sum of the values in the y column gives us 53,170 as the area. The sum of the values of $x\,y$ gives us 792,785 as the first moment, and the sum of the values of $x^2\,y$ gives us 12,143,327 as the second moment. Both moments are reckoned about the Y-axis.

These quantities must be equated to the area and first and second moments of the assumed theoretical curve, to get which we must use a little integral calculus. The area of a small vertical slice of height y and width $d\,x$ is $y\,d\,x$ and since,

$$y = a\,x + b\,x^2 + c\,x^3,$$

$$y\,d\,x = a\,x\,d\,x + b\,x^2\,d\,x + c\,x^3\,d\,x.$$

The integral of this expression is the area of the curve, or

$$A = \int_{x_1}^{x_2} a\,x\,d\,x + b\,x^2\,d\,x + c\,x^3\,d\,x,$$

where x_1 and x_2 are the limits between which the integration is to be performed (here 8 and 18), or

$$A = \frac{a}{2}(x_2^2 - x_1^2) + \frac{b}{3}(x_2^3 - x_1^3) + \frac{c}{4}(x_2^4 - x_1^4).$$

This, after substituting the values of x_1 and x_2 given above, is placed equal to 53,170.

If we multiply the differential area $y\,d\,x$ by x we get its moment about the Y-axis and its integral will be the first moment of the entire area, or

$$M_1 = \int_{x_1}^{x_2} a x^2\,dx + b x^3\,dx + c x^4\,dx = \frac{a}{3}(x_2^3 - x_1^3) + \frac{b}{4}(x_2^4 - x_1^4) + \frac{c}{5}(x_2^5 - x_1^5).$$

This is placed equal to the measured first moment, or 792,785.

Multiplying the differential area next by x^2, we get its second moment about the Y-axis and its integral will be the second moment of the whole area, or

$$M_2 = \int_{x_1}^{x_2} a x^3\,dx + b x^4\,dx + c x^5\,dx = \frac{a}{4}(x_2^4 - x_1^4) + \frac{b}{5}(x_2^5 - x_1^5) + \frac{c}{6}(x_2^6 - x_1^6),$$

which must be placed equal to 12,143,327.

After substituting the limiting values of x_1 and x_2, which are 8 and 18, we have the three equations

$$130\,a + 1773\,b + 25{,}218\,c = 53{,}170,$$
$$1773\,a + 25{,}218\,b + 371{,}366\,c = 792{,}785,$$
$$25{,}218\,a + 371{,}366\,b + 5{,}624{,}977\,c = 12{,}143{,}327.$$

In solving these equations, while it may not be necessary to run all calculations out to the last figures, it will generally be desirable to carry them out to five or six significant figures, since we often have to take the difference between two numbers of nearly equal magnitude, in which case the last figures may have an important influence on the result. The slide rule is, therefore, absolutely useless for these calculations except as a check against large errors. After the calculations are complete it will generally be safe to throw away all except the first three or four significant figures in order to simplify the formula for practical use.

The solution of the above equations gives

$$a = 422.8,\ b = -77.98,\ c = 5.4115,$$

making the equation read

$$y = 422.8\,x - 77.98\,x^2 + 5.4115\,x^3,$$

or, more simply,

$$y = 423\,x - 78\,x^2 + 5.41\,x^3.$$

By the method of selected points we got

$$y = 481\,x - 88.5\,x^2 + 5.853\,x^3.$$

The agreement is as close as could be expected and is really closer than the appearance of the equation might lead us to suppose.

This is shown in Fig. 43, where the equation just obtained is plotted with the previous one by selected points. As the curves run pretty close together, I have not attempted to draw the last one, but have simply indicated the value of y for each even knot by a small circle.

The difference between the two curves is quite small, the last one being possibly slightly nearer the curve we are trying to compensate than the first. So small a difference would hardly make it worth while, as a rule, to use the more laborious method of moments if we knew that the results were going to come out this way beforehand. We have no means of knowing this, however, and there is generally an added feeling of safety in using it on account of the larger number of points which are taken account of. We should probably have obtained a closer approximation to the original curve by using a larger number of ordinates in getting our area and moments. Whether or not this would be desirable would have to be determined after an inspection of the calculated curve to see if its deviation from the original was within the desired limits of accuracy.

This method is of very general application and may be used for any equation of integrable form.

An Alinement Chart Method.

The next method I propose to discuss is one based on the alinement chart described in Chapter II.

The method is due to Captain Batailler, of the French artillery service, who describes the process in the *Revue d'Artillerie* of December, 1906. Those who are interested are referred to it if they desire fuller information than can be given in this brief outline. The process depends on the alinement of a series of points taken from the data or from a curve which is assumed to represent them.

It is not easy to explain the method in a simple manner, but I hope that I shall at least be able to make the practical application clear. This I think can be best done by working out a practical example, explaining each step as it is taken.

The example chosen will be the data given by Prof. R. T. Stewart as the results of his experiments on the collapsing pressure of bessemer-steel tubes, and published in the *Transactions* of the American Society of Mechanical Engineers for 1906. Professor Stewart showed his results in chart form by laying off the values of the thickness of the tube divided by the diameter, or t/d, on the X-axis and the corresponding collapsing pressures as ordinates. He found that a smooth curve drawn through these points was difficult to represent by any simple formula and, therefore, took two bites at it, so to speak, and derived two formulas limited in their application to different parts of the field. This is a very common and useful expedient where the experimental curve is rebellious to representation by a simple formula.

Let us see what can be done toward getting the whole range of results into one equation. To start with, the averages from the tabulated results have been plotted in Fig. 46, and are indicated by the small circles. In doing this I have interchanged the ordinates and abscissas as they appear in Professor Stewart's chart since, with the form of equation I wish to try, there might otherwise be some confusion of nomenclature.

Then a smooth curve was passed through these points so as to represent as nearly as possible a good general average. This is not strictly necessary for the first process I am going to describe, but I have done it in order to have something definite to work toward as a measure of the success of the method, and also because a definite curve is more suggestive of the type of equation than a number of scattered points. Professor Stewart assumed that the greater part of the curve is a straight line with a sharp bend toward the origin as the lower values are approached.

He was probably justified in doing this, as the small number of observations among the higher values make the direction of the curve in that region somewhat uncertain.

In my chart I have drawn the line with a reversal of curvature to permit it to pass closer to the higher-value observations and thus get a somewhat closer agreement with the actual tests. At the lower end of the curve, according to Professor Stewart, the collapsing pressure seems to vary as $(t/d)^3$, or t/d is proportional to the cube root of the collapsing pressure. Acting on this hint, we will use $\sqrt[3]{x}$ in our equation (x being taken to represent the collapsing pressure).

Now, the curve as I have drawn it reverses its direction of curvature as it moves away from the origin. This effect could be brought about by the use of some power of x in the equation in addition to the root.

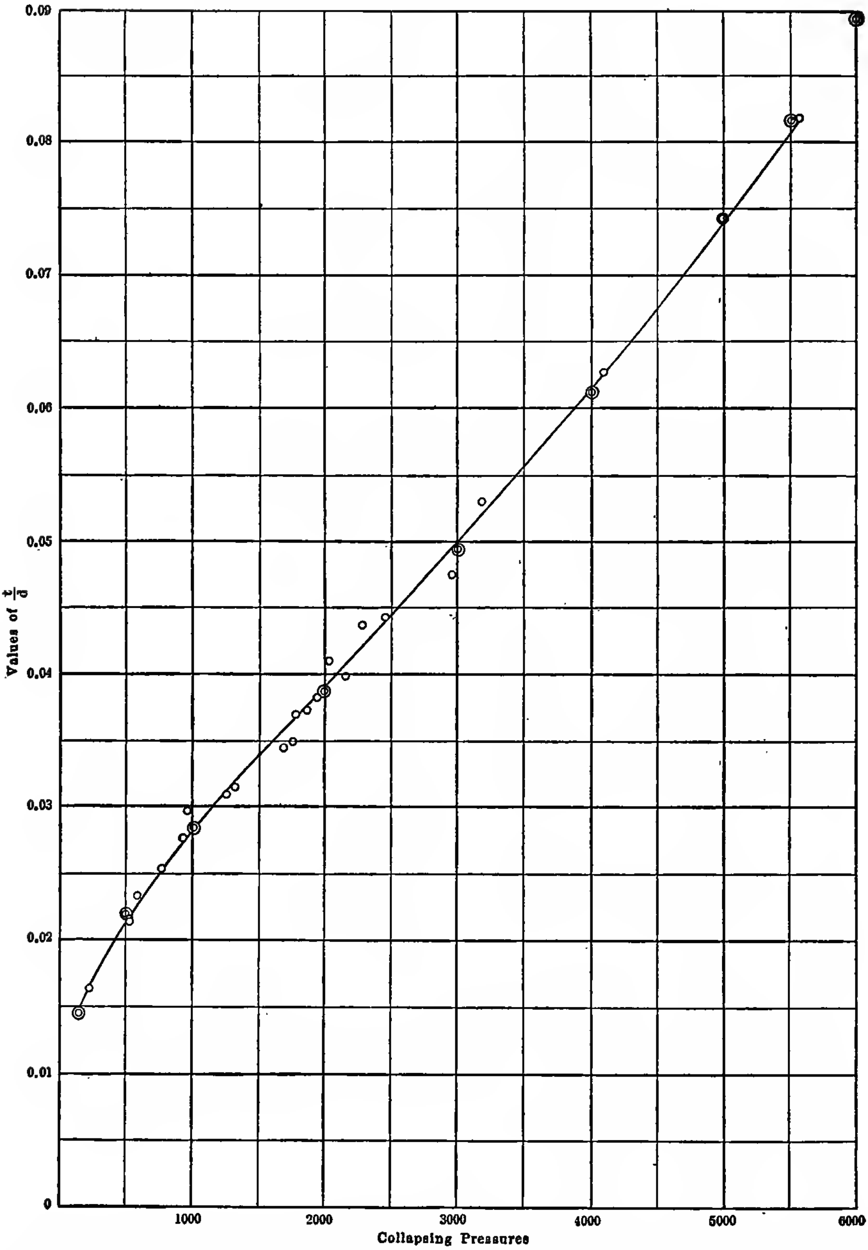

FIG. 46.—Chart showing relations between collapsing pressure of bessemer steel tubes and the ratio of thickness to diameter. Equation of curve is $\frac{t}{d} = 0.00274\sqrt[3]{P} + 0.0000000011P^2$.

The power will have but little influence on the shape of the curve for the lower values of x where the predominant effect of the root is felt, but as we get to the higher values the power will overbalance the effect of the root and cause the reversal we wish. A high power is evidently not indicated as the bend upward is comparatively small, hence (as it is easily calculated) we will try the second.

Let our trial equation then take the form

$$y=A\sqrt[3]{x}+Bx^2$$

where y represents t/d and x the collapsing pressure.

For convenience in handling let us express these pressures in units of 1000 pounds.

The general form of equation used in the discussion of the alinement diagram was

$$au+bv=c,$$

where u and v represent measured distances on the U- and V-axes. If u and v are kept constant while a, b, and c vary, we get a series of points lying along the straight line joining u and v. Hence, if this line can be determined, its intersection with the U- and V-axes should fix the values of u and v.

In our assumed formula A and B are constants, therefore let us consider that they replace the quantities u and v in the alinement equation. Now $\sqrt[3]{x}$, x^2, and y may be given various values, hence let us suppose that they take the place of a, b, and c in the alinement equation.

In order to get the position of the points lying on the line joining u and v or, as we now call them, A and B, we make use of formulas (7) and (8) developed in Chapter II. There, in order to locate our points, we used rectangular coördinates of which the Y-axis was parallel to, and midway between, the U- and V-axes and the X-axis was the line joining the zero points on these same axes.

The formulas for the coördinates of the various points on the third line of the diagram were then found to be

$$X=d\frac{b-a}{b+a},\quad Y=\frac{c}{b+a},$$

d being the half distance between the U- and V-axes.

In these equations we replace a by $\sqrt[3]{x}$, b by x^2, and c by y.

Now, x and y are the coördinates either of the points representing the observations or of the chosen points on the curve. We will in this instance consider them as belonging to points on the curve.

Below are tabulated the quantities we shall require, x and y being read from the curve and x being given in 1000 pound units:

y	x	x^2	$\sqrt[3]{x}$
0.015	0.15	0.0225	0.531
0.0215	0.5	0.25	0.794
0.0284	1.0	1.0	1.0
0.0392	2.0	4.0	1.26
0.05	3.0	9.0	1.44
0.0616	4.0	16.0	1.59
0.074	5.0	25.0	1.71
0.0808	5.5	30.25	1.77

Substituting in the equations for X and Y we have for $x=0.15$,

$$X=d\frac{0.0225-0.531}{0.0225+0.531}=d\frac{-0.5085}{0.5535}=-0.9188\,d,$$

$$Y=\frac{0.015}{0.5535}=0.0271.$$

$$\begin{array}{lll} x=0.5 & X=-0.521\,d & Y=0.02060 \\ x=1.0 & X=0.0 & Y=0.01420 \\ x=2.0 & X=0.521\,d & Y=0.00745 \\ x=3.0 & X=0.724\,d & Y=0.00479 \\ x=4.0 & X=0.819\,d & Y=0.00350 \\ x=5.0 & X=0.872\,d & Y=0.00277 \\ x=5.5 & X=0.889\,d & Y=0.00252 \end{array}$$

These points are now plotted on the alinement chart, shown in Fig. 47. In this chart the distance between the U- and V-axes has been made 20; hence $d=10$, and the values calculated for X will be multiplied by this before laying them off. The values of Y are laid off to any convenient scale which will give clear readings. The measurements on the U- and V-axes are to this scale.

The points with the exception of the first two seem to be in nearly perfect alinement, which leads us to infer that the formula chosen is a good one. If they fail to line up in a satisfactory manner it is useless to go further, as this is an indication that the wrong equation is being used. Of course, if the ordinates taken from the observations themselves had been used instead of the points on the curve, we could not expect them to fall so nearly on a straight line, but they should be grouped close enough to one to make it evident that the axis of the group is straight and not curved.

The line extended cuts the U-axis at 0.0274 and the V-axis at 0.0011, which are, therefore, the desired values of A and B. Before using them in the equation, however, we shall have to modify them slightly to take account of the change in size of the pressure unit which is really 1000 times that which we have been working with. Thus A will have to be divided by $\sqrt[3]{1000}$, or 10, and B will have to be divided by 1000^2, or 1,000,000, and our final formula becomes, after substituting t/d for y and P for x,

$$t/d = 0.00274\,\sqrt[3]{P} + 0.0000000011\,P^2.$$

The formula was now solved for a series of values of P, and the results are shown by the double circles on the chart. The curve could not be drawn in a satisfactory manner as it lies very close to the original for a considerable portion of its length, and this closeness is a good indication of the success of the method.

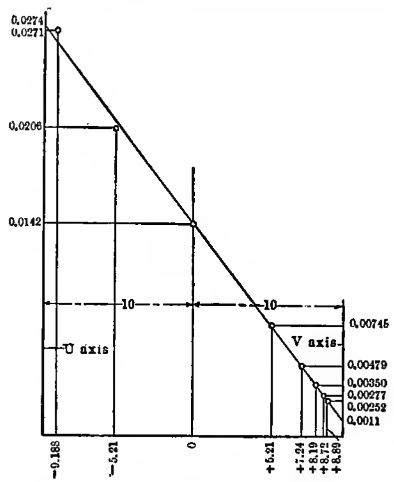

FIG. 47.—Alinement diagram for testing points found in determining equation for curve of Fig. 46.

Lest I be misunderstood, let me say here that I make no pretense at having obtained a better mathematical expression for his results than Professor Stewart. The scarcity of data in the region of higher values renders it extremely unsafe to say whether the line there is straight or curved. What interested me mainly in this problem was the possibility of expressing the entire series of results by one formula. This, I believe, has been accomplished with a very fair degree of success and by the use of a comparatively simple equation.

The method we have been investigating is generally quite sensitive, and if the equation is not a good one for the purpose the points will depart markedly from the straight line. Thus the possibility of forcing an unsuitable equation into the appearance of an agreement with the original curve, which may be done with most of the other methods, is largely absent here. Often a portion of the points will lie along a straight line while the others depart from it. In this case it indicates that in a limited field the compensation is possible and may be good, a fact which it is sometimes desirable to ascertain.

In the example just explained, we have assumed that not only the general type of the compensating equation was known, but also the values of the exponents of x.

Some guide to the choice of the exponents is evidently much to be desired, since if we rely upon guesswork we may consume a great deal of valuable time in hunting for them, and may even then not hit upon the best values.

Another Illustration of the Alinement-chart Method.

The Bätailler method just described may be extended to do this for many types of equation in a manner which is comparatively simple in operation, though a little difficult to explain. The additional one or two constants which may thus be determined are not limited to exponents, but may also be coefficients.

As before, I shall make the explanation while working out a problem. The example chosen will be taken from Rateau's "Flow of Steam Through Nozzles," and is the diagram shown in Plate IV of that book for Hirn's experiments on the flow of air through thin plate orifices. I have redrawn the curve for this in Fig. 48. In it the abscissas represent the ratio of back pressure p to initial pressure P, and the ordinates the ratio of observed discharge to the maximum discharge. The abscissas on the X-axis are numbered from 1 to 0.4, but I have reversed the numbering in order to avoid confusion and will afterward insert the quantities as they appear in the original diagram. My numbers will then be 0, 0.1, 0.2, 0.3, etc., instead of 1, 0.9, 0.8, 0.7, etc.

The problem will be to see how nearly we can compensate this curve by an equation of the type

$$y = Ax^p + Bx^q$$

in which A, B, p, and q are all unknown and are to be evaluated.

The first step is to differentiate the curve and obtain its first and second derivatives, y' and y''. Then A and B are eliminated from these equations, and p and q determined by a process analogous to that last described.

The equation and its first and second derivatives are:

$$y = Ax^p + Bx^q$$
$$y' = Apx^{p-1} + Bqx^{q-1}$$
$$y'' = Ap(p-1)x^{p-2} + Bq(q-1)x^{q-2}$$

To eliminate A and B from these equations and put them in the necessary form for use, I am constrained to use determinants. Any other method lands us in such a snarl of equations as to be very objectionable,

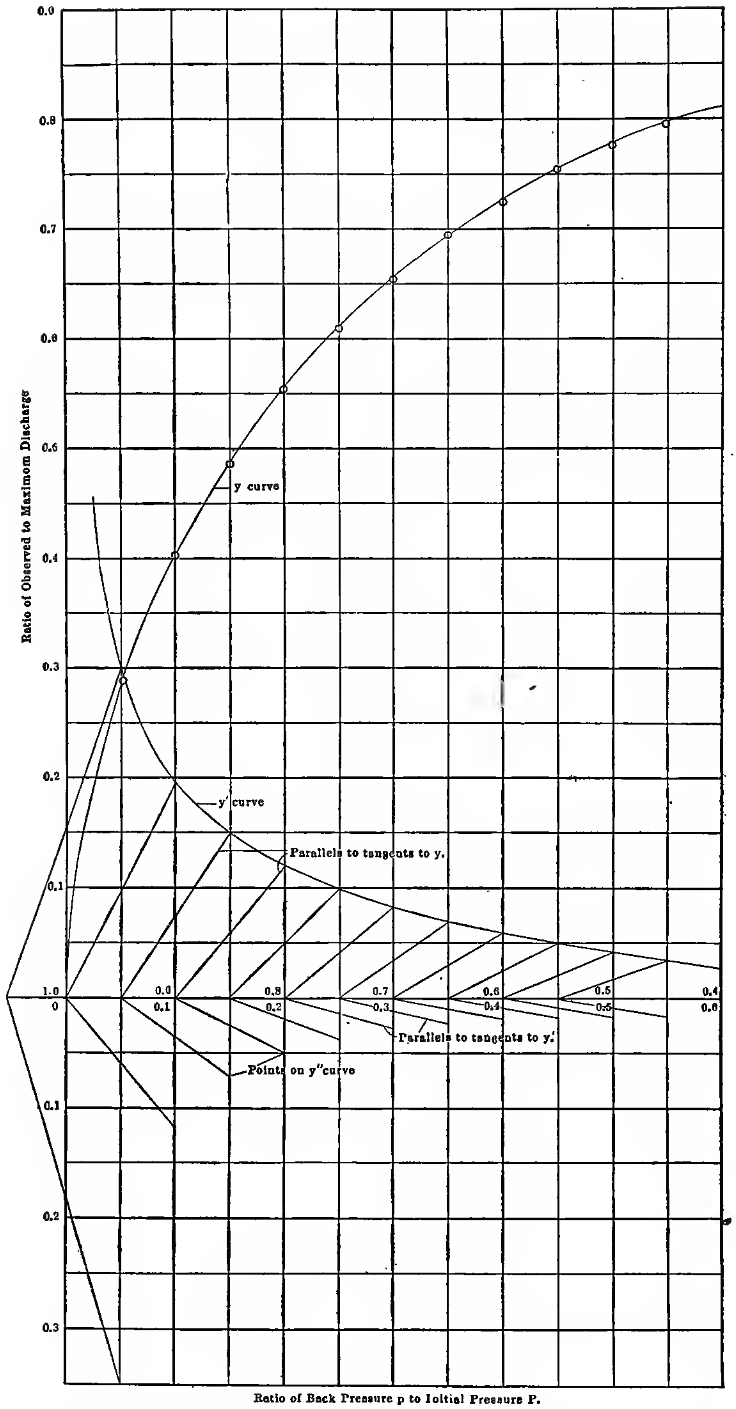

FIG. 48.—Chart showing relation of ratio of back pressure to initial pressure, and ratio of observed to maximum discharge of air through thin plate orifices.

while by determinants we can reach the desired results by a comparatively simple process. All forms of equation will not demand this treatment, and each case must be looked upon as more or less of a special problem.

The three equations may be written in the determinant form as follows:

$$\begin{vmatrix} y & Ax^p & Bx^q \\ y' & Apx^{p-1} & Bqx^{q-1} \\ y'' & Ap(p-1)x^{p-2} & Bq(q-1)x^{q-2} \end{vmatrix} = 0$$

Divide the second column by Ax^p and the third column by Bx^q and we have:

$$\begin{vmatrix} y & 1 & 1 \\ y' & px^{-1} & qx^{-1} \\ y'' & p(p-1)x^{-2} & q(q-1)x^{-2} \end{vmatrix} = 0$$

Then multiply the second row by x and the third by x^2, giving:

$$\begin{vmatrix} y & 1 & 1 \\ xy' & p & q \\ x^2y'' & p(p-1) & q(q-1) \end{vmatrix} = 0$$

Taking the three columns as the coefficients of three equations of the alinement type, we have:

$$y = xy'u + x^2y''v$$
$$1 = pu + p(p-1)v$$
$$1 = qu + q(q-1)v$$

The first equation is affected only by x or its functions y, y', and y'', the second by p only, and the third by q only.

Three supports for an alinement diagram may be constructed from these three equations on the same U- and V-axes, giving us three curves, one for x, one for p, and one for q.

If we join up some point on the p-line with another on q, the connecting line will cut the x-line in what must be looked upon as a corresponding value. But according to the original assumption p and q were constants in the equation and remain so whatever the value of x. If this is true, the desired values of p and q must be so located that the line joining them will cut every value of x on the x-line. This can only be possible by having the support for x a straight line joining these constant values of p and q.

Our next step is to plot the support for x from the equation:

$$y = xy'u + x^2y''v$$

The coördinates for the points on this line will be found from the equations used in the previous example, which will read here

$$X = d\,\frac{x^2y'' - xy'}{x^2y'' + xy'},$$

$$Y = \frac{y}{x^2y'' + xy'}$$

Here x and y are, of course, the coördinates of any points on the observation curve; y' and y'' must, however, be determined from this primary curve. As is well known, the tangent to a curve at any point corresponds to the first derivative. If we get the tangents at a sufficient number of points their values may be plotted into a second curve of which the ordinates are y'. Similarly by drawing tangents to this second curve, we get the values of the quantity y''.

These values of y' and y'' are then to be substituted in the equations for X and Y.

The chief and only difficulty connected with this process is in drawing the tangents to the curves. The "curve of error" is sometimes recommended for this purpose but is, in my opinion, too cumbersome for practical use where any considerable number of points is to be operated on.

My own preference is for taking two ordinates at equal distances on either side of the point at which the tangent is desired and draw the chord of the curve between them. If the curve is flat these side ordinates may be considerably separated, but if not they must be closer. The slope of the chord will be nearly equal to that of the tangent. The greatest care must be exercised in this part of the process, but if this is done the method will yield results of a very satisfactory character.

To get the numerical value of the tangent, or y', a parallel to the chord is drawn from a point on the base line at unit distance to the left of the foot of the ordinate we are operating on and its intersection with the ordinate gives the desired value of y' when measured by the same scale as was used for the ordinates of the original curve.

If this unit distance is inconveniently small or large we may increase or diminish it to a more suitable value but must remember that the reading on the ordinates must then be changed to correspond.

In Fig. 48 the primary curve and its first and second derivatives are shown, the last being represented only by points and the actual curve omitted as unnecessary. It was convenient in laying out these secondary curves to use a base unit on the X-axis equal to 1/10; hence the readings on the y' curve must be multiplied by 10 to get their true value and those of the y'' curve by 100 (since we have used the 1/10 base unit twice).

In the accompanying table are given the quantities necessary for our calculations, the values of y, y' and y'' being read directly from the curves; y'', it will be noted, has the minus sign prefixed throughout, as its points all lie below the X-axis.

x	y	y'	y''	x^2	x^2y''	xy'
0.05	0.283	3.00	−34.5	0.0025	−0.0863	0.150
0.10	0.402	1.94	−11.8	0.0100	−0.1180	0.194
0.15	0.488	1.50	− 7.1	0.0225	−0.1598	0.225
0.20	0.556	1.19	− 5.0	0.0400	−0.2000	0.238
0.25	0.613	1.00	− 3.7	0.0625	−0.2313	0.250
0.30	0.658	0.83	− 2.85	0.0900	−0.2565	0.249
0.35	0.695	0.70	− 2.35	0.1225	−0.2879	0.245
0.40	0.728	0.605	− 2.0	0.1600	−0.3200	0.242
0.45	0.757	0.50	− 1.9	0.2025	−0.3848	0.225
0.50	0.780	0.42	− 1.7	0.2500	−0.4250	0.210
0.55	0.800	0.33	− 1.6	0.3025	−0.4840	0.1815

Now, substituting from the first row in the equations given above for X and Y, we have

$$X=d\frac{-0.0863-0.15}{-0.0863+0.15}=d\frac{-0.2363}{0.0637}=-3.71\ d,$$

$$Y=\frac{0.283}{0.0637}=4.45,$$

and for the remaining values of x,

$x=0.10$	$X=-\ 4.11d$	$Y=\ 5.3$
$x=0.15$	$X=-\ 5.91d$	$Y=\ 7.5$
$x=0.20$	$X=-11.55d$	$Y=14.62$
$x=0.25$	$X=-25.75d$	$Y=32.8$
$x=0.30$	$X=67.5d$	$Y=-87.8$
$x=0.35$	$X=12.45d$	$Y=-16.2$
$x=0.40$	$X=\ 7.21d$	$Y=-\ 9.35$
$x=0.45$	$X=\ 3.81d$	$Y=-\ 4.74$
$x=0.50$	$X=\ 2.96d$	$Y=-\ 3.63$
$x=0.55$	$X=\ 2.19d$	$Y=-\ 2.64$

Slide-rule calculations are usually of sufficient accuracy for this purpose, and after a start is once made they may be run off quite rapidly.

These values must next be plotted on the alinement diagram, as shown in Fig. 49. Since we make no use of the U- and V-axes here, we will omit them and indicate only the X- and Y-axes from which the above quantities are laid off. The half distance d between the U- and V-axes appears in the equation for X, but since they are not shown, its only function will be that of a scale unit, which we may make any size we please. Here it

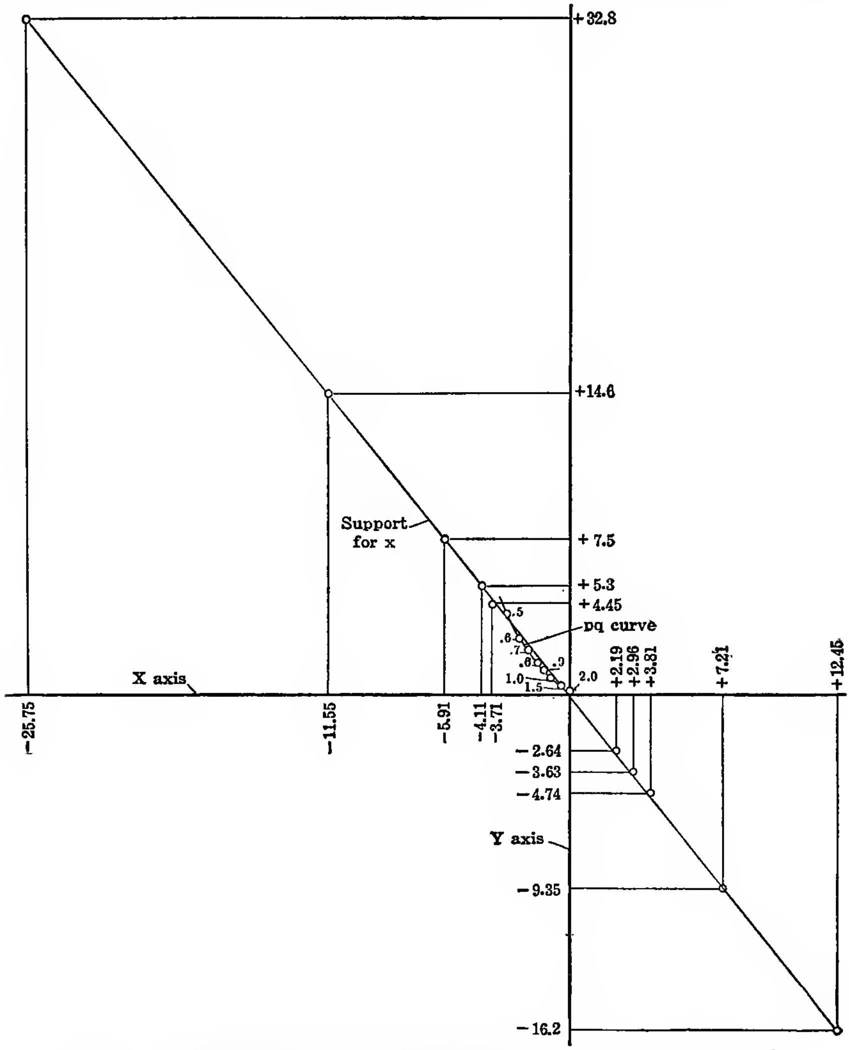

FIG. 49.—Alinement diagram for testing points found in determining an equation for curve of Fig. 48.

was made of such a size that all of the points given above could be plotted within the limits of Fig. 49, except the one corresponding to $x = 0.30$.

We do not need this point, however, as there are enough other points without it to determine the alinement.

An examination of Fig. 49 shows that, although the plotted points do not lie exactly on any straight line, they are in very close agreement with the one shown. Exact agreement is never expected, of course, and there will generally be more divergence than shown here. The alinement of the points indicates that the type of equation chosen is good for the purpose. If it had not been the points would have scattered badly, or would have had a curve as their locus.

Now, we must plot the curves for p and q for such values of these quantities as we suppose to lie near the line just drawn.

The p and q equations for this type of formula are identical, hence they will be represented by only one curve instead of by two, as is the case with other types where p and q are not symmetrically disposed. In order to get a value each for p and q, we must, therefore, have two intersections between the x support and the suppport for p q.

The alinement equation for p as given previously is

$$1 = p\,u + p\,(p - 1)\,v.$$

The coördinates for the various points on the p support will then be

$$X = d\,\frac{p(p-1)-p}{p(p-1)+p} = d\,\frac{p^2-2p}{p^2} = d\left(1-\frac{2}{p}\right),$$

$$Y = \frac{1}{p(p-1)+p} = \frac{1}{p^2}.$$

These equations have been solved for p = 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.5, and 2, and the points thus found plotted in Fig. 49.

A larger scale for the drawing would have made things clearer, but it can be seen that the x-support cuts this curve at two points, one of them exactly at 1.5 and the other at a point between 0.5 and 0.6, which I have called 0.53. It would have required but a slight shift of the x-support to have made the intersections at the points 0.5 and 2. If convenience in use were an important factor these latter values could probably be employed with but little less accuracy. However, we will use the original more exact figures and call p 1.5 and q 0.53. It is immaterial at this stage which quantity is assigned to which letter.

Having found p and q, our equation now reads

$$y = Ax^{1.5} + Bx^{0.53}$$

and our next step is to proceed exactly as we did in the previous example to find A and B, using the equations

$$X = d\,\frac{x^{0.53}-x^{1.5}}{x^{0.53}+x^{1.5}}, \qquad Y = \frac{y}{x^{0.53}+x^{1.5}}.$$

in order to locate the points on the test line.

As there is nothing novel in the process the details will be omitted and reference merely made to Fig. 50, where the points have been plotted and where they are seen with but one exception to lie almost exactly upon a straight line, again indicating the adaptability of the formula to the portion of the curve we have operated on. This line extended to the U- and V-axes is found to cut them at the points -0.59 on U and $+1.425$ on V.

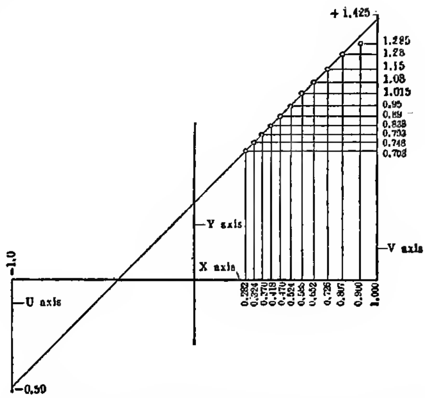

FIG. 50.—Alinement diagram for testing points found in determining an equation for curve of Fig. 48.

Then A is -0.59 and B, 1.425, and our equation reads

$$y = -0.59\, x^{1.5} + 1.425\, x^{0.53}.$$

Reverting now to the first diagram where, as was stated, the numbering on the X-axis was altered, we see that we can get the original numbers by putting

$$x = 1 - \frac{p}{P},$$

p/P being the ratio of the back to the initial pressure and the final equation may then be written

$$y = -0.59\,(1 - p/P)^{1.5} + 1.425\,(1 - p/P)^{0.53}.$$

A series of points has been calculated from this formula and plotted on the chart in Fig. 48, as indicated by the small circles, and the agreement will be seen to be quite good.

The lengthy description which I have given of the Batailler method has, I know, a somewhat formidable sound, but in practical operation "It is," as Bill Nye observed of Wagner's music, "much better than it sounds."

The only operation which presents much difficulty is the graphical differentiation which must be done with great care, or the results will be poor. Otherwise the work is all of a simple character, and may be carried out very expeditiously as compared with some of the other processes. While other types of equation are developed on the same general lines as the one explained, there are differences of detail which have to be looked out for, and which could not even be touched upon here without lengthening this discussion beyond reasonable limits. The equation we have worked on is, perhaps, the one most commonly met with in practice and shows as well as any other the very decided advantage of this process for certain classes of work.

CHAPTER VII.

STEREOGRAPHIC CHARTS AND SOLID MODELS.

THREE DIMENSIONAL CHARTS.

Two dimensional charts for the representation of mathematical equations or experimental data are in very common use nowadays and are everywhere recognized as valuable devices for giving a clear conception of the manner in which the variables are related.

Their application is generally restricted, however, to cases where there is but one variable and its function, if the variation to be shown is continuous. Nevertheless cases often arise in which there are two variables and a function to be represented and where it is desirable to show a continuous variation for all three.

A simple and logical extension of the two-dimensional chart, in which the variation is represented by a plane curve, leads us to the idea of a solid, three-dimensional chart in which the variation is shown by a surface.

It has received some attention at the hands of a number of writers on engineering matters and graphics, but for some reason, probably the labor and expense involved in its construction, its actual use has been rather limited. Where it has been used it has in some instances been fruitful in good results and has thrown much light upon obscure phenomena. In this connection its chief value has probably come from the facility with which we are able to detect maximum and minimum conditions and rates of change among variables whose relationship is complex or unknown. Often we must deal with conditions where no known equations will connect our experimental results and where a mere tabulation of figures will not yield the desired information without much tedious study. The well recognized superiority of any graphical representation over an equation or table in conveying a clear impression to the mind of the way in which a set of variables is related will often in itself be a sufficient justification for the use of this type of chart.

Between the solid model and the plane chart there is a borderland occupied by types which do not truly belong to either and which are really plane projections of solid models. They may be orthographic, isometric, perspective or, generally, axonometric, according to the taste of the maker or the exigencies of the subject.

The orthographic projection here referred to is the topographic map projection in which the relief of the model is indicated by a series of contour lines. Each line passes through a series of points at the same elevation and is numbered to show this elevation. Only a slight effort of the imagination is required to give a very good idea of the undulating surface which they represent. The familiar weather map is a good example of such a chart. Here points of equal barometric pressure are connected by curved lines called isobars. Charts of this description have been much used to record tidal phenomena, magnetic observations, etc., and also in the presentation of vital and financial statistics.

Axonometric projections will usually be found superior to the topographic in bringing out clearly the shape of the surface and are not at all difficult to construct. The special case where the projection is isometric was very fully dealt with by Prof. Guido Marx in the *American Machinist*, Volume 31, Part 2, page 701.

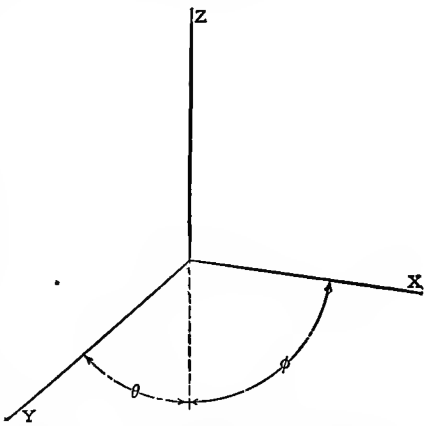

FIG. 51.—Axes and angles for axonometric projection.

Any of the other well-known methods of rectangular axonometry or of perspective may, of course, be applied to these figures. As these methods are generally understood or may be found described in almost any good book on projection or descriptive geometry, no attempt will be made to discuss their principles here.

The accompanying table may, however, be convenient for reference as indicating the proper choice of angles for the axes to conform to the scale units most commonly used.

TABLE OF RATIO OF UNIT LENGTHS ON THE AXES AND ANGLES OF THE AXES FOR AXONOMETRIC PROJECTIONS

Ratio of unit lengths $u_x : u_y : u_z$	*Tan.* ϕ	*Tan.* θ
1 : 1 : 1 Isometric	$\phi = \theta$	= 60°
2 : 1 : 2	8 : 1	8 : 7
3 : 1 : 3	18 : 1	18 : 17
4 : 1 : 4	32 : 1	32 : 31
5 : 4 : 6	5 : 1	3 : 1
9 : 5 : 10	11 : 1	25 : 8

The letters in the table refer to the same symbols in Fig. 51 and the scale values, designated by u, represent the ratio of sizes for a unit length on each of the axes.

The question of scales in the projection of such figures as we are now considering is, however, of relatively little importance since the units used on the different axes have generally no relation which makes any special scale ratio necessary. The angles for the axes, on the other hand, should be so chosen as to agree, approximately at least, with those given in the table, otherwise the figure may have an awkward and unnatural appearance.

Axonometric Charts.

But one simple illustration will be given for this type of chart which will, however, show some interesting and rather unusual features. It is taken from the *Zeitschrift des Vereines Deutscher Ingenieure* for December 27, 1902, and occurs in an article by O. Lasche on the friction of journals with high surface velocities.

Fig. 52 was redrawn from a chart given in this paper with a few unimportant modifications to render it better adapted to purposes of illustration. The chart was constructed from data obtained from experiments on a nickel-steel journal running in a white-metal bearing and is intended to show the relation between the temperature of the bearing in degrees Centigrade, the surface velocity in meters per second and the heat generated per square centimeter of effective projected area, expressed in heat units, and also in meter-kilograms per second.

The experiments were made at a specific pressure of 6.5 kilograms per square centimeter, but since, with the lubrication used, the product of the specific pressure and the coefficient of friction was sensibly constant over a considerable range, the results are said to be applicable to any specific pressure from 3 to 15 kilograms per square centimeter.

In laying out a chart of this description the three coördinate axes and their planes are first drawn and the former properly graduated between the limits set by the experiments. From the graduations on the ground plane axes perpendiculars, lying in the ground plane, are drawn, thus giving a checkered surface on which points may be located as is done with ordinary section paper. At the points thus found perpendiculars are erected to the ground plane, their height being so taken as to represent the value of the third variable. The tops of these lines are now connected by suitable curves, which must lie in the surface we are seeking and which are assumed to represent it.

In the chart under discussion five different curves were drawn parallel to the temperature-heat plane, and then, to bind them together and render the shape of the surface more apparent, three more curves were drawn at right angles to the first and parallel to the velocity-heat plane. Taking one of these latter curves, that corresponding to 50 degrees, we see that at this constant temperature the heat generated by friction increases with increasing velocity, not exactly in direct ratio with it, however, since

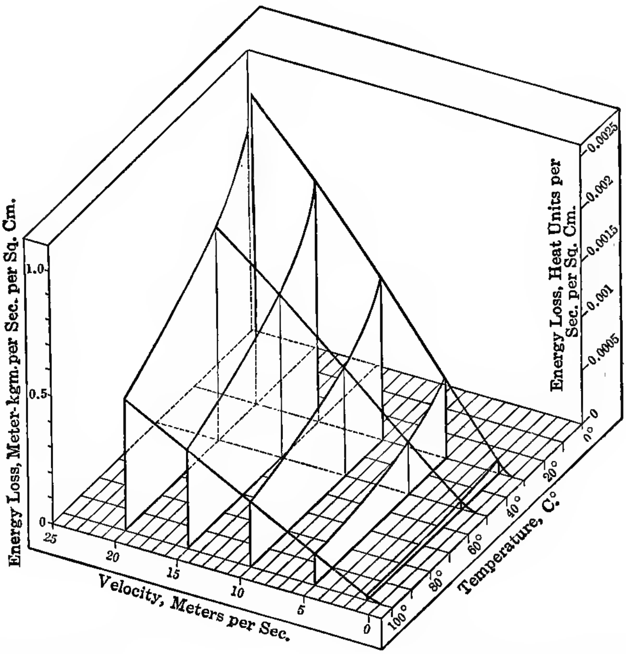

FIG. 52.—Axonometric chart showing relation between journal bearing temperature, surface velocity and heat generated.

the coefficient of friction does not remain quite constant as the velocity changes. Keeping the velocity constant and varying the temperature, we see that the heat generated by friction decreases as the temperature rises, rapidly at first and then more slowly.

It is evident from the chart that with the journal in question the heat produced by friction will be greatest when it is starting up and the temperature is low. Also that at this temperature the radiation to the surrounding atmosphere will be small on account of the small temperature difference. The heat produced therefore goes to warm the bearing, but as its temperature rises the heat generated becomes less and the radiation greater until we reach a point where the radiation just balances the heat production and the temperature remains stationary.

The question naturally arises as to whether it is possible to tell where this point will be. If the necessary experimental data are at hand it may be done on the chart. Suppose we have this data and from it construct a second chart on the same heat, temperature and velocity axes as before. See Fig. 53. It shows the capacity for heat radiation per square centimeter of effective projected area for the bearing we are considering and is

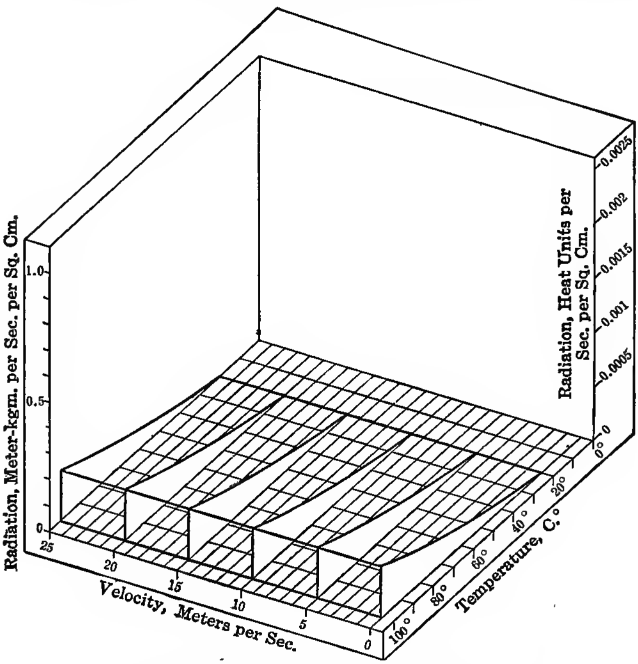

FIG. 53.—Axonometric chart showing capacity for heat radiation per unit of effective projected area of journal bearing.

constructed for a room temperature of 20 degrees Centigrade. When the bearing has this temperature its radiation is, of course, zero. The radiation is independent of the velocity of the journal and this is indicated by the fact that the surface is a ruled one composed of straight lines parallel to the velocity axis. Increasing bearing temperature means, of course, increasing radiation.

Next suppose these two charts to be combined as in Fig. 54. It is apparent that the two surfaces will intersect along some line as *h c j*, the location of which is easily found by the rules governing this form of projection.

Any point on this line will correspond to some temperature and velocity at which the radiation just equals the heat production, the necessary condition for constant temperature. This line projected to the ground plane gives us the line *i d j*. Any point in the ground plane, projected from the temperature and velocity axes, which falls in front of the line will indicate that under these conditions the radiation is greater than the

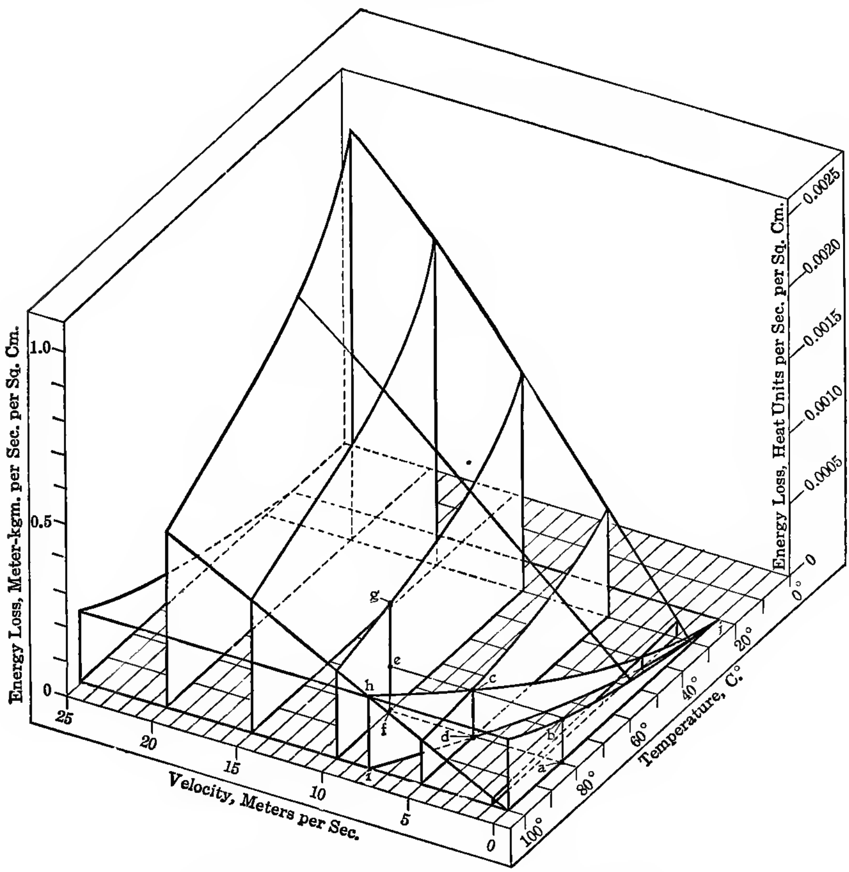

FIG. 54.—Chart combining charts of Figs. 52 and 53.

heat generation or that natural cooling will be effective to keep the bearing below the chosen maximum temperature. Points which fall beyond this line correspond to conditions where artificial cooling must be resorted to.

Suppose, for instance, we take some point, such as *c* on the line *h c j*. Project it to the ground and we find that it falls at the intersection of ordinates from 80 degrees on the temperature axis and 5 meters per second on the velocity axis. Under these conditions the temperature will be steady.

Next suppose that we arbitrarily fix the upper limit for temperature at 80 degrees, and that we have a velocity of 10-meters per second.

Entering the radiation diagram on the 80-degree line at *a* we run up till we reach its surface at *b*. Then, following the surface along the line *bc*, we find its intersection with the 10-meter plane of the friction diagram at *e*. Through this point a perpendicular is drawn to the ground. The length *fg* on this perpendicular measures the heat generated by friction, *ef* is the amount carried off by radiation, while *eg* represents the remainder which must be artificially removed by circulating a current of water, oil, or air around the bearing.

The writer of the article from which I take this illustration goes on to show how, after measuring *eg* in heat units, a very simple calculation will give the amount of cooling fluid.

It will be apparent from the foregoing description that the axonometric projection has some advantages over its solid prototype from the facility with which we can project *through* the figure in case of need. Special attention should also be directed to the use which has been made of the line of intersection of the two surfaces. It is a rather novel feature and one which should prove valuable in many engineering problems.

The Solid Model.

Next let us consider the true solid model. It has received attention at the hands of several eminent writers, among them the late R. H. Thurston. He published a number of articles explaining its uses and advantages, among which articles may be cited one on glyptic models in the *Transactions*, American Society of Mechanical Engineers, for 1898. He appears to have been much impressed by the possibilities it offered for the solution of a certain class of problems and he illustrates its application by a number of examples.

In spite of his optimistic views as to its value, the solid model has never seemed to "take" well; at least there are relatively few recorded instances of its use. This may be partly due, as was observed before, to the labor involved in its construction, but possibly, also, to a lack of sufficient exploitation.

These models may be made in various ways. Wood is a suitable material where the surface to be produced is sufficiently regular, but this is not often the case. Ruled surfaces may be produced by strings stretched on suitable frames, but the material most generally used is plaster of Paris. After the first model is made, replicas may, of course, be cast in any suitable metal or material. Cardboard, as will be shown later, is a cheap and convenient substitute for some of the above-named materials.

In making the plaster-of-Paris model, we first stretch a sheet of section paper on a board and lay off on it the points corresponding to two of the variables in the usual manner. At these points we next insert vertical

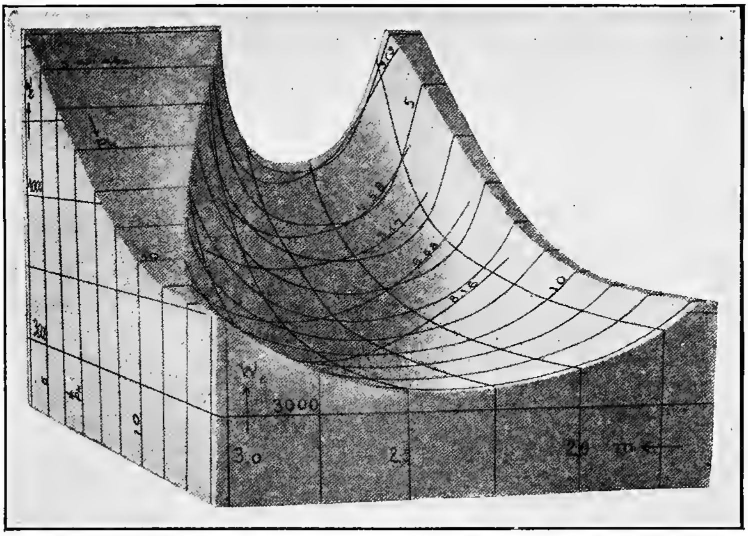

FIG. 55.—Solid model showing relation between heat units per hour per brake horsepower, compression pressure and volume of gas mixture for a gas engine.

wires which are cut off at heights corresponding to the third variable. A box is then formed around the whole and wet plaster of Paris is poured into it until all the wires are covered. After it has set, the upper surface is carefully cut and smoothed away until the tops of the wires are exposed and the resulting surface is taken as the graphical representation of the law of connecting the variables.

A comparatively recent example of such a model is found in an article on the mixture ratio for gas engines in the *Zeitschrift des Vereines Deutscher Ingenieure* for September 14, 1907.

This model is represented in Fig. 55. It is based on data obtained from a gas engine running at four horsepower on producer gas, and is intended to show the relation between the heat units per hour per brake horsepower, the compression pressure in atmospheres and the volume of mixed gas and air per 1000 heat units of lower heating value.

The front horizontal axis is graduated to represent the cubic meters of mixture per 1000 heat units and extends from about 1.7 cubic meters to 3. Perpendicular to this axis, and also horizontal, is the axis for compression pressures graduated from back to front between 4 and 13 atmospheres.

The vertical axis is used for the heat units required per brake horsepower-hour, the graduations beginning at 2500 at the ground plane and running up to 5000.

The hollowed surface in the middle of the model covers the range within which the experiments were conducted, and the cut-off portions at the sides of the hollow have no meaning.

Without in any way attempting to discuss the conditions under which the experimental results were obtained, we will take the model as it stands, and see what conclusions may be reached from a simple inspection of it.

The bottom of the valley indicates the lowest heat consumption per horsepower-hour in any given locality.

The intersection of the valley with the back vertical plane is a curve somewhat resembling a parabola with steeply rising sides. As we come toward the front the curves cut by parallel vertical planes flatten out and the vertex of the curve becomes lower, indicating a smaller heat consumption as the compression increases. At the back of the model the lowest part of the curve is tangent to a horizontal line at about 4100, while in front it touches 3100.

It will also be noted that the slope of the bottom of the valley is steepest in the rear and is nearly horizontal in front, indicating a more rapid gain in heat economy from increased compression when the original compression is low than when high.

The bottom of the valley shows a tendency to drift to the left as we come forward, indicating that with increased compression the best economy was obtained by increasing the dilution of the mixture. The flattening out of the front part of the valley indicates that as compression increases the necessity for an exact mixture ratio for good economy becomes progressively less important.

These points are all interesting, and while they might have been discovered from an inspection of a series of curves or of the tabulated data, it is clear that the model has greatly simplified the process of deduction, and has thus justified its construction.

CARDBOARD SUBSTITUTE FOR THE SOLID MODEL.

Reference has been made above to the cardboard model as a cheap substitute for the solid type. The next illustration will be an example showing its construction.

If we assume one of three variables to have different constant values, we get a series of plane curves connecting the other two. Then, by doing the same with one of the other variables, we get a second series of curves

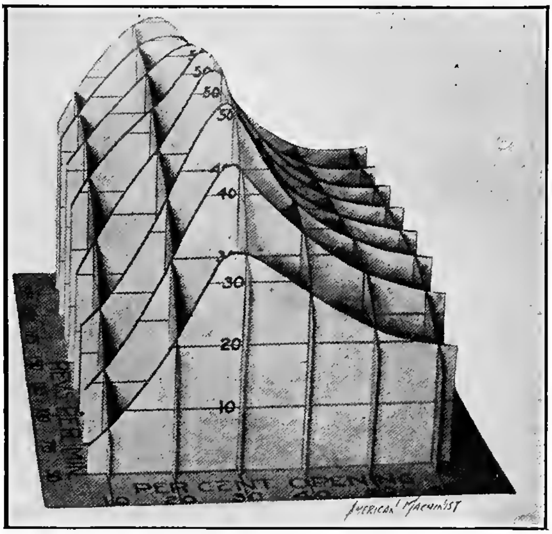

FIG. 56.—Cardboard substitute for a solid model.

for planes at right angles to the first. Each of these curves is cut from a piece of cardboard and slit half way up or down the lines of intersection with the cards at right angles to it. They are then fitted together something on the principle of an egg box, and the result will be a series of plane curves, properly spaced with reference to each other, all of which lie in the surface we are trying to represent. It is evidently closely related to the axonometric projection previously described.

Such a model is shown in Fig. 56. It was constructed from the curves given in a paper in the *Transactions*, American Society of Mechanical Engineers, for 1904, by E. S. Farwell, entitled "Tests of a Direct Connected Eight Foot Fan and Engine." These curves were chosen chiefly on account of the irregular hilly character of the surface to which they belong as affording a good test of the method. They occur in Fig. 39, of the article referred to, and are supposed to show the relation between the efficiency of the fan and the area of the outlet opening for various speeds, the area of opening being designated as a percentage of the product of the fan diameter by the width of periphery.

Eight different curves are given for eight different speeds, which advance by steps of 25 from 50 revolutions per minute to 225 revolutions per minute. The curves shown in the figure had the efficiencies plotted as ordinates, and the outlet-opening percentages as abscissas, and were used as they stood for one set of cards. Then taking the intersection of these curves with one of the perpendiculars to the outlet axis we get a series of lengths which we use as equally spaced ordinates for a curve at right angles to the plane of the original drawing, the ordinates again representing efficiencies while the abscissas this time are velocities. As many similar curves as were deemed necessary were taken from the other perpendiculars. All were cut out and fitted together, forming the model shown in the photograph. Such a model may be applied to many, if not most, of the purposes for which the solid type is used and has a decided advantage in simplicity of construction.

The Tri-axial Model.

Before leaving this subject a brief reference must be made to an ingenious form of solid chart described by Professor Thurston in several of his articles. It is called the tri-axial model. By its use it is possible to take into account four different variables instead of three as was previously the case. It is a necessary condition, however, that for each set of corresponding variables three of them should add up to a constant value, generally 100 per cent. The fourth is unrestricted. These models have been very useful in representing the properties of ternary alloys, furnace slags, etc. If we have an equilateral triangle as shown in Fig. 57, and from any point, *O*, within it we drop perpendiculars to the three sides, geometry tells us that the sum of these perpendiculars is constant wherever the point may be located.

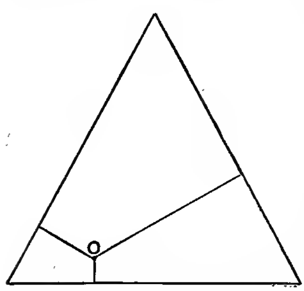

Fig. 57.—Diagram illustrating principle of tri-axial solid model.

Therefore, if we wish to study the alloys composed of, say, copper, tin, and zinc, with reference to any property such as strength, ductility, hardness, or melting point, a large number of experiments are made with specimens of varying composition and the value of the quality we are studying tabulated with the composition of the alloy.

This composition is expressed for each constituent as a percentage, and the three percentages necessarily add up to 100.

Laying out a triangle whose altitude to some scale is 100, we designate one side as copper, another as tin, and the third as zinc. Parallels are then drawn to the sides, distant from them by amounts corresponding to the percentage of each metal in the specimen. The scale in which these distances are measured is the same as that which was used in laying off the altitude of the triangle. These three parallels must meet in a point which is taken to represent the alloy in question. Perpendicular to the ground plane at this point we insert a wire whose length represents the value of the quality we are studying. When all the wires are fixed the whole is covered with plaster of Paris, as explained before, which is then pared down to the tops of the wires.

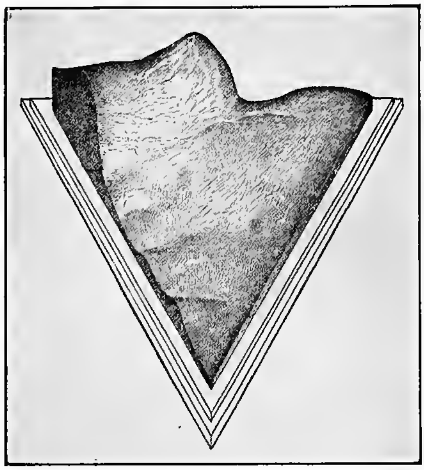

FIG. 58.—Professor Thurston's solid tri-axial model for copper alloys.

The resulting model is shown in Fig. 58, and from it Thurston found that the strongest alloy had a composition of Cu = 55 per cent., Zn = 43 per cent., and Sn = 2 per cent.

Models of this description are evidently of especial value in the study of metallurgical problems and are by no means uncommon, particularly in that field of work.

Often, however, instead of the solid model, a topographical chart of it with the necessary contour lines is plotted, which answers many purposes almost equally well and commends itself for use in a great many cases.