数学教科書
英文

筆算手引 一

竹窓の筆写せし表紙

法朗西文典 児童手引 一

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321 per cent

examples.

1. a merchant wishes to remit $4888.40 from new york to london, and the exchange is at a premium of 10 per cent: he finds that he can remit to paris at 5 francs 15 centimes to the dollar, and to hamburg at 35 cents per marc banco. now, the exchange between paris and london is 25 francs 80 centimes for £1 sterling, and between hamburg and london $13\frac{3}{4}$ marcs banco for £1 sterling: how had he better remit?

operation.

1st. to london direct.

$$\$4888.40 \times \frac{1}{4.88\frac{8}{9}} = £1000.$$

2d. through paris.

$$\$4888.40 \times \frac{\overset{1.03}{\cancel{5.15}}}{1} \times \frac{1}{\underset{5.16}{\cancel{25.80}}} = £975.7852 = £975\ 15s.\ 8\tfrac{1}{4}d.$$

3d. through hamburg.

$4888.40 × $\frac{1}{35}$ × $\frac{1}{13.75}$ = £1015.771 = £1015 15s. 5d.

hence, the best way to remit is through hamburg, then direct: and the least advantageous, through paris.

2. A merchant in new york wishes to transmit $1500 to vienna, through london and hamburg: what will be the value when received, if £1 = $4.86, £1 = 14 marcs banco, and 6 marcs banco = 8 florins?

3. A merchant at natchez wishes to pay $10000 in boston by transmitting through new orleans and new york. from natchez to new orleans exchange is $\frac{1}{2}$% premium, from new orleans to new york $\frac{5}{8}$% discount, and from new york to boston $\frac{1}{4}$% discount: by this exchange, what amount at natchez will pay the debt?

4. A, of london, draws a bill of £862 10s. on B, of cadiz, and remits the same to C, of havre, who in turn, remits to D, of amsterdam, and D remits to B, of cadiz: how much will pay the bill,

if 1 spanish dollar = 2 florins 15 stivers, 12 florins = 26 francs, and 24 f. 15 c. = £1?

involution.

371. a power of a number is any product which arises from multiplying the number continually by itself.

the root, or simple factor, is called the first power:

the second power is the product of the root by itself:

the third power is the product, when the root is taken 3 times as a factor:

the fourth power is the product, when it is taken 4 times:

the fifth power is the product, when it is taken 5 times.

372. the number denoting how many times the root is taken as a factor, is called the exponent of the power. it is written a little at the right and over the root: thus, if the equal factor or root is 3,

$3^1 = 3$, the 1st power, root, or base.

$3^2 = 3 \times 3 = 9$, the 2d power of 3.

$3^3 = 3 \times 3 \times 3 = 27$, the 3d power of 3.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the fourth power of three.

373. Involution is the operation of finding the powers of numbers.

Note.—1. There are three things connected with every power: 1st, the root; 2d, the exponent; and 3d, the power or result of the multiplication.

2. In finding any power, one multiplication gives the 2d power: hence, the number of multiplications is 1 less than the exponent.

Rule.—Multiply the number into itself as many times less 1 as there are units in the exponent, and the last product will be the power.

Examples.

Find the powers of the following numbers:

1. the square of 4?
2. the square of 15?
3. the square of 142?
4. the square of 463?
5. " " " 1340?
6. " " " 24.6?
7. " " " .526?
8. " " " 3.125?
9. " " " .0524?
10. " " " $\frac{3}{4}$?
11. " " " $\frac{6}{7}$?
12. " " " $\frac{7}{9}$?
13. " " " $\frac{35}{84}$?
14. " " " $\frac{125}{247}$?
15. " " " $7\frac{5}{8}$?
16. " " " $15\frac{9}{11}$?
17. " " " $225\frac{9}{10}$?
18. the cube of 6?
19. the cube of 24?
20. the cube of 125?
21. the cube of 136?
22. " 4th power of 12?
23. " 5th " " 9?
24. " value of $(4.25)^3$?
25. " " " $(1.8)^4$?
25. " " " $(.45)^5$?
27. " " " $(\frac{15}{16})^3$?
28. " cube of $(\frac{5}{6})$?
29. " 4th power of $\frac{3}{8}$?
30. " value of $(2\frac{1}{4})^5$?
31. " " " $(\frac{25}{27})^4$?
32. " " " $(24\frac{3}{5})^3$?
33. " " " $(.25)^6$?
34. " " " $(142.5)^3$?

Evolution.

374. Evolution is the operation of finding the root of a number; that is, of finding one of its equal factors.

375. The square root of a number is the factor which, multiplied by itself once, will produce the number.

Thus, 8 is the square root of 64, because $8 \times 8 = 64$.

The sign $\sqrt{}$ is called the radical sign. When placed before a number, it denotes that its square root is to be extracted:

Thus, $\sqrt{36}=6$.

376. The cube root of a number is the factor which, mul-
tiplied by itself twice, will produce the number.

Thus 3 is the cube root of 27, because $3\times3\times3=27$.

We denote the cube root by the sign $\sqrt{\ }$, with 3 written over it: thus $\sqrt[3]{27}$ denotes the cube root of 27, which is equal to 3. The small figure 3, placed over the radical, is called the index of the root.

The terms power and root, are dependent on each other: thus, the power is the product of equal factors; and the root is one of the equal factors.

Extraction of the square root.

377. The square root of a number is one of its two equal factors. To extract the square root is to find this factor.

The first ten numbers and their squares are:

1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

The numbers in the first line are the square roots of those in the second. The numbers 1, 4, 9, 16, 25, 36, &c., having two exact equal factors, are called perfect squares.

A perfect square is a number which has two exact equal factors.

Note.—The square root of a number less than 100 will be less than 10; while the square root of a number greater than 100 will be greater than 10: hence, the square root of a number expressed by one or two figures, is a number expressed by one figure.

378. To find the law of the square of a number.

Any number expressed by two or more figures may be regarded as composed of tens and units.

1. What is the square of 36 = 3 tens + 6 units?

Analysis.—The square of 36 is found by taking 36, thirty-six times. This is done

Operation.

by first taking it 6 units times, and then 3 tens times, and adding the prod- ucts. 36 taken 6 unit times, gives $6^2+3\times3$; and taken 3 tens times, gives $3\times6+3^2$; and their sum is, $3^2+2(3\times6)+6^2$: that is,

$$\begin{array}{r} 3+6 \\ 3+6 \\ \hline 3\times6+6^2 \\ 3^2+3\times6 \quad \\ \hline 3^2+2(3\times6)+6^2 \end{array}$$

Rule.—The square of a number is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

377. To find the square root of any number.

1. Let it now be required to extract the square root of 2025.

Analysis.—Since the number contains more than two pl- aces of figures, its root will contain tens and units. But as the square of one ten is one hundred, it follows that the square of the tens of the required root must be found in the figures on the left of 25. Hence, beginning at the right, we point off the number into periods of two figures each.

We then find the root contained in 20 hun- dreds, which is 4 tens or 40. We then square 4 tens, which gives 16 hundred, and then pla- ce 16 under the first period, and subtract; this takes aw- ay the square of the tens, and leaves 425, which is twice the product of the tens by the units plus the square of the units.

Operation

$$\begin{array}{r|l} \dot{20}\ \dot{25} & 45 \\ 16 \quad & \\ \hline 85)\ 4\ 2\ 5 & \\ 4\ 2\ 5 & \end{array}$$

If, now, we double the tens, and then divide the remain- der, exclusive of the right-hand figure (since that figu- re cannot enter into the product of the product of the tens by the units), by it, the quotient will be the units figu- re of the root. If we annex this figure to the root and to the augmented divisor, and then multiply the whole di- visor thus increased by it, the product will be twice the tens by the units, plus the square of the units: and hence, we have found both figures of the root.

Rule.—I. Separate the given number into periods of two figures each, by writing a dot over the place of units, a second over the place of hundreds, and so on for each alternate figure to the left:

II. Note the greatest square contained in the period on the left, and place its root on the right, after the manner of a quotient in division. Subtract the square of this root from the first period, and to the remainder bring down the second period for a dividend:

III. Double the root thus found for a trial divisor, and place it on the left of the dividend. Find how many times the trial divisor is contained in the dividend, exclusive of its right-hand figure, and place the quotient in the root, and also annex it to the divisor:

IV. Multiply the divisor thus increased, by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend:

V. Double the whole root thus found, for a new trial divisor, and continue the operation as before, until all the periods are brought down.

Examples.

1. What is the square root of 425104?

Analysis.—We first place a dot over the 4, making the right-hand period 04. We then put a dot over the 1, and also over the 2, making three periods.

Operation.

```
      42 51 04 (652
      36
125)  6 51
      6 25
1302)   26 04
        26 04
```

The greatest perfect square in 42 is 36, the root of which is 6. Placing 6 in the root, subtracting its square from 42, and bringing down the next period 51, we have 651 for a dividend; and by doubling the root, we have 12 for a trial divisor. now, 12 is contained in 65, 5 times. place 5 both in the root and in the divisor; then multiply 125 by 5; subtract the product, and bring down the next period.

We must now double the whole root 65 for a new trial divisor; or we may take the first divisor, after having doubled the last figure 5; then dividing, we obtain 2, the third figure of the root.

notes.—1. the left-hand period may contain but one figure, each of the others will contain two.

2. if any trial divisor is greater than its dividend, the corresponding root figure will be a cipher.

3. if the product of the divisor by any figure of the root exceeds the corresponding dividend, the root figure is too large, and must be diminished.

4. there will be as many figures in the root as there are periods in the given number.

4. there will be as many figures in the root as there are periods in the given number.

5. if the given number is not a perfect square, there will be a remainder after all the periods are brought down. in this case, periods of ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

2. What is the square root of 758692?

Analysis.—after using all the periods of the given number, we annex periods of decimal ciphers, each of which gives one decimal place in the quotient.

operation.

```
          75 86 92(871.029+.
          64
     167)11 86
         11 69
    1741)17 92
         17 41
  174202)510000
         348404
 1742049)16159600
         15678441
           481159 rem.
```

What are the square roots of the following numbers:

3. square root of 49? | 9. $\sqrt{19000}$ = What no.?

4. Square root of 144?
5. Square root of 225?
6. Square root of 2304?
7. Square root of 7994?
8. Square root of 6275025?

10. $\sqrt{19000}$ = What no.?
11. $\sqrt{35754}$ = What no.?
12. $\sqrt{1000000}$ = What no.?
13. $\sqrt{96728}$ = What no.?
14. $\sqrt{30225}$ = What no.?

388. To extract the square root of a fraction.

1. What is the square root of .6?

Analysis.—We first annex one cipher, to make even decimal places; for, one decimal multiplied by itself will give two places in the product. We then extract the root of the first period, and to the remainder annex a decimal period; and so on, till we have found a sufficient number of decimal places.

Operation.

$$
\begin{array}{r|l}
 & .60(.774+ \\
 & 49 \\
147) & 1100 \\
 & 1029 \\
1544) & 7100 \\
 & 6176 \\
 & 924 \text{ rem.}
\end{array}
$$

2. What is the square root of $\frac{16}{25}$?

Analysis.—The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

Operation.

$$\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}.$$

3. What is the square root of $\frac{3}{4}$?

Analysis.—When the terms are not perfect squares, reduce the common fraction to a decimal, and then extract the square root of the decimal.

Operation.

$$\frac{3}{4} = .75;$$

$$\sqrt{\frac{3}{4}} = \sqrt{.75} = .8545+$$

Rule.—I. If the fraction is a decimal, point off the periods from the decimal point to the right, annexing ciphers if necessary, so that each period shall contain two places, and then extract the root as in integral numbers;

II. If fraction is a common fraction, and its terms are perfect squares, extract the square root of the numerator and denominator separately;

III. If, after being reduced to their lowest terms, the numerator and denominator are not perfect squares, reduce the fraction to a decimal, and then extract the square root of the result.

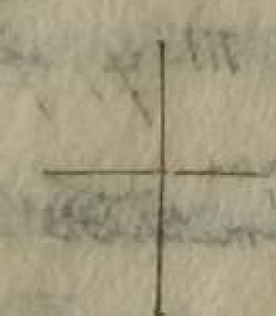
examples

4. square root of $\frac{36}{81}$?
5. square root of $\frac{225}{2304}$?
6. square root of .0196?
7. square root of 6.25?
8. square root of 278.89?
9. square root of .205209?
10. square root of $\frac{7}{8}$?
11. square root of $\frac{15}{16}$?
12. square root of $\frac{1}{40}$?
13. square root of $5\frac{4}{9}$?
14. square root of .7994?
15. value of $\sqrt{222\frac{2}{9}}$?
16. square root of .60794?
17. value of $\sqrt{.022201}$?
18. value of $\sqrt{25.1001}$?
19. value of $\sqrt{196.425}$?
20. value of $\sqrt{1.5}$?
21. value of $\sqrt{\frac{2809}{8241}}$?
22. value of $\frac{\sqrt{9}}{\sqrt{49}}$?
23. value of $\sqrt{\frac{2}{25}}$?
24. value of $\sqrt{135}$?
25. value of $\sqrt{.784}$?
26. square root of 5647.5225?
27. square root of 160048.0036?

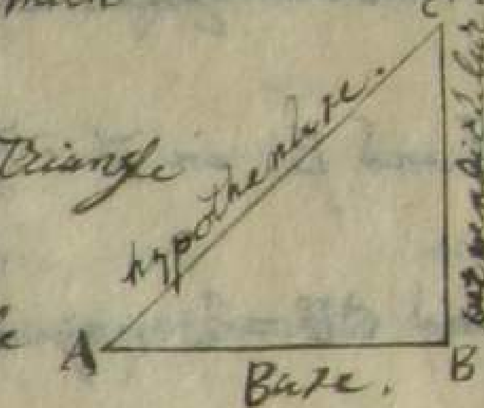
applications in square root.

381. A triangle is a plain figure which has three sides and three angles.

If a straight line meets another straight line, making the adjacent angles equal, each is called a right angle; and the lines are said to be perpendicular to each other.



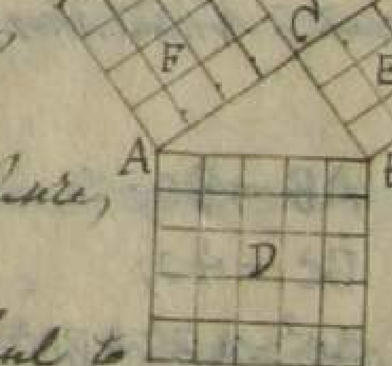
382. A right-angled triangle is one which has one right angle. In the right-angled triangle ABC, the side AC, opposite the right angle B, is called the hypothenuse; the side AB, the base; and the side BC, the perpendicular.



383. A square is a figure bounded by four equal sides, at right angles to each other.

384. In a right-angled triangle the square described on the hypothenuse is equal to the sum of the squares described on the other two sides.

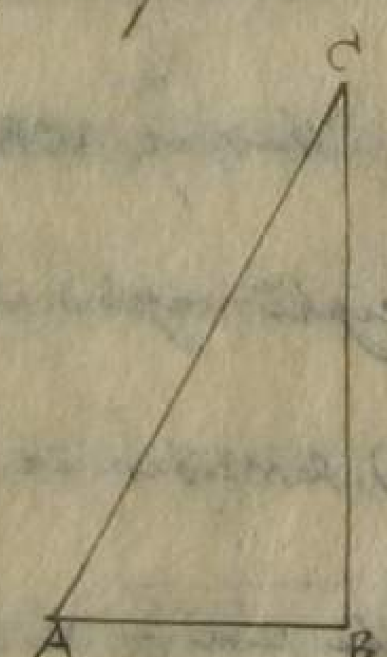
Thus, if ACB be a right-angled triangle, right-angled at C, then will the large square, D, described in the hypothenuse AB, be equal to the sum of the squares F and E, described on the sides AC and CB. This is called the carpenter's theorem. By counting the small squares in the large square D, you will find their



number equal to that contained in the small squares F and E. in this triangle the hypothenuse AB=5, AC=4, and CB = 3. any numbers having the same ratio, as 5, 4, and 3, such as 10, 8, and 6; 20, 16, and 12, &c, will represent the sides of a right-angled triangle.

385. when the base and perpendicular are known, to find the hypothenuse.

Analysis.—wishing to know the distance from a to the top of a tower, I measured the height of the tower, and found it to be 40 feet; also the distance from A to B, and found it 30 feet: what was the distance from A to C?



$AB = 30;\qquad AB^2 = 30^2 = 900$

$BC = 40;\qquad BC^2 = 40^2 = 1600$

$AC^2 = AB^2 + BC^2 = 900 + 1600$

$AC = \sqrt{2500} = 50$ feet.

rule.—square the base and square the perpendicular, add the results, and then extract the square root of their sum.

386. to find one side, when we know the hypothenuse and the other side.

1. the length of a ladder which will reach from the middle of a street 80 feet wide to the eaves of a house, is 50 feet: what is the height of the house?

Analysis.—since the square of the length of the ladder is equal to the sum of the squares of half the width of the street and the height of the house, the square of the length of the ladder diminished by the square of half the width of the street, will be equal to the square of the height of the house: hence,

rule.—square the hypothenuse and the known side and take the difference; the square root of the difference will be the other side.

examples.

1. A general having an army of 117649 men, wished to fo,

to lay out in

em them into a square: how many should he place on each front?

2. in a square piece of pavement there are 48841 stones, of equal size, one foot square: what is the length of one side of the pavement?

3. in the center of a square garden, there is an elliptical circular pond, covering an area of 810 square feet, which is $\frac{1}{10}$ of the whole garden: how many rods of fence will inclose the garden?

4. let it be required to lay out 67A. 2R. of land in the form of a rectangle, the longer side of which is to be three times as great as the less: what is its length and width?

5. A farmer wishes to set out an orchard of 3200 dwarf pear-trees. he has a field twice as long as it is wide, which he appropriates to this purpose. he sets the trees 12 feet apart, and in rows that are likewise 12 feet apart:

line 線幹

how many rows will there be, how many trees in a row, and how much land will they occupy?

6. there is a wall 45 feet high, built upon the bank of a stream 60 feet wide: how long must a ladder be that will reach from the one side of the stream to the top of the wall on the other?

7. a boy having lodged his kite in the top of a tree, finds that by letting out the whole length of his line, which he knows to be 225 feet, it will reach the ground 180 feet from the foot of the tree: what is the height of the tree?

8. there are two buildings standing on opposite sides of the street, one 39 feet, and the other 49 feet from the ground to the eaves. the foot of a ladder 65 feet long rests upon the ground at a point between them, from which it will touch the eaves of either building: what is the width of the street?

9. A tree 120 feet high was broken off in a storm, the top striking 40 feet from the root, and the broken end resting upon the stump: allowing the ground to be a horizontal plane, what was the height of the part standing?

10. What will be the distance from corner to corner, through the center of a cube, whose dimensions are 5 feet on a side?

11. Two vessels start from the same point, one sails due north at the rate of 10 miles an hour, the other due west at the rate of 14 miles an hour: how far apart will they be at the end of 2 days, supposing the surface of the earth to be a plane?

12. How much more will it cost to fence 10 acres of land, in the form of a rectangle, the length of which is four times its breadth, than if it were in the form of a square, the cost of the fence being $2.50 a rod?

13. What is the diameter of a cylindrical reservoir containing 9 times as much water as one 25 feet in diameter, the height being the same?

Note.—If two volumes have the same altitude, their contents will be to each other in the same proportion as their bases; and if the bases are similar figures (that is, of like form), they will be to each other as the squares of their diameters, or other like dimensions.

14. If cylindrical cistern eight feet in diameter will hold 120 barrels, what must be the diameter of a cistern of the same depth to hold 1500 barrels?

15. If a pipe 3 inches in diameter will discharge 400 gallons in 3 minutes, what must be the diameter of a pipe that will discharge 1600 gallons in the same time?

16. What length of rope must be attached to a halter 4 feet

long, that a horse may feed over 2½ acres of ground?

17. three men bought a grindstone, which was 4 feet in di, ameter: how much of the radius must each grind off to use up his share of the stone?

Cube root.

327. the cube root of a number is one of its three equ, al factors.

thus, 2 is the cube root of 8; for, $2 \times 2 \times 2 = 8$: and 3 is the cube root of 27; for, $3 \times 3 \times 3 = 27$.

to extract the cube root of a number, is to find one of its three equal factors.

1,	2,	3,	4,	5,	6,	7,	8,	9,
1	8	27	64	125	216	343	512	729

the numbers in the first line are the cube root of the corresponding numbers of the second. the numbers of the second line are called <u>perfect</u> <u>cubes</u>.

a perfect cube is a number which has three exact equal factors. by examining the numbers in the two li, nes, we see,

1st. that the <u>cube</u> of <u>units</u> <u>cannot</u> <u>give a higher</u> <u>order</u> than <u>hundreds</u>:

2d. that since the cube of one ten (10) is 1000, and the cube of 9 tens (90), 729,000, <u>the cube of tens will not give a lower denomination than thousands, nor a higher</u> than ~~denomination~~ <u>than hundreds of thousands</u>.

hence, if a number contains more than three figures, its cube root will contain more than one; if it contains more than six, its root will contain more than two, and so on; every additional three figures giving one additional figure in the root, and the figures which remain at the le, ft hand, although less than three, will also give a figure in the root. this last explains the reason for pointing off into periods of three figures each.

388. Let us see how the cube of any number, as 16, is formed. Sixteen is composed of 1 ten and 6 units, and may be written, 10+6. To find the cube of 16 = 10 + 6, we must multiply the number by itself twice.

To do this we place the number thus,	16 = 10 +			6
	10 +			6
product by the units, - - - -	60 +			36
" " " tens, - - - -	100 + 60			
square of 16, - - - -	100 + 120 +			36
multiply again by 16, - - - -		10 +		6
product by the units, - - -	600 + 720 +			216
" " " tens, - - -	1000 +	1200 +	360	
cube of 16, - - - -	1000 +	1800 +	1080 +	216

1. By examining the parts of this number, it is seen that the first part 1000 is the cube of the tens; that is,

$$10 \times 10 \times 10 = 1000:$$

2. The second part 1800 is three times the square of the tens multiplied by the units; that is.

$$3 \times (10)^2 \times 6 = 3 \times 100 \times 6 = 1800:$$

3. The third part 1080 is three times the square of the units multiplied by the tens; that is,

$$3 \times 6^2 \times 10 = 3 \times 36 \times 10 = 1080:$$

4. The fourth part is the cube of the units; that is,

$$6^3 = 6 \times 6 \times 6 = 216.$$

1. What is the cube root of the number 4096?

Analysis.—Since the number contains more than three figures, we know that the root will contain at least units and tens.

operation.

$$\dot{4}\,09\dot{6}\,(16$$
$$1$$
$$1^2 \times 3 = 3)\,30 \quad (9\text{-}8\text{-}7\text{-}6$$
$$16^3 = 4\,096$$

Separating the three right-hand figures from the 4, we know that the cube of the tens will be found in the 4; and 1 is the greatest cube in 4.

Hence, we place the root 1 on the right, and this is the tens of the required root. We then cube 1, and subtract the result from 4, and to the remainder we bring down

4th =

the first figure 0 of the next period.

We have seen that the second part of the cube of 16, viz., 1800, is three times the square of the tens multiplied by the unit; and hence, it can have no significant figure of a less denomination than hundreds. it must, therefore, make up a part of the 30 hundreds above. but this 30 hundreds also contains all the hundreds which come from the 3d and 4th parts of the cube of 16. if it were not so, the 30 hundreds, divided by three times the square of the tens, would give the unit figure exactly.

forming a divisor of three times the square of the tens, we find the quotient to be ten; but this we know to be too large. placing 9 in the root, and cubing 19, we find the result to be 6859. then trying 8, we find the cube of 18 still too large; but when we take 6, we find the exact number. Hence, the cube root of 4096 is 16.

387. hence, to find the cube root of a number:

Rule.—I. separate the given number into periods of three figures each, beginning at the right, by placing a dot over the place of units, a second over the place of thousands, and so on over each third figure to the left: the left-hand period will often contain less than three places of figures:

II. note the greatest perfect cube in the first period, and set its root on the right, after the manner of a quotient in division. subtract the cube of this number from the first period, and to the remainder bring down the first figure of the next period for a dividend:

III. take three times the square of the root first found for a trial divisor, and see how often it is contained in the dividend, and place the quotient for a second figure of the root. then cube the figures of the root thus found, and if their cube be greater than the first two periods of the given number, diminish the last figure; but if it be less, subtract it from the first two periods, and to the remainder bring down the first figure of the next period for a new dividend:

IV. take three times the square of the whole root for a second trial divisor, and find a third figure of the root as before. cube the whole root thus found, and subtract the result from the first three periods of the given number when it is less than that number, r; but if it is greater, diminish the last figure of the root: proceed in a similar way for all the periods.

examples.

1. What is the cube root of 20796875?

operation.

20 796 875 (275

$2^3 = 8$

$2^2 \times 3 = 12)\ 127$

first two periods, - - - - 20 796

$27)^3 = 27 \times 27 \times 27 =$ 19 683

$3 \times (27)^2 = 2187)\ 11\ 138$

first three periods, - - - 20 796 875

$(275)^3 = 275 \times 276 \times 275 =$ 20 796 875

find the cube root of the following numbers:

1. cube root of 1728?
2. cube root of 117649?
3. cube root of 46656?
4. cube root of 15069223?
5. cube root of 5735339?
6. cube root of 48228544?
7. cube root of 84604519?
8. cube root of 28791029248?

390. to extract the cube root of a decimal fraction.

rule.—annex ciphers to the decimal, if necessary, so that it shall consist of 3, 6, 9, &c., decimal places. then put the first point over the place of thousandths, the second over the place of millionths, and so on over every third place to the right; after which, extract the root as in fraction =

whole numbers.

notes.—1. there will be as many decimal places in the root as there are periods of decimals in the given number.

2. if, in extracting the root of a number, there is a remainder after all the periods have been brought down, periods of ciphers may be annexed by considering them as decimals.

examples.

find the cube roots of the following numbers:

1. cube root of 8.343?	5. cube root of 387420489?
2. " " " 1728.729?	6. " " " .000003375?
3. " " " .0125?	7. " " " .0066592?
4. " " " 19683.46656?	8. value of $\sqrt[3]{81.729}$?

341. to extract the cube root of a common fraction.

rule.—1. reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms:

11. extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal, and extract the root as in the last case.

examples

find the cube root of the following fractions:

1. cube root of $\frac{64}{125}$?	6. cube root of $\frac{729}{15625}$?
2. cube root of $\frac{343}{729}$?	7. cube root of $\frac{17533}{262144}$?
3. " " " $31\frac{35}{343}$?	8. " " " $\frac{13824}{42875}$?
4. " " " $91\frac{1}{8}$?	9. " " " $7\frac{6}{7}$?
5. " " " $\frac{343}{512}$?	10. " " " $56\frac{3}{5}$?

applications:

1. what must be the dimensions of a cubical bin, that its volume or capacity may be 19683 feet?

2. if a cubical body contain 6859 cubic feet, what is the length of one side? what the area of its surface?

3. the volume of a globe is 46656 cubic inches; what would be the side of a cube of equal solidity?

barrels

cents

and breadth

4. A person wishes to make a cubical cistern, which shall hold 150 barrels of water: what must be its depth?

5. A farmer constructed a bin that would contain 1500 bushels of grain; its length & width were equal, and each half the height: what were its dimensions?

6. What is the difference between half a cubic yard, and a cube whose edge is half a yard?

7. A merchant paid $911.25 for some pieces of muslin. He paid as many cents a yard as there were yards in each piece, and there were as many pieces as there were yards in one piece: how many yards were there, and how much did he pay a yard?

8. If a sphere 3 feet in diameter contains 14.1372 cubic feet, what are the contents of a sphere 6 feet in diameter?

$3^3 : 6^3 :: 14.1372 : 113.0976$. ans.

9. If a ball $2\frac{1}{2}$ inches in diameter weighs 8 pounds, how much will one of the same kind weigh, that is 5 inches in diameter?

plate cellar

10. What must be the size of a cubical bin, that will contain 8 times as much as one that is 4 feet on a side?

11. How many globes, 6 inches in diameter, would be required to make one 12 inches in diameter?

12. If a ball of silver, one inch in diameter, is worth $8, what will be the value of one $5\frac{1}{2}$ inches in diameter?

13. If a plate of silver, 6 inches long, 3 inches wide and $\frac{1}{2}$ inch thick, is worth $100, what will be the dimensions of a similar plate, of the same metal, worth $800?

14. If a man can dig a cellar 12 feet long, 10 feet wide, and $4\frac{1}{2}$ feet deep, in 3 days, what will be the dimensions of a similar cellar, requiring 24 days to dig it, working at the same rate, and the ground being of the same degree of hardness?

15. If 1 foot 2 tons of hay in a stack 10 feet high, how high

must a similar stack be to contain 16 tons?

16. four women bought a ball of yarn 6 inches in diameter, and agreed that each should take her share separately from the outer part of the ball: how much of the diameter did each wind off?

arithmetical progression.

392. an arithmetical progression is a series of numbers in which each is derived from the one preceding, by the addition or subtraction of the same number.

the common difference is the number which is added or subtracted.

393. when the series is formed by the continued addition of the common difference, it is called an increasing series; and when it is formed by the subtraction of the common difference, it is called a decreasing series: thus,

2, 5, 8, 11, 14, 17, 20, 23, is an increasing series
23, 20, 17, 14, 11, 8, 5, 2, is a decreasing series.

the several numbers are called terms of the progression the first and last terms are called the extremes, and the intermediate terms are called the means.

394. in every arithmetical progression there are five parts, any three of which being given or known, the remaining two can be determined. they are,

1st, the first term;
2d, the last term;
3d, the common difference;
4th, " number of terms;
5th, " sum of all the terms.

case I.

395. having given the first term, the common difference, and the number of terms, to find the last term.

1. the first term of an increasing progression is 4, the common difference 3, and the number of terms 10: what is the last term?

Analysis.—by considering the manner in which the increasing progression is formed, we see that the 2d term is obtained by add, ing the common difference to the 1st term; the 3d, by adding the common difference to the 2d; the 4th, by adding the common diff- erence to the 3d, and so on; the number of additions, in every case, being one less than the number of terms found. instead of making the additions, we may multiply the common difference by the number of additions, that is, by 1 less than the number of terms, and add the first term to the product.

operation.

$$\begin{array}{rl} 9 & \text{no. less 1} \\ 3 & \text{com. diff.} \\ \hline 27 & \\ 4 & \text{1st term.} \\ \hline 31 & \text{last term.} \end{array}$$

rule.—multiply the common difference by 1 less than the nu- mber of terms: if the progression is increasing, add the prod- uct to the first term, and the sum will be the last term; if it is decreasing, subtract the product from the first term, and the difference will be the last term.

examples.

1. what is the 18th term of an arithmetical progression, of whi- ch the first term is 4, and the common difference 5?

2. a man is to receive a certain sum of money in 12 pay- ments: the first payment is $300, and each succeeding payment is less than the previous one by $20: what will be the last payment?

3. what will $200 amount to in 15 years, at simple interest, the increase being $14 for the first year, $28 for the second, and so on?

4. mr. jones has 12 children. he gives, by will, $1000 to the youngest, $50 more to the next older, and so on to each next older $50: how much did the oldest receive?

5. a man has a piece of land 35 rods in length, which tapers to a point, and is found to increase ½ rod in width, for eve- ry rod in length: what is the width of the wide end?

6. james and john have 100 marbles. it is agreed between them that john shall have them all, if he will place them in a stra- ight line half a foot apart, and so that he shall be obliged to travel 300 feet to get and bring back the farthest marble; and

also, if he will tell, without measuring, how far he must t, 行夫
ravel to bring back the nearest. how far?

case II.

396. knowing the two extremes of an arithmetical progress,
增減 ion, and the number of terms, to find the common difference.

1. the two extremes of a progression are 4 and 68, and the n,
umber of terms 17: what is the common difference?

analysis.—since the common difference multiplied by 1 less than the number of terms gives a product equal to the difference of the extremes, if we divide the difference of the extremes by 1 less than the number of terms, the quotient will be the comm, on difference: hence,

operation.

$$17-1=16)\overset{\displaystyle 64}{\underline{\overset{4}{64}}}(4$$

rule.—subtract the less extreme from the greater, and di, vide the remainder by 1 less than the number of terms: the q, uotient will be the common difference.

examples.

1. a man started from chicago and traveled 15 days, each day's journey was longer than that of the preceding day by the distance which he traveled the first day: what was his daily increase if he traveled 75 miles the last day? 距離

2. a merchant sold 14 yards of cloth, in pieces of 1 yard ea, ch, for the first yard he received $\$\frac{1}{2}$, and for the last $\$26\frac{1}{2}$: what was the difference in the price per yard?

3. a board is 17 feet long; it is $2\frac{1}{2}$ inches wide at one end, and $14\frac{1}{2}$ at the other: what is the average increase in width per foot in length?

4. the fourth term of a series is 12, and the eleventh is 33: find the intermediate terms.

case III.

397. to find the sum of the terms of an arithmetical progression.

1. what is the sum of the series whose first term is 2, common

difference 3, and the number of terms 8?

given series,	2	5	8	11	14	17	20	23
same, order inverted,	23	20	17	14	11	8	5	2
sum of both series,	25 +	25 +	25 +	25 +	25 +	25 +	25 +	25

analysis.—The two series are the same; hence, their sum is equal to twice the given series. but their sum is equal to the sum of the two extremes, 2 and 23, taken as many times as there are terms; and the given series is equal to half this sum, or to the sum of the extremes multiplied by half the number of terms.

rule.—add the extremes together, and multiply their sum by half the number of terms; the product will be the sum of all the terms.

examples.

1. what debt could be discharged in a year, by weekly payments in arithmetical progression, the first payment being $5, and the last $100?

2. a person agreed to build 56 rods of fence; for the first rod he was to receive 6 cents, for the second, 10 cents, and so on: what did he receive for the last rod, and how much for the whole?

3. if a person travels 30 miles the first day, and a quarter of a mile less each succeeding day, how far will he travel in 30 days?

4. if 120 stones be laid in a straight line, each at a distance of a yard and a quarter from the one next to it, how far must a person travel who picks them up singly, and places them in a heap, at the distance of 6 yards from the end of the line and in its continuation?

case IV.

398. having given the first and last terms, and the common difference, to find the number of terms.

1. the first term of an arithmetical progression is 5, the common difference 4, and the last term 41: what is the number

of terms?

analysis.—since the last term is equal to the first term added to the product of the common difference, by one less than the number of terms (Art. 395), it follows that, if the first term be taken from the last term, the difference will be equal to the product of the common difference by 1 less than the number of terms: if this be divided by the common difference, the quotient will be 1 less than the number of terms.

operation.

$41-5=36$

$4)36(=9$

$9+1=10$ no. terms

Rule.—Divide the difference of the two extremes by the common difference, and add 1 to the quotient: the sum will be the number of terms.

examples.

1. a farmer sold a number of bushels of wheat; it was agreed that, for the first bushel, he should receive 50 cents, and an increase of 9 cents for each succeeding bushel, and for the last, he received $500: how many bushels did he sell?

2. a person proposes to make a journey, and to travel 15 miles the first day, and 33 miles the last, with a daily increase of $1\frac{1}{2}$ miles: in how many days did he make the journey, and what was the whole distance traveled?

3. I owe a debt of $2325, and wish to pay it in equal installments, the first payment to be $575, the second, $500, and decreasing by a common difference, until the last payment, which is $200: what will be the number of installments?

geometrical progression.

399. a geometrical progression is a series of terms, each of which is derived from the preceding one, by multiplying it by a constant number. the constant multiplier, is called the ratio of the progression.

400. an increasing series is one whose ratio is greater than 1:

a decreasing series is one whose ratio is less than 1. thus,

1, 2, 4, 8, 16, 32, &c.—ratio 2—increasing series:

32, 16, 8, 4, 2, 1, &c.—ratio $\frac{1}{2}$—decreasing series.

the several numbers resulting from the multiplication, are called <u>terms</u> of the progression. the first and last terms are called the <u>extremes</u>, and the intermediate terms are called <u>means</u>.

401. in every geometrical, as well as in every arithmetical progression, there are five parts:

1st, the first term;

2d, the last term;

3d, the common ratio;

4th, the number of terms;

5th, the sum of all the terms;

if any three of these parts are known, or given, the remaining ones can be determined.

Case 1.

402. having given the first term, the ratio, and the number of terms, to find the last term.

1. the first term is 4, and the common ratio 3: what is the 5th term?

Analysis.—the second term is formed by multiplying the first term by the ratio; the third term, by multiplying the second term by the ratio, and so on; the number of multiplications being <u>1 less than the number of terms</u>: thus,

operation.

$$\begin{array}{r} 3\times3\times3\times3=81 \\ 4 \\ \text{Ans. } \overline{324} \end{array}$$

$4 = 4$, 1st term,
$3\times4=12$, 2d term,
$3\times3\times4=36$, 3d term,
$3\times3\times3\times4=108$, 4th term,
$3\times3\times3\times3\times4=324$, 5th term,

therefore, <u>the last term is equal to the first term mul-</u>

tiplied by the ratio raised to a power whose exponent is 1 less than the number of terms.

rule.—raise the ratio to a power whose exponent is 1 less than the number of terms, and then multiply this pow, er by the first term.

examples.

1. the first term of a decreasing progression is 2187; the r, atio is $\frac{1}{3}$, and the number of terms 8: what is the last term?

2. the first term of an increasing geometrical series is 8 the ratio 5: what is the 9th term?

3. the first term of a decreasing geometrical series is 729, the ratio $\frac{1}{3}$: what is the 10th term?

4. if a farmer should sell 15 bushels of wheat, at 1 mill for the first bushel, 1 cent for the second, 1 dime for the th, ird, and so on; what would he receive for the last bushel?

5. A man dying left 5 sons, and bequeathed his estate in

the following manner: to his executors, $100; to his youn, gest son twice as much as to the executors, and to each 1, on double the amount of the next younger brother: wh, at was the eldest son's portion?

6. a merchant engaging in business, trebled his capital once in 4 years: if he commenced with $2000, what shoul, ld his capital amount to at the end of the 12th year?

7. a farmer wishing to buy 16 oxen of a drover, finally agreed to give him for the whole the cost of the last ox only. He was to pay 1 cent for the first, 2 cents for the second, and doubling on each one to the last: how much would they cost him?

8. what is the amount of $500 for 3 years at 6 per cent. compound interest?

note.—the ratio is 1.06.

case 11.

403. knowing the two extremes and the ratio, to find the sum of the terms.

1. what is the sum of the terms of the progression 2, 6, 18, 54, 162?

operation.

$$
\begin{array}{r}
6+18+54+162+486 = 3 \text{ times.} \\
2+6+18+54+162 \qquad\;\; = 1 \text{ times.} \\
\hline
486-2 = 2 \text{ times}
\end{array}
$$

$$\frac{486-2}{2} = \frac{484}{2} = 242 \text{ sum.}$$

analysis.— if we multiply the terms of the progression by the ratio 3, we have a second progression, 6, 18, 54, 162, 486, which is 3 times as great as the first. if from this we subtract the first, the remainder, 486−2, will be 2 times as great as the first; and if this remainder be divided by 2, the quotient will be the sum of the terms of the first progression.

but 486 is the product of the last term of the given progression multiplied by the ratio; 2 is the first term; and the divisor 2, 1 less than the ratio: hence,

rule.— multiply the last term by the ratio; take the difference between this product and the first term, and divide the remainder by the difference between 1 and the ratio.

note.— when the progression is increasing, the first term is subtracted from the product of the last term by the ratio, and the divisor is found by subtracting 1 from the ratio. when the progression is decreasing, the product of the last term by the ratio is subtracted from the first term, and the ratio is subtracted from 1.

examples.

1. the first term of a progression is 4, the ratio 3, and the last term 78722: what is the sum of the terms?

2. the first term of a progression is 1024, the ratio $\frac{1}{2}$, and the last term 4: what is the sum of the series?

Nail

3. What debt can be discharged in one year by monthly payments, the first being $2, the second $8, and so on to the end of the year; and what will be the last payment?

4. A gentleman being importuned to sell a fine horse, said that he would sell him on the condition of receiving 1 cent for the first nail in his shoes, 2 cents for the second, and so on, doubling the price of every nail: the number of nails in each shoe being 8; how much would he receive for his horse?

5. A laborer agreed to thresh 64 days for a farmer, on the condition that he should him 1 grain of wheat for the first day's labor, 2 grains for the second, and double each succeeding day: what number of bushels would he receive, supposing a pint to contain 7680 grains; and what number of ships, each carrying 1000

tons burden, might be loaded, allowing 40 bushels to a ton?

Analysis

404. An analysis is an examination of the separate parts of a proposition, and of the connection of those parts with each other.

In analyzing, we generally reason from a <u>given number</u> <u>to its unit</u>, and then from this unit to the <u>required number</u>.

The process is indicated by the relations which exist between the given and the required numbers, and pursued, step by step, independently of set rules.

1. If 12 yards of cloth cost $48.36, what will 7 yards cost?

Analysis.—One yard of cloth will cost $\frac{1}{12}$ as much as 12 yards: since 12 yards cost $48.36, one yard will cost $\frac{1}{12}$ of $48.36 = $4.03; 7 yards will cost 7 <u>times as much</u> as 1 yard, or 7 times $\frac{1}{12}$ of $48.36 = $28.21; therefore, if 12 yards of cloth cost

\$48.36, 7 yards will cost \$28.21.

operation.

$\frac{1}{12}$ of 48.36 = \$4.03 = price of 1 yd.,

4.03 × 7 = \$28.21 = price of 7 yd., or $\left\{\frac{48.36 \times 7}{12}\right. = \$28.21$

2. if 27 pounds of butter will buy 45 pounds of sugar, h[ow] much butter will 36 pounds of sugar buy?

analysis.—one pound of sugar will buy $\frac{1}{45}$ of 27 lb. of butter, and 36 lb. of sugar will buy 36 times $\frac{1}{45}$ of 27 lb.

operation.

$\frac{1}{45}$ of 27 = $\frac{27}{45}$ = value of 1 lb. of sugar, or $\left\{\frac{27 \times 36}{45}\right. = 21\frac{3}{5}$ lb.

$\frac{27}{45} \times 36 = 21\frac{3}{5}$ lb. = value of 36 lb.

3. What will $6\frac{3}{4}$ cords of wood cost, if $2\frac{3}{8}$ cords cost \$$4\frac{1}{2}$?

analysis.—price divided by quantity, or $4\frac{1}{2} \div 2\frac{3}{8} = \frac{57}{8} \div \frac{19}{8}$ = \$3 = price of 1 cord; \$3 × $6\frac{3}{4}$ = \$$20\frac{1}{4}$ = cost of $6\frac{3}{4}$ cords.

operation.

$4\frac{1}{2} \div 2\frac{3}{8} \times 6\frac{3}{4} = \frac{57}{2} \times \frac{8}{19} \times \frac{27}{4} = \frac{81}{4} = 20\frac{1}{4}$ = \$20.25. Ans.

4. a farmer sold a number of cows, and had 12 left, which

÷ = 除

規律 普通

22. Règle generale. On forme le pluriel des substantifs en ajoutant une s au singulier: un roi, des rois; une ville des villes.

普通の規律

規則ノ外

Exception.

23. 1re Exception. Les substantifs terminés au singulier par s, x, z, n'ajoutent rien au pluriel: un héros, des héros; une voix, des voix; une nez, des nez.

則チ單數ニ於テス又ズヲ以テ終ル名詞ノ
複數ニ於テ何モ加ヘズ

24. 11e Exception. Les substantifs terminés au singulier par au et par eu prennent x au pluriel: un étau, des étaux; un tableau, des tableaux; un cheveu, des cheveux; un jeu, des jeux.

則チ單數ニ於テヲー及ヒヲヂヲ以テ終ル名詞ハ
複數ニ於テエキスヲ取ル