



J. L. del.

AN EXPLANATION  
OF THE METHOD OF ADJUSTMENT  
OF THE  
*BACK HORIZON GLASS*  
OF  
HADLEY'S QUADRANT,  
BY TWO NEAR OBJECTS:  
ALSO A DESCRIPTION.  
OF A PROJECTED ADDITION TO THE QUADRANT,  
FOR REFLECTING THAT ADJUSTMENT ACCORDING TO THE METHOD OF  
MR. BLAIR,  
BY THE REV. JAMES LITTLE.

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Read, January 28th, 1811.

**H**OW desirable as well as difficult it is, to adjust on every occasion the Back Horizon Glass of *Hadley's* Quadrant with necessary precision, is declared by the many different contrivances which have been suggested for that purpose; and this I hope will procure an indulgent approbation of the present, as well as the future, attempts that may be made for that end, till it shall be accomplished in every manner desirable. The mode of its adjustment, by two near objects, has been described by the late *Rev. Mr. Ludlam* in his treatise on the quadrant; and it may by this be accurately performed, if executed with due and intelligent attention to the

requisite circumstances: but as neither *Mr. Ludlam*, nor any other person that I know, has explained the grounds of the directions he has given; and as these directions will probably be applied in an unskilful and negligent manner, unless it be generally understood and impressed of what importance they are: as moreover this is the method, at least the most generally practicable, of adjusting the back horizon glass, as well as of trying the accuracy of the construction of the quadrant for effecting it in *Mr. Blair's* method; and is also subservient to the contrivance hereafter mentioned for accomplishing it in the same way; it is necessary, before I proceed to the description of it, to state the principles on which *Mr. Ludlam's* judicious instructions are founded.

He directs that the back horizon glass may be adjusted at right angles to the index glass, by the means of two near objects, such as two lines sustaining plummets in water, or two candles \*, &c. lying in the plane of the quadrant placed horizontal, and in a line joining the objects equidistant from the quadrant; one of them being before, and the other behind the observer; by reversing the instrument by turning it half round in its own plane, and shifting its position laterally on either side, till the images of the two objects are seen, through the back sight vane, to coincide, when each of them alternately is viewed by the observer, by di-

\* When plummets are used, they must be placed at opposite doors or windows against the light of the sky: and if candles be employed, their light should be seen through a small slit in a screen placed before each.

rect vision after a half turn of the instrument; the index fixed at  $o$ , and the back horizon glass shifted, till the images are brought to coincide, whichever of them be viewed directly. The quadrant is to be supported on a moveable stand, on the points of two erected pins fixed on the stand, inserted into two conical holes made in the middle of the heads of the screw pins in the back of the instrument, which fasten the central pins supporting the index and the back horizon glass; the placing the respective glasses alternately on these points, in the manner represented in fig. 1., will reverse the quadrant, by giving it just a semicircular motion in its own plane. The manner of performing this adjustment has been fully described by *Mr. Ludlam*, to whom I refer; but as I have seen no demonstration of its accuracy, I give the following proof of it; assuming the established optical principles.

Let  $A P Q$  (fig. 1.), be the octant, fixed on two points under the centres of the index glass  $A$ , and the back horizon glass  $B$ , or any two other fixed points; and let  $C$  and  $c$  be the two candles or objects by which the glass  $B$  is to be adjusted. The image of the object  $C$  will be seen by the eye  $E$ , looking through the sight vane, coincident with the object  $c$ , when the stand of the quadrant is properly placed, by the ray  $E B$ , parallel to  $A C$ , if the glasses are at right angles. Let the quadrant now be turned half round, and placed on the points in the position  $a p q$ ; and if the back horizon glass is properly adjusted, then the eye looking

through the vane at  $e$ , will see the reflected image of the object  $c$  coincident with the object  $C$ : because in these different positions of the quadrant, the incident rays become the reflected ones, and *vice versa*; and the index glass in the 2nd position  $a$ , will be parallel to the same, as it was in the 1st position  $A$ ; as also the horizon glass  $b$  to  $B$ .

But if the speculums  $A$  and  $B$  were not rightly adjusted at right angles to each other, the reflected ray  $BE$  in the former position of the quadrant would not be parallel to the incident ray  $CA$ , but these rays would make an angle, equal, (suppose) to  $EBM$  (or  $EBN$ ); and consequently this  $BM$  (or  $BN$ ) is the reflected ray, by which, and in the direction of which, the image of the object  $C$  is seen: then the object  $c$  must be placed at  $m$  (or  $n$ ) in order to coincide with the image of  $C$ , which appears only in the direction of  $MB$  (or  $NB$ ). Let them coincide in  $m$ ; and let the quadrant now be turned half round, and put into the position  $a p q$ ; in which the glasses  $a$  and  $A$ , and  $b$  and  $B$  are parallel: the angle of incidence is now one half of the angle  $mad$  greater than half  $caA$  or  $CAB$  by the angle  $cam$ ; so that the reflected ray  $ad$  will fall without the angle  $cab$ ; and will therefore either fall quite without the horizon glass  $b$ , or at least at a distance from its centre: in the former case the image of  $c$  would not be seen by the eye at  $e$  at all, unless the index glass were so long, and the object  $c$  so near, that a ray  $mg$  could fall on it in an angle so much less than the half of  $mad$ , as that the reflected ray

$g h$  would fall on the glass  $b$ , and be again reflected to the eye at  $e$ .\*

But if the rays forming the image are reflected from the middle part only of the mirrors, the image of  $m$  or  $n$  could not be seen to coincide with  $C$  by the eye at  $e$ ; for if the incident ray were different from  $c a$ , as suppose  $m a$ , the re-

\* In order to understand the theory of the reflection of the rays forming the image seen in the back horizon glass, the following circumstances are to be considered :

1st, Because the speculum  $b$  is parallel to  $B$ , whatever inclination  $B$  has, which diverts the image of the object  $C$  from the point  $c$  to  $m$ , the same inclination  $b$  also has, tending to divert the image of  $c$  to  $o$ , let the reflected rays  $g b$  or  $a d$  fall where they will on the speculum, or with whatever inclination.

2d, If the mirrors were at right angles, the rays  $m a$ ,  $m g$  would be reflected from the mirror  $b$ , in a direction parallel to themselves, *i. e.* the ray  $m a$  falling on the point  $d$  in the mirror  $b$ , would be reflected in  $d o$  parallel to  $m a$ ; and the ray  $m g$  falling on  $h$ , in the mirror  $b$ , would be reflected in  $h i$  parallel to  $m g$ : but when the mirrors are inclined to each other, the rays  $d o$ , and  $h i$  will decline from such parallelism in an angle equal to  $c a m$  double the inclination of the mirrors.

3d, When the angle  $c a m$  exceeds the angle  $a m g$  subtended by half the length of the mirror  $a$ , by a difference equal to, or exceeding the angle, subtended by half the length of the horizon glass at the middle of the index glass; all the rays reflected from the latter will fall without the horizon glass, and not be reflected by it; but when the angle  $c a m$  is less than this, some of the rays incident on the index mirror  $a$ , will fall from it on the mirror  $b$ , and be again reflected; and since the object  $c$  or  $m$  is so near that there is a considerable difference in the incidences of the rays diverging from it on the mirror  $a$ , from the point  $a$  to  $g$ , (the greatest difference equal to  $a m g$ ), and the same in the reflections from the mirror  $b$ , the image of the object  $c$  placed at  $m$ , may be seen in different places by some of the rays diverging from this mirror.

4th, The difference of the incidences of the rays  $m a$  and  $m g$  is that of their reflections in  $a d$  and  $g h$ ; and the difference of the incidences of  $a d$  and  $g h$  will be that of their reflections from the mirror  $b$ ; and the angular motion of the speculum  $b$  will be half of either these or those, in order to its reflecting one of these rays in the same direction in which the other had been reflected.

flected ray would be different from  $a b$ ; *i. e.* it would not fall on the speculum in the point  $b$ , nor consequently be seen (by the eye at  $e$ ), to coincide with  $C$ ; but would fall without  $a b$  as at  $d$ , and would be reflected in  $d o$ ; in which direction the image would be seen, and would be painted in the bottom of the eye, in a different place from that of the direct image of  $C$ ; so that these images would be divaricated; and it would be necessary to make them unite, by giving such a motion to the little mirror, as would have made the first reflected ray  $B E$  parallel to the incident ray  $C A$ ; by which the first image would be transferred from  $m$  to  $c$ , and the second image from  $o$  to the eye at  $e$ .

In the same manner it may be shewn, that if the second reflected ray tended to any point  $N$  on the other side of the line  $B E$ , from an inclination of the speculum  $B$  on the other side, there would be a divarication of the images to the eye at  $e$ , till such inclination of the glass was removed. It also appears that the objects  $C$  and  $c$ , by which the adjustment is made, may be placed very near the instrument, provided the reflection be made from the middle part only of the glasses, especially of the index-glass, the incidence of rays being different in different parts of it; for unless the sight vane or eye hole for the little mirror, be large, so as that the eye could shift across the vane the axis of vision, which ought to be fixed, or the hole be very near the mirror; the image, if reflected from a part of it distant from the middle, would yet not appear coincident with the object seen

directly through the middle, so as to prevent the separation of the images; because when the rays which form both images, cross one another, and proceed in different directions, though they should even cross in the same point in the mirror, yet they will penetrate the eye diverging, and form different images on the retina. But if the image may be seen by reflection from any part of the index-glass  $a$ , the angle of incidence of a ray  $m g$ , (of the near object  $c$  removed to  $m$ ,) falling on that glass at a point  $g$  distant from its middle point  $a$ , will be less than that of a ray incident on the point  $a$ , by the angle  $a m g$ : (for if the line  $a g$  were produced, the external angle at  $g$  would be equal to the angle at  $a$  and also to  $a m g$  together; and therefore as much as the external angle at  $g$  is increased above that at  $a$ , as falling toward a perpendicular from  $m$  to the line  $a g$  (El. 1. 32. cor.) the internal one, or the angle made with the mirror, is diminished;) but if the incidences on  $a$  and  $g$  were equal, the reflections would be so too; i. e. both  $m a$  and  $m g$ , and also  $ad$  and  $gh$  would be parallel; which is the case when the object is very remote, the angle  $a m g$  then vanishing. Also since by reflection from any number of plane mirrors, the direction of the rays is changed, but not their inclination to each other, the ang.  $a m g$  made by the rays incident on the index-glass, will be likewise the measure of their divergence reflected from the horizon glass.\* If therefore the glasses

\* The angle  $b a d$  is equal to  $c a m$ , and the angle made by  $e b$  and  $o d$  is equal to either; and an angular motion of the speculum  $b$  equal to half of any of these



are uncovered, the eye may see the image of  $m$ , (by rays incident on  $a$  and  $g$  in the mirror  $a$ , and on  $d$  and  $h$  in the mirror  $b$ ;) in different places, whose angular distance is  $amg$ ; and if the sight vane or hole at  $e$  be of any breadth, the second reflected image may be seen in two extreme places, whose distance will be as near to that angle as the breadth of the vane and of the horizon glass will allow, and it may be also seen coincident with the direct image  $C$ , because the unsilvered part of the glass  $b$  extends across its whole breadth, so that in whatever part of it the reflected image appears, the sight may be directed through that part to  $C$ .\*

angles would make  $do$  issue parallel to  $be$ ; but if the speculums were uncovered, the difference only of the angles  $amg$  and  $cam$  would require to be corrected by an angular motion of the speculum  $b$ , which would be half of this difference; and this being done the image of  $m$  would be seen in the direction  $be$  by the ray  $mg$ , while the same image would be visible at an angular distance equal to  $amg$ ; by the ray  $ma$ ; so that the image of  $c$  or  $m$  might be seen in different places under the same inclination of the glasses; i. e. the adjustment would be uncertain.

\* To shew that what is here stated is applicable to observations made with the quadrant, let  $d$  (fig. 2.) be a luminous body, from which light falls on the mirror  $ab$  with an angle of incidence  $dfe$ : its image will be visible to an eye at  $e$  in the direction  $ef$ , when the angle  $efc$  is equal to  $dfd$ . Let the mirror be turned on its axis  $f$ , carrying the perpendicular  $fc$  with it: when this has arrived to the position  $fg$ , the angle of incidence will be increased by  $cfg$ ; and the angle of reflection must be augmented by the same, so as now to be equal to  $dfg$ : if therefore the image is to be seen still in the point  $f$ , and no other point in the speculum, the eye must be placed at  $h$ ; when  $gh$  will be equal to  $gfd$ ; in which case the angle  $efh$  will be equal to twice the angular motion  $afi$  of the speculum, or of its perpendicular  $cf$ , which is the same; i. e.  $efh$  will be equal to twice  $cfg$ . If the eye may be shifted from the place  $h$  to a different place, as  $n$ , by looking through a hole or vane, whose breadth is equal to the interval  $hn$ , the image of the object  $d$  may be seen by reflection from the mirror  $ik$  in a different place or di-

From this it appears, that to adjust the horizon glass properly by two near objects, the face of both mirrors should be covered, except the middle parts only, or means must be used to view the images by those rays only, which are incident on the middle of both mirrors. But if according to *Mr. Ludlam's* direction, the object *C* be seen directly through the middle of the glass *b*, and if the image of no other part of the glass *a*, but its middle part also, can be seen by reflection from the middle of the mirror *b*; then no rays incident on any other part *g* of the index glass could be seen to coincide with the object *C*. Suppose this to be effected as *Mr. Ludlam* directs, by covering the index glass with a piece of card-paper, equal in size to itself, and lying close to it, having a black line marked on the middle perpendicular to the plane of the instrument: and the whole card to be made visible in the horizon glass *b*, and the black line to appear in

recession *n m*, visible in the mirror not in the place *f*, but in *m*, by a ray *d m*, reflected in *m n*, making an angle with the former line of vision *f h* equal to the angle *f d m*; and as the eye shifts along the interval *h n*, carrying with it the axis of vision through the different points in that interval, the line of direction of the image, or its visible place will also shift through the interval *f m* in the mirror with an angular motion finally equal to the angle *f d m*. Hence the place of a very near object seen by reflection from a mirror through a vane, also very close to the mirror as in the back observation for this adjustment, may be very inaccurately determined, unless it be seen only in that place or spot in the mirror from which spot the image had been reflected in a reversed position of the quadrant in the adjustment. When the object *d* is so remote, that the angle *f d m* becomes insensibly small, then the apparent place of the image will be the same, in whatever part of *f m* in the mirror it is seen reflected from: but when the object is near, since the axis of vision cannot be fixed by contracting the eye-hole to a point, the images must be seen in the same place in the mirror.

appear in the middle of *b*, through which the object *C* is seen directly. As the whole card covering the mirror *a*, is seen equidistant from the extremities of *b*, every point in the surface of *a*, and consequently every ray reflected from such point, must be in the same manner seen to preserve their relative positions, and as the picture of *a* seen in *b*, should occupy nearly its whole surface; the extremities of *a*, or any rays reflected from such extremities, could not be seen in the centre of *b*; but if the objects *C* and *c*, being small, could not subtend at the eye so great a space as the whole mirror *a*, the image of *c* would cover but a small part of the image of *a*; and if that image proceeded by reflection not from the centre, but the extremity of *a*, it would be visible in the extremity of the image of *a* as seen in *b*; i. e. at a distance from the centre of *b*, (and consequently remote from the image of *C*;) if it were seen in the centre of *b*, it must be reflected from the centre of *a*; but if the whole surface of *a* were not apparently coincident with that of *b*, this might not be the case.

Hence appears the justness of *Mr. Eudlam's* direction, that the centres of both mirrors should be seen to coincide in the horizon glass with the object seen directly; for the images can appear thus, in both positions of the quadrant coincident only under a certain and invariable position of the specula, though their whole surfaces were uncovered; it is hard however to distinguish by the eye what is the

middle part of the back horizon glass. \* By the glass herein after proposed to be used for Mr. Blair's adjustment, instead of the polished edge of the index glass, the beam of light is reflected to the eye undivided, which will allow the axis of vision to pass through the axis of the back-horizon glass; as it ought to do, whether for adjustment of this glass, or for taking angles; and as the axis of vision cannot be the same

\* This glass lies so oblique to the eye, that I think it yet remains to be enquired what is to be considered as its middle part, whether the middle of the fore or back surface, or the middle of its substance, or lastly that point in the same, which is the vertex of the angle made by the incident ray with the same refracted by its fore surface after reflection from its silvered surface. It would appear to me of little moment, which of the two beams of light, proceeding singly from the middle of the index-glass, and reflected double from the two surfaces of the horizon glass, be chosen for adjustment as the fixed axis of vision, (for both cannot be indiscriminately used, as emerging from different parts of the glass,) provided the reflected image be seen only by the same beam, issuing from the same part of the horizon glass in all reversals of the quadrant; were it not that the axis of vision ought to pass through the middle or axis of the glass, for the convenience of direct as well as reflex vision; according to which the reflected ray cannot, in the oblique position of the glass, impinge on the middle of either surface; but must be made (by turning the instrument in its plane, and placing the sight vane properly,) to fall on its fore surface between the middle of it, and the edge next the eye, if the reflection is to be made from the fore surface; and between the middle and the remote edge, if the image reflected from the back surface is to be seen. The proper place for reflection in the designed axis of vision, may be marked on the face of the glass, by sticking to it a fine waxed thread; and then the black line on the card before mentioned, covering the face of the index-glass, (or such another thread fixed along the middle of it,) must be made to coincide with this thread in every position of the quadrant for this adjustment; and as two images of the line will appear from the two surfaces, one only of them must be invariably used: the card to be removed in order to view the objects, when the line on it is made coincident with the thread.

for both these, since the incidence of the rays from the middle, and of those from the normal edge of the index glass on the horizon glass, is different; so the position of the back sight vane, and the position and direction of a telescope, (if one be used,) must be altered for these different purposes. The vane may, without moving its support, have its position changed, by having the eye hole made in a little moveable plate fastened on the support; but a complicated motion would be requisite for the telescope, to place it in the best manner for each of the above intentions. If it is expected to answer by only a circular motion of its upright stand, changing its direction, without moving it from its place, the stand should be placed as near as possible to the back horizon glass; for the farther it is removed from it, the more distant in one of its two positions will its axis be from the axis of that glass.

To ascertain the direction of the sight and of the telescope in making an observation by the edge of the index-glass, or of the glass here to be proposed for the same purpose, let a moveable rule or square, perpendicular to the face of the quadrant, be applied to the farther side of the quadrant opposite to the back horizon glass; and when the direct and reflected images are brought to unite, as the eye looks through the axis of the glass, let the rule be shifted, till its edge is made to appear in the place of their coincidence. If then a mark be made on the side of the quadrant at the edge of the rule, a line drawn from the mark through the axis of

the horizon glass, will point out the axis of vision and direction of the telescope. If the position of the latter be wrong, the observations will be erroneous, unless *Mr. Hadley's* correction be applied. \*

If the object which by the eye at *E* is seen in *m*, i. e. the object *c* removed to *m*; were to be brought to appear to the eye at *e* to coincide with *C*, by giving the mirror *b* an angular motion sufficient for this, such motion would be too great; for then the incident ray *ma*, and the reflected ray *be*, would not be parallel, nor consequently the glasses perpendicular: only half this motion must be given, and then the stand changed, or the object *m* moved to *c*, till the object and image are made to unite; (it being the same in effect, whether the stand be moved toward the object, or line joining the objects, or the object toward the stand); and then the quadrant must be turned half round to its first position, and the images brought half way together by turning the horizon glass and united as before: this to be repeated at every semirevolution, so often as necessary, till the adjustment of the horizon glass is perfected.

When the objects *C* & *c* are very distant, a small removal of the quadrant to the right or left of a line joining the objects,

\* Whether the eye, which is itself a telescope, and with a large aperture, ever requires a correction of this sort, when it looks through a sight vane, is not questioned; nor whether it views any thing obliquely; i. e. whether its axis be always the axis of its vision; but enough is said here to shew the errors that may arise in some cases, from looking through an eye-hole or vane of too great magnitude; and these errors would not be corrected by using a telescope, unless *Mr. Hadley's* correction, (in his 5th corollary,) were applied.

will make no sensible difference in the angle of incidence and reflection of the rays, nor consequently alter the place of the images, as would be the case if the objects were near.

If the quadrant, instead of being turned half round from the position  $A P Q$  to  $a p q$ , were to be so inverted, that the index and horizon glasses  $A$  and  $B$  should be placed on the lines  $E c$ ,  $C e$ , the adjustment could not be made, unless the objects were so remote, that the interval between the glasses would make an insensible angle at either of the objects, and that any little motion on either side of a line joining the objects, which might accidentally be given in reversing the quadrant, would cause likewise only an imperceptible divariation of the images. For if the quadrant were to be turned upside down, and so that the centres of the mirror would fall on the lines  $C e$ ,  $E c$  as before, the centre  $A$  on  $D$ , and  $B$  on  $F$ ; then the angle of incidence of a ray falling from  $C$  on  $D$ , would be different from that of a ray from  $C$  on  $A$ ; it would therefore not be reflected to  $E$ ; so that it would be necessary to turn the instrument in its plane, in order to make the image of  $C$  be visible in the horizon glass; by which the glasses in the 2d. position would not be parallel to themselves as they were in the 1st. nor is there any certain position in which they could be placed, as this will depend on the distance of the objects. So that the horizon glass cannot be adjusted by reversing the face of the octant, unless the objects by which this is to be done, are so far removed, that the distance between the glasses subtends at them an imper-

ceptible angle; which *Mr. Ludlam* says will be, when they are removed at least half a mile off: \* and for the same reasons, the adjustment cannot be made by the observer's turning himself half round with the instrument, without reversing it, unless the objects are at a distance as great as this, if it be not fixed on the same points, as above directed; by which alone the parallelism of the glasses is preserved, and also the same incidences and reflections, which are only exchanged one for the other by a half turn of the instrument; so that when the horizon glass is rightly adjusted, the direct and reflected images are reciprocally visible and coincident.

By this mode of adjusting the back horizon glass, by placing the quadrant on two fixed points between two near objects, a contrivance is made practicable, of using with full advantage the excellent method proposed by *Mr. Blair* of adjusting it at all times, by placing it parallel to a reflecting plane perpendicular to the index glass: for ascertaining which perpendicularity, the above mode of adjustment is necessary; as without knowing and making allowance for any deviation from it, in all observations taken, they would all be erroneous; which circumstance, as also this adjustment being the test of the accuracy of the addition, which I am to propose to the furniture of the quadrant, is the reason why I have been so diffuse in the explanation of this method.

The reflecting plane *Mr. Blair* proposed to be formed of

† This depends on the magnifying power of the telescope, and the smallness of the ang. it will render discernable.



the lower edge of the index glass itself, by grinding and polishing this edge perpendicular to the plane of the glass. The adjustment would be thus rendered admirably easy and certain, if the edge of the glass be formed perfectly plane and truly at right angles to it's face; were it not that this edge is necessarily so narrow, as not to afford a sufficient field of view to the observer, for distinguishing the object by which the adjustment is to be made: for the rays fall on the edge of the mirror so obliquely (making an angle with the plane of the edge, of no more than about 21 or 22 degrees, and forming on the back horizon glass an image equal in breadth, on its oblique surface, to the edge), that if the index glass were so great as half an inch in thickness, its edge would subtend at the eye near the horizon glass an optic angle of about 85 minutes; and if its thickness be, as usual,  $\frac{1}{8}$ th of an inch, it would take in a field of only about 20 minutes; which is too small to distinguish with ease the terrestrial objects to be viewed, though it would serve with difficulty for adjustment by the contact of the edges of the direct and reflected images of the sun or moon: this however it would do with all facility, if the thicknesses of the index and horizon glasses were such, and so proportioned to each other, that the image of the former might be reflected from the fore and back surfaces of the back horizon glass, single, so as to form one image of double breadth, by the double reflection: for which purpose the back horizon glass must be very thin, and the index glass too thick; as otherwise the image from the under surface of

the former would emerge at a distance from that reflected from its face; and the interval would to the naked eye appear like a shaded list, preventing the contact of the images observed from being seen by the double reflection, and confining the field to one of the images emerging from one surface of the glass; which will be as contracted as above stated.\* However, as it will always be easier and more

\* This will readily appear on inspection of Fig. 3: in which *A* is the index glass, and *B* the back horizon glass, placed at right angles to each other; each glass being  $\frac{1}{4}$ th of an inch in thickness: on which a beam of light *a b*, proceeding from a remote object *S*, is incident on the edge of the mirror *A*, in an angle with the plane of the edge of about 22 degrees, being the complement of the angle of incidence on the same; which in the quadrant is generally about at least 68 degrees: from which edge it is reflected to the glass *B*, and reflected again from both surfaces of the same; the extreme rays *a* and *b* of the beam of light, being throughout its progress, distinguished by the same letters; and those reflected from the back surface marked *a 2*, and *b 2*: their course (as the fig. itself will shew), is traced with sufficient exactness; from which it appears, that the beam of light *a b*, contracted by reflection from the mirror *A* to the  $\frac{1}{5}$ th part of an inch in breadth, preserves the same dimension till it enters the eye; both in the beam *x*, reflected from the anterior surface of the glass *B*, and in the beam *z* reflected from its back surface: for though this latter is diffused when it has penetrated the surface of the glass, it is again contracted on emerging from it; and is, as reflected from both surfaces, become a double and divided beam, the interval between both its parts being almost the thickness of each of them, which is equal to the sine of 22 degrees to a radius  $\frac{1}{4}$ th of an inch: and if the thickness of the index glass were to that of the horizon glass, as the sine of the refraction of the rays to its cosine, the interval between the beams would be equal to the breadth of either. To fill up the vacuity of the reflected light in this interval, by making the beams *x* and *z* issue contiguous, the thickness of the index glass must be to that of the horizon glass, as double the sine of refraction, to the cosine: this may be made evident as follows.

Let the beam of light *a b* (fig. 4.), be reflected from the edge of the index glass *A* to

pleasant to adjust by *Mr. Blair's* method, when the eye takes in a sufficient field of view ; and moreover as not every where a quadrant can be procured, furnished with an index

the horizon glass *B*, in the same manner, and with the same incidences, refractions and reflections as in fig. 3 : on the mirror *C* it will occupy a space *Ii*, equal to the breadth of the edge of the mirror *A*; and will cover the equal space *Rr*, on the back surface of the mirror *B*; after reflection from which, it will be refracted in the surface *I k*, emerging in the beam *z*; the several rays in this beam issuing at distances from *I* toward *k*, equal to the distances of their first incidence from *i* toward *I*; the last ray *b i* emerging coincident with the ray *I a*: so that if the beam *x* did not fill the space *i I*, the beam *z* would not fill the space *k I*, but would leave an interval next to *I* equal to the deficiency toward *i*. Let the line *p r* be drawn perpendicular to the mirror, bisecting the line *I i*, and the angle of incidence and reflection *I r i*, and parallel to *c s* the cosine of the angle of refraction *n I s*, which angle is equal to *c s I*. In the similar triangles *s c I*, *r p i*, the side *r p*, the thickness of the mirror *B*, is to *p i* half the thickness of the mirror *A*, as *s c* the cosine of the angle of refraction, to *c i* or *s n* the sine of the same; so that when the thickness of the mirror *B*, is *p r* the cosine of refraction, the thickness of the mirror *A* must be double of *p i* the sine of the same angle. Now to make the index mirror of so great thickness may produce a small inaccuracy, when angular distances are to be taken between very near objects, at which a small part of the length of this mirror would subtend a perceptible angle; for the thicker the glass is, and the greater the complement of the angle observed, the greater intervals on its surface will there be between the places of incidence and emergence of the rays forming the reflected images; which will therefore be seen, sometimes by rays issuing from the middle of this mirror, and sometimes by rays distant from the same: from which variation I have above stated the errors that may arise: and because every minutia in the construction or use of this admirable instrument is deserving attention, it may be worth while to shew the manner in which this happens.

Let *I G* (fig. 5.) be the index glass, in its position when the index is at *e*, and *H* the horizon glass at right angles to it; its adjustment being made by the reflected image of an object *S*, seen by the eye at *E* to coincide with another opposite object visible in the direction *E H*. The image of *S* is conveyed to the eye by the ray *S A* refracted in *A C*,

mirror, whose edge is ground accurately at right angles to its plane, and the edge also set up perpendicular to the plane of the instrument; (for which the purchaser must generally rely on

reflected from  $C$  to  $B$ , thence refracted again in  $BH$ , and reflected by the mirror  $H$  in  $HE$  parallel to  $SA$ . Since the mirror  $H$ , and the axis of vision  $EH$  are fixed, the ray  $BH$  is also fixed, in all observations taken; and every object must be seen by rays ultimately coincident with  $BH$ . Suppose it be required to find the angular distance of another object  $s$ , from the object seen directly in the line  $EH$ ; and that for this purpose, and to make the image of  $s$  appear in  $EH$ , the index is moved to the position  $ig$ , through half the angular distance  $SA s$  of the objects, (the lines  $SA$  and  $EH$  being supposed the same, and the interval  $AH$  to be accounted for): then the image will be seen by the ray  $sa$ , inflected, as before traced, in the lines  $ac$ ,  $cb$ ,  $bH$ , and  $HE$ ; and the thickness of the index glass being moderate, there will be an interval between the place of incidence on it of the rays  $SA$  and  $sa$ , so small as to be imperceptible, and to occasion no error. But if the thickness of this mirror were great, as  $AD$  or  $ad$ , and the rays to be reflected from  $D$  and  $d$ ; the image of  $S$  would be visible by the ray  $Rp$ , proceeding in  $pD$ ,  $DB$ ,  $BH$ , and  $HE$ ; and the image of  $s$  by the ray  $te$ ,  $ed$ ,  $db$ ,  $bH$ ,  $HE$ . So that when the adjustment was made, by the ray  $Rp$  incident at  $p$ ; the object  $s$  would afterward be seen, and the angle  $sAS$  measured, by the ray  $te$ , incident at  $e$ , considerably distant from  $p$ . If the object  $s$  was very remote, the rays  $sa$  and  $te$  would be as it were parallel, and their incidences and course the same; but if the object  $s$  were near, as at  $r$ , then the incidences would differ, and the error of observation be equal to the angle  $era$ , so much the greater, as the object is nearer, or as the complement of the angular distance observed is greater: and the same will be the case in the fore as well as in the back observation; which latter may be made as true as the former, if the line of direction of the sight be accurately fixed, by a long eye-tube, or telescope rightly placed, and if the other requisites above mentioned be observed.

Thus though in observing remote objects, and for nautical uses, no inconvenience will arise from the thickness of the index glass; (which if it be duly proportioned, as here stated, to that of the horizon glass, and its edge truly formed, is doubtless the best and surest mechanical organ for adjusting the latter); and though no error can hence arise in performing the above described adjustment; wherein the position of the index glass

the maker; and few artists can be furnished with the exquisite apparatus, which must be employed for effecting the former); I think it may be to many desirable to have an

to the object is not changed, nor consequently the incidences of the rays on it; yet in observing very near objects, as the height or angular distances of buildings, offsets in surveys, bearings, &c. a great thickness of the index glass will produce a variable error, which though trifling, is unsatisfactory in an instrument, whose general excellence would make one wish it to be exempt from even the smallest imperfection. And this error can be diminished only by choosing such a certain position for the index glass with respect to its centre of motion, as would cause a part of the field of view to be lost in measuring angles but little exceeding  $90^\circ$ , when the rays fall very obliquely on the index glass, and when also the error increases, as does the complement of the observed angle to 180 degrees. For the point  $e$  (in fig. 5.), can be made to approach to the point  $p$ , only as the triangle  $e d b$ , which is of given dimensions, shifts toward the mirror  $H$ , by its angle  $b$  advancing toward it in the line  $b H$ , the triangle being moved parallel to itself; by which the point  $b$  would fall beyond the end of the mirror  $i g$  at  $g$ , and the field would be contracted. But the face of the mirror  $e b$ , and consequently the triangle  $e b d$ , will be elevated, by advancing toward  $H$ , more or less, as the centre of motion of the index is placed farther from, or nearer to, the line  $p B$  the face of the index glass: so that the point of incidence of the ray  $t e$  cannot fall nearer to that of the ray  $R p$ , without causing a part of the field to be lost, and this where it is most contracted.

It has been made evident there, that if the thickness of the index and horizon glasses be equal, or as formerly in use, there will be an interval between the beams of light reflected from the opposite surfaces of the latter; and in this interval the reflected image is not visible to the observer; who can only see there the object directly through the glass: and if he is to view both images coincident, he can only do so in the space filled by the beams; as in  $x$  or  $z$  fig. 3; for if he attempted to make the extremities of the images to coincide at the internal edge of either of the reflected beams, he could not hold the quadrant steady enough to keep them there; for which purpose it would require to be absolutely immoveable.

On these accounts, if advantage is to be taken of the double reflection, (without which the narrowness of either beam of light, and the evanescence of the reflected

easy contrivance to be substituted, where required, instead of this operation on the edge of the mirror; and which can be executed with little additional labour by any instrument-maker; so as to afford a sufficient field of view, with a capacity for accurate adjustment.

This I have effected by the contrivance of a second small index mirror; requiring only one plane surface, and fixed on the index at right angles to the great mirror; being totally free and detached from the index mirror, and capable of every adjustment for itself, without interfering with, or impeding any motion requisite for that purpose for the index glass, or altering its position. The following description of such a one, which I have made, will shew that it is a very simple and easily fabricated addition to the quadrant.

image in their interval of separation, will make the observation with the naked eye uncertain and troublesome); it is necessary that the light from both surfaces of the glass, should be contiguous, having no interval; which can only be effected, either by making the index glass almost one quarter of an inch thick, or by reducing the thickness of the horizon glass to less than  $\frac{1}{12}$ th of an inch; or by such a mutual compensation of both, as would still leave one as much too thick as the other would be too thin, for the uses above stated: and though this may be remedied, while yet the glasses remain of their due and proper dimensions, by using a telescope, whose aperture is large enough to take in both beams of the reflected light with the interval of their separation; yet in ordinary quadrants, of simpler construction and more moderate price, not designed to be furnished with a telescope, or mirror with a polished edge, I cannot but think that an easy and cheap substitution for both, would, if found to answer, be very useful; as securing at all times the advantages of *Mr. Blair's* invention for the back observation, (at least for taking altitudes); to those navigators, who do not furnish themselves with a more perfect and expensive instrument; as well as to those who on land desire, in surveys, to ascertain large angular distances by the quadrant; or by an artificial horizon

The ichnographical plan and position of both the mirrors is represented in fig. 6. as they are fixed on the head of the index.

*B* is the great mirror, and *b* is the cock supporting it, with its case; *D* the wing or adjusting lever of the cock; *f* and *g* the screws for erecting it perpendicular to the plane of the instrument in the usual manner; and *e e* are the steady pins in the index fastening the cock; *d d* are two pins on which the edge of the mirror rests.

*A* is a round brass plate, with a milled edge of the same size with the head of the index, and with the fig. both being three inches in diameter, and screwed fast to it, concentric with the index, by the four screws *s s s s* screwed into the index. *C* is the little mirror screwed to the plate by the screw *h* passing through the wing *E* of its cock *c*: it is erected and fastened perpendicular by means of the screws *h* and *i*, in the same manner as the mirror *B* is by the screws *f* and *g*: *m* is a steady pin fastened in the cock *c*, inserted into a hole in the plate *A*; and *k* another strong steady pin rivetted in the plate, the upper part of which, being cylindrical passes up through a hole in the strong bar or wing *E* of the cock *c*, which hole it exactly fills, but allowing the cock to be elevated or depressed a little for adjustment of the mirror, without any angular mo-

to take altitudes or angles exceeding 45 degrees, to find the latitude, &c., for which the back horizon glass must be used. It is also desirable for the interests of science and navigation, that quadrants of sufficient performance should be made capable of being fabricated in different places.

tion about the centre of the index. Thus by the screws *h* and *i*, and the steady pins *k* and *m*, the little mirror is made erect and fixed on the plate *A*: it is also set at right ang<sup>s</sup>. to the index mirror *B*, by loosening a little the screws *s s s s*, and turning the whole plate *A* by its milled edge, round its centre on either side, so far as necessary; and when this is found to be accurately effected, the screws *s s s s* are to be again made fast; when the little mirror will be perpendicular both to the plane of the instrument, and also to that of the great mirror, and cannot, without suffering violence, alter its position.

This circular motion of the plate *A*, and of the little mirror fastened to it, is permitted, without communicating any motion or even contact of it, to the index-glass, its cock, steady pins, or screws, by the following contrivance.

Through the plate *A* are cut long holes or slits, formed as represented in the fig. concentric with it, at the places of the screws *s s s s*, *f*, and *g*, and also at the pins *d d* and *e e*. these slits are made just so wide, as that the screws and pins will not touch them, and that the heads of the screws will, when screwed down, press upon the edges of the slits: the slits at *e e* are not represented in the fig. to avoid confusion: the slits at *s* and *g* should be so long as to allow the plate *A* to turn through the space about  $\frac{1}{10}$ th of an inch on each side of the screws fixed erect; and the slits at *d d*, *e e*, and *f*, may be shorter, according as they lie nearer the centre, each slit bounded within the same sector of a circle.



Through these slits in the plate *A*, the screws *s s s s*, *f*, and *g* are inserted, and all except *g* fastened in the head of the index; in which latter, the pins *e e* penetrate also through the wing *D* of the cock of the index-glass, to steady it.

The edge of the index-glass *B* rests on the pins *d d*, which project only so far above the surface of the plate *A*, as to keep the mirr. and the wing *D* of its cock clear of it, so as that the plate can turn about under both without touching them: and the bar or wing *E* of the cock *c* lies about  $\frac{1}{4}$ th of an inch above the wing *D*, so as to be quite clear of it, and permit adjustment by raising or depressing the wing. It may be supposed that the cock *c* and its wing *E* must at first be so formed that the mirror *C*, when fastened to them, will be nearly at right angles to the mirror *B*, when the bar *E* lies parallel to the index mirror, and the screws, &c. are in the middle of the slits in the plate *A*; that a small motion of the plate *A* on either side, will suffice for an exact adjustment of the mirror *C*.

There is a round hole in the middle of the plate *A*, a good deal wider than the head of the pin, about which the index turns; and the plate is made to turn concentric with the pin by a little ring or socket *R*, brazed in the hole in the plate, or by an annular ledge formed on it projecting downward below the under surface of the plate about  $\frac{1}{2}$ th part of an inch. The outside of this projecting ring is to be exactly fitted into a circular groove or cavity formed in the index, so

far distant from the head of the pin, that the ring will not touch it, nor affect its motion or position.

The cock *c* and its cap are formed with an indenture in them, (as in the fig.) at the end of the index-glass, in order that the mirror *C* may lie nearer to it, which will allow the little mirror to be made broader, without enlarging the brass plate *A* and the head of the index; while the part of the cock not indented may, as well as its wing *E*, be made so massive and strong, as not to be bent and strained easily by any accident: a notch is cut in the side of the bar *E* at the screw *f*, to allow this screw to be turned, without touching the bar: thus both the mirrors may be adjusted independent on each other.

The little mirror *C* requires not to be silvered on the back, and consequently its opposite surfaces need not be parallel, so that it may be made of a piece of well polished and plane looking glass, but the polish must be taken away from its back surface by grinding it on a plate with fine emery and water; and the surface thus made rough should be smeared over with a feather dipt in oil of turpentine mixed with lampblack, to prevent all reflection from that surface.

The addition of this mirror to the index adds no trouble to the business of fixing the index-glass: the extra work required is that of the little mirror and of the plate *A*; and the fabrication of this plate will be greatly facilitated, if it be cast from a model, in which the slits and perforations, and

the central annular ledge, (the former to be of a size a little less than requisite in the plate when finished,) are already made in their proper places. The plates cast from this model will require no measurement nor piercing, and only want to be filed, turned and polished.

The mirror *C* is to be adjusted by making it, by a fore observation, parallel to the back horizon glass, by turning with the left hand the plate *A* by its milled edge, till the images of the object viewed are seen to coincide; and then fastening the plate *A* to the index by the screws *s s s s*: it follows, that the back horizon glass must itself be for this purpose previously adjusted at right angles to the index-glass, in the way before mentioned, which may always on land be easily performed. And I should imagine that even atsea, when the sea is calm, it might perhaps be practicable, by fixing up two papers, with a black line drawn vertically on each, on the adjacent sides of two masts of the ship; by which lines, the quadrant fixed on two points of a moveable stand, placed on deck between the two masts, may have the back horizon glass adjusted as above, in order by this means to try at any time whether the little mirror *C* has its adjustment altered; which, however, must be very unlikely to happen, especially if the contiguous sides of the plate *A* and of the index be not polished, lest the plate should slide on the index. At other times the back horizon glass is to be adjusted by the mirror *C*, supposed to be itself right in position; and indeed considering that this mirror is not, as the horizon glasses are, rested on the

points of two pins like a lever, and that these glasses are moved and secured, not by the outer edges of the circular plates of their frames, but by the small axes of those plates, very near the centres of their motion; whereas the mirror C is fastened firmly on a broad plate, screwed tight near its margin by 4 screws to the head of the index, it is not easy to conceive how it can alter its adjustment; though from the above mode commonly in use for fixing the horizon glasses, it is not unaccountable why they should frequently do so, not being fastened by the margins of their frames. These frames, however, are sometimes moved by endless screws playing in their ratched edges; and this construction when well executed, is much better than the former.

Read 21st May, 1810.