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PREFACE

TO THE FIRST EDITION.

THE following pages were drawn up for the use of the junior officers of the Royal Engineers, and those of the Honourable East India Company's Service, in their course of instruction in Trigonometrical Surveying and Practical Astronomy at this establishment, of which branch of their studies I have for some time had the superintendence.

My original intention was to have had them lithographed for distribution among the officers; but I have been since led to the resolution of publishing them in their present form, from their having swelled to a size beyond what I at first contemplated; and also from the total want experienced, during the period occupied in compiling them, of any practical English work on Geodesical Operations, extending beyond the mere elementary steps of Land Surveying. Of this class there are several very useful publications, containing instruction in all the necessary detail, to some of which references are made for information respecting the preliminary knowledge of the construction and use of the instruments most generally employed, as well as to the French aruthos on Geodesy, whose works I have consulted.

Of the extensive and scientific Geodesical Operations described in these latter works, the present Treatise professes to give nothing beyond a brief outline, as their detailed account would be far too voluminous to be condensed in so small a compass.

The cadets at Woolwich and Addiscombe are taught the use of the Chain and Theodolite, and to calculate the contents of the different portions into which the ground is divided by natural and artificial boundaries; they are also rendered conversant with Plane Trigonometry and Mensuration, and with sufficient Spherical Trigonometry for the solution of the ordinary cases of Spherical Triangles. Such preliminary knowledge is consequently assumed as being already acquired. It is, however, in the power of any individual to make himself master of the necessary theoretical part of this knowledge, by the study of one or other of the numerous excellent works on Trigonometry and Mensuration; and the practice of Land Surveying can be acquired in a few weeks in the Field, under any competent Instructor, or even without this assistance, by the careful study of some elementary work on the subject.

Royal Engineer Establishment, Chatham,
1839.

PREFACE

TO THE THIRD EDITION.

THIS work having been for some time out of print, a third Edition has been prepared, with many additions, principally to meet the requirements of the Royal Military Academy, Woolwich.

DUBLIN, 1862.

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TRIGONOMETRICAL SURVEY,

ETC.

CHAPTER I.

GENERAL OUTLINE OF THE SYSTEM OF CARRYING ON A TRIGONOMETRICAL SURVEY.

THE basis of an accurate survey undertaken for any *extensive* geodesical operation such as the measurement of an arc of the meridian or of a parallel, or for the formation of a geographical or territorial map showing the positions of towns, villages, &c., and the boundaries of provinces and counties, or a topographical plan for military or statistical purposes, must necessarily be an *extended system of Triangulation*, the preliminary step in which is the careful measurement of a base line on some level plain:—at each extremity of this base, the angles are observed between several surrounding objects previously fixed upon as trigonometrical stations, and also, when practicable, those subtended at each of these points by the base itself. The distances of these stations from the ends of the base line and from each other are then calculated and laid down upon paper forming so many fresh bases from whence other trigonometrical points are determined, until the entire tract of country to be surveyed is covered over with a net-work of triangles of as large a size as is proportioned to the contemplated extent of the survey, and the quality and power of the instruments employed. Within this principal triangulation secondary triangles are formed and laid down in like manner by calculation, and if necessary a series of minor tertiary triangles between them, and the interior detail is filled up between *these points, either entirely by measurement with the chain and theodolite, or by partial measurement [principally of the roads], and by sketching the remainder with the assistance of some portable instrument. The degree of accuracy and minuteness to be* observed in this detail, and the scale upon which the work is to be laid down, will of course determine which of these methods is to be adopted—the latter was practised on the Ordnance Survey of the South of England, which was plotted on the scale of 2 inches to 1 mile, and reduced for publication to that of 1 inch; but on the Survey of Ireland, and that of Scotland and the six Northern Counties of England, sketching has been almost entirely superseded by chain measurement, even in the most minute particulars, and the undulations of the surface of the ground represented with mathematical accuracy by horizontal contour lines, traced by actual levelling at equidistant vertical intervals,* the whole survey being laid down to the scale of 6 inches to 1 mile. In the survey of only a *limited* extent of country there does not exist the same absolute necessity for a triangulation even though a considerable degree of accuracy should be required; this will appear evident from the consideration that in every practical operation some amount of error (independent of the errors of observation) is to be expected—sometimes a definite quantity dependent upon the means employed; sometimes a quantity varying in amount with the extent of the operation.

In all *angular* measurements, the errors to be expected evidently depend upon the power and quality of the instruments made use of, and are altogether irrespective of the *space* over which the work extends. In *linear* measurements, on the contrary, the probable error is some proportional part (dependent upon the circumstances and the means employed) of the *distances measured.* So long then as the extent of the survey, and the scale upon which it is to be laid down, are such that the probable error attendant upon ordinary chain measurement of the largest figures would be *imperceptible on the plan,* no triangulation is necessary on the

* Sketching, in place of tracing contour lines, has been again lately resorted to on the Ordnance Survey for the features of the ground, on account of the greater cost of the latter more accurate system.

score of accuracy alone, though in many cases even of this nature it would be found in the end a saving of both time and expense.

In a new and unsettled country, particularly if flat and thickly wooded, the outlay that would be required, and the time that would be occupied by an accurate triangulation, would probably prevent its being attempted, at all events in the first instance. If only a general map upon a very small scale is required, the latitude and longitude of a number of the most conspicuous stations can be determined by astronomical observations, and the distances between them calculated, to allow of their positions being laid down as correctly as this method will admit of, within which, as within a triangulation, the interior detail can be filled up. In surveying an extended line of coast where the interior is not triangulated, no other method presents itself, and a knowledge of practical astronomy therefore becomes indispensable in this, as in all extensive geodesical operations. A topographical survey further requires that some of the party employed upon it should be practically versed in the general outlines of geology, as a correct description of the soil and mineral resources of the different parts of every country forms one of its most important features. The heights of the principal hills, and of marked points along the ridges, plains, valleys, and watercourses above the level of the sea should also be determined, which in a survey of no great pretensions to correctness in minute detail, may be ascertained with tolerable accuracy by means of the mountain barometer, or aneroid, or even approximately by observing the temperature at which water boils at different stations.

A sketch of a certain tract of country, on a far larger scale than that of most general maps, is constantly required on service for the purpose of showing the military features of the ground, the relative positions of towns and villages, and the direction and nature of the roads and rivers comprised within its limits. This species of sketch, termed a "Military Reconnaissance," approaches in accuracy to a regular survey in proportion to the time and labour that is bestowed upon it. Having thus adverted briefly to the progressive steps in the different species of surveying, they will each be treated of more in detail in their proper order.

The system of forming the "net-work of triangles" alluded to, of as large a size as is consistent with the circumstances under which the survey is undertaken, within and dependent upon which the secondary and tertiary triangulation and all the interior details are included, is to be considered as the working out of a general principle to be borne in mind in all topographical and geodesical operations, the spirit of which is as much as possible to work from *whole to part*, and not from *part to whole*.

By the former method errors are subdivided, and time and labour economised; by the latter, the errors inseparable from even the most careful observations are constantly accumulating, and the work drags on at a slower rate and an increasing expenditure.

CHAPTER II.

MEASUREMENT OF A BASE LINE.

In fixing upon an appropriate site for the measurement of a base line, a level plain should obviously be selected where both ends of the base would be visible from the nearest trigonometrical points. Where extreme accuracy has been required, steel chains, glass, deal, and platinum rods have at different times been used for the purpose of determining its length; but each of these units of measurement, whichever is preferred, must be supported so as to ensure its being laid perfectly level.* The whole thus forms a portion of a great circle, which has ultimately to be reduced to its proper measure at the level of the sea at one mean temperature.

In measuring a base for the topographical survey of any small detached portion of ground, it will be sufficient for ordinary purposes to measure its length carefully two or three times with a chain which has been compared with a standard†, and if rendered necessary by the irregularity of the ground, to take an accurate section along the line (which should be laid out with a theodolite between marks at each extremity), for the purpose of reducing this measurement by calculation to its true horizontal value. The length of a base, which has subsequently to be determined with the most minute accuracy by means of glass rods, compensation bars, or other contrivance, is generally first measured two or three times in this manner.

* Otherwise a correction becomes necessary on account of the difference of level of the two extremities.

† A spiral spring, something like that used in weighing-machines, is attached to the end of a chain used for purposes requiring much accuracy; this indicates the power of tension exerted, which should always be the same as when compared with the standard. The surveyors under the Tithe Commission Act are furnished with this contrivance.

The exact measurement of a base is perhaps the most difficult and the most important part of a trigonometrical survey, as upon its accuracy that of every subsequent proceeding depends. In the account of this operation on the Trigonometrical Survey of England and Wales published in 1801, will be found detailed accounts of the base measured on Hounslow Heath, in 1784, with Ramsden's steel chain, at first intended solely for the purpose of connecting by triangulation the Observatories of Paris and Greenwich, but afterwards made the first step in the trigonometrical survey of England. This base was measured a second time with prepared deal rods*, and again by a combination of these two methods, the mean of the three valuations being 27404·0137 feet at the level of the sea. The details of the base of verification (*i. e.* the actual measurement of the side of a remote triangle, whose length had been previously obtained by calculation) in Romney Marsh, in 1787, are also given in the same work, as well as the remeasurement of the original base on Hounslow Heath, in 1791, and of another base of verification on Salisbury Plain in 1794, which is stated to have corresponded exactly with its mean length as obtained by calculation in three different triangles.

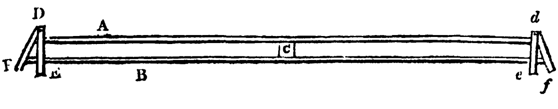
A detailed account was in 1847† drawn up by Colonel Yolland, R.E., of the mode adopted by General Colby to obtain the accurate value of the base measured on the Ordnance Survey of Ireland at Loch Foyle in the county of Londonderry, in which work will also be found a quantity of scientific information con-

* The deal rods were first laid, as it is termed, "in coincidence;" that is, lines drawn across them, near their extremities, were made to coincide most accurately by fine screws, as in the sketch,  but this method occupying a considerable time, their spherical ends were afterwards brought in contact  and the measurement was continued in this manner, so that no decision was arrived at as to the comparative accuracy of the two modes; that by coincidence would, however, appear likely to be more minutely *correct* than the one adopted.

† Many years after the 1st edition of this work; the short popular description of the process of using the bars is however retained. Colonel Yolland has likewise given a description of these bars, with the method of using them, in the 3rd volume of the Woolwich Mathematical Course, under the head of "Geodesy."

nected with the principal triangulation. The principles of the contrivance, in which it differs fr6m all other methods that have preceded it, consist in always preserving by a mechanical compensation obtained by the use of two metals having different powers of expansion and contraction, exactly the same distance between two points at the extremities of the compensation bars, instead of allowing, as had been hitherto done, for this expansion or contraction according to the temperature at which each rod was laid, and also in obtaining a *visual* instead of an *actual* contact of the rods. This will be explained by the following short description of the compensation bars and the method of using them.

Two bars, one of iron and the other of brass, 10 feet long, placed parallel to each other $1\frac{1}{4}$ inches apart, were riveted together at their centres, it having been previously ascertained by numerous experiments, that they expanded and contracted in their transitions from cold to heat, and the reverse, in the proportion of three to five. The latter was coated with some non-conducting substance to equalise the susceptibility of the two metals to change of temperature, and across each extremity of these combined bars was fixed a tongue of iron, with a minute dot of platinum almost invisible to the naked eye, and so situated on this tongue, that under every degree of expansion or contraction of the rods the dots at each end always remained at the constant distance of 10 feet. This will be better understood by reference to the sketch below.



A is the iron bar (about five-eighths of an inch wide and one and a half deep), the expansion of which is represented by three; B the brass bar (of the same size), the expansion of which is five, the two being riveted together at the centre C; D E and *d e* are the iron tongues pinned on to the bars so as to admit of their expansion, with the platina dots at D and *d*. The tongues are by construction made perpendicular to the rods at a mean temperature of 60° Fahrenheit, and the expansion taking place from their common centre, when A expands any quantity which may be expressed by *three*, B expands at the same time a quantity equal *to five, and the position of the tongues is changed to D F, d f,* the *dots D and d remaining unalterably fixed at the exact distance of ten feet. It is evident from this construction, that the dots at the extremities of these bars could not, if desired, be brought either* into actual contact or coincidence; but a more correct plan was adopted, which consisted in laying each rod so that the dot at its extremity should always be at a fixed distance from that at the end of the next rod. This was effected by means of powerful microscopes, attached to the end of similar short compound bars,* 6 inches long, and mounted on a stand, by which means they could be laid perfectly horizontal by a spirit level, the microscopes in these bars occupying the position of the dots on the longer rods. These dots, after the rods had all been carefully levelled, were brought exactly under the microscopes by means of three micrometer screws attached to the box in which each rod was laid, so that it could be moved to either side, backwards or forwards, elevated or depressed, as required, the rods being laid on supports equidistant from the centre of the box, that they might always have the same bearing. The point of starting was a stone pillar, with a platina dot let into its centre, and a transit instrument was placed over it by which the line was laid out with the greatest precision with the assistance of sights at each end of the bars, an average of about 250 feet being completed in one day, and five boxes, giving a length of 52 feet, being levelled and laid together.

About 400 feet of this measured base was across the river Roe, and clumps of pickets were driven at intervals of about 5 feet 3 inches apart from centre to centre by a small pile engine, on the

* This was the usual distance between the foci of the microscopes; but to meet cases where the uneven surface rendered it difficult to bring the short bars to a level at this distance, it was sometimes diminished to one half. Microscopes of *different lengths* were used where the inclination of the ground rendered it necessary to lay the boxes on *different levels*, so that the platina dots might be brought in the focus to each microscope. The old base of verification on Salisbury Plain was afterwards remeasured with these compensation bars.

heads of which the boxes containing the compound rods rested. At the end of each day's work a triangular stone was sunk at the end of the last bar laid with a cast-iron block fitting over it, having a brass plate with a silver disk let into the middle of the brass, which was adjustable by means of screws. This disk was brought exactly under the focus of the extreme microscope, and served as a starting point the following day, a sentinel being always left in charge of this stone which was further secured by a wooden cover screwed over it.

The total length of the measurement of this base amounted to about 8 miles; 2 miles were subsequently added by a method described in page 11, making the entire distance between the two extremities rather more than 10 miles.

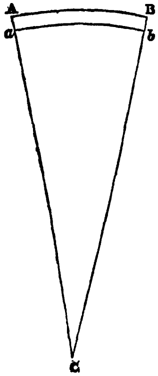
Detailed descriptions of the various methods that have been at different times adopted to insure the correct measurement of base lines on the Continent, may be found in all standard works on geodesical operations.* A popular account of the mode of conducting these measurements, and of the nature of the rods, &c., used, is also given in Mr. Airy's "Figure of the Earth," in the "Encyclopædia Metropolitana," commencing at page 206.

A base measured on any elevated plain is thus reduced to its proper measure at the level of the sea:

* "Recueil des Observations Géodesiques, par Biot et Arago"—"Puissant, Traité de Géodesie"—"Base du Système Métrique decimal;" and the works of Cassini, Francœur, Colonel Lampton, &c.

The bases of the original arc of Mechain and Delambre, described in the "Base du Système Métrique," were measured with rods of platinum two toises long; to each bar was attached at one end a rod of brass. The proportion of the expansion of brass and platinum being known, the expansion of the platinum rod was inferred from the observed difference of expansion of the two rods. The rods were laid in boxes, and placed on trestles; and their ends not brought into contact, but measured with a slider. The temperature was reduced to thirteen degrees of Reaumur. The length of the base of Perpignan was 6006·28 toises; and that of Melun 6075·9 toises. The calculation of Perpignan base of verification from that of Melun differed only eleven inches from its actual measurement on the ground.

These platinum bars are described in page 203, vol. i. Puissant's "Géodesie." Few bases have ever been measured solely for the determination of the value of an arc of the meridian, or of a parallel, but have formed at the same time the foundations of the survey of a country.



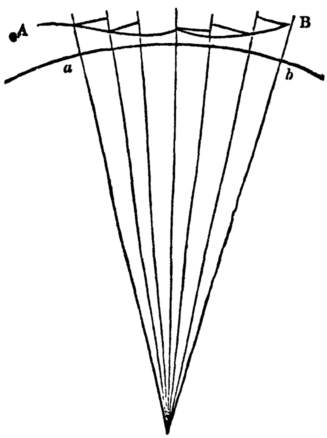
Call A B, the base measured at any elevation A a above the level of the sea . . } B
$a\,b$ its value at this level . . . b
C b the radius of the earth . . . R
And the altitude above the sea A a . h,
as ascertained by levelling, or by the barometer.

Then $R + h : R :: B : b.$ & $b = \frac{R.\,B}{R+h}$

And $B - b$ the difference of the measured and reduced base $= B - \frac{B.\,R}{R+h} = B.\,h$

The radius of the earth may be considered $=21008000$ feet; if then, the log of the base in feet, be added to the log of the altitude, and the log of the sum of the radius and altitude be subtracted therefrom, the remainder will be the log of a number to be deducted from the measured base to reduce it to its value at the level of the sea. This correction, though generally trifling, is not to be neglected when the base is measured upon ground of any considerable elevation.

Mr. Airy, in page 198 of the "Figure of the Earth," in the "Encyclopædia Metropolitana," gives this formula:—"If r be the earth's radius, or the radius of the surface of the sea (which is known nearly enough), and h the elevation, the measured lengths must



be multiplied by the fraction $\frac{r}{r+h}$ or $1\frac{h}{r}$, or they must be diminished by the part $\frac{h}{r}$ of the whole. If the surface slopes uniformly, the mean height may be taken; if it is very irregular, it may be divided into several parts."

The reduced length $a\,b$ of the base A B is thus found, and if the length of the chord* is required, it is found by subtracting $\frac{A\,B}{24r^2}$.

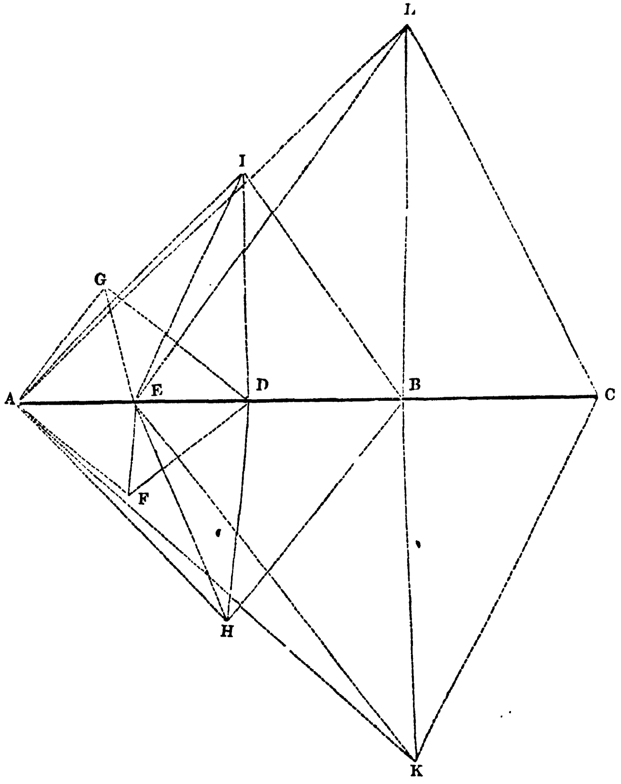
* In ordinary surveying the surface of the earth may be considered a plane, a degree of 69½ English miles not exceeding its chord by more than 25 feet.

Beside the marks at the *extremities* of a base line—which if the base is to form the groundwork of a survey of considerable extent, should be constructed so as to be *permanent*, as well as *minute*—intermediate points should be carefully determined and marked during the progress of the measurement, by driving strong pickets, or sinking stones, into the ground, with dots upon a plate of metal, or some other indication of the exact termination of the chain clearly defined upon them. These marks serve for testing the accuracy of the different portions, and reciprocally comparing them with each other. It has been already remarked, that the length of the base on the Ordnance Survey of Ireland was not obtained *entirely* by measurement, an addition of two miles having been made to its measured length by calculation. This calculation was also contrived to answer the purpose of verifying the measurement of intermediate portions of the base between marks left for the purpose, as previously alluded to, and explained by reference to the figure on next page, in which A B represents the portion of the base actually measured, and B C that to be added by calculation for the purpose of extending the base to C, to obtain a more eligible termination.

The points E and D have been marked during the measurement, and are thus made use of:—

The stations F and G are selected so that the angles at E may be nearly right angles, and the points themselves nearly equidistant from the line, and about equal to A E. Similar conditions determine the positions of H, I, K, and L. At A the whole of the objects visible are most accurately observed with a large theodolite, which is then taken to the other points on the line, as well as those selected on either side of it, where all the angles are measured. From A E and the three observed angles, G E and E F are determined, from *each of which* in the triangles G E D and D E F the side E D is obtained, the distances thus found forming two checks on its measured length; I D and D H are in like manner calculated from A D and also from E D as bases, and each of these again furnish data for the determination of D B. Lastly, B L and B K are found from A B, and also from E B; from the mean results of which B C, the required addition to the measured base, is obtained.

Even if the entire base had been measured, the above is an excellent method of verifying the accuracy of the intermediate component parts, and is also a test of the instrument used for measuring the angles. The stations H, K, L, &c., will also answer for minor trigonometrical points, and will be found useful in the course of the work.



The next process is the Triangulation, which (combined with the measurement of a base line just described) forms the preliminary step not only in a correct trigonometrical survey, but in the more delicate operations of the determination of the difference of longitudes between two meridians such as those of the observatories of Greenwich and Paris, and the measurement of an arc of the meridian to obtain the length of a degree in different latitudes, from whence to deduce the figure and magnitude of the earth.

CHAPTER III.

TRIANGULATION.

The most conspicuous stations are selected as trigonometrical points, and are chosen with reference to their relative positions, as the nearer these triangles approach to being equilateral, the less will be the error in the calculation of the sides resulting from any slight inaccuracy in the observed angles.

The base being generally of trifling length compared with the distances between the points of the principal triangles to be ultimately deduced from it, the sides of these triangles must be from the first gradually increased as rapidly as is consistent with the remark in the previous paragraph, till they arrive at their greatest limit*, determined in an extensive survey by the distance at which these points can be rendered clearly visible. As early as 1822, the reflection of the sun from a plane mirror was employed in Hanover for the purpose of rendering distant stations visible, and this method was adopted by General Colby and Captain Kater in verifying General Roy's triangulation for connecting the meridians of Paris and Greenwich. The station on Hanger Hill tower could not be seen from Shooter's Hill (only 10 miles distant), owing to the dense smoke of London, but was rendered clearly visible by tin plates attached to the signal post, so as to reflect the sun towards

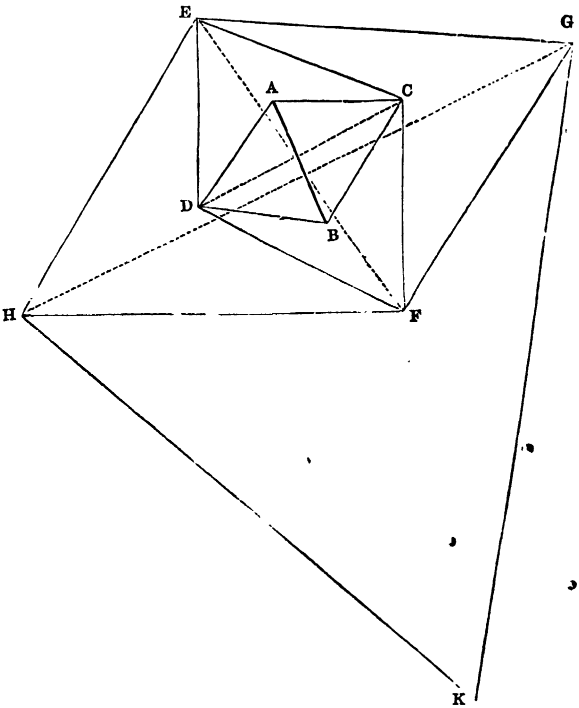
* "Laplace a demontré par le calcul des probabilités qu'il ne faut employer que le moins grand nombre possible de triangles du premier ordre couvrant l'étendue entiere du pays, en leur donnant les plus grandes dimensions permises par les localités, et par la puissance des lunettes des instruments."—*Francœur,* "*Géodesie,*" page 110.

The distances between some of the trigonometrical points on the Ordnance Survey of Ireland exceed 100 miles (the average being about 60) and have been deduced from the original base of about 10 miles. Observations *may be* made on a station which would be hid by intervening high ground were it not elevated above its real place by refraction, but periods should always be chosen for observing angles when *extraordinary refraction* is not remarkable, on account of its very irregular action.

the station at stated times on a certain day. The same plan was tried the following year at the station on Leith Hill, near Dorking, rendering the station visible at the distance of 45 miles, though the hill itself was never once seen. The utility of thus employing the sun's reflected rays being estalished by these results, an instrument was invented by the late Captain Drummond, Royal Engineers, in lieu of the former temporary expedients for directing the rays upon the station to be illuminated, the description of which will be found in his Paper on the means of facilitating the observations of distant stations, published in the "Philosophical Transactions" for 1826, and from whence the above remarks have been taken. In using this "*Heliostat*" it is only necessary for the assistant to keep the mirror adjusted so as to always reflect the rays upon the station from which the observation is being made.* But a contrivance was still wanting to produce a light sufficiently brilliant to answer for distant stations at night. Bengal lights had been used by General Roy, which were succeeded by argand lamps and parabolic reflectors, and these again, by a large plano-convex lens prepared by MM. Fresnel and Arago, and used by the latter gentleman conjointly with General Colby and Captain Kater, by the light of which a station distant 48 miles was observed. The light invented by Captain Drummond, described in the volume of the "Philosophical Transactions" alluded to, far surpassed all previous contrivances in intensity. A ball of lime, about a quarter of an inch in diameter, placed in the focus of a parabolic reflector and raised to an intense heat by a stream of oxygen gas directed through a flame of alcohol, produced a light eighty times as intense as that given by an argand burner. A station on the hill in the barony of Ennishowen, of great importance, could not be seen from Devis Mountain, near Belfast, and this instrument was consequently sent there by General Colby; and in spite of

* This was effected by means of a small brass ring placed 50 or 60 feet in front of the mirror, adjusted to the proper elevation and in the line of direction of the station previously approximately determined. As long as this ring was kept illuminated it was certain that the Heliostat was properly adjusted. For a distance of 40 or 50 miles a mirror of 4 or 5 inches diameter was found sufficient; for 100 miles, one of 8 or 10 inches would be required.

boisterous and hazy weather, the light was brilliantly visible at the distance of 67 miles, and would have been so at a much greater distance. *Drummond's light* might be also made available in determining the difference of longitudes by signals which will be explained hereafter*; but difficulties connected with its management, as well as the cost of the apparatus, prevented its being brought into general use on the Ordnance Survey.

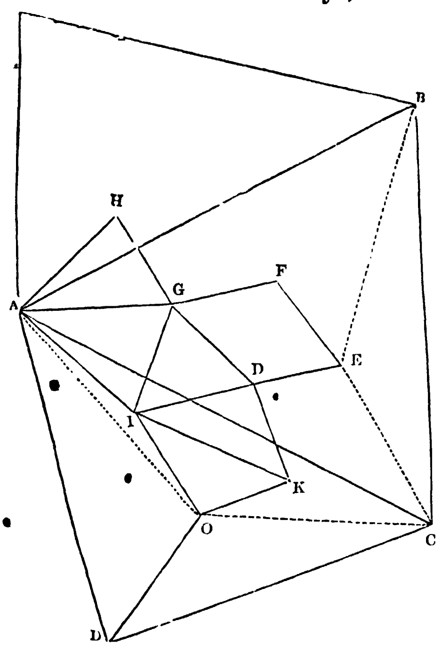


It has been already stated that the sides of the principal triangles should increase as rapidly as possible from the measured base. The accompanying sketch will show how this is to be managed without admitting any *ill - conditioned triangles.*

A B is supposed to be the measured base of 3 miles or any other length, and C and D the nearest trigonometrical points. All the angles being observed, the distances of C and D from the extremities of the base are calculated with the greatest accuracy.

* It is also eminently calculated for those lighthouses where powerful illumination is required. In the "Philosophical Transactions" for 1830 is a paper of Captain Drummond's on this subject, containing the results of a course of experiments carried on by order of the Trinity Board. The lime in these experiments was exposed to streams of oxygen and hydrogen gas from separate gasometers, instead of passing the oxygen gas through a flame of alcohol, which was done on the survey for the convenience of carriage, though at an increased expense.

In each of the triangles D A C and D B C, we have then the two sides and the contained angles to find D C, one calculation acting as a check upon the correctness of the other. This line, D C, is again made the base from which the distances of the trigonometrical stations E and F are computed from D and C; and the length of E F is afterwards obtained in the two triangles D E F and F E C. In like manner the relative positions of the points H G K, &c., are obtained, and this system can be pursued until the trigonometrical stations arrive at the required distance apart.



On the Ordnance Surveys, both of England and Ireland, the largest sized instruments, 3 feet in diameter, were used for fixing the principal stations.* The angles at the vertices of the *secondary* triangles were observed with the second-class theodolites. The sides of these triangles were on an average about 8 or 10 miles long, and the intervals between them were divided into small *tertiary* triangles, with sides of from 1 to 3 miles in length, smaller theodolites of 7, 9, and 10 inches diameter being used for measuring the angles. All points of the secondary order of triangles, which were fixed upon during the progress of the principal triangulation, were *observed with the largest instrument;* and a number of the *minor* stations, mills, churches, &c., were observed with the second-class theodolites from different stations: thus the connection between the three classes of triangles was

* The large class of theodolites used upon an accurate triangulation require some protection from the weather. Light portable frame-work erections, covered with canvas or boarding, are used on the Ordnance Survey.—See the article "Observatory, Portable" in the *Aide Mémoire.*

established, and the positions of many of the minor stations, which had been determined by calculation from a series of small triangles, were checked by being made the vertices of larger triangles, based upon sides of those of the second order.

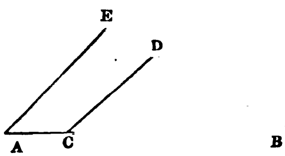
Thus the point E in the figure is determined from the base B C; and O from both D C and A D, forming a connection between the larger and smaller order of triangles, and constituting a series of checks upon the latter.

The length of the sides of the smallest triangles must depend upon the intended method of filling up the interior. If the contents within the boundaries of parishes, estates, &c., are to be calculated, the distances between these points must be diminished to one or two miles for an inclosed country, and two or three, perhaps for one more open. If no contents are required, and the object of the triangulation is solely to ensure the accuracy of a topographical survey, the distances may be augmented according to the degree of minutiæ required and the scale upon which the work is to be laid down.

The direction of one or more of the sides of the principal triangles must also be determined with regard to the meridian. The methods of ascertaining this angle, termed its azimuth, will be described hereafter.

It is also advisable not merely to measure the angles between the different trigonometrical points, but to observe them all with reference to certain stations previously fixed upon for that purpose.

If for any cause it has been found advisable to commence the triangulation before the base has been measured, the sides of the triangles may be calculated from an assumed base, and corrected afterwards for the difference between this imaginary quantity and the real length of the base line; or if the length of the base is subsequently found to have been incorrectly ascertained, the triangulation may be corrected in a similar manner.



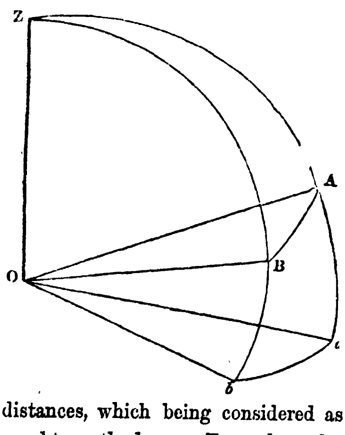
Thus, suppose C B the assumed, and A B the real length of the base—also E B and A E the real distances to the trigonometrical point E, and D B and D C those calculated from the assumed base, then E E evidently $= C D. \frac{A B}{C B}$, and $E B = B D. \frac{A B}{C B}$.

On the Continent, the instrument that has been generally used for measuring the angles of the principal and secondary triangles is Borda's repeating circle*; but the theodolite is universally preferred in England, and those of the larger description, in their present improved state, are in fact portable *Altitude and Azimuth* instruments. The theodolite possesses the great advantage of reducing, *instrumentally*, the angles taken between objects situated in a plane oblique to the horizon, to their horizontal values, which reduction, in any instrument measuring the exact angular distance between two objects having different zenith distances, is a matter of calculation depending upon the zenith distances or co-altitudes of the objects observed†. The formula given by Dr. Pearson for this correction when the obliquity is inconsiderable,

* For a detailed account of this instrument, which is so seldom met with in England, see pages 89 to 99, "Géodesie, par Francœur;" also page 142, vol. i. "Puissant, Géodesie." There is also a very able paper upon the nature of the repeating circle by Mr. Troughton in the first volume of the Memoirs of the Astronomical Society.

The portability of this instrument is one of its great recommendations; but it seems to be always liable to some *constant error*, which cannot be removed by any number of repetitions, and the causes of which are still unknown. With all the skill of the most careful and scientific observers, the repeating circle has never been found to give the accurate results expected from it, though in *theory* the principle of repetition appears calculated to prevent almost the possibility of error; its accuracy is also limited by the small size of the telescopes.

† This will be evident from the figure below, taken from page 220 of Woodhouse's Trigonometry.



Let O be the station of the observer, A and B the two objects whose altitudes above the horizon are not equal; then the angle subtended by them at O is A O B measured by A B; but if Z a, Z b, are each = 90°, then a b, and not A B, measures the angle a Z b, which is the horizontal angle required. The difference, then, between the observed angle A O B and a Z b, is the correction to be applied as the reduction to the horizon. The horizontal distances between these stations of different elevations may be found from having the reciprocal angles of elevation and depression, and the measured or calculated distances, which being considered as the hypotenuse of the *triangle*, the distances sought are the bases. From these the *horizontal* angles may be calculated if required.

which must always be the case in angles observed between distant objects on the horizon, is as follows :—

A being the angle of position observed, H and h the altitudes of the two objects, and $n = \sin^2 (\frac{1}{2} H + h). \tan. \frac{1}{2} A - \sin^2 (\frac{1}{2} H - h). \cot. \frac{1}{2} A$. then x (the correction) $= n. \sec. H. \sec. h$. The value of n is given in tables computed for the purpose of facilitating this calculation, for every minute of H and h, and for every ten minutes of A. When the altitude differs more than 2° or 3° from zero, the following formula is to be used in preference :—

$$\left.\begin{array}{l}\text{Sin } \frac{1}{2} Z \\ \text{the reduced angle}\end{array}\right\} = \frac{\sqrt{(\sin \frac{1}{2} S - \delta) . \sin (\frac{1}{2} S - \delta')}}{\sin \delta, \sin \delta'};$$

S being the sum of the angle observed and the two zenith distances; and δ and δ' the respective zenith distances of the objects.*

All observed horizontal angles are however essentially *spherical angles;* and in every triangle measured on the surface of the earth, the sum of the three angles must therefore, *if taken correctly,* be more than 180°. The lines containing the observed angles are in fact *tangents* to the sphere (supposing the earth to be one), whereas to obtain the three points considered as *vertices of a plane riangle,* the angles must be reduced to the value of those contained between the *chords* of the arcs constituting the sides of the spherical triangle. The correction for this spherical excess, though too minute to be applied to angles observed with moderate-sized instruments (being completely lost in the unavoidably greater errors of observation) should however be calculated in the principal triangles, which is easily done on the supposition that the area of a spherical triangle whose sides are immeasurably small compared with the whole sphere, may be considered iden-

* For the investigation and application of these formulæ, see vol. i. "Puissant, Traité de Géodesie," page 174 ; "Géodesie, par Francœur," pages 128 and 435 ; and Dr. Pearson's "Practical Astronomy," vol. ii. page 505. Hutton's formula is the same, except that it is expressed in terms of the altitude instead of the zenith distances. See also Woodhouse's "Trigonometry," page 220, and the corrections to the observed angles in the first volume of the "Base Métrique."

tical with that of a plane triangle whose sides are of the same length as those of the spherical, and whose angles are each diminished by one-third of the spherical excess; from which theorem, demonstrated by Legendre, and known by his name, is deduced the form $\frac{S}{R}$; or for the excess in seconds, $\frac{S}{R^2}$ R″; where S denotes the area of the triangle, and R the radius of the earth.*

The earth being considered a perfect sphere whose radius is 21,008,000 feet, one second of space = 101·43 feet, and $(101{\cdot}43)^2$ = the square feet in a square second.—R the radius = 206264,8 seconds, and the expression becomes $\frac{\text{area in feet}}{(101{\cdot}43)^2 \times (206264{,}8)^2}$ × 206264,8; or, in logarithms, Log area 4,0123486 − 5,3144251 = Log area — 93267737 for the spherical excess in seconds.†

On the Trigonometrical Survey of England, the spherical excess was constantly calculated, not solely for the purpose of diminishing the observed angles by the amount, but *to correct the observations.* Thus in one of the large triangles in Dorsetshire the sum of the three angles was 0″·5 less than 180°, the calculated spherical excess amounted to 1″·29, showing an error of 1″·79 in the observation, and in many of the triangles this error was more considerable. *One-third of the error* thus found added to each of the angles, corrects them as *angles of a spherical triangle,* and one-third of the spherical excess deducted from each of these corrected spherical angles converts them into the angles of a plane triangle ready for calculation, the sum of whose angles is = 180°, as is seen in the example below.

Observed Angles.	One-third of Error.	Corrected Sph. Angles.	One-third of Sph.Excess.	Rectilinear Angles corrected for calculation.
Maker Heights } 45° 54′ 37″	+ ·597	45° 54′ 37″ ·597	− ·43	45° 54′ 37″ ·167
Bolt head 48 39 24·5	+ ·597	48 39 25 ·097	− ·43	48 39 24 ·667
Butterton 85 25 58	+ ·597	85 25 58 ·597	− ·43	85 25 58 ·167
179 59 59·5		180 0 1 ·29		180 0 0

* R″ may be considered identical with $\frac{1}{\sin 1''}$. See "Puissant," vol. i. page 100.

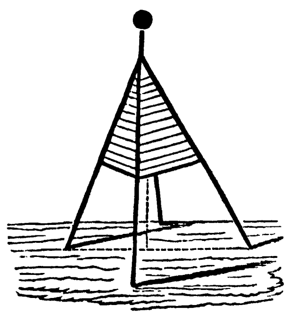
† Woodhouse arrives at the same result at the termination of a long investigation of this correction.—"Trigonometry," page 229.

One-third of the spherical excess has here been deducted from *each* angle, but it might have been calculated for each separately, by reducing the angles of the spherical triangles to the angles formed by *the chords.* (*Woodhouse,* page 239; *Base du Système Métrique,* &c.) Thus there are three modes of solving the large triangles of a survey, first, by calculating them as *spherical triangles* with the *corrected spherical angles;* secondly, by computing them as rectilinear triangles with the *angles of the chords;* and thirdly, by Legendre's more expeditious method of reducing each angle by one-third of the spherical excess. In the "Base du Système Métrique," the sides of the triangles were *computed by all three methods.* On the Ordnance Survey they were formerly mostly calculated by the second, and checked by the third, but latterly the last of these modes, that by Legendre's formula, was the only one used.

This subject is treated at length in Puissant, vol. i. pages 100, 117, and 223, and also in the account of the Trigonometrical Survey, in Professor Young's, and Woodhouse's Spherical Trigonometry; and in various other works.

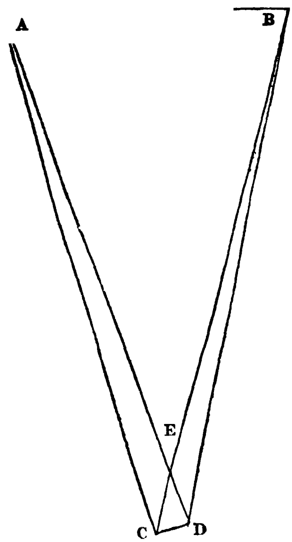
When the theodolite cannot be placed exactly over the station,* a correction for this eccentricity, termed the "*Reduction to the Centre,*" becomes necessary.

In the triangle A B C, suppose C the station where the instrument cannot be set up. If at any convenient point D, the angles



* Where mills, churches, and other marked objects are selected as trigonometrical points, which are otherwise peculiarly well adapted, but on which the theodolite cannot be set up, this reduction becomes necessary if angles are required to be taken from them. Temporary trigonometrical stations are easily formed of three or four pieces of scantling, 10 or 12 feet long, framed together as in the sketch, with a short pole projecting vertically upward from the apex of the pyramid. A plummet suspended from this gives the exact spot on which to set up the theodolite. Long poles, which can be removed when it is required to adjust the theodolite over the station, answer the same purpose. Two circular discs of iron or other metal on the top of a pole, placed at right angles to each other, form very good marks for observation.

A D B and A D C are taken, and the distance C D measured, the angle A C B can be thus determined.



A E B = A C B + C A D.

and A E B = A D B + D B C.

$\because$ A C B + C A D = A D B + D B C, and

A C B = (A D B + D B C) − C A D.

But $\sin DBC = \sin BDC \times \frac{CD}{BC}$,

and $\sin CAD = \sin ADC \times \frac{CD}{AC}$,

and as these angles are exceedingly minute, the sines may be substituted for the arcs, and we have $ACB = ADB + \frac{CD}{BC} \cdot \sin BDC - \frac{CD}{AC} \cdot \sin ADC$* or in seconds

$$ACB = ADB + \frac{CD}{\sin 1''}\left(\frac{\sin BDC}{BC} - \frac{\sin ADC}{AC}\right)$$

The necessity for the above correction is not of common occurrence, as in the principal triangles, stations are generally selected from whence observations can be made; and in those of the secondary order, the measurement of the third angle is not considered imperative.

In observing the angles for triangulation, too much care cannot be bestowed upon the adjustments of the instrument. These are briefly as follows for the 5- or 7-inch theodolites used in fixing points in the interior, and for traversing. The large theodolite 3 feet in diameter, known by the name of its maker, Ramsden,†

* Instead of deducing the angle at the station on which the instrument cannot be set up from that observed at any spot convenient to it, it is often found more expeditious, particularly if there are many observations made, to correct the other angles of the triangles; this latter method is generally now practised on the Ordnance Survey.

† An instrument of the same size has since been made by Messrs. Troughton and Simms for the survey of India, as also another for the Ordnance Survey. A theodolite of 18 inches diameter upon a repeating stand was constructed by General Mudge, with an idea of its superseding the larger theodolite, the weight and size of which rendered its carriage an affair of difficulty; but the advantage *of repetition* (so desirable in single observations) possessed by moderate-sized instruments does not

and liberally lent by the Royal Society to the Ordnance, is fully described in the "Trigonometrical Survey;" and the peculiarities in the construction and management of the other large instruments with which the angles of the principal and secondary triangles were observed, are soon understood by any officer conversant with the adjustment of the smaller class, which he most generally has to work with, and which is therefore the one selected for description.

The first adjustment is for the line of collimation, and consists in making the cross wires* in the diaphragm of the telescope coincide with the axis of the supports in which the telescope rests; the proof of which is their intersection remaining constantly fixed upon some minute, well-defined, distant point, during an entire revolution of the telescope upon its own axis in the Ys, which are left open for the purpose. When this intersection on the contrary forms a circle round the object, the wires require adjusting. They are generally placed crossing each other at an angle inclined to the horizon of about 45°, and the operation is facilitated by first turning the telescope partly round, till they appear horizontal and vertical; half the divergence of each of

appear to compensate for the diminished size of the circumference of the horizontal circle. Theodolites of 24, 18, 12, 10, 9, and 8 inches diameter are also used on the Ordnance Survey, as well as those of smaller dimensions, of 7 and 5 inches.

* Platinum wire is the best adapted for the purpose, though cobwebs are generally used by surveyors; and as they are liable to break from the slightest touch, it is necessary that every person using a theodolite should be able to replace them himself. They must be stretched tight across the diaphragm, and confined in their places (indicated by faint notches on the metal) by gum, or varnish, the latter of which is to be preferred on account of its not being affected by the humidity of the atmosphere. The following simple and ingenious mode of fixing these cobwebs, which to a novice is often a difficult and tedious operation, was mentioned to me by Mr. Simms, who constructs all the mathematical and astronomical instruments for the Ordnance Survey. A piece of wire is bent into a shape something like a fork, the opening *a b* being rather larger than the diameter of the diaphragm. A cobweb being selected, at the extremity of which a spider is suspended, it is wound round the fork in the manner represented in the sketch, the weight of the insect keeping it constantly tight. The web is thus kept stretched ready for use; and when it is required to fix on a new hair, it is merely necessary to put a little gum or varnish over the notches on the diaphragm, and adjust one of the threads to its proper position.

these lines from the point is then corrected by the screws near the eye-piece working in the diaphragm, loosening one screw as that opposite to it is tightened. One or two trials will perhaps be required, the diaphragm being moved in *the contrary direction* to that which in the inverting eye-piece it appears to require.

The second adjustment is for the purpose of setting the *level attached to the telescope* parallel to the optical axis, and to the surface of the cylindrical rings on which it is supported; this is done by simply levelling the telescope by means of the tangent screw to the vertical arc, and then reversing it end for end in the Ys. If the air-bubble does not remain in the centre of the tube after this reversion, it must be corrected, *one half* of the error by the screw attached to one end of the level, and the remainder by the vertical arc. A few trials will be necessary to obtain this adjustment perfectly; and the level should be at the same time adjusted *laterally*, so as to be in the same vertical plane as the line of collimation, if it should be found, on moving the telescope *slightly* on either side, that the bubble becomes deranged from its central position.

The object of the third adjustment is to ensure the verticality of the axis of the instrument, and consequently the horizontal position of the azimuth circle, which is instrumentally at right angles to it. The level of the telescope already adjusted furnishes the means of effecting this. The instrument being placed approximately level, and the lower plate clamped, the upper plate is moved till the axis of the telescope is nearly over two of the opposite plate screws; the bubble of the telescope level is then adjusted by the vertical arc, and the upper plate turned round 180°; if the level is not in adjustment, half the error is to be corrected by the *plate screws*, and half by the tangent screw of the vertical arc. The same operation must be repeated with the telescope over the other pair of plate screws; and when, after several trials, the air-bubble of the level attached to the telescope remains constantly in the centre of the tube in whatever position it is turned, it is only necessary to *adjust the two small levels on the upper plate* to correspond, and they will serve to indicate when the axis of the instrument is vertical, care being taken to verify their adjustment from time to time.

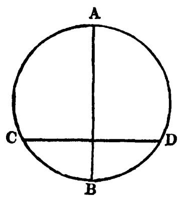
The vernier of the vertical arc is the last adjustment; it should indicate zero when all the above corrections have been made. If it differs from this point, it can be set to zero by releasing the screws by which the arc is held; but if the difference is small, it is better to note it as an *index error* +, *or* —, than to make the alteration.

A better plan of obtaining the index error of the vertical arc with accuracy is by observing reciprocal angles of depression and elevation from two stations about four hundred or five hundred yards distant. If none exists the angles will correspond; otherwise the errors will be equal, but in an opposite direction; and half their difference is the index error.

If the distance selected be too long, it becomes necessary to take into account the corrections for refraction and the curvature of the earth, depending upon the arc of distance, which subjects will be explained hereafter: but for the purpose of ascertaining the index error of the vertical arc of a theodolite, the distance named is quite sufficient.

The mean of all the verniers should invariably be taken,* and each angle repeated six or eight times. The errors of eccentricity and graduation of the instrument are thus almost annihilated, and those of observation of course much diminished. The repetition of angles is also the only means by which they can be measured with *any degree of minuteness by small instruments:* the large 2-feet theodolites used on the Ordnance Survey are in fact

* On the azimuth circle of the large theodolite used on the triangulation of the Ordnance Survey, the original verniers were only at the two opposite points A and B, the mean of the readings at which were, of course, always taken. Subsequently, the verniers at C and D were added, each of them equidistant 120° from A, and also from each other. It has since been sometimes the custom, first to take the mean of A and B, and afterwards the mean of A C and D, and to consider the mean between these two valuations as the true reading of the angle; this method has, however, been objected to as being incorrect in principle, an undue importance being given to the reading of the vernier A, and also in a smaller degree to B. The influence assigned to each vernier is, in fact, as follows :—A . 5; B . 3; C and D, 2 each. A theodolite of the same size and construction has been since made with four equidistant verniers.



portable altitude and azimuth instruments, and a short description of their construction and adjustments will be found at the end of Chapter XI., after the Problems.

The "Vernier" above alluded to is a subsidiary contrivance for measuring minute spaces between the graduated divisions of an arc of any instrument (or of any scale, such as that of the barometer), and consists of a slide moving with the index easily along the arc or scale with which it is in close contact. The space occupied by a certain number of the divisions on the limb of the instrument is equally divided in the vernier into either one more or one less than this number, generally the former, and the value of the intermediate space between any divisions on the limb is obtained by noting the coincidence of any division on the vernier with some other on the limb, which gives the difference between one of each of these two divisions multiplied by the number, as in each coincidence the zero of the vernier has to be moved through a space equal to the difference between one division of the instrument and one of the vernier. Call L the length of one division of the limb, and V that of one division of the vernier, and n the number of equal parts into which the vernier is divided—

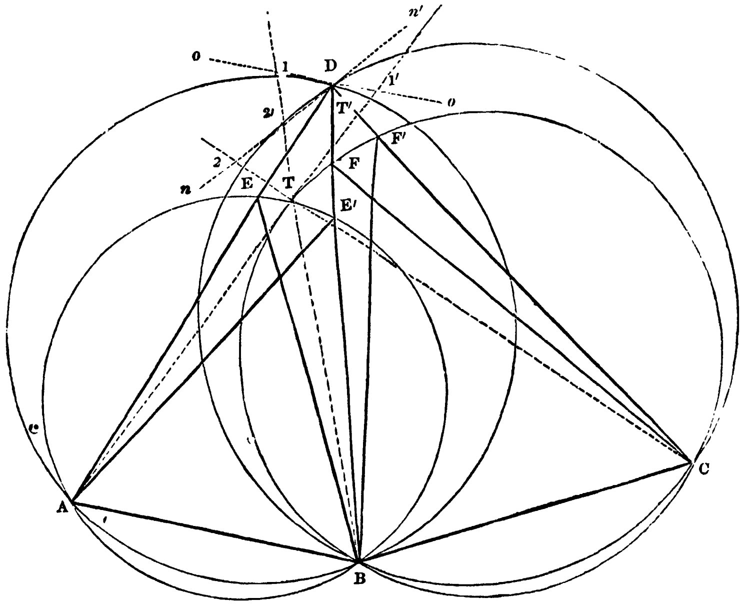
$$\text{then } L\,(n-i) = V n$$
$$\text{or } L n - L = V n$$
$$\text{whence } L n - V n = L$$
$$\text{and } L - V = \frac{L}{n};$$

that is the difference between each of the divisions on the respective scales is equal to $\frac{1}{n}$ of one of the divisions on the limb. As the number of divisions on the limb are limited by the size of the arc, the subdivisions of the intermediate space between each division by means of the vernier is also limited, and for very minute readings the *micrometer microscope* is substituted for it. Where the zero of the vernier corresponds with any division on the limb of the instrument, of course the observed angle is read without the assistance of the vernier. On a 9-inch sextant the arc is generally divided to 20 minutes, and 59 of such equal parts are made equal to 60 divisions of the vernier. In this case $L - V = \frac{20'}{60} = 20''$, which is the limit of the power of the vernier, the total length of which must be at least equal to $19° 40$.

The micrometer microscope consists of a system of lenses similar to those of an ordinary microscope, having across it a rectangular box (whose plane is perpendicular to the optical axis of the microscope) in which is placed the diaphragm, consisting of two parts, one moving over the other. One of these parts, containing the cross wires, is made to move freely with its accompanying index by means of a finely cut screw, the circumference of whose outer head is divided into 60 equal parts (or more if required), acting against the fixed part of the box and turned by a mill head, and the other part of a small comb or set of teeth used for noting the number of revolutions of the milled head, and capable of adjustment to secure the agreement of the zeros of the micrometer and the comb. The micrometer head can also be turned round on the screw, to be made to read zero when the cross wires bisect any division on a graduated scale or arc. The teeth of the comb agree with the divisions on the arc, whether they may be 5, 10, or 15 minutes. The micrometer head when divided into 60 parts represents by each division one second, and 5 revolutions of the micrometer are equal to a space of five minutes on the divided limb.

It is frequently necessary to refer to trigonometrical stations long after the angles have been observed; either for the purpose of fixing intermediate points, or of rectifying errors that may have crept into the work. Large marked stones should therefore be always buried under the principal stations which are not otherwise identified by permanent erections, and a clear description of the relative position of these marks with reference to objects in their vicinity should be always recorded. If, however, any station should be lost, and its site required to be ascertained for ulterior observations, the following method which was adopted by General Colby, will be found to answer the purpose with very little trouble and with perfect accuracy.

Let D be the lost station, the position of which is required. Assume T as near as possible to the supposed site of the point in question (in the figure the distance is much exaggerated to render the process intelligible), and take the angles A T B, B T C; A, B, and C being corresponding stations which have been previously fixed, and the distances of which from D are known. If the angle A T B be less than the original angle A D B, the point T is evidently *without* the circle in the segment of which the stations A and B are situated; if the angle be greater, it is of course *within* the segment. The same holds good with respect to the angles B T C and B D C.



Recompute the triangle A B D, assuming the angle at D to have been so altered as to have become equal to the angle at T, and that the angle at A is the one affected thereby.

Again, recompute the triangle, supposing the angle at B the one affected. In like manner in the triangle B D C recompute the triangle, supposing the angles at B and C to be alternately affected by the change in B D C. These computations will give the triangles A B E, A B E′, B C F, B C F′ calculated with the values of T, as observed at the first trial station (in both the present cases greater than those originally taken at D), and the angles at A, B, and C, alternately increased and diminished in proportion. Produce A T and B T, making T 1 and T 1′ equal respectively to E D and E′D, the differences between the distances just found and the original distances to the point D; and through the points 1 1′, which fall *nearly*, though not *exactly*, in the circumference of the circle passing through A B D, draw the line 0 0′. A repetition of the same process in the triangle B C D gives the points 2 2′, through which draw the line N N′, the intersection of which with 0 0′ gives the point T′, which is *approximately* the lost station required. Only two triangles are shown in the diagram, to prevent confusion, but three at least ought to be employed to verify the intersection at the point T′ if the original observations afford the means for doing so; and where the three lines are found not to meet, but form a small triangle, the centre of this is to be considered the second trial station, from whence the real point D is to be found by repeating the process described above, unless the observations taken from it prove the identity of the spot by their agreeing exactly with the original angles taken during the triangulation.

If the observed angle T′ be less than the original angle, the distances T1, T1′, T2, and T2′, must be set off towards the stations A, B, and C, for the point T′; and these stations should be selected not far removed from D, and forming triangles approaching as near as possible to being equilateral, as the smallest errors in the angles thus become more apparent. If the observations have been made carefully and with due attention to these points, the first intersection will probably give very near the exact site of the original station, or at all events a *third* trial will not be necessary.

To save computation on the ground, it is advisable to calculate previously the difference in the number of feet that an alteration of *one minute* in the angles at A, B, C, &c., would cause respectively in the sides A D, D B, D C, &c. The quantities thus obtained being multiplied by the errors of the angle at T, will give the distances to be laid off from T in the direction AT, BT. And in order also to avoid as much as possible any operations of measurement to obtain the position of the point T′, the distances from the trial station T should be laid down on paper on a large scale in the directions TA, TB, &c. (or on their prolongation), to obtain the intersection T′ of the lines 1 1′ and 2 2′ and from this diagram the angle formed at T with this point T′, and the line drawn in the direction of any of the stations A, B, or C, can be taken, as also the distance T T′; the measurement of one angle and one short line is all that is required on the ground.

The triangulation should never be laid down on paper until its accuracy has been tested by the actual measurement of one or more of the distant sides of the triangles as a base of verification, and by the calculation of others from different triangles to prove the identity of the results. Beam compasses, of a length proportioned to the distance between the stations and the scale upon which the survey is to be plotted are necessary for this operation; when the skeleton triangulation is completed, the next step is the delineation of the roads, &c., and the interior filling in of the country, either entirely or partially by measurement, as has been already stated.

The latitude and longitude of each of the trigonometrical stations were obtained with the most minute exactness on the Ordnance Survey, both by astronomical observations and by computation. For the latitude a zenith sector was used, which was constructed under the directions of the Astronomer Royal, and for which a portable wooden observatory was contrived. The instrument is placed in the plane of the meridian, and the axis, which has three levels attached, made vertical. In observing, the telescope is set nearly for a star, reading the micrometer microscope to the sector, and the observation is completed by the wire micrometer attached to the eye end of the telescope, the level readings and the time being also noted. The instrument is then turned half round, and the observation repeated, completing the bisection on this side by the tangent screw, again noting the levels and times, and lastly, the readings of the micrometer microscopes. The double zenith distance is thus obtained, from whence the latitude is determined, as explained in the Astronomical Problems. The latitudes and longitudes have lately been adapted to the Ordnance Maps publishing on the enormous scale of 6 inches to 1 mile, to *seconds* of latitude and longitude, with a very trifling maximum error, a triumph of practical science that some years since would have been deemed impossible.

CHAPTER IV.

INTERIOR FILLING-IN OF SURVEY, EITHER ENTIRELY OR PARTIALLY, BY MEASUREMENT.

THE more minutely the triangulation has been carried on, the easier and the more correct will be the interior filling-up, whether entirely by measurement with the chain and theodolite, or only partially so, the remainder being completed by sketching; the former of these methods will be first explained.—

On the Ordnance Survey the sides of the tertiary or minor triangles are actually measured with the chain between the nearest trigonometrical points (upon the accuracy of which they depend), the directions of the lines forming the sides of which have been partially selected with reference to the ultimate objects of the delineation of the boundaries of woods, estates, parishes, &c.* Where it is practicable, these lines should connect conspicuous permanent objects, such as churches, mills, &c.; and in all cases the old vicious system of measuring field after field, and patching these separate little pieces together, should be most carefully avoided.† The method of keeping the field-book in

* Great assistance is derived from a rough diagram representing the proposed method of proceeding, with references to the marks left on the measured sides of the triangles to be subsequently connected by cross or check lines, either joining two sides, or extending from one side to the opposite angle; this may appear at first to be a waste of time, but it will soon be found to be the contrary, as the lines will be all run in directions advantageous to the filling-up of the interior. These marks should be made on the ground, so as to be easily recognised, and should be copied in the margin of the field-book.

† Very excellent instructions for the guidance of surveyors employed in forming plans of estates and parishes are to be found in the report from the late Colonel Dawson, Royal Engineers, to the Tithe Commissioners of England and Wales, November, 1836, from which report Mr. Bruff, in his "Engineering Field-book," has extracted a number of valuable directions.

measuring the interior with the chain,* and plotting from its contents, is of course similar to the usual mode of surveying estates, parishes, &c.; and, as stated in the Preface, this preliminary knowledge is supposed to have been already acquired. But on an extensive survey *one general system must of necessity be vigorously enforced*, to insure uniformity in all the detached portions of detail.

Previous to commencing any measurement, the ground should be carefully walked over for the purpose of laying out the work, and marks set up at the average height of a theodolite on the highest parts of the different hills, on the necks of the ridges jutting out from them, and at the level of lakes and rivers in various parts of their course, as well as on the site of permanent objects such as churches, &c. These levelling marks should be all numbered and entered in a separate book, termed a field-levelling book, which also contains reciprocal angles of elevation and depression afterwards taken between them for the calculation of the horizontal values of the measured lines and of their comparative altitudes; these quantities are subsequently reduced to their actual heights above the level of the sea.† During the measurement of the principal lines, suitable points are selected at

* Gunter's chain is always used for surveying, though for mere lineal measurement chains of 50 or 100 feet are more convenient. It is 22 yards or 66 feet long, and divided into 100 links, 7·92 inches each; 10 square chains are equal to one acre, hence the area, when expressed in square chains and decimals, is converted into acres by merely dividing the amount by 10; if any decimals remain they are reduced to rods and perches by multiplying first by 4 and then by 40, thus :—

$$
\begin{array}{lr}
669{\cdot}146 \text{ square chains} = & 66{\cdot}9146 \\
& 4 \\
\hline
& 3{\cdot}6584 \\
& 40 \\
\hline
\text{Giving } 66\text{a. } 3\text{r. } 26{\cdot}336\text{p.} & 26{\cdot}3360
\end{array}
$$

The staff for measuring offsets is also divided into links.

† Among the advantages of connecting a well-arranged series of levels with the plan of any portion of country, is that of rendering it at once available to the engineer in selecting the best trial lines for roads, railroads, or canals. The system of tracing horizontal contour lines at short vertical intervals, instead of sketching the features of the ground, affords not only the means of deciding upon the best trial lines, but actually furnishes data for constructing accurate sections across the country in any direction.

which to connect them by check lines, or on which to base minor triangles, of course with a view to the determination of the natural and artificial boundaries, so that measured lines running near them, the whole of the interior content may be computed from the "Register" made out directly from the field book, the calculation from the *plot* being afterwards made simply as a check upon the other. All trigonometrical points and levelling marks should, if practicable, be measured up to with the chain during the progress of the survey, and their distinctive letters or marks entered in the field-books. Allowance may be made for short distances by holding up one end or portions of the chain till it appears horizontal, and dropping a pointed plummet on the ground in measuring up or down a slope, or by deducting the number of links corresponding to the angle of elevation or depression as marked on the reverse of the vertical arc of the theodolite;* but in all considerable distances this deduction is more correctly obtained by calculation from the data in the *field-levelling book*, kept in the following form:

From	To	Horizontal Reading.	Apparent Elevation or Depression.	Remarks.

The third column, headed "*horizontal reading*," is the reading of the vertical arc when the telescope is levelled, and is in fact the *index error*, which is however best determined by reciprocal angles of elevation and depression as before explained; and under the head of *remarks* are kept horizontal angles to surrounding objects

* The reduction marked on the reverse of the instrument can be made in the field by drawing the chain forward the stated number of links. It is, however, generally the practice upon the Ordnance Survey, to measure horizontal distances at once upon the ground, using in steep slopes only short portions of the chain, by which means all reductions and subsequent calculations are avoided. The forms given above and many of the directions are taken from the original instructions for the Interior Survey of Ireland.

and other collateral details. From the angles thus observed, and the known distances between the places of observation, is made out the following table:—

FORM OF REGISTER OF HORIZONTAL AND VERTICAL DISTANCES.

Plan and Plot.	Measured distances.	Elevation or Depression.	Calculations of Reductions to the Horizon.	Horizontal distances in links.	Calculation of vertical distances.	Relative altitude in feet.	Altitude above low-water mark.	Remarks.
A 2	B						355	Obtained by levelling.
	B̄ 12 54 C	4° 15′ 0″ Ele.	9,9988041 3,0982975 3,0971016	1251,5	9,8195439 8,8698680 3,0982975 1,7877094	61,33	416,33	
	C 984 D	3° 20′ 30″ De.	9,9992609 2,9929951 2,9922560	982,25	9,8195439 8,7655943 2,9929951 1,5781333	37,88	378,45	

This form almost explains itself: the first column refers to the plot or plan in which the points or lines are contained; the second shows the measured length of the line written between the letters marking its extremities; the third gives the mean elevation or depression of the second object deduced from the reciprocal angles in the levelling field-book after applying the correction for the index error in the third column of the same book, and also those for curvature and refraction when very long distances render their effect sensible; the fourth column contains the log. cosine of the angle in the preceding one, and the logarithm of the distance, the natural number answering to the sum of which is entered in the fifth column. The sixth contains the logarithm of 66=9·8195439 (the proportion of one link to one foot), the log. sine of the angle, and the log. of the distance; and the number answering to the sum of these three logarithms gives the relative altitude in feet, which is entered in the seventh column. The eighth column shows absolute altitudes above low-water mark, those that have been previously determined by levelling being entered in red: the others are obtained by the addition or subtraction of the altitudes in the preceding column.

The survey of the roads (though for the sake of saving unnecessary labour it is as much connected with them as possible) is sometimes quite independent of the measured triangles connecting churches or other permanent objects and the minor trigonometrical points, which lines mutually constitute a check upon each other. The term *traversing* is generally applied to this, and indeed to all irregular surveying by the chain and theodolite. On starting from any point in road surveying, the instrument being adjusted and set to zero, the telescope is directed upon one of the most conspicuous stations, and after taking two or three angles to other fixed points, the forward angle is read off in the direction it is intended to pursue, and the upper plate firmly clamped. On arriving at the end of this line the theodolite is set on the flagstaff or picket left at the back station, *the plates remaining still clamped to the last angle;* and the reading on the graduated limb when the telescope is pointed to the next forward station is not the number of degrees contained between these two lines, but the angle that this second line *forms with the first meridian, or the line upon which the theodolite was first set.* This method, now in general use among surveyors, saves the trouble of shifting the protractor at every angle when plotting the work and also insures greater accuracy, as the bearings being laid down from one meridian,* a trifling error in the direction of one line does not affect the next. As the work progresses of course other lines are selected as meridians; and it should be an invariable rule on beginning and ending a day's work, always to take the angles between the back or forward stations and any two or three fixed points that may be visible.

This rigidly mechanical method of surveying the interior evi-

* The readiest way of plotting lines whose directions have all reference to one meridian is by the use of a circular pasteboard protractor, with the centre cut out. A parallel ruler or angle (if the angle and ruler be preferred) is stretched across its diameter to the opposite corresponding angle, the zero having been first laid on the meridian line and moved forward to the point from whence the bearing is to be drawn. For surveys on a very large scale, however, the semicircular brass protractor, with a vernier, is better adapted, as being more minutely accurate.

dently leaves nothing to be afterwards filled up in the field, except the features of the ground, which is effected either by sketching or by tracing horizontal contour lines at fixed vertical intervals. The comparative heights obtained by levelling with the theodolite during the survey, present so many certain points of reference as to the relative command of the ground, and are of course of the greatest assistance in the subsequent delineation of the features upon the outline plan. Where the boundaries of parishes, townlands, &c., are to be ascertained and shown on the plan, there must be persons procured whose local knowledge can be depended upon, and whose authority to point them out to the surveyors is acknowledged.

The most accurate method of calculating the contents contained between the various boundaries of parishes, estates, &c.,* has been already stated to be from the data furnished by the field-book, in which case every measured figure must be either a triangle or a trapezoid. The diagram and the content plot must be first drawn,

* The contents even of the fields and other inclosures can be calculated from the field-book; but if the parishes and larger figures are so determined, the minute subdivisions of the interior may be taken from the plan. On the Ordnance Survey of Ireland, the number of acres in the different parishes, baronies, &c., were calculated, as also those covered by water, and given in a table accompanying the "Index Map" of each county; but the contents of the fields were not computed, though the hedges and other inclosures are shown on the plot. The contents of inclosures can be very quickly ascertained from the plan, by drawing lines in pencil about one or two chains distant across the paper, both longitudinally and transversely, or by laying a piece of transparent paper, so ruled, over it; the number of squares in each field are then counted, and the broken portions either estimated by the eye or reduced to triangles for calculation.

The "computing scale," upon a principle similar to the pediometer described at the end of this work, also affords the means of ascertaining mechanically the acreage of inclosures divided into triangles or trapeziums. It has been for many years in use at the Tithe Commission Office, for the purpose of calculating and checking the contents of plans surveyed under the Act of Parliament, and is productive of a great saving of time and expense. The principle of the construction of the pediometer depends upon the following equation, combined of the sum and difference of a diagonal of the trapezium and the two perpendiculars. Let a represent the diagonal, and b the sum of the two perpendiculars; then the area $\frac{a\,b}{2} = \frac{(\frac{1}{2}a + \frac{1}{2}b)^2 - (}{2}$

Acreages of inclosures, &c., are now obtained on the Ordnance Survey by the Computing Scale to the $\frac{1}{100}$ part of an acre with great rapidity.

in outline, and used as references during the calculation to prevent errors and to assist in filling up the content register, and from this the acreage of the different portions is taken. The annexed example of the field-book, with the diagram content plot and content register, all deduced from it, will better explain the details of this system.

In this specimen of a field-book, all offsets, except those having relation to the boundary lines (supposed to be of townlands, or any division of property, the contents of which are to be calculated from the field-book), are purposely omitted to prevent confusion, the example being given solely to illustrate the method of calculating these larger divisions. The rough diagrams are drawn in the field-book not to any scale, but merely bearing some sort of resemblance to the lines measured on the ground for the purpose of showing at any period of the work their directions and how they are to be connected; and also of eventually assisting in laying down the diagram and content plot. On these rough diagrams are written the distinctive letters by which each line is marked in the field-book, and also its length, and the distances between points marked upon it, from which other measurements branch off to connect the interior. The boundary lines are further distinguished from those run merely for the purpose of taking offsets to the minute subdivision of the property, &c. (and which, as before observed, are omitted in the present instance, both in the field-book and the plot), by dotted lines; so that in plotting the diagram to a scale, their difference is at once perceptible.

The form of keeping the field-book is similar to that practised on the Ordnance Survey, reference to the letters distinguishing former measurements being always made, the letter of the beginning and ending of every line by which it is designated in the diagram, being also written at the top and bottom of its representative in the field-book.

The construction lines all forming triangles, and offsets having reference to the boundaries, are retained in the content plot, and tend to prevent mistakes in the calculation.

In the content plot and diagrams* the trigonometrical points

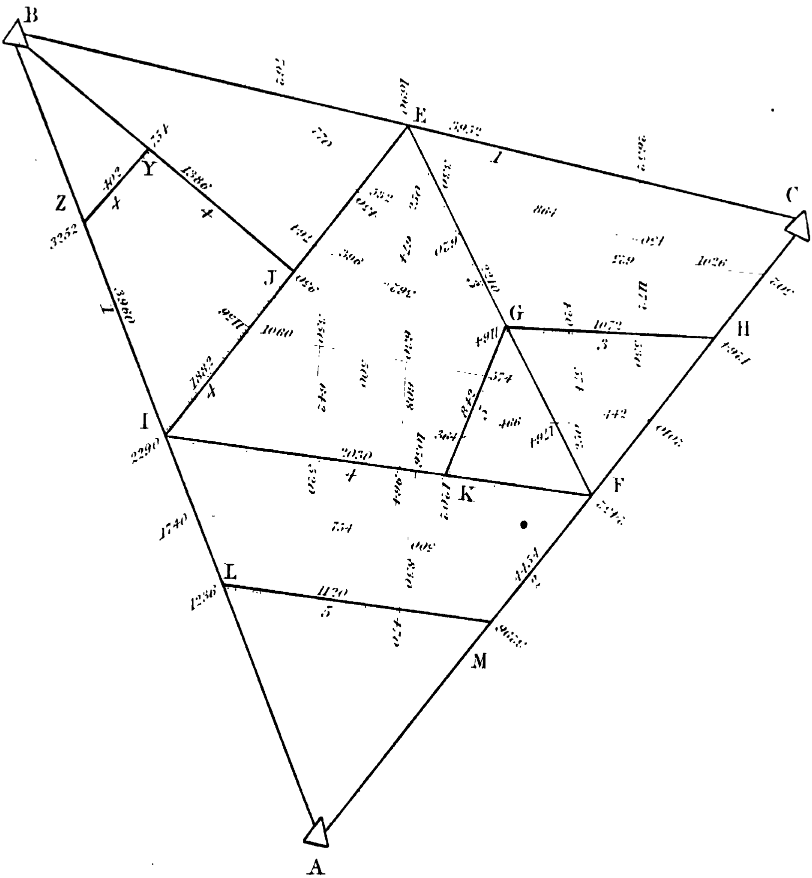
* It is now usual on the Ordnance Survey to combine the information contained in the "Content Plot" and "Diagram" in one drawing.

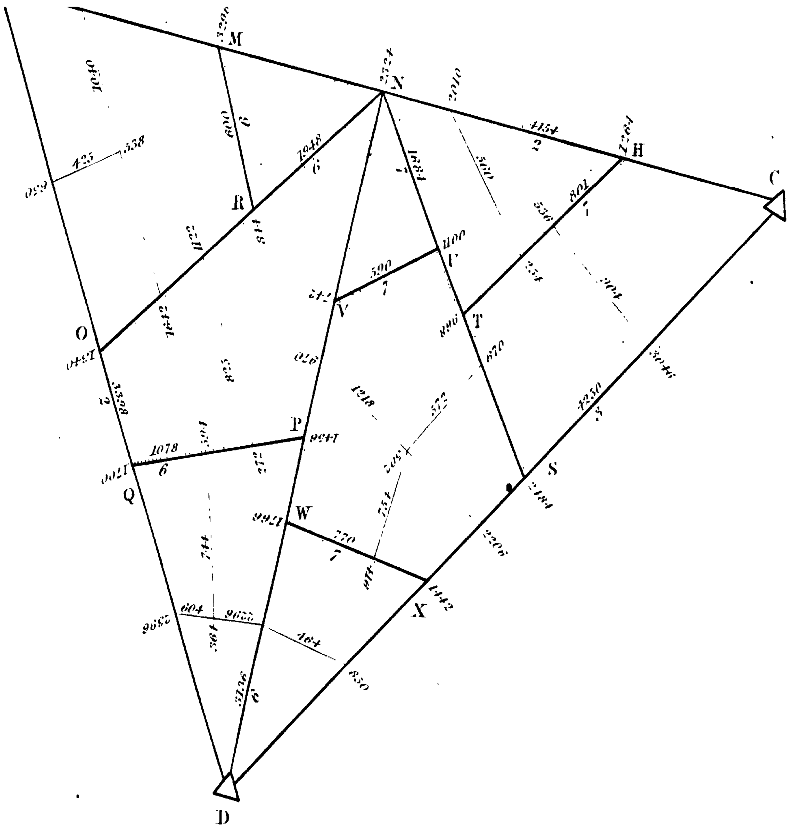
A, B, C, D, are on an average rather more than half a mile apart, so that in reality the same number of divisions of townlands would not occur in the space comprised within them; and, instead of letters, they would be distinguished by the name of the townland or parish.

The large letter B 2 on the diagram of the triangle A B C refers to the *distinctive mark* of the field-book; and the small figures 3, 4, 5, &c., written along the construction lines, to the *different pages* of the same book, to which reference can thus be made at any moment.

The contents only of the *large divisions* are calculated from the field-book. Those of the minute inclosures are (if required) obtained by the computing scale from the plot, from which the contents of townlands and parishes are also computed, for the purpose of checking the previous calculations.

The method of calculating these contents by means of the measured triangles and offsets will be easily comprehended by comparing together the *field-book, content plot,* and *content register,* for the triangle C A D. That for A B C, being on exactly a similar principle, has been omitted, as it could add nothing to the explanation of the system.


B
C
A
E
Y
Z
J
G
H
I
K
F
L
M



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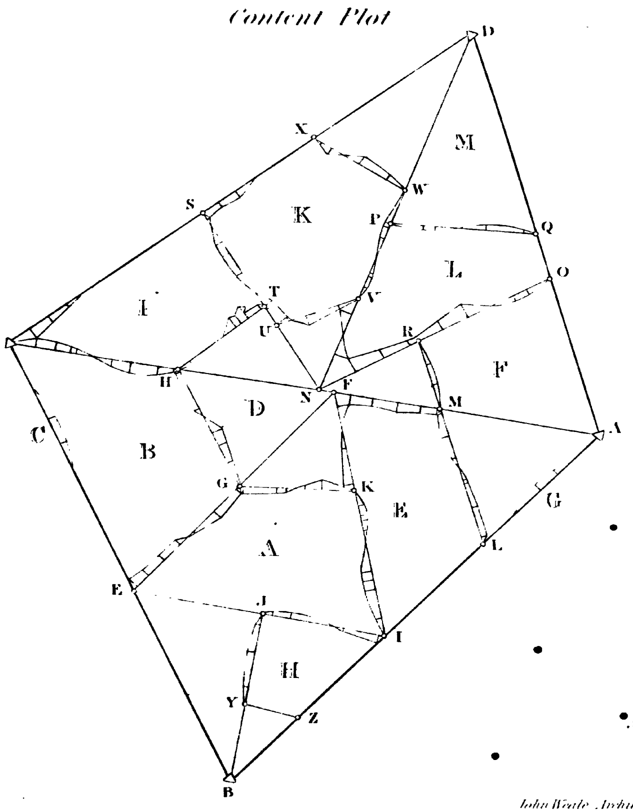

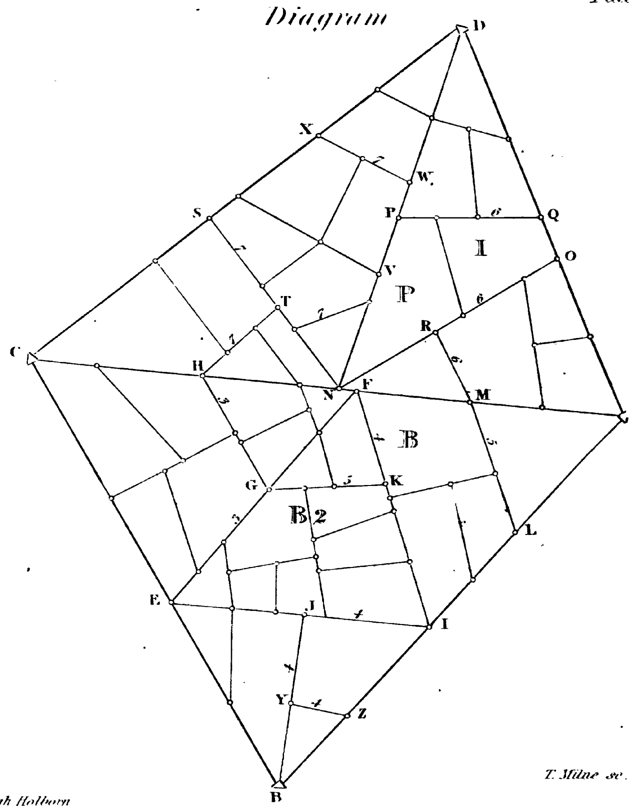

Scale 6 inches to 1 Mile

T. Milne sc.

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CONTENT REGISTER-TRIANGLE C A D.—Plate 4.

Plan and Plots.	Division or Sub-division.	Triangle or Trapezium.	1st Side.	2nd Side.	3rd Side.	Content in Chains.	Content in Statute Acres.
	Triangle.	A C D	4454	3398	4250	679·5032	
		C N S	2324	1766	1684	148·0516	
		X S	—	60	150	·4500	
			60	40	126	·6300	
			40	72	152	·8512	
						1·9312	
	I	S T	—	4	73	·0146	
			4	—	53	·0106	
	Additives.		—	56	150	·4200	
						·4452	
		T H	—	36	166	·2988	
			36	40	144	·5472	
			40	—	151	·3020	
						1·1480	
				Total	Additives	151·5760	
		H T N	801	1028	788	31·1374	
		S T	—	52	164	·4264	
			52	74	96	·6048	
			74	46	174	1·0440	
			46	20	140	·4620	
			20	—	46	·0460	
						2·5832	
		T H	—	40	42	·0840	
			40	46	100	·4300	
			—	30	24	·0360	
			30	—	32	·0480	
			102	76	64	·5696	
			76	90	48	·3984	
			90	—	86	·3870	
	I					1·9530	
	Negatives.	C H	—	82	220	·9020	
			82	70	122	·9272	
			70	50	108	·6480	
			50	34	52	·2184	
			34	—	102	·1734	
						2·8690	
		S C	—	42	190	·3990	
			42	62	160	·8320	
			62	100	160	1·2960	
			100	—	200	1·0000	
						3·5270	
				Total	Negatives	42·0696	
				Total	Additives	151·5760	
					Difference	109·5064	10·95064
	B Additives.	C H	See	above	...... Negatives *none.*	2·8690	·28690

Plan and Plots.	Division or Sub-division.	Triangle or Trapezium.	1st Side.	2nd Side.	3rd Side.	Content in Chains.	Content in Statute Acres.
	J Additives. Negatives.	S C	Page 39	. .	. . *None.*	3·5270	
	D Additives.	H T N T H	Page 39	. .	. .	33·0904	
		N U V	584	742	590	16·8759	
		N R M	844	972	600	25·1184	
		T U	56	54	34	·1870	
			54	98	170	1·2920	
						1·4790	
		U V	—	20	96	·0960	
			20	—	44	·0440	
						·1400	
		N V	—	104	332	1·7264	
			104	—	122	·6344	
						2·3608	
		N R	—	136	320	2·1760	
			136	90	204	2·3052	
			90	46	218	1·4824	
			46	30	68	·2584	
			30	36	34	·1122	
						6·3342	
		R M	—	30	256	·3840	
			30	26	164	·4592	
			26	—	180	·2340	
						1·0772	
				Total	Additives	86·4759	
	D Negatives.	T H	Page 39	. .	. .	1·1480	
		U V	—	90	72	·3240	
			90	48	228	1·5732	
			48	—	150	·3600	
						2·2572	
		N V	—	100	162	·8100	
			100	—	126	·6300	
						1·4400	
				Total	Negatives	4·8452	
				Total	Additives	86·4759	
					Difference	81·6307	8·16307
	F Additives.	A N O	2130	1340	1948	127·8318	
		R O	36	62	176	·8624	
			62	110	230	1·9780	
			110	—	160	·8800	
						3·7204	
				Total	Additives	131·5522	

Plan and Plots.	Division or Sub-division.	Triangle or Trapezium.	1st Side.	2nd Side.	3rd Size.	Content in Chains.	Content in Statute Acres.
		N R M / R M	Page 40	. .	. .	26·1956	
	F	R O	—	36	232	·4176	
			36	50	109	·4687	
	Negatives.		50	—	197	·4925	
						1·3788	
				Total	Negatives	27·5744	
				Total	Additives	131·5522	
					Difference	103·9778	10·39778
		D N O	3136	2058	1948	195·3072	
		N V	Page 40	. .	. .	1·4400	
		R O	See	above	. .	1·3778	
	L	V P	—	52	214	·5564	
			52	50	96	·4896	
	Additives.		50	30	36	·1440	
						1·1900	
		P Q	—	50	174	·4350	
			50	30	292	1·1680	
			30	—	66	·0990	
						1·7020	
				Total	Additives	201·0180	
		D P Q	1680	1698	1078	86·2650	
		N V / N R / R O	Page 40	. .	. .	12·4154	
		V P	—	36	110	·1980	
			36	40	88	·3344	
	L		40	30	30	·1050	
			30	—	140	·2100	
	Negatives.					·8474	
		P Q	—	52	250	·6500	
			52	30	80	·3280	
			30	—	216	·3240	
						1·3020	
				Total	Negatives	100·8298	
				Total	Additives	201·0180	
					Difference	100·1882	10·01882
		D P Q / P Q	See	above	. .	87·5670	
		D W X	1370	1442	770	51·8339	
	M	P W	30	—	310	·4650	
		W X	—	56	114	·3192	
	Additives.		56	36	104	·4784	
			36	—	90	·1620	
						·9596	
				Total	Additives	140·8255	

Plan and Plots.	Division or Sub-division.	Triangle or Trapezium.	1st Side.	2nd Side.	3rd Side.	Content in Chains.	Content in Statute Acres.
		P Q	Page 41	. .	. . .	1·7020	
		W X	—	52	142	·3692	
			52	64	232	1·3456	
	M		64	—	88	·2816	
						1·9964	
	Negatives.			Total	Negatives	3·6984	
				Total	Additives	140·8255	
					Difference	137·1271	13·71271
		D N S	3136	2484	1684	208·1249	
		S T	Page 39	. .	. . .	2·5832	
	K	U V	,, 40	. .	. . .	2·2572	
		V P	,, 41	. .	. . .	·8474	
	Additives.	W X	See	above.	. . .	1·9964	
				Total	Additives	215·8091	
		X S, S T	Page 39	. .	. . .	2·3764	
		T U, U V	,, 40	. .	. . .	1·6190	
	K	V P, P W, W X, D W X	,, 41 ,,	. .	. . .	54·4485	
	Negatives.	N U V	,, 40	. .	. . .	16·8759	
				Total	Negatives	75·3198	
				Total	Additives	215·8091	
		END OF △	A C D		Difference	140·4893	14·04893

INDEX.

	Triangle	A C D	Page 39	. .	. . .	679·5032	
		I, B	,, 39	. .	. . .	109·5064 2·8690	
		J, D	,, 40	. .	. . .	3·5270 81·6305	
		F, L	,, 41	. .	. . .	103·9778 100·1882	
		M, K	See	above.	. . .	137·1271 140·4893	
					Divisions or Townlands	679·3155	67·9315
					Triangle A C D . .	679·5032	67·9503
					Difference	·1877	

It may perhaps be thought that too much stress has been laid upon *forms* in the above description of the details of an extensive survey; but *method* is a most essential part of an undertaking of such magnitude, and without excellent preliminary arrangements to insure uniformity in all the most trifling details, the work never could go on creditably. In topographical surveys on a smaller scale, where the boundaries of parishes, &c., are not to be shown, or the contents of various portions to be calculated, the same rigid attention to minutiæ is not requisite; but before closing this branch of the subject, it is only necessary, as a proof of the mass of valuable statistical and geological information that can be collected during the progress of a national trigonometrical survey, and which is quite out of the reach of any individual, to turn to the first volume of "The Ordnance Survey of the County of Londonderry." If this valuable accompaniment to the field operations could have been continued throughout every county, Ireland would be possessed of more available local knowledge than is on record in any part of the world.

The following brief hints may be found useful in filling in the detail of a survey with the chain and theodolite.

The field-book should be kept in ink in the field, and have a distinctive letter marked on it as a reference; *every day's work* should be dated, and the names of those employed entered. On an extensive survey it is also necessary that every book should be kept on precisely *the same system*, in order that no one person might find any difficulty in plotting from the book of another.

The theodolites should be constantly examined and adjusted, and the chains compared every day with a standard chain, or with marks laid down from one for that purpose, and their errors, if any, either corrected or entered in the field-book, to be allowed for in plotting. The offsets should be numerous, and minute in proportion to the scale upon which the survey is to be plotted, *

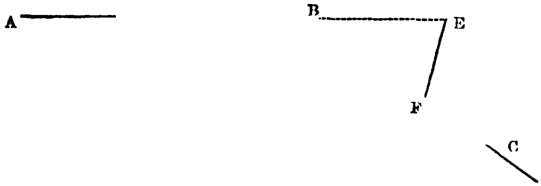
* From one to two chains should be the maximum length of offsets where the contents of inclosures are to be computed, or even laid down on a large scale. These limits must of course be extended in filling in the interior in less accurate surveys, or which are to be plotted on a very small scale. As drawing-paper is very much stretched when mounted on a board, and partially contracts when cut off, and as it is always liable to change from the atmosphere, it is a good precaution to divide the

and the names of all towns, villages, &c., carefully noted, and care taken to insure their correct orthography, and to quote the authority upon which it rests when different from that sanctioned by custom.

In measuring long lines between conspicuous objects, marks should be left, to be afterwards connected by check lines, or on which to base smaller triangles; where impeded by a house or any obstacle, the means of avoiding it and returning again to the measured line are to be found further on.

Irregular inclosures and roads, even where triangles cannot be measured, can still be surveyed by the *chain alone*, but of course not so accurately as with the aid of the theodolite.

This method of "traversing" is managed as follows:—Suppose A B the first line, and B C the direction in which the next is



required to be measured, prolong A B to E, make B F equal to B E, and measure the cord E F, from which data the direction of B C can be laid down.

The dimensions in the field-book may be kept either between two parallel lines running up the page, with the offsets written on the right and left of these lines as in the example facing page 38, or on a species of diagram bearing some sort of resemblance to the outline of the ground to be surveyed, which latter method is supposed to assist in the plotting; but if references to the starting points of the different lines, and their junctions with each other, are entered in the field-book kept according to the first system,

scale for laying off distances from the field-book, on the paper upon which the plot is to be made, as it will then always expand and contract with the outline of the survey; and also to mount the paper *before commencing plotting, or not at all.* If possible, the drawing-room should be kept at the same temperature, and drawing-paper should be kept in it for some time before it is used.

and the angles forward written on the right or left of the ruled lines, according to the direction of the next forward station, there can never be any difficulty in plotting the work, even after a considerable lapse of time, though it should not be delayed longer than is absolutely necessary. It is customary for land-surveyors to compute their work from the plot, adding up the contents of each inclosure for the general total, which is perhaps checked by the calculation of two or three large triangles ruled in pencil so as to correspond nearly to the extreme boundaries whose lengths are taken from the scale; but if the rigid mode of computing everything from the field-book is deemed too troublesome, still the areas of the large triangles, *measured on the ground*, should be calculated *from their dimensions taken from the field-book*, and the contents of the irregular boundaries added to or subtracted from this amount, which constitutes a far more accurate check upon the sum of the contents of the various inclosures than the method in general use. The calculation of irregular portions outside these triangles is much facilitated by the well-known method of reducing irregular polygons to triangles having equivalent areas.

When the contents of fields are to be calculated from the plot, which is most rapidly and easily done by the computing scale, the scale should not be less than twenty, and may be as much as three or four chains to one inch. The former of these two last scales is that on which all plans for railroads submitted to the House of Commons are required to be drawn, and the latter is used for plans of estates, &c.

To return to the second division of this subject, viz. the filling up of the interior partly by measurement and partly by sketching, which is generally the mode adopted in the construction of topographical maps.

The roads, with occasional check lines, are measured as already described, the field-book being kept in the same method as when the entire county is to be laid down by measurement, excepting that all conspicuous objects some distance to the right and left of the lines are to be fixed by intersections with the theodolite, either from the extremities of these lines or from such intermediate points as appear best adapted for determining their positions. These points when plotted, together with the offsets* from the field-book, present so many known fixed stations between the measured lines, and of course facilitate the operation of sketching the boundaries of fields, &c., and also render the work more correct, as the errors inseparable from sketching will be confined within very narrow limits.

In all cases where the compass is used to assist in filling-in the interior (*and it should never be trusted in any more important part of the work*), it becomes of course necessary to ascertain its variation by one of the methods which will be hereafter explained. Independent of the annual change in its deviation, the horizontal needle is subject to a small daily variation, which is greatest in summer, and least in winter, varying from 15′ to 7′. Its maximum on any day is attained to the eastward about 7 A.M., from which time it continues moving west till between 2 and 3 P.M., when it returns again towards the east;† but this oscillation is too small to be appreciable, as the prismatic compass used in the field cannot be read to within one-half, or at the nearest one-quarter, of a degree of the truth. Portions of the work as plotted from the field-book, are then transferred to card-board or drawing-paper, or traced off on thin bank post paper, which latter has this advantage that it is capable of being folded over a piece of Bristol board fitting into the portfolio, and from its large size, it will contain on the same sheet distant trigonometrical points which will constantly be of use in the field. It can be folded

* Mr. Holtzapfell's "Engine-divided Scales," engraved on pasteboard, will be found very useful, and their low price is an additional recommendation. Marquois scales are also adapted for plotting and drawing parallel lines at measured intervals, as well as for other purposes. The offset and plotting scales, introduced by Major Robe on the Ordnance Survey, are as convenient as any that have been contrived. The plotting scale has one bevelled edge; and the scale, whatever it may be, engraved on each side, is numbered each way from a zero line. The offset scale is separate, and slides along the other, its zero coinciding with the line representing the measured distance; the dimensions are marked on the bevelled edge of this short scale to the right and left of zero, so that offsets on either side of the line can be plotted without moving the scales; and from the two being separate, there is not the same chance of their being injured, as in those contrivances where the plotting and offset scales are united.

† See Colonel Beaufoy's experiments on the variation of the needle. Also the article Observatory (Magnetical), *Aide Mémoire*.

over the pasteboard, so as to expose any portion that may be required, and when the work is drawing near to the edge it is only necessary to alter its position. In moist weather prepared paper, commonly termed asses' skin, is the only thing that can be used, as the rain runs off it immediately without producing any effect on the sketch.

The portable instruments generally used in sketching between measured lines and fixed points in the interior, as well as in military sketches made in the exigency of the moment sometimes without any measurement whatever, are a small 4-inch, or box sextant, or some other small reflecting instrument,* and the azimuth prismatic compass. The box sextant is, in its principle and adjustment, nearly similar to the sextant described among astronomical instruments in the opening of Chapter XI.. It is, as its name indicates, inclosed in a box, with a lid which is unscrewed when required for use. The index, instead of moving by the hand, and being adjusted when clamped by the tangent screw, has a motion given to it by a rack and pinion in the box, moved by a milled head. The dark glasses are also within the box, and let down out of the way when not required for use. The only

* In using *reflecting instruments*, avoid *very acute angles*, and do not select any object for observation which is *close*, on account of the parallax of the instrument. The brightest and best defined of the two objects should be the *reflected* one; and if they form a very obtuse angle, it is measured more correctly by dividing it into two portions, and observing the angle each of them makes with some intermediate point. Also if the objects are situated in a plane *very oblique to the horizon*, an approximation to their horizontal angular distance is obtained by observing each of them with reference to some distant mark considerably to the right or left, and taking the difference of these angles for the one required.

The *index error* of a sextant must be frequently ascertained. The measure of the diameter of the sun is the most correct method; but for a box sextant, such as is used for sketching, it is sufficient to bring the direct and reflected image of any well-defined line, such as the angle of a building (not very near) into coincidence—the reading of the graduated line is then the index error. For the adjustment of the box sextant, see Simms on Mathematical Instruments. The less the glasses are moved about the better.

A telescope with a wire or prism micrometer is also a most useful instrument in sketching, as with it distances can be approximately ascertained by observing the angle subtended by any distant object, such as a man, &c. If a measured staff can be set up at these points, the distances thus obtained will of course be more accurate. For a description of Rochon's micrometer, see page 70.

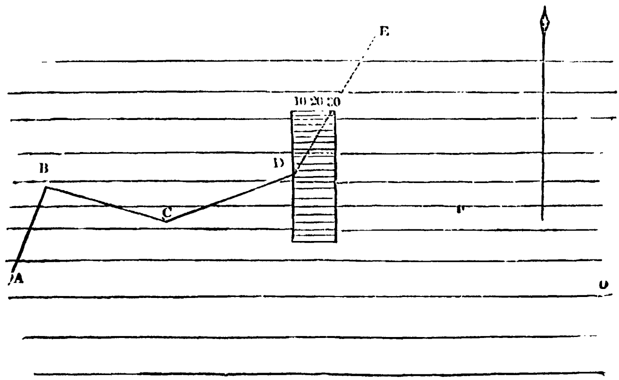
adjustment provided for is that of the horizon glass, which can be set perpendicular to the plane of the instrument, and the index error corrected by a key which is tapped into the box. The small telescope which fits into the case, or is made to slide into the box, is not necessary for very rough observations, which can be made through an aperture in the slide covering the opening for the telescope. The divisions are generally graduated to 30′, and are read by the aid of a magnifying glass, which revolves so as to sweep the whole of the arc.

The Prismatic Azimuth Compass above alluded to is chiefly used for taking bearings with the magnetic meridian in sketching ground for military purposes, or for filling in the interior details of a survey, though it can be made available for observing roughly the azimuth of the sun or a star. The box of the compass is generally about 3 inches diameter, and the divisions on the card, graduated to 30 minutes (in instruments of larger diameter to 15′) are read eastward of the meridian round the whole circle of 360° by means of a prism (whence its name) when the perpendicular thread or wire of the sight bisects the object, which thread appears when viewed through the prism, to be prolonged across the card, marking the division to be read. To the sight is attached a mirror, which slides up and down the frame, and can be set to any angle of inclination for the purpose of reflecting to the eye of the observer the image of any object which is much above or below the horizontal plane, and is indispensable for measuring the azimuth of the sun, for which one or more of the dark glasses attached to the prism must be used. The vibrations of the card are checked by means of a spring under the sight, and the card, with the needle below it, is thrown altogether off the agate point upon which the latter works by a stop at the side, a precaution always to be taken when the instrument is not in use.

In observing, the prism should first be raised or lowered on its slide, to obtain distinct vision of the magnified divisions of the card, and if horizontal angles between any distant objects are required, they are obtained by taking the difference of the observed bearings, though not probably within half a degree.

Few prismatic compasses are found on trial to give precisely similar results, and it is therefore essential that an instrument of this kind should be carefully tested by comparison with a meridian line, noting the difference between its bearing and the known variation as an index error. Any reflecting instrument is certainly capable of observing angles between objects nearly in the same horizontal plane with more accuracy than the compass; and from its observations being instantaneous, and not affected by the movement of the hand, it is better adapted for use on horseback, but it is not so generally useful in filling up between roads, or in sketching the course of a ravine or stream, or any continuous line.

Whichever of these instruments is preferred, of course a scale of chains, yards, or paces, and also a protractor, are required for laying off linear and angular distances in the field.

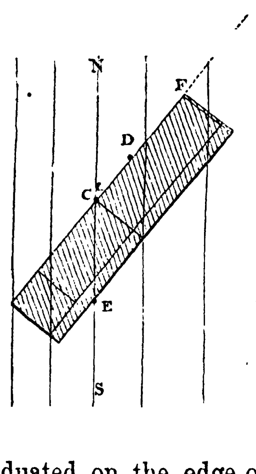


A very convenient method of using the latter for protracting bearings observed with the azimuth compass, is to have lines engraved transversely across the face of the protractor at about a quarter of an inch apart. The paper upon which the sketch is to be made must also be ruled faintly across in pencil *at short unequal distances*, at right angles to the meridian, with which lines one or more of those on the protractor can be made to correspond by merely turning it round on its zero as a pivot, this point being kept in coincidence with the station from whence the bearing is to be drawn. The bevelled edge of the protractor is thus evidently parallel to the meridian, and the observed bearing being marked and ruled from this point is the angle made by the object with the meridian.

For instance, the bearing of a distant object upon which it is required to place, was observed from D to be 30°. The protractor in the sketch is shown in the proper position for laying off this angle, and the dotted line DE is the direction required.

In fixing the position of any point with the compass, by bearings taken *from that point* to two or three surrounding stations whose places are marked on the paper, the zero of the protractor is made to coincide with one of these stations, and its position being adjusted by means of the lines ruled across its face and on the paper, the observed angle is protracted *from this station*, and produced through it. The same operation being repeated at the other points, the intersection of these lines gives the required place of observation.

Instead of the above system of ruling east and west lines across the paper, lines may be drawn *parallel* to the meridian for adjusting the place of the protractor. Thus, suppose from the point D any observed bearing, say 40°, is to be laid down. By placing the zero C of the protractor on any convenient meridian, and turning it upon this point as a pivot until the required angle of 40° at E coincides also with the same meridian N S, it is only necessary to move the protractor, held in this position, slightly up and down upon this line, until its bevelled edge touches the point D; D F is then at once drawn in the required direction. The distances may also be set off from a scale graduated on the edge of the protractor, by merely moving it along this line, D F, until some defined division corresponds with the station D.



By observing with a sextant the angles between three or more known stations, the place of the observer can be ascertained both instrumentally and by calculation, but not so readily as with the compass. The method of thus determining the position of any point will be explained hereafter.

The plane table is perhaps the best contrivance for sketching in the interior detail of a survey with accuracy, but its size renders it too inconvenient to be termed portable, and its use is now almost universally superseded by the portfolio and compass. The little reflecting semicircle invented by Sir Howard Douglas, is so far an improvement on the sextant that it *protracts the angles it observes* by means of a contrivance by which the reflected angle is doubled instrumentally, and the angle is protracted upon the paper by means of a bevelled projection of the radius. Other varieties of small reflecting instruments have also been contrived for the same purpose.

The process of sketching between the fixed points plotted on the paper is similar to surveying with the chain and theodolite as far as the natural and artificial boundaries are concerned; *the distances being obtained by pacing; the offsets (if small) by estimation; and the bearings of the lines by the compass or sextant.** Everything is however here drawn at once upon the paper, instead of being entered in a field-book. The features of the ground are sketched at the same time as the boundaries and other details; and this part of the operation, being less mechanical than the preceding, requires far more practice before anything like facility of execution can be acquired; it is, however, more particularly connected with the subject of the next chapter, where the different methods of delineating ground in the field will be explained.

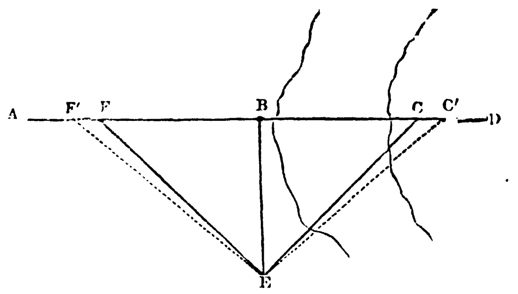
The following are the best practical methods of passing obstacles met with in surveying, and of determining distances which do not admit of measurement by means adapted for use in the field, most of them requiring no trigonometrical calculation. Some of these problems are solved without the assistance of any instrument for observing angles; but as a general rule (subject of course to some few exceptions), it is always better to make use of the theodolite, sextant, or other portable instrument, than to

* A straight walking-stick will be found very useful in sketching, not only for the purpose of getting in line between two objects, which is easily done by laying the stick on the ground, in the direction of one of them, and observing by looking from the other end to which side of the opposite station it cuts, but also for prolonging a line directed on any known point to the rear. A bush or any other mark, observed in the line of the stick, answers as well as another known point for pacing on.

endeavour by any circuitous process to manage without angular measurement.

The measurement of the line A D, supposed to be run for the determination of a boundary, is stopped at B by a river or other obstacle.

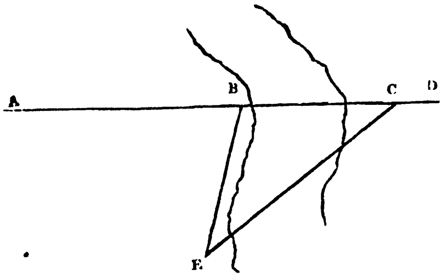
The point F is taken up in the line at about the estimated breadth of the obstacle from B; and a mark set up at E at right angles to AD from the point B, and about the same distance as BF. The theodolite being adjusted at E, the angle BEC is made equal to BEF, and a mark put up at C in the line AD; BC is then evidently equal to the measured distance FB.



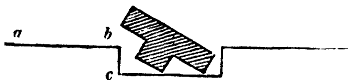
If the required termination of the line should be at any point C′, its distance from B can be determined by merely reversing the order of the operation, and making the angle BEF′ equal to BEC′, the distance BF′ being subsequently measured. There is no occasion in either case to *read* the angles. The instrument being levelled and clamped at zero or any other marked division of the limb, is set on B; the *upper plate* is then unclamped, and the telescope pointed at F, when being again clamped, it is a second time made to bisect B; releasing the plate, the telescope is moved towards D till the vernier indicates zero, or whatever number of degrees it was first adjusted to, and the mark at C has then only to be placed in the line AD, and bisected by the intersection of the cross wires of the telescope.

If it is impossible to measure a right angle at B owing to some local obstruction, lay off any convenient angle ABE, and set up the theodolite at E.

Make the angle BEC equal to *one-half of* ABE, and a mark being set up at C in the prolongation of AB, BC is evidently equal to BE, which must be measured, and which may at the same time be made subservient to the purpose of delineating the boundary of the river.

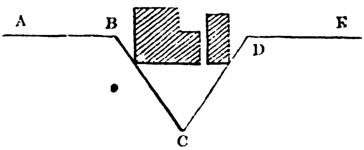


The usual way of avoiding an obstacle of only a chain or two in length,



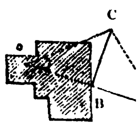
such as a house or barn, is by turning off to the right or left at right angles till it is passed, and then returning in the same manner to the original line.

But perhaps a more convenient method is to measure on a line making an angle of 60° with the original direction, a distance sufficient to clear the obstacle, and



to return to the line at the same angle, making C D = B C ; the distance B D is then equal to either of these measured lines.

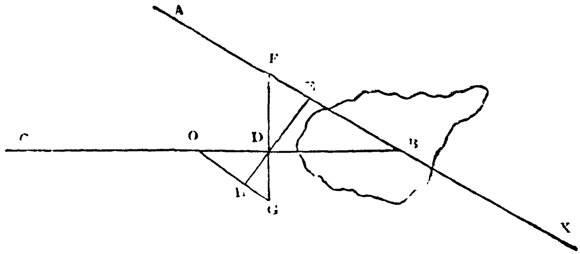
The distance from B on the line A *o*, to the trigonometrical point *o*, which is inaccessible, is determined in the manner explained in the first method in the last page; the point C is taken at right angles to B A from the point B, and the angle B C D made equal to *o* C B, B D is then equivalent to the distance B *o* required. The same object



is attained by laying down the plan of the building and these angles on a large scale, and taking the distance B *o* from the plot.

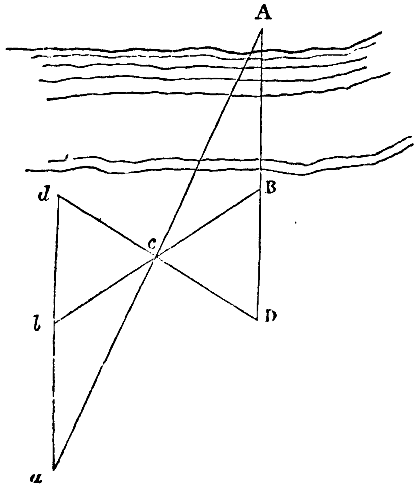
To find the point of intersection of two lines meeting in a lake or river, and the distance D B to the point of meeting:—From any point F on the line A X draw F D, and from any other point

E draw E D, produce both these lines to H and G, making the prolongations either equal to the lines themselves, or any aliquot part of their length, suppose one-half; join H G, and produce it



to O, where it meets the line C B, then O H is one half of E B, and O D equal to half of D B; which results give the point of intersection B, and the distance to it from D.

To find the distance to any inaccessible point, on the other side of a river for instance, without the use of any instrument to

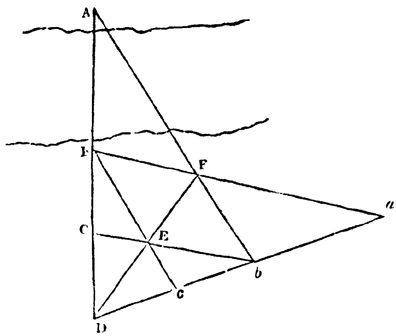


measure angles.—(*This and the two following are taken from the "Aide Mémoire."*) A is any inaccessible point the distance of which from B is required: produce A B to any point D; draw D *d* in any direction bisected in C; join B C and produce it to *b*, C *b* being made equal to B C; join *d b* and produce it to *a*, the intersection of the prolongation of A C, then

$$\left.\begin{array}{r} ab = AB \\ \text{and } ad = AD \end{array}\right\} \text{The proof is evident.}$$

Another method—

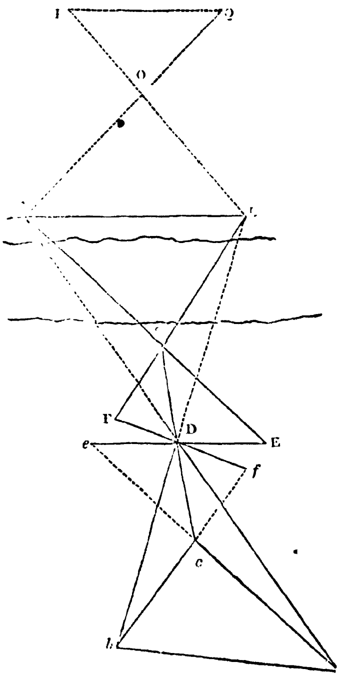
Prolong A B to any point D, making B C equal to C D; lay off the same distances in any direction D c $=c\,b$; mark the intersection E of the line joining B c and $c\,b$; mark also F the intersection of D E produced, and of A b; produce D b, and B F, till they meet in a, and



$$\left.\begin{array}{l} a\,b = \mathrm{A\,B} \\ a\,c = \mathrm{A\,C} \\ a\,\mathrm{D} = \mathrm{A\,D} \end{array}\right\}$$

To measure the distance between A and B, *both* being inaccessible:—From any point C draw any line C c bisected in D; take any point E in the prolongation of A C, and join E D, producing the line to D e=E D; in like manner take any point F in the prolongation of B C, and make Df=F D.

Produce A D and $e\,c$ till they meet in a, and also B D and $f c$ till they meet in b; then $a\,b$= A B.



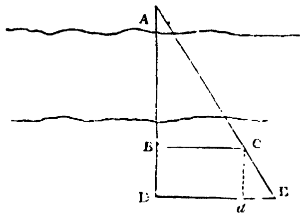
Again, if A B cannot be measured, but the points A and B are accessible, their distances from any point O are determined; and by producing these lines any *aliquot part* of their length, as O P, O Q, the distance P Q will bear the same proportion to A B.

A right angle* can often be laid off when no means of measuring other divisions of the circle are at hand. The distance A B can then be thus obtained:—

B C and D E are both perpendicular to A D, and the points E and C are marked in a line with A; then

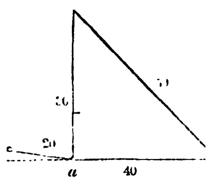
$$AB = \frac{BD.\ BC}{(DE - BC)}.$$

The small triangle C *d* E being similar to A B C.



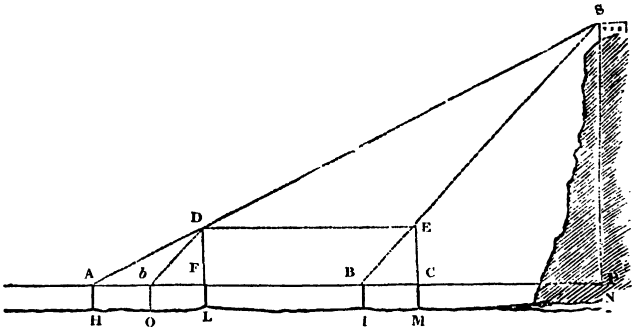
Of course with a sextant, or other means of observing the angle A C B, A B becomes simply the tangent of that angle to the radius B C: a table of natural sines and tangents engraved on the lid of the box-sextant, or on any portable reflecting instrument is often of great service, particularly in sketching ground without any previous triangulation, and in obtaining the distance to an enemy's batteries, &c., on a military reconnaissance. The height

* A perpendicular can be thus laid off with the chain: suppose *a* the point at which it is required to erect a right-angle: fix an arrow into the ground at *a*, through the ring of the chain, marking twenty links; measure *forty* links on the line *a b*, and pin down the *end of the chain* firmly at that spot, then draw out the remaining eighty links as far as the chain will stretch, holding by the centre fifty-link brass ring as at *c*; the sides of the triangle are then in the proportion of three, four, and five, and consequently *c a b* must be a right angle.



An angle equal to any other angle can also be marked on the ground, with the chain only, by measuring equal distances on the sides containing it, and then taking the length of the chord: the same distances, or aliquot parts thereof, will of course measure the same angle.

of a point on an inaccessible hill may also be obtained without the use of instruments, thus:—

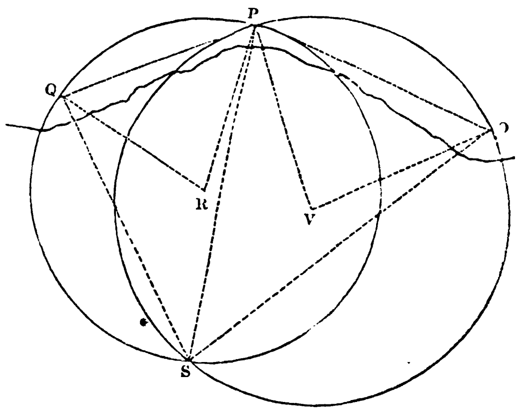


Drive a picket 3 or 4 feet long at H, and another at L, where the top of a long rod FD is in a line with the object S from the point A (the heads of these pickets being on the same level); mark also the point C, where the head of the rod is in the same line with S, from the top of any other picket B, and measure AF and BC; lay off the distance BC from F to b, and the two triangles ADb and ASB are evidently similar, as are also AFD and APS, whence $\frac{PS}{DF}=\frac{AB}{Ab}=\frac{HI}{HO}$ and $\frac{AP}{AF}=\frac{AB}{Ab}=\frac{HI}{HO}$.—PS the height therefore $=DF.\ \frac{HI}{HO}$; and AP the distance$=AF.\ \frac{HI}{HO}$.

A few other methods of ascertaining distances and heights more particularly connected with military reconnaissances, will be found in the next chapter.

Where angles can be taken between *three inaccessible objects*, the relative positions of which are known and can be laid down on paper, the place of the observer can be ascertained either by *calculation*, by *construction*, or by means of an instrument used for that purpose, called a "*station pointer;*" or, what is better still, a piece of thin tracing paper with the observed angles plotted upon it can be shifted about until the point falls into the *only* spot from whence the lines containing these angles pass through the three fixed stations. The case is a very common one in maritime surveying, where the first two methods of solution, *calculation* and *construction*, are seldom thought of; and the last, which is the most simple and sufficiently correct for the purpose, is generally adopted. In a trigonometrical survey, of course, this method would never be thought of for fixing a station, but the calculations for the different cases that may occur of the three points being in *one line*, or forming a triangle *within* or *without which* the observer may happen to be, will be found, with a mass of other information on such subjects, in "Adam's Geometrical Essays," pp. 169 to 177.

The following is the mode of obtaining the position of the observer by *construction* in the case that most commonly occurs, viz. when the three points form a triangle, *without* which the place of observation lies:—O, P, and Q represent the three points



on shore whose positions have been determined by interior triangulation, and S a rock or anchorage whose place is to be determined with relation to the stations above mentioned. Suppose the angle QSP is observed 35°, and PSO=40°, describe a circle passing through Q, S, and P, which is thus done:—Double the angle QSP which=70°; subtract this from 180, leaving 110°; lay off half of this, or 55° at PQR and QPR, and the angle at R is evidently=70°, or double QSP; now the angle at the centre being double that at the circumference, a circle described from R as a centre with the radius RQ, or RP, will pass *through the point* S. In like manner a circle described from V, with the radius VP, will also pass through S, and their intersection gives the spot required.

For the analysis of the calculation of this problem, vide

"Puissant, Géodesie," vol. i. p. 233, and p. 432, vol. iii. of the Woolwich Course.

The method of surveying any tract of country through which a line of railway is projected or has been determined upon, is so similar to that of measuring roads or other continuous lines by "traversing" with the chain and theodolite, that it does not require any peculiar directions. The lines however, being generally very long, must be measured with the greatest exactness, and the angles be observed with proportionate care. Where practicable, also, the work should whilst in progress be tested by reference to known fixed points near which it passes, which can in most cases be obtained from good maps. The existing Standing Orders of Parliament regulate the scale upon which these surveys are required to be plotted in England; and the lateral deviation allowed from the proposed line of rails, with other local causes, determine the breadth required to be embraced in the survey.

For the methods of laying out the lines of railways, the levels of the different portions, and determining the curves, gradients, and slopes of embankments and cuttings, &c., every information can be obtained from the works of Mr. Hascoll and many others; and it would be out of place here to attempt any description of subjects which belong to a most important branch of civil engineering, and embrace such a multitude of details. A few remarks, however, upon the method of taking sections for railways, and the scales upon which they should be plotted, will be found in the chapter upon Levelling.

Before concluding this chapter, a few additional remarks are considered necessary as to the method of laying down the trigonometrical points upon paper, and also of plotting the interior details from the field-book. The sides of the triangles are usually laid down from their calculated distances by means of beam compasses, and the trigonometrical points must be transferred to every sheet upon which the details are afterwards to be plotted, the sides of the minor triangles being on the Ordnance Survey actually measured lines, between which check lines have been run, and on which offsets have to be plotted. Where the scale is very large,—such, for instance, as that adopted for plans of towns,—the trigonometrical points are more conveniently laid down by means of their distances on and from a common meridian, the accuracy of the sides of the triangles thus formed being checked by their calculated distances.

The scales now adopted on the Ordnance Survey are:—

1. Towns, $\frac{1}{500}$, or 10·56 feet (126·72 inches) to 1 mile.

2. Parishes, $\frac{1}{2500}$, or 25·344 inches to 1 mile, in which the English acre is represented by 1 square inch.

3. Counties, $\frac{1}{10560}$, or 6 inches to 1 mile.

4. Kingdom, $\frac{1}{63360}$, or 1 inch to 1 mile.

In different stages of the progress of the survey Scales differing from the above have been used; but at present they are limited to these four, which have been adopted as being best suited to the different descriptions of surveys. The first is perhaps rather in excess, as an equal amount of detail could be shown on a scale of $\frac{1}{1000}$, or 5 feet to 1 mile. The 6-inch scale will also admit of almost the same amount of detail as that of 25 inches. Of their relative cost, the second, that of Parishes, is estimated at 11½*d.* or 1*s.* per acre; the third at 10¾*d.* for cultivated, and 6¼*d.* for uncultivated districts; and the last, 1 inch to 1 mile, at 8*l.* 6*s.* 8*d.* per square mile.

For plotting from the field-books the lines and offsets measured during the progress of the work, "plotting-scales" of various sizes and descriptions are used. One of these contrivances consists of an ivory scale with a dove-tailed groove in the middle running nearly its whole length, in which slides at right angles to the principal scale, a shorter one projecting on each side as far as necessary; for marking the offset distances right and left both have fiducial edges on each side, divided into chains and links according to the scale upon which the survey is to be plotted. Some surveyors prefer the two scales detached, the offset scale merely sliding along the edge of the other.

The whole of Ireland and the rest of the United Kingdom, with the exception of the southern counties of England (surveyed to the scale of 2 inches to 1 mile), were plotted on the scale of 6 inches; these plans, as well as those on the $\frac{1}{500}$ and $\frac{1}{2500}$ scale, had all to be reduced (the two latter first to the 6-inch) to the one uniform scale for the kingdom of 1 inch to 1 mile, the methods of performing which will be explained in Chapter VIII.

CHAPTER V.

MILITARY RECONNAISSANCE, AND HINTS ON SKETCHING GROUND.—GERMAN SYSTEMS OF DELINEATING GROUND.—HORIZONTAL CONTOURS.—GEOLOGICAL MAPS.—CONVENTIONAL SIGNS.

THE sketch of any portion of ground for military purposes should in all cases be accompanied by an explanatory statistical report, and the combination of these two methods of communicating local information constitutes what is termed a *Military Reconnaissance*, in which the importance of the *sketch*, or the *report*, predominates according to circumstances.

The object for which a reconnaissance is undertaken naturally suggests the points to which the attention of the officer should be principally directed; if for example, it is merely to determine the best line of march for troops through a friendly or undisputed country; the state of the communications, the facilities of transport, and possibility of provisioning a stated number of men upon the route, are the first objects for his consideration. If the ground in question is to be occupied, either permanently or for temporary purposes, or if it is likely to become the seat of war, his attention must be directed to its military features, and a sketch of the ground, with explanatory references, together with a full and correct report of all the intelligence he can collect from observation, or from such of the inhabitants as are most likely to be well acquainted with the localities,* and most worthy of credence, will demand the exertion of all his energies, as upon the correct information furnished by this reconnaissance may depend, in a great measure, the fate of the army.

* It is almost needless to point out the incalculable advantages of being a good modern linguist to an officer employed on duty of this nature in an enemy's country

The principal points for observation in a military sketch and report are—

ROADS.—Their direction; nature; liability to injury; facility of repair and of destruction; practicability, in what seasons and for what species of troops; exposure to, and means of security from enfilade; whether bordered or not by hedges, ditches, or banks, or passing through defiles, the nature and extent of which require most careful description and report as to the means of turning, forcing, or defending. Roads are frequently reported impracticable without due consideration of the resources which may be brought to bear upon their improvement, or what difficulties an enterprising enemy may be able to surmount.*

RIVERS.—Their sources, width, depth, velocity of current; facilities for watering horses on an extended front; character of water, whether likely to be frozen in winter, and if so whether the ice is likely to bear the passage of troops; whether subject to sudden or periodical floods, and their effects upon the banks and adjacent country; whether dry or nearly so in summer; facilities for inundation or drainage; profile and nature of banks, whether boggy or wooded; size and nature of vessels and boats employed in the navigation, and their probable number; tributary springs and rivulets; bridges, with their dimensions, nature of construction, and means of destroying or repairing them; fords for infantry or cavalry,† whether permanent or only passable at certain seasons or times of tide, or if exposed to fire, &c.

CANALS.—Means of destruction, or of rendering them of use; construction; depth and width of water, size of locks; how navigated, and on which side the towing path, &c.

MILITARY FEATURES.—Inclination and nature of slopes and all irregularities of ground; accessible or not for cavalry or

* The Austrian staff officers must have reported the passage of the Alps "impracticable," and the French that of the Bohemian mountains, but both were forced. Napoleon's maxim, though perhaps an exaggeration, was, that wherever two men could pass abreast an army could follow.

† A ford should not be deeper than three feet for infantry, four feet for cavalry, and two and a half for artillery and ammunition waggons.—Macauley's "Field Fortification." The nature of the soil at the bottom should always be ascertained, and also if it is liable to shift, which is the case in a mountainous country.

infantry; description of country, open or inclosed; relative command of hills;* ravines; forests; marshes; inundations; barriers; plains; facilities for landing, if on a sea coast with nature of beach and roads within reach; military posts, and fortified towns; good positions, either offensive or defensive; ground suited for encampments; supply of water, &c.

STATISTICAL INFORMATION.—The population and employment of the different towns, villages, and hamlets, contained within the limits of the sketch. Agricultural and other produce; commerce; means of transport; subsistence for men and horses, &c.; with a variety of minute but important details, for which the reader is referred to the excellent essay on this subject, in the fourth volume of the "Mémorial Topographique et Militaire," to the "Aide Mémoire des Officiers du Génie," Macauley's "Field Fortification," &c.

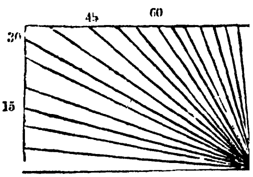
The degree of accuracy of which a sketch of this nature is susceptible depends upon the time that can be allowed, and the means that may be at hand. If a good map of the country can be procured (which is generally the case), the positions of several conspicuous points, such as churches, mills, &c., can be taken from it and laid down on the required scale, and, if the ground to be sketched is extensive, transferred to several sheets of paper to be filled in simultaneously by any requisite number of officers; or a base may be roughly measured, paced, or otherwise obtained by a micrometer or from some known distance, such as that between milestones for instance, and angles taken with a sextant or other instrument from its extremities to different well-defined objects, forming the commencement of a tolerably accurate species of triangulation which may be laid down by the protractor without calculation, within which the detail can be sketched more rapidly and with far more certainty than without such assistance. No directions that can possibly be given will render an officer expert at this most necessary branch of his profession, as practice alone can give him an eye capable of generalising the minute features of the ground, and catching their true military

* If actual differences of level cannot be determined for want of time, still relative command may be obtained, and numbered 1, 2, 3, &c., accordingly.

character, or the power of delineating them with ease, rapidity, and correctness.

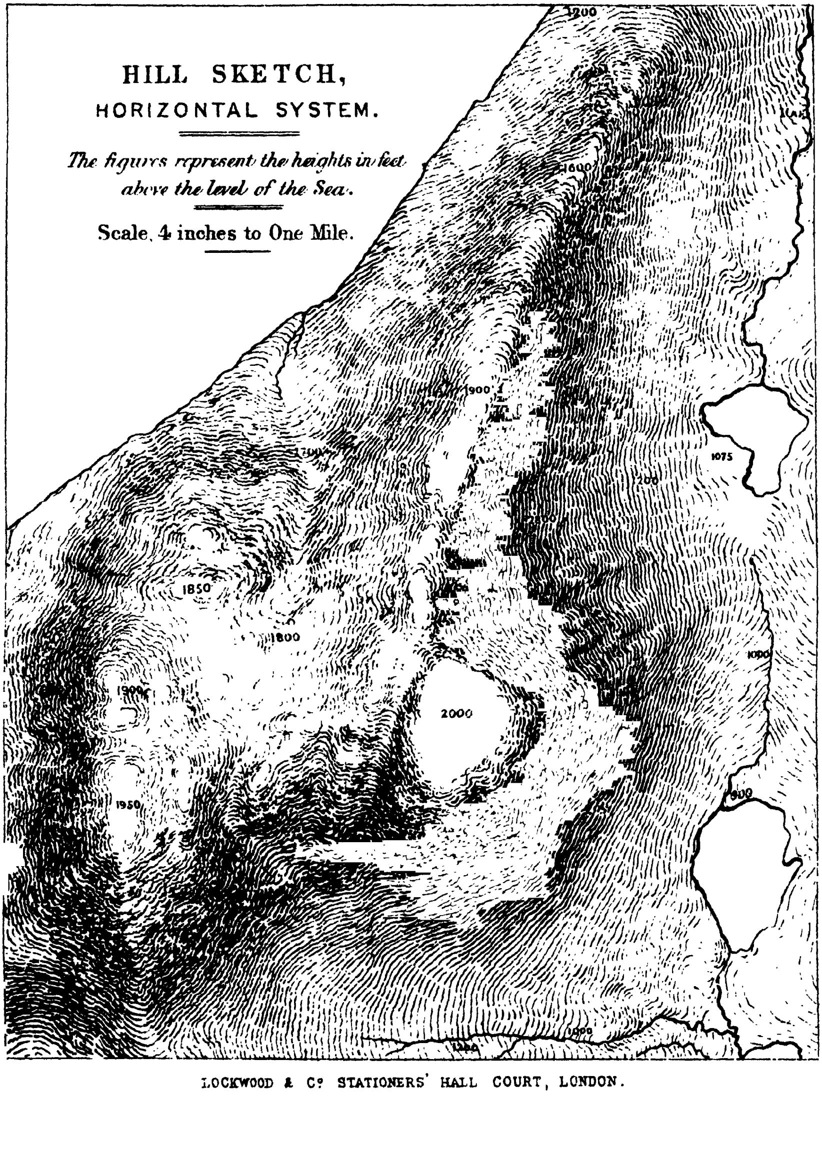
The instruments used in sketching ground have already been alluded to when describing the mode of filling in the detail between measured lines on a regular survey. In addition to the advantages there ascribed to the azimuth compass, it will be found peculiarly well adapted for sketching on a continuous line, such as the course of a road or river, or a line of coast, which *reflecting instruments are not;* and the angles with the magnetic meridian, measured by the compass, can be read off with quite as much accuracy as they can be laid down by the small protractor used in the field. This should have a scale of 6, 4, or 3 inches to one mile (or whatever other proportion may be preferred) engraved on the other bevelled side, and with a sketching portfolio and compass, together with a small sextant and field telescope with a micrometer scale in the diaphragm, comprise all the instruments that can be required by an officer employed on a reconnaissance; and as they can *always* be carried without inconvenience about his person, or strapped in front of his saddle, he need never be driven to the necessity of sketching entirely without their assistance, though the practice of doing so occasionally is decidedly of service, as it teaches him to make *use of his eyes*, and tends to make him a good judge both of linear and angular measurement.*

Sketching such parts of the interior detail as have a decidedly marked outline is comparatively easy, but the delineation of ground, so as to represent the various gentle slopes of the hills and undulations and irregularities of the surface is far more difficult, and attempts have been made both on the Continent and in this country, to establish recognised systems for expressing these features, which should give not merely a general idea of



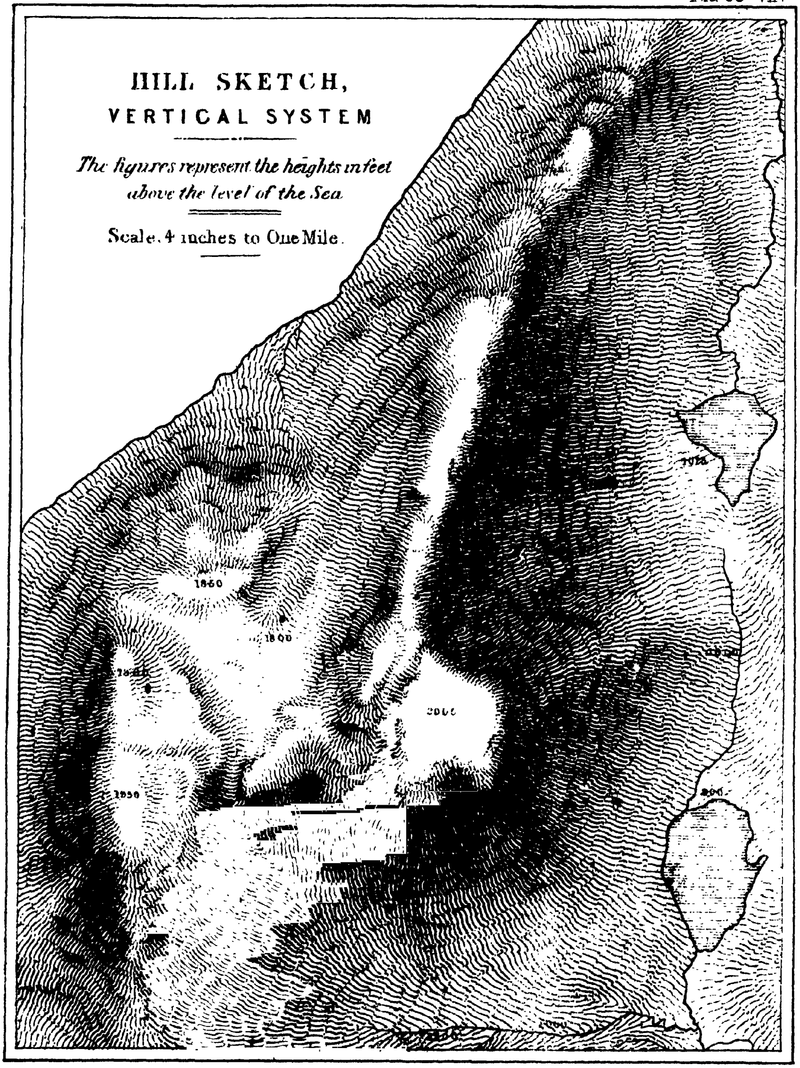
* A protractor (for want of a better) can be made by folding a square or rectangular piece of paper into three, which, when doubled, divides the edge into six portions of fifteen degrees each; these can be again divided into three parts, by which angles of five degrees can be laid down, or even approximately observed, the intermediate degrees being judged by the eye.

Plate VII.



LOCKWOOD & C° STATIONERS' HALL COURT, LONDON.

Plate VII^{a}



LOCKWOOD & Co, STATIONERS' HALL COURT, LONDON.

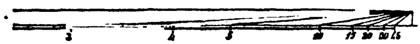
their character, but a *mathematical representation* of their various complicated inclinations; so that the angle of every slope might be evident from a mere inspection of the drawing, or measured from a scale, which would furnish data for constructing sections of the ground in any required direction. This degree of perfection would of course be most desirable in military sketches as well as in finished topographical plans, but the labour and difficulty attending the execution will always prevent its general application excepting in surveys of a national character, or of limited detached portions of ground.

The two methods in general use for representing with a pen or pencil the slopes of the ground are known as the *vertical* and the *horizontal*. In the first of these the strokes of the pencil follow the course that a heavy body or ball would take in running down these slopes; in the second (which is comparatively of late introduction) they represent horizontal lines traced round them, such as would be shown on the ground by water flooding the country at the different stages of its progressive altitude. This last is the mode now generally practised except in very hurried sketches, and it certainly produces a more correct representation of the general character and features of the ground than the vertical method.* Neither of them however when sketched merely by the eye between fixed points and measured lines aspires to the mathematical accuracy which is obtained by tracing with a theodolite or spirit level, horizontal contour lines at equidistant vertical distances over the surface of the ground, the method of doing which will be treated of in the chapter upon Levelling. Systems were introduced into Germany, by Major Lehman, for representing the slopes of the ground by *a scale of shade* consisting of a combination of vertical and horizontal lines, but they have not been adopted in this country. The light in Major Lehman's system, as is generally the case in describing ground with a pen, is supposed to descend in vertical rays, and the illumination received by each

* Specimens of both these styles of sketching hills are given. They are also to be found in Mr. Burr's "Practical Surveying." The vertical is best adapted to a military sketch if pressed for time, as however roughly it may be scratched down a good general idea of the ground is conveyed.

slope is diminished in proportion to its divergence from the plane of the horizon. As vertical rays falling upon a plane inclined at an angle of 45° are reflected *horizontally*, this slope, which is considered the greatest that is ever required to be shown, is also considered the *maximum* in the scale of shade, and is represented by *perfect black*. A horizontal plane reflects all rays upwards, and is, therefore, represented at the other end of the scale by *perfect white;* and the intermediate degrees being divided into nine parts, show the proportion of black in the lines to the white spaces intervening between them for every 5°; which at 5° is 1 to 8; at 10°, 2 to 7; at 15°, 3 to 6, &c. Figure 1 will explain the construction of this scale, and the thickness of the strokes drawn on this principle must be copied till the hand becomes habituated to their formation. In sketching ground the inclinations must be measured, or estimated if the eye is experienced enough to be trusted, and are to be represented by lines of a proportional thickness. To this system is to be objected its extreme difficulty of execution, as well as that of estimating correctly by the eye the angle intended to be represented by the thickness of the lines; though Mr. Siborn, who published a work in 1822 on "Topographical Plan Drawing" founded on this system of Major Lehman's, considers that between 10° and 35° of altitude the slope may be read by mere inspection within 1°, more accurately indeed than it can possibly be measured on the ground by a clinometer, or any portable contrivance of the sort. In Mr. Siborn's work contour lines are recommended to be drawn merely as a guide for the vertical strokes; but the system of tracing these horizontal lines at *fixed vertical intervals*, and drawing between the contours vertical strokes, without any reference to their *thickness* but merely their *direction*, presents a far more easy mode of expressing correctly the actual surface of the ground, and infinitely more intelligible to those who have to make use of the plan. Indeed, if the contour lines are traced at short vertical distances, either fixed or varying according to the nature of the ground, there is no occasion for the vertical strokes whatever, as these always cut the horizontal lines at right angles: this was the method recommended, wherever the ground was required to be shown very accurately, by the committee of French officers of engineers,

appointed, in conjunction with some of the most scientific men of that period, to establish one general system of topographical plan drawing. The combined method of vertical lines and horizontal contours, at one *fixed difference of level*, is described in the German work alluded to, and also in Sir J. C. Smyth's "Topographical Memoir." From the vertical distance being a constant quantity, the angle formed by the slope of the ground is obtained by taking the length of the vertical line between any two of the contours in a pair of compasses, and applying it to a scale constructed upon a simple principle self-evident from the figure. Above 45° the



base, or "*normal*," becomes too short to be appreciable if it has been constructed to suit moderate inclinations of the ground, and if on account of steep declivities the normal is increased in length, it becomes quite unmanageable on gently-inclined surfaces.

By way of obviating this difficulty, and also making the same scale of normals still universally applicable, the vertical distance, where required from the bold nature of particular slopes, is doubled or tripled, and these normals distinguished from others of the same length by being *represented with thicker double or triple lines.* This contrivance, the invention of Colonel Van Gorkum, is most highly extolled by Sir J. C. Smyth in his "Topographical Memoir," in which he strongly recommends the adoption in the British service of some part of the detail of this method of sketching ground, and purposes to omit the horizontal contours, but to take the angles of depression of the hills in sketching and to represent their slopes, not over the whole plan, but occasionally on ground of the most importance, by normals of the proper length corresponding to such a vertical distance as may be judged best suited to the scale employed. On a scale of 4 inches to 1 mile, Colonel Van Gorkum fixes his perpendicular at 24 feet: Sir J. C. Smyth, in the memoir alluded to, has tabulated what he considers best adapted to the four scales in most general use, making it at 6 inches to 1 mile 22 feet; at 4 inches 32 feet; at 2 inches 66 feet; and at 1 inch 132 feet. At 13°, in all these cases he doubles the perpendicular, and at 50° triples it. With all deference to such authority, it is conceived that horizontal contour lines, traced at short *known and generally equal vertical distances* over the ground, afford ample data for the construction of sections in any required directions even more accurate than a model of the features of the ground. The delineation of ground on the Ordnance Survey has been partially effected on this system.* The contours are traced with a spirit level or theodolite at different vertical intervals suited to the character of the surface, but averaging about 100 feet; these are interpolated with intermediate contour lines traced with a water level as being more expeditious, at the constant vertical distance of 25 feet. For the method of tracing these instrumental contour lines, see the chapter on Levelling, to which this subject more particularly belongs.

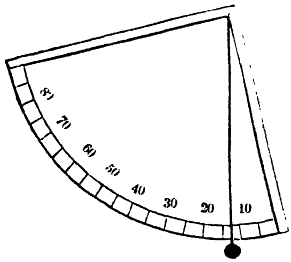
For representing the features of the country in a topographical plan, on a moderate scale, where the surface of the ground is not required to be determined with mathematical precision, the horizontal system of sketching the hills alluded to in page 36, is sufficiently accurate, and has the advantage of being generally intelligible. In addition to the sketch of the ground, a representation of the geological features of the country can be given without at all interfering with or confusing the sketch, by tracing on the back of the paper the divisions of the geological features, the different portions of which are afterwards coloured according to the conventional system of distinguishing the several various formations on geological maps. On holding the sketch against the light these divisions appear clearly visible, though in any other position of the paper they are not in the least perceptible. Geological sections should also be shown on the margin of the sketch, having reference to lines drawn across it.†

The inclination of such slopes as are of peculiar moment are measured with a "Clinometer," and the angles written either on the slopes themselves or as references. This little instrument

* Economy alone has led to the present (it is to be hoped temporary) suspension of the system of contouring on the Ordnance Survey.

† The geological part of the Ordnance Survey is now quite distinct from the geodesical.

can be made by cutting a small quadrant out of pasteboard and roughly graduating the arc. A small shot, suspended by a piece of silk, forms the plummet: and independently of its use in measuring vertical angles, it is of great assistance in tracing level lines in sketching the contours. The instrument sold under this name is made with a spirit level; but the substitute as described above answers the purpose equally well, and moreover, from its being made merely of pasteboard, fits into the pocket of the sketching portfolio.



The slopes most necessary to note on a military sketch are those which relate to the facilities of ascent for artillery, cavalry, and infantry. According to the "Aide Mémoire," a slope of about

60°, or of 4 to 7, is inaccessible for infantry.
45°, or of 1 to 1, difficult.
30°, about 7 to 4, inaccessible for cavalry.
15°, „ 4 to 1, inaccessible for wheel carriages.
5°, „ 12 to 1, easy for carriages.

The leading features of ground are the summit ridges of hills (termed the water-shed lines), and the lowest parts of the valleys down which the rain finds its way to the nearest rivers or pools, called water-course lines. These two directing lines if traced with care will alone give some idea of the surface of the country, and will assist materially in sketching the hills, particularly if drawn on the horizontal system, as the *contour lines always cut the ridges and all lines of greatest inclination at right angles.* It is a very common error in first beginning to sketch ground to regard hills as isolated features as they often appear to the eye. Observation and a slight practical knowledge of geology, inevitably produce more enlarged ideas respecting their combinations, and analogy soon points out where to expect the existence of fords, springs, defiles, and other important features incidental to peculiar formations. Thus appearances that at one time presented nothing but confusion and irregularity, will, as the eye becomes more experienced, be recognised as the results of general and known laws of nature.

The representation of the outline of the hills, and their relative command, is also materially assisted in a topographical plan, and *more particularly in a military reconnaissance*, by a few outline sketches taken from spots where the best general views can be obtained. A series of these topographical sketches running along the length of a range of hills, and a few taken perpendicular to this direction, supply in some degree the place of longitudinal and transverse sections; and give in addition to the information communicated by a mere section, a general idea of the nature of the surrounding country.*

A good judgment of distances is indispensable in sketching ground, for filling up the interior of a survey, and more particularly in a reconnaissance, where there has not been either time or means for measurement or triangulation. Practising for a few days will enable an officer to estimate with tolerable accuracy the length and average quickness of his ordinary pace, as also that of his horse (as on a rapid reconnaissance he must necessarily be mounted); and the habit of judging distances, which can afterwards be verified will tend to correct his eye.† A micrometrical scale, or cross wires in his field telescope, with a table of distances

* A brush and a few water-colours will be found very useful in rendering the various parts of a topographical sketch more intelligible, and save much time and labour. Water—woods—buildings (whether stone or brick, or of wood), can be shown much more clearly and rapidly with a brush than the pen.

† Dr. Brewster's micrometrical telescope is fully described in the second volume of Pearson's Astronomy, and more portable instruments upon nearly the same principle have since been contrived, the best known of which are Cavello's and Rochon's micrometers. The latter consists of a telescope with a double refracting prism attached to a moveable slide working between the object-glass and eye-piece, having a graduated scale with a vernier on the outside of the tube, showing the observed angle and the ratio of the corresponding distance to some assumed distant base such as the height of a man, or any other object the dimensions of which are supposed to be known. This scale is graduated to half minutes, and each of these divisions can be decimally divided by the vernier, but a table is required for all the intermediate divisions, showing the number of times the assumed base must be multiplied to obtain the distance.

If the distance is known the height of any object can be ascertained by reversing the process. If the height of the object and the distance are both unknown, an approximate result can be obtained, if the object, say a column of men or a ship, is advancing or receding, by making two observations, separated by some convenient interval of time, and estimating the distance the object has moved in a *direct line* to corresponding to the angle subtended by some distant object, is also a very useful auxiliary. The gradual blending of colours,

the observer within that period, or by moving in a direct line to a stationary object any measured distance, and observing the two angles subtended by it.

A still more portable instrument for measuring distances, by observing by a prism micrometer the angle subtended by a distant object (Mr. Porro's Longue vue Napoleon III.) has recently been introduced into the French service, and if its results could be relied upon for great distances, its extreme portability, and the facility with which it is used, would leave little to be desired in an instrument based upon this principle, but beyond 500 yards the distances often vary considerably from the truth, and as it is only constructed for a range of about 1000, it evidently, even if perfectly accurate, would not meet the requirements of the service in the present day when such extreme ranges are obtained both by infantry and artillery.

All the above instruments have moreover, from their principle, a source of error which (excepting where the object observed has been measured) no accuracy of construction can remedy, viz. : that the distance sought to be obtained depends upon the correct estimation of the dimensions of a distant object, and an error of only six inches in the estimated height of five feet will produce in the result a difference of 100 yards in a distance of 1000. To obviate the probability of inaccuracy in assuming the dimensions of any distant base, Professor Piozzi Smith invented an instrument upon a directly opposite principle, viz., that it was to carry its *own base*, and that the angle measured should not be that subtended at the station of the observer by a distant uncertain object, but the actual angular measure of the base attached to the telescope at the distance at which that object was situated.

This instrumental base, at right angles to the telescope, has two mirrors or prisms at its extremities, one of which is in the line of the axis of the telescope, through which the object (a *point*) is seen by direct vision through the unsilvered portion of the mirror. The same object is reflected from the other mirror, and the coincidence, or the amount of separation of the two images, furnishes the means of ascertaining the required distance.

To effect this, the index mirror may be made to turn through the necessary angle; or it may be kept at one fixed angle, and the coincidence effected by sliding it along the base; or both mirrors may remain fixed, and the angle measured by the amount of separation of the two images as shown by a wire micrometer and finely divided scale, which latter arrangement has been found the most convenient.

The length of this instrumental base is about two feet, and in a pamphlet by Col. Clerk, R.A., it is stated that its results can be depended upon up to 1000 yards, and that, if the base were increased to five feet, up to 2000 yards, beyond which he conceives it would be necessary to measure a base upon the ground; and for the purpose of obtaining an instrument to be used with artillery for long ranges, he constructed a frame, to be used with a telescope, consisting of a mirror placed at an angle of 45°, with an index arm seven inches long, the indications of which are measured upon an arc graduated to ten seconds. The telescope is fixed upon the distant object, and the angle subtended by a measured base of 100 yards at that point is obtained by the coincidence of the image reflected from the mirror with the and the well-known rate at which sound has been ascertained to travel,* will all materially assist him. According to the "Aide Mémoire," the windows of a large house can generally be counted at the distance of 3 miles; men and horses can just be perceived as points at about 2200 yards; a horse is clearly distinguishable at 1300 yards; the movements of a man at 850 yards; a man's head clearly visible at 400 yards; and partially so between that distance and 700 yards.

These directions however, cannot be considered as infallible, as the power of vision differs so materially; but nothing can be more easy than for an officer to *make a scale of this kind for himself.*

Another easy mode of judging distances is by marking on a scale or pencil held at some fixed distance from the eye, the apparent diameter or height at different measured distances of any objects the dimensions of which may be considered nearly constant; the average height of a man, a house of one or two stories, the diameter of a windmill, &c., will furnish suitable standards; and a short piece of string, with a knot to hold between the teeth, will serve to keep the pencil always at the proper distance. Suppose these scales to have been carefully

vertical wire in the telescope, and measured by the graduated arc, the multiple of the base giving the actual distance obtained by inspection from tables constructed for the purpose. If the distance exceeds 4000 yards the base can be increased to 200 yards, the multiple given by the tables being simply *doubled.*

This instrument, and that previously described, though adapted to the use of artillery or coast batteries are not applicable to general service in the field, as they both require steady level rests, and are not sufficiently portable; and the want is still seriously felt of some description of instrument which shall combine accuracy in the estimation of long distances with facility of use and portability.

* About 1100 feet in one second. A light breeze will increase or diminish this quantity 15 or 20 feet in a second, according as its direction is to or from the observer. In a gale a considerable difference will arise from the effects of the wind. A common watch generally beats five times in one second. See "Philosophical Transactions," 1823. The number of pulsations of a man in health is about 75 per minute. Either of these expedients will serve as a sort of substitute for a seconds watch. The velocity of sound is affected by the state of the atmosphere, indicated by the thermometer, hygrometer, and barometer; according to Mr. Goldingham, $\frac{1}{10}$ of an inch rise in the barometer diminishes the velocity about 9 feet per second. Mr. Baily rates the velocity of sound, at 32° Fahr., at 1090 feet per second, and directs the addition of 1 foot for every degree of increase of temperature above the freezing point.

marked for four or five of these objects, at the distance of 150, 200, 300, &c., yards, they will evidently afford the means of obtaining an approximate distance; but even without this scale, if the pencil *b* be held up to the eye at any distance *a*, and the

height or diameter of any object *h* of known dimensions be observed, then the distance from this object is evidently $\frac{h \times a}{b}$.

In reconnoitring the outline of a work which cannot be approached closely for the purpose of tracing parallels and determining the positions of batteries, the best plan is to mark, if possible, the intersection of the prolongations of the faces and flanks of the line on which the distances are being paced or measured, instead of merely obtaining intersections of the salient and re-entering angles with a sextant. Soon after sunrise, or a little before sunset, are the best times for these observations, as lights and shades are then most strongly marked; in the middle of the day it is often impossible to distinguish anything of the outline of a work of low profile, even at the distance of 200 or 300 yards.

If the perpendicular distance from the angle, or any other point of the face of a work, is required to be ascertained in the field, and the line marked on the ground for the purpose of laying out a battery, it can be readily done by the following method :—

Suppose in each of the figures below, A to be the point from which the distance is required on a line perpendicular to A B; measure any distance C D, in a direction nearly parallel to A B, and take the angles at C and D, formed by the line C D and each of the points A and B; B being some marked object situated anywhere on the line of the work, probably a salient or re-entering angle. From these data ascertain the values of A B, and the angle A B D, either by calculation or by any of the practical methods already described; B E is then the *secant of the angle* A B D *to* radius A B, and the difference D E between this quantity (to be found by means of a table of secants), and the calculated distance B D being laid off either on the line D B from D towards B (as in *fig.* 1), or on the prolongation of this line (as in *fig.* 2), the distance A E becomes the tangent of the same angle

Fig. 1.

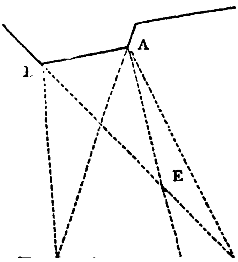
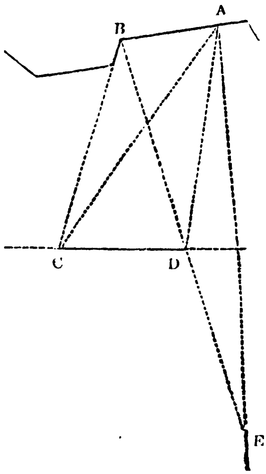
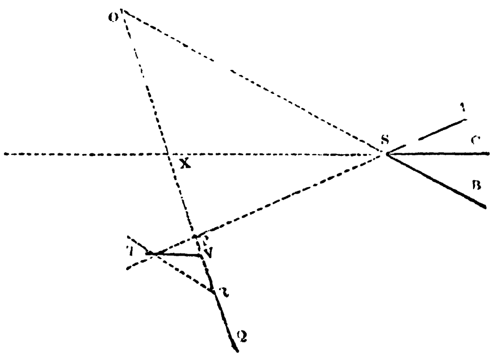


Fig. 2.



also to the radius A B; and the distance required for the battery can therefore be laid off on the ground by increasing or diminishing the length of this line A E.

The direction of the capital of a work, and the distance from its salient, can be thus determined in the field.



On any line O Q, mark two points, O and P, in the prolongation of the faces, the distance between them being measured or paced. Take any other point R, one hundred paces or any convenient distance from P and make the angle P R T equal to that observed at O; T being in the prolongation of S P. The triangles O S P and R T P are therefore similar, and the angle T being bisected by the line T V, it results that R P : P V : : P O : P X; which distance, laid down on the line P O, gives the point X required in the prolongation of the capital. The sides of the small triangle T P R and T V being all capable of measurements, O S, S P, and S X can, if required, be all found by a similar simple proportion.*

It is, however, generally practicable to obtain a plan of any attacked works and of its environs, more or less correct; and on this any perceptible errors discovered during the reconnaissance are marked. On approaching a place *by day*, the officer should be *alone*, so as to attract little attention, but supported at a distance by troops hid from observation by any cover that can be taken advantage of. *By night* he should be accompanied by a strong party; and by advancing as near as possible towards day-break, and retiring gradually, he would be enabled to make more correct observations as to the outline and state of repair of the works than at any other time.

The numerous conventional signs recommended in most continental military works are extremely puzzling, difficult to remember, and are mostly unintelligible. In a little work, the "*Aide Mémoire Portatif*," published in 1834, there are no less than *ten pages* devoted to these signs. Beyond the few that are absolutely necessary and generally understood, it is far better to trust to references written on the face of the sketch, and the explanatory report, than by endeavouring to convey so much information by these conventional symbols and attempts at mathematical representations of the ground, to render a drawing *so confused and difficult to comprehend* that it really becomes of less value than an indifferent sketch with copious and clear remarks.

* With a pocket or prismatic compass this operation may be more easily performed—by taking up a position on the prolongation of each face, and observing their inclination to the magnetic meridian, that of the line bisecting the salient, or the capital of the work, is at once known; for the mean between the two readings will be the bearing of the salient when the observer is upon the capital; and by measuring a base in a convenient situation, the distance may be readily found.

Below are given a few conventional signs, applicable only to military sketches :—

- Artillery Position.
- Cavalry } Dark line shows the front.
- Infantry }
- Sentinel.
- Fort.
- Redoubt.
- Palisades.
- Chevaux de Frise.
- Abatis.
- Telegraph.
- Post-house.
- Mortar Battery.
- Gun Battery.
- Site of an Engagement.
- Passable.
- Impassable for Cavalry.
- Impassable for Infantry.

The following are those of most general use in topographical plan-drawing: the boundary lines are those employed in the Ordnance Survey; a similar arrangement could of course be adopted to mark the divisions of any other country, however they may be designated.

- Smithies. A small horse-shoe with the open side turned towards the road.
- Limekiln.
- Turnpike roads. The side *from* the light shaded.
- Cross roads. Narrower, and both sides alike.
- Railroads. Both sides dark, very narrow, and perfectly parallel.
- Canals. Distinguished from roads by the parallelism of the sides, the locks, and bridges, and by having the side *next* the light shaded like rivers. Canals and navigable rivers to be coloured blue.
- Windmills.
- Bridges.
- Fords.
- Ferry.
- Trigonometrical point.

BOUNDARIES.

Counties.

Baronies.

Parishes.

Townlands.

Counties and Baronies.

Counties and Parishes.

Counties and Townlands.

Baronies and Parishes.

Baronies and Townlands.

Parishes and Townlands.

Counties, Baronies, and Parishes.

Counties, Parishes, and Townlands.

Counties, Baronies, and Townlands.

Baronies, Parishes, and Townlands.

Counties, Baronies, Parishes, and Townlands.

CHAPTER VI.

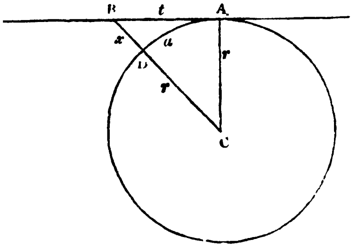
LEVELLING.

The method of ascertaining the difference of level between stations on a trigonometrical survey by means of reciprocal angles of elevation and depression has already been alluded to in the fourth chapter, and detailed sections of ground can be taken in the same manner, though not so conveniently or accurately as with a spirit level. It is however necessary before entering upon this subject, to explain more fully the two corrections that must be applied to all vertical angles when used for the purpose of obtaining relative altitudes between stations a considerable distance apart which were referred to in the chapter upon Triangulation. If they are only separated by a few hundred yards, the corrections are too trifling to have any appreciable effect upon the result.

Considering the earth as a sphere, any number of points upon its surface equidistant from its centre are on the same *true level;* but the *apparent level* (and of course the apparent altitude or depression) is vitiated by these two causes of error, *curvature* and *terrestrial refraction;* the correction for the first of which depends upon the "arc of distance," which is that contained between the two stations at the centre of the earth; and the second upon their comparative elevations above the horizon.

The effect of the curvature of the earth is to depress any object below the spectator's sensible horizon. Every horizontal line is evidently a tangent to the surface of the globe at that spot; and the difference between the *apparent* and *true* level at any distant point B (putting the effect of refraction for the present out of the question) will be seen by reference to the accompanying figure, to be the excess B D, of the secant of the arc A D, above the radius C D.

Putting a for the arc A D, t for the tangent A B (the horizontal line, or line of apparent level), r for the radius A C, or D C; and x for the excess of the secant B C above the radius or the difference between the true and apparent level. Then $(r + x)^2 = r^2 + t^2$. Whence $x\ (2r + x) = t^2$; and owing to the small proportion that any distance measured on the surface must bear to the earth's radius, $2\,r$ may be substituted for $(2r + x)$, and the arc a for the tangent t; $2\,r\,x$ then becomes $= a^2$, and $x = \frac{a^2}{2\,r,}$ which, assuming the mean diameter of the earth at 7916 miles, gives $x = 8{\cdot}004$ inches or ·667 of a foot for one mile; which quantity increases as the square of the distance. Or otherwise,

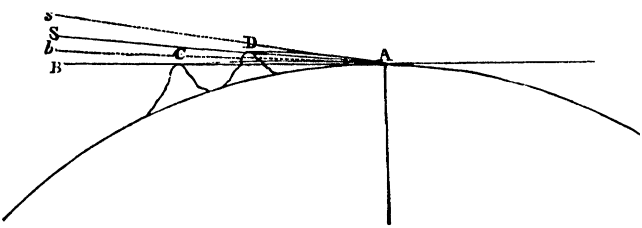


$2\,r + x : t :: t : x,$

or $2\,r : a :: a : x$, x being omitted in the expression $(2\,r + x)$, and a substituted for t; whence $x = \frac{a^2}{2\,r,}$ as before.

A very easily remembered formula derived from the above for the correction for curvature in *feet*, is two thirds of the square of the distance in *miles*; and another, for the same *in inches*, is *the square of the distance in chains divided by* 800.*

The second correction, *terrestrial refraction*, on the contrary, has the effect of *elevating the apparent place of any object above its real* place, and consequently above the sensible horizon. The rays of light bent from their rectilinear direction in passing from a rare into a denser medium, or the reverse, are said to be *refracted;* and this causes an object to be seen in the direction of



* The amount of the correction for curvature at different distances will be found by reference to the tables, and further remarks on Atmospheric Refraction in the chapter on the Definitions of Practical Astronomy.

the tangent to the last curve at which the bent ray enters the eye.

A is any station on the surface of the earth, the sensible horizon of which is A B; C and D are two stations on the summits of hills, of which C is supposed in reality to be situated on the horizontal line A B, and D above it, the angle of elevation of which is B A S. Owing however to the effects produced on the rays from these objects in their passage to the eye, by the atmosphere through which they pass, they are seen in the directions A *s* and A *b*, tangents to the curve described by the rays, and B A *b*, and S A *s*, are the measures of the respective *terrestrial refractions.*

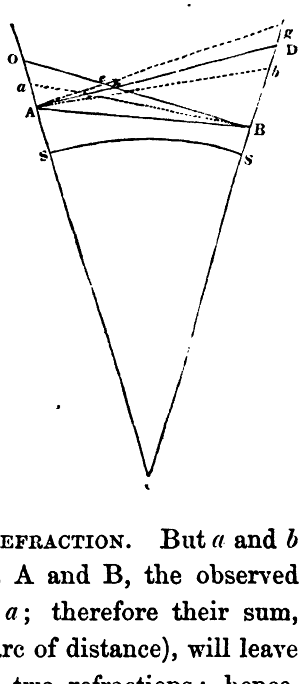
Above eight or ten degrees of altitude, the rate at which the effects of refraction decrease as the altitudes increase (varying with the temperature and density of the atmosphere), is so well ascertained that the refraction of the heavenly bodies for any altitude may be obtained with minute accuracy from any of the numerous tables compiled for the purpose of facilitating the reduction of astronomical observations; but when near the horizon, the refraction, then termed *terrestrial refraction*, is so unequally influenced by the variable state of the atmosphere that no dependence can be placed upon the accuracy of any tabulated quantities.* The rays are sometimes affected laterally, and they have been even seen convex instead of concave. Periods for observing angles of depression and elevation, particularly if the distances between the stations are long, should therefore be selected when this *extraordinary refraction* is least remarkable: morning and evening are the *most* favourable; and the heat of the day after moist weather, when there is a continued evaporation going on, is the *least* so.

It is a common custom to estimate the effects of refraction at some *mean quantity*, either in *terms of the curvature*, or of the *arc of distance.* The general average in the former case is $\frac{1}{7}$ of the curvature, making the correction in feet for *curvature and refraction combined* = $\frac{4}{7}$ D^2, D being the distance in miles as before. In the

* Puissant "Géodesie," vol. i. p. 342; and "Recherches sur les Réfractions Extraordinaires, par Biot." Also, the "Trigonometrical Survey," vol. i. p. 352.

latter the proportion varies considerably;* and General Roy, in the operations of the trigonometrical survey, assumed it at $\frac{1}{10}$, and sometimes at $\frac{1}{11}$, in cases where it had not been ascertained by actual observation of reciprocal angles of elevation or depression, by the following simple method.† These angles should, to insure accuracy, be observed simultaneously, the state of the barometer and thermometer being always noted:—

In the accompanying figure, C represents the centre of the earth, A and B the true places of two stations above the surface S S; A D, B O are horizontal lines at right angles to the radii A C, B C; *a* and *b* are also the *apparent places* of A and B.



In the quadrilateral A E B C, the angles at A and B are right angles, therefore the sum of the angles at E and C are equal to two right angles; and also equal to the three angles, A, E, and B, of the triangle A E B; taking away the angle E common to both, the angle C, or the arc S S, remains = E A B + E B A; or, in other words, *the sum of the reciprocal depressions below the horizontal lines* A D, B O, *represented by* A E B + E B A, *would be equal to the contained arc if there were* NO REFRACTION. But *a* and *b* being the *apparent places* of the objects A and B, the observed angle of depression will be D A *b*, O B *a*; therefore their sum, taken from the angle C‡ (the contained arc of distance), will leave the angles *b* A B, *a* B A, the sum of the two refractions; hence, supposing half that sum to be the true refraction, we have the following rule when the objects are *reciprocally depressed. Subtract the sum of the two depressions from the contained arc, and half the remainder is the mean refraction* :—

* Carr's "Synopsis of Practical Philosophy," articles 'Levelling' and 'Refraction.

† "Trigonometrical Survey," vol. i. p. 175. See also, on the subject of refraction, Woodhouse's "Trigonometry," p. 202.

‡ One degree of the earth's circumference is, at a mean valuation, equal to 365,110 feet, or 69·15 miles; and one second = 101·42 feet.

If one of the points B, instead of being depressed, be elevated suppose to the point g, the angle of elevation being g A D, then the sum of the two angles, e A B and e B A, will be greater than E A B + E B A (the angle C, or the contained arc) by the angle of elevation, e A D; but if from e A B + e B A, we take the depression O B a, there will remain e A B + a B A, the sum of the two refractions; the rule for the mean refraction then in this case is, *subtract the depression from the sum of the contained arc and the elevation, and half the remainder is the mean refraction.**

The refraction thus found must be subtracted from the angle of elevation as a correction, each observation being previously reduced if necessary to the axis of the instrument, as in the following example taken from the Trigonometrical Survey:—At the station on Allington Knoll, known to be 329 feet above low water†, the top of the staff on Tenterden steeple appeared depressed by observation 3′ 51″, and the top of the staff was 3·1 feet higher than the axis of the instrument when it was at that station. The distance between the stations was 61,777 feet, at which 3·1 feet subtend an angle of 10″·4‡, which, added to 3′ 51″, gives 4′ 1″·4 for the depression of the *axis of the instrument*, instead of the top of the staff. On Tenterden steeple, the ground at Allington Knoll

* The formula given in the "Synopsis of Practical Philosophy" is identical with this rule:—

Refraction $= \frac{(A+E)-D}{2}$; E being the apparent elevation of any height; D the apparent reciprocal angle of depression; and A the angle subtended at the earth's centre by the distance between the stations.

† A difference of opinion exists as to the zero from which all altitudes should be numbered. What is termed "Trinity datum" is a mark at the average height of high water at spring-tides, fixed by the Trinity Board, a very little above low-water mark at Sheerness. A Trinity high-water mark is also established by the Board at the entrance of the London Docks, the low-water mark being about 18 feet below this. Again, some engineers reckon from low-water spring-tides; and as the rise of tide is much affected by local circumstances, this latter must, in harbour, and up such rivers as the Severn, where the tide rises to an enormous height, be nearer to the general level of the sea. One rule given for obtaining the *mean level of the sea*, by reckoning *from low-water mark*, is to allow one-third of the rise of the tide at the place of observation. The datum-level referred to in all the maps of the Ordnance Survey of Great Britain is that of the level of mean tide at Liverpool.

‡ At 206,265 feet distant, 1 foot subtends 1″; or at 1 mile it subtends 39″·06 nearly.

was depressed 3′ 35″; but the axis of the instrument, when at this station, was 5·5 feet above the ground, which height subtends an angle of 18″·4: this, taken from 3′ 35″, leaves 3′ 16″·6 for the depression of the axis of the instrument.

Contained arc 61,777 feet = ...	10′ 6″	nearly.
Sum of depression, 4′ 1″·4 + 3′ 16″·6	7 18	
	2 48	
Mean refraction	1 24	

which in this example is nearly $\frac{1}{7}$ of the contained arc.

This, added to the depression at Allington Knoll, 3′ 16″·6, gives 4′40″·6 for the angle corrected for refraction; which, *being* 22″·4 *less than* 5′ 3″, *half the contained arc*, the place of the axis of the instrument at Allington Knoll is evidently above that at the other station by 6·7 feet, the amount which this angle 22″·4 subtends. This, taken from 329, leaves 322·3 feet for its height when on Tenterden steeple, corrected both for *refraction and curvature.* The result would have been the same if these corrections had been applied separately, as before described.

Correction for curvature.

D = 61,777 feet = 11·7 miles, log. 1·0681859
2

136·89 = 2·1363718
2

3)273·78

Curvature = 91·26

Angle of depression, *corrected for refraction:*

Sine 4′ 40″·6 = log. 7·1336617
61777 feet 4·7908268

84·405 1·9244885

Then + 91·26
− 84·405

6·855 feet.

By employing the observation from Tenterden steeple, and estimating the refraction at $\frac{1}{7}$ of the curvature, or using the expression $\frac{4}{7}D^2$ for both corrections, the difference of level between these stations would appear about 12 feet greater; which shows how necessary it is, when accuracy is required, to *ascertain the refraction at the time by reciprocal angles of depression or elevation.* In another example (page 178, vol. i. "Trigonometrical Survey"), where the depression was observed to the horizon of the sea, the dip of the horizon* is calculated from the radius of curvature, and the known length of a degree. The difference between this calculated depression and that actually observed is, of course, *due to refraction.*

To return to the subject of the different methods of taking sections of ground, either—

By angles of elevation and depression with the theodolite.

By the spirit, or water-level; or the theodolite used as a spirit-level.

By the old method of a mason's level and boning-rods, and also by the French reflecting level.

The relative altitude of hills, or their heights above the level of the sea, or other datum, can also be ascertained by a mercurial mountain barometer; the Aneroid; Bourdon's more recently invented metallic barometer; or approximately by the temperature at which water is found to boil at the different stations whose altitudes are sought.

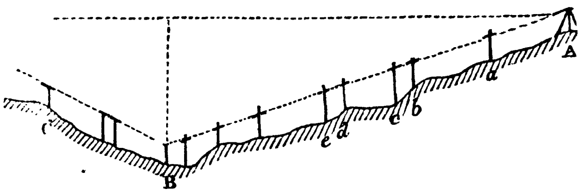
Levelling for sections by angles of elevation and depression with the theodolite is thus performed †:—The instrument is set up at one extremity of the line, previously marked out by pickets at every change of the general inclination of the ground, and a levelling-staff with the vane set to the exact height of the optical

* The dip of the horizon would be equal to the contained arc, when seen fro objects on the spherical surface if there were no refraction, which is therefore equal to the difference between the depression and the contained arc.

† In taking sections across very broken irregular ground intersected by ravines, this system of operation is recommended, as being much more easy and rapid than tracing a series of short horizontal datum lines with the spirit level. Where, however, this latter instrument can be used with tolerable facility, it should always be preferred

axis of the telescope being sent to the first of these marks, its angle of depression or elevation is taken; by way of insuring accuracy, the instrument and staff are then made to change places, and the vertical arc being clamped to the *mean of the two readings*, the cross wires are again made to bisect the vane. The distances may either be chained before the angles are observed, marks being left at every irregularity on the surface where the levelling-staff is required to be placed, or both operations may be performed at the same time, the vane on the staff being raised or lowered till it is bisected by the wires of the telescope, and the height on the staff noted at each place.

The accompanying sketch explains this method:—A and B are the places of the instrument and of the first station on the line



where a mark equal to the height of the instrument is set up; between these points the intermediate positions, *a*, *b*, *c*, *d*, &c., for putting up the levelling-staff, are determined by the irregularities of the ground. The angle of depression from A to B is observed, and if great accuracy is required the mean of this and the reciprocal angle of elevation from B to A is taken, and the vertical arc being clamped to this angle, the telescope is again made to bisect the vane at B. On arriving at B, after reading the height of the vane at *a*, *b*, *c*, &c., and measuring the distances A *a*, &c., the instrument must be brought forward, and the angle of elevation taken to C; the same process being repeated to obtain the outline of the ground between B and C. In laying the section down upon paper, a horizontal line being drawn, the angles of elevation and depression can be protracted, and the distances laid down on these lines; the respective height of the vane on each staff being then laid off from these points in a *vertical direction*, will give the points *a*, *b*, *c*, &c., marking the outline of the groun d A more correct way of course is to calculate the difference of level between the stations, which is the *sine* of the angle of depression or elevation to the hypothenusal distance A B considered as radius, allowing in long distances for curvature and refraction, which may be ascertained sufficiently near by reference to the tables.

The distances, instead of being measured with the chain, may, if only required approximately, be ascertained by means of a micrometer, attached to the eye-piece of the telescope.

Instead of only taking the single angle of depression to the distant station B, and noting the heights of the vane at the intermediate stations, *a*, *b*, *c*, &c., angles may be taken to marks the same height as the instrument set up at *each of these intermediate points*, which will equally afford data for laying down the section; but the former method is certainly preferable.

The details may be kept in the form of a field-book*; but for this species of levelling, the measured distances and vertical heights can be written without confusion on a diagram, leaving the corrections for refraction and curvature (when necessary) to be applied when the section is plotted.

Where a number of cross sections are required, the theodolite is particularly useful, as so many can be taken without moving the instrument. It is also well adapted for *trial sections*, where minute accuracy is not looked for, but where economy both of time and money is an object.

The theodolite is likewise used in running *check levels* to test the general accuracy of those *taken in detail with a spirit level.* Reciprocal angles of elevation and depression, taken between bench marks† whose distances from each other are known, afford a proof of the general accuracy of the work; and if these points of reference are proved to be correct, it may safely be inferred that the intermediate work is so likewise.

* Bruff's "Engineer Field Work," page 122.

† Marks on stumps of trees, mile or boundary stones, &c., or any convenient permanent object on which the staff is placed to obtain the comparative level of these intermediate points of reference. They are useful either for the subsequent laying out of the detail of work, or for comparison in running check or trial sections. Bench marks should be conspicuously marked and clearly described in the field-book, that no doubt may arise as to their identity.

Instead however of observing reciprocal angles of elevation and depression between marks at measured distances, levelling for sections, where minute accuracy is required, is always performed with a spirit level or some instrument capable of tracing *horizontal lines.* The different instruments used for the purpose, and their adjustments will be first described, and the most approved methods of using them, and keeping the field-book, as well as plotting the detail on paper, will be afterwards explained.

The species of level formerly in general use, termed the Y level, owes its name to the supports upon which the telescope rests. This instrument, as well as Mr. Troughton's improved level, and the dumpy level introduced by Mr. Gravatt, are described at length in Mr. Simms' "Treatise on Mathematical Instruments." It is decidedly inferior to the two last mentioned, its only claim to notice when compared with them being the greater ease with which its adjustments are made, though this advantage is again partially negatived by the equal facility with which they are deranged.

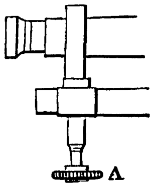
The first adjustment in the Y level is for the *line of collimation;* the method is the same as that described in page 23 for the theodolite, half the error being corrected by the screws acting upon the diaphragm containing the cross hairs.

The second adjustment, that of the spirit level attached to the telescope, is also similar to that for the theodolite.* After the air-bubble has been brought into the centre by the plate-screws, the telescope is reversed in the supports, and if it has moved to either end of the level, it is brought back to its central position, *one half by the screw at one end of the level,* and the other half by the *plate-screws,* there being *no vertical motion* as in the theodolite. This correction will probably require two or three repetitions.

The third adjustment is for the purpose of bringing the Y *supports exactly on the same level* when the previous corrections

* Before adjusting the focus of the object-glass, that of the eye-piece should be always attended to, both in the spirit level and theodolite; it should be drawn out till the cross wires are clearly defined, and there is no instrumental parallax; so that on fixing their intersection on some distant object there may be no displacement of the contact on moving the eye sideways to the right or left.

have been made, so that the optical axis of the telescope may always revolve at right angles to the vertical axis of the instrument. This is effected by first levelling the telescope when placed over two opposite screws, and then turning it round so that



the eye-piece and the object-glass may change places. If in this reversed position the bubble is no longer in the centre, it must be adjusted, one half being done by turning the milled headed screw A, placed directly below one of the Ys, which is thereby raised or lowered in its socket, and the other half by the *plate-screws.* This operation must be repeated with the other pair of plate-screws, and care must be taken that the screw represented by A in the sketch is *never touched* except for the purpose of making this adjustment.

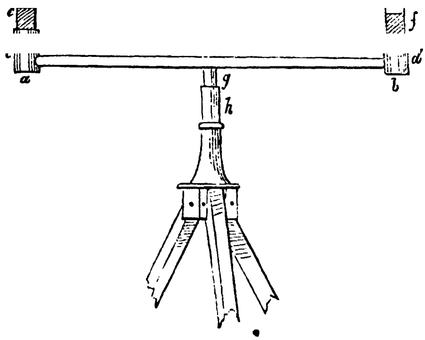
In Troughton's instrument, the spirit level, being fixed to the telescope, has no separate means of adjustment, and the line of collimation must therefore be determined *by its assistance.* The telescope also, being bedded in a sort of frame, cannot be reversed end for end; the level is first adjusted by correcting half the error when turned round, by the screws which act upon the supports, and half by the plate-screws; the line of collimation is then made to agree with the corrected level by noting the height of the intersection of the cross wires on a staff about 200 or 300 yards distant. The instrument and the staff are then made to change places, and if the difference of level remains the same, the optical axis is already correct; if not, *half the difference* of the results must be applied to the observed height of the vane on the staff, and the cross wires adjusted to this height by means of the screws of the diaphragm at the eye-piece of the telescope.

A pool of water furnishes another easy mode of adjusting the line of collimation. A mark being set up at any convenient distance of exactly the same height above the surface of the water as the instrument adjusted for observation, the cross wires have only to be made to intersect each other at this point.

The adjustments of Mr. Gravatt's level (the best of the three) are nearly similar; and will be found described by himself, in Mr. Simms' little work, already quoted.*

* Also in page 137 of Mr. Bruff's "Engineering Field Work."

The French *water level* is much used on the Continent in taking sections for military purposes. It possesses the great advantage of *never requiring any adjustment*, and does not cost one-twentieth part of the price of a spirit level. From having no telescope, it is impossible to take long sights with this instrument, and it is not of course susceptible of *very minute accuracy;* but, on the other hand, no gross errors can creep into the section as may be the case with a badly-adjusted spirit level or theodolite, the horizontal line being adjusted by nature without the intervention of any mechanical contrivance. As this species of level is not generally known in England, the following description is given, which with the assistance of the sketch, will enable any person to construct one for himself without further aid than that of common workmen to be found in every village.*



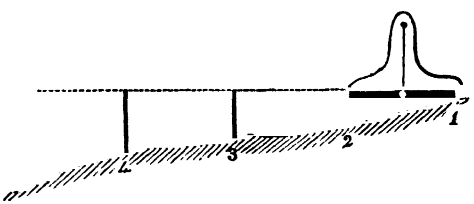
a b is a hollow tube of brass about half an inch in diameter, and about three feet long, *c* and *d* are short pieces of brass tube of larger diameter, into which the long tube is soldered, and are for the purpose of receiving the two small bottles *e* and *f*, the ends of which, after the bottoms have been cut off by tying a piece of string round them when heated, are fixed in their positions with putty or white lead, —the projecting short axis *g* works (in the instrument from which the sketch was taken) in a hollow brass cylinder *h*, which forms the top of a stand used for observing with a repeating circle; but it may be made in a variety of ways so as to revolve on any light portable stand. The tube, when required for use, is filled with water (coloured with lake or indigo), till it nearly reaches to

* The instrument from which the sketch was made was constructed for me by an ironmonger in Chatham; I have tried it against a very good spirit-level, and found the results perfectly satisfactory. This water-level is now constantly used on the Ordnance Survey for interpolating horizontal contours at vertical intervals of 25 feet between the more correct contours traced at greater distances apart by the spirit-level.

the necks of the bottles, which are then corked for the convenience of carriage. On setting the stand tolerably level by the eye, these corks are both withdrawn,* and the surface of the water in the bottles being necessarily on the same level, gives a horizontal line in whatever direction the tube is turned, by which the vane of the levelling-staff is adjusted. A slide could easily be attached to the outside of *c* and *d*, by which the intersection of two cross wires could be made to coincide with the surface of the water in each of the bottles; or floats, with cross hairs made to rest on the surface of the fluid in each bottle, the accuracy of their intersection being proved by changing the floats from one bottle to the other: either of these contrivances would render the instrument more accurate as to the determination of the horizontal line of sight; though one of its great merits, quickness of execution, would be impaired by the first, and its simplicity affected by either of them. For detailed sections on rough ground where the staff is set up at *short distances apart*, it is well qualified to supersede the spirit-level, and is particularly adapted to tracing *contour lines*: which operation will be described in its proper place.

A mason's level and boning-rods also answer very well for taking sections where *no better instruments are at hand*, and are used as described below.

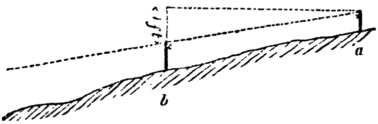
A horizontal line is obtained by driving two pickets (1 and 2) into the ground, and applying a large mason's level to their heads which should be previously cut square. The pickets 2 and 3, 3 and 4, &c., can be levelled in the same manner, as far as may be necessary to obtain a correct horizontal line for a short distance;



but if any considerable length is required, two boning - rods, of about three feet long, with a cross piece at the top, are placed on the heads of any two of the pickets already levelled, and the vane of a staff

* These corks must be drawn carefully, and when the tube is nearly level, or the water will be ejected with violence.

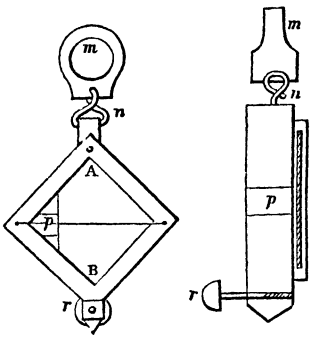
raised or depressed at any required point, till it is on a level with the tops of the boning-rods. The reading of the staff will give the respective depths below the level of the heads of the rods, the heights of which must be subtracted. Boning-rods are chiefly used in laying out slopes in military works, and for setting up profiles to direct working parties. A slope of 5 to 1, for instance, is laid out by measuring 5 feet from *a* towards *b*, and driving the head of the picket at the end nearest *b*, one foot lower than that at *a*; the heads of boning-rods, of equal height, placed on the tops of these pickets, are evidently on a slope of 5 to 1.



The last description of instrument used for levelling is the French "Reflecting Level," invented by Colonel Burel; a description of which is given in the second volume of "Professional Papers of the Royal Engineers."

The principle upon which this instrument acts is implied by its name. In a plane mirror the rays are reflected as though they diverged from a point *behind* the mirror, situated at precisely the *same distance in rear of its surface, as the object itself is in front.* If the mirror be vertical, *the eye and its image are on the same horizontal line;* and any object coinciding with these is necessarily on *the same level.* It appears then only requisite to ensure the verticality of a small piece of common looking-glass set in a frame of wood or metal, to be able without further assistance to trace contour lines in every direction, or to take a section on any given line. The mirror A B, described in the paper alluded to, is only one inch square, fixed against a vertical plate of metal weighing about 1 lb., and suspended from a ring *m*, by a twisted wire *n*, so that it may hang freely, but not turn round on its axis of suspension. It can either be used for sketching in the field, being held by this ring at arm's length; or fixed for greater accuracy in a frame which fits upon the top of the legs of a theodolite, with a bar of metal like a bent lever pressing so slightly against it from below, that it may check any tendency to oscillation, and at the same time not prevent the mirror from adjusting itself vertically by its own weight. The accompanying sketch will render this description more intelligible.

The required verticality of the plane of the mirror is thus ascertained: a level spot of ground is chosen, where it is suspended



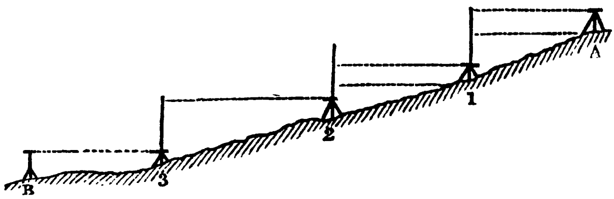
in its frame (or any temporary stand) 40 or 50 yards from a wall, and the prolongation of the line of sight *from the eye to its image*, coinciding with a fine silk thread across the centre of the mirror, is marked on the wall, which is visible through a small opening *p*, in the metal frame. The mirror is then turned round, and the observer, placed between it and the wall, with his back to the latter, notes the spot where the *image of his eye* coincides with the reflected wall *above or below the former mark*. The mean distance between these two points is assumed and marked, and by turning the screw *r*, the centre of gravity of the mirror is altered until the image of the eye coinciding as before with the silk thread agrees also with this central mark on the wall. It would perhaps be a better plan to send an assistant some distance behind the mirror with a levelling-staff the vane of which could be raised or lowered to coincide with the line of sight; on reversing the mirror (the staff remaining stationary) the vane would be again moved, until its reflected zero mark is cut by the thread on a level with the image of the eye, and finally, the mirror adjusted by the screw to the mean between these two heights; this method admits apparently of greater nicety than a chalk mark on a rough wall.

The reflecting-level is not generally known in this country; but for many purposes it is superior to any other description of instrument, particularly for tracing contour lines on the ground in a military sketch. It is peculiarly simple in its construction, is easily managed, easily adjusted, is not liable to have this adjustment deranged, or to be injured by a fall; is from its size, more portable than any other instrument, and can be used either held at an arm's length, or at a distance of several feet, in which position the length of the line of sight ensures the greatest accuracy.

The levelling-staff, a necessary accompaniment to each of the species of levelling instruments described, was formerly made with a sliding vane to move up and down a staff graduated to feet and decimals, or feet and inches; this was effected by a string and pulley, or the staff itself was made in two or three pieces, each of the upper pieces sliding in a groove in the one next below it. For any height less than the length of the first piece (generally about 6 feet) the vane was slid up or down with the hand; but for a greater height, the second piece, with the vane *at the top*, was moved up bodily till the centre of the vane was cut by the line of the optical axis of the instrument, when the height was read on another scale graduated *downwards from the top* on the side of the lower joint of the staff. A description of staff was however introduced some years ago by Mr. Gravatt, and has been since improved upon, on which the divisions (in feet and decimals) are marked so distinctly that they can be read by the *observer without the use of a vane*, or the necessity of trusting to an assistant; the figures are *inverted* to suit the inverting telescopes now generally used, and instead of moving about a heavy iron tripod on which to rest the staff, a species of shoe with a hinge is attached to it which allows the face to be turned round in any required direction without the staff being moved off the ground. Though much more convenient and less liable to mistakes in reading than the old species of staff, the same minute degree of accuracy cannot be obtained with it.

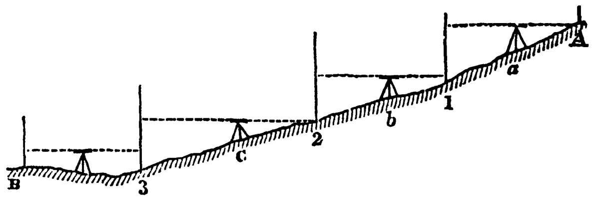
To proceed to the method of using the spirit-level or other instrument for tracing horizontal lines, and also of keeping the field-book in levelling for sections. In the system formerly pursued, the instrument was set up at one end of the line A, of which a section was required; and having ascertained the accuracy of its adjustments, and levelled it by the plate-screws, an assistant was sent forward with the levelling staff to the first station, and the difference between the height of the vane when intersected by the cross wires of the telescope, and the height of the optical axis of the instrument from the ground gave of course the difference of level between these two points. The distance was then measured and entered in the field-book, and the level moved on to the first station, the staff being sent on to the second, where the same process was repeated.

It is self evident that this manner of levelling is vitiated by the errors of *curvature* and *refraction*, which, if not allowed for in a long section, would in the end produce a considerable error.



But the necessity for these corrections is avoided by simply placing the instrument half-way between the two stations, and either in the line of section, or on one side of it.

Thus the level * being set up, as in the figure at *a*, the difference between the reading on the staff set at the back station A, and at



the forward station (1), gives at once the difference of level between the ground at these points without any correction for *refraction* or *curvature*, and also without taking into account the height of the instrument; a slight error in the line of collimation of the telescope also does not impair the results, as the elevation or depression of the optical axis would have the same effect on both staves; whereas in levelling entirely by the *forward station* the least error in the adjustment of the instrument is fatal to the accuracy of the section, being always carried on, whether additive or subtractive. This assertion, however, supposes the instrument to be exactly equidistant from the two stations, which in ground having a great inclination is often impossible; nevertheless, by good management, any reference to the table of curvature and refraction may generally be avoided, and if this correction is

* By having two assistants, with levelling-staves, one for the back and the other for the forward station, much time may be saved.

necessary, it should be made merely for the *difference* between the distances.

In keeping the detail in the field, the horizontal and vertical distances are sometimes written on a sort of rough diagram, as recommended in levelling by angles of elevation and depression with the theodolite; but the most general and best plan is to enter all the dimensions in the field-book, particularly if the distance to be levelled is considerable, and references are made to bench-marks. There are slight differences in the modes in which this field-book is kept, but the following example, with the description below, will show the usual method of entering the details, so as to render them at once available for transferring to paper:*—

Distance in Feet.	Back Sight.	Fore Sight.	+	−	Rise.	Fall.	REMARKS.
250	2·35	14·55	——	12·20	——	12·20	Commenced at bench-mark A.
200	3·56	9·58	——	6·02	——	18·22	
250	10·34	6·21	4·13	——	——	14·09	Crosses hedge into road.
270	14·55	0·25	14·30	——	0·21	——	Bench-mark on oak tree, in hedge close to fourth milestone.
200	9·98	1·67	8·31	——	8·32	——	
250	3·62	14·54	——	10·92		2·40	
B. M.	1·23	13·45	——	12·22		14·62	B. M. on sill of canal lock.
300	2·23	12·05	——	9·82	——	24·44	
250	0·20	13·55	——	13·35		37·79	Mark centre of road.
	48·06	85·85	26·4	64·53			
		48·06		26·74			
		37·79		37·79			

This table almost explains itself: the first column headed "Distances," contains the distances measured between each place where the staff is put up.† The second and third columns are for

* For more detailed instructions on the method of levelling for and plotting sections see Mr. Simms' work. Where very great accuracy is required, the level is always read over a second time, the instrument being thrown out of adjustment and readjusted—a certain amount of difference only is allowed—about ·003 ft. A levelling staff, with an improved vane, is also used, instead of the now common staff without a vane.

† Where only lineal distances or sectional areas are required, a chain of *feet* is the most convenient for use, instead of the Gunter's chain used for determining superficial areas in acres.

the readings of the staff at each back and forward station, the differences between each of which are entered under the fourth and fifth columns, headed + and — : under the two last, headed "Rise" and "Fall," are carried out the total rise or fall of each place where the staff was placed *above or below the starting point.*—The bench-mark at the end of the fourth station *being in the line of the section,* the distance is entered as usual; but that at the seventh, being *out of this line,* and its level merely ascertained for a future reference, there is no dimension entered in the column of "Distances," so that is not plotted in the section. Under the head of "Remarks" are noted the bearings* of the different lines of the sections if required to be laid down on a plan, the references to bench-marks, cross-sections, and other information that may subsequently prove useful. If the instrument is placed in the direct line of the section, it will give an intermediate point on the ground between the staves, by measuring its height; this requires again another column, and leads to confusion, without being of much benefit. The difference of the sum of all the back and forward sights should of course correspond with the difference between the quantities under the head of + and —, and also with the last reduced level, either rise or fall.

In making trial sections with the spirit-level to ascertain the best line for a railway or other work, the same form applies as for sections for more particular purposes, either civil or military; but the distances may be longer, as was observed when speaking of the theodolite. The same bench-marks should be always levelled up to in every trial section.

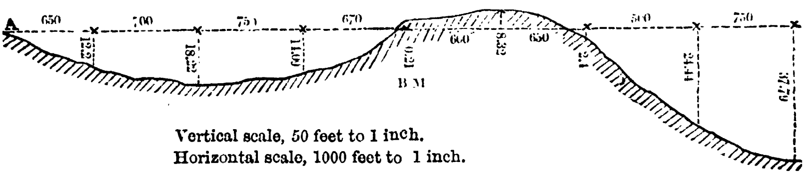
In running check sections, to ascertain the accuracy of former sections, there is generally no occasion for measuring distances: and only a column for "back," and another for "fore" sights, with a third for remarks, are required.

* A separate column is often kept for "Bearings;" and instead of the bearings and distance between each staff, the angles with the meridian, and the distances are sometimes taken between *the instrument and each back and forward station;* which arrangement requires two columns for distances, and two for bearings; or instead of bearings, angles may be taken to some known object.

B. S.	F. S.	Remarks.

At each bench-mark these columns may be added up, and their difference entered under the column of "Remarks." As already stated, check sections are more quickly taken with a theodolite by reciprocal angles of elevation and depression than by the spirit-level.

In laying down a section on paper, particularly if the ground is of gentle slope and the section of considerable length, it is usual to exaggerate the vertical heights for the purpose of rendering the undulations of the surface perceptible, which necessarily produces a distorted representation of the ground. The horizontal scale is usually made an aliquot part of the vertical, that the proportions between them may be at once obvious. Scales of 25, 50, 100 or 150 feet to one inch,* are appropriate for the latter, according to the degree of detail required in the section; and the horizontal scale may be from $\frac{1}{2}$ to $\frac{1}{10}$ of either of them; or even a less proportion if the section is of great length, and the ground generally



Vertical scale, 50 feet to 1 inch.
Horizontal scale, 1000 feet to 1 inch.

flat, as in the figure above, plotted from the specimen of a levelling field-book in page 95.

* The plotting scales, already alluded to, are very convenient for laying down sections; and Mr. Holtzapffell's cardboard Engine-Divided Scales will be found useful where a variety of scales are often required; from their method of construction, they can be sold at the low price of *nine shillings a dozen*, of all descriptions in general use. If the paper is stretched on a rectangular board, and two of these scales are placed along two of the sides at right angles to each other, the horizontal and vertical distances can be laid down with a T ruler and angle without using the compasses.

The horizontal line from which the vertical distances are set off may be either on a level with one end, or some one point of the section; or a datum line may be drawn any number of feet above or below this line, *exceeding the sum of all the vertical heights:* this latter arrangement makes all the dimensions reduced for plotting either *plus* or *minus*. Laying off *intermediate* horizontal and vertical distances should be avoided in plotting sections; the former ought always to be measured from the commencement of the section with as few interruptions as the length of the line will allow, and the latter from the datum line. Both horizontal and vertical distances should, particularly in a working section, be written legibly on the drawing.

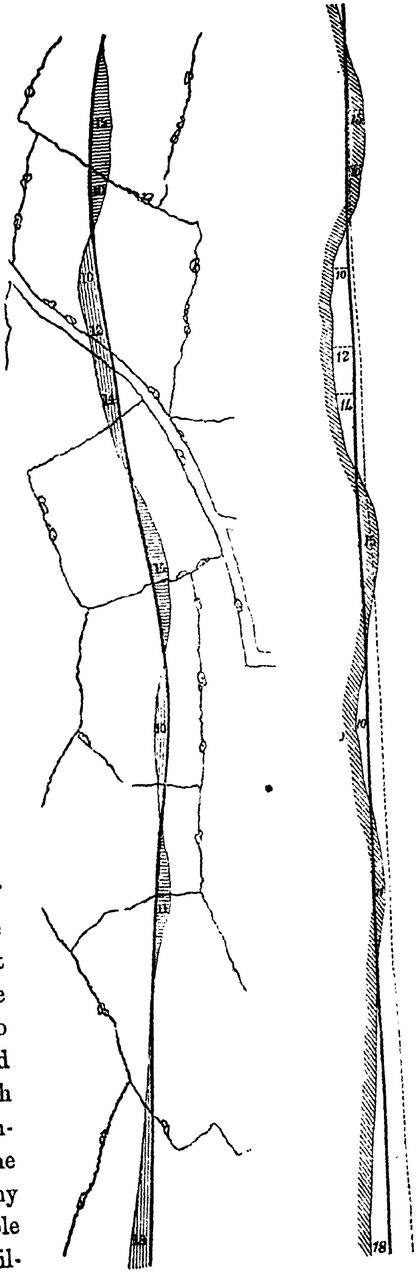
Trial sections that have been run for the purpose of ascertaining the best of several routes for a railroad, canal, or other work, should *invariably* be all plotted on the same scale and paper, and from the same datum line; and commencing at, and having reference to, the same points as bench-marks. By this arrangement their comparison by the eye is facilitated.

Cross or transverse sections are sometimes plotted *above*, and sometimes *below* the longitudinal section: and if only extending a few feet to the right and left, they are occasionally plotted *on* the line of section: but, if numerous, this last method causes a confused appearance in the drawing.

A method of combining plan and section has lately been introduced by Mr. Macneil, for the purpose of giving a popular representation of the quantity of excavation and embankment at any part of the section of a line of railway, the direction of which is shown on the outline plan of the country through which it passes by a thick black line, supposed to represent a vertical section of the rail. From the accurate section previously drawn, the heights of the embankments and depths of excavation at the different parts of the line are transferred to this datum line on the plan; and these quantities being tinted with different colours, or if engraved, represented the one with vertical, and the other with horizontal lines, show at a glance the general relative proportions of *cutting* or *embankment*, as in the annexed figure.

The dark line in both figures represents the surface of the railroad or embankment.

To those unaccustomed to the use of sections, this simple contrivance by which they are rendered intelligible is particularly useful, and has been ordered to be adopted in all plans for railways submitted to the House of Commons. Of course it is only intended to give a general idea of the quantity of work on any line of road, railroad, or canal, and to be explanatory of the report and estimate.



The section which has always to accompany this species of plan must be plotted on a scale, the horizontal distances being *not less* than 4 inches to 1 mile, and the vertical *not less* than 100 feet to 1 inch. A line must also be drawn on the section representing the upper surface of the rails. At each change of inclination the height above some datum plane must be shown, and also the rates of the slopes, and the distances for which these gradients are maintained. The height of the railway over or under any turnpike road, navigable river, canal, or other railway, is likewise to be marked at the crossing. A variety of precautions and regulations are enforced by the "Standing Orders" relative to the construction of railways; and there are numerous other details connected with them, for which reference must be made to some of the numerous excellent practical works devoted solely to this branch of civil engineering.

Numerous transverse sections are required for computing the relative proportions of embankment and excavation* on any work, which operation is much facilitated by the use of Mr. Macneil's ingenious tables, calculated upon the "*Prismoidal Formula,*" which shows the cubic content of any prism to be equal to *the area of each end* + *four times the middle area, multiplied by the length and divided by* 6; whereas the common methods of taking *half the sum of the extreme heights* for a mean height, or of taking *half the sum of the extreme areas* for a mean area, are both erroneous; the first giving too large a result, and the second too little.

Mr. Haskoll also gives very useful tables for the calculation of the areas of cross sections in the 2nd vol. of his "Engineer's Railway Guide;" a book containing full information upon all subjects connected with the laying out and construction of railway works.

The last description of levelling by the spirit-level to be noticed, is the method of tracing instrumentally horizontal sections termed "*contours,*" either round a group of isolated features of ground for the formation of plans for drainage, sanitary, railway, or other engineering purposes—models or plans of comparison for military works, &c.; or over a whole tract of country with the view of giving a mathematical representation of the surface of the ground in connection with a national, or other extensive and accurate survey.

As regards the first of these, the tracing instrumental contour lines round any limited feature, or group of features of ground, the manner of proceeding is very simple. The site must be first

* Of the greatest possible consequence, both for the sake of avoiding unnecessary expense, and of laying out the work to the best advantage. Valuable information upon this subject will be found in Mr. Macneil's work.

carefully examined, and those slopes that best define the configuration of the surface, particularly the ridge and watercourse lines, marked out by rods or long pickets at such distances apart as may appear suited to the degree of minutiæ required and the variety in the undulations of the ground. Where no such marked sensible lines exist, the rods must be placed where they can most readily be observed, being necessary as guides for the levelling staff during the subsequent operations. An accurate survey of the ground on which the positions of these rods are shown is then to be made. This should be laid down upon a scale proportioned to the purposes for which the plan is required, and to the vertical interval by which the contour lines are to be separated.

The scale for towns now adopted on the Ordnance Survey is $\frac{1}{500}$ or 10·56 feet to 1 mile, which is sufficiently large for most engineering and municipal works, but can be increased if necessary for illustrating projects for drainage, or for the supply of water by pipes, &c. Estates are generally laid down upon a scale of 3 or 4 chains to 1 inch. For the larger scales the contour lines may be traced at equidistant vertical intervals of from 2 to 10 feet where the scale of the plan varies from 50 to 500 feet to 1 inch. This plan of the ground should be in the hands of the surveyor on commencing his contouring, as it will be of considerable assistance during the operation, and it is also desirable that sections should be run from the level of some fixed plane of comparison along the principal and best-defined lines marked out by the rods alluded to, leaving pickets at the vertical intervals assigned to the contours. These pickets serve as tests of the accuracy of the work as it progresses and as starting points for fresh contours. The staff is now to be held at one of the pickets, the spirit-level (or theodolite used as a spirit-level) being so placed as to command the best general view of the line of level, and adjusted so that its axis may, when horizontal, cut the staff; and the vane (for a levelling staff of this description is required) raised or lowered till it is intersected by the cross wires of the instrument. The staff with the vane *kept to this height* is then shifted to a point about the same level between the next row of ranging rods not more than 12 or 15 chains distant from the spirit-level, on account of the correction that would otherwise be required for the curvature of the earth (about one inch in 10 chains), and moved up and down the slope till the vane again coincides with the wires, when another picket is driven. This process is continued until it is found necessary to move the level to carry on the contour line to the extent required.

The same operation takes place with the contours above and below that first laid out; and where any bench-marks or points, the level of which may be of importance, come within the scope of the spirit-level, they should be invariably determined.

Where the vertical interval is small, the pickets upon more than one line of contours can often be traced without shifting the position of the instrument if the levelling staff is of sufficient length. Too much should not however, be attempted at one time.

With regard to the second division of this subject, the tracing instrumental contours in connection with a national survey, the best instructions that can be given is a brief outline of the mode followed on the Ordnance Survey.

The ground between each of the trigonometrical stations is carefully levelled with a spirit-level, pickets being left at convenient intervals for the contours to start from. The surveyor to be employed in tracing these contours is furnished with the absolute altitudes of the pickets, or those of bench-marks out of the direct line between the trigonometrical points if they have been so left in preference, from which he has to level up or down to the contour height from whence he is to commence. With a theodolite or spirit-level he then traces the contour lines round the hill features in the manner already described, levelling to certain other bench-marks whose positions have been given to him, but of whose altitudes he is not informed in order that a check may be established upon his work, the position of the contour lines being recorded in a field-book with reference to the measured detail of the houses, fences, &c., in a close country; or by transverse lines in open uncultivated ground.

The whole of the altitudes for the foundation of the contour lines are determined by levelling with the spirit-level, the calculated heights obtained by angles of elevation and depression during the progress of the survey not being considered sufficiently accurate for the work as it is now performed:—the vertical

Fig. 1.

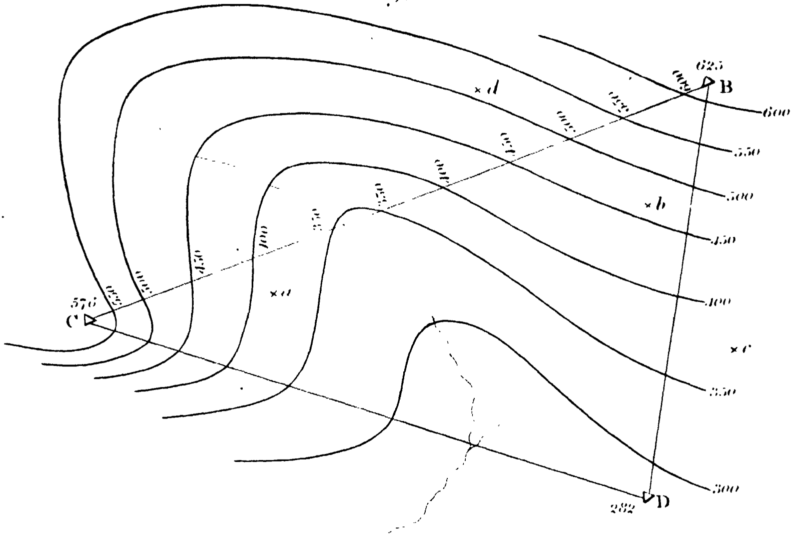


Fig. 2.

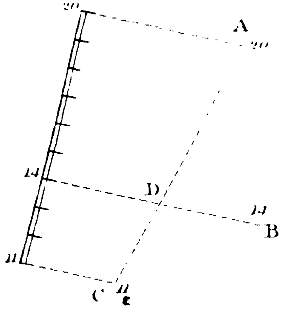
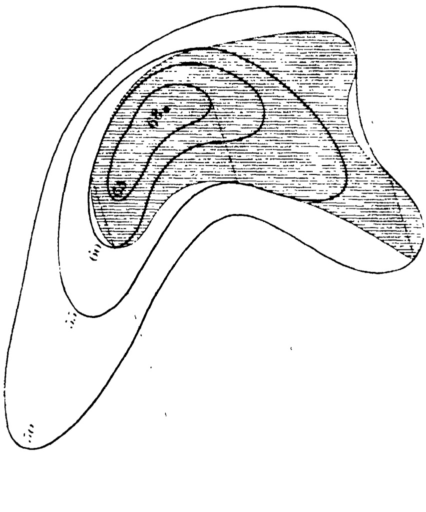


Fig. 3.



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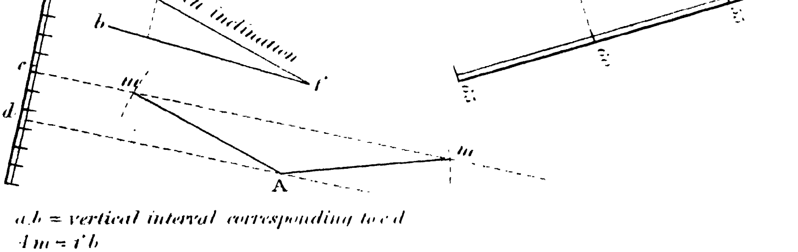
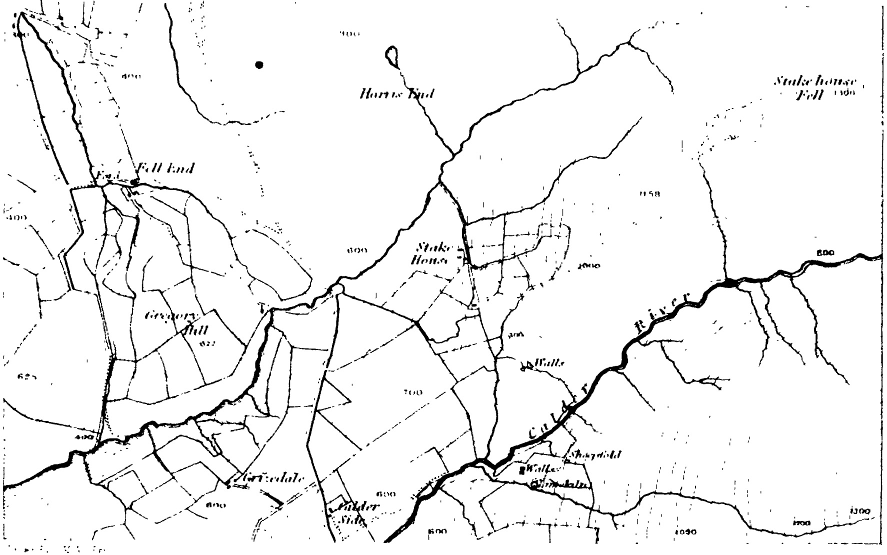


a b = vertical interval corresponding to c d
A m = f' b

These Contours should have been represented by dotted lines

Plate 9



HORIZONTAL CONTOURS

Traced at equidistant vertical intervals of 25 feet.

Scale 2 inches to 1 mile

distances between the contour lines thus traced out on the Ordnance Survey (now published on the scale of 6 inches to 1 mile, or 880 feet to 1 inch), vary according to the altitude above the sea, and as the character of the ground is steep or flat—from 25 to 250 feet. These contours are however interpolated with intermediate horizontal lines run with the water level generally at the constant fixed vertical intervals of 25 feet.

By assuming the level of the sea as the datum plane from which these progressive series of contours are to reckon, the altitudes of the several horizontal sections above that point are at once represented, which is a more useful and practical arrangement than the system adopted by the French (who first introduced this method of delineating ground), of fixing upon some imaginary plane of comparison above the highest parts of the plan, similar to the mode still practised with ordinary sections.

On surveys where pretensions are not made to such extreme mathematical precision, horizontal sections at distant vertical intervals may be traced with the theodolite or spirit-level, and the intermediate contours filled in by the eye; to perform which with tolerable accuracy, with the assistance of the instrumental contours previously marked by pickets on the ground, becomes after a little practice an operation of no great difficulty.

Even in surveys where the delineation of the surface of the ground is to be represented entirely by sketching on the horizontal system, as described in page 65, a few distant instrumental contours very much facilitate the work, and give it a character of truth and certainty that could not otherwise be looked for.

Fig. 1, Plate 8, illustrates the method of tracing and surveying the contour lines when the operation is carried on between the separate secondary triangles on an extensive survey. As has been remarked however, there is no necessity for following this system of working rigidly within the boundary lines of these triangles, as bench-marks established at any convenient spots out of the direct line connecting two trigonometrical stations, answer just as well for checks upon the progress of the work, and for datum points from whence to commence, and upon which to close the work.

Supposing, for instance, the altitudes of the trigonometrical points B, C, D, had been previously ascertained to be respectively 625, 570, and 282 feet above the level of the sea, and that the instrumental contours were required to be marked at equal vertical intervals of 50 feet above that level. Starting from either of these points, say C, in the direction of C B, mark the level of the nearest line of contours, which in this case would be 20 feet below C; and then the points where every difference of altitude of 50 feet would cut the line CB (500, 450, &c.). On arriving at B a check is at once obtained upon the section that has just been run; and the error, if any, can be corrected upon the spot. The other sides of the triangle, B D and D C, are then levelled in the same manner; the connection of the corresponding contour lines cutting each of them traced out by the spirit-level; and their position in plan laid down, either by traversing, or by reference to points and lines already surveyed and plotted. The places of many of these contour pickets can generally be ascertained whilst the levelling is in progress, by measuring their distances from the instrument and observing the angles made by them and the trigonometrical, or other known points. For this and other methods of obtaining their positions in the readiest manner no fixed directions can be given, as they must vary in different localities; and nothing but practice will render a surveyor capable of availing himself of the many opportunities he will constantly meet with of simplifying his operations by the exercise of a little forethought and judgment.

If, instead of confining the process of contouring within triangles, the altitudes of any points, *a*, *b*, *c*, *d*, &c., had been determined by levelling, and given to the surveyor as his starting points; he has only to level from one of them to the required altitude of the nearest contour line, either above or below him, and then proceed to carry this level round the hill features as in contouring isolated surveys. In very hilly or broken ground this system would appear preferable to that of working within the limits of regular figures, as the whole operation is made to depend more upon the marked natural features of the country.

It is hardly necessary to enumerate the advantages of a system of horizontal contours traced thus accurately upon the plans of a national survey. Not only can the best general lines of directions for roads, canals and railways, conduits for the supply of water, drainage pipes, &c., be ascertained without the trouble and expense of trial sections; but *accurate sections*, for whatever purpose required, may be traced to any extent across the country in all directions. Had this system been adopted on the Ordnance Survey of England thirty years ago, an incalculable saving would have been effected on all the trial lines run to ascertain the best practicable directions for the railways that now intersect this country.

Another use to which contour lines traced round any limited extent of ground can be applied is the formation of models for military or other purposes, though the contour plan itself affords far more accurate data for reference than can be obtained from the model, the dimensions of which being derived from the plan, are like all copies more liable to be vitiated by errors than the originals.

To construct these models an outline of the plan is pasted upon a flat board of seasoned wood or other material, the points at which all the vertical heights have been determined being marked upon the orthographic projection. Vertical standards of copper, zinc, or any other metal, are then inserted into the board at these points and cut off at the proper heights. The level of the board forms the lowest horizontal plane—that of the sea at low water, if the ground to be represented is contiguous to the coast;—and the tops of the highest set of rods the superior plane of contours. The intervals between these pieces of wire are filled in with composition or modelling clay which is worked carefully to the level of the tops of the rods, and then with a small flattening tool or the hand, moulded so as to represent as nearly as possible the irregularities of the surface of the ground; the representation will be more or less perfect in proportion to the smallness of the vertical intervals between the successive series of contours.

In some cases, particularly when the scale of the model is small and the character of the country of slight elevation, it is found desirable to increase the vertical scale, making it some multiple of the horizontal; this of course produces an unreal, and more or less exaggerated representation of the ground.

Where the contours have been run at considerable vertical intervals, and the surface sketched by the eye between them, the sketch will be found of much assistance in shaping the surface of the model.

From this model, if a mould in plaster of Paris is made, any required number of casts can be taken, which if properly prepared with isinglass or size, may be coloured and have delineated on their surfaces, references, boundary lines, &c., for geological purposes. These models are eminently useful, but they should be made of small detached pieces, representing the different divisions and characters of the strata.

By the aid of a contoured plan, many problems can likewise be worked out without the aid of vertical sections; from among others the five following are selected as of practical utility: *—

1. *To find the direction of the slope and the inclination of a plane passing through three given points* A B C, *not in the same straight line.—Fig.* 2, *Plate* 8.

Divide the line A C, joining the highest and the lowest of the given points, so that the two parts may bear the same proportion to each other as the numbers expressing the difference of level between the third point and each of the other two; that is, make A D : D C : : A∾B : B∾C; D will then be on the same level as B; and D B will be a horizontal of the plane required.

2. *To find the scale of a plane which shall pass through two given points and have a given inclination.*

This inclination determines the interval in plan between the contours passing through the two given points. With one of these points as a centre, and that interval as radius, describe a circle, the tangent drawn to which from the other point is a horizontal of the plane required. If the distance between the points is less than the necessary interval between the contours, this problem is of course impossible; and when possible it admits of two solutions.

* These problems are taken from a paper on Contour Plans and Defilade, by Colonel Harness, extracted principally from the "Mémorial du Génie."

3. *To find what part of a given surface is elevated above a given plane.*

The intersection of the horizontals of the plane with the contour lines at corresponding levels of the surface above, denotes, as seen in Fig. 3, the portion of the surface rising above the plane.

4. *To find the intersection of two planes.*

Produce until they meet two or more contours, having corresponding levels of each; the line joining the points of meeting will be that of intersection. If the contours of the two planes be parallel, their intersections, being a horizontal of each plane, will be known if one point in it be found.

5. *To find in a plane, given by its scale of slope, a straight line, which, passing through a given point in the plane, shall have a given inclination less than that of the plane* (*Fig.* 4).

Trace a contour of the plane having any convenient difference of level above or below this point. With that point as a centre, and with the base due, with the required inclination of the line to the assumed difference of level as a radius, describe an arc cutting that contour. The line drawn through their intersections and the given point will have the required inclination.

By the above problem a road up the side of a hill represented by contours, can be traced so as not to *exceed in any part a given inclination.*

The application of contours to the object of defilading a work to secure its interior from fire (almost the first use to which they were applied) can hardly be entered upon here. The subject is fully treated by many French authors on fortification; and extracts from Captain Noizet's paper, in the "Mémorial du Génie," will be found in the sixth volume of the Royal Engineers' Professional Papers.*

The method of measuring altitudes by the barometer and the temperature of boiling water is reserved for the next chapter.

* See also the chapter upon Defilade in Captain Macaulay's "Field Fortification."

CHAPTER VII.

LEVELLING CONTINUED.

MOUNTAIN BAROMETER, ETC.

THE Mountain Barometer presents a method of determining comparative altitudes not susceptible of so much accuracy as those already described, but far more expeditious when applied to isolated stations separated from each other by considerable distances. It is also capable of being used extensively by one individual; and the observations if performed with care will in most cases give results very near the truth. The instrument as made at present is very portable, though liable to injury in travelling if the proper precautions are not invariably taken, the most essential of which is that of always *carrying the cistern inverted*, and in this position tightening the screw* at the bottom of the cistern to prevent the oscillations of the mercury breaking the tube. In barometers *considered* of the best construction, and which are the most expensive, the surface of the mercury in the cistern is brought by a screw to the zero of the instrument, which marks the height at which it stood there when the scale was first graduated.† In others, not furnished with the means of effecting this adjustment, and in which the cistern is entirely enclosed from

* Mr. Howlett remarks that, in barometers where the bottom of the cistern is formed by a *leather bag*, the mercury should be forced up nearly to the top of the tube by the bottom screw, whilst the instrument is *held upright*. It should then be carefully inverted, in which position it must always be carried. When required for use, it should again be placed upright before the pressure of the screw against the bag is relaxed; otherwise the bag is liable to be burst.

† It is doubtful if this is any advantage: a barometer of this kind takes a long time to adjust and read; and as a tangent to the surface of the mercury is required, both in the tube and the cistern, there are more chances of error in the observation.

view, an allowance must be made to reduce the reading on the scale to what it would have been if the mercury in the cistern had been adjusted to zero. It is evident that this correction of the height of the column of mercury must be proportioned to the relative capacities of the cistern and the bore of the tube.

Thus, supposing the interior diameter of the tube to be ·1, its exterior ·3, and the diameter of the cistern ·9 inches; the ratio of the areas of the surfaces will be (81—9) 72 to 1.* The difference, then, between the observed reading of the barometer, and that of the "*neutral point*," which is the height at which the mercury stood *in the tube* above the zero mark of the cistern when the instrument was first made (and is always marked N P), is to be diminished in this proportion, and the quotient applied to the observed reading, *additive* when it is above this standard, and *subtractive* when below. The small correction for the capillary attraction of the glass tube is constant and additive, and is generally allowed for by the maker in laying off the neutral point, in which case no further notice need be taken of it. Should air by any means have found its way into the tube, it can if this is of large bore, be nearly got rid of by holding the barometer upright, with the cistern downwards, and turning the screw at the bottom as far as it will go without forcing. The instrument must then be sloped to an angle of about 45°, when more air will rush into the tube. If the screw is now unloosed, and the instrument held with the cistern upwards, at an angle of 45°, and gently tapped, the air will nearly all escape, the test of which is the mercury striking the top with a clear, and not a muffled sound, showing that the vacuum is nearly perfect.

The principle upon which the density of the atmosphere, measured by the height of the column of mercury, is applied to the determination of comparative altitudes is too generally known to need explanation; but the mere comparison of the observed heights of mercury at the places of observation will not suffice for

* This correction, termed the "*capacity*," is generally ascertained by trial. A certain quantity of mercury is first poured into the tube, which it fills to the height, say of 14·4 inches: this same quantity is then transferred to the cistern, and found to rise ·2 inch. The capacity is therefore as 14·4 to ·2, or 72 to 1; and this ratio is always marked by the maker on the instrument.

the purpose, as every change of one degree of temperature of Fahrenheit's thermometer causes an expansion or contraction of the fluid of $\frac{1}{9000}$ of its bulk, and all observation must be corrected on this account if made under different degrees of temperature. The method of using the mountain barometer is shortly as follows: it is carried as before observed inverted until required for use, the cistern being always kept above the horizontal at an angle of at least 45°, when the screw at the bottom of the cistern being first turned until it no longer acts against the end of the tube, the instrument is reversed, and the gauge-point (if there is one) is set to zero. The index is then moved till its lower edge is a tangent to the globular surface of the mercury, the height of which in the tube is read off to $\frac{1}{1000}$ of an inch by means of the index vernier; the thermometer *attached* to *the instrument* showing the temperature of the fluid, and the *detached* thermometer that of the atmosphere at the time of observation, are also noted, together with the heights of the mercury. The following form is convenient, as containing the observations, and leaving a space for the results:—

N.P. = 30·100
Cap. $\frac{1}{68 \cdot 37}$ } Lat. 51° 24′.

Station.	Attd. Ther.	Detd. Ther.	Observed Barometer.	Correction for Capacity.	Corrected Barometer.	Difference of Level.	Remarks.
High-water mark......	61°	58°	30·405	·004	30·409		
Parade, Brompton Barracks..................	60°	57°	30·276	·002	B 30·278	116·6	
Star Mill	67°·5	54°	30·120	——	B 30·120		

It is of course preferable to have two barometers, and to make simultaneous observations, as during changeable weather dependence cannot be placed upon results obtained with only one; particularly if *any considerable interval of time* has elapsed between the comparison of the heights of mercury at the different stations. Even the method that has been suggested by Mr. Howlett of noting the *time* of each observation, ending the day's work at the spot where it was commenced, and then correcting the readings of the barometer and thermometer at each station for the proportion of the total change between the first and last reading due to the respective intervals of time, cannot of course render observations taken with one barometer equal in accuracy to those observed simultaneously with two instruments, unless the rise or fall of the barometer, and particularly of the thermometer, was ascertained to have been *uniformly progressive* during the whole day. Observing however the barometer again at the first station at the close of the day has this advantage, that any great change during the period will be immediately detected, and the degree of dependence to be placed upon the observation made evident. The difference of readings, owing to these changes, will also be *generally* subdivided among a number of observations, though instances *may* occur, where this caution, as *regards the thermometer*, will be productive of error in the result. There are several methods of calculating altitudes from data thus obtained. That according to a formula given by Mr. Bailey, in page 183 of his invaluable "Astronomical Tables and Formulæ," is perhaps the most simple when a table of logarithms is at hand—it is deduced from the rule given by La Place reducing the French measures to English feet, and expressing the temperature by Fahrenheit's thermometer, and becomes by the use of the Table* in the next page $A+C+\log D$. D being $=\log \beta-(\log \beta'+B)$ where

t represents the temperature of the air at the lower station.

t' that at the upper.

r the temperature of the mercury at the lower station.

r' that at the upper.

A the correction for temperature dependent upon $t+t'$.

B that for the temperature of the mercury dependent upon $r-r'$, and

C the correction for the latitude of the place.

* In Mr. Bailey's table, the column B is calculated on the supposition that the thermometer is always the *highest* at the *lowest* station, which in *great* altitudes will be the case; but as the barometer may be used with advantage in a comparatively flat country, this omission has been remedied in a table published by Mr. Howlett, in the "Professional Papers" of the Royal Engineers, from which the column B has been taken. The *more accurate method* is to correct the barometer *for temperature*, independently of the tables.

TABLE

FOR DETERMINING ALTITUDES WITH THE MOUNTAIN BAROMETER.

Thermometer in open air.				Thermometers to the Barometers.				
A				B			C	
$t+t'$		$t+t'$		$r-r'$	Highest at Lowest Station.	Lowest at Lowest Station.	Latitude.	
40	4·76891	110	4·80229					
42	4·76989	112	4·80321					
44	4·77089	114	4·80412	0	0·00000	0·00000	0	0·00117
46	4·77187	116	4·80504	1	0·00004	9·99995	5	0·00115
48	4·77286	118	4·80595	2	0·00009	·99993	10	0·00111
50	4·77383	120	4·80687	3	0·00013	·99987	15	0·00100
52	4·77482	122	4·80777	4	0·00017	·99982	20	0·00090
54	4·77579	124	4·80869	5	0·00022	·99978	25	0·00075
56	4·77677	126	4·80958	6	0·00026	·99974	30	0·00058
58	4·77774	128	4·81048	7	0·00030	·99970	35	0·00040
60	4·77871	130	4·81138	8	0·00035	·99965	40	0·00020
62	4·77968	132	4·81228	9	0·00039	·99961	45	0·00000
64	4·78065	134	4·81317	10	0·00043	·99956	50	9·99980
66	4·78161	136	4·81407	11	0·00048	·99952	55	9·99960
68	4·78257	138	4·81496	12	0·00052	·99948	60	9·99942
70	4·78353	140	4·81585	13	0·00056	·99943	65	9·99925
72	4·78449	142	4·81675	14	0·00061	·99940	70	9·99910
74	4·78544	144	4·81763	15	0·00065	·99935	75	9·99900
76	4·78640	146	4·81851	16	0·00069	·99930	80	9·99890
78	4·78735	148	4·81940	17	0·00074	·99926	85	9·99885
80	4·78830	150	4·82027	18	0·00078	·99922	90	9·99883
82	4·78925	152	4·82116	19	0·00083	·99917		
84	4·79019	154	4·82204	20	0·00087	·99913		
86	4·79113	156	4·82291	21	0·00091	·99910		
88	4·79207	158	4·82379	22	0·00096	·99904		
90	4·79301	160	4·82466	23	0·00100	·99900		
92	4·79395	162	4·82553	24	0·00104	·99895		
94	4·79488	164	4·82640	25	0·00109	·99891		
96	4·79582	166	4·82727	26	0·00113	·99887		
98	4·79675	168	4·82813	27	0·00117	·99882		
100	4·79768	170	4·82900	28	0·00122	·99878		
102	4·79860	172	4·82986	29	0·00126	·99874		
104	4·79953	174	4·83072	30	0·00130	·99869		
106	4·80045	176	4·83158	31	0·00134	·99865		
108	4·80137	178	4·83234					

Make $D = \log. \beta - (\log. \beta' + B)$ then the log. of the differences of altitudes in feet $= A + C + \log. D$.

The following example taken from page 110 will explain the method of computation :—

$\beta = 30·409 - \beta' = 30·278$; latitude 51° 24′.

$t + t' = 115$ —	from the table,	A =	4·80458
$r - r' = 1$ —	„ „	B =	0·00004
	„ „	C =	9·99974
log β 30·409	. . .		1·48300
log β' 30·278	1·48113		1·48117
+ B	·00004		
		D =	0·00183

log D — 7·26245
A — 4·80458
C — 9·99974

2·06677 = 116·6 altitude in feet.

By a section taken with a spirit level, this altitude was found to be exactly 115 feet.*

Altitudes are also very easily (but not always so correctly) obtained by the tables in a pamphlet, entitled "A Companion to the Mountain Barometer," published by Mr. Jones, and sold with the instruments made by him. The barometrical observations are first brought to the same temperature by applying to the coldest a correction found in the first table for the difference† of the attached thermometers. The approximate height is then obtained by inspection, taking the difference between the numbers

* As a proof, however, that the results given by the barometer are not always to be depended upon when extended to very great distances, the observations consequent upon which occupy a considerable time; it may be mentioned that Professor Parrott who was employed in determining by barometrical measurement the level of the Black Sea above that of the Caspian, made this quantity by a series of the most careful *simultaneous* observations in 1811 exactly 300 feet; the same operation repeated by him in 1830 gave a result of only 3 or 4 feet. In 1837 this altitude was determined geodesically by the Russian Government to be 83·6, and was afterwards made by a French observer between 60 and 70 feet.

† In Mr. Jones's Pamphlet the centigrade thermometer is supposed to be used (the comparison of which with Fahrenheit's is given in Table 19). The centigrade, or centesimal thermometer, derives its name from the interval between *freezing and boiling water* being divided into *one hundred parts*. It is adapted to the decimal system of measurement, and since the Revolution has been very generally used in France. Its zero, like that of Reaumur's, commences at the freezing point.

corresponding to the corrected readings of the barometer from the second table.

Lastly, the correction in the third table opposite to this result, multiplied by the mean of the detached thermometers, and added to the approximate height, gives the true difference of altitude. The same example as before is worked out by means of these tables, the temperatures being converted from Fahrenheit to the centigrade scale to correspond with the tables.

	Fahr.	Cent.
	60 = . .	15·6
	61 = . .	16·1
		·5
Table first . . .		·0060
Correction applied		·0030
to coldest barom.		30·276
		30·281

Fahr.	Cent.	
58 =	14·4	
57 =	13·9	
	2)28·3	
	14·15	
	·45	From Table III.,
		for approximate
	7075	altitude 110 ft.
	5660	
	6·3675	

In Table II. opposite 30·281 is	611
opposite 30·409	501
Approximate diff. of alt. . .	110
Add correction table . .	6·3
True difference of altitude .	116·3

Dr. Hutton's rule for the calculation of altitudes by the barometer is as follows:—First correct the heights of the mercury, or reduce them to the same temperature, increasing the colder, or diminishing the warmer, by $\frac{1}{9600}$ part for every degree of difference between them as shown by the attached thermometer.

2nd. Take the difference of the common logarithms of the heights of the barometer thus corrected, setting off four figures from the left hand for integers, which will be an approximate height in *fathoms*.

3rd. Correct the number last found for the atmospheric temperature shown by the detached thermometers as follows:— For every degree that the mean of the two differs from 31°, take so many $\frac{1}{435}$ parts of the fathoms above found, and add them if the temperature be above 31°, but subtract them if below, for the true difference of altitude in fathoms.* The same example as before is thus solved by this rule:

$\frac{30,278}{9600}$ =	·003		.57
	30·278 add		58
	30·281 logs.		57·5 mean.
30·409	= =	1·4830021	31· subtract.
30·281	. .	1·4811702	
			26·5
Approximante alt. fathoms		18·319	
		1·116	$\frac{18\cdot3 \times 26\cdot5}{435} = 1\cdot116$
True altitude in fathoms		19·435	
		6	
Or in feet		116,61	

Where no table of logarithms is at hand, the following rule is given in Mr. Howlett's paper for the altitude:—

$$a = \text{diff. bar.} \times \frac{48820 + 58\cdot4 \times \text{sum detached thermometers}}{\text{sum of barometers.}}$$

Approximate altitude $= a - a\,(\cdot00006 \times \text{lat. in degrees})$.

This is nearly correct up to 2500 ft.; for a greater altitude apply the following correction:—

$$\text{True alt.} = \text{approx. alt.} + \tfrac{1}{3}\ \text{approx. alt.} + \left(\frac{\text{diff. bar.}}{\text{sum. bar.}}\right)$$

* In this rule of Dr. Hutton's, as in Jones's tables, there is no correction for latitude. One of the latter, I have also been informed, is erroneous; but they will, at all events, give good approximate results, which is all that is generally required of the mountain barometer.

The computation of heights from barometrical observations without the use of logarithms is also much facilitated by the aid of the following tables constructed by Mr. J. O. Farrell of the Ordnance Survey, which are applicable to all ordinary cases.

TABLE I.

Inches.	Mean Reading of Barometer.										Proportional Part for Hundreds. Subtract.			
	Tenths.													
	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	·02	·04	·06	·08
25	1004·9	999·9	995·0	990·1	985·3	980·5	975·8	971·1	966·5	961·9	1·0	1·9	2·9	3·8
26	957·4	952·9	948·4	944·0	939·7	930·4	931·1	926·9	922·8	918·6	0·9	1·7	2·6	3·4
27	914·4	910·5	906·5	902·5	898·6	894·7	890·8	887·0	883·3	879·5	0·8	1·5	2·3	3·1
28	875·8	872·1	868·5	864·9	861·3	857·8	854·3	850·8	847·4	844·0	0·7	1·4	2·1	2·8
29	840·6	837·2	833·9	830·6	827·3	824·1	820·9	817·7	814·5	811·4	0·6	1·3	1·9	2·6
30	808·3	808·2	802·1	799·0	796·0	793·0	790·0	787·0	784·1	781·2	0·6	1·2	1·8	2·4

TABLE II.

Difference of Attached Thermometers.	Mean of Detached Thermometers. Corrections in Feet.			Prop. Parts for Difference of Attached Thermometers.	
	40°	60°	80°	Difference, Att. Therm.	Prop. Parts.
°	°	°	°	°	Ft.
10	24	25	26	4	10·
20	48	50	52	5	12·5
30	71	74	77	6	15·
40	95	99	103	7	17·5
50	119	124	129	8	20·
60	143	149	155	9	22·5

To use these tables add the tabular number from Table I. corresponding to the half sum of the readings of the barometers (corrected for instrumental errors) to the sum of the readings of the detached thermometers, and multiply this sum by the difference of the barometers—then from the product thus found subtract (or *add* if the reading of the upper attached thermometer be the greater) the correction from Table II. corresponding to the difference of the attached thermometers found in the column headed "Mean of detached Thermometers" which most nearly corresponds with their mean reading.

The following example being the same as that worked out by Mr. Bailey's formula in page 110, shows the application of this simple rule.

Barometers.		Attached Therms.	Detached Therms.
At High-Water Mark . . .	30·409	61	58 } mean 57·5.
Parade, Brompton Barracks . .	30·278	60	57 }
	2)60·687	Diff. 1°	
½ sum . . .	30·343	Add from Table I.	797·8
		Sum . .	912·8
Difference. Barometers . .	0·131	Multiply by .	·131
	Product.	Product .	119·5768
Diff. Attached Therms. .	1°· }	Correction, Table II. Subt.	2·5
Mean Detached Therms. .	57·5 }		
		Altitude	117·0 feet.
		True altitude by levelling . . .	115·

This process is still further simplified by adding or subtracting as the case may be, 2½ times the difference of the attached thermometers in lieu of the correction found in Table II.

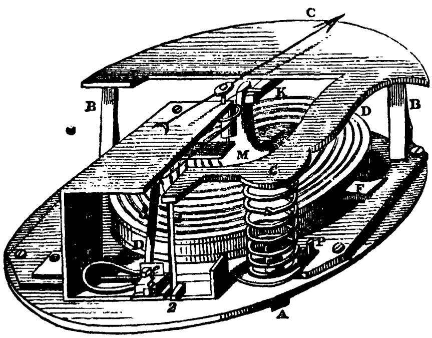
The well-known description of barometer, termed an *Aneroid*, would if more accuracy and minuteness could be introduced into the mode of reading off the graduation of the dial by the indication of the hand, be found a most valuable substitute for the mercurial barometer in the determination of moderate * altitudes, being much more portable, and not subject to the same derangement and risk of fracture by carriage as the other more delicate instrument. The pressure of the atmosphere is also the motive power in this invention; but its application is totally different from that of the barometer, as it is made to act not on the surface of a fluid, but upon the top of a shallow cylindrical metal box, from which the air has been exhausted and a small quantity of gas introduced into what otherwise would have been a vacuum, for the purpose of compensating (by its expansion with the increase of temperature) for the tendency to collapse consequent upon the loss of elasticity thereby caused in the metal. The top and bottom of the box are forcibly separated and kept in this state of tension by a plate acting as a lever, the end farthest from the central point by which the box is supported resting upon a spiral spring. The increase or diminution of the atmospheric pressure

* The very limited range of the instrument, as at present constructed—only 2·5 inches below 30°—confines its power of measuring altitudes to about 2000 feet above the sea.

upon the surface of the box depresses or elevates this end of the lever, with which two other levers are connected; the last acting by means of a piece of watch-spring on the roller upon the axis of which is fixed the hand that indicates upon the dial the degree of pressure; a flat spiral spring also acts slightly upon this roller, always against the levers; and thus keeps the hand, which would otherwise remain stationary after being propelled to its full distance, in constant unison with the varying fluctuations of the atmosphere.

In measuring altitudes by the aneroid the same rules for calculating the heights hold good as with the barometer; but in the present imperfect state of the instrument the precaution appears necessary to be attended to of ascertaining by trial the actual value in feet of the graduations on the dial; and also the effect produced upon these results by any change of temperature, as different instruments will be found to vary in these particulars.

The sketch below of the interior of the aneroid, the dial plate being supposed to have been removed, is taken from an extract from Mr. Dent's treatise on the instrument in the "Aide



D D is the cylindrical vacuum box; C C the lever, to the end of which is attached the vertical rod *i*, connecting it with the other levers acting by means of a piece of watch spring upon the roller carrying the index hand. An alteration in the distance of leverage to regulate the movement of this hand, so as to correspond with the scale of a mercurial barometer, is managed by means of the screws *e* and *b*.

The position of the hand is made to coincide with the indication of a barometer by means of the screw A (to be touched for *no other purpose*), which effects the object by raising or depressing the lever C.

Hitherto there was no prospect of improvement in this instrument, as it could only be made by the patentee; but as the period for which this patent was granted has expired it is to be hoped it will be taken in hand by some of our best mathematical instrument makers, and rendered capable of supplying the place of the mercurial mountain barometer, at all events under circumstances where the latter would be liable to injury or even destruction.

Still more recently Mr. Bourdon has introduced another substitute for the barometer, now sold under the name of the Metallic Barometer, which is even more sensitive than the aneroid, but would probably be more liable to injury in travelling. The theory of this instrument is that a bent hollow tube, the transverse section of which is not a *perfect* circle, cannot expand transversely when under pressure from the atmosphere without also opening outwards in the whole curve; one end of this being fixed, the other has a certain amount of play varying with the pressure, and to this end is attached the machinery for moving the index. This same principle has been also applied most successfully by Mr. Bourdon to the steam gauge and other purposes.

A contrivance for measuring altitudes was proposed by Sir John Robinson, Secretary to the Royal Society of Edinburgh, at one of the meetings of the British Association at Newcastle.* The instrument consisted of a glass tube, about one and a quarter inch in diameter and fourteen inches long with a small bulb at the end, the capacity of which was three or four times that of the inside of the tube, and the graduations on the stem of the tube were formed experimentally by the maker in the following manner:—

The instrument was suspended within the receiver of an air-pump over a cup containing water at the temperature of 62°, the mercurial barometer standing at 30 inches. The air in the receiver being exhausted to a degree of *rarefaction corresponding to twenty-nine inches* of the barometer, the lower end of the instrument was immersed in a cup of water; and air being admitted into the receiver, the exhaustion was repeated until the barometer gauge indicated a pressure equal to *twenty-eight inches* when a corresponding mark was made on the tube, the air being in like manner admitted after its re-immersion. By the repetition of this

* A description of this instrument is given in the "Mechanics' Magazine," for October, 1839.

process the graduation of the stem was carried on as far as was necessary.

With several tubes thus graduated, an observer in a hilly country may ascertain the density of the atmosphere on the summits of different elevations, by sending an assistant to each with one of these tubes and a tin case containing water. They are taken up with the stems open, and (the air within each partaking of the density of that at the station) the mouth of the tube is put into the water, *and left in it as the assistant descends.* The water will rise in the stem as the density of the atmosphere increases, and will indicate by its height the degree of rarefaction of the air at the upper station—a correction being made for the variation of the barometer from the standard height, and also for that of the *temperature* of the atmosphere.

This substitute for the expensive and delicate mercurial mountain barometer would, from its portability and simplicity, be useful in roughly determining comparative altitudes in a mountainous country, but of course much accuracy cannot be expected from it.—Another method of obtaining approximate differences of altitude is by a comparison of the *temperatures* of boiling water (which vary with the pressure of the atmosphere), upon which a paper was some years since published by Colonel Sykes, who practised it extensively in India.*

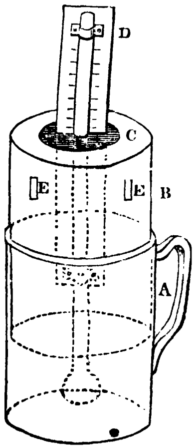
As the necessary apparatus is exceedingly simple, and the instrument not so liable to injury as the barometer, and much more portable and easily replaced, I have taken from this paper, (which will be found in the 8th number of the "Geographical Journal,") the tables computed by Mr. Prinsep to facilitate the computation of altitudes, and also the examples given by Colonel Sykes, which render their application evident without further explanation.

The results deduced from the use of these tables appear *always rather less* than those obtained from careful barometrical observations, and also less than those calculated from the different

* I ascertained lately the approximate altitudes above the sea of a number of places in Australia by this method; many of these were afterwards tested by the triangulation, and the results proved even more satisfactory than I had anticipated.

formulæ, which have been arranged for the determination of altitudes by this method, but which do not all agree. The results of a number of careful observations made with the thermometer compared with those obtained at the same time with the barometer; or which have been ascertained by levelling, or trigonometrically, will afford the means of making any necessary corrections in the tables, which however, giving so close an approximation, deserve to be more generally known and made use of.

The accompanying sketch and explanation, taken from Col. Sykes's pamphlet, show the whole apparatus required:



A. A common tin pot, 9 inches high by 2 in diameter.

B. A sliding tube of tin, moving up and down in the pot: the head of the tube is closed, but has a slit in it, C, to admit of the thermometer passing through a collar of cork, which shuts up the slit where the thermometer is placed.

D. Thermometer,* with as much of the scale left out as may be desirable.

E. Holes for the escape of steam.

The pot is filled four or five inches with *pure* water; the thermometer fitted into the aperture in the lid of the sliding tube by means of a collar of cork; and the tin sliding tube pushed up or down to admit of the bulb of the thermometer being about two inches from the bottom of the pot.

Before using a thermometer for this purpose, it is necessary to ascertain if the boiling point is correctly marked for the level of the sea by a number of careful observations, and the difference, if any, must be noted as an *index error*. It is always desirable to have two or more thermometers which have been thus tested; and in all observations the temperature of the air at the time should be noted.

* Thermometers are made expressly for this method of determining altitudes, the graduations being on a very large scale, and extending only a limited number of degrees above and below 212° Fahrenheit. Any common brewer's thermometer, with a metal pot or saucepan, will however answer the purpose when in want of the apparatus described above.

TABLE I.

TO FIND THE BAROMETRIC PRESSURE AND ELEVATION CORRESPONDING TO ANY OBSERVED TEMPERATURE OF BOILING WATER BETWEEN 214° AND 180°.

Boiling Point of Water.	Barometer Modified from Tredgold's Formula.	Logarithmic Differences or Fathoms.	Total Altitude from 30·00 in. or the Level of the Sea.	Value of each Degree in Feet of Altitude.	Proportional Part for one-tenth of a Degree.
°			Feet.	Feet.	Feet.
214	31·19	00·84·3	−1013	−505	...
213	30·59	84·5	507	−507	...
212	30·00	84·9	0	+509	...
211	29·42	85·2	+509	511	51
210	28·85	85·5	1021	513	...
209	28·29	85·8	1534	515	...
208	27·73	86·2	2049	517	...
207	27·18	86·6	2566	519	52
206	26·64	87·1	3085	522	...
205	26·11	87·5	3607	524	...
204	25·59	87·8	4131	526	...
203	25·08	88·1	4657	528	...
202	24·58	88·5	5185	531	53
201	24·08	88·9	5716	533	...
200	23·59	89·3	6250	536	...
199	23·11	89·7	6786	538	...
198	22·64	90·1	7324	541	54
197	22·17	90·5	7864	543	...
196	21·71	91·0	8407	546	...
195	21·26	91·4	8953	548	...
194	20·82	91·8	9502	551	55
193	20·39	92·2	10053	553	...
192	19·96	92·6	10606	556	...
191	19·54	93·0	11161	558	...
190	19·13	93·4	11719	560	56
189	18·72	93·8	12280	563	...
188	18·32	94·2	12843	565	...
187	17·93	94·8	13408	569	57
186	17·54	95·3	13977	572	...
185	17·16	95·9	14548	575	58
184	16·79	96·4	15124	578	...
183	16·42	96·9	15702	581	...
182	16·06	97·4	16284	584	...
181	15·70	97·9	16868	587	...
180	15·35		17455		59

The Fourth Column gives the Heights in Feet.

TABLE II.

TABLE OF MULTIPLIERS TO CORRECT THE APPROXIMATE HEIGHT FOR THE TEMPERATURE OF THE AIR.

Temperature of the Air.	Multiplier.	Temperature of the Air.	Multiplier.	Temperature of the Air.	Multiplier.
°		°		°	
32	1·000	52	1·042	72	1·083
33	1·002	53	1·044	73	1·085
34	1·004	54	1·046	74	1·087
35	1·006	55	1·048	75	1·089
36	1·008	56	1·050	76	1·091
37	1·010	57	1·052	77	1·094
38	1·012	58	1·054	78	1·096
39	1·015	59	1·056	79	1·098
40	1·017	60	1·058	80	1·100
41	1·019	61	1·060	81	1·102
42	1·021	62	1·062	82	1·104
43	1·023	63	1·064	83	1·106
44	1·025	64	1·066	84	1·108
45	1·027	65	1·069	85	1·110
46	1·029	66	1·071	86	1·112
47	1·031	67	1·073	87	1·114
48	1·033	68	1·075	88	1·116
49	1·035	69	1·077	89	1·118
50	1·037	70	1·079	90	1·121
51	1·039	71	1·081	91	1·123

When the water (with the thermometer immersed) has been boiled at the foot and at the summit of a mountain, nothing more is necessary than to deduct the number in the column of feet opposite the boiling point below, from that opposite the boiling point above: this gives an approximate height, to be multiplied by the number opposite the *mean* temperature of the air in Table II., for the correct altitude.

	°	Feet.
Boiling point at summit of Hill Fort of Púrundhur, near Púna	204·2	=4027
Boiling point at Hay Cottage, Púna . .	208·7	=1690
Approximate height		2337

Temperature of the air above . 75°
ditto ditto below . 83

Mean 79—Multiplier 1·098

Correct altitude 2566 feet.

When the boiling point at the upper station alone is observed, the lower being the level of the sea, or the register of a distinct barometer, then the barometric reading had better be converted into feet by the usual method of subtracting its logarithm from 1·47712 (log. of 30 inches) and multiplying by 6, as the differences in the column of "*barometer*" vary more rapidly than those in the "*feet*" column.

	Feet.
Example.—Boiling point at upper station . . .	185°=14548
Barometer at Calcutta (at 32°) 29·75	
Then 1·47712−1·47349=·00363	
Setting off four figures gives 36·3 fathoms, which × 6	218
Approximate height . . .	14330
Temperature, upper station, 76°	
ditto lower, 84	
Mean temperature . . . 80 Multiplier Table II.	1·100
True altitude	15763

Assuming 30·00 inches as the average height of the barometer at the level of the sea (which is however too much), the altitude of the upper station is at once obtained by inspection in Table I., correcting for temperature of the stratum of air traversed, by Table II.

In moderate elevations, the difference of *one degree* in the temperature at which water boils, indicates a change of level of *about* 500 *feet*, nearly equivalent to what would be shown by a difference of 0·6 of an inch in a mercurial barometer.

CHAPTER VIII.

COPYING, REDUCING, SHADING, AND ENGRAVING TOPOGRAPHICAL PLANS.

ALLUSION was made at the end of Chapter IV (p. 60), to the reduction of the surveys of towns and parishes plotted on the larger scales to that of 6 inches to the mile, for their insertion in their proper places in the county maps, and also the further general reduction of all these to the one-inch scale; these reductions were until lately made generally by the pentagraph, in preference to using proportional compasses, or drawing squares in pencil of the required proportional size over the original and the paper for the intended reduced copy,* and it is only very recently that they have been effected by photography, from which a vast saving of time and expense has resulted.

In the pentagraph, the extreme range of reduction is in the proportion of 12 to 1, whereas there is hardly any limit to that by which plans may be reduced by photography; the reduction from the $\frac{1}{500}$ to the 6 inch scale is in the proportion of 21 to 1, and consequently if made by the pentagraph would require two separate processes.

The plans of the towns, parishes, and counties, containing a much greater quantity of detail than could be crowded into the

* Plans may be either reduced or enlarged by means of a sheet of vulcanised India rubber. In the former case the sheet is first stretched upon an expanding frame, and the plan copied on its surface with prepared ink. It is then allowed to collapse to the required scale, and the impression transferred for printing off either upon stone or zinc. When an enlarged copy is wanted, the plan is drawn upon the India rubber in its normal condition, which is then stretched to the required proportional scale, and the expanded copy transferred for printing. Copies of plans upon the *same* scale are usually made by the method of equal squares,—with proportional compasses,—by tracing paper,—or with the tracing glass mounted upon a frame.

scale of 1 inch to 1 mile, it is usual in the reduction by the pentagraph, or any other method, to omit in the reduced drawings such portions of this minutiæ as appeared necessary or advisable. A difficulty on this score arose in the adaptation of photography which was met by making tracings of these plans on prepared calico omitting the superfluous detail, from which tracings the copy by photography was made; and more recently by printing the whole impression in *white ink*, and colouring with yellow such portions as were to be retained (which alone are taken up by the instrument), exaggerating as is necessary the breadth of the roads, buildings, &c. An immense saving of time and labour was thus effected, as by the old process three copies or tracings of the reduction by hand were requisite, one for the engraver and two on which to fill in the features of the ground, contours, &c.; whereas by the present method it is only necessary to print off two more of the photographic impressions.

The principal use at present made of photography, upon the Ordnance Survey is confined to the reduction of the towns on the $\frac{1}{500}$ scale, to that of the parishes on the $\frac{1}{2500}$, and of the latter to the scale of 6 inches to 1 mile for the counties; and though it may probably be much extended, it is not contemplated to apply it to the purpose of multiplying plans for publication, in which respect it cannot bear comparison with impressions on copper or zinc, either as regards economy or rapidity of execution.

With respect to accuracy, although theoretically owing to the small distortion of the lens, photographic copies and reductions of plans can never be perfectly true, they are practically more so than those obtained by the pentagraph or any known method.

Another proposal for further reducing labour by means of photography has reference to the hill sketching drawn upon one of the outline copies on the 6 inch scale, obtained as described above.

The present mode, as described in his report on the reduction of plans, by Capt. Scott, R.E., who states that with the engraver it requires three different persons, each possessing a certain amount of artistic skill, is as follows:—

"The first receives a 6 inch impression with contours (if these have been traced), which he takes into the field, and sketches on it the ground in pen and ink by horizontal shading.

"The second receives this hill sketch, and on a 1 inch outline impression with contours in pencil, he produces with brush and Indian-ink a reduced copy of the 6 inch field sketch.

"The third engraves on copper the hills from this shaded drawing, rendering the character given by the brush by means of vertical hachures. It is now proposed to have the field sketch made on the ground in so complete a manner, that the second process of making a finished drawing with the brush may be altogether dispensed with, and the hill sketch on the 6 inch scale reduced by photography to the 1 inch scale, as a sufficient guide for the engraver.

Attempts are now being made to substitute a new style of engraving more characteristic of the real features of the ground, for that of the vertical hachures, which has been adopted on the Ordnance Survey in preference to the horizontal system used in the field, partly on account of its greater facility of execution, and partly it is believed from the supposed interference of horizontal lines with the outline details, particularly of the roads, the general directions of which when laid down on a map on a small scale run nearly parallel to contour lines; and from specimens produced by Mr. Duncan, the Chief Engraver at the Ordnance Survey Office in Dublin, it would appear that he has succeeded in this altogether new style of engraving, to which he has given the name of Trio-tinto, because it combines the effect of the three methods of Mezzo-tinto, Aqua-tinto, and Stippling.

The inventor appears confident of being able to bring this method so completely within the control of the engraver, that the same "scale of shade" shall without difficulty be invariably adapted to its corresponding angle of inclination, which in a contoured plan can be ascertained with mathematical certainty by the mere application of a scale to the drawing, the distance measured being the base, and the number of contours within that distance multiplied by the vertical distance between each contour the perpendicular. The establishment of a scale of shade has frequently been attempted on the Continent, as before stated, by Major Lehman, and several French engineers, as well as by Mr. Burr at Sandhurst; and its realisation in an easy and graphic system of hill-engraving would certainly supply a want long felt in topographical drawing, more particularly if the cost of engraving should, as it is supposed, not exceed $\frac{1}{3}$ or $\frac{1}{2}$ of that of the present system of vertical hachures.

It was observed above that it was not proposed to multiply plans by photography for publication, that is, not to print off a number of photographic copies from a negative upon glass; but most successful attempts have been made by Sir H. James to transfer photographic prints of manuscripts, maps, and line engravings, to zinc and stone, and to etch in the impression by a weak solution of phosphoric acid, copies from which are printed off in the usual manner.*

In a pamphlet published by him, entitled "Photo-zincography," the whole of this (termed the "Anastatic") process is described, and it will no doubt be extensively applied, not only to the multiplication of maps, but of old manuscripts and other documents now almost inaccessible,† at a much lower cost than by any other method, and with unerring fidelity.

To return to the subject of representing the features of the ground with the brush and Indian-ink upon a topographical plan, either for the purpose of producing a drawing giving the physical relief of the ground, or for that of guiding the engraver.

The different disposition of the light affords the means of varying the system of shading hills. Where it is supposed to descend in parallel vertical rays upon the ground, each slope evidently receives less light, or relatively speaking *more shade* in proportion to its deviation from a horizontal plane on which the maximum of light falls. Mr. Burr, in his "Practical Surveying," devotes a chapter to the *scale of shade* to be applied to plans finished on this supposition, which however he candidly acknowledges to be an impracticable theory; but it leads him to the very just conclusion, that hills are generally shaded *much too dark* to give anything like a natural representation of their various slopes, which defect has also the additional fault of confusing the appearance of the drawing, and impairing the accuracy and

* The specimens of hill sketching given in plates 7 and 7 *a* were thus obtained by photography from pen and ink sketches.

† The Doomsday Book for instance, which Sir H. James is now printing by photography.

distinctness of the outline. The slopes drawn upon this system have evidently no light or dark sides, which causes a monotonous effect; and yet, on the same plan, both trees and houses are constantly represented with shadows.

The other system of supposing the light to fall obliquely upon the ground (as in nature), either at one fixed angle or at an angle proportioned to the general character of the slopes,* is decidedly favourable to the talent of an artist; but there are two objections to its general adoption in plans of an extended survey: first the difficulty of execution; and secondly, its ambiguity even when correctly drawn, except to those accustomed to the style. The slopes directly opposed to the light would evidently receive a greater portion of illumination than the summits of the highest hills; and, in fact, the whole arrangement of the disposition of the shades is quite different from what it would be under a vertical light, as is seen by exposing a model of any portion of ground to a strong light from a partially-closed window. The practice of copying the effects of light and shade from models is the best introduction to this system of shading ground, and is in fact indispensable before attempting to finish a plan.†

The method now most generally practised in topographical plan-drawing partakes of both these systems;‡ the light is considered as falling *nearly vertical*, but sufficiently oblique to allow of a decided light and shade to the slopes of the hills, trees, &c. The hills are shaded, *not as they would really appear in nature*, but on the *conventional system of making the slopes darker in proportion*

* Mr. Burr proposes an angle of about 15° for a flat country, and 40° for mountainous districts; the angle of oblique light ranging between these two extremes according to the nature of the ground.

† The late Mr. Dawson, whose talents and energy did so much towards bringing the sketching and shading plans of the Ordnance Survey to its present state of perfection, was the principal advocate of this system of oblique light; and some of the copies, from models of large tracts of country drawn by Mr. Carrington, at the Ordnance Map-office, in the Tower, are hardly to be distinguished from the models themselves, when they are both placed in the proper light.

‡ These and the preceding remarks apply solely to shading with the *brush;* the methods of delineating slopes by *the pen and pencil* having been explained in the last chapter. The Ordnance Surveys are finished on this system for the engraver, even though the ground may have been instrumentally contoured.

to their steepness; the summits of the highest ranges being left white. This arrangement, though obviously incorrect in theory, has the advantage of being more generally understood by those not accustomed to plan-drawing, and is also easy of execution: it is that now adopted in finishing the plans of the Ordnance Survey, and from which the features of the ground are engraved on the vertical system of etching, as being much the easiest, although not so for sketching in the field.

In finishing detailed plans on a large scale, stone or other permanent buildings are generally coloured red (lake or carmine). Wooden or temporary structures are tinted with a shade of Indian ink. Water is always coloured blue. Where distinctions between public and private buildings or property are required to be shown, different colours must be used and explained by references on the drawing; the same remark applies to the distinction between buildings erected and those only contemplated. The most usual conventional signs have already been alluded to in pages 76 and 77.

Trials have been made to render the patent process of engraving by a machine, known by the name of "Anaglyptograph," which answers so beautifully for giving a correct representation of a cast, or basso-relievo, available for topographical designs. A surprising relief is produced by this method of engraving, but it renders the general surface of the plan so dark as to obscure the accuracy of the outline, and as it is necessary that a model should be previously made of the feature to be represented, it is only suited to small portions of irregular ground.

Any lengthened description of the method of engraving upon copper the plans of the Ordnance Survey would be foreign to the objects of this work; the process is of course nearly similar to that of all copperplate engraving, but a considerable portion of the writing, and all flat shades of water, &c., are done by machine; the parks and sands are also ruled by machinery by a steel dotting wheel, the interval between the dots being regulated according to the required tint.

Woods, figures, rocks, &c., are engraved by steel punches, by which a vast saving of labour is effected.

As copperplates will only bear a certain number of impressions

being taken off without evincing a deterioration in the impressions, the plan of renewing these plates by the electrotype process has been resorted to with perfect success.

Before a plate shows symptoms of wear, generally after a fixed number of copies have been taken, it is placed in a large bath of sulphate of copper and diluted sulphuric acid, with a sheet of crude copper to supply the waste, and submitted to the action of a very strong galvanic battery. A copper matrix is gradually formed upon the engraved plate by the decomposition of the copper and its deposit on the plate; and when sufficiently formed the matrix itself is removed and submitted to the same process, until a fac-simile of the original copper engraving is perfected, which is then used for producing further impressions, which may thus be perpetuated for ever.

Another use to which this process is applied is that of taking impressions of the copperplate at different stages of its progress of engraving, so that one copy is obtained with the contours, boundaries, &c., another with the hill features, a third with geological lines, &c.

CHAPTER IX.

COLONIAL SURVEYING.

The preceding chapters will, it is believed, be found to contain all necessary information connected with the survey of any tract of country, whatever degree of accuracy or detail may be required; but in a newly-established colony, or one only partially settled, the primary object in view in commencing an undertaking of this nature is not the same as in that of a thickly peopled and cultivated country. In the latter case, the surveyor aims at obtaining, by the most approved methods consistent with the time and means at his disposal, data for the formation of a territorial map showing the position and extent of all roads, towns, provinces, counties; and where the scale is large, parishes, and even the boundaries of property and cultivated or waste land, as well as the features of the surface of the ground, and all natural and artificial divisions, together with the collection of a variety of other useful geological and statistical information. In a *new country* only the natural lines and features exist;—the rest has all to be created.

The first operations then required in a perfectly new settlement, are, the division into sections of such size as may be considered best adapted to the wants of settlers, of the land upon which they are to be located; and the marking out the plan of the first town or towns, the sizes and positions of which will of course be regulated by local circumstances and advantages; whilst the first rural sections will naturally be required either in their immediate vicinity, or contiguous to the main lines of communication leading to the different portions of the province whose local importance is the earliest developed.

In the case of a small settlement established upon the coast of any country for the immediate reception of settlers who require to be put in possession directly upon their arrival of a certain stipulated amount of land for agricultural or other purposes, the simplest form of survey must necessarily be adopted; that described in the late Col. Dawson's Report upon the Survey of New Zealand for instance, which consists simply in marking methodically upon the ground the angles of a continued series of square or rectangular figures, leaving even the roads which are intended to surround each block of sections, to be laid off at some future period,—would answer the purpose of putting impatient emigrants in possession of a homestead containing about the number of acres to which they might be entitled. But this system could not be carried out extensively with any degree of accuracy even in a comparatively level country, and not at all in a mountainous or irregular one. In fact, it is not a survey; and though perhaps it may sometimes be necessary to adopt what Mr. F. Wakefield, in his recently-published pamphlet upon Colonial Surveying, terms this "make-shift process," * the sooner a regular survey takes its place the better for the colony, even on the score of the ultimate saving that would be effected by getting rid of the necessity of incessant alterations and corrections; to say nothing of the amount of litigation laid up in store by persevering in a system necessarily entailing an incorrect division of property, upon which there is no check during the progress of the survey, and for which there is no remedy afterwards.

Excepting in some isolated instances such as described above, where everything is required to give way to the imperative necessity of at once locating the first settlers upon land for which payment has been received (for by the present system of colonization no land is alienated from the Crown otherwise than by purchase, the greater portion of the proceeds of the sale being devoted to the purpose of further emigration), the first step to be undertaken at the commencement of the survey of a new country, is a careful and laborious exploration within the limits

* For an explanation of the details of this species of surveying, see Mr. Kingston's Statements, page 33, Third Report of the South Australian Commissioners, 1838; and Col. Dawson's Report on the Survey of New Zealand, 1840.

over which its operations are to extend; during which would be collected for subsequent use a vast amount of practical information as to the number and physical condition of the aboriginal natives (if any); the geological character of the soil; its resources of all kinds; sources and directions of rivers; inland lakes and springs; the probable sites of secondary towns; the most apparent, practicable, and necessary main lines of communication; prominent sites for trigonometrical stations, &c., &c. A sketch of the country examined, rough and inaccurate doubtless, but still sufficient for future guidance, is at the same time obtained; the positions of many of the most important points for reference being determined by astronomical observation, and the altitudes of some of them by the mountain-barometer or aneroid, or by the temperature of boiling water, by methods already explained.

The next step should be (if this question has not been already determined by strongly-marked local advantages, or previous settlement) the position of the site of the first principal township, a nucleus being immediately required where fresh arrivals may be concentrated prior to their dispersion over the country. The size* and figure of the town will of course vary according to circumstances; and the principal general requirements that should suggest themselves to any one charged with a decision of this nature are,—facilities of drainage; plentiful supply of good water; easy access both to the interior of the country, and if not situated on the coast, to the adjacent port; the apparent salubrity of the site; facility of procuring timber and other building materials, such as sand, lime, brick-earth, stone, &c.; security from predatory attacks, and vicinity to sufficient tracts of land suited to agricultural and pastoral purposes.

The site of the town, with its figure and extent, being decided upon after a careful investigation of the above and a variety of other minor considerations, the best main lines of road diverging

* The size of the lots into which the township is to be divided may vary from a quarter of an acre to one acre; half an acre would be found generally sufficient. It is customary to give to the *first* purchasers of rural sections one town lot in addition for every such section, the remaining lots to be sold either by auction, or at some fixed price.

from it in all the palpably-required directions should be marked out, and upon these main lines should abut the sections to be first laid out for selection. Errors of judgment will doubtless be subsequently found to have been made in the directions of some of these roads; but this is certainly productive of less injury to the colony than the plan of systematically marking out the land without providing for any *main lines of communication at all*, leaving them to be afterwards forced through private property under the authority of separate acts of the colonial legislature; a system entailing discontent, litigation, delay, and expense. The marked natural features of the ground, such as the lines of the coast, or the banks of lakes or rivers of sufficient importance to constitute the division of property, and the main lines of roads alluded to, will, where practicable, guide the disposition of the lines forming the boundaries of the sections to be now marked out. Where no such natural or artificial frontages exist, the best directions in which these rectangular figures can be laid out are perhaps those of the cardinal lines, excepting in cases where the nature, inclination, and general form of the ground evidently point out the advantage of a deviation from this rule.

The size of these sections is a question to be determined by that of the minimum average number of acres which it is supposed is best adapted to the *means* and *wants* of the settler; the latter being in a great measure regulated by the apparent capabilities of the soil. Land divided into very large farms is placed beyond the reach of settlers of moderate capital; and if subdivided into *very* small portions, the expense of the survey is enormously increased, and labourers are tempted to become at once proprietors of land, very much to their own real disadvantage, as well as that of the colony. In South Australia, 80 acres has been adopted as the average content. In parts of New Zealand * and elsewhere, 100 acres. In Canada,† generally more than double

* In the Canterbury Settlement, on the Middle Island, New Zealand, 50 acres has been fixed as the minimum size; the maximum is unlimited; as in South Australia; no reservation is made of coal and other minerals; the purchaser being put in possession of all that is on and under the surface.

† The rude and inaccurate mode in which land has been marked out in Canada by the chain and compass, and the little value that has been set upon waste land which

that quantity. Whatever size may be determined upon, it is advisable to adhere to as nearly as possible, in all general cases; though, where special application is made for rather larger blocks, there has been found no mischief in departing from the average size, provided this deviation is not so extreme as to prevent fair competition for any peculiarly valuable locality. In such cases, it is however always necessary to guard particularly against the monopoly of surface water within the area of the section, or of any extended valuable frontage; as well as against any impediment that might be placed in the way of forming roads through the property. Where the main lines of communication have not been previously laid out, it is requisite, especially in *large blocks* of land, to reserve to the government at all events for a limited number of years, a right of forming such roads as are evidently for the public benefit, making of course compensation for any damage that may be thereby done, though this can generally be met by a previous allowance of a certain number of acres in excess of the proper content of the block.* Indeed, if proper precautions could be taken to prevent its being abused, it would be advisable to reserve this power of making such general roads as are clearly advantageous to the community through all sections of land of whatever size; with the right of taking stone and timber, for making and repairing these roads and the bridges erected along their line, though all such interference with private rights should as much as possible be obviated by previous careful examination of the country.

The rapid settlement of a newly-formed colony being an object always to be fostered, the sections marked out for sale should be so arranged as to conduce as much as possible to this desideratum; to attain which end the surveys should at all events at first, be kept well in advance of the demand for land, for the purpose of giving the most ample choice of selection to intended purchasers. By the opposite system of selling land in advance of the survey,

used to be alienated from the Crown in grants of extensive size, renders the survey of that country not a fair point of comparison with that of more modern colonies.

* Two or three per cent. upon the average, is proved amply sufficient in small or moderate-sized sections. In very large blocks, one per cent. would perhaps be as much as could be required.

an unfortunate emigrant not unfrequently finds the greater part of his section occupied by the bed of a salt lagoon or swamp, and experiences no slight dismay in discovering that he is not even in possession of the number of acres for which he has paid, and to which perhaps he has no access with any sort of wheeled vehicle, in consequence of the occupation roads being marked down upon the ground to correspond with straight lines previously drawn upon paper; so that they lead, without any controlling power in the surveyor to alter their course, up and down almost inaccessible ravines, or probably for several hundred yards at a stretch along the bed of a stream.

In marking out these sections, the following remarks* will direct attention to the different local peculiarities which require a deviation from established rules, and to the general system of conducting the work in the field; the mechanical practice of surveying being of course supposed to be already known.

Sections laid out with frontages upon main lines of road, rivers, or wherever increased value is thereby conferred upon the land, should have their frontage reduced one-half, or even one-third of the depth of the section, so as to distribute this advantage among as many as can participate in it without rendering the different sections too elongated in figure to be advantageously cultivated as a farm.

In addition to this contraction of frontage, easy access by roads must be provided from the country in the rear leading to this water or main road; without which precaution the owners of the front lots would, by blocking up the land behind them, virtually obtain possession of it, for at least pastoral purposes, without payment. These roads should occur at intervals proportioned to their requirement, generally between every third or fourth section.

Every section should have an available road on one of the four sides forming its boundaries, by which the proprietor is secured access to the main lines of communication; its breadth may vary from half a chain to one chain, according to circumstances; in square or rectangular sections of 80 or 100 acres each, roads

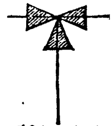
* Partly extracted from the instructions issued to the surveyors employed in South Australia.

surrounding each block of six or eight sections have been found amply sufficient; but in a country at all broken or irregular, some of the roads so laid out would often be found quite impracticable; in such cases, it is necessary either to trace and mark on the ground along the ridges of the secondary features, or wherever the ground may offer fewest impediments, cross roads leading into the main lines, and to lay off the sections fronting upon them; or to make these by-roads run *through* the sections; which is to be avoided as much as possible on account of their cutting up small properties, and entailing a very considerable expense in the increased quantity of fencing required.

In parts of the country where water is scarce, the greatest care should be taken to prevent its monopoly by individuals. Springs and permanent water-holes should in such localities be enclosed within a small block of land (one or two acres), and reserved for the use of neighbouring flock-owners and the public generally; and practicable roads must be arranged leading to these reserves, without which, excellent and extensive tracts of land would often be comparatively valueless.

As it would evidently very much increase the cost of laying out sections having broken and irregular frontages, if they were required each to contain *exactly* the same number of acres; the nearest approximation that can be made to the established size by the judgment of the surveyor should be adopted, and the section afterwards sold according to the quantity of land it is found to measure.

For the purpose of giving to settlers seeking for land upon which to locate, every facility for acquiring information respecting its capabilities, and the positions of the different surveyed portions, the freest access to the statistical reports of the surveyors, and to the plans of the different districts deposited in the Survey Office, should be given. In addition to which, the sections themselves should be marked so distinctly upon the ground by short pickets driven at intervals regulated by the comparative open and level character of the country, as to enable any person to follow up their boundary lines without difficulty. The *angular* pickets should be much larger, and squared at the head, on which the number of the section, and that of all the contiguous sections, should be marked. Adjacent roads should also be designated by the letter R. Independent of the corners of sections being pointed out by these pickets, they should be deeply trenched with a small spade or pick, showing not only the angle formed by contiguous sections, but also the directions of their boundary lines.



Such marks remain easily recognised for years, and are not injured either by bush fires or by the constant passage of herds of cattle, by both of which means many of the wooden pickets are soon destroyed.

It has been generally considered expendient, that roads should be reserved if not actually marked on the ground, (excepting in cases where they would interfere with the erection of wharves, mills, &c.,) along the banks of all navigable rivers, the borders of lakes, and along the lines of a coast. This regulation, if stringently applied without reference to peculiar circumstances in different localities, would often be found oppressive and mischievous. Very frequently roads laid out with judgment to the various points on the margins of these waters which are best adapted for the purposes of fisheries, watering flocks, establishment of ferries, building or launching boats, &c., with a sufficient space reserved for the use of the public at these spots, would prove of more general utility.

As a general rule, as many sections as possible should be laid out in the same locality, if the land is of a nature to be soon brought into cultivation. Whilst greater choice of selection is thus given, the comparative cost per acre of the survey is diminished; of course this remark applies only to situations the rapid settlement of which is anticipated.

In marking the boundaries of sections on the ground, all natural features crossed by the chain should be invariably noted in the field-book, on the outlines plotted from which are drawn the general character of the contours of the hills, the different lines proposed for roads, directions of native paths, wells, springs, and every other object tending to mark the nature and resources of the country. Copies of these plans* should always be trans-

* Two inches to one mile is found a very convenient scale for plans of these sections, intended for the information of the public.

mitted to the principal Survey Office, accompanied by a rough diagram, showing, for future reference, the construction lines of the work, and the contents and length of the sides of all sections, also the measure of the angles when not right angles, and by an explanatory report describing the nature of the soil, description of timber, &c., upon each section, and the facilities for making and repairing roads and bridges, and peculiar geological formations of the different districts. A collection of botanical and mineralogical specimens from all parts of the province will also contribute materially to the early development of its natural resources; and surveyors should not be deterred from giving their attention to this subject by ignorance of these sciences, as the specimens can be afterwards weeded and arranged, and afford invaluable statistical information.

At the head Survey Office a meteorological register* is of course supposed to be kept. It is also very desirable that each of the surveyors employed in any large district should be furnished with a good thermometer, rain-gauge, and a mountain-barometer or aneroid, for the purpose of registering daily observations to be forwarded periodically to the general office for comparison with those obtained from different parts of the province, between which the difference of peculiarities of climate will be thus arrived at.

Surveyors working on a line of coast should be particular in noting all phenomena connected with the rise and fall of the tides, and in obtaining soundings, laid down with reference to established and easily-recognised marks on shore, of all creeks and harbours, whenever this may be in their power. The depths and velocities of all rivers should also be noted at different points in their course, as well as the periods of floods, and their observed influence upon the volume of water in the river.

In laying out sections up narrow rocky ravines, or in situations where creeks or any other natural features present obstacles to the continuance of the methodical rectangular form adopted as the standard figure, a deviation from this form becomes of course necessary, and the contents of some of the sections thus often unavoidably differ from the established average. Care should

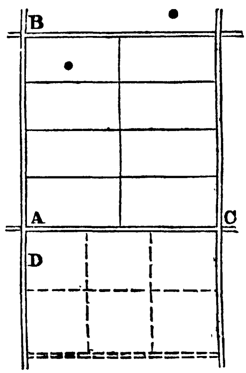
* A simple form adapted for this is given at the end of the Astronomical Tables.

however be taken in such cases, to make the outline of these irregular figures as simple as the ground will admit of, both on account of the additional trouble and time lost in their survey, and the increased cost of subsequent fencing by the purchaser.

Attention has already been drawn in page 137 to the necessity of guarding against the monopoly of road or water frontage. The same sort of precaution is also required in marking out land in rich narrow valleys, or in spots valuable on account of minerals. As a general rule, from which no deviation whatever should be allowed, it may be laid down that no section should ever be permitted to enclose an undue proportion of land, unusually valuable from whatever cause, by *extending its length in the direction in which that valuable portion of land runs;* whether it be a rich agricultural valley, a mineral lode, a stream, or watercourse.

As regards the actual marking out of the sections upon the ground when the figure is of a square or rectangular form, the process is a very simple one, whether the true meridian, or the direct line of some main road, or a line forming any angle with the meridian that may be found better adapted to the local peculiarities of the district, be adopted as the guiding line of direction.

A spot being fixed upon for the starting point, represented by A in the accompanying figure,* the normal line A B is carefully marked out by a good theodolite in the required direction; if intended to correspond, or to form any fixed angle with the meridian, this must be determined by one of the methods explained in the next chapter. The right angle B A C is then set off, which angle should be observed on both sides of A B (produced on purpose to D), and the chain measurement along these lines A B and A C, and afterwards along the parallels to A C, may, if two parties



are employed together which can generally be managed under the

* This figure represents rectangular sections of 80 acres, as laid out in South Australia, the length of which bore to their breadth the proportion of 2 to 1—occupation roads one mile apart, enclosing eight sections. They were, however, frequently laid out square, according to the nature of the ground.

charge of one efficient surveyor with an intelligent assistant, be carried on simultaneously, the points of junction at the angles of the blocks forming in some measure checks upon the accuracy of the work as it proceeds. The size of these sections, and the intervals between the parallel sectional roads, will depend of course upon local regulations. The operation would evidently be simplified by running all the measured lines in the middle of these roads, leaving half their breadth to be afterwards set off on each side by the proprietors of the land, but the palpable objections to this are too serious to be compensated by the trifling saving thereby effected. In fact, the real boundaries of *no one section* would by this plan be marked on the ground by the surveyor; and constant disputes and encroachments would be the consequence of adopting it.

It must be obvious to every practical surveyor, that it would be impossible for him to continue this mechanical system of marking a series of rectangular figures on the ground to any great extent without being liable to constantly-increasing errors, which could not be guarded against by any degree of care in the operation, and of the amount of which he could never be aware without establishing some check altogether independent of the chain measurement of the sections themselves, and this is only to be accomplished by combining with it a triangulation of the country, more or less accurate according to the nature of the survey. Whilst then, this methodical division of the land is in progress, it is advisable if anything like accuracy is required, and if the detached portions of settled country are to be laid down upon a general map, that the sites of the trigonometrical stations should be decided upon, and the stations themselves (however roughly they may be constructed) erected, in order that they may throughout be made use of as guides and checks upon the measurements. The triangulation indeed would be found of the greatest service, if carried on rather in advance of the detail, as in the survey of old countries. Any great accumulation of error could be then easily guarded against, by the angles observed at different parts of the chain survey, subtended by three or more of the trigonometrical stations, and in very many instances these stations could be actually measured up to, which should be done wherever practicable; by which means the marking out of the sections answers the same purpose that is obtained in ordinary surveys by the measurement of check lines, and traversing along the roads, by which the interior detail is mostly filled in. Angles of depression and elevation should also be taken to these trigonometrical points (whose altitudes are all obtained by the triangulation), from various parts of the chain survey, the heights of which positions above the level of the sea are thus obtained with tolerable accuracy.

As to the mode of conducting this triangulation, all necessary instructions have already been given in the third chapter. The degree of accuracy with which the base is measured, and the angles observed, will depend evidently upon various contingencies; for instance—the extent over which the triangulation is to be carried, the time and expense that can be bestowed upon it, the degree of minutiæ required in the maps, &c., &c. On the survey of South Australia the base was measured upon a nearly level plain very little elevated above the sea, with a standard chain, the operation being repeated several times, to obtain a more correct mean value: the angles were observed with a very excellent 7-inch theodolite and the result was found sufficiently accurate for the purpose of connecting all the detached blocks of surveyed land, and laying down the work to the scale of 2 inches to 1 mile.

In addition to the above use of the triangulation, it is found, in the survey of a wild country, peculiarly serviceable in enabling the Government to define, with the aid of marked natural features, the boundaries of the extensive tracts of land leased to different individuals for pasturage, until, with the increase of population and civilisation, more convenient and better-defined demarcations are substituted. Some of the principal natural landmarks of a country also, such as chains of mountains and rivers, traverse the wildest parts of the land, where chain surveying would never penetrate. Many of these landmarks are made the boundaries of counties, and other internal territorial divisions; and their positions in different parts of their course are often only to be determined by reference to the trigonometrical stations, which likewise serve as guides for ascertaining and laying down upon paper the directions of roads through extensive, barren, and uninhabited tracts of country.

Most of the foregoing remarks have been made under the supposition that a number of detached surveying parties are distributed over different parts of the country, all working under the directions of, and reporting to, a central Survey Establishment. As the population becomes distributed over a wider extent, and applications are constantly made for the survey of small irregular blocks of land to complete and consolidate properties, some alterations will be required in the method of carrying on the measurement of land to meet these new demands.* It could evidently be only by an increased expenditure of time and money that surveying parties could be kept constantly moving from one distant spot to another, to lay out perhaps, only a very limited number of acres at each; and the division of the country into *Districts*, for the purposes of the survey, becomes almost imperative. Copies of the plans of sections open for selection, and other information of a similar character, would be thus placed more within reach of distant settlers, and their wants could more readily and rapidly be met without augmented expense.

Portions of the work might also at this advanced stage of progress be filled in by contract, subject to careful and rigid examination; the triangulation, and the previous chain measurement connected with it, affording sufficient checks for this purpose; without which, surveying by contract should be most carefully avoided, especially in new communities where but little competition can be expected, and where it would be unreasonable to expect to find competent surveyors distributed over the remote parts of the colony.

The rate of progress and cost per acre of a sectional survey such as has been described, must vary considerably, according to

* These subsequent wants and demands do not affect the first stage of the survey in a new country; it is only as it becomes gradually settled that they are felt. The first survey evidently cannot be a *complete* one, unless it could embrace every acre of land that might by possibility be required; it is constantly demanding *extension* in every direction, therefore the more imperatively necessary is it, that the first land surveyed and laid down on the maps should be based upon a triangulation sufficiently accurate to allow of this extension, without the certainty of accumulating error.

the nature of the country, the prices of labour and provisions, and the minuteness of the divisions. If the size of the sections is small, 80 or 100 acres for instance, the number of lineal miles to be measured is of course very much greater in proportion than would be the case with blocks of a larger area, and the progress must bear an inverse ratio to the increased expense. The facility of transport is another item that materially influences both these questions, as also the system of marking out patches of land in whatever locality they may be applied for, instead of carrying the survey regularly forward, embracing all the available land in its progress. On an average the division of the land in South Australia into sections containing generally about 80 acres each, costs* including the marking out the roads surrounding the different blocks to which each section had access as well as all other roads through the settled districts, the close picketing of the boundary lines of each section, and marking and trenching the corner posts; with all other details relative to the survey of such portions of the natural features of the ground as came within the limits of the chain survey, from 3*d.* to 4*d.* per acre; and each party, consisting of a non-commissioned officer of Sappers, with four or five labourers, according to the difficulties of the country, marked out on an average perhaps about 30,000† acres per annum; a very large proportion of their time, particularly towards the close of the work, being occupied in moving from one distant part of the colony to another to meet the varying demands for land.

The triangulation of the settled parts of the province, and in some directions far beyond this, did not amount to ½*d.* per acre; including, as did also the average of the sectional survey, all expenses of transport of men, provisions, and camp equipage, with the wear and tear of the latter, and that of the necessary instruments; in fact, all expenses excepting those connected with the central establishment, where the plans were drawn and ex-

* This average has no reference to the first settlement of the province in 1838; it applies more particularly to the period between the years 1842 and 1848 inclusive.

† Occasionally, under favourable circumstances, *three times* this average was produced for limited periods.

hibited, and where the preliminary business of the land sales was conducted.

Even had this cost been doubled or increased in a still greater proportion, it would have been false economy to have shrunk from it, and have put the settlers in possession, or rather to have allowed them to take possession, of land the boundaries and contents of which could not have been relied upon or subsequently verified. The expense of the surveys in all new colonies is now defrayed out of the proceeds of the sales of land; and proof of the recognition of the advantages of the accurate delineation of the boundaries of property, features of the ground, and main lines of roads, &c., is given by the system adopted by the New Zealand Association in the establishment of the "Canterbury Settlement," of charging for all land the uniform price of 3*l.* per acre* (instead of the 1*l.* fixed as the lowest upset price in the other Australian colonies where the plan of selling land by auction is in force), to provide funds for a superior nature of survey, and a variety of works of a public character; the proportions being, 10*s.* per acre as the price of the waste land, 10*s.* per acre for the cost of the surveys, formation of roads, and other miscellaneous expenditure; 20*s.* per acre to be devoted to the purposes of emigration; and another 20*s.* per acre to ecclesiastical and educational purposes.

The boundaries of what in the Australian colonies are termed "*Runs*," for depasturing sheep and cattle, are not generally marked out during the survey, but are described by reference to the trigonometrical stations or other known fixed points, the approximate distances and bearings of the lines being stated. As portions of this land are at all times liable to be purchased by individuals, after a due stipulated notice to the occupier of the run, who pays yearly a trifling sum for his licence, it would of course be a waste of labour to mark out such temporary divisions;

* Formerly land used to be sold in South Australia at the uniform fixed price of 1*l.* per acre. The system of selling by auction was introduced by the Australian Waste Lands Act, in the year 1843. There are various opinions as to the comparative merits of these opposite systems, the first of which was introduced by Mr. E. G. Wakefield; and its advantages are strongly set forth in the pamphlet upon Colonial Surveying published by his brother, Mr. F. Wakefield.

but the settlers themselves very frequently define their respective limits, either by blazing the trees in a wooded country, or by running a plough line across it in an open one.

As regards the interior division of a colony into Counties, &c., the following general regulations, established many years since, are still in use:—

Counties are to contain, as nearly as may be, 400 miles square; hundreds, 100 square miles; and parishes, 25 square miles.

Natural divisions, such as rivers, streams, highlands, &c., to constitute as much as possible these boundaries; and for the purpose of obtaining a well-defined natural boundary, a smaller or greater quantity than the above averages is permitted; but not to exceed or fall short of such established areas by more than one-third of each.

Reserves are allowed to be made for all necessary public roads and other internal communications, either by land or water; also for the sites of towns, villages, school-houses, churches, and other purposes of public utility and convenience.

When the division between Provinces or Counties, or other lines of territorial demarcation, is represented, either altogether or in part, by a meridian line, or by a line having any fixed angle with the meridian, or by a portion of the arc of a parallel (as is the case in many of the Australian provinces); it is of course necessary to be able to determine and mark upon the ground with accuracy such meridian or parallel, directions for which are given in the last chapter on Practical Astronomy. Most useful practical information upon this subject will also be found in the narrative of the survey, and marking of the boundary between the British possessions in North America and the United States of America, in 1842, published by Major Robinson, Royal Engineers, in the second and third volumes of the "Corps Papers."

Operations of this nature, if conducted with the very great care and precision that were bestowed upon the boundary alluded to, involve the perfect knowledge of the manner of using and adjusting the transit, and altitude and azimuth instruments; and also the management of chronometers. The boundary line between South Australia and what now constitutes the province of Victoria, (the 141st degree of east longitude) was however determined (and since marked on the ground for a considerable distance), under the New South Wales Government by one of their surveyors,* with only a sextant, a pocket chronometer, and a small 3½-inch theodolite; but though the work was performed with the greatest care and attention, and with probably as great a degree of accuracy as could be obtained with these imperfect instruments, the result can of course only be looked upon as an approximation far too vague for the determination of a division of importance. The North American boundary, on the other hand, may perhaps have been defined with more precision than was absolutely necessary in a line of demarcation running for its whole length through a wild uncleared country.

Having now gone through the method of dividing the land into minute sections for occupation, and its further division for territorial purposes, this chapter will conclude with a short reference to the objects to be held in view in conducting exploring expeditions beyond the bounds of the settled districts for the purpose of adding to the geographical knowledge of the country and developing its resources; which objects are very similar in character to those described in page 134, when treating of the preliminary operations of a survey in a newly formed colony.

The nature of the country to be traversed will, as far as this is known, indicate the method of travelling that must of necessity be adopted. Extensive inland water communication as in the Canadas, points to the canoe as the readiest mode of transport; comparatively open and generally grassy land, as in Australia and Southern Africa, requires the use of horses and oxen; whilst in many other countries the thick underwood can, in parts, be traversed only on foot; and barren deserts by the aid of camels. These different modes of locomotion evidently all require different preliminary arrangements. The objects in view, however, are much the same in all cases; † viz. a knowledge of the climate,

* Mr. Tyers.

† Expeditions for one single definite object, such as tracing the sources of a river, &c., are not intended to be here referred to.

soil, native population, geological formation, botanical character, of the country, and its resources of all kinds; as well as the delineation (as perfect as the time and means that are available will admit) of the natural features of the ground.

All points known as portions of the settled country being soon left behind, the explorer has to trust to his own judgment as to the best directions in which to conduct his party; to his own energy in overcoming the natural obstacles that he will be certain to encounter; and his own practical skill in fixing at proper intervals his different positions by means of astronomical observations, and mastering rapidly the general massive features of the ground for the purpose of making a rough sketch of the country passed over, showing more particularly the directions of the principal ranges of hills, and of rivers, and watercourses.

In a large party these labours may often be subdivided advantageously; but the leader must remember that the *entire responsibility* still rests with him; and if he does not actually participate in every portion of the work, he must nevertheless exert a general influence over the whole.

As regards the fixing, with as much accuracy as may be attainable, the various positions of encampments, the directions and sources of rivers, and all marked prominent features; much assistance is to be obtained by carrying on, as far as it can be done, a species of rough triangulation (with a sextant or other portable instrument) from the extreme trigonometrical stations, or from any prominent landmarks the positions of which are known and represented on the plans. This may however very soon become impracticable from the nature of the country or other causes, and the traveller then finds himself much in the same predicament as at sea, having little beyond his dead reckoning to trust to for the delineation on paper of his day's work. In this position he must look to the heavens for his guide; and hence the necessity for his becoming himself, or having with him, a good and rapid observer.

At sea, the latitude is always obtained at noon by a meridian altitude of the sun* (when visible); "*sights*," as they term observa-

* For the method of calculating the latitude from a meridian altitude, see Chapter XI.

tions of single altitude for time, having been taken three or four hours before. The latitude obtained at noon is then reduced by dead reckoning to what it would have been at the time and place of the morning observation (using the traverse table), and with this deduced latitude the hour angle is computed,* and the equation of time *plus* or *minus* applied for the mean local time; which, when compared with the Greenwich time, shown by the chronometer (allowing for its rate and error), gives the longitude east or west of Greenwich *at the time of the morning observation.*

By applying, by dead reckoning, the change in longitude between that time and noon, the longitude of the ship at noon is obtained,—the latitude has already been found by direct observation,—and the two determinations afford the means of recording upon the chart the position of the ship at *noon* on that day.

Somewhat similar to the above proceeding must be that of the explorer in a wild unknown tract of country. He would not probably find it convenient always to obtain his latitude at noon; but he can generally do so (and more correctly) at night† by the meridian altitude of one or more of the stars of the first or second magnitude, whose right ascension and declination are given in the Nautical Almanac. His local time can, immediately before or after, be ascertained by a single altitude of any other star out of the meridian (the nearer to the prime vertical the better); and if he carries a pocket chronometer upon which any dependence can be placed, he has thus the means, by comparison with his local time, of obtaining his approximate longitude, and of laying down his position upon paper.

In travelling, the rate of the chronometer will probably be found to vary, and as frequent halts of two or three days are likely to occur, these opportunities should never be lost of ascertaining the change of rate. The longitude should also be obtained occasionally by lunar observations on both sides of the meridian; or by some of the other methods given in the last chapter.

The results deduced from such observations must not be relied upon within eight or ten miles, but a careful observer should rarely exceed these limits; and his latitude ought always to be

* See Chapter XI.

† See Chapter XI. on Practical Astronomy.

within half a mile, or under the most unfavourable circumstances, one mile of the truth.

With these all-important data, enabling him to fix with approximate accuracy point after point * in his onward course, the explorer can have no difficulty in interpolating by angles taken with a sextant or with an azimuth compass, all strongly-marked prominent features, or in laying down his route upon paper correctly enough for the purposes of identifying particular spots, and giving a faithful general representation of the features of the ground he has travelled over. The value of this sketch will be much enhanced by its having recorded on it, as nearly as they can be ascertained by the mountain barometer or aneroid,† or by the temperature at which water is found to boil,‡ the altitudes of the most important positions, as the summits of hills, the levels of plains, and sources of springs and rivers.

Daily meteorological observations, even of the most simple character, such as merely recording the readings of the thermometer and barometer at stated times, will also prove of essential service as illustrative of the climate; and these will be of additional value if accompanied by a record of the quantity of rain fallen on different days should any portion of the party be stationary for sufficient length of time at any one spot, to make these observations. If not provided with a rain gauge of a better description, a tin pipe with a large funnel, the area of the top of which bears a certain proportion to that of the tube, will answer perfectly to measure the quantity of water fallen. A light graduated wooden rod is fixed in a cork float, and indicates, above the level of the top of the funnel, the number of inches;—the graduations of the rod of course being proportioned to the ratio between the areas of the surface of the funnel and that of the tube. Thus, if the proportion is ten to one, the measuring rod will be lifted 10 inches for every inch of rain.

* The distances between positions, the latitudes and longitudes of which have been determined, can be easily calculated in the manner described in the next Chapter; by which means they can be laid down with more accuracy, if the extent of ground travelled over is not very great.

† See Chapter XI. ‡ See page 121.

CHAPTER X.

GEODESICAL OPERATIONS CONNECTED WITH A TRIGONOMETRICAL SURVEY.

In the words of Sir J. Herschel, "Astronomical Geography has for its objects the exact knowledge of the form and dimensions of the earth, the parts of its surface occupied by sea and land, and the configuration of the surface of the latter regarded as protuberant above the ocean, and broken into the various forms of mountain, table land, and valley."

The form of the earth is popularly considered as a sphere, but extensive geodesical operations prove its true figure to be that of an oblate spheroid, flattened at the poles, or protuberant at the equator; the polar axis being about $\frac{1}{300}$ part shorter than the equatorial diameter.* This result is arrived at by the measurement of arcs of the meridian in different latitudes, by which it is

* The exact determination of arcs of the meridian measured in France, and also the comparison of the three portions into which the arc of the meridian between Clifton and Dunnose was divided, presenting the same anomaly of the degrees appearing to diminish as they approach the pole, are opposed to the figure of the earth *being exactly a homogeneous or oblate ellipsoid;* but its approximation to that figure is so close that calculations based upon it are not affected by the supposed slight difference. The proximity of the extreme stations to mountainous districts was supposed to have been partly the cause of this discrepancy, as the attraction of high land, by affecting the plummet of the Zenith Sector, might have vitiated the observations for the difference of latitude between two stations. A survey was undertaken by Dr. Maskeylene solely to establish the truth of this supposition, the account of which is published in the "Philosophical Transactions" for 1775. A distance of upwards of 4000 feet was accurately measured between two stations, one on the north and the other on the south side of a mountain in Perthshire. The difference of latitude between these extremities of the measured distance was, from a number of most careful observations, determined to be 54″·6. *Geodesically* this arc *ought to have been* only 42″·9, showing an error of 11″·7 due to the deflection of the plummet.

ascertained beyond the possibility of doubt, that the length of a degree at the equator is *the least that can be measured*, and that this length increases as we advance towards the pole; whence the greater degree of curvature at the former, and *the flattening* at the latter, is directly inferred.

Our "diminutive measures" can only be applied to comparatively small portions of the surface of the earth in succession; but from thence we are enabled, by geometrical reasoning, to deduce the form and dimensions of the whole mass.

There are two difficulties attending the measurement of any definite portion of the earth's circumference (such as one degree, for instance*) in the direction of the meridian, independent of those caused by the distance along which it is to be carried: the first is, the necessity of an undeviating measurement in the *true direction of a great circle;* and the second, the determination of the *exact spot where the degree ends.*

The earth having on its surface no landmarks to guide us in such an undertaking, we must have recourse to the heavens; and though by the aid of the stars† we can ascertain *when we have accomplished exactly a degree*, it is far more convenient to fix upon two stations as the termini of the arc to be measured, *having as nearly as possible the same longitude*, and to calculate the length of the arc of the meridian contained between their parallels from a series of triangles connected with a measured base, and extending along the direction of the arc. From the value thus obtained, compared with the difference between the latitudes of the two termini determined by a number of accurate astronomical observations,

* More than an entire degree (about 100 miles) was actually measured on the ground in Pennsylvania, by Messrs. Mason and Dixon, with wooden rectangular frames, 20 feet long each, laid perfectly level, without any triangulation. Page 10, "Discours Préliminaire, Base du Système Métrique," and "Philosophical Transactions" for 1768.

† The stars whose meridional altitudes are observed for the determination of the latitude should be selected among those passing through, or near, the zenith of the place of observation, that the results may be as free as possible from any uncertainty as to the amount of refraction. With proper care and a good instrument, the latitude for so important a purpose ought to be determined within one *second of space*, unless local causes interfere to affect the result.

can be ascertained of course the length of one degree in the required latitude.

The measurement of an arc of the meridian, or of a parallel, is perhaps the most difficult and the most important of geodesical operations, and nothing beyond a brief popular description of the modes of proceeding which have been adopted in this country, and elsewhere, can be here attempted. For the details of the absolute measurement of the bases from which the elements of the triangles were deduced, as well as the various minute but necessary preliminary corrections, and the laborious analysis of the calculations by which the length of the arcs were determined from these data, reference must be made to the standard works descriptive of these operations.

At the end of the second volume of the "Account of the Operations on the Trigonometrical Survey of England and Wales," will be found all the details connected with the measurement of an arc of the meridian, extending from Dunnose in the Isle of Wight, to Clifton in Yorkshire. The calculations are resumed at page 354 of the third volume; the length of one degree of the arc resulting from which, in latitude 52° 30′ (about the centre of England), being equal to 364,938 feet.

An arc of a parallel was also measured in the course of the trigonometrical survey between Beachy Head and Dunnose, in 1794, but fault has been since found with the triangulation, and corrections have been applied to the longitudes deduced therefrom, which are alluded to in "The Chronometer Observations for the Difference of the Longitudes of Dover and Falmouth," by Dr. Tiarks, published in "The Phil. Trans. for 1824," and in Mr. Airy's paper "On the Figure of the Earth."

The arc measured by Messrs. Mechain and Delambre between the parallels of Dunkirk and Barcelona, described in detail in the Base du Système Métrique Décimal," had for its object (as the title of the work implies) not only the determination of the figure of the earth, but also that of some certain standard, which, being an aliquot part of a degree of the meridian in the mean latitude of 45°, might be for ever recognised by all nations as the *unit of measurement.* To have any idea of the labour and science devoted to this purpose, it is necessary to refer to the work itself, in which will be found the reasons for preferring a portion of the measurement of the surface of the globe involving *only the consideration of space*, to the length of a pendulum vibrating seconds having reference *both to time and space.* In addition to the determination of this standard of linear measurement, which was denominated the "metre," and defined to be the ten-millionth part of a quarter of a great circle passing through the poles,* the Committee, consisting of all the most distinguished scientific men on the Continent, agreed also upon a *standard of weight derived* from the same source. A cube, each side $\frac{1}{10}$ part of the metre, or a "*decimetre*" (chosen on account of its convenient size), was supposed to be filled *with distilled water of the temperature of ice just melting*; and the weight of the fluid constituted the "*killogramme.*" This temperature was selected as being *pointed out by nature*, and independent of any artificial gradations; and also as being the point at which the *density of water is nearly a maximum, as it expands immediately on solidifying, although down to about* 40° it continues gradually to condense. No other substance either liquid or solid combines so many recommendations; but the difficulty that arose was to construct a *solid mass representing this weight of water* which might be kept as a standard; their method of overcoming this is explained at pp. 563, 626, and the following pages of the third volume. "Bodies of *unequal* specific gravities may weigh equally in one state of the atmosphere, but not so in one of either greater or less density, and a vacuum was therefore of necessity resorted to." In the words of the Report

* The French Commissioners, however, having in their calculations employed $\frac{1}{334}$ as their value of the earth's compression, now known to be incorrect, the métre, strictly speaking, can no longer be so defined. The determination of the value of the English tandard,—the yard,—has been recommended by the commissioners appointed in 1841 for the restoration of the standards of weight and measures after the injury done to the original standard by the burning of the House of Commons in which it was deposited, to be effected by joint reference to the three standards extant upon which most reliance can be placed; viz., those belonging to the Royal Society, the Royal Astronomical Society, and the Board of Ordnance, instead of having recourse to the standard previously established by Act of Parliament, of the length of a pendulum vibrating seconds at a fixed temperature in the latitude of London. Mr. Baily states this length at the level of the sea, in vacuo, at the temperature of 62° Fahr., by Sir G. Shuckburgh's scale, to be 39·1393 inches.

(vol. iii. p. 565,) "C'est au poids du decimètre cube d'eau distillée, à sa plus grande densité, qu'on doit faire égal le poids d'une masse solide donnée, tous les deux étant supposés dans le vide; voilà a quoi se reduisoit la question de la fixation de l'unité de poids." In the end, cylinders of platinum and of brass were constructed, of precisely the same weight as the killogramme of water, both weighed in *a vacuum.* These two, from the difference of their masses, evidently *would not weigh alike* in the air. A brass cylinder (of which several were made) was kept as a standard for public use; the platinum presented to the "Institut," to be deposited there as "le représentatif d'une mass d'eau prise à son maximum de condensation, contenue dans le cube du decimètre, et pesée dans le vide."

During the progress of these operations, observations were made by Borda (whose repeating circles of 16 and 16½ inches diameter were used in triangulation), on the length of a pendulum vibrating seconds at the level of the sea, in the latitude of 45°, at one determinate temperature. The length of this pendulum (of platina) was ascertained in *millimetres,* and was declared by the Committee to be so accurate, as to serve, in case of any accident happening to the standard, to construct again the *unit of measurement* without another reference to an arc of the meridian.

The prolongation of the measurement of this arc from Barcelona to Formentera, the most southerly of the Balearic Isles, and its connection with England and Scotland, was published in 1821 by Messrs. Biot and Arago (under whom the operations were conducted), in a work entitled "Recueil des Observations Géodesiques, Astronomiques, et Physiques." The whole arc measured amounted nearly to 12½°, and was crossed at about half its length by the mean parallel of 45°.

The following table, taken from Mr. Airy's "Figure of the Earth," published in the Encyclopædia Metropolitana," shows the length of the principal arcs of meridian and parallel that have been measured in different latitudes:

ARCS OF MERIDIAN.	Latitude of Mid. Point.			Amplitude of Arc.			Length in Eng. ft.
Peruvian Arc, calculated by Delambre .	1°	31′	0″	3°	7′	3″·1	1131057
Maupertuis' Swedish Arc	66	19	37	0	57	30·4	351832
French Arc, by Lacaille and Cassini .	46	52	2	8	20	0·3	3040605
Roman Arc, by Boscovich	42	59	0	2	9	47	787919
Lacaille's Arc, near the Cape of Good Hope	33	18	30	1	13	17·5	445506
American Arc, by Mason and Dixon .	39	12	0	1	28	45	538100
French Arc, from Formentera to Dunkirk	44	51	2	12	22	12·6	4509402
Svanberg's Swedish Arc	66	20	10	1	37	19·3	593278
English Arc, from Dunnose to Burleigh Moor	52	35	45	3	57	13·1	1442953
Lambton's first Indian Arc	12	32	21	1	34	56·4	574368
Lambton's second Indian Arc, as extended by Everest	16	8	22	15	57	40·2	5794599
Piedmontese Arc, by Plani and Carlini .	44	57	30	1	7	31·1	414657
Hanoverian Arc, by Gaufs	52	32	17	2	0	57·4	736426
Russian Arc, by Struve	58	17	37	3	35	5·2	1309742

ARCS OF PARALLEL.	Latitude.			Extent in Longitude.			Length in Eng. ft.
Arc across the mouth of the Rhone, by Lacaille and Cassini	43°	31′	50″	1°	53′	19″	503022
General Roy's Arc, between Beachy Head and Dunnose	50	44	24	1	26	47·9	336099
Arc from Dover to Falmouth	50	44	24	6	22	6	1474775
Arc from Padua to Marennes . . .	45	43	12	12	59	3·8	3316976

The detailed accounts of the measurements of these arcs are to be found in the works of Puissant, Cassini, Biot, Arago, Borda; in Colonel Lambton's papers in the "Philosophical Transactions" (1818 and 1823); and in the works of Captain Everest, published in 1839; and a popular description of the different methods adopted for the measurement of the bases, in each of these operations, is given in the paper "On the Figure of the Earth," in the "Encyclopædia Metropolitana," from which the foregoing table was extracted.

The conclusion drawn by Professor Airy from the above measures is, that "the measured arcs may be represented nearly enough *on the whole*, by supposing the earth's surface at the level of the sea, or at the level at which water communicating freely with the sea, would stand, to be an ellipsoid of revolution whose polar semi-axis is 20853810 English feet, or 3949·583 miles; and whose equatorial radius is 20923713 feet, or 3962·824 miles. The ratio of the axis is 298·33 to 299·33: and the ellipticity (measured by the quotient of the difference of the axis by the smaller) is $\frac{1}{298\cdot33}$, or ·003352. The meridional quadrant is 32811980 feet, and one minute = 6076·2777 feet."

Mr. Baily assumes the proportion between the polar axis and the equatorial diameter to be as 304 to 305, whence the compression amounts to $\frac{1}{325}$.

The most general valuation of the compression is $\frac{1}{300}$, and in the numerous tables of compression, given by Dr. Pearson in his invaluable work on Practical Astronomy, it varies from $\frac{1}{300}$ to $\frac{1}{325}$.

Instructions for conducting the measurement of arcs of the meridian will be found in Francœur, page 148, and also in Puissant's "Géodesie," vol. i. p. 242, and in the 12th chapter of "Woodhouse's Trigonometry." Below is given a popular account of the methods of procedure.

The line A X in the figure annexed (*fig.* 1) represents a portion of an arc of the meridian on which it is required to measure the length of one degree. A and L are the two stations selected as the extreme points to be connected by a series of triangles A B C, B C D, D C E, &c., running along the direction of the meridian which passes through A. The vertices of these triangles, *particularly the station* L, are purposely chosen as near as possible to this meridian line; and the distance from A to X, the intersection of a perpendicular to the meridian drawn through L (the distance L X being short), or more correctly to X′, the point of intersection with this meridian of the *parallel drawn through* L becomes the distance to be attained by calculation. The length of A B, or of any other side, is first accurately determined with reference to some measured base, and the angles at the vertices of all the triangles observed with the most rigid accuracy; and after the necessary corrections for spherical excess have been made, with the reductions to the centre and to the horizon if required,* the sides of the triangles are calculated from these data, as if *projected on the surface of the globe, at the mean level of the sea.* The azimuths of all these sides also require to be known, that is, the angles they respectively make with the meridian, which can be calculated from C A X,

* Francœur's "Géodesie," p. 132; Airy's "Figure of the Earth," p. 199.

or any other azimuth which has been observed, and the latitudes of the two extreme stations must be ascertained with all the minuteness of which the best instruments are capable* for comparison with the distance obtained by calculation between them. The first method that was adopted of ascertaining from

Fig. 1.

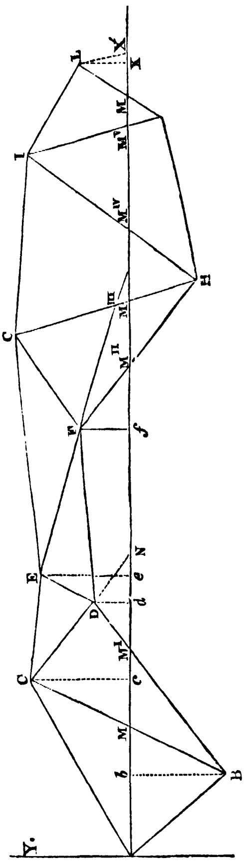
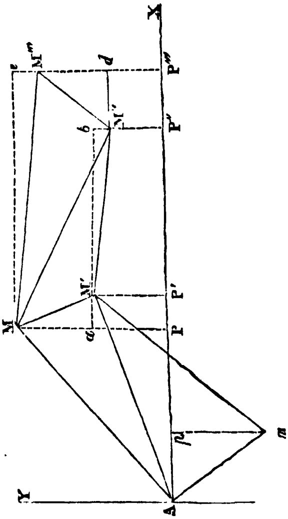


Fig. 2.



these data the required length of A X, is termed that of *oblique-angled triangles*, described in Francœur's "Géodesie," p. 151; in "Puissant," vol. i. p. 243; in the "Base du Système Métrique;" and in p. 277 of Woodhouse's "Trigonometry."

* No less than 3900 observations were made for the determination of the latitude of Formentera.

It consists in calculating the distances A M, M M′, &c., on the meridian line between the intersections of the sides of these triangles, or their prolongations, as at N; their sum evidently gives the total length A X.

The preliminary steps of the second method are the same; but instead of finding the distances A M, M M′, &c., the perpendiculars to the meridian* B *b*, C *c*, D *d*, are calculated (p. 246, Puissant's "Géodesie," vol. i.), the azimuths of all the sides being known; and from thence are obtained the distances on the meridian A *b*, A *c*, *c* N, &c., and of course the total length A X. This method was introduced by Mr. Legendre, and has been partly adopted in the calculation of the arc measured between Dunkirk and Barcelona described in the "Base du Système Métrique," as also on that between Dunnose and Clifton, it being considered not only more expeditious, but also more correct. Another advantage of this method is (if all the triangles are intersected by the meridian) that by calculating the various portions of which the arc is composed from the right-angled triangle formed on each side of the meridian separately, one result serves as a check upon the other.

A modification of this method is described in Puissant's "Géodesie," p. 248, which consists in constructing through the vertices of the triangles *parallels both to the meridian* A X *and the perpendicular* A Y, without taking any account of the spherical excess. The intersections of these lines form, with the sides of the triangles, right-angled triangles, of which those sides are the hypothenuses; and the azimuth of each being known, all the elements can be ascertained, as is evident by reference to *fig.* 2. In this manner, the distances of several places from the perpendicular, and the meridian passing through the observatory of Paris, were calculated by Cassini.

The third method ("Puissant," vol. i. p. 316) of ascertaining the length of the arc A X is by determining the geographical positions of the vertices of the triangles extending along the

* Perpendiculars to the meridian in a sphere cut the equator in two points diametrically opposite, but not in an ellipsoid of revolution, or in an irregular spheroid.

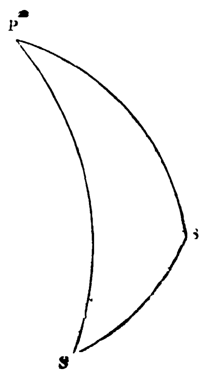
meridian, and calculating the difference of their parallels of latitude projected on the meridian, the sum of these being the measure of the arc.

The measure of an arc of *a parallel* is calculated by a similar process, which is described at p. 319 of the same work.

The methods of calculating geodesically the latitudes, longitudes, and azimuths of the different stations from one meridian with the rigid accuracy required in such operations as the measurement of an arc of the meridian or parallel, will be found fully explained in the 12th chapter of Woodhouse's "Trigonometry;" in the 18th chapter of Puissant's "Géodesie;" and in "Francœur." Their determination by astronomical observations will be treated of hereafter.

On the supposition that the earth is a sphere, the calculations are resolved into the solution of spherical triangles.

The accurate length of the arc on the surface of the earth between two very distant places whose latitude and longitude have been determined, is on account of the spheroidal figure of the globe a problem of great difficulty, and of no real practical utility; —it is fully investigated in Puissant's "Géodesie," vol. i., p. 296.* Between stations however within the limits of triangulation, it is often useful to calculate the distance as a check upon the geodesical operations; and in the length of an extended line of coast, or in a wild country, where triangulation may be, from local obstacles or want of means quite impossible, the solution of this problem is of great importance for the purpose of laying down upon paper the positions of a certain number of fixed stations, between which the interior survey has to be carried on; and it is, within such bounds, one of easy application, particularly in the latter case where the observations themselves are generally taken with portable instruments, and not with minute accuracy.



Inthe accompanying figure, P is the pole of the earth (considered as a sphere), and S and S′ the two stations, whose latitude and longitude are determined; the angle S P S′ is

* See also Francœur's "Géodesie," p. 208.

evidently measured by the difference of their longitude, and P S and P S′ are their respective latitudes; the solution of the spherical triangle P S S then gives the length of the arc S S′.

If it is possible, when observing at S and S′, to determine the *azimuths of these stations from each other*, that is, the angles P S S′ and P S′S, a more accurate result will be obtained, as these angles can be determined with precision, whereas the angle P depends upon the correctness of the observations for longitude at each station, which with portable instruments is always at best but a close approximation;* and the errors in the determination of each may lie in the same, or in different directions. In geodesical operations, if it be possible, the reciprocal azimuths of stations should *always* be observed, as well as the angles contained between them and other trigonometrical points.

From these reciprocal azimuths, with the astronomical latitudes of each station, the difference of their longitudes, or the angle of inclination of their meridians, is found by Dalby's method of solution which is applicable to spheroids. This mode of determining the difference of longitudes by observations of reciprocal azimuths was practised on the Ordnance Survey, and the analysis of the theorem is given at length in p. 214 of Airy's "Figure of the Earth." In the course of the investigation it is proved, that the spherical excess in a spheroidal triangle is equal to that in a spherical triangle whose vertices have the same astronomical latitudes and the same difference of longitude; from whence results the following simple rule—

$$\tan \tfrac{1}{2} \text{ diff. longitudes} = \frac{\cos \tfrac{1}{2} \text{ diff. lat.}}{\sin \tfrac{1}{2} \text{ sum of lat.}} \times \cot \tfrac{1}{2} \text{ sum of azimuthal angles.}$$

Generally, a small error in the latitudes produces no sensible error in the determination, but in the azimuths, accuracy is of vital importance; when the latitudes are *small*, their correctness becomes of consequence, and the method is not therefore well adapted for stations near the equator.

The angle at the pole formed by the two meridians being thus obtained, the distance S S′ between the stations can be found

* In cases where the *difference* of longitude between the two stations can be ascertained by means of signals, or by the interchange of chronometers, as explained in the next chapter, the measure of the angle P may be obtained with great accuracy.

nearly in the triangle P S S′; this arc, however, must be converted into its corresponding value in distance on the surface of the earth; and if its spheroidal figure be taken into account, the radius of curvature must be ascertained for the middle latitude $\frac{1}{2}$ $(l - l')$.

On the other hand, to obtain *geodesically* the latitudes, longitudes, and azimuths of stations from others whose positions on the surface of the globe have been determined by triangulation, it is necessary to be able to convert any measured or calculated distances on the earth's surface into arcs; for which purpose the *radius of curvature* of the arc in question is required, to obtain an accurate result. In a paper published by Mr. Galbraith, in the 51st number of the "Edinburgh New Philosophical Journal," tables are given to facilitate this preliminary computation, whether the arc be in the direction of a meridian, of a perpendicular to the meridian, or forming an oblique angle with it—as also those for the azimuths, latitudes, and longitudes, and convergence of meridians.

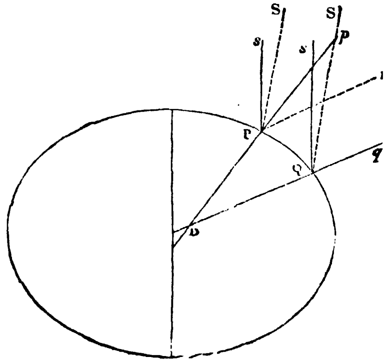
The formula given in the "Synopsis of Practical Philosophy" for the radius of curvature at any point of the *terrestrial meridian*, supposing the earth to be an oblate spheroid, is as follows, a and b being the equatorial and polar semi-axes, l the latitude, $c = (a - b)$ the compression:—

$$r = a - 2c + 3c.\ sin\ 2l.$$

$$\text{or} = a - \frac{c}{2} - \frac{3c}{2}\ cos\ 2l$$

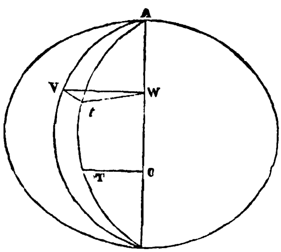
At page 192 of Mr. Airy's "Figure of the Earth," the following method is given for determining the radius of curvature:—

"The latitudes of the places P and Q, whether on the *same meridian* or not, are the complements of the angles p P s, q Q s respectively, which are included by the verticals at the places, and the lines drawn to the celestial pole. And if S be any star which can be observed at both places, the angle s P p = s P S + S P p, and s Q q = s Q S + S Q q; considering, therefore, the angles S Q s, s P S as equal, the difference of



latitudes is the same as the difference of S P *p*, S Q *q*; that is, it is the same as the difference of the zenith distances of the same star at the two places, and can therefore be easily found. Now, if the places P and Q be on the same meridian, their verticals will intersect in some point D; and the difference of latitudes, which is the difference of *s* Q *q* and *s* P *p*, or (P *r* being parallel to Q *q*) the difference of *s* P *r* and *s* P *p*, is equal to *r* P *p* or Q D P, the angles contained by the verticals. The length P Q being known from measurement, and the angle P D Q, or the difference of latitude, being found by observations of the zenith distances of a star, the length of P D or Q D, *or the radius of curvature*, is found.

"Again, if T and V be two places on *different meridians*, and if planes be drawn through these places, and through the axis, A C, of the earth, the angle made by these planes (or the difference of the longitudes) may be determined astronomically. Now, instead of T we have a place *t*, whose latitude is the same as that of V; and if we draw V W, *t* W perpendicular to the axis, the angle between the planes will be the same as the angle V W *t*. The distance V *t* being measured (or otherwise obtained), and the angle V W *t*, or the difference of longitude being found, the length of V W, or *t* W, or the *radius of a parallel*, will be found. *Either* of the measures will give this line, which will materially assist in determining the earth's form and dimensions, but they cannot easily be combined: the difference of latitude can be ascertained with so much greater accuracy than the difference of longitude, that measures of the former kind have generally been relied upon."



This subject is still further pursued in the work from which the above extract has been made.

It may also be required to calculate with the greatest exactness the azimuths or true bearings of two distant stations from each other, the latitudes and difference of longitudes of these points having been determined by observation; as was the case in marking the North American boundary in 1845, when one line sixty-four miles in length was cut through the dense Canadian forest upon bearings from each of the extremities computed by the following directions and formulæ furnished by Mr. Airy.

Convert the difference of longitude found in time into arc.

From the latitudes of the stations compute the following formulæ :—

Tan ½ sum of spherical azimuths

$$= \frac{\cos \frac{1}{2} \text{ diff. colat.}}{\cos \frac{1}{2} \text{ sum colat.}} \times \text{cotan } \frac{1}{2} \text{ difference longitudes.}$$

Tan ½ difference spherical azimuths

$$= \frac{\text{sine } \frac{1}{2} \text{ diff. colat.}}{\text{sine } \frac{1}{2} \text{ sum colat.}} \times \text{cotan } \frac{1}{2} \text{ difference longitudes.}$$

The larger azimuth (at the place where the latitude is greatest)

$= \frac{1}{2}$ sum azimuths $+ \frac{1}{2}$ diff. azimuths.

The smaller

$= \frac{1}{2}$ sum azimuths $- \frac{1}{2}$ diff. azimuths.

These azimuths, found for a *sphere*, are thus corrected for the earth's spheroidal form.

From the above spherical azimuths find the spherical amplitudes by taking the difference between each of them and 90°; for each case find an angle, a, by the formula *

$$\sin a = \frac{\text{sine colatitude}}{\sqrt{75}}.$$

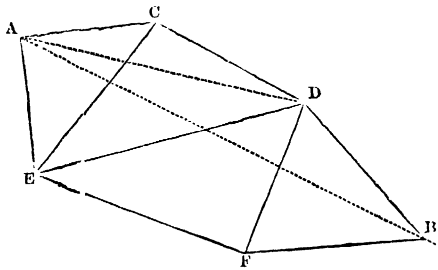
Then the tangent of each of the true *spheroidal* amplitudes = cos a × tangent spherical amplitude; the azimuths being obtained by applying to these 90°, additive or subtractive, according to the case.

If, instead of determining astronomically and by the transmission of chronometers the absolute latitudes and the difference of longitudes of these distant stations, they had been connected by a series of triangles, and that from this triangulation it was

* The steps by which this formula is arrived at are shown at page 346 of the "Corps Papers," where also will be found examples of azimuths calculated by it on the survey of the boundary alluded to.

required to obtain the true bearings of each point from the other for the purpose of running a straight line between them, the following is the simple process :—

Supposing A and B to be the two stations, connected as in the figure by a series of triangles ; assume one side as a standard, say A C ; compute C E as in a plane triangle; from this compute C D, D E ; from D E compute D F ; from D F compute D B. With the two known sides A C and C D, and the angle A C D, compute A D and the angle C D A ; subtract this from the sum



of the three angles C D E, E D F, and F D B, and you have the angle A D B ; with this angle and the two sides, A D and D B, compute the angle D B A ; this is the difference between the bearing of A from B, and that of D from B. The latter is known, or can be directly observed ; whence the former is deduced.

In the same manner the azimuth of the line A B, or the bearing of B from A, can be ascertained.

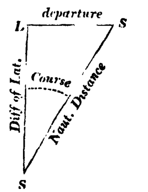
On the North American boundary the azimuths were laid off with an altitude and azimuth instrument, and the line prolonged with a portable transit by which the party sent on in front to take up the rough alignement for cutting a track through the dense forest were directed. A torch of birch bark was moved to the right or left as required by concerted signals made from the transit, by flashing small quantities of gunpowder in an open pan, both the lighted torches and the flashes of gunpowder being visible for far greater distances * than were ever required.

By daylight heliostats were used for keeping the advanced party in the right direction.

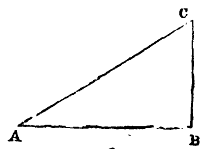
* Major Robinson states as much as 40 miles. See the narrative of his operations, 2nd and 3rd Numbers of the "Corps Papers."

The true bearings of the line of 64 miles in length were in this operation determined so accurately, that when the parties employed in marking it out from each extremity met about midway, the sum of their joint deviation from the true line was exactly 341 feet; equal, as Mr. Airy observes, to "only one-quarter of a second of time in the difference of the longitudes, or only one-third of the error which would have been committed if the *spheroidal form of the earth had been neglected.*" This slight error was corrected by running offsets at certain points along each line, proportioned of course to the distances from the extreme end.

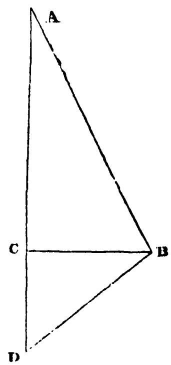
The distances between two places of a ship at sea are generally resolved by *plane* trigonometry; the difference of latitude S L, and the azimuth represented by the angle S′ S L and termed the *course*, forming a right-angled triangle, in which S S′, the *nautical distance*, is determined; the other side S′ L, termed the *departure*, being the sum of all the meridional distances passed over.



Again, in the triangle A B C: let A B represent the meridian distance (or departure), and the angle B A C be equal to the latitude, then A C, the hypothenuse, will be equal to the difference of longitude.



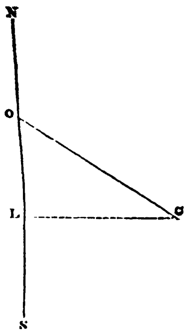
Also, if D B represent the nautical distance, and C D the difference of latitudes, then B C D will be a right angle, and B C the departure, *nearly* equal to the meridian distance in the middle latitude. If then, in the triangle A B C the angle A B C be measured by that *middle latitude*, A B the hypothenuse will be *nearly* equal to the difference of longitude between D and B.



For further information on this subject, no better work can be consulted than Riddle's "Navigation."

By the use of "Mercator's Projection," most of these questions can be solved without calculation. In this ingenious system the globe is conceived to be so projected on a plane that the meridians are *all parallel lines*, and the *elementary parts* of the meridians and parallels bear in all latitudes the same proportion to each other that they do upon the globe. The uses to which this species of projection can be applied, and the vast benefit its invention has proved to the navigator, will be evident by reference to any work on navigation.

The latitude and longitude of any place being known, that of any other station within *a short distance* can be determined by plane trigonometry. Suppose the latitude and longitude of G for instance to be known, from whence that of O, an adjacent station, is to be determined; the distance O G must be measured, or obtained by triangulation, and the azimuth N O G observed; then the difference of longitude G L between the stations is the sine of the angle L O G to radius O G; and O L, the difference of latitude, is the cosine to the same angle and radius. The following example will show the application of this simple method:—



The distance of a station O′, 238 feet due south of the Rl. Engr. Observatory at Chatham from Gillingham Church, was ascertained to be 7547·4 feet, and the angle S O G, the supplement of the azimuth, = 78° 55′ 55″; Gillingham Church being situated in 51° 23′ 24″·12 north latitude, and 0° 33′ 49″·41 east longitude.

Then cos 78° 55′ 55″—9·283243
log 7547·4—3·877796

1448·9—3·161039 Diff. of latitude (north), in feet.

And sine 78° 55′ 55″—9·991846
log 7547·4—3·877796

7407· —3·869642 Diff. of longitude (west), in feet.

The lengths of one second of latitude and longitude in latitude 51° 23′ are—

Latitude 102·02 feet.
Longitude 63·41 feet.

$$\therefore \frac{1448{\cdot}9 + 238}{102{\cdot}02} = 16''{\cdot}53. \text{ Difference of latitude in arc,}$$

$$\text{and } \frac{7407}{63{\cdot}41} = 116''{\cdot}8 = 1'\,56''{\cdot}8. \text{ Difference of longitude in arc.}$$

		Latitude.		Longitude.
Gillingham Church	N.	51° 23′ 24″·12	E.	0° 33′ 49″·41
Difference N. . .	+	16 ·53	W.	1′ 56 ·8
Observatory . . .		51° 23′ 40 ·65		0° 31′ 52 ·6

It is always necessary to ascertain the variation of the compass before plotting any survey, for the purpose of protracting such parts of the interior details as have been filled in by magnetic bearings, and also of marking the direction of the magnetic meridian upon detached plans. The laws of this variation are at present but little known; and it is only by accumulating a vast number of observations at different places, and at different periods, that the position of the magnetic poles and the annual variation and dip can be ascertained with anything like certainty

A meridian line being once marked on the ground, the bearing of this line by the compass is of course the variation east or west. It can be traced with an altitude and azimuth instrument, or even a good theodolite, by observing equal *altitudes and azimuths* of the sun, or a star, on different sides of the meridian. With the latter object *no correction* whatever is required: the cross hairs are made to thread the star exactly (by following its motion with the tangent screws) two or three hours before its culmination; the vertical arc is then clamped to this altitude, and the azimuth circle read off. On the star descending to the same altitude, at the same interval of time after its transit, it is again bisected by the cross hairs, and the mean between the two readings of the azimuth circle gives the direction of the true meridian, which being marked out on the ground, its bearing is then read with the compass.

When the *sun* is the object observed, the altitude taken may be that of either the *upper* or *lower*, and the azimuth that of the *leading* or *following limb;* the mean of the readings of the azimuth circle does not necessarily therefore in this case give the true meridian, as correction must be applied for the change in the sun's declination during the interval of time between the observations.

If the sun's meridian altitude is increasing, as is the case from midwinter to midsummer, his lower limb when descending will have the same altitude at a greater distance from the meridian than *before* apparent noon, and the reverse when it is decreasing. The following formula for this correction is taken from Dr. Pearson :—

$x = \frac{1}{2} D \times$ sect. lat. $\times$ cosect. $\frac{1}{2}$ T, where D is the change of declination* in the interval of time expressed by T.

Example:—In latitude 51° 23′ 40″ N. on May 12, 1838, the upper limb of the sun had equal altitudes.

At 9h.	54m.	26·8s.	A.M.	} By chronometer.
2	5	46	P.M.	

And the readings of the azimuth circle at these times were—

311° 47′ 20″ morning observation.
47 45 50 afternoon do.

	h.	m.	s.			
	12	0	0	360°	0′	0″
	9	54	26·8	311	47	20
Distance from noon, A.M.	2	5	33·2	48	12	40

	h.	m.	s.				
	2	5	33·2	48°	12′	40″	azimuth A.M.
	2	5	46	47	45	50	ditto P.M.
T=	4	11	19·2		2)26	50	diff.
½T=	2	5	39·6				
or in space	31°	24′	54″		13	25	
				360	0	0	
				359	46	35	reading of approximate meridian.

* The sun's change of declination is given for *every hour* in the first page of each month in the Nautical Almanac.

The sun's change of declination in one hour of mean time on May 12 appears, by the Nautical Almanac, $=37''\cdot53$, therefore for 2h. 5·6m., the half interval, it is $= 78''\cdot5$.

$\frac{D}{2} =$	$78''\cdot5$	log.	1·8948697
$L =$	$51^\circ\ 23'\ 40''$	sec.	0·2048465
$\frac{T}{2} =$	$31\ \ 24\cdot54$	cosec.	0·2829690
	$4'\ \ 1''\cdot37$	. .	2·3826852

Middle point	$359^\circ\ 46'\ 35''$
Correction	$4\ \ 1\cdot4$
Correct reading of true meridian	$359\ \ 42\ \ 33\cdot6$

The magnetic bearing of the pole star, or of any circumpolar star at its upper or lower culmination, gives at once the variation of the compass; a meridian may likewise be traced by *observing the azimuths of a star at its greatest elongations*, and taking the mean.

If only *one elongation* is observed, the sine of the angular distance $= \frac{\textit{sin polar distance of star}}{\textit{cosine latitude}}$, which added to, or subtracted from the observed azimuth, gives the direction of the meridian.

The time at which any star is at its greatest elongation is thus found. The cosine of the hour angle in space = tan polar dist. × tan lat. This hour angle divided by 15 gives the interval in sidereal time.

The other methods of finding the variation of the compass by the amplitude of the sun at sunrise or sunset, and by his azimuth at any period of the day, requiring more calculation, will be found among the Astronomical Problems.

A meridian line can be marked on the ground, without the aid of any instrument, with sufficient accuracy to obtain the variation of the needle for common purposes, by driving a picket vertically into the ground on a perfectly level surface. At three or four hours before noon, measure the length of its shadow on the ground, and from the bottom of the picket, as a centre, describe an arc with this distance as radius. Observe when the shadow intersects this arc about the same time in the afternoon, and the middle point between these and the picket gives the line of the meridian. It is of course better to have three or four observations at different periods before and after noon; and these several middle points afford means of laying out the line more correctly.

The method hitherto described of laying down stations by triangulation, or by means of distances calculated from astronomical observation, is however only applicable *within certain limits;* as, on account of the spherical figure of the earth, the relative positions of places on the globe cannot be represented by any projection in geographical maps embracing very large portions of its surface except by altering more or less their real distances, the content of various tracts of territory, and in fact, *distorting* the whole appearance, when compared with the different portions of the same country represented as plane surfaces.

Either a true projection or some arbitrary arrangement of the meridians and parallels is therefore necessarily adopted, and each place is marked on this skeleton according to its relative latitude and longitude. Those *projections* should be preferred in which the geographical lines are most easily traced, and whose *arrangement distorts as little as possible the linear and superficial dimensions.**

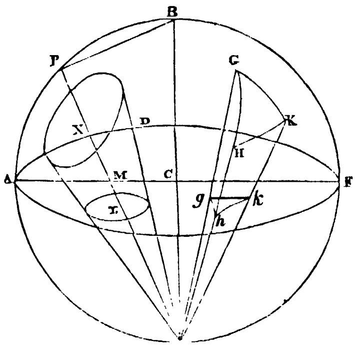
Descriptions of various projections will be found in the works of Puissant, Francœur, and other authors on the subject; and some very useful explanations of the projections of the sphere in a treatise on "Practical Geometry and Projection," published by the Society of Useful Knowledge.

The following short but clear definition of the three species of projection commonly used in maps, viz., the *orthographic, stereo-*

* Sir Henry James states that the one-inch maps of the Ordnance Surveys of Scotland and Ireland are laid down on a modification of Flamstead's projection, so that the sheets of each will, when put together, form one map. England has not been laid down upon any projection, but by the method of parallels and perpendiculars to meridians in different parts of the country.

graphic, and Mercator's, is taken from Sir J. F. Herschel's "Astronomy:"

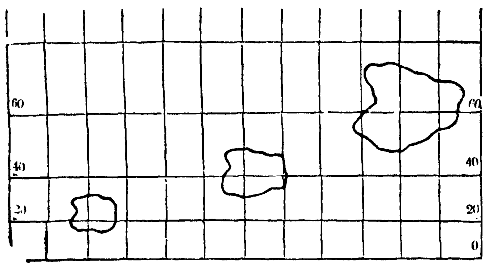
"In the *orthographic* projection every point of the hemisphere is referred to its diametral plane or base, by a perpendicular let fall on it, so that its representation, thus mapped on its base, is such as it would actually appear to an eye placed at an infinite distance from it. It is obvious that in this projection only the *central* portions are represented in their true forms, while the exterior is more and more distorted and crowded together as it approaches the edges of the map. Owing to this cause, the orthographic projection, though very good for *small portions* of the globe, is of little service for large ones.



"The *stereographic* projection is in a great measure free from this defect. To understand this method, we must conceive an eye to be placed at E, one extremity of a diameter E C B of the sphere, and to view the concave surface of the sphere, every point of which, as P, is referred to the diametral plane A D F perpendicular to E B by the visual line P M E. The *stereographic* projection of a sphere, then, is a true perspective representation of its concavity on a diametral plane; and as such it possesses some singular geometrical properties, of which the following are two of the principal:—first, all circles on the sphere are represented by circles in the projection; thus the circle X is projected into *x*: only great circles passing through the vertex B are projected into straight lines traversing the centre C; thus B P A is projected into C A.

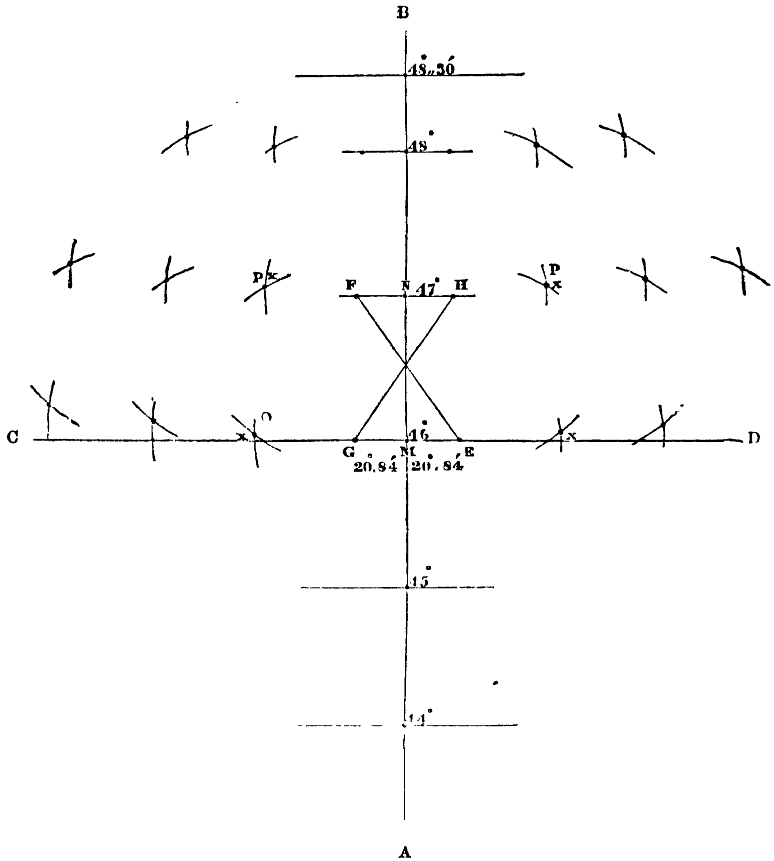
"Secondly, every very small triangle G H K on the sphere is represented by a *similar* triangle *ghk* in the projection. This valuable property ensures a general similarity of appearance in the map to the reality in all its parts, and enables us to project at least a hemisphere in a single map, without any violent distortion of the configurations on the surface from their real forms. As in the *orthographic* projection, the *borders* of the hemisphere are unduly crowded together; in the *stereographic*, their projected dimensions are, on the contrary, somewhat enlarged in receding from the centre."

Both these projections may be considered *natural ones*, inasmuch as they are really *perspective representations of the surface on a plane;* but Mercator's projection is entirely artificial, representing the sphere as it *cannot be seen from any one point, but as it might be seen by an eye carried successively over every part of it.* The degrees of longitude are assumed equal, and of the value of those at the equator. The degrees of latitude are extended each way from the equator, retaining always their proper proportion to those of longitude; consequently the intervals between the parallels of latitude increase from the equator to the poles. The equator is conceived to be extended out into a straight line, and the meridians are straight lines at right angles to it, as in the figure. Altogether the general character of maps on this projection is not very dissimilar to what would be produced by referring every point in the globe to a circumscribing cylinder, by lines drawn from the centre, and then unrolling the cylinder into a plane. Like the *stereographic* projection, it gives a true representation as to *form* of every particular small part, but varies greatly in point of *scale* in its different regions—the polar regions, in particular, being extravagantly enlarged; and the whole map, even of a single hemisphere, not being comprisable within any finite limits.



The following simple directions are given by Mr. Arrowsmith for a projection adapted to a map to comprehend only a limited portion of the globe; for instance, that between the parallels of 44° and 48° 30′ north latitude, and longitudes 9° and 18° east of Greenwich.—Draw a line A B for a central meridian; divide it into the required number of degrees of latitude ($4\frac{1}{2}$); through one of these points of division (say 46°) draw C D intersecting the meridian at right angles, and likewise draw lines through the other points parallel to C D.

Take the breadth in minutes of a degree of longitude in lat. 46° = 41·63 ; from M towards C and D, set off each way one-half



of this, 20·84 (M E . M G). Again, from N lay off on each side one-half of the length of a degree in lat. 47° = 40·92 — N F, N H. Measure the diagonals G H, E F, and putting one point of the compasses successively on F, G, H, and E, describe the arcs, *x x x x*.

Take 41·68, the whole measurement of a longitudinal degree in lat. 46°, and lay off the distance, G O, E O, intersecting the arcs

$x\,x\,x$ at O O. Again, take the value of a degree in latitude 47° 40·92, and lay off the distances E P, H P.

This process continued until the parallels of 46° and 47° are completed, the whole projection may be carried on in the same manner, the two parallels first drawn furnishing the respective points of each meridian.

It would occupy too much space to pursue the subject further; explanations of all the most useful projections will be found in the sixth chapter of Francœur's "Géodesie," and in other works of the same character.

CHAPTER XI.

PRACTICAL ASTRONOMY.

Before proceeding to the solution of the few simple problems by which the latitude, longitude, and time, as also the variation of the compass and the azimuth of any celestial body can be determined under different circumstances, it is considered advisable to explain the meaning of such terms as are most constantly met with in practical astronomy, and the corrections necessary to be applied to all observations.

The Sextant,* Reflecting Circle, or Dollond's Repeating Circle, with the Artificial Horizon and Chronometer, the Barometer and Thermometer, are the description of *portable* instruments generally used in taking astronomical observations. The Nautical Almanac for the year, as well as tables of logarithms and refraction are also necessary. In an observatory, or for any extensive geodesical operation, instruments are required of firmer construction, and admitting from their size of more minute graduation. The principal Observatory Instruments are the Transit, Sidereal Clock, Equatorial, Altitude and Azimuth Instrument; to which may be added the Zenith Sector and Mural or Transit Circle.

In *all* reflecting instruments the angle formed by the planes of the two mirrors is only half the observed angle, but the arc or circle is graduated to meet this effect of the principle of their construction; thus an angle of 60° is marked on the limb of the sextant 120°; and the entire circle reads 720°.

* The altitude and azimuth of any celestial body can also of course be taken with the Theodolite,—indeed a large Theodolite of the best construction is a portable Altitude and Azimuth Instrument.

Full descriptions, with plates, of the methods of using and adjusting the sextant and reflecting circle are given in Mr. J. Simms' "Treatise on Mathematical Instruments," which little work is, or should be, in the hands of every *observer;* but a short descriptive outline of the management of these instruments is given below, as well as of the "repeating circle,"* which is at all events in theory, the most perfect of the class of reflecting instruments.

The Sextant—The arc is generally graduated to 10 minutes, and these divisions are subdivided by the vernier to 10 seconds, one-half of which can by aid of the reading microscope attached to the index arm be easily estimated, so that the instrument may be said to read to 5 seconds. The index is clamped to the arc when the observation is approximately made, which is then perfected by the tangent screw. At the central point of the arc on which the index arm revolves is fixed the silvered mirror termed the "index" glass, perpendicular to the plane of the instrument. To the frame is attached a second glass, termed the "horizon" glass, the lower half only of which is silvered, the upper being left for the purpose of direct sight of the object through it, and the plane of this second glass must be parallel to that of the index glass, when the index is at zero on the limb, any deviation from which constitutes the "index error." The telescope, also parallel to the plane of the instrument, is carried by a ring attached to the other radius part of the frame, and can be

* The repeating circle here spoken of, is a *reflecting circle,* having the power of repetition. For the determination of latitudes and longitudes on surveys of the magnitude of the Ordnance Survey of Great Britain, or for very important and delicate geodesical operations, the Zenith Sector, Altitude and Azimuth Instrument, and Portable Transit are employed. This latter, though properly an observatory instrument, can be used upon a stand formed by the stump of a large tree, or by three or four strong posts driven into the ground, supporting a top on which the transit is placed. A rough pedestal of masonry or brick-work of course answers the same purpose, great care being taken to secure its steadiness, and prevent its being affected by the movement of those about it, to ensure which, a sort of detached platform upon posts will be found efficient. Solid rock is considered not so suited for the foundation of this sort of pedestal as sand, or other species of earth, on account of its more readily conveying tremulous vibrations to the instrument. Transits of from 20 to 30 inches focal length were thus used upon the survey (in 1845) of the North American Boundary, a tent made of fine canvas being contrived to protect the lights from the wind.

raised or lowered bodily in order to enable the observer to make the object seen by direct vision and its image by reflection nearly equally distinct, and for observations of the sun the shades attached to the frame, or one of the dark glasses to the eye-piece, must be made use of.

In observing the altitude of the sun or a star at sea, the eye of the observer is directed to the line of the horizon immediately below the object, the reflected image of which is brought down to the horizon by moving forward the index. With the artificial horizon on shore, the two images are brought into contact on the surface of the mercury, and the angle observed in this case is double that of the former.

The index glass is but little liable to derangement, and provision is seldom made for altering its adjustment, the accuracy of which can be tested by moving the index to about the middle of the limb, and looking obliquely down the glass by observing if the circular arc, as seen by direct vision and by reflection, appears as it ought to do in one continuous curve without being broken at the point of contact.

The horizon glass is set perpendicular to the plane of the sextant by a small screw at the lower end of the frame, and it is known to be in adjustment when the reflected image of the object passes directly over that by direct vision when brought into contact by the index arm.

The amount of the index error (caused by the want of parallelism in the two glasses) is ascertained by observing if when the zero of the vernier is set to 0 on the limb, the direct and reflected images do or do not coincide. If they do, the index error is nothing; if not, the angle the index has to move through to make the coincidence perfect represents the index error. The usual and most correct method of ascertaining this error is by measuring the diameter of the sun in the following manner: clamp the index at about 30 minutes from zero on the graduated arc and perfect the contact of the two images of the sun by the tangent screw, reading the angle; then put the index to about the same number of minutes on the "arc of excess," which is a small portion of the arc graduated on the other side of zero, and complete the contact as before, again reading the number of minutes and seconds. One half of the difference of these two readings is the index error, + when that on the arc of excess is the greatest, and — when the contrary, thus,

Reading on the arc . . .	33′	20″
Do. on the arc of excess .	32	40
Difference	0	40
Index error	—	20

If these observations have been correctly made, ¼ of the sum of the two readings should correspond with the semi-diameter of the sun as given in the Nautical Almanac for that day. In most instruments there is no means of rectifying the index error, and it is necessary frequently to ascertain if it has at all varied and to allow for it + or — in every observation.

The parallelism of the line of collimation of the telescope to the plane of the sextant is not often deranged, and is known to be correct when two objects having an angular distance of 90° or more are brought into contact on the wire nearest the plane of the instrument and found to maintain this contact when the position of the sextant is altered to make the contact appear on the other wire. The adjustment, if any is found necessary, is made by the two screws which hold the collar into which the telescope screws.

The Reflecting Circle is in principle and use the same as the sextant, but instead of merely an arc, the observer has an entire circle to work upon; it has three verniers, one of which carries the clamp and tangent screw, moving round the same centre as the index glass which is situated on the opposite face of the instrument. There are two handles fixed parallel to the plane and a moveable one at right angles, which can be screwed into either of the others when observing horizontal angles.

The advantage of the reflecting circle over the sextant is that all index errors are negatived by the observations being taken on each side, and also all errors of centering, as the verniers are read at three equidistant points of a circle. Its scope is also far greater, as angles may be measured with it up to 150°, which will allow of the sun's double altitude being taken with it, with an artificial horizon when within 15° of the zenith. Like the sextant these three adjustments are required to be perfect.

1. The index glass being perpendicular to the plane of the circle, which when so fixed by the maker is seldom deranged.

2. The horizon glass to be also perpendicular in the same plane.

3. The line of collimation of the telescope parallel to the plane of the circle.

The Repeating Circle—Set the vernier, which moves on the circumference of the inner circle (as do also the horizon glass and telescope at the extremities of arms having one common centre), to zero (or 720°), on the graduated outer circle, and clamp it. Unclamp the vernier at the end of the arm carrying the index glass, which, when the two glasses are parallel, should read zero. Take the required altitude or angular distance by moving the index forwards till a perfect contact is obtained, and clamp it to the outer circle, noting the time if required, but merely reading approximately the angle.

Unclamp the arm to which the telescope is attached, and, reversing the instrument, make the contact again on the other side, by moving forward this arm concentric with that carrying the horizon glass, (which can be done very rapidly by setting it nearly to the approximate angle already read, but on the other side of the zero of the inner circle which is graduated each way to 180°,) and perfect the observation by the tangent screw. The angle now read on the outer circle is evidently *double* that observed for the mean of the times, freed from any index error by the reversal of the instrument. This process may be repeated over and over again all round the circle as often as required, and the *last angle* shown by the vernier of the horizon glass is the only one which requires to be read, and divided by the number of observations for the mean angular measurement answering to the mean of the times.

Instead of setting the vernier at first to 720°, it may be read off at any angle as with the theodolite; but the method described above is preferable.

The Artificial Horizon alluded to as being a necessary adjunct to the use of all reflecting instruments, consists generally of a smal oblong box, with a glass cover (usually made to fold back into the box to render the whole portable), into which mercury strained quite free from impurities is poured when required for observation. To avoid the use of mercury, which is inconvenient to carry, various other contrivances have been tried, among others a circular plate of glass blackened behind, adjusted to a horizontal plane by means of a spirit level and screws, but it is not to be equally trusted. Oil or treacle would answer for the purpose in place of mercury, but would not give so clear an image.

The artificial horizon when used must be placed on some solid support not liable to vibration, (otherwise the surface of the mercury will never be quite at rest,) in such a position that the reflected image of the object can be clearly seen; and when the object has a low altitude, it must be raised nearly to the level of the eye of the observer.

In observing, the image of the object reflected from the index to the horizon glass must be brought down by a gentle forward movement of the index until it is nearly in contact with that seen on the surface of the mercury, when the index arm is clamped and the observation completed by the tangent screw.

When the *lower* limbs of the sun are in apparent contact, the image reflected from the index glass appears uppermost; when the upper limb, of course the reverse. With the inverting telescope generally used with reflecting instruments, the lower limb seen in the mercury always appears the upper, and *vice versâ;* and the index must be moved in the contrary direction to that the object appears to take in order to keep it in the field of view.

The angle observed with an artificial horizon is, as has been before stated, always double the angle of elevation above the sensible horizon.

The terms answering to *terrestrial longitude* and *latitude*, when referred to the celestial sphere, are *right ascension* and *declination;* the former being measured on the equinoctial (or the plane of the equator produced to the heavens) commencing from the first point of Aries, which for many reasons has been taken as the conventional point of departure in the celestial sphere; and the latter on great circles perpendicular to the equinoctial and meeting at the poles, being reckoned north or south of this plane.

A confusion is caused, often puzzling to beginners, by the introduction of the terms longitude and latitude in the celestial nomenclature, having a different meaning from the same expressions as applied to the situation of places on the earth; they have reference to the *ecliptic* instead of the *equinoctial;* celestial longitudes commence also from the intersection of these two planes.

This point has a constant gradual irregular retrograde motion on the ecliptic caused by the combined varying influences of the sun and moon upon the bulging equatorial portion of the spheroidal form of the earth, whereby the intersection of the planes of the equator and ecliptic recedes annually in a direction contrary to the signs at the rate of about 50·3″. The result of this is a slow circular motion of the pole of the equator round that of the ecliptic, the revolution being completed in about 25,800 years. The influence exerted by the sun in this phenomenon is to that of the moon in the ratio of nearly 2 to 5, but not always in the same direction, and the result given above is due to the combination of those disturbing causes termed solar precession and nutation. A full and lucid description of these and all other irregularities in the earth's motion will be found in Woodhouse and Sir J. Herschel's astronomy. The angle of inclination of the planes of the equator and ecliptic, termed the "obliquity of the ecliptic," is about 23° 27′ 30″, and varies slowly at the rate of 0·4755″ per annum, but its maximum variation cannot exceed 2° 42′. The longitudes as well as the right ascensions and declinations, even of the fixed stars, are constantly undergoing a slight change, though imperceptible to measurement in short intervals of time. The corrections for their places on this account, as well as on that of their *aberration*,* are allowed for in the "catalogue of the hundred principal stars," given in the Nautical Almanac for every tenth day, and can easily be calculated for any particular period of time.

* The light from any celestial body is propagated by a progressive motion at the rate of 200,000 miles per second. If the observer were stationary the result would be the same as if it were instantaneous; but as he participates in the motion of the revolution of the earth, the point in space in which the object is actually situated is not that in which it appears at the time of observation in the axis of the telescope which has in the interval moved forwards with the rotary motion of the earth. This difference between the real and apparent place of the star is termed its "Aberration."

Great circles perpendicular to the horizon, and meeting in the zenith and nadir, are called *vertical circles;* on these the altitudes of objects above the horizon are measured; the complements to these altitudes are termed *zenith distances;* and the arc of the horizon contained between a vertical circle, passing through any object and the plane of the meridian, is termed the azimuth of that object. The altitude and azimuth of any object being known, its place in the visible heavens *at that moment* is determined; whereas the latitude and longitude, or the right ascension and declination, fix its place in the celestial sphere.

The right ascension and declination of any celestial object can evidently be determined from its latitude and longitude, and *vice versâ;* the *obliquity of the ecliptic,* or the angle it forms with the equinoctial, being known.

The *sensible horizon* is an imaginary plane tangential to the earth at the place of the observer; whereas the *rational horizon* (to which all altitudes must be reduced by the correction for parallax) is a plane parallel to the former, passing through the centre of the globe: altitudes require also another correction for the effects of *refraction,** which, it has been already explained in Chap. VI., causes the *apparent place* of any object to be always elevated above its *real place;* the correction is therefore subtractive.

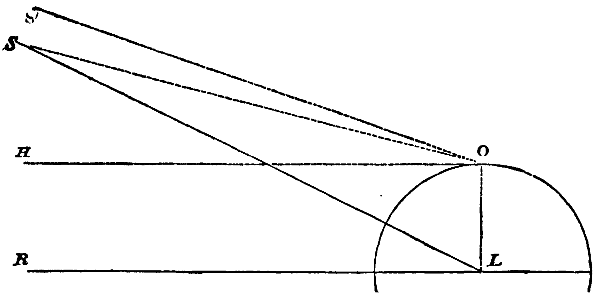
The first correction alluded to, that for *parallax,*† is *always additive.* This term, as applied in its limited sense to altitudes of celestial objects, is meant to express the angle subtended by the semi-diameter of the earth at the distance of the object observed. Altitudes of the moon, from her proximity to the earth, are most affected by parallax: it is also always to be taken into account in observing altitudes of the sun, or any of the planets; but the fixed stars have no *appreciable* parallax, owing to their immeasurable ‡ distance from our globe.

* See the tenth chapter of Woodhouse's "Astronomy" for the explanation of the method of obtaining the *constant* of refraction, and the different values of this quantity, generally estimated at 57″.

† For a further explanation of Parallax in a more general sense, see Sir J. F. Herschel's "Astronomy," p. 47.

‡ At *least* 5000 million times the diameter of the globe.

In the figure below, H O is the *sensible*, and R L the *rational* horizon; S the real place of the object, and S′ its apparent place, elevated by refraction; S′ O H is the angle observed; S O H the altitude corrected for refraction, and S L R the same altitude corrected both for refraction and parallax, being equal to the angle S O H + O S L, the *parallax*.



It is evident that the *equatorial parallax* of any object (which is that given in the Nautical Almanac), being subtended by the semi-diameter of the earth at the equator, is always the *greatest*, and that at the poles the *least*. The diminution, according to the latitude of the place of observation, can be obtained from tables constructed for the purpose. The parallax in any latitude is also *greatest at the horizon*, and diminishes as the object approaches the zenith, where it vanishes.

Another correction that must be applied to the observed altitudes of the sun or moon is that for their semi-diameters, *plus* or *minus*, according as the upper or lower limb has been taken:* this quantity is found for each day of the month in the Nautical Almanac.

When observations are made at sea, an allowance must be made for the height of the eye above the horizon: this correction, termed the *dip*, is evidently always *subtractive;* and in observing with a sextant, it is always necessary to ascertain and apply its *index error*, which term is meant to express the deviation of the reading of the instrument from zero, when the direct and reflected images of an

* When several observations are taken, the necessity for this correction can be obviated by observing alternately the upper and lower limb.

object are made *exactly to coincide*, in which case the horizon and index glasses are parallel.

The usual method of ascertaining the amount of this error of the instrument in astronomical observations, is by measuring the diameter of the sun on different sides of the true zero, and is done as follows:—set the vernier at about half a degree from zero on the graduated limb, and perfect the contact of the *two limbs* with the tangent screw,* noting the reading: unclamp the index, and set the vernier again to about the same distance on the *other* side of zero, termed the *arc of excess* (which is divided for a few degrees for this purpose), observing also this reading, when the contact has been again perfected; half the difference will evidently be the index error, + when the reading of the arc of excess is the greatest, and − when that of the limb: thus,

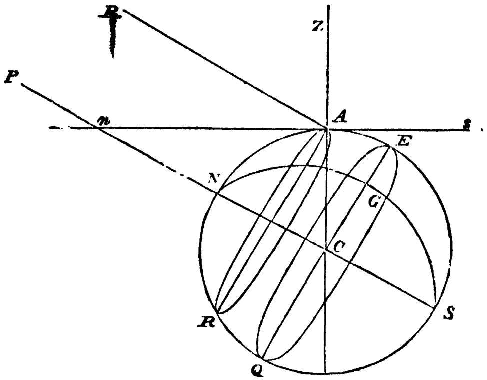
Reading on the arc	32′	10″
On arc of excess	33	20
	2) 1	10
Index error +	0	35

These definitions are rendered more evident by reference to the figure opposite, taken from Sir J. Herschel's Treatise on Astronomy, published in the Cabinet Cyclopædia.

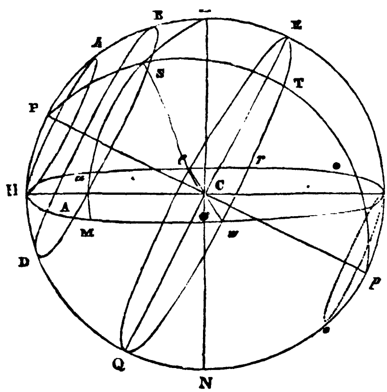
"Let C be the centre of the earth, N C S its *axis;* then are N and S its *poles;* E Q its *equator;* A R the *parallel of latitude* of the station A on its surface; A P, parallel to S C *n*, the direction in which an observer at A will see the *elevated* pole of the heavens; and A Z, the prolongation of the terrestrial radius C A, that of his zenith; N A E S will be his *meridian;* N G S that of some fixed station, as Greenwich; and G E, or the spherical angle G N E, his *longitude*, and E A his *latitude.* Moreover, if *n s* be a plane touching the surface in A, this will be his *sensible* horizon; *n* A *s*,

* In using the tangent screw, a perceptible difference is found between a *progressive* and a *retrograde* motion—the latter had better always be avoided. A difference is also found in *different parts* of the length of the screw.

marked on that plane by its intersection with his meridian, will be his meridian line, and *n* and *s* the north and south points of his horizon."



"Again, neglecting the size of the earth, or conceiving him stationed at its centre, and referring everything to his *rational* horizon, let the next figure represent the sphere of the *heavens*; C the spectator; Z his *zenith*; and N his *nadir*; then will H A O, a great circle of the sphere whose poles are Z and N, be his *celestial horizon*; P *p* the *elevated* and *depressed* poles of the heavens; H P the *altitude* of the pole; H P Z E O his meridian; E T Q, a great circle perpendicular to P*p*, will be the *equinoctial*; and if *r* represent



the equinox, *r* T will be the *right ascension*, T S the *declination*, and P S the *polar distance* of any star or object S, referred to the equinoctial by the hour circle P S T *p*; and B S D will be the diurnal circle it will appear to describe about the pole. Again, if we refer it to the horizon by the vertical circle Z S M; H M will be its *azimuth*, M S its *altitude*, and Z S its *zenith distance*. H and O are the north and south, and *e* and *w* the east and west points of the horizon, or of the heavens. Moreover, if H *h*, O *o*, be small circles, or *parallels of declination* touching the horizon in its north and south points, H *h* will be the circle of *perpetual apparition*, between which and the elevated pole the stars *never set;* O *o* that of *perpetual occultation*, between which and the depressed pole they *never rise*. In all the zone of the heavens between H*h* and O *o* they rise and set; any one of them, as S, remaining above the horizon in that part of its diurnal circle represented by A B *a*, and below it throughout all that represented by *a* D A."

From these figures it is evident that the altitude of the elevated pole is equal to the latitude of the spectator's geographical station, for the angle P A Z in the first figure, which is the *co-altitude* of the pole, is equal to N C A; C N and A P being parallels whose vanishing point is the pole. But N C A is the co-latitude of the place A, whence the altitude of the pole must be equal to the latitude. The equinoctial intersects the horizon in the east and west points, and the meridian in a point whose *altitude is equal to the co-latitude of the place.*

The *natural standards* of the measurement of time are the *tropical year* and the *solar day*, and these are in a manner forced upon us by nature, though, from their "*incommensurability* and *want of perfect uniformity*," they occasion great inconvenience, and oblige us, while still retaining them as *standards*, to have recourse to other artificial divisions. In all measures of *space* the subdivisions are aliquot parts; but a year is no exact number of days, or even an integer with an exact fractional part; and before the introduction of the *new style* into England in 1752, an error of as much as 11 days had thus crept into the calendar. By the present arrangement, every year whose number is not divisible by 4 without remainder, consists of 365 days; every year which is so divisible, but is not by 100, consists of 366 days; every year again, which is divisible by 100, but not by 400, consists of only 365 days; and every year divisible by 400, of 366. The possibility of error is thus so far guarded against, that it cannot amount to *one day* in the course of 3000 years, which is sufficient for all civil reckoning, of which, however, astronomy is perfectly independent.

The three divisions of time for civil and astronomical purposes are the *apparent solar*, *mean solar*, and *sidereal day*. The apparent solar day is the interval between two successive transits of the sun over the same meridian; and from the path of the sun lying in the ecliptic inclined at an angle to the equator upon the poles of which the earth revolves, and the earth's orbit not being circular, it follows that the length of this day is constantly varying; so that, although it is the *only solar time which can be verified by observation*, it is quite unfit for application to general use.

The mean solar day, which is purely a conventional measure of time, is derived from the preceding, and is the average of the length of all the apparent solar days in the year, as nearly as it can be divided; and this is the measure of all civil reckoning. Mean time is in fact that which would be shown by the sun if he moved in the *equator instead of the ecliptic, with his mean angular velocity*.

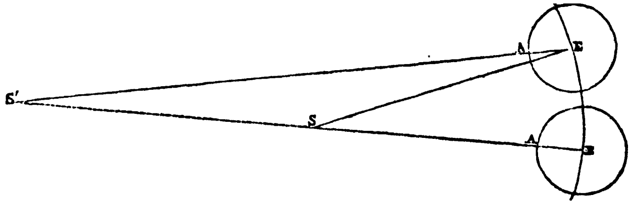
The difference on any day between *apparent* and *mean* time is termed the *equation of time*, and is given for every day of the year at mean and apparent noon in the first and second pages of each month in the Nautical Almanac, additive or subtractive, according to the relative positions of the real, and the imaginary mean sun.*

A sidereal day is the time employed by the earth in revolving on its own axis from one star to the same star again; or the interval between two successive transits of any fixed star, which is always *so nearly* the same length, that no difference can be perceived except in long intervals of time,† particularly in stars situated near the equator. A sidereal is $3^m\ 55^s{\cdot}91$ shorter than a mean solar day, and is also less than the shortest apparent solar day, as must be evident from the figure in next page, where the earth, moving in its orbit, and revolving on its own axis, after any point on its surface, A, has by its revolution brought the star S′

* For a most lucid explanation of this varying equation, see Woodhouse's "Astronomy," chap. xxii., commencing at page 537; and also Vince's "Astronomy," &c.

† For the causes of this almost imperceptible variation in the length of a sidereal day, see Woodhouse, page 106; there is, *in fact*, a *mean* and an *apparent* sidereal day.

again on its meridian, must move also through the angle S′E S, before the arrival of the sun S on the same meridian.



Both *sidereal* and *apparent solar* time are measured on the equinoctial, the former being at any particular instant the angle at the pole between the *first point of Aries* and the meridian of the observer; and the latter, that contained between this meridian and the meridian where *the sun is at the moment of observation*, both reckoned westward; hence the apparent solar time added to the sun's right ascension is the sidereal time, and when any object is *on the meridian*, the sidereal time, and the apparent right ascension of that object, are the same.

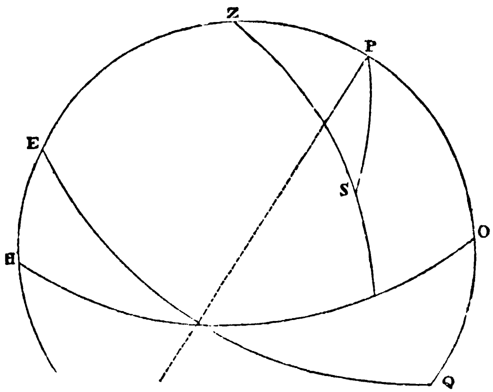
It is evident that the difference between the *time* at any two places on the earth's surface is measured by the same arc of the equator, which measures the *difference of their longitudes*, the circumference of the circle representing 360 degrees or 24 hours; making 15 degrees of longitude = one hour of time. To find the difference of longitude then between any two places, only requires us to be able to determine *exactly the local time at each place, at the same instant;* for which purpose chronometers whose rates are known, and which have been set to, or compared with, Greenwich mean time, are used particularly at sea where other means more to be depended upon, cannot, from the motion of the ship, and the constant change of place, be always resorted to.

From these explanations it will easily be seen that of the *five* following quantities, any *three* being given, the other two can be found by the solution of a spherical triangle, viz.:

1. The latitude of the place.
2. The declination of the celestial object observed.
3. Its hour angle east or west from the meridian.
4. Its altitude.
5. Its azimuth.

Thus in the triangle P Z S, named from its universal application the *astronomical triangle*—

P is the elevated pole, Z the zenith, and S the star or object observed; and the five quantities above mentioned, or their complements, constitute the sides and angles of the spherical triangle Z P S; P Z being the co-latitude, P S the co-declination, or north polar distance, Z S the co-altitude or zenith distance, the angle Z P S the hour angle, and P Z S the azimuth.



The further application of this triangle will be seen in the astronomical problems.

In all the ordinary observations made for the determination of the latitude, local time, &c., the object observed may be either the sun, or a star whose declination and right ascension are known: the latter is wherever practicable, to be preferred, as the use of the sun always involves corrections for semi-diameter and parallax; also in many observations of the sun, such as those of equal altitudes for time, or for determining the direction of a meridian line, or circum meridian altitudes for finding the latitude,—still further corrections are requisite on account of the change of the sun's declination during the period embraced by the observations; which corrections are avoided by using a star.

The bisection of a star is likewise more to be depended upon than the observed tangent of the sun's limb. At sea, where minute accuracy is neither sought, nor to be obtained, and where at night the horizon is generally obscured, and often not to be discerned at all, this advantage is either not material, or not often to be taken advantage of; but on shore an artificial horizon is always used with reflecting instruments, and upon this the darkness of the night has no effect.

In all observations of a star, the clock or chronometer, if not already so regulated, must be reduced to *sidereal time;* with the sun on the contrary, the timekeeper must be brought to *mean solar* time whether the local or Greenwich time be required.

PROBLEMS.

PROBLEM I.

TO CONVERT SIDEREAL TIME INTO MEAN SOLAR TIME, AND THE REVERSE.

This problem is of constant use wherever the periods of solar observations are noted by a clock regulated to sidereal time, or those of the stars by a chronometer showing mean time. A simple method of solution is given in the "explanation" at the end of the Nautical Almanac, which has the advantage of not requiring a reference to any other work, and also of all the quantities being additive.

The three tables used in this method are those of *equivalents;* the *transit of the first point of Aries* in the 22nd; and the *sidereal time at mean noon*, in the 2nd page of each month.

To convert sidereal into mean solar time:—

To the mean time at the *preceding sidereal noon*, *i. e.* the transit of the first point of Aries, in Table XXII., add the *mean interval* corresponding to the given sidereal time, taken from the table of equivalents.

To convert mean solar into sidereal time:—

To the sidereal time at the *preceding mean noon*, found in Table II., add the *sidereal interval* corresponding to the given mean time also from the table of equivalents.

The mean right ascension of the meridian, or the sidereal time at mean noon given in the Nautical Almanac, is calculated for the *meridian of Greenwich*, and must, therefore, be corrected for the difference of longitudes between that place and the meridian of the observer

One of Mr. Baily's formulæ for the solution of the same problem is—

$$M = (S - Æ) - a$$
$$\text{and } S = Æ + M + A$$

Where M represents the mean solar time at the place of observation, S the corresponding sidereal time, Æ the mean right ascension of the meridian at the *preceding mean noon*, found under the head of "*sidereal time*" in page 2 of each month; *a*, the *acceleration* of the fixed stars given in Baily's Table VI. for the interval denoted by (S — Æ); and A the acceleration shown in his 7th table for the time denoted by M.

Examples.

Convert $8^h\ 1^m\ 10^s$ sidereal time, March 6, 1838, longitude $2^m\ 21{\cdot}5^s$ east, into mean solar time.

	H.	M.	S.
Mean time at preceding sidereal noon Greenwich (Table XXII.)	1	4	44·19

Correction for Longitude:

M. S.	S.				
2 21·5 or	141·5	2·1507564			
*·0027305		$\bar{3}$·4362422			
	·3863	$\bar{1}$·5869986			·3863
			1	4	44·5763

Table of Equivalents:—

H.	M.	S.	H.	M.	S.	H.	M.	S.
8	0	0	7	58	41·3635			
0	1	0	0	0	59·8362	7	59	51·1724
0	0	10	0	0	9·9727			
Mean time required . . .						9	4	35·7487

* ·0027305 is the change in time of sidereal noon in one second; and ·0027379 is the change in the sun's mean right ascension in one second of time, or 9·8565 in one hour.

Again, to convert $9^h\ 4^m\ 35{\cdot}748^s$ mean solar into sidereal time.

☉ right ascension at mean noon Greenwich, under head of "Sidereal Time," Table II. . . . $22^h\ 55^m\ 5{\cdot}18^s$

Correction for Longitude E:

141·5	2·1507564			
*·0027379	3·4374176			
·3874	1·5881740			·3874
		22	55	4·7926
$9^h\ 4^m\ 35{\cdot}748^s$ solar time, equivalent sidereal .		9	6	5·2112
Sidereal time required . .		8	1	10·0038

The same examples by Mr. Baily's formula:—

		H.	M.	S.
	S =	8	1	10
	Æ =	22	55	4·79
		9	6	5·21
A (Table VI., Baily)		—	1	29·46
	M =	9	4	35·75

Again S = Æ + M + A

		H.	M.	.
	M =	9	4	35·75
As above,	Æ =	22	55	4·79
		7	59	40·54
A (Table VII., Baily)	=	+	1	29·46
	S =	8	1	10·00

PROBLEM II.

TO DETERMINE THE AMOUNT OF THE CORRECTIONS TO BE APPLIED TO OBSERVATIONS FOR ALTITUDE, ON ACCOUNT OF THE EFFECTS OF ATMOSPHERIC REFRACTION, PARALLAX, SEMI-DIAMETER, DIP OF THE HORIZON, AND INDEX ERROR.

THE formula given by Bradley for computing the value of atmospheric refraction is $r = a.\tan(Z - br)$, where Z represents the zenith distance of the object, and a and b constants determined by observation; a, the average amount of refraction at an apparent zenith distance of 45° being assumed $= 57''$; and $b = 3''{\cdot}2$.

The formula of Laplace is $\cdot 99918827 \times c \tan Z - \cdot 001105603 \times c \tan^3 Z$, where c is assumed $= 60''{\cdot}66$.

The tables constructed from these formulæ are of course not *exactly* similar on account of the difference of the constants which are also slightly varied in the tables of Bessel, Groombridge, &c. *They all* suppose a mean temperature, and mean pressure of the atmosphere, corrections being in all cases required on account of the deviation of the thermometer and barometer from these assumed standards. These corrections are however rendered perfectly simple in operation, by the use of any of the numerous tables of refraction; those by Dr. Young being given in Table IV. in this volume.

The rate of the increase of refraction is evidently, from the above formula, nearly as the tangent of the apparent angular distance of the object from the zenith in *moderate altitudes.* In *very low* altitudes (which should always be avoided on this account), the refraction increases rapidly and irregularly, being at the *horizon* as much as 33′—more than the diameter of the sun or moon.

The next correction is for *parallax*, the explanation of which term has been given in page 184. The sine of its value in any altitude decreases as the cosine of that altitude; but the parallax in altitude may be obtained from the *horizontal* parallax without computation, by the aid of tables.

The parallax given in any ephemeris is the equatorial, which has been shown in page 185 to be always the greatest. The first correction, where great accuracy is required, is on account of the latitude of the place of observation, but this is seldom necessary except in altitudes of the moon. The *mean horizontal parallax* of the sun is assumed = $8''\cdot6$; but as our distance from this luminary is always varying in different parts of the earth's orbit, this value must be corrected for the period of the year. In Table VIII., the sun's horizontal parallax is given for the first day of every month, which will facilitate this reduction, the proportional parts being found for any intermediate day. In the Nautical Almanac, however, this quantity is given more correctly for every tenth day. The *parallax in altitude*, corresponding to this horizontal parallax, can also be ascertained by inspection, from the same general table.

The parallaxes of the planets are given for every fifth day, in the Nautical Almanac; but of those likely ever to be found useful in observation, Venus and Mars are the only planets to whose parallaxes any correction need be applied in observing with small instruments. The horizontal equatorial parallax of the moon is to be found for mean noon and midnight of every day in the year, in the third page of each month, in the Nautical Almanac. The corrections for its reduction for the latitude of the place, and the moon's altitude, require from their magnitude more care than those of any other celestial body; but in observations at sea the former correction is generally neglected, and the latter is much facilitated by the use of tables giving the reduction for every 10′ of the moon's altitude.* The example given in this case will explain the method of making these corrections.

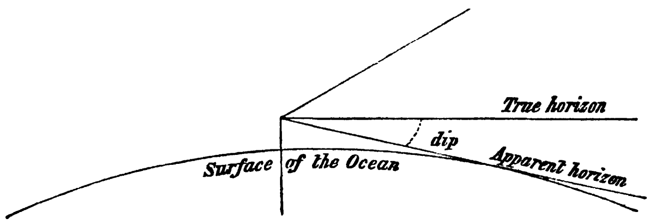
The *semidiameter*† of the sun is given for mean noon on every day of the year, in the second page of every month of the Nautical Almanac; that of the moon in the third page of each month for

* See Table VIII. of Lunar Tables, page 188 of Dr. Pearson's "Astronomy." Riddle's Table, page 154, includes the corrections both for *Parallax* and *Refraction*, and is useful for "clearing the lunar distance," to be hereafter explained.

† *All* quantities in the Nautical Almanac are calculated for *Greenwich* time; allowance must therefore be made, where necessary, for difference of longitude, which is the same as difference of time.

both mean noon and midnight; and those of the planets (which are seldom required) in the same table as their parallaxes. The correction for semidiameter is obviously to be applied, additive or subtractive, wherever the lower or upper limb of any object has been observed, to obtain the apparent altitude of its centre ;—the moon's semidiameter *increasing* with her altitude, from the observer being brought nearer to her as she approaches his meridian, must be *corrected for altitude*, which can be done by the aid of Table VII.*

The dip of the horizon is a correction only to be applied at sea, and is necessary on account of the height of the eye above the



sensible horizon (on shore an *artificial horizon* is always used). A larger angle is evidently always observed; and this correction, which can be taken from Table XI., is always subtractive.

The correction for the index error has already been explained.

EXAMPLE I.

On March 15, 1838, the observed *double* altitude of *the sun's upper limb*, taken with a sextant, was 42° 37′ 15″, the thermometer at the time standing at 42°,† and the barometer at 29·98 inches. Required the altitude, corrected for semidiameter, refraction, and parallax.

* The augmentation of the moon's semidiameter for every degree of altitude is given in Table VII. of Dr. Pearson's "Lunar Tables." Altitudes taken with an artificial horizon are obviously *double* those observed above the sensible horizon.

† In rough altitudes, such as those taken at sea for latitude, no correction is made on account of the state of the thermometer or barometer.

Index Error.

	′	″
Reading on the arc	33	40
Arc of excess	30	40
	2) 3	0
Index Error.	— 1	30

Refraction.

21°, Table IV.	2	30·5
1′·8		·2
	2	30·3
Thermometer	+	2·4
	2	32·7
Barometer		— ·1
Corrected Refraction	2	32·6

Parallax.

At 21°, March 15, Table VIII. —	+	8·1
Correction for refraction and parallax	2	24·5

Observed double altitude	42	37	15
Index error	—	1	30
	2) 42	35	45
Apparent altitude $\bar{\odot}$	21	17	52·5
Semidiameter	—	16	5·5
Apparent altitude $\ominus$	21	1	47
Correction for refraction and parallax . .	—	2	24·5
Altitude of sun's centre	20	59	22·5

EXAMPLE II.

On April 6, 1838, at 9 P.M., Greenwich time, in latitude 51° 30′, the double altitude of *the moon's lower limb* was observed 97° 21′ 50″. Index error of sextant, 50″. Thermometer 54°. Barometer, 30·1 inc. Required the correct altitude.

Semidiameter.

	′	″
Horizontal, 9 P.M.	14	42·8
Augmentation for 48° 40′·5	+	10·9
	14	53·7

Refraction.

48°	0	52·3
55′·4	—	1·7
Thermometer		
	50	·2
Barometer	+	·2
		50·4

Parallax.

Horizontal equatorial, 9 P.M		53	59·7
Corr. for Latitude, 51° 30′		—	6·4
Reduced horizontal parallax		53	53·3
Sin, 53′ 53″·3		=8·1952030	
Cos, 48° 54′·33		=9·8177337	
Parallax in altitude*	35′ 25″	8·0129367	

	°	′	″
Observed double altitude	97	21	50
Index error		—	50
	2)97	21	0
	48	40	30
Semidiameter	+	14	53·7
	48	55	23·7
Refraction		—	50·4
	48	54	33·3
Parallax	+	35	25
Corrected altitude required	49	29	58·3

* This might have been obtained at once by inspection, by using the tables of Parallax.

In these examples no allowance has been made for the *dip of the horizon*, as the observations were made with an artificial horizon: with the fixed stars no correction is required for semidiameter or parallax.

PROBLEM III.

TO DETERMINE THE LATITUDE.

Method 1st.—By observations of a circumpolar star at the time of its upper and lower culminations.

This method is independent of the declination of the star observed: the altitudes are observed with any instrument fixed in the plane of the meridian, or (not so accurately of course) with a sextant or other reflecting instrument at the moments of both the upper and lower transits of the star; or a number of altitudes may be taken immediately before and after its culminations, and *reduced to the meridian*, as will be explained. In either case, let Z denote the observed or reduced meridional zenith distance of the star at its lower culmination, and r its refraction at that point; also let Z′ and r' denote the zenith distance and refraction at its upper culmination. Then the correct zenith distance of the pole, or the *co-latitude of the place*, will be $= \frac{1}{2}(Z + Z') + \frac{1}{2}(r + r')$.

According to Baily, a difference of about half a second may result from using different tables of refraction.

Method 2nd.—By meridional altitude of the sun, of a star whose declination is known, involving, when several observations are made on each side of the meridian, a reduction to the meridian.

The altitude of the sun or star being determined at the moment of its superior transit, as before explained, and corrected for refraction, and also for parallax and semidiameter when necessary, the latitude required will be—

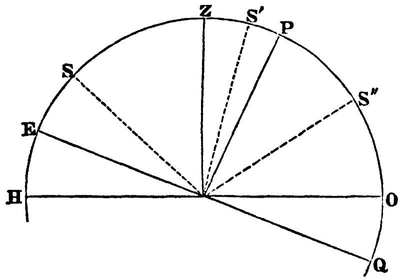
$Z + D$, if the observation is to the south of the zenith.

$D - Z$, if to the north above the pole.

$180 - (Z + D)$ to the north below the pole.

Z being put to denote the meridional zenith distance, and D the declination (— *when south*).

This is evident from the figure below, E S, E S′, and Q S″ being the respective declinations of the objects S, S′, and S″; and P O or Z E the latitude of the place of observation, which is equal to (Z S + E S) in the case of the star being to the south of the zenith Z; or E S′ − Z S′, when to the north *above* the pole P; and to 180 − (Q S″ + Z S″) when to the north *below* the pole.



Perhaps the rule given by Professor Young for the two first cases is more simply expressed thus:—Call the zenith distance north or south, according as the zenith is north or south of the object. If it is of the same name with the declination, their *sum* will be the latitude; if of different names, their *defference;* the latitude being of the same name as the greater

EXAMPLE I.

On April 25, 1838, longitude 2ᵐ 30ˢ east, the meridional double altitude of the sun's upper limb was observed with a sextant 104° 3′ 57″; index error 1′ 52″; thermometer 56°; barometer 29·04. Required the latitude of the place of observation.

Refraction and Parallax.

		″
51°		47·10
45′	−	1·23
		45·87
Barometer	−	1·52
		44·35
Thermometer	−	0·56
Corrected refraction	−	43·79
Parallax	+	5·29
	−	38·5

Declination.

	°	′	″
Apparent noon at Greenwich	13	8	9·30
Change for 2^m 30^s longitude . . . . —	0	0	2·04
	13	8	7·26

	°	′	″
Observed double altitude	104	3	57
Index error . . . . —	0	1	52
	2)104	2	5
	52	1	2·5
Semidiameter	0	15	54·4
Apparent altitude	51	45	8·1
Correction for refraction and parallax . .	0	0	38·5
True altitude	51	44	29·6
	90	0	0
Zenith distance	38	15	30·4
Declination North	13	8	7·3
Latitude North	51	23	37·7

EXAMPLE II.

On March 31, 1838, at 5^h 12^m 57^s by chronometer, the meridian altitude of the moon's upper limb was observed 67° 1′ 5″; the index error of instrument being — 1′ 0″; barometer 30·1 inc.; thermometer 51°; the approximate north latitude was estimated 52°, and longitude 2^m 21′ 5″ E. Required the latitude.*

* The number of corrections required, and the necessary dependence upon Lunar tables, render an altitude of the moon less calculated for determining the latitude than one either of the sun or a star.

Semidiameter.

		′	″	′	″
Horizontal, noon		15	26·2		
Do. midnight		15	19·3		
Difference in 12 hours		0	6·9		
Proportion for 5h 12m 57s, and 2m 21·5s longe		0	3·0		
Horizontal semidiameter at time of observation				15	23·2
Augmentation for altitude	+			0	14·3
Corrected semidiameter				15	37·5

Refraction.

		″
66° =		25·9
44′·5 =		0·9
		25·0
Barometer	+	·09
Thermometer	—	·06
Refraction		25·03

Parallax.

	′	″
Horizontal equatorial parallax, noon	56	38·9
midnight	56	13·6
Difference in 12 hours	0	25·3
Corrected for 5h 12m 57s, and 2m 21·5s E. longitude	56	27·9
Correction for latitude (see 4th Lunar Table, Pearson)	0	7·1
Horizontal parallax	56	20·8

Log. sin semidiameter	56′ 20″·8	8·2146006
Log. cos altitude	66° 44′ 2″·5	9·5966081
Parallax in altitude	22′ 15″·5	7·8112087

	′	″
Or by the use of Table VIII., Dr. Pearson	22	15·1

Declination.

			h.	m.	s.
Time of observation			5	12	57
Longitude (in time)			0	2	21·5
Corresponding Greenwich mean time			5	10	35·5

		°	′	″
At 5h	N	28	29	37·7
10m·5	+	0	0	16·97
		28	29	54·67

		°	′	″
Apparent altitude ☽		67	1	5
Index error	—	0	1	0
		67	0	5·0
Semidiameter	—	0	15	37·5
Apparent altitude		66	44	27·5
Refraction	—	0	0	25·0
		66	44	2·5
Parallax	+	0	22	15·5
Corrected altitude		67	6	18
		90	0	0
Zenith distance		22	53	42
Declination	+	28	29	54·67
Latitude required		51	23	36·67

An observer not furnished with a mural circle, or other instrument fixed in the plane of the meridian with which to measure meridional altitudes, can obtain his latitude more correctly than by observing a single approximate meridional altitude with a sextant or other reflecting instrument, by taking a number of altitudes of the sun or a star near to, or on each side of the meridian, and from thence determining the correct altitude of the object at the time of its culmination.

This method, termed that of "circum-meridian altitudes," to the mean of which altitudes is to be applied a correction for its "*reduction to the meridian,*" is susceptible of great accuracy; and the repeating circle, already described, is peculiarly adapted for these observations, on account of the rapidity with which they can be taken. The distance of the sun or star from the meridian (in time) is noted at the moment of each observation, by a chronometer when the former is the object, and by a sidereal clock (if there is one) when the latter, to save the conversion of one denomination of time into the other. The formula given by Mr.

Baily, freed from the second part of the equation which it is seldom necessary to notice, is—

$$x = A \times \frac{\cos L. \cos D}{\sin Z} \text{ where}$$

x represents the required correction in *seconds*.

L, the latitude (known approximately).

D, the declination (minus when south).

Z, the meridional zenith distance, also known approximately from the above.

A, a quantity depending on the horizontal angle of the object, and given in Table XIII. under the head of "Reduction to the meridian," being $= \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$ where P = the horary angle at the pole as shown by a well-regulated clock ; which angle will change its sign after the meridional passage of the star.

Among the instructions drawn up by Mr. Airy for the guidance of the officers employed upon the survey of the North American Boundary, this method of determining the latitude with the altitude and azimuth instrument is recommended, and was constantly practised with stars near the meridian. The axis of the instrument is to be adjusted nearly vertical, and the cross axis nearly horizontal (great accuracy is not required), the telescope made to bisect the star upon its middle horizontal wire, and the time noted. Then read the large divisions with the pointer, and the two microscopes A and B ; read also the level *right hand* and *left hand*.

Turn the instrument 180° in azimuth, and repeat these observations—revert to the first position, and continue this process as often as may be thought necessary—note the barometer and thermometer—then add together

Reading of A.

Reading of B.

And equivalent for left-hand level.

Subtract equivalent for right-hand level.

Divide the remainder by 2, and apply the pointer reading of A for the uncorrected circle reading for the first observation.

The same process is repeated for the second and all the other observations.

For each observation correct the chronometer time for rate and error, and convert this into (if not already showing) *sidereal time;* take the difference between the sidereal time and the star's right ascension for the *star's hour angle,* which reduce to *seconds* of time and call *p*.

Then compute for each observation the number

$$\left(\frac{225}{2} \sin 1''\right) \times \frac{\text{cos Lat.} \times \text{cos Star's Declination}}{\text{sin Star's Zenith Distance}} \times p^2,$$

which is the correction in seconds of arc to the observed zenith distance to bring it to the true *meridian* zenith distance, and is always subtractive, except the star is below the pole. In applying this correction however to the circle readings, it will be additive, or subtractive, according as by the construction of the circle, increasing readings represent increasing or decreasing zenith distances.

Half the difference of two corrected readings in opposite positions of the instrument is the star's apparent zenith distance on the meridian; or the mean of all the observations in one position may be compared with the mean of all those in the other, and half their sum is the zenith point.

To this zenith distance add the correction for refraction, taking into consideration the readings of the thermometer and barometer, and apply the star's declination for the day (from the Nautical Almanac) for the latitude.

The above instructions* apply only to stars observed *near* the meridian. The latitude can, however, be obtained by similar observations of stars situated very far from the meridian, though this method would very seldom be resorted to.

When the sun is the object observed, a further correction must be made on account of the change in declination during the time occupied by the observation, which is expressed by

$$- \text{S} \times \frac{\text{E} - \text{W}}{n}$$

* See "Corps Papers," vol. iii. page 328, where will also be found examples worked out in detail, of latitudes thus obtained on the survey of the North American Boundary.

S being the change of declination in one minute of time, *minus* when decreasing.

E the sum of the horary angles observed to the east, expressed *in minutes* of time, and considered *as integers.*

W their sum to the west, and

n the number of these observations.

When a star is the object observed, and the time is noted by a chronometer, regulated to *mean time,* the value of A must be multiplied by 1·0054762. Also, if the clock does not keep its rate either of sidereal or mean time accurately, a further correction is imperative; and A must be multiplied by $1 + \cdot0002315\, r$, where r denotes the daily rate of the clock in seconds, *minus* when *gaining,* and *plus* when *losing.*

EXAMPLE.

On March 8, 1837, the following observations were taken, with a sextant, the chronometer being fast $9^{m}\ 16^{s}$; index error of sextant, $-\ 1'\ 20''$; barometer, 29·54; thermometer, 50°.

		H.	M.	S.	°	′	″
1	$\overline{\odot}$	12	9	48	68	3	0
2	$\underline{\odot}$	0	10	53	66	51	20
3	$\overline{\odot}$	0	12	9	68	5	0
4	$\odot$	0	13	15	67	0	25
5	$\overline{\odot}$	0	14	46	68	6	10
6	$\underline{\odot}$	0	15	54	67	1	50
7	$\overline{\odot}$	0	19	32	68	7	30
8	$\underline{\odot}$	0	21	3	67	2	10
9	$\overline{\odot}$	0	22	25	68	7	20
10	$\underline{\odot}$	0	23	55	67	1	5
11	$\overline{\odot}$	0	24	53	68	7	10
12	$\underline{\odot}$	0	26	54	67	0	40
13	$\overline{\odot}$	0	27	57	68	5	40
14	$\underline{\odot}$	0	29	32	66	58	0
				Sum of altitudes	945	37	20

	° ′ ″
	14) 945 37 20
	67 32 40
Index error	− 1 20
	2) 67 31 20
Mean apparent altitude	33 45 40
*Refraction and parallax	− 1 18·5
True mean altitude	33 44 21·5
	90 0 0
Zenith distance	56 15 38·5
Apparent noon	12 0 0
Equation of time	+ 0 11 1·32
Mean time at apparent noon	12 11 1·32
Error of chronometer	+ 0 9 16
Time shown by chronometer at apparent noon	12 20 17·32

Observ.	Distance from noon. M. S.	Value of A, Table XIII.
1	10 29·3 E.	216·1
2	9 24·3	173·8
3	8 8·3	130·2
4	7 2·3	97·3
5	5 31·3	59·9
6	4 23·3	37·9
7	0 45·3	1·2
8	0 45·7 W.	1·2
9	2 37·7	13·5
10	3 37·7	25·9
11	4 35·7	41·3
12	6 6·7	73·4
13	7 39·7	115·2
14	9 14·7	167·8
		7) 1154·7
		2) 164·95
Mean value of A =		82·5

* The process by which this and other corrections are obtained is omitted, having been fully explained by the preceding examples.

Approximate zenith distance	56° 15″
Declination	− 4 50
Approximate latitude	51 25

Cos L = ...	9·7949425
Cos D = ...	9·9984465
Ar. comp. sin. Z = ...	0·0800783
Log A, 82·5 = ...	1·9164539
(x) − 61″·6 = ...	1·7899212

	m
Angles East = ...	45·7
Angles West = ...	34·6
14)	11·1
	·8

S = − ·97
·8

− ·776 Correction for change of sun's declination.

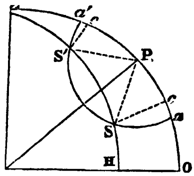
	° ′ ″
Mean observed zenith distance	56 15 38·5
Correction x	— 1 1·6
Ditto for declination	— 0 0·7
Correct meridian zenith distance . .	56 14 36·2
Declination south	− 4 50 34·9
Latitude	51 24 1·3

*Method 3rd.—By the altitude of the pole star, at any time of the day.**

If the altitude of the pole star can be taken when *on* the meridian, its polar distance either added to, or subtracted from, the altitude, gives at once the latitude; and when observed *out of*

* This of course is only applicable to northern latitudes. In the southern hemisphere there is no star sufficiently near to the south pole to be made available in thus determining the latitude.

the meridian, as at the point S or S′ in the figure, the latitude can be easily obtained, as follows:—



Let Z P O represent the meridian, Z the zenith, P the pole, and a S a' the circle described by the polar star S, at its polar distance P S. The star's horary angle Z P S, or Z P S′, is evidently the difference between its right ascension and the sidereal time of observation; and in the spherical triangle Z P S (or Z P S′) we have Z S, P S, and the angle Z P S; to find Z P, the co-latitude. The result may be obtained with almost equal accuracy by considering P S c as a plain right-angled triangle, of which P c is the cosine of the angle c P S to radius P S, the distance P c thus found is to be added to, or subtracted from, the altitude H S, according as the star is above or below the pole, which is thus ascertained:—If the angle Z P S′ be less than 6, or more than 18 hours, the star is above the pole, as at S′; if between 6 and 18 hours, it is below the pole, as at S.

By the tables given in the Nautical Almanac, the solution is even more easy, and has the advantage of not requiring any other reference. The rule is as follows:

1st. From the corrected altitude subtract 1′.

2nd. Reduce the mean time of observation at the place to the corresponding sidereal time.

3rd. With this sidereal time take out the *first correction* from Table I., with its *proper sign*, to be applied to the altitude for an *approximate latitude.*

4th. With this approximate latitude and sidereal time take out from Table II. the *second correction;* and with the day of the month and the same sidereal time take from Table III. the *third correction.* These are to be *always added* to the approximate latitude for the latitude of the place.

EXAMPLE.

On Oct. 26, 1838, the double altitude of Polaris, observed with a repeating circle, at $11^h\ 55^m\ 30^s$ mean time, was 105° 44′ 63″, the barometer standing at 29·8; thermometer, 50°. Required the latitude of the place of observation.

By the method given in the Nautical Almanac,—

		h m s
Mean time		11 55 30
Corresponding sidereal time		2 15 6·78
		° ′ ″
Observed altitude	2)	105 44 53
		52 52 26·5
Refraction	−	0 0 44
Corrected altitude		52 51 42·5
Subtract		0 1 0
		52 50 42·5
Correction 1st for sidereal time		1 28 21·7
		51 22 20·8
Correction 2nd	+	0 9·6
		51 22 30·4
Correction 3rd	+	0 1 10·5
Latitude required		51 23 40·9

The same example by spherical trigonometry:—

	° ′ ″	
Corrected altitude	52 51 42·5	
Zenith distance	37 8 17·5	ZS
Declination	88 27 7·6	
N. P. distance	1 32 52·4	PS
	h m s	
Sidereal time	2 15 6·78	
R. A. Polaris	1 2 12·94	
Hour angle past meridian	1 12 53·84	
Equal in space to	18° 13′ 27″	

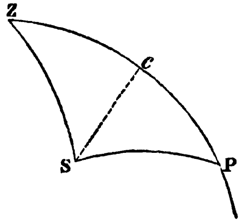
Then in the triangle Z P S, we have—

	°	′	″
Z S=	37	8	17·5
P S=	1	32	52·4
Angle P=	18	13	27

To find Z P, the co-latitude.

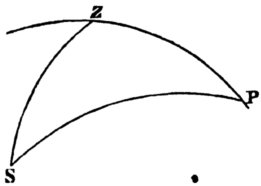
The solution of which triangle gives—

	°	′	″
Z P=	38	36	21
And latitude	51	23	39



By considering S P c as a plane right-angled triangle, in which P c, the correction to be subtracted, is the cosine of P to radius P S, the latitude is found by plane trigonometry within a few seconds of the above results.

Method 4th.—By an altitude of the sun, or of a star, out of the meridian, the correct time of observation being known.



By reference to the figure, it will be seen that this method simply involves the solution of the spherical triangle Z P S already alluded to, formed by the zenith, the pole, and the object at the time of observation; of which Z S, the zenith distance, P S, the polar distance, and the angle at P are known, and Z P, the co-latitude, is the quantity sought.

The formula given by Baily, for finding the third side, when the other two sides and an angle opposite to one of them are given, is

$$\tan a' = \cos \text{given angle} \times \tan \text{adjacent side}$$

$$\cos a'' = \frac{\cos a' \times \cos \text{side opp. given angle}}{\cos \text{side adjacent given angle}}$$

and $x = (a' \pm a'')$, which formula is used in the following examples :—

EXAMPLE I.

On May 4, 1838, the observed altitude of the sun's upper limb at $5^h\ 47^m\ 15^s$ by chronometer was $14°\ 44'\ 58''$. The index error of the sextant being 28″, and the watch 3^m $34^s.4$ too fast. Barometer 29·9; thermometer 61; required the latitude.

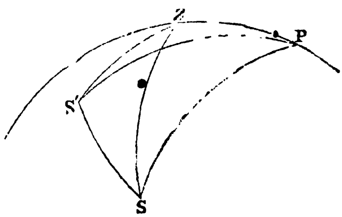
		° ′ ″
Apparent altitude ☉̲ (lower limb)		14 44 58
Index error	−	0 0 28
		14 44 30
Semidiameter	−	0 15 52·2
Apparent altitude ⊕		14 28 37·8
Refraction and parallax	−	0 3 28·1
Altitude		14 25 9·7
		90 0 0
Zenith distance (Z S)		75 34 50·3
Declination		15 59 14
		90 0 0
North Polar distance (P S)		74 0 46
		h m s
Mean time of observation		5 43 40·6
Equation of time	+	3 24·46
Apparent time		5 47 5·06
In space	5^h =	75 0 0
	47^m =	11 45 0
	5.06^s =	0 1 15·9
Angle P		86 46 15·9

Cos P	86 46 15·9	8·7506671
Tan P S	74 0 46	0·5428692
Tan a'	11 7 17	9·2935363
Cos a'	11 7 17	9·9917668
Cos Z S	75 34 50·3	9·3962296
Ar. comp. P.	74 0 46	0·5599998
Cos a'' =	27 28 58·6	9·9479962
a'	11 7 17	
a'' +	27 28 58·6	
P Z =	38 36 15·6	
	90 0 0	
Latitude required	51 23 44·4	

When the sun is the object observed, the hour angle P (as in the last example) is the *apparent time* from *apparent noon* at the place of observation, converted into space; but with a star, it is its distance from the meridian, either to the east or west according as it has or has not come to its culmination; and this angle is simply the sum or difference of the *star's right ascension*, and the time of the observation converted into *sidereal time;* to be multiplied by 15 for its conversion into space.

Method 5th.—By two observed altitudes of the sun, or a star, and the interval of time between the observations.

This problem is of importance, as its solution, though long, does not involve a knowledge of the correct time at the place of observation; and the short interval of time can always be measured with sufficient accuracy by any tolerable watch. Various methods have been devised to shorten the calculation of "double altitudes" by tables formed for the purpose, one of which may be found at p. 231 of Riddle's "Navigation;" but the direct method by spherical trigonometry is most readily understood and easily followed.



Let S and S′ represent the places of the object at the times of the two several observations, (they may be on different sides of the meridian, or, as in the figure, both on the same side); Z S and Z S′ then are their respective zenith distances, and P S and P S′ their polar distances; S P S′ being the hour angle observed.

First—in the triangle P S S′, the two sides P S and P S′ are given with the included angle at P to find S S′ and the angle P S S′. Again, in the triangle Z S S′, we have the three sides to find the angle Z S S′, which taken from P S S′ just found, leaves the remaining angle P S Z. Lastly—in the triangle P S Z we have P S, Z S, and the angle P S Z, to find P Z, the co-latitude sought. In a similar manner the latitude may be found by *simultaneous* *altitudes of different stars*, the difference of their right ascensions giving the angle S P S, without the use of a watch. Tables have been calculated by Dr. Brinkley, from which the distance S S′ can be obtained by inspection (allowing for the change in the right ascension of the stars after any long interval), and the calculation is thus considerably abridged. By an azimuth and altitude instrument, the latitude may also be found by the two altitudes, and the *difference or sum of the observed azimuths* of the sun or star.

Equal altitudes of the same star on different sides of the meridian, with the interval of sidereal time between the observations, also furnish the means of ascertaining the latitude, and this method is most useful in a perfectly unknown country. The hour angle, east or west, will evidently be measured by *half the elapsed interval of time;* and in the triangle Z P S, we have this hour angle Z P S, the polar distance P S, and the co-altitude Z S, to find Z P the co-latitude; moreover the hour angle being known, and also the right ascension of the star, the point of the equinoctial which is on the meridian, and consequently the *local sidereal* time is determined, from which the *mean* time can be deduced.

The latitude may likewise be ascertained, independently of the graduation of the instrument, by placing the *axis* of the telescope of an altitude and azimuth circle* due north and south, so that the vertical circle shall stand east and west. The observations of the two moments T and T′ (in sidereal time), in which the star passes the wire of the telescope, will give the latitude from the following formula.

$$\text{Cot L} = \text{cot declination} \times \cos \tfrac{1}{2}\,(\text{T} - \text{T}')$$

If a chronometer set to *mean* time is used, the interval (T − T′) must be multiplied by 1·0027379, or the value corresponding to

* A portable transit placed in the plane of the prime vertical, instead of that of the meridian, of course affords the same facility for thus determining the latitude. The stars selected should have their declinations less than the latitude of the place, but by as small a quantity as possible.

the interval, found in the table for converting mean into sidereal time, must be added.*

The accuracy of this method depends upon the correctness of the *tabulated declination of the star*, but a slight error in this will not affect the *difference of latitude* between two places, thus found. By observing on following days with the axis *reversed*, and taking the mean of the observations, any error in the instrument is corrected; this method is particularly recommended by Mr. Baily for adoption in geodesical operations, as the *difference of latitude* of two stations is obtained almost independently of the declination of the star, and the only material precaution to be taken is in *levelling* the axis of the telescope, which should be one of very good quality.

PROBLEM IV.

TO FIND THE TIME.

Method 1st.—From single, or absolute, altitudes of the sun, or a star whose declination is known, as also the latitude of the place.

This problem is solved by finding the value of the horary angle P, in the same "astronomical triangle" Z P S, whose elements have already been described. In this case, the three sides, viz. the co-latitude, the zenith, and polar distances, are given to find the hour angle P, which, when the sun is the object observed, will (as was explained in page 215) be the apparent time from apparent noon at the place of observation; and it is converted into mean time by applying to it the *equation* of time with its proper sign. In *the case of a star*, it will denote its distance in time from the meridian, which being *added* to its right ascension, if the observation be made to the westward of the meridian, or *subtracted* from the right ascension (increased by 24 hours if necessary) if to the eastward, will give the *sidereal* time, to be converted into mean solar time if required.

* Table VII. Baily's Astronomical Tables and Formulæ.

A simple formula for finding the angle of a spherical triangle whose three sides is given is $\sin^2 \frac{1}{2} P = \frac{\sin (\frac{1}{2} S - c) (\sin \frac{1}{2} S - b)}{\sin. c. \sin b}$ where S denotes the sum of the three sides *a*, *b*, and *c*; of which *a* is assumed as the one *opposite the required angle.* In the present case *a* represents the co-altitude or zenith distance; *b* the co-declination, or polar distance; and *c* the co-latitude.

EXAMPLE.

Observed altitude of the upper limb of the sun on May 4, 1838, was 14° 44′ 58″ at 5^h 47^m 15^s by chronometer; latitude 51° 23 40″; longitude 2^m $21{\cdot}5^s$ east; index error of sextant 28″.

Thermometer . .	61°	Required the error of the watch.
Barometer . .	29·9	

Observed altitude $\bar{\odot}$ . .		14° 44′ 58″	
Index error	—	0 0 28	
		14 44 30	
Semidiameter at 6· 75^h . .		0 15 52·2	
Apparent altitude $\ominus$ . .		14 28 37·8	
Correct. refractn. and parallax	—	0 3 28·1	
True altitude		14 25 9·7	
		90 0 0	
Zenith distance (Z S) . .		75 34 50·3	
Latitude		51°23′ 40″	
		90 0 0	
Co-latitude (P Z) . . .		38 36 20	
Declination			15 59 14·2
			90 0 0
N P. Distance (P S)			74 0 45·8

(*a*) ZS	=	75°34′50″·3		
(*b*) PS	=	74 0 45 ·8		
(*c*) PZ	=	38 36 20		
S	=	188 11 56 ·1		
½S	=	94 5 58		
c	=	38 36 20	sin. ar. comp.	0·2048465
½S−*c*	=	55 29 38	sine	9·9159620
b	=	74 0 45 ·8	sin. ar comp.	0·0171307
½S−*b*	=	20 5 12 ·2	sine	9·5358540
		sin² ½P	. . 2)	19·6737932
		½P. 43°23′8″·5		9·8368966
		2		
*Hour angle P	=	86 46 16 ·2		

Equivalent in apparent time to	5 47 5·06
Equation of time at time of observation . .	0 3 24·46
Mean time	5 43 40·60
Time by chronometer	5 47 15
Chronometer fast	0 3 34·40

Method 2nd.—From equal altitude of a star or the sun, and the interval of time between the observations.

If a star is the object observed, it is evident that half the interval of time elapsed between its returning to any observed altitude after its culmination, will give the moment of its passing the meridian without any correction, from whence the error of the clock or chronometer is at once found. But with regard to the sun, there is a correction to be applied to this half interval,

* The most favourable time for observing single, (or absolute,) altitudes of the sun or stars to obtain the local time, is when they are on or near the *prime vertical*, since their motion in altitude is then most rapid, and a slight error in the assumed latitude is not of so much consequence. The corrections for the refraction, however, are then often considerable. The same observation will of course give the azimuth Z, and also the variation of the needle, if the magnetic bearing of the star or of either limb of the sun, is taken by another observer at the same moment as the altitude. This will be further explained.

on account of his constant change of declination. From midwinter to midsummer the sun gradually approaches the North Pole, and therefore a longer period will intervene *after, than before noon,*—between the sun's descent to the same altitude in the evening as at the morning observation: and the reverse takes place from midsummer to midwinter. The amount of this correction depends partly upon the change of declination (proportioned to the interval of time on the day of observation); and partly upon the latitude of the place.—The difference of the sun's horary angles at the morning and afternoon observations is easily calculated by the following formula of Mr. Beily's:—

$x = \mp A\,\delta \tan L + B\,\delta \tan D$, where

T = the interval of time expressed in *hours;*

L, the latitude of the place, *minus when south;*

D, the declination at noon, also *minus when south;*

δ the *double* daily variation in declination *in seconds*, deduced from the noon of the preceding day to that of the following, *minus* when the sun is proceeding to the south; and

x = the required correction in *seconds*, A* being *minus* when the time of noon is required.

The result is of course *apparent* noon, to which must be applied the *equation of time*, in order to compare a chronometer with *mean* noon.

In an observatory, or wherever a transit or other instrument is fixed in the plane of the meridian, the easiest and most accurate mode of obtaining the true local time is by observing the transit of the sun or a star over the vertical wires of the telescope. With the sun, the mean of the times of both leading and following limbs gives the transit of the sun's centre, which is apparent noon, to which the equation of time + or − has to be applied to obtain the mean time.

When the transit of a star is obtained, the sidereal time at preceding mean noon taken from the Nautical Almanac, corrected for longitude (see page 194) is to be subtracted from the star's R A (to which 24 hours is to be added, should this latter be the smaller

* The logs of A and B will be found in table 14.

quantity), and the difference is the interval of sidereal time after mean noon, which interval corrected into its equivalent of mean time gives the true local time. For an example of the method of recording these transits, see the form pages 248 and 251.

The same result can be *approximately* obtained with a sextant by noting the moment when the upper or lower limb of the sun, or a star, appears to cease rising.—The observation must be commenced before the time of the object's culmination, and the reflection of the images kept in contact by the gradual forward motion of the tangent screw until the images tend to overlook instead of receding.

If the *rate* only of a chronometer is required, it can be obtained by observing the transits of a star on successive days, or by equal altitudes of the same star, on the same side of the meridian, on different evenings; as a star attains the same altitude after each interval of a sidereal day, which is $3^m\ 56{\cdot}91^s$ less than a mean solar day; but if the refraction is not alike on the days of observation, a correction will be required.

By reading the *azimuths*, when the sun or a star has equal altitudes we obtain the true meridian line, which will be again alluded to. Very frequently the afternoon altitude cannot be observed on account of intervening clouds, but the time can still be calculated from the observed single altitude, as in the last problem.

PROBLEM V.

TO DETERMINE THE LONGITUDE.

The usual method of finding the longitude at sea is by comparing the local time, found by observation, with that shown by a chronometer whose error and rate for Greenwich mean time are known. The accuracy of the result depends of course upon the chronometer maintaining a strictly equal rate under all circumstances, which cannot always be relied upon,* and various methods

* It is usual to have several chronometers on board, and to take the mean of those most to be depended upon. If one varies considerably from the others it is rejected.

have been resorted to, to render the solution of this most important problem independent of such uncertain data, or at all events to afford frequent and certain checks upon its correctness. Any celestial phenomenon which should be visible at the same predicted instant of time in different parts of the globe would of course furnish the necessary standard of comparison, and all the methods in use for determining the longitude are based upon this foundation; but they are not generally practicable at sea, with the exception of that derived from the observed angular distances between the moon and the sun, or certain stars, which are calculated for every three hours of Greenwich time, and which *lunar distance* is measured with a sextant, or other reflecting instrument.—*Artificial signals* have been resorted to as a means of ascertaining the difference of longitude, with considerable success, between places not separated from each other by any very considerable distance.

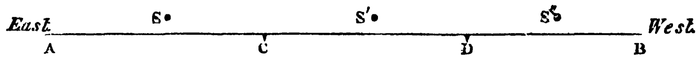
In the Philosophical Transactions for 1826 is an account drawn up by Sir J. Herschel of a series of observations made in the summer of 1825, for the purpose of connecting the royal observatories of Greenwich and Paris, undertaken by the Board of Longitude, in conjunction with the French Minister of War. The signals were made by the explosion of small portions of gunpowder* fired at a great elevation by means of rockets, from three stations, two on the French, and one on the English side of the Channel; and were observed at Greenwich and Paris, as well as at two intermediate places, Legnieres, and Fairlight-Downs near Hastings. The difference of longitude thus obtained, 9′ 21·6″, is supposed by Sir J. Herschel to be correct within one tenth of a second, and the observations were taken with such care that those of the French and English observers at the intermediate stations only differed one-hundreth part of a second.

At page 198 also, of Francœur's "Géodésie," is a description of similar operations for the purpose of ascertaining the difference of longitude between Paris and Strasbourg. In operations of this

* Flashes of gunpowder upon a metal plate are visible at night for a very considerable distance, upwards of 40 miles,—this method is far superior to firing rockets,—the quantity may be from 4 to 16 drachms or more for moderate distances, and a quarter of a pound for long ones.

nature, it is only necessary that the *rates* of the chronometers used should be uniform for the short period of time occupied by the transmission of the signals.

Suppose A and B are two places, whose difference of longitude is required, and that they are too far distant to allow of one signal being seen from each—



C and D are taken as intermediate stations, and the first signal made at S, is observed from A and C, and the times noted; the second signal at S′, is observed from C and D, some fixed number of minutes after; and then that at S″ from D and B. Suppose these two intervals to have been five minutes each, then the difference of longitude is equal to the difference between the local time at A + ten minutes, and that observed at B at the moment of the last signal.

Everything in this operation depends upon the correct observation of the times, which should be kept in sidereal intervals, or reduced to such if observed with a chronometer regulated to mean time. When, instead of the two or three chronometers generally taken on board every ship, a *number* of these instruments, whose rates and errors have been previously carefully ascertained, are conveyed from one meridian to another, the comparison of the mean of the time shown by the chronometers, with the local time at each place, affords the means of determining with considerable accuracy the difference of their longitudes; this mode is much practised at present on board surveying vessels,* for measuring the respective meridian distances between a number of maritime towns, ports, and other places on the sea-coast of distant countries. On shore the difference of longitude between two stations can also be determined with precision by the transmission of pocket chronometers between them; provided the errors of the box chronometers or clocks at these stations on sidereal time, and

* On board H. M. S. Beagle, employed as a surveying vessel principally on the coasts of Australia and Van Diemen's Land, there were at one time as many as *twenty-one* first-rate chronometers.

their rates, have been carefully ascertained by transit observations. Where the distance is not very considerable, the operation consists simply of comparing several pocket chronometers with the standard instrument at one of the stations, and then sending them * with the greatest care to be compared with the clock or chronometer at the other station, to be returned immediately for another comparison at the starting point; which process of transmission should be repeated several times.

When the time occupied by this operation is considerable, more than four or five days for instance, the accuracy of the result will be increased by stationing a careful assistant at a post midway between the two extreme stations with a box chronometer with which the transmitted pocket chronometers are to be compared. Mr. Airy recommends commencing from this central position, sending the pocket chronometers (divided into two batches) simultaneously for comparison to the two principal extreme stations, and comparing them again on their return, at nearly the same time, at the intermediate point; by which modification, the time through which reliance is placed upon the pocket chronometers is diminished one-half, and very little dependence is made to rest upon the steadiness of the performance of the box chronometer at the central place of observation.

This method of obtaining the difference of longitudes of two distant places would, it is imagined, seldom be resorted to where the distance was *very great*, and where an intermediate station was found necessary. On the North American Boundary Survey the second method was never tried, but the first and more simple process of direct transmission and comparison between the two stations was constantly practised with great success. One example has been selected from Major Robinson's report, calculated according to the directions drawn up by Mr. Airy, each of the three comparisons recorded being the mean of *six* observations.

* This should be done directly after the error of the standard chronometer has been tested by observations with the transit instrument.

Calculation for Difference of Longitude between St. Helen's Island, Montreal, and St. Regis.

First Comparison.

	H.	M.	S.
Standard Chronometer (943) .	2	42	43·18
Pocket Chronometer (2187) .	20	44	0
Difference . . .	5	58	43·18
Difference on Return . .	6	6	19·76
Do. at first Comparison . .	5	58	43·18
	0	7	36·58

Second Intermediate Comparison.

	H.	M.	S.
Standard Chronometer (341) .	5	43	56·82
Pocket Chronometer . .	23	43	0
Difference . . .	6	0	56·82

	D.	H.	M.
Date of Return . . .	24	20	50
Date of first Comparison . .	22	20	44
	2	0	6

Third Comparison (on return).

	H.	M.	S.
Standard Chronometer (943) .	2	56	19·76
Pocket Chronometer . .	20	50	0
Difference . . .	6	6	19·76

	D.	H.	M.
Intermed. date of Comparison .	23	23	43
Date of first Comparison . .	22	20	44
Intermediate interval .	1	2	54

H.	M.		M.	S.		H.	M.		M.	S.
48	6	:	7	36·58	::	26	59	:	4	16·12
60			60			60				
2886			456·58			1619				

	H.	M.	S.
Difference at first Comparison	5	58	43·18
Add proportional part of intermediate interval .	0	4	16·12
	6	2	59·30

	H.	M.	S.
No. 943 faster than 2187	6	2	59·30
No. 341 ,, 2187	6	0	56·82
No. 943 ,, 341	0	2	2·48

	H.	M.	S.
Reading of 943	2	42	43·18
,, 943 slow	0	1	24·85 +
Rate, losing 2s·17 per diem for 32 hours	0	0	2·89 +
True Sidereal Time by 943	2	44	10·92

	H.	M.	S.
Corresponded to Reading of 341	2	40	40·70
,, 341 fast	0	0	56·33 —
Rate, gaining 1s·19 per diem for 59 hours .	0	0	2·92 —
True Sidereal Time by 2187	2	39	41·45

	H.	M.	S.
Sidereal Time by 943	2	44	10·92
,, ,, 2187	2	39	41·45
Difference of Longitude . . .	0	4	29·47 St. Regis, west of St. Helen's Station.

In comparing chronometers, two persons are generally employed, one of whom watches the seconds hand of one instrument until it arrives at some convenient division, such as the commencement of a minute, or one of the ten seconds, when he gives the signal to "*stop*" to the other, whose attention has been meanwhile fixed upon the seconds hand of the other chronometer. Where one person alone makes the comparison, his only plan is to register the seconds, and then the minutes and hour of one instrument, commencing to count the beats 1, 2, 3, &c., from the moment selected by him (whilst he is writing down the time observed), and then to transfer his eye to the other chronometer, continuing to count the beats until he observes its second hand opposite some marked number of seconds, when he stops; writing down first the number of beats counted, and then the seconds, minutes, and hour of the second chronometer; the number of beats is of course to be subtracted from this for the comparison of the time shown by the first instrument.

When a chronometer adjusted to mean solar time is to be compared with one going sidereal time, or with a sidereal clock, the only correct method with one observer is by the coincidence of their beats, in the manner described by Mr. Airy.

When the chronometer going mean solar time has a half-second beat, and the other instrument or the clock a second's beat, they will appear at the end of every second to beat (after some little time) almost simultaneously. Select one that appears perfectly coincident, and commence counting the beats 1, 2, 3, &c., of the clock or sidereal chronometer, writing down at the same time the second, minutes, and hour of the solar one; then turn your eye to the seconds hand of the clock or other chronometer, continuing counting till the seconds hand is at some conspicuous place, and then stop. Write down first the number of seconds you have counted; then the seconds on the clock face at which you stopped; and lastly, the minutes and hour; then the comparison will stand thus:—the time observed by the first chronometer = time observed by the second (or the clock as it may be), *minus* the number of beats counted.

When the solar time chronometer and the sidereal have both half-second beats, the process is the same, counting every *alternate* beat of the sideral instrument. With a chronometer going mean solar time, and having a beat of five times in two seconds (a very common one, particularly in pocket chronometers), the beats will only coincide with the divisions upon the dial every *alternate second,* each beat being equivalent to $0^s{\cdot}4$; the process of comparison is however much the same as that already detailed, but it will be facilitated by marking distinctly with ink upon the face of the chronometer every other second, unless this has been originally so divided as to render the precaution unnecessary.

The following example shows the method of deducing the error of a chronometer going mean solar time, by comparison with a sidereal clock whose rate and error are known by transit observations.

R. E. Observatory, Jan. 24, 1849.

Clock's error . . . . . . $44^s{\cdot}41$ slow.
Rate . . . . $0{\cdot}43$ losing.

H.	M.	S.	
20	11	46·90	Sidereal time. Greenwich mean noon.
0	0	0·35	Correction for longitude $2^m\ 9^s$ east.
20	11	46·55	Sidereal time at mean noon at place of observation.
17	13	0	Clock at time of comparison.
2	58	46·55	
1	59	40·34	Equivalents in mean solar time for above difference.
0	57	50·49	
0	0	45·87	
0	0	0·54	
2	58	17·24	Mean interval from noon by clock.
12	0	0	
9	1	42·76	Mean time A.M. by clock.
9	0	5	Time by chronometer.
0	1	37·76	Chronometer slow (relatively).
0	0	44·41	Clock slow.
0	2	22·17	Error of chronometer, slow.

The eclipses of Jupiter's satellites are phenomena of very frequent occurrence, the precise instants of which can be calculated with certainty for Greenwich time*; but a telescope magnifying at least forty times is required for their observation; and those of different powers are found to give such different results as to the moment of immersion or emersion, that the method is not susceptible of the accuracy it would appear to promise, and is moreover almost impracticable at sea. In determining the longitude by this method, the local time must be found by observations of one or more fixed stars, unless it is known from a chronometer whose error and rate has been previously ascertained.

The eclipses of the sun and moon also enable us to determine the longitude; the former with considerable accuracy; but their rare occurrence renders them of little or no practical benefit, and the results obtained by the eclipses of the moon are generally unsatisfactory, owing to the indistinct outline of the shadow of the earth's border.

The three methods upon which the most dependence can be placed, are—1st, by a "*lunar observation*," which, as before stated, possesses the great advantage of being *easily taken at sea;* 2ndly, by the meridional transits of the moon, compared with those of certain stars previously agreed on, which are given in the Nautical Almanac under the head of "*Moon Culminating Stars;*" and 3rdly, by *occultations of the fixed stars by the moon.*—The two latter methods are the most accurate of any, but the first of them requires the use of a transit instrument, and the latter a good telescope; both involve also long and intricate calculations, which will be found fully detailed in the works of Dr. Pearson, and in Chapter XXXVII. of Woodhouse's Astronomy. The methods

* The *time occupied by light* in travelling from the sun to the earth is also ascertained by means of the eclipses of Jupiter's satellites.

The *difference* of distance the light has to travel from Jupiter to the earth, on the occasion of an eclipse of one of the satellites, happening when they are in *opposition* or in *conjunction*, is evidently the major axis of the earth's orbit. This difference has been ascertained to be $16^{m}\ 26^{s}{\cdot}4$, which gives $8^{m}\ 13^{s}{\cdot}2$ for the time occupied by light in passing from the sun to the earth.

The *distance* of the sun from the earth was determined by means of the transit of Venus over the sun's disc.

given in the following pages considerably shorten the labour of the more accurate computations, and are the same as those in Mr. Riddle's "Navigation."

Method 1st.—By a Lunar Observation.

The observations for this method of ascertaining the longitude of any place can be taken by one individual; but as there are *three* elements required as data, which, if not obtained simultaneously, must be reduced to what they would have been if taken at the same moment of time, it is better, if possible, to have that number of observers.

The *lunar distance,* which is of the first importance, is measured by bringing the *enlightened* edge of the moon and the star, or the edge of the moon and either limb of the sun, in *perfect contact.* The other observations required are, the altitudes of the moon, and that of the other object, whether it be the sun, a fixed star, or a planet*; and as these are only taken for the purpose of *correcting the angular distance,* by clearing it from the effects of parallax and refraction, they do not require the same accuracy, or an equal degree of dexterity in observing. When the observations are made consecutively by one person, the two altitudes are first taken (noticing of course the times); then the lunar distance repeated any number of times, from whence a mean of the times and distance is deduced; and afterwards the altitudes again in reverse order, which altitudes are to be reduced to the same time as that of the mean of the lunar distances.

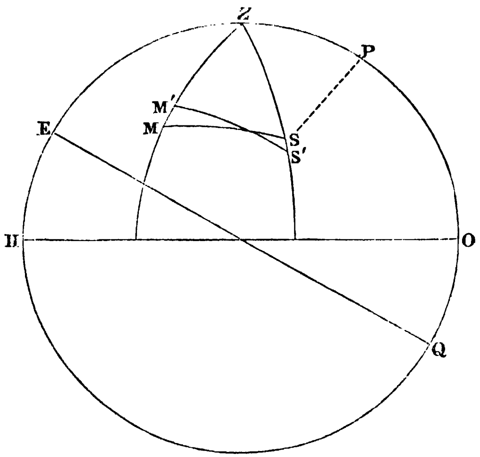
It being of great moment to simplify and render easy the solution of this problem which is of the most vital importance at sea, a number of celebrated practical astronomers have turned their attention to the subject, and tables for "*clearing the lunar distance*" are to be found in all works on Nautical Astronomy, by the use of which the operation is undoubtedly very much shortened†; but as none of these methods show the steps by

* These altitudes, if not observed, can be *calculated* when the latitude is known; by which method more accurate results are obtained.

† Dr. Pearson enumerates no less than *twenty-four* astronomers who have published different methods of facilitating the "Clearing the Lunar Distance."

which this object is attained, the example given below is worked out by spherical trigonometry, and the process will be rendered perfectly easy and intelligible by the following description :—

In the accompanying figure Z represents the zenith, P the pole, M the observed place of the moon, and S that of the sun or star. The data given are M S, the measured angular distance; and Z M and Z S the two zenith distances (or co-altitudes) from whence the angle M Z S is found, the value of which is evidently



not affected by refraction or parallax, which, acting in vertical lines, cause the true place of the moon to be *elevated above* its apparent place (the parallax, from her vicinity to the earth, being a greater quantity than the correction for refraction), and that of the sun or star, to be *depressed below its* apparent place. Let M′ and S′ represent the corrected places of these bodies, and we have then Z M′ and Z S′—the zenith distances *corrected for refraction and parallax*—and the angle Z′ before found, to find the true lunar distance M′S′ in the triangle Z M′S′. The apparent time represented by the angle Z P S may be found in the triangle Z P S, having S S, P S, and Z P the co-latitude, if the exact error of the chronometer at the moment is not already known; and this time, compared with the Greenwich time at which the lunar distance is found from the Nautical Almanac to be the same, gives the difference of longitude east or west of the meridian of that place. The example below will show all the steps of the operation.

On May 4, 1838, at $10^h\ 41^m\ 45^s{\cdot}8$ by chronometer, the following observations were taken in latitude 51° 23′·40 north, to find the longitude; the chronometer having been previously ascertained the same evening to be $3^m\ 34^s$ too fast.

Double altitude—☽ 74° 42′ 35″, taken with a sextant; index error—22″.

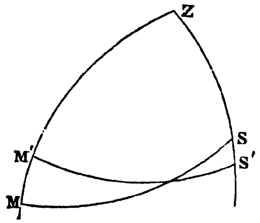
Altitude Spica Virginis 28° 15′ 50″—with alt. and az. inst.; index error—28″.

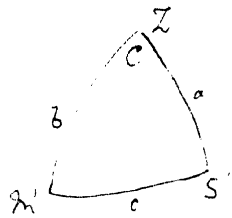
Distance ☽—✳ 31° 25′ 55″—with repeating circle.

Barometer standing at 29″·9, and thermometer at 61°.

Double altitude—	74	42	35
Index error sextant	0	0	22
2)	74	42	13
	37	21	6·5
Semidiameter	0	14	53·8
Apparent altitude ☽	37	6	12·7
	90	0	0
ZM, Apparent zenith distance	52	53	47·3
Altitude Spica Virginis	28	15	50
Index error	0	0	28
Apparent altitude	28	15	22
	90	0	0
Z S, Apparent zenith distance	61	44	38
Observed distance ✳—☽	31	25	55
Moon's semidiam.—$10^h\ 7^m$—14′ 45″·31; Augmentation for 37° 6′ . 8·49 . .		14	53·8
M S, Apparent lunar distance	31	11	1·2

1st—Then in the triangle Z M S we have the three sides to find the angle M Z S.





	°	′	″		
(a) M S =	31	11	1·2		
(b) Z S =	61	44	38·	ar. comp. sin	0·0551028
(c) Z M =	52	53	47·3	ar. comp. sin	0·0982439
S =	145	49	26·5		
$\frac{1}{2}$ S =	72	54	43·25		
($\frac{1}{2}$ S$-b$) =	11	10	5·25	sin.	9·2871039
($\frac{1}{2}$ S$-c$) =	20	0	55·95	sin.	9·5343750
					2) 18·9748256
	17	53	25	=	9·4874128
			2		
Angle M Z S	35	46	50		

Then to correct the zenith distances for refraction and parallax:

	°	′	″
Apparent zenith distance Z M = . . .	52	53	47·3
Refraction	+	1	14·1
	52	55	1·4
Parallax	0	43	7·4
Z M′, Corrected zenith distance	52	11	54·
Z S, Spica Virginis apparent zenith distance .	61	44	38
Refraction	+	1	45
Z S′, Corrected zenith distance	61	46	23

Then in the triangle Z M′ S′, we have

$$\left.\begin{array}{r} ZM' = 52\ 11\ 54 \\ ZS' = 61\ 46\ 23 \\ \text{and angle } Z = 35\ 46\ 50 \end{array}\right\} \text{to find } M'S' \text{ the corrected lunar distance.}$$

$$\text{Formula, } \tan a' = \cos Z \times \tan ZM'$$

$$a'' = ZS' - a'$$

$$\cos M'S = \text{cosine } ZM' \times \frac{\cos a''}{\cos a'},$$

cos	35	46	50	9·9091613
tan	52	11	54	0·1102916
a' =	46	16	58	0·0194529
Z S′ =	61	46	23	
(Z S′ − a') =	15	29	25 = a''	

	°	′	″		
cos Z M′ =	52	11	54		9·7874110
cos a'' =	15	29	25		9·9839310
cos a' =	46	16	58	ar. com.	0·1604593
M′ S′ =	31	16	34		9·9318013

The corrected lunar distance.

By the Nautical Almanac, it appears that the Greenwich mean time answering to this distance, must be *between* 9 P.M. *and midnight*—the difference of distance answering to this interval of 3 hours, being . . . . . 1° 28′ 52″ Prop. log. 3065*

Lunar dist. at 9 P.M. Greenwich	32	3	55	
Corrected distance found above	31	16	34	
		47	21	Prop. log. 5800
Interval of time past 9 . .	1	35	54	—— 2735
	9	0	0	
Greenwich mean time . .	10	35	54	
Mean time at place of observation	10	38	11·8	
Longitude east	0	2	17·8	
Or in space	0	34	27	

* The interval of time past 9 P.M. might of course have been found by a common proportion, without the aid of prop. logarithms.

The difference between the prop. log. at 9 and midnight being 0, the correction of 2nd differences is nothing.

Mr. Baily's formula for a lunar observation for longitude is as follows:—

x the true lunar distance required,

H the apparent } altitude of the moon,
and H′ the true }

h apparent } altitudes of sun or star,
h' true }

and Δ the apparent distance.

$$\text{Make } \beta = \tfrac{1}{2}(\Delta + \text{H} + h)$$

$$\text{Since} = \frac{\left(\cos\beta\cos(\beta - \Delta)\dfrac{\cos \text{H}'\cos h'}{\cos \text{H}\cos h}\right)^{\frac{1}{2}}}{\cos\tfrac{1}{2}(\text{H}' + h')}$$

$$\text{then } \sin\tfrac{1}{2}x = \cos\tfrac{1}{2}(\text{H}' + h)\cos a.$$

The following example will also show the method of working out a lunar observation, by Dr. Young's formula, all the terms of which are *cosines*:—

		°	′	″
Given apparent altitude	S K = ...	7	48	1
	☽ M H = ...	35	45	4
	⊙ ☽ M S = ...	95	50	53
True altitude ⊖	S′K	7	41	31
	☽ M′H	36	27	54

	°	′	″
S Z = ...	82	11	59
M Z	54	14	56
S′Z	82	18	29
M′Z	53	32	6

Required M′S′ the true distance.

By Dr. Young's formula,

$$\text{Cos M}'\text{S}' = \left\{ \frac{2 \cos \frac{1}{2}(\text{MH} + \text{SK} + \text{MS}) \cos \frac{1}{2}(\text{MH} + \text{SK} - \text{MS}) \cos \text{M}'\text{H} \cos \text{S}'\text{K}}{\cos \text{MH} \cos \text{SK}} \right\}$$
$$- \cos (\text{M}'\text{H} + \text{S}'\text{K}).$$

MS =	95 50 53		
MH =	35 45 4	ar. comp. cos	0·090678
SK =	7 48 1	ar. comp. cos	0·004037
2)	139 23 58		
½ Sum	69 41 59	cos	9·540254
½ (MH + SK − MS)	26 8 54	cos	9·953110
M′H	36 27 54	cos	9·905375
S′K	7 41 31	cos	9·996074
			9·489528
Log 2		. .	0·301030
			9·790550
			nat. cosine = 0·617387

	36 27 54		
	7 41 31		
M′H + S′K =	44 9 25	nat. cosine . .	0·717434
	nat. cos. M′S′ 95° 44′ 31″	. . .	0·100047

the true lunar distance.

The same example, by Mr. Riddle's first method, which will be found in his "Navigation," gives 95° 44′ 29″ for the corrected lunar distance.

By Mrs. Taylor's method, which requires the use of her "Tables," the true distance is obtained as follows:—

Table 1 . .	⊙ 1·3873 . .	☽ ·7533
Table 2 . . .	− 0·5077 . . .	− 1·4997
	− 1·8950	− 2·2530
Table 3 { . .	− 7′ 25″	
Table 3 { . .	− 3 15	
„ 4 . .	+ 4 22	
„ 5 . . .	− 0 2	
Total corrections .	− 6 20	
Appt. distance .	95 50 53	
True distance .	95 44 33	

The apparent altitudes and distance are first obtained from those observed, by correcting them for semidiameter and dip if necessary. Then in Table I. find the log of the corrections for the altitudes on account of the moon's parallax.

From Table II. take the logs of the effect of the moon's horizontal parallax upon the distance.

Table III. gives the minutes and seconds answering to these logarithms.

From Table IV., find the effect of the refractions of both objects on the observed distance.

And from Table V., if the sun is one of the objects observed, the effect of his parallax.

These corrections, applied, with their proper signs, to the apparent distance, give the true distance as above. Mr. Airy makes the following remarks upon the effect of errors of observation in taking lunar distances and lunar transits. A certain error of time produces that same error in the deduced longitude; and an error in the *measure* of one second produces about two seconds of time in the longitude.

An error of one *second of time* in a lunar transit produces about 30 seconds error in the longitude.

An error of one *second of time* in a lunar zenith distance will produce at least 30 seconds of time error in longitude—sometimes considerably more. An error of one second in *zenith distances* produces at least two seconds of time in longitude. An error of one second of time in an occultation produces one second of time in the longitude.

The same with eclipses of Jupiter's satellites.

Instead of measuring the distance between the moon and a star, for a comparison with the time at which the same distance is obtained by calculation for the meridian at Greenwich; altitudes may be taken simultaneously of the moon and a star, from the latter of which, its right ascension and declination being accurately known, the *right ascension of the meridian* can be computed. This right ascension applied to the moon's distance from the meridian (the angle P in the astronomical triangle) gives the right ascension of the moon, to be compared with the time at Greenwich at which it is identical, for the difference of longitudes.

Another method, applicable particularly to low latitudes,* is to select, when the moon is on or near the prime vertical, any star whose right ascension and declination are known; it being at the time within 8° or 10° of the zenith.

Take the distance between this star and the moon; also the moon's altitude, and apply the moon's correction in altitude with a contrary sign as the correction in distance; then, with the corrected distance as a base, and the co-declinations as containing sides, the difference of right ascension, and consequently the moon's right ascension, and Greenwich time, are found.

If a star answering to the above conditions is not available, select any star having the same or nearly the same azimuth as the moon, and not less than 30° or 40° distant; the sum or difference of the corrections in altitude would then evidently be the correction in distance. If the star happened to be one of those given in the lunar distance, the Greenwich time is at once found; if not, with the corrected distance as a base, the problem is worked out as before.

The objection to both these methods is, that the moon's declination is required to be known accurately as an important part of the data, to compute which, it is necessary to know the longitude correctly (the very thing sought), except in cases where the moon's declination on either side of the equinoctial is nearly a maximum, and consequently for some time comparatively stationary. Under these circumstances a good result may be expected from the last method when the moon is on, or nearly on, the prime vertical.

BY THE METHOD OF MOON CULMINATING STARS.

The proper motion of the moon causing a difference in the interval of time between her transit, and that of any star, over different meridians, affords another method of determining the longitude.† The times of transit (or apparent right ascension) of

* Obtained from Mr. E. K. Horn.

† The time of the moon's transit compared with that observed at, or calculated for, another meridian, would be sufficient data for ascertaining differences of longitude; but by making *a fixed star the point of comparison*, we obviate any error in the position of the instrument, and also of the clock.

the moon's enlightened edge, and that of certain stars *varying but little from her in declination*, are calculated for Greenwich mean time, and given among the last tables in the Nautical Almanac. The transits of the moon's limb, and of one or more of these stars, are observed at the place whose longitude is required, and from the comparison of the differences of the intervals of time, results a most easy and accurate determination of the difference of meridians;* of which the following example is sufficiently explanatory.

EXAMPLE.

At Chatham, March 9, 1838, the transit of α Leonis was observed by chronometer at $10^h\ 52^m\ 46^s$, and of the moon's bright limb, at $10^h\ 20^m\ 7^s$; the gaining rate of chronometer being $1^s{\cdot}5$.

Eastern Meridian Chatham—observed transits.

		h	m	s
α Leonis		10	52	46
☽|		11	20	7·5
		0	27	21·5
On account of rate of chronometer . . .	−	0	0	0·03
		0	27	21·47
Equivalent in sidereal time			27	25·96

Western Meridian Greenwich—apparent right ascensions.

		h	m	s
α Leonis		9	59	46·18
☽|		10	27	16·76
		0	27	30·58
Observed transits		0	27	25·96
Difference of sidereal time between the intervals		0	0	4·62
Due to change in time of moon's semidiameter passing the meridian	+	0	0	0·01
Difference in ☽'s right ascension		0	0	4·63

* For a more rigid method of computing the difference ot meridians by lunar transits, see Baily's Formulæ and Problems, pp. 239 to 247.

The variation of ☽'s right ascension in 1 hour of terrestrial longitude is, by the Nautical Almanac, 112·77 seconds. Therefore as $112{\cdot}77^{s} : 1^{h} :: 4{\cdot}63^{s} : 147{\cdot}80, = 2' \; 27''{\cdot}8$, the difference of longitude.

But when the difference of longitude is considerable, instead of using the figures given in the list of moon-culminating stars for the variation of the moon's right ascension in one hour of longitude, the right ascension of her centre at the time of observation should be found, by adding to, or subtracting from the right ascension of her bright limb at the time of Greenwich transit, the observed change of interval, and the sidereal time in which her semidiameter passes the meridian. The Greenwich mean time corresponding to such right ascension being then taken from the Nautical Almanac, and converted into sidereal time will give, by its difference from the observed right ascension, the difference of longitude required. For instance, in the above example:—

	h	m	s		h	m	s
☾ Right ascension at Greenwich transit . .					10	27	16·76
Sidereal time of semidiameter passing meridian of place				+	0	1	2·26
☽ Right ascension at Greenwich transit . .					10	28	19·02
Observed difference . . .					0	0	4·62
☽ Right ascension at the time, and sidereal time at the place, of observation . . .					10	28	14·40
Greenwich mean time corresponding to the above right ascension. Page 7, Nautical Almanac.	11	17	0·5				
Or sidereal time at Greenwich					10	25	46·5
Difference of longitude . .					0	2	27·9

BY OCCULTATIONS OF FIXED STARS BY THE MOON.

The rigidly accurate mode of finding the longitude from the occultation of a fixed star by the moon, involves a long and intricate calculation, an example of which will be found in the 37th chapter of Woodhouse's "Astronomy:" and the different methods of calculating occultations, are analysed at length by Dr. Pearson in his "Practical Astronomy," commencing at page 600, vol. ii.

The following rule, however, taken from Riddle's "Navigation," will give the longitude very nearly, without entering into so long a computation:—

Find the Greenwich *mean* time from knowing the local time and the approximate longitude, and for that time take, with the *greatest* exactness, from the Nautical Almanac the sun's right ascension, and the moon's polar distance, semidiameter, and parallax, *applying all corrections.*

To the *apparent* time, add the sun's right ascension, and the difference between this sum, and the star's right ascension, will be the *meridian distance* of the latter. Call this distance P; the star's polar distance p; its right ascension R; the reduced co-latitude l; the moon's polar distance m; her reduced horizontal parallax H; and her semidiameter s.

Then add together sec $\frac{l+p}{2}$, cos $\frac{l \sim p}{2}$, and cot $\frac{P}{2}$, and the sum, rejecting twenty, will be the tangent of arc a, of the same affection as $\frac{l+p}{2}$.

Add together cosec $\frac{l+p}{2}$, sin $\frac{l \sim p}{2}$, and cot $\frac{P}{2}$, and the sum, rejecting twenty, will be the tan of arc b (*always acute*). When l is greater than p, $a + b =$ arc c; and when l is less than p, $a - b =$ arc c.

Add together tan c, cosec l, cosec P, and prop. log H, and the sum, rejecting the tens, is prop. log of arc d. When arc c is obtuse, $p + d =$ arc e; and when c is acute, $p - d =$ arc e.

Add together cosec l, cosec P, prop. log H; and with the sum S, and p, take the correction from the subjoined table, and applying it with its proper sign to e, call the sum or the remainder e'. The difference of m and e' is arc f.

To S add sin e', and the sum, rejecting the tens, is the prop. log of arc g.

To the prop. logs of $s + f$, and $s - f$, add twice the sine of arc e, and half the sum, rejecting the tens, is the prop. log of arc h.

Then the moon's right ascension = R ± g ± h, where g is additive west of the meridian, and subtractive east; and h is additive at an *emersion*, and subtractive at an *immersion*.

Having found the moon's right ascension, the corresponding Greenwich time is to be found from the Nautical Almanac, the comparison of which with the *local* time gives the longitude of the place of observation.

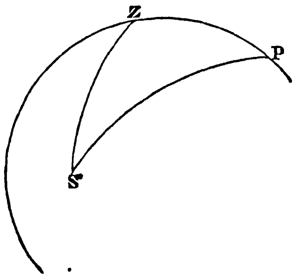
TABLE FOR CORRECTION OF *e*.

S	Star's Polar Distance *p*.						
	60° +	65° +	70° +	75° +	80° +	85° +	90° −
	″	″	″	″	″	″	″
·50	16·5	13·2	10·3	7·5	5·0	2·5	·0
·55	13·0	10·5	8·2	6·0	4·0	2·0	·0
·60	10·3	8·3	6·5	4·7	3·2	1·5	·0
·65	8·2	6·6	5·1	3·8	2·5	1·2	·0
·70	6·5	5·2	4·1	3·0	2·0	1·0	·0
·75	5·1	4·2	3·2	2·4	1·5	·8	·0
·80	4·1	3·2	2·6	1·9	1·2	·6	·0
·85	3·2	2·6	2·0	1·5	·9	·5	·0
·90	2·6	2·1	1·6	1·1	·8	·4	·0
·95	2·1	1·7	1·3	1·0	·6	·3	·0
1·00	1·6	1·3	1·0	·7	·4	·2	·0
1·10	1·0	·9	·6	·5	·3	·1	·0
1·20	·6	·5	·4	·3	·2	·1	·0
1·30	·4	·3	·3	·2	·1	·0	·0
1·50	·2	·1	·1	·0	·0	·0	·0
1·80	·0	·0	·0	·0	·0	·0	·0
S	120°	115°	110°	105°	100°	95°	90°
	Star's Polar Distance *p*.						

PROBLEM VI.

TO DETERMINE THE DIRECTION OF A MERIDIAN LINE* AND THE VARIATION OF THE COMPASS.

In the spherical triangle Z P S, already alluded to as the *astronomical triangle;* and in which the co-latitude Z P, and the time represented by the angle P, were ascertained by the method of absolute altitudes in pages 213 and 217; the *azimuth* of any celestial body S is measured by the angle Z, which is found from knowing either the time, or the latitude, in addition to the observed altitude. This calculated *azimuth* compared with the magnetic bearing of the object observed at the same instant, and determined with reference to some well-defined terrestrial mark, affords the means of laying down a meridian line, and gives the variation of the compass.



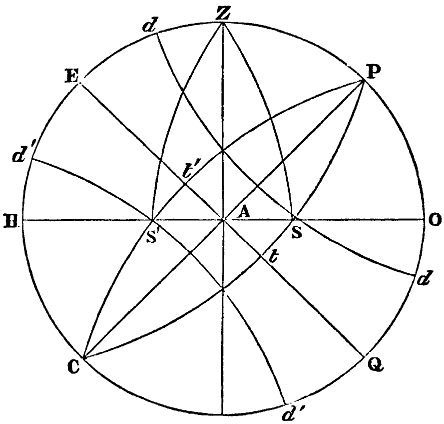
Another mode is by calculating the *amplitude* of the sun at his rising or setting for any day in any latitude, and comparing it with his *observed* bearing when on the horizon, or rather when he is 34 minutes, or about his own diameter, above it, as his disc is elevated that amount above its true place by refraction.

In the accompanying figure H O is the horizon, P the pole, E Q the equator, P A C the six o'clock hour circle, P E C the meridian, Z the zenith and *d d* or *d′ d′* the circle of declination of the sun, either north or south of the equator, and supposed to be drawn through his place at the time of sunrise, which is approximately known.

S or S′ then, the intersection of this declination circle with the horizon, is the position of the sun at rising in the first case *before* arriving at the 6 o'clock hour circle, and in the second *after* having passed it.

* The method of ascertaining the direction of the meridian with an altitude and azimuth instrument, or a large theodolite, has been already described at page 169.

In the triangles A S t or A S′ t' then, t S or t' S′ is the sun's declination, and the angle S A t, S′ A t' the co-latitude of the place; from whence we obtain A S or A S′, *the amplitude*, and also A t or A t', the angular distance before or after 6 o'clock for *the time of*



sunrise.—In the same way can be obtained the sun's amplitude at sunset; as also the time, allowing for the change in declination.—If the meridian is to be marked on the ground, it is necessary, as before stated, to observe some object with reference to the magnetic bearing.

A transit instrument* placed in the plane of the meridian, of course affords the means of marking out at once a meridian line on the ground; the following short description, abridged from Dr. Pearson's "Practical Astronomy," explains the method of adjusting a portable transit approximately in this plane, and of verifying its position when so placed.

1st. *The adjustment of the level, and of the axis of the telescope.*—These two adjustments may be carried on at the same time; as when the level is made horizontal and parallel to the axis, the axis must be horizontal also.—Apply the level to its proper place

* In an Observatory, the principal uses to which a transit is applied are the *obtaining true time*, and the determination of *right ascensions*—very excellent directions for using and adjusting a portable transit for the determination of longitudes, &c., drawn up by Mr. Airy, will be found in the Narrative of the North American Boundary, by Major Robinson, from which one example is given at the end of this chapter, to show the form there adopted for recording transit observations.

on the pivots of the axis, and bring it horizontal by the foot-screws of the instrument; reverse the level, and mark the difference as shown on the scale attached to it—half this difference must be corrected by the screw of the level, and half by the foot-screws which operation will probably want repeating—if by previous observation, the level has been ascertained to be correct, the foot-screws alone must be used in the correction, and if on reversing the instrument in its Y s, the level is still correct, the pivots of the axis are of equal size; if not, the instrument should be returned to its maker as imperfect.

2nd. The next object will be to *place the spider lines truly vertical, and to determine the equatorial value of their intervals.*

Suspend a thick white plumb-line on a dark ground, at a distance from the telescope; then the middle wire may be made to coincide with it to insure its verticality, and if a motion in altitude be given to the telescope, and the coincidence continues unaltered by change of elevation, the axis has been truly levelled.

The equatorial value of the intervals between the wires may be determined by counting the time in seconds and parts occupied by the passage of an *equatorial* star over all the intervals, taken separately and collectively, by several repetitions on or near the meridian. If the star observed has any declination, the value of an interval obtained from its passage may be reduced to its equatorial value by multiplying the seconds counted, by the cosine of the star's declination; before this method can be used, the telescope must have been placed nearly on the meridian.

3rd. *Collimation in azimuth.*—When the preceding adjustments have been made, the telescope should be directed to a distant object, the middle spider line brought to bisect it, and the axis then turned end for end. If, after this reversion, the same point be again bisected by the wire, it is a proof that a line passing from the middle spider line to the optical centre of the object glass is at right angles to the axis of the telescope's motion. But if, after this reversion of the axis, the visible mark be found on one side of the middle line, half the error thus found must be corrected by the screw that moves the Y s in azimuth, and the other half by the screw for adjusting the wires; several reversions must be made to ensure accuracy.—The verification of this adjustment may be proved by the passage of the pole star;—note the time at the *preceding* and at the *middle* wire, then reverse the axis, and note the passage over what *was* the preceding, but is now the *following* wire; half the difference of the intervals before and after reversion will show how much the position of the centre wire has been altered by reversion.

4th. *Collimation in altitude.*—When the telescope is directed to the pole star at the time of its crossing the meridian, or to any well-defined distant point by daylight, read the vernier of the altitude circle while the bubble of the level is at zero. The axis of the telescope must then be reversed, and the horizontal line again brought to bisect the star; and when the bubble is made to stand at zero as before, the reading of the vernier must be again noted; half the sum of these readings will be the true altitude, and half the difference the error of collimation in altitude. This error may consist of two parts: the spider line may be out of the optical centre of the field of view; and the level (supposing it previously adjusted to reverse properly in position) may not be in its true position as regards the zero of the circle's divisions; half, therefore, of the error arising from the half difference of altitudes must be adjusted by the screws carrying the spider lines, and the other half by the screw that alters the level.

5th. The last and most difficult of all the adjustments is that by which the instrument is placed *in the plane of the meridian of the place of observation.* There are many modes of accomplishing this, both by direct and indirect means; but the most convenient and most generally practised are those in which a circumpolar star is employed; or in which *two* circumpolar stars, differing little in declination, but nearly twelve hours in right ascension; or in which two stars, differing considerably in altitude, and but little in right ascension, are successively observed; but in whatever way the adjustment may be made, the clock that gives the times of transit must have its rate previously well determined.

The approximate position of the instrument may be ascertained by calculating the solar time of the pole star's passage over the meridian for any given day; and then the telescope levelled and pointed at it at the computed time will require but little adjustment. Subsequent observations of *circumpolar*, or of *high and low* stars, will gradually rectify the position, provided all the adjustments previously directed continue unaltered for a sufficient length of time; and a meridian mark, capable of adjustment, may be placed at a convenient distance north or south, until their places are definitely fixed by some of the following methods. At 95·49 yards from the object end of the telescope, *one inch* will subtend 1′ or 60″, and a scale may be made accordingly, varying of course inversely as the distances; so that when the transit is found to be any number of seconds, say thirty, too much to the east or west, a corresponding distance on the scale shows how much the instrument is to be moved in azimuth, by the proper screws, to effect the correction required.

Method 1st.—By a circumpolar star.

$$a = \frac{t - t' - 12^h}{2 \cos L} \text{ cotan } \delta$$

where a = azimuthal deviation in *seconds* at the horizon,
t = the time at upper transit,
t' = at lower passage,
L = the latitude,
δ = the declination :
by multiplying by 15, a is converted into space if required.

If the *western* semicircle is passed through in less time than the *eastern*, the object end of the telescope points to the *west* of the true meridian. The clock must be a *good one* for this method, as it supposes no change of rate for twelve hours.

Method 2nd.—By a pair of circumpolar stars.

$$a = \frac{(t - t' - 12^h) - (T - T' - 12^h) \sin \Delta \sin \Delta'}{2 \cos L - \sin (\Delta' - \Delta)}$$

where Δ and Δ' = the star's polar distances, L = the latitude, t and t' the times of the first star's upper and lower passages, T and T′ the times of the contrary passages of the second star, following the other at an interval of nearly 12 hours in right ascension; or this formula, omitting the 12 hours,

$$a = \frac{(t - r) - (t' - r') \sin \Delta \sin \Delta'}{2 \cos L - \sin (\Delta' - \Delta)}$$

when $(t - t' - 12^h)$ is a greater interval than $(r - r' - 12^h)$ the hori zontal deviation a will be towards the *east*, and *vice versâ;* or when $(t'-r')$ is greater than $(t-r)$ the deviation is also to the *east.*

Method 3rd.—By high and low stars.

$$a = \frac{(D-D')\cos\delta.\cos\delta'}{\cos L\sin(\delta'-\delta)}.$$

where $D=(t-t')$ the difference of the observed times of passage, and $D' = (Ra - Ra')$ the difference of the apparent right ascension of the two given stars, δ' the declination of the higher star, and δ that of the lower. The stars for this method ought to be removed from each other at least 40° in declination. When $(D-D')$ is *positive*, the horizontal deviation is to the *east* of the *south point* in *northern* latitudes; and the contrary when *negative*. Tables are formed to facilitate the computation of the above formulæ. The times are all supposed to be *sidereal*; if, therefore, solar time is used in the observations, the acceleration must be added.

The following example is given of the last method, in which, if the difference of the times of the observed passages be exactly equal to the difference of the computed right ascensions of the two stars, the instrument will necessarily be already in the plane of the meridian.

On June 20, 1838, in latitude 51° 23′ 40″, the transits of a Corona Borealis, and of Antares, were observed.

		Transits. H. M. S.	Rt. Ascensions. H. M. S.
a Corona Borealis . . .		9 25 31·5	15 27 52·25
Antares		10 17 17·8	16 19 31·93
	D	− 51 46·3	− 51 39·68
	D′	− 51 39·68	
		+ 6·62	

6·62 . . .	log.		·8208580
Cos δ′ . . .	27′ 15 46		9·9488603
Cos δ . . .	26 4 1		9·9534124
Cos L . . .	51 23 40	ar. com.	0·2048465
Sin 6+6′ . .	53 19 47	ar. com.	0·0957794
$a = 10^{s}·562$			1·0237566

FORM FOR RECORDING OBSERVATIONS MADE WITH A PORTABLE TRANSIT.

(Date, Place, and Name of Observer.)

Approximate Solar Time.	H. M. 9 47	H. M. 10 40	H. M. 10 45
Object.	12 Canum Venaticorum.	η Ursæ Majoris.	η Bootis.
	H. M. S.	H. M. S.	H. M. S.
1st Wire	13 2 41·5	13 55 6·5	14 1 26·0
2nd ,, ...	3 17·0	55 49·5	Lost
3rd ,, ...	3 51·5	56 31·0	2 23·0
4th ,, ...	4 27·0	Lost	2 53·0
5th ,, ...	5 1·0	57 56·0	3 21·0
Sum . . .	19 18·0	225 23·0	10 3·0
Mean of Wires	13 3 51·6	13 45 4·6	14 2 0·6
Correction for wires lost	......	11 26·70	0 22·94
True Transit on Instruments . .	13 3 51·6	13 56 31·30	14 2 23·54
Azimuthal Error + 20 × { + 0·009, − 0·008, + 0·031 }	+ 0·18	− 0·16	+ 0·62
True Transit over Meridian . . .	13 3 51·78	13 56 31·14	14 2 24·16
Star's Right Ascension . . .	12 48 48·97	13 41 28·49	13 47 21·19
Error of Chronometer	15 2·81	15 2·65	15 2·97

The above is one of the sets of observations made by Major Robinson at St. Helen's Island, Upper Canada, in 1845. Reference was also made to the particular transit and chronometer used; stating also if the error of collimation had been determined, and the transit levelled immediately before the observation, and whether the east or west end of the axis was illuminated.

In the transit books used on this occasion, made of four or five quires of letter paper bound up in a strong cover, the right-hand page was printed in the above form, leaving the other blank, for recording levels, calculating the azimuthal errors, &c., &c.

The form for registering transit observations in a permanent Observatory is of course different from the above: that at present in use at the Royal Engineer Observatory at Chatham, taken from the "Corps Papers," is given as an example in page 251.

The altitude and azimuth instrument alluded to in page 177, as one of the standard instruments of an Observatory, can be used when fixed in the plane of the meridian as a transit, for which purpose the diaphragm is provided with 5 vertical and 5 horizontal wires, the central intersection of which in the axis of the telescope is used for other observations. An instrument thus placed is of itself competent for almost all the requirements of an Observatory excepting those requiring an equatorially mounted telescope, as by it may be obtained right ascensions and true time, as well as altitudes or zenith distances and azimuths. When constructed to be portable it becomes pre-eminently useful in all geodesical operations, more particularly when placed on a repeating stand as was done with those made for the Ordnance Survey (generally known as 2-feet theodolites).

This instrument is fully described at page 412 of the 3rd vol. of the Woolwich Course, from which the following outline of its construction and adjustments has been taken:—The telescope is of 2 feet 3 inches focal length; the vertical circle 15 inches diameter divided to 5′, and the intervening space between each division read by two micrometers. The horizontal circle is of 2 feet in diameter, also divided to 5′, and read by six micrometers. The repeating table of three radiating arms is in two parts: the lower rests upon three levelling screws, and the upper carries the instrument.

The adjustments are as follows: the repeating table is first levelled by the three foot-screws by means of a short spirit level screwed on to its upper plate. The table is then turned 180°, and the level adjusted, one half by the screw of the spirit level, and the other half by the two foot-screws which are parallel to the level. The table is then turned 90°, and the air-bubble of the spirit level brought to the centre of its tube by the remaining screw of the repeating table. This process must be repeated until the table is perfectly level.

The second process is the centering of the repeating table, which is effected by removing the spirit level from the upper plate and screwing the large centering microscope to the centre of the table. A fine circular dot upon a wafer is placed immediately below the centre at the level of the bottom of the triangular stand, and the cross hairs of the centering microscope made to bisect it, one half of the adjustment being made by the centering screws, and the other half by altering the collimating screws at the upper end of the microscope. This operation is repeated with the table turned 180°, and subsequently again at right angles.

The instrument itself is then placed with its three foot-screws in the brass cups at the three arms of the repeating table, and levelled by its own screws; it is then centered by the *centering* microscope, so as to be used as a repeating theodolite, the operation being repeated with the instrument turned 180° and 90° from its original position.

The next adjustment is that of the telescope level. The telescope is placed in its Ys as nearly horizontal as possible, the vertical circle clamped, and the air bubble of the suspended level made by the tangent screw to bisect the centre of its tube. The level is then reversed on its supports, and one half of the error thus caused corrected by its adjusting screws and the other half by the tangent screw of the vertical arc, repeating the operation until no error exists.

The last process is the levelling of the horizontal axis. The instrument is first carefully levelled by the telescope level, and the axis level suspended from the two pivots of the telescope, the exact readings of the ends of the air bubbles on the level scale being noted. The level is then reversed on the pivots, and the difference of the two readings of the ends of the air bubbles on the scale is corrected one half by the adjusting screw of the level, and the other half by the screw attached to the moveable Y that receives one of the pivots of the telescope, repeating the operation until the adjustment is perfect. The correction of the line of collimation and of the micrometer microscopes are the same as for the theodolite.

FORM FOR REGISTERING TRANSIT OBSERVATIONS.

No. of Observation.	Date.	Illuminated End, East or West.	Inclination of Axis. E end highest— W end +		Object	Zenith Distance.	North or South.	Telescope Wires.					Mean.	Corrections.				True Passage.	Clocks.		No. of Days.	Remarks.
			Inclination of Level.	Inequality of Pivots.				1	2	Centre.	4	5		To Centre Wire.	Collimation.	Inclination.	Azimuth.		Error.	Rate.		
533	1849 Jan. 22	E	—·2	+·04	Aldebaran.	° ′ 35 11	S	s. 12	s. 30	H. M. S. 4 25 47	s. 5	s. 23	H. M. S. 4 25 47·4		—·06	—·13	—1·39	H. M. S. 4 15 45·82	—90·71			
534	...	E	Level	+·04	δ Orionis.	51 47	S	16	33	5 22 49·5	6·5	23·5	5 22 49·7		—·06	0	—1·82	5 22 47·82	—90·89			
			–·05					M. S. [illegible]5 56	M. S.		M. S.	M. S.										
			—·15						58 3													
535	...	E	—·15	+·04						5 0 9												
			—·10								2 15											
535			—·10		ε Ursæ Minoris.	46 20	N					4 22	5 0 9		+·47	+·35	—12·43	4 59 57·3	—92·88			Below the Pole.
536		E	Level	+·04	ε Orionis.	52 40	S	s. 31	s. 48 5	5 27 5·5	s. 22	s. 39	5 27 5·2		—·06		—16·7	5 27 3·47	—90·67	+·10	7	

TABLE I.

FOR CONVERTING SIDEREAL INTO MEAN SOLAR TIME.

Hours.			Minutes.				Seconds.			
	M.	S.		S.		S.		S.		S.
1	0	9·330	1	0·164	31	5·079	1	0·003	31	0·085
2	0	19·659	2	0·328	32	5·242	2	0·005	32	0·087
3	0	29·489	3	0·491	33	5·406	3	0·008	33	0·090
4	0	39·318	4	0·655	34	5·570	4	0·011	34	0·093
5	0	49·148	5	0·819	35	5·734	5	0·014	35	0·096
6	0	58·977	6	0·983	36	5·898	6	0·016	36	0·098
7	1	8·807	7	1·147	37	6·062	7	0·019	37	0·101
8	1	18·636	8	1·311	38	6·225	8	0·022	38	0·104
9	1	28·466	9	1·474	39	6·389	9	0·025	39	0·106
10	1	38·296	10	1·638	40	6·553	10	0·027	40	0·109
11	1	48·125	11	1·802	41	6·717	11	0·030	41	0·112
12	1	57·955	12	1·966	42	6·881	12	0·033	42	0·115
13	2	7·784	13	2·130	43	7·044	13	0·036	43	0·118
14	2	17·614	14	2·294	44	7·208	14	0·038	44	0·120
15	2	27·443	15	2·457	45	7·372	15	0·041	45	0·123
16	2	37·273	16	2·621	46	7·536	16	0·044	46	0·126
17	2	47·103	17	2·785	47	7·700	17	0·047	47	0·128
18	2	56·932	18	2·949	48	7·864	18	0·049	48	0·131
19	3	6·762	19	3·113	49	8·027	19	0·052	49	0·134
20	3	16·591	20	3·277	50	8·191	20	0·055	50	0·137
21	3	26·421	21	3·440	51	8·355	21	0·057	51	0·140
22	3	36·250	22	3·604	52	8·519	22	0·060	52	0·142
23	3	46·080	23	3·768	53	8·683	23	0·063	53	0·145
24	3	55·909	24	3·932	54	8·847	24	0·066	54	0·148
			25	4·096	55	9·010	25	0·068	55	0·150
			26	4·259	56	9·174	26	0·071	56	0·153
			27	4·423	57	9·338	27	0·074	57	0·156
			28	4·587	58	9·502	28	0·076	58	0·159
			29	4·751	59	9·666	29	0·079	59	0·161
			30	4·915	60	9·830	30	0·082	60	0·164

The quantities opposite the [different numbers of hours, minutes, and seconds, are to be subtracted, to obtain the equivalent interval of mean solar time for any period.

TABLE II.

FOR CONVERTING MEAN SOLAR INTO SIDEREAL TIME.

Hours.			Minutes.				Seconds.			
	M.	S.		S.		S.		S.		S.
1	0	9·856	1	0·164	31	5·092	1	0·003	31	0·085
2	0	19·713	2	0·329	32	5·257	2	0·005	32	0·087
3	0	29·569	3	0·493	33	5·421	3	0·008	33	0·090
4	0	39·426	4	0·657	34	5·585	4	0·011	34	0·093
5	0	49·282	5	0·821	35	5·750	5	0·014	35	0·096
6	0	59·139	6	0·986	36	5·914	6	0·016	36	0 098
7	1	8·995	7	1·150	37	6·078	7	0·019	37	0·101
8	1	18 852	8	1·314	38	6·242	8	0·022	38	0·104
9	1	28·708	9	1·478	39	6·407	9	0·025	39	0·106
10	1	38·565	10	1·643	40	6·571	10	0·027	40	0·109
11	1	48·421	11	1·807	41	6·735	11	0·030	41	0·112
12	1	58·278	12	1·971	42	6·900	12	0 033	42	0·115
13	2	8·134	13	2·136	43	7·064	13	0·036	43	0·118
14	2	17·991	14	2·300	44	7·228	14	0·038	44	0·120
15	2	27·847	15	2·464	45	7·392	15	0·041	45	0·123
16	2	37 704	16	2·628	46	7·557	16	0·044	46	0·126
17	2	47·560	17	2·793	47	7 721	17	0·047	47	0·128
18	2	57·416	18	2·957	48	7·885	18	0·049	48	0·131
19	3	7·273	19	3·121	49	8·050	19	0·052	49	0·134
20	3	17·129	20	3·285	50	8·214	20	0·055	50	0·137
21	3	26·986	21	3·450	51	8·378	21	0·057	51	0·140
22	3	36·842	22	3·614	52	8·542	22	0·060	52	0·142
23	3	46·699	23	3·778	53	8·707	23	0·063	53	0·145
24	3	56·555	24	3·943	54	8·871	24	0·066	54	0·148
			25	4·107	55	9·035	25	0·068	55	0·150
			26	4·271	56	9·199	26	0·071	56	0·153
			27	4·436	57	9·364	27	0·074	57	0·156
			28	4·600	58	9·528	28	0·076	58	0·159
			29	4·764	59	9·692	29	0·079	59	0·161
			30	4·928	60	9·856	30	0·082	60	0·164

The quantities opposite the different numbers of hours, minutes, and seconds are to be added, to obtain the equivalent interval of sidereal time for any period.—Vide *Table of Equivalents*, page 489 of the Nautical Almanac. This Table, and the preceding, are calculated from the ratio of a sidereal to a mean solar day—twenty-four hours of mean time being equivalent to $24^h\ 3^m\ 56^s{\cdot}5554$ sidereal time.

TABLE III.

FOR CONVERTING SPACE INTO TIME, AND *VICE VERSA.*

SPACE INTO TIME.

To convert degrees and parts of the Equator into Sidereal Time; or to convert degrees and parts of Terrestrial Longitude into Time.

°	h. m.	′	m. s.	″	s.
1	0 4	1	0 4	1	0·066
2	0 8	2	0 8	2	0·133
3	0 12	3	0 12	3	0·200
4	0 16	4	0 16	4	0·266
5	0 20	5	0 20	5	0·333
6	0 24	6	0 24	6	0·400
7	0 28	7	0 28	7	0·466
8	0 32	8	0 32	8	0·533
9	0 36	9	0 36	9	0·600
10	0 40	10	0 40	10	0·666
11	0 44	11	0 44	11	0·733
12	0 48	12	0 48	12	0·800
13	0 52	13	0 52	13	0·866
14	0 56	14	0 56	14	0·933
15	1 0	15	1 0	15	1·000
16	1 4	16	1 4	16	1·066
17	1 8	17	1 8	17	1·133
18	1 12	18	1 12	18	1·200
19	1 16	19	1 16	19	1·266
20	1 20	20	1 20	20	1·333
25	1 40	21	1 24	21	1·400
30	2 0	22	1 28	22	1·466
35	2 20	23	1 32	23	1·533
40	2 40	24	1 36	24	1·600
45	3 0	25	1 40	25	1·666
50	3 20	26	1 44	26	1·733
55	3 40	27	1 48	27	1·800
60	4 0	28	1 52	28	1·866
65	4 20	29	1 56	29	1·933
70	4 40	30	2 0	30	2·000
75	5 0	31	2 4	31	2·066
80	5 20	32	2 8	32	2·133
90	6 0	33	2 12	33	2·200
100	6 40	34	2 16	34	2·266
110	7 20	35	2 20	35	2·333
120	8 0	36	2 24	36	2·400
130	8 40	37	2 28	37	2·466
140	9 20	38	2 32	38	2·533
150	10 0	39	2 36	39	2·600
160	10 40	40	2 40	40	2·666
170	11 20	41	2 44	41	2·733
180	12 0	42	2 48	42	2·800
190	12 40	43	2 52	43	2·866
200	13 20	44	2 56	44	2·933
210	14 0	45	3 0	45	3·000
220	14 40	46	3 4	46	3·066
230	15 20	47	3 8	47	3·133
240	16 0	48	3 12	48	3·200
250	16 40	49	3 16	49	3·266
260	17 20	50	3 20	50	3·333
270	18 0	51	3 24	51	3·400
280	18 40	52	3 28	52	3·466
290	19 20	53	3 32	53	3·533
300	20 0	54	3 36	54	3·600
310	20 40	55	3 40	55	3·666
320	21 20	56	3 44	56	3·733
330	22 0	57	3 48	57	3·800
340	22 40	58	3 52	58	3·866
350	23 20	59	3 56	59	3·933
360	24 0	60	4 0	60	4·000

TIME INTO SPACE.

To convert Sidereal Time into degrees and parts of the Equator; or to convert Time into degrees and parts of Terrestrial Longitude.

h.	°
1	15
2	30
3	45
4	60
5	75
6	90
7	105
8	120
9	135
10	150
11	165
12	180
13	195
14	210
15	225
16	240
17	255
18	270
19	285
20	300
21	315
22	330
23	345
24	360

Tenths.

s	″
·1	1·5
·2	3·0
·3	4·5
·4	6·0
·5	7·5
·6	9·0
·7	10·5
·8	12·0
·9	13·5
1·0	15·0

Hundredths.

s	″
·01	0·15
·02	0·30
·03	0·45
·04	0·60
·05	0·75
·06	0·90
·07	1·05
·08	1·20
·09	1·35
·10	1·50

Thousandths.

s	″
·001	0·015
·002	·030
·003	·045
·004	·060
·005	·075
·006	·090
·007	·105
·008	·120
·009	·135
·010	·150

m.	° ′	s.	′ ″
1	0 15	1	0 15
2	0 30	2	0 30
3	0 45	3	0 45
4	1 0	4	1 0
5	1 15	5	1 15
6	1 30	6	1 30
7	1 45	7	1 45
8	2 0	8	2 0
9	2 15	9	2 15
10	2 30	10	2 30
11	2 45	11	2 45
12	3 0	12	3 0
13	3 15	13	3 15
14	3 30	14	3 30
15	3 45	15	3 45
16	4 0	16	4 0
17	4 15	17	4 15
18	4 30	18	4 30
19	4 45	19	4 45
20	5 0	20	5 0
21	5 15	21	5 15
22	5 30	22	5 30
23	5 45	23	5 45
24	6 0	24	6 0
25	6 15	25	6 15
26	6 30	26	6 30
27	6 45	27	6 45
28	7 0	28	7 0
29	7 15	29	7 15
30	7 30	30	7 30
31	7 45	31	7 45
32	8 0	32	8 0
33	8 15	33	8 15
34	8 30	34	8 30
35	8 45	35	8 45
36	9 0	36	9 0
37	9 15	37	9 15
38	9 30	38	9 30
39	9 45	39	9 45
40	10 0	40	10 0
41	10 15	41	10 15
42	10 30	42	10 30
43	10 45	43	10 45
44	11 0	44	11 0
45	11 15	45	11 15
46	11 30	46	11 30
47	11 45	47	11 45
48	12 0	48	12 0
49	12 15	49	12 15
50	12 30	50	12 30
51	12 45	51	12 45
52	13 0	52	13 0
53	13 15	53	13 15
54	13 30	54	13 30
55	13 45	55	13 45
56	14 0	56	14 0
57	14 15	57	14 15
58	14 30	58	14 30
59	14 45	59	14 45
60	15 0	60	15 0

TABLE IV.

Barometer, 30 in. + when above. — when below.	TABLE OF REFRACTIONS.	Thermometer, 50°. — when above. + when below.

App. Alt.	Refr. B. 30. Th. 50°.	Difference to be allowed for.			App. Alt.	Refr. B. 30. Th. 50°.	Difference to be allowed for.		
		1′ Alt.	+ 1 B	—1°Th			1′ Alt.	+ 1 B	— 1° Th.
° ′	′ ″	″	″	″	° ′	′ ″	″	″	″
0 0	33 51	11·7	74	8·1	3 0	14 35	3·2	30	2·3
5	32 53	11·3	71	7·6	5	14 19	3·1	29	2·2
10	31 58	10·9	69	7·3	10	14 4	3·0	29	2·2
15	31 5	10·5	67	7·0	15	13 50	2·9	28	2·1
20	30 13	10·1	65	6·7	20	13 35	2·8	28	2·1
25	29 24	9·7	63	6·4	25	13 21	2·7	27	2·0
30	28 37	9 4	61	6·1	30	13 7	2·7	27	2·0
35	27 51	9·0	59	5·9	35	12 53	2·6	26	2·0
40	27 6	8·7	58	5·6	40	12 41	2·5	26	1·9
45	26 24	8·4	56	5·4	45	12 28	2·4	25	1·9
50	25 43	8·0	55	5·1	50	12 16	2·4	25	1·9
55	25 3	7·7	53	4·9	55	12 3	2·3	25	1·8
1 0	24 25	7·4	52	4·7	4 0	11 52	2·2	24·1	1·70
5	23 48	7·1	50	4·6	10	11 30	2·1	23·4	1·64
10	23 13	6·9	49	4·5	20	11 10	2·0	22·7	1·58
15	22 40	6·6	48	4·4	30	10 50	1·9	22·0	1·53
20	22 8	6·3	46	4·2	40	10 32	1·8	21·3	1·48
25	21 37	6·1	45	4·0	50	10 15	1·7	20·7	1·43
30	21 7	5·9	44	3·9	5 0	9 58	1·6	20·1	1·38
35	20 38	5·7	43	3·8	10	9 42	1·5	19·6	1·34
40	20 10	5·5	42	3·6	20	9 27	1·5	19·1	1·30
45	19 43	5·3	40	3·5	30	9 11	1·4	18·6	1·26
50	19 17	5·1	39	3·4	40	8 58	1·3	18·1	1·22
55	18 52	4·9	39	3·3	50	8 45	1·3	17·6	1·19
2 0	18 29	4·8	38	3·2	6 0	8 32	1·2	17·2	1·15
5	18 5	4·6	37	3·1	10	8 20	1·2	16·8	1·11
10	17 43	4·4	36	3·0	20	8 9	1·1	16·4	1·09
15	17 21	4·3	36	2·9	30	7 58	1·1	16·0	1·06
20	17 0	4·1	35	2·8	40	7 47	1·0	15·7	1·03
25	16 40	4·0	34	2·8	50	7 37	1·0	15·3	1·00
30	16 21	3·9	33	2·7	7 0	7 27	1·0	15·0	0·98
35	16 2	3·7	33	2·6	10	7 17	·9	14·6	·95
40	15 43	3·6	32	2·6	20	7 8	·9	14·3	·93
45	15 25	3·5	32	2·5	30	6 59	·8	14·1	·91
50	15 8	3·4	31	2·4	40	6 51	·8	13·8	·89
55	14 53	3·3	30	2·3	50	6 43	·8	13·5	·87

Young's Refractions have been selected from among those by different eminent Astronomers, given in Dr. Pearson's Tables.

TABLE IV.—*continued.*

Barometer, 30 in. + when above. — when below.		TABLE OF REFRACTIONS.						Thermometer, 50°. — when above. + when below.	
App. Alt.	Refr. B. 30. Th. 50°.	Difference to be allowed for.			App. Alt.	Refr. B. 30. Th. 50°.	Difference to be allowed for.		
		1′ Alt.	+ 1 B.	—1°Th.			1′ Alt.	+ 1 B.	—1°Th.
° ′	′ ″	″	″	″	° ′	′ ″	″	″	″
8 0	6 35	·7	13·3	·85	14 0	3 49·9	·28	7·70	·469
10	6 28	·7	13·1	·83	10	3 47·1	·28	7·61	·464
20	6 21	·7	12·8	·82	20	3 44·4	·27	7·52	·458
30	6 14	·7	12·6	·80	30	3 41·8	·26	7·43	·453
40	6 7	·7	12·3	·79	40	3 39·2	·26	7·34	·448
50	6 0	·6	12·1	·77	50	3 36·7	·25	7·26	·444
9 0	5 54	·6	11·9	·76	15 0	3 34·3	·24	7·18	·439
10	5 47	·6	11·7	·74	30	3 27·3	·22	6·95	·424
20	5 41	·6	11·5	·73	16 0	3 20·6	·21	6·73	·411
30	5 36	·6	11·3	·71	30	3 14·4	·20	6·51	·399
40	5 30	·5	11·1	·71	17 0	3 8·5	·19	6·31	·386
50	5 25	·5	11·0	·70	30	3 2·9	·18	6·12	·374
10 0	5 20	·5	10·8	·69	18 0	2 57·6	·17	5·98	·362
10	5 15	·5	10·6	·67	19 0	2 47·7	·16	5·61	·340
20	5 10	·5	10·4	·65	20 0	2 38·7	·15	5·31	·322
30	5 5	·5	10·2	·64	21 0	2 30·5	·13	5·04	·305
40	5 0	·5	10·1	·63	22 0	2 23·2	·12	4·79	·290
40	4 56	·4	9·9	·62	23 0	2 16·5	·11	4·57	·276
11 0	4 51	·4	9·8	·60	24 0	2 10·1	·10	4·35	·264
10	4 47	·4	9·6	·59	25 0	2 4·2	·09	4·16	·252
20	4 43	·4	9·5	·58	26 0	1 58·8	·09	3·97	·241
30	4 39	·4	9·4	·57	27 0	1 53·8	·08	3·81	·230
40	4 35	·4	9·2	·56	28 0	1 49·1	·08	3·65	·219
50	4 31	·4	9·1	·55	29 0	1 44·7	·07	3·50	·209
12 0	4 28·1	·38	9·00	·556	30 0	1 40·5	·07	3·36	·201
10	4 24·4	·37	8·86	·548	31 0	1 36·6	·06	3·23	·193
20	4 20·1	·36	8·74	·541	32 0	1 33·0	·06	3·11	·186
30	4 17·3	·35	8·63	·533	33 0	1 29·5	·06	2·99	·179
40	4 13·9	·33	8·51	·524	34 0	1 26·1	·05	2·88	·173
50	4 10·7	·32	8·41	·517	35 0	1 20·0	·05	2·78	·167
13 0	4 7·5	·31	8·30	·509	36 0	1 20·0	·05	2·68	·161
10	4 4·4	·31	8·20	·503	37 0	1 17·1	·05	2·58	·155
20	4 1·4	·30	8·10	·496	38 0	1 14·4	·05	2·49	·149
30	3 58·4	·30	8·00	·490	39 0	1 11·8	·04	2·40	·144
40	3 55·5	·29	7·89	·482	40 0	1 9·3	·04	2·32	·139
50	3 52·6	·29	7·79	·476	41 0	1 6·9	·04	2·24	·134

TABLE IV.—*continued.*

Barometer, 30 in. + when above. — when below. | TABLE OF REFRACTIONS. | Thermometer, 50°. — when above. + when below.

App. Alt.	Refr. B. 30. Th. 50°.	Difference to be allowed for.			App. Alt.	Refr. B. 30. Th. 50°.	Difference to be allowed for.		
		1′ Alt.	+ 1 B.	—1°Th.			1′ Alt.	+ 1 B.	—1°Th.
°	′ ″	″	″	″	°	′ ″	″	″	″
40	1 9·3	·040	2·32	·139	70	0 21·2	·020	·71	·043
41	1 6·9	·040	2·24	·134	71	0 19·9	·020	·67	·040
42	1 4·6	·038	2·16	·130	72	0 18·8	·019	·63	·038
43	1 2·4	·036	2·09	·125	73	0 17·7	·018	·59	·036
44	1 0·3	·034	2·02	·120	74	0 16·6	·018	·56	·033
45	0 58·1	·034	1·94	·117	75	0 15·5	·018	·52	·031
46	0 56·1	·033	1·88	·112	76	0 14·4	·018	·48	·029
47	0 54·2	·032	1·81	·108	77	0 13·4	·017	·45	·027
48	0 52·3	·031	1·75	·104	78	0 12·3	·017	·41	·025
49	0 50·5	·030	1·69	·101	79	0 11·2	·017	·38	·023
50	0 48·8	·029	1·63	·097	80	0 10·2	·017	·34	·021
51	0 47·1	·028	1·58	·094	81	0 9·2	·017	·31	·018
52	0 45·4	·027	1·52	·090	82	0 8·2	·017	·27	·016
53	0 43·8	·026	1·47	·088	83	0 7·1	·017	·24	·014
54	0 42·2	·026	1·41	·085	84	0 6·1	·017	·20	·012
55	0 40·8	·025	1·36	·082	85	0 5·1	·017	·17	·010
56	0 39·3	·025	1·31	·079	86	0 4·1	·017	·14	·008
57	0 37·8	·025	1·26	·076	87	0 3·1	·017	·10	·006
58	0 36·4	·024	1·22	·073	88	0 2·0	·017	·07	·004
59	0 35·0	·024	1·17	·070	89	0 1·0	·017	·03	·002
60	0 33·6	·023	1·12	·067					
61	0 32·3	·022	1·08	·065					
62	0 31·0	·022	1·04	·062					
63	0 29·7	·021	·99	·060					
64	0 28·4	·021	·95	·057					
65	0 27·2	·020	·91	·055					
66	0 25·9	·020	·87	·052					
67	0 24·7	·020	·83	·050					
68	0 23·5	·020	·79	·047					
69	0 22·4	·020	·75	·045					

TABLE V.

Contraction of Semidiameters of ☉ and ☽ from Refraction.

Inclin. of Semid. to Horizon.	App. Alt. of ☉ or ☽.						
	° 7	° 10	° 12	° 14	° 20	° 32	° 20
°	"	"	"	"	"	"	"
0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0
15	2	1	0	0	0	0	0
24	3	1	1	1	0	0	0
30	4	2	1	1	1	0	0
36	5	3	2	1	1	0	0
42	6	4	2	2	1	0	0
48	8	4	3	2	1	0	0
54	9	5	4	3	1	1	0
60	11	6	4	3	2	1	0
66	12	6	5	3	2	1	0
72	13	7	5	4	2	1	0
90	14	8	5	4	2	1	0

TABLE VI.

☉ Semidiameter.

Days.	Jan.		July.	
	'	"	'	"
1	16	18	15	46
11	16	17	15	46
21	16	17	15	46
	Feb.		August.	
1	16	15	15	47
11	16	13	15	49
21	16	11	15	51
	March.		Sept.	
1	16	10	15	53
11	16	7	15	56
21	16	4	15	58
	April.		Oct.	
1	16	1	16	1
11	15	58	16	3
21	15	55	16	7
	May.		Nov.	
1	15	53	16	9
11	15	51	16	12
21	15	49	16	14
	June.		Dec.	
1	15	48	16	16
11	15	46	16	17
21	15	46	16	18

TABLE VII.

AUGMENTATION OF ☽'s SEMIDIAMETER ACCORDING TO HER INCREASE IN ALTITUDE.

The Moon's horizontal semidiameter is found in page 3 of each month in the Nautical Almanac, for every day at mean noon and midnight at Greenwich; and the Sun's in page 2, for every mean noon.

Moon's app. Altitude.	Horizontal Semidiameter.					
	14′ 30″	15′ 0″	15′ 30″	16′ 0″	16′ 30″	17′ 0″
°	″	″	″	″	″	″
0	0·00	0·00	0·00	0·00	0·00	0·00
3	0·71	0·75	0·80	0·86	0·92	0·97
6	1·41	1·50	1·60	1·71	1·83	1·94
9	2·11	2·25	2·40	2·56	2·73	2·90
12	2·81	3·00	3·20	3·41	3·63	3·86
15	3·50	3·74	3·99	4·25	4·52	4·80
18	4·17	4·46	4·76	5·07	5·39	5·73
21	4·84	5·18	5·52	5·89	6·26	6·65
24	5·49	5·88	6·27	6·68	7·11	7·54
27	6·13	6·56	7·00	7·46	7·93	8·42
30	6·75	7·23	7·71	8·22	8·74	9·28
33	7·35	7·88	8·40	8·96	9·52	10·12
36	7·93	8·50	9·07	9·67	10·28	10·92
39	8·49	9·10	9·72	10·36	11·02	11·66
42	9·03	9·68	10·34	11·02	11·72	12·44
45	9·55	10·23	10·93	11·65	12·39	13·15
48	10·05	10·76	11·49	12·25	13·03	13·83
51	10·52	11·26	12·02	12·81	13·63	14·46
54	10·95	11·72	12·52	13·34	14·19	15·06
57	11·35	12·15	12·98	13·83	14·72	15·62
60	11·72	12·55	13·40	14·29	15·20	16·13
63	12·06	12·91	13·79	14·70	15·64	16·60
66	12·37	13·24	14·14	15·08	16·04	17·03
69	12·64	13·53	14·46	15·41	16·39	17·40
72	12·88	13·79	14·73	15·70	16·70	17·73
75	13·08	14·01	14·96	15·95	16·96	18·01
78	13·24	14·18	15·15	16·15	17·18	18·24
81	13·37	14·32	15·30	16·31	17·35	18·42
84	13·46	14·42	15·41	16·42	17·47	18·55
87	13·52	14·48	15·47	16·49	17·54	18·62
90	13·54	14·50	15·49	16·51	17·57	18·65

TABLE VIII.

PARALLAX OF THE SUN ON THE FIRST DAY OF EACH MONTH, THE MEAN HORIZONTAL PARALLAX BEING 8″·60.

Altitude.	Jan.	Feb. Dec.	March Nov.	April Oct.	May Sept.	June Aug.	July.
°	″	″	″	″	″	″	″
90	0·00	0·00	0·00	0·00	0·00	0·00	0·00
85	0·76	0·76	0·76	0·75	0·74	0·74	0·74
80	1·52	1·52	1·51	1·49	1·48	1·47	1·47
75	2·26	2·26	2·25	2·23	2·21	2·19	2·19
70	2·99	2·98	2·97	2·94	2·92	2·90	2·89
65	3·70	3·69	3·67	3·63	3·60	3·58	3·57
60	4·37	4·36	4·34	4·30	4·26	4·24	4·23
55	5·02	5·01	4·98	4·93	4·89	4·86	4·85
50	5·62	5·61	5·58	5·53	5·48	5·45	5·44
45	6·19	6·17	6·13	6·08	6·03	5·99	5·98
40	6·70	6·68	6·64	6·59	6·53	6·49	6·48
35	7·17	7·15	7·11	7·04	6·99	6·94	6·93
30	7·58	7·56	7·51	7·45	7·39	7·34	7·33
25	7·93	7·91	7·86	7·79	7·73	7·68	7·67
20	8·22	8·20	8·15	8·08	8·01	7·97	7·95
15	8·45	8·43	8·38	8·30	8·24	8·19	8·17
10	8·62	8·59	8·54	8·47	8·40	8·35	8·33
5	8·73	8·69	8·64	8·56	8·50	8·44	8·42
0	8·75	8·73	8·67	8·60	8·53	8·48	8·46

The Sun's Horizontal Parallax is also given for every ten days, in the Nautical Almanac, immediately before the ephemeris of the planets.

The Sun's Parallax in Altitude, for every degree, is given in the last of Dr. Pearson's "Solar Tables," vol. i. page 180.

TABLE IX.

REDUCTION OF THE MOON'S EQUATORIAL HORIZONTAL PARALLAX TO THE HORIZONTAL PARALLAX IN ANY LATITUDE.

Latitude.	HORIZONTAL PARALLAX.				
	54′	56′	58′	60′	62′
°	″	″	″	″	″
0	0·0	0·0	0·0	0·0	0·0
8	0·2	0·2	0·2	0·2	0·2
16	0·8	0·8	0·9	0·9	0·9
20	1·3	1·3	1·4	1·4	1·5
24	1·8	1·9	1·9	2·0	2·0
28	2·4	2·5	2·6	2·6	2·7
32	3·0	3·1	3·3	3·4	3·5
36	3·7	3·9	4·0	4·1	4·3
40	4·5	4·6	4·8	5·0	5·1
44	5·2	5·4	5·6	5·8	6·0
48	6·0	6·2	6·4	6·6	6·8
52	6·7	7·0	7·2	7·4	7·6
56	7·4	7·7	8·0	8·2	8·5
60	8·1	8·4	8·7	9·0	9·3
64	8·7	9·1	9·4	9·7	10·0
68	9·3	9·6	10·0	10·3	10·6
72	9·8	10·1	10·4	10·8	11·2
76	10·2	10·6	10·9	11·3	11·7
84	10·7	11·1	11·5	11·9	12·0
90	10·8	11·2	11·6	12·0	12·4

The Moon's Horizontal Parallax, given in the third page of each month in the Nautical Almanac for noon and midnight, is the equatorial parallax for Greenwich mean noon and midnight; from thence it is to be deduced for the time and place of observation. The correction for latitude, on account of the spherical figure of the earth, is seldom thought of at sea, but can be made from the table above. Thus, supposing the hor. equat. par. to be 58′; the hor. par. in lat. 52° would be 58″·2 = 57′ 52″·8.

This reduced hor. par. is to be farther corrected for altitude by means of tables for that purpose (see Pearson, vol. i. pages 188 to 196: and Riddle, pages 156* to 173); or by the following rule:—sin hor. par. × cos alt. = sin par. in alt.

* Riddle's tables are for clearing the lunar distance, and the corrections are for both parallax and refraction.

TABLE X.

PARALLAX OF THE PLANETS IN ALTITUDE.

App. Alt.	PLANET'S HORIZONTAL PARALLAX.															
	″ 1	″ 3	″ 5	″ 7	″ 9	″ 11	″ 13	″ 15	″ 17	″ 19	″ 21	″ 23	″ 25	″ 27	″ 29	″ 31
°	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
10	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	30
20	1	3	5	7	9	10	12	14	16	18	20	22	24	25	27	29
25	1	3	5	6	8	10	12	14	15	17	19	21	23	24	26	28
30	1	3	4	6	8	10	11	13	15	16	18	20	22	23	25	27
33	1	2	4	6	8	9	11	13	14	16	18	19	21	23	24	26
36	1	2	4	6	7	9	11	12	14	15	17	19	20	22	23	25
39	1	2	4	5	7	9	10	12	13	15	16	18	19	21	23	24
42	1	2	4	5	7	8	10	11	13	14	16	17	19	20	22	23
45	1	2	4	5	6	8	9	11	12	13	15	16	18	19	21	22
48	1	2	3	5	6	7	9	10	11	13	14	15	17	18	19	21
51	1	2	3	4	6	7	8	9	11	12	13	14	16	17	18	20
54	1	2	3	4	5	6	8	9	10	11	12	14	15	16	17	18
57	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17
60	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
63	0	1	2	3	4	5	6	7	8	9	10	10	11	12	13	14
66	0	1	2	3	4	4	5	6	7	8	9	9	10	11	12	13
69	0	1	2	3	3	4	5	5	6	7	8	8	9	10	10	11
72	0	1	2	2	3	3	4	5	5	6	6	7	8	8	9	10
75	0	1	1	2	2	3	3	4	4	5	5	6	6	7	8	8
78	0	1	1	1	2	2	3	3	4	4	4	5	5	6	6	6
81	0	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5
84	0	0	1	1	1	1	1	2	2	2	2	2	3	3	3	3
87	0	0	0	0	0	1	1	1	1	1	1	1	1	1	2	2
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The Parallaxes and Semidiameters of the Planets are given in the "Nautical Almanac."

TABLE XI.

DIP OF THE SEA HORIZON.

Height of the Eye in Feet.	Dip.	Height of the Eye in Feet.	Dip.	Height of the Eye in Feet.	Dip.	Height of the Eye in Feet.	Dip.
	′ ″		′ ″		′ ″		′ ″
1	0 59	18	4 11	35	5 49	86	9 8
2	1 24	19	4 17	38	6 4	89	9 17
3	1 42	20	4 24	41	6 18	92	9 26
4	1 58	21	4 31	44	6 32	95	9 36
5	2 12	22	4 37	47	6 45	98	9 45
6	2 25	23	4 43	50	6 58	101	9 54
7	2 36	24	4 49	53	7 10	104	10 2
8	2 47	25	4 55	56	7 22	107	10 11
9	2 57	26	5 1	59	7 34	110	10 19
10	3 7	27	5 7	62	7 45	113	10 28
11	3 16	28	5 13	65	7 56	116	10 36
12	3 25	29	5 18	68	8 7	119	10 44
13	3 33	30	5 24	71	8 18	122	10 52
14	3 41	31	5 29	74	8 28	125	11 0
15	3 49	32	5 34	67	8 38	128	11 8
16	3 56	33	5 39	80	8 48	131	11 16
17	4 4	34	5 44	83	8 58	134	11 24

TABLE XII.

DIP OF THE SEA HORIZON AT DIFFERENT DISTANCES FROM IT.

Distance in Miles.	Height of the Eye in Feet.					
	5	10	15	20	25	30
	′	′	′	′	′	′
0·25	11	22	34	45	56	68
0·5	6	11	17	22	28	34
0·75	4	8	12	15	19	23
1·0	4	6	9	12	15	17
1·25	3	5	7	9	12	14
1·5	3	4	6	8	10	12
2·0	2	3	5	6	8	10
2·5	2	3	5	6	7	8
3·0	2	3	4	5	6	7
3·5	2	3	4	5	6	6
4·0	2	3	4	4	5	6
5·0	2	3	4	4	5	5
6·0	2	3	4	4	5	5

TABLE XIII.

FOR THE REDUCTION OF THE MERIDIAN,

Showing the value of $A = \dfrac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$.

Sec.	0m.	1m.	2m.	3m.	4m.	5m.	6m.	7m.	8m.	9m.	10m.	11m.	12m.	13m.	14m
	″	″	″	″	″	″	′	″	″	″	″	″	″	″	″
0	0·0	2·0	7·8	17·7	31·4	49·1	70·7	96·2	125·7	159·0	196·3	237·5	282·7	331·8	384·7
1	0·0	2·0	8·8	17·9	31·7	49·4	71·1	96·7	126·2	159·6	197·0	238·3	283·5	332·6	385·6
2	0·0	2·1	8·1	18·1	31·9	49·7	71·5	97·1	126·7	160·2	197·6	239·0	284·2	333·4	386·6
3	0·0	2·2	8·2	18·3	32·2	50·1	71·9	97·6	127·2	160·8	198·3	339·7	285·0	334·3	387·5
4	0·0	2·2	8·4	18·5	32·5	50·4	72·3	98·0	127·8	161·4	198·9	240·4	285·8	335·2	388·4
5	0·0	2·3	8·5	18·7	32·7	50·7	72·7	98·5	128·3	162·0	199·6	241·2	286·6	336·0	389·3
6	0·0	2·4	8·7	18·9	33·0	51·1	73·1	99·0	128·8	162·6	200·3	241·9	287·4	336·9	390·2
7	0·0	2·4	8·8	19·1	33·3	51·4	73·5	99·4	129·3	163·2	200·9	242·6	288·2	337·7	391·1
8	0·0	2·5	8·9	19·3	33·5	51·7	73·9	99·9	129·9	163·8	201·6	243·3	289·0	338·6	392·1
9	0·0	2·6	9·1	19·5	33·8	52·1	74·3	100·4	130·4	164·4	202·2	244·1	289·8	339·4	393·0
10	0·1	2·7	9·2	19·7	34·1	52·4	74·7	100·8	131·0	165·0	202·9	244·8	290·6	340·3	393·9
11	0·1	2·7	9·4	19·9	34·4	52·7	75·1	101·3	131·5	165·6	203·6	245·5	291·4	341·2	394·8
12	0·1	2·8	9·5	20·1	34·6	53·1	75·5	101·8	132·0	166·2	204·2	246·3	292·2	342·0	395·8
13	0·1	2·9	9·6	20·3	34·9	53·4	75·9	102·3	132·6	166·8	204·9	247·0	293·0	342·9	396·7
14	0·1	3·0	9·8	20·5	35·2	53·8	76·3	102·7	133·1	167·4	205·6	247·7	293·8	343·7	397·6
15	0·1	3·1	9·9	20·7	35·5	54·1	76·7	103·2	133·6	168·0	206·3	248·5	294·6	344·6	398·6
16	0·1	3·1	10·1	20·9	35·7	54·5	77·1	103·7	134·2	168·6	206·9	249·2	295·4	345·5	399·5
17	0·2	3·2	10·2	21·2	36·0	54·8	77·5	104·2	134·7	169·2	207·6	249·9	296·2	346·4	400·5
18	0·2	3·3	10·4	21·4	36·3	55·1	77·9	104·6	135·3	169·8	208·3	250·7	297·0	347·2	401·4
19	0·2	3·4	10·5	21·6	36·6	55·5	78·3	105·1	135·8	170·4	208·9	251·4	297·8	348·1	402·3
20	0·2	3·5	10·7	21·8	36·9	55·8	78·8	105·6	136·3	171·0	209·6	252·2	298·6	349·0	403·3
21	0·2	3·6	10·8	22·0	37·2	56·2	79·2	106·1	136·9	171·6	210·3	253·0	299·4	349·8	404·2
22	0·3	3·7	11·0	22·3	37·4	56·5	79·6	106·6	137·4	172·2	211·0	253·6	300·2	350·7	405·1
23	0·3	3·8	11·2	22·5	37·7	56·9	80·0	107·0	138·0	172·9	211·7	254·4	301·0	351·6	406·0
24	0·3	3·8	11·3	22·7	38·0	57·3	80·4	107·5	138·5	173·5	212·3	255·1	301·8	352·5	407·0
25	0·3	3·9	11·5	22·9	38·3	57·6	80·8	108·0	139·1	174·1	213·0	255·9	302·6	353·3	408·0
26	0·4	4·0	11·6	23·1	38·6	58·0	81·3	108·5	139·6	174·7	213·7	256·6	303·5	354·2	408·9
27	0·4	4·1	11·8	23·4	38·9	53·3	81·7	109·0	140·2	175·3	214·4	257·4	304·3	355·1	409·9
28	0·4	4·2	11·9	23·6	39·2	58·7	82·1	109·5	140·7	175·9	215·1	258·1	305·1	356·0	410·8
29	0·5	4·3	12·1	23·8	39·5	59·0	82·5	110·0	141·3	176·6	215·8	258·9	305·9	356·9	411·7
30	0·5	4·4	12·3	24·0	39·8	59·4	83·0	110·4	141·8	177·2	216·4	259·6	306·7	357·7	412·7
31	0·5	4·5	12·4	24·3	40·1	59·8	83·4	110·9	142·4	177·8	217·1	260·4	307·5	358·6	413·6
32	0·6	4·6	12·6	24·5	40·3	60·1	83·8	111·4	143·0	178·4	217·8	261·1	308·4	359·5	414·6
33	0·6	4·7	12·8	24·7	40·6	60·5	84·2	111·9	143·5	179·0	218·5	261·9	309·2	360·4	415·5
34	0·6	4·8	12·9	25·0	[illegible]	60·8	84·7	112·4	144·1	179·7	219·2	262·6	310·0	361·3	416·5
35	0·7	4·9	13·1	25·2	41·2	61·2	85·1	112·9	144·6	180·3	219·9	263·4	310·8	362·2	417·5
36	0·7	5·0	13·3	25·4	41·5	61·6	85·5	113·4	145·2	180·9	220·6	264·1	311·6	363·1	418·4
37	0·7	5·1	13·4	25·7	41·8	61·9	86·0	113·9	145·8	181·6	221·3	264·9	312·5	364·0	419·3
38	0·8	5·2	13·6	25·9	42·1	62·3	86·4	114·4	146·3	182·2	222·0	265·7	313·3	364·8	420·3
39	0·8	5·3	13·8	26·2	42·5	62·7	86·8	114·9	146·9	182·8	222·7	266·4	314·1	365·7	421·3
40	0·9	5·4	14·0	26·4	42·8	63·0	87·3	115·4	147·5	183·5	223·4	267·2	315·0	366·6	422·2
41	0·9	5·6	14·1	26·6	43·1	63·4	87·7	115·9	148·0	184·1	224·1	267·9	315·8	367·5	423·2
42	1·0	5·7	14·3	26·9	43·4	63·8	88·1	116·4	148·6	184·7	224·8	268·7	316·6	368·4	424·2
43	1·0	5·8	14·5	27·1	43·7	64·2	88·6	116·9	149·2	185·4	225·5	269·5	317·4	369·3	425·1
44	1·1	5·9	14·7	27·4	44·0	64·5	89·0	117·4	149·7	186·0	226·2	270·3	318·3	370·2	426·1
45	1·1	6·0	14·8	27·6	44·3	64·9	89·5	117·9	150·3	186·6	226·9	271·0	319·1	371·1	427·0
46	1·2	6·1	15·0	27·9	44·6	65·3	89·9	118·4	150·9	187·3	227·6	271·8	319·9	372·0	428·0
47	1·2	6·2	15·2	28·1	44·9	65·7	90·3	118·9	151·5	187·9	228·3	272·6	320·8	372·9	429·0
48	1·3	6·4	15·4	28·3	45·2	66·0	90·8	119·5	152·0	188·5	229·0	273·3	321·6	373·8	429·9
49	1·3	6·5	15·6	28·6	45·5	66·4	91·2	120·0	152·6	189·2	229·7	274·1	322·4	374·7	430·9
50	1·4	6·6	15·8	28·8	45·9	66·8	91·7	120·5	153·2	189·8	230·4	274·9	323·3	375·6	431·9
51	1·4	6·7	15·9	29·1	46·2	67·2	92·1	121·0	153·8	190·5	231·1	275·6	324·1	376·5	432·8
52	1·5	6·8	16·1	29·4	46·5	67·6	92·6	121·5	154·4	191·1	231·8	276·4	325·0	377·4	433·8
53	1·5	7·0	16·3	29·6	46·8	68·0	93·0	122·0	154·9	191·8	232·5	277·2	325·8	378·3	434·8
54	1·6	7·1	16·5	29·9	47·1	68·3	93·5	122·5	155·5	192·4	233·2	278·0	326·7	379·3	435·8
55	1·6	7·2	16·7	30·1	47·5	68·7	93·9	123·1	156·1	193·1	234·0	278·8	327·5	380·2	436·7
56	1·7	7·3	16·9	30·4	47·8	69·1	94·4	123·6	156·7	193·7	234·7	279·5	328·4	381·1	437·7
57	1·8	7·5	17·1	30·6	48·1	69·5	94·8	124·1	157·3	194·4	235·4	280·3	329·2	382·0	438·7
58	1·9	7·6	17·3	30·9	48·4	69·9	95·3	124·6	157·8	195·0	236·1	281·1	330·0	382·9	439·7
59	1·9	7·7	17·5	31·1	48·8	70·3	95·7	125·1	158·4	195·7	236·8	281·9	330·9	383·8	440·6

Table XVIII. of Mr. Baily extends to 36 minutes from the meridian.

TABLE XIV.

TO COMPUTE THE EQUATION OF EQUAL ALTITUDES.

Interval.	Log. A.	Log. B.	Interval.	Log. A.	Log. B.	Interval.	Log. A.	Log. B.	Interval.	Log. A.	Log. B.
H. M.			H. M.			H. M.			H. M.		
2 0	7·7297	7·7146	4 0	7·7447	7·6823	6 0	7·7703	7·6198	8 0	7·8072	7·5062
2	·7298	·7143	2	·7451	·6815	2	·7708	·6184	2	·8079	·5036
4	·7300	·7139	4	·7454	·6807	4	·7713	·6170	4	·8086	·5010
6	·7302	·7136	6	·7458	·6800	6	·7719	·6156	6	·8094	·4983
8	·7304	·7132	8	·7461	·6792	8	·7724	·6142	8	·8101	·4957
10	·7305	·7128	10	·7464	·6784	10	·7729	·6127	10	·8108	·4930
12	·7307	·7125	12	·7468	·6776	12	·7735	·6113	12	·8116	·4902
14	·7309	·7121	14	·7472	·6768	14	·7740	·6098	14	·8123	·4874
16	·7311	·7117	16	·7475	·6759	16	·7745	·6083	16	·8130	·4846
18	·7313	·7113	18	·7479	·6751	18	·7751	·6068	18	·8138	·4818
20	·7315	·7109	20	·7482	·6743	20	·7756	·6053	20	·8145	·4789
22	·7317	·7105	22	·7486	·6734	22	·7762	·6038	22	·8153	·4760
24	·7319	·7101	24	·7490	·6726	24	·7767	·6023	24	·8160	·4731
26	·7321	·7097	26	·7494	·6717	26	·7773	·6007	26	·8168	·4701
28	·7323	·7092	28	·7497	·6708	28	·7779	·5991	28	·8176	·4671
30	·7325	·7088	30	·7501	·6700	30	·7784	·5975	30	·8183	·4640
32	·7327	·7083	32	·7505	·6691	32	·7790	·5959	32	·8191	·4609
34	·7329	·7079	34	·7509	·6682	34	·7796	·5943	34	·8199	·4578
36	·7331	·7075	36	·7513	·6673	36	·7801	·5927	36	·8206	·4546
38	·7333	·7070	38	·7517	·6663	38	·7807	·5910	38	·8214	·4514
40	·7336	·7065	40	·7521	·6654	40	·7813	·5894	40	·8222	·4482
42	·7338	·7061	42	·7525	·6645	42	·7819	·5877	42	·8230	·4449
44	·7340	·7056	44	·7529	·6635	44	·7825	·5860	44	·8238	·4415
46	·7342	·7051	46	·7533	·6626	46	·7831	·5843	46	·8246	·4381
48	·7345	·7046	48	·7537	·6616	48	·7836	·5825	48	·8254	·4347
50	·7347	·7041	50	·7541	·6606	50	·7842	·5808	50	·8262	·4312
52	·7349	·7036	52	·7545	·6597	52	·7848	·5790	52	·8270	·4277
54	·7352	·7031	54	·7549	·6587	54	·7854	·5772	54	·8278	·4241
56	·7354	·7026	56	·7553	·6577	56	·7860	·5754	56	·8286	·4205
58	·7357	·7021	58	·7557	·6567	58	·7867	·5736	58	·8294	·4168
3 0	·7359	·7015	5 0	·7562	·6556	7 0	·7873	·5717	9 0	·8302	·4131
2	·7362	·7010	2	·7566	·6546	2	·7879	·5699	2	·8311	·4093
4	·7364	·7005	4	·7570	·6536	4	·7885	·5680	4	·8319	·4055
6	·7367	·6999	6	·7575	·6525	6	·7891	·5661	6	·8328	·4016
8	·7369	·6993	8	·7579	·6514	8	·7898	·5641	8	·8336	·3977
10	·7372	·6988	10	·7583	·6504	10	·7904	·5622	10	·8344	·3937
12	·7374	·6982	12	·7588	·6493	12	·7910	·5602	12	·8353	·3896
14	·7377	·6976	14	·7592	·6482	14	·7916	·5582	14	·8361	·3855
16	·7380	·6970	16	·7597	·6471	16	·7923	·5562	16	·8370	·3813
18	·7383	·6964	18	·7601	·6460	18	·7929	·5542	18	·8378	·3771
20	·7386	·6958	20	·7606	·6448	20	·7936	·5522	20	·8387	·3728
22	·7388	·6952	22	·7610	·6437	22	·7942	·5501	22	·8396	·3684
24	·7391	·6946	24	·7615	·6425	24	·7949	·5480	24	·8404	·3639
26	·7394	·6940	26	·7620	·6414	26	·7955	·5459	26	·8413	·3594
28	·7397	·6934	28	·7624	·6402	28	·7962	·5437	28	·8422	·3548
30	·7400	·6927	30	·7629	·6390	30	·7969	·5416	30	·8430	·3501
32	·7403	·6921	32	·7634	·6378	32	·7975	·5394	32	·8439	·3454
34	·7406	·6914	34	·7638	·6366	34	·7982	·5372	34	·8448	·3406
36	·7409	·6908	36	·7643	·6354	36	·7989	·5350	36	·8457	·3357
38	·7412	·6901	38	·7648	·6342	38	·7995	·5327	38	·8466	·3007
40	·7415	·6894	40	·7653	·6329	40	·8002	·5304	40	·8475	·3256
42	·7418	·6888	42	·7658	·6317	42	·8009	·5281	42	·8484	·3205
44	·7421	·6881	44	·7663	·6304	44	·8016	·5258	44	·8493	·3152
46	·7424	·6874	46	·7668	·6291	46	·8023	·5234	46	·8502	·3099
48	·7428	·6867	48	·7673	·6278	48	·8030	·5211	48	·8511	·3045
50	·7431	·6859	50	·7678	·6265	50	·8037	·5186	50	·8520	·2989
52	·7434	·6852	52	·7683	·6252	52	·8044	·5162	52	·8530	·2933
54	·7437	·6845	54	·7688	·6239	54	·8051	·5137	54	·8539	·2876
56	·7441	·6838	56	·7693	·6225	56	·8058	·5112	56	·8548	·2817
58	·7444	·6830	58	·7698	·6212	58	·8065	·5087	58	·8558	·2758
4 0	7·7447	7·6823	6 0	7·7703	7·6198	8 0	7·8072	7·5062	10 0	7·8567	7·2697

In Table XVI. of Mr. Baily, the Equation of equal Altitudes is given for the entire interval of 24 hours, but it is seldom required beyond the above limits.

TABLE XV.

LENGTH OF A SECOND OF LATITUDE AND LONGITUDE IN FEET ON THE SURFACE OF THE EARTH, THE COMPRESSION BEING TAKEN AS $\frac{1}{300}$.

Lat.	Seconds of Longitude.	Seconds of Latitude.	Lat.	Seconds of Longitude.	Seconds of Latitude.	Lat.	Seconds of Longitude.	Seconds of Latitude.
0	101·42	101 42	25	91·97	101·60	50	65·32	102·02
1	101·40		26	91·21		51	63·95	
2	101·36		27	90·43		52	62·57	
3	101·28		28	89·62		53	61·17	
4	101·17		29	88·77		54	59·75	
5	101·03	101·43	30	87·90	101·67	55	58·30	102·11
6	100·87		31	87·01		56	56·84	
7	100·67		32	86·09		57	55·37	
8	100·44		33	85·14		58	53·87	
9	100·18		34	84·17		59	52·36	
10	99 89	101·45	35	83·17	101·75	60	50·84	102·19
11	99·57		36	82·15		61	49·30	
12	99·22		37	81·10		62	47·74	
13	98·84		38	80·02		63	46·17	
14	98·43		39	78·92		64	44·58	
15	97·99	101·49	40	77·80	101·84	65	42·98	102·26
16	97·52		41	76·65		66	41·37	
17	97·02		42	75·48		67	39·74	
18	96·49		43	74·29		68	38·10	
19	95·44		44	73·07		69	36·45	
20	95·36	101·54	45	71·83	101·93	70	34·80	102·40
21	94·74		46	70·57		71	33·12	
22	94·09		47	69·29		72	31·43	
23	93·41		48	67·99		73	29·74	
24	92·70		49	66·66		74	28·04	

One second of time, at the Equator = 1521·3 feet, or 507 yards.

Puissant, calculating the compression from the measurement of the great arc in France, obtains different results on different sides of the Meridian of Paris, making it as low as $\frac{1}{250}$ on the side of the Atlantic, and $\frac{1}{300}$ to the Eastward; which latter quantity is generally assumed on the Continent.

TABLE XVI.

CORRECTIONS FOR CURVATURE AND REFRACTION.

Showing the difference of the Apparent and True Level in Feet, and Decimal parts of Feet, for Distances in Feet, Chains, and Miles.

Distances in Feet.	Correction in Feet.			Distances in Chains.	Correction in Feet.			Distances in Miles.	Correction in Feet.		
	For Curvature.	For Refraction.	For Curvature and Refraction.		For Curvature.	For Refraction.	For Curvature and Refraction.		For Curvature.	For Refraction.	For Curvature and Refraction.
100	·00024	·00004	·00020	1·0	·00010	00001	·00009	¼	·0417	·0060	·0357
150	·00054	·00008	·00046	1·5	·00024	·00003	·00021	½	·1668	·0238	·1430
200	·00096	·00013	·00083	2·0	·00042	·00006	·00036	¾	·3752	·0536	·3216
250	·00149	·00021	·00128	2·5	·00065	·00009	·00056	1	·6670	·0953	·5717
300	·00215	·00031	·00184	3·0	·00094	·00013	·00081	1½	1·5008	·2144	1·2864
350	·00293	·00042	·00251	3·5	·00128	·00018	·00110	2	2·6680	·3811	2·2869
400	·00383	·00055	·00328	4·0	·00167	·00024	·00143	2½	4·1688	·5955	3·5733
450	·00484	·00069	·00415	4·5	·00211	·00030	·00181	3	6·0030	·8561	5·1469
500	·00598	·00085	·00513	5·0	·00261	·00037	·00224	3½	8·1708	1·1673	7·0035
550	·00724	·00103	·00621	5·5	·00315	·00045	·00270	4	10·6720	1·5246	9·1474
600	·00861	·00123	·00738	6·0	·00375	·00054	·00321	4½	13·5468	1·9295	11·5773
650	·01010	·00144	·00866	6·5	·00440	·00063	·00377	5	16·6750	2·3821	14·2929
700	·01172	·00167	·01005	7·0	·00511	·00073	·00438	5½	20·1769	2·8824	17·2945
750	·01345	·00192	·01153	7·5	·00586	·00084	·00502	6	24·0120	3·4303	20·5817
800	·01531	·00219	·01312	8·0	·00667	·00095	·00572	6½	28·1809	4·0258	24·1551
850	·01728	·00247	·01481	8·5	·00753	·00108	·00645	7	32·6830	4·6690	28·0143
900	·01938	·00277	·01661	9·0	·00844	·00121	·00723	7½	37·5190	5·3599	32·1591
950	·02159	·00308	·01851	9·5	·00940	·00134	·00806	8	42·6880	6·0997	36·5883
1000	·02392	·00333	·02059	10·0	·01042	·00149	00893	8½	48·1910	6·8844	41·3066
1050	·02638	·00377	·02261	10·5	·01149	·00164	·00985	9	54·0270	7·7181	46·3089
1100	·02895	·00414	·02481	11·0	·01261	·00180	·01081	9½	60·1971	8·5996	51·5975
1150	·03164	·00452	·02712	11·5	·01378	·00197	·01181	10	66·7000	9·5286	57·1714
1200	·03445	·00492	·02953	12·0	·01501	·00214	·01287	11	80·7070	11·5296	69·1774
1250	·03738	·00534	·03204	12·5	·01628	·00233	·01395	12	96·0480	13·7211	82·3269
1300	·04043	·00578	·03465	13·0	·01761	·00252	·01509	13	112·7230	16·1033	96·6197
1350	·04361	·00623	·03738	13·5	·01899	·00271	·01628	14	130·7320	18·6760	112·0560
1400	·04689	·00670	·04019	14·0	·02043	·00292	·01751	15	150·0750	21·4393	128·6357
1450	·05030	·00719	·04311	14·5	·02191	·00313	·01878	16	170·7520	24·3931	146·3589
1500	·05383	·00769	·04614	15·0	·02345	·00335	·02010	17	192·7630	27·5376	165·2254
1550	·05748	·00821	·04927	15·5	·02504	·00358	·02146	18	216·1086	30·8727	185·2359
1600	·06125	·00875	·05250	16·0	·02668	·00381	·02287	19	240·7870	34·3981	206·3889
1650	·06514	·00931	·05583	16·5	·02837	·00405	·02432	20	266·8000	38·1143	228·6857
1700	·06914	·00988	·05926	17·0	·03012	·00430	·02582				
1750	·07327	·01047	·06280	17·5	·03192	·00456	·02736				
1800	·07752	·01107	·06645	18·0	·03377	·00482	·02895				
1850	·08188	·01170	·07018	18·5	·03567	·00509	·03058				
1900	·08637	·01234	·07403	19·0	·03762	·00537	·03225				
1950	·09098	·01300	·07798	19·5	·03963	·00566	·03397				
2000	·09570	·01367	·08203	20·0	·04169	·00596	·03573				

TABLE XVII.

REDUCTION IN LINKS AND DECIMALS UPON EACH CHAIN'S LENGTH FOR THE FOLLOWING VERTICAL ANGLES.

Angle.	Reduction.	Angle.	Reduction.	Angle.	Reduction.	Angle.	Reduction.
° ′		° ′		° ′		° ′	
3 0	·137	7 15	·800	11 45	2·095	16 0	3·874
3 15	·161	7 30	·856	12 0	2·185	16 15	3·995
3 30	·187	7 45	·913	12 15	2·277	16 30	4·118
3 45	·214	8 0	·973	12 30	2·370	16 45	4·243
4 0	·244	8 15	1·035	12 45	2·466	17 0	4·370
4 15	·275	8 30	1·098	13 0	2·553	17 15	4·498
4 30	·308	8 45	1·164	13 15	2·662	17 30	4·628
4 45	·343	9 0	1·231	13 30	2·763	17 45	4·760
5 0	·381	9 15	1·300	13 45	2·866	18 0	4·894
5 15	·420	9 30	1·371	14 0	2·970	18 15	5·030
5 30	·460	9 45	1·444	14 15	3·077	18 30	5·168
5 45	·503	10 0	1·519	14 30	3·185	18 45	5·307
6 0	·548	10 15	1·596	14 45	3·295	19 0	5·448
6 15	·594	10 30	1·675	15 0	3·407	19 15	5·591
6 30	·643	10 45	1·755	15 15	3·521	19 30	5·736
6 45	·693	11 0	1·837	15 30	3·637	19 45	5·882
7 0	·745	11 15	1·921	15 45	3·754	20 0	6·031
		11 30	2·008				

TABLE XVIII.

RATIO OF SLOPES FOR THE FOLLOWING VERTICAL ANGLES.

Angle.	To one perpendicular.	Angle.	To one perpendicular.	Angle.	To one perpendicular.	Angle.	To one perpendicular.
° ′		° ′		° ′		° ′	
0 15	229	3 35	16	8 8	7	18 26	3
0 30	115	3 49	15	8 45	6½	19 59	2¾
0 45	76	4 6	14	9 27	6	21 48	2½
1 0	57	4 24	13	9 52	5¾	23 58	2¼
1 15	46	4 45	12	10 18	5½	26 34	2
1 30	39	5 0	11½	10 47	5¼	29 44	1¾
1 45	33	5 12	11	11 19	5	33 42	1½
2 0	28	5 27	10½	11 53	4¾	38 40	1¼
2 15	25	5 42	10	12 32	4½	45 0	1
2 30	23	6 0	9½	13 15	4¼	53 8	¾
2 45	21	6 21	9	14 2	4	63 28	½
3 0	19	6 43	8½	14 55	3¾	75 58	¼
3 15	18	7 7	8	15 56	3½	78 41	⅕
3 28	17	7 36	7½	17 6	3¼		

TABLE XIX.

COMPARATIVE SCALE OF FAHRENHEIT'S, REAUMUR'S, AND THE CENTESIMAL THERMOMETERS.

Fah.	Reau. —	Cent. —	Fah.	Reau. —	Cent. —	Fah.	Reau. +	Cent. +	Fah.	Reau. +	Cent. +
°	″	″	°	″	″	°	″	″	°	″	″
0	14·2	17·8	25	3·1	3·9	50	8·0	10·0	75	19·1	23·9
1	13·8	17·2	26	2·7	3·3	51	8·4	10·6	76	19·6	24·4
2	13·3	16·7	27	2·2	2·8	52	8·9	11·1	77	20·0	25·0
3	12·9	16·1	28	1·8	2·2	53	9·3	11·7	78	20·4	25·6
4	12·5	15·6	29	1·3	1·7	54	9·8	12·2	79	20·9	26·1
5	12·0	15·0	30	0·9	1·1	55	10·2	12·8	80	21·3	26·7
6	11·6	14·4	31	0·4	0·6	56	10·7	13·3	81	21·8	27·2
7	11·1	13·9	32	0·0	0·0	57	11·1	13·9	82	22·2	27·8
				+	+						
8	10·7	13·3	33	0·4	0·6	58	11·6	14·4	83	22·7	28·3
9	10·2	12·8	34	0·9	1·1	59	12·0	15·0	84	23·1	28·9
10	9·8	12·2	35	1·3	1·7	60	12·4	15·6	85	23·6	29·4
11	9·3	11·7	36	1·8	2·2	61	12·9	16·1	86	24·0	30·0
12	8·9	11·1	37	2·2	2·8	62	13·3	16·7	87	24·4	30·6
13	8·4	10·6	38	2·7	3·3	63	13·8	17·2	88	24·9	31·1
14	8·0	10·0	39	3·1	3·9	64	14·2	17·8	89	25·3	31·7
15	7·6	9·4	40	3·6	4·4	65	14·7	18·3	90	25·8	32·2
16	7·1	8·9	41	4·0	5·0	66	15·1	18·9	91	26·2	32·8
17	6·7	8·3	42	4·4	5·6	67	15·6	19·4	92	26·7	33·3
18	6·2	7·8	43	4·9	6·1	68	16·0	20·0	93	27·1	33·9
19	5·8	7·2	44	5·3	6·7	69	16·4	20·6	94	27·6	34·4
20	5·3	6·7	45	5·8	7·2	70	16·9	21·1	95	28·0	35·0
21	4·9	6·1	46	6·2	7·8	71	17·3	21·7	96	28·4	35·6
22	4·4	5·6	47	6·7	8·3	72	17·8	22·2	97	28·9	36·1
23	4·0	5·0	48	7·1	8·9	73	18·2	22·8	98	29·3	36·7
24	3·6	4·4	49	7·6	9·4	74	18·7	23·3	99	29·8	37·2

The following formula will serve for the comparison of these Thermometers:—

$$F = \tfrac{9}{5}\,C + 32 = \tfrac{9}{4}\,R + 32$$
$$C = \tfrac{5}{9}\,(F - 32) = \tfrac{5}{4}\,R$$
$$R = \tfrac{4}{9}\,(F - 32) = \tfrac{4}{5}\,C$$

	Freezing point.	Boiling point.
Fahrenheit	32°	212°
Reaumur	0	80
Centigrade	0	100

The Logarithms answering to every degree of the graduations of the above Thermometers will be found at page 296, vol. i., of Dr. Pearson's "Practical Astronomy."

TABLE XX.

COMPARATIVE SCALE OF BAROMETERS.

English.	French.		
Inches.	Inches.	Lines.	Millimètres.
29·0	27	2·53	736·6
29·1	27	3 65	739·1
29·2	27	4·78	741·7
29·3	27	5·90	744·2
29·4	27	7·03	746·8
29·5	27	8·16	749·3
29·6	27	9·28	751·8
29·7	27	10·41	754·4
29·8	27	11·53	756·9
29·9	28	0·66	759·5
30·0	28	1·79	762·0
30·1	28	2·91	764·5
30·2	28	4·04	767·1
30·3	28	5·16	769·6
30·4	28	6·29	772·2
30·5	28	7·42	774·7
30·6	28	8·55	777·3
30·7	28	9·67	779·8
30·8	28	10·80	782·3
30·9	28	11·93	784·8
31·0	29	1·05	787·4

TABLE XXI.

FORM FOR REGISTERING DAILY METEOROLOGICAL OBSERVATIONS.

Year and Month.	Barometer. Height.			Barometer. Temp. of Mercury.			Thermometer.			Wet Thermometer.	Self-registering Thermometer.		Rain Gauge.	Remarks, including the direction and force of winds, nature of clouds (cumulus, cirrhus, &c.), and all remarkable atmospheric phenomena.
	A.M. 8	P.M. 2	P.M. 8	A.M. 8	P.M. 2	P.M. 8	A.M. 8	P.M. 2	P.M. 8	P.M. 2	Max.	Min.	Inches.	
1st	30·136	30·102	30·024	59	62	60	61	66	63	58	68	53	0·53	
2nd														
3rd														

ABSTRACT OF METEOROLOGICAL REGISTER FOR THE MONTH ENDING

Station.
Height above level of the sea.

Period. From	Period. To	State.	Barometer. A.M. 8	Barometer. P.M. 2	Barometer. P.M. 8	Thermometer. A.M. 8	Thermometer. P.M. 2	Thermometer. P.M. 8	Wet Thermometer. P.M. 2	Self-registering Thermometer. Max.	Self-registering Thermometer. Min.	Rain Gauge. Inches.	General Remarks.
...	...	Highest	30·241										Summary of all most remarkable atmospheric phenomena and prevailing winds during the different periods.
		Lowest	30·124										Barometer: highest, 30·245— 9th.; lowest, 29·521—15th.
...	...	Highest	30·262										State of winds, moon's age, &c.
		Lowest	29·994										
Mean for the Month . .			30·092	30·105	29·999								

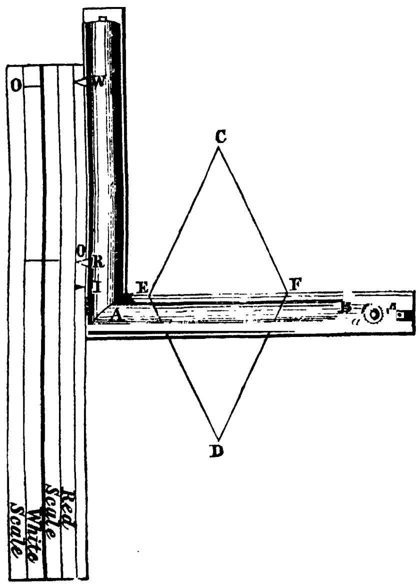
These tabular forms can of course be varied to suit different localities, where different instruments are used, or where other periods of observation may be preferred. Those given above are recommended by Sir J. Herschel where only three daily readings are recorded.

The barometer should be fixed in a good light, but one not exposed to the rays of the sun, drafts of wind, or very sudden changes of temperature. Its *actual* reading should always be recorded, leaving all corrections, for index errors, temperature of the mercury (for which a column is provided), capacity, &c., to be subsequently applied. The thermometer should be hung for observation out of doors, in a perfectly-shaded situation, but otherwise fully exposed, care being taken that it is not so placed as to be affected by reflected rays from water, buildings, or light-coloured hard soil, or by radiated heat from its proximity to the ground. The self-registering thermometer should be fastened so as to admit of one end being detached and lifted up, to allow the indexes to slide down to the extremities of the fluid columns, which is better than using a magnet for the purpose. These instruments are unfortunately very liable to get out of order.

DESCRIPTION OF THE PEDIOMETER AND COMPUTING SCALE.

THESE instruments are used for determining the areas of Plans without calculation—whereby a saving is effected of more than half the time consumed in computation, and the liability to error is very materially diminished.

THE PEDIOMETER.



The instrument consists of a square, and a graduated scale, corresponding with that of the survey.

a—The milled head, by turning which, motion is given to the brass slider B, and the two pointers R and W.

I—The index to be placed in coincidence with the — division upon the scale.

When the brass slider B is in contact with A, I coinciding with – division, and R and W pointing to O upon their respective scales, the instrument is in adjustment.

When deranged, restore it, by opening R and W to the proper distance, and then moving A and I, the former into contact with B, and the latter into coincidence with –

Required the content of the trapezium E C F D.

1st.—Place the edge A upon the point E, and open B to the point F.

2nd.—Press the square firmly down with the right hand, and with the left place the scale against the edge of it, as shown in the figure.

3rd.—Now press the scale firmly, and slide the square *up*, until the edge A B is upon the point C.

4th.—Press the square firmly, and slide the scale against its edge until – coincides with I.

Finally.—Press the scale and slide the square *down* until the edge A B is upon the point D, and taking out the numbers to which W and R point, subtract the latter from the former, and the contents in acres and decimal parts of an acre will at once be given.

The red pointer directs to the numbers that are to be taken from the red scale, and the white one to those upon the white scale.

When the pointers fall exactly upon the line engraved on the ivory edge of the scale, the folding leaf is to be doubled down to the left hand; but when the pointers fall between any two of the lines on the ivory edge, the folding leaf must then be doubled over to the right hand before the numbers are read off.

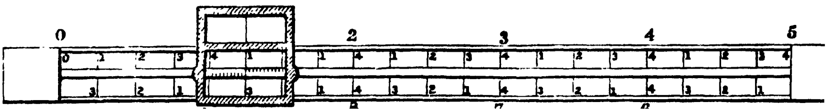
For instance, when the leaf is turned to the left and the red pointer falls between the two lines which refer to ·008 and ·013, turn the folding leaf to the right hand, and the pointer will read 0·10.

It will be found most convenient and most accurate in practice to take the shortest diagonal for the line E F.

THE COMPUTING SCALE.

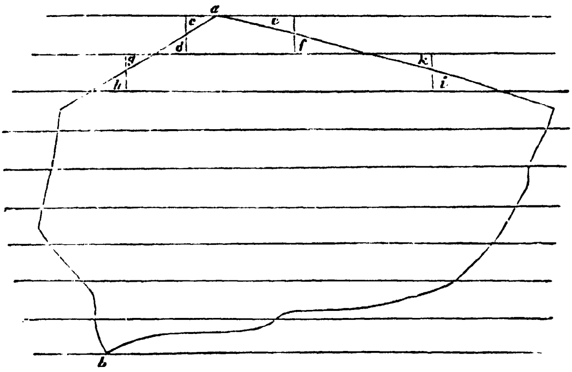
This instrument answers the same purpose of giving mechanically the contents of enclosures as the Pediometer, but is more simple in its construction and principle of operation.

It consists of a scale divided for its whole length from the zero point into divisions, each representing 2½ *chains*, and is used with a sheet of transparent tracing paper, ruled with parallel lines at equidistant intervals of *one chain*.

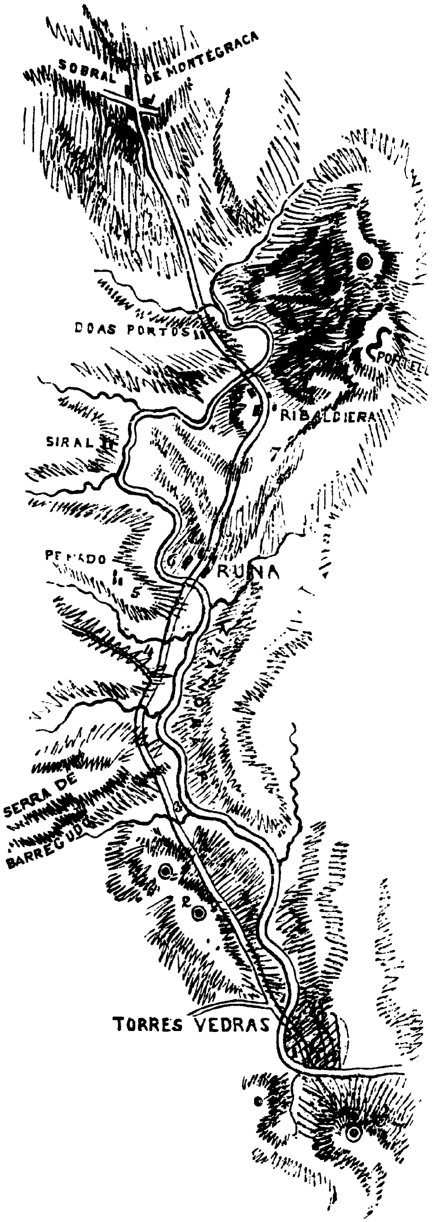


The slider B, which moves along the scale, has a wire drawn across its centre at right angles to its line of motion; and on each side of this wire a distance equal to one of the primary divisions of 2½ chains is laid off, and divided into 40 parts. It is evident, then, that during the passage of the slider over one of the divisions of 2½ chains, *one rood* has been measured between two of the parallel lines on the tracing paper; and that one of the smaller divisions would measure between the same parallels *one perch*. Four of the larger divisions give *one acre;* and the scale itself, generally made long enough to measure at once five acres, is thus used. Lay the transparent paper over the enclosure the content of which is required, in such a position that two of the ruled lines shall touch two of the exterior points of the boundaries, as at *a* and *b*.

Lay the scale, with the slider set to zero, over the tracing paper, in a direction parallel to the lines, and so placed that the portions *c* and *d* are estimated by the eye as equal to each other. Holding the scale steady, move on the sliding frame until the equality of the portions *e* and *f* are also estimated. With the slider kept at this mark, move the scale bodily down the space of one of the ruled lines (one chain), and commencing again at the left hand, estimate the equal areas of *g* and *h*, sliding the frame on to *k* and *l*. When the whole length of the scale, denoting 5 acres, is run out, commence at the right hand side, and work backwards to the left, reading the lower divisions, by which the instrument is made to measure up to 10 acres. By a continuation of this process, the contents of any sized enclosures can be obtained without calculation, and with sufficient accuracy



for general purposes if the scale is tolerably large. It would, however, expedite the measurement if the tracing paper was divided into *squares* of one chain each; the application of the computing scale need then only be made to the portions *outside* the squares, and the content added to that of the squares themselves, which is obtained by simply counting them. Where the wire of the slider coincides with any portion of the boundary between two of the parallels, no *equalisation* is of course necessary



7
6
5
4
3
2
1

Sketch of the Road between Torres Vedras and Sobral de Montegraca.

*Scale, — inches to 1 mile.**

Places on and near the Road.	Distanc in mile Inter-mediate.	Total

This form is nearly similar to that used by the Quartermaster-General's department n the Peninsula. Where more information is required to be tabulated, columns can be dded; but generally it is better to embody all other statistical details in the Report hat accompanies the sketch of the road. On a hasty reconnaisance, the object of which s principally to ascertain the practicability of any route for different arms of the service, he five last columns can be omitted. In a sketch of this nature, the ROAD is evidently he feature of paramount importance, and the ground contiguous to it is only of material onsequence in those spots that present positions for disputing its passage or embarrasing its free occupation. In calculating the number of men a village or hamlet would ontain for *one night*, five men may be allowed per house; for a longer period a coniderable reduction must be made. In the country the best guides from whom to btain information are obviously those who, from their pursuits, must be possessed of uch local knowledge, such as shepherds, pedlars, poachers, &c. In towns, reference hould be made to the local authorities for all statistical information. In addition to he field sketch of the road, a few outline sketches of the principal marked positions, ith references to the spot from which they were taken, would often prove of great ervice. These positions would, if of importance, require a separate sketch and report.

When the routes for different columns to arrive at any fixed spot at any required ime have been decided upon, separate sketches of the ground will be requisite for heir guidance. The annexed form for the "Detail of March" is taken from Captain Macauley's "Treatise on Field Fortifications."

COLUMN OR DIVISION.	GENERAL INSTRUCTIONS.
Sketch of ground, showing, by a dotted line, the route.	State the hour of marching, the formation; and describe the route by reference to letters on the sketch. Give the distances in infantry or cavalry paces, the times allowed for halts, and the time at which the column should arrive. State also the employment of the troops on arrival, &c.

Accommodation.				Water.	Forage.		Provisions		Observations.
Permanent.		On a march.							
Men.	Horses.	Men.	Horses.		Hay.	Corn.	Meat.	Bread.	
									4
									3
									2
									1

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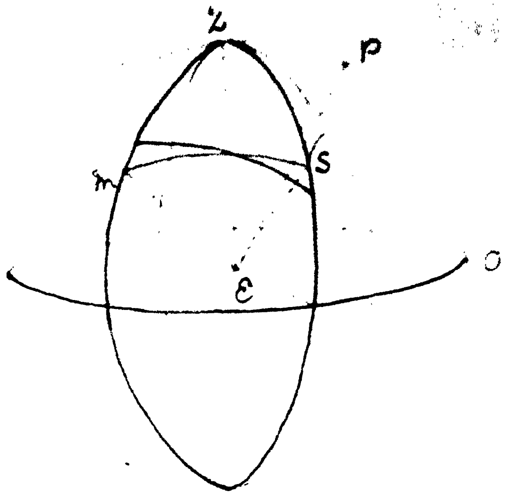
z
p
s
m
ε
o