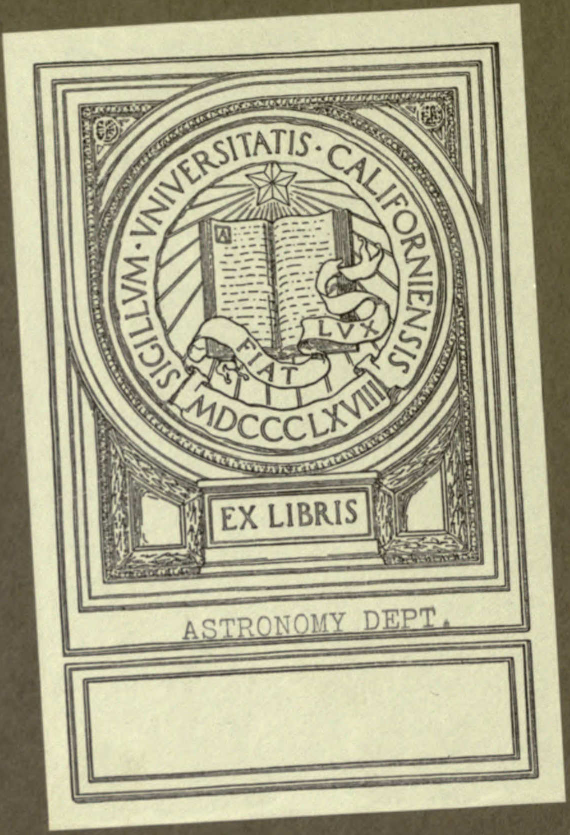

Determinations of Stellar Parallax

BY

HENRY NORRIS RUSSELL

Professor of Astronomy at Princeton University

Based upon Photographs taken at the Cambridge Observatory
by Arthur R. Hinks and the Writer

With Magnitudes and Spectra Determined at the Harvard College
Observatory under direction of Professor E. C. Pickering

WASHINGTON, D. C.
Published by the Carnegie Institution of Washington
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Determinations of Stellar Parallax

INTRODUCTION.

The observations upon which the present volume is based were made at the Cambridge Observatory (England) in the years 1903-1905, according to plans prepared for the stellar parallax work of that observatory by Mr. Arthur R. Hinks, chief assistant at the observatory, and the writer, who had been appointed a research assistant of the Carnegie Institution of Washington for the prosecution of the work whose results are here given.

It was originally planned that all the observations should be made by the writer, but in the autumn of 1904 he was disabled by serious illness, and Mr. Hinks very kindly undertook their continuance. Upon the writer's recovery, his term as a research assistant had practically expired and he received a call to America for the following year. It was then arranged that Mr. Hinks should complete the photographic work (for which he and the writer are therefore jointly responsible in nearly equal proportions—43 per cent being done by the former and 57 per cent by the latter).

The methods of observation and measurement, detailed in Chapter I—so far as they contain anything new—are also the result of collaboration between Mr. Hinks and the writer.

For the measurement and reduction of the plates, the discussion of the results, and the conclusions deduced therefrom (which together occupy the remaining chapters of this volume) the writer is alone responsible.

The magnitudes and spectra of the stars have been determined at the Harvard College Observatory, under the direction of Prof. E. C. Pickering, to whom the writer is very greatly indebted for this valuable and generous contribution.

It is also a pleasure to him to express his gratitude to Sir Robert Ball, Director of the Cambridge Observatory, and to the Observatory Syndicate, who did everything in their power to facilitate his work; to the members of the Observatory staff, for their cordial interest, and in particular to Mr. Hinks, for valuable comment and criticism and many kindnesses in addition to the invaluable collaboration already described; and finally to the authorities of Princeton University, for the time and instrumental means for completing the work there.

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CHAPTER I.

METHODS OF OBSERVATION AND MEASUREMENT.

§1. *General Policy.**

The object of the present work is the determination of the parallax of certain selected stars relative to the mean of a group of comparison-stars in their neighborhood, and incidentally of the several comparison-stars relative to their mean. In preparing plans for it two main objects were constantly in view:

a. To eliminate at any cost all known sources of systematic error.

b. To secure the most advantageous relation, consistent with the first condition, between the amount of labor to be expended and the probable accuracy of the results to be obtained.

§2. *Reasons for taking Separate Plates.*

After careful consideration, it was decided not to adopt Professor Kapteyn's plan of making exposures upon the same plate at three successive epochs of maximum parallactic displacement separated by approximately six months each. The reasons for this decision were as follows:

(1) The identification and preservation of the undeveloped plates and their readjustment upon the telescope in exactly the right position (so that the later series of star-images shall neither interfere with the earlier ones nor be too far from them) seriously increase the labor of observation.

(2) Failure to obtain satisfactory results from the exposures of any one epoch renders the whole plate useless, at least for discussion according to Kapteyn's method.

(3) It is necessary with this method to wait at least a year after beginning observations before any plates can be measured. (As the writer's appointment as a research assistant of the Carnegie Institution was for two years only, this was in itself a conclusive argument.)

That these anticipations were in accordance with the facts appears from the recently published work of Kostinsky, who finds also, in results obtained by this method, clear evidence of a systematic error (affecting the deduced parallaxes and varying with the magnitude of the stars) between the results from plates on which the first exposure was made at a maximum and at a minimum of parallactic displacement.† The source of this error appears to be the alteration of the latent image of a star during the long interval between the earlier exposure and the time of development‡ and also the influence of neighboring images, at least for the bright stars.§

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*Compare the discussion by Hinks and Russell in Monthly Notices, LXV, pp. 775–787, upon which much of this chapter is based and which is often quoted *verbatim.*

†Publ. de l' Obs. Cent. Nicolas, Série II, vol. 17, part 2 (1905), p. 138. ‡Ditto, p. 69.

§Tikhoff; Pulkovo Mittheilungen, No. 18, p. 101.

The experience of these observers therefore justifies the decision, reached before the publication of their results, with respect to the present work, namely:

> RULE I: Take separate plates at each epoch, and develop them at once.

In this case it is usually possible, when a plate turns out to be bad, to duplicate it within a few days; and, under favorable conditions, several plates can be obtained at one epoch, even under the severe restrictions demanded by parallax work.

If no observations at all can be secured before the star becomes unobservable, those of a year later will fill the gap, with no inconvenience beyond the delay. It is in fact desirable to have some stars under observation at every available season for a considerable time, as in this way the uniformity of the instrumental conditions throughout the period can be controlled.

§3. *Hour-angle Error and its Elimination.*

Of the various systematic errors which may affect the positions of photographic star-images, the most serious, from our standpoint, are those included by Professor Kapteyn* in the category of "hour-angle error." Under this head come all errors which tend to alter the relative coördinates of the images, when the same stars are observed at different hour-angles. One cause of this may be optical distortion varying with the hour-angle, which is especially to be feared when, as at Cambridge, a mirror forms part of the optical train. This may displace bright stars relatively to faint ones, or stars in one part of the plate relatively to those in another. Still more serious, because certainly unavoidable, is the influence of atmospheric dispersion. The refraction-constant for different stars varies with the mean photographically effective wave-length of their light and, as Kapteyn has suggested and Bergstrand† has recently shown by experiment, the latter varies not only with the spectral type of the stars, but with their brightness and even with the length of exposure for the same star. Hence there arise relative displacements of the star-images varying with the zenith-distance, and consequently with the hour-angle.

It is therefore of prime importance (as was first pointed out by Kapteyn‡) that in investigations of stellar parallax all the photographs of a given region shall be taken at the same hour-angle, or at least that the mean hour-angles for the epochs of positive and negative parallactic displacement shall be the same.

To observe a given star only at a fixed hour-angle means, however, that the morning observations must be begun later or the evening ones finished earlier than the most favorable dates (and in most cases both). This involves a considerable loss of parallax factor and consequently of

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*Groningen Pub. No. 1, p. 67.

‡Ibid., p. 68.

†Astr. Nachrichten 3999 and 4240.

the weight of the determination of parallax; but this is a much less serious matter than the introduction of systematic error.

The question next arises. Should this hour-angle be different for different stars, or the same for all (in which case they should obviously be observed on the meridian)?

The parallax factor in *x*, that is, in right ascension, for any star, is independent of its declination and depends only on its right ascension and the sun's longitude. Its maximum value varies slightly with the right ascension of the star, from 1.00 in 6^h and 18^h to 0.92 in 0^h and 12^h. But when the parallax factor in *x* is a maximum, the star is in quadrature with the sun, and it is usually impossible to observe it on the meridian. The available parallactic displacement for meridian observations is therefore diminished by an amount which varies with the season of the year at which the observations are made, and depends upon the star's right ascension and on the latitude of the observer.

Table 1[*] is computed for Cambridge and represents, on account of the high latitude, a somewhat extreme case. It gives for different right ascensions:

(1) The dates between which a star can be observed on the meridian, while the sun is more than 10° below the horizon.

(2) The parallax factors in *x* for these limiting dates.

(3) The total available parallactic displacement in *x*, in terms of the star's heliocentric parallax.

(4) The dates and amounts of the maximum parallactic displacements in *x* in each direction, in terms of the same unit.

TABLE 1.—*Observations on the Meridian.*

R. A.	Limiting dates.		Parallax factor.		Available displacement.	Maximum displacement.		
	Morning: after.	Evening: before.	Morning.	Evening.		Morning.	Evening.	Amount.
0^h	Aug. 2	Jan. 3	+0.65	−0.89*	1.57	June 22	Dec. 22	0.92
2	Aug. 23	Jan. 25	0.77	0.94*	1.71	July 22	Jan. 20	0.94
4	Sept. 14	Feb. 16	0.89	0.98	1.87	Aug. 22	Feb. 18	0.98
6	Oct. 4	Mar. 9	0.96	0.97	1.93	Sept. 23	Mar. 21	1.00
8	Oct. 24	Mar. 30	0.97	0.90	1.87	Oct. 25	Apr. 21	0.98
10	Nov. 15	Apr. 20	0.94*	0.79	1.73	Nov. 24	May 22	0.94
12	Dec. 9	May 9	0.88*	0.69	1.61	Dec. 22	June 22	0.92
14	Jan. 5	May 29	0.88*	0.57	1.51	Jan. 20	July 22	0.94
16	Feb. 17	June 21	0.98*	0.47	1.45	Feb. 18	Aug. 22	0.98
17	Mar. 20	July 9	0.95	0.52	1.47	Mar. 5	Sept. 7	1.00
18	Apr. 30	Aug. 4	0.77	0.71	1.48	Mar. 21	Sept. 23	1.00
19	June 3	Sept. 16	0.54	0.94	1.48	Apr. 6	Oct. 9	1.00
20	June 20	Oct. 25	0.48	0.98	1.46	Apr. 21	Oct. 25	0.98
22	July 14	Dec. 6	0.55	0.90*	1.49	May 22	Nov. 24	0.94

Sometimes one of these maxima falls between the limiting dates. In such cases the parallax factor at one of the limiting dates is not the greatest available, and is marked with an asterisk in the table.

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[*] Compare Hinks, Monthly Notices, LVIII, p. 440; and the similar tables by De Sitter for different latitudes in Groningen Pub. No. 15, pp. 5–10; which, however, are constructed for the Sun 12° below the horizon.

It will be noticed that the parallax factors for the morning and evening observations are usually unequal. By making the observations off the meridian, in an hour-angle constant for each star, but different for stars of different right ascension, symmetrical parallax factors for the morning and evening observations can be obtained.*

Table 2 gives the hour-angle for such symmetrical observations and the available parallactic displacement in terms of the heliocentric parallax:

TABLE 2.—*Observations off the Meridian.*

R. A. of star.	Hour-angle.	Sidereal time of observation.	Available displacement.	Available displacement for observation on meridian.
0^h	$-2^h\ 22^m$	$21^h\ 38^m$	1.72	1.57
2	−1 44	0 16	1.80	1.71
4	−0 42	3 18	1.89	1.87
6	0 0	6 0	1.93	1.93
8	+0 42	8 42	1.89	1.87
10	+1 44	11 44	1.80	1.73
12	+2 22	14 22	1.72	1.61
14	+2 5	16 5	1.65	1.51
16	+1 10	17 10	1.55	1.45
18	0 0	18 0	1.47	1.48
20	−1 10	18 50	1.55	1.46
22	−2 5	19 55	1.65	1.49

It appears upon comparison with the preceding table that there is on the average very little gained as regards available parallactic displacement by going off the meridian. On the other hand the stars in the neighborhood of 18^h right ascension are inconveniently crowded together as regards the sidereal time of observation, and hence also as regards the time of the year which they must be observed—which depends upon this in just the same way as in the case of meridian observations. It was therefore decided to follow the simple rule:

> RULE II: All photographs are to be taken as nearly as possible on the meridian.

§4. *Observation of Bright Stars. Color Screen.*

In almost all forms of precise astronomical observation systematic errors are to be feared when it is necessary to determine the relative position of stars of very different brightness. In observing with the meridian circle or heliometer this source of error is removed by the use of screens placed before the object-glass (or one-half of it), which reduce the apparent brightness of the brighter star to approximate equality with the fainter ones.

In photographic work a screen, if used at all, must clearly be placed near the plate, so as to affect the light of the bright star alone, while allowing unobstructed passage to that of the faint stars. At the Greenwich Observatory† this has been accomplished by the use of an occulting shutter,

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*Kapteyn, Groningen Pub. 1, p. 70.

†Monthly Notices, LIX, p. 501.

immediately in front of the plate, which is lifted to give a series of short exposures, of small aggregate duration, upon the bright star, during the progress of the long exposure upon the faint stars; and somewhat similar devices have been used by Schlesinger at the Yerkes Observatory.*

The screen designed by the writer for use in the present work is of a different character: In the plate-holder, directly in front of the sensitive plate, is inserted a screen of clear glass, upon which, in the optical center of the field, is a small patch of gelatin stained with a yellow dye. The image of the bright star is made to fall upon this patch, and suffers a large reduction in photographic brightness—depending upon the absorption of the dye used—which may amount to several magnitudes.

An experimental screen, with a gelatin film stained as above, was found to cut down the photographic brightness of a star by rather more than six magnitudes, while the sharpness of the images was satisfactory, although the screen was nothing more than a selected piece of plate glass.

It is well known that the principal effect of interposing a plane-parallel transparent plate, of thickness t and refractive index μ, in the path of a pencil of converging rays, is to set back their focus along the normal to the plate through the distance $\frac{\mu-1}{\mu}t$. A plane-parallel glass screen therefore sets back the focal plane of the telescope by about one-third of its own thickness (the refractive index of glass being about 1.5). There is in addition a very small distortion proportional to the thickness of the plate and to the cube of the angle of incidence. For a plate of refractive index 1.5 and thickness of 5 mm. the amount of the distortion is less than one ten-thousandth mm. for angles of incidence less than 2.5°. It may therefore be safely neglected in practice.

The center of projection of the field is shifted by the action of the screen by exactly the same amount as the focal plane, so that the interposition of the screen has no effect on the scale-value.

The effect of irregularities of the surfaces of the screen may easily be calculated. We may safely disregard the variations in the thickness of the screen and regard the angle between the normal to the surface at any point and the normal to the surface of a true plane-parallel plate as a small quantity, whose square may be neglected.

To this degree of approximation, the position of the focal plane is unaltered by the irregularities of the surface and the effects of irregularities of the two surfaces are independent of one another. If the thickness of the plate is t, its refractive index μ, and the distance of its inner (nearest) surface from the focal plane is s, a deflection of the normal to the outer surface by an angle θ_1 produces a displacement of the image in the focal plane of magnitude $\theta_1(\mu-1)\left(s+\frac{t}{\mu}\right)$ in the direction in which the inward normal to the

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*Schlesinger, Science, N. S., vol. xxv, p. 568.

plate is displaced, while a deflection of the normal to the inner surface by an angle θ_2 displaces the image by the amount $\theta_2(\mu-1)s$ in the direction in which the outward normal to the plate at this surface is displaced. If we assume

$$\mu=1.5 \qquad t=1.5\text{ mm.} \qquad s=0.5\text{ mm.}$$

(which closely represent the conditions for the screen actually used) we find that the displacement of the image due to the irregularities of the outer surface will be less than $\frac{1}{10,000}$ mm. if $\theta_1<\frac{1}{7500}$, that is, if $\theta_1<30''$ approximately, while for the second surface we must have $\theta_2<\frac{1}{2500}$ or $\theta_2<1'30''$ approximately. Thus it appears that the surface of the screen need be by no means optically perfect in order to avoid sensible distortion.

As the absorbing film can be made exceedingly thin, its presence will not influence these results. It should, however, be put on the side of the screen which is nearest the sensitive plate.

The method above described appears to be more convenient than the use of an occulting shutter to obscure the image of the bright star, and also than the method of dyeing the film of the sensitive plate itself (which may introduce distortion of the film). It is very easy to set the color-screen in exactly the right place by removing the back of the plate-holder and looking at the images of the stars with a low-power eye-piece. In this way it is possible to equalize the images of a bright star and a faint companion 30″ distant.

The chief disadvantage of this method—which it shares with all others depending upon the use of colored absorbing media—is that the effective wave-length of the light of the bright-star and the comparison-stars is different, and hence atmospheric dispersion comes into play with unusual force. But this is completely eliminated along with the hour-angle errors by taking all photographs upon the meridian. The rule was therefore adopted:

> RULE III: All stars brighter than the fifth magnitude are to be photographed with a color screen.

The screen which was used in the present work was made of worked glass by Messrs. Sanger, Shepherd & Co., of London. A thin film of gelatin was coated on the surface and dyed a deep orange; it was then cut down to a patch about 3 mm. square. This screen diminished the light of a star about six magnitudes. It gave very satisfactory definition (the images photographed through it being quite as sharp as those of faint stars obtained in the usual way) and left nothing to be desired as regards convenience of manipulation. It proved, however, to have one serious defect: it lacked permanence. The makers reported that they had some difficulty in getting the small gelatin patch to adhere to the worked glass surface. They finally succeeded so well that after rather more than twelve months' use (in March 1905) the patch, contracting, pulled off the face of the glass. The observations of bright stars were thus interrupted, but not before several series had been completed whose results afford a convincing proof of the accuracy and usefulness of the method. In the light of this experience it appears that color-screens of this sort should either be made of some less perishable material or be sealed in between two plates of glass. A complete outfit should consist of several screens of graduated degrees of absorption, differing by about two magnitudes. With such an equipment, the parallax of all stars, even the brightest, could be determined photographically with equal accuracy and convenience.

§5. *Other Systematic and Quasi-Systematic Errors.*

Systematic error may also arise from change of the adjustments of the instrument with which the photographs are made. For example, if the plate is not perpendicular to the optical axis of the telescope, the images near the center of the field will be displaced, relatively to those at the edge, to an extent depending upon the amount and direction of the tilt of the plate. This and other similar adjustments were controlled carefully during the progress of the work and remained satisfactory throughout.

We may now pass to those errors which, while of at least approximately random character for different plates, affect all the images of a given star on one plate alike. Perhaps the most important of these "plate-errors" is that called "guiding error" by Kapteyn.*

If the clock is not driving correctly it is probably going pretty regularly fast or slow, and the stars on the plate are constantly trailing a little in one direction and being brought back by the action of the control. Under these conditions a bright star-image is displaced relatively to a faint by an amount which becomes quite sensible before the distortion of the disk is apparent on inspection. A good electric control will correct errors as soon as they amount to two or three tenths of a second of arc, and possibly the best visual guiding may do the same. The automatic control has this advantage, that one can set the clock regulator so that the accelerating and retarding trains come into operation with nearly equal frequency, which eliminates guiding error, properly so called. A slight continuous trail in one direction has little or no effect on the relative position of the star-images.

When two plates of the same field were taken on the same night, the adjustment of the control pendulum was usually slightly changed between them to make the residual influence of the guiding error different for the two.

Distortion of the gelatin film, so far as it affects any large region of the plate, is eliminated by the use of the réseau in measurement. All that need be feared is *local* distortion, too local to be regarded as uniform inside a single réseau-square 5 mm. on a side. A bad plate of this sort can almost certainly be detected by irregularities of the réseau lines, and rejected upon mere inspection. The remedy for these errors is to separate the different exposures well and not to take too many on one plate.

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*Groningen Pub. No. 1, p. 67.

§6. *The Photographic Observations.*

The photographs were taken with the Sheepshanks Equatorial of the Cambridge Observatory, a coudé telescope of the polar siderostat type. As the instrument is of unusual design, a brief description is appropriate here.*

The main tube of the telescope is mounted in bearings near the top and at the bottom, and forms the polar axis. Toward its lower end it carries the declination axis, upon which turns a short tube carrying the object glass. Upon an axis concentric with the declination axis is borne a plane mirror, which is geared so as to bisect the angle between the polar axis and the objective tube. The light of any star toward which the latter is directed is thus, after reflection, brought to a focus at the upper end of the polar tube, which passes through the wall of the building in which the telescope is installed into a closed observing room. The rest of the telescope is protected from the weather by a light cover, which is moved away to the southward during observations, leaving the whole sky available, except for the region near the pole, which is obscured by the main building.

The mounting, which was constructed by Sir Howard Grubb, is very massive and stable, owing especially to the position of the eye-end in the axis of rotation. It is possible to strike the eye-end a smart blow with the hand without causing any displacement of the guiding star perceptible with a high power.

The object glass, by Cooke and Son of York, is a triple photo-visual combination, of 12.5 inches aperture (reduced in practice to 12 inches) and 19.3 feet focal length. It is practically perfectly achromatic. The image of a bright star is quite free from the violet glare familiar in ordinary instruments, and its spectrum, observed with a direct-vision prism over the eye-piece, is linear throughout its extent.

The plane mirror, 18 inches in diameter, was figured by the late Dr. Common, and gives very perfect definition. As an illustration of this it may be mentioned that, under good atmospheric conditions, such double stars as OΣ 156 (0″.60) and OΣ 175 (0″.65) are easily divided, the dark interval being apparently half the diameter of the disks, while ζ Boötis was repeatedly seen elongated (its distance at the time being 0″.35).

The driving clock and electric control (of the Grubb mouse-wheel pattern) are at the upper end of the polar tube, directly under the eye of the observer—an inestimable advantage in practical work. The latter not only corrects errors in the rate of the driving clock, but automatically sets it right if it gets more than a few hundredths of a second fast or slow, compared with the controlling pendulum. The rate of the latter can be varied at will, even during the exposure, to correct drift of the guide-star due to refraction, etc.

The plate-holder is mounted on a double-slide carrier, as suggested by Dr. Common, which also bears a guiding eye-piece, outside the plate, which

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*Compare the description by Sir Robert Ball in Monthly Notices, LIX, pp. 152–155, from which much of the present account is taken.

is adjustable in both coördinates and is furnished with divided scales, so that it is usually easy to find a suitable guide-star with any given object at the center of the field and, once the scale-readings for this are recorded, to set the telescope again on the same center within a few seconds of arc. Corrections to the guiding in either coördinate may be made by hand, by moving the plate-holder in its slides by the screws provided for the purpose. The performance of the clock and control was, however, so uniformly excellent that no attempt was made to guide by hand, and these screws were only used to displace the plate between the successive exposures.

The plate-holder itself is of brass, and the plate (two of whose edges are ground smooth) is held in place by springs, against metal stops, to avoid possible displacement. The proper adjustment of the plate-holder with respect to the telescope is assured by three contact pieces, which engage with a conical hole, a slot and a flat surface on the end-plate of the telescope, and is maintained by strong spring clamps.

The focal length of the objective varies considerably with the temperature, depending not only upon its value at the moment, but on its course for some time previously. It was therefore necessary to determine the focal setting for every evening of observation, and sometimes to change it during the evening. A scale attached to the guiding eye-piece made this an easy matter. The variations in the scale-value of the plates are mainly due to this cause. The mirror was dismounted and re-silvered three times during the period covered by the observations, on July 27, 1904, February 21, 1905, and about July 10, 1906. There is no evidence that this affected the accuracy of the observations in any way.

The adjustments of the instrument, and especially the perpendicularity of the plate to the optical axis, were tested from time to time and remained throughout satisfactory.

The plates are of the Barnet "Rocket" brand and are coated on "patent-plate" glass. They are of the standard Astrographic size, 6.25 inches square, which with the Cambridge telescope gives a field of 1° 28′.

Four exposures were usually made on each plate. In the intervals the plate was moved 0.5 mm. by means of the screw of the double-slide plate carrier, in the direction of the y axis (declination), except for a few double stars, for which, to avoid confusion of images, it was necessary to displace in x, or in both coördinates. The length of the exposures varied from 2 to 10 minutes, according to the brightness of the stars under investigation and the state of the sky. Most of the exposures were of 4 or 5 minutes' duration.

A standard Gautier réseau (No. 88) was impressed on all plates before development, using divergent light, from a source whose optical distance from the plate was equal to the focal length of the telescope. This procedure eliminates any error due to curvature of the plate.*

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*Hinks, Monthly Notices, LIX, p. 532.

The developer was metol, made up according to the makers' formula, and development was continued until the plates began to show traces of fog. They were fixed in a simple hypo-solution, thoroughly washed in running water, and dried in a vertical position, with the x axis horizontal.

With the normal exposure of 4 or 5 minutes, the faintest stars of the Bonn Durchmusterung are usually visible (though not well measurable) on the plates. On fields lying in or near the Milky Way, many more stars are shown on the plates than appear in the Bonn Durchmusterung; but this is not usually the case for other parts of the sky.

The working list was in the form of a card catalogue. Each star has a card on which the parallactic ellipse is drawn from the tables given by Sir David Gill (Annals of the Cape Observatory, vol. VIII). On the ellipse are marked the places corresponding to the dates up to which evening observations and from which morning observations are possible. A glance at the card shows whether circumstances are favorable or unfavorable; whether the evening observations should be put off to the last moment or may be made with equal advantage any time in the preceding month; and whether the morning observations must be got immediately after the earliest possible date or may be delayed without damage. The conditions vary so much from star to star, especially in a latitude like that of Cambridge, which is really too high for convenient parallax work, that to have diagrams always in sight is really necessary. They are made complete by a tracing from the Bonn Durchmusterung chart to identify faint stars, and by the necessary miscellaneous instructions.

§7. *Measurement of the Plates.*

Two instruments have been used for measuring the plates—the measuring machines of the Cambridge Observatory and of the Princeton University Observatory. A full description of the first has been given by Mr. Hinks in Monthly Notices, LXI, pp. 444–458. The second, though somewhat simpler in construction, is identical with it in all essential features, which alone need be described here.

The star-image is referred to the adjacent lines of the photographed réseau. Its distances from these are measured by means of a finely divided glass scale, which is itself movable by means of a micrometer screw.

A real image of the réseau-square with its contained stars, of magnification unity, is formed in the plane of the scale by an objective, and the whole is viewed by an eye-piece magnifying 20 diameters (which is equivalent to a telescopic magnifying power of 460). The divisions of the scale are 0.05 mm. apart, so that 100 of them equal the réseau-interval of 5 mm. The scale alone, therefore, makes it possible to read directly the distance of a star from the réseau lines to one one-hundredth of their interval. By moving the scale by means of the screw (which has a pitch of 0.5 mm. and a head divided into 100 parts) it is possible to measure the distance through which the scale must be moved, in order to bring first a réseau line, and later the star-image, into the middle of the nearest spaces on the scale.* The excess of the distance from the line to the image above the integral number of scale-divisions between these two spaces can thus be measured, by estimating tenths of a division on the micrometer head, to 0.0005 mm. or one one-hundredth of a scale-interval. By taking readings on both the adjacent réseau-lines, the error of "runs" due to the lack of exact agreement between a réseau-interval and 100 scale-divisions is allowed for, and the coördinates are thus obtained to 0.0001 of this interval.

Two settings were usually made on each star-image and one on each of the adjacent réseau-lines. The accidental error of setting on either is so small that it is not usually worth while to make more. When, however, the first two settings on the star-image differed by more than 0.0004 R (which corresponds to 0″.07) further settings were made; and the number of settings on réseau and image was doubled for the "parallax stars."

The whole number of réseau-intervals is read on scales attached to the frame in which the plate is carried, which is movable so that any desired réseau-square can be quickly brought into the field of the microscope. The x coördinates range from 6 to 36, and the y coördinates from 7 to 37. The measured star coördinates have therefore as a rule six significant figures. This numbering was adopted so that the optical center of the field (which is not quite at the geometrical center) might have the coördinates 20, 20. The images of the star under special investigation were always made to fall near the latter point.

§8. *Errors of the Measuring Apparatus.*

It is clear that coördinates thus measured are liable to errors of several kinds, arising from inaccuracies in the réseau, scale, or screw, from optical distortion in the measuring machine, and from personal equation of the measurer. Errors of the réseau will affect the determination of absolute places of their stars to their full amount. But the present work is purely differential and all the images of a given star are not only in a given réseau-square, but in the same part of it—their coördinates on different plates seldom varying by as much as one-fourth of a réseau-interval. The maximum influence which such errors can have on the deduced parallaxes or proper-motions is therefore but a small fraction of the division-errors of two adjacent réseau-lines—which, in view of the uniformly high accuracy of Gautier réseaux, may safely be neglected.

The micrometer screw is used only to measure distances of the order of 0.1 mm. Its progressive errors need therefore hardly be feared, but periodic errors might be troublesome. The division errors of the scale, on the contrary, may affect the measurements with their full value.

The errors of the scale and screw of the Cambridge machine have been investigated by Mr. Hinks,† and those of both machines independently by

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*The spaces, not the divisions, of the scale are numbered.

†Monthly Notices, LXI, pp. 456–457.

the writer. The screws (which were made by Messrs. Brown and Sharpe, of Philadelphia) appear in both cases to be free from sensible errors, either progressive or periodic.

The division errors of the Cambridge machine average less than 0.0005 mm.—that is, less than the least reading estimated in a single measurement—and may therefore be neglected.

The first scale furnished with the Princeton machine was much less satisfactory, one end being seriously in error. It was found, however, that if this end was not used, the error due to the scale would on the average be only 0.0004 mm., which as before may be neglected. The scale with which the makers of the instrument replaced the first one was quite satisfactory, its errors throughout its length averaging but 0.0003 mm. As different parts of the scale are used, almost at random, in setting on different stars and réseau lines, the division errors will at most do little more than increase the accidental error of the measured coördinates.

The graduations of the glass scale are on the side farthest from the eye-piece. As the rays from the plate and from the scale have identical paths from this point to the observer's eye, optical distortion in passing through the glass on which the scale is ruled, or in the eye-piece, can not affect the measures, but optical distortion due to the objective which forms the image of the plate on the scale may do so.

This distortion of the field was investigated for both machines by the writer, by measuring the distance between the same pair of réseau lines in different parts of the field, using the same scale divisions. In neither case is there any evidence of true optical distortion, such as has been discovered in machines of similar type at Greenwich and Oxford.* In the latter machines the objective is a single achromatic lens, while in the others it is a doublet (a "rapid rectilinear" camera lens), which explains their freedom from error.

In the Cambridge machine, however, the distance between the two objects, as measured by the scale, appears to change uniformly as the objects are moved across the field in the direction of the line joining them. This can not be due to optical distortion, which would produce effects of equal magnitude at equal distances on each side of the optical center. Its cause was found in a slight tilt of the glass scale, which was not parallel to the focal plane of the objective. The divisions of that part of the scale which is nearest the objective, if a scale of equal parts on the plate was projected on them, would appear too long, and those of the opposite ends too short. This error is uniformly progressive along the scale, so that the apparent lengths of the (really equal) scale-divisions form an arithmetical progression, and the apparent division errors of the scale are proportional to the square of the distance of a division from any given point of the scale.

It is easily shown that the influence of this error upon the measured coördinate of a star-image, which has been referred to the two adjacent réseau

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*Monthly Notices, LXIV, pp. 632, 643.

lines, correcting for "runs," is independent of the part of the scale used and depends only on the position of the star relative to the lines, varying as the product of its distances from the two. When the plate is turned through 180° in its own plane and remeasured (which is necessary for other reasons) the scale-reading for the star-image is affected to the same extent and in the same direction as before; but as in one case the readings increase, and in the other decrease, with increasing coördinates on the plate, the errors in the measured coördinates will be equal and of opposite sign, and their mean will be free from this error. It is therefore of no practical importance.

The Princeton machine is free from sensible error of this sort—special care having been taken by the makers (the Cambridge Scientific Instrument Company) to avoid it. It thus appears that neither measuring machine has any errors which can sensibly influence the results obtained with it, and therefore no corrections for instrumental errors are necessary.

It is, however, well known that most measurers have a systematic tendency to set farther to one side when bisecting a round star-image (or when setting it between two wires or scale-divisions) than when setting similarly on a réseau line. This error may vary with the brightness of the stars and the character of their images, and it is universally recognized that it must be got rid of, as far as possible, by turning the plate through 180° in its own plane and measuring again. In the case of symmetrical images at least, it seems safe to assume that the errors in the measured coördinates will be equal and opposite in sign in these two cases and that the mean of the two measures will be free from error.

§9. *Economy of Measurement.*

At first all the images were measured in both positions of the plate, as described above. But later it was suggested by Mr. Hinks that if this error was really constant for images of a given intensity and appearance it should be eliminated from the mean of the measures of several similar images of the same star by measuring half of them in one position only of the plate and the rest in the other. Examination of the measures of 1304 star-images on 31 plates showed that this was substantially the case. The mean coördinates of the stars, derived from measures of all four images in both positions of the plate, would on the average have been altered by only 0.00009 réseau-intervals ($0''.015$) if the first two images had been measured only in one position of the plate and the last two in the other. This is less than one-third of the average probable error of such a mean coördinate. It follows that doubling the labor of measurement leads to only a small increase of accuracy,* which does not at all repay the additional work. The rule was therefore adopted:

> RULE IV: Measure half the images upon each plate in the "direct" position in the machine and the other half in the "reversed."

When there are but three measurable exposures, it is necessary to measure one in both positions. In the few cases where, owing to gathering

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*For further details see Chapter IV, page 64.

haze or passing clouds, the successive images of the same star were not equally intense, care should be taken that the images measured in opposite positions should be as nearly comparable as possible. The work of measurement—one of the heaviest parts of the whole—was thus halved, with very little loss of accuracy.

It is possible to economize measurement still further. From diagrams of the card catalogue it appears that the available parallactic displacement for observations on the meridian is rarely more than half as great in the y coördinate (declination) as in x, so that for parallactic purposes the latter have on the average more than four times the weight of the former. It is therefore desirable to confine exact measurements to the x's alone. To be sure, we need approximate values of the y's in the reduction of the plates, but these can be obtained from rough measures of a single plate—in fact, of a single exposure—in perhaps one-twentieth of the time that it would take to measure the y's completely. It is worth while to measure the y's on a second plate belonging to the epoch farthest removed from the first, as a control, and in order to detect any large proper-motion in declination among the comparison stars. So the rule was adopted:

> RULE V: Measure the x's accurately on all plates, but the y's approximately on two plates only.

This again saves half the work of measurement and reduces it to a moderate proportion of the whole. If the weight of the y's warrants it they may be measured later, in the few cases where it is worth while.

By making the plate-carrier adjustable in position-angle it would be practicable to set the x axis in such a direction for each star that the whole available parallactic displacement would be in this coördinate, and thus get the greatest possible return as regards accuracy for a given amount of time spent in measurement. This was unfortunately impossible in the case of the present work.

§10. *Working List.*

The stars selected for observation belong to two classes:

I. Stars for which any information as to the parallax, even if it be only a superior limit, is valuable, such as visual binaries, pairs of stars with common proper-motion, variable stars, and others for which a knowledge of the parallax, even approximately, affords information of astrophysical value.

II. Stars likely to have larger parallaxes than the stars in general, such as those of large proper-motion, binaries whose apparent separation is great in proportion to their period, and the like.

A number of stars, mostly of class II, were also included, for which the investigations of previous observers gave discordant results, or which had never been investigated by methods of the highest precision.

The working lists of several observers, whose results have appeared while the present research was in progress (notably the great series of heliometer determinations at the Yale Observatory), were independently constructed on much the same plan. In consequence, an unusually large proportion of the stars here investigated have previously been observed elsewhere. In the writer's opinion, this is by no means to be regretted. The present state of the problem of determining stellar parallax is such that the greatest hope of advance, as well as the best test of the absolute accuracy of the work, lies in the comparison of results obtained by as many and as different methods as are capable of giving satisfactory precision; while in the case of individual stars the mean result of short series of observations by several observers, using different methods, whose agreement shows them to be free from serious systematic error, is entitled to more consideration than that of an extended and elaborate investigation by a single observer.

§11. *The Magnitudes and Spectra.*

The photometric and spectroscopic data contained in this work were obtained at the Harvard College Observatory. Prof. E. C. Pickering—whose generosity in offering this entirely unsolicited and very valuable information has been already recorded—describes the methods of observation as follows:

> "All the photometric magnitudes, measured for you this year, were obtained with the 12-inch meridian photometer.* An artificial star, formed by allowing the light of a Welsbach burner to shine through a small hole, is reduced to equality with the real star by a wedge of shade glass. The scale is reduced to that of the 4-inch photometer, by measuring five stars taken from H. A. 54, before and after each of your groups. The magnitudes of bright stars are taken from H. A. 50, or occasionally from H. A. 54."
>
> [The difference of magnitude of the components of the double stars in the list was determined from measures by Mr. Wendell.]
>
> "The spectrum of each star was estimated by Mrs. Fleming independently on two plates. The differences are, as you see, insensible, although it is necessary for such faint stars to use spectra only about 2 mm. long."

In some cases only one plate of the region was available, but independent estimates on two plates are recorded for 164 stars. For 134 of these the two estimates differ by less than half the interval between two adjacent spectral classes (A, F, G, K, etc.). In 25 cases the difference equals half a unit, in two cases 0.8, and it exceeds one unit in three cases, in all of which one of the estimates is recorded as doubtful. It is therefore evident that the probable error of an estimate is but a small fraction of the interval between adjacent classes. For this reason no distinction has been made in the tables between the results based on one and on two plates.

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*The observer being Professor Pickering himself.

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"The photometric magnitudes were measured by comparison with Polaris and with the meridian photometer.* An artificial star, [illegible] had been taken [illegible] each plate. The scale is referred to that of the [illegible] were taken from H. A. [illegible] before and after each of four groups. The magnitudes of bright stars are taken from H. A. [illegible] from H. A. [illegible]

The difference of magnitude of the [illegible] was determined from [illegible] by Mr. Wendell.

The spectrum of each star was estimated by Mrs. Fleming [illegible] of two plates. The difference [illegible] although it is necessary [illegible] only about [illegible]

In some cases only one plate of the region was available. For independent estimates, two plates are required for any star. For six of these the two estimates differ by more than half the interval between two adjacent spectral classes, A, F, G, K, and M. In no case is the difference equal to a whole class, and it generally is less. In all these cases, in all of which one of the estimates is regarded as doubtful. It is therefore evident that the probable error of an estimate is but a small fraction of the interval between adjacent classes. For this reason no distinction has been made in the tables between the results based on one and on two plates."

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* [illegible]

CHAPTER II.

REDUCTION OF THE MEASURES.*

§1. *Formulæ and Standard.*

The first step in the determination of the star's parallax from its measured coördinates and those of the comparison-stars is to reduce the rectangular coödinates measured on the different plates to some uniform standard. We have to correct not only for those causes—aberration, refraction, etc.—which alter the stars' apparent places in the sky, but for the inevitable errors of centering, scale-value, and orientation of our individual plates. It is, however, a well-known advantage of working in rectangular coördinates that all these corrections can be combined into one very simple expression.

It was first shown by Turner† that, except for photographs taken at great zenith distances, all the necessary corrections are sensibly linear functions of the measured coördinates. It may be added that, for plates having the same center among the stars and taken at a fixed hour-angle (as is the case here), the small non-linear terms (which arise from the differential refraction) are practically constant for each star and affect the star-places alone, not the parallaxes.

If then x and y are the measured coördinates of a star on any plate, and ξ and η what may be called its ideal coördinates, cleared of refraction, aberration, etc., upon a plate of correct orientation and predetermined centering and scale-value, we should have for every star on the plate relations of the form

$$\xi=x+ax+by+c \quad . \quad . \quad . \qquad \eta=y+dx+ey+f \quad . \quad . \quad . \qquad (1)$$

where a, b, c, d, e, f, are constants for the whole plate.

Owing to errors of the measured star-positions, these relations will not be rigorously true; but the errors are in practice so small that their squares and products, and also their products by the plate-constants a, b, d, e (which are themselves small), may be neglected.

Under these conditions we may assume, without sensible error, that the difference of the measured coördinates of the same star on any two plates (barring errors of measurement and proper-motion) is a linear function of its coördinates upon either one of them, or upon any other plate with the same center, or even of the x of one such plate and the y of another.

It might seem desirable to compute standard coördinates for our stars from their catalogued right ascensions and declinations; but there are usually not enough catalogue stars on the plates; the errors of their tabular places

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*The greater part of this chapter, so far as it contains new material, represents the unpublished investigation of the writer, referred to in Monthly Notices, LXV, p. 781.

†Monthly Notices LIV, p. 11.

are much larger than those of the photographs; and the necessary calculations take a good deal of time. The photographs themselves are free from all these objections. But a standard derived from them may differ slightly in scale value and orientation from the assumed values of these constants which we use in computing the parallax factors and in reducing our results to seconds of arc.

These differences, however, are in practice so small that they can not sensibly affect the deduced value of the parallax. The scale-value never differs by as much as one part in a thousand from the assumed constant ($175\overset{''}{.}8$ per réseau interval). This will affect the deduced parallax in the same proportion. The effect of error in orientation is to change the parallax factor in x by an amount equal to the corresponding factor in y, multiplied by the orientation constant b. As b is almost always less than 0.01 and the parallactic displacement in y averages less than half that in x, the error so introduced will be at most about one two-hundredth part of the whole displacement. Both these effects may therefore be neglected. The best procedure is therefore:

> RULE VI: Choose any plate, or the mean of any number of plates, as a standard, and reduce the others to this.

It is not even necessary that the x and y coördinates shall be derived from the same plates.

If we have to reduce any plate to the standard, we must assume as a first approximation that our comparison-stars have no sensible parallax or proper-motion. Each of them gives us an equation of condition of the form:

$$a\xi + b\eta + c - (x - \xi) = v \qquad (2)$$

where x, y are the coördinates on the plate; ξ, η are the standard coördinates; the quantities in parentheses are the observed data; and v represents the inevitable residuals. These equations must be solved for the plate-constants a, b, and c.

It should be observed:

1. That as we are dealing with the x's only, we are not concerned with such debated questions as the identity of the scale-value of our plates in the two coördinates, and no *a priori* relations can be assumed between the three constants which have to be determined.

2. That by expressing $x - \xi$ as a linear function of ξ, η rather than of x, y the equations of condition for different plates differ only in their absolute terms.

3. That the values of η need be known only approximately. Since b may be as much as 0.01 they should be carried to two decimal places less than those of x and ξ.

4. That in case a star has moved sensibly between the epochs of the plate and the standard we must correct its standard coördinates ξ, η for the amount of this motion before calculating the expression $a\xi + b\eta + c$. This correction is sensible only for very rapidly moving stars.

§2. *Dyson's Method of finding the Plate-Constants.*

These equations of conditions might be solved by least-squares. But by a suitable choice of comparison-stars a simpler method of solution may be used without loss of accuracy. This method was first described by Dyson,* and proceeds as follows:

Arrange the equations of condition in order according to the values of ξ; divide them into two groups, one with large and one with small ξ, and take the mean of each group.

This gives two equations in which the coefficients of a are considerably different and those of c are unity. If the stars are distributed over the plate with tolerable uniformity the mean values of η for the first two groups, and hence the coefficients of b, will be nearly the same.

Subtracting the second of these from the first we obtain an equation in which the coefficient of a is large, that of b is small, and c does not appear at all. Arranging our equations now according to the values of η, and proceeding similarly, we reach an equation with a small coefficient of a and a large one for b. These two equations give us a good determination of a and b. The value of c can then be found by substitution in any one of the four mean equations already constructed. All four will give the same value of c, as they are not independent (since from either the first pair or the last pair we may deduce the equation obtained by adding together all the original equations of condition.)

This method is an extension of the familiar one of forming quasi-normal equations by altering the signs of the equations of condition so that all the coefficients of a given unknown are positive, and then adding. Like this, it sometimes fails of application. For example: if all the stars on a plate were confined to its northeast and southwest quarters, large ξ and η coördinates would always go together, and vice versa, so that the two pairs of mean equations would be identical, and also the final equations between a and b, which could not be determined at all from them. If, however, such a plate were divided into halves by its diagonals instead of by parallels to its sides, and the equations of condition grouped accordingly, a good determination of a and b could usually still be made. With liberty to draw the dividing lines across the plate at any angle, the method would fail only when all the stars were in one straight line; and in that case the least-square solution also becomes illusory. This indicates that Dyson's method, so modified, ought to be generally applicable.

§3. *Accuracy of the Method.*

Treating the matter analytically, we have to represent our observed quantities, $x-\xi$, by a linear function of ξ, η. Instead of these we may use any rectangular or oblique coördinates p, q, on the plate, represent our observations by a linear function of these, and transform to ξ, η when necessary.

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*Monthly Notices, LV, p. 61; LVI, pp. 118–119. See also Turner, Monthly Notices, LIV, p. 489; Hinks, Astronomical Journal, No. 475.

If there were no errors of observation, either Dyson's method or that of least-squares would lead us to the exact expression which would satisfy all the equations

$$x-\xi=ap+bq+c \tag{3}$$

We have therefore to concern ourselves only with the effects of such errors upon the plate-constants, as determined by the two methods.

Suppose that we have n equations of condition, and that the errors of their absolute terms are $\Delta_1\ \Delta_2\ .\ .\ .\ \Delta_n$. In applying Dyson's method we divide these equations into two groups. Let n_1 and n_2 be the number of equations in the first pair of groups, and let the sign $(p)_1$ denote the sum of the p's for all the n_1 equations of group 1, etc. Similarly let n_3 and n_4 be the number of equations in the second pair of groups. Then

$$n_1+n_2=n_3+n_4=n$$

Finally let (p) denote the sum of the p's for all n equations. Then we shall have the following equations to determine the errors δa, δb, δc of the plate-constants:

$$\begin{array}{ll}(p)_1\,\delta a+(q)_1\,\delta b+n_1\,\delta c=(\Delta)_1 & \left.\begin{array}{l}(p)_3\,\delta a+(q)_3\,\delta b+n_3\,\delta c=(\Delta)_3\\(p)_4\,\delta a+(q)_4\,\delta b+n_4\,\delta c=(\Delta)_4\end{array}\right\}\\(p)_2\,\delta a+(q)_2\,\delta b+n_2\,\delta c=(\Delta)_2 & \end{array} \tag{4}$$

These four equations are not independent, for the sum of either the first or the last pair gives

$$(p)\ \delta a+(q)\ \delta b+nc=(\Delta)$$

It follows from these equations that, when the plate has been reduced to standard by this method, the sum of the outstanding residuals must vanish for each of the four groups; and hence that the sums of the residuals for the stars in each of the four quarters into which the dividing lines cut the plate must be numerically the same, but of opposite signs in adjacent quarters (since each group is composed of two adjacent quarters). This affords in practice a simple and complete control of the numerical work.

We may simplify the equations (4) by a proper choice of axes. Each of the four groups of comparison-stars has a *centroid* whose coördinates are the mean of those of the stars of the group. If the p-axis passes through the centroid of groups 3 and 4, and the q-axis through those of groups 1 and 2, the origin will fall at the centroid of the whole system of comparison-stars. The sum of the p's will vanish in groups 3 and 4, and that of the q's in groups 1 and 2, and the equations (4) will become

$$\begin{array}{ll}(p)_1\,\delta a+n_1\,\delta c=(\Delta)_1 & \left.\begin{array}{l}(q)_3\,\delta b+n_3\,\delta c=(\Delta)_3\\(q)_4\,\delta b+n_4\,\delta c=(\Delta)_4\end{array}\right\}\\(p)_2\,\delta a+n_2\,\delta c=(\Delta)_2 & \end{array} \tag{5}$$

while the sum of either pair gives simply

$$n\ \delta c=(\Delta) \tag{6}$$

If the probable error of one of the n quantities Δ is r, that of δc will therefore be $\frac{r}{\sqrt{n}}$, whatever the choice of the groups 1, 2, 3, 4. Those of δa and δb will depend upon the arrangement of the stars in these groups.

It is usually possible to choose the comparison-stars and group them so that the p-axis—that is, the line joining the centroids of groups 3 and 4—completely separates the stars of groups 1 and 2, and vice versa. In this case p is negative for all the stars in group 1, and positive for all those in group 2; and the other two groups have the same property with regard to q.

If then P denotes the mean, without regard to sign, of the p-coördinates of all the stars, we shall have

$$(p)_2 = -(p)_1 = \frac{nP}{2} \tag{7}$$

Introducing this into (5) and eliminating δc, we find

$$\frac{nP}{2}(n_1+n_2)\,\delta a = n_1(\Delta)_2 - n_2(\Delta)_1$$

The probable error of the second member is $r\sqrt{n_1^2 n_2 + n_2^2 n_1}$. Since $n_1 + n_2 = n$ this reduces to $r\sqrt{n\,n_1 n_2}$, and the probable error of δa is

$$r_a = \frac{r}{P}\sqrt{\frac{4\,n_1 n_2}{n^3}} \tag{8}$$

But $4n_1\,n_2$ is at most equal to n^2. Hence we have

$$r_a \leqq \frac{r}{P\sqrt{n}} \tag{9}$$

Proceeding similarly, we find $r_b \leqq \frac{r}{Q\sqrt{n}}$, where Q is the mean of the absolute values of q for all the stars.

We may now compare these with the results of the least-square solution. Using the same notation, but capital letters to distinguish the plate-constants found by this method, the normal equations are

$$(p^2)\delta A + (pq)\delta B = (p\Delta) \qquad (pq)\delta A + (q^2)\delta B = (q\Delta) \qquad n\delta c = (\Delta) \tag{10}$$

The last equation is identical with that for δc in Dyson's method. The two methods therefore give identical values of c.

The values of a and b given by the two methods will in general be different. The probable errors of the constants determined by least-squares will be given by the equations

$$r_A^2 = \frac{(q^2)r^2}{(p^2)(q^2) - (pq)^2} \qquad r_B^2 = \frac{(p^2)r^2}{(p^2)\ (q)^2 - (pq)^2}$$

Since the sums of the values of p, corresponding to positive and negative values of q, separately vanish (and vice versa) the summation (pq) will be small compared with the others, and we shall have

$$r_A \geqq \frac{r}{\sqrt{(p^2)}} \qquad r_B \geqq \frac{r}{\sqrt{(q^2)}} \tag{11}$$

the actual values usually approaching closely to equality. Now, unless all the p's are numerically equal $(p^2) > nP^2$. We may set $(p^2) = KnP^2$, where $K > 1$. If the stars are uniformly distributed over the plate, $K = \frac{4}{3}$. They can hardly in practice be so irregularly grouped that K is greater than 2. We then have $r_A \geqq \frac{r}{P\sqrt{nK}}$, whence by (9) $r_a \leqq r_A\sqrt{K}$, and similarly $r_b \leqq r_B\sqrt{K}$.

The diminution in the probable errors of these plate-constants, upon passing from the approximate method to that of least-squares, is therefore usually small.

For the actual difference between the two values of a, we have, in the case where $(pq) = 0$

$$\delta A - \delta a = \frac{(p\Delta)}{(p_2)} - \frac{2n_1(\Delta)_2}{n^2P} - \frac{2n_2(\Delta)_1}{n^2P} \tag{12}$$

The most probable value of this difference is obviously zero. Its probable error R_a may be obtained by squaring the second member of (12), substituting r^2 for the square of each of the Δ's, and zero for all product terms. We thus obtain

$$R_a^2 = \frac{r^2}{(p^2)} + \frac{4(n_1^2n_2 + n_2^2n_1)r^2}{n^4P^2} - \frac{4n_1(p)_2r^2}{n^2P(p^2)} + \frac{4n_2(p)_1r^2}{n^2P(p^2)}$$

By means of (7), (8), and (11) this reduces to

$$R_a^2 = r_A^2 + r_a^2 - \frac{2(n_1 + n_2)r^2}{n(p^2)} = r_a^2 - r_A^2 \leqq r_A^2(K - 1) \tag{13}$$

When (pq) is not zero, the same final equation is reached, after a slightly longer reckoning.

Now it has already been shown that $K - 1$ is usually considerably less than unity. The equation (13) may therefore be stated verbally as follows:

> *The probable differences between the values of a and b obtained by Dyson's method and by least-squares (i. e., the limits which these differences are as likely as not to exceed) are, under all ordinary circumstances, considerably less than the probable errors of the latter.*

It remains to consider the probable error of the calculated correction which must be applied to the measured coördinates to reduce them to the standard system. This correction is $ap + bq + c$. Since the three plate-constants have been determined independently of one another, the probable error r_1 of this expression will be given by the equation

$$r_1^2 = r_a^2 p^2 + r_b^2 q^2 + r_c^2 \leqq \frac{r^2}{n}\left(1+\frac{p^2}{P^2}+\frac{q^2}{Q^2}\right) \qquad (14)$$

The correction is most accurately determined for a point at the centroid of the comparison-stars. They should therefore be so chosen that this point falls as near as possible to the parallax-star which is the main object of the investigation.* It is almost always possible to choose them so that for the parallax star $\frac{p}{P}$ and $\frac{q}{Q}$ are less than 0.3, and they can usually be made very much less. The uncertainties of a and b then contribute less than one-sixth of the whole uncertainty of the calculated correction. These are the only quantities which could be more accurately found by a least-squares solution, and this would usually diminish their uncertainty by only about one-third, and hence of the whole correction to standard by only about one-twentieth of its whole amount.

The error of the final reduced coördinate of the parallax star will be the resultant of the error of its own measured coördinate and of the correction just discussed. If the accuracy of the measures of the comparison-stars is comparable with that of the parallax star, it is clear from (14) that the latter will be much more accurately determined than the former—its relative weight being proportional to the number of comparison-stars. As in the present work there are always at least five of the latter, it is clear that the uncertainty of this correction will contribute less than one-fifth of the whole uncertainty of a reduced coördinate (allowing something for the inferior accuracy of the measures of the comparison-stars).

To determine the plate-constants by least-squares would therefore increase the weight of the reduced coördinates of the parallax-star by less than 1 per cent, which would be far from repaying the additional labor. The rule has therefore been adopted:

> RULE VII: Choose the comparison-stars so that their centroid falls as near as possible to the parallax-star, and use Dyson's method of reduction.

In dividing the comparison-stars into groups, in applying this method, the principles laid down on pp. 21, 23 must be observed.

§4. *Number of Comparison-Stars.*

The result just obtained shows that very little accuracy is gained by increasing the number of comparison-stars, so far as the accidental errors of observation are concerned.

If we wish to estimate the number n of comparison-stars which will give the most favorable relation between the work expended on a plate

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*Plummer, Monthly Notices, LXIV, p. 646.

and the weight of the resulting coördinate of the parallax star, we must express both these quantities in terms of n.

If r_0 is the probable error of a measured coördinate for the comparison-stars and r_1 that for the parallax-star, the square of the probable error of the reduced coördinate of the latter will be $r_1^2+\frac{r_0^2}{n}\left(1+\frac{p^2}{P^2}+\frac{q^2}{Q^2}\right)$ and its weight will be inversely proportional to this expression.

The work of taking, measuring, and reducing the plates may be divided into two parts, one of which is independent of n, and the other proportional to it. It may therefore be set proportional to $n+k$. The value of k can only be roughly estimated, on a basis of experience: for the present work it seems certain that it is not much greater than 8. The ratio $\frac{\text{Work}}{\text{Weight}}$ is therefore proportional to

$$(k+n)\left\{r_1^2+\frac{r_0^2}{n}\left(1+\frac{p^2}{P^2}+\frac{q^2}{Q^2}\right)\right\}$$

This expression is a minimum when

$$n^2=k\,\frac{r_0^2}{r_1^2}\left(1+\frac{p^2}{P^2}+\frac{q^2}{Q^2}\right)$$

From the values given in Chapter IV it appears that the second factor in this expression averages about 1.7. The third is only slightly greater than unity. The most favorable value of n will therefore usually be 4. This reasoning, however, tacitly assumes that it is always as easy to take more plates at a given epoch as to measure more stars on the plates—which is very far from the truth. There is also the objection that with so few comparison-stars it is impossible to detect a large parallax or proper-motion among them.

Suppose that one of them has moved, since the epoch of the standard plate, by an amount which is large compared with the ordinary errors of observation. If there are but three comparison-stars, there will be only three equations of condition for a, b, and c. The calculated values of these constants will satisfy them exactly, leaving no residuals, and the motion can not be detected; nor can unusually large errors of observation, or even numerical mistakes in the coördinates to be reduced. If there were four, and the reduction is made by Dyson's method, the residuals for all four stars will be numerically equal (two positive and two negative), so that we can not pick out the moving star. Something of the same sort happens whenever the moving star is alone in the quarter of the plate where it lies.

It is therefore desirable to have two comparison-stars in each quarter of the plate, or eight in all. This also makes it much easier to satisfy the condition of Rule VII concerning their centroid. The usual number of comparison-stars chosen in the present work is eight. In a few cases it was necessary to reduce this number to seven or six, and in one exceptional instance to five, while in a few others it was increased to nine or ten.

For eight comparison-stars, setting $k=8$ in the formula above, we find that about one-eighth more work must be expended in order to get results of equal weight for the parallax-star than if there were only four. But the certainty that the results are not modified by the existence of a considerable parallax in one of the comparison-stars compensates for this—to say nothing of the information concerning the parallaxes of the comparison-stars themselves that can be deduced from their residuals.

§5. *Solution for the Parallax and Proper-Motion.*

When all the plates of a field have been reduced, the residuals for the parallax-star are converted into seconds of arc (using the standard value $175''.8$ for the réseau interval) reduced to a common epoch with the catalogued proper-motion of the star (or if need be with an approximate value deduced from the plates themselves) and discussed by least-squares in the ordinary fashion, the unknowns being the star's parallax and corrections to its assumed x coördinate and proper-motion in x.

It is not worth while to make such an elaborate discussion of the residuals for the comparison-stars, because any parallax or proper-motion in any one of them will produce systematic changes in the calculated plate-constants, and hence in the residuals for all the other stars, so that the results for the different stars are not independent.

By taking means of the equations of condition for each parallactic epoch and combining these with due regard to their weights, results practically identical with those of the least-square solution can be obtained in much less time. The parallaxes and proper-motions so computed for the comparison-stars will usually be almost wholly due to errors of observation. If any star has a large one this can be at once detected. Such a star should be rejected as a comparison-star.

It is a good illustration of the advantages of photographic work in rectangular coördinates that this can be done very easily indeed. Owing to the linear character of all the equations used in reducing the plates to standard and determining the parallax and proper-motion, it follows that even a large proper-motion in one comparison-star will be without influence upon the computed parallaxes of the stars, but will affect only their calculated proper-motions. The extent of this influence for each star depends upon its position on the plate, and also in some degree upon the way in which the stars are divided into groups in using Dyson's method; but it can be readily calculated when these are known. We can then determine at once what proper-motion we must assume the suspected star to possess in order to make the mean of those of the other stars vanish (that is, its proper-motion relative to the others, considered as fixed) and also those of all the other stars referred to this new standard. A suspected large parallax may be treated in the same way. In neither case is it necessary to recompute plate-constants and the like, or to deal with any quantities saving those in direct need of correction. The experience of the present work indicates that such cases are rare in practice. An example will be given in the next chapter.

Even when the computed parallaxes and proper-motions of the comparison-stars appear to be almost wholly due to errors of observation, they furnish information of much value, serving to detect the existence, or prove the absence, of systematic errors—depending, for example, upon a star's position on the plate, or upon its brightness—and showing also how the accuracy of the measures is affected by these conditions.

If the weight of the y coördinates justifies their being discussed at all, they may be handled as follows. Choose three of the comparison-stars (already found to have no sensible proper-motion or parallax) so that their centroid falls as near as may be to the parallax-star. Measure the y's of these four stars accurately and discuss them as above. The reductions may be made very short. If the subscripts 1, 2, 3, a, denote quantities belonging to the three comparison-stars and the parallax-star, and we determine three constants α, β, γ by the equations

$$\alpha\xi_1+\beta\xi_2+\gamma\xi_3=\xi_a \qquad \alpha\eta_1+\beta\eta_2+\gamma\eta_3=\eta_a \qquad \alpha+\beta+\gamma=1$$

then if f denotes any linear function of ξ, n

$$f_a=\alpha f_1+\beta f_2+\gamma f_3$$

The correction to reduce the place of the parallax-star to standard may then be derived immediately from the differences from standard for the comparison-stars.

The parallactic displacement in y is usually so small that even this short method of discussion does not repay the labor.

CHAPTER III.

THE OBSERVATIONS.

§1. *General Summary.*

The numerical data and results of the work are contained in table C (at the end of the volume). Summarizing them, it appears that the present work depends on 254 plates, of 37 different fields; of these, 109 (43 per cent) were taken by Mr. Hinks and 145 (57 per cent) by the writer; on these plates were made 976 exposures, of an average length of $4^m\ 22^s$ and an aggregate duration of about 71 hours, and upon them were measured 9232 images of 338 different stars, 52 of which are "parallax-stars," and the remaining 286 comparison-stars. The parallax was determined for all the former and for 242 of the latter, excluding only those in the incomplete Series XXXII to XXXVII, which were observed at two epochs only, owing to the accident to the color-screen (see page 8), making a total of 294 stars whose parallaxes are given in this work.

§2. *Description of Table C.*

The observational data and the necessary details of their reduction and discussion are given in table C, pages 104–142, each section of which gives the results from one series of plates. At the top of the page is the current number of this series and those which the star or stars especially observed for parallax bear in the final table of results; then follow the designations of these stars, their approximate places for 1900, and their proper-motions—in right-ascension, declination, and on a great circle. The latter are taken from Boss's Preliminary General Catalogue, when the stars appear in the latter; otherwise, usually from Bossert's Catalogue of stars of large proper-motion. The designations of the stars are for the most part those employed by the latter authority.

The upper half of each table gives the necessary data concerning the individual stars and plates, and also the measured coördinates, the constants necessary to reduce them to the common standard, and the results of this reduction. The last two (or occasionally three) also give proper-motions and parallaxes of all these stars, resulting from the approximate discussion described below. These last quantities are given in thousandths of a second of arc; the remainder are in réseau intervals of $175''.8$.

The first five columns of the table deal with the stars observed. The parallax-stars are denoted by letters, and the comparison-stars by numbers (in order of increasing right ascension), which are given in the first column; the second gives the Durchmusterung numbers of these stars, and the third gives their magnitudes from the same source. For a few stars which do not appear in the Bonn Durchmusterung, the second column is left blank and the third contains a rough estimate of magnitude based on the photographs.

The next two columns contain the photometric magnitudes and spectra, determined at the Harvard College Observatory. For some of the faintest stars, and a few others which lie near bright ones, these data are not available and the corresponding spaces are vacant.

The next two columns, headed "Standard," contain the system of coördinates of these stars, used as the standard to which all the plates were reduced. The coördinates η (given to three decimal places) depend upon measures of one or two images of each star on a single plate. Their probable error is less than a unit of the third decimal place, as is shown by comparison with similar measures upon a plate of the last epoch of each series. (These comparisons were undertaken primarily to detect any possible large proper-motions in declination. As none were found, it is needless to give further details here.)

The coördinates ξ (which are given to five decimal places) are sometimes derived from the same plate as the η's and sometimes are mean values including other plates as well. This latter policy was at first adopted in order to secure as accurate a standard as possible; but it was afterwards realized that, owing to the strictly differential character of the work, this led to no real advantage, and the practice was discontinued.

It should be noted that where the ξ-coördinates are the mean of two or more plates they are not exactly rectangular with the η-coördinates. As the latter are used only to compute the reductions to standard, this is of no importance for the purpose in hand. If a set of rectangular coördinates are desired for any purpose, they can be obtained by using, with the tabulated η's, the x's of the corresponding plate.

At the bottom of these columns are given the proper-motions of the parallax-star or stars, in réseau intervals per year, to three places of decimals. These are necessary in the case of rapidly moving stars, because in calculating the expression $a\xi+b\eta+c$, used in reducing any plate to the standard, the values of ξ and η should be those for the date of observation. These may easily be computed from the tabular data, and this correction has been made whenever its effects are sensible.

§3. *Data for Individual Plates.*

The following double columns each contain the data for one plate. At the top is the current number, from the observing books; next, the (astronomical) day of observation; then the number of exposures (those actually measured, ignoring any defective ones), the average exposure-time in minutes, the hour-angle of the principal star at the mean of the times of these exposures, and finally the initial of the observer (H = Mr. A. R. Hinks, R = the writer).

Then follow the x-coördinates of the stars. Only the decimal part is given, as the whole number is usually identical with the standard ξ, and, when it differs by a unit, the true value is obvious upon inspection (since the differences from standard are nearly the same for all the stars). These quantities are the means of the x's of all the images of the star which were measured on the plate. The measures for the individual images are carried to four decimal places only, but the means are taken to five, to avoid errors due to neglected decimals in the reduction.

The results for the individual images are not given, partly because of their bulk, but mainly because (since each image was measured in but one position of the plate) they are affected by the personal error of bisection (page 15) which is only eliminated when the mean is taken.

When the number of measurable images of each star was odd, one was measured in both positions, and when one image was unlike the others (*e.g.* fainter) care was taken that this should be the one.

In certain cases the images of the parallax-star, and occasionally of other stars, were remeasured, to investigate apparent discordances. In almost all cases the original measures were closely confirmed, showing that the trouble was in the actual position of the image, and not in the measures. The letter B, at the foot of this column, denotes that all the images were measured in both positions of the plate.

It is next necessary to explain the meaning of the quantities given at the foot of the column for each plate, under the caption "Average residual." The successive exposures on one plate differ of course in centering. They may possibly differ also in orientation (owing to changes in refraction, etc.), but they must be practically identical in scale-value. If the orientation is constant, the differences between the x-coördinates of the stars for a given exposure and the mean x's for the same stars for all the exposures should be the same all over the plate. If the orientation differs, these differences should be of the form $by+c$.

The actual differences for individual images will not agree with these theoretical values, owing to errors of measurement and of the real position of the star images; and the residuals obtained by comparing them with theory afford a measure of these errors.

For the plates first measured, the assumption of differences of orientation was made, and the residuals determined graphically. The results did not confirm the reality of such differences; and later in the work changes of centering alone were supposed to exist. The residuals found on the first assumption will naturally average somewhat less than on the second.

The average value, regardless of sign, of these residuals for all the star-images on each plate is the quantity given in the tables. The values resulting from the graphical process are distinguished by an asterisk. The calculation of these residuals gives a valuable control of the numerical work up to this point and serves to detect any serious error of measurement or any grossly bad images. Their numerical average is evidently an indication of the general quality of the plate.

The quantities a, b, c, at the foot of each column, are the plate-constants, determined by Dyson's method. If x is the measured coördinate, and ξ and η the standard coördinates, then $x=\xi+a\xi+b\eta+c$.

These constants are expressed in units of the fifth decimal place and are given with accuracy sufficient to insure the correctness of the last figure of the reduced value of x. It should be observed that a and b are abstract numbers, while c is proportional to the réseau interval. Thus, for example, for Plate 188 (Series II) we have

$$x=\xi+0.0000968\,\xi-0.0018755\eta+0.05562\mathrm{R}$$

The four groups into which the comparison-stars are divided by wide spacing correspond to the four quarters into which the plate is divided in using Dyson's method. To obtain one pair of the quasi-normal equations, take means for the stars of the first and second, and for those of the third and fourth groups; to get the other pair, combine the first and the fourth groups, and the second and third. For Series XV the stars were divided into three groups, each of which gives directly one equation between the plate-constants.

The column following the measured x's gives the residuals resulting from their reduction to standard, in the sense Plate *minus* Standard, in units of the fifth decimal place of a réseau interval. Negative residuals are in **boldface type.**

For the comparison-stars, the sum of the residuals should be numerically equal for each of the four groups, but of alternating sign. The residuals for the parallax-stars often show conspicuous evidence of proper-motion, even in an interval of a few months.

In two cases (plate 398, Series XIII and plate 436, Series XVIII) the center of the field is, through some error in setting, at some distance from its usual position. In these cases the measured coördinates require a slight correction for the effect of the inclination between the planes of the plate and of the standard.

If x, y are the coördinates of any star, referred to the standard plate-center; X, Y those of the center of the plate under discussion; and F the focal length expressed in réseau intervals, it is easy to show that, if the corrections

$$\delta x=+x\left(\frac{Xx+Yy}{F^2}\right) \qquad \delta y=+y\left(\frac{Xx+Yy}{F^2}\right)$$

are applied to the measured coördinates, the results will be connected with the standard by linear relations and may be reduced as usual. This "non-linear" correction is given immediately after the measured coördinates.

§4. *Approximate Solution for all Stars.*

The last two (or occasionally three) columns of the upper part of the table ("Approximate Solution") give the values of the proper-motion μ, and parallax π, of all the stars, as derived from the approximate solution described on p. 27, and illustrated below.

When one comparison-star gives evidences of real proper-motion, the values obtained by excluding it as an object of reference (see p. 27) are given in the third column headed μ'. Except in this case, the computed parallaxes and proper-motions of the comparison-stars (being linear functions of the residuals of the individual plates) must have the same properties as these with regard to their sum by groups. The values for the parallax-stars are included to test the accuracy of the approximate method employed.

§5. *Least-squares Solution for the Principal Stars.*

The lower half of each table contains the least-squares discussion for the parallax-stars. In this, all observational quantities are expressed in thousandths of a second of arc. The value of the réseau interval is taken as 175″.8—which was derived by Mr. Hinks, in the course of reduction of the Cambridge plates of Eros, from comparison of numerous photographs with meridian places of the stars.

The first column gives the number of the plate; the second (headed "Observed") gives the residual (Plate minus Standard) for the star in question, and the third the correction for proper-motion necessary to reduce each observation to a common epoch. This epoch is given at the head of the column, and the assumed value of the annual proper-motion at its foot. The latter often differs slightly from the more accurate values at the top of the page, which were not available when the reductions were begun.

The following column (headed "Corrected") gives the sum of the quantities in the two which precede it. If the assumed proper-motion were correct, and the star had no parallax, the numbers in this column should be identical. An assumed value, Δx is given at the foot of the column. By subtracting this from the individual entries, we obtain the absolute terms of the equations of condition.

When there is more than one parallax-star, and their proper-motions are different, the data for the second star follow those for the first (as in Series II and XXXI). But when, as is more often the case, they have nearly or quite the same proper-motion, the observed residuals for both stars are first given, then the proper-motion correction common to both, and lastly the corrected values (as in Series VII, XV, XIX, etc.). When the assumed proper-motion is zero, the corrected values are identical with the observed, which alone are given. Following them come the *equations of condition.* These are in the form

$$x + ay + p\pi = n$$

where x represents the correction to the assumed value of Δx, and y to the assumed proper-motion, already given, π is the star's parallax, and n the observed quantity.*

The coefficients of y are simply the interval in years between the assumed epoch and the date of observation. Those of π (which are practically the parallax-factors in right ascension) are computed by the formula

$$p = R \cos D \sin (A - \alpha)$$

where α is the star's right ascension, and A, D, and R are the sun's right ascension, declination, and distance. By forming a table of log ($R \cos D$) for every five days throughout the year, the computation was made very simple. Four-place logarithms were used, and the results were checked by calculation in duplicate.†

The parallax-factors in y, when required, were computed by the formula

$$p_y = R \sin D \cos \delta - R \cos D \cos (A - \alpha) \sin \delta$$

where δ is the star's declination.

Following the equations of condition are the residuals, in the sense $(O - C)$, derived from the various solutions. At the foot of each column of residuals is the weighted sum of their squares. (pvv)

The last column gives the weights assigned to the equations. Usually all plates were given unit weight. Some poor plates were assigned weight ½ and a few very poor ones ¼. The reasons for giving low weight to a plate were: (1) small number of exposures; (2) bad observing conditions (according to the notes in the observing book); (3) bad character of the images or réseau (noted by the measurer before reduction); (4) discordance of the measures of the different exposures on the plate (shown by the unusually large "average residual").

In all, 29 plates out of 255, or about one-ninth of the whole, received diminished weight. In only one case (Plate 207, Series v) was a plate given reduced weight *a posteriori* (because of discordance appearing upon reduction, and otherwise unaccountable). In two other cases, Plates 455, (Series xxvii) and 318 (Series xxxvii), images of one star upon an otherwise good plate were entirely rejected for similar discordance.

At the bottom of the tables are the normal equations and their solution. The coefficients of the former have been checked by duplicate computation. The absolute terms are given in thousandths of a second of arc.

The first solution gives for each parallax-star the values of x, y, and π, with their probable errors and weights, and also r_0, the probable error of one equation of condition of unit weight. When it seemed doubtful whether the

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*These equations are written with the observed quantities in the second member, in conformity with the usage of other works on stellar parallax. Compare Cape Annals, vol. viii, part ii, p. 11 B. Yale Transactions, vol. ii, part ii, p. 219.

†Errors in the parallax factors so small that they obviously would not influence the deduced results by more than 0."001 were not corrected.

computed values of y or π were real, additional solutions were made, in which the doubtful quantities were assumed to be zero, or occasionally to have some value otherwise determined. The resulting values of the remaining unknowns, the residuals, and the probable errors are given in their appropriate places.

For two series (XI and XV) the y coördinates were measured. The results are given as Series XI*a* and XV*a*. They are arranged similarly to the other series, except that the data already given in the previous table are not repeated and that, instead of the plate-constants, the three reduction constants α, β, γ are given (see p. 28), enabling us to find the correction to reduce star A to the standard, from the differences from standard for the three selected comparison-stars.

The y coördinates for the plate formerly chosen as the standard (*e.g.*, Plate 191, Series XI) differ slightly from the approximate values given in the preceding table, because they are means including a larger number of exposures than do the latter.

§6. *Stars Observed at but Two Epochs.*

The stars of Series XXXII to XXVII are those for which observations were interrupted by the accident to the color-screen (see p. 8), so that photographs could be obtained at only two parallactic epochs. For all of these except η Geminorum (Series XXXII) measurement and reduction were under way before the series were cut short, and the results are presented in tables similar to those for the majority of the stars, while in the remaining case the rapid reduction, with but three comparison-stars, was employed.

The reduction of the residuals to a common epoch and the equations of condition are presented just as in the preceding tables.

It is of course impossible to determine the proper-motion and parallax independently from these equations. The course adopted has been to form mean equations for each of the two epochs, and from these to determine x and π in terms of y (the correction to the catalogued proper-motion). As these are all stars of well-determined proper-motion the terms involving y are presumably small.

The values of y necessary to reduce the assumed proper-motions to those of Boss's Preliminary General Catalogue (which appeared after the first discussions had been completed) and the corresponding values of π are given at the bottom of the table. These values of the parallax have been taken as final.

The probable error of one plate has been determined as usual; that of π by dividing the probable error of the difference of the two epoch-means by the difference of the parallax factors. As it can be shown (see Chapter IV, p. 56) that there are no sensible systematic errors, this process must give a close approximation to the truth. The solutions that have sometimes been made, assuming $\pi = 0$, serve to show more clearly what reliance can be placed on the observed parallaxes. The parallaxes and proper-motions of the comparison-stars can not of course be determined for these series.

At the bottom of the page are found certain notes.

The "Observer's Notes" are taken from the observing books and deal mainly with the atmospheric conditions and the performance of the driving clock and electric control. The "Measurer's Notes" are taken from the sheets of measures and deal principally with the character of the images and réseau lines. The reasons for assigning reduced weight to any plate (other than a large "average residual") can usually be found in these notes.

Sundry other notes, which occasionally appear, are self-explanatory.

§7. *Success in Eliminating Hour-angle Error.*

It is desirable to extract from table C and to summarize the data which show the degree of success attained in eliminating hour-angle error. The form of this which is most to be feared in photographic work is atmospheric dispersion. Its influence may be calculated as follows.

Suppose that the refraction constant β differs by $\delta\beta$ for two stars of different spectral types. By Turner's well-known formula, the increase of the x coördinate by refraction is βX (+ small terms) when X is the coördinate of the zenith on the plate. The relative displacement of the two stars is therefore $\delta x = X\delta\beta$

For a plate whose center is in declination δ, taken at hour-angle t, in latitude φ,

$$X = \frac{\sin t \cos \varphi}{\cos \delta \cos \varphi \cos t + \sin \delta \sin \varphi}$$

For plates taken near the meridian, t is small. Expressing it in *minutes of time*, neglecting its square and higher powers, and introducing the latitude of Cambridge ($+52°12'$), this equation becomes

$$X = +0.0027 t \sec (\delta - \varphi)$$

If now the weighted mean hour-angles at which morning and evening observations of the stars are made are t_1 and t_2, and the corresponding parallax factors in x are p_1 and p_2, the resulting error $\delta\pi$ in the derived relative parallax will be

$$\frac{X_1 - X_2}{p_1 - p_2} \delta\beta$$

whence

$$\frac{\delta\pi}{\delta\beta} = +0.0027 \frac{t_1 - t_2}{p_1 - p_2} \sec (\delta - \varphi)$$

If the observations of different years are made at different hour-angles, there will also be some error in the derived proper-motion; but the preceding expression for the influence on the parallax will still be approximately correct.

Table 3 gives the values of these quantities for each of the 37 series of plates.

TABLE 3.

Series.	t_1-t_2	p_1-p_2	$\frac{\delta\pi}{\delta\beta}$	Series.	t_1-t_2	p_1-p_2	$\frac{\delta\pi}{\delta\beta}$	Series.	t_1-t_2	p_1-p_3	$\frac{\delta\pi}{\delta\beta}$
	m				m				m		
1	+ 1	+1.38	+0.002	14	0	+1.56	0.000	26	+12	+1.24	+0.029
2	+ 4	1.45	+0.008	15	− 5	1.49	−0.015	27	−21	1.34	−0.046
3	− 6	1.51	−0.011	16	+ 3	1.39	+0.007	28	+13	1.37	+0.033
4	− 5	1.74	−0.014	17	+ 7	1.42	+0.017	29	− 6	1.38	−0.012
5	−13	1.41	−0.026	18	+ 8	1.39	+0.018	30	+ 8	1.36	+0.016
6	+ 2	1.39	+0.004	19	+ 4	1.26	+0.023	31	− 4	1.34	−0.013
7	0	1.49	0.000	20	− 5	1.34	−0.012	32	0	1.78	0.000
8	− 2	1.40	−0.004	21	0	1.39	0.000	33	− 8	1.75	−0.013
9	−23	1.63	−0.038	22	− 8	1.07	−0.031	34	+ 1	1.35	+0.003
10	− 1	1.52	−0.002	23	−27	1.44	−0.051	35	− 5	1.40	−0.010
11	+ 1	1.50	+0.002	24	−25	1.12	−0.089	36	− 6	1.28	−0.013
12	+21	1.46	+0.039	25	−25	1.01	−0.067	37	− 1	1.08	−0.003
13	0	1.49	0.000								

The average value of t_1-t_2 regardless of sign, is $7^{\text{m}}.3$, and that of p_1-p_2 is 1.40. It follows that the average influence of the hour-angle error upon the parallaxes should be equal to the relative displacement of the star-images due to a change of 5.2 minutes in hour-angle, or (since most of the stars are in considerable north declination) an actual motion of the telescope through about one degree, in following the stars—which is almost certain to be insensible. The average value of $\frac{\delta\pi}{\delta\beta}$, without regard to sign, is 0.018; with regard to sign it is −0.008.

The whole change in β, from the visual to the photographic rays, is about 0″.8. Between any two stars photographed on the same plate it must be much less. For 61 Cygni—a reddish star which should give a large value of $\delta\beta$—Bergstrand finds $\delta\beta = 0''.12$ (mean for the two components).* If it is equally great for all the parallax-stars the average error of one parallax, due to this cause, would be 0″.002. The greatest value of $\frac{\delta\pi}{\delta\beta}$ is −0.09, for Series 24. To produce an error of 0″.01 in the parallax, $\delta\beta$ must in this case be 0″.11. It is not likely to be much greater.

For the stars observed with the color-screen, it is uncertain whether green or violet light did most to make the photographic image. If the former alone was effective $\delta\beta$ would be about −0″.8. For these stars the average value of $\frac{\delta\pi}{\delta\beta}$, regardless of sign, is 0.011, so that, even upon the most unfavorable hypothesis, the average error of these parallaxes due to this cause would be only 0″.009.

It therefore appears that the residual errors, due to atmospheric dispersion, can hardly in any case exceed 0″.01. The means adopted for its elimination have therefore been successful. What they have cost, in loss of

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*Astronomische Nachrichten, 3999.

weight, may be seen from table 4, which gives the mean values of $p_1 - p_2$, grouping the stars according to their right ascension. (The first group includes all stars between 0^h and 4^h, &c.)

TABLE 4.

Mean R. A.	Mean $p_1 - p_2$.	No. of series.
2^h	1.47	8
6	1.72	3
10	1.51	5
14	1.38	8
18	1.21	8
22	1.36	5

The available parallactic displacements, actually realized, as here shown, average about 12 per cent less than the theoretically available values given in Chapter I, page 5. The general mean for all the series is 1.40.

By removing the restriction of observations to the meridian the average available displacement could have been increased to about 1.90 and the nominal weight with which the parallax was determined nearly doubled; but the average difference between the hour-angles at morning and evening observations would have been more than three hours, and the systematic error would have been increased thirty-fold.

§ 8. *Examples of Details of Measurement and Reduction.*

In conclusion it may be well to give examples of the details of measurement and reduction. For this purpose Series 11 may be selected. Here there are nine comparison-stars and two parallax-stars—A, originally chosen on account of its large proper-motion, and B, added because it happened to appear on the plates, which is too bright to give images of the best quality. As an illustration of the way in which the measures were made and recorded, we may take Plate 351, on which, according to the usual plan, two of the four exposures were measured in one position of the plate and two in the other.

The record of the measures of the first star (with certain explanations added in parenthesis) is as follows:

TABLE 5.

Star 1.	Exp. 1.	Exp. 2.	Exp. 3.	Exp. 4.
(Integral part of x) 13	(Réseau) 17600	17597	8238	8235
(Integral part of y) 21	(Star) 11298	11255	14606	14645
	(Star) 11302	11250	14607	14655
	(Star)	11255	....	14640
Faint.	(Star)	....	....	14648
	(Réseau) 7614	7611	18235	18240
(Resulting x).	13.6309	13.6353	13.6370	13.6409

Here the first line gives the reading on the réseau-line adjacent to the star in the direction of decreasing x coördinates on the plate (which in the position of the latter for the measures of the first two exposures was apparently that of increasing readings on the eye-piece scale, but opposite for the last two, after the plate had been reversed). This reading, obtained by adding together the readings of the scale and micrometer screw, is given in hundredths of a scale-division.

The last line gives the similar reading on the réseau line adjacent to the star in the opposite direction. These two readings should differ by just 10,000 if the scale stood in exactly the assumed ratio to the réseau-interval. The differences from this value give the error of "runs," which is allowed for in the usual way.

The zero from which these readings are taken is an arbitrary one, depending on the position of the plate at the moment under the microscope. It is nearly the same for neighboring images measured in immediate succession, and would be exactly so if the slides in which the plate-carrier moves were geometrically perfect (which would have involved a quite useless expense). But for the measures in the reversed position of the plate it is wholly different.

The intervening lines give the similar readings on the star-images. Where more than two are made, the reason is the discordance of the first two. The settings on this star are unusually discordant, probably because, as noted at the time, it was faint. The differences between the mean of the readings on the image and that on the first réseau line, corrected for "runs," gives the fractional part of the star's *x* coördinate. The integral part, taken from the setting scales, is recorded at the side of the sheet.

When the whole plate has been measured, the mean of the coördinates of the four images of each star is taken. These are given in table C. The differences from this mean are then formed for each image and arranged in columns for each exposure; the mean is taken for each column, and the individual differences are used to determine the "average residual," as described above. For this same plate the resulting table is as follows:

TABLE 6.

Star.	Exposure 1.		Exposure 2.		Exposure 3.		Exposure 4.	
	d	*v*	*d*	*v*	*d*	*v*	*d*	*v*
2	−37	+12	+ 3	+11	− 2	−6	+36	−17
3	−44	+ 6	−12	− 4	0	−4	+56	+ 2
1	−51	− 2	− 7	+ 1	+10	+6	+49	− 4
4	−57	− 7	−20	−12	+11	+7	+66	+12
5	−54	− 5	− 9	− 1	+ 5	+1	+57	+ 4
8	−56	− 6	0	+ 8	− 5	−9	+61	+ 7
6	−56	− 7	+ 5	+13	+ 5	+1	+47	− 6
7	−44	+ 6	− 4	+ 4	− 1	−5	+49	− 5
9	−46	+ 3	−18	−10	+10	+6	+54	+ 1
A	−48	+ 2	−15	− 7	+ 7	+3	+56	+ 2
B	−53	− 3	− 9	− 1	+ 3	−1	+57	+ 4
Mean	−49.5		− 7.9		+ 4.0		+53.5	
Sum for *v*		59		72		49		64

Average residual 5.7.

For each image, *d* denotes the difference of the measured *x* from the mean for the star, and *v* the difference of this from the mean for the exposure. Both are expressed in units of the fourth decimal place of a réseau interval. It is clear that any serious error in the original coördinates, however arising, can not escape detection in such a table.

The sum of the quantities v, either by rows or by columns, must vanish (within the error arising from neglected decimals). Their sums, regardless of sign, are given at the foot of each column and afford a measure of the accuracy of the coördinates. The "average residual," given in table C, is the numerical mean of the values of v for the whole plate. For this plate it is about 20 per cent greater than the average, showing that it is of rather poor quality.

Still taking the same plate as an illustration, each star gives an equation of condition for the plate-constants, of the form $a\xi+b\eta+c=x-\xi$. For example: star 2 gives $14.578a+9.738b+c=-0^{\mathrm{R}}.34302$. Taking the means of these for the first two and the last two groups of stars, we obtain

$$14.871a+17.965b+c=-0.31325$$
$$25.587a+21.098b+c=-0.29142$$

whence, subtracting,

$$10.716a+\ 3.133b\qquad=+0.02183 \tag{1}$$

Similarly, the means of the first and fourth and the second and third groups give

$$21.998a+13.104b+c=-0.32344$$
$$19.358a+27.958b+c=-0.27323$$

whence

$$-2.640a+14.854b\qquad=+0.05021 \tag{2}$$

Solving the equations (1) and (2) we have

$$a=+0.0009969 \qquad\qquad b=+0.0035573$$

and substituting in the four original equations, we find $c=-0.39198$. The substitution should be made in all four, to detect possible errors of arithmetic.

For any other plate, the absolute terms only of equations (1) and (2) will be different. If we call these c_1 and c_2 we find once for all

$$a=+0.088709c_1-0.018710c_2 \qquad\qquad b=+0.015766c_1+0.063996c_2$$

and their determination becomes very convenient. Having found the plate-constants, we calculate for each star the quantities $\xi+a\xi+b\eta+c$. The excess of the observed values over these represents the combined effect of errors and of the star's motion. For example, for stars A and B, on the above plate, we have the results shown in table 7.

TABLE 7.

	A.	B.
ξ	19.81751	22.58960
$a\xi$	+ 0.01977	+ 0.02251
$b\eta$	+ 0.07208	+ 0.05606
c	− 0.39198	− 0.39198
Sum	19.51738	22.27619
Measured x	19.53016	22.27571
$o-c$	+ 0.01278	− 0.00048

In practice, only the decimal parts are written down. In carrying out the numerical work of reducing the plates and solving the equations resulting from them, much use was made of an arithmometer at Cambridge and of a large cylindrical slide-rule belonging to the Department of Civil Engineering at Princeton. The latter proved to be especially convenient in solving the systems of linear equations which continually present themselves.

§9. *Example of Approximate Solution.*

As an example of the approximate solution for parallax and proper-motion, we may take Series XXII, where we have eleven equations, for five parallactic epochs. Taking means for each epoch, and remembering that Plate 496 has half-weight, we have the equations

$$\begin{array}{lc} & \text{Weight.} \\ x-0.68y+0.71\pi=+\ \ 9 & 2 \\ x-0.50y-0.33\pi=-118 & 3 \\ x+0.35y+0.55\pi=-345 & 2 \\ x+1.33y+0.65\pi=-641 & 1\frac{1}{2} \\ x+1.55y-0.60\pi=-768 & 2 \end{array}$$

where the absolute terms are those for star A and are expressed in units of the fifth decimal place of a réseau interval, and where y represents the whole proper-motion in the x coördinate.

Taking the means of the first two and the last three of these equations, we find

$$x-0.59y+0.19\pi=-54 \qquad x+1.08y+0.20\pi=-585$$

whence

$$+1.67y+0.01\pi=-531$$

Taking means of the equations in which the coefficients of π are of the same sign, we have

$$x+0.33y+0.60\pi=-326 \qquad x+0.52y-0.47\pi=-443$$

whence

$$-0.19y+1.07\pi=+117$$

whence we find

$$y=-318 \qquad \pi=+52$$

or in seconds of arc

$$y=-0''.559 \qquad \pi=+0''.092$$

The values derived from the least-square solution are

$$y=-0''.554\pm0''.007 \qquad \pi=+0''.095\pm0''.012$$

so that in both cases the agreement is well within the probable error of the latter (definitive) values.

When the different epoch-mean equations are of different weight, this fact should be borne in mind in combining them. If a little discretion is shown in this, the results of the approximate solution agree very closely with those obtained by least-squares. This is a consequence of the approximate equality of all the positive and all the negative coefficients of π in each set of equations of condition. It affords a valuable control on the arithmetical work.

§10. *Case when a Comparison-Star has Sensible Proper-Motion.*

It remains to give an example of the discussion of a case when a comparison-star shows evidences of proper-motion (see p. 27). For this we may take Series II, where the computed proper-motions of the stars, in thousandths of a second of arc, are as shown in table 8.

The numerical equality of the sums for the four groups (within the errors of reckoning) shows that the arithmetical work is correct; but the value of y for star 8 is so large as to indicate real proper-motion rather than errors of observation. This motion will alter all this star's coördinates except the standard. The differences $x-\xi$ will be affected, and hence the derived plate-constants and the residuals, not only for star 8, but for all the others as well. To correct all these would be very laborious, but is fortunately quite unnecessary.

Since the equations for the reduction of all the plates are linear, and differ only in their absolute terms, a change, of magnitude z, in the x coördinate of a given comparison-star on any plate will produce changes in the plate-constants and in the deduced residuals for any star (whether this or another) which bear a fixed ratio to z.

If z is proportional to the time, the changes in the residuals for all the stars will be so; that is, they will be indistinguishable from their own proper-motion. Hence a real proper-motion in any comparison-star will introduce spurious alterations into the computed proper-motions of all the stars on the plate, but will be without influence on any independent quantity (*e.g.*, their computed parallaxes).

TABLE 8.

Star.	y	Sums by groups.
2	− 15	+86
3	+ 101	
1	− 10	−85
4	− 75	
5	− 137	+83
8	+ 220	
6	+ 65	−87
7	− 119	
9	− 33	
A	+2794	
B	− 106	

Let us then suppose that the standard x of star 8 is changed by z, while the others are unaltered. Applying Dyson's method (as above) to the resulting coördinates, we find very easily the plate constants

$$a=+0.013306z \qquad b=+0.01915z \qquad c=-0.538z$$

The assumed differences from standard and those computed by means of these plate-constants are as follows:

TABLE 9.

Star.	Assumed.	Computed.	Residual (A−C).
2	0	$-0.162z$	$+0.162z$
3	0	$-0.139z$	$+0.139z$
1	0	$+0.067z$	$-0.067z$
4	0	$+0.234z$	$-0.234z$
5	0	$+0.327z$	$-0.327z$
8	z	$+0.374z$	$+0.626z$
6	0	$+0.135z$	$-0.135z$
7	0	$+0.054z$	$-0.054z$
9	0	$+0.111z$	$-0.111z$
A	0	$+0.101z$	$-0.101z$
B	0	$+0.059z$	$-0.059z$

The numbers in the last column represent the spurious proper-motions of stars, which will appear as a result of the calculations, when star 8 alone has a real proper-motion, of magnitude z. Their sums by groups are numerically equal, as they should be.

If now we wish to find the proper-motions of our stars, referred to the other comparison-stars as a standard, rejecting star 8, we may proceed as follows:

The rejection of star 8 leaves star 5 alone in its "group." Had we determined plate-constants, etc., anew by Dyson's method, we would then have the resulting proper-motion for this star equal to the sum of those for stars 2 and 3, and also to that for either of the other two groups, with its sign changed. But we can do this at once by the choice of a suitable value of z.

Correcting the original proper-motions by subtracting from them the spurious proper-motions just given, we find for star 5 the value $-137+0.327z$, while the sums for the other three groups are

Stars.	
2, 3	$+86-0.301z$
1, 4	$-87+0.301z$
6, 7, 9	$-87+0.300z$

We have thus, to determine z, the equation

$$-137+0.327z=+86-0.301z$$

whence $z=+355$.

To correct our individual proper-motions we must add to each the quantity given in the third column of table 9 (which represents the influence of the change of plate-constants). The results are as shown in table 10.

TABLE 10.

Star.	2	3	1	4	5	8	6	7	9	A	B
Previous values..	−15	+101	−10	−75	−137	+220	+ 65	−119	−33	+2794	−106
Correction.......	−58	− 49	+24	+83	+116	+133	+ 48	+ 19	+40	+ 36	+ 21
New values......	−73	+ 52	+14	+ 8	− 21	+353	+113	−100	+ 7	+2830	− 85
Sums by groups excluding No. 8.	−21		+22		−21		+20				

We have now a set of proper-motions which satisfy (within the errors of reckoning) exactly the same conditions that would result from a complete new solution. But since these conditions are linear, there is only one way of satisfying them. Our results must therefore be identical with those of the more complicated process. They are given in table C, Series II, under the heading μ'.

The average value, regardless of sign, of the computed proper-motions of the remaining comparison-stars is reduced, by the exclusion of star 8, from 69 to 48, while the resulting value for the proper-motion of star 8 comes out more than three times as large as any of the others. Its reality is thus confirmed, but its amount is not great enough to justify the rejection of this star as a comparison-star for parallax.

CHAPTER IV.

DISCUSSION OF THE OBSERVATIONS.

I. ABSENCE OF SYSTEMATIC ERRORS.

§1. *Errors of Observation almost wholly Accidental in Character.*

In the course of the reduction of the 31 series, which form the principal part of the present work, the parallaxes of 242 comparison-stars have been determined. If any systematic errors affect the results, this large amount of material should suffice for their detection, provided that it is true, as is now generally believed, that the individual differences of parallax among such stars are practically insensible. To see whether this is the case, the numbers of observed parallaxes lying between different numerical limits were counted with the results shown in table 11.

The second column gives the number of parallaxes which lie between the given limits, while the third column shows the distribution resulting from the "law of errors" with probable error ±0″.0283.

It is at once manifest that the observed parallaxes of the comparison-stars are almost wholly due to errors of observation and that they furnish no information at all about the real parallax of *individual* stars.

TABLE 11.

Limits.	Observed.	Theory.
> +0″.12	1	1
+0″.12 to +0.09	4	3
+0.09 to +0.06	16	15
+0.06 to +0.03	41	39
+0.03 to 0.00	61	63
0.00 to −0.03	64	63
−0.03 to −0.06	36	39
−0.06 to −0.09	14	15
−0.09 to −0.12	2	3
< −0.12	3	1

But this is just the condition under which they are suitable for the investigation of systematic errors. Such errors may depend: (*a*) upon the star's position upon the plate. (*b*) upon its brightness. (*c*) upon its spectral type. (*d*) upon the season of the year in which observations are made.

§2. *Search for Systematic Error depending on position on the Plate.*

To investigate possible errors of class (*a*) a diagram was prepared, showing the position of each of the comparison-stars in the field, and its observed parallax. The field was then divided into sixteen regions, containing nearly the same number of stars, by means of the réseau lines $x = 20$, $y = 20$, which pass through the optical center, and the lines five réseau intervals (14′.5) distant from these on each side.

The results may best be represented graphically. In the diagrams which follow the *x*-coördinates increase toward the left, and the *y* coördinates upward. Diagram A shows the number of stars in each region, and B their average parallax, regardless of sign, in thousandths of a second of arc. Diagram C gives the average parallax, taking account of sign, for each region, and D the numerical values which this might be expected to have, if due to accidental errors alone (which are obtained by dividing the numbers in B by the square root of the number of stars in each region, given in A)—all in thousandths of a second of arc.

A.

9	18	19	17
20	11	18	13
20	13	11	15
17	12	15	14

B.

49	27	23	37
32	53	26	31
34	38	22	34
35	31	34	39

C.

+20	− 9	− 1	+14
−10	+15	− 2	− 1
+12	− 5	−11	− 1
+13	− 9	−20	− 4

D.

16	6	5	9
7	16	6	9
8	11	7	9
8	9	9	10

Of the sixteen quantities in C, seven are less than might be expected, one is equal to expectation, and eight are greater. The numerical mean of all the observed quantities is 0″.0092, and of those predicted by the theory of errors 0″.0090. It would therefore appear that the observed quantities are almost wholly due to accidental error.

Taking means for the four middle regions, the four corner ones, and the eight others at the sides, the results are given in table 12:

TABLE 12—*Mean Parallax.*

	Without regard to sign.	With regard to sign.	Expectation.	No. of stars.
Corners	0″.039	+0″.010	0″.005	57
Sides	0.031	−0.004	0.003	132
Middle	0.033	−0.001	0.004	53
All	0.0332			242

The fourth column gives the values which the quantities in the third column might be expected to have if due to accidental error only. It appears that there is no systematic error, unless perhaps a very small one for the stars at the extreme corners of the field, which in any case must be less than one-hundredth of a second of arc. The average value of the parallax, regardless of sign, is 25 per cent greater for this group than for the others, showing that in this region the measures become less accurate.

§3. *Search for Error depending on Magnitude.*

To investigate possible errors of type (*b*), depending on magnitude, the stars were grouped according to their photometric magnitudes (a few stars which were not observed photometrically being distributed among the groups on the basis of their Bonn Durchmusterung magnitudes).

The results are as follows:

TABLE 13.

No. of stars.	Mean magnitudes.		Mean parallax.		Expectation.
	Photometric.	Bonn Durchmusterung.	Without regard to sign.	With regard to sign.	
25	7.22	7.39	0″.037	+0″.006	0″.007
59	8.64	8.55	0.030	−0.001	0.004
106	9.51	9.07	0.034	−0.002	0.003
52	10.35*	9.46*	0.032	+0.003	0.004

Here again there is clearly no sensible systematic error. The observed mean parallaxes are smaller than might be expected. This is probably to be explained by the uneven distribution of stars of a given magnitude among the different fields. The sum of the parallaxes of all the comparison-stars in a given field necessarily vanishes; and if all of them, or all but one or two, fall in the same magnitude-group they will make an unduly small contribution to the total for this group. This effect should be least for the groups which contain fewest stars (to which each field will contribute but one or two members); and this is in accord with the facts.

The values of the average parallax, regardless of sign, show that the measures are less accurate for the brightest stars than for the others, for which they are almost equally good.

§4. *Search for Error depending on Spectral Type.*

It is possible in the present work to investigate the errors of class (*c*) (depending on the spectral type) directly.

The whole number of comparison-stars whose spectral type was determined at Harvard is 216. These are divided almost equally among the four principal classes. In table 14 all spectra from A6 to F5, inclusive, are counted as F, and so on, except that the three stars of type M are included with those of type K.

TABLE 14.

Spectrum.	No. of stars.	Mean photometric magnitude.	Mean parallax.		Expectation.
			Without regard to sign.	With regard to sign.	
A............	40	9.03	0″.031	−0″.005	0″.005
F............	53	9.48	0.033	+0.008	0.005
G............	65	9.09	0.033	+0.003	0.004
K............	58	8.85	0.036	−0.002	0.005
Not observed.	26	9.92	0.033	−0.011	0.006

*Several of the faintest stars do not appear in the Bonn Durchmusterung, and were not observed at Harvard. If they were included, the mean magnitudes in the last line would be slightly lower. They are included in the mean for parallax.

Once more it is clear that the systematic error must be quite insensible The slight deficiency in parallax for the (very faint) stars whose spectra were not observed is not confirmed by the result previously found for the whole of the faintest stars, and it is doubtless due to accidental errors of observation.

§5. *Search for Errors depending on Right Ascension.*

There may also exist systematic errors arising from change in the instruments, or in the other conditions of observation, from time to time, and especially with the seasons. The influence of such errors upon a star's observed parallax will be a function of its right ascension. In studying such errors it is no longer legitimate to combine stars in all parts of the sky, as has previously been done. They have accordingly been divided into groups, covering 4^h in right ascension, and for each of these the difference of the mean parallaxes has been determined: (*a*) for stars in the inner and outer parts of the field; (*b*) for the brighter and fainter stars; and (*c*) for those of "earlier" or "later" spectral type (which may temporarily for convenience be called "white" and "red"), dividing the stars of each field into groups as nearly equal as possible. The results are as follows:

TABLE 15.—*Mean difference of parallax.*

R. A.	Inner *minus* Outer.		Bright *minus* Faint.		White *minus* Red.	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
0^h	+0″.002	0″.008	+0″.018	0″.008	+0″.008	0″.009
4^h	+0.005	0.009	−0.001	0.009	−0.008	0.010
*						
12^h	−0.003	0.009	+0.010	0.009	+0.017	0.010
16^h	+0.005	0.011	−0.008	0.011	+0.014	0.014
20^h	+0.002	0.011	−0.016	0.011	−0.003	0.011
Average	0.003	0.010	0.011	0.010	0.010	0.011

The expectation has been calculated as usual. It is somewhat larger in the last column, because fewer stars were observed for spectrum than for magnitude or position. The last line contains the average, regardless of sign, of the quantities in the preceding lines.

It is clear that there is practically no systematic error of any of these kinds, depending on the right ascension.

As an additional test, the differences of mean parallax were computed for each field separately, and compared with the corresponding expectations. The averages, without regard to sign, for all the fields, are shown in table 16.

TABLE 16.

	Average.	Expectation.
Inner *minus* Outer.	0″.025	0″.024
Bright *minus* Faint.	0.028	0.024
White *minus* Red..	0.024	0.026

The observed differences of parallax for groups of stars differing in position on the plate or in spectral type are practically identical with those which

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*No stars lying between 6^h and 10^h were observed at three or more epochs.

might be expected if they had been chosen at random. It follows that the distortion of the field and the errors depending on the color of the stars are quite insensible.

Bright and faint stars show somewhat greater differences of parallax than groups chosen at random. There appears to be some real cause at work here. Its average influence for any given series, as determined from the difference of the squares of the observed and expected values, is 0″.014. This, however, should hardly be called a systematic error, for it appears to vary quite at random from one series to another.

Of the 31 differences "Bright *minus* Faint," 17 are positive and 14 are negative; 16 are less than the probable error computed from their numerical average and 15 greater. If they are arranged in order of right ascension there are 18 changes and 13 permanences of sign.

The source of this error presumably lies in the individual plate. It is very probably "guiding error"—due to imperfect following—which can not quite be eliminated by even the best instrumental means. Since the average weight of a parallax is 2.98, an average displacement of the bright stars, relative to the faint, of 0″.024, taking place quite at random from plate to plate, is sufficient to produce the observed result. Such an error for any given star will simply increase the accidental error of observation by a small amount; and the probable error, as determined by comparison of different plates, will include its full effect. It need not, therefore, be further considered, as it will introduce no real systematic error into the final parallaxes.

§6. *Conclusion: Systematic Errors Apparently Insensible.*

It may be concluded from the preceding discussion that there are no systematic errors affecting a star's observed parallax, dependent either upon its magnitude, its position in the field, the character of its own light, or upon seasonal or instrumental changes, of greater magnitude than a few thousandths of a second of arc. In other words—

> *The observed parallaxes appear to be altogether free of sensible systematic error.*

This conclusion is all the more satisfactory because serious doubts have been expressed concerning the possibility of obtaining photographic positions of high precision with instruments in which a mirror forms part of the optical train. It is clear that such fears may be laid aside—at least under the circumstances of the present work. It may be observed, however, that these are much more favorable to accuracy than are the conditions in many of the most familiar instances in which mirrors are used.

Comparison with solar observations, where the mirrors must be exposed to direct sunlight, would be manifestly unfair. As compared with the ordinary reflecting telescope, the coudé has the advantage that its mirror is flat. Mere linear expansion or contraction, without deformation, does not affect its definition at all, while a parabolic mirror, under similar circumstances, varies in focal length. It is therefore not surprising that the temperature effects, which are often troublesome in such instruments, are here practically absent.

II. THE PROPER-MOTIONS.

§7. *Comparison-stars with Sensible Proper-motion.*

The real differences of parallax among the comparison-stars are too small to be detected; is the same true of the proper-motions? The individual proper-motions of the comparison-stars are usually less than the errors of observation, but there are a few stars which show pretty clear evidence of motion. Those whose proper-motion in *x* exceeds 0″.20 are as follows:

TABLE 17.

Bonn Durchmusterung.		Photometric magnitude.	Sp.	Proper-motion in *x*.	Weight.	Observed parallax.	Series.
+43°	55	9.72	*K*	+0″.35	1.1	−0″.082	2
+30°	619	10.29	*K*?	−0.38	0.9	+0.001	5
+49°	1956	10.26	*G*2	−0.37	1.8	−0.029	10
+11°	4776	8.10	*G*2	+0.23	3.9	+0.043	28
+43°	4434	9.53	*A*	−0.37	1.2	+0.030	30
+ 1°	4784	10.06	....	−0.21	1.1	+0.004	31

The above are the proper-motions referred to the mean of the remaining comparison-stars. The weights are those found in the least-squares solutions for the parallax-stars of the same series. As will appear below, the probable error of the unit of weight is about ±0″.057. Their mean observed parallax is −0″.005 and the average, without regard to sign, is 0″.031. If the first star, which is clearly affected to an unusual degree by observational error, be excluded, the mean parallax is +0″.010. By Kapteyn's formulæ the mean parallax of these stars, relative to the comparison-stars, should be +0″.018.

§8. *Average Proper-Motion of the Rest.*

Excluding these six stars, the proper-motions of the remaining comparison-stars may be investigated as follows. If the 31 complete series are arranged according to the weight with which the proper-motion is determined, they fall naturally into four groups constituted as follows:

I. Series 1–6, 15, 23–26, 30, 31.
II. Series 10, 11, 14, 17, 18, 20.
III. Series 7–9, 28.
IV. Series 12, 13, 16, 19, 21, 22, 27, 29.

The first consists of the fields photographed at three epochs only, the second of those observed at four consecutive epochs; while the remaining series fall into two groups, in one of which the weight of *y* is between 3 and 4, and in the other from 7 to 12.

The average value of the observed parallax and proper-motion, without regard to sign, for the stars of these groups, together with their mean weights, as found in the least-square solutions for the parallax stars, are given in table 18.

TABLE 18.

Group.	No of stars.	Average parallax.	Weight of parallax.	Average proper-motion.	Weight of P. M.	Expectation.	True proper-motion.
I	101	0″.034	2.58	0″.059	0.93	0″.057	0″.015
II	47	0.034	3.12	0.051	1.70	0.046	0.022
III	29	0.029	2.96	0.031	3.39	0.027	0.016
IV	59	0.032	3.57	0.022	10.52	0.019	0.012

The observed proper-motions decrease rapidly with increasing weight, showing that they are principally due to accidental error. If they were entirely due to this cause, the mean proper-motion and parallax for each group should be in the inverse ratio of the square roots of their weights. In this way the quantities in the column headed "Expectation" are derived from the observed mean parallaxes. These are uniformly smaller than the observed proper-motions. The most reasonable explanation of the discrepancy is that the stars really have small but sensible proper-motions. If these are distributed in a random manner, the square of the observed average proper-motion will be equal to the sum of the squares of the average real proper-motion and the accidental error. Thus the "true" average proper-motions given in the last column are determined.

The observed proper-motions, *by themselves*, can be represented as the results of accidental error about as well as the parallaxes. The average error of a single plate can be found from each of the tabular quantities by multiplying it by the square root of its weight. This gives the results shown in table 19.

But the two values of the average error, derived in this way, are inconsistent. There is some systematic cause of discrepancy at work, which causes the plates to agree better when the proper-motions of the stars are *eliminated* than when they are merely assumed to be negligible; and this must almost certainly be real proper-motion.

TABLE 19.—*Average error of one plate.*

	From the parallaxes.	From the proper-motions.
Group I........	0″.055	0″.061
Group II.......	0.060	0.066
Group III......	0.049	0.058
Group IV.......	0.061	0.072
Weighted mean.	0.057	0.064

If each of the tabular values of the real motion be given a weight proportional to the product of the number of stars in the group by the weight of an individual proper-motion, the mean is 0″.014. With this value of the average real motion, and the accidental errors given above, the observed proper-motions are represented as shown in table 20, the agreement being very satisfactory.*

TABLE 20.

	Observed.	Computed.
Group I....	0″.059	0″.059
Group II...	0.051	0.048
Group III..	0.031	0.031
Group IV...	0.022	0.023

The mean proper-motions of the stars of each spectral type may be derived in the same fashion. To find the "expected" values of the proper-motion it is here necessary to calculate the average value of $\frac{1}{\sqrt{p}}$ (where p is the weight of the determination) for the stars of each group separately. The influences of accidental error upon the observed parallaxes and proper-motions will be in the ratio of the resulting means.†

The results, including all the comparison-stars whose spectra were determined, are as follows:

TABLE 21.

Spectrum.	No. of stars.	Observed parallax.	Average $\frac{1}{\sqrt{p}}$	Observed proper-motion.	Average $\frac{1}{\sqrt{p}}$	Expectation.	True average proper-motion.
A	40	0″.031	0.60	0″.065	0.86	0″.044	0″.048
F	53	0.033	0.58	0.043	0.75	0.042	0.010
G	65	0.033	0.57	0.046	0.69	0.039	0.024
K	58	0.036	0.59	0.047	0.75	0.046	0.011

The mean, weighted according to the number of stars, is 0″.026 for Type I and 0″.018 for Type II. In view of the inevitable uncertainties of the method, no stress can be laid on the differences between these individual values, but it is noteworthy that all four types show evidence of real proper-motion. The mean for all the stars is 0″.0215. This includes five stars (listed above) whose average proper-motion is 0″.34. Excluding these, the mean proper-motion of the remainder is 0″.014, in satisfactory agreement with the result previously found with a different grouping.

Since the observed proper-motions, upon which the above calculations are based, satisfy three conditions for every field (their sum for the north-and-south and the east-and-west halves of the plate vanishing) the real proper-motion of the stars will, on the average, be greater than the tabular values in the ratio $\sqrt{n} : \sqrt{n-3}$, where n is the number of comparison-stars on the plate. The average value of n is 7.8. The average proper-motion of the comparison-stars, excluding the six already mentioned, is therefore $0''.014 \times \sqrt{\frac{7.8}{4.8}}$ or 0″.017, in x. Including the six stars, the average proper-

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*These and many similar quantities have been calculated with one more place of decimals than appears in the tables—which accounts for some apparent inconsistencies.

†It will not do in this case to calculate mean weights and then take the square root, because of the great range in the individual weights of the proper-motions.

motion in x is 0″.025. Their average proper-motion on a great circle (if the proper-motions are distributed at random on the celestial sphere) must be $\frac{4}{\pi}$ times as great—that is 0″.032.

§9. *Proper-Motion of the Parallax-Stars. Reality of the Observed Corrections.*

The corrections derived from the plates for the catalogued proper-motions of the parallax-stars may be similarly discussed. To render the material homogeneous, Bossert's proper-motions have been used whenever possible.*

Forming means by groups as above (but combining the two middle groups on account of the smaller number of stars contained in them), the results are:

TABLE 22.

Group.	No. of stars.	Average correction (without regard to sign).	Average. weight.	Average probable error of y.	Average probable error of one plate.
I..........	18	0″.045	0.96	±0″.049	±0″.048
II and III..	13	0.046	2.33	±0.031	±0.043
IV........	13	0.042	10.75	±0.012	±0.039

It should be noticed that the average probable error of one plate (arithmetical mean) decreases steadily with increasing weight of y, that is, with increasing length of time covered by the observations. This may be partly due to chance, but it affords a strong presumption against the existence of any serious changes in the instrumental conditions with the time, which would show themselves by an increase. The results for the comparison-stars (see p. 51) confirm this conclusion.

In this case, however, there is no such conspicuous decrease of the corrections with increasing weight as was apparent among the uncorrected proper-motions of the comparison-stars. In the first group 11 out of 18 corrections are less than their probable error; in the second group, 4 out of 13, and in the third group none at all. Of values greater than three times their probable error, there are none in the first group, two in the next, and eight in the last. It would therefore appear that many of these corrections must have a real meaning. They are no doubt due in part to errors in the catalogued proper-motions and in part to proper-motion of the comparison-stars. The relatively small average for the first group may be explained by the fact that it contains all the bright stars, observed with the color-screen, whose catalogued proper-motions are the most accurate. This is confirmed by the comparison of the proper-motions used above with the very accurate ones of Professor Boss's Preliminary General Catalogue (which he very kindly communicated to the writer, in advance of publica-

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*These corrections therefore differ from the values of y found in the least-square solutions, when the proper-motion used in preparing the observations for the latter was different from that mentioned above. The orbital motion of η Cassiopeiæ has been allowed for.

tion, for the 18 stars which appear in the table). The average correction to the tabular values, regardless of sign, is 0″.008 for the five bright stars and 0″.025 for the thirteen others. The proper-motions of the remaining stars, not included in the above catalogue, are presumably known with less accuracy, but the assumption that their errors are great enough to account for the whole of the discrepancies revealed by the plates is obviously violent.

That these are due in many instances to proper-motion of the comparison-stars is shown by discussion of the cases where proper-motion was evident in one of the latter. Table 23 gives the corrections, resulting from the plates, to the catalogued proper-motion, before and after the rejection of the comparison-stars known to be in motion.

TABLE. 23.

Star.	Correction to tabular proper-motion.		Probable error.
	Before rejection.	After rejection.	
Groombridge 34	−0″.013	+0″.023	±0″.018
26 Andromedae*	−0.124	−0.083	±0.068
ρ Persei*	+0.093	+0.049	±0.068
Groombridge 1646*	−0.108	−0.140	±0.031
83^1 Leonis*	+0.064	+0.016	±0.017
83^2 Leonis*	+0.023	−0.026	±0.018
Lalande 25372	+0.069	+0.086	±0.024
Lal. 43492	−0.035	−0.013	±0.012
Lal. 45755	+0.046	+0.001	±0.034
Lal. 46650	+0.015	+0.008	±0.022
Lam. 32805	+0.027	−0.024	±0.049

In seven cases out of eleven the discordance is diminished by the rejection of the moving comparison-star. In two others it is slightly increased numerically, but not to an amount unreasonable in view of its probable error. In the two remaining cases the discordance is increased, and is large compared with its probable error. For Lal. 25372 it may be partly due to error of tabular proper-motion; but this can not be the case for Groombridge 1646, whose proper-motion, according to Professor Boss, is very well determined and shows no evidence of being variable. Proper-motion of the comparison-stars could produce so large a correction, without evidence of marked relative motion, only if a considerable number of them were moving together. It may be that the difficulty is, after all, due to some concealed form of error of observation; but the possibility remains that their proper-motions are really variable. As both stars have sensible parallaxes, the actual velocities which need be assumed are not great, and if the orbital motion was of short period the amplitude would be small enough to escape detection from the meridian observations of the star. Whatever may be the explanation, the fact remains that the catalogued proper-motions of the parallax-stars differ from their motions relative to the stars chosen for comparison by amounts which are

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*The "tabular" proper-motions for these stars are those of Boss's Preliminary General Catalogue.

often much too great to be disregarded. Those solutions of the equations of condition in which these differences are taken into account have therefore been regarded throughout as definitive.

III. THE "TWO-EPOCH" PARALLAXES.

§ 10. *Reliability of these Results. Their Probable Errors.*

There are six series (XXXII to XXXVII) for which observations could be secured at but two parallactic epochs (owing to the accident to the color-screen) and from which the parallaxes of the principal stars were derived with the aid of the catalogued proper-motions.* The value of the results may be tested by applying the same process to the stars of the series which were fully observed—confining the discussion to those for which at least two plates are available at each of the first two epochs (as is the case for all the incomplete series).

A comparison of the results with those of the least-square solutions (including proper-motion corrections) is given in the following table. In this p_1-p_2 denotes the difference of the mean parallax-factors for the first and second epochs (which is positive when the star's conjunction with the Sun, negative when its opposition, falls in the interval). P denotes the parallax derived from the observations of these two epochs and the catalogued proper-motion, and π denotes the final parallax.

TABLE 24.

Star.	p_1-p_2	P	$P-\pi$
2	+1.43	+0″.245	−0″.005
3	+1.43	−0.075	−0.049
5	+1.76	+0.126	−0.010
6	+1.41	+0.130	+0.047
7	+1.41	−0.007	−0.014
11	+1.63	+0.081	+0.003
13	−1.58	+0.356	+0.010
14	+1.40	+0.111	−0.052
17	−1.57	+0.111	+0.011
18	−1.44	+0.046	−0.026
19	−1.44	+0.040	−0.014
21	−1.47	+0.198	−0.023
22	−1.40	+0.059	0.000
23	−1.40	+0.044	+0.056
28	−1.36	+0.029	+0.009
29	−1.36	+0.082	+0.023
30	−1.03	+0″.110	+0″.015
31	−1.51	+0.282	−0.009
32	−1.51	+0.314	+0.008
33	−1.21	+0.087	+0.011
34	−0.93	−0.004	+0.007
35	−0.93	+0.045	−0.030
36	−1.20	−0.011	+0.007
37	+1.31	+0.473	+0.067
38	+1.31	+0.415	+0.054
39	+1.31	+0.021	+0.056
40	+1.37	+0.004	−0.017
41	+1.30	+0.320	+0.062
42	+1.35	+0.060	+0.023
43	+1.34	+0.218	+0.007
44	+1.34	−0.007	+0.015

The mean value of $P-\pi$ (taking account of sign) is +0″.012 for the 15 stars for which p_1-p_2 is positive, and +0″.003 for the 16 for which it is negative, or +0″.008 for all together. The average value, regardless of sign, is 0″.027, so there is no evidence of systematic difference between the two groups.

Some difference between P and π should be caused by the accidental errors of observation. On the assumption (which is approximately true) that an error has the same numerical influence on the observed parallax, no matter on what plate it occurs, this effect may be calculated.

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*Pages 8, 35.

Let a be the mean of m independent quantities of probable error r, b that of n others, and c that of all together. Then the probable error of $(a-b)$ will be $r\sqrt{\frac{1}{m}+\frac{1}{n}}$, and that of c, $r\sqrt{\frac{1}{m+n}}$. But $(a-c)=\frac{n}{m+n}\,(a-b)$, whence its probable error is $r\sqrt{\frac{m}{m\,(m+n)}}$, which is $\sqrt{\frac{n}{m}}$ of that of c. If then P depends upon m plates, and π upon these and n additional ones, the probable error of $P-\pi$ should be $\sqrt{\frac{n}{m}}$ of that of π.

The average values of m and n for the stars of table 24 are 4.65 and 2.90; the average probable error of π is ±0″.0277; and the corresponding average error, regardless of sign, is 0″.0328.

The accidental errors of observation will therefore account for a discrepancy between P and π, of average amount $0''.0328\times\sqrt{\frac{2.90}{4.65}}$ or 0″.026—as against 0″.027 observed. There is, therefore, no sensible systematic difference between P and π.

The parallaxes derived from observations at two epochs and the catalogued proper-motions are therefore entitled to confidence.* They are less accurate than those derived from longer series, but the difference is mainly due to the increased number of observations in the latter and the corresponding diminution of accidental error.

The probable errors deduced from the residuals are, however, not those of π, but of $(\pi+by)$, where b is a numerical coefficient, whose values (given in table C) range in absolute magnitude from 0.37 to 0.17, and y is the excess of the proper-motion, relative to the comparison-stars, above the catalogued proper-motion.

The values of this quantity for the five stars observed at three epochs with the color-screen (using Boss's proper-motions) are:

β Cassiopeiæ	−0″.053
η "	+0.033
ρ Persei	+0.093
β "	−0.041
γ Virginis (mean)	+0.094

The mean without regard to sign is 0″.063, corresponding to a probable error of ±0″.053. This is greater than the actual probable error arising from y, for it includes the effects of accidental error of observation. If we take this as the probable error of y, and combine the probable errors of $(\pi+by)$ and by as if they were independent, we will certainly obtain a sufficiently large value for that of π. To avoid all possibility of understatement, this has been done in the final table of parallaxes for these stars.

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*This might not be the case where there were but two comparison-stars, and no check upon the proper-motions of these.

IV. ACCIDENTAL ERROR OF STAR-PLACES.

§11. *Types of Error. Notation.*

It has already been seen that there is no evidence of sensible systematic error in the measured star positions—at least of such a character as to affect the deduced parallaxes. Their accidental errors, however, deserve study.

The error of the x-coördinate of a star (such as is given in Table C), derived from the mean of the measures of several exposures on one plate, arises from several sources. These are

(1) Error of measurement proper—giving rise to differences between successive measures of the same image and réseau-lines.

(2) Errors peculiar to the individual image—which may be due to many causes—bad seeing, unequal sensitiveness of the plate, etc. These affect the real position of the center of the image or réseau-lines, but vary in a random manner from one image to another.

(3) Errors peculiar to the individual plate, which may arise from guiding error, systematic distortion of the film, etc. These affect all the images of the same star on this plate to the same extent, but for different plates may be regarded as of random character.

(4) Errors due to changes in the instrumental conditions from time to time. These would cause the agreement of two plates, taken at a long interval, to be worse than that of two taken a few days apart, when allowance was made for the motion of the stars in the interval. Only errors of this type can give rise to systematic errors in the results of observation. From the evidence already obtained, it appears that they must be very small in the case of the present work.

(5) In addition to these, there is the personal error of bisection, differing systematically for stars and réseau-lines and depending also on the appearance of the images. This has been eliminated from the mean results for each plate by measuring half the images with the plate in one position and the other half with the plate turned through 180°. (See Chapter I, § 9, pp. 15, 16.)

Before proceeding to determine the magnitude of these errors, it is well to fix a notation for them. Let the average value, without regard to sign, of the measurement-error (1) be m, that of the image-error (2) be n, of the plate-error (3) be p, and of the error (4) due to instrumental changes be t. All these are presumably independent; that is, they may be expected to show little or no correlation, and their combined effect may be found upon the principles of the theory of errors.

The personal error of bisection (5) is, by hypothesis, eliminated by reversal of the plate, and is the same for all the images of the same star.* For

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*Any error of bisection that fails to satisfy these two conditions will be combined with the image-error or the measurement-error, respectively.

different stars it may be different. Let B be its mean value for any plate, and b the average variation from star to star. The first is a constant; the second may be regarded as varying at random unless the stars are especially selected.

It will be convenient to express the average value of these errors (and the observed quantities from which they are derived) in linear measure on the plate as well as in the corresponding angular values. As they are all small, the most convenient unit is the micron. In terms of the quantities previously used we have $1.0\mu = 0.00020$ réseau-intervals—which corresponds to $0''.0352$.

§ 12. *Error of Measurement.*

To find the value of the measurement-error m, it is necessary to take plates on which each image was measured in both positions. The difference between the mean of the measures of two images of each star in the "direct" position of the plate and of the other two "reversed," and that of the measures of the same images in opposite positions, will be due wholly to the measurement-error (since the image-error is independent of the direction of measurement and the bisection-error is eliminated in the mean).

The average effect of this error upon each mean will be $\frac{1}{2}m$, and upon their difference, $\frac{m}{\sqrt{2}}$.

From the measures of 319 stars on 31 plates, the average difference between such means is 0.86μ. The corresponding value of m is

$$1.22\mu, \text{ or } 0''.043$$

The average difference between the mean result when each star-image is measured only once, in the manner described above, and that of measuring each star in both positions is only $0.43\,\mu$, or $0''.015$. This is practically negligible in comparison with the other errors of the plates, and on this account the shorter method of measurement was used for the rest of the work.

The personal error of bisection B can be found from the differences of the "direct" and "reversed" measures of each image—the errors of measurement, which influence individual results, practically disappearing from the mean. The average value of this difference for the 31 plates is $+4.85\mu$. Since B changes sign upon reversal its value is half this: that is, 2.43μ or $0''.085$. This is one twenty-first of a division of the scale with which the measures were made. The individual values for different plates show, however, a much larger range of variation than the errors of measurement will account for (the average difference, regardless of sign, between an individual value and the general mean being 22 per cent of the latter). It therefore appears that the personal error of setting is subject to considerable fluctuations—depending not only upon the appearance of the images, but on the physical condition of the measurer, and perhaps upon such factors as the illumination of the field as well.

§13. *Error Peculiar to the Individual Image.*

The image-error n may be found from the "average residual" for the plates. For the 31 plates already mentioned the mean of the two measures of each image is free from bisection-error, but is affected by measurement and image error. The resultant of these may be called the internal error (since it is derived from the "internal agreement" of the measures on the plate) and its average value denoted by i. Then since each image was measured twice, $i^2 = n^2 + \frac{1}{2}m^2$.

The "average residual" for these plates was obtained as follows:* The differences between the measured coördinates of each star for a given exposure and their mean for all the exposures were taken, and compared graphically with an expression of the form $by + c$. The average of the residuals, without regard to sign, is the tabular quantity.

Since, on the average, there are ten stars on each plate, this process involves the representation of 40 observed coördinates by 18 unknowns derived from them (10 mean coördinates of the stars and 8 constants b and c). The "average residual" must therefore be multiplied by $\sqrt{\frac{40}{40-18}}$ to find the true value of i.

For these 31 plates the mean "average residual" is 1.58μ. Hence $i = 2.14\mu$, or $0''.075$, and with the value of m found above $n = 1.96\mu = 0''.069$. If each image had been measured but once, the average internal error (excluding bisection-error) would have been $\sqrt{m^2+n^2}$, or $2.30\mu = 0''.081$.

On the remaining plates each image was measured but once. It is therefore impossible to find separate values of the measurement and bisection errors for them.

The internal error can, however, be found—but not directly from the "average residual," which in this case includes also the effect of the variable part of the error of bisection (which, though eliminated from the mean of the measures of the four exposures, influences their individual differences from this mean).

The differences between the measures for the two exposures measured in the same position of the plate are, however, free from bisection error. As there is no evidence of any real difference of orientation between the successive exposures on a plate,* these differences should be constant, and their deviation from their mean will measure the internal error of the plate.

These differences were calculated for the suitable plates of every fifth series (*i. e.*, those with four exposures, not already discussed), numbering 35 in all. Their average deviation from the means for each set is 3.17μ. The average error of such a difference should be $i\sqrt{2}$, and the average deviation from the mean of n such differences should be $i\sqrt{\frac{2n-2}{n}}$. The average value of n is in this case 9.3. Hence, for these plates, $i = 2.38\mu$.

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*Chapter III, p. 31.

The "average residual" for these plates is 2.15μ. If the internal error is computed from this the result will not be i, but $\sqrt{i^2+b^2}$, where b is the variable part of the error of bisection. In this case 36 observed coördinates have been represented by 13 unknowns (9 mean coördinates of the stars and 4 means of the differences for each exposure). Therefore

$$\sqrt{i^2+b^2}=2.15\mu\times\sqrt{\frac{36}{23}}=2.68\mu$$

For the remaining plates with four exposures and full weight, 121 in number, the mean average residual is 2.33μ, whence, as above, $\sqrt{i^2+b^2}=2.92\mu$.

If b has the same value for these plates as for the preceding ones, which form a large and apparently typical sample of the whole, then for these last plates $i=2.64\mu$.

The mean of the three values of i, with weights proportional to the number of plates on which each is based, is $i=2.53\mu=0''.089$.

If the measurement-error for all the plates is the same for those upon which it could be determined, then, since $i^2=m^2+n^2$

$$n=2.22\mu=0''.078$$

§14. *Error Peculiar to the Individual Plate.*

The error peculiar to the plate, p, must be found by comparison of pairs of plates taken within a few days of one another, for which the real motions of the stars and the possible instrumental changes are presumably negligible.

The average error k of a mean coördinate derived from four exposures on such a plate will be given by the equation $k^2=p^2+\frac{1}{4}i^2$.

The average discordance between the mean results for the two plates would be $k\sqrt{2}$ if the plate constants used in comparing them were exactly known. But since there are three of these constants, the average discordance for the n comparison-stars used in determining them will be $k\sqrt{\frac{2(n-3)}{n}}$.

For the parallax stars the average discordance will be $k\sqrt{\frac{2(n+1)}{n}}$, since for them the uncertainty of the correction to reduce to standard, whose weight is approximately n, is added to the (independent) error of the measured coördinates.

Since on the average plate there are 7.9 comparison and 1.4 parallax stars, the average discordance will be $1.115k$ for the former and $1.501k$ for the latter, which, considering their numbers, makes the general mean $1.173k$.

Taking only plates of full weight, 86 pairs are available. The average discordance of the coördinates of such a pair, after reduction to standard, is 1.99μ; whence

$$k=1.70\mu=0''.060$$

With the value of i already found

$$p=1.13\mu=0''.040$$

The relative displacement of bright and faint stars, varying at random from plate to plate, whose average amount was estimated in §5[*] as 0".024 (in addition to the other errors which vary without reference to the magnitude of the stars) is of the type of error here considered and will be included in the value of p just obtained. These groups of bright and faint stars contain on the average about four members. Between the means for any two such groups, chosen at random, there should be a difference, owing to the plate-error, of $\frac{p}{\sqrt{2}}$ or 0".028. It therefore appears that a considerable part of the plate-error depends on the magnitude of the stars—which confirms the opinion that guiding error is an important factor in its production—probably exceeding all other causes combined. This being the case, it is apparent that the plate-error would not be wholly eliminated by making all the exposures for a single series at successive epochs on one plate, according to Professor Kapteyn's plan. This would indeed get rid of such errors as arise from distortion of the film or of the réseau-lines; but the guiding-error, which is not influenced at all by the making of previous or subsequent exposures on the same plate, or by the method of measurement, would have the same effect as ever.

§15. *Error Due to Instrumental or Seasonal Changes.*

There remains the error t, due to instrumental changes. The value of this can be found by comparing the average error of a determination of parallax, calculated from the agreement of plates taken at a few days interval, with that actually observed.

The theoretical expression for the former is $\frac{k}{\sqrt{p}}$, where p is the weight of the determination of parallax. The average value of $\frac{1}{\sqrt{p}}$ for the 31 series is 0.601; whence the average error of one parallax if no instrumental errors are present, should be 0".036.

The actual value may readily be found in the case of the parallax-stars. The arithmetical mean of the probable errors of their parallaxes is 0".028. The corresponding average error, without regard to sign (obtained by dividing this by 0.845) is 0".033. But, by the reasoning of the last section, this must be multiplied by $\sqrt{\frac{n}{n+1}}$, or 0.942, to allow for the effect of the errors of the plate constants. The final value for these 44 stars is therefore 0".031.

The average value, without regard to sign, of the observed parallaxes of the 242 comparison stars is 0".033. To allow for the errors of the plate-constants, this must be multiplied by $\sqrt{\frac{n}{n-3}}$, or 1.270, giving 0".042.

If the observed parallaxes were wholly due to errors of observation the average error of one parallax for all the stars would be 0".040, and the part

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[*] Page 49.

of this due to instrumental changes would be 0″.017. But this is undoubtedly too great. The observed parallaxes of the comparison-stars are influenced, in addition to the errors of observation, by (1) the errors of the approximate method used in deriving them, and (2) the real differences in parallax between the stars. The first of these quantities can be determined by comparison of the least-squares and approximate solutions for the parallax-stars. The average difference, without regard to sign, between the values of the parallax obtained in the two ways is 0″.005. The actual differences in parallax among the comparison-stars are more difficult to estimate. An attempt may, however, be made in two ways:

(*a*) The comparison-stars are selected by magnitude alone, without respect to their proper-motion. The only group of stars of known parallax which satisfies the same condition is that of the brightest stars, all of which have been observed. From the table given in the Annals of the Cape Observatory (vol. VIII, part II, p. 142B) it follows that the mean parallax of 22 such stars is 0″.108, while the average value, regardless of sign, of the differences of the individual parallaxes from the mean is 0″.117, and the mean-square value of these differences is 0″.174. This surprising state of affairs results from the fact that a few of the stars—notably α Centauri—have very large parallaxes whose differences from the mean are much greater than the mean itself, while several others have very small parallaxes, so that their residuals are negative and almost numerically equal to the mean.

The errors of observation for these parallaxes are not great enough to have any serious influence on the above values.

According to Kapteyn's formulæ, the mean parallax of the comparison-stars is 0″.0057. By analogy with the bright stars, we might therefore expect the average difference, regardless of sign, of one parallax from the mean to be 0″.006, and the mean-square difference 0″.009.

(*b*) The only direct determination of such differences of parallax among the faint stars appears to be that given by Kapteyn (Groningen Pub. No. 20, p. 27). After a thorough elimination of the accidental errors of observation and the errors depending on magnitude, he finds evidence of real differences of parallax (among 3600 stars in eight areas in different parts of the sky) whose average amount corresponds to a probable error of ±0″.017. The photographs upon which this determination was based were, however, exposed at widely different hour-angles for the morning and evening observations, so that systematic differences of the observed parallax, arising from atmospheric dispersion[*] and depending on the spectral type of the stars, may affect the results.

As no spectroscopic data are available for these stars (most of which are too faint to appear in the Bonn Durchmusterung), such differences can not be distinguished from the real parallaxes, and the quantity just

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[*] Kapteyn calls attention to this and emphasizes the importance of confining the exposures to a fixed hour-angle in future work, a policy which he was the first to propose.

quoted is the resultant of both. It appears, however, from accompanying data, that the quantity given by the observations is approximately $\pi - 0.5\delta\beta$ (where π is the parallax and $\delta\beta$ the difference of the refraction constant, relative to the mean of all the stars on the plate). The coefficients of $\delta\beta$ for the individual plates vary somewhat, without departing far from this average value. As $\delta\beta$ may in some cases be as great as 0″.10, it is clear that its variations may account for the greater part of the real differences among the observed quantities. In the absence of data as to its average amount, it is impossible to derive from these data the real average difference of parallax among these stars.

It is necessary, therefore, to fall back on the estimate (*a*). Taking the mean-square value for the variability of the real parallax (0″.009) and the value 0″.005 for the average error due to the approximate method of solution, the average influence of all errors of observation on an observed parallax of a comparison-star becomes 0″.041, and that for all the stars 0″.039, so that the effect of the instrumental error is 0″.016.

The corresponding average error t of one star-coördinate may be found by dividing this by 0.60, whence

$$t = 0''.75\mu = 0''.026$$

If Kapteyn's value for the real differences of parallax should be adopted without correction for the error depending on spectral type, the average value of this difference, without regard to sign, would be $\frac{0''.017}{0.854}$ or 0″.020. The influence of errors of observation upon the parallax of a comparison-star is then reduced to 0″.037, and for all the stars together to 0″.036, leaving nothing to be explained by instrumental changes; but, for the reasons already stated, it seems that this tempting procedure is not warranted.

§ 16. *Summary of Results. Best Number of Exposures per Plate.*

The average errors already found may be converted into probable errors by multiplying by 0.8543. The results are as shown in table 25. These are average values for all stars and all plates. The error of bisection, which is eliminated by reversal during measurement, is

$$0.0024 \pm 0.0011 \text{ mm. } (0''.085 \pm 0''.037)$$

the probable error measuring its variation from star to star.

The average probable error of one mean star-coördinate, derived from plates with different numbers of exposures, is given in table 26.

Table 25.

Seat of error.	Probable error.	
	In mm.	In arc.
Measurement (*m*)	±0.0010	±0″.036
Image (*n*)	±0.0019	±0.066
Plate (*p*)	±0.0010	±0.034
Instrumental change (*t*)	±0.0006	±0.022

Table 26.

Exposures.	Probable error.
2	±0″.067 or ±0.0019 mm.
4	±0.055 or ±0.0016
6	±0.051 or ±0.0015

Certain important conclusions can be drawn from table 25. The errors m, n, p, t are very nearly in the ratio 3:6:3:2. Their respective contributions to the total error of the mean result from a plate with four exposures are 14, 36, 37, and 16 per cent. That is, the measurement-error contributes but one-seventh of the whole. To have measured all the star-images in both positions of the plate would have decreased the probable error of the mean by only 3 per cent—a ridiculously insignificant return for the labor involved. The policy of measurement adopted for the major part of the work is thus conclusively confirmed.

The number of exposures per plate has an important bearing on the efficiency of the work. To save measurement it must be even. The probable error of the resulting mean coördinates may be divided into two parts, one independent of the number of exposures and the other decreasing with it. If we take the former as our unit (that is, $\sqrt{p^2+t^2}$ or $0''.040$), the probable error of an equation derived from a plate with n exposures will be $0''.040\sqrt{1+\frac{3.5}{n}}$, and its weight proportional to $\frac{2n}{2n+7}$. To determine the efficiency, it is necessary to estimate the relative labor which must be expended in obtaining such equations.

The work of taking the plates and measuring them is very nearly proportional to the number of exposures. That of development, etc., and that of reduction are the same for all. The latter is certainly less than the former. Under the conditions of the present work it may be estimated to be between one-half and three-fourths as great (for a plate of four exposures).*

The total labor cost of a plate of n exposures will therefore be proportional to some quantity between $n+2$ and $n+3$. On these two hypotheses the values of the quotient of *work* divided by *weight* are as shown in table 27.

It is clear that there is a considerable loss in making more than six exposures on one plate. For smaller numbers the efficiency is nearly constant. In practice it is found that to make six exposures on each plate seriously diminishes the number of fields that can be observed each evening, while to make but two exposures (on twice as many plates) is also rather inconvenient. Moreover, when there are but two images of each star on the plate, it is impossible to tell which is wrong, in case of trouble. Thus the policy of making four exposures on each plate is also justified by the results.

TABLE 27.—*Work necessary to secure equal accuracy with different numbers of exposures.*

n	Work proportional to—	
	$n+2$	$n+3$
2	11.0	13.8
4	11.2	13.1
6	12.7	14.2
8	14.4	15.8

The errors due to instrumental changes are almost surprisingly small, when it is considered that they include changes in the réseau, as well as in

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*This estimate, and the results deduced therefrom, might be materially changed in a case where the services of assistants were available for measurement and reduction.

the telescope and its mirror, through more than three years. They contribute less than one-sixth of the whole error of an average plate (with four exposures). About three-fourths of the error of such a plate is inherent in the individual plate and images. This must be the combined effect of many causes—bad seeing, imperfect guiding, unequal sensitiveness of the plate, distortion of the film, etc.—whose relative importance can not now be estimated. But it would be very desirable to test different kinds of plates, with the same instrumental conditions, in the hope of reducing it.

V. PROBABLE ERRORS FOR THE PARALLAX-STARS.

§17. *Loss of Accuracy for Close Double Stars.*

The mean-square value of the probable error of unit weight resulting from the least-square solutions for the 44 parallax-stars is ±0″.048. This is less than the average value just found for all the stars on the plate—which is not surprising, since the parallax-stars are favorably placed at the center of the plate and were measured with special care. The results for different classes of stars are, however, of very different degrees of precision.

There are five pairs of stars* on the list, whose images on the plate lie much closer to one another, in the direction of measurement, than the ordinary separation of the successive exposures (17″.6). The mean-square probable error for these ten stars is ±0″.0596, while for the remaining 34 stars it is ±0″.0437.

It therefore appears that the presence of a comparison-star, of comparable magnitude, within about 10″ (or 0.03 mm. on the plate) seriously diminishes the accuracy of the measures. This might be expected, owing to mutual disturbance of the images, which under ordinary conditions are from 3 to 5 seconds of arc in diameter. This difficulty is due to the duplicity of the stars, and not to the photographic method of observation. When the stars are close, and of comparable brightness, observations with the heliometer (and doubtless with other instruments as well) are diminished in accuracy in the same way (as was found by Gill in the case of α Crucis).†

It is clear that only the isolated stars afford a fair test of the accuracy of the photographs. Three of these, however (Nos. 3, 39, and 44), are at some distance from the center of the fields on which they appear (being observed because they happened to lie on plates taken primarily for some other star). The mean-square probable error for these is ±0″.059, and for the remaining 31 stars is ±0″.042. This is again what might be expected, for the accuracy of the measures, and also of the corrections necessary to reduce them to standard, diminishes toward the edge of the field.

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*Nos. 8–9, 15–16, 18–19, 31–32, 34–35.

†Cape Annals, vol. VIII, part II, p. 55B.

§18. *Dependence of Accuracy upon Photographic Magnitude, and Position on the Plate.*

The probable errors for the remaining stars (including all those observed under normal conditions) are given below, arranged in order of their photographic magnitudes. The latter were obtained by adding to the visual photometric magnitudes the corrections shown in table 28 (given by King, Harvard Annals 59, v, page 152).

TABLE 28.

Spectrum.	Correction.
O	$-0^{m}.4$
A	0.0
F8	+0.6
G	+0.7
G5	+0.9
K	+1.2
K5	+1.4
M	+1.7

For the bright stars observed with the color-screen, the effective photographic magnitude was assumed to be 5.5 magnitudes fainter than the visual magnitude (no account being taken of the spectral type, as it is well known that photographs taken through a yellow screen give results which in this respect agree closely with the visual magnitudes). The effective magnitudes so obtained are inclosed in parentheses in table. The last column gives the average duration of exposure for each group.

TABLE 29.

Star No.	Photographic magnitude.	Probable error of one observation of unit weight.	Mean-square value of groups.	Average exposure.
			″	m
37	7.0	$\pm 0''.051$	±0.048	3.7
12	7.1	42		
17	7.1	54		
11	7.5	24		
26	7.5	61		
40	7.6	±0.023	±0.035	3.9
27	7.6	38		
7	(7.6)	36		
38	7.7	48		
29	7.8	27		
1	(7.9)	±0.014	±0.029	4.0
42	8.2	37		
13	8.6	31		
28	9.0	09		
4	(9.1)	40		
10	9.1	±0.011	±0.035	4.3
6	(9.3)	64		
2	9.4	19		
36	9.4	34		
30	9.5	21		
21	9.7	±0.032	±0.051†	5.0
25	9.8	41		
33	9.9*	80		
14	10.1	30		
5	10.2	70		
24	10.3	31		
43	10.5	±0.023	±0.047	4.6
22	10.6	78		
41	10.7*	26		
23	10.8	48		
20	11.0‡	39		

*Direct estimate from plates. †Or excluding No. 33, $\pm 0''.042$. ‡Estimated (spectrum not determined).

The individual values vary through a wide range. This is not surprising, for the number of observations for a single star (from five to eleven) is really not sufficient to permit a reliable determination of the individual probable errors (especially when three unknowns have to be derived from the observed data). There is little doubt that the very small probable errors found for a few stars are evidence, not mainly of the exceptional accuracy of the observations, but very largely of the fortuitous coincidence of errors of nearly equal magnitude in the two or three observations of each parallactic epoch; and it is equally likely that in some other cases the probable errors in the table are unduly increased by the opposite accident.

The means of groups of five, however, are based on a total of from 34 to 41 observations (that is, at least 19 more than the whole number of unknowns derived from them) and should therefore be reliable indications of the mean error of observation for stars of the corresponding magnitudes.

These show a distinctly systematic variation with the magnitude, the only exception being the last group but one. The mean for this is raised by the presence of one very bad star (No. 33). The original observations show that the abnormally large probable error for this star is almost entirely due to the extreme discordance of a single plate, on which its images were recorded at the time of measurement as excessively faint. It is questionable whether this plate should have been measured at all. In any case, it does not represent the normal probable error for stars of this magnitude. A second mean has therefore been formed for this group, excluding this star. The legitimacy of this process is confirmed by the fact that if the under-exposed plate were rejected the probable error derived for this star from the remaining plates would have been close to the mean for the rest of the group. The means, after this correction, can be closely represented by the formula

$$r^2 = 10 + 4\,(m-9)^2 \qquad (1)$$

where r is the probable error, expressed in hundredths of a second of arc, and m the photographic magnitude—as is shown by table 30.

The average duration of exposure for the different groups is so nearly equal that the extreme magnitudes need be changed by little more than one-tenth of a unit to reduce the results for all the groups to the average exposure of $4^{\mathrm{m}}.3$. The corrections to the effective photographic brightness of the stars on different plates, depending on the clearness of the air, etc., would doubtless be much larger than this. The above formula may therefore be applied with sufficient approximation to all the plates.

TABLE 30.

Photographic magnitude.	Observed.	Computed.
7.2	0″.048	0″.047
7.6	.035	.041
8.6	.029	.032
9.4	.035	.032
10.2	.042	.039
10.8	.047	.047

For the stars not at the center of the field, an additional term may be introduced into the expression (1) proportional to the square of the distance ρ from the center of gravity of the comparison-stars. The material is not sufficient to give a reliable coefficient for this term; but with the approximate expression

$$r^2=10+4\,(m-9)^2+0.3\rho^2 \tag{2}$$

(where ρ is expressed in réseau intervals) the observed data are represented in table 31.

From the formula (14), page 25, setting $r=\pm 0''.055$, $n=7.8$, $P=Q=5$, it appears that the increasing uncertainty of the correction to standard for points remote from the center would alone produce a term $+0.15\rho^2$ in (2). The remainder may be interpreted as showing a decrease in the accuracy of the measures of stars at a distance from the center, which may help to account for the inferior accuracy of the measures of the comparison-stars.

TABLE 31.

Star.	m	ρ	r comp.	r obs.
3	6.0	4	±0″.072	±0″.069
39	9.0	9	0.057	0.055
44	9.0	7	0.049	0.050

If the term depending on the photographic magnitude is assumed to be the same for the close double stars as for the others, the probable errors of observation may be represented by the formula (see table 32)

$$r^2=30+4(m-9)^2 \tag{3}$$

TABLE 32.

Star.	Photographic magnitude.	Observed r.	Mean-square value.	Formula (3).
15	7.5	±0″.059	±0″.057	±0″.058
34	7.8	34		
35	8.0	67		
16	8.7	62		
8	9.0	±0.058	±0.058	±0.055
18	(9.2)	49		
19	(9.2)	62		
9	9.5	61		
31	11.0	±0.056	±0.068	±0.068
32	11.0?	78		

This satisfactorily explains the unusually large probable error for the last pair. The observations of a component of such a double have, under the best conditions, only one-third the weight of those upon an isolated star of the same magnitude.

For the eight stars (Nos. 45 to 52) observed at two epochs only, the mean-square probable error of unit weight, resulting from the observations, is ±0″.053. That computed as above, with the aid of the formulae (1) and (3), remembering that Nos. 46 and 47 form a close pair, is ±0″.041.

In this group there is again one star (No. 50) showing a very large probable error (±0″.092) due to discordance of very faint images on underexposed plates. If this is excluded from the mean, we have for the other stars an observed mean-square probable error of ±0″.044, as against ±0″.042 computed. It therefore appears that, except for the one case aforesaid, the observations of the interrupted series are closely comparable in accuracy with the rest.

§ 19. *Importance of Correct Exposure.*

The results for the isolated stars show the great importance of correct exposure. The range of brightness within which measures of the greatest precision can be made on a given plate is decidedly limited, not exceeding two magnitudes. A departure of one magnitude on either side of the most favorable brightness decreases the weight of an observation to seven-tenths, and one of two magnitudes to less than one-third, of its maximum value. Part of this error is due to the character of the images of bright or faint stars—the former being too large, and often too diffuse at the edge, for accurate measurement, and the latter too ill-defined, often without any definite center to set on. But it is probable that some of the other sources of error are also considerably increased—*e.g.*, guiding error for bright stars, and that arising from unequal sensitiveness of the plate for faint ones.

Over-exposure or under-exposure, sufficient to impair seriously the accuracy of the measures, can often be recognized on inspection of the star-images. With longer series of plates, it would probably be advisable to reject, *a priori*, any on which the images of the parallax-star were defective in this respect. In the present case this would be too drastic; but such plates were given diminished weight.

It would have been well if the exposures for the faint stars had been longer. Those for the bright stars could not well have been much curtailed, owing to the necessity of getting good images of the comparison-stars; but a color-screen of small absorption (two or three magnitudes) would make it possible to get good images of both.

By formula (1) the normal probable error, for an isolated star at the center of the field, of the mean coördinate derived from one plate with four exposures of $4^{m}.3$ each is as shown by table 33.

TABLE 33.

Photographic magnitude.		Probable error.
7.0	11.0	$\pm 0''.051$
7.5	10.5	.044
8.0	10.0	.037
8.5	9.5	.033
9.0	9.0	.032

As it is not certain to what extent the deviations from these values for the individual stars are due to chance, and to what extent they represent real variations in the accuracy of observation, two sets of probable errors are given in the final table of observed parallaxes; the first being that resulting from the individual least-squares solution, while the second is that derived from the weight of the determination of the parallax and the formula (1), (2), or (3) for the probable error of one observation, according as the star is: (1) an isolated star at the center of the field; (2) a similar star in another part of the field; (3) a close double star. The mean-square value of the probable error given by formula (1) for stars within one magnitude of the most favorable brightness may well be taken as a measure of the average accuracy of observation attainable in the present research. This is readily found to be $\pm 0''.0356$, which corresponds to ± 0.00101 mm. on the plates.

§20. *Results for Stars close together only Partially Independent. Explanation.*

The measures of double stars are of much less precision. It has already been shown that special sources of error exist in close pairs. In addition to this, there is reason to fear that when we have two stars of comparable magnitude within a few minutes of arc in the heavens (or a few millimeters on the plates) the effects of the instrumental error (t) and of the plate-error (p)—or at least of the guiding error which forms the greater part of this—will be practically identical for the two stars, as will also be any influence of errors of measurement of the comparison-stars, so that the parallaxes derived for the two from the same plates will not really be independent determinations. Twelve such pairs appear in the list of the stars observed for parallax. In every case the two stars have a considerable common proper-motion, and it is therefore practically certain that their parallaxes are sensibly identical. The difference of the observed values will thus afford a measure of those errors of observation which are not common to the measures of the two stars.

The mean-square value of this difference for the 12 pairs is 0ʺ.043, to which corresponds the probable error ±0ʺ.029 for the difference of the two observed parallaxes and ±0ʺ.021 for each singly, if their error were independent. But the mean-square probable error of a determination of parallax, relative to the comparison-stars, for one of these same stars is ±0ʺ.035. It follows that those errors of observation which differ for the two stars of the pair account on the average for only 36 per cent of the whole, leaving 64 per cent as the contribution of errors common to the two stars on the same plate.

This is of the order of magnitude which was to be expected, for the data of p. 63 show that for plates with four images of a star, each measured with double the usual number of settings (as was the case for the parallax-stars), the errors p and t account for 56 per cent of the whole error of the measured coördinates of this star. The error of the correction, computed from the measures of the comparison-stars, and necessary to reduce the measures to standard, increases the square of the error of the reduced coördinate by about 12 per cent. Hence 61 per cent of the latter is due to known sources of error certainly or presumably common to the two stars of a pair. It is therefore unnecessary to go farther for an explanation of the observed facts.

It follows that the mean of the observed parallaxes of such a pair of stars has by no means twice the weight of an individual determination. According to the data just obtained it has 1.22 times that of one of the components.

It would therefore appear at first sight that the photographic method is in this respect at some disadvantage. But it should be remembered that the measurement of the second component of a pair on the plates involves very little additional work. The same comparison-stars and plate-constants serve for both, and the whole increment of labor after the plates are taken (if there are seven or eight comparison-stars) is hardly over 10 per cent; so that the mean parallax of a pair of stars (provided they are not too close) can be determined with slightly greater accuracy, in proportion to the work expended on it, than that of an isolated star.

§21. *Comparison of the Average Precision Attained by Different Observers.*

It is of interest to compare the accuracy of these results with that of other modern methods of observation. The probable errors found by other photographic or micrometric observers are directly comparable with these; those of heliometer observations must be halved (since in this case any displacement of the central star produces a change twice as great in the measured difference of distances.

Table 34 shows the accuracy with which the relative position of a star has been determined by some of the best modern observers and methods, and the relative weights of an average observation. The unit weight corresponds to a probable error of 0″.05, and the means are in all cases "mean-square."

TABLE 34.

Method or instrument.	Observer.	No. of series.	Mean probable error.	Wt.	References.
Heliometer:					
Yale	Chase (earlier)	83	±0.094	0.3	Yale Transactions, Vol. II, part I, page 193.
	Chase (later)	45	±0.071	0.5	
	Smith	23	±0.072	0.5	
Leipzig	B. Peter	14	±0.048	1.1	Abhd. d. Kgl. Sachs. Gesellsch. der Wissenschaften XXII, XXIV, XXVII, XXX.
Cape (7-inch)	Gill	10	±0.036	2.0	Cape Annals VIII, part II, page 136B.
	Finlay	3	±0.047	1.1	
	de Sitter	4	±0.057	0.8	
Micrometer:					
Yerkes 40-inch	Barnard	1	±0.060	0.7	Monthly Notices, LXVIII, p. 637 (computed from data there given).
Photography:					
Upsala	Bergstrand	1	±0.050	1.0	Astronomische Nachrichten 3999.
Bonn (11-inch)	Küstner Kapteyn	2	±0.034	2.2	Mean-square probable error for stars all over the plate for mean of three exposures at one epoch. Computed from data given in Pub. Ast. Lab. Groningen *23*, pages 55-56, etc.
Yerkes (40-inch)	Schlesinger	1	±0.030	2.8	Probable error of *single exposures* Provisional value. Astrophysical Journal 20, page 126.
Present Work:					
Close doubles		10	±0.060	0.7	
All other stars		34	±0.044	1.3	
Stars with correct exposure, Formula (1)			±0.036	2.0	

The average accuracy of the present work has been considerably diminished by the inclusion of stars observed under unfavorable circumstances.

For isolated stars (which alone furnish a fair basis of comparison) the average of all its results, good and bad, is exceeded in precision by but a single observer using other than photographic methods. The observations made under suitable conditions of exposure (which could always be realized in future work) are seriously surpassed only by the plates taken with the great Yerkes telescope, whose focal length gives it a great advantage over smaller instruments.

In view of its high precision and of the comprehensive evidence obtained of its freedom from perceptible systematic errors (at least in the case of the present work), the photographic method may fairly claim to be established in the front rank as a means of determining stellar parallax.

CHAPTER V.

RESULTS OF OBSERVATION.*

I. RESULTS OF THE PRESENT WORK.

§1. *Description of Table A.*

The observed parallaxes of those stars which have been the special objects of investigation are collected in table A, pp. 76–77.

The first seven columns of this table, giving the current number of the star, its designation, place for 1900, photometric magnitude and spectral type (as determined at Harvard),* and its proper-motion, are practically identical with the corresponding columns in table C and are repeated here for convenience. The last five columns, also taken from table C, show in what series of plates each star appears, the number of comparison-stars, of plates in the series, of exposures measured, and the average exposure time.

The eighth column contains the observed parallax, relative to the comparison-stars, as derived from the least-squares solutions in which proper-motion terms were included. The tenth column gives the probable errors derived directly from these solutions, and the eleventh those derived from the weights of the determinations of parallax and the general expressions for the probable error of one observation, derived in Chapter IV, §19 (page 69), and depending on the photographic magnitude.

The number of plates (*i. e.*, of equations of condition) from which the observed parallaxes are derived is so small that the probable errors derived directly from the residuals are subject to very considerable uncertainty. On the other hand, it is quite possible that the assumption that all observations of stars of the same photographic magnitude are of equal accuracy (on which the formulæ for probable error are based) goes too far. Both sets of values are therefore given. By choosing the greater, one can pretty surely be on the safe side; but this is hardly fair to the observations. In the remainder of this work, the mean-square average of the two has been adopted except in six cases—Nos. 5, 6, 18 (*y* measures), 33, 50, and 52—in which the excess of the directly derived probable error is due to the large discordance of one or two observations in each case. As in such instances it is clear that some

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*For two stars, the Harvard data are lacking, or uncertain. Their spectral types have been estimated from the difference between the known visual magnitude and the photographic magnitude determined by comparison with the other stars on the plates (using for these the visual magnitudes corrected for spectral type according to King's determination). The visual and photographic magnitudes of these stars and the concluded spectral types are shown in the accompanying table. The results are given in parentheses in Table A.

No.	Visual magnitude.	Photographic magnitude.	$P-V$	Spectrum.
20	9.98	11.0	+1.0	G5
41	9.41	10.8±	+1.4±	K5 (Harvard K?)

of the errors of observation must be unusually great, the larger values given by the direct reckoning are retained. These adopted values are given in the ninth column immediately after the parallaxes.

In the case of pairs of stars with common proper-motion, the simple mean of the observed parallaxes is taken as the definitive value for both. The differences in weight corresponding to the directly derived probable errors are for the most part illusory, and even those resulting from the formulæ just mentioned are too great, for, as shown in Chapter IV, §20 (p. 70), the greater part of the error of observation is common to the two stars.

The probable errors of these means are therefore considerably greater than they would be if the two determinations were strictly independent. In accordance with the discussion just quoted, they have been taken as 0.90 times the mean-square average of the probable errors for the two components.

For the stars numbered from 45 to 52, which were observed at two epochs only, the probable errors derived as above have been increased to the extent demanded by the assumption that the assumed values of their proper-motion (relative to the comparison-stars), which were used in deriving the parallaxes, have a probable error of ±0″.053. (See page 56.)

When the difference between the proper-motions previously assumed for these stars and those given in Boss's Catalogue was great enough to have a sensible influence on the deduced parallax, the necessary corrections were made.

In the two cases where the parallax was derived independently from measures of the *x* and *y* coördinates, the resulting values have been combined with weights according to their observed probable errors.

The notes concerning table A explain themselves. They deal mainly with the numerous double stars.

NOTES TO TABLE A.

The data concerning double stars are taken from Burnham's "General Catalogue of Double Stars,"* unless otherwise noted; the remainder are principally from Lewis's memoir on the Struve stars (Mem. R. A. S. LVI). Magnitudes given to two decimal places are derived from photometric measures made at Harvard, some of which may be found in the Harvard Annals, vol. LXIV, No. VI. The relative motion given in the case of double stars is that of the fainter component referred to the brighter. The relative masses in certain binary systems are taken from the notes to Boss's "Preliminary General Catalogue."

(1) Bu. 24; 204°, 22″.6 (1900); Mags. 2.42, 13.7; Optical pair; Companion not shown on plates; Observed with color-screen.

(2) Triple; Mags. 7.73, 10.5, 11.5; (the last two rough estimates).

A B		A C		Date.	Observer.	Remarks.
53°.9	40″.4	.....	....	1866.23	Auwers.†	B is a physical and C an optical companion. The proper-motion of A accounts for the change in the latter. Not in Bu.
56.2	38.9	112°.6	34″.7	1904.98	Plate 391.	
56.2	38.7	118.0	30.2	1906.78	Russell, 1*n*	

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*Referred to hereafter as Bu.

†*Math. Abth. Berlin Akad. der Wissensch.* 1867, p. 23.

(3) Bu 131; OΣ 5; Mags. 6.04, 10; 241°, 6″.1; Fixed; Measured because it was on the plates of Series 11; Companion shown on plates, but not measurable; Proper-motion given ten times too great in *A. G. Bonn.*

(4) Bu. 426; Σ 60; Mags. 3.67, 7.41; 227°, 5″.6 (1904); Binary; Period long and uncertain. Annual motion of B relative to A (1904) from Lewis's diagram, −0″.19 in x, +0″.19 in y. Mass of B 0.76 that of A (Boss); Observed with color-screen; Companion not shown.

(5) Variable (*Mira*); Mag. 1.7 to 9.6; Period 331.6 days; Radial velocity constant, +63 km. (Campbell and Stebbins, *Astrophysical Journal*, vol. 18, page 341). The distant (optical) companion is comparison-star 5; 79°, 116″.9 (1904); Bu. 1209.

(6) Variable; Mag. 3.4 to 4.2; Irregular; Obs. with color-screen.

(7) Variable (Algol) Mag. 2.1 to 3.2; Period $2^d\ 20^h\ 48^m\ 55^s$; Spectroscopic triple; the close pair having the period of the light variation and also revolving about the center of mass of the system in a nearly circular orbit with radius not less than 89,000,000 km. and period 1.899 years (Curtiss, *Science* N. S., vol. 28, page 848). Observations extending over at least two years are necessary to separate the effects of this orbital motion and the annual parallax. The present series, completed before this fact was known, covers only one year, and its results must therefore be regarded as provisional. Three faint and distant companions; Bu. 1565. Observed with color-screen.

(8, 9) Bu. 1854; Σ 443; 48°, 8″.6 (1897); Combined mag. 7.83. Physical pair; Relative motion −0″.014 in x, +0″.003 in y (Lewis).

(11) Lalande 9012.

(15, 16) Bu. 5779; Σ 1540; 150°, 29″.2 (1892); Combined mag. 6.04; Physical pair; Relative motion per year −0″.006 in x, +0″.010 in y (Lewis).

(18, 19) Bu. 6243; Σ 1670; 328°, 5″.9 (1903); Combined mag. 2.91; Binary; Period 182 years, a = 3″.90; Masses equal (Boss). Relative motion of following star +0″.040 in x, −0″.003 in y (Lewis). Observed with color-screen.

(20) Proper-motion from A. G. Berlin A. Comparison of this catalogue with the plates gives the proper-motion relative to four comparison-stars as +0″.030, −1″.85, which is much nearer the value deduced from the plates themselves.

(22, 23) Bu. 6869; 75°, 45″.2 (1904); Relative motion very small; Proper-motion from Porter *Pub. Cincinnati Observatory*, vol. 15, page 100.

(26, 27) Bu. 7162; Σ1919; 10°, 24″.1 (1905); Combined mag. 6.41; Physical pair; Relative motion −0″.001 in x, −0″.006 in y (Lewis).

(29) Bu. 7332; OΣ298; 185°, 1″.2 (1903); Binary; Period 56 years; a = 0″.88; Not separated on the plates. Difference of magnitude between the components 0.3 (Bu.); which makes the individual magnitudes 7.4 and 7.7.

(28, 29) Have common proper-motion and are relatively fixed in 328°, 121″.9.

(31, 32) Bu. 8798; Σ2398; 150°, 17″.1 (1900). Combined mag. 8.87; Binary; Period long; Relative motion −0″.05 in x, −0″.01 in y (from diagram in Bu.).

(34, 35) Cygni 6B.; Bu. 9137; Σ2486; 217°, 9″.2 (1905); Combined mag. 5.97; Physical pair; Relative motion +0″.022 in x, +0″.002 in y (Lewis).

(36) Fifth-type star with hydrogen atmosphere 5″ in diameter. No data regarding proper-motion.

(37, 38) Bu. 10732; Σ2758; 127°, 22″.5 (1904); Combined mag. 5.12; Binary; Period very long; Relative motion 0″.00 in x, −0″.20 in y (Lewis); Masses nearly equal (Boss).

(39) Suspected by Kapteyn to have a parallax of about 0″.1; Proper-motion from comparison of the plates with A. G. Lund, using 9 comparison-stars.

(41) Bu. 11671; Krüger 60; 120°, 3″.3 (1905); Combined mag. 9.43; Difference of magnitude 1.3, according to Barnard's estimates; which makes the individual magnitudes 9.7 and 11.0; Binary; Period probably less than 100 years; Distant optical companion in 59°.6, 40″.2 (1905), of mag. 10.23, used by Barnard as a comparison-star for parallax; Proper-motion variable, owing to orbital motion. Masses of the components comparable. (Barnard, *M. N.*, vol. 68, page 643.) Tabular proper-motion from Barnard, *A. J.*, vol. 23, page 171. Only the principal star measurable on the plates; close companion invisible.

TABLE A.—OBSERVED PARALLAXES.

No	Designation.	Right ascension 1900.0	Declination 1900.0	Magnitude.	Spectrum.	Proper-motion.	Parallax.	Probable error.	P. E. from observations.	P. E. from formula.	Series.	Comparison-stars.	Plates.	Exposures.	Average length.
1	β Cassiopeiae	$0^h\ 3^m.8$	+58°36′	2.42	F5	0″.56	+0″.082	±0″.019	±0″.009	±0″.026	I	9	5	20	$3^m.2$
2	Groombridge 34	0 12.7	+43 27	7.73	Ma	2.80	+0.250	±0.016	0.011	0.020	II	9	6	24	3.8
3	26 Andromedae	0 13.3	+43 15	6.04	A	0.03	−0.026	±0.042	0.041	0.043					
4	η Cassiopeiae	0 43.0	+57 17	3.64	F8	1.24	+0.187	±0.019	0.021	0.017	III	8	7	25	4.1
5	ο Ceti	2 14.3	− 3 26	1.7 to 9.6	Md	0.24	+0.136	±0.035	0.035	0.020	IV	9	7	21	5.2
6	ρ Persei	2 58.8	+38 27	3.4 to 4.2	Mb	0.17	+0.083	±0.040	0.040	0.020	V	9	7	23	5.0
7	β Persei	3 1.7	+40 34	2.1 to 3.2	B8	0.01	+0.007	±0.027	0.025	0.028	VI	8	7	24	4.3
8	Lalande 6888	3 40.2	+41 9	8.35	G	1.38	−0.029	±0.033	0.033	0.032	VII	6	6	24	4.5
9	Lalande 6889			8.89			+0.020	±0.034	0.035	0.032					
	Mean						−0.004	±0.030	0.031	0.029					
10	Lalande 7443	3 56.5	+35 2	8.34	G2	2.20	−0.011	±0.014	0.006	0.019	VIII	8	6	23	4.2
11	Groombridge 884	4 44.4	+45 41	6.83	G	0.68	+0.078	±0.019	0.013	0.023	IX	8	6	21	4.2
12	Groombridge 1646	10 21.9	+49 21	6.50	F8	0.90	+0.049	±0.023	0.021	0.025	X	8	7	29	3.5
13	Lalande 21185	10 57.9	+36 38	7.42	K	4.78	+0.346	±0.015	0.015	0.015	XI	9	8	31	4.8
	From measures of y						+0.335	±0.032	0.031	0.032	XI*a*				
	Mean						+0.344	±0.014	0.013	0.014					
14	Lalande 21258	11 0.5	+44 2	8.63	K5	4.40	+0.163	±0.018	0.015	0.020	XII	6	9	35	4.7
15	83^1 Leonis	11 21.7	+ 3 33	6.36	K	0.74	+0.048	±0.034	0.033	0.035	XIII	5	7	28	3.9
16	83^2 Leonis			7.51	K		+0.057	±0.033	0.035	0.031					
	Mean						+0.052	±0.031	0.031	0.031					
17	Groombridge 1830	11 47.2	+38 26	6.46	G	7.04	+0.100	±0.029	0.030	0.028	XIV	7	8	31	4.1
18	γ^1 Virginis	12 36.6	− 0 54	3.65	F	0.56	+0.054	±0.032	0.034	0.030	XV	6	8	31	4.8
19	γ^2 Virginis			3.68			+0.070	±0.028	0.027	0.030					
18	From measures of y						+0.068	±0.157	0.157	0.067	XV*a*				
19	From measures of y						+0.072	±0.071	0.074	0.067					
	Mean						+0.063	±0.026	0.026	0.026					
20	Berlin A 4999	13 40.2	+18 20	9.98	(G5)	1.89	+0.105	±0.024	0.020	0.027	XVI	7	8	32	4.8
21	Lalande 25372	13 40.7	+15 26	8.30	K5	2.32	+0.221	±0.020	0.019	0.020	XVII	9	8	30	4.6
22	Berlin B 5072	14 21.1	+24 6	9.40	K	1.38	+0.059	±0.036	0.044	0.025	XVIII	7	7	27	5.2
23	Berlin B 5073			9.64			−0.012	±0.027	0.027	0.027					
	Mean						+0.024	±0.029	0.033	0.024					
24	A. Oe. 14318	15 4.7	−15 59	9.09	K	3.75	+0.045	±0.022	0.019	0.025	XIX	7	6	26	5.8
25	A. Oe. 14320	15 4.7	−15 54	8.86	G5		+0.014	±0.023	0.025	0.021					
	Mean						+0.030	±0.021	0.021	0.021					
26	Lalande 27742	15 8.3	+19 39	6.83	G	0.68	−0.039	±0.051	0.058	0.042	XX	8	5	20	4.0
27	Lalande 27743			7.63	A?		−0.077	±0.038	0.036	0.040					
	Mean						−0.058	±0.041	0.045	0.037					
28	W. B. 15^h 716	15 32.4	+40 10	7.78	K	0.48	+0.020	±0.013	0.005	0.017	XXI	8	8	30	4.5
29	W. B. 15^h 720	15 32.5	+40 8	6.83	G5		+0.059	±0.019	0.014	0.021					
	Mean						+0.040	±0.015	0.010	0.018					
30	W. B. 17^h 322	17 20.8	+ 2 14	7.82	Ma	1.36	+0.095	±0.016	0.012	0.019	XXII	7	11	41	4.8
31	Pos. Med. 2164	18 41.7	+59 29	9.33	K5	2.27	+0.291	±0.042	0.038	0.045	XXIII	8	6	23	4.5
32				10.01	K5		+0.306	±0.049	0.052	0.045					
	Mean						+0.298	±0.041	0.041	0.041					
33	Lam. 18180	18 53.1	+ 5 48	9.14	Ma	1.26	+0.076	±0.065	0.065	0.029	XXIV	8	6	24	4.7
34	Groombridge 2789	19 9.5	+49 40	6.84	K	0.64	−0.011	±0.049	0.034	0.061	XXV	7	7	28	4.6
35				6.62			+0.075	±0.063	0.067	0.059					
	Mean						+0.032	±0.050	0.045	0.055					
36	B. D. + 30° 3639	19 30.9	+30 18	9.85	O	......	−0.018	±0.022	0.022	0.022	XXVI	6	6	24	4.8
37	61^1 Cygni	21 2.4	+38 15	5.57	K5	5.27	+0.406	±0.024	0.024	0.024	XXVII	10	11	46	2.7
38	61^2 Cygni			6.28	K5	5.15	+0.361	±0.021	0.023	0.019					
	Mean						+0.384	±0.021	0.021	0.020					
39	B. D. +38° 4362	21 5.2	+38 19	7.68	K	0.06	−0.035	±0.029	0.029	0.029			10	42	2.7
40	Lalande 43492	22 12.3	+12 24	6.97	F8	0.83	+0.021	±0.023	0.015	0.028	XXVIII	8	6	23	3.8
41	Hels-Gotha 13170 (Krüger 60).	22 24.5	+57 12	9.43	(K5)	0.95	+0.258	±0.019	0.013	0.023	XXIX	9	8	32	3.9
42	Lalande 45755	23 16.8	+43 33	7.56	F8	0.68	+0.037	±0.021	0.021	0.020	XXX	8	7	27	3.4
43	Lalande 46650	23 44.0	+ 1 52	9.09	K5	1.40	+0.211	±0.023	0.015	0.028	XXXI	10	6	24	3.7
44	Lam. 32805	23 44.9	+ 2 19	8.28	G	0.44	−0.022	±0.033	0.033	0.022					

TABLE A (Continued). PARALLAXES DERIVED WITH THE AID OF CATALOGUED PROPER-MOTIONS.

No.	Designation.	Right ascension 1900.0	Declination 1900.0	Magnitude.	Spectrum.	Proper-motion.	Parallax.	Probable error	P. E. from observations.	P. E. from formula.	Series.	Comparison-stars.	Plates.	Exposures.	Average length.
45	η Geminorum.	$6^h\ 8^m.8$	+22°32′	3.2 to 4.2	Ma	0″.07	**+0″.034**	**±0″.025**	0″.025	0″.025	XXXII	3	5	19	5.0
46	α_1 Geminorum.	7 28.2	+32 6	2.85	A	0.20	+0.104	±0.036	0.036	0.036	XXXIII	9	6	23	3.3
47	α_2 Geminorum.			1.99	A		+0.102	±0.028	0.022	0.033					
	Mean.........	....	....	....	.....	......	**+0.103**	**±0.029**	0.027	0.031					
48	γ Serpentis.....	15 51.8	+15 59	3.86	F8	1.33	**−0.081**	**±0.024**	0.018	0.028	XXXIV	8	4	16	5.0
49	ζ Herculis.....	16 37.6	+31 47	3.00	G	0.60	**+0.101**	**±0.024**	0.024	0.023	XXXV	8	6	20	5.1
50	η Herculis.....	16 39.5	+39 7	3.61	K	0.10	**+0.014**	**±0.066**	0.066	0.025	XXXVI	8	6	24	5.2
51	μ Herculis A...	17 42.6	+27 47	3.48	G5	0.82	+0.024	±0.028	0.032	0.024	XXXVII	8	7	27	5.0
52	μ Herculis BC .			9.68			+0.051	±0.049	0.049	0.026			6	23	5.0
	Mean.........	....	....	....	.....	......	**+0.038**	**±0.036**	0.036	0.024					

NOTES ON TABLE A—Continued.

(44) Proper-motion from *A. G. Albany*. Measured because it was on the plates of Series XXXI.

(45) Bu. 3239; β 1008; 290°, 1″.0 (1900); Physical pair in slow motion; Principal star variable; Mag. 3.2 to 4.2; Period 231 days; also a spectroscopic binary; Mag. of companion 8.8. Observed with color-screen.

(46, 47) *Castor*. Bu. 4122; Σ1110; 224°, 5″.6 (1904.) Combined mag. 1.58; Binary; Period about 400 years; Relative motion (1904) +0″.050 in x, −0″.032 in y (from Lewis's diagram). Both components spectroscopic binaries, of period 2.93 and 9.22 days. The companion of mag. 9.03 at 164°, 73″ shares the proper-motion and belongs to the system. Observed with color-screen.

(48) Observed with color-screen.

(49) Bu. 7717; Σ2084; 193°, 1″.1 (1904); Mags. 3.0 and 6.5; Binary; Period 34.5 years; a = 1″.4; companion not shown on the plates; Relative motion in 1904 +0″.076 in x, −0″.056 in y; Masses approximately equal (Lewis). Observed with color-screen. Mass of companion given by Boss as 0.43 that of primary.

(50) Bu. 7738; Supposed to be a close double, but undoubtedly single; Observed with color-screen.

(51, 52) Bu. 8162; Σ2220; 245°, 32″.2 (1904); Companion a close pair (A. Clark 7); 68°, 1″.5 (1904); not separated on the plates; Difference of magnitude 1.0 (Lewis); which makes the individual magnitudes 10.0 and 11.0; Close pair binary; Period 44 years; a = 1″.4; Wide pair a physical system in slow motion; −0″.043 in x, +0″.007 in y (Lewis). Bright star observed with color-screen; faint companion outside it.

§2. *Reality of the Results. Negative Parallaxes.*

There are in all fifty-five determinations of parallax, for forty stars or pairs of stars. If these are classified according to the ratio of the observed parallax π to its probable error r, the results are as shown in Table 35, p. 78. In the second column the individual determinations are counted, and in the last the final values for the different stars or systems. The negative parallaxes are in each case just one-fifth of the whole number. If an equal number of the smaller positive results are assumed to be illusory, it follows that 60 per cent. of the stars observed have really sensible parallaxes. The largest negative parallax resulting from the observations is −0″.081. All the stars whose observed parallaxes are positive and numerically greater than this (and also many of the remainder) are therefore presumably nearer us than the comparison-stars.

TABLE 35.

	Determinations.	Stars.
π greater than $4r$	17	14
π between $4r$ and $3r$	5	3
$3r$ and $2r$	4	4
$2r$ and r	11	5
r and 0	7	7
Total number of positive parallaxes	44	33
π negative and between 0 and r	8	5
r and $2r$	1	2
$2r$ and $3r$	1	..
$3r$ and $4r$	1	1
Total number of negative parallaxes	11	8

It is, however, noteworthy that the two largest negative parallaxes shown in table A (Nos. 27 and 48), the only two that considerably exceed their probable errors, are derived from series of but four and five plates (disposed in the latter case in such a manner that the weight of the parallax is unusually small). These series are insufficient for a reliable determination of parallax; but, as circumstances beyond the writer's control prevented their extension, it seemed desirable to give their results among the others, in order that the whole outcome, good and bad, of the work might be clearly exhibited. It is, however, proper to call attention here to their relatively low accuracy.

It should be noticed, too, that the only other series which gives an equally low weight for the parallax contributes another negative value (No. 34) and that the three stars which lie away from the center of the field, in a position unfavorable to accuracy, also give negative results (Nos. 3, 39, and 44).

Of the remaining forty-seven determinations, including all those made under conditions even reasonably favorable to accuracy (star central and weight of determination of parallax greater than 1.5) only four are negative. Nine out of the eleven negative parallaxes are less than their mean errors; hence a better formal representation of the observations could be secured by suppressing the parallax terms in the equations of condition altogether. This seems, however, to be of doubtful propriety, for there is no reason to suppose that the relative parallax in these cases is really exactly zero, and moreover this plan is equivalent to systematically rejecting those results where the errors of observation diminish the parallax beyond a certain limit, and would vitiate the mean values derived from groups of observations.

II. COMPARISON WITH OTHER OBSERVERS.

§3. *Description of Table B.*

As has already been mentioned, a large number of the stars in the above list have been observed elsewhere for parallax—the fact in many cases not being made public until after the present work was under way. In consequence a large amount of material is available for comparison of the results of different methods of observation. The determinations of the present work, being, as far as can be discovered from internal evidence, homogeneous and free from sensible systematic error, may not unreasonably be employed as a standard, not of absolute accuracy, but of comparison, to which the results of others may for the moment be referred.

Table B gives in summary form the principal results of modern observers for the stars of the present list. Such a collection can make no claim to finality in the present rapidly growing state of observation; but comparison with a manuscript list kindly furnished by Professor Kapteyn in return for a summary of the results of the present work, and with the extensive Catalogue of M. Bigourdan in the Bulletin Astronomique (July–December, 1909), gives occasion for the hope that few published determinations of value have been omitted.

The exact limits of exclusion, especially for the older observations, are largely a matter of opinion. It is, however, improbable that moderate differences in this respect would sensibly alter the conclusions hereafter expressed.

The first column gives the current number of the star in table A; the second gives the parallax, and the third the probable error, determined by the observer whose name follows in the fourth column. The probable error is, except when noted, that derived directly from the residuals of the equations of condition, the agreement of the parallaxes derived from various plates, or the like—that is from the "internal" consistency of the observations of the same observer and series. For the three spectroscopic determinations the annexed probable errors are rough estimates by the writer, based on the data given in the notes. In the case of Flint, who has applied systematic corrections to his observed parallaxes, both the observed and corrected values are given. As he makes no estimate of the uncertainty of these empirically derived corrections, the probable error of his corrected results can not be given. It must, however, be greater than that assigned to the observed parallaxes.

The next column, headed Method, shows the general nature of the method of observation; H denoting observations made with the heliometer; M results obtained with the equatorial and filar micrometer; P_1, those derived photographically from series of separate plates; and P_2 those from plates on which exposures are made at three epochs according to Kapteyn's method; S parallaxes derived from spectroscopically observed radial velocities in binary systems; T_1, those determined by means of meridian transits, and T_2 by transits with the equatorial telescope.

The column headed "Reference" gives references to the notes which follow the table. These show the source from which the tabular information has been derived and add occasional remarks.

The last two columns give, in thousandths of a second of arc, the excess of the corresponding determination above that of the present work for the same star or system, and its probable error, calculated by taking the square root of the sum of the squares of the probable errors determined by the two observers separately from the accordance of their own observations—the values used for the present work being those finally adopted in table A.

TABLE B.—RESULTS OF OTHER OBSERVERS.

No.	Parallax.	Probable error.	Observer.	Method.	Reference.	Difference.	Probable error of difference.
1	+0″.17	±0″.029	Flint	T_1	1	+ 88	± 35
	+0.10	....	Flint (corr.)	..	..	+ 18	.....
2	+0.292	±0.025	Auwers	T_2	2	+ 42	± 30
	+0.44	±0.034	Flint	T_1	1	+190	± 38
	+0.31	....	Flint (corr.)	..	..	+ 60	.....
4	+0.18	±0.010	Peter	H	3	− 7	± 22
	−0.02	±0.044	Flint	T_1	1	−207	± 49
	+0.34	....	Flint (corr.)	..	..	+153	.....
7	+0.037	±0.020	Chase	H	4	+ 30	± 34
8, 9	−0.04	±0.040	Flint	T_1	1	− 36	± 50
	−0.14	....	Flint (corr.)	..	..	−136	.....
	+0.035	±0.017	Chase	H	4	+ 39	± 34
	+0.057	±0.017	von Zeipel	P_2	5	+ 61	± 34
	+0.10	±0.02	Rambaut	P_2	6	+104	± 36
10	+0.21	±0.055	Flint	T_1	1	+221	± 57
	−0.02	....	Flint (corr.)	..	..	− 9	.....
	+0.039	±0.018	Chase	H	4	+ 50	± 23
11	+0.117	±0.019	Elkin, Chase	H	4	+ 39	± 27
12	+0.101	±0.026	Kapteyn	T_1	7	+ 52	± 35
13	+0.428	±0.030	Kapteyn	T_1	7	+ 84	± 33
	+0.36	±0.047	Flint	T_1	1	+ 16	± 49
	+0.37	....	Flint (corr.)	..	..	+ 26	.....
	+0.361	±0.023	Jost	T_1	8	+ 17	± 27
14	+0.167	±0.027	Kapteyn	T_1	7	+ 4	± 33
	+0.34	±0.110	Flint	T_1	1	+177	±111
	+0.37	....	Flint (corr.)	..	..	+207	.....
15, 16	−0.039	±0.027	Chase	H	4	− 90	± 41
17	+0.090	±0.025	Brünnow	M	10	− 10	± 38
	+0.139	±0.026	Kapteyn	T_1	7	+ 39	± 39
	+0.02	±0.055	Flint	T_1	1	− 80	± 62
	−0.01	....	Flint (corr.)	..	..	−110	.....
	+0.085	±0.024	Jost	T_1	8	− 15	± 38
18, 19	+0.051	±0.025?	Belopolsky	S	11	− 13	± 36?
21	+0.40	±0.065	Flint	T_1	1	+179	± 68
	+0.43	....	Flint (corr.)	..	..	+209	.....
	+0.174	±0.043	Elkin	H	4	− 47	± 47
26, 27	+0.038	±0.053	Elkin, Smith	H	4	+ 96	± 67
30	+0.18	±0.055	Flint	T_1	1	+ 85	± 58
	+0.17	....	Flint (corr.)	..	..	+ 75	.....
	+0.17	±0.017	Chase	H	4	+ 75	± 23

Table B.—Results of Other Observers—Continued.

No.	Parallax.	Probable error.	Observer.	Method.	Reference.	Difference.	Probable error of difference.
31, 32	+0.353	±0.014	Lamp	M	12	+ 55	± 43
	+0.36	±0.043	Flint	T_1	1	+ 62	± 59
	+0.32	....	Flint (corr.)	..	..	+ 22	.....
	+0.29	±0.021	Kostinsky	P	13	− 8	± 46
	+0.301	....	Bohlin	$P_1^?$	19	+ 3	> 42
	+0.282	±0.004	Schlesinger	P_1	26	− 16	± 42
34, 35	−0.021	±0.008	A. Hall	M	14	− 53	± 51
	−0.027	±0.024	Chase	H	4	− 49	± 55
	+0.04	±0.021	Kostinsky	P_2	13	+ 8	± 54
	+0.064	±0.040	Neander	P_1	15	+ 32	± 64
37, 38	+0.326	±0.035	Kapteyn	P_2	16	− 58	± 41
	+0.21	±0.029	Flint	T_1	1	−174	± 36
	+0.21	....	Flint (corr.)	..	..	−174	.....
	+0.270	±0.010	A. Hall	M	14	−114	± 23
	+0.340	±0.029	Peter	H	17	− 44	± 36
	+0.293	±0.007	Bergstrand	P_1	18	− 91	± 22
	+0.38	±0.015	Kostinsky	P_2	13	− 4	± 26
	+0.320	±0.028	Jost	T_1	8	− 64	± 35
	+0.291	±0.005	Chase	H	20	− 93	± 22
	+0.23	±0.035	Abetti	T_1	21	−154	± 41
39	+0.116	±0.021	Kapteyn	P_2	16	+151	± 36
	+0.04	±0.021	Kostinsky	P_2	13	+ 75	± 36
40	+0.140	±0.106	Chase	H	4, 28	+119	±108
41	+0.249	±0.010	Barnard	M	22	− 9	± 21
	+0.248	±0.009	Schlesinger	P_1	27	− 10	± 21
42	+0.031	±0.016	Chase	H	4	− 6	± 26
43	+0″.39	±0″.092	Flint	T_1	1	+179	± 95
	+0.23	....	Flint (corr.)	..	..	+ 19	.....
	+0.200	±0.084	Elkin	H	4	− 11	± 87
46, 47	−0.14	±0.041	Flint	T_1	1	−243	± 50
	−0.17	....	Flint (corr.)	..	..	−273	.....
	+0.022	±0.010	Smith	H	23	− 81	± 31
	+0.07	±0.03?	Curtis	S	24	− 33	± 42?
48	+0.11	±0.028	Flint	T_1	1	+191	± 37
	+0.05	....	Flint (corr.)	..	..	+131	.....
	+0.090	±0.024	Chase	H	4	+171	± 34
49	+0.172	±0.029	Smith	H	4	+ 71	± 37
	+0.11	±0.025?	Lewis	S	25	+ 9	± 33?
50	+0.21	±0.035	Flint	T_1	1	+196	± 75
	+0.15	....	Flint (corr.)	..	..	+136	.....
51,52	+0.122	±0.028	Chase	H	4	+ 84	± 46

Notes to Table B.

1) *Publications of the Washburn Observatory*, vol. XI (1902).

The parallaxes resulting directly from the observations are found by the author to require systematic corrections, depending on the apparent difference of magnitude between the parallax-star and the mean of the comparison-stars. These corrections were determined empirically by comparison of the parallaxes derived with the aid of individual comparison-stars of different magnitude. Their amount varies with the R. A., the spurious parallax of a star one magnitude fainter than the comparison-stars being +0″.17 at 0^h and 3^h, +0″.10 at 21^h and 6^h, +0″.05 at $19\frac{1}{2}^h$ and $7\frac{1}{2}^h$, 0″.00 at 18^h and 10^h, and −0″.05 at 17^h and 13^h.

The probable error of these systematic corrections must be considerable. Both the uncorrected and corrected values are given in the table.

(2) *Berlin Akad. Abhandlungen*, 1867. (*Math.*), page 18.

(3) *Astr. Nach.*, 3533 (1898).

(4) *Transactions of the Astronomical Observatory of Yale University*, vol. II, Part I (1906). The annexed probable errors are those resulting directly from the observations of each star, as given in the body of the work, and not those given in the final table of results (pages 196–198) which are increased to allow for the effect of a presumed systematic error of $\pm 0''.030$ for an average determination (except for No. 30, where the mean result of several series, given on page 198, is taken).

(5) *Astr. Nach.*, 4188 (1907). Hour-angles for morning and evening observations differ by 7 hours.

(6) *Monthly Notices.*, LXX, page 325. (1910.) Photographs taken near the meridian. *Monthly Notices*, LXVII, page 259. (1907.)

(7) *Annalen der Sternwarte in Leiden*, Bd. VII, p. 119. (1897.)

(8) *Vierteljahrsschrift.* 1906, p. 146.

(10) *Dunsink Observations* II, p. 19. (1873.)

(11) *Astr. Nach.*, 3510. (1898.)

(12) *Astr. Nach.*, 2676 and 2807. (1885, 1887.)

(13) *Publ. de l'Obs. Central Nicolas* (Poulkowa) série II, vol. XVII (1905), pp. 129 ff., 140–141. Hour-angles at morning and evening observations differ by 6 hours or more.

(14) *Washington Obs.* 1883. Appendix II. Observations with 26-inch equatorial.

(15) *Astronomische Jahresbericht* 1907, p. 289. Components not separated.

(16) *Publ. of the Astronomical Laboratory at Groningen.* No. 10, pp. 48, 58. From photographs by Donner at Helsingfors. Hour-angles for morning and evening observation differ by 7½ hours.

(17) From manuscript list sent by Prof. Kapteyn. Original reference undiscoverable.

(18) *Astr. Nach.*, 3999. (1904.) Atmospheric dispersion determined and allowed for.

(19) *Astr. Nach.*, 4365. (1909.) From two epochs only; dispersion taken into account. Bohlin gives, from $\Delta\alpha \cos\delta$, $\pi = +0''.296$; from $\Delta\delta$, $\pi = 0''.306$; from both together $\pi = +0''.251$. The mean of the first two has been taken.

(20) *Astronomical Journal*, 593. (1907).

(21) *Astr. Nach.*, 4270. (1908.)

(22) *Monthly Notices*, LXVIII, p. 637. (1908.) Observations with Yerkes 40-inch equatorial.

(23) *Astronomical Journal*, 594. (1907.)

(24) *Astrophysical Journal*, vol. 23, p. 351. (1906.) From radial velocities of centers of mass of the two components and Doberck's orbit, $\pi = 0''.05$. The two alternative orbits given by Doberck (*Astr. Nach.*, 3970) lead with the same radial velocities to parallaxes of $0''.06$ and $0''.11$. The mean of the three has been taken.

(25) *Memoirs of the Royal Astronomical Society*, vol. LVI, p. 471. From changes in the radial velocity of the bright component, $\pi = 0''.14$. If Boss's result, that the mass of the fainter component is 0.43 times that of the brighter, is taken instead of Lewis's conclusion that their masses are equal, the parallax is reduced to $0''.09$. The mean of the two has been taken.

(26) *Astr. Nach.*, 4365. (1909.)

(27) *Monthly Notices*, LXVIII, p. 637. (1908.) From photographs with Yerkes 40-inch equatorial. Hour-angles limited and isochromatic plates used to minimize atmospheric dispersion.

(28) Corrected to $-0''.047 \pm 0''.046$. *Yale Transactions*, vol. II, p. 295. Corrected difference from result of present work $-0''.068 \pm 0''.052$. Received too late for incorporation into the body of this work.

§4. *Search for Systematic Errors.*

Systematic error in determinations of parallax may be of two kinds:

(1) There may be a constant tendency toward too great or too small values for stars of a given class. Examples of this are the personal equation, depending both on magnitude and right ascension, detected and empirically corrected by Flint in his earlier work, and the influence of atmospheric dispersion, varying with the color of the star, which is to be feared in photographic work when all the exposures are not made at the same hour-angle.

(2) In addition to this, and even in its absence, there may be causes of error at work which bring about discrepancies between the results of different series of observations on the same star, greater on the average than would be expected on the basis of the probable errors derived from the internal agreement of the observations of each series, but varying from series to series in an apparently random manner, or at least without any clearly discernible law. Examples may be found in some of the work of the Yale heliometer.

It is worthy of remark that discordances of this type may be expected (to a greater or less extent) whenever the observations are confined to the minimum number of parallactic epochs necessary to separate the unknowns; for any errors, whether instrumental or personal, which vary slowly with the time, will be practically constant during the few weeks within which the observations at any epoch lie, and so will affect the values of the unknowns without perceptibly affecting the agreement of the observations at any one epoch, or increasing the residuals from which the probable error is derived. Errors of this type are much less serious than those first described, since their action is rather to diminish the accuracy of the resulting parallax than to vitiate it.

If no systematic errors are present (or, at least, if they are identical in sign and magnitude in the two groups of observations compared) the results of the two must agree, on the average, within the limits of error defined by the probable errors derived from the internal agreement of the observations of each series. This may be tested in two ways:

(*a*) The probable error of one difference, deduced in the ordinary way from the mean-square or numerical average values of these differences, must agree with that already derived.

(*b*) The actual distribution of the ratios of these differences to their probable errors must conform to that demanded by theory.

When, as in the present case, the probable errors of the individual determinations vary through a wide range, the second method is preferable to the first (which amounts to giving the greatest weight to the poorest observations, especially if the mean of squares is taken). It has also the advantage that it shows whether any discordance is due to a general prevalence of differences exceeding their probable errors, or to a few large discrepancies.

Applying these methods to the data of table B, the results are as shown in table 36: The photographic observations have been divided into two groups: (*a*) those in which precautions are known to have been taken to eliminate or determine the influence of atmospheric dispersion,* and (*b*) those in which they do not appear to have been considered (mostly of an earlier date, before the importance of the matter was fully realized). The latter, along with Flint's results, have been put by themselves, on account of the possible or probable presence of systematic error.

TABLE 36.

Method.	Excess above present work.				Ratio of excess to its probable error.					
	Average without regard to sign.		Average with regard to sign.		0 to 1	1 to 2	2 to 3	3 to 4	Over 4	Total.
Heliometer	0″.059	±0″.042	+0″.012	±0″.010	6	6½	3	1	1½	18
Micrometer	0.048	±0.035	−0.026	±0.016	2	2	.....	.....	1	5
Photography (*a*)	0.045	±0.033	−0.002	±0.015	3	.....	1	.....	1	5
Spectroscopic	0.018	±0.037	−0.012	±0.021	3	.....	.....	.....	.....	3
Transits	0.052	±0.035	+0.001	±0.012	4	3	1	1	.....	9
All together	0.052	±0.038	+0.001	±0.006	18	11½	5	2	3½	40
Theory	.....	.....	.....	.....	20	13	5	1½	½	40
Photography (*b*)	0.049	±0.042	+0.032	±0.015	4	2	1	.....	1	8
Flint:										
Uncorrected	0.144	±0.059	+0.048	±0.015	2	5	3	.....	5½	15½
Corrected	0.109	(±0.092)	+0.028	(±0.023)	6	7	½	.....	2	15½
Theory	.....	.....	.....	.....	8	5	2	½	.....	15½

The probable errors given in the third column of this table are the simple means of those of the individual differences given in table B, except in the last line but one, which will be explained later. In the absence of systematic error each of these quantities should be less by 15 per cent than that in the preceding column. Those in the fifth column are obtained by dividing the former by the square root of the number of observations combined to form the mean and are (very nearly) what the probable errors of the means in the fourth column would be, in the absence of systematic error.

The two stars (Nos. 26–27 and 48) whose parallaxes are worst determined in the present work (see §2) are given half weight in forming these means and in the counts detailed in the following columns, since in these cases the use of the present work as a standard of comparison, though necessary for homogeneity, is otherwise by no means desirable.

All the results in which there was not good reason to suspect definite systematic errors of known origin are collected in the upper part of table 36. It is clear at a glance that the systematic differences between these and the present work must be small. The mean differences (taking account of sign) never greatly exceed their probable errors, and the magnitudes of the indi-

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*The work of Bergstrand, Bohlin, Rambaut, and Schlesinger.

vidual discordances are distributed in general agreement with the law of chance, when the very small numbers in some of the groups are considered.

When the various groups are combined there appears distinct evidence of something more than random error. This does not, however, consist of a general increase in the number of cases in which the discordances of individual determinations exceed their probable error (which is what might be expected if sources of systematic difference were generally or frequently at work). Instead, there is a small group of large discordances, exceeding four times their computed probable error, while the smaller discordances are distributed in close agreement with theory.

These cases deserve special consideration. One of them arises from the large negative parallax found for star No. 48 (γ Serpentis)—which has already been pointed out as the weakest of all the determinations of the present work, and which for comparison purposes has been given half weight. The other three large discordances are all for the same star, 61 Cygni (Nos. 37, 38), and arise from the disagreement of the result of the present work with those of the exceptionally long and careful series of Hall, Bergstrand, and Chase, which agree closely *inter se*. Here there can be little doubt that the parallax found in the present work is too great by 0″.08 or thereabouts.

As these stars are the brightest which were observed without the color-screen*, and are photographically 2.5 and 2 magnitudes brighter than the average of their comparison-stars, it is very probable that this is a case of guiding error—a supposition which is confirmed by the fact that the parallax of the brighter component comes out 0″.04 greater than that of the fainter.

It is, however, hardly fair to count this one error as three—especially since, by comparison with other observers, this star has already contributed three discordances, all exceeding their probable errors, to our list. If they are reduced to one, the observed and theoretical distribution of the discordances, in terms of their probable errors, compare as shown in table 37.

TABLE 37.

Ratio.	Observed.	Theory.
0 to 1	18	19
1 to 2	11½	12
2 to 3	5	5
3 to 4	2	1½
4 to 5.1	1½	½
Total......	38	38

The agreement with theory is now apparently very close. However, the mean-square value of all the discordances, in terms of their probable errors, is on this calculation 1.77. Reducing this itself to a probable error, by multiplying by 0.6745, we find that the probable error, deduced from the agreement of the results of different observers with the present work, is on the average 1.19 times that derived from the internal agreement of the observations of the series in question—which would indicate the existence of some source of systematic discordance, whose average amount corresponds to a probable error 0.65 times that deduced from the internal agreement of

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*Except No. 3, which was observed only because it happened to be shown on the plates of another star.

the observations. The average value of the latter for the cases in question is ±0″.038. The average systematic error, expressed as a probable error, is therefore ±0″.025, including the combined errors of both observers.

Approaching the matter in a somewhat different way, but with the same restrictions as above, the average value of one discordance, without regard to sign, is found to be 0″.049, to which corresponds a probable error 0.845 times as great, or ±0″.0414. The average, without regard to sign, of the probable errors computed from the internal agreement of the observations is ±0″.0385. This would indicate a systematic discordance of amount corresponding to a probable error of ±0″.015 for the difference between the results of two observers.

It might be argued that mean-square values instead of the arithmetical means ought to be taken; but this would be equivalent to giving the greatest weight to the poorest determinations, unless some means are adopted to reduce all to a uniform standard of weight; and this has already been done in the procedure first followed. If the three large discordances for 61 Cygni are counted separately, the value of the systematic error comes out ±0″.032 by the first method and ±0″.022 by the second; but, for reasons already stated, these are probably somewhat too great.

The mean of all these determinations is ±0″.024, which may be adopted as the probable error corresponding to the average influence of the combined systematic errors of other observers and the present work on the difference of their results.

If these errors were uniformly divided among all the observations, the average systematic error of one observed parallax would be ±0″.017. Those of some observers are doubtless greater than this, and of others less. For example, the observers at Yale, from comparison of the results of successive series of observations on the same star, estimate that the systematic error of a parallax derived from a single such series is ±0″.030.[*]

The average systematic difference from the present work (determined by the second of the methods described above, without weighting down the comparisons for 61 Cygni) is ±0″.028 for the heliometer observations (all but two of which were made at Yale) and ±0″.016 for all the others.

The latter value corresponds to an average systematic error of ±0″.011 in the individual determinations of parallax. If we assume this as the systematic error of the observations of the present work which were compared with the heliometer results, and likewise of the two of these not obtained at Yale, the average systematic error of the Yale results comes out ±0″.027.

If we take into account the fact that four of the latter[†] are the means of two or four series, the average systematic error of a single series becomes ±0″.029, accidentally in almost perfect agreement with the estimate of the observers.

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[*] Yale Transactions, vol. II, part I, p. 194.

[†] For stars Nos. 7, 30, 37, and 46.

The last four lines of table 36 illustrate the effects of systemtic error.

Of Flint's uncorrected results, only one-eighth differ from those of the present work by less than the computed probable errors. The average probable error derived from the differences themselves is more than double that deduced from the internal agreement of the observations. Large systematic errors are clearly present. After the corrections deduced by the observer have been applied, things are much better. It would be unfair in this case to demand agreement within the limits set when the unknown probable errors of the systematic corrections are neglected. It is better to derive the probable error of one difference from their average value, and this has been done in the last line but one of the table. It appears that the probable error of one of the corrected results is, on the average, rather more than half as great again, and the weight rather less than half as great, as the internal agreement of the observations would indicate. If the probable errors of the individual differences given in table B are increased in the same proportion, the distribution of the ratios of one to the other is that given toward the end of the line in question, in table 36.

There is now little evidence of outstanding systematic error. By the application of the corrections, the mean difference (regarding signs) between the two sets of results considered is reduced to little more than half its initial value, and becomes comparable with its probable error, and the distribution of the individual discordances is in fair agreement with that predicted by theory and given in the last line of the table.

There is also evidence, of a somewhat different kind, that systematic error exists in those photographic results in which the influence of atmospheric dispersion was not eliminated, as in the present work and that of other recent observers. Here neither the distribution of the individual discordances nor their average numerical value shows signs that anything is amiss; but the photographic parallaxes come out too great, on the average, by $0''.032$, which is more than twice its probable error.

This in itself would be hardly too much to attribute to chance; but it agrees in sign and magnitude with the error which might be anticipated. The "average spectrum," if the phrase may be used, of the stars under investigation is K, and that of the comparison-stars about F5, so that the effective refraction constant is probably sensibly less for the former. If the difference is $\delta\beta$, the formulæ of Chapter III, §7 (taking as average conditions $\varphi=45^\circ$, $\delta=45^\circ$, $t=-3^h$ for the morning and $+3^h$ for the evening observations), show that an error $-0.6\delta\beta$ will appear in the relative parallax deduced from photographs. The observed discordance therefore demands that $\delta\beta$ shall be about $-0''.05$, which is of the order of magnitude indicated both by our knowledge of the dispersion of air and by direct photographic investigations.

The results of this discussion may be summarized as follows:

Systematic errors of the first kind—*i. e.*, constant errors depending on magnitude, spectrum, and the like—exist in Flint's work (where they have been effectively corrected by the author) and in the photographic observations made at varying hour-angles, but are absent, or at least very small, in the work of other observers. Those of the second kind—which amount, so far as can be determined, to an increase of the probable error of observation—are generally present, but very small. In the observations made at Yale they appear to be somewhat less than was estimated by the observers themselves. For all other observations, including the present work, their average influence on the parallax corresponds to a probable error only very slightly exceeding 0″.01.

The influence of this systematic error may be included in the probable errors given in table A by increasing those less than 0″.020 by 0″.003, those between 0″.020 to 0″.033 by 0″.02, and those greater than this by 0″.001.

III. COMPARISON WITH KAPTEYN'S FORMULAE.

§5. *Parallax of the Comparison-Stars.*

According to Kapteyn,* the mean parallax of all the stars of visual magnitude m is

$$\pi_m = 0''.0160\ (0.75)^{m-5.5} \qquad (1)$$

while that of the group of magnitude m and proper-motion μ is

$$\pi_{m,\mu} = (0.87)^{m-5.5}\ \sqrt[p]{A\,\mu} \qquad (2)$$

where $A = 0.0753$, $p = 1.20$ for stars of the first spectral type, and $A = 0.0316$, $p = 1.47$ for those of the second. (In deriving these formulæ, all the faint stars of large proper-motion were considered to be of the second type—an assumption invariably verified upon investigation.)

Applying the first of these to the comparison-stars (grouped according to their photometric magnitude, as in Chapter IV, §3) the results are as follows:

TABLE 38.

No. of stars.	Mean magnitude.	Computed parallax.	Difference from mean.	Mean observed parallax.
25	7.22	+0″.0098	+0″.004	+0.006
59	8.64	+0.0065	+0.001	−0.001
106	9.51	+0.0051	−0.001	−0.002
52	10.35	+0.0039	−0.002	+0.003
Weighted mean......	....	+0.0057	....	....

The mean parallax of the comparison-stars may therefore be taken as +0″.006. The observed mean parallaxes of the groups of different magnitude, relative to the whole, agree with the theoretical values very closely.

For the six comparison-stars showing the clearest evidence of proper-motion (listed in Chapter IV, §7), the average magnitude and proper-

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*Publications of the Astronomical Laboratory of Groningen, No. 8, p. 24; revised, No. 11, p. 18.

motion are 9.66 and 0″.32; the corresponding parallax is 0″.023. The mean parallax of the comparison-stars for the fields in which one of these stars lie should therefore be taken as 0″.008.

From the data of page 62 it appears that the probable error of the mean parallax of the six or eight comparison-stars of a given field is about one-third of its value. It is therefore possible to pass from the observed relative parallaxes to the absolute parallaxes by adding the amounts just given, without sensibly increasing the probable error derived from the observations.

§6. *Data for the Individual Parallax-Stars.*

The individual results for the stars specially observed for parallax are as follows: The observed parallaxes have been increased by the computed parallax of the comparison-stars, to render them comparable with the theoretical values. The latter have been derived from the formula (2), considering all spectra from O to F5, inclusive, as of Type I, and those from F8 to M as of Type II. For star No. 36 the proper-motion derived from the plates has been employed in the absence of better data. Otherwise the data are those of table A.

For double stars and pairs with common proper-motion, the mean of the observed parallaxes and the computed parallax for the brighter component are given. For the long-period variable Mira (No. 5) the computed parallaxes corresponding to the average maximum and minimum brightness are given. The former, which is nearer the observed value, has been used in the discussion of the results.

Table 39. Comparison with Kapteyn's Formula.

No.	Mag.	Proper motion	Sp.	Observed parallax.	Computed parallax.
1	2.4	0.56	F5	+0″.088	+0″.110
2	7.7	2.80	Ma	+0.258	+0.141
3	6.0	0.03	A	−0.018	+0.006
4	3.6	1.24	F8	+0.193	+0.140
5	3.0 9.5	0.24	Md	+0.142	+0.051 +0.021
6	3.8	0.17	Mb	+0.091	+0.038
7	2.1	0.01	B8	+0.013	+0.005
8, 9	8.3	1.38	G	+0.002	+0.080
10	8.3	2.20	G2	−0.005	+0.109
11	6.8	0.68	G	+0.084	+0.061
12	6.5	0.90	F8	+0.057	+0.078
13	7.4	4.78	K	+0.350	+0.211
14	8.6	4.40	K5	+0.169	+0.169
15, 16	6.4	0.74	K	+0.058	+0.070
17	6.5	7.04	G	+0.106	+0.315
18, 19	3.6	0.56	F	+0.070	+0.095
20	10.0	1.89	(G5)	+0.111	+0.079
21	8.3	2.32	K5	+0.227	+0.114
22, 23	9.4	1.38	K	+0.030	+0.070
24, 25	8.9	3.75	G5	+0.036	+0.147
26, 27	6.8	0.68	G	−0″.052	+0″.060
28, 29	6.8	0.48	G5	+0.046	+0.049
30	7.8	1.36	Ma	+0.101	+0.085
31, 32	9.3	2.27	K5	+0.304	+0.098
33	9.1	1.26	Ma	+0.082	+0.067
34, 35	6.6	0.64	K	+0.038	+0.060
36	9.8	0.03	O	−0.012	+0.005
37, 38	5.6	5.27	K5	+0.390	+0.290
39	7.7	0.06	K	−0.029	+0.010
40	7.0	0.83	F8	+0.029	+0.068
41	9.4	0.95	(K5)	+0.264	+0.053
42	7.6	0.68	F8	+0.045	+0.055
43	9.1	1.40	Ma	+0.219	+0.073
44	8.3	0.44	G	−0.014	+0.035
45	3.5	0.07	Ma	+0.040	+0.021
46, 47	2.0	0.20	A	+0.109	+0.051
48	3.9	1.33	F8	−0.075	+0.144
49	3.0	0.60	G	+0.107	+0.096
50	3.6	0.10	K	+0.020	+0.022
51, 52	3.5	0.82	G5	+0.044	+0.109

§7. *Comparison by Groups. Systematic Differences for Different Spectral Types.*

Grouping together those stars whose computed parallaxes lie between specified limits,[*] the observed and computed values compare as follows, the general agreement being very good:

TABLE 40.

Limits of computed parallax.	Mean parallax.		No. of stars.
	Observed.	Computed.	
>0".20	0".282	0".272	3
0".20 to 0".10	0.104	0.132	9
0.10 to 0.07	0.106	0.082	10
0.07 to 0.05	0.082	0.059	9
<0".05	0.015	0.021	9
Altogether.........	0.093	0.089	40

Taking means for groups of stars of similar magnitude, proper-motion, or spectral type, the results are shown in tables 41 and 42.

TABLE 41.

Limits of—	No. of stars.	Mean mag.	Mean proper-motion.	Mean parallax.		Ratio.	Computed ratio.
				Observed.	Formula.		
Magnitude.							
2.0 to 4.0	12	3.2	0".49	+0".070	+0".073	1.0	1.1
5.6 to 7.0	9	6.4	1.83	+0.079	+0.110	0.7	0.8
7.1 to 8.0	6	7.5	1.75	+0.126	+0.095	1.3	1.3
8.1 to 9.0	6	8.5	2.41	+0.079	+0.109	0.7	0.9
9.0 to 10.0	7	9.4	1.31	+0.143	+0.063	2.3	1.3
Proper-motion.							
Over 3".0	5	7.4	5".05	+0".210	+0.226	0.9	1.1
3.0 to 1.5	5	8.7	2.30	+0.179	+0.108	1.7	1.4
1.5 to 1.0	7	7.3	1.34	+0.079	+0.094	0.8	1.0
1.0 to 0.7	5	6.6	0.85	+0.090	+0.076	1.2	0.9
0.7 to 0.4	9	5.8	0.59	+0.046	+0.069	0.7	0.6
Under 0.4	9	4.6	0.10	+0.040	+0.023	1.7	1.4

TABLE 42.

Spectrum.	No. of stars.	Mean magnitude.	Mean proper-motion.	Mean parallax.		Ratio.
				Observed.	Formula.	
O to F5 } Type I }	6	4.3	0".23	+0".042	+0".045	0.9
F8	5	5.7	1.00	+0.050	+0.097	0.5
G, G2	7	6.9	1.86	+0.032	+0.109	0.3
G5	4	7.3	1.73	+0.059	+0.096	0.6
K	6	6.9	1.28	+0.078	+0.074	1.1
K5	5	8.3	3.04	+0.271	+0.145	1.9
M	7	6.3	1.04	+0.133	+0.068	1.9
Type II	34	6.7	1".62	0".102	0".096	1.1

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[*] It will not do to group them according to the observed parallaxes, for the groups of largest parallax will then contain a disproportionate number of stars whose parallaxes are greater than the average for stars of the same magnitude and proper-motion, and also of those whose parallaxes are *increased* by errors of observation, and similar systematic errors of opposite sign will appear in the groups of smallest parallax.

The last column but one in table 41, and the last column in table 42 give the ratio of the mean observed parallax for each group to the mean of the values predicted by Kapteyn's formula.

For the different spectral types these ratios vary in a strikingly systematic fashion. The first type stars (O to F5) are too few in number to permit of separation into sub-groups. Among the remaining stars, for which Kapteyn's "second-type" formula was used the ratio of the observed to the computed parallax increases rapidly and almost regularly with increasing redness from less than one-half for Type G to more than double for Type M. If the stars under discussion are separated according to magnitude (above and below 7.0) or proper-motion (greater or less than 1″.0) and similar ratios are taken (combining adjacent spectral classes to get enough stars) the results are as follows:

TABLE 43.

Spectrum.	Average proper-motion.	Ratio.	No. of stars.	Average magnitude.	Ratio.	No. of stars.
F8–G2	2″.64	0.3	5	5.3	0.5	7
G5–K	2.94	1.0	4	5.4	0.7	5
K5–M	2.63	1.7	8	4.0	1.7	4
F–G2	0.68	0.6	7	7.9	0.2	5
G5–K	0.47	0.6	6	8.7	1.0	5
K5–M	0.35	3.3	4	8.7	2.0	8

The phenomenon, therefore, appears to persist throughout a considerable range of magnitude and proper-motion.

For the average of all these stars together the observed and computed values are in close agreement, thus confirming Kapteyn's formula for them as a whole. It is, however, evident that, with the more detailed spectroscopic data now available, the accuracy of the prediction of parallax can be considerably increased by taking these differences into account. For example, the ratios of the observed and computed parallaxes of the groups of stars of similar magnitude or proper-motion, given in table 41, vary through a wide range, in a very irregular fashion. If for each of these groups the mean of the ratios corresponding to the spectral types of the individual stars is taken, the values given in the last column of that table, under the head "Computed ratio" are obtained. It is clear that a large part of the irregularity of the observed ratios arises from the irregular distribution of stars of the different spectral types.

The material here discussed is not a large enough part of the whole available information concerning stellar parallax to warrant the derivation of corrections to Kapteyn's formula from its results alone. The remarkable increase in parallax with increasing redness is clearly demonstrated among the stars of considerable proper-motion; but it is quite uncertain whether it will be found to hold good among those of small proper-motion.

It is obvious that the use of a single formula for all the yellow and red stars may give rise to serious systematic errors; but the adoption of the factors of correction here derived, for groups of stars differing greatly in proper-motion from those used in deriving them, might lead to equally erroneous results.

Further investigation of the matter is eminently desirable, especially the determination of the spectral types of all the faint stars which have been observed for parallax and a study of the parallactic motions of the brighter stars of each spectral type separately.

IV. ASTROPHYSICAL DATA.

§8. *Brightness and Cross Velocities of the Individual Stars.*

Table 45 shows what information can be derived from the results of the present work concerning the actual brightness of the stars observed for parallax and their velocities at right angles to the line of sight. The former of these—following Kapteyn—is expressed in terms of the "absolute magnitude," *i. e.*, the magnitude which the star would seem to have if placed at such a distance that its parallax was one-tenth of a second of arc. This is found from the observed magnitude m by the equation

$$M = m + 5 + 5 \log \pi$$

The absolute magnitude of the sun, on this scale, is 31.58 magnitudes fainter than its apparent stellar magnitude. According to the recent determination of Prof. W. H. Pickering,* the latter is −26.83, which would make the Sun's absolute magnitude 4.75. Earlier determinations make it considerably lower: for example, Kapteyn adopts the value 5.5.

The actual brightness, in terms of the Sun, of a star of given absolute magnitude, according to those two determinations, is shown in table 44, the light decreasing tenfold for every 2.5 magnitudes.

TABLE 44.

Absolute. magnitude.	Light, in terms of Sun.		Absolute. magnitude.	Light, in terms of Sun.	
	Kapteyn.	Pickering.		Kapteyn.	Pickering.
2.50	15.85	7.95	4.25	3.16	1.59
2.75	12.59	6.31	4.50	2.51	1.26
3.00	10.00	5.01	4.75	2.00	1.00
3.25	7.95	3.98	5.00	1.59	0.80
3.50	6.31	3.16	5.25	1.26	0.63
3.75	5.01	2.51	5.50	1.00	0.50
4.00	3.98	2.00	5.75	0.80	0.40

The cross-velocities (that is, the component of the velocity, relative to the Sun, which is perpendicular to the line of sight) are given in kilometers per second and are computed by the formula $V = 4.74 \frac{\mu}{\pi}$, where μ is the annual proper-motion on a great circle.

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*Harvard Annals, LXI, part I, p. 69.

TABLE 45.

Star.	Sp.	No.	Absolute magnitude corresponding to the parallax.			Cross velocity corresponding to the parallax.		
			Decreased by probable error.	Observed.	Increased by probable error.	Decreased by probable error. Km./sec.	Observed. Km./sec.	Increased by probable error. Km./sec.
+30°3639...	O	36†	...	...	(4.8)	...	...	...
β Persei....	B8	7*†	...	−2.3	0.1	...	3.5	1.2
26 Androm..	A	3†	...	...	2.9	...	...	6
Lal. 27743...	A?	27	...	...	...	...	...	...
α_1 Gemin...	A	46*†	2.4	3.0	3.4	11	9	7
α_2 Gemin...	A	47*†	1.6	2.2	2.6			
γ Virginis..	F	18*	1.9	2.9	3.6	60	38	28
	F	19*	1.9	2.9	3.6			
β Cass......	F5	1*	1.5	2.1	2.5	40	31	25
η Cass......	F8	4*	4.8	5.1	5.3	34	30	28
		Comp.	8.6	8.8	9.0			
Gr. 1646....	F8	12	4.0	5.2	5.9	134	77	55
Lal. 43492...	F8	40	(1.2)	4.3	5.5	(550)	135	77
Lal. 45755...	F8	42	4.5	5.8	6.7	132	74	49
γ Serpentis..	F8	48*	...	...	...	...	...	...
Lal. 6888....	G	8	...	(0.8)	5.8	...	(3300)	205
Lal. 6889....	G	9	...	(1.4)	6.4			
Lal. 7443....	G2	10	...	...	(3.1)	...	...	(1200)
Gr. 884.....	G	11	5.9	6.4	6.9	50	38	31
Gr. 1830....	G	17	5.8	6.6	7.1	430	315	266
Lal. 27742...	G	26	...	...	...	...	...	...
Lam. 32805..	G	44	...	...	4.7	...	...	110
ζ Herc......	G	49*	2.6	3.2	3.6	34	27	22
Berl. A 4999.	(G5)	20	9.6	10.2	10.6	103	81	66
A. Oe. 14320.	G5	25	4.8	6.7	7.7	1180	495	311
A. Oe. 14318.	K	24	5.0	6.9	7.9			
W.B.15^h716.	K	28	5.2	6.1	6.7	72	48	36
W.B.15^h720.	G5	29 A	4.8	5.7	6.3			
(OΣ 298)..	......	..... B	5.1	6.0	6.6			
μ Herc.....	G5	51* A	(−2.0)	1.7	3.0	(480)	88	48
Companion..	......	52 B	(4.5)	8.2	9.5			
		C	(5.5)	9.2	10.5			
Lal. 21185...	K	13	10.0	10.1	10.2	68	65	62
83 Leonis..	K	15	3.6	5.3	6.2	120	58	38
	K	16	4.7	6.4	7.3			
Berl.B.5072.	K	22	...	6.8	8.2	...	216	105
Berl.B.5073.	......	23	...	7.0	8.4			
Gr. 2789...	K	34	...	4.7	6.5	...	80	35
	K	35	...	4.5	6.3			
+38°4362...	K	39†	...	...	...	...	...	...
η Herc.....	K	50*	...	0.1	3.3	...	24	6
Lal. 21258...	K5	14	9.5	9.8	10.1	138	124	112
Lal. 25372...	K5	21	9.9	10.1	10.3	53	48	44
Pos. M.2164	K5	31	11.4	11.7	12.0	41	35	31
(Σ 2398)...	K5	32	12.1	12.4	12.7			
61 Cygni..	K5	37	8.4	8.5	8.6	67	63	60
	K5	38	9.1	9.2	9.3			
H−G 13170	(K5)	41	11.6	11.8	12.0	18	17	16
(Kruger 60)		Comp.	12.9	13.1	13.3			
Gr. 34......	Ma	2	9.7	9.8	9.9	55	51	48
o Ceti......	Md	5*†	1.9 to 9.8	2.5 to 10.4	2.9 to 10.8	10	8	6
ρ Persei....	Mb	6*†	1.9 to 2.7	3.1 to 3.9	4.0 to 4.8	16	9	6
W. B. 17^h322	Ma	30	7.5	7.8	8.1	76	64	55
Lam. 32805.	Ma	33	5.3	8.7	9.9	350	73	41
Lal. 46650...	Ma	43	10.5	10.8	11.0	34	30	27
η Gemin....	Ma	45*†	−0.8 to 0.2	1.3 to 2.3	2.3 to 3.3	21	8	5

*Naked-eye stars (above the fourth magnitude). †Proper-motion less than 0″.4.

In tabulating the results the stars are numbered as in table A, but arranged in order of spectral type. The mean observed parallax of the two stars of a physical pair is adopted for both. For certain double stars the magnitudes of companions, not shown on the plates, are included, and for some variables the maximum and minimum magnitudes are given. To show the extent of the uncertainty due to errors of observation, the values resulting when the observed parallax is increased or decreased by its probable error are given on each side of those corresponding to the observed parallax which are printed in heavier type. When the parallax used for computation is less than 0".01, the corresponding entry in the table is inclosed in parentheses; and when it is negative the space is left blank.

In examining these results, it should be remembered that it is as likely as not that the true values for any given star lie outside the range of those given in the table, and that in some cases they are doubtless very considerably outside this range. It must also be borne in mind that the stars included in this list are but few in number and have been selected from the general mass according to definite apparent characteristics—practically all having large, or at least considerable, proper-motions, and all lying within the limits of magnitude set by the necessity of correct exposure of the plates (with or without the color-screen).

For example, all but one of the bright stars, observed with the color-screen, appear to be actually brighter than the Sun, while few of the more numerous stars observed without it (which are almost all invisible to the naked eye) appear to equal the Sun in luminosity. There is no doubt that this is due almost entirely to the fact that the latter were selected on account of their large proper-motion, and hence represent stars much nearer us than the average of those of the same magnitude, and necessarily really much fainter.

In any discussion it is therefore desirable to separate these two groups. Is has also seemed best to exclude from the latter group the four stars whose proper-motion is less than 0".40. Three of these (Nos. 3, 36, and 39), with proper-motions less than 0".10 and parallaxes apparently insensible, are clearly in no way comparable with the remaining stars, while the fourth is the long period variable Mira, which ought obviously to be kept by itself.

§9. *Means for Different Spectral Types.*

It is a matter of some difficulty to find really representative mean values for any group of stars. The results derived from the observed parallaxes of each star separately are affected by the errors of observation. Since a given decrease in the assumed parallax of a star increases its computed velocity and brightness more than an equal increase in the assumed parallax diminishes them, the mean of a number of such results may be expected to come out too great; and if any of the observed parallaxes are negative, no satisfactory mean can be found in this way.

If, on the other hand, the means of the observed magnitudes, proper-motions, and parallaxes are taken, and the corresponding cross-velocity and luminosity found, the results may be expected to be too small, on account of the neglect of the real departures of the parallax of individual stars from the mean.* The true values will usually lie between the mean values found in these two ways.

Grouping the stars in this way, according to spectral type, the results are as shown in table 46. The parallax of the comparison-stars has been allowed for and, as usual, only the brighter star of a physical pair is counted. A blank in the last column denotes that some of the observed parallaxes are negative, and no mean of the individual values can be taken.

TABLE 46.

Naked-Eye Stars.

Spectrum.	No. of stars.	Mean mag.	Mean proper-motion.	Mean observed parallax.	Corresponding absolute magnitude.	Corresponding velocity. Km./sec.	Mean of individual absolute magnitudes.	Mean of individual velocities. Km./sec.
B8 to F5...	4	2.6	0″.33	0″.070	1.8	22	1.2	20
F8 to M...	7	3.7	0.62	0.060	2.6	49	....	....
Fainter Stars of Large Proper-motion.								
F8.........	3	7.0	0.80	0.044	5.2	86	5.1	94
G, G2.....	6	7.5	2.07	0.020	4.0	508	....	....
G5........	3	8.6	2.04	0.064	7.6	151	7.3	208
K.........	4	7.4	1.88	0.119	7.8	75	6.7	102
K5........	5	8.3	3.04	0.271	10.5	53	10.3	57
M.........	4	8.4	1.66	0.165	9.5	48	9.3	54

The differences in actual brightness and velocity between the two groups of naked-eye stars are closely parallel to those in their apparent brightness and proper-motion, and may be due largely to arbitrary selection of the stars, some of which were put on the working list for quite different reasons from others.

The faint stars of large proper-motion, on the other hand, are apparently fairly homogeneous, the mean magnitude and proper-motion for all the spectral sub-groups except the first being nearly the same. The discordant values for the stars of spectrum F8, which average a magnitude brighter than the others and have little more than one-third of their mean proper-motion, are undoubtedly due to the manner of their selection. In preparing the working list, stars below the sixth magnitude, and with proper-motion less than 1″ were included only (1) if their spectrum was given as A in the Draper Catalogue; (2) in the case of pairs with common proper-motion. The latter are distributed almost uniformly among the different spectral types; but the former (although the later observations at Harvard modify the original esti-

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*In small groups, accidental irregularities may mask this general tendency.

mates of the spectrum) furnish all the cases of spectrum F8 and one G.* The small average proper-motion of the first group is therefore due to the adoption of a different standard of admission; while their greater average brightness means only that none are included which are too faint to appear in the Draper Catalogue.

The other five groups, though very similar in apparent magnitude and proper motion, show systematic differences in real brightness and velocity of so marked a character that they must have some real significance.

§ 10. *Possible Explanations of the Differences.*

It is worthy of especial attention that if we assume that either the absolute magnitudes or the cross velocities given in table 46 are characteristic of all (or at least of the large majority) of the stars of the corresponding spectral type, then the observed distribution of the other tabular quantity (cross-velocity or absolute magnitude) and of the average parallaxes becomes a necessary consequence of the manner of selection of the stars.

Suppose, for example, that, as indicated by the absolute magnitudes of the table, stars of Type G are on the average 2.5 magnitudes (or ten times) brighter than those of Type K, and these again as much brighter than those of Type M. Then, since our stars have been so chosen that the average apparent brightness of all three groups is nearly the same, those stars of Type G, *which appear in our list*, must on the average be some three times as far off as those of Type K, and those again at three times the distance of those of Type M; and their mean parallaxes will be to one another in the inverse ratio.

Since they have also been selected so that the mean proper-motion of the different groups is nearly the same, the average cross-velocity of those stars *which pass the conditions of admission* must be about three times as great for each type as for the following.

This is roughly what is shown in table 46, the differences being due to departures of the tabular numbers from the simple relations assumed in the above illustration.

In just the same way it follows that if the stars of Type G are actually moving faster, on the average, than those of Type K, the method of selection compels us to choose brighter stars, on the average, in the first case than in the second.

The observed differences in brightness and velocity can not both be results of the method of selection; but either one may (not must) be so, if the other represents a real characteristic of the different spectral types.

Among the various hypotheses thus suggested there can be little doubt which is the most plausible. Recent researches have established a very strong presumption that the spectral type of a star is intimately connected with its surface temperature (the evidence being particularly convincing for just the range of spectral types under discussion).

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*No. 11. Groombridge 884.

The sequence of Types G, K, M is almost certainly one of decreasing temperature, and therefore of diminishing surface-brightness, and the marked diminution in the average luminosity of the stars from type to type is just what might be expected.

On the other hand, it is very hard to see how the velocity of a star in space can be a function of its temperature, especially to the enormous extent demanded by the observations, if they are to be explained in this way.

Certain other facts confirm the former explanation of the observations. The stars of Type F8, on account of the peculiar method of selection described above, are apparently, on the average, considerably brighter and much more slowly moving than the closely related stars of Type G. The difference in the observed mean parallaxes is, however, such as to make the real average brightness of the two groups come out nearly equal, while exaggerating that between the mean velocities. Though the number of stars involved is small, this may be taken as confirmatory evidence that brightness, rather than velocity, is the principal point of similarity between adjacent spectral types and of difference between those widely separated.

Again, the spectroscopic determinations of radial velocity show no such marked progression with the spectral type as is exhibited above, and what there is has the opposite sense—the mean velocity, after allowance is made for the solar motion, being distinctly less for Type B than for the others, and probably increasing slowly from A to M.[*]

(The stars with which such observations deal are for the most part, however, so very different in intrinsic brightness from those here considered that this argument is by itself less conclusive than it might at first appear.)

All things considered, it may be regarded as probable that the differences in absolute magnitude (or actual luminosity) between the stars of different spectral types, revealed in table 46, are real and typical, while those in mean parallax and cross-velocity are consequences of this, together with the way in which the stars were selected for observation.

The remarkable differences from Kapteyn's formula, described in §6, (page 91) may likewise be attributed to this cause.

§11. *Bearing on Stellar Evolution.*

The work of Scheiner and Wilsing at Potsdam makes it possible to estimate what part of the differences in brightness between the stars of different spectral types is due to temperature alone. They find that the distribution of brightness in the visual spectrum of a large number of stars agrees closely with Planck's formula $J_\lambda = \dfrac{c_1 \lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1}$; and with the aid of this formula they determine their effective temperatures.

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[*] Kapteyn, Astrophysical Journal, vol. XXXI, p. 260 (April 1910).

If the actual light emission of the stars is also approximately in agreement with the formula (which seems reasonable) the relative brightness (expressed in stellar magnitudes) of stars of the same diameter, but of different spectral types and temperatures, should on their data* be as follows:

TABLE 47.

Spectral type.		Absolute temp. (C.)	Relative brightness for wave-length.		
Potsdam.	Harvard.		0.600μ	0.500μ	0.400μ
Ia 1	A	9600°	$0^{m}.0$	$0^{m}.0$	$0^{m}.0$
IIa	G	5400	2.2	2.6	3.2
IIa–IIIa	K	4000	3.9	4.6	5.7
III	M	3200	5.6	6.6	8.2

On this hypothesis, the lower temperature accounts for nearly two magnitudes of the difference of absolute magnitude between Types G and K, or K and M, leaving little to be explained by differences in the actual size of the stars. The latter, however, appears to be somewhat smaller, and the density presumably greater, for those of lower temperature. It is probable that these stars are in the later stages of evolution, having passed their maximum temperature, and that the reddest ones are approaching extinction. Evidence that some of them are of considerable density will be given later.

It should be particularly noticed that the foregoing remarks apply only to stars whose actual luminosity is comparable with or less than that of the Sun. The majority of the naked-eye stars of these spectral types (and probably of those of any given visual magnitude) have small proper-motions and are presumably remote and of great intrinsic luminosity—as is shown by direct measurement for such stars as Capella, Arcturus, and Antares. These may reasonably be supposed to be stars in a much earlier stage of evolution, of small density and rising temperature, owing their high luminosity to their great superficial area; and, if this hypothesis is sound, the reddest among them are in the most primitive condition.

Red stars (of Types K5 and M) may therefore be supposed to represent two widely different stages of evolution, one early and one very late. Most if not all of those which are conspicuous to the naked eye belong to the former class; while the latter are to be found exclusively among the fainter stars of large proper-motion.

§ 12. *Masses of Binary Stars.*

Six of the stars observed for parallax are binaries for which more or less satisfactory orbits have been computed. The following table gives for these the semi-major axis a and period t, with the authority from which they are taken; the sum of the masses of the components, computed by the equation $m = \frac{a^3}{\pi^3 t^2}$, and on each side of it the values obtained by increasing or decreas-

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*Potsdam Publications, vol. XIX, part 1, p. 67; the assumed value of c_2 being 14600.

ing the observed parallax* by its probable error; and finally the "hypothetical parallax" p obtained from the above equation on the assumption that the mass is in all cases 2.4 times that of the Sun, the excess of the observed parallax above this, and the probable error of the latter.

TABLE 48.

No.	Star.	a	t years.	Authority.	Mass, corresponding to the parallax.			Comparison of hypothetical and observed parallaxes.		
					Decreased by its probable error.	Observed.	Increased by its probable error.	p	$\pi-p$	$p.\ e.$
4	η Cassiopeiæ	8″.51	233	Lewis (1)	2.2	**1.6**	1.2	0″.168	+0″.025	±0″.019
18, 19	γ Virginis	3.90	182	Lewis (2)	21.2	**5.3**	2.1	0.091	−0.021	±0.026
29	OΣ 298	0.88	56	Celoria (6)	7.4	**2.3**	1.0	0.045	+0.001	±0.015
46, 47	α Geminorum	5.75	347	Doberck (4)	2.6	**1.2**	0.7	0.087	+0.022	±0.029
49	ζ Herculis	1.38	34.5	Doolittle (5)	3.7	**1.8**	1.0	0.098	+0.009	±0.024
52	μ Herculis BC	1.45	43.5	Lewis (3)	(3100)	**19.0**	3.1	0.088	−0.044	±0.036

(1) Mem. R. A. S., LVI, p. 16. (2) Ibid, p. 339. (3) Ibid, p. 506. (4) Astronom. Nach., 3970. (5) Astronomical Jour. 460 (vol. 20, p. 25). (6) Astronom. Nach., 2843.

The mean of the masses corresponding to the observed parallaxes is 5.2 times that of the Sun. Excluding the last star, for which the errors of observation are unusually large, this is reduced to 2.4 times the Sun's mass.

The mean-square difference between the hypothetical parallaxes, computed with this latter value of the mass, and the observed values is 0″.024, while the mean-square probable error of one such difference is ±0″.025, so that a mean-square difference of 0″.037 might have been expected. This remarkably close agreement is doubtless due to chance; but it is clear that the present work, though showing conclusively that the average mass of these systems considerably exceeds that of the Sun, does not enable us to arrange them in order of mass.

According to Professor Boss, the ratio of the mass of the companion to the primary, in three of these systems (Nos. 4, 18, and 49) averages 0.83. It appears, therefore, that on the average the principal star in one of these systems is rather more massive than the Sun, and the companion nearly equal to the latter.

There are also three slow binaries on the list, whose relative motion, though distinctly curved, does not yet cover a long enough arc to permit the calculation of even an approximate orbit. It does not appear to be generally known that in such cases an inferior limit to the mass of each system, and a close approximation to the actual mean mass of a number of them, can be obtained if the parallaxes are known.

The observations, in such a case, enable us to find the coördinates, velocity, and acceleration of one star relative to the other, as projected on a plane tangent to the celestial sphere, for any convenient instant near the middle of the interval which they cover.

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*Corrected for the probable parallax of the comparison-stars.

Since the acceleration must be directed toward the principal star, the motion of the companion may be represented by the expressions

$$s\cos(p-p_0)=x=a+b(t-t_0)-c(t-t_0)^2 \qquad s\sin(p-p_0)=y=a^1+b^1(t-t_0)$$

where p_0 is the position angle at the time, t_0; s and p the distance and position angle at the time t; and the terms involving the cube of the time are neglected.

Now, according to the law of gravitation,

$$\frac{d^2x}{dt^2}=-K(m_1+m_2)\frac{x}{r^3}$$

where r is the distance between the two stars. If the astronomical unit, the year, and the Sun's mass are chosen as units, the constant K is $(2\pi)^2$, or 39.478.

If i_0 is the angle which the line joining the stars makes with the line of sight at the instant t_0, then at this moment $r=a\operatorname{cosec} i_0$ and $\frac{d^2x}{dt^2}=-2c$, whence $(m_1+m_2)\sin^3 i_0=\frac{a^2c}{19.74}$. If a and c are expressed in seconds of arc, and π is the parallax, this becomes

$$(m_1+m_2)\sin^3 i_0=\frac{a^2c}{19.74\pi^3}$$

The mass of the system is thus determined, except as regards the factor $\sin^3 i_0$. The second member of the above equation is evidently the minimum value of the mass. In any individual system more can not be said until the elements of the orbit can be determined; but in the mean of a large number of cases (since there is no reason to suppose any connection between the direction of the line joining the stars and the line of sight) the average value of $\sin^3 i_0$ can be found on principles of geometrical probability—the reasoning being identical with that familiar in the kindred case of spectroscopic binaries. The probability that $\sin i$ is less than any given limit $\sin i_0$ is $1-\cos i_0$, and the theoretical mean value of $\sin^3 i_0$ is $\frac{3\pi}{16}$ or 0.589. The actual mean is likely to be somewhat larger, for when i_0 is small the curvature of the relative path of the two stars will be small, and such cases may escape detection.

Applying this method to the systems mentioned above, the results are as follows. The data for 61 Cygni are those of Bergstrand* modified only by changing t_0 from 1902 to 1857; the others are from least-square discussions of the measures by the writer.

TABLE 49.

No.	Star.	t_0	p_0	a	b	c	a^1	b^1	$(m_1+m_2)\sin^3 i_0$
31, 32	Σ 2398	1870	144°	15″.96	+0″.0624	0″.00094	+0″.03	+0″.0584	0.43
37, 38	61 Cygni	1857	106.4	17.77	+0.0913	0.00048	+0.02	+0.1665	0.13
41	Krüger 60	1900	149	3.16	+0.0070	0.0128	−0.70	−0.192	0.35

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*Nova acta reg. soc. scient. Upsaliensis. Série IV, vol. I, No. 3 (1905).

The average value of $(m_1+m_2)\sin^3 i_0$ for the three systems is 0.30, and the most probable value of m_1+m_2 is one-half the mass of the Sun.

In spite of the uncertainty introduced by the unknown factor $\sin^3 i_0$, there can be no doubt that these systems are considerably less massive than those previously discussed; for otherwise it would be necessary to suppose that the average of three values of $\sin^3 i_0$ was only one-eighth; and the probability that $\sin^3 i_0$ will be so small in a single case is only 0.13.

The average mass of a component of one of these systems is one-fourth that of the Sun. Even if its density is 8 times that of the latter, or 11 times that of water—a somewhat violent assumption—its surface area must be one-tenth that of the Sun. The actual brightness of these stars (using Kapteyn's value for the Sun's light[*]) is as follows:

TABLE 50.

	61 Cygni.	Σ 2398.	Krüger 60.
Primary......	0.063	0.0033	0.0031
Companion...	0.033	0.0018	0.0009

It is evident that, except in the case of 61 Cygni, their surface-brightness must be very small, not exceeding one-thirtieth that of the Sun, and in some cases much less. This affords further evidence of their low temperature in addition to that of their spectral types (K5 and M); and shows also that, unless their surface brightness is considerably less than that previously computed for the average of stars of this type, their density must be very great.

13. *Distribution of Stars of Different Spectral Types.*

The comparison-stars whose magnitudes and spectra were determined at Harvard number over 200, and are widely distributed over the sky. They were chosen upon inspection of the plates, principally with regard to their photographic magnitude, sometimes also on account of favorable position, but quite without knowledge of their spectra, and there appears to be no reason why they should not be regarded as fair samples of the stars of comarable photographic brightness.

As the only published data regarding the relative proportions of the different spectral types among stars equally faint (averaging somewhat below the ninth magnitude) appear to be those derived from a single plate of a rich region in the Milky Way,[†] the information furnished by these stars may be of value. The field in which they lie may be dividher rather sharply into two groups,[‡] according to their distances from the Milky Way—the mean galactic latitude of the centers of the first group being 10°, and of the second 58°, with only two individual latitudes over 20° in the first and under 50° in the

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[*] With Pickering's value, all these numbers would be halved.

[†] Harvard Annals, vol. LVI, No. 1, pp. 14, 21.

[‡] Galactic: Series 1–3, 5–9, 23–27, 29, 30. Non-galactic: Series 4, 10–21, 28. (See table A.)

second. Excluding series 17, 18, 22, 31, for which the spectroscopic observations are very incomplete, there remain 15 galactic and 12 non-galactic fields for discussion.

For comparison with these there are added groups composed (*a*) of the faint stars of large proper-motion among those especially observed for parallax (see page 95) and (*b*) of those stars among the latter whose observed parallaxes exceed 0".10 (after allowance for the parallax of the comparison-stars). We thus obtain four groups, comparable in apparent brightness (especially from the photographic standpoint), but differing in other respects. The first three are mutually exclusive; the fourth a sub-group of the third.

The numbers and percentages of stars of the different spectral types in those groups are as follows. As in the recent Harvard work, G5 is counted with K, K5 with M, etc. The photographic magnitudes have been derived from the visual by means of King's table of corrections for the different spectral types.*

In the case of pairs of stars with common proper-motion, only the brighter component is counted; and in forming the percentages for the first two groups the few stars not observed spectroscopically are ignored.

TABLE 51.

Group.	Mean magnitude.		Number of stars.					Not observed.	Total.	Percentage.				
	Visual.	Photographic.	A	F	G	K	M			A	F	G	K	M
Galactic	9.38	9.84	34	25	16	42	0	4	121	29	21	14	36	0
Non-galactic	8.93	9.70	5	10	36	29	3	5	88	6	12	43	35	4
Large proper-motion	7.89	8.95	0	0	10	7	8	....	25	0	0	40	28	32
Large parallax	8.21	9.58	0	0	1	2	8	....	11	0	0	9	18	73

The steady increase of the percentage of red stars, at the expense of the white, is very striking.

As regards the last two lines, the explanation clearly lies in the fact brought out in § 10 (pages 96, 97)—that, among stars of moderate or small intrinsic brightness, faintness and redness go together.

The stars of the last group—invisible to the naked-eye, and with very large parallaxes—are of necessity intrinsically very faint, and therefore almost all red.

For those of the third group this requirement is less rigorous, and so more yellow stars (Type G) are included.

The region of space within which such stars must lie, in order to appear within given limits of visual magnitude, is several hundred times greater in volume than the corresponding region for the fainter stars of Type M; and it might seem that they ought therefore to be correspondingly numerous. But the conditions of inclusion in group 3, which require also a large proper-

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*Harvard Annals, vol. LIX, No. 5, p. 152.

motion, demand for these stars an actual cross-velocity so great that only a small percentage of the stars within the aforesaid region exceed it; and this is probably the reason why the numbers of proper-motion stars of Types G, K, and M are approximately equal.

This may also explain the absence of faint stars of large proper-motion and spectra A or F. Stars of these spectral types are presumably brighter intrinsically than those of Type G, and, to appear of the required brightness, must lie at such great distances that even the highest velocities occurring among the stars do not change their direction from us by so much as 1″ a year.

If the limits of the group were widened so as to admit stars of smaller proper-motion—or brighter stars of the same proper-motion—we might expect the percentage of stars of Type G to increase, and stars of Type F to appear, followed by those of Type A.

The data for the comparison-stars show the usual excess of stars of Type A in the Milky Way as compared with the regions outside it; but the actual percentage of such stars is surprisingly small. The ratio of the number of stars of this type to that of Types F, G, and K together, is in the present case 0.41 for the galactic fields, and only 0.07 for the non-galactic, whereas the similar ratios for stars of all magnitudes down to 8.2, as determined from counts made on a great number of plates at Harvard,* is 2.10 in the Milky Way and 0.70 outside it.

For the single plate of a rich galactic region on which fainter stars were investigated, the corresponding ratio for stars between the estimated magnitudes 8.5 and 9.5 (*i. e.*, those shown with a longer but not with a shorter exposure) is 2.9. It is not certain, however, that this one region is typical of the whole Milky Way. It is hard to explain so great a discrepancy. There seems to be no reason whatever why there should have been any discrimination against stars of Type A in picking out comparison-stars in fields where all the spectra were wholly unknown. The fact that the selection was made on photographs would favor the whiter stars at the expense of the others; but the same is true of the Harvard plates from which the data just referred to were obtained; and the fact that the estimates of spectral type were made in both cases at the same observatory, and on the same system, makes it very improbable that large systematic differences in classification exist. It is very desirable that the question should be settled by determination of the spectra of a much greater number of faint stars, well distributed over the sky.

It is possible that a diminution of the relative proportion of stars of Type A sets in at about the ninth magnitude, similar to that found for Type B, in passing from the brighter to the fainter naked-eye stars;* but it would be premature to suggest any explanation until the reality of the phenomenon is better assured.

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*Harvard Annals, vol. LVI, No. 1, p. 21.

TABLE C, SERIES I, STAR 1; β Cassiopeiae; $0^h\ 3^m.8$, $+58°\ 36'$; Proper-motion $+0^s.068$, $-0''.18$, $=0''.56$. Observed with Color-screen.

Stars.	Bonn Durchmusterung		Harvard.		Pl. 199 (Stand.) 1903, Dec. 30 Exp. 4, 3^m Hour-angle $+30^m$ Obs. H		Pl. 352 1904, Aug. 17 Exp. 4, 3^m Hour-angle $+21^m$ Obs. R		Pl. 360 1904, Aug. 26 Exp. 4, 4^m Hour-angle $+21^m$ Obs. R		Pl. 393 1904, Dec. 30 Exp. 4, 3^m Hour-angle $+13^m$ Obs. H		Pl. 400 1905, Jan. 5 Exp. 4, 3^m Hour-angle $+16^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	+58° 2700	8.8	9.36	F	15.092	14.50250	60480	18	58444	18	66998	**36**	65389	**13**	**0".044**	0".045
3	58 2704	9.5	......	.....	18.754	16.88101	93695	**6**	92065	31	00285	**3**	98804	51	042	**005**
2	58 2701	9.2	9.53	K	23.747	15.26208	25084	**22**	23995	**13**	32100	**40**	30534	**45**	**074**	011
4	58 1	9.0	9.96	A5	21.656	18.83284	85244	11	83834	**30**	91902	75	90375	5	070	**045**
6	58 8	8.6	9.41	A	23.737	22.75431	74979	1	73789	**1**	81299	**53**	80059	24	**026**	013
7	58 9	8.9	8.93	A	30.908	23.56279	46534	65	46091	33	53229	23	51999	40	056	036
9	58 13	9.5	10.56	F?	20.647	29.10056	14154	**52**	12620	14	19822	**9**	18679	**22**	**026**	**011**
5	58 6	9.4	10.31	A	16.256	22.10821	20109	**40**	18130	**29**	26129	5	24729	**14**	**009**	**034**
8	57 19	8.9	8.84	A	12.013	25.32202	47402	21	44852	**27**	52844	28	51481	**33**	**004**	**003**
A	58 3	2.2	2.42	F5	20.383	21.04789	08838	240	07308	241	15175	255	13818	290	475	080
P. M. of A	....	....	....	.....	—.001	+.003	....	....	....	....	....	....	....	....	....	....
					B											
Average residual.....................					32*		58		41		37		34			
a.....................					0.00		+ 84.90		+ 73.14		− 3.08		+ 29.30			
b.....................					0.00		− 1314.62		− 1205.27		− 1253.82		− 1248.00			
c.....................					0.		+28822.		+25305.		+35753.		+33562.			

Plate.	Observed.	P. M. to 1904.5	Corrected.	Equations of condition.	$o-c_1$	$o-c_2$
199	0".000	+0".261	+0".261	$x-0.506y-0.894\pi=+0''.011$	0".000	+0".025
352	+0.422	−0.066	+0.356	$x+0.128y+0.551\pi=+0.106$	0.000	0.000
360	+0.424	−0.078	+0.346	$x+0.152y+0.431\pi=+0.096$	0.000	0.000
393	+0.449	−0.256	+0.193	$x+0.497y-0.892\pi=-0.047$	−0.020	−0.033
400	+0.510	−0.265	+0.245	$x+0.513y-0.877\pi=-0.005$	+0.021	+0.008
Assumed P. M. and Δx		+0.516	+0.250	..........................(pvv)	841	1778

Normal equations.

$$+5.000x+0.784y-1.681\pi=+0''.161$$
$$+0.784\quad +0.805\quad -0.305\quad =-0.004$$
$$-1.681\quad -0.305\quad +2.854\quad =+0.136$$

Solution I.

		Weight.
x =	$+0''.066\pm0''.007$	3.54
y =	-0.038 ± 0.017	0.68
π =	$+0.082\pm0.009$	2.29
r_0 =	±0.014	

Solution II.

	Weight.
$+0''.060\pm0''.008$	4.01
....	....
$+0.083\pm0.011$	2.29
±0.016	

Observer's notes—360, Drifting clouds; 400, Probably light clouds passing. Measurer's notes—199, Image of A near edge of screen.
All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*
*For explanations of the symbols used in this table, see pp. 29-35.

TABLE C, SERIES II. STAR 2. Groombridge 34; $0^{h}\ 12^{m}.7$, $+43^{\circ}\ 27'$; Proper-motion +0″.257, +0″.40, =2″.80.
3. 26 Andromedae; $0^{h}\ 13^{m}.3$, $+43\ \ 15$; +0.002, −0.01, =0.03.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Pl. 188 1903, Dec. 4 Exp. 4, 3ᵐ Hour-angle −1ᵐ Obs. R		Pl. 198 1903, Dec. 14 Exp. 4, 4ᵐ Hour-angle +3ᵐ Obs. R		Pl. 351 1904, Aug. 11 Exp. 4, 4ᵐ Hour-angle +14ᵐ Obs. R		Pl. 356 1904, Aug. 24 Exp. 4, 4ᵐ Hour-angle +21ᵐ Obs. R		Pl. 391 1904, Dec. 22 Exp. 4, 4ᵐ Hour-angle +27ᵐ Obs. H		Pl. 394 1904, Dec. 30 Exp. 4, 4ᵐ Hour-angle +27ᵐ Obs. H		Approximate solution.		
	No.	Mag.	Mag.	Sp.	η Plate 188.	ξ Mean of 188 and 198.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	μ'	π
2	+42° 43	9.1	9.56	K	9.738	14.57858	61728	**7**	53989	8	23556	**17**	70828	**59**	67518	18	69418	**34**	**0″.015**	**0″.073**	**0″.039**
3	42 44	9.1	9.58	A	10.561	15.11807	15513	**21**	08100	20	77890	17	24080	73	21416	48	23250	71	101	052	006
1	43 37	9.5	10.76	A	22.065	13.93592	95178	27	92006	**27**	63601	**31**	93568	12	02505	**12**	01925	**1**	**010**	014	**007**
4	43 40	9.1	9.20	G8	29.496	15.85101	85288	4	84915	**3**	58010	35	77311	**23**	93405	**57**	91418	**36**	**075**	008	043
5	43 45	7.0	7.33	G2	31.844	19.50112	49950	59	50274	**59**	24181	**5**	40041	**34**	58008	**113**	55768	**51**	**137**	**021**	041
8	43 55	9.2	9.72	K	28.427	28.14370	14786	**08**	13955	89	88090	1	08488	41	22274	179	21012	89	220	353	**082**
6	43 52	9.4	9.61	G5	17.893	25.29510	31966	5	27053	**6**	99274	76	34595	44	38072	31	38918	46	065	113	041
7	42 53	8.9	8.47	F	13.475	25.61601	64908	24	58295	**23**	29718	**33**	71272	**50**	70329	**55**	72042	**89**	**119**	**100**	004
9	42 57	8.0	7.87	A2	13.854	29.38110	41360	1	34859	**2**	06728	**41**	47652	4	46610	**45**	48520	6	**033**	007	**006**
A	43 44	8.1	7.73	Ma	20.261	19.81751	83703	**2**	79799	2	53016	1278	85301	1328	92084	1634	92205	1683	2.794	2.830	250
B	42 48	5.9	6.04	A	15.760	22.58960	61790	5	56129	**6**	27571	**48**	66025	**74**	67789	13	68772	**133**	**106**	**085**	**025**
P. M of A.					+.002	+.016															
							B		B												
Average residual							23*		31*		57		45		42		46				
a							+ 9.68		− 9.68		+ 99.69		+ 58.10		− 57.11		− 6.74				
b							− 187.55		+ 187.55		+ 355.73		− 1056.75		− 61.15		− 264.80				
c							+5562		−5562		−39198		+22481		+11070		+14271				

Star A.				Star B.		Equations of condition.	Star A.			Star B.			
Plate.	Observed.	P. M. to 1904.5	Corrected.	Obs.	P. M.		n	$o-c_1$	$o-c_2$	n	$o-c_1$	$o-c_2$	$o-c_3$
188	−0″.004	+1″.616	+1″.612	+0″.009	0″.000	$x-0.577y-0.837\pi=$	−0″.188	+0″.029	+0″.036	+0″.009	+0″.008	+0″.065	+0″.080
198	+0.004	+1.537	+1.541	−0.010	0.000	$x-0.549y-0.883\pi=$	−0.259	−0.031	−0.023	−0.010	−0.009	+0.045	+0.061
351	+2.247	−0.311	+1.936	−0.084	0.000	$x+0.111y+0.654\pi=$	+0.136	−0.009	−0.011	−0.084	+0.022	+0.022	−0.013
356	+2.335	−0.417	+1.918	−0.130	0.000	$x+0.149y+0.494\pi=$	+0.118	+0.011	+0.011	−0.130	−0.024	−0.028	−0 059
391	+2.872	−1.330	+1.542	+0.023	0.000	$x+0.475y-0.901\pi=$	−0.258	−0.012	−0.018	+0.023	+0.128	+0.077	+0.095
394	+2.959	−1.392	+1.567	−0.234	0.000	$x+0.497y-0.898\pi=$	−0.233	+0.013	+0.006	−0.234	−0.126	−0.179	−0.163
Assumed P. M. and Δx.		+2.800	+1.800	0.000	0.000	...............(pvv)	....	2317	2427	....	33465	45488	49365

Normal equations.

	A	B
$+6.000x+0.106y-2.371\pi=$	−0″.684	−0″.426
$+0.106\ \ +1.141\ \ +0.241\ \ =$	+0 044	−0.131
$-2.371\ \ +0.241\ \ +3.771\ \ =$	+0.975	+0.071

Solution I.

	A	B	Weight.
x =	−0″.015±0″.009	−0″.079±0″.034	4.45
y =	−0.013±0.018	−0.102±0.068	1.11
π =	+0.250±0.011	−0.024±0.043	2.77
r_0 =	±0.019	±0.071	

Solution II, omitting terms in y.

A	B	Weight.
−0″.016±0″.008	−0″.085±0″.034	4.50
....	....	....
+0.249±0.010	−0.034±0.043	2.83
±0.017	±0.072	

Solution III, omitting y and π.

B
−0″.071±0″.027
....
....
±0.067

Measurer's notes—188, 351, Images diffuse; 391, Very good. The proper motions obtained by rejecting star 8 are given under the heading μ'.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES III. Star 4; η Cassiopeiae; $0^h 43^m.0$, +57°17′; Proper-motion $+0^s.139$, $-0''.52$, $= 1''.24$. Observed with Color-screen.

Stars.	Bonn Durchmusterung.		Harvard.		Pl. 200 (Stand.) 1904, Jan. 9 Exp. 4, 4^m Hour-angle $+12^m$ Obs. R		Plate 353 1904, Aug. 17 Exp. 3, 4^m Hr.-ang. $+8^m$ Obs. R		Plate 357 1904, Aug. 24 Exp. 4, 4^m Hr.-ang. $+20^m$ Obs. R		Plate 361 1904, Aug 26 Exp. 4, $4\frac{1}{2}^m$ Hr.-ang. $+10^m$ Obs. R		Plate 392 1904, Dec. 22 Exp. 4, 4^m Hr.-ang. $+25^m$ Obs. H		Plate 395 1904, Dec. 30 Exp. 4, 4^m Hr.-ang. $+24^m$ Obs. H		Plate 401 1905, Jan. 5 Exp. 2, 4^m Hr.-ang. $+13^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
3	+56° 128	8.5	9.18	A	14.490	18.19415	15560	48	25025	**7**	17608	21	29219	10	27828	4	31900	82	0″.044	0″.007
5	56 130	8.7	8.58	G5	10.034	21.79373	78605	**50**	87869	27	79735	61	91325	6	90260	9	94400	**49**	**007**	018
1	57 138	8.5	8.30	Ap	24.630	12.13765	02805	19	13025	32	07139	**15**	18540	15	16603	51	20122	39	062	**012**
2	57 145	8.0	8.31	K	24.132	17.80649	70433	**18**	80329	**48**	74365	**68**	85272	**34**	83630	**67**	87168	**78**	**102**	**010**
4	57 152	9.1	9.33	F	27.946	21.26331	13942	5	23871	20	18504	17	28745	23	27285	37	30675	57	064	**009**
7	57 160	9.5	10.06	G8	23.742	26.55010	45767	**6**	55265	**9**	49295	62	59300	**8**	58224	**25**	61785	**24**	**033**	031
6	57 155	9.4	10.30	F	19.740	22.08511	01517	5	11080	14	04345	**70**	15195	**47**	13985	**4**	17772	34	024	**010**
8	56 142	8.5	8.73	K	16.223	31.42898	38937	**2**	47895	**26**	40666	**10**	50934	31	50295	**9**	54138	**71**	011	**010**
A	57 150	3.8	3.64	F8	20.338	21.02553	95659	577	05335	628	98712	557	09720	687	08358	656	12062	641	1.206	186
P. M. of A.					−.003	+.006														
						B.														
Average residual						21*	44		45		43		30		40		19			
a						0.00	+ 80.89		+ 34.47		+ 29.78		− 65.03		− 2.89		− 3.69			
b						0.00	− 649.46		− 610.39		− 453.89		− 535.38		− 556.23		− 602.43			
c						0.	+4035.		+13842.		+4206.		+18735.		+16522.		+21200.			

Plate.	Observed.	P. M. to 1904.5	Corrected.	Equations of condition.	$o-c_1$	$o-c_2$	Weight.
200	0″.000	+0″.524	+0″.524	$x-0.478y-0.894\pi=-0''.276$	−0″.002	−0″.033	1
353	+1.014	−0.140	+0.874	$x+0.128y+0.682\pi=+0.074$	−0.013	−0.013	1
357	+1.104	−0.161	+0.943	$x+0.147y+0.601\pi=+0.143$	+0.070	+0.070	1
361	+0.979	−0.167	+0.812	$x+0.152y+0.575\pi=+0.012$	−0.057	−0.057	1
392	+1.208	−0.521	+0.687	$x+0.475y-0.888\pi=-0.113$	+0.057	+0.068	1
395	+1.153	−0.545	+0.608	$x+0.497y-0.903\pi=-0.192$	−0.021	−0.009	1
401	+1.127	−0.563	+0.564	$x+0.513y-0.903\pi=-0.236$	−0.066	−0.053	½
Assumed P. M. and Δ*x*	+1.097		+0.800	........................(pvv)	14190	15517	

Normal equations.

$$+6.500x+1.178y-1.278\pi=-0''.470$$
$$+1.178\quad+0.894\quad-0.413\quad=-0.046$$
$$-1.278\quad-0.413\quad+3.968\quad=+0.766$$

Solution I.

		Weight.
x	$=-0''.055\pm0''.018$	4.81
y	$=+0.106\pm0.049$	0.67
π	$=+0.187\pm0.021$	3.66
r_0	$=\quad\pm0.040$	

Solution II.

		Weight.
x	$=-0''.048\pm0''.015$	6.09
y	$=+0.063$.......	(Orbital motion)
π	$=+0.185\pm0.020$	3.70
r_0	±0.038	

Observer's notes—353, Clouds during 3d exposure; 361, Drifting clouds; 401, Flying clouds, very unsatisfactory.
Measurer's notes—353, Exp. 3 faint; 361, Plate splashed with fine drops.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES IV. Star 5; o Ceti; $2^h14^m3, -3°26'$; Proper-motion 0".000, −0".24, = 0".24.

Stars.	Southern Durchmusterung.		Harvard.		Standard.		Plate 203 1904, Jan. 19 Exp. 3, $6\frac{3}{4}^m$ Hr.-ang. $+15^m$ Obs. R		Plate 205 1904, Jan. 21 Exp. 3, 5^m Hr.-ang. $+11^m$ Obs. R		Plate 362 1904, Aug. 26 Exp. 3, 6^m Hr.-ang. -5^m Obs. R		Plate 365 1904, Aug. 28 Exp. 4, 6^m Hr.-ang. $+6^m$ Obs. R		Plate 402 1905, Jan. 9 Exp. 4, 5^m Hr.-ang. -13^m Obs. H		Plate 403 1905, Jan. 9 Exp. 4, 5^m Hr.-ang. $+12^m$ Obs. H		Plate 409 1905, Jan. 10 Exp. 4, 5^m Hr.-ang. -4^m Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 203.	ξ Mean of 203 and 205.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	−3° 346	9.3	10.04	G5	32.544	8.10222	08938	**24**	11505	23	13005	**29**	19992	**27**	27170	**20**	29798	**73**	25700	**49**	**0".084**	0".002
2	−3° 347	9.0	9.59	G	23.920	12.03749	02150	2	05348	**2**	07782	29	14459	11	21515	51	24158	77	19845	106	121	**021**
4	−2° 394	9.1	9.80	K	35.898	17.53374	52528	**6**	54220	6	56225	**18**	63585	**2**	70045	**39**	72770	1	68325	**16**	**035**	002
6	−2° 396	8.4	8.85	K	35.426	21.58289	57563	32	59015	**32**	61418	13	68835	16	75065	14	77722	**3**	73112	**38**	002	014
8	−3° 363	8.9	8.98	G	20.431	31.06364	05130	47	07598	**47**	11713	31	18695	50	24428	**4**	26975	**16**	21980	45	**009**	042
9	−3° 364	9.0	9.81	G	14.735	32.32132	30522	**70**	33742	70	38142	**32**	44850	**64**	50722	33	53230	21	48068	13	047	**063**
3	−4° 375	8.9	9.68	G	9.636	16.03688	01490	32	05887	**31**	09472	**78**	15705	3	22732	98	25198	49	20680	151	153	**089**
5	−3° 355	9.3	9.24	K	20.266	20.11113	09552	15	12675	**14**	16055	126	22699	58	29168	**5**	31772	30	26860	**216**	**028**	102
7	−4° 379	8.5	8.04	F8	8.662	23.34452	32342	**22**	36562	22	40723	**53**	47000	**47**	53368	**133**	55915	**83**	51120	13	**172**	008
A	−3° 353	Var	Var	Md9	20.147	19.45633	44073	39	47192	**40**	50612	182	57171	48	63638	**54**	66255	**15**	61650	22	**056**	135
P. M. of A. . . .	. . . . .	. . . . .	. . . . .	. . . . .	−.001																	
Average residual.							B. 28*		B. 28*		58		44		46		39		64			
a.							+ 26.29		− 26.29		+ 47.60		+ 69.92		+ 2.91		+ 1.03		− 34.68			
b.							+ 51.47		− 51.47		− 116.64		− 72.57		− 85.17		− 78.75		− 69.36			
c.							−3149		+3148		+6224		+11592		+19720		+22204		+18066			

Plate.	Observed.	P. M. to 1904.5.	Corrected.	Equations of condition.	$o-c_1$	$o-c_2$	Weight.
203	+0".069	−0".014	+0".055	$x-0.451y-0.920\pi=+0''.055$	+0".068	+0".085	1
205	−0.070	−0.013	−0.083	$x-0.445y-0.923\pi=-0.083$	−0.069	−0.054	1
362	+0.320	+0.005	+0.325	$x+0.152y+0.846\pi=+0.325$	+0.119	+0.119	1
365	+0.084	+0.005	+0.089	$x+0.158y+0.831\pi=+0.089$	−0.119	−0.119	1
402	−0.095	+0.016	−0.089	$x+0.524y-0.887\pi=-0.089$	−0.050	−0.064	1
403	−0.026	+0.016	−0.010	$x+0.524y-0.887\pi=-0.010$	+0.028	+0.013	1
409	+0.039	+0.016	+0.055	$x+0.526y-0.891\pi=+0.055$	+0.094	+0.081	$\frac{1}{4}$
Assumed P.M. and Δ*x*.	−0".030	0".000	. . . . (pvv)	43200	44368		

Normal equations.

$+6.250x+0.594y-2.163\pi=+0''.300$
$+0.594\ \ +1.068\ \ +0.039\ \ =+0.031$
$-2.163\ \ +0.039\ \ +4.877\ \ =+0.451$

	Solution I.	Weight.	Solution II.	Weight.
$x=$	$+0''.098\pm0''.032$	4.94	$+0''.095\pm0''.028$	5.29
$y=$	$-0''.030\pm0''.070$	1.00	. . . .	. . . .
$\pi=$	$+0''.136\pm0''.035$	4.07	$+0.134\pm0.031$	4.13
$r_0=$	$\pm0''.070$		±0.064	

Observer's notes—362, Clouds at end; 402, 403, Found on closing that the house was hardly far enough south for this star. The central star had clear field at beginning and end of set, but probably not in middle; 409, Clouded up.

Measurer's notes—205, Poor focus; 362, Exp. 1 too faint to measure; 402, 403, Apparently all right; 409, Réseau lines crooked. Very poor.

Approximate magnitudes of A—1904, January 20, 6.9; 1904, August 27, 8.3; 1905, January 9, 6.1 (from light-curves by Nijland, A. N. 4012, 4013.)

All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES V. STAR 6; ρ Persei; $2^h 58^m.8$, $+38° 27'$; Proper-motion +0″.012, −0″.11, =0″.17. Observed with Color-screen.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 206 1904, Jan. 21 Exp. 4, 5^m Hr.-ang. $+12^m$ Obs. R		Plate 209 1904, Jan. 22 Exp. 3, 5^m Hr.-ang. $+26^m$ Obs. R		Plate 368 1904, Sept. 26 Exp. 4, 5^m Hr.-ang. -1^m Obs. H		Plate 374 1904, Oct. 18 Exp. 3, 5^m Hr.-ang. -17^m Obs. H		Plate 414 1905, Jan. 18 Exp. 4, 5^m Hr.-ang. -11^m Obs. H		Plate 416 1905, Jan. 25 Exp. 2, 5^m Hr.-ang. 0^m Obs. H		Plate 422 1905, Jan. 27 Exp. 3, 5^m Hr.-ang. $+3^m$ Obs. H		Approximate solution.		
	No.	Mag.	Mag.	Sp.	η Plate 206.	ξ Mean of 206 and 209.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	μ'	π
1	+38° 619	9.4	10.29	K?	17.566	10.73243	77934	54	68552	**54**	71619	**134**	76242	**21**	81085	**150**	82935	**88**	82517	**87**	**0″.238**	**0″.383**	0″.001
3	37 695	9.4	10.54	A	13.911	17.12916	17522	**29**	08310	29	11658	41	16712	46	17648	94	25732	76	22447	73	164	023	**016**
2	38 623	9.2	10.19	F	23.909	14.78012	82618	**27**	73405	26	76694	**1**	80658	**27**	91286	46	82642	52	87125	15	085	034	**049**
4	38 625	9.4	10.42	G5	23.320	17.37339	41971	1	32707	**1**	36194	90	40227	3	49970	5	42350	**39**	46443	**1**	**014**	**054**	063
9	......	....	......	....	22.034	21.47278	51918	11	42638	**11**	46110	**63**	50533	2	58652	**3**	53270	**90**	56385	**20**	**057**	**084**	**013**
5	38 631	9.4	9.88	A	27.069	22.67743	72401	31	63085	**31**	66652	**56**	70573	**26**	83312	**42**	69658	**47**	76607	**67**	**104**	**070**	**010**
7	38 639	8.6	9.01	A	27.982	30.46043	50650	**15**	41437	16	45304	31	49332	48	62125	**6**	47350	124	54968	76	083	177	008
6	38 634	8.5	8.63	A	15.824	26.88784	93391	**21**	84177	21	87904	82	92953	8	94698	26	99950	29	98215	104	092	034	017
8	38 641	9.1	9.78	F?	15.881	30.63088	67710	**4**	58465	3	62258	7	67437	**33**	68905	25	74148	**14**	72298	**90**	**026**	**059**	**001**
A	38 630	Var.	Var.	Mb	20.671	21.12860	17564	74	08157	**73**	11886	151	16447	227	23260	171	20184	124	22182	142	221	177	091
P.M. of A	......	....	......	....	−.001	+.001																	
Average residual							B 45*		B 30*		38		47		33		34		43				
a							− 0.55		+ 0.55		+ 33.35		+ 60.50		− 38.39		− 4.18		− 6.16				
b							− 0.31		+ 0.31		+ 6.02		− 89.43		+ 850.30		− 817.49		− 37.37				
c							+4648.		−4648.		−1956.		+4011.		−6535.		+24184.		+10083.				

Plate.	Observed.	P. M. to 1904.5.	Corrected.	Equations of condition.	$o-c_1$	$o-c_2$	Weight.
206	+0″.130	+0″.066	+0″.196	$x-0.451y-0.893\pi=-0''.004$	+0″.085	+0″.037	1
209	−0.128	+0.065	−0.063	$x-0.442y-0.908\pi=-0.263$	−0.174	−0.220	½
368	+0.266	−0.035	+0.231	$x+0.237y+0.662\pi=+0.031$	−0.065	−0.066	1
374	+0.399	−0.043	+0.356	$x+0.298y+0.355\pi=+0.156$	+0.079	+0.085	1
414	+0.301	−0.080	+0.221	$x+0.549y-0.892\pi=+0.021$	+0.028	+0.063	1
416	+0.218	−0.083	+0.135	$x+0.568y-0.924\pi=-0.065$	−0.057	−0.021	½
422	+0.250	−0.084	+0,166	$x+0.573y-0.929\pi=-0.034$	−0.026	+0.010	½
Assumed P. M. and Δx.		+0.146	+0.200	......................(*pvv*)	35575	41389	

Normal equations.

$+5.500x+0.982y-2.148\pi=+0''.023$
$+0.982\ \ +1.071\ \ -0.152\ \ =+0.097$
$-2.148\ \ -0.152\ \ +3.427\ \ =+0.225$

	Solution I.	Weight.	Solution II.	Weight.
x =	+0″.022±0″.034	3.47	+0″.039±0″.030	4.16
y =	+0.082±0.068	0.87	....	....
π =	+0.083±0.040	2.53	+0.090±0.038	2.59
r_0 =	±0.064		±0.061	

Observer's notes—209, Very clear and steady; 374, Ground fog forming; 414, Seeing unsteady.

Measurer's notes—209, Very good, exposure 4 of A outside color-screen; 368, Images diffuse; 374, Under-developed, réseau faint; 416, Exposures 1 and 2 of A outside C S.

Plate 209 was given half weight on account of the large unexplained discordance for Star A. If it was entirely rejected, the resulting value of π would be +0″.066±0″.029, and r_0, ±0″.045; but there appears to be no sufficient reason for this.

The proper motions obtained by rejecting star 1 are given under the heading μ'.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES VI. STAR 7; β Persei; $3^h\,1^m.7$, +40° 34′; Proper-motion +0".001, −0".01, =0".01. Observed with Color-screen.

Stars.	Bonn Durchmusterung		Harvard.		Standard.		Plate 214 1904, Feb. 1 Exp. 4, 4^m Hr.-ang. $+5^m$ Obs. R		Plate 215 1904, Feb. 4 Exp. 2, 5^m Hr.-ang. $+8^m$ Obs. R		Plate 369 1904, Sept. 26 Exp. 3, 5^m Hr.-ang. $+27^m$ Obs. H		Plate 375 1904, Oct. 18 Exp. 4, $5\frac{1}{4}^m$ Hr.-ang. $+10^m$ Obs. H		Plate 415 1905, Jan. 18 Exp. 3, 4^m Hr.-ang. $+12^m$ Obs. H		Plate 417 1905, Jan. 25 Exp. 4, 4^m Hr.-ang. $+20^m$ Obs. H		Plate 423 1905, Jan. 27 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. $+22^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 214.	ξ Mean of 214 and 215.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	+39° 703	9.3	10.12	G	9.968	10.85755	83644	46	87865	**47**	85202	39	89405	**28**	94403	28	87904	**5**	89104	12	**0".005**	**0".024**
2	40 667	9.5	10.54	K	18.454	11.33185	31543	**40**	34828	41	32859	1	36942	43	42553	**2**	40634	**13**	41358	**24**	000	053
3	40 668	9.1	9.25	K	30.035	15.74528	73601	**28**	75455	28	74578	**130**	78502	**8**	84827	**55**	89245	**35**	89370	1	**038**	**032**
4	40 671	8.8	8.95	A	22.498	19.58722	57289	21	60155	**21**	58900	96	62912	**9**	68393	30	68864	53	69276	14	045	001
6	40 675	8.6	8.04	G5	27.324	24.48787	47583	2	49990	**3**	49196	12	53250	**26**	58775	**30**	61939	**12**	62092	**14**	**032**	001
8	40 684	7.8	8.26	G	25.170	33.44745	43270	4	46220	**4**	45395	17	49796	48	54528	56	56666	**5**	56864	1	029	033
5	40 674	9.1	10.38	F5	14.159	24.04104	02024	**8**	06183	7	04118	32	08586	28	12923	**32**	09030	**5**	09818	**68**	**058**	070
7	40 681	8.9	10.07	A	17.354	28.90520	88590	1	92450	**1**	90696	**67**	95210	**45**	99608	4	97528	22	98230	82	063	**098**
A	40 673	Var	2.1 to 3.2	B8	20.328	21.01059	99463	23	02655	**23**	01092	**32**	05358	19	10507	15	09759	**50**	10358	**3**	**037**	011
Average residual							B 19*		B 14*		55		41		29		28		35			
a							− 14.570		+ 14.570		+ 33.87		+ 58.31		− 10.82		+ 12.04		+ 3.43			
b							+ 66.269		− 66.269		+ 30.20		+ 0.92		+ 89.04		+ 624.97		+ 572.48			
c							−2660.		+2660.		−1261.		+3036.		+7850.		−4207.		−2406.			

Plate.	Observed.	P. M. to 1904.683	Equations of condition.	$o-c_1$	$o-c_2$	$o-c_3$	Weight.
214	+0".040	0".000	$x-0.598y-0.940\pi=+0''.040$	+0".027	+0".050	+0".047	1
215	−0.040	0.000	$x-0.590y-0.946\pi=-0.040$	−0.053	−0.030	−0.033	½
369	−0.056	0.000	$x+0.054y+0.672\pi=-0.056$	−0.061	−0.056	−0.049	½
375	+0.033	0.000	$x+0.115y+0.366\pi=+0.033$	+0.027	+0.035	+0.040	1
415	+0.026	0.000	$x+0.366y-0.889\pi=+0.026$	+0.046	+0.034	+0.033	1
417	−0.088	0.000	$x+0.385y-0.922\pi=-0.088$	−0.067	−0.078	−0.081	1
423	−0.005	0.000	$x+0.390y-0.928\pi=-0.005$	+0.016	+0.004	+0.002	1
Assumed P. M. and Δx	0.000		........................(pvv)	11585	12999	13208	

Normal equations.

$$+6.000x+0.390y-3.450\pi=-0''.042$$
$$+0.390\ \ +0.980\ \ -0.141\ \ =-0.036$$
$$-3.450\ \ -0.141\ \ +4.191\ \ =+0.037$$

	Solution I.	Weight.	Solution II.	Weight.	Solution III.
x =	−0".001±0".021	3.08	−0".004±0".019	3.16	−0".007±0".013
y =	−0.034±0.037	0.96	....	....	....
π =	+0.007±0.025	2.20	+0.006±0.023	2.20	....
r_0 =	±0.036		±0.034		±0.032

Observer's notes—214, 215, Thin clouds; 369, Clouded up; 375, Fog.
Measurer's notes—215, Exp. 2 faint, 3 and 4 unmeasurable; 369, Poor, exp. 3 unmeasurable; 423, Plate cracked.
All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES VII. STAR 8; Lalande 6888 } 9; Lalande 6889 }; $3^h 40^m.2$, +41°9′; Proper-motion +0″.053, −1″.23, =1″.38.

Stars.	Bonn Durch-musterung.		Harvard.		Pl. 218 (Stand.) 1904, Feb. 5 Exp. 4, 3^m Hour-angle $+10^m$ Obs. R		Plate 370 1904, Sept. 26 Exp. 4, 5^m Hr.-ang. $+33^m$ Obs. H		Plate 376 1904, Oct. 18 Exp. 4, 5^m Hr.-ang. $+5^m$ Obs. H		Plate 450 1905, Oct. 31 Exp. 4, 5^m Hr.-ang. -20^m Obs. H		Plate 467 1906, Feb. 9 Exp. 4, 4^m Hr.-ang. -11^m Obs. H		Plate 468 1906, Feb. 9 Exp. 4, 4^m Hr.-ang. $+19^m$ Obs. H		Approxi-mate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
2	+40° 828	7.8	8.24	F8	9.2345	11.61120	61735	**12**	82525	54	30204	**43**	34807	4	77577	22	**0″.032**	**0″.015**
3	41 744	8.6	8.68	G5	26.9880	16.24872	25079	**16**	29999	**54**	94655	**3**	32734	**32**	32740	**94**	**010**	011
4	41 748	8.5	8.78	G5	25.0675	19.29220	29524	30	36218	2	98971	44	37760	22	38170	71	037	008
8	41 760	9.1	9.58	F5	21.6465	31.09928	10295	**13**	20326	52	79439	**44**	19558	4	20460	21	**032**	**015**
5	40 833	8.9	9.39	A2	11.5745	23.70449	71066	12	89920	27	39621	**19**	83416	**46**	85824	57	**028**	011
7	40 839	9.4	10.01	A?	16.4880	26.54055	54549	0	68970	**79**	23485	60	65308	43	66950	**78**	058	003
A }	41 750	8.2	8.35 }	G	19.9985	19.77382	78001	223	89174	135	47470	573	88226	670	89344	658	595	**029**
B }			8.89 }		20.0310	19.80930	81618	292	92761	204	51081	634	91839	745	92882	664	626	015
P. M. of A and B.					−.007	+.003												
Average residual..........					B 33*		40		39		63		62		103			
a..........							+ 1.69		+ 18.80		− 2.04		− 0.57		− 0.01			
b..........							− 23.81		− 916.42		+ 37.60		− 325.97		− 477.34			
c..........							+836.		+29607.		−31197.		+16700.		+20843.			

Plate.	Star A. Observed.	Star B. Observed.	P. M. to 1905.0	Star A. Corrected.	Star B. Corrected.	Equations of condition.	Star A.				Star B.				Wt.
							n	$o-c_1$	$o-c_2$	$o-c_3$	n	$o-c_1$	$o-c_2$	$o-c_3$	
218	0″.000	0″.000	+0″.545	+0″.545	+0″.545	$x-0.904y-0.940\pi=$	+0″.045	+0″.020	+0″.048	+0″.052	+0″.045	+0″.010	−0″.011	−0″.037	1
370	+0.392	+0.513	+0.158	+0.550	+0.671	$x-0.262y+0.787\pi=$	+0.050	+0.079	+0.055	+0.057	+0.171	+0.083	+0.100	+0.089	1
376	+0.237	+0.359	+0.122	+0.359	+0.481	$x-0.202y+0.515\pi=$	−0.141	−0.119	−0.136	−0.134	−0.019	−0.103	−0.091	−0.101	1
450	+1.007	+1.115	−0.502	+0.505	+0.613	$x+0.833y+0.325\pi=$	+0.005	+0.030	+0.014	+0.012	+0.113	+0.006	+0.016	+0.031	1
467	+1.178	+1.310	−0.668	+0.510	+0.642	$x+1.109y-0.953\pi=$	+0.010	+0.001	+0.019	+0.017	+0.142	+0.052	+0.039	+0.060	1
468	+1.157	+1.167	−0.669	+0.488	+0.498	$x+1.109y-0.953\pi=$	−0.012	−0.021	−0.003	−0.005	−0.002	−0.092	−0.105	−0.084	½
Assumed P. M. and Δx..........			+0.603	+0.500	+0.500	............(pvv)		21924	24386	24354		24570	25682	27580	

Normal equations.			Solution I.				Solution II.			Solution III.		
	A.	B.		A.	B.	Weight.	A.	B	Weight.	A	B	Weight.
$+5.500x+1.128y-0.742\pi=$	−0″.037	+0″.451	$x=$	−0″.009±0″.026	+0″.079±0″.027	5.06	−0″.006±0″.023	+0″.077±0″.024	5.13	−0″.007±0″.020	+0″.082±0″.021	5.50
$+1.128\ +3.466\ -0.774\ =$	−0.018	+0.168	$y=$	−0.008±0.033	+0.027±0.034	3.11	−0.003±0.029	+0.024±0.030	3.24	....	....	
$-0.742\ -0.774\ +3.236\ =$	−0.079	−0.014	$\pi=$	−0.029±0.033	+0.020±0.035	3.02	....	....		....	....	
			r_0	±0.058	±0.061		±0.053	±0.054		±0.047	±0.050	

Observer's notes—370, Start delayed by clouds; 376, Fog; 468, Clock running feebly. Helping it continually.
Measurer's notes—370, Good; 376, Underexposed; 468, Fogged, very bad.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES VIII. Star 10; Lalande 7443; $3^h56^m.5$, $+35°2'$; Proper-motion $+0''.142, -1''.35, = 2''.20$.

Stars.	Bonn Durchmusterung.		Harvard.		Pl. 219 (Stand.) 1904, Feb. 5 Exp. 4, 4^m Hour-angle $+19^m$ Obs. R		Plate 377 1904, Oct. 18 Exp. 3, 5^m Hr.-ang. $+20^m$ Obs. H		Plate 451 1905, Oct. 31 Exp. 4, 4^m Hr.-ang. -9^m Obs. H		Plate 452 1905, Oct. 31 Exp. 4, 4^m Hr.-ang. $+13^m$ Obs. H		Plate 469 1906, Feb. 14 Exp. 4, 4^m Hr.-ang. -8^m Obs. H		Plate 470 1906, Feb. 14 Exp. 4, $4\frac{1}{2}^m$ Hr.-ang. $+18^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
2	+34° 785	9.3	9.41	K	11.176	10.25038	36400	6	11826	**19**	12882	59	26404	**35**	33605	**24**	**0".025**	0".043
4	34 795	8.5	8.42	G5	16.281	18.79032	91590	**64**	65132	**13**	66138	**53**	83339	**26**	87506	0	**012**	**043**
1	35 784	9.4	10.22	F	30.838	9.73818	89273	94	58068	121	59086	**7**	87385	28	82684	**24**	003	085
3	34 791	9.0	9.27	K	20.544	15.28415	41782	**35**	13858	**89**	15016	**4**	35500	34	37078	47	033	**085**
6	35 793	9.1	9.52	G5	29.683	24.85236	00745	**9**	69510	**12**	70721	16	97719	40	93830	59	042	**045**
7	35 794	9.4	10.24	A	21.295	25.61722	75495	**49**	47131	**21**	48256	**7**	68872	**102**	70035	**83**	**079**	046
5	34 800	9.1	9.79	A	15.968	22.26106	38707	**47**	12244	**17**	13181	**135**	30132	**26**	34426	**73**	**044**	**040**
8	34 805	9.5	10.48	A	11.014	29.16442	28360	105	03321	49	04430	125	17350	87	24705	97	081	038
A	34 796	8.5	8.34	G2	19.202	19.81649	95570	678	69085	1720	70172	1732	89761	2014	92159	2016	1.747	**010**
P. M. of A......	......	......	......	......	−.008	+.010												
Average residual..................					B 48*		35		80		83		32		49			
a..............................							+ 25.90		− 0.01		+ 3.14		− 25.39		− 22.33			
b..............................							+ 204.38		− 136.25		− 127.50		+ 616.71		+ 14.63			
c..............................							+8807.		−11669.		−10824.		−5231.		+8656.			

Plate.	Observed.	P. M. to 1905.5	Corrected.	Equations of condition.	$o-c_1$	$o-c_2$
219	0".000	+2".414	+2".414	$x-1.404y-0.929\pi=+0''.014$	+0".008	+0".015
377	+1.192	+1.207	+2.399	$x-0.702y+0.574\pi=-0.001$	+0.016	−0.024
451	+3.024	−0.574	+2.450	$x+0.334y+0.384\pi=+0.050$	−0.002	−0.009
452	+3.045	−0.574	+2.471	$x+0.334y+0.384\pi=+0.071$	+0.019	+0.012
469	+3.541	−1.071	+2.470	$x+0.623y-0.962\pi=+0.070$	−0.006	+0.002
470	+3.544	−1.071	+2.473	$x+0.623y-0.962\pi=+0.073$	−0.004	+0.004
Assumed P. M. and Δx	+1.719		+2.400	........................(pvv)	737	1046

Normal equations.

$$+6.000x-0.192y-1.511\pi=+0''.277$$
$$-0.192\ \ +3.463\ \ -0.041\ \ =+0.111$$
$$-1.511\ \ -0.041\ \ +3.338\ \ =-0.105$$

	Solution I.	Weight.	Solution II.	Weight.
$x =$	$+0''.044\pm0''.005$	5.30	$+0''.047\pm0''.004$	5.99
$y =$	$+0.034\pm0.006$	3.45	$+0.035\pm0.006$	3.46
$\pi =$	-0.011 ± 0.006	2.96	....	
$r_0 =$	±0.011		±0.011	

Observer's notes—377, Sky thick; 470, Clock running badly.
Measurer's notes—219, Rather diffuse; 451, 452, Underexposed; 469, 470, Good.
All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES IX. Star 11; Groombridge 884; $4^h44^m.4$, $+45°41'$; Proper-motion $+0''.037$, $-0''.56$, $=0''.68$.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 226 1904, Feb. 15 Exp. 4, $3\frac{3}{4}^m$ Hr.-ang. $+12^m$ Obs. R		Plate 228 1904, Feb. 18 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. $+9^m$ Obs. R		Plate 378 1904, Oct. 18 Exp. 2, 5^m Hr.-ang. $+9^m$ Obs. H		Plate 379 1904, Oct. 24 Exp. 4, $5\frac{1}{4}^m$ Hr.-ang. $+17^m$ Obs. H		Plate 453 1905, Oct. 31 Exp. 3, 4^m Hr.-ang. -9^m Obs. H		Plate 472 1906, Mar. 7 Exp. 4, 4^m Hr.-ang. $+79^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 226	ξ Mean of 226 and 228	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	+45° 989	9.5	10.38	K	19.672	8.59093	63608	**11**	54578	11	68022	28	67569	19	....	......	98951	43	0".037	0".001
4	45 991	9.4	9.69	F	17.002	17.99319	03598	6	95040	**6**	07549	14	06805	**31**	11722	**90**	39305	5	004	**029**
2	46 920	9.4	9.63	K	30.726	11.25233	30370	8	20095	**9**	38848	**36**	39315	18	40955	128	66938	**19**	**016**	029
3	46 925	8.4	8.50	A	28.461	17.34953	39881	**8**	30025	8	47932	**5**	48136	**7**	50098	**38**	76672	**29**	**025**	000
6	45 996	9.5	10.57	F	21.736	26.18268	22705	**17**	13832	18	28870	25	28466	**11**	31953	**90**	59498	8	**007**	**010**
7	45 998	8.8	9.30	F	20.377	29.16006	20363	13	11648	**14**	26170	17	25664	2	29502	**19**	57272	57	049	**022**
5	45 994	9.5	10.44	G5	14.588	22.45210	49291	**6**	41129	6	52568	**54**	51682	**28**	....	......	85071	**16**	**015**	**028**
8	45 999	7.9	8.03	G	16.809	30.20318	24456	8	16179	**9**	29030	13	28282	40	33123	111	61001	**29**	**025**	061
A	45 992	6.5	6.83	G	20.028	19.82006	86419	**12**	77592	11	91791	218	91331	229	95770	522	23051	474	404	084
P. M. of A........	....	....	....	....	−.003	+.002														
Average residual........							34*		45*		33		39		50		65			
a........							− 10.797		+ 10.797		+ 46.18		+ 41.29		+ 20.24		+ 62.40			
b........							+ 57.084		− 57.084		+418.73		+ 497.42		+ 235.87		+ 157.67			
c........							+3495.6		−3495.6		+266.		−1684.		+8118.		+36177.			

Plate.	Observed.	P. M. to 1905.0	Corrected.	Equations of condition.	$o-c$	Weight.
226	$-0''.021$	$+0''.338$	$+0''.317$	$x-0.877y-0.937\pi=-0''.083$	$-0''.012$	1
228	$+0.019$	$+0.334$	$+0.353$	$x-0.869y-0.952\pi=-0.047$	$+0.024$	1
378	$+0.383$	$+0.078$	$+0.461$	$x-0.203y+0.726\pi=+0.061$	-0.021	1
379	$+0.403$	$+0.071$	$+0.474$	$x-0.185y+0.652\pi=+0.074$	-0.002	1
453	$+0.918$	-0.320	$+0.598$	$x+0.831y+0.570\pi=+0.198$	$+0.098$	$\frac{1}{4}$
472	$+0.833$	-0.455	$+0.378$	$x+1.181y-0.982\pi=-0.022$	-0.013	1
Assumed P. M. and Δx		$+0.385$	$+0.400$	........................(pvv)	3735	

Normal equations.

$$+5.250x-0.745y-1.351\pi=+0''.032 \qquad x=+0''.030\pm0''.011 \quad 4.64$$
$$-0.745\ \ +3.167\ \ +0.342\ \ =+0.103 \qquad y=+0.031\pm0.014 \quad 3.05$$
$$-1.351\ \ +0.342\ \ +3.781\ \ =+0.265 \qquad \pi=+0.078\pm0.013 \quad 3.43$$
$$r_0 \qquad \pm0.024$$

Weight.

Observer's notes—228, Clock ran badly; 378, Sky thick; 379, Seeing very poor;453, Getting thick.

Measurer's notes—228, Images of exp. 3 deformed; 378, Underexposed; 453, Underexposed; exp. 4 unmeasurable; Stars 1 and 5 unmeasurable in the other exposures; Weight $\frac{1}{4}$.

All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

Table C, Series X. Star 12; Groombridge 1646; $10^h21^m.9, +49°19'$; Proper-motion $+0^s.009, -0''.89, =0''.90$

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 182 1903, Nov. 24 Exp. 6, $2\frac{1}{2}^m$ Hr.-ang. $+6^m$ Obs. R		Plate 257 1904, Apr. 16 Exp. 4, 4^m Hr.-ang. $+7^m$ Obs. R		Plate 259 1904, Apr. 19 Exp. 4, 4^m Hr.-ang. $+14^m$ Obs. R		Plate 396 1904, Dec. 30 Exp. 3, 4^m Hr.-ang. $+12^m$ Obs. H		Plate 404 1905, Jan. 9 Exp. 4, 4^m Hr.-ang. -11^m Obs. H		Plate 424 1905, Apr. 15 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. -14^m Obs. H		Plate 425 1905, Apr. 15 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. $+6^m$ Obs. H		Approximate solution.		
	No.	Mag.	Mag.	Sp.	η Plate 257	ξ Mean of 257 and 259	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-x$	x	$o-c$	μ	μ'	π
3	+49° 1958	8.4	8.86	K	11.372	17.97353	10262	29	12136	16	82570	**16**	87832	**2**	88175	21	87220	1	88808	73	0″.015	**0″.021**	0″.005
4	49 1959	8.8	9.21	F	18.364	19.26008	32563	**22**	39259	14	12756	**15**	18292	35	18372	38	19682	68	16205	**18**	070	034	009
1	49 1956	9.5	10.26	G2	17.875	7.98005	05427	**26**	11202	**42**	84809	43	90643	**153**	90889	**79**	91162	**188**	88135	**161**	**224**	**371**	**029**
2	49 1957	8.8	9.30	G	27.982	9.49767	48136	18	60802	11	38732	**11**	45253	118	44972	16	48635	118	38478	107	135	**015**	014
5	49 1960	6.6	6.62	G	23.458	19.95854	97860	11	07998	24	83711	**23**	89402	**4**	89361	55	92260	76	85333	114	090	052	**013**
6	49 1962	9.5	9.97	F	27.854	21.01262	99282	**4**	12428	5	90095	**6**	95947	37	95662	5	99939	**7**	89834	**58**	**003**	**037**	027
7	....	.....	10.43	.....	18.305	21.67600	74050	**88**	80815	**60**	54385	60	59617	**73**	59665	**87**	61105	**69**	57720	**106**	**072**	**093**	**061**
8	49 1964	9.5	10.40	F8	19.918	31.57461	62256	79	70500	28	44422	**28**	49425	39	49351	24	51895	**2**	47472	49	**015**	069	046
A	49 1961	6.2	6.50	F8	20.274	19.97796	02692	54	10596	**23**	84996	23	90522	17	90579	66	92460	40	87661	**32**	**002**	**034**	052
P. M. of A.	......	......	......	.....	−.005																		
							B																
Average residual.........							40*		31*		32*		31		43		46		39				
a..........							− 38.31		+ 9.46		− 9.46		− 59.53		− 66.43		− 0.08		+ 0.45				
b..........							− 894.32		− 220.59		+ 220.59		+ 263.83		+ 230.30		+ 534.94		− 166.98				
c..........							+23739.		+17106.		−17106.		−11450.		−10625.		−16216.5		−6727.0				

Plate.	Observed.	P. M. to 1904.5	Equations of condition.	$o-c$
182	+0″.095	0″.000	$x-0.603y+0.920\pi=+0''.095$	+0″.007
257	−0.040	0.000	$x-0.210y-0.746\pi=-0.040$	−0.045
259	+0.040	0.000	$x-0.202y-0.783\pi=+0.040$	+0.037
396	+0.030	0.000	$x+0.498y+0.746\pi=+0.030$	−0.048
404	+0.116	0.000	$x+0.525y+0.641\pi=+0.116$	+0.044
424	+0.070	0.000	$x+0.787y-0.733\pi=+0.070$	+0.065
425	−0.056	0.000	$x+0.787y-0.733\pi=-0.056$	−0.060
Assumed P. M. and Δx		0.000	..........................(pvv)	15508

Normal equations.

$+7.000x+1.582y-0.688\pi=+0''.255$
$+1.582\ \ +2.211\ \ -0.686\ \ =+0.030$
$-0.688\ \ -0.686\ \ +4.058\ \ =+0.172$

	Weight.
$x=+0''.042\pm0''.018$	5.86
$y=-0.001\pm0.031$	1.78
$\pi=+0.049\pm0.021$	3.84
$r_o=\quad\pm0.042$	

Observer's notes—182, 3 exposures of 3m. and 3 of 2m.
Measurer's notes—182, Short exp. faint, 425. Fogged.
Images of A usually large and somewhat diffuse.
The proper-motions obtained by rejecting star 1 are given under the heading μ'. The proper-motion of Star A may be variable.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XI. STAR 13; Lalande 21185; $10^h\ 57^m.9$, $+36°\ 38'$; Proper-motion $-0''.047$, $-4''.75$, $=4''.78$.

Stars.	Bonn Durchmusterung. No.	Bonn Durchmusterung. Mag.	Harvard. Mag.	Harvard. Sp.	Standard. η Plate 191.	Standard. ξ Mean of 191 and 194.	Plate 191 1903, Dec. 9 Exp. 4, $3\frac{1}{4}^m$ Hr.-ang. $+12^m$ Obs. R x	Plate 191 o−c	Plate 194 1903, Dec. 13 Exp. 4, 5^m Hr.-ang. $+13^m$ Obs. R x	Plate 194 o−c	Plate 258 1904, Apr. 16 Exp. 4, $5\frac{1}{2}^m$ Hr.-ang. $+6^m$ Obs. R x	Plate 258 o−x	Plate 260 1904, Apr. 19 Exp. 4, 5^m Hr.-ang. $+12^m$ Obs. R x	Plate 260 o−c	Plate 268 1904, Apr. 25 Exp. 4, 5^m Hr.-ang. -1^m Obs. R x	Plate 268 o−c	Plate 397 1904, Dec. 30 Exp. 3, 5^m Hr.-ang. $+8^m$ Obs. H x	Plate 397 o−c	Plate 405 1905, Jan. 9 Exp. 4, 5^m Hr.-ang. -14^m Obs. H x	Plate 405 o−c	Plate 426 1905, Apr. 15 Exp. 4, 5^m Hr.-ang. -2^m Obs. H x	Plate 426 o−c	Approximate solution. μ	Approximate solution. π
2	+36°2141	8.6	9.13	G	19.633	10.94751	99402	**19**	90100	19	06051	**57**	90310	49	99550	27	96753	71	05411	32	04686	37	0".084	0".018
5	36 2144	9.1	9.88	G	11.872	17.24770	31680	**21**	17860	21	38366	**13**	22938	17	31110	**8**	29022	7	37341	**6**	33213	**17**	**009**	007
6	36 2146	8.5	8.73	A2	11.688	17.90609	97612	17	83606	**17**	04341	45	88785	**59**	97012	**20**	94872	**33**	03232	**4**	99029	**25**	**023**	007
1	37 2142	8.3	8.72	G	29.371	9.86863	88715	**5**	85010	4	95802	1	79325	**5**	90202	28	85855	0	94913	**17**	98870	64	021	**032**
3	37 2145	6.8	7.31	A	32.133	13.04166	05248	10	03084	**10**	12621	**2**	95914	**2**	07305	**14**	02270	**35**	11451	2	16795	**19**	**026**	000
4	37 2151	7.7	8.38	G	25.178	16.65267	68378	22	62156	**22**	75642	23	59295	**4**	69669	**8**	65478	**12**	74313	**21**	76518	**37**	**042**	**005**
9	37 2153	8.5	8.61	G	32.542	32.70085	71085	**24**	69085	24	79579	**22**	62509	8	74631	**5**	68040	47	76968	35	83515	**6**	048	035
7	36 2150	8.9	9.37	K	14.563	25.99022	05232	25	92811	**26**	12535	44	96736	37	05645	10	02415	12	10768	24	08384	14	009	**019**
8	36 2151	8.8	9.36	G8	9.874	26.09609	17148	1	02070	**1**	24196	**22**	08689	**44**	16892	**4**	14348	**60**	22510	**60**	17950	**7**	**058**	**018**
A	36 2147	7.3	7.42	K	20.286	19.84767	89288	8	80246	**8**	96096	**394**	80011	**425**	89642	**468**	86080	**380**	94740	**342**	94413	**701**	**535**	344
P. M. of A.					−.027	−.003																		
Average residual							B 37*		B 25*		26*		38*		54*		33*		20*		37*			
a							+ 3.54		− 3.54		+ 58.88		+ 40.47		+ 74.09		− 5.52		− 16.56		+ 35.62			
b							− 288.44		+ 288.44		− 241.84		− 307.75		− 142.30		− 302.39		− 264.76		+ 214.06			
c							+10294.		−10294.		+15460.		+1107.		+6759.		+7928.		+16006.		+5305.			

Plate.	Observed.	P. M. to 1904.0	Corrected.	Equations of condition.	$o-c_1$	$o-c_2$
191	+0".014	−0".032	−0".018	$x-0.061y+0.907\pi=-0''.018$	+0".020	+0".022
194	−0.014	−0.027	−0.041	$x-0.051y+0.900\pi=-0.041$	−0.001	+0.001
258	−0.692	+0.154	−0.538	$x+0.291y-0.634\pi=-0.538$	+0.034	+0.035
260	−0.747	+0.158	−0.589	$x+0.299y-0.668\pi=-0.589$	−0.005	−0.004
268	−0.823	+0.167	−0.656	$x+0.315y-0.736\pi=-0.656$	−0.049	−0.048
397	−0.668	+0.529	−0.139	$x+0.999y+0.817\pi=-0.139$	−0.066	−0.068
405	−0.601	+0.543	−0.058	$x+1.026y+0.733\pi=-0.058$	+0.044	+0.042
426	−1.232	+0.683	−0.549	$x+1.288y-0.621\pi=-0.549$	+0.023	+0.020
Assumed P. M. and Δx.		−0.530	0.000	........................(pvv)	10804	10818

Normal equations.

$$+8.000x+4.106y+0.698\pi=-2''.588$$
$$+4.106\ +3.989\ +0.051\ =-1.442$$
$$+0.698\ +0.051\ +4.613\ =+1.349$$

	Solution I.	Weight.	Solution II.	Weight.
x =	$-0''.352\pm0''.017$	3.68	$-0''.354\pm0''.010$	7.89
y =	-0.004 ± 0.023	1.86	....	....
π =	$+0.346\pm0.015$	4.50	$+0.346\pm0.013$	4.55
r_0 =	±0.031		±0.029	

Observer's notes—191, Clouds between exp. 2 and 3.
Measurer's notes—397, Exp. 2 trailed, unmeasurable; 405, Very good.

All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XIa. STAR 13; Lal. 21185. y Co-ordinates.

Stars.	Standard mean of 191 and 194	191	194	258	260	268	397	405	426
2	19.74821	63175	86468	29722	65062	70200	71560	64839	66138
6	11.78118	68525	87711	34156	70130	73778	77020	70158	67678
9	32.59270	53881	64660	20162	56754	58585	62460	54908	46458
A	20.37582	28496	46669	93828	29759	33351	34162	27082	23610
Average residual.		51	31	45	45	66	28	31	25
Residuals for A...		+19	−19	−807	−817	−845	−2835	−2902	−3433

Reduction constants $y_A = +0.33054y_2 + 0.38299y_6 + 0.28647y_9$

Plate.	Observed.	P. M. to 1904.0	Corrected	Equations of conditions.	$o-c_1$	$o-c_2$
191	$+0''.033$	$-0''.289$	$-0''.256$	$x-0.061y-0.294\pi=-0''.256$	$+0''.040$	$+0''.041$
194	−0.033	−0.242	−0.275	$x-0.051y-0.256\pi=-0.275$	+0.009	+0.010
258	−1.419	+1.379	−0.040	$x+0.291y+0.595\pi=-0.040$	−0.041	−0.041
260	−1.436	+1.417	−0.019	$x+0.299y+0.585\pi=-0.019$	−0.018	−0.018
268	−1.486	+1.493	+0.007	$x+0.315y+0.571\pi=+0.007$	+0.013	+0.013
397	−4.984	+4.735	−0.249	$x+0.999y-0.078\pi=-0.249$	−0.024	−0.025
405	−5.102	+4.863	−0.239	$x+1.026y+0.027\pi=-0.239$	−0.049	−0.050
426	−6.035	+6.105	+0.070	$x+1.288y+0.596\pi=+0.070$	+0.070	+0.068
Assumed P. M.		−4.740	....	..........................(pvv)	11732	11704

Normal equations.	Solution I.	Weight.	Solution II.	Weight.
$+8.000x+4.106y+1.749\pi=-1''.001$	$x=-0''.198\pm0''.017$	3.61	$x=-0''.199\pm0''.012$	6.01
$+4.106\ \ +3.989\ \ +1.277\ \ =-0.389$	$y=-0.002\pm0.025$	1.76	....	....
$+1.749\ \ +1.277\ \ +1.540\ \ =+0.169$	$\pi=+0.335\pm0.031$	1.08	$\pi=+0.335\pm0.028$	1.16
	$r_0=\ \pm0.033$		$r_0=\ \pm0.030$	

Measurer's notes—260, Plate spattered with fine drops.

All quantities in this table are in réseau intervals of $175''.8$ unless otherwise stated.

TABLE C, SERIES XII. STAR 14; Lalande 21258; $11^{h}0^{m}.5$, $+44°2'$; Proper-motion $-0''.404$, $+0''.95$, $=4''.40$.

Stars.	Bonn Durchmusterung.		Harvard		Pl.264 (Stand.) 1904, April 20 Exp. 4, 5^m Hr.-ang. -15^m Obs. R		Plate 265 1904, Apr. 21 Exp. 4, 5^m Hr.-ang. -7^m Obs. R		Plate 406 1905, Jan. 9 Exp. 4, 5^m Hr.-ang. $+30^m$ Obs. H		Plate 410 1905, Jan. 12 Exp. 4, 5^m Hr.-ang. $+17^m$ Obs. H		Plate 427 1905, Apr. 15 Exp. 4, 4^m Hr.-ang. $+22^m$ Obs. H		Plate 486 1906, Apr. 25 Exp. 4, 4^m Hr.-ang. -19^m Obs. H		Plate 487 1906, Apr. 25 Exp. 4, 5^m Hr.-ang. $+5^m$ Obs. H		Plate 574 1906, Nov. 30 Exp. 3, 4^m Hr.-ang. $+25^m$ Obs. H		Plate 593 1907, Jan. 3 Exp. 4, 5^m Hr.-ang. $+8^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
2	+43°2079	8.9	9.18	G5	12.7024	16.59119	57260	**43**	73826	2	74362	**16**	67944	15	72476	18	74574	**1**	71840	52	83956	10	0".023	**0".005**
1	44 2048	9.2	9.98	F5	23.2233	15.63419	65548	**13**	78330	55	79140	10	72410	58	76184	93	76255	63	77452	67	79390	78	055	**016**
3	44 2049	9.3	10.05	G?	20.9453	18.11699	13004	57	26296	**55**	27155	5	20510	**73**	24399	**112**	24970	**63**	25093	**119**	29278	**91**	**081**	021
4	44 2050	9.5	10.22	G5	25.7784	20.53678	56564	**133**	68192	7	69088	**45**	62640	51	66174	**5**	65754	0	67658	89	67068	21	047	**010**
6	44 2056	9.2	9.64	G	22.1046	26.09048	10689	89	23106	**4**	24001	29	17826	**37**	21798	23	22014	**1**	22050	**38**	25165	**11**	**025**	005
5	43 2080	9.0	9.12	K	13.7654	22.42478	41009	44	56752	**1**	57375	16	51226	**15**	55721	**19**	57612	1	54775	**52**	65982	**12**	**024**	005
A	44 2051	8.5	8.63	K5	20.0995	19.83802	84749	44	96630	**1690**	97390	**1711**	90172	**2487**	91645	**5020**	92258	**5076**	90713	**6378**	95452	**6618**	**4.396**	170
P. M. of A					+.005	−.025																		
Average residual					51		80		36		34		47		43		41		83		55			
a							− 16.38		− 75.09		− 71.35		− 10.16		− 1.59		− 8.77		− 76.16		− 68.94			
b							+ 374.67		+ 7.52		+ 36.50		+ 10.73		− 63.53		− 255.87		+ 116.36		− 855.50			
c							−6303.		+15856.		+15979.		+8842.7		+14172.		+18852.		+12455.		+36837.			

Plate.	Observed.	P. M. to 1905.0	Corrected.	Equations of condition.	$o-c$	Weight.
264	0".000	−3".052	−3".052	$x-0.701y-0.663\pi=-0''.052$	+0".002	1
265	+ 0.077	−3.040	−2.963	$x-0.698y-0.675\pi=+0.037$	+0.093	½
406	− 2.971	+0.105	−2.866	$x+0.024y+0.744\pi=+0.134$	−0.017	1
410	− 3.008	+0.140	−2.868	$x+0.032y+0.717\pi=+0.132$	−0.014	1
427	− 4.372	+1.254	−3.118	$x+0.288y-0.615\pi=-0.118$	−0.037	1
486	− 8.825	+5.736	−3.089	$x+1.317y-0.723\pi=-0.089$	+0.045	1
487	− 8.924	+5.736	−3.188	$x+1.317y-0.723\pi=-0.188$	−0.054	1
574	−11.214	+8.343	−2.871	$x+1.915y+0.915\pi=+0.129$	+0.017	½
593	−11.631	+8.745	−2.886	$x+2.001y+0.807\pi=+0.114$	+0.022	1
Assumed P. M. and Δx	−4.356	−3.000	……………………(pvv)	11752		

Normal equations.

$$+8.000x+4.886y-0.336\pi=+0''.016$$
$$+4.886\ +10.125\ +1.151\ =-0.017$$
$$-0.336\ +\ 1.151\ +4.229\ =+0.641$$

	Weight.
$x=+0''.030\pm0''.013$	5.45
$y=-0.035\pm0.012$	6.70
$\pi=+0.163\pm0.015$	3.96
$r_0=\quad\pm0.030$	

Observer's notes—486, 487, Not very clear; 593, High wind.

Measurer's notes—264, Images elongated; 265, Stars faint; 574, Images diffuse.

All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XIII. STAR 15; 83¹ Leonis; 16; 83² Leonis; } $11^h 21^m.7, +3°33'$; P. M. $-\ 0^s.048, +0''.17, = 0''.74$

Stars.	Bonn Durchmusterung.		Harvard.		Plate 195 (Stand) 1903, Dec. 13 Exp. 4, 4^m Hr.-ang $+21^m$ Obs. R		Plate 192 1903, Dec. 11 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. $+28^m$ Obs. R		Plate 261 1904, Apr. 19 Exp. 4, 4^m Hr.-ang. $+22^m$ Obs. R		Plate 398 1904, Dec. 30 Exp. 4, 4^m Hr.-ang. $+24^m$ Obs. H			Plate 491 1904 Apr. 30 Exp. 4, 4^m Hr.-ang. $+6^m$ Obs. H		Plate 594 1907, Jan. 3 Exp. 4, 4^m Hr.-ang. $+16^m$ Obs. H		Plate 602 1907 Jan. 17 Exp. 4, 4^m Hr.-ang. -17^m Obs. H		Approximate solution.		
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	Corr.	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	μ'	π
2	+3°2500	9.0	8.70	K	9.052	15.52771	63964	**31**	84190	31	10.23674	4	**10**	82461	22	73531	65	89745	**10**	0″.023	0″.003	**0″.028**
1	4 2461	7.3	6.70	A2	26.328	13.72430	72184	30	67326	**31**	18.35295	24	10	81492	**24**	80565	**63**	90875	8	**023**	**003**	031
3	4 2465	9.0	8.84	F5	30.151	18.05771	03031	**31**	92866	31	12.66935	6	**11**	10475	22	11165	63	19970	**10**	023	003	**030**
4	3 2505	8.2	7.44	.....	16.583	25.30656	37155	38	46639	18	19.98270	15	46	51692	2	45650	**29**	59302	26	**012**	**123**	007
5	3 2506	8.2	7.18	F	15.615	27.77144	84298	**8**	95208	**50**	22.45155	29	**33**	99386	**22**	92784	**32**	06724	**17**	**011**	**148**	021
A	3 2502	7.5	6.36	K	20.257	19.65438	69371	42	73222	**189**	14.30858	0	**369**	81024	**896**	76708	**1204**	89144	**1104**	**669**	**717**	052
B	3 2503	8.0	7.51	K	20.113	19.73661	77648	**1**	81684	**256**	14.39045	0	**472**	89341	**977**	84945	**1294**	97415	**1209**	**711**	**760**	061
P.M. of A and B	.....	.....	.....	.....	+.001	−.004																
Average residual						42	46		43		40			41		41		50				
a						0.00	+ 23.69		+ 44.80		+ 16.65			+ 32.18		− 21.35		− 26.32				
b						0.00	− 663.16		− 2105.80		− 463.07			− 1187.90		− 725.70		− 1076.23				
c						0.00	+16858.		+49753.		−525158.			+39920.		+27597.		+47134.				

Plate.	Observed.		P. M. to mean epoch 1905.368.	Corrected.		Equations of condition.	Star A.		Star B.	
	Star A.	Star B.		Star A.	Star B.		n	$o-c$	n	$o-c$
192	+0″.074	−0″.002	−0″.995	−0″.921	−0″.997	$x-1.422y+0.906\pi=$	+0″.079	+0″.049	+0″.003	+0″.042
195	0.000	0.000	−0.993	−0.993	−0.993	$x-1.418y+0.906\pi=$	+0.007	−0.027	+0.007	+0.047
261	−0.332	−0.450	−0.748	−1.080	−1.198	$x-1.069y-0.592\pi=$	−0.080	−0.054	−0.198	−0.076
398	−0.649	−0.830	−0.258	−0.907	−1.088	$x-0.369y+0.852\pi=$	+0.093	+0.021	−0.088	−0.053
491	−1.575	−1.718	+0.672	−0.903	−1.046	$x+0.960y-0.704\pi=$	+0.097	+0.045	−0.046	+0.067
594	−2.117	−2.275	+1.149	−0.968	−1.126	$x+1.641y+0.831\pi=$	+0.032	−0.122	−0.126	−0.104
602	−1.941	−2.125	+1.176	−0.765	−0.949	$x+1.680y+0.718\pi=$	+0.235	+0.085	+0.051	+0.078
Assumed P.M. and Δx.......			−0.700	−1.000	−1.000	...............(pvv)	....	30621	....	33947

Normal equations.

	A	B
$+7.000x+0.003y+2.917\pi$	$=+0''.463$	$-0''.397$
$+0.003\ +11.749\ -0.360$	$=+0.470$	$+0.065$
$+2.917\ -\ 0.360\ +4.420$	$=+0.332$	$+0.015$

Solution.

	Star A.	Star B.	Weight.
x =	+0″.046±0″.026	−0″.080±0″.028	5.07
y =	+0.042±0.017	+0.007±0.018	11.71
π =	+0.048±0.033	+0.057±0.035	3.19
r_0 =	±0.059	±0.062	

Observers' note—491, Clouds about; 594, High wind.

Measurer's notes—192, Images diffuse; 398, Wrong center, X=+5.32 Y=−3.11, images excellent; 594, Diffuse.

Assumed P. M. derived from the plates themselves.

Proper-motion of star 5 −0″.142 in x, −0″.037 in y (A. G. Albany); P. M. of star 4 −0″.125 in x, +0″.014 in y, (from micrometric comparison with τ Leonis. Burnham's Gen. Cat. 5790). The P. M.'s obtained from the plates when these values are represented as closely as possible are given under the heading μ'.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XIV. STAR 17; Groombridge 1830; $11^h47^m.2$, +38°26′; P. M. +0".341, −5".80, =7".04.

Stars.	Bonn Durchmusterung.		Harvard.		Pl. 196 (Stand.) 1903, Dec. 13 Exp. 4, 4^m Hour-angle $+24^m$ Obs. R.		Pl. 193 1903, Dec. 11 Exp. 4, 4^m Hr.-ang. $+27^m$ Obs. R		Pl. 270 1904, May 2 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. $+20^m$ Obs. R		Pl. 274 1904, May 4 Exp. 4, 5^m Hr.-ang. $+26^m$ Obs. R		Pl. 399 1904, Dec. 30 Exp. 4, 4^m Hr.-ang. $+21^m$ Obs. H		Pl. 407 1905, Jan. 9 Exp. 3, 4^m Hr.-ang. $+20^m$ Obs. H		Pl. 411 1905, Jan. 12 Exp. 4, 4^m Hr.-ang. $+6^m$ Obs. H		Pl. 428 1905, May 8 Exp. 4, 4^m Hr.-ang. $+8^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
2	+38°2284	9.3	10.19	K	8.042	12.68430	63731	9	04601	30	03101	57	15182	43	91760	43	11658	28	18586	169	0".093	**0".055**
4	38 2286	9.3	9.52	G5	12.816	22.25908	22278	20	61839	42	61585	24	71550	29	47643	45	69158	0	74504	**40**	**017**	**002**
1	38 2283	9.2	9.75	F5	19.656	9.38611	36258	**30**	71829	**73**	73102	**81**	81954	**72**	57807	**89**	81656	**27**	82620	**132**	**080**	057
3	....	.....	10.55		30.245	14.84170	84075	**8**	15635	61	19552	31	24902	120	99873	28	27219	14	23878	82	088	**028**
5	38 2287	9.5	10.00	G?	24.570	23.20853	19680	34	54231	13	56926	49	63185	**48**	38570	59	64028	14	64030	47	**008**	**031**
6	38 2288	8.5	8.83	F8	20.617	26.38775	36759	**12**	73271	**12**	75068	56	82255	**70**	57757	**47**	81941	**58**	83970	**122**	**128**	**018**
7	38 2290	9.4	10.54	G?	16.709	30.76200	73398	**19**	11881	**60**	12630	**137**	20934	**2**	96547	**42**	19528	31	23788	**8**	047	074
A	+38 2285	6.5	6.46	Gp	18.961	19.72593	70132	**67**	07634	723	09019	870	18849	2394	94706	2533	18239	2471	21263	3180	4104	103
P. M. of A.....	.....	.....	.....	.....	−.033	+.023																
Average residual................					B. 25*		74*		44*		63*		38		51		38		50			
a................					0.00		+ 7.05		+ 84.10		+ 96.60		+ 23.59		+ 4.29		+ 9.38		+ 96.08			
b................					0.00		+ 207.30		− 221.48		+ 23.76		− 276.91		− 343.33		− 8.34		− 476.05			
c................					0.		−6464.		+36855.		+33198.		+48637.		+25994.		+43148.		+52596.			

Plate.	Observed.	P. M. to 1904.5	Corrected.	Equations of condition.	$o-c_1$	$o-c_2$	Weight.
193	−0".118	+2".247	+2".129	$x-0.556y+0.896\pi=+0''.029$	−0".043	−0".083	$\frac{1}{2}$
196	0.000	+2.224	+2.224	$x-0.551y+0.900\pi=+0.124$	+0.051	+0.012	1
270	+1.271	+0.670	+1.941	$x-0.166y-0.659\pi=-0.159$	−0.099	−0.117	1
274	+1.529	+0.648	+2.177	$x-0.160y-0.681\pi=+0.077$	+0.138	+0.121	$\frac{1}{2}$
399	+4.209	−2.011	+2.198	$x+0.498y+0.881\pi=+0.098$	−0.036	−0.012	1
407	+4.453	−2.124	+2.329	$x+0.525y+0.832\pi=+0.229$	+0.098	+0.124	$\frac{1}{2}$
411	+4.344	−2.155	+2.189	$x+0.533y+0.813\pi=+0.089$	−0.040	−0.015	1
428	+5.590	−3.435	+2.155	$x+0.850y-0.719\pi=+0.055$	+0.058	+0.103	$\frac{1}{2}$
Assumed P. M. and Δx.		+4.040	+2.100	....	32229	37959	

Normal equations.

$$
\begin{array}{llll}
+6.000x & +0.644y & +2.099\pi & =+0''.347 \\
+0.644 & +1.530 & +0.203 & =+0.124 \\
+2.099 & +0.203 & +3.919 & =+0.437
\end{array}
$$

	Solution I.	Weight.	Solution II.	Weight.
$x=$	$+0''.016\pm0''.025$	4.69	$+0''.023\pm0''.024$	4.88
$y=$	$+0.061\pm0.045$	1.46	....	....
$\pi=$	$+0.100\pm0.030$	3.19	$+0.099\pm0.030$	3.19
$r_0=$	±0.054		±0.054	

Observer's notes—270, Occasional light clouds; 274, Sky very thick.
Measurer's notes—193, Fogged, réseau very faint; 407, Exposures growing fainter; exposure 4 unmeasurable; 428, Underexposed and diffuse.
All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XV. STARS 18 and 19; γ Virginis; $12^{h}36^{m}.6$, $-0^{\circ}54'$; P. M. −0″.038, +0″.01, =0″.56. Observed with Color-screen.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 207 1904, Jan. 21 Exp. 3, 4^{m} Hr.-ang. $+15^{m}$ Obs. R		Plate 210 1904, Jan. 28 Exp. 4, 4^{m} Hr.-ang. $+29^{m}$ Obs. R		Plate 279 1904, May 9 Exp. 4, 5^{m} Hr.-ang. $+21^{m}$ Obs. R		Plate 282 1904, May 18 Exp. 4, 5^{m} Hr.-ang. $+11^{m}$ Obs. R		Plate 283 1904, May 18 Exp. 4, 5^{m} Hr.-ang $+38^{m}$ Obs. R		Plate 408 1905, Jan. 9 Exp. 4, 5^{m} Hr.-ang. $+4^{m}$ Obs. H		Plate 413 1905, Jan. 12 Exp. 4, 5^{m} Hr.-ang. $+19^{m}$ Obs. H		Plate 418 1905, Jan. 26 Exp. 4, 5^{m} Hr.-ang. $+7^{m}$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 210.	ξ Mean of 207 and 210.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	−0°2595	8.0	7.08	A	32.411	6.86638	90980	9	82295	**10**	85451	**82**	02046	**9**	00324	**64**	03035	**20**	03529	**25**	03656	**97**	**0″.086**	0″.041
3	−0 2599	9.2	9.36	F2	20.154	18.33214	38208	**11**	28220	11	34582	81	49789	10	49998	64	50681	20	49478	25	52024	97	086	**041**
5	−1 2708	9.2	9.60	K	9.776	20.09284	14797	10	03770	**11**	11992	15	25885	83	27542	**40**	27530	24	24874	74	29095	50	088	**019**
6	−0 2600	9.2	8.90	F	18.796	21.06970	12052	**11**	01888	11	08662	**16**	23768	**83**	24230	40	24524	**24**	23088	**74**	25864	**52**	**090**	020
10	−0 2604	8.5	8.49	Ma	20.486	31.21736	26873	32	16599	**32**	24002	11	39992	31	40041	116	39339	31	38232	**16**	40891	**115**	**057**	**060**
9	−0 2606	8.7	8.44	Ma	19.430	34.87803	92958	**33**	82648	33	90461	**12**	06440	**30**	06447	**116**	05465	**31**	04321	15	07229	115	057	060
A	−0 2601	2.8	3.68	F	20.225	21.07112	12148	10	02075	**11**	08518	**87**	23827	**203**	24023	**120**	24380	**205**	23141	**268**	25631	**290**	**461**	074
B			3.65		20.252	21.05350	10383	8	00316	**9**	06674	**164**	22090	**177**	22383	6	22571	**251**	21369	**279**	23875	**281**	**487**	062
P.M. of A and B	.....		.....	.....	......	−.003																		
							B.		B.															
Average residual							28*		36*		47*		58*		65		25		48		37			
a							+ 8.97		− 8.97		+ 78.20		+ 128.18		+ 117.54		+ 11.71		+ 19.14		+ 38.06			
b							− 46.51		+ 46.51		− 122.15		+ 26.27		− 132.20		− 72.90		+ 73.05		− 94.59			
c							5778.		−5778.		+2316.		+13686.		+17229.		+18700.		+14417.		+19920.			

Plate.	Observed.		P. M. to 1904.0	Corrected.		Equations of condition.	Star A.			Star B.			Wt.
	Star A.	Star B.		A.	B.		n	$o-c_1$	$o-c_2$	n	$o-c_1$	$o-c_2$	
207	+0″.018	+0″.014	+0″.032	+0″.050	+0″.046	$x+0.056y+0.846\pi=$	+0″.050	+0″.012	−0″.054	+0″.046	+0″.015	−0″.039	1
210	−0.019	−0.016	+0.037	+0.018	+0.021	$x+0.075y+0.800\pi=$	+0.018	−0.019	−0.082	+0.021	−0.009	−0.060	1
279	−0.153	−0.288	+0.202	+0.049	−0.086	$x+0.354y-0.582\pi=$	+0.049	+0.081	+0.079	−0.086	−0.067	−0.068	1
282	−0.357	−0.311	+0.221	−0.136	−0.090	$x+0.378y-0.686\pi=$	−0.136	−0.099	−0.096	−0.090	−0.067	−0.065	1
283	−0.211	+0.011	+0.221	+0.010	+0.232	$x+0.378y-0.686\pi=$	+0.010	+0.047	+0.050	+0.232	+0.255	+0.257	½
408	−0.360	−0.441	+0.585	+0.225	+0.144	$x+1.026y+0.893\pi=$	+0.225	+0.077	+0.117	+0.144	+0.024	+0.056	1
413	−0.471	−0.490	+0.589	+0.118	+0.099	$x+1.034y+0.884\pi=$	+0.118	−0.030	+0.011	+0.099	−0.021	+0.012	1
418	−0.510	−0.494	+0.611	+0.101	+0.117	$x+1.072y+0.808\pi=$	+0.101	−0.046	+0.001	+0.117	−0.002	+0.035	1
Assumed P. M. and Δx......			−0.570	0.000	0.000	.... (pvv)	....	26916	40158	....	42817	51500	

Normal equations.

	A	B
$+7.500x+4.184y+2.620\pi=$	+0″.430	+0″.367
$+4.184\ +3.619\ +2.209\ =$	+0.435	+0.359
$+2.620\ +2.209\ +4.690\ =$	+0.504	+0.398

Solution I.

	A	B	Weight.
x =	−0″.029±0″.030	−0″.019±0″.038	2.66
y =	+0.110±0.046	+0.089±0.059	1.14
π =	+0.072±0.027	+0.054±0.034	3.34
r_0 =	±0.049	±0.062	

Solution II.

A	B	Weight.
+0″.025±0″.022	+0″.024±0″.025	6.04
....	....	....
+0.094±0.028	+0.072±0.032	3.78
±0.055	±0.063	

Observer's notes—282, Strong twilight; 418, Sky thick.
Measurer's notes—283, Very diffuse; 413, Diffuse.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

Table C, Series XVa. Stars 18, 19; γ Virginis; y Co-ordinates.

Stars.	Standard mean of 207 and 210.	Plate 207.	Plate 210.	Plate 279.	Plate 282.	Plate 283.	Plate 408	Plate 413.	Plate 418.
3	20.14108	13321	14894	12720	15602	17922	19362	13988	19511
6	18.78534	77862	79206	77400	79884	82538	83988	78195	84126
10	20.48029	47828	48230	48220	49296	53414	54290	47075	54615
A	20.21304	20584	22024	20265	22739	25526	26732	21012	26872
B	20.24018	23294	24742	23029	25547	27951	29491	23592	29639
Average residual ...		55	44	51	37	61	53	40	31
Residuals for A.....		−58	+58	+13	−11	+ 68	−40	+ 5	−87
Residuals for B.....		−61	+61	+64	+81	−247	+ 6	−131	−33

Reduction constants: $y_A=+0.78739y_3+0.00004y_6+0.21257y_{10}$

$y_B=+0.80394y_3-0.01920y_6+0.21526y_{10}$

Plate.	Observed Star A.	Observed Star B.	Equations of condition.	Star A.			Star B.			Weight.
				n	$o-c_1$	$o-c_2$	n	$o-c_1$	$o-c_2$	
207	−0″.102	−0″.107	$x+0.056y-0.324\pi=$	−0″.102	−0″.103	−0″.051	−0″.107	−0″.106	−0″.042	1
210	+0.102	+0.107	$x+0.075y-0.293\pi=$	+0.102	+0.100	+0.151	+0.107	+0.107	+0.169	1
279	+0.023	+0.113	$x+0.354y+0.328\pi=$	+0.023	−0.002	+0.001	+0.113	+0.097	+0.098	1
282	−0.019	+0.142	$x+0.378y+0.361\pi=$	−0.019	−0.044	−0.044	+0.142	+0.126	+0.123	1
283	+0.120	−0.434	$x+0.378y+0.361\pi=$	+0.120	+0.095	+0.095	−0.434	−0.450	−0.453	½
408	−0.070	+0.011	$x+1.026y-0.363\pi=$	−0.070	+0.002	+0.029	+0.011	+0.104	+0.067	1
413	+0.009	−0.230	$x+1.034y-0.352\pi=$	+0.009	+0.081	+0.049	−0.230	−0.138	−0.175	1
418	−0.153	−0.058	$x+1.072y-0.299\pi=$	−0.153	−0.082	−0.119	−0.058	+0.034	−0.010	1
Assumed P. M.....		0.000	..................(pvv)	....	40350	49254	....	180236	192876	

Normal equations.

				A	B
$+7.500x$	$+4.184y$	-0.762π	=	−0″.150	−0″.239
+4.184	+3.619	−0.776	=	−0.202	−0.275
−0.762	−0.776	+0.839	=	+0.094	+0.108

Solution I.

	Star A.	Star B.	Weight.
$x=$	+0″.028±0″.037	+0″.026±0″.079	2.64
$y=$	−0.073±0.057	−0.092±0.121	1.12
$\pi=$	+0.070±0.074	+0.068±0.157	0.67
$r_0=$	±0.061	±0.128	

Solution II.
substituting the value of y given in the
"Fundamental Catalogue."

	Star A.	Star B.	Weight.
$x=$	−0.018±0″.023	−0″.028±0″.046	6.80
$y=$	+0.015	+0.015	
$\pi=$	+0.106±0.071	+0.117±0.139	0.76
$r_0=$	±0.061	±0.121	

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated.

Table C. Series XVI. Star 20; Berlin A 4999; $13^{h}40^{m}2, +18°20'$; P. M. $+0^{s}.027$, $-1''.85$, $=1''.89$.

Star.	Bonn Durchmusterung. No.	Bonn Durchmusterung. Mag.	Harvard. Mag.	Harvard. Sp.	Pl. 204 (Stand.) 1904, Jan. 19 Exp. 4, $4\frac{1}{2}^{m}$ Hour-angle $+37^{m}$ Obs. R η	Pl. 204 ξ	Plate 211 1904, Jan. 28 Exp. 4, 5^{m} Hr.-ang. $+3^{m}$ Obs. R x	Plate 211 $o-c$	Plate 284 1904, May 18 Exp. 4, 5^{m} Hr.-ang. $+14^{m}$ Obs. R x	Plate 284 $o-c$	Plate 419 1905, Jan. 26 Exp. 4, 5^{m} Hr.-ang. -12^{m} Obs. H x	Plate 419 $o-c$	Plate 435 1905, May 22 Exp. 4, 4^{m} Hr.-ang. $+6^{m}$ Obs. H x	Plate 435 $o-c$	Plate 500 1906, May 18 Exp. 4, 5^{m} Hr.-ang. $+17^{m}$ Obs. H x	Plate 500 $o-c$	Plate 598 1907, Jan. 3 Exp. 4, 5^{m} Hr.-ang. $+7^{m}$ Obs. H x	Plate 598 $o-c$	Plate 608 1907, Jan. 17 Exp. 4, 5^{m} Hr.-ang. $+43^{m}$ Obs. H x	Plate 608 $o-c$	Approximate solution. μ	Approximate solution. π
2	+18°2773	7.8	7.92	K	11.336	13.70910	80546	**89**	76071	**123**	77941	**17**	86544	11	88226	**14**	73791	**73**	70142	**34**	**0″.006**	**0″.001**
3	18 2775	9.5	10.21	.....	11.381	16.18471	28181	65	24019	85	25568	28	34282	17	35860	**9**	21408	7	17661	31	**008**	**001**
1	18 2770	9.5	10.84	.....	24.166	9.33689	32714	27	38395	39	41155	**11**	50568	**25**	47158	23	35145	65	31739	**3**	009	004
4	19 2710	7.7	7.91	F	30.940	20.06331	99431	**16**	11618	**3**	14159	25	24838	52	18102	**45**	06774	**20**	03272	14	003	**002**
6	19 2711	8.9	9.46	F5	30.359	23.17680	11225	**9**	23175	**36**	25476	**13**	36246	**26**	29785	20	18145	**45**	14546	**6**	**013**	**001**
5	18 2777	9.0	9.43	K	18.160	22.37945	41728	**52**	43722	20	45310	3	54949	**37**	53559	**4**	40004	22	36142	**6**	020	001
7	18 2780	9.5	10.45	G5	22.123	25.55406	55930	79	61319	17	62922	**13**	73159	12	69996	27	56965	37	53076	9	**010**	005
A	18 2776	9.2	9.98	(G5)	20.042	19.65042	67329	**10**	70519	**37**	72638	188	82352	222	80535	503	67636	767	63860	710	441	106
P. M. of A.....	.....	.....	.....	.....	−.010	+.002																
Average residual.....					44		51		46		48		46		50		34		48			
a.....					0.00		− 16.80		+ 72.78		+ 7.80		+ 67.03		+ 32.14		− 7.93		− 37.54			
b.....					0.00		− 841.79		− 23.26		+ 36.09		+ 122.70		− 291.78		− 124.51		− 106.66			
c.....					0.		+19498.		+4550.		+6532.		+13313.		+20198.		+4474.		+ 980.			

Plate.	Observed.	P. M. to 1905.5	Corrected	Equations of condition.	$o-c$
204	$0''.000$	$+0''.626$	$+0''.626$	$x-1.448y+0.917\pi=+0''.126$	$+0''.047$
211	-0.018	$+0.615$	$+0.597$	$x-1.423y+0.901\pi=+0.097$	$+0.019$
284	-0.065	$+0.486$	$+0.421$	$x-1.125y-0.481\pi=-0.079$	-0.017
419	$+0.331$	$+0.186$	$+0.517$	$x-0.430y+0.907\pi=+0.017$	-0.079
435	$+0.390$	$+0.048$	$+0.438$	$x-0.111y-0.531\pi=-0.062$	-0.013
500	$+0.884$	-0.379	$+0.505$	$x+0.878y-0.473\pi=+0.005$	$+0.030$
598	$+1.348$	-0.651	$+0.697$	$x+1.508y+0.887\pi=+0.197$	$+0.064$
608	$+1.248$	-0.670	$+0.578$	$x+1.545y+0.916\pi=+0.078$	-0.052
Assumed P. M. and Δx		$+.432$	$+0.500$	.......................(pvv)	16969

Normal equations.

$+8.000x-0.606y+3.043\pi=+0''.379$
$-0.606\ +11.018\ -0.061\ =+0''.191$
$+3.043\ -\ 0.061\ +4.839\ =+0''.533$

Solution.	Weight.
$x=+0''.009\pm0''.016$	6.06
$y=+0.018\pm0.012$	10.98
$\pi=+0.105\pm0.020$	3.68
$r_0=\pm0.039$	

Observer's notes—435, Twilight. Measurer's notes—284, 435, Fogged; 608, Plate cracked.

Assumed P. M. from the plates themselves. Auwers' value is $+0''.385$.

All quantities in this table are in réseau intervals of $175''.8$ unless otherwise stated.

Numbers in bold-face type are negative.

TABLE C, SERIES XVII. STAR 21; Lalande 25372; $13^{h}40^{m}.7$, $+15°26'$; P. M. $+0^{s}.125, -1''.47 = 2''.32$

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 208 1904, Jan. 21 Exp. 4, $4\frac{1}{2}^{m}$ Hr.-ang. $+7^{m}$ Obs. R		Plate 212 1904, Jan. 28 Exp. 4, $4\frac{1}{2}^{m}$ Hr.-ang. $+31^{m}$ Obs. R		Plate 286 1904, May 19 Exp. 4, 5^{m} Hr.-ang. $+16^{m}$ Obs. R		Plate 290 1904, May 25 Exp. 4, 5^{m} Hr.-ang. $+14^{m}$ Obs. R		Plate 292 1904, May 30 Exp. 2, 5^{m} Hr.-ang. $+22^{m}$ Obs. R		Plate 420 1905, Jan. 26 Exp. 4, 5^{m} Hr.-ang. $+21^{m}$ Obs. H		Plate 433 1905, May 17 Exp. 4, 4^{m} Hr.-ang. -6^{m} Obs. H		Plate 434 1905, May 17 Exp. 4, $4\frac{1}{2}^{m}$ Hr.-ang. $+21^{m}$ Obs. H		Approximate Solution.		
	No.	Mag.	Mag.	Sp.	η Plate 208.	ξ Mean of 208 and 212.	z	$o-c$	z	$o-c$	z	$o-c$	x	$o-c$	z	$o-c$	z	$o-c$	z	$o-c$	z	$o-c$	μ	μ'	π
1	+15° 2613	9.5	10.08	F5?	16.720	8.59792	61545	23	58039	23	70065	30	63209	107	61882	11	77859	39	70402	73	70751	59	0″.039	0″.024	0″.049
2	15 2617	9.1	9.48	F5?	18.606	16.04685	06264	15	03106	15	14540	8	08214	44	06770	29	21892	0	14799	51	15339	2	014	033	021
4	16 2547	9.3	9.58	......	26.770	18.56113	56996	19	55229	18	61639	43	55024	88	54385	119	69218	33	60205	26	61787	15	007	042	039
5	16 2549	9.3	9.82	......	28.047	19.53258	54050	6	52466	5	58176	26	51436	3	50570	299	65704	15	56471	22	58279	77	128	182	065
8	16 2553	9.5	10.60	......	26.224	24.07321	08239	16	06404	17	13740	106	07292	21	06725	218	20704	10	12391	28	13920	45	105	060	056
7	15 2624	9.3	9.68	F5?	18.278	23.13799	15403	28	12195	28	24452	39	18542	15	16980	65	31246	16	24891	13	25426	49	002	017	035
9	15 2628	9.3	9.85	F7	20.030	31.57063	58491	10	55635	10	67314	76	61965	48	60555	27	73615	55	67751	10	68571	2	023	004	007
3	15 2618	9.2	9.53	......	8.504	16.31992	34345	23	29640	24	47369	72	41781	5	30462	3	54348	7	49921	67	88942	26	061	042	004
6	15 2621	9.4	9.96	......	9.920	19.72212	74451	16	69973	16	87265	110	81674	59	79442	42	93899	32	89320	44	49226	38	032	047	033
A	15 2620	8.5	8.30	K5	18.486	19.78480	80026	19	76934	19	88851	141	82775	172	81368	205	96865	1083	90248	1180	90905	1261	1.879	1.896	224
P. M. of A......					−.008	+.010																			
							B		B																
Average residual......							44°		22°		82°		47°		50		28°		41°		60°				
a......							− 2.00		+ 2.00		+ 80.41		+ 150.99		+ 135.31		+ 8.04		+ 111.35		+ 114.60				
b......							− 80.48		+ 80.48		− 550.77		− 617.50		− 526.89		− 508.86		− 769.04		− 646.89				
c......							+3093.		−3093.		+18821.		+12551.		+9746.		+26545.		+22585.		+20849.				

Plate.	Observed.	P. M. to 1904.5.	Corrected.	Equations of condition.	n	$o-c_1$	$o-c_2$	Weight.
208	−0″.033	+0″.802	+0″.769	$z-0.444y+0.916\pi=$	+0″.269	−0″.019	−0″.045	1
212	+0.033	+0.763	+0.796	$z-0.422y+0.900\pi=$	+0.296	+0.010	−0.014	1
286	+0.248	+0.215	+0.463	$z-0.119y-0.490\pi=$	−0.037	−0.038	−0.070	½
290	+0.302	+0.186	+0.488	$z-0.103y-0.569\pi=$	−0.012	+0.004	−0.030	1
292	+0.360	+0.161	+0.521	$z-0.089y-0.630\pi=$	+0.021	+0.049	+0.015	½
420	+1.904	−1.035	+0.869	$z+0.573y+0.906\pi=$	+0.369	+0.013	+0.057	1
433	+2.088	−1.583	+0.505	$z+0.876y-0.458\pi=$	+0.005	−0.071	−0.035	1
434	+2.217	−1.583	+0.634	$y+0.876y-0.458\pi=$	+0.134	+0.058	+0.094	1
Assumed P.M and Δx		+1.807	+0.500	...............(pvv)	....	10974	18993	

Normal equations.	Solution I.	Weight.	Solution II.	Weight.
$+7.000z+1.251y+0.677\pi=+1''.053$	$z=+0''.117\pm0''.013$	5.84	$+0''.131\pm0''.014$	6.87
$+1.251\ +2.260\ -0.954\ =+0.091$	$y=+0.069\pm0.024$	1.70		
$+0.677\ -0.954\ +3.531\ =+0.793$	$\pi=+0.221\pm0.019$	2.90	$+0.199\pm0.020$	3.47
	$r_0=\ \pm0.032$		±0.038	

Observer's notes—286, Rather thick; 292, Sky thick, clouds at end.

Measurer's notes—286, Images of exp. 3 oval; 292, Only exp. 1 and 2 measurable, and these very faint.

The proper motions obtained by rejecting star 5 are given under the heading μ'.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C. SERIES XVIII. STAR 22; Berlin B 5072; 23; Berlin B 5073; $\}$ $14^h21^m.1$, $+24°6'$; P. M. $\left\{\begin{matrix}+0''.060, -1''.11\\+0.059, -1.11\end{matrix}\right\} = 1''.38$.

Stars.	Bonn Durchmusterung.		Harvard		Standard.		Plate 220 1904, Feb. 8 Exp. 4, 5ᵐ Hr.-ang.+16ᵐ Obs. R		Plate 223 1904, Feb. 11 Exp. 4, 5ᵐ Hr.-ang.+19ᵐ Obs. R		Plate 291 1904, May 25 Exp. 4, 5ᵐ Hr.-ang.+10ᵐ Obs. R		Plate 293 1904, May 31 Exp. 4, 5ᵐ Hr.-ang.+4ᵐ Obs. R		Plate 296 1904, June 3 Exp. 4, 6ᵐ Hr.-ang.+14ᵐ Obs. R		Plate 421 1905, Jan. 26 Exp. 4, 5ᵐ Hr.-ang.+11ᵐ Obs. H		Plate 436 1905, May 22 Exp. 3, 5ᵐ Hr.-ang.+1ᵐ Obs. H			Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 220.	ξ Mean of 220 and 223.	x	o−c	x	o−c	x	o−c	x	o−c	x	o−c	x	o−c	x	Corr.	o−c	μ	π
3	+23°2691	9.3	10.20	F	8.1150	14.37256	35822	6	38690	**6**	42018	**38**	19135	**11**	32164	**74**	34740	12	12.20140	4	**69**	**0".019**	0".062
2	24.2730	9.1	9.74	F	32.7222	11.90261	88338	**5**	92184	5	82235	**3**	83812	**47**	88000	110	86671	**81**	9.72838	13	35	**054**	**074**
4	24.2731	9.1	9.58	F	20.8455	18.61506	59818	**2**	63194	2	59924	43	49931	59	58090	**37**	58375	69	16.44337	0	34	072	010
6	....	10*	.....	....	25.7043	23.32488	30694	**13**	34282	13	28568	30	23568	7	29891	8	28862	**76**	21.15295	0	44	**038**	**066**
7	....	10*	.....	....	23.2154	23.77633	75922	18	79344	**18**	74921	**67**	67520	**17**	74690	**82**	74265	89	21.60325	0	**113**	019	129
5	24.2734	9.5	9.17	K	19.6614	20.99038	97376	1	00699	**1**	98194	71	87040	29	95689	51	95872	56	18.81960	0	86	086	**036**
8	....	10*	.....	....	19.5155	26.24982	23314	**9**	26649	9	24315	**31**	13269	**17**	21849	24	21542	**68**	24.07847	3	**16**	**067**	**025**
A	24.2733	9.0	9.40	K	20.1782	20.10167	08528	34	11806	**34**	08974	24	98518	194	06869	84	07413	462	17.93515	0	528	801	065
B	....	†9.1	9.64	....	20.2485	20.34962	33269	**18**	36656	18	33792	73	23313	142	31696	95	32072	338	18.18333	0	550	712	**011**
P.M. of A and B	....	.....	.....	....	−.007	+.005																	
Average residual							B. 30*		B. 28*		42*		51*		67*		57*		48*				
a							+ 0.11		− 0.11		+ 38.85		+ 76.01		+ 48.75		− 29.97		+ 8.72				
b							− 19.40		+ 19.40		− 517.22		+ 483.39		+ 112.49		− 42.80		− 15.50				
c							−1285.		+1285.		+8440.		−23125		−6632.		−1750.		+222958.				

Plate.	Star A.			Star B.			Equations of condition.	Star A.				Star B.			
	Observed.	P. M. to 1904.5.	Corrected.	Observed.	P. M. to 1904.5	Corrected.		n	$o-c_1$	$o-c_2$	$o-c_3$	n	$o-c_1$	$o-c_2$	$o-c_3$
220	+0".060	+0".339	+0".399	−0".032	+0".319	+0".287	$x-0.395y+0.915\pi=$	+0".199	+0".051	+0".072	+0".125	+0".087	+0".003	+0".033	+0".033
223	−0.060	+0.332	+0.272	+0.032	+0.312	+0.344	$x-0.387y+0.900\pi=$	+0.072	−0.073	−0.054	−0.002	+0.144	+0.061	+0.090	+0.090
291	+0.042	+0.088	+0.130	+0.128	+0.083	+0.211	$x-0.103y-0.428\pi=$	−0.070	−0.121	−0.108	−0.144	+0.011	−0.063	−0.043	−0.043
293	+0.341	+0.074	+0.415	+0.250	+0.069	+0.319	$x-0.086y-0.512\pi=$	+0.215	+0.170	+0.183	+0.141	+0.119	+0.045	+0.065	+0.065
296	+0.148	+0.067	+0.215	+0.167	+0.063	+0.230	$x-0.078y-0.552\pi=$	+0.015	−0.027	−0.014	−0.059	+0.030	−0.044	−0.024	−0.024
421	+0.812	−0.489	+0.323	+0.594	−0.460	+0.134	$x+0.570y+0.931\pi=$	+0.123	+0.031	−0.005	+0.049	−0.066	−0.066	−0.120	−0.120
436	+0.928	−0.761	+0.167	+0.967	−0.716	+0.251	$x+0.887y-0.384\pi=$	−0.033	−0.029	−0.073	−0.107	+0.051	+0.063	−0.003	−0.003
Assumed P. M. and Δx		+0.858	+0.200	....	+0.807	+0.200	..............(pvv)	....	54002	58803	73577	....	19985	30248	30248

Normal equations.	A	B	Solution I.	A	B	Wt.	Solution II. A	B	Wt.	Solution III. A	B	Wt.
$+7.000x+0.408y+0.870\pi=$	+0".521	+0".376	$x =$	+0".070±0".031	+0".060±0".019	6.60	+0".066±0".028	+0".054±0".020	6.78	+0".074±0".028	+0".054±0".018	7.00
$+0.408x+1.442y-0.388\pi=$	−0".078	−0".096	$y =$	−0.058±0.067	−0.087±0.041	1.36	....	....	....	....	....	.....
$+0.870x-0.388y+3.411\pi=$	+0".285	+0".046	$\pi =$	+0.059±0.044	−0.012±0.027	3.17	+0.067±0.040	+0".000±0.029	3.30	....	....	.....
			$r_0 =$	±0.078	±0.048		±0.073	±0.052		±0".075	±0.048	.....

Observer's notes—291, 421, Sky thick; 296, Thin clouds.
Measurer's notes—291, Reseau poor, 296, Diffuse; 436 Exp. 1 trailed, unmeasurable; Wrongly centered, $X=+2.17$, $Y=-0.30$.
Assumed P. M. from comparison of Oxford Astrographic and Cambridge plates.
All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

*Estimated.
†From A. G. Berlin B.

TABLE C, SERIES XIX. STAR 24; A. Oe. 14318; $15^{\rm h}4^{\rm m}.7$, −15°59′; P. M. −0″.067, −3″.64, = 3″.75
25; A. Oe. 14320; $15^{\rm h}4^{\rm m}.7$, −15°54′; P. M. −0″.066, −3″.63, = 3″.74

Stars.	Southern Durchmusterung.		Harvard.		Standard.		Plate 216 1904, Feb. 4 Exp. 4, $5\frac{1}{4}^{\rm m}$ Hr.-ang. $+5^{\rm m}$ Obs. R		Plate 224 1904, Feb. 11 Exp. 4, $5\frac{1}{2}^{\rm m}$ Hr.-ang. $+10^{\rm m}$ Obs. R		Plate 297 1904, June 3 Exp. 4, $7^{\rm m}$ Hr.-ang. $+9^{\rm m}$ Obs. R		Plate 437 1905, May 22 Exp. 4, $5^{\rm m}$ Hr.-ang. $-1^{\rm m}$ Obs. H		Plate 509 1906, June 2 Exp. 4, $5^{\rm m}$ Hr.-ang. $+1^{\rm m}$ Obs. H		Plate 631 1907, Feb. 22 Exp. 6, $6\frac{1}{2}^{\rm m}$ Hr.-ang. $+7^{\rm m}$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 216.	ξ Mean of 216 and 224.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	−16° 4011	9.3	9.73	F2	10.768	8.21728	20745	49	22710	**50**	22134	94	54645	59	21638	126	29889	**18**	0″.005	**0″.142**
4	−15 4044	8.8	9.20	G8	20.630	23.60757	60260	**60**	61255	61	63604	**96**	90878	**20**	65634	**47**	70427	21	024	085
2	−15 4035	9.4	9.80	G	32.916	11.04697	04815	11	04579	**11**	08056	3	30660	**39**	13749	**78**	18214	**3**	**029**	056
5	−15 4047	6.5	6.80	G8	23.041	27.05344	05050	0	05639	1	09002	101	34831	17	11532	30	15369	**2**	**026**	**066**
8	−15 4050	7.8	7.41	K	32.080	30.10324	10496	**11**	10152	11	15191	**103**	37060	21	20476	51	22416	7	056	010
6	−16 4025	7.7	7.91	F8	13.805	27.09840	09099	20	10581	**20**	12184	**55**	42211	13	12101	**84**	17610	49	007	091
7	−15 4048	7.2	7.41	A	17.870	29.54391	53841	**10**	54941	10	57580	58	85482	**53**	58590	5	62916	**52**	**037**	**033**
A	−15 4042	9.0	8.86	G5	20.523	19.64119	63670	19	64568	**19**	66464	**226**	93409	**797**	67370	**1334**	72231	**1750**	**997**	046
B	−15 4041	9.2	9.09	K	18.814	19.62280	61754	28	62806	**28**	64395	**240**	92162	**738**	64812	**1345**	69933	**1785**	**1.008**	008
P. M. of A and B.					−.021	−.006														
Average residual							B. 42*		B. 45		87*		78*		55		85			
a							+ 6.19		− 6.19		+ 90.24		+ 23.72		+ 69.14		− 64.36			
b							+ 50.65		− 50.65		+ 125.87		− 312.59		+ 413.15		+ 249.39			
c							−1628.		+1628.		−1784.		+36029.		−5232.		+6022.			

Plate.	Observed.		P. M. to 1905.5.	Corrected.		Equations of condition.	Star A.			Star B.			Weight.
	Star A.	Star B.		Star A.	Star B.		n	$o-c_1$	$o-c_2$	n	$o-c_1$	$o-c_2$	
216	+0″.033	+0″.049	−1″.406	−1″.373	−1″.357	$x-1.406y+0.946\pi=$	+0″.067	+0″.034	+0″.030	+0″.083	+0″.051	+0.055	1
224	−0.033	−0.049	−1.386	−1.419	−1.435	$x-1.386y+0.947\pi=$	+0.021	−0.012	−0.015	+0.005	−0.026	−0.022	1
297	−0.397	−0.422	−1.078	−1.475	−1.500	$x-1.078y-0.399\pi=$	−0.035	−0.006	−0.027	−0.060	−0.065	−0.043	1
437	−1.401	−1.297	−0.113	−1.514	−1.410	$x-0.113y-0.209\pi=$	−0.074	−0.049	−0.059	+0.030	+0.043	+0.054	1
509	−2.345	−2.365	+0.918	−1.427	−1.447	$x+0.918y-0.376\pi=$	+0.013	+0.049	+0.046	−0.007	+0.030	+0.035	1
631	−3.077	−3.138	+1.646	−1.431	−1.492	$x+1.646y+0.920\pi=$	+0.009	−0.008	+0.012	−0.052	−0.017	−0.040	2
Assumed P. M. and Δx....			−1.000	−1.440	−1.440	..............(pvv)	....	6266	7739	....	10829	12699	

Normal equations.

	A.	B.
$+7.000x+0.225y+2.749\pi=$	+0″.010	−0″.053
$+0.225\ \ +11.336\ \ +0.496\ \ =$	−0.035	−0.240
$+2.749\ \ +0.496\ \ +3.830\ \ =$	+0.124	+0.009

Solution I.

	A.	B.	Weight.
$x=$	−0″.016±0″.014	−0″.012±0″.018	5.00
$y=$	−0.005±0.009	−0.022±0.012	11.27
$\pi=$	+0.045±0.019	+0.014±0.025	2.72
$r_0=$	±0.031	±0.041	

Solution II.
assuming y and π to be the same for A and B.

		Weight.
$x_A=$	−0″.010±0″.015	5.00
$x_B=$	−0.019±0.015	5.00
$y=$	−0.013±0.007	22.53
$\pi=$	+0.030±0.015	5.44
$r_0=$	±0.034	

Observer's notes—216, 224, 297, 437, Clear and very steady; 509, Clear, high wind; 631, Very fine clear night.
Measurers's notes—224, Diffuse; 297, Underexposed; 437, Fogged, diffuse; 509, Good; 631, Plate cracked; images very good.
Assumed proper-motion from the plates themselves.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XX. Star 26; Lalande 27742 } ; $15^{h}8^{m}.2$, +19°39′; P. M. −0″.043, +0″.30, =0″.68.
27; Lalande 27743 }

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 227 1904, Feb. 15 Exp. 4, 4^{m} Hr.-ang.+12^{m} Obs. R		Plate 229 1904, Feb. 18 Exp. 4, 4^{m} Hr.-ang.+15^{m} Obs. R		Plate 295 1904, May 31 Exp. 4, 4^{m} Hr.-ang.+22^{m} Obs. R		Plate 443 1904, June 14 Exp. 4, 4^{m} Hr.-ang.+14^{m} Obs. H		Plate 506 1906, May 31 Exp. 4, 4^{m} Hr.-ang.+22^{m} Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 227.	ξ Mean of 227 and 229.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	η	π
1	+19° 2930	8.2	8.82	F5	14.267	8.30542	33724	**27**	27361	26	27850	**44**	38845	39	19788	36	0″.070	0″.055
4	19 2935	5.9	5.98	Mb	14.017	16.21204	24615	19	17792	**20**	19380	39	30362	**19**	10786	**48**	**077**	**062**
2	19 2933	8.4	8.58	F	23.639	13.48501	44062	15	52940	**15**	42929	37	52731	**18**	39385	13	**021**	**039**
3	19 2934	9.1	9.14	F	22.822	14.99471	95672	**8**	03270	8	94258	**34**	04241	**15**	90309	**5**	026	049
6	20 3080	8.8	8.35	G	31.050	20.15203	04695	9	25711	**9**	07515	38	16616	103	07660	123	075	**052**
7	20 3084	9.4	10.62	F5	29.554	28.06121	96791	**15**	15450	14	99591	**42**	08860	**89**	98558	**136**	**083**	038
5	19 2937	7.4	7.14	K	16.469	19.56725	58111	3	55339	**3**	54279	17	65064	16	46975	69	046	000
8	19 2940	8.8	9.11	G	9.323	29.47940	55132	2	40748	**2**	48936	**14**	60688	**32**	37598	**58**	**039**	009
A }	19 2939	6.7	{6.83	G	20.208	19.73830	72206	45	75455	**44**	69972	**47**	79900	**457**	63820	**744**	**613**	**045**
B }			{7.63	A?	20.344	19.76163	74414	32	77912	**32**	72260	**45**	82254	**372**	66196	**721**	**594**	**075**
P. M. of A and B........			......	.....	+.002	−.003												
Average residual............							28*		40*		32*		48		47			
a............							− 2.416		+ 2.416		+ 87.6440		+ 100.11		+ 57.81			
b............							− 816.033		+ 816.033		− 364.45		− 485.05		+ 145.30			
c............							+14869.2		−14869.2		+1823.6		+14353.		−13343.			

Plate.	Observed.		P. M. to 1905.0.	Corrected.		Equations of condition.	Star A.				Star B.			
	Star A.	Star B.		Star A.	Star B.		n	$o-c_1$	$o-c_2$	$o-c_3$	n	$o-c_1$	$o-c_2$	$o-c_3$
227	+0″.077	+0″.056	−0″.499	−0″.422	−0″.443	$x-0.876y+0.945\pi=$	+0″.078	+0″.069	+0″.090	+0″.069	+0″.057	+0″.054	+0″.033	+0″.013
229	−0.079	−0.056	−0.492	−0.571	−0.548	$x-0.864y+0.936\pi=$	−0.071	−0.081	−0.060	−0.080	−0.048	−0.053	−0.074	−0.094
295	−0.079	−0.083	−0.334	−0.413	−0.417	$x-0.586y-0.341\pi=$	+0.087	+0.041	+0.035	+0.078	+0.083	−0.012	−0.006	+0.037
443	−0.803	−0.684	+0.257	−0.546	−0.397	$x+0.451y-0.540\pi=$	−0.046	−0.060	−0.076	−0.055	+0.103	+0.017	+0.034	+0.057
506	−1.308	−1.268	+0.805	−0.503	−0.463	$x+1.413y-0.330\pi=$	−0.003	+0.029	+0.010	−0.012	+0.037	−0.008	+0.011	−0.009
Assumed P. M. and Δx...			−0.570	−0.500	−0.500	...............(pvv)		17444	18801	20414		6222	7878	13332

Normal equations.	A.	B.	Solution I.	A.	B.	Weight.	Solution II. Assuming y and π the same for A and B. A.	B.	Weight.	Solution III. Assuming $y=0$ $\pi=0$. A.	B.
$+5.000x-0.462y+0.670\pi=$	+0″.045	+0″.232	$x=$	+0″.011±0″.028	+0″.054±0″.017	4.79	+0″.014±0″.021	+0″.051±0″.021	4.79	+0″.009±0″.021	0″.046±0″.017
$-0.462\ +4.056\ -2.147\ =$	−0.083	+0.040	$y=$	−0.040±0.043	−0.025±0.026	2.03	−0.032	±0.022	4.06	....	....
$+0.670\ -2.147\ +2.286\ =$	+0.004	−0.087	$\pi=$	−0.039±0.058	−0.077±0.036	1.11	−0.058	±0.030	2.22	....	....
			$r_0=$	±0.061	±0.038			±0.045		±0.048	±0.039

Observer's notes—506, Probably bits of thin cloud.
Measurer's notes—443, Plate spattered with fine drops; 506, good.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXI. STAR 28; W. B. 15^h, 716; $15^h32^m.4$, $+40°10'$ } ; P. M. $-0''.041$, $+0''.08$, $=0''.48$.
29; W. B. 15^h, 720; $15^h32^m.5$, $+40° 8'$

Stars.	Born Durchmusterung.		Harvard.		Standard.		Plate 217 1904, Feb. 4 Exp. 3, 4^m H.-ang.+10^m Obs. R		Plate 222 1904, Feb. 8 Exp. 4, 4^m H.-ang.+11^m Obs. R		Plate 225 1904, Feb. 11 Exp. 4, 4^m Hr.-ang.+14^m Obs. R		Plate 298 1904, June 3 Exp. 4, 5^m Hr.-ang.+17^m Obs. R		Plate 301 1904, June 18 Exp. 4, 5^m Hr.-ang.+12^m Obs. R		Plate 444 1905, June 14 Exp. 4, 4^m Hr.-ang.+13^m Obs. H		Plate 649 1907, Mar. 13 Exp. 3, 5^m Hr.-ang.+24^m Obs. H		Plate 704 1907, June 22 Exp. 4, 5^m Hr.-ang.+17^m Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 222.	ξ Mean of 222 and 225.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
4	+39°2888	8.8	9.02	A8	12.876	13.61724	67310	69	64578	11	58871	**10**	53111	11	55491	**26**	68325	17	78103	76	71849	51	0".034	0".042
5	40 2902	9.1	9.84	F8	17.870	17.09002	13023	**29**	10961	**24**	07042	23	03541	**54**	05409	**36**	16434	**93**	23815	**125**	17844	**19**	**025**	004
2	40 2900	9.0	9.11	K	22.866	10.11032	13440	**61**	12112	12	09952	**12**	08484	47	09438	101	18838	72	24325	97	18230	31	019	**060**
3	40 2901	9.0	9.48	F8	30.823	12.14202	14317	22	13884	0	14519	**1**	16592	**4**	16426	**38**	23196	6	25048	**48**	19214	**62**	**027**	010
6	40 2906	8.5	8.96	K	30.311	23.22333	22573	**126**	22139	**21**	22526	20	24886	42	25168	**20**	31949	**79**	33680	**98**	27998	**55**	**037**	**050**
8	40 2908	9.0	8.98	G	31.299	31.31035	31355	165	30740	8	31331	**7**	34386	**85**	34931	**42**	41436	1	42523	47	36940	87	045	097
7	39 2892	8.8	9.49	G	9.538	23.40120	46757	8	43574	**24**	36665	23	29911	92	33174	117	47016	92	57798	26	51609	50	**013**	**106**
9	39 2896	8.7	9.02	K	12.118	32.98976	04883	**47**	02086	35	95866	**35**	90590	**50**	93819	**53**	06798	**14**	16227	24	10062	**81**	006	060
A	40 2903	7.1	7.78	K	20.465	19.61494	64797	9	63035	3	59952	**3**	57692	**94**	59269	**99**	69230	**330**	74995	**752**	68911	**845**	**429**	c20
B	40 2904	6.8	6.83	G5	19.879	19.98374	01865	16	00025	17	96724	**16**	94209	**111**	95869	**176**	05992	**390**	11998	**818**	05904	**903**	**450**	060
P.M. of A and B . . .	. . .	. . . .	. . . .	. . . .	. . . . . .	−.002																		
									B.		B.													
Average residual							57		33*		29*		53*		63*		32		31		68			
a							+ 10.73		+ 5.09		− 5.09		+ 39.04		+ 75.61		+ 70.13		+ 35.95		+ 45.67			
b							− 301.42		− 175.72		+175.72		+ 617.20		+ 478.08		+ 139.73		− 298.50		− 274.90			
c							+9253.		+5035.		−5035.		−17104.		−13386.		+3830.		+19658.		+12992.			

Plate.	Observed.		P. M. to 1905.0.	Corrected.		Equations of condition.	Star A.		Star B.	
	Star A.	Star B.		Star A.	Star B.		n	$o-c$	n	$o-c$
217	+0".016	+0".028	−0".383	−0".367	−0".355	$x-0.904y+0.949\pi=$	+0".033	+0".005	+0".045	+0".024
222	+0.005	+0.030	−0.379	−0.374	−0.349	$x-0.895y+0.952\pi=$	+0.026	−0.002	+0.051	+0.031
225	−0.005	−0.028	−0.376	−0.381	−0.404	$x-0.886y+0.956\pi=$	+0.019	−0.009	−0.004	−0.024
298	−0.165	−0.195	−0.245	−0.410	−0.440	$x-0.578y-0.295\pi=$	−0.010	−0.011	−0.040	+0.022
301	−0.174	−0.309	−0.228	−0.402	−0.537	$x-0.537y-0.517\pi=$	−0.002	+0.002	−0.137	−0.061
444	−0.580	−0.686	+0.191	−0.389	−0.495	$x+0.451y-0.457\pi=$	+0.011	+0.020	−0.095	+0.007
649	−1.322	−1.438	+0.932	−0.390	−0.506	$x+2.198y+0.861\pi=$	+0.010	+0.004	−0.106	−0.029
704	−1.485	−1.587	+1.048	−0.437	−0.539	$x+2.473y-0.563\pi=$	−0.037	−0.013	−0.139	+0.029
Assumed P. M. and Δx . . .			−0.424	−0.400	−0.400	. . . . (pvv)	. . . .	820	. . . .	8049

Normal equations.

	A.	B.
$+8.000x+1.322y+1.886\pi$	$=+0''.050$	$-0''.425$
$+1.322\ +14.176\ -1.815$	$=-0.127$	-0.605
$+1.886\ -\ 1.815\ +4.342$	$=+0.103$	$+0.191$

Solution.

	Star A.	Star B.	Weight.
$x=$	$+0''.003\pm0''.003$	$-0''.062\pm0''.010$	6.85
$y=$	-0.007 ± 0.002	-0.029 ± 0.008	12.80
$\pi=$	$+0.020\pm0.005$	$+0.059\pm0.014$	3.57
$r_0=$	±0.009	±0.027	

Observer's notes—217, Clouds about, dawn; 222, Clouds at end; 298, Clear and very steady; 301, Control working badly; 649, High wind, seeing unsteady; 704, Seeing bad.
Measurer's notes—217, Underexposed; 444, Plate spattered with fine drops; 649, Plate badly cracked, and fogged; images good.
All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXII. STAR 30; W. B. 17ʰ, 322; 17ʰ20ᵐ8, +2°14′; P. M. −0″040, −1″22, =1″36.

Stars.	Bonn Durchmusterung.		Harvard.		Plate 271 (Stand.) 1904, May 2 Exp. 4, 5ᵐ Hour-angle +9ᵐ Obs. R		Plate 263 1904, Apr. 19 Exp. 4, 5ᵐ Hr.-ang.+12ᵐ Obs. R		Plate 304 1904, June 21 Exp. 4, 5ᵐ Hr.-ang. +7ᵐ Obs. R		Plate 317 1904, July 6 Exp. 4, 6ᵐ Hr.-ang.+15ᵐ Obs. R		Plate 320 1904, July 8 Exp. 4, 5ᵐ Hr.-ang. +4ᵐ Obs. R		Plate 430 1905, May 8 Exp. 4, 4½ᵐ Hr.-ang.− 9ᵐ Obs. H		Plate 431 1905, May 8 Exp. 4, 4ᵐ Hr.-ang. +18ᵐ Obs. H		Plate 494 1906, Apr. 30 Exp. 3, 5ᵐ Hr.-ang. −5ᵐ Obs. H		Plate 496 1906, May 3 Exp. 2, 5ᵐ Hr.-ang. +8ᵐ Obs. H		Plate 537 1906, July 20 Exp. 4, 4ᵐ Hr.-ang. +9ᵐ Obs. H		Plate 538 1906, July 20 Exp. 4, 4ᵐ Hr.-ang.+30ᵐ Obs. H		Approximate Solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	+1° 3428	9.5	10.77	.....	13.600	8.68954	75179	56	72860	67	76226	46	81920	68	80819	65	85844	48	83842	92	89521	00	78325	40	82934	16	0″011	0″020
2	2 3308	9.3	10.28	.....	27.514	15.41700	34006	**69**	32256	**61**	33278	**121**	31961	**112**	29744	75	32991	**99**	40760	52	40771	109	38972	3	33910	**46**	010	000
3	2 3309	8.6	9.04	G5	28.896	18.22234	13205	9	11570	10	12540	67	10511	44	07955	6	11065	**44**	19627	42	19222	42	18311	**44**	12385	31	**021**	**022**
5	2 3314	8.5	8.78	G2	18.651	22.05382	06395	25	04514	49	07474	40	10785	102	08395	37	12713	74	13995	73	17624	32	10678	48	11685	75	022	**027**
6	2 3318	9.1	9.23	G2?	23.942	25.78386	74144	37	72469	20	74591	15	75151	**29**	72362	31	75908	**19**	80917	28	82454	31	79069	6	76312	69	**009**	051
4	1 3433	9.4	10.25	.....	13.725	20.20560	26404	22	24372	50	28111	**25**	33862	**6**	31888	25	36844	27	34553	76	40220	78	30088	8	34452	**33**	**032**	**041**
7	2 3321	9.0	9.56	G5?	14.939	31.54104	58564	36	56635	**117**	00760	24	65954	**69**	63129	41	67729	**77**	66233	**21**	71454	16	02846	**34**	66294	17	021	022
A	2 3312	8.0	7.82	Ma	19.899	20.07951	07766	18	05766	**76**	08350	**152**	10986	**125**	08524	**343**	12672	**346**	14605	**625**	17724	**674**	11292	**766**	11449	**770**	**559**	092
P. M. of A	.....	.....	.....	.....	−.007	−.003																						
Average residual					46		61		59		58		50		52		49		22		37		54		52			
a					0.00		− 15.54		+ 3.66		+ 42.73		+ 53.50		− 20.43		− 31.00		− 51.16		− 50.40		+ 27.32		+ 13.34			
b					0.00		− 984.41		− 951.17		− 1137.24		− 1645.25		− 1692.52		− 1814.65		− 1103.04		− 1509.00		− 880.62		− 1567.27			
c					0.		+19691.		+16742.		+22322.		+34809.		+34996.		+41790.		+30242.		+41468.		+21070.		+35166.			

Plate.	Observed.	P. M. to 1905.0.	Corrected.	Equations of condition.	n	$o-c$	Weight.
263	+0″032	−0″421	−0″389	$x-0.701y+0.785\pi=$	+0″011	+0″015	1
271	0.000	−0.399	−0.399	$x-0.665y+0.631\pi=$	+0.001	+0.018	1
304	−0.134	−0.317	−0.451	$x-0.529y-0.156\pi=$	−0.051	+0.035	1
317	−0.267	−0.293	−0.560	$x-0.488y-0.399\pi=$	−0.160	−0.053	1
320	−0.220	−0.289	−0.509	$x-0.482y-0.430\pi=$	−0.109	+0.001	1
430	−0.603	+0.210	−0.393	$x+0.350y+0.553\pi=$	+0.007	−0.015	1
431	−0.608	+0.210	−0.398	$x+0.350y+0.553\pi=$	+0.002	−0.020	1
494	−1.099	+0.797	−0.302	$x+1.328y+0.664\pi=$	+0.098	+0.021	1
496	−1.185	+0.802	−0.383	$x+1.336y+0.625\pi=$	+0.017	−0.056	½
537	−1.347	+0.931	−0.416	$x+1.552y-0.598\pi=$	−0.016	+0.015	1
538	−1.354	+0.931	−0.423	$x+1.552y-0.598\pi=$	−0.023	+0.009	1
Asumed P.M and △ x	−0.600	−0.400	...............(pvv)	....	7524		

Normal equations.

$$\begin{aligned} +10.500x+2.935y+1.317\pi &= -0''.232 \\ +\ 2.935\ +9.402\ -0.656\ &= +0.232 \\ +\ 1.317\ -0.656\ +3.345\ &= +0.227 \end{aligned}$$

Solution.

		Weight.
x =	−0″047±0″007	8.82
y =	+0.046±0.007	8.25
π =	+0.095±0.012	3.06
r_0 =	±0.021	

Observer's notes—304, Sky pretty thick; 430, Very clear night; guide star drifts about curiously. (Refraction?) Control good; 537, 538, Occasional small clouds.

Measurer's notes—263, Diffuse; 304, Underexposed; 537, Diffuse, fogged.

All quantities in this table are in réseau intervals of 175″8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C. SERIES XXIII. STAR $\left.\begin{matrix}31\\32\end{matrix}\right\}$; Pos. Med. 2164; $18^h41^m.7$, $+59°29'$; P.M. $-0''.171$, $+1''.87$, $=2''.27$.

Stars.	Bonn Durchmusterung.		Harvard.		Plate 281 (Stand.) 1904, May 17 Exp. 4, $4\frac{1}{2}^m$ Hour-angle -9^m Obs. R		Plate 288 1904, May 19 Exp. 4, 4^m Hr.-ang. $+2^m$ Obs. R		Plate 363 1904, Aug. 27 Exp. 3, 5^m Hr.-ang. $+19^m$ Obs. R		Plate 366 1904, Aug. 29 Exp. 4, 5^m Hr.-ang. $+21^m$ Obs. R		Plate 438 1905, May 27 Exp. 4, $4\frac{1}{2}^m$ Hr.-ang. -21^m Obs. H		Plate 439 1905, May 27 Exp. 4, 4^m Hr.-ang. $+1^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	…………	……	10.02	……	18.039	13.59274	75039	**29**	50410	117	51839	33	63654	6	65756	**16**	0″.005	**0″.086**
4	+59° 1914	9.5	9.65	K?	14.750	18.96609	12762	11	88112	**35**	89694	**13**	01528	29	03640	36	049	039
2	59 1912	9.5	9.83	F	26.118	13.81640	96834	**66**	72373	**88**	73792	**90**	84790	**85**	86949	**67**	**068**	052
3	59 1913	9.4	9.66	F?	19.786	14.98204	14000	85	89292	5	90852	70	02401	55	04518	48	017	**004**
5	59 1916	9.4	9.68	……	21.941	20.57562	73192	**72**	49033	**4**	50472	**45**	61455	**4**	63538	**34**	030	**001**
8	59 1921	7.3	7.34	G6	22.141	27 76620	92536	53	68752	86	70221	65	80601	39	82710	52	030	**049**
6	59 1918	8.7	8.48	K	16.962	25.33434	49576	**7**	25360	**63**	26925	**43**	38091	13	40179	5	028	060
7	59 1919	7.8	7.69	K	19.548	27.36659	52709	25	28722	**21**	30282	21	40912	**49**	43032	**23**	**084**	**010**
A	59 1915	8.9	9.33	K5	20.091	20.08452	24312	47	99590	**347**	00972	**465**	11898	**704**	13962	**752**	**1.256**	295
B			10.01	K5	20.004	20.13265	29195	110	04348	**408**	05828	**429**	16729	**698**	18735	**804**	**1.348**	305
P. M.	of A and B……	……	……	……	+.011	−.007												
Average residual……					44		66		35		42		45		42			
a……					0.00		+ 24.11		+ 80.20		+ 81.82		+ 10.20		+ 7.87			
b……					0.00		− 66.64		− 26.73		− 38.12		− 140.47		− 139.14			
c……					0.		+16668.		−9589.		−7893.		+6769.		+8901.			

Plate.	Observed.		P. M. to 1905.0.	Corrected.		Equations of condition.	Star A.			Star B.			Weight.
	Star A.	Star B.		Star A.	Star B.		n	$o-c_1$	$o-c_2$	n	$o-c_1$	$o-c_2$	
281	0″.000	0″.000	−0″.812	−0″.812	−0″.812	$x-0.624y+0.689\pi=$	+0″.188	−0″.047	−0″.070	+0″.188	−0″.103	−0″.082	1
288	+0.082	+0.193	−0.806	−0.724	−0.613	$x-0.619y+0.664\pi=$	+0.276	+0.047	+0.026	+0.387	+0.104	+0.126	1
363	−0.610	−0.717	−0.449	−1.059	−1.166	$x-0.345y-0.818\pi=$	−0.059	+0.131	+0.129	−0.166	+0.016	+0.019	½
366	−0.818	−0.754	−0.443	−1.261	−1.197	$x-0.340y-0.837\pi=$	−0.261	−0.065	−0.067	−0.197	−0.008	−0.006	1
438	−1.237	−1.226	+0.525	−0.712	−0.699	$x+0.403y+0.561\pi=$	+0.288	+0.043	+0.067	+0.301	+0.095	+0.070	1
439	−1.323	−1.413	+0.525	−0.798	−0.888	$x+0.403y+0.561\pi=$	+0.202	−0.042	−0.018	+0.112	−0.095	−0.118	1
Assumed P. M. and Δx…			−1.302	−1.000	−1.000	……………(pvv)	………	20836	23199	………	39667	41640	

Normal equations.

	A.	B.
$+5.500x-0.949y+1.229\pi=$	$+0''.663$	$+0''.708$
$-0.949\ \ +1.272\ \ +0.037\ \ =$	$+0.008$	-0.095
$+1.229\ \ +0.037\ \ +2.579\ \ =$	$+0.830$	$+0.852$

Solution I.	A.	B.	Wt.	Solution II. A.	B.	Wt.
$x=$	$+0''.063\pm0''.028$	$+0''.053\pm0''.038$	4.18	$+0''.054\pm0''.023$	$+0''.062\pm0''.031$	4.91
$y=$	$+0.044\pm0.054$	-0.044 ± 0.075	1.08	……………	……………	….
$\pi=$	$+0.291\pm0.038$	$+0.306\pm0.052$	2.25	$+0.296\pm0.034$	$+0.302\pm0.045$	2.30
$r_0=$	±0.056	±0.078		±0.051	±0.069	

Observer's notes—363, Thin clouds.
Measurer's notes—288, Diffuse; 363, Images good, but "runs" on réseau very irregular; exposure 4 too faint to measure.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXIV. STAR 33; Lam. 18180; $18^h53^m.1$, $+5°48'$; P. M. $-0^s.016$, $-1''.22$, $=1''.24$

Stars.	Bonn Durchmusterung.		Harvard.		Plate 299 (Stand.) 1904, June 3 Exp. 4, 5^m Hour-angle $+9^m$ Obs. R		Plate 300 1904, June 17 Exp. 4, 5^m Hr.-ang. $+21^m$ Obs. R		Plate 354 1904, Aug. 24 Exp. 4, 5^m Hr.-ang. $+17^m$ Obs. R		Plate 367 1904, Aug. 29 Exp. 4, 5^m Hr.-ang. $+37^m$ Obs. R		Plate 440 1905, June 13 Exp. 4, 4^m Hr.-ang. -20^m Obs. H		Plate 447 1905, June 22 Exp. 4, 4^m Hr.-ang. $+0^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
2	+5° 3989	9.1	9.40	F4	20.159	18.36590	37135	**35**	44900	**16**	37208	**10**	49899	**31**	50866	**41**	**0″.042**	**0″.007**
3	5 3990	9.4	9.78	A	16.488	18.92952	95366	50	00838	74	97518	17	08131	58	09606	102	104	**022**
1	5 3987	8.4	8.13	K	21.831	15.63452	63154	**15**	72012	10	62181	**23**	75990	**16**	76655	**44**	**044**	**006**
4	5 3991	9.1	9.46	A	23.628	19.13139	12058	0	21885	**69**	10129	20	24769	**12**	25359	**12**	**015**	034
5	5 3994	9.3	9.46	A8	21.148	20.02058	02160	**33**	10522	**8**	01616	**79**	14889	**8**	15796	**15**	005	049
7	5 3998	8.6	8.78	A	23.797	23.77398	76375	48	86325	66	74439	85	88940	34	89696	73	053	**078**
6	5 3996	8.8	8.62	G5	15.950	21.70828	73518	11	78508	**69**	76024	**23**	86154	**27**	87734	**32**	**047**	064
8	5 4005	9.0	9.05	A	18.396	27.68831	70422	**27**	76958	8	71681	15	82926	**1**	84389	**33**	**019**	**034**
A	5 3993	9.3	9.14	Ma	20.126	19.76305	77100	172	84632	0	76958	**64**	89580	**68**	90572	**99**	**230**	092
P. M. of A.........	....	....	....	....	−.007	−.001												
Average residual......................					40		53		37		41		40		38			
a......................................					0.00		+ 19.96		+ 4.32		+ 35.71		− 10.97		+ 20.97			
b......................................					0.00		− 483.20		+ 140.60		− 1062.43		− 487.29		− 606.13			
c......................................					0.		+9954.		+5412.		+21392.		+23364.		+26150.			

Plate.	Observed.	P. M. to 1905.0.	Corrected.	Equations of condition.	n	$o-c_1$	$o-c_2$	$o-c_3$	Weight.
299	0″.000	−0″.138	−0″.138	$x-0.578y+0''.498\pi=$	−0″.143	−0″.099	−0″.110	−0″.065	1
300	+0.302	−0.129	+0.173	$x-0.540y+0.282\pi=$	+0.168	+0.228	+0.218	+0.246	½
354	0.000	−0.085	−0.085	$x-0.354y-0.758\pi=$	−0.090	+0.046	+0.045	−0.011	1
367	−0.113	−0.081	−0.194	$x-0.340y-0.810\pi=$	−0.199	−0.061	−0.061	−0.121	1
440	−0.120	+0.107	−0.013	$x+0.449y+0.346\pi=$	−0.018	+0.018	+0.028	+0.061	1
447	−0.174	+0.113	−0.061	$x+0.474y+0.201\pi=$	−0.066	−0.019	−0.009	+0.012	1
Assumed P. M. and Δx		−0.239	+0.005	...............(pvv)	....	42315	42473	53110	

Normal equations.

$+5.500x-0.619y-0.382\pi=-0''.432$
$-0.619\ \ +1.147\ \ +0.429\ \ =+0.099$
$-0.382\ \ +0.429\ \ +1.678\ \ =+0.163$

Solution I.

	Weight.
$x=-0''.071\pm0''.035$	5.15
$y=+0.019\pm0.081$	0.99
$\pi=+0.076\pm0.065$	1.51
$r_0=\ \ \pm0.080$	

Solution II.

	Weight.
$x=-0''.073\pm0''.030$	5.41
....	....
$\pi=+0.080\pm0.054$	1.65
$r_0=\ \ \pm0.070$	

Solution III.

	Weight.
$x=-0''.078\pm0''.030$	5.50
....	
....	
$r_0=\ \ \pm0''.070$	

Observers' notes—300, Control failed for one exposure (not measured); 354, Drifting clouds; 367, Very clear and steady.

Measurer's notes—300, Underexposed; A very faint; 367, Images very good; Star A appears to have a faint companion, in about 250°, 1″.5.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXV. STARS 34, 35; Groombridge 2789; $19^h 9^m.5$, +49°40′; P. M. −0″.019, +0″.62, =0″.64.

Stars.	Bonn Durchmusterung.		Harvard.		Plate 310 (Stand.) 1904, June 28 Exp. 4, 5^m Hour-angle $+14^m$ Obs. R		Plate 305 1904, June 21 Exp. 4, 5^m Hr.-ang. $+19^m$ Obs. R		Plate 355 1904, Aug. 24 Exp. 4, 5^m Hr.-ang. $+41^m$ Obs. R		Plate 364 1904, Aug. 27 Exp. 4, 5^m Hr.-ang. $+25^m$ Obs. R		Plate 441 1905, June 13 Exp. 4, 4^m Hr.-ang. -8^m Obs. H		Plate 442 1905, June 13 Exp. 4, 4^m Hr.-ang. $+12^m$ Obs. H		Plate 448 1905, June 22 Exp. 4, 4^m Hr.-ang. $+8^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
2	+49° 2947	8.0	8.12	K	16.924	12.97972	01119	**2**	04642	0	91463	**46**	09680	**32**	09241	**17**	13461	**33**	**0″.053**	0″.030
4	49 2957	9.2	9.62	A	12.400	18.82909	86869	**42**	90640	83	74840	112	96519	107	96210	81	03421	56	200	**176**
1	50 2734	7.4	7.14	A	31.001	8.66305	66095	**37**	69459	**84**	65172	**70**	71930	**88**	70942	**103**	66290	**1**	**112**	104
3	49 2952	8.3	8.84	A	25.108	17.01889	02871	80	06376	**4**	98598	6	09886	15	09160	37	08259	**34**	**044**	042
6	49 2968	6.3	6.50	G	24.729	30.75718	76010	**43**	80009	87	72398	65	83409	77	82678	63	82388	30	151	**143**
5*	49 2961	8.8	9.19	G8	18.139	22.20768	23162	**5**	26818	**56**	14765	**45**	31495	**126**	31046	**88**	34760	**60**	**170**	062
7	49 2969	7.3	7.40	G8	11.472	34.23615	27156	50	31036	**29**	15120	**22**	36978	51	36719	21	44932	39	023	084
A	49 2959	6.2	6.84	K	20.007	20.13874	15880	**25**	19565	0	08584	**43**	23885	**91**	23265	**154**	25749	**130**	**209**	**011**
B			6.62		20.048	20.17132	19235	84	22809	**4**	11884	**18**	27191	**25**	26655	**1**	28950	**144**	**162**	046
P. M. of A and B	......		......	.....	+.003	−.001														
Average residual					44		50		45		42		33		50		38			
a					0.00		− 48.23		− 27.89		+ 4.20		− 38.88		− 37.74		− 13.38			
b					0.00		− 250.76		− 252.31		+ 384.98		− 440.03		− 476.69		− 1107.76			
c					0.		+8019.		+11302.		−13034.		+19691.		+19844.		+34444.			

*N. f. component of double.

Plate.	Observed.		P. M. to 1905.0.	Corrected.		Equations of condition.	Star A.				Star B.				Weight
	Star A.	Star B.		Star A.	Star B.		n	$o-c_1$	$o-c_2$	$o-c_3$	n	$o-c_1$	$o-c_2$	$o-c_3$	
305	−0″.044	+0″.148	−0″.086	−0″.130	+0″.062	$x-0.529y+0.278\pi=$	−0″.030	−0″.033	+0″.001	−0″.011	+0″.062	+0″.073	+0″.087	+0″.105	½
310	.000	.000	−0.082	−0.082	−0.082	$x-0.509y+0.164\pi=$	+0.018	+0.015	+0.044	+0.037	−0.082	−0.061	−0.050	−0.039	1
355	0.000	−0.007	−0.057	−0.057	−0.064	$x-0.354y-0.711\pi=$	+0.043	+0.039	+0.035	+0.062	−0.064	+0.025	+0.023	−0.021	1
364	−0.076	−0.032	−0.056	−0.132	−0.088	$x-0.345y-0.745\pi=$	−0.032	−0.037	−0.040	−0.013	−0.088	+0.004	+0.002	−0.045	1
441	−0.160	−0.044	+0.072	−0.088	+0.028	$x+0.449y+0.406\pi=$	+0.012	+0.059	+0.046	+0.031	+0.028	+0.049	+0.044	+0.071	1
442	−0.271	−0.002	+0.073	−0.198	+0.071	$x+0.449y+0.406\pi=$	−0.098	−0.052	−0.063	−0.079	+0.071	+0.093	+0.087	+0.114	1
448	−0.229	−0.253	+0.077	−0.152	−0.176	$x+0.474y+0.266\pi=$	−0.052	−0.006	−0.022	−0.033	−0.176	−0.144	−0.151	−0.133	1
Assumed P.M. and Δx....			−0.162	−0.100	0	................(pvv)	....	9880	11331	13734	....	38812	39124	45225	

Normal equations. A.	B.	Solution I. A.	B.	Weight.	Solution II. A.	B.	Weight.	Solution III. A.	B.
$+6.500x-0.100y-0.075\pi=-0''.124$	−0″.280	$x=-0''.020\pm0''.013$	−0″.043±0″.026	6.49	−0″.020±0″.013	−0″.042±0″.023	6.50	−0″.019±0″.013	−0″.043±0″.023
$-0.100\ +1.271\ +0.842\ =-0.069$	+0.041	$y=-0.049\pm0.037$	−0.020±0.074	0.80	....	....		....	....
$-0.075\ +0.842\ +1.528\ =-0.057$	+0.100	$\pi=-0.011\pm0.034$	+0.075±0.067	0.97	−0.038±0.026	+0.064±0.048	1.52	....	....
		$r_0=\quad\pm0.034$	±0.067		±0.032	±0.060		±0.032	±0.059

Observer's notes—305, 364, Thin clouds. Measurer's notes—305, Diffuse, underdeveloped; 364, Fogged, images very good.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXVI. STAR 36; B. D.+30°.3639; 19h30m.9, +30°18′.

Stars.	Bonn Durchmusterung.		Harvard.		Plate 449 (Stand.) 1905, June 22 Exp. 4, 4m Hour-angle +17m Obs. H		Plate 311 1904, Jun 28 Exp. 4, 5m Hr.-ang.+31m Obs. R		Plate 313 1904, July 1 Exp. 4, 5m Hr.-ang.+29m Obs. R		Plate 371 1904, Oct. 3 Exp. 4, 5m Hr.-ang. −8m Obs. H		Plate 372 1904, Oct. 3 Exp. 4, 5m Hr.-ang.+19m Obs. H		Plate 373 1904, Oct. 15 Exp. 4, 5m Hr.-ang.+30m Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	+30° 3624	9.4	10.13	A8	20.4602	13.86889	73798	86	71388	**3**	71371	**7**	71718	**50**	71628	**19**	**0".082**	0".077
2	30 3629	8.7	8.84	A	18.7827	15.22274	08350	**47**	05639	**22**	06005	10	07178	14	06820	5	069	**049**
3	30 3635	9.3	9.83	K	28.3722	18.88850	79288	**36**	79130	23	77065	**4**	73799	35	74622	16	016	**030**
4	30 3641	9.0	9.50	K	28.4995	21.06452	97086	35	96868	**24**	94754	4	91346	**36**	92204	**16**	**014**	028
5	30 3646	8.2	8.30	G6	20.3080	24.09580	96636	**32**	94412	**6**	94114	20	94562	28	94352	56	042	**069**
6	30 3652	9.1	9.90	K	18.8803	27.47349	33910	**6**	31362	29	31206	**24**	32336	8	31835	**41**	**025**	039
A	30 3639	9.3	9.85	O	22.8955	20.30061	18200	34	16552	6	15795	51	14978	**8**	15166	40	**034**	**020**
Average residual					28		55		55		33		40		20			
a					0.00		+ 32.30		+ 43.26		+ 9.39		+ 7.34		− 2.27			
b					0.00		+ 440.84		+ 699.91		+ 465.51		− 0.29		+ 127.53			
c					0.		−22645.		−30418.		−25166.		−15216.9		−17819.			

Plate.	Observed.	P. M. to 1905.0.	Equations of condition.	n	$o-c_1$	$o-c_2$	$o-c_3$
311	+0".060	0".000	$x-0.510y+0.249\pi=$	+0".060	+0".024	+0".012	+0".024
313	+0.011	0.000	$x-0.501y+0.200\pi=$	+0.011	−0.025	−0.036	−0.025
371	+0.090	0.000	$x-0.244y-0.969\pi=$	+0.090	+0.041	+0.052	+0.054
372	−0.014	0.000	$x-0.244y-0.969\pi=$	−0.014	−0.063	−0.052	−0.050
373	+0.070	0.000	$x-0.211y-0.985\pi=$	+0.070	+0.022	+0.034	+0.034
449	0.000	0.000	$x+0.474y+0.346\pi=$	0.000	−0.001	−0.009	−0.036
Assumed P. M. . .		0.000	. . . . (pvv)	. . . .	7336	8085	9069

Normal equations.

$+6.000x-1.236y-2.128\pi=+0''.217$

$-1.236\quad+0.900\quad+0.618\quad=-0.070$

$-2.128\quad+0.618\quad+3.070\quad=-0.125$

Solution I.

		Wt.
x	$=+0''.023\pm0''.017$	3.68
y	$=-0.034\pm0.042$	0.62
π	$=-0.018\pm0.022$	2.26
$r_0=$	±0.034	

	Solution II.	Wt.	Solution III.	Wt.
x =	$+0''.028\pm0''.015$	4.30	$+0''.036\pm0''.012$	6.00
y =	-0.039 ± 0.038	0.64	. . . .	
π =	. . . .		. . . .	
r_0 =	±0.030		$\pm0''.029$	

Observer's notes—371, Control suspicious; 373, Very thin cloud.
Measurer's notes—371, 372, 373, Very good; 449, Images elongated vertically.
All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXVII.

STAR 37; 61^1 Cygni ; } 21^h 2^m.4, + 38° 15′; P. M. { +0″.352, + 3″.24, =5″.27
38; 61^2 Cygni ; } 21^h 2^m.4, + 38° 15′; P. M. { +0.351, + 3.07, =5.15
39; B. D. +38°,4362; 21^h 5^m.2, + 38° 19′; P. M. +0.005, + 0.02, =0.06

Stars.	Bonn Durchmusterung. No.	Bonn Durchmusterung. Mag.	Harvard. Mag.	Harvard. Sp.	Standard. η Plate 180.	Standard. ξ Mean of 173 and 179.	Plate 173 1903, Nov. 18 Exp. 4, 2½^m Hr.-ang. +44^m Obs. R x	o−c	Plate 179 1903, Nov. 23 Exp. 4, 2^m Hr.-ang. +44^m Obs. R x	o−c	Plate 180 1903, Nov. 24 Exp. 6, 2^m Hr.-ang. +16^m Obs. R x	o−c	Plate 322 1904, July 8 Exp. 4, 3^m Hr.-ang. +6^m Obs. R x	o−c	Pl. 324 1904, July 10 Exp. 4, 3^m Hr.-ang +5^m Obs. R x	o−c	Pl. 326 1904, July 11 Exp. 4, 3^m Hr-ang. +11^m Obs. R x	o−c	Pl. 328 1904, July 13 Exp. 4, 3^m Hr.-ang. +4^m Obs. R x	o−c	Pl. 454 1905, Nov. 6 Exp. 4, 3^m Hr.-ang. +2^m Obs. H x	o−c	Pl. 455. 1905, Nov. 6. Exp 4, 3^m Hr.-ang. +20^m Obs. H. x	o−c	Pl. 531. 1906, July 3. Exp. 4, 3^m Hr.-ang. −1^m Obs. H. x	o−c	Pl. 535. 1906, July 14. Exp. 4, 3^m Hr.-ang. −2^m Obs. H. x	o−c	Approximate solution. μ	π
2	+37° 4170	8.5	......	G4	13.529	14.36547	57602	12	15492	12	98641	121	63026	64	59434	34	62145	30	59332	22	72702	67	63351	50	13018	95	61786	6	0″.019	0″.034
5	+37 4173	8.8	......	G5	17.243	17.45302	66885	12	23719	12	03321	50	68584	64	65089	22	67588	60	64898	10	74116	46	68349	56	19696	27	66508	54	023	011
1	+38 4336	8.8	9.40	F5	22.800	14.27876	50275	8	05476	9	80162	22	45946	62	42666	31	45004	10	42066	93	45596	87	45288	72	98979	85	42962	52	047	022
3	+38 4337	9.0	9.42	G	30.672	15.12749	36268	4	89230	4	56660	38	23631	47	20588	21	22810	70	19491	41	14661	43	22111	83	79028	30	19155	3	010	037
4	+38 4339	8.9	9.50	A	31.885	17.09470	33171	15	85769	15	52057	59	19340	6	16346	10	18411	49	15278	19	08945	16	17492	49	75078	9	14605	7	017	005
6	+37 4179	8.6	......	G8	18.400	22.21072	42844	17	90299	18	77941	54	43686	18	39928	66	42590	25	40061	28	47761	20	43132	54	94720	64	41256	26	009	060
8	+38 4363	9.1	9.99	A	22.940	31.26087	48461	19	03714	20	77763	64	45160	50	41329	17	43985	2	41614	13	43835	65	43550	59	97134	21	41481	97	036	021
9	+38 4367	9.0	9.23	K	26.508	33.55353	78231	22	32475	22	03162	61	71315	49	67632	66	70092	17	67708	45	66004	69	69158	99	24310	16	67022	21	016	062
7	+37 4185	9.0	......	A8	12.014	25.29760	50606	7	08914	7	93381	56	58568	82	54434	17	57330	47	55110	109	69354	36	58395	17	07375	55	57036	32	005	051
10	+37 4203	8.3	......	A8	12.501	34.47743	68661	20	26825	20	10574	13	76726	31	72298	10	75286	20	73296	82	86652	77	76125	87	25095	11	74842	21	051	075
A	+38 4343	5.0	5.57	K5	20.922	19.92571	14684	4	70458	4	46848	101	14571	1867	11080	1903	13559	1874	10800	1860	18722	4673	16655	4746	71248	6523	16410	6593	4.163	412
B	+38 4344	5.3	6.28	K5	20.847	20.02796	24919	17	80674	16	57112	74	24802	1795	21270	1797	23835	1850	21102	1856	20089	4658	26921	4706	81452	6455	26708	6576	4.156	371
C	+38 4362	7.8	7.68	K	22.308	31.15078	37436	55	92721	54	67482	14	34791	18	30932	32	33525	12	31208	8	34256	102	33403	159	86556	33	31236	24	054	066
P. M. of A	......	......	......	......	+.018	+.023																								
P. M. of B	......	......	......	......	+.017	+.023																								
Average residual							31°		43°		B. 35°		50°		54		46		52		53°		64		55		42			
a							− 1.09		+ 1.09		− 24.62		+ 73.29		+ 36.75		+ 52.68		+ 92.63		+ 27.17		+ 19.17		+ 7.99		+ 35.95			
b							+ 142.82		− 142.82		− 1068.85		− 918.98		− 880.02		− 910.89		− 938.75		− 2005.14		− 1026.11		− 598.71		− 1099.98			
c							+19151.		−19151.		−22971.		+37913.		+34300.		+37134.		+34176.		+62961.		+40461.		−15449.		+39600.			

Plate.	Observed. Star A.	Observed. Star B.	P. M. to 1905.0.	Corrected. A.	Corrected. B.	Equations of condition.	Star A. n	Star A. o−c$_1$	Star B. n	Star B. o−c$_1$	Star C. n	Star C. o−c$_1$	o−c$_2$	o−c$_3$	o−c$_4$	o−c$_5$
173	− 0″.007	+ 0″.030	+4″.619	+4″.612	+4″.649	$x-1.121y-0.927\pi=$	−0″.388	−0″.053	−0″.351	+0″.009	+0″.097	+0″.052	+0″.091	+0″.072	+0″.091	+0″.072
179	+ 0.007	− 0.028	+4.561	+4.568	+4.533	$x-1.107y-0.905\pi=$	−0.432	−0.107	−0.467	−0.116	−0.095	−0.140	−0.101	−0.119	−0.101	−0.120
180	+ 0.177	+ 0.130	+4.548	+4.725	+4.678	$x-1.104y-0.899\pi=$	−0.275	+0.049	−0.322	+0.027	−0.025	−0.069	−0.032	−0.049	−0.031	−0.050
322	+ 3.283	+ 3.156	+1.986	+5.269	+5.142	$x-0.482y+0.440\pi=$	+0.269	+0.025	+0.142	−0.015	+0.032	+0.053	+0.001	+0.036	+0.014	+0.007
324	+ 3.346	+ 3.159	+1.965	+5.311	+5.124	$x-0.477y+0.411\pi=$	+0.311	+0.079	+0.124	−0.023	+0.056	+0.075	+0.025	+0.059	+0.038	+0.031
326	+ 3.295	+ 3.252	+1.953	+5.248	+5.205	$x-0.474y+0.396\pi=$	+0.248	+0.023	+0.205	+0.064	−0.021	−0.003	−0.052	−0.018	−0.040	−0.046
328	+ 3.270	+ 3.263	+1.928	+5.198	+5.191	$x-0.468y+0.366\pi=$	+0.198	−0.016	+0.191	+0.060	+0.014	+0.030	−0.018	+0.016	+0.005	−0.011
454	+ 8.215	+ 8.189	−3.498	+4.717	+4.691	$x+0.849y-0.949\pi=$	−0.283	−0.013	−0.309	−0.011	+0.179	+0.028	+0.096	+0.096	+0.134	+0.154
455	+ 8.343	+ 8.273	−3.498	+4.845	+4.777	$x+0.849y-0.949\pi=$	−0.155	+0.115	−0.223	+0.075	+0.279	+0.128	+0.196	......	......	......
531	+11.466	+11.348	−6.200	+5.266	+5.148	$x+1.506y+0.517\pi=$	+0.266	−0.084	+0.148	−0.107	+0.058	−0.019	−0.052	+0.007	0.000	+0.033
535	+11.591	+11.561	−6.325	+5.266	+5.236	$x+1.535y+0.362\pi=$	+0.266	−0.022	+0.236	+0.036	−0.042	−0.131	−0.152	−0.099	−0.101	−0.067
Assumed P. M. and Δx......			+4.120	+5.000	+5.000	....	($p\pi$)	45244		41207		71098	96600	46169	50865	54625

Normal equations.

	A.	B.	C.
$+11.000x-0.498y-2.137n=$	+0″.025	−0″.626	+0″.532
$-0.498\ +10.663\ +1.987\ =$	+1.167	+1.086	+0.395
$-2.137\ +1.987\ +5.339\ =$	+2.064	+1.967	−0.367

Solution I.

	A.	B.	C.	Weight.
$x=$	+0″.083±0″.016	+0″.015±0″.015	+0″.036±0″.020	10.14
$y=$	+0.038±0.016	+0.035±0.015	+0.052±0.020	9.92
$n=$	+0.406±0.024	+0.361±0.023	−0.074±0.030	4.59
$r_0=$	±0.051	±0.048	±0.064	

Solution II—Star C.

	Weight.
+0″.050±0″.021	10.98
+0.039±0.021	10.64
....	.....
±0.070	

Star C. B. D. +38° 4362.

In Solution II (which alone is physically possible) the residual for plate 455 is excluded by Pierce's Criterion. The images of C on this plate are recorded as poor. Rejecting it, the normal equations are:

$$+10.000x-1.347y-1.188n=+0''.253$$
$$-1.347\ +9.941\ +2.793\ =+0.159$$
$$-1.188\ +2.793\ +4.438\ =-0.103$$

whence

Solution III.	Weight.	Solution IV.	Weight.
$x=+0''.025\pm0''.018$	9.64	0″.028±0″.017	9.82
$y=+0.029\pm0.019$	8.15	0.020±0.017	9.76
$\pi=-0.035\pm0.029$	3.59	....	....
$r_0=\ \pm0.055$		±0.054	

Solution V.

$x=+0''.025\pm0''.017$
$r_0=\ \pm0.053$
$y=\pi=0.$

Measurer's note—179, Images of exposure 3 elongated in x.; 328, réseau faint at S. W. corner; 454, réseau faint; 531, "Runs" irregular; 454, 455, Images of C oval.
Observer's notes—179, High wind; 180, Hazy; 328, Thin clouds; 454, 455, Sky thick, probably clouds.
Assumed P. M. from Lewis, Mem. R. A. S., LVI, page 635.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXVIII. STAR 40; Lalande 43492; $22^h 12^m.3$, $+12° 24'$; P. M. $+0^s.057$, $+0''.09$, $=0''.83$.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 174 1903, Nov. 18 Exp. 4, 2^m Hr.-ang. $+9^m$ Obs. R		Plate 176 1903, Nov. 19 Exp. 4, 2^m Hr.-ang. -12^m Obs. R		Plate 334 1904, July 17 Exp. 4, 5^m Hr.-ang. $+2^m$ Obs. R		Plate 341 1904, Aug. 2 Exp. 4, 4^m Hr.-ang. $+5^m$ Obs. R		Plate 456 1905, Nov. 6 Exp. 4, 5^m Hr.-ang. -16^m Obs. H		Plate 461 1905, Nov. 14 Exp. 3, 5^m Hr.-ang. -19^m Obs. H		Approximate solution.		
	No.	Mag.	Mag.	Sp.	η Plate 176.	ξ Mean of 174 and 176.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	μ'	π
1	+12° 4790	9.1	9.16	G	21.789	12.11109	12998	11	09220	**11**	60666	22	63996	**4**	65769	27	60677	46	0".032	**0".004**	**0".004**
3	12 4793	7.5	7.50	G5	20.994	16.78257	80376	29	76138	**29**	27765	35	31294	44	33248	21	27620	15	016	017	044
4	12 4794	9.0	9.06	K	21.964	17.96779	98539	**24**	95019	24	46746	**39**	50304	**22**	51434	**7**	46438	**62**	**025**	**014**	**027**
5	12 4796	8.6	8.55	G8	34.160	18.75425	73448	**17**	77402	17	31179	**16**	34515	**19**	25789	**55**	29537	0	**025**	**002**	**012**
7	12 4803	8.5	8.28	K	21.367	27.08712	10623	6	06800	**7**	58979	**60**	62884	**5**	63558	**128**	58395	**35**	**072**	014	**005**
8	11 4776	8.0	8.10	G2	11.636	30.51141	56042	36	46241	**35**	97251	116	01322	43	09718	176	97547	95	120	231	043
2	12 4792	9.5	9.64	K	17.847	14.27299	30313	**50**	24285	50	75099	**32**	78566	**64**	83276	**54**	75448	**10**	**028**	**048**	**048**
6	11 4771	8.7	8.54	K	10.845	23.76520	81689	16	71351	**16**	21675	**24**	25659	22	35115	6	22338	**52**	**020**	035	009
A	12 4797	7.0	6.97	F8	20.496	19.23293	25494	**25**	21092	25	73014	320	76639	335	79365	897	73423	906	800	822	022
P. M. of A						+.005															
							B.		B.												
Average residual							25*		33*		51		44		43		37				
a							− 6.76		+ 6.76		+ 66.01		+ 98.52		+ 12.97		+ 23.15				
b							− 306.54		+ 306.54		+ 468.54		+ 449.79		− 347.64		+ 358.50				
c							+8638.		−8638.		+38528.		+41898.		+62051.		+41432.				

Plate.	Observed.	P. M. to 1905.0	Corrected.	Equations of condition.	n	$o-c$
174	−0".044	+0".936	+0".892	$x-1.121y-0.919\pi=$	−0".008	−0".042
176	+0.044	+0.934	+0.978	$x-1.118y-0.920\pi=$	+0.078	+0.043
334	+0.562	+0.382	+0.944	$x-0.458y+0.559\pi=$	+0.044	+0.002
341	+0.589	+0.346	+0.935	$x-0.414y+0.337\pi=$	+0.035	−0.002
456	+1.577	−0.710	+0.867	$x+0.850y-0.883\pi=$	−0.033	+0.001
461	+1.593	−0.730	+0.863	$x+0.874y-0.915\pi=$	−0.037	−0.002
Assumed P.M. and Δx		+0.835	+0.900	.... (pvv)	....	3626

Normal equations.

$+6.000x-1.387y-2.741\pi=+0''.079$
$-1.387\ +4.373\ +0.112\ =-0.172$
$-2.741\ +0.112\ +3.754\ =+0.035$

Solution I.	Weight.
$x=+0''.014\pm0''.012$	3.61
$y=-0.035\pm0.012$	3.94
$\pi=+0.021\pm0.015$	2.44
$r_0=\ \pm0.023$	

Observer's notes—456, Sky thick; 461, Clouds.

Measurer's notes—334, Plate spattered with fine drops; 461, Diffuse; exp. 4 too faint to measure.

The proper motions obtained by rejecting star 8 are given under the heading μ'.

All quantities in this table are in réseau intervals of 175".8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXIX. STAR 41; Hels.-Gotha 13170 (Krüger 60); $22^h24^m.5$, +57°12′; P. M. −0″.107, −0″.38, =0″.95.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 184 1903, Dec. 2 Exp. 4, 3^m Hr.-ang. 0^m Obs. R		Plate 186 1903, Dec. 4 Exp. 4, 3^m Hr.-ang. $+6^m$ Obs. R		Plate 337 1904, Aug. 1 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. $+24^m$ Obs. R		Plate 343 1904, Aug. 3 Exp. 4, 4^m Hr.-ang. $+6^m$ Obs. R		Plate 541 1906, July 20 Exp. 4, 4^m Hr.-ang. -24^m Obs. H		Plate 544 1906, July 24 Exp. 4, 4^m Hr.-ang. $+14^m$ Obs. H		Plate 567 1906, Nov. 22 Exp. 4, $5\frac{1}{4}^m$ Hr.-ang. $+6^m$ Obs. H		Plate 568 1906, Nov. 22 Exp. 4, 5^m Hr.-ang. $+31^m$ Obs. H		Approximate solution.	
	No.	Mag.	Mag.	Sp.	η Plate 184.	ξ Mean of 184 and 186.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	π
1	+56°2773	9.5	10.14	F5	16.848	10.10236	13902	**3**	06570	3	10369	**114**	99558	23	80421	**126**	78119	**52**	85506	**57**	86475	**92**	**0″.041**	**0″.033**
3	56 2778	8.5	9.06	A8	16.748	16.32861	36550	**14**	29172	14	33935	46	23090	26	03336	82	01019	45	08434	16	09452	35	018	046
2	57 2532	8.8	9.30	F8	27.083	13.89900	91055	13	88746	**12**	86532	65	75238	**55**	55116	23	50978	**14**	57685	57	58724	48	018	**029**
4	57 2535	9.1	9.53	F8	23.618	16.77020	79028	4	75012	**4**	75329	3	64315	3	43986	19	40536	26	47404	**19**	48458	5	004	020
5	57 2545	9.0	9.94	F	27.811	21.66109	67082	**2**	65136	2	63298	**7**	52272	13	30936	**39**	26889	29	33484	3	34512	**12**	**004**	002
7	56 2788	8.5	8.50	K	21.349	22.52427	54982	**21**	49872	21	52300	**33**	41529	33	20569	27	17509	**41**	24731	51	25674	**20**	004	**015**
8	57 2549	9.0	9.01	F5	27.550	25.29487	30540	6	28435	**5**	27196	**23**	16258	4	94468	**34**	90481	3	97050	**97**	98158	**27**	**025**	025
6	56 2787	8.4	8.46	K	11.010	22.03373	08525	19	98221	**19**	07392	**4**	96840	**12**	76649	**27**	75448	**10**	83435	48	84374	19	010	**039**
9	56 2797	9.0	8.98	A	15.244	26.41244	45334	**2**	37155	3	44145	70	33458	**35**	12513	71	10566	13	18176	**7**	19206	41	014	029
A	56 2783	9.0	9.43	(K5)	21.119	19.24264	26884	**6**	21643	5	23795	**78**	12868	**110**	91440	**1044**	88438	**1050**	95218	**1395**	96232	**1393**	**843**	259
P. M. of A.					−.002	−.005																		
Average residual							B. 38*		B. 26*		38		59		40		58		47		41			
a							+ 1.68		− 1.68		+ 118.86		+ 138.31		+ 5.00		+ 17.91		+ 24.89		+ 24.01			
b							− 247.42		+ 247.42		− 403.75		− 433.07		− 502.05		− 674.22		− 751.86		− 747.43			
c							+7820.		−7820.		+5849.		−4802.		+78720.		+79113.		+87743.		+88682.			

Plate.	Observed.	P. M. to 1905.5.	Corrected.	Equations of condition.	n	$o-c$
184	−0″.011	−1″.378	−1″.389	$x-1.582y-0.915\pi=$	−0″.289	−0″.036
186	+0.009	−1.374	−1.365	$x-1.577y-0.910\pi=$	−0.265	−0.007
337	−0.137	−0.799	−0.936	$x-0.917y+0.401\pi=$	+0.164	+0.056
343	−0.193	−0.793	−0.986	$x-0.911y+0.372\pi=$	+0.114	+0.013
541	−1.835	+0.915	−0.920	$x+1.051y+0.576\pi=$	+0.180	−0.037
544	−1.846	+0.924	−0.922	$x+1.061y+0.529\pi=$	+0.178	−0.027
567	−2.453	+1.212	−1.241	$x+1.392y-0.917\pi=$	−0.141	+0.017
568	−2.449	+1.213	−1.236	$x+1.392y-0.917\pi=$	−0.136	+0.022
Assumed P.M. and Δx	−0.871		−1.100	..............(pvv)	....	7521

Normal equations.

$+8.000x-0.091y-1.781\pi=-0''.195$
$-0.091+12.728+0.789=+0.614$
$-1.781+0.790+4.258=+1.065$

Solution.	Weight.
$x=+0''.034\pm0''.010$	7.25
$y=+0.032\pm0.007$	12.57
$\pi=+0.258\pm0.013$	3.82
$r_0=\quad\pm0.026$	

Observer's notes—184, Hazy; 337, Thin clouds, seeing very good; 541, Seeing very bad.

Measurer's notes—567, 568, Réseau faint.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXX. STAR 42; Lalande 45755; $23^{h}16^{m}.8$, $+43°32'$; P. M. $+0^{s}.058$, $+0''.22$, $=0''.68$.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 175 1903, Nov. 18 Exp. 4, 2^m Hr.-ang. -19^m Obs. R		Plate 177 1903, Nov. 19 Exp. 4, 2^m Hr.-ang. -8^m Obs. R.		Plate 342 1904, Aug. 2 Exp. 4, 4^m Hr.-ang. $+11^m$ Obs. R		Plate 347 1904, Aug. 9 Exp. 3, 4^m Hr.-ang. $+6^m$ Obs. R		Plate 349 1904, Aug. 11 Exp. 4, 4^m Hr.-ang. $+12^m$ Obs. R		Plate 387 1904, Dec. 5 Exp. 4, 4^m Hr.-ang. $+11^m$ Obs. H		Plate 389 1904, Dec. 22 Exp. 4, 4^m Hr.-ang. $+24^m$ Obs. H		Approximate solution.		
	No.	Mag.	Mag.	Sp.	η Plate 175	ξ Mean of 175 and 177.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	μ	μ'	π
2	+42°4641	9.2	9.66	F?	8.762	13.67887	54866	**4**	80909	5	44238	**19**	05353	9	12159	34	15169	21	12475	**3**	0″.015	0″.006	0″.003
3	43 4437	9.4	10.10	G5	17.865	14.41528	29960	**16**	53096	16	15832	13	79340	**49**	83188	**25**	86905	26	87096	26	044	**024**	**047**
1	43 4434	9.0	9.53	A	29.884	8.11155	01606	5	20704	**5**	81746	**87**	48875	**1**	48681	**51**	53572	**100**	58080	**108**	**172**	**368**	030
4	43 4438	9.2	9.73	G	22.712	15.67271	56505	15	78036	**16**	40648	91	05470	42	07729	44	11696	52	13376	85	110	017	016
5	43 4443	9.2	9.72	F	24.990	18.31754	21323	7	42185	**7**	04810	**2**	70240	0	71686	**93**	75714	**41**	77892	**40**	**066**	**156**	**006**
8	43 4456	8.5	8.98	A	24.079	31.70862	60131	**26**	81592	25	45732	**2**	10515	**40**	12609	101	15725	88	16569	63	126	138	**039**
6	42 4655	9.4	10.22	F?	12.916	25.40658	28213	7	53103	**7**	17431	**7**	79318	**36**	84815	**71**	87494	**47**	85211	**101**	**121**	**076**	015
7	43 4454	9.2	10.17	F	20.470	31.04198	92929	14	15468	**13**	79861	10	43773	78	46882	64	49714	0	49555	78	062	095	032
A	43 4445	7.3	7.56	F8	19.954	19.76302	65021	**17**	87583	17	51074	343	15014	278	18218	292	21856	417	22301	408	674	629	035
P. M. of A........					+.001	+.004	....	......															
Average residual........							B. 30*		B. 30*		44		37		61		44		42				
a........							− 9.052		+ 9.052		+ 119.20		+ 92.63		+ 101.41		+ 43.29		− 32.54				
b........							+ 161.531		− 161.531		− 238.09		+ 36.95		− 288.58		− 213.22		+ 107.06				
c........							−14308.		+14308.		+23174.		+35866.		+45379.		+48537.		+44098.				

Plate.	Observed.	P. M. to 1904.5.	Corrected.	Equations of condition.	n	$o-c_1$	$o-c_2$
175	−0″.030	+0″.391	+0″.361	$x-0.621y-0.837\pi=$	−0″.039	−0″.030	−0″.054
177	+0.030	+0.389	+0.419	$x-0.618y-0.843\pi=$	+0.019	+0.029	+0.004
342	+0.603	−0.054	+0.549	$x+0.086y+0.578\pi=$	+0.149	+0.073	+0.073
347	+0.489	−0.067	+0.422	$x+0.106y+0.483\pi=$	+0.022	−0.051	−0.049
349	+0.513	−0.070	+0.443	$x+0.111y+0.456\pi=$	+0.043	−0.029	−0.027
387	+0.733	−0.270	+0.463	$x+0.429y-0.903\pi=$	+0.063	+0.027	+0.050
389	+0.717	−0.299	+0.418	$x+0.475y-0.885\pi=$	+0.018	−0.020	+0.005
Assumed P.M. and Δx		+0.630	+0.400	..............(pvv)	....	11641	13916

Normal equations.

$$+7.000x-0.032y-1.951\pi=+0''.275$$
$$-0.032\ +1.207\ +0.386\pi=+0.068$$
$$-1.951\ +0.386\ +3.785\pi=+0.061$$

	Solution I.	Weight.	Solution II.	Weight.
$x=$	$+0''.050\pm0''.015$	5.97	$+0''.051\pm0''.015$	5.99
$y=$	$+0.046\pm0.034$	1.16	....	....
$\pi=$	$+0.037\pm0.021$	3.12	$+0.043\pm0.020$	3.24
$r_0=$	±0.037		±0.036	

Observer's notes—342, Sky thick; 347, Clouds at end.
Measurer's notes—342, Fogged; 347, Exp. 3 faint; 4 invisible; 349, Images diffuse.
The proper motions obtained by rejecting Star 1 are given under the heading μ'.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXXI. STAR 43; Lalande 46650: $23^h44^m.0$, +1°52′; P. M. +0ˢ.065, −1″.00, = 1″.40.
44; Lam. 32805; $23^h44^m.9$, +2°19′; P. M. +0ˢ.026, −0″.20, = 0″.44.

Stars.	Bonn Durchmusterung.		Harvard.		Standard.		Plate 187 1903, Dec. 4 Exp. 4, 3ᵐ Hr.-ang. −3ᵐ Obs. R		Plate 190 1903, Dec. 9 Exp. 4, 3ᵐ Hr.-ang. +9ᵐ Obs. R		Plate 350 1904, Aug. 11 Exp. 4, 4ᵐ Hr.-ang. +12ᵐ Obs. R		Plate 359 1904, Aug. 26 Exp. 4, 4ᵐ Hr.-ang. +5ᵐ Obs. R		Plate 388 1904, Dec. 5 Exp. 4, 4ᵐ Hr.-ang. +17ᵐ Obs. H		Plate 390 1904, Dec. 22 Exp. 4, 4½ᵐ Hr.-ang. +27ᵐ Obs. H		Approximate solution.		
	No.	Mag.	Mag.	Sp.	η Plate 187.	ξ Mean of 187 and 190.	x	o−c	x	o−c	x	o−c	x	o−c	x	o−c	x	o−c	μ	μ′	π
3	+1° 4771	9.5	.....	.....	10.203	12.58109	29821	0	86396	0	93018	**69**	86869	9	92030	8	90558	**46**	**0″.033**	0″.008	**0″.022**
4	1 4773	7.7	6.63	F2	14.011	18.17347	88560	8	46133	**8**	52600	25	48216	**10**	51814	**29**	51758	**8**	**031**	**021**	026
1	2 4710	8.5	8.11	.....	27.765	10.50276	19886	**18**	80666	18	84484	**35**	87455	**34**	86362	**9**	93328	**5**	**012**	**016**	**039**
2	1 4790	8.7	.....	.....	24.924	10.79577	49494	**49**	09660	49	14001	66	15455	36	15328	9	20970	42	045	049	043
5	2 4718	8.5	8.45	G	33.775	19.08723	77611	58	39835	**58**	43360	16	49282	2	45756	22	54846	14	031	**020**	**004**
6	2 4720	9.0	9.87	G	29.896	21.71958	41240	10	02676	**10**	06952	63	10732	**20**	08505	**21**	15585	**15**	**031**	**080**	045
8	1 4784	9.3	10.06	.....	22.700	27.50564	20679	34	80449	**34**	86022	**108**	86111	**19**	86222	**106**	89492	**88**	**167**	**215**	004
9	2 4724	9.0	9.82	.....	28.733	28.95999	65306	**37**	26691	37	31461	0	34615	38	32658	105	38568	57	140	070	**048**
7	1 4782	9.5	10.74	.....	15.112	25.69205	40208	**3**	98203	**3**	04902	**5**	00972	**12**	03972	**6**	03822	1	**003**	**022**	**009**
10	1 4786	7.5	7.04	G	14.522	29.88870	59901	**7**	17838	7	24925	49	20576	11	23672	28	22928	46	064	032	007
A	1 4774	8.7	9.09	Ma	18.346	19.59738	30396	**15**	89080	15	95486	570	93336	540	95386	578	97216	573	993	986	210
B	2 4723	8.3	8.28	G	27.477	24.39435	08989	18	69881	**18**	74754	124	77355	176	75935	189	81765	288	410	359	**017**
P. M. of A	.....	.....	.....	.....	−.006	+.006															
							B		B												
Average residual							34*		25*		51		54		42		36				
a							− 9.09		+ 9.09		+ 67.88		+ 48.43		+ 18.22		− 60.74				
b							− 123.02		+ 123.02		− 33.85		+ 487.55		+ 126.38		+ 594.35				
c							−26824.		+26824.		+34469.		+23168.		+32394.		+27194.				

Plate.	Star A.			Star B.			Equations of condition.	Star A.			Star B.			
	Observed.	P. M. to 1904.5.	Corrected.	Observed.	P. M. to 1904.5.	Corrected.		n	$o-c_1$	$o-c_2$	n	$o-c_1$	$o-c_2$	$o-c_3$
187	−0″.026	+0″.563	+0″.537	+0″.032	+0″.225	+0″.257	$x-0.577y-0.876\pi=$	−0″.213	+0″.020	−0.029	+0″.057	−0″.038	+0″.023	+0″.032
190	+0.026	+0.549	+0.575	−0.032	+0.219	+0.186	$x-0.563y-0.892\pi=$	−0.175	−0.020	+0.012	−0.014	+0.035	−0.049	−0.036
350	+1.002	−0.109	+0.893	+0.218	−0.044	+0.174	$x+0.112y+0.556\pi=$	+0.143	+0.023	+0.025	−0.026	−0.032	−0.032	−0.051
359	+0.949	−0.148	+0.801	+0.309	−0.059	+0.250	$x+0.152y+0.351\pi=$	+0.051	−0.026	−0.024	+0.050	+0.037	+0.039	+0.025
388	+1.016	−0.417	+0.599	+0.332	−0.167	+0.165	$x+0.428y-0.883\pi=$	−0.151	+0.028	+0.035	−0.035	−0.080	−0.069	−0.060
390	+1.007	−0.463	+0.544	+0.506	−0.185	+0.321	$x+0.475y-0.900\pi=$	−0.206	−0.024	−0.016	+0.121	+0.072	+0.086	+0.096
Assuned P. M. and Δx		+0.975	+0.750	....	+0.390	+0.200	..............(pvv)	....	3365	3667	....	16646	17632	18362

Normal equations.

	A.	B.
$+6.000x+0.027y-2.644\pi=$	−0″.551	+0″.153
$+0.027\ +1.095\ +0.317\ =$	+0.084	+0.022
$-2.644\ +0.317\ +3.585\ =$	+0.759	−0.112

Solution I.

	A.	B.	Weight.
$x=$	+0″.001±0″.011	+0″.016±0″.025	3.99
$y=$	+0.015±0.022	+0.027±0.049	1.05
$\pi=$	+0.211±0.015	−0.022±0.033	2.32
$r_0=$	±0.023	±0.050	

Solution II.

A.	B.	Weight.
+0″.002±0″.010	+0″.017±0.022	4.05
....	....	....
+0.213±0.013	−0.019±0.029	2.42
±0.021	±0.045	

Solution III.

B.
+0″.025±0″.017
....
....
±0.041

Measurer's notes; 350, 359, images diffuse. Observer's notes; 359, Drifting clouds. The proper motions obtained by rejecting star 8 are given under the heading μ′.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXXII. STAR 45; η Geminorum; $6^{h}8^{m}.8$, $+22°32'$; P. M. $-0^{s}.005, +0''.02$, $=0''.07$
Observed with Color-screen.

Stars.	Bonn Durchmusterung.		Plate 233 (Stand.) 1904, Mar. 8 Exp. 4, 5^{m} Hour-angle -4^{m} Obs. R		Plate 236 1904, Mar. 10 Exp. 4, 5^{m} Hr.-ang. $+5^{m}$ Obs. R	Plate 380 1904, Oct. 24 Exp. 4, 5^{m} Hr.-ang. -10^{m} Obs. H	Plate 381 1904, Oct. 24 Exp. 3, 5^{m} Hr.-ang. $+15^{m}$ Obs. H	Plate 383 1904, Nov. 7 Exp. 4, $5\frac{1}{4}^{m}$ Hr.-ang. $+3^{m}$ Obs. H
	No.	Mag.	η	ξ				
1	+22° 1237	9.1	18.183	18.73825	74989	86715	88542	86034
2	+22 1247	9.5	25.736	23.91850	94262	09259	07573	04952
3	+22 1250	8.8	13.410	26.38734	38764	48780	52940	50501
A	+22 1241	3.2	20.271	21.04475	05932	18586	19510	16949
Average residual.......			40*		26*	37	25	35
Residuals for A........			0		−24	−29	+30	+ 9

Reduction Constants $x_{A}=+0.5929\,x_{1}+0.3270\,x_{2}+0.0801\,x_{3}$.

Plate.	Observed.	P. M. to 1904.5.	Equations of condition.	n	$o-c_{1}$	$o-c_{2}$	Weight.
233	$0''.000$	$-0''.022$	$x-0.317y-0.963\pi=$	$-0''.022$	$+0''.021$	$-0''.014$	1
236	-0.042	-0.022	$x-0.311y-0.972\pi=$	-0.064	-0.021	-0.056	1
380	-0.051	$+0.022$	$x+0.314y+0.866\pi=$	-0.029	-0.050	-0.021	1
381	$+0.053$	$+0.022$	$x+0.314y+0.866\pi=$	$+0.075$	$+0.054$	$+0.083$	$\frac{1}{2}$
383	$+0.016$	$+0.024$	$x+0.353y+0.725\pi=$	$+0.040$	$+0.024$	$+0.048$	1
Assumed P. M...		−69	...............(pvv)	....	6874	9521	

Epoch mean equations.	Solution I.	Solution II.
$x-0.314y-0.967\pi=-0''.043$	$x=-0''.009$	$x=-0''.008$
$x+0.339\;+0.810\;=+0.019$	$\pi=+0.035-0.36y\;\pm0''.017$	....
	$r_{0}=\pm0.032$	$r_{0}=\pm0''.033$
	With Boss's P. M. $y=+0''.005$	
	$\pi=+0.034$	

Observer's notes—233, Control slow; 380, 381, 382, seeing poor; 382, High wind.

Measurer's notes—233, Images elliptical; 380, 381, Diffuse; 381, Image 4 of A very discordant; exp. 4 therefore rejected.

All quantities in this table are in réseau intervals of $175''.8$ unless otherwise stated.

TABLE C, SERIES XXXIII. STAR 46; α_1 Geminorum, 47; α_2 Geminorum; $7^h28^m.2$, $+32°6'$; P. M. $-0''.014$, $-0''.11$, $=0''.20$. Observed with Color-screen.

Stars.	Bonn Durchmusterung. No.	Mag.	Standard. η Plate 235	Standard. ξ Mean of 235 and 238.	Plate 235 1904, Mar. 8 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang.+11^m Obs. R x	o−c	Plate 238 1904, Mar. 10 Exp. 4, $3\frac{1}{2}^m$ Hr.-ang. +1^m Obs. R x	o−c	Plate 245 1904, Mar. 17 Exp. 4, 3^m Hr.-ang. 0^m Obs. R x	o−c	Plate 382 1904, Oct. 24 Exp. 3, 4^m Hr.-ang.+14^m Obs. H x	o−c	Plate 384 1904, Nov. 7 Exp. 3, 4^m Hr.-ang. +2^m Obs. H x	o−c	Plate 385 1904, Nov. 14 Exp. 4, 3^m Hr.-ang.+20^m Obs. H x	o−c
2	+32° 1575	9.3	18.020	14.13948	22950	10	04945	**11**	32186	**11**	32116	**10**	33027	25	32535	**7**
5	31 1611	8.7	16.214	16.40675	49514	**2**	31836	2	58555	19	59234	**8**	59288	**68**	58885	**18**
1	32 1570	9.0	27.445	7.76551	86415	19	66688	**18**	96640	**13**	92288	**123**	97478	**39**	96679	**115**
3	32 1576	8.7	23.261	15.24318	33831	**9**	14805	9	43438	**14**	40984	20	44370	**32**	43610	1
4	32 1578	8.9	28.362	16.10627	20644	**18**	00609	17	30650	21	25914	121	31827	113	30751	140
6	32 1585	9.3	26.756	24.46826	56804	9	36848	**9**	66224	**61**	62091	85	67570	34	66265	78
9	31 1633	7.5	14.838	33.73677	82598	**2**	64756	2	90825	69	91624	**104**	91885	**77**	90792	**105**
7	31 1624	8.2	11.574	26.94586	03110	14	86062	**14**	11335	31	13951	42	12295	22	11665	10
8	31 1627	8.5	15.772	28.88815	97739	**23**	79891	23	06171	**38**	06832	**24**	07340	21	06408	12
A	32 1581	1.7	20.290	21.02750	12046	**2**	93455	3	21205	23	19960	30	22250	45	21408	63
B			20.267	21.00628	09892	**32**	91365	33	19022	**35**	17776	**39**	20143	65	19282	61
Average residual					42*		30*		42*		52*		34		34	
a					+ 12.01		− 12.01		− 31.22		− 52.21		− 7.12		− 46.92	
b					+ 98.66		− 98.66		+ 175.49		− 281.34		+ 197.96		+ 143.35	
c					+7044.		−7044.		+15528.		+23986.		+15588.		+16674.	

Plate.	Observed. A	Observed. B	P. M. to 1904.5.	Equations of condition.	Star A. n	Star A. o−c	Star B. n	Star B. o−c	Weight.
235	−0″.004	−0″.056	−0″.054	$x-0.316y-0.835\pi=$	−0″.058	−0″.022	−0″.110	−0″.040	1
238	+0.005	+0.058	−0.053	$x-0.310y-0.843\pi=$	−0.048	−0.011	+0.005	+0.076	1
245	+0.040	−0.062	−0.050	$x-0.291y-0.901\pi=$	−0.010	+0.033	−0.112	−0.035	1
382	+0.053	−0.069	+0.054	$x+0.315y+0.965\pi=$	+0.107	−0.041	−0.015	−0.132	1
384	+0.079	+0.114	+0.060	$x+0.353y+0.889\pi=$	+0.139	−0.001	+0.174	+0.064	1
385	+0.111	+0.107	+0.064	$x+0.373y+0.828\pi=$	+0.175	+0.043	+0.171	+0.069	1
Assumed P. M.			−0.172	. . . . (pvv)	. . . .	5225	. . . .	34882	

Epoch mean equations.

	A.	B.
$x-0.306y-0.860\pi=$	$-0''.039$	$-0''.072$
$x+0.347y+0.894\pi=$	$+0.140$	$+0.110$

Solution.

	A.	B.
$x=$	$+0''.049$	$+0''.017$
$\pi=$	$+0.102-0.37y\ \pm0''.011$	$+0.104-0.37y\ \pm0''.029$
$r_0=$	±0.024	±0.063

With Boss's P. M. $y=0''.000$

Observer's notes—235, Sky thick; 245, 384, Seeing very bad; 384, 385, Clock running badly.
Measurer's notes—384, Images of exp. 3 bad, not measured.
Assumed P. M. from Crommelin, M. N. lxvii, p. 141.
All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXXIV. STAR 48; γ Serpentis; $15^h51^m.8, +15°59'$;
P. M. $+0^s.021$, $-1''.30$, $=1''.33$. Observed with Color-screen.

Stars.	Bonn Durchmusterung.		Plate 239 (Stand.) 1904, Mar. 10 Exp. 4, 5^m Hour-angle $+10^m$ Obs. R		Plate 252 1904, Mar. 31 Exp. 4, 5^m Hr.-ang. $+25^m$ Obs. R		Plate 302 1904, June 21 Exp. 4, 5^m Hr.-ang. $+24^m$ Obs. R		Plate 306 1904, June 25 Exp. 4, 5^m Hr.-ang. $+9^m$ Obs. R	
	No.	Mag.	η	ξ	x	$o-c$	x	$o-c$	x	$o-c$
1	+16° 2841	9.5	21.580	8.09101	17504	83	09350	63	12360	119
2	16 2846	9.5	23.293	16.89078	98265	**17**	89900	**32**	93244	**93**
3	16 2847	9.2	24.828	18.37451	47405	20	38344	**62**	42395	53
6	16 2851	8.5	28.040	26.62146	73570	**86**	63739	31	68582	**77**
5	16 2850	9.5	20.972	22.93208	01425	29	94540	**21**	97025	36
8	16 2852	9.4	22.003	31.45126	53915	36	47181	49	49758	**9**
4	15 2931	9.3	14.077	18.43422	48358	2	44554	55	44550	65
7	15 2936	9.0	15.656	29.76559	82278	**66**	78418	**85**	78749	**91**
A	16 2849	3.8	20.315	20.95984	03841	**5**	97296	107	99519	103
Average residual........			43*		31*		60*		68*	
a..................			0.00		+ 10.13		+ 78.13		+ 57.83	
b..................			0.00		+ 464.89		− 11.01		+ 356.47	
c..................			0.		−1795.		−209.		−5021.	

Plate.	Observed.	P. M. to 1904.5.	Equations of condition.	n	$o-c_1$	$o-c_2$
239	0″.000	+0″.090	$x-0.313y+0.912\pi=$	+0″.090	+0″.021	−0″.044
252	−0.009	+0.073	$x-0.252y+0.738\pi=$	+0.064	−0.021	−0.070
302	+0.188	+0.008	$x-0.029y-0.495\pi=$	+0.196	+0.007	+0.062
306	+0.181	+0.005	$x-0.017y-0.553\pi=$	+0.186	−0.008	+0.052
Assumed P. M...		+0.288	...............(pvv)	....	995	13384

Epoch mean equations.
$x-0.282y+0.825\pi=+0''.077$
$x-0.023\ -0.524\ =+0.191$

Solution I.
$x=+0''.147$
$\pi=-0.085+0.19y\pm0''.011$
$r_0=\pm0.015$
With Boss's P. M. $y=+0''.015$
$\pi=-0.081$

Solution II.
$x=+0''.134$
....
$r_0=\pm0''.045$

Observer's notes—302, Sky pretty thick.

All quantities in this table are réseau intervals of 175″.8 unless otherwise stated *Numbers in bold-face type are negative*

TABLE C, SERIES XXXV. STAR 49; ζ Herculis; $16^h37^m.6, +31°47'$; P. M. $-0''.037, +0''.39, = 0''.60$. Observed with Color-screen.

Stars.	Bonn Durch-musterung.		Standard.		Plate 240 1904, Mar. 10 Exp. 4, 5^m Hr.-ang. -4^m Obs. R		Plate 248 1904, Mar. 21 Exp. 4, 5^m Hr.-ang. $+16^m$ Obs. R		Plate 303 1904, June 21 Exp. 4, 5^m Hr.-ang. $+10^m$ Obs. R		Plate 308 1904, June 28 Exp. 3, $5\frac{1}{3}^m$ Hr.-ang. -2^m Obs. R		Plate 315 1904, July 2 Exp. 1, 5^m Hr.-ang. $+8^m$ Obs. R		Plate 319 1904, July 7 Exp. 4, $5\frac{1}{4}^m$ Hr.-ang. $+28^m$ Obs. R	
	No.	Mag.	η Plate 240	ξ Mean of 240 and 248	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$
1	+31° 2882	9.3	19.710	10.48942	46105	**2**	51778	1	39699	22	51912	**64**	47940	**136**	49459	**5**
2	31 2883	9.3	11.591	15.34555	33675	**21**	35435	21	22386	**73**	39678	**48**	35985	103	28434	**76**
3	32 2764	9.0	24.234	16.74274	70458	1	78090	**1**	67356	29	77077	35	73125	8	79430	121
5	32 2768	9.3	25.231	22.41456	37496	21	45416	**21**	35398	22	44685	77	40698	23	47854	**42**
7	31 2892	8.4	21.412	27.53373	50296	**62**	56449	61	46161	**35**	57755	**67**	53788	**129**	56949	**45**
8	32 2775	8.8	25.016	33.82356	78610	38	86102	**38**	77119	**15**	86681	**46**	82895	95	89665	**34**
4	31 2885	9.2	14.379	22.27388	26002	41	28774	**41**	16982	7	32793	80	28900	48	24459	39
6	31 2887	9.2	17.429	23.25716	23568	**16**	27865	17	16642	43	30558	31	26632	**14**	25519	38
A	31 2884	3.0	20.304	21.02430	99584	**7**	05275	6	94154	**119**	06280	**152**	02388	**142**	04254	**218**
Average residual					18*		52*		57*		38*				54*	
a					+ 12.90		− 12.90		+ 82.74		+ 103.11		+ 103.47		+ 95.43	
b					− 235.60		+ 235.60		+ 398.23		− 201.50		− 208.17		+ 865.89	
c					+1674.		−1674.		−17982.		+5925.		+2152.		−17546.	

Plate.	Observed.	P. M. to 1904.5.	Equations of condition.	n	$o-c$	Weight.
240	$-0''.012$	$-0''.160$	$x-0.310y+0.971\pi=$	$-0''.172$	$-0''.022$	1
248	+0.011	−0.144	$x-0.280y+0.926\pi=$	−0.133	+0.021	1
303	−0.209	−0.015	$x-0.029y-0.329\pi=$	−0.224	+0.049	1
308	−0.267	−0.005	$x-0.009y-0.436\pi=$	−0.272	+0.011	1
315	−0.250	+0.000	$x+0.001y-0.496\pi=$	−0.250	+0.039	½
319	−0.383	+0.008	$x+0.016y-0.567\pi=$	−0.375	−0.079	1
Assumed P. M. . .	−0.515	...............(pvv)	....	10448		

Epoch Mean Equations.

$x-0.295y+0.948\pi=-0''.152$

$x-0.006\ \ -0.450\ \ =-0.285$

Solution.

$x=-0''.242$

$\pi=+0.095+0.21y\pm0''.021$

$r_0=\pm0.035$

With Boss's P. M. and mass-ratio. $y=+0''.028$

$\pi=+0.101$

Observer's notes—303, Sky pretty thick; 315, Clouds during second exposure.

Measurer's notes—248, Images diffuse; 303, Star A near edge of color-screen, images good; 308, Exp. 1 of A outside screen (not measured), 319, Images good; réseau rather poor.

Assumed P. M. of A includes the orbital motion, using Lewis's data.

All quantities in this table are in réseau intervals of $175''.8$ unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXXVI. STAR 50; η Herculis; $16^h 39^m.5 + 39°7'$; P. M.$+0^s.003, -0''.10, =0''.10$. Observed with Color-screen.

Stars.	Bonn Durch-musterung.		Standard.		Plate 243 1904, Mar. 11 Exp. 4, 4^m Hr.-ang.$+10^m$ Obs. R		Plate 253 1904, Mar. 31 Exp. 4, 5^m Hr.-ang.$+10^m$ Obs. R		Plate 262 1904, Apr. 19 Exp. 4, 5^m Hr.-ang.$+18^m$ Obs. R		Plate 307 1904, June 27 Exp. 4, $7\frac{1}{4}^m$ Hr.-ang.$+21^m$ Obs. R		Plate 309 1904, June 28 Exp. 4, 5^m Hr.-ang.$+25^m$ Obs. R		Plate 316 1904, July 6 Exp. 4, 5^m Hr.-ang.$+11^m$ Obs. R	
	No.	Mag.	η Plate 253.	ξ Mean of 253 and 262.	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$	x	$o-c$
1	+38° 2814	9.3	14.190	9.03963	14462	**36**	10660	8	97266	**8**	14349	36	04709	**128**	07835	3
2	39 3026	9.3	16.224	13.49625	60329	29	56246	0	43005	0	60154	4	50490	7	53860	**89**
3	39 3027	9.2	35.198	15.06390	20575	85	12948	20	99832	**20**	16382	**11**	05034	185	10589	108
4	39 3028	9.3	24.198	18.29082	40899	**79**	35575	**30**	22589	30	39580	**30**	29102	**63**	33748	**24**
6	39 3033	9.4	19.646	24.81990	92680	**20**	88389	**41**	75590	40	92954	**61**	83019	**19**	87521	13
8	39 3037	8.0	29.638	31.30931	43159	14	37298	51	24565	**50**	42088	102	30931	**103**	36862	**99**
5	38 2822	9.1	11.897	20.32962	42472	4	39458	**33**	26465	32	43965	**28**	34858	85	38184	40
7	38 2830	9.1	15.821	29.38792	48562	9	45199	23	32385	**23**	50172	**12**	40636	37	44948	44
A	39 3029	3.0	20.288	20.84981	96166	146	91475	**3**	78488	4	95648	**124**	85811	90	90073	32
Average residual					82*		35*		39*		98*		40*		49*	
a					− 52.81		− 14.73		+ 14.73		+ 53.85		+ 56.33		+ 111.95	
b					+ 184.91		− 2.95		+ 2.95		− 31.94		− 131.14		− 21.51	
c					+8388.		+6864.		−6864.		+10316.		+2226.		+3162.	

Plate.	Observed.	P. M. to 1904.5.	Equations of condition.	n	$o-c_1$	$o-c_2$	Weight.
243	$+0''.257$	$+0''.010$	$x-0.307y+0.970\pi=$	$+0''.267$	$+0''.204$	$+0''.216$	½
253	-0.005	$+0.008$	$x-0.255y+0.861\pi=$	$+0.003$	-0.058	-0.048	1
262	$+0.007$	$+0.007$	$x-0.200y+0.663\pi=$	$+0.014$	-0.044	-0.037	1
307	-0.218	0.000	$x-0.013y-0.415\pi=$	-0.218	-0.261	-0.269	½
309	$+0.158$	0.000	$x-0.010y-0.430\pi=$	$+0.158$	$+0.115$	$+0.107$	1
316	$+0.056$	0.000	$x+0.012y-0.547\pi=$	$+0.056$	$+0.015$	$+0.005$	1
Assumed P. M. .		$+0.033$	...............(pvv)	....	73618	74655	

Epoch Mean Equations (Weighted).

$x-0.244y+0.803\pi=+0''.060$

$x-0.002\ \ -0.474\ \ =+0.042$

	Solution I.	Solution II.
x	$=+0''.049$	$+0''.051$
π	$=+0.014+0.19y\pm0''.065$	....
r_0	$=\pm0.092$	$\pm0''.082$

With Boss's P. M. $y=0''.000$

Observer's notes—243, Fog; very steady; 307, Very hazy; control bad.
Measurer's notes—243, Underexposed; A very faint; 307, Residuals very irregular.
All quantities in this table are in réseau intervals of $175''.8$ unless otherwise stated. *Numbers in bold-face type are negative.*

TABLE C, SERIES XXXVII. STAR 51; μ Herculis, A
52; μ Herculis, BC}; $17^h 42^m.6$, +27°47′; P. M. −0ˢ.024, −0″.75, =0″.82. A observed with Color-screen; B outside it.

Stars.	Bonn Durchmusterung.		Standard.		Plate 272 1904, May 2 Exp. 4, 5^m Hr.-ang. $+21^m$ Obs. R		Plate 280 1904, May 15 Exp. 3, 5^m Hr.-ang. $+31^m$ Obs. R		Plate 287 1904, May 19 Exp. 4, 5^m Hr.-ang. $+28^m$ Obs. R		Plate 318 1904, July 6 Exp. 4, 5^m Hr.-ang. $+31^m$ Obs. R		Plate 321 1904, July 8 Exp. 4, 5^m Hr.-ang. $+22^m$ Obs. R		Plate 336 1904, Aug. 1 Exp. 4, 5^m Hr.-ang. $+25^m$ Obs. R		Plate 340 1904, Aug. 2 Exp. 4, 5^m Hr.-ang. $+32^m$ Obs. R	
	No.	Mag.	η Plate 272.	ξ Mean of 272 and 280.	x	o−c	x	o−c	x	o−c	x	o−c	x	o−c	x	o−c	x	o−c
1	+27° 2878	9.3	16.731	10.73660	74054	18	73267	**17**	84631	**13**	72872	40	70035	**48**	74555	**81**	70692	**65**
2	27 2881	8.8	14.350	13.26931	27509	**30**	26352	29	40031	**30**	26302	**8**	23370	0	28389	**12**	24488	22
3	28 2822	8.3	30.498	17.36202	35364	34	37040	**34**	35156	28	35392	**45**	34160	43	36498	24	32861	**15**
4	28 2827	8.9	29.253	20.64096	63334	**19**	64858	19	64194	18	63569	15	62171	6	64891	68	61250	57
7	....	9.8	26.076	28.23001	22629	42	23373	**42**	26019	0	23028	43	21441	37	24801	**8**	21082	**20**
8	27 2894	9.5	19.219	32.58656	58845	**54**	58467	54	67765	**44**	59050	**12**	56802	**83**	61501	**86**	57705	**23**
5	27 2887	9.0	10.126	20.75950	77022	37	74878	**37**	92935	**5**	75860	**14**	72698	61	78612	17	74566	6
6	27 2889	9.4	13.604	24.12824	13528	**23**	12120	23	26846	49	12848	**16**	10006	**11**	15539	77	11535	38
A	27 2888	3.5	20.260	20.80711	80835	30	80587	**30**	88651	**94**	80308	**87**	78069	**71**	82265	**86**	78452	**80**
B	....	9.5	20.182	20.64175	64256	**19**	64093	18	72125	**150**	63538	**312**	61485	**100**	65665	**141**	61846	**138**
Average residual..................					54*		50*		46*		58*		43*		56*		57*	
a..................					+ 4.48		− 4.48		+ 16.83		+ 59.19		+ 74.43		+ 100.77		+ 99.37	
b..................					− 92.87		+ 92.87		− 883.88		− 23.97		+ 72.60		− 99.73		− 78.53	
c..................					+1882.		−1882.		+25591.		−1062.		−5591.		+1564.		−2657.	

Plate.	Star A.		Star B.		Equations of condition.	Star A.			Star B.			Weight.
	Observed	P.M. to 1904.5.	Observed	P. M. to 1904.5.		n	$o-c_1$	$o-c_2$	n	$o-c_1$	$o-c_2$	
272	+0″.053	−0″.054	−0″.033	−0″.061	$x-0.165y+0.698\pi=$	−0″.001	+0″.096	+0″.114	−0″.094	+0.039	+0″.075	1
280	−0.053	−0.042	+0.031	−0.048	$x-0.129y+0.523\pi=$	−0.095	+0.006	+0.020	−0.017	+0.125	+0.152	1
287	−0.165	−0.039	−0.264	−0.044	$x-0.119y+0.469\pi=$	−0.204	−0.102	−0.089	−0.308	−0.153	−0.139	1
318	−0.153	+0.004	−0.548	+0.004	$x+0.012y-0.318\pi=$	−0.149	−0.027	−0.037	−0.544	(−0.359)	(−0.375)	1; 0*
321	−0.125	+0.006	−0.176	+0.007	$x+0.018y-0.350\pi=$	−0.119	+0.003	−0.014	−0.169	+0.018	0.000	1
336	−0.151	+0.027	−0.248	+0.031	$x+0.083y-0.692\pi=$	−0.124	+0.007	−0.009	−0.217	−0.012	−0.048	1
340	−0.141	+0.028	−0.243	+0.031	$x+0.086y-0.705\pi=$	−0.113	+0.018	+0.002	−0.212	−0.007	−0.043	1
Assumed P. M.		−0.326	....	−0.369	...............(pvv)	....	20767	22787	....	41072	52203	

Epoch Mean Equations.

	A.	B.
$x-0.138y+0.563\pi=$	−0″.100	−0″.140
$x+0.050$ -0.516 $=$	−0.126	....
$x+0.062$ -0.582 $=$	....	−0″.199*

Solution I.

A.

$x=-0''.114$

$\pi=+0.024+0.17y\pm0''.031$

$r_0=\pm0.044$

B.

$x=-0''.169$

$\pi=+0.051+0.17y\pm0''.047$

$r_0=\pm0.069$

With Boss's P. M. $y=+0''.002$

Solution II, assuming $\pi=0$.

	A.	B.
$x=$	−0″.115	−0″.169
$r_0=$	±0.042	±0.069

Observer's notes—272, Seeing poor; 280, Control magnets weak.

*The image of B on Plate 318 is clearly affected by some very large error, which justifies its complete rejection.

P. M. of B relative to A from micrometer measures.

All quantities in this table are in réseau intervals of 175″.8 unless otherwise stated. *Numbers in bold-face type are negative.*