14.8 Esercizi

14.8.1 Esercizi dei singoli paragrafi

14.2 - Condizioni di esistenza per una frazione algebrica

14.1 (*). Determinare per ciascuna frazione la condizione di esistenza.

a) $\dfrac{-3x^3 + x - 2x^2 + 1}{3x - 6}$;

b) $\dfrac{-x^3 - 8x}{x^2 + 4x + 4}$;

c) $\dfrac{-54}{a^3 b^5 c}$;

d) $\dfrac{a + b - 1}{2a\left(b^2 - 1\right)}$;

e) $\dfrac{3x - 8}{x^2}$;

f) $\dfrac{-3x^3 + x - 2x^2 + 1}{x - 1}$;

g) $\dfrac{a^2 - 3b}{a - b}$;

h) $\dfrac{a + 2ab - 6b}{a + b}$;

i) $\dfrac{-a}{2a - b}$;

j) $\dfrac{b - 1}{3ab}$;

k) $\dfrac{-x^3 - 8y^2}{x^2 + y^2}$;

l) $\dfrac{-8a + 3}{a^3 + 3a^2 + 3a + 1}$.

14.3 - Semplificazione di una frazione algebrica

14.2 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{2xb^2c^3}{4x^3b^2c}$;

b) $\dfrac{36a^4b^3c^2}{48a^2b^4cx}$;

c) $\dfrac{24m^3n^4}{3mn^3}$;

d) $\dfrac{30a^5x^{12}b^7}{2a^7x^{10}b^6}$;

e) $\dfrac{4x^2aby^2}{6x^2ay}$;

f) $\dfrac{20a^3b^2 - 16a^2b^3}{24a^2b}$;

g) $\dfrac{-x^2}{x^3 - xz}$;

h) $\dfrac{a^2 - b^2}{5a + 5b}$;

i) $\dfrac{x^5 - x}{x^3 - x}$.

14.3 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{x^2 - 6x + 9}{x^2 - 9}$;

b) $\dfrac{4x^2 - 4}{8x^2 - 8}$;

c) $\dfrac{ax + x + a^2 + a}{a^2 + 2a + 1}$;

d) $\dfrac{4x^2 - 4 + x^3 - x}{2x + 2}$;

e) $\dfrac{5x + 5y}{3x + 3y + ax + ay}$;

f) $\dfrac{3a^3 - 3a^2 - a + 1}{9a^4 - 1}$.

14.4 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{2x - 2 - ax + a}{x^2 - 2x + 1}$;

b) $\dfrac{6a^2 - 4ab + 3a - 2b}{4a^2 + 4a + 1}$;

c) $\dfrac{4x + 4y}{3x + 3y + ax + ay}$;

d) $\dfrac{a^2 - b^2 - ac + bc}{ab + ac + b^2 - c^2}$;

e) $\dfrac{x^2 + xy}{2x + 2y + ax + ay}$;

f) $\dfrac{3ax + 6a + 3x + 6}{6ax + 6x + 12a + 12}$.

14.5 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{2x^2 - x - 1}{3x^2 - x - 2}$;

b) $\dfrac{2x^2 - 5x + 2}{2x^2 - 7x + 6}$;

c) $\dfrac{a^3 + a^2 + a + 1}{ax + x + 2a + 2}$;

d) $\dfrac{x^2 + 5x + 6}{x^2 + 6x + 9}$;

e) $\dfrac{-2x + 2 + ax - a}{x^2 - 2x + 1}$;

f) $\dfrac{4x^3 - 4x^4 + 8x - 8x^2}{1 - x^2}$.

14.6 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{3a^2 + 6a + 3}{6a + 6}$;

b) $\dfrac{(3x-y)^3 - 2y(3x-y)^2}{(3x-y)^2}$;

c) $\dfrac{14a^2 - 7ax}{10ay - 5xy}$;

d) $\dfrac{ax^2 + x^3}{a^2x^2 - x^4}$;

e) $\dfrac{30a^3b - 30ab^3}{24a^2 - 24b^2}$;

f) $\dfrac{12 - 3x^2}{16 - 2x^3}$.

14.7 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{2x^2 - 3x + 1}{2x^2 - 5x + 3}$;

b) $\dfrac{x^2 + x - 2}{x^2 + 2x - 3}$;

c) $\dfrac{x^2 - 2x + 1}{x^3 - 3x^2 + 3x - 1}$;

d) $\dfrac{6a^2b^3 - 9a^3b^2}{2ab - 3a^2 - 2b + 3a}$;

e) $\dfrac{x^2 + 7x + 12}{x^2 - 9}$;

f) $\dfrac{x^3 - 1}{x^4 + 2x^3 + x^2 - 1}$.

14.8 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{2x^2 + 3x - 2}{2x^2 + x - 6}$;

b) $\dfrac{x^3 - x^2 + x - 1}{2x^2 - x - 1}$;

c) $\dfrac{2x^2 - 4xy}{ax - 2ay + 2x - 4y}$;

d) $\dfrac{8a^5b^5 - 4a^3b^5}{2a^3 - a - 1 + 2a^2}$;

e) $\dfrac{2x^2 - x - 3}{3x^2 + 2x - 1}$;

f) $\dfrac{x^3 + x^2 - 2x - 2}{x^3 + x^2 + 2x + 2}$.

14.9 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{-2a - a^2}{2b + ab + 4 + 2a}$;

b) $\dfrac{x^2 + 3x - 28}{x^2 + 2x - 24}$;

c) $\dfrac{2x^3 - 7x^2 + 7x - 2}{2x^3 - 5x^2 + x + 2}$;

d) $\dfrac{a^2 + a}{ab + b + a + 1}$;

e) $\dfrac{x^2 - x - 6}{x^2 + 2x - 15}$;

f) $\dfrac{x^3 + x^2 - 2x - 2}{x^2 + 2x + 1}$.

14.10 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{-a^2 - a}{ab + b + a + 1}$;

b) $\dfrac{2x^2 - x - 3}{x^3 + 1}$;

c) $\dfrac{4x + 4y}{6x + 6y + 2ax + 2ay}$;

d) $\dfrac{x^3 - x^2 + x - 1}{x^3 - 3x^2 + 3x - 1}$;

e) $\dfrac{x^2 - xy}{2x^2 - 2xy + ax^2 - axy}$;

f) $\dfrac{x^3 - 8}{\left(x^2 + 4\right)^2 - 4x^2}$.

14.11 (*). Semplifica le seguenti frazioni algebriche.

a) $\dfrac{2x^2 - x - 1}{2x^2 + x}$;

b) $\dfrac{2x^3 - x - 1}{ax^2 - ax + x^2 - x}$;

c) $\dfrac{x^6 - 1}{x^4 - 1}$.

14.4 - Moltiplicazione di frazioni algebriche

14.12 (*). Determinate i seguenti prodotti.

a) $\dfrac{3x - 6y}{5xy^3} \cdot \dfrac{2x^2y^2 + xy^3}{4y^2 - x^2}$;

b) $\dfrac{x^4 - 5x^2 + 4}{x^2 - 1} \cdot \dfrac{x}{x^3 - 4x}$;

c) $\dfrac{4x - 2a}{x - a} \cdot \dfrac{3a - 3x}{a - 2x}$;

d) $\dfrac{-1 - 2a - a^2}{1 + a^2 - 2a} \cdot \dfrac{a^3 - 3a^2 + 3a - 1}{a^4 + 2a^3 - 2a - 1}$.

14.13 (*). Determinate i seguenti prodotti.

a) $\dfrac{2a^4 + 6a + 12 + 4a^3}{16 - a^4} \cdot \dfrac{a^2 - 7a + 10}{5a^5 + 15a^2}$;

b) $\dfrac{-45x^7}{y^{-2}} \cdot \dfrac{4y^{-7}}{36x^{-1}}$;

c) $\dfrac{x^2 - 3x + 2}{x^2 - 4} \cdot \dfrac{x^2 + 3x + 2}{x^2 - 2x + 1}$;

d) $\dfrac{x^2 - 4x + 4}{x^3 - 8} \cdot \dfrac{x^2 + 2x + 4}{x^2 - 2x}$.

14.14 (*). Determinate i seguenti prodotti.

a) $\dfrac{x^3 + 3x^2 + 3x + 1}{x^2 + 2x + 1} \cdot \dfrac{ax + x}{x^2 + x}$;

b) $\dfrac{4x^3 - 4x^2 - x + 1}{8x^3 - 1} \cdot \dfrac{4x^3 + 2x^2 + x}{2x^2 - x - 1}$;

c) $\dfrac{x^2 - x - 6}{2x^2 - 8x + 8} \cdot \dfrac{x^2 + x - 6}{x^3 + 2x^2 - 9x - 18}$;

d) $\dfrac{x^4 - 1}{x^2 - 2x + 1} \cdot \dfrac{2x^2 - x - 1}{2x^3 + x^2 + 2x + 1} \cdot \dfrac{2x^2 - 2x + 2}{x^3 + 1}$.

14.15. Determinate i seguenti prodotti.

a) $\dfrac{x^2 - 4}{x^2 + 4x + 4} \cdot \dfrac{2x^2 + 8x + 8}{4x^2 - 16}$;

b) $\dfrac{2x^3 - 2x^2 - 3x + 3}{2x^2 - 4x + 2} \cdot \dfrac{x^2 - 2x + 1}{x^2 - 1}$;

c) $\dfrac{a^2 - b^2}{3x - 3y} \cdot \dfrac{6x^3y - 6xy^3}{a^2x - a^2y + b^2y - b^2x}$;

d) $\dfrac{2x^2 - x - 3}{3x^2 + 2x - 1} \cdot \dfrac{x^3 + 1}{2x^2 - x - 3}$.

14.16. Determinate i seguenti prodotti.

a) $\dfrac{x^2 + x - 2}{x^2 + 2x - 3} \cdot \dfrac{x^2 + 2x - 15}{x^2 - x - 6}$;

b) $\dfrac{2x^2 - 5x - 3}{ax - 3a + x - 3} \cdot \dfrac{2ax + 4a + 2x + 4}{4ax - 4x + 8a - 8}$;

c) $\dfrac{x^3 - x}{x^3 - 2x^2 - x + 2} \cdot \dfrac{x^3 - 8}{(x^2 + 4)^2 - 4x^2}$;

d) $\dfrac{a^3 + a^2 + a + 1}{ax + x + 2a + 2} \cdot \dfrac{x^4 - 5x^2 + 4}{x^2 - 3x + 2}$;

e) $\dfrac{2x^2 - 5x - 3}{ax - 3a + x - 3} \cdot \dfrac{2ax + 4a + 2x + 4}{4ax - 4x + 8a - 8}$;

f) $\dfrac{2ax + 4a + 2x + 4}{4ax - 4x + 8a - 8} \cdot \dfrac{-a - b}{a^2 + ab + a + b}$.

14.5 - Potenza di una frazione algebrica

14.17 (*). Determina, con le dovute condizioni sulle variabili, le seguenti frazioni.

a) $\left(\dfrac{3x^2}{5y^3}\right)^2$;

b) $\left(\dfrac{x + y}{x^2 - y^2}\right)^3$;

c) $\left[\left(\dfrac{12ab}{a^2b - ab^2}\right)^2 \cdot \left(\dfrac{a - b}{2a^2}\right)^{-2}\right]^{-1}$;

d) $\left[\left(\dfrac{x^2 + x}{x^2 + 4x + 3}\right)^2 \cdot \left(\dfrac{2x}{x + 3}\right)\right]^2$;

e) $\dfrac{a^2 - b^2}{a^3 + ab^2 + 2a^2b} \cdot \left(\dfrac{5a^2 - 5ab}{4ab + 4b^2}\right)^{-1}$;

f) $\left(\dfrac{a^2 - 9}{12a^2 - 12a + 3}\right) \cdot \left(\dfrac{12a^3 - 6a^2}{a^2 - 4a + 3}\right)^3$.

14.6 - Divisione di frazioni algebriche

14.18 (*). Semplifica le seguenti espressioni.

a) $\dfrac{x^2 - 5x + 6}{x^2 - 9} : \dfrac{x^2 - x - 6}{x^2 - 4}$;

b) $\dfrac{x^2 + ax - x - a}{x^2 - 1} : \dfrac{x^2 + 2x + 1}{x^2 + x + ax + a}$;

c) $\dfrac{2x^2 - 3x + 1}{x^3 - 3x^2 - x + 3} : \dfrac{4x^2 - 1}{x^2 - 2x - 3}$;

d) $\dfrac{x^4 - 1}{x^4 - 2x^2 + 1} : \dfrac{x^3 - x^2 + x - 1}{x^3 - 3x^2 + 3x - 1}$.

14.19 (*). Semplifica le seguenti espressioni.

a) $\dfrac{xy + x + 2y + 2}{xy + 2x - y - 2} \cdot \dfrac{x^2 - 3x + 2}{x^2 - 5x + 6} : \dfrac{x^2 + 5x + 6}{x^2 - 9}$;

b) $\left(\dfrac{a^3 - a^2}{2a^2 + a - 1} \cdot \dfrac{a^2 - 2a - 3}{a^2 - 2a + 1} \right) : \left(\dfrac{a^2 - 9}{12a^2 - 12a + 3} \cdot \dfrac{12a^3 - 6a^2}{a^2 - 4a + 3} \right)$;

c) $\dfrac{a^2 - b^2 - a - b}{3a^2 - 3b^2} : \left(\dfrac{a^2 - ab}{3a^2} \cdot \dfrac{5a + 5ab - 5a^2}{a^2 - 2ab + b^2} \right)$;

d) $\dfrac{x^3 - x^2 + x - 1}{2x^2 - x - 1} : \dfrac{2x^3 - 7x^2 + 7x - 2}{2x^3 - 5x^2 + x + 2} \cdot \dfrac{2x^2 - 5x + 2}{x^2 - 5x + 6}$.

14.20. Semplifica le seguenti espressioni.

a) $\dfrac{4x^3 - 4x^2 - 8}{4x^2 - 16} : \dfrac{x^2 - 1}{x^2 + x - 2}$;

b) $\dfrac{x^2 + x}{5x - 10} : \dfrac{x + 1}{20x}$;

c) $\dfrac{2x^2 - x - 3}{x^2 - 1} : \dfrac{x^3 + x^2 - 2x - 2}{x^2 + 2x + 1}$;

d) $\left(\dfrac{-2a}{b^3} \cdot \left(\dfrac{-ab}{4} \right)^2 \right) : \left(\dfrac{a^2}{2b^3} \right)^{-2}$.

14.7 - Addizione di frazioni algebriche

14.21 (*). Vero o falso? Se falso calcola il risultato corretto.

a) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{y^2 + x^2}{x^2 + y^2} = 1$ ☐V ☐F

b) $\dfrac{1}{x^2} + \dfrac{1}{x} = \dfrac{1 + x}{x^2}$ ☐V ☐F

c) $\dfrac{1}{x} + \dfrac{1}{x - y} = \dfrac{-y + 1}{x - y}$ ☐V ☐F

d) $\dfrac{1}{x - 1} - \dfrac{1}{1 - x} = \dfrac{2}{x - 1}$ ☐V ☐F

e) $1 + \dfrac{1}{x} = \dfrac{x + 1}{x + 1} = 1$ ☐V ☐F

f) $\dfrac{1}{a - b} + \dfrac{1}{b - a} = \dfrac{1 + 1}{a - b}$ ☐V ☐F

g) $\dfrac{1}{x} + \dfrac{2}{x} = \dfrac{3}{x}$ ☐V ☐F

h) $x - \dfrac{y}{x + y} = \dfrac{x^2 + xy - y}{x + y}$ ☐V ☐F

i) $\dfrac{1}{x + y} + \dfrac{1}{x - y} = \dfrac{2x}{x^2 - y^2}$ ☐V ☐F

j) $\dfrac{x}{y} + \dfrac{y}{y + 1} = \dfrac{x + y^2}{y}$ ☐V ☐F

14.22 (*). Riduci le seguenti somme di frazioni algebriche.

a) $\dfrac{x + 2y}{15} + \dfrac{x - y}{3}$;

b) $\dfrac{a}{2x} + 5 - \dfrac{3a}{4x^2}$;

c) $\dfrac{5a^2 + 1}{12a} - \dfrac{4a^2 - 1}{3a} + \dfrac{5a + a^2}{4a}$;

d) $\dfrac{a}{9b} - \dfrac{2b}{27a} - \dfrac{ab}{3}$.

14.23. Riduci le seguenti somme di frazioni algebriche.

a) $\dfrac{1}{x-2}+1$;

b) $\dfrac{x}{2y}+1-\dfrac{3x}{4y^2}$;

c) $\dfrac{1}{x^2}+\dfrac{1}{x}+\dfrac{1}{2}$;

d) $\dfrac{2}{xy}-\dfrac{1}{xy-1}$;

e) $-\dfrac{1}{x}+\dfrac{2}{3x}-6x$;

f) $-3x+\dfrac{1}{2x}$.

14.24. Riduci le seguenti somme di frazioni algebriche.

a) $\dfrac{5}{x}-x+\dfrac{1}{3}$;

b) $\dfrac{2x+1}{8x}-\dfrac{x-1}{4x^2}$;

c) $\dfrac{2x+1}{3}-\dfrac{1}{x}$;

d) $\dfrac{1}{x+y}-y$;

e) $\dfrac{9}{x^3y}+\dfrac{x^2}{x^2y^2}$;

f) $1-\dfrac{x+1}{x-1}$.

14.25 (*). Riduci le seguenti somme di frazioni algebriche.

a) $\dfrac{1}{x^2y}+\dfrac{1}{xy^2}-\dfrac{1}{x^2y^2}$;

b) $\dfrac{1}{x}+\dfrac{1}{2x}-\dfrac{1}{3x}$;

c) $\dfrac{2}{a}+\dfrac{1}{a^2-a}-\dfrac{1}{a-1}$;

d) $\dfrac{a-1}{a^2-a}+\dfrac{1}{a-2}-\dfrac{2}{a}$;

e) $\dfrac{2}{a-1}+\dfrac{3}{1-a}+\dfrac{a}{a-1}$;

f) $\dfrac{1}{1-x}+\dfrac{1}{x-1}+x$.

14.26 (*). Riduci le seguenti somme di frazioni algebriche.

a) $\dfrac{x+1}{x}-\dfrac{x}{x-1}$;

b) $\dfrac{1}{x-2}+\dfrac{1}{x+2}+\dfrac{1}{x^2-4}$;

c) $\dfrac{1}{x-2}+\dfrac{1}{x-1}+\dfrac{1}{x^2-3x+2}$;

d) $\dfrac{1}{x-1}+\dfrac{1}{x^2-2x+1}$.

14.27 (*). Riduci le seguenti somme di frazioni algebriche.

a) $\dfrac{1}{a-1}+\dfrac{1}{b+1}+\dfrac{1}{a+ab-b-1}$;

b) $\dfrac{1}{2a+1}+\dfrac{1}{a-1}+\dfrac{1}{2a^2-a-1}$;

c) $\dfrac{2x-3}{x}+\dfrac{-2x}{2x+3}-1$;

d) $\dfrac{1}{x-1}-\dfrac{1}{x^3-1}$.

14.8 - Espressioni con le frazioni algebriche

14.28 (*). Semplifica le seguenti espressioni.

a) $\dfrac{2a-b}{3}-\dfrac{3a-b}{6}+\dfrac{4b-a}{8}$;

b) $\dfrac{(x+1)^2+1}{5x^2+10x+5}-\dfrac{3}{10x+10}+\dfrac{x+2}{3x+3}$;

c) $\dfrac{6ab+3}{16ab}-\dfrac{16a^2b^2-3}{36a^2b^2}-\dfrac{ab+9b}{48ab^2}+\dfrac{a^2b^2-4b+6a^2b^3}{48a^2b^3}$;

d) $\dfrac{3x-2y}{3x}+\dfrac{4x-3y}{2y}-\dfrac{6x^2-2y^2}{3xy}$;

e) $\dfrac{(x+y)^2-6y^2}{6y^2(x+y)^2}-\dfrac{x}{8y^2(x+y)}+\dfrac{3x+7y}{4y(x+y)^2}$;

f) $\dfrac{a}{a-1}+\dfrac{3a}{a+1}-\dfrac{2a}{a^2-1}$.

14.29 (*). Semplifica le seguenti espressioni.

a) $\dfrac{a+1}{a-1} + \dfrac{a-2}{a+1} + \dfrac{a^2+3}{1-a^2}$;

b) $\dfrac{3a^2-6a-1}{4-(a-1)^2} + \dfrac{4}{3-a} - \dfrac{2}{a+1}$;

c) $\dfrac{2x-1}{4x+2} + \dfrac{4x-1}{12x-6} - \dfrac{6x-4}{6x+3}$;

d) $\dfrac{3x+2y+6}{3x-2y-6} - \dfrac{24y(x+2)}{9(x+2)^2-4y^2}$.

14.30 (*). Semplifica le seguenti espressioni.

a) $\dfrac{2}{a-2} + \dfrac{3}{a+3} - \dfrac{12-a}{a^2+a-6}$;

b) $\dfrac{1}{x-1} + \dfrac{x}{x+2} - \dfrac{6}{x^2+x-2}$;

c) $\dfrac{x-4}{9-3x} + \dfrac{x^2+1}{2x^2+6x} - \dfrac{27+x^2-8x}{6x^2-54}$;

d) $\dfrac{4-2a-a^2}{8-2a^2} + \dfrac{4-2a+a^2}{4a-a^3} + \dfrac{a^2+2a+4}{2a^2+4a}$.

14.31 (*). Semplifica le seguenti espressioni.

a) $\dfrac{2}{x} + \dfrac{3}{x^3} - \dfrac{5}{x^2}$;

b) $\dfrac{1}{1-x} + \dfrac{1}{x-x^2} + \dfrac{1}{x}$;

c) $\dfrac{a-3}{a+3} + \left(\dfrac{1}{a} - \dfrac{1}{3}\right) : \left(\dfrac{1}{a} + \dfrac{1}{3}\right) - \dfrac{1}{3}$;

d) $\dfrac{x^2-4x+3}{x-1} + \dfrac{2-x}{x^2-4}$.

14.32 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{1}{a-1} + \dfrac{1}{a+1}\right)\dfrac{a^2-1}{2a}$;

b) $\dfrac{1}{a-1} + \dfrac{1}{a+1}\dfrac{a^2-1}{2a}$;

c) $1 - \dfrac{a+b}{a-b}\left(\dfrac{2a-b}{a+b} - \dfrac{a-b}{a}\right)$;

d) $\dfrac{x^2+2x+1}{1-x^2} - \dfrac{x^3-1}{x-1} + \dfrac{2-8x^2}{4x^2-1}$.

14.33 (*). Semplifica le seguenti espressioni.

a) $\dfrac{1}{x-1} + \dfrac{1}{x^2-2x+1} + \dfrac{1}{x^3-3x^2+3x-1}$;

b) $\dfrac{1-x}{(x-1)^2} - \dfrac{x^3+1}{(x+1)^2} + \dfrac{3x^2-4x+1}{1-x^2}$;

c) $\dfrac{1}{2-3x} + \dfrac{2x+2}{2x} + \dfrac{6x+1}{3x-2} - \dfrac{x+2}{3x^2-2x}$;

d) $\dfrac{3x}{x^2-2xy+y^2} - \dfrac{3}{x-y} + \dfrac{9}{2y-2x}$.

14.34 (*). Semplifica le seguenti espressioni.

a) $\dfrac{6x}{x^2-4} + \dfrac{3}{2-x} - \dfrac{1}{x+2}$;

b) $\dfrac{x^2-1}{x^3-1} - \dfrac{1}{x^2+x+1}$;

c) $\dfrac{(x-1)^2}{x^3-3x^2+3x-1} - \dfrac{x-1}{(1-x)^3}$;

d) $\dfrac{1}{2x-1-x^2} - \dfrac{x}{1-x}$.

14.35 (*). Semplifica le seguenti espressioni.

a) $\dfrac{24x}{x^2+3x-4} + \dfrac{x+1}{x^2-3x+2} - \dfrac{18(x-1)}{x^2+2x-8}$;

b) $\dfrac{2}{x^2-9x+20} - \dfrac{2}{25-x^2} - \dfrac{4}{x^2+x-20}$;

c) $\dfrac{4ay-4a^2}{y^3+8a^3} + \dfrac{1}{y+2a} - \dfrac{y-a}{y^2-2ay+4a^2}$;

d) $\dfrac{8x-12}{4x^2-12x+9} - \dfrac{5x}{2x^2+3x} - \dfrac{20x}{9-4x^2}$.

14.36 (*). Semplifica le seguenti espressioni.

a) $\dfrac{x^2-2x+3}{x^3+1} + \dfrac{x-2}{x^2-x+1} - \dfrac{1}{x+1}$;

b) $\dfrac{t^2-1}{4+t^2} - \dfrac{4z-1}{2z+1} + \dfrac{24z-4t^2-2t^2z}{2t^2z+t^2+8z+4}$;

c) $\left(\dfrac{x}{y} + \dfrac{y}{x} - 2\right) : \left(1 - \dfrac{x^2}{y^2}\right) + \dfrac{x-y}{x}$;

d) $\left(\dfrac{x+a}{x-a} - \dfrac{x-a}{x+a}\right) : \left(1 - \dfrac{x-a}{x+a}\right)^2$.

14.37 (*). Semplifica le seguenti espressioni.

a) $\dfrac{x^2-4}{x^2-4x+4} - \dfrac{x^2-5x+6}{x^2-4x+4} + \dfrac{x^3-x}{x^3-2x^2-x+2} - \dfrac{x^3-8}{x^2-4x+4}$;

b) $\dfrac{2x^2-5x-3}{ax-3a+x-3} - \dfrac{2x^3-x-1}{ax^2-ax+x^2-x}$;

c) $\dfrac{b+1}{a^2+ab+a} - \dfrac{1}{a} + \dfrac{a+1-b}{a^2+2a+1-b^2}$;

d) $\dfrac{x^4-x^2a^2}{4x^2a^2+4xa^3+a^4} \cdot \left(\dfrac{2x^2+ax}{2x^2a+xa^2} \cdot \dfrac{2xa^2+a^3}{x^2-ax}\right)$.

14.38 (*). Semplifica le seguenti espressioni.

a) $\dfrac{1}{xy+yz-y^2-xz} - \dfrac{1}{zx+zy-xy-z^2} - \dfrac{1}{xy-x^2-yz+xz}$;

b) $1 - \dfrac{2x(x-2)}{x+2} + \dfrac{2-x^2}{-x-2} + \dfrac{6+(3-x)^2}{x+2}$;

c) $\dfrac{a^2b^2}{a^4-ab^3+a^3b-b^4} : \left(\dfrac{a+b}{a^3-b^3} - \dfrac{1}{a^2-b^2}\right)$;

d) $\left(\dfrac{2a^2+a}{a^3-1} - \dfrac{a+1}{a^2+a+1}\right) \cdot \left(1 + \dfrac{a+1}{a} - \dfrac{a^2+5a}{a^2+a}\right)$.

14.39 (*). Semplifica le seguenti espressioni.

a) $\dfrac{x+2}{x-3} - \dfrac{2-x}{1-x} + \dfrac{x^2+1}{x^2-4x+3} - 1$;

b) $\dfrac{1}{2}\left[\dfrac{2x}{x^2-4} - \left(\dfrac{x}{x+2} - 1\right)\right] : \dfrac{1}{2-x}$;

c) $\left(\dfrac{x^3-x^2}{1-x^2} + x - 1\right) : \left(1 - \dfrac{x}{x+1}\right)$;

d) $\left(\dfrac{1}{z} - \dfrac{1}{z+1}\right) : \left(\dfrac{z^3-z^2}{z-5} : \dfrac{z^5-z^3}{2z-10}\right)$.

14.40 (*). Semplifica le seguenti espressioni.

a) $\dfrac{x+y}{x^2+x+xy+y} - \dfrac{1}{y+1} + \dfrac{x}{x+1}$;

b) $\left(\dfrac{1}{a} + \dfrac{1}{a^2} + \dfrac{1}{a^3}\right) \cdot \left(\dfrac{1}{1-a^3} - 1\right)$;

c) $\left(\dfrac{x}{x-1} + \dfrac{x}{x+1} + \dfrac{2x}{1-x^2}\right) \cdot \dfrac{x^2+2x+1}{4x^2}$;

d) $\left(\dfrac{1}{a} + \dfrac{1}{a^2-a} - \dfrac{1}{a-2}\right) : \dfrac{1+a+a^2}{1-a^3}$.

14.41 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{x^2 - 5}{x^2 + 4x + 4} + \dfrac{1}{2 + x} + \dfrac{6}{4x + 8} \right) \cdot \dfrac{2x + 4}{2x^2 + 5x}$;

b) $\dfrac{x^2}{2} - \dfrac{(1 - x)^2}{x^3 - x} - \dfrac{2}{1 - x} + (x - 3) \cdot \dfrac{2x - x^2 - 1}{(1 - x^2)^2}$;

c) $\left(\dfrac{1}{a^2 - 2a + 1} + \dfrac{1}{a^2 - 3a + 2} \right) : \dfrac{4a^2 - 6a}{1 - a}$;

d) $\left(\dfrac{x}{x - a} - \dfrac{x}{x - 1} \right) \dfrac{ax^2 - ax - a^2x + a^2}{ax - x^2}$.

14.42 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{x + 1}{2x - 2} + \dfrac{5}{2x^2 - 2} - \dfrac{x + 3}{2x + 2} \right) : \dfrac{3}{4x^2 - 4}$;

b) $\dfrac{x^3 + x^2 + x + 1}{x^2 + 2x + 1} - \dfrac{x^3 - 1}{x^2 - 1} + \dfrac{x^2 - 3x - 4}{x^2 + 2x + 1} - \dfrac{2x^2 - x - 1}{x^2 - 1}$;

c) $\dfrac{x^4 - x^2a^2}{4x^2a^2 + 4xa^3 + a^4} : \dfrac{x^2 + ax}{2x^2a + xa^2} : \dfrac{2xa^2 + a^3}{x^2 - ax} : \dfrac{x^2 - 2ax + a^2}{4x^2 + 4ax + a^2}$;

d) $\left(\dfrac{a}{a^2 - 1} - \dfrac{a}{a^2 + 1} \right) \cdot \dfrac{a^3 - a^2 + a - 1}{2a^2} + \dfrac{a}{1 + a}$.

14.43 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{a^2 + 1}{2a} - 1 \right) : \dfrac{a^2 - 3a + 2}{4a} : \dfrac{a^2 + a - 2}{a^2 - 4} + \dfrac{a^2 + 1}{a}$;

b) $\left(x^2 + \dfrac{1}{4} + x \right) \left(\dfrac{1 - 6x}{1 - 2x} + \dfrac{1 - 12x^2}{4x^2 - 1} \right) \left(4 - \dfrac{2}{x} \right) \left(1 + \dfrac{1}{4x^2 - 1} \right)$;

c) $\dfrac{a^2 + b^2 - 2ab}{a - 1} \left(a - b + \dfrac{a^2 + b^2}{b - a} \right) \left(\dfrac{a + 2}{a - 1} - \dfrac{a^2 + 1}{a^2 - a} \right) \dfrac{a - 1}{2b^2 - 2ab}$;

d) $\left(\dfrac{x^2}{x + 1} + 1 - x \right) \left(1 + \dfrac{2}{x} + \dfrac{1}{x^2} \right) \left(\dfrac{2x}{x - 1} + \dfrac{x^2}{(x - 1)^2} + \dfrac{2x^3 - 3x^2 + x}{(x - 1)^3} \right) + \dfrac{3}{x}$.

14.44 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{2a - b}{a - 2b} - 2 \right)^2 \left[\left(\dfrac{4b^2}{a} - a \right) \left(\dfrac{3a + 2b}{a + 2b} - 1 \right) \right]^2$;

b) $\dfrac{\left(\dfrac{x^2 + 2x + 2}{x^3} - \dfrac{1}{x + 2} \right) \left(\dfrac{10 - 3x}{4 - x^2} - \dfrac{1}{2 - x} \right)}{\left(\dfrac{2 + x}{2 - x} - \dfrac{2 - x}{2 + x} \right)^2}$;

c) $\left(\dfrac{a - 1}{a + 4} - \dfrac{a + 1}{a - 4} \right) : \left(\dfrac{a - 1}{4 - a} + \dfrac{a + 1}{a - 4} \right) \cdot \dfrac{a^2 - 16}{5a^2 - 20a}$;

d) $\left(1 - \dfrac{x + 1}{x - 1} \right) : \dfrac{x - 2}{x - 1} + \dfrac{3x + 6}{x^2 - 4} - \dfrac{2}{x^2 - 3x + 2} : \left(\dfrac{x}{x - 3} - \dfrac{x}{x + 3} \right) + \dfrac{2x - 3}{x(x - 2)(x - 1)}$.

14.45 (*). Semplifica le seguenti espressioni.

a) $\dfrac{1}{x^2 + 4x + 4} + \dfrac{1}{x^3 + 2x^2} - \dfrac{1}{x^2} + \dfrac{1}{x^3 + 4x^2 + 4x} - \dfrac{2}{x^4 + 4x^3 + 4x^2};$

b) $\dfrac{4x + 1}{2 - 2x} + \dfrac{3x + 5}{3x - 5} - \dfrac{1}{3x} + 3x \cdot \dfrac{x - 3}{3x^2 - 8x + 5};$

c) $\left(\dfrac{3}{x - 8} - \dfrac{22x - 48}{x^2 - 12x + 32} + \dfrac{4x}{x - 4} \right) : \dfrac{x - 12}{x - 8};$

d) $\dfrac{x^3 - 25x}{x^2 + 8x + 15} : \left(\dfrac{x}{2x + 6} + \dfrac{2}{3 - x} + \dfrac{6 + x}{x^2 - 9} \right).$

14.46 (*). Semplifica le seguenti espressioni.

a) $\dfrac{x - 2}{2} - \left(\dfrac{1}{3x - 6} + \dfrac{1}{2} \right) \cdot \dfrac{4}{2 - x} - \left[\dfrac{1}{2}x^2(x - 6) + 6x - 5 \right] : (x - 2)^2;$

b) $2x^2 \cdot \dfrac{x - 3}{x - 1} + (2x - 1)(1 - x) - \dfrac{1}{1 - x} + \dfrac{1}{9x - 9} + x;$

c) $\dfrac{x + 5}{x^2 - 6x + 5} + \dfrac{3x + 3}{x^2 - 4x + 3} - \dfrac{2(x + 1)}{x^2 - 8x + 15} - \dfrac{2x + 4}{(x - 1)(x - 5)};$

d) $\dfrac{x^2 + x - 6}{27 - 3x^2} + \dfrac{5x - 18}{2x^2 - 18} - \dfrac{11}{4x - 12} + \dfrac{1}{3} - \dfrac{1}{12(x - 3)}.$

14.47 (*). Semplifica le seguenti espressioni.

a) $\left(2a - \dfrac{1}{2} \right) a + \left(\dfrac{2a^2}{2 + a} + 2 + a \right) x + (3a - 2)x - x \left(\dfrac{8}{a + 2} + 6a - 4 \right);$

b) $\left(\dfrac{3x^2 + x - 3}{2 - 6x} + \dfrac{x}{2} \right) : \dfrac{3 + 2x}{3x - 1} + \dfrac{9x - 3}{(6x - 2)(3 + 2x)} - \dfrac{x}{3 + 2x};$

c) $\dfrac{8x^2 - 2a^2}{(a - 1)^2} + \left(\dfrac{2x^2 - 1}{a + 1} + \dfrac{2x^2 + 1}{1 - a} \right)(a + 1) + \dfrac{4x^2 - 1}{a - 1} - \dfrac{16x^2 - 5a^2 + a}{(a - 1)^2};$

d) $\left[\left(\dfrac{x + 6}{2} + \dfrac{3}{a - 1} \right) : \dfrac{x}{a - 1} + 3a + \dfrac{x(1 + 2a)}{2} + \dfrac{x(1 - 7a) + 3}{2x} \right] \cdot \dfrac{2x^2}{x^2 + 3}.$

14.48 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{1}{5}x - \dfrac{x - 1}{x - 5} \right) \left(\dfrac{x + 1}{5} - \dfrac{1}{x + 5} \right) \cdot (25 - x^2) - \dfrac{2}{5}x(x + 5)(7 - x);$

b) $\left(\dfrac{ax}{a^2 - 9} \cdot \dfrac{3a + 9}{b - 3} - \dfrac{ax}{9 - 3b - 3a + ab} \right) \cdot \dfrac{b - 3}{2};$

c) $\left(\dfrac{1}{a} + \dfrac{x - a}{x^2 + ax} - \dfrac{2}{a + x} \right) \cdot \dfrac{x^2 + 3x}{x^2 - a^2} + \left(\dfrac{x^2}{x^2 - a^2} + \dfrac{a - x}{a + x} \right) \cdot \dfrac{x - a}{2ax - a^2};$

d) $\left(\dfrac{a}{a^2 - a} + \dfrac{a}{a^2 + a} + \dfrac{1}{a} - \dfrac{1}{a - a^2} \right) : \dfrac{3a + 1}{2a^2 - 2}.$

14.49 (*). Semplifica le seguenti espressioni.

a) $\dfrac{2}{x^2 - x} + \dfrac{2x}{x^3 - 1} - \dfrac{3}{x^2 + x + 1} - \dfrac{4x + 1}{x^4 - x};$

b) $\dfrac{3a}{a + 1} + \dfrac{2}{a - 1} + \dfrac{a - 5}{a^2 - 1};$

c) $\dfrac{4a^2 - 2 - 2a}{3a^2 + 3 - 6a} - \dfrac{a + 1}{2a - 2} - \dfrac{2a + 1}{3a - 3};$

d) $\dfrac{2 + x}{4x - 2x^2} - \dfrac{2x}{8 - 2x^2} + \dfrac{1}{x^2 - 2x}.$

14.50 (*). Semplifica le seguenti espressioni.

a) $\left[\left(\dfrac{1}{a^2 + 9 - b^2 + 6a} - \dfrac{1}{a^2 + 9 + b^2 + 6a - 2ab - 6b} \right) : \dfrac{-6b}{3a + 9 + 3b} \right]^{-1};$

b) $\left(\dfrac{3}{x^6 - x^3} - \dfrac{1}{9x^3 - 9} \right) : \dfrac{9 + x^2 + 3x}{3x^5 + 3x^3 + 3x^4} + \dfrac{6x - 5}{x - 1} - \dfrac{x - 2}{x + 2} + \dfrac{12}{x^2 + x - 2};$

c) $\left(\dfrac{1}{2b - 2 - a + ab} + \dfrac{1}{1 - b} \right) : \left(\dfrac{1}{b + 1} - \dfrac{1}{2 + a + 2b + ab} \right) + \dfrac{2b^2 - b - 1}{b^2 - 2b + 1};$

d) $\left(\dfrac{x + 2}{x - 2} - \dfrac{2x^2 + ax}{x^2 - 4} + \dfrac{ax - 3x + 2a - 6}{(x + 2)(a - 3)} \right) \cdot \dfrac{x^2 - 4}{a - 4}.$

14.51 (*). Semplifica le seguenti espressioni.

a) $\left[\dfrac{(a - 3b)(b - 2)}{a^2 - 4a + 4} - \left(2 - \dfrac{1}{a + 1} \right) \dfrac{a^2 + 2a + 1}{2a + 1} + \dfrac{3a + b}{2 - a} + \dfrac{a^3 - 4a + 4}{(a - 2)^2} \right] \cdot \dfrac{a - 2}{b};$

b) $\left(\dfrac{x + 1}{a} \right)^3 \cdot \dfrac{6x^2}{-x^3 - x^4} \cdot \left(1 - \dfrac{1}{x + 1} \right) \left(\dfrac{x^2 - 1}{x - 1} \right)^2 : \dfrac{x + 1}{a^3};$

c) $\left(\dfrac{6ab + 6ab^2}{ab - 3a^2b} - \dfrac{3(a - x) + 6b}{1 - 3a} + \dfrac{2 - x}{x} - 1 - \dfrac{2}{x} - \dfrac{1}{1 - 3a} \right) : \dfrac{3}{1 - 3a};$

d) $\left(\dfrac{(a + 2)^2 - 4(1 + ax) - 7}{a^2 - 16} + \dfrac{3ax + a}{a - 4} - \dfrac{5a + 17}{a^2 - 16} + ax - 1 \right) : \dfrac{ax + 1}{a - 4}.$

14.52 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{a^2 + 3ax}{10a^2 - 4a} + \dfrac{x}{2} - \dfrac{2 + ax}{2a} \right) \cdot \dfrac{2a}{a^2 + 4} + \left[\dfrac{2(x + 3)}{2 - 5a} - \dfrac{x}{4 - 10a} + \dfrac{11}{5a - 2} \right] : \dfrac{a^2 + 4}{2a};$

b) $\left(\dfrac{x + 2a}{2 + a} + \dfrac{x - 6}{a + 4} + \dfrac{x - 4}{-a - 2} + \dfrac{2x}{a + 4} + 1 \right) \cdot \dfrac{a + 4}{x + a + 2};$

c) $\left(\dfrac{3ax + 2b}{a^2} + \dfrac{6ax + 8b}{2a^2 - a^2b} + \dfrac{8b - 2b^3}{a^2 \left(b^2 - 4b + 4 \right)} + \dfrac{16b}{a^2b - 2a^2} \right) \cdot \dfrac{b - 2}{b - 4};$

d) $\left(\dfrac{1}{a} - \dfrac{x + a}{x^2 + ax} \right) : \dfrac{x^2 - a^2}{ax^2 + 3ax} + \dfrac{x + a}{2ax - a^2} \left(\dfrac{x^2}{x^2 - a^2} - \dfrac{x - a}{x + a} \right) - \dfrac{4x - ax}{x^2 + ax} + \dfrac{a^2 - 2a}{x^2 - a^2}.$

14.53 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{1 + x - y}{x - y} + \dfrac{1 - x - y}{x + y} - \dfrac{2x + 2y}{x^2 + xy} \right) \cdot \left(y - \dfrac{x^2}{y} \right);$

b) $\dfrac{a^2 - ab - 2b^2}{a^2 - 4ab + 3b^2} \cdot \dfrac{a^2 - 9b^2}{a^2 + 5ab + 4b^2} \cdot \dfrac{a^2 + 3ab - 4b^2}{a^2 + ab - 6b^2};$

c) $\left(1 - \dfrac{b}{a + b} \right) \cdot \left(1 + \dfrac{a}{b} \right) \cdot \dfrac{b}{a};$

d) $\left(\dfrac{a - 2b}{2b} + \dfrac{2a}{a - 2b} + \dfrac{a^3 + 2a^2b}{2a^2b - 8b^3} \right) \cdot \dfrac{a^2 - 4b^2}{2a^2 + 4b^2}.$

14.54 (*). Semplifica le seguenti espressioni.

a) $\dfrac{6x + 1}{x^2 - 4x + 4} + \dfrac{10x - 12}{4x} + 1 + \dfrac{1 - x^2}{x(x - 2)} + \dfrac{9x - 28}{x^2 - 4x + 4};$

b) $\left(\dfrac{4}{x^2 - 4} + \dfrac{x}{x^2 - 2x} + \dfrac{3}{x^2 + 2x} + \dfrac{24}{x^3 - 4x} \right) \cdot \dfrac{x^2 + 2x}{x + 6};$

c) $\left(\dfrac{6x^2 - 26x - 15}{x - 5} : \dfrac{2x^2 + 1}{x^2 - x - 20} \cdot \dfrac{6x}{x + 4} + \dfrac{90x}{2x^2 + 1} \right) : \dfrac{12x^2}{2x^2 + 1};$

d) $\left(\dfrac{6x + 2}{x^2 - 4x + 4} + \dfrac{2}{2x - x^2} \right) \left(1 - \dfrac{2}{x} \right) + \dfrac{x - 1}{x^2 - 2x} + \dfrac{x - 4}{x^3 - 2x^2}.$

14.55 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{a + x + 2}{2a - a^2} + \dfrac{2x}{2 - a} + \dfrac{x}{a} \right) \cdot \dfrac{2 - a}{a + 3} - \dfrac{a + 2}{a^2 + 3a};$

b) $\left[\dfrac{2x + 2(a + 1)}{a^2 - 1} + \dfrac{2 - x}{1 - a} \right] \cdot \dfrac{a + 1}{ax + 3x};$

c) $\left(\dfrac{x^2 + 3x + 9}{x^3 - 27} + \dfrac{1}{9 - 6x + x^2} - \dfrac{1}{3 - x} \right) : \dfrac{2x - 5}{x^2 - 9}.$

14.56 (*). Semplifica le seguenti espressioni.

a) $\left[\dfrac{a + x}{a - x} : \left(\dfrac{2x}{a - x} + 1 \right) - \dfrac{4a^2 + x}{x + 2} \right] \cdot \dfrac{2x^2 - 8}{1 - 2a^2};$

b) $\left(\dfrac{1 - x}{x - a} + \dfrac{3 + 2a}{x + a} - \dfrac{5x}{x + a} - \dfrac{6x^2 - a^2}{a^2 - x^2} \right) : \dfrac{3a + 2}{a^2 - 2ax + x^2};$

c) $\left[\dfrac{a - b + 1}{a^2 - ab} + \dfrac{1}{a^2 + ab} + \dfrac{2b\left(1 - a^2\right)}{a\left(a^2 - b^2\right)} - \dfrac{2a - b^2 + 2b}{a^3 - ab^2} \right] \cdot \dfrac{a^2 - b^2}{2b^2 - b}.$

14.57 (*). Semplifica le seguenti espressioni.

a) $\dfrac{a + 1}{a^2 - 3a + 2} - \dfrac{a}{(4 - a^2)(1 - a)} + a^2 - a;$

b) $\dfrac{x^4 - x^2 a^2}{4x^2 a^2 + 4x a^3 + a^4} : \dfrac{x^2 + ax}{2x^2 a + x a^2} \cdot \dfrac{2x a^2 + a^3}{x^2 - ax};$

c) $\dfrac{1}{4} \left(\dfrac{15}{x + 3} - \dfrac{12}{x + 1} + \dfrac{1}{x - 1} \right) : \left[\dfrac{1}{2} \left(\dfrac{3}{x + 1} - \dfrac{1}{x - 1} \right) \right];$

d) $\left(1 + x^2\right) \left(\dfrac{1}{x - 1} + \dfrac{1}{x + 1} \right) : \left(\dfrac{x}{x - 1} + \dfrac{1}{x + 1} \right).$

14.58 (*). Semplifica le seguenti espressioni.

a) $\left[\dfrac{a + 3}{a^2 + 3a + 2} : \left(\dfrac{1}{a + 2} - \dfrac{2}{a + 1} \right) \right] : \left[\left(\dfrac{1}{2a} + \dfrac{1}{a + 1} \right) : (3a + 1) \right];$

b) $\dfrac{2y + 1}{2y - 1} \left(\dfrac{x + 2y}{x - 2y} + \dfrac{x - 2y}{x + 2y} - \dfrac{2x^2 + 1 + 4y^2}{x^2 - 4y^2} \right) : \dfrac{4y^2 + 1 + 4y}{x^2 - 4y^2};$

c) $(x + 2) \cdot \left(\dfrac{2}{x^2 + 5x + 6} + \dfrac{2}{x + 3} - \dfrac{2}{x + 2} \right)^2;$

d) $2 \left(1 + \dfrac{x}{y} \right) + \left[\left(\dfrac{x}{y} + 1 \right)^2 : \left(\dfrac{x}{y} - 1 \right) \right] \cdot \left(\dfrac{x}{y} - 1 \right)^2 : \left(\dfrac{x}{y} + 1 \right).$

14.59 (*). Semplifica le seguenti espressioni.

a) $\left(\dfrac{x^2+4}{2x}+2\right):\left(\dfrac{x^2+4}{2x}-2\right):\left(\dfrac{x}{x-2}:\dfrac{2}{x+2}\right)^2$;

b) $x^2\left(\dfrac{1+x}{1-x}-\dfrac{1-x}{1+x}\right):\left[\left(\dfrac{1+x}{1-x}-1\right)\cdot\left(1-\dfrac{1}{1+x}\right)\right]$;

c) $\left[\left(m-2+\dfrac{8m}{m-2}\right):\left(m+2-\dfrac{4m+8}{m+2}\right)\right]\cdot\dfrac{m-2}{m+2}$;

d) $\left(1-\dfrac{a}{3-a}\right)\cdot\left(1-\dfrac{3}{a+3}\right)\cdot\dfrac{a-3}{3}\cdot\dfrac{a+3}{a}$;

e) $\left(\dfrac{a-x}{a+x}+1\right)\cdot\left(\dfrac{a^2+x^2}{2ax}-1\right)\cdot\left(\dfrac{a+x}{a-x}-1\right)\cdot\left(\dfrac{a^2+x^2}{2ax}+1\right)$.

14.60 (*). Semplifica le seguenti espressioni.

a) $\dfrac{x+5}{x}\cdot\left(\dfrac{x-2}{x^2+7x+10}-\dfrac{x-3}{x^2+8x+15}\right):\dfrac{x}{x^2+5x+6}$;

b) $\dfrac{2a+3b}{2a-3b}\cdot\left(\dfrac{2a}{2a+3b}+\dfrac{3b}{2a-3b}\right):\left(\dfrac{2a}{2a-3b}-\dfrac{3b}{2a+3b}\right)$;

c) $\dfrac{x-y}{2}\cdot\left(\dfrac{2xy}{x^2-xy+y^2}+1\right):\left[\dfrac{x^3-y^3}{x^3+y^3}\cdot\left(\dfrac{2y}{x-y}+1\right)\right]$;

d) $\dfrac{x+y}{x-y}\cdot\left(\dfrac{x}{xy+y^2}-\dfrac{y}{x^2+xy}\right):\left(\dfrac{x}{xy-y^2}-\dfrac{y}{x^2-xy}\right)$.

14.61 (*). Semplifica le seguenti espressioni.

a) $(a+1)\cdot\left(\dfrac{a-1}{a^2+4a+3}-\dfrac{a-2}{a^2+3a+2}\right):\dfrac{4}{a^2+5a+6}$;

b) $\left(\dfrac{2x}{x+y}+\dfrac{y}{x-y}-\dfrac{y^2}{x^2-y^2}\right):\left(\dfrac{1}{x+y}+\dfrac{x}{x^2-y^2}\right)$;

c) $\left[\dfrac{a^4}{16}-\dfrac{(a+1)^4}{a^4}\right]:\left[\dfrac{a^2}{4}+\dfrac{(a+1)^2}{a^2}\right]+\left(\dfrac{a+1}{a}+\dfrac{a}{2}\right)\cdot\left(\dfrac{a+1}{a}-\dfrac{a}{2}\right)$;

d) $\left(\dfrac{x+y}{a+b}+\dfrac{x-y}{a-b}\right):\left(\dfrac{x+y}{a-b}+\dfrac{x-y}{a+b}\right)$;

e) $(a^2-b^2)\cdot\left(\dfrac{2a+4b}{a^2-b^2}-\dfrac{4}{a-b}\right):\left(\dfrac{2b}{a^2-b^2}-\dfrac{2}{a-b}\right)$.

14.62 (*). Semplifica le seguenti espressioni.

a) $\left(1-\dfrac{2y}{x+y}\right):\left[\left(1-\dfrac{2xy}{x^2+xy+y^2}\right):\dfrac{x^3+y^3}{x^3-y^3}\right]^2$;

b) $\dfrac{2x-3}{x-1}\cdot\left[\left(\dfrac{3x^2-2}{x-1}+\dfrac{6x-2}{x-3}\right)\cdot\dfrac{1}{x-2}-\dfrac{x+13}{x-3}\right]^2$;

c) $\dfrac{a^2-2a+1}{a^2+2a+1}\left(a+\dfrac{a}{a+3}\cdot\dfrac{4}{a+3}\right)\left[\left(\dfrac{2}{a+1}-1+a\right):\dfrac{2+3a+a^2}{a^2+2a-3}\right]^2:\left(\dfrac{2}{a+1}+a-1\right)^2$;

d) $\left(2a+\dfrac{1+4a-8a^3}{4a^2-1}\right):\left(\dfrac{2}{a-1}+\dfrac{4}{2a+1}-2\right)\cdot\left(\dfrac{8a^2}{1+2a}-2a\right)\cdot\left(a-\dfrac{2a}{2a+1}\right)^{-1}$.

14.63 (*). Semplifica le seguenti espressioni.

a) $\left[\left(\dfrac{1}{x-2}-\dfrac{1}{3-x}\right)\cdot\left(\dfrac{5-2x}{x^2+3-4x}\right)^{-1}+\left(\dfrac{1-x}{x-2}\right)^2\right]\cdot\left[\left(\dfrac{x-1}{x-2}\right)^2-\dfrac{x-1}{x^2-4x+4}\right]^{-1};$

b) $\dfrac{2x}{x^2-1}\cdot\left[\dfrac{2}{x^2-1}+\dfrac{1}{3}x\left(1+\dfrac{1}{x-1}+\dfrac{9}{x^3-x^2}\right)\right]:\left[\dfrac{1}{1-x}\left(\dfrac{2}{x+1}+\dfrac{3}{x}\right)-\dfrac{x^2}{3x-3}\right];$

c) $\dfrac{x^3-x}{x^3-2x^2-x+2}+\dfrac{x^3-8}{x^2-4x+4}+\dfrac{x^4-5x^2+4}{x^2-3x+2}+\dfrac{x^2-5x+6}{x^2-4x+4};$

d) $\dfrac{\dfrac{1}{x^2-25+20y-4y^2}-\dfrac{1}{x^2-25+4y^2+4xy}}{\dfrac{x^2-4y^2}{x^2+25+10x-4y^2}}\cdot\left(\dfrac{x}{4}+\dfrac{y}{2}-\dfrac{5}{4}\right):\dfrac{1}{\dfrac{x}{y}-2}.$

14.64 (*). Semplifica le seguenti espressioni.

a) $\dfrac{\dfrac{x^{3n}-y^{3n}}{x^{2n}+2x^ny^n+y^{2n}}}{\dfrac{2x^{2n}-4x^ny^n+2y^{2n}}{x^{2n}-y^{2n}}}+\dfrac{1}{2}\left(x^n-y^n\right)-\dfrac{x^ny^n}{2\left(x^n+y^n\right)};$

b) $\dfrac{\dfrac{x^ny+x^{n+1}+y^{n+1}+xy^n}{x^{n+1}-x^ny-xy^n+y^{n+1}}}{\dfrac{x^n+y^n}{x^n-y^n}}-\dfrac{\dfrac{x^3}{x^3-y^3}}{\dfrac{x^2}{x^2+y^2+xy}};$

c) $\dfrac{\dfrac{x^{n+1}+xy-x^ny-y^2}{x^{2n}-y^2}}{\dfrac{1+\dfrac{y}{x}}{x^{n-1}-\dfrac{y}{x}}}-\left(-\dfrac{a}{a+2}+\dfrac{x}{x+y}-\dfrac{1}{\dfrac{ax+2x+ay+2y}{2y+2x}}\right);$

d) $\dfrac{\left(\dfrac{x^3-b^3}{x^3-3bx^2+3b^2x-b^3}-\dfrac{bx}{x^2-2bx+b^2}+\dfrac{x+b}{b-x}\right):\left(\dfrac{x+b}{x-b}+1\right)}{\dfrac{x^2-bx-6b^2}{x^2+bx-2b^2}}:\dfrac{b}{x}.$

14.65 (*). È vero che $P=\left(\dfrac{4a^2-1}{8a^3b}:\dfrac{2a+1}{4a^4b}\right)\cdot\left(\dfrac{2a^5}{6a-3}:\dfrac{a^2}{27}\right)$ è sempre positiva per qualunque $a\neq0$ e $b\neq0$?

14.66 (*). Data $Q=\dfrac{4-\left(a^2-2ab+b^2\right)}{b-2-a}:\left(\dfrac{4-2a+2b}{3a^2}:\dfrac{2}{a^3}\right)$, quali condizioni dobbiamo porre alla variabile b affinché sia vera la proposizione "Per $a=3$, l'espressione Q assume il valore -1"?

14.8.2 Risposte

14.1. a) $x \neq 2$, b) $x \neq -2$, c) $a \neq 0 \land b \neq 0 \land c \neq 0$, d) $a \neq 0 \land b \neq \pm 1$, e) $x \neq 0$, f) $x \neq 1$, g) $a \neq b$, h) $a \neq -b$, i) $b \neq 2a$, j) $a \neq 0 \land b \neq 0$, k) $x \neq 0 \land y \neq 0$, l) $a \neq -1$.

14.2. a) $\frac{c^2}{2x^2}$, b) $\frac{3a^2c}{4bx}$, c) $8m^2n$, d) $\frac{15x^2b}{a^2}$, e) $\frac{2}{3}by$, f) $\frac{b(5a-4b)}{6}$, g) $\frac{x}{z-x^2}$, h) $\frac{a-b}{5}$, i) $x^2 + 1$.

14.3. a) $\frac{x-3}{x+3}$, b) $\frac{1}{2}$, c) $\frac{x+a}{a+1}$, d) $\frac{x^2+3x-4}{2}$, e) $\frac{5}{3+a}$, f) $\frac{a-1}{3a^2+1}$.

14.4. a) $\frac{2-a}{x-1}$, b) $\frac{3a-2b}{2a+1}$, c) $\frac{4}{a+3}$, d) $\frac{a-b}{b+c}$, e) $\frac{x}{a+2}$, f) $\frac{1}{2}$.

14.5. a) $\frac{2x+1}{3x+2}$, b) $\frac{2x-1}{2x-3}$, c) $\frac{a^2+1}{x+2}$, d) $\frac{x+2}{x+3}$, e) $\frac{a-2}{x-1}$, f) $\frac{4x(x^2+2)}{x+1}$.

14.6. a) $\frac{a+1}{2}$, b) $3(x-y)$, c) $\frac{7a}{5y}$, d) $\frac{1}{a-x}$, e) $\frac{5ab}{4}$, f) $\frac{3(2+x)}{2(4+x^2+2x)}$.

14.7. a) $\frac{2x-1}{2x-3}$, b) $\frac{x+2}{x+3}$, c) $\frac{1}{x-1}$, d) $\frac{3a^2b^2}{a-1}$, e) $\frac{x+4}{x-3}$, f) $\frac{x-1}{x^2+x-1}$.

14.8. a) $\frac{2x-1}{2x-3}$, b) $\frac{x^2+1}{2x+1}$, c) $\frac{2x}{a+2}$, d) $\frac{4a^3b^5}{a+1}$, e) $\frac{2x-3}{3x-1}$, f) $\frac{x^2-2}{x^2+2}$.

14.9. a) $\frac{-a}{b+2}$, b) $\frac{x+7}{x+6}$, c) $\frac{2x-1}{2x+1}$, d) $\frac{a}{b+1}$, e) $\frac{x+2}{x+5}$, f) $\frac{x^2-2}{x+1}$.

14.10. a) $-\frac{a}{b+1}$, b) $\frac{2x-3}{x^2-x+1}$, c) $\frac{2}{a+3}$, d) $\frac{x^2+1}{(x-1)^2}$, e) $\frac{1}{a+2}$, f) $\frac{x-2}{x^2+4-2x}$.

14.11. a) $\frac{x-1}{x}$, b) $\frac{2x^2+2x+1}{x(a+1)}$, c) $\frac{x^4+x^2+1}{x^2+1}$.

14.12. a) $\frac{-3(2x+y)}{5y(x+2y)}$, b) 1, c) 6, d) $-\frac{1}{a+1}$.

14.13. a) $\frac{-2(a-5)}{5a^2(a^2+4)}$, b) $-5\frac{x^8}{y^5}$, c) $\frac{x+1}{x-1}$, d) $\frac{1}{x}$.

14.14. a) $a+1$, b) x, c) $\frac{1}{2(x-2)}$, d) 2.

14.17. a) $\frac{9x^4}{25y^6}$, b) $\frac{1}{(x-y)^3}$, c) $\frac{(a-b)^4}{576a^4}$, d) $\frac{4x^6}{(x+3)^6}$, e) $\frac{4b}{5a^2}$, f) $\frac{72a^6(a+3)(2a-1)}{(a-3)^2(a-1)^3}$.

14.18. a) $\frac{(x-2)^2}{x^2-9}$, b) $\left(\frac{x+a}{x+1}\right)^2$, c) $\frac{1}{2x+1}$, d) $\frac{x-1}{x+1}$.

14.19. a) $\frac{y+1}{y+2}$, b) $\frac{a-3}{2a+6}$, c) $-\frac{1}{5}$, d) $\frac{x^2+1}{x-3}$.

14.21. a) F, b) V, c) F, d) V, e) F, f) F, g) V, h) V, i) V, j) F.

14.22. a) $\frac{2x-y}{5}$, b) $\frac{20x^2+2ax-3a}{4x^2}$, c) $-\frac{8a^2-15a-5}{12a}$, d) $\frac{3a^2-2b^2-9a^2b^2}{27ab}$.

14.25. a) $\frac{x+y-1}{x^2y^2}$, b) $\frac{7}{6x}$, c) $\frac{1}{a}$, d) $\frac{2}{a(a-2)}$, e) 1, f) x.

14.26. a) $-\frac{1}{x^2-x}$, b) $\frac{2x+1}{x^2-4}$, c) $\frac{2}{x-2}$, d) $\frac{x}{(x-1)^2}$.

14.27. a) $\frac{a+b+1}{(a-1)(b+1)}$, b) $\frac{3a+1}{2a^2-a-1}$, c) $\frac{-3(x+3)}{x(2x+3)}$, d) $\frac{x(x+1)}{x^3-1}$.

14.28. a) $\frac{a}{24}$, b) $\frac{13x^2+30x+23}{30(x+1)^2}$, c) $\frac{1}{18}$, d) $-\frac{1}{2}$, e) $\frac{x+22y}{24y^2(x+y)}$, f) $\frac{4a}{a+1}$.

14.29. a) $\frac{a}{a+1}$, b) $\frac{3(1-a)}{a-3}$, c) $\frac{8-2x}{6(2x+1)}$, d) $\frac{3x-2y+6}{3x+2y+6}$.

14.30. a) $-\frac{3}{a+3}$, b) $\frac{x-2}{x-1}$, c) $\frac{1}{6x}$, d) 0.

14.31. a) $\frac{2x^2-5x+3}{x^3}$, b) $\frac{2}{x(1-x)}$, c) $-\frac{1}{3}$, d) $\frac{x^2-x-7}{x+2}$.

14.32. a) 1, b) $\frac{a^2+1}{2a(a-1)}$, c) $\frac{b^2}{a(b-a)}$, d) $\frac{x^3+3x-2}{x-1}$.

14.33. a) $\frac{x^2-x+1}{(x-1)^3}$, b) $\frac{-x^3-x^2+x-1}{x^2-1}$, c) $\frac{3x+2}{x}$, d) $\frac{3(5y-3x)}{2(x-y)^2}$.

14.34. a) $\frac{2}{x+2}$, b) $\frac{x}{x^2+x+1}$, c) $\frac{x}{(x-1)^2}$, d) $\frac{x^2-x-1}{(x-1)^2}$.

14.35. a) $\frac{7(x+1)}{(x+4)(x-1)}$, b) $\frac{22}{(x+5)(x-5)(x-4)}$, c) $\frac{a}{y^2-2ay+4a^2}$, d) $\frac{9}{2x-3}$.

14.36. a) $\frac{x^2-2x}{x^3+1}$, b) $\frac{3-2t^2}{t^2+4}$, c) $\frac{x-y}{x+y}$, d) $\frac{x(a+x)}{a(x-a)}$.

14.37. a) $\frac{x^2+x-1}{2-x}$, b) $-\frac{x+1}{x(a+1)}$, c) 0, d) $\frac{x(x+a)}{a(2x+a)}$.

14.38. a) $\frac{2}{(x-y)(y-z)}$, b) $\frac{15-x}{x+2}$, c) ab, d) $\frac{a-1}{a^2+a}$.

14.39. a) $\frac{10}{x-3}$, b) $\frac{2(1-x)}{x+2}$, c) -1, d) $\frac{1}{2}$.

14.40. a) $\frac{y}{y+1}$, b) $\frac{1}{1-a}$, c) $\frac{x+1}{2x}$, d) $\frac{1}{a-2}$.

14.41. a) $\frac{1}{x+2}$, b) $\frac{x^6+x^5-x^4-x^3+18x^2-2}{2x(x-1)(x+1)^2}$, c) $\frac{-1}{2a(a-1)(a-2)}$, d) $\frac{a(a-1)}{a-x}$.

14.42. a) 6, b) $-\frac{2x+5}{x+1}$, c) $\frac{x^3}{a^3}$.

14.43. a) $\frac{(a+1)^2}{a}$, b) $\frac{8x^2}{2x-1}$, c) $\frac{2a-1}{a-1}$, d) $\frac{4(2x-1)}{(x-1)^2}$.

14.44. a) $36b^2$, b) $\frac{(x-2)^2(2x^2+3x+2)}{8x^5}$, c) -1, d) $\frac{2x+3}{3(x-2)(x-1)}$.

14.45. a) $\frac{-2}{x^2(x+2)}$, b) $\frac{x+2}{6x(x-1)}$, c) $\frac{4x-3}{x-4}$, d) $2(x-3)$.

14.46. a) $\frac{6x-5}{3(x-2)^2}$, b) $\frac{45x-19}{9(1-x)}$, c) $\frac{8(x+2)}{(1-x)(x-3)(x-5)}$, d) $-\frac{4x+111}{6(x+3)(x-3)}$.

14.47. a) $\frac{a(4a-1)}{2}$, b) $\frac{3-2x}{2x+3}$, c) $\frac{-8x^2+a^2+1}{(a-1)^2}$, d) $x(2a+1)$.

14.48. a) $\frac{-x^4+14x^3+35x^2+380x}{25}$, b) $\frac{ax}{a-3}$, c) $\frac{x+a+3}{a(x+a)}$, d) 2.

14.49. a) $\frac{1}{x(x-1)}$, b) 3, c) $\frac{1}{6}$, d) $\frac{1}{2(2+x)}$.

14.50. a) $(a+3-b)^2$, b) $\frac{14x+3}{3(x-1)}$, c) $\frac{b}{b-1}$, d) $-x$.

14.51. a) $\frac{8-3b}{a-2}$, b) $-6(x+1)^2$, c) $a+x+1$, d) $\frac{a^2+3a-8}{a+4}$.

14.52. a) $\frac{1}{5a-2}$, b) 3, c) $\frac{3x}{a}$, d) $\frac{x^2}{(x+a)(x-a)}$.

14.53. a) $-\frac{2y}{x}$, b) 1, c) 1, d) $\frac{a+2b}{2b}$.

14.54. a) $\frac{5x^3-28}{2x(x-2)^2}$, b) $\frac{x+3}{x-2}$, c) $3x-13$, d) $\frac{7}{x-2}$.

14.55. a) $\frac{x}{a}$, b) $\frac{1}{a-1}$, c) $\frac{x+3}{x-3}$.

14.56. a) $4(x-2)$, b) $\frac{(x-a)(2x-a)}{x+a}$, c) $-\frac{a}{b}$.

14.57. c) $\frac{x-3}{x+3}$, d) $\frac{2x(x^2+1)}{x^2+2x-1}$.

14.58. a) $-2a(a+1)$, b) 1.

14.59. a) $\frac{4}{x^2}$, b) 2, c) $\frac{m+n}{m-n}$, d) $\frac{2a-3}{3}$, e) $\frac{a^2-x^2}{ax}$.

14.60. a) $\frac{2}{x}$, b) $\frac{2a+3b}{2a-3b}$, c) $\frac{x-y}{2}$, d) 1.

14.61. a) 1, b) x, c) 0, d) $\frac{ax-by}{ax+by}$ e) a^2-b^2.

14.62. a) $\frac{x+y}{x-y}$, b) $\frac{x-1}{2x-3}$.

14.63. a) $\frac{1}{x-2}$, d) $\frac{1}{x+2y}$.

14.64. a) $\frac{x^{2n}}{x^n+y^n}$, b) $\frac{y}{x-y}$, c) $\frac{x}{x+y}$, d) $\frac{b}{x-3b}$.

14.65. Vero: $P = 9a^4$.

14.66. $b \neq 1 \wedge b \neq 5$.