XVI. *Residual Charge of the Leyden Jar.—Dielectric Properties of different Glasses.*

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I. BEFORE proceeding to comparative experiments on different glasses, it appeared desirable to verify experimentally the two following propositions:—

(*a*) If two jars be made of the same glass but of different thicknesses, if they be charged to the same potential for equal times, discharged for equal times and then insulated, the residual charge will after equal times have the same potential in each. In experiments in which potentials and not quantities of electricity are measured the thickness of the jar may be chosen arbitrarily, nor need any inconvenience be feared from irregularities of thickness.

(*b*) Residual charge is proportional to exciting charge.

These propositions may be included in one law—that superposition of simultaneous forces is applicable to the phenomena of residual charge.

To verify (*a*) two flasks were prepared of the glass afterwards referred to as No. 1. One was estimated to be about 1 millim., the other 6 or 7 millims. thick. These were cleansed and insulated in the usual way by filling with strong sulphuric acid without soiling the neck of the flask. They were placed in the same basin of water, which was electrically connected with the outside of the quadrant electrometer. The interiors of the flasks were respectively connected with the two quadrants; they were also connected together by a wire which could at any instant be removed. One DANIELL's element gave a deflection of 69 scale-divisions. The two flasks were charged together with 48 elements for some minutes, and it was observed that the equal charge of the two quadrants did not deflect the needle. The flasks were discharged for 15 to 20 seconds and insulated, still connected. The connecting wire was then removed, and the subsequent movement of the image observed. If left undisturbed a maximum of about 20 divisions of the scale was attained. But usually the deflection in from 20 to 30 seconds reaching 10 divisions, the thick flask was discharged, and the image was driven from the scale, showing that at that time the potential of either flask was represented by more than 500 scale-divisions, and hence that the difference between them was less than 2 per cent. of either of them. When the charge was negative the error was in favour of the thin flask. This is in complete accord with anomalous results subsequently obtained with the same glass. Correcting for this peculiarity of the glass we may conclude that the law is verified within the limits of these experiments.

The second proposition was confirmed with two different glasses; but the results in one case are not quite accordant, possibly owing to variations of temperature, or to slight unremoved effects of previous chargings; but the irregularities indicate no continuous deviation from the law. In these and all the subsequent experiments the flasks were blown as thin as possible in the body, but with thick necks, the neck being thick that the capacity of any zone might be small.

Flask of optical soft crown, No. 5. The electrometer reads $28\frac{1}{2}$ for one DANIELL'S element. The charging in each case lasted some hours, the discharge 30 seconds. The flask was then insulated and remained insulated; the residual charge was read off from time to time. Column I. gives the time in minutes from insulation; II., III., IV., V., the readings at those times, the exciting electromotive force being respectively that of 48, 48, 24, and 12 elements of the battery.

I.	II.	III.	IV.	V.
1	90	92	46	....
5	218	225	103	51
15	....	344	160	79
30	....	423	197	99
60	478	462	226	114
120	492	....	233	120

Flask of blue glass, No. 2. The reading of the electrometer for one element was 69 divisions. The charge in each case lasted 10 minutes, the discharge 30 seconds; the flask was then insulated. Column I. gives the time from insulation in minutes; II., III., IV., V., the potentials at those times when the batteries which had been employed were respectively of 48, 12, 3, and 1 DANIELL'S elements.

I.	II.	III.	IV.	V.
$\frac{1}{2}$	414	102	$26\frac{1}{2}$	9
Maximum potential.	472	$117\frac{1}{2}$	$30\frac{1}{4}$	10
$1\frac{1}{2}$	456	114	$29\frac{1}{2}$	10
$2\frac{1}{2}$	385	96	$24\frac{3}{4}$	$8\frac{1}{4}$
$4\frac{1}{2}$	256	65	$16\frac{1}{2}$	6
$9\frac{1}{2}$	120	....	$8\frac{1}{2}$	3

The agreement in this case, all the experiments being made on the same day, is fairly satisfactory.

II. The following method of treating the question of residual charge was suggested to the author by Professor CLERK MAXWELL; it is essentially similar to that used by BOLTZMANN for the after-effects of mechanical strain ("Zur Theorie der elastischen Nachwirkung," aus dem lxx. Bande der Sitz. der k. Akad. der Wissensch. zu Wien, II. Abth. Oct. Heft, Jahrg. 1874).

Let L be the couple tending to twist a wire or fibre about its axis, θ_t the whole angle of torsion at time t; then L at time t depends upon θ_t, but not wholly on θ_t, for the torsion to which the wire has been submitted at all times previous to t will slightly affect the value of L. Assume only that the effects of the torsion at all previous times can be superposed. The effect of a torsion $\theta_{t-\omega}$ at a time ω before the time considered, acting for a short time $d\omega$, will continually diminish as ω increases; it may be expressed by $-\theta_{t-\omega}f(\omega)d\omega$, where $f(\omega)$ is a function of ω, which diminishes as ω increases. Adding all the effects of the torsion at all times, we have

$$L=a\theta_t-\int_0^\infty \theta_{t-\omega}f(\omega)d\omega.$$

In the case of a glass fibre BOLTZMANN finds that $f(\omega)=\frac{A}{\omega}$, where A is constant for moderate value of ω, but decreases when ω is very great.

The after-effects of electromotive force on a dielectric are very similar; to strain corresponds electric displacement, to stress electromotive force. Let x_t be the potential at time t as measured by the electrometer, and y_t the surface-integral of electric displacement divided by the instantaneous capacity of the jar; then, assuming only the law of superposition already proved to be true for simultaneous forces, we may write

$$x_t=y_t-\int_0^\infty y_{t-\omega}\phi(\omega)d\omega, \quad \ldots \ldots \ldots \ldots \quad (1)$$

where $\phi(\omega)$ is a function decreasing as ω increases. This formula is precisely analogous to that of BOLTZMANN; but in the case of a glass jar the capacity of which is too small to give continuous currents, it is not easy to measure y_t; hence it is necessary to make x_t the independent variable. From the linearity of the equation (1) as regards $x_t\,y_t$ and the value of $y_{t-\omega}$ for each value of ω, and from the linearity of the equation expressing $x_{t-\omega}$ for each value of ω, it follows that

$$y_t=x_t+\int_0^\infty x_{t-\omega}\,\psi(\omega)d\omega, \quad \ldots \ldots \ldots \ldots \quad (2)$$

where $\psi(\omega)$ decreases as ω increases.

The statement of equations (1) and (2) could be expressed in the language of action at a distance and electrical polarization of the glass, y_t being replaced by the polarization as measured by the potential of the charge which would be liberated if the polarization were suddenly reduced to zero, the jar being insulated. It should be noted that the view of this subject adopted by the author in the previous paper[*] can be included in equation (2) by assuming that $\psi(\omega)$ is the sum of a series of exponentials.

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[*] *Vide* Phil. Trans. vol. clxvi. pt. 2.

If $\psi(\omega)$ is determined for all values of ω, the properties of the glass, as regards conduction and residual charge, are completely expressed.

Suppose that in equation (2) $x_t=0$ till $t=0$, and that after that time $x_t=X$ a constant,

$$y_t=X(1+\int_0^t\psi(\omega)d\omega),$$

$$\frac{dy}{dt}=X\psi(t);$$

now when t is very great, $\frac{dy}{dt}$ is the steady flow of electricity through the glass divided by the capacity. Hence

$$\psi(\infty)=B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

B is the reciprocal of the specific resistance multiplied by 4π and divided by the electrostatic capacity of the substance.

We have no practicable method of determining y_t; but we may proceed thus:—During insulation y_t is constant; we have then

$$x_t=A-\int_0^t x_{t-\omega}\psi(\omega)d\omega; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

x_t and $x_{t-\omega}$ alone can be measured; (4) is, then, the equation by aid of which $\psi(\omega)$ must be determined.

(α) Let x_t be maintained constant=X from time 0 to time t, then insulate; differentiating (4),

$$\left.\begin{aligned}\frac{dx_t}{dt}&=-X\psi t-\int_0^t\frac{dx_{t-\omega}}{dt}\psi\omega d\omega\\&=-X\psi t\\&=-BX \text{ when } t \text{ is very great.}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

To find B, charge for a long time to a constant potential, insulate and instantly observe the rate of decrease of the potential.

(β) Let the flask be charged for a very short time τ and then be insulated; at the instant of insulation we have $\frac{dx_t}{dt}=-X\psi(\tau)$. Hence an approximation may be made to an inferior limit of $\psi(0)$.

(γ) Let x_t be constant =X for a long time from $t=-T$ to $t=0$; discharge and, after a further time t, insulate:—

$$\left.\begin{aligned}x_{t+\tau}&=A-X\int_{t+\tau}^{T+t+\tau}\psi(\omega)d\omega-\int_0^\tau x_{t-\omega+\tau}\psi(\omega)d\omega,\\\frac{dx_t}{dt}&=X(\psi(t)-B) \text{ when } \tau \text{ vanishes.}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

To find $\psi(t)$ in terms of t charge for a very long time, discharge and from time to time insulate and determine $\frac{dx_t}{dt}$.

(δ) Let the charging last during a shorter time τ', then discharge and insulate from time to time as in (γ):—

$$\frac{dx_t}{dt}=X\{\psi(t)-\psi(\tau'+t)\}. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (7)$$

(ε) Charge during time τ', and reverse the charge for time τ'' before discharging:—

$$\frac{dx_t}{dt}=X\{\psi(t)-2\psi(\tau''+t)+\psi(\tau'+\tau''+t)\}. \quad . \ . \ . \ . \quad (8)$$

III. *Glass* No. 1.—This glass is a compound of silica, soda, and lime. In a damp atmosphere it "sweats," the surface showing a crystalline deposit easily wiped off. For a soda glass it is very white. Density 2·46.

When the flask was mounted, connected with the electrometer, the image from which was deflected 70 divisions by one Daniell's element, and insulated, it was found to steadily develop a negative charge, amounting to 11 scale-divisions in 10 seconds, and increasing to a maximum of 25 divisions. The cause of this the author cannot explain. Two other flasks of the same glass behaved in a similar manner—in one case, with the thin flask of § I., the charge rising to 40 divisions, with the thick flask to only 15 divisions. No sensible effect of the same kind was noticed with any other glass. The effect does not appear to be due to the connecting wires (for these were repeatedly removed and replaced by fresh ones), nor to difference between the acid within and that outside the flask, as this also was changed.

Experiment α.—The flask was charged to 500 divisions for half an hour, insulated, and the potential observed after 5, 10, 15, 20 seconds. The mean of several experiments gave for these times 372, 275, 216, 170: hence the loss in 5 seconds is about 25 per cent.; and from this we may readily deduce $\frac{dx}{dt}$, since the percentage of loss is not materially different in the second interval of 5 seconds, $B=3{\cdot}4$, the minute being unit of time.

Experiment β.—An attempt was made to estimate $\psi(0)$. The charging lasted one second. In two seconds from insulation the charge fell from 500 to about 330, which gives $\psi(0)$ certainly greater than 10·2. This can, of course, only be regarded as the roughest approximation.

Experiment γ.—The flask was charged positively for about 19 hours with 48 elements, the electromotive force of which is represented by about 3360 scale-divisions. It was then discharged, and at intervals insulated for 10 seconds, and the residual charge developed in that time observed. Column I. gives the time in minutes from first discharge to the middle of each 10-second period; II. the charge developed in ten seconds; III. the estimated value of $\psi(t)-B$, obtained by correcting for the negative charge which it was found this flask took in 10 seconds, and dividing by 3360.

These results are certainly much below the true values, for the image moved over the scale much more rapidly in the first than in the second five seconds; but their ratios are probably fair approximations.

I.	II.	III.	I.	II.	III.
½	190	0·36	15	17	0·050
1	106	0·21	20	14	0·045
2	57	0·12	30	11	0·040
3	42	0·094	40	7	0·032
4	36	0·084	50	5	0·029
5	30	0·074	60	3	0·025
7	26	0·066	90	0	0·020
10	22	0·060	180	−5	0·011

Experiment δ.—This experiment was tried both with a positive and negative charge The charge lasted 90 minutes. The readings were made as in γ.

I. gives the time in minutes;
II. the readings when the charge was positive;
III. when the charge was negative;
IV. the mean of II. and III.;
V. the value calculated from γ.

I.	II.	III.	IV.	V.
½	180	190	185	190
1	93	120	106	106
2	45	75	60	57
3	31	68	49	42
4	...	53	...	36
5	22	47	34	30
7	16	43	29	26

The same experiments were made, but with time of charging only 5 minutes.

Columns II. and III. give the means in each case of two separate observations, made on different occasions.

I.	II.	III.	IV.	V.
$\frac{1}{2}$	150	170	160	162
1	80	$92\frac{1}{2}$	$86\frac{1}{4}$	79
2	$22\frac{1}{2}$	$41\frac{1}{2}$	32	31
3	$10\frac{1}{2}$	29	$19\frac{3}{4}$	18
4	5	23	14	13
5	0	$20\frac{1}{2}$	$10\frac{1}{4}$	8
7	−4	$18\frac{1}{2}$	7	6

Glass No. 2.—This glass is of a deep blue colour; it is composed of silica, soda, and lime, the quantity of soda being less than in No. 1, but of lime greater. The colour is due to a small quantity of oxide of cobalt. The temperature throughout ranged from 62° F. to 64° F.

Experiment α.—The flask was charged for several minutes, and then insulated. The intensity of the charge before insulation, and at intervals of 5 seconds after, was observed, the whole experiment being repeated three times. The mean is given.

Time	0.	5.	10.	15.	20.	30.	40.
Reading . . .	497	465	433·6	405	379	342	311

$$B=0{\cdot}77.$$

Experiment β.—The charging lasted 2 seconds. The flask was then insulated, and its charge measured at intervals of 5 seconds. The mean of two fairly accordant observations is given.

Time	0.	5.	10.	15.	20.
Reading . . .	490	390	325	$282\frac{1}{2}$	249

Hence $\psi(0)>2{\cdot}4$, probably much greater.

Experiment γ.—The flask was charged with 48 elements for 8 hours in the first experiment, and subsequently for 3 hours 25 minutes for a second experiment, the effect of the previous charging being still considerable when the charging began. After discharge the flask was from time to time insulated for 20 *seconds*, and the residual charge developed in that time was observed.

I. gives the time from discharge to the middle of the periods of insulation;

II. and III. the observations in the two experiments;

IV. the results corrected by a curve from II. and III.;

V. the values of $\psi(t)-\text{B}$, again not corrected for the rapid decrease in $\frac{dx}{dt}$ after each insulation.

It may be remarked that the image in this case moved in 10 seconds about $\frac{3}{5}$ of what it attained in 20 seconds.

I.	II.	III.	IV.	V.
½	470	468	469	0·42
1	300	325	310	0·28
2	178	183	180	0·16
3	134	134	133	0·12
4	105	107	106	0·094
5	89	91	90	0·080
7	68	69	68	0·061
10	52	54	53	0·047
15	39	41	40	0·036
20	32	36	34	0·030
30	...	29	28	0·025
60	...	20	20	0·018
90	16	...	16	0·014
600	...	6	6	0·005

Experiment δ.—The charging with 48 elements lasted 5 minutes. The experiment was tried twice with positive and negative charges respectively. II. and III. give the readings, whilst IV. gives the value calculated from the curve of γ.

I.	II.	III.	IV.
½	...	385	385
1	212	228	232
2	...	110	112
3	66	67	72
4	47	47	50
5	34½	33½	37
10	11½	10	13
15	5	4½	6

Experiment ε.—The flask was for many hours charged negatively, then positively for 5 minutes, and observations of residual charge were made as before.

Column III. are the values calculated from γ by the formula $\frac{dx}{dt}=\psi(t)-2\psi(\mathrm{T}+t)$.

I.	II.	III.
½	−310	−301
1	−168	−154
2	− 48	− 44
4	+ 8	+ 6
5	+ 17	+ 16
10	+ 28	+ 27
15	+ 27	+ 28
20	+ 27	+ 27

Glass No. 3.—Common window-glass, composed of silica, soda, and lime, the quantity of lime being greater than in No. 2. This glass does not "sweat" in a moist atmosphere. The temperature was 68° F.

Experiment α.—The flask was charged to 425 divisions for about 3½ hours, and was then insulated. After ½ minute the charge was 210; 1 minute, 138; 2 minutes, 74; 3 minutes, 50. Hence B is certainly greater than unity, and lies intermediate between the values for glasses 1 and 2.

Experiment γ gives the *observed values* of $\psi(t)-\mathrm{B}$ throughout a little less than in No. 2. As this flask was not very well blown further experiments were not made.

If the values of $\psi(t)-\mathrm{B}$ could be accurately obtained for these three glasses, they would certainly differ less from each other than they appear to do.

Glass No. 4.—*Optical hard crown.* Density 2·48. Composed of silica, potash, and lime. The composition may be regarded as corresponding to a glass intermediate between 1 and 3, with the soda replaced by potash.

The experiments α and β were made by the following modified method:—The whole battery of 48 elements was used, one pole being connected with the case of the electrometer and the exterior of the flask, the other with the interior of the flask by a cup of mercury and also with one electrode of the electrometer. The other electrode was permanently connected with the interior of the flask. It was ascertained that the image remained at zero whether both quadrants were charged equally or both discharged. The potential of the 48 elements was measured by 6 elements at a time; the extremes were 432 and 437, and the total 3475 scale-divisions. Where the charge of each quadrant is considerable and of the same sign, it cannot be assumed that the deflection for a given difference is the same as if the charges were small, or of equal and opposite sign; in fact, if the potentials of the quadrant and the jar of the electrometer are of the same sign, the sensibility of the instrument will be diminished (*vide* Maxwell's 'Electricity and Magnetism,' vol. i. p. 273). On this account the results for $\psi(t)$ given below should be increased by about $\frac{1}{15}$ part of their value. The experiment consisted in insulating the flask from the battery, and observing the difference of potential between the flask and the battery after a suitable interval.

The flask was charged and instantly insulated at 8.25 P.M. The image traversed 164 divisions in 10 seconds. The flask was again connected with the battery, and insulated from time to time.

I. gives the middle of the period of insulation, measured from 8.25; II. the division traversed; III. the duration of insulation; IV. the value of $\psi(t)$.

I.	II.	III.	IV.
5 seconds.	164	10 seconds.	0·28
1 minute.	26	20 ,,	0·022
2 minutes.	14	20 ,,	0·012
3 ,,	11	20 ,,	0·0094
5 ,,	8	20 ,,	0·0069
10 ,,	34	2 minutes.	0·0049
15 ,,	28	2 ,,	0·0040
20 ,,	22	2 ,,	0·0031
30 ,,	36½	4 ,,	0·0026
60 ,,	25	4 ,,	0·0018
15 hours.	11	6 ,,	0·0005

Glass No. 5.—Optical soft crown. Density 2·55. Composed of silica and potash, with lead and lime in small quantity.

Experiments α *and* β.—68 divisions of the electrometer-scale equal one Daniell's element.

The flask was charged for 5 seconds, insulated, and the loss in the subsequent 10 seconds observed. The result may be regarded as giving an approximation to $\psi(\frac{1}{6})$. The mean of two experiments gives a fall from 471 to 452½, or $\psi(\frac{1}{6})=0·23$.

Charging for 45 seconds, and observing the loss during 30 seconds, gave $\psi(1)=0·06$.

The flask was connected with the battery continuously, and only insulated at intervals, and connected with the electrometer for a short time to determine the rate of loss. The following values are thence deduced:—

t	5.	10.	30.	60.	120.	180.	300.
$\psi(t)$ . . .	0·025	0·017	0·012	0·009	0·007+	0·007−	0·006

$\psi(\infty)$ probably does not differ much from 0·005 or 0·004.

Experiment γ.—The flask was charged for three days with 48 elements, equal to 3260 divisions, or thereabouts, then discharged.

I. gives the time from first discharge to the middle of the period of insulation;

II. the scale-divisions traversed;

III. the times of insulation in minutes;

IV. the value of $\frac{dx_t}{dt}$;

V. $\psi t - B$.

I.	II.	III.	IV.	V.
$\frac{1}{2}$	53	$\frac{1}{6}$	318	0·098
1	62	$\frac{1}{3}$	186	0·057
2	64	$\frac{1}{2}$	128	0·039
3	62	$\frac{2}{3}$	93	0·029
5	70	1	70	0·021
10	92	2	46	0·014
15	71$\frac{1}{2}$	2	35·75	0·011
20	63	2	31·5	0·0097
30	48$\frac{1}{2}$	2	24·25	0·0074
60	109	8	13·6	0·0042
90	89	8	11·12	0·0034
125	69$\frac{1}{2}$	8	8·7	0·0027
180	54	8	6·75	0·0021

The results thus obtained agree fairly with those obtained by Experiment β; the differences may be attributed to errors of observation.

Experiment δ.—The charging lasted 5 minutes. The experiment was performed twice, with positive and negative charges respectively.

I. gives the time from first discharge;

II. the period of insulation;

III. and IV. the divisions traversed in that time;

V. their mean;

VI. the value of $\frac{dx_t}{dt}$ thence obtained;

VII. the value of $\frac{dx_t}{dt}$ calculated from the last experiment.

I.	II.	III.	IV.	V.	VI.	VII.
$\frac{1}{2}$	$\frac{1}{6}$	37	...	37	222	252
1	$\frac{1}{3}$	42	43	$42\frac{1}{2}$	127·5	124
2	$\frac{1}{2}$	32	$32\frac{1}{2}$	$32\frac{1}{4}$	64·5	72
5	1	23	$22\frac{1}{2}$	$22\frac{3}{4}$	22·75	24
15	4	23	...	23	5·75	4·25

The differences between VI. and VII. are somewhat large; they may perhaps be in part attributed to the fact that $\frac{dx_t}{dt}$ is deduced from observations on a quantity not uniformly increasing, on the assumption that the increase is uniform, and to the inequality of the times of insulation.

Glass No. 6.—A flint glass containing less lead than No. 7.

Experiments α and β.—66 divisions of the scale equal to one DANIELL'S element.

The flask was continuously connected with the battery, and only insulated for brief periods, to determine the rate of loss, the following values are thence deduced:—

t	1.	5.	15.	120.	240.
$\psi(t)$. . . .	0·013	0·007	0·004	0·0016	0·001

Experiment γ.—The flask was charged for 13 hours with 48 elements, then discharged. The columns are the same as in glass No. 5.

I.	II.	III.	IV.	V.
1	21	$\frac{1}{3}$	63	0·02
5	$37\frac{1}{2}$	2	18·75	0·006
15	48	6	8·0	0·0026
75	60	24	2·5	0·0008

There is a considerable discrepancy between the values of $\psi(1)$ from α and $\psi(1)-B$ from γ; the former may be in error, as it was deduced from the time of traversing 3 divisions only.

Glass No. 7.—Optical "light flint." Density 3·2. Composed of silica, potash, and lead. Almost colourless. The surface neither "sweats" nor tarnishes in the slightest degree. This glass at ordinary temperatures is sensibly a perfect insulator.

A flask was mounted in the usual way on July 15th; it was charged with 48 elements for some hours, the potential being 240 scale-divisions as measured through the "induction-plate" of the electrometer. The charging-wire was then withdrawn. On July 23rd the wire was again introduced and connected with the induction-plate; a charge of 183 scale-divisions still remained, although the temperature of the room was as high as 72° F. The flask was again put away till Aug. 9th, when the charge was found to be 178. On September 14th it was 163. Lastly on October 14th it had fallen to 140.

As might be expected from the last experiment, the residual charge in this glass is small. The flask was charged for nine hours with 48 elements; it was discharged, and after 4 minutes insulated; in 2 minutes the residual charge had only attained $11\frac{1}{2}$ divisions, giving $\psi(5)=0\cdot0017$. It was again insulated after 44 minutes; in 12 minutes the charge was $10\frac{1}{2}$, giving $\psi(50)=0\cdot00026$.

Since the loss by conduction is so small, the flask may be strongly charged by an electrophorus instead of with the battery. If it is left insulated for a considerable time, and then discharged, and the return charge observed, it may be assumed that the exciting charge has been sensibly constant during the latter portion of the period of insulation.

The flask was strongly charged and remained insulated for 3 hours 40 minutes; it was then discharged, and from time to time was temporarily insulated to ascertain the rate of return of charge.

At $\frac{1}{2}$ minute	250	divisions in	$\frac{1}{6}$ minute	=1500	per minute.
5 minutes	247	,,	1 ,,	= 247	,, ,,
10 ,,	285	,,	2 minutes	= $142\frac{1}{2}$	,, ,,
15 ,,	304	,,	3 ,,	= 101	,, ,,
30 ,,	326	,,	6 ,,	= 54	,, ,,

It was immediately charged again, insulated for 70 minutes, and then the observations repeated.

At $\frac{1}{2}$ minute	120	divisions in	$\frac{1}{6}$ minute	=720	per minute.
1 ,,	135	,,	$\frac{1}{3}$,,	=405	,, ,,
2 minutes	125	,,	$\frac{1}{2}$,,	=250	,, ,,
5 ,,	121	,,	1 ,,	=121	,, ,,
10 ,,	142	,,	2 minutes	= 71	,, ,,
15 ,,	106	,,	2 ,,	= 53	,, ,,

The ratios of the numbers in the two experiments agree fairly.

Glass No. 8.—"Dense flint." Density 3·66. Composed of silica, lead, and potash, the proportion of lead being greater than in No. 7.

Experiment α.—The flask was charged for three hours to 500 divisions, and then insulated:—

After 1 minute from insulation	$499\frac{3}{4}$
,, 5 minutes ,, ,,	499
,, 30 ,, ,, ,,	495

hence $\psi(180)=0\cdot0004$.

Experiment β.—The flask was charged for 5 seconds, insulated, and the potential read off at intervals of ½ minutes. The results are the mean of two observations:—

Reading . .	497	479½	475½	474	473	472½	472¼
Time . . .	0	½	1	1½	2	2½	3

from this it may be inferred that $\psi(0)$ is considerably greater than 0·07. An experiment on residual charge gives $\psi(1)-\mathrm{B}=0{\cdot}017$.

Glass No. 9.—Extra dense flint. Density 3·88. Colour slightly yellow. The proportion of lead is somewhat greater than in No. 8. The surface tarnishes slowly if exposed unprotected to the air.

The flask was charged for 10 seconds to 500, and was then insulated.

After	1 minute	the reading was	499
"	3 minutes	" " "	497¾
"	5 "	" " "	495
"	30 "	" " "	486½
"	60 "	" " "	479

The flask was charged with 48 elements for 1½ hour, and the residual charge observed,

$$\psi(2)-\mathrm{B}=0{\cdot}003.$$

An attempt was made to obtain a knowledge of the form of the function $\psi(t)$ in the same manner as for No. 7. The flask was charged from the electrophorus, and allowed to stand insulated for 22 hours; it was then discharged and temporarily insulated at intervals.

At	½ minute	traversed	130	divisions in	⅙ minute	=780	per minute.
"	1 "	"	160	" "	⅓ "	=480	"
"	2 minutes	"	145	" "	½ "	=290	"
"	5 "	"	152	" "	1 "	=152	"
"	10 "	"	189	" "	2 minutes	= 94½	"
"	15 "	"	217	" "	3 "	= 72	"
"	30 "	"	275	" "	6 "	= 46	"
"	60 "	"	360	" "	12 "	= 30	"
"	120 "	"	437	" "	24 "	= 18	"

It will be remarked that in this case $\psi(t)-\mathrm{B}$ deviates further from the reciprocal of the time than in the case of No. 7.

Glass No. 10.—Opal glass. This glass is white and opaque. It is essentially a flint. The reason for examining it was to ascertain if its opacity had any striking effect on its electrical properties.

Experiment α.—The flask was charged to 462 divisions for five hours; on insulation the loss was found to be 4 to 5 divisions in an hour; hence $B=0{\cdot}00016$.

Experiment β.—Charged to 462 for 10 seconds; a loss of 2 in 3 minutes was observed on insulation.

Experiment γ.—The flask was charged with 48 elements, each equal to 67 divisions of the scale, for 5 hours, and was then discharged.

At 1 minute, $4\frac{1}{2}$ divisions in $\frac{1}{3}$ minute.
„ $2\frac{1}{2}$ minutes, 6 „ „ 1 „
„ 5 „ 6 „ „ 2 „

or

$$\psi(1)-B=0{\cdot}004$$
$$\psi(2\tfrac{1}{2})-B=0{\cdot}002$$
$$\psi(5)-B=0{\cdot}001$$

The residual charge is smaller than in any other glass observed.

A few of the results of the preceding experiments are collected in the following Table for the purpose of ready comparison.

I. The greatest value of ψt observed.
II. „ least „ „ „
III. $\psi(1)-B$ as obtained by experiment γ.
IV. $\psi(5)-B$ „ „ „
V. $\psi(60)-B$ „ „ „

Glass.	I.	II.	III.	IV.	V.
1.	10·2	3·4	0·21	0·073	0·025
2.	2·45	0·76	0·28	0·08	0·018
3.	......	1·0	......	0·05	0·01
4.	0·28	0·0005	0·0215	0·0064	0·0013
5.	0·23	0·006	0·057	0·021	0·0042
6.	0·013	0·001	0·02	0·006	1·0008*
7.	......	0·00002	......	0·0017	0·00026†
8.	0·07	0·0004	0·017	......	......
9.	0·002	......	0·003‡	......	......
10.	0·0014	0·00016	0·004	0·001	......

From this Table two classes can at once be selected as having well-marked characters. The soda-lime glasses, although the composition and colour vary widely, agree in

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* $\psi(75)-B$. † $\psi(50)-B$. ‡ $\psi(20)-B$.

possessing small insulating power, but exhibit very great return charge. The values of the function $\psi(t)-\mathrm{B}$ for the three glasses agree almost within the limits of these roughly approximate experiments.

At the opposite extreme are the flints or potash-lead glasses, which have great specific resistance. The experiment does not prove that No. 7 conducts electricity at all; for it is not certain that the very slight loss of charge may not be due to conduction over the surface of the glass; but it is certainly not less than 100,000 times as resistant as No. 1. The flints also have very similar values of $\psi(t)-\mathrm{B}$, much smaller than the soda-lime glasses.

IV. It is known that glass at a moderately high temperature conducts electricity electrolytically. The following experiment shows that with the more conductive glasses electrolytic conduction occurs at the ordinary temperature of the air.

A flask of blue glass, No. 2, was very carefully insulated with strong sulphuric acid within the flask, and was placed in a vessel of caustic potash. Platinum wires dipping in the two liquids communicated with the quadrants of the electrometer. On insulation the acid developed a positive charge as follows:—

In	½	minute	15	divisions	of the	scale.
„	1	„	22½	„	„	„
„	2	minutes	33½	„	„	„
„	5	„	47	„	„	„
„	10	„	55	„	„	„
„	15	„	57	„	„	„

one DANIELL'S element giving 68 divisions of the scale.

The experiment was repeated after the flask had stood some days with the two liquids connected by a platinum wire; the potential developed much more slowly, and in 50 minutes was stationary at 38½ divisions.

Summary.—These experiments are subject to many causes of error. Deducing $\frac{dx_t}{dt}$ from an observation of dx_t in a period of many seconds or even minutes gives values of $\psi(t)-\mathrm{B}$ necessarily too low, in some cases very much too low. No attempt was made to keep the glass at a constant temperature; the temperature of the room was occasionally noted, but is not given here, as no conclusion is based upon it. The experiments were performed irregularly at such times as other circumstances permitted. It will be observed that the discords of the experiments of verification are considerable, but they are irregular. It may, perhaps, be assumed that they are within the limits of error, and we may infer that the fundamental hypothesis is verified, viz. that the effects on a dielectric of past and present electromotive forces are superposable. OHM'S law asserts the principle of superposition in bodies in which conduction is not complicated by residual charge. Conduction and residual charge may be treated as parts of the same phenomenon, viz. an after-effect, as regards electric displacement, of electromotive force. The experiments appear to show, though very roughly, that the principle of OHM's law is applicable to the *whole* phenomenon of conduction through glass.

V. *Effect of Temperature.*

The purpose of the previous experiments being to examine generally the applicability of the formulæ and to compare the values of $\psi(t)$ for different glasses of known composition, no account was taken of temperature, and no attempt made to maintain it constant, although it is well known that changes of temperature greatly affect both conduction and polarization in glass*. It appeared, however, desirable to compare the same glass at different temperatures in the same manner as different glasses at the same temperature.

The flask, carefully filled with sulphuric acid as before, was placed in an earthenware jar containing sulphuric acid, which was in its turn placed in a double cylindrical shell of copper, with oil or water between the cylinders. The jar was covered by two disks of wood, through holes in the centre of which the neck of the flask projected. A cap of sealing-wax, carrying a small cup of mercury for making electrical connexions with the interior, closed the flask. A thermometer dipped into the acid outside the flask for reading the temperature of the glass, whilst a second thermometer was inserted between the cylinders in the oil or water to help the observer in regulating the temperature by means of a spirit-lamp. In the two experiments below freezing-point the earthenware jar was removed from the oil-bath and placed in a freezing-mixture of hydrochloric acid and sulphate of soda. In all cases the temperature was maintained approximately constant for some time before observing. It will be remarked that, as the acid was not stirred, the temperature-readings are subject to a greater probable error than that due to the thermometer itself. But as the changes of temperature of the acid were always very slow, the error thus introduced cannot seriously affect the results. All temperatures are Centigrade. The actual readings are given, and also the temperature, roughly corrected when necessary, for the exposed portion of the stem of the thermometer. The times in these and in most of the previous experiments were taken by ear from a dead-beat seconds clock, the eye being fixed on the image and the scale. In the intervals between the short insulations to determine $\frac{dx}{dt}$, the flask was either connected with the battery or discharged. In all cases the registered time of observation is taken at the middle of the period of insulation; thus, in the experiment at $39\frac{1}{2}°$ below, insulation was made one second before the minute, and the reading one second after. Two glasses were examined, Nos. 2 and 7, selected as extreme cases. The whole of the observations made are given, excepting three manifestly in error, although

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* *Vide* Mr. PERRY, Proceedings of the Royal Society, 1875, p. 468; Prof. CLERK MAXWELL, "Electricity and Magnetism," Art. 271.

only a portion are used. The values of $\psi(5)$ and $\psi(10)$, for glasses 2 and 7 respectively, are taken as sensibly equal to B, and are calculated on the assumption that during the short time of insulation the rate of loss at any instant is proportional to the then charge. The values of $\psi(1)-B$ and $\psi(5)-B$ are deduced as though $\frac{dx}{dt}$ were constant during the time of insulation, and are therefore considerably below the truth in all cases. It will be observed that the battery was not quite constant; but the value of 48 elements may be taken as 3160 scale-divisions without serious error.

Glass No. 2.—Temperature 53°. It was roughly estimated that on insulation $\frac{1}{4}$ of the charge was lost within 1 second. Notwithstanding this high conductivity, the residual charge was capable of rising to more than 400 scale-divisions when the flask had been charged with 48 elements and then discharged for a few seconds. This differentiates the polarization in even highly conductive glass from the electrochemical polarization in a voltameter, in a single element of which no electromotive force can give rise to a return force greater than that due to the energy of combination of the constituents of the electrolyte. Subsequently, considerable residual charges were obtained with the same glass up to 150°; at 180° the residual charge was so rapidly lost that it was hardly sensible.

Temperature 39½°.

h. m.
Time 6 10. Charged with 7 elements.
6 11. From 462 to 350 in 2 seconds.
6 12. „ 463 to 360 „ „
6 17. „ 464 to 350 „ „
6 19. „ 464 to 350 „ „

$$\left.\begin{array}{r} B=10\cdot0 \\ \text{Log } B=\ 1\cdot0 \end{array}\right\} \text{ at } 39\tfrac{1}{2}^\circ.$$

At 6 20. Charged with 48 elements.
Temperature 41°.
6 40. Discharge.
6 41. 50 in 4 seconds.
6 42. 28 „ „
6 43. 18 „ „
Temperature 41°.

$$\psi(1)-B=0\cdot24 \text{ at } 41^\circ.$$

h m
Temperature 33½°.
Time 7 50. Charged with 7 elements.
7 51. 462 to 340 in 4 seconds.
7 52. 463 to 340 „ „
7 55. 465 to 343 „ „
Temperature 33¼°.

$$\left.\begin{array}{r}B=5{\cdot}4\\ \text{Log } B=0{\cdot}73\end{array}\right\}\text{ at } 33\tfrac{3}{8}°.$$

7 56. Charged with 48 elements.
Temperature 35°.
8 30. Discharge.
8 31. 115 in 10 seconds.
8 32. 67 „ „
8 33. 46 „ „
8 35. 29 „ „

$$\left.\begin{array}{r}\psi(1)-B=0{\cdot}22\\ \psi(5)-B=0{\cdot}055\end{array}\right\}\text{ at } 35°.$$

Temperature 27½°.
10 2. Charged with 7 elements.
10 3. 459 to 340 in 5 seconds.
10 4. 460 to 360 „ „
10 7. 461 to 368 „ „
Temperature 27°.

$$\left.\begin{array}{r}B=3{\cdot}2\\ \text{Log } B=0{\cdot}50\end{array}\right\}\text{ at } 27\tfrac{1}{4}°.$$

10 8. Charged with 48 elements.
Temperature 28°.
10 41. Discharge.
10 42. 140 in 10 seconds.
10 43. 77 „ „
10 44. 53 „ „
10 46. 34 „ „

$$\left.\begin{array}{r}\psi(1)-B=0{\cdot}26\\ \psi(5)-B=0{\cdot}064\end{array}\right\}\text{ at } 28°.$$

h m.
Temperature 26°.
Time 8 45. Charged with 7 elements.
8 46. 452 to 350 in 6 seconds.
8 47. 453 to 350 „ „
8 50. 455 to 368 „ „
Temperature 25½°.

$$\left.\begin{array}{r} B=2{\cdot}5 \\ \text{Log } B=0{\cdot}40 \end{array}\right\} \text{at } 25\tfrac{3}{4}^{\circ}.$$

Temperature 24¼°.
9 11. Charged with 7 elements.
9 12. 458 to 345 in 8 seconds.
9 13. 458 to 351 „ „
9 16. 457 to 355 „ „
Temperature 24°.

$$\left.\begin{array}{r} B=2{\cdot}2 \\ \text{Log } B=0{\cdot}34 \end{array}\right\} \text{at } 24\tfrac{1}{8}^{\circ}.$$

Temperature 22½°.
9 38. Charged with 7 elements.
9 39. 455 to 338 in 10 seconds.
9 40. 456 to 340 „ „
9 43. 457 to 352 „ „
Temperature 22¼°.

$$\left.\begin{array}{r} B=1{\cdot}85 \\ \text{Log } B=0{\cdot}27 \end{array}\right\} \text{at } 22\tfrac{3}{8}^{\circ}.$$

9 45. Charged with 48 elements.
Temperature 20½°.
10 15. Discharged.
10 16. 150 in 10 seconds.
10 17. 81 „ „
10 18. 55 „ „
10 20. 33 „ „

$$\left.\begin{array}{r} \psi(1)-B=0{\cdot}28 \\ \psi(5)-B=0{\cdot}062 \end{array}\right\} \text{at } 20\tfrac{1}{2}^{\circ}.$$

h m
Temperature $7\frac{1}{4}°$.
Time 4 40. Charged with 7 elements.
4 41. 466 to 385 in 20 seconds.
4 42. 465 to 397 ,, ,,
4 45. 466 to 411 ,, ,,

$$\left.\begin{array}{r} B=0{\cdot}45 \\ \text{Log } B=\bar{1}{\cdot}65 \end{array}\right\}\text{ at } 7\tfrac{1}{4}°.$$

4 46. Charged with 48 elements.
Temperature $7\frac{1}{4}°$.
5 15. Discharge.
5 16. 250 in 20 seconds.
5 17. 160 ,, ,,
5 18. 110 ,, ,,
5 20. 66 ,, ,,

$$\left.\begin{array}{r} \psi(1)-B=0{\cdot}24 \\ \psi(5)-B=0{\cdot}062 \end{array}\right\}\text{ at } 7\tfrac{1}{4}°.$$

Temperature $-3°$, after standing 30 minutes in the freezing-mixture.
7 19. Charged with 7 elements.
7 20. 457 to 417 in 20 seconds.
7 21. 458 to 427 ,, ,,
7 24. 459 to 438 ,, ,,
7 29. 461 to 442 ,, ,,
Temperature $-3°$.

$$\left.\begin{array}{r} B=0{\cdot}17 \\ \text{Log } B=\bar{1}{\cdot}22 \end{array}\right\}\text{ at } -3°.$$

7 30. Charged with 48 elements.
Temperature $-1\frac{1}{4}°$.
8 3. Discharged.
8 4. 180 in 20 seconds.
8 5. 115 ,, ,,
8 6. 83 ,, ,,
8 8. 56 ,, ,,
Temperature $-1°$.

$$\left.\begin{array}{r} \psi(1)-B=0{\cdot}17 \\ \psi(5)-B=0{\cdot}053 \end{array}\right\}\text{ at } -1\tfrac{1}{8}°.$$

h m
Temperature $-5°$, in a fresh freezing-mixture.
Time 8 48. Charged with 7 elements.
8 49. 463 to 432 in 20 seconds.
8 50. 464 to 438 " "
8 53. 465 to 447 " "

$$\left.\begin{array}{r}B=0{\cdot}14\\ \operatorname{Log} B=\bar{1}{\cdot}15\end{array}\right\}\text{at } -5°.$$

8 55. Charged with 48 elements.
Temperature $-3°$.
9 25. Discharged.
9 26. 176 in 20 seconds.
9 27. 108 " "
9 28. 80 " "
9 30. 53 " "

$$\left.\begin{array}{r}\psi(1)-B=0{\cdot}17\\ \psi(5)-B=0{\cdot}050\end{array}\right\}\text{at } -3°.$$

As in Mr. PERRY'S experiments the results agree closely with the formula

$$\operatorname{Log} B=a+b\theta,$$

where θ is the temperature, and in this case $a=\bar{1}{\cdot}35$ and $b=0{\cdot}0415$. The following Table gives the observed and calculated values:—

Temp.	B observed.	B from formulæ.
$\overset{\circ}{39}\frac{1}{2}$	10	9·8
$33\frac{3}{8}$	5·4	5·4
$27\frac{1}{4}$	3·2	3·0
$25\frac{3}{4}$	2·5	2·6
$24\frac{1}{4}$	2·2	2·2
$22\frac{3}{8}$	1·85	1·9
$7\frac{1}{8}$	0·45	0·46
-3	0·17	0·17
-5	0·14	0·14

The residual charge results do not show so great a degree of regularity, probably because the direct deduction of $\frac{dx}{dt}$ as equal to $\frac{\delta x}{\delta t}$ gives a greater error than the method used for obtaining B. This much is quite certain, that the value of $\psi(1)-B$ and $\psi(5)-B$ is rapidly increasing up to 7°. It appears probable that at higher temperatures these do not increase so rapidly if at all; but this is by no means certain, as although shorter times of insulation were used, the values at higher temperatures are notwithstanding more reduced by conduction than at the lower.

Glass No. 7.—Temperature 119°.

h m
Time 6 21. Charged with 7 elements.
6 22. 463 to 390 in 20 seconds.
6 23. 464 to 399 ,, ,,
6 26. 465 to 412 ,, ,,
6 31. 465 to 419 ,, ,,
Temperature 119°.

$$\left.\begin{array}{r} B=0\cdot370 \\ \operatorname{Log} B=\bar{1}\cdot568 \\ \psi(1)=0\cdot608 \end{array}\right\} \text{at } 120\tfrac{1}{4}^\circ.$$

6 32. Charged with 48 elements.
Temperature 122°.
7 5. Discharged.
7 6. 226 in 20 seconds.
7 7. 141 ,, ,,
7 8. 104 ,, ,,
7 10. 65 ,, ,,

$$\left.\begin{array}{r} \psi(1)-B=0\cdot215 \\ \psi(5)-B=0\cdot062 \end{array}\right\} \text{at } 123\tfrac{1}{4}^\circ.$$

Temperature 107°.
7 51. Charged with 7 elements.
7 53. 466 to 437 ? in 20 seconds.
7 56. 466 to 429 ,, ,,
8 1. 466 to 447 ,, ,,
Temperature 107°.

$$\left.\begin{array}{r} B=0\cdot148 \\ \operatorname{Log} B=\bar{1}\cdot169 \end{array}\right\} \text{at } 108^\circ.$$

8 2. Charged with 48 elements.
Temperature 107°.
8 36. Discharged.
8 37. 162 in 20 seconds.
8 38. 100 ,, ,,
8 39. 76 ,, ,,
8 41. 51 ,, ,,

$$\left.\begin{array}{r} \psi(1)-B=0\cdot155 \\ \psi(5)-B=0\cdot05 \end{array}\right\} \text{at } 108^\circ.$$

h m
Time 9 25. Charged with 48 elements.
Temperature 98°.
10 1. Discharged.
10 2. 110 in 20 seconds.
10 3. 74 „ „
10 4. 56 „ „
10 6. 39 „ „
Temperature $97\tfrac{3}{4}$°.

$$\left.\begin{array}{l}\psi(1)-B=0{\cdot}11\\ \psi(5)-B=0{\cdot}037\end{array}\right\}\text{ at }98\tfrac{3}{4}^\circ.$$

Temperature $172\tfrac{1}{2}$°.
7 25. Charged with 7 elements.
7 26. 461 to 270 in 3 seconds.
7 27. 462 to 272 „ „
7 30. 463 to 277 „ „
7 35. 465 to 281 „ „
Temperature 172°.

$$\left.\begin{array}{r}B=11{\cdot}9\\ \operatorname{Log}B=\ 1{\cdot}076\end{array}\right\}\text{ at }175\tfrac{1}{2}^\circ.$$

7 36. Charged with 48 elements.
Temperature 172°.
7 50. Discharged.
7 51. 100 in 5 seconds.
7 52. 50 „ „
7 53. 28 „ „
7 55. 9 „ „
Temperature $171\tfrac{1}{2}$°.

$$\left.\begin{array}{l}\psi(1)-B=0{\cdot}38\\ \psi(5)-B=0{\cdot}034\end{array}\right\}\text{ at }175^\circ.$$

9 0. Charged with 48 elements.
Temperature 150°.
9 30. Discharged.
9 31. 122 in 5 seconds.
9 32. 125 in 10 seconds.
9 33. 96 „ „
9 35. 64 „ „

$$\left.\begin{array}{l}\psi(1)-B=0{\cdot}46\\ \psi(5)-B=0{\cdot}12\end{array}\right\}\text{ at }152\tfrac{1}{4}^\circ.$$

h m
Temperature 162°.
Time 10 13. Charged with 7 elements.
10 14. 461 to 330 in 3 seconds.
10 15. 462 to 340 " "
10 18. 463 to 346 " "
10 23. 463 to 353 " "
Temperature 161°.

$$\left.\begin{array}{r}B=6{\cdot}42\\ \operatorname{Log} B=0{\cdot}807\\ \psi(1)=7{\cdot}88\end{array}\right\}\text{ at }164^\circ.$$

10 24. Charged with 48 elements.
Temperature 165°.
10 58. Discharged.
10 59. 125 in 5 seconds.
11 0. 74 " "
11 1. 50 " "
11 3. 26 " "

$$\left.\begin{array}{r}\psi(1)-B=0{\cdot}47\\ \psi(5)-B=0{\cdot}098\end{array}\right\}\text{ at }167\tfrac{1}{2}^\circ.$$

Temperature 143°.
4 48. Charged with 7 elements.
4 49. 469 to 400 in 4 seconds.
4 50. 469 to 403 " "
4 53. 470 to 410 " "
4 58. 470 to 420 " "
Temperature 143½°.

$$\left.\begin{array}{r}B=1{\cdot}99\\ \operatorname{Log} B=0{\cdot}300\\ \psi(1)=2{\cdot}82\end{array}\right\}\text{ at }145\tfrac{1}{4}^\circ.$$

5 0. Charged with 48 elements.
Temperature 143°.
5 23. Discharged.
5 24. 190 in 10 seconds.
5 25. 115 " "
5 26. 88 " "
5 28. 55 " "

$$\left.\begin{array}{r}\psi(1)-B=0{\cdot}36\\ \psi(5)-B=0{\cdot}105\end{array}\right\}\text{ at }144\tfrac{3}{4}^\circ.$$

h m

Temperature 127°.

Time 7 8. Charged with 7 elements.

7 9. 465 to 412 in 10 seconds.

7 10. 466 to 416 „ „

7 13. 467 to 427 „ „

7 18. 468 to 428 „ „

$$\left.\begin{array}{r} B=0\cdot63 \\ \operatorname{Log} B=\bar{1}\cdot80 \\ \psi(1)=0\cdot86 \end{array}\right\} \text{at } 128\tfrac{1}{2}^{\circ}.$$

7 20. Charged with 48 elements.

Temperature 126°.

7 58. Discharged.

7 59. 135 in 10 seconds.

8 0. 86 „ „

8 1. 73 „ „

8 3. 47 „ „

$$\left.\begin{array}{r} \psi(1)-B=0\cdot26 \\ \psi(5)-B=0\cdot09 \end{array}\right\} \text{at } 127\tfrac{1}{2}^{\circ}.$$

Temperature 79°.

9 30. Charged with 7 elements.

9 35. 468 to 448 in 2 minutes.

9 40. 468 to 450 „ „

Temperature 79°.

$$\left.\begin{array}{r} B=0\cdot023 \\ \operatorname{Log} B=\bar{2}\cdot36 \end{array}\right\} \text{at } 79\tfrac{1}{2}^{\circ}.$$

5 15. Charged with 48 elements.

Temperature 66°.

5 45. Discharged.

5 46. 55 in 40 seconds.

5 47. 31 „ „

5 48. 22 „ „

5 50. 14 „ „

Temperature 64½°.

$$\left.\begin{array}{r} \psi(1)-B=0\cdot026 \\ \psi(5)-B=0\cdot007 \end{array}\right\} \text{at } 65\tfrac{1}{2}^{\circ}.$$

h m
Temperature 94°.
Time 6 35. Charged with 7 elements.
6 37. 457 to 422 in 1 minute.
6 40. 458 to 432 ,, ,,
6 47. 458 to 433 ,, ,,
Temperature 94½°.

$$\left.\begin{array}{r}B=0{\cdot}066\\ \text{Log } B=\bar{2}{\cdot}82\end{array}\right\}\text{ at } 95^\circ.$$

Temperature 153½°.
8 0. Charged with 7 elements.
8 1. 461 to 340 in 4 seconds.
8 2. 461 to 350 ,, ,,
8 3. 462 to 352 ,, ,,
8 5. 463 to 358 ,, ,,
8 10. 463 to 362 ,, ,,
Temperature 153½°.

$$\left.\begin{array}{r}B=4{\cdot}36\\ \text{Log } B=0{\cdot}64\\ \psi(1)=5{\cdot}39\end{array}\right\}\text{at } 155\tfrac{3}{4}^\circ.$$

Temperature 66°.
10 31. Charged with 7 elements.
10 41. 964½ to 945 in 4 minutes.
Temperature 67°.

$$\left.\begin{array}{r}B=0{\cdot}0126\\ \text{Log } B=2{\cdot}101\end{array}\right\}\text{ at } 66\tfrac{3}{4}^\circ.$$

With this glass the results do not agree so closely with the exponential formula as with glass No. 2. This is perhaps not surprising when it is considered that the temperatures differ more from that of the room, and, consequently, that errors due to unequal heating of the acid, and to exposure of the stem of the thermometer, will be greater.

The observed values of B, and those calculated from the formula $\log B=\bar{4}{\cdot}17+0{\cdot}0283\,\theta$, are given in the following Table :—

θ.	Observed.	Calculated.
175½°	12	14
164	6·4	6·4
155¾	4·4	3·8
145⅛	2·0	1·9
128½	0·63	0·63
120¼	0·37	0·37
108	0·15	0·17
95	0·066	0·073
79½	0·023	0·026
66	0·013	0·011

The values obtained for $\psi(1)$ and B do not in general give a value of $\psi(1)-B$, which agrees very closely with that obtained by residual charge. This is not astonishing, for $\psi(1)$ and B are both subject to a considerable probable error, and do not differ greatly from each other. On the other hand, at high temperatures, the values of $\psi(1)-B$ and $\psi(5)-B$, obtained by residual charge, are undoubtedly much too low. It is interesting to remark, that whereas the values of $\psi(1)-B$ and $\psi(5)-B$ from residual charge do not increase with temperature above 160°, the values of $\psi(1)-B$ obtained by difference show a continually accelerated increase. The observed values of $\psi(1)-B$ and $\psi(5)-B$ are collected in the following Table. The values above 140°, if admitted at all, must be regarded as subject to an enormous probable error.

Temperature.	$\psi(1)-B$.	$\psi(5)-B$.
175°	0·38	0·034
167½	0·47	0·098
152¼	0·46	0·12
144¾	0·36	0·105
127½	0·26	0·09
123¼	0·215	0·062
108	0·155	0·05
98¾	0·11	0·037
65¼	0·026	0·007

It should be mentioned that the temperature experiments were not made on the same flask as flask No. 7 of the previous experiments, but on a flask of the same composition.