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AN ALGORITHM FOR COMPUTING NON-
ISOMORPHIC SEMIGROUPS OF FINITE
ORDER

by

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United States Naval Postgraduate School



THESIS

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June 1969

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An Algorithm for Computing Non-Isomorphic Semigroups
of Finite Order

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE

from the

Naval Postgraduate School

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ABSTRACT

In this paper an algorithm for computing semigroups of finite order is discussed. A computation procedure is developed to generate, for any specified finite order, all semigroups which are distinct up to isomorphism. Additional restrictions are also placed in the generating procedure to produce all groups of the given finite order. The algorithm was placed on the computer and the numerical results for orders one through four obtained.

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ACKNOWLEDGEMENTS

The author would like to thank Professor C. O. Wilde for his helpful suggestions in preparing this paper.

I. INTRODUCTION

In this paper we investigate the problem of computing all possible distinct algebraic systems of a certain type, namely the semigroup, with restrictions on the order only. A semigroup is an algebraic system which is closed and associative, and as such is the simplest algebraic system of significance in mathematics. More complex systems are determined by postulating additional properties, for example commutativity, many of which can easily be placed in a computation procedure. To illustrate this point a procedure was constructed to produce all groups in addition to all semigroups of the specified finite orders. In this paper we consider orders up to and including four only.

II. DEFINITIONS AND PRELIMINARY RESULTS

There are two basic alternatives for defining equivalence of semigroups. One approach is to identify two semigroups if the first is either isomorphic or anti-isomorphic to the second. The resulting collection of distinct semigroups is then described as a collection of non-equivalent semigroups. The alternate approach is to define one semigroup distinct from another if the first is not isomorphic to the second. The resulting collection is then described as a collection of semigroups which are distinct up to isomorphism. We use the latter approach in this paper and will explain the reasons for this choice later.

We begin by recalling the definitions of a binary operation on a set, of some properties a binary operation may possess, and of a semigroup itself.

Definition. A binary operation on a set S is a mapping from $S \times S$ into S .

Definition. A binary operation Q is said to be associative if $Q(s_1, Q(s_2, s_3)) = Q(Q(s_1, s_2), s_3)$ for all $s_1, s_2, s_3 \in S$.

Definition. A binary operation Q is said to be commutative if $Q(s_1, s_2) = Q(s_2, s_1)$ for all $s_1, s_2 \in S$.

Definition. A semigroup is a couple (S, Q) where S is a set on whose elements is defined an associative binary operation Q . The notation $Q(s_1, s_2) = s_3$ is somewhat cumbersome and will be used interchangeably with $(s_1 \cdot s_2) = s_3$ from this point on.

The order of a semigroup will mean the number of elements in the underlying set. For any given positive integer n there exists at least one semigroup of order n . For example, let $S = \{1, \dots, n\}$ and define

$$Q(i,j) = \begin{cases} i + j & \text{if } i + j \leq n \\ i + j - n & \text{if } i + j > n \end{cases}$$

for $i, j \in S$. This is the cyclic semigroup on n elements.

In this paper we compute the distinct (up to isomorphism) binary operations on a set of finite order satisfying the above conditions. One way to specify a binary operation on a finite set is by means of the multiplication table.

Example. Let $S = \{1, 2, 3\}$ and define the binary operation Q by the following table.

	1	2	3
1	1	2	3
2	2	3	1
3	3	1	2

The notation means that $Q(1,1) = 1$, $Q(1,2) = 2$, $Q(1,3) = 3$, $Q(2,1) = 2$, $Q(2,2) = 3$, and so on.

We use the following definitions as the basis for the construction of the isomorphism testing subroutine of the generation procedure.

Definition. Two semigroups (S_1, Q_1) and (S_2, Q_2) are called isomorphic if there exists a one-to-one mapping F of S_1 onto S_2 such that if $s_1, t_1 \in S_1$ and $F(s_1) = s_2$, $F(t_1) = t_2$ with $s_2, t_2 \in S_2$, then $F(Q_1(s_1, t_1)) = Q_2(s_2, t_2)$.

Definition. Two semigroups (S_1, Q_1) and (S_2, Q_2) are called anti-isomorphic if there exists a one-to-one mapping G of S_1 onto S_2 such that if $s_1, t_1 \in S_1$ and $G(s_1) = s_2$, $G(t_1) = t_2$ with $s_2, t_2 \in S_2$, then $G(Q_1(s_1, t_1)) = Q_2(t_2, s_2)$.

Given any semigroup (S, Q) we can in a natural way associate with it a semigroup (S^*, Q^*) defined by letting $S^* = S$, and for $s_1, s_2 \in S^*$ putting

$Q^*(s_1, s_2) = Q(s_2, s_1)$. If G is the identity mapping on S , then $G(Q(s_1, s_2)) = Q^*(s_2, s_1)$ and hence (S, Q) is anti-isomorphic to (S^*, Q^*) .

If a semigroup (S, Q) is commutative, then it is both isomorphic and anti-isomorphic with (S^*, Q^*) since the identity map G is both an isomorphism and an anti-isomorphism. In the journal, Mathematical Algorithms, 1967, the editor remarked that the converse is also valid. However, the converse is not valid in general, as the following example illustrates.

Example. Let (S, Q) be defined by the table

	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	2	1

then (S^*, Q^*) is defined by this table

	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	2
4	1	1	1	1

where the second table was determined from the first by $Q^*(s_1, s_2) = Q(s_2, s_1)$ with the only differences in the multiplication tables being $Q^*(3, 4) = Q(4, 3)$ and $Q^*(4, 3) = Q(3, 4)$. The anti-isomorphism G is the identity mapping, while the mapping F defined by $F(1) = 1$, $F(2) = 2$, $F(3) = 4$, and $F(4) = 3$ is an isomorphism linking (S, Q) with (S^*, Q^*) . It is interesting to note that orders two and three contain no non-commutative semigroup (S, Q) which is both isomorphic and anti-isomorphic to its (S^*, Q^*) , but that order four contains six such semigroups. By computing non-equivalent semigroups the question whether or not a given semigroup is isomorphic to its anti-isomorphic image is left unanswered

unless the semigroup in question is commutative. In computing semigroups distinct up to isomorphism we avoid this difficulty.

III. ALGORITHM FOR COMPUTING SEMIGROUPS OF FINITE ORDER

Using the fact that the only condition on the algebraic system under consideration in addition to closure is associativity, we are able to build a systematic generating procedure. Noting that in any given multiplication table if $Q(i,j) = k$, then $Q(k,m) = Q(i,Q(j,m))$, we construct a procedure to complete any partially completed table. In essence, we utilize the associative law to complete the unfilled portion of the table. The first step consists of placing a few key values in a blank multiplication table. Then as much as possible of the table is completed by the application of the above equation.

Example. Let $S = \{1,2,3,4\}$ and let Q be partially defined by the following table.

	1	2	3	4
1	1	3		
2	2	1	3	
3		4		
4				

Then by applying the above associativity equation to the table we find $(3 \cdot 1) = ((1 \cdot 2) \cdot 1) = (1 \cdot (2 \cdot 1)) = (1 \cdot 2) = 3$ and $(1 \cdot 3) = (1 \cdot (1 \cdot 2)) = ((1 \cdot 1) \cdot 2) = (1 \cdot 2) = 3$ as well as $(1 \cdot 4) = 4$, $(2 \cdot 4) = 4$, $(4 \cdot 1) = 4$, and $(4 \cdot 2) = 3$. We also have $(4 \cdot 3) = (3 \cdot 3)$ and $(4 \cdot 4) = (3 \cdot 4)$. The following table results.

	1	2	3	4
1	1	3	3	4
2	2	1	3	4
3	3	4	x	y
4	4	3	x	y

Of course, in supplying the original six values a check must be made to determine whether or not they satisfy the associative law to make the

completion of the table worthwhile. The table must be checked again when completed since not all associativity conditions need necessarily be used in the completion of it.

The above example points out the difficulties that arise when a partially completed table has values which are not conducive toward further generation. When this happens additional values must be supplied to restart the generation procedure. The choosing of these additional values must follow a pattern and must exhaust all possibilities. In the above example the values 1,2,3, and 4 must be tested in both the x and y positions, which results in a total of 16 additional cases to be checked.

In the case of order four we set initial values in six positions. We chose the positions (1,1), (1,2), (2,1), and (2,2) to facilitate the generation procedure by filling a fourth of the blank multiplication table, while we picked the two additional positions (2,3) and (3,2) to be used as launching points for the completion of the table. We chose an initial six positions for two reasons. First, since every finite semigroup has an idempotent element,¹ then every finite semigroup is isomorphic to some semigroup with the element one (1) in the position (1,1). If in a semigroup of order n the element i is idempotent, then define an isomorphism F such that $F(i) = 1$, $F(1) = i$, and $F(k) = k$ for $k = 2, \dots, i - 1, i + 1, \dots, n$. The second reason is that the remaining five positions have to be filled in a manner that exhausts all possible combinations of the values one (1) through four (4). The number of initial cases, ranging from the values 1,1,1,1,1 to the values

¹Clifford, A. H. and Preston, G. B., The Algebraic Theory of Semigroups, v. 1, p. 20, American Mathematical Society, 1961.

4,4,4,4,4 in the five positions, is 4^5 , or 1024. The addition of any more initial positions would increase this number by a factor of four for each added position.

We developed this generating procedure because of time limitations on the use of the computer. For orders two and three the number of positions set with initial values were four and nine respectively. We exhaustively checked all possible combinations in these two orders since the number of cases to be checked was low. For order two there were only 2^4 cases while for order three there were only 3^9 cases. However, for order four there are 4^{16} , or over four billion, cases to be examined, which proved to be much too time consuming to allow the exhaustive procedure.

IV. IMPLEMENTATION OF ALGORITHM

We divided the algorithm into three basic parts in order to place it on the computer using the Fortran programming system. The first and primary part consists of producing the completed multiplication tables by application of the associative equation, $Q(k,m) = Q(i,Q(j,m))$ when $Q(i,j) = k$, to the initial values of the partially completed tables. The initial values are supplied by use of nested "DO loops." The number of nested "DO loops" used were four, nine, and five for orders two, three, and four respectively. Hence, for orders two and three entire multiplication tables were set with initial values and the algorithm degenerated into an exhaustive test of every possible multiplication table.

For order four we found that seven nested "DO loops" would be the maximum number for practical purposes, that is, any more would result in too much time consumption. We decided on five for the reasons stated before. After the initial values are supplied we check for violations of the associative law. If there are none the computer then applies the associative equation to the initial values and then to generated values until the generation procedure ceases. At this point the number of blanks remaining in the multiplication table determines to which further generation subroutine the computer switches. Once the multiplication table is completed we are finished with the first part of the algorithm.

The second part of the algorithm consists of the main associativity test. We check the entire multiplication table since the generation procedure does not necessarily use every associativity condition. In this associativity test as well as in previous ones we make use of the "LOGICAL IF" statement and self-subscripting capabilities of the Fortran language.

Example. IF (I(KK,I(LL,MM)).NE.I(I(KK,LL),MM)) GO TO 100

Once a multiplication table passes the second part it is given a number and recognized as the representation of a semigroup.

In the third part of the algorithm we take these multiplication tables and determine those which are not isomorphic to any of the others. This select group then represents a collection of semigroups which are distinct up to isomorphism.

As we mentioned before, additional subroutines were added to produce all groups as well as semigroups of the specified finite orders. These subroutines follow the third part of the algorithm.

We include the programs used and the output obtained in the latter part of this paper.

TABLE I

ORDER	1	2	3	4
Number of semigroups distinct up to isomorphism	1	5	24	188
Number of commutative semigroups	1	3	12	58
Number of non-commutative isomorphic anti-isomorphic semigroups	0	0	0	6
Number of groups distinct up to isomorphism	1	1	1	2

SEMIGROUPS OF ORDER TWO DISTINCT UP TO ISOMORPHISM

$\begin{matrix} 1 \\ 1 \end{matrix}$
 $\begin{matrix} 1 \\ 1 \end{matrix}$
 SEMIGROUP NUMBER IS 1
 SEMIGROUP IS COMMUTATIVE

$\begin{matrix} 1 \\ 1 \end{matrix}$
 $\begin{matrix} 1 \\ 2 \end{matrix}$
 SEMIGROUP NUMBER IS 2
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $x = 2$ SUCH THAT
 $xy = y$
 SEMIGROUP HAS RIGHT IDENTITY $z = 2$ SUCH THAT
 $yz = y$
 SEMIGROUP HAS IDENTITY

$\begin{matrix} 1 \\ 2 \end{matrix}$
 $\begin{matrix} 1 \\ 2 \end{matrix}$
 SEMIGROUP NUMBER IS 3
 SEMIGROUP HAS RIGHT IDENTITY $z = 1$ SUCH THAT
 $yz = y$
 SEMIGROUP HAS RIGHT IDENTITY $z = 2$ SUCH THAT
 $yz = y$

$\begin{matrix} 1 \\ 1 \end{matrix}$
 $\begin{matrix} 2 \\ 2 \end{matrix}$
 SEMIGROUP NUMBER IS 4
 SEMIGROUP HAS LEFT IDENTITY $x = 1$ SUCH THAT
 $xy = y$
 SEMIGROUP HAS LEFT IDENTITY $x = 2$ SUCH THAT
 $xy = y$

$\begin{matrix} 1 \\ 2 \end{matrix}$
 $\begin{matrix} 2 \\ 1 \end{matrix}$
 SEMIGROUP NUMBER IS 5
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $x = 1$ SUCH THAT
 $xy = y$
 SEMIGROUP HAS RIGHT IDENTITY $z = 1$ SUCH THAT
 $yz = y$
 SEMIGROUP HAS IDENTITY
 SEMIGROUP IS A GROUP

SEMIGROUPS OF ORDER THREE DISTINCT UP TO ISOMORPHISM

1 1 1
 1 1 1
 1 1 1
 SEMIGROUP NUMBER 1
 SEMIGROUP IS COMMUTATIVE

1 1 1
 1 1 1
 1 1 2
 SEMIGROUP NUMBER 2
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP IS COMMUTATIVE

1 1 1
 1 1 1
 1 1 3
 SEMIGROUP NUMBER 3
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP IS COMMUTATIVE

1 1 1
 1 1 1
 1 2 3
 SEMIGROUP NUMBER 4
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP HAS LEFT IDENTITY $x = 3$
 SUCH THAT $xy = y$

1 1 1
 1 1 1
 3 3 3
 SEMIGROUP NUMBER 5
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)

1 1 1
 1 1 2
 1 1 3
 SEMIGROUP NUMBER 5
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$

1 1 1
 1 1 2
 1 2 3
 SEMIGROUP NUMBER 7
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $X = 3$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1 1 1
 1 2 1
 1 1 3
 SEMIGROUP NUMBER 8
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP IS COMMUTATIVE

1 1 1
 1 2 1
 3 3 3
 SEMIGROUP NUMBER 9
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (1,3)
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y

1 1 1
 1 2 2
 1 2 2
 SEMIGROUP NUMBER 10
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (2,3)
 SEMIGROUP IS COMMUTATIVE

1 1 1
 1 2 2
 1 2 3
 SEMIGROUP NUMBER 11
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (1,3)
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (2,3)
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY X = 3
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 3
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 1 1
 1 2 2
 1 3 3
 SEMIGROUP NUMBER 12
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
 (2,3)
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 3
 SUCH THAT YZ=Y

1 1 1
 1 2 3
 1 2 3
 SEMIGROUP NUMBER 13
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (2,3)
 SEMIGROUP HAS LEFT IDENTITY $X = 2$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS LEFT IDENTITY $X = 3$
 SUCH THAT $XY=Y$

1 1 1
 1 2 3
 1 3 2
 SEMIGROUP NUMBER 14
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (2,3)
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $X = 2$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1 1 1
 1 2 3
 3 3 3
 SEMIGROUP NUMBER 15
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (2,3)
 SEMIGROUP HAS LEFT IDENTITY $X = 2$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1 1 1
 2 2 2
 3 3 3
 SEMIGROUP NUMBER 16
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (2,3)
 SEMIGROUP HAS RIGHT IDENTITY $Z = 1$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$

1 1 3
 1 1 3
 1 1 3
 SEMIGROUP NUMBER 17
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)

1 1 3
 1 1 3
 3 3 1
 SEMIGROUP NUMBER 18
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP IS COMMUTATIVE

1 1 3
 1 2 3
 1 1 3
 SEMIGROUP NUMBER 19
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,2)
 SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
 (1,3)
 SEMIGROUP HAS LEFT IDENTITY $X = 2$
 SUCH THAT $XY=Y$


```

1      1      3
1      2      3
1      3      3
SEMIGROUP NUMBER 20
SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
  (1,2)
SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
  (1,3)
SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
  (2,3)
SEMIGROUP HAS LEFT IDENTITY X = 2
  SUCH THAT XY=Y
SEMIGROUP HAS RIGHT IDENTITY Z = 2
  SUCH THAT YZ=Y
SEMIGROUP HAS IDENTITY

```

```

1      1      3
1      2      3
3      3      1
SEMIGROUP NUMBER 21
SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
  (1,2)
SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
  (1,3)
SEMIGROUP IS COMMUTATIVE
SEMIGROUP HAS LEFT IDENTITY X = 2
  SUCH THAT XY=Y
SEMIGROUP HAS RIGHT IDENTITY Z = 2
  SUCH THAT YZ=Y
SEMIGROUP HAS IDENTITY

```

```

1      2      2
2      1      1
2      1      1
SEMIGROUP NUMBER 22
SEMIGROUP HAS A SUBSEMIGROUP OF ORDER TWO
  (1,2)
SEMIGROUP IS COMMUTATIVE

```

```

1      2      3
1      2      3
1      2      3
SEMIGROUP NUMBER 23
SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
(1,2)
SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
(1,3)
SEMIGROUP HAS A SUBSEMIGROUP CF ORDER TWO
(2,3)
SEMIGROUP HAS LEFT IDENTITY X = 1
SUCH THAT XY=Y
SEMIGROUP HAS LEFT IDENTITY X = 2
SUCH THAT XY=Y
SEMIGROUP HAS LEFT IDENTITY X = 3
SUCH THAT XY=Y

```

```

1      2      3
2      3      1
3      1      2
SEMIGROUP NUMBER 24
SEMIGROUP IS COMMUTATIVE
SEMIGROUP HAS LEFT IDENTITY X = 1
SUCH THAT XY=Y
SEMIGROUP HAS RIGHT IDENTITY Z = 1
SUCH THAT YZ=Y
SEMIGROUP HAS IDENTITY
SEMIGROUP IS A GROUP

```

SEMIGROUPS OF ORDER FOUR DISTINCT UP TO ISOMORPHISM

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
	1	1	1
		SEMIGROUP	NUMBER
		SEMIGROUP	IS COMMUTATIVE
			1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
	1	1	2
		SEMIGROUP	NUMBER
		SEMIGROUP	IS COMMUTATIVE
			2
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
	1	1	4
		SEMIGROUP	NUMBER
		SEMIGROUP	IS COMMUTATIVE
			3
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
	1	2	1
		SEMIGROUP	NUMBER
			4
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
	1	2	2
		SEMIGROUP	NUMBER
			5

1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1	3	4	
	SEMIGROUP		NUMBER	6
1	1	1	1	
1	1	1	1	
1	1	1	1	
1	2	2	4	
	SEMIGROUP		NUMBER	7
1	1	1	1	
1	1	1	1	
1	1	1	1	
1	2	3	4	
	SEMIGROUP	NUMBER		8
	SEMIGROUP	HAS LEFT	IDENTITY	$x = 4$
		SUCH THAT	$xy=y$	
1	1	1	1	
1	1	1	1	
1	1	1	1	
4	4	4	4	
	SEMIGROUP		NUMBER	9
1	1	1	1	
1	1	1	1	
1	1	1	2	
1	1	1	2	
	SEMIGROUP		NUMBER	10
1	1	1	1	
1	1	1	1	
1	1	1	2	
1	1	2	1	
	SEMIGROUP	NUMBER		11
	SEMIGROUP	IS	COMMUTATIVE	

1 1 1 1
 1 1 1 1
 1 1 1 2
 1 1 2 2
 SEMIGROUP NUMBER 12
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 1
 1 1 1 2
 1 1 2 3
 SEMIGROUP NUMBER 13
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 1
 1 1 1 3
 1 1 1 4
 SEMIGROUP NUMBER 14

1 1 1 1
 1 1 1 1
 1 1 1 3
 1 1 3 4
 SEMIGROUP NUMBER 15
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 1
 1 1 1 3
 1 2 1 4
 SEMIGROUP NUMBER 16

1 1 1 1
 1 1 1 1
 1 1 1 3
 1 2 3 4
 SEMIGROUP NUMBER 17
 SEMIGROUP HAS LEFT IDENTITY $x = 4$
 SUCH THAT $xy = y$

1	1	1	1	
1	1	1	2	
1	1	1	2	
1	1	1	4	18
	SEMIGROUP		NUMBER	
1	1	1	1	
1	1	1	2	
1	1	1	2	
1	2	2	4	19
	SEMIGROUP		NUMBER	
	SEMIGROUP		IS COMMUTATIVE	
1	1	1	1	
1	1	1	2	
1	1	1	2	
1	2	3	4	20
	SEMIGROUP		NUMBER	
	SEMIGROUP		HAS LEFT IDENTITY	$X = 4$
		SUCH	THAT $XY=Y$	
1	1	1	1	
1	1	1	2	
1	1	1	3	
1	1	1	4	21
	SEMIGROUP		NUMBER	
	SEMIGROUP		HAS RIGHT IDENTITY	$Z = 4$
		SUCH	THAT $YZ=Y$	
1	1	1	1	
1	1	1	2	
1	1	1	3	
1	1	3	4	22
	SEMIGROUP		NUMBER	
	SEMIGROUP		HAS RIGHT IDENTITY	$Z = 4$
		SUCH	THAT $Y^2=Y$	
1	1	1	1	
1	1	1	2	
1	1	1	3	
1	2	2	4	23
	SEMIGROUP		NUMBER	
	SEMIGROUP		HAS RIGHT IDENTITY	$Z = 4$
		SUCH	THAT $Y^2=Y$	

1 1 1 1
 1 1 1 2
 1 1 1 3
 1 2 3 4
 SEMIGROUP NUMBER 24
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $x = 4$
 SUCH THAT $xy = y$
 SEMIGROUP HAS RIGHT IDENTITY $z = 4$
 SUCH THAT $yz = y$
 SEMIGROUP HAS IDENTITY

1 1 1 4
 1 1 1 4
 1 1 1 4
 1 1 1 4
 SEMIGROUP NUMBER 25

1 1 1 4
 1 1 1 4
 1 1 1 4
 4 4 4 1
 SEMIGROUP NUMBER 26
 SEMIGROUP IS COMMUTATIVE

1 1 1 4
 1 1 1 4
 1 1 1 4
 4 4 4 4
 SEMIGROUP NUMBER 27
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 1
 1 1 2 1
 1 1 1 2
 SEMIGROUP NUMBER 28
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 1
 1 1 2 1
 1 1 1 4
 SEMIGROUP NUMBER 29
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 1
 1 1 2 1
 1 1 2 2
 SEMIGROUP NUMBER 30

1 1 1 1
 1 1 1 1
 1 1 2 1
 4 4 4 4
 SEMIGROUP NUMBER 31

1 1 1 1
 1 1 1 1
 1 1 2 2
 1 1 2 2
 SEMIGROUP NUMBER 32
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 2
 1 1 2 3
 1 2 3 4
 SEMIGROUP NUMBER 33
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $x = 4$
 SUCH THAT $xy = y$
 SEMIGROUP HAS RIGHT IDENTITY $z = 4$
 SUCH THAT $yz = y$
 SEMIGROUP HAS IDENTITY

1 1 1 4
 1 1 1 4
 1 1 2 4
 1 1 1 4
 SEMIGROUP NUMBER 34

1	1	1	4
1	1	1	4
1	1	2	4
4	4	4	1
	SEMIGROUP	NUMBER	35
	SEMIGROUP	IS	COMMUTATIVE

1	1	1	4
1	1	1	4
1	1	2	4
4	4	4	4
	SEMIGROUP	NUMBER	36
	SEMIGROUP	IS	COMMUTATIVE

1	1	1	1
1	1	1	1
1	1	3	1
1	1	1	4
	SEMIGROUP	NUMBER	37
	SEMIGROUP	IS	COMMUTATIVE

1	1	1	1
1	1	1	1
1	1	3	1
1	2	1	4
	SEMIGROUP	NUMBER	38

1	1	1	1
1	1	1	1
1	1	3	1
4	4	4	4
	SEMIGROUP	NUMBER	39

1	1	1	1
1	1	1	1
1	1	3	3
1	1	3	3
	SEMIGROUP	NUMBER	40
	SEMIGROUP	IS	COMMUTATIVE

1	1	1	1	
1	1	1	1	
1	1	3	3	
1	1	3	4	
		SEMIGROUP	NUMBER	41
		SEMIGROUP	IS	COMMUTATIVE
1	1	1	1	
1	1	1	1	
1	1	3	3	
1	1	4	4	
		SEMIGROUP	NUMBER	42
1	1	1	1	
1	1	1	1	
1	1	3	3	
1	2	3	4	
		SEMIGROUP	NUMBER	43
		SEMIGROUP	HAS	LEFT
		SUCH	THAT	IDENTITY
				$X = 4$
				$XY = Y$
1	1	1	1	
1	1	1	1	
1	1	3	4	
1	1	3	4	
		SEMIGROUP	NUMBER	44
1	1	1	1	
1	1	1	1	
1	1	3	4	
1	1	4	3	
		SEMIGROUP	NUMBER	45
		SEMIGROUP	IS	COMMUTATIVE
1	1	1	1	
1	1	1	1	
1	1	3	4	
4	4	4	4	
		SEMIGROUP	NUMBER	46

1	1	1	1	
1	1	1	2	
1	1	3	1	
1	1	1	4	NUMBER 47
1	1	1	1	
1	1	1	2	
1	1	3	1	
1	2	1	4	NUMBER 48
		SEMIGROUP	IS	COMMUTATIVE
1	1	1	1	
1	1	1	2	
1	1	3	3	
1	1	3	4	NUMBER 49
		SEMIGROUP	HAS	RIGHT IDENTITY Z = 4
		SEMIGROUP	SUCH	THAT YZ=Y
1	1	1	1	
1	1	1	2	
1	1	3	3	
1	2	3	4	NUMBER 50
		SEMIGROUP	IS	COMMUTATIVE
		SEMIGROUP	HAS	LEFT IDENTITY X = 4
		SEMIGROUP	SUCH	THAT XY=Y
		SEMIGROUP	HAS	RIGHT IDENTITY Z = 4
		SEMIGROUP	SUCH	THAT YZ=Y
		SEMIGROUP	HAS	IDENTITY
1	1	1	4	
1	1	1	4	
1	1	3	4	
1	1	1	4	NUMBER 51
1	1	1	4	
1	1	1	4	
1	1	3	4	
1	1	4	4	NUMBER 52
		SEMIGROUP	NUMBER	

1 1 1 4
 1 1 1 4
 1 1 3 4
 4 4 4 1
 SEMIGROUP NUMBER 53
 SEMIGROUP IS COMMUTATIVE

1 1 1 4
 1 1 1 4
 1 1 3 4
 4 4 4 4
 SEMIGROUP NUMBER 54
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 1 1
 1 2 3 1
 4 4 4 4
 SEMIGROUP NUMBER 55

1 1 1 1
 1 1 1 1
 1 2 3 3
 1 2 3 3
 SEMIGROUP NUMBER 56

1 1 1 1
 1 1 1 1
 1 2 3 3
 1 2 3 4
 SEMIGROUP NUMBER 57
 SEMIGROUP HAS LEFT IDENTITY $x = 4$
 SUCH THAT $xy=y$

1 1 1 1
 1 1 1 1
 1 2 3 3
 1 2 4 4
 SEMIGROUP NUMBER 58

1	1	1	1	
1	1	1	1	
1	2	3	4	
1	2	3	4	
	SEMIGROUP	NUMBER	59	
	SEMIGROUP	HAS LEFT	IDENTITY	X = 3
	SUCH	THAT	XY=Y	
	SEMIGROUP	HAS LEFT	IDENTITY	X = 4
	SUCH	THAT	XY=Y	

1	1	1	1	
1	1	1	1	
1	2	3	4	
1	2	4	3	
	SEMIGROUP	NUMBER	60	
	SEMIGROUP	HAS LEFT	IDENTITY	X = 3
	SUCH	THAT	XY=Y	

1	1	1	1	
1	1	1	1	
1	2	3	4	
4	4	4	4	
	SEMIGROUP	NUMBER	61	
	SEMIGROUP	HAS LEFT	IDENTITY	X = 3
	SUCH	THAT	XY=Y	

1	1	1	1	
1	1	1	2	
1	2	3	1	
1	1	1	4	
	SEMIGROUP	NUMBER	62	

1	1	1	1	
1	1	1	2	
1	2	3	2	
1	1	1	4	
	SEMIGROUP	NUMBER	63	

1	1	1	1	
1	1	1	2	
1	2	3	3	
1	2	3	4	
	SEMIGROUP	NUMBER	64	
	SEMIGROUP	HAS LEFT IDENTITY	X = 4	
	SUCH	THAT XY=Y		
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 4	
	SUCH	THAT YZ=Y		
	SEMIGROUP	HAS IDENTITY		
1	1	1	4	
1	1	1	4	
1	2	3	4	
1	1	1	4	
	SEMIGROUP	NUMBER	65	
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH	THAT XY=Y		
1	1	1	4	
1	1	1	4	
1	2	3	4	
1	1	4	4	
	SEMIGROUP	NUMBER	66	
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH	THAT XY=Y		
1	1	1	4	
1	1	1	4	
1	2	3	4	
4	4	4	1	
	SEMIGROUP	NUMBER	67	
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH	THAT XY=Y		
1	1	1	4	
1	1	1	4	
1	2	3	4	
4	4	4	4	
	SEMIGROUP	NUMBER	68	
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH	THAT XY=Y		

1 1 1 1
 1 1 1 1
 3 3 3 3
 3 3 3 3
 SEMIGROUP NUMBER 69

1 1 1 1
 1 1 1 1
 3 3 3 3
 3 3 3 4
 SEMIGROUP NUMBER 70

1 1 1 1
 1 1 1 1
 3 3 3 3
 4 4 4 4
 SEMIGROUP NUMBER 71

1 1 1 1
 1 1 1 2
 3 3 3 3
 1 1 1 4
 SEMIGROUP NUMBER 72
 SEMIGROUP HAS RIGHT IDENTITY $Z = 4$
 SUCH THAT $Y^2=Y$

1 1 1 1
 1 1 1 2
 3 3 3 3
 1 1 3 4
 SEMIGROUP NUMBER 73
 SEMIGROUP HAS RIGHT IDENTITY $Z = 4$
 SUCH THAT $Y^2=Y$

1 1 1 1
 1 1 1 2
 3 3 3 3
 1 2 1 4
 SEMIGROUP NUMBER 74
 SEMIGROUP HAS RIGHT IDENTITY $Z = 4$
 SUCH THAT $Y^2=Y$

1 1 1 1
 1 1 1 2
 3 3 3 3
 1 2 3 4
 SEMIGROUP NUMBER 75
 SEMIGROUP HAS LEFT IDENTITY $x = 4$
 SUCH THAT $xy=y$
 SEMIGROUP HAS RIGHT IDENTITY $z = 4$
 SUCH THAT $yz=y$
 SEMIGROUP HAS IDENTITY

1 1 1 1
 1 1 1 2
 3 3 3 3
 3 3 3 4
 SEMIGROUP NUMBER 76
 SEMIGROUP HAS RIGHT IDENTITY $z = 4$
 SUCH THAT $yz=y$

1 1 1 1
 1 1 1 3
 3 3 3 3
 4 4 4 4
 SEMIGROUP NUMBER 77

1 1 1 4
 1 1 1 4
 3 3 3 4
 4 4 4 4
 SEMIGROUP NUMBER 78

1 1 1 1
 1 1 2 2
 1 1 3 3
 1 1 3 3
 SEMIGROUP NUMBER 79

1 1 1 1
 1 1 2 2
 1 1 3 3
 1 1 3 4
 SEMIGROUP NUMBER 80
 SEMIGROUP HAS RIGHT IDENTITY $z = 4$
 SUCH THAT $yz=y$

1	1	1	1
1	1	2	2
1	1	3	3
1	1	4	4
	SEMIGROUP	NUMBER	91
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH THAT	YZ=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 4
	SUCH THAT	YZ=Y	

1	1	1	1
1	1	2	2
1	1	3	3
1	2	3	4
	SEMIGROUP	NUMBER	82
	SEMIGROUP	HAS LEFT IDENTITY	X = 4
	SUCH THAT	XY=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 4
	SUCH THAT	YZ=Y	
	SEMIGROUP	HAS IDENTITY	

1	1	1	1
1	1	2	2
1	1	3	4
1	1	3	4
	SEMIGROUP	NUMBER	83

1	1	1	1
1	1	2	2
1	1	3	4
1	1	4	3
	SEMIGROUP	NUMBER	84
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH THAT	YZ=Y	

1	1	1	4
1	1	2	4
1	1	3	4
1	1	1	4
	SEMIGROUP	NUMBER	85

1 1 1 4
 1 1 2 4
 1 1 3 4
 1 1 4 4
 SEMIGROUP NUMBER 86
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$

1 1 1 4
 1 1 2 4
 1 1 3 4
 4 4 4 1
 SEMIGROUP NUMBER 87
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$

1 1 1 4
 1 1 2 4
 1 1 3 4
 4 4 4 4
 SEMIGROUP NUMBER 88
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$

1 1 1 1
 1 1 2 2
 1 2 3 3
 1 2 3 3
 SEMIGROUP NUMBER 89
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 1 2 2
 1 2 3 3
 1 2 3 4
 SEMIGROUP NUMBER 90
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $X = 4$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 4$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1	1	1	1
1	1	2	2
1	2	3	3
1	2	4	4
	SEMIGROUP	NUMBER	91
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH THAT	YZ=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 4
	SUCH THAT	YZ=Y	

1	1	1	1
1	1	2	2
1	2	3	4
1	2	3	4
	SEMIGROUP	NUMBER	92
	SEMIGROUP	HAS LEFT IDENTITY	X = 3
	SUCH THAT	XY=Y	
	SEMIGROUP	HAS LEFT IDENTITY	X = 4
	SUCH THAT	XY=Y	

1	1	1	1
1	1	2	2
1	2	3	4
1	2	4	3
	SEMIGROUP	NUMBER	93
	SEMIGROUP	IS COMMUTATIVE	
	SEMIGROUP	HAS LEFT IDENTITY	X = 3
	SUCH THAT	XY=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH THAT	YZ=Y	
	SEMIGROUP	HAS IDENTITY	

1	1	1	4
1	1	2	4
1	2	3	4
1	1	1	4
	SEMIGROUP	NUMBER	94
	SEMIGROUP	HAS LEFT IDENTITY	X = 3
	SUCH THAT	XY=Y	

1	1	1	4
1	1	2	4
1	2	3	4
1	1	4	4
	SEMIGROUP	NUMBER	95
	SEMIGROUP	HAS LEFT IDENTITY	X = 3
	SUCH THAT	XY=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH THAT	YZ=Y	
	SEMIGROUP	HAS IDENTITY	

1 1 1 4
 1 1 2 4
 1 2 3 4
 4 4 4 1

SEMIGROUP NUMBER 96
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY X = 3
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 3
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 1 1 4
 1 1 2 4
 1 2 3 4
 4 4 4 4

SEMIGROUP NUMBER 97
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY X = 3
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 3
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 1 1 1
 1 2 1 1
 1 1 3 1
 1 1 1 4

SEMIGROUP NUMBER 98
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 2 1 1
 1 1 3 1
 4 4 4 4

SEMIGROUP NUMBER 99

1 1 1 1
 1 2 1 1
 1 1 3 3
 1 1 3 3

SEMIGROUP NUMBER 100
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 2 1 1
 1 1 3 3
 1 1 3 4
 SEMIGROUP NUMBER 101
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 2 1 1
 1 1 3 3
 1 1 4 4
 SEMIGROUP NUMBER 102

1 1 1 1
 1 2 1 1
 1 1 3 4
 1 1 3 4
 SEMIGROUP NUMBER 103

1 1 1 1
 1 2 1 1
 1 1 3 4
 1 1 4 3
 SEMIGROUP NUMBER 104
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 2 1 1
 1 1 3 4
 4 4 4 4
 SEMIGROUP NUMBER 105

1 1 1 1
 1 2 1 2
 1 1 3 3
 1 2 3 4
 SEMIGROUP NUMBER 106
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $x = 4$
 SUCH THAT $xy = y$
 SEMIGROUP HAS RIGHT IDENTITY $z = 4$
 SUCH THAT $yz = y$
 SEMIGROUP HAS IDENTITY

1 1 1 4
 1 2 1 4
 1 1 3 4
 1 1 1 4
 SEMIGROUP NUMBER 107

1 1 1 4
 1 2 1 4
 1 1 3 4
 1 1 4 4
 SEMIGROUP NUMBER 103

1 1 1 4
 1 2 1 4
 1 1 3 4
 4 4 4 1
 SEMIGROUP NUMBER 109
 SEMIGROUP IS COMMUTATIVE

1 1 1 4
 1 2 1 4
 1 1 3 4
 4 4 4 4
 SEMIGROUP NUMBER 110
 SEMIGROUP IS COMMUTATIVE

1 1 1 1
 1 2 1 1
 3 3 3 3
 3 3 3 4
 SEMIGROUP NUMBER 111

1 1 1 1
 1 2 1 1
 3 3 3 3
 4 4 4 4
 SEMIGROUP NUMBER 112
 SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
 SUCH THAT $YZ=Y$

1 1 1 1
 1 2 1 2
 3 3 3 3
 1 2 1 2
 SEMIGROUP NUMBER 113

1 1 1 1
 1 2 1 2
 3 3 3 3
 1 2 1 4
 SEMIGROUP NUMBER 114
 SEMIGROUP HAS RIGHT IDENTITY Z = 4
 SUCH THAT YZ=Y

1 1 1 1
 1 2 1 2
 3 3 3 3
 1 2 3 4
 SEMIGROUP NUMBER 115
 SEMIGROUP HAS LEFT IDENTITY X = 4
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 4
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 1 1 1
 1 2 1 2
 3 3 3 3
 1 4 1 4
 SEMIGROUP NUMBER 116
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 4
 SUCH THAT YZ=Y

1 1 1 1
 1 2 1 2
 3 3 3 3
 3 4 3 4
 SEMIGROUP NUMBER 117
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 4
 SUCH THAT YZ=Y

1 1 1 1
 1 2 1 4
 3 3 3 3
 1 2 1 4
 SEMIGROUP NUMBER 118

1 1 1 1
 1 2 1 4
 3 3 3 3
 1 4 1 2
 SEMIGROUP NUMBER 119
 SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
 SUCH THAT $YZ=Y$

1 1 1 1
 1 2 1 4
 3 3 3 3
 4 4 4 4
 SEMIGROUP NUMBER 120
 SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
 SUCH THAT $YZ=Y$

1 1 1 4
 1 2 1 4
 3 3 3 4
 4 4 4 4
 SEMIGROUP NUMBER 121
 SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
 SUCH THAT $YZ=Y$

1 1 1 1
 1 2 2 4
 1 2 2 4
 1 2 2 4
 SEMIGROUP NUMBER 122

1 1 1 1
 1 2 2 4
 1 2 2 4
 1 4 4 2
 SEMIGROUP NUMBER 123
 SEMIGROUP IS COMMUTATIVE

1	1	1	1
1	2	2	4
1	2	2	4
1	4	4	4
	SEMIGROUP	NUMBER	124
	SEMIGROUP	IS	COMMUTATIVE

1	1	1	1
1	2	2	4
1	2	2	4
4	4	4	4
	SEMIGROUP	NUMBER	125

1	1	1	4
1	2	2	4
1	2	2	4
1	1	1	4
	SEMIGROUP	NUMBER	126

1	1	1	4
1	2	2	4
1	2	2	4
1	4	4	4
	SEMIGROUP	NUMBER	127

1	1	1	4
1	2	2	4
1	2	2	4
4	4	4	1
	SEMIGROUP	NUMBER	128
	SEMIGROUP	IS	COMMUTATIVE

1	1	1	1
1	2	2	2
1	2	3	3
1	2	3	4
	SEMIGROUP	NUMBER	129
	SEMIGROUP	IS	COMMUTATIVE
	SEMIGROUP	HAS	LEFT IDENTITY $x = 4$
		SUCH	THAT $xy = y$
	SEMIGROUP	HAS	RIGHT IDENTITY $z = 4$
		SUCH	THAT $yz = y$
	SEMIGROUP	HAS	IDENTITY

1	1	1	1	
1	2	2	2	
1	2	3	3	
1	2	4	4	
		SEMIGROUP	NUMBER	130
		SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
		SUCH THAT	Y7=Y	
		SEMIGROUP	HAS RIGHT IDENTITY	Z = 4
		SUCH THAT	Y7=Y	

1	1	1	1	
1	2	2	2	
1	2	3	4	
1	2	3	4	
		SEMIGROUP	NUMBER	131
		SEMIGROUP	HAS LEFT IDENTITY	X = 3
		SUCH THAT	XY=Y	
		SEMIGROUP	HAS LEFT IDENTITY	X = 4
		SUCH THAT	XY=Y	

1	1	1	1	
1	2	2	2	
1	2	3	4	
1	2	4	3	
		SEMIGROUP	NUMBER	132
		SEMIGROUP	IS COMMUTATIVE	
		SEMIGROUP	HAS LEFT IDENTITY	X = 3
		SUCH THAT	XY=Y	
		SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
		SUCH THAT	Y7=Y	
		SEMIGROUP	HAS IDENTITY	

1	1	1	1	
1	2	2	2	
1	2	3	4	
1	4	4	4	
		SEMIGROUP	NUMBER	133
		SEMIGROUP	HAS LEFT IDENTITY	X = 3
		SUCH THAT	XY=Y	
		SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
		SUCH THAT	Y7=Y	
		SEMIGROUP	HAS IDENTITY	

1	1	1	1	
1	2	2	4	
1	2	3	4	
1	2	2	4	
		SEMIGROUP	NUMBER	134
		SEMIGROUP	HAS LEFT IDENTITY	X = 3

1 1 1 1
 1 2 2 4
 1 2 3 4
 1 2 4 4
 SEMIGROUP NUMBER 135
 SEMIGROUP HAS LEFT IDENTITY $X = 3$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1 1 1 1
 1 2 2 4
 1 2 3 4
 1 4 4 2
 SEMIGROUP NUMBER 136
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY $X = 3$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1 1 1 1
 1 2 2 4
 1 2 3 4
 4 4 4 4
 SEMIGROUP NUMBER 137
 SEMIGROUP HAS LEFT IDENTITY $X = 3$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1 1 1 4
 1 2 2 4
 1 2 3 4
 1 1 1 4
 SEMIGROUP NUMBER 138
 SEMIGROUP HAS LEFT IDENTITY $X = 3$
 SUCH THAT $XY=Y$

1 1 1 4
 1 2 2 4
 1 2 3 4
 1 1 4 4
 SEMIGROUP NUMBER 139
 SEMIGROUP HAS LEFT IDENTITY $X = 3$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS RIGHT IDENTITY $Z = 3$
 SUCH THAT $YZ=Y$
 SEMIGROUP HAS IDENTITY

1	1	1	4
1	2	2	4
1	2	3	4
1	4	4	4
	SEMIGROUP	NUMBER	140
	SEMIGROUP	HAS LEFT IDENTITY	$X = 3$
	SUCH	THAT $XY=Y$	
	SEMIGROUP	HAS RIGHT IDENTITY	$Z = 3$
	SUCH	THAT $YZ=Y$	
	SEMIGROUP	HAS IDENTITY	
1	1	1	4
1	2	2	4
1	2	3	4
4	4	4	1
	SEMIGROUP	NUMBER	141
	SEMIGROUP	IS COMMUTATIVE	
	SEMIGROUP	HAS LEFT IDENTITY	$X = 3$
	SUCH	THAT $XY=Y$	
	SEMIGROUP	HAS RIGHT IDENTITY	$Z = 3$
	SUCH	THAT $YZ=Y$	
	SEMIGROUP	HAS IDENTITY	
1	1	1	1
1	2	2	2
1	3	3	3
1	4	4	4
	SEMIGROUP	NUMBER	142
	SEMIGROUP	HAS RIGHT IDENTITY	$Z = 2$
	SUCH	THAT $YZ=Y$	
	SEMIGROUP	HAS RIGHT IDENTITY	$Z = 3$
	SUCH	THAT $YZ=Y$	
	SEMIGROUP	HAS RIGHT IDENTITY	$Z = 4$
	SUCH	THAT $YZ=Y$	
1	1	1	1
1	2	2	4
1	3	3	4
4	4	4	4
	SEMIGROUP	NUMBER	143
	SEMIGROUP	HAS RIGHT IDENTITY	$Z = 2$
	SUCH	THAT $YZ=Y$	
	SEMIGROUP	HAS RIGHT IDENTITY	$Z = 3$
	SUCH	THAT $YZ=Y$	
1	1	1	4
1	2	2	4
1	3	3	4
1	1	1	4
	SEMIGROUP	NUMBER	144

1	1	1	4
1	2	2	4
1	3	3	4
1	4	4	4
	SEMIGROUP	NUMBER	145
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 2
	SUCH	THAT $YZ=Y$	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH	THAT $YZ=Y$	

1	1	1	4
1	2	2	4
1	3	3	4
4	4	4	1
	SEMIGROUP	NUMBER	146
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 2
	SUCH	THAT $YZ=Y$	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH	THAT $YZ=Y$	

1	1	1	1
1	2	3	4
1	2	3	4
1	2	3	4
	SEMIGROUP	NUMBER	147
	SEMIGROUP	HAS LEFT IDENTITY	X = 2
	SUCH	THAT $XY=Y$	
	SEMIGROUP	HAS LEFT IDENTITY	X = 3
	SUCH	THAT $XY=Y$	
	SEMIGROUP	HAS LEFT IDENTITY	X = 4
	SUCH	THAT $XY=Y$	

1	1	1	1
1	2	3	4
1	2	3	4
4	4	4	4
	SEMIGROUP	NUMBER	148
	SEMIGROUP	HAS LEFT IDENTITY	X = 2
	SUCH	THAT $XY=Y$	
	SEMIGROUP	HAS LEFT IDENTITY	X = 3
	SUCH	THAT $XY=Y$	

1	1	1	4	
1	2	3	4	
1	2	3	4	
1	1	1	4	
	SEMIGROUP	NUMBER	140	
	SEMIGROUP	HAS LEFT IDENTITY	X = 2	
	SUCH THAT	XY=Y		
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH THAT	XY=Y		
1	1	1	4	
1	2	3	4	
1	2	3	4	
1	4	4	4	
	SEMIGROUP	NUMBER	150	
	SEMIGROUP	HAS LEFT IDENTITY	X = 2	
	SUCH THAT	XY=Y		
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH THAT	XY=Y		
1	1	1	4	
1	2	3	4	
1	2	3	4	
4	4	4	1	
	SEMIGROUP	NUMBER	151	
	SEMIGROUP	HAS LEFT IDENTITY	X = 2	
	SUCH THAT	XY=Y		
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH THAT	XY=Y		
1	1	1	1	
1	2	3	3	
1	3	2	2	
1	3	2	2	
	SEMIGROUP	NUMBER	152	
	SEMIGROUP	IS COMMUTATIVE		
1	1	1	1	
1	2	3	4	
1	3	2	4	
4	4	4	4	
	SEMIGROUP	NUMBER	153	
	SEMIGROUP	HAS LEFT IDENTITY	X = 2	
	SUCH THAT	XY=Y		
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 2	
	SUCH THAT	YZ=Y		
	SEMIGROUP	HAS IDENTITY		

1 1 1 4
 1 2 3 4
 1 3 2 4
 1 1 1 4
 SEMIGROUP NUMBER 154
 SEMIGROUP HAS LEFT IDENTITY X = 2
 SUCH THAT XY=Y

1 1 1 4
 1 2 3 4
 1 3 2 4
 1 4 4 4
 SEMIGROUP NUMBER 155
 SEMIGROUP HAS LEFT IDENTITY X = 2
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 1 1 4
 1 2 3 4
 1 3 2 4
 4 4 4 1
 SEMIGROUP NUMBER 156
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY X = 2
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 1 1 1
 1 2 3 4
 3 3 3 3
 3 4 1 2
 SEMIGROUP NUMBER 157
 SEMIGROUP HAS LEFT IDENTITY X = 2
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1	1	1	1
1	2	3	4
3	3	3	3
4	4	4	4
	SEMIGROUP	NUMBER	158
	SEMIGROUP	HAS LEFT IDENTITY	X = 2
	SUCH	THAT XY=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 2
	SUCH	THAT YZ=Y	
	SEMIGROUP	HAS IDENTITY	

1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
	SEMIGROUP	NUMBER	159
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 1
	SUCH	THAT YZ=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 2
	SUCH	THAT YZ=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 3
	SUCH	THAT YZ=Y	
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 4
	SUCH	THAT YZ=Y	

1	1	3	3
1	1	3	3
1	1	3	3
1	1	3	3
	SEMIGROUP	NUMBER	160

1	1	3	3
1	1	3	3
1	1	3	3
1	1	3	4
	SEMIGROUP	NUMBER	161

1	1	3	3
1	1	3	3
1	1	3	3
1	2	3	4
	SEMIGROUP	NUMBER	162
	SEMIGROUP	HAS LEFT IDENTITY	X = 4
	SUCH	THAT XY=Y	

1 1 3 4
 1 1 3 4
 1 1 3 4
 1 1 3 4
 SEMIGROUP NUMBER 163

1 1 3 4
 1 1 3 4
 1 1 3 4
 1 3 3 4
 SEMIGROUP NUMBER 164

1 1 3 3
 1 1 3 3
 3 3 1 1
 3 3 1 1
 SEMIGROUP NUMBER 165
 SEMIGROUP IS COMMUTATIVE

1 1 3 3
 1 1 3 3
 3 3 1 1
 3 3 1 2
 SEMIGROUP NUMBER 166
 SEMIGROUP IS COMMUTATIVE

1 1 3 3
 1 2 3 3
 1 1 3 3
 1 1 3 4
 SEMIGROUP NUMBER 167

1 1 3 3
 1 2 3 4
 1 1 3 3
 1 2 3 4
 SEMIGROUP NUMBER 168
 SEMIGROUP HAS LEFT IDENTITY $X = 2$
 SUCH THAT $XY=Y$
 SEMIGROUP HAS LEFT IDENTITY $X = 4$
 SUCH THAT $XY=Y$

1	1	3	4	
1	2	3	4	
1	1	3	4	
1	1	3	4	
				SEMIGROUP NUMBER 169
				SEMIGROUP HAS LEFT IDENTITY $X = 2$
				SUCH THAT $XY=Y$
1	1	3	4	
1	2	3	4	
1	1	3	4	
1	4	3	4	
				SEMIGROUP NUMBER 170
				SEMIGROUP HAS LEFT IDENTITY $X = 2$
				SUCH THAT $XY=Y$
1	1	3	3	
1	2	3	4	
1	3	3	1	
1	4	3	2	
				SEMIGROUP NUMBER 171
				SEMIGROUP HAS LEFT IDENTITY $X = 2$
				SUCH THAT $XY=Y$
				SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
				SUCH THAT $YZ=Y$
				SEMIGROUP HAS IDENTITY
1	1	3	4	
1	2	3	4	
1	3	3	4	
1	4	3	4	
				SEMIGROUP NUMBER 172
				SEMIGROUP HAS LEFT IDENTITY $X = 2$
				SUCH THAT $XY=Y$
				SEMIGROUP HAS RIGHT IDENTITY $Z = 2$
				SUCH THAT $YZ=Y$
				SEMIGROUP HAS IDENTITY
1	1	3	3	
1	2	3	3	
3	3	1	1	
3	3	1	1	
				SEMIGROUP NUMBER 173
				SEMIGROUP IS COMMUTATIVE

1 1 3 3
 1 2 3 3
 3 3 1 1
 3 4 1 1
 SEMIGROUP NUMBER 174
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y

1 1 3 3
 1 2 3 4
 3 3 1 1
 3 3 1 1
 SEMIGROUP NUMBER 175
 SEMIGROUP HAS LEFT IDENTITY X = 2
 SUCH THAT XY=Y

1 1 3 3
 1 2 3 4
 3 3 1 1
 3 4 1 1
 SEMIGROUP NUMBER 176
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY X = 2
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 1 3 3
 1 2 3 4
 3 3 1 1
 3 4 1 2
 SEMIGROUP NUMBER 177
 SEMIGROUP IS COMMUTATIVE
 SEMIGROUP HAS LEFT IDENTITY X = 2
 SUCH THAT XY=Y
 SEMIGROUP HAS RIGHT IDENTITY Z = 2
 SUCH THAT YZ=Y
 SEMIGROUP HAS IDENTITY

1 2 2 2
 2 1 1 1
 2 1 1 1
 2 1 1 1
 SEMIGROUP NUMBER 178
 SEMIGROUP IS COMMUTATIVE

1	2	3	4	
1	2	3	4	
1	2	3	4	
1	2	3	4	
	SEMIGROUP	NUMBER	179	
	SEMIGROUP	HAS LEFT IDENTITY	X = 1	
	SUCH	THAT XY=Y		
	SEMIGROUP	HAS LEFT IDENTITY	X = 2	
	SUCH	THAT XY=Y		
	SEMIGROUP	HAS LEFT IDENTITY	X = 3	
	SUCH	THAT XY=Y		
	SEMIGROUP	HAS LEFT IDENTITY	X = 4	
	SUCH	THAT XY=Y		

1	2	3	1	
2	3	1	2	
3	1	2	3	
1	2	3	1	
	SEMIGROUP	NUMBER	180	
	SEMIGROUP	IS COMMUTATIVE		

1	2	3	1	
2	3	1	2	
3	1	2	3	
1	2	3	4	
	SEMIGROUP	NUMBER	181	
	SEMIGROUP	IS COMMUTATIVE		
	SEMIGROUP	HAS LEFT IDENTITY	X = 4	
	SUCH	THAT XY=Y		
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 4	
	SUCH	THAT YZ=Y		
	SEMIGROUP	HAS IDENTITY		

1	2	3	4	
2	3	1	4	
3	1	2	4	
4	4	4	4	
	SEMIGROUP	NUMBER	182	
	SEMIGROUP	IS COMMUTATIVE		
	SEMIGROUP	HAS LEFT IDENTITY	X = 1	
	SUCH	THAT XY=Y		
	SEMIGROUP	HAS RIGHT IDENTITY	Z = 1	
	SUCH	THAT YZ=Y		
	SEMIGROUP	HAS IDENTITY		

1	2	3	4
1	2	3	4
3	4	1	2
3	4	1	2

SEMIGROUP NUMBER 183
SEMIGROUP HAS LEFT IDENTITY $x = 1$
SUCH THAT $xy=y$
SEMIGROUP HAS LEFT IDENTITY $x = 2$
SUCH THAT $xy=y$

1	2	2	4
2	4	4	1
2	4	4	1
4	1	1	2

SEMIGROUP NUMBER 184
SEMIGROUP IS COMMUTATIVE

1	1	4	4
1	1	4	4
2	2	3	3
2	2	3	3

SEMIGROUP NUMBER 185

1	1	4	4
2	2	3	3
3	3	2	2
4	4	1	1

SEMIGROUP NUMBER 186
SEMIGROUP HAS RIGHT IDENTITY $z = 1$
SUCH THAT $yz=y$
SEMIGROUP HAS RIGHT IDENTITY $z = 2$
SUCH THAT $yz=y$

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

SEMIGROUP NUMBER 187
SEMIGROUP IS COMMUTATIVE
SEMIGROUP HAS LEFT IDENTITY $x = 1$
SUCH THAT $xy=y$
SEMIGROUP HAS RIGHT IDENTITY $z = 1$
SUCH THAT $yz=y$
SEMIGROUP HAS IDENTITY
SEMIGROUP IS A GROUP

1	2	3	4
2	1	4	3
3	4	2	1
4	3	1	2

```

SEMIGROUP NUMBER 198
SEMIGROUP IS COMMUTATIVE
SEMIGROUP HAS LEFT IDENTITY X = 1
      SUCH THAT XY=Y
SEMIGROUP HAS RIGHT IDENTITY Z = 1
      SUCH THAT YZ=Y
SEMIGROUP HAS IDENTITY
SEMIGROUP IS A GROUP

```

SEMIGROUPS OF ORDER FOUR WHICH ARE ISOMORPHIC TO
 THEIR ANTI-ISOMORPHIC IMAGE BUT NOT COMMUTATIVE

1	1	1	1	
1	1	1	1	
1	1	1	1	
1	1	2	1	
	SEMIGROUP		NUMBER	1

1	1	1	1	
1	1	1	1	
1	1	2	1	
1	1	2	2	
	SEMIGROUP		NUMBER	2

1	1	1	1	
1	1	1	1	
1	1	1	3	
1	2	1	4	
	SEMIGROUP		NUMBER	3

1	1	1	1	
1	1	1	2	
1	2	3	1	
1	1	1	4	
	SEMIGROUP		NUMBER	4

1	1	1	1	
1	1	1	2	
1	2	3	2	
1	1	1	4	
	SEMIGROUP		NUMBER	5

1	1	4	4	
1	1	4	4	
2	2	3	3	
2	2	3	3	
	SEMIGROUP		NUMBER	6

GENERATION OF ORDER TWO SEMIGROUPS

```

INTEGER H
DIMENSION I(10,10),M(20,3,3),L(5,5),LP(5,5),LF(5)
N=2
H=1
M6=0
DO 10 I1=1,N
DO 10 J1=1,N
DO 10 K1=1,N
DO 10 L1=1,N
I(1,1)=I1
I(1,2)=J1
I(2,1)=K1
I(2,2)=L1
L4=1
N4=1
I5=1
L5=0
N5=0
DO 15 I2=1,N
K2=N+1
K2N=(N*N)+N
DO 15 J2=K2,K2N
IF (J2.EQ.N+1) L2=I1
IF (J2.EQ.N+2) L2=J1
IF (J2.EQ.N+3) L2=K1
IF (J2.EQ.N+4) L2=L1
15 I(I2,J2)=I(I2,L2)
DO 20 M3=1,N
J3=(M3*N)+1
J3N=(M3*N)+N
DO 20 K3=J3,J3N
DO 20 L3=1,N
I3=((K3-(M3*N))*(N-L3))+((K3-(M3*N)+1)*L3)
IF (I(M3,I3).NE.I(K3,L3)) GO TO 24
20 CONTINUE
WRITE (6,21) H,((I(J,K),K=1,2),J=1,2)
WRITE (7,210) ((I(J,K),K=1,2),J=1,2)
21 FORMAT (///1X'PERMUTATION NUMBER',I5,' IS A SEMI
1 GROUP',//2X,I2,4X,I2,//2X,I2,4X,I2)
210 FORMAT (///44X,//2X,I2,4X,I2,//2X,I2,4X,I2)
M6=M6+1
DO 16 I6=1,N
DO 16 J6=1,N
M(M6,I6,J6)=I(I6,J6)
WRITE (6,17) M6
WRITE (7,17) M6
17 FORMAT (10X'SEMIGROUP NUMBER IS ',I5)
IF (M6.EQ.1) GO TO 19
LF(1)=2
LF(2)=1
DO 18 N6=1,N
DO 18 L6=1,N
DO 18 K6=1,N
IF (M(M6,L6,K6).EQ.N6) L(L6,K6)=LF(N6)
18 CONTINUE
LP(1,1)=L(2,2)
LP(1,2)=L(2,1)
LP(2,1)=L(1,2)
LP(2,2)=L(1,1)
L7=1
11 DO 12 I7=1,N
DO 12 J7=1,N
IF (LP(I7,J7).NE.M(L7,I7,J7)) GO TO 13

```



```

12 CONTINUE
   GC TO 14
13 L7=L7+1
   IF (L7.EQ.M6) GC TO 19
   GC TO 11
14 WRITE (6,8) L7
   8 FORMAT (10X'SEMIGROUP IS ISOMORPHIC TO SEMIGROUP
1 NUMBER',I5)
   GC TO 24
19 DC 22 I4=1,N
   DC 22 J4=1,N
   IF ((I(I4,J4).NE.I(J4,I4)) GC TO 26
22 CONTINUE
   WRITE (6,23)
   WRITE (7,23)
23 FORMAT (10X'SEMIGROUP IS COMMUTATIVE')
26 DC 27 K4=1,N
   IF ((I(L4,K4).NE.K4) GC TO 29
27 CONTINUE
   WRITE (6,28) L4
   WRITE (7,28) L4
   L5=L4
28 FORMAT (10X'SEMIGROUP HAS LEFT IDENTITY X =',I2,
1, ' SUCH THAT XY=Y')
29 L4=L4+1
   IF (L4.GT.N) GO TO 30
   GC TO 26
30 DC 31 M4=1,N
   IF ((I(M4,N4).NE.M4) GC TO 33
31 CONTINUE
   WRITE (6,32) N4
   WRITE (7,32) N4
   N5=N4
32 FORMAT (10X'SEMIGROUP HAS RIGHT IDENTITY Z =',I2
1, ' SUCH THAT YZ=Y')
33 N4=N4+1
   IF (N4.GT.N) GC TO 34
   GC TO 30
34 IF ((N5.NE.L5).OR.((N5.EQ.0).AND.(L5.EQ.0)))
   GC TO 24
   WRITE (6,35)
   WRITE (7,35)
35 FORMAT (10X'SEMIGROUP HAS IDENTITY')
36 DC 37 J5=1,N
   IF ((I(I5,J5).EQ.N5) GC TO 38
37 CONTINUE
   GC TO 24
38 I5=I5+1
   IF (I5.GT.N) GO TO 39
   GC TO 36
39 WRITE (6,40)
   WRITE (7,40)
40 FORMAT (10X'SEMIGROUP IS A GROUP')
24 H=H+1
10 CONTINUE
68 STOP
   END

```

GENERATION OF ORDER THREE SEMIGROUPS

```

INTEGER H
DIMENSION I(15,15),M(400,5,5),LP(5,5),L(5,5),LF(5),
1 LR(5)
N=3
H=1
MF=0
MC=0
DC 10 I1=1,N
DC 10 I2=1,N
DC 10 I3=1,N
DC 10 I4=1,N
DC 10 I5=1,N
DC 10 I6=1,N
DC 10 I7=1,N
DC 10 I8=1,N
DC 10 I9=1,N
L4=1
N4=1
L5=0
N5=0
J6=1
I(1,1)=I1
I(1,2)=I2
I(1,3)=I3
I(2,1)=I4
I(2,2)=I5
I(2,3)=I6
I(3,1)=I7
I(3,2)=I8
I(3,3)=I9
DC 15 J1=1,N
J2=N+1
J2N=(N*N)+N
DC 15 J3=J2,J2N
IF (J3.EQ.N+1) J4=I1
IF (J3.EQ.N+2) J4=I2
IF (J3.EQ.N+3) J4=I3
IF (J3.EQ.N+4) J4=I4
IF (J3.EQ.N+5) J4=I5
IF (J3.EQ.N+6) J4=I6
IF (J3.EQ.N+7) J4=I7
IF (J3.EQ.N+8) J4=I8
IF (J3.EQ.N+9) J4=I9
15 I(J1,J3)=I(J1,J4)
I(J3,J1)=I(J4,J1)
DC 20 M1=1,N
M2=(M1*N)+1
M2N=(M1*N)+N
DC 20 M3=M2,M2N
DC 20 M4=1,N
M5=((M3-(M1*N))*(N-M4))+((M3-(M1*N)+1)*M4))
IF (I(M1,M5).NE.I(M3,M4)) GC TO 24
20 CONTINUE
WRITE (6,21) H,((I(J,K),K=1,3),J=1,3)
21 FORMAT ( '//1X,PERMUTATION NUMBER',I5,' IS A SEMI
1 GROUP', //2X,I2,4X,I2,4X,I2, //2X,I2,4X,I2,4X,I2, //2X,
2 I2,4X,I2,4X,I2)
M9=M9+1
M8=M8+1
DC 50 I10=1,N
DC 50 J10=1,N
50 M(M9,I10,J10)=I(I10,J10)
WRITE (6,51) M9
WRITE (7,51) M8

```

```

51 FORMAT (10X'SEMIGROUP NUMBER',I5)
   IF ((M9.EQ.1).CR.(M9.EQ.400)) GO TC 5
DC 90 I20=1,N
DC 90 I21=1,N
DC 90 I22=1,N
LF(1)=I20
LF(2)=I21
LF(3)=I22
LF(I20)=1
LF(I21)=2
LF(I22)=3
   IF ((I20.EQ.I21).CR.(I20.EQ.I22).CR.(I21.EQ.I22))
1 GO TO C0
74 DC 80 K10=1,N
   DC 80 M10=1,N
   DC 80 N10=1,N
   IF (I(M10,N10).EQ.K10) L(M10,N10)=LF(K10)
80 CONTINUE
   LP(1,1)=L(LR(1),LR(1))
   LP(1,2)=L(LR(1),LR(2))
   LP(1,3)=L(LR(1),LR(3))
   LP(2,1)=L(LR(2),LR(1))
   LP(2,2)=L(LR(2),LR(2))
   LP(2,3)=L(LR(2),LR(3))
   LP(3,1)=L(LR(3),LR(1))
   LP(3,2)=L(LR(3),LR(2))
   LP(3,3)=L(LR(3),LR(3))
79 L20=1
59 DC 60 I11=1,N
   DC 60 J11=1,N
   IF (LP(I11,J11).NE.M(L20,I11,J11)) GO TC 61
60 CONTINUE
   GC TO 65
61 L20=L20+1
   IF (L20.EQ.M9) GO TO 90
   GC TO 59
65 WRITE (6,66) L20
66 FORMAT (10X'SEMIGROUP IS ISOMORPHIC TO SEMIGROUP
1 NUMBER',I5)
   M8=M8-1
   GC TO 10
90 CONTINUE
   DC 500 I500=1,N
   J500=N+1-I500
   DC 501 I501=1,N
   DC 501 I502=1,N
   IF ((I501.EQ.J500).OR.(I502.EQ.J500)) GO TC 501
   IF (I(I501,I502).EQ.J500) GO TO 500
501 CONTINUE
   N501=1
   N502=2
   N503=3
   IF (J500.EQ.1) GC TC 503
   IF (J500.EQ.2) GC TC 504
   IF (J500.EQ.3) GC TC 505
503 WRITE (7,502) N502,N503
   GC TO 500
504 WRITE (7,502) N501,N503
   GC TO 500
505 WRITE (7,502) N501,N502
502 FORMAT (10X'SEMIGROUP HAS A SUBSEMIGROUP OF
1 ORDER TWO',//14X(' ',I1,', ',I1,' '))
500 CONTINUE
5 DO 22 K1=1,N
   DC 22 K2=1,N
   IF (I(K1,K2).NE.I(K2,K1)) GC TO 26
22 CONTINUE
   WRITE (6,23)
23 FORMAT (10X'SEMIGROUP IS COMMUTATIVE')
26 DC 27 K4=1,N
   IF (I(L4,K4).NE.K4) GC TO 29
27 CONTINUE

```

```

WRITE (6,28) L4
WRITE (7,28) L4
L5=L4
28 FORMAT (10X'SEMIGROUP HAS LEFT IDENTITY X =',I2,
1//14X' SUCH THAT XY=Y')
29 L4=L4+1
IF (L4.GT.N) GO TO 30
GC TO 26
30 DC 31 M6=1,N
IF (I(M6,N4).NE.M6) GC TO 33
31 CONTINUE
WRITE (6,32) N4
WRITE (7,32) N4
N5=N4
32 FORMAT (10X'SEMIGROUP HAS RIGHT IDENTITY Z =',I2,
1//14X' SUCH THAT YZ=Y')
33 N4=N4+1
IF (N4.GT.N) GC TO 34
GC TO 30
34 IF ((N5.NE.L5).OR.((N5.EQ.0).AND.(L5.EQ.0)))
1 GO TO 24
WRITE (6,35)
35 FORMAT (10X'SEMIGROUP HAS IDENTITY')
36 DC 37 J5=1,N
IF (I(J6,J5).EQ.N5) GC TO 38
37 CONTINUE
GC TO 24
38 J6=J6+1
IF (J6.GT.N) GC TO 39
GC TO 36
39 WRITE (6,40)
40 FORMAT (10X'SEMIGROUP IS A GROUP')
24 H=H+1
IF (H.GT.20000) GC TO 68
10 CONTINUE
68 WRITE (6,25) H
25 FORMAT (///5X'H =',I5)
STOP
END

```

GENERATION OF ORDER FOUR SEMIGROUPS PART ONE

```

INTEGER H
DIMENSION I(10,10),J(15,10,10)
H=1
N=4
N1=1
N2=1
I13=C
I14=C
I24=C
I31=C
I33=C
I34=C
I41=C
I42=C
I43=C
I44=C
DO 600 K3=1,9
DO 600 K4=1,9
I(K3,K4)=C
600 CONTINUE
DO 100 I11=1,N1
DO 100 I12=1,N
DO 100 I21=1,N
DO 100 I22=1,N
DO 100 I23=1,N
DO 100 I32=1,N
I(1,1)=I11
I(1,2)=I12
I(1,3)=I13
I(1,4)=I14
I(2,1)=I21
I(2,2)=I22
I(2,3)=I23
I(2,4)=I24
I(3,1)=I31
I(3,2)=I32
I(3,3)=I33
I(3,4)=I34
I(4,1)=I41
I(4,2)=I42
I(4,3)=I43
I(4,4)=I44
DO 800 L1=1,3
DO 800 L2=1,3
DO 800 L3=1,3
IF ((I(L1,L2).EQ.9).OR.(I(L2,L3).EQ.9)) GO TO 800
IF (I(L1,I(L2,L3)).NE.I(I(L1,L2),L3)) GO TO 801
GO TO 800
801 IF ((I(L1,I(L2,L3)).EQ.9).OR.(I(I(L1,L2),L3).EQ.9))
1 GO TO 800
GO TO 100
800 CONTINUE
L7=10*H
WRITE (6,16) L7,((I(K,L),L=1,4),K=1,4)
N7=0
N8=C
N12=C
N13=C
900 DO 500 KK=1,14
DO 500 K1=1,N
DO 500 K2=1,N
J(KK,K1,K2)=I(K1,K2)
500 CONTINUE
IF (N7.EQ.0) GO TO 999

```

```

I(3,3)=N7
J(1,3,3)=N7
IF (N12.GE.1) GC TC 852
808 DO 850 LL=1,N
DO 850 MM=1,N
IF (J(1,LL,MM).GE.5) GO TO 851
GC TO 850
851 N13=N13+1
IF (N13.GE.7) GC TO 852
850 CONTINUE
GC TO 902
852 N12=N12+1
IF (N12.GE.5) GC TC 853
J(1,LL,MM)=N12
I(LL,MM)=N12
GO TO 902
853 N13=0
N12=0
GC TO 905
902 J(13,I(1,3),3)=I(1,I(3,3))
J(13,I(2,3),3)=I(2,I(3,3))
J(13,I(3,3),3)=I(3,I(3,3))
J(13,I(4,3),3)=I(4,I(3,3))
J(14,3,I(3,1))=I(I(3,3),1)
J(14,3,I(3,2))=I(I(3,3),2)
J(14,3,I(3,4))=I(I(3,3),4)
J(14,3,I(3,3))=I(I(3,3),3)
IF (N7.NE.4) GO TO 999
J(12,4,1)=I(3,I(3,1))
J(12,4,2)=I(3,I(3,2))
J(12,4,3)=I(3,I(3,3))
J(12,4,4)=I(3,I(3,4))
J(12,3,4)=I(I(3,3),3)
J(12,2,4)=I(I(2,3),3)
J(12,1,4)=I(I(1,3),3)
IF (J(12,4,4).EQ.9) J(12,4,4)=I(I(4,3),3)
GO TO 999
999 IF (I12-3)19,20,21
19 GO TO 22
20 J(1,I12,1)=I(1,I21)
J(1,I12,3)=I(1,I23)
J(1,I12,4)=I(1,I(2,4))
J(1,1,I12)=I(I11,2)
J(2,3,I12)=I(I(3,1),2)
J(1,4,I12)=I(I(4,1),2)
21 J(1,I12,1)=I(1,I21)
J(1,I12,2)=I(1,I22)
J(2,I12,3)=I(1,I23)
J(1,I12,4)=I(1,I(2,4))
J(1,1,I12)=I(I11,2)
J(1,2,I12)=I(I21,2)
J(2,3,I12)=I(I(3,1),2)
J(2,4,I12)=I(I(4,1),2)
N2=2
GC TO 30
22 IF (I22-3)23,24,25
23 GC TO 26
24 J(2,I22,1)=I(2,I21)
J(3,I22,3)=I(2,I23)
J(3,I22,4)=I(2,I(2,4))
J(2,1,I22)=I(I12,2)
J(4,3,I22)=I(I32,2)
J(3,4,I22)=I(I(4,2),2)
25 J(2,I22,1)=I(2,I21)
J(2,I22,2)=I(2,I22)
J(4,I22,3)=I(2,I23)
J(3,I22,4)=I(2,I(2,4))
J(2,1,I22)=I(I12,2)
J(2,2,I22)=I(I22,2)
J(4,3,I22)=I(I32,2)
J(4,4,I22)=I(I(4,2),2)
N2=4

```

```

GO TO 30
26 IF (I21-3)27,28,29
27 GC TO 31
28 J(3,I21,1)=I(2,I11)
   J(5,I21,3)=I(2,I(1,3))
   J(5,I21,4)=I(2,I(1,4))
   J(3,1,I21)=I(I12,1)
   J(6,3,I21)=I(I32,1)
29 J(5,4,I21)=I(I(4,2),1)
   J(3,I21,1)=I(2,I11)
   J(3,I21,2)=I(2,I12)
   J(6,I21,3)=I(2,I(1,3))
   J(5,I21,4)=I(2,I(1,4))
   J(3,1,I21)=I(I12,1)
   J(3,2,I21)=I(I22,1)
   J(6,3,I21)=I(I32,1)
   J(6,4,I21)=I(I(4,2),1)
N2=6
GC TO 30
31 IF (I23-3)32,33,34
32 GC TO 35
33 J(4,I23,1)=I(2,I(3,1))
   J(7,I23,3)=I(2,I(3,3))
   J(7,I23,4)=I(2,I(3,4))
   J(4,1,I23)=I(I12,3)
   J(8,3,I23)=I(I32,3)
   J(7,4,I23)=I(I(4,2),3)
34 J(4,I23,1)=I(2,I(3,1))
   J(4,I23,2)=I(2,I32)
   J(8,I23,3)=I(2,I(3,3))
   J(7,I23,4)=I(2,I(3,4))
   J(4,1,I23)=I(I12,3)
   J(4,2,I23)=I(I22,3)
   J(8,3,I23)=I(I32,3)
   J(8,4,I23)=I(I(4,2),3)
N2=8
30 N3=N2/2
   DO 300 L3=N3,N2
   DO 300 L4=1,N
   DO 300 L5=1,N
   IF (J(L3,L4,L5).NE.9) I(L4,L5)=J(L3,L4,L5)
300 CONTINUE
   IF (N2.EQ.8) GO TO 35
   IF (N2-4)22,26,31
35 IF (I32-3)36,37,38
36 GC TO 39
37 J(5,I32,1)=I(3,I21)
   J(9,I32,3)=I(3,I23)
   J(9,I32,4)=I(3,I(2,4))
   J(5,1,I32)=I(I(1,3),2)
   J(10,3,I32)=I(I(3,3),2)
   J(9,4,I32)=I(I(4,3),2)
38 J(5,I32,1)=I(3,I21)
   J(5,I32,2)=I(3,I22)
   J(10,I32,3)=I(3,I23)
   J(9,I32,4)=I(3,I(2,4))
   J(5,1,I32)=I(I(1,3),2)
   J(5,2,I32)=I(I23,2)
   J(10,3,I32)=I(I(3,3),2)
   J(10,4,I32)=I(I(4,3),2)
N2=10
39 N3=N2/2
   IF (N3.EQ.0) GO TO 18
   DO 301 L3=N3,N2
   DO 301 L4=1,N
   DO 301 L5=1,N
   IF (J(L3,L4,L5).NE.9) I(L4,L5)=J(L3,L4,L5)
301 CONTINUE
18 IF (I12.EQ.1) GC TC 40
   IF (I12.EQ.2) GC TC 41
40 J(6,I12,3)=I(1,I23)
   J(6,I12,4)=I(1,I(2,4))

```

```

J(6,3,I12)=I(I(3,1),2)
J(6,4,I12)=I(I(4,1),2)
41 J(6,I12,4)=I(1,I(2,4))
J(6,4,I12)=I(I(4,1),2)
N6=6
GC TO 50
51 IF (I21.EQ.1) GC TO 42
IF (I21.EQ.2) GC TC 43
42 J(7,I21,3)=I(2,I(1,3))
J(7,I21,4)=I(2,I(1,4))
J(7,3,I21)=I(I32,1)
J(7,4,I21)=I(I(4,2),1)
43 J(7,I21,4)=I(2,I(1,4))
J(7,4,I21)=I(I(4,2),1)
N6=7
GC TO 50
52 IF (I22.EQ.1) GC TO 44
IF (I22.EQ.2) GC TC 45
44 J(8,I22,3)=I(2,I23)
J(8,I22,4)=I(2,I(2,4))
J(8,3,I22)=I(I32,2)
J(8,4,I22)=I(I(4,2),2)
45 J(8,I22,4)=I(2,I(2,4))
J(8,4,I22)=I(I(4,2),2)
N6=8
GC TO 50
54 IF (I23.EQ.1) GC TO 46
IF (I23.EQ.2) GC TC 47
46 J(9,I23,3)=I(2,I(3,3))
J(9,I23,4)=I(2,I(3,4))
J(9,3,I23)=I(I32,3)
J(9,4,I23)=I(I(4,2),3)
47 J(9,I23,4)=I(2,I(3,4))
J(9,4,I23)=I(I(4,2),3)
N6=9
GC TO 50
55 IF (I32.EQ.1) GC TO 48
IF (I32.EQ.2) GC TO 49
48 J(10,I32,3)=I(3,I23)
J(10,I32,4)=I(3,I(2,4))
J(10,3,I32)=I(I(3,3),2)
J(10,4,I32)=I(I(4,3),2)
49 J(10,I32,4)=I(3,I(2,4))
J(10,4,I32)=I(I(4,3),2)
N6=10
50 DO 302 L4=1,N
DO 302 L5=1,N
IF (J(N6,L4,L5).NE.9) I(L4,L5)=J(N6,L4,L5)
302 CCNTINUE
IF (N6-7)51,52,53
53 IF (N6-9)54,55,56
56 DO 400 L4=1,N
DO 400 L5=1,N
IF (I(L4,L5).EQ.4) GC TO 401
IF (I(L4,L5).EQ.3) GC TO 402
GC TO 400
401 J(11,4,1)=I(L4,I(L5,1))
J(11,4,2)=I(L4,I(L5,2))
J(11,4,3)=I(L4,I(L5,3))
J(11,4,4)=I(L4,I(L5,4))
J(11,3,4)=I(I(3,L4),L5)
J(11,2,4)=I(I(2,L4),L5)
J(11,1,4)=I(I(1,L4),L5)
IF(J(11,4,4).EQ.9) J(11,4,4)=I(I(4,L4),L5)
GC TO 400
402 J(15,3,1)=I(L4,I(L5,1))
J(15,3,2)=I(L4,I(L5,2))
J(15,3,4)=I(L4,I(L5,4))
J(15,1,3)=I(I(1,L4),L5)
J(15,2,3)=I(I(2,L4),L5)
J(15,4,3)=I(I(4,L4),L5)
J(15,3,3)=I(I(3,L4),L5)

```



```

IF (J(15,3,3).EQ.9) J(15,3,3)=I(L4,I(L5,3))
400 CCNTINUE
DO 8 L4=1,N
DO 8 L5=1,N
N4=2
IF (J(1,L4,L5)-9)8,10,10
10 J(1,L4,L5)=J(N4,L4,L5)
IF (J(N4,L4,L5)-9)8,11,11
11 N4=N4+1
IF (N4.EQ.13) GO TO 14
IF (N4.EQ.14) GO TO 8
GO TO 10
14 IF (J(13,L4,L5).EQ.9) GO TO 13
GO TO 10
13 J(1,L4,L5)=I(L4,L5)
8 CCNTINUE
WRITE (6,105) N7
105 FORMAT (2CX,I5)
L8=100*H
WRITE (6,16) L8,((J(1,K,L),L=1,4),K=1,4)
GO TO 909
905 IF (N7-1)904,903,903
904 IF (J(1,3,3).GE.5) GO TO 903
GO TO 909
903 IF (N12.NE.0) GO TO 900
N7=N7+1
IF (N7.GE.5) GO TO 102
GO TO 900
909 N5=1
DO 9 L4=1,N
DO 9 L5=1,N
IF (J(1,L4,L5).NE.9) GO TO 15
IF (N5-2)5,6,7
5 J(1,L4,L5)=5
N5=N5+1
GO TO 9
6 J(1,L4,L5)=6
N5=N5+1
GO TO 9
7 IF (N5-4)12,4,3
12 J(1,L4,L5)=7
N5=N5+1
GO TO 9
4 J(1,L4,L5)=15
N5=N5+1
GO TO 9
3 IF (N5-6)2,1,9
2 J(1,L4,L5)=16
N5=N5+1
GO TO 9
1 J(1,L4,L5)=17
GO TO 9
15 CCNTINUE
9 CCNTINUE
DO 200 L1=1,N
DO 200 L2=1,N
IF (J(1,L1,L2).EQ.9) GO TO 201
GO TO 200
201 J(1,L1,L2)=I(L1,L2)
200 CCNTINUE
IF (N9.EQ.0) GO TO 710
WRITE (7,777) ((J(1,K,L),L=1,4),K=1,4)
777 FORMAT (I15,I6I2)
WRITE (6,16) H,((J(1,K,L),L=1,4),K=1,4)
16 FORMAT (10X'PERMUTATION',I5,/,2X,I2,4X,I2,4X,I2,4X,I2,4X,I2,/,
1,/,2X,I2,4X,I2,4X,I2,4X,I2,/,2X,I2,4X,I2,4X,I2,4X,I2,/,
2/2X,I2,4X,I2,4X,I2,4X,I2,/,2X,I2,4X,I2,4X,I2,4X,I2)
710 CCNTINUE
I(1,1)=I11
I(1,2)=I12
I(1,3)=I13
I(1,4)=I14

```

```

I(2,1)=I21
I(2,2)=I22
I(2,3)=I23
I(2,4)=I24
I(3,1)=I31
I(3,2)=I32
I(3,3)=I33
I(3,4)=I34
I(4,1)=I41
I(4,2)=I42
I(4,3)=I43
I(4,4)=I44
IF (N7.FQ.0) GO TO 707
GC TO 905
707 NC=N9+1
IF (N9.GE.2) GO TO 102
GO TO 905
102 CONTINUE
H=H+1
IF (H.EQ.50) GO TO 101
100 CONTINUE
101 CONTINUE
STOP
END

```

GENERATION OF ORDER FOUR SEMIGROUPS PART TWO

```

INTEGER H
DIMENSION I(70,70),J(70,70),J1(5,5,5)
N=4
H=1
DO 10 K9=1,200
READ (5,11) M,((I(L,K),K=1,4),L=1,4)
11 FORMAT (17I4)
IF (M.EQ.9999) GO TO 10
N12=0
DO 60 LL=1,N
DO 60 KK=1,N
IF (I(LL,KK).GE.5) GC TO 61
GC TO 60
61 N12=N12+1
IF (N12.GE.6) GC TO 107
60 CCNTINUE
DO 100 I21=1,N
DO 100 I22=1,N
DO 100 I23=1,N
DO 101 I24=1,N
DO 101 I25=1,N
101 J(I24,I25)=I(I24,I25)
DO 50 I11=1,N
DO 50 I12=1,N
102 IF (I(I11,I12).EQ.5) GO TO 51
IF (I(I11,I12).EQ.6) GO TO 52
IF (I(I11,I12).EQ.7) GO TO 53
GC TO 49
51 J(I11,I12)=I21
GC TO 49
52 J(I11,I12)=I22
GC TO 49
53 J(I11,I12)=I23
49 IF (J(I11,I12).LE.4) GO TO 50
I(I11,I12)=J(I11,I12)
GC TO 102
50 CCNTINUE
333 DO 105 M3=1,N
DO 105 M4=1,N
J1(1,M3,M4)=J(M3,M4)
105 CCNTINUE
DO 12 K1=1,N
DO 12 K2=1,N
DO 12 K3=1,N
IF (J(J(K1,K2),K3).NE.J(K1,J(K2,K3))) GO TO 24
12 CCNTINUE
IF (H.EQ.1) GC TO 103
DO 104 M1=1,N
DO 104 M2=1,N
IF (J1(2,M1,M2).NE.J1(1,M1,M2)) GO TO 103
104 CCNTINUE
GC TO 24
103 WRITE (6,21) H,((J(L,K),K=1,4),L=1,4)
WRITE (7,21) H,((J(L,K),K=1,4),L=1,4)
WRITE (6,22) M
21 FORMAT (I15,16I2)
22 FCORMAT (I60)
24 H=H+1
DO 106 M5=1,N
DO 106 M6=1,N
J1(2,M5,M6)=J1(1,M5,M6)
106 CCNTINUE
100 CCNTINUE
GC TO 10

```

```

107 I(1,1)=1
    DC 120 I21=1,N
    DC 120 I22=1,N
    DC 120 I23=1,N
    DC 120 I24=1,N
    DC 120 I25=1,N
    DC 120 I26=1,N
    DC 109 L8=1,N
    DC 109 L9=1,N
109 J(L8,L9)=I(L8,L9)
    DC 500 I11=1,N
    DC 500 I12=1,N
111 IF (I(I11,I12).EQ.5) GC TO 510
    IF (I(I11,I12).EQ.6) GO TO 520
    IF (I(I11,I12).EQ.7) GC TO 530
    IF (I(I11,I12).EQ.15) GC TO 690
    IF (I(I11,I12).EQ.16) GO TO 700
    IF (I(I11,I12).EQ.17) GC TO 710
    GC TO 490
510 J(I11,I12)=I21
    GC TO 490
520 J(I11,I12)=I22
    GO TO 490
530 J(I11,I12)=I23
    GC TO 490
690 J(I11,I12)=I24
    GC TO 490
700 J(I11,I12)=I25
    GC TO 490
710 J(I11,I12)=I26
490 IF (J(I11,I12).LE.4) GO TO 500
    I(I11,I12)=J(I11,I12)
    GC TO 111
500 CONTINUE
    DC 125 M3=1,N
    DC 125 M4=1,N
    J1(1,M3,M4)=J(M3,M4)
125 CONTINUE
    DC 82 K1=1,N
    DC 82 K2=1,N
    DC 82 K3=1,N
    IF (J(J(K1,K2),K3).NE.J(K1,J(K2,K3))) GO TO 94
82 CONTINUE
    IF (H.EQ.1) GO TO 123
    DC 124 M1=1,N
    DC 124 M2=1,N
    IF (J1(2,M1,M2).NE.J1(1,M1,M2)) GO TO 123
124 CONTINUE
    GO TO 94
123 WRITE (6,91) H,((J(L,K),K=1,4),L=1,4)
    WRITE (7,91) H,((J(L,K),K=1,4),L=1,4)
    WRITE (6,92) M
91 FORMAT (I15,16I2)
92 FORMAT (I60)
94 H=H+1
    DC 126 M5=1,N
    DC 126 M6=1,N
    J1(2,M5,M6)=J1(1,M5,M6)
126 CONTINUE
120 CONTINUE
10 CONTINUE
    STOP
    END

```



```

LF(2,1)=L(LR(2),LR(1))
LP(2,2)=L(LR(2),LR(2))
LP(2,3)=L(LR(2),LR(3))
LP(2,4)=L(LR(2),LR(4))
LP(3,1)=L(LR(3),LR(1))
LP(3,2)=L(LR(3),LR(2))
LP(3,3)=L(LR(3),LR(3))
LP(3,4)=L(LR(3),LR(4))
LP(4,1)=L(LR(4),LR(1))
LP(4,2)=L(LR(4),LR(2))
LP(4,3)=L(LR(4),LR(3))
LP(4,4)=L(LR(4),LR(4))
79 L20=1
59 DC 60 I31=1,N
DC 60 J11=1,N
IF (LP(I31,J11).NE.M(L20,I31,J11)) GO TO 61
60 CONTINUE
GC TO 65
61 L20=L20+1
IF (L20.EQ.M8) GC TO 90
GC TO 59
65 WRITE (6,66) L20
66 FORMAT (10X'SEMIGROUP IS ISOMORPHIC TO SEMIGROUP
M8=M8-1
GC TO 100
90 CONTINUE
5 CONTINUE
DC 500 I500=1,N
DC 500 J500=1,N
K500=N+1-I500
L500=N+1-J500
N501=1
N502=2
N503=3
N504=4
IF (I500.EQ.J500) GC TO 501
DC 502 I502=1,N
DC 502 J502=1,N
IF ((I502.EQ.K500).OR.(I502.EQ.L500).OR.(J502.EQ.K500
1).OR.(J502.EQ.L500)) GC TO 502
IF ((I(I502,J502).EQ.K500).OR.(I(I502,J502).EQ.L500))
1 GC TO 500
502 CONTINUE
M500=K500+L500
IF (M500.EQ.3) GC TO 510
IF (M500.EQ.4) GC TO 511
IF (M500.EQ.5) GC TO 512
IF (M500.EQ.6) GC TO 513
IF (M500.EQ.7) GC TO 514
510 WRITE (6,503) N503,N504
GC TO 500
511 WRITE (6,503) N502,N504
GC TO 500
512 IF ((K500.EQ.1).OR.(L500.EQ.1)) GC TO 515
WRITE (6,503) N501,N504
GC TO 500
515 WRITE (6,503) N502,N503
GC TO 500
513 WRITE (6,503) N501,N503
GC TO 500
514 WRITE (6,503) N501,N502
503 FORMAT (10X'SEMIGROUP HAS A SUBSEMIGROUP OF
1ORDER TWO ',2I2)
GC TO 500
501 DC 504 I504=1,N
DC 504 J504=1,N
IF ((I504.EQ.K500).OR.(J504.EQ.K500)) GC TO 504
IF (I(I504,J504).EQ.K500) GC TO 500
504 CONTINUE
IF (K500.EQ.1) GC TO 506
IF (K500.EQ.2) GC TO 507
IF (K500.EQ.3) GC TO 508

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IF (K500.EQ.4) GC TO 509
506 WRITE (6,505) N502,N503,N504
GC TO 500
507 WRITE (6,505) N501,N503,N504
GC TO 500
508 WRITE (6,505) N501,N502,N504
GC TO 500
509 WRITE (6,505) N501,N502,N503
505 FORMAT (10X'SEMIGROUP HAS A SUBSEMIGROUP OF
1ORDER THREE ',3I2)
500 CONTINUE
DC 22 K1=1,N
DC 22 K2=1,N
IF (I(K1,K2).NE.I(K2,K1)) GC TO 26
22 CONTINUE
WRITE (7,23)
WRITE (6,23)
23 FORMAT (10X'SEMIGROUP IS COMMUTATIVE')
26 DC 27 K4=1,N
IF (I(L4,K4).NE.K4) GC TO 29
27 CONTINUE
WRITE (7,28) L4
WRITE (6,28) L4
L5=L4
28 FORMAT (10X'SEMIGROUP HAS LEFT IDENTITY X =',I2,/
1 THAT XY=Y')
29 L4=L4+1
IF (L4.GT.N) GC TO 30
GC TO 26
30 DC 31 M6=1,N
IF (I(M6,N4).NE.M6) GC TO 33
31 CONTINUE
WRITE (7,32) N4
WRITE (6,32) N4
N5=N4
32 FORMAT (10X'SEMIGROUP HAS RIGHT IDENTITY Z =',I2,
1 THAT YZ=Y')
33 N4=N4+1
IF (N4.GT.N) GO TO 34
GC TO 30
34 IF ((N5.NE.L5).OR.((N5.EQ.0).AND.(L5.EQ.0))) GO TO
WRITE (7,35)
WRITE (6,35)
35 FORMAT (10X'SEMIGROUP HAS IDENTITY')
36 DC 37 J5=1,N
IF (I(J6,J5).EQ.N5) GC TO 38
37 CONTINUE
GC TO 24
38 J6=J6+1
IF (J6.GT.N) GO TO 39
GC TO 36
39 WRITE (6,40)
WRITE (7,40)
40 FORMAT (10X'SEMIGROUP IS A GROUP')
24 CONTINUE
100 CONTINUE
105 CONTINUE
STOP
END

```

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE An Algorithm for Computing Non-Isomorphic Semigroups of Finite Order			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Master's Thesis; June 1969			
5. AUTHOR(S) (First name, middle initial, last name) James Stephen Cullen			
6. REPORT DATE June 1969		7a. TOTAL NO. OF PAGES 79	7b. NO. OF REFS 2
8a. CONTRACT OR GRANT NO.		9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.			
c.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.			
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited. Distribution of this document is unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940	
13. ABSTRACT In this paper an algorithm for computing semigroups of finite order is discussed. A computation procedure is developed to generate, for any specified finite order, all semigroups which are distinct up to isomorphism. Additional restrictions are also placed in the generating procedure to produce all groups of the given finite order. The algorithm was placed on the computer and the numerical results for orders one through four obtained.			

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Algorithm for computing semigroups of finite order						



thesC9256

An algorithm for computing non-isomorphi



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