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THE AEROPLANE.

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THE AEROPLANE.

A CONCISE SCIENTIFIC STUDY.

BY

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PREFACE TO THIRD EDITION.

THAT a third edition of this book is called for within eight months of the publication of the second is gratifying to both Author and Publisher alike. Even in this brief time a few alterations to the text are deemed necessary, together with several new illustrations; whilst to increase the value of the book three Appendices are given, viz. :—

(1) A graphical method of calculating the aerodynamical performance of an aeroplane.

(2) A table of the results of some experiments on the skin friction on various surfaces.

(3) A collection of data which are often needed in aeronautical calculations.

Grateful thanks are expressed for the appreciative reviews which appeared in the technical press, and for the kind welcome accorded both in this country and overseas, which indicate that the book is proving of service.

A. F.

TEDDINGTON, *December* 1916.

PREFACE.

This book has been written to meet the requirements of engineers who are desirous of an introduction to the study of aeronautics. The fundamental principles of mechanics are unalterable, although the many interpretations and practical applications of such laws are the fruit of scientific labour. The new science of aeronautics, which has necessitated a fuller understanding of the dynamics of the air, must now be regarded as a branch of engineering, although each step forward into the realm of aeronautical research seems but to reveal an ever-increasing unexplored region. The many unsound theories often advanced by well-intentioned people, who have had little opportunity to traverse the paths of aeronautical research, rather tend to confuse a new reader. The reports of the several aeronautical laboratories have been drawn upon in the preparation of this book, and, as far as possible, no controversial matter has been discussed. Moreover, sketches and descriptions of aeroplane construction, which are of minor importance compared with a full understanding of the underlying principles of aeronautics, have only been considered briefly. The author is greatly indebted to Mr L. Bairstow of the National Physical Laboratory for his helpful criticism and encouragement, and to Mr A. Landells, who kindly assisted in the laborious task of reading the proofs.

A. F.

Teddington.

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CHAPTER I.

WINDS.

Atmospheric Winds.—A regular wind is one which has a constant velocity (speed and direction of course), and such an ideal wind is a convenient standard for the comparison of atmospheric winds. The artificial wind of a research wind channel may be regarded as the nearest approximation to a regular wind. The fluctuations of the speed of the wind in the wind channel at the National Physical Laboratory are easily maintained within one-half per cent. of the mean wind speed.

The range of speed variation of a "steady" atmospheric wind is roughly proportional to the mean speed of the wind, and positive and negative speed fluctuations of 25 per cent. of the mean speed, with occasional speed excursions of 35 per cent., are to be expected, so that a steady wind of a mean speed of 20 miles per hour will probably have a speed range of 13 miles per hour to 27 miles per hour. A steady wind also has fluctuations of direction, although no definite connection appears to exist between the speed and the direction fluctuations. Low surface winds are usually of a "gusty" nature, chiefly due to the eddy motion created by obstacles.

A horizontal eddy—that is, a revolving roller of air with a horizontal axis—may be encountered at the horizontal edge of a high vertical cliff, if the general wind impinges directly upon the face of the cliff, whilst a well-defined vertical edge of such a cliff will favour the formation of a vertical eddy. If the fluctuations of the wind velocity are much greater than the range of the normal gust excursions of a steady wind, the wind is said to be "squally."

The most dangerous winds encountered in England are termed Line Squalls, and they are usually accompanied by a sudden fall of temperature, a rise of pressure, a veer of the general wind direction, and a squall of hail, rain, or snow. Line squalls, so called because they advance with the characteristic line front of a tidal wave, traverse the surface of the country at a great speed. These squalls, which would have a disastrous action upon air-craft, are probably formed by the spreading of the upper cold air layers before a rising warm air current.

On account of surface irregularities, ground winds are of an entirely different character from the winds of the upper atmospheric regions. The speed and steadiness of a surface wind increase with the height above the earth's surface, until the wind velocity reaches a more or less limiting value. There is evidence to show that the gustiness of surface winds is due to the effect of the ground, and it is convenient to assume that gustiness disappears, and the limiting velocity is reached, in the air strata which are removed from ground influences. The upper layers probably have a regular periodic motion.

A surface wind blowing in from the sea has its velocity continuously diminished as it passes inland, on account of the retarding influences of surface friction and obstacles.

The experience of balloonists and pilots has proved the existence of vertical atmospheric currents. The formation of clouds, as is well known, depends upon the rising of warm moist air currents into the colder upper regions, although the absence of clouds does not necessarily imply the absence of ascending air currents. Occasionally, the velocity of these ascending currents is sufficiently great to support large hailstones. A cumulus cloud is generally the cap of an ascending air current, and a cumulus sky may be regarded as a danger-signal indicating the prevalence of ascending and descending air currents. Rising currents, chiefly formed by the local heating of the earth's surface by the sun's rays, are always accompanied by descending air currents. The low-pressure centre of a cyclonic disturbance is also partial evidence of the existence of an ascending current, and, conversely, the high-pressure core of an anticyclone indicates the presence of a descending air current.

There are no well-pronounced atmospheric signs to indicate the presence of descending air currents, although the bright fine weather which occasionally follows a rainy period is probably due to descending currents. Cyclonic and anticyclonic motions have no resemblance to those perfect columns of rotating air, the product of the imagination. Moreover, most eddies are of short duration, as the low pressure of the core can only be maintained in most exceptional circumstances. Horizontal eddies, which are certainly dependent upon peculiarities of the earth's surface, are of less frequent occurrence than vertical eddies.

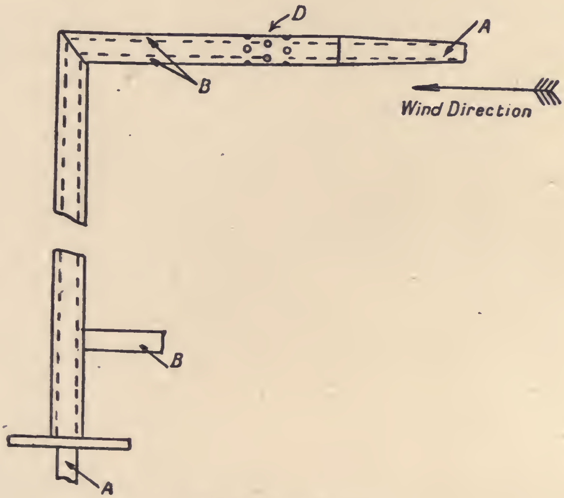


FIG. 1.—Pitot tube.

Wind Velocity.—*Pitot and Static Pressure Tube.*—The velocity of the wind may be determined very accurately by a Pitot and Static Pressure tube. A sketch of such a tube is given in fig. 1. The inner tube or Pitot tube A faces the wind, and is surrounded by a concentric tube B. The annular space C between the two concentric tubes is connected to the outside air by a series of small holes D.

To measure the velocity of the wind, the tube A is connected to one end of a sensitive tilting water gauge and the annular space B to the other end of the gauge, and the pressure difference is balanced by a head of water. Adopting consistent units, let

v = velocity of the wind,
ρ = density of the air,
p = static or barometric pressure of the air,
h = head of water.

Now, it has been proved experimentally that the pressure reading of the annular space equals p, and the pressure reading of the Pitot tube equals $\left(p+\frac{1}{2}\rho v^2\right)$.

Hence, when the pressures have been balanced by the head of water, we have

$$\left(\frac{1}{2}\rho v^2+p\right)-p=h,$$

and
$$\therefore \quad \frac{1}{2}\rho v^2=h.$$

Pitot and Suction Tube.—Fig. 2 is a sketch of a Pitot and Suction tube commonly employed upon an aeroplane. If p_1 be the pressure at the mouth E of the suction tube F, then, adopting the same notation as formerly,

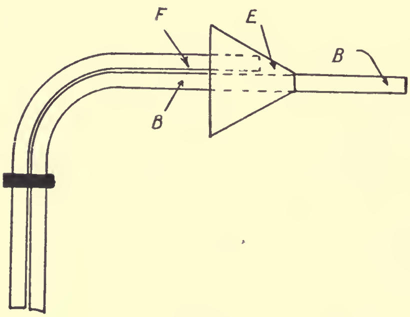


Fig. 2.

$$\left(\frac{1}{2}\rho v^2+p\right)-p_1=h.$$

It has been experimentally established that the difference of the pressure readings of the two tubes F and B is proportional to $\frac{1}{2}\rho v^2$, and hence,

$$h=\mathrm{K}\left(\frac{1}{2}\rho v^2\right),$$

where K is the constant for the instrument.

Also

$$\frac{1}{2}\rho v^2+p-p_1=\mathrm{K}\left(\frac{1}{2}\rho v^2\right),$$

and
$$\therefore \quad (p-p_1)=(\mathrm{K}-1)\left(\frac{1}{2}\rho v^2\right).$$

We see, then, that the difference between the static pressure and the pressure in the cone at the point E is proportional to v^2.

A Comparison of Forces acting upon Similar Bodies.—Let two similar bodies A and B be similarly situated in two different fluids. For the body A let L represent a linear dimension, F represent a force, and V represent the velocity of the body relatively to the fluid. Also let ρ_A=density, and ν_A=coefficient of kinematic viscosity of the fluid surrounding body A. The coefficient of kinematic viscosity is equal to the ordinary coefficient of viscosity divided by the density of the fluid.

Then for the body B let l represent a similar dimension, and f the force corresponding to the force F of the body A, also v the velocity of the body relatively to the fluid. Assume ρ_B to be the density, and ν_B the coefficient of kinematic viscosity of the second fluid.

Now, if
$$\frac{VL}{\nu_A}=\frac{vl}{\nu_B},$$
then
$$\frac{F}{f}=\frac{K\rho_A L^2V^2}{K\rho_B l^2v^2}=\frac{\rho_A L^2V^2}{\rho_B l^2v^2}$$
if we ignore the compressibility of the fluids. This statement may be deduced from a mathematical discussion of dimensional equations.

Moreover, the flow around the body A will be exactly similar to the flow around the body B, and the photographs of the flows, when taken upon plates of the same size, will be exactly the same.

Application of the Preceding Discussion to the Case of Air.—For the same fluid $\rho_A=\rho_B=\rho$, say, and $\nu_A=\nu_B=\nu$, say; and if we make vl=VL, and also ignore compressibility, then

$$\frac{F}{f}=\frac{K\rho L^2V^2}{K\rho l^2v^2}=\frac{L^2V^2}{l^2v^2}.$$

When the wind speed is above 125 miles an hour, compressibility effects, although very small, may have to be considered. The accurate determination of air forces acting upon full-sized machines may be deduced from the results of experiments upon models, if the product of the linear dimension and the velocity for the machine is the same as that of the model.

Pressure upon Square Plates.—The pressure upon a square plate may be written at $P=KV^2$, where P is in lbs. per square foot, and V is in feet per second.

The dependence of the value of K upon the value of *vl* is clearly shown in fig. 3, so that if we required the pressure upon a square plate of side 8 feet, when the wind velocity was 50 feet per second, the value of K would then be ·00153, and the pressure in this case would be $\cdot 00153 \times 2500 = 3\cdot 82$ lbs. per square foot. The values of K given by the graph would need to be modified under abnormal temperature and pressure conditions.

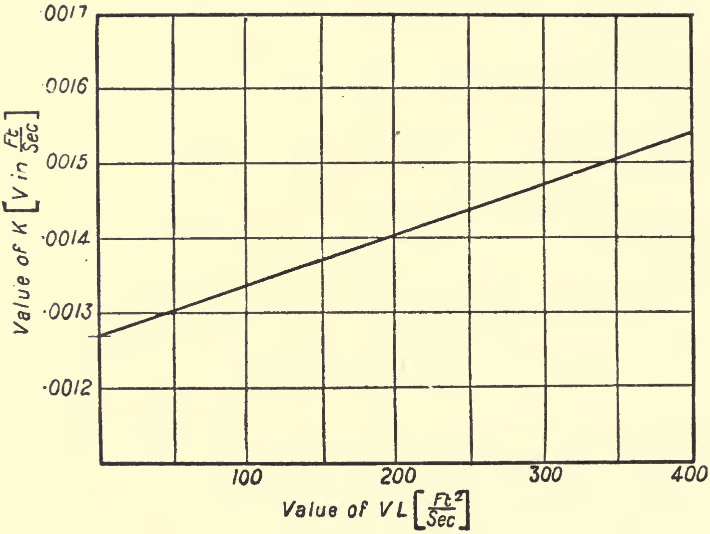


FIG. 3.

The Air Resistance of Smooth Wires.—The resistance of a wire or rope can be conveniently expressed by the equation $F = kDV^2$,

where F = force per unit length of the wire,
D = diameter of the wire,
V = velocity of the wind.

k is not a constant, but a function of (DV), *i.e.* the product of the diameter of the wire and the wind velocity, and also of the density of the air.

The resistances at several wind speeds of a large number of smooth wires have been determined in the wind channel at the National Physical Laboratory, and fig. 4 gives the values of k, *i.e.* $\frac{F}{DV^2}$ plotted against the corresponding values of DV. These experimental results have been reduced to 15·6° C. temp. and 760 mms. pressure. We see, then, that k is not a constant but a function of DV. From a practical point of view the most useful portion of the curve is that between DV=·5 to DV=2·5. Owing to instability of air flow, the values of k for the portion of the curve ranging from DV=·14 to DV=·40 were difficult to determine. However, the errors in these values of K are

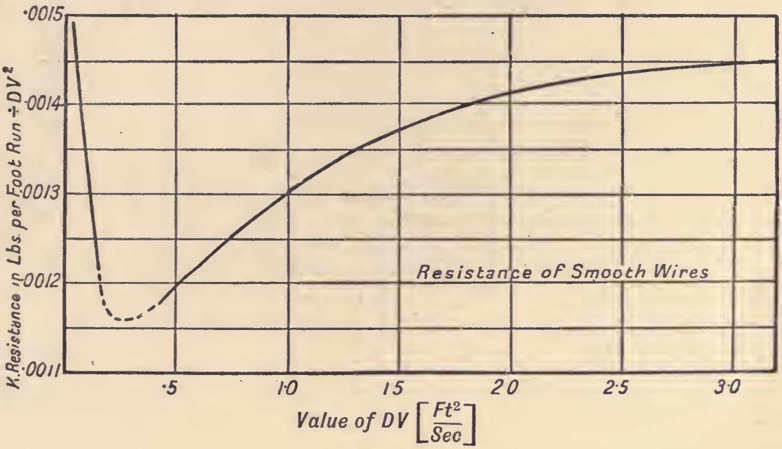


Fig. 4.

considered to be within 10 per cent. of the correct values. The resistance of a wire is not appreciably influenced by a small lateral vibration. The following example illustrates the use of the curve. Suppose that it is desirable to know the resistance per foot run of a 0·1-inch diameter tie wire if the machine has a speed of 81·5 miles per hour.

Now, $D=\frac{1}{120}$ feet,

$V=81\cdot5$ miles per hour $=120$ feet per second,

and $\therefore$ $DV=1$.

From the curve when $DV=1$ the value of k is ·001304,

and $\therefore$ $R=\cdot001304\times\frac{1}{120}\times(120)^2=\cdot1565$ lbs. per foot.

CHAPTER II.

STREAM-LINE BODIES AND STRUTS.

Stream-Line Bodies.—At the outset, it is convenient to define a stream-line body as one which has a gradual change of curvature along any section, and it may be of interest to examine briefly the nature of the flow of an inviscid fluid, *i.e.* one possessing no viscosity, around such a stream-line body.

We may assume the fluid in the vicinity of the body to be divided into a large number of imaginary tubes, graphically represented in fig. 5. Now, imagine any one of these tubes, say, A B C D E, to be isolated from its fellows. Then the reaction upon the walls of this tube due to the centrifugal component of the curvilinear path of the flow, will be as represented by the arrows. If the area of the cross section of the tube at A is equal to the area at B, and the pressures at these sections are also equal, there will be no momentum communicated to the inviscid fluid during its flow through the tube. Naturally, in view of the general static pressure of the fluid, *i.e.* the pressure existing in the stream before the introduction of the body, no tensile stress can exist between the adjacent walls of neighbouring tubes.

From fig. 5 we see that the tubes are crowded together in the vicinity of *c*, and the contraction in the sectional area of each tube is accompanied by an increase of the velocity of flow. In the immediate neighbourhood of the extremities of the body where the stream-lines widen out, the motion of the fluid is actually slower than the general velocity of the stream. Since the increase of velocity across a section of any tube is only achieved at the expense of the pressure energy of the fluid, it follows that the pressure of the fluid within the region of *c* is low. Also, as the pressure of the inviscid fluid at any point within a stream-line tube is not affected by its boundary conditions, we may anticipate, upon the basis of the foregoing remarks, that the pressure distribution over the stream-line body of fig. 5 is as is indicated by the arrows.

Energy Considerations.—By the aid of fig. 5, we will endeavour to examine the mutual transference of energy between the stream-line body and the fluid. It is immaterial whether we consider the body to move through the fluid or the fluid to flow past a stationary body.

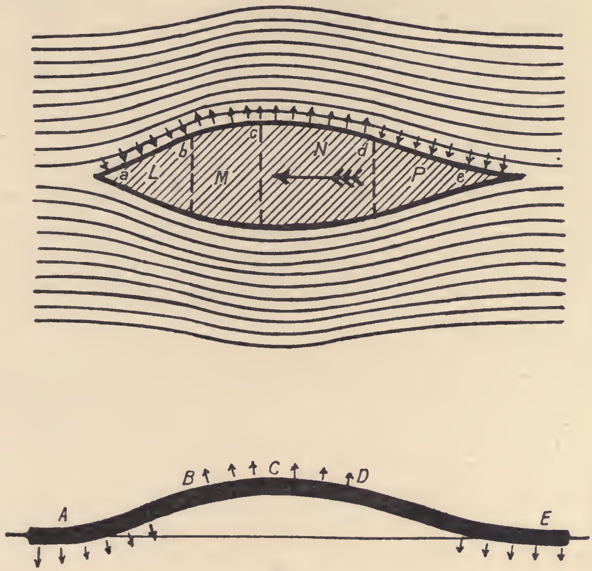


Fig. 5.

The pressure at any point of the surface of the portion L of the body is greater than the static pressure, and therefore the nose of the body does work upon the fluid in contact with it. The fluid which flows over the advancing surface of M gives energy to the body, because the pressure over this area is less than the static pressure of the fluid. The surface of N is also under suction, but the fluid is now receding relatively to the body, and so takes energy from the body. Lastly, the pressure on the surface of P is greater than the static pressure, and hence the fluid does work upon the body. Briefly, the two portions of the body L and N continuously give energy to the fluid, whilst the two remaining portions M and P continuously receive the energy back again, a balance of the energy account indicating a perfect stream-line motion.

The Nature of Stream-Line Flow.—The well-defined stream-lines of fig. 5 are an expression of the perfect harmony which exists between the molecules of the fluid, each distant molecule appearing to be perfectly cognisant of the nature of its future path around the body. A stream-line body acts sympathetically on the fluid, and the approaching stream-lines are gently coaxed to assume shapes which are more or less characteristic of the smooth contours of the body. Eddying and impact motions, which are opposed to a continuity of flow, cannot exist in a stream-line motion. It is thus apparent that sudden changes of curvature in the "body lines" should be avoided if discontinuity of flow, which in a viscid fluid will be accompanied by the formation of a "dead-water" region, is to be discouraged. Further, it should be noted that the direction and magnitude of the motion at any point in an inviscid fluid, flowing around a perfect stream-line body, is only a function of the position of the point relatively to the body, and is entirely independent of time.

Imperfect Stream-Line Bodies in a Viscid Fluid.—Most of the energy which has been taken from an imperfect stream-line body to form eddies is of a rotational nature, a form which is not favourable to any subsequent return of the energy to the body. The high resistance of bodies of imperfect form may be attributed to two disadvantageous conditions: firstly, an excess of pressure upon the surface of presentation of the body; secondly, by virtue of the viscous drag exerted by the stream, at the "surface of discontinuity," upon the mass of dead water there is a diminished pressure in the neighbourhood of the tail of the body. By the "surface of discontinuity" we mean the region separating the mass of dead water from the surrounding non-eddying fluid. Fig. 6 shows clearly a "surface of discontinuity" at the tail of a badly designed strut. The expression "surface of discontinuity" is but a convenient name: the "surface" is rather a stratum, which in all probability is a region of turbulent motion.

On the basis of the foregoing remarks, a stream-line body may be conveniently defined as one which, in its motion through a fluid, does not give rise to a surface of discontinuity.

Practical experience is, however, entirely opposed to the existence of such a body.

Generally, the motion around an ordinary body may be regarded as composed of two currents, which flow in opposite directions. The wake current of the dead-water region has a general translational motion of the same direction as the body, and superposed on the wake current is a rotational motion, the

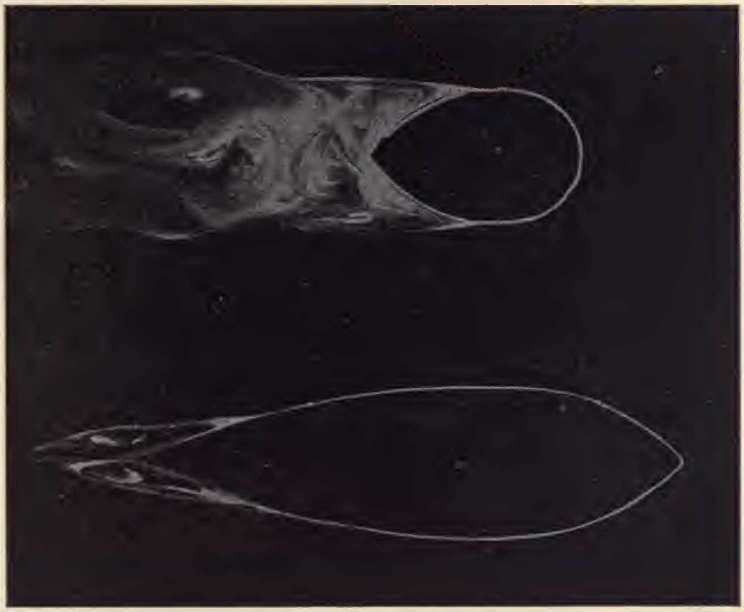


FIG. 6.

magnitude of which is determined by the frictional drag at the surface of discontinuity. It is evident that equilibrium of the fluid can only be satisfied by a flow in the opposite direction to the wake current. Hence the counterwake current which always surrounds the wake current.

Comparison of Body Forms.—The study of the flow of fluid around bodies has been greatly facilitated by the aid of photography. A contrast between a good and a bad type of flow is afforded by fig. 6. Both bodies are models of struts of which the sections B and C are shown in fig. 7. The eddying motion in the region of the tail of strut B, defined clearly in fig. 6, is accompanied by a great increase in the resistance of the strut (see Table Iʙ). The flow around the low resistance strut C, of fig. 6, is characterised by an almost complete absence of eddying motion. Blunt tails should be avoided if the elimination of eddying motion is desired, although a blunt nose may be used, without introducing great disadvantages, upon a body which has some approximation to a stream-line form. It is probable that once eddying motion has commenced, the shape of that portion of the body within the disturbed region has but a small influence upon the total resistance of the body. Care should be taken that the advantages of a good stream-line tail are

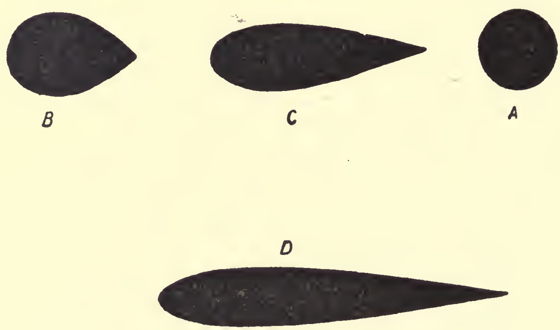


Fig. 7.—Sections of struts.

not diminished by an imperfectly shaped nose or a badly designed "middle region." The economical advantages of the employment, where possible, of bodies approximating to the stream-line form are well demonstrated in the following section.

Distributions of Pressure around a Cylinder and Stream-line Body.—A further insight into the nature of the air flow around a body may be obtained if we know the distribution of pressure around the surface of the body. By the pressure at a point of a body, we mean the difference between the actual pressure at the point when the air is flowing around the body, and the static pressure which would exist at that point if the body were removed but the general velocity of the air remained unaltered. The author has determined, experimentally, the distributions of pressure around the surfaces of a cylinder and of a stream-line body, and the results of these investigations are given below.

The pressure distribution around a cross section of a cylinder of infinite length and diameter 2 inches is graphically represented in fig. 7A. The values of the pressure coefficients at intervals of 10° around the circumference of the section are given in Table I.

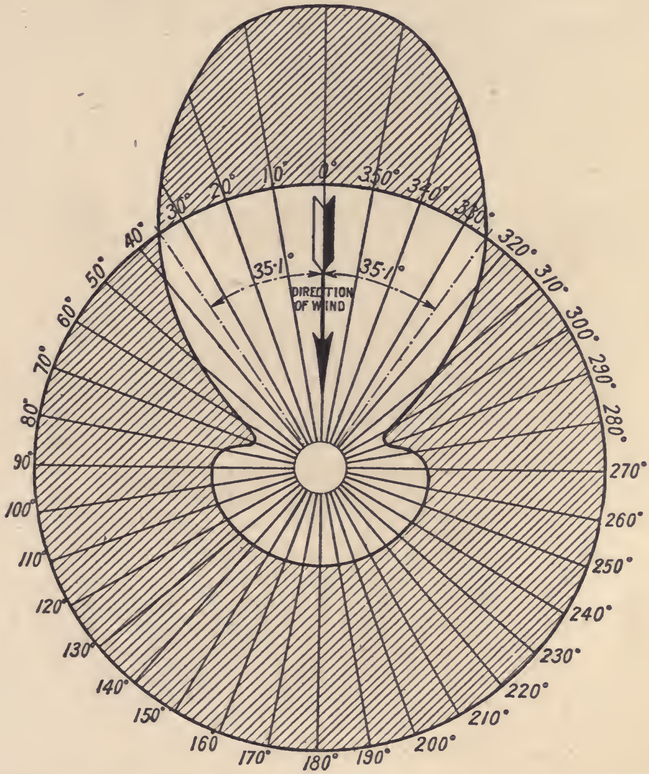


Fig. 7A.—Pressure distribution around a cylinder section 2 inches in diameter. Wind velocity 30 feet per second. Positive pressures measured radially outwards from the circumference, and negative pressures measured radially inwards.

The pressure in lbs. per square foot may be calculated from the expression $P = K_c \rho U^2$.

where U = velocity of the air in feet per second,

and $\rho = \cdot 00237$ at ground level.

From fig. 7A, it is seen that the maximum positive pressure, $\cdot 5 \rho U^2$, occurs at a point of the cylinder directly facing the wind, and also that the pressure changes from positive to negative when $\theta=35 \cdot 1°$. Only a small portion of the front of the cylinder is under a positive pressure, whilst the greater part of the front and the whole of the back of the cylinder is under negative pressure. The maximum negative pressure of $0 \cdot 60 \rho U^2$—numerically greater than the maximum positive pressure—occurs at a point which makes an angle of 70° with the direction of the wind. At the back of the cylinder (from $\theta=90°$ to $\theta=270°$) the negative pressure is fairly uniform, and has the average value of $\cdot 50 \rho U^2$.

TABLE I.

Wind speed at which experiments were made was 30 feet per second.
θ defines the position of a point on the surface. θ is measured from the direction of the wind.

Angle. Degrees. θ.	Value of K_c.	Angle. Degrees. θ.	Value of K_c.
0	+0·500	100	−0·485
10	+0·469	110	−0·485
20	+0·317	120	−0·490
30	+0·120	130	−0·496
40	−0·113	140	−0·503
50	−0·359	150	−0·512
60	−0·541	160	−0·512
70	−0·598	170	−0·512
80	−0·520	180	−0·515
90	−0·487		

When we consider the "heaping up" of air at the front of the cylinder with the resulting outward flow of air from this region, it is not surprising that a large part of the front of the cylinder is under negative pressure. The high negative pressure at the back is due to the want of sympathy between the shape of the cylinder and the surrounding current of air.

A cylinder has a high resistance, although the frictional forces are negligible.

As a contrast to the preceding, we shall now consider the pressure distribution around the surface of a stream-line body. A sketch of the body and a graphical representation of the pressure distribution around its surface are given in fig. 7B.

The data of the pressure experiments are given in Table IA.

To calculate the pressure in lbs. per square foot, when the velocity, U, is in feet per second, ρ (at ground level) must have the value of ·00237.

From the figure we see that the positive pressure gradient at the nose is very great, but only a small portion of the nose

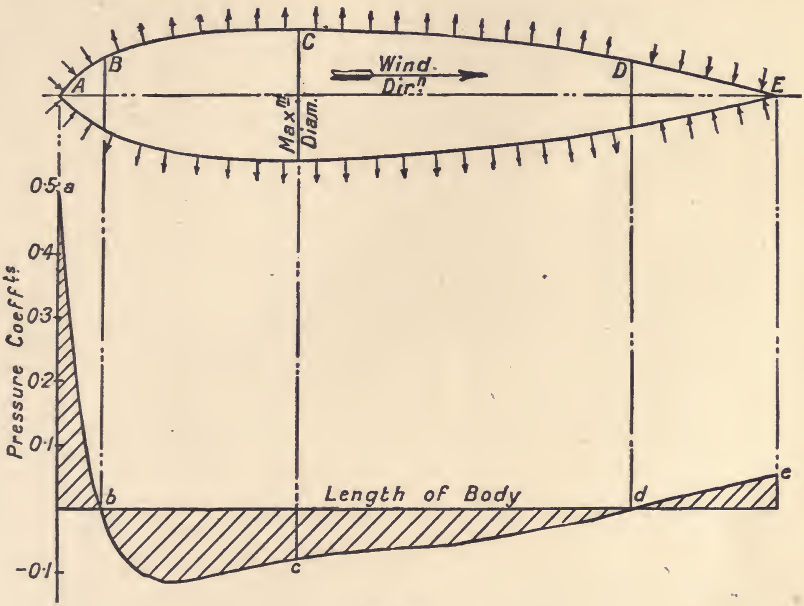


FIG. 7B.

is under a positive pressure. The maximum positive pressure ($\frac{1}{2}\rho U^2$) occurs at the tip of the nose, whilst the maximum negative pressure of $\cdot 11 \rho U^2$ occurs at a point not greatly distant from the nose. The region of positive pressure at the tail is a characteristic of a good stream-line body.

The resistance of this body in a wind of 60 feet per second, as measured on a balance, is about six times greater than the resistance computed from the integration of the pressures around the surface, so that the frictional resistance of the body —in a wind of 60 feet per second—is about 84 per cent. of the total resistance.

TABLE IA.

Length of the body=18 inches.
Maximum diameter of the body=3·17 inches.
Speed of the wind throughout the experiments was 40 feet per second.
The axis of the body was along the direction of the wind, so that the distribution of pressure around any transverse section is uniform.
The pressure at a point on the surface=$K_b \rho U^2$.

Position of the Point at which the Pressure was measured. Distance measured from the Nose.	Value of the Pressure Coefficient. Absolute. K_b.
Inches.	
0	+0·500
0·37	+0·240
0·75	+0·075
1·15	−0·064
2·25	−0·110
3·00	−0·111
4·50	−0·092
6·00	−0·076
7·50	−0·068
9·00	−0·060
10·50	−0·051
12·00	−0·032
13·50	−0·017
15·75	+0·030
18·00	+0·057

Struts.—In a comparison of the relative merits of struts of different sectional shapes, it is reasonable to consider that equal lengths of different struts should be just capable of supporting the same end load. Adopting this standard of comparison, the relative linear scales of different strut sections may be found. The maximum thickness of each strut, fig. 7, has been calculated from a constant value, [·05 (inch)^4], of the least moment of inertia of the section. It is of great importance to realise that any saving in the resistance greatly increases the lifting capacity of a machine. Thus, assume a "theoretical machine," without struts, to have a gliding angle of 1 in 6, *i.e.* the ratio of the lift to the drag in steady flight is 6. Introducing struts of total weight w and resistance r will increase the resistance of the machine by $\frac{1}{6}[w+6r]$ (see Chapter V.), assuming the gliding angle to remain unaltered. The total increase of resistance is least when $(w+6r)$ is a minimum, and hence the quantity $(w+6r)$ may be regarded as a criterion of the total worth of a strut. In such a machine, a saving of 15 lbs. in the resistance of the struts increases the carrying capacity by 90 lbs.

Table Iв has been constructed from data taken from the *Report of the Advisory Committee for Aeronautics*, 1911–12. The sections of the struts are shown in fig. 7. The weight of the wood is taken as 43 lbs. per cubic foot. It may be noted that the total length of the struts employed in a biplane approximates to 100 feet. The table, when studied in conjunction with fig. 6, should enable the student to understand more fully the dependence of the usefulness of a strut upon the shape of its section.

TABLE Iв.

Type of Strut.	Maximum Thickness for Equal Strength.	Resistance of 100 Feet of Strut at 60 Miles per Hour.	Weight of 100 Feet of Strut.	Equivalent Weight of Struts of Equal Strength in a Wind of 60 Miles per Hour $(w+6r)$.
	Inches.	lbs.	lbs.	
A.	1·005	93·0	23·4	581·0
B.	·990	54·0	29·0	353·0
C.	·718	10·6	45·5	109·0
D.	·677	9·2	58·5	113·0

Manufacturers should endeavour to make struts of symmetrical cross section, because small deviations from symmetry are usually accompanied by wide divergences from symmetry in the crosswind and drag forces (see fig. 8). The direction of the crosswind force acting upon a strut occasionally depends upon a critical type of flow, the nature of which is further dependent upon the "fineness ratio" of the strut, *i.e.* upon the ratio of the length of the cross section to the maximum thickness. The crosswind force occasionally acts in the opposite direction to that anticipated from the angle between the wind direction and the axis of the section of the strut. Again referring to fig. 8, the value of θ at which a critical breakdown of air flow occurs, increases with the fineness ratio. When the wind blows directly upon the nose of the strut, the smaller the fineness ratio the larger the resistance of the strut, if due regard be taken of any alteration in the sectional area of the strut. Further, when a strut of low fineness ratio is inclined at small angles to the wind direction, that is, θ is small, the crosswind force usually acts in a negative direction. Positive values of θ and the positive direction of the crosswind force are shown in fig. 8. When struts which have a fineness ratio greater than 5 are acted upon by a cross wind, they may be regarded as the equivalent of positive vertical fins—that is, the crosswind forces

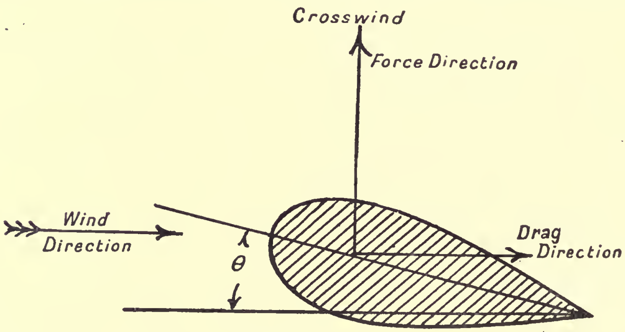


Fig. 8.

act in positive directions for positive values of θ. Struts of small fineness ratio, that is, $2\frac{1}{2}-5$, have uncertain fin action in cross winds. The fin action of such struts may vanish in strong, gusty side winds. The resistance of a strut of fineness ratio less than $2\frac{1}{2}$ is very high, and the fin effect, if present, will probably have an action opposite to that which may be anticipated from the direction of the wind. It appears, then, that a strut of fineness ratio less than $2\frac{1}{2}$ has nothing to recommend it.

The chief woods employed in the manufacture of struts are spruce and ash. Hollow steel tubes, surrounded with wood of good stream-line form, are also in common use. The struts of the future will probably be constructed of steel.

CHAPTER III.

AEROPLANE WINGS.

This branch of aeronautics may be studied most conveniently in two sections: (*a*) an aerodynamic consideration of wings, (*b*) a mechanical consideration of wings. We are greatly indebted to experimental research for its contributions to the science of aeronautics. Certainly most of our present knowledge of the aerodynamic properties of aeroplane wings has been gained from the results of laborious experiments made upon

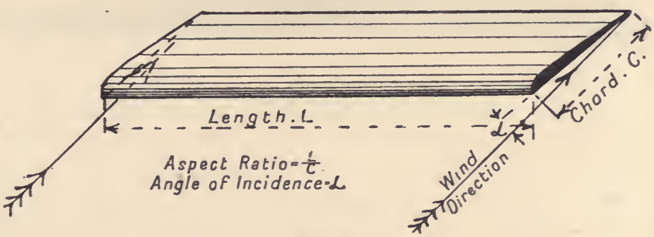


Fig. 9.

small models. Hence, as is most fitting, the greater proportion of the matter of the first section of this chapter has been compiled from the reports of the big aeronautical laboratories, and more especially from the *Annual Reports of the Aeronautical Section of the National Physical Laboratory.* The second section, which deals with wing stresses and wing construction, should be of interest to the designer and workman.

Fig. 9 will be of service in explaining some of the terms employed in this chapter.

Definition of Lift and Drag.—The total air force acting upon the wings of an aeroplane, when the machine is flying normally, may be resolved into two components: (*a*) a force acting at right angles to the general wind direction and to the plane of the wing, which is called the "Lift" component, and (*b*) a force acting along the general wind direction, which is called the "Drag" component.

Section I.—AN AERODYNAMIC CONSIDERATION OF AEROPLANE WINGS.

Nature of the Air Flow around a Wing.—We can conceive that by virtue of the larger number of "flow paths" afforded by the configuration of the wing-tips, the air flow at the end of the wing is of an entirely different nature from that which exists at the middle portion of the wing. The deviation of the behaviour of a wing which has this three-dimensional type of flow at its ends from that of a similarly constructed wing in which the two-dimensional flow at the middle is assumed to extend to the wing-tips is said to be due to "end effects." The nature of the flow around the middle portion of the wing,

Fig. 10.

i.e. the two-dimensional flow region, has been experimentally investigated, and we will now discuss, as briefly as possible, the salient features of this research. Fig. 10 is a sketch of a section of a wing of a modern aeroplane. Firstly, we will consider the distribution of pressure around the upper and lower surfaces of such a wing. By "pressure" we shall mean the difference between the pressure at the point on the wing and the pressure of the undisturbed air of the same locality—that is, the pressure that would be indicated by a barometer.

When the wing has an ordinary or "normal flight" angle of incidence there is a positive pressure upon its lower surface, and a negative pressure upon its upper surface, and both these pressures have a maximum value in the vicinity of the leading edge. Also, the numerical values of both these pressures considerably diminish as we travel towards the trailing edge, and some idea of the nature of this pressure gradient will be realised from the following illustration. Thus, when an aeroplane travels at 60 miles per hour, the pressure normal to the chord at the leading edge of the wing is of the order of 35 lbs. per square foot, whilst the pressure at the trailing edge may be only 2 lbs. per square foot. Under these circumstances we should expect the average normal pressure upon the wing to be about 10 lbs. per square foot. With such a distribution of pressure the point of intersection of the line of the resultant force with the chord of the section, conveniently called the centre of pressure of the section, is well forward of the middle of the chord.

Principle of the Dipping Front Edge.—We will now discuss a phenomenon commonly known as the "Principle of the Dipping Front Edge." Referring to fig. 10, we see that the region in the neighbourhood of A faces the wind, whilst the area around B is sheltered from the wind, and we, in ignorance, would naturally expect a positive pressure at A and a negative pressure at B. We have just read, however, that such is not the case: the pressure at A is most decidedly negative, whilst that at B is positive. Photographs which illustrate the flow of air around model wings show that the wind is deflected upwards as it approaches the leading edge of the wing, so that although the surface of A faces the general wind direction, this surface is screened from the general wind. It is this upward deflection of the air which is responsible for the formation of a general low-pressure region above and a high-pressure region below the wing. The efficient results of a well-designed wing depend chiefly upon the "suction" over the top surface, and, obviously, the greater the suction the greater the lifting force in a direction normal to the chord. Usually the angle of incidence is small, so that the force normal to the chord approximates closely to the lift on the wing. Furthermore, since the negative pressure on the upper surface, in the region of the leading edge, is much greater than that at the trailing edge, the resultant force along the chord due to the pressures on the upper surface of the wing acts in an upwind direction. The lower surface does not materially affect the numerical value of the upwind force. It would appear, then, that the direction of the resultant force upon an efficient wing is between the normal to the chord and the normal to the wind direction, and the smallness of the angle between the direction of the resultant force and a normal to the wind direction may be regarded as a criterion of the aerodynamic efficiency of the wing. Roughly speaking, about 60 per cent. of the wing area may be considered to be subjected to the two-dimensional flow which has been described above.

Pressure Distribution around the Wing-Tips.—The nature of the air flow in the neighbourhood of the wing-tip, usually about 40 per cent. of the wing area, has also been investigated at the National Physical Laboratory. The distributions of pressure upon the upper and lower surfaces of sections, which were situated at several distances along the wing, were found. The

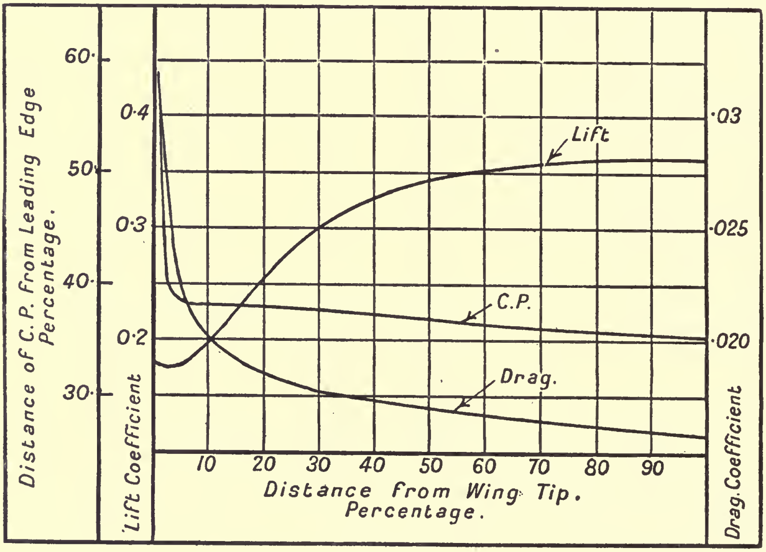


FIG. 11.

distribution of negative pressure upon the upper surface of a section alters progressively as we approach the tip of the wing, until at the tip section we have the greatest suction in the neighbourhood of the trailing edge. The undesirable increase of drag which is introduced by such a distribution of pressure should be quite clear, as the force component parallel to the chord, which has an upwind direction for a middle section, has a downwind direction at the tip section. The portion of the wing in the immediate neighbourhood of the tip is almost completely under suction, and therefore the flow over the lower surface reduces by a small amount the value of the lift developed by the flow over the upper surface. It is of interest to note that any movement of the position of the maximum suction upon the upper surface towards the trailing edge will also be accompanied by a similar backward movement of the centre of pressure of the wing section. Table II., compiled from data taken from a paper dealing with this subject, may illustrate more clearly the relative contributions of the various sections of the wing towards the lift, drag, and centre of pressure of the complete wing. The values of Table II. have been plotted in fig. 11.

In connection with Table II., it should be stated that the lift and drag forces on the wing are $K\rho AU^2$ and $k\rho AU^2$ where K and k are absolute coefficients. The value of an absolute coefficient is independent of the system of units employed, so long as the units themselves are dynamically consistent. Thus, if we use the foot-lb.-second system, and the velocity is in feet per second, the area should be in square feet, the density in lbs. per cubic feet, and the force is then in poundals. The density of the air at the ground level is 0·0763 lb. per cubic foot. If we put ρ equal to $\frac{0{\cdot}0763}{g}$, that is, 0·00237, then the force will be in lbs., when the area is in square feet, and the velocity in feet per second. Care should be taken to use the value of ρ corresponding with the height of the aeroplane.

TABLE II.

Lift on the wing $= K\rho AU^2$,
Drag „ „ $= k\rho AU^2$,
where K and k are constants, when the angle of incidence of the wing has a constant value.

Angle of incidence of wing $=4°$.

Distance of the Section from the Wing-Tip, expressed as a Percentage of the Total Length of the Wing.	Lift Coefficient. K.	Drag Coefficient. k.	$\frac{K}{k}$.	Position of the Centre of Pressure from the Leading Edge. Percentage of Chord.
Tip section				
1·1	0·179	0·0348	5·1	55·5
5·6	0·175	0·0216	8·1	38·5
16·8	0·242	0·0189	12·6	38·5
33·4	0·311	0·0177	17·6	36·9
100·0	0·361	0·0157	23·0	35·0
Body end of wing				

Direct force measurements made upon the complete wing gave the following results :—

Lift coefficient for the complete wing $=0 \cdot 314$,
Drag ,, ,, ,, $=0 \cdot 0210$,

and $\therefore \frac{\text{Lift}}{\text{Drag}}=15.$

Distance of the centre of pressure behind the leading edge was 35·9 per cent. of the chord length.

The Lift, Drag, and Centre of Pressure Curves of a Modern Wing.—The data of Table IIa. are the results of some experiments made at the National Physical Laboratory on a model of a wing of an aeroplane. A sketch of the section of this particular wing is given in fig. 10. The model had square wing-tips and a section of constant shape throughout its length.

TABLE IIa.

Aspect ratio of the aerofoil=6.
Length of the chord=3 inches.
The speed of the wind throughout the experiments was 40 feet per second.

Angle of Incidence of Wing. Degrees.	Lift Coefficient. Absolute. K.	Drag Coefficient. Absolute. k.	$\frac{K}{k}$.	Position of the Centre of Pressure from the Leading Edge of Wing. Percentage of Chord.
−6	−0·148	0·0367	−4·1	..
−4	−0·075	0·0256	−3·0	..
−2	+0·001	0·0181	+0·1	..
0	+0·086	0·0151	5·7	0·575
2	0·197	0·0145	13·6	0·425
4	0·286	0·0184	15·4	0·358
6	0·355	0·0244	14·5	0·329
8	0·425	0·0325	13·1	0·312
10	0·494	0·0423	11·7	0·302
12	0·552	0·0509	10·8	0·292
14	0·587	0·0617	9·5	0·280
16	0·587	0·0765	7·7	0·296

By the " centre of pressure " we mean the point of intersection of the line of the resultant force with the chord of the section.

To find the corresponding values of K and k for a full-scale biplane wing of similar section, the same aspect ratio, and square wing-tips, the results of the experiments on the model need to be corrected for differences of scale and of wind speed, and also the interference between the two wings of a biplane. The data of Table IIB have been deduced from the preceding table, by applying the appropriate corrections to the values of K and k.

TABLE IIB.

Aspect ratio of the biplane wing=6.

Angle of Incidence of Wing. Degrees.	Lift Coefficient. Absolute. K.	Drag Coefficient. Absolute. k.	$\frac{K}{k}$ or $\frac{L}{D}$
−4	−0·040	0·0182	−2·2
−2	+0·025	0·0167	1·5
0	0·100	0·0132	7·6
2	0·180	0·0131	13·7
4	0·260	0·0168	15·5
6	0·325	0·0224	14·5
8	0·390	0·0300	13·0
10	0·445	0·378	11·8
12	0·500	0·0464	10·8
14	0·540	0·0568	9·5
16	0·540	0·0700	7·7
18	0·510	0·1130	4·5

The lift, in lbs., on the wing $=K\rho AU^2$,
The drag ,, ,, $=k\rho AU^2$,

where K and k have the values given in the table.

A = total area, in square feet, of the wings of the biplane.
U = speed of the machine in feet per second,

and ρ has the value of 0·00237 at ground level.

The aerodynamic performance of the above biplane could be improved by (a) staggering the top wing forward of the lower wing, (b) increasing the aspect ratio, (c) fitting rounded wing-tips.

At an angle of 15° (see point C of fig. 12) we observe a big falling-off in the value of the lift coefficient and a marked increase in the value of the drag coefficient. Both these phenomena are the result of a breakdown in the character of the air flow, which results in a sudden alteration of the pressure distribution over the upper surface. The angle at which the breakdown in flow takes place is appropriately called the "critical angle." At angles of incidence greater than the critical angle, the negative pressure upon the upper surface

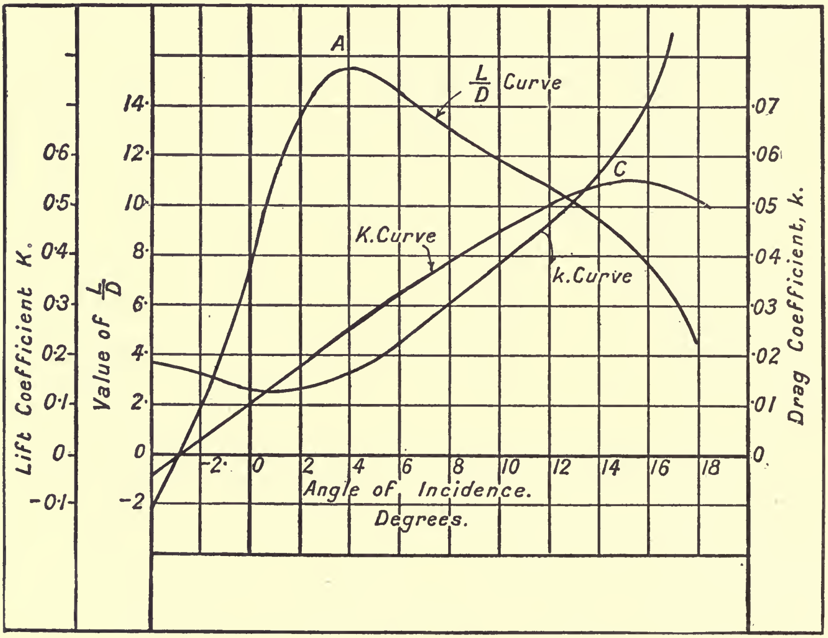


Fig. 12.

tends to become uniform, so that now the negative pressure upon the surface contributes but a small upwind component in the direction of the chord. The increase of drag may be thus partially explained.

The value of the critical angle and the change of force occurring at it are a function of the position of the maximum ordinate. The value of $\frac{L}{D}$ for the wings is a maximum when the angle of incidence is equal to 4°. Neglecting any lift on the other parts of the machine, the best angle of incidence is that which makes the ratio $\frac{\text{Lift of wings}}{\text{Drag of machine}}$ a maximum, so that the curve of body resistance must be estimated before the best angle of incidence of normal flight can be assigned.

Fig. 13 has been plotted from data taken from Table IIA.

Dependence of the Performance of a Machine on the Characteristics of the Wing.—A convenient method of representing the performance of a wing is given in fig. 13A, which shows directly the relationship between the ratio $\frac{\text{Lift}}{\text{Drag}}$ and the lift

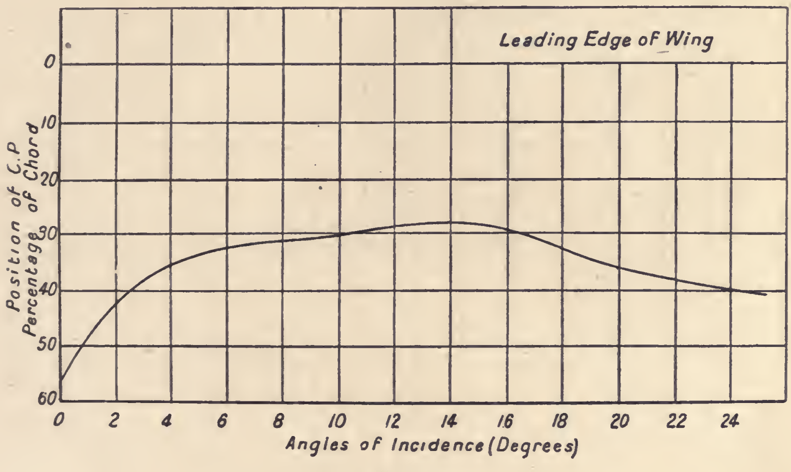


Fig. 13.

coefficient of the wing. During alighting the whole of the weight of the machine must be taken on the wings, whilst at the same time the shocks of landing must be kept within safe limits by reducing as much as possible the speed of the machine. If U_L be the alighting speed, it is seen from the expression $W = \rho KAU^2$ that the best alighting speed is given at the angle of incidence which makes the value of K a maximum. K has a maximum value at the critical angle of incidence of the wing, and for this particular wing the value of $K_{max.}$ is 0·55. Hence $W = \cdot 55 \rho A U_L^2$.

The speed of horizontal flying, U, at any other angle of incidence is given by $U_L \sqrt{\left(\frac{\cdot 55}{K}\right)}$. In fig. 13B the values of $\frac{L}{D}$ are plotted against the corresponding values of $\frac{U_L}{U}$. The maximum landing speed consistent with safety may be taken at 45 miles per hour, so that when the wings are working at the maximum aerodynamic efficiency the speed of horizontal flight will be $\frac{45}{\cdot 68} = 66$ miles per hour. If we desire our machine to be a fast scout of maximum speed of horizontal flight of

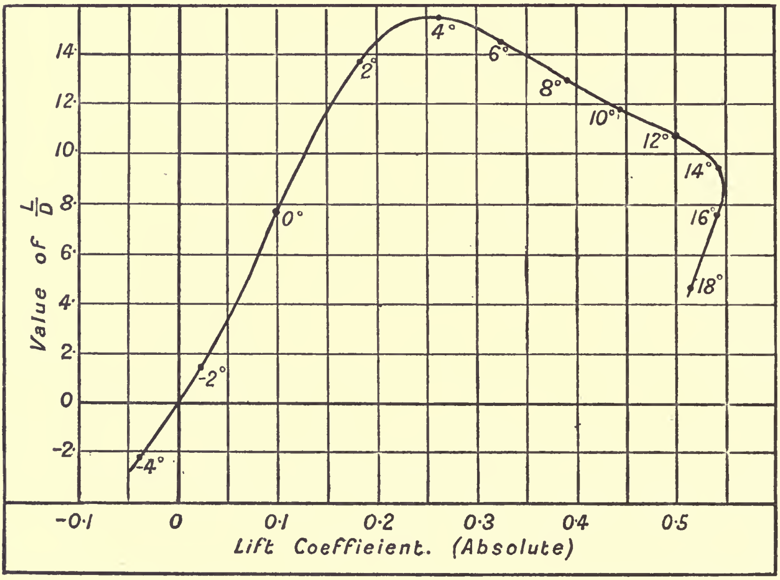


FIG. 13A.

100 miles per hour, say, the $\frac{L}{D}$ for the wings will be only 8·5. The $\frac{L}{D}$ of the machine can only be computed when the drag of the body has been added to the drag of the wings. It appears, then, that a machine can only fly horizontally at a moderately high speed, if the angle of incidence of the wings be appreciably smaller than the angle of incidence which makes $\frac{L}{D}$ for the wing a maximum.

From the foregoing it follows that if a machine is to have a large range of flying speed, the wings must possess the following characteristics :—

(*a*) A high value of the lift coefficient at the critical angle.

(*b*) At a small angle of incidence of the wings, the lift coefficient may be small but the corresponding value of $\frac{L}{D}$ must be large.

(*c*) The wing should have a large value of the maximum

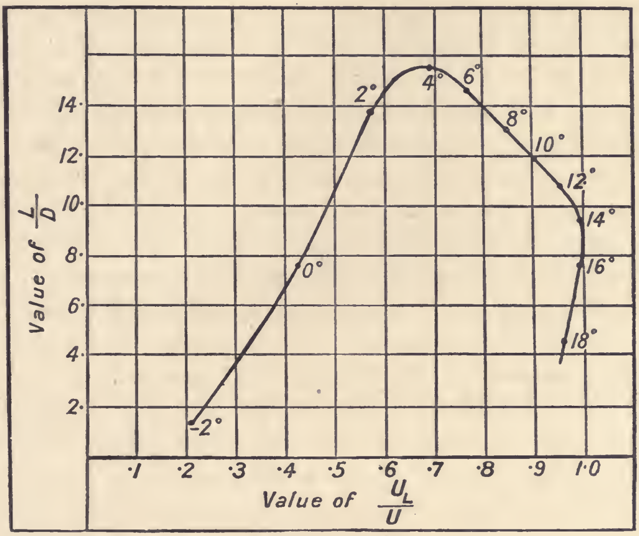


FIG. 13B.

$\frac{L}{D}$, and the ratio of the maximum lift coefficient to the lift coefficient at maximum $\frac{L}{D}$ must be large.

Usually, a wing which has a high maximum value of $\frac{L}{D}$ has also a low lift coefficient at the angle of incidence, which makes $\frac{L}{D}$ a maximum.

Relative Importance and Interdependence of the Two Surfaces of a Wing.—The negative pressures upon the top surface of a wing are much greater, numerically, than the positive pressures upon the lower surface, and hence the greater part of the resultant force normal to the chord is contributed by the pressures upon the upper surface. The force in the direction of the chord depends not only upon the pressures upon a surface, but also upon the projected area of the surface upon a plane which is normal to a chord of the wing. The importance of the upper surface, which has the greater projected area, by virtue of its greater camber, is still further increased.

Generally, we may say that the upper surface of an efficient wing supplies, at least, three-quarters of the total lift. Moreover, the pressure distribution over the upper surface is practically unaffected by changes made in the camber of the lower surface—that is, if the lower surface is not given a convex curvature. The converse is not true, however: the pressure distribution over the lower surface is affected by alterations in the camber of the upper surface, but the forces contributed by the lower surface are small, and hence a considerable percentage change in these forces does not greatly affect the resultant wing forces. Further, the structural advantages of a wing with a flat or slightly concave camber would probably outweigh the small aerodynamic advantages which would be derived by cambering still further the lower surface.

Camber of the Wing.—The camber of a section is measured by the ratio of the length of the maximum ordinate, which can be drawn from a point of the section upon the chord, to the length of the chord. Generally speaking, as the camber of the upper surface decreases, the angle of incidence of the chord of the wing, at no lift and at maximum lift, becomes greater, and also the rate of decrease of the lift coefficient with the angle of incidence, in the region beyond the critical angle, is much less marked. The average camber of the upper surface of the wing of a modern aeroplane is of the order of ·08.

The Position of the Maximum Ordinate.—If the maximum ordinate is placed too close to the leading edge of the wing, low values of the critical angle and the lift coefficient at the critical angle are encouraged. We have already seen that it is advantageous to have a high value of the critical angle, and also a high value of the lift coefficient, at this angle. The maximum ordinate may be shifted forward from the middle of the chord to within a distance of 0·2 of the chord from the nose without greatly influencing the behaviour of the wing at small angles of incidence, say from 0° to 8°. It has been the practice of wing designers to place the maximum ordinate at a distance of one-third of the chord from the leading edge, although it is probable that the more stable type of flow which would result by the placing of the maximum ordinate at a distance of three-eighths of the chord from the leading edge, would more than compensate for any slight decrease in the maximum value of $\frac{L}{D}$. It is of theoretical interest to know that the values of the lift and drift coefficients at angles of incidence greater than the critical angle are greatly dependent upon the position of the maximum ordinate.

Aspect Ratio.—A consideration of the influence of the aspect ratio upon the values of the lift and drag coefficients of a wing (see fig. 9).

Imagine a wing of constant section to be cut by a series of parallel planes which are perpendicular to the length of the wing, and suppose the flow at each of these sections to be photographed. Then if the photographs are exactly similar, the flow around the surface of the wing is two-dimensional. Obviously, if the flow over the surface of the wing were strictly two-dimensional, both the lift and drag coefficients would be independent of the aspect ratio. It is only by preventing an inherent feature of the modern wing—that is, the lateral escape of the air at the wing-tips—that we may hope to approximate to a uniform two-dimensional type of flow. The maximum value of $\frac{L}{D}$ decreases considerably with a decrease of the aspect ratio of a wing. Recent researches at the National Physical Laboratory indicate that a fall of 35 per cent. in the maximum value of $\frac{L}{D}$ is quite probable when the aspect ratio is diminished from 8·0 to 3·0. This great fall in the maximum value of $\frac{L}{D}$ is largely due to the increase in the drag coefficient, which is accompanied by only a slight decrease in the lift coefficient. The limit to the maximum practical value of the aspect ratio must be assigned by a consideration of the weight and the strength of the wing. Obviously, the weight of a wing must be some function of its aspect ratio.

The Leading and Trailing Edges.—The popular notion that aerodynamic advantages may accrue from a thick leading edge has not been substantiated by research. It is even probable that the aerodynamic efficiency of the wing, as measured by the maximum value of $\frac{L}{D}$, may decrease as the nose of the wing becomes thickened. When the nose of the wing is decidedly thick the lift coefficient, although not greatly affected at the normal flight angles of incidence, has a tendency to remain constant at large inclinations of the wing. The thick leading edge certainly has structural advantages, and is not accompanied by any great aerodynamic disadvantages. Moreover, a slight thickening in the neighbourhood of the trailing edge, if reasonably performed, will not greatly interfere with the aerodynamic properties of the wing.

The Dihedral Angle.—The wings of many aeroplanes are not in the same plane, but are slightly inclined upward from the shoulder to the tip, or, to use the technical expression, the wings have a dihedral angle. The dihedral angle is measured by the angle between the lateral axis of the wing and the plane of XOY (see fig. 36, Chapter VI.). Usually the dihedral angle is small, and such a small angle difference does not have any appreciable effect upon the lift and drag forces of the wing. The dihedral angle between the wings has a great influence on the lateral stability of the machine.

Biplane Wings.—We now wish to consider the interaction which takes place between the two superposed wings of a biplane. The wings may be arranged in a "normal" position, so that the chords of each wing are parallel and the longitudinal axis OX of the machine is normal to the plane passing through the leading edges, or the wings may be staggered, in a positive direction, by a forward displacement in the direction of the chord, of the top wing relatively to the lower wing, the wings still remaining parallel. The lifting capacity of a wing and also to a lesser extent the drag is diminished by the presence of a neighbouring wing.

A popular spacing of biplane wings is to make the gap between the wings equal to the chord. With such a spacing, and at the usual angles of incidence, the lift coefficient of each wing is approximately 80 per cent. of the value of the lift coefficient of the single wing, and we may roughly assume that the drag coefficient of either wing is unaffected by the presence of the other. If, now, the top wing is staggered a distance of 0·4 of the chord length forward of the lower wing, an improvement of about 5 per cent. in the value of the lift coefficient may be introduced. A similar improvement is also found in the ratio of $\frac{L}{D}$.

Aerodynamic Behaviour of a Wing which has a Reflex Curvature towards the Trailing Edge.—The navigation of an aeroplane would be greatly facilitated if the position of the centre of pressure of the wings was not influenced by the angle of incidence. This desirable condition may be obtained by giving to that portion of the wing in the neighbourhood of the trailing edge a reversed or "turned-up" curvature of the correct amount.

A stationary position of the centre of pressure can, however, only be obtained at the expense of the other desirable properties of the wing. Thus the maximum value of the ratio $\frac{L}{D}$ may be reduced more than 10 per cent., whilst a diminution of about 25 per cent. in the value of the maximum possible lift may result. Also, since an increase in the curvature at the trailing edge of the wing will increase the angle of incidence necessary to produce a given lift, such a wing could not be employed upon a variable-speed machine.

Section II.—THE DESIGN AND CONSTRUCTION OF A WING.

In the previous section we were entirely concerned with a theoretical discussion of the aerodynamic properties of a wing; we will now study an aeroplane wing from a more practical standpoint.

The framework, or skeleton of a wing, is commonly built up of two main spars, *i.e.* the front and rear spars, which are connected together by equally spaced transverse ribs. The curvatures of the upper and lower surfaces of any rib decide the shape of the section of the wing in that neighbourhood. The front ends of the ribs are connected to a "nose" or leading edge spar. The fabric, sufficiently stretched to give an even surface, is fastened to the upper and lower surfaces of the ribs and spars. The shoulder end of the front or main spar is firmly connected to the body of the machine, but occasionally, to enable warping duties to be performed, the shoulder end of the rear spar has a swivelling connection to the body. Wire bracing is extensively employed to stiffen the wing, and tie wires connect several points in the lower surfaces of the spars to the lower portion of the body. The wings of a biplane are further stiffened by a system of strut bracing. We will now consider in greater detail the several parts of a wing, and the contributions of each part towards the efficient construction of the wing.

Front or Main Spar.—Designers endeavour to arrange that the whole of the drag of the wings shall be taken by the forward pull of the lift wires. Usually the tie wires are connected to points in the front spar, which are an appreciable distance, say 2 inches, below the neutral axis of the spar—a factor which should not be omitted in stress calculations.

The weight of the machine, minus the weight of the wings, is chiefly carried by the lift wires; but although the bending moment at the connection between the front spar and the body may consequently be small, the spar with the shoulder connection is under a great compressive force. The calculation of the stresses in any part of an aeroplane should be made for the particular manœuvre which imposes a maximum stress upon that part. Thus the stress calculations for the front spar should be made when the centre of pressure of the wing is nearest the leading edge, and the wing forces are also a maximum. Warping a wing increases the loading upon it, and the wing loading during a *vol piqué,* although difficult to estimate, is probably very great. Further, side gusts may introduce stresses much greater than the calculated stresses, although such contingencies may be adequately provided for by the choice of a reasonable factor of safety. The spar stresses due to bending are greater than those due to direct compression. Great attention should be given to the initial straining of the wires —or "timing," to use the technical term,—as any interference with the previous alignment of the points of attachment of the wires to the spar, consequent upon the application of the load, may introduce great stresses in the spar.

The breaking of any tie wire endangers the wing, and this especially applies to the end wire. As may be expected from the great difficulty of calculating the maximum stresses possible, a spar should be allowed a very large factor of safety. Although at the present time ash is the material chiefly employed in the making of spars, it is extremely likely that in the future, wood will be superseded by metal. The rear spar, which may take only a small percentage of the total load of the wing, is usually smaller in section than the front spar, although a good deal of the foregoing discussion on the front spar is equally true for the rear spar.

The ribs are commonly made of French poplar, spruce, Honduras mahogany, and occasionally of cotton wood, a light wood which does not readily split. The ribs may be rigidly connected to the spars, but some makers affirm that a loose fit between the ribs and spars prevents any excessive fatigue of the material of the wing structure during an excessive warp. The ties, usually made from piano-wire, should be efficiently connected at their ends, and should have a factor of safety of at least 12.

Aeroplane Fabric.—Linen, of a fine, closely-woven texture, is employed as a covering for the upper and lower surfaces of aeroplane wings. It is most essential that an aeroplane fabric should possess strength, lightness, and durability. The weight of aeroplane fabric is about 4 ounces per square yard, and its strength varies from about 80 lbs. per inch in the warp direction, to 110 lbs. per inch in the weft direction. The warp threads of a fabric are those stretched lengthwise in the loom, and crossing at right angles these long threads are the shorter weft threads. The fabric is attached to the structure of the wing so that the weft threads take the greater strain—that is, the warp threads are in the direction of the ribs. No great difficulty is experienced in the production of a strong and light linen, but a linen which is at the same time airtight and unaffected by moisture has yet to be manufactured. Airtightness and water-tightness are obtained, and also a taut surface given to the wing, by coating the linen with a chemical preparation termed "Dope." Almost all the dopes now on the market contain cellulose acetate in solution, the chief solvents being tetrachlorethane, methyl alcohol, and acetone. The first coat of dope penetrates the threads of the linen and fills the interstices between the threads, and, as the drying proceeds, the gelatinous substance remaining contracts and brings the threads more closely together. Usually four or five coats of dope are applied to the surface of the linen, the increase of weight due to the dope being about 40 per cent. of the weight of the undoped linen. It is desirable that the coating of dope should remain airtight and watertight after long periods of weathering, and should not in any way weaken the fabric. Needless to say, the doped fabric should be non-inflammable. As a rule, doping increases the strength of the fabric by about 20 per cent. The fabric now used in aeroplane construction is sufficiently strong to support any load which the wings may have to bear. If the fabric is punctured, say, by a bullet, its strength is diminished, a cut of length $\frac{3}{4}$ inch causing a reduction of strength of at least 50 per cent. of the strength of the undamaged fabric.

We have read on a previous page that the maximum positive pressure upon the lower surface, and the maximum negative pressure upon the upper surface, occur on areas very close to the leading edge of the wing.

It would follow, then, that the greatest possible pressure which the fabric may have to bear would occur at a point of maximum suction upon the upper fabric when there is a leak at a point of maximum positive pressure in the lower fabric, assuming the remainder of the fabric to be airtight. The value of the maximum pressure in this hypothetical case, when the machine has a normal flying speed of 70 miles per hour, does not greatly differ from 20 lbs. per square foot.

Moreover, assuming the existence of such a leak, the sudden flattening out of the machine after a steep dive, when the speed of the machine is probably of the order of 100 miles per hour and the angle of incidence of the wings about 12°, imposes a maximum stress of 100 lbs. per square foot upon the fabric. Ordinarily, the initial tensions of those threads of the fabric which are transverse to the line of flight have negligible values. Also any small initial tensions in the longitudinal threads of the fabric, such as are introduced during the construction of the wing, are greatly reduced by subsequent weathering. It is assumed then, that, in the absence of initial tensions in the longitudinal threads, the whole of the pressure upon the fabric is transmitted to the framework of the wing at the areas of attachment of the fabric to the ribs. The transverse spacing between the ribs is only a small fraction of the chord length (about $\frac{1}{5}$th), so that during flight the greater proportion of the strain in the bulging fabric is taken by the transverse threads.

At the expense of a small sacrifice of accuracy, the calculation of the stresses in the fabric is facilitated by the reasonable assumption that the transverse fibres support the whole of the pressure upon the fabric. It is comforting to know that, with the abnormal pressure of 100 lbs. per square foot, the value of the maximum stress in the fabric is much less than the limiting stress which would cause the rupture of an unwounded fabric. The fabric of the upper wing of a biplane is more severely stressed than the lower, the upper wing usually carrying a load which is about 30 per cent. greater than the load upon the lower wing. In view of the great air load on the fabric, an efficient connection of the fabric to the ribs is of the greatest importance. The fabric may be fixed to the ribs with small brass tacks and washers, although a more efficient method is to tack upon the outside of the fabric strips of cane or wood. A great improvement is effected by sticking the fabric to the ribs before tacking, and connecting the portions of the upper and lower fabrics, which are in the neighbourhood of the ribs, by knotted sewing, and covering the exposed portions of the sewing with tape. Should such a wing be tested to destruction, it is highly probable that the main spar would fracture before there would be any failure of the attachment of the fabric to the ribs.

CHAPTER IV.

CONSTRUCTION OF AN AEROPLANE.

The Aeroplane.—The author does not desire to consider the general details of the present-day machines, but a broad outline of the construction and function of the several aeroplane parts should be of some interest to the reader. Prominence has been given, when possible, to the good points of any particular type of machine.

The classification of modern aeroplanes favours two groups: (1) monoplanes, (2) biplanes. A monoplane is a machine with a single pair of wings or main planes, whilst a biplane, as its name implies, has two superposed pairs of wings. Multiplaned machines have been constructed, although the successful development of these machines probably belongs to the future. The aeroplane is a heavier-than-air machine, and a prolonged sustentation in the air of such a machine is only possible by virtue of the lift on the wings, the lift depending upon the configuration and the speed of the wings relatively to the air.

Nearly all the British machines are biplanes, since the biplane, with its elaborate system of wing bracing, is usually more strongly constructed than the monoplane, and the additional wing area is in favour of a much larger carrying capacity.

The planes at the tail of an aeroplane may be divided into two groups: (1) those fixed in the machine—such planes are essentially stabilisers; (2) those capable of partial rotations. The rudder, a vertical plane capable of rotation about a vertical axis, partially controls the yawing or turning of the machine. The elevator, a plane turning about a lateral axis, regulates the pitching of the machine. The terms "horizontal" and "vertical" in this connection are not strictly accurate. "Lateral" and "longitudinal" are correct (see diagram, fig. 36, Chapter VI.). The position and the size of the rudder have an important bearing on the lateral stability of the machine. Similarly, the elevator should be regarded as a longitudinal stabiliser. Usually, the leading plane which is fitted to some biplanes has both a directive and stabilising action upon the machine.

At present the two methods of controlling transverse equilibrium are (1) flaps; (2) warping the wings.

Flaps.—The employment of flaps or ailerons, that is, auxiliarly horizontal surfaces which are capable of partial rotation. Fig. 20 gives a general idea of the position and the size of a wing flap. The flaps are cross-connected so that a downward rotation of one flap is accompanied by an upward rotation of the other.

The yawing and rolling moments, which are greatly dependent upon the differences of the drags and lifts of the two wings respectively, are affected by the flap action. If necessary, the yawing moment may be modified by rudder action.

The maximum lift coefficient of a wing may be increased by rotating both wing flaps downward, and so the speed of landing of the machine may be diminished; hence, at a constant speed of landing the weight of the machine may be increased or the area of the wings diminished. Also, the speed of the machine, when running on the ground, may be effectively braked if the plane of the flaps make a large angle with the direction of motion. On the other hand, when the machine is accelerating on the ground, just before an ascent, the resistance of the wings may be diminished by placing the flaps at a small negative angle, and hence at the same horse-power the distance traversed on the ground, before "getting off," may be diminished.

For some positions of the flaps the air flow around the wings is of an uncertain character, and before fitting flaps to a machine experiments should be made to determine the positions of the flaps, relative to the wing and the wind direction, at which instability of flow occurs. The position of the centre of pressure of a wing is very sensitive to changes of the negative angle of the flap (region about $-3°$) and also to a variation of a large positive angle of incidence of the flap (say 60°).

Warping the Wings.—By warping, the angle of incidence of one wing may be increased relatively to the other. Warping is performed by a slight rotation of the rear spar about its attachment to the body, and the various wing sections have small rotations about the main spar. There is a growing tendency nowadays to substitute wing flaps for the wing warp. A brief summary of the disadvantages of the wing warp is given below.

(*a*) The self-warping of the wings imparts a continuous flicking motion of the warping lever. During a long flight, the constant attention to the warping lever imposes a great strain on the pilot.

(*b*) Great attention must be given to the adjustment of warping wings.

(*c*) Warping may introduce stresses in the material of the wings. Certainly, warping cannot be recommended from a mechanical standpoint.

(*d*) "Flapped" wings may be easily dismantled and again set up.

The wings, rudder, elevator, and stabilising fins are fixed in their correct relative positions by a framework of wood which is known as the body or fuselage. The body also contains the engine, propeller, control levers, and seating accommodation. Finally, the aeroplane is completed by the chassis or undercarriage which supports the fuselage when the machine is on the ground.

A consideration of aeroplane wings from a mechanical standpoint has already been given in Chapter III.

The Aeroplane Undercarriage or Chassis.—At the present time the reliability of an aeroplane engine is a factor of uncertain amount, so that the pilot may not always be able to choose a convenient and safe landing-place. The undercarriage, which should not be of cumbrous proportions, must be sufficiently strong to withstand the abnormal stresses of a sudden and, maybe, violent landing. The following points dealing with the construction of an undercarriage should be considered by the designer :—

(1) The machine has to attain its flying speed rapidly, so that the ground resistance when the machine is running along the ground should be small.

(2) Suitable devices to absorb landing and rolling shocks must be fitted.

(3) The machine must be efficiently steered when on the ground, and also the alighting speed must be braked. It has been found that a rim, or a tyre, or a band brake, which acts directly upon the main wheels, readily keeps the machine under control when running on the ground.

(4) The stability of the machine at high ground speeds demands attention. When alighting, the machine should have no tendency to pitch forward on to its nose. Such a disaster may be prevented by fitting a leading wheel or skid.

A simple construction of undercarriage facilitates the repair of accidental breakages, and also minimises weight and head resistance. When the machine rests upon the ground, the main wheels support most of the weight, so that the rear skid or wheel bears only a comparatively small load. An uncomfortable "rebounding" landing is the result of placing the C.G. (centre of gravity) of the machine some distance behind the axis of the main wheels. The pitching moment called into play by the reaction of the ground upon the wheels increases the angle of incidence of the wings, and the increased lift causes the machine to ascend again. Moreover, if the main chassis wheels are placed nearly under the C.G. of the machine, the loading upon the rear skid will be small, so that no undue strengthening of the framework, entailing additional weight, is necessary.

The hubs of the wheels should be sufficiently strong to resist side strains. For reasons of decreased resistance, and probably also from strength considerations, wheels with dished steel sides are preferable to ordinary spoked wheels. A broad wheel base is necessary to ensure that the machine when running on the ground may have sufficient stability.

The working of a land steering gear should not be dependent on air pressures, which are a function of the velocity of the machine. Also steering should not be dependent upon the propeller slip stream. A pivoted rear skid, when operated from the wheel or foot-bar which works the rudder, allows ground steering to be performed efficiently.

Shock absorbers are largely manufactured from rubber cable, a material which has good lasting qualities and a high ultimate stress. Ordinary steel springs and rubbers both dissipate and store the energy of landing and rolling shocks. The stored energy is afterwards imparted to the machine as minor shocks.

Some absorbers do not greatly diminish the energy which is subsequently given to the machine, although by increasing the time of impact the magnitude of the blow is diminished. The real shock absorber—a dashpot, for instance,—would dissipate most of the energy, and only a small fraction of the original energy of the blow would exert a disturbing influence upon the machine. Rubber cable dissipates energy quite rapidly, and has, therefore, a "dashpot" action.

Ash is largely employed in the construction of an undercarriage of an aeroplane. The advantages of ash are (i.) lightness, (ii.) strength, (iii.) cheapness, (iv.) hardness, (v.) a capacity to withstand bending and torsional stresses, (vi.) facility with which a damaged portion may be replaced. Nevertheless, the great advantages of metals—of which strength, the use of strong welded joints and connections, and almost complete indifference to weather conditions, may be mentioned—encourage us to anticipate that the undercarriage of the future will be constructed exclusively of metal.

The Hydro-Aeroplane.—A hydro-aeroplane has been defined as an air-borne craft capable of floating on water. Such a machine must be made sufficiently strong to resist the severe buffeting of a rough sea, although its flying capacity is affected adversely by any undue weight and head resistance. The double-float machine is probably the more seaworthy class of hydro-aeroplane, and is fairly efficient in rough water. The floats should not be too far apart, otherwise the lifting of one float out of the water due to excessive rolling causes the machine to suddenly swing round; and, moreover, the unbalanced forces called into action by the reduction of the resistance of the rising float, and the increase of the resistance of the falling float, are greatly assisted by the leverage between the two floats. On the other hand, when the two floats are close together, the behaviour approximates to that of a single-float machine. The water resistance of the floats appears to be practically independent of their distance apart. To prevent the float driving into the water when the machine alights, and also to keep the nose of the float well out of the water when the machine is at rest on the water, it is desirable that the centre of buoyancy of the float, when the longitudinal axis of the machine is horizontal, should be well forward of the position of the C.G. of the machine. The total buoyancy of the floats should be almost equal to twice the weight of the fully loaded machine. Whilst the float is rising from the water the free access of air to the bottom of the step should not be hindered. The tendency to hop—a characteristic of most floats when running along the surface of the water at high speeds and small buoyancy—is due to the inherent instability of a machine partly supported on a small area. Any tendency to dip the nose of the float under water, when the machine is travelling at high speeds, may be minimised by a low position of the line of the propeller thrust. Floats must be sufficiently strong to resist the severe wave blows experienced during an enforced alighting on a rough sea. The bow of a float, where great buoyancy is required, should not be too narrow,

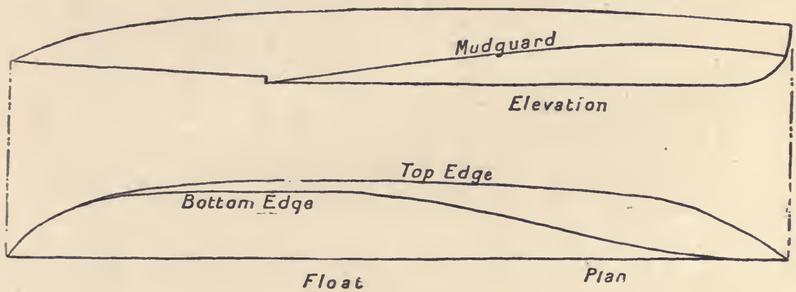


Fig. 15.

otherwise when alighting the required " pick up " off the water is not attained. If bows of the floats are too bluff, the machine has to traverse a long distance on the water before the planing speed is reached. Long and narrow floats with flared sides at the step are very serviceable upon a rough sea. To enable the float to obtain a greater lift, by a partial utilisation of the energy of the bow waves, concave mudguards are fitted from the bow to the bottom of the step. The mudguards also favour the admittance of air to the bottom of the step. Seaworthiness may be increased with a sacrifice of " getting off " efficiency by transverse rounding of the bottom of the float. A low position of the C.G. of the machine is in favour of good floating stability, but may have a disturbing influence upon the behaviour of the machine in flight. A low position of the wings enhances the possibility of damage by a rough sea. A sketch of a good float is shown in fig. 15.

It is thought that descriptions of one or two of the modern machines may be of some interest to the reader. Technical details of construction are omitted, so that the descriptions are necessarily of a brief character.

The Bristol 80 H.P. Tractor Biplane (Two-Seater) (*see* fig. 16).

Weight, light =440 kilos=970 lbs.
Range of flight=5 hours.
Speed =120 to 56 kms. per hour=75 to 35 miles per hour.
Useful weight carried, including pilot and passenger (160 kilos), with petrol and oil (155 kilos)—Total=315 kilos=695 lbs.

Dimensions :—

Length over all=8·9 metres=29·2 feet.
Span =11·5 metres=37·8 feet.
Chord =1·85 metres=6·1 feet.
Total area =39 square metres=362 square feet.

Fuselage.—The fuselage is of a special design adapted to offer a minimum resistance at the flying speed of the machine. It is flat-sided in section in order to provide the vertical surface necessary for directional stability, and is built up of longitudinal members of ash with cross members and struts of spruce, which are connected by steel joints and strongly braced with steel piano-wire.

Wings.—The wings are exceptionally light and strong. Special compression ribs are provided for the internal bracing of the wings. The upper and lower wings are separated by twelve struts of stream-line section, braced together with stranded steel cable.

Engine.—An 80 H.P. Gnome engine (Monosoupape type) is mounted in front of the fuselage. To prevent any oil being thrown back on the pilot, the engine is entirely enclosed in an aluminium shield. The propeller boss is so designed that it is the forward continuation of the lines of the aluminium shield, so that the head resistance is thereby greatly diminished.

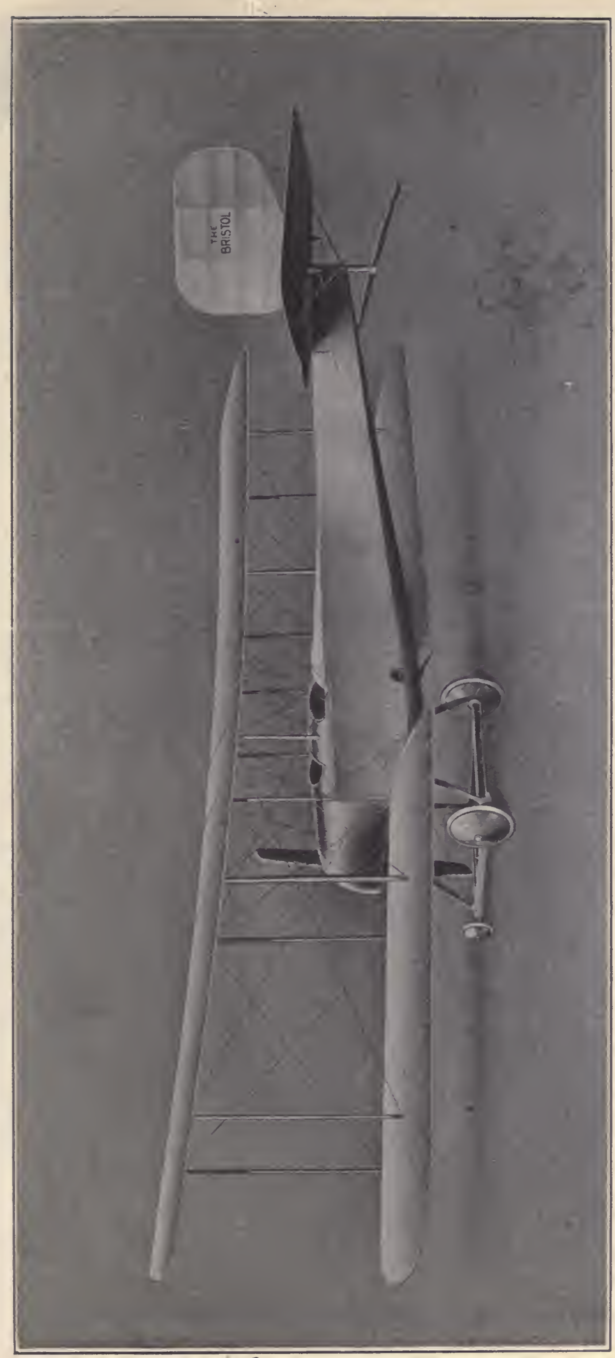


FIG. 16.—Bristol 80 H.P. two-seater biplane (1914).

The seats for the pilot and passenger are in tandem, the passenger, in front, being situated directly over the C.G. of the machine, which arrangement enables the machine to be flown either with or without a passenger. The suspension of the seat allows the height and the angle to be adjustable.

Wind Shield.—The top of the fuselage is covered in by a curved sheet of three-ply wood, designed to deflect the air upwards from the passenger's face. The fuselage is covered with fabric which is laced on, and which can be easily removed.

Tail and Empennage.—The empennage, which has a non-lifting plane as a directive organ, ensures the longitudinal stability of the machine. The plane is arranged at a negative angle and placed midway between the upper and lower surfaces of the fuselage. The levers for the elevator and rudder are of the best-quality steel.

Landing Chassis.—The chassis is of the combined skid and wheel type, in which the skids are connected to the fuselage by stream-like struts of very ample dimensions, with pin joints to give the whole structure more flexibility. At the forward end, the skids carry two small wheels journalled on one axle, sprung with rubber cord. They form, therefore, a resilient member to counteract any tendency of the machine to tip forward, and also to effectively protect the propeller on uneven ground. The machine runs upon a pair of wheels, 66 cms. in diameter, which have wide hubs and stout gauge spokes. The wheels are journalled upon a stout tubular axle which is sprung from the skids by rubber cord. The tail is protected by a tail skid.

Control Gear.—The main control of the machine consists of a hand-wheel mounted on a vertical lever. The longitudinal movement controls the tail plane, and a rotation of the wheel actuates the warping of the wings by means of wires passing round suitably arranged pulleys. The rudder is operated by foot-levers. All the moving parts of the control are of brass or other non-magnetic material, and all the control wires are duplicated.

The coverings of the wings, tail, rudder, and fuselage are treated with dope, which gives them a smooth, taut surface. Upon the instrument board are usually fitted an air-speed indicator, altimeter, engine tachometer, inclinometer, watch, trip watch, petrol gauge, and communicator between pilot and passenger. Fig. 17 is an illustration of a "Bristol" 80 H.P. "Scout."

FIG. 17.—Bristol 80 H.P. single-seater "Scout" biplane (1914).

The Sopwith Bat-Boat.

This machine, a photograph of which is shown in fig. 18, is a particular type of hydro-aeroplane, the pilot and passenger sitting in the boat.

Weight, light	=2200 lbs.
Useful load	=1000 lbs.
Maximum speed	=70 miles per hour.
Minimum speed	=40 miles per hour.

Fig. 18 (1914).

Dimensions :—

Span of upper wing	=54 feet.
Span of the lower wing	=44·5 feet.
Chord of the wings	=6·75 feet.
Total area of the wings	=600 square feet.

The main wing spars are made of spruce. The fuselage is made chiefly of mahogany. The pilot's and passenger's seats are arranged side by side in the roomy cockpit of the boat. The machine is driven by a 200 H.P. Canton-Unné engine. The main petrol tank has a capacity sufficient for four and a half hours' flight, and is situated in the boat behind the occupants. Petrol is forced from this tank to a smaller service tank between the engine and the radiator, whence it is fed by gravity to the engine. Two compressed-air self-starters are carried under the seats, so that the engine may be started by the pilot without the necessity of swinging the propeller.

The four tail booms and their struts are made of spruce. The rudder is of the balanced type, and, to allow sufficient rudder movement, the elevator is divided. The lower wings have a well-pronounced dihedral angle which allows the machine to roll considerably on the water. Floats at the wing-tips are also fitted. The boat is built up of two skins of mahogany over ash stringers.

Fig. 19.—Sopwith 100 H.P. tractor hydro-aeroplane (1914).

It is of the single-stepped type, and air is admitted to the bottom of the step by sheet-brass channels screwed to the sides of the boat. A wheel mounted on a single tube controls the aileron and elevator movements, whilst the rudder is actuated by a pivoted foot-lever. A complete set of instruments is mounted on the instrument board in front of the pilot, and a wireless set driven by a motor-cycle engine is mounted in front of the passenger's seat.

Sopwith 100 H.P. Tractor Hydro-Aeroplane (*see* fig. 19).

The floats of this machine are of the single-step type. The float has five watertight compartments, each fitted with a neat inspection door. The doors are screwed down on rubber seatings with butterfly nuts, thus ensuring a watertight joint. A 100 H.P. Anzani engine is mounted on this machine.

80 H.P. Avro Biplane (*see* fig. 20).

The main planes are staggered, giving the pilot an extremely good view forward and downward. The fuselage is made narrow for the same reason. The engine is carried on fore-and-aft bearings, but at the same time a well-rounded stream-line nose is obtained, which gives the machine an exceptionally clean

Fig. 20.—Avro biplane (1914 type).

appearance. The chassis is extremely light, but at the same time strong, and the machine starts from, or lands on, rough ploughed ground in an easy manner. The centre skid is placed well forward, and thus protects the propeller and minimises the possibility of the machine turning over during alighting. When an 80 H.P. Gnome Monosoupape engine is mounted, the maximum speed may be about 90 miles an hour. A machine of this type has climbed at the rate of 550 feet per minute, when carrying a three hours' fuel supply and a passenger.

The "Avro" Hydro-Aeroplane.

A 100 H.P. "Avro" hydro-aeroplane is illustrated in fig. 21.

Fig. 21.—100 H.P. "Avro" waterplane (1914).

The good characteristics of the "Avro" hydro-aeroplane are:—

(*a*) The floats are mounted well forward, so that the tendency of the machine to overturn on alighting is reduced. The machine is easily manageable on the water in a wind, as the tail float rests on the water and a water rudder is provided.

(*b*) The main floats are sprung internally by rubber cord suspension, giving a very large range of action and a certain amount of "give," both laterally and longitudinally.

(*c*) The engine is fitted up with a crank starting gear, enabling it to be started by the observer.

(*d*) The passenger and pilot are well protected from the weather, and they have an exceptionally easy means of egress and access. The engine is very neatly mounted, and shielded to offer the minimum of head resistance.

(*e*) The bottom wings are made shorter than the top wings in order to give an uninterrupted air space and a greater water clearance laterally. The ailerons on the top plane are very large, giving a powerful lateral control.

(*f*) The floats are very durable, and are rendered absolutely watertight by a covering of stout rubber-proofed cloth.

The Blériot Single-Seater Monoplane.

Span of the wings	=29·3 feet.
Area of the wings	=194 square feet.
Weight, light	=618 lbs.
Useful load	=309 lbs.

The main spars are made of ash. The fuselage is constructed of ash and spruce, and the landing gear of ash and steel. Lateral control is obtained by warping the wings. An 80 H.P., seven-cylinder, Gnome engine is carried. The propeller, of the integral type, is mounted in front of the machine.

Nieuport Single-Seater Monoplane (*see figs.* 22 *and* 23).

Span of the wings	=29 feet.
Chord of the wings	=6 feet.
Area of the wings	=155·5 square feet.
Maximum speed	=84 miles per hour.
Weight, light	=595 lbs.
Useful load	=352 lbs.

The main spars are made of steel. The fuselage is constructed of ash and spruce, and the landing gear of steel. A

FIG. 22.—Nieuport single-seater monoplane (1914).

FIG. 23.—Nieuport (1914).

60 H.P. Le Rhône engine is mounted in front of the pilot. The propeller is of the Régy type.

Vickers Two-Seater Biplane.

Span of the upper wing	=38 feet.
Span of the lower wing	=38 feet.
Area of wings	=400 square feet.
Maximum speed	=70 miles per hour.
Minimum speed	=40 miles per hour.
Weight, light	=850 lbs.
Useful load	=850 lbs.

The main spars are made of steel. The fuselage and landing gear are also constructed of steel. Lateral control is by the aileron system. A 100 H.P. Gnome engine, with nine cylinders, is mounted at the rear of the pilot.

The "Wight" Hydro-Aeroplane.

A two-seater machine built by Samuel White of Cowes, Isle of Wight.

Span of the upper wing	=63 feet.
Span of the lower wing	=59 feet.
Chord	=6·5 feet.
Gap	=5·6 feet.
Area of the wings	=735 square feet.
Weight, light	=2600 lbs.
Useful load	=900 lbs.
Maximum speed	=72 miles per hour.
Minimum speed	=35 miles per hour.

The main spars are made of spruce. The fuselage and landing gear are also constructed of spruce. Lateral control by the aileron system. The machine is driven by a 200 H.P. Canton-Unné engine mounted at the rear of the pilot. A doubled cambered wing section is an interesting feature of this machine. The two main floats are unusually long and are provided with three steps. These floats are built up of three-ply wood on a strong framework of elm. Each float is divided into six watertight compartments, and the whole is strengthened by a longitudinal partition extending throughout the length.

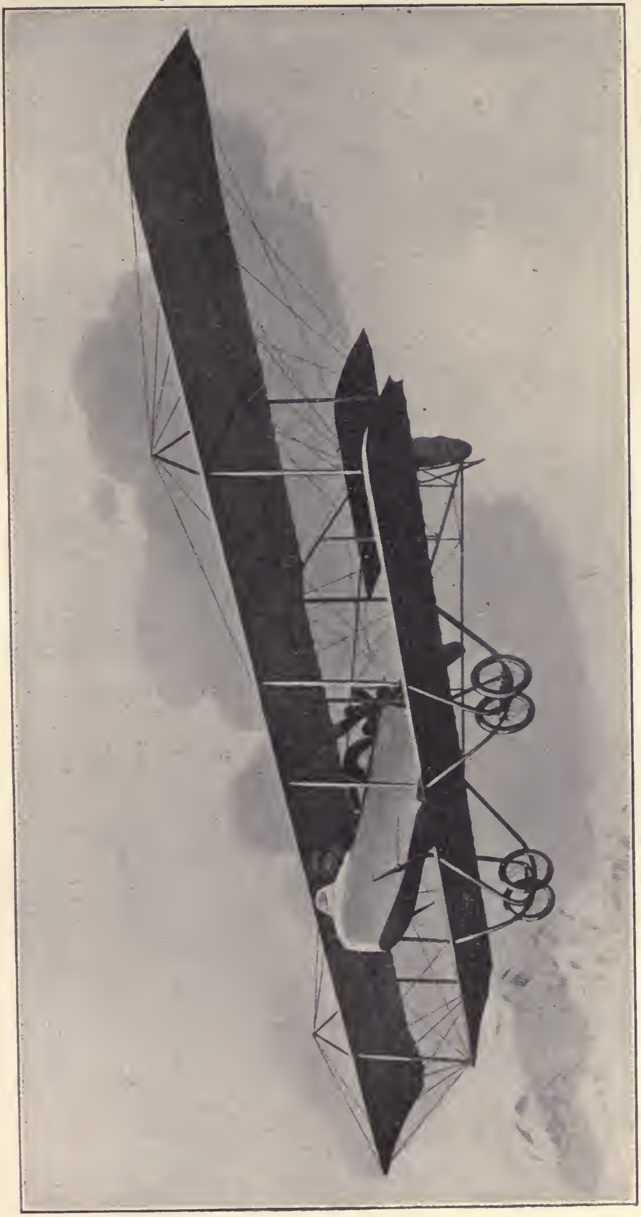


FIG. 24.—Henri Farman (1914).

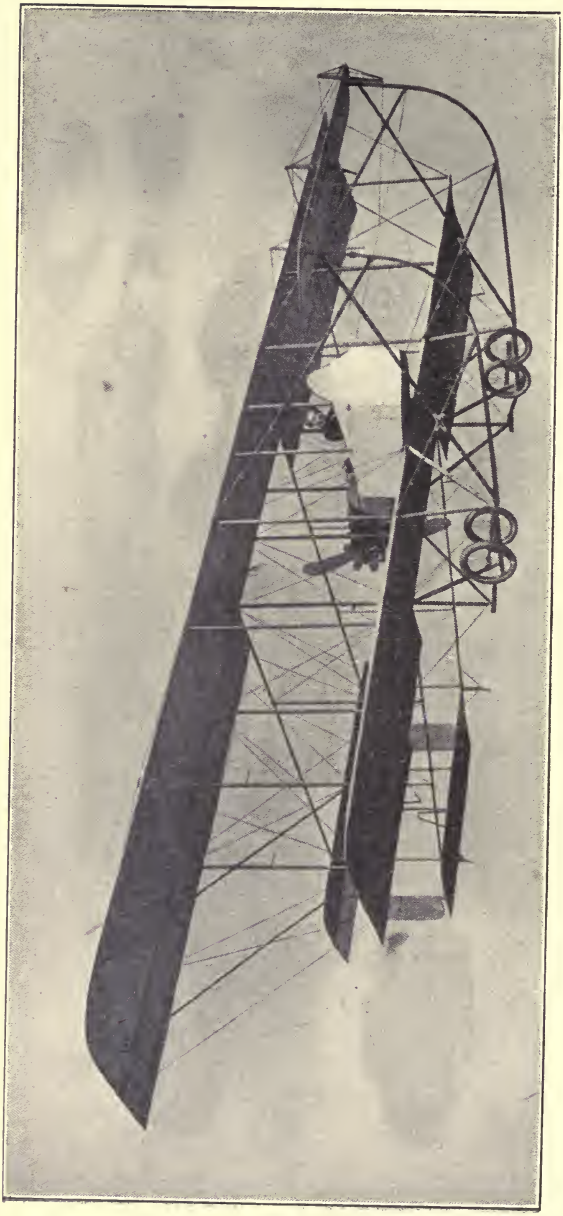


Fig. 25.—Maurice Farman (1914).

The Farman Two-Seater Biplane (*fig.* 24).

Span of the upper wing	=50·8 feet.
Span of the lower wing	=32·8 feet.
Area of wings	=560 square feet.
Weight of the machine, light	=1320 lbs.
Useful load	=660 lbs.

The main wing spars and the fuselage are made of spruce, and the landing gear is constructed of ash and steel. The lateral control is by the aileron system. The machine is driven by a 70 H.P. Renault engine mounted at the rear of the pilot. An automatic machine-gun may be mounted at the forward end of the nacelle—that is, just in front of the passenger's seat, the pilot occupying the rear seat.

The Grahame-White Two-Seater Biplane.

Span of the upper wing	= 37 feet.
Span of the lower wing	= 35 feet.
Chord	= 5 feet.
Gap	= 6 feet.
Area of the upper wing	=183 square feet.
Area of the lower wing	=175 square feet.
Weight of the machine, light	=1000 lbs.
Useful load	=550 lbs.
Maximum speed	= 80 miles per hour.
Minimum speed	= 42 miles per hour.

The main wing spars are made of spruce, and the fuselage is constructed of ash, hickory, and spruce. The landing gear is of steel. Lateral control by the aileron system. A 100 H.P. Gnome engine is mounted at the rear of the pilot. The tanks have a capacity of 45 gallons of petrol and 9 gallons of lubricating oil—that is, sufficient for a flight of about five hours' duration.

CHAPTER V.

THE EQUILIBRIUM OF AN AEROPLANE.

Horizontal Flight.—An aeroplane in ordinary linear flight—that is, a horizontal, a gliding, or a climbing flight—is symmetrical about a vertical plane passing through the longitudinal axis, so that we need only consider its longitudinal equilibrium. If the planes of a machine are fixed relatively to each other, flight is only possible at one particular angle of incidence of the wings. To enable a machine to adopt several flying positions, an elevator—a movable plane capable of rotation about an axis perpendicular to the plane of symmetry—is fitted. In the following discussion we shall assume, and quite rightly, that the position of the centre of gravity of a machine is unaffected by the elevator position. The main functions of the tail are (*a*) to control the attitude, and (*b*) to damp out longitudinal oscillations. It is assumed that in normal flight the tail does not contribute to the total lift of the machine.

The following nomenclature is adopted throughout this chapter :—

- W =weight of the machine.
- A =area of the wings.
- α =angle of incidence of the wings.
- L =lift of the machine—that is, the air force acting on the machine in a direction normal to the wind direction, and to the plane of the wing.
- D =drag of the machine—that is, the air force acting on the machine in the direction of the wind.
- H =propeller thrust, or the tractor pull.
- K_α =absolute lift coefficient of the machine, when the wings have an angle of incidence α.
- k_α =absolute drag coefficient of the machine, when the wings have an angle of incidence α.

U =speed of the machine in the direction of the longitudinal axis.

R =total air force acting on the machine. The propeller thrust is, of course, an air force, although we shall take R to be exclusive of the propeller thrust. Hence R is the resultant of L and D.

Equilibrium Conditions of a Machine in Horizontal Flight.—(1) W, H, L, and D are in equilibrium. Hence, if the propeller thrust acts in the direction of flight, the condition (1) may be rewritten as :—

(*a*) Lift of the wings, L=weight of the machine, W.
and $\therefore W=L=K_a.\rho.AU^2$.

(*b*) Drag of the machine, D=propeller thrust, H,
and $\therefore H=D=k_a.\rho.AU^2$.

(*c*) The algebraic sum of the moments of W, H, L, and D about the C.G. of the machine is zero.

Considering equation (*a*), if the angle of incidence a be constant, then K_a will also be constant, and since A and W are constants of the machine, the speed of the machine in horizontal flight will be constant for a particular value of a. The speed of a machine in horizontal flight only depends, then, upon the angle of incidence of the wings—that is, upon the position of the elevator. To alter the horizontal flight speed of a machine, an alteration of the propeller thrust must be accompanied by a change in the angle of incidence of the wings.

Briefly, in any one machine, the speed of horizontal flight, the propeller thrust, and, consequently, the engine-power, are dependent wholly upon the angle of incidence of the wings.

The drag or total resistance of a machine may be regarded conveniently as composed of—

(*a*) The resistance of the wings.

(*b*) The sum of the resistances of the remaining portions of the machine, which we shall call the body resistance.

Influence of Altitude.—From the relationship $W=K_a.\rho.AU^2$, it follows if the angle of incidence of the wings remain constant, that the speed of a machine in horizontal flight increases with the altitude. The density of the air at an altitude of 6000 feet —pilots usually fly at this altitude—is about 80 per cent. of the density at the ground, and hence if the thrust of the airscrew and the angle of incidence of the wings remain unaltered, the speed of a machine in horizontal flight at this altitude is 1·12 times the speed at ground level.

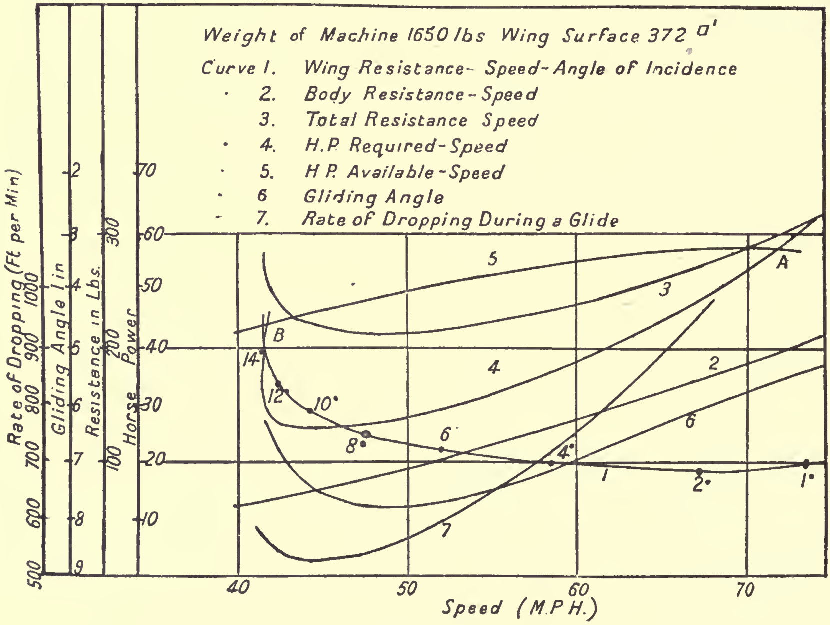


Fig. 26.

If the ratio of the translational speed, U, and the rotational speed, n, of the airscrew has a constant value, the thrust and torque of the airscrew are each proportional to ρU^2 (see Chapter VII.).

The torque of the engine is almost proportional to the density of the air, that is, at an altitude of 6000 feet the torque diminishes 20 per cent., due to the change of density.

Throughout the calculations of this chapter, the machine *is assumed to be at ground level*, so that $\rho = \cdot 00237$. It should be borne in mind, however, that most flying is done at an altitude of several thousand feet.

"Performance" Curves.—Most of the curves of fig. 26 were obtained experimentally at the Royal Aircraft Factory, and they have been taken from the *Report of the Advisory Committee for Aeronautics* (1912–13).

Curve 1 of fig. 26 gives the relation between the wing resistance, angle of incidence, and the speed in horizontal flight of a constant-weight machine. The weight of this particular machine is 1650 lbs., and the area of its wing surface 372 square feet. From the curve it is seen that when the speed of the machine is 60 miles per hour, the wings have a resistance of 97 lbs. and an angle of incidence of slightly less than 4°.

Now, the drag D_w of the wings may be written in the form $D_w=({}_1k_a)\,.\,\rho\,.\,AU^2$, and therefore

$$\left({}_1k_a\right)=\frac{97}{372\times(88)^2\times\cdot00237}=0\cdot0141\ (\alpha=4^\circ\text{ approximately}).$$

Also, if the machine be supported entirely by the lift on the wings,

$$W=K_a\,.\,\rho\,.\,AU^2,\text{ and therefore}$$

$$K_a=\frac{1650}{372\times(88)^2\times\cdot00237}=0\cdot241\ (\alpha=4^\circ\text{ approximately}).$$

Since $D_w=\left(\frac{{}_1k_a}{K_a}\right)W$, the resistance of the wings is a minimum at the angle of incidence which makes $\left(\frac{K_a}{{}_1k_a}\right)$ a maximum. From curve 2 we see that the body resistance R_b is practically proportional to the square of the speed. The body resistance at 60 miles per hour is 137 lbs., and consequently, if we assume the body-resistance law to be $R_b=cU^2$, the value of the constant c is equal to $\frac{137}{(60)^2}$, that is, 0·038. The body resistance may be materially diminished by the adoption, where possible, of stream-line forms, and also by the elimination of unnecessary wires, etc.

The following comparisons of two similar machines—that is, machines built to the same drawing but to different linear scales —are instructive :—

Case I.—Assumption { Linear scale of machine A=2 linear scale of machine B. Weight of A=weight of B.

Then, at similar horizontal flying attitude—

Speed of B is double the speed of A.

Thrust of B is equal to the thrust of A.

Power of engine B is double the power of engine A.

Case II.—Assumption { Linear scale of A=linear scale of B. Weight of A=4 weight of B.

Then, at a similar horizontal flying attitude—

Speed of A is double speed of B.

Thrust of A is four times thrust of B.

Power of engine A is eight times the power of engine B.

A few remarks on the stresses in similar machines may not be out of place. Stresses in like parts of similar machines vary as the linear dimensions when such stresses are entirely dependent on the dead-weight of the machine, and from this point of view the smaller machine is the stronger. The stresses in similar *wings* due to equal loading at similar points are proportional to the reciprocal of the square of the linear dimension. Generally speaking, the smaller machine is the stronger.

The speed of a machine in horizontal flight may be increased (*a*) by decreasing the wing area, the other factors—such as the weight of the machine, the angle of incidence of the wings, etc. —remaining constant ; (*b*) by increasing the weight of the machine, the other factors, with the exception of the propeller thrust, remaining constant. The first method is to be preferred, and nearly all the modern high-speed monoplanes have small wing areas.

From the data of curve 3 of fig. 26—that is, the curve of total resistance—we may calculate the " horse-power required " curve.

Thus the total resistance of the machine at 60 miles per hour is 235 lbs., so that the horse-power required will be

$$\frac{235 \times 88 \times 60}{33,000} = 37 \cdot 6.$$

The high speed of horizontal flight, which occurs at a small angle of incidence of the wings of a constant-weight machine, is only attained if the engine run at a high power, see curve 4, fig. 26. Moreover, a small angle of incidence is not to be encouraged, as should such a machine be struck by a downward vertical gust, the relative wind may strike the wings upon the upper surface.

From curve 4 we see that the smallest horse-power is required at a speed of 44·5 miles per hour, and then the angle of incidence is 10°. What may be occasionally required is the most economical speed for a journey of a certain distance, see fig. 27. Let us assume that the machine is carrying a Chenu engine, in which the fuel consumption per horse-power hour has the constant value of ·54 lbs., and that the machine carries such a large supply of fuel that the consumption of a portion of this fuel will not appreciably affect the weight of the machine. The fuel-consumption-speed curve has been constructed for a distance of 200 miles. The distance taken does not affect the shape of the curve.

From curve 4, fig. 26, it is seen that when the machine travels at 60 miles per hour the horse-power required is 37·6. Also, the time taken to perform the journey is $\frac{200}{60}$ hours, so that the fuel consumption is $\frac{200}{60} \times 37 \cdot 6 \times \cdot 54 = 67 \cdot 5$ lbs. In this way the curve of fig. 27 has been constructed, from which it follows that with this engine the total fuel consumption, over any distance, is a minimum when the aeroplane has a speed of 50 miles per hour approximately.

The following curve, fig. 27, cannot be exceptionally accurate, because the weight of the machine is not constant during a flight; and, further, the fuel consumption per horse-power hour is not a constant but a function of the horse-power.

Generally speaking, a high velocity consistent with safety is desirable, and the greatest speed attainable in horizontal flight is the speed beyond which a diminution in the angle of incidence becomes dangerous. To facilitate starting and climbing, the horse-power of an engine is greater than the power necessary to maintain the normal speeds of horizontal flight, so that in horizontal flight the engine is eased down.

It is desirable that the normal speed of the engine should

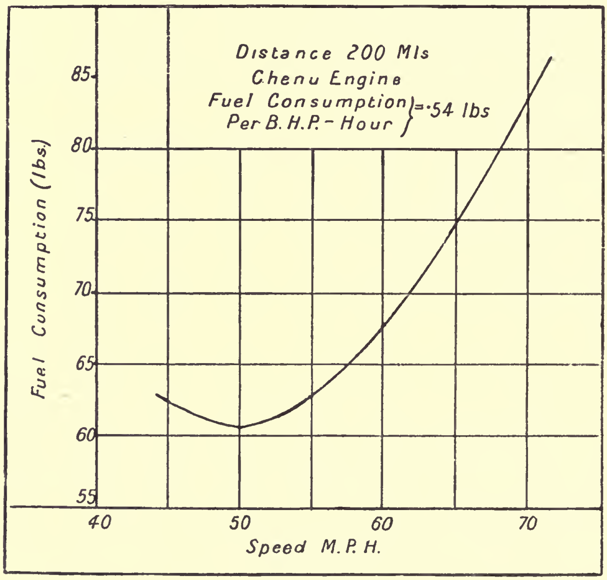


Fig. 27.

approximate to the value designated by the engine designer as the best. Theoretically, the maximum efficiency of the propeller and its gearing should be given at the normal speed of the engine. Usually, however, the propeller runs at a rotational speed greater than that corresponding to its maximum efficiency. Curve 4, the "horse-power required" curve, and curve 5 (fig. 26), the "horse-power available" curve, cut at two points A and B: so that when the engine is running all out, horizontal flight may be maintained at two different angles of incidence and the two different corresponding translational speeds. If an aviator desire horizontal flight at full engine-power, he would prefer the upper speed limit A—that is, if the angle of incidence corresponding to the upper speed is well within the safety limit. From curve 4 we see that horizontal flight at the various speeds ranging from 72 to 42 miles per hour may be obtained by a throttling of the engine, and the necessary adjustment of the elevator position. A vertical distance between the "horse-power available" curve and the "horse-power required" curve gives the horse-power available for climbing at any angle of incidence of the wings.

This particular machine will climb most rapidly when the angle of incidence of the wings is equal to $6\frac{1}{2}°$ approximately. If the machine is flying horizontally with the engine at full power, so that the speed is 72 miles per hour (see point A of curve 4, fig. 26), climbing may be performed by a simple manipulation of the elevator to increase the angle of incidence of the wings. A well-designed machine enables climbing to be performed by the operation of the elevator alone, the power of the engine remaining unaltered. If the horizontal flight speed be less than 72 miles per hour, or greater than 42 miles per hour, the engine has a reserve of power, and occasionally a good reserve of power is necessary to facilitate rapid climbing. Nevertheless, a storage of excess engine-power during a horizontal flight entails a burden of excess engine weight.

Inclined Flight.—The steady flight of an aeroplane is said to be inclined when the direction of its forward motion is not horizontal, and the slope of the flight path is measured by the tangent of the angle between the direction of forward motion and the horizontal. Also when an aeroplane is in steady inclined flight, the angle of incidence of a wing is measured by the angle between a chord of the wing and the direction of the relative wind.

When the direction of the propeller thrust passes through the C.G of the machine, the angle of incidence of the wings in an equilibrium position of the machine depends only upon the position of the elevator, and is independent of the magnitude of the propeller thrust.

As before, the *condition of equilibrium* is that the forces acting upon the machine, L, D, W, H, must be in equilibrium, and this statement includes the fact that the algebraic sum of the moments of these four forces L, D, W, H, about any point, say the C.G. of the machine, must be zero.

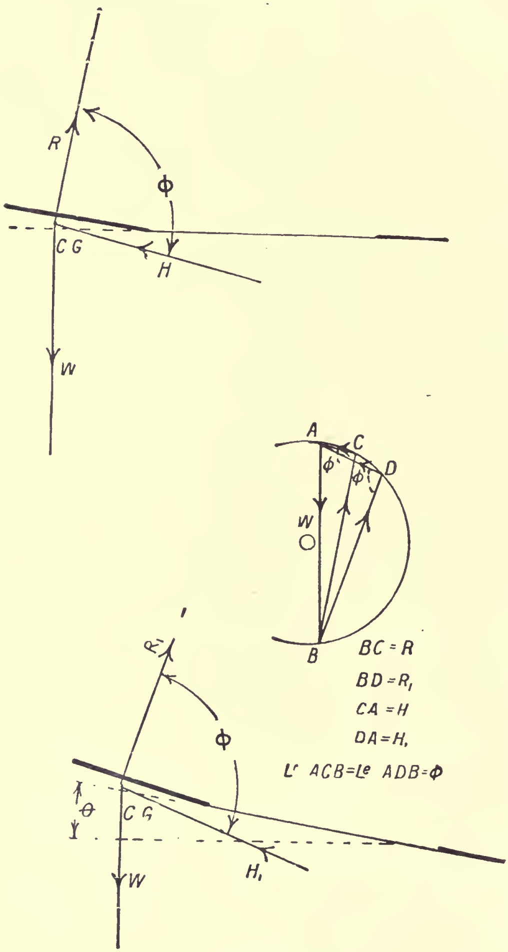


Fig. 28.

In the following two sections the equilibrium of a machine during an inclined flight has been considered, firstly, when the propeller thrust passes through the C.G., and secondly, when the propeller thrust does not pass through the C.G. of the machine.

Case I.—When the direction of the propeller thrust passes through the centre of gravity of the machine. Assume the elevator to be locked in position. Now H and W always pass through the centre of gravity of the machine, so that, for equilibrium, R, the resultant of the air forces L and D must also pass through the centre of gravity (fig. 28). An increase of the value of the propeller thrust does not introduce any moment about the C.G., so that the machine has no tendency to turn about a lateral axis, and hence in an equilibrium position the direction and the point of application of the resultant of the lift and drag forces, both taken relatively to the machine, are constants, and thus the angle of incidence of the wings has also a constant value. The new value of the propeller thrust

H_1 introduces a new air force R_1, so that H_1, R_1, and W are represented in magnitude and direction by the sides of a triangle taken in order (see fig. 28). It follows, then, that if the elevator position be fixed, variations in the magnitude of the propeller thrust will produce alterations in the slope of the flight path, and different speeds of the machine along the flight path, but no alteration of the angle of incidence of the wings.

Case II.—When the direction of the propeller thrust does not pass through the centre of gravity of the machine. In this case, any alteration of the propeller thrust must be accompanied by a slight alteration of the elevator position, if equilibrium is to be maintained.

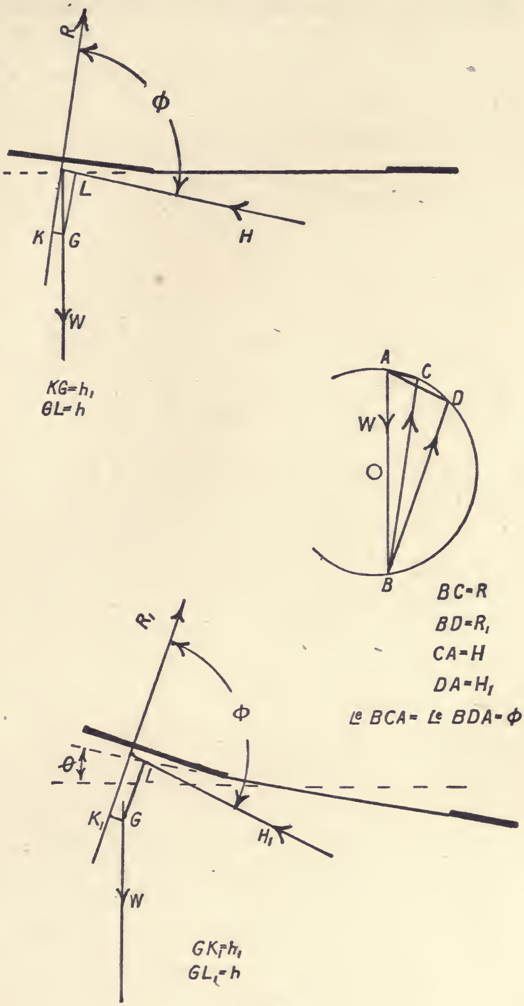


Fig. 29.

Let the fig. 29 represent an equilibrium position, then

(*a*) R, W, and H may be represented in direction and magnitude by the sides of a triangle taken in order.

(*b*) $Rh_1 = Hh$; and $\therefore \dfrac{h}{h_1} = \dfrac{R}{H}$.

Now imagine the elevator locked in position, and the propeller thrust increased to H_1. *Assume* for the new equilibrium position that the angle of incidence of the wings remains constant, and the angle of the flight path has been increased by θ, so that, according to the condition (*a*), R_1 is the new value of the air force.

Also, condition (*b*) now gives $H_1 . h = R_1 \times h_1$;

and therefore $$\frac{R_1}{H_1} = \frac{h}{h_1} = \frac{R}{H},$$

that is $$\frac{R}{R_1} = \frac{H}{H_1}.$$

Also, since $H_1 > H$, it follows from fig. 29 that $\frac{H_1}{H} > \frac{R_1}{R}$, and therefore $\frac{R}{R_1}$ cannot equal $\frac{H}{H_1}$.

Hence the initial assumption that the angle of incidence of the wings remains constant is proved to be incorrect, and any alteration of propeller thrust must be accompanied by a slight alteration of the elevator position, if the machine is to remain in equilibrium.

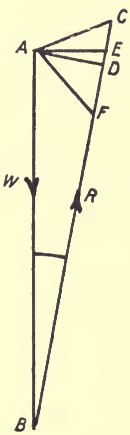


Fig. 30.

It is hardly necessary to add that the flight path of a machine is not necessarily in the same direction as the propeller axis. The magnitude and direction of the minimum propeller thrust necessary to sustain a machine during a horizontal flight may be easily obtained if (*a*) the point of application of the propeller thrust be at the centre of gravity, (*b*) the angle of incidence of the wings be constant. Under these circumstances R will have a constant direction relatively to the aeroplane passing through the centre of gravity. In fig. 30 AB represents the weight of the machine, and BC the constant direction of R. Obviously the direction and magnitude of the minimum thrust is represented by the line DA, the angle ADB being a right angle. It is usually arranged that the direction of the propeller thrust shall be horizontal when the machine is flying horizontally at its normal speed, and in this attitude of the machine the direction of

R does not greatly deviate from the vertical, so that the thrust employed, AE, is not much greater than the minimum thrust AD. If the angle DAB be equal to zero, the forward speed of the machine will be zero, so that equilibrium is only possible when the propeller, which now operates as a helicopter, supports the weight of the machine.

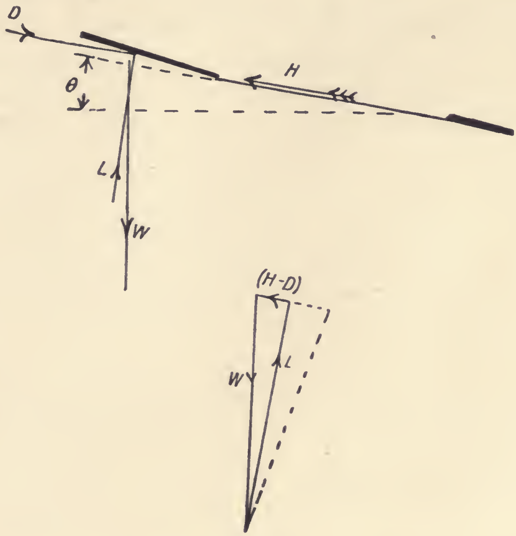


FIG. 31.

Assuming the propeller thrust to act parallel to the longitudinal axis of the machine, it is seen easily from fig. 31 that

(*a*) For a climb θ is positive, and $\therefore$ $H>D$.
(*b*) Horizontal flight $\theta=0$, and $\therefore$ $H=D$.
(*c*) For a fall θ is negative, and $\therefore$ $H<D$.
(*d*) For a glide $H=0$ and $\therefore$ $\theta=\tan^{-1}\left(\frac{D}{L}\right)$.

Gliding Flight.—If the engine of an aeroplane suddenly stop, the machine will commence to glide, and the slope of the glide path will be $\left(\frac{D}{L}\right)$. Hence the gliding angle has its minimum value at the angle of incidence which makes the ratio $\frac{D}{L}$ for the machine a minimum; or, in other words, the angle of incidence corresponding to the minimum drag of horizontal flight. The same gliding path may be followed at two different angles of incidence, the two speeds of the machine corresponding to these angles. This statement is a corollary of a previous statement that, if the propeller thrust be constant, horizontal flight may be maintained at two definite angles of incidence, the machine having the corresponding speeds. At any angle of incidence, the gliding angle is the ratio of the total resistance of the machine in horizontal flight to the weight of the machine. According to the curve 3 (fig. 26), when the angle of incidence of the wings is equal to 6°, the total resistance of the machine in horizontal flight is 215 lbs. This machine would therefore have a gliding angle of $\frac{215}{1650}$, that is, 1 in 7·7, at the same angle of incidence of the wings. As the gliding angle of a modern machine is small, the gliding speed is not greatly different from the speed of horizontal flight, the angle of incidence of the wings remaining unaltered.

Thus

Let U_H = speed of the machine in horizontal flight.
U_G = speed of the machine in gliding flight.
θ = angle of glide.
K_α = lift coefficient at the angle of incidence α.

In horizontal flight $W = L = K_\alpha \cdot \rho \cdot A \cdot U_H^2$. (1)
In gliding flight $W \cos\theta = L_1 = K_\alpha \cdot \rho \cdot A \cdot U_G^2$. (2)

And ∴ from equations (1) and (2)

$$\text{Cos}\ \theta = \left(\frac{U_G}{U_H}\right)^2 .$$

When the gliding angle is equal to $\tan^{-1}\left(\frac{1}{7\cdot 7}\right)$,

$U_H = 52$ miles per hour.
And $U_G = \cdot 99 U_H$,
so that $U_G = \cdot 99 \times 52 = 51\cdot 5$ miles per hour.

Curve 6 (fig. 26) gives the relation between the angle of gliding, the speed of gliding, and the angle of incidence of the wings, and it is seen that a small diminution in the value of a small angle of incidence is sufficient to cause the flight path to approach the vertical rapidly. A high-speed machine in which the wings have a small area, and also a small angle of incidence, has a strong tendency to dive with a diminution of the angle of incidence. If the propeller axis of such a high-speed machine be below the centre of gravity, a sudden stoppage of the engine is accompanied by an automatic decrease of the angle of incidence of the wings, the unfavourable condition for a rapid steep dive.

Consider A and B to be two exactly similar machines which are equal in all respects. If B be now more heavily loaded, by placing a weight at its centre of gravity, then the gliding slopes of both machines are still the same, but the machine B will glide at a greater speed than A. The distance fallen in one second depends upon the angle of gliding and the speed of gliding, and thus the angle of incidence which gives the minimum gliding angle may not give the minimum rate of falling. Curve 7 (fig. 26) may easily be constructed. Thus, when the angle of incidence of the wings is equal to 6° the gliding angle is 1 in 7·7, and the speed of gliding is 4590 feet per minute, so that the distance fallen in one minute is 596 feet. The fall per minute is a minimum when the angle of incidence is equal to 10° approximately, and the corresponding speed is 44·5 miles per hour.

Climbing.—When a machine climbs with uniform velocity, H, D, L, and W are in equilibrium.

That this is so may be readily seen from work considerations.

Adopting the usual notation (see fig. 31), let the machine have a uniform velocity of U feet per second.

Hence,

Work done by the engine in unit time = HU foot-lbs.
Work done against gravity in unit time = $U \sin\theta \, . \, W$ foot-lbs.
Work done against wind resistance in unit time = U . D foot-lbs.

And $\therefore \quad H\,.\,U = U\,.\,\sin\theta\,.\,W + U\,.\,D,$

and $$H = W \sin\theta + D \qquad (1)$$

Furthermore, the lift L does no work, because the velocity of the machine in the direction of L is zero. The lift L merely supports $W \cos\theta$ the component of the weight,

and $$\therefore \quad W \cos\theta = L \qquad (2)$$

Equations (1) and (2) are the equilibrium equations when the forces are resolved along, and perpendicular to the direction of the wind. If U_C be the uniform speed of climbing a slope of inclination θ, and U_H be the horizontal speed corresponding to the same setting of the elevator, then $U_C = U_H \sqrt{\cos\theta}$, and as the value of θ is fairly small, U_C is not greatly different from U_H. A glance at curves 4 and 5 of fig. 26 shows that the aviator may alter the angle of the climbing path by operating the elevator. Thus if the engine is running all out, and the machine is in horizontal flight at the angle of incidence given by the point A, the machine will climb if the angle of incidence be increased, although the engine-power remains constant. The machine climbs most rapidly when the angle of incidence of the wings is equal to $6\frac{1}{2}°$, so that an increase of an angle of incidence which has a greater value than $6\frac{1}{2}°$ will cause the machine to climb more slowly. The maximum limit of the angle of incidence should be that corresponding to the maximum rate of climbing.

Starting.—Initially the elevator is placed in a position which corresponds to a small angle of incidence of the wings, and also small head resistance. When the machine has a sufficiently great ground speed, the pilot adjusts his elevator so that the wings may have a large angle of incidence. The machine now rises, and with the disappearance of the ground friction, coupled with the fact that the wings have a large angle of incidence and that the engine is working at full power, the machine continues to ascend. When a sufficient altitude has been reached, the elevator is adjusted to give the correct angle of incidence for horizontal flight, usually an operation of skill. Obviously, starting and also alighting are most efficiently performed in a head wind.

Alighting.—Firstly, the rate of falling of the machine must be reduced to a minimum. When the engine is running, the descent is easily regulated by a combined use of the elevator and throttle. When the engine is out of action, the angle of incidence of the wings should give the gliding angle which corresponds to the minimum rate of falling.

When the machine is approaching the ground the aviator adjusts either the elevator or the engine, or both, according to his particular fancy, so as to flatten out the flight path.

The ground speed of the machine is gradually diminished by the ground friction of the wheels and skids, and may be further checked by an increase of the angle of incidence of the wings.

Turning.—It is of the greatest importance to realise that the action of the rudder is only effective when the machine has an adequate keelplate. The term "keelplate" includes all the portions of the machine which are capable of offering resistance to lateral motion. Thus, wings which have a dihedral angle assist the keelplate action. To the first order of approximation a turn of the rudder of an aeroplane which has

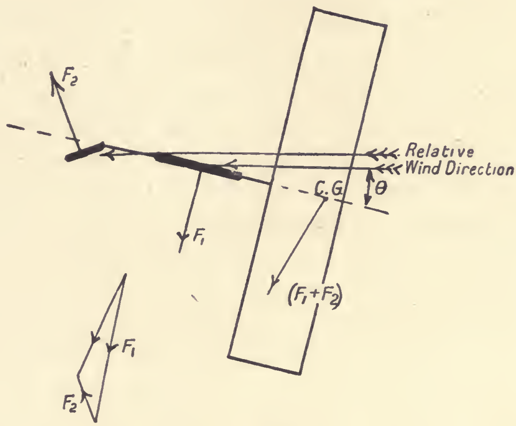


Fig. 32.

no keelplate—an imaginary aeroplane of course—would result in an equal deflection of the longitudinal axis of the machine, with no appreciable alteration in the direction of motion of the centre of gravity of the machine. Briefly, the rudder action on such a machine would be primarily a turning action, and secondarily, a side-slip. A deflection of the rudder of an aeroplane introduces a force at the centre of gravity of the machine, which tends to damp out the original motion. We shall now consider how the keelplate of an up-to-date machine facilitates turning. Fig. 32 represents the forces acting on the machine when the turning action has just commenced, so that the rudder action has had sufficient time to rotate the machine through a small angle about a normal axis. When the rotation of the machine about its normal axis is completed the algebraic sum of the moments of the two forces F_1 and F_2 about the centre of gravity is equal to zero, and we are left with a force (F_1+F_2) acting at the centre of gravity. This force curves the trajectory of the centre of gravity until the centrifugal force thus introduced just balances the centripetal force. Obviously the greater (F_1+F_2), the smaller is the radius of the turn. Ordinarily, the area of the keelplate is large, and its centre of pressure is fairly close to the centre of gravity of the machine, whilst a rudder of small area is placed a long distance from the centre of gravity. If the centre of pressure of the keelplate area coincide with the centre of gravity of the machine, then at the commencement of the turn the angle between the relative wind direction and the longitudinal axis of the machine will be theoretically equal to the angle through which the rudder has been turned. The component of the force (F_1+F_2), which acts in the direction of the longitudinal axis, increases the head resistance, and hence the machine will drop unless the angle of incidence of the wings is increased by the operation of the elevator. It is apparent that any tendency to drop is minimised by the use of a high keelplate: the component of (F_1+F_2) along the longitudinal axis is then above the centre of gravity and its moment increases the angle of incidence of the wings. The above discussion, although interesting and instructive, is not wholly applicable to modern machines. Moreover, a good many machines which have small keelplate areas are not entirely dependent upon the action of the rudder to enable them to turn.

Generally, when a machine turns, the outside wing, which moves at a greater speed than the inside one, rises relatively to the latter, and the machine heels over or banks. The lift component of the air force acting on the wings has now a lateral component which must be added to the centripetal force due to the keelplate action. Furthermore, the increased drift of the outside wing has a tendency to turn the machine in the opposite direction, and such an antagonistic tendency can only be balanced by a suitable manipulation of the rudder. If the machine has an appreciable bank, the vertical component of the lift will be less than the weight. Also when the rudder is deflected for a turn the longitudinal component of (F_1+F_2) increases the drag. To counteract the disastrous consequences of the reduced lift and increased drag, the elevator and engine are adjusted, prior to a turning manœuvre, to the conditions which, under ordinary circumstances, would favour climbing. Turning at a small elevation, unless skilfully performed, is a dangerous operation. At the commencement of this article we ignored the rolling action due to the difference between the relative wind speeds of the wings, and it is now of theoretical interest to consider the banking angle of a machine which has no keelplate.

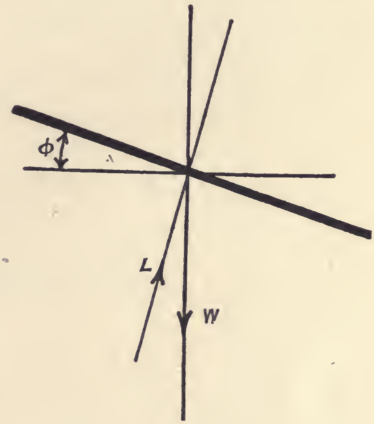


FIG 33.

We shall adopt the bold assumption that the machine turns without dropping. Then, if the turning speed U be uniform, and the angle of bank, ϕ, a constant, we have, if r be the radius of the turn (see fig. 33),

$$L \cos \phi = W \quad . \quad . \quad . \quad . \quad (1)$$

$$L \sin \phi = \frac{WU^2}{rg} \quad . \quad . \quad . \quad . \quad (2)$$

And $\quad \therefore \quad \tan \phi = \frac{U^2}{rg}$; if ϕ be small, $\phi = \frac{U^2}{rg}$.

To keep this theoretical angle of bank within reasonable limits, it is desirable that the turning speed of the machine shall not be too great. Owing to the keelplate action of a well-constructed machine the angle of bank is much less than $\left(\frac{U^2}{rg}\right)$. The gyroscopic action of the propeller has also to be considered during turning operations. Thus, imagine an aeroplane to be driven by a left-handed tractor, then the gyroscopic action of the propeller during a left-handed turn will cause the machine to dive. Similarly, a right-handed turn of this machine will increase the angle of incidence of the wings.

Motion of a Hydro-Aeroplane on the Water.—At any time during the motion of the machine on the surface, the forces acting are shown in fig. 34, where

W = weight of the machine.
H = propeller thrust.
L = lift on the wings.
D = drag of the machine (including air resistance of the floats).
B = buoyancy of the floats.
F = water resistance of the floats.

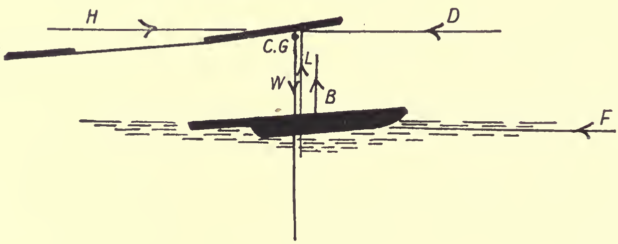


Fig. 34.

The curves of fig. 35 are taken from *The Report of the Advisory Committee for Aeronautics*, 1912–13.

The dependence of the water resistance F of the float upon the area of the wetted surface, and the square of the speed through the water, clearly accounts for the well-defined shape of curve 3.

The Behaviour of an Aeroplane in a Regular Wind.—We shall assume that for any arbitrarily fixed position of the controls, and any arbitrarily fixed value of the propeller thrust, an aeroplane is in stable flight and moving with a velocity U through still air. Now, imagine the dead calm of the atmosphere to be instantaneously converted into a regular wind of velocity u (U and u are not necessarily horizontal). Then, if the position of the controls and the value of the propeller thrust have remained unaltered, the aeroplane, after some time, will be in the same flying configuration, relatively to the air, as formerly, so that the new velocity of the machine relatively to the ground is $(U+u)$. An illustration may further enlighten the reader. Imagine a portion of the atmosphere to be enclosed in an exceedingly large railway carriage. Then once the carriage has attained a uniform velocity, the bodies in and the air of the compartment are not acted upon by any force due to the movement of the carriage, so that a man in the compartment regards the air of the compartment as stationary. An aeroplane flying in the compartment would appear to the man to be manœuvring in a dead-calm air. A spectator outside the compartment knows that the compartment, man, and the aeroplane have the additional velocity of the train; and, furthermore, he regards the air of the compartment as a wind

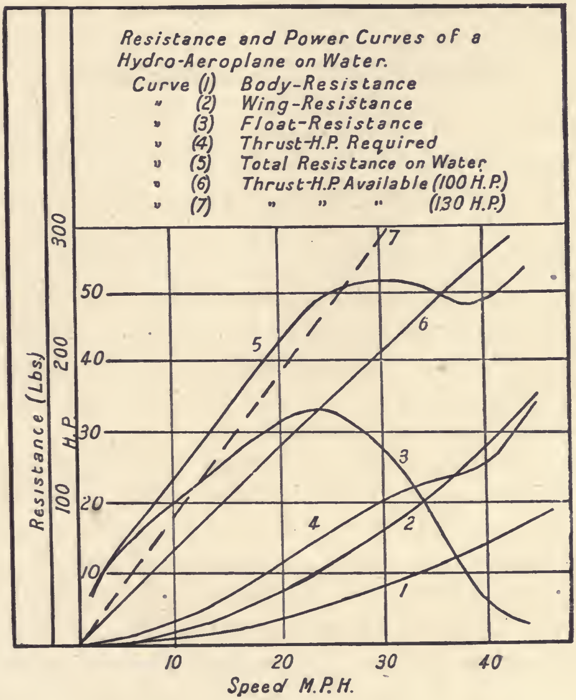


FIG. 35.

of uniform velocity. The velocity of the aeroplane to the spectator standing by the track (that is, its velocity relatively to the earth) will be different from the apparent velocity to the man in the compartment—or, in other words, the velocity of the machine relatively to the air in the compartment—by the velocity of the train.

Hence to an observer on the earth, the gliding angle of a machine in a horizontal following wind will appear to be less than the gliding angle of the same machine in a horizontal head wind.

CHAPTER VI.

STABILITY OF AN AEROPLANE.

Introduction.—At present but little is known of the stability of an aeroplane, and in view of the extreme difficulty of the subject, which necessitates a limited number of investigators, rapid progress in the future is not to be expected. Mathematical ability, coupled with a sound engineering experience, must be the equipment of one who wishes to further develop the present knowledge of aeroplane stability. The successful designer of the future will be able to predict the probable behaviour of his machine, no matter in what adverse circumstances the machine may be placed.

F. W. Lanchester, one of the pioneers of aeronautical research, has done a great deal of useful stability work, although he was undoubtedly handicapped by the lack of experimental evidence and the general apathy of the scientific world towards the study of aeronautics. Later, much excellent work has been done by Bryan, Crocco, Ferber, Bairstow, and others. Bairstow, who has had the advantage of aeronautical research work at the National Physical Laboratory, has written many admirable papers upon the mathematics of stability.

Definition of Stability.—Bairstow has proposed the following definition of a stable machine :—

" A stable machine is one which from any position in the air into which it may have got, either as a result of gusts or the pilot's use of controls, shall recover its correct flying position and speed when the pilot leaves the machine to choose its own course, with free or fixed controls, according to the character of the stability."

The definition presumes a sufficient height above the ground to enable the machine to perform the necessary righting manœuvres. In view of the possible action of gusts when the machine is near the ground, it would appear that a slightly stable machine, which can only right itself by virtue of a great fall, is not all that is desirable. In such a case the pilot would need to operate the controls, so that a combination of the skill of the pilot and the stability characteristics of the machine would secure safety in quite a limited range of action. It is the general impression that the pilot's final corrective manœuvres may be occasionally necessary even if the machine be itself inherently stable. The degree of stability of a machine is dependent, more or less, upon the presence of the desirable aerodynamic qualities which are conducive to the safety of the machine. The relative importance of these "desirable" characteristics will be greatly influenced by the weather conditions, and the various complications of aeroplane manœuvres. The ideal machine enables all safety manœuvres to be successfully performed in an "airworthy" manner, irrespective of the vagaries of the weather. If a stable aeroplane be moving in a uniform manner relatively to the air, and, as a result of a small external temporary disturbance—say, a gust—the machine is made to adopt a new configuration relatively to the air, then the forces and reactions acting upon the machine will tend to restore the original motion. In an unstable machine the new external forces would introduce other forces which encourage a further departure from the initial flying conditions. A machine which has no tendency to return to its original condition, or depart from its slightly displaced new condition, is said to be neutral.

A machine which possesses small instability continually tends to depart from its normal flying condition, the continual application of correcting manœuvres entailing a great strain upon the pilot.

An inherent characteristic of a stable machine is the maintenance of speed, although a sufficient height of the machine above the ground is required. A machine which is over-stable is insensitive to control, and is difficult to manage in gusty weather in consequence of the tendency of the machine to lie in the direction of the relative wind—that is, to adopt an altered attitude relatively to the earth with each new direction of the relative wind. Hence a stable machine turns into the relative wind, and such good "weathercock" stability is only obtained when the longitudinal position of the centre of gravity is well forward of the longitudinal position of the centre of pressure of the machine, when the longitudinal axis of the machine faces down the wind. To prevent discomfort to the pilot, weathercock or directive stability should not be too pronounced.

A machine which possesses good inherent stability does not encourage the development of large oscillations, as the damping forces become more effective with large and rapid oscillations. Moreover, inherent stability is attained without the aid of any moving parts, and depends upon the correct size and position of the several fins and surfaces. The resistances offered by the wings, keelplate, tail, rudder, and fins of a stable machine readily damp out any oscillations. Great inherent stability is not desirable in gusty weather, the big initial disturbances imposing great discomfort upon the pilot.

Automatic Stability.—The automatic devices are only called into play after the oscillations have started, and there is a lag between the cause and the corrective action, so that "hunting" of the machine may be a disastrous consequence. Although automatic stability may be a great aid to the pilot, it should not be regarded as a substitute for inherent stability. It would appear that "automatic controllability" would be a more appropriate term than "automatic stability." Automatic devices should be capable of being rapidly thrown out of action to enable the pilot to perform easily extreme manœuvres.

A serviceable machine for gusty weather should have a small stability factor, with good damping and small restoring couple, so that the pilot can effect a ready recovery after the action of a gust. If the periodic disturbances due to wind gusts synchronise with the periods of the machine, big, and maybe dangerous, oscillations may be set up.

Dynamic stability is dependent upon all the forces acting upon a moving body, whereas statical stability is only concerned with the forces acting upon a stationary body. Generally, the motion of an aeroplane may be considered to be one main translational velocity upon which is superposed small translational and angular velocities. A machine, when slightly displaced from its normal flying position, oscillates, and hence, if the general direction of the wind remain unaltered, each portion of the machine will have a new velocity relatively to the air.

The forces acting on the machine at any instant are thus a function of the motion of the machine, so that when speaking of the stability of an aeroplane, we mean dynamical stability. It is possible for a body which is statically stable to be dynamically unstable. Thus, the well-known experiment of producing unstable oscillations in a pendulum by the action of a succession of small impulses of the same period as the natural period of the pendulum is a case of dynamical instability. Nevertheless, a pendulum is always statically stable.

The Stability of an Aeroplane.—We shall nowproceed to examine the stability of an aeroplane, when the path of the centre of gravity is rectilinear, and in the plane of symmetry of the machine, the plane of symmetry being vertical. Firstly, it is necessary to define the various oscillations common to an aeroplane. Fig. 36 gives a sketch of the three principal axes OX, OY, OZ of an aeroplane. These axes, at right angles to each other, pass through the centre of gravity of the machine, and are fixed in the machine.

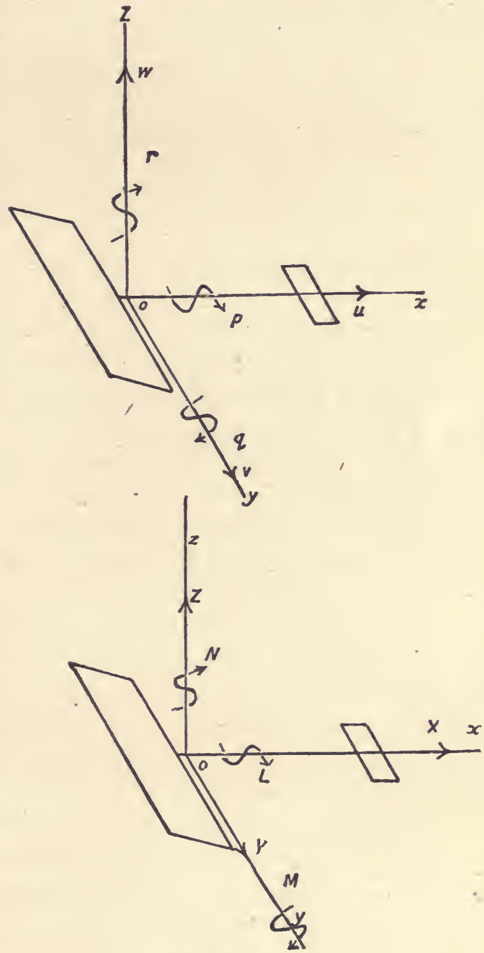


Fig. 36.

The following nomenclature has been adopted at the National Physical Laboratory :—

TABLE III.

Axis.	Name of Axis.	Translational Velocity along Axis.	Rotational Velocity about Axis.	Name of Force Acting along the Axis.	Symbol of Force.	Name of Motion which takes place about Axis.	Symbol for Angle of Rotation.	Name of Moment about Axis.	Symbol of Moment.
x	Longitudinal	u	p	Longitudinal	X	Rolling	ϕ	Rolling	L
y	Lateral	v	q	Lateral	Y	Pitching	θ	Pitching	M
z	Normal	w	r	Normal	Z	Yawing	ψ	Yawing	N

The air forces X, Y, Z, and the air moments L, M, N, are functions of $p, q, r, u, v, w, \theta, \phi, \psi$.

The rigid mathematical discussion of the stability of the motion of an aeroplane is of some difficulty, as the general equations of motion for oscillations about a state of steady motion give rise to a linear differential equation of the eighth order. If the steady motion does not involve rotation, the equation may be split into two fourth-order linear differential equations, one equation dealing with symmetrical or longitudinal oscillation, and the second equation with asymmetrical or lateral oscillation.

The standard methods for the examination of the motions of a machine for small oscillations as given by Routh, and as first applied by Bryan, are used in the following investigation. Bairstow has investigated the stability of the motion of a modern machine, and the following treatment of the subject has been based upon his work.

It has been assumed that the forces and couples acting on the machine are functions only of the position of the machine relative to the vertical, and of the linear and angular velocities of the machine.

For convenience of reference we state now the conventions adopted throughout the investigation.

(a) The equations of motion are referred to rectangular axes OX, OY, OZ fixed in the body of the aeroplane. The axis OX passes through the centre of gravity of the machine parallel to the thrust of the airscrew.

(b) The linear velocities u, v, w, and the angular velocities p, q, r, are referred to the body axes.

(*c*) The angular velocity of steady motion about the vertical is denoted by $\dot{\psi}$.

(*d*) The position of the aeroplane relative to the vertical is completely defined by the direction cosines N_1, N_2, N_3, of the vertical relative to the body axes. To be consistent, the upward direction of the vertical is taken as positive.

(*e*) Θ, Φ, Ψ are the angular co-ordinates of Euler, and show how the position of the aeroplane in space can be reached by a definite succession of rotational movements.

(*f*) The mathematical conventions as to the signs of the linear and angular velocities are indicated in fig. 36.

(*g*) The gravitational forces are defined by the position of the vertical relative to the body axes.

(*h*) The air forces are dependent entirely on the linear and angular velocities of the machine relative to the air.

(*i*) A capital letter with a suffix 0 denotes an "equilibrium" value of a variable.

(*j*) A, B, C are the moments of inertia about the axes OX, OY, OZ.

(*k*) D, E, F are the products of inertia about the axes OX, OY, OZ. [D=F=0.]

(*l*) Since in actual flight, the sign of U_0 is always negative, it is convenient to write ${}_1U_0 = -U_0$.

At any instant of steady flight, not necessarily horizontal flight, the positions of the body axes are given by OZ_0, OX_0, OY_0. (See fig. 36A.) Now imagine the machine to have a slight disturbance, so that after any small interval of time the body axes take the positions OZ, OY, OX. The new position of the aeroplane relative to the vertical can readily be specified by Euler's angular co-ordinates. If, initially, it be assumed that the body axes of the machine coincide with the gravitational axes OX_1, OY_1, OZ_1, then the final position of the body is obtained, firstly, by a rotation of the machine about OY_1, through an angle $(\Theta+\theta)$, the positions of the body axes are then given by OX, OY_1, Oz; and, secondly, by a rotation of the machine about OX through an angle ϕ, the final positions of the body axes being OX, OY, OZ.*

* There is no need in this investigation to introduce the Eulerian co-ordinate Ψ, that is, a rotation about the vertical, since Ψ merely changes the orientation of the machine. The gravitational forces and the air forces are unaffected by a change of the value of Ψ.

The direction cosines between the body axes OX, OY, OZ, and the axes OX_1, OY_1, OZ_1, are given in Table IIIA.

TABLE IIIA.

	OX.	OY.	OZ.
OX_1	$\cos(\Theta+\theta)$	$\sin\phi\sin(\Theta+\theta)$	$\cos\phi\sin(\Theta+\theta)$
OY_1	0	$\cos\phi$	$-\sin\phi$
OZ_1	$-\sin(\Theta+\theta)$	$\sin\phi\cos(\Theta+\theta)$	$\cos\phi\cos(\Theta+\theta)$

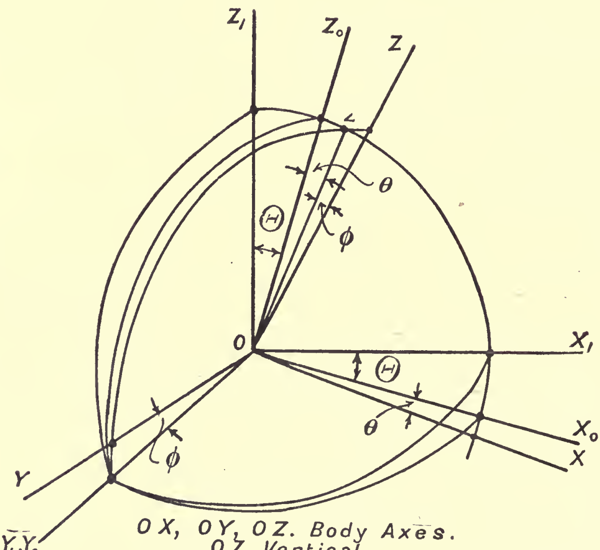


FIG. 36A.

In steady motion $\theta=0$, $\phi=0$, so that the directional cosines of OZ_1 with respect to the axes OX, OY, OZ are

$$-\sin\Theta,\ 0,\ \cos\Theta.$$

The conditions of equilibrium are

$$Z_0-g\cos\Theta=0 \quad . \quad . \quad . \quad . \quad (1)$$

$$X_0+g\sin\Theta=0 \quad . \quad . \quad . \quad . \quad (2)$$

$$Y_0=0 \text{ (no lateral force on machine)} \quad . \quad (3)$$

$$L_0=M_0=N_0=0 \text{ (no moment on machine)} \quad . \quad . \quad (4)$$

The machine in a disturbed position has linear velocities along the body axes of U_0+u, v, W_0+w, and angular velocities about these axes of p, q, r.

Since the disturbance is assumed to be quite small, each of the quantities $u, v, w, p, q, r, \theta, \phi$ has a small order of magnitude. The machine after such a disturbance performs small oscillations, and it is assumed, therefore, that p, q, r, u, v, w are each proportional to $e^{\lambda t}$.

Hence $$\frac{du}{dt}=\lambda u, \text{ etc.}$$

The angular velocities of the machine about OX, OZ_1, OY_1 are $\dot{\phi}$, $\dot{\psi}$, $\dot{\theta}$ respectively.

So that
$$p=\dot{\phi}-\dot{\psi}\sin\Theta \qquad (5)$$
$$q=\dot{\theta}\cos\phi+\dot{\psi}\sin\phi\cos\Theta \qquad (6)$$
$$r=\dot{\psi}\cos\phi\cos\Theta-\dot{\theta}\sin\phi \qquad (7)$$

From the above relationships we have, if ϕ be small,

$$\dot{\theta}=\lambda\theta=q \qquad (8)$$
$$\dot{\phi}=\lambda\phi=p+r\tan\Theta \qquad (9)$$

Since the air forces X, Y, Z, and the air moments L, M, N, are functions of the velocity components U_0+u, v, W_0+w, p, q, r, we may as a first approximation write,

$$X=X_0+uX_u+vX_v+wX_w+pX_p+qX_q+rX_r \qquad (10)$$

And so on for the other forces and moments.

X_u, Y_u, Z_u, . . . are known as Resistance Derivatives.

Hence the resistance derivative X_u, say, may be defined as a quantity proportional to the rate of change of X with u. There are thirty-six resistance derivatives, since each of the six quantities X, Y, Z, L, M, N may be expressed as a linear function of u, v, w, p, q, r, but from the symmetry of an aeroplane it follows that eighteen derivatives vanish. Thus, X, Z, M do not occur with suffixes v, p, r, and Y, L, N do not occur with suffixes u, w, q.

A Theoretical Consideration of the Values of the Several Resistance Derivatives.—It is now proposed to discuss the nature of the dependence of the resistance derivatives on the several parts of a machine, and to calculate numerical values. The values of the resistance derivatives substituted in the stability equations were calculated—with one or two exceptions—from experiments made on a model aeroplane in a wind channel.

The velocity of the wind relative to the machine $= -U_0 = {}_1U_0$.

Also we may write with good accuracy

$$mZ_0 = K_1 \,.\, A \,.\, (-U_0)^2 \,.\, a_0 = mg \cos \Theta,$$

where m = mass of the machine,

A = area of the wings,

and K_1 is a constant.

(mZ_0 is approximately equal to the lift on the machine.)

Z_u. The Rate of Change of Z with *u*.—If U_0 be increased to $U_0 + u$, the new value of Z_0 becomes

$$\frac{K_1 \,.\, A}{m}\left(-U_0 - u\right)^2 \,.\, a_0 = \frac{K_1 \,.\, A \,.\, (-U_0)^2 a_0}{m}\left(1 + \frac{2u}{U_0}\right)$$

$$= \left(1 + \frac{2u}{U_0}\right) g \cos \Theta.$$

And $\therefore \quad Z_u = \frac{2g}{U_0} \cos \Theta = -\frac{2g \cos \Theta}{{}_1U_0}.$

${}_1U_0$ has a value ranging from 60 feet per second to 150 feet per second, the corresponding values of Z_u in horizontal flight being $-1{\cdot}0$ and $-{\cdot}4$.

X_u. The Rate of Change of X with *u*.—The variation of the thrust of the airscrew is not considered. At the normal angle of incidence the ratio of $\frac{X_0}{Z_0}$, is the same as the ratio of drag to lift, and varies from $\frac{1}{6}$ to $\frac{1}{8}$.

We have then $\frac{X_u}{Z_u} = \frac{X_0}{Z_0}$, and X_u may be between $-{\cdot}20$ and $-{\cdot}05$, the higher value corresponding with a low-speed machine.

M_u. Variation of Pitching Moment with Forward Velocity.—As the direction of the wind undergoes no change, the pitching moment M will remain zero.

Z_w. The Variation of the Normal Force due to a Normal Velocity of the Machine relative to the Wind.—A small upward velocity w reduces the angle of incidence of the wings by $\tan^{-1}\left(\frac{w}{{}_1U_0}\right)$, that is by $\left(\frac{w}{{}_1U_0}\right)$.

The new value of mZ is

$$K_1 \,.\, A \,.\, ({}_1U_0)^2\left(a_0 - \frac{w}{{}_1U_0}\right) = mg\cos\Theta\left(1 - \frac{w}{a_0 \cdot {}_1U_0}\right).$$

And $$\therefore \quad Z_w = -\frac{g\cos\Theta}{{}_1U_0 \,.\, a_0}.$$

The value of a_0 varies from $\frac{1}{8}$ to $\frac{1}{20}$ radian, the lower value corresponding with a high-speed machine.

The value of Z_w varies from $-1\cdot5$ to $-4\cdot5$.

X_w. Variation of the Longitudinal Force due to Normal Velocity.—The longitudinal force is the algebraic sum of the components along the axis of X, of the lift and drag forces on the machine.

If the machine be flying at the angle of minimum drag, the change of force is $Z_0\frac{w}{{}_1U_0}$, *i.e.* $X_w = \frac{Z_0}{{}_1U_0} = \frac{g\cos\Theta}{{}_1U_0}$, and varies from 0·5 to 0·2.

X_w depends also on the body resistance. For actual machines X_w will probably be between 0 and 0·4.

M_w. Variation of Pitching Moment with Normal Velocity.—The pitching moment M chiefly depends on the elevator and tail planes, although the pitching moment due to the movement of the centre of pressure of the wings and fin action of the body has also to be considered. High-speed machines, which probably have a small angle of incidence of the wings, have very large tails to counterbalance any pitching moment due to pitching introduced by the displacement of the centre of pressure of the wings. Usually the area of the tail is about one-tenth the area of the wings, and with such an aeroplane the pitching moment due to the wings is small compared with that due to the tail.

As before, $mZ_0 = K_1 \,.\, a_0 \,.\, A \,.\, {}_1U_0^2$.

The increase of the angle of incidence of the tail due to an upward normal velocity $w = +\frac{w}{U_0} = -\frac{w}{{}_1U_0}$.

If the area of the tail is $\frac{A}{10}$, the increase of the upward normal force on the tail $= -K_1 \,.\, \frac{w}{{}_1U_0} \,.\, \frac{A}{10} \cdot {}_1U_0^{\,2}$.

$$= -\frac{g \,.\, \cos\Theta \,.\, w}{10 \,.\, a_0 \cdot {}_1U_0}.$$

Moreover, if the distance of the centre of pressure of the tail from the centre of gravity of the machine be 15 feet,

$$M_w = +\frac{15g \cdot \cos\Theta}{10 \cdot a_0 \cdot {}_1U_0}.$$

M_w usually varies from 2·0 to 8·0.

During a *vol-piqué* the angle of incidence of the wings is small, and even a small variation in such a small angle would appreciably affect the position of the centre of pressure of the wings, so that a slight modification of the above discussion may be necessary.

Z_q. The Variation of Normal Force with Pitching.—This coefficient is of small importance in the stability equation, and depends mainly on the up-and-down movement of the tail.

If the distance of the centre of pressure of the tail from the centre of gravity of the machine be 15 feet, the increase of the normal force on the tail due to a small pitching velocity q is $+\frac{15 \cdot q \cdot g \cdot \cos\Theta}{10 \cdot {}_1U_0 \cdot a_0}$. So that

$$Z_q = \frac{15 \cdot g \cdot \cos\Theta}{10 \cdot {}_1U_0 \cdot a_0}.$$

X_q. The Variation of Longitudinal Force due to Pitching.—This is of small importance.

M_q. The Variation of the Pitching Moment due to Pitching.—This derivative has a large value, and depends upon the action of the wings, tail and fin area of the body.

The value of M_q due to the tail action is easily seen to be

$$\frac{-22{\cdot}5\, g \cdot \cos\Theta}{{}_1U_0 \cdot a_0}.$$

If the movement of the centre of pressure of the wing be large, the value of M_q may be $3\left[\frac{-22{\cdot}5\, g \cdot \cos\Theta}{{}_1U_0 \cdot a_0}\right]$.

M_q varies from -100 to -300.

Y_v. Variation of the Lateral Force due to a Side Slip or a Side Wind.—The value of Y_v depends upon the area of the body, rudder, vertical fins, etc., and also upon the dihedral angle of the wings. The total area affected by a side slip is about 70 square feet.

Y_v varies from $-0{\cdot}1$ to $-0{\cdot}4$.

L_v. Variation of Rolling Moment due to Side Slip.—L_v depends mainly on the dihedral angle, a side wind increasing the lift on one wing, and diminishing the lift on the other.

Referring to fig. 37, let

β = half the dihedral angle.

a_0 = Angle of incidence of the wings.

The relative wind now veers through an angle $\left(\frac{v}{{}_1U_0}\right)$,

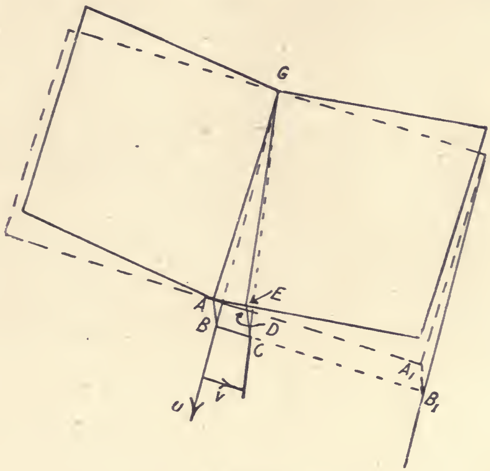


Fig. 37.

and $\therefore$ Angle $BGC = \frac{v}{{}_1U_0}$.

Angle $AGB = a_0$.

Angle $EAD = \beta$.

Hence the new angle of incidence = angle $EGC = \frac{ED + DC}{GC}$

$$= \frac{AD \cdot \beta + BG \cdot a_0}{BG} = a_0 + \frac{BG \cdot \frac{v}{{}_1U_0} \cdot \beta}{BG} = a_0 + \frac{v}{{}_1U_0}\beta.$$

Similarly, the angle of incidence of the second wing is equal to $a_0 - \frac{v}{{}_1U_0}\beta$.

Increase of lift on the wing towards which side slipping is taking place $= \frac{g \cos \Theta \cdot \beta \cdot v}{2 \cdot {}_1U_0 \cdot a_0}$.

Decrease of lift on the other wing $=\dfrac{g\cos\Theta\,.\,\beta\,.\,v}{2\,.\,{}_1U_0\,.\,a_0}$.

Hence, rolling moment $=\dfrac{g\cos\Theta.\beta\,.\,v\,.\,d}{2\,.\,{}_1U_0\,.\,a_0}$ approximately, where d = length of one wing.

And $\therefore\quad L_v=\dfrac{g\cos\Theta\,.\,\beta\,.\,d}{2\,.\,{}_1U_0\,.\,a_0}$.

L_v varies from 0·4 to 2·0 when $\beta=2°$.

N_v. Variation of Yawing Moment with Side Slip.—This coefficient depends upon the position of the centre of pressure of the rudder with respect to the centre of gravity of the machine, and is rather difficult to determine.

Y_p. Variation of Lateral Force due to Rolling.—A roll increases the angle of incidence of the falling wing and decreases the angle of incidence of the rising wing.

Assume that b = breadth of the wings. Now consider an element of the wing, of area $(b\Delta y)$, at a distance y from the longitudinal axis.

Increased lift on the falling element due to a rolling velocity p

$$=K_1\,.\,\frac{py}{{}_1U_0}\,.\,{}_1U_0^2\,.\,(b\,dy).$$

And $\therefore$ the increase of lateral force due to this element

$$=K_1\,.\,\frac{py}{{}_1U_0}\,.\,(b\,dy)_1U_0^2\beta=\frac{Z_0\,.\,b\,dy\,.\,\beta\,.\,py}{a_0\,.\,A\,.\,{}_1U_0},$$

so that increase of lateral force Y due to the rolling of the wings

$$=2\int\frac{Z_0\,.\,b\,dy\,.\,\beta\,.\,py}{a_0\,.\,A\,.\,{}_1U_0}=\frac{2Z_0\,.\,p\,.\,\beta}{a_0\,.\,A\,.\,{}_1U_0}\int y\,b\,dy,$$

$$=\frac{Z_0\,.\,\beta\,.\,d\,.\,p}{2\,.\,a_0\,.\,{}_1U_0}=\frac{g\cos\Theta\,.\,d\,.\,\beta\,.\,p}{2\,.\,a_0\,.\,{}_1U_0}\text{ approximately,}$$

since $$\int y\,b\,dy=\frac{A}{2}\,.\,\bar{y}=\frac{A\,.\,d}{4}.$$

A theoretical value of Y_p is, therefore, $\dfrac{g\cos\Theta\,.\,d\,.\,\beta}{2\,.\,a_0\,.\,{}_1U_0}$.

Y_p is of small importance and varies from 1 to 3.

L_p. Variation of Rolling due to Rolling.—Proceeding as in the case of Y_p, it is obvious that a rolling velocity p introduces a rolling moment

$$-2\int\frac{Z_0 \, . \, b\,dy \, . \, yp \, . \, y}{a_0 \, . \, A \, . \, {}_1U_0} = -\frac{2Z_0 \, . \, p}{a_0 \, . \, {}_1U_0 \, . \, A}\int y^2\, b\,dy$$

$$= -\frac{Z_0 \, . \, d^2 \, . \, p}{3 \, . \, a_0 \, . \, {}_1U_0} \text{ approximately,}$$

since $$\int y^2\, b\,dy = \frac{A \, . \, d^2}{6}.$$

Hence $$L_p = -\frac{d^2 \, . \, g \, . \, \cos\Theta}{3 \, . \, a_0 \, . \, {}_1U_0}.$$

The L_p of an actual machine varies from -200 to -400.

N_p. Variation of Yawing Moment due to Rolling.—N_p is proportional to the slope of the longitudinal force curve. The rolling of the body and rudder has a small effect on the yawing moment.

The limits of N_p are $+40$ and 0.

Y_r. Variation of the Lateral Force due to Yawing.—Working upon the same lines as previously, we find the increase of lift of the element $b\,dy$ of the wings due to a yawing velocity r to be

$$\frac{Z_0 \, . \, 2yr \, . \, b\,dy}{{}_1U_0 \, . \, A}.$$

Therefore, the increase of Y due to a small yawing velocity r

$$= -2\int\frac{Z_0 \, . \, 2yr \, . \, b\,dy}{{}_1U_0 \, . \, A}\beta = -\frac{Z_0 \, . \, \beta \, . \, d \, . \, r}{{}_1U_0} \text{ approximately.}$$

And $$\therefore \quad Y_r = \frac{-g\cos\Theta \, . \, \beta \, . \, d}{{}_1U_0}.$$

Y_r varies from -1 to -4.

The rudder will also influence the value of Y. Stability conditions are not greatly affected by the value of the lateral force due to yawing.

L_r. Variation of the Rolling Moment due to Yawing.—Increase of the rolling moment L due to the wings

$$= 2\int\frac{Z_0 \, . \, b \, . \, dy \, . \, 2yr \, . \, y}{A \, . \, {}_1U_0} = \frac{4 \, . \, Z_0 \, . \, r}{A \, . \, {}_1U_0}\int y^2\, b\,dy$$

$$= \frac{2 \, . \, Z_0 \, . \, d^2 \, . \, r}{3 \, . \, {}_1U_0},$$

so that $$L_r = +\left(\frac{2 \, . \, Z_0 \, . \, d^2}{3 \, . \, {}_1U_0}\right) = \frac{2g\cos\Theta \, . \, d^2}{3 \, . \, {}_1U_0}.$$

L_r for modern machines varies from $+50$ to $+100$.

N_r. Variation of Yawing Moment due to Yawing.—The part of N_r due to the wings may vary from -5 to -20, whilst the part due to the body and rudder is very variable.

N_r for the whole machine varies from -20 to -60.

Resistance Derivatives due to the Airscrew. X_u.—An approximate value of X_u is $\frac{H}{{}_1U_0 \cdot m}$, where H=thrust of the airscrew, and m=mass of the aeroplane.

If H be taken as -250 lbs., X_u varies from $-0{\cdot}12$ to $-{\cdot}04$.

Y .—A side wind introduces a lateral force on the airscrew. If the airscrew, as viewed from the pilot's seat, be a left-handed tractor, then, when the machine has a positive sideslip, the angle of incidence of the top blade is increased, and the angle of incidence of the bottom blade diminished. A side wind, then, introduces fluctuations in the values of the torque and the lateral force, with no appreciable change of thrust.

If the speed of the aeroplane be 80 feet/second, $Y_v = -{\cdot}01$ for an airscrew 8 feet diameter, rotating at 1150 R.P.M., and giving a thrust of 250 lbs.

N_v.—If the airscrew be 6 feet ahead of the centre of gravity of the machine, $N_v = -6Y_v = {\cdot}06$.

N_r.—The sideway velocity of the airscrew due to yawing $= -6r$, so that $N_r = -0{\cdot}4$.

Other derivatives may be calculated similarly.

We shall now commence a mathematical analysis of the stability of the motion of an aeroplane.

The general equations of motion * of the aeroplane are—

$$m[\dot{u} + (W+w)q - vr] = mX \quad . \quad . \quad . \quad (11)$$
$$m[\dot{v} + (U+u)r - (w+W)p] = mY \quad . \quad . \quad (12)$$
$$m[\dot{w} + vp - (U+u)q] = mZ \quad . \quad . \quad . \quad (13)$$
$$\dot{h}_1 - rh_2 + qh_3 = mL \quad . \quad . \quad . \quad . \quad (14)$$
$$\dot{h}_2 - ph_3 + rh_1 = mM \quad . \quad . \quad . \quad . \quad (15)$$
$$\dot{h}_3 - qh_1 + ph_2 = mN \quad . \quad . \quad . \quad . \quad (16)$$

where

$$\left.\begin{aligned} h_1 &= pA - qF - rE \\ h_2 &= qB - rD - pF \\ h_3 &= rC - pE - qD \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (17)$$

* These equations of motion are to be found in any standard text-book on Rigid Dynamics.

Neglecting quantities of the second order of magnitude and writing F=D=0, we have

$$\dot{u}+\mathrm{W}q=\mathrm{X} \quad . \quad . \quad . \quad . \quad (11\text{A})$$
$$\dot{v}+\mathrm{U}r-\mathrm{W}p=\mathrm{Y} \quad . \quad . \quad . \quad . \quad (12\text{A})$$
$$\dot{w}-\mathrm{U}q=\mathrm{Z} \quad . \quad . \quad . \quad . \quad (13\text{A})$$
$$\dot{p}\mathrm{A}-\dot{r}\mathrm{E}=m\mathrm{L} \quad . \quad . \quad . \quad . \quad (14\text{A})$$
$$\dot{q}\mathrm{B}=m\mathrm{M} \quad . \quad . \quad . \quad . \quad (15\text{A})$$
$$\dot{r}\mathrm{C}-\dot{p}\mathrm{E}=m\mathrm{N} \quad . \quad . \quad . \quad . \quad (16\text{A})$$

Making use of the preceding data, the equations for small oscillations may now be written down. Equation (11A) becomes

$$\lambda u+\mathrm{W}q=g\sin(\Theta+\theta)+\mathrm{X}_0+u\mathrm{X}_u+w\mathrm{X}_w+q\mathrm{X}_q\,.$$
$$=\theta\,.\,g\cos\Theta+u\mathrm{X}_u+w\mathrm{X}_w+q\mathrm{X}_q\,.$$
$$=\frac{q}{\lambda}\,.\,g\,.\cos\Theta+u\mathrm{X}_u+w\mathrm{X}_w+q\mathrm{X}_q \quad . \quad (11\text{B})$$

Equation (12A) becomes

$$\lambda v+\mathrm{U}r-\mathrm{W}p=-g\,.\,\phi\,.\cos\Theta+\mathrm{Y}_0+v\mathrm{Y}_v+p\mathrm{Y}_p+r\mathrm{Y}_r\,.$$
$$=\frac{-g\cos\Theta}{\lambda}(p+r\tan\Theta)+v\mathrm{Y}_v+p\mathrm{Y}_p+r\mathrm{Y} \quad (12\text{B})$$

Equation (13A) becomes

$$\lambda w-\mathrm{U}q=\mathrm{Z}_0-g\cos(\Theta+\theta)+u\mathrm{Z}_u+w\mathrm{Z}_w+q\mathrm{Z}_q\,.$$
$$=g\,.\,\frac{q}{\lambda}\sin\Theta+u\mathrm{Z}_u+w\mathrm{Z}_w+q\mathrm{Z}_q \quad . \quad . \quad (13\text{B})$$

Equation (14A) becomes

$$\lambda p\,.\,k_{\mathrm{A}}^2-\lambda r k^2_{\mathrm{E}}=\mathrm{L}_0+v\mathrm{L}_v+p\mathrm{L}_p+r\mathrm{L}_r\,.$$
$$=v\mathrm{L}_v+p\mathrm{L}_p+r\mathrm{L}_r \quad . \quad . \quad (14\text{B})$$

Equation (15A) becomes

$$\lambda q\,.\,k_{\mathrm{B}}^2=\mathrm{M}_0+u\mathrm{M}_u+w\mathrm{M}_w+q\mathrm{M}_q\,.$$
$$=u\mathrm{M}_u+w\mathrm{M}_w+q\mathrm{M}_q \quad . \quad . \quad . \quad (15\text{B})$$

Equation (16A) becomes

$$\lambda r\,.\,k_{\mathrm{C}}^2-\lambda p\,.\,k_{\mathrm{E}}^2=\mathrm{N}_0+p\mathrm{N}_p+r\mathrm{N}_r+v\mathrm{N}_p$$
$$=p\,.\,\mathrm{N}_p+r\mathrm{N}_r+v\mathrm{N}_v \quad . \quad (16\text{B})$$

Collecting the equations of motion and writing them in a convenient order we have—

$$\left.\begin{array}{llll}(\lambda-X_u)u & -wX_w & +\left(W_0-X_q-\frac{g}{\lambda}\cos\Theta\right)q & =0\\ -Z_u u & +(\lambda-Z_w)w & +\left(-U_0-\frac{g}{\lambda}\sin\Theta-Z_q\right)q & =0\\ -M_u u & -M_w w & (\lambda k_B{}^2-M_q)q & =0\end{array}\right\}\quad(17)$$

$$\left.\begin{array}{llll}(\lambda-Y_v)v & +\left(\frac{g\cos\Theta}{\lambda}-Y_p+W_0\right)p & +\left(U_0+\frac{g}{\lambda}\sin\Theta-Y_r\right)r & =0\\ -L_v v & +(\lambda k_A{}^2-L_p)p & +(-\lambda k_E{}^2-L_r)r & =0\\ -N_v v & +(-\lambda k_E{}^2-N_p)p & +(\lambda k_C{}^2-N_r)r & =0\end{array}\right\}\quad(18)$$

The generality of the above equations is unaffected and the solution is simplified if we write $W=0$.

Equations (17), which contain only the variables u, w, q, represent the longitudinal oscillations, whilst the lateral oscillations are given by the equations (18), which contain the variables v, p, r.

Hence the solution of the equations of the longitudinal oscillation is obtained from the determinant

$$\begin{vmatrix}\lambda-X_u, & -X_w, & -\lambda X_q-g\cos\Theta\\ -Z_u, & \lambda-Z_w, & -\lambda U_0-Z_q\lambda-g\sin\Theta\\ -M_u, & -M_w, & \lambda(\lambda k_B{}^2-M_q)\end{vmatrix}=0 \quad . \quad (19)$$

And, similarly, the solution of the equations of the lateral oscillation may be deduced from the determinant

$$\begin{vmatrix}\lambda-Y_v, & g\cos\Theta-Y_p\lambda, & \lambda U_0-Y_r\lambda+g\sin\Theta\\ -L_v, & \lambda(\lambda k_A{}^2-L_p), & \lambda(-\lambda k_E{}^2-L_r)\\ -N_v, & \lambda(-\lambda k_E{}^2-N_p), & \lambda(\lambda k_C{}^2-N_r)\end{vmatrix}=0 \quad . \quad (20)$$

Longitudinal Stability.—The solution of the determinantal equation (19) may be written as

$$A_1\lambda^4+B_1\lambda^3+C_1\lambda^2+D_1\lambda+E_1=0 \quad . \quad . \quad (21)$$

The motion will be stable if the real roots and the real parts of the complex roots of this algebraic equation of the fourth order are negative. These conditions are fulfilled if A_1, B_1, C_1, D_1, E_1, and Routh's Discriminant $(B_1C_1D_1-A_1D_1{}^2-B_1{}^2E_1)$ are each positive.

The values of the coefficients of the biquadratic for longitudinal oscillations are easily seen to be

$$A_1 = k_B^2.$$

$$B_1 = -(M_q + X_u k_B^2 + Z_w k_B^2)$$

$$C_1 = \begin{vmatrix} Z_w, & U_0 + Z_q \\ M_w, & M_q \end{vmatrix} + \begin{vmatrix} X_u, & X_q \\ M_u, & M_q \end{vmatrix} + k_B^2 \begin{vmatrix} X_u, & X_w \\ Z_u, & Z_w \end{vmatrix}$$

$$D_1 = - \begin{vmatrix} X_u, & X_w, & X_q \\ Z_u, & Z_w, & U_0 + Z_q \\ M_u, & M_w, & M_q \end{vmatrix} - g \begin{vmatrix} M_u, & -\sin\Theta \\ M_w, & \cos\Theta \end{vmatrix}$$

$$E = -g \begin{vmatrix} X_u, & X_w, & \cos\Theta \\ Z_u, & Z_w, & \sin\Theta \\ M_u, & M_w, & 0 \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

We now proceed to work out the longitudinal stability of the motion of a modern machine, which is in horizontal flight ($\Theta = 0$). The following data have been taken :—

Weight of machine = 1300 lbs.
Flying speed 80 feet per second = 55 miles per hour.
Area of wings about 300 square feet.

The values of the resistance derivatives are calculated from experiments on a model of the machine. The mass of the machine is taken as $\frac{1300}{g} = 40$. The forces are in lbs., and the moments in ft.-lbs. And thus there is no necessity to convert into poundals and ft.-poundals.

The values of the longitudinal resistance derivatives are

$$X_u = -0{\cdot}14 \qquad Z_u = -0{\cdot}80 \qquad M_u = 0$$
$$X_w = +0{\cdot}19 \qquad Z_w = -2{\cdot}89 \qquad M_w = 2{\cdot}66$$
$$X_q = \pm 0{\cdot}5 \qquad Z_q = 9{\cdot}0 \qquad M_q = -210{\cdot}0$$

also $k_B^2 = 25{\cdot}0$ and $\Theta = 0$.

Substituting the above values in equation (22), the equation of the longitudinal motion is

$$\lambda^4 + 11{\cdot}4\lambda^3 + 33{\cdot}6\lambda^2 + 5{\cdot}72\lambda + 2{\cdot}72 = 0 \ . \quad . \qquad (23)$$

All the coefficients are positive, and since Routh's discriminant has a positive sign, the longitudinal motion of this machine is stable.

Equation (23) may be written

$$(\lambda+5{\cdot}62\pm0{\cdot}45i)(\lambda+0{\cdot}0747\pm0{\cdot}283i)=0 \quad . \quad (24)$$

The reader will notice that some of the resistance coefficients have quite a small influence on the values of the coefficients A_1, B_1, C_1, D_1, and E_1. Considering only the important parts of each coefficient, approximate values of the coefficients are

$$\begin{aligned}
A_1 &= k_B{}^2 \\
B_1 &= -(M_q+Z_w k_B{}^2) \\
C_1 &= (Z_w M_q - U_0 M_w) \\
D_1 &= -M_q(X_u Z_w - X_w Z_u)+M_w(U_0 X_u - g\sin\Theta) \\
E_1 &= -gM_w(Z_u\cos\Theta - X_u\sin\Theta) \quad . \quad . \quad . \quad (25)
\end{aligned}$$

Further, if $B_1 < C_1$

$$A_1E_1 < \frac{1}{20}C_1{}^2, \text{ approximately,}$$

$$A_1D_1 < \frac{1}{20}B_1C_1 \text{ approximately,}$$

the biquadratic equation $A_1\lambda^4+\beta_1\lambda^3+C_1\lambda^2+D_1\lambda+E_1=0$ may be written approximately as

$$\left(\lambda^2+\frac{B_1}{A_1}\lambda+\frac{C_1}{A_1}\right)\left(\lambda^2+\left(\frac{D_1}{C_1}-\frac{B_1E_1}{C_1{}^2}\right)\lambda+\frac{E_1}{C_1}\right)=0 \quad . \quad (26)$$

Such approximations reduce the biquadratic equation to a much simpler form, and the influence of each resistance derivative on the stability of the motion of the aeroplane is more readily understood. Moreover, stability of any desired amount may be given to the machine by a suitable alteration of the principal derivatives. When the values of the resistance derivatives have been decided upon, the approximate solutions may be discarded, and a rigid solution of the biquadratic obtained.

Substituting the approximate values of A_1, B_1, and C_1, the first factor $\lambda^2+\frac{B_1}{A_1}\lambda+\frac{C_1}{A_1}=0$ of the biquadratic equation becomes $\lambda^2-\left(\frac{M_q}{k_B{}^2}+Z_w\right)\lambda+\frac{1}{k_B{}^2}\left(Z_wM_q-U_0\,.\,M_w\right)=0 \quad . \quad . \quad (27)$

The roots of this equation are

$$\frac{1}{2}\left[\left(\frac{M_q}{k_B{}^2}+Z_w\right)\pm\sqrt{\left(\frac{M_q}{k_B{}^2}-Z_w\right)^2+4\frac{U_0M_w}{k^{B2}}}\right],$$

that is, $(-5{\cdot}62\pm0{\cdot}45i)$, so that equation (27) represents a very heavily damped oscillation of a period of 14 seconds.

The resistance derivatives M_q and Z_w are always negative, and hence $\left(\frac{M_q}{k_B^2}+Z_w\right)$ has a negative sign.

If the term under the radical sign be positive and numerically greater than $\left(\frac{M_q}{k_B^2}+Z_w\right)$, there will be a positive root. This, however, is improbable, because M_w must be positive, and therefore $4U_0\frac{M_w}{k_B^2}$ is negative. As previously stated, M_q and Z_w are each negative, which makes $\left(\frac{M_q}{k_B^2}-Z_w\right)^2<\left(\frac{M_q}{k_B^2}+Z_w\right)^2$.

Under these circumstances it is highly improbable that the real parts of the roots of the equation will be positive, so that the resulting longitudinal motion must be stable. The reader will notice that g does not enter into the above roots, and also that the terms involved correspond with a motion which produces a change in the attitude of the machine relative to the wind. The motion is almost independent of variations of the flight speed.

The second factor of the biquadratic equation, namely,

$$\lambda^2+\left(\frac{D_1}{C_1}-\frac{B_1E_1}{C_1^2}\right)\lambda+\frac{E_1}{C_1}=0,$$

becomes for this machine

$$(\lambda^2+\cdot1495\lambda+\cdot0855=0),$$

that is,

$$\lambda+0\cdot0747\pm0\cdot283i=0 \quad . \quad . \quad . \quad (28)$$

It is seen that this motion is a slightly damped oscillation of period 22 seconds.

With good accuracy we may write the value of $\frac{E_1}{C_1}$ as

$$\frac{-g\,.\,Z_u\,.\,M_w}{Z_wM_q-U_0\,.\,M_w},$$

and since $Z_u=\frac{2g}{U_0}=-\frac{2g}{{}_1U_0}$,

$$\frac{E_1}{C_1}\text{ becomes equal to }\frac{2g^2}{{}_1U_0\cdot\frac{Z_w}{M_w}M_q+{}_1U_0^2}.$$

Lanchester, in his "phugoid" analysis, assumed $M_q=0$, so that in this case the period of oscillation $\frac{2\pi \cdot {}_1U_0}{\sqrt{2}g}$ is, that is, 11 seconds.

A good approximate condition for stability is $D_1C_1>B_1E_1$, that is,

$$(Z_wM_q+{}_1U_0M_w)(-M_q.X_u.Z_w+M_q.X_w.Z_u-M_w.{}_1U_0.X_u) > gM_wZ_u(M_q+k_B{}^2Z_w).$$

To favour discussion, assume $M_q=0$. (M_q is quite important, however.)

Hence $$-{}_1U_0{}^2 . M_w . X_u>g . Z_u . Z_w . k_B{}^2,$$

that is, $$\frac{{}_1U_0{}^2}{g}>-k_B{}^2 . \frac{Z_u}{X_u} . \frac{Z_w}{M_w} .$$

and $$\therefore \quad 4g>-\frac{k_B{}^2 . Z_u{}^3 . Z_w}{X_u . M_w} \quad . \quad . \quad . \quad (29)$$

$\frac{Z_u}{X_u}$ is approximately the ratio of lift to drag, so that the greater the aerodynamic efficiency of a machine, the more unstable will it become.

The importance of the righting moment due to the tail is shown by the presence of M_q. The motion given by the second factor of the biquadratic equation is a function of the variations of both the forward and normal speeds, the aeroplane rising above and falling below its mean flight path, with the slight alterations of the speed. The period of these oscillations is about 22 seconds. Longitudinal stability can always be obtained by using a sufficiently large tail, but the tail of a machine of a high speed range must not be too large, otherwise at high speeds the machine is rather difficult to control.

It is not a difficult matter to design a machine which shall be longitudinally stable, and we have seen that if such a stable machine suffer a slight disturbance, the trajectory of its centre of gravity is a damped oscillatory curve of a slow period, say, 20 seconds. In addition to this slow-period oscillation there is a second longitudinal oscillation, usually heavily damped, about the lateral axis of the machine. Comparatively slow oscillations may be anticipated by the pilot, and the correct manœuvre performed. Longitudinal stability also depends on the inclination of the flight path to the horizontal, and a modern machine is more stable when gliding than when climbing.

Also, it is important to realise that longitudinal stability increases with the speed of the machine. Nevertheless, a high speed is not conducive to comfort, unless the machine has been especially designed for speed, that is, the wings should have small area, and a normal angle of incidence.

Lateral Stability of the Motion of an Aeroplane in Horizontal Flight $[\Theta=0]$.—The values of the resistance derivatives, which influence the lateral stability of a machine, are

$$
\begin{array}{lll}
Y_v=-0{\cdot}25 & L_v=+0{\cdot}83 & N_v=-0{\cdot}54 \\
Y_p=1{\cdot}0 & L_p=-200{\cdot}0 & N_p=+28{\cdot}0 \\
Y_r=-3{\cdot}0 & L_r=65{\cdot}0 & N_r=-37{\cdot}0
\end{array}
$$

also $$k_A^2=25,\ k_C^2=35,\ k_E^2=0.$$

The solution of the determinant (20) may be written

$$A_2\lambda^4+B_2\lambda^3+C_2\lambda^2+D_2\lambda+E_2=0 \quad . \quad . \qquad (30)$$

where

$$A_2=\begin{vmatrix} k_A^2, & -k_E^2 \\ -k_E^2, & k_C^2 \end{vmatrix}$$

$$B_2=\begin{vmatrix} Y_v, & 0, & 0 \\ L_v, & -k_A^2, & -k_E^2 \\ N_v, & k_E^2, & k_C^2 \end{vmatrix} - \begin{vmatrix} k_A^2, & L_r \\ -k_E^2, & N_r \end{vmatrix} - \begin{vmatrix} L_p, & -k_E^2 \\ N_p, & k_C^2 \end{vmatrix}$$

$$C_2=\begin{vmatrix} Y_v, & Y_p, & 0 \\ L_v, & L_p, & -k_E^2 \\ N_v, & N_p, & k_C^2 \end{vmatrix} - \begin{vmatrix} Y_v, & 0, & Y_r-U_0 \\ L_v, & -k_A^2, & L_r \\ N_v, & k_E^2, & N_r \end{vmatrix} + \begin{vmatrix} L_p, & L_r \\ N_p, & N_r \end{vmatrix}$$

$$D_2=-\begin{vmatrix} Y_v, & Y_p, & Y_r-U_0 \\ L_v, & L_p, & L_r \\ N_v, & N_p, & N_r \end{vmatrix} +g\cos\Theta \begin{vmatrix} L_v, & -k_E^2 \\ N_v, & k_C^2 \end{vmatrix}$$

$$+g\sin\Theta \begin{vmatrix} L_v, & -k_A^2 \\ N_v, & k_E^2 \end{vmatrix}$$

$$E_2=-g\cos \begin{vmatrix} L_v, & L_r \\ N_v, & N_r \end{vmatrix} +g\sin\Theta \begin{vmatrix} L_v, & L_p \\ N_v, & N_p \end{vmatrix}$$

The equation of the lateral stability is

$$\lambda^4+9{\cdot}31\lambda^3+9{\cdot}81\lambda^2+10{\cdot}15\lambda-0{\cdot}161=0 \quad . \quad (31)$$

Although Routh's discriminant is positive, the negative sign of the coefficient E_2 shows that the motion is unstable.

Good approximate values of the coefficients are

$$A_2=k_A{}^2 \,.\, k_C{}^2 \,.$$

$$B_2=\begin{vmatrix} L_p, & k_A{}^2 \\ N_r, & -k_C{}^2 \end{vmatrix} .$$

$$C_2=\begin{vmatrix} L_p, & L_r \\ N_p, & N_r \end{vmatrix} +k_C{}^2 \,.\, Y_v \,.\, L_p+k_A{}^2 \,.\, N_v \,.\, U_0$$

$$D_2=-Y_v\begin{vmatrix} L_p, & L_r \\ N_p, & N_r \end{vmatrix} +U_0\begin{vmatrix} L_v, & L_p \\ N_v, & N_p \end{vmatrix} +g\cos\Theta \,.\, L_v \,.\, k_C{}^2$$

$$E_2=-g\cos\Theta\begin{vmatrix} L_v, & L_r \\ N_v, & N_r \end{vmatrix} +g\sin\Theta\begin{vmatrix} L_v, & L_p \\ N_v, & N_p \end{vmatrix}$$

If E_2 be less than $\frac{1}{20}$th of either B_2 or D_2, and $(B_2D_2-C_2{}^2)$ be less than $\frac{1}{20}$th of $C_2{}^2$, then an approximate solution to equation (30) is

$$\left(\lambda+\frac{E_2}{D_2}\right)\left(\lambda+\frac{B_2{}^2-A_2C_2}{A_2B_2}\right)\left(\lambda^2+\left(\frac{C_2}{B_2}-\frac{E_2}{D_2}\right)\lambda+\frac{B_2D_2}{B_2{}^2-A_2C_2}\right)=0 \quad (32)$$

The solution of equation (31) is

$$(\lambda-0{\cdot}0157)(\lambda+8{\cdot}265)(\lambda+0{\cdot}526\pm0{\cdot}984i)=0 \quad . \quad (33)$$

We shall now consider the first factor, $\left(\lambda+\frac{E_2}{D_2}\right)=0$, of the equation for lateral stability.

In all machines D_2 will be positive, and so the stability depends upon the sign of E_2. Considering the signs of the various derivatives, it follows that for stability L_vN_r should be numerically greater than L_rN_v. That is

$$\frac{L_v}{N_v} > \frac{L_r}{N_r}.$$

If $\frac{L_v}{N_v} < \frac{L_r}{N_r}$, spiral instability will result.

L_v, the rolling moment due to a side-slip, is dependent mainly upon the dihedral angle, whilst L_r, the rolling moment due to yawing, is due to the increased lift upon the other wing when turning. L_v may be increased by an increase of the dihedral angle, but L_r is difficult to control. If the machine has negative wing-tips, L_r will be diminished. The term $\frac{L_r}{N_r}$ may be better reduced by increasing N_r. N_r may be increased without affecting N_v by putting equal areas in front of and behind the centre of gravity of the machine. N_v can be reduced by using a smaller rudder, but there will be some loss of controllability.

The second factor $\left(\lambda + \frac{B_2^2 - A_2C_2}{A_2B_2}\right) = 0$ becomes for this machine $(\lambda + 8{\cdot}265) = 0$. It is extremely unlikely that $\left(\frac{B_2^2 - A_2C_2}{A_2B_2}\right)$ will be negative. The motion represented by this factor is a very heavily-damped subsidence, the damping being due to L_p, that is, rolling due to rolling.

The third factor of the equation is

$$\lambda^2 + \left(\frac{C_2}{B_2} - \frac{E_2}{D_2}\right)\lambda + \frac{B_2D_2}{B_2^2 - A_2C_2} = 0.$$

Owing to the particular relationships which exist between the values of A_2, B_2, C_2, D_2, and E_2, this equation may be written approximately $\lambda^2 + \frac{C_2}{B_2}\lambda + \frac{C_2^2}{B_2^2} = 0$, and the motion is a damped oscillation of period $2\pi\sqrt{\frac{4B_2^2}{3C_2^2}}$, that is, about 6 seconds, and damping $\frac{C_2}{2B_2}$.

There is not much doubt that with modern machines A_2, B_2, C_2, and D_2 will be positive, although some difficulty may be experienced in making E_2 positive.

Effect of the Gyroscopic Action of the Airscrew and Engine on the Stability of an Aeroplane.—When the airscrew is revolving the lateral and longitudinal oscillations are not independent of each other, although there is good reason to believe that the gyroscopic action has small influence on the stability of the motion of an aeroplane. A machine which is laterally stable without the airscrew running will remain stable when gyroscopic actions are considered.

The converse is also true, that is, an unstable machine, the stability of which has been calculated from resistance derivatives which ignore the gyroscopic action of the airscrew, cannot be made stable by rotating the airscrew. Owing to the gyroscopic interconnection of the longitudinal and lateral oscillations a machine longitudinally stable and laterally unstable, when the airscrew is not running, becomes also longitudinally unstable upon rotating the airscrew.

Although a complete investigation has not yet been made, it would appear that if a machine be inherently stable during

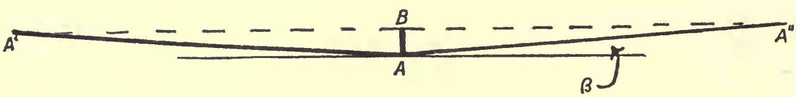


FIG. 38.

gliding flight, no trouble is to be anticipated from the gyroscopic action of the airscrew and the engine.

The centre of pressure of the keelplate, vertical longitudinal fins, rudder, etc., should be above the longitudinal axis passing through the centre of gravity. The dihedral angle, if suitably chosen, largely assists the lateral stability. It is worthy of note, that the dihedral angle A′AA″ (see fig. 38) gives much greater rolling and yawing moments, due to rolling and yawing, than the fin AB. A consideration of one case—that is, rolling due to rolling—may be regarded as typical of the rest.

Assume, as before, that the angular velocity of rolling $=p$. Increase of rolling moment due to the dihedral angle of the wings

$$=\frac{-Z_0 \cdot p \cdot d^2}{a_0 \cdot {}_1U_0 \cdot 3} \text{ approximately} \quad . \quad . \quad (34)$$

Now, the height of the fin AB $=d \cdot \beta$,

and the area of the fin AB $=\frac{A}{2} \cdot \beta$.

Hence, increase of rolling moment due to an element dA of this fin, when the distance of the element from the longitudinal axis $= z$,

$$= -K_1 \frac{zp}{{}_1U_0} dA \cdot {}_1U_0^2 \times z.$$

And $\therefore$ increase of rolling moment due to the fin

$$= \int -K_1 \frac{zp}{{}_1U_0} dA \cdot {}_1U_0^2 z = \frac{-K_1 \cdot {}_1U_0^2 \cdot p}{{}_1U_0} \cdot \int dA \cdot z^2$$

$$= -\frac{K_1 \cdot {}_1U_0^2 \cdot p}{{}_1U_0} \times \frac{A}{2}\beta \times \frac{d^2 \cdot \beta^2}{3} = -\frac{Z_0 \cdot p \cdot d^2 \cdot \beta^3}{a_0 \cdot 6 \cdot {}_1U_0} \qquad (35)$$

Expression (35) is very much less than expression (34). The other cases may be worked out similarly.

The position of the centre of gravity of the machine has very little influence upon the lateral stability, chiefly because of the important behaviour of the wings. The lateral oscillations of an aeroplane, and also any skidding, are readily damped out by the dynamic action of the wings, rudder, tail, keelplate, vertical fins, etc., and the magnitude of the dynamic righting moment of any one of these parts depends upon its area, position, and the rapidity of the oscillation. If a machine side-slip inward during turning, a righting moment, due to the skidding, is introduced if the lateral position of centre of pressure of the machine as a whole is above the position of the centre of gravity. If the moments of inertia of the machine about the longitudinal and normal axes be small, the yawing and rolling oscillations will be rapid and the damping will be heavy. Nevertheless, if the period of an oscillation be large the pilot has ample time to perform the correct manœuvre to swamp the oscillation. A machine which has great lateral stability in calm weather may be exceptionally dangerous and uncomfortable in gusty weather. On the other hand, if a machine has indifferent lateral stability the pilot, with the aid of controls, may introduce the necessary righting couples. The management of such a machine imposes a severe strain upon the pilot. Nevertheless, an indifference to yawing, and possibly to rolling, is regarded favourably in many aeronautical circles. Machines usually side-slip inwards when turning, so that a rolling moment, tending to reduce the banking, is called into play. But the rolling moment due to the yawing, or turning, tends to increase the banking. The banking must not be excessive if spiral instability is to be avoided.

Lateral Stability of a Machine at Low Speeds.—At the usual speeds of horizontal flight, the angle of incidence of the wing is less than the critical angle of incidence, but occasionally at low speeds the machine becomes "stalled," that is, the angle of incidence of the wing is greater than the critical angle. A pilot who unconsciously "stalls" his machine finds the machine behaving in an uncanny manner, as a control movement which, under normal circumstances, increases the lift on the wings, now only diminishes the lift (see fig. 12). The lateral stability of the motion of a "stalled" machine is of sufficient importance to be considered at some length. We have already read that the lateral stability is partly dependent on (*a*) L_p, rolling due to rolling (*b*) L_v, rolling due to side-slipping, and we shall now consider the dependence of these two coefficients on the angle of incidence of the wings. On p. 90 it is seen that a roll increases the angle of incidence of the falling wing and decreases the angle of incidence of the rising wing. At ordinary angles of incidence, the lift coefficient of the wing increases with an increase of the angle of incidence, so that the couple due to the increased lift on the falling wing and the decreased lift on the rising wing will tend to damp out the roll. If, however, the lift coefficient decrease with the angle of incidence, as is the case if the angle of incidence be greater than the critical angle, the rolling moment due to the forces on the wing aggravates the roll, and the sign of L_p is such as to produce instability. In the present discussion we are more concerned with the critical angle at the wing-tips, as the value of a rolling moment due to the wings is dependent mainly on the air forces of the wing-tip area. The critical angle at the region of the wing-tips has not yet been experimentally determined, but the present discussion is not in any way invalidated. L_v, the rolling due to side-slipping, is a function of the dihedral angle of the wings and of the area and position of the longitudinal fin surfaces. For stability, the sign of L_v must be such that the air forces acting on the machine tend to raise the wing towards which side-slipping is taking place. The magnitude and sign of the part of L_v due to the wings is dependent on the value of the dihedral angle, and the variation of the lift coefficient with the angle of incidence. If, therefore, at ordinary flight speeds this part of L_v be a damping coefficient, when the machine is stalled the sign of L_v will change, owing to the change of sign of the slope of the "Lift Coefficient—Angle of Incidence" curve. The lateral instability due to stalling should not be dangerous at high altitudes; the machine may pitch, roll, and side-slip, but a skilled pilot should readily regain control if he has sufficient air room.

A machine may be made moderately safe at low speeds, if it possess the following features:—

(*a*) A partial "washout" towards the wing-tip of the angle of incidence and the camber.

By "washing out" the angle of incidence, the critical angle of the wing-tip will be delayed. Also wings of a small camber have a high value of the critical angle of incidence.

(*b*) The centre of pressure of the longitudinal fin system should be above the centre of gravity of the machine. The part of L_v due to the fin system, which is uninfluenced by the angle of incidence of the wings, will then always be positive.

A Gust Problem.—It is a matter of common experience that a pilot of a variable-speed machine prefers to travel at his lowest speed in gusty weather. We have already seen that the increases in X and Z due to a vertical gust are greater by about three to ten times those due to a horizontal gust of the same magnitude. Moreover, a horizontal gust does not alter the value of M.

In a variable-speed machine the lift is a constant, and therefore $U_0^2 a_0$ is constant approximately where the value of a_0 ranges from $\left(\frac{1}{7} \text{ to } \frac{1}{20}\right)$.

Now, the increases in the values of X, Z, and M, due to a downward vertical gust, vary as $\frac{w}{a_0 U_0}$, that is, as wU_0, and hence the effect of a vertical gust increases with the speed of the machine. It is to minimise the effect of vertical gusts that the pilot prefers to travel at his lowest speed. It appears that the velocity of a gust is not a function of its direction.

Longitudinal Instability.—Generally speaking, a machine which is longitudinally unstable will, as the motion proceeds, rise, fall, and pitch with increasing violence.

Longitudinal instability may be divided into three sections : (*a*) catastrophic instability, (*b*) rapid oscillations, (*c*) phugoid oscillations.

(*a*) *Catastrophic Instability.*—The steady flight of a machine, under certain conditions, may be completely changed by the action of a powerful vertical gust. The pitching moment may be sufficiently great to cause the machine to turn over upon its back, so that the new flight attitude may be upside down. Catastrophic instability is avoided by ensuring that for any position of the controls there shall be only one flying attitude, relatively to the wind, which makes the pitching moment about the centre of gravity of the machine zero.

(*b*) *Rapid Oscillations.*—It has been shown mathematically that rapid oscillations are almost entirely dependent upon air forces and couples, and that they are comparatively uninfluenced by gravity. These rapid oscillations very rarely persist, and in some cases give rise to a dead-beat motion.

(*c*) *Phugoid Oscillations.*—These oscillations are of a comparatively slow period. The amplitude of successive oscillations may be diminished if a sufficiently large tail and elevator are fitted to the machine, whilst the oscillations are increased if the machine has a large moment of inertia and a small tail area.

Spiral Instability (a Spiral Glide).—A spirally unstable machine will probably be fitted with a large rudder. Imagine such a machine to be performing a right-handed turn. Then the increased lift upon the left wing, and the decreased lift on the right wing, give the machine a bank. Incidentally, the yawing couple due to the drag forces tends to diminish the turn. Initially, the centrifugal force of the turn has caused a side-slip outwards, but now, by virtue of the bank, gravity causes the machine to side-slip inwards. A large turning moment is now given to the machine by the action of the side-slip upon the large rudder, and thus the banking increases. After a time the rolling moment due to side-slipping will tend to increase the banking. Hence, as the motion proceeds, the banking increases and the radius of the turn diminishes, so that a recovery becomes hopeless.

The conditions favourable to the elimination of spiral instability are :—

(*a*) Rolling due to side-slipping, great. Hence, a good dihedral.

(*b*) Yawing due to side-slipping, small. This moment is greatly dependent upon the rudder.

(*c*) The rolling due to yawing, small. Negative wing-tips may favour this condition.

(*d*) Yawing due to yawing, great.

Big Yawing and Rolling Oscillations.—A large vertical longitudinal fin in the front of the machine produces another form of instability.

Imagine such a machine to side-slip towards the left. The air pressure upon the large leading fin gives the machine the bank for a right-hand turn, and also a right-hand yawing moment. Under the action of gravity the machine now side-slips to the right. A reverse motion will then occur, so that finally big unstable yawing and rolling oscillations are set up in the machine.

Much lateral stability work remains to be done, such as the effect of negative wing-tips, turned-up wing-tips, turned-down wing-tips, and it is possible that the near future may see startling changes in the design of machines.

Gusts.—In the preceding stability work, the calculations have been based upon the assumption that the direction of any gust is along a principal axis of the machine, but the gusts which a pilot encounters are not so well defined. Nevertheless, any gust may be resolved along the three well-defined directions, and the separate effect of each of these gusts upon the behaviour of the machine ascertained. The assumption that a gust strikes each part of the machine simultaneously is not correct, although, if the machine be of reasonably small size, theoretical calculations upon such an assumption are of great practical value. We have already seen that the initial effect of a gust is to change the velocity of the machine relatively to the air. The new air forces impart a linear acceleration to the centre of gravity and angular accelerations about the three principal axes of the machine, but these linear and angular accelerations are not uniform. Further, the time-action of a gust is very small. A pilot is only conscious of changes of acceleration, and the body sensations caused by the variable linear and angular accelerations inform the pilot that his machine has been deflected from its course, although, should he be in a cloud, and therefore unable to see the ground, he will have no sense of the new flight path of the machine.

CHAPTER VII.

THE AERIAL PROPELLER.

THE many propeller theories now in existence, some of a more or less conflicting nature, and not a few of an unsound character, rather tend to confuse the elementary reader. Obviously, any theory which is not substantiated by experimental evidence is valueless to the practical engineer. It is not the author's intention to adhere rigidly to any one of these theories, although the problem has been treated in an exceedingly interesting, if not absolutely exact, manner by S. Drzewiecki, who appears to have further developed the excellent work of F. W. Lanchester. A research upon propellers which have been constructed upon the Drzewiecki assumptions, indicates that, although his method is not accurate, the discrepancy between the actual behaviour of a propeller and the assumed behaviour is of no great importance. In fact, this theory may be accepted with modifications, and is quite as sound as many of our so-called "engineer's theories." In any case, it is to be hoped that the following remarks may aid the student considerably to appreciate the aerodynamical behaviour of a propeller.*

Description of Propeller.—Fig. 39 is an engineer's drawing of the blade of an aerial propeller. The line through A, perpendicular to the plane of the paper, is called the axis of the propeller, and the horizontal line AB may be conveniently termed the axis of the blade. Imagine the surfaces of cylinders, which have the axis of the propeller as their common axis, to cut the blade. The developments of the sections of the blade so cut are shown in the figure, 0_1, 0_2, 0_3 . . . being the points

* It should be noted that a propeller is an air screw which pushes, a tractor being an air screw which pulls. At the time the book was written it was customary to use the general term propeller.

of intersection of the cylinders with the axis of the blade, whilst C_1C_1, C_2C_2, C_3C_3 . . . are the lines of intersection of the surfaces of the cylinders with the plane containing the propeller axis and the blade axis. D_1D_1, D_2D_2, D_3D_3 . . . are the developments of the lines of intersection of the surfaces of the cylinders with the horizontal plane passing through the axis of the blade. We should now be able to picture in our mind's eye the true shape of the propeller blade.

Propeller Terms.—Next, it is desirable to have a clear conception of the several "propeller" expressions, so that we may immediately proceed to the study of some experimental curves.

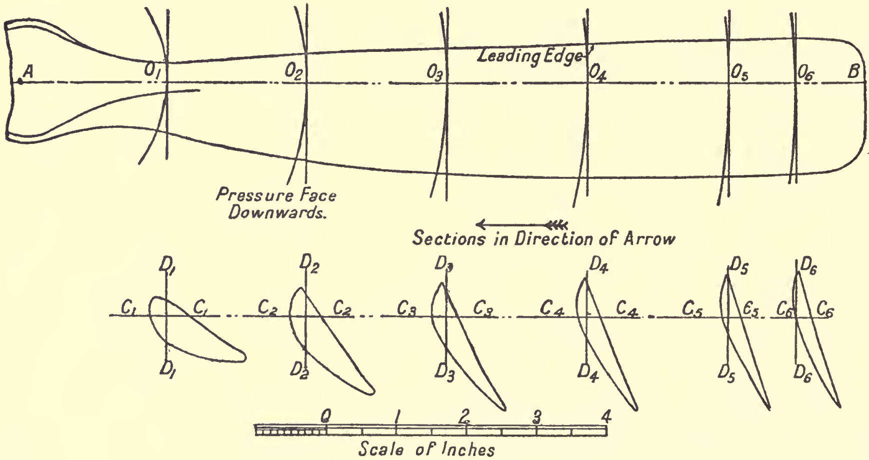


FIG. 39.

The time-honoured marine terms "pitch" and "slip," although adhered to when speaking of aerial propellers, have undergone slight changes in their meanings.

By "pitch" of an aerial propeller we shall mean the axial advance of the propeller during one revolution, when the thrust has a zero value. The various elements of a marine propeller usually have the same pitch, but this is not the case for the aerial propeller, which is generally designed from different considerations.

It is better to determine the pitch of an aerial propeller experimentally, rather than from theoretical considerations, and the term "pitch" should then strictly be termed the "experimental mean pitch" of the propeller.

The "slip-ratio" of a propeller will be defined as $\left[1-\frac{V}{np}\right]$, where V is the translational speed of the propeller, n is the rotational speed, and p is the pitch.

The author does not wish to discuss the physical meaning of "slip-ratio," but prefers rather to regard it as a convenient non-dimensional expression which greatly facilitates a concise presentment of the performance of a propeller. "Efficiency" has the usual meaning, that is, the ratio of the useful work done—the product of the thrust and translational speed—to the total work expended: $2\pi \times$ torque $\times$ rotational speed.

Discussion of the Performance of a Propeller.—Propellers may be most favourably compared when they drive the same aeroplane at the same forward speed—that is, when they develop the same thrust at the same translational speed. The large number of variables which influence the behaviour of a propeller renders the exact appreciation of the experimental performance of the propeller a more or less difficult matter. A model of the propeller shown in fig. 39 has been tested by F. H. Bramwell and the author, and the anticipated performance of the full-size propeller, as deduced from these experiments, is given in figs. 40–43.

The pitch of the propeller = 8·1 feet.

Diameter of the propeller = 8·0 feet.

A static test of a propeller—that is, a test when the translational speed of the propeller is zero—has little practical value. From these static curves (figs. 40 and 41) we see that the thrust and the torque of the propeller approximately vary as the square of the rotational speed. The curves of figs. 42 and 43 completely define the performance of the propeller at a translational speed of 50 miles per hour. It should be noted that the thrust and torque of a propeller, at the same slip, practically vary as the square of the translational speed, or, what is the same thing, as the square of the rotational speed. A maximum efficiency of 68 per cent. is reached at a slip of 24 per cent., and the slip may vary from 18 per cent. to 32 per cent. without any appreciable variation in the efficiency of the propeller. The following simple calculation shows how a small increase in the rotational speed of the propeller greatly increases the thrust of the propeller, with no appreciable alteration in the efficiency. Thus

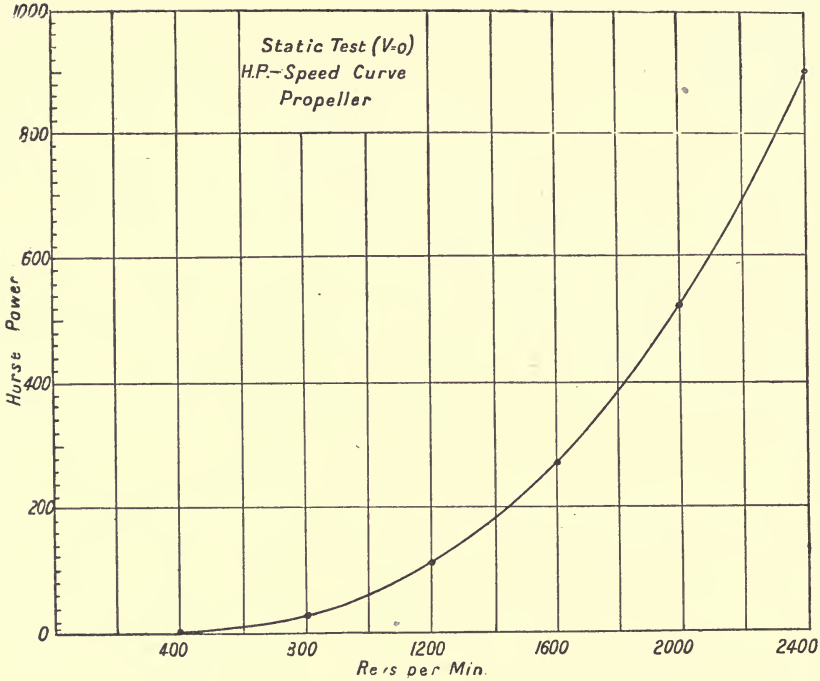

Static Test (V=o)
H.P.-Speed Curve
Propeller
Horse Power
Revs per Min.
1000
800
600
400
200
0
400
800
1200
1600
2000
2400

FIG 40.

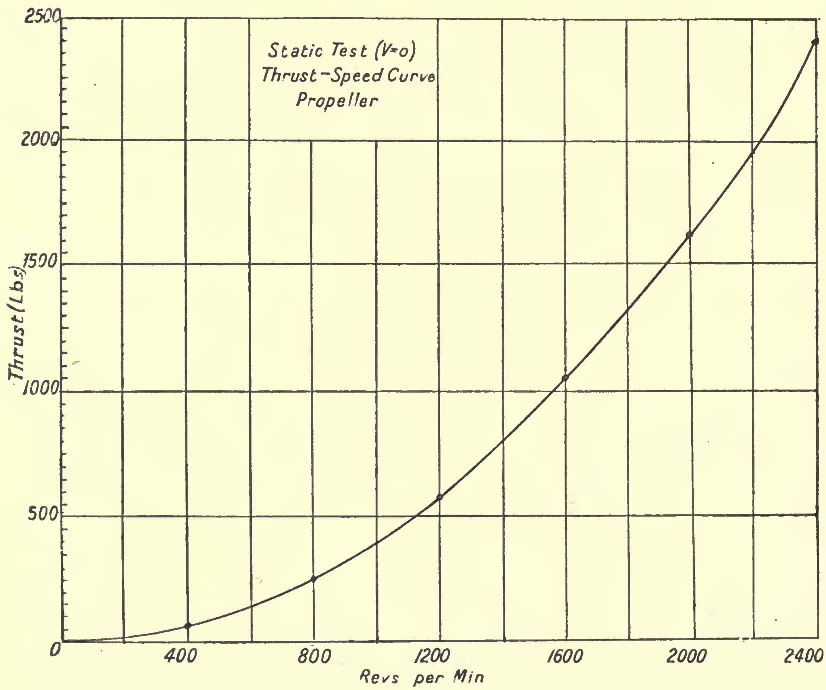

Static Test (V=o)
Thrust-Speed Curve
Propeller
Thrust (Lbs)
Revs per Min
2500
2000
1500
1000
500
0
400
800
1200
1600
2000
2400

FIG. 41.

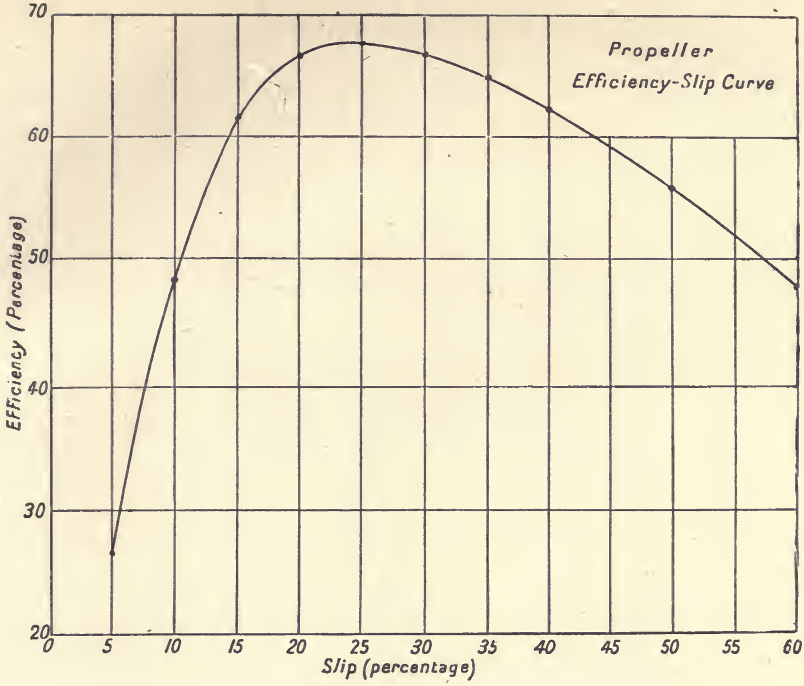

Propeller
Efficiency-Slip Curve
Efficiency (Percentage)
70
60
50
40
30
20
0
5
10
15
20
25
30
35
40
45
50
55
60
Slip (percentage)

FIG. 42.

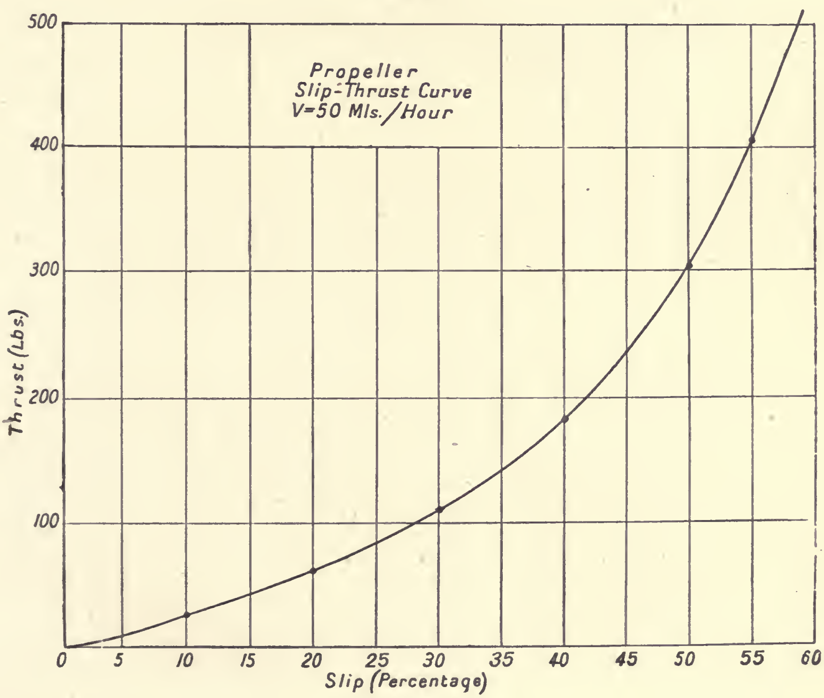

Propeller
Slip-Thrust Curve
V=50 Mls./Hour
Thrust (Lbs.)
500
400
300
200
100
0
5
10
15
20
25
30
35
40
45
50
55
60
Slip (Percentage)

FIG 43.

we shall assume that to drive an aeroplane at 50 miles per hour, our propeller must have a thrust of 237 lbs. and a rotational speed of 970 revolutions per minute. Hence slip of the propeller = 44 per cent. A glance at fig. 43 shows that when the rotational speed is 970 revolutions per minute, the thrust is only 222 lbs. If, by slightly opening the throttle valve of the engine, we increase the speed of our propeller to 990 revolutions

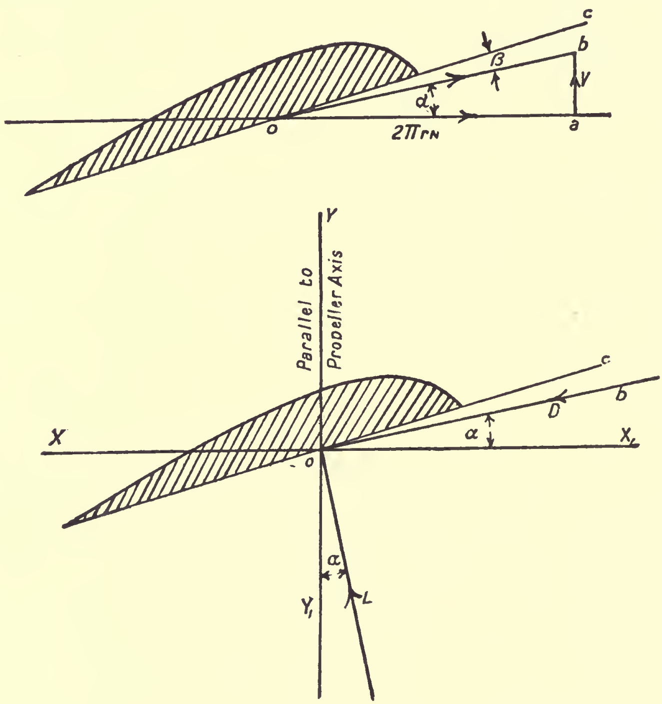


FIG. 44.

per minute, so that the slip is now 45 per cent., we enable the propeller to give the required thrust of 237 lbs.; and, further, the efficiency remains practically unaltered. In other words, reasonable discrepancies between designed and calculated values are not of great importance, as a small variation in the engine speed is sufficient to bring the thrust up or down, as the case may be, to the required value.

A Propeller Theory.—We may now consider the flow of air around an element of a blade, which is at a distance r feet from the propeller axis. The good aerofoil contour of such an element, or, rather, of the development of the section made by an imaginary cylinder of radius r, is shown clearly in fig. 44. In fact, we may consider the blade of a propeller to be an aerofoil of such suitable configuration as enables it to glide in a helical path. If the rotational speed of our propeller be represented by n revolutions per minute, and its translational speed by V feet per minute, any element of the blade will have a forward translational velocity (in the direction of the propeller axis) of V feet per minute, and also a translational velocity of $2\pi rn$ in the direction OX_1, where OX_1 is at right angles to OY and also to the axis of the blade (see fig. 44). The element is therefore moving through the air in the direction *ob* with a velocity of V_1, where

$$V_1 = \sqrt{(V^2 + (2\pi rn)^2)}.$$

We should now recognise that we are on familiar ground, and we will call β—that is, the angle between the chord of the element and the wind direction—the angle of attack of the section. Also the wind forces D and L, which act upon the element, have their usual signification. For our present purpose, it is convenient to resolve the resultant force upon the section into the two directions OX and OY. The component of the resultant force in the direction OY will then be $[L \cos a - D \sin a]$, whilst the component in the direction OX is $[L \sin a + D \cos a]$. The useful work done by the section will then be $[L \cos a - D \sin a]V$, and this work is accomplished by a work expenditure of $[L \sin a + D \cos a]2\pi r . n$. The efficiency of the element is represented by

$$\eta = \frac{V[L \cos a - D \sin a]}{2\pi rn[L \sin a + D \cos a]}.$$

If the ratio $\frac{D}{L}$ be represented by $\tan \phi$, the efficiency is concisely represented by the expression—

$$\frac{V}{2\pi rn} \cot (\phi + a).$$

The importance of the function ϕ is clearly demonstrated by such an expression; and to ensure a small value of $\tan \phi$, the section should have a good shape and should be inclined to the wind direction at the best value of the angle of attack β. Moreover, if for the arbitrary values of V and n, the values of L and D for each section are known, we may, by integration, obtain the thrust, torque, and efficiency of the propeller.

It has been thought desirable to summarise the prominent features of the foregoing remarks :—

(*a*) Each element of the blade has an aerofoil shape.

(*b*) The air flow around each element is assumed to be uninfluenced by the presence of the neighbouring portions of the blade.

(*c*) The forces upon each element are due to the resultant of the axial and circumferential velocities of the element, and the air into which the element moves is *assumed* to be *motionless.*

(*d*) Since $\tan \alpha = \frac{V}{2\pi rn}$, where V and n are arbitrary constants for all the elements of the blade, the value of α for each element will depend upon the distance of the element from the axis of the propeller.

(*e*) Under such conditions the efficiency, thrust, and torque of a propeller may be determined theoretically when the aerodynamic properties of the elements are known.

A series of experiments was conducted at the National Physical Laboratory by F. H. Bramwell and the author to ascertain the degree of accuracy of the foregoing assumptions, and also to afford a comparison between the calculated and experimental values of the various factors which define the performance of a propeller. In the light of this research, the hypothesis, that the air flow around the blade element is not influenced by the presence of adjacent elements, is not strictly accurate. Furthermore, the value of the best angle of attack of an aerofoil may undergo a change when a section of the aerofoil is employed in the construction of a propeller blade. Nevertheless, in the hands of a skilled designer, this theory, although not accurate, furnishes a convenient basis for the design of a propeller. Experimental evidence rather encourages the adoption of the theory, if the working conditions of the propeller are not greatly different from those upon which the design has been based. We found that the calculated values of the efficiencies were less than the experimental values, and these discrepancies were chiefly due to the non-agreement of the torque values. The calculated and experimental values of the thrusts were not greatly different. We anticipate that the discrepancy between the calculated and experimental values is mainly due (*a*) to the initial motion of the air into which the propeller moves, and (*b*) to the radial velocity of that portion of the air in the neighbourhood of the blades, such a velocity being partly attributed to centrifugal force.* The frictional contact between the air and the blades must influence the character of the air flow, and it is a matter for surprise that we found the calculated and experimental values in such close agreement.

A brief survey of the tests performed upon a large number of model propellers reveals the astonishing fact that the maximum efficiencies of almost all the propellers lie within a range of 60 per cent. to 75 per cent.

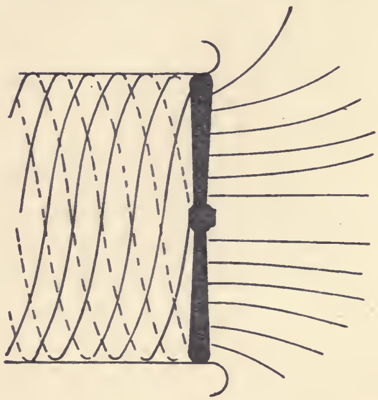


FIG. 45.

The Flow of Air around a Propeller. — The air which is drawn in at the front of the propeller is discharged in practically a cylindrical jet. The axial velocity of the discharged air increases from the tip of the blade to a short distance inwards, and then gradually decreases until the velocity has a zero or small negative value in the neighbourhood of the boss of the propeller. The rotational motion of the discharged air has the same direction as that of the propeller. When this motion is superposed on the translational motion, the usual spiral flow results (see fig. 45). The area of a cross section of the discharged stream is less than the area of a section of the incoming air column, and it is assumed to be equal to the area swept out by the propeller blades. The calculation of the thrust of the propeller from the axial momentum of the discharged air can only be performed when we know the distribution of velocity across the area of the discharged stream, and is therefore a matter of great difficulty. Furthermore, our ignorance of the

* No account was taken of the eddying flow at the tip of the blade.

nature and magnitude of the rotational motion renders the exact calculation of the torque, and therefore of the propeller efficiency, an impossibility.

The thrust T of a propeller may be represented by $\Sigma(mv)$, where m = the mass of an elemental volume of air in the discharge stream. Every particle of this volume is supposed to be moving with the same axial velocity v.

The kinetic energy of translation wasted by the discharge stream $=\Sigma\left(\frac{1}{2}mv^2\right)$.

Similarly, the torque $Q=\Sigma(m.r^2.\omega)$, where m = the mass of an elemental volume of air in the discharge stream. Every particle of this volume is supposed to be moving with an angular velocity ω about, and also to be at a distance r from the axis of the propeller.

The kinetic energy of rotation dissipated in the discharge stream $=\Sigma\left(\frac{1}{2}mr^2w^2\right)$.

It would follow, then, that to develop a given thrust, it is advantageous to discharge a large mass of air at small axial and rotational velocities.

The kinetic energy of rotation is only a fraction of the total kinetic energy of the discharged stream.

It is supposed that the air which is in the region of the high-speed propeller tips does not undergo any significant change in density.

The Position of the Propeller.—The position of the propeller upon the machine is usually determined by constructional exigency. In monoplanes, the propeller—or, more correctly, the "tractor screw"—is conveniently placed at the front of the machine. At the same translational speed of a machine, by virtue of the viscous drag exerted upon the air by the aeroplane wings, the tractor screw has a greater velocity of entry into the air than a propeller situated at the rear of the wings. Now, to get the necessary thrust, a propeller works with a slip-ratio well above the value which corresponds to the maximum efficiency of the propeller. Consequently if the speed of the machine and the thrust of the propeller be the same, a shift of the airscrew from the front of the wings to the back will necessitate the airscrew running at a greater slip, and hence at a smaller efficiency (see figs. 42 and 43). Furthermore, the wings which are situated in the slip stream of a tractor screw have a greater lifting capacity and also, unfortunately, a greater drag. It is credible that if these forces have a large value, a sudden failure of the engine will greatly interfere with the equilibrium of the machine. We may venture to assume that the efficiency of a tractor screw will be increased by the near presence of the aeroplane body and wings, but the inevitable increase of head resistance of the machine will tend to counteract this advantage. With the advent of a reliable make of engine, the utilisation of the energy in the discharge stream for navigation and stabilising purposes will be worthy of serious consideration. When flying, the reaction torque of the air upon the propeller, which will be transmitted to the machine, is usually counterbalanced by a wing warp of the requisite amount combined with a judicious employment of the rudder. Two similar propellers symmetrically situated, but rotating at the same speed in opposite directions, introduce no disturbing torque upon the machine.

The Balance of the Propeller.—A simple calculation may demonstrate the importance of the employment of a well-balanced propeller. Thus, assume that a propeller of weight 30 lbs. and diameter 8 feet has a rotational speed of 1200 r.p.m. Assume, further, that its C.G. is displaced a distance of $\frac{1}{10}$ inch from the propeller axis.

The centrifugal force due to this eccentricity

$$=\frac{mv^2}{rg}=\frac{30\times 1600\pi^2}{32\times 120}=125 \text{ lbs. approximately.}$$

An unbalanced force of this magnitude and period may introduce unpleasant, if not dangerous, oscillations in the machine.

The Gyroscopic Action of a Rotary Engine and its Propeller.—Before we proceed to the study of the gyroscopic behaviour of a rotary engine and propeller upon an aeroplane, we shall discuss briefly some principal features of gyroscopic action. When an extraneous couple acts on the machine, the axis of rotation of the propeller and engine, if the engine be a rotary one, tends to place itself in line with the axis of this couple, so that the direction of the rotation of the engine and propeller is the same as that of the applied couple. Thus, assume an aeroplane, which is driven by a right-handed tractor

—that is, the tractor has a clockwise direction of rotation when it is viewed from the pilot's seat—is to turn to the right, so that the applied air couple acts about a vertical axis. In this case the machine will tend to dive. Further, to consider another illustration, let us assume that the applied air couple tends to turn the nose of the machine downwards. Then, due to the influence of the rotating propeller and engine, the machine will tend to make a horizontal left-handed turn. The mathematics of gyroscopic action is simple, if we accept the following statement. The magnitude of the gyroscopic couple is equal to the angular momentum of the rotating masses multiplied by the angular velocity of the precessional movement. In the above illustrations we have dealt with two precessional movements—firstly, one of pitching; secondly, one of turning. Now, for a practical illustration, let us imagine our machine to precess to the right with an angular velocity of $\frac{2\pi}{16}$ radians per second—that is, one turn in 16 seconds; also that

The weight of the rotary engine	$=200$ lbs.
The weight of the propeller	$=30$ lbs.
Radius of gyration of the engine	$=\cdot 8$ foot.
Radius of gyration of the propeller	$=2\cdot 5$ feet.
Rotational speed of engine and propeller	$=1200$ r.p.m.

Angular momentum of engine

$$=\left[\frac{200\times\cdot 64\times 40\pi}{32}\right]=160\pi \text{ (Ft.-lb.-sec. units).}$$

Angular momentum of propeller

$$=\left[\frac{30\times 6\cdot 25\times 40\pi}{32}\right]=234\pi.$$

The angular velocity of turning $=\frac{2\pi}{16}$ radians per second. Hence the pitching couple exerted upon the machine due to the gyroscopic action of the engine and propeller

$$=[160\pi+234\pi]\frac{\pi}{8} \text{ lbs.-feet}=490 \text{ lbs.-feet.}$$

Also suppose the area of the elevator is 20 square feet, and that its C.P. is situated at a distance of 15 feet from the point about which the machine turns. To balance such a couple the pressure of the air on the elevator in a direction normal to the longitudinal axis of the machine will need to be $\frac{490}{300} = 1\cdot6$ lbs. per square foot. Such an air-pressure loading cannot be considered of great magnitude, especially as the pilot has already anticipated the required elevator position for such a turn. Furthermore, the inertia of the machine, and the damping action of the wings and tail, must tend to minimise the disturbing influence of the gyroscopic couple.

Design of an Aerial Propeller.—The following design presumes a knowledge of the matter of the foregoing pages, and the author hopes that this treatment of the subject may familiarise the reader with the aerodynamical behaviour of a propeller. The design has been based upon the teachings of F. W. Lanchester and Drzewiecki, but only in so far as their theories have been substantiated by experimental evidence. The non-mathematical student should glean some useful information from the various conclusions and observations which are interspersed amongst the "troublesome" calculations.

Data necessary for the Design of a Propeller.*—(*a*) The thrust of the propeller at the known translational speed of horizontal flight.

(*b*) The rotational speed of the propeller when the machine is in horizontal flight. This speed will be a function of the speed of the engine and the type of gearing employed.

(*c*) The diameter of the propeller. This is determined, more or less, from a consideration of constructional exigencies. We shall assume that the following data have been supplied to the designer:—

The "propeller" to be of the tractor type.
Translational speed of machine, and therefore of the tractor = 65 miles per hour.
Pull of the tractor screw = 385 lbs.
Rotational speed of the tractor = 1100 r.p.m.

The diameter of the tractor must not be greater than 10

* This design would need to be modified in the case of the helicopter.

feet. Hence take the diameter as 9 feet. This will allow a clearance of 6 inches.

Use the same notation as previously. (Also see fig. 44.)

The lift upon an aerofoil $= K_a \times \rho \times \text{area} \times (\text{velocity of wind})^2$, where K_a = absolute lift coefficient at an angle of incidence equal to a.

Let l_1 = chord length of a blade section at a radius r.

The lift upon a unit length of the blade, assuming any cross section of this unit length to have the same shape and size as that of the cross section of the blade at a radius r

$$= K_a \times \rho \times l_1 \times V_1^2 = K_a \times \rho \times l_1[V^2 + (2\pi rn)^2].$$

Similarly the drag $D = k_a\rho \times l_1 \times V_1^2 = k_a\rho l_1[V^2 + (2\pi rn)^2]$.

As before, the thrust contributed by such a unit length of the blade

$$= K_a\rho l_1 V_1^2\left[\cos a - \frac{D}{L}\sin a\right]$$

$$= K_a\rho l_1 \frac{V^2}{\sin^2 a} \times \frac{1}{\cos \phi}\cos(a+\phi),$$

where $\frac{D}{L} = \tan \phi$.

In like manner the torque for such a unit length

$$= \frac{K_a \cdot \rho \cdot l_1 \cdot V^2}{\sin^2 a} \times \frac{1}{\cos \phi}\left[\sin(a+\phi)\right]r.$$

From the above two expressions it would appear advantageous to make ϕ as small as possible.

The values of a for the elements of the blade may be directly calculated from the expression, $\tan a = \left[\frac{V}{2\pi rn}\right]$,

which gives $$a = \tan^{-1}\left[\frac{5720}{2\pi \times 1100 \times r}\right] = \tan^{-1}\left[\frac{\cdot 828}{r}\right].$$

In the light of the researches conducted upon aerofoils of multiform sections, the best angle of attack, *i.e.* β, for the aerofoil sections suitable for propeller-blade construction may be taken as 4°. It should be remembered that by "angle of attack" we mean the angle between the wind direction and the direction of the line fixed upon the aerofoil section which gives the wind direction when there is no lift upon the section. The calculated values of $(\beta+\alpha)$, *i.e.* $(\alpha+4°)$, are given in Table IV.

TABLE IV.

Section.	Radius. Feet.	tan α.	α. Degrees.	$\alpha+4°$. Degrees.
1	4·0	·207	11·7	15·7
2	3·5	·241	13·6	17·6
3	30	·276	15·4	19·4
4	2·5	·331	18·3	22·3
5	2·0	·414	22·5	26·5
6	1·5	·552	28·9	32·9
7	1·0	·828	39·6	43·6

Adopting a suitable linear scale, we then design what appears to be a good plan form, taking great care that the blade widths at radii 2½ feet, 3 feet, 3½ feet, 4 feet have the maximum widths consistent with a suitable plan form. From the known values of $(\beta+\alpha)$ we may also draw, in position, the lines which will eventually become the "no-lift" lines of the various sections. Now select some good aerofoil sections, of which the aerodynamical properties are known, and sketch them in position about the no-lift lines. Such sections may be found in the *Annual Reports of the Advisory Committee for Aeronautics* and the published works of Eiffel. Also a designer may, if he so wish, have aerofoil sections tested at the National Physical Laboratory.

The following points should be borne in mind :—

(1) The chord lengths of the sections have already been fixed by the plan form.

(2) The sections at radii 4 feet, 3½ feet, 3 feet, and probably 2½ feet, should be good aerofoil shapes.

(3) The maximum ordinate of each section should be situated at a distance from the nose of the section of about one-third the chord.

(4) Sections at the boss of the propeller should be designed chiefly from considerations of strength.

(5) That the centre of area of each section, which may be decided upon quite accurately without calculation, should be as nearly as possible on the blade axis.

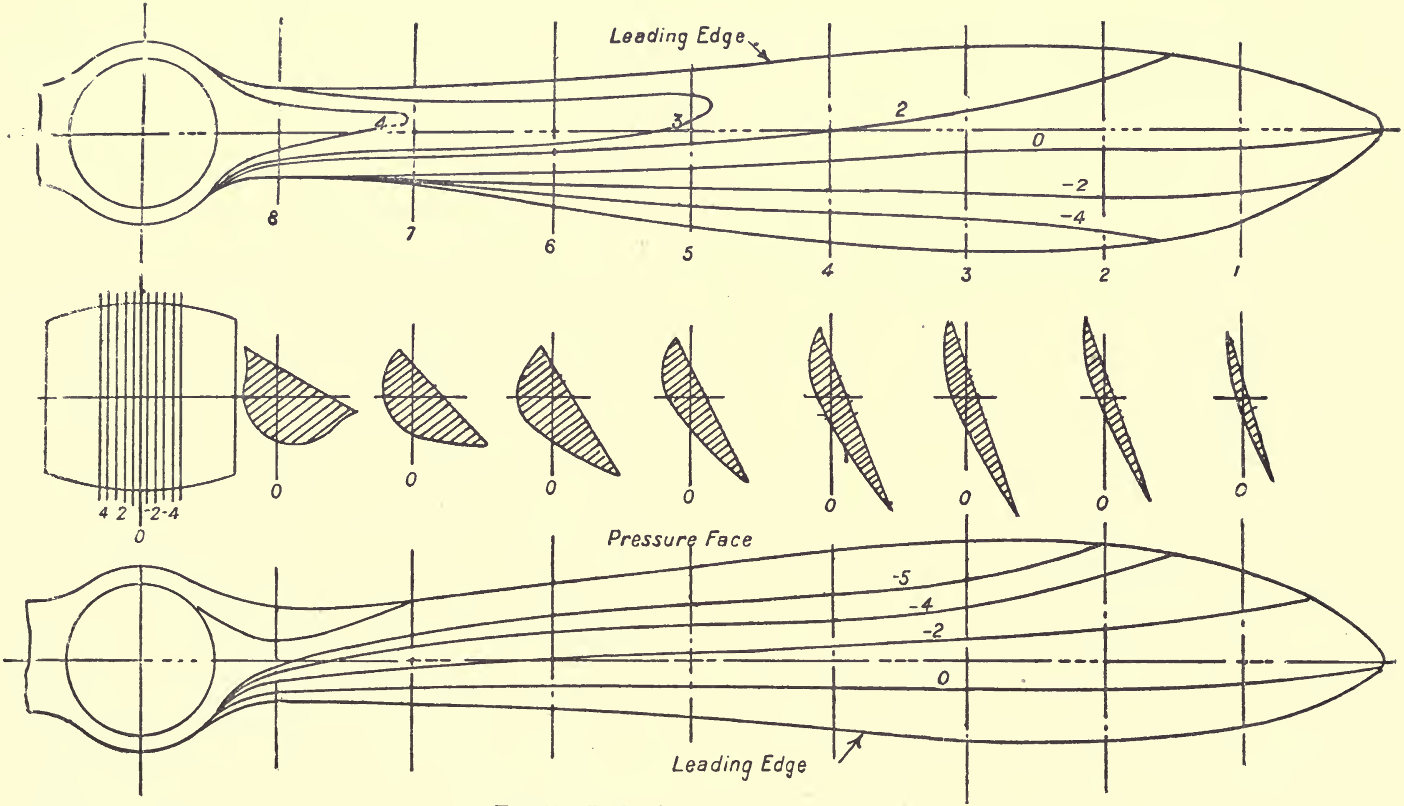
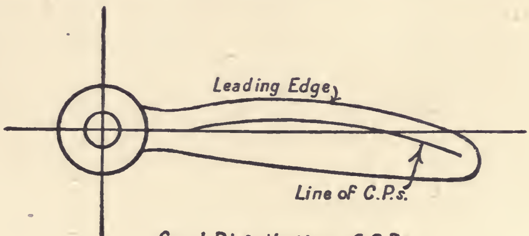


FIG. 46.—Design for propeller. Scale 1″=8″.

(6) That the centres of pressure of the various sections should be so placed with respect to the blade axis that the resultant air pressure on the blade passes through the blade axis. Fig. 47 gives a good distribution of the centres of pressure on the several sections of a blade.

The conditions (5) and (6) may be occasionally antagonistic.

From these sections we may now construct the contour lines of the plan form (see fig. 46). After "fairing" the contour lines it may be necessary to "refair" the sections. By such a tentative method of fairing—that is, constructing the contour lines of the plan form from the sectional shapes, and *vice versa*—we obtain eventually a smooth blade surface. Whilst fairing we should have constantly in mind the conditions above stated. Also, the contour lines should not suddenly undergo any great



Good Distribution of C.P.s.

FIG. 47.

changes of curvature, otherwise the propeller blades may, by virtue of their great twist, present an unsightly appearance. The success of the above operation, like all fairing processes, depends greatly upon the experience and the practical instinct of the designer. A study of the characteristics of the many aerofoil sections which have been tested at the big aeronautical laboratories should enable the designer to approximate with some accuracy to the aerodynamical properties of those sections, the shapes of which have been slightly modified by the fairing. It has been the author's experience that the fairing of the blade surfaces, if well performed, does not greatly alter the shapes of the blade sections. Further, the author ventures to suggest that a deduction of the aerodynamical properties of the blade sections after fairing—the alterations of the sections must be small—may be productive of better results than the utilisation of data, which, although accurate for the initial blade sections, cannot be applicable to the sections of the finished propeller.

Table V. gives the aerodynamical properties of aerofoils having the same shapes as the several blade sections.

Let the scale of blade widths be

1 inch represents k feet.

TABLE V.

Section.	Length of Chord of Section. Feet.	$\frac{L}{D}$.	ϕ. Degrees.	Lift Coefficient K at Maximum $\frac{L}{D}$. Absolute.
1	0·87k	14	4·1	·152
2	1·07k	14	4·1	·162
3	1·15k	13	4·5	·172
4	1·05k	11	5·2	·186
5	0·95k	9	6·3	·196
6	0·8k	9	6·3	·098
7	0·72k	4	14·0	·049

Now, from the two expressions,

$$T = \frac{K_\alpha \cdot \rho \cdot l \cdot V^2}{\sin^2 \alpha} \times \frac{1}{\cos \phi} \cos(\alpha + \phi),$$

$$Q = \frac{K_\alpha \cdot \rho \cdot l \cdot V^2}{\sin^2 \alpha} \times \frac{r}{\cos \phi} \sin(\alpha + \phi),$$

we may calculate the values given in the following table :—

TABLE VI.

Section.	T. Lbs. per foot-length.	Q. Lbs.-ft. per ft.-length.
1	144·5k	164·0k
2	138·0k	154·0k
3	122·0k	132·5k
4	83·0k	90·0k
5	51·5k	56·5k
6	12·7k	13·5k
7	2·4k	1·8k

If the values of T—that is, the thrust upon unit length of the blade, assuming the distribution of thrust over this unit length to be uniform and of the same magnitude as the actual thrust distribution on the blade section at radius r—be plotted

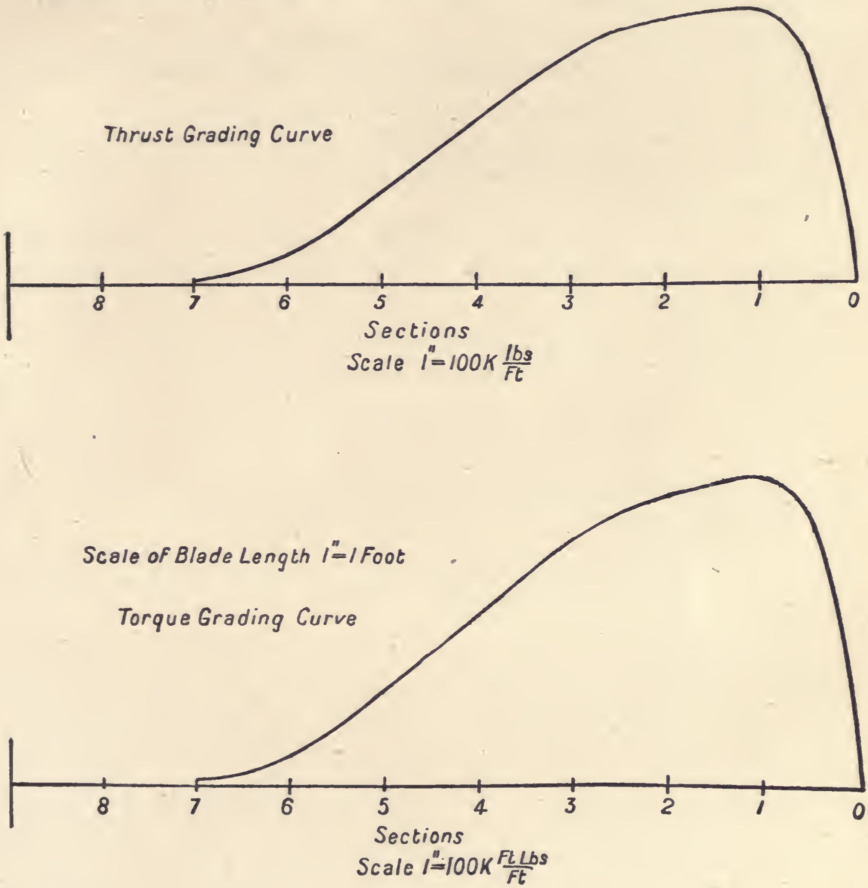


FIG. 48.

against r, we obtain the "thrust grading curve" (fig. 48). The area of this curve gives the total thrust contributed by the blade. A study of the curve confirms a remark made formerly, that the portion of the blade between the radii 1 foot and 2 feet is of small aerodynamical importance, since it only contributes about 6 per cent. to the total thrust of the blade, whilst the remainder of the blade from $r=2$ feet to the tip, contributes 94 per cent. Integrating the area of the thrust grading curve, we find that the thrust contributed by a single blade $=289k$ lbs. The value of k may be deduced from the expression $289k \times n =$ thrust of propeller, that is 385 lbs., where $n =$ number of blades.

Aerodynamically, a four-bladed propeller is probably more efficient than a two-bladed one, but the former is more difficult to make, and is usually weaker at the boss. Other advantages of a four-bladed propeller are: (*a*) good balance, (*b*) small interference forces are introduced by the action of side gusts upon the blades.

We shall assume a two-bladed propeller, and, therefore, $k = \frac{2}{3}$, so that the linear scale of blade widths becomes—1 inch represents $\frac{2}{3}$ ft.

Similarly, integrating the area of the torque grading curve (fig. 48), we find that the torque of the propeller

$$= 318 \times k \times 2 = 424 \text{ lbs.-feet.}$$

$$\text{The propeller efficiency} = \left[\frac{\text{V}\,.\,\text{T}}{2\pi\ \text{Q}n}\right] 100 \text{ per cent.}$$

$$= \frac{100 \times 65 \times 88 \times 385}{2\pi \times 424 \times 1100} \text{ per cent.} = 75 \text{ per cent.}$$

The values of the thrust and torque as calculated from this theory are greater than those given in practice, chiefly because no account has been taken of the flow of air into the airscrew. Roughly speaking, the ratio of the thrust as calculated from the above theory to the thrust actually given by the airscrew has a mean value of 1·4 over the working range of the airscrew, the corresponding ratio for the torque being 1·3. The author has been working on a theory in which the effect of the inflow velocity is considered, and he hopes to publish the results of this investigation at a more opportune time.

Calculation of the Stresses in the Propeller.—A propeller blade is usually under the combined action of (*a*) a twisting couple, (*b*) a bending moment, (*c*) the direct tension due to centrifugal force, and obviously a rigid calculation of the stresses at any point of the blade would be one of complex difficulty. However, from the independent calculations of the centrifugal and bending stresses some approximation to the stresses at any section of the blade may be obtained.

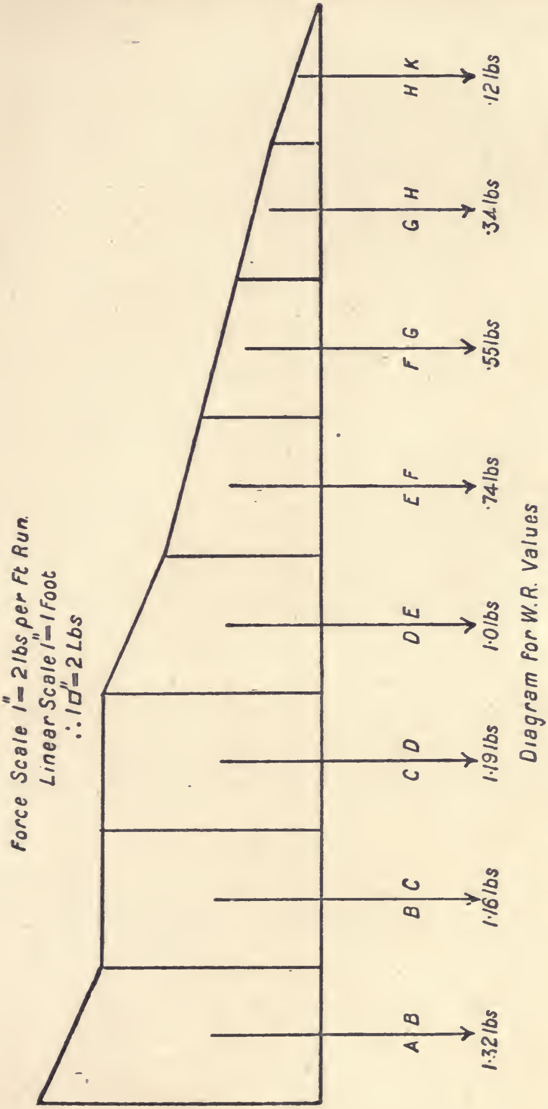


Fig. 49.

We shall ignore the stresses due to the twisting couple, because in a well-designed propeller these should be quite small. The calculation of the centrifugal stresses (see Table VII. and fig. 49) is a matter of small difficulty, and the student should readily understand the method adopted.

In this calculation the weight of the wood was taken as 35 lbs. per cubic foot.

TABLE VII.

Section.	Area of Section. Sq. ins.	Values of *w*. Lbs. per foot.	Values of W. Lbs.	Values of *r*. Feet.	Values of *r*W. Foot-lbs.
0	0	0			
			0·12	4·17	0·50
1	1·99	0·48			
			0·34	3·72	1·30
2	3·55	0·86			
			0·55	3·23	1·78
3	5·40	1·31			
			0·74	2·73	2·05
4	6·72	1·63			
			1·00	2·23	2·23
5	9·60	2·38			
			1·19	1·75	2·08
6	9·60	2·38			
			1·16	1·25	1·48
7	9·30	2·26			
			1·32	0·74	1·00
8	12·30	3·00			

w is the weight of a foot-length of the blade in lbs. when the sectional area is assumed to be constant, and the same as at the section under consideration.

W = weight in lbs. of a portion of the blade between any two consecutive sections.

TABLE VIII.

Section.	Centrifugal Force acting across Section. Lbs.	Stress (tensional). Lbs. per square inch.
1	205	105
2	745	210
3	1485	275
4	2340	345
5	3260	340
6	4120	430
7	4740	510
8	5150	420

Calculation of Stresses due to Air Pressure upon the Blade. —Assumptions :—

(1) The calculation to obtain the bending moment at a section, assumes the resultant air forces over the elements of the blade to be parallel, and also in the same plane.

(2) Afterwards, when calculating the stresses, the bending moment was assumed to act about an axis parallel to the chord passing through the C.G. of the section.

The value R (see Table IX.) gives the total load per foot-length of the blade, assuming the distribution of load over the unit length to have the same magnitude as the actual load distribution at the section at radius r.

Hence $R=\sqrt{\left(T^2+\left(\frac{Q}{r}\right)^2\right)}$, and makes an angle ψ with the direction of T, where $\psi=\tan^{-1}\left(\frac{Q}{Tr}\right)$.

A study of columns 3 and 4 of Table IX. justifies the two assumptions previously made. Column 3 gives the angle at any section between the direction of R and the direction of translational motion of the blade. The angular change in the direction of R as we pass from section 1 to section 5 is only 13°.

Column 4 gives the angular displacement of the direction of R forward of the normal to the chord of the section, and it is seen that R is practically normal to the chord of the section.

TABLE IX.

Section.	R. Lbs. per foot.	Column 3. ψ. Degrees.	Column 4. Degrees.
1	100·0	15·8	0·1
2	97·0	17·7	0·1
3	66·5	20·0	0·6
4	60·5	23·5	1·2
5	39·0	28·7	2·2
6	10·7	35·3	2·4
7	2·0	37·0	3·4

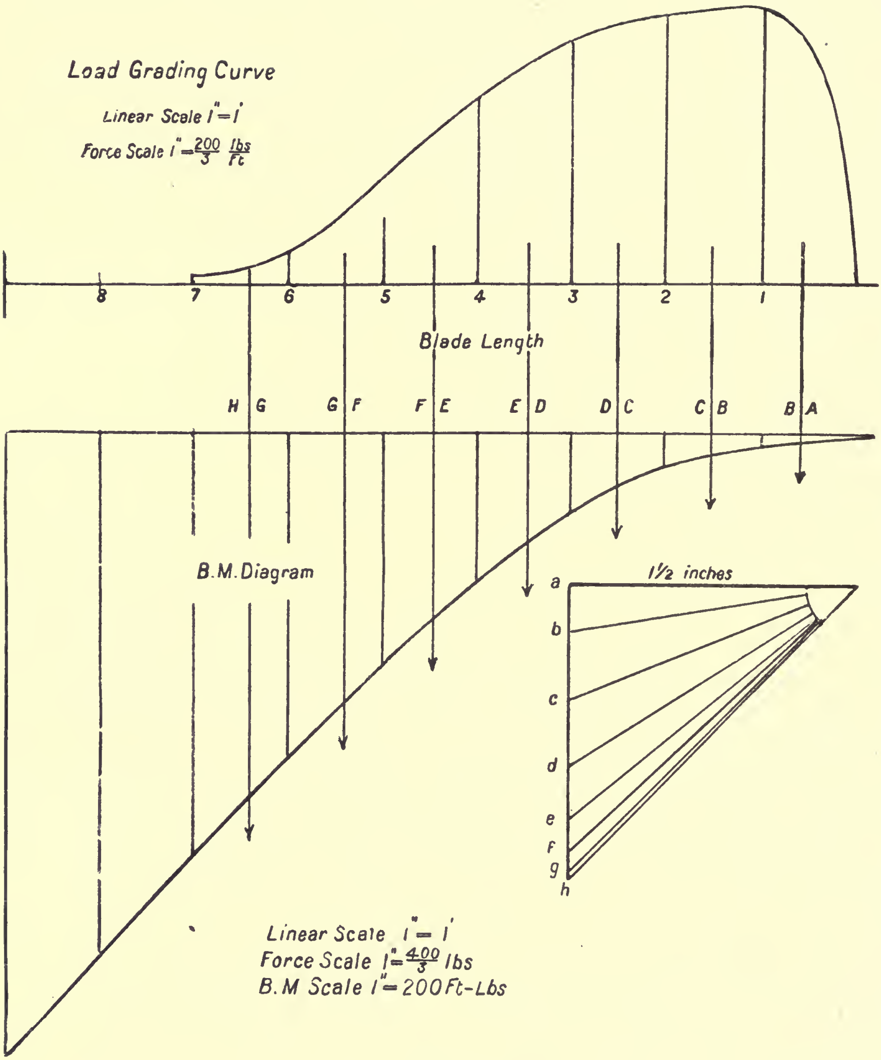


Fig. 50.

We may now construct a load grading curve from which the bending moment at any section may be calculated (see fig. 50).

TABLE X.

Section.	Bending Moment, M, at Section. Inch-lbs.	$\frac{Y}{I}$ for Section Compression. [Inch]$^{-3}$.	$\frac{Y}{I}$ for Section Tension. [Inch]$^{-3}$.	Maximum Compression. Lbs. per sq. in.	Maximum Tension. Lbs. per sq. in.
1	72	8·4	5·60	607	407
2	442	4·2	2·80	1865	1240
3	1080	1·77	1·18	1914	1280
4	1950	1·08	0·72	2108	1410
5	3050	0·57	0·38	1740	1145
6	4240	0·41	0·27	1735	1145
7	5460	0·39	0·26	2130	1420
8	6720	0·25	0·17	1680	1120

The maximum stresses at the sections may be approximated to, by adding the centrifugal and bending moment stresses (see Table IX.).

TABLE XI.

Section.	Maximum Compressive Stresses due to Bending and Centrifugal Force. Lbs. per sq. in.	Maximum Tensile Stress due to Bending and Centrifugal Force. Lbs. per sq. in.
1	502	512
2	1655	1450
3	1639	1555
4	1763	1755
5	1400	1485
6	1305	1575
7	1620	1930
8	1260	1540

The best woods for the construction of a propeller are (1) walnut, (2) Honduras mahogany, (3) spruce. Spruce, when in tension, is the weakest of the three, but it retains its shape well. Mahogany has a tendency to warp, and walnut is a very heavy wood. Assuming a working load for mahogany and walnut of 2000 lbs. per square inch, we see that the above stresses are well within the safety limit.

CHAPTER VIII.

AERONAUTICAL ENGINES.

The author presumes that the reader has some knowledge of internal combustion engines, and hence only the first few pages are devoted to a general study of petrol engines. A detailed and complete study of the petrol engine may readily be obtained from the many excellent text-books entirely devoted to this subject.* The author has endeavoured to treat the subject more especially from an aeronautical standpoint, and, as far as possible, prominence has been given to the essential characteristics of the aerial engine.

The Otto, or Four-Stroke Cycle.—An indicator diagram, taken from a four-cycle engine, is shown in fig. 51. With such an engine, the whole cycle of operations in each cylinder repeats itself after four strokes of the piston—that is, two revolutions of the crankarm. At the commencement of the cycle the inlet valve, usually of the poppet type and operated by a cam, is opened, and, as the piston descends, a mixture of petrol vapour and air is sucked from the carburettor into the cylinder. When the crankarm has slightly passed the lower dead centre the inlet valve closes, and the rising piston compresses the charge. Just before the end of the compression stroke an electric spark fires the mixture, but no appreciable combustion occurs until the piston reaches the top of the cylinder. At this point, however, the combustion proceeds very rapidly, and the piston descends under the pressure of the burning gases, the work performed upon the piston appearing as a turning moment about the crankshaft. When the piston has almost reached the bottom of the cylinder, the exhaust valve opens, and an escape

* *Aero Engines,* by G. A. Burls, M.Inst.C.E.; London: C. Griffin & Co., Ltd., 1915.

of the products of combustion occurs, the pressure within the cylinder falling to approximately that of the atmosphere. During the fourth stroke the remaining exhaust gases, with the exception of the gases which will eventually remain in the clearance space, are expelled into the atmosphere.

The Two-Stroke Cycle Petrol Engine.—We will now consider very briefly the two-stroke engine—that is, an engine which has one working stroke during each complete revolution of the crankarm. An indicator diagram from this type of engine is

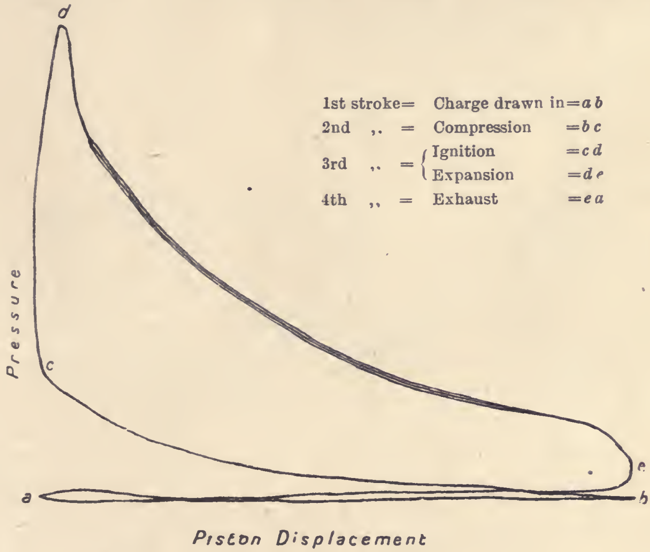


FIG. 51.—Indicator diagram of four-stroke engine.

given in fig. 52. Imagine the piston at the top of its stroke, and that the petrol mixture in the combustion space has just been ignited. When the descending piston is near the end of its working stroke, it uncovers an exhaust valve and allows the products of combustion to escape into the atmosphere. During the further descent of the piston a port connected to the crankcase is opened, and a fresh mixture, which has been slightly compressed in the crankcase by the descending piston, is allowed to flow into the cylinder. The inflowing gases are deflected upwards, usually by baffle plates, and drive the burnt gases out through the exhaust port. At the commencement of the return stroke the piston first closes the port connected to the crankcase, and afterwards the exhaust port, so that the further rise of the piston compresses the gases into the combustion space. Further, at the end of the second stroke the uncovering of a port enables a fresh charge of mixture to pass from the carburettor into the crankcase.

Magnetos.—High-tension magnetos are largely employed to ignite the charge of an aerial engine, and these are quite reliable at the ordinary speeds of rotation. To diminish the possibility of misfiring of the charge of a high-speed aerial engine, the employment of two synchronised magnetos is to

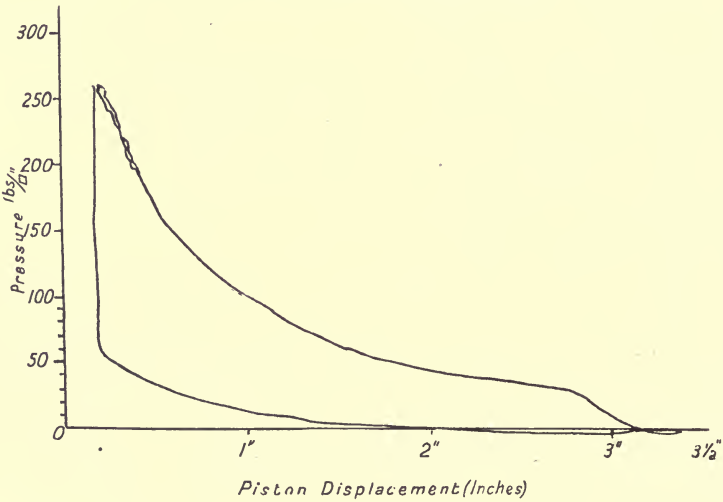


Fig. 52.—Indicator diagram of two-stroke engine.
Mean pressure = 62·81 lbs. per square inch. Speed of engine = 900 r.p.m.

be recommended, even although the weight of the engine is thereby slightly increased.

Carburettors.—A sketch of a jet carburettor is shown in fig. 53. The petrol, which passes through a pipe G to the small tank K, is maintained at a constant level, normally a little below the jet, by means of a float A and a needle valve B. The air drawn in at C passes a constriction D on its way to the cylinder. The pressure of the air surrounding the jet at D is therefore lower than atmospheric pressure, and the petrol is projected or sucked into this air, partly as a fine spray and partly as a vapour. The area of the petrol orifice may be regulated by the needle valve L. If we assume the area of the petrol orifice is adjusted to give the correct mixture strength at low engine speeds, the mixture, as regulated by the needle valve, would be too rich at any higher speed. This disadvantage is partly overcome by fitting an air inlet valve F. The area of this inlet opening and the air flow depend upon the difference between the suction pressure at F and the atmospheric pressure. This extra air, which mixes with the mixture flowing up the pipe E, tends to make the mixture strength more or less independent of the speed of the engine.

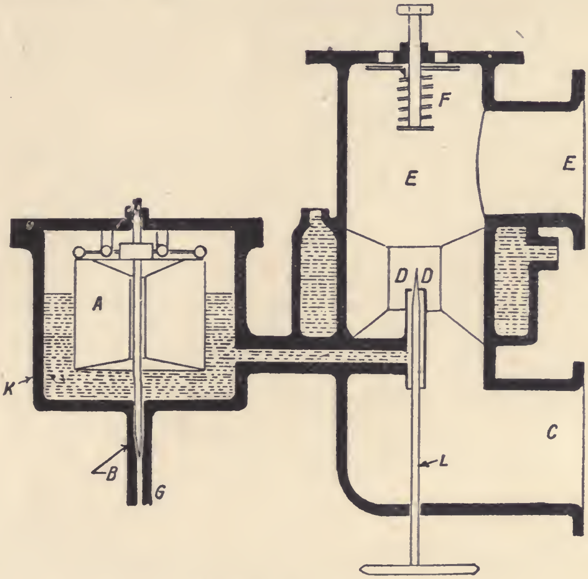


FIG. 53.—Carburettor.

The weight of the petrol flowing from the orifice depends upon :—

(*a*) The pressure of the air at D.

(*b*) The area of the petrol orifice.

(*c*) The viscosity of the petrol.

(*d*) The difference of head h between the petrol in the tank and the petrol orifice.

Assume—

V = velocity of the air at D.

ρ = density of the air at D.

V_1 = velocity of the petrol in the pipe at D.

ρ_1 = density of the liquid petrol in the pipe.

Then the available head which produces the flow of the petrol $=\frac{V^2\rho}{2}-h\rho_1$.

The resistance to the flow of petrol in the pipe $=KV_1$ approximately.

K is a constant, and depends upon (1) the viscosity of the petrol, (2) the shape and size of the petrol jet and pipe.

The "carburettor equation" will then be

$$\frac{V^2\rho}{2}-h\rho_1=\frac{V_1^2\rho_1}{2}+KV_1.$$

If the petrol jet area be large, V_1 may be small, and therefore KV_1 will be more important than $\frac{\rho_1 V_1^2}{2}$.

It is desirable that KV_1 may be negligibly small, because K, which depends upon the viscosity of the petrol, is a factor of uncertain amount.

Assume that in a particular carburettor $h=0$ and KV_1 is negligibly small. The above equation then reduces to $V^2\rho=V_1^2\rho_1$.

Now let $A=$area of the air orifice at D,

$A_1=$area of the petrol orifice,

then the weight ratio of the air to petrol

$$=\frac{A\rho V}{A_1\rho_1 V_1}=\frac{A}{A_1}\sqrt{\frac{\rho}{\rho_1}}.$$

If such a carburettor were fitted to an aerial engine, we see that the petrol-air ratio, which is proportional to $\sqrt{\frac{\rho_1}{\rho}}$, would be dependent upon the height of the machine in the air.

Dependence of the Performance of a Petrol Engine upon the Compression Ratio ***r***.—The percentage of the heat of combustion which is wasted by the escape into the atmosphere of the exhaust gases may be reduced by increasing the compression ratio r. This may be demonstrated by the aid of a hypothetical diagram, see fig. 54.

Thus, assume the charge to be drawn into the cylinder at atmospheric pressure P and absolute temperature t_0. At the end of the adiabatic compression, $pv^\gamma=$a constant, the volume, pressure, and temperature are $\frac{V}{r}$, Pr^γ, and $t_0 r^{\gamma-1}$ respectively. The rise of temperature due to the complete combustion of the mixture, say t, may be assumed to be independent of the compression ratio. The state of the mixture at C (see diagram) is then represented by a volume $\frac{V}{r}$, a temperature $(t_0 r^{\gamma-1}+t)$, and a pressure $Pr\left[r^{\gamma-1}+\frac{t}{t_0}\right]$. After adiabatic expansion to the original volume V, the temperature and pressure become $t_0\left[1+\frac{t}{t_0 r^{\gamma-1}}\right]$ and $P\left[1+\frac{t}{t_0 r^{\gamma-1}}\right]$. We may now complete our imaginary cycle by cooling the volume V of the products of combustion to the

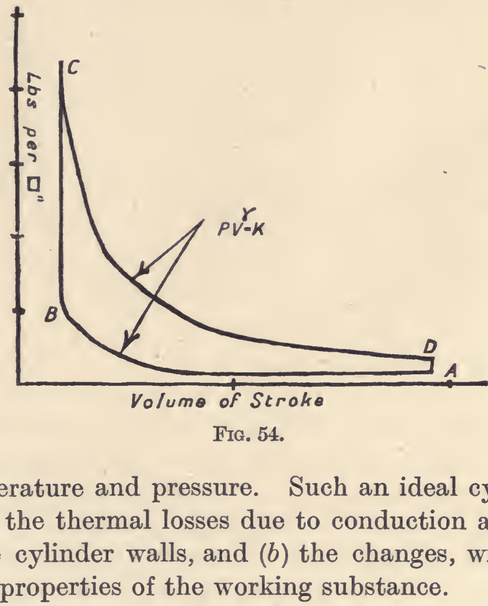


FIG. 54.

initial temperature and pressure. Such an ideal cycle does not consider (*a*) the thermal losses due to conduction and radiation through the cylinder walls, and (*b*) the changes, with temperature, of the properties of the working substance.

The efficiency of this cycle $=\frac{\text{Work done}}{\text{Heat taken in}}$

$$=\frac{\text{Heat taken in}-\text{Heat given out}}{\text{Heat taken in}}=\frac{K_v t-K_v\left[t_0+\frac{t}{r^{\gamma-1}}-t_0\right]}{K_v t}$$

$$=1-\left(\frac{1}{r}\right)^{\gamma-1}$$

where K_v=specific heat at constant volume.

The value of γ for air $=1{\cdot}408$.

The temperature of the products of combustion at D, that is, at the commencement of the exhaust stroke, is $\left[t_0+\frac{t}{r^{\gamma-1}}\right]$, and therefore a large value of the compression ratio encourages an economical use of the fuel. The maximum value of r is limited, however, by the maximum pressure reached during the combustion of the working substance, a large value of r greatly increasing the size, weight, and initial cost of the engine.

From the point of view of fuel economy, the employment of a heavy, strong engine, which has a large compression ratio, is to be encouraged. To prevent pre-ignition of the petrol mixture, the temperature at the end of the compression stroke should be less than 700° C. Pre-ignition is encouraged by the presence of hot particles of carbon or drops of lubrication oil on the cylinder walls, and for this reason the cylinders of aerial engines are thoroughly examined and cleaned at frequent periods. Compression ratios common in aerial engines vary from $3\frac{1}{2}$–5 .

The Horse-Power developed by an Engine.—The work done per stroke is the product of the thermal efficiency of the engine and the heat of combustion supplied at each working stroke.

Let the thermal efficiency of the engine be denoted by η. Also if V = stroke volume in cubic feet,

let λV = volume of the mixture drawn in per stroke, measured at the pressure and temperature of the atmosphere,

and h = heating value of a cubic foot of this mixture measured at the same temperature and pressure,

then the work done during the working stroke $= \eta \cdot \lambda \cdot V \cdot h$,

and the mean effective pressure during the stroke $= \eta \cdot \lambda \cdot h$.

The Volumetric Efficiency λ.—The value of the volumetric efficiency is always less than unity, and this may be chiefly attributed to the following causes :—

(*a*) The incoming mixture has its temperature increased by contact with the hot cylinder walls and valves, and also by further contact with the high-temperature exhaust gases of the clearance spaces.

(*b*) The pressure within the cylinder is lower than the atmospheric pressure.

(*c*) By virtue of the drop of pressure at the supply valve, usually about 2 lbs. per square inch, work is done upon the mixture as it passes from the supply pipe to the cylinder. This work is proportional to the square of the mixture velocity at the valve opening, and may be lessened by an increase in the area of the valve opening.

(*d*) To enable the frictional resistance of the valve and pipes to be overcome, work is performed upon the incoming mixture, and this work increases the temperature of the mixture.

A Consideration of *h*.—The heating value h of a cubic foot of the mixture depends partly upon its density, and therefore upon the temperature and pressure. The fluctuation of h with the altitude of the engine is a question of importance. The pressure of the air at a height of x feet $= P_0 e^{-\frac{x}{26250}}$, where P_0 is the pressure at the earth's surface. It follows, then, that at a height of 5000 feet the pressure may be reduced by about 18 per cent. Usually the air is warmed before passing through the carburettor, so that any fall of temperature with altitude is of less consequence. Assuming that the absolute temperature of the air varies as $(\text{pressure})^{\cdot 28}$—that is, the ordinary adiabatic relation,—the fall in the absolute temperature at a height of 5000 feet would be about 7 per cent. Now, h varies as the ratio of pressure to absolute temperature, and hence the value of h at a height of 5000 feet would be about 13 per cent. less than its value at the earth's surface. Hence the torque of an engine diminishes with an increase of altitude.

Considerations of η.—We may express the thermal efficiency of an actual engine by $R\left[1-\left(\frac{1}{r}\right)^{\cdot 4}\right]$, where R, the relative efficiency, is defined as the ratio of the actual efficiency of the engine to the efficiency of an ideal engine working with the same compression ratio. We shall base the following investigation into the dependence of η upon the size of the engine, upon the assumption that the heat loss furnishes a criterion of the work wasted per stroke. The losses due to conduction of heat through the cylinder walls of similar engines are proportional to the area, whilst the radiation losses are a function of the cylinder volume. Hence, in similar engines, the heat losses may be conveniently expressed by $a_1L^2+b_1L^3$, where a_1 and b_1 are constants and L is the linear scale of the engine. The ratio of the heat dissipated during the working stroke to the total heat of combustion is then proportional to $\frac{a_1}{L}+b_1$. If R_0 represent the relative efficiency of an engine of which the cylinder walls are impervious to heat flow (this is not an ideal engine), the relative efficiency of a similarly designed engine, but with conducting walls, may be expressed by $R_0\left[1-\frac{a}{L}-b\right]$. The experimental value of R_0 is about ·8. It is apparent, then, that the proportionally greater heat loss during the working stroke of a small engine will affect adversely the value of η.

Mechanical Efficiency.—The frictional losses in similar engines which are run at the same piston speed are proportional to area, since the pressure between similar parts and the velocity of rubbing between these parts have constant values. The pump losses are also proportional to area, and hence both the frictional and pump losses of similar engines may be accounted for by a constant reduction from the mean effective pressure. The mechanical efficiency of an engine depends greatly upon the workmanship and the method of lubrication employed. It is a matter of common experience that an engine gives smoother running and a higher mechanical efficiency when it has been "warmed up," and this is probably due to the lower viscosity of the lubricant at the higher temperature. Forced lubrication ensures a steady supply of lubricating oil at a constant temperature, with the maintenance of an oil film of sufficient viscosity between the bearing surfaces, and these advantages more than compensate for the additional cost of the engine. The feeding-in of lubricating oil by centrifugal force, a characteristic of many air-cooled engines, is usually a most wasteful process, and does not encourage a uniform supply of lubricant to the vital working parts of the engine.

Engine Cooling.—The necessity for efficient cooling of the cylinder walls is manifest when we consider that a temperature of 2000° C. is probably reached during the combustion of the petrol mixture. The pre-ignition of a fresh charge can only be prevented if the temperature of the cylinder walls be below 700° C. The temperature and density of the gas have maximum values at the commencement of the working stroke, and to keep the temperature of the inside of the cylinder walls within safe limits, by the maintenance of a great heat flow, the cooling of the cylinder head must be very efficient. Usually air-cooling depends upon the position and speed of the engine, and hence cannot ensure uniform cooling of the working parts. Moreover, any unequal expansion of the cylinder walls, due to a lack of uniformity in cooling, introduces frictional losses between the piston and cylinder walls.

The lightness of air-cooled engines is only attained with a sacrifice of reliability, and, furthermore, in rotary engines such as the Gnome, to overcome the air resistance created by the rotating cylinders, there is a further sacrifice of the engine-power.

Dependence of the Ratio $\frac{\textbf{Engine Weight}}{\textbf{Horse-power}}$ upon (*a*) **Size of Cylinder,** (*b*) **Piston Speed.**—We now propose to compare the performances of engines built from the same drawings but to different linear scales when running at the same piston speed. Assume, further, that the same strength of mixture is employed, and that similar engines have the same efficiency. The latter assumption will need further qualification.

The horse-power of a four-cycle petrol engine is equal to

$$\frac{\text{Mean effective pressure} \times \text{Piston area} \times \text{Piston velocity}}{4 \times 33{,}000}.$$

Under the above conditions, the horse-powers of similar engines will vary as area—that is, as L^2. It is interesting to note that the Royal Automobile Club Taxation Rule is H.P.$= \cdot 4D^2$, where D, the bore, is measured in inches.

The inertia forces of the rotating masses vary as $\frac{\text{Mass} \times (\text{Piston velocity})^2}{\text{Linear dimension}}$—that is, as L^2; and those of reciprocating masses vary as (Mass $\times$ Acceleration)—that is, as L^2; and therefore the stresses in like parts of similar engines, due to inertia forces, have constant values. In addition, the stresses in like parts due to the gas pressures have also constant values, and hence from such considerations similar engines have the same factor of safety.

Under the above conditions, the ratio $\frac{\text{Weight}}{\text{Horse-power}}$ then varies as L.

The foregoing discussion needs to be modified if we consider the decrease in the efficiency of a small engine due to the proportionally greater heat loss during the working stroke.

The data furnished below were obtained from a test upon a 72·5 H.P. Renault engine. We desire to find the probable values of (*a*) $\frac{\text{Weight}}{\text{Horse-power}}$ ratio, and (*b*) $\frac{\text{Fuel}}{\text{B.H.P. hour}}$ ratio, of an engine of the same B.H.P. and piston speed, but a larger number of smaller cylinders.

The First Renault Engine.

Bore $=3\cdot8$ inches. Stroke $=4\cdot75$ inches. No. of cylinders $=8$.

B.H.P. $=72\cdot5$. $\frac{\text{Weight of engine}}{\text{B.H.P.}}=5\cdot25$ lbs.

Piston speed $=1420$ feet per minute. Efficiency $\eta=\cdot77$.

Mean effective brake pressure $=76\cdot4$ lbs. per square inch.

Petrol consumption per B.H.P. hour $=\cdot665$ lb.

The Second Renault Engine.

For the purpose of illustration we shall assume that the bore of each cylinder $=\frac{3\cdot8}{2}=1\cdot9$ inches, and since the cylinders are similar the stroke $=\frac{4\cdot75}{2}=2\cdot375$ inches.

The value of η may be expected to be about $\cdot50$, and $\therefore$ Mean effective brake pressure $(\eta \,.\, \lambda \,.\, h)$

$$=\frac{\cdot50}{\cdot77}\times76\cdot4=49\cdot5 \text{ lbs. per square inch.}$$

$$\text{H.P. per cylinder}=\frac{49\cdot5\times\pi(1\cdot9)^2\times1420}{4\times4\times33{,}000}=1\cdot51,$$

so that the number of cylinders $=\frac{72\cdot5}{1\cdot51}=48$.

$$\text{The ratio } \frac{\text{Weight of engine}}{\text{B.H.P.}}=\frac{5\cdot25}{8}\times\frac{1}{2^3}\times\frac{48}{1}=3\cdot9 \text{ lbs.}$$

$$\text{Fuel used by the engine}=\frac{\cdot75}{\cdot50}\times\cdot665=1 \text{ lb. per B.H.P. hour.}$$

We thus see that the saving of $\left[\frac{5\cdot25-3\cdot9}{5\cdot25}\right]100=25\cdot7$ per cent. in the $\frac{\text{Weight of engine}}{\text{B.H.P.}}$ ratio is only accomplished by the large increase of $\left[\frac{1-\cdot665}{\cdot665}\right]100=50$ per cent. in the $\frac{\text{Petrol}}{\text{B.H.P. hour}}$ ratio.

(*b*) **Piston Speed.**—An increase in the piston speed V will be accompanied by a rise in the temperature of the engine, since the heat of combustion has less time to escape from the cylinder. Also, if the cylinder be very hot η has a high value and λ a low value. The maximum value of the piston velocity is limited by a consideration of the inertia stresses of the reciprocating and rotating masses, and, obviously, these stresses have to be taken in conjunction with those dependent upon the pressure of the burning gas. The whipping of the connecting rod of a high-speed engine may introduce large bending-moment stresses. Furthermore, the great wear and tear between the moving parts of a high-speed machine are not favourable to smoothness nor reliability of running.

The Weight of an Engine.—By the weight of an engine we mean the sum of the weights of all the parts, such as flywheel, silencer, fuel and lubrication tanks, cooling water, radiators, piping, etc., which are essential for a prolonged run. The weights of fuel and of lubricating oil carried by an aeroplane are not included in the engine weight. A factor of great importance to the aeronaut is the sum of the weights of the engine and of the fuel and lubricating oil which must be carried for a flight of known duration. The curves of fig. 55 have been plotted from the data of Table XII., and give for several well-known engines the sum of the weights of the engine and of the fuel and lubricating oil consumed during a flight of known time. From these curves it is apparent that the fuel economy is practically unimportant for a short-distance flight, whilst, on the other hand, a strong, heavy engine, with a good fuel and oil economy, is most suitable for a long-distance one.

It is of the greatest importance that the lightness of an engine should not be due to a weakening of the engine parts, but the design should be such as to favour a minimum number of light efficient working parts. The saving of weight which may be achieved by rotating the cylinders, and so adopting air cooling, has been dealt with elsewhere, the detrimental effects of such a practice, however, must not be ignored. Undue lightness in the piston and cylinder greatly interferes with the heat flow, so that pre-ignition may probably result from the high temperatures of these parts. An economic distribution of the stresses in the working parts by a judicious

Table XII.

Weight of tanks taken as 10 per cent. weight of fuel.
Specific gravity of petrol = 0·725 approximately.

Engine.	Method of Cooling.	Number of Cylinders.	R.P.M.	B.H.P.	Stroke. Mms.	Bore. Mms.	Pints per B.H.P. Hour.		Initial Weight per nominal H.P. Lbs.	Starting Weight for a Run of Six Hours per nominal H.P. Lbs.
							Fuel.	Oil.		
120 H.P. Argyll, sleeve valve. Vertical	Water	6	1200	116·6	175·0	125·0	0·635	0·046	6·6	11·0
125 H.P. British Anzani. Radial	Water	10	1110	100·2	155·0	115·0	1·06	0·150	4·9	10·6
120 H.P. Beardmore Austro-Daimler. Vertical	Water	6	1275	129·0	175·0	130·0	0·60	0·022	—	—
200 H.P. Salmson (Dudbridge). Radial	Water	14	1307	185·0	140·0	120·0	0·678	0·029	5·00	9·3
100 H.P. British Gnome, Monosoupape. Rotary	Air	9	1206	99·3	150·0	110·0	0·764	0·162	3·60	9·1
100 H.P. Green. Vertical	Water	6	1242	103·5	152·0	140·0	0·70	0·048	5·5	9·8
135 H.P. Sunbeam	Water	8	1023	130·9	150·0	90·0	0·591	0·038	6·1	9·9

use of hollow shafts and rods, H-section connecting rods, etc., is a favourable method of reducing engine weight. Also, where possible, simply constructed engine parts should be employed.

Some idea of the materials employed in the construction of the principal parts of an aeronautical engine is given in Table XIII.

General Remarks.—A water-cooled engine, if at the front of the machine, should be enclosed in a good stream-line casing,

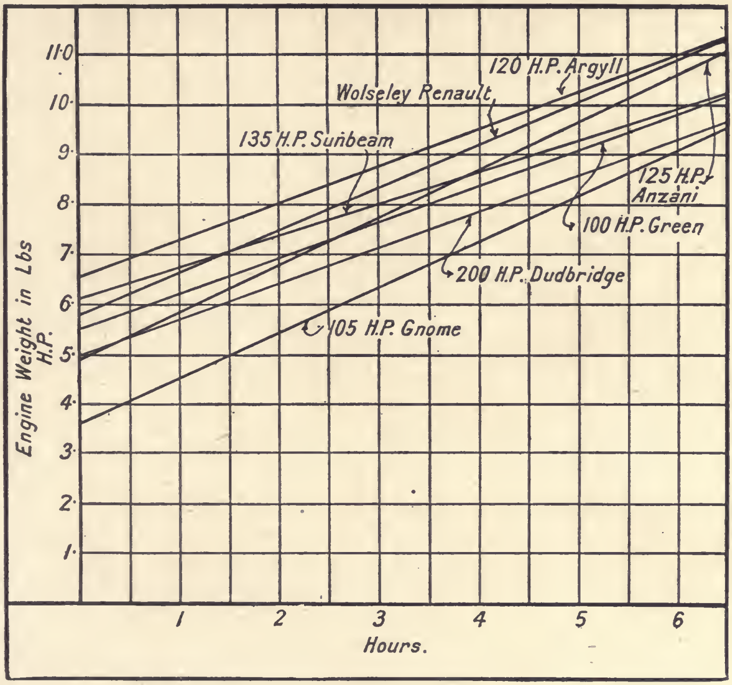


FIG. 55.

so that the power expended in overcoming air resistance, which may be considerable at a high velocity, may be reduced to a minimum. The air-cooling of an engine introduces a large air resistance, so that a reduction in the useful H.P. of the engine is inevitable. In the absence of a flywheel, the non-uniformity of torque of a well-balanced stationary motor may give rise to unpleasant vibrations.

The absence of chattering amongst the working parts of the valve mechanisms, and a silent exhaust-gas escape, are inestimable qualities of a land engine, but in view of the great noise created by the rotating propeller they are, more or less, of small importance in an aeronautical engine.

Where possible, the burnt gases and unburnt oil should be exhausted at a convenient place, so that the motor and machine may not present an unduly dirty appearance. The valves and vital working parts of the engine must be cleaned

TABLE XIII.

Name of Engine Part.	Material and Design.
Cylinder . . .	Cast iron (stationary engines). Nickel steel (rotary engines). *Nickel steel to be preferred.*
Cylinder jacket . .	Jacket may be: (*a*) integral with the body; (*b*) steel, shrunk or welded on, nickel-plated or copper electrolytically deposited on the inside; (*c*) aluminium. (*a*) *is not to be recommended because of the unequal expansion of the cylinder and jacket.*
Piston	Pressed steel. Steel. Semi-steel. Cast iron.
Piston rings . .	Cast iron.
Connecting rods . .	H-section chiefly—nickel steel, special steel, chrome vanadium steel, nickel chrome steel.
Crankshaft . .	Nickel chrome. Chrome vanadium steel. Nickel steel. Special steel.
Main bearings . .	White metal between each crank. Ball bearings.
Crankcase . . .	Aluminium alloy.

frequently, so that accessibility to these parts is a factor to be considered.

The following classification of aeronautical engines has been suggested by J. S. Critchley, M.I.M.E. :—

Class (A).—Engines which retain the approved design of the automobile motor, but which have slight modifications in favour of an increase of reliability and mechanical strength, and a reduction of weight. This is the largest class, and includes the following well-known engines: (*a*) Green, (*b*) Wolseley, (*c*) E.N.V., (*d*) Mercédès, (*e*) Renault, (*f*) Gobron-Brillie, (*g*)

Korting Brothers, (*h*) N.A.G., (*i*) Austro-Daimler, (*j*) Curtiss, (*k*) Sturtevant.

Class (B).—Engines characterised by a star-wise or a radial arrangement of cylinders. Engines of this class are : (*a*) Anzani, (*b*) R.E.P., (*c*) Salmson.

Class (C).—The design of this class of engine is essentially a complete departure from that of the preceding classes. These engines have the star-wise disposition of the cylinders of Class (B), but the cylinders themselves rotate, the crankshaft remaining stationary. The important rotary engines are : (*a*) Gnome, (*b*) Burlat, (*c*) Gyro, (*d*) Clerget.

Class (A).—A popular design of this class is the diagonal or V type of motor, in which the centre lines of the cylinders make 90° with each other. Furthermore, compactness and lightness are favourable characteristics of the V type of engine. The balance of an eight-cylinder V type of engine is thoroughly satisfactory, when the connecting rods of two opposite pistons are connected to the same pin of the crankshaft.

Class (B).—The economic distribution of stress is one of the fundamental reasons for the adoption of a star-wise arrangement of cylinders. Constancy of torque and smoothness of running are achieved by an increase in the number of cylinders. Also, by connecting all the pistons to the same crankpin, the crankarm and pin are continuously under maximum stress, and the efficient use of this material is ensured. When the cylinders are arranged in a vertical plane, the successful lubrication of the set of cylinders below a central horizontal plane is a matter of great difficulty. These lubrication difficulties may be successfully overcome by rotating the lower set of cylinders through 180°, about an imaginary axis passing through the crankshaft. We have now a two-throw crank engine—that is, the two cranks are 180° apart—and the balance of such an engine is less perfect than when the cylinders were arranged uniformly in a vertical plane.

It is worthy of notice that a uniform series of impulses can only be obtained during a complete engine cycle—that is, two revolutions of the crankshaft—when the engine has an odd number of cylinders.

Class (C).—The motors of this class were designed to meet the special requirements of aeronautical engines. Undoubtedly the small value of the ratio $\frac{\text{Engine weight}}{\text{H.P.}}$ and the compactness of this type of engine have only been achieved with a sacrifice in other important factors, such as fuel and oil economy, cooling, and, to a lesser extent, reliability. Nevertheless, the balance of these engines is exceptionally good, and the revolving cylinders perform adequately the functions of a flywheel. The Gnome engine, the most popular rotary engine, is described on p. 155.

A Consideration of some Modern Aeroplane Engines.—Table XII. gives the performances during a six-hour run of some good British engines. The data of this table have been plotted in the curves of fig. 55.

During a recent competition the following tests were performed on each engine :—

(*a*) Two runs of six hours each, at full power or throttled down. Each engine was placed in an inclined position—not exceeding 15°—for short special runs.

(*b*) The consumptions of fuel and lubricant were measured.

(*c*) Each engine was dismantled between tests.

(*d*) Other tests were imposed to bring out the relative merits of competing engines.

For the benefit of engine makers, the desirable attributes of an aeroplane engine were classified thus :—

(*a*) Total light weight. (*b*) Economy of consumption. (*c*) Absence of vibration. (*d*) Smooth running, whether in an inclined or normal position, and whether at full power or throttled down. (*e*) Slow running under light load. (*f*) Workmanship. (*g*) Silence. (*h*) Absence of deterioration after running. (*i*) Simplicity of construction. (*j*) Suitable shape to minimise head resistance. (*k*) Precautions against accidental stoppage, that is, dual ignition. (*l*) Adaptable for starting otherwise than by airscrew swinging. (*m*) Accessibility of parts. (*n*) Freedom from risk of fire. (*o*) Absence of smoke or ejections of oil and petrol. (*p*) Convenience of fitting in an aeroplane. (*q*) Relative invulnerability to small-arm projectiles. (*r*) Economy (in bulk, weight, and number) of minimum spare parts. (*s*) Excellence of material. (*t*) Reasonable price. (*u*) Satisfactory running under climatic variations of temperature.

We shall now give descriptions of several well-known aeronautical engines.

120 H.P. Argyll Engine.—The engine has six separate vertical steel cylinders on which are welded steel water jackets. A single cast-iron sleeve valve is fitted between the piston and the cylinder walls, which alternately covers and uncovers the inlet and exhaust valves in the cylinder walls. Thus the valves of ordinary type are not fitted. The cylinders, which are attached to an aluminium crankcase, are provided with detachable water-cooled heads in which two plugs are fitted. The pistons are made of steel and are fitted with two rings. The crankshaft has seven journals running in seven white-metal bearings, a large double ball-bearing taking the thrust of the airscrew. The water is circulated by a centrifugal pump, and a honeycomb radiator is fitted. The oil is pumped under a pressure of 25 lbs. per square inch to the crankshaft bearing, sleeve-valve actuating gear, and other important parts. A second pump sucks the surplus oil from the bottom of the oil chamber and returns it to the tank, which is at the rear of the engine. A high-tension two-spark Bosch magneto is fitted and is connected to both plugs. Two Zenith carburettors are carried, each supplying three cylinders. A hand starting-gear enables the engine to be turned without the pilot leaving his seat.

120 H.P. Beardmore Austro-Daimler Engine.—A photograph of this engine is given in fig. 56.

The six vertical cylinders are each fitted with an electrically deposited copper water-jacket, the water being circulated by a centrifugal pump, whilst the cooling takes place in a light honeycomb radiator. The pistons are of steel, and the crankshaft, as in the Argyll engine, runs in seven main bearings. The valves, which are situated in the head of the cylinder, are operated by push rods and overhead rockers. A Bosch high-tension double-spark magneto is fitted. The two synchronised carburettors supplied with the engine are fitted with hot-water jackets. The lubricating oil is forced to the working parts by valveless piston pumps.

The 200 H.P. Salmson Engine (Canton-Unné System).—A photograph of this engine is given in fig. 57.

This engine is manufactured at the Dudbridge Ironworks. The cylinders are of forged steel, machined all over, and each cylinder may be readily removed without parting the crankcase. The water jackets are of spun copper, brazed on. The water circulation is maintained by a centrifugal pump driven by gearing, which draws the water through the radiators and forces it through the water jackets from bottom to top. When the engine is in motion, the momentum and perfect balance of the moving parts are sufficient to keep the engine running steadily, even when the airscrew is removed.

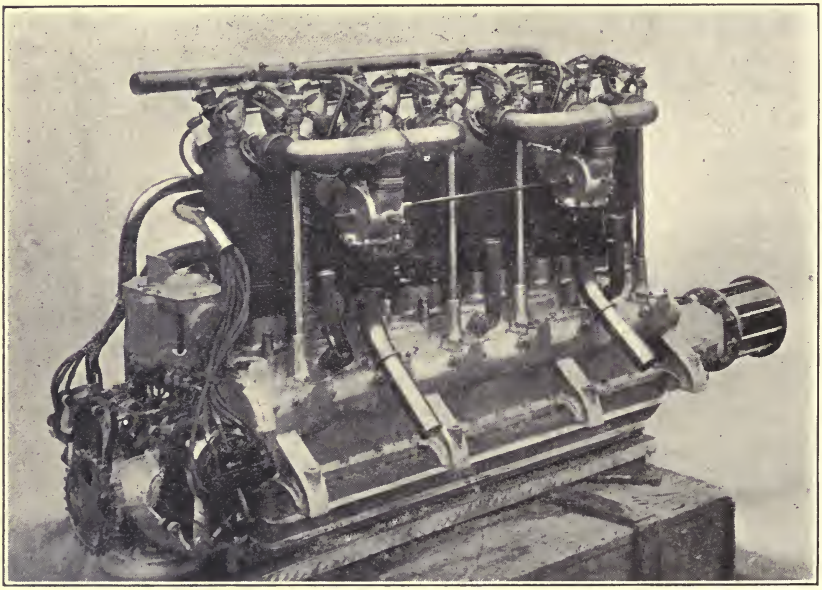


FIG. 56.—120 H.P. Beardmore Austro-Daimler Engine.

The pistons are of cast iron of a special quality, and are each fitted with three rings, and the resulting slight increase of weight is fully compensated for by the greater reliability of the engine.

The connecting rods are made from high-grade steel, machined all over, fitted with phosphor-bronze bushes. The connecting rods are connected to a central collar, which is carried on a crankpin by two ball-bearings.

By means of a patent planetary gearing all the pistons have exactly the same stroke.

The casing is in two parts : one carries the brackets for fixing the engine, and the other the valve gear. These two parts, when bolted together, hold the cylinders in position, but in such a manner that the clamping of the cylinders does not produce any distortion.

The valves are of special steel, and are all mechanically

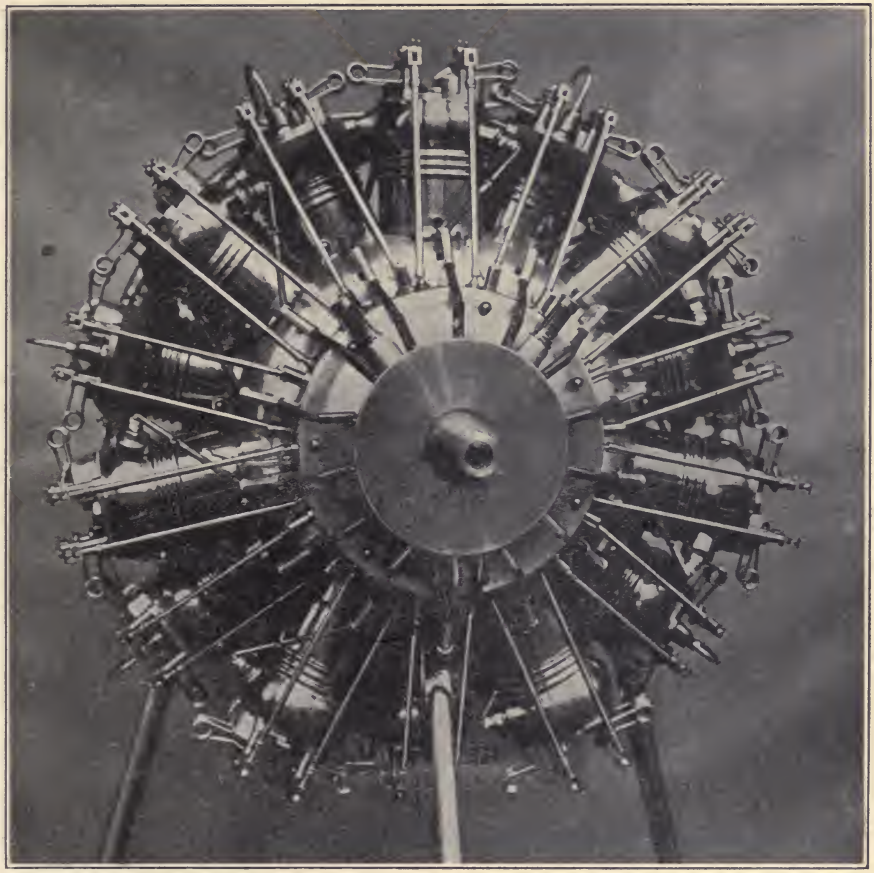


Fig. 57.—200 H.P. Salmson Engine.

operated. They are of large dimensions, and are held on their seats by double springs. These springs are so designed and arranged that their active parts are practically isolated from the hot parts of the cylinders. The valves are operated by means of push rods and rocking levers, bushes of tempered steel being fitted between the parts in contact.

Two Zenith carburettors are fitted at the extremities of a horizontal diameter of an annular space, which is cast integral with the rear half of the crankcase. From this passage short induction pipes are led to each cylinder. Each carburettor is heated by exhaust gases. Ignition by two Bosch high-tension magnetos. A compressed air self-starting arrangement is provided.

Efficient lubrication is assured by a double oil pump situated at the lower part of the engine. The first pump forces the oil under pressure into two single feeds, one of which leads to the centre of the crankshaft, which is hollow, and conducts the oil to the principal parts to be lubricated ; the second to the valve gear. The oil which collects at the lower part of the engine is taken up again by the second pump and returned to the tank. The cylinders are arranged in such a way that the oil which flows along the walls cannot flood the lower cylinders.

100 H.P. Gnome Engine, Monosoupape Type.—A photograph of this engine is shown in fig. 58. This rotary engine is made from the design of Laurent Sequin, and is manufactured by the Gauthier Company. The cylinders, nine in number, are arranged in one plane radially round a single fixed crank. Each cylinder is provided with an exhaust valve in the cylinder head, and the inlet ports are in the cylinder walls in such a position that they are uncovered by the piston as it approaches the bottom of its stroke. Communication is made from the interior of the crankcase to the inlet port. The cycle of operations is as follows :—

After firing the mixture, the piston descends, and at $\frac{3}{4}$ stroke, approximately, the exhaust valve opens. The inlet ports are uncovered just before the end of this stroke, but centrifugal force prevents the exhaust gases passing into the crankcase. During the return stroke the inlet ports are covered by the piston, and the remaining burnt gases are driven out of the exhaust valve. During the third stroke air is drawn into the cylinder through the exhaust valve, which does not close until the middle of the stroke. At the end of this stroke the inlet ports are again uncovered and a rich mixture is drawn into the cylinder. The explosive mixture, composed of the pure air drawn in the exhaust valve and the rich mixture which passed through the inlet valve, is compressed during the fourth stroke. The rich mixture in the crankcase is obtained by spraying petrol into the front half of the hollow crankshaft, the air necessary for the carburation of the fuel passing through this shaft. The exhaust valves are operated by long push rods from the cam-box situated in the front of the crankcase, the cams operating these rods through tappet rollers and rocking levers. The rocking levers are carried in a separate housing, which can be rotated about

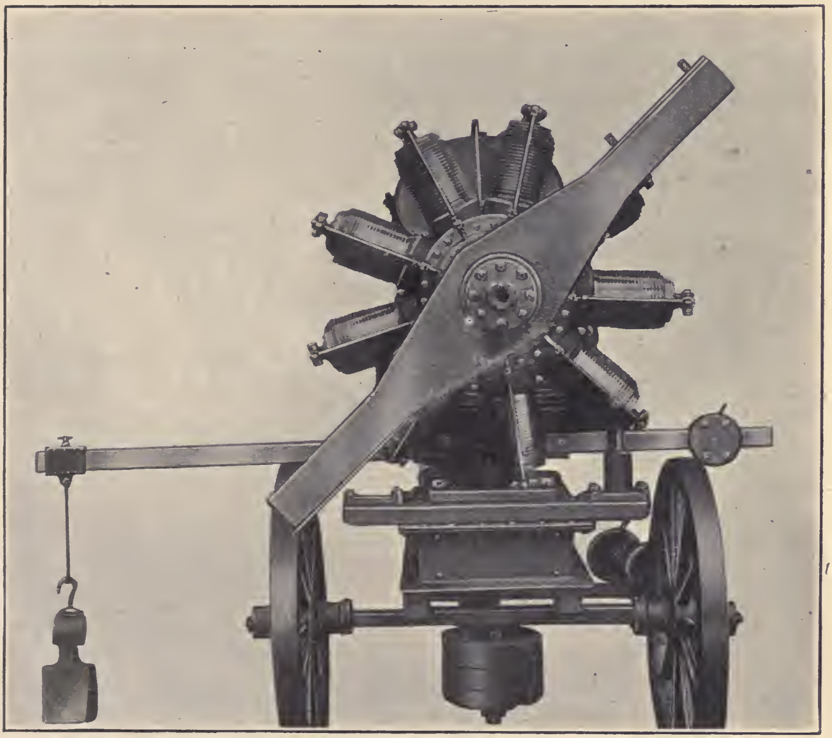


Fig. 58.—100 H.P. Gnome Engine, Monosoupape type.

the centre of the cam-shaft by means of bell-cranks and a worm situated in the rear part of the crankshaft. The worm can be operated by a hand-wheel placed near the pilot's seat. By means of these movable rocking levers, the lift of the exhaust valves is adjustable, and thus the H.P. of the engine may be varied. Each cylinder is machined out of a solid bar of steel.

A drip-feed lubricator is provided, which delivers oil into the centre of the crankcase, whence the lubricant is distributed to the cylinders by centrifugal force.

100 H.P. Green Engine.—This engine has six separate vertical cylinders, which are bolted down to the top half of an aluminium crankcase by five bolts, four of which are carried right through the crankcase and form supports for the main bearing caps. The cylinders are of cast-steel, machined inside and outside; spun copper water jackets are fitted. At the botton of the jacket a rubber ring fitting into a groove takes up the difference of expansion of the copper jacket and the steel cylinder, at the same time forming a watertight joint. The valves are of the usual mushroom type, and are fitted in cages side by side in an inverted position on the cylinder head. The valve springs are enclosed under a small dome through the centre of which the valve stem projects. The valves are actuated by short rocking levers from one cam-shaft which is enclosed in a small aluminium casing made in sections. The cam-shaft is driven by a vertical encased shaft, which is rotated from the crankshaft by skew gearing. The rocking levers are pivoted in an extension of the cam-shaft casing in such a manner that they can be swung clear of the cylinder heads. Two interconnected Zenith carburettors are fitted. A water pump maintains the circulation of the cooling water, the design of the cylinder heads ensuring the efficient cooling of the valves. Two spiral tube radiators are fitted on each side of the engine.

The crankshaft runs in white-metal bearings, and has a large double-thrust bearing at the airscrew end. The cam-shaft bearings are of phosphor bronze. The main oil channel is cast solid with the crankcase, and oil is forced from this by a small gear pump through hollow columns, through which the holding-down bolts pass, and thence to the main bearings and the crankshaft, the latter being hollow. Hence no separate lubricating pipes are employed on the engine. The oil pump sucks oil from a tank placed immediately under the crankcase.

300 H.P. Green Engine.—This powerful engine has twelve cylinders arranged in V formation (stroke 172 mms., bore 142 mms.). The design of these cylinders is similar to that of the 100 H.P. engine. The controls of the carburettors, four in number, are synchronised. The oil-pump unit and the water-pump unit are duplicated, although the engine may be run with only a single set of units. The most economical speed of running is 1300 R.P.M., but the engine runs smoothly when throttled down to 160 R.P.M.

The petrol consumption is about ·63 pint per B.H.P. hour, the oil consumption during an hour being about 1 gallon. The

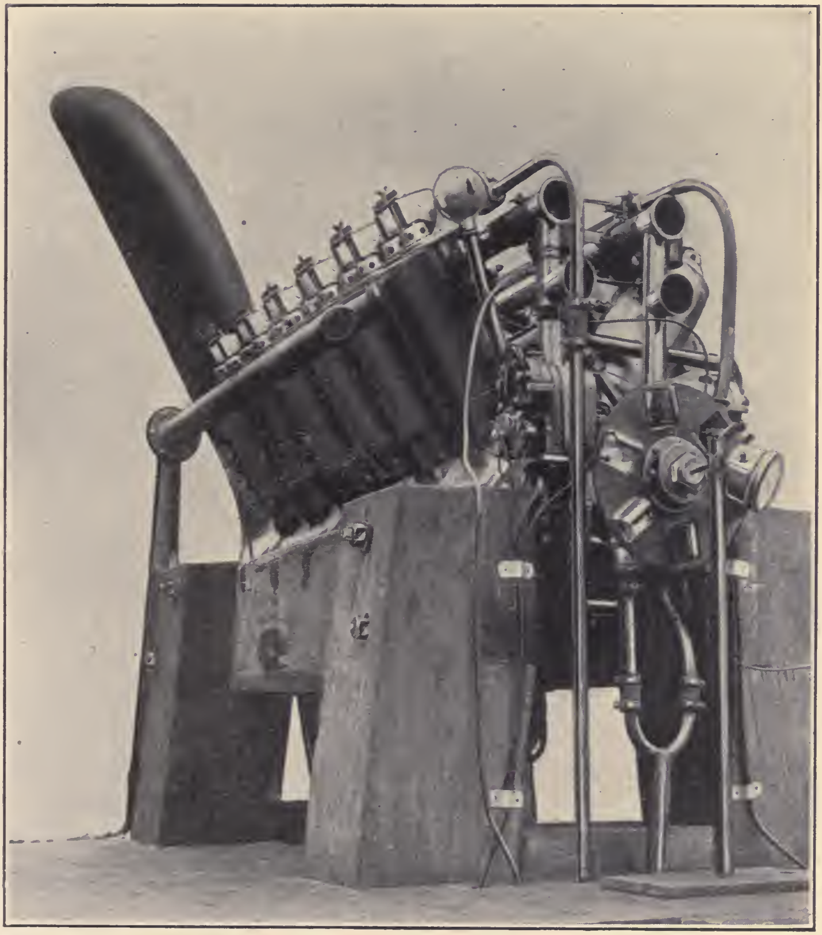


FIG. 59.—300 H.P. Green Engine.

weight of the engine is 900 lbs., and the weight of radiator and connections 150 lbs.

135 H.P. Sunbeam Engine (90° V Type).—This engine has eight cylinders in two monoblock castings of four each. The cylinders are water-cooled, the jackets being of copper electrically deposited, the water being circulated by a centrifugal pump. The valve-caps are water-cooled by means of a manifold placed along the tops of the cylinders. The crankshaft has five journals running in white-metal bearings, with phosphor-bronze liners housed in the webs of the crankcase. The crankcase is made of an aluminium alloy. The lubricating oil is forced to the crankshaft journals and pins, but splashed to the cylinders and gudgeon pins.

APPENDIX I.

A GRAPHICAL METHOD OF CALCULATING THE AERO-DYNAMICAL PERFORMANCE OF AN AEROPLANE.[1]

THE machine for which the calculations are made is a tractor biplane of weight 2100 lbs. and wing area 370 square feet. It is assumed that the weight of the machine is taken on the wings alone, and no allowance is made for the lift on other parts of the machine. Consideration is taken of the extra resistance of those parts of the machine in the slip-stream of the airscrew.

It is convenient to consider the resistance of the machine as made up of two parts: (*a*) the resistance of the wings, and (*b*) the resistance of the remaining parts of the machine—body, under-carriage, tail plane, etc.—which is called the "extra-to-wing" resistance.

The data from which the calculations are made are given below.

TABLE XIV.

Area of wings = 370 square feet.

Angle of Incidence of Wings. Degrees.	Absolute Lift Coefficient. K_a.	$\frac{\text{Absolute Lift Coefficient}}{\text{Absolute Drag Coefficient}}$. $\frac{K_a}{k_a}$.
−2	0·105	10·0
0	0·172	16·8
2	0·243	19·5
4	0·315	19·4
6	0·385	17·9
8	0·450	15·9
10	0·517	13·6
12	0·567	11·4
14	0·595	9·0
16	0·600	7·0

$$\text{Lift, in lbs., on the wings} = K_a A \rho V^2,$$
$$\text{Drag, in lbs., on the wings} = k_a A \rho V^2,$$

where A = area of wings in square feet,

V = wind speed in feet per second,

ρ = density of the air ($\rho = 0·00237$ at the ground level).

[1] A method of calculating the aerodynamical performance of an aeroplane is given in Chapter V.

Table XIV. gives the aerodynamical properties of the wings, which were deduced from experiments on a model of a single wing, the appropriate corrections having been applied for (*a*) "biplane" effect, (*b*) "scale-speed" effect, (*c*) gap, (*d*) stagger, (*e*) aspect ratio, (*f*) wing tips.

At a wind speed of 100 feet per second the "extra-to-wing" resistance is 130 lbs., of which 65 lbs. is assumed in the slip-stream of the airscrew. The increase of resistance of the machine due to the slip-stream of the airscrew may be expressed approximately by the expression $\frac{2R_0T}{(100)^2\rho A}$, where R_0 is the resistance of that portion of the machine affected by the slip-stream, in a wind of speed 100 feet per second and of density ρ. T lbs. is the thrust of the airscrew, and A square feet is the disc area of the airscrew.

For the machine under consideration, $R_0 = 65$ lbs., and $A =$ 65 square feet, so that $\frac{2R_0T}{(100)^2\rho A} = 0{\cdot}08T$ approximately, and the "extra-to-wing" resistance is $\left[130\left(\frac{V}{100}\right)^2 + {\cdot}08T\right]$ lbs.

Airscrew Data.—Diameter, D, of the airscrew $= 9' \; 1'' = 9{\cdot}08'$. The translational speed of the airscrew, V, is measured in feet per second, and the rotational speed, n, in revs. per second.

The performance of the airscrew is expressed by the data of the following table:—

TABLE XV.

$\frac{V}{nD}$.	T_C.	Q_C.
1·0	0·050	0·0120
0·9	0·102	0·0200
0·8	0·180	0·0310
0·7	0·300	0·0470
0·6	0·495	0·0695

The thrust, T, in lbs. $= T_C \rho D^2 V^2$.

The torque, Q, in lbs.-ft. $= Q_C \rho D^3 V^2$.

And the horse-power put into the airscrew $= \frac{2\pi Q_C \rho D^3 V^2 n}{550}$.

The interference of the body on the airscrew has been neglected.

Engine Data.—The relationship between the speed of rotation and the horse-power of the engine is given in Table XVI.

TABLE XVI.

Speed of Engine. R.P.M.	Horse-power.
1800	105·5
1700	100·6
1600	95·2
1500	89·5
1400	83·2
1300	77·1
1200	70·6

The airscrew runs slower than the engine, the gearing ratio being 2.

Calculation of the Performance of the Machine at Ground Level ($\rho = 0·00237$).—Table XVII. shows the relationship between the resistance of the machine and the speed of horizontal flight. The table needs no explanation if we remember that, when a machine is in horizontal flight, $W = K_a \rho A V^2$, that is,

$$V = \sqrt{\left(\frac{2100}{0·00237 \times 370 \times K_a}\right)} = \frac{49}{\sqrt{K_a}}.$$

TABLE XVII.

Angle of Incidence of Wings. Degrees.	K_a of Wings.	Speed of Horizontal Flight. Feet per second.	$\frac{K_a}{k_a}$ of Wings.	Resistance of Wings. $\frac{Wk_a}{K_a}$. Lbs.	"Extra-to-wing" Resistance, not including Resistance due to Slip-stream. Lbs.	Total Resistance of Machine, ignoring Slip-stream. Lbs.	Extra Resistance due to Slip-stream. ·08T.	Thrust of Airscrew. T. Lbs.
−2	0·105	151·0	10·0	210	298	508	45	553
0	0·172	118·0	16·8	125	182	307	27	334
2	0·243	99·5	19·5	108	129	237	21	258
4	0·315	87·2	19·4	108	99	207	18	225
6	0·385	78·9	17·9	118	81	199	18	217
8	0·450	73·0	15·9	132	70	202	18	220
10	0·517	68·0	13·6	154	60	214	19	233
12	0·567	65·0	11·4	185	55	240	21	261
14	0·595	63·6	9·0	234	52	286	25	311
16	0·600	63·2	7·0	300	52	352	31	383

The data of the above table are shown graphically in fig. 60.

Using the data of Table XV., we next construct the curves of fig. 61, each member of the family showing the variation of the thrust of the airscrew with the speed of rotation, at a constant value of the translational speed. Similarly, the "constant-velocity" lines of fig. 62 give the relationship between the horse-power put into the airscrew and its speed of rotation.

Any point on curve A of fig. 61, which is constructed from information obtained from curve D of fig. 60, gives the thrust

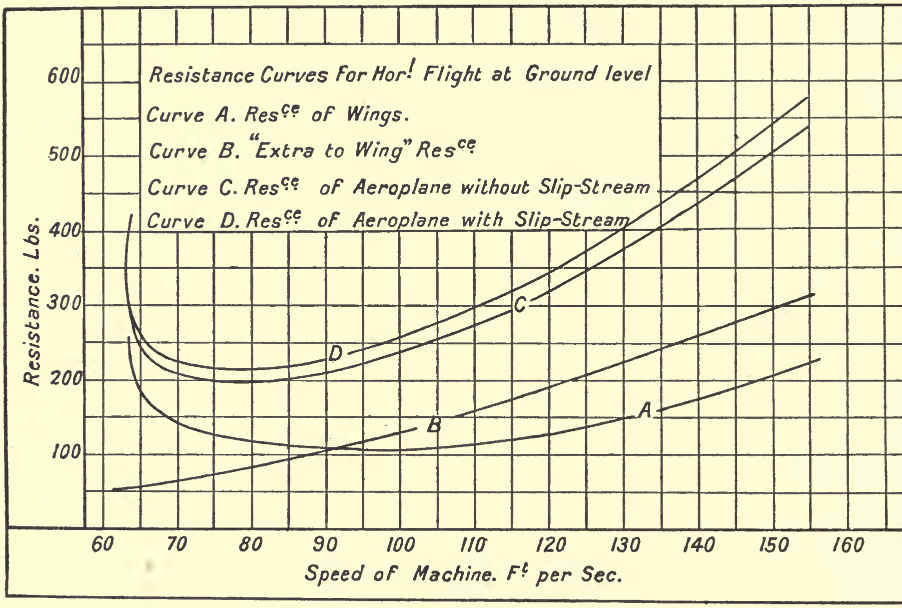


FIG. 60.

and the rotational speed of the airscrew at the corresponding speed of horizontal flight. Thus, from fig. 60 we see that at a speed of horizontal flight of 100 feet per second the resistance of the machine is 260 lbs., and plotting this resistance on the "100 ft.-per-sec." line of fig 61 we find the rotational speed of the airscrew to be 772 R.P.M. Similarly, transferring the curve A of fig. 61 to the "constant-velocity" lines of fig. 62, we obtain curve A, from which we can read directly the horse-power put into the airscrew at any speed of horizontal flight. In curve B of fig. 62 the maximum horse-power developed by the engine is plotted against the rotational speed of the *airscrew*, so that for

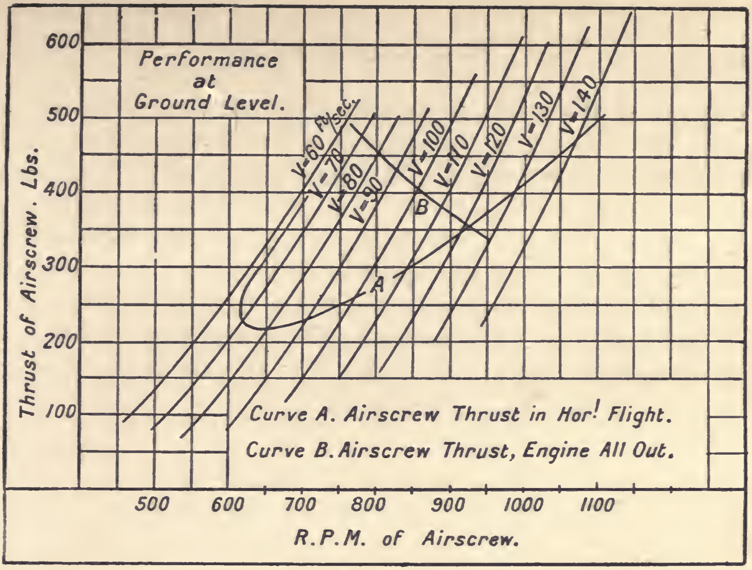

Performance
at
Ground Level.
600
500
400
300
200
100
Thrust of Airscrew. Lbs.
V=60 ft/sec.
V=70
V=80
V=90
V=100
V=110
V=120
V=130
V=140
A
B
Curve A. Airscrew Thrust in Hor! Flight.
Curve B. Airscrew Thrust, Engine All Out.
500
600
700
800
900
1000
1100
R.P.M. of Airscrew.

FIG. 61.

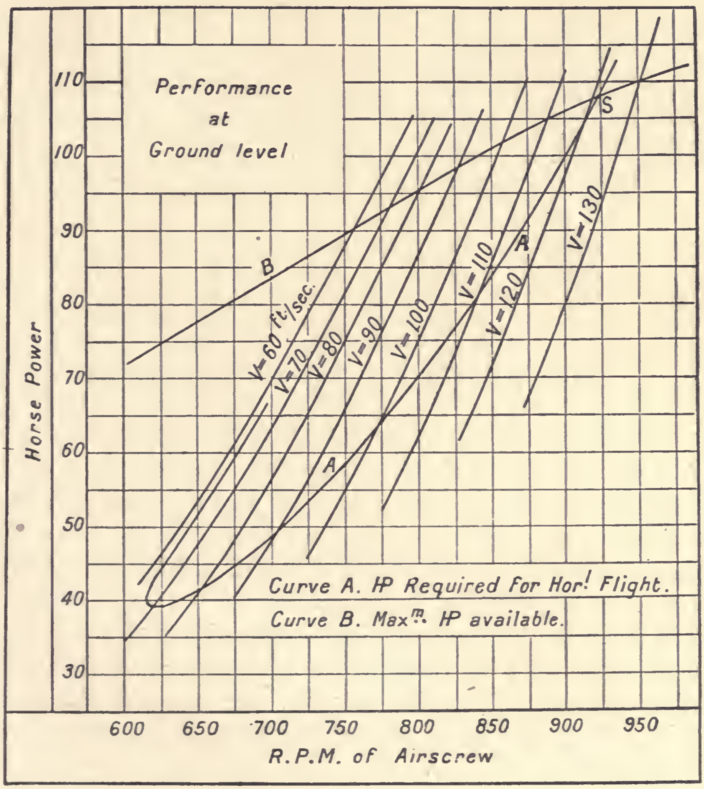

Performance
at
Ground level
110
100
90
80
70
60
50
40
30
Horse Power
B
A
A
S
V=60 ft/sec.
V=70
V=80
V=90
V=100
V=110
V=120
V=130
Curve A. HP Required for Hor! Flight.
Curve B. Maxm. HP available.
600
650
700
750
800
850
900
950
R.P.M. of Airscrew

FIG. 62.

any speed of the machine we may obtain the speed of rotation of the airscrew when the engine is developing its maximum horse-power at that speed. The maximum speed of horizontal flight is given by the point S, where the curves A and B intersect. Curve B of fig. 61, which is plotted from the data of curve B of fig. 62, gives for any speed of the machine the maximum thrust which the airscrew can develop, and its speed of rotation.

A calculation of the maximum rate of climbing at any speed of the machine is given in Table XVIII. Obviously, the rate of climbing is $\frac{(T-R).V.60}{W}$ ft. per min., where

T lbs. = thrust of the airscrew,
R lbs. = resistance of the machine,
W lbs. = weight of the machine,
and V ft. per sec. = speed of the machine.

TABLE XVIII.

Speed of Machine. Feet per second.	Airscrew Speed in R.P.M. when Engine is working all out. (See curve B of fig. 62.)	Airscrew Thrust, T lbs., when Engine is working all out. (See curve B of fig. 61.)	Resistance of Machine, ignoring Slip-stream Resistance. Lbs.	Extra Resistance due to Slip-stream ·08T. Lbs.	Total Resistance of Machine. R. Lbs.	Rate of Climbing. Feet per minute.
120	919	358	318	29	347	38
110	888	383	275	31	306	242
100	857	407	235	33	268	397
90	827	432	210	35	245	480
80	803	456	198	37	235	506
70	781	476	207	38	245	462
65	770	485	245	39	284	374

A small correction should be applied to the rate of climbing as calculated above, because, at the same speed, the attitude relatively to the air of a machine during a climb is slightly different from that of horizontal flight. This correction can, however, be neglected with safety.

The data of the first and last columns of Table XVIII. have been plotted in fig. 63, from which it is seen that the maximum speed of horizontal flight is 121·5 ft. per sec., and also that the machine has a maximum rate of climbing of 510 ft. per min. at a speed of 79 ft. per sec.

The above calculations give the performance of the machine at ground level. Obviously, the performance at any altitude may be similarly calculated, if ρ be given its correct value. It must be remembered that the maximum power developed by an engine falls as the altitude of the machine increases, an

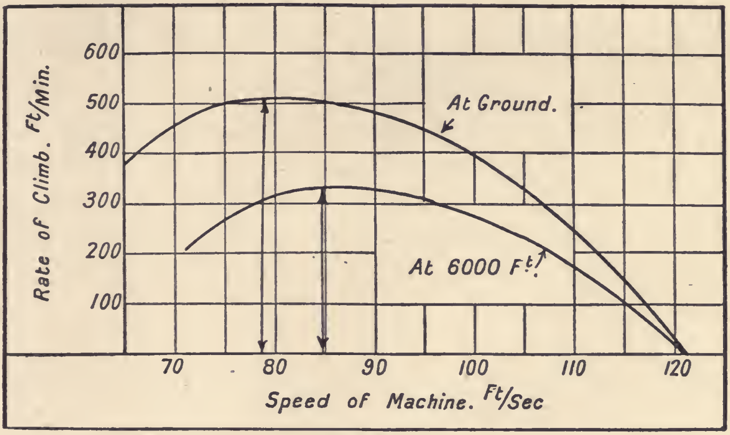


FIG. 63.

approximate rule being that for any given speed of the engine and for the same setting of the throttle and petrol mixtures the horse-power developed is proportional to the density of the outside atmosphere. Table XIX. gives the performance of the machine at an altitude of 6000 feet, where the density of the air is 0·81 of the density at the ground.

TABLE XIX.—*Calculated Performance at an Altitude of* 6000 *feet.*

Speed of Machine. V. Feet per second.	Rate of Climbing. Feet per minute.
115	108
110	173
100	272
90	327
80	313
75	268

The above values have been plotted in fig. 63. The maximum speed of horizontal flight is 121 ft. per sec., and the maximum rate of climbing is 330 ft. per min. at a speed of 85 ft. per sec.

APPENDIX II.

SKIN FRICTION OF FLAT SURFACES IN A UNIFORM CURRENT OF AIR.

SOME of the first experiments to determine the friction of air flowing over even surfaces were made by Dr A. F. Zahm, of the United States, who measured the frictional forces on smooth, plane, rectangular boards at various wind speeds. It was found that for any one board the surface friction due to the flow of air could be expressed in the form

$$F = KV^{1·85},$$

where $F \equiv$ the frictional force per unit of area,
$V \equiv$ the wind speed,
and K is a numerical constant.

A summary of the data obtained from these experiments on smooth boards is given in Table XX.

TABLE XX.—*Skin Friction for various Lengths of Rectangular Surface at a Wind Speed of* 10 *Feet per Second.*

The length of the board is in the direction of the wind.

Length, *l*, of board measured in feet	2	4	8	12	16
Average friction, *f*, measured in lbs. per square foot	0·000524	0·000500	0·000475	0·000467	0·000457

It can easily be found that the law connecting the values of "l" and "f" of the above table is $f = 0·000551 l^{-0·07}$, and hence the law for skin friction on rectangular boards is given by the expression

$$\frac{F}{A} = 0·000551\left(\frac{V}{10} \times \frac{88}{60}\right)^{1·85} l^{-0·07},$$

that is, $$F = 0·0000158 A V^{1·85} l^{-0·07},$$

where $V \equiv$ wind velocity measured in miles per hour,
$A \equiv$ area of the surface measured in square feet,
$l \equiv$ length of the board measured in feet.

Zahm observed the same friction whether the board was covered with glazed cambric, or sheet zinc, or dry varnish, or wet sticky varnish, or sprinkled with water, or covered with calendered or uncalendered paper; but the friction was increased from 10 to 15 per cent. when the board was covered with coarse buckram having sixteen meshes to the inch. He came to the conclusion that all even surfaces have approximately the same coefficient of skin friction, whereas uneven surfaces have a greater coefficient of skin friction, the resistance increasing approximately as the square of the wind speed. From Zahm's expression for skin friction, it is seen that at a wind speed of 60 miles per hour the skin friction per unit of area of an average size of plank is 0·027 lb., and for practical purposes it is safe to assume that the frictional resistance varies as the square of the wind speed.

Some further experiments on skin friction of various surfaces in air have been made by Willis A. Gibbons in the wind channel of the Washington Navy Yard, U.S.A.[1] Plate glass was used as the standard or ideal surface, which could be roughened by the attachment of various fabrics. As in Zahm's experiments, the skin friction has been expressed by the relationship $F = KV^n$, where

$F \equiv$ frictional force in lbs. per square foot,
$V \equiv$ wind speed in miles per hour,
and K and n are constants for any one surface.

The values of K and n for various surfaces are given in Table XXI. The table has been made more complete by the inclusion of the surfaces 18–24, the values of K and n, for air, of these surfaces having been deduced from Froude's experiments.

The surfaces marked by * are classified as rough.

The values of K and n of the table are plotted in fig. 64, from which it is seen that there are two distinct types of surface:—

(a) Smooth surfaces, which are more or less continuous, even, and free from nap. The best surface in this group is smooth glass, whilst fabric surfaces of fine threads closely woven and free from nap mark the other extremity.

[1] First Annual Report of the National Advisory Committee for Aeronautics, United States of America, 1915.

TABLE XXI.—*Results of Experiments on Surface Friction, in Air, of Various Surfaces.*

Reference Number of Surface.	Nature of Surface.		Value of n.	Value of [K 10^7]	Skin Friction in Lbs. per square foot at a Wind Speed of 60 miles per hour.
1	Plate glass	. . . . . .	1·81	166	0·0276
2	Fine linen	1 coat of aero varnish, 1876	1·84	163	0·0309
3	,,	3 coats of aero varnish, 1876	1·89	129	0·0295
4	,,	3 coats of aero varnish, 1876; 1 coat of spar varnish	1·84	153	0·0287
5	,,	uncoated . . .	1·94	128	0·0361
6	,,	1 coat of varnish, 1877	1·85	149	0·0292
7	,,	3 coats of varnish, 1877	1·85	149	0·0288
8	,,	3 coats of varnish, 1877; 2 coats of spar varnish	1·84	157	0·0286
9*	Balloon fabric	No. 3 double parallel, cotton surface	1·90	219	0·0518
10	,,	No. 3 double parallel, singed	2·05	96·5	0·0426
11	,,	No. 3 double parallel; 1 coat of varnish, 1876	1·95	123	0·0362
12	,,	No. 3 double parallel; 3 coats of varnish, 1876	1·85	153	0·0300
13*	,,	No. 3 bias, cotton surface	1·95	207	0·0494
14	,,	No. 6 bias, special, cotton surface	2·03	99·7	0·0412
15	,,	No. 6 bias, special, cotton, singed	2·05	82 5	0·0368
16	Aeroplane fabric	Rubber surface . .	1·83	165	0·0299
17	,,	Rubber, aluminium surface	1·83	166	0·0299
18	Varnish	These results for air were deduced from Froude's experiments.	1·85	156	0·0304
19	Paraffin		1·94	126	0·0355
20	Tin foil		1·99	101	0·0351
21*	Calico		1·92	261	0·0672
22*	Fine sand		2·00	209	0·0753
23*	Medium sand		2·00	223	0·0802
24*	Coarse sand		2·00	255	0·0917

(*b*) Surfaces roughened by nap. Usually these surfaces have high indices (1·9 to 2·0). Cotton, although of fine weave, has a higher resistance than linen, this being probably due to the smooth, wiry nature of the linen yarn.

From the above experimental data it is seen that the frictional force per unit area of plate glass, at a wind speed of 60 miles per hour, is 0·0276 lb.

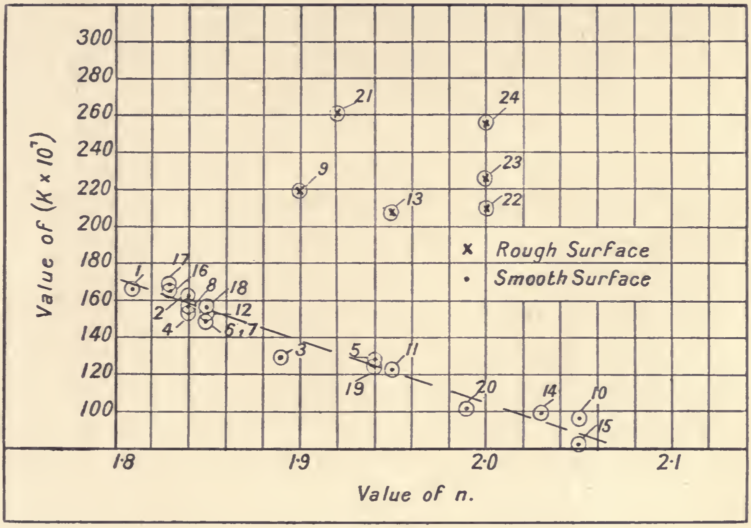


Fig. 64.

A general expression for the frictional resistance of a plane, which takes into consideration the linear size of the plane and the coefficient of kinematic viscosity of the fluid, is $F = K\rho\nu^{2-n}L^nV^n$, where the force F, the density of the fluid ρ, the velocity of the plane relatively to the fluid V, and the coefficient of kinematic viscosity ν, are all measured in any system of consistent units, K and n being absolute coefficients. The values of K and n may be calculated from the measurement of forces on similar surfaces which are similarly situated relatively to the fluid.

APPENDIX III.

A COLLECTION OF USEFUL DATA.

TABLE XXII.—*Table showing some Properties of Metals and Alloys.*

Material.	Weight. Lbs. per cubic foot.	Strength to resist Tension. Tons per square inch.	Strength to resist Compression. Tons per square inch.	Young's Modulus of Elasticity, E. Tons per square inch.
Steels:				
Ordinary mild steel (0·2 per cent. of carbon)	490	28–32	...	13,000–14,000
Specially mild steel . .	...	24–36	...	...
Spring steel, annealed .	...	45–50	...	...
,, tempered .	...	60–70	...	...
Steel castings . . .	...	15–45	...	...
,, annealed .	...	25–35	...	...
Ordinary steel wire . .	...	70	...	...
Steel wire, tempered .	...	100	...	...
Pianoforte steel wire .	...	120–150	...	...
Nickel steel, 5 per cent. nickel, annealed	...	40	...	...
Nickel steel, 12 per cent. nickel, annealed	...	90	...	...
Chrome steel . . .	...	80	...	...
Tungsten steel . . .	...	72	...	...
Wrought iron (good quality)	480	24–29	16–20	12,500–13,500
Cast iron	430–470	5–15	25–65	4,500–7,000
Aluminium, sheet . .	165	12	...	6,000
,, cast . . .	160	8	...	5,600
Duralumin, wire . . .	175	40	...	...
,, rolled . .	...	25	30–35	4,700
Naraltum, rolled . . .	155	25	...	...
,, annealed . .	155	14	...	...
,, cast . . .	...	9	...	...
Copper, cast . . .	550	8–12	...	5,000–6,000
,, rolled . .	...	13–16	...	5,500–7,500
,, wire, hard-drawn .	...	26–30	...	...
Ordinary yellow brass, cast .	520	10–12	5	5,000–6,500
(66 per cent. copper, 34 per cent. zinc), rolled	530	15–24	...	...
Gun-metal	540	12–17	...	5,000
Phosphor bronze . . .	535	16–18	...	6,000
,, wire, hard-drawn	...	45–70	...	...
Aluminium bronze (90 per cent. copper)	480	25–30	58	6,500
Zinc, cast	450	1–3	...	6,100
,, rolled	...	7–10	...	...
Lead	710	1 about	3	320
Tin	465	1–2·5	6·7	2,050
Soft solder	...	3	...	...

TABLE XXIII.—*Table showing the Strength and Weight of Common Woods.*

Name of Wood.	Tensile Strength. Lbs. per square inch.		Compressive Strength. Lbs. per square inch.	Modulus of Elasticity. E. Lbs. per square inch.	Weight. Lbs. per cubic foot.
	With Grain.	Across Grain.			
Ash	12,000–17,000	2000–3000	8000–10,000	1,600,000	43–47
Beech	11,000–22,000	1500–2000	8000–9000	1,350,000	43
Birch	15,000	1900	3000–6000	1,500,000	44–45
Cedar, West Indian	5,000	2000	5700	500,000	47
American	10,800	1000	6000	500,000	35
Lebanon	11,000	1000	5800	500,000	30
Elm, English	13,000–14,000	800–1100	6000–10,000	1,000,000	34–36
Canadian	13,000	2000	7500	1,000,000	45
Larch	9000–10,000	...	3000–5500	1,000,000	34–35
Mahogany, Honduras	20,000	1900	8000	1,500,000	35
Spanish	15,000	1300	8200	1,500,000	53
Oak, English	10,000–19,000	1600	6000–10,000	1,450,000	48–58
African	21,000	2500	9000	2,280,000	62
Pine, Red	12,000–14,000	1200–1400	5500–7500	1,850,000	36–41
White	8,700–11,000	1300	4000–6500	1,100,000	27–34
Yellow	13,000	1200	5300	1,600,000	32
Dantzic	8,000	1400	5400	...	40
Pitch	12,000	1800	8000	1,600,000	38
Riga	4,500	1400–1600	4000	...	34
Spruce	10,000	600	6500	1,800,000	32
Poplar	7,500	1600	4800	...	24
Teak	7,000–15,000	2000	8000–12,000	2,400,000	46–50

TABLE XXIV.—*Table showing the Average Values of the Pressure, Temperature, and Density of the Air in Regions of High and Low Pressure.*

Height above Ground.		Region of High Pressure.				Region of Low Pressure.			
Feet.	Kilo-metres.	Pressure in Lbs. per square inch.	Temperature, Absolute. A°.	Density. Grammes per cubic metre.	Density. Ratio to Density at Ground.	Pressure in Lbs. per square inch.	Temperature, Absolute. A°.	Density. Grammes per cubic metre.	Density. Ratio to Density at Ground.
0	0	14·94	282	1270	1·000	14·25	279	1226	1·000
3,281	1	13·32	279	1137	0·895	12·60	275	1100	0·898
6,562	2	11·70	277	1012	0·796	11·10	269	992	0·810
9,843	3	10·32	272	911	0·717	9·78	263	893	0·729
13,124	4	9·09	267	818	0·644	8·56	255	807	0·659
16,406	5	8·00	261	736	0·580	7·47	248	724	0·590
19,685	6	7·00	254	662	0·521	6·50	240	652	0·532
22,966	7	6·11	247	595	0·469	5·62	232	583	0·475
26,247	8	5·30	240	531	0·418	4·85	227	514	0·419
29,528	9	4·59	233	474	0·373	4·17	226	444	0·363
32,809	10	3·95	226	421	0·332	3·58	225	382	0·312

$A° = C° + 273.$

$\frac{F° - 32}{9} = \frac{C°}{5}.$

A° ≡ Absolute temperature.
C° ≡ Centigrade temperature.
F° ≡ Fahrenheit temperature.

Table XXV.—*British Units of Measurement and their Equivalents.*

Measurement of	Equivalents.
Length .	1 inch = 2·54 centimetres. 1 yard = 0·9144 metre. 1 mile = 1609·3 metres.
Area . .	1 square inch = 6·4516 square centimetres. 1 square foot = 929·03 square centimetres. 1 square yard = 8361·3 square centimetres.
Volume .	1 cubic inch = 16·387 cubic centimetres. 1 cubic foot = 28·317 litres = 28,317 cubic centimetres. 1 pint = 0·5682 litre. 1 gallon = 4·5460 litres.
Mass . .	1 ton = $1·01605 \times 10^6$ grammes. 1 pound = 453·59 grammes. 1 ounce (Avoir.) = 28·3495 grammes.
Angle .	1 radian = $\frac{180}{\pi}$ degrees = 57·296 degrees.
Density .	1 pound per cubic foot = 0·01602 gramme per cubic centimetre.
Velocity .	1 foot per second = 3·048 metres per second. 1 mile per hour = $\frac{88}{60}$ feet per second = 0·44704 metre per second. 1 knot = 0·5145 metre per second = 1·15 miles per hour.
Acceleration	1 centimetre per second per second = 0·03281 foot per second per second.
Force .	1 dyne = weight of 0·00102 gramme. 1 pound weight = $4·45 \times 10^5$ dynes. 1 poundal = 13,825 grammes.
Work and Energy ($g = 981$)	1 foot-pound = 13,825 grammes per centimetre. = $1·3562 \times 10^7$ ergs. 1 foot-poundal = $4·2139 \times 10^5$ ergs. 1 Joule = 10^7 ergs.
Rate of Working	1 horse-power = 33,000 ft.-lbs. = 746 watts = $7·46 \times 10^9$ ergs per second. 1 watt = 10^7 ergs per second. 1 force de cheval = $7·36 \times 10^9$ ergs per second.
Stress .	1 pound per sq. inch = 70·31 grammes per sq. centimetre. 1 pound per inch = 17·86 kilogrammes per metre.

TABLE XXVI.—*Miscellaneous Data.*

Weight of a cubic foot of pure water	= 62·3 lbs.	
,, ,, ,, salt water	= 64·0 lbs.	
,, ,, ,, petrol	= 44·0 lbs. (average)	
,, ,, ,, lubricating oil	= 58·0 lbs.	
,, ,, ,, rubber	= 60·0 lbs.	
,, ,, ,, glass	= 172·0 lbs.	
,, ,, ,, asbestos	= 135–180 lbs.	
,, ,, ,, celluloid	= 85 lbs.	
,, ,, ,, glycerine	= 78·5 lbs.	
,, ,, ,, cork	= 14–16 lbs.	

1 gallon of pure water weighs 10 lbs.

A head of 1 foot of water gives a pressure of 0·4325 lb. per square inch.

1 square yard of undoped aeroplane fabric weighs 3·5–4·5 ozs.

1 square yard of doped aeroplane fabric weighs 5–6·5 ozs.

Density of air under a pressure of 760 mm. of mercury at a temperature of 0° C. = 1·2928 grammes per litre.

Density of hydrogen under a pressure of 760 mm. of mercury at a temperature of 0° C. = 0·08987 grammes per litre.

Density of carbon monoxide under a pressure of 760 mm. of mercury at a temperature of 0° C. = 1·2504 grammes per litre.

Density of carbon dioxide under a pressure of 760 mm. of mercury at a temperature of 0° C. = 1·9768 grammes per litre.

Specific heat of air at constant pressure = 0·238.

Specific heat of air at constant volume = 0·169.

Ordinary viscosity of water at 15° C. = 0·01142 C.G.S. units.

Coefficient of kinematic viscosity at 15° C. = 0·01144 C.G.S. units.

Ordinary viscosity of air at 15° C. = 0·000181 C.G.S. units.

Coefficient of kinematic viscosity at 15° C. = 0·1476 C.G.S. units.

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