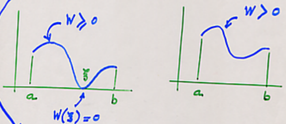
Mtg 5: Tue, 12 Jan 10

HW: a) Prove IMVT p. 2-3 for $W(\cdot)$ non-neg. i.e, $W \geq 0$

b) Another version of IMVT
$W(x) \neq 0 \quad \forall x \in [a, b]$



Either $W > 0$ (done in (a) w/ $W \geq 0$)
or $W < 0$

Taylor series cont'd p. 2-2

HW: Use IMVT to show
(5) p. 2-2 ⟹ (1) p. 2-3

Pf of Taylor series: Similar tech. used in error analysis later.

(1) $f(x) = f(x_0) + \int_{x_0}^{x} \underbrace{f^{(1)}(t)}_{\text{"}} dt$

$f'(t) = \frac{df(t)}{dt}$

Clearly, $\int_{x_0}^{x} f^{(1)}(t)\,dt = \left[f(t)\right]_{x_0}^{x}$ (2)

Int. by parts: $\int_{x_0}^{x} \underbrace{1}_{u'} \cdot \underbrace{f^{(1)}(t)}_{v} dt$

(3)

$= \left[uv\right]_{x_0}^{x} - \int uv'$

$= \left[t\, f^{(1)}(t)\right]_{t=x_0}^{t=x} - \underbrace{\int_{x_0}^{x} t\, f^{(2)}(t)\,dt}_{\alpha}$

$= x\, f'(x) - x_0 f^{(1)}(x_0) - \alpha$

$+ x f^{(1)}(x_0) - x f^{(1)}(x_0)$

$= \underbrace{\left[x f^{(1)}(x) - x f^{(1)}(x_0)\right]}_{\int_{x_0}^{x} x f^{(2)}(t)\,dt} + (x - x_0) f^{(1)}(x_0) - \alpha$

Use (3) p. 5-2 in (1) p. 5-2

$$f(x) = f(x_0) + \frac{(x-x_0)}{1!} f^{(1)}(x_0)$$

$$+ \int_{x_0}^{x} (x-t)\, f^{(2)}(t)\, dt$$

HW: Repeat int. by parts to reveal

$$\frac{(x-x_0)^2}{2!} f^{(2)}(x_0) + \frac{(x-x_0)^3}{3!} f^{(3)}(x_0)$$

plus remainder.

Next, assume (4)-(5) p. 2-2 true, do int. by parts one more time.

HW: