INTERMEDIATE BOOK

CAJORI

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SCHOOL ARITHMETICS

INTERMEDIATE BOOK

BY

FLORIAN CAJORI

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PREFACE

As in the Primary Arithmetic, so in this Intermediate Arithmetic, the aim is to render the subject attractive to the pupil, without sacrifice of serious intent. The pupil's self-activity is encouraged. By the selection, so far as possible, of problems bearing on the practical life of to-day, the pupil is made to feel that he is engaged in studies that are truly worth his while.

Our constant aim has been to lay emphasis upon fundamental operations. Frequent reviews enable the pupil to hold in mind the new knowledge he has acquired.

As in the Primary Arithmetic, so here, the technique of arithmetic is simplified, with the aim of securing greater economy of effort. Thus the subject of ratio is robbed of some of its terrors by its identification with a "common fraction." A proportion expresses the "equality of two common fractions." There is no need of the terms "antecedent" and "consequent." Again, there is given, as an alternative, a simplified method of reading decimal fractions. After the theory of decimal fractions is understood, .425 is read "Point, four,

two, five." It is recognized as a great convenience to omit the denominator in *writing* decimal fractions; why not enjoy the same convenience in *reading* decimal fractions?

The author takes pleasure in acknowledging the help he has received in the preparation of this series of texts from several teachers in the public schools of Colorado Springs, particularly from Mrs. L. D. Coffin, Mrs. S. J. Lewis, Miss Minnie L. McCall, and Miss Edna Kinder.

FLORIAN CAJORI.

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PART ONE

NOTATION AND NUMERATION

Review — Oral

1. 1. Read the following numbers:

125	3,604	40,587	987,500
360	1,234	18,356	198,799
405	1,034	97,876	897,747
987	1,204	99,999	534,689

2. Tell how many units there are in units' place, in tens' place, in hundreds' place, in thousands' place, in ten-thousands' place, in hundred-thousands' place, in each of the numbers in this exercise.

Written Exercise

2. Write as one number:

$10+7$	$200+40+5$	$3000+200+50+8$
$20+8$	$300+60+7$	$4000+500+60+7$
$30+9$	$400+90+0$	$7000+600+50+4$
$40+6$	$700+00+8$	$8000+900+10+0$
$70+7$	$100+10+1$	$9000+000+90+9$

Written Exercise

3. 1. Make a number chart like the illustration.

Orders of Whole Numbers									Orders of Decimals		
Millions			Thousands			Units or Ones			Thousandths		
Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units or Ones	Tenths	Hundredths	Thousandths
3d Period			2d Period			1st Period			1st Period		

Oral Exercise

Based on the Number Chart

4. What place is always occupied:

1. By the figure that stands for ones?
2. By the figure that stands for tens?
3. By the figure that stands for thousands?
4. By the figure that stands for hundred-thousands?
5. By the figure that stands for hundreds?

6. By the figure that stands for millions?

Name:

7. The order of whole numbers.

8. The order of decimals.

Oral Exercise

5. 1. How many ones make 1 ten?

2. How many tens make 1 hundred?

3. How many hundreds make 1 thousand?

4. How many times greater is each one than the one in the next place or order to the right?

5. How many times greater is one of thousands' order than one of tens' order?

6. How many times greater is one in millions' place than one in thousands' place?

7. How many times greater is one in ten-thousands' place than one in hundreds' place?

8. How many ones of the thousands' order are equal to one of the hundred-thousands' order?

Since each number is ten times greater than the next unit to the right, the system of writing numbers is called the **decimal system**. The word *decimal* comes from the Latin word meaning ten.

It is easier to read numbers if a comma is placed between the orders of hundreds and thousands and the orders of hundred-thousands and millions.

Oral Exercise

6. Separate the following numbers into their orders and read the numbers:

1. 4,327

PROCESS AND EXPLANATION

4,327 = 4 thousands + 3 hundreds + 2 tens + 7 ones
= 4000 + 300 + 20 + 7

2. 125	3. 360	4. 405	5. 987
6. 542	7. 1,234	8. 4,034	9. 7,204
10. 9,560	11. 5,610	12. 18,356	13. 78,347
14. 92,701	15. 72,079	16. 40,679	17. 198,765
18. 819,675	19. 576,198	20. 918,567	21. 765,951

Written Exercise

7. Write as one number:

10 + 7	200 + 40 + 5	3,000 + 1,600 + 300 + 50 + 8
90 + 8	100 + 90 + 6	40,000 + 5,000 + 700 + 90 + 4
60 + 9	300 + 63 + 4	30,000 + 6,000 + 200 + 70 + 5

Written Exercise

8. Fill in the missing orders and write the numbers:

1. 2 hundreds + 5 ones.

PROCESS AND EXPLANATION

2 hundreds + 5 ones = 2 hundreds + 0 tens + 5 ones
= 205

2. 2 ten-thousands + 7 thousands + 6 hundreds + 7 ones.

3. 3 hundred-thousands + 9 hundreds.

4. 5 millions + 6 hundred-thousands + 4 ten-thousands.

5. 3 hundred-millions + 2 ten-millions + 5 millions + 5 tens.

6. 1 million + 4 ten-thousands.

7. 6 ten-millions + 5 hundred-thousands + one.

8. 4 hundred-millions + 5 ten-thousands + 6 tens + one.

Oral Exercise

9. Read the following numbers:

7,123	10,000	673,854
4,275	67,431	705,239
7,155	10,500	900,432
1,279	30,006	821,006
10,000,000		9,234,567

For what does 0 stand in each of these numbers?

REVIEW OF FUNDAMENTAL OPERATIONS

Addition

ORAL EXERCISE

10. 1. Beginning with 8, add by 8 to 96.

2. Beginning with 2, add by 7 to 100.

3. Beginning with 1, add by 9 to 100.

4. Beginning with 101, add by 6 to 200.

5. Beginning with 0, add by 5 to 100.

6. Beginning with 0, 1, 2, 3, 4, 5, add by 6 to 100.

7. Beginning with 0, 1, 2, 3, 4, 5, 6, add by 7 to 100.

8. Beginning with 0, 1, 2, 3, 4, 5, 6, 7, add by 8 to 100.

9. Beginning with 0, 1, 2, 3, 4, 5, 6, 7, 8, add by 9 to 100.

Drill on Difficult Combinations

11. Extend each set of operations and drill:

1. $\begin{array}{rrrr} 7 & 7 & 7 & \text{etc.} \\ \underline{8} & \underline{18} & \underline{28} & \end{array}$

2. $\begin{array}{rrrr} 8 & 8 & 8 & \text{etc.} \\ \underline{8} & \underline{18} & \underline{28} & \end{array}$

3. $\begin{array}{rrrr} 9 & 9 & 9 & \text{etc.} \\ \underline{8} & \underline{18} & \underline{28} & \end{array}$

4. $\begin{array}{rrrr} 7 & 7 & 7 & \text{etc.} \\ \underline{7} & \underline{17} & \underline{27} & \end{array}$

5. 8 8 8 etc.
   9 19 29

6. 9 9 9 etc.
   7 17 27

7. 7 17 27 etc.
   7 7 7

8. 7 17 27 etc.
   8 8 8

9. 7 17 27 etc.
   9 9 9

10. 8 18 28 etc.
    7 7 7

11. 8 18 28 etc.
    8 8 8

12. 8 18 28 etc.
    9 9 9

13. 9 19 29 etc.
    7 7 7

14. 9 19 29 etc.
    8 8 8

15. 9 1 29 etc.
    9 9 9

12. Add rapidly:

1. 65 2. 67 3. 26 4. 37 5. 48 6. 77
   46 43 75 65 59 54

7. 58 8. 79 9. 46 10. 36 11. 86 12. 74
   47 48 98 67 45 75

Written Exercise

13. Add and check:

PROCESS

1.

103
9,737
456
8,354
10,605
25
13
21
17
1
29,255

EXPLANATION. — The columns may be added separately and the results added to find the sum. In this process there is no carrying. The process is somewhat simpler if sums of ten are grouped together. 3 and 7 make a group of ten. Indicate the other groups of ten in these examples. In adding the first column of Ex. 1, say: five, fifteen, twenty-five.

2.

317
943
862
198
94,628

3.

875
465
878
760
432

4.

9,762
4,348
7,489
4,765
3,484

5.

2,769
8,441
4,798
7,452
3,000

6.

$ 13.49
3.71
90.76
87.33
108.74
98.77

7.

9,876 ft.
987
7,654
1,063
9,616
1,007

8.

3,075 lb.
64,072
985
70,308
7,293
2,187

In adding dollars and cents, place the decimal points under each other.

Written Exercise

14. Copy and add:

1. $475.83 + $437.75 + $789.85.
2. $1,198.95 + $364.20 + $375.98.
3. $754 + $7,689.50 + $5,000.
4. $376.95 + $1,234.25 + $1,990.05.

Written Exercise

15. Add by columns and by lines:

1. $9{,}234 + 8{,}920 + 356 + 9{,}076 =$
   $8{,}456 + 7{,}122 + 3{,}738 + 5{,}951 =$
   $7{,}078 + 6{,}324 + 394 + 5{,}230 =$
   $6{,}910 + 6{,}256 + 404 + 4{,}545 =$ ____
   $+ \quad + \quad + \quad =$

2. $16{,}263 + 9{,}790 + 9{,}920 + 11{,}002 =$
   $7{,}465 + 8{,}610 + 8{,}934 + 9{,}003 =$
   $7{,}067 + 828 + 6{,}945 + 8{,}004 =$
   $6{,}869 + 7{,}485 + 5{,}967 + 7{,}112 =$ ____
   $+ \quad + \quad + \quad =$

3. $\$40.12 + \$57.28 + \$4.23 + \$35.60 =$
   $33.14 + 32.93 + 4.45 + 25.78 =$
   $15.16 + 21.32 + 46.47 + 15.96 =$
   $7.18 + 3.34 + 4.89 + 6.01 =$ ____
   $+ \quad + \quad + \quad =$

4. $\$50.71 + \$58.74 + \$49.89 + \$43.14 =$
$47.23 + 38.88 + 19.90 + 34.15 =$
$17.45 + 18.90 + 8.90 + 16.17 =$
$7.68 + 9.01 + 6.01 + 10.19 =$ _____
$+ \quad + \quad + \quad =$

Subtraction

16. 1. Beginning with 50, subtract by 2 to 0.
2. Beginning with 41, subtract by 2 to 1.
3. Beginning with 60, subtract by 3 to 0.
4. Beginning with 71, subtract by 3 to 2.
5. Beginning with 80, subtract by 4 to 0.
6. Beginning with 81, subtract by 4 to 1.
7. Beginning with 83, subtract by 4 to 2.
8. Beginning with 90, subtract by 5 to 0.
9. Beginning with 96, subtract by 5 to 1.
10. Beginning with 94, subtract by 5 to 4.

Continue the exercise.

Drill on Difficult Combinations

17. Drill until habits of accuracy and rapidity are established.

1. 17 27 37 47 57 etc.
7 7 7 7 7

2.	17	27	37	47	57 etc.
	8	8	8	8	8
3.	17	27	37	47	57 etc.
	9	9	9	9	9

Do likewise with 18 and 7, 19 and 7, 18 and 8, 19 and 8, 19 and 9.

18. Subtract rapidly:

1. 67	2. 60	3. 75	4. 92	5. 80
25	28	46	43	34
6. 55	7. 45	8. 99	9. 91	10. 83
27	32	35	57	45
11. 74	12. 95	13. 52	14. 73	15. 45
32	26	23	24	39

Written Exercise

19. Subtract and check:

PROCESS

1. 40,735
39,564
1,171 *Ans.*

EXPLANATION. 4 and 1 are 5; 6 and 7 are 13; 5 and 2 (1 and 1) are 7; 9 and 1 are 10.

2. 107,864	3. 987,603	4. 367,890
72,895	367,809	176,805
5. 134,578	6. 3,764,001	7. 500,897,431
90,387	1,987,373	176,354,897

8.	9.	10.
$754.37	$7,689.50	$7,989.50
376.95	1,234.25	1,990.35

Written Problems

20. 1. A man had $1,000 in a bank. He drew out the following amounts: $95.50, $180.65, $75.05, $96.75, $7.85, $1.25, $60.73.

How much money remained in the bank?

2. A man earned $30.45 in January, $40.26 in February, $42.19 in March, $61.34 in April, $53.50 in May, $39.29 in June, $27.00 in July, $47.45 in August, $40.10 in September, $50.24 in October, $55.75 in November. During the month of December he was idle. If his expenses during the year were $240, how much did he save?

3. The Panama Canal is about 50.4 mi. long. The first 8 mi. are a sea-level channel. The next 24 mi. are through a lake above sea level. From this point the channel passes $7\frac{1}{2}$ mi. through a cut in the Culebra Hill. The channel then passes through a lake 5 mi. long. The rest of the distance is a sea-level channel. How long is the last section of the canal?

4. The aggregate population of 25 cities of the United States in 1910 was 11,042,500. In 1900 they had an aggregate population of 8,273,482; in 1890 they had 6,213,583. What was the increase

in population from 1890 to 1900? What was the increase in population from 1900 to 1910? How does the increase between 1890 and 1900 compare with the increase between 1900 and 1910? Which is the greater? How much greater is it?

Multiplication

ORAL DRILL

21. How much is

1. 7×8	**2.** 6×9	**3.** 9×7	**4.** 8×8
5. 9×4	**6.** 9×6	**7.** 7×9	**8.** 8×7
9. 6×12	**10.** 12×7	**11.** 12×9	**12.** 8×12
13. 7×11	**14.** 11×8	**15.** 9×11	**16.** 11×6

Oral Exercise

22. Multiply the numbers in the upper line by each number in the lower line:

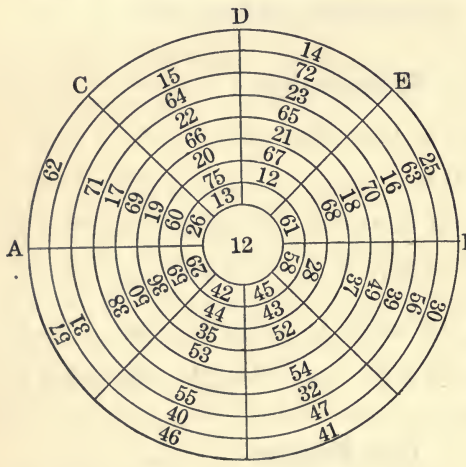
3	7	6	5	2	4	11	8	12	10	9
×	4	7	9	3	12	6	5	11	8	2

3	7	6	5	2	4	11	8	12	10	9
				×	10	100	1000			

Make devices that provide drill upon the products in which you fail.

Drill — Magic Circle

23. 1. If the 8 vacant spaces are properly filled, we get circles, called "magic circles." Lines radiating from the center divide the circles into 8 parts. Fill the empty spaces so that the sum of all 8 numbers in each part, plus the number in the middle, is 360.



2. If the empty spaces are properly filled, then the sum of the 8 numbers in any one ring, together with the number in the middle, is 360. Add and show that this is true.

3. Show that the sum of the numbers in any half ring, above the line AB or below it, together with half the middle number, is 180.

4. Show that the sum of any four numbers, each next to the other three, together with half the middle number, is equal to 180. Thus, 20, 67, 75, 12 are four such numbers, also 14, 25, 72, 63.

Written Exercise

24. Find the product:

1. 648
   14

PROCESS

$$\begin{array}{r} 648 \\ 14 \\ \hline 2592 \\ 648 \\ \hline 9072 \end{array}$$

EXPLANATION. — To multiply 648 by 14 means to multiply 648 by 4 and 10. The process of multiplying 648 by 4 is to multiply by 4 *ones*. It remains to multiply 648 by 1 *ten*. This gives 648 *tens*. Since the number is tens, the first figure, 8, is written in tens' place. The vacant place is sometimes filled in with a 0. This is not necessary.

2. 754 24	3. 423 36	4. 295 32	5. 798 16
6. 581 24	7. 649 52	8. 959 27	9. 764 24
10. 953 21	11. 825 65	12. 647 68	13. 939 44
14. 558 28	15. 275 27	16. 821 97	17. 478 84
18. 589 76	19. 643 87	20. 824 48	21. 7,468 47
22. 7,245 58	23. 9,398 69	24. 8,556 76	25. 5,729 87

26. 5,749	27. 3,128	28. 1,847	29. 3,985
89	96	97	98
30. 9,346	31. 9,098	32. 9,009	33. 9,999
99	78	89	86

Written Problems

25. Find the cost of:

1. 64 gallons of oil at 17¢ per gallon.
2. 47 pounds of tea at 48¢ a pound.
3. 6 dozen penknives at $8.75 a dozen.
4. 25 barrels of flour at $4.25 a barrel.
5. 50 dozen eggs at 27¢ a dozen.
6. 620 pounds of chicken at 19¢ a pound.
7. 72 packages of crackers at 15¢ a package.
8. 6 firkins of butter, each containing 100 pounds, at 26¢ a pound.

Multiplication by more than Two Digits

26. Multiply:

1. 1,854 by 237.

PROCESS

1,854
237
12978
5562
3708
439,398

EXPLANATION. — Multiply by 7 *ones*. Write the product so that its right-hand digit is in *ones*' column, under the 7.

Multiply by 3 *tens*. Write the product so that its right-hand digit is in *tens*' column.

Then multiply by 2 hundreds. Write the product so that its right-hand digit is in *hundreds*' column. Add the three products.

2. 175 by 206.

PROCESS

175
206
1050
305
31,550

EXPLANATION. — In this exercise it is necessary to multiply by six *ones* and two *hundred*. Multiplying by two hundred may be done by multiplying by two and moving the product two places to the left. The vacant places are shown under the 5 and the cipher.

3. 312 by 620.

PROCESS

312
620
6240
1872
193,449

EXPLANATION. — Multiply first by two *tens*, then by six *hundreds*. Fill in the first place with a cipher. There are no *ones*. The first partial product is 312 times 2 tens.

Written Exercise

27. Multiply:

1. 123 by 123
2. 497 by 132
3. 568 by 231
4. 759 by 322
5. 897 by 432
6. 575 by 405
7. 612 by 740
8. 1,019 by 305
9. 765 by 600
10. 896 by 500
11. 1,324 by 901
12. 897 by 700
13. 678 by 509
14. 437 by 987

Division

28. Give the answer:

1. $56 \div 8$
2. $63 \div 9$
3. $72 \div 8$
4. $48 \div 6$

5. $54 \div 9$ 6. $64 \div 8$ 7. $63 \div 7$ 8. $49 \div 7$
9. $81 \div 9$ 10. $84 \div 7$ 11. $45 \div 9$ 12. $55 \div 5$
13. $60 \div 12$ 14. $72 \div 12$ 15. $84 \div 12$ 16. $99 \div 11$

Oral Exercise

29. Divide each number in the upper line by each number in the lower line.

1.

60	24	36	72	48	144	96
÷	2	3	6	4	12	9

2.

1,000	10,000	17,000	90,000
÷	10	100	1,000

Division by Two Digits

30. Divide:

1. 43,641 by 71.

PROCESS

$$\begin{array}{r} 614\overset{\times}{4}\frac{7}{71} \\ 71\overline{)43641} \\ 426 \\ \hline 104 \\ 71 \\ \hline 331 \\ 284 \\ \hline 47 \end{array}$$

EXPLANATION. — Indicate above the dividend by a small cross mark, ×, the last or right-hand figure in the first partial dividend.

436 is the first partial dividend. (Try $436 \div 70$.) $436 \div 71 = 6$ and a remainder. Place the 6 above the last figure in the partial dividend. $6 \times 71 = 426$; $436 - 426 = 10$, the remainder. Bring down the 4, the next figure in the dividend. 104 is the second partial

dividend. (Try $104 \div 70$.) $104 \div 71 = 1$ and a remainder. $1 \times 71 = 71$; $104 - 71 = 33$, the remainder. Bring down the 1, the next figure in the dividend. (Try $331 \div 70$.) $331 \div 71 = 4$ and a remainder. $4 \times 71 = 284$; $331 - 284 = 27$, the last remainder.

The quotient is 614 and the remainder 47, or $614\frac{47}{71}$. *Ans.*

2. $158.22 by 54.

PROCESS

$$
\begin{array}{r}
\$\,2\,93 \\
\times \\
\hline
54)\$\,158.22 \\
108 \\
\hline
502 \\
486 \\
\hline
162 \\
162 \\
\hline
\end{array}
$$

EXPLANATION. — The first partial dividend is 158. Trial division indicates that 54 is contained in 153, 3 times. $3 \times 54 = 162$. 162 is greater than the first partial dividend. $158 \div 54 = 2$ and 50 remainder. The second partial dividend is 502. $502 \div 54 = 9$ and 16 remainder. The third partial dividend is 162. $162 \div 54 = 3$.

$2.93 *Ans.*

3. 2,790 by 29.

PROCESS

$$
\begin{array}{r}
96\frac{6}{29} \\
\times \\
\hline
29)2790 \\
261 \\
\hline
180 \\
174 \\
\hline
6
\end{array}
$$

EXPLANATION. — 279 is the first partial dividend. Use 30 for trial divisor. $279 \div 30 = 9$. $9 \times 29 = 261$; $279 - 261 = 18$ remainder. The next partial dividend is 180. $180 \div 30 = 6$. $6 \times 29 = 174$; $180 - 174 = 6$ remainder. $96\frac{6}{29}$ *Ans.*

When the ones' figure is 9, as 29, 39, etc., it is generally easier to use 30, 40, etc., as a trial divisor.

Written Exercise

31. Divide:

1. 4,875 by 31
2. 2,849 by 42
3. 18,785 by 23
4. 54,632 by 77
5. 64,751 by 78
6. 60,543 by 89
7. 49,790 by 96
8. 38,000 by 67
9. 50,000 by 65
10. $657.50 by 24
11. $785.60 by 26
12. $6000.00 by 33

Division by more than Two Digits

32. Divide:

1. 6,084 by 234.

PROCESS

```
   26
   ×
234)6084
  468
 1404
 1404
```

EXPLANATION. — 234 is contained in 608, 2 times, and a remainder. Write the 2 of the quotient. 234 into 1404 goes 6 times. Write the 6 of the quotient. There is no remainder. The answer is 26.

2. 8,499 by 293.

PROCESS

```
   29
   ×
293)8499
  586
 2639
 2637
    2
```

EXPLANATION. — Since 293 is nearly 300, it is convenient to use 300 as our trial divisor to find the first figure in the quotient. 300 is contained in 849, 2 times. Write the 2 of the quotient over the 9, the last figure of the partial dividend, 849. 300 into 2639 goes nearly 9 times. Try 9.

293×9 is less than 2639. Hence 9 is the second figure of the quotient.

The quotient is 29, the remainder is 2.

Written Exercise

33. Divide and check:

1. 19,623 by 211	2. 17,347 by 209
3. 40,260 by 915	4. 52,288 by 817
5. 15,022 by 406	6. 33,744 by 703
7. 20,262 by 614	8. 16,302 by 429.
9. 37,700 by 725	10. 50,255 by 529.
11. 19,277 by 663	12. 28,644 by 682
13. 20,000 by 607	14. 50,000 by 600
15. 60,000 by 705	16. 75,000 by 850

Divisors 10, 20, 100, 600

34. Divide:

1. 87 by 10.

PROCESS

$$10\overline{)87} \quad 8\tfrac{7}{10}$$

EXPLANATION. — 87 = 8 tens + 7 ones. 10 is contained in 8 tens + 7 ones 8 times with 7 as a remainder or $8\frac{7}{10}$.

A short method of dividing a number by 10 (when the dividend does not end with a **0**) is to separate the tens from the ones by a vertical line and divide.

2. 7,300 by 100.

PROCESS

$$100\overline{)7300} \quad 73$$

EXPLANATION. — 7,300 = 73 hundreds. 1 hundred is contained in 73 hundreds 73 times.

73 is the quotient.

A short method of dividing by 100 a number that ends with two 0's is to cancel the two 0's at the right of the divisor and the dividend, and divide.

2. 3,660 by 200.

PROCESS

$$\begin{array}{r} 18\frac{60}{200} \\ 2\not{0}\not{0})\overline{36\,|\,60} \end{array}$$

EXPLANATION. — Write the explanation.

Explain a short method.

3. 4,575 by 300.

PROCESS

$$\begin{array}{r} 15\frac{75}{300} \\ 3\not{0}\not{0})\overline{45\,|\,75} \end{array}$$

EXPLANATION. — Write the explanation.

Canceling a cipher at the right of a number divides the number by what?

Canceling two ciphers at the right of a number divides the number by what?

Written Exercise

35. Divide:

1. $30\overline{)127}$	2. $40\overline{)4,326}$	3. $50\overline{)6,534}$
4. $60\overline{)4,563}$	5. $70\overline{)7,564}$	6. $80\overline{)6,574}$
7. $90\overline{)9,046}$	8. $30\overline{)2,147}$	9. $40\overline{)8,435}$
10. $80\overline{)3,064}$	11. $90\overline{)6,847}$	12. $70\overline{)9,037}$
13. $100\overline{)8,400}$	14. $200\overline{)7,800}$	15. $400\overline{)97,600}$
16. $500\overline{)64,500}$	17. $600\overline{)9,600}$	18. $700\overline{)8,890}$
19. $800\overline{)89,700}$	20. $900\overline{)54,600}$	21. $600\overline{)93,310}$
22. $700\overline{)33,970}$	23. $800\overline{)828,240}$	24. $900\overline{)565,830}$

Oral Problems

36. 1. What is the cost of 20 lb. of beef at 23¢ a pound?

2. A clerk in a store sold 30 sets of books at $6 per set. How much did he receive altogether?

3. How far will a train travel in 12 hours at the rate of 40 miles an hour?

4. At 4 miles per hour, how long will it take a man to walk 52 miles?

5. How many inches are there in 30 feet? In 40 feet?

6. How many feet are there in 240 inches?

7. At 6¢ a quart, how many quarts of milk can be bought for 78¢? 90¢?

8. What is the cost of eggs a dozen, if 7 dozen cost $2.80?

9. What is the cost of 9 dozen eggs at 40¢ a dozen?

10. How many square feet in 288 sq. in.?

Written Problems

37. 1. Find the number of minutes in 24 hours.

2. How many hours in the month of July?

3. Find the number of hours in 365 days.

4. How many feet in 76 miles? 1 mi. = 5280 ft.

5. What is the weight in pounds of $15\frac{1}{2}$ tons of coal? 1 T. = 2000 lb.

6. How many street cars are necessary to carry 901 passengers, if 53 passengers are put into each car?

7. A train runs 984 miles in 24 hours. How many miles does it run an hour?

8. On a city street there are 139 houses. Each house contains 4 families and each family 6 persons. How many persons live on the street?

9. A booklet has 65 pages. There are 30 lines on each page and in each line 43 letters. How many letters are there in the book?

10. 4,368 oranges are to be packed in 56 boxes of equal size. How many oranges must be put into each box?

11. What number multiplied by 73 will give 4,526?

12. How many bushels of potatoes will 54 acres yield, if each acre yields 243 bushels?

13. 35 acres yield 8,295 bushels of potatoes. How many bushels is this per acre?

14. It is 85 miles from Chicago to Milwaukee. How many rods is this?

15. From New York to Buffalo it is 411 miles. How many miles does a man travel in going 18 times from one city to the other?

16. A locomotive has been run 75 times between Chicago and New York. The distance between these cities is 908 miles. How many miles has this locomotive traveled?

17. If 29 acres of land cost $3,828, what is the cost of 1 acre?

18. If an automobile travels 23 miles an hour, how far will it go in 78 hours?

19. At 27 miles an hour, how long will it take an automobile to go 351 miles?

20. At 57¢ a bushel, what is the cost of 125 bushels of potatoes?

21. The President of the United States has a salary of $75,000 per year. How much does he receive per month? Per day, counting 365 days to the year?

22. A man started on a journey of 620 miles. After he had traveled 15 hours at the rate of 39 miles an hour, how far was he from his journey's end?

23. How many pounds of coffee, at 60¢ a pound, will cost as much as 120 gallons of molasses at 57¢ a gallon?

24. A merchant makes five payments of $1,275 each on a debt and finds that he still owes $785. How much was the debt?

25. A store has 475 boxes of soap, each box containing 175 cakes. What is the entire number of cakes?

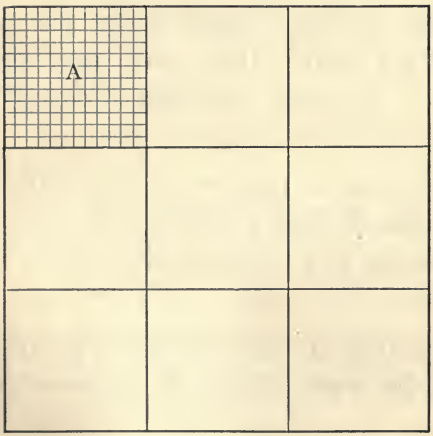
26. A factory made 2,748 suits during a season. At $19 a suit, how much was received for them?

27. A man sold a farm for $5,775 and gained $1,200. What would have been the selling price if he had gained $1,354?

28. If 525 gallons of milk sell for $84, what is the rate per gallon?

Oral Exercise

38. 1. Let the drawing represent 1 sq. yd.



2. Then the 9 smaller divisions represent what?

3. How many square feet in 1 sq. yd.?

4. What does each small division in A represent?

5. How many small squares in one row in A?

6. How many rows of small squares in A are there?

7. How many small squares in A?

8. Make the table of square measure.

Construction Exercise

39. 1. Draw a square, one inch long and one inch wide. Call it a square inch.

2. Draw a rectangle 4 in. long and 2 in. wide. The picture shows the rectangle smaller than it really is. How many square inches are there in one row? In the two rows? What is the area of the rectangle? Tell how to find the area of a rectangle.

3. How many square inches are there in a square that is 12 in. long and 12 in. wide? How many square inches make a square foot?

4. How many square feet are there in a square, 3 feet long and 3 feet wide? How many square feet make a square yard?

The area of a rectangle is obtained by multiplying its length by its width.

Written Exercise

40. 1. How many square feet in 139 sq. yd.?

2. How many square inches in 15 sq. ft.?

3. How many square yards in 1 sq. mi.?

4. How many square yards in 180 sq. mi.?

5. How many feet in 108 mi.?

Written Problems

41. 1. Find the area of a floor, the dimensions of which are 6 yd. by 5 yd.

2. Find the number of square yards in the surface of a wall 9 ft. high and 15 ft. long.

3. A garden is 48 ft. by 120 ft. There is a gravel walk 3 ft. wide around the outside of the garden. Make a drawing of the garden (scale, 1 in. = 1 ft.) and show the walk.

4. What is the area of the garden in square feet?

5. What is the area of the walk in square feet?

6. What is the area of the garden and walk?

7. A piece of land, 16 rd. long and 10 rd. wide, is divided into 4 equal parts by lines 4 rd. apart. What are the dimensions of each part?

8. How many square rods in each part?

PROPERTIES OF NUMBERS

Factors

42. A **factor** of a whole number is an integer that is an exact divisor of that number.

Thus, the number 24 has, besides itself and 1, the factors 2, 3, 4, 6, 8, 12.

A **prime number** is an integer that has no factors, except itself and 1.

2, 3, 7 are examples of prime numbers. It is evident that 7 is not exactly divisible by any integer, except itself and 1. Hence it is a prime number. The same is true of 2 and 3.

A **prime factor** is a factor that is a prime number.

2 and 3 are prime factors of 24.

Oral Exercise

43. 1. Is 8 a prime number? Why?

2. Is 11 a prime number? Why?

3. There are 4 prime numbers between 1 and 10. Find them.

4. There are 4 prime numbers between 10 and 20. Find them.

5. Find the prime numbers between 20 and 30.

6. Name the prime factors of 4, 6, 8, 9, 10, 12.

Divisibility of Numbers

44. A whole number is exactly divisible by 2, if the digit in ones' place is 2, 4, 6, 8, or 0.

A whole number is exactly divisible by 5, if it ends in 5 or 0.

A whole number is exactly divisible by 3, if the sum of its digits is divisible by 3.

For example, 432 is exactly divisible by 3, because the sum of its digits $(4+3+2)$ is 9, and 9 is exactly divisible by 3.

A whole number is exactly divisible by 6, if it is even and exactly divisible by 3.

Thus, 168 is even; $1+6+8=15$; hence 168 is exactly divisible by 6.

A whole number is exactly divisible by 4, if the number made up of the two right-hand digits is so divisible. Thus, 3148 is exactly divisible by 4, because 48 is so divisible.

A whole number is exactly divisible by 8, if the number made up of the three right-hand digits is so divisible. Thus, 94,128 is exactly divisible by 8, because 128 is so divisible.

Oral Exercise

45. 1. What numbers multiplied together give the following products:

33	16	26	27	32	30	34
35	36	42	44	45	48	49

2. What whole numbers will exactly divide:

44	46	63	64	54	56
72	81	84	88	96	108

3. Name two factors of 12.

PROCESS

$12 = 3 \times 4$
or
2×6

EXPLANATION.—3 and 4 are called factors of 12.
2 and 6 are also factors of 12.

4. Name two factors of each of the following:

65	35	42	50	72	48
24	18	36	45	33	27

Oral Exercise

46. 1. Name two factors of each of the following:

6	10	15	21	26	33
35	39	49	55	25	22

2. Name three factors of each of the following:

8	12	16	18	24	27
30	36	42	45	48	54

3. Name all the factors of each of the following:

33	56	64	49	72	81
63	54	96	108	120	144

4. Which of the following numbers are exactly divisible by 2? By 3? By 4? By 5? By 6? By 8?

15, 18, 24, 30, 42, 45, 48, 62, 170, 212, 312, 330, 450, 790, 1,012, 3,618, 7,544, 6,908, 4,345, 7,545, 10,000.

5. The prime factors of 12 are 2, 2, and 3. What are the prime factors of 15? Of 18? Of 24?

6. Name the prime factors of 14; of 28; of 30; of 42; of 45; of 49; of 50.

Finding Prime Factors

47. 1. Find the prime factors of 36.

PROCESS

$$\begin{array}{r} 2)\overline{36} \\ 2)\overline{18} \\ 3)\ \overline{9} \\ 3)\ 3 \end{array}$$

$36 = 2 \times 2 \times 3 \times 3.$

EXPLANATION. — 36 is exactly divisible by 2. Why?

18 is exactly divisible by 2. Why?

9 is exactly divisible by 3.

The prime factors of 36 are $2 \times 2 \times 3 \times 3$.

2. Find the prime factors of 450.

PROCESS

$$\begin{array}{r} 5)\overline{450} \\ 5)\overline{90} \\ 3)\overline{18} \\ 3)\ \overline{6} \\ 2)\ \overline{2} \\ \quad 1 \end{array}$$

EXPLANATION. — 10, 9, 5, and 3 are all factors of 540.

In finding the prime factors it is better to factor by the prime factors. Therefore divide 450 by 5, 5, 3, 3, and 2.

The prime factors of 450 are:

$2 \times 3 \times 3 \times 5 \times 5.$

Written Exercise

48. Find the prime factors of

1.	72	75	81	96	108
2.	120	125	128	144	150
3.	201	195	333	444	234
4.	340	570	690	640	729

CANCELATION

49. 1. Divide 12×15 by 8×10.

PROCESS BY CANCELATION

$$\frac{12 \times 15}{8 \times 10} = \frac{\overset{3}{\cancel{12}} \times \overset{3}{\cancel{15}}}{\underset{2}{\cancel{8}} \times \underset{2}{\cancel{10}}} \text{ (Dividend)} \quad \text{(Divisor)}$$

$$\frac{\overset{3}{\cancel{12}} \times \overset{3}{\cancel{15}}}{\underset{2}{\cancel{8}} \times \underset{2}{\cancel{10}}} = \frac{3 \times 3}{2 \times 2} = \frac{9}{4} = 2\tfrac{1}{4}$$

EXPLANATION.— Take the common factor 4 out of 12 in the dividend and 8 in the divisor, leaving the factors 3 and 2. Take the common factor 5 out of 15 in the dividend, and 10 in the divisor leaving the factors 3 and 2. Find the product of 3×3, the remaining factors in the dividend, and 2×2, the remaining factors in the divisor and divide.

2. Divide, using cancelation, $\frac{12 \times 15}{10 \times 18}$.

PROCESS

$$\frac{12 \times 15}{10 \times 18} = \frac{\overset{2}{\cancel{12}} \times \overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times \underset{3}{\cancel{18}}} = \frac{6}{6} = 1$$

EXPLANATION. — Reject the common factors 6 and 5 from both the dividend and the divisor.

Divide the product of the factors in the dividend by the product of the factors in the divisor.

3. Divide, using cancelation, $\frac{12 \times 15 \times 42}{8 \times 9 \times 24}$.

PROCESS

$$\frac{12 \times 15 \times 42}{8 \times 9 \times 24} = \frac{\overset{3}{\cancel{12}} \times \overset{5}{\cancel{15}} \times \overset{14}{\cancel{42}}}{\underset{2}{\cancel{8}} \times \underset{3}{\cancel{9}} \times \underset{8}{\cancel{24}}} =$$

$$\frac{\overset{3}{\cancel{12}} \times \overset{5}{\cancel{15}} \times \overset{7}{\cancel{14}}}{\underset{2}{\cancel{8}} \times \underset{3}{\cancel{9}} \times \underset{8}{\cancel{24}}} = \frac{35}{8} = 4\tfrac{3}{8}$$

EXPLANATION. — Reject the common factors 4, 3, and 3 from both the dividend and the divisor.

Reject the common factors 3 and 2 from the new dividend and divisor.

Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

PRINCIPLE TO BE REMEMBERED

Dividing both dividend and divisor by the same factor does not change the value of the quotient.

Written Exercise

50. Divide, using cancelation:

1. $24 \times 49 \times 18$ by $12 \times 21 \times 36$
2. $25 \times 35 \times 56$ by $15 \times 28 \times 49$
3. $32 \times 108 \times 100$ by $64 \times 36 \times 25$
4. $39 \times 28 \times 72$ by $35 \times 52 \times 24$
5. $16 \times 40 \times 24$ by $20 \times 8 \times 48$

6. $\dfrac{30 \times 32 \times 36 \times 40}{50 \times 16 \times 20 \times 18}$

7. $\dfrac{350 \times 120 \times 72}{500 \times 63 \times 6}$

8. $\dfrac{625 \times 49 \times 81}{75 \times 210 \times 140}$

9. $\dfrac{1728 \times 99 \times 100}{1440 \times 108 \times 25}$

10. $\dfrac{625 \times 1728 \times 121 \times 1000}{2500 \times 720 \times 99 \times 24}$

COMMON FRACTIONS

Review

51. Read the fractions. Which number is the numerator? Which the denominator?

1. $\frac{2}{3}$ $\frac{7}{8}$ $\frac{5}{9}$ $\frac{12}{20}$ $\frac{20}{31}$ $\frac{45}{54}$ $\frac{108}{250}$

2. $\frac{9}{4}$ $\frac{11}{2}$ $\frac{13}{5}$ $\frac{17}{9}$ $\frac{105}{25}$ $\frac{212}{10}$ $\frac{875}{125}$

3. $3\frac{1}{2}$ $4\frac{2}{3}$ $5\frac{7}{8}$ $9\frac{1}{2}$ $25\frac{1}{4}$ $135\frac{3}{11}$ $97\frac{11}{12}$

In which of these fractions is the numerator less than the denominator?

In which of these fractions is the numerator greater than the denominator? One or more of the equal parts of a unit is called a **fraction.**

The **denominator** shows into how many equal parts the unit is divided.

The **numerator** shows how many of the equal parts have been taken.

The numerator and denominator are sometimes called **terms** of the fraction.

A **proper fraction** is one whose numerator is less than its denominator. For example, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator. For example, $\frac{8}{8}$, $\frac{9}{8}$, $\frac{5}{4}$.

A **mixed number** is an integer and a common fraction taken together. $3\frac{3}{4}$ is a mixed number.

Oral Exercise

52. Name the unit in each of these fractions:

1. $\frac{3}{4}$ bu. $\frac{4}{5}$ mi. $\frac{9}{10}$ lb. $\frac{5}{8}$ ft.

2. $\frac{2}{3}$ of an apple; $\frac{3}{4}$ of a circle; $\frac{4}{5}$ of a rectangle.

3. Into how many equal parts has each of these units been divided?

4. What term of the fraction shows into how many equal parts the unit has been divided?

5. How many equal parts of the unit have been taken in each of these fractions?

6. What term of the fraction shows how many of the equal parts have been taken?

7. $\frac{4}{6}$ means that 1 unit is divided into 6 equal parts and that 4 parts are taken.

$\frac{4}{6}$ may be explained also as meaning $6\overline{)4}$. The dividend being smaller than the divisor, we indicate the division by writing $\frac{4}{6}$. The simpler form is $\frac{2}{3}$.

Drill Exercise

53. 1. $100¢ = \$1$ $1¢ = \$\frac{1}{100}$
$50¢ = \$\frac{1}{2}$ $5¢ = \$\frac{1}{20}$
$20¢ = \$\frac{1}{5}$ $25¢ = \$\frac{1}{4}$
$10¢ = \$\frac{1}{10}$ $75¢ = \$\frac{3}{4}$

2. 12 in. $= 1$ ft. 1 in. $= \frac{1}{12}$ ft.
6 in. $= \frac{1}{2}$ ft. 2 in. $= \frac{1}{6}$ ft.
3 in. $= \frac{1}{4}$ ft. 8 in. $= \frac{2}{3}$ ft.

3. 16 oz. = 1 lb. 1 oz. = $\frac{1}{16}$ lb.
8 oz. = $\frac{1}{2}$ lb. 2 oz. = $\frac{1}{8}$ lb.
4 oz. = $\frac{1}{4}$ lb. 12 oz. = $\frac{3}{4}$ lb.

Oral Exercise

54. 1. What part of a dollar is 1 dime?

2. What part of a dollar are 5 dimes? 2 dimes?

3. What part of a dollar is 1 nickel? What part are 2 nickels? 4 nickels? 5 nickels? 10 nickels?

4. What part of a dollar are 2 nickels and 1 dime?

5. What part of a dollar are 2 nickels and 4 dimes?

6. What part of a dollar are 2 dimes and 1 nickel?

7. What part of a foot are 3 inches and 1 inch?

8. Three books, each 1 inch thick, are placed upon a library shelf. What part of a foot of shelf-room do they occupy?

Reduction of Fractions

55. 1. Examine the drawing of the ruler.

2. How many half inches in one inch?

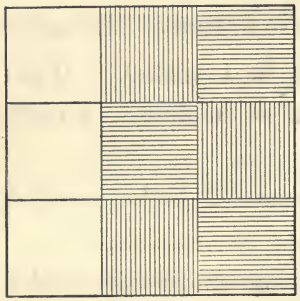
3. How many fourths of an inch in one inch?

4. How many eighths of an inch does $\frac{5}{8}$ in. lack to make 1 in.?

5. How many fourths make one half?

6. Compare $\frac{1}{8}$ in. in length with $\frac{1}{4}$ in.

7. How many eighths in one fourth?



8. Examine the drawing of the square.

9. How many thirds of the whole square are shaded?

10. How many ninths of the whole square are shaded? The illustration shows that $\frac{6}{9} = \frac{2}{3}$.

11. Divide both terms of the fraction $\frac{6}{9}$ by 3, thus: $\frac{\overset{2}{\cancel{6}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$. To divide both terms of the fraction $\frac{6}{9}$ by 3, cancel the common factor 3 and reduce $\frac{6}{9}$ to lower terms, $\frac{2}{3}$.

$\frac{4}{8}$ is reduced to $\frac{1}{2}$ by canceling the common factor 4, thus: $\frac{\overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}}} = \frac{1}{2}$.

12. How is $\frac{2}{4}$ reduced to its equal, $\frac{1}{2}$? Explain by diagram.

Changing the forms of fractions without changing their value is called **reduction of fractions.**

PRINCIPLE TO BE REMEMBERED

Dividing both the numerator and the denominator of a fraction by the same number does not change the value of the fraction. This process reduces the fraction to lower terms.

Reduction of Fractions to Lower Terms

56. Reduce to lower terms:

1. $\frac{6}{9}$.

PROCESS

$$\frac{\overset{2}{\cancel{6}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$$

EXPLANATION. — Cancel from the numerator and the denominator the common factor 3. This leaves the factor 2 in the numerator and 3 in the denominator. The fraction is thus reduced to lower terms.

2. $\frac{4}{8}$ **3.** $\frac{3}{15}$ **4.** $\frac{4}{6}$ **5.** $\frac{2}{4}$ **6.** $\frac{2}{12}$

7. $\frac{9}{12}$ **8.** $\frac{8}{12}$ **9.** $\frac{10}{12}$ **10.** $\frac{3}{9}$ **11.** $\frac{6}{9}$

12. $\frac{6}{8}$ **13.** $\frac{2}{8}$ **14.** $\frac{7}{14}$ **15.** $\frac{3}{12}$ **16.** $\frac{4}{12}$

Written Exercise

57. 1. Reduce to lowest terms. Cancel from the numerator and the denominator all the common factors.

$\frac{10}{18}$ $\frac{10}{12}$ $\frac{10}{16}$ $\frac{10}{20}$ $\frac{8}{16}$ $\frac{12}{16}$ $\frac{14}{16}$

$\frac{8}{12}$ $\frac{14}{32}$ $\frac{16}{42}$ $\frac{18}{42}$ $\frac{20}{32}$ $\frac{12}{22}$ $\frac{12}{18}$

$\frac{14}{28}$ $\frac{20}{48}$ $\frac{22}{88}$ $\frac{26}{39}$ $\frac{28}{56}$ $\frac{25}{50}$ $\frac{42}{56}$

2. Cancel from the numerator and the denominator all the common factors.

$\frac{10}{8}$	$\frac{20}{12}$	$\frac{22}{12}$	$\frac{16}{12}$	$\frac{20}{8}$	$\frac{25}{10}$
$\frac{20}{10}$	$\frac{40}{20}$	$\frac{50}{25}$	$\frac{60}{30}$	$\frac{30}{15}$	$\frac{45}{15}$

A fraction is in its **lowest terms** when the numerator and the denominator do not contain a common factor.

Reduction of Improper Fractions

58. 1. How many bushels in $\frac{10}{4}$ bu.? How many whole bushels? What fraction is left over?

$$\frac{10}{4} \text{ bu.} = 2\frac{2}{4} \text{ bu.} = 2\frac{1}{2} \text{ bu.}$$

In the same manner reduce

2. $\frac{11}{4}$ in.	3. $\frac{12}{4}$ yd.	4. $\frac{5}{3}$ lb.	5. $\frac{12}{10}$ bu.
6. $\frac{9}{8}$ da.	7. $\frac{11}{7}$ hr.	8. $\frac{15}{6}$ lb.	9. $\frac{27}{13}$ T.
10. $\frac{35}{20}$ mi.	11. $\frac{60}{15}$ yr.	12. $\frac{65}{15}$ mo.	13. $\frac{70}{8}$ bbl.
14. $\frac{75}{7}$ wk.	15. $\frac{72}{6}$ sec.	16. $\frac{75}{6}$	17. $\frac{42}{20}$
18. $\frac{44}{12}$	19. $\frac{66}{11}$	20. $\frac{61}{12}$	21. $\frac{61}{15}$

To what whole number and what fraction is each equivalent?

Oral Problems

59. 1. John has 21 dimes in the bank. How many dollars has he?

2. In a bin there are 9 pk. of wheat. How many bushels are there in the bin?

3. A board is 38 in. long. What is its length in feet?

4. A roll of paper is 43 ft. long. How many yards long is it?

5. How many strips of cloth, each 10 in. wide, may be cut from a piece of cloth 36 in. wide?

6. A straight section of road is 4 rd. wide and 100 rd. long. How many square rods in the section?

7. A roll of paper is $\frac{3}{4}$ of a yard wide. How many feet wide is it?

8. A roll of paper is 27 in. wide. How many strips each 8 ft. long must be taken from the roll in order to paper a wall 16 ft. long and 8 ft. high?

To Reduce a Fraction to a Mixed Number

60. Reduce to a mixed number:

1. $\frac{121}{13}$.

PROCESS

$$13\overline{)121}\quad 9\tfrac{4}{13}$$
$$\underline{117}$$
$$4$$

EXPLANATION. — In one unit there are $\frac{13}{13}$. In the fraction $\frac{121}{13}$ there are as many units as 13 is contained in 121, or 9 and 4 remainder. The remainder is written as a fraction $\frac{4}{13}$.

2. $\frac{385}{15}$	3. $\frac{496}{12}$	4. $\frac{578}{20}$	5. $\frac{75}{10}$
6. $\frac{897}{25}$	7. $\frac{901}{26}$	8. $\frac{891}{27}$	9. $\frac{1050}{67}$
10. $\frac{675}{29}$	11. $\frac{837}{27}$	12. $\frac{912}{48}$	13. $\frac{647}{19}$

To Reduce Whole Numbers to Improper Fractions

Oral Exercise

61. 1. How many halves in 1 unit?

2. How many halves in 2 units?

3. How many halves in 5 units?

4. How many thirds in 1 unit?

5. How many thirds in 2 units?

6. How many thirds in 4 units?

7. How many fourths in one?

8. How many fourths in two?

9. How many fourths in five?

10. Tell how to reduce a whole number to a fraction having a given denominator.

Written Exercise

62. Reduce:

1. 16 to halves.

PROCESS

$1 = \frac{2}{2}$

$16 = 16 \times \frac{2}{2}$

$= \frac{32}{2}$

EXPLANATION. — In one unit there are two halves. In 16 units there are 16 times 2 halves, or 32 halves, or $\frac{32}{2}$.

2. 24 to halves.

3. 25 to halves.

4. 27 to halves.

5. 29 to halves.

6. 36 to halves.

7. 39 to thirds.

8. 38 to thirds.

9. 45 to thirds.

10. 47 to thirds.

11. 49 to thirds.

12. 56 to fourths.
13. 57 to fourths.
14. 58 to fourths.
15. 59 to fourths.
16. 64 to fourths.
17. 75 to eighths.
18. 87 to tenths.
19. 99 to twelfths.
20. 117 to elevenths.
21. 100 to tenths.

Oral Problems

63. 1. How many quarters will a person receive in change for a $2.00 bill?

2. How many dimes will be received in change for a $5.00 bill?

3. How many eighths of an inch are there in 12 in.?

4. How many sixteenths of a pound are there in 2 lb.?

5. How many eighths of a gallon are there in 2 gal.?

To Reduce a Mixed Number to an Improper Fraction

Oral Exercise

64. 1. How many halves in one? In $1\frac{1}{2}$?

2. How many thirds in one? In $2\frac{2}{3}$?

3. How many fourths in one? In $3\frac{3}{4}$?

4. How many fifths in one? In $2\frac{2}{5}$?

5. How many eighths in one? In $4\frac{1}{8}$?

Written Exercise

65. Reduce to an improper fraction:

1. $4\frac{3}{5}$.

PROCESS

$4\frac{3}{5} = 4 + \frac{3}{5}$

$4 = 4 \times \frac{5}{5} = \frac{20}{5}$

$4\frac{3}{5} = \frac{20}{5} + \frac{3}{5} = \frac{23}{5}$

EXPLANATION. — The denominator of the fraction in the mixed number $4\frac{3}{5}$ is 5. Reduce 4 to fifths. Add the fraction $\frac{3}{5}$. The mixed number $4\frac{3}{5} = \frac{23}{5}$.

2. $1\frac{4}{5}$	3. $2\frac{1}{2}$	4. $3\frac{1}{4}$	5. $4\frac{1}{3}$
6. $5\frac{1}{2}$	7. $6\frac{3}{4}$	8. $7\frac{2}{3}$	9. $8\frac{7}{8}$
10. $12\frac{1}{2}$	11. $10\frac{1}{3}$	12. $30\frac{3}{4}$	13. $60\frac{2}{3}$
14. $10\frac{1}{6}$	15. $20\frac{1}{5}$	16. $40\frac{1}{12}$	17. $7\frac{8}{12}$

Written Exercise

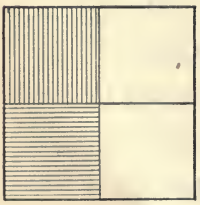
66. Change to improper fractions:

1. $15\frac{1}{3}$	2. $120\frac{3}{5}$	3. $107\frac{4}{5}$	4. $63\frac{1}{2}$	5. $13\frac{1}{2}$
6. $89\frac{7}{10}$	7. $60\frac{1}{5}$	8. $63\frac{5}{6}$	9. $125\frac{2}{3}$	10. $75\frac{5}{12}$

11. Change to sixths: 25, 63, 75, $76\frac{1}{6}$.

12. Change to eighths: 14, 24, 48, $32\frac{1}{8}$.

To Reduce a Fraction to Higher Terms



Oral Exercise

67. 1. Examine the drawing of the square.

2. Show by diagram that $\frac{1}{2} = \frac{2}{4}$.

3. Make a diagram to show that $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{2} = \frac{4}{8}$, $\frac{1}{2} = \frac{5}{10}$.

4. To change $\frac{1}{2}$ to sixths, make the denominator of the new fraction 6. To make the denominator of the fraction 6, multiply both terms of $\frac{1}{2}$ by 3. Thus: $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$.

5. By what number must both terms of $\frac{1}{2}$ be multiplied to change the fraction to eighths? $\frac{1 \times ?}{2 \times ?} = \frac{?}{8}$.

Oral Exercise

68. Give the answers:

1. $\frac{1}{2} = \frac{?}{4} = \frac{?}{6} = \frac{?}{8} = \frac{?}{10}$
2. $\frac{1}{3} = \frac{?}{6} = \frac{?}{9} = \frac{?}{12} = \frac{?}{15}$
3. $\frac{2}{3} = \frac{?}{6} = \frac{?}{9} = \frac{?}{12} = \frac{?}{15}$
4. $\frac{1}{4} = \frac{?}{8} = \frac{?}{12} = \frac{?}{16} = \frac{?}{20}$
5. $\frac{3}{4} = \frac{?}{8} = \frac{?}{16} = \frac{?}{12} = \frac{?}{24}$
6. $\frac{2}{5} = \frac{?}{10} = \frac{?}{15} = \frac{?}{20} = \frac{?}{25}$
7. $\frac{3}{5} = \frac{?}{10} = \frac{?}{20} = \frac{?}{12} = \frac{?}{15}$
8. $\frac{4}{5} = \frac{?}{10} = \frac{?}{20} = \frac{?}{30} = \frac{?}{15}$
9. $\frac{1}{6} = \frac{?}{12} = \frac{?}{24} = \frac{?}{18} = \frac{?}{30}$
10. $\frac{5}{6} = \frac{?}{12} = \frac{?}{24} = \frac{?}{18} = \frac{?}{30}$
11. $\frac{1}{8} = \frac{?}{16} = \frac{?}{32} = \frac{?}{24} = \frac{?}{40}$
12. $\frac{3}{8} = \frac{?}{16} = \frac{?}{32} = \frac{?}{40} = \frac{?}{24}$
13. $\frac{5}{8} = \frac{?}{24} = \frac{?}{32} = \frac{?}{40} = \frac{?}{16}$
14. $\frac{7}{8} = \frac{?}{40} = \frac{?}{24} = \frac{?}{16} = \frac{?}{32}$

Written Exercise

69. Change to the required denomination:

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, 1, 2 to sixths.
2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, 3 to twelfths.
3. $\frac{5}{6}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{4}{6}$, 1 to twelfths.
4. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{2}$, $\frac{3}{4}$, $\frac{7}{2}$, $\frac{5}{4}$ to eighths.
5. $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{5}{8}$ to twenty-fourths.

Fractions having the same denominator are called **similar fractions.**

In order to compare the values of fractions, to add or to subtract fractions, we must first reduce the fractions to similar fractions.

To Reduce a Fraction to Higher Terms

Oral Exercise

70. 1. Change $\frac{3}{4}$ to eighths.

PROCESS

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

EXPLANATION. — To change fourths to eighths, make the denominator of the new fraction 8. To make the denominator 8, multiply both terms of the fraction by 2.

2. Change $2\frac{1}{5}$ to tenths.

PROCESS

$$2\tfrac{1}{5} = \tfrac{11}{5}$$

$$\frac{11}{5} = \frac{11 \times 2}{5 \times 2} = \frac{22}{10}$$

EXPLANATION. — Reduce the mixed number to an improper fraction.

To change fifths to tenths, make the denominator of the new fraction ten. Then multiply both terms by the factor that will give the required denominator.

AN IMPORTANT PRINCIPLE IN FRACTIONS

Multiplying both terms of a fraction by the same number does not change the value of the fraction.

Written Exercise

71. Reduce:

1.	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{3}$	$1\frac{1}{3}$	$2\frac{2}{3}$	to sixths.
2.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	to eighths.
3.	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$4\frac{2}{3}$	to twelfths.
4.	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	to tenths.
5.	$1\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{2}{5}$	$3\frac{2}{5}$	$5\frac{1}{5}$	to tenths.
6.	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{8}$	to sixteenths.
7.	$4\frac{1}{8}$	$12\frac{1}{4}$	$5\frac{5}{8}$	$16\frac{1}{2}$	$10\frac{7}{8}$	to sixteenths.
8.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{3}{10}$	to twentieths.
9.	$5\frac{4}{5}$	$10\frac{2}{5}$	$4\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{1}{10}$	to twentieths.
10.	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	to hundredths.
11.	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$	to hundredths.
12.	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{4}$	$\frac{3}{4}$	to hundredths.
13.	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{7}{20}$	$\frac{5}{20}$	$\frac{8}{20}$	to hundredths.
14.	$\frac{4}{25}$	$\frac{7}{25}$	$\frac{16}{25}$	$\frac{10}{50}$	$\frac{20}{50}$	to hundredths.

Least Common Denominator

72. Heretofore easy fractions were considered, the least common denominator of which could be told at sight.

Now we proceed to explain the process of discovering the least common denominator when it cannot be readily recognized at sight.

1. Find the least common denominator (l. c. d.) of $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{12}$.

Find the prime factors of each denominator.

Thus:

$$4 = 2 \cdot 2$$
$$8 = 2 \cdot 2 \cdot 2$$
$$12 = 2 \cdot 2 \quad \cdot 3$$

The l. c. d. must contain the prime factor 2 three times, or it would not be exactly divisible by 8.

The l. c. d. must contain the factor 3 once, or it would not be exactly divisible by 12.

Hence the l. c. d. is $2 \times 2 \times 2 \times 3 = 24$.

Factor each denominator and take each prime factor the greatest number of times it occurs in any denominator.

2. Find the l. c. d., if the given denominators are 4, 6, 16.

We have:

$$4 = 2 \cdot 2$$
$$6 = 2 \quad \cdot 3$$
$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

The l. c. d. must contain the factor 2 four times.

The l. c. d. must contain the factor 3 once.

Hence the l. c. d. is $2 \times 2 \times 2 \times 2 \times 3 = 48$.

3. If the denominators are 4, 9, 12, find the l. c. d.

$$4 = 2 \cdot 2$$
$$9 = \quad 3 \cdot 3$$
$$12 = 2 \cdot 2 \cdot 3$$

Hence the l. c. d. $= 2 \times 2 \times 3 \times 3 = 36$

Written Exercise

73. Find the l. c. d. of the following denominators:

4. 2, 6, 36.
5. 3, 4, 16, 20.
6. 9, 24, 36.
7. 10, 20, 25.
8. 15, 8, 30.
9. 20, 15, 30.
10. 7, 49, 6.
11. 8, 6, 9, 36.
12. 12, 16, 36.
13. 16, 18, 12.

Written Exercise

74. Find the common denominator:

1. $\frac{2}{3}, \frac{3}{5}, \frac{5}{8}$
2. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
3. $\frac{3}{5}, \frac{1}{2}, \frac{1}{8}$
4. $\frac{1}{4}, \frac{5}{9}, \frac{2}{3}$
5. $\frac{5}{8}, \frac{3}{4}, \frac{5}{6}$
6. $\frac{7}{8}, \frac{3}{10}, \frac{2}{5}$
7. $\frac{5}{12}, \frac{5}{6}, \frac{7}{8}$
8. $\frac{3}{10}, \frac{5}{6}, \frac{7}{20}$
9. $\frac{5}{21}, \frac{4}{7}, \frac{1}{2}$
10. $\frac{4}{10}, \frac{3}{100}, \frac{27}{1000}$
11. $\frac{5}{100}, \frac{25}{1000}, \frac{80}{10000}$

Oral Exercise

75. 1. What are similar fractions?

2. Change $\frac{1}{2}$ and $\frac{1}{4}$ to fractions having a common denominator.

3. Change $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ to fractions having a common denominator.

4. Change $\frac{1}{2}$ and $\frac{1}{3}$ to fractions having a common denominator.

5. Change $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ to fractions having a common denominator.

6. Change $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ to fractions having a common denominator.

7. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$ to fractions having a common denominator.

8. Tell how to change fractions to fractions having a common denominator.

Study Exercise

76. 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to similar fractions; that is, to fractions having the same denominator.

PROCESS

$\frac{1}{2} = \frac{6}{12}$
$\frac{2}{3} = \frac{8}{12}$
$\frac{3}{4} = \frac{9}{12}$

EXPLANATION.—By inspection we find that these fractions may all be reduced to twelfths.

$\frac{1}{2} = \frac{6}{12}$
$\frac{2}{3} = \frac{8}{12}$
$\frac{3}{4} = \frac{9}{12}$

2. Reduce to similar fractions: $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{12}$, $\frac{9}{16}$

PROCESS

Find the least common denominator

$3 = 3$
$4 = 2 \times 2$
$12 = 2 \times 2 \times 3$
$16 = 2 \times 2 \times 2 \times 2$
l. c. d. $= 2 \times 2 \times 2 \times 2 \times 3$
The least common denominator is 48
Then $48 \div 3 = 16$ $\quad \frac{1}{3} \times \frac{16}{16} = \frac{16}{48}$

$48 \div 4 = 12 \qquad \frac{3}{4} \times \frac{12}{12} = \frac{36}{48}$

$48 \div 12 = 4 \qquad \frac{5}{12} \times \frac{4}{4} = \frac{20}{48}$

$48 \div 16 = 3 \qquad \frac{9}{16} \times \frac{3}{3} = \frac{27}{48}$

EXPLANATION. — 48 is the l. c. d. Change the denominator of the fraction to 48. Multiply both terms of the fraction $\frac{1}{3}$ by 16. Multiplying both terms of the fraction by 16 does not change the value of the fraction.

Proceed in like manner with the fractions $\frac{3}{4}$, $\frac{5}{12}$, and $\frac{9}{16}$.

3. Reduce $1\frac{2}{3}$, $\frac{7}{10}$, $\frac{4}{5}$ to similar fractions.

PROCESS

$1\frac{2}{3} = \frac{5}{3} = \frac{50}{30}$

$\frac{7}{10} = \frac{7}{10} = \frac{21}{30}$

$\frac{4}{5} = \frac{4}{5} = \frac{24}{30}$

EXPLANATION. — Reduce the mixed number, $1\frac{2}{3}$, to the improper fraction, $\frac{5}{3}$. Find by inspection or by factoring the least common denominator (l. c. d.). The l. c. d. = 30.

The fractions must be reduced to thirtieths.

$1\frac{2}{3} = \frac{50}{30}$

$\frac{7}{10} = \frac{21}{30}$

$\frac{4}{5} = \frac{24}{30}$

These fractions have a common denominator; hence, they are **similar fractions.**

Tell how to add similar fractions.

Written Exercise

77. Reduce to similar fractions:

1. $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{4}$
2. $\frac{1}{6}$, $\frac{7}{8}$, $\frac{9}{16}$
3. $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{8}$
4. $\frac{7}{12}$, $\frac{19}{72}$, $\frac{13}{18}$

5. $\frac{5}{8}$, $\frac{7}{4}$, $\frac{2}{3}$

6. $\frac{7}{24}$, $\frac{5}{8}$, $\frac{7}{12}$

7. $\frac{5}{6}$, $\frac{5}{8}$, $\frac{3}{4}$

8. $\frac{1}{15}$, $\frac{7}{12}$, $\frac{13}{10}$, $\frac{1}{4}$

9. $\frac{7}{9}$, $\frac{4}{5}$, $\frac{8}{15}$

10. $\frac{9}{10}$, $\frac{23}{30}$, $\frac{7}{15}$, $\frac{19}{20}$

11. $\frac{3}{10}$, $\frac{7}{25}$, $\frac{3}{5}$

12. $\frac{7}{1}$, $\frac{5}{16}$, $\frac{3}{4}$, $\frac{5}{8}$

13. $\frac{3}{8}$, $\frac{5}{6}$, $\frac{1}{9}$, $\frac{2}{3}$

14. $\frac{7}{16}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{12}$

15. $\frac{2}{5}$, $\frac{7}{10}$, $\frac{5}{6}$, $\frac{9}{20}$

16. $\frac{9}{20}$, $\frac{7}{30}$, $\frac{8}{15}$, $\frac{11}{12}$

Oral Exercise

78. Reduce to similar fractions and compare. Which is the larger?

1. $\frac{2}{5}$ and $\frac{1}{3}$

2. $\frac{3}{10}$ and $\frac{1}{4}$

3. $\frac{7}{8}$ and $\frac{5}{6}$

4. $\frac{5}{8}$ and $\frac{2}{3}$

5. $\frac{7}{12}$ and $\frac{6}{10}$

6. $\frac{3}{4}$ and $\frac{4}{5}$

7. $\frac{3}{7}$ and $\frac{4}{9}$

8. $\frac{1}{2}$ and $\frac{2}{3}$

9. $\frac{4}{5}$ and $\frac{8}{9}$

Addition of Fractions

79. 1. Add $\frac{1}{4}$ and $\frac{3}{4}$, $\frac{1}{4}$ and $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$.

2. What is the common denominator of $\frac{1}{2}$ and $\frac{1}{3}$? $\frac{1}{4}$ and $\frac{1}{2}$? $\frac{1}{3}$ and $\frac{1}{6}$?

3. Change $\frac{1}{4}$ and $\frac{3}{4}$ to similar fractions.

4. Change $\frac{1}{4}$ and $\frac{1}{2}$ to similar fractions.

5. Change $\frac{1}{3}$ and $\frac{1}{6}$ to similar fractions.

6. Change $\frac{1}{2}$ and $\frac{1}{3}$ to similar fractions.

7. Tell how to add $\frac{1}{4}$ and $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$, $\frac{1}{2}$ and $\frac{1}{3}$.

8. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? $\frac{3}{6}$ and $\frac{2}{6}$?

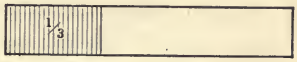
9. Name four common denominators of $\frac{1}{2}$ and $\frac{2}{3}$.

10. What is the least common denominator of $\frac{1}{2}$ and $\frac{2}{3}$?

Written Exercise

80. 1. Show by the diagram that $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$.

2. $\frac{1}{2}=\frac{?}{6}$; $\frac{1}{3}=\frac{?}{6}$;
$\frac{1}{2}+\frac{1}{3}=\frac{?}{6}$.



3. Can you show by a diagram that $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$?

4. Can you show by diagram that $\frac{1}{3}+\frac{1}{2}+\frac{1}{6}=1$?

5. Can you show by diagram that $\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$? $\frac{1}{2}+\frac{2}{5}=\frac{9}{10}$?

Written Exercise

81. Add:

1. $\frac{5}{6}$ and $\frac{7}{8}$.

PROCESS

(1) The common denominator is 24.

(2) $\frac{5}{6}=\frac{5\times 4}{6\times 4}=\frac{20}{24}$

$\frac{7}{8}=\frac{7\times 3}{8\times 3}=\frac{21}{24}$

(3) $\frac{20}{24}+\frac{21}{24}=\frac{41}{24}$

$24\overline{)41}\quad 1\frac{17}{24}$

$1\frac{17}{24}$ *Ans.*

EXPLANATION. — (1) Find the common denominator by inspection or by factoring.

(2) Reduce the fractions.

(3) Add the numerators of the similar fractions. Place the sum over the common denominator.

(4) Reduce this fraction to a whole or a mixed number.

What other common denominator might have been used? Why is the least common denominator the best denominator to use? Why is it called the least common denominator?

Written Exercise

82. Add, using pencil only when necessary:

1. $\frac{1}{2}+\frac{1}{3}$	2. $\frac{2}{3}+\frac{1}{6}$	3. $\frac{1}{2}+\frac{1}{6}$	4. $\frac{1}{2}+\frac{1}{4}$
5. $\frac{3}{4}+\frac{1}{8}$	6. $\frac{3}{4}+\frac{5}{8}$	7. $\frac{1}{3}+\frac{3}{4}$	8. $\frac{1}{3}+\frac{7}{12}$
9. $\frac{3}{5}+\frac{1}{10}$	10. $\frac{2}{3}+\frac{2}{9}$	11. $\frac{1}{2}+\frac{3}{8}$	12. $\frac{1}{4}+\frac{1}{6}$
13. $\frac{3}{4}+\frac{5}{6}$	14. $\frac{1}{2}+\frac{3}{5}$	15. $\frac{1}{2}+\frac{7}{10}$	16. $\frac{1}{4}+\frac{3}{8}$

Written Exercise

83. Add:

1. $\frac{3}{4}+\frac{2}{3}$	2. $\frac{5}{6}+\frac{3}{4}$	3. $\frac{3}{8}+\frac{7}{10}$
4. $\frac{4}{5}+\frac{9}{20}+\frac{3}{4}$	5. $\frac{5}{8}+\frac{5}{12}+\frac{3}{4}$	6. $\frac{2}{5}+\frac{9}{10}+\frac{3}{4}$
7. $15\frac{5}{24}+11\frac{1}{18}+\frac{5}{12}$	8. $\frac{4}{7}+\frac{2}{3}+\frac{5}{6}$	9. $\frac{5}{6}+\frac{4}{5}+\frac{7}{10}$
10. $17\frac{7}{24}+11\frac{1}{30}+\frac{8}{15}+\frac{9}{10}$		11. $\frac{7}{10}+\frac{4}{5}+11\frac{1}{15}+\frac{2}{3}$

Written Problems

84. 1. A boy earned $\$\frac{2}{5}$ one day, $\$\frac{3}{5}$ another day, and $\$\frac{1}{2}$ the third day. How much did he earn in the three days?

2. During the forenoon four observations of the thermometer were made. At the first reading the temperature was 50°; twenty minutes later it showed an increase of 1°; twenty minutes later it had increased $\frac{2}{3}°$ more; and at the last reading it had raised $\frac{3}{4}°$ more. What was the temperature at the last reading?

3. If the length of a book is $\frac{1}{2}$ ft. and the width $\frac{1}{3}$ ft., what is the length of both sides and both ends of the book?

4. A farmer gathers $\frac{1}{4}$ doz. eggs the first day, $\frac{2}{3}$ doz. on the second day, and $\frac{1}{2}$ doz. on the third day. How many eggs did he gather in the three days?

5. A clerk cut 3 pieces from a roll of ribbon. The first piece was $\frac{7}{8}$ yd., the second $\frac{1}{2}$ yd., and the third $\frac{3}{4}$ yd. What was the total length cut from the roll?

Addition of Mixed Numbers

85. Add at sight:

1. $10\frac{2}{3}$ $6\frac{1}{3}$	2. $30\frac{1}{4}$ $15\frac{3}{4}$	3. $50\frac{5}{6}$ $25\frac{1}{6}$	4. $70\frac{5}{8}$ $10\frac{3}{8}$
5. $90\frac{1}{2}$ $10\frac{1}{4}$	6. $20\frac{1}{2}$ $10\frac{1}{3}$	7. $30\frac{1}{3}$ $10\frac{1}{6}$	8. $40\frac{1}{2}$ $15\frac{1}{6}$
9. $80\frac{3}{4}$ $10\frac{1}{4}$	10. $67\frac{2}{3}$ $15\frac{1}{6}$	11. $59\frac{1}{4}$ $11\frac{3}{8}$	12. $36\frac{1}{2}$ $14\frac{1}{6}$

13. $46\frac{2}{3}$	14. $83\frac{1}{3}$	15. $144\frac{2}{3}$	16. $760\frac{3}{4}$
$5\frac{1}{6}$	$8\frac{1}{6}$	$67\frac{1}{6}$	$187\frac{1}{8}$
17. $265\frac{1}{4}$	18. $472\frac{1}{3}$	19. $34\frac{2}{3}$	20. $144\frac{5}{8}$
$321\frac{1}{8}$	$69\frac{1}{2}$	768	793
$87\frac{1}{2}$	$967\frac{1}{6}$	$25\frac{1}{6}$	$47\frac{1}{4}$

Written Exercise

86. Add:

1. $43\frac{3}{5}$ and $19\frac{2}{3}$.

PROCESS

$43\frac{3}{5} = 43\frac{9}{15}$
$79\frac{2}{3} = 79\frac{10}{15}$
$123\frac{4}{15}$

EXPLANATION. — Add the fractions. The least common denominator is 15.

$\frac{3}{5} + \frac{2}{3} = \frac{19}{15} = 1\frac{4}{15}$. Write $\frac{4}{15}$.

Add integers.

Find the sum of the integers and the fractions.

2. $14\frac{3}{4}$	3. $113\frac{4}{5}$	4. $76\frac{1}{2}$	5. $123\frac{5}{6}$
$5\frac{2}{3}$	$19\frac{3}{4}$	$47\frac{1}{6}$	$11\frac{1}{4}$
6. $30\frac{7}{10}$	7. $706\frac{3}{4}$	8. $100\frac{3}{4}$	9. $737\frac{11}{12}$
$24\frac{1}{2}$	$69\frac{1}{3}$	$79\frac{2}{5}$	$94\frac{2}{3}$
10. $43\frac{19}{20}$	11. $111\frac{11}{15}$	12. $875\frac{13}{16}$	13. $405\frac{1}{2}$
$34\frac{7}{20}$	$22\frac{2}{3}$	$69\frac{1}{3}$	$13\frac{1}{3}$
14. $763\frac{4}{5}$	15. $108\frac{3}{4}$	16. $95\frac{1}{10}$	17. $78\frac{2}{3}$
$45\frac{1}{5}$	$45\frac{1}{4}$	36	$97\frac{4}{5}$

Written Exercise

87. Add:

1. $75\frac{3}{4}$	2. $79\frac{3}{4}$	3. $76\frac{1}{2}$	4. $701\frac{4}{5}$	5. $101\frac{1}{2}$
$46\frac{1}{4}$	$63\frac{3}{8}$	$40\frac{1}{3}$	$67\frac{1}{5}$	$67\frac{3}{8}$
6. $46\frac{2}{3}$	7. $79\frac{3}{4}$	8. $101\frac{1}{5}$	9. $780\frac{3}{4}$	10. $167\frac{1}{4}$
$23\frac{1}{2}$	$63\frac{1}{5}$	$67\frac{1}{10}$	$63\frac{3}{4}$	98
11. $29\frac{1}{2}$	12. $65\frac{2}{3}$	13. $49\frac{4}{5}$	14. $987\frac{3}{4}$	15. $787\frac{4}{5}$
$38\frac{1}{3}$	$79\frac{3}{4}$	$58\frac{3}{4}$	$769\frac{2}{3}$	$868\frac{5}{8}$
$47\frac{3}{4}$	$87\frac{5}{6}$	$67\frac{7}{10}$	$898\frac{5}{6}$	$979\frac{3}{4}$
$56\frac{2}{3}$	$98\frac{1}{2}$	$86\frac{2}{5}$	$654\frac{4}{5}$	$697\frac{7}{10}$

Written Problems

88. 1. Mr. Edwards sold $10\frac{1}{2}$ bu. of wheat to one man and $4\frac{3}{4}$ bu. to another. How many bushels did he sell to both?

2. An agent's expenses were $\$ 4\frac{3}{4}$ the first day, $\$ 3\frac{1}{2}$ the second day, and $\$ 3\frac{3}{4}$ the third day. What is the total amount of his expenses for the three days?

3. Robert worked $\frac{2}{5}$ of the day Monday, $\frac{1}{3}$ of the day Tuesday, $\frac{1}{2}$ of the day Wednesday, and $\frac{5}{6}$ of the day Thursday. How many days did he work?

4. A girl studied $2\frac{1}{2}$ hours Monday, $1\frac{1}{4}$ hours Tuesday, $3\frac{1}{2}$ hours Wednesday, $2\frac{5}{8}$ hours Thursday, and $\frac{1}{2}$ hour Friday. How many hours did she study in all?

5. A jointed fishing pole has 3 sections. The first section is $2\frac{1}{2}$ ft. long, the second is $2\frac{3}{4}$ ft., and the third is 3 ft. How long is the pole?

6. A field is $80\frac{3}{4}$ rd. long and $40\frac{7}{8}$ rd. wide. How many rods of fence are required to inclose the field?

7. The ice for a family weighs $20\frac{1}{2}$ lb., $22\frac{3}{4}$ lb., $18\frac{1}{4}$ lb., $25\frac{5}{8}$ lb. Find the total weight of ice used in four days.

8. A painter, working by the hour, works $6\frac{3}{4}$ hr. the first day, $8\frac{1}{2}$ hr. the second day, $8\frac{1}{3}$ hr. the third, $7\frac{5}{8}$ hr. the fourth, and 4 hr. the fifth day. How many hours did he work in the five days?

Subtraction of Fractions

89. 1. What is the difference between $\frac{1}{2}$ and $\frac{1}{4}$? $\frac{3}{4}$ and $\frac{1}{2}$? $\frac{2}{3}$ and $\frac{1}{6}$? $\frac{2}{3}$ and $\frac{1}{2}$?

2. Change $\frac{1}{2}$ and $\frac{1}{4}$ to similar fractions.

3. Change $\frac{2}{3}$ and $\frac{1}{6}$ to similar fractions.

4. Change $\frac{2}{3}$ and $\frac{1}{2}$ to similar fractions.

5. Tell how to subtract $\frac{1}{4}$ from $\frac{3}{4}$.

6. Tell how to subtract $\frac{1}{6}$ from $\frac{5}{6}$.

7. Tell how to subtract $\frac{1}{2}$ from $\frac{2}{3}$.

8. Tell how to subtract $\frac{2}{3}$ from $\frac{3}{4}$.

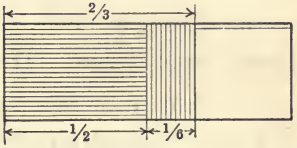
9. Explain how to subtract similar fractions.

10. Make a rule for subtracting fractions that are not similar.

Written Exercise

90. 1. Show by the diagram that $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.

2. $\frac{2}{3} = \frac{?}{6}$; $\frac{1}{2} = \frac{?}{6}$;
$\frac{2}{3} - \frac{1}{2} = \frac{?}{6}$.



3. Can you show by diagram that $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$?

4. Can you show by diagram that $1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$?

5. Can you show by diagram that $\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$?

Oral Exercise

91. Subtract:

1. $1 - \frac{1}{4}$	2. $1 - \frac{3}{4}$	3. $1 - \frac{1}{2}$	4. $1 - \frac{5}{8}$
5. $1 - \frac{1}{3}$	6. $1 - \frac{2}{3}$	7. $1 - \frac{5}{6}$	8. $1 - \frac{7}{10}$
9. $\frac{1}{2} - \frac{1}{4}$	10. $\frac{3}{4} - \frac{1}{2}$	11. $\frac{5}{8} - \frac{1}{4}$	12. $\frac{7}{8} - \frac{3}{4}$

Oral Problems

92. 1. What is the difference between $\frac{1}{2}$ of an apple and $\frac{1}{3}$ of an apple?

2. What is the difference between $\frac{1}{3}$ lb. and $\frac{1}{4}$ lb.?

3. In a bin there are $1\frac{3}{4}$ bu. of potatoes. If $\frac{1}{2}$ bu. is taken from the bin, how many bushels remain?

4. From a roll of cloth $12\frac{1}{2}$ yd. long a salesman cut $4\frac{1}{4}$ yd. How many yards remained in the roll?

5. The distance between two villages is 4 mi. If a house is $1\frac{1}{2}$ mi. from one village, how far is it from the other?

6. From a piece of cheese containing $3\frac{3}{4}$ lb. a grocer sold $2\frac{1}{2}$ lb. How many pounds remained?

7. A farmer has two fields. One field contains $20\frac{1}{4}$ A. The other field contains $4\frac{1}{8}$ A. less. How many acres in the smaller field?

8. From a piece of steak weighing $4\frac{1}{2}$ lb. a butcher cut $\frac{3}{4}$ lb., $\frac{1}{4}$ lb., and $\frac{1}{2}$ lb. How many pounds of steak has he left?

9. From a gasoline tank containing 20 gal., $4\frac{1}{2}$ gal., $2\frac{1}{4}$ gal., and 3 gal. were drawn. How many gallons were left in the tank?

10. A carpenter had a piece of molding $4\frac{1}{2}$ ft. long, from which he cut two pieces, one $2\frac{1}{4}$ ft. and the other $1\frac{1}{8}$ ft. long. How long is the piece that is left?

Oral Exercise

93. Reduce to similar fractions and subtract:

1. $\frac{1}{2}-\frac{1}{8}$	2. $\frac{1}{2}-\frac{1}{4}$	3. $\frac{1}{2}-\frac{1}{3}$	4. $\frac{1}{2}-\frac{5}{12}$
5. $\frac{1}{2}-\frac{2}{5}$	6. $\frac{1}{2}-\frac{1}{6}$	7. $\frac{1}{3}-\frac{1}{4}$	8. $\frac{1}{3}-\frac{1}{5}$
9. $\frac{1}{3}-\frac{1}{6}$	10. $\frac{1}{3}-\frac{1}{12}$	11. $\frac{1}{4}-\frac{1}{5}$	12. $\frac{1}{4}-\frac{1}{6}$
13. $\frac{1}{4}-\frac{1}{8}$	14. $\frac{1}{3}-\frac{2}{9}$	15. $\frac{3}{4}-\frac{7}{12}$	16. $1\frac{1}{4}-\frac{7}{8}$
17. $1\frac{1}{12}-\frac{3}{4}$	18. $1\frac{1}{8}-\frac{3}{4}$	19. $1\frac{1}{9}-\frac{8}{9}$	20. $1\frac{1}{2}-\frac{3}{4}$

Written Exercise

94. Subtract:

1. $27\frac{1}{2}$
   $12\frac{1}{4}$

PROCESS

Arrange the work as follows:

$27\frac{1}{2} = 27\frac{2}{4}$
$12\frac{1}{4} = 12\frac{1}{4}$

EXPLANATION. — $\frac{1}{2} = \frac{2}{4}$.
$27\frac{1}{2} = 27\frac{2}{4}$
$27\frac{2}{4} - 12\frac{1}{4} = 15\frac{1}{4}$

2. $105\frac{1}{2}$	3. $63\frac{1}{2}$	4. $57\frac{3}{4}$	5. $67\frac{5}{8}$
40	$15\frac{1}{2}$	$19\frac{1}{4}$	$23\frac{3}{8}$
6. $25\frac{1}{2}$	7. $75\frac{1}{2}$	8. $48\frac{5}{8}$	9. $54\frac{7}{8}$
$13\frac{1}{3}$	$69\frac{1}{6}$	$23\frac{1}{4}$	$24\frac{3}{4}$
10. $105\frac{5}{8}$	11. $302\frac{3}{4}$	12. $514\frac{1}{2}$	13. $524\frac{3}{4}$
$97\frac{1}{4}$	$67\frac{3}{8}$	$86\frac{3}{8}$	$76\frac{1}{2}$
14. $574\frac{2}{3}$	15. $733\frac{7}{8}$	16. $100\frac{7}{9}$	17. $1000\frac{1}{3}$
$125\frac{1}{6}$	$47\frac{1}{4}$	$60\frac{1}{3}$	$901\frac{1}{6}$
18. $700\frac{2}{3}$	19. $487\frac{3}{4}$	20. $527\frac{1}{3}$	21. $800\frac{1}{10}$
$549\frac{1}{2}$	$100\frac{7}{8}$	$200\frac{1}{4}$	$400\frac{1}{4}$
22. $501\frac{3}{5}$	23. $1010\frac{3}{8}$	24. $1626\frac{4}{5}$	25. $2000\frac{1}{2}$
$109\frac{3}{4}$	$700\frac{4}{5}$	$448\frac{3}{8}$	$1111\frac{2}{3}$

Study Exercise

95. Reduce to similar fractions and subtract:

1. $\frac{1}{2}$ from $\frac{2}{3}$.

PROCESS

(1) l. c. d. is 6

(2) $\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$

(3) $\frac{1}{2} = \frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

$\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

EXPLANATION. — To reduce to similar fractions multiply both terms of the fraction $\frac{1}{2}$ by 3 and both terms of the fraction $\frac{2}{3}$ by 2. Then subtract.

Multiplying both terms of a fraction by the same number does not change the value of the fraction. The difference between $\frac{4}{6}$ and $\frac{3}{6}$ is $\frac{1}{6}$, the answer.

2. $\frac{2}{3}$ from $\frac{3}{4}$.

PROCESS

$\frac{3}{4} = \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$

$\frac{2}{3} = \frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

EXPLANATION. — The l. c. d. is 12. Multiplying both terms of the fraction by the same number does not change the value of the fraction.

$\frac{3}{4} = \frac{9}{12}$

$\frac{2}{3} = \frac{8}{12}$

$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

3. $15\frac{3}{4}$ from $26\frac{2}{3}$.

PROCESS

$26\frac{2}{3} = 26\frac{8}{12} = 25\frac{20}{12}$

$15\frac{3}{4} = 15\frac{9}{12} = 15\frac{9}{12}$

$10\frac{11}{12}$

EXPLANATION. — Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to similar fractions. We obtain $\frac{8}{12}$ and $\frac{9}{12}$. We cannot subtract $\frac{9}{12}$ from $\frac{8}{12}$. Take 1 from 26, and add it to $\frac{8}{12}$; $\frac{8}{12} + \frac{12}{12} = \frac{20}{12}$. $25\frac{20}{12} - 15\frac{9}{12} = 10\frac{11}{12}$, the answer.

Written Exercise

96. Subtract:

1. $5\frac{1}{4}$	2. $6\frac{1}{4}$	3. $8\frac{1}{3}$	4. $10\frac{2}{5}$	5. $7\frac{1}{2}$
$2\frac{3}{4}$	$3\frac{1}{2}$	$4\frac{2}{3}$	$3\frac{4}{5}$	$4\frac{3}{4}$
6. $8\frac{1}{3}$	7. $3\frac{1}{2}$	8. $12\frac{1}{5}$	9. $10\frac{1}{2}$	10. $19\frac{1}{4}$
$5\frac{1}{2}$	$1\frac{2}{3}$	$3\frac{5}{8}$	$6\frac{7}{8}$	$12\frac{3}{8}$
11. $23\frac{1}{4}$	12. $18\frac{1}{4}$	13. $14\frac{1}{4}$	14. $15\frac{1}{4}$	15. $27\frac{3}{4}$
$13\frac{5}{8}$	$9\frac{7}{8}$	$10\frac{1}{3}$	$10\frac{2}{3}$	$11\frac{11}{12}$

Written Problems

97. 1. A farmer had $30\frac{1}{2}$ bu. of apples. He sold $17\frac{3}{4}$ bu. How many bushels has he left?

2. If I have $\$79\frac{1}{4}$ and spend $\$51\frac{3}{4}$, how many dollars have I left?

3. A table is $3\frac{11}{12}$ ft. long and $2\frac{7}{12}$ ft. wide. How much greater is the length than the width? Find the perimeter.

4. A farmer sold $105\frac{3}{4}$ bu. of his potato crop, kept $9\frac{1}{2}$ bu. for planting, and used $55\frac{1}{4}$ bu. for cooking. How many bushels in the crop?

5. A $4\frac{2}{3}$-in. spike is driven through a $2\frac{1}{4}$-in. board into a post. How far is it driven into the post?

6. The time required to travel from A to C is $28\frac{3}{4}$ hr. The time required to travel from A to B is $10\frac{5}{6}$ hr. How much longer does it require to travel from A to C than from A to B?

7. Along one side of a field 80 rd. long, there is, for a distance of $37\frac{3}{4}$ rd., a stone fence. The remaining distance is fenced with wire. How long is the wire fence?

8. A loaded truck weighs $2\frac{4}{5}$ T. The load consists of two parts. The first part weighs $1\frac{1}{4}$ T. the second $\frac{2}{5}$ T. Find the weight of the truck.

Multiplication of Fractions

98. Give answers rapidly:

1. How many fourths are 5 times $\frac{3}{4}$?
2. 7 times $\frac{2}{5}$ are how many fifths?
3. $6 \times \frac{3}{5} = ?$
4. $10 \times \frac{2}{3} = \frac{20}{3} = 6\frac{?}{3}$
5. $6 \times \frac{2}{3} = ?$
6. $10 \times \frac{2}{5} = ?$
7. $\frac{1}{2} \times 6 = ?$
8. $\frac{1}{3} \times 7 = ?$
9. $\frac{1}{7} \times 2 = ?$
10. $\frac{1}{3} \times 3 = ?$
11. $\frac{1}{2} \times \frac{1}{3} = ?$
12. $1\frac{1}{2} \times 4 = ?$
13. $8 \times 1\frac{3}{4} = ?$
14. $12 \times 1\frac{1}{4} = ?$
15. $10 \times 1\frac{1}{5} = ?$
16. $12 \times 1\frac{1}{2} = ?$

Oral Problems

99. 1. William earns $\$\frac{3}{4}$ daily, or $\$$ —— in 6 days.

2. John's step is $\frac{4}{5}$ of a yard. How far does he go in 6 steps?

3. George paces the width of his tennis court, taking 10 steps. How wide is the court, if his step is $\frac{5}{6}$ of a yard long?

4. A father gives $\$\frac{3}{4}$ to each of his 5 children. How much does he give them all together?

5. A man takes a run of $\frac{2}{3}$ of a mile every day. How many miles does he run in 6 days?

To Multiply a Fraction by a Whole Number

100. 1. Multiply $\frac{7}{8}$ by 64.

PROCESS

$$\frac{7}{8} \times 64 = \frac{7 \times \overset{8}{\cancel{64}}}{\cancel{8}} = 56$$

EXPLANATION. — 64 times $\frac{7}{8}$ may be written $\frac{64 \times 7}{8}$. Cancel the common factor. Why?

2. Multiply $\frac{7}{12}$ by 64.

PROCESS

$$\frac{7}{12} \times 64 = \frac{7 \times \overset{16}{\cancel{64}}}{\cancel{12}_{3}} = \frac{7 \times 16}{3} = \frac{112}{3} = 37\frac{1}{3}$$

EXPLANATION. — 64 times $\frac{7}{12}$ may be written $\frac{64 \times 7}{12}$. Cancel the common factor and reduce to a mixed number.

To Multiply a Fraction by a Whole Number

Multiply the numerator of the fraction by the whole number and divide the product by the denominator of the fraction. Cancel when possible.

Written Exercise

101. Solve, using pencil only when necessary:

1. $22 \times \frac{7}{11}$	2. $6 \times \frac{3}{8}$	3. $7 \times \frac{3}{14}$
4. $8 \times \frac{3}{16}$	5. $120 \times \frac{2}{5}$	6. $360 \times \frac{1}{6}$
7. $72 \times \frac{5}{6}$	8. $75 \times \frac{2}{5}$	9. $840 \times \frac{2}{3}$
10. $960 \times \frac{11}{12}$	11. $42 \times \frac{3}{2}$	12. $35 \times \frac{3}{5}$

The sign × may mean *times* or *multiply by*. In this exercise it should be read *times*.

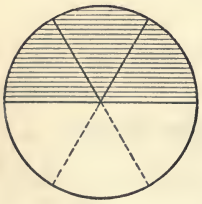
To Multiply a Whole Number by a Fraction

102. Multiply and explain:

1. 64 by $\frac{7}{8}$	2. 96 by $\frac{7}{9}$
3. 64 by $\frac{7}{12}$	4. 88 by $\frac{7}{10}$
5. $\frac{6}{7}$ times 24	6. $\frac{5}{26} \times 38$
7. $\frac{4}{5}$ times 48	8. $\frac{7}{11}$ times 20
9. $\frac{3}{4}$ times 54	10. $\frac{5}{9}$ times 42

To Multiply a Fraction by a Fraction

103. 1. What part of the circle is shaded? Show $\frac{1}{3}$ of the part that is shaded. What part is this of the whole circle?

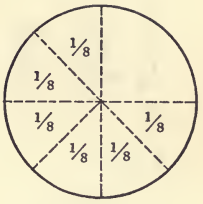


2. Show $\frac{2}{3}$ of the part that is shaded. What part of the whole is this? $\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{3} \times \frac{1}{2} = ?$

3. Draw a circle and shade $\frac{1}{2}$ of it. Draw lines from the center of the circle dividing the shaded

part into 4 equal parts. What part of the whole circle is one of these parts? $\frac{1}{4}$ of $\frac{1}{2} = ?$

4. A boy takes $\frac{3}{4}$ of an apple and cuts each quarter into two equal parts. What fraction of the whole apple is each part?



5. Make a rule for the multiplication of a fraction by a fraction.

$\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$ $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$ $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

6. In these examples, the word *of* indicates multiply. $\frac{1}{2}$ of $\frac{3}{4}$ means $\frac{1}{2} \times \frac{3}{4}$, or $\frac{3}{4}$ multiplied by $\frac{1}{2}$.

The fraction $\frac{1}{2}$ of $\frac{3}{4}$ is sometimes called a **compound fraction.**

Written Exercise

104. 1. Find $\frac{3}{4}$ of $\frac{5}{6}$.

PROCESS

$$\frac{3}{4} \text{ of } \frac{5}{6} = \frac{3}{4} \times \frac{5}{6}$$

$$= \frac{\overset{1}{\cancel{3}} \times 5}{4 \times \underset{2}{\cancel{6}}}$$

$$= \frac{1 \times 5}{4 \times 2}$$

$$= \frac{5}{8}$$

EXPLANATION. — Write the fractions in the form for multiplication. Cancel common factors from the numerator and denominator. Write the product of the remaining factors in the numerator and the denominator.

2. Find $\frac{7}{12}$ of $\frac{16}{21}$.

PROCESS

$$\frac{\cancel{7}^{1} \times \cancel{16}^{4}}{\cancel{12}^{3} \times \cancel{21}^{3}} = \frac{4}{9}$$

EXPLANATION. — Write the explanation.

Find:

3. $\frac{1}{2}$ of $\frac{1}{2}$
4. $\frac{1}{3}$ of $\frac{1}{2}$
5. $\frac{1}{3}$ of $\frac{1}{3}$
6. $\frac{1}{3}$ of $\frac{1}{4}$
7. $\frac{1}{3}$ of $\frac{3}{4}$
8. $\frac{1}{4}$ of $\frac{3}{2}$
9. $\frac{1}{2}$ of $\frac{7}{8}$
10. $\frac{4}{5}$ of $\frac{5}{6}$
11. $\frac{5}{6}$ of $\frac{7}{8}$

Tell how to multiply a fraction by a fraction. Make a rule for the multiplication of a fraction by a fraction.

TO MULTIPLY A FRACTION BY A FRACTION

Cancel factors common to the numerators and denominators and multiply the remaining factors in the numerators for the new numerator, and the remaining factors in the denominators together for the new denominator.

Oral Exercise

105. Solve, using pencil only when necessary:

1. $\frac{5}{8}$ of $\frac{4}{5}$
2. $\frac{3}{4}$ of $\frac{2}{3}$
3. $\frac{2}{3}$ of 16
4. $\frac{4}{5} \times \frac{5}{4}$
5. $\frac{4}{5} \times \frac{17}{2}$
6. $\frac{4}{3} \times \frac{9}{5}$
7. $120 \times \frac{5}{6}$
8. $12 \times \frac{11}{15}$
9. $15 \times \frac{7}{10}$

10. $63 \times \frac{1}{14}$	11. $16 \times \frac{7}{12}$	12. $\frac{7}{8} \times 48$
13. $\frac{2}{5} \times \frac{1}{6}$	14. $\frac{3}{10} \times \frac{7}{10}$	15. $\frac{5}{6} \times \frac{3}{5}$
16. $\frac{25}{24} \times \frac{3}{5}$	17. $\frac{24}{3} \times \frac{3}{8}$	18. $\frac{9}{16} \times \frac{5}{3}$
19. $\frac{7}{10} \times \frac{5}{14}$	20. $\frac{3}{5} \times \frac{2}{3}$	21. $2 \times \frac{3}{4}$
22. $\frac{22}{5} \times \frac{10}{11}$	23. $5 \times \frac{3}{4}$	24. $\frac{3}{2} \times \frac{8}{3}$
25. $\frac{7}{6} \times \frac{18}{5}$	26. $\frac{11}{8} \times 72$	27. $\frac{3}{5} \times \frac{5}{12}$
28. $\frac{11}{5}$ of $\frac{25}{11}$	29. $\frac{4}{5}$ of $\frac{55}{12}$	30. $\frac{7}{8} \times \frac{72}{6}$

Explain how the multiplication of a whole number by a fraction or a fraction by a whole number may be performed as if it were the multiplication of a fraction by a fraction.

Oral Problems

106. 1. How much does a boy earn in 7 days if he makes $\$\frac{1}{2}$ in a day?

2. A man charges $\$\frac{1}{2}$ an hour. How much does he earn in $\frac{3}{4}$ of an hour?

3. A boy rides his bicycle at the rate of 12 mi. an hour. How far does he go in $\frac{2}{3}$ of an hour?

4. Find the cost of 6 chairs at $\$1\frac{3}{4}$ each.

5. Make a bill for 6 days' wages at $\$2\frac{1}{4}$ per day.

6. What is the cost of a piece of dress goods containing 7 yd. if the material sells at $\$1\frac{3}{8}$ per yard?

7. A roll of wall paper is $\frac{3}{4}$ yd. wide. How many yards can be covered with 6 strips?

8. A man's expenses were $\$4\frac{3}{4}$ per day. What were his expenses for 20 da.?

To Multiply a Mixed Number by a Whole Number or a Whole Number by a Mixed Number

107. 1. Multiply $4\frac{3}{4}$ by 8.

PROCESS

$$8 \times 4\tfrac{3}{4} = 8 \times \tfrac{19}{4}$$
$$= \frac{\cancel{8}^{2} \times 19}{\cancel{4}_{1}}$$
$$= 38$$

EXPLANATION. — Reduce the mixed number to an improper fraction. Proceed as in multiplication of a fraction by a whole number.

2. Multiply 26 by $4\frac{2}{3}$.

PROCESS

$$4\tfrac{2}{3} \times 36 = \tfrac{14}{3} \times 36$$
$$= \frac{14 \times \cancel{36}^{12}}{\cancel{3}_{1}}$$
$$= 168$$

EXPLANATION. — Reduce the mixed number to an improper fraction. Proceed as in multiplication of a fraction by a whole number.

3. Multiply 39 by $4\frac{4}{5}$.

PROCESS

$$4\tfrac{4}{5} \times 39 = \tfrac{24}{5} \times 39$$
$$= \frac{24 \times 39}{5}$$
$$= 187\tfrac{1}{5}$$

EXPLANATION. — Reduce the mixed number to an improper fraction. Proceed as in multiplication of a fraction by a whole number.

Written Exercise

108. Multiply, using pencil only when necessary:

1. $2\frac{3}{4} \times 4$	2. $3\frac{7}{8} \times 8$	3. $4\frac{9}{10} \times 5$
4. $12 \times 1\frac{5}{6}$	5. $14 \times 3\frac{2}{3}$	6. $17 \times 4\frac{5}{6}$
7. $25 \times 3\frac{2}{5}$	8. $48 \times 4\frac{9}{10}$	9. $7\frac{2}{5} \times 15$
10. $27\frac{1}{3} \times 20$	11. $48\frac{4}{5} \times 40$	12. $56\frac{8}{9} \times 60$
13. $100 \times 2\frac{2}{3}$	14. $450 \times \frac{7}{8}$	15. $600 \times 5\frac{5}{9}$

To Multiply a Mixed Number by a Fraction or a Fraction by a Mixed Number

Written Exercise

109. Multiply:

1. $\frac{3}{4}$ by $20\frac{2}{3}$.

PROCESS

$$\frac{3}{4} \times 20\frac{2}{3} = \frac{\cancel{3}}{\cancel{4}_{2}} \times \frac{\overset{31}{\cancel{62}}}{\cancel{3}_{1}}$$

$$= \frac{31}{2}$$

$$= 15\frac{1}{2}$$

EXPLANATION. — Reduce the mixed number to an improper fraction and multiply. Cancel wherever possible.

2. $2\frac{2}{3} \times \frac{3}{8}$	3. $6\frac{1}{4} \times \frac{3}{5}$	4. $3\frac{3}{4} \times \frac{7}{5}$
5. $3\frac{3}{5} \times \frac{1}{6}$	6. $2\frac{2}{3} \times \frac{1}{8}$	7. $3\frac{2}{5} \times \frac{1}{2}$
8. $4\frac{5}{9} \times \frac{3}{4}$	9. $7\frac{5}{8} \times \frac{2}{3}$	10. $5\frac{3}{7} \times \frac{7}{8}$
11. $\frac{11}{12} \times 10\frac{1}{2}$	12. $\frac{21}{25} \times 3\frac{3}{4}$	13. $\frac{14}{15} \times 6\frac{7}{8}$
14. $\frac{9}{4} \times 4\frac{2}{3}$	15. $\frac{12}{5} \times 7\frac{3}{4}$	16. $\frac{10}{3} \times 20\frac{1}{2}$

To Multiply a Mixed Number by a Mixed Number

110. Multiply:

1. $9\frac{3}{4}$ by $15\frac{1}{3}$.

PROCESS

$$9\tfrac{3}{4} \times 15\tfrac{1}{3} = \frac{\overset{13}{\cancel{39}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{23}{\cancel{46}}}{\underset{1}{\cancel{3}}}$$

$$= \frac{299}{2}$$

$$= 149\tfrac{1}{2}$$

EXPLANATION. — Reduce the mixed number to improper fraction and multiply. Cancel where possible.

2. $3\frac{1}{4} \times 4\frac{1}{3}$
3. $5\frac{1}{4} \times 1\frac{1}{3}$
4. $2\frac{1}{4} \times 5\frac{1}{3}$
5. $7\frac{7}{8} \times 6\frac{4}{9}$
6. $17\frac{4}{5} \times 11\frac{2}{3}$
7. $19\frac{4}{5} \times 28\frac{2}{11}$
8. $48\frac{2}{3} \times 24\frac{7}{8}$
9. $33\frac{1}{3} \times 34\frac{7}{10}$
10. $16\frac{1}{2} \times 16\frac{1}{2}$
11. $66\frac{2}{3} \times 33\frac{1}{3}$
12. $\frac{1}{8} \times 16\frac{2}{3}$
13. $33\frac{1}{3} \times 3\frac{3}{10}$

Written Problems

111. 1. Theodore earns $\$4\frac{1}{2}$ a week for 7 weeks. How many dollars does he earn?

2. James works $13\frac{1}{2}$ hr. at $30¢$ an hour. How much does he earn?

3. A plumber charges $\$\frac{3}{4}$ per hour for his time. How much does he get for 3 hours' work?

4. An engineer charged $\$5\frac{1}{2}$ a day for work that occupied him $13\frac{1}{2}$ days. How much was his bill?

5. What is the cost of $13\frac{1}{2}$ doz. eggs at $\$\frac{1}{4}$ a dozen?

6. John is $5\frac{1}{3}$ ft. tall. James is only $\frac{3}{4}$ as tall as John. How tall is James?

7. For 9 years a boy has spent $\frac{3}{4}$ of every year in school. How many years has he spent in school?

8. If pepper sells at $15\frac{1}{3}¢$ a pound, find the cost of $1\frac{1}{2}$ bags, weighing 120 pounds each.

9. A boy invests $\$ 26\frac{1}{4}$ in pigeons. At the end of a year he gains $\frac{1}{3}$ of his investment. What is his gain?

10. When wheat is $85\frac{1}{5}¢$ a bushel, how much will 205 bushels bring?

11. Theodore's wages are $\frac{2}{5}$ of his father's. What does Theodore receive, if his father earns $\$ 22$ a week? If his father earns $\$ 17\frac{1}{2}$ a week?

12. If a cord of wood cost $\$ 3\frac{1}{2}$, what will $5\frac{1}{2}$ cords cost?

13. William has spent $\frac{1}{2}$ of his weekly allowance. He has $\$ 2\frac{1}{2}$ left. What is his weekly allowance?

14. At $5¢$ per square foot what is the cost of painting an advertisement upon a wall, $13\frac{1}{2}$ ft. by $5\frac{1}{3}$ ft.?

15. A grocer having $16\frac{1}{2}$ crates of berries sold $\frac{2}{3}$ of them, or —— crates.

16. Find the cost of 125 tons of lignite coal at $\$ 5\frac{2}{5}$ a ton.

Practical Problems — Area

112. 1. How many square feet in the area of a rectangle $10\frac{2}{3}$ ft. long and $3\frac{3}{4}$ ft. wide?

PROCESS

$$\frac{\overset{8}{\cancel{32}} \times \overset{5}{\cancel{15}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{4}}} = \frac{40}{1}$$

$$= 40$$

The answer is 40 sq. ft.

EXPLANATION. — In finding the product of the base and the altitude, *indicate* the operations, and then cancel factors common to the numerator and denominator. Not until after this is done should the multiplications be performed. In this example, the indicated area is $\frac{32 \times 15}{3 \times 4}$ sq. ft. Cancel the factors and multiply.

2. Find the area of a rectangle $7\frac{1}{2}$ ft. long and $7\frac{1}{3}$ ft. wide.

3. A table is $4\frac{1}{4}$ ft. long and half as wide. How wide is it?

4. The area of a garden is $30\frac{3}{4}$ sq. yd. If an area $\frac{2}{3}$ as large as the garden be added to it, how many square yards larger will it be?

5. A rectangle has a base 64 ft. long and an altitude $36\frac{1}{2}$ ft. Find its area.

6. The base of a rectangle is $\frac{5}{4}$ of a foot, its height is $\frac{2}{3}$ of a foot. Find its area.

7. How many square feet are there in the surface of a trunk 4 feet long, $\frac{3}{2}$ of a foot wide, and $\frac{5}{4}$ of a foot high?

8. A border is $\frac{2}{5}$ of a yard wide and 4 yd. long. What is its area in square yards?

9. A room is 25 ft. long and $17\frac{1}{5}$ ft. wide. What is the area of the floor?

10. A room is 44 ft. by $20\frac{1}{4}$ ft. What is the area of the floor?

11. A room is 8 ft. high, 16 ft. long, and 12 ft. wide. What is the area of the four walls? What is the area of the floor and ceiling?

Written Exercise

113. 1. Find the product of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{8}{9}$.

PROCESS

$$\frac{\cancel{3}\times 5\times \overset{2}{\cancel{8}}}{\underset{2}{\cancel{4}}\times\cancel{6}\times 9}=\frac{5}{9}\ \textit{Ans.}$$

EXPLANATION.—In multiplication of fractions, it is best merely to indicate the operations at first, then to cancel equal factors in the numerator and denominator.

2. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$

3. $\frac{3}{2}\times\frac{3}{4}$ of $\frac{4}{9}$

4. $\frac{3}{5}$ of $\frac{5}{3}$ of $\frac{7}{8}$

5. $\frac{\cancel{10}}{\cancel{11}}\times\frac{\overset{2}{\cancel{2}}}{\underset{5}{\cancel{5}}}\times\frac{1}{5}$

6. $\frac{\cancel{11}}{\cancel{12}}\times\frac{4}{5}\times\frac{\cancel{5}}{\cancel{22}}$

7. $\frac{1}{3}\times\frac{6}{7}\times\frac{\cancel{21}}{\cancel{2}}$

8. $\frac{\cancel{11}}{\cancel{12}}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$

9. $\frac{\cancel{10}}{\cancel{14}}$ of $\frac{7}{8}$ of $\frac{\cancel{12}}{\cancel{5}}$ of $\frac{3}{4}$

10. $\frac{\cancel{12}}{\cancel{15}}\times\frac{5}{6}$ of $\frac{\cancel{20}}{\cancel{12}}\times\frac{1}{5}$

A fraction of a fraction is called a **compound fraction.** $\frac{3}{4}$ of $\frac{4}{5}$ and $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ are compound fractions.

Written Exercise

114. Find the product without reducing to an improper fraction:

1. 15 times $7\frac{2}{3}$.

PROCESS

$$\begin{array}{r} 7\frac{2}{3} \\ \underline{15} \\ 10 \\ 105 \\ \underline{} \\ 115 \end{array}$$ *Ans.*

EXPLANATION

$15 \times \frac{2}{3} = 10$
$15 \times 7 = 105$
$15 \times 7\frac{2}{3} = 115$

2. $6\frac{3}{4}$ times 18.

PROCESS

$$\begin{array}{r} 18 \\ \underline{6\frac{3}{4}} \\ 13\frac{1}{2} \\ 108 \\ \underline{} \\ 121\frac{1}{2} \end{array}$$

EXPLANATION

$\frac{3}{4} \times 18 = \frac{54}{4} = 13\frac{1}{2}$
$6 \times 18 = 108$
$6\frac{3}{4} \times 18 = 121\frac{1}{2}$

Written Exercise

115. Find the products:

1. $16\frac{1}{4} \times 24$
2. $73\frac{1}{3} \times 35$
3. $726\frac{1}{4} \times 8$
4. $78\frac{1}{3} \times 9$
5. $97\frac{3}{4} \times 12$
6. $987\frac{2}{3} \times 15$
7. $6\frac{2}{3} \times 15$
8. $8\frac{3}{10} \times 24$
9. $40\frac{4}{5} \times 60$
10. $37\frac{1}{2} \times 33\frac{1}{3} \times 16\frac{2}{3}$
11. $12\frac{1}{8} \times 125\frac{3}{4} \times 20\frac{2}{3}$
12. $1078 \times 30\frac{3}{5} \times 8\frac{2}{11} \times 20\frac{5}{14}$

In the multiplication of mixed numbers, it is usually the best plan to reduce the mixed numbers to improper fractions. Then multiply, canceling wherever possible.

To Divide a Fraction by an Integer

116. 1. Divide $\frac{3}{4}$ by 3.

Process	Explanation
$\frac{3}{4} \div 3 = \frac{1}{4}$	$\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{4}$

2. Divide $\frac{1}{2}$ by 5.

Process

$$\frac{1}{2} \div 5 = \frac{1}{2} \text{ of } \frac{1}{5}$$
$$= \frac{1}{2} \times \frac{1}{5}$$
$$= \frac{1}{10}$$

Explanation

$\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{5}$ times $\frac{1}{2}$
$\frac{1}{5}$ times $\frac{1}{2}$ is $\frac{1}{10}$

3. In the exercise, $\frac{3}{4} \div 3$, tell how to obtain the answer.

4. Give the answers:

$\frac{2}{3} \div 2$	$\frac{4}{5} \div 4$	$\frac{8}{9} \div 4$
$\frac{10}{11} \div 5$	$\frac{10}{11} \div 2$	$\frac{12}{7} \div 3$

5. In the exercise, $\frac{1}{2} \div 5$, tell how to obtain the answer.

To divide a fraction by an integer, divide the numerator of the fraction by the integer, or multiply the denominator of the fraction by the integer.

In dividing $\frac{12}{5}$ by 4, we may multiply the denominator by 4 and get $\frac{12}{20}$, or we may divide the numerator by 4 and obtain $\frac{3}{5}$. Which is the better way? Why?

In dividing $\frac{3}{5}$ by 4, which is the better way? Why?

Oral Problems

117. 1. A mother divides $\frac{1}{2}$ of a cake equally among 3 children. What portion of the whole cake does each receive?

2. Mary cuts $\frac{3}{4}$ of a yard of ribbon into 2 equal parts. How long is each part?

3. Three boys are given $\frac{3}{4}$ of a pound of dates. How much is each boy's share?

4. Four baskets of coal weigh $\frac{1}{10}$ of a ton. What is the weight of 1 basket?

5. John jumps $\frac{11}{3}$ yd. in 3 jumps. What is the distance covered in one jump?

Oral Exercise

118. Answer at sight:

1. $\frac{3}{4} \div 2$	2. $\frac{3}{4} \div 3$	3. $\frac{3}{4} \div 2$	4. $\frac{2}{3} \div 3$
5. $\frac{1}{4} \div 3$	6. $\frac{1}{3} \div 2$	7. $\frac{1}{5} \div 3$	8. $\frac{1}{5} \div 4$
9. $\frac{3}{4} \div 3$	10. $\frac{2}{3} \div 2$	11. $\frac{3}{5} \div 3$	12. $\frac{4}{5} \div 4$
13. $\frac{1}{2} \div 3$	14. $\frac{1}{2} \div 6$	15. $\frac{1}{3} \div 4$	16. $\frac{4}{3} \div 4$
17. $\frac{1}{10} \div 2$	18. $\frac{3}{10} \div 3$	19. $\frac{14}{10} \div 7$	20. $\frac{11}{5} \div 11$

Written Exercise

119. Solve:

1. $\frac{21}{33} \div 12$
2. $\frac{21}{32} \div 14$
3. $\frac{28}{33} \div 15$
4. $\frac{75}{108} \div 96$
5. $\frac{87}{107} \div 48$
6. $\frac{24}{49} \div 21$
7. $\frac{56}{25} \div 24$
8. $\frac{35}{18} \div 14$
9. $\frac{150}{64} \div 30$
10. $\frac{100}{56} \times \frac{64}{75} \div 24$
11. $\frac{125}{8} \times \frac{16}{25} \times \frac{3}{2} \div 3$

To Divide a Mixed Number by an Integer

120. Divide:

1. $5\frac{1}{5}$ by 3.

PROCESS

$$5\tfrac{1}{5} \div 3 = \tfrac{26}{5} \div 3$$
$$= \tfrac{26}{5} \times \tfrac{1}{3}$$
$$= \tfrac{26}{15}$$
$$= 1\tfrac{11}{15}$$

EXPLANATION. — Reduce the mixed number to an improper fraction. $\frac{26}{5} \div 3$ is the same as $\frac{1}{3}$ of $\frac{26}{5}$. Solve $\frac{1}{3}$ of $\frac{26}{5}$.

2. $3\frac{3}{4} \div 5$
3. $17\frac{1}{3} \div 13$
4. $25\frac{1}{3} \div 19$
5. $12\frac{1}{2} \div 5$
6. $29\frac{7}{8} \div 9$
7. $49\frac{4}{5} \div 17$
8. $68\frac{3}{11} \div 21$
9. $79\frac{2}{3} \div 18$
10. $48\frac{7}{8} \div 39$

To Divide an Integer by a Fraction

Oral Exercise

121. 1. Draw a line 4 in. long. Divide it into parts, each $\frac{1}{4}$ in. long. How many parts are there? How many fourths in 1? In 4?

2. $4 \div \frac{1}{4} = ?$

3. Draw a line 6 in. long. Divide it into parts each $\frac{1}{2}$ in. long. How many are there? How many halves in 1? In 6?

4. $6 \div \frac{1}{2} = ?$

5. Draw a line 2 in. long and divide it into parts each $\frac{1}{8}$ in. long. How many parts are there?

6. $2 \div \frac{1}{8} = ?$

7. Would there be less parts if the divisor were $\frac{3}{8}$?

8. Tell how to divide an integer by a fraction.

To divide an integer by a fraction, invert the fraction and then multiply.

Written Exercise

122. Divide:

1. 6 by $\frac{7}{8}$.

PROCESS

$6 \div \frac{7}{8} = 6 \times \frac{8}{7}$
$= \frac{48}{7}$
$= 6\frac{6}{7}$

EXPLANATION.—Invert the divisor $\frac{7}{8}$. Multiply.

2. 9 ft. $\div \frac{3}{4}$
3. \$6 $\div$ \$$\frac{3}{5}$
4. $5 \div \frac{2}{3}$
5. 8 yd. $\div \frac{3}{4}$
6. 75 mi. $\div \frac{6}{7}$
7. 98 lb. $\div \frac{3}{4}$
8. $128 \div \frac{4}{5}$
9. $200 \div \frac{3}{11}$
10. $1000 \div \frac{5}{9}$
11. $400 \div \frac{7}{8}$
12. $2000 \div \frac{8}{9}$
13. $5000 \div \frac{5}{7}$

To Divide a Fraction by a Fraction

123. Divide:

1. $\frac{1}{2}$ by $\frac{2}{3}$.

PROCESS

$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2}$
$= \frac{3}{4}$

EXPLANATION. — Invert the divisor and multiply.

2. $\frac{1}{3})\overline{\frac{1}{2}}$ 3. $\frac{1}{2})\overline{\frac{1}{3}}$ 4. $\frac{1}{4})\overline{\frac{1}{2}}$ 5. $\frac{1}{4})\overline{\frac{1}{3}}$

6. $\frac{1}{3})\overline{\frac{2}{3}}$ 7. $\frac{1}{3})\overline{\frac{2}{5}}$ 8. $\frac{1}{2})\overline{\frac{3}{4}}$ 9. $\frac{1}{5})\overline{\frac{4}{5}}$

10. $\frac{1}{3})\overline{\frac{5}{6}}$ 11. $\frac{1}{2})\overline{\frac{4}{5}}$ 12. $\frac{2}{3})\overline{\frac{2}{3}}$ 13. $\frac{1}{2})\overline{\frac{1}{4}}$

14. $7)\overline{\frac{7}{12}}$ 15. $\frac{1}{4})\overline{\frac{1}{12}}$ 16. $4)\overline{\frac{4}{5}}$ 17. $5)\overline{\frac{3}{4}}$

To divide an integer or a fraction by a fraction, invert the terms of the divisor and proceed as in multiplication of fractions.

To Divide an Integer by a Mixed Number

124. Divide:

1. 12 by $2\frac{2}{3}$.

PROCESS

$12 \div 2\frac{2}{3} = 12 \div \frac{8}{3}$
$12 \times \frac{3}{8} = \frac{9}{2} = 4\frac{1}{2}$

EXPLANATION. — Reduce the mixed number to an improper fraction. Divide.

2. 12 by $3\frac{1}{2}$ 3. 25 by $6\frac{1}{3}$ 4. 49 by $5\frac{1}{4}$

5. 69 by $30\frac{1}{5}$ 6. 84 by $42\frac{2}{3}$ 7. 76 by $12\frac{2}{3}$

8. 100 by $41\frac{2}{3}$ 9. 600 by $84\frac{3}{5}$ 10. 1000 by $96\frac{1}{2}$

To Divide a Mixed Number by a Mixed Number

125. Divide:

1. $11\frac{1}{3}$ by $8\frac{1}{2}$.

PROCESS

$$11\tfrac{1}{3} \div 8\tfrac{1}{2} = \frac{2 \times \cancel{34}^{2}}{\cancel{17} \times 3} = \frac{4}{3} = 1\tfrac{1}{3}$$

EXPLANATION. — Reduce the mixed numbers to improper fractions and divide.

2. $7\frac{2}{5}$ by $2\frac{2}{5}$
3. $4\frac{1}{2}$ by $3\frac{1}{3}$
4. $2\frac{1}{3}$ by $3\frac{3}{7}$
5. $4\frac{3}{8}$ by $5\frac{2}{3}$
6. $7\frac{3}{10}$ by $14\frac{3}{5}$
7. $12\frac{2}{3}$ by $16\frac{5}{6}$
8. $5\frac{5}{9}$ by $25\frac{1}{2}$
9. $24\frac{3}{4}$ by $10\frac{1}{2}$
10. $100\frac{1}{2}$ by $50\frac{1}{4}$

Written Problems

126. 1. How many caps can be purchased with $\$2\frac{1}{2}$, if each cap costs $\$\frac{1}{2}$?

2. If a book costs \$2, how many books can you buy with \$4? If a book costs $\$\frac{2}{5}$, how many books can you buy with \$2?

3. Mrs. Jones spent \$18 for ribbon, paying $\$\frac{3}{5}$ a yard. How many yards did she buy?

4. How many yards of lace can be bought for \$25 at $\$\frac{3}{8}$ a yard?

5. If it takes $\frac{3}{5}$ lb. of flour for each loaf of bread, how many loaves can be made from one barrel of flour weighing 195 lb.?

6. Mary uses $\frac{3}{2}$ lb. of sugar for a cake. How many cakes will 27 lb. of sugar make?

7. In a schoolroom $\frac{3}{8}$ of a box of chalk is used each school day. How many days will 9 boxes last?

8. A bootblack uses $\frac{1}{10}$ of a box of blacking for three pairs of shoes. How many pairs can he black with 3 boxes? How many boxes does he need for 60 pairs of shoes?

9. If $2\frac{1}{3}$ yd. of cloth are needed for a coat, how many coats can be made from 35 yd.? How many yards are needed for 12 coats?

10. During the month of July a laborer was idle $\frac{1}{7}$ of the time. How many days was he idle?

11. At $\$ 2\frac{1}{4}$ per volume, how many books can be bought for $\$ 18$?

12. Find the cost of 37 electric globes at $\$ \frac{1}{5}$ apiece. How many globes can be purchased for $\$ 5$?

13. A certain postage stamp is $\frac{8}{9}$ in. by $\frac{3}{4}$ in. Give its area. How many stamps of this size will it take to cover completely a page 7 in. by 6 in.?

14. If 5 bu. of wheat cost $\$ 3\frac{3}{4}$, what is the cost of 1 bu.?

15. If 6 boys earn $\$ 1\frac{1}{5}$ in 1 hr., what part of a dollar does each earn?

16. Charles has $\$ 2\frac{1}{4}$. How many railroad tickets at $\$ \frac{1}{4}$ each can he purchase?

17. What is the cost of 21 books at $\$ \frac{1}{5}$ each?

18. What is the cost of one pencil, if 6 cost 25¢?

19. If one electric globe costs $\$\frac{1}{4}$, how many dollars will 7 globes cost?

20. How many electric globes, at $\$\frac{1}{4}$ apiece, can be bought for $\$ 2\frac{3}{4}$?

21. How many pencils, at $3\frac{1}{2}¢$ apiece, can be purchased for $7¢$? For $21¢$?

22. What is the cost of 8 pencils at $3\frac{1}{2}¢$ apiece?

23. Mary buys 6 notebooks and pays $50¢$. What is the price of each?

24. At $2\frac{1}{2}¢$ apiece, what is the cost of 5 lemons?

25. How many lemons, at $2\frac{1}{2}¢$ apiece, can be bought for $20¢$?

26. How many sheets of paper, at $\frac{1}{2}¢$ a sheet, can you get for $11¢$?

27. How many sticks, $\frac{2}{3}$ yd. long, can you saw from a pole 2 yd. in length? Draw a diagram of the pole and show the points of division.

28. A peddler has $17\frac{3}{4}$ pecks of peanuts. How many times can he fill a measure that holds $\frac{1}{4}$ of a peck?

29. How many tons of coal, at $\$ 5\frac{1}{4}$ a ton, can be bought for $\$ 57.75$?

30. If it takes $20\frac{3}{4}$ yd. of canvas to make a tent, how many yards are needed for 7 tents?

31. If $\frac{3}{4}$ of a sack of flour will last a family 1 week, how many weeks will $6\frac{3}{4}$ sacks last the family?

REVIEW

Written Exercise

127. 1. Multiply each of the following by $1\frac{1}{2}$: 22, 46, 50, 48, 64, 68.

2. Divide each of the following by $\frac{2}{3}$: 76, 66, 84, 140, 126, 114.

3. Reduce to mixed numbers: $\frac{11}{2}$, $\frac{121}{7}$, $\frac{67}{8}$, $\frac{125}{4}$, $\frac{493}{11}$, $\frac{150}{12}$.

4. Change to improper fractions: $1\frac{7}{8}$, $2\frac{11}{12}$, $3\frac{1}{5}$, $13\frac{1}{5}$, $24\frac{2}{3}$, $75\frac{1}{2}$.

5. Which is larger, $\frac{131}{3}$ or $42\frac{1}{2}$?

6. Multiply each of the following by 12: $21\frac{1}{2}$, $3\frac{1}{4}$, $6\frac{1}{3}$, $10\frac{2}{3}$, $25\frac{3}{4}$, $11\frac{1}{6}$.

7. Divide each of the following by 6: $3\frac{1}{3}$, $8\frac{1}{2}$, $7\frac{1}{5}$, $21\frac{1}{3}$, $14\frac{1}{2}$, $6\frac{2}{3}$.

8. Perform the following operations: $2\frac{1}{3} \div 5\frac{1}{2}$, $6\frac{1}{2} \div 3\frac{1}{4}$, $1\frac{7}{8} \div \frac{3}{4}$, $\frac{7}{8} \div \frac{14}{4}$.

Find the cost of:

9. $2\frac{1}{3}$ lb. of cheese at 15¢ a pound.

10. $2\frac{1}{2}$ gal. of molasses at 50¢ a gallon.

11. $7\frac{1}{4}$ lb. of coffee at 40¢ a pound.

12. $4\frac{3}{4}$ yd. of ribbon at 20¢ a yard.

13. 24 shovels at $\$\frac{3}{4}$ each.

14. 5 tons of coal at $\$5\frac{1}{5}$ a ton.

15. $1\frac{1}{2}$ bu. of apples at $\$\frac{3}{4}$ a bushel.

16. $10\frac{1}{2}$ A. of land at \$200 an acre.

Tell the quantity of goods purchased:

17. $\$1\frac{1}{2}$ worth of vinegar at $\$\frac{1}{4}$ a gallon.

18. $51¢$ worth of berries at $8\frac{1}{2}¢$ a quart.

19. $85¢$ worth of milk at $8\frac{1}{2}¢$ a quart.

20. Lard at $12\frac{1}{2}¢$ a pound, and pay \$1.

21. Butter at $20\frac{1}{3}¢$ a pound, and pay $63¢$.

22. Candy at $60¢$ a pound, and pay $15¢$.

23. Tomatoes at $16\frac{2}{3}¢$ a can, and pay $50¢$.

24. Oranges at $45¢$ a dozen, and pay \$1.35.

25. Silk at $\$2\frac{1}{2}$ a yard, and pay $\$1\frac{1}{4}$.

Problems

128. 1. An envelope is $6\frac{3}{4}$ in. by $3\frac{1}{2}$ in. How much greater is the length than the width? Find the perimeter.

2. A wagon and its load of coal weigh $3\frac{4}{5}$ tons. The empty wagon weighs $\frac{3}{5}$ ton. Find the weight of the coal.

3. A ranchman sells $\frac{3}{8}$ of his corn crop and then $\frac{1}{4}$ of it. What part has he left?

4. Find the perimeter of an envelope $7\frac{1}{2}$ in. long and 5 in. wide. What is its area in square inches?

5. John buys $\frac{1}{2}$ lb. of cheese and gives $\frac{1}{3}$ of it to James. What part of a pound does James get?

6. If a boy sells $\frac{3}{4}$ and $\frac{1}{6}$ of his marbles, what part has he left?

7. Draw a figure and show that $\frac{3}{4}$ of $\frac{1}{2}$ an inch is $\frac{3}{8}$ of an inch.

8. A carpenter cuts board, $\frac{3}{4}$ ft. long, into pieces $\frac{1}{4}$ ft. long. How many pieces does it make?

9. If a boy earns $\$\frac{3}{4}$ a day, and spends $\$\frac{2}{3}$ a day, in how many days can he save $\$1$?

10. Draw a figure and show that $\frac{2}{3}$ in. divided by $\frac{1}{3}$ in. gives 2 as the answer.

11. It took Louise $\frac{3}{4}$ of an hour to embroider 2 leaves. How long did it take her to embroider 1 leaf?

12. George picked 7 qt. of berries and sold them for $\$\frac{7}{5}$. What did he get for each quart?

13. Albert earns $\$5\frac{1}{2}$ a week for 6 weeks. How much does he earn?

14. James has $1\frac{2}{3}$ lb. of candy and is allowed to eat $\frac{1}{3}$ of a pound a day. How many days will the candy last?

15. If 2 tons of coal cost $\$10\frac{1}{4}$, what will 6 tons cost?

16. What is the cost of $7\frac{1}{3}$ yd. of silk at $\$1\frac{1}{2}$ a yard?

17. How many yards, at $\$\frac{3}{4}$ each, can you buy for $\$8\frac{1}{4}$?

18. How many times larger is \$25 than \$5? \$123 than \$7?

19. What must you multiply $1\frac{1}{3}$ by to obtain $7\frac{1}{2}$?

20. If James earns $\$2\frac{1}{4}$ a day, how many days must he work to earn \$30?

21. By what must you divide $10\frac{1}{2}$ to obtain $7\frac{1}{2}$?

22. A traveler spends $\$16\frac{1}{4}$ in $6\frac{1}{4}$ days. How much does he spend each day?

23. Find the cost of $20\frac{1}{2}$ lb. of sugar at $5\frac{1}{2}¢$ a pound.

24. The length of a fourpenny nail is $1\frac{3}{8}$ in., a sixpenny nail 2 in. If 4 nails, 2 of each kind, are placed in a line, end to end, how long a line will they make?

Oral Exercise

129. 1. What does the denominator of a fraction indicate? The numerator?

2. Make a drawing and show that $\frac{1}{3}=\frac{2}{6}$.

3. Reduce to sixths:

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{2}$ $\frac{2}{2}$ $1\frac{1}{2}$ $1\frac{1}{3}$ $1\frac{2}{3}$.

4. Reduce to eighths:

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $1\frac{1}{4}$ $2\frac{3}{4}$ $3\frac{1}{2}$.

5. Reduce to ninths:

$\frac{2}{3}$ $\frac{1}{3}$ 5 $1\frac{1}{3}$ $4\frac{2}{3}$.

6. Reduce to twelfths:

$\frac{3}{4}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{2}$ $1\frac{3}{4}$ $2\frac{2}{3}$.

7. Reduce to integers or mixed numbers:

$\frac{15}{3}$ $\frac{15}{4}$ $\frac{17}{2}$ $\frac{19}{5}$ $\frac{23}{4}$ $\frac{25}{6}$ $\frac{32}{12}$,
$\frac{18}{5}$ $\frac{18}{6}$ $\frac{32}{5}$ $\frac{41}{8}$ $\frac{49}{7}$ $\frac{81}{8}$ $\frac{90}{12}$.

8. Reduce to improper fractions:

$13\frac{1}{2}$ $12\frac{1}{4}$ $15\frac{2}{5}$ $10\frac{1}{9}$ $11\frac{5}{8}$ $20\frac{2}{3}$,
$50\frac{1}{2}$ $60\frac{1}{3}$ $100\frac{1}{4}$ $11\frac{1}{6}$ $12\frac{5}{6}$ $10\frac{7}{8}$.

9. Reduce to their lowest terms:

$\frac{6}{12}$ $\frac{8}{12}$ $\frac{4}{12}$ $\frac{4}{6}$ $\frac{5}{15}$ $\frac{3}{12}$ $\frac{8}{24}$,
$\frac{5}{20}$ $\frac{6}{24}$ $\frac{4}{16}$ $\frac{3}{15}$ $\frac{2}{16}$ $\frac{7}{28}$ $\frac{6}{68}$.

Oral Exercise

130. 1. Multiply each by $\frac{1}{2}$:

$\frac{1}{2}$ $\frac{4}{5}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{4}{9}$ $1\frac{7}{2}$.

2. Find $\frac{1}{2}$ of each:

1 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{4}{5}$.

3. How much is $\frac{1}{3}$ of each:

2 $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{3}$ $\frac{4}{5}$ $1\frac{1}{2}$ $2\frac{1}{2}$?

4. Take $\frac{2}{3}$ of each:

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{3}{2}$ $\frac{5}{6}$ 4 $\frac{1}{4}$.

5. Find $\frac{1}{4}$ of each:

4 $\frac{1}{2}$ $\frac{3}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$.

6. How much is $\frac{3}{4}$ of each:

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $1\frac{1}{3}$ $2\frac{1}{3}$ $\frac{5}{6}$?

7. Find $\frac{1}{6}$ of each:

$\frac{2}{3}$ $\frac{3}{4}$ $1\frac{1}{2}$ 7 $4\frac{1}{2}$ 13 $\frac{1}{2}$.

8. Find $\frac{5}{6}$ of each:

$\frac{7}{8}$ $\frac{8}{9}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{7}{12}$ $\frac{7}{10}$ $\frac{5}{9}$.

Oral Exercise

131. 1. Divide $\frac{1}{2}$ by each of the following:

$\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{5}{6}$ $\frac{5}{12}$.

2. Divide $\frac{2}{3}$ by each of the following:

$\frac{3}{4}$ $\frac{4}{3}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{7}{6}$.

3. Divide $\frac{3}{4}$ by each of the following:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $1\frac{1}{2}$ $1\frac{1}{3}$.

4. Divide $\frac{4}{3}$ by each of the following:

$\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{8}{9}$ $\frac{6}{7}$ $\frac{12}{13}$ $\frac{11}{12}$.

Oral Problems

132. 1. A grocer sold $1\frac{1}{2}$ dozen eggs to one person and $\frac{3}{4}$ dozen to another. How many dozen eggs did he sell?

2. A boy spent $\frac{1}{10}$ of a dollar for paper and $\$\frac{3}{5}$ for an arithmetic. How much did he spend for both?

3. A man goes to market and spends $\frac{2}{3}$ of his money for goods and $\frac{1}{12}$ for dinner. What part of his money is left?

4. If a pound of tea costs $\frac{1}{2}$ a dollar, what will $1\frac{1}{2}$ pounds cost?

5. A man owned $\frac{3}{4}$ of a farm and sold $\frac{1}{3}$ of his share. What part of the farm did he sell?

6. Of $\frac{3}{4}$ of an acre of land, bordering on the Mississippi River, $\frac{1}{3}$ was washed away. What part of an acre was washed away?

7. A clerk has $1\frac{1}{2}$ months' vacation, $\frac{2}{3}$ of which is spent in Colorado. How long was he in Colorado?

8. A piece of ribbon $3\frac{1}{2}$ ft. long is divided into strips $\frac{1}{2}$ ft. long. How many strips are there?

9. How many pieces of paper, each $\frac{2}{3}$ in. long, can be cut from a paper 4 in. long?

10. Letters are mailed at the rate of 2¢ an ounce, books at the rate of $\frac{1}{2}$¢ an ounce. How much more expensive is letter postage than book postage?

11. How many pounds of tea can be purchased with \$ 6 at the rate of \$ $\frac{3}{5}$ a pound?

Written Problems

133. 1. A boy earned \$ $\frac{3}{10}$ one day, \$ $\frac{1}{2}$ the next day, and \$ $\frac{4}{5}$ the third day. How much did he earn in the 3 days?

2. A father earns \$ $4\frac{3}{4}$ a day; his son earns \$ $1\frac{2}{5}$ a day. How much more does the father earn in a day than his son?

3. A road is built $1\frac{7}{8}$ mi. along level ground, $2\frac{3}{4}$ mi. along rising ground, and $5\frac{1}{6}$ mi. along ground sloping downward. How long is the entire road?

4. During three days the sun shone $9\frac{7}{8}$, $8\frac{1}{3}$, and $7\frac{5}{6}$ hr., respectively. How many hours of sunshine were there in all?

5. A train travels $12\frac{1}{3}$ mi. faster per hour than a steamboat. How many miles farther than the boat does the train travel in 16 hr.?

6. A flower bed is $40\frac{1}{2}$ ft. long and $5\frac{1}{3}$ ft. wide. How many square feet in its area?

7. A farmer sold $235\frac{2}{5}$ lb. of maple sugar at $17\frac{3}{4}¢$ a pound. How much did he receive?

8. A room is $15\frac{1}{3}$ ft. long and $12\frac{1}{2}$ ft. wide. What is the cost of a molding extending entirely around it at $5\frac{1}{2}¢$ a foot?

9. A ship is worth \$100,000. A man owns $\frac{5}{16}$ of it. If he sells $\frac{2}{5}$ of his share, what is the value of what he still owns?

10. How many yards of cloth can be bought for \$140 at $\$1\frac{3}{4}$ a yard?

11. What is the price of hay, when $5\frac{3}{4}$ tons are worth \$69?

Written Exercise

134. Reduce to the least common denominator:

1. $\frac{1}{2}, \frac{2}{3}, \frac{3}{24}$
2. $\frac{4}{5}, \frac{2}{3}, \frac{7}{15}$
3. $\frac{1}{3}, \frac{3}{4}, \frac{5}{6}$
4. $\frac{1}{12}, \frac{7}{8}, \frac{5}{6}$
5. $\frac{3}{4}, \frac{1}{3}, \frac{7}{8}$
6. $\frac{11}{12}, \frac{3}{2}, \frac{13}{8}$

Written Exercise

135. Perform the operations indicated:

1. $\frac{3}{4} + 1\frac{1}{2} + \frac{2}{3} = ?$
2. $10 - 2\frac{5}{6} - 3\frac{7}{8} = ?$
3. $1\frac{3}{4} + 1\frac{2}{3} - \frac{5}{6} = ?$
4. $11\frac{1}{3} - \frac{2}{9} + \frac{5}{6} = ?$
5. $10\frac{1}{6} + 11\frac{3}{4} - \frac{7}{9} = ?$
6. $20\frac{2}{5} + 7\frac{5}{8} - 6\frac{3}{4} = ?$
7. $2\frac{3}{4} \times \frac{14}{11} \times \frac{5}{6}$
8. $6\frac{1}{8} \times 16 \times 2\frac{1}{7}$
9. $144 \times 8\frac{5}{6} \times \frac{1}{4}$
10. $5\frac{1}{4} \times 3\frac{1}{5} \times \frac{7}{16}$
11. $80 \times 3\frac{1}{12} \times \frac{1}{2}$
12. $\frac{1}{9} \times 16\frac{1}{5} \times 3$
13. $9\frac{3}{7} \div 4\frac{5}{7}$
14. $\frac{23}{26} \div \frac{7}{13}$
15. $27\frac{2}{3} \div 16\frac{3}{5}$
16. $\frac{31}{4} \times \frac{2}{3} \div 1\frac{1}{16}$
17. $14\frac{1}{2} \div 5\frac{4}{5}$
18. $6750\frac{2}{3} \div 33\frac{1}{3}$

136. **Drill Device — Magic Squares**

$\frac{5}{6}$	$\frac{5}{3}$	$\frac{1}{2}$
$\frac{2}{3}$	1	$\frac{4}{3}$
$\frac{3}{2}$	$\frac{1}{3}$	$\frac{7}{6}$

$\frac{1}{2}$	$\frac{11}{12}$	$\frac{1}{3}$
$\frac{5}{12}$	$\frac{7}{12}$	$\frac{3}{4}$
$\frac{5}{6}$	$\frac{1}{4}$	$\frac{2}{3}$

		$\frac{1}{2}$
$\frac{3}{4}$		$\frac{7}{4}$

1. In the first magic square, find the sum of the three numbers in each line, each column, and each diagonal.

2. Do the same in the second magic square.

3. In the third square, fill the vacant places, so that the sum of the three digits in each line, column, and diagonal will be $\frac{15}{4}$.

Suggestive Questions

137. 1. A man pays a nickel carfare twice a day. How can you find the fare he pays in a month?

2. If you know a man's salary per month, how can you find his yearly salary?

3. If you know the number of days in each month, how can you find the number of days in a year?

4. Each member of a class needs a new pencil every month. How can you find the number of months a gross of pencils will last?

5. If a telegrapher knows the number of words he can send per minute, how can he figure the time it takes him to send a given number of words?

6. John has a certain amount of money. If he knows the cost of writing tablets, how is he to find how many tablets he can buy with the whole of his money?

7. A merchant sells shoes at a price that enables him to double his money. How can we find what they cost him per dozen pairs of shoes?

8. A confectioner has a number of pounds of candy which he wishes to put up into boxes, all of the same size, each holding a given fraction of a pound. How is he to find how many boxes to order for the candy?

9. If you know how fast a train travels, how can you determine the time it takes the train to go a given distance?

10. If you know how much a plant grows in a month, how can you find the average amount of growth per day?

11. If James can read a certain number of pages per hour, how can you find the number of pages he can read in a week, when he reads a fixed number of hours each day?

12. If you know the number of stories in a building and the height of each story, how can you ascertain the height of the building?

Drill Exercise

138. 1. Find the sums of the three fractions in each line, each column, and each diagonal.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{5}$
$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{5}$
$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{10}$

2. Add $\frac{2}{3}$ to each of the nine fractions. Add $\frac{11}{12}$ to each.

3. Subtract each fraction from $\frac{7}{12}$. From 2.

4. Multiply together the pairs of fractions in the first two columns. The pairs of fractions in the last two columns.

5. Divide each fraction in the first line by the fraction below.

6. Divide each fraction in the second line by the fraction below.

7. Divide each fraction in the first column by the fraction to its right.

8. Multiply the fractions along each diagonal by $\frac{3}{4}$.

9. Subtract each fraction along the diagonals from $\frac{7}{8}$.

10. Add each fraction along the diagonals to $1\frac{1}{3}$.

QUANTITY AND COST

Written Exercise

139. Find the quantity and cost:

1. 2 bbl. of flour, each 196 lb., @ $4.65 per barrel.

2. 7 tubs of butter, each 60 lb., @ $27\frac{1}{2}¢$ per pound.

3. 6 bbl. of pork, each 285 lb., @ $8.50 per hundredweight.

4. $24\frac{1}{2}$ lb. boxes of apricots, each 12 lb., @ $9\frac{1}{2}¢$ per pound.

5. 6 boxes of dates, each 30 lb., @ $2.25 per box.

6. 75 bbl. of pork, each 200 lb., @ $5\frac{1}{2}¢$ a pound.

7. 125 bales of cotton, each 4 tons, @ $8\frac{1}{2}¢$ a pound.

8. 30 sacks of grain, each 200 lb., @ $80¢$ per hundredweight.

9. If $\frac{1}{3}$ of a box of peaches cost $45¢$, what is the cost of a whole box?

10. If $\frac{2}{3}$ of a barrel holds $66\frac{2}{3}$ gallons of pine tar, how many gallons will the barrel hold?

Written Problems

140. 1. If Robert can solve 9 problems in half an hour, how many can he solve in $1\frac{1}{2}$ hr., at the same rate?

2. How many miles can James walk in 3 hr., if he walks 7 mi. in 2 hr.?

3. John paces off the length of a fence and finds it to be 36 paces. If 3 of his steps measure 8 ft., how many feet long is the fence?

4. If 5 lb. of apples can be bought for 15¢, how many pounds can be bought for 45¢?

5. Josephine reads 50 pages of history in 3 hours or —— pages in 27 hours.

6. If two boys working together, at the same rate, can do a piece of work in $7\frac{3}{4}$ hours, in what time can one do it?

7. If a certain supply of provisions lasts 2 men 18 days, how long will it last 9 men?

8. The Atlantic liner *Lusitania* went from Queenstown to New York in 4 days and 19.9 hours, at an average speed of 24 miles an hour. How far is it between Queenstown and New York?

9. The *Mauretania*, a sister ship, attained an average speed of 25.83 miles an hour. How much farther will she travel in 10 hours than the *Lusitania?*

10. One year the skating record for 100 yd. was $11\frac{1}{5}$ sec. or —— yd. per second, or —— ft. per second.

11. The skating record for 400 yd. was $46\frac{1}{5}$ sec. or —— yd. per second or —— ft. per second.

12. The distance of 880 yards in skating was made in 1 minute, 36 seconds. How many yards per second is this?

13. In the year 1903 the United States produced 234,000 tons of cane sugar and 214,825 tons of beet sugar. In 1906 it produced 243,000 tons of cane sugar and 431,796 tons of beet sugar. By how many tons did the increase in the production of beet sugar exceed the increase in the production of cane sugar?

14. Colorado produced 2974 million pounds of beets in one year, from which 334 million pounds of sugar were manufactured. How many pounds of beets did it take to yield one pound of sugar?

15. A merchant buys 10 doz. pairs of shoes for $\$360$ and sells them at $\$4$ a pair. What is his profit?

16. A grocer buys 20 doz. eggs for $\$.25$ a dozen and sells them at $\$.35$ a dozen. How much does he gain?

17. If 6 doz. oranges cost $\$2.40$, what will 1 doz. cost? 5 doz.?

18. A dealer buys knives at $.35 and sells them at $.50. What is his profit on 100 knives?

19. Find the cost of 10 lb. of butter at $\$\frac{1}{4}$ a pound.

20. Find the cost of $5\frac{1}{3}$ lb. of butter at $\$\frac{1}{4}$ a pound.

21. What is the value of $2\frac{1}{3}$ acres of land at $18 an acre?

22. A carpenter earns $.45 in $\frac{5}{6}$ of an hour. How much does he earn in an hour?

23. Only $\frac{6}{7}$ of a class are present. If 30 are present, how large is the class?

24. If $\frac{4}{5}$ of a flock of sheep are 100 sheep, how many sheep in the entire flock?

25. $\frac{3}{4}$ of the distance between two cities is 27 mi. What is the distance?

26. The distance between two towns is 55 mi. How much is $\frac{3}{5}$ of that distance?

27. A train runs 35 miles in $\frac{5}{7}$ of an hour. What is the rate per hour?

28. James buys 10 tons of soft coal and pays $62\frac{1}{2}$. What is the cost per ton? (Use business fractions.)

29. What is the cost of 16 notebooks at $.12\frac{1}{2}$ apiece? (Use business fractions.)

30. Find the cost of 24 doz. eggs at $.37\frac{1}{2}$ a dozen.

31. How much must you pay for 40 yd. of cloth at $\$.37\frac{1}{2}$ a yard?

32. A merchant buys 1000 yd. of lining at $\$.03$ a yard and sells at $\$.05$ a yard. Find the profit.

33. A real estate man buys a lot for $\$1,200$. He divides it up into 3 small lots and sells each lot for $\$600$. How much does he make?

34. Another real estate man buys a lot for $\$2,500$, pays $\$100$ taxes, and then sells it for $\$2,750$. How much does he gain?

35. At $\$.08$ a pound for rice and $\$.05$ a pound for sugar, how much will $10\frac{1}{2}$ lb. of each cost?

36. A woman buys 3 boxes of starch for $\$.50$ and a bag of flour for $\$.75$. She gives the clerk a $\$5$ bill. How much change does she receive?

37. What is the cost of 15 hair brooms at $\$3.25$ each and 3 mops at $\$5.60$ per doz.?

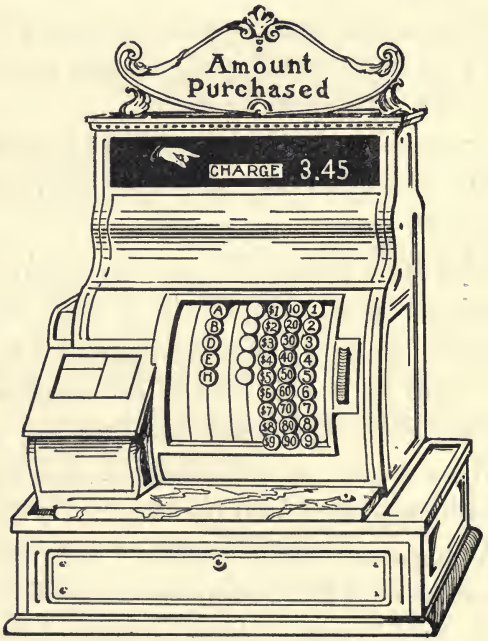
38. A business house orders 7 quartered-oak flat-top desks at $\$19.85$ each and 7 revolving arm-chairs at $\$9.33$ each. What is the total cost of this order?

39. A student buys 3 books at $\$1.25$ each and a student's notebook for 37 cents. He gave in payment a $\$10$ bill. How much change should he receive?

40. Find the cost of $4\frac{1}{2}$ yards of calico at 6 cents a yard; $5\frac{1}{3}$ yards of gingham at 24 cents a yard.

Oral Exercise — Cash Register

141. This picture shows a cash register. If a purchaser pays for goods amounting to $3.45, the



A Cash Register

clerk presses the keys marked $3, 40¢, and 5¢. This prints the amount of the purchase on a slip of paper within, pushes up cards, showing the purchaser the sum paid, and opens the cash drawer.

Suppose the purchaser has given the clerk a $5 bill. The clerk puts the money into the drawer.

He may take out the following change: a 5-cent piece, a 50-cent piece, and one dollar. He hands the change to the customer, in the same order, and says, "Three fifty, four, five dollars."

1. What keys must be pressed to indicate the payment of each of the following sums: $1.50, $1.46, $2.35, $4.75, $3.05, $2.14?

2. What amount of money is shown by the card register when the following keys are pressed:

Dollars	Cents	Cents	Dollars	Cents	Cents
1	20	5	1	60	7
2	30	6	2	90	8
3	80	5	4	10	5

3. If a purchase is $1.65 and the purchaser gives the clerk $2, what change should he receive?

4. State what keys the clerk must press, what coins he may take out of the drawer, and what he will say to the purchaser in each of the following cases:

Amount of Purchase	Money Given in Payment	Amount of Purchase	Money Given in Payment
$1.75	$2.00	$3.85	$4.00
$2.55	$3.00	$1.97	$2.00
$2.95	$5.00	$4.05	$5.00

5. Let some of the pupils of the class act as clerks, while other pupils make supposed purchases. Which pupils were able to act as clerks without making mistakes?

Written Problems

142. PRICES OF HOUSEHOLD GOODS

Tea Kettle	$ .49	Range	$ 42.50
Bread Box	.55	Sideboard	15.65
Water Bottle	.23	Refrigerator	12.75
Broom	.15	Kitchen Table	3.50
Teapot	.53	Dining-room Chair	2.75
Towels, per doz.	2.15	Kitchen Chair	1.25
Oilcloth, per yd.	.30	Dining Table	24.65
Linoleum, per yd.	1.20	China Closet	19.75

1. Mary bought a tea kettle, a water bottle, two brooms, a dozen towels. She pays her bill with a $ 5 bill. How much change does she receive?

2. Mr. James ordered a china closet, a dining table, a kitchen table, and a range. How much was his bill?

3. Mrs. Hathaway has $ 45.60 in the bank. How much does she lack in order to purchase 10 yd. of linoleum, 2 doz. towels, a dining-room chair, a sideboard, and a china closet?

4. Mother sends Martha with a $ 10 bill to buy 2 yd. of oilcloth, a kitchen table, a teapot, a bread box, and a kitchen chair. How much change should she bring home?

5. Mr. Newman purchases a sideboard, a refrigerator, a dining table, and a bread box. He has $ 99.15 in the bank, but wishes to keep at least $ 50 on deposit in the bank. How much can he pay on account?

6. Purchase any six articles you like and find the cost.

7. A man earns $100 a month. His monthly expenses are $45.60. Is the balance sufficient to pay for a range, a kitchen table, two dozen towels, a teakettle, and a bread box?

8. Mrs. Grant purchases a sideboard, 10 yd. of linoleum, 3 brooms, a kitchen table, 2 kitchen chairs, a refrigerator. She pays $30 on account. How much did she have charged?

9. Cash sales were $125.35 on Monday, $130 on Tuesday, $175.25 on Wednesday, $43.40 on Thursday, $150.10 on Friday, and $247.65 on Saturday. How much were the sales for the week?

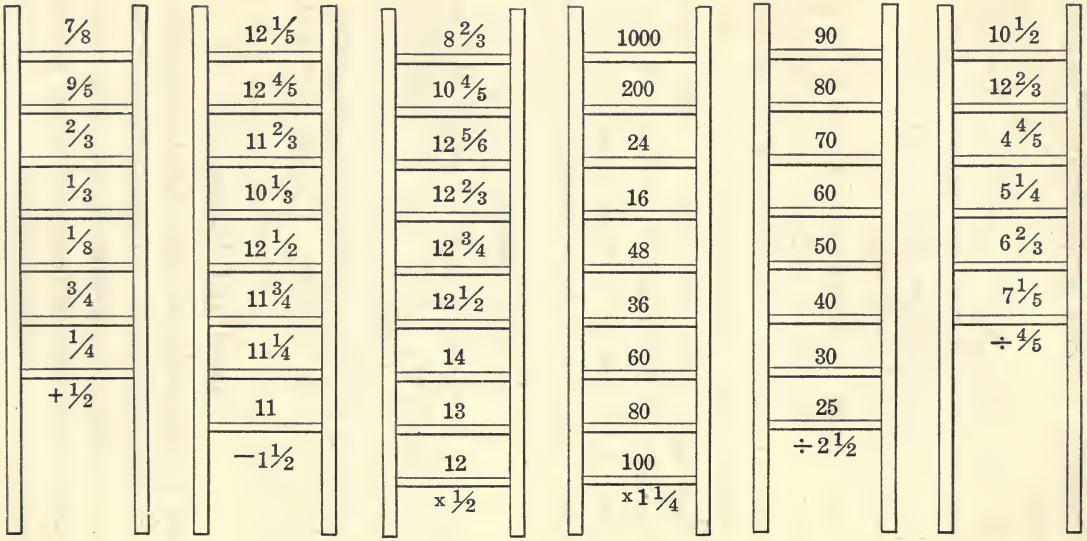
Drill Device

143. At the completion of the study of fractions use the drill device. Continue the drill daily for short periods of rapid accurate work. Vary the numbers to be added and subtracted and to be used as multipliers and divisors. Change the numbers on the drill.

Speed and Accuracy

Speed and accuracy should be developed in the fundamental operations. Practice should be continued on drills of this kind, in fractions, decimals, and percentage until a speed of about 60 correct answers a minute can be obtained.

DRILL DEVICES



DECIMALS

Review

144. Study the three ways of writing decimal fractions.

1. One tenth	$\frac{1}{10}$	.1
2. One hundredth	$\frac{1}{100}$	.01
3. One thousandth	$\frac{1}{1000}$	.001
4. Five tenths	$\frac{5}{10}$	.5
5. Six hundredths	$\frac{6}{100}$	.06
6. Seven thousandths	$\frac{7}{1000}$	.007
7. Twenty-five hundredths	$\frac{25}{100}$	.25
8. Seven hundred fifty-six thousandths	$\frac{756}{1000}$	.756

A fraction whose denominator is 10, 100, 1000, or 1 with any number of ciphers annexed, is called a **decimal** or a **decimal fraction.**

A decimal fraction may be written in three ways:

In words, as, one tenth, six hundredths.

As a common fraction, as, $\frac{5}{10}$, $\frac{25}{100}$.

By using the decimal point, as, .1, .06, 756.

Decimal fractions or decimals are usually written with a decimal point.

Written Exercise

145. 1. Make a chart like the illustration.

Orders of Whole Numbers									Orders of Decimals		
Millions			Thousands			Units or Ones			Thousandths		
Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units or Ones	Tenths	Hundredths	Thousandths
3d Period			2d Period			1st Period			1st Period		

2. Count the orders of integers shown on the chart.

3. Count the orders of decimals.

4. Name the orders of each.

5. Write on the chart the following numbers:

Four and five tenths.

Seven and six tenths.

Five and thirty-six hundredths.

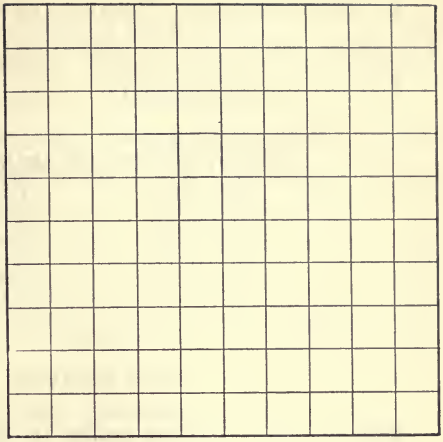
Six and seven hundredths.

Ten and one hundred twenty-five thousandths.

In these numbers *and* indicates the position of the decimal point.

Study Exercise

146. 1. Into how many small squares is the large square divided? One of the smaller squares is $\frac{?}{100}$ of the large square. Write this as a decimal fraction.



2. How many of these smaller squares in $\frac{1}{2}$ of the large square?

Then, $\frac{1}{2} = \frac{?}{100} = .50 = .5$.

3. In $\frac{1}{4}$ of the large square there are —— small squares.

Then, $\frac{1}{4} = \frac{?}{100} = .25$ of the large square.

4. Shade the smaller squares which together stand for $\frac{7}{100}$ or .07. For $\frac{10}{100}$ or .10 or .1. For $\frac{13}{100}$ or .13.

5. The entire large square $= \frac{?}{2} = \frac{?}{4} = \frac{?}{5} = \frac{?}{100}$.

6. Tell from the drawing $\frac{10}{100} = \frac{?}{10}$, $\frac{20}{100} = \frac{?}{10}$, $\frac{30}{100} = \frac{?}{10}$.

7. $.75 = \frac{?}{4}$, $.50 = \frac{?}{100} = \frac{?}{4} = \frac{?}{2}$, $\frac{20}{100} = \frac{?}{5}$, $\frac{40}{100} = \frac{?}{5}$, $\frac{60}{100} = \frac{?}{5}$, $\frac{80}{100} = \frac{?}{5}$.

8. Tell from the illustration what decimal fractions are equal to the following common fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{50}$.

FRACTIONS TO BE MEMORIZED

$\frac{1}{2} = .5$	$\frac{1}{5} = .2$
$\frac{1}{4} = .25$	$\frac{2}{5} = .4$
$\frac{3}{4} = .75$	$\frac{1}{10} = .1$

Oral Exercise

147. 1. How many cents in \$.5? In \$.2? In \$.4? In \$.1?

2. How much more is \$.5 than \$.25?

3. How much more is \$.75 than \$.5?

In writing money \$.5 is written \$.50.

4. Mary buys candy worth \$.35. She gives \$$\frac{1}{2}$ to the clerk. How much change does she receive?

5. A matchbox contains 100 matches. One match is $\frac{?}{100}$ of the entire number.

6. Frank takes $\frac{1}{2}$ of all the matches, or —— matches.

7. Had he taken only $\frac{1}{4}$ of all the matches, he would have taken —— matches. $\frac{1}{4}$ is equal to what decimal fraction?

8. Write as a decimal fraction the part of all the matches you would have taken, if you took 9 matches. 11 matches. 37 matches.

9. Albert had 100 old five-cent stamps in an envelope. He loses .25 of all he had. How many does he lose? What part of all still remains?

10. Henry has 100 tin soldiers. He gives away $\frac{1}{2}$ of them to George and $\frac{1}{4}$ of them to James. How many has he left? What part of the whole is this?

Oral Exercise

148. Read 97.58.

There are two ways of reading 97.58.

(1) Ninety-seven and fifty-eight hundredths.

(2) Ninety-seven, point, five, eight.

The second way is generally preferred because it is shorter.

Read 1.4, 100.75, 45.05, 3.275, 005.

Read 0.014, 7.09, 18.71, 19.1, 1.004.

Write in the form of decimal fractions:

$\frac{3}{10}$, $\frac{3}{100}$, $\frac{5}{100}$, $1\frac{1}{10}$, $4\frac{7}{10}$, $4\frac{7}{100}$, $5\frac{55}{100}$, $6\frac{875}{1000}$.

Write in the form of common fractions:

.3, .13, .05, .75, 4.65, 3.03, .575, 005, 1.006, 7.01, 8.10, 10.101.

Which is larger

9.7 or 9.07?	9.86 or 9.68?
5.8 or 8.5?	3.6 or 3.60?
4.8 or 5.8?	4.320 or 4.32?

Addition of Decimals

149. Copy, add, and check:

1.	2.	3.
$ 3.75	$ 735.15	$ 1095.64
16.85	176.45	46.75
195.43	714.36	9.49
236.78	895.85	9864.25
900.00	100.25	897.68
736.45	236.70	1098.45
$	$	$

In adding money, the decimal points are put in a column so as not to confuse dollars and cents. Precisely the same thing is done in adding *all* decimal fractions. The decimal points are kept in a column. Do not forget to write the decimal point in the sum.

4.	5.	6.
101.13	973.1	796.125
709.09	79.175	15.471
1000.15	100.65	210.005
793.75	452.456	75.6
434.14	75.5	400.06
11.25	176.47	750.005
125.17	55.125	66.543

Observe the groups of 10 when such groups occur.

7. Copy the numbers in each column. Add vertically and horizontally:

12.34	+ 21.22	+ 916.	+ 567.8	=
56.70	+ 232.4	+ 928.3	+ 5.432	=
88.90	+ 25.26	+ 89.76	+ 4.678	=
21.26	+ 272.8	+ 9.476	+ 48.65	=
	+	+	+	=

8.

12.14	+ 2.905	+ 99.15	+ 364.	=
13.73	+ 3.062	+ 88.25	+ 98.9	=
16.27	+ 31.35	+ 77.5	+ 897.5	=
17.85	+ 33.35	+ 66.25	+ 6.432	=
18.05	+ 353.6	+ 5.125	+ 78.23	=
	+	+	+	=

9.

8.479	+ 698.9	+ 795.3	+ 478.79	=
584.7	+ 89.67	+ 93.57	+ 7879 6	=
49.76	+ 875.4	+ 6.932	+ 43.715	=
75.42	+ 196.8	+ 58.75	+ 791.45	=
66.53	+ 789.15	+ 4.676	+ 89.176	=
78.90	+ 67.817	+ 798.	+ 458.9	=
634.5	+ 819.43	+ .95	+ 45.61	=
4.571	+ 278.9	+ 638.7	+ 79.05	=
	+	+	+	=

10. Copy and add the answers to 7, 8, and 9.

11. Separate Ex. 9 into two parts. Add and find the sum of the answers. Compare with the answer to 9.

Subtraction of Decimals

150. Copy and subtract:

1. $\$10.75 - 9.28$
2. $11.25 \text{ lb.} - 7.45 \text{ lb.}$
3. $119.45 \text{ yd.} - 73.15 \text{ yd.}$
4. $120.6 \text{ mi.} - 14.75 \text{ mi.}$

Since 6 *tenths* are equal to 60 *hundredths*, we may write 120.60 in place of 120.6. Then, $120.60 - 14.75 = 105.85$.

5. $\$419.1 - \$296.7$.
6. $\$786.55 - \$654.6$.
7. $786.4 \text{ T.} - 6.98.7 \text{ T.}$
8. $1000 \text{ lb.} - 736.45 \text{ lb.}$
9. Subtract 1.795 from 12.
10. 2.364 from 5.7.

Process	Process
$12.000 = 12.$	$5.700 = 5.7$
$1.795 = 1.795$	$2.364 = 2.364$
$10.205 = 10.205$	$3.336 = 3.336$

How do you check subtraction?

11. Subtract 69.7895 from 79.123.

Multiplication of Decimals

Oral Exercise

151. 1. If a boy earns \$1.25 a day, he earns \$12.5 or \$12.50 in 10 days, and \$125 in 100 days.

In multiplying by 10, move the decimal point

one place to the right. How do we multiply by 100? By 1000?

2. Multiply the following by 10. Also by 100:

$7.35, 7.75 T., 95.5 lb., 73.47 ft., .1 in., 75 bu., $2.335, 791.9, 78.732 mi., .05 oz.

3. Multiply 35.2 by 200. Multiply by 2. Then multiply the product by 100.

4. Multiply each by 10. Multiply each by 20:

1.11	3.4	4.33	3.23	10.2	10.24
10.15	2.05	2.44	2.25	.25	.04

5. Multiply each by 100. Multiply each by 300:

$ 1.11	2.22 lb.	30.3 ft.	3.2 yd.	2.2 doz.
10.5	1.05	3.23	2.21 lb.	.25

6. Multiply each by 10. Multiply each by 40:

$1.21	2.11	11.2 T.	10.2 yd.	4.1 pk.
3.02	1.22	1.05	.25	.5

Oral Problems

152. 1. If the wages of each man in a factory employing 100 men are $2.25, what is the amount of the payroll for one day?

2. If a train goes .75 mile a minute, how far will it go in 100 minutes? In 200 minutes?

3. A boy walks 112.5 yards per minute. How many yards in 10 minutes?

To Multiply a Decimal by an Integer or an Integer by a Decimal

153. 1. Multiply .75 by 9.

PROCESS

.75
9
6.75

EXPLANATION. — If we were multiplying 75 by 9, the product would be 675.

But we are multiplying $\frac{75}{100}$ by 9. Hence the product is $\frac{675}{100}$, or 6.75.

2. Multiply 64 by .8.

PROCESS

64
.8
51.2

EXPLANATION. — If we were multiplying 64 by 8, the product would be 512.

But we are multiplying 64 by $\frac{8}{10}$. Hence the product is $\frac{512}{10}$, or 51.2.

TO MULTIPLY A DECIMAL BY A WHOLE NUMBER

I. Multiply as in whole numbers.

II. Point off as many decimal places in the product as there are decimal places in the multiplier.

Written Exercise

154. Multiply:

1. .15 by 8
2. .25 by 9
3. .09 by 15
4. 3 by $1.15
5. 13 by $2.25
6. 11 by .95
7. 12 by 10.5
8. 21.75 ft. by 29
9. 2579 lb. by .87
10. 458 by .009
11. .043 by 798
12. 457987 by .0009

Written Exercise

155. Multiply each number in columns *B*, *C*, *D*, and *E*, by the corresponding number in column *A*.

	A	*B*	*C*	*D*	*E*
1.	19	.62	1.05	10.36	6.45
2.	109	.705	8.25	19.75	14.95
3.	28	.901	9.28	109.6	121.5
4.	307	.025	7.01	203.5	906.7
5.	412	.909	8.75	100.9	976.5
6.	505	.097	1.89	23.45	8.756
7.	620	.202	3.333	89.01	45.67
8.	136	1.012	.963	75.75	44.44
9.	809	8.97	7.09	45.45	67.5
10.	879	.009	.008	.007	.006

To Multiply a Decimal by a Decimal

156. **1.** Multiply .64 by .8.

PROCESS AND EXPLANATION

If we were multiplying 64 by 8, the product would be 512.

But we are multiplying $\frac{64}{100}$ by $\frac{8}{10}$.

Hence the product is $\frac{512}{1000}$, or .512.

2. Multiply 4.28 by 1.15.

PROCESS AND EXPLANATION

If we were multiplying 428 by 115, the product would be 49,220.

But we are multiplying $\frac{428}{100}$ by $\frac{115}{100}$.
The product is $\frac{49220}{10000}$, or 4.9220, or 4.922.

TO MULTIPLY A DECIMAL BY A DECIMAL

I. Multiply as in whole numbers.

II. Point off as many decimal places in the product as there are decimal places in both factors.

Oral Exercise

157. Answer at sight:

1. $.9 \times .2$	2. $.8 \times .3$	3. $.6 \times .4$
4. $.5 \times .5$	5. $.4 \times .7$	6. $.3 \times 8$
7. $.2 \times .8$	8. $10 \times .8$	9. $10 \times .3$
10. $.4 \times 10$	11. $1.1 \times .2$	12. $1.2 \times .3$
13. $.4 \times 2.1$	14. $.5 \times 1.2$	15. 3×3.1
16. $.3 \times 3.1$	17. $.5 \times 1.1$	18. $5 \times .11$
19. $.11 \times .1$	20. $.17 \times 2$	21. .2 of .4
22. .4 of .8	23. 1.1 of .9	24. 7 of .12
25. 12×12	26. 1.2×12	27. 1.2×1.2
28. $1.2 \times .12$	29. 1.1×12	30. 11×1.2
31. $1.1 \times .12$	32. $.11 \times .12$	33. 13×11
34. 13×1.1.	35. $13 \times .11$	36. $1.3 \times .11$

Written Exercise

158. 1. Find the product of 195 and 32. Also of 19.5 and 3.2.

2. Which of the three products is the largest: 1.2×1.8, 12×18, $.12 \times 1.8$?

Multiply:

3. 2.5×3.2	4. 6.7×7.5	5. 77.7×6.6
6. 6.3×8.5	7. 9.8×1.2	8. $37.5 \times .05$
9. 16.5×12.9	10. 3.5×2.06	11. $1.07 \times .35$
12. $.01 \times .5$	13. $19.1 \times .01$	14. $.05 \times .96$
15. .7 of 35.6	16. $1.25 \times .7$	17. $.1 \times 7.5$
18. 7×6.8	19. .9 of 10.7	20. $12.5 \times .85$
21. .2 of 7.3	22. 7×6.8	23. .6 of 11.3
24. $9.17 \times .65$	25. .3 of 9.4	26. 91×87
27. .05 of 9.8	28. $8.06 \times .54$	29. .03 of 8.5
30. 9.1×8.7	31. .15 of 125	32. $0.25 \times .25$
33. .33 of 6.1	34. $.64 \times .75$	35. 92.34×81.4
36. 80.43×76.4	37. 764.1×17.9	38. 63.01×70.6
39. 19.84×1.25	40. 670.9×15.2	41. 99.9×8.88

Written Problems

159. 1. The wheat yield of a field of 19.6 acres is at the rate of 27.5 bushels per acre. What is its value at \$ 1 a bushel?

2. If 1 ft. of lead piping weighs 1.82 lb., what is the weight of a piece of piping 22.5 ft. long?

3. If an aëroplane travels at the rate of 67.5 mi. an hour, how far will it travel in 3.4 hr.?

4. At 25.3 mi. per hour, how far can you ride in 4.5 hr.?

5. A carpenter earns $4.25 a day. How many days will he have to work in order to earn $180?

6. A farm consists of 120 acres. It is valued at $7500.50. What is the value per acre?

7. To find the diameter of a circle, divide the length of the circle by 3.1416. What is the diameter of a circle, if its length is 25 ft.?

8. What is the diameter of a circle, if its length is 50 ft.?

Division of Decimals

To Divide Integers and Decimals by Integers

160. 1. 20 divided by 5 may be written $5\overline{)20}$, $\frac{20}{5}$, or $20 \div 5$.

2. Since $1.56 \times 10 = 15.6$, we have $10\overline{)15.6} = ?$

3. Since $1.565 \times 100 = 156.5$, we have $100\overline{)156.5} = ?$

TO DIVIDE A NUMBER BY 10

Move the decimal point in the dividend one place to the left.

In multiplying by 10, the decimal point is moved one place to the right.

In dividing by 10, the decimal point is moved one place to the left.

4. Tell how to divide a number by 100. By 1000.

Written Exercise

161. 1. Divide 222.4 by 200.

Divide 222.4 by 100, and the result by 2.

$$100\overline{)222.4} = 2.224, \quad 2\overline{)2.224}\ 1.112$$

2. $10\overline{)35.4}$ $100\overline{)35.4}$ $10\overline{)3.7}$ $100\overline{)3.7}$

3. $100\overline{)5.6}$ $20\overline{)4.4}$ $20\overline{)44}$ $20\overline{).44}$

4. $100\overline{)115.6}$ $100\overline{)22.2}$ $200\overline{)22.2}$ $200\overline{)222}$

5. $10\overline{)55.5}$ $50\overline{)55.5}$ $30\overline{)36.3}$ $40\overline{)4.8}$

6. $90\overline{)3600}$ $400\overline{)3200}$ $300\overline{)7200}$ $30\overline{)60.6}$

Written Exercise

162. 1. Divide 427.2 by 24.

PROCESS

$$\begin{array}{r} 17.8 \\ \times \\ 24\overline{)427.2} \\ 24 \\ \hline 187 \\ 168 \\ \hline 192 \\ 192 \\ \hline \end{array}$$

EXPLANATION. — Divide as in whole numbers. 24 into 42 tens goes 1 ten and a remainder. Write the 1 above the dividend in the tens' column and so on. Place the decimal point in the dividend and quotient under each other.

2. $13\overline{)1599}$	3. $13\overline{)159.9}$	4. $23\overline{)538.2}$
5. $21\overline{)51.03}$	6. $53\overline{)424}$	7. $58\overline{)5.22}$
8. $62\overline{)43.4}$	9. $64\overline{)38.4}$	10. $127\overline{)88.9}$
11. $131\overline{)10.48}$	12. $139\overline{)834}$	13. $151\overline{)120.8}$
14. $17\overline{)212.5}$	15. $23\overline{)632.5}$	16. $97\overline{)145.5}$
17. $83\overline{)207.5}$	18. $39\overline{)48.75}$	19. $76\overline{)538.84}$
20. $17\overline{).85}$	21. $16\overline{)1.28}$	22. $19\overline{).57}$
23. $19\overline{).285}$	24. $16\overline{).272}$	25. $14\overline{).182}$

Written Problems

163. 1. A dozen handkerchiefs sell for $2.88. What is the price of one handkerchief?

2. Nine boys hire rowboats and pay $2.25. What is each one's share of the expense?

3. On a football trip the expense of 13 men was $14.95. How much did each pay?

4. If the fare of 35 pupils on an excursion is $15.75, what is the fare of one?

5. A town in Colorado had 226.3 hr. of sunshine during a month of 31 da. What was the daily average?

6. A potato patch contains 67.5 sq. yd. It is in the form of a rectangle 9 ft. long. How wide is it?

7. A white ash, 29.9 in. thick, was 115 yr. old. Find the average yearly growth.

8. A tree grows to a height of 77.9 ft. in 41 yr. What was its average gain in height per year?

9. A train travels 213.6 miles in 6 hours. How many miles is this an hour?

10. In a long-distance race one man ran 24 miles in 2 hours, 57.6 minutes. He ran 1 mile in —— minutes.

To Divide a Decimal by a Decimal

164. 1. $\frac{6}{3} = ?$ $\frac{60}{30} = ?$ Compare the quotients.

2. $\frac{9}{3} = ?$ $\frac{18}{6} = ?$ $\frac{27}{9} = ?$ $\frac{36}{12} = ?$

3. $\frac{2.5}{.5} = \frac{?}{5}$ $\frac{3.6}{.6} = \frac{?}{6}$ $\frac{1.25}{.05} = \frac{?}{5}$

Instead of $\frac{2.5}{.5}$ we may take $\frac{25}{5}$, which is easier to divide.

4. $\frac{2.5}{.5} = ?$ $\frac{3.6}{.6} = ?$ $\frac{4.5}{.5} = ?$ $\frac{1.25}{.05} = ?$

5. $1.5 \div .5$ $.25\overline{).75}$ $.9\overline{)4.5}$ $.9\overline{)6.3}$

6. $.6\overline{)4.2}$ $.7\overline{)5.6}$ $.2\overline{)1.4}$ $.7\overline{)2.1}$

7. $\frac{.45}{.5} = ?$ $.8\overline{)7.2}$ $.9\overline{)4.5}$ $.6\overline{)7.2}$

PRINCIPLE TO BE REMEMBERED

Multiplying the dividend and divisor by the same number does not change the quotient.

Written Exercise

165. Answer the following:

1. $.5\overline{)9.5}$	2. $1.2\overline{)7.2}$	3. $1.1\overline{)1.21}$
4. $.11\overline{)1.21}$	5. $.9\overline{).81}$	6. $.09\overline{).81}$
7. $.8\overline{)6.4}$	8. $.08\overline{).64}$	9. $.12\overline{)1.44}$
10. $1.2\overline{)1.44}$	11. $12\overline{)1.44}$	12. $12\overline{)14.4}$
13. $25\overline{)75}$	14. $2.5\overline{)7.5}$	15. $2.5\overline{)75}$
16. $2.5\overline{).75}$	17. $.12\overline{)1.32}$	18. $1.3\overline{)910}$

Written Exercise

166. Divide:

1. 1.872 by .13.

PROCESS

$$
\begin{array}{r}
14.4 \\
\times \\
13\overline{)187.2} \\
13 \\
\hline
57 \\
52 \\
\hline
52 \\
52 \\
\hline
\end{array}
$$

EXPLANATION. — The divisor becomes an integer, if we multiply by 100. We have $.13\overline{)1.872}$ equal to $13\overline{)187.2}$.

2. 77 by 2.5.

PROCESS

$$
\begin{array}{r}
30.8 \\
\times \\
\hline
25)\overline{770.0} \\
75 \\
\hline
200 \\
200 \\
\hline
\end{array}
$$

Check:

$$
\begin{array}{r}
30.8 \\
2.5 \\
\hline
1540 \\
616 \\
\hline
77.00
\end{array}
$$

EXPLANATION. — $770 \div 25$ gives the quotient 30 and the remainder 20. If the division is carried one step farther, in order to secure greater accuracy, then write the dividend 770.0 instead of 770. The next digit in the quotient is 8. There is no remainder. The exact quotient is 30.8. Check by multiplying the quotient by the divisor. What should the product be?

3. 224 by 2.4
4. 1718.64 by .62
5. $775 by $.25
6. 74.256 by .34
7. $8955 by 4.5
8. 10548.72 by .39
9. 1718.64 by 9.3
10. 150.696 by 2.8
11. 6715.1756 by .085
12. 566.351 by 6.7

Written Problems

167. 1. What number, multiplied by 1.2, gives the product 14.4?

2. Multiplying a certain number by 1.3 yields the product 11.7. What is the number?

3. The product is $10.35, the multiplier is 2.3; what is the multiplicand?

4. If a man earns $26.25 in 7.5 days, how much does he earn in one day?

5. John earns $3.60 a week. His father earns $18 a week. How many times greater than the son's are the father's wages?

Written Exercise

168. Divide, carrying the quotient to 3 decimal places.

1. 9.2 by 1.3.

PROCESS

$$
\begin{array}{r}
7.076 \\
\times \\
\hline
13)\overline{92.000} \\
91 \\
\hline
100 \\
91 \\
\hline
90 \\
78 \\
\hline
12
\end{array}
$$

EXPLANATION. — When we divide 92 by 13, there will always be a remainder, however far we carry the division. When it is not necessary to know the fractions of a cent, we write the answer, $7.07+.

The plus sign shows that the quotient 7.07 is not exact and that the true answer is a little larger.

2. $73 \div 2.9$
3. $97 \div .41$
4. $107 \div 5.3$
5. $4.55 \div 9.7$
6. $2.55 \div 1.07$
7. $.75 \div 8.9$
8. $\$1.8 \div 6.4$
9. 7.95 in. $\div 12$
10. $272.5 \div 2.72$
11. 7.63 lb. $\div 17$

Written Problems

169. 1. The area of a drawing board is 483.5 sq. in., its length is 23.1 in. Compute its width to the tenth part of an inch.

2. A place in Arizona had 305.4 hr. of sunshine during January. What is the daily average?

3. The salary of the President of the United States is $75,000. What is his salary for a day in a year of 365 days?

4. A man travels 1,896 miles in his automobile during the month of July. How many miles a day does he average?

5. During March, a few years ago, 406.1 tons of dynamite were used for blasting the Panama Canal. On an average, how many tons of dynamite were used a day? How many pounds of dynamite were used a day?

To Change a Decimal to a Common Fraction

170. 1. Change .625 to a common fraction in its simplest terms.

PROCESS

$.625 = \frac{625}{1000}$

$\frac{125}{200} = \frac{25}{40} = \frac{5}{8}$ *Ans.*

EXPLANATION.—Write in the form of a common fraction. Cancel. What common factors were canceled?

2. .8	3. 25	4. .75	5. .125	6. .64
7. .05	8. .2	9. .08	10. .35	11. .28
12. .95	13. .96	14. .55	15. .175	16. .16
17. .48	18. .225	19. .175	20. .125	21. .725

To Change a Common Fraction to a Decimal

171. 1. Change $\frac{3}{4}$ to a decimal.

PROCESS

$$\frac{3}{4} = 4\overline{)3.00}\quad .75$$

EXPLANATION

$\frac{3}{4}$ means $3 \div 4 = .75$ *Ans.*

2. Change $\frac{8}{25}$ to a decimal.

PROCESS

$$25\overline{)8.00}\quad .32$$

EXPLANATION

$\frac{8}{25} = 8 \div 25 = 8.00 \div 25 = .32$ *Ans.*

3. Change $\frac{1}{3}$ to a decimal.

PROCESS

$$3\overline{)1.00}\quad .33+$$

EXPLANATION. —In changing $\frac{1}{3}$ to a decimal, we find that the division does not come out exact. $\frac{1}{3}$ cannot be exactly expressed as a decimal. In such a case carry the answer to the second or third decimal place by adding as many ciphers to the dividend as there are to be decimal places in the answer. Write plus after the last figure in the quotient to show the omission of the rest.

Written Exercise

172. Reduce to decimals:

1. $\frac{7}{8}$	**2.** $\frac{7}{4}$	**3.** $\frac{15}{8}$	**4.** $\frac{12}{16}$	**5.** $\frac{15}{12}$	**6.** $\frac{19}{14}$
7. $\frac{13}{40}$	**8.** $\frac{19}{20}$	**9.** $\frac{11}{25}$	**10.** $\frac{125}{50}$	**11.** $\frac{11}{10}$	**12.** $\frac{97}{4}$
13. $\frac{2}{3}$	**14.** $\frac{5}{6}$	**15.** $\frac{4}{7}$	**16.** $\frac{5}{9}$	**17.** $\frac{7}{11}$	**18.** $\frac{18}{13}$

In examples **13–18** carry the answer out to 3 decimal places.

Written Exercise

173. Reduce to mixed numbers:

1. $\frac{75}{8}$	**2.** $\frac{24}{5}$	**3.** $\frac{413}{20}$	**4.** $\frac{783}{12}$
5. $\frac{96}{21}$	**6.** $\frac{87}{19}$	**7.** $\frac{79}{27}$	**8.** $\frac{98}{46}$
9. $\frac{875}{17}$	**10.** $\frac{379}{28}$	**11.** $\frac{100}{99}$	**12.** $\frac{81}{80}$

174. Reduce to a decimal, carrying the answer out four places:

1. $\frac{427}{41}$.

PROCESS

$$
\begin{array}{r}
10.4146^{+} \\
\times \\
41\overline{)427.0000} \\
41 \\
\hline
170 \\
164 \\
\hline
60 \\
41 \\
\hline
190 \\
164 \\
\hline
260 \\
246 \\
\hline
\end{array}
$$

EXPLANATION. — Add ciphers in the dividend to make four decimal places. Indicate the first partial dividend.

Divide:

Indicate the incomplete answer by the $^{+}$ sign.

2. $\frac{573}{59}$	**3.** $\frac{648}{47}$	**4.** $\frac{829}{81}$	**5.** $\frac{956}{79}$
6. $\frac{448}{98}$	**7.** $\frac{875}{89}$	**8.** $\frac{492}{42}$	**9.** $\frac{119}{27}$
10. $\frac{216}{489}$	**11.** $\frac{307}{501}$	**12.** $\frac{2001}{877}$	**13.** $\frac{5091}{654}$

REVIEW

Decimal Fractions

175. 1. Read: .27, 3.01, 4.025, .007, .726.

2. In .32, the unit is divided into —— equal parts, and —— of these are taken.

3. How can we tell the denominator of a fraction when it is written in the decimal form?

4. Arrange in the ascending order of value: 5.05, 5.51, 5.5, 5.005.

Oral Exercise

176. Add at sight:

1. 3.2 4.5	2. 7.8 2.4	3. 3.7 4.8	4. 10.3 20.8
5. 10.7 15.01	6. .07 .14	7. .03 .145	8. 6.44 1.4
9. 2.07 .155	10. 7.01 2.10	11. 9.09 3.46	12. 7.993 1.007

Oral Exercise

177. Subtract at sight:

1. 7.5 2.5	2. 7.4 2.5	3. 10.6 4.7	4. 7.5 1.05

5.	8.7	6.	9.7	7.	.100	8.	.200
	8.65		7.9		.001		.005
9.	1.9	10.	3.75	11.	7.111	12.	5.000
	.05		1.7		3.01		.001

Oral Exercise

178. 1. Change to common fractions and simplify: .4, .8, 1.2, .25, .75, .5, .55

2. Change to decimal fractions:

$\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$

3. Express in dollars and cents:

$\$1\frac{1}{4}$, $\$1\frac{1}{3}$, $\$4\frac{1}{2}$, $\$1\frac{2}{5}$, $\$2\frac{3}{4}$, $\$3\frac{2}{3}$

Compare and determine which is the larger:

$7\frac{1}{4}$ and 7.2	8.5 and $8\frac{3}{5}$
7.4 and 7.40	4.4 and $4\frac{1}{5}$
3.3 and $3\frac{1}{3}$	$2\frac{2}{3}$ and 2.6

Drill Exercise

179. 1. Add the numbers in each column.

2. Subtract the lesser numbers from the greater in each column.

3. Find the product of the two numbers in each exercise.

4. Divide the first number by the second.

	A	*B*	*C*	*D*
1.	$\frac{2}{3}$, $\frac{3}{4}$	$\frac{1}{10}$, $\frac{2}{5}$	$\frac{1}{2}$, $\frac{1}{5}$	1.1, 2.2
2.	$\frac{3}{4}$, .1	.5, .25	$\frac{1}{4}$, $\frac{2}{3}$	$\frac{4}{5}$, .5

	A	*B*	*C*	*D*
3.	$1\frac{1}{2}$, $1\frac{1}{2}$	$2\frac{1}{2}$, 1	$2\frac{1}{2}$, $\frac{1}{2}$	$2\frac{1}{2}$, $\frac{1}{3}$
4.	2, .4	.3, 3	$1\frac{1}{3}$, $\frac{2}{3}$	$\frac{2}{3}$, $\frac{5}{6}$
5.	$\frac{3}{4}$, $\frac{4}{5}$	$\frac{4}{5}$, $\frac{1}{6}$	$\frac{1}{100}$, .5	.25, $\frac{1}{10}$
6.	$10\frac{1}{2}$, 2	$9\frac{1}{2}$, $\frac{1}{3}$	$3\frac{1}{2}$, 7	$3\frac{1}{3}$, .7
7.	$3\frac{1}{3}$, .1	$5\frac{1}{2}$, $\frac{1}{10}$	$5\frac{1}{2}$, 11	$\frac{1}{24}$, $\frac{1}{12}$
8.	$\frac{3}{24}$, $\frac{1}{8}$	$\frac{4}{12}$, $\frac{2}{3}$	$\frac{4}{8}$, $2\frac{1}{3}$	$\frac{6}{5}$, $\frac{1}{6}$
9.	$\frac{1}{5}$, $\frac{4}{5}$	$\frac{2}{5}$, $\frac{6}{5}$	$\frac{4}{3}$, $\frac{1}{6}$	$\frac{3}{4}$, .25
10.	$.33\frac{1}{3}$, $.66\frac{2}{3}$	.75, .25	.5, $12\frac{1}{2}$	$.12\frac{1}{2}$, .25
11.	$.16\frac{2}{3}$, $\frac{5}{6}$	$\frac{4}{5}$, .5	$\frac{2}{3}$, $.66\frac{2}{3}$	.75, $\frac{1}{2}$
12.	$\frac{7}{8}$, $\frac{1}{4}$	.99, 2	.49, 1	1, $\frac{3}{4}$
13.	.5, .6	4.5, 2	8.6, 2	1.2, 3
14.	24, .2	36, $\frac{1}{10}$	3.6, $\frac{1}{100}$	4.7, .1
15.	$2\frac{1}{3}$, $3\frac{1}{3}$	21, $\frac{2}{5}$	1, .01	.9, $\frac{9}{10}$
16.	.11, .11	.12, .12	.01, .05	1.3, 13
17.	$2\frac{1}{3}$, $1\frac{1}{3}$	$\frac{1}{2}$, $\frac{2}{3}$	$\frac{1}{3}$, $\frac{3}{4}$	$\frac{1}{4}$, $\frac{4}{5}$
18.	$\frac{1}{5}$, $\frac{5}{6}$	1, $.33\frac{1}{3}$	200, 300	250, 350
19.	$2\frac{1}{4}$, $\frac{1}{2}$	$\frac{1}{4}$, $1\frac{1}{4}$	$\frac{1}{10}$, $\frac{99}{10}$	$\frac{1}{5}$, $\frac{19}{5}$
20.	$3\frac{1}{20}$, $\frac{2}{3}$	1.1, .1	2.2, .2	3.3, .3

Oral Exercise

180. Find the product:

1. $.7 \times .6$	2. 12×1.2	3. $7 \times .111$
4. 8×1.2	5. 1.2×1.2	6. $.8 \times .12$
7. $.1 \times .1$	8. 1.2×12	9. 8×1.2
10. $.25 \times 5$	11. $.15 \times 5$	12. $.05 \times 5$

Oral Exercise

181. Find the quotient:

1. $.74 \div 10$	2. $1.4 \div 10$	3. $14 \div 10$
4. $62 \div 100$	5. $6.2 \div 100$	6. $620 \div 100$
7. $14 \div .1$	8. $1.5 \div .1$	9. $2.03 \div .1$
10. $24 \div .2$	11. $2.4 \div .2$	12. $.240 \div .2$

Written Exercise

182. Perform the operations indicated. Check the results.

1. $17 \times 18.5 = ?$
2. $6.4 \times 7.5 = ?$
3. $175 \times 1.5 = ?$
4. $12.5 \times .25 = ?$
5. $17.28 \div 144$
6. $95.2 \div 3.4$
7. $28.7 \div 109$
8. $67.67 \div 67$
9. $28.71 \div 9.9$
10. $151.3 \div 39$
11. $144 \div 7$
12. $194 \div 9$
13. $226 \div 5$
14. $742 \div 11$
15. $7.23 \div .4$
16. $17.69 \div 7$
17. $45.6 \div .13$
18. $18.78 \div .17$
19. How much is $\frac{1}{8}$ of 123?
20. How much is $\frac{1}{8}$ of 560?
21. How much is $\frac{1}{8}$ of 1780?
22. Find $\frac{1}{6}$ of 75
23. Of 39.9
24. Of 947
25. Of 9003
26. How much is 81.7 less $\frac{1}{12}$ of 900?
27. From $\frac{1}{10}$ of 964 take $\frac{1}{5}$ of 435.
28. The quotient is 9.36, the divisor is 7.3. Find the dividend.

29. Find the divisor when the dividend is 21.42 and the quotient 6.3.

30. Find the quotient to 3 decimal places, when the dividend is .074 and the divisor is .3.

Written Problems

183. 1. If a man saves $32 a month, how long will it take him to save $432?

2. If a car conductor earns $2.25 a day, how long will it take him to earn $776.25?

3. If 9.75 tons of coal cost $47.26, what is the price of 1 ton?

4. What is the cost of 12 bales of cotton at $.12 a pound, if each bale weighs 410.8 pounds?

5. Two motor cars start from the same place and travel in opposite directions, one at 15.3 miles an hour, the other at 18.4 miles an hour. How far apart are they at the end of 6.3 hours?

6. The daily wages of each employee in a factory were increased $.18. The daily total amount paid in wages was thereby increased $84.06. Find the number of employees.

7. A torpedo boat has a speed of 38.4 knots an hour. What is its speed in miles, if 1 knot is 1.15 miles?

8. If 9.8 inches of snow when melted make 1 inch of water, how much snow is necessary to make .55 inch of water?

9. If a merchant buys 125 suits of clothes at $13.25 each and sells them at $22.50 each, what is his profit?

10. Find the product of the sum and the difference of 43.25 and 13.76.

184.

FRACTIONS FREQUENTLY USED IN BUSINESS		
$10¢ = \$\frac{1}{10}$	$25¢ = \$\frac{1}{4}$	$12\frac{1}{2}¢ = \$\frac{1}{8}$
$50¢ = \$\frac{1}{2}$	$75¢ = \$\frac{3}{4}$	$16\frac{2}{3}¢ = \$\frac{1}{6}$
$20¢ = \$\frac{1}{5}$	$40¢ = \$\frac{2}{5}$	$33\frac{1}{3}¢ = \$\frac{1}{3}$

Oral Exercise

185. Find the cost. Use fractions.

Quantity	Rate	Quantity	Rate	Quantity	Rate
1. 32 lb.	$12\frac{1}{2}¢$	**6.** 40 oz.	20¢	**11.** 36 doz.	$33\frac{1}{3}¢$
2. 18 lb.	$16\frac{2}{3}¢$	**7.** 80 bu.	25¢	**12.** 96 yd.	$12\frac{1}{2}¢$
3. 16 lb.	75¢	**8.** 32 bu.	$1.25	**13.** 60 bbl.	75¢
4. 24 lb.	$12\frac{1}{2}¢$	**9.** 24 ft.	$12\frac{1}{2}¢$	**14.** 46 yd.	$1.50
5. 15 yd.	$66\frac{2}{3}¢$	**10.** 66 yd.	$1.66\frac{2}{3}$	**15.** 32 bu.	$1.12\frac{1}{2}$

Written Problems

186. 1. How long will it take a person to earn $63.25, if he earns $2.75 a day?

2. A train runs at the rate of 36.5 mi. per hour. How long will it take to run 277.4 mi.?

3. The area of a floor is 396.8 sq. ft.; its length is 25.6 ft. Find its width.

4. How long will it take a man to walk 8035 mi., if he walks 3.75 per hour?

5. A factory employs 305 men who work 8 hr. per day. If the daily payroll is $512.40, what is the average wage paid per hour?

6. A farmer paid three men $72 to shock his corn, wages being $1.50 a day. How many days did each man work?

7. A steel rail, 30 ft. long, weighs 72 lb. per yard. How many men are needed to carry it, if each man carries 120 lb.?

8. A city lot containing 950 sq. ft., and 190 ft. deep, sells for $1560. What was the price per foot of frontage?

9. How many states the size of Delaware, 2050 sq. mi., could be made from Texas, 265,780 sq. mi.?

10. A man takes 6 acres of city land, at $560 per acre, in exchange for 55 acres of farm land. What is the value of the farm land per acre?

Drill Table

187. Multiply:

1. $\frac{5}{9} \times \frac{12}{25}$
2. $\frac{18}{15} \times \frac{25}{3}$
3. $\frac{24}{25} \times \frac{5}{8}$
4. $\frac{15}{24} \times \frac{36}{45}$
5. $2\frac{1}{2} \times 3$
6. $3\frac{1}{3} \times 5$

7. $5\frac{5}{8} \times 7$
8. $5\frac{4}{5} \times 9$
9. $3\frac{3}{5} \times 12\frac{1}{2}$
10. $4\frac{4}{7} \times 16\frac{1}{3}$
11. $2\frac{2}{5} \times 6\frac{1}{4}$
12. $5\frac{5}{8} \times 2\frac{2}{3}$
13. 32.5×7
14. 45.62×29
15. 4.55×789
16. 14.15×875
17. $16.5 \times .081$
18. $3.7 \times .079$
19. $.087 \times .097$
20. $.095 \times .008$
21. $6.663 \times .63$
22. $2.34 \times .96$
23. $.83\frac{1}{2} \times .9\frac{1}{4}$
24. $1.5 \times .33\frac{1}{3}$

Divide:

25. $\frac{5}{8} \div \frac{1}{4}$
26. $\frac{7}{8} \div \frac{3}{4}$
27. $\frac{24}{25} \div \frac{8}{15}$
28. $\frac{4}{5} \div \frac{2}{3}$
29. $\frac{9}{10} \div \frac{2}{3}$
30. $10 \div 2\frac{2}{5}$
31. $20 \div 3\frac{1}{4}$
32. $36 \div \frac{9}{20}$
33. $35 \div 3\frac{1}{5}$
34. $3\frac{3}{5} \div 9$
35. $4\frac{4}{5} \div 20$
36. $6\frac{2}{3} \div 12$
37. $12\frac{2}{3} \div 12\frac{1}{2}$
38. $9\frac{3}{8} \div 6\frac{1}{4}$
39. $4\frac{4}{5} \div 5\frac{1}{4}$
40. $35.5 \div .75$
41. $6.3 \div .25$
42. $.33\frac{1}{3} \div .12\frac{1}{2}$
43. $.66\frac{2}{3} \div .16\frac{1}{6}$
44. $.75 \div .87\frac{1}{2}$
45. $.625 \div .25$
46. $42.15 \div .625$
47. $.625 \div .87\frac{1}{2}$
48. $.897 \div .789$
49. $3.75 \div .0375$
50. $.0062 \div .0012$

DENOMINATE NUMBERS

Study Exercise

188. All numbers are either **abstract** or **concrete.**

A **concrete number** is one that refers to particular objects, as, 25 sheep, 46 feet, 72 bushels.

An **abstract number** is one that does not refer to particular objects, as, 7, 8, 13, 16.

A **denominate number** is a particular kind of concrete number expressing measure of size or weight, as, 7 feet, 4 pounds.

Thus, 25 *houses* is a number that is concrete but not denominate.

24 *days* is a number that is both concrete and denominate.

A denominate number of one denomination is called a simple denominate number. If it has two or more denominations it is called a compound denominate number. 2 ft. is a simple denominate number. 2 ft. 4 in. is a compound denominate number.

Oral Exercise

189. 1. Name six abstract numbers.

2. Name six concrete numbers.

3. Name a concrete number that is not denominate. Name one that is denominate.

Tables

190. Memorize

LINEAR MEASURE

12 inches (in. or ″)	= 1 foot (ft. or ′)
3 feet	= 1 yard (yd.)
16.5 feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
1760 yards	= 1 mile

TIME

60 minutes (min.)	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days or 12 months (mo.)	= 1 year

DRY MEASURE

2 pints	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
32 quarts	= 1 bushel

WEIGHT

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)

Reduction of Denominate Numbers

191. 1. Reduce $9\frac{1}{3}$ yd. to inches.

PROCESS

$9\frac{1}{3} \times 3 = 28$ ft.

$28 \times 12 = 336$ in.

EXPLANATION

Since 1 yd. = 3 ft.

$9\frac{1}{3}$ yd. = $9\frac{1}{3}$ times 3 ft.

= 28 ft.

And since 1 ft. = 12 in.

28 ft. = 28 × 12 in.

= 336 in.

2. Reduce 234 in. to yards.

Process		Explanation
$19\frac{1}{2}$ ft.		Since 12 in. = 1 ft.
12)234 in.		234 in. = 234 ÷ 12 ft.
12		= $19\frac{1}{2}$ ft.
114	$6\frac{1}{2}$ yd.	Since 3 ft. = 1 yd.
108	3)$19\frac{1}{2}$ ft.	$19\frac{1}{2}$ ft. = $19\frac{1}{2}$ ÷ 3 yd.
6		= $6\frac{1}{2}$ yd.

3. How many inches in 17 ft.? In $\frac{3}{4}$ ft.? In $1\frac{2}{3}$ ft.?

4. Change to a fraction of a foot: 9 in., 6 in., 2 in., 8 in., 10 in.

5. How many feet in 73 yd.? In $4\frac{1}{3}$ yd.? In $\frac{3}{4}$ yd.?

6. Change to rods: 121 yd., 11 yd., 66 yd.

Oral Exercise

192. Reduce to inches:

1. 2 ft., $2\frac{1}{2}$ ft., 2 ft. 3 in., 3 ft. 2 in.
2. $2\frac{1}{3}$ ft., $4\frac{1}{3}$ ft., $3\frac{1}{4}$ ft., $10\frac{1}{12}$ ft.
3. $3\frac{2}{3}$ ft., $2\frac{3}{4}$ ft., $4\frac{5}{6}$ ft., $\frac{7}{12}$ ft.
4. $5\frac{1}{6}$ ft., $1\frac{2}{6}$ ft., $7\frac{3}{6}$ ft., $6\frac{4}{6}$ ft.
5. $4\frac{2}{12}$ ft., $2\frac{13}{12}$ ft., $3\frac{4}{12}$ ft., $4\frac{5}{12}$ ft.
6. $5\frac{6}{12}$ ft., $6\frac{7}{12}$ ft., $\frac{8}{12}$ ft., $4\frac{11}{12}$ ft.
7. Change to ounces: 4 lb., 2 lb., 10 oz, $2\frac{1}{2}$ lb., $3\frac{1}{4}$ lb., $1\frac{3}{4}$ lb.

8. How many hours in: 2 da.? $2\frac{1}{2}$ da.? $1\frac{3}{4}$ da.? $2\frac{1}{6}$ da.? $\frac{5}{6}$ da.?

9. Reduce to minutes: $1\frac{1}{2}$ hr., $2\frac{1}{10}$ hr., $3\frac{1}{6}$ hr., $\frac{1}{92}$ hr., $\frac{4}{15}$ hr.

10. Reduce to seconds: $\frac{7}{15}$ min., $\frac{8}{15}$ min., $\frac{9}{15}$ min., $\frac{11}{15}$ min.

11. How many days in $2\frac{1}{7}$ wk.? in $3\frac{2}{7}$ wk.? in $3\frac{4}{7}$ wk.?

12. In reducing from larger or higher units to smaller or lower units, do you multiply or divide?

13. How many bushels or parts of bushels in 8 pk.? in 10 pk.? in 12 pk.? in 14 pk.?

14. Change to pecks or parts of pecks: 8 qt., 16 qt., 12 qt., 20 qt.

15. Reduce to quarts: 8 pt., 9 pt., 10 pt., 11 pt.

16. In reducing from smaller or lower units to larger or higher units, do you multiply or divide?

Written Exercise

193. 1. A grocer buys apples at \$1.25 a bushel and sells them at 50¢ a peck. Find his profit on one bushel.

2. A bushel of peanuts costing \$1.35 is sold at 5¢ a pint. What is the gain?

3. A coal dealer sells coal by the sack (weighing 100 lb.) at the rate of 25¢ each. What is his profit per ton, if he buys the coal at \$3 a ton?

4. A man works 8 hr. daily; at this rate how many hours does he work in 3 weeks (omitting Sundays)?

5. A man's business office is 420 rd. from his home. If he walks to his office and back once every day, how many miles does he walk in 16 days?

6. If pears cost $1.35 a bushel and are sold at 60¢ a peck, what is the profit on 2 bu.?

7. A dealer sells peanuts at 3¢ a bag. If two bags hold 3 pints, how much does he get in selling 1 pk. of peanuts?

8. How much cheaper is it to buy a pound of candy at 30¢ a pound than at the rate of 10¢ for 4 ounces?

Tables

194. Memorize.

SQUARE MEASURE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet (sq. ft.) = 1 square yard (sq. yd.)
640 acres (A.) = 1 square mile (sq. m.)

LIQUID MEASURE

2 pints (pt.) = 1 quart (qt.)
4 quarts (qt.) = 1 gallon (gal.)

COUNTING

12 units = 1 dozen (doz.)
12 dozen = 1 gross

Study Exercise

195. 1. Draw a diagram and explain why it is that, while 3 ft. = 1 yd., it takes 9 sq. ft. to make 1 sq. yd.

2. In the same way, explain why it takes 144 sq. in. to make 1 sq. ft.

3. Pace the distance of a little over 200 ft. ($208\ \frac{7}{10}$ ft.). Then imagine a square that long and wide. This square covers *one acre.* How many feet of fence are needed to inclose this square?

Oral Exercise

196. 1. How many square inches in 2 square feet?

2. How many square feet in 13 square yards?

3. How many acres in 100 square miles?

4. How many quarts in 12 gallons, liquid measure?

5. How many units in one gross?

6. How many square yards in 27 square feet?

7. How many square miles in 6400 acres?

8. How many dozen in 84 units?

Study Exercise

197. Reduce 435 in. to feet and inches (avoiding fractions).

PROCESS

$$\begin{array}{r} 36 \text{ ft.} \\ 12\overline{)435 \text{ in.}} \\ 36 \\ \hline 75 \\ 72 \\ \hline 3 \text{ in.} \end{array}$$

EXPLANATION. — The answer is 36 ft. 3 in. Notice that the remainder 3 is inches.

The dividend and remainder are always the same kind of measure.

Written Exercise

198. 1. Reduce 1179 sq. ft. to sq. yd.

2. Reduce 1290 sq. ft. to sq. yd. and sq. ft. (avoiding fractions).

3. If 5280 ft. make a mile, how many square feet make a square mile?

4. Reduce 37 gallons to pints.

5. A small farm contains 160 acres. What part of a square mile is this? How many such farms can there be on a square mile?

6. A town lot is 20 yd. wide and 180 ft. deep. How many yards of fence will inclose it? What is its area in square yards?

7. A farmer bought 60 acres of timber land in South Texas at $46.75 an acre. Then he paid $12.25 an acre for clearing the land. How much was his total outlay?

8. On 39 acres he plants rice and the yield is worth $45 an acre; 10 acres planted with corn yield 40 bushels per acre, worth 78¢ a bushel; 1 acre yields strawberries that sell for $150. Find the total value of the crops.

9. Multiply $5\frac{1}{2}$ by $5\frac{1}{2}$. Then explain why it is that $30\frac{1}{4}$ sq. yd. make 1 sq. yd.

10. Reduce $20\frac{1}{2}$ bu. to quarts. To pecks.

11. How many quarts in 7 bu. 3 pk. and 7 qt.? The liquid quart is less than the dry quart. 4

liquid quarts make 1 gal., but it takes 8 dry quarts to make 1 pk.

12. If 31.5 gal. make a barrel, and 2 barrels a hogshead, how many gallons are there in $13\frac{1}{2}$ hogsheads?

13. How many bushels in 564 pecks? In 164 dry quarts?

14. A merchant buys blackberries at $12\frac{1}{2}$ cents a quart and sells them at \$.23 a quart. What is his profit on 68 quarts?

15. A milkman buys 8 gallons of milk at \$.15 a gallon and sells it at $\$.07\frac{1}{4}$ a quart. Find his profit.

16. How many pounds and ounces are there in 450 oz.?

17. How many hundredweight are there in $7\frac{1}{2}$ tons? How many pounds?

18. Which is more, 45 hundredweight or $2\frac{1}{5}$ tons? How much more?

19. What is the difference in weight between a gross ton and the ordinary ton?

20. A man sells $7\frac{1}{2}$ tons of coal at \$ 3.25 a ton. Find the selling price.

21. Change 26 lb. 3 oz. to ounces.

22. Change 8000 lb. to hundredweight, also to tons.

23. How many pounds in 5 T. $13\frac{1}{2}$ cwt.?

Written Problems

199. Study the daily army ration.

Daily army rations to a United States soldier in garrison comprise the following articles, measured in *ounces:*

Fresh beef	20	Prunes	1.28
Flour	18	Coffee	1 12
Baking powder	.08	Sugar	3.2
Beans	2.4	Evaporated milk	.5
Potatoes	20	Salt	.64
Black pepper	.04	Lard	.64
Cinnamon	.014	Butter	.5

Flavoring extract .014

1. How many soldiers can be rationed for one day on 96 oz. salt?

PROCESS

$96 \div .64 = 150$ *Ans.*

EXPLANATION. — .64 oz. salt supply one soldier for one day. 96 oz. salt supply as many soldiers for one day as .64 is contained in 96.

2. How many soldiers can be rationed for one day from each amount named below?

48 oz. salt	7 oz. cinnamon
12 oz. black pepper	6 lb. lard
10-lb. keg of butter	3 lb. flavoring extract
1 cwt. of sugar	70 lb. coffee
100 lb. potatoes	64 lb. prunes
96 lb. beans	16 lb. baking powder
180 lb. flour	500 lb. beef

3. How many days can 10 soldiers be rationed from each amount named below?

11 lb. sugar	8.4 oz. baking powder
$183\frac{3}{4}$ lb. flour	252 lb. 8 oz. beef
$13\frac{3}{4}$ lb. beans	$65\frac{5}{8}$ lb. potatoes
77.3 lb. prunes	76.4 lb. coffee
46 lb. 15 oz. milk	120 lb. 13.6 oz. salt
$13\frac{3}{4}$ lb. black pepper	23.96 oz. cinnamon
6.9 lb. lard	212 lb. 3 oz. butter

PRACTICAL EXERCISES AND PROBLEMS

Written Problems

200. 1. One year there passed through the canals at the "Soo" about 90 million bushels of wheat and 60 million bushels of other grain. How many million bushels of grain were shipped through the "Soo"?

2. How much more wheat is carried than other grain?

3. Duluth and Superior have together 27 grain elevators with a joint capacity of 35 million bushels. Buffalo has 28 elevators with a joint capacity of 23 million bushels. How much smaller is the average capacity of a Buffalo elevator?

4. If it takes $4\frac{3}{4}$ bu. of grain for one barrel of flour, how many million barrels of flour are obtained from 150 million bushels of grain? Does the answer exceed $31\frac{1}{2}$ million barrels?

5. Besides the grain there passed through the "Soo" locks about 71 million barrels of flour. Add this to the $31\frac{1}{2}$ million barrels.

6. If one barrel of flour yields 250 one-pound loaves of bread, how many million loaves can be made from 39 million barrels of flour?

7. Allowing each individual 1 loaf a day, how many days would this supply New York City, which has a population of $3\frac{1}{2}$ million?

8. How many common years and days is this?

9. Allowing each individual 1 loaf a day, how many days would this amount of bread supply the 90 million inhabitants of the United States?

10. How many months and days would this be, counting 30 days to a month?

11. 16 million tons of coal were transported in one year on Lake steamers. If this coal were sold in equal amounts to each individual in a city of 500,000 inhabitants, how many tons would each person get?

12. If a vessel carries a cargo of 10,000 T. of coal, how many vessels would be required to carry 16 million T.?

13. If such a cargo of coal can be loaded on a ship in 12 hours, how many tons are loaded per hour?

14. Find the cost of transporting 8 million T. of coal on the Lakes from the East to Duluth, a dis-

tance of about 1000 mi., at $.35 a ton. What is the cost of transportation per mile?

15. On an average, one large freight vessel passes from Lake Huron through the Detroit River to Lake Erie every 12 minutes, day and night, during the 8 months of navigation. How many vessels pass in 8 months (of 30 days each)?

16. One year there were 571 steel ships carrying freight on the Great Lakes. What was the average tonnage, if the tonnage of all ships was 2 million?

Written Problems

201. 1. Study and compare the height of tall buildings in New York City.

Building	Height in Feet	No. of Stories
Trinity Church	234	
Flatiron	286	20
Pulitzer	375.5	22
Park Row	382	26
Times	419	28
Singer	617	42
Metropolitan	700	46
Woolworth	750	51
New Equitable (plan)	909	62

2. How much higher will the New Equitable building be than the Trinity Church, Flatiron, and Pulitzer taken together? Than the Park Row and Times combined?

3. Compute to a tenth of a foot the average height of a story in each building. Which has the highest stories?

4. How many times higher will the New Equitable be than Trinity Church? Than the Times building?

5. The flagpole of the New Equitable will extend 150 feet above the top of the building. How far above the street will the flag float?

6. The Eiffel tower in Paris is 75 feet higher than the New Equitable will be. How much higher than the Eiffel tower will the flag on the New Equitable be?

7. The New Equitable will have 8 passenger elevators running all the way to the top. What is the combined length of the 8 elevator shafts?

8. If the New Equitable elevators travel 600 feet a minute, how long will it take an elevator to travel, without stops, from the ground floor to the top?

9. The Metropolitan is built on ground 75 ft. square; that is, 75 ft. long and 75 ft. wide. The Singer on ground 65 ft. square. By how many square feet does the former ground exceed the latter?

10. The Metropolitan Tower has a clock with a dial $25\frac{1}{2}$ feet in diameter. The circumference of a circle is about 3.14 times the length of the diameter.

How far is it around this dial? How many feet must the extremity of the gigantic minute hand move every minute?

Gas Meter

202. A gas meter has three dials. By reading these dials we can find the number of cubic feet of

MARCH 4, 1914

APRIL 3, 1914

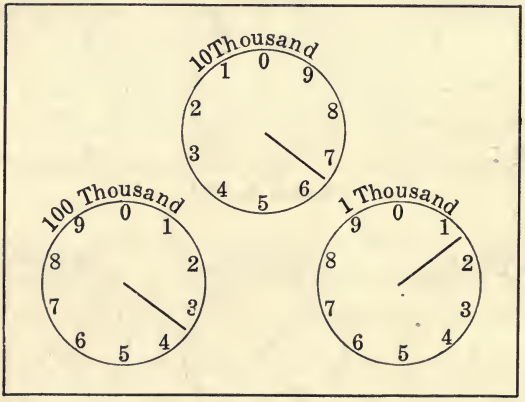
gas used. The figures on the dial at the right denote hundreds of cubic feet, the figures of the middle dial denote the thousands of cubic feet; the figures on the dial at the left denote ten thousands of cubic feet.

While the hand of the right dial makes one revolution, the hand of the middle dial moves through one division; while the hand of the middle dial

makes one revolution, the hand of the left dial moves through one division.

The dials are read from *left* to *right* by taking the figures which the hands have just passed.

Thus, the meter in the picture above gives the figures 3, 6, 1. This means 30,000 cu. ft. plus 6,000 cu. ft. plus 100 cu. ft. = 36,100 cu. ft.



Notice that it is necessary only to write 361 and annex two zeros.

1. The figure shows the meter in Mr. Jackson's house on March 4, 1914, and on April 3, 1914. Read the meter for each date.

2. How many cubic feet of gas were consumed from March 4 to April 3?

What was the cost of the gas at $1 per 1000 cu. ft.?

3. At $1 per 1000 cu. ft., what is the cost of 4,320 cu. ft.? Of 800 cu. ft.? Of 76,800 cu. ft.?

4. The following is a gas bill sent to Mr. A. Hart:

To Consolidated Gas Company of New York, Dr.

Branch Office, 132 EAST 15th STREET, near Irving Place

TELEPHONE 4901 STUYVESANT

Mr. A. Hart
25 Western Avenue
New York City

For Gas Supplied from Jan. 1, to Feb. 1, 1915 Arrears.....191		$	
Present State of Meter	76,800		
Previous State of Meter	74,900		
This charge is made in conformity with the opinion and decree of the Supreme Court of the United States rendered in the suit brought on May 1, 1906, by the Consolidated Gas Company of New York in the United States Circuit Court.	1,900 cubic feet of Gas at 80c per M	1	52

Received payment.....for the Company.

Make a bill for Example 1 similar to the one shown here.

5. If an ordinary gas burner consumes 6 cu. ft. of gas per hour and a Welsbach burner consumes 4 cu. ft. per hour, how many cubic feet will both consume in $17\frac{1}{2}$ hr.?

6. If gas costs $1 per 1000 cu. ft., how much is saved in a month of 30 da. by using the Welsbach burner?

7. At various times read the gas meter in your home or the home of a friend, and make out bills for gas consumed, using the rate actually charged.

203. Time Tables

CONDENSED TIME, CHICAGO TO OMAHA

Read Down				*Read Up*	
Limited Daily	Express Daily	Mi.	Stations Rock Island Lines	Limited Daily	Express Daily
9:15 A.M.	10:32 P.M.	0	Lv....Chicago...Ar.	4:59 P.M.	7:25 A.M.
10:21 A.M.	11:44 P.M.	40	.....Joliet.....	z	6:20 A.M.
12:10 P.M.	1:37 A.M.	114	.....Bureau.....	2:10 P.M.	4:25 A.M.
2:00 P.M.	2:25 A.M.	183	.....Davenport...	12:25 P.M.	2:35 A.M.
8:45 P.M.	8:40 A.M.	358	Ar. Des Moines Lv.	7:35 A.M.	9:42 P.M.
8:50 P.M.	8:45 A.M.	358	Lv. " " Ar.	7:20 A.M.	9:30 P.M.
11:05 P.M.	1:10 P.M.	503	Ar. ...Omaha... Lv.	3:00 A.M.	4:40 P.M.

z Trains stop to let off passengers from Colorado and Points west.

1. What are the terminal points given in this table?

2. What other cities are given?

3. How many trains are given in this table? Which run west? Which run eastward?

4. Name the time the former leave Chicago; also the time the latter leave Omaha.

5. How long a stop do the trains make in Des Moines?

6. How far is it from Des Moines to Omaha? From Joliet to Davenport?

7. In what time does the westbound express run from Bureau to Davenport? From Des Moines to Omaha?

8. In what time does the morning train run from Omaha to Davenport?

9. What is the fare from Chicago to Des Moines at 2¢ a mile? At 3¢ a mile?

10. Can a passenger from Colorado, passing through Omaha in the morning, get off at Joliet?

11. In what time does each of the four trains travel between Chicago and Des Moines? Between Des Moines and Omaha?

12. What is the distance of each city from Chicago?

Areas

204. 1. Review square measure, Art. 194.

2. A plane figure is a part of a plane bounded by straight or curved lines.

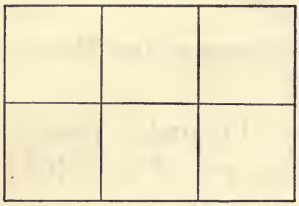
3. Construct with ruler a plane figure bounded by four straight lines. How many sides has the figure? How many corners or angles has it?

4. A plane figure bounded by four straight lines is called a **quadrilateral**.

5. Construct a quadrilateral that has square corners, or right angles. What is this figure called?

6. A **rectangle** is a quadrilateral whose angles are right angles.

7. In the figure how many squares in each row? How many rows are there? If each square is one square inch, how many square inches in the area of the figure?



8. How is the area of any rectangle found?

9. **The area of a rectangle** is the base times the altitude.

10. A **triangle** is a plane figure bounded by 3 straight lines.

11. The line on which a rectangle or a triangle stands is called its **base.**

12. How does the shaded triangle A compare in area with the rectangle A?

13. How does the shaded part in B (triangle B) compare in area with the rectangle B?

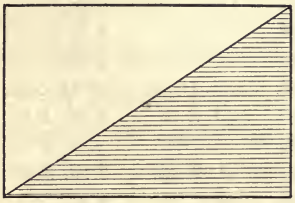
14. In A the rectangle and the triangle have the same base and the same height. Is this true of B? A triangle is exactly half the rectangle of the same base and height.

15. **The area of the triangle** is the base times the altitude, divided by 2.

16. If rectangle A is 10 sq. in., what is the area of triangle A?

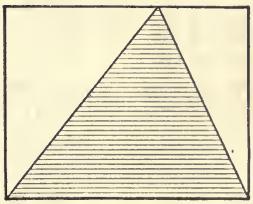
17. If rectangle B measures 16 sq. in., what is the area of triangle B?

18. If rectangle B is 6 in. long and 4 in. high, what is its area? What is the area of triangle B?



RECTANGLE A

TRIANGLE A (THE SHADED PART).



RECTANGLE B

TRIANGLE B (THE SHADED PART).

19. The perimeter of a triangle or rectangle is the sum of the lengths of its sides.

Written Exercise

205. Find the areas:

RECTANGLES

BASE	HEIGHT	BASE	HEIGHT
1. 36 ft.	105 ft.	**11.** 175 ft.	$348\frac{1}{3}$ ft.
2. 79 ft.	437 ft.	**12.** 175 ft.	$17\frac{1}{2}$ yd.
3. 63 ft.	84 in.	**13.** 96 ft.	$2\frac{1}{2}$ rd.
4. 48 in.	4 ft.	**14.** 180 rd.	$29\frac{2}{3}$ rd.
5. 96 in.	5 yd.	**15.** $274\frac{3}{4}$ rd.	$48\frac{3}{4}$ yd.
6. 100 in.	6 rd.	**16.** $29\frac{2}{3}$ yd.	$17\frac{1}{2}$ ft.
7. 50 yd.	10.5 ft.	**17.** 75.79 rd.	18.80 rd.
8. 127 yd.	37.75 ft.	**18.** 96.37 ft.	29.98 ft.
9. 40 rd.	19.66 yd.	**19.** 48.24 yd.	3.48 rd.
10. 80 rd.	28.8 rd.	**20.** .75 in.	.8 in.

TRIANGLES

	BASE	HEIGHT		BASE	HEIGHT
1.	$4\frac{2}{3}$ ft.	$5\frac{5}{10}$ ft.	**11.**	1 ft. 4 in.	3 ft. 7 in.
2.	$7\frac{2}{5}$ in.	$6\frac{3}{4}$ in.	**12.**	5 ft. 8 in.	4 ft. 6 in.
3.	$8\frac{1}{2}$ yd.	$7\frac{2}{3}$ ft.	**13.**	25 ft. 11 in.	16 ft. 9 in.
4.	25.5 rd.	12.8 rd.	**14.**	12 ft. 10 in.	20 ft. 5 in.
5.	36.87 rd.	14.24 rd.	**15.**	3 yd. 2 ft.	4 yd. 1 ft.
6.	$5\frac{2}{5}$ yd.	6.8 yd.	**16.**	5 yd. 1 ft.	1 yd. 2 ft.
7.	28.48 in.	$2\frac{3}{4}$ yd.	**17.**	8 rd. 2 yd.	2 rd. 2 yd.
8.	18.5 ft.	18.5 ft.	**18.**	3 rd. 2 ft.	4 rd. 6 ft.
9.	$12\frac{3}{4}$ in.	$12\frac{3}{4}$ in.	**19.**	5 rd. 1 ft.	4 rd. 1 ft.
10.	10.5 in.	20.5 in.	**20.**	10 rd. 10 ft.	10 rd. 10 ft.

Written Problems

206. **1.** The pages of a book are $5\frac{1}{2}$ in. wide and 10 in. high. How many square inches in the area of the page?

2. If one of these pages is cut in two from corner to corner, what is the area of each part?

3. An envelope is 4 in. by $5\frac{1}{2}$ in. How many square inches in its area?

4. What is the area of a postage stamp $\frac{4}{5}$ in. by $\frac{9}{10}$ in.?

5. Which covers the greater surface, a triangle with a base of 8 in. and an altitude of 7 in., or a triangle with a base of 6 in. and an altitude of 9 in.?

6. The first baseman, second baseman, and catcher are on the corners of a triangle. Call the

line from the home plate to first base the base of the triangle and the line from the first to the second base its height. The base and height are each 90 ft. Find the area of the triangle in square feet. Also in square yards.

7. A mason has 1000 tiles, each $\frac{1}{3}$ ft. square (that is, $\frac{1}{3}$ ft. long and $\frac{1}{3}$ ft. wide). How many square feet of floor can he lay with them?

8. A cement walk $35\frac{1}{2}$ ft. long and 6 ft. wide costs 10¢ a square foot. What is its total cost?

9. A rug is 7 yd. by $6\frac{1}{2}$ yd. How much larger is this than another rug 6 yd. by 7 yd.?

10. How many square yards of wall can be covered with a roll of paper $\frac{4}{5}$ yd. wide containing 85 yd.?

11. A public hall is 150 ft. long and 20 yd. wide. Find its floor space in square feet.

12. A rectangular garden is $17\frac{1}{2}$ yd. long and $10\frac{1}{2}$ yd. wide. How many yards of fencing would it take to inclose it? How many square yards of ground in the garden?

13. Which is more, a piece of ground $25\frac{4}{5}$ rd. by 10 rd., or one containing two acres?

14. How many acres of land are there in $\frac{1}{8}$ sq. mi.?

15. One flower bed is 35 ft. by 20 ft., another is 70 ft. by 10 ft. Find their perimeters. Find their areas. Can two rectangles have equal areas but different perimeters?

16. A teacher orders a slate blackboard with a wooden molding around it. If the blackboard is to be $2\frac{1}{2}$ yd. long and $1\frac{1}{2}$ yd. wide, how many square yards of slate must be ordered? How many yards of molding?

17. In making boxes, pieces are cut from sheets of pulp board so as to waste as little as possible. How many pieces, each $2'' \times 2''$, can be cut from a sheet $14''$ by $16''$? Can waste be avoided?

18. How many pieces, each $3'' \times 4''$, can be cut from a sheet $15'' \times 16''$? Can waste be avoided?

19. How many pieces, each $4'' \times 5''$, can be cut from a sheet $16'' \times 18''$? Can waste be avoided?

20. Make similar problems of your own in which there is waste. In which there is no waste.

21. From one sheet, make a box, without cover, of the dimensions $5'' \times 3'' \times 2''$. How large a sheet is needed?

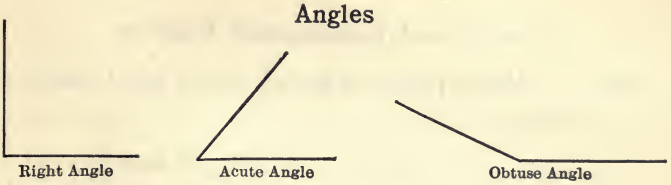
22. Draw a similar figure showing the sheet needed in making a box $7'' \times 4'' \times 3''$. How much is the waste?

23. How much is the waste in making from one square sheet a box $4'' \times 4'' \times 4''$? How large a sheet is needed?

24. How much is the waste in making a box $5'' \times 3'' \times 2''$ from a sheet $9'' \times 8''$?

25. How many covers the size of this book cover can be made from a piece of cloth 1 yd. square?

Angles



207. 1. What kind of angle do the hands of a clock make at 9 o'clock? At 3 o'clock?

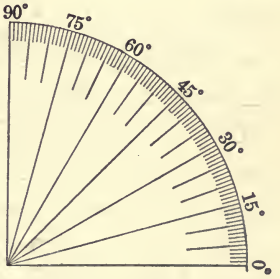
2. What kind of angle do the hands of a clock make at 10 o'clock? At 11 o'clock?

3. When do the hands form the larger angle, at 11 o'clock or at 2 o'clock?

4. What kind of an angle do the hands of a clock form at 4 o'clock? At 5 o'clock?

5. Is the angle formed by the hands larger at 8 o'clock than at 7 o'clock? At 9 o'clock than at 8 o'clock?

6. A right angle has been divided into 90 equal parts, called **degrees.** Surveyors and draftsmen usually give the size of angles by telling the number of degrees which they contain.

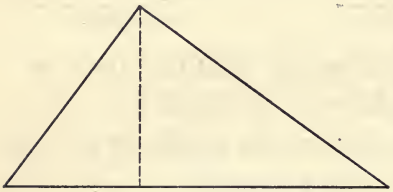


90 degrees (90°) = 1 right angle

How many degrees are there in 2 right angles? In 3? In $\frac{1}{2}$ of a right angle? In $\frac{1}{3}$? In $\frac{2}{3}$?

Drawing and Construction Exercise

208. 1. Measure the sides of this triangle to find its perimeter.



2. Measure the height of this triangle. Find its area in square inches.

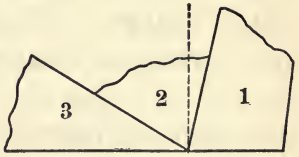
3. Find the area of a triangle which is twice as high and has a base twice as long as that of the triangle shown in the figure.

4. Construct 5 triangles. Measure the base and the altitude to find the perimeter and the area of each.

5. Construct a triangle with one angle a right angle.

6. Draw a large triangle similar to the triangle in Example 1 of this exercise.

Cut off the 3 angles of the triangle and set them together on one side of a straight line as illustrated. Then draw a line as indicated.



7. To how many right angles are the three angles of a triangle equal?

8. If a right angle is equal to 90°, how many degrees in all the angles of a triangle?

Written Exercise

209. 1. If the three angles of a triangle are equal, what part of a right angle is each?

2. If two angles of a triangle are each just $\frac{1}{2}$ of a right angle, how big is the third?

3. If one angle is $\frac{5}{8}$ of a right angle, and another angle is $\frac{3}{8}$ of a right angle, how much is the third angle?

4. If two angles are each $\frac{5}{8}$ of a right angle, how large is the third angle?

5. If one angle of a triangle is $\frac{1}{2}$ of a right angle and another is $\frac{3}{4}$ of a right angle, what is the third angle?

6. Find in the walls and ceiling of your room two lines forming a right angle.

7. Express in degrees the sum of the angles of a triangle.

8. Two of the angles of a triangle are 65° and 73°. What is the third angle?

9. How many right angles will exactly fill the space about a point in a plane?

10. A wheel has 8 spokes. What is the angle between two neighboring spokes?

11. Through how many degrees does the minute hand turn in an hour? In half an hour? In 15 minutes? In 5 minutes?

12. What angle do the hour and minute hands form at 1 o'clock? At 2 o'clock? At 5 o'clock?

13. How many degrees in each angle of an equiangular triangle?

14. How many degrees are there in the four angles of a rectangle taken together?

Time

Exercises and Problems

210. 1. Compute the number of seconds in one day.

2. Reduce 90,630 seconds to days, hours, minutes, and seconds (avoiding fractions).

3. Find the number of seconds in $\frac{1}{8}$ of an hour.

4. Reduce 400 seconds to a fraction of an hour.

5. Change 114 hours to days.

6. How many weeks in 192 days? In 200 days?

7. Which months have 31 days? Which 30? How many has February?

8. A clock ticks 138 times every minute. How many times in 10 seconds? In one hour?

9. A man in walking takes 3 steps every 2 seconds. How many steps does he take in an hour?

10. If he takes 90 steps per minute, how many does he take per hour?

11. If he takes 5,400 steps per hour, and each step measures 3 ft., how many feet does he travel per hour?

12. A man walks at the rate of 4 mi. an hour. At this rate, how many rods will he walk in 20 min.?

13. In a 10-acre nursery of white ash trees there were 63 rows of trees, 630 trees in each row. In 9 weeks gophers killed $\frac{1}{2}$ of the trees by gnawing their roots. How many trees were destroyed in 9 weeks? In 1 week? In 1 day?

14. A gopher is known to have dug an underground burrow 480 ft. long in 2 days, throwing up hillocks of loose earth at intervals of about 4 ft. How many hillocks were thrown up at this rate in 2 days?

The Difference in Time between Two Dates

211. 1. Find the exact number of days from June 16 to Aug. 24.

In June there are 14 days remaining; in July there are 31 days; in August there are 24 days to Aug. 24. Add 14, 31, 24, to obtain the answer.

2. Find the number of months and days from March 4 to July 10.

Find the whole number of months and then the number of days remaining. From March 4 to July 4 are 4 months. From July 4 to July 10 are 6 days. The answer is 4 months and 6 days.

3. How many days are there from July 7 to July 31? From Feb. 3 to Feb. 27? From March 16 to April 25?

4. Find the exact number of days from May 3 to July 22.

5. How many months are there from Jan. 5 to June 5?

6. How many even months and how many days over from Feb. 12 to Sept. 25?

7. School opens Sept. 12 and closes June 5. How many even months and how many days over are there between these dates?

8. How many years, months, and days between Sept. 15, 1875 and Nov. 24, 1910?

From Sept. 15, 1876, to Sept. 15, 1910, are 35 years; from Sept. 15 to Nov. 15 are 2 months; from Nov. 15 to Nov. 24 are 9 days.

Answer: 35 years, 2 months, 9 days.

9. What was Benjamin Franklin's age at the time of his death, born Jan. 17, 1706, and died April 17, 1790?

10. What was George Washington's age at the time of his death, born Feb. 22, 1732, and died Dec. 14, 1799?

11. What was Henry W. Longfellow's age at the time of his death, born Feb. 27, 1807, and died March 24, 1882?

12. What was Abraham Lincoln's age at the time of his death, born Feb. 12, 1809, and died April 15, 1865?

13. What was James R. Lowell's age at the time of his death, born Feb. 22, 1819, and died Aug. 12, 1891?

14. How many years elapsed between the birth of Washington and of Lowell?

15. How long did Washington live after the death of Franklin?

16. How old was Lowell when Lincoln died?

17. How old was Longfellow when Lowell was born?

Review Exercise

212. 1. How many dimes make $1? Then what part of $1 is a dime? What part are 2 dimes? 3 dimes?

2. How many cents make $1? Then what part of $1 is a cent? What part are 3 cents? 7 cents?

3. In $2.35 there are how many whole dollars? How many dimes? How many cents besides?

4. In $4.44 what does each digit stand for?

5. What part of a dollar is $0.10? $0.60? $0.70?

6. What part of a dollar is $0.05? $0.09? $0.06?

7. What part of a dollar is 5¢? 15¢? 65¢?

8. $35¢ = \frac{\$35}{?}$, $55¢ = \frac{\$55}{?}$, $\$0.45 = \frac{\$45}{?}$.

9. What does each digit stand for in $\$15.678$?

10. How does the value of each digit in $\$6.666$ compare with the digit to its right? Which digit has the least value? Which the most?

Oral Exercise

213. Tell how many dollars, cents, and mills there are in:

1. $\$4.765$	2. $\$10.075$	3. $\$0.457$
4. $\$0.043$	5. $\$111.111$	6. $\$10$
7. $\$.01$	8. $\$.001$	9. $\$10.01$
10. $\$100.01$	11. $\$1000.101$	12. $\$1000.01$
13. $\$1001.001$	14. $\$1.001$	15. $\$8.663$

Making Change

214. If you owe $85¢$ and pay the debt with a dollar, the storekeeper gives you 5 cents and 10 cents, saying *85 and 5 is 90 and 10 is a dollar.*

Make change

1. For $\$1$, when some one pays $70¢$.
2. For $\$1$, when some one pays $65¢$.
3. For $50¢$, when some one pays $35¢$.
4. For $\$5$, when some one pays $\$2.75$.

Make change for $\$1$, $\$2$, and $\$5$ with each of the following numbers:

5.	75¢	55¢	45¢	35¢	85¢	45¢	65¢
6.	77¢	67¢	15¢	31¢	47¢	66¢	18¢
7.	34¢	62¢	82¢	78¢	23¢	21¢	56¢
8.	77¢	36¢	54¢	52¢	48¢	34¢	71¢

Oral Exercise

215. 1. At 25¢ a yard, how many yards can be bought for \$1? For \$4? For \$17?

2. At $33\frac{1}{3}$¢ a bushel, how many bushels can be purchased for \$2? For \$10? For $\$15\frac{1}{3}$?

3. How many yards of calico at 20¢ a yard can be bought for \$920?

4. \$10 will buy how many pounds at $12\frac{1}{2}$¢ a pound?

5. \$30 will buy how many dozen at $33\frac{1}{3}$¢ a dozen?

6. How many yards of silk costing $\$1.12\frac{1}{2}$ per yard can be bought for \$243?

7. How many articles at \$1.25 each can be bought for \$75?

8. How many articles costing $33\frac{1}{3}$¢ each can be bought for \$20? \$33? $\$10\frac{2}{3}$?

9. How many $12\frac{1}{2}$¢ articles can be bought for \$2? \$7? \$15? $\$7\frac{1}{8}$?

10. How many 25¢ articles can be had for \$75? \$10? \$10.75?

11. How many $66\frac{2}{3}$¢ articles can be purchased for \$10? \$14? \$76?

THE ANALYSIS AND SOLUTION OF PROBLEMS

216. 1. If 2 lamp chimneys cost 12¢, what will 3 cost?

ANALYSIS

If 2 chimneys cost 12¢,
1 chimney will cost $\frac{1}{2}$ of 12¢, or 6¢.
3 chimneys will cost 3×6¢, or 18¢.

2. What is the cost of 5 drinking glasses, if 2 of them cost 22¢?

3. If 3 cakes of soap cost 15¢, what is the cost of 2 cakes?

4. If 2 sugar bowls are sold for 38¢, what are 3 sold for?

5. If 3 coffee pots cost $1.20, what will 5 of them cost?

6. What is the cost of 2 water bottles, if 5 of them cost $1.25?

7. What do we pay for 7 boxes of matches, if 2 boxes sell for 4¢?

8. What is the price of 9 brooms, if 2 brooms cost 30¢?

9. Trout is advertised at 30¢ for 2 lb. Find the cost of 5 lb. Of 10 lb. Of 11 lb.

10. If 5 lb. of salmon cost $1, what will 9 lb. cost?

Written Problems

217. Analyze and solve:

1. If 21 gallons of gasoline cost $3.54, what will 49 gallons cost?

SOLUTION

$$1 \text{ gallon costs } \frac{3.54}{21} \text{ dollars.}$$

$$49 \text{ gallons cost } \frac{3.54 \times 49}{21} \text{ dollars.}$$

Cancel factors common to both terms, thus:

$$\frac{\overset{1.18}{\cancel{3.54}} \times \overset{7}{\cancel{49}}}{\underset{1}{\cancel{21}}} = \frac{1.18}{1} \times 7 = 8.26. \quad \textit{Ans.}$$

The process is shorter if the indicated division is simplified but not performed. Thus, $\frac{3.54}{21}$ is $\frac{354}{2100}$. Then $\frac{3.54 \times 49}{21} = \frac{354 \times 49}{2100}$. By so doing we often escape the necessity of dividing. In any case, it is better not to carry out the operations until the last step.

2. What is the cost of 35 lb. of catfish at $2.67 for 15 lb.?

3. If 2 gallons of benzine sell for 25¢, what is the cost of 25 gallons?

4. If 20 A. of land sell at $310, what will 59 A. sell at?

5. If 2 acres of strawberries yield a crop worth $259.65, what will $12\frac{1}{2}$ acres yield?

6. At $16.50 per half dozen pairs of gloves, what will $2\frac{1}{2}$ dozen pairs cost?

7. John's salary is $700 a year. What is it for 7 months?

8. If $\frac{3}{4}$ of an acre of garden land sells for $375, what will $\frac{1}{4}$ of an acre sell for? 1 acre?

9. At $5 a ton of 2000 lb., what will 3000 lb. of coal cost? 3500 lb.?

10. Mutton is quoted at $4.60 a hundred pounds. How many pounds can be purchased for $230?

11. At $5.75 a hundred pounds, find the cost of 275 lb. of lamb.

12. Calves sell at $5.10 per hundred pounds. How much will calves weighing 2700 lb. bring?

13. Find the price of cows weighing 3200 lb. at $3.25 a hundred pounds.

14. At $30 a hundred pounds, find the cost of 1875 lb. of creamery butter.

15. At $9.75 a hundred pounds, $7\frac{1}{2}$ tons of timothy hay cost $——.

16. If 3 men can do a piece of work in 6 days, how long would it take 1 man to do this work?

17. How long will it take 10 masons to build a certain foundation for a house, if 3 masons can do it in 20 days?

18. If 50 ft. of garden hose sell at $5.75, what do 125 ft. sell for?

19. When asked his age, a man replied, "$\frac{3}{8}$ of my age is $22\frac{1}{2}$ years." How old was he?

SUGGESTION

$\frac{1}{8}$ of his age is $\frac{45}{2 \times 3}$; $\frac{8}{8}$ of it is $\frac{45 \times 8}{2 \times 3}$. Then cancel.

20. If $8\frac{1}{4}$ yd. of calico cost $.66, what will 57 yd. cost?

21. If the rent for a house for 9 mo. was $319.50, what is the rent for a year?

22. If a certain sum of money brings $198 in 15 months, what will it bring in 20 months?

23. If 65% of a certain sum of money, put out at interest, yields $390 annually, how much would the entire sum yield at the same rate of interest?

24. At $14.50 a hundredweight, what will $7\frac{1}{2}$ lb. cost? $17\frac{1}{2}$ lb.?

25. If 5 men can do a piece of work in 12 da., how long will it take 15 men to do the same work?

SOLUTION AND EXPLANATION

Will 1 man do the work in a longer or a shorter time than 5 men? To find the time it takes 1 man to do the work must you *multiply* 12 days by 5, or *divide* 12 days by 5?

If it takes 1 man 12×5 days, must this product be *multiplied* by 15 or divided by 15, to find the time it takes 15 men to complete the work?

26. If 12 dredging machines can clear a certain channel in 18 days, how long will it take 16 to do the same work?

27. If 4 persons eat 5 packages of breakfast food in 15 days, how many persons will eat 6 packages in 8 days?

SOLUTION AND EXPLANATION

If 5 packages are eaten in 15 da. by 4 persons, then 5 packages are eaten in 1 da. by 4×15 persons; 1 package is eaten in 1 da. by $\frac{4 \times 15}{5}$ persons; 6 packages are eaten in 1 da. by $\frac{6 \times 4 \times 15}{5}$ persons, and 6 packages are eaten in 8 da. by $\frac{6 \times 4 \times 15}{5 \times 8}$ persons. Since $\frac{36 \times 2 \times 15}{5 \times 8} = 9$, the answer is 9 persons.

In this method of solution, the concrete number of the kind required in the answer is put last.

In this example, the required number is "persons." We arrange the statement so that "persons" comes last.

28. If 2 launches require 10 gallons of oil to travel 7 hours, how many launches can travel on 15 gallons for 3 hours?

29. If it takes 2 boys 5 days to build a pigeon house, how long will it take 3 boys working at the same rate?

30. If 880 bricks are needed for a wall 10 ft. long, 2 ft. wide, and 2 ft. high, how many bricks are needed for a wall 12 ft. long, 1 ft. wide, and 5 ft. high?

31. A clothier invests $368.55 in boys' coats. How many does he buy, if each coat costs $1.89?

32. A man fails in business. He owes $7900, and his creditors receive 65% of this. What amount do they receive?

33. If $\frac{3}{5}$ of a man's money is $2430, how much has he?

34. How long is a pole, if $\frac{7}{8}$ of it is 40 ft.?

35. If 10 yd. of gingham cost $.75, how much will 87 yd. cost?

36. If 3 teams of horses can plow a field in 16 days, how long will it take 4 teams to plow the same field?

37. If 5 men can do a piece of work in 12 days, how long will it take 7 men to do the same work?

38. If 1.8 yd. of silk cost $3.24, find the price of $7\frac{1}{2}$ yd.

39. The earth moves in its path around the sun at the rate of 1110 miles a minute. How many times faster does it move than a train which travels 54 miles an hour or —— miles a minute?

40. If a train travels 75 miles in $1\frac{3}{4}$ hours, how far will it travel in 7 hours?

41. Skimming milk by hand, only $\frac{4}{5}$ of the cream is obtained. In a week a farmer obtained 270 qt. of cream. How much would he have obtained if he had used a separator, which extracts all the cream?

42. William earns $750 a year, which is $\frac{3}{7}$ as much as his father earns. How much does his father earn?

43. If $\frac{3}{11}$ of a certain number is 42, how much is $\frac{2}{3}$ of that number?

44. If a stable has enough oats to feed 30 horses 45 days, how long will the oats feed 20 horses?

45. If a clock gains $1\frac{1}{2}$ minutes in 24 hours, how much time will it gain in 40 hours?

46. At the rate of 3 miles an hour, I can walk a certain distance in 2 hours 30 minutes. What is my rate when I walk this distance in 3 hours?

47. If a certain weight of sheet iron, $\frac{3}{8}$ in. thick, covers 45 sq. ft., how many square feet will the same weight of sheet iron only $\frac{1}{4}$ in. thick cover?

48. Using 4 electric lights, the electric bill is \$ 5 a month. What is the bill for 6 months when 3 electric lights are used?

49. If the interest on a certain sum of money is \$ 120 for 9 months, what is the interest on that sum for 25 months?

50. A farmer raised 1,125 bushels of beets on 3 acres. At this rate, how many bushels could he have raised on 11 acres?

Unitary Analysis

218. **1.** If 3 packages of rolled oats sell for 24¢, what is the cost of 4 packages?

ANALYSIS

In problems of this kind it is often easier to find the cost of *one* unit, then the cost of the required units. In this case find the cost of 1 package, then the cost of 4. Thus,

If 3 packages sell for 24¢,
1 package sells for $\frac{1}{3}$ of 24¢, or 8¢.
4 packages sell for $4 \times 8¢ = 32¢$. *Ans.*

2. If 276 is $\frac{4}{5}$ of a number, what is the number?

ANALYSIS

If $\frac{4}{5}$ of the number = 276, then
$\frac{1}{5}$ of the number = $\frac{1}{4}$ of $276 = \frac{276}{4} = 69$.
$\frac{5}{5}$ of the number $= 5 \times 69 = 345$. *Ans.*

Oral Exercise

219. Analyze, explaining the process in each case:

1. If 51 is $\frac{3}{5}$ of a number, what is the number?
2. If 15 is $\frac{3}{7}$ of a number, what is the number?
3. Of what number is 12 the six sevenths part?
4. 54 is $\frac{9}{11}$ of what number?
5. 56 is $\frac{4}{5}$ of what number?
6. If 48 is $\frac{3}{4}$ of a number, what is the number?
7. If 121 is $\frac{11}{3}$ of a number, what is the number?
8. If 3 men together earn \$12 in one day, what will 7 men earn daily at the same rate?
9. If 14 men pay \$56 for board per week, what will 11 men pay at the same rate?
10. If 4 yd. cost 22¢, what will 10 yd. cost?
11. 35 is $\frac{5}{7}$ of what number?
12. 27 is $\frac{3}{4}$ of what number?
13. 100 is 20% of what number?
14. 21 is $1\frac{1}{2}$ times what number?
15. 40 is .20 of what number?
16. 24 is $\frac{1}{3}$ less than what number?
17. 16 is $\frac{1}{5}$ less than what number?
18. 48 is $\frac{1}{5}$ more than what number?
19. 36 is $\frac{1}{5}$ more than what number?
20. 75 is 75% of what number?

APPROXIMATIONS

Oral Exercise

220. As a check against absurd results it is desirable that pupils accustom themselves to giving approximate answers. Whenever possible, the approximate results should be found orally. Suppose a pupil wishes to find the cost of 2.8 lb. at $1.05 a pound and writes his answer $29.40. By a brief mental computation he should detect the error at once. 2.8 lb. is nearly 3 lb., and $1.05 is nearly $1. 3 lb. at $1.00 gives $3.00 as an approximate answer. Hence, $29.40 is absurd. The correct answer is $2.94.

221. Give the approximate answer. Check with the correct answer.

1. 28 handkerchiefs at $.24.
2. 22 yards at $.29.
3. 13 wool blankets at $4.95.
4. 5 gingham skirts at $.98.
5. 11 suits of clothes at $12.75.
6. 98 yd. of silk at $.79.
7. Find cost of 1 pair of shoes when 15 pairs cost $74.25.

8. Cost of 23 linen suits at $3.98 each.

9. Cost of 105 pairs of shoes at $2.45 a pair.

10. At 44¢ a pound, find cost of $2\frac{2}{7}$ lb.

11. Cost of 1 hammock, 31 cost $96.72.

12. The monthly salary of a man who receives $2350 a year.

13. Cost of 52 books of fiction at $.39.

14. Cost of 19 sacks of potatoes at $1.25.

15. Suggest problems for testing the ability to estimate answers that are approximately correct.

AVERAGES

Introduction

222. We have used the term **average** in our problems and our discussions.

The average of 6, 7, 11 is found by adding them and dividing the sum by 3.

The average is $\frac{6+7+11}{3}=\frac{24}{3}=8$.

In all cases, find the total of the items, then divide the sum by the number of the items.

Statistics are, to a large extent, averages.

We speak of the average attendance at school, the average temperature, the average number of days of sunshine per month, the average rations for man and animals, the average height of men, the average crop, and so on.

What problem can you make requiring that the average be found?

Problems in Averages

223. 1. John is 11 years old, James 12, Harry 10, Wallace 15. Find the average age.

2. A driver earned on successive days \$3, \$5, \$4, \$1. What is his average daily earnings?

3. A merchant's receipts for 3 consecutive days were \$ 200, \$ 200, \$ 500. Find his average daily receipts.

4. At 6 o'clock on 4 mornings the thermometer stood 61°, 59°, 50°, 70°. Find the average temperature.

5. A man earns 50¢ an hour. He works 6 hr. on Monday, 2 on Tuesday, 7 on Wednesday, 10 on Thursday, 8 on Friday and 3 on Saturday. What were his average daily earnings?

Written Problems

224. 1. What is the average weight of 5 bales of cotton weighing 450, 460, 475, 455, 457 lb.?

2. What should ground feed, made from an equal number of bushels of oats @ 28¢, barley @ 78¢, and corn @ 59¢, be sold per bushel, in order to yield a profit of 20 % on the cost?

3. The weekly salary list of 5 employees in a store is \$ 25, \$ 20, \$ 18, \$ 17.75, \$ 15.50. What is the average weekly salary?

4. The cyclometer on an automobile shows that the distances traveled in 4 da. are 130.4 mi., 82.27 mi., 90.01 mi., 207.54 mi. What is the average distance traveled per day?

5. A pleasure launch uses $5\frac{1}{2}$ gal. gasoline the first day, $4\frac{3}{4}$ gal. the second day, $6\frac{1}{8}$ the third day,

and $4\frac{7}{8}$ the fourth day. What is the average daily consumption of gasoline?

225. TABLE FOR REFERENCE

HEIGHT		AGES						
Ft.	In.	15–24	25–29	30–34	35–39	40–44	45–49	50–54
		lb.	lb.	lb.	lb.	lb.	lb.	lb.
5	0	120	125	128	131	133	134	134
5	2	124	128	131	133	136	138	138
5	4	131	135	138	140	143	144	145
5	6	138	142	145	147	150	151	153
5	8	146	151	154	157	160	161	163
5	10	154	159	164	167	170	171	172
6	00	165	170	175	179	180	183	182

Problems Based on the Table

226. 1. The weights given in this table are the averages obtained by weighing 74,162 applicants for life insurance. From this table complete the average weight of men between the ages of 15 and 24, which are not less than 5 ft. nor more than 6 ft. tall.

2. Do the same for each of the six other age periods in the table.

3. Find the average weight of men 5 ft. tall, between the ages of 15 and 54 years.

4. Do the same for each of the other heights given in the table.

5. Make 5 problems based on the table.

THE DIRECT METHOD OF SOLUTION

227. It frequently happens that the solution of problems may be performed in a direct method by eliminating useless operations. The results are more speedily obtained in this way and there is less likelihood of error.

Oral Problems

1. If butter sells at $18.50 per 100 lb., what is the cost of 300 lb.?

SOLUTION. — In examples like this, where 300 lb. is exactly divisible by 100 lb., it is easier *not* to find the cost of 1 lb. Since 300 lb. is 3 times 100 lb., the cost of 300 lb. will be 3 times $18.50, or $55.50. This is called the *direct method of solution.*

2. At $3.50 for 200 lb. of lignite, what is the cost of 6000 lb.?

3. Find the cost of 120 eggs at $23\frac{1}{5}¢$ per dozen.

4. Find the cost of 300 bbl. of flour at $575 a 100 bbl.

5. If 10 men earn $175 a week, what will 40 men earn in the same time?

6. If 10 men can do a piece of work in 36 days, how long will it take 40 men to do the work?

7. How much will 150 gal. of molasses cost at $12.50 for 50 gal.?

8. Mary bought 10 yd. of cheviot at $3.95. If Lucy bought 30 yd. of cheviot, how much did she pay?

Written Problems

228. Solve, using direct method wherever possible:

1. In a city of 200,000 inhabitants, 93 inhabitants out of every 1000 are foreign born. What is the total foreign population?

2. In the same city there are $2\frac{1}{4}$ grocery stores for every 6 inhabitants, or how many grocery stores in all?

3. If a circle, 16 in. in diameter, has a circumference of 50.2 in., what is the circumference of a circle whose diameter is 112 in.?

4. If 56 lb. of rye make a bushel, how many bushels in 1120 lb.?

5. What will 8000 bu. of barley weigh, if 40 bu. weigh 1920 lb.?

6. If 1 cu. ft. of cast iron weighs 446 lb., what will 144 cu. in. weigh?

7. What is the cost of floor tiles for a room 35 ft. by 40 ft., at $15 per 100 sq. ft.?

8. A certain mantel tiling costs 40¢ per square foot. If a man purchases tiling to the amount of $8, how many square feet can he cover?

Oral Exercise

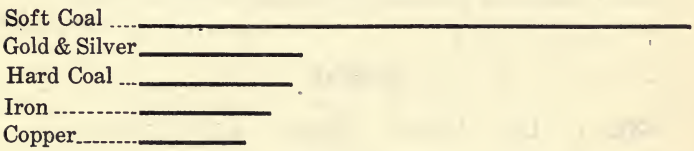
229. Find the cost:

	Articles	Rate	Articles	Rate	Articles	Rate
1.	7	3 for 12¢	5	2 for 24¢	6	5 cost 55¢
2.	10	4 for 20¢	6	7 cost 84¢	3	4 cost 52¢
3.	3	2 cost 34¢	2	3 cost 27¢	12	7 cost 28¢
4.	30	$33\frac{1}{3}$ cost 100¢	12	$12\frac{1}{2}$ cost 100¢	5	$16\frac{2}{3}$ cost 100¢
5.	6	4 cost 32¢	5	6 cost 72¢	21	30 cost 60¢
6.	5	3 cost 36¢	9	5 cost 75¢	10	4 cost 36¢
7.	12	4 cost $44	7	6 cost $36	100	2 cost $3.50
8.	100	3 cost $3.60	100	7 cost $2.10	100	5 cost $5.50
9.	20	13 for $2.60	200	14 for $4.20	30	11 for $1.21
10.	50	5 for 15¢	10	6 for $4.20	4	3 for $3.30

PROBLEMS THAT MAY BE ILLUSTRATED BY SIMPLE GRAPHS

Industry

230. 1. Much gold and silver is taken from mines in Colorado, California, and Nevada. Coal is found in many places in the United States. Is the coal taken out of the earth in one year worth as much as the gold and silver? The lines show the value of several minerals produced in the United States during one year. One inch stands for 120 million dollars. Which mineral represents the greatest value? Which the least? How long is the line representing soft coal? Measure the line to the nearest tenth of an inch and see how nearly correct you are.



2. How many million dollars stand for the value of the soft coal for last year?

3. How long is the line for gold and silver, and how many million dollars does it represent?

4. How many million dollars of hard coal were produced?

5. How many millions more soft coal than hard coal?

6. How many millions more of soft coal than gold and silver?

7. About how much more of gold and silver than hard coal?

8. How many million dollars of iron were mined?

9. How many million dollars of copper were mined?

10. The limestone production was valued at 30 million dollars How long a line stands for that value?

11. Draw a line representing a mineral production worth 150 million dollars.

12. Construct similar problems of your own.

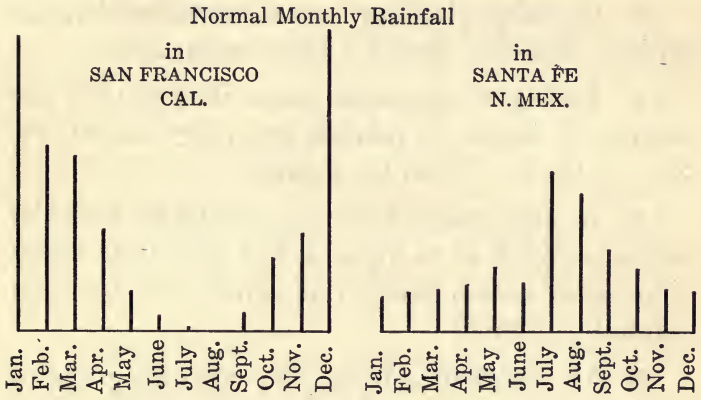
Rainfall

231. 1. The United States government has gauges in different parts of the country by which the exact amount of rain or melted snow is measured. From this diagram, show during which two months San Francisco has the least rain and Santa Fe the most.

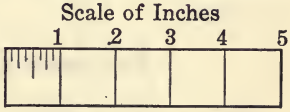
2. During which months has San Francisco the most rain and Santa Fe the least?

3. On a strip of paper copy carefully the scale of inches.

You can measure easily to the nearest $\frac{1}{4}$ of an inch or, if you are careful, to the nearest $\frac{1}{8}$ of an



inch. If you apply the scale to the line showing the rainfall in San Francisco for January, you find the line $4\frac{5}{8}$ in. long.



That means that if a tub is left out of doors to catch the rain and snow during January, the rain and melted snow in the tub at the end of the month is, in an average year, $4\frac{5}{8}$ in. deep.

The tub must not leak. It must be covered during fine weather, to prevent evaporation.

4. Find the number of inches of rainfall for a year in San Francisco. Also in Santa Fe.

5. In which place is the total amount of rainfall for one year greater?

6. Which place has a more even distribution of the rainfall throughout the year?

7. In which place can you more easily dispense with an umbrella during a summer vacation?

8. During 6 successive years in Santa Fe the number of inches of rainfall per year was 20, 14, 20, 12, 10, 16. Find the average.

9. In some parts of the Panama Canal zone the annual rainfall is as high as 120 in. How many times more is this than 15 in., which is the annual rainfall in Denver?

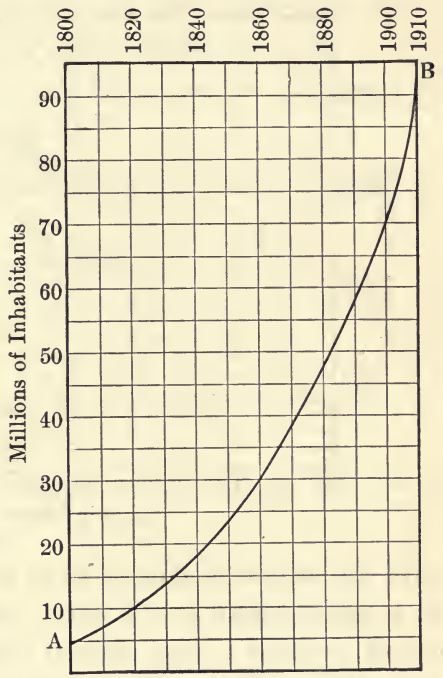
10. At Cheyenne, Wyo., the monthly rainfall (in inches), beginning with January, is as follows: $\frac{4}{10}$, $\frac{5}{10}$, $\frac{8}{10}$, $1\frac{5}{10}$, $2\frac{4}{10}$, $1\frac{5}{10}$, 2, $1\frac{5}{10}$, $\frac{9}{10}$, $\frac{7}{10}$, $\frac{4}{10}$, $\frac{3}{10}$. Find the rainfall per year, also the monthly average.

Population of the United States

232. 1. The curve AB shows the increase in population in the United States during 110 years. Years are marked off from left to right; the population from the bottom line up. One space up stands for 5 million inhabitants. The point A indicates the population in the year 1800. This point is about one space up and stands, in round numbers, for how many inhabitants?

2. At the year 1820 the curve is —— spaces up. This indicates about —— inhabitants.

3. At the year 1840 the curve is a little more than 3 spaces up, or about 3.4 spaces. The population was, therefore, about 3.4 times 5 million, or —— million.



4. In this way estimate the population for 1860, 1880, 1900, and 1910.

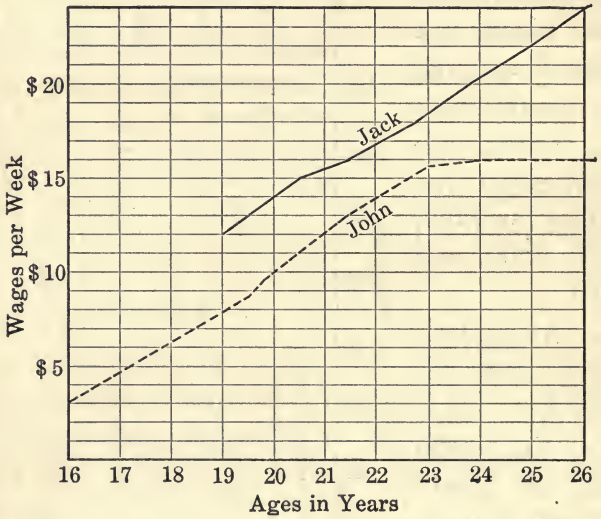
5. About how many more inhabitants were there in 1820 than in 1800? In 1900 than in 1880?

6. Estimate the population for the years 1810, 1830, 1850, 1870, and 1890.

7. Make a new graph to show about what the population of the United States is in 1915. What will be the approximate population of the United States in 1920?

The Money Value of Training

233. The *St. Nicholas* (Vol. 31, p. 57) gives statistics of two typical boys. The wages of John, who enters a shop at the age of 16, are compared



with the wages of Jack, who at the age of 16 goes to a trade school for 3 years' training, and then enters a shop. The dotted line shows John's wages; the full line shows Jack's wages.

1. How much does John earn weekly when he is 16 years old? When he is 17? When he is 18? Each space from the lowest line up stands for 1. When John is 16 years old, the dotted line is 3 spaces up. His weekly wages are then $ 3.

2. Does Jack earn anything when he is 16, 17, 18 years old?

3. At the age of 19, how much does each earn? How much more does Jack earn?

4. At the age of 20, how much does each earn? Find the difference in their wages.

5. At what age are John's wages more nearly equal those of Jack?

6. At what age does John reach his highest wage?

7. Do Jack's wages steadily increase?

8. At what age does John earn as much as Jack earns at 19?

9. At what age does Jack earn as much as John earns at 24?

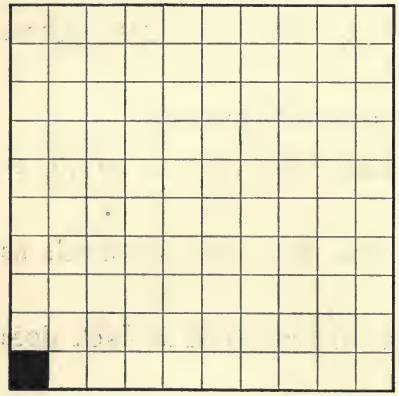
10. Does John ever earn as much as Jack does at the age of $21\frac{1}{2}$?

PART TWO

PERCENTAGE

Introduction

234. When we say that 5 **per cent** of the enrolled pupils are absent, we mean that 5 out of every 100 pupils, or 5 **hundredths** of them, are absent.



In this Illustration 1% of the Surface is Shaded

Per cent means hundredths.

Five per cent is written 5 %. It means 5 *hundredths*, or $\frac{5}{100}$, or .05.

1. If a school has 200 pupils enrolled, and 4 % are absent, what is the number of pupils that are absent?

4 % means $\frac{4}{100}$, or .04.

To find 4 % of 200 we *multiply* 200 by $\frac{4}{100}$, or .04.

We have $200 \times .04 = 8$, the answer.

2. How many little squares are there in this drawing? Shade 3 % of them. Shade 6 %; 8 %.

3. How many squares must be shaded to make them 25 % of the whole?

4. How much is

$\frac{1}{100}$ of 100?	1 % of 100?
$\frac{2}{100}$ of 100?	2 % of 100?
$\frac{5}{100}$ of 100?	5 % of 100?
$\frac{1}{100}$ of 200?	1 % of 200?
$\frac{2}{100}$ of 200?	2 % of 200?
$\frac{4}{100}$ of 500?	4 % of 500?
$\frac{6}{100}$ of 800?	6 % of 800?
$\frac{8}{100}$ of 1000?	8 % of 1000?

5. Study and compare

a. $\frac{5}{100} = .05 = 5\,\%$	*b*. $\frac{4}{100} = .04 = 4\,\%$
c. $\frac{6}{100} = .06 = 6\,\%$	*d*. $\frac{8}{100} = .08 = 8\,\%$
e. $\frac{12}{100} = .12 = 12\,\%$	*f*. $\frac{20}{100} = .20 = 20\,\%$

Reduction of Decimals and Common Fractions to Per Cents

235. Express with the per cent sign:

1. .06.

PROCESS

$.06 = \frac{6}{100} = 6\,\%$

2. .05	3. .07	4. .09	5. .04	6. .02
7. .1	8. .11	9. .12	10. .13	11. .14
12. .20	13. .30	14. .40	15. .50	16. .80

Tell how to express a decimal as a per cent.

Written Exercise

236. Express with the per cent sign:

1. $\frac{6}{10}$.

PROCESS

$$\frac{6}{10} = \frac{60}{100} = 60\,\%$$

2. $\frac{7}{100}$ 3. $\frac{19}{100}$ 4. $\frac{74}{100}$ 5. $\frac{1}{10}$ 6. $\frac{7}{10}$
7. $\frac{3}{10}$ 8. $\frac{31}{100}$ 9. $\frac{42}{100}$ 10. $\frac{6}{10}$ 11. $\frac{6}{100}$
12. $\frac{7}{10}$ 13. $\frac{7}{100}$ 14. $\frac{22}{100}$ 15. $\frac{5}{10}$ 16. $\frac{5}{100}$
17. $\frac{8}{10}$ 18. $\frac{8}{100}$ 19. $\frac{9}{10}$ 20. $\frac{9}{100}$ 21. $\frac{11}{10}$

Tell how to express a common fraction as a decimal.

Written Exercise

237. Express as a decimal:

1. 8 %.

PROCESS

$$8\,\% = \frac{8}{100} = .08$$

2. 10 % 3. 9 % 4. 7 % 5. 6 % 6. 5 %
7. 2 % 8. 1 % 9. 20 % 10. 30 % 11. 40 %
12. 50 % 13. 80 % 14. 25 % 15. 75 % 16. 66 %

Tell how to express per cent as a decimal.

Written Exercise

238. Write as common fractions in their lowest terms:

1. 10 %.

PROCESS

$$10\,\% = \frac{10}{100} = \frac{1}{10}$$

2. 6 % 3. 4 % 4. 12 % 5. 16 % 6. 18 %
7. 20 % 8. 25 % 9. 50 % 10. 40 % 11. 75 %
12. 48 % 13. 64 % 14. 24 % 15. 72 % 16. 96 %

Drill Exercise

239. Memorize these relations:

$50\% = \frac{1}{2}$	$100\% = 1$	$33\frac{1}{3}\% = \frac{1}{3} = .33\frac{1}{3}$
$25\% = \frac{1}{4}$	$75\% = \frac{3}{4}$	$66\frac{2}{3}\% = \frac{2}{3} = .66\frac{2}{3}$
$20\% = \frac{1}{5}$	$10\% = \frac{1}{10}$	$12\frac{1}{2}\% = \frac{1}{8} = .12\frac{1}{2}$
$2\% = \frac{1}{20}$	$62\frac{1}{2}\% = \frac{5}{8}$	$37\frac{1}{2}\% = \frac{3}{8} = .37\frac{1}{2}$
$1\% = \frac{1}{100}$	$80\% = \frac{4}{5}$	$16\frac{2}{3}\% = \frac{1}{6} = .16\frac{2}{3}$

Written Exercise

240. Write as common fractions in their lowest terms:

1. $12\frac{1}{2}\%$.

PROCESS

$$12\tfrac{1}{2}\% = .12\tfrac{1}{2} = \tfrac{1}{8}$$

2. $33\frac{1}{3}\%$ 3. $66\frac{2}{3}\%$ 4. $37\frac{1}{2}\%$ 5. $87\frac{1}{2}\%$

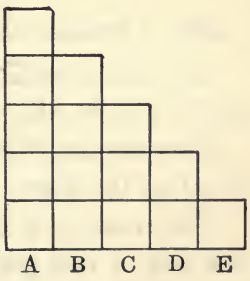
Comparison

241. 1. How many fifths of A is B? C? D? E?

2. How many hundredths of A is E? D? C? B?

3. What per cent of A is C? B? D? E?

4. What per cent of D is E?



5. What per cent of C is E? D?

6. What per cent of B is E? D? C?

7. What part is 50 % of A? 20 % of A?

8. What part is 50 % of B? 25 % of B?

9. What part is $33\frac{1}{3}$ % of C? $66\frac{2}{3}$ % of C?

10. What part is 50 % of D? 100 % of D?

Oral Exercise

242. 1. How many hundredths of an inch is $\frac{1}{8}$ in.? $\frac{2}{8}$ in.? $\frac{3}{8}$ in.? $\frac{4}{8}$ in.? $\frac{5}{8}$ in.? $\frac{6}{8}$ in.? $\frac{7}{8}$ in.? $\frac{8}{8}$ in.?

2. What per cent of 1 in. is $\frac{1}{8}$ in.? $\frac{2}{8}$ in.? $\frac{3}{8}$ in.? $\frac{4}{8}$ in.? $\frac{5}{8}$ in.? $\frac{6}{8}$ in.? $\frac{7}{8}$ in.? $\frac{8}{8}$ in.?

3. What per cent of 2 ft. are 6 in.? 3 in.? 2 in.? 1 in.? 4 in.? 5 in.? 9 in.?

4. What part of 1 inch is 50 % of it? 25 % of 1 inch?

5. What part of $\frac{1}{2}$ an inch is 25 % of it? 50 % of $\frac{1}{2}$ inch?

Study Exercise

243. Consider the exercise 5 % of 400 is 20.

In this exercise we call 400, the base; 5 %, the rate; and 20, the percentage.

The number upon which the percentage is found is called the **base.**

The number of hundredths to be taken is called the **rate** or **rate per cent.**

The result obtained by finding a certain per cent of the base is called **percentage.**

When you have the product of two numbers, how do you check your result?

If you divide the percentage by the base, what is the quotient?

Or, if you divide the percentage by the rate (expressed decimally), what is the quotient?

We have, therefore, the following principles:

The percentage is equal to the base multiplied by the rate.

The rate is equal to the percentage divided by the base.

The base is equal to the percentage divided by the rate.

1. Given the base \$ 2300, the rate 8 %, compute the percentage.

2. Given the rate 11 %, the base 3587 lb., find the percentage.

3. When you know the base and the rate, how do you find the percentage?

If we write b for base, r for rate, and p for percentage, we can indicate these principles more briefly by means of equations or formulas, thus,

$$p = b \times r, \quad r = \frac{p}{b}, \quad b = \frac{p}{r}$$

Find the Percentage

Written Exercise

244. 1. Find 5 % of 100 bu.

PROCESS

$5\,\%$ of 100 bu. $= \frac{5}{100}$ of 100 bu.
$= 100 \times \frac{5}{100}$
$= 5$ bu.

EXPLANATION. — Apply the formula $p = b \times r$. Change the per cent into a decimal fraction. The exercise then becomes an exercise in fractions. Solve by fractions.

In the exercise what is the base? What is the rate? What is the percentage? Perform the operation by fractions when it is possible to do so to advantage.

2. Find 5 % of 600 lb.
3. 6 % of 50 yd.
4. 20 % of 60 oz.
5. 30 % of 400 bu.
6. 12 % of 900 doz.
7. 3 % of 800 mi.
8. 5 % of 80¢.
9. 16 % of 20 T.
10. 50 % of 40 men.

Written Exercise

245. Apply the formula. Solve and explain.

Find 6 % of:

100	400	150	250	300
200	1000	900	500	700

Find 8 % of each of the following:

200	700	900	150	250
450	500	550	600	750

What is 10 % of:

350 A.	130 T.	145 bu.	700 men	870 bu.
750 bricks	930 ft.	670 lambs	360 da.	750 ft.

Written Problems

246. Read the problems carefully. Select the formula to be used. Decide whether to use fractions or decimals in the solution. Then solve the problem.

1. A farmer has 200 sheep, of which 5 out of every hundred, or 5 %, are black. How many sheep are black?

In this problem it is required to find a per cent of a number. What is the base? What is the rate? What is the formula to be used?

PROCESS

$$5\,\% = \frac{5}{100}$$

$$\frac{5}{100} \text{ of } 200 = \frac{5}{\cancel{100}} \times \cancel{200} = 10$$

2. John buys a $2.50 hat at 10 % off. How much does he save by this reduction in price?

PROCESS BY COMMON FRACTIONS

$$10\% \text{ of } \$2.50 = \tfrac{1}{10} \text{ of } \$2.50$$
$$= \$2\tfrac{1}{2} \times \tfrac{1}{10}$$
$$= \frac{\$\,\cancel{5}}{\cancel{2}} \times \frac{1}{\cancel{10}\,\cancel{2}}$$
$$= \frac{\$\,1}{4}$$
$$= \$.25$$

EXPLANATION. — Change the per cent to a common fraction. Change the number of dollars to an improper fraction. Multiply, using cancelation. Express the fraction of a dollar in cents.

PROCESS BY DECIMALS

$$10\% \text{ of } \$2.50 = .10 \text{ of } \$2.50$$
$$= \$2.50 \times .10$$
$$= \$.25$$

EXPLANATION. — Change the per cent to a decimal fraction. Multiply as in decimals.

Written Problems

247. 1. A suit of clothes, marked \$15.50, is sold at 10% off. How much in dollars and cents was saved by the buyer?

2. A merchant offers a reduction of 15% for cash purchases. Mary's mother buys goods to the amount of \$150. How much does she save by paying cash? How much does she actually pay?

3. A school has 360 pupils enrolled. Of these 5% are absent. How many pupils are absent? How many are present?

4. A man sold a bicycle that cost him $60, and lost 15% of the cost. What was the selling price?

5. Last year a man earned $1500. This year he earned 8% more. How much more does he earn this year than last?

6. An agent sells a lot for $1250 and receives 2% of this sum for selling it. How much does he receive?

7. A real estate man bought a house for $5750 and then sold it at a profit of 6%. What was the selling price?

8. A farmer buys 32 cows at $55 each. For what sum must he sell his stock to realize 10% on the sum paid?

9. A boy bought a bicycle for $25 and sold it at a loss of 40%. How much did he lose?

10. A ranchman lost 5% of his herd of 4560 sheep. How many sheep were left?

11. A farm costing $6000 was sold at a gain of 7%. What was the amount gained?

12. An orchard has 8 rows of trees with 10 trees in each row. Five per cent of the trees are dead. How many trees are alive?

13. At a city election there were cast 5600 votes, of which the successful candidate received 59%. How many votes did he receive?

Written Exercise

248. 1. Find 36 % of $ 32.50. Check the answer.

PROCESS

32.50
.36
19500
97500
11.7000

EXPLANATION.—36% = .36. Hence, 36 % of $32.50 is the same as .36 of $ 32.50, or .36 × $32.50 = $11.70, the answer.

Check: 1170 ÷ 36 = 32.50

2. 15 % of $ 279.60
3. 75 % of $ 988
4. $16\frac{1}{2}$ % of $ 960
5. 12 % of $ 304
6. 7 % of $ 1234
7. $37\frac{1}{2}$ % of $ 875.50
8. $24\frac{3}{4}$ % of $ 1758.97
9. $87\frac{1}{2}$ % of $ 565.50
10. $3\frac{1}{2}$ % of $ 75.60

Find the Rate

Written Exercise

249. 1. If the base is $ 375 and the percentage $ 52.50, find the rate.

PROCESS

$$\begin{array}{r} .14 \\ \times \\ 375\overline{)52.50} \\ 37\,5 \\ \hline 15\,00 \\ 15\,00 \\ \hline \end{array}$$

EXPLANATION. — Apply the formula $r = p \div b$.

Write the quotient as a per cent.

$.14 = \frac{14}{100} = 14\%$ *Ans.*

Find the rate:

2. Base $ 83.40, percentage $ 4.17.
3. Base $ 66, percentage $ 4.62.
4. Base $ 37.50, percentage $ 13.50.
5. Percentage 540 lb., base 4500 lb.
6. Percentage 5.88 bu., base 36.75 bu.
7. Percentage 559.5 mi., base 746 mi.
8. Percentage $ 882, base $ 840.

Written Problems

250. Use pencil only when necessary.

1. Herbert writes 200 words, but misspells 20 of them. What per cent does he miss?

The problem is to find what per cent one number is of another. What is the base? What is the percentage? What formula to be used?

PROCESS

$$20 \div 200 = \frac{2\cancel{0}}{2\,0\cancel{0}} = \frac{1}{10} = .1 = 1\,\%$$

2. A fruit raiser planted 200 orange trees, 7 % of which died. How many trees died? Which is the base? Which the rate? Which the percentage?

3. A fruit raiser planted 200 orange trees, 14 of which died. What per cent died?

4. Of 300 children enrolled in a school, 5 % are absent. How many are absent?

5. From Problem 3 make up an example in which the base and the percentage are given, and the rate is to be found.

6. With the numbers \$ 500 and 5 %, make up a problem, to find the percentage; then a problem to find the rate.

7. A boy had 200 stamps, but lost 6. What per cent did he lose? In other words, 6 is what per cent of 200?

8. The loss in weight of 800 lb. of wheat in drying was 16 lb. What was the rate of shrinkage?

9. A small army of 600 men has 66 officers. What per cent of the army are officers?

10. A tank filled with 2500 lb. of salt water taken from Great Salt Lake contains 475 lb. of salt. What per cent of salt is there in the lake water?

11. 5000 lb. of water from the Atlantic Ocean contains 180 lb. of salt. What per cent of salt is there in the water?

12. A man at the seashore allows 6600 lb. of salt water to evaporate and he finds that 264 lb. of salt remain behind. What per cent of salt is there in the water?

13. A man with a yearly salary of \$ 1500 spends \$ 112.50 on clothes. What rate per cent of his salary is thus spent?

Find the Base

251. 1. If the percentage is $122.45 and the rate 16 %, what is the base?

PROCESS

$765.31+ *Ans.*

.16)$122.45.00

112

104

96

85

80

50

48

20

16

EXPLANATION. — Apply the formula $b = p \div r$. Change 16% to .16 and multiply both dividend and divisor by 100 in order to remove the decimal point from the divisor. This multiplication may easily be performed by moving the point two places to the right. Why? Check.

Check the answer thus:
$765.31 × .16 = ?

Find the base:

2. Percentage 80.75 T., rate 17 %.
3. Rate $6\frac{1}{2}$ %, percentage 32.5.
4. Rate $5\frac{1}{2}$ %, percentage 41.47.
5. Rate 23 %, percentage 107.18 lb.
6. Rate .06, percentage $30.
7. Percentage 11.4, rate .12.
8. Percentage 1139.45, rate $\frac{13}{100}$.

Written Problems

252. 1. Mr. Jones borrows $400 and pays 6% interest a year. How much interest does he pay yearly?

2. Mr. Jones pays $24. interest a year, which is 6 % of the money he borrowed. How much did he borrow?

SUGGESTION

$$\$24 \div .06 = \$400$$

3. How many pupils in a school of 200 pupils are absent if 3 % are absent?

4. From Problem 3 make an example in which the percentage and the rate are given, and the base is to be found.

5. A man paid $18 interest a year, which was 6 % on the money borrowed. How much did he borrow?

6. If 6 % of a number is 24, what is the number?

7. A merchant sold a motorcycle for 80 % of its cost and received $160. How much did the machine cost?

8. Mary spends 15¢ at a fair, which is 10 % of what she had. How much did she have?

9. In a battle an army lost 1815 men, which was 3 % of the number of men engaged. How many men took part in the battle?

10. An orchardist has 75 orange trees, which is 60 % of his number of lemon trees. How many lemon trees has he?

11. A merchant saves $875 a year, which is 35 % of his earnings. Find his earnings.

12. An agent collected money for me, and I paid him $14.40 for his services. This was 6 % of what he collected. How much did he collect?

Oral Problems

253. 1. Albert has 48¢ and spends 25 % of it for writing paper and 50 % of it for firecrackers. How much does he spend on writing paper? How much on firecrackers?

2. William has 96¢ and spends 50 % of it for entertainment. How many cents has he left? Had he spent only 25 %, how much would be left?

3. A farmer bought a house for $200 and sold it at a loss of $7\frac{1}{2}$ %. What did he receive for it?

4. A ranchman bought 400 sheep from one man, and 75 % as many sheep from another. How many did he buy all together?

5. A merchant buys suits at $20 each and wants to sell them so as to make 40 % on the cost. How high must he mark each?

6. A boy gains $5 by selling a bicycle. This is a gain of 10 % of the cost. What is his per cent of profit?

7. A real estate dealer buys a lot for $2000 and sells it for $2100. What is his per cent profit?

8. If I sell my watch at a gain of $6, I gain 25%. Find the cost and the selling price of the watch.

9. A boy buys a dozen stamps for 25¢ and sells them for 30¢. What is his per cent of profit?

10. A girl had a number of roses. She gave away 10 of them. This was 50% of the whole number. How many roses had she in all?

Written Problems

254. 1. In a ward $12\frac{1}{2}$% of the registered voters did not vote at the last election. There were 70 who did not vote. What was the total registration?

2. Of the votes cast, 55% were for one party and 45% for the other. The winning party had 56 more votes. How many voted?

3. Of the 5800 registered voters in a city, 899 fail to vote. What per cent fail to vote?

4. 65% of the blossoms on a small apple tree failed to develop into fruit. The tree bore 77 apples. How many blossoms did it have?

5. A poultry raiser set 150 eggs. 12% failed to hatch. How many eggs did hatch?

6. A farmer raised 921 bu. of wheat. He sold $33\frac{1}{3}$% of them at $1.05 a bushel and the rest for 95¢. What did he get for the whole crop?

7. If unseasoned lumber is 18% water, what will 125 T. of grain lumber weigh after it is seasoned?

8. An acre of land produces 12,750 lb. of sugar beets. If the beets are $12\frac{1}{2}$% sugar, how many pounds of sugar were obtained from these beets?

9. If a beef weighing 1200 lb. contains 192 lb. of tallow, what per cent of the whole weight is tallow?

10. On an experimental farm 200 seeds were planted to test them. Of these only 105 sprouted. What per cent of the seed was good?

11. If an ounce of flower-seed costs 30¢, and 50% of the seed is good, what is the price per ounce of the good seed?

12. A fruit dealer buys a crate of oranges for $2.50 and sells them at 2¢ each, making a profit of 40%. How many oranges are there in the crate?

APPLICATION OF PERCENTAGE

Oral Problems

255. 1. Sugar costing a merchant 5¢ a pound is sold by him for 6¢ a pound. How much is his profit on 37 lb.? What per cent of the cost is his profit?

2. If a merchant buys sugar at 4¢ a pound and sells it at 5¢, what per cent of the cost is his profit?

3. If an automobile is bought for $1200 and sold later at a loss of 5 %, what is the selling price?

4. A dealer in stationery buys pencils at 40¢ a dozen and sells them at 5¢ apiece. What is his per cent of profit?

Notice that the per cent of gain or loss is always figured on the cost of the goods or on the sum invested.

5. A grocer buys grapefruit at 7¢ each and sells them at 10¢ each. What per cent of profit does he make?

6. He buys lemons at 25¢ a dozen and sells them at 35¢. What per cent is his profit?

Exercises and problems of this character are sometimes classified under the heading Gain or Loss.

Written Problems

256. 1. A city lot was bought for $600, and sold at a loss of 15 %. What was the loss? What was the selling price?

2. A dealer paid $450 for a pair of horses and sold them at a profit of 25 %. Find the selling price.

3. A merchant sold a piano and gained $50. If it cost him $400, what was his per cent of gain?

4. A merchant has $15,000 invested in a store. His profit this year is $3000. What is his per cent of profit?

5. What is the per cent of profit, when you buy at $600 and sell at $700?

6. If I buy table water at $1.10 a dozen bottles and sell it at 2 bottles for 25 ¢, what is my per cent of profit?

7. If I buy pickles at $25.92 for a gross of bottles, and sell at 25 ¢ a bottle, what is my per cent of profit?

8. Clothes that cost $2000 were damaged by fire and sold at a loss of 17 %. How much was lost?

9. What is the gain on bank stock bought at 88 and sold at 96?

Written Exercise

257. Find the selling price, the per cent gain or loss, or the cost:

1. The cost is \$40, the loss is 5 %.
2. Loss 7 %, cost \$1000.
3. The cost is \$16, the gain 25 %.
4. Gain $12\frac{1}{2}$ %, cost \$250.
5. The selling price \$80, the cost \$75.
6. The selling price \$120, the cost \$100.
7. The selling price \$225, the cost \$180.
8. The selling price \$240, the cost \$250.
9. The selling price \$850, cost \$100.
10. The selling price \$1275, the cost \$1500.

Make problems to illustrate these relations.

Discount

258. Merchants, manufacturers, and business houses frequently make a reduction or a discount from the catalogue price or the list price to those who buy goods in large quantities and to those who pay cash for their goods. Discounts are often offered by merchants in order to increase trade.

1. A merchant offered a reduction of 5 % on purchases amounting to \$25 or more. One customer bought \$30 worth of goods. What reduction was made?

Any reduction made from a fixed price is called a **discount.** Discounts are usually reckoned as so many per cent of the **fixed** or **list price.** The price after the discount is taken off is often called the **net price.**

2. A suit of clothes, marked at $35, is offered at 10 % off. How much is the discount and how much is the selling price?

3. Goods damaged by fire were sold at the following discounts:

Marked price:	$15	$39	$60	$120	$24	$77
Discount:	20 %	10 %	5 %	1 %	$12\frac{1}{2}$ %	50 %

Find the reduced prices.

Discount is simply an application of percentage. It involves no new principles.

The marked price or list price corresponds to the base.

The **rate of discount** is the rate per cent.

The **discount** expressed as a sum is the percentage.

Oral Exercise

259. Find the discount and the reduced selling price when the fixed or list price and the rate of discount are:

1.	30¢	10 %	2.	$44	25 %	3.	75¢	50 %
4.	40¢	15 %	5.	$40	75 %	6.	48¢	25 %
7.	56¢	$12\frac{1}{2}$ %	8.	$50	6 %	9.	$1	2 %

Written Problems

260. 1. What is the net price on a set of books, listed at $25, when a discount of 20 % is allowed? When a discount of 5 % is allowed?

2. A shipment of bananas was slightly damaged in transit. It had been valued at $120, but the buyer agreed to take it at a discount of 25 %. How much was the discount in dollars? How much the selling price? What loss did the shipper sustain, if the bananas cost him $95?

3. A lawn mower, listed at $5.40, is sold at 10 % off. Find the selling price.

4. The marked price on a piano being $500 and the rate of discount 20 %, how much is the discount in dollars? If you were given the rate of discount and the amount of the discount, how would you find the list price? If you were given the list price and the discount, how would you compute the rate of discount?

5. An automobile is listed at $1750. From this price there is a discount of 10 % and still another discount of 6%. What is the net price of the automobile?

Written Problems

261. 1. If a set of books, listed at $36, sells for $27, find the rate of discount.

Find the first discount and price. Then find the second or net price.

PROCESS

$\$36 - \$27 = \$9$

$36)\overline{9.00}\ .25$

72

180

180

EXPLANATION. — The discount is $\$36 - \$27 = \$9$.

$9 \div 36 = \frac{1}{4} = 25\,\%$, the rate of discount.

Solve by fractions when possible.

Thus: $9 \div 36 = \frac{9}{36} = \frac{1}{4} = 25\,\%$.

2. Find the net price on goods listed at $\$3360$, when a discount of $33\frac{1}{3}\,\%$ is allowed.

3. A merchant offers cloth at $\$1.50$ a yard, subject to a discount of $16\frac{2}{3}\,\%$. What is the net price a yard? How many yards can be bought for $\$45$? What part of $\$1$ are $16\frac{2}{3}¢$?

4. Find the net price of a table listed at $\$36$, discount $16\frac{2}{3}\,\%$.

5. A store sells desks at $30\,\%$ off. What is the selling price of one marked $\$25.75$?

6. A hardware merchant gives an order for 350 lb. of nails at $24¢$ a pound, discount $35\,\%$. How much was his bill?

7. A man buys 50 ft. of garden hose at $9\frac{1}{2}¢$ a foot, discount $15\,\%$. How much does he pay?

8. A merchant advertises $25\,\%$ discount on cash purchases. A lawn mower, listed at $\$7.65$, can be bought for what sum?

9. An article, marked $2.50, sells for $2.25. What is the rate of discount?

10. If an article, listed at $3.80, sells at $3.42, find the rate of discount.

262. Find the numbers that belong in the blank spaces:

List Price	Rate of Discount	Net Price	List Price	Rate of Discount	Net Price
1. $4.50	16⅔%		11. $ 525.75		$ 515.23
2. 3300	33⅓%		12. 278.40	4%	
3. 4760	12½%		13. 979.97	5%	
4. 860		$731	14. 275.50		259.07
5. 950	16%		15. 478.88		431.00
6. 200		150	16. 9648.72	8%	
7. 540		351	17. 4887.75	10%	
8. 1750	4%		18. 8425.25	16⅔%	
9. 1800		90	19. 9637.75		7510.20
10. 2175	5%		20. 7475.40	25%	

Commission

263. When a person is engaged as an agent to transact business for another, he usually is paid a certain per cent for his services. The amount thus paid is called **commission**.

1. An agent sells a lot for me at $800 and I pay him 5% of this sum for his services. How much was his commission?

2. John Smith sells for John Brown 10 A. of land at $50 an acre at a commission of 3%. How

much was Smith's commission? After taking out his commission, how much more does he turn over to Brown?

Commission is one of the applications of percentage. The sum collected, the value of the goods or property bought or sold, the sum mentioned in the contract, corresponds to the base.

The **rate** of commission is the rate per cent.

The commission expressed as a sum is the percentage.

Oral Problems

264. 1. If an agent charges 8% of one month's rent for his services, what is his commission for finding tenants for a house that rents for $60 a month? For a house that rents for $75 a month?

2. A lawyer collects a debt of $500 and receives 10% of it for his services. How much does he send his employer after deducting his commission?

3. The contract for building a house is $6000. The architect is allowed 5% for supervising the construction. What is his commission?

4. An agent sells books on commission. He sells a $40 set and receives a commission of 25%. What is his commission?

5. A doctor gave his accounts to a collector. If the collector receives 20% for collections, what will he receive on a $25 account?

Written Problems

265. 1. I bought through an agent 10 bags of coffee, each containing 120 lb., at 15¢ a pound. The agent charged 4% commission. How much did I pay the agent? How much did the coffee cost me, including the commission?

2. Acting as an agent, I buy for W. Burgess & Co., 75 barrels of flour at $4.50 a barrel. I charge a commission of 3%. How much commission do I receive?

3. A real estate agent receives for his services 5% of the rents collected on three houses, one renting for $40 a month, another for $75 a month, and the third for $87 a month. How much does he receive a year for his services?

4. A merchant gave bills, aggregating $360, to an agent for collection. The latter succeeded in collecting only $240 and reported the remainder uncollectible. How much does the merchant lose if he pays the agent 10% on the amount collected? What per cent of the $360 did the merchant lose?

5. An agent for farm machinery sells to a farmer a mowing machine for $250. After deducting his commission of $12\frac{1}{2}\%$ he sends the balance to the manufacturers. How much does he send?

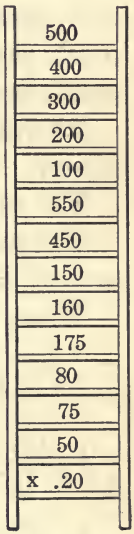
6. An auctioneer received $23.40 for selling $588 worth of goods. What was his rate of commission?

7. I, as agent, sold a carload of fruit for $476. If I retained 7% for my services, how large a sum did I transmit to my employers?

8. A commission merchant sold 60 boxes of oranges at $2 a box. Find his commission at 5% and the amount sent to his employer.

9. An agent collects house rent for the owner of the house and charges 5% of the rent of his services. If the rent is $50 a month, how much does the agent get a year?

10. Find the commission on the sale of $700 worth of goods at the rate of 7% on the selling price.



Drill Exercise

266. Use drill device for Percentage, Commission, and Discount. Change the rate. Use 5, 10, 50, 75, etc. Change the numbers.

INTEREST

Introduction

267. If I borrow money from John Smith, not only must I return the money to him after a certain period of time mutually agreed upon but I must pay him for the use of his money. This extra sum that I pay him is called **interest.**

Interest is the money paid for the use of money.

The sum borrowed is called the **principal.**

The principal and the interest, added together, give the **amount.**

When we say that the rate of interest is 4 % or 6 %, we mean that the rate per annum (by the year) is 4 % or 6 %.

1. What is the interest on $300 for 1 yr. at 6 %? at 3 %?

2. What is the interest on $500 for 1 yr. at 5 %? at 4 %?

3. What is the interest on $600 for 6 mo. at 5 %? In finding the interest for 6 mo. you first find the interest for 1 yr. and then divide by 12.

Oral Exercise

268. Find the interest for 1 mo.:

1. \$500 at 7%
2. \$60 at 6%
3. \$1000 at 6%
4. \$7000 at 4%
5. \$700 at 8%
6. \$70 at 4%
7. \$800 at 5%
8. \$900 at 7%

Find the interest on each of these sums for 6 mo.

Tell how to find the interest on \$100 for 6 mo. at 6%.

Tell how to find the interest on a sum of money for 3 mo.

Oral Exercise

269. Find the interest for 6 mo.; for 3 mo.:

1. \$200 at 6%
2. \$800 at 5%
3. \$300 at 8%
4. \$900 at 4%
5. \$500 at 5%
6. \$1000 at 6%
7. \$700 at 8%
8. \$800 at 9%
9. \$400 at 7%
10. \$2000 at 3%

Tell how to find the interest on a sum of money for 3 mo.; for 1 mo.; for any number of months.

Written Exercise

270. 1. Find the interest on \$350 at 6% for 6 mo.; for 4 mo.; for 3 mo.

PROCESS

$$\overset{7}{\cancel{\$350}} \times \frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{100}}} \times \frac{1}{\cancel{2}} = \frac{21}{2} = 10\tfrac{1}{2} = \$10.50$$

EXPLANATION. — Multiply the sum at interest by the rate of interest expressed as a decimal fraction and by the number of months expressed as part of a year. Use cancelation when possible.

2. What is the interest on $475 at 5% for 6 mo.? for 3 mo.? for 4 mo.?

3. Reckon the interest on $465.50 for 2 yr. at 6%.

4. Compute the interest on $505.50 for 3 yr. at 5%.

5. Find the interest on $675.25 for 5 yr. at 3%.

6. Find the interest on $325.40 at 4% for 1 yr. and 6 mo.

7. By how much does the yearly interest on $375 at 5% exceed the yearly interest on $400 at 4%?

8. How much are the principal and interest together on $575 at 5% at the end of one year? What is the amount?

9. Find the amount at the end of two years of $565.50 at 4%.

10. Find the amount of $1000 for 3 yr. at 6%.

Oral Exercise

271. 1. What is the interest on \$120 at 5% for 1 yr.? 1 mo.?

2. Find the interest on \$240 at 10% for 1 yr. 1 mo.

3. Compute the interest on \$200 at 6% for 1 yr.; for 1 mo.

4. How much interest must you pay on \$300 at 8% for 1 yr. 1 mo.?

5. What is the interest on \$300 at 6% for 2 mo.? 2 mo. are what part of 12 mo.?

6. What is the amount of \$200 at 6% for 1 yr. and 1 mo.?

7. Find the amount of \$600 at 4% for 1 yr. and 1 mo.

8. Find the interest on \$120 at 5% for 1 mo.; for 7 mo.; for 11 mo.

9. What is the interest on \$240 at 10% for 11 mo.?

10. What is the interest on \$500 for 2 yr. 8 mo. at 3%?

Written Exercise

272. Find the interest:

	Prin.	Rate	Time		Prin.	Rate	Time
1.	\$300,	6%,	2 mo.	2.	\$200,	3%,	2 mo.
3.	\$100,	5%,	3 mo.	4.	\$50,	6%,	4 mo.

	Prin.	Rate	Time		Prin.	Rate	Time
5.	$150,	6%,	6 mo.	6.	$500,	6%,	4 mo.
7.	$600,	4%,	5 mo.	8.	$400,	4%,	3 mo.
9.	$430,	5%,	6 mo.	10.	$650,	6%,	9 mo.
11.	$1000,	4%,	8 mo.	12.	$1560,	5%,	8 mo.
13.	$2575,	6%,	10 mo.	14.	$8975,	7%,	7 mo.

15. $6875, 8%, 8 mo.

Study Exercise

273. Find the interest and the amount:

1. $525.75 for 2 yr. and 5 mo. at 3%.

Process

Int. for 1 yr.
$525.75
× .03
$15.77

Int. for 2 yr.
$15.77
× .02
$31.54

Int. for 1 mo.
$1.31
12)$15.77

Int. for 5 mo.
$1.31
× 5
$6.55

$31.54 Int. for 2 yr.
6.55 Int. for 5 mo.
$38.09 Int. for 2 yr. 5 mo.
$525.75 Principal
38.09 Interest
$563.84 Amount

Explanation. — Find the interest on the sum at the required rate for one year. Find the interest for the number of years. Find the interest for one month, $\frac{1}{12}$ the interest for 1 year. Find the interest for the number of months. Add the interest for the years and the months. Add the principal to the interest to get the amount.

Written Exercise

274. Find the interest and the amount:

1. \$307.25 at 6% for 5 yr.
2. \$906 at 5% for $1\frac{1}{2}$ yr.
3. \$175.40 at 4% for $1\frac{1}{3}$ yr.
4. \$263 at 7% for 1 yr. 3 mo.
5. \$650 at 5% for 1 yr. 1 mo.
6. \$780 at 4% for 2 yr. 9 mo.
7. \$690 at 7% for 3 yr. 8 mo.
8. \$360 at 5% for 4 yr. 7 mo.
9. \$1200 at 9% for 5 yr. 5 mo.
10. \$94.60 at 6% for 1 yr. 6 mo.

Written Exercise

275. Find the interest and the amount:

1. \$785.45 for 1 yr. 2 mo. at $4\frac{1}{2}$%.

PROCESS

\$785.45 × .045 = 392725 + 314180 = \$35.34525 (1 yr.)

6)\$35.35 = \$5.89 (2 mo.)

\$35.34 + 5.89 = \$41.23 *Ans.*

EXPLANATION

$4\frac{1}{2}\% = .045$

2 mo. $= \frac{1}{6}$ of 1 yr.

Use \$35.35 as interest for 1 yr.

2. \$375.35 for 3 yr. 4 mo. at $5\frac{1}{2}$%.
3. \$750.45 for 2 yr. 6 mo. at $4\frac{3}{4}$%.
4. \$4850.75 for 5 yr. 4 mo. at $5\frac{1}{4}$%.

5. $7525.25 for 1 yr. 8 mo. at $2\frac{7}{8}\%$.

6. $8759.40 for 4 yr. 9 mo. at $2\frac{5}{8}\%$.

7. $2002.60 for 1 yr. 10 mo. at $4\frac{3}{8}\%$.

8. $5000 for 1 yr. 1 mo. at $6\frac{1}{8}\%$.

Written Problems

276. 1. I borrow $375 at 7% for 1 yr. and 3 mo. How much must I pay back at the end of that time?

2. I borrow $2000 at 6% for 2 yr. With this money I buy a small lot and build upon it a cottage. At the end of two years, I sell the cottage and lot for $2500. Do I gain or lose by this transaction? How much?

3. A real estate agent buys a house for $1500. He must pay $50 every year for taxes, insurance, and repairs. For how much per month must he rent the house to make 6% on the cost of the house?

4. I borrow $60 from Mr. Brown for 9 months at 6%. How much interest must I pay him?

5. Mr. James borrows $75 from a friend and promises to pay it back in 4 months at 6%. How much is the interest for that time? Find the amount that Mr. James must pay at the end of the 4 months.

Interest Accounts

277. Any person ten years of age or over may deposit money at interest at the post office, with the security of the United States government for repayment. Money may also be deposited in banks, savings banks, and trust companies where it will draw interest according to the rules of the institution.

In postal savings banks, interest is allowed at the rate of 2% for each full year that the money remains on deposit. The interest is computed from the first day of the month following the day on which the money is deposited.

In savings banks and trust companies the interest rate varies. It is usually 3% or $3\frac{1}{2}$%. The interest is usually computed from the first day of the month following the day on which the money is deposited. In these institutions interest is usually paid quarterly or semiannually.

Written Exercise

278. Compute the interest on deposits in postal savings banks as follows:

1. $50, deposited May 6, 1914, withdrawn June 5, 1915.

2. $70, deposited Jan. 4, 1914, withdrawn Dec. 17, 1914.

3. $95, deposited April 29, 1915, withdrawn May 1, 1917.

4. $75, deposited June 15, 1914, withdrawn June 1, 1916.

5. $64, deposited Sept. 3, 1913, withdrawn Oct. 1, 1915.

Written Exercises

279. 1. Find out how often interest is paid on accounts in the local savings bank.

2. What rate of interest is paid by the local bank on savings accounts?

3. Find out how interest is computed on savings bank accounts.

4. Make an account of savings deposited in a savings bank. Can you compute the interest on the account at the end of one year?

5. How is interest computed using the interest table?

BILLS AND CHECKS

Introductory

280. 1. Mrs. James Downs makes several purchases at the store of Milton Bros. & Co. The articles are shown in the following bill:

SAN FRANCISCO, Sept. 15.

MRS. JAMES DOWNS,

BOUGHT OF MILTON BROS. & CO.,
165 Market St. Phone 204

	2½ lb. steak	28¢	$	70	$	
	1 lb. tea	45¢		45		
	4 lb. butter	35¢	1	40		
	10 lb. beet sugar	5¢		50	3	05

What are the articles purchased?

What is the price of each article?

How much is the entire bill?

Where and when were the articles sold?

Who is the buyer? The seller?

After the bill was paid by Mrs. Downs, the clerk wrote the words "Received payment, Milton Bros. & Co. per J. F." The words show that the bill is paid. The bill is now called a "receipted bill."

"J. F." are the initials of the clerk's name.

The receipted bill should be kept by the buyer to show that the bill has been paid.

2. Mr. J. W. Jones goes to the bookstore of Burgess & Co. and buys books and stationery. He receives a bill and pays it. What does the clerk in the store write on the bill to show that it has been paid?

The **receipted bill** is as follows:

MADISON, WIS., Dec. 1.

MR. J. W. JONES,

BOUGHT OF BURGESS & CO.,

105 State St. Phone 714

Nov.	12	1 arithmetic	$	45	$	
		1 geography		85		
		3 lead pencils 5¢		15	1	45
		Received payment Burgess & Co. per B. C.				

3. Mrs. A. Smith orders from Gerry & Co.:

4 lb. coffee at 30¢ a pound

18 lb. sugar at $5\frac{1}{2}$¢ a pound

5 gal. molasses at 60¢ a gallon

Make out the bill which Gerry & Co. send Mrs. Smith. How much does Mrs. Smith owe? Indicate what must be written on the bill to show that it has been paid.

4. Make out bills of the following items, using your father's name as buyer and the name of your local merchant as seller:

2 bags salt	@	$.10	15 yd. silk	@	$1.50
3 bu. potatoes	@	.75	12 yd. muslin	@	.09
10 lb. prunes	@	.12½	9 yd. lace	@	.65

5. Make out 4 bills for goods purchased of the local dealers.

6. If a person has money deposited in a bank, he may pay a bill by **check**, thereby ordering the bank to pay the sum specified.

FORM OF A CHECK

No. 39 **Bank of Wisconsin**

Madison, Wis., July 19, 1915.

Pay to the order of James Ward $35.45

Thirty five and 45/100 *Dollars*

Donald Gregg.

Name the sum paid. Who ordered it paid?

To whom is the amount paid?

In what bank does Donald Gregg have his money deposited?

How much does this check reduce the amount of Donald Gregg's deposit in the bank?

When James Ward presents the above check at

the bank, he writes his name on the back of the check and either receives the $ 35.45 from the bank in cash or has it added to his own deposit in the bank. The writing of his name across the back is called indorsing the check and is evidence that the sum has been received by him. The bank returns the indorsed check to Donald Gregg, who keeps it as evidence that the sum has been paid.

7. John Ward has his money deposited in the First National Bank of Lawrence, Kansas. He wished to pay Richard March of that city the sum of $ 105.25. Write the check.

8. Write out checks of your own on the supposition that you had money deposited in the Second National Bank of Liliputia.

PROBLEMS ON INDUSTRY

281. Table for Reference in Agricultural Problems

Kind of Crop	Average Amount of Fertilizers per Acre	Average Amount of Seed per Acre	Average No. of Weeks to Maturity
Corn	10 T.	10 qt.	$15\frac{1}{2}$
Fall wheat	18 T.	2 bu.	42
Oats	$7\frac{1}{2}$ T.	$5\frac{1}{2}$ pk.	13
Potatoes	$17\frac{1}{2}$ T.	14 bu.	16
Turnips	10 T.	2 lb.	10

Problems Based on the Table

282. 1. How much more fertilizing material is used per acre for fall wheat than for corn? What per cent of 10 T. is this excess? Comparing potatoes and corn, what per cent of 10 T. is the excess?

2. How much less fertilizing material per acre is used for oats than corn? What per cent of 10 T. is this difference?

3. What per cent of the fertilizers used for corn is that used for wheat?

4. How much fertilizing material is needed for $50\frac{1}{2}$ A. of potatoes? 7 A. of oats?

5. How many acres of land for wheat can be fertilized from 50 loads of material, $\frac{3}{4}$ T. per load?

6. How many quarts more of wheat than of corn are needed on 8 A.?

7. How many pecks less of oats than of wheat are seeded on 10 A.?

8. How many more acres can be seeded with 154 bu. of potatoes than with 20 lb. of turnips?

Written Problems

283. Refer to the table on page 237 for facts necessary to the solution of the problem.

1. When may we expect corn planted on May 15th to mature?

PROCESS

May 16 da.
June 30 da.
July 31 da.
Aug. 31 da.
108 da.

EXPLANATION. — Count off the weeks on a calendar, or proceed thus: $15\frac{1}{2}$ wk. $= 108\frac{1}{2}$ da.; to Sept. 1 are 108 da. Therefore the required date is Sept. 1 or 2.

2. How many more acres can be seeded with 493 bu. of potatoes than with 63 bu. of turnips?

3. About what date may we expect wheat seeded Sept. 25th to mature?

4. How many bushels and pecks less of oats are seeded on $74\frac{1}{3}$ A. than of wheat over an equal area?

5. How much fertilizing material is needed for $210\frac{1}{2}$ A. of potatoes and $27\frac{3}{4}$ A. of oats?

6. What per cent of the fertilizer per acre on a wheat field are the fertilizers commonly put on a 1-acre potato patch?

7. What per cent of the fertilizers for 4 A. of oats are the fertilizers needed for 2 A. of wheat?

8. Mr. Jones agrees to cover $12\frac{1}{2}$ A. of corn land with fertilizers at \$.75 a load, and to allow 5% discount for payment within 10 days. How much will he be paid, if he allows the discount?

9. A ranchman buys potatoes at \$.60 a bushel for planting $50\frac{1}{2}$ acres and is allowed a discount of 7%. How much does he pay?

10. In seeding tobacco, 1 oz. of seed is applied to 6 sq. rd. How many ounces is this for 1 sq. rd.? For 160 sq. rd. or 1 A.?

11. What fraction of an ounce of seed is that for 1 sq. yd.?

12. In case of tomatoes, $6\frac{1}{2}$ oz. of seed usually suffice per acre. What part of an ounce is needed for 1 sq. rd.?

13. Tomatoes seeded in the Southern states ordinarily ripen in 16 weeks, but this year they take 5 days longer than the usual time. On what date should they be picked if they were seeded Feb. 15?

14. A farmer plans to harvest his oats on July 25. What would seem to be the most favorable time for putting in the seed?

15. In planting sweet potatoes $11\frac{1}{2}$ bu. are usually needed per acre. How many bushels must be procured for 17.6 A., if 5% more than the amount named are to be planted per acre?

Written Problems

284. 1. In the spring and summer the salmon leave the sea and proceed up the rivers to places where the river water is cool. There they deposit their eggs and then die. Young fish develop from the eggs and float downstream into the ocean. Salmon starting up the Columbia River early in March move at first only about $2\frac{1}{2}$ miles a day. How many rods a day is this? How many yards a day? How many feet a day?

2. If they travel 13,200 ft. a day, how far do they go in an hour on an average?

3. Later the salmon move faster, reaching, perhaps, a speed of 6 mi. a day. How far can they go at that rate in the month of May? What per cent of 6 mi. is the former rate of $2\frac{1}{2}$ mi.?

4. Some salmon go up the Columbia River 1000 mi. to tributaries fed by melting snow in Idaho. If a fish left the ocean March 1, traveled 1000 mi., and reached the cold waters in Idaho on Oct. 1, how many days was it traveling? How many miles did it average a day. If it made 4.7 mi. a day, how many feet a day did it travel?

5. When salmon come to a low waterfall, they leap atop of it. A salmon jumps atop of one 12 ft. high. How many times higher is this than Harry's running-high-jump record of 4 ft. 6 in.?

6. While ascending streams the salmon eat nothing and consequently lose weight. If a salmon weighed 20 lb. at the beginning of its trip and 16 lb. at the end of it, what per cent of its original weight did it lose?

7. Another salmon weighed at first 30 lb. and then lost 16 %. What was its final weight?

8. The young fish float down the river, tail foremost. If it takes them 11 months to descend 1000 miles to the ocean, how many miles do they make a month?

9. If a young salmon is 4 in. long when it first reaches the ocean, and is $2\frac{1}{2}$ ft. long when later it ascends the river, how many times has it increased its length while in salt water?

10. No other fish is canned so extensively as the salmon. The Columbia River yielded during the 6 yr. ending 1880 about 150,000,000 lb. of salmon. If the fish averaged 30 lb., how many fish were killed?

11. In one year 31,500,000 pound cans of salmon were shipped from the Oregon coast, valued at $3,300,000. Compute to cents and mills the value per can.

12. The value of the entire salmon catch on our west coast, including Alaska, exceeds $13,000,000 annually. If 65% of this is from canning the species known as chinook salmon, how many dollars come from other species of salmon?

13. A firm desire to expend $75,000 upon a salmon-canning factory, but have only $36,000 in cash. They borrow the balance at $3\frac{1}{2}\%$. How much interest must they pay annually?

14. An agent sells 50,000 cans of salmon for this firm, at $.09 a can, and charges 2% commission. How much money does he send to the firm after deducting his commission?

15. It is feared that the salmon will be exterminated before many years. One year Oregon yielded 39,500,000 lb. of salmon. If the year following 10% less was canned, how many pounds were canned?

16. Salmon are known to have reached the weight of 100 lb. What per cent is this of the average weight of 25 lb.?

17. In Monterey Bay salmon are caught by trolling, the hook being baited with sardine; 25 fish by one line is a big day's catch. If the fish caught average 19 lb. and sell at $.05 a pound, what is the value of a day's catch?

18. If the speed in trolling is 4 mi. an hour, and a man trolls $7\frac{2}{5}$ hr., how far will he have gone?

19. If 5 gallons of oil are used in moving the boat during 7 hours, oil costing \$.16 a gallon, what is the cost of oil for a 21-hour cruise?

20. One observer found the average weight of salmon in the Columbia River to be 22 lb. and in the Sacramento River 16 lb. What per cent was the latter of the former?

21. In trolling in Monterey Bay, fishermen let out about 150 ft. of line. Through what distance in yards must the line be drawn in while catching 24 fish?

Written Problems

285. This diagram shows the standard method of cutting meat. Suppose the weights and pieces of the different kinds to be as follows:

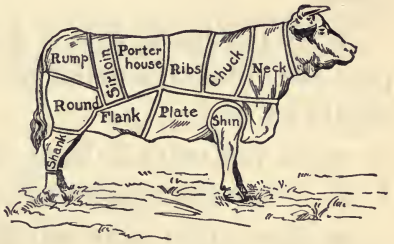


Table for Reference

Neck, 27 lb. at \$.13	Porterhouse, 95 lb. at \$.30
Chuck, 139 lb. at \$.15	Sirloin, 41 lb. at \$.25
Ribs, 74 lb. at \$.18	Flank, 49 lb. at \$.15
Shin, 63 lb. at \$.06	Rump, 41 lb. at \$.15
Plate, 120 lb. at \$.13	Round, 123 lb. at \$.20
Shank, 27 lb. at \$.06	

Problems Based on the Table

286. 1. Find the cost of 11 lb. of each kind of meat.

2. Find the cost of 12 lb. of each kind of meat.

3. What is the cost of $10\frac{1}{2}$ lb. of neck? of round? of chuck?

4. What must you pay for $2\frac{1}{2}$ lb. of ribs? of shin? of flank? If the answer has $\frac{1}{2}$ cent, figure 1 cent in place of $\frac{1}{2}$.

5. What is the cost of 2 lb. of porterhouse and 3 lb. of plate?

6. Find the cost of 5 lb. of rump and 3 lb. of chuck.

7. A purchaser gets 3 lb. of porterhouse and pays the bill with a dollar bill. How much change should he get?

8. If you order 3 lb. of sirloin and hand over \$.75, how much change will you receive?

9. How much more will 30 lb. of ribs cost than 35 lb. of chuck?

10. Mr. Jones buys a piece of flank weighing $30\frac{1}{2}$ lb. and a piece of shin weighing $20\frac{1}{2}$ lb. How much does he pay for both pieces?

11. How much more or less than the piece indicated above does a man pay for all the porterhouse and all the sirloin of that one animal, if he buys at the rate of \$.22 a pound?

Written Problems

287. 1. In 1888 the steamship *Philadelphia* was built, 560 ft. long and 63.3 ft. broad. How many times the breadth was the length? What per cent of the length was the breadth?

2. The *Columbia*, built in 1901, was 503 ft. long and 56 ft. broad. Was this broader than the *Philadelphia*, in proportion to its length, or less?

3. If you divide the length by the breadth, you find what is often called the "ratio" between the length and the breadth. Find this ratio for each of the following steamers of the Atlantic Transport Line:

Mesaba, length 482.1 ft., breadth 52.2 ft., depth 31.6 ft.
Minnehaha, length 600.7 ft., breadth 65.5 ft., depth 43.3 ft.
Minnewask, length 616 ft., breadth 66 ft., depth 44 ft.

4. In each case the ratio was found to be not far from ——. It is found that steamers built on this ratio make the best travelers.

5. Find the ratio between the length and depth of the steamers in Exercise 3. Is this as fixed as the other ratio?

6. The *Lusitania*, built in 1906, was 790 ft. long, 88 ft. broad, and 60.6 ft. deep. How many feet longer is this than the *Minnewask*? What per cent of the length of the *Minnewask* is this excess?

7. The *Olympic* is 882 ft. long. What per cent is this of the length of the *Mesaba*?

8. The *Lusitania* has a horse power of 70,000, the *Minnewask* of 1616. What is the ratio of the former to the latter?

9. The largest locomotive for moving trains has 4000 horse power. What per cent is this of the *Lusitania's* horse power?

10. A knot used in indicating distances on the ocean is equal to 6086 ft. How far is this in miles?

11. In 1856 the *Persia* made the trip between New York and Queenstown, England, in 9 da. 15 hr. 45 min. In 1908 the *Lusitania* made this trip in 4 da. 15 hr. Could the *Lusitania* have gone to Queenstown and back to New York in the time it took the *Persia* to go one way? What is the difference in time of the two steamers for the trip one way?

Written Problems

288. TABLE FOR REFERENCE

PLACES	RATE		PLACES	RATE	
	Day Message	Night Message		Day Message	Night Message
Alabama	60–4	50–3	Louisiana	60–4	50–3
California	1.00–7	1.00–7	New Hampshire	35–2	25–1
Colorado	75–5	60–4	New York City	20–1	20–1
Connecticut	25–2	25–1	Tennessee	50–3	40–3
Dist. of Columbia	30–2	25–1	Wisconsin	50–3	40–3
Kentucky	50–3	40–3	Yukon, Dawson	4.25–29	4.25–29

EXPLANATION. — These rates are between New York City and the places named. A day rate, "60–4," means 60¢ for 10 words and 4¢ for each additional word. A night rate, 50–3, means 50¢ for 10 words and 3¢ for each additional word. Address and signature are free.

A **Night Letter** is different from a night message. The standard day rate for 10 words is charged for the transmission of 50 words or less, and $\frac{1}{5}$ of such standard day rate for 10 words is charged for each additional 10 words or less.

A **Day Letter** of 50 words or less is transmitted at one and one-half times the standard night letter rate.

Problems

289. 1. Find the cost of sending a 15-word day message from New York City to Montgomery, Ala. How much cheaper is a night message?

2. What is the cost of a 50-word night letter from New York City to Montgomery? What is the cost, if sent as a day letter?

3. What is the cost of a 12-word message from New York City to Dawson?

4. A man in New York City sends a 20-word telegram to a friend in a distant part of that city. How much does he pay for it? Does he save anything by sending it as a night message?

5. Find the cost of wiring the following message from New Haven, Conn., to a home in New York City: "Yale-Harvard football game a tie."

6. What is the cost of a 60-word night letter sent from New York City to Denver, Col.?

7. Find the cost of sending a 15-word telegram from New York City to New Orleans, La., at day rates. How much cheaper is a night message? What per cent of the cost of a day message is this saving?

8. A 25-word day message sent from New York City to Madison, Wis., costs how much more than a night message? What per cent of the cost of the day message is saved by sending a night message?

THE WESTERN UNION TELEGRAPH COMPANY

1.45 P.M.

Chicago, Feb. 15, 1911.

To The Macmillan Company,
64-66 Fifth Avenue, New York City.

Send by express ten copies Tarr's New Physical Geography, twenty copies Coleman's The People's Health.

John Lake.

9. If the rates for Illinois are the same as for Wisconsin, what is the cost of this telegram, night rate?

10. Write telegrams to some of your acquaintances, and compute the cost of sending each.

11. What per cent of the rate to Louisiana is the rate to New Hampshire for 10 words? For 20 words?

12. In 1870 the Western Union Telegraph Co. operated 54,000 miles of telegraphic lines; in 1908 this had increased to 209,000 miles. What per cent of the miles in 1870 is the increase in miles shown in 1908? On an average, what was the per cent of increase per year?

13. The number of messages sent in 1870 was 9 million; that sent in 1908 was 63 million or sevenfold the former number. Which increased at a more rapid rate, the number of messages or the number of miles of line?

14. Write messages and letters, then ascertain the cost of sending them from your town to neighboring towns.

REVIEW

Miscellaneous Problems

290. 1. One year the Bureau of Engraving in Washington printed over 35,000 postage stamps a minute. If these stamps are $\frac{7}{8}$ in. long and are placed end to end, how many inches long will the line thus formed be? How many feet long will it be? Yards?

2. If these stamps are $\frac{7}{8}$ in. long and $\frac{5}{8}$ in. wide, how many square inches of area can be covered with them? How many square feet? How many square yards?

3. It is found that, owing to the wear of the stream upon the rock, Niagara Falls recedes at the rate of 4 ft. a year. How far back will it move, at this rate, in 100 yr.? How far has it moved during *your* lifetime?

4. If 1 cu. ft. of anthracite coal weighs 93.5 lb., how many cubic feet will weigh 3 T.?

5. Pressed brick weighs 140 lb. per cubic foot. Find the weight in tons of a brick wall 30 ft. long, 12 ft. high, and $1\frac{1}{2}$ ft. thick.

6. A cubic foot of water weighs 62.4 lb., a cubic foot of ice weighs 57.4 lb. What is the

difference in weight of a cubic yard of ice and an equal volume of water?

7. A building for a poultry show is 200 ft. by 152 ft. How many feet is it around the building? How much floor space is there in the building? If there are over 100 exhibitors and half of the entire floor space is given over to them, how much floor space may be assigned to each?

8. A trough is 6 ft. long, 2 ft. wide, and 1 ft. deep. How many cubic feet of water does it hold?

9. A trunk is 3 ft. long, 2 ft. wide, and 1.5 ft. deep. Find its capacity in cubic feet.

10. A box is 6 in. long, 3 in. wide, and 2 in. deep. How many blocks, of the shape and size of cubic inches, may be packed into this box?

11. George rode his motor cycle for 3 hr. at an average speed of 27.75 mi. an hour. How far did he go?

12. A schoolroom is 9.75 yd. long and 8 yd. wide. How many square feet in its floor area?

13. Mr. Jones has 500 chickens on a plot of ground 150 ft. square. How many feet of fence has he around the plot? How many square feet of area are there in the plot? How many square feet is this for every chicken?

14. One season a baseball team made 18 trips between Chicago and Milwaukee. The railroad

fare amounted in all to $275.40. What was the fare for the team for each trip? What was the fare per individual, if there were 9 men on the team?

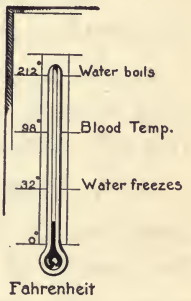
15. If the fare is $2.15 to a place 86 mi. distant, what is the fare per mile?

16. Martha spent $20.15 during the month of May. How much was this a day?

POSITIVE AND NEGATIVE NUMBERS

Introduction

291. The thermometer commonly used in America is the Fahrenheit (F.), in which the freezing point of water is marked 32°, and the boiling point 212°.



1. How many degrees are there between the freezing and the boiling points of water?

2. How many degrees are there between the freezing point of water and the temperature of the blood?

3. At how many degrees above blood temperature does water boil?

Problems in Temperature

292. 1. How much must the blood temperature be reduced to reach the freezing point?

2. If at 2 o'clock P.M. the thermometer reads 91° and is steadily falling at the rate of $4\frac{1}{2}$° an hour, at what time will the temperature be 64°?

3. Between 8 A.M. and 1 P.M. the mercury rose from 65° to 88°. At what average rate per hour did it rise?

4. Temperatures below zero are designated by $-$ (minus). For six mornings in succession the temperatures were at 6 o'clock: $-7°$, $6°$, $-1°$, $-10°$, $4°$, $-3°$. What was the average temperature for these six mornings?

5. In freezing ice cream the mixture of ice and salt reduces the temperature from 60° F. to 28° F. in 12 minutes. What was the average drop of temperature per minute?

6. An animal was killed and the carcass put in cold storage, where its temperature was reduced $69\frac{2}{5}\%$. What is the temperature of cold storage?

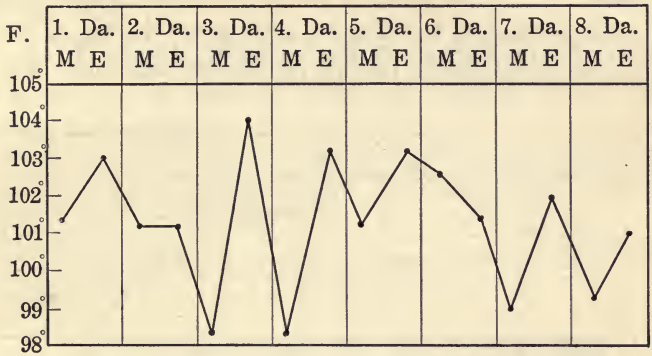
7. At Mobile, Ala., the average temperature one year for the month of January was 50°, and for the month of July 80°. The hottest day in the same year reached 102°, the coldest $-1°$. Find the difference between the mean temperatures for January and July. Find the difference between the extreme temperatures for the year.

8. Read the thermometer each hour of the school day. Record and tabulate the readings. Find the average temperature.

Temperature Chart

293. This chart shows the temperatures of a patient, taken mornings and evenings for 8 successive days. Such records often furnish important information to the physician. This chart shows

not only how much the temperature changed, but also how rapidly.



Exercise Based on the Chart

294. 1. Read the temperature for each day; estimate the fractions of degrees.

2. How many hours intervene between two successive readings of temperature?

3. On what day do you see the greatest difference in temperature?

4. On what day the least?

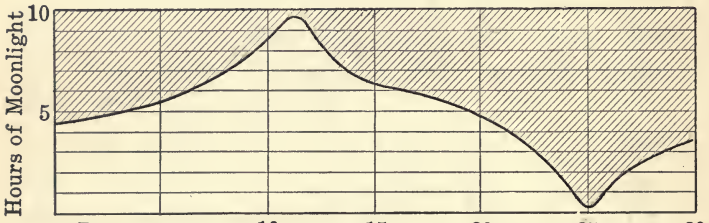
5. When the temperature changes rapidly during 12 hr., is the line connecting the two temperatures steep or not?

6. The line connecting the two temperatures of the second day is level. What does this show?

7. Which line is steeper, the one for the first day or the one for the third? What does this show?

Moonlight Chart

295. This chart shows the hours of moonlight on cloudless nights in Boston, one year during June. One small space to the right stands for one day;



MOONLIGHT CHART FOR JUNE

one small space up stands for one hour. Spaces from the bottom line up to the heavy top line show the hours of moonlight. There are about 9 spaces in the heavy top line, showing that the nights were 9 hr. long. On June 5 there were nearly 5 spaces up to the curved line, showing that there were nearly 5 hr. of moonlight during the night, if cloudless.

Problems Based on the Chart

296. 1. During what part of the night following June 5 did the moon shine?

2. Find the number of hours of moonshine during the nights following June 10, June 15, June 20, June 25, June 30.

3. Express by a common fraction the part of the night that was lighted by the moon on each of these dates.

4. For each of these dates express in per cent the part of the time of the night during which the moon was shining.

5. On what date was nearly the whole night illuminated by the moon?

6. On what date was there practically no moon?

7. On what dates did the moon shine half the night?

8. An electric light company agrees to supply street lights in a Boston suburb during the hours of the night when the moon did not shine, as shown by our chart. How many hours were the streets lighted on the 5th, 10th, 15th, 20th, 25th, of June?

9. If a certain district was lighted at the rate of $1.25 an hour, what was the cost of lighting for each date named?

10. If a rival company offered to supply the same kind of light at 15% less, what would have been the charge for June 5?

11. On what night was there no electric light? On what night was the electric light on all the time?

12. How much was saved on the night of June 20 by turning off the electric light during moonlight?

13. By looking at the chart, do you judge that the expense of lighting is reduced to about $\frac{1}{2}$, by following the moonlight schedule instead of an all-night schedule?

DOMESTIC POSTAGE

297. To all parts of the United States, including Hawaii, Porto Rico, and the Philippine Islands, also to Canada, Mexico, and the Republic of Panama,—First-class matter: Letters or sealed matter 2¢ an ounce or fraction thereof; this rate holds also for letters to and from England and Germany, postal cards 1¢ each; with paid reply card 2¢ each.

Second-class matter: Newspapers and other periodical publications; when sent by publishers or news agents, 1¢ per pound or fraction thereof; when sent by others, 1¢ for each 4 ounces or fractional part thereof.

Third-class matter: Books, circulars, pamphlets, proof sheets, or other printed matter, 1¢ for each 2 ounces or fractional part thereof, sent to a single address.

Registered matter: 10¢ in addition to the regular postage.

Special delivery: 10¢ in addition to the regular postage of first-class matter.

Written Problems Based on Rates of Postage

298. 1. How much postage is required for domestic letters weighing, respectively, $\frac{2}{5}$ oz., $5\frac{1}{8}$ oz., $1\frac{3}{4}$ oz., 2 oz.?

2. A newspaper with wrapper weighs 3 oz. How much postage must the publishers pay on 8000 copies?

3. John Smith remails one of these papers. How many cents postage must be put on?

4. George sends to a friend a book weighing, with wrapper, 35 oz. How much is the postage?

5. How many cents postage are needed in all for mailing to friends in Canada 5 letters weighing, respectively, $\frac{1}{2}$ oz., $\frac{3}{4}$ oz., $\frac{2}{5}$ oz., 1 oz., $\frac{4}{3}$ oz.?

6. The Sunday edition of a paper weighs 7 oz. How much must the publishers pay for an edition of 18,000?

7. Mrs. Brown sends a sealed package, weighing $10\frac{1}{2}$ oz., by special delivery. She pays —— postage.

8. Mary sends a registered letter, weighing $1\frac{3}{4}$ oz., to a friend in Hawaii. How much does she pay?

9 John mails at Christmas a book weighing $21\frac{3}{4}$ oz., a registered letter weighing $\frac{3}{4}$ oz., and a package of newspapers weighing 9 oz. How much must he pay in all?

10. A bicycle agent sends out 1000 circulars advertising his machines. Each circular weighs $2\frac{1}{4}$ oz. How much postage must he pay altogether?

11. What is the postage on a letter weighing 3 oz., sent by special delivery as registered mail?

The Parcel Post

299. 1. Is Springfield, Illinois, nearer to Chicago than to Indianapolis?

2. Do these distances seem more than 200 miles each? Measure the distances as nearly as you can on the map, using the scale of miles indicated on the map.

3. Is Frankfort nearer to Nashville than to Columbus (Ohio)?

4. Estimate the following straight-line distances. Then measure to see how close you come.

Chicago to Nashville Louisville to Wheeling
Chicago to Cincinnati Toledo to Indianapolis

5. Make a list of the important articles sold in the place in which you live. Describe the methods by which these articles are distributed to neighboring and distant places. To what extent is parcel post used in this distribution?

Merchandise, and, in general, all matter not classified in the United States postal service as first, second, or third-class matter may be sent by United States **parcel post** in parcels not greater in size than 72 in. in length and girth combined, nor exceeding in weight 50 lb. for the first and second zones, and 20 lb. for the other zones. The postage on parcels varies with the distance sent, as is shown by the following table:

TABLE OF RATES

Weight	First Zone		Second Zone Rate	Third Zone Rate	Fourth Zone Rate	Fifth Zone Rate	Sixth Zone Rate	Seventh Zone Rate	Eighth Zone Rate
	Local Rate	Zone Rate							
1 pound	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 pounds	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 pounds	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 pounds	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 pounds	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 pounds	.08	.10	.10	.16	.27	.38	.49	.61	.72
7 pounds	.08	.11	.11	.18	.31	.44	.57	.71	.84
8 pounds	.09	.12	.12	.20	.35	.50	.65	.81	.96
9 pounds	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10 pounds	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11 pounds	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12 pounds	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13 pounds	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56
14 pounds	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68
15 pounds	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16 pounds	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17 pounds	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18 pounds	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19 pounds	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20 pounds	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
25 pounds	.17	.29	.29						
30 pounds	.20	.34	.34						
40 pounds	.25	.44	.44						
50 pounds	.30	.54	.54						

The local rate is applicable to parcels intended for delivery at the office of mailing or on a rural route starting therefrom.

The circles drawn on our map have Chicago as their centers and indicate the first, second, third, and fourth zones of distances from Chicago to other

places. Similar circles, drawn upon the map of the entire United States, showing all together 8 zones, are used by clerks in the Chicago post office to determine quickly the distances from Chicago to other post offices.

The post-office clerk in Nashville has the same map of the United States, but the circles are drawn with their centers at Nashville, so as to indicate distances from Nashville to other places.

Exercise Based on the Rules

300. Which of these parcels are mailable by parcel post?

1. 10-lb. box, 20″ by 18″ by 10″.
2. 9-lb. box, 18″ by 15″ by 10″.
3. 12-lb. box, 25″ by 10″ by 8″.
4. 11-lb. box, 30″ by 12″ by 9″.
5. $8\frac{1}{2}$-lb. box, 14″ by 14″ by 14″.

Exercise Based on the Table

301. Compute the rate:

1. 8 lb. from Chicago to Columbus (Ohio).
2. 11 lb. from Chicago to Louisville.

3. 3 lb. from Nashville to Bowling Green.

4. Get a rate card or table for the local city.

5. Make 5 exercises based on that table.

Written Problems

302. 1. What is the postage on a parcel weighing 8 lb., sent from Chicago to Nashville?

2. What is the charge for sending 10 lb. from Chicago to Cincinnati? To Aurora?

3. 9 lb. from any post office or delivery on a rural route starting from that office?

4. How much must a boy pay in all for the following parcels which he mails in Chicago:

8 lb. to be sent to Pittsburgh, Pa.
4 lb. to be sent to Nashville, Tenn.
10 lb. to be sent to Springfield, Ill.

5. What is the combined length and girth of a box 20 in. long, 15 in. wide, 10 in. deep? Is it mailable as a parcel, if it does not exceed the limit of weight?

6. A parcel is 30 in. long and has a girth of 35 in. Is it mailable?

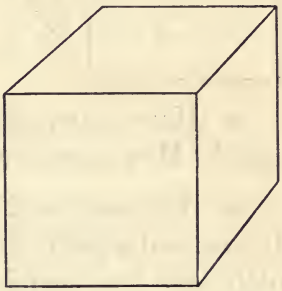
7. Make problems based on the rates and the size of packages received by the school.

8. Make 5 problems based on parcel post deliveries for local industries.

VOLUME OR CONTENT

Introduction

303. 1. The illustration represents a **cube.** It is 1 inch long, 1 inch wide, and 1 inch high. Its contents or volume is 1 cubic inch. It has 6 faces. Is each face a square? How large a square?



2. Two such cubes put together form a prism, 2 in. long, 1 in. wide, and 1 in. high. Its volume is 2 cubic inches.

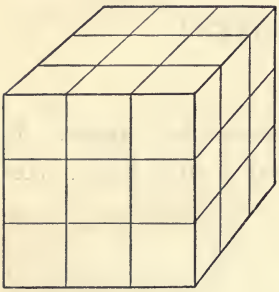
3. If three such cubes are put together in one row, they make a prism 2 in. long, 1 in. wide, and 1 in. high. Its volume is 2 cubic inches.

4. If 2 such prisms, each 3 in. long, are put side by side, they form together a new prism, 3 in. long, 2 in. wide, and 1 in. high. How many cubic inches in that new prism?

5. The edge stands for 1 yard. How many feet does $\frac{1}{3}$ of that line stand for?

6. This drawing is a cube 1 yd. or 3 ft. long, wide, and high. How many square feet are there in

one face of this cube? The cube is divided into smaller cubes, each 1 ft. on a side and each a cubic foot. How many cubic feet do you see in the front layer of the large cube?



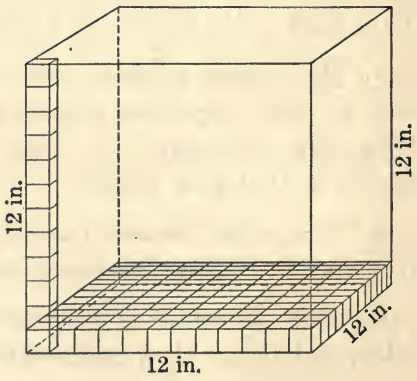
7. If there are 9 cubic feet in the front layer, how many are in the layer just back of it? How many in the last layer?

8. How many cubic feet in the entire large cube? How many cubic feet in a cubic yard?

9. You see that 27 is the product of 3, 3, and 3. If you had a prism 3 in. long, 2 in. wide, and 3 in. high, how many cubic inches would there be in it?

10. How many 1-inch cubes can be packed in a box, 4 in. long, 3 in. wide, and 2 in. deep?

11. A merchant has boxes 1 ft. each way. How many such boxes can be packed in a trunk 3 ft. long, 2 ft. wide, and 2 ft. deep?



12. The drawing, as a whole,

stands for a cubic foot. How many inches long, wide, and high is a cubic foot? Imagine it made up of 1-inch cubes. How many cubic inches in the bottom layer?

13. The other layers are not drawn. Imagine them drawn. How many layers are there, including the bottom layer?

14. If there are 12 layers, and each layer contains 144 cubic inches, how many cubic inches are there in a cubic foot?

Cubic Measure

304. Memorize the table.

TABLE OF CUBIC MEASURE	
1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

In this table these relations are established:

Length		Width		Height		Volume
in.	×	in.	×	in.	=	cu. in.
ft.	×	ft.	×	ft.	=	cu. ft.
yd.	×	yd.	×	yd.	=	cu. yd.

Oral Problems

305. 1. How many 1-inch cubes can be packed into a box 12 in. on an edge?

2. The contents of a tank is 1 cu. yd. A bucket can hold exactly 1 cu. ft. of water. How many bucketfuls will fill the tank?

3. A bin is 2 ft. by 3 ft. by 4 ft. Does it hold more than a cubic yard, or less?

4. A box is 20 in. by 10 in. by 9 in. Does it hold more than 1 cu. ft., or less?

5. How many cubic yards of air can a schoolroom 10 yd. long, 10 yd. wide, and 3 yd. high hold?

6. A man starts to dig a trench 5 ft. by 3 ft. by 2 ft. How many cubic feet of earth must he remove?

7. Take a candy box and estimate how long, wide, and deep it is. How many *cubic* inches in its contents? How many square inches in the bottom? The sides? The ends?

8. Make a paper box 2 in. long, 1 in. wide, and 1 in. deep. How many 1-inch cubes will it hold?

9. A coal bin is 10 ft. by 4 ft. by 3 ft. How many *cubic* feet of coal will it hold? How many *square* feet of boards are there in each of the 2 sides of the bin? In each of the 2 ends of the bin? In both sides and both ends taken together?

10. Make an oral problem based on the table.

Written Exercise

306. Find the volume of a prism:

	1.	2.	3.	4.	5.
Length	12 in.	15 ft.	20 in.	17 in.	10 ft.
Breadth	7 in.	5 ft.	7 in.	12 in.	9 ft.
Thickness	6 in.	$3\frac{1}{3}$ ft.	$5\frac{4}{5}$ in.	$4\frac{3}{4}$ in.	$5\frac{2}{3}$ ft.

Written Problems

307. 1. A bin is 7 ft. by 4 ft. by 3 ft. How much more will it hold than 2 cu. yd.?

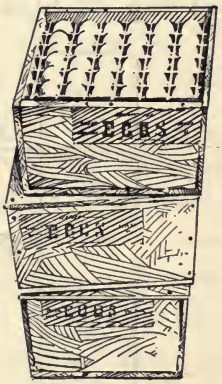
2. A box is 12 in. long, 9 in. wide, and 5 in. deep. How much less than 1 cu. ft. does it hold?

3. How many cubic feet of earth were removed in digging a cellar 15 ft. long, 13 ft. wide, and 7 ft. deep?

4. How much larger is a box 10 ft. by 5 ft. by 3 ft. than a box 6 ft. by 6 ft. by 4 ft.?

5. Which holds more, a box 6 in. by 4 in. by 2 in. or one 5 in. by 4 in. by 3 in.?

6. How many 1-foot cubes are equal to a 2-foot cube (a cube 2 ft. on an edge)? To a 3-foot cube? To an 11-foot cube?



7. How many eggs are there in the upper layer of the top box?

8. Each box has two such layers of eggs. How many eggs in each box? How many in all three boxes?

9. How many dozen eggs in one box? In all three?

10. What is each box worth when eggs are $22\frac{1}{2}$¢ a dozen?

11. A merchant received 13 such boxes and sold them at $28\frac{1}{2}¢$ a dozen eggs. How much money did he take in?

12. A brick is 8 in. long, 4 in. wide, and 2 in. thick. How many cubic inches in it?

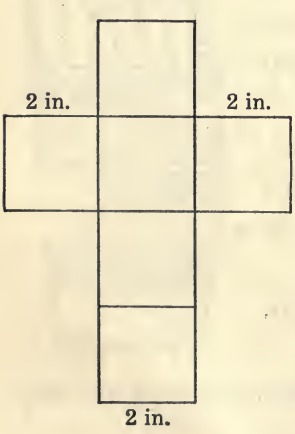
13. How many such bricks does it take for a cubic foot?

14. A brick wall is 5 ft. by 3 ft. by 1 ft. How many bricks are there in it?

15. How long a wall 4 ft. high and $1\frac{1}{2}$ ft. thick can be built with 972 bricks?

16. A tinsmith is making a water tank, without cover. The tank is 6 ft. long, 4 ft. wide, and 5 ft. deep. How many square feet of tin does he use?

17. How many cubic feet will it hold? Allowing $62\frac{1}{2}$ lb. to a cubic foot of water, how many pounds of water will the tank hold? How much more is it than 3 T.?



18. Find the number of square inches in the entire surface of a 2-inch cube.

Cut thin cardboard into the shape of the drawing shown here. Make a 2-inch cube. Paste together the free edges by using stickers for mounting stamps.

19. Find the number of cubic inches in this 2-inch cube.

20. Find the area of the surface of a box and cover 15 in. long, $5\frac{1}{2}$ in. wide, and $2\frac{1}{2}$ in. high. Find also its volume.

21. A strawberry box is 6″ long, 4″ wide, and 4″ deep. How many such boxes can be packed in a case $2\frac{1}{2}'$ long, $1\frac{1}{2}'$ wide, and $\frac{2}{3}'$ deep?

22. A rectangular water trough is 4 ft. long, 1 ft. wide, and $\frac{1}{2}$ ft. deep. How many gallons of water will it hold, if there are 231 cu. in. in a gallon?

23. One cubic yard of earth makes one load. How many loads are necessary to fill up an old cellar 15 ft. by 10 ft. by 6 ft.?

24. A wagon box is 9 ft. long, 3 ft. wide, and 1 ft. deep. How many times will this be filled with earth in digging a cellar 21 ft. by 9 ft. and 7 ft. deep?

25. A boy split enough kindling to make a pile 6 ft. long, 1 ft. wide, and 3 ft. high. What part of a cord did he split?

26. A water trough holds 15 gal. of water. It is 14 in. wide and 11 in. deep. Find its length.

27. If 36 cu. ft. of coal weigh a ton, what is the weight of a wagon load 9 ft. long, $3\frac{1}{3}$ ft. wide and 3 ft. deep?

REVIEW

Oral and Written Problems

308. Study each problem. Write an answer that is approximately the answer to that problem. Solve the problems and notice what answers are approximately correct.

1. If it takes 7 men $35\frac{3}{4}$ da. to do a piece of work, how long will it take 1 man?

2. Add $15\frac{3}{4}$, $10\frac{4}{7}$, $25\frac{1}{3}$, 30.6, 20.2.

3. How many feet in 120.56 yd.?

4. Express as a decimal fraction $67\frac{1}{2}\%$.

5. Express as a common fraction 175%.

6. Find percentage on \$2609 at 19%.

7. A fruit raiser planted 260 trees. 13 died. What per cent died?

8. Of 360 pupils, 18 are absent. What per cent are absent?

9. 1000 lb. of sea water contains 36 lb. of salt. What per cent is salt?

10. Mr. Hunt borrows money at 6% and pays \$25 interest annually. How much did he borrow?

11. If 5 per cent of a number is 8.3, what is the number?

12. What per cent of a pound are 9 ounces?

13. A man spends $10\frac{1}{2}\%$ of his earnings for board. He pays $\$260$ a year for board. How much does he earn?

14. A merchant sold a piano for $\$450$ that cost him $\$400$. What per cent of the cost did he gain?

15. 93.08 is 26% of ——.

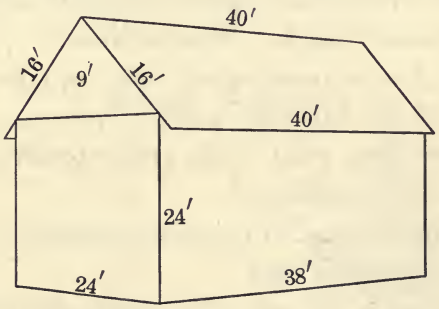
16. 1753 is what per cent of 3506?

17. What per cent of a square foot are 80 sq. in.?

18. What per cent of a mile are 81 rods?

Written Problems

309. 1. How long and wide is one side of the roof of this barn?



2. Snow 2 ft. deep has a weight of about 12 lb. per square foot. How much pressure will 2 ft. of snow exert on the two sides of the roof of this barn?

3. It takes about 1000 shingles to make 100 sq. ft. of roof. If a bunch of shingles contains 250 shingles, how many bunches are needed for this roof? For a fraction of a bunch, take a whole bunch.

4. If a bunch of shingles sells at 97¢, find the cost of the shingles for this roof.

5. If more of each shingle is exposed to the weather, so that only 800 shingles are needed per 100 sq. ft., what is the cost of the shingles for this roof?

6. When a roof is to be covered with slates 6 in. by 12 in., builders use about 500 slates per 100 sq. ft. How many slates will be needed for this roof?

7. The triangular part of each end of the building, close to the roof, called the gable, is 24′ along the base and 9′ high. Find its area.

8. If a very strong wind blows squarely against the end of the barn and exerts a pressure of 14 lb. per square foot, what is the entire pressure against the end of the building?

9. Find the cost of painting the sides of the barn at 23¢ per square yard.

10. What is the cost of painting the roof at 19¢ per square yard?

11. Would the bill have been more or less, if the painting of the whole barn had been done at $21\frac{1}{2}$¢ a square yard?

12. If the building had been erected of brick, and 14 bricks had been used for each square foot of outside surface, what would have been the bill for brick, at $6 per thousand brick?

13. How much greater would the expense have been, if the brick wall had been built thicker, so as to require 22 bricks, instead of 14, for a square foot of outside surface? In what ratio is the new cost to the old?

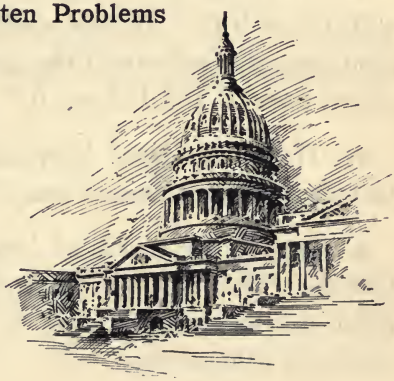
14. How many cubic yards of space are there in this barn from the ground floor up to the gables?

15. If well-settled timothy hay runs about 360 cu. ft. to the ton, how many tons of hay will the barn hold when filled up to the gables?

16. If loose timothy hay runs about 450 cu. ft. to the ton, how many tons of it will the barn hold when filled up to the gables?

Written Problems

310. 1. The corner stone of the capitol at Washington was laid Sept. 18, 1788. Exactly how long ago was that?



2. The old dome was removed in 1856 and the present

dome completed in 1865. Find the age of the new dome.

3. The capitol stands on a plateau 88 ft. above the level of the Potomac River. The height of the dome above the ground is 287 ft. 5 in. The statue of Freedom is 19 ft. 6 in. tall. How high above the ground is the head of the statue? How far above the Potomac is it?

4. The Metropolitan Life Insurance Building in New York City is 700 ft. 3 in. high. How much higher is it than the National Capitol?

5. The Metropolitan Life is 275 ft. 3 in. long and 123 ft. 5 in. wide. The area covered by the United States capitol building is 153,112 sq. ft. How many square feet larger is the area of the latter? How many times larger is it?

6. The capitol is 751 ft. 4 in. long, its maximum width is 350 ft. If this width were uniform, how many square feet would the area exceed its present actual area of 153,112 sq. ft.?

7. The dome is of iron and weighs 8,009,200 pounds. Reduce this to tons.

8. The bronze statue of Freedom weighs 14,985 pounds. If bronze is 8.45 times heavier than wood, what would be the weight of a wooden statue of the same dimensions?

9. The Senate chamber is 113 ft. long, 80 ft. wide, and 36 ft. high. Find the number of cubic yards of space in it.

10. The Representatives' Hall is 139 ft. long, 93 ft. wide, and 36 ft. high. How many cubic yards of space in it?

11. The capitol covers an area of 153,112 sq. ft. How many acres is this in round numbers?

12. The building of the Library of Congress covers $3\frac{3}{4}$ acres. Is this more than the area of the capitol? Find the difference in square feet.

13. The site of the library building is 10 acres. What per cent of it is taken up by the building itself? Find the ratio of the part of the site occupied by the building to the whole site.

14. The floor space in all parts of the building, taken together, is 326,195 sq. ft. How many square feet less than 8 A. is this?

15. The book stacks contain 56 mi. of shelving. How many shelves, each 4 ft. long, furnish this amount of shelving?

16. How long would it take you to travel 56 miles on a bicycle at the rate of 9 miles an hour?

17. The service of the library consists of 236 employees in the library proper, 70 for copyright, 25 for distribution of cards, 5 for law indexing,

127 for care of building and grounds. How much less is this than the total number of the members of Congress, there being 96 senators and 391 representatives?

Written Exercises and Problems

311. 1. How many cubic inches in a peck, if there are 2150.4 cu. in. in a bushel?

2. How many cubic inches in a pint, if there are 231 cu. in. in a gallon?

3. A liquid quart is less than a dry quart, containing only $57\frac{3}{4}$ cu. in., while the dry quart contains $67\frac{1}{5}$ cu. in. Find the difference between the two.

4. Find the area of a table 5 ft. 7 in. long and 4 ft. 5 in. wide.

5. The area of a drawing board is 7 sq. ft. 72 sq. in. Its length is 3 ft. 4 in. Find its width.

312. Find the missing part in each of the following surfaces:

	Figure	Base	Height	Area
1.	Rectangle	2 ft. 3 in.	1 ft. 4 in.	——
2.	Rectangle	10 ft. 2 in.	1 ft. 6 in.	——
3.	Rectangle	5 ft. 6 in.	2 ft. 3 in.	——
4.	Rectangle	2 ft. 6 in.	——	3 sq. ft. 18 sq. in.
5.	Triangle	3 ft. 6 in.	2 ft. 4 in.	——
6.	Triangle	4 ft. 7 in.	3 ft. 2 in.	——
7.	Rectangle	3 ft. 6 in.	——	11 sq. ft. 96 sq. in.

Written Problems

313. 1. Find the volume in cubic feet of a box 3 ft. 2 in. long, 2 ft. 1 in. wide, and 1 ft. 6 in. high.

2. Find the number of cubic feet of space in a trunk 2 ft. 6 in. long, 1 ft. 6 in. wide, and 1 ft. 8 in. deep.

3. Which trunk holds more, one that is 2 ft. 8 in. long, 1 ft. 8 in. wide, and 1 ft. 7 in. deep, or one that is 2 ft. 6 in. long, 1 ft. 11 in. wide, and 1 ft. 4 in. deep? What is the difference in volumes?

4. If a ton of coal occupies 36 cu. ft., how many tons of coal will a bin $8' \times 4' \times 6'$ hold?

5. Coal is bought by the long ton (2240 lb.) and sold by the short ton (2000 lb.). How many long tons does it take to gain one short ton?

Table for Reference

314. Weight and Cost of Railroad Cars

Type of Car	Weight in Lb.	Capacity	Length	Width	Height	Cost
Wood Box	37,000	80,000 lb.	36 ft.	8 ft. 6 in.	8 ft.	$1,100
Steel Coal	42,000	100,000 lb.	31 ft.	9 ft. 4 in.	7 ft. 6 in.	1,200
Flat	32,000	80,000 lb.	41 ft.	9 ft. 2 in.		950
Day Coach	85,000	68 passengers	60 ft.	8 ft. 10 in.	9 ft.	9,000
Parlor Car	105,000	34 passengers	70 ft.	8 ft. 6 in.	9 ft. 6 in.	15,500
Sleeping Car	115,000	27 berths	72 ft. 6 in.	8 ft. 6 in.	9 ft. 6 in.	19,000

Problems Based on the Table

315. 1. Nine empty wood box cars are disconnected from a freight train. In their place 8 empty steel coal cars are added on. How much heavier is the train now?

2. Passengers weigh 150 lb. on an average. Which is heavier, a day coach full of passengers or a parlor car full of passengers? What is the difference in weight?

3. If two trains travel on the same route and with the same speed, which needs a more powerful locomotive, the train with 4 flat cars, 5 steel cars, and 10 wood box cars, or the train with 11 flat cars, 4 steel coal cars, and 7 wood box cars, each car in both trains being loaded to its full capacity?

4. Find the floor area of a wood box car.

(Instead of reducing feet to inches and then finding the number of square inches, it is easier in this case to take 8 ft. 6 in. (8.5 ft.) and compute the number of square feet.)

5. How much less is the floor area of a coal car than of a box car?

6. How much greater is the floor area of a parlor car than the floor area of a day coach?

7. Which has a larger capacity, a coal car or a box car?

(Take 8 ft. 6 in. = 8.5 ft.; 7 ft. 6 in. = 7.5 ft.; 9 ft. 6 in. = 9.5 ft.)

8. Find the number of cubic feet of space for each passenger in a day coach.

9. Find the dimensions of your schoolroom, also the number of pupils in it. See whether the space allowance for each pupil exceeds that of a passenger in a day coach. What is the difference?

10. Why does a passenger in a parlor car pay more than one in a day coach? How many cubic feet of space are allowed for each in a parlor car?

11. How many cubic feet are allowed for each berth in a sleeping car?

12. Compute the cost of the car and locomotive equipment of a train made up of 8 day coaches, 2 parlor cars, and 5 sleepers, and pulled by a Pacific type locomotive costing $18,700.

13. If a Mogul locomotive costs 28% less than one of the Pacific type, find the cost of the former, when the latter is $18,700.

14. What is the ratio of the cost of a coal car to that of a box car?

15. What is the ratio of the cost of a day coach to the cost of a sleeping car?

Since $9000 : 19000 = 9 : 19$, we may say that the ratio required is as 9 is to 19.

16. Show that the cost of a parlor car is to the cost of a flat car as 310 is to 19, or, approximately, as 16 is to 1.

17. An electric locomotive used on one road weighs 160,000 lb.; that used on another road weighs 180,000 lb. Find the ratio of the first weight to the second.

Written Exercise

316. Write in the Arabic notation:

1. IV, V, VI, VII, VIII, IX, XI, XII.

2. XIII, XVI, XIV, XXI, XXV, XXIV, XXIX.

3. LX, LXV, LXIV, MC, MCC, MCCC.

Write in the Roman notation:

4. 13, 14, 17, 18, 19, 20, 30.

5. 36, 47, 54, 69, 99, 95, 96.

6. 94, 200, 500, 600, 400, 1100.

GENERAL REVIEW

Addition Combinations

317. Add quickly. Practice until the additions can be made perfectly without hesitation.

1. Group I.	2	4	9	2	5	3	6	8	6	9
	2	9	3	3	4	6	2	2	3	4
2. Group II.	4	2	8	7	5	5	8	5	7	3
	4	4	6	3	8	2	7	5	6	5
3. Group III.	2	6	9	8	9	3	5	9	7	9
	7	6	7	4	9	4	7	1	7	2
4. Group IV.	6	5	8	1	8	5	1	2	8	7
	9	6	8	7	9	9	8	1	3	1

When these combinations are perfectly mastered, the addition of larger numbers may be made familiar by adding 10 to each number in the lower line. Then 20, 30, and so to 100 may be added. Thus in the second group the lower row of figures will be 14, 14, 16, etc.

Oral Exercise

318. Add columns of numbers from 3 to 10 and higher. These columns may be extended to correspond with the usual length of columns in business.

1.	2.	3.	4.	5.	6.	7.	8.	9.
					6	5	4	3
				2	2	6	3	7
			4	3	9	3	8	6
		9	3	4	3	4	5	8
		3	9	7	5	6	2	2
	3	8	3	5	4	2	7	1
4	6	2	4	2	6	9	4	8
8	4	5	7	9	3	2	5	3
5	3	8	2	3	6	3	4	9
2	4	3	6	4	2	8	5	3

When this exercise has been drilled sufficiently, begin with 12, 14, 23, 26, . . . 98, 103, etc., in place of the 2, 4, 3, 6, etc., the first line in each column, to train in the addition of larger numbers.

Another step is necessary to secure the practical skill required in daily transactions.

319. Since members of two or more figures are met in everyday business, the ability to *carry* in addition is indispensable. Teachers may form such columns of numbers of two, three, four, and more figures, and train pupils to add them with accuracy at optimum rate of speed.

Later it is well to follow usual method of adding (and subtracting) in making change in money transactions. It is not too much to expect pupils to add numbers of two and even three figures, in

money terms, as high as $100.00. Thus pupils will readily learn to add: 13 + 12 + 25 + 10 + 15 + 25 as 13, 25, 50, 60, 75, 100. They do this in money and should readily carry the power over to related exercises. Exercises should be formed to complete as far as possible pupils' skill in such addition. The following columns may be suggestive.

10.	11.	12.	13.	14.
			42	18
	16	47	26	65
22	28	32	43	44
13	35	24	72	51
74	52	46	25	36
26	68	24	46	72
18	27	35	87	39

15.	16.	17.	18.
27	46	507	213
52	73	351	625
24	32	112	143
62	45	214	432
27	81	315	107
43	36	421	634
56	65	137	210

320. Exercises may be brought into close agreement with actual practice in money transactions by making problems involving numbers as follows:

1.	2.	3.	4.	5.
		$2.00	.25	$1.00
.50	$10.00	2.00	.25	2.00
.25	.50	.25	.15	.50
.10	.25	.25	.03	.25
.15	.25	.50	.32	1.25

6.	7.	8.	9.
.50			
.25	$5.00	$15.00	$25.00
.05	1.00	.25	12.50
.01	.25	2.75	.05
2.19	3.75	3.00	.20

Addition Exercise

321. Add the columns beginning at the bottom, test by adding down, time both methods:

1.	2.	3.	4.
25	96	93	48
37	81	33	84
42	25	62	53
67	37	41	79
66	44	67	67
89	66	88	75
23	75	72	56
34	89	12	94

5.	6.	7.	8.
	21	57	13
69	46	22	42
32	59	45	86
45	60	73	47
67	83	19	23
33	49	64	55
71	27	38	63
90	68	43	92
64	45	37	77

Make new exercises by rearranging the addends.

Addition Exercise

322. Add by columns, then by rows across the page. Find the total sum of the numbers by each method.

1.	2.	3.
$3.25	.75	.23
.76	1.50	1.48
1.19	2.20	2.25
.29	.65	.55
.98	9.35	1.75
.85	6.89	6.80
7.63	2.19	.49
.84	3.20	1.98
1.25	1.15	8.75

Practice addition across the page. It is a form that occurs often in business.

4.	5.	6.
12.65	75.15	31.15
23.40	6.27	16.25
69.25	8.95	6.15
46.37	6.45	2.50
6.55	45.60	50.67
14.75	16.27	25.18
42.20	24.15	7.75
6.49	6.95	1.50
7.25	35.45	62.35

Addition Exercise

323. Add. Try to shorten time by practice:

1.	2.	3.	4.
11	65	26	73
22	32	55	84
64	14	14	23
71	23	27	46
19	57	64	17
36	91	86	29
49	38	95	31
53	45	37	72
27	63	84	18
82	72	25	65
14	49	60	94
15	77	42	73

5.	6.	7.	8.
18	132	346	456
27	640	220	634
33	309	614	581
48	246	325	790
62	125	675	688
12	670	806	857
19	87	642	435
21	75	614	506
67	42	126	783
85	369	350	658
94	401	623	743
33	625	847	387
72	72	637	961
24	155	949	203

Dictation Exercise

324. Write from dictation and add. Time yourself in addition.

Test accuracy and speed by adding from top to bottom.

1.	2.	3.	4.
423	231	186	301
576	147	724	810
962	376	476	256
128	312	352	100
634	732	471	403
246	673	524	719
107	132	289	322

5.	6.	7.	8.
523	2376	8361	34,562
879	2837	5496	18,649
391	1549	8067	43,786
418	3412	2952	83,245
573	5835	1694	52,837
726	8043	7209	29,004
607	1979	3573	37,561

9.	10.	11.
$ 5640.71	$ $183\frac{1}{2}$	$ $45\frac{3}{8}$
3763.89	376	$125\frac{1}{2}$
2163.25	$214\frac{3}{4}$	$212\frac{1}{8}$
7189.66	$865\frac{1}{6}$	320
3548.75	$461\frac{1}{4}$	$404\frac{3}{4}$
2762.57	329	$516\frac{1}{2}$
1836.12	$105\frac{1}{3}$	75

Addition Exercise

325. Add. Test by adding down columns.

12.	13.	14.
96,423	47,485	76,423
8,402	3,689	89,845
6,075	62,549	6,505
8,941	8,632	92,861
6,243	4,815	14,650
8,655	6,842	19,425
75,242	95,586	6,307
68,115	3,704	25,642

15.	16.	17.
96,500	98,424	156,742
6,270	16,821	68,045
8,964	42,461	784,924
53,825	53,119	61,075
6,403	55,427	42,740
8,947	43,862	312,625
13,425	31,465	61,842
6,854	55,408	37,896
1,521	87,623	896,420
680,425	62,576	7,256,748
36,017	119,214	847,223
62,422	35,560	67,895
119,627	125,225	967,425
869,421	67,580	5,564,261

Subtraction Exercise

326. Subtract:

1.	2.	3.	4.
$156.47	75.89	97.14	76.15
3.29	15.62	38.25	6.27

5.	6.	7.	8.
86.05	69.11	1915	1914
14.20	15.27	1492	1775

9.	10.	11.	12.
1912	1492	1900	1912
1366	850	1399	1792

Subtraction Exercise

327. Subtract the number at bottom of each column, from each of the numbers of the column. Time yourself for each column. Repeat until you can do them unhesitatingly and accurately. How much time can you gain from practice?

1.	2.	3.
487,205	125,786	6.14
67,481	78,543	115.
695,560	86,491	69.75
72,897	196,325	4.689
846,423	477,584	32.12
952,843	673,962	6.23
65,896	425,759	82.04
712,872	153,861	3.006
49,753	75,708	2.375

4.	5.	6.
9.2	72.3	175.00
14.03	150.605	87.82
6.152	53.715	325.45
35.005	29.45	4575.00
8.141	43.007	500.00
6.007	21.4	50.00
25.340	115.008	375.00
15.261	62.5	400.40
155.01	314.1	100.20
6.3	27.8	1000.10
4.75	18.075	25.50

Multiplication Exercise

328. Multiply:

1. 347 by 900
2. 568 by 36
3. 425 by 17
4. 456 by 305
5. 46,420 by 37
6. $4.75 by 9
7. $0.85 by 24
8. $.026 by 136
9. $6.483 by 320
10. 39.5 by 42
11. .036 by 500
12. 8.072 by 4000
13. 5.672 by 8.2
14. .0375 by .05
15. .0076 by .082

329. Multiply:

Multiplication Exercise

1.	2.	3.	4.
$25.65	$69.75	$172.50	$67.58
14	213	250	182
5.	**6.**	**7.**	**8.**
$96.47	$84.55	67	881
319	86	125	2036
9.	**10.**	**11.**	**12.**
475	9423	6987	527
86	515	484	8643

Division Exercise

330. Divide:

1. $38,642 \div 8$
2. $67,479 \div 9$
3. $25,800 \div 60$
4. $750,000 \div 25,000$
5. $182,000 \div 40,000$
6. $90,750 \div 250$

7.	8.	9.	10.	11.
42)365	870	920	6486	1452
12.	**13.**	**14.**	**15.**	
38)6758	4237	60086	42118	
16.	**17.**	**18.**	**19.**	
79)55,869	125,000	640,050	98,425	
20.	**21.**	**22.**	**23.**	
152)5650	8240	36,472	805,590	
24.	**25.**	**26.**	**27.**	
.25)125.60	$48.75	$360.10	$450	

28. $68{,}429 \div .025$

29. $1326.14 \div 3.45$

30. $80.742 \div 375$

31. $8.60 \div 48$

32. $\$84.00 \div 112$

33. $365.702 \div 36$

34. $.8386 \div 46$

35. $72{,}862 \div 5000$

36. $306{,}125 \div 2500$

37. $3.649 \div 28$

38. $90.362 \div .345$

39. $6724.06 \div 3.32$

40. $32{,}624.6 \div .516$

41. $8885 \div .85$

42. $.36429 \div .035$

43. $6305.5 \div .007$

44. $3460 \div .025$

Exercises in Fractions

331. Add:

1. $\frac{3}{7}, \frac{6}{7}, \frac{5}{8}, \frac{3}{4}$.

2. $\frac{3}{8}, \frac{2}{11}, \frac{1}{4}, \frac{6}{7}$.

3. $\frac{2}{5}, \frac{1}{15}, \frac{3}{12}, \frac{5}{8}$.

4. $\frac{4}{9}, \frac{6}{18}, \frac{5}{14}, \frac{3}{8}$.

5. $16\frac{2}{3}, 25\frac{4}{5}, 67\frac{1}{4}, 81\frac{5}{8}, 12\frac{1}{2}$.

6. $38\frac{1}{2}, 67\frac{5}{8}, 72\frac{8}{9}, 96\frac{2}{3}, 46\frac{1}{5}, 13\frac{7}{8}$.

Subtract:

7. $\frac{8}{9} - \frac{5}{6}$.
8. $\frac{7}{8} - \frac{2}{12}$.
9. $\frac{4}{7} - \frac{3}{18}$.
10. $\frac{6}{8} - \frac{5}{7}$.
11. $\frac{4}{15} - \frac{2}{3}$.
12. $\frac{6}{21} - \frac{3}{8}$.
13. $3\frac{1}{2} - 2\frac{5}{7}$.
14. $6\frac{4}{5} - 2\frac{2}{3}$.
15. $6\frac{8}{9} - 1\frac{1}{2}$.
16. $8\frac{4}{7} - 1\frac{5}{6}$.
17. $6\frac{1}{2} - 1\frac{7}{8}$.
18. $9\frac{1}{12} - 3\frac{7}{8}$.
19. $18\frac{2}{3} - 6\frac{1}{5}$.
20. $27\frac{5}{8} - 1\frac{1}{9}$.
21. $67\frac{1}{2} - 8\frac{4}{11}$.
22. $16\frac{5}{8} - 1\frac{2}{3}$.
23. $25\frac{3}{4} - 6\frac{5}{7}$.
24. $469\frac{3}{4} - 67\frac{5}{8}$.
25. $315\frac{8}{11} - 6\frac{9}{25}$.
26. $46\frac{2}{7} - 3\frac{1}{2}$.
27. $56\frac{3}{25} - 14\frac{3}{8}$.

Find the value:

28. $\frac{3}{5}$ of 275.
29. $\frac{5}{8}$ of 416.
30. $\frac{11}{12}$ of 4800.
31. $\frac{4}{9}$ of 618.
32. $320 \times \frac{3}{8}$.
33. $4800 \times \frac{5}{16}$.
34. $2800 \times \frac{5}{7}$.
35. $36{,}400 \times \frac{4}{11}$.
36. $24.45 \times \frac{4}{5}$.
37. $9.375 \times \frac{2}{3}$.
38. $360 \times 2\frac{1}{4}$.
39. $250 \times 1\frac{3}{8}$.
40. $450 \times 2\frac{3}{5}$.
41. $340\frac{1}{2} \times 12$.
42. $73\frac{1}{3} \times 36$.
43. $2.48 \times 1\frac{3}{4}$.
44. $84.5 \times 2\frac{3}{5}$.
45. $16\frac{3}{4} \times .8$.

Find the value:

46. $25 \div \frac{3}{4}$.
47. $16 \div \frac{4}{5}$.
48. $\frac{15}{18} \div 12$.
49. $25 \div 2\frac{1}{2}$.
50. $16\frac{2}{3} \div \frac{1}{3}$.
51. $33\frac{1}{3} \div 4$.
52. $12\frac{5}{6} \div 60$.
53. $40 \div 8\frac{1}{3}$.
54. $8\frac{1}{3} \div 5$.
55. $\frac{5}{16} \div \frac{3}{8}$.
56. $7\frac{1}{2} \div \frac{3}{4}$.
57. $83\frac{1}{3} \div 6\frac{1}{4}$.
58. $3.75 \div \frac{1}{2}$.
59. $.084 \div \frac{1}{5}$.
60. $64.8 \div \frac{1}{4}$.

Combined Fundamental Processes

332. Perform the operations indicated:

1. $2 \times 6 + 8 - 4 =$
2. $18 + 6 \times 4 \div 3 =$
3. $24 \div 3 \times 6 + 8 =$
4. $11 + 6 \times 2 + 8 \div 4 =$
5. $27 \div 3 \times 4 + 6 - 7 =$
6. $2 \times 143 \div 6 + 8 - 17 =$
7. $78 \div 13 + 26 \times 4 =$
8. $65 - 9 \times 4 + 16 \times 2 =$
9. $(14 \times 2) - 16 + 2 \times 4 \div 3 =$
10. $3 + 7 \times 5 \div 5 \times 3 =$
11. $(2 \times 8 - 4) \div 3 + (30 - 8 \times 3) - 18 \div 5 =$
12. $(235 - 78 \div 6) - 14 \times 11 \div (186 \div 31 \times 2 + 5) - 4 =$
13. $(\frac{1}{2} \times \frac{4}{7}) + \frac{2}{3} \div \frac{5}{6} =$
14. $25 \div \frac{5}{7} \times 5\frac{1}{4} - \frac{3}{8} =$
15. $16\frac{2}{3} \div 6 + 2\frac{1}{8} - 3\frac{1}{12} \times \frac{4}{5} =$
16. $10 \times \frac{3}{8} \div (\frac{2}{5} \times \frac{3}{7} \times \frac{7}{8}) - 3\frac{5}{16} =$

DENOMINATE NUMBERS

333. Tables

Linear

12 inches (in. or ″)	= 1 foot (ft. or ′)
3 feet	= 1 yard (1 yd.)
16.5 feet	= 1 rod (rd.)
320 rods	= 1 mile (1 mi.)
1760 yards	= 1 mile
5280 feet	= 1 mile
6086 feet	= 1 knot

Liquid

2 pints (pt.)	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

Dry

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
32 quarts	= 1 bushel

Value

10 mills	= 1 cent (¢)
10 cents	= 1 dime
10 dimes	= 1 dollar ($)
100 cents	= 1 dollar

Time

60 minutes (min.)	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week
320 days or	(wk.)
12 months (mo.)	= 1 year (yr.)

Counting

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross
30 units	= 1 score

Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 gross ton
7000 grains	= 1 pound

Angle

90° = 1 right angle (rt. ∠)
180° = 1 straight angle (st. ∠)

Square

144 square inches (sq. in)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Cubic

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord

TESTS OF MATHEMATICAL ABILITY FOR PUPILS OF THE FIFTH AND SIXTH GRADES

Written tests like the ones proposed below, but with changes in the numbers and also slight changes in the wording, should be given, without previous notice, limiting the time to 15 or 20 minutes and requiring pupils to work the tests in the given order. Assign more questions than pupils are able to answer in the given time. One examination may be on Part I, another on Part II. As has been suggested by other writers, either or both of two systems of marking may be adopted, (1) marking each example 1 or 0, according as it is right or wrong, or (2) marking each example on the basis of the total number of steps involved.

PART I

1. Add 37.5
405.7
90.4
380.5
986.1

2. Subtract 980651.19
786438.255

3. Multiply 71.98 by 23.5.

4. Divide 17.89 by 3.5. Carry answer to 3 decimal places.

5. $2\frac{1}{2}+\frac{3}{4}+\frac{11}{12}-\frac{1}{3}=?$

6. Multiply $5\frac{1}{7}$ by $\frac{7}{9}$ and simplify the answer.

7. Divide $\frac{7}{8}$ by $\frac{5}{4}$.

8. Divide $10\frac{3}{5}$ by $4\frac{2}{3}$.

9. 15 % of 48760.

10. 2.5 % of 178.65.

PART II

1. What is the cost of eggs a dozen, if 7 dozen cost $ 2.73?

2. A locomotive has been run 57 times between Chicago and New York. If the distance between

these cities is 908 miles, how many miles has this locomotive traveled?

3. Which of the following numbers are exactly divisible by 6: 46872, 176509, 37932?

4. A loaded truck weighs $6\frac{1}{4}$ T. The load consists of two parts, one weighing $2\frac{1}{3}$ T., the other weighing $3\frac{1}{2}$ T. Find the weight of the truck.

5. The weekly wage list of 5 employees is \$ 27, \$ 30, \$ 32, \$ 29, \$ 40. What is the average wage per week?

6. A merchant saves \$ 475 a year, which is 35 % of his earnings. Find his earnings.

7. A man buys 40 ft. of garden hose at $9\frac{1}{2}$¢ a foot, discount 15 %. How much does he pay?

8. Find the interest on \$ 375 at 6 % for 1 yr. and 6 mo.

9. Find the number of cubic feet of space 2 ft. 6 in. long, 1 ft. 6 in. wide, and 1 ft. 5 in. deep.

10. If 3 tons of coal cost \$ 14.25, what will 4 tons cost?

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THE following pages contain advertisements of a few of the Macmillan books on kindred subjects

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Professor of Education, University of Wisconsin; Author of "Dynamic Factors in Education," etc., and

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