

Accounts of Books. (1.) *Le grand & fameux Probleme de la Quadrature du Cercle resolu Geometriquement par le Cercle & la Ligne droite, par Monsieur Mallement de Messange. A Paris, in 12° 1686. With a Refutation of the same, by Mr. D. Cluverius. Reg. S. Soc.*

THIS Author is one of those unhappy Geometricians, who without having acquired a through Understanding of the Principles, have yet thought themselves able to master the abstrusest Difficulties in this nice Mathematical Science, where the least oversight or mistake subverts the whole superstructure. Hence it is, that the true Quadrature of the Circle here pretended to, is lost upon the same Rock with those many others, which the less knowing and more opinionated of their own Skill have produced, in this and the last Century: But briefly to shew wherein the Paralogism of our Author consists, we must first lay down the construction, whereby he pretends to do the Business: In Tab. 2. Fig. 2. let $f k z k$ be a Circle, $f a z$ the Diameter, a the Center, $k z k$ an equilateral Triangle inscribed, $B b$ a line equal to the three sides of that Triangle, and dividing the Arch $f k$ equally in i , the line $i e$ will be half the side of a Hexagon inscribed, which side taken 6 times, is the line $e E =$ to the circumference of the Hexagon; and dividing the Arch $i f$ in b , the sine $b d$ is half the side of the Dodecagon inscribed, and $D d = 24 b d$ is the circumference of the Dodecagon; and proceeding after the same manner, the circumferences of Polygons of 24, 48, 96 sides, &c. may be found, approaching still nearer and nearer to the circumference of the Circle, which at length will be equal to the line $f F$ in the Tangent; but how to find the Point F is all the Skill: Here our Author tells us, that the Points $B E D F$ are all in the Arch of a Circle, whose center is in the line

H h f z

fz continued; but to contract his Work into a little compass, he assures us that it is all one, if instead of the whole circumferences Bb , Ee , Dd , Ff he take the lines bq , ep , do , fm , each a third part of its correspondent, and that in this case too, the Circle whose Center is L , taken in the Diameter fz continued, shall pass through the Points o , p , q and intersect the Tangent Ff in the Point m , so as to leave $fm =$ to a third of the circumference of the Circle; which supposition being proved to be groundless and erroneous, all the consequences drawn from thence must be so too. If our Author had but considered what the intermediate Points of the *Curve* between o & p , p & q , q & t ought to express, he could not but have discovered the fallacy himself, for the lines od , pe , qb are each proportioned to fm , as the lines kb , ie , bd to their respective Arches kf , if , bf , and so of all the rest between o and m . This would have taught him, that the *Curve* he has occasion to use, did universally express the Proportion of the Arches to their Sines, by that of the line fm to its respective ordinates; that it was a sort of *Linea Quadratrix*, to be reckoned among the *Lineæ Geometricæ Irrationales*, or such whose relation between the parts of the Diameter and the Ordinates, are not generally expressible by any one Equation; that this *Curve* did intersect the Circle in the middle of the Arch kz , and continge it in the point z . This *Curve* will be better understood in *Fig. 3*, where it is drawn as it ought, and wherein the proportion of the line fm to the lines qb , is as any Arch kf is to its Sine kb .

Tis evident, that this *Curve* is not the Circle $mopqt$ in *Fig. 2*, yet tis not apparent but that a Circle passing through the point m , may intersect it in several points, as o , p , q : (but to suppose it to pass through all the extremities of the Circumferences of the infinite Polygons between the Circle and Triangle, or their thirds, is to make it coincident with that *Curve*.) It remains therefore to shew, that the Circle passing through p and q , whose center is in the Line fz , does not pass

pass through the point *o*, which from the following Considerations will be made evident.

First let it be required, by the extremities of the lines *a, b, c*, or *b, g, f* (in *Fig. 4*) parallel one to another and *d, e*, or *k, l*, given as parts or segments of the Axis or Diameter of the Figure, to determine what curvity passeth through their extremities, according to the conditions of the five Conical Sections.

First if it be found that $\frac{bb - aa}{de} \frac{cc + aa}{de + ee}$ is equal to $1 = \frac{cc - aa}{dd + ee} \frac{cc + bb}{ed}$, then it is the Characteristic of a Circle, the

Lines *a, b, c* being disposed in an uniform increasing order :

But if *c* the biggest stands in the middle, then $\frac{cc - aa}{dd + ee} + \frac{cc - bb}{ee + de} = 1 = \frac{cc - bb}{de} + \frac{bb - aa}{de + dd}$ will shew the same. If the

Lines *d, e* be segments of a line drawn parallel to the *Axis*, then transposing and ordering the foregoing Equations, Rules also may be found accordingly. If $\frac{c - b}{b - a} = \frac{e}{d}$, or $\frac{c - a}{b - a} \frac{e}{d} = 1$, then the Line passing through the Extremities is right:

If $\frac{cc - bb}{bb - aa} = \frac{e}{a}$, then a Parabola is designed. The Characteristic of an Hyperbola or Ellipsis differs not from that of a

Circle, but only by a Relation to the inequality of the *Axes*, and the alteration of the Signs + and -.

Secondly, out of *a, b, d*, or *b, c, e*, lines given, that stand

in the Arch of a Circle, to find the Distance from the Center = to *m*, or *m + e*, and to determine the *Radius*. There is a little variety in the case, when the given lines are in the same

Quadrant or otherwise : but there being only occasion for this first Case, the Rule is this, $\frac{cc - bb}{2e} + \frac{1}{2}e = e + m$: And

$\sqrt{\frac{cccc - 2ccbb + bbbb}{4ee} + \frac{1}{4}ee + \frac{1}{2}bb + \frac{1}{2}cc} = \text{Radius.}$

Thirdly, in a Circle, having *a, b, d* and *e*, to find *c* : The

H h 2 Equa-

Equation is $\frac{b^2 e - a^2 e}{d} de - e^2 + b^2 = c^2$. Fourthly: To inscribe

Polygons in a continual double Progression within a Circle, many different Rules may be given: the following will serve, which is the same with that, how to find the subtense of an Arch, out of the subtense of a double Arch. The Rule is thus; $2 R^2 - \sqrt{4 R^2 - A^2} R^2 = B^2$; Supposing A to be the Chord of a double Arch, and B of a single Arch. From hence it is easily deduced, that $\sqrt{3} R^2$ being the side of an equilateral Triangle inscribed, the side of a Hexagon will be R: of the Dodecagon $\sqrt{un} \frac{3}{2} R^2 - \sqrt{\frac{1}{2} R^2}$: and so for the rest. Now reducing according to these Equations the Lines to Numbers, it will be found that in *Fig. 2 Tab. 2*

$$\begin{array}{ll} b q = 173205,08 & a b = 50000,00 \\ e p = 200000,00 & e b = 36602,54 \quad f m = 209439,51 \\ d o = 207055,23 & d e = 9990,04 \end{array}$$

But supposing, as our Authour will have it, that *d o* stands in the same Circle with *b q* and *e p*, it follows that the square of *d o* = 422638679 &c. whereas it should have been equal to 428718707 &c. Square of *d o* in the Table. The Square of the Tangent *f m* is also a great deal too small, and the whole Quadrature too little: All which make it appear, that the Glory of *Lewis* the Great is not (as this Book pretends) much advanced by the Atchivements of this Author; who would have done well, in a Matter that so little needed it, to have forbore to make use of the sacred Words of our Saviour, *Math. 11th. 25th.*

II. *Voiage de Siam des Peres Jesuites envoyez par el Roy aux Indes & a la Chine. A Paris 1686. 4°.*

THIS is a second Relation of the *Voiage* and *Embassy* of the *French* to the King of *Siam* in the Year 1685, and being a more particular Account than the former, an extract of this, 'twas thought, might suffice for both : That was composed by *le Chevalier Chaumont* the *Embassadour*, and now this by *le Pere Tachart* *Jesuite*, who was one of six *Fathers* of his Order, that went with the *Embassadour*, as *Missionaries* to *China*. The whole being much interspersed with matters of Religion and Ceremony, I shall only take notice of such things as relate to Arts and Sciences, and particularly of the *Astronomical Observations* made at the *Cape of Good Hope* and at *Siam* ; whereby the *Longitudes* of those places are stated : following herein the Authors method.

He divides his *Treatise* into six *Books*, whereof the first contains the *Voiage* from *Brest* to the *Cape of Good-Hope*. Here he gives the reasons and motives of sending this *Embassy*, as likewise the six *Jesuites* who are *Mathematicians* and by the Kings *Letters Patents* are so stiled ; Their *Instructions* being, besides their *Spiritual Function*, to prosecute the business of the *Royal Academie* of *Paris* (of which they are admitted *Members*) by accurately observing the curious things in *Art* and *Nature*, and particularly to make *Observations* for discovering the *Longitudes* of the *Places* where they pass ; for which purpose they are well provided with *Instruments*. They sailed from *Brest* on the third of *March* st. n. and arrived at the *Cape of Good-Hope* the last day of *May*, taking notice by the way of the several *remarkables* in that *Voiage*, which are here too well known to need repeating ; But mentioning the *faultiness* and *rectification* of the *Southern Constellations*, our Author is not willing to take notice of what has been done in that matter by a member
of

of the *Royal Society* of *London*, tho' his Catalogue of those Stars hath been translated from *Latine* into *French* and Printed at *Paris*, and an Account thereof is in the *Journal des Scavans* of *Aug. 7. & Sep. 4. 1679*; but speaks of it as a thing not done, wishing they had had the opportunity to augment the Science of *Astronomy*, by observing them themselves.

The second Book is entituled the Voyage from the *Cape* of *Good-Hope* to the Iland *Java*; but is chiefly taken up with the description of the *Colonie* of the *Hollanders* there, the *Natives*, and the *Astronomical* Observations they made there during their stay, by which they have determined the Longitude of the *Cape* of *Good-Hope* 18 degrees to the *East* of *Paris*: (but we here must begg leave to make a Remark) He mentions 7 several Nations of the *Natives*, viz. the *Hottentots*, whom he describes at larg, the *Namaquaas*, (of these two there are the Figures) the *Ubiquaas*, the *Gouriquaas*, *Ilassiquaas*, *Sousiquaas* and the *Odiquaas*: and here he relates a Voiage made in the Year 1685, as farr as the *Tropick*, by the Governour of the *Cape*, *Mr. Vanderstell*; who is said to have found about the Latitude of 27 degrees and about 10 or 12 leagues from the shore, a Nation of *Natives* that are very Musical, who have long Hair flowing on their shoulders, some of the Men as White as *Europeans*, and their Women Naturally very White, but they Blacken themselves to please their Husbands: This Nation seems to have much more Intelligence than their Neighbours, but some circumstances seem to argue it Fabulous.

Here are likewise the Figures of a *Stagg* with Horns like a *Goat*, of the *Zembra*, the *Sea Cow*, the *Cerastes* or Horned *Serpent*, the *Cameleon*, and two sorts of uncommon *Lizards*, whereof one is made to have 3 fair Crosses on his back: Speaking of *Elephants* he says he was told by creditable persons, that they had seen the foot steps of *Elephants* two foot and half diameter; and that there are *Rhinocerotes* there as bigg as ordinary *Elephants*, but, by I know not what mistake, he makes the *Rhinocerote* a two horned Animal. The Voi-
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age from the *Cape* to *Java* (in fight whereof they arrived *Aug. 5th. ft. n.*) contains nothing very Extraordinary.

The third Book is the *Voiage* from *Java* to the Kingdome of *Siam*: which is chiefly taken up with what occurred at *Bantam* and *Batavia*, and at their arrival on the Coast of *Siam*, here are represented the Roads of *Bantam* and *Batavia*, together with the Plan of the City and Fortrefs of *Batavia*.

The fourth Book describes the Entry and Audience of the *French* Embassadour at the Court of *Siam*, who, as they say, was received with more Honour and Respect, than was ever yet shewn to any Ambassadour whatever; and that even those of *Persia*, the *Mogul* and the *Tartar* Emperour of *China*, (tho' his neighbour, and by much the most Potent Monarch of the Universe) present themselves before the King of *Siam* on their Knees, whereas Mr. *Chaumont* the *French* Ambassadour made his harangue, sitting with his Hat on his Head. Here are described the *Baloons* or *Barges* of State which are used at *Siam*, which are of a very odd Figure, as of *Serpents* or *Sea-Horses*, but which by their sharpness and number of Oars are of an incredible Swiftnes: here likewise 'tis related that the old White *Elephant* of the King of *Siam* is near upon 300 Years Old; as also that there are *Tumblers* there, of an extraordinary Agility, as that they would stand upon one Foot on the top of a *Bamboo* of 80 or 100 Foot high, and then turn themselves, and stand on their Heads thereon, and afterwards hang, by the Chin only, on the top of the same, and then descend by a Ladder down right, with an incredible Swiftnes, working their Bodies all the while through the Rounds of the Ladder.

The fifth Book is entituled the Return of the *Voiage* of *Siam*, and first relates several notable Shews presented to entertain the Ambassadour; as the fight of two *Elephants*, who were only suffered to twist each others Teeth, as Bulls do their Horns; the fight of an *Elephant* and a *Tiger* or rather a *Panther* according to the description; and the manner of catching the wild

wild *Elephants*, by alluring them into an inclosure by the means of a Female tame *Elephant*, and the like. Next are related several Observations of the variation of the Magnetical Needle, which was found towards the end of the Year 1685, to be about half a degree West, at *Louvo* near *Siam*; as likewise the Observation of an Eclipse of the Moon on the 10th. December st. n. post medium noctem, made at a place near *Louvo* called *Tblee-Pouffonne*, in the presence of the King. It begun about 15b. 20m. the total Darkneſs at 16b. 23m. 45f. the Emerſion or end of Total Darkneſs at 18b. 2m. 3^lf. or rather, as is there ſaid, at 18b. 10m. 25f. whereby the Longitude of this place is found 98 degrees and half from *Paris*, and about 6b. 45m. to the Eaſt of *London*, as may be ſeen by comparing this Observation with the Observations thereof made at *Dantzick*, *Nurenburg* and *Liſbon*, Published in *Philof. Transact.* Num. 178, 182 and 184. And whereas tis here ſaid that ſome Charts have made the Longitude of *Siam* above 20 degrees more than it is, tis to be underſtood only of the Charts of *Sanſon*, which in this particular are the worſt extant: But that this Correction is juſt, we are fully ſatisfied, by the like errors in thoſe parts, diſcovered and publiſhed in the *Philoſoph. Colleſt.* of Feb. 1681, and in *Philof. Transf.* of June 1683. The relation of the homeward bound Voyage (which was of about 6 Months) is ſhort and contains very few conſiderable Remarks.

The ſixth and laſt Book is of the manners and Religion of the *Siammers*, where is a ſhort deſcription of the Kingdome and Capital City of *Siam*: Next the habits of the People, and their uſe of *Betele*, *Arek*, and *Tea* is deſcribed, as likewise the Root *Ginſeng* ſo much eſteemed in the Eaſt, with its Vertues and Uſes. As to the Religion of this people, which is here deſcribed at large, I ſhall ſay little to it, as not falling under our Argument, only one principal point therein is the *Metemphychoſis* of *Pythagoras* and the *Bramines*, and they hold the Eternity of the World, but on the contrary they ſuppoſe God Mortal, Corporeal and produced in time: their preſent God they call *Sommonokbodom*.

A Remark concerning the Longitude of the Cape of Good Hope.

IN the second Book of this Voiage are related two Observations of the *Satellites* of *Jupiter*, capable, if well made, to ascertain the Longitude of the *Cape of Good Hope*. The first was there made *June 2d. st. n. 1685*, when at *11h. 29m. 20s.* the first or innermost *Satellite* touched the *Western* edge of *Jupiter* and at *11h. 30m. 50s.* it appeared no more: this Observation is said to be made with an excellent *Telescope* of twelve Foot: The other was on *June 4th.* following *st. n.* when the *Emerfion* of the same *Satellite* was observed at *9h. 37m. 40s.* from which latter is concluded, that the Longitude of the *Cape* is *18 gr.* to the East of *Paris*, for that the said *Emerfion*, according to the *Calculus* of *Sigr. Cassini*, in the Meridian of *Paris* ought to have happened at *8h. 26m.* This same *Emerfion* is computed by *Mr. Flamsteed* at *8h. 19m.* at *London*, that is, *3m.* later than by *Sigr. Cassini*; and considering that neither is verified by Observation in *Europe*, the Longitude hence deduced is doubtful at least *3 minutes*, if this had been the only Observation: But the former being considered will yet shew that there is a much greater doubt still remaining: For from certain *Astronomical* principles the *parallax* of the *Orb*, or difference between the place of *Jupiter* seen from the *Sun* and *Earth* was, at the time of the first Observation, *9 gr. 19m.* which Arch that *Satellite* moves in *1h. 6m.* and the utmost duration of an Eclipse thereof in this position of *Jupiter* being scarce *2h. 20m.* (as appears by the accurate Observations of *Mr. Cassini* and *Mr. Flamsteed*) it will follow, that from the Immersion behind *Jupiters* Western Edg, to the Emerfion out of the shadow, there could not be full *3h. 26m.* wherefore the Emerfion out of the shadow, on *June 2d.* ought, according to the time of the Immersion, to be at *14h. 56m.* at latest, at the *Cape*; which by *Mr. Flamsteeds Calculus* was at *London 13h. 51m.* or according to *Sigr. Cassini* at

13^b. 58^m. at *Paris*. Hence the Longitude of the *Cape* will be found but 14 *degrees* and half at most to the East of *Paris*; so that these 2 Observations will differ in the result about a quarter of an hour; which is a little too much. However there are some reasons that seem to argue for this latter Longitude rather than the former; for it is much easier to observe what becomes of a luminous Object that appears, than to wait upon the first appearance of a Star Eclipsed: and tis probable that the *Satellite* might, in the latter time, be several minutes Emerged out of the shadow, when they might first perceive it; but they could not but see the application to the Body of *Jupiter* in the former, if we may suppose their *Telescopes* so good as they are said to be: And that the *Cape of Good Hope* is not more than an hour to the East of *Paris*, is proved by the constant consent of our Navigators, who find by their Reckonings that the Island of *St. Helena* is about 22 or 23 degrees of Longitude to the Westward of the *Cape*: (and that Sailing both backward and forwards tis the same, which takes away the Objection of Currents) now by accurate Observations made at *St. Helena*, and compared with others made in *Europe* at the same time, the Longitude of that Isle is certainly about $8\frac{1}{2}$ degrees to the West of *Paris*: It follows therefore that the *Cape* cannot be much more than 14 or 15 degrees to the East of *Paris*; and undoubtedly it must be less than 18, for 3 degrees is much too great an Error to be committed in so short a distance Sailing.

