New Method For Finding Prime & Composite Numbers

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Abstract:

The idea behind this paper is to create a method for finding Prime Numbers, as all Prime numbers are odd numbers (except for number 2) so finding a formula to generate all odd composite numbers was important. This paper introduced the method for finding Prime & Composite number between any two numbers, by first finding all odd composite number between any two numbers using formula & given recursive relation then obviously all remaining odd numbers are prime.

Introduction:

In about 200BC the Greek Eratosthenes devised an algorithm for calculating Primes called the Sieve of Eratosthenes. By the time Euclid's Elements appeared in about 300BC, several important results about Primes had been proved. In Book IX of the Elements, Euclid proves that there are infinitely many Prime numbers. With time it was being worked on, but no complete solution was found so far.

In this paper, a formula and recursive relation is given to find all odd composite numbers b/w any two number and remaining odd number are Prime numbers.

Methodology

Step1

By creating formula & recursive relation for finding all odd composite numbers between any two numbers.

Step2

When all odd composite numbers will be found between any two numbers then obviously the remaining odd numbers are prime.

Explanation:

1) Take two numbers A & Z in between we have to find all odd composite numbers.

2) Next take the square root of Z.

3) then take all Prime numbers between 3 and the integral value of the square root of Z & denoted these values as a set P_i.

&

$3 \leq P_i \leq$ square root Z

$P_i = p_1, p_2, p_3 \ldots . p_n$,i=0,1,2,3…………………n

So,

$P_i \leq$ square root Z

As not every time Z have perfect square root. It can be a number with integral and fractional part as we just have to take it's integral part so it may be composite or prime

That's why we can say

P_i=square root Z………(iii)

e.g. Z=1708

Square root Z= 41.327

Square root Z = 41 which is prime.

4) Putting these values of P_i one by one in this formula

C_i= (int.X) P_i

Where X= A/P_i

Implies that int. X=integral part of A/P_i

For this, a time we have to change our range A & Z, so in this regard, we will take our Z whose square root value ≤ A/2

So,

A≥2 square root Z

A is the 1st value and P_i is the Particular Prime.

Note that X will be different for each P_i

The first composite number for a particular P_i is calculated by the formula described above

C_i= (int.X) P_i

If the first composite number found by the above formula is odd then for calculating all other odd composite direct use the recursion relation.

$C_{i+1}=C_i+2P_i$

But, if the first composite number is even then using the recursion relation.

First use this

$C_{i+1}=C_i+P_i$

By this, we find an odd composite no. Then all other odd composite numbers can be found by recursion relation described above

$C_{i+1}=C_i+2P_i$

5) When we find all odd composite b/w any two numbers will be found then all remaining odd numbers are prime.

Theoretical Proof:

Problem:

Let A & Z be two numbers in between we must prove that all numbers computed by using this formula are the composite number.

C_i =(int.X) p_i

Where X=A/P_i

Where i=1,2, 3....n

Where A is the 1st no. And P_i are prime numbers,

(int.X mean integral part of X)

&

3≤P_i≤square root Z

$P_i = p_1, p_2, p_3....p_n$

So,

P_i ≤ square root Z

&

A ≥ 2square root Z

Also, prove that

$C_{i+1} = C_i+P_i$

&

$C_{i+1} = C_i +2P_i$

Both are the composite number.

Also, prove that when C_i is even then

$C_{i+1}=C_i + P_i$

is an odd composite number

If C_i is odd then

$C_{i+1}=C_i + 2P_i$

Is an odd composite number.

Proof:

Given is that

C_i= (int. X) P_i......................(i)

Where X=A/P_i...........................(ii)

P_i =P_1, P_2, P_3.............P_n

As P_i is a prime number of ranges

3≤P_i≤square root Z

As not every time Z has a perfect square root. It can be a number with integral and fractional parts as we just must take its integral part so it may be composite or prime.

As

P_i≤square root Z

That's why we can say

P_i=squareroot Z………(iii)

e.g. Z=1708

Square root Z= 41.327

Squareroot Z = 41 which is prime.

&

A≥2 square root Z.............(iv)

We must prove that every number computed by the formula in eq (i) must be a composite number.

As we know the product of any two numbers greater than or equal to 2 is always a composite number. So, we use this concept to prove it.

C_i =(int.X) P_i……. (i)

As P_i is a prime number of range $3 \le P_i \le$ square root Z, which is greater than 2

So, we just must prove that, int. $X \ge 2$

So, for this

As given in eq. (ii)

$X = A/P_i$

Which implies $A = XP_i$...................(v)

Now by putting the value of A using eq. (v) and value of square root Z using eq. (iii) in eq. (iv)

$A \ge 2$ square roots Z……… eq. (iv)

We get

$XP_i \ge 2P_i$

$X \ge 2$

implies that

Int. $X \ge 2$

So, C_i is a composite number

Now, as C_i is a composite number. It may be even composite or odd composite.

Case 1:

1) If it is even composite.

If we find the first composite even, then for finding our 1st odd composite number, we use the relation

$C_{i+1} = C_i + P_i$.........................(vi)

And after getting our first odd composite, then all other odd composite numbers can be calculated by this relation

$C_{i+1} = C_i + 2P_i$...................(vii)

Now we must prove that

$C_{i+1} = C_i + P_i$

&

$C_{i+1} = C_i + 2P_i$

Both are composite

So,

I) $C_{i+1} = C_i + P_i$

As C_i=(int.X) Pi

So C_{i+1}=(int.X)P_i + P_i

C_{i+1}=P_i [int.X+1]

As already prove that

Int.X ≥2

implies that

(Int.X+1)>2

And, P_i >2

As the product of two number is greater than or equal to 2 is composite. So, $C_{i+1}$is a composite.

Hence

C_{i+1}= C_i +P_i is composite.

Now we will prove that it must be odd if C_i is even.

[as in method we take prime numbers P≥3 so all are odd]

& C_i is even.

Then obviously

C_{i+1}=C_i+P_i= even +odd

C_{i+1}=odd

Case 2

if it is odd composite number

C_{i+1}=C_i+2p_i

C_{i+1}=(int.X) Pi+2P_i

$C_{i+1} = P_i[\text{int}.X+2]$

As int .X ≥2

implies that

(Int.X+2)>2

&

as Pi >2

So, C_{i+1} is a composite number.

Now we must prove that

$C_{i+1}=C_i + 2P_i$ is always odd composite if C_i is odd

As C_i is odd (given) & $2P_i$ is always even, because multiple of 2 is always even so,

$C_{i+1}=C_i+2P_i$

C_{i+1} =odd+ even

C_{i+1} =odd

This complete the proof

Proof with Example:

1) Take two numbers A & Z in b/w we have to find all odd composite no.

A=100, Z=200

2) Next, we have to take the square root of Z.

As Z=200, square root of Z =14.142

But just take its integral part as the square root of Z=14.

3) Now take all prime numbers between 3 and the integral part of the square root of Z. And denotes these values as a set P_i so, P_i=3,5,7,11,13

4) Put these values of P_i one by one in this formula

$C_i=(\text{int}.X)P_i$

And then we use recursive relation (vi) and (vii) according to requirement.

As P_i=3,5,7,11,13

For i=0, P_0=3

As A=100, P_0=3

So int. X=int.(A/P_0) =int (100/3) =33

Using eq.(i)

$C_i=(int.X)P_i$

$C_0= (33)3$

$C_0=99$

As 1st composite is odd so direct, we can use the recursive relation (vii) for computing all other odd composite numbers.

As $C_{i+1}=C_i+2P_i$

i=0,1,2,3, 4…..........

$C_1=C_0+2P_0$

$C_1=99+2(3)$

$C_1=105$

$C_2=C_1+2P_0$

$C_2=105+2(3)$

$C_2=111$

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C_{17} =201

So, we get

99,105,111,117,123,129,135,141,147,153,159,165,171,177,183,189,195,201

Our range should between A & Z

For i=1, P_1=5

As A=100, P_1=5

So int. X=int.(A/P_1) =int (100/5) =20

Using eq.(i)

$C_i=(int.X)P_i$

$C_0= (20)5$

$C_0=100$

As 1st composite is even so first, we use the recursive relation (vi) for finding a first odd composite number.

As $C_{i+1}=C_i+P_i$

i=0,1,2,3, 4…..........

$C_1=C_0+P_1$

$C_1=100+5$

$C_1=105$

As we have found our first odd composite so now, we have to use recursive relation (vii) for computing all other odd composite numbers

As $C_{i+1}=C_i+2P_i$

i=0,1,2,3, 4…..........

$C_2=C_1+2P_1$

$C_2=105+2(5)$

$C_2=115$

$C_3=C_2+2P_1$

$C_3 =115+2(5)$

$C_3 =125$

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C_{11} =205

So, we get

105,115,125,135,145,155,165,175,185,195,205

For i=2, P_2=7

As A=100, P_2=7

So int X=int.(A/P) =int (100/7) =14

Using eq.(i)

C_i=(int.X)P_i

C_0= (14)7

C_0=98

As 1st composite is even so first, we use the recursive relation (vi) for finding the first odd composite number.

As $C_{i+1}=C_i+P_i$

i=0,1,2,3, 4…..........

$C_1=C_0+P_2$

C_1=98+7

C_1=105

As we have found our first odd composite now, we have to use recursive relation (vii) for computing all other odd composite numbers

As $C_{i+1}=C_i+2P_i$

i=0,1,2,3, 4…..........

$C_2=C_1+2P_2$

$C_2=105+2(7)$

$C_2=119$

$C_3=C_2+2P_2$

$C_3 =119+2(7)$

$C_3 =133$

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$C_8=203$

So, we get

98,105,119,133,147,161,175,189,203

For i=3, P_3=11

As A=100, P_3=11

So int X=int.(A/P_3) =int (100/11) =9

Using eq.(i)

$C_i=(int.X)P_i$

$C_0= (9)11$

$C_0=99$

As 1st composite is odd so direct, we can use the recursive relation (vii) for computing all other odd composite numbers.

As $C_{i+1}=C_i+2P_i$

$i=0,1,2,3,4$…..........

$C_1=C_0+2P_3$

$C_1=99+2(11)$

$C_1=121$

$C_2=C_1+2P_3$

$C_2=121+2(11)$

$C_2=143$

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$C_5=209$

So, we get

99,121,143,165,187,209

For $i=4$, $P_4=13$

As $A=100$, $P_4=13$

So int $X=\text{int.}(A/P_4)=\text{int}(100/13)=8$

Using eq.(i)

$C_i=(\text{int.}X)P_i$

$C_0=(8)13$

$C_0=104$

As 1st composite is even so first, we use the recursive relation (vi) for finding the first odd composite number.

As $C_{i+1}=C_i+P_i$

$i=0,1,2,3,4$…..........

$C_1=C_0+P_4$

$C_1=104+13$

$C_1=117$

As we have found our first odd composite so now, we have to use recursive relation (vii) for computing all other odd composite numbers.

As $C_{i+1}=C_i+2P_i$

n=0,1,2,3, 4…..........

$C_2=C_1+2p_4$

$C_2=117+2(13)$

$C_2=143$

$C_3=C_2+2P_4$

$C_3=143+2(13)$

$C_3=169$

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$C_6=221$

So, we get

104,117,143,169,195,221

STEP 2

Remaining odd numbers are obviously prime, which are 101,103,107,109,113,127,131,137,139,149,151,157,

163,167,173,179,181,191,193,197,199

Conclusion:

By comparison with other methods, it is concluded that in any other method we need a lot of calculations to find prime numbers. But in this method, we need a small no. of calculations.

If Z is large Z=1000

Then the square root of Z =32

Under 32, there are 11 primes, so P is set of 11 members. Just 11 basic calculations will be needed and all other numbers can be calculated by recursive relation and if Z is too large about 1 lac

Suppose Z=100000

Then the square root of Z=316

So, under 316, there will be about 100 primes so for finding composite and prime between 1 lac there just need 100 basic calculations and some other preliminary calculation by using recursive relation. We can develop an algorithm for computing these calculations. This will make it easy to find within minutes.

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