ELEMENTS

OF

MACHINE DESIGN

BY

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PREFACE

The purpose of the author, in preparing this book, has been to present in fairly complete form a discussion of the fundamental principles involved in the design and operation of machinery. An attempt is also made to suggest or outline methods of reasoning that may prove helpful in the design of various machine parts. The book is primarily intended to be helpful in the courses of machine design as taught in the American technical schools and colleges, and it is also hoped that it may prove of service to the designers in engineering offices.

Since a text on machine design presupposes a knowledge of Strength of Materials and Mechanics of Machinery, a chapter reviewing briefly the more important straining actions to which machine parts are subjected is included as well as a chapter discussing briefly the properties of the common materials used in the construction of machinery. Furthermore, throughout the book, the question of the application of mechanical principles to machines and devices has not been overlooked, and many recent devices of merit are illustrated, described and analyzed. A considerable amount of the material in this book was published several years ago in the form of notes which served as a text in the courses of machine design at the University of Illinois.

In the preparation of the manuscript the author consulted rather freely the standard works on the subject of machine design, the transactions of the various national engineering societies and the technical press of America and England. Whenever any material from such sources of information was used, the author endeavored to give suitable acknowledgment. The numerous illustrations used throughout the book have been selected with considerable care and in the majority of cases they represent correctly to scale the latest practice in the design of the parts of modern machines. At the close of nearly every chapter a brief list of references to sources of additional information is given.

Through the generosity of various manufacturers, drawings illustrating the prevailing practice in America were placed at the author's disposal thus making it possible to use scale drawings for illustrating the various machine parts. To all such manufacturers he is especially indebted. The author is also indebted to Mr. H. W. Waterfall of the College of Engineering of the University of Illinois for the many helpful suggestions and criticisms received during the preparation of the manuscript. To his friend and colleague Professor G. A. Goodenough, the author is deeply indebted for much valuable advice and the many suggestions received in preparing the manuscript, also for the critical reading of the proof.

O. A. L.

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MACHINE DESIGN

CHAPTER I

STRESSES AND STRAINS IN MACHINE PARTS

1. Forces.—The object of a machine is to transmit motion through its various links to some particular part where useful work is to be done. The transmission of this motion gives rise to forces which must be resisted by the parts of the machine through which the forces are acting.

The forces acting upon the various machine parts may be classified as follows:

(*a*) *Useful forces.*—In doing the useful work for which the machine is intended, the various parts are subjected to certain forces; for example, the parts of a shaper must transmit the forces produced by the resistance offered to the tool by the material to be cut.

(*b*) *Dead weight forces.*—Dead weight forces are those due to the weights of the individual parts in a machine. Generally these forces are not considered in the design of a machine, except in cranes and machines having large gears and flywheels. In the design of roof trusses, bridges, structural steel towers, floors, etc., the dead weights are always considered, because they form a considerable part of the total load coming upon the members.

(*c*) *Frictional forces.*—Forces called forth by the frictional resistances between the machine parts are designated as frictional forces. In certain classes of machinery such as hoists employing screws, a large amount of work is consumed by friction; hence the various machine elements must transmit this energy in addition to energy required for useful work.

(*d*) *Forces due to change of velocity.*—Frequently the motion of machine parts changes in direction, thus causing forces that must be considered; for example, the rim of a rapidly rotating flywheel or pulley is subjected to rather heavy centrifugal forces. Another example is given by the whipping action of the connecting rod of a high-speed engine; the stresses arising from the reversal of direction may be far in excess of those due to the steam pressure on the piston. In general, whenever the velocity of a machine part changes rapidly heavy stresses are set up due to the accelerating and retarding action. Forces due to a change of velocity are frequently called inertia forces.

(*e*) *Forces due to expansion and contraction.*—In certain structures, as boilers, the forces due to the expansion and contraction caused by heat must be considered. Not infrequently heavy bending stresses are induced in members by expansion and contraction.

(*f*) In addition to the various forces discussed in the preceding paragraphs others exist, such as the following: (1) *forces due to the reduction of area caused by the deterioration of the material;* (2) *force due to the use of non-homogeneous material;* (3) *forces due to poor workmanship.*

Some of the above-mentioned conditions need not be considered at all, but it is well that they all be kept in mind when undertaking the design of a new machine. It is evident from this brief discussion, that before the designer can select a suitable material or determine the proportions of the various elements, he must make a careful analysis of the external forces and their effects.

2. Principles Governing Design.—The design of machine parts may be approached by either of the following methods:

(*a*) Strength alone is the basis of design; that is, the parts are made strong enough to resist the stresses developed in them, and as long as no rupture occurs the machine parts designed in this way fulfill their purpose. As an illustration, the design of a gear transmitting a given horse power at a certain speed is in general satisfactory so long as the various component parts of the gear, such as the teeth, rim and arms, do not rupture.

(*b*) Stiffness in addition to strength is taken into consideration; that is, machine parts must be made rigid enough to perform their function without too much distortion. Stiffness is essential in the design of all the important elements of a machine tool, as without rigidity the machine is not capable of producing work having the desired degree of accuracy. A grinding machine is a good illustration in the design of which the consideration of stiffness is paramount.

The criterion discussed in (*b*) is important, and whenever possible a study of the deflections of the various members of the machine should be attempted. This study, in the majority of cases, is very difficult to carry out, as the deflections are not readily calculated; as a matter of fact, in many cases it is impossible to calculate such deflections with our present knowledge of the subject of "Strength of Materials." Frequently the determination of the stresses induced in certain machine parts is beyond calculation and in such cases as well as those mentioned above, experience together with precedent must be relied upon to suggest the proper proportions to be used.

STRESS, STRAIN, AND ELASTICITY

3. Stress and Strain.—The external forces or loads coming upon the members of a machine cause the latter to undergo a deformation or change of form, the amount of which is called a *strain*. Now within the member that is thus deformed, a certain internal force is produced in the material which will resist this strain. This internal force is called a *stress*, and may be defined as the internal resistance which the particles of the material offer to the external force. A designer should evidently have a knowledge of the stresses and strains induced in a material subjected to an external force, and without such knowledge it is impossible for him to produce a well-designed machine. Information pertaining to stresses and strains is derived from tests of materials. The following articles give briefly some of the results of such tests.

4. Stress-strain Diagram.—The relation existing between the unit stresses and unit strains for any particular material is shown best by means of a diagram. This diagram is based upon the observations and calculations derived from experiments, and is constructed by plotting upon rectangular coördinates the unit strains against the unit stresses, the latter as ordinates and the former as abscissæ. In Fig. 1 is shown such a diagram. The plot shown represents the results of a tensile test on a soft grade of machinery steel. The results of a compression test on any material may be plotted in a similar manner.

An inspection of the plot in Fig. 1 shows that up to a certain point B the stress-strain diagram is practically a straight line; that is, *unit stress is proportional to unit strain.* The law just stated is known as Hooke's Law. The stress corresponding to the point B is known by the term, "*limit of proportionality*" or better still "*proportional elastic limit.*" At the point C there is a well-defined break in the diagram, thus showing a sudden and considerable increase of strain without an appreciable increase of stress; in other words, this point indicates a change in the condition of the material, namely from one of almost perfect elasticity to one of considerable plasticity. The point C is called the *yield point*, and is found only on the stress-strain diagrams of the ductile materials. In testing ductile materials the stress corresponding to the yield point is obtained by observing the load on the scale beam of the machine at the instant the beam takes a sudden drop.

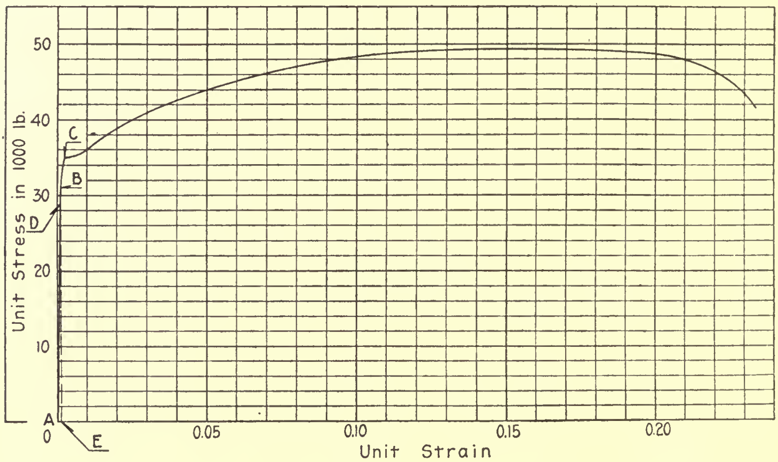

FIG. 1.

Another term used considerably and frequently applied to the stress corresponding to the point B in Fig. 1 is the *elastic limit.* Various definitions have been proposed for this term, and the following is considered about the best: By the *elastic limit* is meant the unit stress below which the deformation or strain disappears completely upon removal of the stress; in other words, no permanent set can be detected. The determination of the elastic limit experimentally requires instruments of high precision, and due to the repeated application and release of the stress that is necessary, such tests require a great amount of time. In general it is assumed that there is but little difference between the elastic limit and the stress corresponding to the *limit of proportionality;* and since the latter can be determined more readily, it may be used by designers as a means of getting at the probable elastic limit of a material.

Referring again to Fig. 1, it is evident that as the stress increases, the deformation increases, until finally rupture of the test piece occurs. The external load required to break the test piece divided by the original area of cross-section of the bar is called the *ultimate strength.*

5. Modulus of Elasticity.—In order to determine the strain for any known load acting upon a given material, it is convenient to make use of the so-called *modulus* of *elasticity.* This is defined as

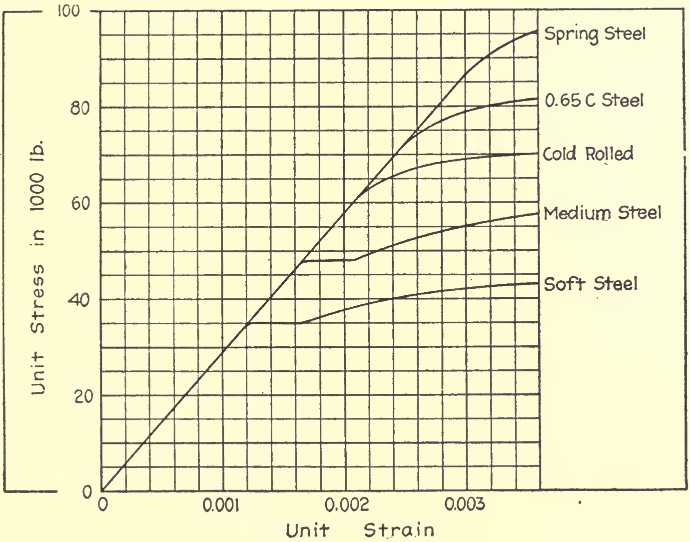

FIG. 2.

the *ratio of unit stress to unit strain,* a value of which is readily obtained from that part of the stress-strain diagram below the point B; in other words, the slope of the line AB gives the value of the modulus of elasticity. Representing this modulus of elasticity for tension by the symbol E_t, the statement just made may be expressed algebraically by the following equation:

$$E_t = \frac{S_t}{\delta}, \tag{1}$$

in which S_t denotes the unit stress and δ the unit deformation; hence E_t is some quantity expressed in the same units as S_t, namely in pounds per square inch.

In Fig. 2 are shown stress-strain diagrams for several grades of steel, which seem to indicate that the modulus of elasticity is practically the same for all grades of steel. According to the results obtained by various authorities the numerical value of the modulus of elasticity for steel varies from 28,000,000 to 32,000,000 pounds per square inch. The modulus of elasticity is also a measure of the *stiffness* or rigidity of a material, and from Fig. 2 it is evident that a machine part made of soft steel will be just as rigid as if it were made of an alloy steel, provided the stresses in the member due to the external load are kept below the limit of proportionality. However, the part when made of high-carbon steel will be much stronger than that made of soft steel. It has been suggested by certain machine-tool builders that excessive deflections of spindles and shafts may be reduced by the use of an alloy steel in place of a 25-point carbon open-hearth steel, but upon actual trial it was found that the trouble was not remedied. The failure of the alloy steel to decrease the deflection, is due to the fact that the modulus of elasticity and not the strength of the steel is the measure of its rigidity.

6. Poisson's Ratio.—When a bar is extended or compressed the transverse dimension as well as the length are changed slightly. Experimental data show that the ratio of the transverse unit strain to the unit change in length is practically constant. This ratio is called *Poisson's ratio,* average values of which, collected from various sources, are given in Table 1.

TABLE 1.—POISSON'S RATIO

Material		Poisson's ratio
Cast iron		0.270
Wrought iron		0.278
Steel	Hard	0.295
	Mild	0.303
Copper		0.340
Brass		0.350

7. Resilience.—Referring to Fig. 1, it is evident that the area under the complete curve represents the work done in rupturing the test specimen, while that under the diagram up to any assumed point on the curve represents the work done in stretching the specimen an amount equal to the deformation corresponding to the assumed point. If this assumed point be taken so that the stress corresponding to it is equal to the elastic limit, then the area under that part of the diagram represents the work done in producing a strain corresponding to that at the elastic limit. The energy thus spent is called *resilience,* and is represented in Fig. 1 by the triangular area AED. Since the area of this triangle is $\frac{1}{2}(AE \times ED)$, it follows that

$$\text{Resilience} = \frac{AS_e}{2} \times \frac{S_e L}{E_t} = \frac{ALS_e^2}{2\,E_t}, \tag{2}$$

in which A denotes the cross-sectional area of the test specimen, L its length, and S_e the stress at the elastic limit.

If the specimen has a cross-sectional area of one square inch and a length of one inch, then the second member of (2) reduces to $\frac{S_e^2}{2\,E_t}$. This magnitude is then the unit of resilience and is called the *modulus of resilience*, a quantity which is useful for comparing the capacity various materials have for resisting shock. As mentioned in Art. 5, the modulus of elasticity in tension is practically constant for the various kinds of carbon and alloy steels; hence it follows from (2), that the modulii of resilience of two steels are to each other as the squares of the stresses at their elastic limits. From this fact it is apparent that the higher carbon steels have a greater capacity for resisting shock than those of lower carbon content, since their elastic limits are higher as shown in Fig. 2.

Unfortunately writers on "Strength of Materials" have paid but little attention to the actual values of the modulus of resilience, and consequently information pertaining thereto is not plentiful. The values given in Table 2 were calculated by means of the expression for the modulus, and may serve as a guide in the proper selection of a shock-resisting material. The stresses at the elastic limit given in Table 2 were collected from various sources, and in all probability in the majority of cases, the yield point instead of the elastic limit is referred to. The error introduced by substituting the stress at the yield point for that at the elastic limit is not of great consequence since the values given in Table 2 serve merely as a guide.

It should be noted that the preceding discussion of resilience applies to the stress-strain diagram given in Fig. 1 which represents the result of a tensile test; however, the formula for the modulus of resilience applies also to direct compressive or shearing stresses, provided the modulus of elasticity and the stress at the elastic limit are given their appropriate values.

TABLE 2.—MODULI OF RESILIENCE FOR STEEL IN TENSION

Type of steel			Elastic limit	Modulus of resilience
Open-hearth carbon steels	0.08 per cent. C		25,000	10.4
	0.15 per cent. C		30,000	15.0
	0.30 per cent. C		35,000	20.4
	0.40 per cent. C		41,000	28.0
	0.50 per cent. C		47,500	37.6
	0.60 per cent. C		63,500	67.2
	0.70 per cent. C		70,500	82.8
	0.80 per cent. C		75,000	93.8
Alloy steels	Nickel steel, 2.85 per cent. Ni.	Annealed	52,000	45.1
		Oil-tempered	121,000	244.0
	Chrome steel, oil-tempered		127,500	271.0
	Carbon vanadium, oil-tempered		136,000	308.0
	Nickel vanadium, oil-tempered		126,250	266.0
	Chrome vanadium	Annealed	63,700	67.5
		Oil-tempered	170,000	482.0

SIMPLE STRESSES

The external forces acting upon a machine part induce various kinds of stresses in the material, depending upon the nature of these forces. The different kinds of stresses with which a designer of machines comes into contact will now be discussed briefly.

8. Tensile Stress.—A machine member is subjected to a tensile stress when the external forces acting upon it tend to pull it apart. Using the notation given below, the relations existing between stress, strain and the external forces for the case of simple tension are derived as follows:

Let A = cross-sectional area of the member.
E_t = modulus of elasticity.
L = length of the member
P = the external force.
S_t = unit tensile stress.
Δ = total elongation.

The area of cross-section of the member multiplied by the unit stress gives the total stress induced in the section, and since the total stress induced is that due to the pull of the force P, it follows that

$$S_t = \frac{P}{A} \tag{3}$$

From the definition of the modulus of elasticity given in Art. 5, or from (1), we get

$$E_t = \frac{S_t L}{\Delta}, \tag{4}$$

from which the following expression for the total elongation is obtained:

$$\Delta = \frac{PL}{AE_t} \tag{5}$$

By means of (5), it is possible to determine the probable elongation of a given member subjected to a load P. This is a very desirable thing to do for all tension members of considerable length, as very frequently such elongation is limited by the class of service for which the proposed machine is intended.

9. Compressive Stress.—A compressive stress is induced in a member when the external forces tend to force the particles of the material together. For a short member, in which no buckling action is set up by the external forces, the various relations deduced in Art. 8 apply also in this case, provided the appropriate values are substituted for the various symbols. If, however, the length of the member exceeds say six times the least diameter, the stresses induced must be determined by the column formulas, which are discussed in Art. 15.

A kind of compressive stress met with extensively in designing machinery is that caused by two surfaces bearing against each other; for example, the edges of plates against rivets or pins, or keys against the sides of the key-way or key-seat. This kind of a stress is usually spoken of as a *bearing stress.*

10. Shearing Stress.—A shearing stress is one that is produced by the action of external forces whose lines of action are parallel and in opposite direction to each other. The relation existing between the external force P, area of cross-section A, and the shearing stress S_s, is similar to (3), or

$$S_s = \frac{P}{A} \tag{6}$$

If a machine member is twisted by a couple, the stress induced in that member is a pure shear, or as it is commonly called, a *torsional* stress. The following discussion establishes the relations existing between stress, strain and the external forces for a member having a circular cross-section.

Equating the external moment T to the internal resisting moment, we obtain

$$T = \frac{2\,S_sJ}{d}, \tag{7}$$

in which J represents the polar moment of inertia and d the diameter of the member. For any given section the value of J may be obtained by means of the relation:

$$J = I_1 + I_2, \tag{8}$$

in which I_1 and I_2 represent the rectangular moments of inertia of the section about any two axes at right angles to each other, through the center of gravity. For a circular cross-section $J = 2\,I_1 = \frac{\pi d^4}{32}$, hence (7) becomes

$$T = \frac{\pi d^3 S_s}{16} \tag{9}$$

The relation between the twisting moment T and the angular deflection θ of a circular member having a length L is derived in the following manner:

$$E_s = \frac{360\,S_sL}{\pi\,d\theta} \tag{10}$$

Substituting in (9) the value of S_s from (10), we obtain

$$T = \frac{\theta d^4 E_s}{584\,L} \tag{11}$$

The expression given by (9) is to be used when the member must be designed for strength, while (11) is used to proportion the member for stiffness.

11. Stresses Due to Flexure.—Machine members may be subjected to transverse forces which produce stresses of several kinds. Such members must be designed by considering the effect produced by the combination of these several stresses. A simple illustration of a member in which several kinds of stresses are induced is an ordinary beam supported at its ends and carrying a load W at a distance x from the left-hand support. Due to the load W, the beam will bend downward producing a *compressive stress* on the upper or concave side, a *tensile stress* on the lower or convex side, and a *shearing stress* at right angles to the tensile and compressive stresses just mentioned. In calculations pertaining to beams, the magnitude of the shearing stress is generally small relative to the tensile and compressive stresses, and may then be neglected altogether; however, cases may arise when the shearing stress must be considered.

The relation existing between the bending moment produced in the beam by the load W, the stress S and the dimensions of the cross-section of the beam, is obtained by equating the external moment to the internal stress moment; thus

$$M = \frac{SI}{c}, \tag{12}$$

in which I represents the moment of inertia of the beam's cross-section, and c the distance from the center of gravity of the section to the outermost fiber. This formula is applicable for determining the strength of the beam, provided S is kept within the elastic limit.

Whenever a beam is to be designed for stiffness the following general formula may be used:

$$M = EI\frac{d^2y}{dx^2} \tag{13}$$

The expression given by (13) is the fundamental equation by means of which the deflection of any beam may be obtained. The method of procedure is to determine, for the case considered, an expression for the bending moment M in terms of x, and after substituting it in (13), integrate twice and solve for the vertical deflection y of the beam.

COMBINED STRESSES

12. Flexure Combined with Direct Stress.—In structures such as bridges and roofs, the members are, in general, pieces that are acted upon by equal and opposite forces. There being no motion at the joints, it is properly assumed that such members are centrally loaded, thus producing a uniformly distributed stress in the material. When, however, we deal with machine parts, central loading is the exception rather than the rule. Even in the ordinary connecting link used merely to transmit motion, the friction between the pin and its bearing in the link causes a shifting of the line of action by an appreciable amount, thus subjecting the link to a flexural stress in addition to the direct stress.

In order to determine the distribution of stress in any right section of a member subjected to flexure combined with direct stress, and thence to find the maximum intensity of stress, the following analysis and discussion is recommended. Attention is called to the fact that the expressions given are strictly applicable only to the following types of members:

(*a*) Short as well as long tension members that are straight.

(*b*) Short and straight compression members.

13. Straight Prismatic Bar.—In Fig. 3 is shown a straight prismatic bar so loaded that the line of action of the external force P is parallel to the axis AB and at a distance e from it. We are to determine the distribution of stress in any right section as CD and thence to find the maximum intensity of stress. Consider the portion of the bar above CD as a free body, and at the center O of the section insert two opposite forces OM and ON acting along the axis AB, and equal to the external force P. These forces being equal and opposite, do not affect the equilibrium of the system. We have thus replaced the single external force by the central force OM and a couple consisting of the equal and opposite forces P and ON. The arm of the couple is e and its moment is Pe. The single force OM must be balanced by a stress in the section CD; and since OM has the axis AB as its line of action, this stress is uniformly distributed over the cross-section. Denoting the intensity of this stress by S'_t, and the area of the section by A, we have

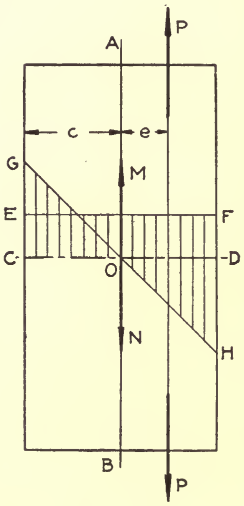

FIG. 3.

$$S'_t = \frac{P}{A} \tag{14}$$

If the intensity S'_t, be denoted in Fig. 3 by CE, the line EF parallel to CD will indicate graphically the uniform distribution of stress over the section.

The couple of moment Pe tends to give the body under consideration a counter-clockwise rotation. Evidently this couple must be balanced by a stress with an equal moment and of opposite sense. The fibers to the right of AB will be subjected to tensile stress and those to the left to compressive stress. Denoting the intensity of the flexural stress at D by S_t'', and the section modulus of the section by $\frac{I}{c_t}$, then

$$S_t'' = \frac{Pec_t}{I} \tag{15}$$

Denoting the intensity of the flexural stress at the point C by S_c'', and the section modulus of the section by $\frac{I}{c_c}$, we get

$$S_c'' = \frac{Pec_c}{I} \tag{16}$$

The law of distribution of the stress induced by the couple Pe is represented graphically by the line GH. Evidently the maximum intensity of tensile stress occurs at the point D, and its magnitude is obtained by adding (14) and (15), or

$$S_t = \frac{P}{A}\left[1 + \frac{Aec_t}{I}\right] \tag{17}$$

The maximum compression stress occurs at the point C, and its magnitude is given by the following expression:

$$S_c = \frac{P}{A}\left[\frac{Aec_c}{I} - 1\right] \tag{18}$$

Equations (17) and (18) are not strictly exact, since the flexural stresses S_t'' and S_c'' do not represent actual direct stresses and therefore should not be combined directly with the true direct stress S_t'. The difference between these stresses may be considerable for materials in which the rates of deformation due to tension and compression are not equal, as in cast iron, brass, and wood. A better method would be to express S_t'' and S_c'' in terms of S_t' before combining them with the latter. In general, to express a stress due to flexure in terms of a direct stress, multiply the former by the ratio that the direct stress of the given material bears to the transverse stress.

In the analysis just given the external force P produces a direct tensile stress over the area A; however, the various formulas derived above apply to the condition when the force P is reversed, namely producing a direct compressive stress, providing the proper symbols are used.

14. Offset Connecting Link.—A case of frequent occurrence in the design of machine parts is the offset connecting link shown in Fig. 4. The circumstances are such that it is not practicable to make the link straight, and the axis of a cross-section, as *CD*, lies at a distance e from the line *AB*, which joins the centers of the pins and is, therefore, neglecting friction, the line of action of the external forces. Let b_0 and h_0 denote the dimensions of the rectangular cross-section of the link if *straight* and *centrally loaded;* and let b and h denote the corresponding dimensions of the *eccentrically loaded* section at *CD*.

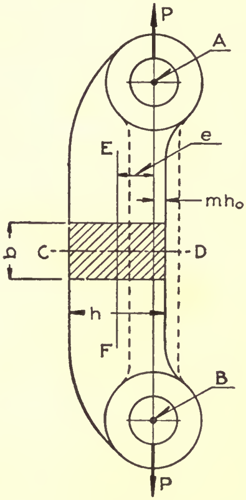

FIG. 4.

For the straight link the intensity of the uniformly distributed stress is

$$S_0 = \frac{P}{b_0 h_0} \tag{19}$$

For the offset link the maximum intensity of stress in the section *CD* as calculated by means of (17) is

$$S_t = \frac{P}{bh}\left[\frac{6\,e}{h} + 1\right] \tag{20}$$

If we impose the condition that S_t shall not exceed S_0, we have

$$bh \geq b_0 h_0 \left[\frac{6e}{h} + 1\right] \tag{21}$$

Let mh_0 denote the distance of the right-hand edge of the cross-section *CD* from *AB*, the line of action of the external forces; this is to be taken positive when measured from *AB* to the right, that is, when *AB* cuts the section in question, and negative when measured from *AB* to the left. Then the eccentricity is

$$e = \frac{h}{2} - mh_0 \tag{22}$$

Substituting this value in (21), we have finally

$$bh \geq b_0 h_0 \left[4 - \frac{6\,mh_0}{h}\right] \tag{23}$$

A discussion of (23) leads to some interesting results. For given values of b_0h_0 and m, we may vary b and h as we choose, subject to the restriction expressed by (23). Economy of material is obtained by making the product bh and, therefore, the expression $\left[4 - \frac{6\,mh_0}{h}\right]$ as small as possible. If m is positive, that is, if the section is cut by the line of action of the forces, this requirement is met by making h as small as possible; on the other hand, if m is negative, that is, if the section CD lies wholly outside of the line of action of P, the product bh is made a minimum by making h as large as possible. In other words, when m is positive, keep the width h as small as possible and increase the area of the section by increasing the thickness b; when m is negative, keep the thickness b small and add to the area of the section by increasing the width h. This principle is of importance in the design of the **C**-shaped frames of punches, shears, presses and riveters.

When $m = 0$, that is when the edge of the section coincides with the line of action AB, (23) reduces to $bh \geqq 4\,b_0h_0$. The area of section bh must be at least four times the area of section b_0h_0, independent of the relative dimensions of the section.

15. Stresses in Columns.—As stated in Art. 9, the formulas for short compression members are not applicable to centrally loaded compression members whose length is more than six times its least diameter. Due to the action of the external load, such a member will deflect laterally, thus inducing bending stresses in addition to the direct stress.

(*a*) *Ritter's formula.*—Many formulas have been proposed for determining the permissible working stress in a column of given dimensions. Some of these are based upon the results obtained from tests on actual columns, while others are based on theory. In 1873, Ritter proposed a rational formula, by means of which the value for the mean intensity of permissible compressive stress in a long column could be determined. This formula, given by (24) is used generally by designers of machine parts:

$$S_c' = \frac{P}{A} = \frac{S_c}{1 + \dfrac{S_eL^2}{nE\,\pi^2r^2}}, \tag{24}$$

in which

A = area of cross-section.

E = coefficient of elasticity.

L = the unbraced length of the column in inches.
P = the external load on the column.
S_c = the greatest compressive stress on the concave side.
S_e = unit stress at the elastic limit.
n = a constant.
r = least radius of gyration of the cross-section.

The strength of a column is affected by the condition of the ends, that is the method of supporting and holding the columns. In (24) this fact is taken care of by the factor n, which may have the following values, taken from Merriman's "Mechanics of Materials."

1. For a column fixed at one end and free at the other, $n = 0.25$.
2. For a column having both ends free but guided, $n = 1$.
3. For a column having one end fixed and the other guided, $n = 2.25$.
4. For a column having both ends fixed $n = 4$.

(*b*) *Straight line formula.*—A formula used very extensively by structural designers is that proposed by Mr. Thos. H. Johnson, and is known as the *straight line* formula. It is not a rational formula, but is based on the results of tests. Using the same notation as in the preceding article, Johnson's straight line formula for the mean intensity of permissible compressive stress is

$$S'_c = \frac{P}{A} = S_c - \frac{CL}{r}, \tag{25}$$

in which C is a coefficient whose value may be determined by the following expression:

$$C = \frac{S_c}{3}\sqrt{\frac{4\,S_e}{3\,n\pi^2 E}} \tag{26}$$

The factor n in (26) has the same values as those used in connection with Ritter's formula given above.

The straight line formula has no advantage over the Ritter formula as far as simplicity is concerned, except possibly in a series of calculations in which the value of C remains constant, as, for example, in designing the compression members of roof trusses in which the same material is used throughout. For a more complete discussion of the above formulas the reader is referred to Mr. Johnson's paper which appeared in the *Transactions* of the American Society of Civil Engineers for July, 1886.

16. Eccentric Loading of Columns.—Not infrequently a designer is called upon to design a column in which the external force P is applied to one side of the gravity axis of the column; in other words, the column is loaded eccentrically. A common method in use for calculating the stresses in such a column consists of adding together the following stresses:

(*a*) The stress due to the column action as determined by means of the Ritter formula, or $\frac{P}{A}\left[1 + \frac{S_e L^2}{nE\pi^2 r^2}\right]$.

(*b*) The flexural stress due to the eccentricity, namely $\frac{Pec}{Ar^2}$; in which c is the distance from the gravity axis of the column to the outer fiber on the concave side, and e is the eccentricity of the external force P, including the deflection of the column due to the load. For working stresses used in designing machine members, the deflections of columns having a slenderness ratio $\frac{L}{r}$ of less than 120 are of little consequence and for that reason may be neglected, thus simplifying the calculations.

By adding the two stresses we find that the expression for the maximum compressive stress in an eccentrically loaded column is

$$S_c = \frac{P}{A}\left[1 + \frac{S_e L^2}{nE\pi^2 r^2} + \frac{ce}{r^2}\right] \tag{27}$$

17. Shearing Combined with Tension or Compression.—Many machine members are acted upon by external forces that produce a direct tensile or compressive stress in addition to a direct shearing stress at right angles to the former. The combination of these direct stresses produces similar stresses, the magnitudes of which may be arrived at by the following expressions taken from Merriman's "Mechanics of Materials:"

$$\text{Maximum tensile stress} = \frac{S_t}{2} + \sqrt{S_s^2 + \frac{S_t^2}{4}} \tag{28}$$

$$\text{Maximum compressive stress} = \frac{S_c}{2} + \sqrt{S_s^2 + \frac{S_c^2}{4}} \tag{29}$$

$$\text{Maximum shearing stress} = \sqrt{S_s^2 + \frac{S_t^2}{4}} \text{ or } \sqrt{S_s^2 + \frac{S_c^2}{4}} \tag{30}$$

These formulas will be found useful in arriving at the resultant stresses in machine members subjected to torsion combined with bending or direct compression. Such cases will be discussed in the chapter on shafting.

18. Stresses Due to Suddenly Applied Forces.—In studying the stresses produced by suddenly applied forces, two distinct cases must be considered.

(*a*) An unstrained member acted upon by a suddenly applied force having no velocity of approach.

(*b*) An unstrained member acted upon by a force that has a velocity of approach.

Case (*a*).—For the case in which the suddenly applied force P has no velocity before striking the unstrained member, the external work done by this force is $P\Delta$, in which Δ represents the total deformation of the member. If the stress S induced in the member having an area A does not exceed the elastic limit, then the internal work is represented by the following expression:

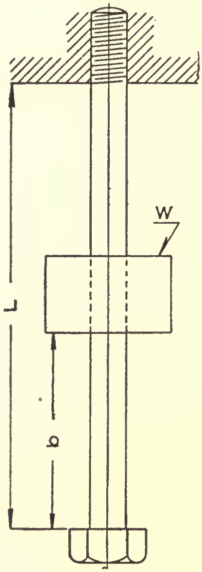

Fig. 5.

$$\text{Internal work} = \frac{A\Delta S}{2}$$

Equating the external to the internal work, we obtain

$$S = \frac{2P}{A} \tag{31}$$

That is, the stress produced in this case by the suddenly applied force P is double that produced by the same force if it were applied gradually.

Case (*b*).—To derive the expression for the magnitude of the stress induced in an unstrained member of area A by a force P that has a velocity of approach v, we shall assume a long bolt or bar having a head at one end and the other end held rigidly as shown in Fig. 5. Upon the bolt a weight W slides freely, and is allowed to fall through a distance b before it strikes the head of the bolt.

As soon as the weight W strikes the head, the bolt will elongate a distance Δ, from which it is evident that the external work performed by W is $W(b + \Delta)$. The stress in the bolt at the instant before W strikes the head is zero, and after the bolt has been elongated a distance Δ the stress is S; hence the work of the variable tension during the period of elongation is $\frac{A\Delta S}{2}$, assuming that S is within the elastic limit. To do this internal work, the weight W has given up its energy; hence equating the external to the internal work and solving for S, we get

$$S = \frac{2W}{A\Delta}(b + \Delta) \tag{32}$$

From Art. 5, the elongation

$$\Delta = \frac{SL}{E},$$

Substituting this value of Δ in (32), and collecting terms

$$S = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2\,bAE}{WL}}\right] \tag{33}$$

If in (33), the distance b is made zero, so as to give the conditions stated in case (a) above, we find that $S = \frac{2\,W}{A}$, which agrees with results expressed by (31).

REPEATED STRESSES

19. Repeated High Stresses.—It is now generally conceded that in a machine part subjected to repeated stress there is some internal wear or structural damage of the material which eventually causes failure of the part. In June, 1915, Messrs. Moore and Seely presented before the American Society for Testing Materials a paper, in which they gave an excellent analytical discussion of the cumulative damage done by repeated stress. The application of the proposed formula gives results that agree very closely with the experimental results obtained by the authors themselves as well as those obtained by earlier investigators. For a range of stress extending from the yield point to a stress slightly below the elastic limit, Messrs. Moore and Seely derived the following formula as representing the relation existing between the fiber stress and the number of repetitions of stress necessary to cause failure:

$$S = \frac{a}{(1 - q)N^b}, \tag{34}$$

in which

N = the number of repetitions of stress.
S = maximum applied unit stress (endurance strength).
a = constant depending upon the material.

b = constant based upon experiment.

$$q = \frac{\text{minimum unit stress}}{\text{maximum unit stress}}.$$

For a complete reversal of stress, $q = -1$, and when the range is from zero to a maximum, $q = 0$.

20. Repeated Low Stresses.—The formula expressing the relation between the fiber stress and the number of repetitions of low stress, according to the above-mentioned paper, is as follows:

$$S = \frac{a}{(1 - q)N^b}(1 + cN^e), \tag{35}$$

in which c and e are constants, the values of which must be obtained by means of experiments. The factor $(1 + cN^e)$ is called

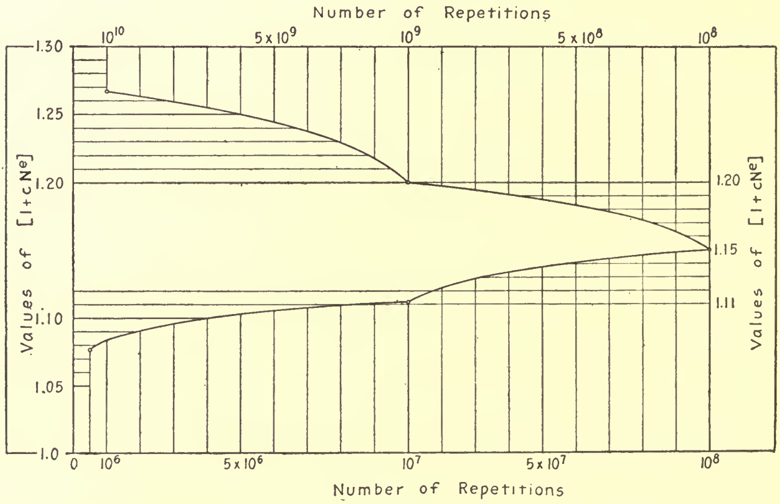

FIG. 6.

by the authors a probability factor, and its numerical value depends altogether upon the judgment of the designer. In Fig. 6 are plotted the values of $(1 + cN^e)$, as proposed by the authors, for use in determining the magnitude of the stress S in any part, the failure of which would not endanger life. For parts, the failure of which would endanger life, this probability factor should be assumed as equal to unity. In Table 3 are given values of a for various materials, as determined from existing data of repeated stress tests.

Table 3.—Values of Constant a

Material	a	Material	a
Structural steel............	250,000	Spring steel.......	400,000 to 600,000
Soft machinery steel........	250,000	Hard-steel wire....	600,000
Cold-rolled steel shafting ...	400,000	Gray cast iron.....	100,000
Steel (0.45 per cent. carbon)	350,000	Cast aluminum....	80,000
Wrought iron.............	250,000	Hard-drawn copper wire.	140,000

The value of q, the ratio of minimum to maximum stress is usually known, or may be established from the given data. According to the authors, if the stress is wholly or partially reversed, q must be taken as negative, having a value of -1 when there is a complete reversal of stress. In cases where the value of q approaches $+1$, it is possible that the endurance stress calculated by means of (35), will be in excess of the safe static stress, in which case the latter should govern the design.

For the exponent b, Messrs. Moore and Seely recommend that it should be made equal to $\frac{1}{8}$, this value being derived from a careful study of data covering a wide range of repeated stress tests.

21. Safe Endurance Stress.—As stated in a preceding paragraph, the formula given applies only to stresses up to the yield point of the material; hence whenever the endurance strength calculated by (35) is less than the yield point, a so-called *factor of safety* must be introduced, in order to arrive at a *safe endurance stress*. This may be accomplished in the following two ways:

(a) By applying the factor of safety to the stress.

(b) By applying the factor of safety to the number of repetitions.

The latter method is recommended by Moore and Seely, and the method of procedure is to multiply the number of repetitions a machine is to withstand by the factor of safety, and then determine the endurance stress for this new number of repetitions.

TEMPERATURE STRESSES

22. Deformation Due to Temperature Change.—It is important that certain machine members be so designed that expansion as well as contraction due to a change in temperature may take place without unduly stressing the material. Now before we can determine the magnitude of such stresses, we must arrive at the deformation caused by the rise or drop in temperature. The amount that a member will change in length depends upon the material and the change in temperature, and may be expressed by the following formula:

$$\Delta = \alpha t L, \tag{36}$$

in which L represents the original length, t the change in temperature in degrees Fahrenheit, and α *the coefficient of linear expansion.* For values of α consult Table 4.

TABLE 4.—VALUES OF COEFFICIENT OF LINEAR EXPANSION

Material	Range of temperature	Coefficient α
Cast iron.......	32 to 212	0.00000618
Wrought iron	32 to 212	0.00000656
	32 to 572	0.00000895
Steel casting....	32 to 212	0.00000600
Soft steel.......	32 to 212	0.00000630
Nickel steel.....	32 to 212	0.00000730
Brass casting...	32 to 212	0.0000104
Bronze.........	32 to 212	0.0000100
Copper......	32 to 212	0.00000955
	32 to 572	0.00001092

23. Stress Due to Temperature Change.—Due to the deformation Δ discussed in the preceding article, the machine member subjected to a change in temperature will be stressed, if its ends are constrained so that no expansion or contraction may occur. Knowing the magnitude of Δ, the unit strain is $\frac{\Delta}{L}$, from which we may readily determine the intensity of stress due to a change t in temperature, by applying the definition of the modulus of elasticity given in Art. 5; hence

$$S = \alpha t E \tag{37}$$

WORKING STRESSES

24. Factor of Safety.—In general, the maximum stress induced in a machine part must be kept well within the elastic limit so that the action of the external forces is almost perfectly elastic. The stress thus used in arriving at the size of the part is called the *working stress*, and its magnitude depends upon the following conditions:

(*a*) Is the application of load steady or variable?

(*b*) Is the part subjected to unavoidable shocks or jars?

(*c*) Kind of material, whether cast iron, steel, etc.

(*d*) Is the material used in the construction reliable?

(*e*) Is human life or property endangered, in case any part of a machine fails?

(*f*) In case of failure of any part, will any of the remaining parts of the machine be overloaded?

(*g*) Is the material of the machine part subjected to unnecessary and speedy deterioration?

(*h*) Cost of manufacturing.

(*i*) The demand upon the machine at some future time.

As usually determined, the working stress for a given case is obtained by dividing the ultimate strength by the so-called *factor of safety*, which factor should really represent a product of several factors depending upon the various conditions enumerated above. In general, larger factors of safety are used when a piece is made of cast metal, than when a hammered or rolled material is used. The selection of a larger factor of safety for cast metals is due to the fact that cast parts may contain hidden blow holes and spongy places. In many cases the material may be stressed an unknown amount due to unequal cooling caused by the improper distribution of the material, no matter how careful the moulder may be in cooling the casting after it is poured.

Again, live loads require much larger factors of safety than dead loads, and loads that produce repetitive stresses that change continually from tension to compression, for example, also require large factors of safety, the magnitudes of which are difficult to determine. For the latter case, the equations of Arts. 19, 20 and 21 may serve as guide.

Table 5.—Suggested Factors of Safety

Material	Kind of stress		
	Steady	Varying	Shock
Hard steel.....	5	6	15
Structural steel.	4	6	10
Wrought iron..	4	6	10
Cast iron......	6	10	20
Timber........	6	10	15

In Table 5 are given suggested *factors of safety* based on the ultimate strength of the material. It must be remembered that the skill and judgment of the designer should play an important part in arriving at the proper working stresses for any given set of conditions.

TABLE 6.—AVERAGE PHYSICAL PROPERTIES OF PRINCIPAL MATERIALS

Material		Wt. per cu. ft.	Tension		Compression		Shearing		Modulus of elasticity		Remarks
			Ultimate	Yield point	Ultimate	Yield point	Ultimate	Yield point	Direct	Transverse	
Cast iron	Gray iron	450	15,000	....	90,000	....	15,000	....	10,000,000	4,000,000	
	Better grade		30,000	....	180,000	....	30,000	....	14,000,000	6,000,000	
Mall. cast iron	Good	450	35,000	16,000							
	Best		48,000	22,000							
Wrought iron	Best	480	42,000	27,000	36,000	....	35,000	....	26,000,000	9,500,000	Tank iron.
	D'ble best		52,000	34,000	45,000	....	44,000	....	28,000,000	10,000,000	Boiler plate.
Steel casting	Soft		60,000	35,000							
	Medium	490	72,000	40,000	....	....	....	....	29,000,000		
	Hard		80,000	45,000							
Carbon steel	0.05 % C.		50,000	25,000	....	40,000	45,000	22,000	27,000,000	13,000,000	Rivets.
	0.10 % C.		60,000	30,000	....	65,000	54,000	28,000	28,000,000	13,500,000	Boiler plate.
	0.25 % C.	490	70,000	35,000	....	80,000	60,000	32,000	29,000,000	14,000,000	Structural.
	0.50 % C.		80,000	40,000	....	90,000	70,000	38,000	30,000,000	14,000,000	Machinery.
	1–2 % C.		150,000	....	....	....	120,000	....	32,000,000	16,000,000	Tools.
Copper	Cast	550	22,000	....	....	....	....	....	12,000,000		
	Rolled		32,000	....	....	....	....	....	15,000,000	6,000,000	
Brass	Cast	500	24,000	....	....	....	....	....	9,500,000		
	Rolled		43,000	....	....	....	....	....	11,500,000	5,000,000	
Bronze			36,000	....	....	....	....	....	10,000,000		
Manganese bronze		530	70,000	33,000	....	30,000					
Phosphor bronze			56,000	26,000	....	....	....	....	14,000,000		
Aluminum bronze		470	77,000	24,000	....	25,000					
Gun metal		550	36,000	18,000	....	14,500					
Aluminum	Cast	165	14,000	7,500	11,500	....	....	....	9,000,000		
	Rolled		20,000	13,500	....	....	....	....	10,000,000		
Monel metal	Cast	550	65,000	32,500							
	Rolled		80,000	45,000							

For ultimate strengths and various other physical properties of the more common metals used in the construction of machinery, consult Table 6.

References

Mechanics of Materials, by Merriman.
The Strength of Materials, by E. S. Andrews.
Mechanical Engineers' Handbook, by L. S. Marks.
Elasticität und Festigkeit, by C. Bach.

CHAPTER II

MATERIALS USED IN THE CONSTRUCTION OF MACHINE PARTS

The principal materials used in the construction of machine parts are cast iron, malleable iron, steel casting, steel, wrought iron, copper, brass, bronze, aluminum, babbitt metal, wood and leather.

CAST IRON

25. Cast Iron.—Cast iron is more commonly used than any other material in making machine parts. This is because of its high compressive strength and because it can be given easily any desired form. A wood or metal pattern of the piece desired is made, and from this a mould is made in the sand. The pattern is next removed from the mould and the liquid metal is poured in, which on cooling assumes the form of the pattern.

Crude cast iron is obtained directly from the melting of the iron ore in the blast furnace. This product is then known as pig iron, and is rarely ever used except to be remelted into cast iron, or to be converted into wrought iron or steel. Cast iron fuses easily, but it cannot be tempered nor welded under ordinary conditions. The composition of cast iron varies considerably, but in general is about as follows:

	Per cent.
Metallic iron	90.0 to 95.0
Carbon	1.5 to 4.5
Silicon	0.5 to 4.0
Sulphur	less than 0.15
Phosphorus	0.06 to 1.50
Manganese	trace to 5.0

(*a*) *Carbon.*—Carbon may either be united chemically with the iron, in which case the product is known as white iron, or it may exist in the free state, when the product is known as *gray iron.* The white iron is very brittle and hard, and is therefore but little used in machine parts. In the free state the carbon exists as graphite.

(*b*) *Silicon.*—Silicon is an important constituent of cast iron because of the influence it exerts on the condition of the carbon present in the iron. The presence of from 0.25 to 1.75 per cent. of silicon tends to make the iron soft and strong; but beyond 2.0 per cent. silicon, the iron becomes weak and hard. An increase of silicon causes less shrinkage in the castings, but a further increase (above 5 per cent.) may cause an increase in the shrinkage. With about 1.0 per cent. silicon, the tendency to produce blow holes in the castings is reduced to a minimum.

(*c*) *Sulphur.*—Sulphur in cast iron causes the carbon to unite chemically with the iron, thus producing hard white iron, which is brittle. For good castings, the sulphur content should not exceed 0.15 per cent.

(*d*) *Phosphorus.*—Phosphorus in cast iron tends to produce weak and brittle castings. It also causes the metal to be very fluid when melted, thus producing an excellent impression of the mould. For this reason phosphorus is a desirable constituent in cast iron for the production of fine, thin castings where no great strength is required. To produce such castings, from 2 to 5 per cent. of phosphorus may be used. For strong castings of good quality, the amount of phosphorus rarely exceeds 0.55 per cent., but when fluidity and softness are more important than strength, from 1 to 1.5 per cent. may be used.

(*e*) *Manganese.*—Manganese when present in cast iron up to about 1.5 per cent. tends to make the castings harder to machine; but renders them more suitable for smooth or polished surfaces. It also causes a fine granular structure in the castings and prevents the absorption of the sulphur during melting. Manganese may also be added to cast iron to soften the metal. This softening is due to the fact that the manganese counteracts the effects of the sulphur and silicon by eliminating the former and counteracting the latter. However, when the iron is remelted, its hardness returns since the manganese is oxidized and more sulphur is absorbed. The transverse strength of cast iron is increased about 30 per cent., and the shrinkage and depth of chill decreased 25 per cent., while the combined carbon is diminished one-half by adding to the molten metal, powdered ferromanganese in the proportion of 1 pound of the latter to about 600 pounds of the former.

26. Vanadium Cast Iron.—The relatively coarse texture of cast iron may be much improved by the addition of 0.10 to 0.20

per cent. of vanadium, and at the same time the ultimate strength is increased from 10 to 25 per cent. Cast iron containing a small percentage of vanadium is tougher than ordinary gray iron, thus making it an excellent material for use in steam- and gas-engine cylinders, piston rings, liners, gears and other similar uses. Some of the larger railway systems have now adopted this material for their cylinder construction. In machining vanadium cast iron, it is possible to give it a much higher finish than is possible with gray iron.

27. Pig Iron.—Pig iron is the basis for the manufacture of all iron products. It is not only used practically unchanged to produce castings of a great variety of form and quality, but it is also used in the manufacture of wrought iron and steel. For each special purpose, the iron must have a composition within certain limits. It follows, therefore, that pig iron offers a considerable variety of composition. The practice of purchasing pig iron by analysis is generally followed at the present time. In Table 7 are give the specifications for the various grades of pig iron used by one large manufacturer.

TABLE 7.—SPECIFICATIONS OF PIG IRON

Class	Total carbon not under, per cent.	Silicon, per cent.	Sulphur not over, per cent.	Phosphorus, per cent.	Manganese not over, per cent.
1	3.0	1.5 to 2.0	0.040	0.20 to 0.75	1.0
2	3.5	2.0 to 2.5	0.035	0.20 to 0.75	1.0
3	3.5	2.5 to 3.0	0.030	0.20 to 0.75	1.0
4	3.5	2.0 to 2.5	0.040	1.00 to 1.50	1.0
5	3.0	4.0 to 5.0	0.040	0.20 to 0.80	1.0

In general, an analysis is made from drillings taken from a pig selected at random from each four tons of every carload as unloaded. The right is reserved to reject a portion or all of the material which does not conform to the above specifications in every particular.

In a general way, the specified limits for the composition of the chief grades of pig iron are given in Table 8.

According to use, pig iron may be divided roughly into two classes. The first class includes those grades used in the production of foundry and malleable irons, while the second includes those used in the manufacture of wrought iron and steel. In the process of remelting or manufacturing, the first class undergoes little if any chemical change, while the second class undergoes a complete chemical change.

Table 8.—General Specifications of Pig Iron

Grade of iron	Silicon, per cent.	Sulphur, per cent.	Phosphorus, per cent.	Manganese, per cent.
No. 1 foundry	2.5 to 3.0	Under 0.035		
No. 2 foundry	2.0 to 2.5	Under 0.045	0.5 to 1.0	
No. 3 foundry	1.5 to 2.0	Under 0.055		
Malleable	0.7 to 1.5	Under 0.050	Under 0.2	Under 1.0
Gray forge	Under 1.5	Under 0.100	Under 1.0	
Bessemer	1.0 to 2.0	Under 0.050	Under 0.1	
Low phosphorus	Under 2.0	Under 0.030	Under 0.3	
Basic	Under 1.0	Under 0.050	Under 1.0	
Basic Bessemer	Under 1.0	Under 0.050	2.0 to 3.0	1.0 to 2.0

28. Malleable Casting.—Malleable castings are made by heating clean foundry castings, preferably with the sulphur content low, in an annealing furnace in contact with some substance that will absorb the carbon from the cast iron. Hematite or brown iron ore in pulverized form is used extensively for that purpose. The intensity of heat required is on the average about 1,650°F. The length of time the castings remain in the furnace depends upon the degree of malleability required and upon the size of the castings. Usually light castings require a minimum of 60 hours, while the heavier ones may require 72 hours or longer.

The tensile strength of good malleable cast iron lies somewhere between that of gray iron and steel, while its compressive strength is somewhat lower than that of the former. Good malleable castings may be bent and twisted without showing signs of fracture, and for that reason are well adapted for use in connection with agricultural machinery, railroad supplies, and automobile parts.

29. Chilled Casting.—Chilled castings are those which have a hard and durable surface. The iron used is generally close-grained gray iron low in silicon. A chilled casting is formed by making that part of the mould in contact with the surface of the casting to be chilled of such construction that the heat will be withdrawn rapidly. The mould for causing the chill usually consists of iron bars or plates, placed so that their surfaces will be in contact with the molten iron. These plates abstract heat rapidly from the iron, with the result that the part of the casting in contact with the cold surface assumes a state similar to white iron, while the rest of the casting remains in the form of gray iron. The withdrawal of heat is hastened by the circulation of cold water through pipes, circular or rectangular in cross-section, placed near the surface to be chilled. Chilled castings offer great resistance to crushing forces. The outside or "skin" of the ordinary casting is in fact a chilled surface, but by the arrangement mentioned above, the depth of the "skin" is greatly increased with a corresponding increase in strength and wearing qualities. Car wheels, jaws for crushing machinery, and rolls for rolling mills are familiar examples of chilled castings. Car wheels require great strength combined with a hard durable tread. The depth of the chill varies from 3/8 to 1 inch.

It has been found that with the use of vanadium in chilled castings, a deeper, stronger and tougher chill can be produced. This chill, however, is not quite as hard as that found on ordinary chilled cast iron, and hence has the advantage that such castings can be filed and machined more easily.

30. Semi-steel.—The term *semi-steel* is applied to a metal that is intermediate between cast iron and malleable iron. The meaning of the term as used at the present time is vague and for that reason its use is questioned. The so-called semi-steel is produced in the cupola by mixing from 20 to 40 per cent. of low-carbon steel scrap with the pig iron and cast scrap. This mixture, if properly handled in the cupola as well as in pouring the mould, produces a clean close-grained tough casting that may be machined easily and that has an ultimate tensile strength varying from 32,000 to 42,000 pounds per square inch. Its transverse strength is also considerably higher than that of ordinary gray iron. However, the material produced by such a mixture as given above has none of the distinctive properties of steel and in reality it is nothing more than a high-grade gray-iron casting. Semi-steel has been used very successfully for cylinders, piston rings, cylinder liners, gears, plow points, and frames of punches and shears.

WROUGHT IRON

31. Wrought Iron.—Wrought iron is formed from pig iron by melting the latter in a puddling furnace. During the process of melting, the impurities in the pig iron are removed by oxidation, leaving the pure iron and slag both in a pasty condition. In this condition the mixture of iron and slag is formed into *muck balls* weighing about 150 pounds, and is removed from the furnace These balls are put into a *squeezer* and compressed, thereby removing a large amount of the slag, after which it is rolled into bars. The bars, known as "muck bars," are cut into strips and arranged in piles, the strips in the consecutive layers being at right angles to each other. These piles are then placed into a furnace and raised to a welding heat and are then rolled into *merchant bars.* If the quality of the iron is to be improved and the last-mentioned process is repeated, we obtain what is known as "*best iron,*" "*double best*" and "*treble best,*" depending upon the number of repetitions. The merchant bar finally produced is the ordinary wrought iron of commerce. At the present time wrought iron is not used as extensively as in the past, steel to a great extent having taken its place; however, it still is used in the manufacture of pipes, boiler tubes, forgings, parts of electrical machinery, small structural shapes, and crucible steel.

STEEL CASTING

32. Manufacturing Processes.—Castings similar to iron castings may be formed in almost any desired shape from molten steel. They are produced by four distinct methods as follows:

(*a*) *Crucible process.*—When it is desired to produce very fine and high-grade castings, not very large, the *crucible process* is used.

(*b*) *Bessemer process.*—This method is used chiefly for producing small castings.

(*c*) *Open-hearth process.*—The *open-hearth process* is used extensively for the production of steel castings either small or extremely large in size. The castings produced by this method are considered superior to those produced by the Bessemer process.

(*d*) *Electric-furnace method.*—The *electric furnace* which is now being introduced into this country is capable of producing the very best grades of steel castings.

In texture, the castings produced by the common processes in use today are coarse and crystalline, since the steel has been permitted to cool without drawing or rolling. In order to improve the grain structure, and at the same time remove some of the internal stresses, all steel castings must be annealed before machining them. Formerly trouble was experienced in obtaining good sound steel castings; but by great care and improved methods in the production of moulds, first-class castings may now be obtained. In general, steel castings are used for those machine parts requiring greater strength than is obtained by using gray-iron castings.

33. Manganese-steel Castings.—Manganese-steel castings are produced by adding ferro-manganese to open-hearth steel, and the average chemical composition of such castings is about as follows: Manganese, 12.5 per cent.; carbon, 1.25 per cent.; silicon, 0.3 per cent.; phosphorus, 0.08 per cent.; sulphur, 0.02 per cent.; iron, 85.85 per cent. The average physical properties of this kind of steel casting are about as follows:

Tensile strength.........	110,000	pounds per square inch.
Elastic limit.............	54,000	pounds per square inch.
Elongation in 8 inches....	45	per cent.
Reduction of area........	50	per cent.

Manganese steel is in general free from blow holes, but is difficult to cast on account of its high shrinkage, which is about two and one-half times as great as that of cast iron. As originally cast it is extremely hard and brittle and it is possible to pulverize it under the blows of a hammer. The fact that this metal is brittle when it comes from the mould makes it possible to break off the risers and gates remaining on the casting, which could not be done were the original casting as tough as the finished product. As mentioned, manganese-steel casting possesses great hardness which is not diminished by annealing, and in addition it has a high tensile strength combined with great toughness and ductility. These qualities would make this steel the ideal metal for machine construction, were it not for the fact that its great hardness prevents it from being machined in any way but by abrasive processes, which at best are expensive. Again, the very property of hardness, combined with great toughness, also limits its use to the rougher class of castings, or such that require a minimum amount of finish.

The toughness of the finished casting is produced by the annealing process. In this process the brittle castings are placed in annealing furnaces, in which they are heated gradually and carefully. After remaining in these furnaces from three to twenty-four hours, depending upon the type of casting treated, the castings are removed from the furnace and quenched in cold water. It is evident that great care must be exercised by the designer to distribute the metal properly in large and complicated castings in order that all the parts may cool at approximately the same rate. It has been found by experience that the heat treatment just described cannot be made to extend through a section thicker than 5 or 5½ inches. In general, thicknesses exceeding 3½ inches are not found in well-designed casting.

34. Applications of Manganese-steel Castings.—Due to the fact that manganese-steel casting is the most durable metal known as regards ability to resist wear, it is well adapted to the following classes of service:

(*a*) For all wearing parts of crushing and pulverizing machinery, such as rolls, jaws and toggle plates, heads, mantels, and concaves.

(*b*) In all classes of excavating machinery; for example, the dipper and teeth of dipper dredges, the buckets of placer dredges, the cutter head and knives of ditching machines.

(*c*) The impellers and casings of centrifugal pumps are frequently made of manganese-steel casting. In this connection it is of interest to note that soft-steel inserts are cast into the casing at proper places to permit the drilling and tapping of holes for the various attachments.

(*d*) In connection with hoisting machinery such parts as sheaves, drums, rollers, and crane wheels made of manganese-steel casting are not uncommon. It is claimed that the life of a rope sheave or roller made of this material is about thirty times that of one made of cast iron.

(*e*) In mining work, the wheels of coal cars and skips, also the head sheaves, are made of manganese steel. In the latter application, the rim only is made of manganese steel and is then bolted to the wrought-iron spokes, which in turn are bolted to the cast-iron hub.

(*f*) In conveying machinery where the parts are subjected to severe usage, as for example a conveyor chain in a cement mill, both the chains and the sprockets are made of manganese-steel casting.

(*g*) In railway track work, manganese-steel casting has given excellent service for crossings, frogs, switches, and guard rails.

(*h*) Another very important use of manganese-steel casting is in the construction of safes and vaults; for this purpose it is particularly well adapted since it cannot be drilled nor can its temper be drawn by heating.

STEEL

Steel is a compound in which iron and carbon are the principal parts. It is made from pig iron by burning out the carbon, silicon, manganese and other impurities, and recarbonizing to any degree desired. The principal processes or methods of manufacturing steel are the following: (*a*) the Bessemer; (*b*) the open-hearth; (*c*) the cementation.

35. Bessemer Process.—In the Bessemer process, several tons, usually about ten, of molten pig iron are poured into a converter, and through this mass of iron a large quantity of cold air is passed. In about four minutes after the blast is turned on, all the silicon and manganese of the pig iron has combined with the oxygen of the air. The carbon in the pig iron now begins to unite with the oxygen, forming carbon monoxide, which burns through the mouth of the converter in a long brilliant flame. The burning of the carbon monoxide continues for about six minutes, when the flame shortens, thus indicating that nearly all of the carbon has been burned out of the iron and that the air supply should be shut off. The burning out of these impurities has raised the temperature of the iron to a white heat, and at the same time produced a relatively pure mass of iron. To this mass is added a certain amount of carbon in the form of a very pure iron high in carbon and manganese. The metal is then poured into moulds forming ingots, which while hot are rolled into the desired shapes.

The characteristics of the Bessemer process are: (*a*) great rapidity of reduction, about ten minutes per heat; (*b*) no extra fuel is required; (*c*) the metal is not melted in the furnace where the reduction takes place.

Bessemer steel was formerly used almost entirely in the manufacture of wire, skelps for tubing, wire nails, shafting, machinery steel, tank plates, rails, and structural shapes. Open-hearth steel, however, has very largely superseded the Bessemer product in the manufacture of these articles.

36. Open-hearth Process.—In the manufacture of open-hearth steel, the molten pig iron, direct from the reducing furnace, is poured into a long hearth, the top of which has a firebrick lining. The impurities in the iron are burned out by the heat obtained from burning gas and air, and reflected from this refractory lining.

The slag is first burned, and the slag in turn oxidizes the impurities. The time required for purifying is from 6 to 10 hours, after which the metal is recarbonized, cast into ingots and rolled as in the Bessemer process.

The characteristics of the open-hearth process are: (*a*) relatively long time to oxidize the impurities; (*b*) large quantities, 35 to 70 tons, may be purified and recarbonized in one charge; (*c*) extra fuel is required; (*d*) a part of the charge, steel scrap and iron ore added at the beginning of the process, are melted in the furnace.

Open-hearth steel is used in the manufacture of cutlery, boiler plate, and armor plate in addition to the articles mentioned in Art. 35.

37. Cementation Process.—In this process of manufacturing steel, bars of wrought iron imbedded in charcoal are heated for several days. The wrought iron absorbs carbon from the charcoal and is thus transformed into steel. When the bars of iron are removed they are found to be covered with scales or *blisters.* The name given to this product is *blister steel.* By removing the scales and blisters and subjecting the bars to a cherry-red heat for a few days, a more uniform distribution of the carbon is obtained.

Blister steel when heated and rolled directly into the finished bars, is known as *German steel.* Bars of blister steel may be cut up and forged together under the hammer, forming a product called *shear steel.* By repeating the process with the shear steel, we obtain *double-shear steel.*

38. Crucible Steel.—Crucible steel, also called cast steel, is very uniform and homogeneous in structure. It is made by melting blister steel in a crucible, casting it into ingots and rolling into bars. By this method is produced the finest crucible steel. Another method of producing crucible-cast steel is to melt Swedish iron (wrought iron obtained from the reduction of a very pure iron in the blast furnace in which charcoal instead of coke for producing the puddling flame is used) in contact with charcoal in a sealed vessel, the contents of which are poured into a large ladle containing a similar product from other sealed vessels. This mixing insures greater uniformity of material. The metal in this large ladle is cast into ingots, which are subsequently forged or rolled into bars. By far the greater part of crucible steel is produced by this method.

39. Cold-rolled Steel.—The so-called cold-rolled steel is rolled hot to approximately the required dimensions. The surface is then carefully cleaned, usually by chemical means, and rolled cold to a very accurately gauged thickness between smooth rollers. The rolling of metal when cold has two important advantages as follows: when steel is rolled hot the surface of the steel oxidizes and forms a scale, while with cold rolling no such action takes place, thus making it possible to produce a bright finish. Furthermore, since no scale is formed the bar or plate to be rolled can be made very accurate. The cold-rolling process has the effect of increasing the elastic limit and ultimate strength, but decreases the ductility. It also produces a very smooth and hard surface. Its principal use is for shafting and rectangular, square and hexagonal bars, as well as strip steel which of late is in demand for use in the manufacture of pressed-steel products. For the latter class of work the absence of scale, already referred to, has a marked effect on the life of the dies, as experience in press working of hot-rolled metal shows that the scale on the latter is exceedingly hard on the dies.

ALLOY STEELS

The term *alloy steels* is applied to all steels that are composed of iron and carbon, and one or more special elements such as nickel, tungsten, manganese, silicon, chromium, and vanadium. In general, alloy steels must always be heat treated, and should never be used in the natural or annealed condition, since in the latter condition the physical properties of the material are but little better than those of the ordinary carbon steels. The heat treatment given to alloy steels causes a marked improvement in the physical properties. A few of the principal alloy steels are discussed in the following paragraphs.

40. Nickel Steel.—Nickel added to a carbon steel increases its ultimate strength and elastic limit as well as its hardness and toughness. It tends to produce a steel that is more homogeneous and of finer structure than the ordinary carbon steel, and if the percentage of nickel is considerable the material produced resists corrosion to a remarkable degree. The percentage of nickel varies from 1.5 to 4.5, while the carbon varies from 0.15 to 0.50 per cent., both of these percentages depending upon the grade of steel desired. Nickel steel has a high ratio of elastic limit to ultimate strength and in addition offers great resistance to cracking. The latter property makes this type of steel desirable for use as armor plate. Nickel steel is also used for structural shapes and for rails; the latter show better wearing qualities than those made from Bessemer or open-hearth steel. On account of its ability to withstand heavy shocks and torsional stresses, nickel steel is well adapted for crankshafts, high-grade shafting, connecting rods, automobile parts, car axles and ordnance.

41. Chrome Steel.—Chrome steel is produced by adding to high-carbon steel (0.8 to 2.0 per cent.) from 1 to 2 per cent. of chromium. The steel thus produced is very fine-grained and homogeneous, is extremely hard, and has a high ratio of elastic limit to ultimate strength. Due to its extreme hardness, chrome steel may be used for ball and roller bearings, armor-piercing shells, armor plate, burglar-proof safes, and vaults. The element chromium is also used in the manufacture of the best high-speed tool steels.

42. Vanadium Steel.—Vanadium steel is produced by adding to carbon steel, a small amount of vanadium, generally between 0.15 and 0.25 per cent. This alloy steel is used as a forging or machinery steel, and should be heated slowly when preparing it for a forging operation. The effect of the vanadium is to increase the elastic limit as well as the capacity for resisting shock. Vanadium is used more in conjunction with chromium or nickel steel than with ordinary carbon steel. Carbon vanadium steel containing from 0.60 to 1.25 per cent. carbon and over 0.2 per cent. vanadium may be tempered, and due to its toughness, is well adapted for punches, dies, rock drills, ball and roller bearings, and other similar uses.

43. Nickel-chromium Steel.—Nickel-chromium steel is used chiefly in automobile construction, where a high degree of strength and hardness is demanded. At the present time this type of steel is also being used for important gears on machine tools. In the automobile industry, three types of nickel-chromium steels are commonly used. These are known as low nickel-, medium nickel-, and high nickel-chromium steels.

In general, nickel-chromium steels having a carbon content up to 0.2 per cent. are intended for case hardening; those having 0.25 to 0.4 per cent. carbon are used for the structural parts of automobiles, while the higher-carbon steels may be used for gears or other important parts.

44. Chromium-vanadium Steel.—Chromium-vanadium steel is tough and capable of resisting severe shocks, and has an exceedingly high elastic limit in proportion to its ultimate strength. This type of steel is used for springs, gears, driving shafts, steering knuckles, and axles in the automobile industry. It is also used for spindles and arbors for machine tools, locomotive driving axles, piston rods, side and connecting rods, and locomotive and car-wheel tires.

For high-duty shafts requiring a high degree of strength and a moderate degree of toughness, the grade of chromium-vanadium steel containing about 0.4 per cent. carbon should be selected. For springs and gears the carbon content should be from 0.45 to 0.50 per cent. Chromium-vanadium steels having a high carbon content of 0.75 to 1.0 per cent. may be tempered and used for tools. In addition to being hard it is tough, and for that reason has been used successfully for dies, punches, ball-bearing races, rock drills, and saws.

45. Silicon-manganese Steel.—A combination of silicon and manganese in moderate amounts added to steel increases its capacity for resisting shock, thus making it particularly suitable for all kinds of springs and to some extent for gears. For each class of service mentioned the steel must be given a proper heat treatment.

46. Tungsten Steel.—Tungsten steel is an alloy of iron, carbon, tungsten and manganese, and sometimes chromium. The element which gives this steel its peculiar property, self or air hardening, is not tungsten but manganese combined with carbon. The tungsten, however, is an important element, since it enables the alloy to contain a larger percentage of carbon. On account of its hardness, this steel can not be easily machined, but must be forged to the desired shape. Its chief use is for high-speed cutting tools.

ALLOYS

Alloys may be made of two or more metals that have an affinity for each other. The compound or alloy thus produced has properties and characteristics which none of the metals possess. The principal alloys used in machine construction may be obtained by combining two or more of the following metals: copper, zinc, tin, lead, antimony, bismuth, and aluminum.

47. Brass.—Brass is an alloy of copper and zinc; however, many of the commercial brasses contain small percentages of lead, tin, and iron. Brass for machine parts may be put in two general classes, namely, cast brass and wrought brass.

(*a*) *Cast brass.*—Cast brass is intended for parts not requiring great strength, and as usually made has a zinc content of about 35 per cent., and the remainder copper with traces of iron, lead and tin. In order to make cast brass free-cutting for machining purposes 1 to 2 per cent. of lead is added. A typical specification for cast brass as used by the Bureau of Steam Engineering of the United States Navy Department is as follows: copper, 59 to 63 per cent.; tin, 0.5 to 1.5 per cent.; iron, not exceed 0.06 per cent.; lead, not exceed 0.60 per cent.; zinc, remainder.

(*b*) *Wrought brass.*—Wrought brass may be of two kinds as follows: 1. That which contains approximately 56 to 62 per cent. of copper and the remainder zinc may be rolled or forged while hot. *Muntz metal* containing 60 per cent. of copper and 40 per cent. zinc is a well-known wrought brass which at one time was used very extensively for ship sheathing. The so-called *Tobin bronze* is another type of wrought brass that may be worked while hot, but it differs from Muntz metal in that it contains very small percentages of iron, tin, and lead, in addition to the copper and zinc. Its ultimate tensile strength is about equal to that of ordinary steel, while its compressive strength is about three times its tensile strength. Tobin bronze resists corrosion and for that reason meets with favor in naval work.

2. The second kind of wrought brass contains approximately 70 per cent. of copper and 30 per cent. of zinc, and not infrequently a small percentage of lead is introduced to facilitate machining. Brass having the composition just stated may be drawn or rolled in the cold state. The cold drawing or rolling changes the structure of the metal, increasing its strength and brittleness, and consequently the original ductility must be restored by an annealing operation.

48. Bronze.—Bronze is an alloy of copper and tin. Zinc is sometimes added to cheapen the alloy, or to change its color and to increase its malleability.

(*a*) *Commercial bronze.*—Commercial bronze is acid-resisting and contains 90 per cent. of copper and 10 per cent. of tin. This metal has been used successfully for pump bodies, also for thrust collars subjected to fairly high pressures. Another bronze which has proven very serviceable for gears and worm wheels where noiseless operation is desired, contains 89 per cent. of copper and 11 per cent. of tin. A form of bronze known as *gun metal* has the following approximate composition: 88 per cent. of copper; 10 per cent. of tin; and 2 per cent. of zinc. It is used for high-grade bearings subjected to high pressures and high speeds.

(*b*) *Phosphor bronze.*—Phosphor bronze varies somewhat in composition, but in general is about as follows: 80 per cent. copper; 10 per cent. tin; 9 per cent. lead; and 1 per cent. phosphorus. It is easily cast and is as strong or stronger in tension than cast iron. It is a very serviceable bearing metal and is used for bearings subjected to heavy pressures and high speeds; for example, locomotive cross-head bearings, crankpin bearings, and bearings on grinders and blowers.

A phosphor bronze intended for rolling into sheets or drawing into wire contains about 96 per cent. of copper, 4 per cent. of tin, and sufficient phosphorus to deoxidize the mixture. The tensile strength of such a phosphor bronze is equal to that of steel.

(*c*) *Manganese bronze.*—By the term manganese bronze, as commonly used, is meant an alloy consisting largely of copper and zinc with small percentages of other elements such as aluminum, tin and iron. In reality many of the so-called manganese bronzes are not bronzes at all, but brasses; however, there are several compositions in use in which the proportion of zinc is small compared to the amount of tin and these are, strictly speaking, bronzes. Many of the commercial manganese bronzes contain no manganese whatever, the latter being used merely as a deoxidizing agent.

Due to its high tensile strength and ductility, manganese bronze is well adapted for castings where great strength and toughness are required. The hubs and blades of propellers and certain castings used in automobile construction are frequently made of this alloy. It is not nearly as satisfactory as phosphor bronze when used for bearings. A manganese bronze made of 56 per cent. of copper, 43.5 per cent. zinc and 0.5 per cent. aluminum possesses high tensile strength and is suitable for the service just mentioned. Manganese bronze may also be rolled into sheets or bars, or drawn into wire.

(*d*) *Aluminum bronze.*—Aluminum bronze is formed by adding not to exceed 11 per cent. of aluminum to copper, thus producing an alloy having great strength and toughness. An alloy containing 90 per cent. of copper and 10 per cent. aluminum with a trace of titanium has given very satisfactory service when used for machine parts requiring strength and toughness, and at the same time subject to wear; for example, a worm wheel. The last-named composition produces an alloy that has an ultimate tensile strength equal to that of a medium-carbon steel. According to tests made at Cornell University, the coefficient of friction of this type of aluminum bronze is 0.0018, thus making it suitable for bearings, and experience has shown that for accurately fitted bearings, the results are very satisfactory. The titanium in the above composition is added to insure good solid castings. In addition to the uses mentioned above, this type of bronze, due to its ability to resist corrosion, may be used for parts exposed to the action of salt water, tanning and sulphite liquids.

49. Monel Metal.—Monel metal is a combination of approximately 28 per cent. of copper, 67 per cent. of nickel and small percentages of manganese and iron. It has a high tensile strength, is ductile, and has the ability to resist corrosion. It may be used to produce castings having an ultimate strength of 65,000 pounds per square inch. When used for rolling into sheets or bars, the strength is increased from 25 to 40 per cent.

Monel metal presents no difficulties in machining, nor in forging operations if worked quickly. Like copper, it is impossible to weld it under the hammer, but it can be welded by means of the oxy-acetylene flame or by electricity. Since this alloy is non-corrodible it is used largely for propeller blades, pump rods, high-pressure valves, and steam-turbine blading.

50. Aluminum.—Within the last few years aluminum alloys have been used rather extensively for many different machine parts. Pure aluminum is very ductile and may be rolled into very thin plates or drawn into fine wire. It may also be cast, but the casting produced has a coarse texture and for that reason pure aluminum is used but little for castings. For good commercial casting, aluminum alloys are used. The alloys recommended by the Society of Automobile Engineers are the following:

(*a*) *Aluminum copper.*—The aluminum copper alloy contains not less than 90 per cent. of aluminum, 7 to 8 per cent. of copper, and the impurities consisting of carbon, iron, silicon, manganese and zinc shall not exceed 1.7 per cent. This is a very light material; is tough, possesses a high degree of strength, and may be used for castings subjected to moderate shocks.

(*b*) *Aluminum-copper-zinc.*—An alloy, made of not less than 80 per cent. of aluminum, from 2 to 3 per cent. of copper, not more than 15 per cent. of zinc, and not to exceed 0.40 per cent. of manganese, gives a light-weight, close-grained material that can be cast easily and will be free from blow holes. The castings produced are very strong and are capable of resisting moderate shocks.

(*c*) *Aluminum zinc.*—The alloy containing 65 per cent. aluminum and 35 per cent. zinc is intended for castings subjected to light loads. It is quite brittle and is used for footboards and other similar parts of an automobile. It is about the cheapest aluminum alloy that is now in use.

Due to the excessive shrinkage to which all aluminum castings are subjected, great care must be exercised in their design. Thick sections should never join thin sections on account of cracks that are very likely to show up in the finished castings. In order to obtain the best results, all parts should be given as nearly a constant or uniform section as is practical, and strength combined with light weight may be obtained by proper ribbing.

Cast aluminum is used successfully in the construction of the framework of automobile motors, thus saving in weight and aiding in cooling the motor. It is also used for the construction of gear cases, pistons and clutch parts. For machine tools such as planers, pulleys are frequently made of cast aluminum so as to decrease the inertia of the rotating parts. The bodies or framework of jigs are occasionally made of cast aluminum, thus making them easier to handle.

51. Babbitt Metal.—The term *babbitt metal* generally refers to an alloy consisting of copper, tin and zinc or antimony and in which the tin content exceeds 50 per cent.

(*a*) *Genuine babbitt metal.*—The alloy containing copper, tin, and antimony is usually called *genuine babbitt metal.* According to the Society of Automobile Engineers, the follownig specifications will produce a high grade of babbitt that should give excellent results when used for such service as connecting-rod bearings, automobile motor bearings, or any other machine bearings subjected to similar service: copper, 7 per cent.; antimony, 9 per cent.; tin, 84 per cent. There are a large number of commercial grades of babbitt metals, many of which have a high percentage of lead and consequently sell at a low price.

(*b*) *White brass.*—The alloy called *white brass* is in reality a babbitt metal, since its tin content exceeds 50 per cent., as the following specifications adopted by the Society of Automobile Engineers show: 3 to 6 per cent. of copper; 28 to 30 per cent. of zinc; and not less than 65 per cent. of tin. This alloy is recommended for use in automobile engine bearings and generous lubrication must be provided to get the best results.

Another well-known alloy of this type is that known as Parsons white brass, containing 2.25 per cent. of copper, 64.7 per cent. of tin, 32.9 per cent. of zinc and 0.15 per cent. of lead. It gives excellent service in bearings subjected to heavy pressures such as are found in marine and stationary engine practice; also in connection with high-speed service such as prevails in saw-mill machinery, paper and pulp machinery and in electric generators.

As a rule, white brass is hard and tough, and to get the best results it must be poured at a very high temperature, and should then be peened or hammered all over before machining the bearing.

HEAT TREATMENTS

52. Heat-treating Processes.—The term *heat treatment* is applied to all processes of heating and cooling steel through certain temperature ranges in order to improve the structure, and at the same time produce certain definite and desired characteristics. The processes involved in heat treatments are as follows:

(*a*) *Annealing.*—The object of annealing steel is to remove the internal stresses due to cooling as well as to produce a finer texture in the material. In general, annealing reduces hardness and increases the tensile strength and elongation of the steel.

(*b*) *Hardening.*—Steel is hardened so as to produce a good wearing surface or a good cutting edge. The effect of hardening is to raise the elastic limit and the ultimate strength of the steel and at the same time reduce its ductility. When the carbon content of the steel is 0.5 per cent. or over, the metal becomes brittle due to the stresses induced by the sudden quenching.

(*c*) *Tempering.*—The process of tempering consists of reheating the hardened steel in order to restore some of the ductility and softness lost in the hardening process. This means that the elasticity and tensile strength are reduced below the values for the hardened steel, but are higher than those prevailing in the original material.

(*d*) *Case-hardening.*—By the process of *case-hardening,* the outer shell or skin of a piece of steel is converted into a high-carbon steel while the material on the inside remains practically unchanged. The type of steel to which this process is applied generally has a carbon content of 0.10 to 0.20 per cent., and ordinarily should not contain more than 0.25 per cent. manganese or the case produced becomes too brittle. Case-hardening may also be applied to nickel steel, chrome steel, chrome-nickel steel, or chrome-vanadium steel.

To case-harden, the pieces are packed in carbonaceous material in special boxes that are air-tight. These boxes with their contents are then placed in a furnace in which the temperature is brought up to about 1,500°F., and maintained at that temperature for a definite time so as to produce the desired result. The pieces, after receiving this first treatment, may be quenched and are ready for use. In order to get better results, the boxes and their contents are allowed to cool in the furnace or in the air to about 1,200°, and are then again subjected to a high temperature after which the contents are quenched. A few of the materials used for packing the steel in the boxes are as follows: crushed bone; charred leather; barium carbonate and charcoal; wood charcoal and bone charcoal.

The above method requires considerable time, and very often it is desirable to produce quickly a case-hardening effect which need not penetrate the material very far. This may be accomplished by the use of a mixture of powdered potassium cyanide and potassium ferrocyanide, or a mixture of potassium ferrocyanide and potassium bichromate. In general, case-hardening is used when a machine part must have a very hard surface in order to resist wear or impact, and when the interior of the piece must be tough so as to resist fracture.

53. S. A. E. Heat Treatments.—In January, 1912, the Society of Automobile Engineers adopted a series of so-called heat treatments which they recommend for use with the various types of steel employed in automobile construction. Each heat treatment is designated by a letter and at the present time seventeen different treatments are included in the above-mentioned list. The specifications are complete as may be seen from the following, taken from the Report of the Iron and Steel Division of the Standards Committee of the above-mentioned society.

Treatment A.—For screws, pins and other similar parts made from 0.15 to 0.25 per cent. carbon steel, and for which hardness is the only requirement, the simple form of case-hardening designated as *Heat Treatment A*, will answer very well. After the piece has been forged or machined, treat it as follows:

1. Carbonize at a temperature between 1,600° and 1,750°F.
2. Cool slowly or quench.
3. Reheat to 1,450° to 1,500°F. and quench.

Treatment C.—Steel containing from 0.25 to 0.35 per cent. carbon is used for axle forgings, driving shafts, and other structural parts, and in order to get better service from this grade of steel, the parts after being forged or machined should be heat-treated, the simplest form of which is given by the following specifications:

1. Heat to 1,475° to 1,525°F.
2. Quench.
3. Reheat to 600° to 1,200°F. and cool slowly.

In the third operation, namely that of drawing, each piece must be treated individually; for example, if considerable toughness with no increase in strength is desired, the upper drawing temperatures must be used; while with parts that require increased strength and little toughness, the lower temperatures will answer.

Treatment K.—Treatment K, specifications for which are given below, is applicable to parts made of nickel and nickel-chromium steels, in which extremely good structural qualities are desired:

1. Heat to 1,500° to 1,550°F.
2. Quench.
3. Reheat to 1,300° to 1,400°F.
4. Quench.
5. Heat to 600° to 1,200°F. and cool slowly.

In reality this is a double heat treatment, which produces a finer structure of the material than is possible with only one treatment.

Treatment V.—Springs made from silicon-manganese steel are treated as follows:

1. Heat to 1,650° to 1,750°F.
2. Quench.
3. Reheat to a temperature between 600° and 1,400°F., and cool slowly.

PREVENTION OF CORROSION

To prevent corrosion of iron and steel it is necessary to protect the surfaces by means of some form of coating which may be either of a non-metallic or metallic nature. In the non-metallic method, the parts are coated with a paint, enamel or varnish, the efficiency of which depends on its being more or less air-tight. This method is far from satisfactory due to the chemical changes causing the coating to peel off or to become porous. In the metallic method the parts are coated with some other metal, generally zinc, though sometimes copper or aluminum, is used. There are three distinct processes of putting a zinc coating on iron and steel, as follows: *hot-galvanizing*, *electro-galvanizing*, and *shererdizing*.

54. Galvanizing.—(*a*) *Hot-galvanizing* consists in dipping the parts, which have been cleaned previously, into molten spelter having a temperature of from 700° to 900°F. To cause the spelter to adhere to the surfaces of the articles, a soldering flux (metallic chlorides) is used. The zinc deposited on the parts is not chemically pure, and the impurities increase with continued use of the molten spelter. Due to these impurities, the coating is more or less brittle and will crack easily. The thickness of the coating is far from uniform. The process just described is the oldest known for coating iron and steel with zinc.

(*b*) *Electro-galvanizing*.—This process is also known as cold-galvanizing and consists in depositing zinc on the parts, previously cleaned, by means of electrolysis. By this method any size of article may be treated, and it is claimed that the deposit consists of chemically pure zinc. The thickness of the coating is more easily controlled by this process than by the one discussed in the previous paragraph.

55. Shererdizing.—In the process known as shererdizing, the articles, after they are cleaned, are packed with zinc dust in an air-tight drum. To prevent the oxidation of the zinc by the air inside of the drum, a small amount of pulverized charcoal is mixed with the contents of the drum. The drum, after being sealed, is placed into a specially constructed oven in which it is brought up to a temperature approximately 200° below the melting point of zinc. To get an even distribution of the heat and at the same time to produce an even coating on the articles, the drum is rotated continually. By means of this process it is possible to produce a homogeneous deposit of zinc, the thickness of which depends upon the length of time the articles are allowed to remain in the oven.

CHAPTER III

FASTENINGS

RIVETS AND RIVETED JOINTS

56. Rivets.—The most common method of uniting plates, as used in boilers, tanks and structural work, is by means of rivets. A *rivet* is a round bar consisting of an upset end called the *head*, and a long part called the *shank*. It is a permanent fastening, removable only by chipping off the head. Rivets should in general be placed at right angles to the forces tending to cause them to fail, and consequently the greatest stress induced in them is either that of shearing or of crushing. If rivets are to resist a tensile stress, a greater number should be used than when they are to resist a shearing or a crushing stress.

Rivets are made of wrought iron, soft steel, and nickel steel. They are formed in suitable dies while hot from round bars cut to proper length. The shank is usually cylindrical for about one-half its length, the remaining portion tapering very slightly. In applying rivets, they are brought up to a red heat, placed in the holes of the plates to be connected, and a second head is formed either by hand or machine work. Generally speaking, machine riveting is better than hand work, as the hole in the plates is nearly always filled completely with the rivet body, while in hand work, the effect of the hammer blow does not appear to reach the interior of the rivet, and produce a movement of the metal into the rivet hole.

57. Rivet Holes.—For the sake of economy rivet holes are usually punched. There are two serious objections to thus forming the holes. The metal around the holes is injured by the lateral flow of the metal under the punch; however, this objection may be obviated by punching smaller holes and then reaming them to size. Secondly, the spacing of the holes in the parts to be connected is not always accurate in the case of punching, so it becomes necessary to ream out the holes, in which case the rivets may not completely fill the holes thus enlarged, or to use a drift pin. The drift pin should be used only with a light-weight hammer.

The diameter of the rivet hole is about $1/16$ inch larger than that of the rivet. This rule is subject to some variation, depending upon the class and character of the work. The clearance given the rivet allows for some inaccuracy in punching the plate and in addition permits driving the rivet when hot. Drilling the holes is the best method for perforating the plates. The late improvements in drilling machinery have made it possible to accomplish this work with almost the same economy as in punching. The metal is not injured by the drilling of holes; indeed, there are tests which show an increase in the strength of the metal between the rivet holes.

58. Forms of Rivets.—Rivets are made of a very tough and ductile quality of iron or steel. They are formed in dies from

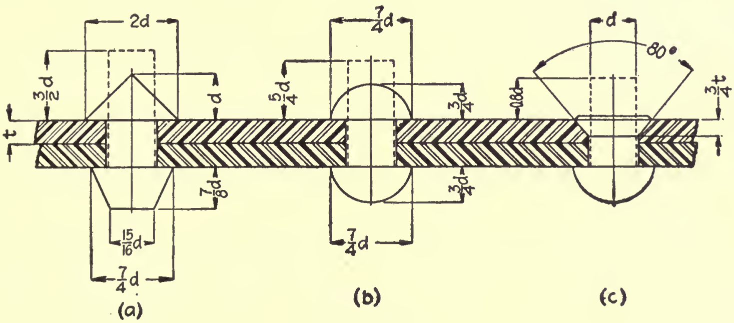

FIG. 7.

the round bar while hot, and in this condition are called *rivet blanks*. For convenience, the head which is formed during the process of driving is called the *point*, to distinguish it from the head that is formed in making the rivet blank. The amount of shank necessary to form the point depends upon the diameter of the rivet. Since the length of the rivet is measured under the head, the length required is equal to the length of shank necessary to form the point plus the *grip* or thickness of plates to be riveted together. The various forms of rivet points and their proportions, as used in riveted joints, are illustrated in Fig. 7. In addition to the proportions for the points, the figure also gives the length of shank required to form these points. The style of point shown in Fig. 7(*a*) is called the *steeple point;* that illustrated by Fig. 7(*b*) is known as the *button point,* while the *countersunk point* is represented by Fig. 7(*c*). The lengths of rivets should always be taken in quarter-inch increments on account of stock sizes. Any length up to five or six inches, however, may be obtained, but the odd sizes will cost more than the standard sizes.

59. Forms of Heads.—Rivets with many different forms of heads may be found in mechanical work, but the ones in general use in boiler work are only three, namely, cone head, button head and countersunk head. These are shown in Fig. 7(*a*), (*b*), and (*c*), respectively. The proportions advocated by different manufacturers vary somewhat; those given in Fig. 7 are used by the Champion Rivet Company. The steeple point, Fig. 7(*a*), is one easily made by hand driving and is therefore much used. This form, however, is weak to resist tension and should not be used on important work.

The cone head, Fig. 7(*a*) is one of great strength and is used a great deal in boiler work. It is not generally used as a form for the point on account of difficulty in driving. The button-head type, Fig. 7(*b*), is widely used for points and may be easily formed in hand driving by the aid of a snap. The countersunk point weakens the plate so much that it is used only when projecting heads would be objectionable, as under flanges of fittings. Its use is sometimes imperative for both heads and points, but it should be avoided whenever possible. The countersink in the plate should never exceed three-fourths of the thickness of the plate, and for that reason, the height of the rivet point is generally from $1/16$ to $1/8$ inch greater than the depth of the countersink. The point then projects by that amount, or if the plate is required to be perfectly smooth, the point is chipped off level with the surface.

RIVETED CONNECTIONS

There are three general groups of riveted connections or joints: the first of these includes all types of joints met with in the construction of tanks and pressure vessels; the second group, commonly called structural joints, includes those that are common to cranes, structures, and machinery in general; the third group includes those joints used in the construction of the hulls of ships. It is evident, that in the first group, in addition to forming a rigid connection between two or more members, the joint must also be made secure against leakage. In the third group mentioned, strength, stiffness and durability are the important points desired, as well as proof against leakage; however, due to the low pressures the question of leakage presents no serious difficulties.

60. Types of Joints.—Generally speaking, the following arrangements used in connecting plates by means of rivets are equally well adapted to the three groups of connections mentioned in the preceding paragraph.

(*a*) *Lap joints.*—By a *lap joint* is meant an arrangement which consists of overlapping plates held together by one or more rows of rivets. If one row of rivets is used as shown in Fig. 8(*a*), the arrangement is called a single-riveted lap joint, and with two rows as represented in Fig. 9, it is called a double-riveted

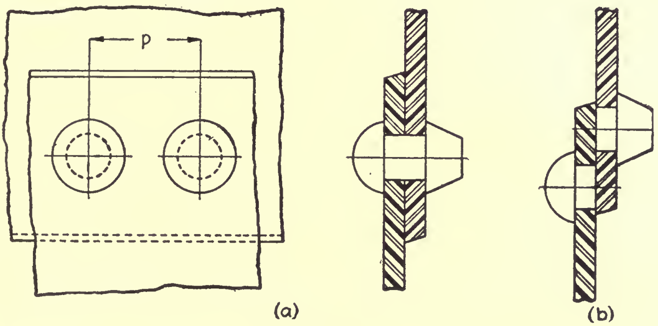

FIG. 8.

lap joint. In the latter form of joint, the rivets may be arranged in two ways, namely *staggered* as shown in Fig. 9(*a*), or the so-called *chain* riveted, illustrated in Fig. 9(*b*).

It is apparent that a load producing a tensile stress in a lap joint tends to distort the joint so that the two connected plates are practically in the same plane, thus inducing a bending stress in the plate as well as tensile and shearing stresses in the rivet. This distortion is not quite so marked in double-riveted lap joints, due to the additional stiffness given by the greater width of the overlap.

(*b*) *Butt joints.*—When plates butt against each other and are joined by overlapping plates or straps, the connection is called a *butt joint.* Such a joint may have one plate on the outside, or one on the outside and another on the inside, as shown in Fig. 10. As in lap joints, the rivets may be grouped in one or more rows on each side of the joint, and in either the chain or staggered

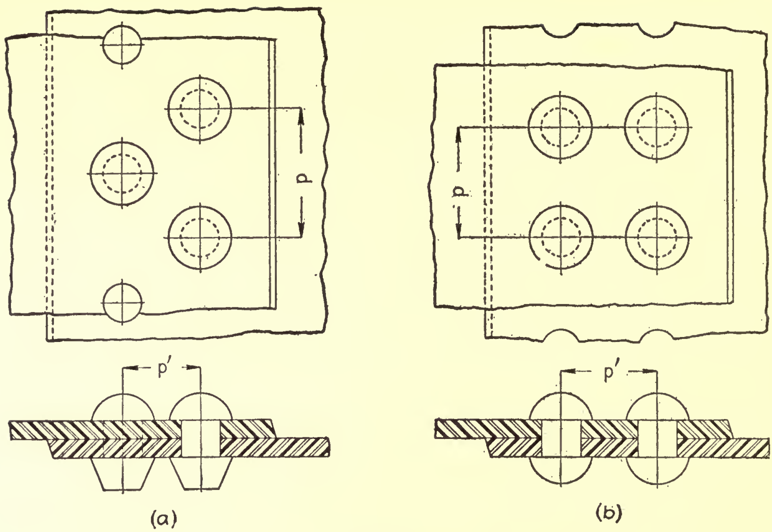

FIG. 9.

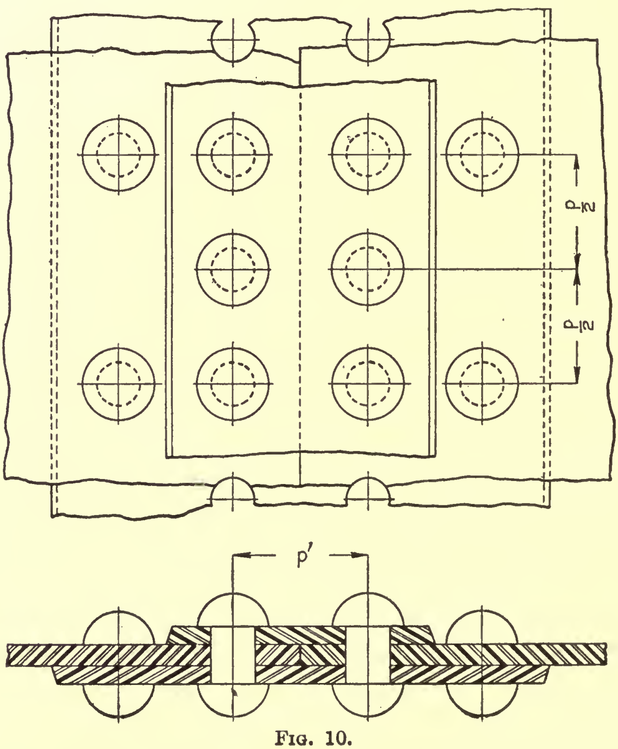

FIG. 10.

riveted arrangement, illustrated by Figs. 10 and 12, respectively. Butt joints having two cover plates are not subjected to the excessive distortion found in lap joints, though poor workmanship may cause a small bending stress in the plates and a tension on the rivet.

61. Failure of Joints.—In arriving at the intensity of stress in any of the types of joints discussed in Art. 60, we shall assume that the unit stress is uniform over the area of the resisting section, which, of course, is not absolutely correct for joints subjected to bending nor for those containing two or more rows of rivets. Furthermore, in the following discussion no allowance will be made for the additional holding power of riveted joints due to the friction between the plates. American designers pay no attention to this, as experiments made at the United States Arsenal at Watertown seem to indicate that the joints will slip a slight amount at loads considerably less than those due to the working pressures. According to experiments made by Bach, the frictional resistance of a riveted joint may be taken approximately equal to 15,000 pounds per square inch of rivet area.

Experience has shown that riveted joints may give way in any one of the following ways:

(*a*) *Shearing of the rivet.*—In all lap joints and butt joints with one strap, the rivets tend to fail along one section; while in butt joints with two straps, failure tends to take place along two sections. Thus in Fig. 8(*a*), the tendency would be for the rivet to fail along the line where the plates come into contact, and after failure, the condition would be represented by Fig. 8(*b*). Such a rivet is said to be in single shear, and in case two sections resist the shearing action, the rivet is in double shear. If P represents the force transmitted by one rivet, and d the diameter of the rivet after driving, then

$$P = \frac{\pi d^2 S_s}{4} \qquad (38)$$

(*b*) *Crushing of the plate or the rivet.*—If the rivet be strong enough to resist the shearing force, the plate or the rivet itself may fail by crushing, as shown at A in Fig. 11. The force upon the rivet is distributed over a semi-cylindrical area causing a distribution of pressure upon this area about which very little is known. In the design of riveted joints it is customary to consider only the component of this pressure which is parallel to the force upon the rivet, and to assume that it is distributed over the projected area of the rivet.

The unit stress indicated by this crushing action is called a *bearing stress,* and representing it by S_b, it is evident that the force transmitted by one rivet is

$$P = dtS_b, \tag{39}$$

in which t represents the thickness of the plate. From (39), it follows that for any particular size of rivet and load P, the bearing stress depends upon the thickness of the plate; hence it is possible to have different bearing stresses in one joint when two or more plates of different thicknesses are connected together.

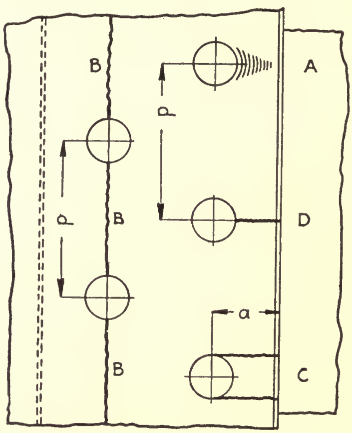

FIG. 11.

(*c*) *Tearing of the plate.*—In a riveted joint subjected to a tension, the plates may be pulled apart along the line of rivets as shown at B in Fig. 11. Evidently the least area of the plate resisting this tension is the net section between consecutive rivets. If p represents the pitch of the rivets, then the force transmitted by each rivet is

$$P = (p - d)tS_t \tag{40}$$

(*d*) *Failure of the margin.*—By the term margin, also called lap, is meant the distance from the edge of the plate to the center of the line of rivets nearest the edge, as shown by the dimension a in Fig. 11. Failure of the margin may occur by shearing of the plate along the lines in front of the rivet as shown at C in Fig. 11. With actual joints in use, failure in this way is not likely to occur. It follows that the shearing resistance offered by the plate is $2\,atS_s$; hence, the force each rivet is capable of transmitting is

$$P = 2\,atS_s \tag{41}$$

The margin may also fail by tearing open as shown at D in Fig. 11. This failure no doubt is due to the fact that in a joint subjected to tension, the material in front of the rivet behaves very much like a beam loaded at the center, thus causing the plate to fail by breaking open on the tension side, usually near the center. The truth of the above statement has been borne out by numerous experiments. A rule used considerably by designers is to make the margin never less than one and one-half times the diameter of the rivet, and experience has proven that joints designed in this manner seldom fail due to a weak margin. The Association of the Master Steam Boiler Makers recommends that for boiler joints the margin be made twice the diameter of the rivet. This marginal distance has proven very satisfactory in that no trouble has been experienced in making such a joint steam-tight by caulking.

62. Definitions.—In the investigation of the stresses in a riveted joint, it is convenient to take a definite length of the joint as the basis for our calculations. This length may or may not be equal to the *pitch*. In joints having two or more rows of rivets, the distance between the rows is commonly called the *back pitch*, and its magnitude is approximately 70 per cent. of the pitch. An examination of Figs. 10 and 12 shows that there are certain groups or arrangements of rivets which are repeated along the entire length of the joint, and for convenience such a group of rivets may be called a *repeating group* and the length occupied by it *a unit length of a riveted joint*. In the analysis of any type of riveted joint, the force transmitted by such a repeating group generally forms the basis of all calculations. Another term used to a considerable extent in connection with riveted joints is the so-called *efficiency*, by which is meant the ratio that the strength of a unit length of a joint bears to the same length of the solid plate.

RIVETED JOINTS IN BOILER CONSTRUCTION

63. Analysis of a Boiler Joint.—One of the objects desired when designing an efficient boiler joint is to make the joint equally strong against failure by shearing, bearing and tension; however, certain modifications are necessary for economic reasons and, as a result, the actual joint as finally constructed in the shop will have a slightly lower efficiency than the one having uniform strength. In order to illustrate the method that may be followed in designing a joint having its resistance to shearing, bearing and tension approximately the same, assume the double-riveted lap joint shown in Fig. 9. From this figure it is evident that the length of a repeating group is p, the pitch of the rivets. We shall assume that the two plates are of the same thickness t, and that the margin was made of sufficient length to insure against its failure.

The resistance P due to the shearing of the rivets in a unit length of the joint is

$$P = \frac{\pi d^2 S_s}{2} \tag{42}$$

The resistance due to crushing of the plate and the rivets is

$$P = 2\, dtS_b \tag{43}$$

The area resisting tension is $(p - d)t$, and multiplying this by the unit stress, S_t, the total resistance against tension is

$$P = (p - d)tS_t \tag{44}$$

The three equations just determined may now be solved simultaneously if it is desired to make the joint of equal strength. Combining (42) and (43), we obtain

$$d = \frac{4\, tS_b}{\pi S_s} \tag{45}$$

Equating (42) and (44), the pitch becomes

$$p = d + \frac{\pi d^2 S_s}{2\, tS_t} \tag{46}$$

Equating (43) and (44), it follows that

$$p = d + \frac{2\, dS_b}{S_t} \tag{47}$$

Basing the size of the rivet upon (45) would lead to odd diameters that are not obtainable, since the commercial sizes vary by 1/16-inch increments from 1/8 inch to 1 5/8 inches in diameter. Hence, with the use of commercial sizes of rivets, it is impossible to make the joint equally strong against the three methods of failure discussed above. Furthermore as the thickness of the plate increases, the diameter d calculated by (45) becomes excessively large, thus introducing serious difficulties in driving such a rivet. Having decided upon the size of rivet, the pitch may be determined by means of (46) and (47), but it may be necessary to modify the calculated pitch so as to insure a steam-tight joint. From this discussion it is apparent that the group of theoretical formulas derived above serves merely as a guide.

In general, the method of procedure to be used in designing riveted joints is as follows:

(*a*) Determine expressions for the various methods of failure.

(*b*) Select a commercial size of rivet, so that it may be driven readily.

(*c*) Having selected the size of rivet, determine whether the rivet will fail by shearing or by crushing.

(*d*) Determine the pitch by equating the expression for the tearing of the plate to that giving the rivet failure.

(*e*) Determine the probable efficiency of the joint.

64. Efficiency of the Joint.—The efficiency of a riveted joint is defined as the ratio that the strength of a unit length of a joint bears to the same length of the solid plate. In the analysis of the double-riveted lap joint, it developed that there were three distinct ways that the joint could fail; hence, the efficiency of that joint depends upon the expression that gives the minimum value of P. In a double-riveted butt and double-strap joint, there are six ways that failure may occur and whichever is the weakest determines the probable efficiency of the joint.

The strength of the solid plate of thickness t and unit length L is $t\,LS_t$; hence, the general expression for the efficiency of a riveted joint becomes

$$E = \frac{\text{minimum } P}{tLS_t} \qquad (48)$$

The range of values for the efficiency E for the various types of joints used in boiler design is given in Table 9. These values may serve as a guide in making assumptions that are necessary when designing joints for a particular duty. In case the actual or calculated efficiency does not agree closely with the assumed value, the joint will have to be redesigned, until a fair agreement is obtained.

Table 9.—Efficiency of Boiler Joints

Type of joint		Efficiency Min.	Efficiency Max.
Lap joint.	Single-riveted	45	60
	Double-riveted	60	75
	Triple-riveted	65	84
Butt joint with two cover plates	Single-riveted	55	65
	Double-riveted	70	80
	Triple-riveted	75	88
	Quadruple-riveted	85	95

65. Allowable Stresses.—In order to design joints that will give satisfactory service in actual use, considerable attention must be given to the selection of the proper working stresses for the materials used. At the annual meeting of the American Society of Mechanical Engineers held in December, 1914, a committee appointed by that society presented an extensive report in which the question of the selection of the material is discussed very fully. The recommendations are as follows:

Table 10.—Ultimate Shearing Stresses in Rivets

Kind of rivet	Ultimate shearing	
	Single shear	Double shear
Iron.......	38,000	76,000
Steel.......	44,000	88,000

Table 11.—Thickness of Shell and Dome Plates after Flanging

Diameter of shell	Minimum thickness
36 and under ...	1/4
36 to 54.......	5/16
54 to 72.......	3/8
72 and over....	1/2

(*a*) In the calculations for steel plates when the actual tensile strength is not stamped on the plates, it shall be assumed as 55,000 pounds per square inch.

(*b*) The ultimate crushing strength of steel plate shall be taken at 95,000 pounds per square inch.

(*c*) In rivet calculations, the ultimate shearing strengths given in Table 10, and based on the cross-sectional area of the rivet after driving, shall be used.

Table 12.—Thickness of Butt Joint Cover Plates

Thickness of shell plates	Thickness of cover plates
1/4 to 11/32 inclusive.....	1/4
3/8 and 13/32...........	5/16
7/16 and 15/32..........	3/8
1/2 to 9/16 inclusive......	7/16
5/8 and 3/4............	1/2
7/8....................	5/8
1 and 1 1/8............	3/4
1 1/4..................	7/8

(*d*) To obtain the allowable working stresses, the ultimate strengths given above must be divided by the so-called *factor of safety,* the value of which should never be less than five.

66. Minimum Plate Thickness.—According to recommendations made by the Boiler Code Committee of the American Society of Mechanical Engineers, no boiler plate subjected to pressure should be made less than 1/4 inch thick, and the thicknesses given in Table 11 for various shell diameters may serve as a guide in designing work.

For the thicknesses of the cover plate for butt joints, the recommendations of this committee are given in Table 12.

Table 13.—Recommended Size of Rivet Holes

<table>
<tr><th rowspan="3">Plate thickness</th><th colspan="6">Diameter of rivet holes</th></tr>
<tr><th colspan="3">Lap joint</th><th colspan="3">Double-strap butt joint</th></tr>
<tr><th>Single-riveted</th><th>Double-riveted</th><th>Triple-riveted</th><th>Double-riveted</th><th>Triple-riveted</th><th>Quadruple-riveted</th></tr>
<tr><td>1/4</td><td colspan="3">5/8</td><td rowspan="2">11/16</td><td colspan="2">9/16</td></tr>
<tr><td>9/32</td><td>11/16</td><td colspan="2">5/8</td><td colspan="2">5/8</td></tr>
<tr><td>5/16</td><td>3/4</td><td colspan="2">11/16</td><td colspan="3" rowspan="2">3/4</td></tr>
<tr><td>11/32</td><td rowspan="2">7/8</td><td>3/4</td><td>11/16</td></tr>
<tr><td>3/8</td><td colspan="2" rowspan="2">13/16</td><td rowspan="2">13/16</td><td rowspan="2">13/16</td><td>3/4</td></tr>
<tr><td>13/32</td><td>15/16</td><td rowspan="2">7/8</td></tr>
<tr><td>7/16</td><td>1</td><td rowspan="2">15/16</td><td>7/8</td><td rowspan="2">7/8</td><td rowspan="2">15/16</td></tr>
<tr><td>15/32</td><td rowspan="2">1 1/16</td><td rowspan="2">15/16</td><td rowspan="4">15/16</td></tr>
<tr><td>1/2</td><td>1</td><td>15/16</td><td rowspan="4">1 1/16</td></tr>
<tr><td>17/32</td><td></td><td></td><td></td><td></td></tr>
<tr><td>9/16</td><td></td><td></td><td></td><td></td></tr>
<tr><td>5/8</td><td></td><td></td><td></td><td></td><td>1 1/16</td></tr>
<tr><td>11/16</td><td></td><td></td><td></td><td></td><td colspan="2" rowspan="2">1 3/16</td></tr>
<tr><td>3/4</td><td></td><td></td><td></td><td></td></tr>
<tr><td>13/16</td><td></td><td></td><td></td><td></td><td colspan="2" rowspan="2">1 5/16</td></tr>
<tr><td>7/8</td><td></td><td></td><td></td><td></td></tr>
<tr><td>15/16</td><td></td><td></td><td></td><td></td><td rowspan="2">1 7/16</td><td>1 5/16</td></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td><td>1 7/16</td></tr>
</table>

In Table 13 are given the diameters of rivet holes for different plate thicknesses and various types of joints, as determined from a study of actual joints used in the construction of boilers and pressure tanks.

67. Design of a Boiler Joint.—It is required to design a triple-riveted double-strap butt joint for the longitudinal seam of a boiler 66 inches in diameter, assuming the working pressure as

150 pounds per square inch, and the ultimate tensile strength of the plates as 60,000 pounds per square inch. For the factor of safety, and shearing and crushing stresses, use the values recommended by the American Society of Mechanical Engineers.

(*a*) The first step in the solution of this problem is to assume the probable efficiency of the joint, which according to Table 9 may be taken as 85 per cent.

(*b*) Determine next the thickness of the shell plates making proper allowances for the decrease in the strength of the shell due to the joint. The formula for the plate thickness is determined by considering the boiler a cylinder with thin walls subjected to an internal pressure, whence

$$t = \frac{P'D}{2ES_t} = \frac{150 \times 66}{2 \times 0.85 \times 12{,}000} = 0.485 \text{ inch.}$$

Selecting the nearest commercial size, the thickness of the shell plates will be made ½ inch.

(*c*) The cover plates or straps of a triple-riveted butt joint for a ½-inch shell should be 7/16 inch thick, according to Table 12, and the diameter of the rivet hole as given in Table 13 will be 1 1/16 inch, thus calling for 1-inch rivets.

(*d*) According to the recommendations of the American Society of Mechanical Engineers, the following ultimate stresses will be used; $S_s = 44{,}000$ and $S_b = 95{,}000$, from which the following unit values are obtained:

$$\frac{\pi d^2 S_s}{4} = 39{,}000 \text{ pounds.}$$

$$dt'S_b = 1\tfrac{1}{16} \times \tfrac{7}{16} \times 95{,}000 = 44{,}150 \text{ pounds.}$$

$$dtS_b = 1\tfrac{1}{16} \times \tfrac{1}{2} \times 95{,}000 = 50{,}470 \text{ pounds.}$$

(*e*) Having arrived at the proper plate thicknesses and the diameter of the rivets, the resistances to failure of the joint must be investigated in order to establish the probable pitch of the rivets. A triple-riveted double-strap butt joint similar to that shown in Fig. 12 may fail in any one of the following ways:

1. *Tearing of the plate between the rivet holes in the outer row.*—Using the notation prevailing in preceding articles, the magnitude of the resistance to failure by tearing of the plate between the rivet holes is

$$P = (p - d)tS_t \qquad (49)$$

2. *Tearing of the plate between the rivet holes in the second row, combined with the failure of the rivet in the outer row.*— An inspection of Fig. 12 shows that before the plate could fail between the rivets in the second row, the rivet in the outer row would have to fail either by shearing or by crushing, hence for this case two separate resistances are obtained as follows:

$$P = (p - 2\,d)tS_t + \frac{\pi d^2}{4} S_s \tag{50}$$

$$P = (p - 2\,d)tS_t + dt'S_b \tag{51}$$

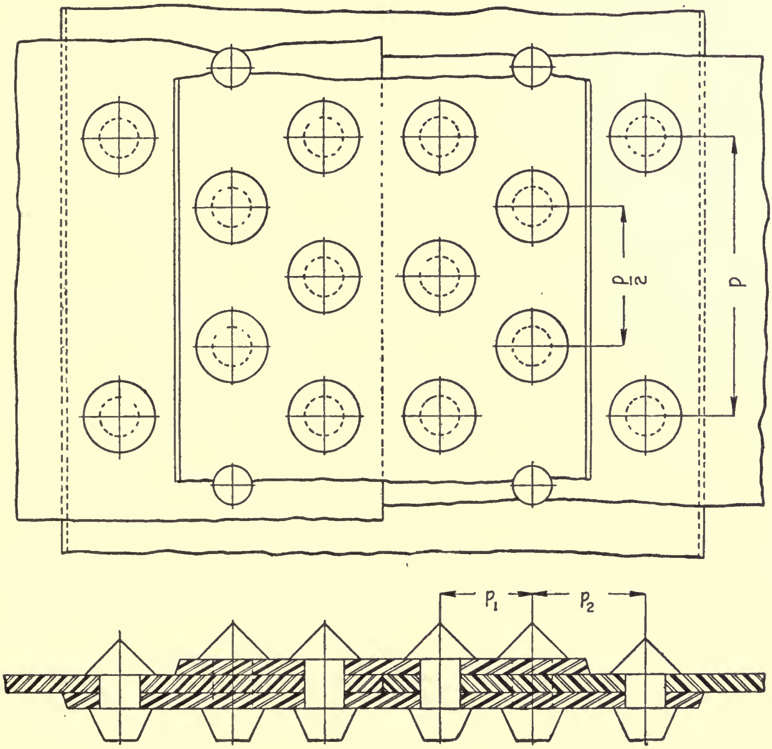

Fig. 12.

3. *Shearing of all the rivets.*—It is evident that in the triple-riveted butt joint shown in Fig. 12, four rivets are in double shear and one in single shear; hence, the magnitude of the resistance to failure is

$$P = \frac{9\,\pi d^2}{4} S_s \tag{52}$$

4. *Crushing of all the rivets.*—There are five rivets resisting crushing; hence, the expression for the resistance to crushing is

$$P = (4\,t + t')dS_b \tag{53}$$

5. *Combined crushing and shearing.*—The joint may also fail by the crushing of the four rivets on the inner and second rows, and the shearing of the rivet in the outer row; hence, the combined resistances of these rivets is

$$P = 4\,dtS_b + \frac{\pi d^2 S_s}{4} \tag{54}$$

A joint of the type discussed above should be designed so that the strength of the critical sections increases as these sections approach the center of the joint. This condition is fulfilled by making the values of P obtained from (50) and (51) greater than that obtained by the use of (49); that is

$$(p - 2\,d)tS_t + \frac{\pi d^2 S_s}{4} > (p - d)tS_t \tag{55}$$

$$(p - 2\,d)tS_t + dt'S_b > (p - d)tS_t \tag{56}$$

From (55) it follows that the diameter of the rivet hole becomes

$$d > \frac{4\,tS_t}{\pi S_s} \tag{57}$$

and simplifying (56), we find that

$$t' > \frac{tS_t}{S_b} \tag{58}$$

The expressions given by (57) and (58) must be satisfied, if it is desired to make the triple-riveted butt joint shown in Fig. 12 stronger along the inner rows than at the outer rows. Having satisfied these equations by choosing proper values for d, t and t', the pitch p is determined by equating the minimum value of P, obtained by evaluating (52), (53) and (54), to that obtained from (49), and solving for p.

Applying the principles just established to the data given above, we find that according to (57), the minimum value of d is 0.87 inch, and from (58) the minimum value of t' is 0.32 inch; hence it is evident that the values assumed above will insure increased strength of the joint along the inner rows.

An inspection of the above formulas indicates that (54) gives the minimum value of P, and, after substitution, we find that $P = 240{,}880$ pounds. Inserting this value in (49) and determining the magnitude of the pitch, we get $p = 9.09$ inches, say 9 inches.

The strength of the solid plate is $9 \times \frac{1}{2} \times 60{,}000 = 270{,}000$ pounds; hence, from (48), the efficiency

$$E = \frac{240{,}880}{270{,}000} = 0.892 \text{ or } 89.2 \text{ per cent.}$$

RIVETED JOINTS FOR STRUCTURAL WORK

The design of riveted joints for structural work generally calls for the selection of the economical size of the members required to transmit the given force, in addition to the determination of the proper size and number of rivets to be used. In structural joints the size of the rivet depends in a general way upon the size of the connected members, but the usual sizes are $\frac{5}{8}$, $\frac{3}{4}$ and $\frac{7}{8}$ inch in diameter. Rivets larger than $\frac{7}{8}$ inch cannot be driven tight by hand and since in structural work many of the joints must be put together in the field by hand riveting, it is evident that $\frac{7}{8}$ inch is the limiting size for this class of work. Tables giving the maximum size of rivets that can be used with the various sizes of structural shapes may be found in the hand books published by the several steel companies.

68. Rivet Spacing.—In the spacing of rivets the following points must be considered:

(*a*) If rivets are spaced too closely, the material between consecutive rivets may be injured permanently.

(*b*) Too close spacing might interfere with the proper use of the snap or set during the driving operation.

(*c*) Rivets that are spaced far apart prevent intimate contact between the members; water and dirt may collect and the joint may thus deteriorate by rusting.

(*d*) Rivets are usually spaced according to rules dictated by successful practice, as the following will indicate. The minimum pitch between rivets is approximately three times the diameter of the rivet, and the maximum is given as sixteen times the thickness of the thinnest plate used in the joint.

(*e*) For gauge lines used in connection with the various structural shapes, the steel companies hand books should be consulted.

69. Types of Joints.—In general, it may be said that the various lap and butt joints used in structural work are very similar to those discussed in Art. 60. In addition to lap and butt joints, there are a great variety of riveted joints in which the several forms of structural shapes are joined together, either with or without the use of connecting plates commonly called gusset plates. Several common forms of such joints will be discussed.

The following order of calculations is common to practically all structural joints:

(*a*) From the magnitude of the load to be transmitted, determine the size of the member.

(*b*) In general the diameter of the rivets to be used in the connection depends upon the size of the connected members.

(*c*) Determine the number of rivets required in each member to transmit the load in that member. This number depends upon the shearing and bearing stresses, whichever determines the method of failure.

(*d*) The rivets in the joint must be arranged or spaced in such a manner that in the case of a tension member the stress along a section through a rivet does not exceed the allowable stress. To determine the net area in such a case it is customary to consider the size of the rivet hole to be $\frac{1}{8}$ inch larger than the diameter of the rivet. For compression members, the area of the rivet hole is never considered in determining the net area of the member.

70. Single Angle and Plate.—A very common method of connecting a single angle, either in tension or compression, to a plate is shown in Fig. 13(*a*). It is apparent that the connection of one leg of the angle to the gusset plate will cause the angle to be loaded eccentrically; this eccentricity increases the stress considerably over that due to central loading. The determination of the additional stress due to the moment does not complicate the problem to any great extent, and for that reason the analysis necessary to determine the size of the angle in any given case should be made as complete as possible. The following problem will serve to illustrate the method of procedure in any given case.

It is desired to determine the size of an angle and the number and size of rivets required in a connection similar to that represented in Fig. 13, in which the force P acting on the member e is 16,800 pounds. Assume the allowable stresses in tension, shearing and bearing as 16,000, 10,000, and 20,000 pounds per square inch, respectively and the thickness of the gusset plate as $\frac{1}{4}$ inch.

The net area of the cross-section of the required angle, assuming central loading, must be $\frac{16,800}{16,000} = 1.05$ sq. in. This condition would be met by a 3 by $2\frac{1}{2}$ by $\frac{1}{4}$-inch angle having a net area of 1.09 square inches after making allowance for a $\frac{3}{4}$-inch rivet. Taking account of the eccentric loading, we will try a $3\frac{1}{2}$ by 3 by

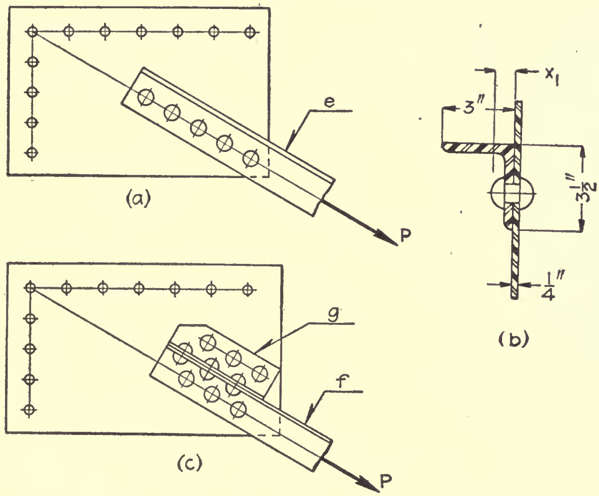

FIG. 13.

$\frac{3}{8}$-inch angle, having a gross area of 2.30 square inches and a net area of 1.97 square inches. From a table of properties of structural angles, we find that the distance x_1 in Fig. 13(*b*) is 0.83 inch, thus making the eccentricity of the load P equal to 0.955 inch. Hence applying (17) the maximum tensile stress in the angle is

$$\frac{16,800}{1.97} + \frac{16,800 \times 0.95 \times 0.83}{1.85} = 16,430 \text{ pounds per square inch}$$

which is assumed as sufficiently close to the allowable stress given above.

To obtain the number of rivets in the joint, determine whether the rivet is stronger in shear or in bearing. For the case considered, the bearing resistance is the smaller, having a value of 3,750 pounds per rivet; hence five $\frac{3}{4}$-inch rivets are required.

From the above analysis, it is apparent that the stress due to the eccentricity of the load P cannot be disregarded, and further that economy of material is obtained by loading the angle centrally. The latter condition is considered fulfilled when both legs are connected to the gusset plate. Such a connection is effected by the use of a clip angle g as shown in Fig. 13(c), provided the rivets are divided equally. Assuming that the joint is made similar to that shown in Fig. 13(c), the data given in the above problem calls for a 3 by 2½ by ¼-inch angle. Each angle must be connected to the gusset plate by means of three rivets, and the same number must be used for connecting together the two angles. Tests made on steel angles having a clip-angle connection with the gusset plate, as illustrated in Fig. 13(c), do not confirm the analysis just given, since the results seem to indicate that very little is gained by the use of such angles.

71. End Connections for Beams.—The rivets in the connections used on the ends of beams are subjected to a secondary shearing stress in addition to the direct stress due to the load on the joint, as the following analysis will show:

According to the steel manufacturer's handbook the standard connection for a 12 by 40-pound I-beam consists of two 6 by 4 by ⅜-inch angles 7½ inches long, as shown in Fig. 14(a). Furthermore, the same source of information gives 8.2 feet as the minimum length of span for which the connection is considered safe when used with a beam loaded uniformly to its full capacity. The uniform load that the beam will carry without exceeding a fiber stress of 16,000 pounds per square inch is

$$W = \frac{8 \times 16{,}000 \times 41.0}{8.2 \times 12} = 53{,}330 \text{ pounds}$$

This gives a reaction R at the end connection of 26,665 pounds, as shown in Fig. 14(a). It is evident from an inspection of the figure that this reaction tends to rotate the connecting angles about the center of gravity of the rivet group, thus causing each rivet to be subjected to a shear due to the turning moment, in addition to the direct shear caused by the reaction.

Due to the reaction R, the direct shear coming upon each rivet in the group has a magnitude of $\frac{26{,}665}{5}$ or 5,333 pounds. Due to the turning moment, the shearing stress produced in any rivet in the group is proportional to the distance that the rivet is from the center of gravity of the group; hence, the resisting moment of each rivet about the center of rotation varies as the square of this distance. Letting S_s' represent the secondary shear in the rivet nearest to the center of gravity, and l_1, l_2, etc., the distances from the center of gravity G to the rivets 1, 2, etc., respectively, as shown in Fig. 14(*b*), then the external moment M, being equal to the summation of the resisting moments due to the rivets, is given by the following expression:

$$M = \frac{S_s'}{l_1}\left[l_1^2 + l_2^2 + l_3^2 + l_4^2 + l_5^2\right] \tag{59}$$

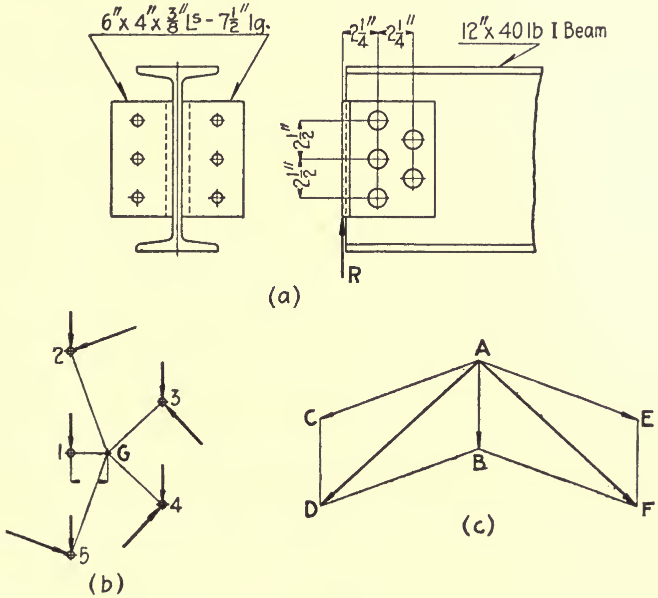

FIG. 14.

From Fig. 14(*b*), the values l_1, l_2, etc., may be calculated, and since M is known, the magnitude of S_s' is readily obtained. For the data at hand $S_s' = 3{,}490$ pounds; hence, the shears coming upon the various rivets are as follows:

Secondary shear on rivet 1 = 3,490 lb.
Secondary shear on rivet 2 = 10,300 lb.
Secondary shear on rivet 3 = 7,140 lb.
Secondary shear on rivet 4 = 7,140 lb.
Secondary shear on rivet 5 = 10,300 lb.

To determine the resultant shear upon each rivet, the direct and secondary shears must be combined. This may be done by algebraic resolution, or graphically as shown in Fig. 14(*c*). It is evident that rivets 2 and 5 are subjected to the heaviest stress, the magnitude of which scaled from Fig. 14(*c*) is 13,150 pounds; whence the unit shearing stress in each of these ¾-inch rivets is 14,880 pounds per square inch. Since the web thickness of the 12-inch by 40-pound beam is 0.56 inch, the bearing stress coming upon rivets 2 and 5 is 31,300 pounds per square inch. This problem shows the importance of determining the actual stresses in the rivets of eccentrically riveted connections.

In the later editions of the steel manufacturer's hand books, it is of interest to note that the "End Connections for Beams and Channels" have been redesigned and for the size of beam given in the preceding problem two 4 by 4 by 7/16-inch angles 8½ inches long are now recommended instead of those mentioned above, and furthermore only three ¾-inch rivets are used.

72. Double Angle and Plate.—A form of connection met with occasionally is shown in Fig. 15. It is desired to determine the load P that this form of connection will safely carry, assuming that all rivets are ¾-inch in diameter and that the following stresses shall not be exceeded: $S_t = 15{,}000$; $S_s = 10{,}000$; $S_b = 20{,}000$.

The connection may fail in the following ways:

(*a*) The rivets in the outstanding leg of the lug and girder angles may fail due to tension.

(*b*) The rivets may shear off or crush in the vertical legs of the lug angle.

(*c*) The rivets may shear off or crush in the angles A.

(*d*) The lug angles may fail by combined tension and bending.

The specifications for structural steel work do not recognize the ability of rivets to resist tension; however, for secondary members it is not unusual to assume the permissible stress in rivets subjected to tension as equivalent to the permissible shearing stress. Upon this assumption, the eight rivets in the outstanding legs of the lug angles are capable of supporting safely a load of $8 \times 0.442 \times 10{,}000 = 35{,}360$ pounds. From the details shown in Fig. 15, it is apparent that the rivets in the vertical legs of the lug angles and those in the angles A are of equal strength, hence the safe load that they are capable of supporting, as measured by their resistance to crushing, is $3 \times \frac{3}{4} \times \frac{3}{8} \times 20{,}000$ or 16,875 pounds.

To determine the bending stress in the lug angles it is assumed that the outstanding legs of these angles are equivalent to cantilevers having the load applied at the center of the rivets. Upon this assumption, the maximum bending moment occurs in the vertical leg, and its magnitude in this case is determined as follows: Let $\frac{P}{2L}$ be the vertical load coming upon each inch of length of the lug angle; then since this load is considered as applied at the center of the rivet, the magnitude of the bending moment M per inch of length of the angle is

$$M = 0.75 \frac{P}{L} \tag{60}$$

Equating this moment to the moment of resistance per inch

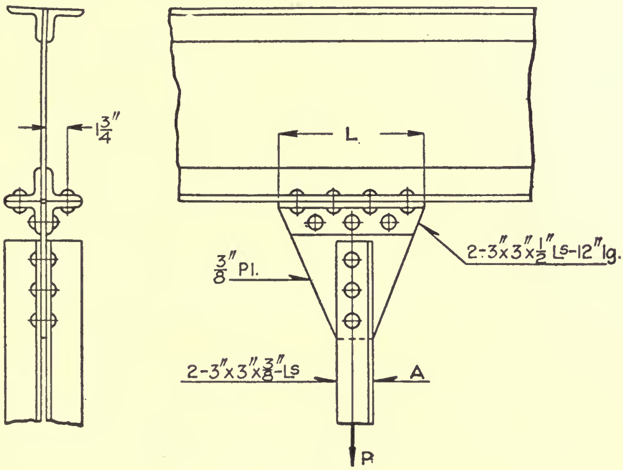

Fig. 15.

of length, we obtain the following relation between the bending stress S_t'' and M:

$$S_t'' = \frac{18P}{L} \tag{61}$$

In addition to this flexural stress there is a direct stress S_t', the magnitude of which is

$$S_t' = \frac{P}{L} \tag{62}$$

The summation of the stresses given by (61) and (62), according to the conditions of the problem should not exceed 16,000; therefore

$$P = \frac{16{,}000\,L}{19} \tag{63}$$

Since L = 12 inches, the maximum safe load that the angle will stand is, according to (63), equal to 10,100 pounds.

Comparing this load with those determined for the other methods of failure, it is evident that the 10,100 pounds is the maximum load that can be supported safely by the connection represented in Fig. 15.

73. Splice Joint.—In Fig. 16 is shown a form of joint used in the bottom chord of a Fink roof truss. Four members are joined together by means of a vertical gusset plate *e* and a splice plate *f* underneath the outstanding legs of the bottom chord angles. Due to the fact that a Fink truss is generally shipped in four

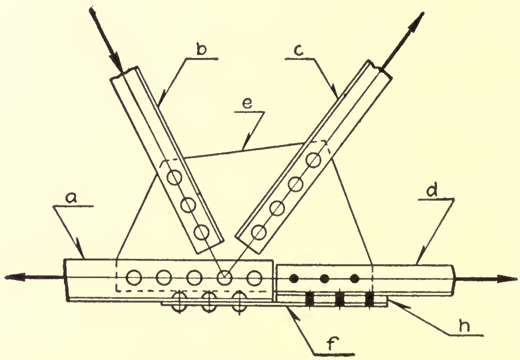

FIG. 16.

pieces, the splice joint is made in the field. In the joint shown in Fig. 16, the magnitude of the loads upon the members *a*, *b*, *c*, and *d* are 30,100, 11,700, 13,000 and 17,700 pounds respectively; it is required to design the complete connection assuming the same working stresses as used in the problem of Art. 72, and furthermore, that no plate shall have a thickness less than ¼ inch.

(*a*) *Size of members.*—In Table 14 are given the steps that are necessary in arriving at the sizes of the tension members *a*, *c* and *d*. Attention is called to the fact that the sizes of the members *a* and *c* are established by the loads given in Table 14 and not by those given above. This is because certain members of light trusses are made continuous. According to certain specifications, the minimum size of angles used is 2 by 2 by ¼ inch while according to others, the minimum is 2½ by 2 by ¼ inch. In the present case the latter size is adopted, as this choice permits the use of ¾-inch rivets through the 2½-inch leg.

Table 14.—Tension Members

Truss member	Max. load	Allow. stress	Required. area	Section selected			
				No.	Size	Area	
						Gross	Net
a	36,600		2.29	2	$3\frac{1}{2}\times2\frac{1}{2}\times\frac{1}{4}$	2.88	2.45
c	19,600	16,000	1.22	2	$2\frac{1}{2}\times2\ \times\frac{1}{4}$	2.14	1.70
d	17,700		1.11	2	$2\frac{1}{2}\times2\ \times\frac{1}{4}$	2.14	1.70

The size of the compression member *b* is arrived at in a general way by determining the allowable unit compressive stress by means of (25), having assumed a probable cross-section for the member in question. The area of the assumed section is then compared with that obtained by dividing the load on the member by the calculated unit stress. If the former area is equal to or slightly greater than the latter, the section assumed is safe. In determining the area of a compression member no reduction is made for the rivet hole, as it is assumed that the rivet in filling up the hole does not weaken the section.

The allowable unit compressive stress is given by the following expression derived directly from (25):

$$S_c = 16,000 - 70\frac{l}{r}, \tag{64}$$

in which l denotes the length of the member in inches and r the least radius of gyration in inches. Generally the length of the compression members in roof trusses should not exceed 125 times the least radius of gyration. If, as in a roof-truss problem, it is required to determine the size of a series of compression members, the best method of procedure is to arrange the calculations in tabular form. In the above problem, the length of the member *b* is 93.8 inches, and the thickness of all plates will be assumed as ¼ inch.

Assume the member *b* to be made of minimum size angles, namely, two 2½ by 2 by ¼ inch having an area of 2.14 square inches. The least radius of gyration r is 0.78 inch when the angles are arranged back to back with a ¼-inch plate between them. This gives a ratio of l to r as 120 which is safe. The allowable working stress calculated by means of (64) is 7600 pounds per square inch; hence, the required area is $\frac{11,700}{7600}$ or 1.54 square inches. Since the area of the members chosen is in excess of the calculated area, our assumption is on the side of safety.

(*b*) *Number of rivets.*—The number of rivets required to fasten each of the members *b* and *c* to the gusset plate *e* is determined as explained in Art. 70, while the number required in the members *a* and *d* depends upon various assumptions that may be made. Among these are the following:

1. The sum of the horizontal components of the forces in the members *b* and *c*, which is equal to the difference between the forces acting on the members *a* and *d*, is transmitted through the gusset plate *e* to the member *a*; hence, the number of rivets required to fasten *a* to the gusset plate is based on this force. It follows that the splice plate *f* and the rivets contained therein must be designed to transmit the total force in *d*. The vertical legs of the member *d* must also be riveted to the plate, but these rivets are not considered as a part of the splice.

2. Consider that all of the rivets in the connection are effective, that is, the total number of rivets required in each of the members *a* and *d* must be based on the load transmitted by these members. This is equivalent to making the gusset plate transmit a certain part, say approximately one-half, of the load in *d*, and the remainder is taken up by the splice plate. Due to the fact that the splice plate is riveted to *a* and *d* by an even number of rivets, it frequently happens that the loads taken up by the splice and gusset plates are far from being equal. The method of procedure is shown by the following problem:

The size of the members will permit the use of ¾-inch rivets throughout, except in the splice plate, where ⅝-inch rivets must be used. We shall assume that four ⅝-inch rivets are used at each end of the splice plate, and these are capable of transmitting $4 \times 2{,}045$ or 8,180 pounds, or 46 per cent. of the load in the member *d*. If six ⅝-inch rivets are used, the splice plate will then transmit 69 per cent. of the load in *d*. The former combination is the one selected, as by its use the entire joint can be made up with fewer rivets than would be required if the second scheme were used. Now the remaining load in *d*, or 9,520 pounds, is transmitted through the gusset plate. The load in the member *a* minus the load transmitted by the splice plate is 21,920 pounds; this load must be transmitted through the gusset plate and requires $\frac{21{,}920}{3{,}750}$ or 6 shop rivets. The number of field rivets in the vertical legs of the member *d* is $\frac{9{,}520}{2{,}500}$ or 4.

The member b requires $\frac{11,700}{3,750}$ or 3 shop rivets while the member c needs $\frac{13,000}{3,750}$ or 4 shop rivets.

74. Pin Plates.—Not infrequently in structural work forces are transmitted from one member to another by means of pins, and in such cases the bearing area between the pin and members must be sufficient to transmit the load safely. A common case is that of channels through the webs of which passes a pin. In order to prevent the crushing of the webs, reinforcing or pin plates must be riveted to them. In arriving at the thickness of such pin plates, it is assumed that the load is distributed uniformly over the total bearing area, and that each plate is capable of taking a load equal to the total load multiplied by the ratio that the thickness of the plate bears to the total thickness. Knowing the load coming upon each plate, the number of rivets required to fasten it to the web of the channel is readily obtained. Another example of the use of pin plates is shown in the reinforcing of the side plates of crane blocks.

75. Diagonal Boiler Brace.—In Fig. 17 is shown a form of boiler brace used for connecting the unsupported area of the head

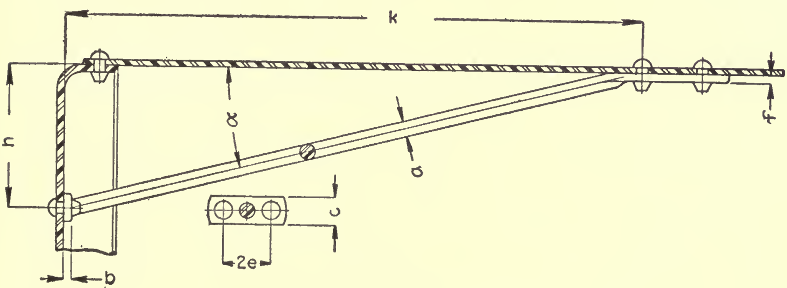

FIG. 17.

to the cylindrical shell. It consists of a round rod having flanged or flattened ends by means of which the brace is riveted to the head and shell. Due to the action of the steam pressure, the brace may fail in any one of the following ways: (1) The body of the brace may fail by tension; (2) the flanged ends at the head may fail due to flexure, while the forged end at the shell may fail due to combined bending and direct tensile stresses; (3) the rivets may fail at the head end; (4) the rivets may fail at the shell end.

(*a*) *Failure of the brace body.*—Letting P represent the force exerted upon the brace due to the pressure on the area supported by the brace, then the component of this force along the rod is $P \sec\alpha$. Hence the stress in the rod is given by the following expression:

$$S_t = \frac{4P\sec\alpha}{\pi a^2} \tag{65}$$

(*b*) *Failure of the brace ends.*—1. *Head End.*—The end attached to the boiler head may fail by bending of the outstanding legs. If $2e$ represents the distance between the two rivets as shown in the figure, then the stress in the sections adjacent to the rod is

$$S_t' = \frac{3P}{2cb^2}(2e - a) \tag{66}$$

As usually constructed the type of brace shown in Fig. 17 is considerably stronger at the flanged ends than in the body.

2. *Shell end.*—At the shell end it is customary to investigate the brace merely for direct tension. Representing the width of the flanged end by g and its thickness by f, then the tensile stress is

$$S_t'' = \frac{P}{f(g - d)} \tag{67}$$

(*c*) *Failure of the rivets at the head end.*—The rivets at the head end of the brace are subjected to direct tensile, shearing, and bending stresses, the latter two of which are generally not considered in actual calculations. The force causing the tensile stress in the rivets is the total force P minus the area $(l \times c)$ multiplied by the steam pressure. However, since the shearing and bending stresses are not considered, it is customary to take the total force P as coming upon the two rivets. Hence the tensile stress in the rivets is

$$S_t = \frac{2P}{\pi d^2} \tag{68}$$

The shearing stress coming upon the rivets is

$$S_s' = \frac{2P\tan\alpha}{\pi d^2} \tag{69}$$

If it is desired to find the resultant stress due to the combined effect of the two stresses just discussed, use the equations given in Art. 17.

(*d*) *Failure of the rivets at the shell end.*—Due to the pull of the brace, the rivets at the shell end are subjected to shearing, tensile, and bending stresses. The first of these stresses is generally the only one considered, since in the majority of cases the direct tensile and bending stresses are small. The component of the force in the rod at right angles to the rivets has a magnitude of P; hence, the shearing stress in the rivets, assuming that two rivets are used to fasten the brace to the shell, is

$$S_s = \frac{2P}{\pi d^2} \tag{70}$$

(*e*) *Allowable stresses.*—The allowable shearing stresses in the rivets vary from 5,000 to 8,000 pounds per square inch, while the permissible tensile stresses in the diagonal brace proper vary from 6,000 to 10,000 pounds per square inch. For the rivets in tension, the allowable stress should not exceed that given for shearing.

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CHAPTER IV

FASTENINGS

BOLTS, NUTS, AND SCREWS

76. Forms of Threads.—The threads of screws are made in a variety of forms depending upon the use to which the screws are to be put. In general, a screw intended for fastening two or more pieces together is fitted with a thread having an angular form, while one intended for the transmission of power will have the threads either square or of a modified angular form.

Two common forms of threads used for screw fastenings are the well-known V and the Sellers or United States Standard threads, shown in Fig. 18(*a*) and (*b*), respectively. Both forms are strong and may be produced very cheaply. Furthermore, due

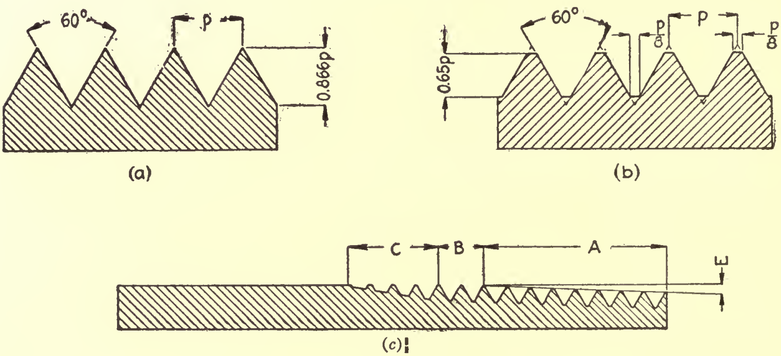


Fig. 18.

to their low efficiency, they are well adapted for screw fastenings. The proportions of these threads are given in the figures, and, as shown, the angle used is 60 degrees. The symbol p denotes the *pitch*, by which is meant the axial distance from a point on one thread to the corresponding point on the next thread; or in other words, the pitch is the distance that the nut advances along the axis of the screw for each revolution of the nut. Evidently, the *number of threads per inch* of length is equal to the reciprocal of the pitch for a single-threaded screw.

(*a*) *Sellers standard.*—The form of thread shown in Fig. 18(*b*) is recognized as the standard in the United States, though the sharp V form is still in use. Due to the flattening of the tops and bottoms of the V's in the Sellers standard, this form is much stronger than the sharp V thread. In Table 15 are given the proportions of the various sizes of bolts and nuts up to 3 inches in diameter, based on the Sellers standard. The Sellers system with some modifications has been adopted by the United States Navy Department. Instead of using different proportions for finished and unfinished bolt heads and nuts, the Navy Department adopted as their standard those given for rough work, thus permitting the same wrench to be used for both classes of bolts. In addition to this change, the Navy Department has adopted a pitch of $\frac{1}{4}$ inch for all sizes above $2\frac{3}{4}$ inches, which does not agree with the Sellers system.

(*b*) *Standard pipe thread.*—In Fig. 18(*c*) is shown a section of a standard pipe thread which may also be considered a form of fastening, though not for the same class of service as those discussed above. It will be noticed that the total length of the thread is made up of three parts. The first part designated as A in Fig. 18(*c*) has a full thread over a tapered length of $\frac{0.8\,D + 4.8}{n}$, in which D represents the outside diameter of the pipe and n the number of threads per inch. The second part B has two threads that are full at the root but imperfect at the top and not on a taper. The part C includes four imperfect threads. The total taper of the threads is $\frac{3}{4}$ inch per foot, or the taper designated by the symbol E is 1 in 32. It should be remembered that gas pipe goes only by inside measurement, that is, by the nominal diameter. The actual inside diameter varies somewhat from the nominal, but only the latter is used in speaking of commercial pipe sizes.

(*c*) *Square thread.*—Three forms of screw threads that are well adapted to the transmission of power are shown in Fig. 19. The *square thread* shown in Fig. 19(*a*) is probably the most common, and its efficiency is considerably higher than that obtained by the use of V threads. It has serious disadvantages in that it is very difficult to take up any wear that may occur, and furthermore, it costs considerably more to manufacture. The proportions of square threads have never been standardized, but the

TABLE 15.—UNITED STATES STANDARD BOLTS AND NUTS

Size	Threads per inch	Diam. at root	Area at root	Rough heads and nuts: Diam. of heads and nuts, Short	Rough heads and nuts: Diam. of heads and nuts, Long, Hex.	Rough heads and nuts: Diam. of heads and nuts, Long, Square	Rough heads and nuts: Thickness, Head	Rough heads and nuts: Thickness, Nut	Finished heads and nuts: Short diam.	Finished heads and nuts: Long diam.	Finished heads and nuts: Thickness
1/4	20	0.185	0.026	1/2	37/64	23/32	1/4	1/4	7/16	1/2	3/16
5/16	18	0.240	0.045	19/32	11/16	27/32	19/64	5/16	17/32	39/64	1/4
3/8	16	0.294	0.068	11/16	51/64	31/32	11/32	3/8	5/8	23/32	5/16
7/16	14	0.345	0.093	25/32	29/32	1 7/64	25/64	7/16	23/32	53/64	3/8
1/2	13	0.400	0.126	7/8	1 1/64	1 1/4	7/16	1/2	13/16	15/16	7/16
9/16	12	0.454	0.162	31/32	1 1/8	1 3/8	31/64	9/16	29/32	1 1/64	1/2
5/8	11	0.507	0.202	1 1/16	1 15/64	1 1/2	17/32	5/8	1	1 5/32	9/16
3/4	10	0.620	0.302	1 1/4	1 29/64	1 25/32	5/8	3/4	1 3/16	1 21/64	11/16
7/8	9	0.731	0.419	1 7/16	1 43/64	2 1/32	23/32	7/8	1 3/8	1 19/32	13/16
1	8	0.837	0.551	1 5/8	1 7/8	2 19/64	13/16	1	1 9/16	1 13/16	15/16
1 1/8	7	0.940	0.693	1 13/16	2 3/32	2 9/16	29/32	1 1/8	1 3/4	2 1/64	1 1/16
1 1/4	7	1.065	0.889	2	2 5/16	2 53/64	1	1 1/4	1 15/16	2 15/64	1 3/16
1 3/8	6	1.16	1.056	2 3/16	2 17/32	3 3/32	1 3/32	1 3/8	2 1/8	2 29/64	1 5/16
1 1/2	6	1.284	1.293	2 3/8	2 3/4	3 23/64	1 3/16	1 1/2	2 5/16	2 43/64	1 7/16
1 5/8	5 1/2	1.389	1.515	2 9/16	2 31/32	3 5/8	1 9/32	1 5/8	2 1/2	2 57/64	1 9/16
1 3/4	5	1.491	1.745	2 3/4	3 3/16	3 57/64	1 3/8	1 3/4	2 11/16	3 7/64	1 11/16
1 7/8	5	1.616	2.049	2 15/16	3 25/64	4 5/32	1 15/32	1 7/8	2 7/8	3 5/16	1 13/16
2	4 1/2	1.712	2.300	3 1/8	3 39/64	4 27/64	1 9/16	2	3 1/16	3 17/32	1 15/16
2 1/4	4 1/2	1.962	3.021	3 1/2	4 3/64	4 61/64	1 3/4	2 1/4	3 7/16	3 31/32	2 3/16
2 1/2	4	2.176	3.716	3 7/8	4 31/64	5 31/64	1 15/16	2 1/2	3 13/16	4 13/32	2 7/16
2 3/4	4	2.426	4.62	4 1/4	4 29/32	6 1/64	2 1/8	2 3/4	4 3/16	4 27/32	2 11/16
3	3 1/2	2.629	5.428	4 5/8	5 11/32	6 17/32	2 5/16	3	4 9/16	5 3/32	2 15/16

practice of Wm. Sellers and Co., exhibited in Table 16, may serve as a guide.

TABLE 16.—PROPORTIONS OF SELLERS SQUARE THREADS

Size	Threads per inch	Root diam.	Size	Threads per inch	Root diam.	Size	Threads per inch	Root diam.
¼	10	0.1625	1¼	3½	1.0	3	1¾	2.5
⅜	8	0.2658	1½	3	1.2084	3¼	1¾	2.75
½	6½	0.3656	1¾	2½	1.4	3½	1⅝	2.962
⅝	5½	0.466	2	2¼	1.612	3¾	1½	3.168
¾	5	0.575	2¼	2¼	1.862	4	1½	3.418
⅞	4½	0.6806	2½	2	2.0626			
1	4	0.7813	2¾	2	2.3126			

(*d*) *Trapezoidal thread.*—The *trapezoidal* or *buttressed thread* shown in Fig. 19(*b*) is occasionally used for the transmission of power in one direction only. The driving face of the thread is at

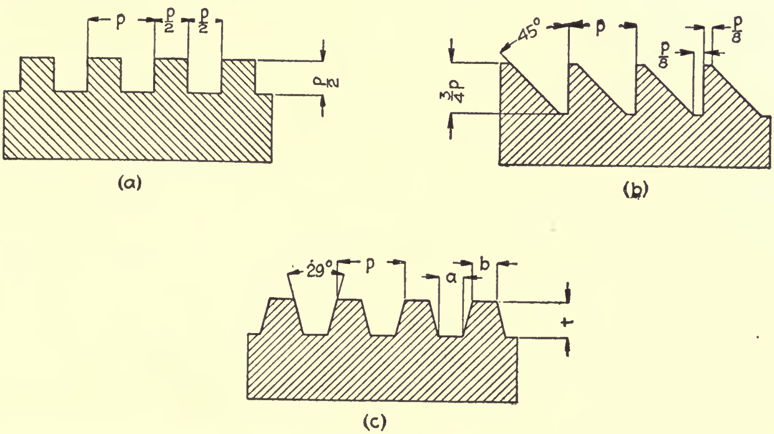


FIG. 19.

right angles to the axis of the screw, while the back face makes an angle of 45 degrees, as shown in the figure. It is evident that the efficiency of this form of thread is the same as for a square thread, while its strength is practically that of the V thread. No standard proportions have ever been devised or suggested, except those given in Fig. 19(*b*).

(*e*) *Acme thread.*—The *Acme thread* is now recognized as the standard form of thread for lead screws and similar service, since the wear can readily be compensated for by means of a nut split lengthwise. Its efficiency is not quite as high as that of a square thread, but its cost of production is less since dies may be used in its manufacture. The form of the standard Acme thread is shown in Fig. 19(*c*), and in Table 17 are given the various dimensions indicated in the figure.

TABLE 17.—PROPORTIONS OF ACME STANDARD THREADS

Threads per inch	*a*	*b*	*t*	Threads per inch	*a*	*b*	*t*
10	0.0319	0.0371	0.0600	3	0.1183	0.1235	0.1767
9	0.0361	0.0413	0.0655	2½	0.1431	0.1483	0.2100
8	0.0411	0.0463	0.0725	2	0.1801	0.1853	0.2600
7	0.0478	0.0529	0.0814	1½	0.2419	0.2471	0.3433
6	0.0566	0.0618	0.0933	1¼	0.2914	0.2966	0.4100
5	0.0689	0.0741	0.1100	1	0.3655	0.3707	0.5100
4	0.0875	0.0927	0.1350	½	0.7362	0.7414	1.0100

SCREW FASTENINGS

In general, screw fastenings are used for fastening together either permanently or otherwise two machine parts. To accomplish this end, the following important forms are met with in machine construction; (*a*) bolts; (*b*) cap screws; (*c*) machine screws; (*d*) set screws; (*e*) studs; (*f*) patch bolts; (*g*) stay bolts.

77. Bolts.—A *bolt* is a round bar one end of which is fitted with a thread and nut, while the other end is upset to form the head. Bolts are well adapted for fastening machine parts rigidly; but at the same time they allow the parts to be easily disconnected. Whenever conditions or surroundings will permit, bolts should be used for fastening machine parts together.

(*a*) *Machine bolts.*—What is known as a *machine bolt* has a rough body, but the head and the nut may be rough or finished, as desired. Commercial forms of machine bolts are shown in Fig. 20(*a*) and (*b*). The heads and nuts may be square as shown in Fig. 20(*a*), or the hexagonal form shown in Fig. 20(*b*) may be used. The standard lengths of machine bolts as given in the manufacturers' catalogs are as follows:

1. Between 1 and 5 inches the lengths vary by ¼-inch increments.

2. Between 5 and 12 inches, the lengths vary by ½-inch increments.

3. Above 12 inches, the lengths vary by 1-inch increments.

Any length of bolt, however, may be obtained, but odd lengths cost more than standard lengths. The length of the threaded part is from three to four times the height of the nut.

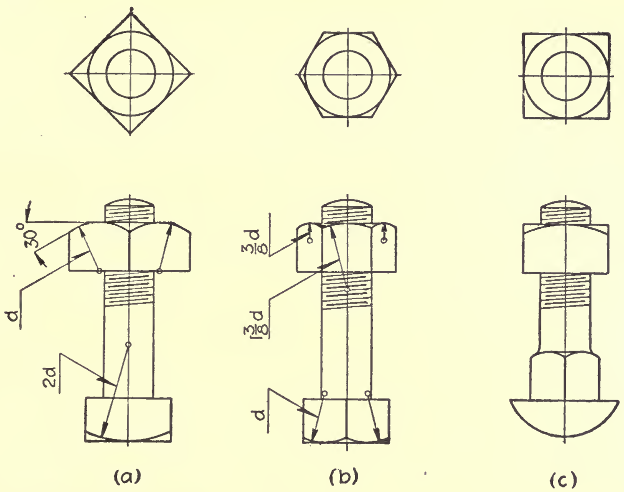


FIG. 20.

The proportions of the heads and nuts as used on standard machine bolts are given in Table 15.

(*b*) *Carriage bolts.*—The carriage bolt, another form of through bolt, is shown in Fig. 20(*c*). Its chief use is in connection with wood construction.

(*c*) *Coupling bolts.*—A *coupling bolt* is merely a machine bolt that has been finished all over, so that it may be fitted into reamed holes of the same diameter as the nominal diameter of the bolt. Coupling bolts are intended for use in connection with the best forms of construction. They are more expensive to produce and at the same time are more costly to fit into place. In Table 18 are given the dimensions of commercial sizes of such bolts. According to the manufacturers'

TABLE 18.—COUPLING BOLTS

Size	Threads per inch	Head and nut		Stock lengths
		Short diam.	Thickness	
½	13	⅞	½	2 –4¾
⅝	11	1 1/16	⅝	2 –5
¾	10	1¼	¾	2¼–5¼
⅞	9	1 7/16	⅞	2½–5½
1	8	1⅝	1	2¾–5¾
1⅛	7	1 13/16	1⅛	3 –6
1¼	7	2	1¼	3¼–6

lists, coupling bolts are made with hexagonal heads and nuts only, and the lengths for the sizes listed in Table 18 run from 2 inches up to 6 inches, varying by quarter-inch increments.

(*d*) *Automobile bolts.*—The various bolts discussed in the preceding paragraphs have not been found satisfactory in automobile construction, and in order to fulfill the requirements of strength and space limits demanded in this class of work, the Society of Automobile Engineers has adopted a special standard. The design of this type of bolt is shown in Fig. 21, and the data

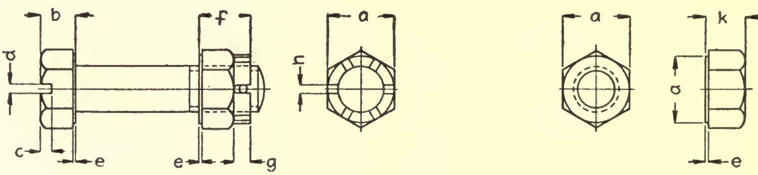


Fig. 21.

in Table 19 give the various detail dimensions for the different sizes that have so far been standardized. It will be noticed that the heads and nuts are hexagonal, and that the thread is the

Table 19.—S. A. E. Standard Bolts and Nuts

Size	Threads per inch	Head of bolt					Castellated nut				Plain nut
		a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	Cotter pin	*k*
1/4	28	7/16	3/16	3/32	1/16	1/64	9/32		5/64	1/16	7/32
5/16	24	1/2	15/64	7/64			21/64	3/32			17/64
3/8	24	9/16	9/32	1/8	3/32		13/32	1/8	1/8	3/32	21/64
7/16	20	5/8	21/64				29/64				3/8
1/2	20	3/4	3/8				9/16	3/16			7/16
9/16	18	7/8	27/64				39/64		5/32	1/8	31/64
5/8	18	15/16	15/32				23/32	1/4			35/64
11/16	16	1	33/64				49/64				19/32
3/4	16	1 1/16	9/16				13/16				21/32
7/8	14	1 1/4	21/32				29/32				49/64
1	14	1 7/16	3/4				1				7/8
1 1/8	12	1 5/8	27/32	7/32	5/32		1 5/32	5/16	7/32	11/64	63/64
1 1/4	12	1 13/16	15/16				1 1/4				1 3/32
1 3/8	12	2	1 1/32	1/4	3/16		1 13/32	3/8	1/4	13/64	1 13/64
1 1/2	12	2 3/16	1 1/8				1 1/2				1 5/16

Sellers standard. Instead of using the same pitch as that recommended in the Sellers system, a finer pitch has been adopted; furthermore, the heads and nuts are made somewhat smaller. The heads are slotted for a screw driver and the nuts are recessed or castellated so they may be locked to the bolt by means of cotter pins.

78. Screws.—Screws, unlike bolts, do not require a nut, but screw directly into one of the pieces to be fastened, either the head or the point pressing against the other piece. The types of screws that hold the pieces together by the pressure exerted by

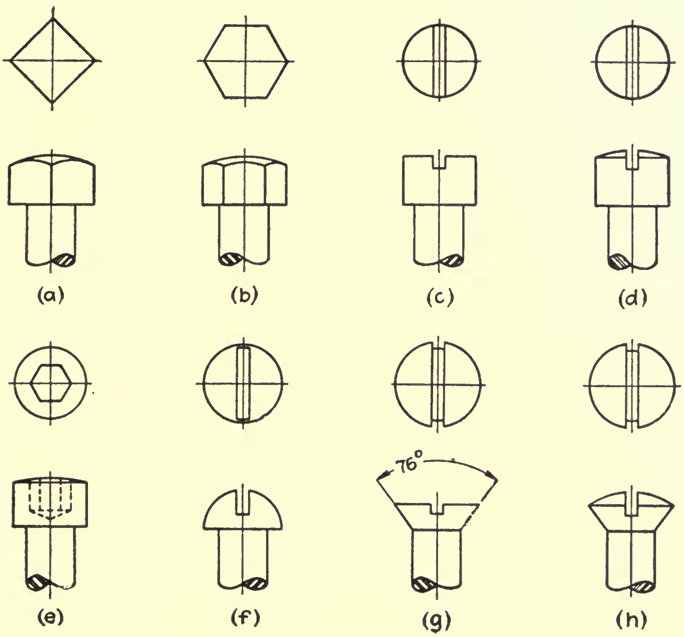


Fig. 22.

the head of the screw are called *cap screws* and *machine screws*, while those whose points press against a piece and by friction prevent relative motion between the two parts are called *set screws*. By the term *length* of a screw is always meant the length under the head.

(*a*) *Cap screws.*—Cap screws are made with square, hexagonal, round or filister, flat and button heads, and are threaded either United States Standard or with V threads. The various forms for heads are shown in Fig. 22, and in Table 20 are given the general dimensions of the commercial sizes that are usually kept in stock. All cap screws, except those with filister heads, are threaded three-fourths of the length for one inch in diameter or less and for lengths less than four inches. Beyond these dimensions, the threads are cut approximately one-half the length. The lengths of cap screws vary by quarter-inch increments between the limits given in Table 20.

Cap screws, if properly fitted, make an excellent fastening for machine parts that do not require frequent removal. To insure a good fastening by means of cap screws, the depth of the tapped hole should never be made less than one and one-half times the diameter of the screw that goes into it. In cast iron, the depth should be made twice the diameter.

Table 20.—Standard Cap Screws

Size	Threads per inch	Square head			Hexagon head			Socket head		
		Short diam.	Thickness	Length	Short diam.	Thickness	Length	Diam.	Thickness	Length
1/4	20	3/8	1/4		7/16	1/4		3/8	1/4	3/4–3 1/2
5/16	18	7/16	5/16	3/4–3	1/2	5/16	3/4–3	7/16	5/16	3/4–3 3/4
3/8	16	1/2	3/8		9/16	3/8		9/16	3/8	3/4–4
7/16	14	9/16	7/16	3/4–4	5/8	7/16	3/4–4	5/8	7/16	3/4–4 1/4
1/2	13	5/8	1/2		3/4	1/2		3/4	1/2	3/4–6
9/16	12	11/16	9/16	1 –4	13/16	9/16	1 –4	13/16	9/16	1 –6
5/8	11	3/4	5/8	1 –4 1/2	7/8	5/8	1 –4 1/2	7/8	5/8	
3/4	10	7/8	3/4	1 1/4–4 3/4	1	3/4	1 1/4–4 3/4	1	3/4	1 1/4–6
7/8	9	1 1/8	7/8	1 1/2–5	1 1/8	7/8	1 1/2–5			
1	8	1 1/4	1	1 3/4–5	1 1/4	1	1 3/4–5			
1 1/8	7	1 3/8	1 1/8	2 –5	1 3/8	1 1/8	2 –5			
1 1/4	7	1 1/2	1 1/4		1 1/2	1 1/4				

Size	Threads per inch	Round and filister head			Button head			Flat head		
		Diam.	Thickness	Length	Diam.	Thickness	Length	Diam.	Thickness	Length
1/8	40	3/16	1/8	3/4–2 1/2	7/32	7/64	3/4–1 3/4	1/4		3/4–1 3/4
3/16	24	1/4	3/16	3/4–2 3/4	5/16	5/32	3/4–2	3/8		3/4–2
1/4	20	3/8	1/4	3/4–3	7/16	7/32	3/4–2 1/4	15/32		3/4–2 1/4
5/16	18	7/16	5/16	3/4–3 1/4	9/16	9/32	3/4–2 1/2	5/8		3/4–2 3/4
3/8	16	9/16	3/8	3/4–3 1/2	5/8	5/16	3/4–2 3/4	3/4		3/4–3
7/16	14	5/8	7/16	3/4–3 3/4	3/4	3/8	3/4–3	13/16		1 –3
1/2	13	3/4	1/2	3/4–4	13/16	13/32	1 –3	7/8		1 1/4–3
9/16	12	13/16	9/16	1 –4 1/4	15/16	15/32	1 1/4–3	1		1 1/2–3
5/8	11	7/8	5/8	1 1/4–4 1/2	1	1/2	1 1/2–3	1 1/8		1 3/4–3
3/4	10	1	3/4	1 1/2–4 3/4	1 1/4	5/8	1 3/4–3	1 3/8		2 –3
7/8	9	1 1/8	7/8	1 3/4–5						
1	8	1 1/4	1	2 –5						

(b) *Machine screws.*—Machine screws are strictly speaking cap screws, but the term as commonly used includes various forms of small screws that are provided with a slotted head for a screw-driver. The sizes are designated by gauge numbers instead of by the diameter of the body. The usual forms of machine screws are shown in Fig. 23, and in Table 21 are given the dimensions of stock sizes.

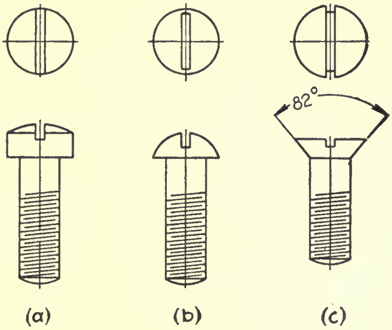


FIG. 23.

There are no accepted standards, each manufacturer having his own. It should also be observed that machine screws have no standard number of threads, hence in dimensioning these screws, always give the number of the screw, the number of threads and the length, thus No. 30 – 16 × 1½ inches M. Sc. It may be noted that machine screws larger than No. 16 are not used extensively in machine construction; for larger diameters than No. 16, use cap screws.

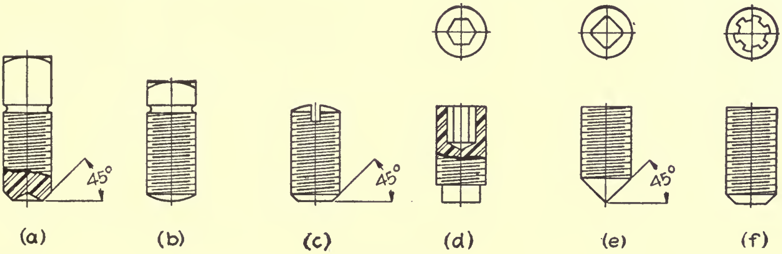


FIG. 24.

The American Society of Mechanical Engineers has adopted a uniform system of standard dimensions for machine screws, but as yet they are not in universal use in this country. The report of the committee which was appointed to draw up such standards may be found on page 99 of volume 29 of the *Transactions*.

(c) *Set screws.*—Set screws are made with square heads or with no heads at all, and may be obtained with either United States Standard or V threads. The short diameter of the square

TABLE 21.—STANDARD MACHINE SCREWS

Size		Threads per inch	Filister head					Button head				Flat head				Stock lengths
			Head			Slot		Head		Slot		Head		Slot		
No.	Diam.		Diam.	Crown	Length	Width	Depth	Diam.	Length	Width	Depth	Diam.	Length	Width	Depth	
2	0.0842	64, 56, 48	0.1350	0.0126	0.0675	0.030	0.0338	0.1544	0.0672	The same as on filister heads	0.0403	0.1631	0.0454	The same as on filister heads	0.0151	1/8–7/8
3	0.0973	56, 48	0.1561	0.0146	0.0780	0.032	0.0390	0.1786	0.0746		0.0448	0.1894	0.0530		0.0177	
4	0.1105	40, 36, 32	0.1772	0.0166	0.0886	0.034	0.0443	0.2028	0.0820		0.0492	0.2158	0.0605		0.0202	1/8–2
5	0.1236		0.1984	0.0186	0.0992	0.036	0.0496	0.2270	0.0894		0.0536	0.2421	0.0681		0.0227	1/8–2 1/4
6	0.1368	36, 32, 30	0.2195	0.0205	0.1097	0.039	0.0549	0.2512	0.0968		0.0580	0.2684	0.0757		0.0252	
7	0.1500	32, 30	0.2406	0.0225	0.1203	0.041	0.0602	0.2754	0.1042		0.0625	0.2947	0.0832		0.0277	1/8–3
8	0.1631	36, 32, 30	0.2617	0.0245	0.1308	0.043	0.0654	0.2996	0.1116		0.0670	0.3210	0.0908		0.0303	
9	0.1763	32, 30, 24	0.2828	0.0265	0.1414	0.045	0.0707	0.3238	0.1190		0.0714	0.3474	0.0984		0.0328	3/16–3 1/2
10	0.1894		0.3040	0.0285	0.1520	0.048	0.0760	0.3480	0.1264		0.0758	0.3737	0.1059		0.0353	
12	0.2158	24, 20	0.3462	0.0324	0.1731	0.052	0.0866	0.3922	0.1412		0.0847	0.4263	0.1210		0.0403	1/4–3 1/2
14	0.2421	24, 20, 18	0.3884	0.0364	0.1942	0.057	0.0971	0.4364	0.1560		0.0936	0.4790	0.1362		0.0454	1/4–4
16	0.2684	20, 18, 16	0.4307	0.0403	0.2153	0.061	0.1077	0.4806	0.1708		0.1024	0.5316	0.1513		0.0504	5/16–4
18	0.2947		0.4729	0.0443	0.2364	0.066	0.1182	0.5248	0.1856		0.1114	0.5842	0.1665		0.0555	3/8–4
20	0.3210	18, 16	0.5152	0.0483	0.2576	0.070	0.1288	0.5690	0.2004		0.1202	0.6368	0.1816		0.0605	
22	0.3474		0.5574	0.0520	0.2787	0.075	0.1314	0.6106	0.2152		0.1291	0.6895	0.1967		0.0656	
24	0.3737	18, 16, 14	0.5996	0.0562	0.2998	0.079	0.1499	0.6522	0.2300		0.1380	0.7421	0.2118		0.0706	
26	0.4000	16, 14	0.6419	0.0601	0.3209	0.084	0.1605	0.6938	0.2448		0.1469	0.7421	0.1967		0.0656	
28	0.4263		0.6841	0.0641	0.3420	0.088	0.1710	0.7354	0.2596		0.1558	0.7948	0.2118		0.0706	
30	0.4526		0.7264	0.0681	0.3632	0.093	0.1816	0.7770	0.2744		0.1646	0.8474	0.2270		0.0757	3/4–4

heads as well as the height of the heads is made equal to the diameter of the body of the screw. The commercial lengths of set screws having heads vary from $\frac{3}{4}$ to 5 inches by quarter inches. The headless set screws shown in Fig. 24(*d*) to (*f*) are made only in the following sizes: $\frac{3}{8}$ by $\frac{3}{8}$ inch; $\frac{1}{2}$ by $\frac{9}{16}$ inch; $\frac{5}{8}$ by $\frac{11}{16}$ inch; and $\frac{3}{4}$ by $\frac{7}{8}$ inch.

The principal distinguishing feature of set screws is the form of the point. The points are generally hardened. Only cup and round point set screws (see Fig. 24(*a*) and (*b*)) are regular, all other types being considered special. Set screws used as fastenings are not entirely satisfactory for heavy loads, and hence should only be used on the lighter loads. The cup point shown in Fig. 24(*a*) has a disadvantage in that it raises a burr on the shaft thus making the removal of the piece, such as a pulley, more difficult. In place of the cup point, the conical point shown in Fig. 24(*e*) is frequently used, but this necessitates drilling a conical hole in the shaft, which later on may interfere with making certain desirable adjustments.

To obtain the appropriate size of set screw for a given diameter of shaft, the following empirical formula based upon actual installations may be found useful:

$$\text{diameter of set screw} = \frac{d}{8} + \frac{5}{16}\text{ inch,} \tag{71}$$

in which d represents the diameter of the shaft.

The question of the holding capacity of set screws has received little attention and about the only information available is that published by Mr. B. H. D. Pinkney in the *American Machinist* of Oct. 15, 1914. His results are based upon some experiments with $\frac{1}{4}$- and $\frac{1}{2}$-inch set screws, in which he found that the latter size had a capacity of five times the former. With this fact as a basis, Mr. Pinkney calculated the data given in Table 22. Experience with the headless variety of set screws seems to indicate that due to the difficulty of screwing up, the holding power is somewhat less than for the cup and flat point type.

TABLE 22.—SAFE HOLDING CAPACITIES OF SET SCREWS

Size	Capacity, pounds	Size	Capacity pounds
$\frac{1}{4}$	100	$\frac{5}{8}$	840
$\frac{5}{16}$	168	$\frac{3}{4}$	1,280
$\frac{3}{8}$	256	$\frac{7}{8}$	1,830
$\frac{7}{16}$	366	1	2,500
$\frac{1}{2}$	500	$1\frac{1}{8}$	3,288
$\frac{9}{16}$	658	$1\frac{1}{4}$	4,198

(*d*) *Studs.*—A stud is a bolt in which the head is replaced by a threaded end, as shown in Fig. 25. It passes through one of the parts to be connected, and is screwed into the other part, thus remaining always in position when the parts are disconnected. With this construction the wear and crumbling of the threads in a weak material, such as cast iron, are avoided. Studs are usually employed to secure the heads of cylinders in engines and pumps.

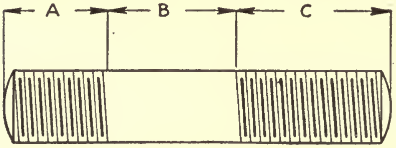


Fig. 25.

There is no standard for the length of the threaded ends of studs; hence, the length must always be specified. Studs may be obtained finished at *B* or rough, and the ends threaded either with United States Standard or V threads. The commercial lengths carried in stock vary from 1¼ to 6 inches by quarter inches for the finished studs. For the rough studs, the lengths vary from ½ to 4 inches by quarter inches, and from 4 to 6 inches by half inches. Usually one end is made a tight fit, while the other is of standard size.

(*e*) *Patch bolts.*—A form of screw commonly called a patch bolt is shown in Fig. 26(*a*); its function is that of fastening patches on the sheets of boilers. The application of a patch

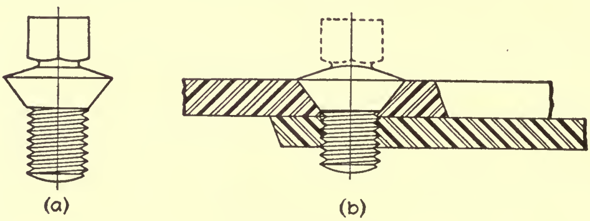


Fig. 26.

bolt is illustrated in Fig. 26(*b*). Patch bolts should be used only when, due to the location of the patch, it is impossible to use rivets, as for example on the water leg of a locomotive boiler. As shown in Fig. 26(*b*), patch bolts are introduced from the side exposed to the fire and are screwed home securely. The head, by means of which they are screwed up, is generally twisted off in making the fastening. Instead of having the form and number of threads according to the United States Standard, all stock sizes have 12 threads per inch of the sharp V type.

79. Stay Bolts.—(*a*) *Stay bolts* are fastenings used chiefly in boiler construction. Due to the unequal expansion and contraction of the two plates that are connected, stay bolts are subjected to a peculiar bending action in addition to a direct tension. As a result of the relative motion between the two connected plates, stay bolts develop small cracks near the inner edge of the sheets. These cracks eventually cause complete rupture, though it may not be noticed until the plates begin to bulge. Three types of stay bolts are shown in Fig. 27, the first

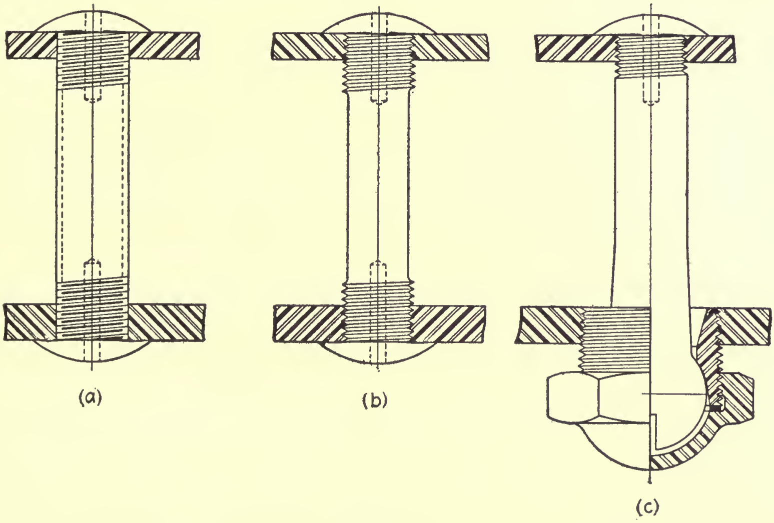


Fig. 27.

of which is used extensively on small vertical and locomotive types of boilers. To provide some slight degree of flexibility and thereby decrease the danger of cracking near the plates, stay bolts are made as shown in Fig. 27(*b*).

According to the Code of Practical Rules, covering the construction and maintenance of stationary boilers, recently adopted by the American Society of Mechanical Engineers, "each end of stay bolts must be drilled with a $\frac{3}{16}$-inch hole to a depth extending $\frac{1}{2}$ inch beyond the inside of the plates, except on small vertical or locomotive-type boilers where the drilling of the stay bolts shall be optional." The object of these holes is to give some indication of a rupture by the leakage of the fluid.

In Fig. 27(*c*) is shown one of the various types of so-called flexible stay bolts used in locomotive boilers.

(*b*) *Stresses in stay bolts.*—The area of the surface supported by a stay bolt depends principally upon the thickness of the plates and the fluid pressure upon the surface. Quoting from the Code of Rules adopted by the American Society of Mechanical Engineers, "the pitch allowed for stay bolts on a flat surface and on the furnace sheets of an internally fired boiler in which the external diameter of the furnace is over 38 inches, except a corrugated furnace, or a furnace strengthened by an Adamson ring or equivalent," may be determined by the following formula, but in no case should it exceed $8\frac{1}{2}$ inches:

$$p = \sqrt{\frac{C(t+1)^2}{P}} + 6, \qquad (72)$$

in which

C = constant having a value of 66.
P = working pressure in pounds per square inch.
t = thickness of plate in sixteenths of an inch.

In addition to the formula just given, the above-mentioned Code of Rules contains tables and other formulas pertaining to the subject of staying surfaces that may be found useful in designing pressure vessels.

Having determined the pitch of the stay bolts, a simple calculation will give the magnitude of the load coming upon each bolt. Dividing this load by the allowable stress, the result is the area at the root of the thread. For mild-steel or wrought-iron stay bolts up to and including $1\frac{1}{4}$ inches in diameter, the American Society of Mechanical Engineers recommends that the allowable stress shall not exceed 6,500 pounds per square inch, and for larger diameters 7,000 pounds per square inch is recommended. The majority of screwed stay bolts have 12 threads per inch of the V type, though the United States Standard form is also used.

80. Nut Locks.—Since nuts must have a small clearance in order to allow them to turn freely, they have a tendency to unscrew. This tendency is especially evident in the case of nuts subjected to vibration. In order to prevent unscrewing, a great many different devices have been originated, a few of which are shown in Figs. 28 to 30 inclusive.

(*a*) *Lock nut.*—The cheapest and most common locking device is the lock nut shown in Fig. 28. Two nuts are used, but it is not necessary that both of these shall be of standard thickness, as frequently the lower nut is made only one-half as thick as the upper one. Some engineers maintain that the lower nut should be standard thickness while the upper one could be thinner. The following analysis, due to Weisbach, shows that conditions might arise for which the first arrangement would answer, while for other conditions, the second arrangement would be the proper one to use.

We shall assume that the lower nut in Fig. 28(*a*) has been screwed down tight against the cap *c* of some bearing. Denote the pressure created between the nut *b* and the cap *c* by the symbol P. Now screw down the upper nut *a* against *b* as tightly as the size of the stud or bolt *d* will permit, thus developing a pressure between the two nuts, which at the same time produces a tensile stress in that part of the stud *d* that comes within the limits of action of the two nuts. Designate the magnitude of this pressure between the nuts by the symbol P_0. Considering the

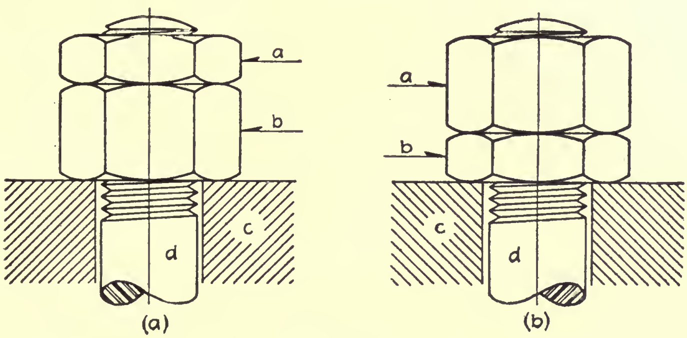


FIG. 28.

forces acting upon the nut *b*, it is evident that the force P_0 acts downward, while the force P acts upward, and the resultant force having a magnitude $P - P_0$ acts upon the threads of the stud. Now the direction of this resultant depends upon the magnitudes of P and P_0. If $P > P_0$ the resultant force on the threads of the stud is upward, or in other words the upper surfaces of the threads in the nut *b* come into contact with the lower surfaces of the threads on the stud. From this it follows that when $P > P_0$, the nut *b* should be of standard thickness as shown in Fig. 28(*a*), since it alone must support the axial load.

Let us consider the case when $P_0 > P$; we shall find that the resultant force on the threads is downward, thus indicating that the lower surface of the threads in the nut *b* bear on the upper surfaces of the threads on the stud; hence, the upper nut must take the axial load and for that reason should be made of standard thickness as shown in Fig. 28(*b*).

Now consider another case might arise, namely in which $P_0 = P$. It is evident that the resultant is zero, thereby showing that no pressure exists on either the upper or lower surfaces of the thread; hence, the nut *a* carries the axial load *P*.

On the spindles of heavy milling machines and other machine tools, the double lock nut is used to a great extent. The nuts are made circular rather than hexagonal and are fitted with radial slots or holes for the use of spanner or pin wrenches.

(*b*) *Collar nut.*—The collar nut, shown in Fig. 29(*a*), has been used very successfully in heavy work. The lower part of the nut

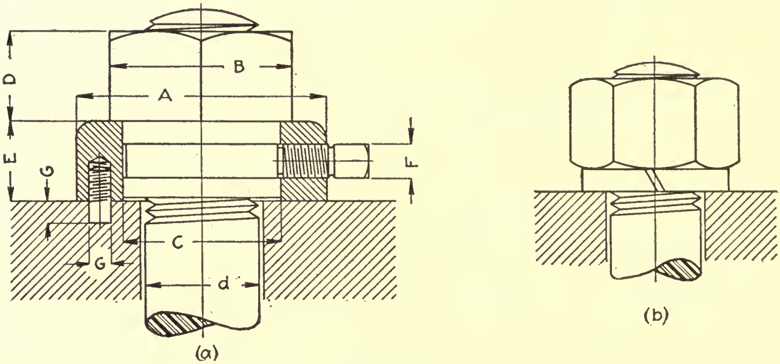


FIG. 29.

is turned cylindrical, and upon the surface a groove is cut. The cylindrical part of the nut fits into a collar or recess in the part connected. This collar is prevented from turning by a dowel pin as shown in the figure. A set screw fitted into the collar prevents relative motion between the latter and the nut. In connecting rods of engines, for example, where the bolt comes near the edge of the rod, the bolt hole is counterbored to receive the cylindrical part of the nut, and the set screw for locking the nut is fitted directly into the head of the rod.

The following formulas have proved satisfactory in proportioning collar nuts similar to that shown in Fig. 29(*a*):

$$\left.\begin{aligned} A &= 2.25\,d + \tfrac{3}{16}\text{ inch} \\ B &= 1.5\,d \\ C &= 1.45\,d \\ D &= 0.75\,d \\ E &= 0.55\,d \\ F &= 0.2\,d + \tfrac{1}{16}\text{ inch} \\ G &= 0.1\,d + 0.1\text{ inch} \end{aligned}\right\} \qquad (73)$$

(*c*) *Castellated nut.*—Another effective way of locking nuts, used extensively in automobile construction, is shown in Fig. 21. It is known as the *castellated nut*, and the commercial sizes correspond to the sizes of automobile bolts discussed in Art. 77(*d*). Attention is directed to the fact that, due to the necessity of turning the nut through 60 degrees between successive locking positions, it may be impossible to obtain a tight and rigid

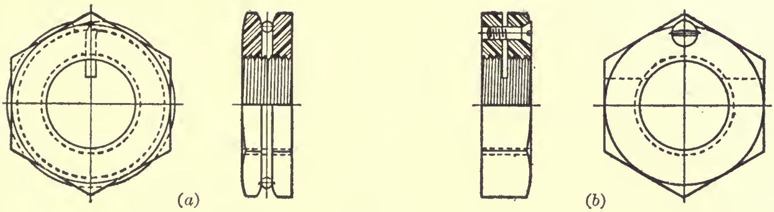


Fig. 30.

connection without inducing a high initial stress in the bolt. The general proportions of the standard castellated nuts approved and recommended by the Society of Automobile Engineers are given in Table 19.

(*d*) *Split nut.*—The double nut method of locking is not always found convenient due to restricted space, and in such places, the forms of nut locks shown in Fig. 30 have been found very satisfactory. In Fig. 30(*b*) is illustrated a hexagonal nut having a saw cut extending almost to the center. By means of a small flat-head machine screw fitted into one side of the nut, the slot may be closed in sufficiently to clamp the sides of the thread. The nut, instead of being hexagonal in form, may be made circular, and should then be fitted with radial slots or holes for a spanner wrench.

(*e*) *Spring wire lock.*—The spring wire lock shown in Fig. 30 (*a*) is another locking device adapted to a restricted space. This is a very popular nut lock for use with the various types of ball bearings. The spring wire requires the drilling of a hole in the shaft, and in case any further adjustment is made after the nut is fitted in place, it requires drilling a new hole. A series of such holes will weaken the shaft materially.

(*f*) *Lock washer.*—A nut lock used considerably on railway track work, and within recent years in automobile work, is shown in Fig. 29(*b*). It consists essentially of one complete turn of a helical spring placed between the nut and the piece to be fastened. When the nut is screwed down tightly, the washer is flattened out and its elasticity produces a pressure upon the nut, thereby preventing backing off. In Table 23 is given general information pertaining to the standard light and heavy lock washers adopted by the Society of Automobile Engineers.

TABLE 23.—PLAIN LOCK WASHERS

Size of bolt	Section of washer			
	Light service		Heavy service	
	Width	Thickness	Width	Thickness
3/16			1/16	3/64
1/4			1/8	1/16
5/16			1/8	1/16
3/8	5/32	1/16	5/32	3/32
7/16	5/32	1/16	5/32	3/32
1/2	5/32	1/16	5/32	3/32
9/16			3/16	3/32
5/8	3/16	3/32	1/4	3/16
3/4	3/16	3/32	1/4	1/4
7/8	1/4	1/8	1/4	1/4
1	1/4	1/8	1/4	1/4

81. Washers.—The function of a washer is to provide a suitable bearing for a nut or bolt head. Washers should not be used unless the hole through which the bolt passes is very much oversize, or the nature of the material against which the nut or bolt head bears necessitates their use. For common usage with machine parts, wrought-iron or steel-cut washers are the best. When the material against which the nut bears is relatively soft, such as wood for example, the bearing pressure due to the load carried by the bolt should be distributed over a considerable area. This is accomplished by the use of large steel or cast-iron washers.

Washers are specified by the so-called nominal diameter, by which is meant the diameter of the bolt with which the washer is to be used.

82. Efficiency of V Threads.—Before discussing the stresses induced in bolts and screws due to the external loads and to screwing up, it is necessary to establish an expression for the probable efficiency of screws.

Let N = unit normal pressure.
Q = axial thrust upon the screw.
d = mean diameter of the screw.
p = pitch of the thread.
α = angle of rise of the mean helix.
β = angle that the side of the thread makes with the axis of the screw.
μ' = coefficient of friction between the nut and screw.
η = efficiency.

Consider a part of a V-threaded screw, as shown in Fig. 31, in which the section CDE is taken at right angles to the mean helix AO. The line OF represents the line of action of the normal

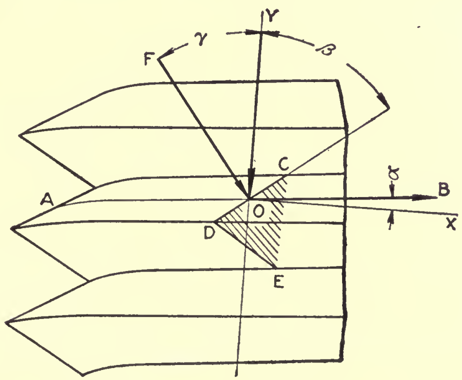


FIG. 31.

pressure N acting upon the thread at the point O, and OY is drawn parallel to the axis of the screw.

The vertical component of the normal pressure N acts downward and has a magnitude of $N \cos \gamma$. The vertical component of the force of friction due to the normal pressure N acts upward, and its magnitude is $\mu' N \sin \alpha$. The algebraic sum of these two vertical forces gives the magnitude of the component of Q acting at the point O. Thus

$$Q = \Sigma N (\cos \gamma - \mu' \sin \alpha),$$

from which the total normal component is

$$\Sigma N = \frac{Q}{\cos \gamma - \mu' \sin \alpha} \tag{74}$$

In one revolution of the screw, the applied effort must be capable of doing the useful work Qp and overcoming the work of friction. Denoting the total work put in by the effort in one revolution of the screw by the symbol W_t, we find that

$$W_t = Qp + \frac{\pi\mu' dQ \sec \alpha}{\cos \gamma - \mu' \sin \alpha} \tag{75}$$

Substituting the value of $p = \pi d \tan \alpha$ in (75), we get

$$W_t = \pi dQ \left[\tan \alpha + \frac{\mu' \sec \alpha}{\cos \gamma - \mu' \sin \alpha}\right] \tag{76}$$

Now to determine a relation between the angles α, β and γ, we make use of a theorem in Solid Analytical Geometry, namely,

$$\cos^2 \gamma + \cos^2 \beta + \cos^2 \left[\frac{\pi}{2} - \alpha\right] = 1,$$

from which

$$\cos \gamma = \sqrt{\cos^2 \alpha - \cos^2 \beta} \tag{77}$$

Substituting (77) in (76), the following expression for the total work required per revolution of the screw, in order to raise the load Q, is obtained:

$$W_t = \pi dQ \left[\tan \alpha + \frac{\mu' \sec \alpha}{\sqrt{\cos^2 \alpha - \cos^2 \beta} - \mu' \sin \alpha}\right] \tag{78}$$

By definition, the efficiency is the ratio of the useful work to the total work; hence, for the V-threaded screw

$$\eta = \frac{\tan \alpha}{\tan \alpha + \dfrac{\mu' \sec \alpha}{\sqrt{\cos^2 \alpha - \cos^2 \beta} - \mu' \sin \alpha}} \tag{79}$$

Very often it is desirable to determine the magnitude of the effort P required at the end of a lever or wrench. Representing the length of the lever by L, and equating the work done by P in one revolution to the total work done, we find that

$$P = \frac{Qd \tan \alpha}{2 L\eta} \tag{80}$$

STRESSES IN SCREW FASTENINGS

To arrive at the proper dimensions of bolts, screws and studs used as fastenings, it is important to consider carefully the following stresses:

(*a*) Initial stresses due to screwing up.
(*b*) Stresses due to the external forces.
(*c*) Stresses due to combined loads.

83. Stresses Due to Screwing Up.—The stresses induced in bolts, screws and studs by screwing them up tightly are a tensile stress due to the elongation of the bolt, and a torsional stress due to the frictional resistance on the thread. To determine the magnitude of the resultant stress induced in a fastening subjected to these stresses, combine them according to Art. 17. For screws less than $\frac{3}{4}$ inch in diameter, the stresses induced by screwing up depend so much upon the judgment of the mechanic that it is useless to attempt to calculate their magnitude.

Experiments on screws and bolts have been made with the hope that the results obtained would furnish the designer some idea as to the magnitude of the stresses due to screwing up. As might be expected, the results varied within rather wide limits so that no specific conclusions could be drawn; however, all such tests seemed to show that the stresses are high, generally higher than those due to the external forces and very frequently running up to about one-half of the ultimate strength of the bolt.

84. Stresses Due to the External Forces.—(*a*) *Direct stress.*—Bolts, screws, and studs, as commonly used for fastening machine parts, are subjected to a direct tensile stress by the external forces coming upon them; but occasionally the parts fastened will produce a shearing action upon the fastening.

Assuming that a certain force Q causes a direct tensile stress in a bolt or screw, it is evident that the weakest section, namely that at the root of the thread, must be made of such a diameter that the stress induced will not exceed the allowable tensile stress. Calling the diameter at the root of the thread d_0, we obtain from (3)

$$d_0 = \sqrt{\frac{4Q}{\pi S_t}} \qquad (81)$$

Table 15 gives the values of d_0 for the various sizes of the Sellers standard threads. Since this table also gives the area at the root of the thread, the calculations for the size of a bolt for a given load Q is considerably simplified by finding the ratio of Q to S_t which is really the root area of the required size of bolt; then select from Table 15 the diameter corresponding to the area.

Screws subjected to a shearing stress should be avoided as far as possible. However, such an arrangement can be used successfully by the use of dowel pins fitted accurately into place after the screws have been fitted. There are many places where dowel pins cannot be used, and for such cases it is suggested that the body of the bolt or screw be made an accurate fit in the holes of the parts to be fastened.

Assuming as above that the external force coming upon the bolt is Q, and that the allowable shearing stress is S_s, then it follows that

$$d = \sqrt{\frac{4Q}{\pi S_s}} \tag{82}$$

(*b*) *Tension due to suddenly applied loads.*—The loads producing the stresses discussed in the preceding paragraphs were considered as steady loads; however, bolts and screws are used in many places where the loads coming upon them are in the

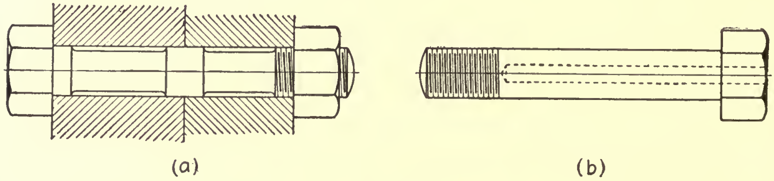


FIG. 32.

nature of shocks, as for example in the piston rod of a steam hammer, and in the bolts of engine connecting rods. Such bolts must then be designed so as to be capable of resisting the shocks due to the suddenly applied loads without taking a permanent set. Now since the energy of the suddenly applied load must be absorbed by the bolt, and as the measure of this energy is the product of the stress induced and the total elongation, it is evident that the stress may be reduced by increasing the elongation. Increasing the elongation may be accomplished in several ways, among which are the following:

1. Turn down the body of the bolt so that its cross-sectional area is equal to the area at the root of the thread; then since the total elongation of the bolt depends upon the length of this reduced section it follows that the length of the latter should be made as great as possible. Such a bolt is weak in resisting torsion and flexure, and instead of fitting the hole throughout its

length, it merely fits at the points where the body was not turned down, as shown in Fig. 32(*a*). Low cost of production is the chief advantage.

The tie rods used in bridge and structural work are generally very long, and the prevailing practice calls for upset threaded ends, which is merely another way of making the cross-section of the body of the rod practically the same as the area at the root of the thread. No doubt in this class of work the object of making the rods as thus described is to save weight and material; however, it should be pointed out that the capacity for resisting shocks has also at the same time been increased.

2. Instead of turning down the body, the cross-sectional area may be reduced by drilling a hole from the head of the bolt toward the threaded end, as shown in Fig. 32(*b*). This method no doubt is the best, as the bolt fits the hole throughout its length, and the hollow section is well-adapted to resist flexural as well as torsional stresses. The cost of production may be excessive for long bolts and for the latter the method of Fig. 32(*a*) may be employed.

Actual tests were made by Prof. R. C. Carpenter at the Sibley College Laboratory on bolts 1¼ inches in diameter and 12 inches long, half of which were solid and the remainder had their bodies reduced in area by drilling a hole as shown in Fig. 32(*b*). Two of these bolts, tested to destruction, showed that the solid or undrilled bolt broke in the thread with a total elongation of 0.25 inch. Additional tests in which similar bolts were subjected to shock gave similar results.

85. Stresses due to Combined Loads.—Having discussed the individual stresses induced in bolts and screws by screwing them up and by the external loads coming upon them, it is in order next to determine the stress induced by the combined action of these loads. This resultant stress depends upon the rigidity of the parts fastened as well as upon the rigidity of the screw itself.

(*a*) *Flanged joint with gasket.*—In general it may be said that for an unyielding or rigid bolt or screw fastening two machine parts that will yield due to screwing up, the stress in the bolt is that due to the sum of the initial tension due to screwing up and the external load, as the following analysis will show. In actual fastenings used for machine parts, neither the bolt nor the parts fastened fulfill the above conditions absolutely; however, the conditions are very nearly approached when some semi-elastic material like sheet packing is used to make a tight joint, as in steam and air piping. In a joint such as illustrated in Fig. 33(*a*) the packing acts like a spring, and tightening the nut will compress the packing a small amount, thus causing a stress in the bolt corresponding to this compression. Assuming that an external load due to some fluid pressure acts upon the flange *a*, its effect will be to elongate the stud thereby increasing the stress, and at the same time reduce slightly the pressure exerted upon the stud by the packing; hence, it follows that for this case, the load upon the stud may for all practical purposes be considered as equivalent to the sum of the two loads.

(*b*) *Flanged joint without gasket.*—The next case to be considered is that type of fastening in which the stud, bolt or screw

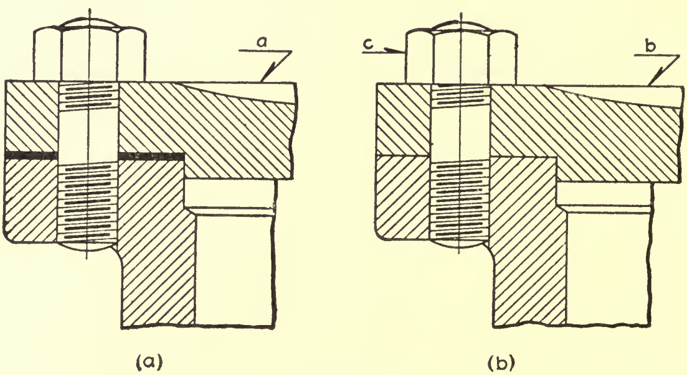


FIG. 33.

yields far more than the connected parts. This case is represented by two flanges having a ground joint, as shown in Fig. 33(*b*). Due to screwing up of this joint, the stud which now elongates, in other words acts like a spring, will be subjected to a stress corresponding to this elongation. If, as in the preceding case, we now introduce a pressure upon the flange *b* which tends to pull the fastening apart, it is evident that the resultant pressure at the ground joint is the difference between the pressures exerted by the nut *c* upon the outside of the flange *b* and that due to the fluid pressure on the inside of *b*. As long as the pressure on the inside of *b* does not exceed that due to the screwing up of the nut, the stud will remain unchanged in length; hence the stress induced is that due to the initial tension and not that due to the external load. If, however, the pressure on the inside is sufficient to overcome that due to the nut, the joint will separate, causing the stud to elongate; hence the stress in the latter is that due to the external load.

86. Fastening with Eccentric Loading.—(*a*) *Rectangular base.* —In Fig. 34(*a*) is shown the column of a drill press bolted to the cast-iron base by cap screws. Due to the thrust P of the drill which tends to overturn the column, these screws are subjected to a tensile stress which is not the same for each screw, as the analysis below will show.

To determine the maximum load that may come upon any screw we shall assume that the column, which is rigid, is fastened

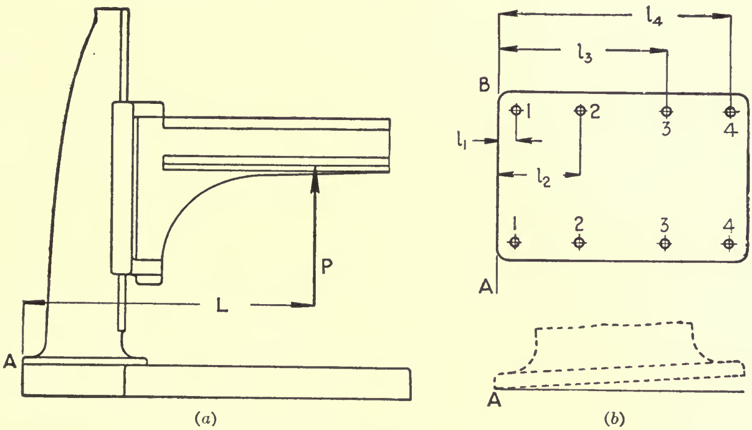


FIG. 34.

to the rigid base by means of eight screws, as shown in Fig. 34(*b*). Due to the thrust P, the column will tip backward about the point A, thus stretching each screw a small amount depending upon its distance from the axis AB, Fig. 34(*b*). Since the stresses induced in the screws vary directly as the elongations, it is evident that the loads upon the screws vary.

Now the moment of the thrust P must be balanced by the sum of the moments of the screw loads about the axis AB; hence, representing the loads upon the screws by Q_1, Q_2, etc., and their moment arms by l_1, l_2, etc., it follows that

$$PL = 2(Q_1l_1 + Q_2l_2 + Q_3l_3 + Q_4l_4) \tag{83}$$

The subscripts used correspond to the number of the screw as shown in Fig. 34(*b*). Since the stresses induced in any screw vary directly as the elongation produced, we obtain the following relations:

$$Q_2 = Q_1 \frac{l_2}{l_1} \qquad Q_3 = Q_1 \frac{l_3}{l_1} \qquad Q_4 = Q_1 \frac{l_4}{l_1} \tag{84}$$

Substituting these values in (83), the expression for the external moment becomes:

$$PL = \frac{2\,Q_1}{l_1}\left[l_1^2 + l_2^2 + l_3^2 + l_4^2\right] \tag{85}$$

From the preceding discussion, it is apparent that the maximum stresses occur in the screws labeled 4, and the magnitude of this maximum stress is given by the following expression:

$$Q_4 = \frac{PLl_4}{2(l_1^2 + l_2^2 + l_3^2 + l_4^2)} \tag{86}$$

Knowing the various dimensions, as well as the thrust P, the magnitude of Q_4 is readily determined, and from this the size of screw for any allowable fiber stress.

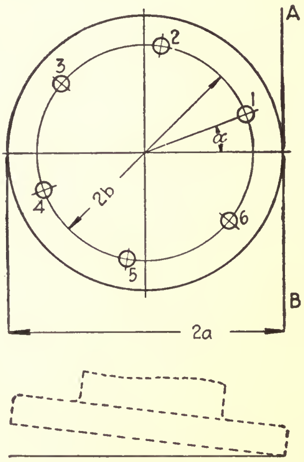


FIG. 35.

(*b*) *Circular base.*—Instead of having a rectangular base as discussed above, columns or machine members are frequently made with a circular base similar to that shown in Fig. 35, in which $2a$ represents the outside diameter of the column flange, and $2b$ the diameter of the bolt circle. For the case under discussion six bolts or screws numbered from 1 to 6 inclusive are used. Adopting a notation similar to that used in the preceding analysis, we have that the external moment due to the load P is

$$PL = \frac{Q_1}{l_1}(l_1^2 + l_2^2 + l_3^2 + l_4^2 + l_5^2 + l_6^2) \tag{87}$$

From the geometry of the figure

$$\left.\begin{aligned} l_1 &= a - b\cos\alpha \\ l_2 &= a - b\cos(60 + \alpha) \\ l_3 &= a + b\cos(60 - \alpha) \\ l_4 &= a + b\cos\alpha \\ l_5 &= a + b\cos(60 + \alpha) \\ l_6 &= a - b\cos(60 - \alpha) \end{aligned}\right\} \tag{88}$$

Substituting these values in (87), it follows that

$$PL = Q_1 \left[\frac{6\,a^2 + 3\,b^2}{a - b \cos \alpha}\right],$$

from which the magnitude of Q_1 is given by the following expression:

$$Q_1 = PL \left[\frac{a - b \cos \alpha}{6\,a^2 + 3\,b^2}\right] \tag{89}$$

Now to determine the maximum value of Q_1 for a given moment PL and dimensions a and b, it is evident from (89) that this occurs when $\cos \alpha$ is a minimum, *i.e.*, $\cos \alpha = -1$, which is the case when the angle α is 180 degrees. Hence

$$\text{max. } Q_1 = \frac{PL}{3}\left[\frac{a + b}{2\,a^2 + b^2}\right] \tag{90}$$

Knowing the maximum load, the size of the bolts or screws must be proportioned for this load.

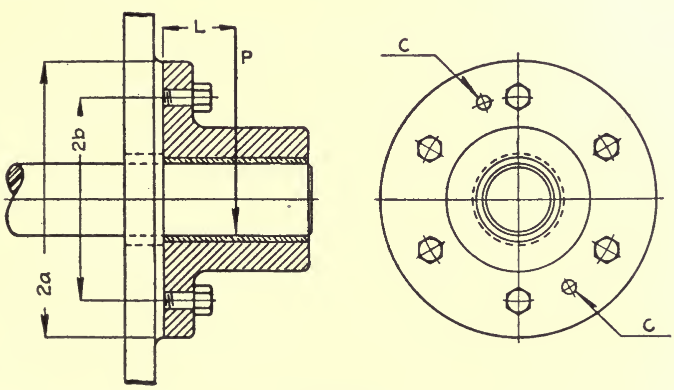


FIG. 36.

By means of an analysis similar to the above, the stresses in any number of bolts or screws may be arrived at.

87. Common Bearing.—In machinery, many forms of fastenings are used in which the bolts or screws are subjected to shearing stresses in addition to tensile stresses. A very simple form of such a fastening is shown in Fig. 36, which represents a solid cast-iron flanged bearing frequently found on heavy machine tools. Due to the power transmitted by the gears located on the shaft, the bearing is subjected to a pressure P which tends to produce a shearing stress in each of the screws. For convenience, all of the screws are assumed to be stressed equally. As mentioned in Art. 83, dowel pins may be used as shown in Fig. 36, and if these are fitted correctly they will, to a great extent if not altogether, relieve the screws from a shearing action.

Due to the eccentric location of P, relative to the supporting frame, the bearing is subjected to an external moment PL, which must be balanced by an equal moment due to the tension set up in the screws. For the bearing shown in Fig. 36 having six screws on a bolt circle of diameter $2b$, the relation between the external moment and the moment of the screw loads may be obtained from (90).

Now assume a diameter of screw, and determine the direct shearing stress, if no dowel pins are used, also the tensile stress caused by the external moment. To arrive at the maximum intensity of stress, combine the two separate stresses by means of (28); the result should not exceed the assumed safe working stress.

POWER SCREWS

Three forms of threads adapted to the transmission of power are shown in Art. 76; of these the square thread is looked upon with the greatest favor due to its higher efficiency. Instead of having single-threaded screws, it is not unusual to employ screws having multiple threads, an example of which is shown in the friction spindle press illustrated in Fig. 125. In connection with multiple-threaded screws, attention is called to the terms *lead* and *divided pitch*. By the former is meant the distance that the nut advances for one revolution of the screw, and by the latter, the distance between consecutive threads; hence a triple-threaded screw of one and one-half inch lead has a divided pitch of one-half inch.

88. Efficiency of Square Threads.—Referring to Fig. 37, let d represent the mean diameter of the screw. The action of the thread upon the nut is very similar to the action of a flat pivot upon its bearing, and hence we shall assume that the pressure between the screw and the nut may be considered as concentrated at the mean circumference of the thread.

(*a*) *Direct motion.*—Representing the average intensity of pressure between the screw and its nut by the symbol q, we get for the total pressure on a small area δA of the surface of the thread $q\delta A$. If the screw is rotated so that the axial load Q is raised, as for example in a screw jack, the pressure $q\delta A$ will act along the line OB making an angle φ' with the normal OA. The symbol φ' represents the angle of friction for the surfaces in contact. Now since the normal OA is inclined to the axis of the screw by the angle α, the angle of rise of the mean helix, it is evident that the components of the pressure $q\delta A$ parallel to the axis and at right angles thereto, are as follows:

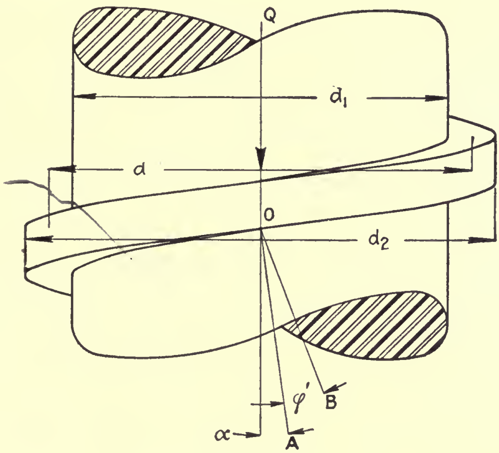


FIG. 37.

$$\text{Parallel component} = q\delta A \cos(\alpha + \varphi')$$
$$\text{Right angle component} = q\delta A \sin(\alpha + \varphi')$$

Hence $$Q = q \cos(\alpha + \varphi') \Sigma \delta A$$
or $$Q = qA \cos(\alpha + \varphi'), \qquad (91)$$

in which A represents the total surface of the thread in actual contact.

The torsional moment of the component at right angles to the axis, about the axis, is

$$\delta T = \frac{qd\,\delta A}{2} \sin(\alpha + \varphi')$$

Summing up for the entire surface in contact,

$$T = \frac{qAd}{2} \sin(\alpha + \varphi') \qquad (92)$$

Since q and A are generally unknown quantities, it is desirable to derive an expression for T in terms of the load Q. This may be done by combining (91) and (92), whence

$$T = \frac{Qd}{2} \tan (\alpha + \varphi') \tag{98}$$

In order to obtain an expression for the efficiency of the square-threaded screw, determine the torsional moment T_0 required when friction is not considered, and divide this moment by T. Without friction $\varphi' = 0$; hence, from (93), it follows that

$$T_0 = \frac{Qd}{2} \tan \alpha \tag{94}$$

Hence, the efficiency is

$$\eta = \frac{T_0}{T} = \frac{\tan \alpha}{\tan (\alpha + \varphi')} \tag{95}$$

The expression given by (95) could have been obtained directly from (79) by making $\beta = 90°$.

Very often it is more desirable to have the expressions for T and η in terms of the coefficient of friction and the dimensions of the screw. Letting p represent the pitch of the screw, then $\tan \alpha = \frac{p}{\pi d}$; also $\tan \varphi' = \mu'$. Substituting these values in (93) and (95), the resulting expression for T is

$$T = \frac{Qd}{2}\left[\frac{p + \pi\mu' d}{\pi d - \mu' p}\right] \tag{96}$$

and that for the efficiency is

$$\eta = \frac{\pi d p - \mu' p^2}{\pi d p + \pi^2 \mu' d^2} \tag{97}$$

The value of the coefficient of friction varies greatly with the method of lubrication and the quality of the lubricant. Very little experimental information on threads is available; probably the most reliable being the results obtained by Prof. A. Kingsbury from an extended series of tests on square-threaded screws made of various materials, such as mild steel, case-hardened mild steel, wrought iron, cast iron, and cast bronze. The results of this investigation, some of which are given in Table 24, were presented in a paper before the American Society of Mechanical Engineers by Prof. Kingsbury, and form a part of volume 17 of the *Transactions* of that society. In the second last column of Table 24 are given the mean values of the coefficient of friction

Table 24.—Coefficients of Friction for Square-threaded Screws

Material used for screw	Pres. per sq. in.	Material used for nut				Aver. coef.	Lubricant used
		Cast iron	Wrought iron	Mild steel	Cast brass		
Cast iron	3,000	0.1400	0.1570	0.1500	0.1200	0.143	Heavy machinery Oil
Wrought iron		0.1500	0.1600	0.1500	0.1170		
Mild steel		0.1320	0.1560	0.1470	0.1270		
Mild steel case-hard.		0.1675	0.1775	0.1550	0.1325		
Cast bronze		0.1300	0.1300	0.1270	0.1400		
Cast iron	10,000	0.1190	0.1390	0.1250	0.1710		
Wrought iron		0.1380	0.1400	0.1390	0.1470		
Mild steel		0.1360	0.1600	0.1410	0.1360		
Mild steel case-hard.		0.1300	0.1430	0.1330	0.1930		
Cast bronze		0.1720	0.1350	0.1240	0.1320		
Cast iron	10,000	0.1050	0.0710	0.1075	0.0590	0.07	Heavy machinery oil and graphite
Wrought iron		0.0750	0.0700	0.0890	0.0550		
Mild steel		0.0650	0.0675	0.1110	0.0400		
Mild steel case-hard.		0.0700	0.0550	0.1275	0.0350		
Cast bronze		0.0440	0.0450	0.0710	0.0360		
Cast iron	10,000	0.0950	0.1000	0.1000	0.1100	0.11	Lard oil
Wrought iron		0.1000	0.1075	0.1125	0.1200		
Mild steel		0.1000	0.1050	0.1200	0.1100		
Mild steel case-hard.		0.1050	0.0975	0.1175	0.1375		
Cast bronze		0.1100	0.1000	0.1150	0.1325		

for the various lubricants as determined by Prof. Kingsbury, and these values are applicable to square-threaded screws running at very slow speeds and upon which the bearing pressure does not exceed 14,000 pounds per square inch, provided the screw is lubricated freely before the pressure is applied.

(*b*) *Reverse motion.*—For the reverse motion of the screw, the line of action of the pressure $q\delta A$ is inclined to the axis of the screw at the angle $(\alpha - \varphi')$; hence the moment required to turn the screw is

$$(T) = \frac{Qd}{2} \tan (\alpha - \varphi')$$

$$= \frac{Qd}{2}\left[\frac{p - \pi\mu' d}{\pi d + \mu' p}\right] \tag{98}$$

For the common screw jack and screws for elevating the cross rail on planers, boring mills, and large milling machines, the angle of friction φ' exceeds the angle α, thus making (T) negative; that is, the lowering of the load Q requires an effort or in other words, the screw is said to be self-locking. If, however, $\varphi' < \alpha$, the moment is positive; that is, an effort must be applied to resist the tendency of the load to descend.

From the above discussion, it is evident that in a self-locking screw, the limiting value of α is φ'. Substituting $\alpha = \varphi'$ in (95), the maximum efficiency of a self-locking screw is

$$\eta = \frac{1 - \tan^2 \varphi}{2}; \tag{99}$$

that is to say η in this case can never exceed 50 per cent.

89. Stresses in Power Screws.—Screws used for the transmission of power are subjected to the following stresses: bearing, tensile or compressive, and shearing.

(*a*) *Bearing stresses.*—In order that the thread of a screw may be capable of transmitting the required power without an undue amount of wear, the unit pressure upon the surfaces in contact must be kept low, especially if the rubbing speeds are high. Instead of giving this permissible pressure in terms of the normal pressure per square inch of actual contact, it is generally quoted as so many pounds per square inch of projected area. To determine an expression for this quantity in terms of the load on the screw, proceed as follows: Using the notation of Art. 88, the projected area of the total thread surface in actual contact between the nut and its screw is $\frac{\pi n}{4}(d_2^2 - d_1^2)$, hence

$$Q = \frac{\pi n}{4}(d_2^2 - d_1^2)S_b, \tag{100}$$

in which n and S_b represent the number of threads in contact and the permissible pressure per square inch of projected area, respectively.

The values of S_b given in Table 25 were determined from actual screws in service, and may serve as a guide in future calculations.

TABLE 25.—BEARING PRESSURES ON POWER SCREWS

Service	Material		Bearing pressures			Remarks
	Screw	Nut	Min.	Max.	Mean	
Jack screw.....	Steel	Cast iron	1,800	2,600	2,200	Slow speed
Hoisting screw.	Steel	Cast Iron	500	1,000	750	Medium speed
Hoisting screw.	Steel	Brass	800	1,400	1,100	Medium speed

(*b*) *Tensile or compressive stresses.*—The method of mounting the screw, and the manner of transmitting the desired power, determine the kind of stress induced in the screw by the action of the direct load. The magnitude of this stress is equivalent to the load divided by the area at the root of the thread, provided the length of the screw if subjected to compression does not exceed six or eight times the root diameter. If a screw subjected to a compression has a length exceeding the limits just given, it must be treated as a column, and the stresses determined according to the formulas given in Art. 15. It is good practice to neglect any stiffening effect that the threads may have.

(c) *Shearing stresses.*—A torsional or shearing stress is induced in the screw by the external turning moment applied, though a part of the latter may also be used in overcoming the friction of bearings, depending upon the arrangement of the screw and nut. In general, the magnitude of the moment causing the shearing is never less than that given by (93) or (96), and hence the shearing stress induced in this case is

$$S_s = \frac{16\,T}{\pi d_1^3} \tag{101}$$

(d) *Combined stresses.*—Having determined the magnitude of the separate stresses induced in the screw, their combined effect must be determined by the principles explained in Art. 17.

CHAPTER V

FASTENINGS

KEYS, COTTERS, AND PINS

KEYS

The principal function of keys and pins is to prevent relative rotary motion between two parts of a machine, as of a pulley about a shaft on which it fits. In general, keys are made either straight or slightly tapering. The straight keys are to be preferred since they will not disturb the alignment of the parts to be keyed, but have the disadvantage that they require accurate fitting between the hub and shaft. The taper keys by taking up the slight play between the hub and shaft are likely to throw

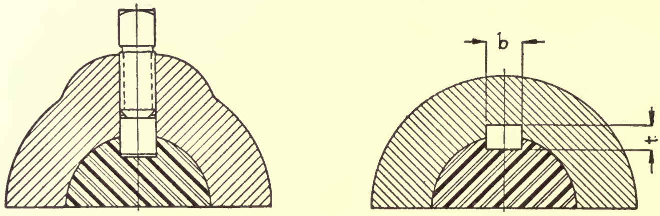


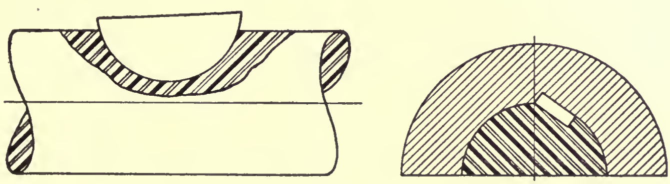
FIG. 38.

the wheels or gears out of alignment, but they have the advantage that any axial motion between the parts is prevented due to the wedging action. Keys may be divided into three classes as follows: (*a*) sunk keys; (*b*) keys on flats; (*c*) friction keys.

90. Sunk Keys.—The types of sunk keys used most in machine construction are those having a rectangular cross-section, though occasionally round or pin keys are used.

(*a*) *Square key.*—The so-called square key is only approximately square in cross-section and has its opposite sides parallel. As shown in Fig. 38(*a*), this type of key bears only on the sides of the key seats, and, being provided with a slight clearance at the top and bottom, the key has no tendency to exert a bursting pressure upon the hub. To prevent axial movement of the hub, set screws bearing upon the key, or other means must be provided. The square key is used where accurate concentricity of the keyed parts is required, also when the parts must be disconnected frequently, as in machine tools. It is suitable for heavy loads, provided set screws are used to prevent tipping of the key in its seat. For a list of commercial sizes of square keys see Table 29 and Fig. 45(*a*), to which the dimensions in the table refer.

(*b*) *Flat key.*—The flat key has parallel sides, but its top and bottom taper. As shown in Fig. 38(*b*), its thickness *t* is considerably less than its width *b*; furthermore, it fits on all sides, thus tending to spring the connected parts and at the same time introducing a bursting pressure upon the hub. The flat key is used for either heavy or light service in which the objections just mentioned are not serious.



(a) (b)

Fig. 39.

(*c*) *Feather key.*—The feather key, sometimes called *spline*, is a key fitted only on the sides, thus permitting free axial movement of the hub along the shaft. Its thickness is usually greater than its width, thereby increasing the contact surface and at the same time decreasing the wear. The feather key is fastened to either the hub or the shaft, while the key-way in the other part is made a nice sliding fit. The key may be secured to the shaft by countersunk machine screws or by pins riveted over; or when it is desired to fasten the key to the sliding hub, dovetailing or riveting may be resorted to. Quite frequently two feather keys set 180 degrees apart are used. The stresses are thereby equalized, and at the same time it is easier to slide the hub along the shaft.

(*d*) *Woodruff key.*—The Woodruff key shown in Fig. 39(*a*) is a modified form of the sunk key. It is patented and is manufactured by the Whitney Mfg. Co. of Hartford, Conn. The key-

TABLE 26.—DIMENSIONS OF WOODRUFF KEYS

No.	1	2	3	4	Key length	No.	1	2	3	4	Key length
1 2 3	1/2	1/16 3/32 1/8	3/64		1/2	23 F	1 3/8	5/16 3/8	3/32		1 3/8
4 5 6 61	5/8	3/32 1/8 5/32 3/16	1/16		5/8	24 25 G	1 1/2	1/4 5/16 3/8	7/64		1 1/2
7 8 9 91	3/4	1/8 5/32 3/16 1/4	1/16		3/4	126 127 128 129	2 1/8	3/16 1/4 5/16 3/8	21/32	5/32	1 3/8
10 11 12 A	7/8	5/32 3/16 7/32 1/4	1/16		7/8	26 27 28 29	2 1/8	3/16 1/4 5/16 3/8	17/32	3/32	1 23/32
13 14 15 B 152	1	3/16 7/32 1/4 5/16 3/8	1/16		1	Rx Sx Tx Ux Vx	2 3/4	1/4 5/16 3/8 7/16 1/2	25/32	0.1625	2
16 17 18 C	1 1/8	3/16 7/32 1/4 5/16	5/64		1 1/8	R S T U V	2 3/4	1/4 5/16 3/8 7/16 1/2	5/8	1/8	2 5/16
19 20 21 D E	1 1/4	3/16 7/32 1/4 5/16 3/8	5/64		1 1/4	30 31 32 33 34 35 36	3 1/2	3/8 7/16 1/2 9/16 5/8 11/16 3/4	13/16	3/16	2 7/8
22	1 3/8	1/4	3/32		1 3/8						

seat in the hub is of the usual form, but that in the shaft has a circular outline and is considerably deeper than the ordinary key-way. The extra depth, of course, weakens the shaft, but the deep base of the key precludes all possibility of tipping.

The freedom of the key to adjust itself to the key-seat in the hub makes an imperfect fit almost impossible, while with the ordinary taper key a perfect fit is very difficult to obtain. In securing long hubs, the depths of the key-way may be diminished by using two or more Woodruff keys at intervals in the same key-seat.

In Table 26 are given the stock sizes of Woodruff keys, also the various dimensions referred to in Fig. 40.

To aid the designer in selecting the suitable size of Woodruff key for any given diameter of shaft, the information contained in Table 27 may be found convenient.

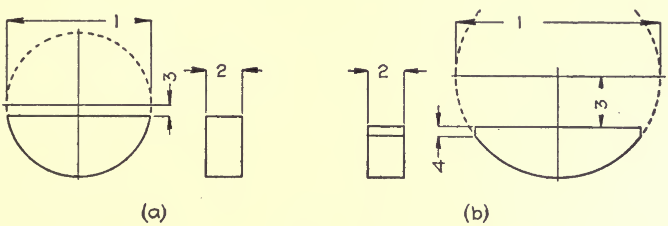


Fig. 40.

Table 27.—Diameters of Shafts and Suitable Woodruff Keys

Shaft diam.	Key No.	Shaft diam.	Key No.	Shaft diam.	Key No.
5/16–3/8	1	7/8–15/16	6, 8, 10	1 3/8–1 7/16	14, 17, 20
7/16–1/2	2, 4	1	9, 11, 13	1 1/2–1 5/8	15, 18, 21, 24
9/16–5/8	3, 5	1 1/16–1 1/8	9, 11, 13, 16	1 11/16–1 3/4	18, 21, 24
11/16–3/4	3, 5, 7	1 3/16	11, 13, 16	1 13/16–2	23, 25
13/16	6, 8	1 1/4–1 5/16	12, 14, 17, 20	2 1/16–2 1/2	25

(*e*) *Lewis key.*—The type of sunk key shown in Fig. 39(*b*) was invented by Mr. Wilfred Lewis. This key is subjected practically to a pure compression in the direction of its longest cross-sectional dimension, and for that reason the location of this key relative to the direction of driving is very important. The Lewis key is rather expensive to fit and probably due to that fact is not used so extensively, though at the present time one manufacturer uses it on large engine shafts. Frequently two such keys are used on one hub.

(*f*) *Barth key.*—Some years ago Mr. C. G. Barth invented the type of key shown in Fig. 41(*a*). It consists of an ordinary rectangular key with one-half of both sides beveled off at 45 degrees. With this form of key it is not necessary to make a tight fit, since the pressure tends to force the key into its seat.

Furthermore, there is no tendency for the key to turn in its seat, since the pressure upon it produces a compression. With respect to the stresses produced, this key is similar to the Lewis key, but has the advantage over the latter that it costs less to fit. The Barth key may also be used as a feather key; in many cases it has replaced troublesome rectangular feather keys and has always given excellent service.

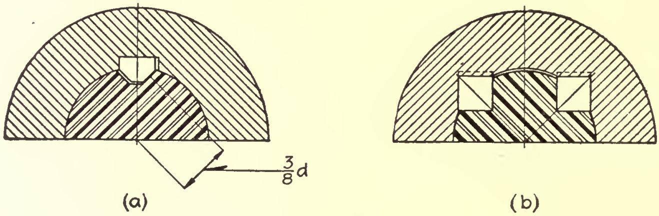


FIG. 41.

(*g*) *Kennedy keys.*—Another system of keying, which has given excellent service in heavy rolling-mill work, is shown in Fig. 41(*b*). This system, known as the Kennedy keys, is similar to that in which two Lewis keys are used in one hub. The two keys are located in the hub in such a manner that the diagonals pass through the center of the shaft as shown in the figure. The dimensions of the key at the smaller end are made approximately one-fourth of the diameter of the shaft, and the taper is made

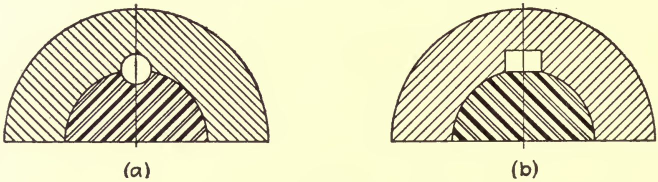


FIG. 42.

⅛ inch per foot. The key should form a driving fit at the top and bottom. The following method of fitting a hub with Kennedy keys represents the practice of a well-known manufacturer, and when thus fitted, such keys have always given good results. "The hub of the gear after being bored for a press fit with its shaft is rebored by offsetting the center approximately $\frac{1}{64}$ inch, thus producing the clearance shown in the figure. The keys are fitted on the eccentric side of the bore and hence when driven home pull the hub into its proper place." The reboring operation is not essential to insure good results, but it facilitates erection of the parts.

(*h*) *Round or pin key.*—A round or pin key gives a cheap and accurate means of securing a hub to the end of a shaft. This form of fastening, shown in Fig. 42(*a*), was originally intended only for light and small work, but if properly designed and constructed will also prove satisfactory for heavy work. The pin, either cylindrical or tapering, is fitted halfway into the shaft and hub as shown in the figure. For heavy duty, the Nordberg Mfg. Co. of Milwaukee uses the proportions given in Table 28, the total taper of the reamer being 1/16 inch per foot.

For light duty when taper pins are used, it is advisable to make use of the so-called "standard taper pins," as they may be purchased for less money than it is possible to make them. In Table 28 are given the proportions of such pins, also information pertaining to the reamers for these pins. The standard taper is 1/4 inch per foot.

TABLE 28.—ROUND KEYS AND TAPER PINS

Nordberg round keys			Standard taper pins and reamers						
Shaft diameter	Reamer		Pins				Reamer		
	Small diam.	Length of flutes	No.	Large diameter: Actual	Large diameter: Approx.	Stock lengths	No.	Small diam.	Length of flutes
2 15/16–3	3/4	4 1/4	0	0.156	5/32	3/4–1 3/4	0	0.135	1 5/16
3 7/16–3 1/2	7/8	4 1/2	1	0.172	11/64	3/4–2	1	0.146	1 9/16
2 7/8–4	1	4 7/8	2	0.193	3/16	3/4–2 1/4	2	0.162	1 13/16
4 3/8–4 1/2	1 1/8	5	3	0.219	7/32		3	0.183	2 1/16
5	1 1/4	4 5/8	4	0.250	1/4	3/4–3	4	0.208	2 3/8
5 1/2	1 3/8	4 7/8	5	0.289	19/64		5	0.240	2 7/8
6	1 1/2	6 1/8	6	0.341	11/32	3/4–4	6	0.279	3 5/8
7, 8, 9	1 5/8	6 7/8, 8	7	0.409	13/32	1 –4	7	0.331	4 7/16
10, 11, 12	2	10 1/4	8	0.492	1/2	1 1/4–4 1/2	8	0.398	5 1/4
13, 14, 15	2 9/16	12	9	0.591	19/32	1 1/2–5 1/4	9	0.482	6 1/8
16, 17, 18	3 1/8	13	10	0.706	23/32	1 1/2–6	10	0.581	7
19, 20, 21	3 11/16								
22, 23, 24	4 1/4	14 1/4							

91. Keys on Flats.—A key on the flat of a shaft has parallel sides with its top and bottom slightly tapering, and is used for transmitting light powers. Fig. 42(*b*) shows this form of fastening. The proportions of keys on flats are about the same as those used for the flat key described in Art. 90(*b*).

92. Friction Keys.—The most common form of friction key is the saddle key shown in Fig. 43(*a*), the sides of which are parallel, and the top and bottom, slightly tapering. The bottom fits the shaft and the holding power of the key is due to friction alone. This form of key is intended for very light duty, or in some cases for temporary service, as in setting an eccentric.

93. The Strength of Keys.—Keys are generally proportioned by empirical formulas, and in almost all cases such formulas are based upon the diameter of the shaft. Neither the twisting moment on the shaft nor the length of the key is considered in arriving at the cross-section. Since a key is used for torsion alone, the twisting moment to be transmitted and not the diameter of the shaft should fix its dimensions. In the majority of cases the shaft must also resist a bending stress in addition to the torsional stress, and a larger shaft is required than would

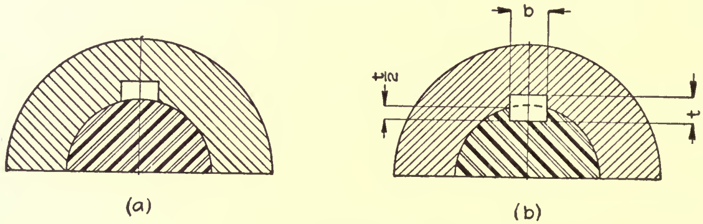


FIG. 43.

be necessary for simple torsion. The empirical formula therefore give a larger key than is really needed, thereby increasing the cost and at the same time decreasing the effective strength of the shaft. The length of the key should be considered in determining its crushing and shearing resistance.

In arriving at the dimensions of the key, the size of the shaft should not be disregarded altogether, or the result might be a key too small to be fitted properly, or one that is too large. In other words, calculate the dimensions of the required key and if necessary modify these dimensions to suit practical considerations. It is generally supposed that keys fail by cross-shearing, but this is seldom the case. A large number of failures are due to the crushing of the side of the key or key-seat, and for that reason the crushing stress should always be investigated.

(*a*) *Crushing strength.*—To determine the crushing stress on the side of a key-seat, let T represent the torsional moment transmitted, l the length of the key, and b and t, the dimensions indicated in Fig. 43(b). Then the crushing resistance of the key is $\frac{tlS_b}{2}$, and its moment about the center of the shaft, whose diameter is d, is approximately $\frac{tldS_b}{4}$. Equating this moment to the torsional moment, and solving for S_b, we have

$$S_b = \frac{4T}{tld} \tag{102}$$

Assuming S_b and having given values for T, t and d, (102) may be used for calculating the required length of the key.

Occasionally a key is required to transmit the full power of the shaft; hence, making its strength equal to that of the shaft, we get

$$\frac{tldS_b}{4} = \frac{\pi d^3 S_s}{16},$$

from which

$$t = \frac{\pi d^2 S_s}{4\,lS_b} \tag{103}$$

(b) *Shearing strength.*—The shearing stress in a key is found by equating the torsional moment T to the product of the radius of the shaft and the stress over the area exposed to a shear; whence

$$S_s = \frac{2T}{bld} \tag{104}$$

Equating the value of T from (102) to that obtained from (104)

$$t = 2b\frac{S_s}{S_b} \tag{105}$$

If $S_b = 2S_s$, as is generally assumed, (105) calls for a square key. To facilitate fitting, the width of the key is frequently made greater than its depth, which has the effect of decreasing S_s relative to S_b. From this it follows that investigations for the crushing stress are more essential than those for the shearing stress, as in actual practice the latter takes care of itself.

94. Friction of Feather Keys.—As stated in Art. 90(c), it is possible to equalize the pressure coming upon the hub by using two feather keys placed 180 degrees apart, thereby reducing materially the force required to slide the hub along the shaft. The following analysis will serve to show that the statement is practically true.

(*a*) *Hub with one feather key.*—In Fig. 44(*a*) is shown a hub which is made an easy sliding fit on the shaft and key, the latter being fastened securely to the shaft. We shall assume that the hub drives the shaft in the direction indicated by the arrow; hence the torsional moment T transmitted produces the two forces P_1, one of which acts on the key and the other, having the same magnitude, causes a pressure on the shaft. These forces being parallel form a couple whose moment P_1a must equal the torsional moment T; hence, the magnitude of the force P_1 is

$$P_1 = \frac{T}{a} \tag{106}$$

(*b*) *Hub with two feather keys.*—In place of a single feather, suppose the shaft is equipped with two keys upon which the hub slides as shown in Fig. 44(*b*). Assuming the direction of rotation

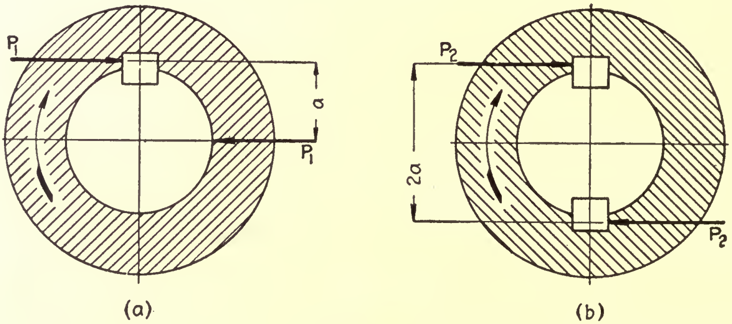


FIG. 44.

shown in the figure, the forces upon the hub are the two equal forces P_2 forming a couple whose moment is $2\,P_2a$. Since the magnitude of this couple is a measure of the torsional moment T, it follows that

$$P_2 = \frac{T}{2\,a} \tag{107}$$

Comparing (106) and (107), it is quite evident that the force producing the frictional resistance in case (*b*) is only one-half as great as that in case (*a*), assuming the same values of T and a, thus showing the advantages gained by the use of two feather keys.

It is important to note that the hub with two feather keys requires very accurate fitting in order to produce the action assumed in the above analysis.

95. Gib-head Key.—The gib-head or hook-head key is shown in Fig. 45, and is nothing more than a flat or square key with the head added. This form of key is used in places where it is inconvenient or practically impossible to drive out a key from the small end. It should be borne in mind, however, that a projecting head is always a source of danger and for that reason many engineers condemn its use. In Table 29 are given the dimensions of a series of sizes of gib-head keys indicated in Fig.

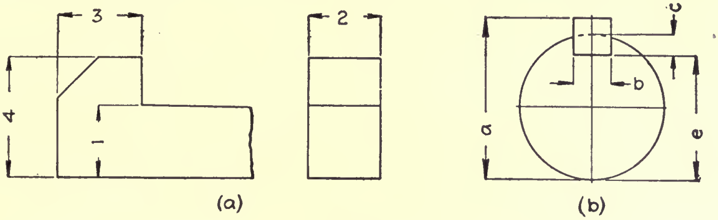


Fig. 45.

Table 29.—Dimensions of Gib-head Keys

1	2	3	4	1	2	3	4
1/8	1/8	7/32	1/4	1 5/8	1 5/8	1 7/8	2 3/4
3/16	3/16	9/32	5/16	1 11/16	1 11/16	1 15/16	2 7/8
1/4	1/4	11/32	15/32	1 3/4	1 3/4	2	3
5/16	5/16	13/32	9/16	1 13/16	1 13/16	2 1/16	3 1/8
3/8	3/8	15/32	11/16	1 7/8	1 7/8	2 1/8	3 3/8
7/16	7/16	17/32	3/4	1 15/16	1 15/16	2 3/16	3 5/8
1/2	1/2	19/32	7/8	2	2	2 1/4	3 3/4
9/16	9/16	21/32	1	2 1/16	2 1/16	2 7/16	3 7/8
5/8	5/8	23/32	1 1/8	2 1/8	2 1/8	2 1/2	4
11/16	11/16	25/32	1 3/16	2 3/16	2 3/16	2 9/16	4 1/8
3/4	3/4	7/8	1 1/4	2 1/4	2 1/4	2 5/8	4 1/4
13/16	13/16	15/16	1 5/16	2 5/16	2 5/16	2 11/16	4 3/8
7/8	7/8	1	1 1/2	2 3/8	2 3/8	2 3/4	4 1/2
15/16	15/16	1 1/16	1 5/8	2 7/16	2 7/16	2 13/16	4 5/8
1	1	1 1/8	1 3/4	2 1/2	2 1/2	2 7/8	4 3/4
1 1/16	1 1/16	1 3/16	1 13/16	2 9/16	2 9/16	2 15/16	4 7/8
1 1/8	1 1/8	1 5/16	1 7/8	2 5/8	2 5/8	3	5
1 3/16	1 3/16	1 3/8	1 15/16	2 11/16	2 11/16	3 1/16	5
1 1/4	1 1/4	1 7/16	2	2 3/4	2 3/4	3 1/8	5 1/8
1 5/16	1 5/16	1 1/2	2 1/8	2 13/16	2 13/16	3 3/16	5 1/8
1 3/8	1 3/8	1 9/16	2 1/4	2 7/8	2 7/8	3 1/4	5 1/4
1 7/16	1 7/16	1 5/8	2 3/8	2 15/16	2 15/16	3 5/16	5 1/4
1 1/2	1 1/2	1 3/4	2 1/2	3	3	3 1/2	5 3/8
1 9/16	1 9/16	1 13/16	2 5/8				

45(*a*). The keys listed in this table are square in cross-section at the head end, and have a taper of ⅛ inch per foot.

96. Key Dimensions.—In Fig. 45(*b*) are shown the dimensions that will prove most convenient for the shop man in order to machine the key-seats in the hub and shaft. The dimension *a* is the one used for arriving at the proper depth of the key-seat in the hub. To arrive at the depth of the key-seat in the shaft, the majority of the workmen prefer to have given the dimension *c*, as that is by far the most convenient dimension when the key-seat is cut on a milling machine. Some mechanics prefer to use the dimension *e* in place of *c* thus enabling them to use calipers.

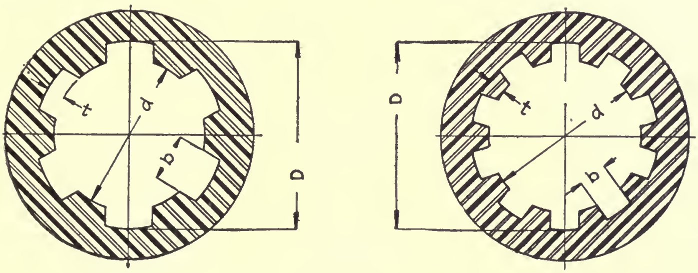
97. Integral Shaft Splines.—With the development of the automobile, the defects of the inserted keys in circular shafts became apparent, and finally the old key construction was discarded almost altogether, in particular in the sliding-gear construction and rear-axle transmissions. Due to the weakening of the shaft by the inserted key, the square shaft was at first introduced, and this met with considerable success. The square shaft, however, is considerably heavier than a circular shaft of the same strength, so in order to keep the weight down and at the same time provide greater key-bearing area, the automobile designer developed what is now called the *integral spline shaft.* Such a shaft is simply a round shaft in which the splines are produced by milling out the metal between them.

At first the integral spline shafts were produced on the milling machine, but at present they can be produced more cheaply on the hobbing machine. The splined holes through the hubs of the gears which slide over such shafts are produced very accurately and cheaply on a broaching machine. It is claimed by some manufacturers that the cost of hobbing a multiple-spline shaft and broaching the hub to fit the shaft is considerably less than the combined cost of turning the circular shaft, cutting the key-way in it, boring the gear to fit the shaft, cutting the key-way in the gear, and fitting the key.

The automobile manufacturer is not the only one that is using integral spline shafts; the advantages of such shafts are so apparent that a considerable number of machine tool builders are now using them in connection with their sliding change-gear mechanisms. As now used in the various classes of service, the integral spline shafts are constructed with from four to ten splines. In

Fig. 46(a) and (b) are shown the cross-sections of a hub containing six and ten splines respectively, the former being used for the sliding gears, while the latter is applied to the rear axle. The proportions of the two types shown in Fig. 46 have been standardized by the Society of Automobile Engineers. Each of these types is made in three different sizes, A, B and C, and the following formulas give the dimensions of the various parts of the bore, while the corresponding parts of the shaft are made one-thousandth of an inch less on the smaller shaft diameters and two one-thousandths on the larger sizes.

For the six-spline type shown in Fig. 46(a), the formula for the width b of the spline is the same for all three sizes; the other dimensions, however, vary.



(a) (b)

FIG. 46.

$$\left.\begin{array}{ll} \text{For 6-}A, & d = 0.90\ D \\ & b = 0.25\ D \\ & t = 0.05\ D \\ \text{For 6-}B, & d = 0.85\ D \\ & t = 0.075\ D \\ \text{For 6-}C, & d = 0.80\ D \\ & t = 0.10\ D \end{array}\right\} \qquad (108)$$

As in the case of the six splines, the width b for the three sizes of the ten-spline fitting shown in Fig. 46(b) is kept constant. The various proportions are given by the following formulas:

$$\left.\begin{array}{ll} \text{For 10-}A, & d = 0.91\ D \\ & b = 0.156\ D \\ & t = 0.045\ D \\ \text{For 10-}B, & d = 0.86\ D \\ & t = 0.07\ D \\ \text{For 10-}C, & d = 0.81\ D \\ & t = 0.095\ D \end{array}\right\} \qquad (109)$$

COTTER JOINTS

A cotter is a cross-key used for joining rods and hubs that are subjected to a tension or compression in the direction of their axis, as in a piston rod and its cross-head; valve rod and its stem; a strap end and its connecting rod.

98. Analysis of a Cotter Joint.—In Fig. 47 is shown one method of joining two rods through the medium of a cotter, the rod being loaded axially as shown. The joint may fail in any one of the ten ways discussed below.

(*a*) *Rods may fail in tension.*—The relation between the external force P and the internal resistance of the rod is given by the following formula:

$$P = \frac{\pi d^2 S_t}{4} \tag{110}$$

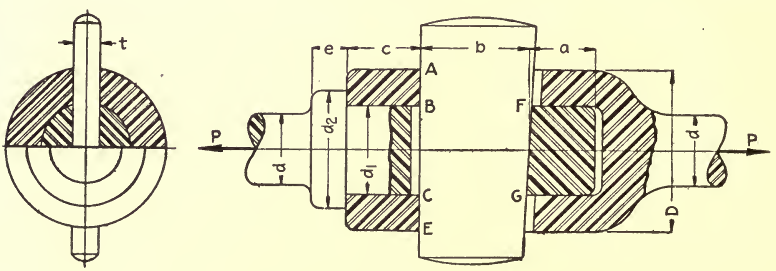


FIG. 47.

(*b*) *Failure of the rod across the slot.*—Equating the external force to the tension in the rod across the slot, we get

$$P = \left[\frac{\pi d_1^2}{4} - td_1\right] S_t \tag{111}$$

(*c*) *Failure of the socket across the slot.*—Equating the external force to the internal resistance due to the tension in the socket across the slot, we find that

$$P = \left[\frac{\pi}{4}(D^2 - d_1^2) - (D - d_1)t\right] S_t \tag{112}$$

(*d*) *Cotter may shear.*—Due to the force P, the cotter may fail by double shearing; hence the relation between the load and stress is as follows:

$$P = 2\,btS_s \tag{113}$$

(*e*) *Rod end may shear.*—To prevent the rod end from failing due to double shearing through the length a, the following expression may be used to determine the minimum value of a:

$$P = 2ad_1S_s \tag{114}$$

(*f*) *Socket end may shear.*—The dimension c must be made long enough so that the end of the socket will not fail by double shearing. Equating the internal resistance to the force P, we get

$$P = 2c(D - d_1)S_s \tag{115}$$

(*g*) *Socket or cotter may crush.*—The external force may crush either the cotter or the socket along the surfaces AB and CE; hence, liberal surfaces must be provided. The following expression gives the relation between the load and stresses:

$$P = t(D - d_1)S_b \tag{116}$$

(*h*) *Rod or cotter may crush.*—To prevent the rod or cotter from crushing along the surface FG, the relation expressed by the following formula must be fulfilled:

$$P = td_1S_b \tag{117}$$

The cotter joint illustrated by Fig. 47 may also be used for a class of service in which the force P may be reversed in direction, thus producing a compression in the rod in place of a tension. Such a loading will then call for an investigation of the collar.

(*i*) *Collar may shear off.*—Due to the compression in the rods, the collar may shear off; whence

$$P = \pi d_1eS_s \tag{118}$$

(*j*) *Collar may crush.*—To prevent crushing of the collar, the surface in contact must be made large enough so that the following relation between the load and stress is satisfied:

$$P = \frac{\pi}{4}(d_2^2 - d_1^2)S_b \tag{119}$$

The taper on the cotter should not be made excessive, or trouble may be experienced due to the loosening of the cotter when the joint is under load. To prevent such loosening, the cotter may be provided with a set screw. A practical taper is ½ inch per foot, but this may be increased to 1½ inches per foot, provided some locking device is applied to the cotter. The cotter instead of being made square-ended as shown in Fig. 47, is more often made with semi-circular edges. This method of making the cotter possesses the following advantages:

1. Sharp corners that are liable to start cracks are avoided.
2. The shearing area at the sides of the slots is increased considerably.
3. The slots with semi-circular ends cost less to make.

PIN JOINTS

In Art. 90(h), the use of round and taper pins in the form of keys was discussed, and in the following articles additional uses of pins will be taken up. These uses are as follows:

(a) For rigid fastenings in which the pins are so placed, that they are either in single or double shear due to the external force.

(b) For joining two rods which require a certain amount of motion at the joint.

99. Taper Pins.—Taper pins properly fitted form a cheap and convenient means of fastening light gears, hand wheels and levers

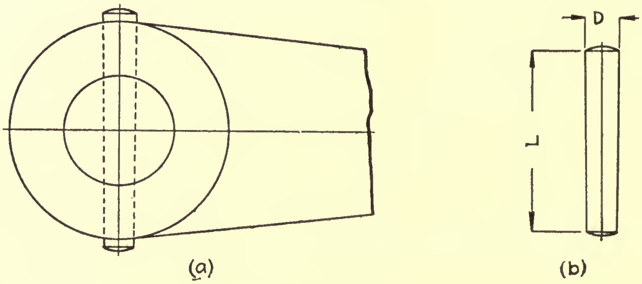


FIG. 48.

to shafts that transmit a small amount of power. They may also be used for making a connection between two rods, similar to the cotter joint described in Art. 98. The common method of applying taper pins is illustrated in Fig. 48(a); but this method is applicable to the transmission of a torque in only one direction. If the machine parts are subjected to alternating stresses, as would be the case in a coupling between the valve rod and the valve stem, the taper pins should be given a slight clearance similar to that provided for the cotter in Fig. 47.

Another very important application of taper pins is their use as dowel pins on bearing flanges, and all forms of brackets and attachments on machine frames. The main function of dowel pins is to form a convenient means of locating accurately a bearing or bracket, since cap screws and studs cannot be relied on for that purpose. If the taper pins are fitted correctly and located properly, no trouble is experienced in reassembling the machine parts after being dismantled for repair or other purposes. In Fig. 36 is shown the application of two taper dowel pins *c* on the flange of a solid bearing. It should be observed that these pins are not diametrically opposite, though in this case they could have been located symmetrically, since the location of the oil hole in the bearing would insure the correct assembling. However, many symmetrical castings or brackets are used, and the location of the dowel pins as illustrated in Fig. 36 may obviate a lot of unnecessary work. Another function of dowel pins, which in many cases is of great importance, is to make these pins take the shearing action due to the external load, thus relieving the cap screws or studs from such action.

As mentioned in Art. 90(*h*), standard taper pins cost but little, and the various sizes and lengths listed in Table 28 are carried regularly in stock by the various manufacturers. The taper adopted by the manufacturers is one-fourth inch per foot. These standard taper pins have no provision on the head or point that will allow for upsetting the ends, if desired. Provisions for upsetting can be made by having the heads and points tapered, which would also facilitate the driving of the pin into the machine part as well as its removal. For removing large dowel pins such as are used in locating the housings on planers and heavy milling machines, the taper pin is provided at the large end with a threaded shank which is fitted with a nut; hence to remove the dowel pin merely back out the pin by screwing up the nut.

Occasionally a threaded shank is provided at the small end of the pin, which if fitted with a nut forms an effective means of retaining a pin having a steep taper. When the taper pin is used as a fastening similar to that shown in Fig. 48(*a*), the large diameter D of the pin is made from one-fourth to one-third of the diameter of the rod or shaft through which the pin passes. The length L Fig. 48(*b*) is chosen so that the pin projects a small amount on each side of the hub, though not enough to make it dangerous. Table 28 also contains information pertaining to the standard reamers that are used with the standard taper pins.

100. Rod and Yoke Ends.—Various forms of pin joints are used for connecting together two or more rods and at the same

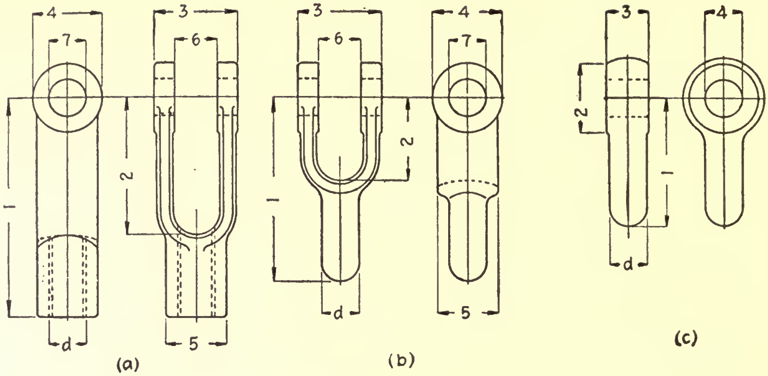


FIG. 49

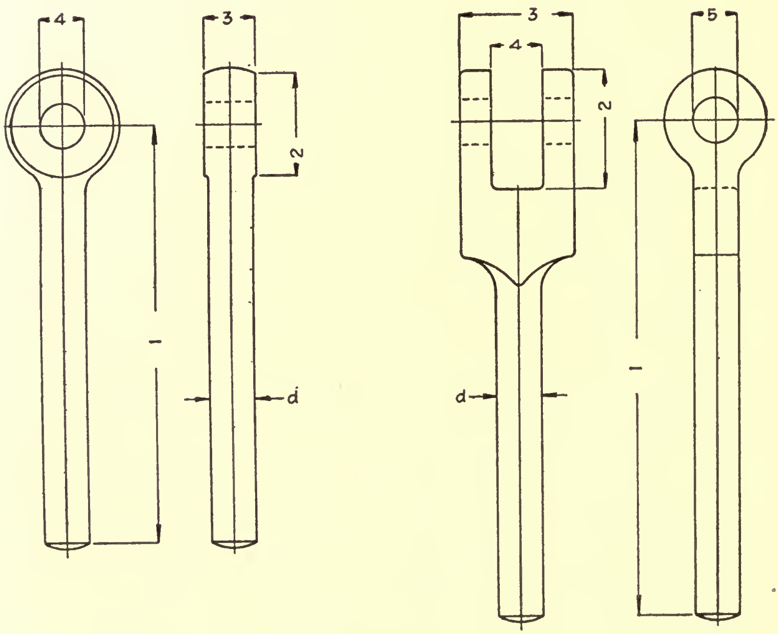


FIG. 50.

time permitting a certain amount of motion at the joint. Such joints are called *rod and yoke ends* or *knuckle joints* and are used in practically all classes of machinery. In Fig. 49 are shown the standard drop-forged rod and yoke ends adopted by the Society of Automobile Engineers, and the proportions thereof are included in Table 30. It should be noticed that the yoke ends are made in two types, namely, the adjustable and the plain, illustrated by Fig. 49(*a*) and (*b*), respectively.

The sizes of yoke and rod ends used in the automobile industry do not cover a wide range, and in order to meet the demand for yoke and rod ends adapted to general use, several manufacturers of drop forgings carry such parts in stock. In Fig. 50 are shown finished plain rod and yoke ends that are a standard product of The Billings and Spencer Co. of Hartford, Conn. The dimensions indicated in Fig. 50 are included in Table 31. The plain shanks of these forgings are made of sufficient length to permit welding them on to rods of any desired length.

The type of rod end just discussed has no provision whatever for taking up wear at the joint, and in the class of service for which they are intended, it is not customary to make such provision. There are, however, many places where the wear on the pin or its bearing must be taken up and

TABLE 30.—S.A.E. DROP FORGED ROD AND YOKE ENDS

d	Adjustable yoke end								Plain yoke end							Plain rod end			
	1	2	3	4	5	6	7	Threads per inch	1	2	3	4	5	6	7	1	2	3	4
3/16	1 9/16	1	7/16	3/8	5/16	3/16	3/16	32	1 1/4	7/16	7/16	3/8	5/16	3/16	3/16	1 1/4	3/8	3/16	3/16
1/4	2	1 1/4	5/8	1/2	7/16	9/32	1/4	28	1 3/4	5/8	5/8	1/2	7/16	9/32	1/4	1 1/4	1/2	9/32	1/4
5/16	2 1/4	1 7/16	3/4	19/32	1/2	11/32	5/16	24	2	3/4	3/4	19/32	1/2	11/32	5/16	1 3/8	19/32	11/32	5/16
3/8	2 1/2	1 5/8	7/8	11/16	5/8	7/16	3/8	24	2 1/8	27/32	7/8	11/16	5/8	7/16	3/8	1 1/2	11/16	7/16	3/8
7/16	2 7/8	1 7/8	1	13/16	23/32	1/2	7/16	20	2 1/4	1	1	13/16	23/32	1/2	7/16	1 1/2	13/16	1/2	7/16
1/2	3	1 7/8	1 1/8	15/16	13/16	9/16	1/2	20	2 1/2	1 1/8	1 1/8	15/16	13/16	9/16	1/2	1 3/4	15/16	9/16	1/2

hence the design of such rod ends requires some knowledge of bearing and journal design. Such machine parts are discussed in detail in Chapter XIX.

TABLE 31.—B. & S. DROP-FORGED ROD AND YOKE ENDS

No.	*d*	Rod end				Yoke end				
		1	2	3	4	1	2	3	4	5
0	1/4	3 15/16	1 1/16	5/16	5/16	4 1/2	2 5/32	3/4	5/16	5/16
1	5/16	4 1/16	1 3/16	3/8	5/16	4 3/4	1 5/16	7/8	3/8	5/16
2	3/8	4 1/4	7/8	7/16	3/8	5	1 1/16	1	7/16	3/8
3	7/16	4 7/16	1	1/2	7/16	5 1/4	1 5/32	1 1/8	1/2	7/16
4	1/2	4 5/8	1 1/8	9/16	1/2	5 1/2	1 5/16	1 1/4	9/16	1/2
5	9/16	4 13/16	1 1/4	5/8	9/16	5 3/4	1 7/16	1 3/8	5/8	9/16
6	5/8	5 3/16	1 1/2	3/4	5/8	6 1/4	1 5/8	1 5/8	3/4	5/8
7	3/4	5 9/16	1 3/4	7/8	3/4	7	1 15/16	1 7/8	7/8	3/4
8	7/8	6 3/32	2	1	7/8	7 3/4	2 7/32	2 3/16	1	7/8
9	1	6 1/2	2 1/4	1 1/8	1	8 1/2	2 17/32	2 1/2	1 1/8	1
10	1 1/8	7 1/32	2 9/16	1 1/4	1 1/8	9 1/4	2 29/32	2 13/16	1 1/4	1 1/8
11	1 1/4	7 9/16	2 7/8	1 3/8	1 1/4	10	3 7/32	3 1/8	1 3/8	1 1/4

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CHAPTER VI

CYLINDERS, PLATES AND SPRINGS

CYLINDERS

In the following discussion, cylinders will be divided into two general classes, as follows:

(*a*) Those having thin walls, as for example, steam and water pipes, boiler shells and drums.

(*b*) Those having relatively thick walls.

101. Thin Cylinders.—In analyzing the stresses induced in the walls of thin cylinders by an internal pressure, we shall assume, first, that the stresses are distributed uniformly over the cross-section of the cylinder; and second, that the restraining action of the heads at the ends of the cylinder is zero. Considering a cylinder having its ends closed by heads, the internal pressure against these heads produces a longitudinal stress in the walls; the magnitude of which is

$$S_t = \frac{pd}{4\,t}, \tag{120}$$

in which d represents the inner diameter of the cylinder, p the unit internal pressure and t the thickness of the cylinder walls.

Assuming that the above cylinder is cut by a plane through its axis, the resultant internal pressure on a section of either half cylinder having a length L as pdL; hence, the magnitude of the tangential or hoop stress is

$$S_t' = \frac{pd}{2\,t} \tag{121}$$

Comparing (120) and (121), it is apparent that the longitudinal stress S_t is one-half of the tangential stress; however, the true tangential stress is even less than that given by (121). Assuming that Poisson's ratio has a value of 0.3, the effective tangential stress is

$$S_t'' = \frac{pd}{2\,t} - \frac{0.3\,pd}{4\,t} = \frac{0.425\,pd}{t} \tag{122}$$

Designers never use formula (122); they prefer (121) since the thickness of the walls obtained by the latter, for any assumed set of conditions, is always greater.

102. Thick Cylinders.—In a cylinder having walls that are thick when compared to the internal diameter, the stresses induced by an internal pressure p cannot be considered uniformly distributed as in the preceding case. The tangential stress, or hoop tension as it is frequently called, varies along the wall thickness, having its greatest magnitude at the interior of the cylinder and its minimum at the exterior surface. Several investigators have proposed formulas that are applicable to the design of thick cylinders, among the most prominent of these being Lamé, Clavarino, and Birnie.

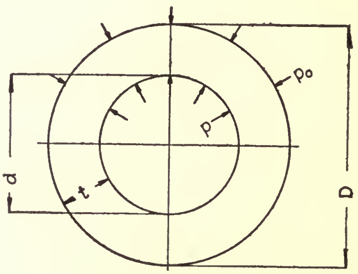

FIG. 51.

(a) *Lamé's formula.*—In the case of a cylinder subjected to both internal and external pressure, as shown in Fig. 51, the tangential and radial stresses at the variable radius r are given, according to Lamé, by the following expressions:

$$S_t = M + \frac{N}{r^2} \tag{123}$$

$$S_r = M - \frac{N}{r^2}, \tag{124}$$

in which

$$M = \frac{pd^2 - p_0 D^2}{D^2 - d^2} \tag{125}$$

and

$$N = \frac{d^2D^2}{4}\left[\frac{p - p_0}{D^2 - d^2}\right] \tag{126}$$

In order to derive a formula that is applicable to thick cylinders subjected only to internal pressure, we make $p_0 = 0$ in (125) and (126); then the maximum tangential stress occurs on the inner surface of the cylinder, and its magnitude is

$$S_t = p\left[\frac{D^2 + d^2}{D^2 - d^2}\right] \tag{127}$$

This is one of the forms in which the Lamé formula may be used, but very often it is found that another form is more convenient. This may be derived by clearing (127) of fractions and substituting $(2t + d)$ for D, whence

$$t = \frac{d}{2}\left[\sqrt{\frac{S_t + p}{S_t - p}} - 1\right] \tag{128}$$

(*b*) *Clavarino's and Birnie's formulas.*—In the preceding discussion, Poisson's ratio of lateral contraction was not introduced, and for that reason (127) and (128) are only approximate.

According to the maximum-strain theory proposed by Saint Venant, the effective tangential and radial stresses are as follows:

$$\left.\begin{aligned} S_t &= E\delta_t \\ S_r &= E\delta_r \end{aligned}\right\} \tag{129}$$

in which δ_t and δ_r represent the unit tangential and radial strains. It is evident that these strains or deformations depend on the longitudinal stress in the walls of the cylinder. Two cases may occur, namely, a cylinder may have its ends open or the ends may be closed.

1. *Cylinder with open ends.*—In a cylinder having open ends, the longitudinal stress is zero; and assuming the cylinder to be under an internal pressure, the maximum tangential stress is

$$S_t = (1 - m)\,M + (1 + m)\,\frac{N}{r^2}, \tag{130}$$

in which m represents Poisson's ratio.

Substituting the values of M and N from (125) and (126) in (130), we get finally

$$S_t = \frac{p}{D^2 - d^2}\,[(1 - m)d^2 + (1 + m)D^2] \tag{131}$$

Substituting in (131), the value of D in terms of d and t, we have

$$t = \frac{d}{2}\left[\sqrt{\frac{S_t + (1 - m)\,p}{S_t - (1 + m)\,p}} - 1\right] \tag{132}$$

This formula is that due to *Birnie* and applies only to cylinders having open ends.

2. *Cylinder with closed ends.*—The second case mentioned above is the one of most frequent occurrence, namely, that in which the ends of the cylinder under internal pressure are closed. For this condition, the magnitude of the maximum effective

tangential stress is given by the expression

$$S_t = (1 - 2m)\,M + (1 + m)\,\frac{N}{r^2}, \tag{133}$$

from which

$$S_t = \frac{p}{D^2 - d^2}[(1 - 2m)\,d^2 + (1 + m)\,D^2] \tag{134}$$

If an expression for t is desired, it may be obtained from (134) by substituting for D its value in terms of t and d; whence

$$t = \frac{d}{2}\left[\sqrt{\frac{S_t + (1 - 2m)\,p}{S_t - (1 + m)\,p}} - 1\right] \tag{135}$$

This expression is known as Clavarino's formula and applies to all cylinders, under internal pressure, having closed ends.

For values of m to be used in the above formulas, refer to Table 1.

PLATES

The various formulas in common use for determining the strength of flat plates subjected to various methods of loading are generally based upon some arbitrary assumption regarding the critical section or the reactions of the supports. Grashof, Bach, Merriman, and others have treated this subject from a mathematical standpoint, and the various formulas proposed by these investigators give results that agree fairly well with the experimental results obtained by Bach, Benjamin, Bryson, and others. Flat plates subjected to various methods of loading are of frequent occurrence in machines, and the formulas in the following articles are those proposed by Prof. Bach. They are reliable and are comparatively easy to apply to any given set of conditions. It should be understood, however, that these formulas apply only to the plain flat plate and not to plates having a series of reenforcing ribs that are commonly used when the plates are cast.

103. Rectangular Plates.—In arriving at formulas for the strength of rectangular plates, the critical section is taken as passing through the center of the plate, and the part to one side of this section is treated as a cantilever beam. The location of the critical section is determined by experiments, and for rectangular plates made of homogeneous material, Bach found that failure does not always occur along a diagonal as in the case with square plates. However, in establishing a general formula, it is usually assumed that the line of maximum stress lies along the diagonal.

(*a*) *Uniformly loaded.*—Consider a rectangular plate of thickness t, of length a and of breadth b, as supported at the periphery and subjected to a pressure p that is uniformly distributed; then, according to Bach,

$$t = b\sqrt{\frac{pK}{S\left[1 + \left(\frac{b}{a}\right)^2\right]}}, \tag{136}$$

in which K is a coefficient depending upon the method of supporting the periphery of the plate, the condition of the surface of the plate, the initial force required to make a tight joint, and the material used for making the tight joint. The values of K for cast iron and mild steel for various conditions of supporting the loaded plate are given in Table 32.

TABLE 32.—VALUES OF COEFFICIENTS K, K_1, K_2 AND K_3

Material	Condition of support	K	K_1	K_2	K_3
Cast iron	Free......	0.565	2.6–3.0	0.282	1.3–1.5
	Fixed.....	0.375	....	0.187	....
Mild steel	Free......	0.360	....	0.180	....
	Fixed.....	0.240	....	0.120	....

(*b*) *Central loading.*—Suppose now that a rectangular plate having the same dimensions as the one discussed previously be supported at the periphery and loaded centrally by a load Q; then the thickness may be determined by the following expression:

$$t = \sqrt{\frac{abQK_1}{S(a^2 + b^2)}} \tag{137}$$

For a cast-iron plate supported freely, the value of K_1 as determined experimentally by Bach varies from 2.6 to 3.0.

104. Square Plates.—For similar conditions of loading and supporting the plate, the formulas for the thickness of square plates may be derived directly from the corresponding formula pertaining to rectangular plates. Therefore, for *uniformly distributed pressure,* the thickness is

$$t = a\sqrt{K_2\frac{p}{S}} \tag{138}$$

For a square plate *centrally loaded*, it is

$$t = \sqrt{K_3 \frac{Q}{S}} \tag{139}$$

For values of K_2 and K_3, consult Table 32.

105. Circular Plates.—(*a*) *Pressure uniformly distributed.*—The thickness of a circular plate having a diameter a, and which is supported around its circumference and subjected to a uniformly distributed pressure, is determined by the following formula:

$$t = a\sqrt{K_4 \frac{p}{S}}, \tag{140}$$

and its deflection is given by the expression

$$\Delta = \frac{a^4 p K_5}{Et^3} \tag{141}$$

In (140) and (141), K_4 and K_5 represent coefficients which depend upon the method of support as well as the method and materials used in making the joint tight. Values of these coefficients are given in Table 33.

TABLE 33.—VALUES OF COEFFICIENTS K_4, K_5, K_6 AND K_7

Material	Condition of support	K_4	K_5	K_6	K_7
Cast iron	Free......	0.30	0.038	1.43	0.1–0.125
	Fixed.....	0.20	0.010	....	....
Mild steel	Free......	0.19	....	....	....
	Fixed.....	0.13	....	....	....

(*b*) *Central loading.*—For a flat circular plate supported freely around the circumference and subjected to a load Q at the center which is considered as distributed uniformly over the area $\frac{\pi d^2}{4}$, the thickness is given by the following formula:

$$t = \sqrt{K_6 \left[1 - \frac{2}{3}\frac{d}{a}\right] \frac{Q}{S}} \tag{142}$$

The deflection caused by the load Q may be determined by the relation

$$\Delta = \frac{a^2 Q K_7}{Et^3} \tag{143}$$

For values of the coefficients K_6 and K_7, consult Table 33.

According to Grashof, the thickness of a circular plate fixed rigidly around the circumference and loaded centrally by the load Q may be calculated by the relation

$$t = \sqrt{\frac{0.435\,Q}{S}\log_e\frac{a}{d}} \tag{144}$$

If the deflection is desired, the following expression may be used:

$$\Delta = \frac{0.055\,Pa^2}{Et^3} \tag{145}$$

106. Flat Heads of Cylinders.—(*a*) *Cast heads.*—In the case of a cast-iron cylinder having the flat head cast integral with the sides, as shown in Fig. 52, the allowable stress in the head, according to Bach, is given by the relation

$$S = 0.8\left[\frac{r}{t} + \left(\frac{a - r\left(1 + \frac{2r}{a}\right)}{2t}\right)^2\right] \tag{146}$$

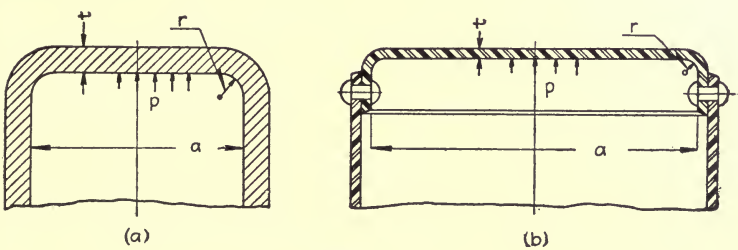

FIG. 52.

(*b*) *Riveted heads.*—The stress in the flat head riveted to a cylindrical shell, according to Bach, is

$$S = p\left[\frac{r}{2t} + 0.38\left(\frac{a - r\left(1 + \frac{2r}{a}\right)}{2t}\right)^2\right], \tag{147}$$

in which the various symbols have the same meaning as above.

107. Elliptical Plates.—(*a*) *Pressure uniformly distributed.*—Plates having an elliptical form are frequently met with in engineering designs; for example, handhole plates and covers for manholes in pressure vessels. The following formula, due to Bach, gives the thickness of an elliptical plate subjected to a uniformly distributed pressure, and whose major and minor axes are a and b, respectively:

$$t = K_8 b \sqrt{\frac{p}{S\left[1 + \left(\frac{b}{a}\right)^2\right]}} \tag{148}$$

The values of K_8 for cast iron and mild steel, and for two conditions of supporting the plate, are given in Table 34.

TABLE 34.—VALUES OF COEFFICIENTS K_8 AND K_9

Material	Condition of support	K_8	K_9
Cast iron	Free.......	0.82	0.85
	Fixed......	0.58	0.77
Mild steel	Free.......	0.60	....
	Fixed......	0.46	....

(*b*) *Central loading.*—The thickness of an elliptical plate supported around the periphery and subjected to a load Q at the center is given by the following expression:

$$t = \sqrt{K_9 \left[\frac{8 + 4c^2 + 3c^4}{3 + 2c^2 + 3c^4}\right]\frac{cQ}{S}}, \tag{149}$$

in which c represents the ratio of the minor axis b to the major axis a. For values of K_9, for various conditions of loading, consult Table 34.

SPRINGS

Springs are made in a variety of forms, depending upon the class of service for which they are intended. Among the common forms used to a considerable extent in connection with machinery, are the following: (*a*) Helical springs; (*b*) spiral springs; (*c*) conical springs; (*d*) leaf springs.

108. Helical Springs.—Helical springs are used chiefly to resist any force or action that tends to lengthen, shorten, or twist them. The wire or bar used to make this type of spring may have a circular, square, or rectangular cross-section. The stresses induced in the material of a helical spring subjected to an extension or a compression consist of a tension combined with secondary stresses, such as tensile and compressive due to a bending action. The latter stresses are generally not considered in the development of suitable formulas for the permissible load and the deflection.

(*a*) *Circular wire.*—The method of procedure in arriving at the relations existing between the axial deflection and the axial load for a helical spring made of round wire is as follows:

Let D = mean diameter of the coils.
E_s = torsional modulus of elasticity.
Q = axial load on the spring.
d = diameter of the wire.
n = number of coils.
p = pitch of coils.
Δ = total axial deflection.

The stresses at any section of the bar at right angles to the axis of the spring are those due to the torsional moment $\frac{QD}{2}$ and to the bending action, the effect of the latter being disregarded. Applying the formula for torsional stress from Art. 10, we have

$$S_s = \frac{8\,QD}{\pi d^3} \tag{150}$$

In determining the safe stress for any given case by means of (150), the magnitude of Q must be taken as the greatest load that will ever come upon the spring. Frequently (150) is used for calculating the safe load that a spring will carry, or it may be used for arriving at the size of the wire required for a given load, safe working stress, and diameter of coil.

The total length of the bar required to make the spring is $n\sqrt{\pi D^2 + p^2}$ or approximately πnD, and according to Art. 10, the angular deflection of a bar having the length just given, is

$$\theta = \frac{360\,nDS_s}{dE_s} \tag{151}$$

The axial deflection of the spring is evidently given by the following formula:

$$\Delta = \frac{\pi nD^2S_s}{dE_s} \tag{152}$$

Substituting the value of S_s from (150) in (152)

$$\Delta = \frac{8\,nQ}{dE_s}\left[\frac{D}{d}\right]^3 \tag{153}$$

Having given the load Q and the corresponding deflection Δ, (153) will be found useful for determining the required number of coils n, by assuming values for the size of wire and the diameter of the coils. The formula for the deflection as given by (152) may be used for calculating the safe deflection.

In designing helical springs, the following method of procedure is suggested:

1. By means of (150), determine the diameter of the coil required for the given load and assumed values of the fiber stress and size of wire. The results obtained may have to be rounded out so as not to get an odd size of arbor upon which the spring is made.

2. Having arrived at a proper dimension for the diameter of the coil, the deflection may be determined by means of (153), provided we know the number of coils required, or if the deflection is fixed by the surrounding conditions, the number of coils required may be calculated by means of (153).

(*b*) *Bar having rectangular cross-section.*—For helical springs made of a wire or bar having a rectangular cross-section $b \times h$, as shown in Fig. 53, the relation between the fiber stress S_s and

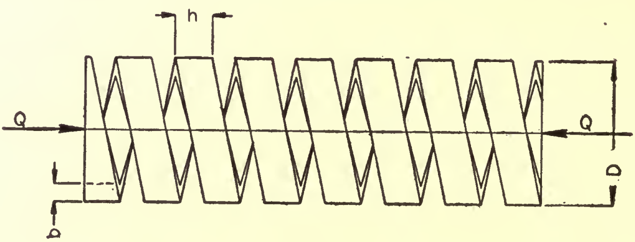

Fig. 53.

the external load Q is obtained by equating the external moment to the moment of resistance; whence

$$S_s = \frac{9\,QD}{4\,b^2h} \tag{154}$$

This formula is used to establish the size of the wire for any given load and safe stress, or it may be used to check the stress having given the load and size of wire.

According to the Mechanical Engineers' Handbook, the axial deflection of the spring may be calculated by the following formula:

$$\Delta = \frac{2.83\,nQD^3\,(b^2 + h^2)}{b^3h^3E_s} \tag{155}$$

If an expression for the axial deflection is desired in terms of the safe stress S_s, the value of Q obtained from (154) is substituted in (155); whence

$$\Delta = \frac{1.26\, D^2 S_s\, (b^2 + h^2)\, n}{bh^2 E_s} \tag{156}$$

The method of procedure to be used in the design of a helical spring constructed of a rectangular bar, as shown in Fig. 53, is the same as that suggested in (*a*) above.

(*c*) *Bar having square cross-section.*—In many installations requiring helical springs, square wire is preferred to the rectangular. By making $b = h$ in (154), (155) and (156), we obtain the desired equations necessary for designing springs constructed of square wires.

For a given load and assumed fiber stress, the size of the wire or bar may be calculated by means of the following formula:

$$S_s = \frac{9\, QD}{4\, b^3} \tag{157}$$

The axial deflection may be determined from

$$\Delta = \frac{5.65\, nQD^3}{b^4 E_s} \tag{158}$$

or from

$$\Delta = \frac{2.52\, nD^2 S_s}{bE_s} \tag{159}$$

For the method of procedure, the suggestions given in (*a*) above may be followed.

109. Concentric Helical Springs.—The springs used in many automobile clutches, as well as those used on railway trucks, consist of two concentric helical coils, both of which are necessarily deflected equal amounts, since their free and solid lengths are made equal. The springs used on railway trucks are generally made of round bars, while those used for automobile clutches are made of round, rectangular and square stock. In actual construction, the adjacent coils of concentric springs are wound right and left hand so as to prevent any tendency to bind. In the design of concentric springs in which the same grade of material is employed, an attempt should be made to get approximately the same stresses in the various coils. With the use of round wire, the latter condition is met by making the ratio $\frac{D}{d}$ the same for all coils, as the following analysis shows:

Using the same notation as before and representing the solid length of the spring by H, but adopting the subscripts 2 and 1 to the various d mensions of the inner and outer coils respectively, it follows from (152) that the stress in the material of the inner coils of a double helical spring is

$$S_s = \frac{\Delta_2 E_s}{\pi H_2} \left(\frac{d_2}{D_2}\right)^2 \tag{160}$$

and that in the outer coils

$$S_s = \frac{\Delta_1 E_s}{\pi H_1} \left(\frac{d_1}{D_1}\right)^2 \tag{161}$$

Now, assuming that the deflections and the solid heights are to be the same for the two coils, it is evident that for equal stresses

$$\frac{D_1}{d_1} = \frac{D_2}{d_2}.$$

Since the ratio $\frac{D}{d}$ is the same for both coils, it follows that the lengths of the bars from which the separate coils are made will be the same.

110. Helical Springs for Torsion.—Helical springs are also used to resist a torsional moment T by having one end held rigidly while the other is relatively free. Such springs are invariably made from bars having a rectangular or square cross-section. The material of the spring is subjected to a bending stress having a magnitude as follows:

$$S = \frac{6T}{hb^2}, \tag{162}$$

in which h is the width and b the radial thickness of the spring stock.

The linear deflection according to the Mechanical Engineers' Handbook is

$$\Delta = \frac{TLD}{2EI} = \frac{LSD}{Eb}, \tag{163}$$

in which the total length L of the bar may be assumed equal to πnD, as in Art. 108(a).

For springs made of square wire, the formulas for stress and deflection may be derived from (162) and (163) by making $h = b$.

111. Spiral Springs.—The spiral spring is used but little in machine construction, and then only for light loads. It constitutes what is commonly called a torsional spring and the material used in its construction is subjected to a bending stress. Letting h represent the width and b the radial thickness of the spring material, the moment of the external force Q must equal the internal resistance; hence

$$S = \frac{3\,QD}{hb^2} \tag{164}$$

The following expression for the linear deflection Δ of a spiral spring is that given in the Mechanical Engineer's Handbook.

$$\Delta = \frac{QLD^2}{4\,EI} = \frac{LSD}{bE}, \tag{165}$$

in which L represents the length of the straightened spring and the other symbols are as in the preceding articles.

112. Conical Springs.—Conical springs are generally used to resist a compression and are made of round or rectangular stock. They are applicable where the space is limited, and where there is no necessity for great deflections. The following formulas derived from the Mechanical Engineers' Handbook may serve for determining the proportions of such springs:

For a conical spring made of *round stock* and loaded as shown in Fig. 54, the shearing-stress in the material is as follows:

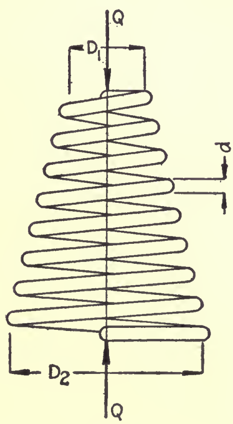

FIG. 54.

$$S_s = \frac{8\,QD_2}{\pi d^3} \tag{166}$$

The axial deflection for n turns or coils is given by the following expression:

$$\Delta = \frac{2\,nQ}{d^4E_s}(D_2^3 + D_2^2D_1 + D_1^2D_2 + D_1^3) \tag{167}$$

If the expression for the deflection is desired in terms of the safe stress, we have

$$\Delta = \frac{\pi n S_s}{4\,dD_2E_s}(D_2^3 + D_2^2D_1 + D_1^2D_2 + D_1^3) \tag{168}$$

A conical spring made of rectangular stock is shown in Fig. 55.

The torsional stress in the material of such a spring may be calculated by the formula

$$S_s = \frac{9\,QD_2}{4\,b^2h} \tag{169}$$

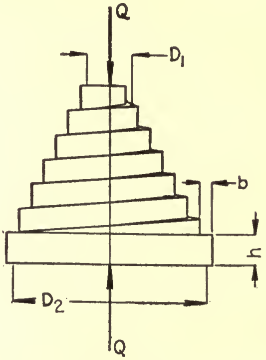

FIG. 55.

The axial deflection in terms of the load Q is

$$\Delta = \frac{0.71\,nQ\,(b^2 + h^2)\,(D_2^3 + D_2^2D_1 + D_2D_1^2 + D_1^3)}{b^3h^3E_s} \tag{170}$$

In terms of the safe stress, the axial deflection is

$$\Delta = \frac{0.315\,n\,(b^2 + h^2)\,(D_2^3 + D_2^2D_1 + D_2D_1^2 + D_1^3)}{bh^2D_2E_s} \tag{171}$$

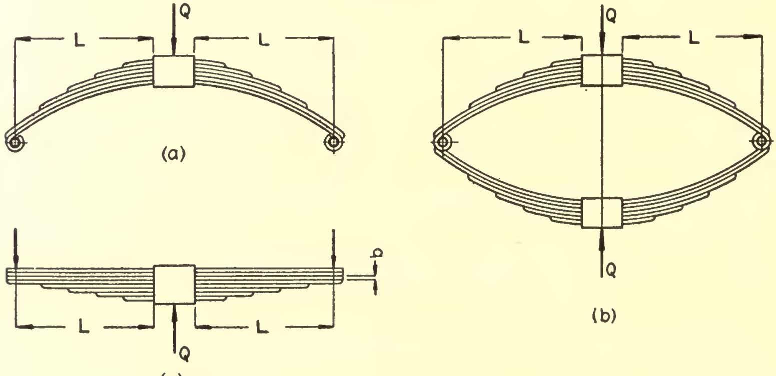

FIG. 56.

113. Leaf Springs.—Leaf springs are made in various forms some of which are shown in Fig. 56. The first form shown is called the full elliptic, the second semi-elliptic and the ordinary flat leaf spring is represented by Fig. 56(*c*). In all of the forms shown, the various leaves are banded tightly together, and, as usually constructed, each type has one or more full-length leaves, sometimes called master leaves, while the remaining leaves are graduated as to length. With this construction it is evident that the master leaves held rigidly by the band constitute a cantilever beam of uniform cross-section, while the remaining leaves form approximately a cantilever beam of uniform strength. From the theory of cantilever beams we find that the deflection of the graduated leaves for the same load and fiber stress will be 50 per cent. greater than that of the master leaves. Furthermore, when the leaves are banded together without any initial stress, the master leaves and the graduated leaves will deflect equal amounts, thus subjecting the former to a higher fiber stress. It is possible to make the fiber stresses in the two parts of the spring approximately equal by separating them by a space equal to the difference between the two deflections before putting the band in place; hence, when the band is in place and the spring is unloaded an initial stress is set up in the leaves. It is customary to consider one of the master leaves as a part of the cantilever beam of uniform strength.

114. Semi-elliptic Springs.—The following analyses and formulas pertaining to semi-elliptic springs are due to Mr. E. R. Morrison, who probably was the first to take into account the effect of the initial stress due to the band located at the middle of elliptic and semi-elliptic springs as used in automobile construction.

Let Q = total load on the spring.
Q_g = load coming upon one end of the graduated leaves.
Q_m = load coming upon one end of the master leaves.
S_g = maximum fiber stress in the graduated leaves.
S_m = maximum fiber stress in the master leaves
n = total number of leaves in the spring.
n_g = total number of graduated leaves.
n_m = total number of master leaves.

(*a*) *Initial space between leaves.*—From a study of cantilever beams, it is evident that in order to satisfy the condition of equal stress in the graduated and master leaves, the following equation will result:

$$\frac{6\,LQ_g}{hb^2n_g} = \frac{6\,LQ_m}{hb^2\,n_m}, \tag{172}$$

from which

$$\frac{Q_g}{n_g} = \frac{Q_m}{n_m} \tag{173}$$

The difference between the deflections of the graduated and master leaves is given by the following expression:

$$\Delta_g - \Delta_m = \frac{6\,L^3Q_g}{hb^3En_g} - \frac{4\,L^3Q_m}{hb^3En_m} = \frac{2\,L^3Q_m}{hb^3En_m} \tag{174}$$

Since $\frac{Q_m}{n_m} = \frac{Q}{2n}$, it follows that the depth of the space which must be provided between the two parts of the spring before they are banded together is

$$\Delta_g - \Delta_m = \frac{QL^3}{nhb^3E} = \frac{SL^2}{3\,bE} \qquad (17$$

(*b*) *Pressure due to the central band.*—If the total pressure exerted by the central band upon the leaves is Q_b, then the deflection of the graduated leaves due to $\frac{Q_b}{2}$, which is the pressure exerted by the band upon each cantilever, is as follows:

$$\Delta'_g = \frac{3\,L^3Q_b}{hb^3En_g} \tag{176}$$

The pressure $\frac{Q_b}{2}$ also produces a deflection in the master leaves, the magnitude of which is

$$\Delta'_m = \frac{2\,L^3Q_b}{hb^3En_m} \tag{177}$$

Combining (176) and (177), we have

$$\Delta'_g = \frac{3\,n_m}{2\,n_g}\,\Delta'_m \tag{178}$$

Since the total deflection produced by the band is equal to the depth of the space provided between the two parts of the spring, it follows that

$$\Delta'_g + \Delta'_m = \frac{QL^3}{nhb^3E}$$

from which

$$\Delta'_m = \frac{2\,n_g}{3\,n_m + 2\,n_g}\;\frac{L^3Q}{nhb^3E} \tag{179}$$

Combining (177) and (179), we get the following expression for the magnitude of the pressure exerted by the band:

$$Q_b = \frac{n_g n_m Q}{n\,(3\,n_m + 2\,n_g)} \tag{180}$$

The expression for Q_b just derived may be simplified by letting $n_m = kn$. Since $n = n_g + n_m$, it follows that $n_g = n(1 - k)$. Substituting these values of n_g and n_m in (180), we get

$$Q_b = \frac{k(1-k)Q}{2+k} \tag{181}$$

(*c*) *Deflection of spring due to Q.*—The deflection Δ of the spring due to Q is determined by taking the difference between the total deflection of the graduated leaves and that due to the band as given by (178); whence

$$\Delta_g - \Delta'_g = \frac{6L^3Q_g}{hb^3En_g} - \frac{3n_m}{3n_m + 2n_g}\,\frac{L^3Q}{nhb^3E}$$

or

$$\Delta = \frac{6}{k+2}\left[\frac{L^3Q}{nhb^3E}\right] \tag{182}$$

Now since $Q = 2(Q_g + Q_m) = \frac{nhb^2S}{3L}$, we get finally that the deflection Δ due to the load Q is

$$\Delta = \frac{2}{k+2}\left[\frac{L^2S}{Eb}\right] \tag{183}$$

In the above discussion, the effect of friction between the leaves was not considered.

(*d*) *Full elliptic springs.*—The analysis given for the semi-elliptic springs also applies to the full elliptic type, except that the total deflection Δ will be double that of a semi-elliptic spring.

115. Materials for Springs.—The majority of springs in common use are made from a high-grade steel, though frequently brass and phosphor bronze are found more desirable. In Chapter II are given the specifications of several grades of steel that are well-adapted for the making of springs. The permissible fiber stress varies with the thickness or diameter of the material used in the construction of the spring, being higher for the smaller thicknesses and diameters than for the larger. According to Kimball and Barr's Machine Design, the maximum allowable stress used by an Eastern railway company in the design of steel leaf springs may be determined from the following formula:

$$S = 60{,}000 + \frac{7{,}500}{b}, \tag{184}$$

in which b represents the thickness of the leaves.

Quoting again from Kimball and Barr, the following formula, based upon an experimental investigation of springs made in the Sibley College Laboratories, may be used for arriving at the probable working stress for round stock, such as is used in the construction of helical springs:

$$S_s = 40{,}000 + \frac{15{,}000}{d}, \qquad (185)$$

in which d represents the diameter of the stock.

The coefficient of elasticity E for all steels may be assumed as 30,000,000, while that for torsion or E_s may be taken at 13,000-000.

The allowable working stresses and coefficients of elasticity for phosphor bronze and high brass spring stock are not well-established, and in the absence of definite knowledge relating to the physical constants of these materials, the following values obtained from various sources may be used:

For phosphor bronze, S_s varies from 20,000 to 30,000 pounds per square inch.

For high brass, S_s varies from 10,000 to 20,000 pounds per square inch.

For high brass and phosphor bronze E = 14,000,000.

For high brass and phosphor bronze E_s = 6,000,000.

In general, when springs are subjected to vibrations or heavy shock, the stresses given above for the various materials must be decreased from 15 to 25 per cent.

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CHAPTER VII

BELTING AND PULLEYS

BELTING

The transmission of power by means of belting may be accomplished satisfactorily and efficiently when the distances between the pulleys are not too great. When the power to be transmitted is not large, round or V-shaped belts are used, the latter form also being used for drives with short centers. The materials used in the construction of belting are leather, rubber, cotton, and steel.

116. Leather Belting.—The highest grade of leather belting is obtained from the central portion of the hide. This central area is cut into strips which are cemented, sewed, or riveted together to form the desired thickness and width of belt. The thicknesses vary from a single hide thickness to that of four, the former being known as a *single leather belt* and the latter as a *quadruple belt.* The terms *double* and *triple belt* are used when two or three thicknesses are employed in the construction. The hides from which leather belts are made may be tanned by different processes. For ordinary indoor installations, the regular oak-tanned leather belting is well-adapted. For service in which the belt is exposed to steam, oil or water, a special chrome-tanned leather is recommended. This special tanning process is more or less secret and is guarded by patents. The users of this process claim that a more durable leather is produced, due to the fact the fibrous structure of the hide is preserved and not weakened as may result in the oak-tanning process. Leather belting weighs on an average about 0.035 pounds per cubic inch.

(*a*) *Commercial sizes.*—Leather belting is made in the following widths:

From one-half to one inch, the widths advance by ⅛-inch increments.

From one to four inches, the widths advance by ¼-inch increments.

From four to seven inches, the widths advance by ½-inch increments.

From seven to thirty inches, the widths advance by 1-inch increments.

From thirty to fifty-six inches, the widths advance by 2-inch increments.

From fifty-six to eighty-four inches, the widths advance by 4-inch increments.

The thickness of a single belt varies from 0.16 to 0.25 inch, while that of a double belt runs from 0.3 to 0.4 inch.

(*b*) *Strength of leather belting.*—The ultimate strength of oak tanned leather runs from 3,000 to 6,000 pounds per square inch, the former figure applying to the lower grades of leather and the latter to the high-grade product. According to tests made on chrome-tanned leather, the ultimate strength varies from 7,500 to 12,000 pounds per square inch. Table 35 contains information pertaining to the strength of leather belting, as given by Mr. C. J. Morrison, page 573 of *The Engineering Magazine,* July, 1916.

117. Rubber Belting.—Rubber belting is made by fastening together several layers of woven duck into which is forced a rubber composition which subsequently is vulcanized. Belting of this description is used to some extent in damp places, as for example in paper mills and saw mills.

A material resembling rubber, known as balata, is now used extensively in the manufacture of an acid- and water-proof belt. Balata is made from the sap of the boela tree found inVenezuela and Guiana. It does not oxidize or deteriorate as does rubber. The body of the belt, consisting of a heavy woven duck, is impregnated and covered with the balata gum, producing a belting material which is acid- and water-proof, and according to tests is about twice as strong as good leather. It is claimed that the heating of the belt due to excessive slippage softens the balata and thereby increases its adhesive properties. Due to this fact, it appears that balata belting is unsuitable for installations where temperatures of over 100°F. prevail.

The weight of rubber belting is about 0.045 pound per cubic inch.

Table 35.—Results of Test on Leather Belting

Mfr.	Sample	Bel Type	Bel Size	Breaking strength	Ultimate strength	Stretch in 2 inches Actual	Stretch in 2 inches Per Cent.
A	1	Double Belt	2×0.406	4,000	4,930	0.25	12.5
	2			3,800	4,680	0.23	11.5
	3			3,200	3,940	......	......
	4		2×0.375	3,430	4,575	0.27	13.5
	5		2×0.344	3,240	4,700	0.25	12.5
	6		2×0.3125	3,240	5,190	0.22	11.0
	7	Single Belt	2×0.266	2,230	4,200	0.23	11.5
	8			1,880	3,540	0.21	10.5
	9			2,240	4,226	0.07	3.5
	10		2×0.25	2,210	4,420	0.25	12.5
	11		2×0.219	1,840	4,200	0.23	11.5
	12		2×0.1875	2,440	6,500	Too small to measure	
B	1	Double Belt	2×0.344	2,280	3,320	0.17	8.5
	2			2,460	3,580	0.27	13.5
	3		2×0.281	2,300	4,100	0.26	13.0
	4			2,310	4,120	0.24	12.0
	5	Single Belt	2×0.219	2,880	6,550	Too small to measure	
	6		2×0.172	1,700	4,980	0.20	10.0
	7		2×0.1875	1,500	4,000	0.25	12.5
	8		2×0.172	2,180	6,380	0.18	9.0
C	1	Triple	2×0.50	4,510	4,510	0.45	22.5
	2	Double Belt	2×0.4375	4,070	4,650	0.30	15.0
	3		2×0.375	3,010	4,020	....	....
	4	Single Belt	2×0.250	2,000	4,000	0.25	12.5
	5			850	1,700	0.15	7.5
	6			2,750	5,500	....	....
D	1	Double Belt	2.5×0.344	3,920	4,558	0.30	15.0
	2		2.5×0.3125	3,740	4,800	0.24	12.0
E	1	Double Belt	2×0.344	2,730	3,970	....	....
	2			2,810	4,090	0.23	11.5
	3		2×0.50	2,600	2,600	0.20	10.0
	4		2×0.375	3,240	4,300	0.21	10.5
	5	Single Belt	2×0.188	2,010	5,360	0.20	10.0
	6			920	2,450	0.27	13.5
	7			1,420	3,790	0.30	15.0

(*a*) *Commercial sizes.*—According to one large rubber-belt manufacturer, the standard widths run from 1 to 60 inches as follows:

From one inch to two inches, the widths advance by ¼-inch increments.

From two inches to five inches, the widths advance by ½-inch increments.

From five inches to sixteen inches, the widths advance by 1-inch increments.

From sixteen inches to sixty inches, the widths advance by 2-inch increments.

The standard thicknesses run from two to eight plies.

(*b*) *Strength of rubber belting.*—Practically no experimental information is available on the strength of rubber belting, though it is claimed by the manufacturers that a three-ply rubber belt is as strong as a good single-thickness leather belt. According to information obtained from the catalog of The Diamond Rubber Co., the following values may be used as representing the net driving tensions per inch of width for a rubber belt having an arc of contact of 180 degrees.

For a three-ply belt use 40 pounds per inch of width.
For a four- and five-ply belt use 50 pounds per inch of width.
For a six-ply belt use 60 pounds per inch of width.
For a seven-ply belt use 70 pounds per inch of width.
For an eight-ply belt use 80 pounds per inch of width.
For a ten-ply belt use 120 pounds per inch of width.

118. Textile Belting.—Textile belts are made by weaving them in a loom or building them up of layers of canvas stitched together. The woven body or strips of canvas are treated with a filling to make them water-proof, and in some cases oil-proof. Generally, belts treated with a cheap filling are very stiff and hence do not conform to the pulley, making it more difficult to transmit the desired power. Textile belts are used more for conveyor service than for the transmission of power.

(*a*) *Commercial sizes.*—The sizes of oiled and stitched duck belting are as follows:

Four-ply is made in widths from 1 inch to 48 inches.
Five- and six-ply are made in widths from 2 inches to 48 inches.
Eight-ply is made in widths from 4 inches to 48 inches.
Ten-ply is made in widths from 12 inches to 48 inches.

From one to five inches, the widths vary by ½-inch increments; from five to sixteen, by 1-inch increments; and from sixteen to forty-eight, by 2-inch increments.

White cotton belting is made in the following sizes:

Three-ply having a width from 1½ inches to 24 inches.
Four-ply having a width from 2 inches to 30 inches.
Five-ply having a width from 4 inches to 30 inches.
Six-ply having a width from 6 inches to 30 inches.
Eight-ply having a width from 6 inches to 30 inches.

The widths of the cotton belting vary as follows: from one and one-half to six inches, by ½-inch increments; from six to twelve, by 1-inch increments; and from twelve to thirty, by 2-inch increments.

119. Steel Belting.—The transmission of power by means of steel belts was first introduced in 1906 by the Eloesser Steel Belt Co. of Berlin, Germany, and at the present time this method of transmitting power is recognized by many German engineers as being superior to that in which leather belting or ropes are used.

The steel belt is used in the same manner as the leather belt, except that it is narrow, thin and of very light weight. It is put on the pulley with a fairly high initial tension and hence runs without sag. The material used in making steel belts is a charcoal steel, prepared and hardened by a secret process. After rough rolling at a red heat, the metal band is allowed to cool and later is finished to exact size. The thicknesses vary from 0.2 to 1 millimeter (0.0079 to 0.039 inch), and the widths range from 30 to 200 millimeters (1.18 to 7.87 inches). The ultimate tensile strength of the finished material is approximately 190,000 pounds per square inch.

The pulleys upon which these belts run are preferably flat, and are covered with layers of canvas and cork so as to increase the coefficient of friction. A crowned pulley may be used, provided the crown does not exceed approximately 33 ten-thousandths of the width of the belt. Steel belts are not adapted to tight and loose pulleys, but crossed belts will work satisfactorily, provided the distance between the shafts is about seventy times the width of the belt.

In case the power transmitted is large, so that a single belt of sufficient width to give the required cross-sectional area cannot be obtained, two or more belts are run side by side. In putting steel belts on pulleys, a special clamp is used in order to measure correctly the initial tension and at the same time to facilitate fitting the special plates necessary to make the joint. The design of a proper fastening for steel belts presented a difficult problem, but after considerable experimental work D. Eloesser, now head of the firm that bears his name, perfected a joint that has proven very satisfactory. His first design was made of one piece and the ends of the belt had to be soldered in place at the installation. The latest design, shown in Fig. 57, consists of several parts fastened together by screws *e* that are removable. The ends of the steel band are soldered to the main parts of the joint and the small screws *f* and *g* passing through the triangular-shaped steel pieces *c* and *d* give added strength to the fastening. The plates *a* and *b* that form the main parts of the joint are curved, the curvature depending upon the size of the pulley upon which the belt is to run.

(*b*) *Experimental conclusions.*—The following conclusions were derived from a study of a large number of tests on steel belts made in actual service.

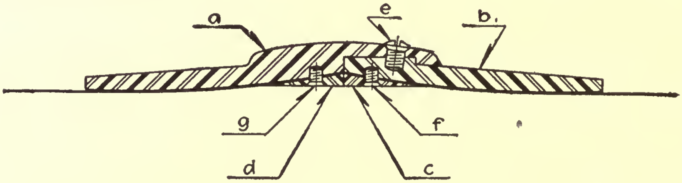

Fig. 57.

1. Steel belts do not stretch after being placed on the pulleys, hence there is no necessity for taking up slack.

2. Steel belts are not affected by variations in temperature and may be used satisfactorily in damp places.

3. Steel belts will transmit the same horse power as leather belts having a width two to four times as great.

4. Due to the decrease in width over leather belts transmitting the same power, narrower-face pulleys may be used, thus effecting a considerable saving in the cost of the pulley and in space due to a reduction in the general dimensions of machinery.

5. It is claimed that the first cost of steel belting is less than that of leather or rubber belting.

6. Steel belts are more sensitive and hence the pulleys, as well as the shafting, require more accurate alignment.

7. Speeds as high as 19,500 feet per minute have been attained, and the slip at this speed was only 0.15 of 1 per cent.

8. Due to the small slip, steel belts transmit power virtually without loss.

9. Steel belts do not wear, and, if properly installed, are said to have a useful life exceeding five years.

10. As the tension in steel belts is only a fraction, about one-tenth, of that used in a leather belt of the same capacity, the pressures on the bearings are less, thus reducing the frictional losses.

11. Steel belts weigh much less than leather belts of equal capacity, and hence reduce the frictional losses still more.

12. Due to the extreme thinness of steel belts and the high speeds used, they might prove dangerous if the drive is not enclosed by proper guards.

(*c*) *Results of tests.*—The following results, collected from the various reports recorded in several German technical journals, are given to show what actually has been accomplished in the transmission of power by means of steel belting.

1. Under ordinary running conditions, a 4-inch steel belt is equivalent to an 18-inch leather belt or six manila ropes 1¾ inches in diameter.

2. In a particular installation, a 4-inch steel belt transmitted 250 horse power, having replaced a 24-inch leather belt.

3. Two steel belts each 5.9 inches wide were used to transmit 450 horse power, which formerly required 12 cables.

4. A 6-inch steel belt 0.024 inch thick is capable of transmitting 200 horse power, and with two such belts placed side by side on the same pulley, 440 horse power has been transmitted.

5. Three 4¾-inch steel belts were used to transmit 1300 horse power at 500 revolutions per minute of the driven pulley. The distance between the 122-inch driving and 63-inch driven pulleys was 46 feet.

6. In another installation, 75 horse power was transmitted by a 6-inch steel belt running over pulleys 108 and 51 inches in diameter, located on 76-inch centers.

(*d*) *American experiments on steel belting.*—In 1911 or 1912, the General Electric Co. made a series of experiments with steel belts, and came to the conclusion that they were not entirely satisfactory. The thicknesses of the belts used in these experiments varied from 0.007 to 0.018 inch. A ⅝-inch belt 0.01 inch thick was capable of transmitting 150 horse power continuously for 17 hours at a speed of 20,000 feet per minute. This belt was made of cold-rolled steel and the initial tension put on the belt in order to give the above results was 90,000 pounds per square inch. The General Electric Co. found that steel belts will not run satisfactorily on the ordinary steel pulleys, and the best results were obtained with a leather-faced pulley. No doubt the following are some of the reasons why the results obtained by the General Electric Co. from their investigation on steel belting were not as promising as those found by the German engineers:

1. Not as good a grade of steel available for making the band.
2. Probably during the early stages of preparing the band, improper treatment gave rise to scale troubles.
3. Difficulty in the process of annealing.
4. Lack of time for further research work.

120. Belt Fastenings.—Fastenings of various forms are used for joining the ends of a belt, but none of them is as strong and durable as the scarfed and glued splice, which when made carefully is but little weaker than the belt proper. Of necessity, the scarfed and glued joint or cemented splice is adapted to installations in which the slack of the belt is taken up by mechanical means, and where careful attention is given to belting by competent workmen. Probably the oldest form of fastening, as well as that used most commonly, is to join the ends of a belt by means of rawhide lacing. Not infrequently belts are laced together with wire, and such joints run very smoothly, especially if made with a machine, and are considerably stronger than the rawhide laced joint, as is indicated in Table 36. Patented metal fasteners in the form of hooks, studs, and plates are also in use and have the advantage that they are cheap and applied very easily and quickly. Some of the metal fasteners are too dangerous to be used on belts that must be touched by hand, and for that reason some states have legislated against their use.

Tests of belt joints.—Tests of various types of belt joints were made at the University of Wisconsin, also at the University of Illinois. In *The Engineering Magazine of* July, 1916, Mr. C. J. Morrison presented a valuable article entitled "Belts—Their Selection and Care," in which he gives considerable information pertaining to the strength of leather belts and the joints used with such belting. In Table 36 is given information pertaining to the strengths and efficiencies of the various types of leather belt joints tested by Mr. Morrison. It should be understood that the term "efficiency" in this case is used in the sense as when applied to riveted joints.

TABLE 36.—STRENGTH OF LEATHER BELT JOINTS

Type of joint		Breaking load, pounds	Efficiency, per cent.
Cemented splice	Cement only	2,440	100.0
	Cement and shoe pegs	2,430	99.6
	Cement and small copper rivets	2,170	88.9
	Cement and small copper rivets	2,060	84.4
	Cement and large copper rivets	2,040	83.6
Wire, machine-laced		5,850	90.0
Wire, hand-laced		5,330	82.0
Rawhide with small holes		4,100	63.0
Rawhide with large holes		3,200	49.0
Metal hooks		2,270	35.0
Metal studs		1,950	30.0

STRESSES IN BELTING

121. Tensions in Belts.—A belt transmits power due to its friction upon the face of the pulley. This transmitting capacity depends upon the following important factors:

(*a*) The allowable net tension in the belt.

(*b*) The coefficient of friction existing between the belt and pulley.

(*c*) The speed at which the belt is running.

Net tensions.—The net tension represents the capacity of the belt and depends upon the maximum allowable tension, the coefficient of friction, the angle of contact that the belt makes with the pulley, the material of both the belting and the pulley, the diameter of the pulley, and the velocity of the belt. The net tension is not a constant as is frequently assumed, but it varies with the speed. Let two pulleys be connected by a belt as shown in Fig. 58, and assume that no power is being transmitted, except that required to overcome the frictional resistance on the bearings due to the initial tension with which the belt was placed on the pulleys. Due to this initial tension, which is the same on both the running on and off sides of the pulleys, the belt exerts a pressure upon the face of the pulleys. This pressure in turn induces a frictional force on the rim capable of overcoming an equivalent resistance, tending to produce relative motion between the belt and pulley. The tensions in the two parts of the belt will change as soon as power is transmitted, say from *a* to *b*, causing that in the pulling side to increase and that in the running off side to decrease. Representing these tensions by the symbols T_1 and T_2, we see that the force causing the driven pulley *b* to rotate is the difference of these tensions, or $T_1 - T_2$. This difference is known as the net tension.

It is evident that due to this difference in tension in the various sections of the belt, a unit length of the belt in running from the point *A* to *B*, decreases in length due to its elasticity. From this it follows that the driver *a* delivers a shorter length of belt at

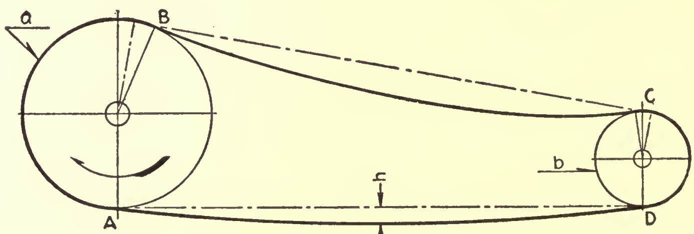

FIG. 58.

B than it receives at *A* and furthermore, that the velocity of the pulley face and that of the belt are not equal. A similar action occurs on the pulley *b*. This action is known as *belt creep* and results in some loss of power.

122. Relation between Tight and Loose Tensions.—The horse power delivered by a belt may be determined as soon as the net tension and the speed are established; hence it is important to derive the relation existing between the tight and loose tensions.

Let A = cross-sectional area of the belt in square inches.
C = centrifugal force of an elementary length of belt.
S = allowable working stress of the belt.
b = width of belt.
t = thickness of belt.
v = velocity of belt, in feet per second.
w = weight of belt, pounds per cubic inch.
μ = coefficient of friction.
θ = total angle of contact, expressed in radians.

In Fig. 59 a short portion of the belt has an arc of contact subtending the angle $\Delta\theta$ at the center of the pulley. Let the tension at one end be T and at the other $(T + \Delta T)$; evidently each of these tensions makes an angle $\left[\frac{\pi}{2} - \frac{\Delta\theta}{2}\right]$ with the vertical center line. The pressure between the portion of the belt and the pulley rim is designated by the symbol N, and the force of friction between them is μN. In addition to these forces, we have the centrifugal force C acting radially as shown in the figure. The magnitude of the centrifugal force is given by the following expression:

$$C = \frac{12\,\Delta\theta w A v^2}{g} \tag{186}$$

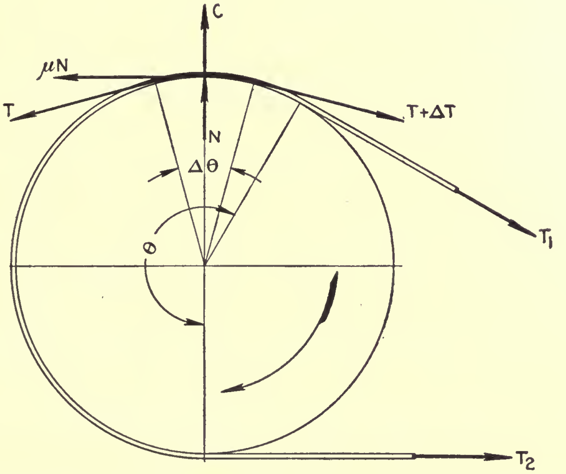

FIG. 59.

The piece of belt referred to above is held in equilibrium by the five forces T, $(T + \Delta T)$, N, μN, and C. The summation of the horizontal and vertical components, respectively, gives the following equations:

$$-\Delta T \cos\frac{\Delta\theta}{2} + \mu N = 0 \tag{187}$$

$$(2\,T + \Delta T)\sin\frac{\Delta\theta}{2} - N - C = 0 \tag{188}$$

Eliminating N

$$\mu\,(2\,T + \Delta T)\sin\frac{\Delta\theta}{2} - \Delta T\cos\frac{\Delta\theta}{2} - \mu C = 0$$

Dividing through by $\frac{\Delta\theta}{2}$, and passing to the limit, we get

$$\mu \lim (2T + \Delta T) \lim \frac{\sin \frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}} - \frac{24\mu A w v^2}{g} = 2 \lim \frac{\Delta T}{\Delta\theta} \lim \cos \frac{\Delta\theta}{2}$$

whence

$$\frac{dT}{d\theta} = \mu (T - k) \tag{189}$$

where

$$k = \frac{12 A w v^2}{}$$

Separating the variables

$$\int_{T_2}^{T_1} \frac{dT}{T - k} = \mu \int_0^\theta d\theta.$$

Integrating, we find that the relation between the tight and loose tensions is as follows:

$$\frac{T_1 - k}{T_2 - k} = e^{\mu\theta} \tag{190}$$

From (190), we find that the net tension is

$$T_1 - T_2 = (T_1 - k)\left[\frac{e^{\mu\theta} - 1}{e^{\mu\theta}}\right] \tag{191}$$

Substituting in (191) the value of T_1 in terms of b, t and S, we have

$$T_1 - T_2 = bt\left[S - \frac{12\, wv^2}{g}\right]\left[\frac{e^{\mu\theta} - 1}{e^{\mu\theta}}\right]$$

Denoting the terms $\left[S - \frac{12\, wv^2}{g}\right]$ and $\left[\frac{e^{\mu\theta} - 1}{e^{\mu\theta}}\right]$ by the symbols m and n, respectively, we get finally

$$T_1 - T_2 = mnbt \tag{192}$$

Having determined the magnitude of the net tension from (192) and knowing the speed v, the horse power delivered may be calculated from the relation

$$H = \frac{v}{550}(T_1 - T_2) \tag{193}$$

123. Coefficient of Friction.—There is much diversity of opinion regarding the working coefficient of friction, but in general it depends upon the material of the belt and the condition of the belt, the permanent slip, whether the load is steady or fluctuating, the diameter of the pulley and the material of which it is made, and the speed of the belt. In view of the foregoing, the coefficient of friction cannot be assumed as an average for all speeds, as is so frequently done in belting calculations. It is practically impossible to derive an expression for μ in terms of all of the factors mentioned above, but the following formula proposed by Mr. C. G. Barth has been found to give fairly satis-

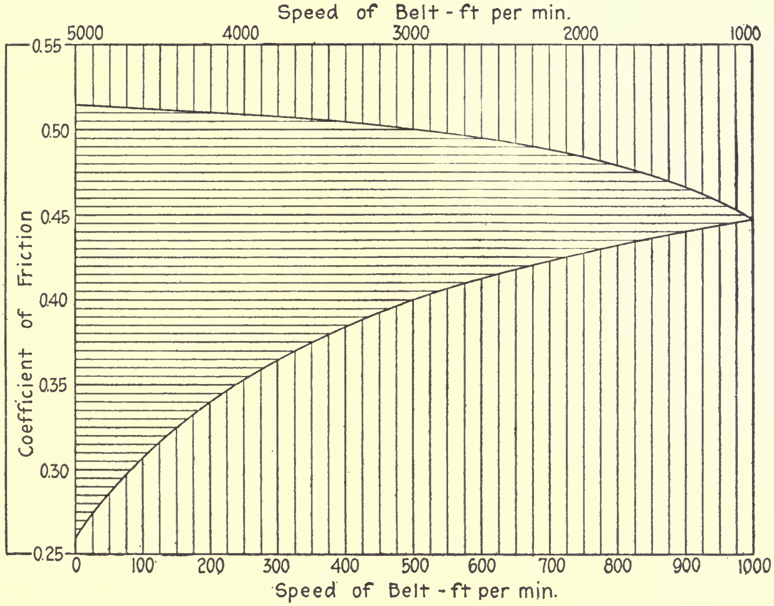

Fig. 60.

factory results in practice for leather belting on cast-iron or steel-rim pulleys.

$$\mu = 0.54 - \frac{140}{500 + V}, \tag{194}$$

in which V represents the velocity of the belt in feet per minute

The Barth formula for μ, as given by (194), has been evaluated for various values of V, and the results obtained are shown in graphic form in Fig. 60.

124. Maximum Allowable Tension.—The maximum allowable tension that may be put upon a belt depends upon the quality of the material, the permanent stretch of the belt, the imperfect elasticity of the belting material, and the strength of the joints in the belts. In Table 37 are given the average values for the ultimate strengths of leather belting, as given by Morrison in his article referred to previously. To arrive at the magnitude of the allowable working stress S for leather, multiply the ultimate strength by the so-called efficiency of the joint and divide the product thus obtained by the assumed factor of safety. As an aid in the solution of belt problems, the several factors just mentioned, as well as the allowable working stresses for the important joints used in connection with leather belting, are given in Table 38.

TABLE 37.—AVERAGE ULTIMATE STRENGTH OF LEATHER BELTING

Mfr.	No. of samples	Best	Poorest	Average	Remarks
A	12	6,500	3,549	4,611	
B	8	6,550	3,303	4,614	
C	6	5,500	1,700	4,062	Poorest broke in the splice.
C	5	5,500	4,013	4,532	Omitting 1,700 sample.
D	2	4,800	4,558	4,679	
E	7	5,360	2,453	3,800	

TABLE 38.—WORKING STRESSES FOR LEATHER BELTING

Type of joint	Ultimate strength	Efficiency of joint	Factor of safety	Working stress S
Cemented	4,300	0.98	10	420
Wire, Machine-laced	4,300	0.88	10	380
Wire, Hand-laced	4,300	0.80	10	340
Rawhide laced	4,300	0.60	10	260

125. Selection of Belt Size.—Having arrived at the allowable working stress in a belt, and knowing the magnitude of the net driving tension P as well as the angle of contact θ and the coefficient of friction μ, the area of the belt may be calculated by means of (192). From the conditions of the problem, either the width of the belt or its thickness may be established; hence the remaining dimension may be determined. Now the selection of the proper belt thickness is, in general, determined by the diameter of the smallest pulley used in the transmission. If the belt is thick relative to the diameter of the smallest pulley, the result will be an unsatisfactory drive, due to the excessive slippage and belt wear, as well as the excessive loss of power. In addition to the points just mentioned, the result of running a thick belt over a small pulley will be a considerable decrease in the life of the belt.

Satisfactory belt service, as well as long life, is secured if the diameter of the smallest pulley in the transmission is made not less than 12 inches if a double belt of medium or heavy weight is used; for a triple belt, the minimum diameter of pulley should be 20 inches, and for a quadruple belt, 30 inches. The selection of a belt thickness may also be influenced to a certain degree by the fact that good reliable single belts are hard to obtain in widths exceeding 12 to 15 inches. A rule occasionally used for the limiting size of a single belt is as follows: "A single belt should never be used where the width is more than four-thirds the diameter of the smallest pulley."

126. Taylor's Experiments on Belting.—In volume XV of the *Transactions* of the American Society of Mechanical Engineers, Mr. F. W. Taylor reports "A Nine Years' Experiment on Belting" carried on at the Midvale Steel Co. This paper gives some valuable data on the actual performance of belts, and a satisfactory abstract of it is impossible in this chapter. The conclusions, thirty-six in number, given in the paper are based upon the cost of maintaining the belts in good condition, including time lost in making repairs, as well as other considerations. The following are some of the conclusions:

(*a*) Thick narrow belts are more economical than thin wide ones.

(*b*) The net driving tension of a double belt should not exceed 35 pounds per inch of width, but the initial tension may be double that value.

(*c*) The most economical belt speed ranges from 4,000 to 4,500 feet per minute.

(*d*) For pulleys 12 inches in diameter or larger double belts are recommended.

For pulleys 20 inches in diameter or larger triple belts are recommended.

For pulleys 30 inches in diameter or larger quadruple belts are recommended.

(*e*) The joints should be spliced and cemented rather than laced with rawhide or wire, or joined by studs or hooks.

(*f*) Belts should be cleaned and greased every five or six months.

(*g*) The best distance between centers of shafts is from twenty to twenty-five feet.

(*h*) The face of a pulley should be 25 per cent. wider than the belt.

127. Tandem-belt Transmission.—Not infrequently two belts, one placed on top of the other, are used to transmit power from one pulley to two separate pulleys. This arrangement is known as a tandem-belt drive. The outside belt travels at a somewhat higher speed than the inner, and this fact must not be lost sight of when a tandem-belt transmission is being designed in which the speeds of the two driven pulleys must be the same. Experience with tandem-belt drives has shown that the best results are obtained when both belts are of the same thickness, preferably of double thickness, and are placed upon the pulleys with the same initial tension. Due to the higher coefficient of friction between leather and leather, practically all the slip will occur between the pulley and the inner belt. To arrive at the proper size of a belt required for a tandem drive, proportion each belt according to the power it must transmit.

128. Tension Pulleys.—Whenever possible, it is well to provide means of releasing the initial tension in belts during extended periods of idleness. In some cases, as in electrical machinery, this is accomplished by mounting the machines on rails, thus providing means for changing the distance between the centers of the pulleys. To a certain extent, the practice of making the loose pulley on machine drives smaller in diameter, will relieve the belt tensions. There are, however, many belting installations where neither of these methods could be used, and in many of these cases tension pulleys designed and installed properly will improve the transmission.

Lenix system.—In the Lenix system, the tension pulley is placed on the slack side of the belt as near to the smaller pulley in the transmission as is practicable. The general features of this system are shown in the two radically different installations represented in Figs. 61 and 62. The tension pulley is carried on an arm pivoted on the axis of the small driving pulley, and by means of a weight the required tension may be put on the slack belt. In the installation shown in Fig. 61, the tension on the belt is changed by increasing or decreasing the leverage of the

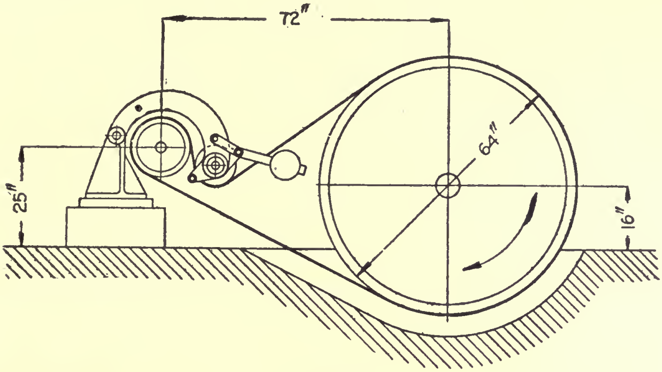

FIG. 61.

tension weight. It is evident from an inspection of Figs. 61 and 62, that a large arc of contact is obtained by means of this system and for that reason the tension in the belt may be reduced.

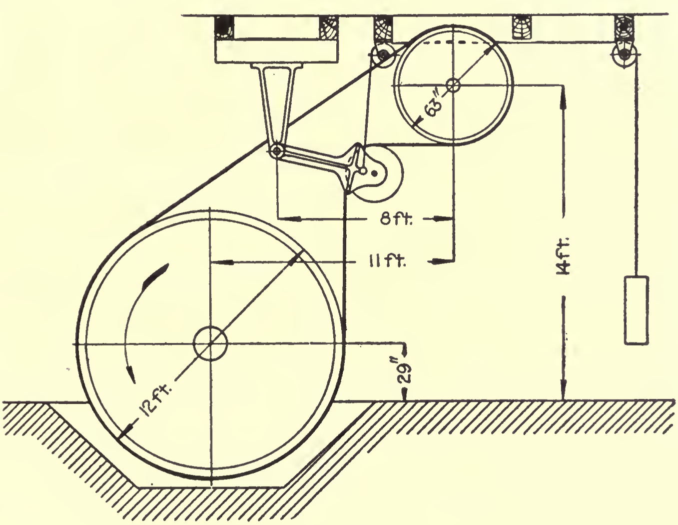

FIG. 62.

The diameter of the tension pulley should never be made less than that of the smallest pulley in the drive. The only losses chargeable to the tension pulley are those due to journal friction, which, if the apparatus is properly designed and erected, are small and have practically no effect on the efficiency of the transmission. Some additional advantages of tension pulleys are as follows: (1) the initial tension of the belt may be regulated very accurately and may be maintained at the proper magnitude; (2) during periods when the drive is not in use the belt may be relieved of the initial tensions.

PULLEYS

129. Types of Pulleys.—(*a*) *Cast-iron pulleys.*—Pulleys are made from various kinds of materials, cast iron, however, being the most common. As far as the cost of manufacture is concerned, cast iron is ideal since it can be cast in any desired shape, though precautions must be taken in the foundry when light-

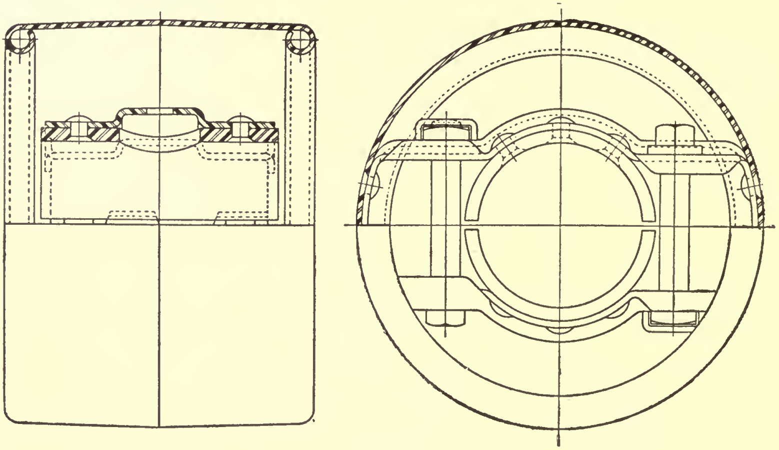

FIG. 63.

weight pulleys are cast. If the metal in the various parts of the pulley is not distributed correctly, shrinkage stresses due to irregular cooling are likely to reduce the useful strength of the material. To partly overcome this trouble, pulleys are split in halves. Careless moulding in the foundry generally produces pulleys having rims that are not uniform in thickness, thus causing them to run out of balance. This defect is rather serious in a high-speed transmission, though the pulley can be balanced by attaching weights at the lightest points. The centrifugal force due to these weights will set up severe stresses in the weak rim and may cause it to burst.

(b) *Steel pulleys.*—A type of pulley introduced to overcome some of the defects of cast-iron pulleys consists of a cast-iron hub and arms to which is riveted a steel rim. Pulleys built in this way are lighter than cast-iron ones for the same duty, but trouble may result with the fastenings as they may work loose due to the heavy loads transmitted. Pulleys built entirely of steel are also used, and are looked upon with favor by many engineers. In Figs. 63 and 64 are shown the designs of a small and large pulley as manufactured by The American Pulley Co. of Philadelphia. An inspection of Figs. 63 and 64 shows that the construction adopted for these pulleys gives a maximum strength for a minimum weight, and furthermore, the windage effect at high speeds is small.

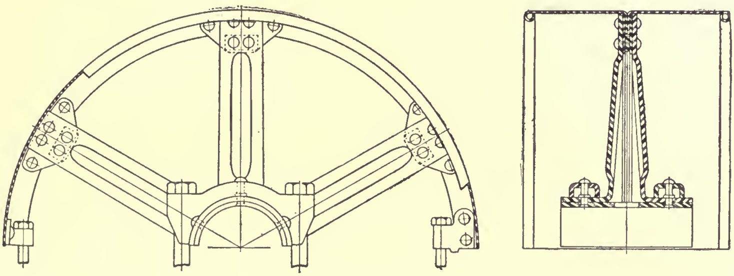

FIG. 64.

(c) *Wood pulleys.*—Wood pulleys in the smaller sizes generally consist of a cast-iron hub upon which is fastened a wood rim built up of segments of well-seasoned maple. In the larger sizes, they are always made in the split form and are built entirely of wood. Due to atmospheric conditions, wood pulleys are very likely to warp or distort, which may cause trouble at high speeds.

(d) *Paper pulleys.*—Pulleys made of paper are also in common use. As shown in Fig. 65, such a pulley consists of a web and rim built up of thin sheets of straw fiber cemented together and compressed under hydraulic pressure. To secure additional strength in the rims, wooden dowel pins extend through the rim and web as shown in the figure. The webs are clamped securely between the flanges of the cast-iron hub as shown.

(e) *Cork insert pulleys.*—Frequently pulleys are lagged with leather or cotton belting in order to increase the coefficient of friction between the belt and pulley. However, such lagging wears out quickly and must be renewed, thus increasing materially the cost of upkeep of the transmission. It has been found by an extended series of experiments, conducted by Prof. W. M. Sawdon of Cornell University, that the transmitting capacity of practically any type of pulley can be increased by fitting cork inserts into the face. The corks are pressed into the face and allowed to protrude above the surface of the material of the face not to exceed $\frac{1}{32}$ inch. These cork inserts dc not wear down nearly as rapidly as the lagging; however, the first cost is considerably more.

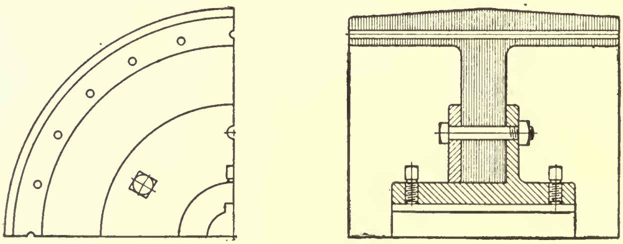

FIG. 65.

130. Transmitting Capacity of Pulleys.—In September, 1911, before the National Association of Cotton Manufacturers, Prof. W. M. Sawdon read a paper entitled "Tests of the Transmitting Capacities of Different Pulleys in Leather Belt Drives," in which he presented the results of an extended investigation on the trans-

TABLE 39.—COMPARATIVE TRANSMITTING CAPACITIES OF PULLEYS

Type of pulley	Relative capacities at various slips		
	1 per cent.	1½ per cent.	2 per cent.
1 Cast iron	100.0	100.0	100.0
2 Cast iron with corks projecting 0.04 inch.	133.5	119.7	107.0
3 Cast iron with corks projecting 0.015 inch.	139.3	124.1	112.0
4 Wood	136.3	118.8	105.6
5 Wood with corks projecting 0.075 inch	130.7	116.8	104.8
6 Wood with corks projecting 0.03 inch	130.7	118.2	104.8
7 Paper	160.7	151.7	137.3
8 Paper with corks projecting 0.087 inch	149.0	135.5	122.0
9 Paper with corks projecting 0.015 inch	150.2	145.3	133.0

mitting capacities of pulleys. In this paper, Prof. Sawdon gave a table of relative capacities based on the same arc of contact and the same belt tensions, which may prove useful in the solution of belt problems. The data given in Table 39 were derived from this paper. In using the table it should be kept in mind that the figures are relative and, strictly speaking, apply only to the conditions of operation prevailing during the tests. However, the results may be used tentatively until further data pertaining to this subject are available.

131. Proportions of Pulleys.—(*a*) *Arms.*—It is very seldom that a designer is called upon to design cast-iron pulleys except

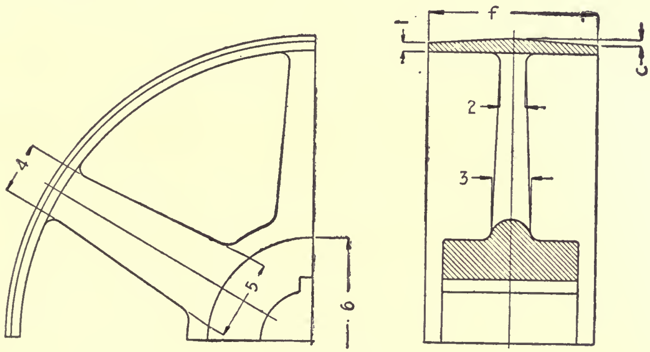

FIG. 66.

for an occasional special purpose, and for that reason it is best to leave the general design of standard pulleys to the pulley manu-

TABLE 40.—PROPORTIONS OF EXTRA-HEAVY CAST-IRON PULLEYS

Diam.	Dimensions					
	1	2	3	4	5	6
12	0.38	13/16	31/32	1 9/16	1 3/4	4
15	0.40	7/8	1 1/32	1 5/8	1 13/16	4 1/4
18	0.42	27/32	1 1/16	1 11/16	1 27/32	4 1/2
24	0.46	1 1/32	1 1/4	2	2 5/16	4 3/4
30	0.50	1 5/32	1 7/16	2 3/8	2 7/8	7
36	0.54	1 1/4	1 17/32	2 9/16	3 1/8	7
42	0.58	1 5/16	1 5/8	3 1/8	3 15/16	8
48	0.62	1 13/32	1 13/16	3 11/16	4 5/8	8
54	0.66	1 1/2	2	4 1/8	5 3/16	9 1/2
60	0.70	1 19/32	2 3/16	4 1/2	5 3/4	9 1/2

facturer. In Fig. 66 is represented an ordinary cast-iron pulley, and the proportions of various sizes of extra-heavy double-belt pulleys given in Table 40 may serve as a guide in the design of special pulleys.

A series of tests made on various kinds of pulleys by Prof. C. H. Benjamin, the results of which were published in the *American Machinist* of Sept. 22, 1898, proved rather conclusively that the rim of a pulley does not distribute the torsional moment equally over the arms as is so frequently assumed. In every test made, the two arms nearest the tight side of the belt gave way first and in almost all cases rupture of the arm occurred at the hub. As a result of these tests, Prof. Benjamin suggests that the hub end of the arm should be made strong enough so that it is capable of resisting a bending moment equivalent to

$$M = \frac{D}{n}(T_1 - T_2), \qquad (195)$$

in which

D = diameter of the pulley in inches.
n = the number of arms.

This means that one-half of the arms are considered as effective. The dimensions of the arm at the rim should be made such that the sectional modulus is only one-half of that at the hub.

The various manufacturers differ as to the number of arms to be used with the different sizes of pulleys, but the following suggestions may be found useful:

Use webs for pulleys having a diameter of 6 inches or less.

Use 4 arms for pulleys having a diameter ranging from 7 to 18 inches.

Use 6 arms for pulleys having a diameter ranging from 18 to 60 inches.

Use 8 arms for pulleys having a diameter ranging from 60 to 96 inches.

When the face of a pulley is wide, a double set of arms should always be provided.

The working stress to be used in calculating the dimensions of the arms by means of (195) varies within very wide limits. An investigation of the arms of pulleys having a diameter of from 12 to 96 inches and a face of 4 to 12 inches gave stresses varying from 200 to 1,500 pounds per square inch. The latter stress is obtained in the smaller pulleys and the former with the larger diameters.

(*b*) *Rim.*—According to Mr. C. G. Barth, the face of the pulley should be considerably wider than the belt that is to run on it, and in order to establish uniform proportions, he proposed the following formulas:

$$f = 1\frac{3}{16}b + \frac{3}{8} \text{ inch.} \tag{196}$$

$$f = 1\frac{3}{32}b + \frac{3}{16} \text{ inch.} \tag{197}$$

Formula (196) is the one that should be used wherever possible, but occasionally due to certain restrictions as to available space, (197) may have to be used. In connection with these formulas, Mr. Barth recommends that the height of the crown should be determined by the formula

$$c = \frac{f^{2/3}}{32} \tag{198}$$

For proportions of the thickness of the rim, the data given in Table 40 may be of service.

132. Tight and Loose Pulleys.—In his consulting work, Mr. Barth has found the need of well-designed tight and loose pulleys. After a thorough study of the conditions under which such pulleys must operate, he developed the design shown in Fig. 67. Furthermore, he standardized the design, and the formulas below give well-proportioned sleeves and pulleys for shaft diameters from 1½ to 4 inches, inclusive. The face and height of crown for these pulleys are based on formulas (196) to (198) inclusive. The formulas giving the proportions of the pulley hub and sleeve *a* are based on the diameter *d* of the shaft.

$$\left.\begin{aligned} d_1 &= 1.5\,d + 1.5 \text{ inches} \\ d_2 &= 1.5\,d + 1 \text{ inch} \\ d_3 &= 1.375\,d + 0.75 \text{ inch} \\ d_4 &= 1.25\,d + 0.25 \text{ inch} \\ e &= \frac{d}{16} + 0.125 \text{ inch} \\ m &= \frac{d}{6} + 0.75 \text{ inch} \end{aligned}\right\} \tag{199}$$

The formulas listed below give proportions of the loose pulley rim, and are based upon the width of the belt running on the pulleys. The belt width as given by Mr. Barth varied from 2 to 6 inches, inclusive.

$$\left.\begin{aligned} f &= 1\frac{3}{16}b + \frac{3}{8}\text{ inch} \\ g &= \frac{b}{16} + 1\frac{1}{8}\text{ inch} \\ L &= f + 2g \\ k &= \frac{b}{16} + \frac{3}{16}\text{ inch} \end{aligned}\right\} \quad (200)$$

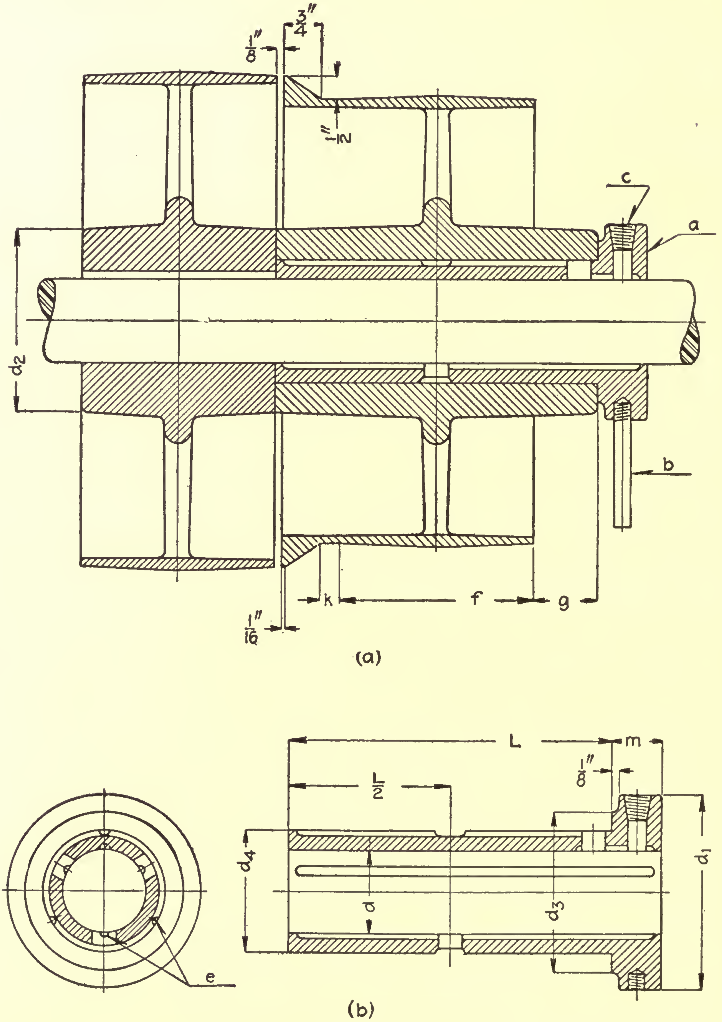

FIG. 67.

The common tight and loose pulleys that are used in the majority of installations differ considerably from the design discussed above in that both pulleys are generally made alike, and in many cases neither pulley is crowned.

V BELTING

133. Types of V Belts.—As stated in the first part of this chapter, V belts are used when it is desired to transmit light power; for example, in driving the cooling fan and generator on

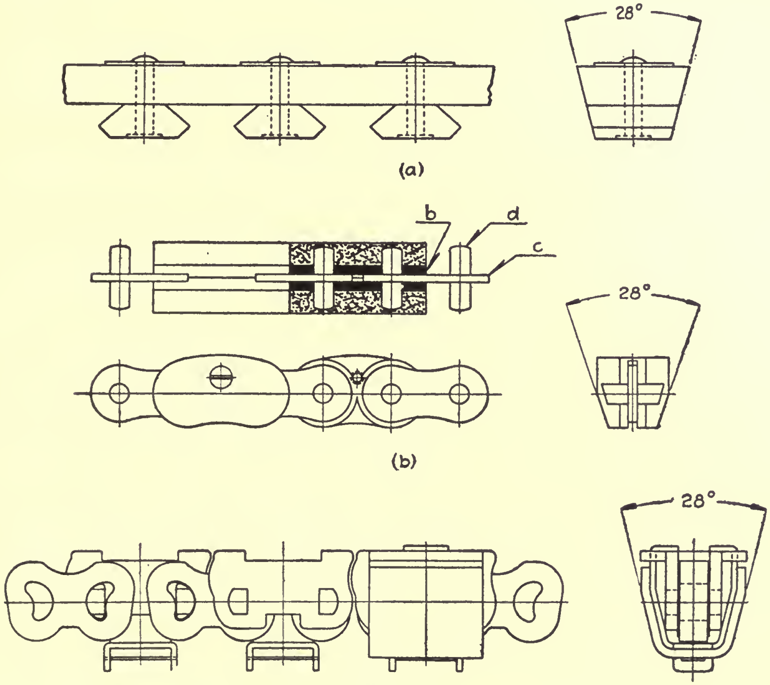

FIG. 68.

automobiles, and transmission drives on motorcycles. It is also used for belting electric motors to pumps and ventilating fans, when the distances between the shafts are short. Several forms of V belting are shown in Fig. 68.

(*a*) *Block type.*—The construction used in the block type of V belt is shown in Fig. 68(*a*). It consists of a plain high-grade and very pliable leather belt to which are cemented and riveted equally spaced V blocks, also made of leather. For light loads, a single belt is used; and for heavy service, a wide belt is fitted with several rows of V blocks. The angle adopted in this design is 28 degrees, and according to the manufacturers of this belt, the maximum speed should not exceed 3,000 feet per minute. The belt shown in Fig. 68(*a*) is also used successfully on high pulley ratios, though the best results are obtained if the ratio does not exceed 6 or 7 to 1. In addition to giving good service on high-ratio pulleys, the block type of V belt also works successfully on pulleys located close together. The following recommendations were furnished by the Graton and Knight Mfg. Co.:

1. For a 2, 3, or 4 to 1 ratio, the minimum center distance equals the diameter of the larger pulley plus twice the diameter of the smaller one.

2. For a 5, 6, or 7 to 1 ratio, the minimum center distance equals the diameter of the larger pulley plus three times the diameter of the smaller one.

3. For a 8, 9, or 10 to 1 ratio, the minimum center distance equals the diameter of the larger pulley plus four times the diameter of the smaller.

(*b*) *Chain type.*—The construction shown in Fig. 68(*b*) is of the chain type, and consists of double links made of oak-tanned sole leather connected together by central links *c* made of steel. The steel links are fitted with short pins *d* to which the leather links are attached. To add strength to the belt as well as to afford a fair bearing for the pins *d*, vulcanized fiber links *b* are used between the leather and steel links. An ordinary wood screw clamps the two sets of double links together, as illustrated in the figure. All the driving is done by the leather links, and the angle used is 28 degrees.

Another construction of the chain type V belt made entirely of steel, except the part coming into contact with the pulley, is shown in Fig. 68(*c*). The material used for lining the steel driving members is not leather but a specially treated asbestos fabric.

134. Force Analysis of V Belting.—To determine the relation existing between the tight and loose tensions in a V-belt power transmission, we may follow the method given in Art. 122.

Let w = weight per foot of belt.

2β = total angle of the V groove.

C, v, μ and θ same meaning as in Art. 122.

Referring to Fig. 69 and taking the summation of the horizontal and vertical components, respectively, of all forces acting upon a small portion of the belt, we get

$$\Delta T \cos \frac{\Delta\theta}{2} - 2\,\mu N = 0 \tag{201}$$

$$(2\,T + \Delta T) \sin \frac{\Delta\theta}{2} - 2\,N \sin \beta - C = 0 \tag{202}$$

The magnitude of the centrifugal force C in this case is given by the following equation:

$$C = \frac{\Delta\theta w v^2}{g} \tag{203}$$

Eliminating N in (201) and (202) and taking the limits of the resultant expression, we finally get

$$\frac{dT}{T - \dfrac{wv^2}{g}} = \frac{\mu}{\sin \beta}\, d\theta \tag{204}$$

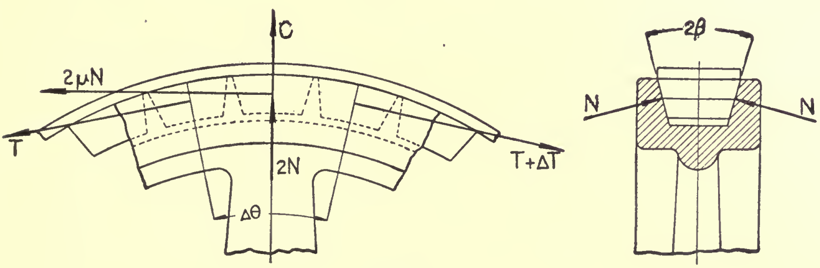

FIG. 69.

Integrating (204) between the proper limits for T and θ, we obtain

$$\frac{T_1 - \dfrac{wv^2}{g}}{T_2 - \dfrac{wv^2}{g}} = e^{\frac{\mu\theta}{\sin\beta}} \tag{205}$$

The net driving tension of the belt is

$$T_1 - T_2 = \left[T_1 - \frac{wv^2}{g}\right]\left[\frac{e^{\frac{\mu\theta}{\sin\beta}} - 1}{e^{\frac{\mu\theta}{\sin\beta}}}\right] \tag{206}$$

To determine the horse power transmitted, substitute the magnitude of the net driving tension obtained from (206) in (193).

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CHAPTER VIII

MANILA ROPE TRANSMISSION

Ropes used in engineering operations are made of a fibrous material such as manila, hemp and cotton, or of iron and steel. As to the kind of service, ropes may be classed as follows: (*a*) those used for the hoisting and transporting of loads; (*b*) those used for the transmission of power.

FIBROUS HOISTING ROPES

135. Manila Hoisting Rope.—Manila rope is manufactured from the fiber of the abaca plant, which is found only in the Philippine Islands. It has a very high tensile strength, tests made at the Watertown Arsenal showing that it exceeds 50,000 pounds per square inch. In making the rope, the fibers are twisted right-handed into yarns; these yarns are then twisted in the opposite direction forming the strands, and to form the finished rope a number of strands are twisted together, again in the right-hand direction.

Practically all manila rope used for hoisting purposes has four strands except the sizes below 7/8 inch, which are made with three strands. For drum hoists using manila ropes, the maximum speed attained under load seldom exceeds 1,000 feet per minute, generally being nearer 300 feet per minute. The permissible working loads of the various sizes of manila ropes used for hoisting service are given in Table 41.

136. Sheave Diameters.—A rope in passing over sheaves is subjected to a considerable amount of internal wear, due to the fibers sliding upon each other. The smaller the diameter of the sheave the greater this sliding action becomes; hence to decrease the wear, large sheaves should be used. In addition to the internal wear there is also wear on the outside of the rope due to the friction between it and the sides of the grooves of the sheave. It is evident, therefore, that the grooves should be finished very smooth. Again, the arrangement of the various elements that make up the hoisting apparatus may be such that an excessive number of bends is introduced, thus increasing the wear.

Table 41.—Manila Rope

Diameter in inches	For hoisting			For Transmission			
	Weight per foot	Ultimate strength	Minimum sheave diam.	Weight per foot	Ultimate strength	Maximum allowable tension	Minimum sheave diam.
1/4	0.018	620					
5/16	0.024	1,000					
3/8	0.037	1,275					
7/16	0.055	1,875					
1/2	0.075	2,400					
9/16	0.104	3,300					
5/8	0.133	4,000					
3/4	0.16	4,700		0.21	3,950	112	28
7/8	0.23	6,500		0.27	5,400	153	32
1	0.27	7,500	8	0.36	7,000	200	36
1 1/8	0.36	10,500	9	0.45	8,900	253	40
1 1/4	0.42	12,500	10	0.56	10,900	312	46
1 3/8	0.55	15,400	11	0.68	13,200	378	50
1 1/2	0.61	17,000	12	0.80	15,700	450	54
1 5/8	0.75	20,000	13	0.92	18,500	528	60
1 3/4	0.93	25,000	14	1.08	21,400	612	64
2	1.09	30,000		1.40	28,000	800	72
2 1/4	1.5	37,000		1.80	35,400	1,012	82
2 1/2	1.71	43,000		2.20	43,700	1,250	90

Experience has shown that manila ropes give good service and will last a reasonable length of time in hoisting operations when the sheaves for the various sizes of ropes are made according to the diameters given in Table 41.

137. Stresses in Hoisting Ropes.—In hoisting operations ropes are wound upon drums, and sheaves are used for changing the direction of the rope. In passing over sheaves or onto drums, the rigidity of the rope offers a resistance to bending which must be overcome by the effort applied to the pulling side of the rope. To determine the relation that exists between the effort P and the resistance Q for a rope running over a guide sheave, the following method may be used:

Let D = pitch diameter of the sheave.
d = diameter of the sheave pin.
μ = coefficient of journal friction.
η = efficiency.

On the running-on side of the sheave shown in Fig. 70, the outer fibers, due to the bending of the rope, are in tension while the inner fibers are in compression. These tensile and compressive stresses when combined with the tension distributed uniformly over the section will produce a resultant which has its

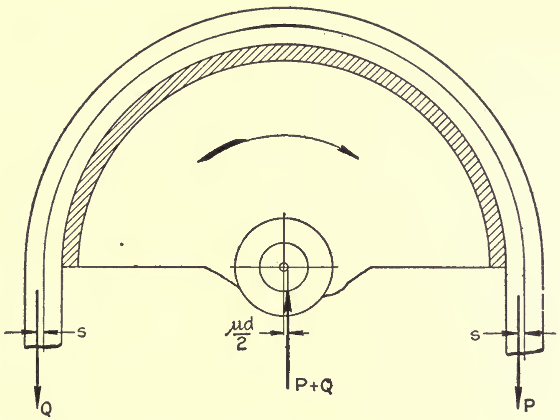

FIG. 70.

point of application to the left of the center line of the rope, a distance designated by the symbol s. The resultant must be equal to Q, from which it follows that rope stiffness may be considered as having the same effect as increasing the lever arm of the resistance Q.

By applying the same line of reasoning to the running-off side, it may be shown that the rigidity of the rope has the effect of decreasing the lever arm of the effort P by an amount which may be taken as approximately equal to s. Introducing friction at the sheave pin and taking moments about the line of action of the resultant pressure upon this pin, we obtain

$$P = \left[\frac{D + \mu d + 2s}{D - \mu d - 2s}\right] Q = CQ \tag{207}$$

Since the efficiency of a mechanism is defined as the ratio of the useful work done to the total work put in, it is evident that in the case of the ordinary rope guide sheave

$$\eta = \frac{Q}{P} = \frac{1}{C} \tag{208}$$

138. Analysis of Hoisting Tackle.—Analyses of systems of hoisting tackle or so-called pulley blocks are readily made with the aid of the principle discussed in the preceding article. The application of this principle will be shown by an example.

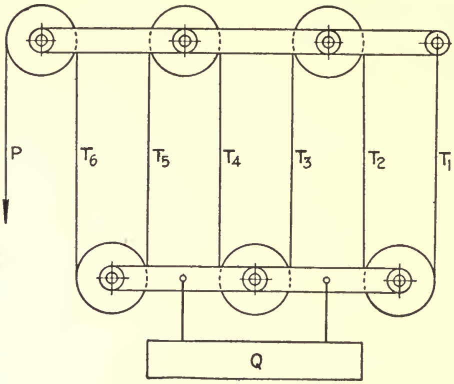

FIG. 71.

Common block and tackle.—The common block and tackle consists of two pulley blocks, each block having a series of sheaves mounted side by side on the same axle or pin. The number of sheaves varies in ordinary hoisting operations from two to four, but when used in connection with wire rope on hydraulic elevators or on cranes these numbers are exceeded. For convenience of analysis, we may assume the sheaves of each block to be placed on separate pins as shown in Fig. 71. Beginning with the end of the rope fastened to the upper block, let the successive tensions in the parts of the rope supporting the load Q, be denoted by T_1, T_2, etc.; then

$$T_2 = CT_1; \quad T_3 = C^2T_1$$
$$T_4 = C^3T_1; \quad T_5 = C^4T_1; \; T_6 = C^5T_1$$
$$P = C^6T_1 \tag{209}$$
$$Q = T_6 + T_5 + T_4 + T_3 + T_2 + T_1$$
$$= T_1\left[\frac{C^6 - 1}{C - 1}\right] \tag{210}$$

Substituting the value of T_1 from (209) in (210), we obtain

$$P = C^6\left[\frac{C - 1}{C^6 - 1}\right]Q \tag{211}$$

Without friction, the effort required to raise the load Q is

$$P_0 = \frac{Q}{6} \tag{212}$$

Hence the efficiency for the tackle shown in Fig. 71 is

$$\eta = \frac{C^6 - 1}{6C^6(C - 1)} \tag{213}$$

In general when the block and tackle has n sheaves and n lines supporting the load Q, we get as the general expression for the effort

$$P = C^n \left[\frac{C - 1}{C^n - 1}\right] Q, \tag{214}$$

and for the efficiency

$$\eta = \frac{C^n - 1}{nC^n(C - 1)} \tag{215}$$

139. Experimental Data on Hoisting Tackle.—Experimental data on hoisting tackle reefed with manila rope are meager, so in order to obtain some information as to the efficiency of such

Table 42.—Hoisting Tackle Reefed with Manila Rope

Size of rope	Block and tackle data				Ratio Q/P	Value of C	
	Sheave diam.	Pin diam.	No. of sheaves	No. of lines		Test	Mean
1¼	7¾	⅞	1	2	1.92	1.087	1.13
			2	3	2.68	1.125	
			3	4	3.37	1.127	
			4	5	3.95	1.135	
			5	6	4.48	1.13	
			6	7	4.92	1.14	
1½	9⅜	1	1	2	1.91	1.098	1.14
			2	3	2.67	1.125	
			3	4	3.36	1.134	
			4	5	3.93	1.14	
			5	6	4.45	1.141	
			6	7	4.89	1.143	
			7	8	5.28	1.143	
			8	9	5.61	1.143	
1¾	10¾	1⅛	2	3	2.64	1.136	1.15
			3	4	3.30	1.142	
			4	5	3.84	1.155	
			5	6	4.33	1.155	
			6	7	4.72	1.158	
			7	8	5.08	1.162	
			8	9	5.37	1.16	
2	13	1⅜	4	5	3.87	1.15	1.15
			5	6	4.37	1.15	
			6	7	4.78	1.153	
			7	8	5.14	1.152	
			8	9	5.45	1.153	

apparatus, the American Bridge Co. made an extended series of tests at the Pencoyd plant. These tests were made with standard types of manila and wire rope blocks, and an attempt was made to reproduce as nearly as possible actual conditions under which such apparatus is used in practice. The results of these tests were reported by S. P. Mitchell in a paper entitled "Tests on the Efficiency of Hoisting Tackle" and were presented before the American Society of Civil Engineers in September, 1903. That part of the data pertaining to manila ropes is given in Table 42. In the last two columns of this table are given the values of C as determined by means of equation (214).

FIBROUS TRANSMISSION ROPE

Leather belting, while excellent for transmitting power for short distances under cover, is not suitable for transmitting power to long distances out of doors, and for this class of service, manila and cotton ropes are used. Cotton rope, however, is not used to any extent in this country. The construction of the manila rope used for the transmission of power is similar to that discussed in Art. 135.

The transmission of power by means of manila rope gives satisfactory results for distances between shafts as great as one hundred and seventy-five feet without the use of carrying pulleys, while with the carriers, the distance may be increased almost indefinitely. Manila rope is also well-adapted to short distances. By the use of properly located guide pulleys power may be transmitted from one shaft to another, no matter what the relative positions of the shaft. There are two systems of rope driving in use, and each has its advocates. The two systems are commonly called the *Multiple* or *English System* and the *Continuous* or *American System.*

140. Multiple System.—The multiple system, which is the simpler of the two, uses separate ropes each spliced into an endless belt and running in a separate groove on each sheave wheel; thus each rope is absolutely independent of any other and carries its proportion of the load. The last statement is only true if the ropes are spliced carefully and the initial tension in each rope is made the same. The multiple system may be used for heavy loads and is recommended where the drive is protected from the weather and when the shafts are parallel or approximately so, as in installations where the power from a prime mover has to be distributed to the several floors of a building. This system also finds favor for rolling mill service, in which service it is common practice to install several more ropes than are absolutely necessary to transmit the power so that the mill need not be closed down even if several of the ropes should fail or jump off.

The advantages possessed by the multiple system are as follows:

1. It is practically secure against breakdowns, and if a rope should break it may be removed and replaced at some convenient time.

2. The power transmitted may be increased by adding extra ropes.

3. Power may be more easily transmitted to the different floors of an establishment.

4. The life of a rope is greater than in the continuous system, since it always bends in the same direction.

5. It is cheaper to install.

Among the disadvantages are the following:

1. It has more slippage than the continuous system.

2. It is not well-adapted to quarter turn drives nor where the shafts are at an angle with each other.

141. Continuous System.—In the continuous system one continuous rope passes around the driving and driven sheaves several times, in addition to making one loop about a tension pulley located on a traveling carriage. Since a single rope is used, it is evident that some device is required that will lead the rope from the outside groove of the driving sheave to the opposite outside groove of the driven sheave. This device is the tension pulley. Other functions of this traveling tension pulley are to maintain continually a definite uniform tension in the rope, and to take care of the slack due to the stretching of the rope. In Fig. 72, is shown one way of taking care of the slack by means of a tension carriage.

The continuous system is well-adapted to vertical and quarter turn drives, and to installations having shafts that are at an angle to each other. It also gives better service in places where the rope is exposed to the weather. The following are some of the disadvantages:

1. A break in the rope shuts down the whole plant until the rope is spliced and again placed on the sheaves.

2. All of the ropes are not subjected to the same tension; that is, the rope leading from the tension carriage has a greater tension than the center ropes.

142. Manila Transmission Rope.—For the transmission of power, the four or six-strand ropes are used on all sizes above ¾-inch. For the ¾-inch size, which is the smallest transmission rope made, the three-strand type gives good service. The four- and six-strand ropes of both hoisting and transmission types have the strands laid around a core which has been treated with a lubricant. A lubricant is used also on the inner yarns of each

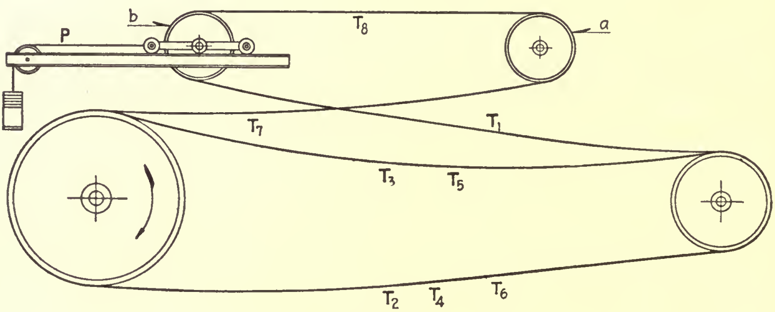

FIG. 72.

strand, thus insuring proper lubrication of the rope. For transmission purposes experience shows that the best results are obtained when the speed of the rope is approximately 4,500 feet per minute. Higher speeds are used, but the life of the rope is decreased due to excessive wear.

143. Sheaves.—The diameter of a sheave, used in the transmission of power by means of manila ropes, should be made forty times the diameter of the rope when space and speeds permit. Sometimes it is necessary, due to constructive reasons, to make the diameter less than that called for by the above rule. This reduction of the diameter decreases the life of the rope very materially and it is well to keep the minimum diameter above thirty-six times the diameter of the rope.

Form of groove.—The forms of the grooves used in the two systems of transmission discussed in the preceding articles differ somewhat, although in the angle used by some of the manufacturers, they are similar. Experience seems to show that an angle of 45 degrees gives the best results for both systems. However, there are one or two manufacturers of rope transmissions that recommend an angle of 60 degrees. In Fig. 73 are shown the forms of grooves recommended for the *continuous system*, (*a*) and (*b*) being used for the driver as well as the driven, and (*c*) for the idler sheaves. As illustrated in the figure, the grooves are not made deep since the rope is kept taut in order to

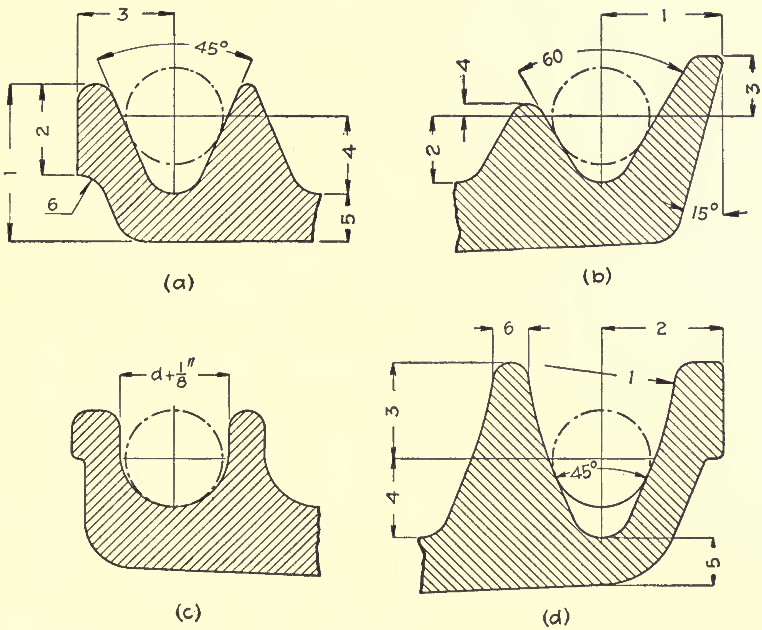

FIG. 73.

decrease the tendency for it to jump out. The type of groove shown in Fig. 73(*a*) is used by the Allis-Chalmers Co.; for proportions thereof consult Table 43. For proportions of the form of groove used by the Dodge Mfg. Co. illustrated in Fig. 73(*b*) consult Table 43.

The form of groove commonly used in the *multiple system*, and occasionally in the continuous system, is shown in Fig. 73(*d*), and in Table 44 are given the proportions of this groove for the various sizes of transmission ropes. The form of the groove used on idlers with the *multiple system* is deeper than that shown in Fig. 73(*c*), but in other details it is about the same.

Table 43.—Dimensions of Grooves for Manila Rope Sheaves
All dimensions in inches

Size of rope	Allis-Chalmers standard							Dodge Mfg. Co. standard				
	Pitch	1	2	3	4	5	6	Pitch	1	2	3	4
3/4								1 1/4	1	9/16	1/2	
7/8								1 3/8	1 1/8	5/8	9/16	
1	1 1/2	1 9/16	15/16	1	3/4	1/2	5/16	1 1/2	1 1/4	11/16	5/8	
1 1/8								1 5/8	1 3/8	3/4	11/16	1/8 (for sizes 3/4 to 1 1/2)
1 1/4	1 3/4	1 7/8	1 1/4	1 3/16	15/16	5/8	1/4	1 3/4	1 1/2	13/16	3/4	
1 3/8								1 7/8	1 5/8	7/8	13/16	
1 1/2	2	2 3/16	1 1/2	1 5/16	1 1/8	3/4	1/4	2	1 3/4	1	13/16	
1 3/4	2 1/2	2 11/16	1 13/16	1 9/16	1 3/8	7/8	1/4	2 1/2	2	1 1/8	15/16	3/16 (for sizes 1 3/4 and 2)
2	2 3/4	2 3/4	1 13/16	1 3/4	1 3/8	1	1/4	2 3/4	2 1/4	1 1/4	1 1/16	

Table 44.—Dimensions of Grooves for Manila Rope Sheaves
All dimensions in inches

Size of rope	Engineers standard						
	Pitch	1	2	3	4	5	6
3/4	1 3/8	2 15/16	15/16	3/4	5/8	3/8	1/4
7/8	1 1/2	2 15/16	1 1/16	7/8	3/4	7/16	1/4
1	1 7/8	3 7/8	1 1/4	1	13/16	1/2	3/8
1 1/8	2	3 7/8	1 3/8	1 1/8	15/16	9/16	3/8
1 1/4	2 1/8	4	1 1/2	1 1/4	1	5/8	3/8
1 3/8	2 1/8	3 1/8	1 9/16	1 3/8	1 1/8	11/16	3/8
1 1/2	2 1/4	3 5/16	1 11/16	1 1/2	1 1/4	3/4	3/8
1 3/4	2 1/2	3 3/8	1 15/16	1 3/4	1 7/16	7/8	3/8
2	2 3/4	3 9/16	2 1/8	2	1 5/8	1	3/8

144. Relation between Tight and Loose Tensions.—In order to calculate the horse power transmitted by a manila rope at a given speed, it is necessary to know the net tension on the ropes, and to get this we must determine the relation existing between the tight and loose tensions. Due to the wedging action of the rope in the groove of the sheave, the friction between the sheave and the rope is considerably greater than for the case of plain belting. The ratio between the tensions may be derived by the same method as that given in Art. 134. Using the same notation as in the discussion of the V belting, and considering a short length of the rope having an arc of contact subtending the angle

$\Delta\theta$ at the center of the sheave, we get for the summation of the horizontal and vertical components, respectively

$$\Delta T \cos \frac{\Delta\theta}{2} - 2\,\mu N = 0 \tag{216}$$

$$(2\,T + \Delta T) \sin \frac{\Delta\theta}{2} - 2\,N \sin\beta - C = 0 \tag{217}$$

Proceeding as in Art. 134, we finally obtain

$$\frac{T_1 - \dfrac{wv^2}{g}}{T_2 - \dfrac{wv^2}{g}} = e^{\frac{\mu\theta}{\sin\beta}} = e^{\mu'\theta} \tag{218}$$

With the usual conditions under which manila ropes run, the coefficient of friction μ may be assumed as 0.12, and the angle $2\,\beta$ as given in Art. 143 may be either 45 or 60 degrees. Using these coefficients, the values of $\dfrac{\mu}{\sin\beta}$ are as follows:

For 45-degree groove, $\mu' = 0.314$.
For 60-degree groove, $\mu' = 0.24$.

Horse power.—As in the case of belt transmission, the horse power is given by the formula

$$H = \frac{v}{550}(T_1 - T_2) \tag{219}$$

From (218), the net driving tension is given by the following expression:

$$T_1 - T_2 = \left[T_1 - \frac{wv^2}{g}\right]\left[\frac{e^{\mu'\theta} - 1}{e^{\mu'\theta}}\right] \tag{220}$$

Therefore

$$H = \frac{v}{550}\left[T_1 - \frac{wv^2}{g}\right]\left[\frac{e^{\mu'\theta} - 1}{e^{\mu'\theta}}\right] \tag{221}$$

It is important to note that there is a rope speed that makes the horse power transmitted a maximum, and beyond which the horse power decreases. An expression for the speed corresponding to the maximum horse power may be determined by equating the first derivative of H with respect to v to zero, and solving for v. Thus from (221)

$$\frac{dH}{dv} = K\left[T_1 - \frac{3\,wv^2}{g}\right]$$

whence for maximum H

$$v = \sqrt{\frac{gT_1}{3\,w}} \tag{222}$$

The general form of the curve expressing the relation between the horse power and the rope speed is shown in Fig. 74. The full line applies to a 1½-inch rope running on a sheave having a 45-degree groove, while the broken line applies to the same size of rope using a 60-degree groove. In plotting these graphs, it

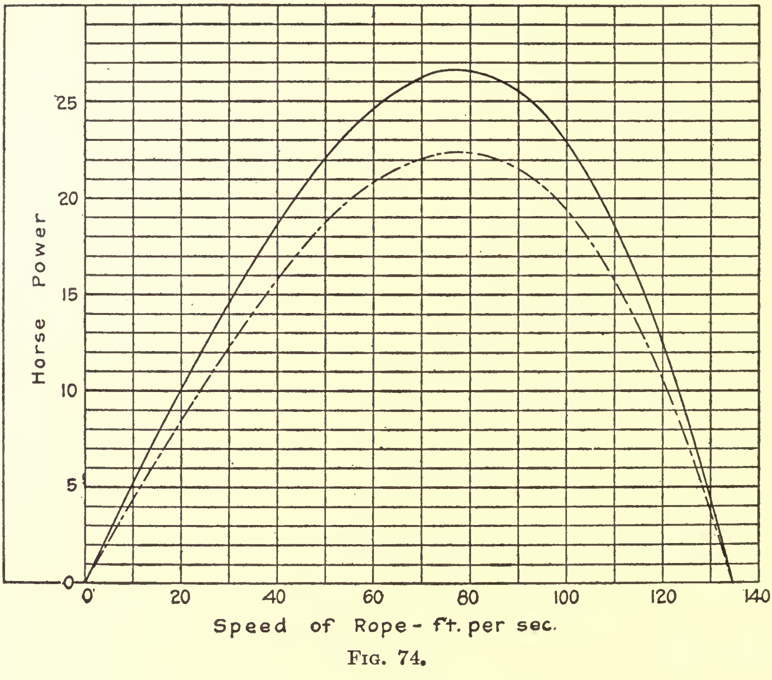

FIG. 74.

was assumed that the coefficient of friction was the same for both cases.

145. Force Analysis of a Manila Rope Transmission.—As stated in Art. 141, one of the functions of the tension carriage is to produce a uniform tension in the ropes, but the following analysis will disclose that such a condition is not realized in the continuous system. In Fig. 72 is shown diagrammatically what is known as the *American Open Drive.* It should be noticed that the tension carriage is located just off the driving sheave. From the discussion in Art. 137 and 144, we readily arrive at the following relations:

$$P = T_1 + T_8 = (k + 1)T_8 \qquad (223)$$

$$\left.\begin{aligned}
\frac{T_2}{T_1} &= e^{\mu'\theta_2} \therefore T_2 = kT_8 e^{\mu'\theta_2} \\
\frac{T_2}{T_3} &= e^{\mu'\theta_1} \therefore T_3 = kT_8 \frac{e^{\mu'\theta_2}}{e^{\mu'\theta_1}} \\
\frac{T_4}{T_3} &= e^{\mu'\theta_2} \therefore T_4 = kT_8 \frac{e^{2\mu'\theta_2}}{e^{\mu'\theta_1}} \\
\frac{T_4}{T_5} &= e^{\mu'\theta_1} \therefore T_5 = kT_8 \frac{e^{2\mu'\theta_2}}{e^{2\mu'\theta_1}} \\
\frac{T_6}{T_5} &= e^{\mu'\theta_2} \therefore T_6 = kT_8 \frac{e^{3\mu'\theta_2}}{e^{2\mu'\theta_1}} \\
\frac{T_6}{T_7} &= e^{\mu'\theta_1} \therefore T_7 = kT_8 \frac{e^{3\mu'\theta_2}}{e^{3\mu'\theta_1}}
\end{aligned}\right\} \qquad (224)$$

The total net tension on the driving sheave is the difference of the sum of the tensions on the tight and slack sides, or

$$T = T_2 + T_4 + T_6 - T_3 - T_5 - T_7 \qquad (225)$$

Now combining (224) with (225), the net tension T may be obtained in terms of T_8 and known constants; hence, the magnitude of T_8 is fully determined since the horse power transmitted and the rope speed are known. Knowing T_8, (223) enables us to establish the magnitude of the tension P.

By comparing the expressions for T_2, T_4 and T_6 it is evident that these tensions are not of the same magnitude, but that each successive tension on the tight side is smaller than the one preceding it. The same is true on the slack side. To overcome this inequality in the tension of the various ropes running over sheaves of unequal diameter, the above analysis shows that either one of the following methods could be used:

1. By using sheaves of different materials, thus changing the coefficient of friction μ so that $\mu_1\theta_1 = \mu_2\theta_2$.

2. By using the same material for both sheaves, but changing the angle of the grooves so that $\mu_1\theta_1 = \mu_2\theta_2$.

The latter method is the more practical and installations using this scheme are in successful operation. Mr. Spencer Miller was probably the first one to advocate using different groove angles on driving and driven sheaves of unequal diameters. The subject was discussed by Mr. Miller in a paper read before the

American Society of Civil Engineers in June, 1898, and reported in volume 39, page 165 of the *Transactions* of that society.

146. Sheave Pressures.—The series of equations given by (224) above enables us to determine the approximate pressures coming upon the shafts of the sheaves due to the rope tensions. The pressure upon the shaft of the driving sheave, assuming the tight and slack side to be practically parallel, is

$$Q_1 = T_2 + T_3 + T_4 + T_5 + T_6 + T_7 \qquad (226)$$

The pressure upon the shaft of the driven sheave is

$$Q_2 = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 \qquad (227)$$

The pressures upon the shafts of the idler sheaves a and b are respectively

$$Q_3 = T_7 + T_8, \qquad (228)$$

and

$$Q_4 = T_1 + T_8 \qquad (229)$$

The horse power absorbed by the friction of the bearings on the shafts, due to the pressure just determined, is considerable and may be estimated by the following expression:

$$H_f = \frac{\mu_3}{63{,}030}(Q_1N_1r_1 + Q_2N_2r_2 + Q_3N_3r_3 + Q_4N_4r_4), \qquad (230)$$

in which N denotes the number of revolutions per minute of the sheave, r the radius of the sheave shaft, and μ_3 the coefficient of journal friction.

147. Sag of Rope.—In practically all rope transmissions it is important to determine the approximate sag of the ropes. In arriving at a formula by means of which the probable sag may be estimated, no serious error is introduced by assuming that the rope hangs in the form of a parabola instead of a catenary. In Fig. 75 is shown a rope suspended over two sheaves, the line ABC representing approximately the curve assumed by the rope. From the equation of the parabola we have

$$\frac{L_1^2}{h_1} = \frac{L_2^2}{h_2} \qquad (231)$$

Substituting the value of $L_2 = L - L_1$ in (231) and reducing the expression to the simplest form, we finally get

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \qquad (232)$$

In a similar manner

$$L_2 = \frac{L\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} \tag{233}$$

The horizontal tension in the rope at the lowest point B is

$$T = \frac{wL_1^2}{2\,h_1} = \frac{wL_2^2}{2\,h_2} \tag{234}$$

The difference in the tensions at any two points of a rope forming a catenary is equal to the difference in elevation of these points multiplied by the weight per unit length of rope. Treating the rope ABC in Fig. 75 as if it formed a catenary and applying the property just mentioned, the tension T_a at A is

$$T_a = T + wh_1 = w\left[\frac{L_1^2}{2\,h_1} + h_1\right] \tag{235}$$

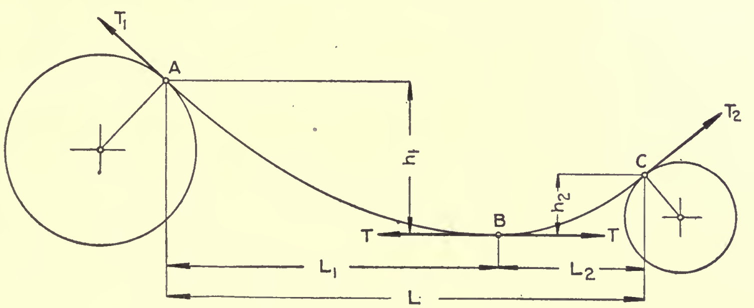

FIG. 75.

and the tension at C is

$$T_c = T + wh_2 = w\left[\frac{L_2^2}{2\,h_2} + h_2\right] \tag{236}$$

From (235), it follows that the magnitude of the sag h_1 is given by the following expression:

$$h_1 = \frac{1}{2w}(T_a \pm \sqrt{T_a^2 - 2\,L_1^2w^2}) \tag{237}$$

and from (236), the sag h_2 is

$$h_2 = \frac{1}{2w}(T_c \pm \sqrt{T_c^2 - 2\,L_2^2w^2}) \tag{238}$$

By means of (237) and (238) the sag of the ropes on either the tight or slack side of the transmission may be estimated by substituting the proper values for the tension. From an inspection of (237), it is evident that for the same tension T_a in the rope at A there are two different values of h_1; however, in rope-transmission problems the smaller value is the correct one to use. The statement applies equally well to (238).

It is important to note that the above discussion applies to the rope standing still. The sag of a rope transmitting power may be determined approximately by means of (234) by substituting the proper value of the tension T.

A special formula may be deduced for the case in which the transmission is horizontal having sheaves of the same diameter. By substituting for $L_1 = \frac{L}{2}$ in either (237) or (238), the amount of sag h is given by the following expression:

$$h = \frac{1}{2w}\left[T \pm \sqrt{T^2 - \frac{L^2w^2}{2}}\right] \qquad (239)$$

In general the bottom rope should form the driving side, as with this arrangement the sag of the slack rope on top increases the arc of contact.

148. Efficiency of Manila Rope Drives.—The efficiency of manila rope transmission is generally high according to several series of experiments performed both in this country and abroad. During the latter part of 1912, the Dodge Mfg. Co. of Mishawaka, Ind. conducted a series of experiments to obtain some information relating to the efficiencies of four general plans of manila rope driving. The four plans investigated were as follows:

1. Open drive using the American or continuous system, as shown in Fig. 72.
2. Open drive using the English or multiple system.
3. American "up and over" drive.
4. English "up and over" drive.

In the tests upon these various plans of rope driving, from one- to eight-ropes, operating at speeds ranging from 2,500 to 5,500 feet per minute were used. High-grade manila ropes one inch in diameter, treated with a rope dressing so as to make them moisture-proof and to preserve the surface, were used throughout the tests. The sheave grooves were in accordance with accepted Dodge practice, namely a 60-degree angle for the American system and a 45-degree angle for the English system. All idler sheaves used in the various arrangements were provided with U-shaped grooves.

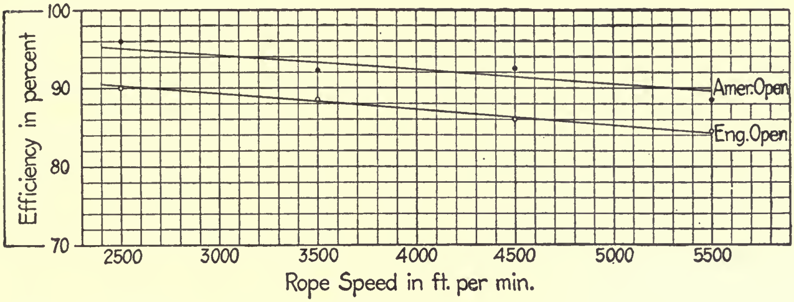

FIG. 76.

Altogether about seven hundred tests were made, the general results of which were published in a paper presented by Mr. E. H. Ahara before the American Society of Mechanical Engineers.

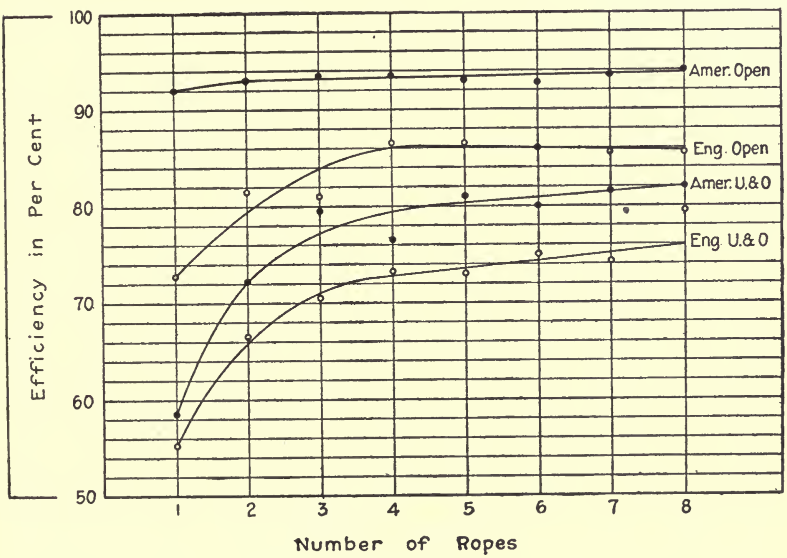

FIG. 77.

An analysis of the results published seemed to indicate that the efficiency for low rope speeds was higher than that obtained at the high rope speeds. This result is shown clearly in Fig. 76, which represents the results obtained from the tests on both systems of open drive operating with six ropes at three-quarters load, the distance between the centers of the sheaves being fifty feet. Furthermore, the tests showed that the efficiency was not affected materially by varying the distances between the driving and driven sheaves. The tests also showed that the efficiency at half load was but very little less than that obtained at full load. For the size of rope used in the experiments, namely one inch, the American system had considerable more capacity as well as a higher efficiency than the English system. In Fig. 77 is represented the relation existing between the efficiency and the number of ropes used for the four plans of driving.

149. Selection of Rope.—Manila ropes for transmission purposes are seldom less than one inch in diameter, and due to the resistance offered to bending over the sheaves, ropes exceeding one and three-quarter inches in diameter are not in general use. For heavy loads such as are met with in rolling-mill installations, ropes two inches in diameter and larger are used.

In order to arrive at the proper number and size of ropes required to transmit a given horse power, the size of both the driving and driven sheaves should be decided, as the smallest sheave in the proposed installation will determine in a general way the largest rope that may be used. If possible, the diameters of these sheaves should be such that the rope will operate at somewhere near its economical speed, which, as stated in Art. 142, has been found in practice to be about 4,500 feet per minute. To obtain a reasonable length of service from a given rope, its diameter should not exceed one-fortieth of the diameter of the smallest sheave. According to the American Manufacturing Co. of Brooklyn, N. Y., it is considered good practice to use a small number of large ropes instead of a large number of small ropes, notwithstanding the fact that the first cost of the sheaves for the former exceeds that required for the smaller ropes. In an installation using a small number of large ropes the number of splices is smaller; hence, the number of shutdowns due to the failure of splices is decreased; furthermore, since the large rope has a greater wearing surface, its life is increased.

150. Cotton Rope Transmission.—The transmission of power by means of cotton rope is not used to any extent in this country, but in England it is used extensively in all kinds of installations.

The strength of good cotton rope is about four-sevenths of that of high-grade manila rope, and its first cost is about 50 per cent. more. Due to the soft fiber, the cotton rope is more flexible than the manila rope, and for that reason smaller sheaves may be used for the former. According to well-established English practice, the diameters of the sheaves are made equal to thirty times the diameter of the rope. The cotton rope, as generally used, is composed of three strands, and being somewhat soft, it is wedged into the grooves of the sheave.

According to some of the American rope manufacturers, a manila rope of a given size will transmit considerably more power than the same size of cotton rope. In view of this statement it is interesting to compare the power that a given size of both manila and cotton rope, say 1½ inches in diameter, will transmit at a speed of 4,500 feet per minute. According to a well-known American manufacturer, the manila rope under the above conditions will transmit 29.1 horse power. According to a table published by Edward Kenyon in the *Transactions* of the South Wales Institute of Engineers, a 1½-inch cotton rope will transmit 33.4 horse power at the same speed. This result represents an increase of 14.7 per cent. in the power transmitted, and also indicates that higher tensions are permissible with cotton rope. As stated above, cotton rope is not as strong as manila rope; hence, these higher tensions must be due to the structure of the rope. The fibers of cotton rope being soft and more flexible do not cut or injure each other when the rope is subjected to bending under a tension, as is the case with the manila fiber; the grooves of the cotton rope sheave are so formed that the rope is wedged into the groove angle; hence, the effect of centrifugal force is not so marked as with manila rope transmission. The inference is clear that it is possible to employ high speeds with cotton rope; and such is the case, as English manufacturers recommend speeds up to 7,000 feet per minute.

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CHAPTER IX

WIRE ROPE TRANSMISSION

The present-day application of wire rope is chiefly to hoisting, haulage, and transporting service, and but little to the actual transmission of power. In this chapter, wire rope will be discussed under two general subheads as follows: (*a*) wire rope hoisting, and (*b*) wire rope transmission.

WIRE ROPE HOISTING

For haulage service, the six-strand seven-wire rope, generally written 6×7, is used, while for hoisting a 6×19, 8×19, or 6×37 construction is employed. The rope last mentioned is the most flexible and may be used with smaller sheaves than either of the others, but the wires are much smaller; hence it should not be subjected to excessive external wear. The 6×19 and 8×19 ropes are recommended for use on cranes, elevators of all kinds, coal and ore hoists, derricks, conveyors, dredges, and steam shovels. The 6×37 rope, which is extra flexible, is used on cranes, special hoists for ammunition, counterweights on various machines, and on dredges.

A hoisting rope under load is subjected to the following principal stresses:

(*a*) Stresses due to the load raised.
(*b*) Stresses due to sudden starting and stopping.
(*c*) Stresses due to the bending of the rope about the sheave.
(*d*) Stresses due to slack.

151. Relation between Effort and Load.—In hoisting machinery calculations, it is necessary to know the relation existing between the effort and the resistance applied to the ends of the rope running over a sheave. The rigidity of the rope and the friction of the sheave pin increase the resistance that the effort applied to the running off side must overcome. By applying the same line of reasoning as used in Art. 137, we obtain a relation which is similar to (207), namely

$$P = \frac{D + \mu d + 2\,s'}{D - \mu d - 2\,s'}\,Q = CQ \tag{240}$$

The efficiency of the ordinary guide sheave, obtained by applying the usual definition of efficiency, is as follows:

$$\eta = \frac{1}{C} \tag{241}$$

152. Stresses Due to Starting and Stopping.—A rope whose speed changes frequently, as in the starting and stopping of a load, is subjected to a stress which in many cases should not be neglected. This stress depends upon the acceleration given to the rope, and its magnitude is determined by the well-known relation, force is equal to the mass raised multiplied by the acceleration. In the calculation of the size of rope for mine hoisting or for elevator service, the stress due to acceleration assumes special importance.

153. Stresses Due to Bending.—The stresses due to the bending of the rope about sheaves and drums are of considerable magnitude and should always be considered in arriving at the size of a rope for a given installation. Several formulas for calculating these stresses have been proposed by various investigators, but they are all more or less complicated. The simplest of these is the following:

$$S_b = E\frac{\delta}{D}, \tag{242}$$

in which D represents the pitch diameter of the sheave, E the modulus of elasticity of the rope, S_b the bending stress per square inch of area of wires in the rope, and δ the diameter of the wire in the rope. This formula was adopted by the American Steel and Wire Co. To determine the value of E the company conducted a series of experiments on some six-strand wire rope having a hemp center. This investigation seemed to show conclusively that the modulus of elasticity for a new rope does not exceed 12,000,000. Using this value in (242), a series of tables was calculated and published in the company's Wire Rope Hand Book. From these data the curves shown in Figs. 78, 79 and 80 were plotted. They show the relation between the bending

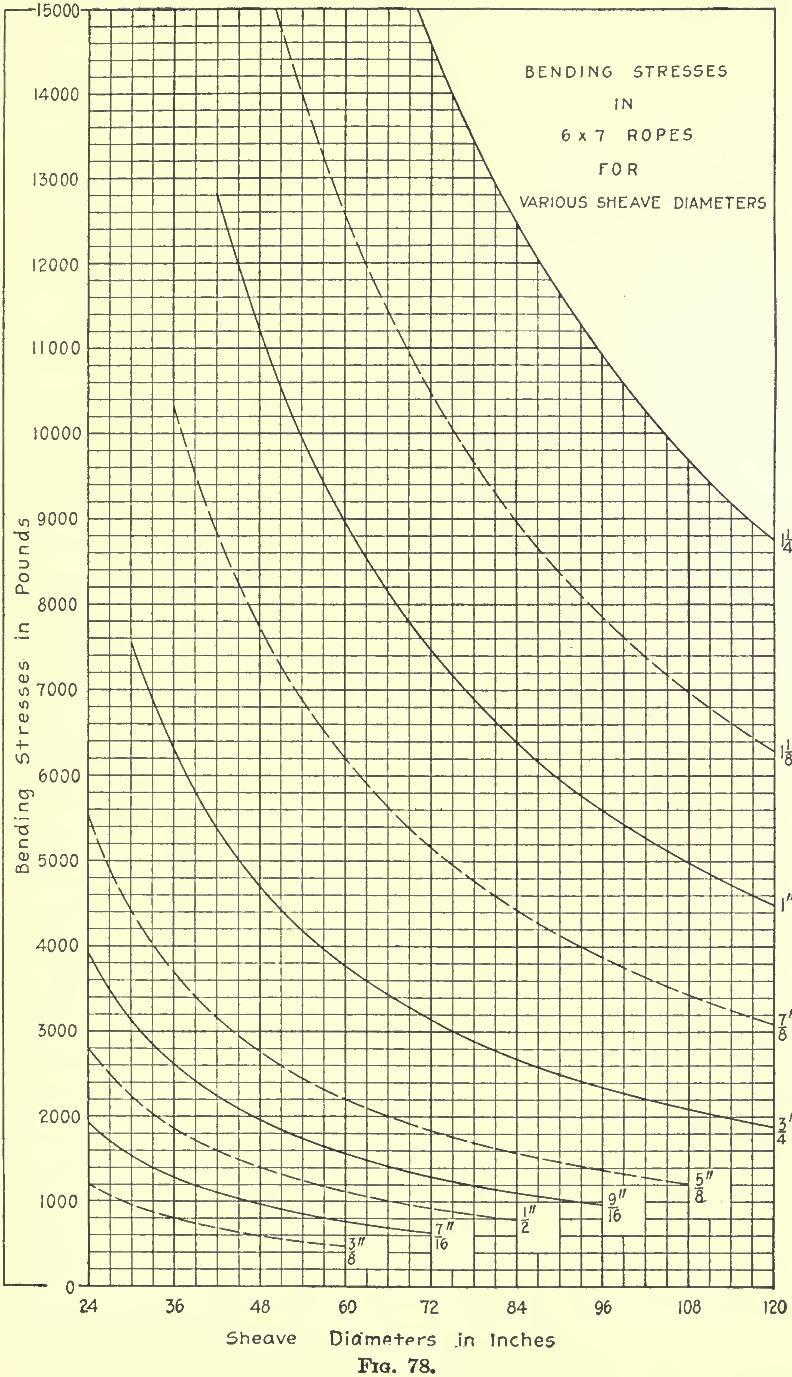
BENDING STRESSES
IN
6 x 7 ROPES
FOR
VARIOUS SHEAVE DIAMETERS
Bending Stresses in Pounds
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
24
36
48
60
72
84
96
108
120
Sheave Diameters in Inches
1 1/4″
1 1/8″
1″
7/8″
3/4″
5/8″
9/16″
1/2″
7/16″
3/8″

Fig. 78.

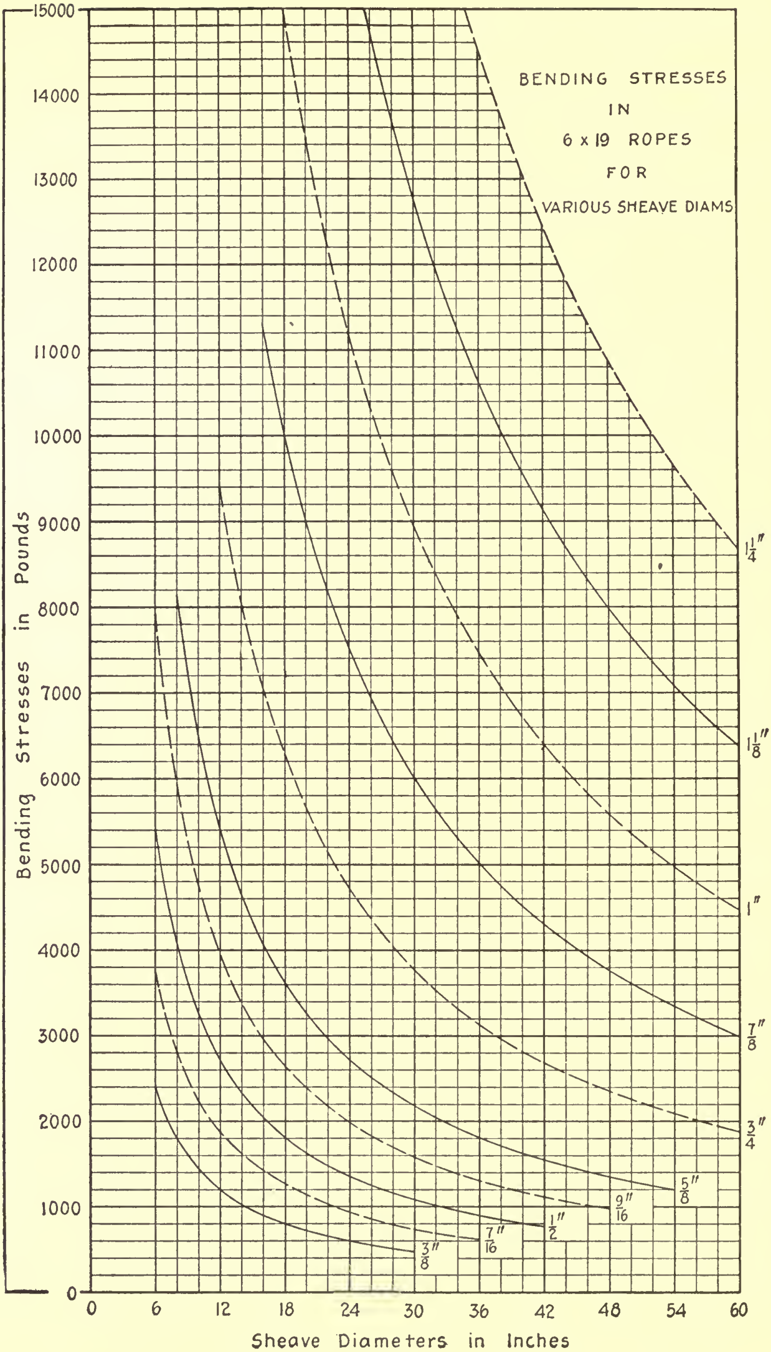
BENDING STRESSES
IN
6 x 19 ROPES
FOR
VARIOUS SHEAVE DIAMS
Bending Stresses in Pounds
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
6
12
18
24
30
36
42
48
54
60
Sheave Diameters in Inches
1 1/4"
1 1/8"
1"
7/8"
3/4"
5/8"
9/16"
1/2"
7/16"
3/8"

Fig. 79.

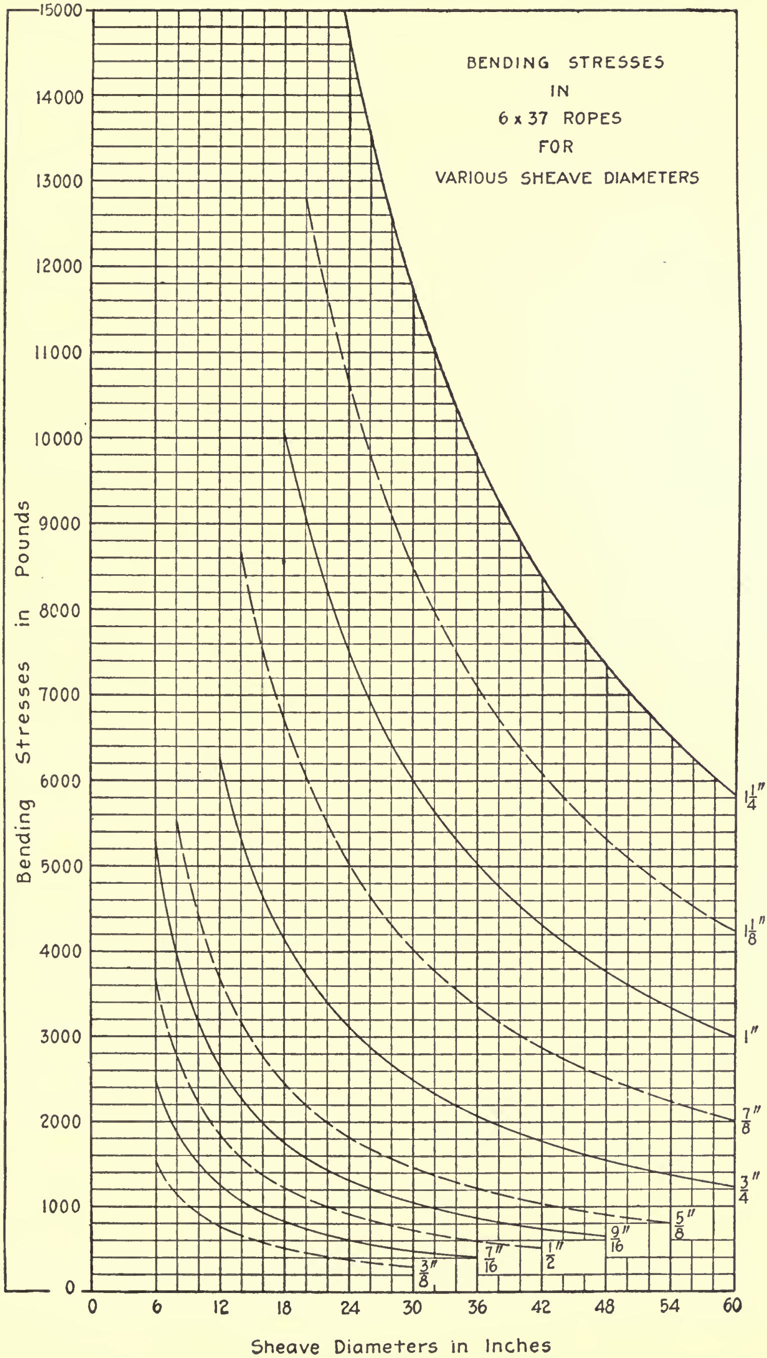
BENDING STRESSES
IN
6 x 37 ROPES
FOR
VARIOUS SHEAVE DIAMETERS
Bending Stresses in Pounds
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
6
12
18
24
30
36
42
48
54
60
1 1/4″
1 1/8″
1″
7/8″
3/4″
5/8″
9/16″
1/2″
7/16″
3/8″
Sheave Diameters in Inches

FIG. 80.

stresses and various diameters of sheaves or drums for the more common sizes of 6 × 7, 6 × 19 and 6 × 37 wire ropes. However, instead of using the ton as a unit, all stresses are reduced to pounds.

154. Stresses Due to Slack.—In any kind of hoisting operation it is important that the rope shall have no slack at the beginning of hoisting, else the load will be suddenly applied and the stress in the rope will be much in excess of that due to the load raised. The results of various dynamometer experiments in this connection are exhibited in Table 45.

TABLE 45.—TENSIONS DUE TO SLACK AS SHOWN BY DYNAMOMETER

Slack in inches	Weight of cage and load			
	3,672	6,384	11,312	11,310
0	4,032	6,720	11,542	11,525
5	5,600	11,200	19,040	19,025
6	8,960	12,320	23,520	25,750
12	12,520	15,680	28,000	28,950

The theoretical relation between the tension in the rope and the load raised may be deduced as follows:

Let W = load to be raised.
T = tension in the rope corresponding to the maximum elongation.
a = acceleration of the rope at the beginning of hoisting.
b = elongation of the rope due to the load W.
c = maximum elongation of the rope.
e = amount of slack in the rope.

The raising of the rope through the distance e, so as to take up the slack, may be considered as producing the same effect as dropping the load W through the distance e, assuming the acceleration in both cases as constant and equal to a. Letting v denote the velocity at the instant when the slack e is taken up, we have $v^2 = 2\,ae$. From this it follows that the kinetic energy of the load W at the instant the rope is taut, is

$$\frac{Wv^2}{2\,g} = \frac{Wae}{g} \tag{243}$$

Due to this loading the rope elongates a distance c, the final tension being T. Hence W in moving through this distance c

does work equal to Wc. Immediately preceding the elongation of the rope, the tension therein is zero and at the end of the elongation the tension has a magnitude T; therefore, the work of the variable tension during the period of rope elongation is $\frac{Tc}{2}$. To do this internal work, the load has given up its kinetic energy $\frac{Wae}{g}$ and the work Wc; hence

$$\frac{Tc}{2} = \frac{Wae}{g} + Wc \tag{244}$$

Assuming that Hooke's Law will hold approximately in the case of a rope, we get

$$T = \frac{Wc}{b} \tag{245}$$

Substituting this value of T in (244), and solving for c, we finally get

$$c = b \pm b\sqrt{1 + \frac{2\,ae}{bg}} \tag{246}$$

The conditions of the problem indicate that the positive sign is the proper one; hence, substituting the value of c in (245),

$$T = W\left[1 + \sqrt{1 + \frac{2\,ae}{bg}}\right] \tag{247}$$

If the slack e is zero (247) shows that $T = 2\,W$; that is, the tension is double the load, which fact was established in Art. 18. The amount of slack simply has the effect of increasing the ratio $\frac{T}{W}$, which, as shown, cannot be theoretically less than two.

It is not to be expected that experiments would give exactly the theoretical values, on account of the fact that wire rope differs materially from a rigid rod, and a certain amount of stretch not according to Hooke's Law will come into play before the actual elongation of the material begins. This fact in a measure, relieves the "suddenness," so to speak, of the action, and we would expect the tensions measured by the dynamometer to be less than those given by (247). To get the experimental values by means of (247), it will be necessary to introduce a coefficient K in the equation, making it

$$T = W\left[1 + K\sqrt{1 + \frac{2\,ae}{bg}}\right] \tag{248}$$

This coefficient must of course be determined by experiments, and will doubtless vary with the construction of the rope and quite likely with the load W and the slack e. Unfortunately, in the experiments quoted in Table 45 no attempt was made to determine the acceleration of hoisting, and as a consequence, one essential factor is lacking; hence it is impossible to arrive at probable values of the coefficient K unless an assumption regarding the ratio a to b is made.

155. Selection of Rope.—The maximum stress coming upon a rope is the summation of the separate stresses that may be present in any installation. These separate stresses have been discussed in the preceding articles, and having determined their intensities, the magnitude of the maximum is readily obtained. The next step is to determine the ultimate strength of the probable size of rope to be used, by multiplying the maximum stress by a factor commonly called the *factor of safety*. This factor varies with the class of service for which the rope is intended, and the following values may serve as a guide in the solution of wire rope problems:

For elevator service the factor of safety varies from 8 to 12.
For hoisting in mines the factor of safety varies from 2½ to 5.
For motor driven cranes the factor of safety varies from 4 to 6.
For hand power cranes the factor of safety varies from 3 to 5.
For derrick service the factor of safety varies from 3 to 5.

Having calculated the ultimate strength, select the size of rope that is strong enough. In practically all hoisting rope calculations, it will be found that two or more wire ropes of different sizes and quality will satisfy the conditions of the problem; for example, from Table 46 it is evident that a ⅞-inch crucible steel rope and a ¾-inch plow steel rope of the 6 × 19 construction have the same ultimate strength; hence, either of these ropes could be selected. In the example just quoted, the ¾-inch plow steel rope would be preferable to the ⅞-inch crucible steel rope, since the smaller sheave called for by the former size would effect a saving of space as well as in the first cost. In a preceding paragraph, the uses of the various types of wire rope were discussed briefly. In Table 46 is given information pertaining to the ultimate strengths and weights of rope, as well as the minimum diameter of sheaves recommended by the manufacturer.

156. Hoisting Tackle.—The analysis of blocks and tackles reefed with wire rope is similar to that given in Art. 138 for manila rope, and the formulas deduced there also apply in the present case, provided a proper value is assigned to the coefficient C.

TABLE 46.—STEEL WIRE ROPE

Diameter in inches	6 × 7 construction				6 × 19 construction			
	Weight per foot	Minimum sheave diam.	Ultimate strength		Weight per foot	Minimum sheave diam.	Ultimate strength	
			Crucible steel	Plow steel			Crucible steel	Plow steel
¼					0.10	12	4,400	5,300
5/16	0.15	27	7,000	8,800	0.15	15	6,200	7,600
⅜	0.22	33	9,200	11,800	0.22	18	9,600	11,500
7/16	0.30	36	11,000	14,000	0.30	21	13,000	16,000
½	0.39	42	15,400	20,000	0.39	24	16,800	20,000
9/16	0.50	48	20,000	24,000	0.50	27	20,000	24,600
⅝	0.62	54	26,000	32,000	0.62	30	25,000	31,000
¾	0.89	60	37,200	46,000	0.89	36	35,000	46,000
⅞	1.20	72	48,000	62,000	1.20	42	46,000	58,000
1	1.58	84	62,000	76,000	1.58	48	60,000	76,000
1⅛	2.00	96	74,000	94,000	2.00	54	76,000	94,000
1¼	2.45	108	92,000	120,000	2.45	60	94,000	116,000
1⅜	3.00	120	106,000	144,000	3.00	66	112,000	144,000
1½	3.55	132	126,000	164,000	3.55	72	128,000	164,000
1⅝					4.15	78	144,000	188,000
1¾					4.85	84	170,000	224,000
1⅞					5.55	96	192,000	254,000
2					6.30	96	212,000	280,000
2¼					8.00	108	266,000	372,000
2½					9.85	120	340,000	458,000
2¾					11.95	132	422,000	550,000

Diameter in inches	8 × 19 construction				6 × 37 construction			
	Weight per foot	Minimum sheave diam.	Ultimate strength		Weight per foot	Minimum sheave diam.	Ultimate strength	
			Crucible steel	Plow steel			Crucible steel	Plow steel
⅜	0.20	12	9,320		0.22	12	9,300	10,600
7/16	0.27	14	12,600		0.30	14	12,700	15,000
½	0.35	16	16,000	19,000	0.39	16	16,500	19,500
9/16	0.45	18	20,200	24,000	0.50	18	21,000	25,000
⅝	0.56	21	24,800	30,000	0.62	21	25,200	32,000
¾	0.80	22	35,200	44,000	0.89	22	38,000	46,000
⅞	1.08	26	46,000	56,000	1.20	26	50,000	58,000
1	1.42	30	59,400	72,000	1.58	30	64,000	74,000
1⅛	1.80	34	76,000	92,000	2.00	34	78,000	92,000
1¼	2.20	38	94,000	112,000	2.45	38	100,000	116,000
1⅜	2.70	42	114,000	136,000	3.00	42	122,000	142,000
1½	3.19	45	132,000	160,000	3.55	45	142,000	168,000
1⅝					4.15		158,000	190,000
1¾					4.85		190,000	226,000
1⅞					5.55		212,000	250,000
2					6.30		234,000	274,000
2¼					8.00		300,000	368,000
2½					9.85		374,000	450,000
2¾					11.95		466,000	556,000

Experimental data on wire-rope hoisting tackle.—Some years ago the American Hoist and Derrick Co. of St. Paul, Minn., conducted a series of experiments on three standard sizes of blocks reefed with wire rope. The results of these tests are given in Table 47. By using the relation between P and Q in terms of C for the various combinations listed, it is possible to calculate the value of C. This was done by the author and the values are tabulated in the last two columns of Table 47.

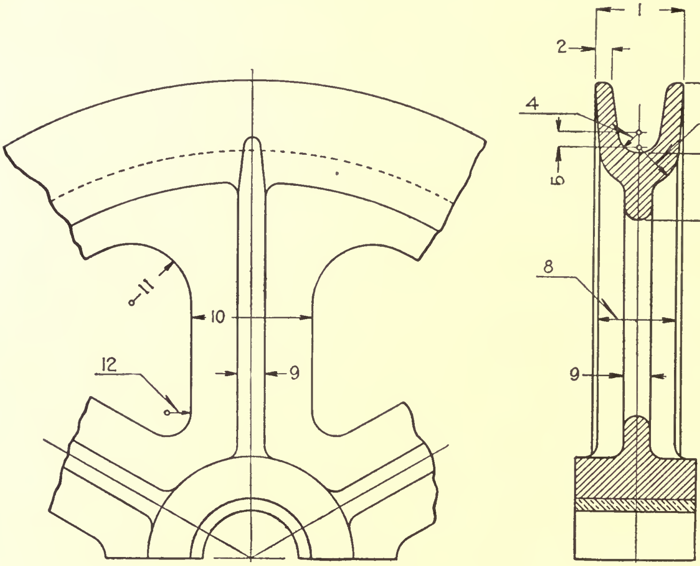

FIG. 81.

An inspection of the values of C given in this table shows that for a given size of rope the coefficient C may safely be assumed as constant.

157. Hoisting Sheaves and Drums.—(*a*) *Sheaves.*—The sheaves used for hoisting purposes vary considerably in their design. For crane work the sheaves are usually constructed with a central web in place of arms and in order to reduce the weight, openings may be put into this web. Such a sheave is shown in Fig. 81 and in Table 48 are given some of the leading dimensions pertaining to the design shown in Fig. 81. As a rule, sheaves of this class are bushed with bronze or some form of patented bushing, and run loose on the pin. For very heavy crane service, the sheaves are frequently made of steel casting, cast iron being used for the medium and lighter class of service.

Table 47.—Hoisting Tackle Reefed with Wire Rope

Size of rope	Block and tackle data				Ratio P/Q	Value of C	
	Sheave diam.	Pin diam.	No. of sheaves	No. of lines		Test	Mean
½	9	1¼	1	2	0.518	1.075	1.076
			2	2	0.559	1.078	
				3	0.358	1.076	
			3	3	0.385	1.076	
				4	0.278	1.076	
			4	4	0.298	1.075	
				5	0.230	1.076	
			5	5	0.247	1.076	
				6	0.198	1.076	
			6	6	0.213	1.076	
⅝	11⅛	1½	1	2	0.516	1.068	1.064
			2	2	0.549	1.066	
				3	0.355	1.066	
			3	3	0.376	1.063	
				4	0.273	1.063	
			4	4	0.291	1.064	
				5	0.225	1.063	
			5	5	0.240	1.064	
				6	0.193	1.064	
			6	6	0.206	1.064	
¾	13¾	1½	1	2	0.513	1.053	1.054
			2	2	0.541	1.055	
				3	0.351	1.054	
			3	3	0.369	1.053	
				4	0.270	1.054	
			4	4	0.284	1.053	
				5	0.221	1.054	
			5	5	0.233	1.054	
				6	0.189	1.054	
			6	6	0.199	1.053	

For heavy high-speed hoisting as found in mining operations, the arms consist of steel rods cast into the hub and rim, as shown in Fig. 82. Sheaves of this class are not bushed as in crane service, but are keyed to the shaft.

The grooves of all hoisting sheaves should be finished smooth

TABLE 48.—GENERAL DIMENSIONS OF WIRE ROPE SHEAVES

Size of rope	Dimensions in inches											
	1	2	3	4	5	6	7	8	9	10	11	12
½	1¾	¼	1	5/16	¼	⅝	......	.....	⅜			
⅝	1⅞	¼	1¼	⅜	5/16	11/16	......	1⅝	½			
¾	2	5/16	1½	7/16	5/16	13/16	1⅜	1¾	9/16	2⅜	1	½
⅞	2¼	⅜	1¾	½	⅜	⅞	1½	2	⅝	2⅝	1¼	¾
1	2½	7/16	2	9/16	⅜	1	1¾	2¼	11/16	2⅞	1½	¾
1⅛	2¾	7/16	2¼	⅝	7/16	1 1/16	2–2¼	2½	¾	3¼–4	1¾	1

so as to protect the individual wires of the rope. The radius of the bottom of the groove should be made slightly larger than the radius of the rope, so that the latter will not be wedged into

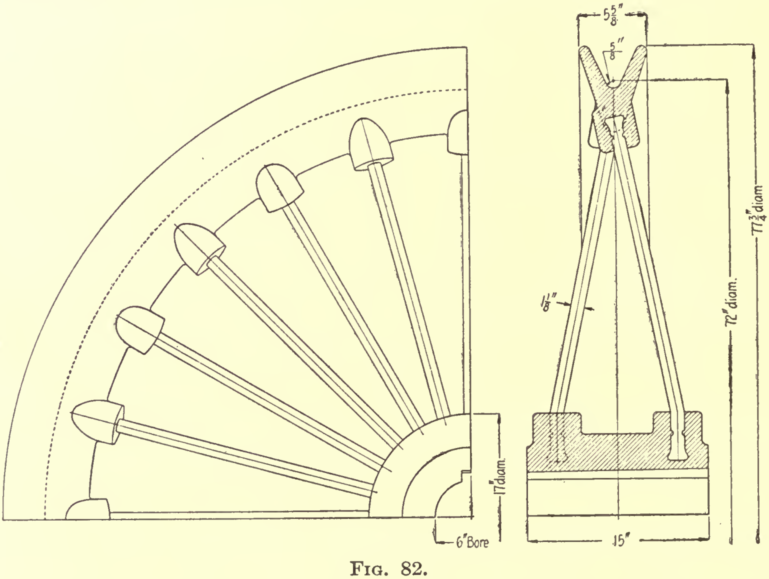

FIG. 82.

the groove. It is important that the alignment of sheaves be the best possible, otherwise the rope will slide on the sides of the groove and cause an undue amount of wear on both the rope and sheave. The diameter of the sheave should be made as large as possible to keep down the bending stresses. In Table 46 are given the minimum sheave diameters recommended by the wire rope manufacturers. It is customary for crane builders to use much smaller sheaves. By using sheaves having a diameter of from eighteen to twenty times the diameter of the rope, a considerable saving in space may result but at the same time the life of the rope is decreased materially.

(*b*) *Drums.*—In hoisting machinery, the drums are usually grooved to receive the rope and their lengths should be sufficient to hold the entire length of rope in a single layer. The use of the plain ungrooved drum should be avoided unless it is lagged. If the drum is grooved, the pitch of the grooves must be made slightly larger than the diameter of the rope so that the successive coils do not touch when the rope is wound onto the drum. In Fig. 83 is shown the form of groove used by several crane builders,

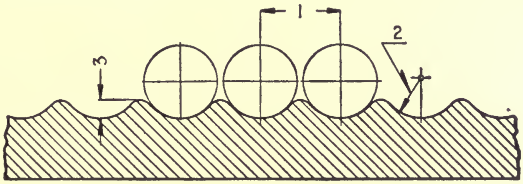

Fig. 83.

and in Table 49 are given the various dimensions required to lay out these grooves.

Table 49.—Dimensions of Grooves for Wire Rope Drums

Dimension	Size of wire rope							
	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
1	7/16	1/2	9/16	5/8	11/16	13/16	15/16	1 1/16
2	7/32	1/4	9/32	5/16	11/32	13/32	15/32	17/32
3	3/32	7/64	1/8	9/64	5/32	3/16	7/32	1/4

The diameters of the drums are usually made larger than those of the sheaves for a given size of rope in order to keep down the length to a reasonable dimension. In general, the diameters of crane drums vary from twenty to thirty times the diameter of the rope. The speed of hoisting, the load to be raised, and the life of the rope should be considered in arriving at the proper diameter of the drum. In order to relieve the rope anchor on the drum, always add about two extra coils to the calculated number, so that the extra coils of rope remain unwound on the drum.

158. Design of Crane Drums.—A simple design of a plain hoisting drum running loose on the shaft is shown in Fig. 84.

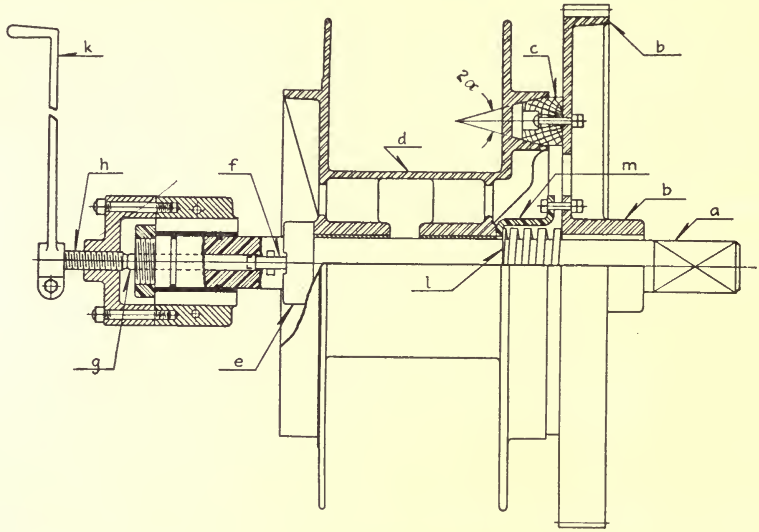

FIG. 84.

The shaft *a* is driven by means of the gear *b*, to the web of which are bolted the double conical friction blocks *c*. These blocks fit into the clutch rim, which in this case is integral with the drum

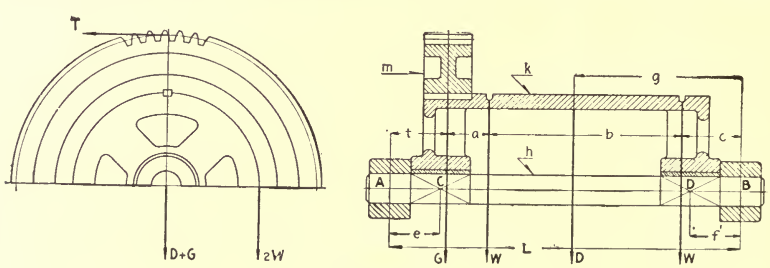

FIG. 85.

d. To rotate the drum with the gear, the clutch is engaged by sliding the drum along the shaft *a* by means of the operating mechanism shown at the left. This design is used on light hoisting engines manufactured by the Clyde Iron Works of Duluth, Minn.

A good design of a crane drum is shown in Fig. 85. In this case the shaft *h* is held stationary, the drum hubs being bushed with bronze as shown. The driving gear *m* is keyed rigidly to the drum *k*, which in this case is scored for a hoisting chain although the same design of drum may be used with rope. Frequently the shaft, instead of being stationary, is cast into the drum and the whole combination rotates on the outer bearings.

The correct stress analysis for a hoisting drum is a complicated problem, and the following approximate method is generally used in arriving at, or for checking, the thickness of the metal below the bottom of the groove:

1. Determine the bending stresses by treating the drum as a hollow cylindrical beam supported at the ends. Assume the maximum rope loads as concentrated at or near the middle, depending upon the scoring on the drum.

2. Determine the crushing stress due to the tension in the coils of rope about the drum. The rope tension varies from coil to coil, and since maximum values are sought, consider only the first coil, namely, the one supporting the load.

3. Determine the shearing stress due to the torsional moment transmitted. As a rule this stress is very small and is usually not considered.

4. Combine the stresses calculated in (1) and (2) above. Drums thus designed have sufficient strength, and in general the weight is not excessive.

159. Conical Drums.—In mine hoists, it is a usual practice to employ drums having varying radii for the successive coils of the rope. The object of such an arrangement is to obviate the variations in the load on the drum due to the varying length of rope. Theoretically, the net moment of the rope pull about the drum axis should be a constant in order that the motors or engines coupled to the drum may operate economically. This condition would require a drum of curved cross-section, a form that would be difficult to construct. In practice, the section of each half of the drum is given the form of a trapezoid, and for that reason it is possible to balance the moments on the drum at but two points of the hoist, namely at the top and bottom.

(*a*) *Relation between R_2 and R_1.*—To determine the relation existing between the large and small diameters R_2 and R_1 of the drum, so as to fulfil the condition just mentioned, we may proceed as follows:

Let C = weight of cage and empty car.
H = depth of mine in feet.
Q = weight of ore in car.
w = weight of the hoisting rope, pounds per foot.

Neglecting the inertia forces, the moment of the rope tension at the beginning of the hoisting period is

$$M_1 = (C + Q + wH)(1 + \mu)R_1 - C(1 - \mu)R_2, \quad (249)$$

in which the symbol μ represents a friction coefficient that may be assumed as equivalent to 0.05 for vertical mine shafts.

The moment at the end of a trip is

$$M_2 = (C + Q)(1 + \mu)R_2 - (C + wH)(1 - \mu)R_1 \quad (250)$$

Equating these moments and solving for the radius of the drum at the large end, we find

$$R_2 = \left[\frac{Q + 2C + 2wH + \mu Q}{Q + 2C + \mu Q}\right] R_1 = mR_1 \quad (251)$$

Evidently the greater the depth of the mine shaft, the greater is wH relative to Q and C, and the greater the value of the factor m.

(*b*) *Length of the conical drum.*—The conical drum must be provided with spiral grooves to receive the rope, and the number required to hoist from a depth H is

$$n = \frac{H}{\pi(m + 1)R_1} \quad (252)$$

Several extra turns are required, so that at the beginning of hoisting the rope will be coiled several times around the drum. The same number should be added at the end of hoisting. This number of extra turns is fixed by state mining laws.

If L denotes the length of the drums and p the horizontal pitch of the grooves, then

$$L = \left[\frac{H}{\pi(m + 1)R_1} + n'\right] p, \quad (253)$$

in which n' represents the extra number of coils added.

The length of a conical drum is necessarily great, and for that reason the drum must be located at a considerable distance from the mine shaft to reduce as much as possible the angular displacement of the rope from the center line of the head sheave.

This displacement is called the *fleet angle* and should not exceed one and one-half degrees on each side of the center line, or a total displacement of three degrees. When it is impossible to locate the drum far enough back from the head sheave to keep the fleet angle within these limits, it is necessary to guide the rope onto the head sheaves by means of rollers or auxiliary sheaves.

(*c*) *Composite drum.*—For deep mines, another form of drum called the composite drum is frequently substituted for the plain conical type. This consists of a cylindrical center portion and conical ends. One rope is wound from one end up the cone and over the cylindrical portion, while the other is unwound from the cylindrical part and down the other cone. This form of drum has the advantage of decreased diameter and shorter length, but possesses the disadvantage of not entirely balancing the effect of the rope.

160. Flat Wire Ropes.—In the preceding articles, the round wire rope has been discussed more or less in detail, and the various points brought out are applicable in general to the flat rope. This type of rope consists of a number of round wire ropes, called *flat rope strands,* placed side by side. Its principal uses are for mine hoisting; for operating emergency gates on canals; for operating the spouts on coal and ore docks; and in elevator service for counterbalancing the hoisting ropes. The individual strands, composed of four separate strands containing seven wires each, are of alternate right and left lay and are sewed together with soft Swedish iron or steel wire. The sewing wires, being much softer than the wires that compose the strands, serve as a cushion for the strand and at the same time will wear out much faster. Flat wire rope with worn out sewing may be resewed with new wire, and in case any particular strands are damaged, they may be replaced by new ones. Flat ropes are made in thicknesses varying from $\frac{1}{4}$ inch to $\frac{7}{8}$ inch, and widths ranging from $1\frac{1}{2}$ to 8 inches. The material used in the construction of flat ropes may be either crucible cast steel or plow steel, the former being more common. The following are some of the advantages flat ropes possess over round ropes.

1. In hoisting from deep mines it is desirable to use a rope that has no tendency to twist and untwist. This tendency is obviated by the use of a flat rope.

2. The reels required for coiling up the flat rope occupy less space and are much lighter and cheaper to construct than large

cylindrical and conical drums. The decrease in bulk and weight is especially important when the mines are located in places accessible only by pack train.

At the present time flat ropes are used but little for mine hoisting and hence the field of application of such ropes is more or less restricted.

WIRE ROPE TRANSMISSION

Wire rope as a medium for transmitting power is used where the distances are too great for manila ropes. The recent development of electrical transmission is gradually crowding out the wire rope, though for distances of from 300 to 1,500 feet it is considered a cheap and simple method of transmitting power. Two systems are used, namely, the *continuous or endless rope* used in operating cableways, haulage systems and tramways, and the *single loop*, the latter being simply a modification of belt driving.

161. Single Loop System.—To transmit power by means of a single loop with a minimum amount of slippage, a certain amount of pressure between the surfaces in contact is necessary. This pressure depends upon the weight and the tension of the rope. Therefore, for short spans it is frequently necessary to use a large rope in order to get the proper weight, although the tension may be increased by resplicing or by the introduction of a tightener. The last two methods are not considered good practice as the rope may be strained too much, and in addition, the filling in the bottom of the grooves of the sheaves wears away too rapidly.

Experience has shown that transmitting power by means of wire rope is generally not satisfactory when the span is less than 50 to 60 feet. This is due to the fact that the weight of the rope is not sufficient to give the requisite friction without the use of tighteners. When the distance between the shaft centers exceeds 400 feet, successive loops are used; that is, the driving sheave of the second loop is keyed fast to the shaft of the driven sheave of the first loop, or double-groove sheaves may be used.

162. Wire Transmission Rope.—Wire rope used for transmitting power consists of six strands laid around a hemp or wire core, each strand containing seven wires. The rope with a hemp core is more pliable and for that reason is generally preferred for power transmission. As mentioned in a preceding paragraph, large ropes are occasionally required to get a satisfactory drive, and in such installations a six-strand nineteen-wire rope is to be preferred. The 6 × 7 construction of rope, having much larger wires, will stand more wear than the 6 × 19 construction, but requires much larger sheaves. The material used in the manufacture of wire transmission rope is iron, crucible cast steel, and plow steel. In Table 46 is given information pertaining to two kinds of high-grade transmission rope.

163. Transmission Sheaves.—The sheaves for transmission rope are quite different from those used with manila rope, as will be seen by consulting Fig. 86. The grooves are made V shape with a space below, which is filled with leather, rubber, or hardwood blocks. One prominent manufacturer uses alternate layers of leather and blocks of rubber for a filling. The function of this filling is to increase the friction between the rope and sheave and at the same time reduce the wear of the rope to a minimum. The filling should have a depression so that the rope will run central and not come into contact with the iron sides of the grooves. The speed of the rim of the sheave should not exceed 5,000 feet per minute.

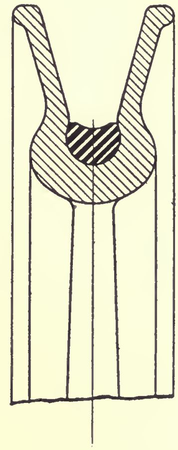

Fig. 86.

The diameter of the sheave should be made as large as practicable consistent with the permissible rim speed.

Large sheaves decrease the bending stresses and at the same time increase the transmitting power of the rope. In Table 46 are given the minimum diameter of sheaves that should be used with the various sizes of 6 × 7 and 6 × 19 transmission rope.

164. Stresses in Wire Rope.—The maximum stress in a wire rope due to the power transmitted should always be less than the difference between the maximum allowable stress and that due to the bending of the rope. For the magnitude of the bending stress in a rope running over a sheave, consult Fig. 78. As the bending stress decreases, the load stress may be increased, but the sum of these two separate stresses should never exceed from one-third to two-fifths of the ultimate strength of the rope given in Table 46. No provision is made, however, for the weakening effect of a splice in the rope. To prevent slippage between the rope and the sheave, the ratio of the tight to the loose tension must have a value given by the following expression, which may be derived directly from (218) by making the angle β equal to 90 degrees. The symbols used have the same meaning as assigned to them in Art. 144.

$$\frac{T_1 - \frac{wv^2}{g}}{T_2 - \frac{wv^2}{g}} = e^{\mu\theta} \tag{254}$$

For the coefficient of friction μ, Mr. Hewitt in his treatise published by the Trenton Iron Co., recommends the values given in Table 50.

TABLE 50.—COEFFICIENTS OF FRICTION FOR WIRE ROPE

Type of groove	Condition of rope		
	Dry	Wet	Greasy
Plain groove	0.170	0.085	0.070
Wood-filled	0.235	0.170	0.140
Rubber- and leather-filled	0.495	0.400	0.205

To determine the horse power capable of being transmitted by a given size of wire rope use (221), substituting for μ' in that equation the proper value from Table 50. As in the case of manila ropes, there is a speed that makes the horse power transmitted a maximum and beyond which the horse power decreases. To determine the speed corresponding to the maximum horse power, use (222).

165. Sag of Wire Rope.—The question of sag was discussed in Art. 147 in connection with manila ropes and the various formulas deduced also apply in the present discussion. It is desirable, whenever possible, to make the lower rope of a transmission do the driving; the upper or slack rope sags, thereby increasing the angle of contact on both sheaves and, at the same time, the transmitting capacity of the installation. According to the Trenton Iron Co., the sag of the tight or lower rope should be about one-fiftieth of the span, and that of the slack rope about double this amount.

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CHAPTER X

CHAINS AND SPROCKETS

The various types of chains found in engineering practice may, according to their use, be grouped into the following classes:

(*a*) Chains intended primarily for hoisting loads.

(*b*) Chains used for conveying as well as elevating loads.

(*c*) Chains used for transmitting power.

HOISTING CHAIN

166. Coil Chain.—The kind of chain used on hoists, cranes, and dredges is shown in Fig. 87(*a*) and is known as *coil chain.*

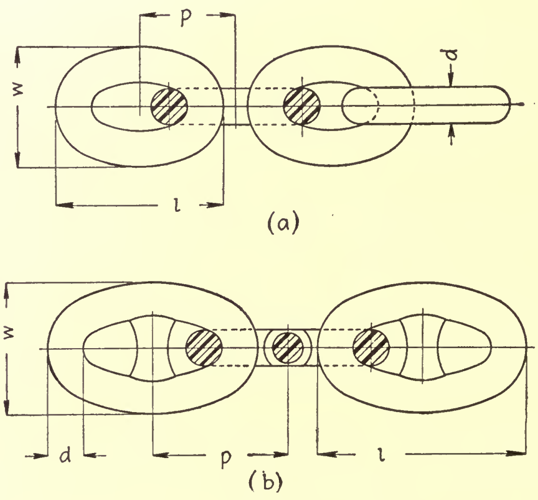

Fig. 87.

The links are made either of an elliptical shape or with the sides parallel, and the material used should be a high-grade refined wrought iron or an open-hearth basic steel. A chain made of the latter material has a higher tensile strength and at the same time stands greater abrasive wear than one made of wrought iron. In order to insure flexibility in a chain, the links should be made small. A small link has the added advantage that the bending action at the middle and at the end of the link due to the pull between adjacent links is decreased. In Table 51 are given the general dimensions of the link, weight per foot, and the approximate breaking strength of the commercial sizes of dredge and crane chains.

Table 51.—Hoisting Chains

Size	Pitch	Weight per foot	Outside length, in.	Outside width, in.	Dredge and shovel	BBB crane
					Approx. breaking load, lb.	
1/4	25/32	0.75	1 5/16	7/8	5,000	4,000
5/16	27/32	1.00	1 1/2	1 1/16	7,000	6,000
3/8	31/32	1.50	1 3/4	1 1/4	10,000	9,000
7/16	1 5/32	2.00	2 1/16	1 3/8	14,000	13,000
1/2	1 11/32	2.50	2 3/4	1 11/16	18,000	17,000
9/16	1 15/32	3.25	2 5/8	1 7/8	22,000	20,000
5/8	1 23/32	4.00	3	2 1/16	27,000	26,000
11/16	1 13/16	5.00	3 1/4	2 1/4	32,500	30,000
3/4	1 15/16	6.25	3 1/2	2 1/2	40,000	36,000
13/16	2 1/16	7.00	3 3/4	2 11/16	42,000	40,000
7/8	2 3/16	8.00	4	2 7/8	48,000	44,000
15/16	2 7/16	9.00	4 3/8	3 1/16	54,000	50,000
1	2 1/2	10.00	4 5/8	3 1/4	61,000	57,000
1 1/16	2 5/8	12.00	4 7/8	3 5/16	69,000	65,000
1 1/8	2 3/4	13.00	5 1/8	3 3/4	78,000	72,000
1 3/16	3 1/16	14.50	5 9/16	3 7/8	88,000	80,000
1 1/4	3 1/8	16.00	5 3/4	4 1/8	95,000	88,000
1 5/16	3 3/8	17.50	6 1/8	4 1/4	104,000	96,000
1 3/8	3 9/16	19.00	6 7/16	4 9/16	114,000	104,000
1 7/16	3 11/16	21.16	6 11/16	4 3/4	122,000	116,000
1 1/2	3 7/8	23.00	7	5	134,000	124,000
1 9/16	4	25.00	7 3/8	5 5/16	142,000	132,000
1 5/8	4 1/4	28.00	7 3/4	5 1/2	154,000	144,000
1 3/4	4 3/4	31.00	8 1/2	5 7/8	166,000	
1 7/8	5 1/4	35.00	9 1/4	6 3/8	190,000	
2	5 3/4	40.00	10	6 3/4	216,000	
2 1/4	6 3/4	53.00	11 1/2	7 5/8	273,000	
2 1/2	7	65.00	12 1/4	8 3/8	337,000	
2 3/4	7 1/4	73.00	13	9 1/8	387,000	
3	7 3/4	86.00	14	9 7/8	436,000	

167. Stud-link Chain.—A type of chain known as the *stud-link chain* is shown in Fig. 87(*b*). It is used mainly in marine work in connection with anchors and moorings. The chief advantage of the stud-link chain is that it will not kink nor entangle as readily as a coil chain. Experiments show that for the same size of link, the addition of the stud results in a decrease of the ultimate strength of the chain. An analysis of the stresses in chains shows, however, that within the elastic limit the stud-link chain will carry a much greater load than the open-link chain. See *Bulletin* No. 18 Univ. of Illinois Experiment Station, G. A. Goodenough and L. E. Moore.

168. Chain Drums and Anchors.—In practically all cases where short-link chains are used for heavy service, as on cranes and dredges, drums are used for winding up the chain. Such

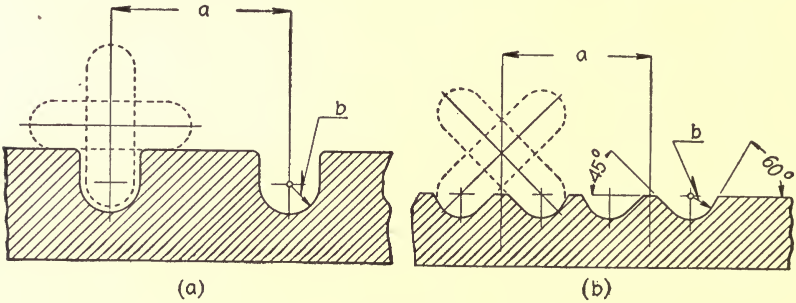

FIG. 88.

drums should always be provided with machined grooves. Two forms of such grooves are shown in Fig. 88, and the dimensions given in Table 52 will be found convenient for layout purposes.

TABLE 52.—DIMENSIONS OF GROOVES FOR CHAIN DRUMS

Type	Dimension	Size of chain										
		3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
(*a*)	*a*	1 1/2	1 11/16	1 7/8	2 1/16	2 5/16	2 1/2	2 11/16	2 7/8	3 1/8	3 5/16	3 1/2
	b	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	17/32	9/16
(*b*)	*a*	1 1/4	1 7/16	1 9/16	1 3/4	1 7/8	2 1/16	2 3/16	2 3/8	2 1/2	2 11/16	2 13/32
	b	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2

The diameter of the drum depends upon the speed of hoisting, the loads to be raised, and the life of the chain. For close-link chain, it has been found by experience that the drum diameter should not be made less than twenty times the thickness of the chain material, and it is better to make it about thirty times the thickness of material. If the drum is made small in diameter relative to the size of the chain, the bending action on the link

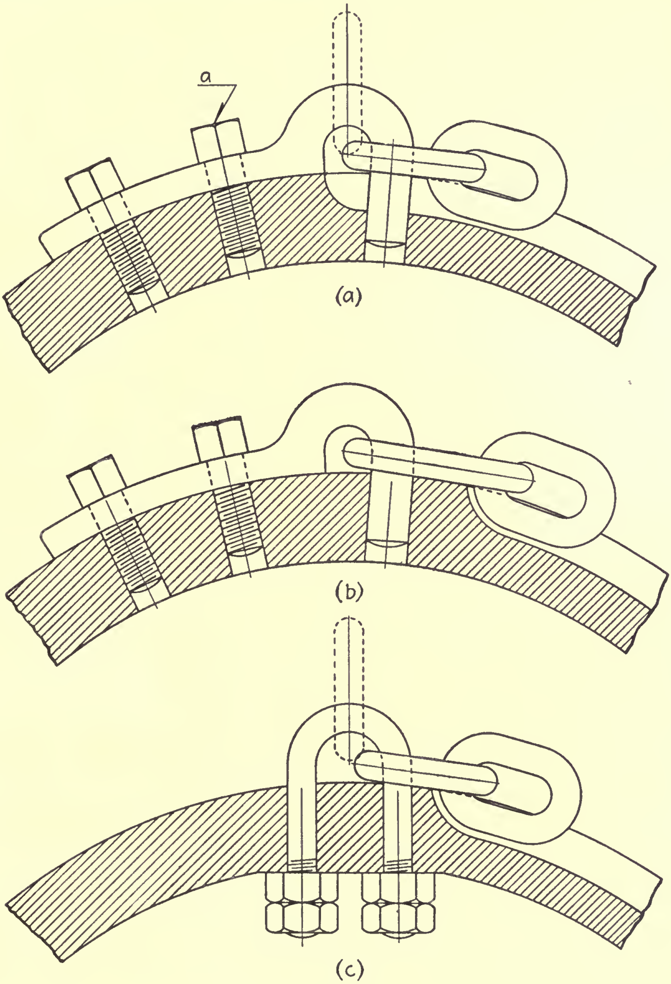

Fig. 89.

referred to in Art. 166, will be excessive, thus decreasing the life of the chain.

The drum should always be made of sufficient length so that the required length of chain may be wound upon it in a single layer. It is considered good design to have one or two coils of chain remaining on the drum when the load is in its lowest position, thus reducing the stress coming upon the anchor. The correct stress analysis for a hoisting drum is rather complex, and the approximate method outlined in Art. 158 for a drum using wire rope is applicable to chain drums.

Anchors.—The method of anchoring the free end of the chain to the drum should be given attention. In Fig. 89 are shown three designs taken from the practice of several crane builders. The method shown in Fig. 89(*a*) is faulty for the following reasons: (1) The hole for the tongue of the anchor is

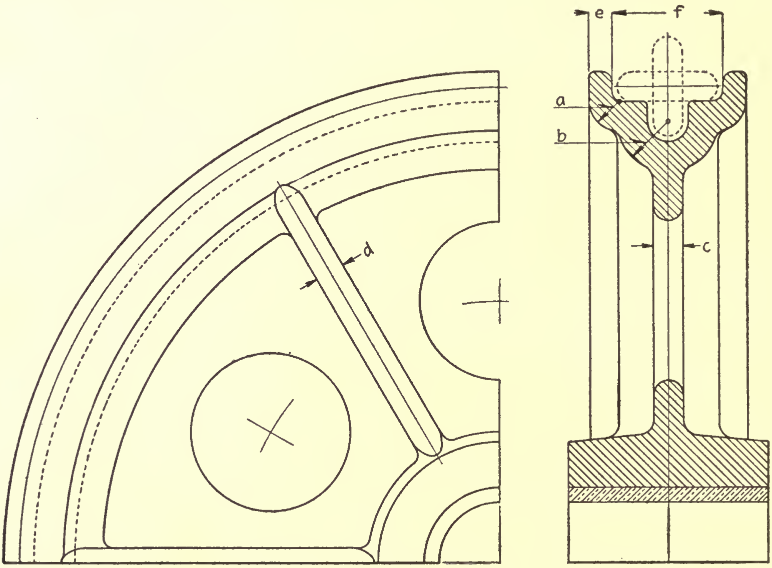

FIG. 90.

drilled in the chain groove, thus increasing the bending action on the tongue. (2) In case the chain should ever assume the position indicated by the dotted lines, the cap screw *a* will receive the greatest load instead of the tongue of the anchor.

The design shown in Fig. 89(*b*) overcomes the first objection in that the tongue of the anchor is placed in a hole drilled into the solid metal. The second objection, however, also applies to this design. In Fig. 89(*c*) is shown a construction that is cheap to make and at the same time overcomes both objections. In certain designs of drums it may not always be as convenient to attach an anchor of this type as one of the first two types.

169. Chain Sheaves.—Sheaves are of two classes, namely those that merely guide the chain as in changing direction, and those that are fitted with pockets to receive the links of the chain. The latter class is used extensively in chain hoists in place of drums, also for transmitting power under certain conditions.

(*a*) *Plain sheaves.*—Designs of plain sheaves, referred to as the first class, are shown in Figs. 90 and 91. The proportions of the two types of sheaves are given in Table 53. For sheaves of large diameters, arms are used, while with small sheaves the web center has given better satisfaction. The web centers may be plain,

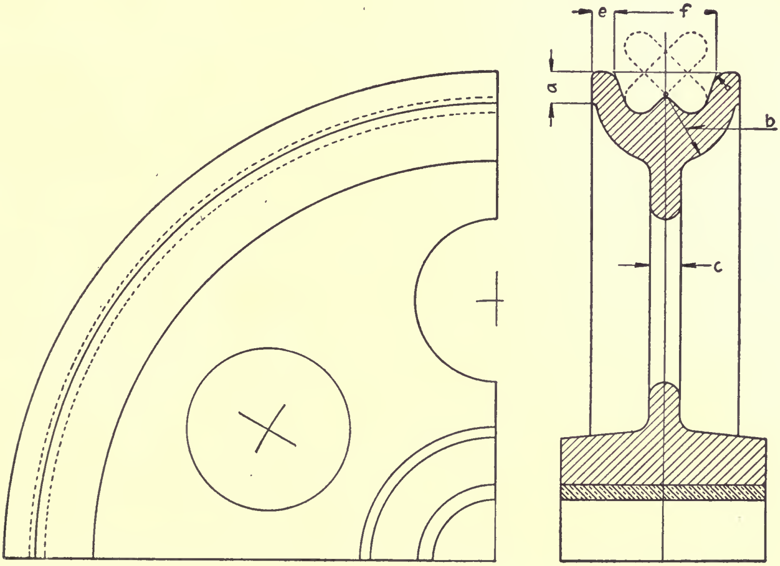

FIG. 91.

TABLE 53.—DIMENSIONS OF PLAIN CHAIN SHEAVES

Size of chain	Type—Fig. 90					Type—Fig. 91				
	a	b	c	e	f	a	b	c	e	f
3/8	15/32	21/32	15/32	9/32	1 9/16	3/8	7/8	15/32	9/32	1 1/4
1/2	19/32	27/32	19/32	5/16	1 15/16	1/2	1 1/16	19/32	5/16	1 5/8
5/8	3/4	1 1/16	3/4	3/8	2 3/8	5/8	1 5/16	3/4	3/8	2
3/4	7/8	1 1/4	7/8	7/16	2 3/4	3/4	1 9/16	7/8	7/16	2 7/16
7/8	1	1 7/16	1	1/2	3 1/4	7/8	1 13/16	1	1/2	2 3/4
1	1 1/8	1 5/8	1 1/8	9/16	3 3/4	1	2 1/16	1 1/8	9/16	3 1/8

or if it is desirable to decrease the weight, round holes may be cut out as shown in Figs. 90 and 91. To give stiffness to the center web, the side ribs as shown in Fig. 90 are added.

(*b*) *Pocket sheaves.*—Sheaves similar to the one shown in Fig. 92 are called *pocket sheaves* and are used principally on chain hoists in place of drums. The horizontal links fit into pockets cast in the periphery of the sheave, while the vertical links fall

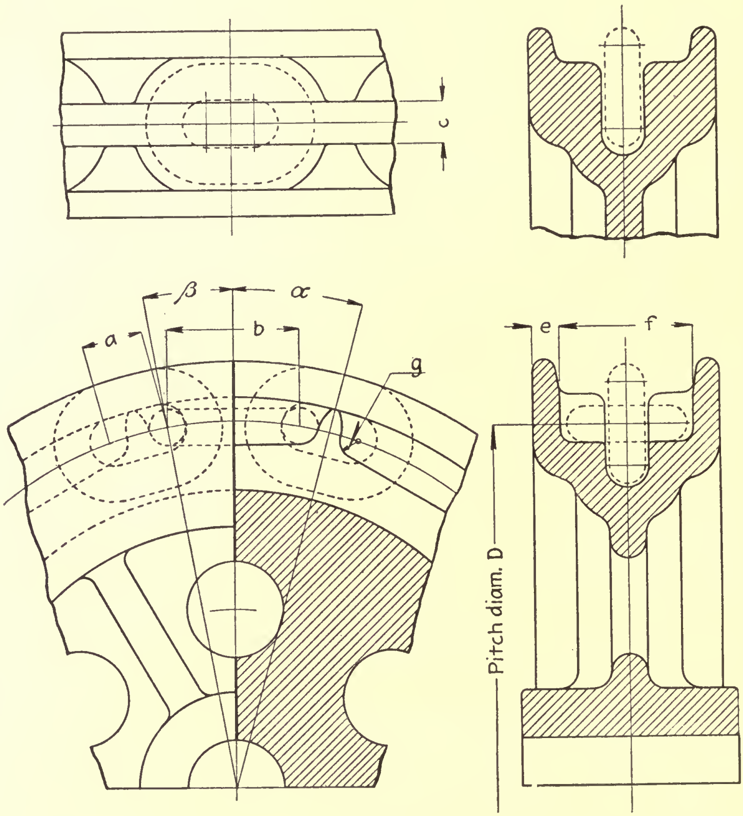

Fig. 92.

into a central groove as shown. In order to design a sheave of this type, the various calculations involving the formulas given below must be carried out with considerable accuracy. The dimensions of the chain and the number of pockets or teeth T desired enable one to derive a formula for the so-called pitch diameter D.

From Table 51, the dimensions a and b of Fig. 92 for the chosen size of chain are established, and from the number of pockets T, we find for the angle α,

$$\alpha = \frac{180°}{T} \tag{255}$$

From the geometry of the figure, the following equations are obtained:

$$\sin(\alpha - \beta) = \frac{a}{D} \tag{256}$$

$$\sin \beta = \frac{b}{D} \tag{257}$$

Eliminating D and solving for β, we obtain

$$\tan \beta = \frac{\sin \alpha}{\cos \alpha + \frac{a}{b}} \tag{258}$$

Since the dimensions a and b as well as the angle α are known in any given case, (258) is used to determine the angle β; having determined this angle, the pitch diameter D of the sheave is found by means of (257).

The rim of the pocket sheave may be proportioned by the following empirical formulas, in which d and w denote the dimensions of the chain links as given in Fig. 87. Referring to Fig. 92:

$$\left.\begin{aligned} c &= d + (\tfrac{1}{8}'' \text{ to } \tfrac{5}{16}'') \\ e &= \tfrac{3}{4}\, d \\ f &= w + (\tfrac{1}{8}'' \text{ to } \tfrac{5}{16}'') \\ g &= \tfrac{1}{2}\, d \end{aligned}\right\} \tag{259}$$

The thickness of the web should not be made less than the diameter of the material in the chain, and the diameter of the hub should be approximately twice the diameter of the pin supporting the sheave.

Having determined the pitch diameter and the general proportions of the rim, web, and hub of the sheaves, the next step is to make a full-size drawing with the pockets from ⅛ to ¼ inch longer than the link. The layout of the tooth shown in Fig. 92 represents the tooth form at the center line of the sheave and not at the side of the central groove where it should begin. However, since the pockets are to be made somewhat longer than the links, the tooth may be given this form at the side of the central groove and it will be found that sufficient clearance is thus provided in the majority of cases. The face of the tooth, or that part lying above the pitch circle, may be drawn with a radius equal to three or four times the diameter of the chain material.

170. Relation between P and Q.—When a chain is wound on, or unwound from, a sheave, the relative motion between the links on the running-on and -off sides introduces frictional resistances. The turning of one link in another is similar to that of a journal running in its bearing, and the relation between the applied effort P and the resistance Q will be based upon the theory of journal friction.

In Fig. 93 is shown diagrammatically a chain raising a load Q by means of an effort P.

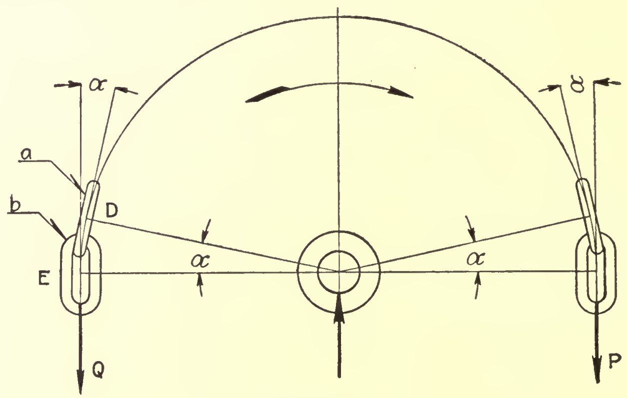

FIG. 93.

Let D = pitch diameter of the sheave.
D_0 = diameter of the sheave pin.
d = size of the chain.
μ = coefficient of journal friction.
μ_c = coefficient of chain friction.

On the load side, the link a in moving from the position E to that at D turns through the angle α relative to the link b. To overcome the frictional resistance between these links requires an amount of work to be done by the effort P equivalent to $\mu_c Q \frac{d}{2} \alpha$. At the same time that the link a is moving from E to D, a link on the effort side is running off, the frictional resistance of which requires work to be done equivalent to $\mu_c P \frac{d}{2} \alpha$. In addition to these resistances, the friction of the sheave pin must be overcome. For the loading shown in Fig. 93, the pressure on the pin is $(P + Q)$; therefore, the work required to overcome the friction of the pin for an angular displacement α of the sheave is equivalent to $\mu\alpha\left(P + Q\right)\frac{D_0}{2}$.

The useful work done is $\frac{QD\alpha}{2}$; hence, the total work required by the effort P to raise the load Q is

$$\frac{PD\alpha}{2} = \frac{QD\alpha}{2} + \mu\alpha\left(P + Q\right)\frac{D_0}{2} + \mu_c\alpha\left(P + Q\right)\frac{d}{2} \qquad (260)$$

from which

$$P = \left[\frac{D + \mu D_0 + \mu_c d}{D - \mu D_0 - \mu_c d}\right] Q = KQ \qquad (261)$$

The value of K varies from 1.04 to 1.10, the first value applying to lubricated chains and the latter to chains running dry.

Efficiency of chain sheave.—By applying the definition of efficiency to the case discussed above, we find that the efficiency is

$$\eta = \frac{1}{K} \qquad (262)$$

Introducing the values of K given above, it follows that the efficiency of a chain sheave having the chain lubricated is 96 per cent., while the same sheave with the chain running dry has an efficiency of approximately 91 per cent.

171. Analysis of a Chain Block.—With the aid of the principle discussed in Art. 170, it is a simple matter to analyze blocks reefed with chains. As an example, it is required to determine the magnitude of the effort P that is required to raise a load Q by means of a differential chain block, similar to the one shown diagrammatically in Fig. 94. As indicated in the figure, an endless chain is reefed around the compound sheave ab and the lower sheave c. As constructed, this block is always made self-locking, except occasionally when the chain becomes very greasy, the load will run down. Due to its self-locking property, the efficiency is rather low.

(*a*) *Raising the load.*—During one revolution of the compound sheave, the part d of the chain rises a distance $2\pi R$, while the part e descends a distance $2\pi r$; hence the sheave c and the load

Q rise a distance $\pi(R - r)$. Without friction, the work of the effort is $2\pi P_0 R$; hence

$$2\pi P_0 R = \pi Q(R - r),$$

from which

$$P_0 = \frac{Q}{2}(1 - n), \tag{263}$$

in which n denotes the ratio $\frac{r}{R}$. Evidently any desired reduction may be obtained by varying the difference $(R - r)$.

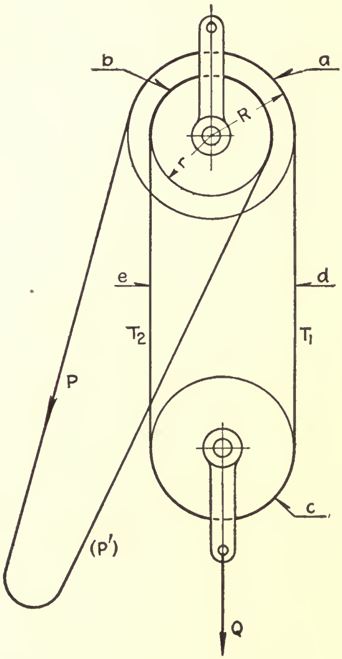

FIG. 94.

Considering the lower sheave c, it is evident from (261) that the relation between the tensions in the running-off and running-on chains is $T_1 = K_1 T_2$, where K_1 depends upon the size of the chain and the diameter of the sheave c. Furthermore, $Q = T_1 + T_2$; hence, the following expressions are obtained:

$$\left.\begin{aligned} T_1 &= \frac{K_1 Q}{1 + K_1} \\ T_2 &= \frac{Q}{1 + K_1} \end{aligned}\right\} \tag{264}$$

Now the compound sheave ab is held in equilibrium by the forces P, T_1, T_2, the pin reaction, and the friction forces. Taking account of friction, we then have

$$PR + T_2 r = K_2 T_1 R \tag{265}$$

where K_2 depends upon the size of the chain and the diameter of the sheave a.

Combining (264) and (265), the magnitude of the effort becomes

$$P = \left[\frac{K_1 K_2 - n}{1 + K_1}\right] Q \tag{266}$$

Replacing K_1 and K_2 by an average value denoted by K, (266) becomes

$$P = \left[\frac{K^2 - n}{1 + K}\right] Q \tag{267}$$

The efficiency for the differential chain hoist is given by the expression

$$\eta = \left[\frac{1 - n}{2}\right]\left[\frac{1 + K}{K^2 - n}\right] \tag{268}$$

(*b*) *Lowering the load.*—When the load is lowered, the frictional resistances all act in the opposite sense, and the analysis is given by the following equations:

$$\left.\begin{aligned} T_1 &= \frac{Q}{1 + K} \\ T_2 &= \frac{KQ}{1 + K} \\ T_1R &= K(P)R + KT_2r \end{aligned}\right\} \tag{269}$$

in which (P) represents the pull required on the chain so as to prevent running down of the load. Combining the three equations given in (269), we obtain the following expressions for the effort (P) and the efficiency (η) for reversed motion:

$$(P) = \frac{Q}{K}\left[\frac{1 - nK^2}{1 + K}\right] \tag{270}$$

$$(\eta) = \frac{2}{K}\left[\frac{1 - nK^2}{(1 - n)(1 + K)}\right] \tag{271}$$

(*c*) *Conditions for self-locking.*—Whether the hoist shown in Fig. 94 is self-locking or not depends upon the values of K and n. For self-locking, it is apparent that $(P) < 0$; hence the critical value of n at which the self-locking property commences is given by the equation,

$$(P) = \frac{Q}{K}\left[\frac{1 - nK^2}{1 + K}\right] \leqq 0,$$

from which it follows that

$$1 - nK^2 \leqq 0 \tag{272}$$

Therefore, the critical value of the ratio $\frac{r}{R}$ is

$$n = \frac{1}{K^2} \tag{273}$$

For a self-locking hoist, $$n > \frac{1}{K^2} \tag{274}$$

(*d*) *Experimental data.*—An investigation of six sizes of differential chain blocks having capacities from 500 to 6,000 pounds, inclusive, gave actual efficiencies varying from 28 per cent. for the larger capacities to 38 per cent. for the smaller sizes. Furthermore, it was found that the value of K as determined from equation (267) or (268) varied from 1.054 to 1.09.

CONVEYOR CHAINS

For the purpose of conveying and elevating all kinds of material, various types of chains are used. These chains may be adapted very readily to a wide range of conditions by using special attachments, such as buckets and flights. The chains used for this class of service may in general be grouped into the following two classes: (*a*) detachable or hook-joint, and (*b*) closed-joint.

172. Detachable Chain.—The detachable, or hook-joint chain shown in Fig. 95 is used very extensively, and under favorable conditions gives good service. The chain shown is made of

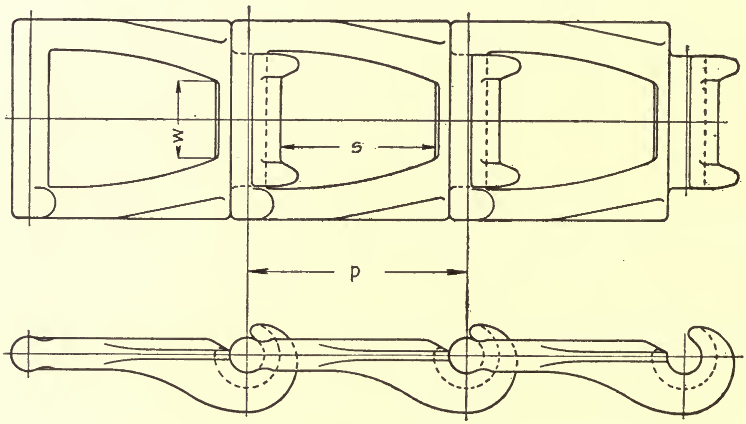

Fig. 95.

malleable iron; but there is a form of hook chain now obtainable that is made of steel. Since the joints between the links are of the hook or open type, this kind of chain is not well adapted to the elevating and conveying of gritty bulk material; however, if the joints are properly protected, slightly gritty material may be handled. In addition to this class of service, hook-joint chains are frequently used for power-transmission purposes at moderate speeds, say not to exceed 600 feet per minute for the Ewart chain shown in Fig. 95 and a considerably higher figure for the lock steel chain. For elevating and conveying service, the speeds seldom exceed 200 feet per minute.

173. Strength of Detachable Chain.—In Table 54 is given general information pertaining to the standard sizes of Ewart detachable chain, manufactured by the Link Belt Co. In addition to the sizes listed, a large number of special sizes are made. In order that a chain drive may be durable, a proper working load

Table 54.—Ewart Detachable Chain

Chain No.	Approx. links per ft.	Aver. pitch	Weight per ft.	Ultimate strength, lb.
25	13.30	0.902	0.239	700
32	10.40	1.154	0.333	1,100
33	8.60	1.394	0.344	1,190
34	8.60	1.398	0.387	1,300
35	7.40	1.630	0.370	1,200
42	8.80	1.375	0.570	1,500
45	7.40	1.630	0.518	1,600
51	10.40	1.155	0.707	1,900
52	8.00	1.506	0.848	2,300
55	7.40	1.631	0.740	2,200
57	5.20	2.308	0.832	2,800
62	7.30	1.654	1.022	3,100
66	6.00	2.013	1.158	2,600
67	5.20	2.308	1.196	3,300
75	4.60	2.609	1.311	4,000
77	5.20	2.293	1.456	3,600
78	4.60	2.609	1.909	4,900
83	3.00	4.000	1.944	4,950
85	3.00	4.000	2.400	7,600
88	4.60	2.609	2.438	5,750
93	3.00	4.033	2.670	7,500
95	3.00	3.967	3.000	8,700
103	3.90	3.075	4.087	9,600
108	2.55	4.720	3.570	9,900
110	2.55	4.720	4.437	12,700
114	3.70	3.250	5.180	11,000
122	2.00	6.050	7.000	15,000
124	3.00	4.063	6.666	12,700
146	2.00	6.150	6.240	14,400

must be used. This depends upon the speed and the class of service for which the chain is used. After a considerable number of years of experimental work, the Link Belt Co. has established a series of factors that may be used for arriving at the proper working stresses at various speeds. In Fig. 96, the factors just referred to have been plotted so as to bring them into more convenient form for general use. To determine the working stress for any particular size of chain, multiply the ultimate strength as given in Table 54 by the *speed coefficient* obtained from the graph in Fig. 96.

174. Closed-joint Chains.—As the name implies, this type of chain has a closed joint; because of this fact it is well adapted to the elevating and conveying of gritty and bulk material, as well

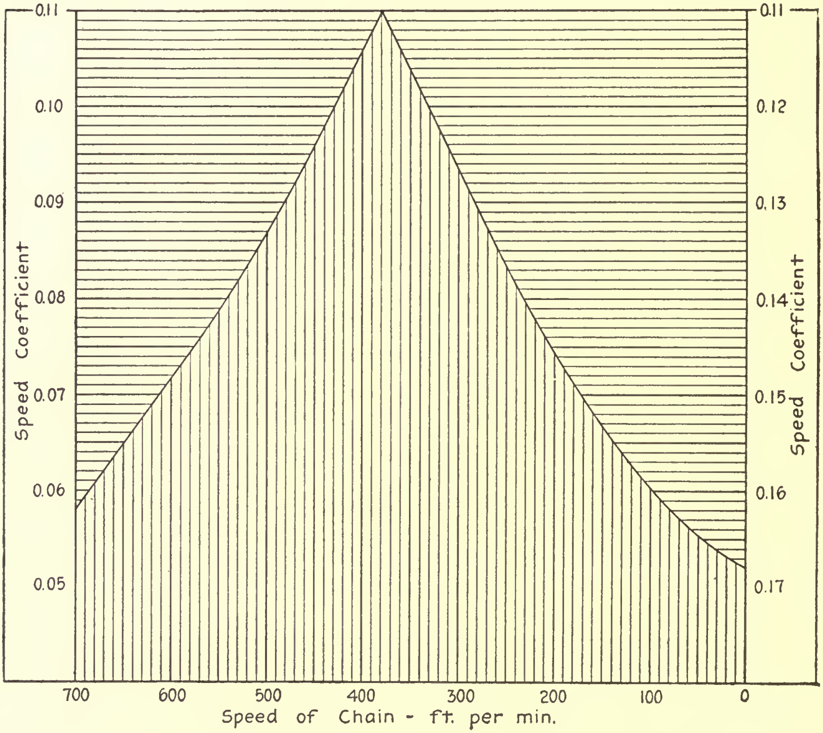

FIG. 96.

as transmitting power at moderate speeds. A large number of different types of closed-joint chain are now manufactured. In Figs. 97, 98 and 99 are shown three types, the first two being made of malleable iron and the third of steel. The closed-joint chains are made in the same sizes as the detachable chains; hence the sprockets are interchangeable. In the better grades of closed-joint chains, the pins and bushings used are frequently made of hardened steel.

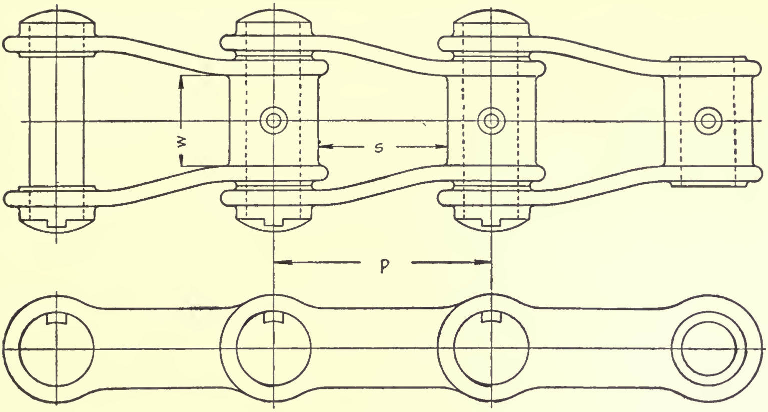

Fig. 97.

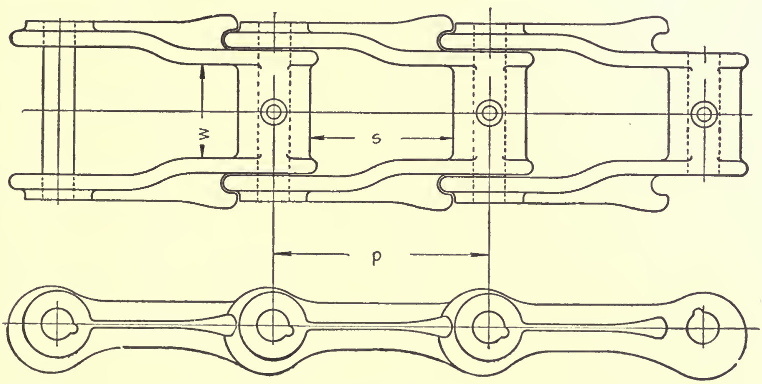

Fig. 98.

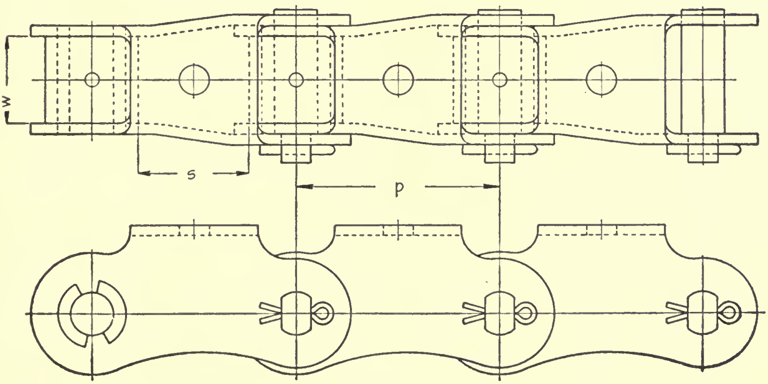

Fig. 99.

175. Strength of Closed-joint Chain.—The information given in Table 55 pertains to the chains shown in Figs. 97 and 98. The chain shown in Fig. 97 is manufactured by the Link Belt Co. and is known as the "400 Class Closed-end Pintle Chain." To determine the proper working load for this chain, the ultimate strength given in Table 55 must be multiplied by the so-called speed coefficient mentioned in Art. 173, values of which may be obtained from Fig. 96.

Table 55.—Closed-joint Conveyor and Power Chains

Link Belt Co's. "400" Class					
Chain No.	Approx. links per ft.	Aver. pitch	Weight per ft.	Maximum speed, ft. per min.	Ultimate strength, lb.
434	8.6	1.398			3,600
442	8.8	1.375	1.470		5,900
445	7.4	1.630	1.428		5,900
445½	7.4	1.630			3,600
447	7.4	1.630			5,900
452	8.0	1.506			7,000
455	7.4	1.630	1.783		7,300
462	7.3	1.634	2.314		9,000
467	5.2	2.308	1.336		6,200
4,072	7.3	1.654	2.445		9,000
477	5.2	2.293	1.960		10,000
×477	5.2	2.308	1.825		7,300
483	3.0	4.000			15,000
488	4.6	2.609	2.769		12,000
488½	4.6	2.609			13,000
4,103	3.9	3.075	5.398		20,000
4,124	3.0	4.100			33,000

Jeffrey-Mey-Obern type					
Chain No.	Approx. links per ft.	Aver. pitch	Weight per ft.	Maximum speed, ft per min.	Ultimate strength, lb.
25	13.30	0.903	0.416		1,000
33	8.60	1.395	0.525		2,200
34	8.60	1.395	0.758	700	2,650
42	8.75	1.370	1.090		3,000
45	7.40	1.623	1.090		3,300
50	12.00	1.000	0.548		1,900
52	8.00	1.517	1.480		4,750
55	7.40	1.630	1.400		4,925
57	5.20	2.307	1.460		5,800
62	7.30	1.647	1.920		5,850
67	5.20	2.308	1.680	600	6,000
75	4.60	2.619	1.970		7,350
77	5.20	2.311	1.740		6,500
77½	5.20	2.311	2.320		8,300
78	4.60	2.620	2.170		8,050
78½	4.60	2.620	2.880		9,900
83	3.00	3.970	3.100		11,425
85	3.00	3.960	3.950	500	13,500
88	4.60	2.610	2.630		8,300
103	4.00	3.058	5.290		13,530
108	2.55	4.751	5.180	400	14,800
121	2.06	6.042	3.600	500	16,600
122	2.00	6.109	8.510	300	24,980
124	3.00	4.074	11.770	300	40,000
146	2.00	6.215	8.120	300	23,500

The chain shown in Fig. 98 is manufactured by The Jeffrey Mfg. Co. and is known as the "Mey-Obern" type. The proper working stress for any particular speed may be found by using the speed coefficients given in Fig. 96.

The chain shown in Fig. 99 differs from those shown in Figs.

97 and 98 in that the body of the link is stamped and formed from one piece of steel, the sides being connected across the top by a bridge as shown. This chain is manufactured by The Union Chain and Mfg. Co. of Seville, Ohio, and is made in two types, namely, the bushing type and the roller type. The information contained in Table 56 pertains to the roller type shown in Fig. 99, the upper part of the table showing the commercial sizes used mainly for power transmission, while the lower part gives the sizes

Table 56.—Union Steel Chains

	Chain No.	Pitch	Rollers		Ultimate strength, lb.
			Length	Diameter	
Driving chains	3R	3/4	3/8 1/2 5/8	15/32	3,500
Driving chains	4R	1	1/2 5/8 3/4	5/8	5,000
Driving chains	5R	1 1/4	5/8 3/4 1	3/4	7,500
Driving chains	6R	1 1/2	3/4 1	7/8	10,500
Driving chains	7R	1 3/4	1	1	14,000
Driving chains	8R	2	1 1 1/4	1 1/8	18,000

	Chain No.	Approx. links per ft.	Average pitch	Rollers		Ultimate strength, lb.
				Length	Diameter	
Standard chain belting	14	8.0	1.50	3/4	....	8,000
Standard chain belting	15R	7.4	1.62	3/4	13/16	6,000
Standard chain belting	16R	6.0	2.00	1	1 1/8	12,000
Standard chain belting	17R	5.2	2.31	1	1 1/8	10,000
Standard chain belting	18R	4.6	2.61	1 1/8	1 3/16	12,000
Standard chain belting	19	3.9	3.07	1 5/16	1 1/4	15,000
Standard chain belting	21	3.4	3.51	1 1/4	1 5/8	22,000
Standard chain belting	22	3.0	4.00	1 1/2	1 3/4	30,000
Standard chain belting	30	2.0	6.00	1 3/4	1 7/8	40,000

that have been designed to run on standard sprockets used for detachable chain. The latter type of chain is used for either power or conveyor service. To arrive at the working stress for a given speed, multiply the ultimate strength given in Table 56 by the speed coefficient taken from the graph in Fig. 96.

176. Sprockets for Detachable Chains.—Cast sprockets are generally inaccurate due to shrinkage and rapping of the pattern; hence in order to get satisfactory service they should be made a

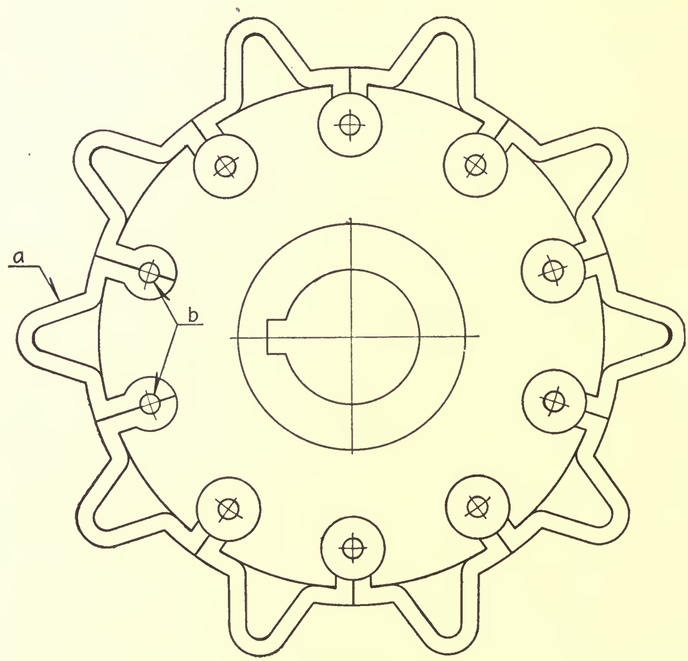

FIG. 100.

trifle large and then ground to fit the chain. The sprockets made from ordinary cast iron give good service, especially if both the chain and the face of the sprockets are lubricated with a heavy oil or a thin grease. For severe service manufacturers furnish sprockets having chilled rims and teeth, while the hubs are soft for machining purposes.

Armor-clad sprocket.—Another form of sprocket that is intended to give great durability is shown in Fig. 100. It consists of a cast-iron central body in the periphery of which are milled slots. Into these slots are fitted the teeth *a*, which are formed from special steel strips. To fasten the teeth rigidly in the body, the ends are expanded by means of a steel pin *b*, and lateral displacement is prevented by washers and riveting. The teeth are heat treated and may be removed very readily. This design of sprocket is used by The Union Chain and Mfg. Co.

It is claimed that these sprockets, and also sprockets having chilled rims and teeth, are more economical since they last considerably longer, although they cost approximately 50 per cent. more than the gray iron sprockets.

177. Relation between Driving and Driven Sprockets.—Theoretically the pitch of the sprocket teeth and that of the chain should be exactly the same; but as chains may vary a trifle from

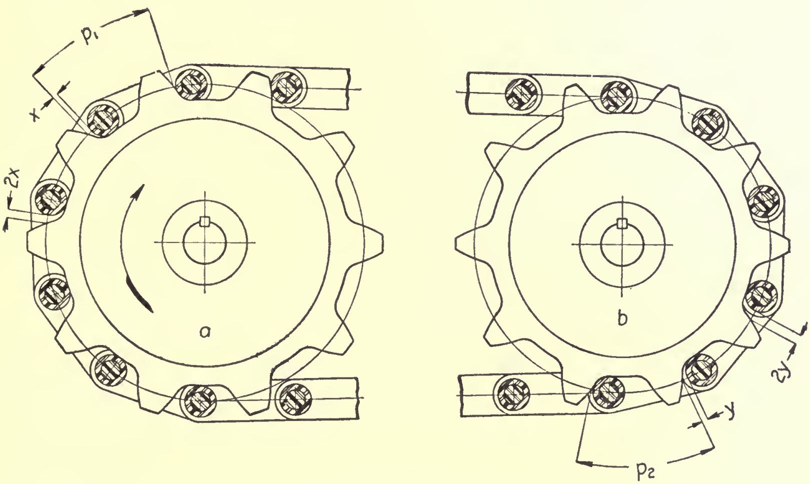

FIG. 101.

the exact pitch, and as the wear of the joints tends to lengthen the pitch, some provision must be made to take care of this elongation or the chain will ride on the teeth of the sprocket. To overcome this riding action, the teeth of sprockets are given back clearance; that is, their thickness on the pitch circle is made less than the dimension *s* shown in Figs. 95, 97, 98, and 99. Furthermore, the pitch of the teeth is increased or decreased, depending upon whether the sprocket is the driving or driven member of the transmission.

In Fig. 101 are shown two sprockets transmitting power, *a* being the driver and *b* the driven. This figure shows the correct chain action, and it should be noted that on each sprocket the entire load transmitted by the chain comes upon one tooth, namely upon the one at the point where the links run off the sprocket. Referring to Fig. 101, it is evident that the loaded tooth on the driving sprocket in pushing the chain forward permits the disengaging link to roll out to the tip of this tooth and at the same time the chain creeps backward a distance equal to the increment x. By the time the driving tooth is completely disengaged, the following tooth of the wheel is seated firmly against the following link; hence it follows that the chordal pitch p_1 of the sprocket is greater than the pitch of the chain. A similar analysis of the action of the chain on the driven sprocket shows that the disengaging link in rolling out on the loaded tooth creeps ahead a distance equal to the increment y, thus bringing the following link and tooth into intimate contact. It is evident, therefore, that the chordal pitch p_2 of the sprocket b should be less than the pitch of the chain. The condition may also be met by making the chordal pitch p_2 of a new sprocket equal to the chain pitch, and as soon as the wear appears the links creep away from the teeth producing the action just discussed.

Sprockets laid out as shown in Fig. 101 are likely to show excessive wear since one tooth must carry the entire load transmitted by the chain. According to information furnished by The Jeffrey Mfg. Co., the amount that the driving sprocket is made larger than the theoretical size depends upon the pitch of the chain, the size of roller or hook of the link, the strength of the chain, and the number of the teeth in the sprocket.

178. Tooth Form.—From the discussion given in Art. 177, it is evident that the teeth of sprockets must be given considerable clearance so as to permit the chain to elongate due to the load as well as the wear on the pins and not permit it to ride on the flanks of the teeth. If the chain transmission is designed properly, each tooth comes into action only once per revolution of the sprocket; hence, in sprockets having large numbers of teeth, the wear on the tooth flanks is distributed over more teeth, and for that reason the thickness of the tooth at the pitch line may be made less than

TABLE 57.—SPROCKET TEETH FACTORS

No. of teeth	Factor
8 to 12	0.75 to 0.80
13 to 20	0.70
21 to 35	0.65
36 to 60	0.55 to 0.60

in smaller sprockets. The data included in Table 57 will serve as a guide in laying out the teeth of sprockets. To obtain t, the thickness of the tooth at the pitch line, for any given size of chain multiply the length of the available tooth space in the link by the factors given in the table. These factors represent the practice of the Link Belt Co. and are based upon experience with chains in service. By the available tooth space in the link is meant the dimension s in Figs. 95, 97, and 98.

Having decided upon the size of chain and the number of teeth in the sprocket for the particular case under consideration, deter-

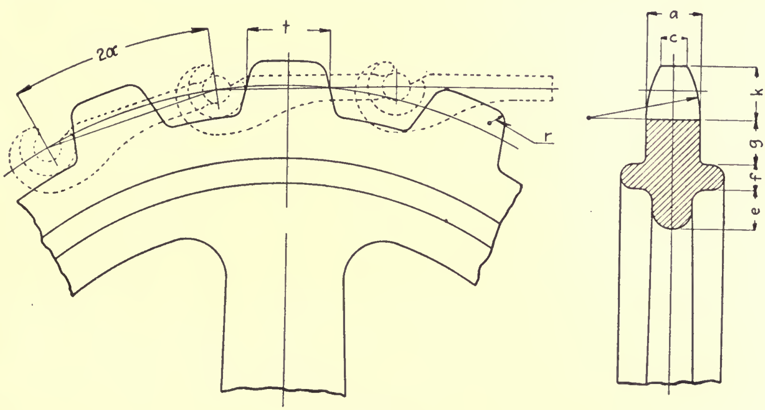

Fig. 102.

mine the pitch diameter of the sprocket by the following expression:

$$D = \frac{p}{\sin \alpha}, \tag{275}$$

in which D denotes the pitch diameter, p the pitch of the chain, and α equals 180 degrees divided by the number of teeth in the sprocket.

Having calculated the pitch diameter, the sprocket teeth may be laid out as shown in Fig. 102. The root circle diameter, as shown in the figure, is fixed by the dimensions of the link. An examination of a considerable number of sprockets made by leading manufacturers seems to indicate that the outline of the tooth may be made a straight line between the root circle and the rounded corner at the top. The radius r of this corner varies from $\frac{1}{16}$ inch for small chains to about $\frac{3}{8}$ inch for the larger chains. The inclination of this line must provide sufficient clearance to prevent interference between the tooth and the link when the latter is entering or leaving the sprocket. The flank of the tooth is joined at the root circle by a fillet having a radius less than that of the hook of the link.

179. Rim, Tooth, and Arm Proportions.—(*a*) *Rim and tooth.*—The rim of the sprocket may be proportioned in a general way by the following empirical formulas taken from Halsey's Handbook for Machine Designers and Draftsmen. In these formulas the dimensions denoted by p and w are obtained from the size of the chain under consideration.

$$\left.\begin{aligned} c &= 0.5\,w \\ a &= \begin{cases} 1\tfrac{5}{16}\,w \text{ for small chains} \\ w - \tfrac{1}{8}'' \text{ for large chains} \end{cases} \\ e &= \tfrac{7}{6}\,w \\ f &= \tfrac{1}{3}\,w \\ g &= 0.7\,w \\ k &= 1.25\;(p - s) \end{aligned}\right\} \qquad (276)$$

(*b*) *Arm proportions.*—Sprockets are made with a web center or with arms. For very small pitch diameters, solid web centers having a thickness determined by the dimension a in Fig. 102 should be used. For larger diameters up to, say, 12 or 15 inches, web centers with holes may be used; but in these cases the web thickness should be made equal to approximately six-tenths of the dimension a as determined by means of (276). For diameters exceeding 12 or 15 inches, the sprockets should be constructed with arms, the dimensions of which may be obtained by the following analysis:

Let W = breaking load of the chain.
S = permissible working stress for the material.
b = thickness of the arm at the center of the shaft.
h = depth of the arm at the center of the shaft.
n = number of arms, 4 to 6.

To be on the safe side, the arm of the sprocket is designed for a load exceeding that coming upon the chain. This condition is met by assuming that one-fifth of the breaking load of the chain comes upon the arms. Equating the bending moment per arm to its resisting moment, considering the arm to be extended to the center of the shaft, we have, assuming the arm to have an elliptical cross-section,

$$\frac{WD}{10\,n} = \frac{\pi Sbh^2}{32},$$

from which

$$bh^2 = \frac{WD}{nS}\text{(approximately)} \tag{277}$$

The arms of sprockets are generally made with a cross-section approximating an ellipse having a ratio between the major and minor axis of about 2.5 to 1 at the center of the sprocket. At the rim, the major and minor axes are made 0.8 and 0.3, respectively, of the major axis at the center.

An investigation of actual sprockets based upon the above assumptions showed that S varied from 2,500 to 3,300 pounds per square inch, in round numbers. As an average value use 3,000. Letting $b = 0.4\,h$, (277) becomes

$$h = \sqrt[3]{\frac{2.5\,WD}{nS}} \tag{278}$$

POWER CHAINS

The types of chains discussed in the preceding articles of this chapter are not well adapted to any service requiring speeds

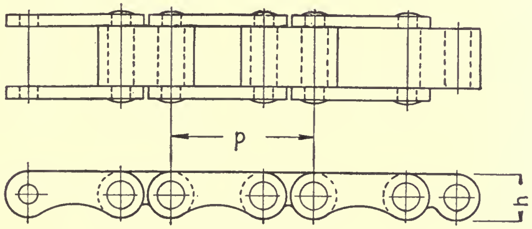

FIG. 103.

above 600 feet per minute, and for that reason they are not suitable for the transmission of power where the speed exceeds this limit. For this class of service special forms of chains, all parts of which are machined fairly accurately, have been devised. These may be classified as follows: (*a*) block chains; (*b*) roller chains; (*c*) silent chains.

180. Block Chains.—As the name implies, the block chain, shown in Fig. 103, consists of solid steel blocks shaped like the letter **B** or the figure **8,** to which the side links are fastened by hardened steel rivets. Block chains have proven very satisfactory for light power transmission where the speeds do not exceed 800 to 900 feet per minute. Table 58 gives the commercial sizes of the block chains manufactured by the Diamond Chain and Mfg. Co.

TABLE 58.—DIAMOND BLOCK CHAINS

Chain No.	Pitch	Dimensions			Width of block	Diam. of rivet	Weight per foot	Ultimate strength
		a	*b*	*h*				
102	1	0.400	0.600	0.325	¼ 5/16 ⅜ ½	11/64 3/16 (for 5/16, ⅜, ½)	0.33 0.38 0.42 0.50	1,500 1,600 1,800 2,000
103	1	0.400	0.600	0.325	¼ 5/16 ⅜ ½	11/64 3/16 (for 5/16, ⅜, ½)	0.33 0.38 0.42 0.50	2,200 2,300 2,400 2,500
105	1½	0.564	0.936	0.532	½ ⅝	0.265	0.89 1.03	5,000

181. Sprockets for Block Chains.—In Fig. 104 is shown a design of a block chain sprocket, the rim part being made of steel plate bolted on to a cast-iron hub. Instead of using the built-up construction, the sprocket may be made completely of cast iron with a central web, or with arms, if the sprocket is large in diameter. Denoting the pitch diameter of the sprocket by the symbol D, the number of teeth in the sprocket by T, and the pitch of the chain by p, then the magnitude of the angle α shown in Fig. 104 is given by the following expression:

$$\alpha = \frac{180°}{T} \tag{279}$$

From the geometry of the figure, it follows that

$$\sin(\alpha - \beta) = \frac{a}{D} \tag{280}$$

and

$$\sin\beta = \frac{b}{D}, \tag{281}$$

Deriving an expression for β by eliminating D, we have

$$\tan \beta = \frac{\sin \alpha}{\cos \alpha + \frac{a}{b}} \tag{282}$$

To obtain the pitch diameter of the sprocket for any desired number of teeth and given size of chain, determine the angle α and substitute this angle in (282) in order to establish the angle β. Knowing β, the pitch diameter D may be found by means of (281). To get satisfactory service from sprockets, the minimum

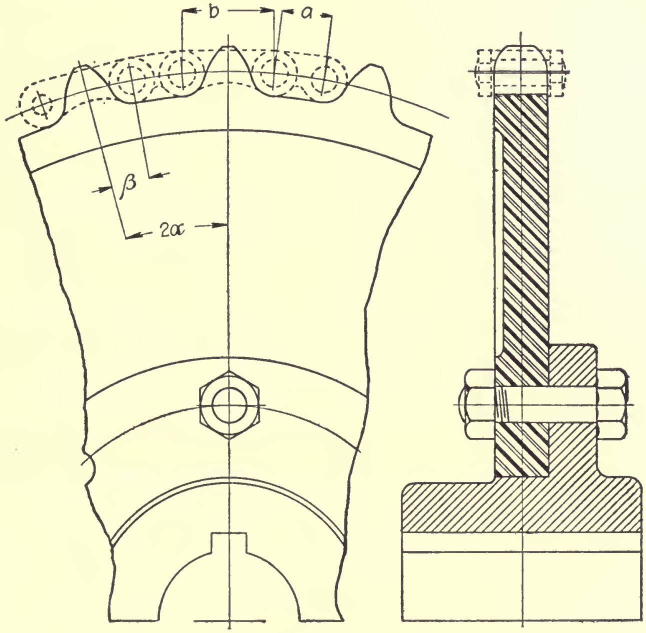

FIG. 104.

number of teeth should be limited to 15, unless the rotative speed of the sprocket is low. The teeth of small sprockets have a tendency to wear hook-shaped, thus causing noise and at the same time decreasing the life of the installation.

The height of the tooth is usually made slightly greater than the dimension h in Table 58. It should be noted that the space between the teeth is made somewhat longer than the overall length of the block, in order to provide for the stretching of the chain due to wear on the rivets.

182. Selection of Block Chains.—A careful study of the operation of chains of the block and roller type conducted by the Diamond Chain and Mfg. Co. indicates that the noisy operation and the rapid wear of a chain are due chiefly to the impact between the sprocket and the rollers or blocks as the latter seat themselves. The effect of impact is more marked when a long pitch chain runs over a sprocket having a high rotative speed. As a result of this study, the following empirical formulas and rules have been proposed by the Diamond Chain and Mfg. Co.:

$$\left.\begin{aligned} \text{max. } p &= \left(\frac{900}{N}\right)^{3/2} \\ \text{max. } N \text{ of small sprocket} &= \frac{900}{\sqrt{p^3}} \end{aligned}\right\} \qquad (283)$$

(*a*) In an installation in which the load on the chain is fairly uniform, the permissible chain pull should not exceed one-tenth of the ultimate strength of the chain as given in Table 58.

(*b*) As a further check on the chain load, the equivalent pressure per square inch of projected rivet area should not exceed 1,000 pounds for general service. When slow chain speeds prevail, this pressure may run as high as 3,000 pounds, although the latter value should be considered the upper limit.

(*c*) When the chain is subjected to sudden fluctuations of load, the permissible chain pull may only be $\frac{1}{30}$ or $\frac{1}{40}$ of the ultimate strength.

(*d*) In selecting a block or roller chain for a given duty it is well to give preference to a light chain rather than a heavy one, provided the former has sufficient rivet area as well as strength to transmit the power. As stated above, long life and quiet running are secured more easily by selecting a short pitch chain. As a rule, a narrow chain is more satisfactory than a wide one except in places where the sprockets are not always in proper alignment; for example, in an electric motor drive or in motor-truck service.

183. Roller Chains.—A typical roller chain is shown in Fig. 105. This type of chain is used to some extent in motor-vehicle service, especially on trucks, as well as for general power transmission. Chain speeds as high as 1,400 feet per minute have been used successfully on light loads; but for general use with proper lubrication 1,200 feet per minute should be the limit. Occasionally double roller chains are used and if properly installed they give good service. In Table 59 are given the commercial sizes and other information pertaining to the roller chain made by the Diamond Chain and Mfg. Co. Instead of the ultimate strength of the chain, the normal and maximum allowable loads are given. The normal loads are based on a bearing pressure of 1,000 pounds per square inch of the projected area of the rivet, while the maximum load is approximately three times the normal but in no case will it exceed one-tenth of the ultimate strength of the chain.

In arriving at the size of a roller chain required for a particular duty, the various points mentioned in Art. 182 apply equally well in the present case.

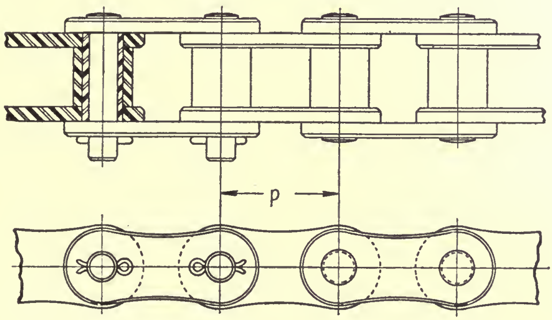

Fig. 105.

184. Sprockets for Roller Chains.—As in the case of block chains, the sprockets used with high-grade roller chains are always made with cut teeth. The forms given to the teeth by the various manufacturers of roller chains differ considerably.

(*a*) *Old-style tooth form.*—In Fig. 106 is shown a tooth form that is faulty in that it makes no provision for the stretching of the chain due to wear on the pins or rivets. If the space between the teeth were made wider, as shown in Fig. 108(*a*), giving the roller more clearance, the chain drive would be satisfactory. At the present time cutters that give a clearance approximating one-tenth of the radius of the chain roller are used in the manufacture of sprockets. As the chain runs on or off the sprocket, the curve described by the roller is an involute of the pitch circle, from which it would appear that the face of the tooth should be made an involute. This, however, is not done as the face of the tooth is generally made an arc of a circle a trifle inside of the involute in order that the roller will have no contact with the tooth on entering or leaving the sprocket. The length of the addendum of the tooth is arbitrarily taken as one-half of the diameter of the roller. The pitch diameter of the sprocket is obtained by the use of formula (275) derived for the common detachable chain in Art. 178.

TABLE 59.—DIAMOND ROLLER CHAINS

Chain No.	Pitch	Roller		Diam. of rivet	Weight per foot	Allowable load		Remarks
		Length	Diam.			Normal	Maximum	
75	1/2	1/8	0.306	11/64	0.280	44	120	Single roller
		3/16			0.300	55		
		1/4			0.320	65		
147–149	5/8	1/4	0.4	0.200	0.475	83	250	Single roller
		3/8			0.619	108	325	
151	1 1/4	1/2	5/8	0.312	1.580	253	760	Single roller
		5/8			1.690	292	877	
		3/4			1.800	331	994	
153	3/4	5/16	0.469	0.220	0.710	106	317	Single roller
		3/8			0.760	120	359	
		1/2			0.860	147	442	
		5/8			0.960	175	500	
		1/2			1.450	295	750	Double roller
154	1	1/2	5/8	0.312	1.680	253	760	Single roller
		5/8			1.810	292	877	
		3/4			1.940	331	994	
		5/8			3.290	585	1,700	Double roller
155	1	3/8	9/16	0.281	1.070	176	527	Single roller
		1/2			1.170	211	632	
		5/8			1.270	246	738	
		1/2			1.840	422	1,200	Double roller
157	1 1/2	3/4	3/4	0.375	2.410	396	1,189	Single roller
		1			2.740	492	1,476	
160	1 1/4	5/8		0.375	2.540	350	1,049	Single roller
		3/4			2.690	396	1,189	
		1			2.990	492	1,476	
		3/4			4.850	793	2,400	Double roller
162	1 1/2	3/4	7/8	0.437	3.890	520	1,560	Single roller
		1			4.150	629	1,888	
164	1 3/4	1	1	0.500	4.960	720	2,160	Single roller
					8.750	1,440	4,000	Double roller
168	2	1 1/4	1 1/8	0.5625	6.320	975	2,925	Single roller
					11.560	2,231	6,000	Double roller

(b) *Diamond tooth form.*—In Fig. 107 is shown the method used by the Diamond Chain and Mfg. Co. for laying out their latest type of sprocket. The information given in the figure as well as the formula below were kindly furnished by Mr. G. M. Bartlett, mechanical engineer for the firm. In the following formulas p represents the pitch of the chain as shown in Fig. 105.

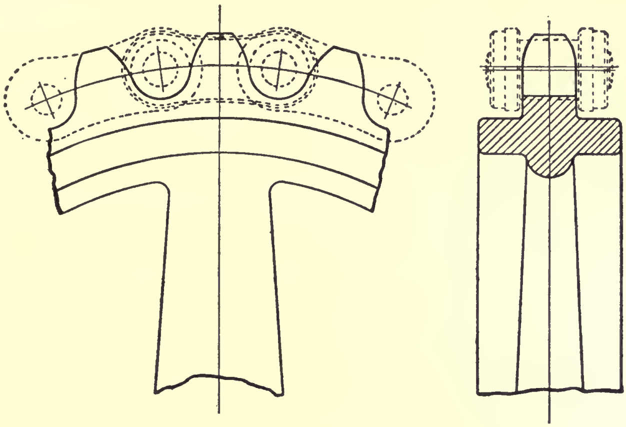

FIG. 106.

$$\left.\begin{aligned} a &= \text{chain width} - 0.045\,p \\ b &= 0.545\,p \\ c &= 0.3\,p \\ d &= \text{diameter of roller} \end{aligned}\right\} \tag{284}$$

The angle of pressure between the roller and the tooth is 20 degrees, as shown in the figure.

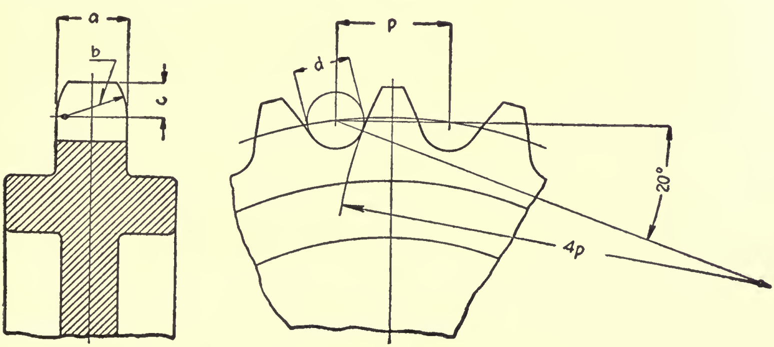

FIG. 107.

(*c*) *Renold tooth form.*—Another recent design of sprocket tooth form is illustrated in Fig. 108(*b*). It represents the results of many years of experience with roller chains as well as several years of special research work by Mr. Hans Renold, a prominent

English chain manufacturer. The results of his work were presented before the American roller chain manufacturers in the spring of 1914. The form of the tooth, which is not protected by patents, has a distinct advantage over the older forms still used by some chain makers, in that the stretch of the chain is taken care of by the rollers rising on the tooth flanks. The tooth is thus prevented from wearing into a hook form and a smooth-running transmission is insured.

The space between the teeth is made an arc of a circle having a radius equal to the diameter of the roller or a few thousands of an

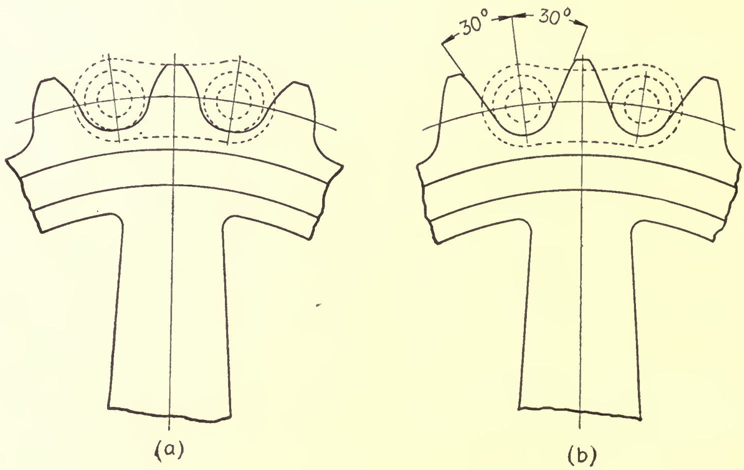

FIG. 108.

inch larger. The straight lines forming the teeth are tangent to this arc and make an angle of 60 degrees with each other as shown in the figure. The face of the tooth is relieved near the top by a circular arc. The height of the tooth thus formed is greater than that used with other tooth designs.

185. Length of Roller Chain.—It is evident that a chain cannot have a fractional number of pitches or links; hence in all cases the next whole number above the calculated number must be selected, and if the distance between the centers of the driving and driven sprocket will permit a slight change, the number chosen should be an even number. An odd number of pitches will necessitate the use of an offset link for joining the ends of the chain. The following formula used by the Diamond Chain and

Mfg. Co. gives the chain length in pitches and has been found to give accurate results:

$$\left.\begin{array}{l}\text{Chain length}\\ \text{in pitches}\end{array}\right\} = 2L + \tfrac{1}{2}(T_1 + T_2) + \frac{0.0257}{L}(T_1 - T_2)^2, \quad (285)$$

in which L denotes the distance between the centers of the two sprockets, and T_1 and T_2 the number of teeth on the large and small sprocket, respectively. If it is desirable to determine the length of the chain in inches, merely multiply the pitch by the chain length obtained from (285).

186. Silent Chains.—The best forms of chain capable of transmitting power at high speeds are those designated as *silent chain*. An installation of such a chain if properly designed and constructed will be just as efficient as a gear drive for the same conditions of operation. At the present time there are in use several designs of silent chain, having in general the same form of link and differing only in the type of joint used. With silent chains, the load transmitted is distributed equally between all of the sprocket teeth in contact with the chain, and is not carried by a single tooth as is the case in some of the chains heretofore discussed.

Silent chains are well adapted for transmitting power economically at speeds of 1,200 to 1,500 feet per minute. The lower speed holds for chains having a pitch greater than one inch and the higher value for small chains. If the speed is in excess of 1,500 feet per minute, chains are liable to be noisy unless they are enclosed and run in oil. With properly designed gear cases and with the use of good lubricants, the smaller sizes of chains may be run at 2,000 feet per minute and the larger sizes at 1,500. It should be borne in mind, however, that these speeds are attained at the cost of reduced life of the chain. Where a positive drive is essential, as in direct-connected motor-driven machinery, and where the shafts are too far apart for gearing, silent chains are used extensively. Chains transmitting power in dusty and dirty surroundings should always be enclosed in an oil-tight case.

187. Coventry Chain.—The Coventry chain shown in Fig. 109 is manufactured in England, but is used to a considerable extent in America. It consists of links of special form assembled in pairs and held together by the hardened steel bushes b. Various widths of chains are produced by assembling these double links alternately on hardened steel pins; for example, the chain shown in Fig. 109 is called a 1 × 2 combination. The links themselves are not hard, and their shape is such that the load is distributed equally over all the teeth on the sprocket in actual contact with

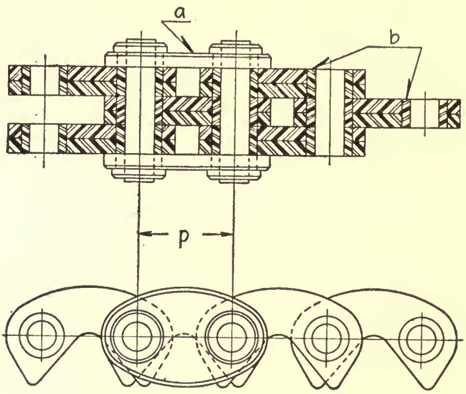

FIG. 109.

the chain. This action is illustrated in Fig. 110, which also shows the form of tooth used on such sprockets.

188. Whitney Chain.—The chain illustrated by Fig. 111 is an American design, manufactured by The Whitney Mfg. Co. of

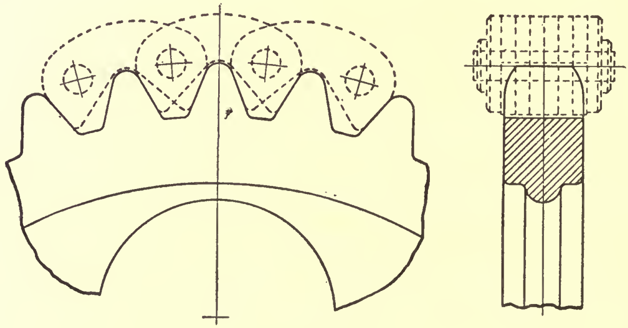

FIG. 110.

Hartford, Conn. The shape of the links in this chain is similar to that used on the Coventry chain, and hence the action of the links on the sprocket teeth is practically the same. The individual load links turn on the outside of the hard steel bushes b which are fastened securely into the guide plates a. The hardened steel pins turning within the bushings are forced into outside steel plates shaped like the figure eight. The function of the outside plates is to increase the tensile strength of the chain.

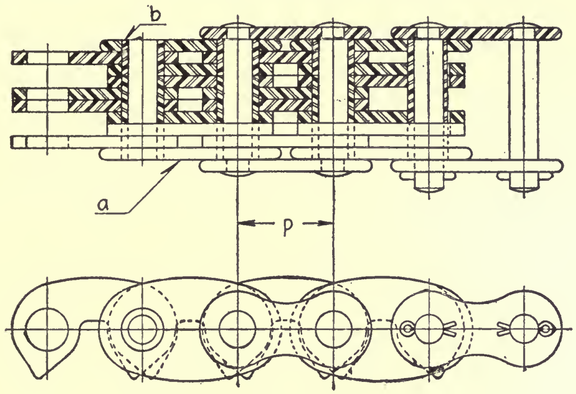

FIG. 111.

189. Link Belt Chain.—The Link Belt Co., after manufacturing for several years a plain pin-joint silent chain patented by Hans Renold of England, finally introduced the chain illustrated in Fig. 112. The joint consists of a case-hardened steel pin having a bearing on two case-hardened steel bushes *b* and *c*. These

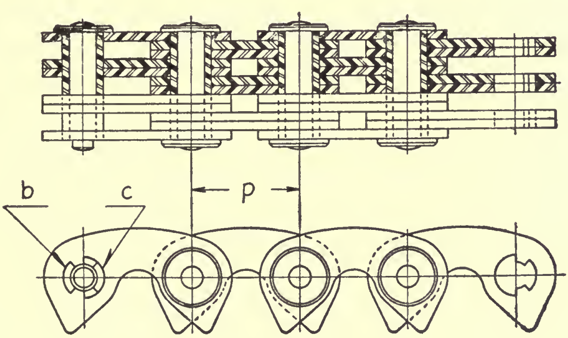

FIG. 112.

bushes are segmental in shape and are fitted into broached holes in the links, as shown in the figure. This type of joint increases the bearing area on the pin over that obtained in the original Renold chain that had no bushes at all. This chain is not provided with guide plates, so special provisions must be made on the sprocket for retaining it.

190. Morse Chain.—In the Morse chain shown in Fig. 113, the joint is of a peculiar construction in that it introduces rolling friction in place of the sliding friction common to all the types of silent chains discussed in the preceding articles. The joint consists of two hardened steel pins *b* and *c* anchored securely in their respective ends of the link. The pin *b* has a plane surface against which the edge of the pin *c* rolls as the chain runs on or off the sprocket. The wear all comes upon the two pins and these may be easily renewed. When the chain is off the sprocket the load upon the joints for that part of the chain between the sprockets

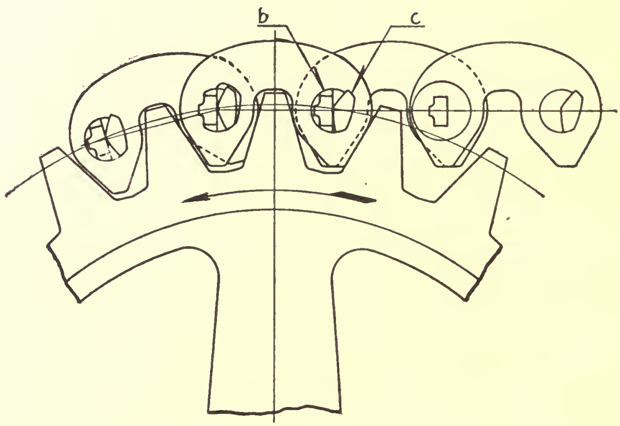

FIG. 113.

is taken by relatively flat surfaces and not by the edge of the pin *c*. It is probable that the Morse chain will give better service in dusty places than any other type of silent chain, due to the fact that the rocker joint used requires less lubrication than the cylindrical pin joints.

191. Strength of Silent Chains.—The life of a silent chain depends upon the bearing area of the pins or bushings and not so much upon the ultimate strength. For minimum wear of the chain and for maintained efficiency, the working load under normal conditions approximates one-thirtieth of the ultimate strength, while under severe fluctuations of load at the maximum speed it is taken as one-fiftieth of the ultimate strength. Some manufacturers limit the bearing pressure on the pins to 650 pounds per square inch of projected pin area. Since the strength of a chain can be increased by merely adding to its width, it is evident that for the same load conditions, chains of different pitches and widths may be selected; for example, a 1-inch pitch chain 4 inches wide and a $1\frac{1}{4}$-inch pitch chain 3 inches wide are capable of transmitting approximately the same horse power at the same speed. Experience dictates that the width should range from two to six times the pitch.

The first cost of narrow chains having a long pitch is less than wide ones of a shorter pitch. The longer pitch chains require larger sprockets, but are to be preferred when the distance between the connected shafts is great. Frequently it is found desirable to run two chains side by side in order to transmit the desired horse power.

In Table 60 is given information pertaining to the Morse chain, which will serve for making the preliminary study of a silent-chain installation. This information was kindly furnished by the Morse Chain Co. of Ithaca, N. Y. Table 61 contains useful data relating to the Whitney chain, while Table 62 applies to the Link Belt chain.

192. Sprockets for Silent Chains.—An inspection of the figures illustrating the various types of silent chains shows that the shapes of the individual links are all about alike. The angle included between the working faces of the link is made 60 degrees by all of the manufacturers; hence it follows that the angle included between the flanks of alternate teeth will always be 60 degrees irrespective of the number of teeth in the sprocket. However, the angle included by the flanks of the same tooth will change, being small for lower numbers of teeth. As this angle decreases rapidly for sprockets having small numbers of teeth, the manufacturers try to limit the number of teeth in small sprockets to 15. Whenever the installation permits, and when very quiet operation is desirable, the lower limit is placed at 17. Again, since the angle between the flanks of the same tooth increases with the number of the teeth in the sprocket, it is found necessary to limit the number of teeth to about 120 or 130 on account of the liability of the chain to slide over the teeth. The tooth form for any particular make of chain is determined best by laying it out to conform to the dimensions of the links to be used.

The so-called pitch diameter of the sprocket for silent chain

TABLE 60.—DESIGN DATA FOR MORSE SILENT CHAIN DRIVES

		Pitch of chain								
		3/8	1/2	5/8	3/4	0.9	1.2	1.5	2	3
Minimum number of teeth in sprockets	Driver	13	13	13	13	15	15	17	17	17
	Driven	13	17	17	21	25	29	29	31	35
Desirable number of teeth in driving sprockets		15–17	15–17	17–21	17–21	17–23	17–23	17–27	17–31	19–31
Maximum number of teeth in sprockets		75	99	109	115	125	129	129	129	131
Desirable number of teeth in driven sprockets		35–45	55–75	55–75	55–85	55–95	55–105	55–115	55–115	55–115
To find pitch diameter, multiply number of teeth by		0.1193	0.159	0.199	0.239	0.2865	0.382	0.477	0.636	0.955
2 × addendum, see Note 1 below		......	0.100	0.120	0.15	0.18	0.24	0.30	0.40	0.60
Maximum rev. per min		6,000	2,400	1,800	1,200	1,100	800	600	400	250
Tension per inch width of chain	Small sprocket driver	60	80	100	120	150	200	270	450	750
	Small sprocket driven	60	65	80	95	120	160	210	350	600
Radial clearance beyond tooth required for chain		0.375	0.5	0.62	0.75	0.90	1.2	1.5	2.0	3.0
Weight per foot of chain per inch of width		0.75	1.00	1.20	1.50	1.80	2.50	3.00	4.00	6.00
Coefficient for determining weight of	Solid sprocket	0.003	0.0045	0.0063	0.009	0.013	0.023	0.035	0.058	0.145
	Armed sprocket	0.10	0.16	0.2500	0.35	0.45	0.7	1.0	2.0	4.0

Approx. weight of solid sprocket = $CT^2(F + 1)$
Approx. weight of armed sprocket = CTF

C = coef. given in above table.
T = number of teeth.
F = face in inches.

Add 25 per cent. for split sprocket.
Add 50 per cent. for split spring sprocket.

NOTE 1.—When $T \leqq 32$, exact outside diam. D of sprocket = pitch diam. When $T > 32$, exact outside diam. D = pitch diam. + 2 × addendum.
NOTE 2.—Use sprockets having an odd number of teeth whenever possible.
NOTE 3.—When specially authorized, a larger number of teeth than shown may be cut in large sprocket.
NOTE 4.—Thickness of sprocket rim including teeth should be at least 1.2 times the chain pitch.
NOTE 5.—The number of grooves in the sprocket, their width and distance apart varies according to pitch and width of chain. In every case leave the designing and turning of these grooves to the M. Ch. Co.
NOTE 6.—The width of the sprockets should be 1/8 to 1/4 inch greater than the nominal width of the chain.
NOTE 7.—An even number of links in the chain and an odd number of teeth in the wheels are desirable.
NOTE 8.—Horizontal drives preferred; tight chain on top desirable for short drives without center adjustment.
NOTE 9.—Adjustable wheel centers desirable for horizontal drives are necessary for vertical drives.
NOTE 10.—Avoid vertical drives.
NOTE 11.—Allow a side clearance for chain equal to the pitch.
NOTE 12.—Maximum linear velocity for commercial service 1,200 to 1,600 ft. per min.

having pin joints may be determined by the formula used for roller chains, namely

$$D = \frac{p}{\sin \alpha},$$

in which p denotes the pitch of the chain and the angle α is equal to 180 degrees divided by the number of teeth. Whenever possible an odd number of teeth should be used for the pinion so that the wear may be distributed more evenly. Sprockets should be made as large as possible to relieve the wear on the chain, as in passing around small sprockets the angular displacements of each link on the pins or bushes is greater than in the case of large sprockets.

TABLE 61.—WHITNEY SILENT CHAINS

Chain No.	Pitch	Width between guide links	Weight per ft.	Maximum speed, ft. per min.	Ultimate strength, lb.	Chain No.	Pitch	Width between guide links	Weight per ft.	Maximum speed, ft. per min.	Ultimate strength, lb.
1201		½	0.56		2,800	1265		1¾	3.22		14,400
1202		¾	0.74		3,400	1266		2	3.59		15,600
1203	⅜	1	0.92		4,000	1267		2¼	3.96		16,800
1204		1¼	1.10		4,600	1268		2½	4.33		18,000
1205		1½	1.28		5,200	1269		2¾	4.70		19,200
						1270		3	5.07		20,400
1221		½	0.83		4,900	1271	¾	3¼	5.44		21,600
1222		¾	1.08		5,800	1272		3½	5.81		22,800
1223		1	1.33		6,700	1273		3¾	6.18		24,000
1224	½	1¼	1.58		7,600	1274		4	6.55		25,200
1225		1½	1.83		8,500	1275		4¼	6.92		26,400
1226		1¾	2.08		9,400	1276		4½	7.29		27,600
1227		2	2.33		10,300						
						1281		1	3.24		17,200
1241		¾	1.35		7,100	1282		1½	4.25		20,100
1242		1	1.66		8,000	1283		2	5.26		23,000
1243		1¼	1.97		8,900	1284		2½	6.27		25,900
1244		1½	2.28		9,800	1285		3	7.28		28,800
1245		1¾	2.59		10,700	1286		3½	8.29		31,700
1246		2	2.90		11,600	1287		4	9.30		34,600
1247	⅝	2¼	3.21		12,500	1288	1	4½	10.31		37,500
1248		2½	3.52		13,400	1289		5	11.32		40,400
1249		2¾	3.83		14,300	1290		5½	12.33		43,300
1250		3	4.14		15,200	1291		6	13.34		46,200
1251		3¼	4.45		16,100	1292		6½	14.35		49,100
1252		3½	4.76		17,000	1293		7	15.36		52,000
						1294		7½	16.37		54,900
1261		¾	1.74		9,600	1295		8	17.38		57,800
1262		1	2.11		10,800						
1263	¾	1¼	2.48		12,000						
1264		1½	2.85		13,200						

Table 62.—Horse Power Transmitted By Link Belt Silent Chain

Pitch of chain	Width of chain	Speed of chain in ft. per min.										
		500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500
	½	0.58	0.66	0.72	0.78	0.82	0.88	0.91	0.95			
	¾	0.87	0.98	1.07	1.16	1.22	1.30	1.38	1.42			
	1	1.16	1.31	1.43	1.55	1.63	1.73	1.82	1.89			
⅜	1¼	1.45	1.64	1.79	1.91	2.04	2.18	2.28	2.36			
	1½	1.74	1.97	2.15	2.30	2.45	2.60	2.73	2.83			
	2	2.32	2.62	2.86	3.08	3.27	3.46	3.64	3.78			
	3	3.48	3.91	4.28	4.61	4.89	5.22	5.46	5.67			
	½	0.84	0.95	1.04	1.11	1.19	1.27	1.33	1.38	1.42		
	¾	1.26	1.40	1.56	1.70	1.79	1.91	1.99	2.07	2.13		
	1	1.68	1.89	2.08	2.25	2.34	2.54	2.65	2.76	2.84		
½	1½	2.52	2.91	3.12	3.44	3.57	3.88	3.98	4.14	4.25		
	2	3.37	3.82	4.17	4.48	4.77	5.10	5.30	5.52	5.68		
	3	5.05	5.73	6.25	6.75	7.15	7.60	7.95	8.29	8.50		
	4	6.73	7.64	8.30	9.00	9.53	10.10	10.60	11.10	11.30		
	1	2.22	2.51	2.74	2.96	3.15	3.33	3.50	3.64	3.75		
	1¼	2.77	3.15	3.41	3.71	3.93	4.18	4.37	4.54	4.70		
	1½	3.33	3.76	4.12	4.43	4.72	5.00	5.25	5.45	5.62		
⅝	2	4.43	5.02	5.47	5.91	6.30	6.67	7.00	7.28	7.50		
	3	6.65	7.52	8.22	8.88	9.45	10.00	10.50	10.90	11.20		
	4	8.86	10.00	10.90	11.80	12.60	13.30	14.00	14.50	15.00		
	6	13.30	15.00	16.40	17.70	18.90	20.00	21.00	21.80	22.50		
	1	2.85	3.22	3.51	3.78	4.05	4.37	4.48	4.65	4.82		
	1¼	3.56	3.98	4.39	4.70	5.06	5.30	5.60	5.78	6.02		
	1½	4.27	4.85	5.27	5.67	6.10	6.40	6.72	6.98	7.23		
	2	5.68	6.42	7.03	7.56	8.10	8.55	8.95	9.31	9.63		
¾	3	8.55	9.63	10.50	11.40	12.10	12.80	13.40	14.00	14.50		
	4	11.40	12.80	14.00	15.10	16.30	17.30	17.90	18.60	19.30		
	5	14.20	16.10	17.60	18.90	20.30	21.30	22.40	23.30	24.10		
	6	17.10	19.30	21.10	22.80	24.30	25.70	26.80	27.90	28.90		
	2	7.00	7.91	8.65	8.33	10.00	10.50	10.90	11.40	11.80		
	2½	9.00	10.10	11.10	12.00	12.90	13.50	14.10	14.70	15.20		
	3	11.00	12.40	13.60	14.60	15.70	16.50	17.20	18.00	18.60		
1	4	15.00	16.90	18.60	20.00	21.50	22.50	23.50	24.60	25.40		
	5	19.00	21.50	23.50	25.20	27.20	28.70	29.70	31.10	32.10		
	6	23.00	26.00	28.50	30.50	32.90	34.50	36.00	37.60	38.90		
	8	31.00	34.90	38.40	41.20	44.30	46.30	48.50	50.70	52.40		
	2	9.70	11.00	11.90	13.00	13.80	14.60	15.30	15.90	16.40	16.7	
	3	15.30	17.30	18.70	20.30	21.70	22.90	24.20	25.00	25.70	26.5	
	4	20.80	23.50	25.50	27.60	29.60	31.20	32.60	34.10	35.10	36.2	
1¼	5	26.30	29.80	32.30	35.10	37.50	39.70	41.60	43.20	44.50	45.8	
	6	31.80	36.20	39.10	42.70	45.30	48.20	50.30	52.20	53.80	55.5	
	8	42.80	48.50	52.70	57.20	61.20	64.00	67.80	70.30	72.50	74.6	
	10	54.10	61.30	66.50	72.20	77.10	81.20	85.60	88.70	91.40	94.1	

Table 62.—Horse Power Transmitted by Link Belt Silent Chain (*Cont.*)

Pitch of chain	Width of chain	Speed of chain in ft. per min.										
		500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500
1½	3	20.10	22.70	24.70	26.90	28.70	30.30	31.80	33.00	34.00	35.	35.7
	4	27.50	31.10	33.70	36.60	39.10	41.20	43.40	45.00	46.40	48.0	48.7
	5	34.80	39.30	42.70	46.30	49.50	52.30	55.00	57.00	58.70	60.7	61.6
	6	42.20	47.60	51.80	56.30	60.00	63.40	66.50	69.00	71.10	73.5	74.7
	8	56.70	64.20	69.70	75.70	81.00	85.20	89.70	93.00	95.80	99.0	101.0
	10	71.40	80.70	87.70	95.20	102.00	107.00	113.00	117.00	121.00	124.0	127.0
	12	86.00	97.30	106.00	115.00	123.00	129.00	136.00	141.00	145.00	150.0	152.0
2	6	56.10	63.50	69.00	75.00	80.00	84.30	88.80	92.00	94.80	97.5	99.6
	8	75.70	85.60	93.00	101.00	108.00	114.00	120.00	124.00	128.00	131.0	134.0
	10	95.20	107.00	117.00	126.00	136.00	143.00	151.00	156.00	161.00	165.0	169.0
	12	114.00	129.00	141.00	153.00	164.00	172.00	182.00	188.00	194.00	199.0	204.0
	14	134.00	152.00	165.00	179.00	191.00	201.00	212.00	220.00	227.00	233.0	240.0
	16	154.00	174.00	189.00	205.00	220.00	231.00	243.00	252.00	260.00	267.0	273.0
2½	6	73.00	82.70	90.00	98.00	104.00	110.00	116.00	120.00	124.00	127.0	130.0
	8	100.00	113.00	123.00	133.00	143.00	150.00	158.00	164.00	169.00	174.0	178.0
	10	126.00	143.00	155.00	168.00	180.00	190.00	200.00	207.00	213.00	220.0	224.0
	12	153.00	173.00	188.00	204.00	218.00	230.00	242.00	251.00	259.00	266.0	272.0
	14	179.00	204.00	220.00	240.00	255.00	270.00	284.00	294.00	303.00	313.0	318.0
	16	206.00	235.00	253.00	274.00	294.00	310.00	326.00	338.00	348.00	359.0	365.0

The several makes of so-called silent chains require different types of sprockets in order to keep the chain from running off, as

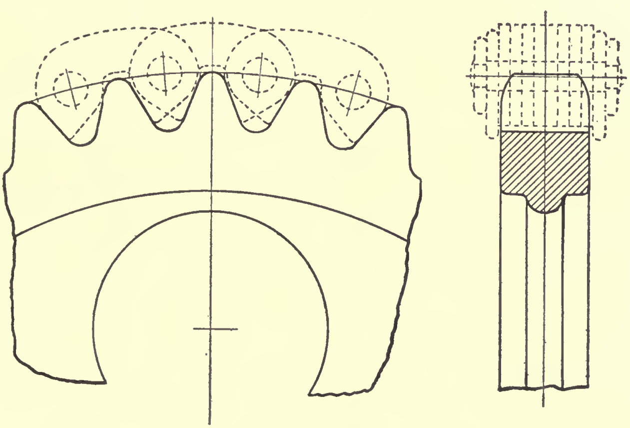

Fig. 114

may be noticed by consulting Figs. 114 and 115. The so-called outside-guided chain shown in Figs. 109 and 111 require plain sprockets, since the guide links prevent it from running off. The Link Belt chain, having no guide links, depends upon flanged sprockets of one form or another. One design of such a sprocket, as used by the Link Belt Co., is shown in Fig. 115, and in Table 63 are given some general proportions pertaining thereto. The Morse chain is always provided with central guide links; hence, the sprocket teeth are provided with one or more central grooves in which the guide plates run. A design of this description is shown in Fig. 117.

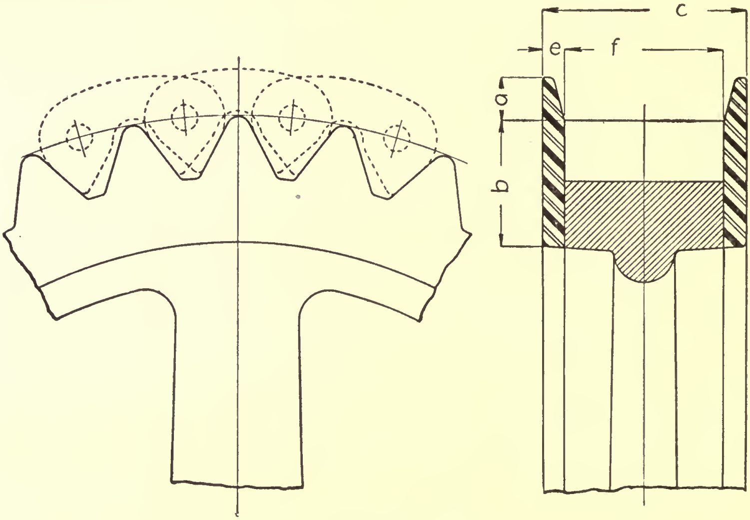

FIG. 115.

TABLE 63.—GENERAL PROPORTIONS OF LINK BELT SPROCKETS

Chain pitch	Dimensions				
	a	*b*	*c*	*e*	*f*
3/8	3/16	1/2	2e + f	3/32	This dimension is made from 1/4" to 1" wider than the chain width.
1/2	0.2	9/16		1/8	
5/8	0.25	11/16			
3/4	0.3	7/8		5/32	
1	0.4	1 1/8			
1 1/4	0.5	1 3/8		7/32	
1 1/2	0.6	1 5/8			
2	0.85	2 1/16		5/16	
2 1/2	1.25	2 1/2		7/16	

193. Spring-cushioned Sprockets.—In a power transmission subjected to shocks due to intermittent and irregular loads, it is considered good practice to use a form of sprocket that is capable, of absorbing these shocks thereby relieving the chain. In general, such a device (see Fig. 116 or 117) consists of an inner hub a keyed to the shaft, and upon this hub is mounted the sprocket rim e. Between the lugs b, cast integral with a, and the lugs d on the inside of the rim e are placed the compression springs c, through which the driving load must be transmitted. The design shown in Fig. 116 is furnished with a cover plate f to make it dustproof, and is representative of the practice of the Link Belt

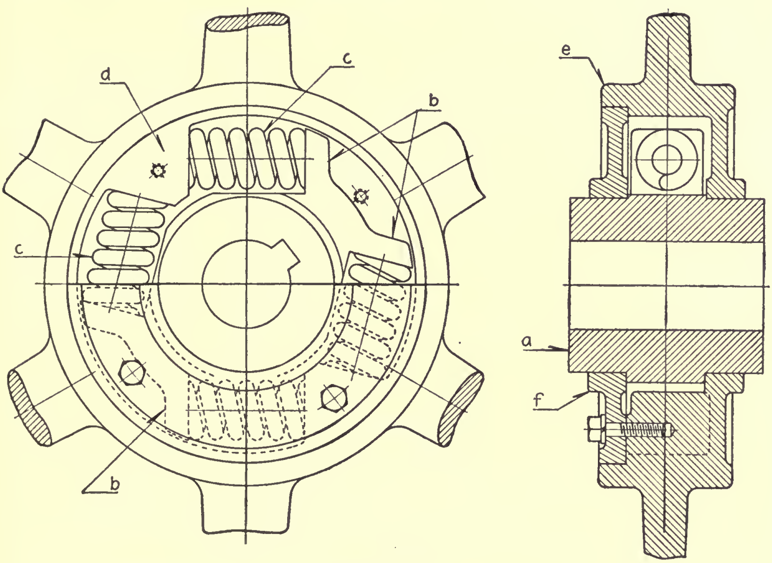

FIG. 116.

Co. The Morse Chain Co. spring-cushion sprocket, shown in Fig. 117, is also dustproof but the split-rim construction is used.

It is suggested that spring-cushioned sprockets are well adapted to such service as is met with in driving air compressors, pumps, metal planers and shapers, and punching and shearing machinery; however, they are not used to any extent in such places, no doubt due to the additional cost.

Whenever two chains are used side by side to transmit a given horse power, a "*compensating sprocket*" should be used unless the transmission is horizontal and the distance between the shafts is considerable so that quite a little weight of chain is between the sprockets. A *compensating sprocket* may be made by mounting two spring-cushioned sprockets side by side on one central hub, thus dividing the load equally between the chains.

In the design of cushioned sprockets for intermittent work, for example, driving reciprocating pumps not subjected to a water-hammer or excessive overloading, the compressive load on the springs should be based on a chain load two and one-half to three times the actual load. In installations where water-hammers on pumps, or other heavy additional loads, would come upon the springs, the latter should be designed for loads from four to five times the actual load on the chain.

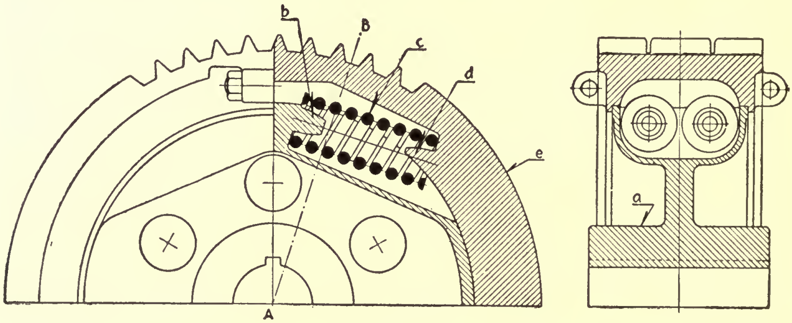

Fig. 117.

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CHAPTER XI

FRICTION GEARING

Friction gearing is employed when the positiveness of relative motion is either unnecessary or not essential. The wheels depend for their driving value upon the coefficient of friction of the composition wheel against its iron mate, and their actual driving capacity becomes a function of the pressure with which they are held in contact. This pressure is limited by the ability of the composition surface to endure it without injury. The composition wheel should never be used as the driven member of a pair of wheels, since, being of a softer material, its surface would be injured and eventually ruined by the occasional rotation of the iron wheel against it under pressure before starting it from rest, or after an excessive load has brought it to a standstill. Friction gearing may be used for transmitting power between shafts that are parallel or between those that intersect.

194. Experimental Results.—Several years ago an extended series of experiments on friction gearing was made at the laboratory of Purdue University, the results of which were reported by Prof. Goss in a paper before the American Society of Mechanical Engineers. These experiments were made upon compressed strawboard driving wheels approximately 6, 8, 12 and 16 inches in diameter in contact with a turned cast-iron follower 16 inches in diameter. The pressures per inch of face varied from 75 to more than 400 pounds, and the tangential velocity from 400 to 2,800 feet per minute. The following are some of the conclusions derived from these tests:

(*a*) Slippage increases gradually with the load up to 3 per cent., and when it exceeds this value it is liable to increase very suddenly to 100 per cent., or in other words, motion ceases.

(*b*) The coefficient of friction varies with the slip, and becomes a maximum when the slip lies between 2 and 6 per cent.

(*c*) The coefficient of friction seems to be constant for all pressures up to a limit lying between 150 and 200 pounds per inch of face, but decreases as the pressure increases.

(*d*) The coefficient of friction is not affected by variations in the tangential velocity between the limits 400 and 2,800 feet per minute.

(*e*) The coefficient of friction for the 6-inch wheel was about 10 per cent. less than for the others.

(*f*) A coefficient of friction of 20 per cent. is readily obtained with wheels 8 inches in diameter and larger.

In December, 1907, Prof. Goss presented before the American Society of Mechanical Engineers, a second paper on the subject of friction drives, in which he reported the results of another extensive series of tests. The values of the coefficient of friction and permissible working pressure per inch of face for the various materials experimented with are given in Table 64. Pressures exceeding 150 pounds per inch of face may be used providing the conditions under which the wheels are working are known definitely, or where experience has proven their use permissible. Several manufacturers now make wheels that allow the use of working pressures of 250 pounds or more.

Table 64.—Experimental Data Pertaining to Friction Gearing

Material	Coefficient of friction-working values			Safe working pressure
	Cast iron	Aluminum	Type metal	
Leather	0.135	0.216	0.246	150
Wood	0.150	.....	.....	150
Tarred fiber	0.150	0.183	0.165	240
Cork composition	0.210	.....	.....	50
Straw fiber	0.255	0.273	0.186	150
Leather fiber	0.309	0.297	0.183	240
Sulphite fiber	0.330	0.318	0.309	140

SPUR-FRICTION GEARING

195. Plain Spur Frictions.—The simplest form of friction gearing consists of two plain cylindrical wheels held in contact with each other by properly constructed bearings. Such wheels, shown in Fig. 118, are known as spur frictions. To determine the least pressure that must be applied at the line of contact in order that the gears may transmit a given horse power, the following method may be used:

Let H = the horse power transmitted.
V = the mean velocity of the gears in feet per minute.
f = face of the gears.
p = permissible pressure per inch of face.
μ = coefficient of friction.

Evidently, the total radial pressure between the two wheels at the line of contact is fp, and the tangential force due to this pressure is μfp. Now this force must at least equal the tangential resistance or

$$T = \frac{33{,}000\,H}{V} \tag{286}$$

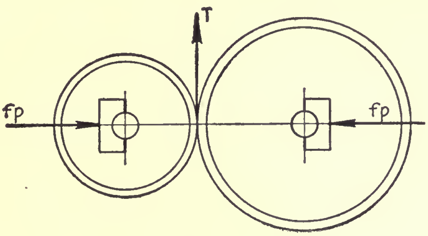

FIG. 118.

Therefore, the least pressure required between the two spur frictions, so that H horse power may be transmitted is

$$fp = \frac{33{,}000\,H}{\mu V} \tag{287}$$

196. Applications of Spur Frictions.—Plain spur-friction gearing is used for driving light power hoists, coal screens, gravel washers, and various forms of driers. Another useful and interesting application of spur frictions is found in friction-board drop hammers used in the production of all kinds of drop forgings. Two designs, differing somewhat in the method of driving the friction rolls, are shown in Figs. 119 and 120. The methods of operation and control of the hammer are similar in the two designs. In Fig. 119, the friction rolls b and c are keyed rigidly to their respective driving shafts d and e and may be brought into contact with the board a, at the lower end of which is fastened the ram. It will be noticed that the friction rolls are brought into contact with the board a by rotating the eccentric bearings in which the driving shafts are supported. The bearings are rotated slightly by the rod f, which in turn is tripped by the descending hammer. The ram and the various operating accessories are not included in the figure.

The function of the friction rolls b and c is to return the ram to its initial position after a blow has been struck. As soon as the ram returns to its initial position, it lifts the rod f by means of a suitable mechanism, and consequently the friction board will again drop unless it is held by the pawls g and h. These pawls are controlled by the operator through a treadle.

The design shown in Fig. 119 is that used by the Billings and Spencer Co., and differs from the other in that both shafts d and e are mounted on eccentric bearings, each shaft being driven by a belt and pulley. Fig. 120 shows the general details of the design used by the Toledo Machine and Tool Co. The driving pulleys are keyed to the shaft e, which has mounted upon it the roll c and a spur gear m. The latter meshes with the gear n which is fastened to the roll b, both being mounted with a running fit on the shaft d. The shaft d is supported on eccentric bearings by means of which the two rolls are brought in contact with the board a.

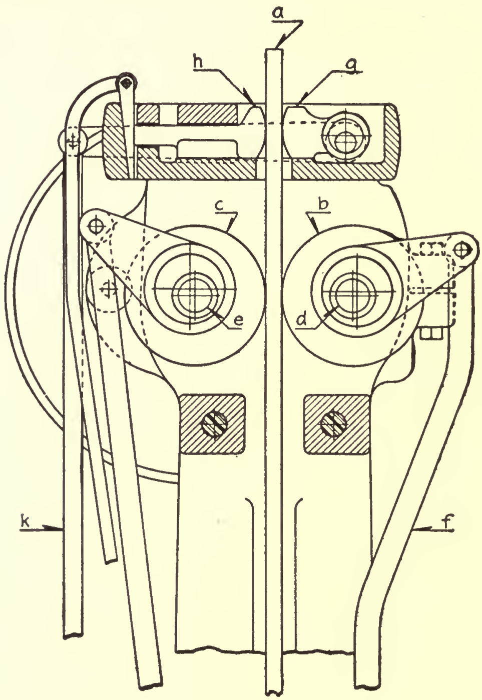

FIG. 119.

In some designs of drop hammers, the teeth on the gears m and n are made of the buttressed type, since they transmit power in only one direction and at the same time are subjected to a considerable shock.

197. Analysis of a Drop Hammer.—The total lifting force T exerted on the friction board by the driving rolls must exceed the weight Q of the ram so that it is possible to accelerate the latter at the beginning of the hoisting period.

Let t_1 = number of seconds required to accelerate the ram.

t_2 = number of seconds during which the ram moves upward at constant velocity.

t_3 = number of seconds required to bring the ram to rest after releasing the rolls.
v = maximum velocity of ram during hoisting period.

The hoisting period is really made up of three separate periods, namely: (1) the period during which the ram is accelerated; (2) the constant-speed period; (3) the period immediately following the releasing of the driving rolls, during which the ram gives up its kinetic energy.

Using the above notation and assuming uniformly accelerated motion, the distance h travelled by the ram in its upward travel is given by the following expression:

$$h = \frac{vt_1}{2} + vt_2 + \frac{v^2}{2g} \quad (288)$$

Denoting the ratio of T to Q by the symbol c and disregarding the frictional resistances, it is evident that the accelerating force is

$$Q(c - 1) = \frac{Qv}{gt_1},$$

from which

$$t_1 = \frac{v}{g(c - 1)} \quad (289)$$

Substituting (289) in (288), and simplifying,

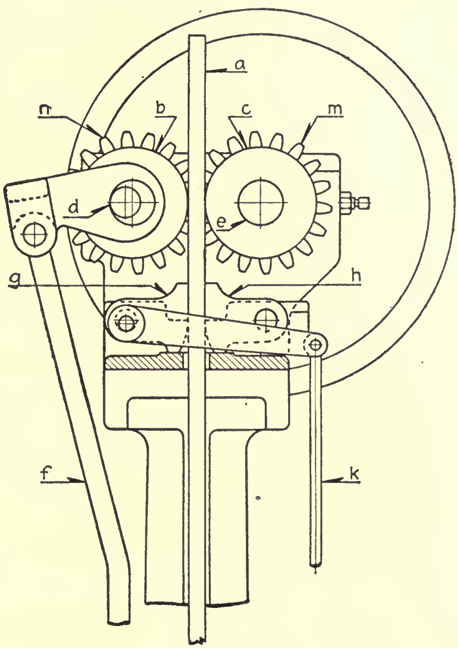

FIG. 120.

$$h = \frac{v^2}{2g}\left[\frac{c}{c - 1}\right] + vt_2,$$

from which

$$t_2 = \frac{h}{v} - \frac{v}{2g}\left[\frac{c}{c - 1}\right] \quad (290)$$

The total number of seconds required for the hoisting period is

$$t_1 + t_2 + t_3 = \frac{v}{2g}\left[\frac{c}{c - 1}\right] + \frac{h}{v} \quad (291)$$

The number of seconds required by the ram to fall through the distance h is

$$t_4 = \sqrt{\frac{2h}{g}} \tag{292}$$

Hence the time required for a complete cycle may be readily determined.

During the accelerating period, the work W_1 expended by the friction rolls upon the board of the ram is

$$W_1 = \frac{Tv^2}{g(c-1)} \tag{293}$$

and during this same period, the useful work done is

$$W = \frac{Qv^2}{2g(c-1)} \tag{294}$$

Hence the lost work is

$$W' = \frac{Qv^2}{2g}\left[\frac{2c-1}{c-1}\right] \tag{295}$$

The work W' represents the loss due to slippage which will tend to produce excessive temperatures, thereby charring the board of the ram; hence its magnitude must be kept down by using a speed v that is not too high, and by making c relatively large. In actual hammers, c varies from 1.2 to 2.

The total lifting force T is produced by the pressure of the rolls upon the board and is given by the relation

$$T = 2\mu P, \tag{296}$$

in which P denotes the normal pressure between each roll and the board and μ the coefficient of friction, which may be assumed to vary from 0.25 to 0.35.

198. Grooved Spur Frictions.—In the case of plain spur frictions, the pressures upon the shafts are excessive for large powers, thus causing a considerable loss of power due to the journal friction. To decrease this loss of power by decreasing the pressure upon the shafts, a form of gearing known as grooved spur frictions is used. Fig. 121(a) shows how such gears are formed. It is desired to determine the relation between the horse power transmitted and the total radial pressure between the frictions.

Let P = radial thrust upon one projection or groove.
R = total reaction on each side of projection or groove.

T = tangential resistance on each projection or groove.
n = number of projections or grooves in contact.
2α = angle of the grooves.

In Fig. 121(b) are shown the various forces acting upon one of the projections. From the force triangle ABC it follows that

$$P = 2R \sin (\alpha + \varphi), \qquad (297)$$

in which φ denotes the angle of friction as shown in the figure.

In any type of friction gearing the tangential resistance T is equivalent to the coefficient of friction multiplied by the total normal pressure at the line of contact, hence for the case under discussion

$$T = \frac{33{,}000\, H}{nV} = 2R\sin\varphi, \qquad (298)$$

in which H and V have the same meaning as in Art. 195.

Eliminating the factor $2R$ by combining (297) and (298), the least total pressure nP between the two grooved friction gears is given by the following expression:

$$nP = \frac{33{,}000\, H}{\mu V}(\sin \alpha + \mu \cos \alpha) \qquad (299)$$

From an inspection of Fig. 121 (a), it is evident that along the lines of contact between the two gears, the so-called pitch point is the only one at which the two gears have the same peripheral speed. At all other points there is a difference in speed between the gears, and hence there must be slippage, as a result of which excessive wear might be expected. In order to make this difference in speed small and at the same time decrease the resultant wear, the projections must be made com-

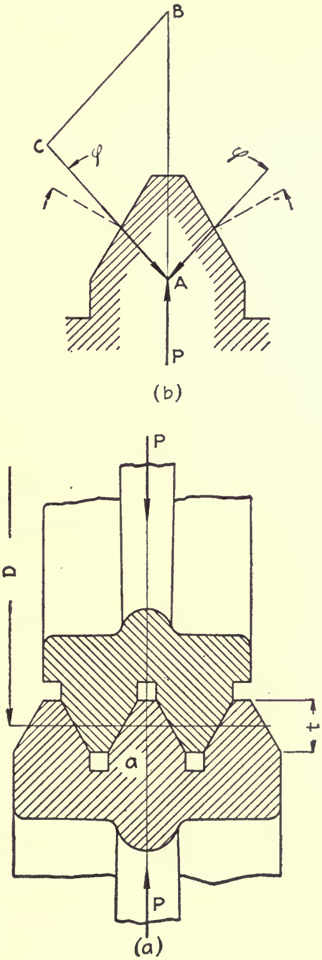

FIG. 121.

paratively short. Furthermore, the normal pressure per inch of side of groove or projection should not, according to Bach, exceed 3,200 pounds. When a considerable number of grooves are used, it is necessary that they be machined very accurately or excessive wear due to high contact pressure will result. The angle 2α of the grooves varies from 30 to 40 degrees.

BEVEL-FRICTION GEARING

Bevel frictions are used when it is desired to transmit power by means of shafts that intersect. Such gears are shown in Fig. 122. Referring to Fig. 122, the gear marked 2 is keyed rigidly to its shaft while the gear 1 is splined to its shaft. By means of a specially designed thrust bearing operated by a lever or other means, the bevel gear 1 is brought into contact with gear 2 and held there under pressure. In designing a bevel-friction transmission, both the starting and running conditions should be investigated.

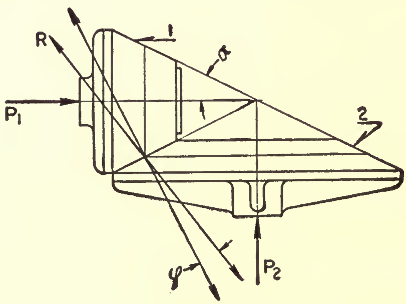

FIG. 122.

199. Starting Condition.—In the following analysis it is assumed that the transmission is to be started under full load, a condition met with frequently in connection with hoisting machinery. At the instant of starting, due to the relative motion between the surfaces in contact, the reaction R instead of being normal is inclined away from the normal by the angle of friction φ, as shown in Fig. 122.

As in the preceding cases, the tangential force that can be transmitted by the two gears is equal to the product of the total normal pressure and the coefficient of friction; thus

$$T = \mu R \cos \varphi = 33{,}000 \frac{H}{V} \tag{300}$$

From the geometry of the figure it is evident that

$$R = \frac{P_1}{\sin(\alpha + \varphi)} = \frac{P_2}{\cos(\alpha + \varphi)} \tag{301}$$

Combining (300) and (301), the expressions for the least axial thrusts that come upon the gears are as follows:

$$P_1 = \frac{33{,}000\,H}{\mu V}(\sin\alpha + \mu\cos\alpha) \tag{302}$$

$$P_2 = \frac{33{,}000\,H}{\mu V}(\cos\alpha - \mu\sin\alpha) \tag{303}$$

200. Running Condition.—After the transmission gets up to speed, the relative motion between the gears along the line of contact ceases; hence the reaction between the two surfaces in contact is normal. Calling this reaction R', we have the relations

$$T' = \mu R' = 33{,}000\frac{H}{V}$$

$$R' = \frac{P_1'}{\sin\alpha} = \frac{P_2'}{\cos\alpha}$$

Combining these equations and solving for P_1' and P_2' we obtain the following:

$$P_1' = \frac{33{,}000\,H\sin\alpha}{\mu V} \tag{304}$$

$$P_2' = \frac{33{,}000\,H\cos\alpha}{\mu V} \tag{305}$$

The expressions just derived may be put in slightly different form by substituting for $\sin\alpha$ and $\cos\alpha$ their equivalents in terms of the diameters D_1 and D_2 of the gears. The resulting forms are

$$P_1' = \frac{33{,}000\,H}{\mu V}\left[\frac{D_1}{\sqrt{D_1^2 + D_2^2}}\right] \tag{306}$$

$$P_2' = \frac{33{,}000\,H}{\mu V}\left[\frac{D_2}{\sqrt{D_1^2 + D_2^2}}\right] \tag{307}$$

Equations (306) and (307) give the least thrusts required along the shafts of the transmission in order to transmit the given horse power.

CROWN-FRICTION GEARING

Crown-friction gearing is used to transmit power by means of shafts that intersect and are at right angles to each other. A simple form of this type of transmission as applied to the driving of a light motor car is shown in Fig. 123. In slightly modified form this same mechanism has been applied to machine tools for varying feeds. Within recent years crown frictions have been used successfully in automobile, motor-truck, and tractor transmissions, as well as in the driving of screw power presses and sensitive drilling machines.

The wheel *c* in Fig. 123 is generally faced with compressed paper, vulcanized fiber, leather, or other suitable friction material,

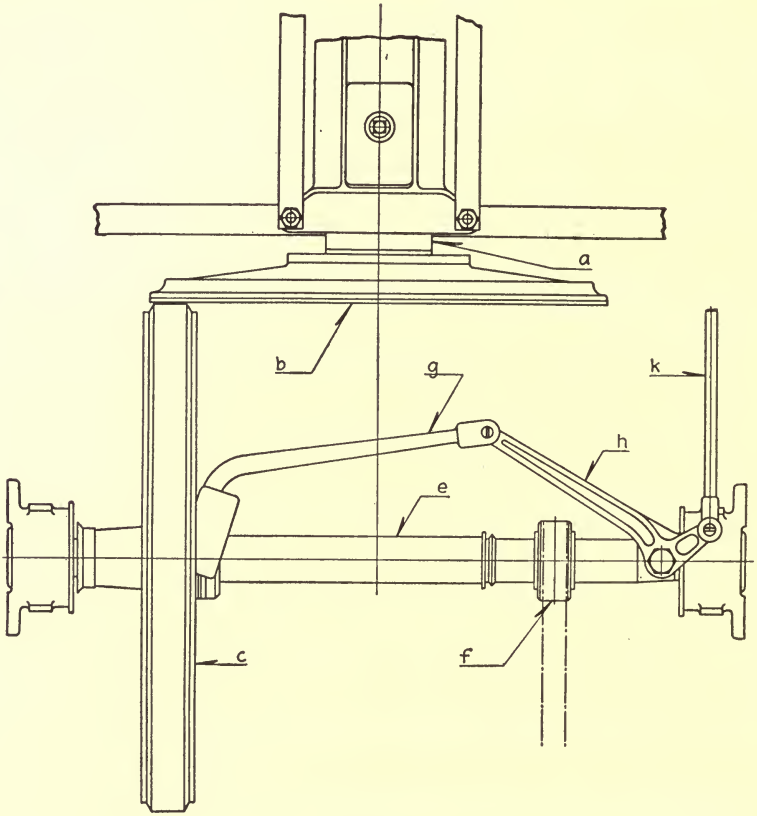

FIG. 123.

and is slightly crowned in order to decrease the slipping action which takes place, due to the varying speeds of the points in contact. Now since wheel *c* is made of a softer material than that used on the disc *b*, it should act as the driver so that its surface will not be worn flat at spots by the rotation of the disc against it under pressure. However, this is not the usual method of mounting a crown-friction transmission. As now installed, the disc serves as the driving member and in practically all cases its face is plain cast iron. In the design just mentioned, the speed of the wheel c may be varied by simply moving c across the face of the disc, while the direction of rotation of the wheel may be reversed by moving it clear across the center of the disc.

201. Force Analysis.—To determine the forces acting upon the various members of a crown-friction transmission similar to that shown in Fig. 123, the following method may be used:

The twisting moment on the driving shaft is $63{,}030 \frac{H}{N}$, hence the tangential forces acting upon the driven wheel for the two limiting speeds are as follows:

$$\left.\begin{aligned} \text{At the minimum speed, } T_1 &= \frac{126{,}060\,H}{ND_1} \\ \text{At the maximum speed, } T_2 &= \frac{126{,}060\,H}{ND_2} \end{aligned}\right\} \qquad (308)$$

in which D_1 and D_2 denotes the minimum and maximum diameters of the driving disc, respectively.

The thrusts that must be applied to the disc for the two speeds are obtained by dividing the values of T_1 and T_2 by the coefficient of friction μ, giving

$$\left.\begin{aligned} P_1 &= \frac{126{,}060\,H}{\mu ND_1} \\ P_2 &= \frac{126{,}060\,H}{\mu ND_2} \end{aligned}\right\} \qquad (309)$$

The forces actually available on the chain sprocket f for the two cases considered are as follows:

$$\left.\begin{aligned} W_1 &= \frac{\eta T_1 D}{D_3} \\ W_2 &= \frac{\eta T_2 D}{D_3} \end{aligned}\right\} \qquad (310)$$

in which D_3 denotes the diameter of the sprocket f. The efficiency η may be taken as 60 per cent. at the low speeds and 80 per cent. at the high speeds.

To determine the magnitude of the force F required to shift the driven wheel c, multiply the thrust P exerted by the disc b upon the wheel c by the coefficient of friction μ and add to this the force required to overcome the frictional resistance between the wheel c and its shaft e. Whence

$$F = P(\mu + \mu_1), \tag{311}$$

in which μ_1 denotes the coefficient of friction between the wheel c and its shaft e.

202. Pressures on the Various Shaft Bearings.—(*a*) *Driving shaft.*—The various forces discussed in Art. 201 produce pressures upon the several bearings used in the transmission. The same type of crown friction drive shown in Fig. 123 is represented

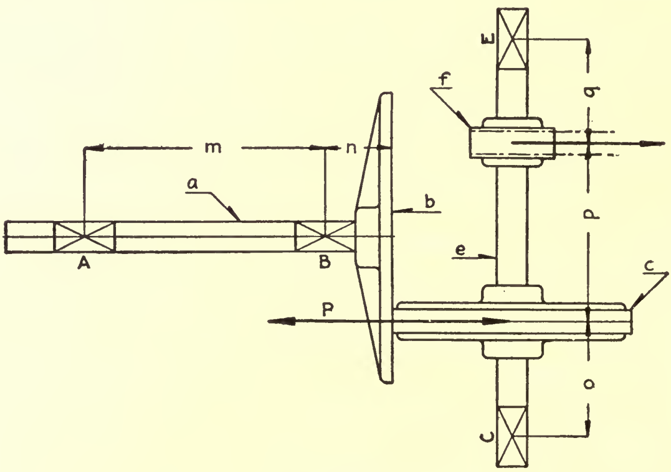

FIG. 124.

diagrammatically in Fig. 124. The journal A on the driving shaft a, due to the tangential force T_1 and the thrust P at the point of contact is subjected to the following pressures:

1. A horizontal force due to P, having a magnitude of $\frac{PD_1}{2m}$. This result is obtained by taking moments about axis of the journal B.

2. A vertical force, due to T_1, the magnitude of which is obtained as in the preceding case. This pressure acts in the same direction as the tangential force T_1 and its magnitude is $\frac{n}{m}T_1$.

By a similar analysis the following forces acting upon the journal B are determined:

3. A horizontal force equal to $\frac{PD_1}{2m}$.

4. A vertical force equal to $\frac{T_1}{m}(m + n)$.

From the analysis of the forces acting upon the shaft a, it is evident that at the point B this shaft is subjected to a bending stress in addition to a torsional stress. There may also be a compressive stress due to the thrust P, but this can be avoided by a careful arrangement of the thrust bearing at that point. To determine the size of the shaft, use the principles discussed in the chapter on shafting.

(*b*) *Driven shaft.*—The driven shaft e is subjected to a combined torsion and bending between the wheel c and the sprocket f. The wheel c is acted upon by the two forces P and T_1, the former producing pure bending of the shaft and the latter combined torsion and bending. After having calculated the load on the sprocket f, the pressures upon the bearings C and E may be obtained by an analysis similar to that used above.

203. Friction Spindle Press.—The so-called friction spindle press used to a large extent in Germany is an excellent application of crown-friction gearing. In this country, the Zeh and Hahnemann Co. of Newark, N. J. are about the only manufacturers that have introduced friction gearing on presses for forging and stamping operations. One of their designs is shown in Fig. 125. The friction wheel d is really a heavy flywheel fastened rigidly to the vertical screw e. The face of the flywheel is grooved, and into this groove is fitted a leather belt which serves as the friction medium. The driving shaft a is equipped with two plain cast-iron discs b and c, which may be brought into contact with the friction wheel d by moving the entire shaft endwise. It should be understood that the function of the friction drive is merely to accelerate the flywheel d, and the energy stored up during the accelerating period does the useful work. To accelerate the flywheel, the driving shaft is moved endwise against the action of the spring f until b is in contact with d, thus causing the screw to rotate. This rotation causes the screw and attached flywheel to move downward, increasing its rotative speed as well as that in a downward direction. It should be noted that the flywheel generates a spiral on the face of the disc b. At the end of the working stroke of the screw a suitable tappit, located on the crosshead at the lower end of the screw, operates a linkage which disengages *b* and *d*, thus permitting the spring *f* to force the disc *c* against the flywheel *d* causing it to return to the top of the stroke.

This type of press is especially adapted for work in which a hard end blow is required. It is not suitable for work requiring a heavy pressure through a considerable part of the stroke, such as is required in the manufacture of shells, for example.

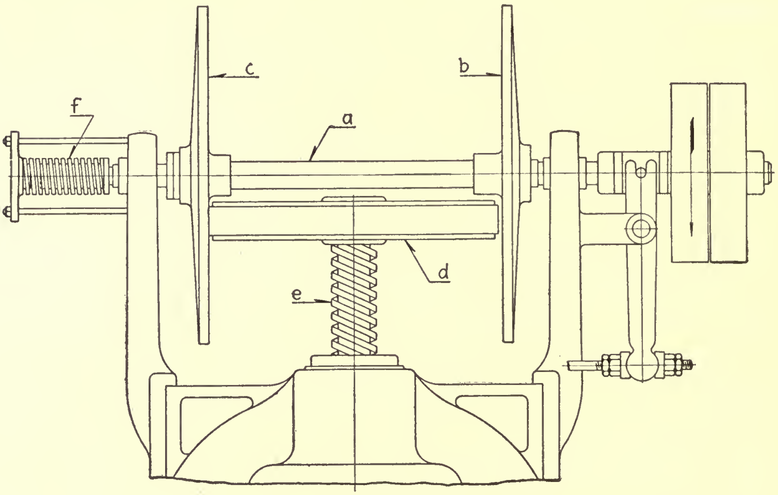

FIG. 125.

204. Curve Described by the Flywheel.—In discussing the action of the friction spindle press, it is of interest to investigate the nature of the path or curve described by the flywheel on the face of the friction disc. It is apparent that the tangential velocity v_t of the flywheel rim is proportional to the radius of the driving disc; hence at any point a distance r from the center of the driving shaft, the magnitude of this velocity is

$$v_t = cr \tag{312}$$

The velocity v_s of the screw in a direction parallel to its center line also is proportional to the radius r; hence

$$v_s = kr \tag{313}$$

Combining (312) and (313), it follows that the ratio of v_s to v_t is a constant, the value of which is readily determined. Representing the diameter of the flywheel by D and the lead of the screw by p, both being expressed in inches, the relation existing between v_s and v_t, is

$$v_s = \left(\frac{p}{\pi D}\right) v_t,$$

from which

$$\frac{v_s}{v_t} = \frac{p}{\pi D} = K \tag{314}$$

Let ABC of Fig. 126 represent a part of the curve described by the flywheel on the surface of the driving disc; then

$$\tan \alpha = \frac{r d\theta}{dr} = \frac{1}{K};$$

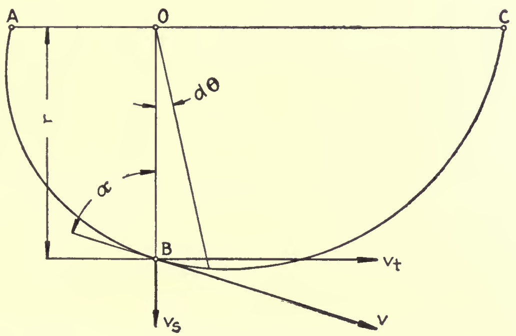

FIG. 126.

since the velocity v makes a constant angle with the radius vector. Hence, we get by integration

$$\log_e r = K\theta$$

or

$$r = e^{K\theta} \tag{315}$$

It appears that the curve described by the flywheel in moving across the face of the driving disc is an equiangular or logarithmic spiral.

205. Pressure Developed by a Friction Spindle Press.—(*a*) *Working stroke.*—Beginning with the ram at the top of the down stroke, the friction wheel d being at rest will tend to assume the same velocity as the driving disc b, but due to slippage this condition will not prevail until the screw and flywheel have moved downward a certain distance. During the next period the wheel d is accelerated with practically no slippage, and when the tool strikes the work, the friction wheel, the screw and ram have accumulated a certain amount of energy which is given out in performing useful work during the remainder of the stroke. It should be noted that the driving disc is thrown out of contact with d about the same instant that the tool strikes the work; hence the driving force is not considered as doing any useful work, but is used merely to accelerate the moving system. It is evident, therefore, that the pressure developed during the latter part of the stroke depends upon the energy stored up by the flywheel, screw, and ram, and the distance through which the ram moves in doing the work.

Assuming that the flywheel d is r_2 inches from the center of the driving shaft when the tool strikes the work, the kinetic energy in the flywheel and screw due to the tangential velocity v_t is given by the following expression:

$$E_1 = \frac{W_1 v_t^2}{2g},$$

in which W_1 is the equivalent weight of the wheel and screw reduced to the outside radius of the rim having a velocity of

$$v_t = \frac{\pi r_2 N}{360},$$

in which N denotes the revolutions per minute made by the driving shaft.

Denoting the actual weight of the wheel, screw and ram by the symbol W_2, we find that the energy stored up in these parts due to the velocity v_s is

$$E_2 = \frac{W_2 v_s^2}{2g},$$

in which v_s, according to (314), is

$$\frac{pv_t}{\pi D} = \frac{pr_2 N}{360\,D}$$

Now in coming to rest the moving mass W_2 also does external work, the magnitude of which is

$$E_3 = \frac{W_2}{12}(r_3 - r_2)$$

in which r_3 denotes the distance between the center line of the driving shaft and the flywheel at the end of the downstroke.

Summing up, we find that the theoretical amount of work that can be done is

$$E = E_1 + E_2 + E_3 \tag{316}$$

and multiplying this by the efficiency η, the external or useful work that can be done is ηE.

The average pressure Q upon the tool multiplied by the distance $\frac{r_3 - r_2}{12}$ through which this force acts must be equivalent to the work done by the moving system; hence

$$Q = \frac{12\eta E}{r_3 - r_2} \tag{317}$$

(*b*) *Return stroke.*—On the return stroke, the driving disc *c* is brought into contact with the wheel *d*, and since the latter is at rest for a short interval of time, we have the same conditions to contend with that prevailed at the beginning of the working stroke, namely, that the flywheel will slip until it attains the same speed as the driving disc. After the flywheel has attained the speed as the driving disc, this condition will continue until the disc *c* is released and the disc *b* is again applied.

206 Double-crown Frictions.—An interesting variable-speed friction drive used on the Albany sensitive drill press is shown in Fig. 127. It consists of two crown friction wheels, one of which is mounted on the drive shaft *a*, and the other on the spindle *k* of the drill press. A hemisphere *c*, made of cast iron and bushed with bronze, is mounted on a shaft *d*, which is pivoted on the adjustable spindle *e*. By means of the handle *g*, the shaft *d* and the hemisphere *c* may be moved in a vertical plane. The speed of rotation of the hemisphere, and the speed of the driven wheel *h* are thus varied. The contact pressure between the hemisphere and the friction wheels may be increased or decreased by means of the adjusting nut *f*. Ball bearings are used in all of the important bearings on the machine, as shown in Fig. 127.

207. Efficiency of Crown-friction Gearing.—A study of the action of crown-friction gearing shows clearly that the points on the disc *b* in contact with the inner and outer edges of the driven wheel *c* will travel unequal distances per revolution of the disc (see Fig. 124). From this it follows that there is slippage between the wheel and the disc at the line of contact. Denoting by f the width of the face of the wheel c, then the difference between the distances traveled per revolution of the disc by the extreme points in contact is $2\pi f$.

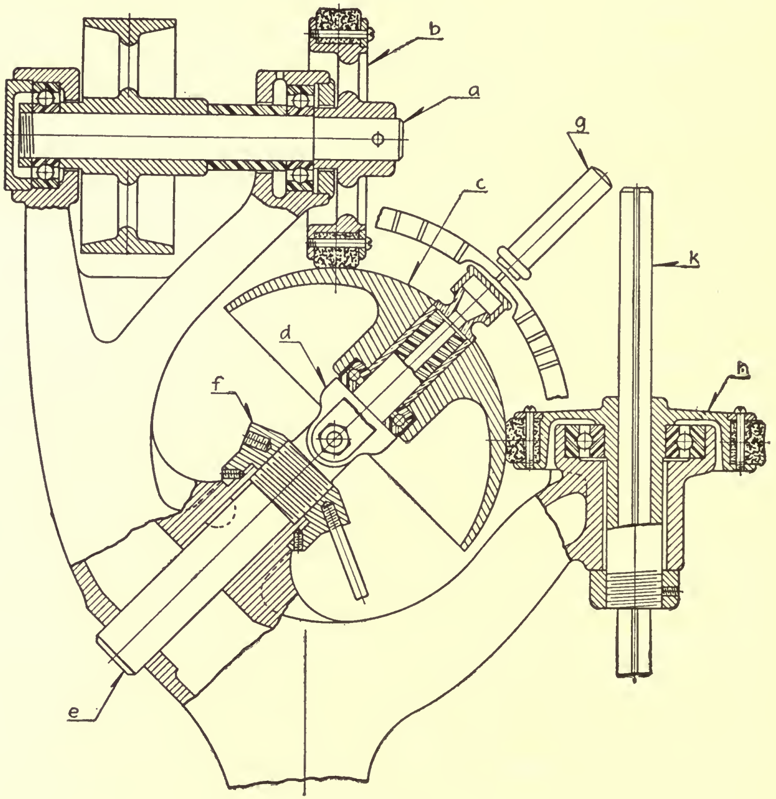

FIG. 127.

To determine the work lost per revolution due to slippage multiply the average slip πf by the tangential resistance between the wheel and the disc; thus

$$W_s = \mu \pi f P \tag{318}$$

The output per revolution of the disc is $\mu \pi P D$; hence the total work put in, exclusive of that required to overcome the frictional resistances of the various bearings, is given by the expression

$$W = \mu \pi P (D + f) \tag{319}$$

Since the efficiency of any machine is equal to the output divided by the input, we obtain in this case

$$\eta = \frac{D}{D + f} \tag{320}$$

According to (320), the efficiency of crown-friction gearing is independent of the diameter of the driven wheel. Furthermore, the efficiency increases as the face of the crown wheel is decreased, and as the line of contact between the disc and the wheel is moved farther from the center of the disc.

MOUNTING FRICTION GEARING

In general, friction gearing must be mounted in such a manner that the pressure required between the surfaces in contact in order to transmit the desired horse power can readily be produced. This result is obtained by equipping one of the shafts with a special bearing or set of bearings.

208. Thrust Bearings for Friction Gearing.—(*a*) *Bearings for spur and grooved frictions.*—In the case of spur and grooved friction gearing, the pinion shaft is mounted on eccentric bearings, the constructive details of which are shown clearly in Fig. 128. The gears themselves should be located close to the bearings in order to insure rigidity, thus obviating undue wear on the gears as well as on the bearings.

(*b*) *Thrust bearings for bevel frictions.*—For engaging a pair of bevel gears, and taking up any wear that may occur, two types of bearings are in common use. The first type, which may be called a quick-acting end-thrust bearing, is shown in Fig. 129. It is used in connection with bevel frictions requiring frequent throwing in and out of engagement. The inner sleeve *a* forming the bearing for the shaft has a helical slot into which the turned end of the adjusting screw *b* is fitted. It is evident that rotating the sleeve *a* in the proper direction will cause the sleeve to advance in an axial direction, thus engaging the gears.

The second type of end thrust bearing works on the same general principle. The inner sleeve, instead of being fitted with a helical slot, is threaded as shown in Fig. 130. This design is well adapted to installations in which the friction gears are not engaged or disengaged very frequently.

(c) *Thrust bearings for crown frictions.*—For engaging crown frictions, the same type of bearings as those shown in Figs. 129

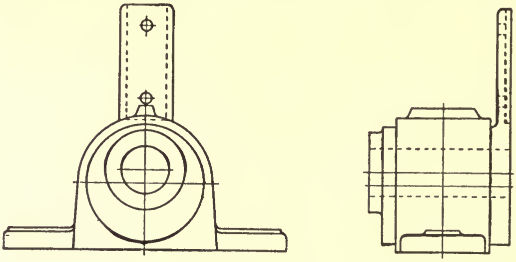

FIG. 128.

and 130 may be used. Occasionally, spring thrust bearings are used in place of those just mentioned.

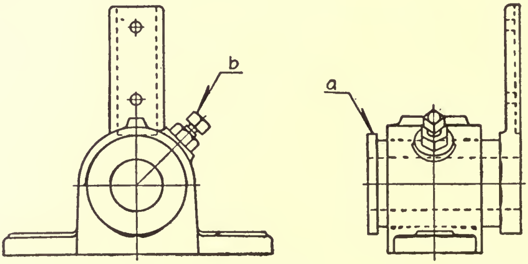

FIG. 129.

209. Starting Loads.—As stated in Art. 194, the coefficient of friction is a maximum when the slip between the friction gears

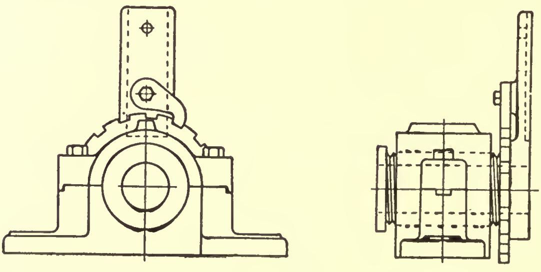

FIG. 130.

lies between 2 and 6 per cent. Experiments have also shown that the coefficient of friction decreases gradually as the slip increases; hence when a friction transmission is started under load, the pressure that must be applied to the surfaces in contact is from two to three times as great as that required for normal operation. This is due to the decrease in the coefficient of friction caused by the excessive slippage during the period of starting. From this discussion it follows that the bearings described in the preceding paragraphs must be designed for the starting conditions. After the transmission is once started the thrusts on the gears may be reduced considerably, thus eliminating excessive wear and lost work.

References

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Paper Friction Wheels, *Trans.* A. S. M. E., vol. 18, p. 102.

Friction Driven Forty-four Foot Pit Lathe, *Trans.* A. S. M. E., vol. 24, p. 243.

Power Transmission by Friction Driving, *Trans.* A. S. M. E., vol. 29, p. 1093.

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CHAPTER XII

SPUR GEARING

Friction gearing, as has been stated, is not suitable for the transmission of large amounts of power, nor where it is desirable that the velocity ratio between the driving and driven members be absolutely positive. For such a transmission it becomes necessary to provide the surfaces in contact with grooves and projections, thus providing a positive means of rotation. The original surfaces of the frictions then become the so-called pitch surfaces of the toothed gears, and the projections together with the grooves form the teeth. These teeth must be of such a form as to satisfy the following conditions:

(*a*) The teeth must be capable of transmitting a uniform velocity ratio. The condition is met if the common normal at the point of contact of the tooth profiles passes through the pitch point, *i.e.*, the point of tangency of the two pitch lines.

(*b*) The relative motion of one tooth upon the other should be as much a rolling motion as possible on account of the greater friction and wear attendant to sliding. With toothed gearing, however, it is impossible to have pure rolling contact and still maintain a constant velocity ratio.

(*c*) The tooth should conform as nearly as possible to a cantilever beam of uniform strength, and should be symmetrical on both sides so that the gear may run in either direction.

(*d*) The arc of action should be rather long so that more than one pair of teeth may be in mesh at the same time.

210. Definitions.—Before taking up the discussion of the various types of tooth curves, it is well to familiarize ourselves with the meaning of different terms and expressions used in connection with gearing of all kinds.

(*a*) By the term *circular pitch* is meant the distance from one tooth to a corresponding point on the next tooth, measured on the pitch circle. The circular pitch is equal to the circumference of the pitch circle divided by the number of teeth in the gear.

(*b*) The *diametral pitch* is equal to the number of teeth in the gear divided by the pitch diameter. It is not a dimension on the gear, but is simply a convenient ratio.

(*c*) The term *chordal pitch* may be defined as the distance from one tooth to a corresponding point on the next measured on a chord of the pitch circle instead of on the circumference. This pitch is used only in making the drawing or by the pattern maker if the teeth are to be formed on a wood pattern.

(*d*) The *thickness* of the tooth is the thickness measured on the pitch line, as illustrated in Fig. 131.

(*e*) By the *tooth space* is meant the width of the space on the pitch line.

(*f*) The term *backlash* means the difference between the tooth space and the thickness of the tooth.

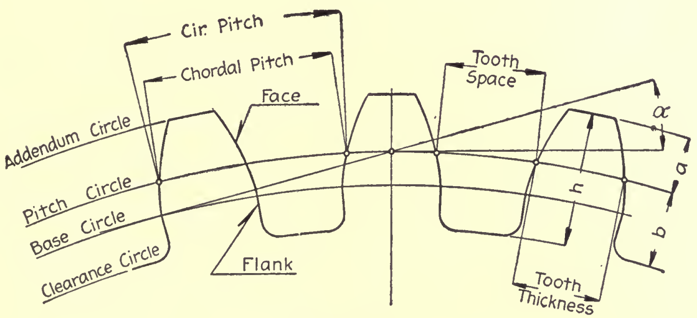


Fig. 131.

(*g*) By the term *addendum* is meant the distance from the pitch circle to the ends of the teeth, as dimension *a* in Fig. 131.

(*h*) The distance *b* between the pitch circle and the bottom of the tooth space is called the *dedendum.*

(*i*) The *clearance* is the difference between the addendum and the dedendum, or in other words, the amount of space between the root of a tooth and the point of the tooth that meshes with it.

(*j*) As shown in Fig. 131, the *face of the tooth* is that part of the tooth profile which lies between the pitch circle and the end of the tooth.

(*k*) The *flank of the tooth* is that part of the tooth profile which lies between the pitch circle and the root of the tooth, as represented in Fig. 131.

(*l*) The *line of centers* is the line passing directly through both centers of a pair of mating gears.

(*m*) The *pitch circles* of a pair of gears are imaginary circles, the diameters of which are the same as the diameters of a pair of friction gears that would replace the spur gears.

(*n*) The *base circle* is an imaginary circle used in involute gearing to generate the involutes which form the tooth profiles. It is drawn tangent to the line representing the tooth thrust, as shown in Fig. 131.

(*o*) The *describing circle* is an imaginary circle used in cycloidal gearing to generate the epicycloidal and hypocycloidal curves which form the tooth profiles. There are two describing circles, one inside and one outside of the pitch circle, and they are usually of the same size.

(*p*) By the *angle of obliquity of action* is meant the inclination of the line of action of the pressure between a pair of mating teeth to a line drawn tangent to the pitch circle at the pitch point, as represented in Fig. 131 by the angle α or the angle *DCF* in Fig. 132.

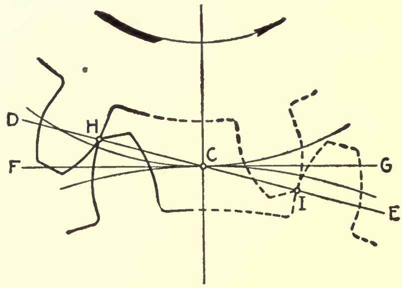


FIG. 132.

(*q*) The *arc of approach* is the arc measured on the pitch circle of a gear from the position of the tooth at the beginning of contact to the central position, that is, the arc *HC* in Fig. 132.

(*r*) The *arc of recess* is the arc measured on the pitch circle from the central position of the tooth to its position where contact ends, that is, the arc *CI* in Fig. 132.

(*s*) The *arc of action* is the sum of the arcs of approach and recess.

(*t*) By the term *velocity ratio* is always meant the ratio of the number of revolutions of the driver to the number of revolutions of the driven gear.

211. Tooth Curves.—There are many different types of curves that would serve as profiles for teeth and satisfy the condition of constant velocity ratio, with sufficient accuracy for all practical purposes; but there are in actual use only two, namely, the *involute* and the *cycloidal.* As regards strength and efficiency the two forms are practically on a par. However, the involute tooth has one decided advantage over the cycloidal, namely, that the distance between centers may be slightly greater or less than the theoretical distance without affecting the velocity ratio. The cycloidal tooth, also, has one important advantage over the involute, namely, that a convex surface is always in contact with one that is concave. Although the contact is theoretically a line, practically it is not; consequently, the wear is not so rapid as with involute teeth where the surfaces are either convex or straight.

212. Methods of Manufacture.—Gear teeth are formed in practice by two distinct processes, moulding and machine cutting. Formerly all gears were cast and the moulds were formed from complete patterns of the gears. Of late years, however, gear moulding machines are used to a considerable extent, and the results obtained are superior to the pattern-moulded gear. Even with machine moulding, however, the teeth are somewhat rough and warped out of shape, so that the gears always run with considerable friction and are not suited to high speeds. At the present time gears of ordinary size are almost always cut, except those used in the cheaper class of machinery. The method which is commonly used is to cut the teeth with a milling cutter that has been formed to the exact shape required. There are also two styles of gear planers, one of which generates mathematically correct profiles by virtue of the motion given to the cutter and the gear blank, and the other forms the outlines by following a previously shaped templet. Another method of producing machine cut teeth is by the stamping process now used extensively in the manufacture of gears for clocks, slot machines, etc.

A method of generating spur and helical gear teeth by means of a hob is now recognized and accepted as the best way of producing accurate teeth. In this generating process a hob, threaded to the required pitch, is rotated in conjunction with the gear blank at a ratio dependent upon the number of teeth to be cut. The cross-section of the thread is a rack that will mesh correctly with the gear to be cut. One important advantage of this process is that only one hob is required for cutting all numbers of teeth of one given pitch. Another advantage of the hobbing system is that gears can be produced more cheaply than by any other system.

SYSTEMS OF GEAR TEETH

213. Involute System.—In the involute system of gearing the outline of the tooth is an involute of a circle, called the base circle. However, when the tooth extends below the base circle that portion of the profile is made radial. The simplest conception of an involute is as follows: if a cord, which has previously been wound around any given plane curve and has a pencil attached to its free end, is unwound, keeping the cord perfectly tight, the pencil will trace the involute of the given curve. The *base circle* may easily be obtained by drawing through the pitch point a line making an angle with the tangent to the pitch circle at this point, equal to the angle of obliquity of action; then the circle drawn tangent to this line will be the required base circle.

In order to manufacture gears economically, it is essential that any gear of a given pitch should work correctly with any other gear of the same pitch, thus making an interchangeable set. To accomplish this end, standard proportions have been adopted for the teeth.

(*a*) *Angle of obliquity.*—The angle of obliquity of action which is generally accepted as the standard for cast teeth is 15 degrees, although in cases of special design this angle is often made greater. When the angle of obliquity is increased, the component of the pressure tending to force the gears apart and producing friction in the bearings is increased; but on the other hand, the profile of the tooth becomes wider at the base and consequently the strength is correspondingly greater. Such gears, having large angles of obliquity, are used where the conditions are unusual and where the standard tooth form is not suitable. In England, teeth of greater obliquity of action and less depth than the standard are quite common, and at present there is a tendency in that direction in America. For cut teeth now used in motorcar construction as well as in machine tools, the manufacturers have adopted what is called the *stub tooth*, having an angle of obliquity of 20 degrees. The proportions of the teeth as used for this service are given in Art. 223. In designing teeth of the stub-tooth form, care must be taken to make the arc of action at least as great as the circular pitch; otherwise the teeth would not be continuously in mesh and would probably come together in such a way as to lock and prevent further rotation. The standard angle of obliquity of action, adopted by manufacturers of gear cutters and used almost exclusively before the advent of the stub teeth, is slightly at variance with the usual standard for cast teeth, being 14° 28′ 40″, the sine of which is 0.25.

(*b*) *Smallest number of teeth.*—The smallest involute gear of standard proportions that will mesh correctly with a rack of the same pitch contains 30 teeth; however, this difficulty is met by slightly correcting the points of all the teeth in the set, so that a gear of 12 teeth may mesh with any of the other gears of the same pitch. The profiles of the teeth may be drawn accurately by means of circular arcs having their centers on the base circle *B*, as shown in Fig. 133. The value of these radii for a15-degree involute have been carefully worked out by Mr. G. B. Grant of the Grant Gear Works and are given in Table 65.

214. Laying Out the Involute Tooth.—To apply the tabular values given in Table 65, draw the pitch, addendum and clearance circles in the usual way, and space off the pitch of the teeth on the pitch circle. The base circle is constructed next. This may be done as described in a preceding article or by making the distance *a* in Fig. 133 equal to one-sixtieth of the pitch diameter. With the base line *B* as a circle of centers, draw that part of the tooth profile above the pitch line *A*, generally called the face of the tooth, by using the face radius *b* given in Table 65. Next draw in that part of the tooth profile between the pitch line *A* and the base circle, using the flank radius *c* given in Table 65. To finish the tooth, that part lying between the base circle and the fillet at the root of the tooth is made a radial line, as shown in Fig. 133. It should be noticed that the values of *b* and *c* given in Table 65 are for 1 diametral pitch or 1 inch circular pitch. For any other pitch divide or multiply the tabular values by the given pitch as directed in the table.

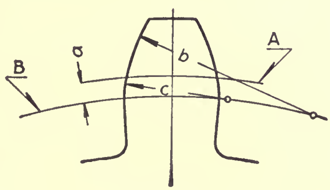


Fig. 133.

It will be noted that the tabular values in this table are for 15-degree involutes and therefore do not apply to the standard form of cut teeth. The forms given, however, may be used on the drawing, because in cutting a gear the workman needs to know only the number of teeth in the gear, and either the number of the cutter or the pitch of the hob. All that is required on a drawing is an approximate representation of the tooth profile. The table also gives values down to a 10-tooth gear, while the standard cut gear sets run down to 12 teeth only. This is theoretically the smallest standard involute gear that will have an arc of action equal to the circular pitch; however, in the 10- and 11-tooth gears the error is so slight that it is practically unnoticeable.

TABLE 65.—RADII FOR 15-DEGREE INVOLUTE TEETH ACCORDING TO G. B. GRANT

No. of teeth	Divide by the diametral pitch		Multiply by the circular pitch		No. of teeth	Divide by the diametral pitch		Multiply by the circular pitch	
	Rad. *b*	Rad. *c*	Rad. *b*	Rad. *c*		Rad. *b*	Rad. *c*	Rad. *b*	Rad. *c*
10	2.28	0.69	0.73	0.22	28	3.92	2.59	1.25	0.82
11	2.40	0.83	0.76	0.27	29	3.99	2.67	1.27	0.85
12	2.51	0.96	0.80	0.31	30	4.06	2.76	1.29	0.88
13	2.62	1.09	0.83	0.34	31	4.13	2.85	1.31	0.91
14	2.72	1.22	0.87	0.39	32	4.20	2.93	1.34	0.93
15	2.82	1.34	0.90	0.43	33	4.27	3.01	1.36	0.96
16	2.92	1.46	0.93	0.47	34	4.33	3.09	1.38	0.99
17	3.00	1.58	0.96	0.50	35	4.39	3.16	1.39	1.01
18	3.12	1.69	0.99	0.54	36	4.45	3.23	1.41	1.03
19	3.22	1.79	1.03	0.57	37–40	4.20		1.34	
20	3.32	1.89	1.06	0.61	41–45	4.63		1.48	
21	3.41	1.98	1.09	0.63	46–51	5.06		1.61	
22	3.49	2.06	1.11	0.66	52–60	5.74		1.83	
23	3.57	2.15	1.13	0.69	61–70	6.52		2.07	
24	3.64	2.24	1.16	0.71	71–90	7.72		2.46	
25	3.71	2.33	1.18	0.74	91–120	9.78		3.11	
26	3.78	2.42	1.20	0.77	121–180	13.38		4.26	
27	3.85	2.50	1.23	0.80	181–360	21.62		6.88	

(*b*) *Laying out the rack tooth.*—It was found necessary to devise a separate means of drafting the rack. The tooth is drawn in the usual manner, the sides from the root line to a point midway between the pitch and the addendum lines making angles of 75 degrees with the pitch line. The outer half of the face is formed by a circular arc with its center on the pitch line and its radius equal to 2.10 inches divided by the diametral pitch, or 0.67 multiplied by the circular pitch. The radius of the fillet at the root of the tooth is taken as one-seventh of the widest part of the tooth space.

215. Standard Involute Cutters.—Brown and Sharpe, the leading manufacturers of formed gear cutters in this country, furnish involute cutters in sets of eight for each pitch, as shown in Table 66.

Table 66.—Brown and Sharpe Standard Involute Cutters

Cutter No. 1 will cut gears from	135 teeth to	a rack.	
Cutter No. 2 will cut gears from	55 teeth to	134 teeth.	
Cutter No. 3 will cut gears from	35 teeth to	54 teeth.	
Cutter No. 4 will cut gears from	26 teeth to	34 teeth.	
Cutter No. 5 will cut gears from	21 teeth to	25 teeth.	
Cutter No. 6 will cut gears from	17 teeth to	20 teeth.	
Cutter No. 7 will cut gears from	14 teeth to	16 teeth.	
Cutter No. 8 will cut gears from	12 teeth to	13 teeth.	

When more accurate tooth forms are desired they also furnish to order cutters of the half sizes, making a set of fifteen instead of eight cutters.

All of the above cutters are commonly based on the diametral pitch and are made in the following sizes:

From 1 to 4 diametral pitch, the pitch advances by quarters.

From 4 to 6 diametral pitch, the pitch advances by halves.

From 6 to 16 diametral pitch, the pitch advances by whole numbers.

From 16 to 32 diametral pitch, the pitch advances by even numbers.

Then 36, 38, 40, 44, 48, 50, 56, 60, 64, 70, 80, and 120 diametral pitch.

At a slightly greater cost, cutters based on circular pitch may be obtained, and the sizes vary as follows:

From 1 to 1½-inch circular pitch, the pitch advances by ⅛-inch increments.

From 1½ to 3-inch circular pitch, the pitch advances by ¼-inch increments.

216. Action of Involute Teeth.—Fig. 134 illustrates the action of a pair of involute teeth. Let the circles *a* and *b* represent the base circles of a pair of involute gears, the pitch circles of which would be the circles described about the centers *A* and *B* with radii of *AC* and *BC* respectively. Imagine a cord attached to *a* extending around the circumference to a point *D*, from there directly across to *E* and around the circumference of *b*. Let the central point of the cord be permanently marked in some manner and be denoted by *C*. Now rotate *a* in the direction of the arrow and trace the path of the point *C* on the surface of *a* extended, on the surface of *b* extended, and also its actual path in space. It is evident that these three curves will be *CG*, *CH*, and *CJ*, and that *CG* and *CH* will be parts of the involutes of the two base circles *a* and *b*. Now reverse the rotation of *B* and rewind the string on *b* until *C* reaches the point *K*. During this motion it will complete the tooth forms *CF* and *FI*. Bearing in mind that *C* is always the point of contact of the teeth, its path is evidently *JK* and coincides exactly with the line of pressure between the teeth, since the line *CD* is always normal to the involute curve it

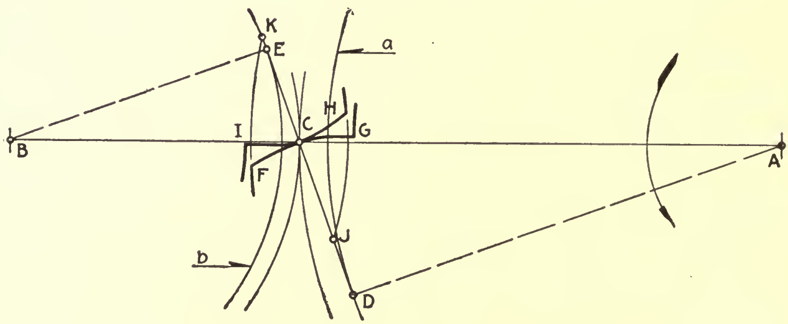


Fig. 134.

is generating. If the centers *A* and *B* should be misplaced slightly on account of wear in the bearings or journals, a uniform velocity ratio would still be transmitted because the normals would still pass through the point *C*. The shifting of the centers will result in a change of the obliquity of the pressure on the teeth, and the length of the arc of contact. The outlines of the teeth would not be changed in the least.

217. Cycloidal System.—The cycloidal system, although the oldest, is not so popular as the involute system and seems to be gradually going out of use. Mr. Grant in his "Treatise on Gear Wheels" says: "There is no more need for two different kinds of tooth curves for gears of the same pitch than there is need for different kinds of threads for standard screws, or of two different kinds of coins of the same value, and the cycloidal tooth would never be missed if it were dropped altogether. But it was the first in the field, is simple in theory, is easily drawn, has the recommendation of many well-meaning teachers, and holds its position by means of human inertia; or the natural reluctance of the average human mind to adopt a change, particularly a change for the better." This view is probably a little biased, but nevertheless there is a great deal of sound truth in it. The proportion of machine cut cycloidal teeth to machine cut involute teeth is very small, but in some classes of work, and especially when the loads are heavy, the cycloidal forms are still used extensively.

218. Form of the Cycloidal Tooth.—The outline of a cycloidal tooth is made up of two curves. The faces of the teeth are epicycloids and the flanks are hypocycloids, with two exceptions, namely, internal gearing and racks. In the former case, the faces are hypocycloids and the flanks are epicycloids, while in the latter both curves are plain cycloids. When a circle rolls on a fixed straight line, the path generated by an assumed point of the circle is a cycloid; should the circle roll on the outside of another circle, the path of this point would be an epicycloid, and should it roll on the inside of another circle, it would be a hypocycloid.

These rolling circles are generally spoken of as describing circles, and their size determines the form of the tooth, the arc of contact, and the angle of obliquity of action. The angle of obliquity in the cycloidal system is constantly changing; but its average value, when the proportions of the teeth are standard, is about 15 degrees, the same as in involute gearing. The circle upon which the describing circles are rolled is the pitch circle. When the diameter of the rolling circle is equal to the radius of the pitch circle, the flanks of the teeth are undercut. In addition to the objection that undercut teeth are weak, the amount of undercut must be very slight if the teeth are to be cut with a rotating cutter.

The same describing circle must always be used for those parts of the teeth which work together, *i.e.*, the faces of a tooth on the one gear must be formed by the same describing circle as the flanks of the tooth it meshes with. In interchangeable sets it is desirable to use the same size describing circle for both the faces and the flanks of all the gears of the same pitch, and the size of the describing circle which is generally accepted as standard is one whose diameter is equal to the radius of a 12-tooth gear of the same pitch. Here again, the manufacturers of gear cutters are at variance, and use a 15-tooth gear as the base of the system. This does not mean that the 15-tooth gear is the smallest gear in the set, but simply means that smaller gears will have undercut flanks.

219. Laying out the Cycloidal Tooth.—The profiles of cycloidal teeth, as in the case of involute teeth, may be very accurately represented by circular arcs. In Table 67 are given the radii of these arcs, also the radial distances from their centers to the pitch line as determined by Mr. Grant. In laying out the profiles of

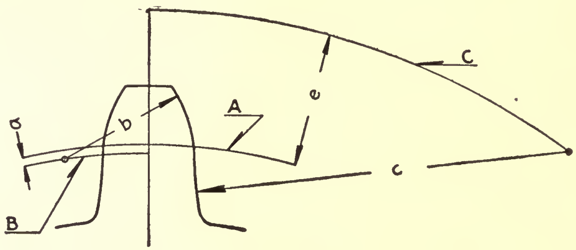


Fig. 135.

cycloidal teeth, draw the pitch, addendum and clearance circles, and space off the pitch of the teeth on the pitch circle. Next draw the circle B as shown in Fig. 135 at a distance a inside of the pitch circle A, also the circle C at a distance e outside of the pitch line. The former is the circle of face centers and the latter, the

Table 67.—Radii for Cycloidal Teeth According to G. B. Grant

Number of teeth		Divide by the diametral pitch				Multiply by the circular pitch			
Exact	Approx.	Rad. b	Dist. a	Rad. c	Dist. e	Rad. b	Dist. a	Rad. c	Dist. e
10	10	1.99	0.02	−8.00	4.00	0.62	0.01	−2.55	1.27
11	11	2.00	0.04	−11.05	6.50	0.63	0.01	−3.34	2.07
12	12	2.01	0.06	∞	∞	0.64	0.02	∞	∞
13½	13–14	2.04	0.07	15.10	9.43	0.65	0.02	4.80	3.00
15½	15–16	2.10	0.09	7.86	3.46	0.67	0.03	2.50	1.10
17½	17–18	2.14	0.11	6.13	2.20	0.68	0.04	1.95	0.70
20	19–21	2.20	0.13	5.12	1.57	0.70	0.04	1.63	0.50
23	22–24	2.26	0.15	4.50	1.13	0.72	0.05	1.43	0.36
21	25–29	2.33	0.16	4.10	0.96	0.74	0.05	1.30	0.29
33	30–36	2.40	0.19	3.80	0.72	0.76	0.06	1.20	0.23
42	37–48	2.48	0.22	3.52	0.63	0.79	0.07	1.12	0.20
58	49–72	2.60	0.25	3.33	0.54	0.83	0.08	1.06	0.17
97	73–144	2.83	0.28	3.14	0.44	0.90	0.09	1.00	0.14
290	145–300	2.92	0.31	3.00	0.38	0.93	0.10	0.95	0.12
Rack		2.96	0.34	2.96	0.34	0.94	0.11	0.94	0.11

circle of flank centers. The tooth profile may now be drawn using the face and flank radii *b* and *c* given in Table 67 for the number of teeth to be used in the gear. The values given for *a*, *b*, *c* and *e* in Table 67 are for 1 diametrical pitch or 1 inch circular pitch. For any other pitch, divide or multiply the tabulated values by the given pitch as directed in the table.

The smallest gear in the set is again one having ten teeth, while the smallest one for which standard cutters are manufactured is one having 12 teeth. The tooth form obtained by using the tabular values as directed above differs slightly from that obtained by the use of standard cutters on account of the difference in the describing circles, but as in the case of involutes, the discrepancy is small and for that reason Grant's tabular values may be used for representing the tooth form on a drawing.

220. Standard Cycloidal Cutters.—The Brown and Sharpe Mfg. Co. furnish sets of cycloidal cutters based on the diametral pitch only, and the sizes vary as follows:

From 2 to 3 diametral pitch, the pitch varies by quarters.

From 3 to 4 diametral pitch, the pitch varies by halves.

From 4 to 10 diametral pitch, the pitch varies by whole numbers.

From 10 to 16 diametral pitch, the pitch varies by even numbers.

Each set consists of 24 cutters, as indicated in Table 68.

TABLE 68.—BROWN AND SHARPE STANDARD CYCLOIDAL CUTTERS

Cutter A for gears having 12 teeth.
Cutter B for gears having 13 teeth.
Cutter C for gears having 14 teeth.
Cutter D for gears having 15 teeth.
Cutter E for gears having 16 teeth.
Cutter F for gears having 17 teeth.
Cutter G for gears having 18 teeth.
Cutter H for gears having 19 teeth.
Cutter I for gears having 20 teeth.
Cutter J for gears having 21 to 22 teeth.
Cutter K for gears having 23 to 24 teeth.
Cutter L for gears having 25 to 26 teeth.
Cutter M for gears having 27 to 29 teeth.
Cutter N for gears having 30 to 33 teeth.
Cutter O for gears having 34 to 37 teeth.
Cutter P for gears having 38 to 42 teeth.
Cutter Q for gears having 43 to 49 teeth.
Cutter R for gears having 50 to 59 teeth.

TABLE 68.—BROWN AND SHARPE STANDARD CYCLOIDAL CUTTERS.—(*Continued.*)

Cutter S for gears having 60 to 74 teeth.
Cutter T for gears having 75 to 99 teeth.
Cutter U for gears having 100 to 149 teeth.
Cutter V for gears having 150 to 249 teeth.
Cutter W for gears having 250 or more.
Cutter X for gears having rack.

221. Action of Cycloidal Teeth.—The action of a pair of cycloidal teeth is illustrated in Fig. 136. Let the circles *a* and *b* represent the pitch circles of a pair of gears having cycloidal teeth, and let the circles *d* and *e* represent the describing circles. Let *C* be the pitch point, and C_d and C_e be the points on the circles *d* and *e* which coincide with *C* when the teeth are in the position shown in the figure. Now let the centers of the circles *a*, *b*, *d*, and *e* be fixed and rotate *a* in the direction indicated by the arrow.

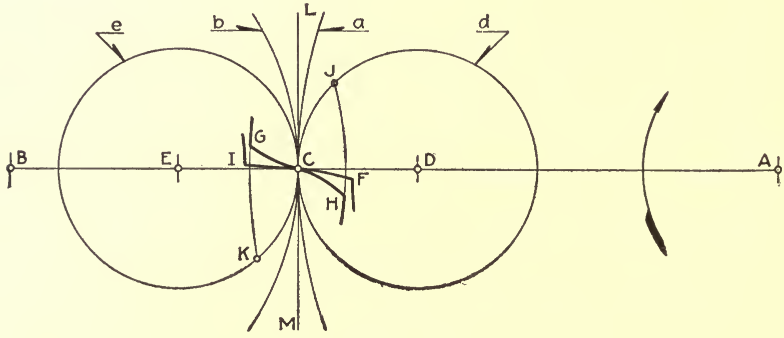


FIG. 136.

Let the contact at *C* be so arranged that the circles *b*, *d*, and *e* are driven with the same peripheral speed as *a*. Trace the path of the point C_d on the surface of *a* extended, on the surface of *b* extended, and also its actual path in space. These paths will evidently be the hypocycloidal flank *CF*, the epicycloidal face *CH* of the meshing tooth, and the path of the point of contact *CJ*. Now replace the mechanism in its original position, rotate *a* in the opposite direction and trace the path of C_e in the same manner. The curves *CG*, *CI* and *CK*, are thus formed and they complete the two tooth forms and the path of contact. As the line of pressure between the teeth, which of course coincides with the common normal at the point of contact, must always pass through the point C in order to transmit a uniform velocity, the angle of obliquity varies from the angle JCL to zero during the arc of approach, and from zero to the angle KCM, which equals the angle JCL, during the arc of recess. In order to show that with this form of tooth the normal to the tooth profile at the point of contact always passes through the pitch point C, let us study Fig. 137. It is evident that the generating point C_e, as well as every other point on the rolling circle, is at any given instant rotating about the point of contact C of the rolling circle with the pitch line. Therefore, at the instant in question the line CC_e is a radius for the point C_e and is consequently normal at that point to the curve which C_e is generating. Now referring again to Fig. 136, the point at which the rolling circle is always in contact with the pitch circle is evidently the pitch point, and therefore the common normal at the point of contact always passes through it.

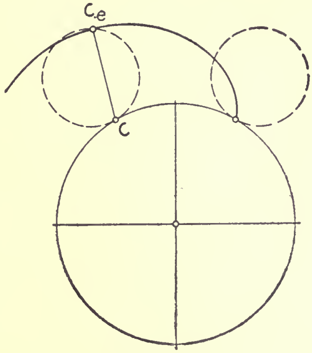


Fig. 137.

STRENGTH OF SPUR GEARING

Having determined the proper form of a gear tooth, the next step is to determine its proportions for strength. Owing to the inaccuracy of forming and spacing the teeth, it is customary to provide sufficient strength for transmitting the entire load by one tooth, rather than considering the load as distributed over the whole number of teeth in theoretical contact.

The load on a single tooth, when the gears are cast from wood patterns, is often concentrated at some one point, usually an outer corner, on account of the draft on the teeth and the natural warp of the castings. The same result is liable to be produced when the shaft is weak or when the gears are not supported on a rigid framework or foundation. However, in the case of well-supported machine-moulded or cut gears, the load may be considered as uniformly distributed along the tooth. For the reason just stated, the subject of the strength of teeth will be discussed under two heads as follows: (*a*) strength of cast teeth; (*b*) strength of cut teeth.

222. Strength of Cast Teeth.—In deriving the formula for the maximum load that a gear with cast teeth will transmit, it will be sufficiently accurate to consider the shape of the tooth as rectangular, and the load as acting at the outer end. The load may, however, be concentrated at one corner or uniformly distributed along the length of the tooth.

(*a*) *Load at one corner.*—With the load concentrated at an outer corner as shown in Fig. 138, it is probable that rupture would occur along a section making some angle α with the base of the tooth. Equating the bending moment about the critical section due to W to the resisting moment of the section, we have

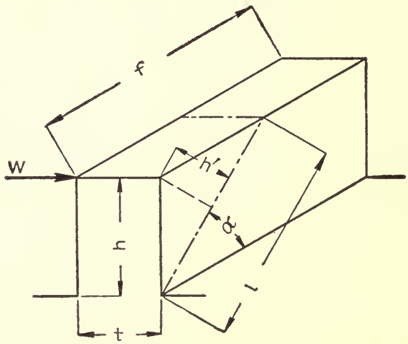


FIG. 138.

$$Wh\cos\alpha = \frac{Sht^2}{6\sin\alpha},$$

in which S denotes the allowable working stress in the material. From this we get

$$S = \frac{3W\sin 2\alpha}{t^2} \tag{321}$$

The stress S is maximum when $\sin 2\alpha$ is maximum, or when α is equal to 45 degrees; therefore,

$$\text{Max. } S = \frac{3W}{t^2} \tag{322}$$

(*b*) *Load uniformly distributed.*—When the load is uniformly distributed along the length of the tooth, we have by equating the bending moment at the base of the tooth to the resisting moment,

$$Wh = \frac{Sft^2}{6},$$

from which

$$S = \frac{6\,Wh}{ft^2} \tag{323}$$

(*c*) *Equal strength.*—Assuming that a tooth is equally strong against both methods of failure, the relation existing between the height h and the face f is found by equating the stresses given by (322) and (323). Hence

$$f = 2\,h = 1.4\,p', \tag{324}$$

where $h = 0.7\,p'$ and p' denotes the circular pitch of the gear.

Although, as shown by (324), the theoretical length of face at which the teeth will be of equal strength for both cases of loading is $1.4\,p'$, a well-known American engineer, C. W. Hunt, taking his data from actual failures in his own work, states that the face should be about $2\,p'$ in order to satisfy this condition.

The seeming discrepancy between theory and actual results may be easily explained when one takes into consideration the fact that even if the load may be entirely concentrated at the corner at the beginning of application of the load, it is very probable that before the full pressure is brought to bear a slight deflection of the outer corner will cause the load to be distributed along a considerable length of the face. Another condition which adds to the length of the face is that of the proper proportions for wearing qualities, and in some cases the faces are made extra long for that purpose alone. It is customary in American practice to make the face of cast teeth two to three times the circular pitch, the length of the face being increased as the quality of the work is improved.

(*d*) *Common proportions of cast teeth.*—The proportions of cast gear teeth as used by the different manufacturers of transmission gears vary somewhat, but for ordinary service the following proportions in terms of the circular pitch have proven satisfactory in actual practice:

Pressure angle or angle of obliquity = 15 degrees.
Length of the addendum = $0.3\,p'$.
Length of the dedendum = $0.4\,p'$.
Whole depth of the tooth = $0.7\,p'$.
Working depth of the tooth = $0.6\,p'$.
Clearance of the tooth = $0.1\,p'$
Width of the tooth space = $0.525\,p'$.
Thickness of the tooth = $0.475\,p'$.
Backlash = $0.05\,p'$.

(*e*) *Allowable working load for cast teeth.*—Assuming the proportions of the teeth as given above, we find from (323) that the allowable working load on cast gear teeth has a magnitude given by the following expression:

$$W = 0.054\,Sp'f \tag{325}$$

This formula has the same general form as the well-known Lewis formula given in Art. 223. The magnitude of the safe working stress depends upon the material, the class of service, and the speed at which the gears are operated. If the gears are subjected to heavy shocks, due allowance must be made for such shocks. To obtain the probable safe working stress for a given speed and material, use (330) and Table 72.

223. Strength of Cut Teeth.—In 1893, Mr. Wilfred Lewis presented at a meeting of the Engineers' Club of Philadelphia an excellent method of calculating the strength of cut gear teeth. His investigation was the first one to take into consideration the form of the tooth profile and the fact that the direction of the pressure is always normal to the tooth profile. The Lewis method has since that time been almost universally adopted for calculating the strength of teeth when the workmanship is of high grade, as in the cut gears, and not infrequently for machine-moulded teeth.

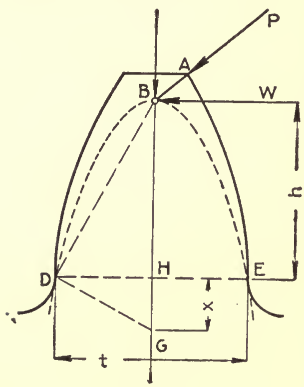


FIG. 139.

In this investigation, Mr. Lewis assumed that at the beginning of contact the load was concentrated at the end of the tooth, with its line of action normal to the tooth profile in the direction AB as shown in Fig. 139. The actual thrust P was then resolved at the point B into two components, one acting radially producing pure compression, and the other, W, acting tangentially. When the material of which the gears are made is stronger in compression than in tension, the radial component adds to the strength of the tooth, and when the tensile and compressive strengths are approximately equal, it is a source of weakness. However, in either case the effect is not marked, and in the original investigation was neglected altogether.

The strength of the tooth may now be determined by drawing through the point B, Fig. 139, a parabola which is tangent to the tooth profile at the points D and E. This parabola then encloses a cantilever beam of uniform strength as the following analysis shows.

A beam of uniform strength is one in which the fiber stress due to bending is constant. For the case under discussion, by equating the external moment to the moment of resistance, we obtain

$$Wh = \frac{Sft^2}{6}, \tag{326}$$

from which

$$t^2 = \frac{6\ Wh}{Sf} = Kh; \tag{327}$$

thus proving that a beam of uniform strength has a parabolic outline.

Since the actual tooth and the inscribed parabola have the same value of t as shown in Fig. 139, it is evident that the parabolic beam must be a measure of the strength of the gear tooth, and that the weakest section of the tooth must lie along DE.

The problem now is to find an expression for the load W in terms of the dimensions of the tooth, the safe fiber stress and a constant. From the similar triangles shown in Fig. 139, it follows that

$$t^2 = 4\ hx \tag{328}$$

Combining (326) and (328), we find

$$W = \tfrac{2}{3}\ Sfx$$

Table 69.—Lewis Factors for Gearing

No. of teeth	Involute		Radial flank	Cycloid	No. of teeth	Involute		Radial flank	Cycloid
	15°	20°				15°	20°		
12	0.067	0.0780	0.0520		40	0.1070	0.1312	0.0674	
13	0.071	0.0840	0.0530		45	0.1080	0.1340	0.0682	
14	0.075	0.0890	0.0540		50	0.1100	0.1360	0.0690	
15	0.078	0.0930	0.0550		55	0.1120	0.1375		
16	0.081	0.0970	0.0560		60	0.1130	0.1390	0.0700	
17	0.084	0.1000	0.0570	Same	65	0.1140	0.1400		Same
18	0.086	0.1030	0.0580	values	70	0.1144	0.1410		values
19	0.088	0.1060	0.0590	as	75	0.1150	0.1420	0.0710	as
20	0.090	0.1080	0.0600	for	80	0.1155	0.1426		for
21	0.092	0.1110	0.0610	15°	90	0.1164	0.1440		15°
22	0.093	0.1130	0.0615	invo-	100	0.1170	0.1450	0.0720	invo-
23	0.094	0.1140	0.0620	lute	120	0.1180	0.1460		lute
24	0.096	0.1160	0.0625		140	0.1190	0.1475		
26	0.098	0.1190	0.0635		160	0.1197	0.1483		
28	0.100	0.1220	0.0643		180	0.1202	0.1490		
30	0.101	0.1240	0.0650		200	0.1206	0.1495	0.0730	
33	0.103	0.1260	0.0657		250	0.1213	0.1504		
36	0.105	0.1290	0.0665		300	0.1217	0.1510	0.0740	
39	0.107	0.1306	0.0672		Rack	0.1240	0.1540	0.0750	

Dividing and multiplying by p', the circular pitch,

$$W = Sp'fy, \tag{329}$$

in which $y = \frac{2x}{3p'}$ is a factor depending upon the pitch and form of the tooth profile. The value of this factor must be obtained from a layout of the tooth, provided a table of such factors is not available. For convenience, the factor y will hereafter be known as the "*Lewis factor*" and in Table 69 are given the values of this

TABLE 70.—VALUES OF y IN LEWIS' FORMULA FOR STUB-TOOTH GEARS.

No. of teeth	Fellows system								Nuttall system
	4/5	5/7	6/8	7/9	8/10	9/11	10/12	12/14	
12	0.096	0.111	0.102	0.100	0.096	0.100	0.093	0.092	0.099
13	0.101	0.115	0.107	0.106	0.101	0.104	0.098	0.096	0.103
14	0.105	0.119	0.112	0.111	0.106	0.108	0.102	0.100	0.108
15	0.108	0.123	0.115	0.115	0.110	0.111	0.105	0.103	0.111
16	0.111	0.126	0.119	0.118	0.113	0.114	0.109	0.106	0.115
17	0.114	0.129	0.122	0.121	0.116	0.116	0.111	0.109	0.117
18	0.117	0.131	0.124	0.124	0.119	0.119	0.114	0.111	0.120
19	0.119	0.133	0.127	0.127	0.122	0.121	0.116	0.113	0.123
20	0.121	0.135	0.129	0.129	0.124	0.123	0.118	0.115	0.125
21	0.123	0.137	0.131	0.131	0.126	0.125	0.120	0.117	0.127
22	0.125	0.139	0.133	0.133	0.128	0.126	0.122	0.118	0.128
23	0.126	0.141	0.134	0.135	0.129	0.128	0.123	0.120	0.130
24	0.128	0.142	0.136	0.136	0.131	0.129	0.125	0.121	0.131
25	0.129	0.143	0.137	0.138	0.133	0.130	0.126	0.123	0.133
26	0.130	0.145	0.139	0.139	0.134	0.132	0.128	0.124	0.134
27	0.132	0.146	0.140	0.140	0.135	0.133	0.129	0.125	0.136
28	0.133	0.147	0.141	0.141	0.136	0.134	0.130	0.126	0.137
29	0.134	0.148	0.142	0.143	0.137	0.135	0.131	0.127	0.138
30	0.135	0.149	0.143	0.144	0.138	0.136	0.132	0.128	0.139
32	0.137	0.150	0.145	0.146	0.140	0.137	0.134	0.130	0.141
35	0.139	0.153	0.147	0.148	0.143	0.139	0.136	0.132	0.143
37	0.140	0.154	0.149	0.149	0.144	0.141	0.138	0.133	0.145
40	0.142	0.156	0.151	0.151	0.146	0.142	0.140	0.135	0.146
45	0.145	0.159	0.154	0.154	0.149	0.145	0.142	0.138	0.149
50	0.147	0.161	0.156	0.156	0.151	0.147	0.144	0.140	0.151
55	0.149	0.162	0.157	0.158	0.152	0.149	0.146	0.141	0.153
60	0.150	0.164	0.159	0.159	0.154	0.150	0.148	0.143	0.154
70	0.153	0.166	0.161	0.161	0.156	0.152	0.150	0.145	0.157
80	0.155	0.168	0.163	0.163	01.58	0.154	0.152	0.147	0.159
100	0.158	0.171	0.166	0.166	0.160	0.156	0.154	0.150	0.161
150	0.162	0.174	0.170	0.169	0.164	0.160	0.158	0.154	0.165
200	0.164	0.176	0.172	0.171	0.166	0.162	0.160	0.156	0.167
Rack	0.173	0.184	0.179	0.176	0.172	0.170	0.168	0.166	0.175

factor as worked out by Mr. Lewis for the several systems of gearing. In Table 70 are given the values of the Lewis factor for the two systems of stub-tooth gearing in common use. These factors were derived and tabulated by Mr. L. G. Smith under the direction of the author, and formed a part of a thesis submitted by Mr. Smith.

(*b*) *Proportions of cut teeth.*—The proportions of cut teeth as recommended by several manufacturers of gear-cutting machinery vary considerably, as may be noticed from an inspection of the formulas given in Table 71. No doubt the formulas proposed by the Brown and Sharpe Co. for the common system of gearing are used more extensively than any other and are generally recognized as the standard. The formulas due to Hunt apply to short teeth, while those proposed by Messrs. Logue and Fellows apply to the well-known stub systems of gear teeth. No formulas are given in Table 71 for the Fellows stub teeth since this system is discussed more in detail in Art. 230 (*d*). It should be noted that the proportions recommended by Messrs. Hunt and Logue agree on all points except the pressure angle.

TABLE 71.—PROPORTIONS OF CUT TEETH

	Brown and Sharpe	Hunt	Logue	Fellows
Pressure angle	14½°	14½°	20°	20°
Length of addendum	0.3183 p'	0.25 p'	0.25 p'	
Length of dedendum	0.3683 p'	0.30 p'	0.30 p'	
Whole depth of tooth	0.6866 p'	0.55 p'	0.55 p'	
Working depth of tooth	0.6366 p'	0.50 p'	0.50 p'	
Clearance		0.05 p'		
Width of tooth space		0.50 p'		
Thickness of the tooth		0.50 p'		
Backlash		0		

Another important fact shown in the table is that for cut teeth the backlash is zero.

224. Materials and Safe Working Stresses.—(*a*) *Materials used in gears.*—The factor S in the Lewis formula depends upon the material used in the construction of the gears. The materials used for gear teeth are various grades of alloy steels, machine steel, steel casting, semi-steel, cast iron, bronze, rawhide, cloth, fiber, and wood. Machine-steel pinions are used with large cast-iron gears; the use of the stronger material makes up for the weakness of the teeth on the pinion, due to the decreased section at the root. At the present time the majority of the gears used in motor-car construction are made of steel and are then subjected to a heat treatment, the effect of which has been discussed in Arts. 52 and 53. Many gears on modern machine tools and electric railway cars are made of steel and then given a heat treatment. Steel gears heat treated are stronger and are capable of resisting wear much better than untreated gears.

Steel casting is used when the gears are of large size. This material is well adapted for resisting shocks and, being much stronger than cast iron, it is used for service in which heavy loads prevail. Semi-steel, which is nothing more than a high-grade cast iron, is also used for large gears where the shocks and loads are not so severe. Cast iron probably is used more frequently than any other material, and in many cases the manufacturers of gears use a special cupola mixture that will produce a tough and close-grained metal.

Bronze is frequently used for spur pinions meshing with steel or iron gears, and when the teeth are properly cut the gears may be run at fairly high speeds. In worm-gear installations, the gear is generally made of bronze and the worm of a good grade of steel, in many cases heat treated. Several manufacturers are now making special gear bronzes that are adapted for a particular type of service. Some of these bronzes are discussed more or less fully in the chapter on worm gearing. In general, bronze is much stronger than ordinary cast iron when applied to gear teeth.

Rawhide, cloth, and fiber gears are used when quiet and smooth-running gears, free from vibration, are desired. Rawhide gears are stronger and are preferable to ordinary fiber ones. The New Process Rawhide Co. claims its gears to be equally as strong as cast-iron gears of the same dimensions. Such gears are furnished with or without metal flanges and bushings, and the teeth are cut the same as in a metal gear. As ordinarily constructed, the flanges and hub of the smaller gears are made of brass or bronze and for the larger ones cast iron or steel may be used. In the case of large gears only the teeth and rim are of rawhide, the center being of cast iron. As a rule, however, rawhide gears are of small size. They are often used as the driving pinions on motor drives, and the fact that rawhide is a non-conductor is in this service a marked advantage.

The cloth or so-called "Fabroil" gears introduced several years ago by the General Electric Co. consist of a filler of cotton or similar material confined at a high pressure between steel flanges held together by either threaded rivets or sleeves, depending upon the size of the gears. After cutting the teeth in the blank, the gear is subjected to an oil treatment making it moisture-proof as well as vermin-proof. In strength, Fabroil gears are the equal of other non-metallic gears, and according to the manufacturer they may be used in practically any service where cast iron gears are used.

Recently the Westinghouse Electric and Manufacturing Co. placed upon the market a non-metallic or fibrous material, called Bakelite Micarta-D, that is suitable for gears and pinions. It is especially adapted for installations where it is desirable to transmit power with a minimum amount of noise. This material possesses good wearing qualities, is vermin-proof, absorbs practically no oil or water, and is unaffected by atmospheric changes and acid fumes. Furthermore, gears made of this material may be run in oil without showing any signs of injury; in fact, the manufacturers specify that a good lubricating oil or grease is essential in order to obtain good results. According to recorded tests, the ultimate tensile strength of Bakelite Micarta-D is approximately 18,000 pounds per square inch with the grain, while its compressive strength across the grain is 40,000 pounds per square inch.

(*b*) *Safe working stress.*—The factor S in (329) depends upon the kind of material used, the conditions under which the gears run and the velocity of the gears. If the gears are subjected to severe fluctuations of load or to shock, or both, due allowance must be made. To provide against the effect of speed, Mr. Lewis published a table of allowable working stresses for a few types of materials. Some years later Mr. C. G. Barth originated an equation giving values for S which agree very closely with those recommended by Mr. Lewis. The Barth formula is generally put into the following form:

$$S = \left[\frac{600}{600 + V}\right] S_0, \tag{330}$$

in which S_0 denotes the permissible fiber stress of the material at zero speed and V the pitch line velocity in feet per minute. In Table 72 are given values of S_0 for the various materials discussed.

Table 72.—Values of S_0 for Various Materials

	Materials	S_0
1	Chrome nickel steel, hardened	100,000
2	Chrome vanadium steel, hardened	100,000
3	Alloy steel, case-hardened	50,000
4	Machinery steel	25,000
5	Steel casting	20,000
6	Special high-grade bronze	16,000
7	Ordinary bronze	12,000
8	High-grade cast iron (semi-steel)	15,000
9	Good cast iron	10,000
10	Ordinary cast iron	8,000
11	Fabroil	8,000
12	Bakelite Micarta-D	8,000
13	Rawhide	8,000

GEAR CONSTRUCTION

The constructive details of gears depend largely upon the size, and to some extent upon the material used as well as upon the machine part to which the gears are fastened. Small metal gears are generally made solid, but when the diameter gets too large for this type of construction thus producing a heavy gear, the weight of such gears can be materially decreased by recessing the sides thus forming a central web connection between the rim and the hub. Not infrequently round holes are put through the web, thus effecting an additional saving in weight and at the same time giving the gear an appearance of having arms. Gear blanks having a central web are usually produced by casting, or by a drop forging operation.

225. Rawhide Gears.—Rawhide gears, as mentioned in the preceding article, are always provided with metal flanges at the side as illustrated in the various designs shown in Figs. 140 and 141. For spur gears up to and including 9 inches outside diameter, the metal flanges are fastened together by means of rivets with countersunk heads as shown. For larger outside diameters either rivets or through bolts are used, depending largely upon surrounding conditions.

The design shown in Fig. 140(*a*), having the plates extending almost to the roots of the teeth, produces a very quiet running gear which gives good service for light and medium loads. In this case the flanges are merely used for supporting the key If a stronger rawhide gear is desired than that just described, the flanges must be extended to the ends of the teeth, thus forming the combination shown in Fig. 140(*b*). The flanges may or may not

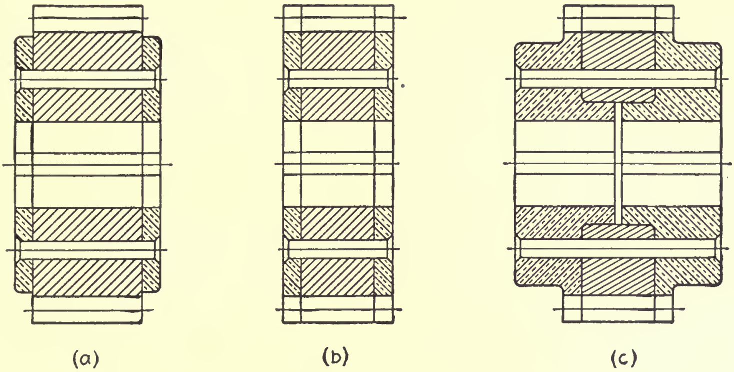


Fig. 140.

form a part of the working face. If the working face does not include the flanges, the rawhide filler must be made ¼ inch wider than the face of the engaging gear; furthermore, if this gear is used as a motor pinion, the rawhide face must be considerably

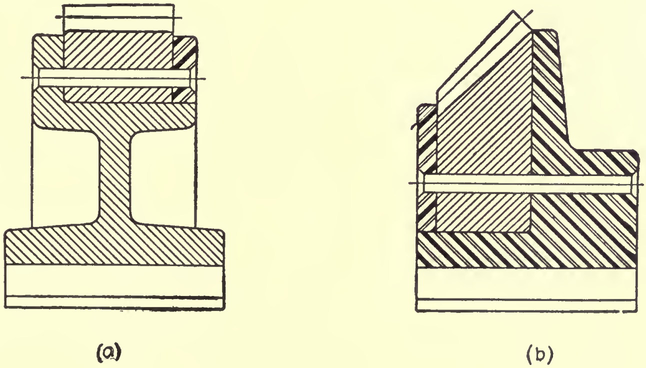


Fig. 141.

wider than the face of the mating gear in order to compensate for the floating of the armature shaft. The object of extending the flanges to the tops of the teeth is to prevent the outer layers of rawhide from curling over and thus eventually ruining the whole gear.

The design shown in Fig. 140(*c*) is intended for severe service. Quiet operation is obtained by eliminating the metal to metal contact, and this is accomplished by making the rawhide face somewhat wider than the face of the engaging gear. The construction shown in Fig. 141(*a*) is that used for large gears, thus effecting a considerable saving of rawhide by using the cast-iron spider to which the rawhide rim is fastened as shown. The flanges may or may not extend to the tops of the teeth. When the face of such a gear is 4 inches or more, through bolts are generally used in place of rivets, unless the projecting heads and nuts are found objectionable.

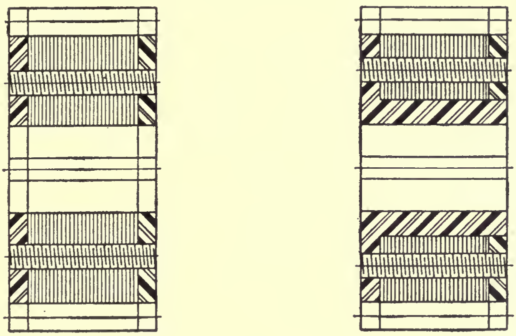
For the constructions shown in Fig. 140(*a*) and (*b*), the thickness of the plates may be made according to the dimensions given in Table 73. This table also gives the size of rivets to be used for a given pitch of tooth and for the ordinary length of face, namely, about three times the circular pitch. The last two columns given in the table refer to the minimum radial thickness of the rawhide blank when used without and with a metal spider.

Table 73.—Data Pertaining to Rawhide Gears

Diametral pitch	Flange thickness	Diameter of rivet	Thickness of rawhide rim	
			Without metal spider	With metal spider
12	1/8	3/32	0.445	0.415
10			0.550	0.505
9		1/8	0.590	0.545
8			0.640	0.590
7		9/64	0.725	0.670
6	1/4	3/16	0.890	0.800
5		1/4	1.100	0.975
4			1.275	1.150
3	3/8		1.675	1.500
2 3/4		5/16	1.780	1.610
2 1/2			1.905	1.735
2 1/4	1/2		2.140	1.980
2		3/8	2.330	2.175

The information included in Table 73 was kindly furnished by the New Process Gear Corporation and represents their practice in the ordinary designs of rawhide gears.

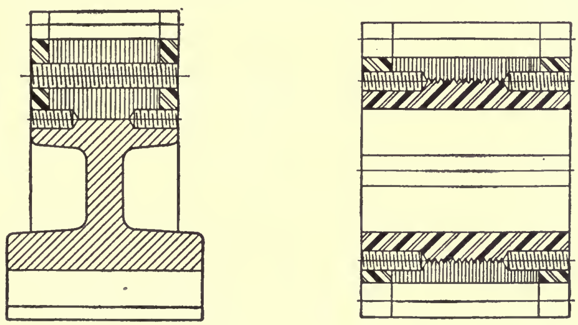
226. Fabroil Gears.—The general constructive features of Fabroil gears are very much the same as those used for rawhide gears. In the usual construction as recommended by the General Electric Co., the flanges are made of steel and threaded studs are



(a) (b)

Fig. 142.

used for clamping the flanges together. Four different designs of such gears are shown in Figs. 142 and 143. The first of these, Fig. 142(*a*), shows the standard construction without a metal bushing or spider. For gears of a larger size, a flanged bushing



(a) (b)

Fig. 143.

made of machine steel or steel casting, depending upon the size of the gear, is employed, as shown in Fig. 142(*b*). This form of construction is used for the sake of economy of material. The two flanges are locked together by the threaded studs, and in addition the removable flange is locked to the bushing by several studs tapped half into the flange and half into the central bushing.

The design shown in Fig. 143(*a*) represents the construction used for large gears when it is desirable to save material. The Fabroil rim is pressed over a metal spider and locked in place by the system of threaded studs shown in the figure. The threaded sleeve construction shown in Fig. 143(*b*) is used for small pinions where there is not sufficient room for the threaded studs used in the standard construction. The end flanges are locked to the threaded sleeve in the manner shown in the figure.

227. Bakelite Micarta-D Gears.—For ordinary service, in which the face of the Bakelite Micarta-D is made equal to or less than the face of the mating gear, no end flanges are required since the material is self-supporting. However, end flanges are recommended when it is desired to transmit heavy loads or when the diameter of the gear is more than four times its face. Bakelite Micarta-D is obtainable in the form of plates up to 36 inches square and in thicknesses varying from $\frac{1}{16}$ inch to 2 inches; hence the largest gears that can be made are limited to 36 inches outside diameter, but the face may be made any width whatsoever by riveting together two or more plates. For economy of material, large-diameter gears are made with a metal center similar to that of the Fabroil gear shown in Fig. 143(*a*).

228. Large Gears.—Gears of medium diameter are cast in one piece, either of cast iron or of steel casting depending upon the class of service for which they are intended. Quite often the gear is cast in one piece and in order to relieve the shrinkage stresses, due to excessive metal in the hub, the latter is split and the halves are then utilized for clamping the gear to the shaft. Such a gear is shown in Fig. 178 of Chapter XIV. Frequently it is desirable to use a split construction, by which is meant the gear is made in two halves that are bolted together. This is a common form of construction in railway motor gears, and as usually made, the joint comes along an arm.

In Fig. 144 is shown the design of a triple-staggered-tooth spur gear designed and constructed by the Mesta Machine Co. of Pittsburg. The design is quite different from that ordinarily used, in that the face of the gear is built up of three separate rims bolted together with the teeth in the three sections arranged in a staggered order. The gear is actually constructed of six separate parts. The central part or spider is split through the hub and rim between two arms and has bolted to it the separate outside rims, each of which consists of two halves. The gear is used for driving a sheet mill and is capable of transmitting 1,600 horse power at a pitch line speed of 2,000 feet per minute. The gear contains 154 teeth of 5½-inch circular pitch and has a face of 38 inches, and the pinion driving the gear has 20 teeth. Because of the high pitch line speed the drive is arranged to run in an oil bath.

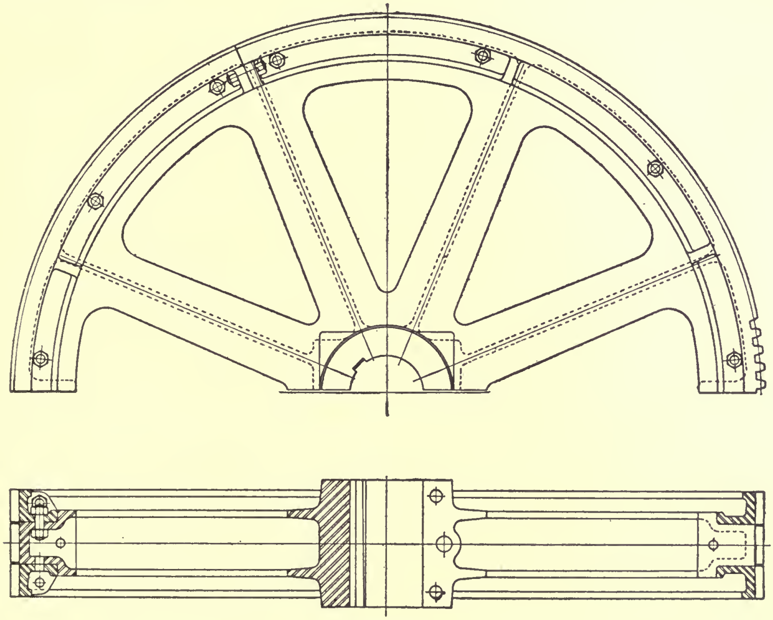


Fig. 144.

Another interesting design of a large gear is that in which a separate spider, consisting of hub and arms, has bolted to it a rim built up in sections. For an illustration of this type, as well as other designs of large gears, consult Chapter XIV.

229. Gear-wheel Proportions.—(*a*) *Arms.*—An exact analysis of the stresses produced in gear arms is exceedingly difficult, and as far as the author is aware, no such analysis has ever been presented. In arriving at a formula by means of which the dimensions of the arm may be calculated, we shall assume that the rim is of sufficient thickness that the load on the teeth is distributed equally among the arms. No doubt this assumption is justifiable, since the rim must be made so rigid that it is subjected to no bending between the arms.

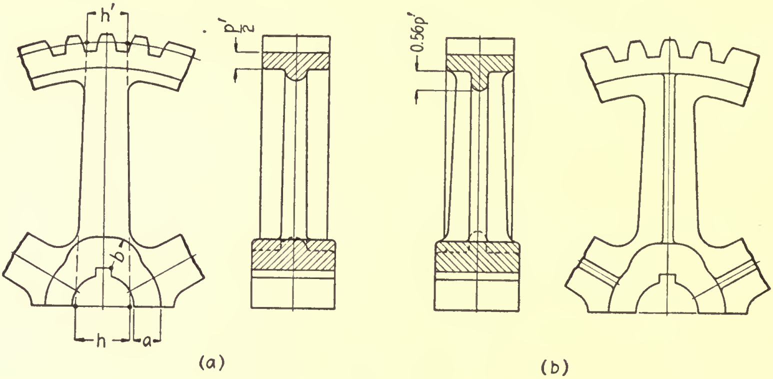


FIG. 145.

With this assumption, we get the following expression for the section modulus of the arm at the center of the hub:

$$\frac{I}{c} = \frac{WR}{nS}, \tag{331}$$

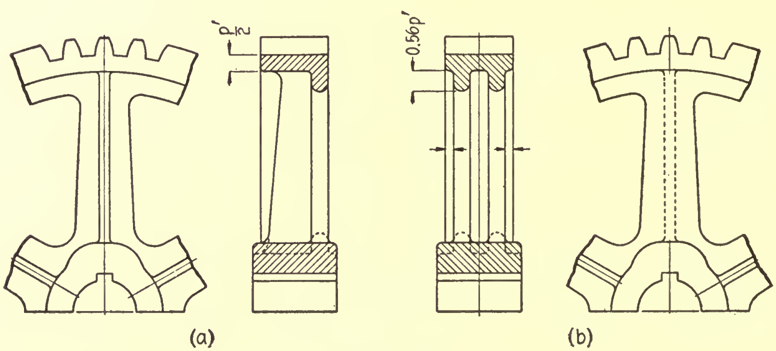


FIG. 146.

in which W is the load on the gear; R the length of the arm or, in this case, the radius of the gear in inches; S the allowable fiber stress in the material, and n the number of arms in the gear.

By means of (331), the value of the section modulus $\frac{I}{c}$ may be determined, from which the dimensions of the adopted arm section may be obtained. In Figs. 145 and 146 are shown four types of arms that are used in gear construction. Of these, the designs shown by Fig. 145(b) and Fig. 146(a) and (b) are used chiefly for large and heavy gears, while the elliptical arm shown in Fig. 145(a) is intended for lighter service, although very often it is also used for heavy work. The proportions of the various arm sections illustrated in the above figures are given in Fig. 147. The dimensions of the arm at the pitch line are generally made approximately seven-tenths of those at the center. Since the elliptical cross-section is used largely for the ordinary gears, we shall derive a formula for that section assuming the proportions

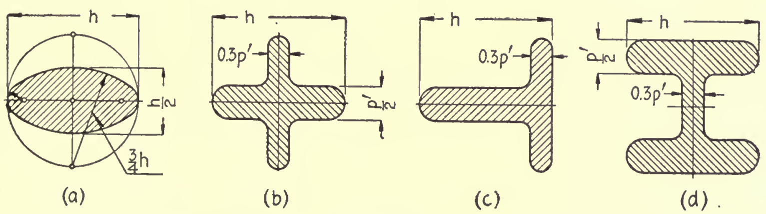


Fig. 147.

given in Fig. 147(a). The section modulus for an ellipse, having the proportions referred to above, is $\frac{\pi h^3}{64}$; hence from (331), we get

$$h = \sqrt[3]{\frac{20\,WR}{nS}} \tag{332}$$

The number of arms in gears varies with the diameter, and the following represents the prevailing practice:

1. Four or five arms for gears up to 16 or 20 inches in diameter.
2. Six arms for gears from 16 to 60 inches in diameter.
3. Eight arms for gears from 60 to 96 inches in diameter.
4. Ten or twelve arms for gears above 96 inches in diameter.

Web centers are used for smaller gears, and the thickness of the web approximates one-half of the circular pitch. Sometimes stiffening ribs are introduced between the hub and rim, and the thickness of such ribs is generally equal to the web thickness.

(b) *Rim.*—Calculations for the rim dimensions are of little value, and in actual designing empirical formulas are resorted to.

As shown in Figs. 145 and 146, the minimum thickness of the rim under the teeth is made about one-half the circular pitch and should taper to a slightly greater thickness where the arms join the rim. Good design dictates that the rim should be supplied with a central rib or bead, as illustrated in Figs. 145 and 146.

(*c*) *Face of gears.*—The width of the face of a gear depends in general upon the type of gear, whether it has cast or cut teeth, the class of service, and the location of the gear. If the gear is located between rigid bearings, the face may be made wider than when the gear is a considerable distance from the bearings, since in the latter case the deflection of the shaft due to the load on the gear might seriously affect the distribution of the load across a wide face. For cast teeth it is good practice to make the face from two to three times the circular pitch, while for cut teeth the face is made from two and one-half to six times the circular pitch, three to four being a fairly good average.

(*d*) *Hubs.*—The hubs of gears are made either solid or split, as stated in the preceding article. In either type of hub good design calls for a reinforcement of metal over the key, and this condition is met if the hubs are proportioned according to suggestions offered in Figs. 145 and 146. The object of a split hub is to reduce the cooling stresses in the gear and at the same time permit any desired adjustment of the gear on the shaft. Keys should always be placed under an arm in the case of a solid hub, and in a split hub approximately at right angles to the center split or hub joint. The diameters and lengths of the hub may be made in accordance with the formulas given in Table 74, in which d denotes the bore of the hub. These formulas were published by Herman Johnson in the *American Machinist* of Jan. 14, 1904, and represent the actual practice of four large manufacturers.

TABLE 74.—DIMENSIONS OF GEAR HUBS

Type of service	Diameter		Length
	Cast iron	Steel casting	
Heavy load and great shock	$2d$	$1.75d+0.125''$	
Medium load and medium shock	$1.75d+0.125''$	$1.625d+0.1875''$	$1.75d$ to $2\frac{1}{4}d$
Light load and no shock	$1.625d+0.125''$	$1.5d+0.25$	

230. Methods of Strengthening Gear Teeth.—Occasionally it is desirable to have the teeth of a gear extra strong, and to obtain additional strength any one of the seven following methods may be used: (*a*) shrouding; (*b*) use of short teeth; (*c*) increase of the angle of obliquity; (*d*) use of stub teeth; (*e*) use of unequal addendum teeth; (*f*) use of buttressed teeth; (*g*) use of helical teeth.

(*a*) *Shrouding.*—The gain in strength due to shrouding depends upon the face of the gear, the effect being more marked in the case of a narrow face than in a wider one. Wilfred Lewis considers shrouding bad practice. However, when the face is two and one-half times the circular pitch, he has demonstrated by an approximate theoretical investigation that single shrouding similar to that illustrated by Fig. 148(*a*) will increase the strength of the tooth at least 10 per cent. and double shrouding

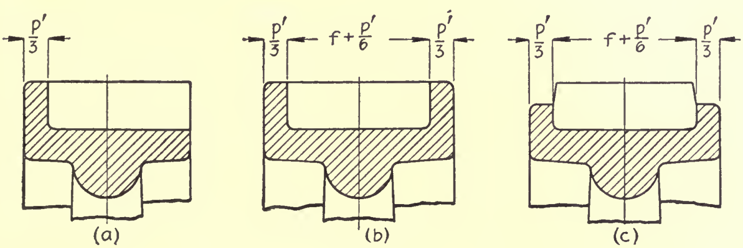


Fig. 148.

as shown in Fig. 148(*b*), at least 30 per cent. In many cases shrouding of gears is necessary, and the proportions given in Fig. 148 for the three methods of shrouding will serve as a guide.

(*b*) *Short teeth.*—Gear teeth whose heights are less than those given by common proportions are considerably stronger, and furthermore, they run with less noise. In America, C. W. Hunt advocates this type of tooth, and the following proportions for cast involute teeth are those he has successfully used on gears for coal-hoisting engines and similar machinery.

$$\left.\begin{aligned} \text{Addendum} &= 0.2\,p' \\ \text{Face of gear} &= 2\,p' + 1'' \\ \text{Clearance} &= 0.05\,(p' + 1'') \end{aligned}\right\} \qquad (333)$$

In Table 75 are given the working, as well as maximum, loads recommended by Mr. Hunt for a cast-iron spur gear having 20 teeth, which is the smallest gear he uses. For proportions of short cut teeth recommended by Mr. Hunt, see Table 71.

TABLE 75.—STRENGTH OF GEAR TEETH USED BY C. W. HUNT

Circular pitch	Load in pounds		Circular pitch	Load in pounds	
	Working	Maximum		Working	Maximum
1	1,320	1,650	$2\frac{1}{4}$	6,700	8,300
$1\frac{1}{4}$	2,300	2,600	$2\frac{1}{2}$	8,300	10,500
$1\frac{1}{2}$	3,000	3,700	$2\frac{3}{4}$	10,000	12,500
$1\frac{3}{4}$	4,100	5,000	3	12,000	14,800

(*c*) *Increase of the angle of obliquity.*—The gain in strength due to an increase of the angle of obliquity is shown in Fig. 149. This figure shows the left half of a tooth having a $22\frac{1}{2}$-degree angle of obliquity and the right half of a tooth having the same pitch but with an angle of obliquity equal to 15 degrees. Now the factor y which appears in the Lewis formula is equal to $\frac{2x}{3p'}$, from which it is evident that an increase of x, when the circular pitch p' remains constant, will result in an increase of y and consequently an increase in the strength of the tooth. This increase of x is shown in the figure.

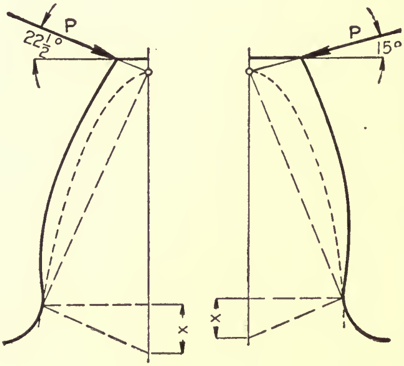


FIG. 149.

A further advantage aside from the increase of strength lies in the fact that the size of the smallest pinion which will mesh with a rack without correction for interference diminishes rapidly as the angle of obliquity increases. Thus with an angle of obliquity of 15 degrees, the 30-tooth pinion is the smallest one that can be used without correction, while with an obliquity of $22\frac{1}{2}$ degrees, the smallest gear in an uncorrected set has theoretically 14 teeth, but practically this number may be reduced to 12.

(*d*) *Stub teeth.*—Another method of strengthening gear teeth, which is now being used extensively in automobile transmission gears and in gears used in machine tools and hoisting machinery consists of a combination of (*b*) and (*c*). This combination gives what is known as the stub tooth. There are two systems of stub teeth, differing in the detail dimensions of the teeth, as shown below, but agreeing on the choice of the angle of obliquity, namely 20 degrees.

In one of these systems, originated by Mr. C. H. Logue, the proportions given in Table 71 are used.

To the second system, that recommended by the Fellows Gear Shaper Co., the tooth dimensions listed in Table 76 apply.

TABLE 76.—DIMENSIONS OF THE FELLOWS STUB TEETH

Pitch	Thickness on the pitch line	Addendum	Dedendum
4/5	0.3925	0.2000	0.2500
5/7	0.3180	0.1429	0.1785
6/8	0.2617	0.1250	0.1562
7/9	0.2243	0.1110	0.1389
8/10	0.1962	0.1000	0.1250
9/11	0.1744	0.0909	0.1137
10/12	0.1570	0.0833	0.1042
12/14	0.1308	0.0714	0.0893

(*e*) *Unequal addendum gears.*—Both in Europe and in the United States certain manufacturers have advocated the use of a system of gearing in which the addendum of the driving pinion is made long, while that of the driven gear is made short, as shown in Fig. 150. Some of the advantages claimed for this system of gearing, after several years of actual experience with it, are the following:

1. This form of tooth obviates interference, thus doing away with undercut on the gears having the smaller numbers of teeth, and at the same time it increases the strength of such gears.

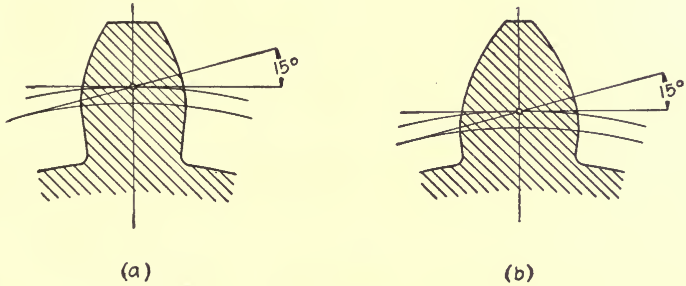


FIG. 150.

2. The sliding friction between the flanks of the teeth is decreased, since the arc of approach is shortened; hence the wear of the teeth is diminished.

3. High-speed gears equipped with unequal addendum teeth run more quietly than standard addendum gears.

4. With this system of gearing it is possible to make the teeth

of the pinion and gear of equal strength, while with the standard system this is impossible without resorting to the use of different materials for the pinion and the gear.

The tooth profile of a 15-tooth pinion having a tooth of standard proportions is shown in Fig. 150(*a*), while Fig. 150(*b*) illustrates the tooth outline of a pinion having the same number of teeth and the same pressure angle, but with the addendum and dedendum based on the Gleason standard given in a following paragraph. An inspection of these profiles shows clearly how the teeth of a pinion are strengthened by means of this system of gear teeth.

From the above discussion it is apparent that the use of unequal addendums is desirable for gears having a high velocity ratio. At the present time unequal addendum gears are used extensively on the rear axle drive of automobiles, as quite a number of manufacturers have now adopted this system of teeth for their bevel gears. In America up to the present time, the unequal addendum teeth are used chiefly with bevel gears, but there is no reason why they should not be used to advantage in certain spur gear drives on machine tools and in other classes of machinery. As yet very little progress has been made in this direction.

TABLE 77.—CONSTANTS FOR DETERMINING TOOTH THICKNESS FOR GLEASON UNEQUAL ADDENDUM TEETH

Angle of tooth thrust	Constant for	
	Pinion	Gear
14½°	0.5659	0.4341
15°	0.5683	0.4317
20°	0.5927	0.4073

Gleason standard.—The Gleason Works have adopted as their standard for high ratio bevel gears the following proportions for unequal addendum teeth.

$$\left.\begin{array}{l}\text{Addendum for pinion} = 0.7 \text{ working depth} \\ \text{Addendum for gear} = 0.3 \text{ working depth}\end{array}\right\} \quad (334)$$

The working depth is assumed to be twice the reciprocal of the diametral pitch, or the circular pitch multiplied by the factor 0.3183.

To determine the thickness of the tooth on the pitch circle, when these formulas are used, multiply the circular pitch by the constants given in Table 77.

(*f*) *Buttressed tooth.*—The buttress or hook-tooth gear can be used in cases where the power is always transmitted in the same direction. The load side of the tooth has the usual standard profile, while the back side of the tooth has a greater angle of obliquity as shown in Fig. 151. To compare its strength with that of the standard tooth, use the following method: Make a drawing of the two teeth and measure their thicknesses at the tops of the fillets; then the strength of the hook tooth is to the standard as the square of the tooth thickness is to the square of the thickness of the standard tooth.

(*g*) *Helical teeth.*—Properly supported gears having accurately made helical teeth will run much smoother than ordinary spur gears. In the latter form of gearing there is a time in each period of contact when the load is concentrated on the upper edge of the tooth, thus having a leverage equal to the height of the tooth.

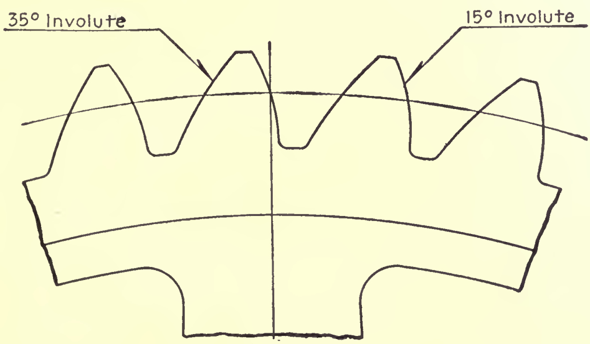


Fig. 151.

With helical gearing, however, the points of contact at any instant are distributed over the entire working surface of the tooth or such parts of two teeth in contact at the same time. Therefore, the mean lever arm with which the load may act in order to break the tooth cannot be more than half the height of the tooth. It follows that the helical teeth are considerably stronger than the straight ones. The subject of helical gears will be discussed more in detail in Chapter XIV.

231. Special Gears.—Specially designed gears, differing radically from those discussed in the preceding articles, are used when it is desired to provide some slippage so as to protect a motor against excessive overload, or to prevent breakage of some part of the machine. Special gears are also required where heavy shocks must be absorbed, thus again protecting the machine against possible damage. The first type of gear mentioned is known as a *slip gear* and the second, as a *flexible* or *spring cushioned gear.*

(*a*) *Slip gears.*—A slip gear is a combination of a gear and a friction clutch, the latter being so arranged that it is always in engagement, but will slip when an extra heavy load comes upon the gear. Slip gears are used to some extent in connection with electric motor drives, and in such installations they really serve

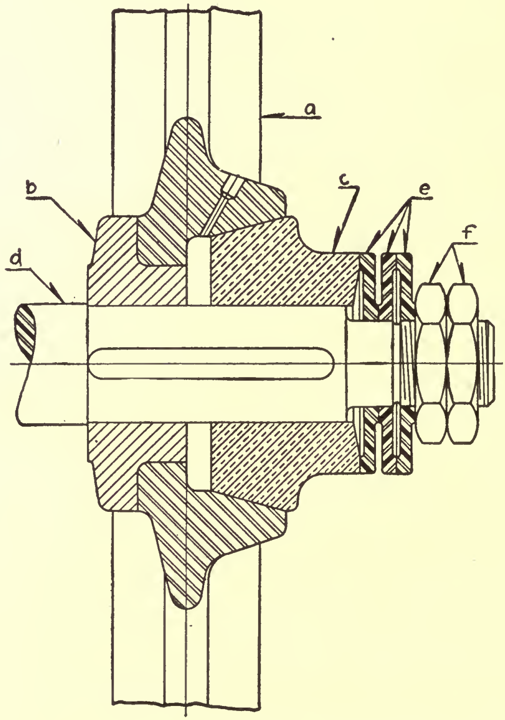


FIG. 152.

as safety devices by protecting the motor from dangerous overloads. Two rather simple designs of slip gears are illustrated in Figs. 152 and 153.

1. *Pawlings-Harnischfeger type.*—The design shown in Fig. 152 is used by the Pawlings-Harnischfeger Co. in connection with some of their motor-driven jib cranes. The gear *a*, meshing directly with the motor pinion, is mounted upon the flanged hub *b* and the bronze cone *c*, both of which are keyed to the driven shaft *d* as shown in the figure. By means of the three tempered

steel spring washers *e* and the two adjusting nuts *f*, the desired axial force may be placed on the clutch members *b* and *c*. In reality the combination *a*, *b*, and *c* is nothing more than a combined cone and disc clutch, the analysis of which is given in detail in Chapter XVI. The angle that an element of the cone makes with the axis is 15 degrees for the design shown in Fig. 152.

2. *Ingersoll type.*—A second design of slip gear differing slightly from the above is shown in Fig. 153. It is used by the Ingersoll Milling Machine Co. on the table feed mechanism of their heavy milling machines. Its function is to permit the pinion *d* to slip when the load on the cutter becomes excessive. The

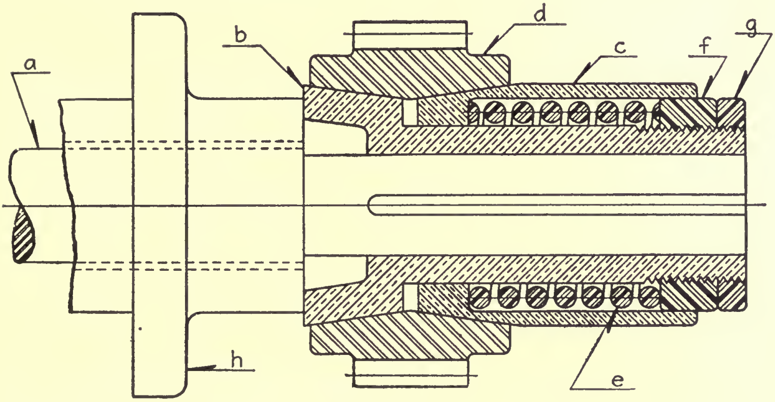


FIG. 153.

driving shaft *a* has keyed to it a bronze sleeve *b* upon which slides the sleeve *c*, also made of bronze. As shown, a part of the length of the sleeves *b* and *c* is turned conical so as to fit the conical bore of the steel pinion *d*. The frictional force necessary to operate the table is obtained by virtue of the pressure of the spring *e* located on the inside of the sleeve *c*. By means of the adjusting nuts *f* and *g*, the spring pressure may be varied to suit any condition of operation.

(*b*) *Flexible gears.*—The so-called flexible or spring-cushioned gear is used on heavy electric locomotives, and its chief function is to relieve the motor and entire equipment from the enormous shocks due to suddenly applied loads. In Fig. 154 is shown a well-designed gear of this kind as made by The R. D. Nuttall Co. The gear consists of a forged-steel rim *a* on the inner surface of which are a number of short arms or lugs *b* as illustrated in Fig.

154(*b*), which represents a section through the gear along the line *OB*. The gear rim *a* is mounted upon the steel casting hub *c* which is equipped with projecting arms *d*. These arms are double, as shown in the section through *OB*, and are provided with sufficient clearance to accommodate the projecting lugs *b*. The heavy springs *e* form the only connection between the double arms *d* and the lugs *b*; hence, all of the power transmitted from the rim to the hub must pass through the springs *e*. Special trunnions or end pieces are used on the springs to give a proper bearing on the lugs and arms. The cover plate *f* bolted to the hub *c* affords a protection to the interior of the gear against dust and grit. It is evident from this description that the springs

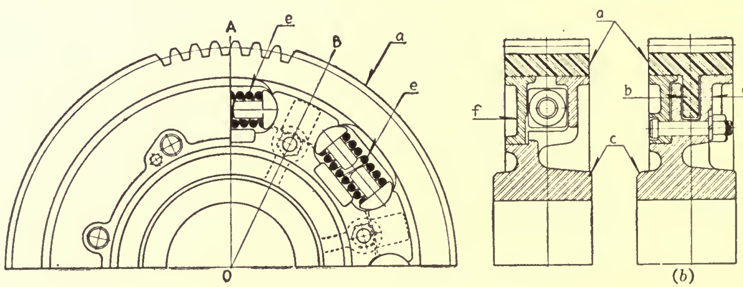


Fig. 154.

provide the necessary cushioning effect required to absorb the shocks caused by suddenly applied overloads. The remarks relating to the design of spring-cushioned sprockets, as given in Art. 193, also apply in a general way to the design of flexible gears.

EFFICIENCY OF SPUR GEARING

There is probably no method of transmitting power between two parallel shafts that shows a better efficiency than a pair of well-designed and accurately cut gears. So far as the author knows, no extensive investigation has ever been made of the efficiency of spur, bevel, and helical gearing; at least, very little information has appeared in the technical press on this important subject. It is generally assumed that the efficiency of gearing becomes less as the gear ratio increases, and the correctness of this assumption is proved by a mathematical analysis proposed by Weisbach.

232. Efficiency of Spur Gears.—By means of the analysis following, it is possible to arrive at the expression for the amount of work lost due to the friction between the teeth. Knowing this lost work, also the useful work transmitted by the gears, we have a means of arriving at the probable efficiency of a pair of gears.

In Fig. 155 are shown two spur gears 1 and 2 transmitting power. In this figure the line MN, making an angle β with the common tangent CT, represents the line of action of the tooth thrust between the gears.

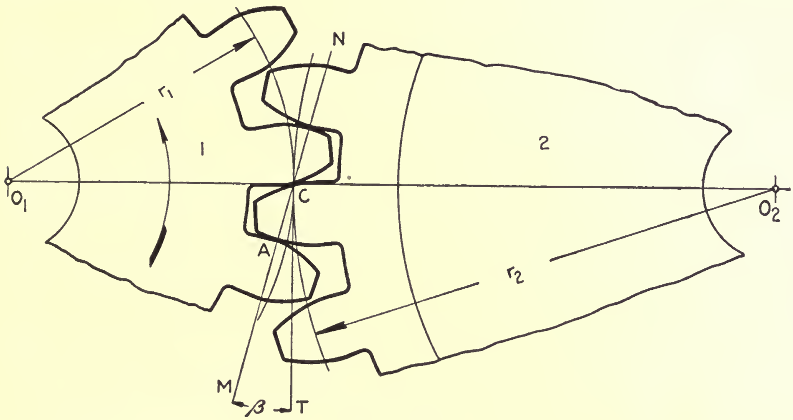


FIG. 155.

Let n_1 and n_2 denote the revolutions per second of the gears.

T_1 and T_2 denote the number of teeth in the gears 1 and 2, respectively.

ω_1 and ω_2 denote the angular velocity of the gears 1 and 2, respectively.

p' = the circular pitch.

s = the distance from the pitch point C to the point of contact of two teeth.

μ = coefficient of sliding friction.

To find the velocity of sliding at the point of contact of two teeth, we employ the principle that the relative angular velocity of the gears 1 and 2 is equal to the sum or difference of the angular velocities ω_1 and ω_2 of the wheels relative to their fixed centers O_1 and O_2; thus if ω denotes this relative angular velocity,

$$\omega = \omega_1 \pm \omega_2,$$

the minus sign being used when one of the wheels is annular and ω_1 and ω_2 have the same sense. The velocity v' with which one tooth slides on the other is then the product of this angular velocity ω and the distance s between the point of contact and the pitch point, which is the instantaneous center of the relative motion of 1 and 2; that is,

$$v' = s\,(\omega_1 \pm \omega_2) \tag{335}$$

The distance s varies; at the pitch point it is zero, and when the teeth quit contact it has a value of 0.7 to 0.9 p' with teeth having the ordinary proportions. The average value of s may be taken as 0.4 p'.

Since P is the normal pressure between the tooth surfaces, the force of friction is μP, and the work of friction per second is

$$W_t = \mu P v' \tag{336}$$

The formula for W_t may be put into more convenient form by combining (335) and (336), and substituting in the resulting equation the following values of ω_1, ω_2 and n_2:

$$\omega_1 = 2\,\pi n_1;\ \omega_2 = 2\,\pi n_2;\ n_2 = n_1\frac{T_1}{T_2}$$

Hence,

$$W_t = 0.8\,\mu\pi P v\left[\frac{1}{T_1} \pm \frac{1}{T_2}\right], \tag{337}$$

in which v represents the velocity of a point on the pitch line.

The component of P, in the direction of the common tangent CT to the pitch circles of the gears, is $P\cos\beta$; hence the work per second that this force can do is

$$W_0 = Pv\cos\beta \tag{338}$$

Adding (337) and (338), it is evident that the work W' put into the gears, omitting the friction on the gear shafts, is

$$W' = W_0 + W_t \tag{339}$$

The component of P in a radial direction is $P\sin\beta$. The total pressure upon the bearings of each shaft is P; hence the work lost in overcoming the frictional resistances of these bearings is as follows:

$$W_b = \pi\mu' P(n_1 d_1 + n_2 d_2), \tag{340}$$

in which d_1 and d_2 represent the diameters of the shafts, and μ' the coefficient of journal friction.

With friction considered, it follows that the total work required to transmit the useful work W_0 is

$$W = W_0 + W_t + W_b \tag{341}$$

The efficiency of the pair of gears including the bearings is therefore

$$\eta = \frac{W_0}{W} \tag{342}$$

If it is desirable to estimate the efficiency of the gears exclusive of the bearings, the following expression may be used:

$$\eta' = \frac{W_0}{W'} = \frac{1}{1 + 2.51\,\mu \sec\beta\left[\frac{1}{T_1} \pm \frac{1}{T_2}\right]} \tag{343}$$

For gears having cast teeth, the coefficient of friction μ may vary from 0.10 to 0.20, while for cut gears the value may be less than one-half those just given. However, even with the larger coefficient of friction quoted, the loss due to friction is small. It is apparent from (343) that the efficiency is increased by employing gears having relatively large numbers of teeth.

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CHAPTER XIII

BEVEL GEARING

When two shafts which intersect each other are to be connected by gearing, the result is a pair of bevel gears. Occasionally, however, the shafts are inclined at an angle to each other but do not intersect, in which case the gears are called skew bevels. The form of tooth which is almost universally used for bevel gears is the well-known involute. This is probably due to the fact that slight errors in its form are not nearly so detrimental to the proper running of the gears as when the tooth curves are cycloidal.

233. Methods of Manufacture.—Bevel gears may be either cast or cut. The process of casting is not materially different from that used in spur gearing, but the process of cutting is much more difficult on account of the continuously changing form and size of the tooth from one end to the other.

As in the case of spur gearing, there are several different methods of cutting the teeth, some of which form the teeth with theoretical accuracy, while others produce only approximately correct forms. Three of the methods give very accurate results, but they require expensive special machines and are used only when very high-grade work is desired. The three methods are: the *templet-planing* process, represented by the Gleason gear planer; the *templet-grinding* process, now used but little, represented by a machine manufactured by the Leland and Faulconer Co.; and the *moulding-planing* process, represented by the Bilgram bevel gear planer.

In each of these processes the path of the cutting tool passes through the apex of the cone, that is, the point of intersection of the two shafts, and consequently the proper convergence is given to the tooth. With a formed rotating cutter, it is impossible to produce the proper convergence and in many cases the teeth have to be filed after they are cut, before they will mesh properly. Nevertheless, the milling machine is very commonly used for cutting bevel gears, for the simple reason that the equipment of most shops includes a milling machine, while comparatively few shops do enough bevel gear cutting to justify the purchase of an expensive special machine for that purpose.

234. Form of Teeth.—When the gears are plain bevel frictions, it is evident that the faces of the gears must be frustums of a pair of cones whose vertices are at the point of intersection of the axes. These cones may now be considered the pitch cones of a pair of tooth gears, and the teeth may be generated in a manner analagous to the methods used for spur gearing. In discussing

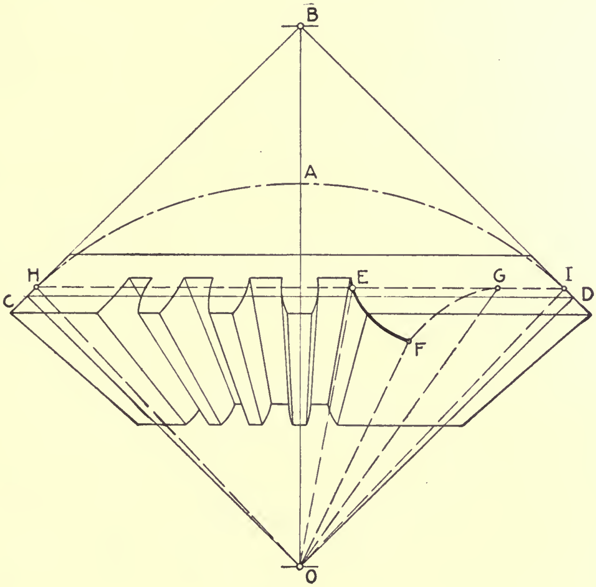

Fig. 156.

the method of forming the teeth, the involute system only will be considered, since the cycloidal forms are seldom used.

In Fig. 156, let the cone *OHI* represent the so-called base cone of the bevel gear shown, from which the involute tooth surfaces are to be developed. In order to simplify the conception of the process of developing, imagine the base cone to be enclosed in a very thin flexible covering which is cut along the line *OE*. Now unwrap the covering, taking care to keep it perfectly tight; then the surface generated by the edge or element *OF* is the desired involute surface. The point *E*, while it evidently generates an involute of the circle *HI*, is also constrained to remain a constant distance from O equal to OE, or in other words, it travels on the surface of a sphere HAI. For that reason the curve EF is called a spherical involute. The spherical surface which should theoretically form the tooth profile is a difficult surface to deal with in practice on account of its undevelopable character, and as is shown in the figure no appreciable error is introduced if the conical surface CBD is substituted for the spherical surface CAD. The cone CBD, which is called the back cone, is tangent to the

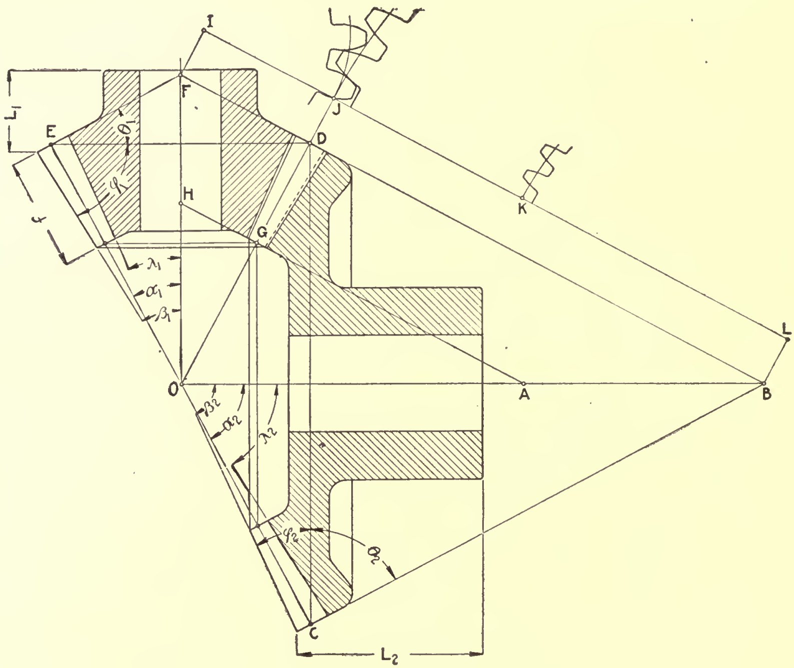

Fig. 157.

sphere at the circle CD, and the pitch distance practically coincides with the sphere for the short distance necessary to include the entire tooth profile. When it is desired to obtain the form of the teeth, as is necessary in case a wood pattern or a formed cutter is to be made, the back cone is developed on a plane surface as shown in Fig. 157. It is evident that the surface which contains the tooth profile has a radius of curvature equal to BD, so the profile must be laid off on a circle of that radius in precisely the same manner as that used for spur gearing. However, this profile is correct for one point only, namely, at the large end. In order to determine the form of the tooth for its entire length, it is necessary to have the profile of the tooth at each end. This may be obtained by developing the back cone *AOG* and proceeding as before. The two profiles just discussed are laid out from the line *LI* as shown. The back cone radius *LK* is equal in length to *AG*, and *LJ* is equal to *BD*. If a wood pattern is to be made, templets are formed of the exact profile of the tooth at the large and small ends. These templets are then wrapped around the gear blank and the material is cut out to the shape of the templets.

235. Definitions.—(*a*) By the expression *back cone radius* is meant the length of an element of the back cone, as for example the line *IJ* in Fig. 157.

(*b*) The *edge angle* is the angle between a plane which is tangent to the back cone and the plane containing the pitch circle. In Fig. 157 this angle is designated by the symbols θ_1 and θ_2 for the pinion and gear, respectively.

(*c*) The *center angle* is the angle between a plane tangent to the pitch cone and the axis of the gear. For the pinion and gear shown in Fig. 157, the center angle is designated as α_1 and α_2, respectively. From the geometry of the figure it is evident that $\theta_1 = \alpha_1$, and $\theta_2 = \alpha_2$.

(*d*) The *cutting angle*, represented by the symbols λ_1 and λ_2 in Fig. 157, is the angle between a plane tangent to the root cone and the axis of the gear.

(*e*) By the term *face angle* is meant the angle between the plane containing the pitch circle and the outside edge of the tooth, as represented by the symbols φ_1 and φ_2 in Fig. 157.

(*f*) *Backing* is the distance from the addendum at the large end of the teeth to the end of the hub, as represented by the dimensions L_1 and L_2 in Fig. 157.

(*g*) The expression *formative number of teeth* is the number of teeth of the given pitch which would be contained in a complete spur gear having a radius equal to the back cone radius. This number of teeth is used in selecting the proper cutter for cutting the gear and also for obtaining the value of the Lewis factor when calculating the strength of the bevel gear.

BEVEL-GEAR FORMULAS

The following formulas, expressing the relations existing between the various dimensions and angles of bevel gears, are important and are necessary for determining the complete dimensions required to manufacture such gears. In arriving at these formulas, two general types of bevel gears must be considered:

1. That type in which the angle between the intersecting shafts is less than 90 degrees, as shown in Fig. 158.

2. That type having an angle between the shafts greater than 90 degrees, an illustration of which is shown in Fig. 159.

Having obtained the formulas for either of these types of gears, those for the more common case, namely when the shafts make an angle of 90 degrees, may readily be derived.

236. Acute-angle Bevel Gears.—In Fig. 158 is shown a pair of bevel gears in which the angle θ between the shafts is less than 90 degrees. In deriving the desired relations, the following notation will be used:

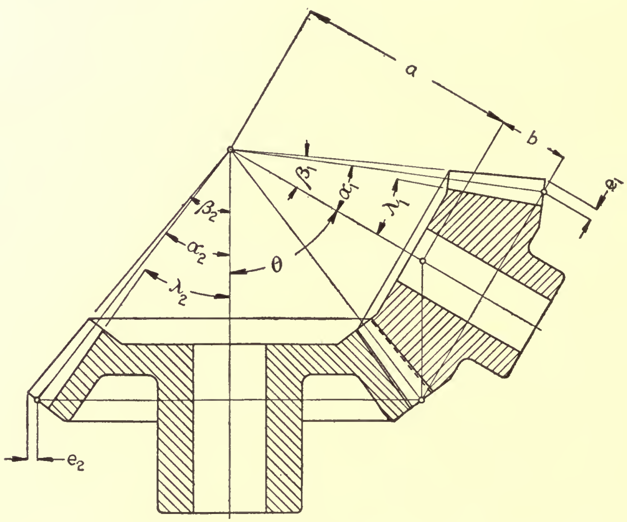

FIG. 158.

D = the pitch diameter.
D' = the outside diameter.
T = the number of teeth.
e = the diameter increment.
c = the clearance at the top of the tooth.
p = the diametral pitch.
p' = the circular pitch.
s = the addendum.

In this discussion the subscripts 1 and 2, when applied to the various symbols, refer to the pinion and gear, respectively. From the geometry of the figure, we obtain the following relations:

$$a = \frac{D_2}{2 \sin \theta}$$

$$b = \frac{D_1}{2 \tan \theta}$$

$$\tan \alpha_1 = \frac{D_1}{2(a+b)} = \frac{D_1 \sin \theta}{D_2 + D_1 \cos \theta}$$

$$= \frac{\sin \theta}{\frac{T_2}{T_1} + \cos \theta} \qquad (344)$$

The equation just established enables us to determine the magnitude of the *center angle* of the pinion. Subtracting α_1 from the angle θ included between the two shafts gives the magnitude of the center angle α_2 of the gear.

If it is desired to determine the angle α_2 by means of calculations, the following formula, derived in the same manner as (344), may be used:

$$\tan \alpha_2 = \frac{\sin \theta}{\frac{T_1}{T_2} + \cos \theta} \qquad (345)$$

Determining the magnitudes of α_1 and α_2 by means of (344) and (345), the calculations may be checked very readily, since $\alpha_1 + \alpha_2 = \theta$.

To determine the angle β_1 of the pinion, we must find the *angle increment*, by which is meant the angle included between the pitch cone element and the face of the tooth. Thus

$$\tan (\beta_1 - \alpha_1) = \frac{2s}{D_1} \sin \alpha_1, \qquad (346)$$

from which the angle increment may be obtained. The addition of $(\beta_1 - \alpha_1)$ to the center angle gives the magnitude of the angle β_1.

The angle decrement $(\alpha_1 - \lambda_1)$ may be determined from the following relation:

$$\tan (\alpha_1 - \lambda_1) = \frac{2(s+c)}{D_1} \sin \alpha_1 \qquad (347)$$

By subtracting $(\alpha_1 - \lambda_1)$ from the center angle the magnitude of the cutting angle λ_1 is found.

Since the angle increment and angle decrement of the pinion are exactly the same as the corresponding angles of the gear, the face and cutting angles of the latter may be found.

In turning the blanks, it is necessary that the outside diameter of both the pinion and the gear be known. These diameters are obtained by adding twice the diameter increment to the pitch diameters. The diameter increment is calculated by the following equations:

$$\left.\begin{aligned}\text{For the pinion, } e_1 &= s\cos\alpha_1\\ \text{For the gear, } e_2 &= s\cos\alpha_2\end{aligned}\right\} \qquad (348)$$

From these relations we get

$$\left.\begin{aligned}D_1' &= D_1 + 2\,s\cos\alpha_1\\ D_2' &= D_2 + 2\,s\cos\alpha_2\end{aligned}\right\} \qquad (349)$$

The length of the face of the pinion measured parallel to the axis is $F\cos\beta_1$ and the corresponding dimension for the gear is $F\cos\beta_2$.

237. Obtuse-angle Bevel Gears.—By the expression obtuse-angle bevel gearing is meant a gear and pinion in which the angle between the shafts is more than 90 degrees. It is evident from this that the following three forms of such gearing are possible:

(*a*) In the first form, which is more common than either of the other two, the center angle α_2 of the gear is made less than 90 degrees. For convenience of reference, we shall call this form the *regular obtuse-angle bevel gear.*

(*b*) In the second form, which is rarely used, the center angle α_2 of the gear is made 90 degrees. In this case the pitch cone becomes a plain disc; such a gear is then known as a *crown gear.*

(*c*) In the third form, which should be avoided whenever possible, the center angle α_2 of the gear is greater than 90 degrees. In such a gear the teeth must be formed on the internal conical surface, thus giving it the name of *internal bevel gear.* An internal bevel gear can generally be avoided without changing the positions of the shafts by using an acute-angle gear set, in which the angle between the shafts is made equal to the supplement of the original angle between the shafts.

Using the same notation as in the preceding article, the important formulas for the bevel gears illustrated in Fig. 159, in which the angle θ is greater than 90 degrees, are as follows:

For the pinion

$$\left.\begin{aligned}\tan \alpha_1 &= \frac{\sin (180 - \theta)}{\dfrac{T_2}{T_1} - \cos (180 - \theta)}\\ &= \frac{\sin \theta}{\dfrac{T_2}{T_1} + \cos \theta}\end{aligned}\right\} \qquad (350)$$

Generally speaking, the first form of equation (350) is preferred by most designers and shop men, although the second form, which is the same as (344), is really more convenient. In the solution of any problem pertaining to obtuse-angle bevel gearing, it is well to determine what form of obtuse bevel gear is being ob-

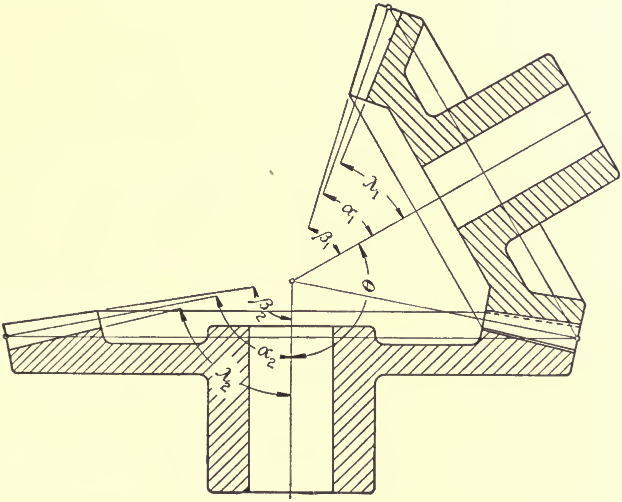

FIG. 159.

tained before proceeding with the calculations, as forms (*b*) and (*c*) discussed above require special formulas. To find out what form of gear is being obtained proceed in the following manner: To the magnitude of α_1, obtained from (350), add 90 degrees and if the sum thus obtained is in excess of the given angle θ, then the resulting gears will be of form (*a*), namely ordinary obtuse bevel gears. If, however, the sum $\alpha_1 + 90$ is equal to the given angle θ, the result will be a crown gear and pinion. An internal bevel gear will result when $(\alpha_1 + 90) < \theta$.

For the ordinary obtuse-angle bevel gear, the center angle α_2

of the gear, if desired, may be determined by means of the following formula:

$$\tan \alpha_2 = \frac{\sin (180 - \theta)}{\frac{T_1}{T_2} - \cos (180 - \theta)} = \frac{\sin \theta}{\frac{T_1}{T_2} + \cos \theta} \qquad (351)$$

The remaining calculations for the ordinary obtuse-angle gears are made by means of the formulas given in the preceding article.

238. Right-angle Bevel Gears.—The great majority of the bevel gears in common use in machine construction have their shafts at right angles to each other as shown in Fig. 157. The formulas in this case may be derived directly from those in Art. 236, by substituting for θ its magnitude 90 degrees; hence (344) and (345) reduce to the following simple forms:

$$\left.\begin{aligned} \tan \alpha_1 &= \frac{T_1}{T_2} \\ \tan \alpha_2 &= \frac{T_2}{T_1} \end{aligned}\right\} \qquad (352)$$

The remaining formulas given in Art. 236 will apply to the present case without change or modification.

STRENGTH OF BEVEL GEARING

239. General Assumptions.—As in the case of spur gearing, formulas for the strength of bevel-gear teeth will be derived for the following two cases: (*a*) When the teeth are cast; (*b*) when the teeth are cut. In analyzing the strength of both kinds of teeth, we shall assume that the gear is supported rigidly and that the load coming upon it will not distort the teeth. Distortion of the teeth means that the elements of the tooth form will no longer intersect at the apex of the pitch cone. The above assumption also means that the distribution of the load on the tooth produces equal stresses at all points along the line of the weakest section. The last statement may be proved by the following analysis: From Fig. 160 or 162 it is evident that the dimensions of the cross-section of the tooth, at any section, are proportional to the distance that the section is from the apex O; hence we obtain the following series of equations:

$$\left.\begin{aligned} t &= \frac{t_1 l}{l_1} \\ h &= \frac{h_1 l}{l_1} \end{aligned}\right\} \qquad (353)$$

Furthermore, the deflection Δ of the tooth at the point where the line of action of the force dW intersects the center line of the tooth is also proportional to the distance l.

The deflection of the small section dl is given by the expression

$$\Delta = \frac{h^3 dW}{3\,EI} = kl \tag{354}$$

Substituting in (354) the value of h from (353) and the value of I in terms of the dimensions of the section, it follows that

$$\frac{dW}{dl} = \frac{klEt_1^3}{4\,h_1^3} = Kl \tag{355}$$

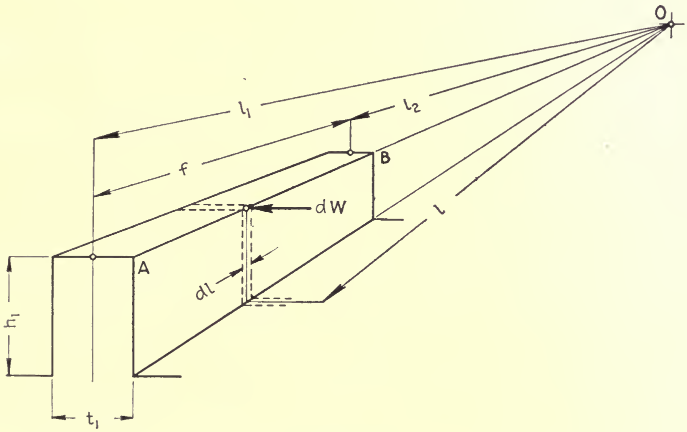

FIG. 160.

Applying the formula for flexure to the elementary cantilever beam, we obtain

$$hdW = \frac{St^2 dl}{6} \tag{356}$$

Combining (353) and (356), we find

$$\frac{dW}{dl} = \frac{St_1^2 l}{6\,l_1 h_1} = ClS \tag{357}$$

Comparing (355) and (357), it follows that

$$S = \frac{Kl}{Cl} = \text{constant} \tag{358}$$

240. Strength of Cast Teeth.—It is sufficiently accurate to consider the cast bevel gear tooth as a cantilever beam, the cross-sections of which are rectangular and converge toward the apex of the pitch cone. Furthermore, the load to be transmitted is assumed as acting tangentially at the tip of the tooth. The formula for the strength of cast teeth based upon the above assumption, as well as that given in the preceding article, may be derived as follows:

By equating the bending moment on a small element dl of the tooth to its moment of resistance and solving for the elementary force dW, we have from (356) that

$$dW = \frac{St^2 dl}{6h}$$

Also, from (357) we get

$$dW = \frac{St_1^2 l dl}{6 l_1 h_1} \tag{359}$$

Now the moment of the elementary force dW about the apex O is ldW; hence the elementary moment

$$dM = \frac{St_1^2 l^2 dl}{6 h_1 l_1}$$

Integrating this expression between the limits l_1 and l_2, we obtain

$$M = \frac{St_1^2}{18\, h_1 l_1}(l_1^3 - l_2^3) \tag{360}$$

Since M represents the total turning moment about the apex O of the pitch cone, we may readily determine the magnitude of the force acting at any point, as for example at the large diameter of the gear, by merely dividing M by the distance from that point to the apex. Let W_1 denote the force which, if applied at the large end of the tooth, will produce a turning moment equal to M; then

$$W_1 = \frac{St_1^2}{18\, h_1}\left[\frac{l_1^3 - l_2^3}{l_1^2}\right] \tag{361}$$

Substituting in (361) the value of $l_2 = l_1 - f$, and simplifying the resulting equation,

$$W_1 = \frac{St_1^2 f}{18\, h_1}\left[3 - \frac{3f}{l_1} + \frac{f^2}{l_1^2}\right] \tag{362}$$

The proportions of cast bevel gear teeth are the same as those given for cast spur gears in Art. 222, namely $h_1 = 0.7\, p'$ and

$t_1 = 0.475\,p'$. Substituting these values in (362), we get

$$W_1 = 0.018\,Sp'f\left[3 - \frac{3f}{l_1} + \frac{f^2}{l_1^2}\right] \qquad (363)$$

Letting m denote the quantity $0.018\left[3 - \frac{3f}{l_1} + \frac{f^2}{l_1^2}\right]$, (363) reduces to the following form:

$$W_1 = Sp'fm \qquad (364)$$

A study of the prevailing practice among manufacturers of cast bevel gears shows that the face of such gears is made from

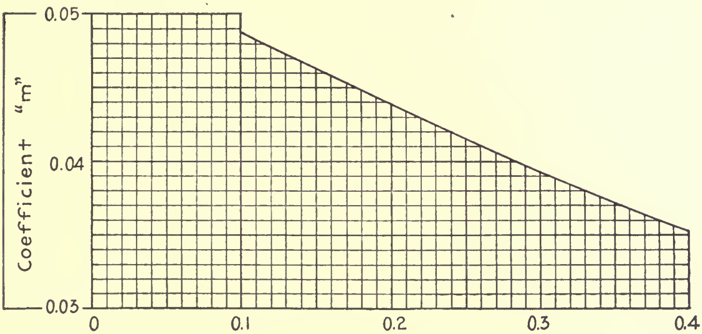

FIG. 161.

two to three times the circular pitch, depending upon the diameters of the gear and pinion. Another rule that should be observed is that the ratio of f to l_1 should not exceed one-third. For values of the permissible stress S for the various grades of cast materials, equation (330) and Table 72 should be used. To facilitate the use of (364), the values of the coefficient m for various ratios of f to l_1 are put into the form of a graph, shown in Fig. 161.

241. Strength of Cut Teeth.—The formula generally adopted by designers for calculating the strength of cut bevel teeth is the one proposed by Mr. Wilfred Lewis. The assumptions regarding the distribution of the tooth pressure made in the preceding article will also hold in the discussion of the cut teeth; hence, the equations (359) to (362) inclusive will hold in the present case. As in the analysis of the cut spur gears, Mr. Lewis considered the tooth as equivalent to a beam of uniform strength, that is, one having a parabolic cross-section as indicated in Fig. 162. From the geometry of the figure, it is evident that

$$t_1^2 = 4\, h_1 x_1$$

and substituting this value in (362), and multiplying and dividing through by p', we get

$$W_1 = \frac{2\, x_1}{3\, p'} Sp'f \left[1 - \frac{f}{l_1} + \frac{f^2}{3l_1^2}\right] \qquad (365)$$

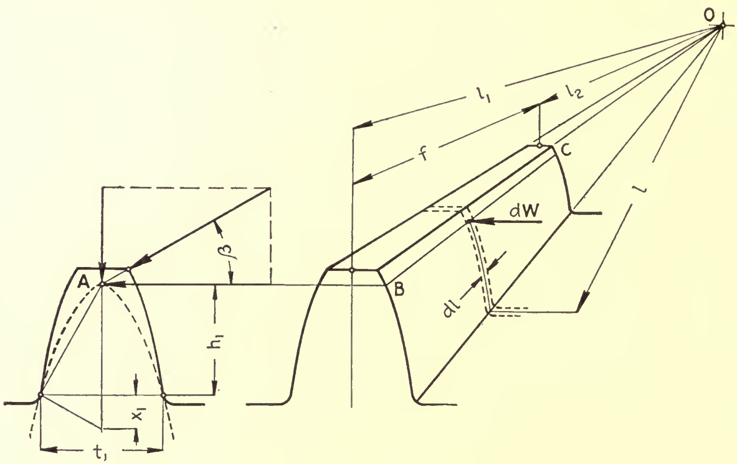

FIG. 162.

Now the ratio of $2x_1$ to $3p'$ is simply the so-called Lewis factor discussed in detail in Art. 223; hence, replacing it by the symbol y and denoting the factor $\left[1 - \frac{f}{l_1} + \frac{f^2}{3l_1^2}\right]$ by the symbol n, equation (365) may be written

$$W_1 = Sp'fyn \qquad (366)$$

As stated in the discussion of cast teeth, the ratio of f to l_1 should not exceed one-third and the face of the gear is usually from two to three times the circular pitch. Equation (330) and the data contained in Table 72 should be used for arriving at the permissible fiber stress for the given material and given condition of operation. It is important to note that the coefficient n represents the ratio that the strength of the bevel gear bears to the strength of a spur gear of the same face and pitch. The graph given in Fig. 163 shows the relation existing between n and the ratio of f to l_1, and will serve as a time saver in the solution of bevel gear problems.

The proportions of cut bevel teeth for the various systems in general use are the same as those given in Table 71 and in Art. 230(e).

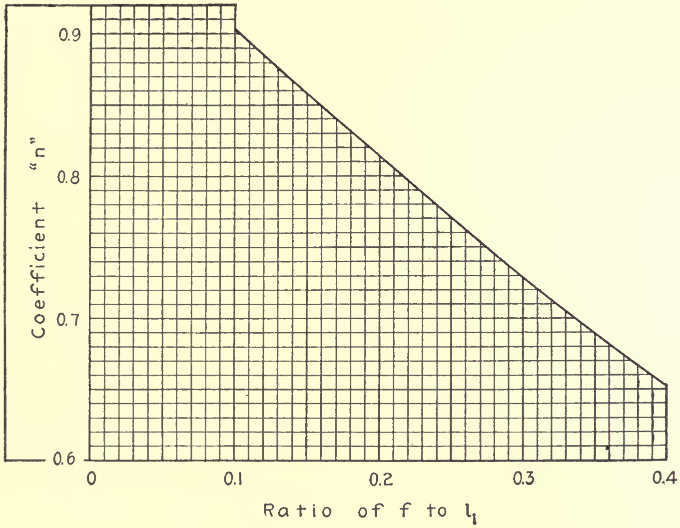

Fig. 163.

242. Method of Procedure in Problems.—In order to save time, the following method may be used in determining the strength of cut bevel gear teeth.

(a) Since y is the form factor, its magnitude cannot be based upon the actual number of teeth in the gear, but must be based upon the so-called formative number of teeth as explained in Art. 235. To obtain the formative number of teeth in the pinion shown in Fig. 164, multiply the actual number of teeth by the ratio $\frac{2\,l_1}{D_1}$. For the gear, the formative number is equal to the ratio $\frac{2\,l_1}{d_1}$ multiplied by the actual number of teeth in the gear. From Tables 69 and 70, determine the factor y corresponding to the formative number.

(b) From Fig. 163 determine the magnitude of the coefficient n for the assumed ratio of f to l_1.

(*c*) For the given material and the speed of the gears determine the magnitude of the stress S.

(*d*) Knowing p' and f and having established values for y, n and S, the load transmitted by either the gear or pinion may be calculated.

243. Resultant Tooth Pressure.—The formulas derived in Arts. 240 and 241, instead of giving the resultant load on the gear tooth, merely give an equivalent load at the large end of the tooth. The resultant normal tooth pressure, as well as its point of application, must be determined before it is possible to analyze the bearing loads and thrusts due to the action of bevel gears. Re-

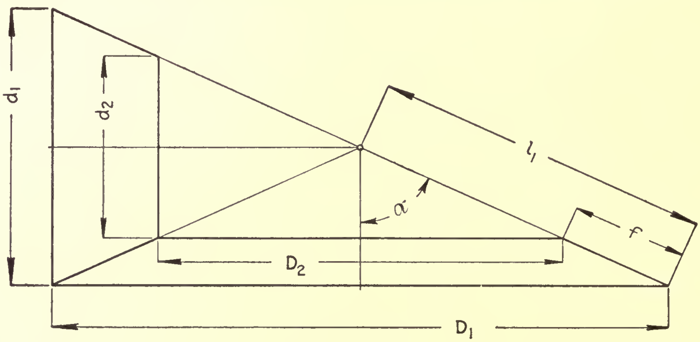

FIG. 164.

ferring to the bevel-gear tooth shown in Fig. 162, the normal tooth pressure is considered as acting along the outer edge of the tooth as shown in the end view of the tooth. The line of action of the normal pressure intersects the center line of the tooth at the point A, at a distance h_1 above the weakest section of the tooth.

To determine the magnitude W of the resultant of all the elementary tooth pressures dW, as well as the point of application of this resultant, proceed as follows:

From (357) it is evident that

$$W = CS \int_{l_2}^{l_1} l\,dl = \frac{CS}{2}(l_1^2 - l_2^2) \tag{367}$$

Taking moments of dW about the apex O, we get

$$dM = l\,dW = CSl^2dl$$

whence

$$M = \frac{CS}{3}(l_1^3 - l_2^3) \tag{368}$$

The distance that the point of application of W is from the apex O is found by dividing M by W; hence

$$l_0 = \frac{M}{W} = \frac{2}{3}\left[\frac{l_1^2 + l_1 l_2 + l_2^2}{l_1 + l_2}\right] \tag{369}$$

To simplify the determination of l_0, it is best to put (369) in terms of the radius R_0 shown in Fig. 165. From the geometry of the figure, it follows that

$$R_0 = l_0 \sin \alpha \tag{370}$$

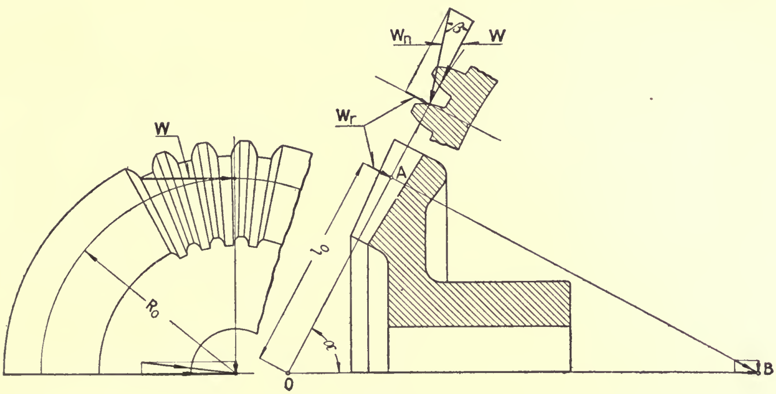

FIG. 165.

Substituting in (369) the value of $l_2 = l_1 - f$, and reducing, we get

$$l_0 = \frac{D_1}{3 \sin \alpha}\left[\frac{3 - \frac{3f}{l_1} + \frac{f_2}{l_1^2}}{2 - \frac{f}{l_1}}\right] \tag{371}$$

Combining (370) and (371), we have

$$R_0 = \frac{D_1}{3}\left[\frac{3 - \frac{3f}{l_1} + \frac{f^2}{l_1^2}}{2 - \frac{f}{l_1}}\right]$$

$$= Z\, D_1 \tag{372}$$

in which Z denotes the factor $\frac{1}{3}\left[\frac{3 - \frac{3f}{l_1} + \frac{f^2}{l_1^2}}{2 - \frac{f}{l_1}}\right]$

To facilitate the use of the formula for R_0, the coefficient Z was determined for various values of the ratio f to l_1. These values were then plotted in the form of a graph, as shown in Fig. 166. By means of the graph and (372), the value of R_0 may easily be calculated, since the angle α is known for any particular gear.

Now the magnitude of the resultant tooth pressure W can be calculated by means of (367), but since the latter is more or less involved a more direct method for finding W is desirable. This is

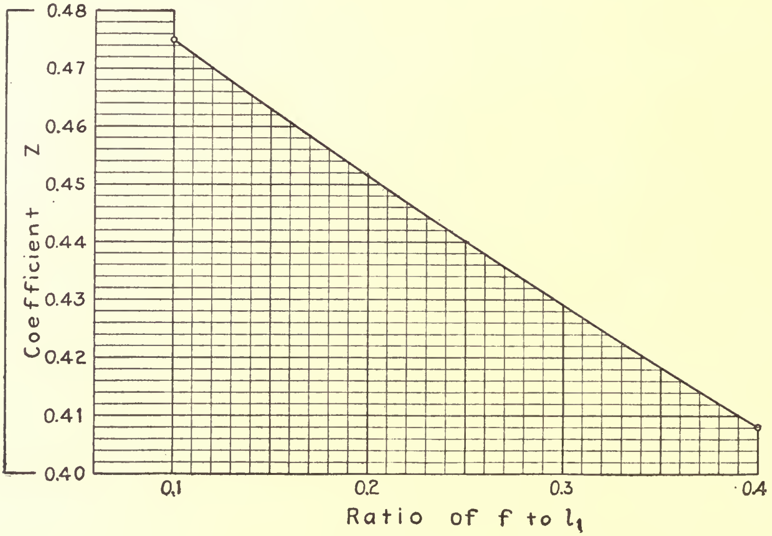

FIG. 166.

obtained by dividing the torsional moment T on the gear by the radius R_0; whence

$$W = \frac{T}{R_0} \tag{373}$$

244. Bearing Pressures and Thrusts.—Having determined the resultant tooth pressure W as well as its point of application, we are now prepared to discuss the pressures and thrusts coming upon the bearings of the supporting shaft. Letting W_n in Fig. 165 represent the *resultant normal tooth pressure;* then resolving W_n along the tangent to the pitch circle, we get the resultant tangential tooth pressure

$$W = W_n \cos\beta \tag{374}$$

The component of W_n at right angles to the element of the pitch cone, namely, that along the line AB in Fig. 165, is

$$W_r = W_n \sin\beta = W \tan\beta \qquad (375)$$

The component W produces a lateral pressure upon the supporting bearings but no thrust along the shaft of the gear. The component W_r produces both lateral pressure and end thrust, the magnitudes of which are given by the following expressions:

$$\left.\begin{aligned} \text{Lateral pressure due to } W_r &= W_r \cos\alpha = W \tan\beta \cos\alpha \\ \text{Thrust due to } W_r &= W_r \sin\alpha = W \tan\beta \sin\alpha \end{aligned}\right\} \qquad (376)$$

To obtain the resultant lateral pressure upon the bearings, the two separate components must be combined, either algebraically or graphically, and in order to arrive at the exact distribution of the resultant pressure, the location of the bearings relative to the gear must be established.

Graphical methods may also be employed to determine W, W_r, and their various components, as shown in Fig. 165.

BEVEL-GEAR CONSTRUCTION

In general, the constructive features of bevel gears are similar to those used for spur gears. Small pinions are made solid as shown in Fig. 158, and for economy of material larger pinions are made with a web. Examples of the latter construction are shown in Figs. 158 and 159. Not infrequently the webs are provided with holes in order to decrease the weight of such gears. Large bevel gears are made with arms, the design of which will be discussed in the following article. Bevel gears are seldom made in extremely large sizes, and for that reason split or built-up gears are used but little.

245. Gear-wheel Proportions.—(*a*) *Arms.*—In bevel gears the T-arm is remarkably well adapted for resisting the stresses that come upon it, and for that reason is used rather extensively in gears of large size. In small gears, however, the greater cost of the arm construction more than offsets the saving of material; therefore for such gears the web and solid centers are in common use. Fig. 167 shows a bevel gear with a T-arm.

The rib at the back of the arm is added to give lateral stiffness, that is, to take care of the load component W_r discussed in Art. 244. This rib adds practically nothing to the resistance of the arm to bending in the plane of the wheel, and for that reason, in deriving the formula for the strength of the arm, the effect of the rib is not considered. As in the case of spur gears, the arm is treated as a cantilever beam under flexure, and it is assumed that each arm will carry its proportionate share of the load transmitted by the gear.

Denoting the thickness and width of the arm by b and h, respectively, and equating the external moment to the resisting moment, we get

$$\frac{W_1 D_1}{2n} = \frac{Sbh^2}{6},$$

in which n denotes the number of arms, and W_1 and D_1 are the

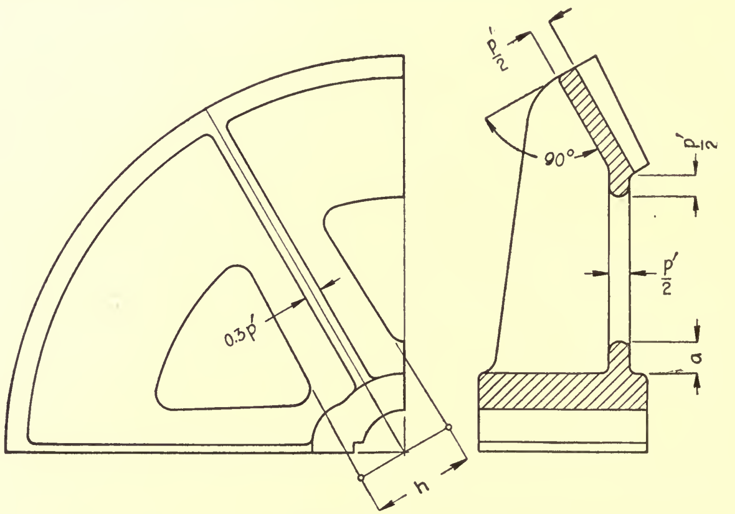

FIG. 167.

equivalent load and pitch diameter, respectively, at the large end of the tooth.

Solving for h, we have

$$h = \sqrt{\frac{3 W_1 D_1}{nbS}} \qquad (377)$$

The dimension b is generally made equal to about one-half of the circular pitch, as shown in Fig. 167. The permissible stress for cast iron varies from 1,500 to 3,000 depending upon the size of the gear. The thickness of the rib on the back of the arm proper is made as shown in the figure.

(*b*) *Rim and hub.*—For the proportions of the rim and the reinforcing bead on the inside of the rim, consult Fig. 167. The hub is made similar to those used for spur gears, proportions of which are given in Table 74.

246. Non-metallic Bevel Gears.—Frequently where noiseless operation is desirable bevel gears made of rawhide and Fabroil are used. In Fig. 141(*b*) is shown the design of a rawhide gear that has given excellent service. The same general constructive feature would be used when a Fabroil filler is employed; but in place of the plain rivets, the threaded type should be used, as recommended by the manufacturer of such gears. In general, the discussion of non-metallic gears given in the preceding chapter applies also to bevel gears.

247. Mounting Bevel Gears.—To obtain good service from an installation of bevel gears, it is important that the material used for the pinion and gear be chosen with some care and that the teeth be formed and cut accurately. These two factors alone, however, do not necessarily make a successful drive, as poorly designed mountings are frequently the source of many bevel-gear failures. The following important points should be observed in designing the mountings of a bevel-gear drive:

1. Make the bearings and their supports rigid, and so that all parts may be easily assembled.

2. Make provisions for taking care of the end thrust caused by the component W_r discussed in Art. 244.

3. Make provisions for lubricating the bearings and if necessary the gears themselves.

4. Provide the gears with a dustproof guard, thus protecting the gears and at the same time protecting the operator of the machine.

5. The shafts supporting the gears should be made large, so as to provide the necessary rigidity. Slight deflections of bevel-gear shafts produce noisy gears and cause the teeth to wear rapidly.

(*a*) *Solid bearing.*—A rigid construction used to a considerable extent on machine tools is the solid bearing construction, two designs of which are shown in Figs. 168 and 169. In both of these designs the end thrusts are taken care of by the use of bronze washers, as shown. The bearings throughout are bronze bushed. The type of bevel-gear drive illustrated in Fig. 169 is used when the pinion is splined to its shaft. In such cases the hub of the pinion is made long, so that it may serve as a bearing. The heavy thrust is taken care of by the self-aligning steel washers, between which is located one made of bronze. In place

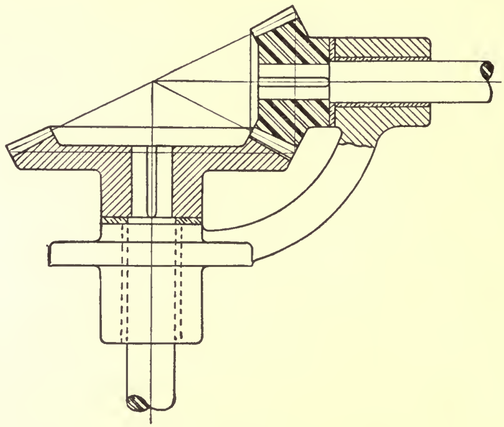

FIG. 168.

of the bronze thrust washers shown in Figs. 168 and 169, ball thrust bearings may be used. The latter type of bearings are more expensive than the bronze washers, and unless they are

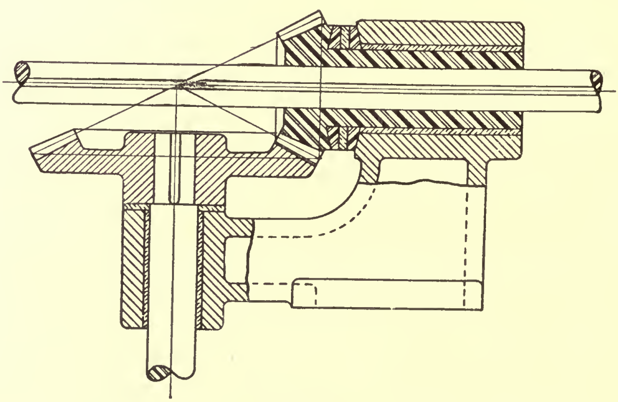

FIG. 169.

designed correctly they are liable to be troublesome. Of late, the type of radial ball bearing that is capable of taking a certain amount of thrust, in addition to the transverse load, is being used in connection with bevel-gear drives. The conical roller bearing is also adapted for use with bevel-gear transmissions.

(*b*) *Ball bearing.*—In Fig. 170 is shown a design of a bevel-gear drive in which ball bearings are used throughout. This form of drive is used on a drill press and the details were worked out by The New Departure Mfg. Co., makers of ball bearings. The double-row ball bearings take both radial loads and thrusts, while the single-row ball bearing having a floating outer race takes only a transverse load. The double-row ball bearing on the horizontal driving shaft is mounted in a shell or housing which is adjustable, thus providing means for getting the proper tooth engagement between the pinion and the gear. Necessarily,

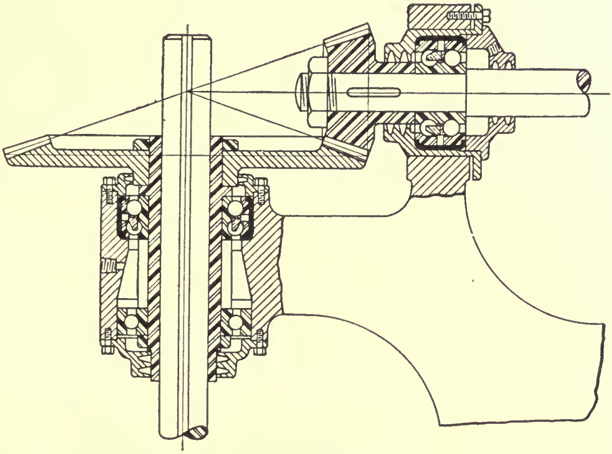

Fig. 170.

this form of construction will call for a bearing having a floating outer race at the farther end of the drive shaft.

When it is desired to support the bevel pinion between two bearings, the design shown in Fig. 171 will give good results. The drive illustrated in this figure is one that is used on the rear axle of an automobile. The arrangement and selection of the various bearings were worked out by the Gurney Ball Bearing Co. The duplex bearing back of the pinion, having a thrust capacity of one and one-half times the radial load, is mounted rigidly in an adjustable cage. The bearing at the other end of the pinion shaft is of the radial type and, as shown, is mounted so as to permit a movement lengthwise of the shaft. The advantage of using the cage construction just mentioned is that the pinion with its shaft and bearings may be assembled on the bench as a unit. The bearing to the left of the bevel gear is of a type capable of taking a thrust equal to the transverse load. The bearing supporting the other end of the differential housing to which the bevel gear is fastened, is also of the combined radial thrust type; but in this case the thrust capacity is equivalent to one-half of the radial load. In Fig. 171, the differential bevels and the two axles are not shown, in order to bring out more clearly the other important

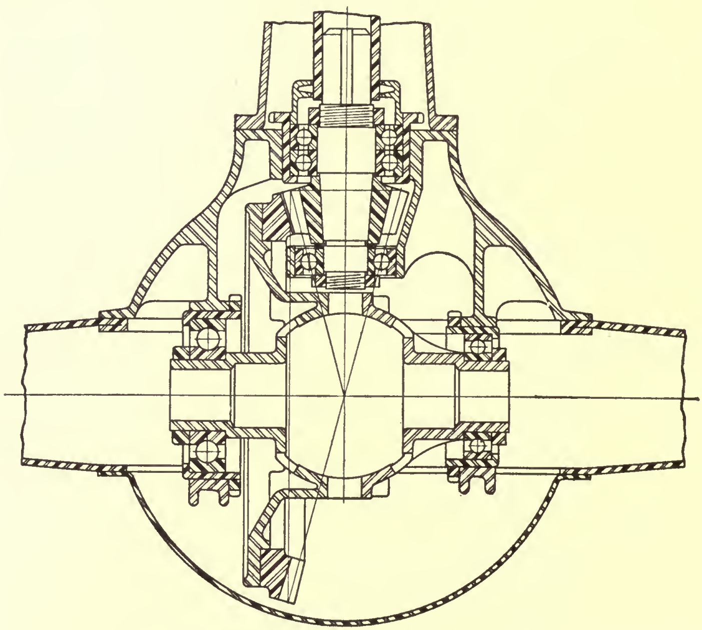

FIG. 171.

details. The type of bevel gearing used in the design just discussed is the so-called "spiral bevel" which will be discussed in the following article.

SPECIAL TYPES OF BEVEL GEARS

248. Spiral Bevel Gears.—A special type of bevel gears called "*spiral bevels*" is now used extensively for driving the rear axles of automobiles. No doubt within a short time manufacturers of machine tools and other classes of machinery will begin to use spiral bevels, since they possess certain advantages over the straight-tooth gears. The teeth of these gears are curved on the arc of a circle if produced by the well-known Gleason spiral bevel-gear generator, or they are helical if produced on a generating-gear planer. An illustration of the former type is shown in Fig. 172.

In discussing spiral bevel gears, one should be familiar with certain terms or expressions that are now in common use. These are as follows:

(*a*) *Angle of spiral.*—By the *angle of spiral* is meant the angle that the tangent AB to the tooth at the center of the gear face

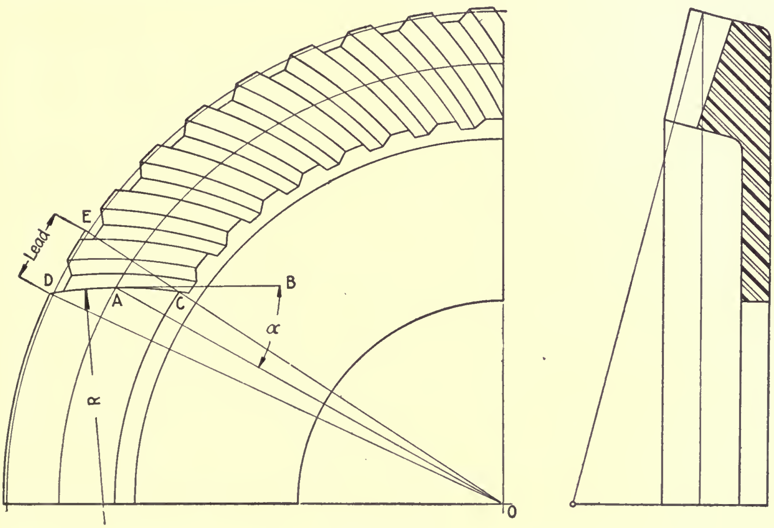

Fig. 172.

makes with the element OA of the pitch cone. In Fig. 172 this angle is designated by the symbol α.

(*b*) *Direction of spiral.*—The *direction of the spiral* is designated as right or left hand, based upon the direction of the spiral on the pinion; thus, by left-hand spiral is meant left hand on the pinion and right hand on the gear.

(*c*) *Lead.*—By the term *lead* is meant the distance that the spiral advances within the face of the gear, as shown in Fig. 172.

249. Advantages and Disadvantages.—(*a*) *Advantages.*—Among the advantages claimed for spiral and helical bevel gears are the following:

1. Due to the curvature of the teeth their engagement is gradual, thus tending to eliminate noise. The best results, according to the Gleason Works, are obtained when the lead of the spiral is made equal to one and one-quarter to one and one-half times the pitch of the teeth.

2. The wear on the teeth of spiral bevel gears is no more than on the teeth of the common type of bevel.

3. It has been found in practice that spiral bevel pinions permit of greater endwise adjustment than straight-tooth bevels, without producing excessive noise or causing bearing troubles.

4. There is practically no difference between the load-carrying capacity of spiral and helical bevel gears, when compared with those having straight teeth.

5. Spiral and helical bevel gears are better adapted to high-gear ratios, 5 and 6 to 1 giving satisfactory service, while with straight teeth 4½ to 1 seems to be about the dividing line between quiet and noisy gears when run at high speeds such as are common in automobile transmissions.

(*b*) *Disadvantages.*—The chief disadvantages resulting from the use of spiral or helical bevel gearing is the provision that must be made to take care of the additional thrust coming upon the bearings. In installations where the direction of rotation is reversed, the end thrust on the bearing must be taken care of in both directions, as the analysis given in the following article will show. Due to the additional end thrust on the bearing, it is probable that the efficiency of a spiral bevel-gear drive is slightly less than that obtained from a common bevel-gear drive.

250. Bearing Loads and Thrusts.—The following analysis, applied to the spiral bevel gear, is based on the assumption that this form of tooth may be treated in a manner similar to a straight tooth having a spiral angle equal to the spiral angle measured at the center of the face, as defined in Art. 248. Furthermore, the friction of tooth contact will not be considered. To arrive at expressions for the bearing loads and thrusts, proceed as follows:

(*a*) *Direct rotation.*—The spiral bevel gear, shown in Fig. 173, has an angle of spiral designated by α, and an angle of obliquity of tooth pressure equal to β. Resolving the resultant normal tooth pressure, acting at G and represented by the vector AB, into three components, we have:

1. The component DF perpendicular to the plane of the paper

and also equal to W, the tangential force acting on the gear at G, is given by the following expression:

$$W = AB\cos\alpha \cos\beta \tag{378}$$

2. The component acting along the element of the pitch cone is represented by EF and its magnitude is

$$EF = HG = W\tan\alpha \tag{379}$$

3. The component at right angles to the element of the pitch

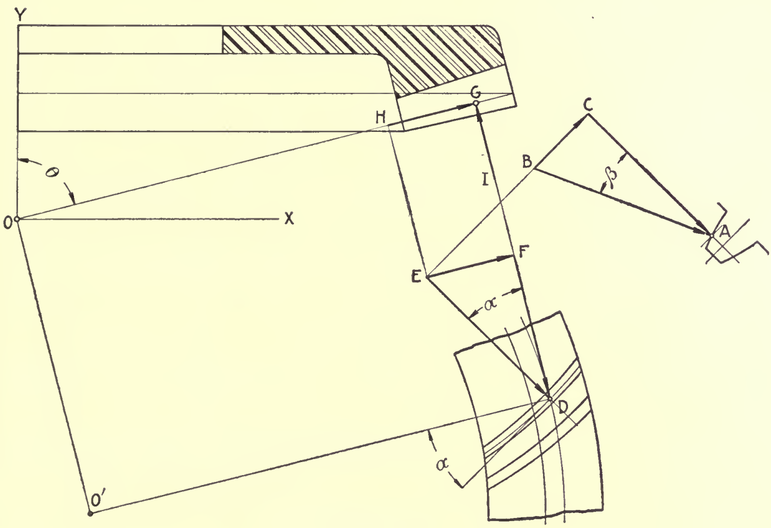

Fig. 173.

cone is represented by the vector BC or GI, the magnitude of which is

$$BC = GI = AB\sin\beta$$

$$= W\frac{\tan\beta}{\cos\alpha} \tag{380}$$

Resolving the three forces DF, HG, and GI into components whose lines of action are along the center line of the shaft and at right angles thereto we obtain for the thrust along the shaft of the gear

$$F_y = HG\cos\theta + GI\sin\theta$$

$$= \frac{W}{\cos\alpha}(\sin\alpha\cos\theta + \tan\beta\sin\theta) \tag{381}$$

and for the thrust along a line at right angles to the shaft of the gear, or in other words along the shaft of the pinion

$$F_x = HG \sin \theta - GI \cos \theta$$

$$= \frac{W}{\cos \alpha} (\sin \alpha \sin \theta - \tan \beta \cos \theta) \qquad (382)$$

It follows that the thrust exerted by the pinion upon its shaft has a magnitude given by (382), but its direction is opposite to that of F_x.

(*b*) *Reversed rotation.*—Supposing now that the direction of rotation of the gear is reversed, the component along the element of the cone, given by (379), reverses in direction, or in other words, it acts toward the point O in Fig. 173; thus

$$EF = GH = -W \tan \alpha \qquad (383)$$

Furthermore, the component BC or GI at right angles to the cone element remains as in the preceding case.

Resolving DF, GH, and GI, as in the preceding case, we get for the thrust along the shaft of the gear

$$F_y = GI \sin \theta + GH \cos \theta$$

$$= \frac{W}{\cos \alpha} (\tan \beta \sin \theta - \sin \alpha \cos \theta) \qquad (384)$$

In a similar manner, the magnitude of the thrust along the pinion shaft is found to be

$$F_x = HG \sin \theta - GI \cos \theta$$

$$= -\frac{W}{\cos \alpha} (\sin \theta \sin \alpha + \tan \beta \cos \theta) \qquad (385)$$

If the spiral of the teeth is reversed for the case just discussed, the equations deduced for the preceding case will hold.

251. Experimental Results.—In order to determine the actual thrusts upon the bevel pinion of automobile drives, the Gleason Works made an extensive series of tests upon various types of bevel gears. The results were published in *Machinery*, vol. 20, p. 690. Table 78 gives the various dimensions and angles of the gears and pinions, and the average pinion thrusts per 100 pounds of load on the tooth. The pinion thrusts have been calculated by substituting in (382) and (385) the value of W and the values of the functions of the various angles. Comparison of these calculated values with the actual pinion thrusts observed in the tests show good agreement.

TABLE 78.—EXPERIMENTAL DATA PERTAINING TO BEVEL GEARING

					Type of bevel gearing		
					Common	Spiral tooth	
1	Number of teeth			Pinion	15	14	15
2				Gear	53	53	53
3	Pressure angle—β				14½ degrees		
4	Pitch cone angle—θ			Pinion	15°–48′	14°–48′	15°–48′
5				Gear	74°–12′	75°–12′	75°–12′
6	Spiral angle—α				0	19°–45′	31°–21′
7	Pinion thrust in pounds per 100 pounds of tooth load		Direct drive	Actual	7.34	−28.70	−49.50
8				Calculated	7.06	−27.6	−50.3
9			Reverse drive	Actual	7.62	45.00	73.82
10				Calculated	7.06	41.6	66.9

252. Skew Bevel Gears.—Another form of special bevel gear, known as a *skew bevel gear*, has no common axes plane, and hence the face and cutting angles of the pinion and gear do not converge to a common apex. This fact introduces more or less involved mathematical calculations in arriving at the various angles required to lay out such gears. Because of the more involved calculations required and the greater cost of manufacture, skew bevel gears are rarely used in machine construction. Strictly speaking, there are two distinct types of skew bevel gears, as follows: (1) Those in which the oblique teeth are confined to the gear, and the mating gear or pinion is really a straight tooth bevel; (2) those in which the teeth of both gear and pinion are oblique.

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Bearing Pressures Due to the Action of Bevel Gears under Load, *Mchy.*, vol. 20, p. 639.

Gleason Spiral Type Bevel Gear Generator, *Mchy.*, vol. 20, p. 690.

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Laying out Skew Bevel Gears, *Mchy.*, vol. 23, p. 32.

CHAPTER XIV

SCREW GEARING

The term screw gearing is applied to all classes of gears in which the teeth are of screw form. Screw gearing is used for transmitting power to parallel shafts as well as to non-parallel and non-intersecting shafts. The following two classes of screw gearing are used considerably in machine construction: (*a*) helical gearing; (*b*) worm gearing.

HELICAL GEARING

253. Types of Helical Gears.—Helical gearing may be used for the transmission of power to shafts that are parallel, or to shafts

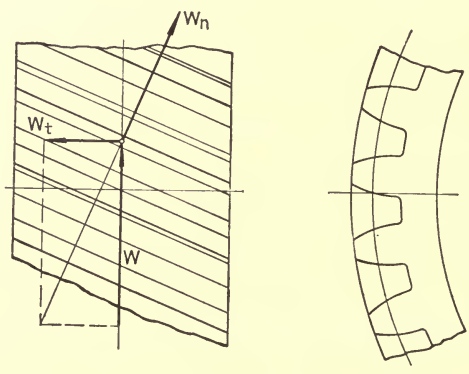
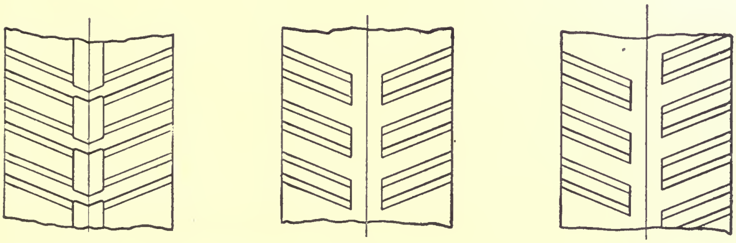


Fig. 174.

that are at right angles to each other and do not intersect, or to shafts that are inclined to each other and do not intersect. The teeth of helical gears used for connecting shafts that are parallel have line contact, while those used for connecting non-parallel, non-intersecting shafts have merely point contact and for that reason are not used much for the transmission of heavy loads. From Fig. 174, it is evident that the normal component of the tangential load W on the teeth of a pair of helical gears connecting two parallel shafts produces an end thrust on each shaft. To overcome this objectionable end thrust, two single helical gears having teeth of opposite hand are sometimes bolted or riveted together, forming what is called the *double-helical* or *herringbone gear.* Due to improved methods of cutting helical teeth, herringbone gears are not now constructed to any great extent from two single-helical gears, but are cut directly from the solid blank. Herringbone gears are also produced by casting them in a properly constructed mould.

There are two general types of double-helical gears, as follows:

(*a*) The ordinary herringbone gear in which the two teeth meet at a common apex at the center of the face, as shown in Fig. 175(*a*). A modification of this type, in which the central part has been removed, is shown in Fig. 175(*b*).



(a) (b) (c)

FIG. 175.

(*b*) The type known as the Wuest gear in which the teeth instead of coming together at a common apex at the center of the face do not meet at all, but are staggered as shown in Fig. 175(*c*).

In the types illustrated by Fig. 175(*b*) and (*c*), a groove is turned into the face as shown, so as to provide clearance for the cutters used in cutting the teeth. In gears having teeth cast approximately to shape, the center part where the two teeth come together is cast somewhat undersize on both sides of the teeth, also at the bottom of the space between the teeth as shown in Fig. 175(*a*).

254. Advantages of Double-helical Gears.—When compared with a spur gear, a double-helical gear has the following advantages:

(*a*) The face of the gear is always made long so that more than one tooth is in action; in other words, the continuity of tooth action depends upon the face of the gear and not upon the number of teeth in the pinion as with spur gearing.

(*b*) Due to the continuity of action, the load is transferred from one tooth to another gradually and without shock, thus eliminating to a great extent noise and vibration.

(*c*) In helical gearing, the load is distributed across the face of the gear along a diagonal line, thus decreasing the bending stress in the teeth.

(*d*) In well-designed helical gearing all phases of engagement occur simultaneously, hence the load is transmitted by surfaces that are partly in sliding contact and partly in rolling contact. Such action has a tendency to equalize the wear all over the teeth, consequently the tooth profile is not altered.

(*e*) Actual tests on double-helical gears show that they have much higher efficiencies than those obtained from spur gears. Efficiencies of 98 to 99 per cent. are not unusual with properly designed transmissions.

(*f*) Gear ratios much higher than those used with spur gearing may be employed.

(*g*) Due to the absence of noise and vibration, double-helical gears may be run at much higher pitch line speeds than is possible with spur gearing.

255. Applications of Double-helical Gears.—Cut double-helical gears have been applied successfully to many different classes of service. The following examples of applications give some idea of the extent of the field in which such gears may be used.

(*a*) *Drives for rolling mills.*—Gears used for driving rolling mills operate under very unfavorable conditions, such as heavy overloads, the magnitudes of which are difficult to determine; furthermore, these overloads are applied suddenly and are constantly repeated. The gears are also subject to excessive wear due to the dirty surroundings. Double-helical gears are now installed for rolling-mill drives, and, due to the continuous tooth engagement, such gears readily withstand the suddenly applied loads. Whenever it is possible, the gears should be enclosed by a casing and run in oil, thereby eliminating all noise.

(*b*) *Drives for reciprocating machinery.*—Gears for motor-driven reciprocating pumps and air compressors are required to transmit a torsional moment which varies between rather wide limits, several times per revolution. Due to the load fluctuation, an ordinary spur-gear drive is noisy and is subject to considerable vibration, while a double-helical gear drive runs quietly, without vibration, and at the same time is more efficient.

(*c*) *Drives for hoisting machinery.*—In connection with motor-driven hoists such as are used in mines, double-helical gears are especially well adapted, since the high gear ratios possible simplify the drives. High-ratio double-helical gears are more efficient and run more quietly than spur gears having the same ratio. Such high-ratio helical gears are also being introduced on modern high-speed traction elevators, with excellent results.

(*d*) *Drives for machine tools.*—Double-helical gears used on motor-driven machine tools produce a noiseless drive free from vibration, and are better adapted to the high speeds that are now common in machine-tool drives.

(*e*) *Drives for steam turbines.*—Gears used for reducing the speed of a steam turbine to that required by a centrifugal pump, fan, or generator must be made accurately, as the pitch line velocity is likely to be from 3,000 to 5,000 feet per minute. Due to the high efficiency and quiet running obtainable by the use of double-helical gears, the latter are used extensively in steam-turbine drives. In such installations the pinions are always made from an alloy-steel forging, and after being machined they are heat treated.

256. Tooth Systems.—Several of the more prominent manufacturers of double-helical gears agree fairly well on the following points relating to the proportions of the teeth:

1. The tooth profile should be formed by a 20-degree involute curve, thus making the tooth-pressure angle 20 degrees.
2. The tooth should be made shorter than the old standard used with spur gears.
3. The angle of the helix, more commonly called the angle of inclination of the tooth, should be 23 degrees.
4. The diametral pitch standard should prevail for all cut teeth.
5. The unequal addendum system should be used on all pinions having few teeth.

(*a*) *Tooth proportions.*—The proportions for the teeth and gear blank given in Table 79 are those proposed and recommended by Mr. P. C. Day of The Falk Co. of Milwaukee, Wis. It should be noted that according to these formulas the pitch and outside diameters of gears having less than 20 teeth are made slightly larger than those of a standard gear. This is done to avoid undercutting of the teeth. If a pinion proportioned in this way meshes with a gear having less than 40 teeth, then the distance between the shafts must be increased by an amount equal to one-half of the increase in the pinion diameter. If the gear, meshing with a small pinion has more than 40 teeth the normal center distance may be used by decreasing the pitch diameter of this gear by the same amount that the pinion diameter was increased. Gears made according to the above suggestions have teeth of standard depth but unequal addendums.

Table 79

1. Tooth profile....................Involute.
2. Pressure angle....................20 degrees.
3. Angle of helix....................23 degrees.
4. Length of addendum $= \frac{0.8}{p} = 0.2546p'$.
5. Length of dedendum $= \frac{1.0}{p} = 0.3183\,p'$.
6. Full height of tooth $= \frac{1.8}{p} = 0.5729\,p'$.
7. Pitch diameter, when $T < 20 = \frac{0.95\,T + 1}{p}$.
8. Pitch diameter, when $T \geqq 20 = \frac{T}{p}$.
9. Outside diameter, when $T < 20 = \frac{0.95\,T + 2.6}{p}$.
10. Outside diameter, when $T \geqq 20 = \frac{T + 1.6}{p}$.

Table 80.—Proportions of Teeth for Cut Double-helical Teeth, Fawcus Machine Co.

Pitch		Addendum	Dedendum	Minimum face
Dia.	Cir.			
8.00	0.393	0.100	0.125	2.5
6.00	0.524	0.133	0.167	3.5
5.00	0.628	0.160	0.200	4.0
4.00	0.785	0.200	0.250	5.0
3.50	0.898	0.229	0.286	5.5
3.00	1.047	0.267	0.333	6.5
2.50	1.257	0.320	0.400	7.5
2.00	1.571	0.400	0.500	9.5
1.75	1.795	0.457	0.572	11.0
1.50	2.094	0.533	0.667	12.5
1.25	2.513	0.640	0.800	15.0
1.00	3.142	0.800	1.000	19.0

In Table 80 are given the commercial pitches, tooth proportions, and minimum lengths of face recommended by the Fawcus Machine Co. for double-helical gears having a pressure angle of 20 degrees and a helix angle of 23 degrees.

257. Strength of Double-helical Teeth.—Various formulas have been proposed for determining the working load that a cut double-helical gear will transmit; probably the most reliable are those given by Mr. W. C. Bates and Mr. P. C. Day.

(*a*) *Bates' Formula.*—In an article entitled "The Design of Cut Herringbone Gears," published in the *American Machinist*, Mr. W. C. Bates, mechanical engineer of the Fawcus Machine Co., proposed a formula for the permissible working load for a double-helical gear, which is really an adaptation of the well-known Lewis spur-gear formula given in Art. 223. The author introduces two additional factors, one of which depends upon the condition of the load, whether it is constant or variable, and the second takes into consideration the lubrication necessary to prevent wear. In addition to these factors, higher fiber stresses than those commonly used with the Lewis formula are recommended. The formula as proposed by Mr. Bates is as follows:

$$W = \tfrac{6}{7}\, Sp'fy\, CK, \tag{386}$$

in which the factors p', f, and y have the same meaning as assigned to them in Art. 223.

The factor C depends upon the ratio of the maximum load to the average load during a complete operating cycle. If the load is fairly uniform, that is, if the ratio of maximum to average load is practically unity, then C is given its maximum value, namely unity. If, however, the load on the gear varies, say from zero to a maximum twice in a revolution, as, for example, when the gear drives a single-cylinder pump or compressor, then C must be given some value less than unity. Experience should dictate the magnitude of the factor C, and the following values, obtained from information furnished by Mr. Bates, will serve as a guide in the selection of the proper value for any particular class of service.

1. For reciprocating pumps of the triplex type, C usually is taken as 0.7.
2. For mine hoists running unbalanced, C is taken as 0.57.
3. For rolling mill drives in which the flywheels are located on the pinion shaft, the factor C varies from 0.50 to 0.66, depending upon the rapidity with which the energy in the flywheel is given up.

The factor K depends for its value upon the effectiveness of the lubricating system used with the gears; in other words, the wearing conditions of the gear depend upon K. When the gears are encased so that the lower part of the gear runs in oil, thus carrying a continuous supply of oil to the mating pinion, the factor K may be assumed as unity. It is claimed that with such a system of lubrication double-helical gearing may be operated successfully at speeds of 2,000 to 2,500 feet per minute. Experience seems to indicate that with speeds exceeding 2,500 feet per minute considerable oil is thrown off the gears due to centrifugal action, and in such installations it is suggested that the oil, under a low pressure, be sprayed against the teeth on the entering side near the line of engagement. For other systems of lubrication, the values of K given in Table 81 are recommended.

TABLE 81.—VALUES OF K AS RECOMMENDED BY W. C. BATES

Types of lubricating systems	Value of K		
	Min.	Max.	Mean
Continuous supply of oil	1	1	1
Thorough grease lubrication	0.83	0.91	0.87
Scanty lubrication, but frequent inspection	0.80	0.87	0.835
Indifferent lubrication	0.77	0.83	0.80

The permissible fiber stress S may be determined by means of the formula

$$S = S_0 \frac{1{,}200}{1{,}200 + V}, \tag{387}$$

which is similar in form to the expression given for the safe stress in the case of spur gearing. The values of S_0 given in Table 72 may also be used for this class of gearing.

The values of the factor y as recommended by Bates are those worked out by Lewis for the 15-degree involute teeth. For commercial pitches and corresponding gear faces, see Table 80.

(*b*) *Formula for Wuest gears.*—In a comprehensive paper before the American Society of Mechanical Engineers, Mr. P. C. Day of The Falk Co. gave a simple formula for determining the safe working load on the teeth of Wuest helical gears. The formula is empirical, as it is based upon the results obtained from several years of experience with such gears. Using as far as possible the notation given in the preceding discussion, the safe working load is as follows:

$$W = 0.4\, Sp'f, \tag{388}$$

in which the factor S represents the shearing stress on a section taken at the pitch line. This shearing stress varies with the pitch-line speed, as shown in Fig. 176. The length of the total face of the gear should be at least five times the circular pitch, and for average conditions six times the pitch gives satisfactory service. When the gear ratio is high, the face may be made ten times the circular pitch, provided the pinion and gear are mounted on rigid bearings located close together.

When the load transmitted by the gears fluctuates from a minimum to a maximum, as in the case of single-acting pumps and mine hoists, the gears should be designed for a load which represents an average between the maximum and mean loads. The gears used in connection with motor-driven machine tools should

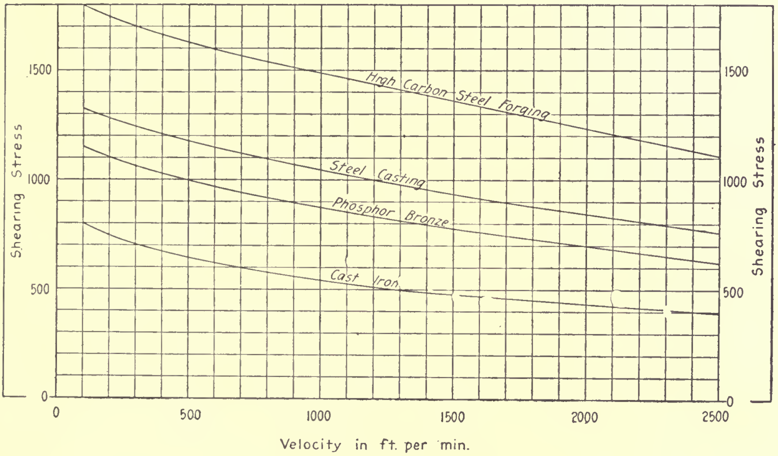


FIG. 176.

be designed to transmit a load equivalent to the rated output of the motor at a speed which is taken as the mean between the maximum and minimum revolutions per minute. The design of high-ratio and rolling-mill transmissions must receive special consideration, and should be left to the engineers of the company that manufacture such gears.

258. Materials for Helical Gearing.—In general, soft materials such as rawhide, fiber, and cloth should never be used for the pinion. Some manufacturers do not consider it good practice in high-ratio transmissions to use cast iron for cut double-helical pinions, claiming that a forged-steel pinion will cost but little more, and, due to its better wearing qualities, will give increased life to the transmission. When the tooth pressures are moderate, cast iron or semi-steel is preferred to steel casting for gears of large diameter; but when the loads are heavy, steel casting is generally more economical. The carbon content of the grade of steel casting used ordinarily for gears varies from 0.25 to 0.30 per cent. When the gear and pinion are both made of steel, the best results are obtained by making the pinion of a different grade of steel than that used for the gear; for example, with a gear made of steel casting having a carbon content of 0.25 to 0.30 per cent., the pinion should be made of a 0.40 to 0.50 per cent. carbon-steel forging. For high-pitch line velocities, alloy-steel pinions subjected to a heat treatment are recommended. Frequently the pinion teeth are cut integral with the shaft.

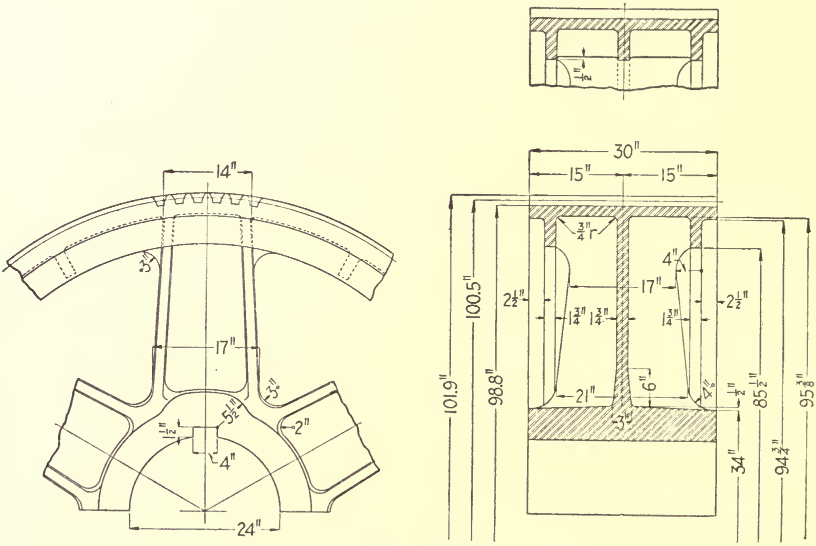


FIG. 177.

259. Double-helical Gear Construction.—(*a*) *Rim.*—For large gears, The Falk Co. has found that whenever possible the rim should be made solid, and when the diameter of the gear exceeds 7 feet the hub should be split. The split in the hub should be placed midway between two arms; thus when six arms are used, as is their usual practice, two of these arms are perpendicular to the split. The Falk Co. has found that with this arrangement the casting will contract very evenly, so that the rough gear blank on leaving the sand is practically round. It is claimed that such a construction, when used with eight arms, produces a casting that is distorted. Figs. 177 and 178 show two large gears made of steel casting and built by The Falk Co.

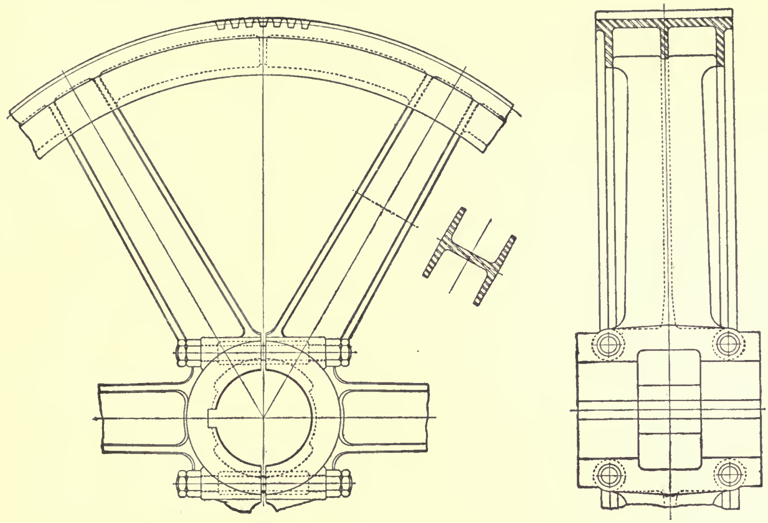


FIG. 178.

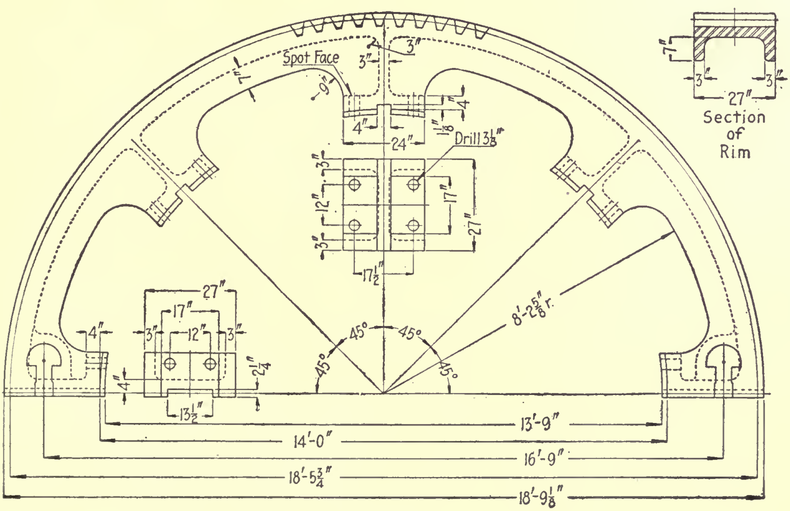


FIG. 179.

Large double-helical gears transmitting heavy loads are frequently made with a steel-casting rim, cast in halves and bolted to a cast-iron spider. The rims of such gears are shown in Figs.

179 and 180, and the cast-iron spider for the latter is shown in Fig. 181. In order to relieve the coupling bolts between the rim and the spider of all shearing action, large heavy keys are fitted

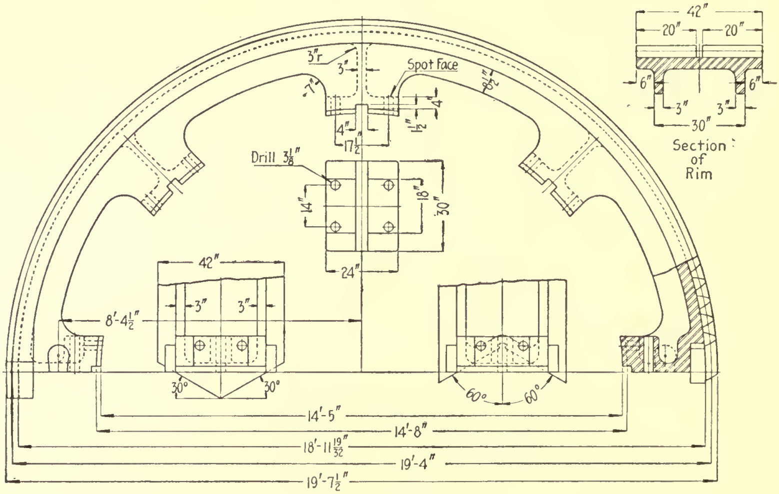


FIG. 180.

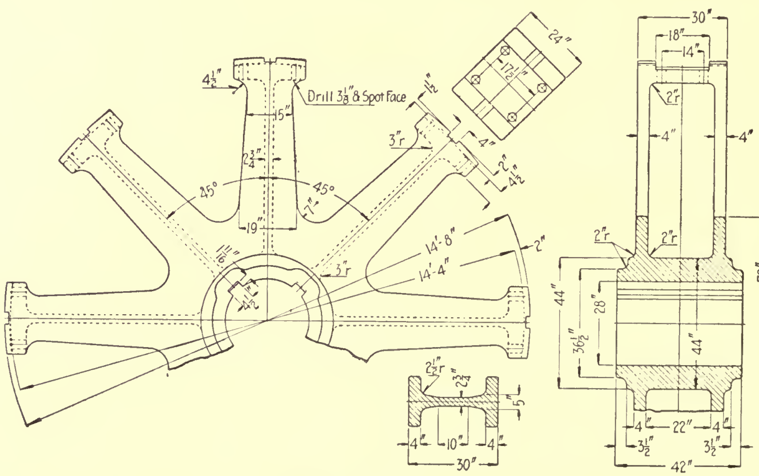


FIG. 181.

between the rim and the arms of the spider. The rim, being made in halves, has the joints split parallel to the tooth angle. These rim joints should always be located between two teeth as shown in Fig. 182. Joints made in this manner do not weaken the teeth, nor do they interfere with the smooth operation of the gear. Bolts and shrink links as shown in Fig. 182 are used for fastening together the two halves of the rim.

Another design of a rim joint is shown in Fig. 183, and as in the design just described, the steel-casting rim is fastened to a cast-iron spider by means of bolts and shrink links. This joint, however, differs from the one shown in Fig. 182 in that a tongue and groove are used, the tendency of which is to weaken the tooth along the joint, as is evident from an inspection of Fig. 183.

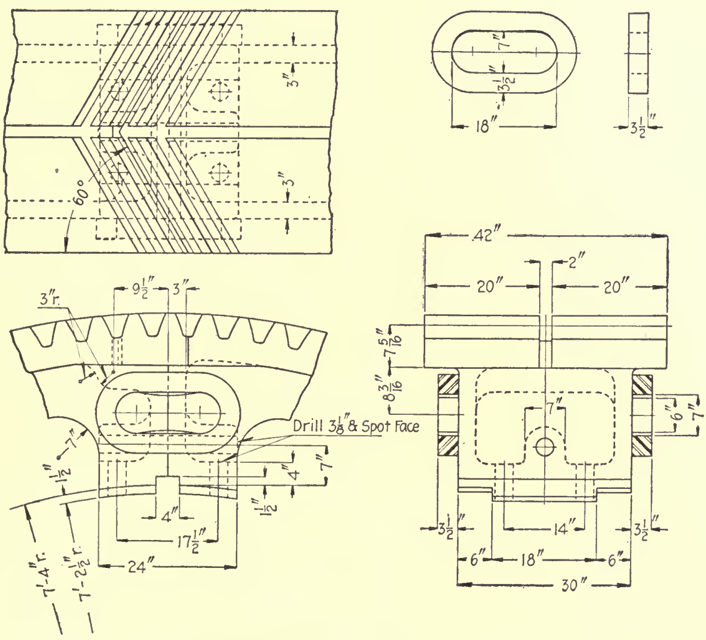


FIG. 182.

In Fig. 184 is shown an excellent design of a heavy steel casting double-helical gear, cast in halves. The joint is made through the arms, and a series of studs as shown hold the two halves of the gear together. The studs in the arms are fitted accurately into reamed holes, while those in the hub and under the rim are fitted very loosely, because it is impossible to ream these holes. The split in the rim is made between two teeth and parallel to the teeth.

The rim sections in common use are illustrated in the various

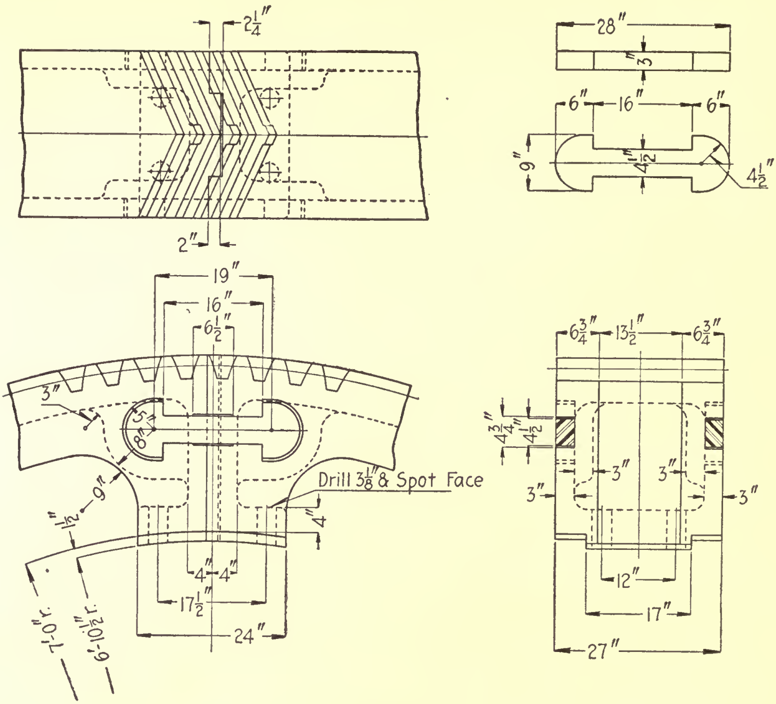


FIG. 183.

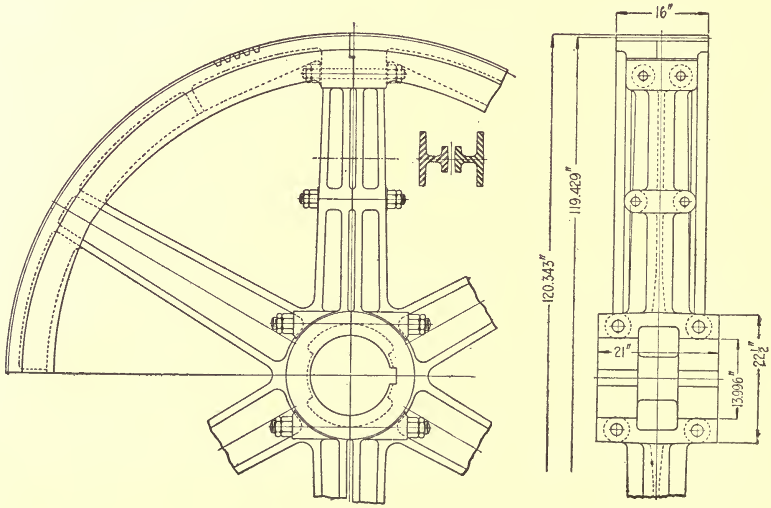


FIG. 184.

figures mentioned in the preceding discussion. According to Bates, the finished rim thickness under the teeth of cut double-helical gears may be arrived at by the following empirical formula:

$$\text{Rim thickness} = \frac{2}{p} + \frac{1''}{2} \qquad (389)$$

In Fig. 185 is shown a double-herringbone pinion, the teeth of which are cut integral with the shaft. This shaft with the double pinion is used for driving two large gears of a rolling-mill drive.

(*b*) *Arms.*—Arms of elliptical cross-section should never be used for double-helical gearing for the reason that they lack rigidity at right angles to the direction of rotation. For gears not exceeding 40 inches in diameter, and having a length of face

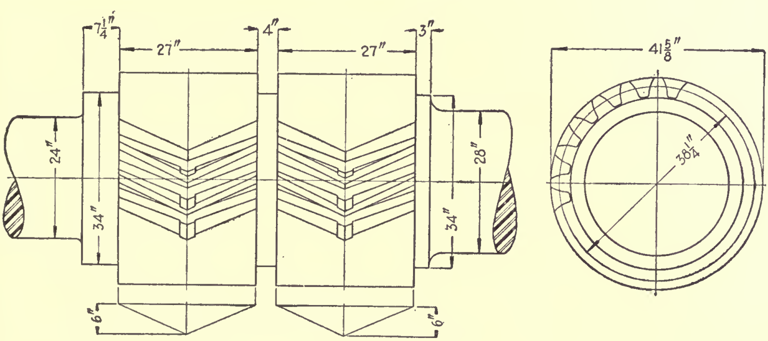


Fig. 185.

approximating one-sixth to one-eighth of the diameter, Bates recommends the use of cross-shaped arms. With gears having wider faces than those just mentioned, the H-section similar to those shown in Figs. 178 and 181 should be used. Furthermore, according to the same authority, the face of cut gears should never be made less than one-tenth of the pitch diameter, if the gear is to possess sidewise rigidity and no vibration is to be set up in the transmission. For heavy rolling-mill drives, the face of the gears is unusually long, and for such gears The Falk Co. recommends the use of double arms of U cross-section. In general, the section of the arms should be made considerably heavier where they join the hub so as to insure sound castings.

260. Mounting of Double-helical Gears.—Due to the high speeds at which double-helical gears are used, the frames and bearings supporting such gears must be made heavy and rigid. The shafts must all be in true alignment, and the pinion and gear must have the supporting bearings located close up to the hubs. The gear with its mating pinion should be aligned correctly so as to eliminate all end thrust. Means for lubricating the transmission must be provided, and the whole arrangement should be made accessible for inspection. For a high-ratio transmission running at a high rotative speed, the pinion is generally integral with its shaft, and the latter is driven by the prime mover or motor through the medium of a flexible coupling.

261. Circular Herringbone Gears.—Several years ago, the R. D. Nuttall Co. developed and introduced a new form of generated tooth gear to which the term *circular herringbone* was applied. Such a gear has continuous teeth extending across its face in the form of circular arcs. The teeth are generated by two cutters, one for each side of the tooth. The profile of these cutters is an involute rack tooth, and the pressure angle for the middle section of the gear tooth is 20 degrees. This angle, however, varies slightly for all the other sections of the tooth, increasing as the sections approach the end of the gear face. The Nuttall Co. has adopted as a standard for these gears a short tooth having the following proportions:

1. The tooth profile is made a 20-degree involute.
2. The length of the addendum is made $0.25\,p'$.
3. The clearance is made $0.05\,p'$.
4. The whole depth of the tooth is made $0.55\,p'$.
5. The radius of curvature of the tooth and that of the face of the gear are made equal, and should never be less than twenty-four divided by the diametral pitch.

According to the manufacturers, the circular herringbone gears have all the advantages of double-helical gears, and in addition two special advantages are claimed.

1. Due to the fact that the tooth is continuous and not grooved at the center, it is stronger and at the same time the rim is reinforced.

2. The lubrication is applied more readily, since the curved tooth acts like a cup.

WORM GEARING

The type of screw gearing commonly called worm gearing is used for transmitting power and obtaining high speed reductions between non-intersecting shafts making an angle of 90 degrees with each other. There are two classes of worm gearing in common use, each of which possesses certain advantages over the other.

262. Straight Worm Gearing.—The class of worm gearing most frequently used is that in which the worm is straight or of a cylindrical shape. The threads of such a worm have an axial pitch that is constant for all points between the top and the root of the threads. Strictly speaking, there are two types of straight worm gearing. In the first of these types, generally called the ordinary worm and gear, the hob used for machining the worm gear is of constant diameter and is fed radially to the proper depth into the blank, both hob and blank being rotated in correct relation to each other. The teeth produced are not theoretically correct in shape. In place of a cylindrical hob, one that tapers may be used, and by feeding it into the gear blank longitudinally at right angles to the axis of the blank instead of radially as in the preceding case, the worm gear produced has teeth that approach very closely the theoretical form. Gears cut by the latter method have given much better service and higher efficiencies than similar gears cut by the first method.

Due to the higher grade of product obtained by the use of a taper hob, the second type of worm and gear is employed to a considerable extent in the rear axle drives of auto-trucks and motor cars. The efficiency and load-carrying capacity are practically the same as for the hollow-worm type of gearing described in the following article.

263. Hindley Worm Gearing.—In the second class of worm gearing, the worm has a shape similar to that of an hour glass. It was introduced by Hindley in connection with his dividing engine, and worms having a hollow face are generally called *Hindley worms.* As may be seen from Fig. 186, the worm is made smaller in the center than at its ends, so that it will conform to the shape of the gear. Since there is a larger contact surface between the mating teeth than in the straight worm class, the wear is reduced and it is possible to use a smaller pitch and face of gear for a given transmission. In the Hindley worm the axial pitch varies at every point, since the angle of the helix changes constantly throughout the length of the worm. At the center of the worm, the helix angle is much greater than at the ends, as is evident from an inspection of Fig. 186.

Hindley worm gearing is produced by the hobbing process, but since the shape of the worm is made to conform to the circumference of the gear, it follows that such worms are not interchangeable. In other words, a worm intended for a gear containing 36 teeth of a given pitch will not mesh correctly with a gear having 54 teeth of the same pitch. In order to obtain good results with the use of Hindley worm gearing, the following requirements must be met:

1. The center distance between the worm and gear must be exact.

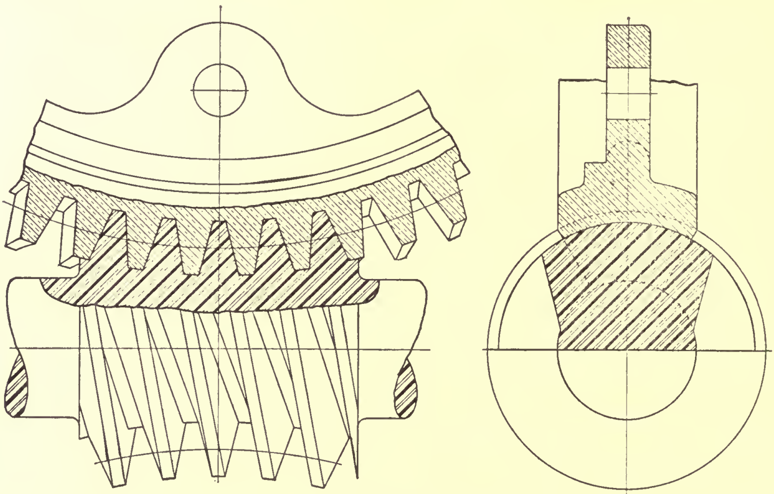


Fig. 186.

2. The center of the worm must conform exactly with the center of the gear so as to avoid any longitudinal displacement of the worm.

3. The worm axis must be in proper alignment, relative to the gear.

Experiments conducted on well-designed and properly mounted worm gears, as used in motor-car work, show that the efficiency and load-carrying capacity of the hollow worm are slightly greater than those obtained by means of the straight worm, although the difference is small.

264. Materials for Worm Gearing.—In general, a worm gear transmission gives satisfactory service when the worm is made of a low-carbon steel and the gear of a good grade of bronze. The steel for the worm should have a carbon content that will permit of heat-treatment without producing serious distortion of the worm. The heat-treatment that is generally used is one of carbonizing or case-hardening. For this purpose some manufacturers prefer a nickel steel with a low carbon content, while others specify an open-hearth high-carbon steel. In Table 82 are given six different gear bronzes that the Wm. Cramp and Sons Ship and Engine Building Co. has found to be satisfactory for the various classes of service indicated.

TABLE 82.—CRAMP'S GEAR BRONZES

Bronze No.	Tensile strength	Elastic limit	Wt. per cu. in.	Range of load	Permissible r.p.m. of worm	Class of service
1	40,000	20,000	0.316	not over 1,500	1,500	Light loads and high speeds.
2	40,000	20,000	0.319	3,000 to 4,000	1,000	Moderate loads and speeds.
3	45,000	22,000	0.321	3,000 to 4,000	1,000	Moderate loads and speeds when excessive wear is expected.
4	30,000	15,000	0.300	5,000 to 25,000	200 to 400	For continuous moderate loads with intermittent heavy load.
5	35,000	18,000	0.325	3,000 1,000 to 1,500	200 600 to 900	For average running conditions of light loads and moderate speeds with heavy starting torque.
Parson's Man. Bronze.	65,000	30,000	0.305	10,000 to 50,000	200	For heavy loads and slow speeds under excessive strain and shock.

From the preceding statements it should not be understood that steel and bronze are the only materials that are satisfactory for worm gearing. A carbonized steel worm and a gear made of a high-grade semi-steel casting will give good service for moderate loads and speeds. For light loads and low speeds, a carbonized-steel worm with a gear made of close-grained cast iron will prove satisfactory.

265. Tooth Forms.—(*a*) *Straight worm.*—The standard form of tooth used for the ordinary worm gearing is that proposed and adopted as a standard by the Brown and Sharpe Mfg. Co. As shown in Fig. 187, the sides of the worm thread make an angle of 29 degrees with each other, or in other words, the pressure angle is 14½ degrees. This form of worm thread is produced by a straight-sided tool having flat ends, and for the various pitches in use, the proportions may be taken from Table 83.

The teeth on the gear which mesh with a worm having teeth according to the proportions shown in Table 83 are given an involute form, and, according to the Brown and Sharpe Mfg. Co., such gears should always have more than 31 teeth in order to avoid undercutting of the teeth.

TABLE 83.—STANDARD 29° WORM THREADS

Circular pitch	Threads per inch	Tooth height above pitch line	Total height of tooth	Width of tooth at	
				Top	Bottom
1/4	4	0.0796	0.1716	0.0838	0.0775
2/7	3 1/2	0.0909	0.1962	0.0957	0.0886
1/3	3	0.1061	0.2288	0.1117	0.1033
2/5	2 1/2	0.1273	0.2746	0.1340	0.1240
1/2	2	0.1592	0.3433	0.1675	0.1550
2/3	1 1/2	0.2122	0.4577	0.2233	0.2066
3/4	1 1/3	0.2387	0.5150	0.2512	0.2325
1	1	0.3183	0.6866	0.3350	0.3100
1 1/4	4/5	0.3979	0.8583	0.4187	0.3875
1 1/2	2/3	0.4775	1.0299	0.5025	0.4650
1 3/4	4/7	0.5570	1.2016	0.5862	0.5425
2	1/2	0.6366	1.3732	0.6708	0.6200

In modern manufacturing, the so-called straight worms are no longer turned on a lathe, but are milled. With the use of the 29-degree thread, there is some difficulty in milling such a worm when the helix angle approaches 28 degrees. To obviate any difficulty that

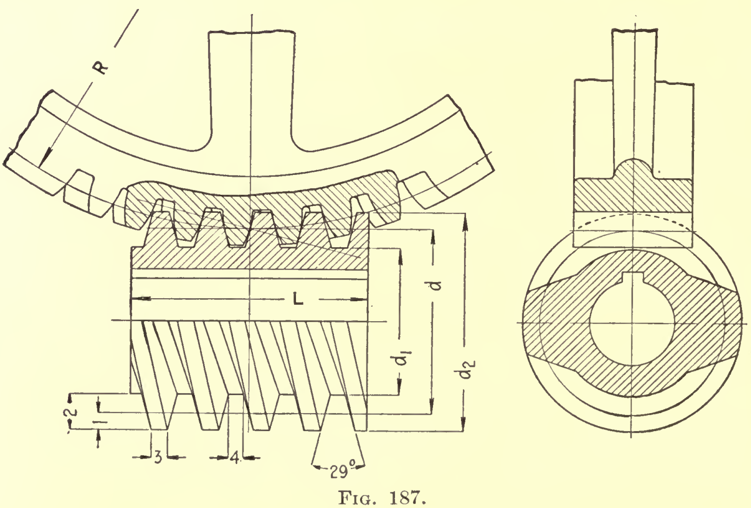


FIG. 187.

may arise, the angle between the sides of the tooth is made larger than 29 degrees. Some designers have adopted an angle of 60 degrees, while others vary the angle for different helix angles.

(*b*) *Hindley worm.*—According to the practice of the Keystone-Hindley Gear Co., the angle included between the sides of the teeth varies considerably, as is shown by the following:

1. For single-threaded worms, the angle is made 29 degrees.
2. For double-threaded worms, the angle is made 35 degrees.
3. For triple-threaded worms, the angle is made 35 degrees.
4. For quadruple-threaded worms, the angle is made 37½ degrees.
5. For worms of small diameter having from two to four threads, the tooth angle is made as high as 52 degrees.

Furthermore, this same company has no uniform depth of tooth, as it varies from 75 to 100 per cent. of the circular pitch, with an average of about 85 per cent.

In the Lanchester worm gearing, which is probably one of the most efficient types of Hindley gearing in use, the side of the tooth is given a slope of 1 in 2.

266. Load Capacity.—The permissible load upon the worm-gear teeth depends more upon the heating effect and wear produced than upon the strength of the teeth. If the oil film between the teeth in contact breaks down, due to high pressure or to thinning of the lubricant caused by high temperatures, excessive heating and wear will result. If not remedied, this will in a short time destroy the gear or worm, or both. The formulas in use for determining the permissible load on worm-gear teeth are all of an empirical nature, having the following form:

$$W = Cfp', \tag{390}$$

in which f and p' denote the face and circular pitch, respectively, and C is a coefficient depending upon the speed, pressure, and temperature. This coefficient must be determined by means of experiments.

In 1902, Prof. C. Bach and E. Roser made an experimental investigation of a triple-threaded soft-steel worm and bronze worm gear running under various conditions. The pitch diameter of the worm was a trifle over 3 inches and the lead was 3 inches, thus giving a helix angle of 17 degrees 34 minutes. The worm gear contained 30 teeth of involute profile having a pressure angle of 14½ degrees. The results of these tests were published in the *Zeitschrift des Vereins deutscher Ingenieure* of Feb. 14, 1903, also in the *American Machinist*, July 16 and 23,

1903. The expression for the allowable load on the worm drive as proposed by Bach and Roser is more or less involved, and since it is based upon the investigation of a single worm transmission, its adoption as a working formula may be questioned. The Bach and Roser formula, assuming continuous service, is as follows:

$$W = (mt + n)f'p', \tag{391}$$

in which f' denotes the face of the worm gear measured in inches on an arc at the base of the teeth; p' denotes the divided pitch of the worm or the circular pitch of the worm gear; t denotes the rise in degrees F. in the temperature of the oil in the reservoir; m and n are experimental coefficients depending upon the velocity of the teeth. The relations existing between the velocity V in feet per minute and the coefficients m and n are given by the following expressions:

$$\left.\begin{aligned} m &= \frac{934}{V} + 30 \\ n &= \frac{305{,}520}{V + 542} - 356 \end{aligned}\right\} \tag{392}$$

For ordinary working conditions, the temperature rise t in (391) may be assumed to vary from 80° to 100°F. If the drive is to be installed in a place where the prevailing temperature is high, the magnitude of t should be based upon the temperature at which the lubricant used in the drive loses its lubricating qualities. In view of the fact that formula (391) is based upon continuous service, it seems reasonable that for intermittent service the permissible load as determined by (391) may be increased; in other words, instead of designing the drive for the maximum load, the average load might be used in arriving at the safe dimensions of the worm-gear teeth.

267. Strength of Worm-gear Teeth.—It may occasionally be necessary to investigate the teeth of the worm gear for strength, and in such cases the formulas derived for spur gearing may be used by making the following modifications:

(*a*) For cast gearing, the load W should be considered as coming upon a single tooth.

(*b*) For cut gearing, assume the load W as equally distributed among all the teeth in actual contact as given by (408).

(*c*) For the magnitude of f in the spur-gear formula, determine the actual length of the gear tooth at the base of the tooth.

268. Force Analysis of Worm Gearing.—In order to arrive at the probable pressure coming upon the various bearings used in the mounting of a worm-gear drive, it is necessary to determine the relation existing between the turning force on the worm and the tangential resistance on the worm gear. Having established this relation, the magnitudes of the various components of the tangential resistance may then be determined, and from these components the pressures upon the bearings may be found.

(*a*) *Relation between effort and load.*—The relation between the equivalent turning force P on the worm and the tangential load W upon the worm gear may be obtained as follows:

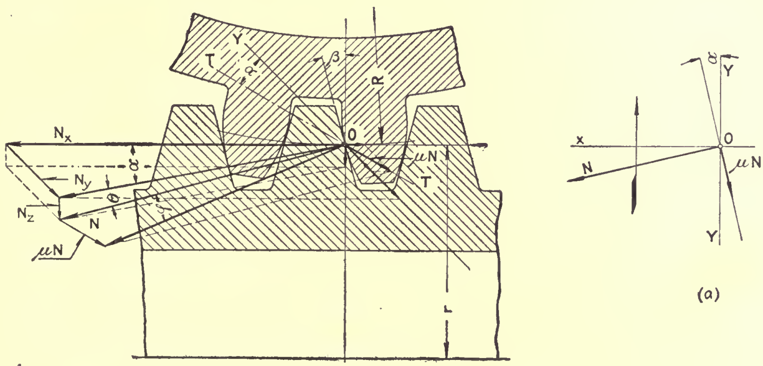


FIG. 188.

Referring to Fig. 188, the vector N represents the normal reaction between the teeth at the point of contact O. The symbol r denotes the pitch radius of the worm; α the angle of the helix of the worm; β the pressure angle or the angle the side of the thread makes with a line at right angles to the center line of the worm. The angle φ is the angle of friction for the materials in contact.

Disregarding the frictional resistances, the components of the normal force N along the X, Y, and Z axes are, respectively,

$$N_x = N\cos\theta\cos\alpha$$
$$N_y = N\cos\theta\sin\alpha$$
$$N_z = N\sin\theta$$

Assuming that the worm shown in Fig. 188 rotates in the direction as indicated in Fig. 188(*a*), the force μN due to friction upon the worm acts along the tangent to the helix. This force of friction tends to increase or decrease the components found above; hence resolving μN along the X and Y axes, we obtain the remaining components:

$$N'_x = \mu N \sin \alpha$$
$$N'_y = \mu N \cos \alpha$$

Each of the five components is shown in Fig. 188. Now adding the components along the same lines of action, we obtain the following expressions:

The magnitude of the tangential force exerted by the worm gear upon the worm teeth is

$$W = N_x - N'_x = N(\cos \theta \cos \alpha - \mu \sin \alpha) \tag{393}$$

The magnitude of the turning force P required at the pitch radius of the worm is obtained by adding N_y and N'_y thus

$$P = N_y + N'_y = N(\cos \theta \sin \alpha + \mu \cos \alpha) \tag{394}$$

The force S, causing a downward pressure upon the worm shaft or an upward pressure upon the worm-gear shaft, has a magnitude given by N_z above, namely,

$$S = N_z = N \sin \theta \tag{395}$$

The relation between P and W may now be obtained by combining (393) and (394); thus

$$P = W\left[\frac{\cos \theta \sin \alpha + \mu \cos \alpha}{\cos \theta \cos \alpha - \mu \sin \alpha}\right] \tag{396}$$

Denoting the ratio of μ to $\cos \theta$ by $\tan \varphi'$, (396) reduces to a simple form of expression which is similar to that derived for screws, namely,

$$P = W \tan(\alpha + \varphi') \tag{397}$$

Letting p' denote the lead of the worm and writing $\mu' = \tan \varphi'$, (397) may be put into the following form:

$$P = W\left[\frac{p' + 2\mu'\pi r}{2\pi r - \mu' p'}\right] \tag{398}$$

From Fig. 188, it is evident that

$$\tan \theta = \tan \beta \cos \alpha$$

Hence
$$\mu' = \frac{\mu}{\cos \theta} = \mu\sqrt{1 + \cos^2 \alpha \tan^2 \beta} \tag{399}$$

Now μ' may be considered a "new coefficient of friction" peculiar to worm gearing and its magnitude may be obtained by means of (399).

Combining (393) and (395) and reducing to a simple form, the magnitude of the force S in terms of W is given by the following expression:

$$S = W\left[\frac{\tan \beta}{1 - \mu' \tan \alpha}\right] \tag{400}$$

(*b*) *Efficiency of worm gearing.*—An expression for the efficiency of a worm and gear may now be determined. In the ideal transmission, namely, one having all of the frictional resistances eliminated, it is apparent that the effort P_0 required at the pitch radius of the worm is as follows:

$$P_0 = W \tan \alpha \tag{401}$$

Hence the efficiency of the worm and gear, not taking into consideration the frictional resistances of any of the bearings used in the mounting, is given by the following formula:

$$\eta = \frac{P_0}{P} = \frac{\tan \alpha}{\tan (\alpha + \varphi')} \tag{402}$$

269. Bearing Pressures.—(*a*) *Worm shaft.*—The worm shaft is generally supported on two bearings, each of which must be

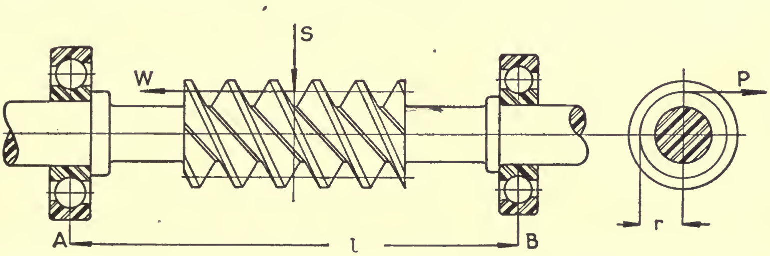


FIG. 189.

capable of withstanding the pressure coming upon it due to the forces P, W, and S. In addition to the transverse forces, the worm shaft is also subjected to a thrust, and for that reason a thrust bearing must be provided. When ball bearings are used for mounting the worm shaft, it is possible to select a type of radial bearing that is capable of taking care of a certain amount of end thrust in addition to the transverse load. Such a bearing makes the installation of a special thrust bearing unnecessary.

In Fig. 189 is shown a worm shaft mounted on radial bearings that are capable of taking an end thrust equivalent to one and one-half times the radial load. Assuming that the turning force P and the downward pressure S are applied midway between the bearings A and B, each of these bearings is subjected to a pressure equal to one-half of these forces. Since S is at right angles to P, the components of these forces at the bearings are at right angles to each other. The tangential force W, in addition to causing an end thrust upon the bearing A, also produces a pressure equal to $\frac{Wr}{l}$ upon each bearing, the one at A acting downward and that at B upward. Hence the resultant pressure upon the bearing A is as follows:

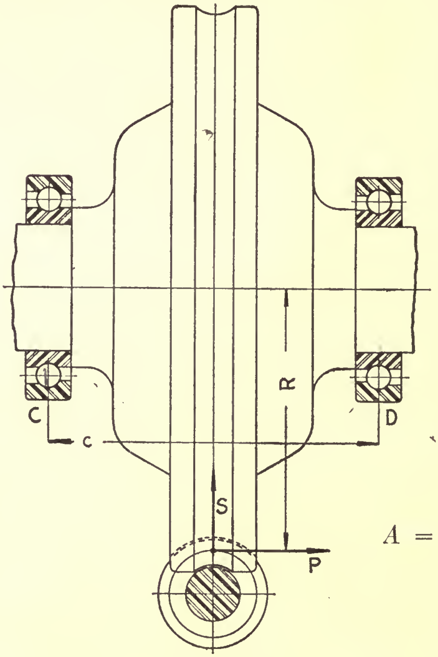


FIG. 190.

$$A = \sqrt{\frac{P^2}{4} + \left[\frac{S}{2} + \frac{Wr}{l}\right]^2} \qquad (403)$$

and the resultant pressure upon the bearing B is

$$B = \sqrt{\frac{P^2}{4} + \left[\frac{Wr}{l} - \frac{S}{2}\right]^2} \qquad (404)$$

Having determined the magnitudes of the bearing pressures and thrusts, the size of bearing may now be selected from tables furnished by the manufacturers of such bearings.

(*b*) *Worm-gear shaft.*—The pressure exerted upon the bearings supporting the worm gear depend upon the magnitudes of the forces P, W, and S, as well as upon the method of mounting the gear. The transmission illustrated in Fig. 190 has the gear supported on ball bearings mounted on the extended hubs of the gear. In some installations, the gear is keyed to a shaft which in turn is supported on proper bearings. If the bearings C and D in Fig. 190 are located symmetrically with respect to the center plane of the worm gear, the pressures upon them due to the forces

S and W will be equal to one-half of these forces. The force P tends to move the gear along its axis, thus producing a thrust on the bearing D, and at the same time this force introduces a transverse pressure upon both of the bearings. The transverse pressures due to P have a magnitude $\frac{PR}{c}$, the one acting upward on the bearing C and the other downward on the bearing D. Since P causes an end thrust, it is necessary that the radial ball bearings used for supporting the gear be of a type that is capable of supporting a thrust in addition to the radial load. Proceeding as in the case of the worm shaft, the following expressions are obtained:

The resultant radial load on the bearing C is

$$C = \sqrt{\frac{W^2}{4} + \left[\frac{S}{2} - \frac{PR}{c}\right]^2} \tag{405}$$

and the resultant radial load on the bearing D is

$$D = \sqrt{\frac{W^2}{4} + \left[\frac{S}{2} + \frac{PR}{c}\right]^2} \tag{406}$$

270. Worm and Gear Construction.—In many worm gear transmissions, the worm is made integral with the shaft as shown in Figs. 186 and 194 to 197, inclusive. However, occasionally in machine tools using worm drives, it is desirable to make the worm separate from the shaft and fasten it to the latter by means of keys or taper pins as shown in Figs. 187 and 191.

Worm gears made of cast iron, semi-steel, or steel casting are constructed in the same way as ordinary spur or helical gearing. If the gear is relatively small the solid or web construction shown in Fig. 191 is used. With gears of large diameter considerable material may be saved by the use of arms in place of a web. The dimensions of the arms may be determined by the formulas given in Art. 229. When bronze is used for the gear the cost may be kept down by making the rim of bronze, as shown in Fig. 186, and bolting it to a spider made of cast iron, semi-steel, or steel casting. An example of a worm gear having a bronze rim bolted to a cast-iron spider is shown in Fig. 197.

(*a*) *Length of worm.*—In the worm and gear, shown diagrammatically in Fig. 192, the symbol D denotes the pitch diameter of the gear, and a the addendum of the teeth. The intersections of the addendum line of the worm with the addendum circle of the gear are the extreme points of available tooth contact; thus the chord AB represents the minimum length of the straight

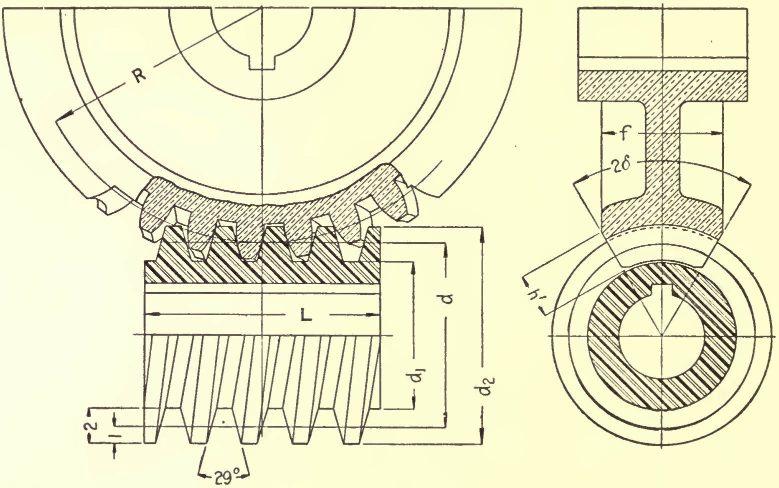


FIG. 191.

type of worm in order that complete tooth action may be obtained. The expression for the length AB is as follows:

$$A = 2\sqrt{2\,aD} = (D + 2\,a)\sin\theta \qquad (407)$$

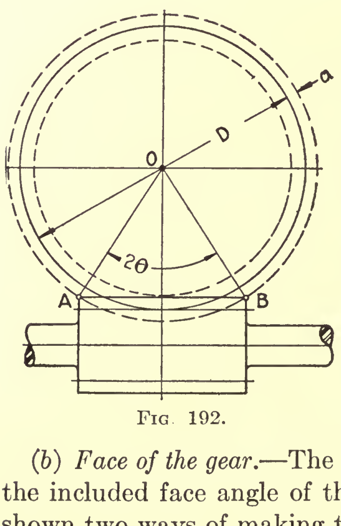


FIG. 192.

For worms of the Hindley type, the length as recommended by Lanchester is such that the difference between the maximum and minimum diameters is approximately 7 to 8 per cent. of the latter.

Having determined the length of the chord AB by means of (407), the number of gear teeth in actual contact with the worm is then given by the formula

$$T' = \frac{AB}{p'} \qquad (408)$$

(b) *Face of the gear.*—The face of the worm gear depends upon the included face angle of the worm. In Figs. 191 and 193 are shown two ways of making the face of worm gears. The design shown in Fig. 191 is used considerably for all ordinary worm gears. The face angle 2δ is chosen arbitrarily, and 60 degrees seems to answer very well for all common proportions, although occasionally 75 degrees may be preferred.

The large diameter D_2 of the gear blank is given by the following expression, provided the corners of the teeth are left sharp:

$$D_2 = D_1 + (d - 2a)(1 - \cos\delta) \qquad (409)$$

in which D_1 denotes the so-called throat diameter and is equal to the pitch diameter D plus twice the addendum of the worm teeth.

The design illustrated by Fig. 193 is intended chiefly for worm-gears having a large angle of lead. According to the practice of one manufacturer of such gears, the magnitude of the face angle 2δ may be obtained from the formula

$$\cos\delta = \frac{d - 3a}{d}, \qquad (410)$$

in which d denotes the pitch diameter of the worm, as shown in the figure. The outside diameter D_2 of the gear blank represented in Fig. 193 is made equal to the pitch diameter plus three times the addendum. The throat diameter D_1 is made equal to the pitch diameter plus twice the addendum.

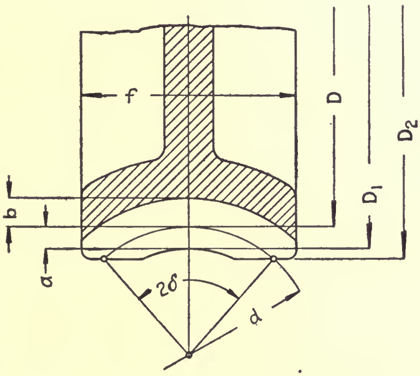


FIG. 193.

271. Sellers Worm and Rack.—On planers and large milling machines, the table is driven by a worm and rack. The teeth of the rack are cut straight across and not at an angle; hence the axis of the worm must be set over through an angle equal to the helix angle. The worm runs in an oil bath and proper thrust bearings are provided to take care of the thrust in either direction. This form of worm and rack drive was introduced by the Wm. Sellers Co. on its planers and later on it was adopted by several manufacturers of large milling machines.

272. Worm-gear Mounting.—Generally speaking, all worm-gear transmissions should be mounted in a dustproof casing which permits either the worm or the gear to run in an oil bath. In many installations the worm is located below the gear, while in others it must be located above. In the former case the worm runs in oil, and experience seems to indicate that such a mounting gives the least trouble and lasts longer than the second type. There are, however, many installations in which the worm must be mounted above the gear, and in such cases the proper lubrication depends upon the amount of oil carried to the worm by the gear, the lower segment of which runs in the oil bath. Many such drives, provided with the proper kind of a lubricant, are in successful use.

From the discussion of the various forces acting upon the several elements of a worm-gear drive, it is evident that the thrust along both the worm and worm-gear shafts must be taken

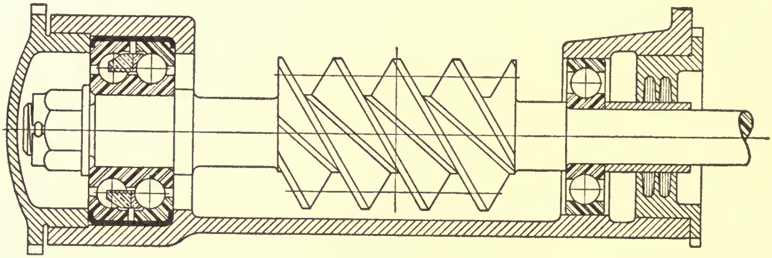


Fig. 194.

care of by suitable thrust bearings. Figs. 189, 190, and 194 to 196, inclusive, show several ways of taking care of the thrusts upon the shafts of a worm-gear transmission. In a drive in which the efficiency is low or of little consequence, the thrust along the worm shaft is taken up by one or more loose washers made of bronze or fiber. If more than one washer is necessary, then alternate washers of steel and bronze give satisfactory service. The shaft bearings of a drive of this kind are generally made of bronze, but a good grade of babbitt may also be used. On the worm-gear shaft bronze or babbitted bearings may be employed, depending upon the magnitudes of the loads coming upon the bearings.

In a drive in which the efficiency must be made as high as possible, ball or roller bearings must be used. In Figs. 194 and 195 are shown two examples of a motor-truck rear-axle worm mounting in which ball bearings are used. The end thrust upon the worm shaft, in the design illustrated by Fig. 194, is taken by the double-row ball bearing, and, at the same time, this bearing takes its share of the transverse loads upon the shaft. The double-row ball bearing is mounted rigidly as shown, while the single-row bearing has its outer race floating, thus making pro-

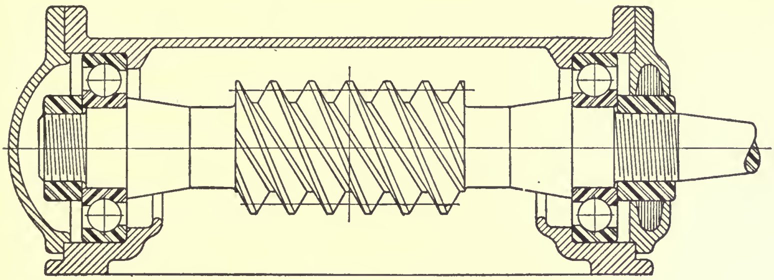


FIG. 195.

vision for expansion of the worm shaft. The design just described was originated by The New Departure Mfg. Co.

The worm-shaft mounting illustrated by Fig. 195 employs the type of radial ball bearing that is capable of taking a thrust, the magnitude of which is equal to or greater than the radial

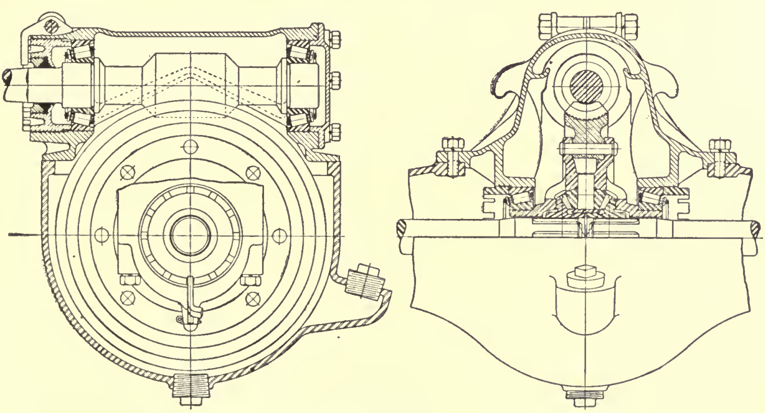


FIG. 196.

load coming upon them. Another feature worthy of attention is the fact that the worm shaft is always in tension, no matter in which direction the thrust of the worm gear acts.

In Fig. 196 is shown another good example of a rear-axle worm-gear transmission, in which Timken conical roller bearings are used throughout. An inspection of the figure shows that the worm shaft is always in compression, and with the rigid mounting of the roller bearings on this shaft, no provision is made for taking care of any expansion that may occur. A mounting similar to that shown in Fig. 195, but using conical roller bearings in place of the ball bearings, will prove satisfactory. Not infrequently, the worm shaft is mounted upon ordinary radial ball or roller bearings and the thrust is taken by a double-thrust ball bearing. A combination of radial and thrust bearings is efficient, but is more or less complicated and at the same time is more expensive than the mountings discussed above.

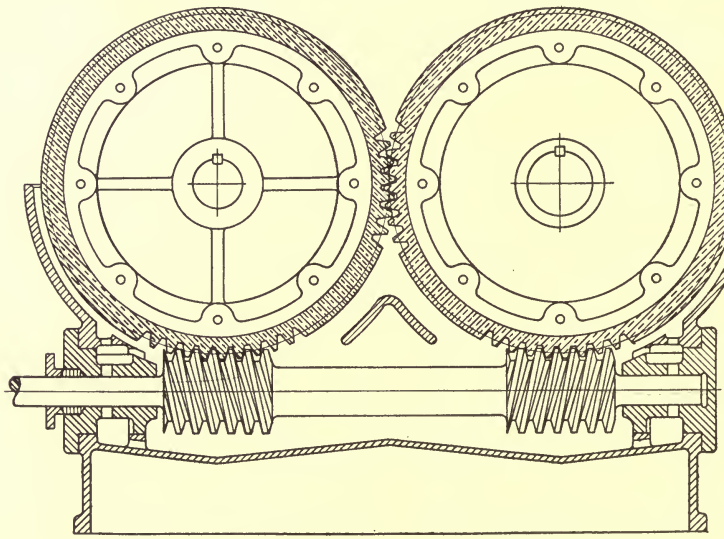


FIG. 197.

273. Tandem Worm Gears.—In heavy-duty elevators, the drum or traction sheave is driven by means of double worm gearing, the arrangement of which is shown in Fig. 197. Such a drive consists of right- and left-hand worms cut integral with the shaft and mounted below the bronze worm gears with which they mesh. The worm gears are, strictly speaking, helical gears and since they are cut right and left hand of the same pitch, they readily engage with each other. One of these worm gears is connected to the hoisting drum or sheave. It is evident that a combination of this description practically eliminates all end thrust on the worm shaft, thus simplifying the arrangement of the bearings on this shaft. The part of the shaft between the worms is subjected either to a tension or a compression, depending upon the loading on the hoisting drum.

274. Experimental Results on Worm Gearing.—A considerable number of tests of worm gearing have been made by various investigators in order to determine the probable efficiency of such gearing, also to determine the relation existing between the coefficient of friction and the sliding velocity of the teeth in contact. Evaluating equation (402) for a given coefficient of friction and various angles of lead, it will be found that the efficiency varies but little for angles between 30 and 60 degrees. The results obtained from the well-known experiments on worm gearing made by Wilfred Lewis agree very closely with those determined by means of (402).

The value of the coefficient of friction for any particular condition of speed and tooth pressure is somewhat difficult to determine. The experimental results obtained by Lewis, Stribeck, Bach, Roser, and other investigators seem to lead to the following conclusions: (1) The coefficient of friction appears to have its greatest value at low speeds, also at high speeds. (2) The coefficient of friction has its lowest values at medium speeds (200 to 600 feet per minute). (3) The coefficient of friction varies but little for different tooth pressures. In Table 84 are given some of the results obtained by Stribeck from a series of tests on a cast-iron worm and gear having the following dimensions: The gear was $9\frac{1}{2}$ inches in diameter and had 30 teeth. The outside diameter of the single-thread worm was approximately $3\frac{3}{4}$ inches, and the tangent of the helix angle was given as 0.1.

Table 84.—Results of Tests on Cast-iron Worm Gearing by Stribeck

Velocity, ft. per min.	Pressure, pounds	Coef. of friction at 60°C.
98	1,100	0.061
196	1,100	0.051
294	1,100	0.047
392	880	0.040
586	550	0.030
784	350	0.025

In the design of high-efficiency worm gearing as used in motor cars, one authority recommends that μ may be taken as low as 0.002; however, this value appears rather low for general use and it is believed that 0.01 will give safer results. For designing single-thread worms of the irreversible or self-locking type, the coefficient of friction may be assumed as 0.05.

The actual efficiencies of well-constructed and properly mounted worms and gears, as used on motor cars, are in general high, running above 95 per cent. in many cases.

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CHAPTER XV

COUPLINGS

A coupling is a form of fastening used for connecting adjoining lengths of shafting so that rotation may be transmitted from one section to the other. Couplings may be divided into the following general groups: (*a*) permanent couplings; (*b*) releasing couplings.

PERMANENT COUPLINGS

A permanent coupling is generally so constructed that it is necessary to partially or wholly dismantle it in order to separate the connected shafts. Hence, it is evident that permanent couplings are only used for joining shafts that do not require frequent disconnection. Permanent couplings may be grouped into the following classes:

(*a*) Couplings connecting shafts having axes that are parallel and coincident.

(*b*) Couplings connecting shafts having axes that are parallel but not coincident.

(*c*) Couplings connecting shafts having axes that intersect.

(*d*) Couplings connecting shafts having inaccurate alignments.

COUPLINGS FOR CONTINUOUS SHAFTS

Some of the requisites of a good coupling for connecting continuous shafts are as follows:

1. It must keep the shafts in perfect alignment.
2. It must be easy to assemble or dissemble.
3. It must be capable of transmitting the full power of the shafts.
4. The bolt heads and nuts, keys and other projecting parts should be protected by suitable flanges, rims, or cover plates.

275. Flange Coupling.—One of the most common as well as most effective type of permanent coupling for continuous shafts is the plain flange coupling shown in Fig. 198. In order to insure positive shaft alignment, one shaft should project through its flange into the bore of the companion flange. Another effective way of accomplishing the same purpose is to allow a part of the one flange to project into a recess in the other, as shown in Fig. 198. The coupling bolts must be fitted accurately, generally a driving fit, so that each one will transmit its share of the torsional moment on the shaft. The size of the bolts should be such that their combined shearing resistance will at least equal the torsional strength of the shaft. In certain installations requiring accurate alignment of the shafts, the flanges of the coupling are forced on the shaft and are then faced off in place.

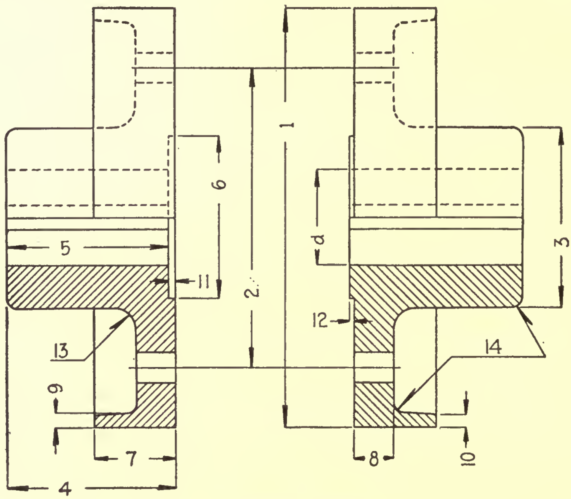

FIG. 198.

Analysis of a flange coupling.—A flange coupling may fail to transmit the full torsional moment of the shaft from the following causes: (1) The key may fail by shearing or by crushing. (2) The coupling bolts may fail by shearing or by crushing. (3) The flange may shear off at the hub.

1. *Failure of the key.*—To prevent the key from shearing, its moment of resistance about the axis of rotation must at least equal the torsional strength of the shaft. Using the notation given in Art. 93, the relation between the shearing strength of the key and torsional moment T according to (104) may be expressed as follows:

$$bl \geq \frac{2T}{dS_s} \tag{411}$$

To prevent crushing of the key, the moment of the crushing resistance of the key about the axis of rotation must exceed slightly the torsional moment T; whence, from (102)

$$lt \geq \frac{4\,T}{dS_b} \tag{412}$$

2. *Failure of the bolts.*—In a flange coupling located at a considerable distance from the bearings supporting the shaft, the bolts are generally subjected to bending stresses in addition to crushing and shearing stresses. It is evident, therefore, that couplings should be located near the bearings. In the following analysis it will be assumed that the coupling bolts are not subjected to a cross-bending, but only to shearing and crushing stresses. Equating the shearing resistance of the bolts to the load coming upon them, we obtain the relation

$$a \geq 2\sqrt{\frac{2\,T}{\pi neS_s}}, \tag{413}$$

in which a denotes the diameter of the bolts, n the number of bolts used in the coupling, and e the diameter of the bolt circle.

Instead of failing by a shearing action, the bolts as well as the flange may fail by crushing; whence we obtain the relation

$$af \geq \frac{2\,T}{neS_b}, \tag{414}$$

in which f denotes the thickness of that part of the flange through which the bolts pass.

3. *Shearing off of the flange.*—The coupling may fail due to the shearing of the flange where the latter joins the hub. To prevent this failure the moment of the shearing resistance of the flange must at least equal the torsional moment transmitted by the shaft. Hence, it follows that

$$c^2f \geq \frac{2\,T}{\pi S_s'}, \tag{415}$$

in which c denotes the diameter of the hub, and S_s' the allowable shearing stress in the material of the coupling.

4. *Proportions of flange couplings.*—In order that a flange coupling may transmit the full torsional strength of the shaft to which it is connected, the various relations derived above must be satisfied. The analysis of the stresses just referred to is only made in special or unusual cases. For the common flange coupling used on

line- and counter-shafts, it is unnecessary to make an investigation of the stresses in the various parts, as the proportions of such couplings have been fairly well established by several manufacturers. However, no uniform proportions of flange couplings have as yet been proposed for adoption as a standard. In Table 85 are given the proportions of a series of flange couplings recommended by the Westinghouse Electric and Mfg. Co., and these represent good average practice. The dimensions listed in Table 85 refer to the flange coupling shown in Fig. 198.

276. Marine Type of Flange Coupling.—The type of flange coupling shown in Fig. 199 is used chiefly in marine work where great strength and reliability are of the utmost importance. The fitting of this form of coupling is done with considerable care; for example, the bolt holes are always reamed after the flanges are placed together, thus insuring perfectly fitted bolts, each of which will transmit its full share of the torsional moment upon the shaft. The method of analyzing the stresses and arriving at the dimensions of the various parts of a marine flange coupling is similar to that given for the common flange coupling.

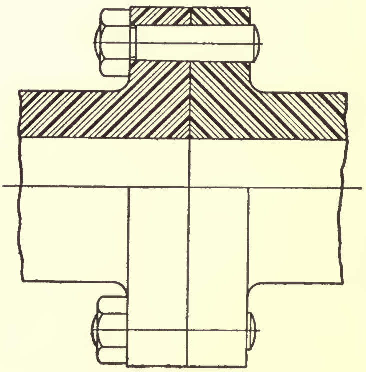

FIG. 199.

277. Compression Coupling.—(*a*) *Clamp coupling.*—A form of coupling used extensively at present on shafts of moderate diameter, say up to approximately 5 inches, is shown in Fig. 200. It is commonly called a *compression* or *clamp coupling.* The two halves of the clamp coupling are planed off, and after the bolt holes are drilled, the halves are bolted together with strips of paper between them and bored out to the desired size. After the boring operation, the strips of paper are removed. When the coupling is fastened to the shaft, the small opening between the two halves, due to the removal of the paper, permits the drawing up of the bolts, and a clamping action on the shaft is thus produced. The square key used in connection with a clamp coupling is generally made straight and is fitted only at the sides. This coupling may be put on and removed very easily, and it has no

TABLE 85.—PROPORTIONS OF WESTINGHOUSE FLANGE COUPLINGS

d	Dimensions														Bolts		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Diam.	Length	No.
1	5 5/8	3 1/8	2 1/8	2 1/4	2 5/32	2	1 1/8	9/16	1/4	3/16	3/32	1/16			3/8	1 1/2	3
1 1/4	6 1/2	4 1/2	2 1/2	2 1/2	2 13/32	2 5/16	1 3/8	5/8	1/4	3/16	3/32	1/16	3/8	3/16	1/2	1 3/4	3
1 1/2	7 1/2	5 3/16	3	2 7/8	2 3/4	2 5/8	1 9/16	11/16	9/32	7/32	1/8	3/32			5/8	2	3
2	8 3/4	6 1/4	3 3/4	3 1/2	3 3/8	3 3/8	1 11/16	13/16	5/16	1/4	1/8	3/32	7/16	7/32	5/8	2 1/4	4
3	11 1/4	8 1/4	5 3/8	4 3/4	4 19/32	4 3/4	2 1/8	1 1/16	13/32	5/16	5/32	1/8	1/2	1/4	3/4	2 7/8	4
4	13 1/2	10 1/8	7	6	5 13/16	6	2 1/2	1 5/16	15/32	3/8	3/16	5/32			7/8	3 1/2	4
5	16	12 1/8	8 5/8	7 1/4	7	7 1/2	2 15/16	1 1/2	17/32	7/16	1/4	3/16	9/16	9/32	1	4	6
6	19	14 3/8	10 1/4	8 1/2	8 1/4	8 3/4	3 3/8	1 3/4	5/8	1/2	1/4	3/16	5/8	5/16	1 1/4	4 3/4	6
7	21 1/4	16 1/4	12	9 3/4	9 7/16	10 1/2	3 5/8	2	3/4	5/8	5/16	1/4	11/16	11/32	1 1/4	5 1/4	6
8	24	18 3/8	13 1/2	11	10 5/8	11 1/2	4 3/16	2 1/4	7/8	11/16	3/8	5/16	3/4	3/8	1 1/2	6	8
10	28	21 7/8	16 3/4	13 1/2	13 1/16	14 1/4	4 11/16	2 3/4	1 1/16	7/8	7/16	5/16	7/8	7/16	1 1/2	7	8
12	33 1/4	26	20	16	15 1/2	17	5 9/16	3 1/2	1 1/4	1	1/2	3/8	1	1/2	1 3/4	8 1/4	8

projecting parts that are liable to injure workmen. In Table 86 are given the general dimensions of a series of sizes of the clamp coupling illustrated in Fig. 200.

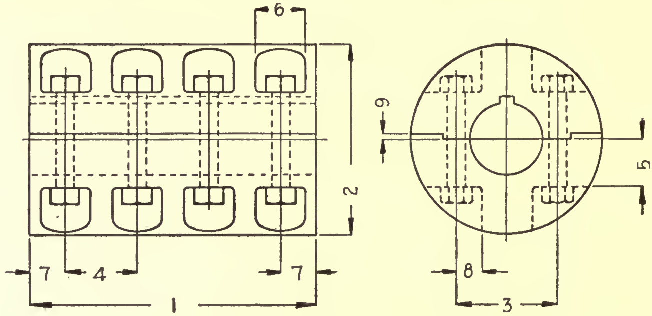

Fig. 200.

Table 86.—Dimensions of Clamp Shaft Couplings

Shaft diameter	Dimensions									Diam. of bolts	Key
	1	2	3	4	5	6	7	8	9		
1 7/16	6	4 1/4	2 1/2		1	1 7/8	1 1/2	1 5/16	1/8	5/8	5/16
1 11/16	7	4 7/8	3		1 3/16		1 3/4		3/16		7/16
1 15/16	8	5 1/2	3 1/4		1 3/8	2 1/8	2	1 1/16		3/4	
2 3/16	9	6 1/8	3 1/2		1 1/2		2 1/4				
2 7/16	10	6 3/4	3 7/8		1 5/8	2 3/8	2 1/2	1 3/16		7/8	9/16
2 11/16	11	7 3/8	4	2 3/4	1 7/8		1 3/8				
2 15/16	12	8	4 1/4	3	2	2 1/8	1 1/2	1 1/16	1/4	3/4	
3 3/16	13	8 5/8	4 1/2	3 1/4	2 1/8		1 5/8				1 1/16
3 7/16	14	9	4 3/4	3 1/2	2 1/4	2 3/8	1 3/4	1 3/16		7/8	
3 11/16	15	9 3/4	5	3 3/4	2 1/2		1 7/8				
3 15/16	16	10 1/4	5 1/2	4	2 3/4	2 5/8	2	1 5/16		1	
4 7/16	18	11 1/2	6	4 1/2	3		2 1/4				1 3/16
4 15/16	20	12 5/8	6 3/4	5	3 3/8	3 1/8	2 1/2	1 9/16		1 1/4	

(*b*) *Nicholson compression coupling.*—Another form of the so-called compression coupling is shown in Fig. 201. This coupling requires no cutting of keyways in the shafts that are to be connected together. It consists of two flanged hubs having tapered bores which do not run clear through the hub, but terminate a short distance from the outer end as shown in the figure. Double-tapering steel jaws are fitted into the tapered bore and held in proper position by the key-seats or slots cut into the end of the hub. These jaws are machined on the inner faces to a radius a trifle less than the radius of the shaft, thus forming a positive grip on the shaft when the two flanges are drawn together by the bolts. The adjustment of the coupling is always concentric and parallel. No keys are required, thus saving the cost

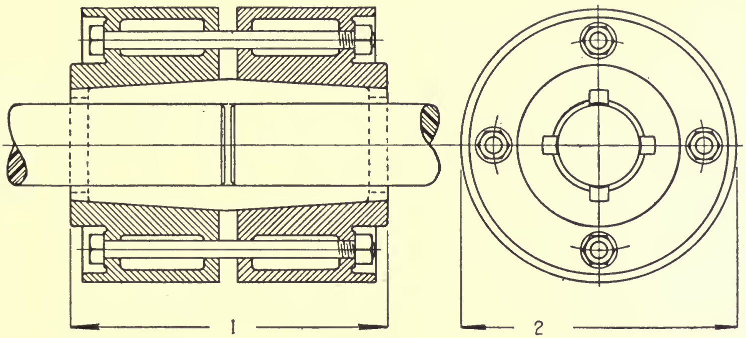

Fig. 201.

of cutting the key-seat in the shaft and of fitting the key. The coupling illustrated in Fig. 201 is manufactured by W. H. Nicholson and Co. of Wilkes-Barre, Pa.

278. Roller Coupling.—In Fig. 202 is shown a form of shaft coupling in which steel rollers are used for gripping the shaft. As

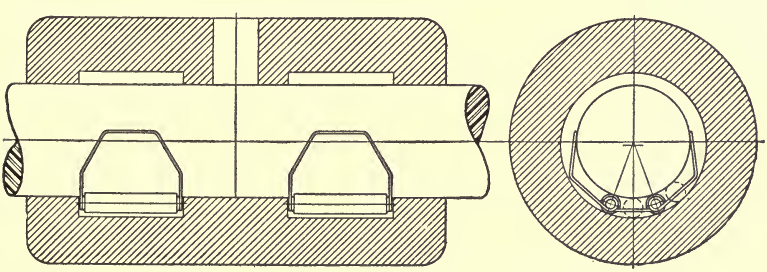

Fig. 202.

shown in the figure, the coupling consists of a cylindrical sleeve with two eccentric chambers on the inside. Each of these chambers contains two steel rollers, held parallel to each other by a light wire frame. With the rollers located in the largest part of the eccentric chambers, the coupling may easily be slipped over the end of the shaft. A slight turn of the coupling in either direction forces the rollers up the inclined sides of the eccentric chamber thereby locking the coupling to the shaft. Since no screws, bolts, pins, or keys are used with this coupling, no tools are needed in applying it to a shaft. Due to the smooth exterior, the roller coupling shown in Fig. 202 insures freedom from accident to workmen.

COUPLINGS FOR PARALLEL SHAFTS

279. Oldham's Coupling.—When two shafts that are parallel, but whose axes are not coincident, are to be used for transmitting power, a form of connection known as *Oldham's coupling* is used.

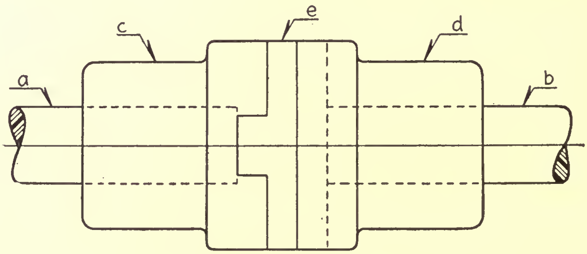

FIG. 203.

The constructive features of such a coupling are shown in Fig. 203. It consists of two flanged hubs *c* and *d* fastened rigidly to the shafts *a* and *b*. Between these flanges is a disc *e*, which engages each flanged hub by means of a tongue and groove joint, thus forming a sliding pair between them. With this form of coupling, the angular velocity of the shafts *a* and *b* remains the same.

Parallel shafts may also be connected by two universal joints in place of an Oldham's coupling.

COUPLINGS FOR INTERSECTING SHAFTS

280. Universal Joint.—For shafts whose axes intersect, a form of connection known as *Hooke's coupling* is frequently used. A more familiar name for this coupling is *universal joint*. In Figs. 204 to 207, inclusive, are shown four types of universal joints. The type of joint illustrated by Fig. 204 consists of two U-shaped yokes which are fastened to the ends of the shafts that are to be connected together. Between these yokes is located a cross-shaped piece, carrying four trunnions which are fitted into the bearings on the U-shaped yokes. The joint shown in Fig. 204 is manufactured by the Baush Machine Tool Co., and is well

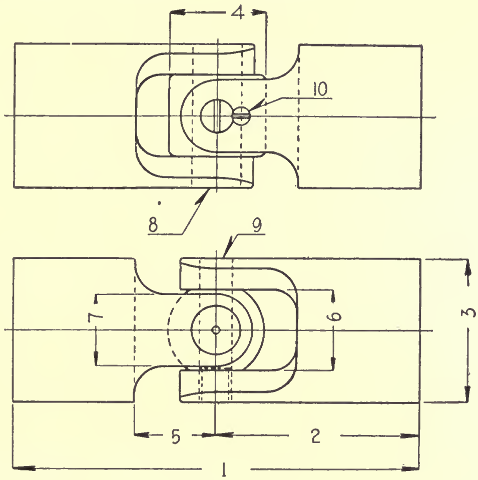

FIG. 204.

TABLE 87.—PROPORTIONS OF BOCORSELSKI'S UNIVERSAL JOINT

Size	Dimensions							Diameters		
	1	2	3	4	5	6	7	8	9	10
1/4	1 1/4	5/8	1/4	3/16	5/32	9/64	1/8	0.076	0.0465	
3/8	1 3/4	7/8	3/8	17/64	7/32	7/32	3/16	0.1065	0.0595	
1/2	2	1	1/2	11/32	9/32	9/32	1/4	0.167	0.096	
5/8	2 1/4	1 1/8	5/8	13/32	11/32	11/32	5/16	7/32	5/32	
3/4	2 11/16	1 11/32	3/4	1/2	13/32	7/16	3/8	1/4	11/64	
7/8	3 1/4	1 5/8	7/8	19/32	15/32	1/2	7/16	9/32	3/16	
1	3 3/8	1 11/16	1	11/16	9/16	9/16	1/2	5/16	7/32	1/8
1 1/4	3 3/4	1 7/8	1 1/4	7/8	11/16	11/16	5/8	3/8	1/4	1/8
1 1/2	4 1/4	2 1/8	1 1/2	1	27/32	27/32	3/4	1/2	11/32	1/8
1 3/4	4 1/2	2 1/4	1 3/4	1 3/16	15/16	31/32	7/8	9/16	3/8	1/8
2	5 7/16	2 23/32	2	1 7/16	1 3/32	1 3/16	1 1/16	11/16	7/16	1/8
2 1/2	7	3 1/2	2 1/2	1 3/4	1 13/32	1 7/16	1 5/16	3/4	1/2	1/8
3	9	4 1/2	3	2 1/8	1 21/32	1 5/8	1 7/16	7/8	5/8	1/4
4	10 5/8	5 5/16	4	2 3/4	2 3/16	2 1/16	1 7/8	1 1/16	3/4	1/4

adapted for machine-tool service as found on multiple drills. In Table 87 are given general dimensions of the Bocorselski's patent universal joint shown in Fig. 204.

The coupling shown in Fig. 205 is intended for heavy service, as the two yokes and the center cross are made of hard bronze, while the screws are made of nickel steel. The maximum angular displacement of this joint is limited to 25 degrees.

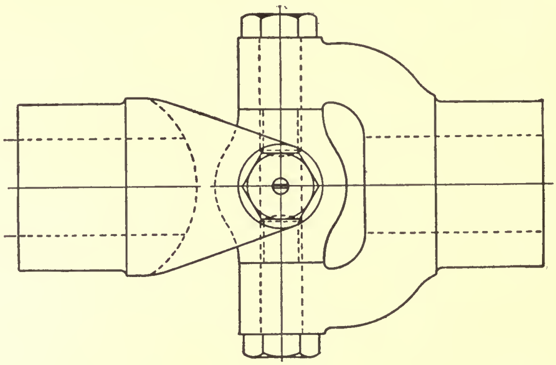

FIG. 205.

The universal joint in one form or other is used extensively in motor-car construction. In Fig. 206 is shown a joint designed by the Merchant and Evans Co. of Philadelphia, Pa. The coupling

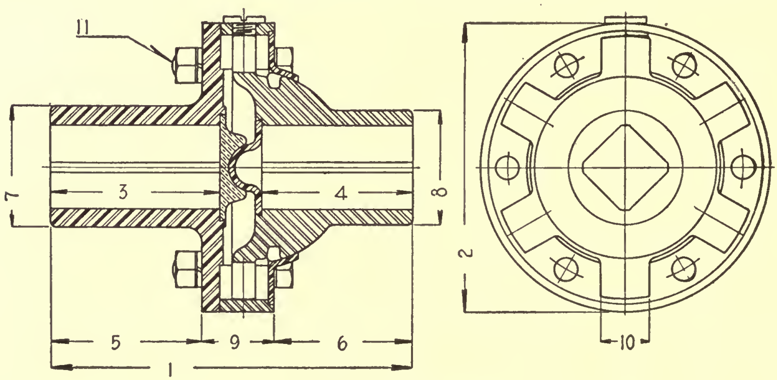

FIG. 206.

TABLE 88.—DIMENSIONS OF MERCHANT AND EVANS UNIVERSAL JOINTS

Horse power rating	Size of shaft	Dimensions										
		1	2	3	4	5	6	7	8	9	10	11
35	1½	5⅞	4⅛	3	2⅜	2¾	1¾	2⅛	2	1⅜	¾	⅜
35–80	1¾	7½	6	3½	3⅛	3⅛	2⅞	2½	2⅜	1½	1	½

consists of a flanged hub to which is attached a ring having radial slots. The flanged hub is made of machine steel and the slotted ring of a high-carbon steel. Into the radial slots of the ring are fitted the projecting arms or teeth of the spider which is also made from a high-carbon steel. On the enlargement of the hub of the spider is formed a spherical surface which fits accurately into a housing, the latter being fastened by bolts to the slotted ring and the flanged hub. Spherical centering caps are fitted to the inside faces of the flanged hub and spider. All of the spherical surfaces have the same center, which, for the design shown, is located on the common center line of the two shafts. The maximum movement out of true alignment that is permissible with the style of coupling shown in Fig. 206 is plus or minus 4 degrees. Table 88 gives general dimensions of two sizes of this coupling, the smaller of which is capable of transmitting 35 horse power and the larger, 80 horse power.

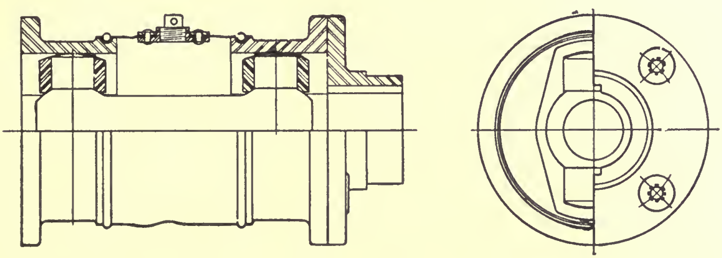

Fig. 207.

In Fig. 207 is shown another design of universal coupling frequently found on motor cars. The constructive details are shown more or less clearly in the figure and hence no further description is necessary.

COUPLINGS FOR SHAFTS HAVING INACCURATE ALIGNMENTS

Frequently it is necessary to connect shafts in which slight deviations in alignment must be taken care of, as for example in connecting a prime mover to a generator, or an electric motor to a centrifugal pump, blower, or generator. For a satisfactory connection, *flexible couplings* are used. Several forms of flexible couplings are now used by various manufacturers, and the following are selected as typical illustrations of the different types.

281. Leather-link Coupling.—In Fig. 208 is shown a leather-link flexible coupling manufactured by The Bruce Macbeth Engine Co. of Cleveland, Ohio. It consists of two flanged hubs connected together by leather links as shown in the figure. The links are held securely by bolts, which in turn are fastened to the flanges so that one end of the links is anchored to the one flange while the other end is anchored to the other flange. The torque of one shaft is transmitted to the other through the combination of flanges, links, and bolts. In order to obtain the desired flexibility, alternate holes in the flanges are made larger so as to permit sufficient play for the enlarged washers used on the bolts.

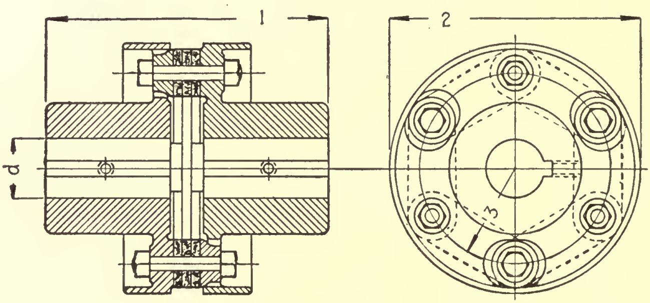

FIG. 208.

TABLE 89.—DATA PERTAINING TO LEATHER LINK COUPLINGS

Bore d	Max. h.p. at 100 r.p.m.	Maximum r.p.m.	Weight, lb.	Dimensions		
				1	2	3
¾	1.5	2,400	15	5	5⅝	2
1³⁄₁₆	2	2,000	25	6	7	2½
1¹¹⁄₁₆	6	1,800	65	8	10½	4
1¹⁵⁄₁₆	10	1,600	110	10	16	6
2³⁄₁₆	15	1,500	210	13	20	8
2¹⁵⁄₁₆	30	1,250	335	15	24	10
3⁷⁄₁₆	50	1,000	560	18	29	12
4⁷⁄₁₆	100	850	1,270	26	34	14
5½	200	750	1,790	30	40	16

The leather used for the links is made from selected hides and is treated by a special tanning process so as to increase the strength and flexibility. According to one prominent manufacturer of leather-link couplings the working stress for the links may be taken as 400 pounds per square inch. Due to the low first cost of leather-link couplings, the General Electric Co. recommends their use on all shafts up to and including 2 inches in diameter. For shafts from 2 to 3½ inches in diameter, either the link type or the leather-laced type may be used. In Table 89 are given general dimensions and other data pertaining to the coupling shown in Fig. 208.

282. Leather-laced Coupling.—The leather-laced flexible coupling shown in Fig. 209 consists of two cast-iron flanges upon which are bolted steel rings. An endless leather belt is laced

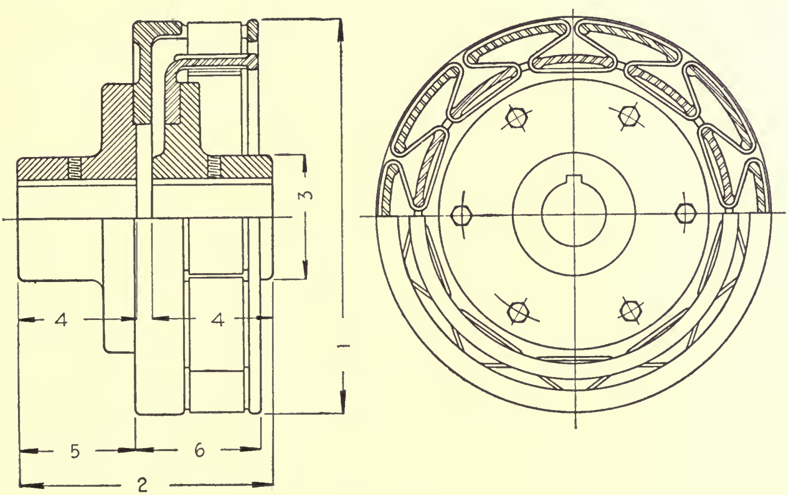

Fig. 209.

through a series of slots that are formed in the rim of these steel rings. The construction used offers a ready means of disconnecting the machines without unlacing the belt. As may be seen in Fig. 209, disconnection is accomplished by simply removing the cap screws that fasten the outer steel ring to the central flange. According to the General Electric Co., the manufacturers, this coupling is recommended when the shafts to be connected are more than 3½ inches in diameter. The belting used is made from a specially prepared leather capable of carrying a working stress of 400 pounds per square inch of section. In Table 90 are given general dimensions and other data pertaining to the laced-belt coupling shown in Fig. 209.

TABLE 90.—DATA PERTAINING TO LEATHER LACED COUPLINGS

Bore *d*	Max. h.p. at 100 r.p.m.	Max. r.p.m.	Weight, lb.	Dimensions						Key
				1	2	3	4	5	6	
2½	16	1,200	160	15½	10	5	4 13/16	4 11/16	4¾	½×½
3 3½	27.7	900	263 256	18½	12	6	5 13/16	5 11/16	5 1/16	¾×¾
4 4½	66	750	494 482	24½	14	8	6 13/16	6 11/16	6¼	1×1
5 5½	128	600	883 868	30½	16	10	7 13/16	7 11/16	7¼	1¼×1¼
6 6½	222	450	1,329 1,307	37	18	12	8 13/16	8 11/16	7⅝	1½×1½
7 7½	352	350	2,076 2,046	43	20	14	9 13/16	9 11/16	8⅜	
8 8½	526	300	2,767 2,727	49	24	16	11 13/16	11 11/16	9⅛	1½×1¾
9 9½	748	250	3,917 3,865	55	28	18	13 13/16	13 11/16	9 11/16	1¾×2
10	1,027	200	5,120	61	32	20	15 13/16	15 11/16	10 5/16	

In general, flexible couplings using leather as the connecting medium are not recommended for places where dampness or oil would affect the leather. Neither should they be used when flying dust or grit are liable to injure the leather links or lacing. It is generally assumed that the leather connectors afford sufficient insulation between the two halves of the coupling when the latter is used in connection with electric motors or generators.

283. Francke Coupling.—The type of flexible couplings discussed in the two preceding articles transmit power from the driving to the driven member by means of a fibrous material. Couplings having soft rubber buffers between interlocking arms of two cast-iron spiders have also been used successfully. Recently a form of coupling known as the Francke flexible coupling in which a pair of flanges are connected by flexible steel pins was placed on the market. The constructive details of this coupling are shown clearly in Fig. 210. The so-called pins are built up of a series of tempered-steel plates having a slotted hole at each end through which a hardened-steel pin passes. By means of these pins, the ends of the tempered plates are held in steel yokes which are fastened to the rims of the flanges by means of cap screws, as shown in Fig. 210. In the smaller sizes of the Francke coupling, the ends of the steel yokes and the inner surfaces of the coupling flanges have grooves into which steel rings are sprung, thus holding the tempered plates in a radial position.

Any flange coupling connecting two shafts that are out of alignment will run open on the one side and closed on the other.

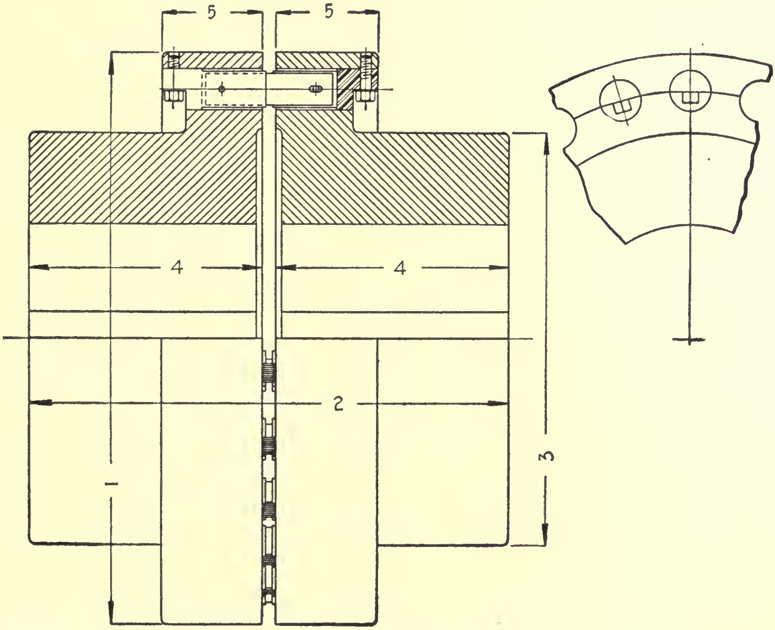

FIG. 210.

The endwise motion due to this opening and closing action of the flanges is provided for, in the Francke coupling, by the slotted holes near the ends of the tempered-steel plates.

In Table 91 are given general dimensions, net weights, permissible speeds, and approximate horse powers pertaining to the commercial sizes of the coupling shown in Fig. 210. The following directions for selecting the proper size of coupling for any desired service are recommended by the manufacturers of the Francke coupling.

(*a*) From Table 91, select the smallest coupling having a maximum bore large enough to receive the largest shaft to be connected.

(*b*) For the installation under consideration, determine the horse power transmitted per 100 revolutions per minute.

TABLE 91.—DATA PERTAINING TO THE FRANCKE COUPLING—HEAVY PATTERN

Size No.	Max. bore	Max. h.p. at 100 r.p.m.	Max. r.p.m.		Weight	Dimensions					Key
			Cast iron	Steel		1	2	3	4	5	
3 1/2	7/8	1.33			8.5	3 1/2	4 7/8	1 11/16	2 3/16	1 5/32	3/16 × 3/16
4	1 1/4	2	4,000	10,000	11	4	5 1/8	2 3/16	2 5/16		1/4 × 1/4
4 1/2	1 3/8	2.75			14	4 1/2	5 1/4	2 1/2	2 3/8		
5	1 5/8	3.75	3,500	8,500	20	5	5 3/8	3	2 7/16	1 7/16	5/16 × 5/16
6	2	6.5	3,100	7,600	35	6	5 5/8	3 3/4	2 9/16		
7	2 1/2	9	2,500	6,400	45	7	6 1/8	4 3/4	2 13/16		3/8 × 3/8
8 1/2	3	28	2,150	5,400	70	8 1/2	7 5/8	5 1/2	3 9/16	1 3/4	1/2 × 1/2
10	3 1/2	65	1,800	4,600	115	10	8 3/8	6 1/2	3 15/16		5/8 × 5/8
12	4 1/2	91	1,500	3,800	210	12	9 3/8	8 1/4	4 7/16		3/4 × 3/4
15	6	145	1,200	3,000	385	15	11 1/8	11	5 5/16	2 3/8	7/8 × 7/8
18	7 3/4	210	1,000	2,500	555	18	13 7/8	13 3/4	6 11/16		1 × 7/8
22	10	300	800	2,000	1,000	22	16 7/8	17 3/4	8 3/16		1 1/8 × 1
24	9	750	750	1,900	1,250	24	18 1/2	16 1/2	9	4 3/4	
27	11	1,000	700	1,700	1,650	27	22 1/2	19 3/4	11		
33	14	2,500	575	1,400	3,330	33	26 1/2	24	13	5 3/4	

TABLE 92.—FACTORS FOR VARIOUS CLASSES OF SERVICE

Class of service	Factor
Steam turbines connected to centrifugal pumps and blowers	1.25
Turbines and motors connected to generators	1.33
Motors connected to centrifugal pumps and blowers	1.5
Motors connected to wood-working machinery	1.67
Motors connected to grinders, conveyors, screens, and beaters with no pulsations	2
Motors connected to crushers, tubemills, and veneer hogs	3 to 4
Gas and steam engines connected to machines carrying a uniform load	3 to 5
Engines connected to fans	6 to 8
Motors connected to single-cylinder compressors	6
Rolling mills	4
Motors connected to mine hoists, elevators or cranes	4 to 8

(*c*) From Table 92, select the factor for the class of service for which the coupling is intended and multiply it by the horse power transmitted per 100 revolutions per minute.

(*d*) Compare the horse power determined in (*c*) with the horse power rating of the coupling selected in (*a*) above. In case the latter is less than the former, select a larger coupling having the desired rating.

(*e*) If the required speed is in excess of that listed for the cast-iron coupling, use a steel coupling.

284. Nuttall Coupling.—The Nuttall coupling illustrated in Fig. 211 differs considerably from those discussed in the preceding articles, in that the power is transmitted through the medium

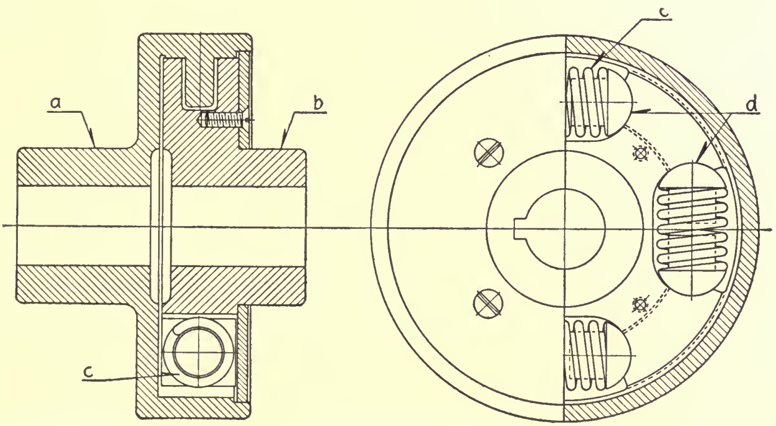

Fig. 211.

of helical springs *c*. These springs with the inserted case-hardened plugs *d* are fitted into pockets between the twin-arms of the spider *b*. The casing *a* is provided with a series of lugs that fit loosely in the twin-arms of the spider and also bear against the spring plugs *d*. It is evident that with the construction shown in the figure this coupling can transmit power in either direction, and, furthermore, that the springs are always in compression. The clearance between the ends of the spring plugs is made slightly less than the maximum deflection of the spring; therefore, a sudden overload cannot break the springs. The coupling has a smooth exterior, hence there is not much danger of injury to workmen.

285. Clark Coupling.—An interesting form of flexible coupling that was placed upon the market recently is the Clark coupling shown in Fig. 212. It consists of two hubs upon the flanges of which are cut a number of special teeth. Over these teeth is fitted a roller chain as shown in the figure. The teeth are cut

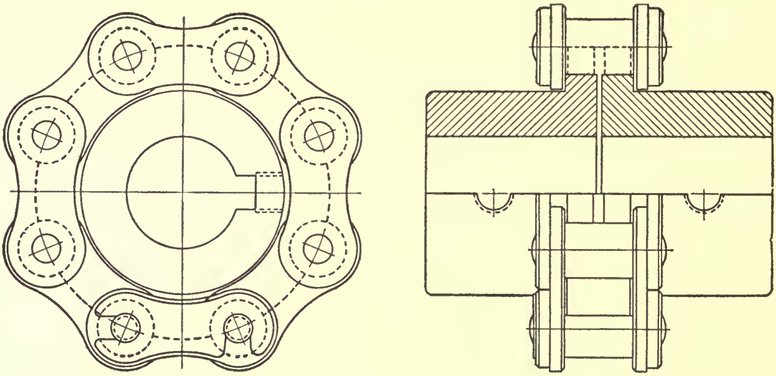

FIG. 212.

accurately so that all of the rollers in the chain are in contact with the teeth, thus insuring an equal distribution of the load transmitted by the coupling. Side clearance is provided between the chain and the teeth, thus permitting the two halves of the coupling to take care of any slight angular displacement of the

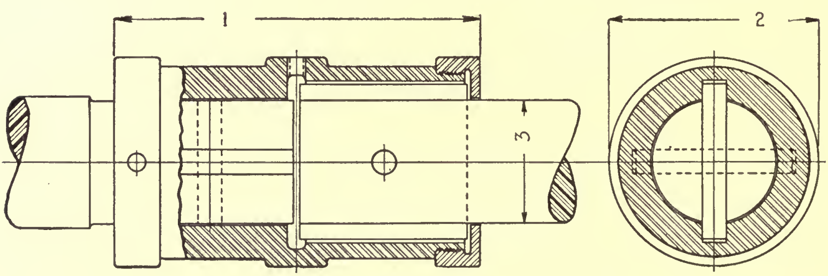

FIG. 213.

shafts. The chain is provided with a master link which may be removed quickly in case it is desired to run each shaft independently.

286. Kerr Coupling.—A type of flexible coupling particularly well adapted to very high rotative speeds is that shown in Fig. 213. It was developed by Mr. C. V. Kerr for use in connecting steam turbines to centrifugal pumps and blowers. In order to make it possible to use this coupling at high speeds, the dimensions are all kept down to a minimum by making the various parts of crucible cast steel. The through keys or cotters are made of tool steel and tempered. Due to the arrangement of the through keys at right angles to each other, the two shafts to be connected may be out of alignment to a considerable extent. To prevent serious wear of the various parts and to eliminate excessive noise, the coupling is filled with a heavy machine oil, or grease and graphite. To design a coupling of this kind the following method of procedure is suggested:

(*a*) Design the shaft so that it will readily transmit the required horse power at the specified speed.

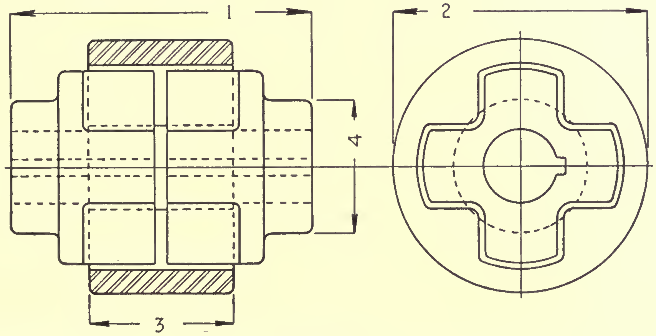

Fig. 214.

(*b*) Design the cross-key so that it will be amply strong against failure due to crushing, shearing, and bending.

(*c*) Design the shell so that it will transmit the torsional moment of the shaft. The key-ways in the shell should be investigated for crushing.

287. Rolling-mill Coupling.—Frequently, flexible couplings are required in places where considerable grit, water, steam, etc., are present, and where noise is not objectionable; for example, in a rolling mill. For such and other heavy service, the rolling-mill type of flexible coupling shown in Fig. 214 is recommended. When the load transmitted is practically constant, a rolling-mill coupling will not be excessively noisy and good results may be expected.

RELEASING COUPLINGS

A releasing coupling, or clutch, as it is commonly called, is so constructed that the connected shafts may be disengaged at will. From this statement it should not be inferred that clutches are used for connecting shafts exclusively, as they are also used for engaging pulleys, gears and other rotating parts. Clutches may be divided into two classes namely: (*a*) Positive clutches; (*b*) friction clutches. The latter class will not be discussed in this chapter, but will be taken up in detail in the following chapter.

288. Positive Clutch.—The simplest form of positive clutch is the jaw clutch shown in Fig. 215(*a*). One part of the clutch is keyed or pinned rigidly to the shaft while the other part is splined, thus permitting it to be engaged with, or disengaged from, the first part by sliding it along the shaft. The interlocking jaws upon the abutting faces of the clutch may have various

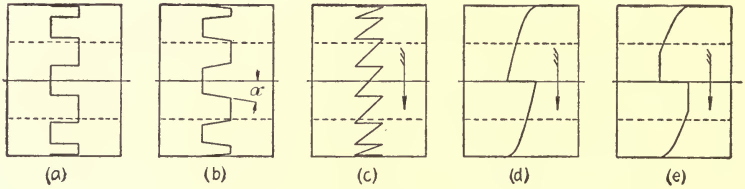

FIG. 215.

forms, as shown in Fig. 215. The jaws of the type shown in (*b*) engage and disengage more freely than square jaws. The jaws illustrated by (*c*), (*d*), and (*e*) are intended for installations where it is necessary to transmit power in only one direction. In punching and shearing machines the types of jaws shown by (*a*) and (*e*) are used considerably.

289. Analysis of Jaw Clutches.—Having decided upon the type of clutch to be used for a particular installation, the next step calls for the determination of the dimensions of the several parts. In general, jaw clutches are designed by empirical rules, and consequently the resultant proportions are liberal. However, if it is desired to arrive at the proportions of a jaw clutch capable of transmitting a certain amount of power, the following analysis is suggested:

(*a*) *Bore of the sleeves.*—The bore of the sleeves is fixed by the size of the shaft required to transmit the required power.

(*b*) *Length of the sleeves.*—If keys are used for fastening the sleeves to the shafts, the lengths of the sleeves are fixed, in a general way, by the length of keys required to transmit the desired power. In connection with punching and shearing machinery, where the clutch sleeve is occasionally fitted onto a squared shaft, the length of the sleeve may be assumed approximately equal to the diameter of the shaft.

(*c*) *Outside diameter of sleeves.*—The outside diameter of the sleeves must be such that the safe shearing strength of the jaws will exceed the pressure coming upon them. The pressure upon the jaws should be calculated on the assumption that it is concentrated at the mean radius of the jaws.

Let A = area of the jaw at the root.
D = outside diameter of the clutch sleeve.
S_s = permissible shearing stress of the material.
T = torsional moment to be transmitted by the clutch.
d = bore of the clutch sleeve.
n = number of jaws on the clutch sleeve.

Equating the torsional moment T, to the moment of the shearing resistance of the jaws, and solving for the total required shearing area, we obtain the following expression:

$$nA = \frac{4T}{(D + d)S_s} \tag{416}$$

Without introducing any appreciable error, the area nA may be taken as equivalent to one-half the area between the circles having diameters equal to the outer and inner diameters of the clutch sleeve. Substituting for nA an expression for the equivalent area in terms of D and d, we arrive at the following relation:

$$(D^2 - d^2)(D + d) = \frac{32\,T}{\pi S_s} \tag{417}$$

In determining the outer diameter D by the use of (417), considerable time may be saved by solving this equation by trial.

(*d*) *Number and height of jaws.*—The number of jaws on clutches depends upon the promptness with which a clutch must act. In punching and shearing machinery, the number of jaws varies from two to four, while in other classes of machinery the number of jaws may run as high as twenty-four.

The height of the jaws must be such that the pressure coming upon them does not exceed the safe crushing strength of the material used in the clutch. The distribution of the pressure upon the face of the jaws depends upon the grade of workmanship put upon the clutch parts. On clutches found on the modern machine tools, we may safely say that the workmanship is of such a quality that the pressure upon the jaws may be assumed as uniformly distributed.

Denoting the area of the engaging face of one jaw by the symbol A_c, and the permissible crushing stress of the material by S_c, we obtain the following relation by equating the torsional moment T to the moment of the resistance to crushing:

$$A_c = \frac{4\,T}{n(D + d)S_c} \tag{418}$$

Having determined the area required to prevent crushing of the jaw, the height h of the latter is given by the following expression:

$$h = \frac{2\,A_c}{D - d} \tag{419}$$

Frequently, the height of the jaw as determined by (419) is so small that it must be increased in order that the mating jaws will hook together sufficiently and not be disengaged by any jarring action. Good judgment should play an important part in arriving at the various dimensions of the parts of a jaw clutch.

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CHAPTER XVI

FRICTION CLUTCHES

290. Requirements of a Friction Clutch.—The object of a friction clutch is to connect a rotating member to one that is stationary, to bring it up to speed, and to transmit the required power with a minimum amount of slippage. In connection with machine tools, a friction clutch introduces what might be termed a safety device in that it will slip when the pressure on the cutting tool becomes excessive, thus preventing the breakage of gears or other parts.

In designing a friction clutch, the following points must be given careful consideration:

(*a*) The materials forming the contact surfaces must be selected with care.

(*b*) Sufficient gripping power must be provided so that the load may be transmitted quickly.

(*c*) In order to keep the inertia as low as possible, a clutch should not be made too heavy. This is very important in high-speed service, such as is found in motor cars.

(*d*) Provision for taking up wear should be made.

(*e*) Provision should be made for carrying away the heat that is generated at the contact surfaces.

(*f*) A clutch should be simple in design and contain as few parts as possible.

(*g*) The construction should be such as to facilitate repair.

(*h*) The motion should be transmitted without shock.

(*i*) A clutch should disengage quickly and not "drag."

(*j*) A clutch transmitting power should be so arranged that no external force is necessary to hold the contact surfaces together.

(*k*) A clutch intended for high-speed service must be balanced carefully.

(*l*) A clutch should have as few projecting parts as possible, and such parts as do project should be covered or guarded so that workmen cannot come into contact with them.

291. Materials for Contact Surfaces.—In order that a material may give satisfactory service as a frictional surface, it must fulfill the following conditions: (1) The material must have a high coefficient of friction. (2) The material must be capable of resisting wear. (3) The material must be capable of resisting high temperatures, caused by excessive slippage due to frequent operation of the clutch.

Among the materials met with in modern clutches are the following:

(*a*) *Wood.*—In many clutches used on hoisting machinery, as well as in some used for general transmission purposes on line- and counter-shafts, the contact surfaces are made of wood and cast iron. Among the kinds of wood that have proven satisfactory in actual service are basswood, maple, and elm.

(*b*) *Leather.*—The majority of the cone clutches in use on motor cars are faced with leather. Some manufacturers use oak-tanned, while others prefer the so-called chrome leather. To obtain the best service from a leather facing, it should be treated by soaking it in castor oil or neat's-foot oil, or boiling it in tallow. Before applying the facing to the clutch, the treated leather should be passed between rolls so as to remove the excess oil or grease. Leather facings should never be allowed to become dry or hard, or the clutch will engage too quickly. Leather that has become charred due to excessive slippage has very little value as a friction material.

(*c*) *Asbestos fabric.*—At the present time there are upon the market several patented asbestos fabrics consisting mainly of asbestos fiber. To give it the necessary tenacity the asbestos fiber is woven onto brass or copper wires. Among the well-known asbestos fabrics used for clutches, as well as for brakes, are Raybestos, Thermoid, and Non-Burn. The first two may be obtained in thicknesses varying from 1/8 to 1/4 inch, inclusive, and in widths of 1 to 4 inches, inclusive. Non-Burn is made in thicknesses up to and including 1 inch, and in widths up to and including 24 inches. Asbestos fabric facings are used to a limited extent on cone clutches, and on a large number of modern disc clutches. When it is used on the latter type of clutch, the fabric may be riveted to the driving or to the driven discs, whichever is the more economical.

The Johns-Manville Co. manufactures an asbestos-metallic block that is giving excellent service on clutches and brakes.

The block is constructed of long-fiber asbestos, reinforced with brass wire and moulded under an enormous pressure into any desired shape.

The main advantages claimed for the use of wire-woven asbestos fabric and asbestos-metallic block are the following:

1. Slightly higher coefficient of friction.
2. Ability to withstand high temperatures.
3. May be run dry or with oil.
4. Not affected by moisture.
5. Ability to resist wear.

(*d*) *Paper.*—Compressed strawboard may be used as a friction surface on clutches in which the speeds and the pressures coming upon the contact surfaces are low. If excessive slippage occurs, the strawboard is liable to become charred rather rapidly. Vulcanized fiber, which is nothing more than a form of paper treated chemically, gives fairly good service as a friction material in clutches. It is capable of withstanding medium pressure, as well as considerable slippage.

(*e*) *Cork inserts.*—Cork is never used alone as a friction material, but always in connection with some other material either of a fibrous or a metallic nature. It is frequently used on leather-faced cone and metallic disc clutches, and is generally in the form of round plugs or inserts. The surface covered by these cork inserts varies from 10 to 40 per cent. of the total frictional area. Due to the higher coefficient of friction of cork, a motor-car clutch equipped with cork inserts is capable of transmitting a little more power for the same spring pressure than a similar clutch lined with leather; or for the same power, the spring pressure in the former is less than in the latter type of clutch. Cork inserts are also used on hoisting-drum cone clutches having wood blocks, and on common transmission clutches of the disc type. Experience has shown that they give excellent service. In general, the cork inserts are operative only at low pressures, as in engaging the clutch. In combination with the cork, the metal, leather, or wood in which it is imbedded forms the surface in contact after full engagement. Cork inserts also aid in keeping the surfaces lubricated.

(*f*) *Metallic surfaces.*—The materials discussed above are all of a fibrous nature, and are always used in conjunction with a metal, such as cast iron, steel casting, steel, or bronze. Frequently, cone clutches used on machine tools have both cones made of cast iron, while in other cases cast iron and steel casting are used. Disc clutches using hard saw-steel discs running in oil are advocated by some manufacturers; others use steel against bronze, cast iron against bronze, and cast iron against cast iron. In all of the clutches using the metal-to-metal surfaces, a liberal supply of oil is furnished by some means or other.

292. Classification of Friction Clutches.—According to the direction in which the pressure between the contact surfaces is applied, friction clutches may be divided into two general classes, as follows:

(*a*) *Axial clutches,* which include all those having the contact pressure applied in a direction parallel to the axis of rotation. This class includes all types of cone and disc clutches.

(*b*) *Rim clutches,* which include all clutches having the contact pressure applied upon a rim or sheave in a direction at right angles to the axis of rotation.

AXIAL CLUTCHES

A study of the designs of the clutches manufactured by the various builders of transmission machinery, machine tools, and motor cars, shows that axial clutches are made in a variety of forms. Such a study leads to the following classification of axial clutches: (1) cone; (2) disc; (3) combined conical disc.

CONE CLUTCHES

The cone clutch is without doubt the simplest form of friction clutch that can be devised, and if properly designed will give entire satisfaction. Two types of cone clutches are commonly met with, as follows: (1) single-cone; (2) double-cone.

293. Single-cone Clutch.—The elements of a simple cone clutch are shown clearly in Fig. 216. The clutch consists of a cone b keyed rigidly to the shaft a, while a second cone d is fitted to the shaft c by means of the feather key e. This key permits the cone d to be engaged with the cone b, thus transmitting the power from one shaft to the other. The hub of the cone d is fitted with a groove f, into which is fitted the shifter collar operated by the engaging lever.

(*a*) *Machine-tool cone clutch.*—A good example of the use of a simple cone clutch, applied to a machine tool, is shown in Fig.

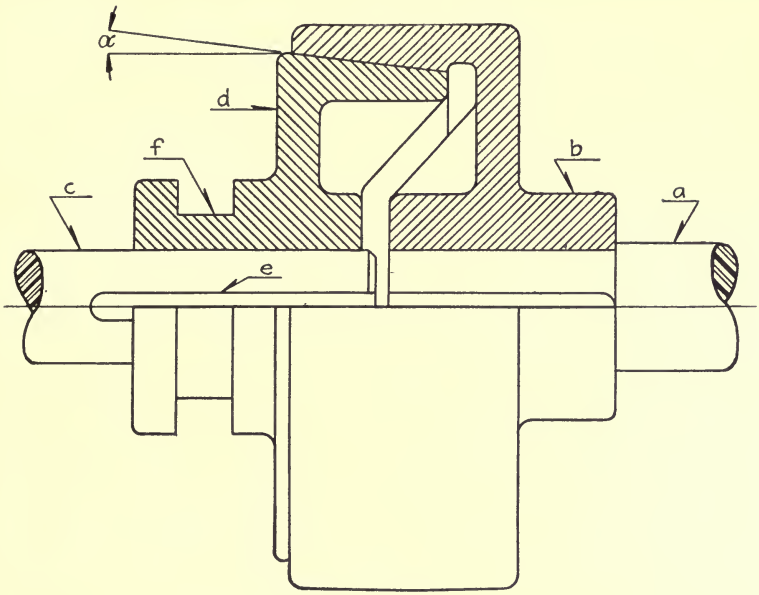

Fig. 216.

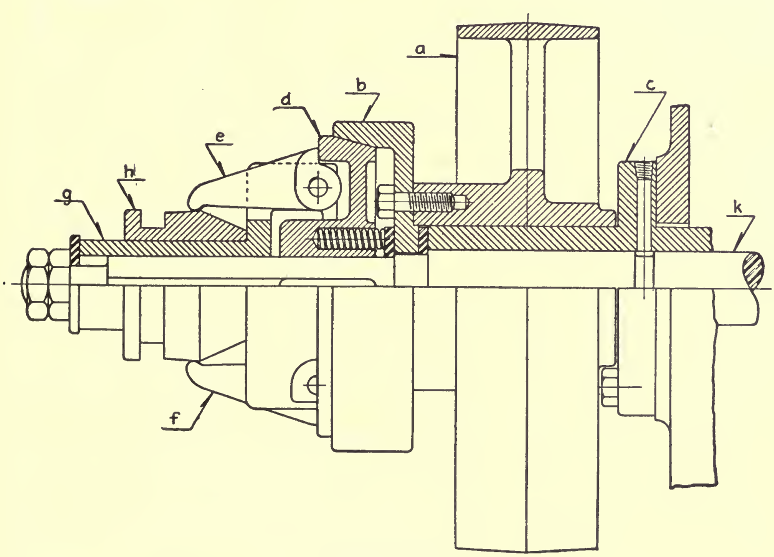

Fig. 217.

217. The design shown is that used on the main driving pulley of the Lucas boring machine. The driving pulley *a* runs loose on the hub of the main bearing *c*, and has bolted to it the cast-iron cone *b*. The sliding cone *d* is fitted into *b* and is keyed to the main driving shaft *k* by a feather key. The cones are engaged by means of the sliding spool *h* and the levers *e* and *f*. Several helical springs, one of which is shown in Fig. 217, are placed into holes drilled into the hub of the cone *d;* the function of these springs is to disengage the two cones when the spool *h* is moved toward the end of the sleeve *g*. It is quite evident that this clutch fulfills the important requirement met with in machine tools, namely, compactness and simplicity of design and ease of operation.

(*b*) *Motor-car cone clutch.*—With the development of the modern automobile, the design of cone clutches was given more attention, and at the present time approximately 40 per cent. of the pleasure-car manufacturers are equipping their cars with clutches of the single-cone type. In motor cars the clutch is used to connect the motor to the transmission, and normally is held in engagement by a spring pressure. This spring pressure must be released by the pedal when the car is to be stopped or when speed changes are made by shifting the gears.

National clutch.—In Fig. 218 is shown a design of a cone clutch used on the National motor car. The cone *a* with its various attachments is forced into the conical bore of the fly-wheel rim by the pressure of the helical spring. To decrease the weight of the clutch, the cone *a* is made of aluminum having its periphery faced with leather. The small flat springs *b*, with which the cone is fitted at various points along its periphery, provide the smooth and easy engagement so desirable in motor cars. To prevent spinning, the sliding sleeve *c* has fastened to it a small brake sheave *d* upon which a brake block *e* acts. The brake block is fitted to an operating lever *f* which is depressed when the clutch is disengaged.

Cadillac clutch.—The clutch shown in Fig. 219 is that used on the old four-cylinder Cadillac motor car. It differs considerably from the National clutch discussed above. The cone *a* is made of pressed steel, and the flywheel instead of having its rim bored conical has a special rim *b* fastened to it. The pressure forcing the cone *a* into the cone *b* is produced by a series of springs in place of a single central spring as in the preceding case. A possible advantage of this arrangement is that the adjustment for wear may be made more easily; also the pressure may be distributed more uniformly over the surface in contact.

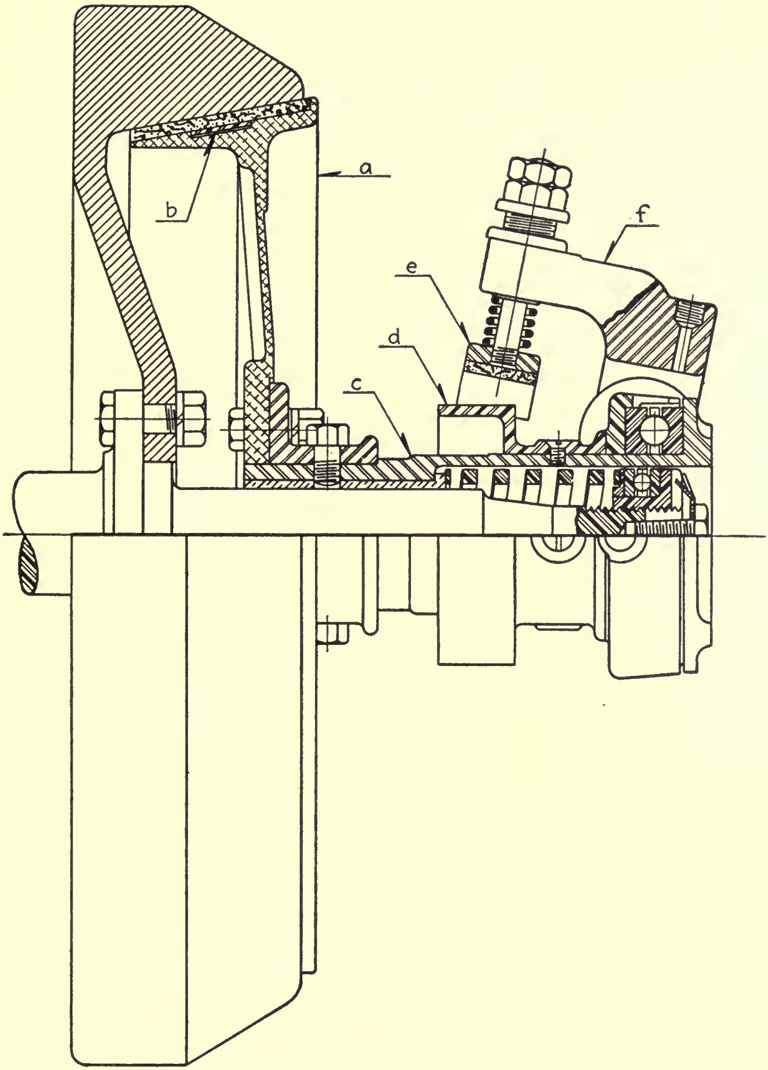

FIG. 218.

294. Double-cone Clutch.—The clutches described in Art. 293 were all of the single-cone type. In connection with hoisting machinery, machine tools, and motor cars, it is not unusual to find double-cone clutches.

(*a*) *Clyde clutch.*—The design of a double-cone clutch used on hoisting drums manufactured by the Clyde Iron Works of

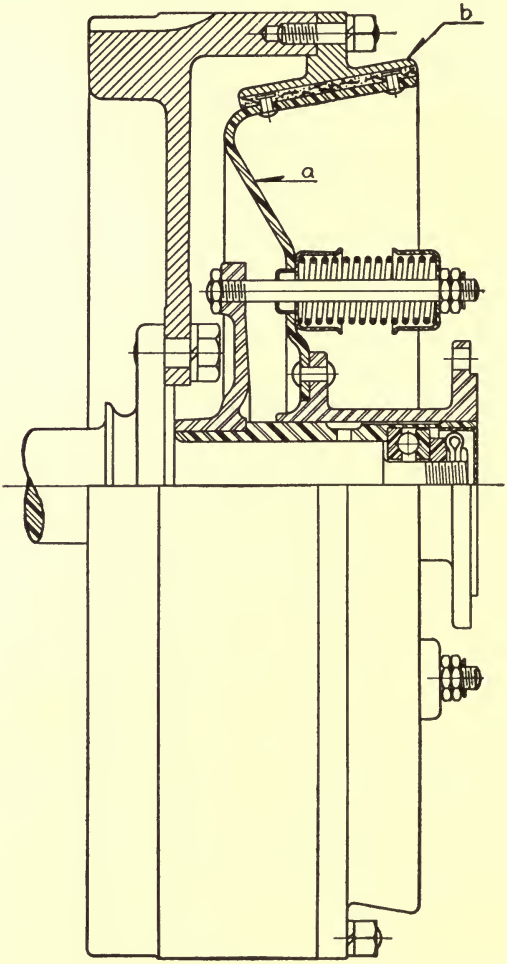

Fig. 219.

Duluth, Minn., is shown in detail in Fig. 223. The friction blocks *c* forming one member of the clutch are fastened to the gear *b*, which is keyed to the shaft *a*. The clutch is engaged by moving the drum *d* along the shaft *a* by means of the combination of lever k, screw h, thrust pin g, cross-key f, and collar e. A spring l, located between the drum and the gear, automatically disengages the clutch when the thrust on the cross-key is released.

In place of using a double-cone clutch, several manufacturers of hoisting drums employ one of the single-cone type, operated practically in the same manner as explained in the preceding paragraph. The Ingersoll slip gear, described in Art. 231, is nothing more than a form of double-cone clutch.

295. Force Analysis of a Single-cone Clutch.—In the following analysis of a single-cone clutch, we shall assume that the outer cone is the driving member while the inner cone is the member having an axial motion. In Fig. 220 are shown the various forces acting upon the inner cone. It is required to determine an expression for the moment M that the clutch is capable of transmitting for any magnitude of the axial force P.

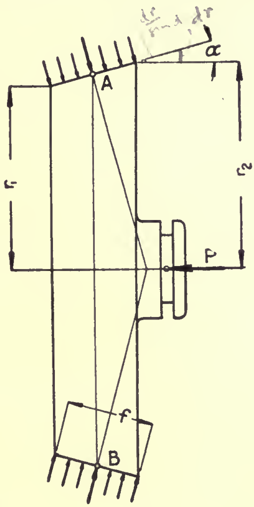

FIG. 220.

Let p = the unit normal pressure at the surface in contact.

r_1 = the minimum radius of the cone.

r_2 = the maximum radius of the cone.

μ = the coefficient of friction.

The maximum moment that the clutch will transmit is equivalent to the moment of the frictional resistance between the inner and outer cones. The normal force acting upon an elementary strip of the surface in contact is $2\pi rp\dfrac{dr}{\sin\alpha}$. The component of this normal pressure parallel to the line of action of the axial force P is $2\pi rpdr$. The summation of these components over the entire surface in contact must equal P; hence

$$P = 2\pi\int prdr \tag{420}$$

The force of friction upon the elementary strip is $2\pi\mu rp\dfrac{dr}{\sin\alpha}$, and its moment about the axis is $2\pi\mu r^2p\dfrac{dr}{\sin\alpha}$. Therefore the moment M is given by

$$M = \frac{2\pi\mu}{\sin\alpha}\int pr^2dr \tag{421}$$

With our present knowledge of friction, it is impossible to determine a correct expression for the moment of the frictional resistance between the two elements of a cone clutch. From (421) it is evident that an expression for the moment of friction depends upon the distribution of the pressure between the contact surfaces as well as the variation of the coefficient of friction. When the clutch is new and the surfaces are machined and fitted correctly, it is probable that the pressure is nearly uniformly distributed. However, after the clutch has been in service for a period of time, there will be a redistribution of the pressure due to the unequal wear caused by the different velocities along the surfaces in contact. This variation in velocity no doubt results in a change in the value of the coefficient of friction. In view of the fact that no experimental data are available, we shall assume that the coefficient of friction remains constant, and further, that the normal wear at any point is proportional to the work of friction. Denoting the normal wear at any point by n, the law just stated may be expressed by the relation

$$n = kpr \tag{422}$$

Assuming that the surfaces in contact remain conical, it follows that the normal wear is constant; hence

$$p = \frac{C}{r}, \tag{423}$$

in which C denotes the ratio of the constants n and k. Substituting the value of p from (423) in (420) and (421), and integrating between the proper limits, we obtain the following relations:

$$P = 2\pi C(r_2 - r_1) \tag{424}$$

$$M = \frac{\pi\mu C}{\sin\alpha}(r_2^2 - r_1^2) \tag{425}$$

Eliminating C between (424) and (425), we get finally

$$M = \frac{\mu PD}{2\sin\alpha}, \tag{426}$$

in which D represents the mean diameter AB of the cone shown in Fig. 220.

To determine the horse power that a cone clutch will transmit, substitute the value of M from (426) in the formula

$$H = \frac{MN}{63{,}030} \tag{427}$$

whence

$$H = \frac{\mu PDN}{126{,}060 \sin \alpha}, \tag{428}$$

and the axial force is

$$P = \frac{126{,}060\, H \sin \alpha}{\mu PDN} \tag{429}$$

The total normal pressure is given by the following expression:

$$P_n = \frac{2\,\pi C}{\sin \alpha}\int_{r_1}^{r_2} dr = \frac{2\,\pi C}{\sin \alpha}\,(r_2 - r_1)$$

Eliminating C by means of (424), it is evident that

$$P_n = \frac{P}{\sin \alpha} \tag{430}$$

The total normal pressure is also equal to the average intensity of unit normal pressure multiplied by the total area in contact; or

$$\frac{P}{\sin \alpha} = \pi D f p' \tag{431}$$

Combining (428) and (431), and solving for H, we get

$$H = \frac{\mu p' f N D^2}{40{,}120} \tag{432}$$

Denoting the product of μ and p' by the symbol K, (432) becomes

$$H = \frac{K f N D^2}{40{,}120} \tag{433}$$

By means of (433) it is possible to determine values of the design constant K for cone clutches in actual service. Such values, if based on clutches in successful operation, will prove of considerable help in the design of new clutches. The analysis used in deriving (433) is similar to that first proposed by Mr. John Edgar in the *American Machinist* of June 29, 1905, though he applied his formulas to expanding ring clutches.

Another design constant that may be found useful in arriving at the proportions of a cone clutch is that which represents the number of foot-pounds of energy per minute that can be transmitted per square inch of contact surface of the clutch. Denoting this constant by the symbol K_1, we find that

$$K_1 = \frac{10{,}500\,H}{fD} \tag{434}$$

296. A Study of Cone Clutches.—Through the generosity and coöperation of about forty automobile manufacturers, information pertaining to a large number of cone clutches was obtained. Some of the clutches that were analyzed were faced with leather, others with asbestos fabric, and a few were equipped with cork inserts.

From the information furnished by the various manufacturers, it was possible to determine for each clutch the magnitude of the design constant K and the intensity of the unit normal pressure p'. With K and p' known, the probable value of the coefficient of friction μ was calculated The values of K, p', and μ were found to vary with the mean velocity of the surface in contact. In this, as well as in all other analyses of motor-car clutches, the values of K are based upon the horse power and speed corresponding to the maximum torque of the motor, and not upon the maximum horse power transmitted and the speed corresponding thereto. It should be remembered that clutches must be designed for the maximum loads coming upon them, and in the case of motor cars, the loads are greatest when the motor transmits the maximum torque.

(*a*) *Leather-faced cone clutches.*—For the leather-faced cone clutches analyzed, the values of K and p' were plotted on a speed base, and the curves shown in Fig. 221 represent the average results. From this figure, it is apparent that the magnitude of K decreases with an increase in the mean velocity of the surfaces in contact. The intensity of the unit normal pressure p' also decreases with an increase in the velocity. The value of the coefficient of friction was also plotted on a speed base, and an average curve passed through the series of points. For all practical purposes, the average value of μ may be represented by a straight line parallel to the velocity axis, giving a constant value of μ equal to 0.2 for all speeds. The μ curve is not shown in Fig. 221.

(*b*) *Cone clutches faced with asbestos fabric.*—At the present time there are only a few motor-car builders using cone clutches faced with asbestos fabric. From an investigation of six such clutches, K was found to vary from 1.95 to 4.77. The intensity of the unit normal pressure p' varied from 9.5 to 17 pounds per square inch. Until such a time as more information pertaining to asbestos fabric facing is available, it is suggested that the values of K given in Fig. 221 be used, and that the coefficient of friction be assumed as 0.30.

(*c*) *Cone clutches with cork inserts.*—It was impossible to get information pertaining to a large number of cone clutches having cork inserts, since very few motor-car builders are using them at the present time. Four such clutches were analyzed and the

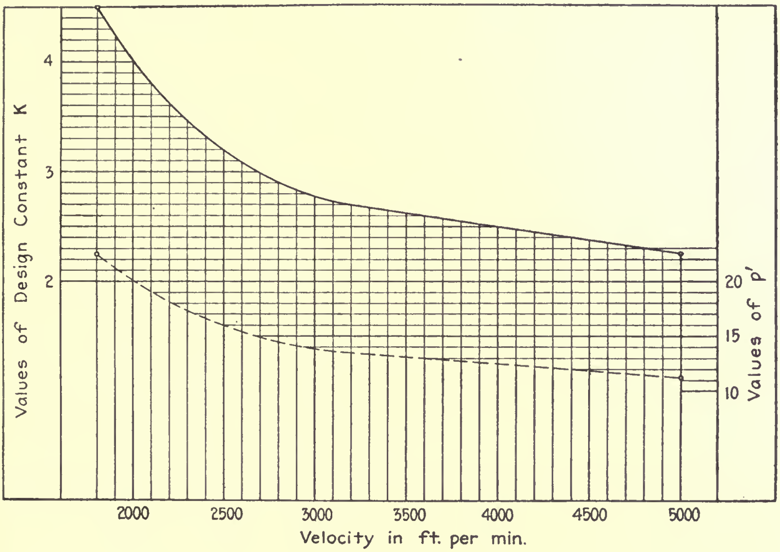

Fig. 221.

design constant K was found to vary from 2.2 to 3.1. Until such a time as sufficient information is available for a more extended analysis, it would seem advisable to use the values of K given for the leather-faced cones when making calculations for cork insert clutches. The coefficient of friction may be assumed as 0.25.

(*d*) *Cone-face angle.*—In the study of the motor-car cone clutches, it was found that for a leather facing the face angle α varied from 10 to 13 degrees. The majority of the manufacturers are using $12\frac{1}{2}$ *degrees which is now recommended as a standard by the Society of Automotive Engineers.* With an asbestos-fabric facing, the angle α varied from 11 to $14\frac{1}{2}$ degrees, and for the cork insert clutches, from 8 to 12 degrees.

297. Experimental Investigation of a Cone Clutch.—In the *Zeitschrift des Vereines deutscher Ingenieure,* for Dec. 15, 1915, Prof. H. Bonte of Karlsruhe presented an article in which he gave the results of an experimental investigation of a cone clutch. The two halves of the clutch were made of cast iron, and during

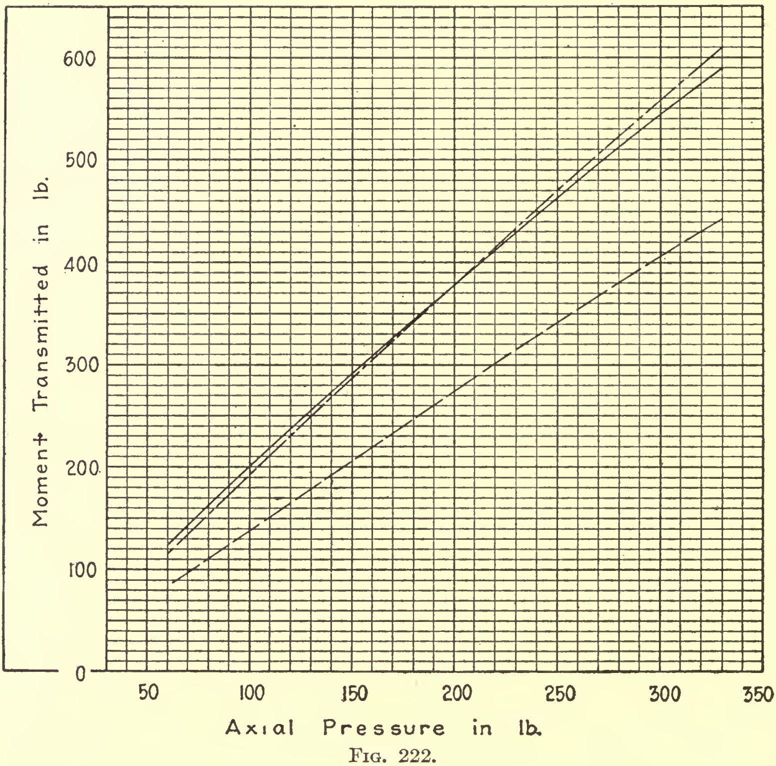

FIG. 222.

the test the surfaces in contact were lubricated. The main object in undertaking the investigation was to determine which of the following two formulas, generally quoted in technical works, was the correct one to use in designing cone clutches:

$$M = \frac{\mu PD}{2 \sin \alpha} \tag{435}$$

or

$$M = \frac{\mu PD}{2 (\sin \alpha + \mu \cos \alpha)} \tag{436}$$

In Fig. 222 are plotted the results of Prof. Bonte's experimental investigation on a clutch having an angle $\alpha = 15$ degrees. In this figure are included the results obtained by evaluating (435) and (436). The results obtained by the use of (435) are represented by the dot and dash line, and those obtained by the use of (436) are represented by the dash line. The following conclusions may be arrived at from the results published by Bonte.

(*a*) When the angle α is 15 degrees, the error introduced by using (436) is large, while the agreement between the experimental results and those obtained by using (435) is very close.

(*b*) When the angle α is 30 degrees, the experimental results lie between those obtained by (435) and (436). The curve representing the results obtained by (435) lies above, but is much closer to the experimental curve than that obtained by (436).

(*c*) For the angle 45 degrees and 60 degrees, the experimental points lay above those obtained by (435), which in turn lay above the points obtained by (436).

(*d*) Apparently, the coefficient of friction is not constant as generally assumed but varies slightly with the pressure.

As a result of this experimental investigation, Prof. Bonte makes a plea that (436), which is apparently incorrect, should no longer be used in designing cone clutches.

298. Analysis of a Double-cone Clutch.—For the double-cone clutch shown in Fig. 223, it is required to determine an expression for the force F that must be applied at the end of the lever k in order to engage the clutch; also, to determine the maximum moment that the clutch is capable of transmitting.

Let D_1 = the mean diameter of the smaller cone.
D_2 = the mean diameter of the larger cone.
D_3 = the mean diameter of the thrust collar e.
D_4 = the mean diameter of the spring cage m.
L = the length of the lever arm k.
P = the axial force holding the drum against the V blocks.
S = the spring force.
d = the mean diameter of the screw h.
β = the angle of rise of the mean helix of the screw.
φ' = the angle of friction for the screw.
μ_0 = the coefficient of friction between the drum and blocks c.
μ = the coefficient of friction between the drum and collar e and cage m.

Consulting Fig. 223, it is evident that the axial pressure that the screw h must produce is $P + S$ plus the force required to move the drum with its load along the shaft. The latter force is relatively small and may be accounted for by considering it equivalent to a certain percentage of the total pressure produced. Calling Q the total pressure due to the screw, we find that its magnitude may be expressed by the formula

$$Q = \frac{P + S}{\eta}, \tag{437}$$

in which η may be assumed as equal to 0.97. From this it follows that the force F required on the operating lever k, in

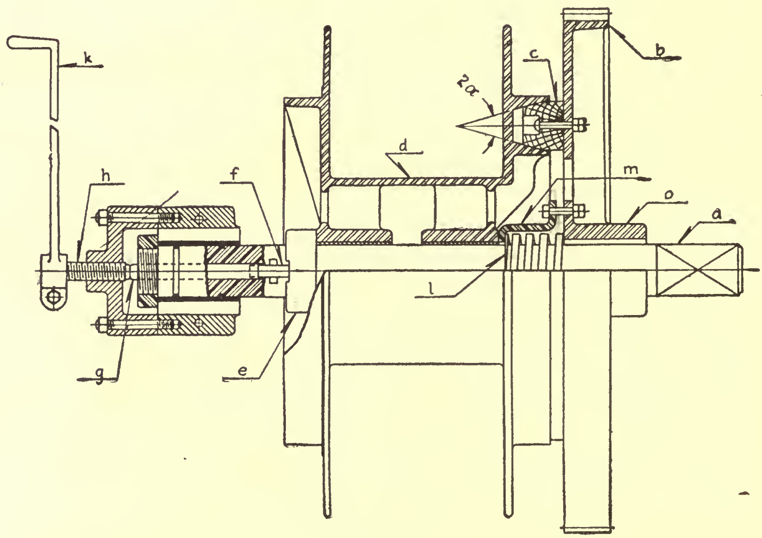

FIG. 223.

order to produce the axial pressure Q, must have a magnitude given by the expression

$$F = \frac{Qd}{2L}\tan(\beta + \varphi') \tag{438}$$

The total moment that the drum will transmit is equivalent to the sum of the moments of friction of the double cone c, of the thrust collar e, and of the spring cage m. The last two moments just mentioned are usually small when compared with the first, and frequently are not considered at all. The moment transmitted by the double cone is equivalent to the sum of the moments of the two cones taken separately, or

$$M_1 + M_2 = \frac{\mu_0 P}{4 \sin \alpha}(D_1 + D_2) \tag{439}$$

The sum of the moments transmitted by the collar e and the spring cage m is

$$M_3 + M_4 = \frac{\mu D_3}{2\eta}(P + S) + \frac{\mu D_4 S}{2} \tag{440}$$

Adding (439) and (440), we find that the total moment transmitted by the drum has the magnitude

$$M = M_1 + M_2 + M_3 + M_4 \tag{441}$$

299. Smoothness of Engagement of Cone Clutches.—In motor-car service, it is very desirable that the car be started

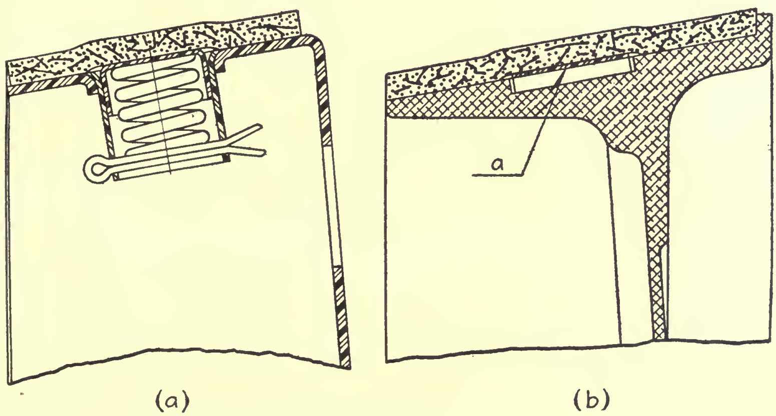

FIG. 224.

without jerks. In order to secure smooth clutch engagement, the designers of clutches were compelled to originate devices that insured evenness of contact between the friction surfaces. A few such devices, as applied to cone clutches, are shown in Figs. 224 to 227, inclusive. In general, it may be said that the function of these devices is to raise slightly the cone facing at intervals around the periphery, so that upon engagement only a small portion of the friction surface comes into contact with the flywheel rim. As soon as the full spring pressure is exerted, the facing is depressed and the entire surface of the cone becomes effective. One disadvantage of the attachments just discussed is that they tend to increase the spinning effect due to the extra weight added to the periphery of the cone.

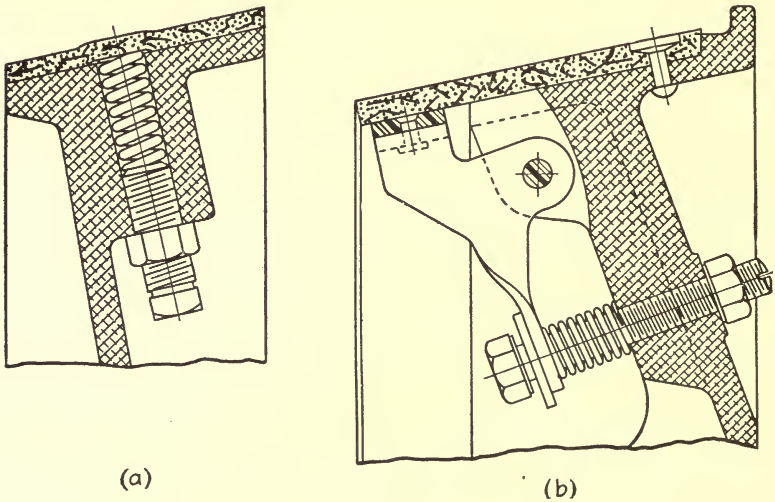

FIG. 225.

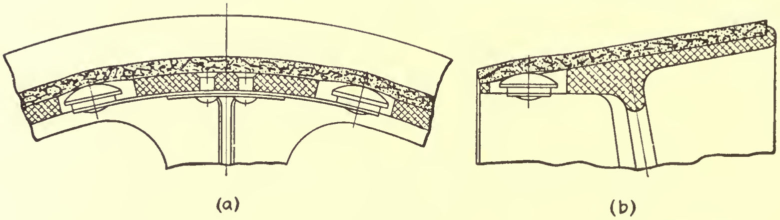

FIG. 226.

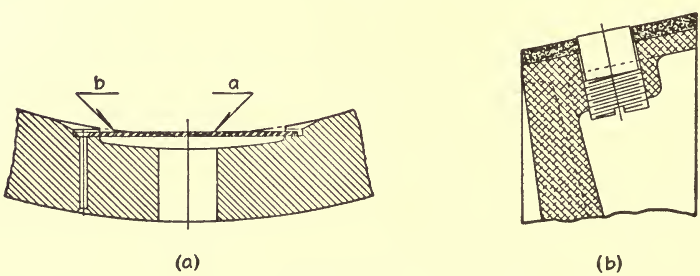

FIG. 227.

A few manufacturers are using cork inserts in connection with their leather-faced cone clutches. It is claimed that in addition to increasing the coefficient of friction between the surfaces in contact, the cork inserts have the effect of producing smooth and easy engagement of the clutch. Obviously, cork inserts have another advantage in that the weight of the cone is actually decreased, thereby decreasing the spinning effect. Fig. 227(*b*) shows one method of holding cork inserts in the facing of a cone clutch.

300. Clutch Brakes.—In addition to securing smooth and easy clutch engagement, some means must be provided to prevent the "spinning" of the clutch when it is disengaged. By keeping the size and weight of the clutch down to a minimum, spinning may be reduced slightly. However, to overcome the spinning action completely, small brakes that are brought into action when the pedal is depressed must be provided A cone clutch equipped with such an auxiliary brake is shown in Fig. 218, and in Figs. 229, 236, and 241 are shown disc clutches equipped with such brakes.

DISC CLUTCHES

In general, a disc clutch consists of a series of discs arranged in such a manner that each driven disc is located between two driving discs. Disc clutches are made in various forms, as a study of the designs used in connection with various classes of machinery will show. For convenience, disc clutches will be classified as follows:

(*a*) Single-disc type, in which a single disc serves as the driven member.

(*b*) Multiple-disc type, in which two or more discs act as the driven member.

301. Single-disc Clutch.—In Figs. 228 to 233, inclusive, are shown six designs of single-disc clutches, the first two representing the practice of two motor-car builders, and the third and fourth showing the details of two clutches used for general power-transmission purposes. The remaining two, namely, those shown in Figs. 232 and 233, are intended for special purposes. As in the case of the cone clutches, the development of the automobile is responsible to a large extent for the advances made in the design of disc clutches.

(*a*) *Knox clutch.*—The disc clutch shown in Fig. 228 is that used on the old Knox motor cars. The discs *a* and *b* are fastened to the flywheel while the driven disc *c* is fastened to the flange *d*, which in turn is splined to the transmission shaft *e*. Due to the action of a series of springs located in the rim of the flywheel, the driven disc *c* is clamped between the two driving discs. The clutch is released by overcoming the spring force upon the discs *b*, through the medium of the sliding sleeve *f*, lever *g*, and plunger

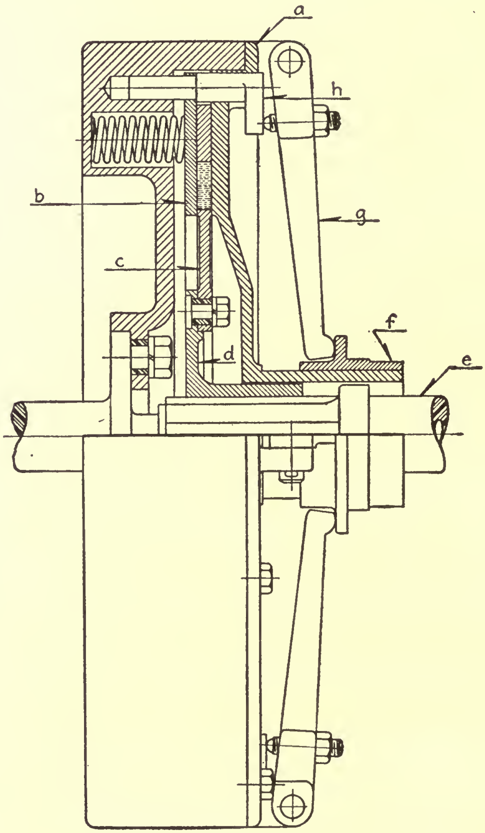

Fig. 228.

h. All of the discs used in this clutch are made of cast iron. In order to obtain smooth engagement and to increase the coefficient of friction between the surfaces in contact, the driven disc *c* is fitted with cork inserts as shown.

(*b*) *Velie clutch*.—The type of single-disc clutch used on the Velie motor cars is shown in Fig. 229. Instead of having two driving discs as in the Knox clutch, this design has only one driving disc *b*, but the web of the flywheel serves the same purpose as a second disc. The steel driven disc *c* is riveted to the flange of the clutch drive shaft *d*. The clutch is kept in engagement by the conical spiral spring pressing upon a bronze sleeve, which in turn transmits the pressure to the wedge *f* by means of

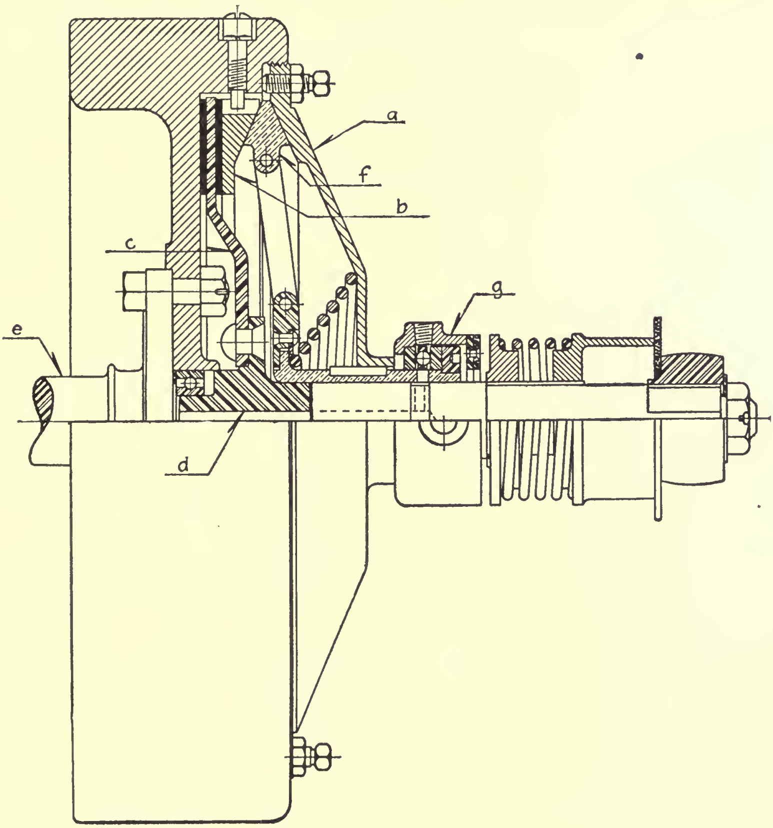

FIG. 229.

suitable links. The back face of the driving disc *b*, as well as the inside face of the cover plate *a*, is bored conical to fit the wedge *f*. The cover plate screws into the flywheel and is locked to it by means of the set screws shown. To release the clutch, the wedge is withdrawn slightly by forcing the bronze sleeve back against the action of the spring. The treadle operates the releasing collar *g* by means of a system of links and levers. In the Velie clutch, the driven disc c is faced on both sides with an asbestos fabric, called Raybestos. Attention is directed to the small disc brake which prevents excessive spinning when the clutch is released.

(*c*) *Plamondon clutch.*—A sectional view of the Plamondon disc clutch as applied to a pulley running loose on a shaft a, is shown in Fig. 230. The disc c, which is faced with hard maple segments, is made in halves so that it can be removed in case the friction blocks require renewal. The flange d slides on the flanged hub e, which is keyed to the shaft a. By means of the compound

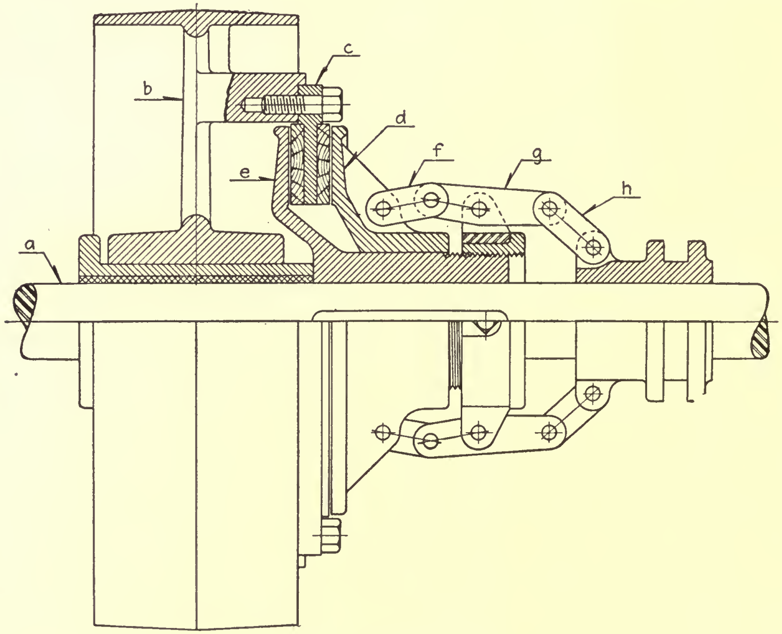

FIG. 230.

toggle levers, f, g, and h, the flanges d and e are pressed against the disc c, thereby transmitting the power from the pulley to the shaft, or vice versa. Attention is called to the simplicity of this clutch and also to the ease with which adjustments for wear may be made.

(*d*) *E. G. I. clutch.*—In Fig. 231 is shown another design of a single-disc clutch, but in this case the pressure upon the discs is produced by a system of rollers and levers instead of springs or toggle joints. The cast-iron discs c and d are made to rotate with the casing *b* by means of the three bolts *e*. The casing *b* is fastened to the shaft *a* by set screws or keys. Between the sliding discs *c* and *d* is located a third disc *l*, to the hub of which may be fastened a gear or pulley. The pressure exerted by the sliding discs upon the disc *l* is produced by shifting the sleeve *f* inward. This movement causes the levers *h* to assume a position perpendicular to the shaft, thereby forcing apart the disc *c* and the casing *b*, and at the same time creating a considerable pressure upon the disc *l*. Upon disengagement of the clutch, the springs

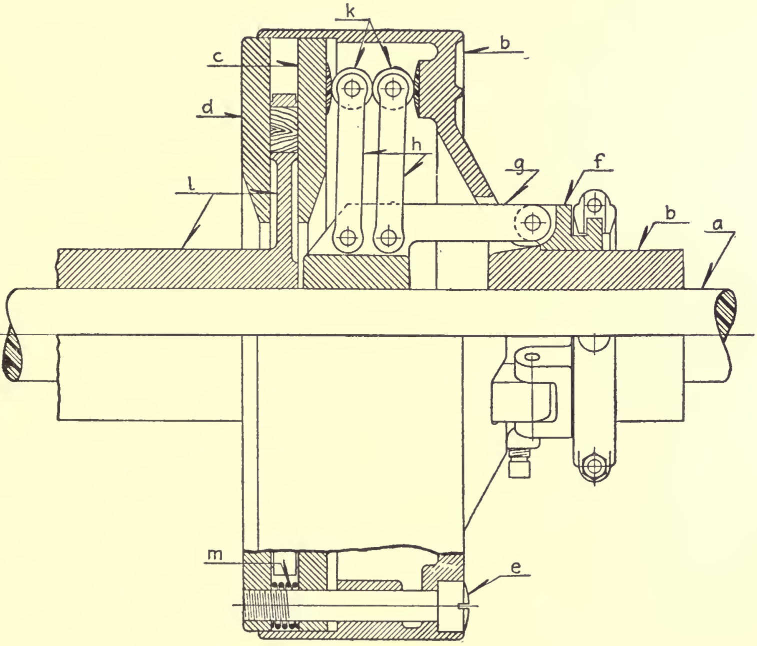

Fig. 231.

m spread the discs *c* and *d*. The disc *l* is fitted with a series of wooden plugs, as shown in the figure.

302. Hydraulically Operated Disc Clutch.—In such naval vessels as torpedo boat destroyers, it has been found that a combination of reciprocating engines with turbines gives better economy over a wide range of speed than turbines alone. The engines are used for cruising speeds only, and exhaust into the low-pressure turbines. At the higher speeds, the ship is propelled by turbines only. According to the machinery specifications drawn up by the Navy Department for some of the latest types of destroyers, the installation of turbines and cruising engines called for must fulfill the following conditions:

(*a*) That the engines and turbines should be capable of operating in combination on cruising speed.

(*b*) That the turbines should be capable of operating alone, the engines standing idle.

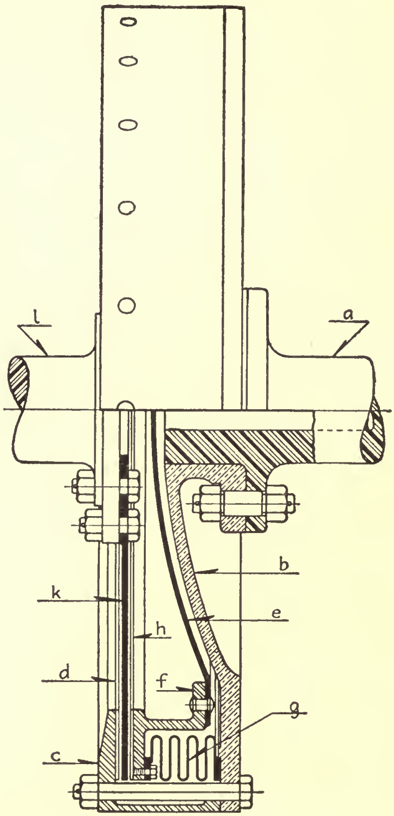

FIG. 232.

(*c*) That means should be provided whereby the cruising engines may be connected to or disconnected from the turbine shafts without stopping the propelling machinery.

It is evident from the above specifications that some form of reliable clutch is necessary to fulfill condition (*c*), and in order to meet this requirement, Mr. J. F. Metten, Chief Engineer of the Wm. Cramp and Sons Ship and Engine Building Co., developed and patented the single-disc clutch shown in Fig. 232. The hollow crankshaft *a* of the reciprocating engine has connected to it the head *b*, which in combination with the steel frame *c* forms the driving member of the clutch. The inner face of the frame *c* is lined with an asbestos fabric. Inside of this driving member and attached to it, is located a movable member consisting of the spherical steel-plate head *e*, ring *f* faced with asbestos fabric, and the flexible ring *g*. The shaft *l*, which is an extension of the main turbine shaft, has bolted to its flange a steel-plate disc *k*, ¼ inch thick. When oil under pressure is forced through the hollow crankshaft into the pressure chamber formed between the heads *b* and *e* and rings *f* and *g*, the disc *k* is gripped by the friction surfaces *d* and *h*. As shown in

Fig. 232, the clutch is disengaged. In order to insure quick disengagement of the clutch, the flexible ring *g* is so designed that its contraction upon release of the oil pressure will force the oil out of the pressure chamber.

The axial force available for creating the frictional resistance on the disc *k* is that due to the fluid pressure upon the combined unbalanced areas of the head *e* and ring *f*, minus the resistance that the flexible ring *g* offers to extension. Having determined the axial force, and knowing the inner and outer diameters of the contact surfaces *d* and *h*, the probable horse power that the clutch is capable of transmitting may readily be determined.

303. Slip Coupling.—In many installations, it is desirable to place between a motor and the driven machine or mechanism some form of coupling that will slip when the load is excessive, thus protecting the motor against overloads. The details of such a coupling, called a slip coupling, are shown in Fig. 233, which represents the design used by the Illinois Steel Co. and others on the drives of rolling mill tables. A modification of this design is also used on the furnace-charging machines found in steel works. The slip coupling illustrated in Fig. 233 is nothing more than a single-disc friction coupling. The flanged hub *a* is keyed to the driving shaft, and has bolted to its rim a plate *b*. Between *a* and *b*, and separated from them by fiber discs, is the flanged hub *c* which is keyed to the driven shaft. The bolts connecting the plate *b* with the hub *a* are provided with springs which create a pressure on both faces of the hub *c*. The torsional moment transmitted by the coupling depends directly upon this spring pressure, which may be varied by merely adjusting the nuts of the coupling bolts. In Table 93 are given the general proportions of a series of sizes of the slip coupling shown in Fig. 233, and these proportions represent the practice of the Illinois Steel Co.

304. Multiple-disc Clutches.—In Figs. 234 to 237, inclusive, are shown four designs of multiple-disc clutches, the first two of which represent the practice of two manufacturers of transmission clutches, and the last two show the type of multiple disc clutches used on motor cars.

(*a*) *Akron clutch.*—The Akron clutch shown in Fig. 234 is a double-disc clutch employing an ingenious roller toggle for producing the pressure between the discs. The clutch consists of a casing *a* upon the hub of which gears, sprockets, or pulleys may be keyed. Into the casing *a* is screwed a head *b* having a series of notches on its periphery, into which the locking set screw *c* projects. This combination of screwed head and set screws affords a simple and effective means of making adjustments for wear. The inner face of the head *b* and that of the casing *a* are

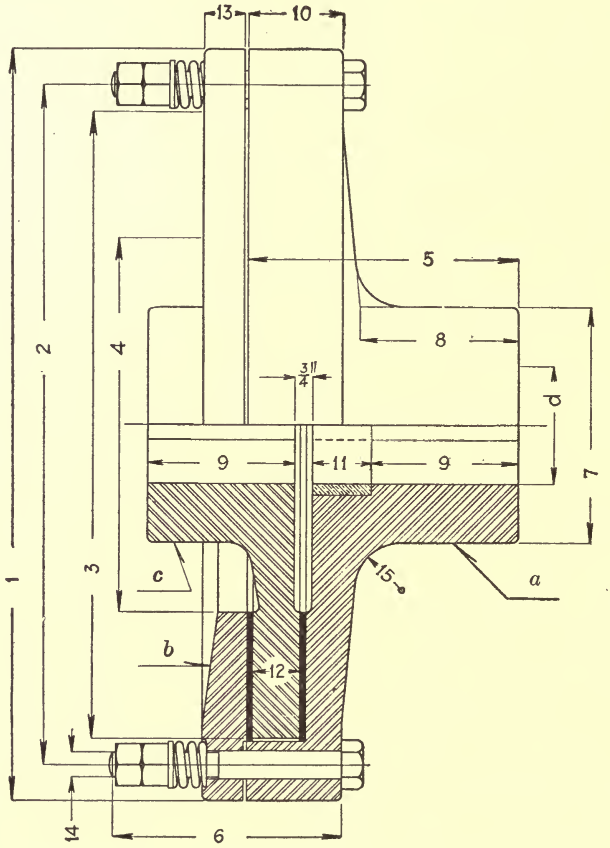

FIG. 233.

machined and serve as contact surfaces for the discs *d* and *e*, respectively. The discs are splined to the hub *f*, which in turn is keyed to the shaft *g*. To engage the clutch, the sliding sleeve *h* is moved outward, thus pulling the forked levers *k* with it, and as a result of the action of the roller toggle, forcing the discs

TABLE 93.—PROPORTIONS OF SLIP COUPLINGS

d max.	h.p. at 100 r.p.m.	Dimensions															Spring				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	No.	Outside diam.	Length	Size of wire	Pitch of coils
3	5	18½	15½	14	11	7⅝	7⅞	6	3⅞	3¾	3¼	1⅝	1½	1¼	¾	1¼	6	2	1¾	¼	½
	6																	1¾			
3½	7	20½	17½	16	12	8½	8⅛	7	4⅝	4½	3⅜	1¾		1⅜		1½	7				
	8																6	1½			
4	10	21½	18½	17	13	9⅜	8⅜	8	5⅜	5	3½	2	1⅝	1½		1¾	7				
	15	23½	20½	19													10				
	20	23¾	20¾			9½	8⅞		5¼		3⅝		1¾	1⅝	⅞		8	1¾		5/16	9/16
	25	24¾	21¾	20													10				
4½	30	26¾	23¾	22	15	10½		9	6¼	5¾		2¼					12	2			
	35	27¾	24¾	23													12	1¾			
	40	28¾	25¾	24													13				
	45	29	26			10¾	9⅞	10	6		4	2½	2	1¾	1⅛	2	10	2		⅜	
5	50	30	27	25	16	11½			6¾	6¼							12				
	60	32	29	27																	
	70	33	30	28	17												13				
	80	34½	31	29		11¾	11¼		6½		4½		2¼	1⅞			12	2½	2½	7/16	⅝
5½	90	35½	32	30	18	12¼	11⅜	11	6⅞	6¾	4⅝						13				
6	100	36½	33	31	19	13¼	11⅞	12	7½	7½	5		2½	2			12	2¼			

d and *e* apart. The clutch is lubricated effectively by having the casing partially filled with oil, and hence the wear on the friction surfaces is reduced to a minimum.

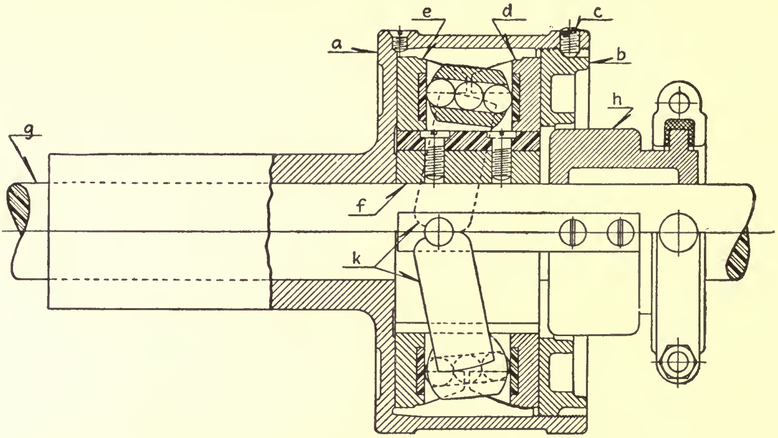

FIG. 234.

(*b*) *Dodge clutch.*—The multiple-disc clutch shown in Fig. 235 is used for general transmission service. The cylindrical casing *c* with its hub *b* is keyed to the shaft *a*, and may serve

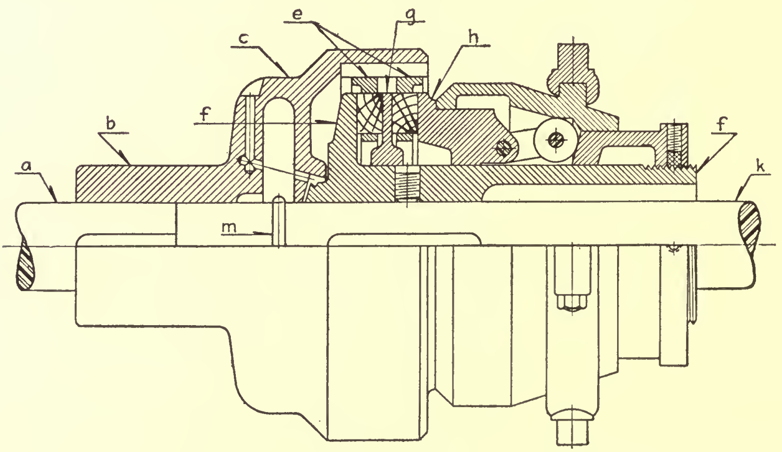

FIG. 235.

either as the driving or the driven member. The discs *e*, fitted with wood blocks, rotate with *c* and at the same time may be moved in an axial direction. The flanged hub *f* is keyed to the shaft k and has splined to it the two discs g and h, the outer one of which may be moved forward by the roller toggle operating mechanism. The axial movement given to h clamps the various discs together, thus transmitting the desired power. It should be noted that means for taking up wear on the discs are provided, and that the clutch is self-lubricating. An oil ring m revolves

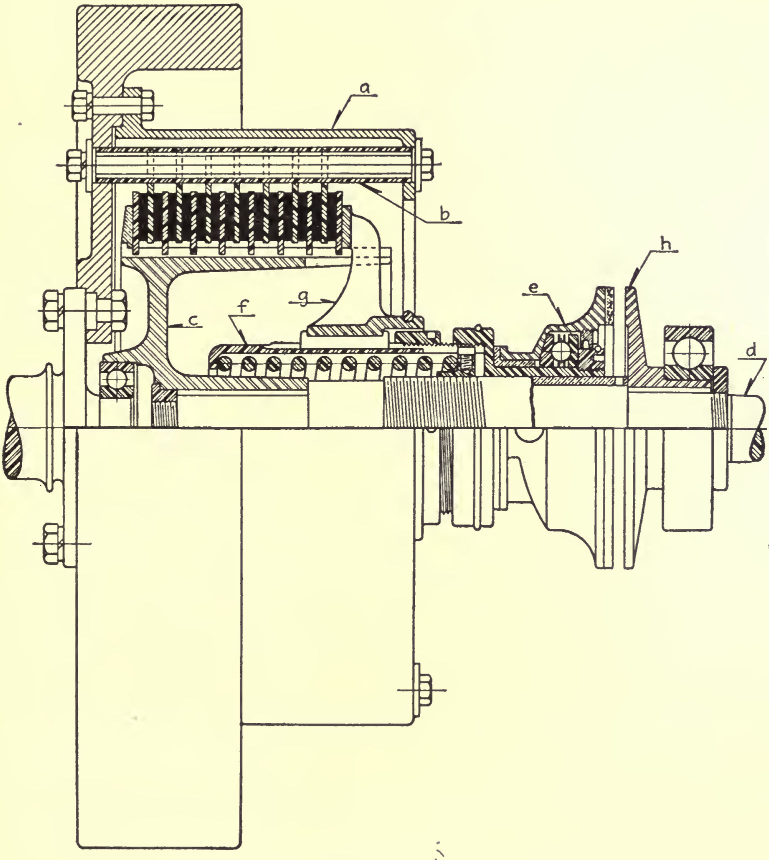

Fig. 236.

upon the shaft and carries a continuous supply of lubricant from the oil reservoir below to all parts of the sleeve.

(*c*) *Alco clutch.*—The multiple-disc clutch used on the Alco car, formerly manufactured by the American Locomotive Co., is shown in Fig. 236. As shown in the figure, the driving discs are connected to the flywheel through the hollow pin b and the drum *a*, while the driven discs are splined to the inner hub *c* which is keyed to the clutch shaft *d*. Both driving and driven discs are so mounted that they must rotate with the member to which they are connected, and at the same time these discs may move in an axial direction. To disengage the clutch, the collar *e* is moved to the right carrying with it the sleeve *f* and the spider *g*, thus releasing the pressure between the two sets of discs. As soon as the treadle is released, the spring will engage the clutch. Both sides of the driving discs are faced with Raybestos.

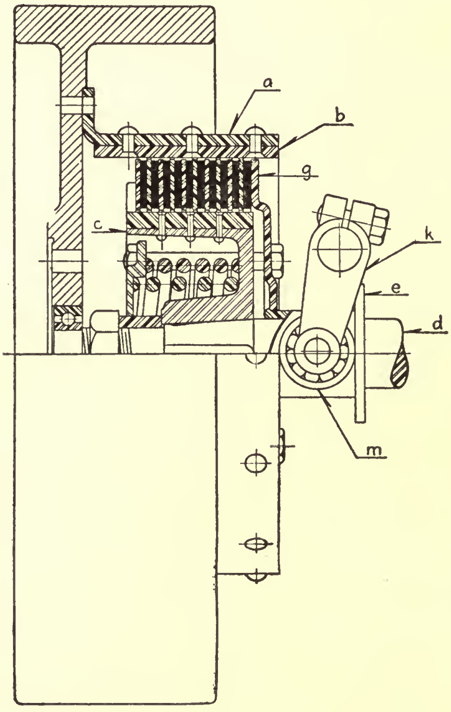

FIG. 237.

(*d*) *Pathfinder clutch.*—Another form of multiple-disc clutch is shown in Fig. 237, and as in the Alco clutch, both sides of the driving discs are faced with asbestos fabric. The latter clutch is much shorter in length than the former, and in place of a single spring to create the axial pressure upon the discs, a double concentric spring is used. The pressure upon the treadle is transmitted to the collar *e* on the transmission shaft *d* by the shipper arm *k*, through the medium of a ball bearing *m*, as shown in the figure. In general, the description and operation of the Pathfinder clutch is similar to that of the Alco clutch.

305. Force Analysis of a Disc Clutch.—It is required to determine an expression for the moment M that the clutch is capable of transmitting for a given magnitude of the axial force P. We shall assume, in the following analysis, that the law expressed by (422) will hold for disc clutches. This is approximately true, especially for clutches having very narrow contact surfaces.

Let D = the mean diameter of the discs.
r_1 = the minimum radius of the discs.
r_2 = the maximum radius of the discs.
s = the number of friction surfaces transmitting power.

The general expressions deduced for the conical clutch may be applied to the disc clutch by making the angle $\alpha = 90$ degrees. Substituting this value in (426), we get for the moment for each contact surface

$$M_1 = \frac{\mu PD}{2}$$

Hence for s surfaces, the total moment becomes

$$M = \frac{s\mu PD}{2} \tag{442}$$

Substituting (442) in (427), we find that the horse power transmitted by a disc clutch is given by the expression

$$H = \frac{s\mu PDN}{126{,}060}, \tag{443}$$

from which the axial force is

$$P = \frac{126{,}060\ H}{s\mu DN} \tag{444}$$

The total axial pressure P is also given by the product of the area of contact of one disc and the average intensity of normal pressure p', that is

$$P = \pi\ (r_2^2 - r_1^2)\ p' \tag{445}$$

Combining (444) and (445), and solving for H, we obtain the following expression

$$H = \frac{\mu p' s D^2 f N}{40{,}120}, \tag{446}$$

in which f denotes the face of the contact surface, or $(r_2 - r_1)$. Replacing $\mu p'$ by the symbol K_2, as was done in Art. 295, (446) becomes

$$H = \frac{sfND^2K_2}{40{,}120} \tag{447}$$

In disc clutches, as in cone clutches, it may be desirable to know the number of foot-pounds of energy the clutch will transmit per minute per square inch of actual contact surface. Representing this factor by the symbol K_3, we get

$$K_3 = \frac{10{,}500\ H}{sfD} \tag{448}$$

306. A Study of Disc Clutches.—(*a*) *Motor-car clutches.*—A study of disc clutches used on motor cars discloses the fact that the majority of such clutches have steel discs in contact with asbestos-fabric-faced steel discs. Among other combinations that are used for the friction surfaces, the following may be

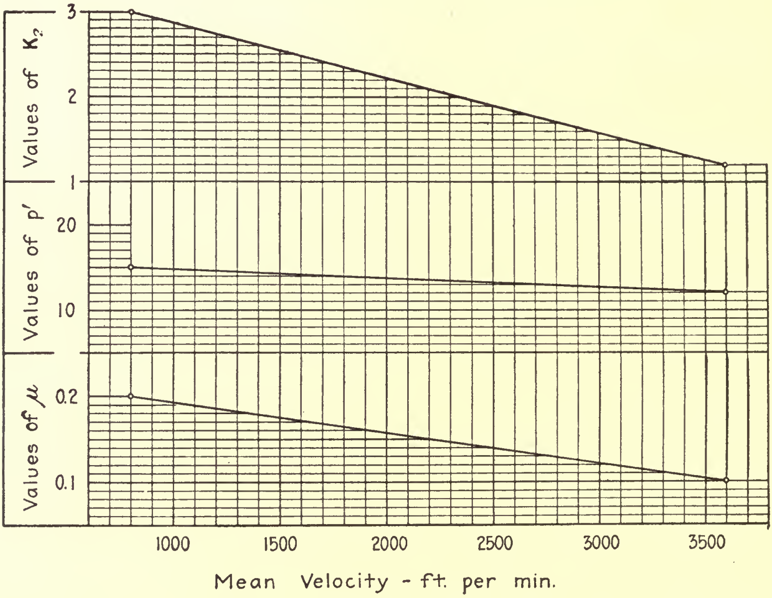

FIG. 238.

mentioned: (1) steel against steel; (2) steel against steel with cork inserts; (3) steel against bronze.

An analysis, similar to that of cone clutches, was made of a large number of different types of disc clutches used on motor cars. The information required for such an analysis was furnished by the various motor-car manufacturers. The graphs plotted in Fig. 238 represent the average results obtained for the asbestos-fabric-faced disc clutches running dry, and are based upon an investigation of at least thirty-five different clutches. The values of K_2 were obtained by evaluating (447), while those of p' were determined by means of (445). The graph for the coefficient of friction μ was established from the relation $K_2 = \mu p'$.

For clutches employing the other friction surfaces mentioned in a preceding paragraph, it was thought best not to represent the results graphically, since there was not sufficient information available to warrant definite conclusions. However, in Table 94 are exhibited the minimum, maximum, and mean values of the design constant K_2, of the average intensity of normal pressure p', and of the coefficient of friction for the various types of motor-car disc clutches investigated.

(*b*) *Transmission clutches.*—Through the generosity of several manufacturers of transmission clutches, considerable information was obtained which made it possible to carry out an analysis similar to that on motor-car clutches mentioned above. Since no information regarding the axial pressure upon the discs was available, it was impossible to determine the probable values of p' and μ, and consequently only the relation between the design constant K_2 and the mean velocity of the friction surfaces at 100 revolutions per minute of the clutch was calculated. The reason for selecting the mean velocity

Table 94.—Data Pertaining to Various Types of Disc Clutches

Type of clutch	Friction surfaces		Values of K_2			Values of p			Values of μ			Remarks
	Driver	Driven	Max.	Min.	Mean	Max.	Min.	Mean	Max.	Min.	Mean	
Motor car..	Steel	Steel faced with asbestos fabric	3.96	0.41	See Chart	26.1	3.52	See Chart	0.450	0.076	See Chart	Dry
			2.37	1.31	1.84	20.40	10.90	15.6	0.120	0.116	0.118	Lubricated
	Steel....	Steel....................	2.46	1.33	1.90	38.20	26.90	32.5	0.092	0.035	0.064	Lubricated
	Steel....	Steel and cork inserts............	1.48	0.88	1.18	14.24	9.73	12.0	0.104	0.082	0.093	Lubricated
	Bronze .	Steel....................	1.89	0.62	1.26	15.00	7.21	11.1	0.091	0.086	0.089	Lubricated

at 100 revolutions per minute of the clutch as one of the variables is the fact that all of the manufacturers rate their clutches at this speed.

The disc clutches investigated were fitted with the following combinations of friction surfaces: cast iron against wood; cast iron against compressed paper and wood; cast iron against cast iron; cast iron against cast iron with cork inserts.

1. For clutches equipped with cast-iron discs in contact with wood-faced discs, it was found that the design constant K_2

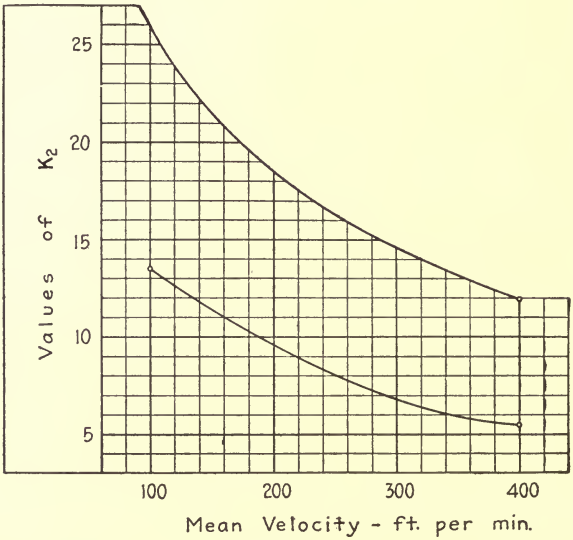

Fig. 239.

varied between wide limits. This variation is clearly shown in Fig. 239, in which the two curves represent the maximum and minimum results obtained.

2. For clutches having cast-iron discs in contact with discs faced with compressed paper, the relation existing between K_2 and the mean velocity at 100 revolutions per minute of the clutch is represented by the graph of Fig. 240.

3. For clutches having cast-iron friction surfaces, the relation between the design constant K_2 and the mean velocity may be expressed by the following formula:

$$K_2 = 18 - \frac{V}{50}, \tag{449}$$

in which V denotes the mean velocity of the friction surfaces at 100 revolutions per minute of the clutch.

4. For clutches having cast-iron discs in contact with cast-iron discs fitted with cork inserts, the relation between K_2 and the mean velocity of the friction surfaces is given by the following expression:

$$K_2 = 17 - \frac{V}{150} \tag{450}$$

COMBINED CONICAL-DISC CLUTCHES

By a combined conical-disc clutch is meant one in which the contact surfaces of the disc or discs are conical. Several designs of conical-disc clutches are available, the most important of which are described briefly in the following paragraphs.

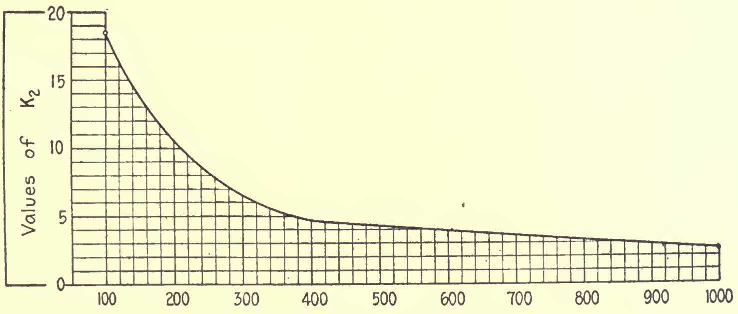

FIG. 240.

307. Hele-Shaw Clutch.—A sectional view of the Hele-Shaw multiple conical-disc clutch as used on motor cars is shown in Fig. 241. The driving and driven discs have a V-shaped annular groove, the sides of which form the surfaces in contact. The phosphor-bronze driving discs are provided with notches on the outer periphery which engage with suitable projections b on the pressed steel clutch casing a. The mild steel driven discs have notches on the inner bore which engage with the corresponding projections on the steel spider c. This spider is splined to the driving shaft, as shown in Fig. 241. The V groove in the discs permits a free circulation of oil, and at the same time insures fairly rapid dissipation of the heat generated when the clutch is allowed to slip. The details of the mechanism used for operating the clutch are shown clearly in the figure.

Analysis of the Hele-Shaw clutch.—Since the surfaces of contact are frustums of cones, the action of the Hele-Shaw clutch is similar

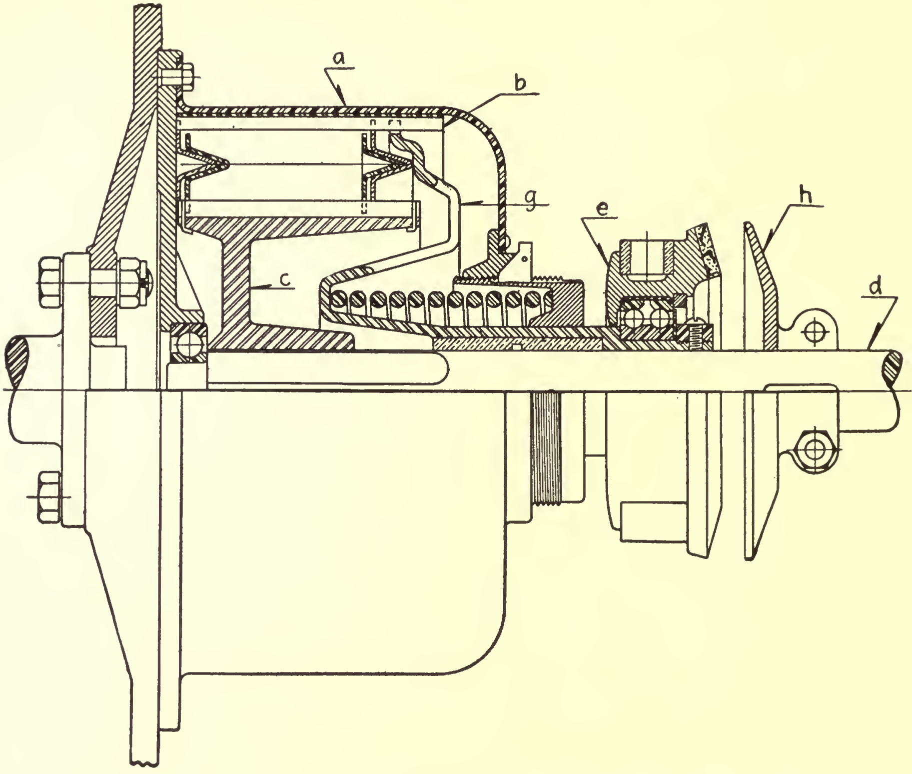

FIG. 241.

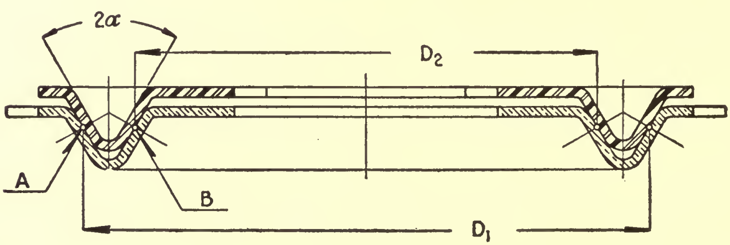

FIG. 242.

to that of an ordinary cone clutch; hence the formulas derived in Art. 295 are applicable. In Fig. 242 are shown a pair of discs as used in the Hele-Shaw clutch. Applying the principles discussed in Art. 295, we find that the magnitude of the moment of friction M_1 for the frustum of the outer cone is

$$M_1 = \frac{\mu P D_1}{4 \sin \alpha} \tag{451}$$

and that on the inner cone is

$$M_2 = \frac{\mu P D_2}{4 \sin \alpha} \tag{452}$$

The moment of friction for one friction surface is the sum of M_1 and M_2, and for s surfaces the total torsional moment that the clutch is capable of transmitting is given by the following expression:

$$M = \frac{s\mu P}{4 \sin \alpha} \left(D_1 + D_2\right) \tag{453}$$

As now constructed, the number of discs used in the standard sizes of Hele-Shaw motor-car clutches is always odd, ranging from 15 to 33, and s in (453) *is always one less than the total number of discs used.*

The horse power transmitted by a Hele-Shaw clutch may be calculated by means of the following formula:

$$H = \frac{s\mu PND}{126{,}060 \sin \alpha}, \tag{454}$$

in which D denotes the mean diameter of the discs as shown in Fig. 242, and N denotes the revolution per minute.

308. Ideal Multi-cone Clutch.—A clutch of the conical-disc type having but one disc was recently placed on the market by The Akron Gear and Engineering Co., of Akron, O. This clutch is shown in Fig. 243 in the form of a friction coupling, connecting shafts a and h. The driving shaft a has keyed to it a sleeve b to which the steel casting cone c is keyed. The internal surface of cone c comes in contact with the cone d, while the outer surface comes in contact with the conical bore of the casing g. The part of the clutch casing marked f is screwed onto the casing g, and is equipped with lugs on the inner surface. These lugs cause the cone d to rotate with f, and at the same time permit d to be moved in an axial direction by the operating mechanism. To provide adjustment for wear at the contact surfaces, the cone d is screwed onto the ring e. This ring, held central by the casing f, is provided with a series of slots on its periphery, into which the set screws l may be inserted after the adjustment for wear has been made.

The axial pressure forcing the cones *d*, *c*, and *g* together is that due to a series of roller toggles that are operated by the sliding sleeve *m*. In disengaging the clutch, the rollers *n* are moved towards the center of the shaft and come in contact with the raised part of the lugs *o*, which are cast integral with the ring *e*. As a result, the cone *d* is pulled out of engagement. Since the casing stands idle when the clutch is disengaged, it may be partially filled with oil, thus causing the driving cone *c* to run in oil and insuring good lubrication at the surfaces in contact.

309. Moore and White Clutch.—In Fig. 244 is shown a friction coupling in which the disc *c* is fitted with hardwood blocks, the

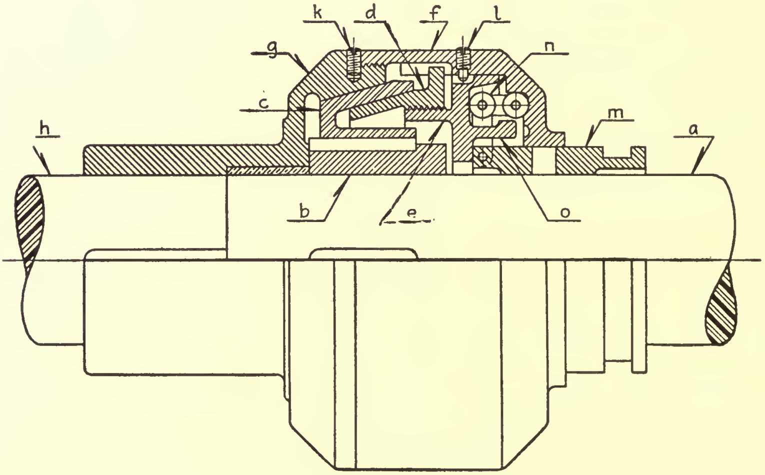

FIG. 243.

ends of which are brought into contact with the flanged hub *d* and ring *e* through the operation of the double toggle mechanism. Suitable lugs on the disc *c* engage corresponding recesses on the flange *b*, thus causing *c* to rotate with the latter, and at the same time permitting it to move in an axial direction. The surface of the wooden blocks in contact with *d* is flat, while the end in contact with the ring *e* is in the form of a double cone, as shown in the figure. The clutch is provided with a series of springs between the hub *d* and ring *e*, which prevent excessive wear of the friction surfaces when the clutch is disengaged.

Another form of combined conical-disc clutch, known as a slip gear, is shown in Fig. 152, and a description of it is given in Art. 231.

The relation between the design constant K_2 and the mean velocity of the friction surfaces for the type of clutch illustrated in Fig. 244 is shown graphically in Fig. 245.

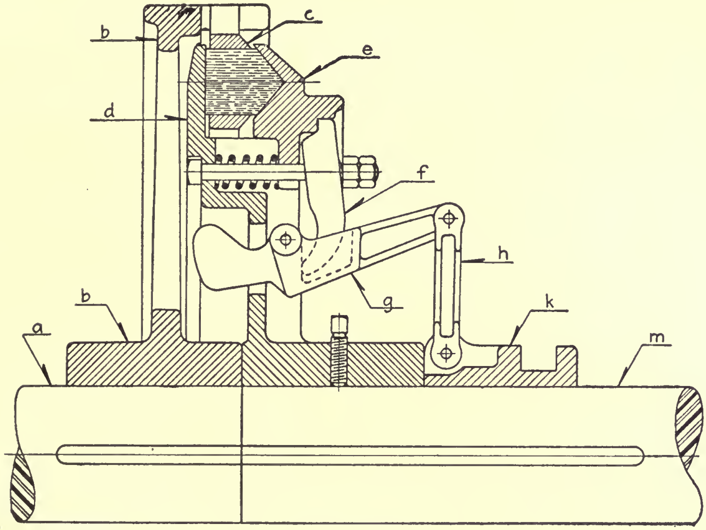

FIG. 244.

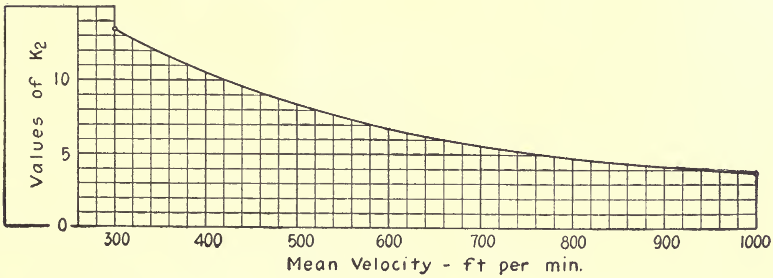

FIG. 245.

RIM CLUTCHES

A large number of different forms of rim clutches are manufactured, and apparently they vary only in the form of the rim or in the method of gripping the rim. A study of commercial rim clutches leads to the following classification: (*a*) block; (*b*) split-ring; (*c*) band; (*d*) roller.

BLOCK CLUTCHES

Block clutches are used chiefly on line shafts and countershafts, although there are several designs that have given good service on machine tools. Examples of the former type are shown

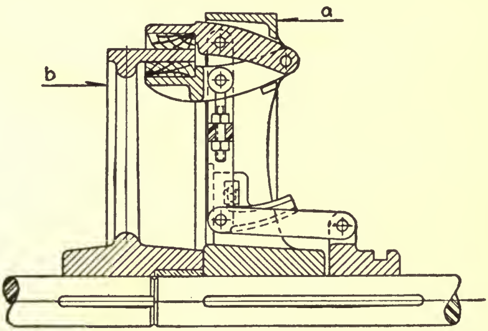

Fig. 246.

in Figs. 246, 247, and 248, while the latter type is represented in Fig. 249.

310. Transmission Block Clutches.—(*a*) *Ewart clutch*—In Fig. 246 are shown the constructive features of the well-known

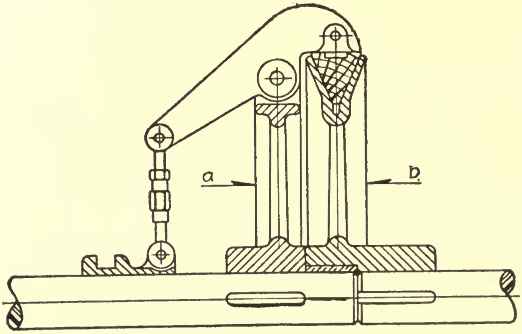

Fig. 247.

Ewart clutch. The levers that move the friction blocks are located inside of the clutch rim *a*, thus decreasing the air resistance at high speeds, and at the same time making it less dangerous to workmen than the type of clutch in which the operating levers and links are exposed. The Ewart clutch is fitted with either two, four, or six friction blocks, depending upon the power that is to be transmitted.

(*b*) *Medart clutch.*—The type of clutch coupling shown in Fig. 247 differs from the Ewart clutch in that the friction blocks are of V shape. Furthermore, the operating levers are exposed, thus making this clutch more or less dangerous. For transmitting large powers, the Medart clutch is made as illustrated in Fig. 247, while for small powers, the clutch rim *b* is made flat.

(*c*) *Hunter clutch.*—The Hunter clutch coupling, shown in Fig. 248, has two cast-iron shoes *c* and *d* which are made to clamp

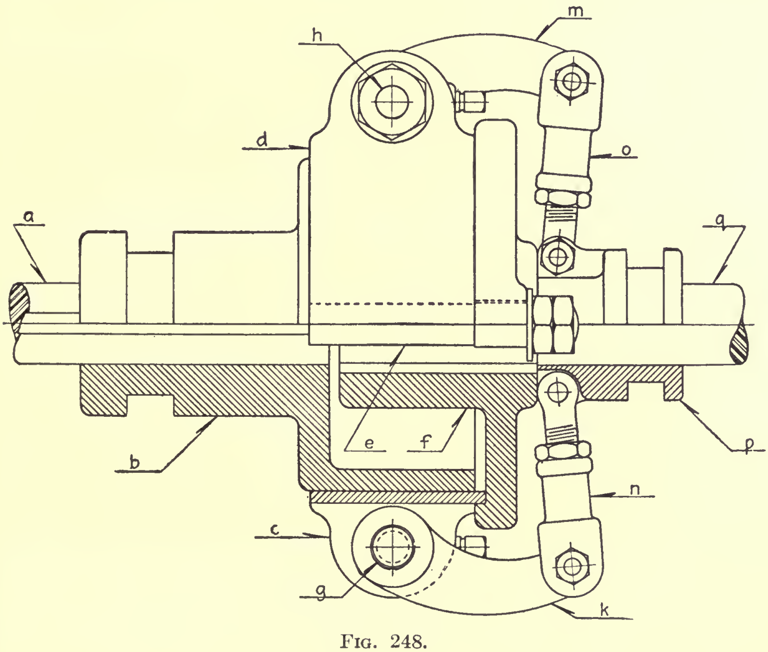

FIG. 248.

the drum *b* when the screws *g* and *h* are rotated by the levers *k* and *m*. Each of the shoes is fitted with a driving pin, by means of which the shoes *c* and *d* are made to revolve with the flanged hub *f* and the shaft *q*. The holes in the flange *f*, through which the driving pins pass, are elongated in order that the shoes may move freely in a radial direction. The drum *b* is fastened to the shaft *a* by a feather key, thus permitting it to be drawn out of contact with the shoes *c* and *d* when the coupling is not transmitting power. The levers k and m are operated by the usual links and sliding sleeves as shown in Fig. 248.

(*d*) *Machine-tool block clutch.*—In general, a block clutch used on machine tools consists of a shell running loose on the shaft, into which are fitted two brass or bronze shoes. These shoes are fastened loosely to a sleeve, which in turn is splined to the shaft. The shoes are pressed against the inner surface of the shell by means of an eccentric, screw, or wedge. Due to the compactness of such clutches, they are well adapted for use where the space is limited, as for example between the reversing bevel gears of a feed mechanism as shown in Fig. 249. In the design illustrated by Fig. 249, the enlarged bore of the bevel gears a forms the shell against which the shoes c and d are pressed by

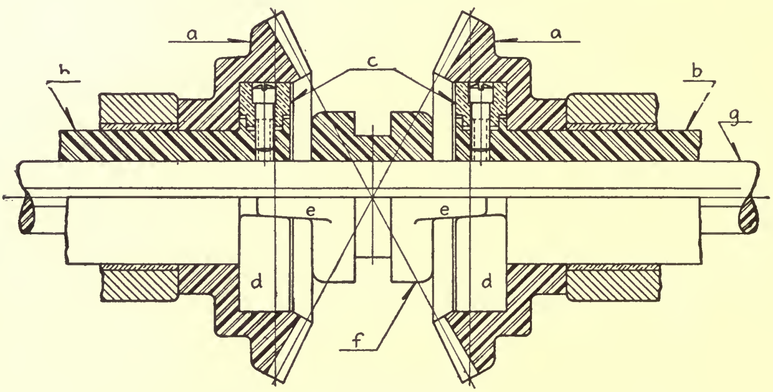

Fig. 249.

the sliding sleeve f. This sleeve is integral with the double wedges e that are fitted to slide along the inclined surface of the shoes c and d, as shown in the figure. The friction shoes are fastened by filister head machine screws to the sleeves b and therefore rotate with them. Sleeve f is fastened to the shaft by means of a feather key. The shaft g may serve either as the driving or the driven member.

311. Analysis of Block Clutches.—In order to arrive at an expression for the moment of the frictional resistance of a block clutch, some assumption regarding the distribution of the contact pressure, as well as the variation in the coefficient of friction, must be made. As in the case of axial clutches, experimental data are lacking, and in what follows, we shall assume that *the normal wear at any point of the contact surface is proportional to the work of friction, and that the coefficient of friction remains constant.*

(*a*) *Grooved rim.*—For our discussion, we shall assume a block clutch in which the rim is grooved as shown in Fig. 250. The total moment that a clutch of this type will transmit is equal to the number of blocks in contact multiplied by the frictional moment of one block, the magnitude of which may be determined as follows:

In Fig. 250 is shown a grooved clutch rim against which a single block is held by the force P; hence, the normal force acting

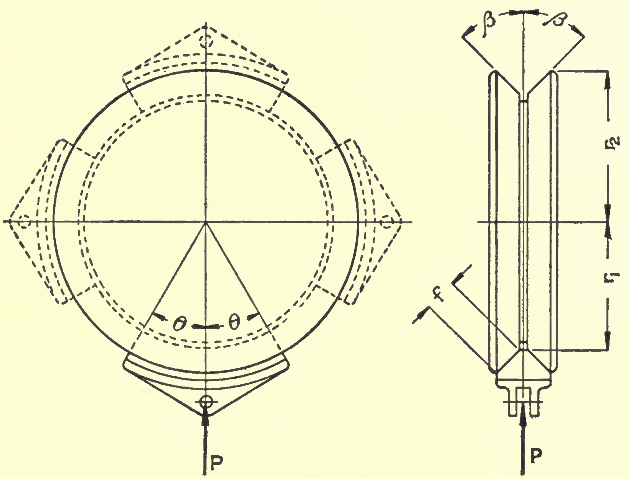

FIG. 250.

upon an elementary area of the surface in contact is $prd\theta df$, and the component of this pressure parallel to the line of action of the radial force P is given by the expression

$$dP = pr \sin\beta \cos\theta \, d\theta \, df$$

Hence $$P = 2 \sin\beta \int\int pr\cos\theta \, d\theta df \tag{455}$$

Since the normal wear n at any point is assumed to be proportional to the work of friction, we get

$$n = kpr$$

If the surfaces in contact remain conical, it follows that the wear

h in a direction parallel to the line of action of P is constant; hence, the normal wear is

$$n = h \sin \beta \cos \theta$$

Combining the two values of n just given, we obtain the relation

$$p = \frac{C \cos \theta}{r}, \tag{456}$$

in which $C = \dfrac{h \sin \beta}{k}$. Substituting (456) in (455), and integrating between the proper limits, the following expression for P is found:

$$P = 2\, Cf \sin\beta\, (\theta + \sin \theta \cos \theta) \tag{457}$$

The moment of the force of friction acting upon the elementary area is

$$dM = \mu p r^2 d\theta df = \frac{\mu C}{\cos \beta}\, r \cos\theta d\theta dr$$

The total moment per block is therefore:

$$M = \frac{2\, \mu C \sin \theta}{\cos \beta}\, (r_2^2 - r_1^2) \tag{458}$$

Combining (457) and (458) in order to eliminate the constant C, we obtain the following expression for the total moment per block:

$$M = \frac{\mu PD}{\sin \beta} \left[\frac{\sin\theta}{\theta + \sin \theta \cos \theta}\right] \tag{459}$$

Since (459) gives the magnitude of the frictional moment that each block will transmit, assuming that the radial force per block is P, the total moment that the clutch is capable of transmitting is obtained by multiplying (459) by the number of blocks.

(*b*) *Flat rim.*—The majority of the block clutches in common use have a flat rim; hence making $\beta = 90$ degrees in (459), the frictional moment transmitted by each block becomes

$$M = \mu PD \left[\frac{\sin \theta}{\theta + \sin \theta \cos \theta}\right] \tag{460}$$

The total moment is obtained in the same manner as outlined in the preceding paragraph.

To facilitate using (459) and (460), the function in the brackets may be evaluated for different angles and the results thus obtained may be plotted. Fig. 251 gives values of $\dfrac{\sin\theta}{\theta + \sin\theta \cos\theta}$ for various values of θ.

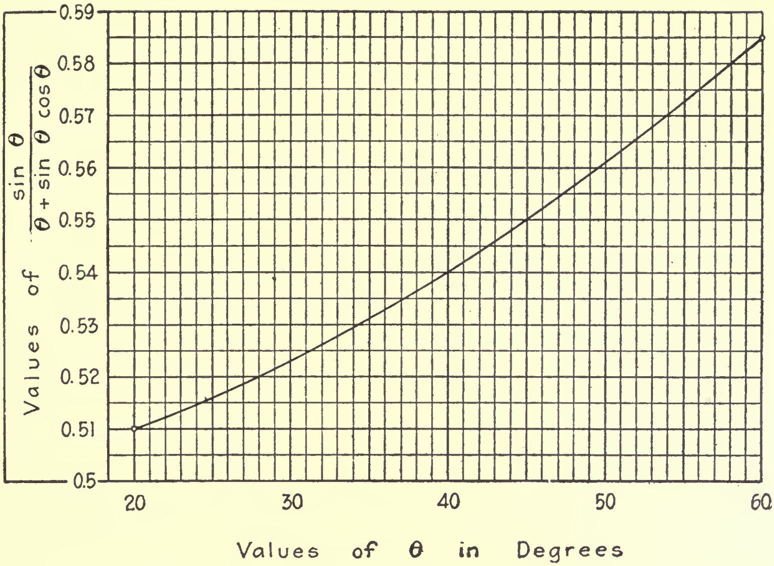

Fig. 251.

SPLIT-RING CLUTCHES

312. Machine-tool Split-ring Clutches.—Split-ring clutches are used for all classes of service but their greatest field of application appears to be in connection with machine tools, or in places where the diameter of the clutch as well as the space taken up by

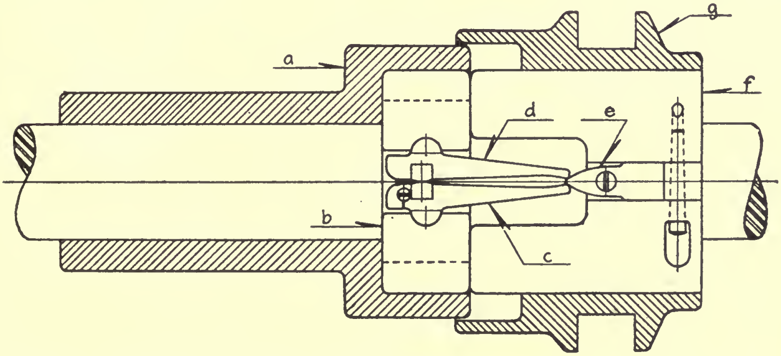

Fig. 252.

the clutch is limited. Such clutches are shown in Figs. 252 to 254, inclusive. An inspection of these figures shows that in

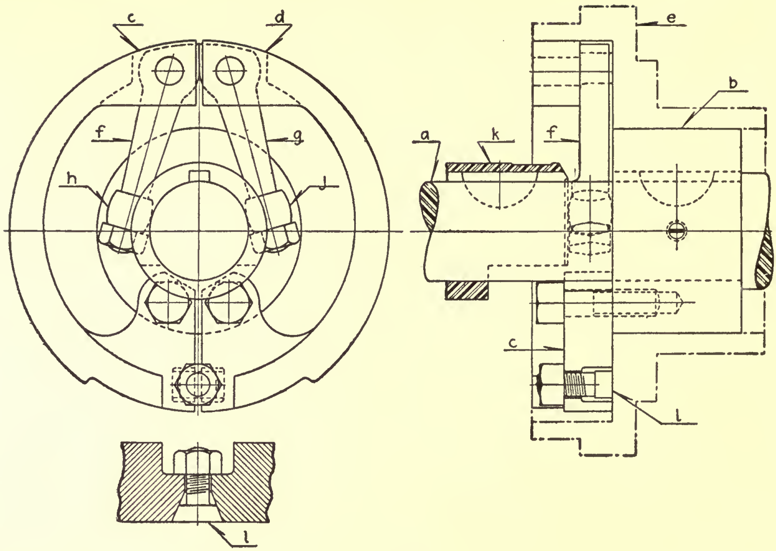

FIG. 253.

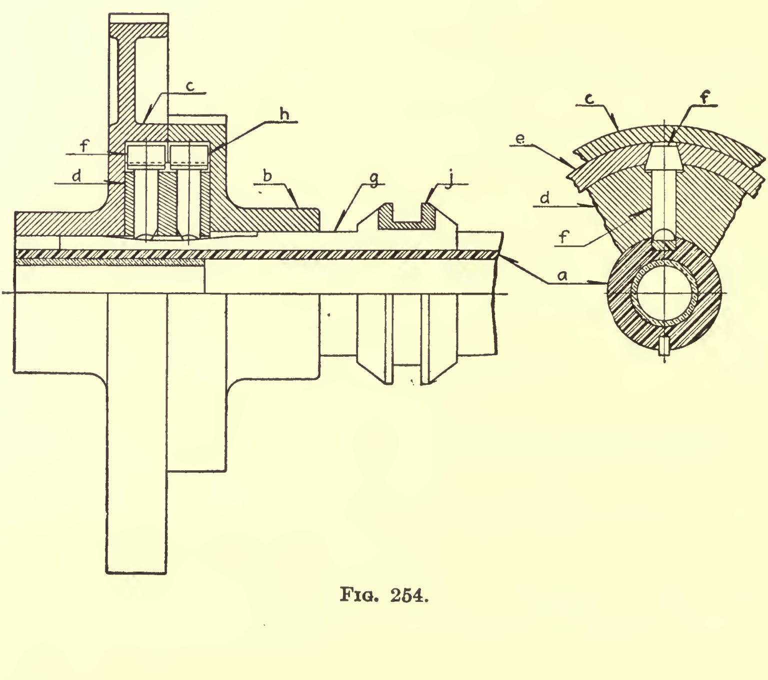

FIG. 254.

general a split-ring clutch consists of an outer shell running loose on a shaft or sleeve; into this shell is fitted a split ring. The latter may be expanded by the action of a pair of levers as shown in Figs. 252 and 253, or by means of a wedge as shown in Fig. 254. A sliding sleeve, operated by a suitable lever, forms a convenient means of engaging the split ring with the outer shell. The outer shell may be in the form of a gear as shown in Figs. 253 and 254, or it may form part of a pulley.

The well-known Johnson clutch shown in Fig. 252 is used on countershafts and on machine tools. It has been adopted by several manufacturers of machine tools and other classes of machinery. The clutch shown in Fig. 253 is that used by the Greaves Klusman Tool Co. on their all-geared-head lathe. The split clutch represented in Fig. 254 is that used by the American Tool Works on the double back-gear of their high-duty lathe.

313. Analysis of a Split-ring Clutch.—(*a*) *Moment of friction.*—For a split-ring clutch, it seems reasonable to assume that the pressure exerted by the ring upon the clutch shell is uniformly distributed over the area in contact; hence, the expression for the moment of the force of friction acting upon the elementary area is

$$dM = \frac{\mu p f D^2 d\theta}{4}, \tag{461}$$

in which D denotes the diameter of the split ring, f its face, and p the normal pressure per unit of area of the ring.

The split ring has an angle of contact with the shell of somewhat less than 360 degrees, but for all practical purposes we may assume it as equal to 360 degrees. Assuming the coefficient of friction μ as constant, the total torsional moment transmitted by the clutch is obtained by integrating (461). Thus

$$M = \frac{\mu \pi p f D^2}{2} \tag{462}$$

(*b*) *Horse power transmitted.*—The horse power transmitted by the clutch at N revolutions per minute is

$$H = \frac{\mu \pi^2 p f N D^2}{396{,}000} \tag{463}$$

Since μ and p are constant for any given case, their product may be denoted by a new constant, as K_4. Hence

$$H = \frac{K_4 f N D^2}{40{,}120} \tag{464}$$

(c) *Force required to spread the split ring.*—The inside diameter of the shell of a split-ring clutch is generally made $\frac{1}{64}$ to $\frac{1}{32}$ inch larger than the diameter of the ring. Due to this fact, a certain part P_1 of the force P exerted by the operating mechanism is used in spreading the ring. As soon as the ring comes into contact with the shell, a force P_2 is required which will press the ring against the shell, thereby causing the frictional moment necessary to transmit the desired power. The sum of P_1 and P_2 must evidently equal the magnitude of the force P.

1. *Determination of* P_1.—In the following analysis we shall assume that the thickness of the ring is small relative to its radius, and that the ring will readily conform to the bore of the shell.

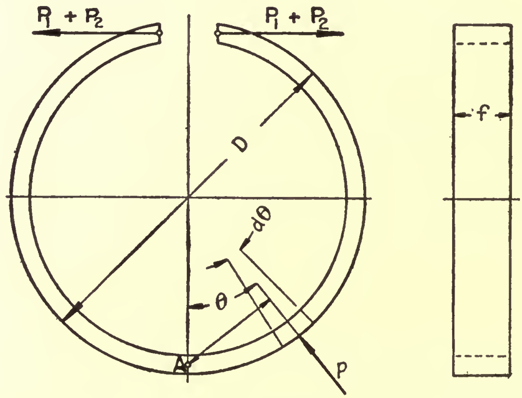

FIG. 255.

According to Bach's "Elasticität und Festigkeit," the moment of the force P_1 about the section at A in Fig. 255 is given by the following expression:

$$2\,P_1 r_1 = EI\left[\frac{1}{r_2} - \frac{1}{r_1}\right], \tag{465}$$

in which r_1 and r_2 denote respectively the original and final radii of the ring. Therefore, the magnitude of P_1 is

$$P_1 = \frac{EI}{2\,r_1}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \tag{466}$$

2. *Determination of* P_2.—The pressure upon an elementary length of the ring is $\frac{pfD\,d\theta}{2}$, and the moment of this pressure about the section at A in Fig. 255 is

$$dM_A = \frac{pfD^2 \sin \theta \, d\theta}{4}.$$

Integrating between the proper limits, we find that the bending moment upon the ring at the section through A has the following magnitude:

$$M_A = \frac{pfD^2}{2} \qquad (467)$$

Since this bending moment must equal that due to the force P_2, it is evident that

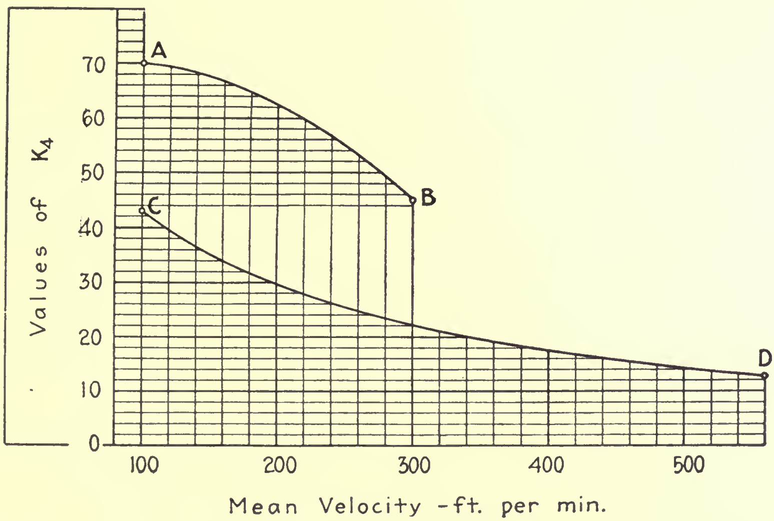

FIG. 256.

$$P_2D = \frac{pfD^2}{2},$$

from which

$$P_2 = \frac{pfD}{2} \qquad (468)$$

Combining (462) and (468), the magnitude of P_2 in terms of M, μ, and D is as follows:

$$P_2 = \frac{M}{\mu\pi D} \qquad (469)$$

314. Study of Split-ring Clutches.—From a study of a considerable number of split-ring clutches of different types, it was found that in nearly all cases the ring and shell are made of cast iron. In the majority of the designs, the ring is of the expanding type shown in Figs. 252 to 254, inclusive. The contracting-ring type is also used, but not to any great extent. An analysis, similar to that made of the cone and disc clutches, was made of a number of split-ring transmission clutches. From the information furnished by two manufacturers, it was possible to determine the value of the design constant K_4 for the various clutches. The graph AB of Fig. 256 represents the results obtained on five clutches of the contracting split-ring type made by one manufacturer. The graph CD represents the results obtained on eleven clutches of the same type as the others, but made by another manufacturer.

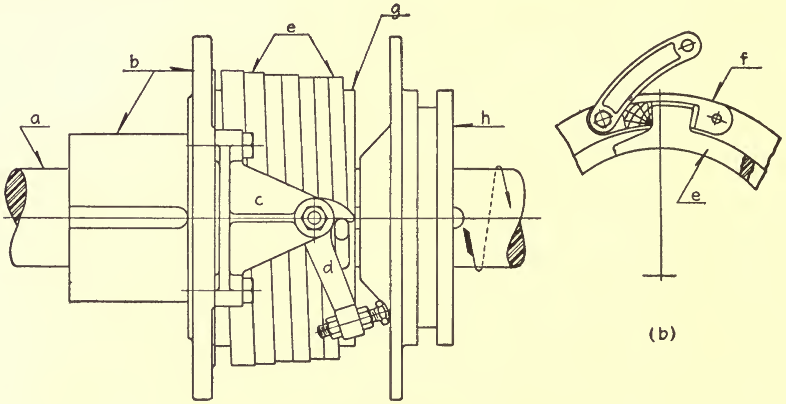

Fig. 257.

BAND CLUTCHES

Band clutches are usually installed when it is necessary to transmit heavy loads accompanied by shocks, as for example, in the drives of rolling mills and heavy mine hoists. In general, a band clutch consists of a flexible steel band, either plain or faced with wood or asbestos fabric, one end of which is fixed and the other is free to move in a circumferential direction. Due to the pull exerted by the operating mechanism on the free end of the band, the latter is made to grip the driving or driven member.

315. Types of Band Clutches.—(*a*) *Farrel clutch.*—A band clutch in which the band is given several turns around the driving drum is shown in Fig. 257. In this design, the driving drum is keyed rigidly to the shaft a and both rotate in the direction indicated by the arrow. The unlined steel band e is given approximately six and one-half turns around the drum *g*. One end of this band is fastened to the flanged hub *b* in the manner shown in Fig. 257(*b*), and the free end is operated by the special lever *d* through the medium of the conical ended shipper sleeve *h*.

(*b*) *Wellman-Seavers-Morgan clutch.*—Another form of single band clutch, installed on heavy mine hoists by the Wellman-Seavers-Morgan Co., is shown in Fig. 258. The band is lined

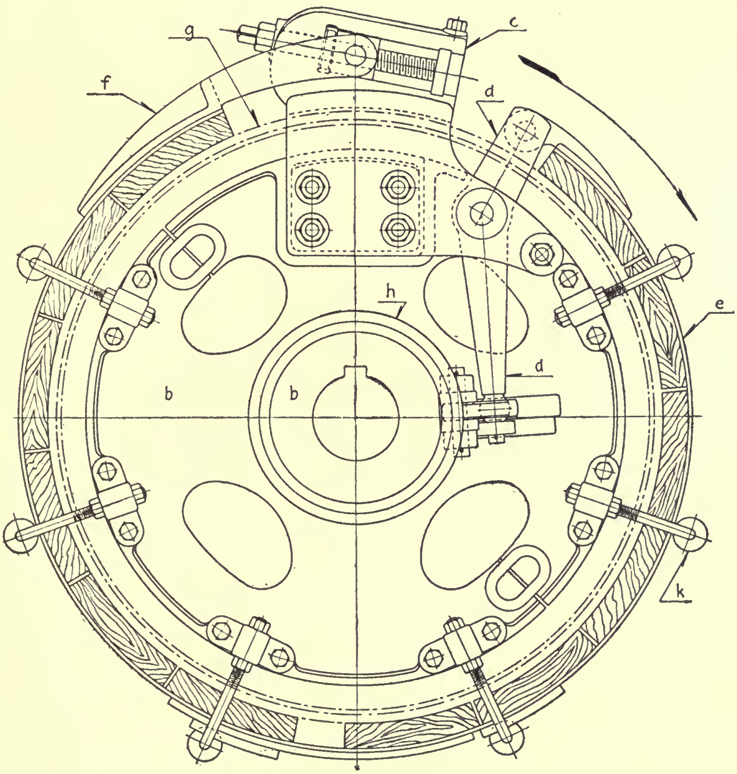

FIG. 258.

with wood and has an angle of contact on the clutch ring *g* of approximately 300 degrees. The flanged hub *b*, upon which the various parts of the clutch proper are mounted, is keyed to the driving shaft, while the hoisting drum, to which the clutch ring is bolted, runs loose on the shaft.

(*c*) *Litchfield clutch.*—In Fig. 259 is shown a two-band clutch designed by the Litchfield Foundry and Machine Co. for use on mine hoists. The bands are lined with wood and each band has an arc of contact with the drum g approximating 140 degrees.

316. Analysis of a Band Clutch.—The principles underlying the design of a band clutch are similar to those employed in de-

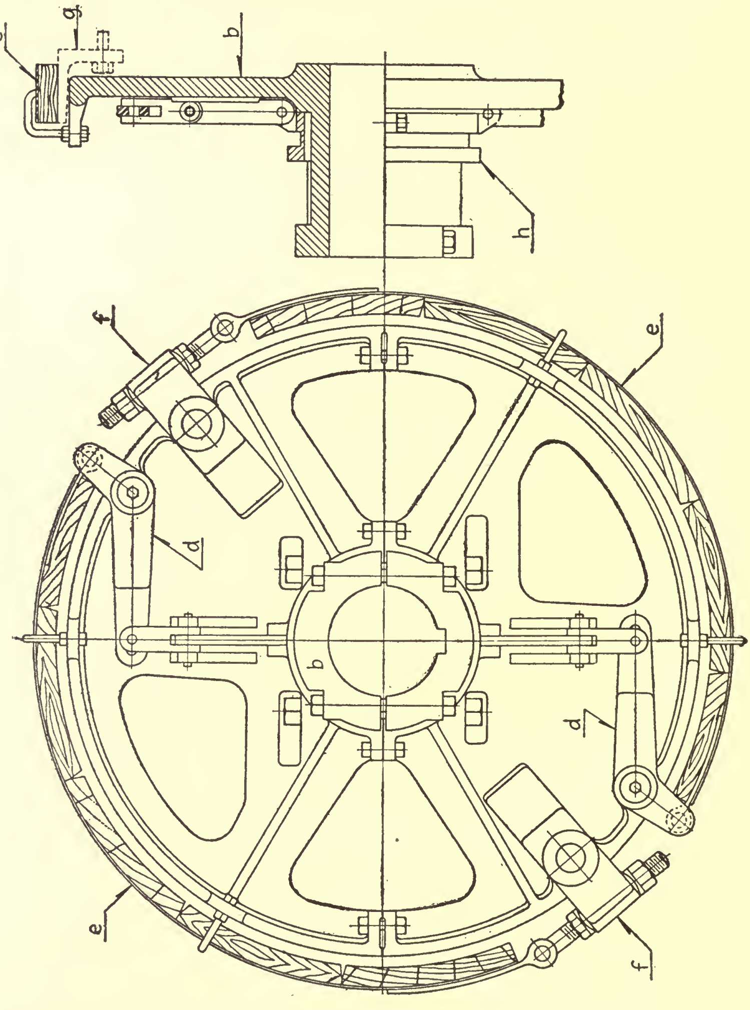

FIG. 259.

termining the power transmitted by a belt. In other words, the ratio of the tight to the loose tensions in the band or bands is given by the following expression:

$$\frac{T_1}{T_2} = e^{\mu\theta}, \tag{470}$$

in which T_1 and T_2 denote the tight and loose tensions, respectively, μ the coefficient of friction, and θ the angle of contact. The value of the coefficient of friction μ for a steel band on a cast-iron drum may be assumed as 0.05 when a lubricant is used, and 0.12 when no lubricant is used. For a wood-faced band, μ may be assumed as 0.3.

ROLLER CLUTCH

317. Horton Clutch.—The type of rim clutch shown in Fig. 260 is known as the Horton roller clutch, and is used to some extent

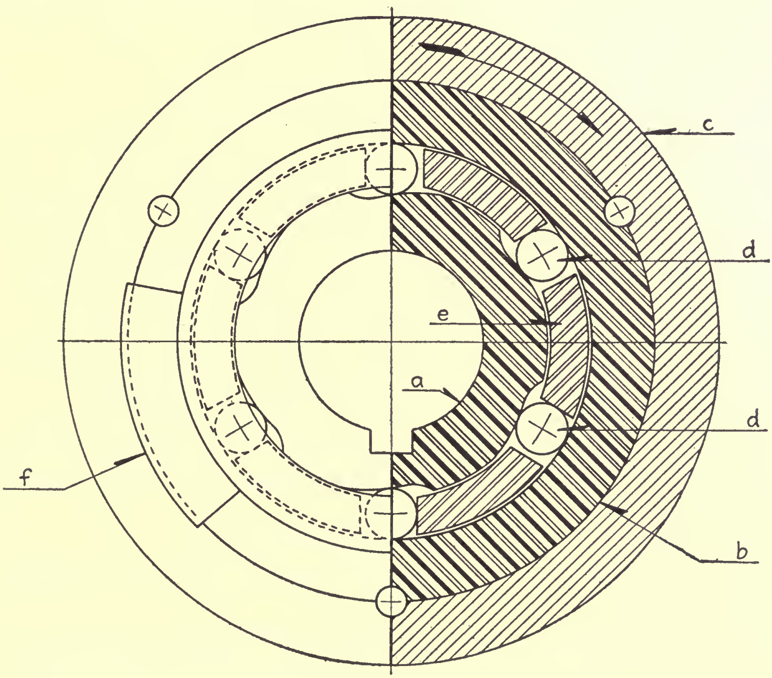

FIG. 260.

on punching presses. The cam a is keyed to the crankshaft, and upon its circumference are cut a number of recesses which form inclined planes. The rollers d, rolling up these inclined planes due to the action of the shell e, wedge themselves between a and the clutch ring b, thus causing the crankshaft to rotate with the flywheel. The ring b is keyed to the flywheel or the driving gear. The rollers are held in place and controlled by the shell e, which is connected with the crankshaft by means of a spring. The latter is not shown in the figure. The lug *f* on the shell *e* engages a latch or buffer which is operated by the treadle on the machine.

The method of operation of this roller clutch is as follows: At the instant the treadle releases the shell *e*, the spring rotates the latter around the shaft a short distance, carrying the rollers with it. This action causes the rollers to wedge between the cam *a* and the ring *b*, thus forming a rigid connection between the flywheel and the crankshaft. To disengage the clutch, the treadle is released and it in turn causes the latch or buffer to strike the lug *f*, thus forcing the cage and rollers back into the original position, and permitting the flywheel to rotate freely again.

CLUTCH ENGAGING MECHANISMS

318. Requirements of an Engaging Mechanism.—From the descriptions of the various types of clutches given in the preceding articles of this chapter, it is evident that clutches are engaged by a lever or shipper arm through the medium of an engaging mechanism which is capable of increasing the leverage rather rapidly toward the end of the lever displacement. In the analysis of the various types of clutches, the force required at the end of the operating lever was not discussed, since its magnitude depends directly upon the engaging mechanism.

In Figs. 223, 248, 261, and 262 are shown four types of engaging mechanisms in which the following important requirements are fulfilled:

(*a*) The arc of lever movement is not excessive.

(*b*) The leverage increases rapidly toward the end of the lever displacement.

(*c*) The engaging mechanism is self-locking, and therefore no pawl and ratchet are necessary to hold the clutch in engagement.

(*d*) The force required at the end of the operating lever in order to engage the clutch is not excessive. The magnitude of this force may be assumed to vary from 15 to 20 pounds in the case of an overhead clutch installation. For large clutches these values may have to be increased somewhat.

319. Analysis of Engaging Mechanisms.—In the majority of engaging mechanisms, graphical methods are generally found more convenient for determining the magnitude of the force required at the end of the operating lever. In Fig. 261(*b*) is given the graphical analysis of the forces acting on the mechanism shown in Fig. 261(*a*). The vector *AB* represents the force *Q* exerted upon the spool *b* by the operating lever. Assuming that the clutch is provided with two toggle levers *c*, only one being shown in Fig. 261(*a*), the force exerted by *b* upon *c* is represented by the vector *AC*. The lever *c* is acted upon by three forces as shown in the figure. The magnitude of *P* is represented by the vector *CD*. The analysis of the forces acting upon the second toggle lever is given by the triangle *BCE*, in which the vector *EC* represents the magnitude of the force corre-

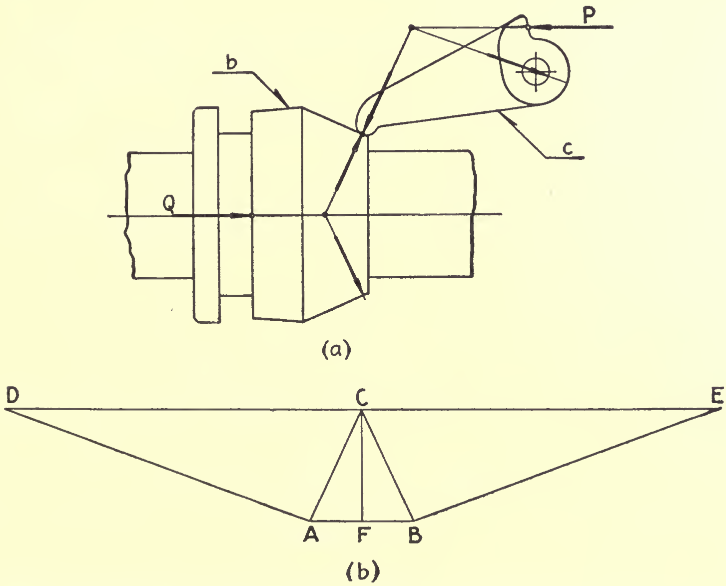

FIG. 261.

sponding to *P*. The magnitude of the axial force produced is given by the vector *ED*.

Analytical methods sometimes are found more convenient than graphical methods, as the following analysis will show. It is required to determine an expression for the force *P* in terms of *Q* in the case of the mechanism shown in Fig. 262(*a*). This mechanism is used on the split-ring clutch shown in Fig. 254. The sliding key *g* is acted upon by three forces as follows: *Q*, *a on g*, and *f on g*. In this case, *Q* denotes the force that the operating lever exerts upon the collar *j* shown in Fig. 254.

Taking components of Q and f *on* g in a direction at right angles to a *on* g, we obtain the relation:

$$Q \cos \varphi = (f \text{ on } g) \sin (\alpha + 2 \varphi)$$

from which

$$f \text{ on } g = \frac{Q \cos \varphi}{\sin (\alpha + 2 \varphi)} \tag{471}$$

At the upper end of the sliding wedge f, the ring e produces a force that will act vertically downward as shown in Fig. 262. This force has a magnitude given by the expression:

$$F = 2P \tan (\beta + \varphi) \tag{472}$$

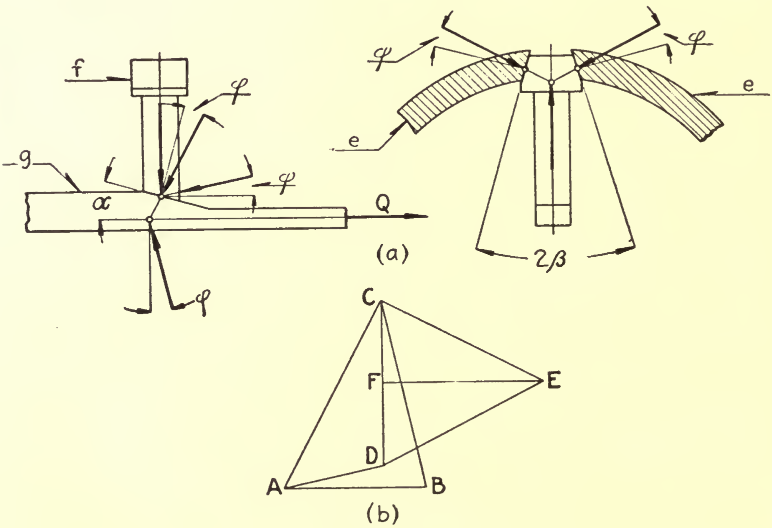

FIG. 262.

in which P denotes the magnitude of the force tending to spread apart the ring e. Taking components of F and f *on* g in a direction at right angles to d *on* f, we have

$$F \cos \varphi = (f \text{ on } g) \cos (\alpha + 2 \varphi)$$

from which we find that

$$F = \frac{Q}{\tan (\alpha + 2 \varphi)} \tag{473}$$

Combining (472) and (473), we obtain the following relation between P and Q:

$$P = \frac{Q}{2 \tan (\alpha + 2 \varphi) \tan (\beta + \varphi)} \tag{474}$$

The graphical analysis for the mechanism illustrated by Fig. 262(*a*) is shown in Fig. 262(*b*). The vector *AB* represents the magnitude of the force *Q*. The force *F* is represented by *CD*, and *P* by the vector *FE*.

For the analysis of a screw operated mechanism, consult Art. 298.

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CHAPTER XVII

BRAKES

The function of a brake is to absorb energy by the creation of frictional resistance, and thereby reduce the speed of a machine or bring the machine to a state of rest. The absorbed energy must equal that given up by the live load and all moving parts that are being retarded. Friction in bearings and between other moving parts always helps a brake.

320. General Equations.—The energy absorbed by a brake is made up of the following factors: (1) The work given up by the live load; (2) the energy given up by the rotating parts. To determine an expression for the tangential force required on the brake sheave so as to bring a load to rest, we shall assume the case of a geared hoisting drum lowering a load.

Let D = diameter of the drum.
W = the load on the drum.
T = tangential force on the brake sheave.
d = diameter of the brake sheave.
n = ratio of the gearing between the drum and the brake sheave.
t = number of seconds the brake is applied.
v = linear velocity of the load in feet per second.
η = efficiency of the mechanism.

To bring the live load to a stop in t seconds requires an expenditure of $\frac{Wv}{2}\left[\frac{v}{g} + t\right]$ foot-pounds of work at the drum. In addition to absorbing the work due to the live load, the brake in bringing the machine to a stop must also absorb the kinetic energy of all of the rotating parts. The energy due to the rotating parts is $\frac{1}{2} I\omega^2$, in which I is the moment of inertia of the rotating parts referred to the axis of the brake, and ω is the angular velocity of these parts in radians per second. It is usually possible to obtain the value of $\frac{1}{2} I\omega^2$ for a rotating mass having a complicated form. The body may be divided into elements in such a way that the kinetic energy of each element is easily calculated; then by summation the total kinetic energy is obtained.

Taking into account the internal friction of the machine, the total energy to be absorbed by the brake in t seconds is given by the following expression:

$$E = \eta \frac{Wvt}{2} + \frac{Wv^2}{2g} + \frac{I\omega^2}{2} \tag{475}$$

The work done by the tangential force T in t seconds is $\frac{ndtvT}{2D}$ foot-pounds. Since the energy given up by the load and rotating parts must equal that absorbed by the brake, we have

$$T = \frac{D}{nd}\left[\eta W + \frac{Wv}{gt} + \frac{I\omega^2}{tv}\right] \tag{476}$$

The minimum value of the force T on the brake sheave occurs when the load has been brought to a state of rest and its magnitude is evidently

$$T_0 = \frac{\eta DW}{nd} \tag{477}$$

321. Classification.—Brakes are made in a variety of forms and no definite classification can be given. The order in which they are discussed in this chapter is as follows:

(*a*) Block brakes.
(*b*) Band brakes.
(*c*) Axial brakes.
(*d*) Mechanical load brakes.

BLOCK BRAKES

322. Single- and Double-block Brakes.—The common form of block brake has a single block pressing against the sheave, thus causing an excessive pressure upon the shaft bearings. Such a brake is shown in Fig. 263. The pressure upon the shaft bearings caused by the block in this form of brake may be practically eliminated by the use of two brake blocks located diametrically opposite to each other. An arrangement of this kind is used on cranes, elevators, and mine hoists. In Figs. 264 to 267, inclusive, are shown various designs of double-block brakes all of which are drawn to scale. The brakes illustrated by Figs. 266 and 267 are used on mine hoists, and are commonly called *post brakes.* The first of these post brakes was designed and built by an English manufacturer for a large mine hoist; while the second, designed and built by the Nordberg Engineering Co., is used on a large

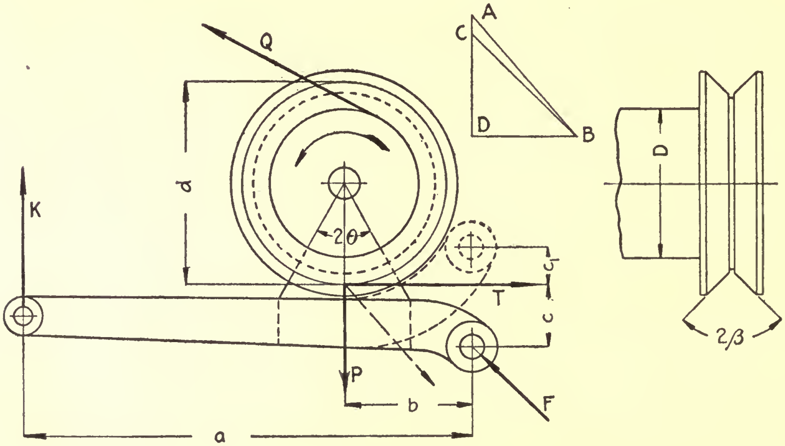

Fig. 263.

hoist installed at the Tamarack mine at Calumet, Mich. As shown in Fig. 267, the posts of the Nordberg brake are held in

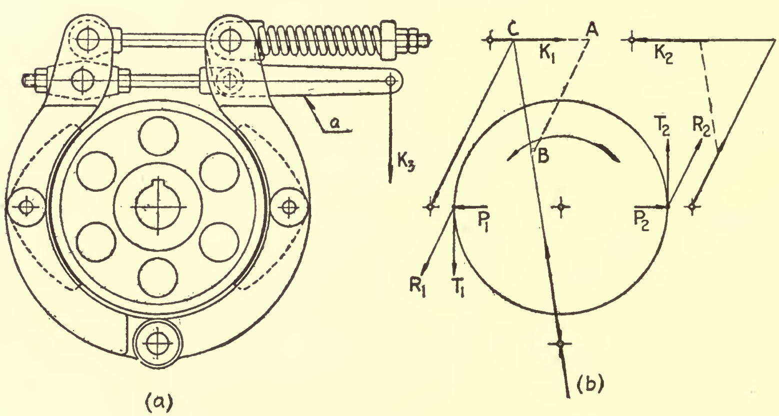

Fig. 264.

position by the swinging links *m*, *n*, and *o*. The blocks or shoes are made of steel casting, instead of wood as in the English design shown in Fig. 266.

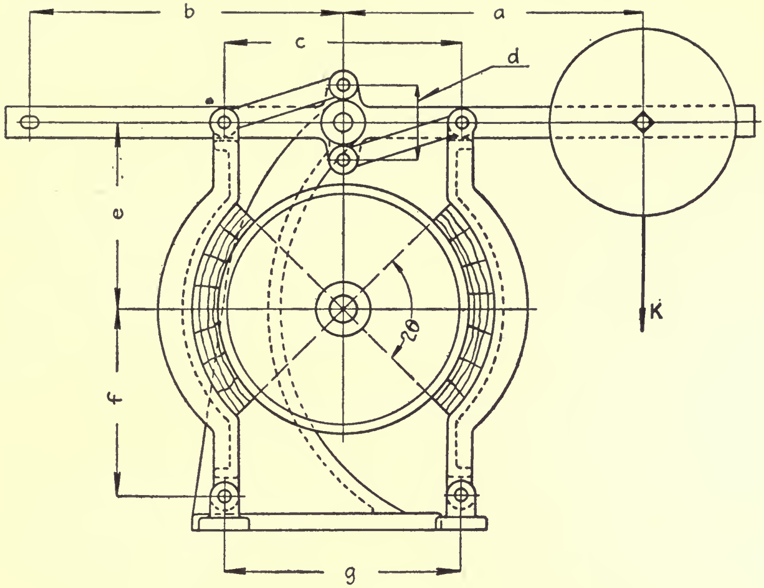

FIG. 265.

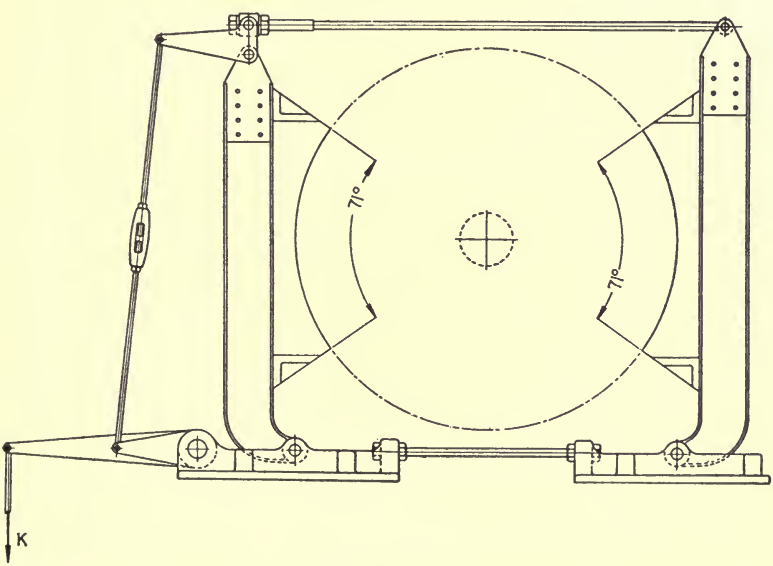

FIG. 266.

323. Analysis of Block Brakes.—In determining the magnitude of the forces coming upon the various members of a block brake, either algebraic or graphical methods may be used. Frequently the latter save considerable time.

Let F = the force acting at the fulcrum of the operating lever.
K = the force applied at the end of the operating lever.
P = the radial force exerted by the sheave upon the block.
T = the tangential resistance upon the block.
μ = the coefficient of friction.

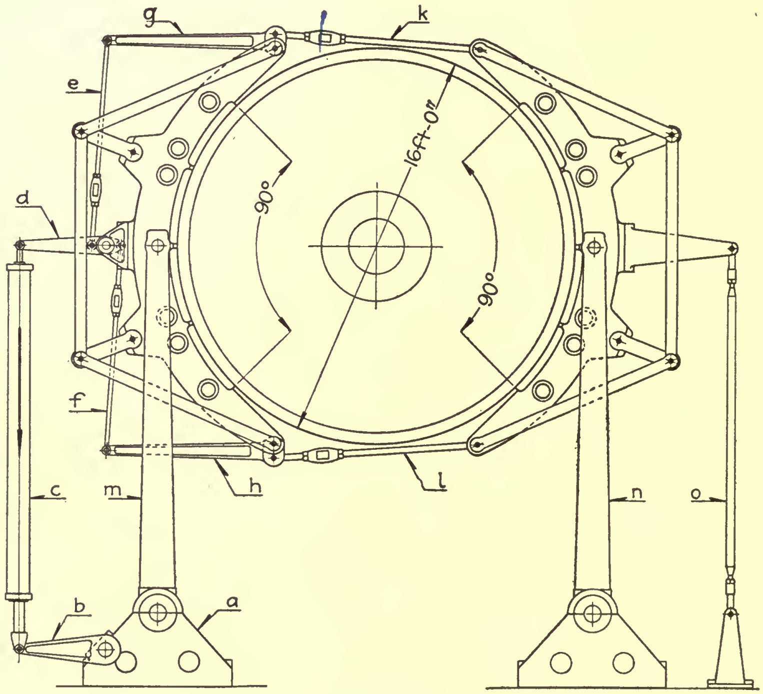

FIG. 267.

Since the action of the block upon the brake sheave is similar to the action between the shoes and sheave of a block clutch, the various formulas derived in Art. 311 may be applied directly. Two cases will be considered, the first involving the action of a grooved sheave and the second the action of a flat sheave.

(*a*) *Grooved sheave.*—A block brake having a grooved sheave is shown in Fig. 263. The moment of the frictional resistance due to the radial force P is given by the following expression:

$$M = \frac{\mu P d}{\sin \beta}\left[\frac{\sin \theta}{\theta + \sin \theta \cos \theta}\right] \tag{478}$$

The product of the factors μ and $\frac{\sin \theta}{\theta + \sin \theta \cos \theta}$ may be treated as a new factor denoted by the symbol μ'. This factor might be termed the *apparent coefficient of friction* between the brake block and sheave.

To determine an expression for the tangential resistance T, divide the moment M by the radius of the sheave, thus

$$T = \frac{2\,\mu' P}{\sin \beta} \tag{479}$$

To determine an expression for the force K applied at the end of the operating lever, treat the latter as the free body and take moments about the fulcrum. Thus for the rotation indicated in Fig. 263, we have

$$K = \frac{Pb - Tc}{a} \tag{480}$$

To calculate the size of the pin at the fulcrum, we must determine the magnitude of the force F coming upon the pin. In general, the graphic analysis affords the most direct means of determining the magnitude of F. Treating the brake lever as the free body, the magnitudes of the various forces are readily obtained by drawing the force diagram $ABDCA$, in which the vectors BC, CA, AD, and DB represent the forces F, K, P, and T, respectively. Having determined the magnitude of the force F, the pin at the fulcrum must be proportioned so that it is capable of resisting the bending moment and bearing pressure coming upon it.

In the above analysis, the frictional resistance on the brake shaft was not considered. The error in any case is small and the method given is the one commonly used. However, when designing the bearings on the brake shaft, the pressure due to the force P should not be neglected.

(*b*) *Flat sheave.*—In the majority of installations, the brake sheave is made with a straight or flat face; hence in the preceding expressions for M and T the angle β becomes 90 degrees. Substituting this value in (479), we get

$$T = 2\,\mu' P \tag{481}$$

The magnitudes of K and F for this case are obtained in a manner similar to that given above.

To facilitate the calculations for the value of the apparent coefficient of friction μ', the graph given in Fig. 251 will be found useful.

324. Graphical Analysis of a Double-block Brake.—It is required to determine graphically the magnitude of the force required to apply the double-block crane brake shown in Fig. 264, assuming that the total moment of the frictional resistance on the brake sheave is known. The following method of procedure is suggested:

(*a*) Determine the relation between T and P for each of the blocks by means of (481); thus

$$\frac{T_1}{P_1} = \frac{T_2}{P_2} = 2\,\mu' \tag{482}$$

Having calculated the value of μ', for the brake under discussion, the actual lines of action of the resultant of T and P on each block may be laid off as shown in Fig. 264(*b*). In addition to the resultant of T and P, each brake block is acted upon by another force which is equal to the resultant but acts in the opposite direction. From this it is evident that each brake block exerts a pressure upon its lever equal to the resultant.

(*b*) Each brake lever is acted upon by three forces, as follows: (1) A pressure due to the spring shown in the figure; (2) the force due to the brake block; (3) the reaction at the fulcrum of the lever. Of the three forces just mentioned, the lines of action of the first two are known, also the magnitude of the second. Since the point of application of the third force is known, the remaining unknown properties of the forces acting upon the levers may readily be determined. Applying the "triangle of forces" to the left-hand lever, the magnitudes of the various forces acting thereon are represented by the sides of the triangle ABC of Fig. 264(*b*). The vector CA represents the magnitude of the spring pressure K_1 upon this lever, as well as upon the right-hand lever. The vector CB represents the magnitude of the pressure exerted upon the fulcrum by the left-hand lever.

Applying the conditions of equilibrium to the forces acting upon the right-hand lever, we find that a couple is necessary to produce equilibrium. Since $K_1 = K_2$ and $R_1 = R_2$, it is evident that the resultant pressure upon the fulcrum, due to the right-hand lever, is equal to that produced by the left-hand lever, but it acts in the opposite direction as shown in Fig. 264(*b*).

(c) Having determined the pressures upon the fulcrum, the dimensions of the pin may be calculated.

(d) The operating lever a, which is actuated by a solenoid, is used to disengage the brake. Knowing the spring pressure, the magnitude of the force K_3 is readily obtained by applying the "triangle of forces."

BAND BRAKES

In a band brake an iron or steel band, lined with wood, leather, or asbestos-fabric encircles the brake sheave and is so arranged that it may be tightened or released. There are three types of

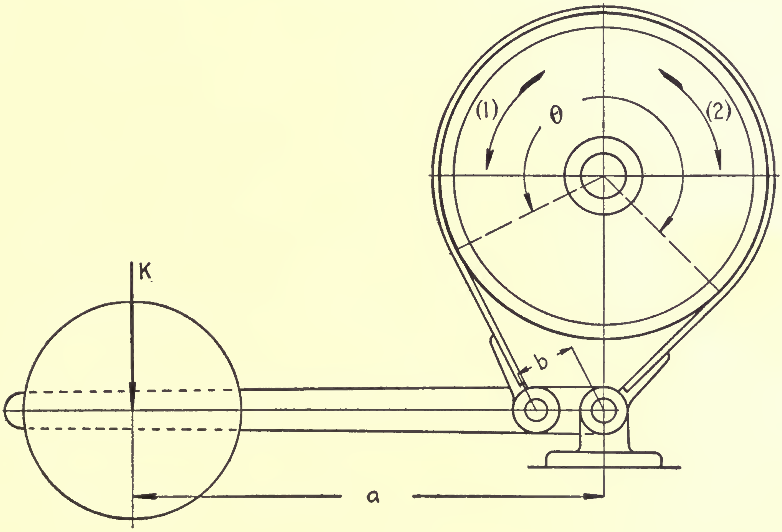

FIG. 268.

band brakes, as follows: (a) simple; (b) band brake for rotation in both directions; (c) differential.

325. Simple Band Brakes.—Two different designs of simple band brakes are shown in Figs. 268 and 269. In general, brakes of this type should be designed so that the heaviest pull will always come upon the anchorage. A band brake arranged in this manner requires a comparatively small pull at the free end of the band to apply the brake. In the brake shown in Fig. 269, the band makes more than a complete turn about the sheave and for that reason the effort required to apply such a brake is small.

Force analysis.—The following method of procedure for determining the force K required at the end of the operating lever is common to all of the simple band brakes.

In Fig. 268 assume that the brake sheave rotates in the direction indicated by the arrow (1). From Art. 122, the ratio of the tight to the loose tension is

$$\frac{T_1}{T_2} = e^{\mu\theta}, \tag{483}$$

in which μ and θ denote the coefficient of friction and angle of contact, respectively. The net tension on the brake sheave is

$$T = T_1 - T_2 = T_2(e^{\mu\theta} - 1) \tag{484}$$

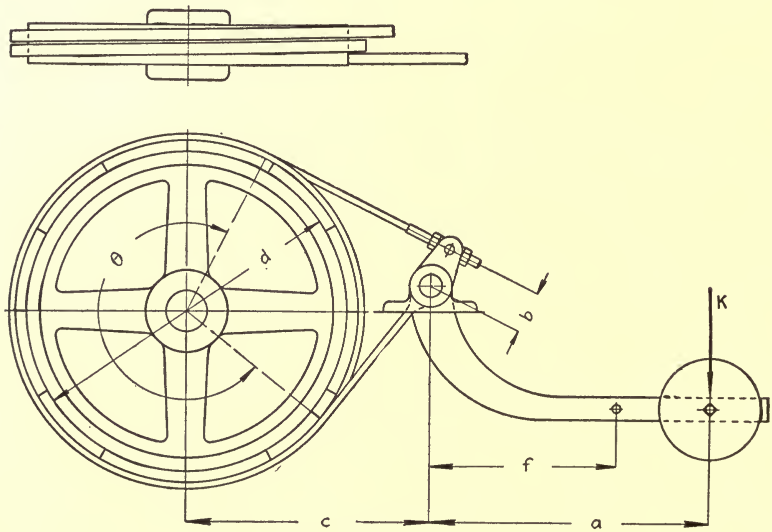

FIG. 269.

Treating the operating lever as the free body, and taking moments about the fulcrum, we get

$$K = \frac{T_2 b}{a} = \frac{Tb}{a(e^{\mu\theta} - 1)} \tag{485}$$

To determine the *net cross-sectional area* A of the band, divide the maximum tension T_1 by the permissible stress S of the material used; or

$$A = \frac{T}{S}\left[\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right] \tag{486}$$

In calculating the dimensions of the band, the thickness should not be made so great that a considerable part of the force on the operating lever is used in overcoming the resistance to bending of the band.

The pin at the anchorage or fulcrum must be made of ample size to resist the bending moment and pressures coming upon it.

The analysis for the rotation indicated by the arrow (2) is similar to that just given, and is left to the student.

326. Band Brakes for Rotation in Both Directions.—As mentioned in the preceding article, the heaviest loaded band should

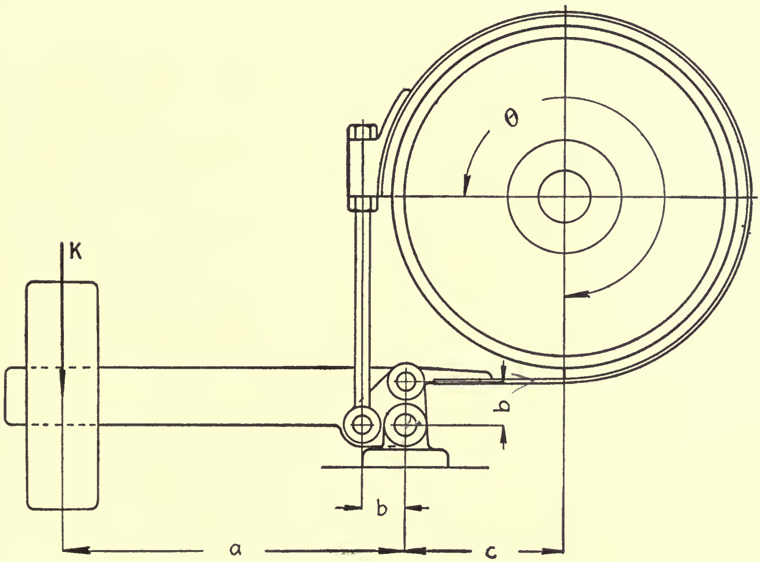

FIG. 270.

always be connected to the anchorage. This condition can readily be fulfilled provided the load acts continuously in one direction. When the load reverses in direction, as in mine hoists, cranes, and elevators, the condition cannot be fulfilled. The design of a band brake, used on mine hoists and on the armature shafts of motors direct-connected to hoisting drums, is shown in Fig. 270. The two ends of the band are connected to the operating lever at points which are equidistant from the fulcrum, as shown by the dimensions *b*. The force analysis of this type of brake is similar to that given in Art. 325.

327. Differential Band Brakes.—The general arrangement of a differential band brake is shown in Fig. 271. As in the brake illustrated by Fig. 270, both ends of the band are connected to the operating lever, but at different distances from the fulcrum.

(*a*) *Force analysis.*—Assuming counterclockwise-rotation, the magnitude of the force K is given by the following expression:

$$K = \frac{T}{a}\left[\frac{c - be^{\mu\theta}}{e^{\mu\theta} - 1}\right] \tag{487}$$

If in (487) the dimension c is made less than the product $be^{\mu\theta}$, the force K has a negative value and the brake is applied automatically. This special condition is used to a considerable extent in connection with automatic crane brakes. In such installations, the automatic band brake is used to take the place of the ordinary ratchet and pawl mechanism. In Figs. 277 and 278 are shown automatic band brakes fulfilling the function of a ratchet and pawl by permitting rotation of the brake sheave in only one direction.

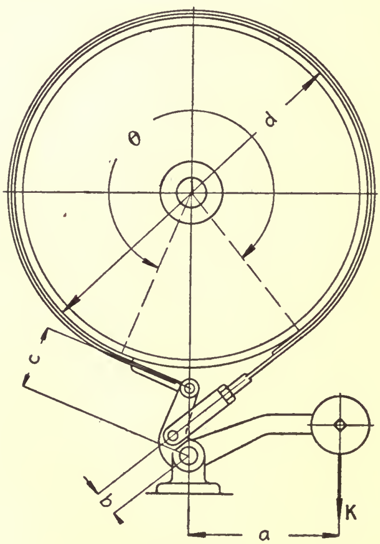

Fig. 271.

AXIAL BRAKES

In the so-called axial brakes a frictional surface of revolution is forced against a corresponding surface, the pressure being applied in a direction parallel to the axis of rotation. According to the form of the surfaces of revolution in contact, axial brakes may be divided into the following types: (*a*) conical brakes; (*b*) disc brakes.

328. Conical Brakes.—One of the simplest forms of axial brake is the conical type, the constructive features of which are shown in Fig. 272. The magnitude of the force K at the end of the operating lever may be determined as follows.

We shall assume that the outer cone forms a part of, or is attached to, the frame work of the machine, while the inner cone is splined to the rotating shaft. The inner cone is acted upon by the axial force Q and the pressure exerted by the outer cone upon the conical surface. The action of the conical brake is similar to the action of the cone clutch discussed in Art. 295. According to (426), the moment of friction that the brake is capable of absorbing is given by the expression

$$M = \frac{\mu Q D}{2 \sin \alpha} \tag{488}$$

from which the tangential resistance upon the cone becomes

$$T = \frac{\mu Q}{\sin \alpha} \tag{489}$$

Treating the operating lever as the free body and taking moments about the fulcrum, we get

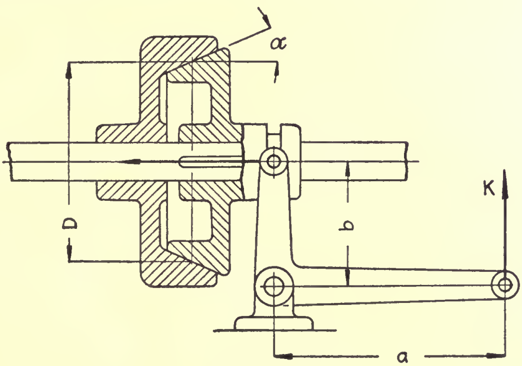

Fig. 272.

$$K = \frac{Qb}{a} = \frac{Tb \sin \alpha}{\mu a} \tag{490}$$

In some conical brakes the two cones are made of cast iron, while in others, such as are used on the armature shafts of crane motors, the outer cone is of cast iron and the inner cone is faced with wood. The angle α varies from 10 to 18 degrees, and the coefficient of friction may be assumed as 0.12 to 0.25. The former coefficient is to be used when both friction surfaces are made of cast iron, and the latter when one of the surfaces is of wood and the other of cast iron.

329. Disc Brakes.—The disc brake is simply a special form of the conical brake having the cones opened out into plane discs. In practically all installations of disc brakes, there are more than two surfaces in contact. A disc brake having several contact surfaces is commonly called a Weston washer brake. Fig. 273

shows one form of multiple-disc brake. The pinion *a*, having a faced surface at *b*, is bushed and runs loose on the shaft *c*. The flange *d*, keyed to the shaft, has a faced surface similar to that at *b*. The shaft *c* carries a ratchet wheel which engages with a pawl and permits rotation in but one direction. Neither the ratchet wheel nor the pawl are shown in the figure. Between the faced surfaces of the pinion and the flange *d*, a series of friction discs is arranged in such a way that alternate discs rotate with *a* and *d*. The discs shown in Fig. 273 are of fiber and steel; the former are keyed to the pinion by the feather keys *e* and the latter are fitted to the squared hub of the flange *d*. Frequently alternate discs of brass or bronze and steel are used.

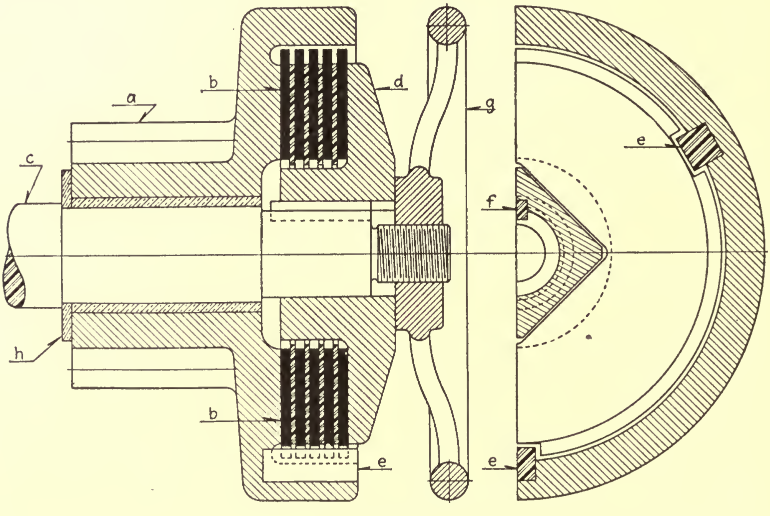

Fig. 273.

The disc brake shown in Fig. 273 is used in connection with hoisting machinery when it is required to lower a load rapidly and have it under the control of the operator. For this reason it is frequently called a "dispatch brake." The shaft being held from running backward by the ratchet and pawl, the operator may lower the load by merely unscrewing the handwheel *g*. This action decreases the friction between the discs and at the same time releases the pinion. By screwing up the handwheel so as to increase the frictional resistance between the discs, the speed of the load may easily be controlled. In hoisting the load the handwheel, the flange d, and the pinion are locked together; in other words, the brake is converted into a clutch.

Force analysis.—To determine the axial thrust Q and the force F on the rim of the handwheel that are necessary to apply the brake, the following analysis may be used.

From (442) the moment of the frictional resistance of the discs is

$$M = \frac{s\mu QD}{2}, \tag{491}$$

in which s denotes the number of friction surfaces and D the mean diameter of these surfaces. The axial thrust required to set the brake is

$$Q = \frac{2\,M}{s\mu D} \tag{492}$$

To produce the axial thrust by means of the screw and handwheel, it is necessary to apply a force F on the rim of the latter. The moment of the force F must exceed the frictional moment of the screw plus the moment of friction between the flange d and the handwheel. Hence

$$\frac{FD'}{2} \geqq \frac{Qd}{2} \tan{(\alpha + \varphi')} + M', \tag{493}$$

in which D' denotes the diameter of the handwheel; α the angle of thread in the screw; φ' the angle of friction of the thread; and M' the frictional moment between d and g.

The moment M_a due to the load on the pinion, must overcome the pivot friction between the pinion and the thrust washer h, the journal friction between the pinion and the shaft c, and the moment of friction of the discs. Denoting the moment of pivot friction by M_1 and that of journal friction by M_2, the magnitude of M_a is given by the following expression:

$$M_a = M + M_1 + M_2 \tag{494}$$

Substituting the value of M from (494) in (492), we may calculate the magnitude of the thrust Q. In order to determine the force F substitute the value of Q in (493).

For values of the coefficient of friction to be used in designing disc brakes those given in Art. 334(*f*) are recommended.

MECHANICAL LOAD BRAKES

Mechanical load brakes are used chiefly in connection with chain hoists, winches, and all types of crane hoists. In general, the functions of a mechanical load brake are as follows:

(*a*) The brake must permit the load to be raised freely by the motor.

(*b*) It must be applied automatically by the action of the load as soon as the lifting torque of the motor ceases to act in the hoisting direction.

(*c*) It must permit the lowering of the load when the motor is reversed. Reversing the motor releases the frictional resistance and allows the load to descend by gravity.

Mechanical load brakes, also called automatic brakes, are made in a variety of forms, but the greater number used on modern cranes are of the disc type.

330. Worm-gear Hoist Brakes.—In Fig. 274 are shown the constructive features of two forms of load brakes used on the worm shaft of German types of worm-geared chain hoists. These brakes are necessary to prevent the running down of the load, as the steep thread angle used on worms brings the efficiency of the hoist above 60 per cent.; hence the worm and its gear are no longer self-locking. Brakes similar to the one shown in Fig. 274(*a*) are also used on some American worm-geared types of drum hoists.

(*a*) *Lüder's brake.*—In Fig. 274(*a*) is shown a sectional view through Lüder's automatic disc brake. The flanged hub *b* and the cap *c* are keyed to the end of the worm shaft. Between *b* and *c*, and rotating upon the hub of the latter, is a hollow bronze ratchet wheel *e* which engages with a pawl. The latter is not shown in the figure. The ratchet wheel is made hollow so as to form a convenient reservoir for the lubricant. Between *b* and *e* a leather or fiber friction disc is used.

In raising the load the friction between the contact surfaces of *e*, due to the thrust of the load on the worm, is greater than that on the hard steel pivot *f*, and as a result the parts *b*, *e*, and *c* rotate with the shaft *a*. In lowering the load a pawl engages the ratchet wheel and holds it stationary, while the collar *b* and the cap *c* rotate with the shaft, thus introducing extra friction on both sides of *e*. The moment of the frictional resistance on *e* is made of such a magnitude as to prevent the overhauling of the load and still not make the pull on the hand chain too excessive.

(*b*) *Becker's brake.*—The constructive features of Becker's automatic conical brake are shown in Fig. 274(*b*). In raising the load the friction between the cones *b* and *c* due to the thrust of the worm shaft *a* is greater than that between the screw *f* and the cap *c*; hence the latter rotates with the shaft, and the moment of friction is reduced to a minimum. In lowering the load, a pawl, not shown in the figure, engages the ratchet teeth *e* and prevents the cap *c* from turning; thus the moment of friction caused by the thrust on the shaft *a* is that due to the two cones *b* and *c*. The moment of friction of these cones must be made sufficient to prevent the running-down of the load, and very little effort is required to lower the load by means of the pendant hand chain.

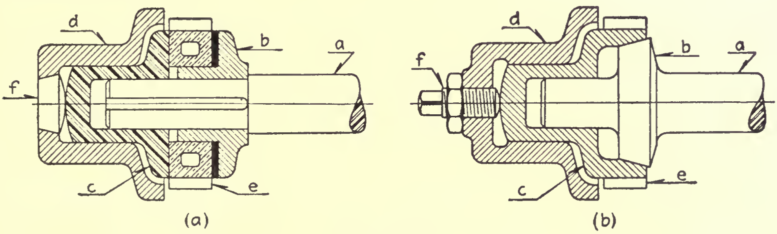

FIG. 274.

(*c*) *Force analysis of Becker's brake.*—It is required to determine an expression for the mean diameter D of the cone in order that the hoist shall be self-locking, and further to determine an expression for the moment $(P)R$ that is required on the hand sheave in order to lower the load.

Let Q = the tangential load on the worm gear.
R = the radius of the hand sheave.
d = the mean diameter of the worm.
α = the angle of the mean helix of the worm.
θ = the half cone angle.
φ' = the apparent angle of friction for the worm.

To prevent the running-down of the load, the moment of the frictional resistances on the worm shaft must exceed the moment on the shaft a due to the load Q; hence

$$\frac{\mu QD}{2\sin\theta} + M_1 \geq \frac{Qd}{2}\tan(\alpha - \varphi'), \qquad (495)$$

in which M_1 denotes the moment of friction on the shaft bearings. The magnitude of M_1 may be determined provided the diameter of the shaft and the distance between the bearings are known. However, this moment is generally small and for practical purposes may be neglected. Solving for D in (495), we have

$$D \geqq \frac{d \tan(\alpha - \varphi') \sin\theta}{\mu} - \frac{2 M_1 \sin\theta}{\mu Q} \tag{496}$$

The relation for D given by (496) must be fulfilled if the hoist is to be self-locking.

The moment on the hand chain sheave required to lower the load, assuming the hoist as self-locking, is

$$(P)R = \frac{\mu Q D}{2 \sin\theta} - \frac{Qd}{2} \tan(\alpha - \varphi') + M_f, \tag{497}$$

in which M_f denotes the frictional moment of the shaft bearings. The magnitude of M_f may be determined approximately if the diameters of the shaft bearings are known. If ball bearings are used on the worm shaft, the loss due to the journal friction will probably not exceed 3 per cent. of the total work expended. Upon the latter assumption (497) reduces to

$$(P)R = \frac{Q}{1.94}\left[\frac{\mu D}{\sin\theta} - d \tan(\alpha - \varphi')\right] \tag{498}$$

331. Crane Disc Brakes.—(*a*) *Niles brake.*—In Fig. 275 is shown the design of a mechanical load brake used on cranes manufactured by the Niles-Bement-Pond Co. The spur gear *a* meshes directly with the motor pinion and is keyed to the sleeve *b*, which rotates freely upon the shaft *c*. The one end of this sleeve *b* is in the form of a two-jaw helical clutch mating with corresponding helical jaws on the collar *h*. The latter is keyed to the shaft, and to prevent it from sliding along the shaft *c* an adjustable thrust collar *l* is provided. The other end of the sleeve *b* is faced and bears against the phosphor-bronze disc *f*. A similar disc *g* is located between the ratchet wheel *d* and the flange *e*. The latter is keyed to the brake shaft. The ratchet wheel *d* is bronze bushed and is free to rotate during the period of hoisting the load, but pawls, not shown in the figure, prevent rotation of *d* during the period of lowering the load. The pinion *p* meshes with the drum gear.

To hoist the load, the motor rotates the gear *a* and the sleeve *b* in the direction indicated by the arrow, while the shaft *c*, due to the action of the load, tends to turn in the opposite direction. The relative motion between the helical jaws formed on *b* and *h* forces the sleeve *b* toward the flange *e*, thus locking the whole mechanism to the driving sleeve *b*. To lower the load, the motor rotates the sleeve *b* in a direction opposite to that indicated by the arrow, thereby reducing the pressure between the disc and the ratchet wheel. Releasing the thrust on the discs *f* and *g* permits the load to descend by gravity. As soon as the speed of the shaft *c* and the collar *h* exceeds that of the sleeve *b*, the relative motion between the helical jaws will cause an increase in the axial thrust between the discs and the ratchet wheel, which in turn locks the brake, since the wheel *d* is held by pawls.

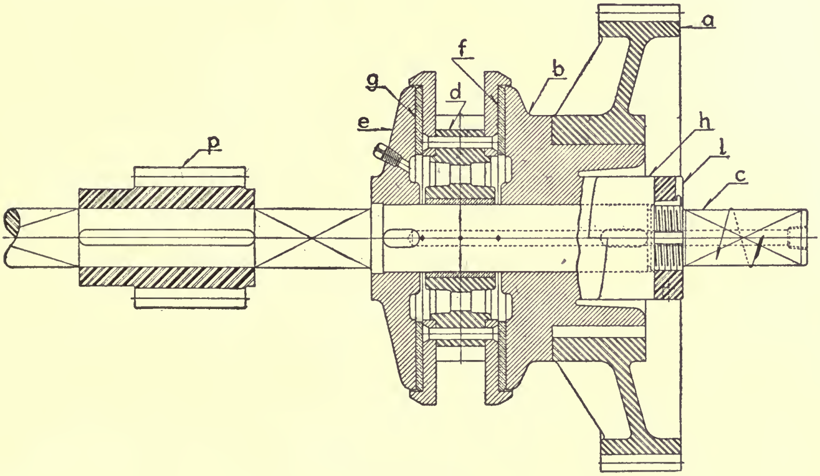

Fig. 275.

(*b*) *Pawlings and Harnischfeger brake.*—The constructive features of a load brake used by the Pawlings and Harnischfeger Co. is shown in Fig. 276. It differs from the Niles brake in that the friction discs *f* and *g* are made of fiber instead of bronze, and instead of using a helical jaw clutch to produce the thrust upon the friction surfaces, the shaft *c* is threaded as shown. The driving spur gear *a*, which meshes with the motor pinion, has the bore of its hub *b* threaded so as to form a good running fit with the thread upon the shaft *c*. In general, the description and method of operation given for the Niles brake in the preceding paragraphs also apply to the brake shown in Fig. 276.

(*c*) *Case brake.*—Several crane manufacturers are using load brakes equipped with more than two friction discs. In Fig. 277 is shown the design used by the Case Crane Co. The spur gear

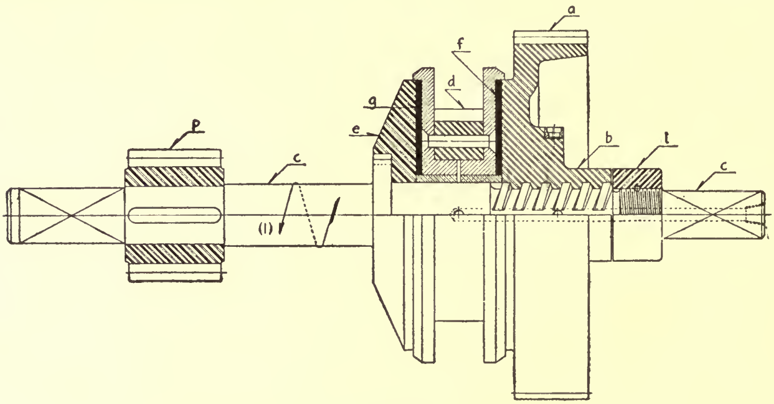

FIG. 276.

a meshes directly with the motor pinion and is keyed to a flanged sleeve *b*, the bore of which is threaded so as to form a good work-

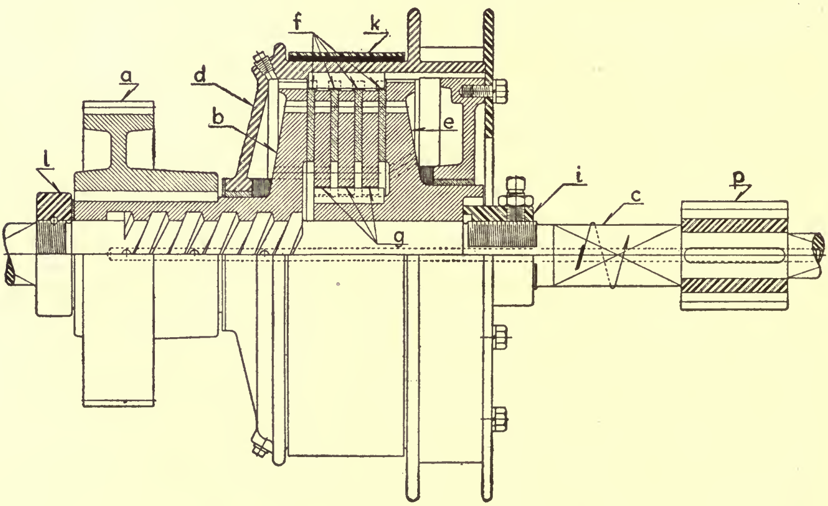

FIG. 277.

ing fit with the thread on the shaft. The flange of the sleeve *b* bears against the first of the bronze friction discs *f*. The flanged hub *e* is keyed to the shaft *c* and bears against the last of the bronze discs. The cast-iron friction discs g are keyed loosely to the hub e, while the discs f are keyed loosely to the shell d. The latter rotates freely during the hoisting period, but during the lowering of the load a properly proportioned differential band brake k prevents rotation.

To hoist the load, the motor rotates the gear a and the sleeve b in the direction indicated by the arrow, while the shaft c, due to the action of the load, tends to turn in the opposite direction. Due to this relative motion, the threaded sleeve b will tend to screw up on the shaft, thus clamping the flanges b and e to the shell d. In this manner the whole mechanism is locked to the driving gear a, thus transmitting the required power to the pinion p. To lower the load, the motor rotates the gear a in a direction

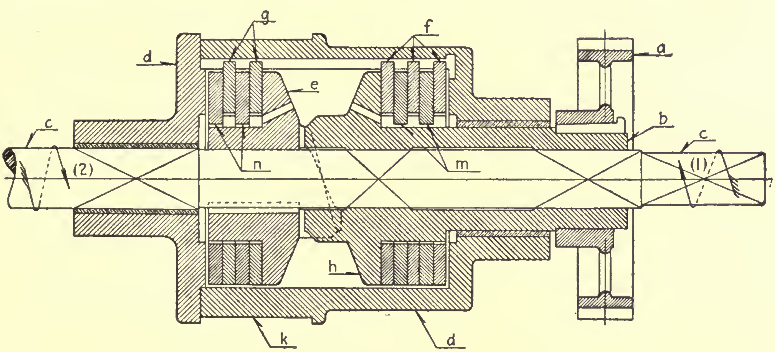

Fig. 278.

opposite to that indicated by the arrow, thus tending to reduce the axial thrust on the discs f and g and permitting the load to descend by gravity. Should the speed of the shaft c, due to the action of the load, exceed that of the gear a, the resultant relative motion will cause the sleeve b to screw up on the shaft and lock the brake, since the reverse rotation of the shell d is prevented by the differential band brake k.

The closed shell d is made oil tight, thus assuring lubrication of the friction surfaces, since the discs run in an oil bath. In order to distribute the oil to the engaging surfaces all of the discs as well as the flanges b and e are provided with holes and grooves. Effective means of lubricating the screw threads are also provided, as shown in the figure.

(*d*) *Shaw brake.*—In Fig. 278 is shown the design of an automatic multiple-disc brake used by the Shaw Electric Crane Co.

The spur gear *a* meshes directly with the motor pinion and is keyed to the sleeve *b*, which rotates freely on the shaft *c*. One end of the sleeve *b* is in the form of a two-jaw helical coupling mating with corresponding helical jaws formed on the flanged hub *e*. Cast integral with the sleeve *b* is the flange *h*, the inner face of which bears against the first of the cast-iron friction discs *f*. The flanged hub *e* is keyed to the shaft *c* and bears against the first of the cast-iron discs *g*. The discs *f* and *g* have lugs upon their outer circumferences which fit into recesses in the shell *d* and hence must rotate with *d*. The discs *m* and *n* have lugs upon their inner circumferences which fit into recesses on the sleeve *b* and hub *e*, respectively. The shell *d* rotates freely during the hoisting period, while during the lowering of the load a differential band brake located on the part *k* prevents rotation.

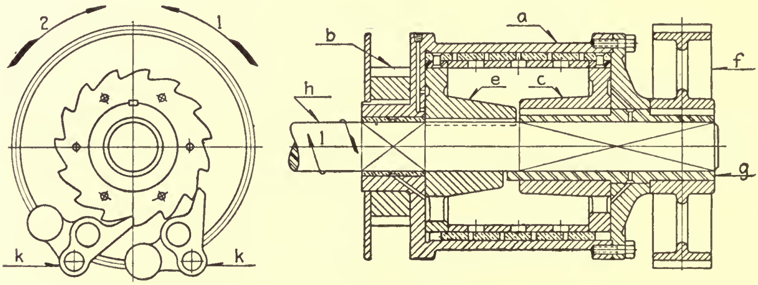

FIG. 279.

The operation of the Shaw brake is similar to that given in detail for the Case brake. An inspection of Fig. 278 shows that the engaging frictional surfaces may be run in an oil bath, and hence no trouble should be experienced as far as lubrication is concerned.

332. Crane Coil Brakes.—A form of automatic coil brake using a continuous shaft is shown in Fig. 279. This design has been used successfully on cranes made by Niles-Bement-Pond Co. It consists of a shell *a* carrying at its closed end a ratchet wheel *b* engaging the pawls *k*. One end of the bronze coil *d* is fixed by means of lugs to the driving head *c*, and the other end is fixed to the driven head *e*. The driving head *c*, as well as the driving gear *f*, is keyed to the sleeve *g* which rotates freely on the shaft *h*. The driven head *e* is keyed to the shaft *h* and is provided with a lug that may engage with a similar lug on the sleeve *g*. These lugs perform the function of establishing a positive drive between the sleeve *g* and the shaft *h* in case the bronze coil *d* wears down too far or in case the coil breaks.

In hoisting the load, the gear *f* meshing directly with the motor pinion rotates the head *c* as shown by the arrow (1), while the driven head *e* and shaft *h* under the action of the load tend to turn in the opposite direction, thus expanding the coil *d* against the inner surface of the shell *a*. As a result of expanding the coil *d*, the whole mechanism is locked to the driving head *c*. The motor, in lowering the load, pulls one end of the coil until the contact surface between *a* and *d* is reduced sufficiently to enable the load to overcome the frictional resistance, thus permitting the load to descend by gravity. It should be remembered that the shell *a* is prevented from rotating in the reverse direction by the ratchet and pawls. The speed of lowering cannot exceed that due to the motor or the coil will expand and apply the brake.

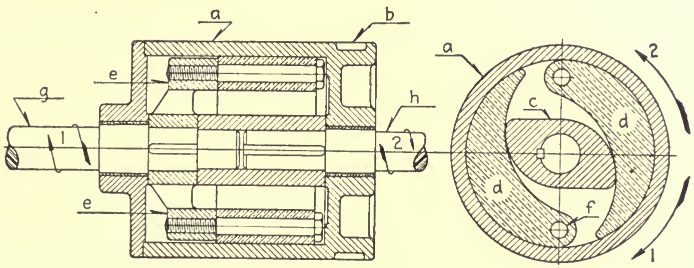

Fig. 280.

333. Cam Brake.—The automatic cam brake shown in Fig. 280 was designed to replace a troublesome coil brake of the two-shaft type. The shell *a* runs free on both of the shafts *g* and *h*. Upon the closed end of the shell is formed the ratchet wheel *b*, and a suitable pawl prevents rotation of the shell *a* when the load is lowered. The bronze coil originally used was replaced by two brass wings *d*, each of which has an arc of contact with the shell of about 165 degrees. A spider *e*, to which the wings are pivoted, is keyed to the driving shaft *g*, and the cam *c* which engages with these wings is keyed to the driven shaft *h*.

In hoisting, the shaft *g* rotates as shown by the arrow (1), while the pinion shaft *h* under the action of the load tends to rotate in the opposite direction, thus causing the cam *c* to force the wings *d* outward against the shell and thereby locking the complete mechanism to the driving shaft *g*. In lowering, the rotation of the driving shaft is reversed, thus tending to release the wings d from between the casing a and the cam c, and permitting the load to descend by the action of gravity. As soon as the load tends to run down too fast, the cam forces the wings outward and automatically applies the brake.

334. Force Analysis of an Automatic Brake.—In determining the relations existing between the external forces and the internal resistances acting on an automatic brake, the following analysis applied to the multiple-disc brake shown in Fig. 278 may serve as a guide.

Let D = the mean diameter of the friction discs.
I = the moment of inertia of the rotating parts located between the load and the brake, referred to the shaft of the latter.
Q = the axial thrust on the helical jaws during hoisting period.
(Q) = the axial thrust on the helical jaws during lowering period.
R = the pitch radius of the hoisting drum.
W = the load on the hoisting drum.
a = the acceleration of the load while hoisting.
(a) = the acceleration of the load while lowering.
d = the mean diameter of the helical jaws.
n = the gear ratio between the brake and the drum.
α = the angle of the helical surface on the jaws.
φ' = the angle of friction for the helical surfaces.
μ = the coefficient of friction for the discs.
η = the efficiency of the transmission between the brake and the load.

(*a*) *Axial thrust on helical jaws for hoisting.*—During the hoisting period the action of the brake is similar to that of a clutch; hence the moment M required on the gear a in order to raise the load is

$$M = \left[W + \frac{Wa}{g}\right]\frac{R}{n\eta} + \frac{Ian}{R} \qquad (499)$$

Equating this moment to that of the internal resistance of the discs f and m, we get

$$M = \frac{5\mu QD}{2} + \frac{Qd}{2}\tan(\alpha + \varphi') \qquad (500)$$

Combining (499) and (500), we obtain the following expression for the axial thrust:

$$Q = \frac{\frac{2R}{n\eta}\left[W + \frac{Wa}{g}\right] + \frac{2Ian}{R}}{5\,\mu D + d\tan(\alpha + \varphi')} \qquad (501)$$

(*b*) *Condition for self-locking.*—When the power is shut off, the load W tends to run the brake and motor in the reverse direction. To prevent reversed rotation it is necessary that the moments of the frictional resistances of all of the discs and the several journals shall exceed by a small amount the moment due to the load. The moment of the load for the running down condition is $\frac{WR\eta}{n}$, and this must be somewhat less than the moment of friction of the discs f and g and the shell d, or

$$\frac{WR\eta}{n} \leqq 5\,\mu Q'D, \qquad (502)$$

in which Q' denotes the axial thrust on the helical jaws. The magnitude of Q' may be determined from (501) by making the acceleration a equal to zero; hence

$$Q' = \frac{2\,WR}{n\eta(5\,\mu D + d\tan(\alpha + \varphi'))} \qquad (503)$$

To determine the relation that must exist between the dimensions of the helical jaws and the friction discs so as to satisfy the condition of self-locking, combine (502) and (503); whence

$$d\tan(\alpha + \varphi') \leqq \frac{5\,\mu D}{\eta^2}(2 - \eta^2) \qquad (504)$$

The relation expressed by (504) must be satisfied if the brake is to hold the load from running down.

(*c*) *Axial thrust on helical jaws for lowering.*—If the power is shut off while the load is being lowered, the moment of the descending load plus that due to the rotating parts tends to lock the brake. In locking the brake, the external moment just mentioned must overcome the frictional resistance of the helical jaws and that between the discs g and n. The magnitude of the internal frictional moments is $\frac{(Q)}{2}(5\,\mu D + d\tan(\alpha + \varphi'))$ Letting M_1 denote the moment due to the rotating parts and the inertia of the load, we obtain the following expression for the external moment:

$$\frac{WR\eta}{n} + M_1 \geqq \frac{(Q)}{2}\left[5\,\mu D + d\tan(\alpha + \varphi')\right],$$

from which
$$(Q) = \frac{2\left[\frac{WR\eta}{n} + M_1\right]}{5\,\mu D + d\tan(\alpha + \varphi')} \qquad (505)$$

The thrust (Q) becomes a minimum when the load comes to rest slowly, or in other words, when the inertia forces become small and their effect may be neglected. Making $M_1 = 0$ in (505), the minimum value of (Q) is given by the following expression:

$$(Q_1) = \frac{2\,WR\eta}{n(5\,\mu D + d\tan(\alpha + \varphi'))} \qquad (506)$$

For all practical purposes, we may assume that (506) gives the magnitude of the axial thrust upon the helical jaws during the lowering period.

The thrust (Q) becomes a maximum when the motor stops suddenly. The magnitude of (Q) for this case is given by (505), in which

$$M_1 = \frac{WR\eta(a)}{gn} + \frac{In(a)}{R} \qquad (507)$$

(*d*) *Condition of self-locking for lowering.*—Assuming that the brake is to be self-locking for all loads, the most unfavorable condition arises when the axial thrust is a minimum, as given by (506). The resistances that actually hold the load from running down, assuming the brake as self-locking, are those upon the discs f, g, m, and n. Equating the external moment, due to the load W, to the frictional moment of the discs, we have

$$\frac{WR\eta}{n} \leqq 5\,\mu D(Q_1) \qquad (508)$$

Combining (506) and (508),

$$d\tan(\alpha + \varphi') \leqq 5\,\mu D \qquad (509)$$

The relation given by (509) must be fulfilled if the brake is to be self-locking during the lowering period. By comparing (504) and (509), it follows that if the latter is fulfilled, the former is also satisfied.

(*e*) *Moment required to release the brake.*—Again assuming that the brake is self-locking, the motor must release the brake in order that the load may descend by gravity. The moment (M) required to release the brake must exceed by a small amount the sum of the frictional resistance of the helical jaws and that on the discs f and m, or

$$(M) \geqq \frac{Q'}{2}\left[5\,\mu\,D + d\tan(\alpha - \varphi')\right] \qquad (510)$$

The moment (M) becomes a maximum when Q' is maximum, which occurs directly after hoisting the load. To determine this maximum value of Q', make $a = 0$ in (501) and we obtain the relation expressed by (503). Substituting (503) in (510), the following expression for (M) is obtained:

$$(M) = \frac{WR}{n\eta}\left[\frac{5\,\mu D + d\tan(\alpha - \varphi')}{5\,\mu D + d\tan(\alpha + \varphi')}\right] \qquad (511)$$

(*f*) *Design constants and coefficients.*—For design purposes, the coefficient of friction μ for various combinations of materials may be assumed as follows:

Wood against cast iron-μ varies from 0.25 to 0.35.

Cast iron against cast iron lubricated-μ varies from 0.08 to 0.12.

Cast iron against bronze lubricated-μ varies from 0.06 to 0.10.

Cast iron against fiber lubricated-μ varies from 0.10 to 0.20.

For screws and helical jaws that are well lubricated, the angle of friction φ' may be assumed as 5 degrees.

The angle α varies from 5 to 17 degrees.

The axial thrust per square inch of projected disc area varies between rather wide limits. An analysis of twelve brakes of various capacities showed that this pressure varied from 17 to 270 pounds per square inch of disc area.

The axial thrust per square inch of projected area of the screw thread or helical jaw for the above-mentioned twelve brakes varied from 90 to 1,800 pounds.

335. Disposal of Heat.—The frictional resistance produced by a brake generates a certain amount of heat which is equivalent to the energy absorbed by the brake. Due to this fact, the brake should be designed so that the heat generated may be easily dissipated by conduction and radiation. Unfortunately, many brakes prove troublesome for the simple reason that the heat generated is not dissipated readily.

The rise in the temperature of the brake sheave depends upon the amount of energy the brake is required to absorb every time it is applied and upon the frequency with which the brake is applied, as well as upon the weight of the rim and the specific heat of the material. In general, the effect of the arms and hub of the brake sheave is neglected in calculating the rise in temperature for a given case.

Prof. Nichols, in his "Laboratory Manual of Physics," gives the following formula for determining the rise in temperature due to radiation:

Let A = the area of the radiating surface in square inches.
T = the number of minutes the brake is at rest.
c = the mechanical equivalent of the specific heat.
k = the radiation factor.
t_1 = the lower temperature of the brake sheave.
t_2 = the higher temperature of the brake sheave.
w = the weight of the brake sheave rim.

Then

$$\log (t_2 - t_1) = \frac{0.434\ kAT}{cw} \tag{512}$$

The energy absorbed by the rim is $cw\ (t_2 - t_1)$, and equating this to the energy given up by the load and the rotating parts as given by (475), we obtain the following expression:

$$E = cw\ (t_2 - t_1) \tag{513}$$

By means of (512), the approximate rise in the temperature of the brake may be determined, provided the factor k is known. Mr. E. R. Douglas, in an article entitled "The Theory and Design of Mechanical Brakes," published in the *American Machinist* of Dec. 19 and 26, 1901, states that "k generally lies between 0.4 and 0.8 of a foot-pound of energy per minute for each square inch of surface and each degree Fahrenheit which that surface is above the temperature of the surrounding air." The lower temperature t_1 of the brake sheave may be assumed to vary from 90° to 110°, while the temperature t_2 should not exceed 140° to 200°, depending upon the material forming the contact surfaces. In order to prevent charring of the wood blocks or leather and fiber facings, the temperature t_2 should not exceed 150°.

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CHAPTER XVIII

SHAFTING

336. Materials.—Shafts for practically all classes of service are subjected to shocks and jars. During each revolution the stresses in a shaft change from a maximum tension to maximum compression, provided the rotating shaft is subjected to cross-bending. It is evident that the material for shafting must be tough and ductile. The common materials used for shafting are as follows.

(*a*) *Wrought iron.*—In the past, engineers considered a good grade of wrought iron as the only material suitable for making shafts; but at present, due to its excessive cost of manufacture, wrought iron is used only in exceptional cases. Its strength is not as high as that of the modern steel that displaced it.

(*b*) *Bessemer steel.*—Machinery steel made by the Bessemer process is used quite extensively for certain classes of machine shafting. It is cheap, and modern methods of manufacture give it sufficient ductility and toughness in the "mild grades" so that it is suitable for making shafts. One disadvantage of Bessemer steel is that it may contain hidden flaws or defects, though this is not a very common occurrence; hence Bessemer steel will fulfill all the ordinary requirements in a large number of cases.

(*c*) *Open-hearth steel.*—Steel made by the open-hearth process is more reliable in that it is more uniform than the Bessemer steel, and for this reason open-hearth steel is specified for many machine parts, such as armature shafts, engine shafts, shafts and spindles of machine tools, etc.

(*d*) *Alloy steels.*—Many of the special steels described in Chapter II are used for making shafts for all classes of service, especially when great strength is desired. Attention is again directed to the fact that shafts made from alloy steels possess no greater rigidity than the same size of shaft made of ordinary machinery steel. The shafts used on motor cars are made of high-grade alloy steels. The main shafts of marine and large hoisting engines are usually made of a high-grade nickel steel. In general, shafts made from alloy steels are more expensive than those made from common grades of steel.

337. Method of Manufacture.—Commercial shafting may be classified into the following groups: (*a*) Turned; (*b*) cold-rolled or drawn.

(*a*) *Turned shafting.*—The ingot of steel, while hot, is rolled into bar stock having a diameter 1/16 inch greater than required for the finished shaft. The bar is then turned down in a lathe and polished accurately to size. It is evident that the diameter of the turned shafting is always 1/16 inch less than the so-called "nominal diameter." Large shafts are forged from an ingot, and turned down and finished accurately in a lathe.

(*b*) *Cold-rolled or drawn shafting.*—To produce cold-rolled shafting, hot-rolled bar stock, previously treated with an acid so as to clean the outer skin, is passed through special rolls under great pressure, or drawn through special dies. This cold-rolling or drawing process renders the shaft fairly uniform in size. The surface acquires a polished appearance and becomes hard and tough. Experiments on cold-rolled and drawn shafting show that the strength of the material is increased, but at the same time the ductility is reduced.

A disadvantage of cold-rolling or drawing lies in the fact that a considerable amount of skin tension is induced in the material of the shaft. This tension is relieved when a key seat is cut, thus causing the shaft to warp, and it must be trued up before it can be used. It is quite evident, therefore, that neither cold-rolled nor cold-drawn shafting is well adapted for use in high-grade machinery where accuracy is desirable. However, for the cheaper grades of machinery such shafts are used extensively.

338. Commercial Sizes of Shafting.—Formerly, when wrought iron was used for shafting, the stock sizes of the hot-rolled bars from which the shafts were made varied by 1/4-inch increments. Since these bars were reduced 1/16 inch in finishing, the commercial sizes of shafts thus established varied by 1/4-inch increments but were always 1/16 inch less than each even 1/4 inch in diameter. Later on, when steel replaced wrought iron, the list of stock sizes was increased. According to some of the prominent manufacturers of power-transmission machinery, it is possible to obtain turned shafting in the following sizes:

From 3/4 to 2 inches, the diameters vary by 1/16-inch increments.

From 2 to 6 inches, the diameters vary by 1/8-inch increments. Sizes which are 1/16 inch less than the even 1/4 inch in diameter are also obtainable.

Shafts larger than 6 inches in diameter are usually forged to order.

All of the sizes which are $\frac{1}{16}$ inch under the even $\frac{1}{4}$ inch in diameter are generally accepted as standard for such appurtenances as couplings, hangers, pillow blocks, etc.

Cold-rolled or drawn steel shafting may be obtained in sizes from $\frac{3}{16}$ inch and up, the diameters varying by $\frac{1}{16}$-inch increments.

SHAFT CALCULATIONS

The straining actions to which shafting may be subjected are as follows: (*a*) simple bending; (*b*) simple twisting; (*c*) combined twisting and bending; (*d*) combined twisting and compression.

339. Simple Bending.—In many classes of machinery, shafts are used that transmit no torsional moment, but merely support certain machine parts. Such shafts may revolve or remain stationary. In the latter case, the rotating machine parts supported by the shaft are generally bronze-bushed or mounted on ball or roller bearings. The hoisting drum shown in Fig. 284 is supported by a stationary shaft which is held rigidly by the supporting pedestals *A* and *B*. In the common car axle we have a good illustration of a rotating shaft subjected to a bending moment.

(*a*) *Strength.*—The diameter of a shaft subjected to simple bending may be determined by equating the external moment M to the moment of resistance of the shaft. Thus

$$M = \frac{\pi d^3 S}{32},$$

from which

$$\frac{M}{S} = \frac{\pi d^3}{32}. \tag{514}$$

To facilitate making calculations, the second member of (514) may be evaluated for various diameters and the results arranged in chart form as shown in Figs. 281 and 282. The determination of the magnitude of the bending moment M depends upon the number of bearings supporting the shaft, and the distribution of the forces coming upon the shaft. The method of procedure in any given case is similar to that used in the case of beams. The value of the permissible fiber stress S varies from 5,000 to 35,000 pounds per square inch and depends upon the material used for making the shaft.

(*b*) *Stiffness.*—In many machines the question of the stiffness of a shaft is of greater importance than that of its strength. In other words, for a shaft subjected to bending only the transverse deflection may have to be limited. These deflections depend upon the method of supporting the shaft as well as the distribution of the forces acting on the shaft. To calculate the deflections in a given case the formulas used in connection with beams will apply. No definite values are available for the transverse deflections of machine shafts, as they depend upon the service for

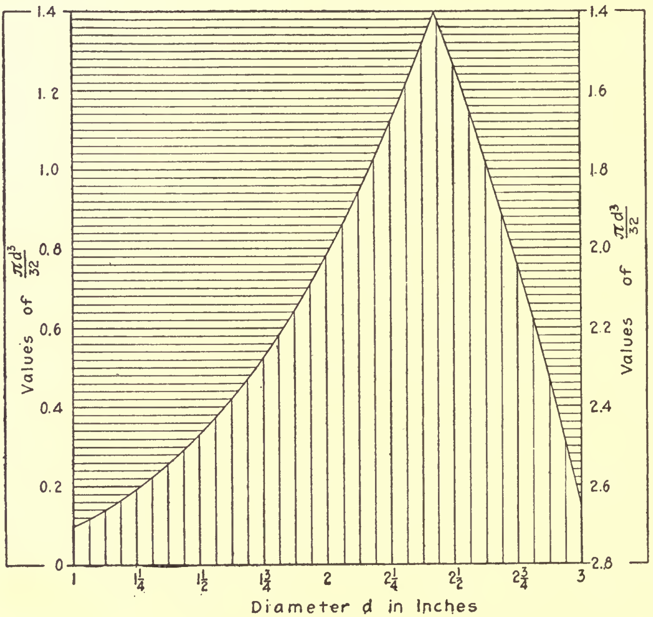

FIG. 281.

which the machine is intended. For line- and counter-shafts, a transverse deflection of 0.01 of an inch per foot of length is considered good practice.

340. Simple Twisting.—Shafting is very rarely subjected to simple twisting, since the weights of pulleys and gears, belt and chain pulls, and gear tooth pressures cause bending stresses. Frequently such bending stresses are difficult to determine beforehand, and due to the fact that the calculations become more or less complicated, many designers omit them in calculating the diameter of shafts. To make allowances for such unknown bending moments that are omitted, a low fiber stress is generally used in establishing the shaft diameter. Such a method of procedure should seldom be used.

(*a*) *Strength.*—A long line- or counter-shaft having the pulleys, gears, or sprockets located near the bearings is generally considered as a shaft transmitting a simple torsional moment. Ordinarily in such a shaft, the belt and chain pulls are not excessive and the bending moment caused by them may be omitted in the calculations for the diameter of the shaft. Hence, equating the torsional moment of the load to the moment of resistance, we have

$$T = \frac{\pi d^3 S_s}{16},$$

from which

$$\frac{T}{S_s} = \frac{\pi d^3}{16}. \qquad (515)$$

The graphs of Figs. 281 and 282 may be found convenient in the solution of problems involving the use of (515), but it should be remembered that for the same diameter of shaft $\frac{M}{S} = \frac{T}{2\,S_s}$. The magnitude of the permissible shearing stress S_s varies from 2,500 up.

Substituting in (515) the value of T expressed in terms of the horse power transmitted and the revolutions per minute of the shaft, we obtain the following expression for the diameter of the shaft:

$$d = \sqrt[3]{\frac{321{,}000\,H}{NS_s}}, \qquad (516)$$

in which H and N denote the horse power and revolutions per minute, respectively. According to the formulas recommended by several prominent manufacturers of power transmission machinery, the shearing stress S_s may be assigned the following values:

1. For well-supported head shafts carrying main driving pulleys, sheaves, or gears and transmitting heavy loads, S_s is approximately 2,600.

2. For regular line shafts supported on bearings every 8 feet, S_s = 4,300.

3. For light duty line shafts supported on bearings every 8 or 10 feet, S_s = 6,400.

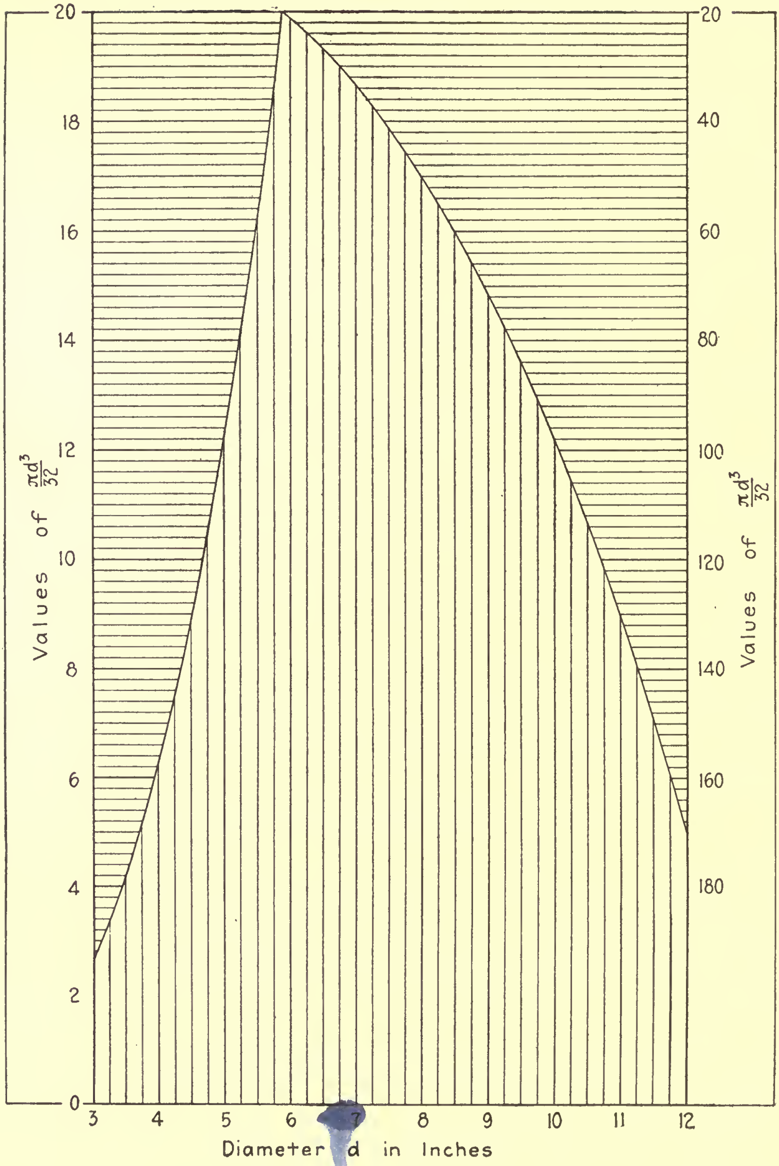
20
18
16
14
12
10
8
6
4
2
0
Values of $\frac{\pi d^3}{32}$
20
40
60
80
100
120
140
160
180
Values of $\frac{\pi d^3}{32}$
3
4
5
6
7
8
9
10
11
12
Diameter d in Inches

FIG. 282.

(b) *Stiffness.*—In machine tools it is necessary that the important drive shafts be made stiff so that they will not "wind up" like a spring. Such angular deflection must be limited in machine tools, while in other classes of machinery it need not be considered at all. To determine the relation between the torsional moment T and the angular deflection θ, the following method may be used.

Since the torsional modulus of elasticity E_t represents the ratio of the unit stress to the unit deformation, we get

$$E_t = \frac{S_s l}{x}, \tag{517}$$

in which l and x denote the length of the shaft and the deflection measured on the surface of the shaft, respectively. Both x and l are measured in inches.

The length of the arc x is $\frac{\theta \pi d}{360}$, and substituting this value in (517), we obtain the following expression for the angular deflection:

$$\theta = \frac{360\, l S_s}{\pi d E_t} \tag{518}$$

Substituting in (518) the value of S_s obtained from (515), we have

$$\theta = \frac{584\, l T}{d^4 E_t} \tag{519}$$

For ordinary shafts, it is common practice to limit the angle θ to 1 degree in a length of shaft equivalent to 20 diameters.

341. Combined Twisting and Bending.—A rotating shaft carrying pulleys, sprockets, sheaves, and gears is subjected to both bending and twisting when used for the transmission of power. Calculating the diameter of the shaft by means of either of the formulas (514) or (515), and ignoring the other, would result in a weak shaft. In designing shafts subjected to combined bending and torsion, several formulas based upon different theories are advocated by various investigators. The theories upon which these formulas are based are as follows: (a) the maximum normal stress theory; (b) the maximum strain theory; (c) the maximum shear theory.

(a) *Maximum normal stress theory.*—The maximum normal stress or Rankine's theory is based upon the assumption that the yield point depends upon the maximum normal stress, and not upon the shear or other stresses acting at right angles to it. The resulting maximum stress is calculated by the following formula:

$$\text{Max. normal stress } S_e'' = \frac{S}{2} + \sqrt{S_s^2 + \frac{S^2}{4}} \tag{520}$$

To facilitate the use of (520) when designing shafts, it has been found convenient to employ what is generally called the "equivalent twisting moment" T_e'', an expression for which may be derived as follows: Substituting in (520) the values of S and S_s in terms of the diameter d, we obtain

$$S_e'' = \frac{16}{\pi d^3}(M + \sqrt{M^2 + T^2}), \tag{521}$$

from which

$$T_e'' = \frac{\pi d^3 S_e''}{16} = M + \sqrt{M^2 + T^2} \tag{522}$$

The so-called equivalent twisting moment T_e'' will produce the same maximum normal stress as is produced by the combined action of M and T. In using (522), it is important to remember that S_e'' is a tensile or compressive stress, and not a shearing stress.

Some designers prefer to use an expression for the "equivalent bending moment" M_e'' in place of (522). Multiplying and dividing (521) by 2, we obtain the following expression:

$$S_e'' = \frac{32}{\pi d^3}\left[\frac{M}{2} + \frac{1}{2}\sqrt{M^2 + T^2}\right],$$

from which

$$M_e'' = \frac{\pi d^3 S_e''}{32} = \frac{1}{2}(M + \sqrt{M^2 + T^2}) \tag{523}$$

The equivalent bending moment M_e'' will produce the same maximum normal stress as M and T acting together. The allowable stress S_e'' must be the same as that used with (522).

(*b*) *Maximum strain theory.*—The maximum strain theory, generally credited to Saint-Venant, is based upon the assumption that yielding of the material will not occur until a certain deformation has been produced. To determine the stress that produces yielding according to this theory, the following formula must be used:

$$\text{Max. normal stress } S_e = (1 - m)\frac{S}{2} + (1 + m)\sqrt{S_s^2 + \frac{S^2}{4}}, \tag{524}$$

in which the symbol m denotes Poisson's ratio, values of which are given in Table 1. For steel, m may be assumed as 0.3. Substituting this value in (524), and introducing the values of S and S_s in terms of d, we get an expression for the equivalent twisting moment T_e, as follows:

$$T_e = \frac{\pi d^3 S_e}{16} = 0.70\,M + 1.3\sqrt{M^2 + T^2} \qquad (525)$$

The equivalent bending moment M_e becomes

$$M_e = \frac{\pi d^3 S_e}{32} = 0.35\,M + 0.65\,\sqrt{M^2 + T^2} \qquad (526)$$

To decrease the numerical work involved in applying (525) or (526) to any particular problem, let $\frac{M}{T} = k$; then (525) and (526) may be written

$$T_e = \frac{\pi d^3 S_e}{16} = T(0.7\,k + 1.3\,\sqrt{k^2 + 1}) \qquad (527)$$

and

$$M_e = T(0.35\,k + 0.65\,\sqrt{k^2 + 1}) \qquad (528)$$

To find the diameter of a shaft suitable for the combined moments M and T, substitute the value of T_e for T and S_e for S_s in (515), and use the graphs of Figs. 281 and 282 as directed in Art. 340(*a*). If (528) is preferred, substitute the value of M_e for M and S_e for S in (514), and consult the graphs of Figs. 281 and 282 for the diameter of the shaft corresponding to the calculated ratio $\frac{M_e}{S_e}$.

(*c*) *Maximum shear theory.*—Up to the year 1900, the two theories just discussed were the only ones in use; at that time Prof. Guest reported in the *Philosophical Magazine* the results of his investigations upon the behavior of ductile materials subjected to combined stresses. His conclusion was that the yield point depends upon the maximum shearing stress; that is, the material yields when the greatest resultant shear reaches a certain limit. The formula for calculating the maximum shearing stress that produces yielding is as follows:

$$\text{Max. shear } S_e' = \sqrt{S_s^2 + \frac{S^2}{4}} \qquad (529)$$

To determine the equivalent twisting moment T_e' for this theory, substitute in (529) the values of S and S_s; hence

$$T'_e = \frac{\pi d^3 S'_e}{16} = \sqrt{M^2 + T^2}$$

$$= T\sqrt{k^2 + 1} \tag{530}$$

To determine the diameter of the shaft by means of (530) in any particular problem, substitute T'_e for T and S'_e for S_s in (515), and consult the graphs of Figs. 281 and 282 as directed in Art. 340(*a*).

342. Method of Application.—In order to determine the diameter of a shaft subjected to combined twisting and bending, we must decide which of the theories discussed in Art. 341 should be used. It should be noted that the maximum strain theory

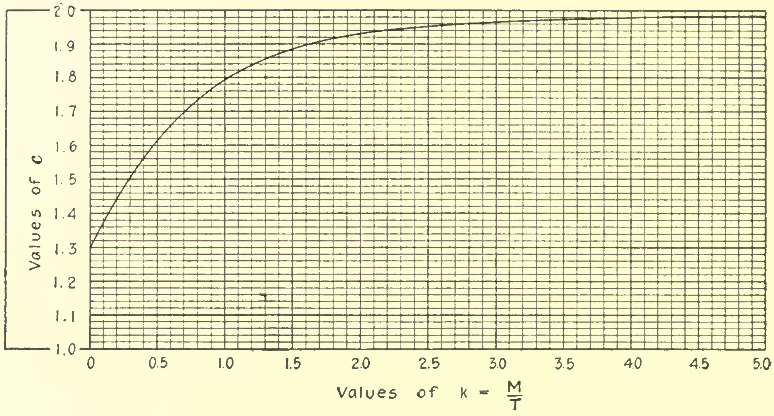

FIG. 283.

is really nothing more than a refinement of the maximum normal stress theory, and for that reason is more accurate.

Comparing (527) and (530), it is evident that for equal diameters d, the following relation must exist between the allowable stresses and k:

$$\frac{S_e}{S'_e} = \frac{0.7\,k + 1.3\sqrt{k^2 + 1}}{\sqrt{k^2 + 1}} = c \tag{531}$$

The relation expressed by (531) may be represented graphically as shown in Fig. 283. Every point on the curve represents simultaneous values of k and the ratio c for which (527) and (530) will give the same shaft diameter. It is evident that if a point represented by the coördinates c and k does not lie on the curve, one of these formulas will give a diameter of shaft which is larger than that given by the other. The object of representing (531) by the graph of Fig. 283 is to show at a glance which formula or theory must be used in a given case in order to obtain the maximum shaft diameter. If the coördinates c and k locate the point below the curve, the maximum strain theory must be used; that is, use formula (527) or (528). If the point lies above the curve, the maximum shear theory or formula (530) must be used.

According to C. A. M. Smith, an English investigator, the ratio of the working stresses for mild steel in tension and shear is practically 2 to 1 instead of 5 to 4, as usually quoted in text books. In Table 95 are given the fiber stresses at the elastic limit for tension and shear as determined by Prof. Hancock.

TABLE 95.—FIBER STRESSES AT THE ELASTIC LIMIT

Material	Tension	Shear	Ratio $\frac{S_e}{S_e'}$
Mild carbon steel..................	47,000	30,500	1.54
Nickel steel........................	76,500	38,000	2.01

343. Combined Twisting and Compression.—Shafts subjected to a twisting moment combined with a compression are frequently met with in machinery. Among the most common examples of such shafts are those used for driving worm gearing and the propeller shafts of ships. Occasionally vertical shafts carrying heavy rotating parts are subjected to combined twisting and compression. However, in many cases of worm gearing the worm can be mounted so that very little of the thrust comes upon the shaft proper.

(*a*) *Short shaft.*—The first case to be discussed is one in which the part of the shaft subjected to compression is so short that it may be considered as a simple compression member so far as the action of the thrust is concerned.

The intensity of compressive stress for a solid shaft is $S_c = \frac{4P}{\pi d^2}$, in which P denotes the thrust. From (515), the intensity of shearing stress due to the twisting moment on the shaft is $S_s = \frac{16\,T}{\pi d^3}$. The resultant maximum stress due to the combined action of S_c and S_s may be found by substituting the values of the latter in (520); whence

$$\text{Max. compressive stress} = \frac{2}{\pi d^2}\left[P + \sqrt{P^2 + \frac{64\,T^2}{d^2}}\right] \qquad (532)$$

To determine the diameter d of a shaft having given the magnitudes of the thrust P, the torsional moment T, and the allowable compressive stress, assume a trial value for d somewhat larger than that required for the twisting moment alone and evaluate (532). If the calculated value of the stress does not come near the allowable maximum make a second calculation, and so on.

(*b*) *Long shaft.*—The second case to be considered is that of a shaft in which the part subjected to a thrust is so long that it is liable to buckle; in other words the shaft must be considered as a long column. According to Art. 15, the mean intensity of permissible compressive stress in a long column having a circular cross-section and subjected to a thrust P is as follows:

$$S_c' = \frac{4P}{\pi d^2} = \frac{S_c}{1 + \dfrac{S_e L^2}{n\pi^2 r^2 E}} \qquad (533)$$

Assuming that the coefficient of elasticity E has an average value of 30,000,000, and that n may be taken as unity, (533) reduces to the following form:

$$S_c' = \frac{4\,P}{\pi d^2} = \frac{S_c}{1 + \dfrac{S_e L^2}{18{,}500{,}000\,d^2}} \qquad (534)$$

The stress calculated by (534) is the mean intensity of compressive stress which corresponds to a maximum compressive stress S_c in the long shaft; hence a short shaft having the same diameter as the longer one is capable of withstanding a thrust P' which is greater than P in the ratio of S_c to S_c'. It is evident that the magnitude of the thrust P' is given by the following expression:

$$P' = P\frac{S_c}{S_c'} \qquad (535)$$

To determine the diameter of the shaft necessary to support the thrust P and twisting moment T, use (532) as before, but substitute therein for P the magnitude of P' as calculated by (535).

344. Bending Moments.—In calculating the bending moments coming upon a shaft supported on the ordinary type of bearings, it seems reasonable to assume that the clearance between the bearings and the shaft will permit the latter to deflect up to the middle of the bearings. Therefore, in such cases the moment arms should be measured to the middle of the bearings, and a shaft designed upon this assumption will generally be of ample size so far as strength is concerned.

Whenever a gear, flywheel, or other machine part is forced or shrunk upon a shaft, it is practically impossible for the shaft to fail at the center of the hub. However, the shaft may fail along a section near either end of the hub, since any bending of the shaft would tend to localize the crushing at those sections. According to Mr. C. L. Griffin, the critical sections may be assumed to lie from ½ to 1 inch inside of the hub.

The majority of machine designers assume the moment arms as extending to the center of hubs and bearings, probably because

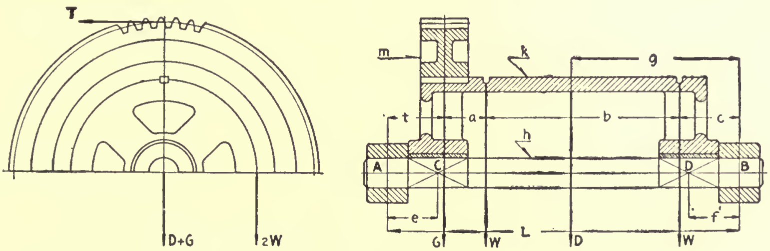

Fig. 284.

the method is simple and the results obtained are on the side of safety.

SPECIAL PROBLEMS

The problems discussed in the following articles will serve to illustrate the general method of procedure that may be used in calculating the bending moments coming upon a shaft and finally in determining the diameter of the shaft required in any given case.

345. Crane Drum Shaft.—In Fig. 284 is shown a crane drum running loose on the stationary shaft. With this construction no twisting moment is transmitted through the shaft. The first step to be taken in calculating the diameter of the shaft is to determine the magnitude and location of the maximum bending moment coming upon the shaft due to the loading shown in Fig. 284.

Let D = the weight of the drum.
G = the weight of the gear.
T = the tooth thrust due to the driving pinion.
W = the load on each hoisting rope.

Taking moments about the center of the supporting pedestal B, we have for the horizontal load at A the following expression:

$$A_h = \frac{T}{L}(L - t), \tag{536}$$

and for the vertical load at A, we have

$$A_v = \frac{G(L - t) + W(b + 2c) + Dg}{L} \tag{537}$$

Combining these loads in the usual manner, we obtain the following expression for the resultant pressure at A:

$$A = \sqrt{A_h^2 + A_v^2} \tag{538}$$

Taking moments about the center of the supporting pedestal A, the horizontal load coming upon pedestal B is

$$B_h = \frac{Tt}{L}, \tag{539}$$

and the vertical load at B is

$$B_v = \frac{Gt + W(2t + 2a + b) + D(L - g)}{L} \tag{540}$$

Hence, the resultant pressure at B is

$$B = \sqrt{B_h^2 + B_v^2} \tag{541}$$

The bending moment at the center of the bearing C is Ae, and that at the center of the bearing D is Bf; whichever of these moments is the greater must be used in calculating the diameter of the shaft by means of (514).

346. Shaft Supporting Two Normal Loads between the Bearings.—A shaft supported on two bearings and carrying two or more gears, sprockets, or pulleys is of common occurrence in machinery. In some cases the gears are located between the bearings as shown in Fig. 285, while in others they are arranged as shown in Fig. 288. Furthermore, the loads coming upon the gears or pulleys produce bending moments that are either coplanar or in planes inclined to each other.

(*a*) *Diameter of shaft required for strength.*—It is desired to determine the diameter of the shaft shown in Fig. 285, assuming the horse power H is transmitted at N revolutions per minute. The first step to be taken in the solution of this problem is to determine, by means of the following formula, the torsional moment T transmitted by the shaft:

$$T = 63{,}030 \frac{H}{N} \tag{542}$$

Knowing the torsional moment T, we may readily calculate the magnitudes of the effort P and the resistance W, since

$$T = PR = Wr \tag{543}$$

Having determined the forces P and W, we may treat the shaft as a simple beam and determine the bending moments at important points along the shaft. Since P and W act in planes that are at right angles to each other, the problem may be simplified by

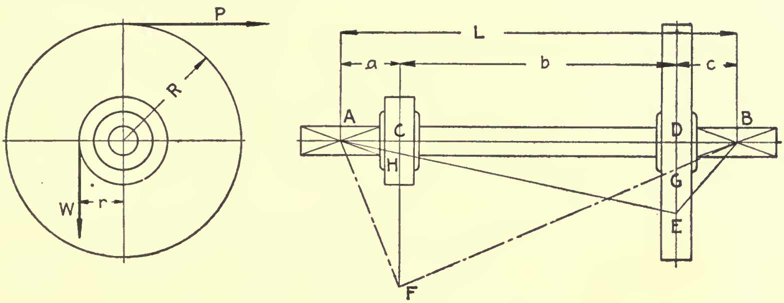

Fig. 285.

constructing the bending moment diagram for each of these forces and later combining these diagrams in order to determine the maximum moment. In Fig. 285 the triangle AEB represents the bending moment diagram for the force P, and AFB represents a similar diagram for the force W. In other words, at the point D the shaft is subjected to two non-coplanar bending moments; the one due to P is represented by the vector DE and the other due to Q is represented by the vector DG. These bending moments are in planes at right angles to each other; hence the resultant moment at D is equal to the vector sum of DE and DG, or

$$M_D = \sqrt{\overline{DE}^2 + \overline{DG}^2} \tag{544}$$

In a similar manner the resultant bending moment at C is the vector sum of CF and CH, or

$$M_C = \sqrt{\overline{CF}^2 + \overline{CH}^2} \tag{545}$$

For the shaft shown in the figure, it is evident that the maximum moment occurs under the pinion, namely, at the point C. Having determined the magnitude of the maximum bending moment M, calculate the value of the ratio $k = \frac{M}{T}$. For the particular material used in the shaft determine the ratio $\frac{S_e}{S_e'}$, and by means of the graph of Fig. 283 ascertain which formula must be used to calculate the diameter of the shaft. The permissible stress S_e or S_e' depends upon the nature of the transmission and the material, and ordinarily it may be assumed as from 20 to 40 per cent. of the stress at the elastic limit.

(*b*) *Diameter of the shaft required for stiffness.*—It is required to determine the deflections at various points of the shaft shown in Fig. 285. Either the analytical or the graphical method may be used for ascertaining the deflections, but since the loads coming upon the shaft are non-coplanar the former method will prove to be the simpler. From the theory of a simple beam supporting a load Q, the deflection Δ_1 of the beam at any point x_1 inches from the left-hand support is given by the following expression:

$$\Delta_1 = \frac{Qbx_1}{6\,EIL}(L^2 - x_1^2 - b^2) \tag{546}$$

The deflection Δ_2 of the beam at any point x_2 inches from the right-hand support may be calculated by a formula similar to (546), namely,

$$\Delta_2 = \frac{Qax_2}{6\,EIL}(L^2 - x_2^2 - a^2) \tag{547}$$

The symbols used in the above formulas have the following significance: L denotes the distance between the supports; a the distance from the left-hand support to the load; b the distance from the right-hand support to the load.

Since the shaft shown in Fig. 285 may be treated as a simple beam, the deflections due to the force P may be calculated by means of (546) and (547). The deflections due to the load W may be determined in the same manner. Using the length of the shaft as a base line, the values determined by (546) and (547) may be plotted, thus giving the deflection curve for each load. To determine the resultant deflection of the shaft at any point, due to the combined effect of P and W, find the vector sum of the separate deflections corresponding to the point under consideration. If the resultant deflection is considered too great for the particular class of service, increase the diameter of the shaft.

347. Shaft Supporting Two Normal Loads with One Bearing Between the Loads.—(*a*) *Diameter of the shaft required for strength.*—A shaft, carrying gears or sprockets, supported on two bearings as shown in Fig. 286 is frequently used in machinery. One of the gears is located outside of the bearing B, thus causing a bending moment at the center of this bearing. The magnitude of the bending moment at any point along the shaft, due to the force P, may be obtained by measuring the ordinate between AD and

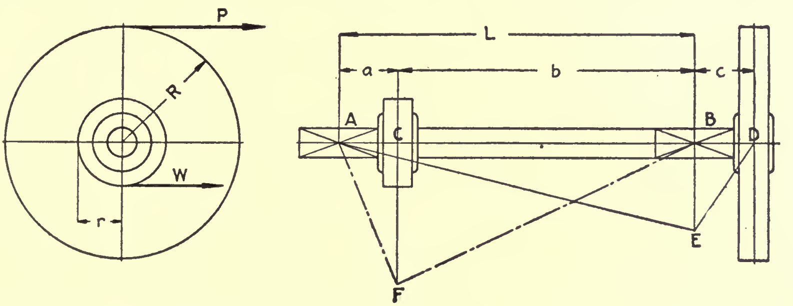

Fig. 286.

the lines AE and ED. Thus the magnitude of the moment at B is represented by the vector BE. In a similar manner the bending moment at any point along the shaft, due to the force W, may be determined by scaling the ordinate between AB and the lines AF and FB.

Since the coplanar forces P and W are located on opposite sides of the bearing B and act in the same direction, it is evident that the magnitude of the resultant bending moment at any point along the shaft is given by the ordinate between the lines $AFBD$ and AED. Combining the maximum resultant bending moment with the torsional moment transmitted, the diameter of the shaft may readily be determined by the method outlined in Art. 342.

(*b*) *Diameter of the shaft required for stiffness.*—Having determined the diameter of the shaft for the consideration of strength, it should be investigated for stiffness. For the shaft shown in the figure, the analytical method of determining the deflections will prove simpler than the graphical method. The formulas for the deflection of the shaft, due to the action of the force P, are as follows:

For any point on the shaft at a distance x_1 to the right of the bearing B

$$\Delta_1 = \frac{Px_1}{6\,EI}(3cx_1 - x_1^2 + 2\,cL) \tag{548}$$

For a point between the bearings at a distance x_2 to the left of the bearing B

$$\Delta_2 = -\frac{Pcx_2}{6\,EIL}(L - x_2)(2\,L - x_2) \tag{549}$$

The minus sign in (549) indicates that the deflection of the shaft between the bearings is in the opposite direction to the deflection of that part of the shaft overhanging the bearing.

The shaft deflections due to the force W are in a direction opposite to those caused by the force P; hence, the resultant deflection at any point is the difference between the deflections due to the loads W and P. The deflections between the bearings due to W may be calculated by means of (546) and (547), while those to the right of the bearing B are given by the following expression:

$$\Delta_1' = -\frac{Wabx_1}{6\,EIL}(L + a) \tag{550}$$

These deflections may be represented graphically as suggested in the preceding article, thus showing at a glance the location of the maximum. If the maximum deflection exceeds the permissible value the shaft diameter must be increased.

348. Shaft Supporting One Normal and One Inclined Load between the Bearings.—In Fig. 287 is shown a shaft supported on two bearings carrying a spur and bevel friction gear. Due to the normal pressure P_n between the contact surfaces of the friction gears, the shaft is subjected to an axial compression and bending moment in addition to the bending and torsional moments caused by the tangential forces on the two gears. The bending moments due to P, P_n, and W may be calculated by the algebraic method or they may be determined graphically. Since P_n and W are coplanar forces, it is unnecessary to consider each of them separately in determining the bending moments. The force and funicular polygons shown in Fig. 287(b) and (c) are obtained in the usual manner. Drawing the dividing ray OD parallel to the closing string od, we obtain in the vectors CD and DA the reactions at the bearings B and A due to the combined action of P_n and W. The force P_n represented by BF may be resolved into two components BC and CF as shown in the force polygon. The component CF produces a compression in the shaft and a thrust upon the bearing A. The magnitude of the component BC must equal the difference between the magnitudes of the pressures BE and EC produced upon the shaft at or near the

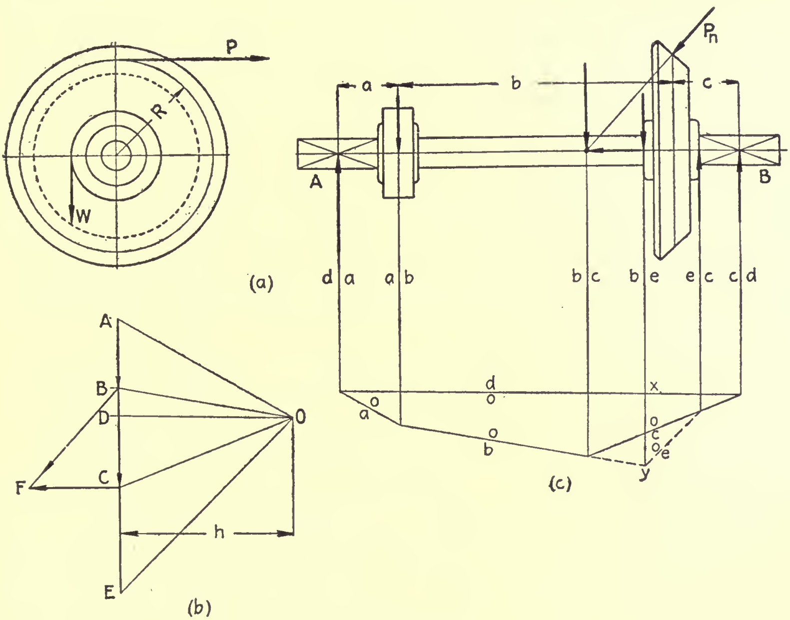

FIG. 287.

ends of the hub of the bevel friction gear by the inclined force P_n. For all practical purposes the lines of action of the pressures BE and EC may be assumed as shown in Fig. 287(a) The magnitudes of BE and EC may be determined as follows: Produce the string ob until it intersects the line of action be, and join this intersection with that of the string oc and the line of action ec; through the pole O draw the ray OE parallel to the string oe, thus establishing the magnitudes of BE and EC.

To determine the bending moment at any section of the shaft, as for example, along the line of action of the pressure BE, multiply the ordinate xy of the funicular polygon by the pole distance h. It should be remembered that the ordinate xy must be measured to the scale of the space diagram, while the pole distance h represents a force and hence must be measured to the scale of the force diagram.

The tangential force P causes bending moments which are at right angles to those caused by the force W and P_n; hence, the method given in Art. 346 must be used to determine the maximum resultant moment coming upon the shaft. Instead of finding the bending moments due to the force P by means of the graphical method, less labor is involved by using the algebraic method. If the shaft is long relative to the diameter, it is necessary either to treat it as a long column or to change the location of the bearing B. In other words, locating the bearing B adjacent

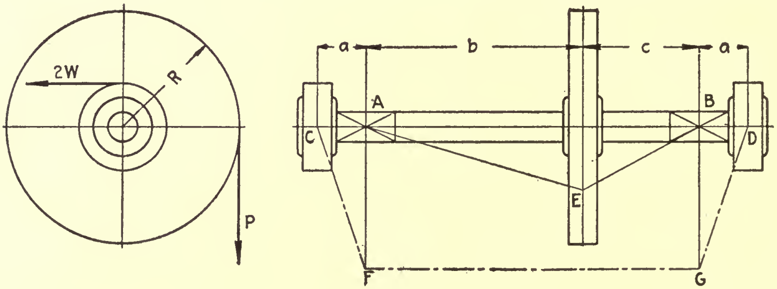

Fig. 288.

to the back of the bevel friction relieves the shaft from all column action, since the axial component would be absorbed by the bearing. Such an arrangement of bearings would in general be preferred to that shown in Fig. 287(a). However, with the suggested change of bearings a force analysis different from that given above would be necessary.

349. Two-bearing Shaft Supporting Three Loads.—Frequently shafts supported on two bearings and carrying more than two gears, sheaves, or sprockets are required. Such a shaft supporting three loads is shown in Fig. 288. The bending moment diagram due to the load P on the large driving gear is represented by the triangle AEB, while that due to the loads W acting on the overhanging pinions is represented by $CFGD$. Since P and W are non-coplanar, the method of procedure for determining the required diameter of the shaft is similar to that given in

Art. 346. Having calculated the shaft diameter necessary for strength, the deflection must be investigated. For the shaft under consideration the deflections at several points along the shaft are readily obtained by the algebraic method, after which the deflection curves for the two systems of loads may be plotted. The maximum resultant deflection at any point may be determined from the curves, and if this is found excessive the diameter of the shaft must be increased.

The deflection at any point between the overhanging load and the adjacent bearing, due to the action of the two equal loads W, may be calculated by the expression

$$\Delta_1'' = \frac{Wx_1}{6\,EI}\,(3\,a(L + x_1) - x_1^2), \tag{551}$$

in which x_1 denotes the distance from the bearing to the point under discussion. For the part between the bearings, the deflection at any point due to the loads W is

$$\Delta_2'' = -\frac{Wax_2}{2\,EI}(L - x_2), \tag{552}$$

in which x_2 denotes the distance from the bearing to the point considered.

350. Hollow Shafts.—In any shaft the outer fibers of the material are more useful in resisting the bending and twisting than the fibers at or near the center; hence the material may be distributed more efficiently by making the shafts hollow. Furthermore, the weight of such a shaft is diminished in greater proportion than its strength.

It is evident that the strength of a hollow shaft is equivalent to the strength of the solid shaft minus the strength of the shaft having a diameter equal to the diameter of the hole. In determining the strength of the shaft having a diameter equal to that of the hole, the fiber stress to be used must be that produced in the solid shaft at a point whose distance from the center is equal to the radius of the hole. Letting S_s denote the shearing stress produced in the outer fiber of the shaft having a diameter d_1, the stress produced at a distance $\frac{d_2}{2}$ from the center of the shaft is $\frac{d_2}{d_1}\,S_s$.

(*a*) *Torsional strength.*—For a hollow shaft the relation between the twisting moment and the diameters of the shaft is

$$T = \frac{\pi d_1^3 S_s}{16} - \frac{\pi d_2^4 S_s}{16\, d_1} = \frac{\pi S_s}{16\, d_1}(d_1^4 - d_2^4) \qquad (553)$$

Denoting the ratio of d_2 to d_1 by u, the expression for the large diameter of a hollow shaft becomes

$$d_1 = \sqrt[3]{\frac{5.1\, T}{S_s(1 - u^4)}} \qquad (554)$$

(*b*) *Torsional stiffness.*—The angular deflection, in degrees, caused by a given torsional moment T may be calculated by means of the following formula, obtained from (518) and (553) by eliminating S_s:

$$\theta = \frac{584\, lT}{E_t(d_1^4 - d_2^4)} \qquad (555)$$

(*c*) *Transverse strength.*—Occasionally it may be desired to calculate the diameter of a hollow shaft subjected to a given bending moment M. This may be done by the use of the following formula, which is obtained by equating M to the moment of resistance of the hollow shaft and solving for d_1:

$$d_1 = \sqrt[3]{\frac{10.2\, M}{S(1 - u^4)}} \qquad (556)$$

(*d*) *Hollow and solid shafts of equal strength.*—It is required to determine the relation between the diameter of a solid shaft and that of a hollow shaft of the same strength. Equating the moments of resistance to twisting for the two shafts, we obtain

$$\frac{\pi d^3 S_s}{16} = \frac{\pi S_s}{16\, d_1}(d_1^4 - d_2^4),$$

or

$$d_1 = \frac{d}{\sqrt[3]{1 - u^4}} \qquad (557)$$

For the same strength, a hollow shaft is much lighter than a solid one. The per cent. saved in weight is given by the following formula:

$$\text{per cent. gain} = \left[\frac{d^2 - d_1^2 + d_2^2}{d^2}\right]100 \qquad (558)$$

EFFECT OF KEY-SEATS ON SHAFTING

The effect of a key-seat in a shaft is to decrease slightly both its strength and stiffness. In order to obtain some knowledge as to the extent of this change in strength and stiffness, Prof. H. F. Moore of the University of Illinois made a series of tests, the results of which were reported in *Bulletin* No. 42, University of Illinois Experiment Station. The shafts used in these tests varied in diameter from 1¼ to 2¼ inches inclusive. Both cold-rolled and turned shafts made of soft steel were tested. The key-seats cut into these shafts were of common proportions.

351. Effect upon Strength.—According to the results obtained by Prof. Moore, the ultimate strength of a key-seated shaft is practically the same as the ultimate strength of the solid shaft. Furthermore, very little difference was observed between the strength of shafts with short key-seats and of similar shafts having long key-seats. The tests, however, showed conclusively that a key-seat has a decided influence upon the elastic strength of the shaft. In order to put the results of these experiments into usable form, the so-called "efficiency of the shaft" was determined for each size of shaft tested. By the term "efficiency" is meant the ratio of the elastic strength of the shaft with the key-seat to the elastic strength of the solid shaft. The following equation for the efficiency is suggested by Prof. Moore as representing fairly well the results he obtained:

$$E_1 = 1.0 - 0.2\,w - 1.1\,h, \tag{559}$$

in which w denotes the ratio of the width of the key-seat to the shaft diameter, and h, the ratio of the depth of the key-seat to the diameter of the shaft.

352. Effect upon Stiffness.—A number of tests were also made to determine the effect of key-seats upon the angular stiffness of shafts. The following equation for the ratio of the angle of twist of the key-seated shaft to the angle of twist of the solid shaft is due to Prof. Moore, and may serve as a guide in determining the probable weakening effect the key-seat has upon the torsional stiffness of the shaft.

$$E_2 = 1.0 + 0.4\,w + 0.7\,h \tag{560}$$

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CHAPTER XIX

BEARINGS AND JOURNALS

BEARINGS

353. Types of Bearings.—Bearings may be divided into two general classes: (*a*) sliding; (*b*) rolling.

(*a*) *Sliding bearings.*—The sliding bearings in common use in machinery are of three types. The first type, called *right line bearing,* is one in which the motion is parallel to the elements of the sliding surfaces. These sliding surfaces may be flat, as the guides on engine crossheads and the ways of large planers and milling machines, or they may be angular as the ways on small planers and lathes. Circular guides are also in use for the crossheads of engines and spindles of boring and drilling machines.

The second type of bearing, called a *journal bearing* consists of two machine parts that rotate relatively to each other. The part which is enclosed by and rubs against the other is called the *journal,* and the part which encloses the journal is called the *box* or less specifically the *bearing.* In the more common form of journal bearings, the journal rotates inside of a fixed bearing. In some cases, as in a loose pulley or a hoisting drum, the journal is fixed and the bearing rotates, while in other cases both the journal and the bearing have a definite motion, as for example, a crankpin and its bearing in the connecting rod.

The *thrust bearing* is the third type of sliding bearing. It is used to take the end thrust in bevel and worm gearing, or in general any force acting in the direction of the shaft axis. Thrust bearings are of two kinds: (1) The so-called *step-* or *pivot-bearing,* which supports the weight of a vertical shaft and its attached parts. The shaft terminates in the bearing. (2) The *collar thrust bearing,* which is used on propeller shafts, spindles of drill presses, and shafts carrying bevel and worm gears. In such cases the shaft generally extends through and beyond the bearing.

(*b*) *Rolling bearings.*—Rolling bearings include all bearings in which rolling elements are used for supporting the rotating members. This class of bearings is discussed in detail in Chapter XX.

JOURNAL BEARING CONSTRUCTION

354. General Considerations.—In designing bearings the following important points must be given due consideration.

(*a*) The proper bearing material must be selected with respect to the load coming upon the bearing and the material used for the journal.

(*b*) Provision must be made for anchoring the bearing material to the bearing proper.

(*c*) Provision must be made for taking up any wear that is liable to occur.

(*d*) In many cases means must be provided for preserving the alignment of the bearings.

(*e*) Proper clearance between the journal and its bearing must be provided.

(*f*) Means must be provided for lubricating the bearing.

(*g*) Bearings running at high speeds and subjected to high pressures must be provided with some means of dissipating the heat that is generated by friction.

(*h*) The dimensions of the bearing, that is, the diameter and the length, are fixed by the journal with which the bearing is to run. The proportions of the journal are determined from a consideration of strength, rigidity, rubbing speed, and the permissible pressure per square inch of projected area.

355. Selection of Bearing Materials.—As a rule bearings give the best service when the material of the bearing and that of the journal are unlike. No satisfactory explanation has ever been offered why unlike materials are better, but it is claimed that with like materials the frictional resistance and the wear are greater. However, there are exceptions, as under certain conditions hardened steel against hardened steel, and cast iron in contact with cast iron have given excellent service. Bearing surfaces are made of many different substances depending largely upon the class of service for which the bearing is intended. The following is a list of some of the materials that are used for bearing surfaces: babbitt metal; various grades of bronzes; cast iron; mild, case-hardened, and tempered steel; wood; fiber graphite.

The main requirements for a good bearing metal are the following: (1) It should possess sufficient strength to prevent squeezing out of the bearing when subjected to a load. (2) It should not heat rapidly and should have a high melting point. (3) It should be able to resist abrasion but should not score the journal. (4) It should be uniform in texture and possess a low coefficient of friction.

(*a*) *Babbitt metal.*—Babbitt metal is used more extensively than any other bearing metal. One reason is that the metal is easily melted in a common ladle and poured into the bearing. Babbitt bearings require an outer shell to which the metal is anchored. Generally the shell is made of cast iron although steel casting and bronze are sometimes used. Shells made of bronze have the advantage that in case the babbitt metal melts and runs out of the bearing, the journal will not be damaged so readily. The babbitt lining is made about $\frac{3}{16}$ inch thick in small bearings and from $\frac{3}{8}$ to $\frac{1}{2}$ inch thick in large bearings. To prevent rotation of the babbitt lining, the shell must be provided with some form of anchor. These anchors may consist of dove-

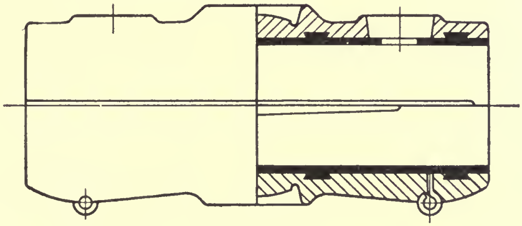

Fig. 289.

tailed grooves as shown in Fig. 289, or cored or drilled holes into which the babbitt may flow when the bearing liner is cast.

Babbitt metals having various degrees of hardness are in use. A so-called hard babbitt is suitable for bearings subjected to heavy pressure or severe shock, while a soft babbitt is better adapted to a light load and high speed. Babbitt metal is used for the main bearings of engines and air compressors, on steam turbines, centrifugal pumps and blowers, motors and generators, in wood-working machinery, in bearings for line- and countershaft, and in many machine bearings of the split type. For the compositions of several grades of babbitt metals see Art. 51, Chapter II.

(*b*) *Bronzes.*—Next to babbitt metal, bronze is considered the most important bearing material. It is commonly used in the form of a one-piece bushing forced under pressure into the shell or framework of the bearing. Frequently the bushing is split into halves each of which is fastened by suitable means to a part of the bearing shell. The thickness of these bushings varies with the diameter and the length of the journal. There are upon the market a large number of different kinds of bronzes, many of which are giving excellent service. For the composition and other information pertaining to several grades of commercial bronzes see Art. 48, Chapter II.

(*c*) *Cast iron.*—Cast-iron bearings running with steel journals have met with considerable success and eminent engineers have advocated their use, claiming that the surface will in a short time wear to a glassy finish and run with very little friction. However, if for any reason lubrication fails and heating begins, the result is liable to be either serious injury or total destruction to both bearing and journal. Several machine-tool builders use cast-iron bearings that are constantly flooded with oil and they experience no bearing troubles. In general it may be said that cast-iron bearings will prove satisfactory when the pressure and speed coming upon the bearing are not excessive and where sufficient lubrication is insured.

356. Provisions for Lubrication.—The object of any system of lubrication is to form and maintain a uniform film of oil between the journal and its bearing. By the term *system of lubrication* is meant the method used for bringing the lubricant to the bearing and its distribution in the bearing. To distribute the oil and assist in the formation of a uniform oil film, the bearing is generally provided with a series of *oil grooves.* These grooves should start at the point of supply and lead diagonally outward in the direction of rotation. For journals rotating in either direction, the bearing is provided with a symmetrical arrangement of grooves. To insure the formation of the oil film, the edges of the oil grooves must be bevelled or rounded off. The lubricant is delivered to the bearing or to the journal in various ways among which are the following: (1) Drop-feed lubrication; (2) wick lubrication; (3) saturated-pad lubrication; (4) chain or ring lubrication; (5) flooded lubrication; (6) forced lubrication; (7) grease lubrication.

(*a*) *Drop-feed lubrication.*—The most common method of oiling a bearing is by means of the drop-feed method. In its simplest form it consists of an open hole in the bearing through which oil is introduced. In many cases the hole is tapped to receive a closed oil cup, thus preventing dirt and grit from entering the bearing.

(*b*) *Wick lubrication.*—In bearings used on line- and countershafts and occasionally in machinery, the oil is transferred by capillary action from a small reservoir in the cap to the bearing surfaces by means of a wick as shown in Fig. 290. This method of lubrication is satisfactory when the bearing pressures and the speed are not excessive.

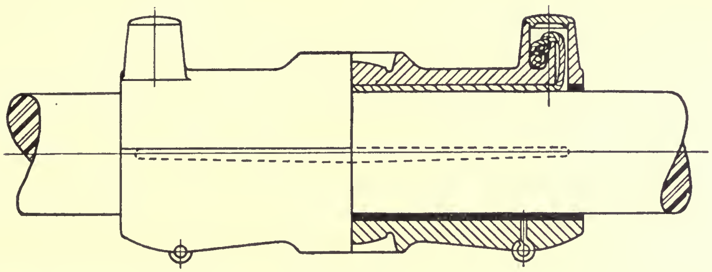

Fig. 290.

(*c*) *Saturated-pad lubrication.*—An effective way of lubricating line- and countershaft bearings is by means of wooden blocks containing a series of saw-cuts through which the oil rises. The blocks, generally two in number, are located in the lower half of the bearing and are held in contact with the shaft by means of springs. The lower ends of these blocks project into the oil

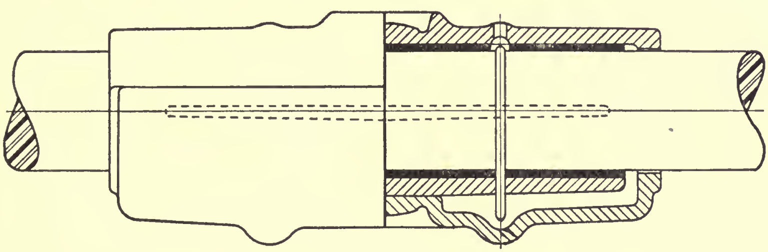

Fig. 291.

reservoir, thus permitting the lubricant to rise from the reservoir to the shaft by means of capillary action.

(*d*) *Ring* or *chain lubrication.*—Ring or chain lubrication is considered one of the best methods of supplying a bearing with oil. It is used on bearings for all classes of machinery. An application of a ring oiler to a line- and countershaft bearing is shown in Fig. 291, and in Fig. 296, 297, 300, 310 and 311 are shown various designs such as are used on machine tools, centrifugal pumps, etc. The quantity of oil delivered to the bearing by a ring depends upon the size and speed of the ring and upon the viscosity of the oil. The diameter of the oil ring should be made approximately double the diameter of the shaft, and the ring may be made solid or split. The former construction is used for small bearings and the latter for larger bearings. An inspection of a considerable number of ring-oiling bearings used on line-shafts, electrical machinery, and centrifugal pumps seems to indicate that an oil ring cannot be expected to supply proper lubrication over a length of bearing exceeding approximately 4 inches on each side of the ring. In electrical machinery the rings are usually made of brass or bronze in order to avoid magnetic difficulties. In general the rings should be perfectly round, they should have no sharp corners, and they should be well balanced.

On the main bearings of high-speed engines a form of bearing similar to the ring-oiling type is occasionally used, but in place of the ring a sash chain is used.

(*e*) *Flooded lubrication.*—In flooded lubrication the oil is supplied to the bearing by means of a pump or from an overhead reservoir, but at practically no pressure. This system has been used to some extent on machine tools.

(*f*) *Forced lubrication.*—In forced lubrication the oil is supplied to the bearing at a considerable pressure by means of a pump. Generally the oil pressure varies from 15 to 25 pounds per square inch; however, the pressure may run up to 600 pounds per square inch as in the case of the step bearing used on Curtis vertical steam turbines.

(*g*) *Grease lubrication.*—Grease lubrication is well adapted for use on bearings subjected to heavy pressures and in which the speeds are relatively low. Grease is introduced into the bearing by any one of the various forms of grease cups obtainable on the market.

Very few of the systems of lubrication discussed above produce a perfect oil film. According to Axel K. Pederson, analytical expert of the General Electric Co., the various systems given above may be arranged into the following three classes:

1. Those systems which produce an imperfect oil film; for example, drop-feed, wick, and grease lubrication.

2. Those systems which produce a semi-perfect oil film, for example, saturated-pad and ring or chain lubrication.

3. Those systems which produce a perfect oil film; for example, flooded and forced lubrication.

357. Adjustments for Wear.—(*a*) *Split bearing.*—In the majority of bearings some means of taking up wear must be provided. The adjustment for wear may be made in various ways, but the

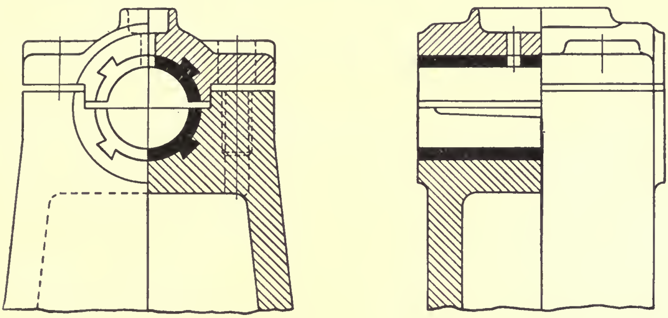

Fig. 292.

most common method is by the use of a split bearing the parts of which are bolted together. The wear is taken up by simply removing some of the metal or paper shims and tightening the bolts in the bearing cap. In split bearings the line of division

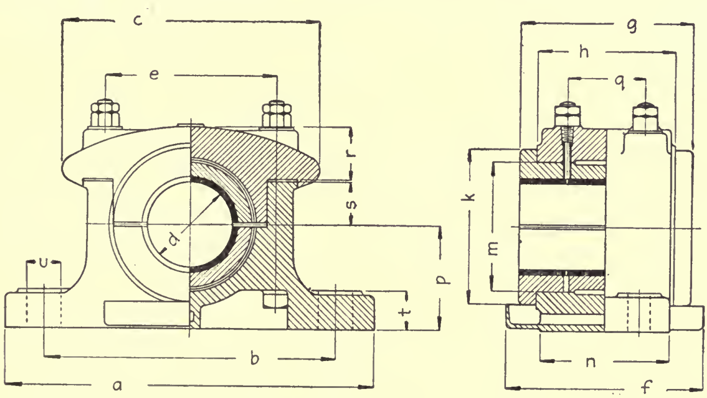

Fig. 293.

should be made with an offset as shown in Fig. 292 and 293 for two reasons: (1) When made with an offset, the cap will prevent the bearing under pressure from springing together at the sides and gripping the shaft. (2) The offset will, to a certain extent, prevent the escape of the lubricant.

(*b*) *Four-part bearing.*—The main bearings of steam and gas engines are generally of the four-part type similar to the design shown in Fig. 294. The babbitt-lined side shells are provided with adjusting wedges which extend the full length of the bearing. The bottom shell is also lined with babbitt metal and rests in a spherical seat in the engine frame, thus keeping the shaft in good alignment at all times. By raising the shaft sufficiently to relieve the bearing of its load, the bottom shell may be rolled out and inspected.

As shown in Fig. 294, the bearing cap is of heavy construction and is not babbitted the entire length of the bearing, but merely for a short distance at each end. The cap is placed over the

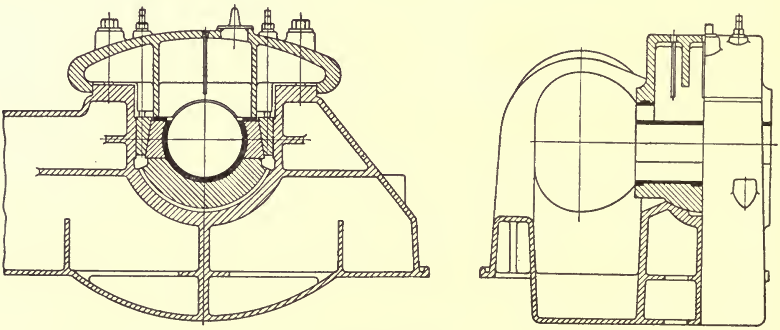

FIG. 294.

jaws of the engine frame with a driving fit. It is evident that a four-part bearing permits making adjustments for wear in a more nearly correct manner than is possible with a common split bearing; hence it is well adapted for installation where the line of action of the resultant bearing pressure changes with the rotation of the shaft.

(*c*) *Solid bearing.*—No doubt the simplest form of bearing is that known as the solid type, designs of which are shown in Figs. 295 to 298 inclusive. The solid bearing has no provision for taking up wear except by removing the worn-out bushing or liner and replacing it with a new one. The bronze bushed bearings shown in Figs. 295 and 296 have been used successfully on heavy machine tools. They have ample provisions for lubrication, but none for wear except by renewal of the bushing. Such bushings are replaced very readily at a small cost. In Fig. 297 is shown another design of ring oiling solid bearing consisting of a cast-iron shell lined with babbitt metal. This type of bearing has been found to give good service on the small and medium sizes of centrifugal pumps. The shell and brass oil ring are fitted into a suitable housing; the shell is held in place by a special headless screw projecting into the hole shown in the figure.

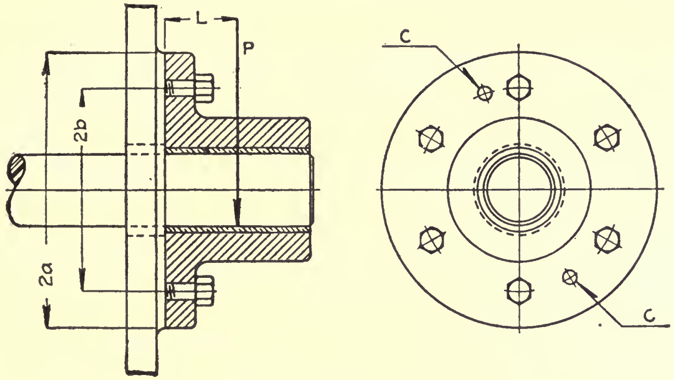

Fig. 295.

The design of a solid bearing shown in Fig. 298 is used in places where the pressure upon the bearing is always in the same direction, as for example on the shaft used for supporting the overhead sheaves of an elevator. As shown, merely the lower half of the

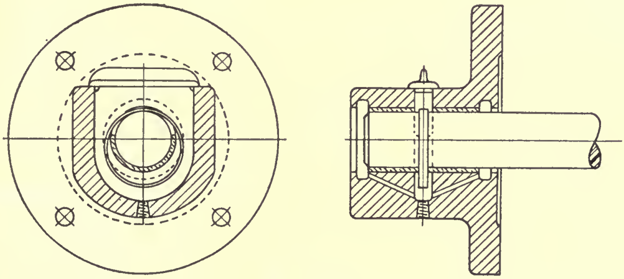

Fig. 296.

bearing, which in this case takes the entire pressure, is lined with babbitt metal. The central part of the bearing shell is made spherical so that it will fit into the spherical seat in the pedestal, thus keeping the shaft in proper alignment.

A design of a solid bearing used on the spindles of heavy milling machines made by the Ingersoll Milling Machine Co. is shown in

Fig. 299. The conical journal of the spindle *a* is fitted with a bronze bushing *b*, the latter being forced in the sleeve *g*. Somewhere near the middle of its length, the spindle has keyed to it a removal conical journal *c*. The latter fits into the conical bronze bushing *f*, which is forced into the sliding sleeve *g*. Practically all of the wear comes upon the bearing *b* and may be taken up by

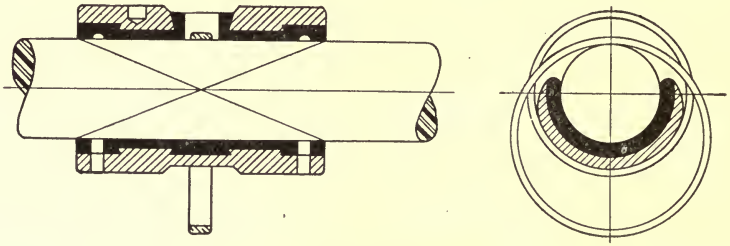

Fig. 297.

means of the adjusting nut *e* and the special washer provided between the end of *b* and the enlarged head of the spindle. Due to the use of the conical bearing, the alignment of the spindle is not disturbed by an appreciable amount when an adjustment for wear is made.

On the spindles of certain machine tools the bearings are made with a bronze bushing having a straight bore and is turned conical

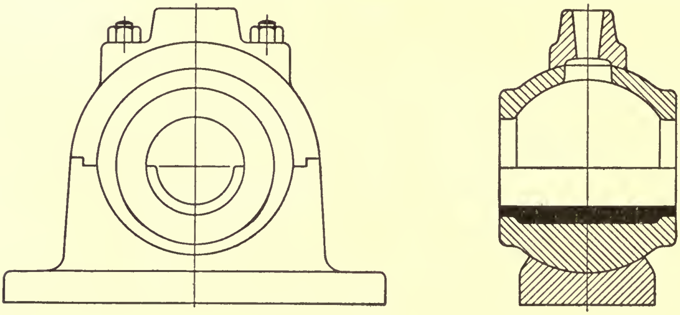

Fig. 298.

on the outside as shown in Fig. 300. The bushing is threaded at each end and is provided with a slit extending through the entire length. It is evident that this bearing may readily be adjusted for wear by means of the adjusting nuts at the ends of the bearing. The oil ring and oil reservoir provided in the framework of the bearing insure proper lubrication of the bearing at all times.

(*d*) *Connecting-rod bearings.*—The bearings used on connecting rods differ somewhat from those discussed in preceding paragraphs. In Figs. 301 to 303 inclusive are shown three designs that have proven satisfactory. The first two are used on the crankpin end of the rod while the third is intended for the crosshead end,

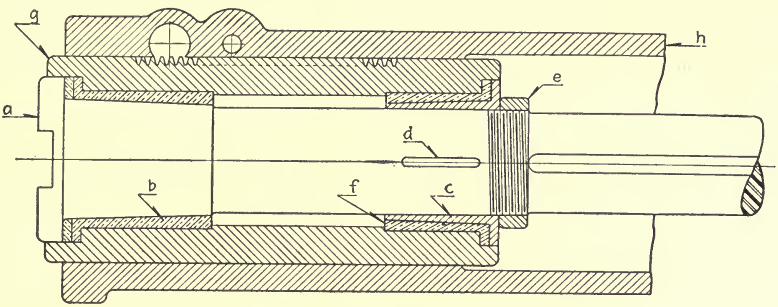

Fig. 299.

although a similar design is frequently used for the crankpin end. In all three cases the adjustments for wear are made by means of a wedge and suitable cap screws.

The design shown in Fig. 301 consists of two half bearings

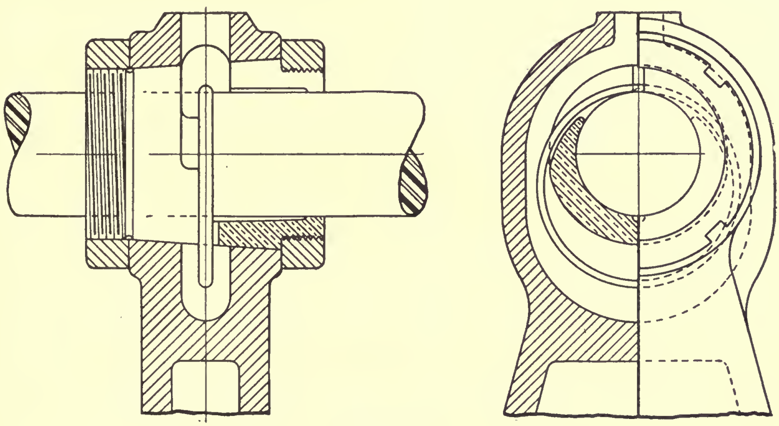

Fig. 300.

around which a steel stirrup or strap is placed, the latter being fastened rigidly to the rod end by two through bolts. The adjusting wedge with its screws is located between the strap and the front half of the bearing. Taking up wear by means of this

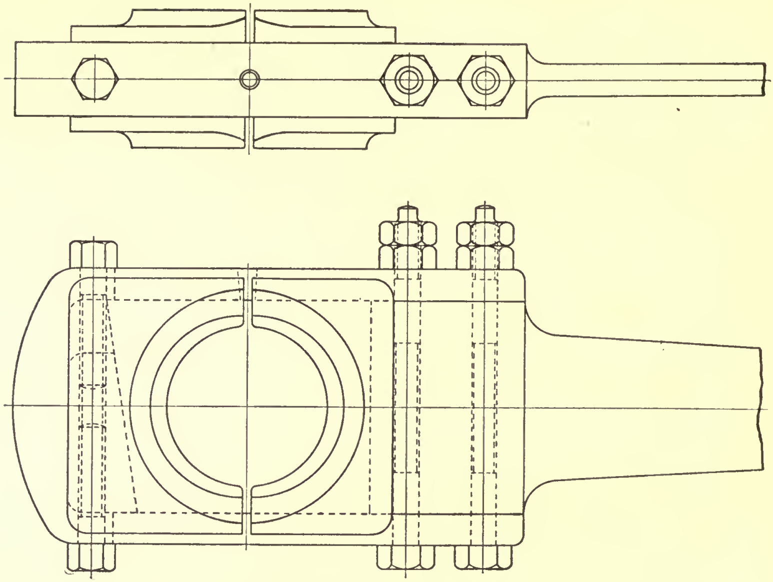

Fig. 301.

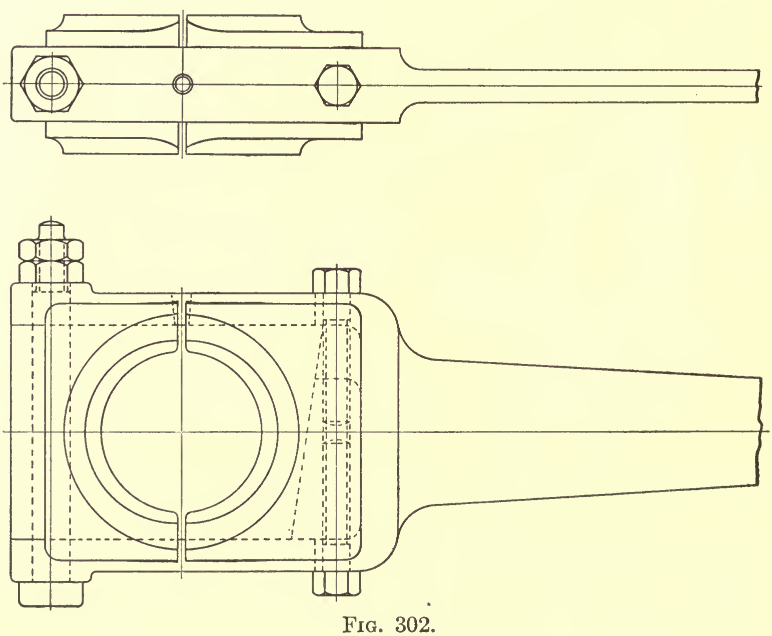

Fig. 302.

wedge tends to shorten the rod, hence the bearing at the other end of the rod should be equipped with an adjustment which tends to counteract the former, thus maintaining a constant distance between the two bearings. For economy of material the two halves of the bearing are made of steel casting lined with babbitt metal. Sometimes brass is used in place of the steel casting.

In Fig. 302 is shown an open rod end into which are fitted the two halves of the bearing; one of these halves is movable and the other is fastened rigidly to the rod by a through bolt. The adjusting wedge and screws are located between the back bearing

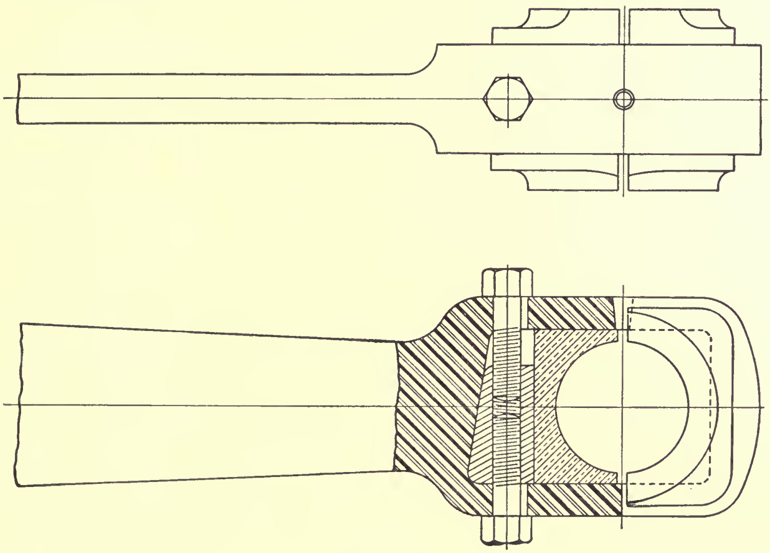

Fig. 303.

and the rod, thus the tendency is to lengthen the rod when the wear is taken up. As in the design shown in Fig. 301, the two halves of the bearing are made of steel casting lined with babbitt metal.

The design shown in Fig. 303 is called a closed-rod end. The adjustments for wear are made in the same manner as in the preceding designs. It is evident from the figure that taking up wear tends to lengthen the connecting rod, hence this design would be a proper one to use in connection with that shown in Fig. 301 since the length of the rod would remain practically a constant. The two parts of the bearing used with the closed-rod end of Fig. 303 are generally made of bronze though occasionally babbitt-lined bearings are used.

358. Adjustments for Alignment.—In addition to provisions for taking up wear, many bearings are provided with means for aligning the shaft. Bearings that are out of line tend to heat and produce wear. Some of the bearings discussed in Art. 357 meet the provisions for alignment by having the bearing divided into parts that can be adjusted vertically or horizontally, while others are provided with spherical seats thus making them self-aligning. In many cases the bearing and its housing are

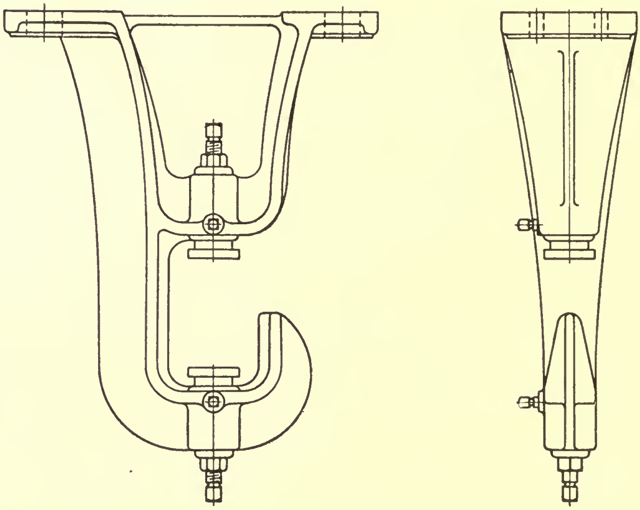

Fig. 304.

mounted on supports which permit the adjustments necessary to line up the shaft. The horizontal adjustment in such cases is generally provided for by elongating the holes through which the housing is bolted to the support.

Hangers.—For lining up the bearings of line- and countershafts various forms of hangers are used. As shown in Figs. 289 to 291 inclusive, line-shaft bearings are made in two parts each of which is provided with a spherical seat which fits into a corresponding seat on the sockets of the hangers. A design of a cast-iron single-brace ball and socket drop hanger is shown in Fig. 304. From this figure it is evident that the two-ball seated sockets provide the vertical adjustment while the slotted holes in the supporting flanges of the hanger take care of the horizontal adjustment. When greater rigidity is required than is furnished by the hanger shown in Fig. 304, a double-brace design similar to that represented in Fig. 305 is used. However, the hanger shown in the latter figure is made entirely of pressed steel, the parts being riveted or bolted together as shown. Set screws are used for giving the desired adjustments.

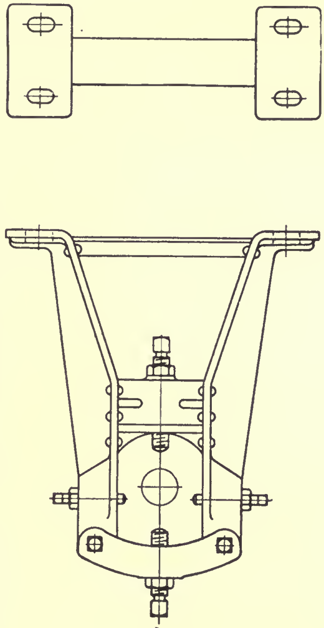

FIG. 305.

DESIGN OF BEARINGS AND JOURNALS

359. Bearing Pressures.—In order to maintain an oil film between the journal and its bearing, the pressure must not exceed the so-called *critical pressure*, by which is meant the limiting pressure at which a perfect film between the journal and the bearing is maintained. This pressure depends upon the speed of the journal, the viscosity of the oil, the temperature of the bearing, the closeness of the fit between the journal and its bearing, and the degree of finish given to the surfaces in contact. As yet no test results are available to show the relation existing between the pressure, viscosity, and temperature. According to H. F. Moore, the relation existing between the critical pressure p_c and the speed of the journal is given by the following formula:

$$p_c = 7.47\sqrt{V}, \tag{561}$$

in which V denotes the peripheral speed of the journal in feet per minute. The Moore formula is based upon the results obtained from a series of experiments on a steel journal running on a white metal bearing. The pressure carried on the bearing varied from 10 to 80 pounds per square inch of projected area, and the speed did not exceed 140 feet per minute.

The following formula for the permissible bearing pressure based on Stribeck's results is taken from Smith and Marx's

"Machine Design," and is recommended for use when the speed does not exceed 500 feet per minute:

$$p_c = 10\sqrt{V} \tag{562}$$

TABLE 96.—ALLOWABLE BEARING PRESSURES

Type of bearing		Pressure p
Punching and shearing machinery	Main journals	2,000–3,000
	Crankpins	5,000–8,000
Engine crankpins	High speed	250–600
	Low speed	850–1,500
	Locomotives	1,500–1,700
	Marine	400–600
	Air compressors, center crank	250–500
	Auto gas engines	350–450
Engine crosshead pins	High speed	900–1,700
	Low speed	1,000–1,800
	Locomotives	3,000–4,000
	Marine	1,000–1,500
	Air compressors, center crank	400–800
	Auto gas engine	800–1,000
Engine main bearings	High speed	180–240
	Low speed	160–220
	Marine	200–400
	Air compressors, center crank	150–250
	Auto gas engine	350–400
Locomotive driving journals	Passenger	190
	Freight	200
	Switching	220
Engine crossheads	Stationary	25–40
	Marine	50–100
Car journals		300–500
Motors and generators		40–80
Horizontal steam turbines		40–60
Eccentric sheaves		80–100
Hoisting machinery shafting		70–90
Propeller shaft thrust bearings.	Freight steamer	40–55
	Passenger steamer	60–80
	Large naval vessels	70–90
	Light naval vessels	110–130

For speeds exceeding 500 feet per minute, the same authorities suggest the formula

$$p_c = 30\sqrt[3]{V} \tag{563}$$

In L. P. Alford's book on "Bearings" is given a chart showing the relation between the maximum safe bearing pressure and the rubbing speed for perfect film lubrication. This chart represents the practice of the General Electric Co. in designing the bearings used on motors and generators. The following expression gives values of the maximum safe bearing pressure which agree very closely with those obtained from the chart.

$$p_m = 15.5\sqrt[3]{V} \tag{564}$$

In addition to the formulas given in this article, the allowable bearing pressures, in pounds per square inch of projected area, contained in Table 96 will serve as a guide in designing bearings and journals. These values are based upon current practice and were collected from various sources.

TABLE 97.—RELATION BETWEEN LENGTH AND DIAMETER OF BEARINGS

Type of bearing		Ratio l/d Min.	Ratio l/d Max.	Type of bearing		Ratio l/d Min.	Ratio l/d Max.
Marine engine	Main bearing	1.00	1.50	Steam turbines		2.0	3.0
	Crankpin bearing	1.00	1.50	Generators and motors		2.0	3.0
High-speed engine	Main bearing	2.00	3.00	Centrifugal pumps		2.0	2.5
	Crankpin bearing	1.00	1.00	Centrifugal fans		2.0	3.0
	Crosshead pin bearing	1.40	1.60	Machine tools		2.0	4.0
Slow-speed engine	Main bearing	1.75	2.25	Hoisting drums		1.5	2.0
	Crankpin bearing	1.00	1.25	Hoisting sheaves for cranes		1.0	2.0
	Crosshead pin bearing	1.20	1.50	Wood working machinery		2.5	4.0
Stationary gas engine	Main bearing	2.00	2.50	Shaft hangers	Rigid	2.5	3.0
	Crankpin bearing	1.00	1.50		Self-adjusting	3.0	4.0
	Crosshead pin bearing	1.50	1.75				
Auto gas engine	Main bearing	1.00	1.75	Pillow blocks	Plain	2.5	3.5
	Crankpin bearing	1.20	1.40		Ring-oiling	4.0	5.0
	Crosshead pin bearing	1.70	2.25				

360. Relation between Length and Diameter.—The ratio of the length of a bearing to its diameter is fairly well established for the different classes of machinery. In Table 97 are given the values of this ratio for a considerable number of different types of bearings, the majority of which were obtained from a study of actual installations.

361. Radiating Capacity of Bearings.—The capacity that a bearing has for radiating the heat generated by the friction between the journal and its bearing depends upon the mass of metal used in the construction of the bearing and upon the condition of the surrounding air. The following formula due to Axel K. Pedersen may be used for determining the amount of heat carried away:

$$Q = \frac{(T_0 + 33)^2}{K}, \tag{565}$$

in which Q denotes the heat radiating capacity of a bearing expressed in foot-pounds per second per square inch of projected area; T_0 denotes the difference between the temperature of the bearing and that of the cooling medium; K denotes an experimental constant the magnitude of which depends upon the method used for cooling the bearing. The following values of K derived by Pedersen from Lasche's and the General Electric Co.'s experiments may be safely used in designing bearings:

1. For bearings of light construction located in still air—$K = 3{,}300$.
2. For bearings of heavy construction and well ventilated—$K = 1{,}860$.
3. For General Electric Co.'s well-ventilated bearings—$K = 1{,}150$.

362. Coefficient of Friction.—The coefficient of friction between the bearing and its journal depends upon the bearing pressure, the speed of the journal, the temperature of the bearing, the specific gravity of the lubricant, and the method used for lubricating the bearing. The laws governing the coefficient of friction in a bearing provided with a limited supply of lubricant are generally assumed the same as those governing ordinary sliding friction. However, when the bearing is provided with a copious supply of lubricant, the coefficient of friction depends upon the laws of friction in a fluid, that is, the resistance the lubricant offers against shearing.

In an article entitled "Bearing Design Constants" which appeared in *Power*, Feb. 22, 1916, Mr. Louis Illmer gives several formulas for the coefficient of friction which are based upon the experimental researches of Tower, Lasche, Thomas, Maurer and Kelso. The coefficient of friction according to the Tower tests is given by the expression

$$\mu = \frac{2}{p}\sqrt{\frac{V}{T}}, \tag{566}$$

in which p denotes the bearing pressure in pounds per square inch of projected area; V the speed of the journal in feet per minute; T the virtual temperature head of the oil, which may be assumed as the temperature of the bearing less 60°. According to Illmer this formula is applicable to bearings having a pressure range of 100 to 500 pounds per square inch of projected area, and in which the speed does not exceed 500 feet per minute.

The Lasche experiments were made on a steel journal running in a ring oiling babbitt lined bearing, and the results obtained lead to the following expression for the coefficient of friction:

$$\mu = \frac{4.5}{p\sqrt{T}} \tag{567}$$

This formula is applicable to bearings subjected to pressures of 15 to 225 pounds per square inch of projected area, and in which the speed may range from 500 to 3,500 feet per minute. The temperature of the bearing may vary from 85° to 210°F.

From the results of experiments made by Thomas, Maurer and Kelso on babbitt-lined hanger bearings, Illmer derived the following formula for the coefficient of friction:

$$\mu = \frac{\sqrt[3]{V}}{20\sqrt{pT}} \tag{568}$$

The use of this formula is limited to bearings in which the pressures vary from 33 to 100 pounds per square inch of projected area, and in which the speed of the journal ranges from 100 to 300 feet per minute.

The experiments of Lasche, as well as some made at Cornell University, seem to indicate that the coefficient of friction is practically independent of the speed when the latter exceeds 500 feet per minute. Upon this assumption, (568) may be simplified by substituting for V the critical value 500, whence

$$\mu = \frac{0.4}{\sqrt{pT}} \tag{569}$$

Equation (569) proposed by Illmer, gives values of the coefficient of friction that may reasonably be expected in the operation of well-designed bearings lined with babbitt metal and lubricated with a generous supply of mineral engine oil.

Mr. William Knight, in the *American Machinist* of Nov. 16, 1916, suggests that (569) be modified by introducing in the numerator a factor s denoting the specific gravity of the oil when compared to water. Thus the revised form of (569) becomes

$$\mu = \frac{0.4\, s}{\sqrt{pT}} \tag{570}$$

Knight bases his suggestion upon an investigation of the results obtained by A. L. Westcott from a series of tests made at the University of Missouri on greases and oils. Furthermore (570) gives values of μ that agree fairly well with the results obtained by Lasche for pressures between 120 and 240 pounds per square inch.

363. Design Formulas.—Having given the ratio between the length of the bearing and its diameter, we may readily develop working formulas for the diameter of the bearing in terms of the load P, the revolutions per minute, and certain constants. The resultant formulas will be based upon equations (562) and (563); hence they will only apply to bearings receiving a copious supply of lubricant and to those in which the speed remains within the range given in Art. 359. For a bearing having a diameter d and length l and subjected to a total pressure P, the pressure per square inch of projected area is

$$p = \frac{P}{cd^2}, \tag{571}$$

in which c denotes the ratio of l to d. Equating the value of p to the limiting pressure given by (562), we obtain

$$P = 10\, cd^2\sqrt{V} \tag{572}$$

Introducing the value of V in terms of d and N, the number of revolutions per minute of the journal, we obtain the following expression for d *for speeds below* 500 *feet per minute:*

$$d = 0.52 \sqrt[5]{\frac{P^2}{c^2N}} \tag{573}$$

By a similar procedure, using (563) in place of (562), we obtain the following formula for d *for speeds exceeding* 500 *feet per minute:*

$$d = 0.282 \sqrt[7]{\frac{P^3}{c^3N}} \tag{574}$$

By means of (573) or (574), whichever applies to the problem under discussion, the diameter of the bearing may be calculated. Knowing d, the length of the bearing may be determined since $l = cd$. Furthermore, the magnitude of p may be determined by means of (571).

364. Temperature of Bearings.—Frequently it is desirable to determine the probable temperature of the bearing due to the heat generated. If the temperature becomes too high the oil is liable to lose its lubricating qualities, hence it may be necessary to redesign the bearing or resort to artificial cooling. The work of friction expressed in foot-pounds per second per square inch of projected area is

$$W_f = \frac{\mu p V}{60}, \tag{575}$$

and this must equal the quantity of heat radiated or carried away as expressed by (565); hence

$$\frac{\mu p V}{60} = \frac{(T_0 + 33)^2}{K}, \tag{576}$$

from which the limiting speed of the bearing for a given final temperature is

$$V = \frac{60}{\mu p K}(T_0 + 33)^2 \tag{577}$$

Equation (577) may also be used to calculate the probable temperature of a well-lubricated bearing running under given conditions of load and speed. To determine this temperature the following method of procedure is suggested: From (568) or (570), depending upon the speed, determine the value of μ in terms of T. Substitute this value of μ as well as the magnitudes of p, V, and K in (577) and determine the probable temperature of the bearing. The maximum temperature of a bearing depends upon the lubricant used, and since bearing oils begin to show signs of losing their lubricating qualities at a temperature of approximately 250°F. it is considered good practice to limit the maximum temperature, as determined by (577), to 180°F.

365. Strength and Stiffness of Journals.—In the majority of cases the journal is integral with and forms a part of the shaft, the diameter of which has been calculated according to the methods given in Chapter XVIII. The dimensions of the journals of important shafts are not generally based on calculations for strength and stiffness but on the liability of heating, that is, the conditions which govern the oil supply. However, the stresses in a journal should always be investigated in order to make sure that the dimensions are ample so far as strength and rigidity are concerned.

(*a*) *Strength of end journals.*—End journals are generally considered cantilever beams loaded uniformly. Equating the bending moment to the moment of resistance and solving for the diameter d, we obtain

$$d = 1.72 \sqrt[3]{\frac{Pl}{S}} \tag{578}$$

Having given the dimensions of the end journal, and the load coming upon it, the magnitude of the stress may be determined by (578), or by means of the formula

$$S = 5.1\, pc^2, \tag{579}$$

in which p and c have the same meaning as assigned to them in Art. 363. The working stress S, due to the fatigue of the material, should not exceed 4,000 to 5,000 pounds per square inch.

(*b*) *Stiffness of end journals.*—In designing journals the question of stiffness is an important one, and should be given the proper consideration. For an end journal loaded uniformly, the deflection Δ is calculated by the formulas

$$\Delta = \frac{2.55\, Pl^3}{Ed^4} \tag{580}$$

whence

$$d = 1.26 \sqrt[4]{\frac{pl^3}{\Delta E}} \tag{581}$$

For common end journals good engineering practice dictates that the value of Δ should not exceed 0.01 of an inch.

366. Design of Bearing Caps and Bolts.—The cap of a bearing should never be subjected to a heavy load; however, there are cases in which the circumstances are such that a considerable pressure comes upon the cap. In such cases the cap is generally regarded as a beam supported by the holding down bolts or screws and loaded at the center, as shown in Fig. 306. As in the journal, the cap should be investigated for both strength and stiffness.

(*a*) *Strength of cap.*—Assuming the dimensions of the cap as represented in Fig. 306, we obtain the following expression for the thickness b, by equating the bending moment to the moment of resistance:

$$b = \sqrt{\frac{3\,Pa}{2\,Sl}} \tag{582}$$

(*b*) *Stiffness of cap.*—In order that the cap will have ample rigidity, the thickness b should be calculated by the following

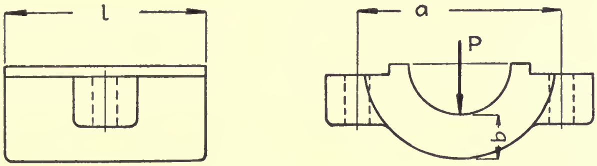

FIG. 306.

expression based upon the formula for the deflection Δ of a simple beam loaded as shown in Fig. 306:

$$b = 0.63\,a\,\sqrt[3]{\frac{P}{l\Delta E}} \tag{583}$$

For the cap of a common end journal or a marine end connecting rod, good engineering practice limits the deflection Δ to 0.01 of an inch.

(*c*) *Holding-down bolts.*—The bolts, screws, or studs that are used for holding down the cap are generally assumed to be subjected to a simple tension, and as a rule each bolt is designed for a load equivalent to $\frac{4\,P}{3n}$, in which n denotes the number of bolts used for holding down the cap.

367. Work Lost Due to the Friction on a Cylindrical Journal.—With our present state of knowledge of the subject of friction, we are unable to determine a correct expression for the work lost due to journal friction. In deriving an expression for the moment of journal friction, it is generally assumed that the coefficient of friction is constant for a given speed and further that the pressure between the surfaces in contact is uniformly distributed, or that the wear of the journal and its bearing is uniform and proportional to the work of friction. The assumption of uniform distribution of pressure is hardly warranted in the case of a "worn-in" journal and bearing; but for a new journal and bearing having perfect contact over the entire bearing surface, it is probable that the pressure is uniformly distributed. In the following analysis, formulas based on both assumptions will be derived.

(*a*) *Pressure uniformly distributed.*—Assuming that the pressure between the journal and its bearing is uniformly distributed over the contact surface, the intensity of pressure p is equal to the load P on the journal divided by the projected area of the journal. This may be shown as follows:

The pressure on a longitudinal strip of width ds and length l is $plds$. Let the direction of the pressure p make an angle θ with the vertical center line of the journal, and assume that the load P acts in vertical direction; then the component of p parallel to the line of action of P is

$$dP = pl\cos\theta ds = \frac{pld}{2}\cos\theta d\theta, \tag{584}$$

from which

$$P = pld,$$

or

$$p = \frac{P}{ld} \tag{585}$$

The force of friction on the elementary strip lds is $\mu plds$, and the moment of this force about the axis of the journal is

$$dM = \frac{\mu pld}{2} ds;$$

whence by integration

$$M = \frac{\pi\mu Pd}{4} \tag{586}$$

The work, in foot-pounds, lost per minute due to the friction is given by the formula

$$W_f = \frac{\mu\pi^2 NPd}{24}, \tag{587}$$

in which the diameter d is expressed in inches, and N denotes the revolutions per minute.

(*b*) *Uniform vertical wear.*—The statement that the normal wear is proportional to the work of friction is equivalent to saying that the normal wear n is equal to the product of a constant

k, normal pressure p, and the diameter d of the journal, since work is proportional to the product of p and d. Hence

$$n = kpd \tag{588}$$

It is evident that the normal wear of a journal and bearing is greatest at the bottom and becomes zero at the sides. If the journal and bearing remain cylindrical after being worn it is apparent that the vertical wear h is constant, and the normal wear at any point of the surface in contact will be given by the relation

$$n = h \cos\theta \tag{589}$$

Combining (588) and (589)

$$p = C \cos\theta, \tag{590}$$

in which the constant $C = \frac{h}{kd}$. Substituting (590) in (584), we get

$$dP = \frac{Cld}{2} \cos^2\theta d\theta,$$

from which the total load upon the bearing is

$$P = \frac{\pi Cld}{4} \tag{591}$$

The moment of the force of friction about the axis of the journal is

$$dM = \frac{\mu Cld^2}{4} \cos\theta d\theta,$$

whence

$$M = \frac{\mu Cld^2}{2} \tag{592}$$

Eliminating C by combining (591) and (592), we get

$$M = \frac{2\mu}{\pi} Pd \tag{593}$$

The energy, in foot-pounds, lost per minute is given by the expression

$$W_f = \frac{\mu NPd}{3} \tag{594}$$

368. Work Lost Due to the Friction on a Conical Journal.—The expressions for the moment of friction and the work lost due to the friction on a conical journal, having the dimensions shown in Fig. 307, are determined in a manner similar to that used in Art. 367.

(*a*) *Pressure uniformly distributed.*—On the assumption of uniform distribution of pressure, the vertical component of the normal pressure on an elementary area is given by the expression

$$dP = \frac{pr}{\tan \alpha} \cos\theta d\theta dr \tag{595}$$

from which

$$P = \frac{p}{\tan \alpha} \left[r_2^2 - r_1^2\right] \tag{596}$$

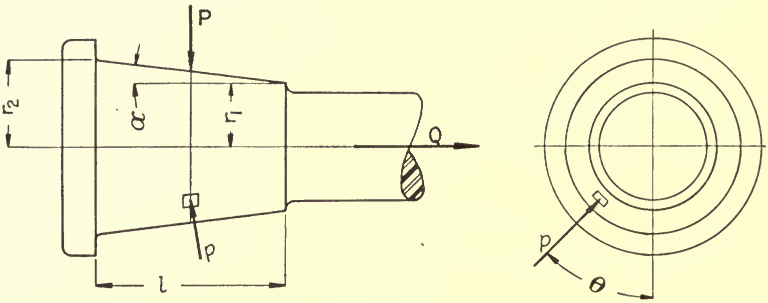

FIG. 307.

The force of friction on the elementary area is $\frac{\mu prdld\theta}{\cos \alpha}$, and the moment of this force about the axis of the journal is

$$dM = \frac{\mu pr^2}{\sin \alpha} drd\theta; \tag{597}$$

whence

$$M = \frac{\mu \pi p}{3 \sin \alpha} \left[r_2^3 - r_1^3\right]$$

$$= \frac{\mu \pi P}{3 \cos \alpha} \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \tag{598}$$

The energy, in foot-pounds, lost per minute is

$$W_f = \frac{\mu \pi^2 NP}{18 \cos \alpha} \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \tag{599}$$

(*b*) *Uniform vertical wear.*—Assuming that the vertical wear

h of the journal and bearing remains constant for all points, the normal wear at any point is

$$n = h \cos \alpha \cos \theta \tag{600}$$

Since the normal wear is proportional to the work of friction, it is evident that

$$n = kpr \tag{601}$$

Combining (600) and (601), we get

$$p = \frac{C \cos\theta}{r} \tag{602}$$

in which the constant $C = \frac{h \cos \alpha}{k}$. Substituting the value of p in (595), we get

$$dP = \frac{C}{\tan \alpha} \cos^2\theta d\theta dr;$$

whence the total load P upon the journal becomes

$$P = \frac{\pi C}{2 \tan \alpha} (r_2 - r_1) \tag{603}$$

To determine an expression for the moment of friction, substitute (602) in (597); whence

$$dM = \frac{\mu C}{\sin \alpha} r \cos\theta d\theta dr$$

Integrating

$$M = \frac{\mu C}{\sin \alpha} \left[r_2^2 - r_1^2 \right] \tag{604}$$

Combining (603) and (604) in order to eliminate C, the magnitude of the moment of friction of the conical journal is given by the expression

$$M = \frac{2 \mu P d}{\pi \cos \alpha}, \tag{605}$$

in which d denotes the mean diameter of the conical journal.

To calculate the energy lost due to friction, the following formula may be used:

$$W_f = \frac{\mu N P d}{3 \cos \alpha} \tag{606}$$

369. Proportions of Journal Bearings.—In general the dimensions of the various parts of a bearing are determined by means of empirical formulas which are based upon the diameter of the shaft. Such formulas usually give a well-proportioned bearing having an excess of strength.

(*a*) *Common split bearings.*—The empirical formulas given below are based upon a series of dimensions obtained from several sizes of common split bearings similar to the type represented by Fig. 292. The cap is held down by either two or four bolts, studs or cap screws, the number depending upon the length of the bearing. In the following formulas the symbols d and l denote respectively the diameter of the shaft and the length of the bearing:

$$\left.\begin{aligned} \text{Outside diameter of bearing} &= 1.75\,d + 0.5'' \\ \text{Span of bolts} &= 1.7\,d + 0.7'' \\ \text{Distance between bolts} &= 0.5\,l \\ \text{Size of bolts} &= \tfrac{3}{16}\,d + 0.25'' \\ \text{Thickness of babbitt} &= \tfrac{1}{16}\,d + 0.125'' \end{aligned}\right\} \quad (607)$$

(*b*) *Pedestal bearings.*—The pedestal bearing shown in Fig. 293 is provided with removable bearing shells which are made alike so that they are reversible. The shells are lined with babbitt metal that is peened, then bored and scraped to exact size. This type of bearing is manufactured by the Stephens-Adamson Mfg. Co. in six sizes ranging from $3\tfrac{15}{16}$ to $9\tfrac{1}{2}$ inches in diameter. The empirical formulas given below were derived from dimensions furnished by the manufacturer, and the various symbols used in these formulas apply to the key drawing of Fig. 293.

$$\left.\begin{aligned} a &= 3.7\,d + 3'' & m &= 1.47\,d + 0.25 \\ b &= 3.1\,d + 1.75'' & n &= 1.5\,d \\ c &= 3\,d + 0.8'' & p &= 0.88\,d + 1.8'' \\ e &= 1.9\,d + 0.5'' & q &= 0.7\,d + 0.75'' \\ f &= g + 1.5'' & r &= 0.75\,d - 0.5'' \\ g &= 2\,d & s &= 0.38\,d + 0.5'' \\ h &= 1.7\,d - 0.3'' & t &= 0.5\,d + 0.1'' \\ k &= 1.43\,d + 1.3'' & u &= 0.28\,d + 0.4'' \end{aligned}\right\} \quad (608)$$

$$\left.\begin{aligned} \text{Diam. of bolts for base} &= 0.14\,d + 0.45'' \\ \text{Diam. of bolts for cap} &= 0.2\,d - 0.08'' \\ \text{Thickness of babbitt} &= 0.025\,d + 0.18'' \end{aligned}\right\} \quad (609)$$

(*c*) *Rigid post bearings.*—In Fig. 308 is shown a form of babbitt-lined split bearing that is used for carrying line- or countershafts when it is necessary to support the latter from posts and

Table 98.—Dimensions of Rigid Post Bearings

Size	Dimensions																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
15/16	8 5/8	6	3	3	...	2 5/8	2	1 5/8	1 3/8	2 3/8	....	1 5/8	1	1 5/16	11/16	9/16	1/2	1/4	3/16
1 3/16	9 7/8	7	3 3/4	3 1/2	...	3	2 3/8	1 15/16	1 3/4	2 9/16	....	1 7/8	1 1/4	1 7/16	13/16	3/4	9/16	5/16	1/4
1 7/16	11 1/8	8 1/8	4 1/2	4	...	3 1/4	2 5/8	2 9/32	2	2 3/4	....	2	1 1/4	1 1/2	15/16	13/16	5/8	3/8	1/4
1 11/16	12 1/2	9 1/8	5 1/4	4 3/8	...	3 3/4	3	2 5/8	2 3/8	3	....	2 1/4	1 1/2	1 11/16	1	15/16	11/16	7/16	5/16
1 15/16	13 3/4	10 1/4	6	4 7/8	...	4	4	3 1/16	2 3/4	3 1/4	....	2 1/2	1 3/4	1 3/4	1 1/8	1 1/8	3/4	1/2	3/8
2 3/16	15 1/8	11 3/8	6 3/4	5 3/8	...	4 1/2	3 5/8	3 5/16	3 1/8	3 1/2	....	2 3/4	2	1 7/8	1 3/8	1 1/8	13/16	9/16	3/8
2 7/16	16 1/4	12 3/8	7 1/2	5 3/4	6	4 5/8	4	3 5/8	3 1/2	3 3/4	....	3	2 1/4	1 15/16	1 7/16	1 3/8	7/8	5/8	1/2
2 11/16	17 5/8	13 1/2	8 1/4	6 1/4	...	5 1/8	4 1/8	4	3 3/4	4	2 7/8	3 1/8	2 1/4	2 1/16	1 1/2	1 7/16	15/16	11/16	1/2
2 15/16	18 7/8	14 1/2	9	6 3/4	...	5 3/8	4 1/2	4 5/16	4 1/8	4 3/16	3 1/8	3 3/8	2 1/2	2 3/16	1 1/2	1 1/2	1	3/4	9/16
3 3/16	20 1/8	15 5/8	9 3/4	7 1/8	...	5 5/8	4 7/8	4 5/8	4 1/2	4 3/8	3 3/8	....	2 1/2	2 1/4	1 9/16	1 9/16	1 1/16	13/16	5/8
3 7/16	21 3/8	16 5/8	10 1/2	7 5/8	...	6	5 1/4	5	4 3/4	4 5/8	3 3/4	....	2 1/2	2 3/8	1 3/4	1 5/8	1 1/8	7/8	11/16
3 11/16	22 3/4	17 3/4	11 1/4	8	...	6 3/8	5 5/8	5 3/8	5 1/8	4 7/8	3 7/8	....	2 1/2	2 1/2	1 7/8	1 7/8	1 3/16	15/16	3/4
3 15/16	24	18 7/8	12	8 1/2	...	6 3/4	5 7/8	5 9/16	5 1/2	5 1/8	4 1/4	....	2 3/4	2 9/16	1 15/16	1 15/16	1 1/4	1	13/16

columns. Bearings for a similar service but furnished with oil rings are also obtainable. In Table 98 are given the dimensions pertaining to the different sizes of the type of rigid post bearing illustrated in Fig. 308.

THRUST BEARING CONSTRUCTION

The journal bearings discussed in the preceding articles are not suitable for supporting vertical shafts carrying heavy rotating

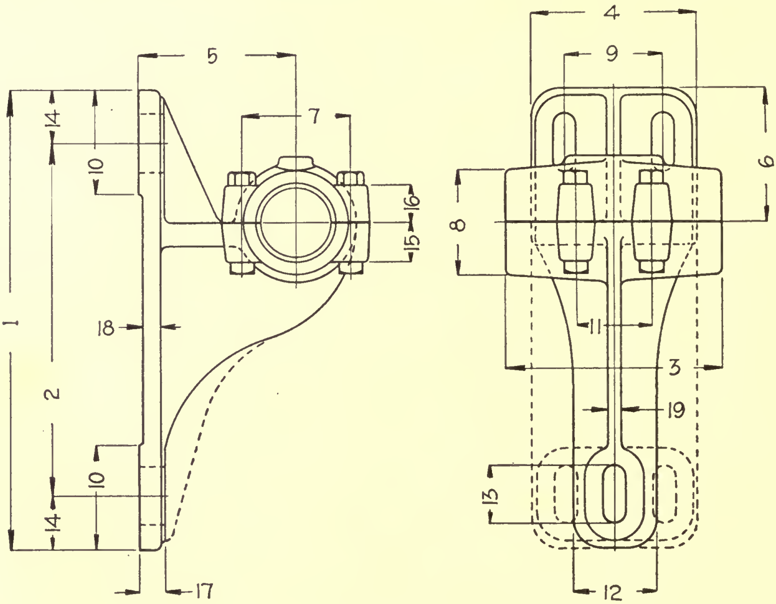

Fig. 308.

parts or horizontal shafts subjected to heavy pressures acting in a direction parallel to the shaft. However, in many installations of horizontal shafts subjected to axial loads, as for example, thrusts due to bevel and worm gearing, the ordinary journal bearing is used and the axial pressures are taken care of by means of one or more loose washers located between the supporting bearing and a suitable collar on the shaft.

In all thrust bearings the speed of the surfaces in contact is a maximum at the outer edge, and at the axis, theoretically, it is zero. At any point of contact the wear is proportional to the work of friction; namely, the product of the pressure at the point and the velocity of the point. The exact distribution of the pressure existing between the contact surfaces is not known definitely, but very likely it is maximum at the center, and for that reason a well-designed flat pivot bearing should have ring contact. This form of contact surface is produced by merely removing some of the metal at the center.

370. Solid Bearing with Thrust Washers.—A solid bearing provided with thrust washers and used for supporting a bevel-gear transmission is shown in Fig. 309. The thrust of the gear is taken up by a single bronze washer, while that of the pinion is taken up by three washers, the two outer ones being made of steel and the other of bronze. The steel washers have spherical faces which fit into the spherical seats furnished on the hub of the

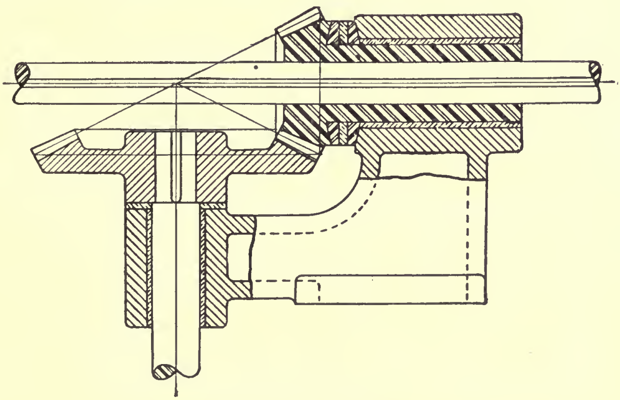

Fig. 309.

pinion and the end of the bearing. Frequently bearings of this description are furnished with a casing for enclosing the gear and pinion thus permitting the gearing and washers to run in an oil bath. In addition to providing an effective means of lubrication, the casing also protects the workmen from coming into contact with the gearing. The thrust due to worm gearing is frequently taken care of by an arrangement similar to that shown in Fig. 309, but plain washers are used in place of spherical seated ones.

371. Collar Thrust Bearings.—(*a*) *Marine thrust bearings.*—For shafts subjected to a considerable end thrust, as for example a screw propeller shaft, or the shaft of a centrifugal pump or blower, the axial load is generally absorbed by a special type of thrust bearing, commonly called a *collar thrust bearing.* Instead of transmitting the axial load to the end of the bearing, the shaft is provided with a series of collars cut integral with it, which distribute the pressure over the length of the bearing.

In modern marine practice the rings that come into contact with the collars on the shaft are made in the shape of a horseshoe. Such a construction permits their removal without disturbing any other part of the bearing. The lower part of the bearing housing is provided with a reservoir containing oil, and in order that the temperature of the oil may not become excessive, a water coil is fitted into this reservoir. Each end of the housing is

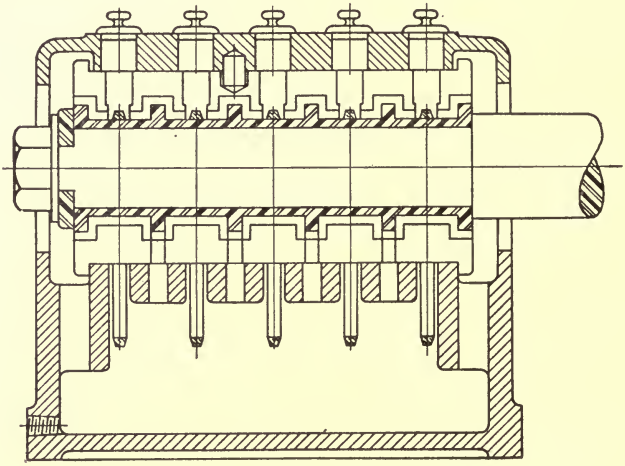

FIG. 310.

equipped with a stuffing box so that the level of the oil in the reservoir may be carried slightly above the lower line of the shaft, thus insuring ample lubrication. Each of the bearing rings has an independently controlled circulation of water, thus making it possible to maintain a uniform temperature throughout the bearing.

(*b*) *De Laval thrust bearing.*—For the high rotative speeds used on certain classes of centrifugal pumps, the De Laval Steam Turbine Co. developed a babbitt-lined ring-oiled collar thrust bearing, the details of which are clearly shown in Fig. 310. The collars, instead of being integral with the shaft, are formed on a removable steel or cast-iron sleeve which is fitted to the impeller shaft and held in place by a special collar and lock nut. The babbitt-lined bearing shells are split vertically, while the pedestal or housing into which these shells are fitted is split horizontally. From Fig. 310 it is evident that the cap of the bearing is not subjected to a thrust, but the entire axial load comes upon the pedestal. Vertically split shells such as those used on the De Laval bearing are easily removed, and since the two shells are alike they may be interchanged. The oil rings are made of bronze and a sufficient number are provided to insure ample lubrication.

(*c*) *Bearing for combined radial and axial loads.*—In Fig. 311 is shown a design of a combined ring-oiling journal and collar

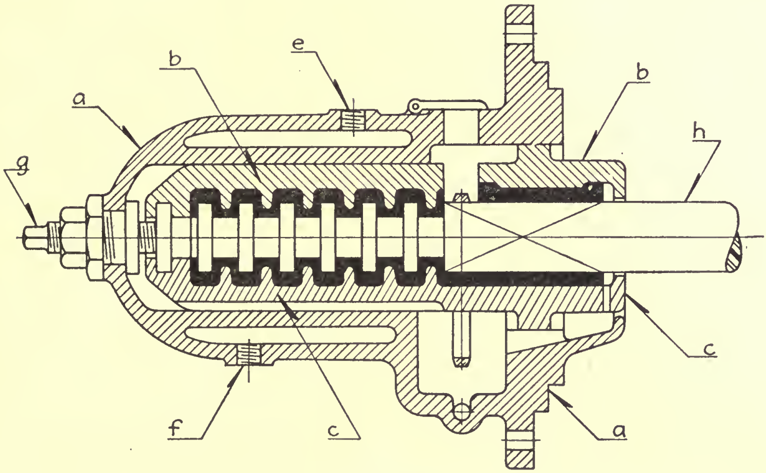

Fig. 311.

thrust bearing that is used on a single suction multistage turbine pump. The bearing is babbitt-lined throughout and ample lubrication is furnished by means of an oil ring. The housing or bracket *a*, into which the combined bearing shells *b* and *c* are fitted, is cored out so that water may be circulated through it in order to keep the bearing cool. The tapped hole *e* at the top is connected to the discharge side of the first stage while the hole *f* at the bottom is connected to the pump suction. The shell of the bearing is split horizontally. To provide means for taking up the wear of the thrust collars, an adjusting screw *g* and lock nut is provided. This adjusting screw is also used for locating the propeller shaft *h* in its correct position relative to the guide vanes of the pump.

The thrust bearings discussed in the preceding paragraphs may be located at any convenient point along the shaft, but precautions should be taken that the part of the shaft subjected to a thrust will not be too long or it may tend to fail by a buckling action similar to a long column.

372. Step Bearings.—(*a*) *Single-disc type.*—Frequently a form of thrust bearing is used in which all the thrust must be taken up at the end of the shaft, as for example a vertical transmission

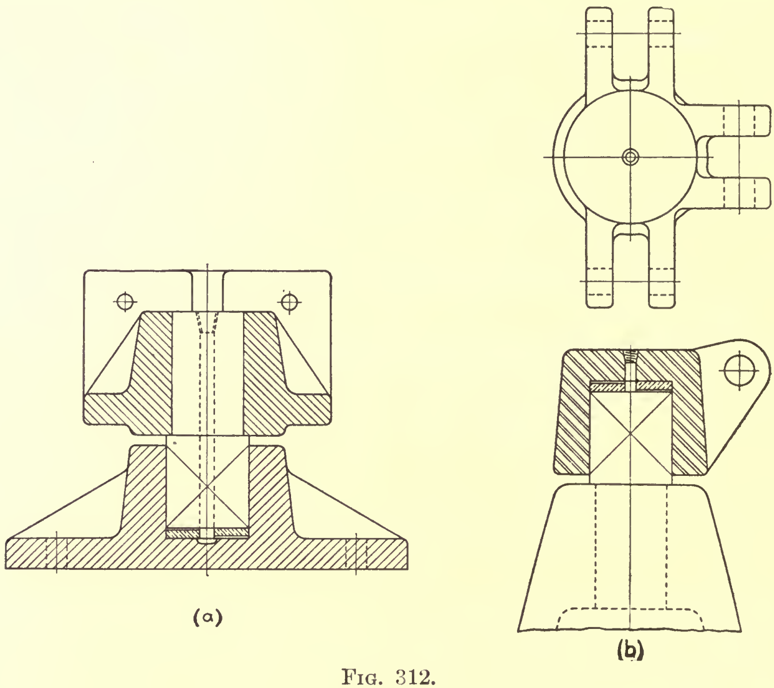

FIG. 312.

shaft. For slow speeds such as prevail in rotary cranes of the jib and pillar types, the thrust due to the load and weights of moving parts are usually taken care of by an ordinary flat pivot or step bearing similar to the designs shown in Fig. 312 and 313.

The thrust bearing illustrated in Fig. 312(*a*) is used on jib cranes and is frequently called a *pintle bearing.* The pintle or pin is subjected to a radial load in addition to the axial thrust. The pin in the design represented by Fig. 312(*b*) is also subjected to a combined radial and axial load and is used on pillar cranes.

(*b*) *Multiple-disc type.*—The wear upon a pivot may be reduced materially by introducing several discs between the end of the pin or shaft and the housing of the bearing. Alternate discs are generally made of bronze or brass and steel. The lower disc should be fastened to the bearing proper, and the upper one should be fastened to the shaft, while the intermediate ones must be free. It is evident that each disc is subjected to the same unit pressure, hence the effect of such a combination of discs is to reduce the wear, since the relative velocity between adjacent discs is decreased. In order to lubricate the various disc surfaces, oil is introduced through a central hole and radial grooves cut into the faces of the discs serve as distributors.

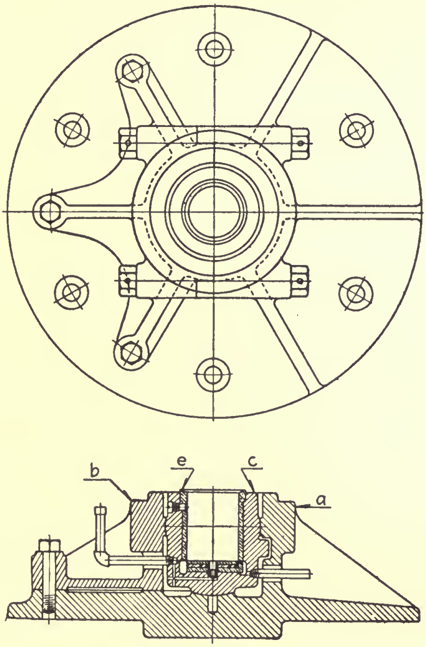

Fig. 313.

An application of the use of loose thrust discs is shown in Fig. 313 which illustrates a special step bearing designed by the Pawlings Harnischfeger Co. and used for supporting a heavy cantilever jib crane. In bearings of this kind the base casting is usually made in one piece, but in this case it is made in two parts, the base *a* and the cap *b*, which are bolted together by heavy stud bolts and special cap screws. The bronze bushed bearing shell *c* is provided with two spherical seats, the centers of which are located at the center of the bearing, as shown in the figure, thus insuring proper alignment at all times. The thrust due to the load upon the crane and the weight of the crane comes upon the discs, two of bronze and one of steel, and is transmitted through the end of the shell *c* to the spherical seated pivot bearing in the base *a*. The horizontal pressure due to the load and weight is transmitted to the spherical journal bearing in the base *a* and cap *b*. The method of lubricating the loose discs is clearly shown in the figure, also the method used for fastening the bushing *e* in the shell *c*.

In Fig. 314 is shown a form of an adjustable step bearing that is intended for use at the base of a vertical shaft. It is evident from the construction that such a bearing cannot take care of heavy radial loads. Solid or split bearings must be provided for the radial loads. The bearing shell is babbitted and the axial load comes upon two hardened steel discs having spherical faces as shown in the figure. The housing containing the bearing is large and provides ample reservoir capacity for the lubricant.

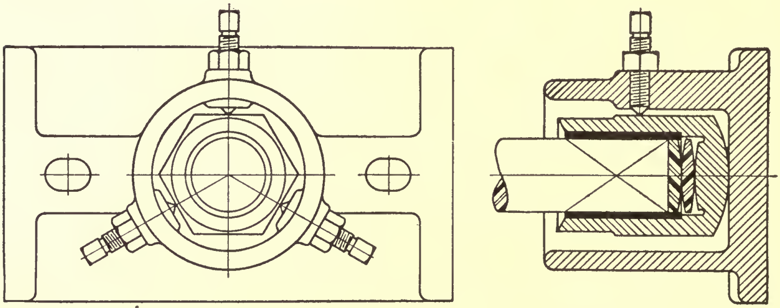

FIG. 314.

373. Work Lost due to Pivot Friction.—*General equations.*—For the general case we shall assume the pivot to be some surface of revolution, as shown in Fig. 315(*a*), the equation of the curve being unknown. Assume any point C at a distance x from the axis AB. If p denotes the intensity of the normal pressure at the point C, the total pressure on an annular strip of width ds and radius x will be $2\pi xpds$. Since the normal to the surface at the point C makes an angle θ with the axis AB, the vertical component of the pressure on the annular strip is

$$dP = 2\pi px\cos\theta ds \tag{610}$$

If r_1 and r_2 denotes respectively the smaller and larger radii of the pivot, the integration of (610) between these limits will give the sum of the vertical components and this sum must be equal to the axial load or thrust P; that is

$$P = 2\pi\int pxdx \tag{611}$$

The value of the integral will depend upon the law of variation of the normal pressure p.

The force of friction upon an annular strip of width ds is $2\pi\mu pxds$, in which μ denotes a coefficient of friction; hence the moment of this frictional resistance about the axis of rotation is

$$dM = 2\,\pi\mu px^2ds, \tag{612}$$

from which we obtain the following general expression for the moment of friction of a pivot:

$$M = 2\,\pi\int\mu px^2ds \tag{613}$$

From (613) it is evident that the value of M depends upon the following important considerations:

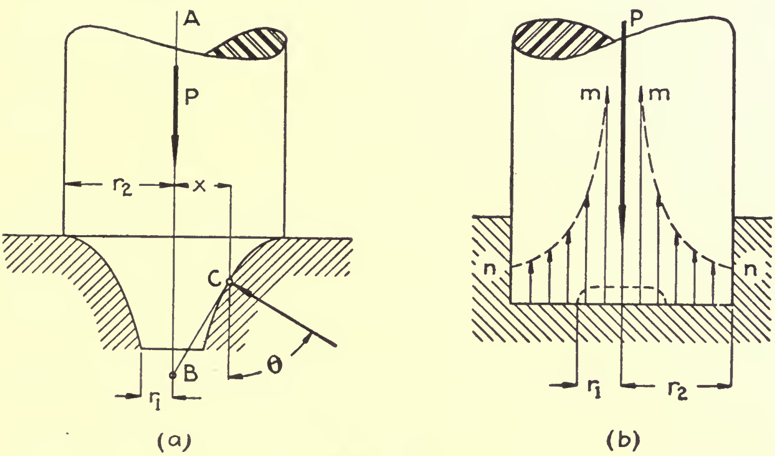

FIG. 315.

1. Upon the form of the pivot, that is upon the equation of the bounding curve.
2. Upon the law of variation of the normal pressure p.
3. Upon the law of variation of the coefficient of friction μ.

In any given case the form of the pivot is known, but the laws of variation for p and μ are not known. But little experimental work has been carried on to establish such laws. A common method of dealing with pivots is to assume *that the coefficient of friction remains constant* and that the pressure is uniformly distributed. The assumption of uniform pressure distribution may represent fairly well the condition existing when the pivot and its bearing are new, but would seem unwarranted in the case of a pivot that has been worn in. A more reasonable supposition is that *the normal wear at any point is proportional to the work of friction.*

374. Work Lost in a Collar Thrust Bearing.—(*a*) *Pressure uniformly distributed.*—With the assumption that the normal pressure p is the same at all points of the surfaces in contact, the magnitude of the thrust P upon a collar pivot according (611) is given by the following expression:

$$P = \pi p(r_2^2 - r_1^2) \tag{614}$$

From (614) it is apparent that the uniformly distributed pressure p is equal to the thrust P divided by the area of the collar.

Substituting in (613) the value of p obtained from (614) and integrating, assuming μ as constant, we obtain the following expression for the moment of friction of a collar bearing:

$$M = \frac{2\,\mu P}{3}\left[\frac{r_2^2 + r_2r_1 + r_1^2}{r_2 + r_1}\right] \tag{615}$$

The work, in foot-pounds, lost per minute by a collar pivot according to the above assumption may be determined by the formula

$$W_f = \frac{\mu\pi NP}{9}\left[\frac{r_2^2 + r_2r_1 + r_1^2}{r_2 + r_1}\right], \tag{616}$$

in which the dimensions r_2 and r_1 are expressed in inches and N denotes the revolution of the collar per minute.

(*b*) *Uniform vertical wear.*—Letting n denote the normal wear of the collar, the statement "the normal wear at any point is proportional to the work of friction" may be expressed by the relation

$$n = kpx,$$

from which

$$p = \frac{n}{kx} \tag{617}$$

Substituting this value of p in (611) and (613) and assuming μ as constant, we obtain the following expression for P and M:

$$P = \frac{2\pi n}{k}\,(r_2 - r_1) \tag{618}$$

$$M = \frac{\pi\mu n}{k}\,(r_2^2 - r_1^2) \tag{619}$$

Eliminating n and k between (618) and (619), we find that

$$M = \frac{\mu P}{2}(r_2 + r_1), \tag{620}$$

which shows that the moment of friction of a collar pivot is the same as that of a ring of infinitesimal breadth, with a diameter equal to the mean diameter of the pivot.

Upon the foregoing assumption, the work, in foot-pounds, lost per minute by a collar pivot is

$$W_f = \frac{\pi \mu N P}{12}(r_2 + r_1) \tag{621}$$

375. Analysis of a Flat Pivot.—It is of interest to consider briefly the theoretical distribution of pressure in the case of a pivot in which the surface in contact is not a ring but a complete circle. From (617) we have

$$p = \frac{n}{kx}$$

The normal wear n may practically be assumed as constant, hence the pressure p at any point varies inversely as the distance of that point from the axis of the pivot. Theoretically the pressure at the axis is infinitely great. While this is not the actual state of affairs, there is doubtless a great intensity of pressure at the axis and this produces a crushing of the material as experience with flat pivots seems to show. It is a good plan therefore to cut out the material at the center of the pivot as shown in Fig. 315(*b*) thus changing its surface of contact to that of a ring, as in the case of a collar pivot. The curve mn in Fig. 315(*b*) is an equilateral hyperbola whose equation is $px = \frac{n}{k}$, and it also shows graphically how the pressure upon the contact surfaces varies.

376. Tower's Experiments on Thrust Bearings.—(*a*) *Collar bearings.*—In the *Proceedings* of the Institution of Mechanical Engineers, 1888, p. 173, Mr. Beauchamp Tower reported the results of a series of experiments on a collar thrust bearing 14 inches outside diameter and 12 inches inside diameter. The surfaces in contact consisted of a mild-steel ring located between two rings made of gun metal. Table 99, giving the values of the coefficient of friction for the various speeds listed, was compiled from the results published in the original report. The coefficients

TABLE 99.—COEFFICIENTS OF FRICTION FOR COLLAR THRUST BEARINGS —TOWER

Pressures		Revolutions per minute					Aver. values
Total	lb./sq. in.	50	70	90	110	130	
600	14.7	0.0450	0.0646	0.0433	0.0537	0.0642	0.0541
1,200	29.4	0.0375	0.0481	0.0496	0.0489	0.0475	0.0463
1,800	44.1	0.0357	0.0399	0.0361	0.0357	0.0371	0.0369
2,400	58.8	0.0286	0.0375	0.0361	0.0373	0.0410	0.0361
2,700	66.1	0.0354	0.0334	0.0346	0.0361	0.0378	0.0354
3,000	73.5	0.0347	0.0341	0.0348	0.0352	0.0356	0.0348
3,300	80.8	0.0337	0.0322	0.0348	Bearing		0.0336
3,600	88.2	0.0312	0.0444		seized		0.0378

of collar friction as determined by Tower are based upon the assumption that the force of friction was concentrated at the end of the mean radius of the collars. From the results given in Table 99 it is apparent that the coefficient of friction is practically independent of the speed and that it tends to decrease as the load on the bearing is increased.

(*b*) *Step bearings.*—In Table 100 are given the results of a series of experiments, made by Tower, on a flat steel pivot 3 inches in diameter running on a manganese bronze step bearing. To insure proper lubrication of the contact surfaces, the oil was supplied to the center of the pivot and distributed by a single diametrical groove which extended to within $\frac{1}{16}$ inch from the circumference of the pivot. At the slower speeds the oil circulation, which was automatic due to the centrifugal action, was

TABLE 100.—COEFFICIENTS OF FRICTION FOR STEP BEARINGS—TOWER

Pressures, lb. per sq. in.	Revolutions per minute				
	50	128	194	290	353
20	0.0196	0.0080	0.0102	0.0178	0.0167
40	0.0147	0.0054	0.0061	0.0107	0.0096
60	0.0167	0.0053	0.0051	0.0078	0.0073
80	0.0181	0.0063	0.0045	0.0064	0.0063
100	0.0219	0.0077	0.0044	0.0056	0.0057
120	0.0221	0.0083	0.0052	0.0048	0.0053
140	......	0.0093	0.0062	0.0046	0.0053
160	......	0.0113	0.0068	0.0044	0.0054

somewhat restricted, varying from 20 to 56 drops per minute, while at the higher speed the bearing was flooded. Due to the more effective lubrication of the step bearing used in these experiments, the coefficients of friction are much less than those obtained with the experimental collar bearing. The coefficients of friction as given in Table 100 were determined from the moments of friction by means of a formula based on the assumption that the pressure is uniformly distributed. In other words the moment of friction is $M = \frac{2}{3} \mu Pr$.

In a second series of experiments, a white metal step bearing was used in place of the manganese bronze bearing, and the results obtained gave coefficients of friction slightly greater than those given in Table 100. For all practical purposes the coefficients given in Table 100 may also be used for white metal bearings.

377. Schiele Pivot.—It is possible to design a pivot with a surface of such a nature that the pressure between the pivot and its bearing shall be the same at all points of contact. From Fig. 315(a), it is evident that the relation between the normal wear n and the vertical wear h is

$$n = h \cos\theta \tag{622}$$

Combining (617) and (622), we obtain the following general expression for the normal pressure:

$$p = \frac{C \cos \theta}{x} \tag{623}$$

in which C denotes the ratio of the constants h and k.

Assuming that p is to remain constant at all points of contact, it follows that $\cos \theta$ is proportional to x, that is

$$\cos \theta = Kx \tag{624}$$

Since θ is the angle that the normal to the bounding curve of the pivot makes with the axis of the pivot, we have

$$dy = \tan\theta dx$$

Differentiating (624)

$$Kdx = - \sin\theta d\theta,$$

whence

$$Kdy = \left[\frac{\cos^2\theta - 1}{\cos\theta}\right] d\theta \tag{625}$$

Integrating (625), we get

$$Ky = \sin \theta - \log_e (\sec \theta + \tan \theta) + F$$

Eliminating θ by means of (624)

$$Ky = \sqrt{1 - K^2x^2} - \log_e\left[\frac{1 \pm \sqrt{1 - K^2x^2}}{Kx}\right] + F \qquad (626)$$

This is the equation of the *tractrix* or sometimes wrongly called, the *antifriction curve.*

From Fig. 315(*a*), $\cos\theta = \frac{x}{BC}$, and combining this with (624), we find $K = \frac{1}{BC}$. From the construction of the tractrix $BC = r_2$, therefore

$$K = \frac{1}{r_2} \qquad (627)$$

Moment of friction.—The moment of friction, about the axis of the pivot, of the normal pressure p acting on an annular strip of width ds is

$$dM = 2\mu\pi p r_2 x dx$$

Assuming as in the preceding discussions that the coefficient μ remains constant, we obtain by integration the following expression for the moment of friction:

$$M = \mu\pi p r_2 (r_2^2 - r_1^2) \qquad (628)$$

For uniform distribution of pressure it was shown that

$$p = \frac{P}{\pi (r_2^2 - r_1^2)}$$

Substituting this value of p in (628), we have

$$M = \mu P r_2 \qquad (629)$$

Comparing (629) with the expression for the moment of friction for a collar pivot as given by (620), it is evident that the moment of friction of a Schiele pivot is always the greater. The Schiele pivot has one advantage in that it keeps its shape as it wears and it is self-adjusting. Due to its excessive cost of manufacture it is used but little.

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CHAPTER XX

BEARINGS WITH ROLLING CONTACT

378. Requirements of Rolling Contact.—A bearing having a rolling contact is one in which the journal is supported on rollers or balls, thereby decreasing the frictional resistance, since rolling friction seldom exceeds sliding friction under the same conditions of load and operation. Due to the use of improved machinery for producing the rolling elements, bearings with rolling contact are now used for all classes of service. A bearing of this kind to be commercially successful must fulfill the following conditions:

(*a*) The arrangement of the rolling elements should be such that sliding is reduced to a minimum.

(*b*) The rolling elements must all be of the same size, and accuracy in form is absolutely essential.

(*c*) The rolling elements must be extremely hard and their surfaces must be polished very smooth.

(*d*) The rolling elements must be so arranged that they will not run off their guides or raceways.

(*e*) The rolling elements must not be overloaded, as they may become distorted thus changing the conditions entirely.

(*f*) The pressure should be approximately normal to the surface of contact.

379. Classification.—Bearings with rolling contact may be divided, according to the kind of rolling element used, into the following classes:

(*a*) *Roller bearings,* in which either cylindrical or conical rollers are placed between the journal and its bearing.

(*b*) *Ball bearings,* in which hardened steel balls are used in place of the rollers.

Each of the above classes may be subdivided into the following types: (1) *radial bearings*; (2) *thrust bearings.*

ROLLER BEARINGS

380. Radial Bearing having Cylindrical Rollers.—(*a*) *Mossberg bearing.*—The simplest form of roller bearing for a journal consists of a sleeve surrounded by a series of cylindrical rollers, rolling inside of a bored casing or outer race. Fig. 316 shows such a bearing made by the Standard Machinery Co. and known as the Mossberg bearing. The sleeves, rollers, and outer casings must have true cylindrical surfaces and for proper working the axes must always remain parallel to each other. To keep the rollers *d* in the desired position, they are placed in a cage *c* having its outside diameter slightly less than the inner diameter of the outer race *e*, and its internal diameter a trifle greater than the diameter of the sleeve *b*. The cage, made from a good tough bronze or steel, is provided with a series of slots reamed to size,

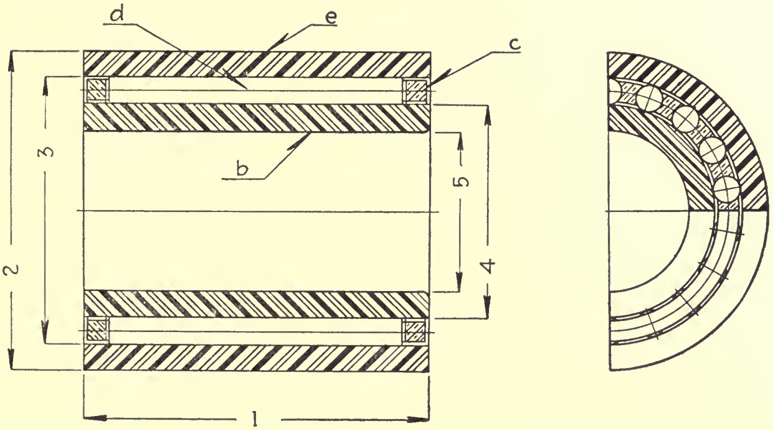

Fig. 316.

into which the steel rollers are placed as shown in Fig. 316. No doubt there is a certain amount of sliding between the roller and the cage, but actual tests seem to show that this sliding action reduces the efficiency of the bearing but little.

(*b*) *Norma bearing.*—A form of roller bearing shown in Fig. 317 has been recently developed and placed on the market. It consists of an outer race *e* having a convex or ball-shaped interior surface, against which the rollers *d* bear. The sleeve or race *b* is cylindrical and is fastened to the shaft or journal. The cylindrical rollers, which are short, are held in alignment by the specially constructed steel cage *c*. As may be seen in Fig. 317, the outer race *e* is open-sided thus facilitating the assembling, mounting or dismantling of the bearing. The manufacturers of this bearing, which originated in Stuttgart, Germany, claim that the Norma bearing, as it is called, is capable of supporting greater loads than ball bearings having the same dimensions and running at the same speed. These bearings are made so that they are interchangeable with ball bearings, thus providing for their application in cases where ball bearings fail under the applied load.

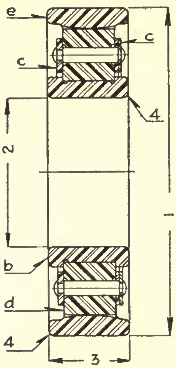

Fig. 317.

Due to inaccuracies of the rolling element or to wear, the rollers in an ordinary roller bearing may acquire a tendency to move lengthwise, thus causing more or less end pressure on the cage. To eliminate this end pressure, holes are drilled in the ends of the rollers, or in the cage, and steel balls are inserted.

381. Radial Bearings having Conical Rollers.—The rollers instead of being cylindrical may be conical as shown in the bearing illustrated in Fig. 318. In its general construction this bearing is similar to the plain roller bearing. It consists of a series of conical rollers *d* located between the inner and outer cones *b* and *e*. The cage, consisting of two rings *c* and *f* made of high-carbon steel, is provided with sockets for holding the ends of the rollers. These rings are held together by stay rods *g*, shown by the dotted lines. The ends of the rollers are beveled

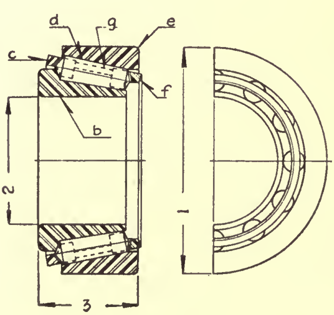

Fig. 318.

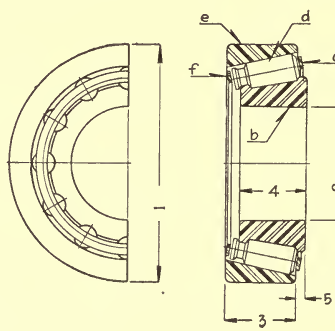

Fig. 319.

to a slight angle and bear against corresponding shoulders on the cage and inner cone *b*. To insure true rolling in this type of bearing it is necessary that all the axes of the rollers intersect the axis of the journal in a common point. Bearings having two sets of conical rollers are also made at the present time.

Timken roller bearing.—Another successful roller bearing using tapered rollers is shown in Fig. 319. It is used rather extensively in automobile construction and differs from the bearing just described in minor details only. The Timken bearing is made in various styles and that shown in Fig. 319 is known as the "short series."

One important advantage possessed by conical roller bearings over any other form of roller bearing lies in the provision for taking up the wear if there is any. It is merely necessary to force the inner cone and the rollers further into the outer cone or cup.

382. Radial Bearings having Flexible Rollers.—Due to inaccuracy in manufacturing, slight deflections of the shaft, yield-

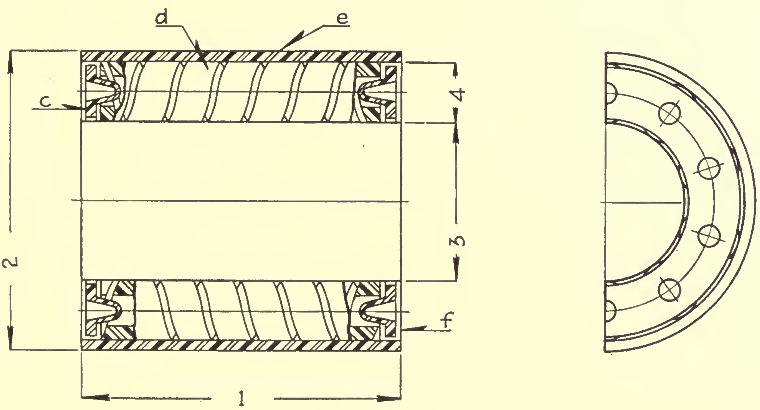

FIG. 320.

ing of the supports or mounting of a roller bearing, a roller may move out of its correct position and cause the line of contact with the sleeves or races to become curved instead of straight. Such a condition would cause a long roller of brittle material to break and the whole bearing would thereby be ruined. To overcome this difficulty the flexible roller has been devised and is now used for all classes of service. A form of bearing, known as the Hyatt bearing, using flexible rollers is shown in Fig. 320. The Hyatt rollers are made of a strip of steel wound into a coil or spring of uniform diameter. Due to its flexibility, the roller will adjust itself to any irregularity of the bearing such as imperfect alignment; furthermore, the distribution of the load along the entire length of the roller will be practically uniform, thus permitting the use of commercial shafting, hardened and ground journals not being essential except under extreme conditions of loading. Due to its construction the lubrication of the Hyatt bearing is very effective, since the center of each roller is really a large oil reservoir. The Hyatt bearing is also made with a hardened steel inner sleeve which may be fastened to a soft steel shaft.

383. Thrust Bearings having Cylindrical Rollers.—In Fig. 321 is shown a thrust bearing using cylindrical rollers. It is

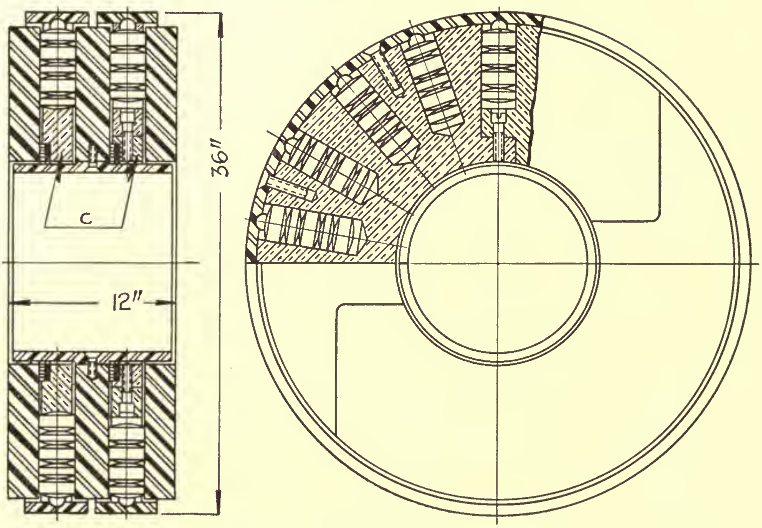

Fig. 321.

claimed by the manufacturer of this bearing, that while theoretically a thrust bearing having conical rollers is better than one having cylindrical rollers, the theory is not borne out in actual operation. The explanation no doubt lies in the mechanical inaccuracy of the various parts in contact. To reduce the tendency of the cylindrical rollers to groove the discs, the rollers should travel in different paths. In some designs this is accomplished by placing the slots in the cage at different distances from the shaft center. Another scheme used for preventing the formation of grooves is shown in Fig. 321, and consists of rollers having different widths. The thrust of the rollers in a radial direction may be taken up by a ball, as shown in the figure.

Fig. 321 shows a roller thrust bearing installed under a 5,500-horse-power turbine generator of the Niagara Falls Power Co. The maximum load coming upon this bearing is 190,000 pounds and the normal load is 156,000 pounds. The normal speed is 250 revolutions per minute, and the maximum may reach 500 if the governor fails. By consulting Fig. 321 it will be noticed that on the under side of each cage *c* and near its bore is located an auxiliary roller bearing, the function of which is to support the weight of the cage. The cages of both the main and auxiliary bearings are made of bronze, while the thrust discs or washers are made of case-hardened machinery steel.

384. Thrust Bearings having Conical Rollers.—A common form of thrust bearing using conical rollers consists of a cage *c*, a series of steel rollers *d*, two steel thrust discs *e* and *b*, and an external ring *f* as shown in Fig. 322. The cage *c*, generally made of one solid piece of metal, is provided with tapered holes into which are placed the conical rollers *d*. Due to the action of the load upon the bearing, the rollers tend to move radially outward, and to reduce this tendency, the apex angle is made relatively small. One prominent manufacturer makes this angle 6 degrees. The thrust discs may both be made conical, or either may be flat and the other coned; in other words, the axes of the rollers need not necessarily be at right angles to the axis of the shaft. However, to obtain pure rolling, the vertices of the rollers and of the conical thrust surfaces must be in a common point on the axis of the shaft. In the thrust bearing shown in Fig. 322, the radial thrust of the rollers is taken care of by the tool-steel ring *f*. In some designs the end thrust of the rollers against the cage is taken by a ball located between two cupped surfaces.

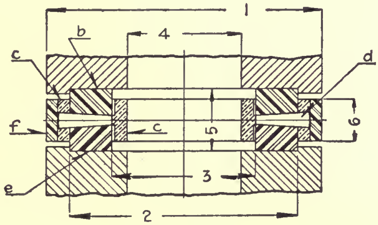

FIG. 322.

385. Allowable Bearing Pressures and Coefficients of Friction.—The intensity of pressure coming upon the elements of a roller bearing should not exceed the elastic limit of the material or permanent deformation will occur. Such deformation ruins either the rollers or the bearing surfaces, or both.

In 1898 Prof. Stribeck, the well-known head of the Technical Laboratories in New Babelsberg carried on extensive investigations on sliding, roller and ball bearings. Among the tests made was a series investigating the relations existing between the coefficient of friction, specific load, and the speed for many types and sizes of bearings. The following are a few of the conclusions mentioned in a report submitted by Prof. Stribeck.

(*a*) That the load coming upon either a roller or ball bearing may be considered as supported by one-fifth the number of rollers or balls in the bearing. This distribution of the load is not uniform over each of the carrying rollers or balls.

(*b*) That the ball bearing has a load carrying capacity much in excess of that of the roller bearing.

(*c*) That the roller bearing has a higher coefficient of friction than the ball bearing for similar conditions of speed and loading.

(*d*) That the coefficient of friction for ball bearings is practically a constant for a wide range of speed and load.

(*e*) That the chief advantage of roller bearings over plain bearings lies in the lower coefficient of friction.

By the term "*specific load*" is meant the pressure per unit of carrying element. For a plain bearing the carrying element is considered the projected area of the journal. For roller bearings the carrying element is considered equivalent to one-fifth of the number of rollers times the product of the length by the diameter of the rollers. For ball bearings, the product of one-fifth of the number of balls and the square of the diameter is considered as equivalent to the carrying element.

386. Roller Bearing Data.—Roller bearings for motor car service have been standardized to such an extent by several manufacturers that they may be interchanged with ball bearings of similar capacity.

(*a*) *Norma bearings.*—In Table 101 are given the various dimensions of the medium and heavy-duty Norma roller bearings. The symbols denoting the dimensions refer to the key drawing of Fig. 317. The load capacity as given in this table is based on a steady load and slow speed. To obtain the rating at any particular speed, multiply the rating given in Table 101 by the speed coefficient obtained from Fig. 323. The chart plotted in this figure is based upon data deduced from the load ratings given in the trade publication issued by The Norma Co. of America. In addition to the types of Norma bearings listed in the table, a

Table 101.—Data Pertaining to Norma Roller Bearings

Medium-duty series						Heavy-duty series					
Size	Dimensions				Load at 10 r.p.m.	Size	Dimensions				Load at 10 r.p.m.
	1	2	3	4			1	2	3	4	
NM 25	2.4410	0.9842	0.67	0.04	1,650	NS 25	3.1496	Same as dimension 2 for medium-duty series	0.83	0.08	3,410
NM 30	2.8346	1.1811	0.75	0.08	2,150	NS 30	3.5433		0.91	0.08	3,850
NM 35	3.1496	1.3779	0.83	0.08	2,750	NS 35	3.9370		0.98	0.08	4,400
NM 40	3.5433	1.5748	0.91	0.08	3,520	NS 40	4.3307		1.06	0.08	5,940
NM 45	3.9370	1.7716	0.98	0.08	4,400	NS 45	4.7244		1.14	0.08	7,260
NM 50	4.3307	1.9685	1.06	0.08	5,280	NS 50	5.1181		1.22	0.08	8,580
NM 55	4.7244	2.1653	1.14	0.08	6,600	NS 55	5.5118		1.30	0.12	9,460
NM 60	5.1181	2.3622	1.22	0.08	7,700	NS 60	5.9055		1.38	0.12	11,200
NM 65	5.5118	2.5590	1.30	0.12	8,360	NS 65	6.2992		1.45	0.12	12,100
NM 70	5.9055	2.7559	1.38	0.12	10,120	NS 70	7.0866		1.65	0.12	16,060
NM 75	6.2992	2.9527	1.46	0.12	11,880	NS 75	7.4803		1.77	0.12	18,040
NM 80	6.6929	3.1496	1.54	0.12	12,980	NS 80	7.8740		1.88	0.12	18,260
NM 85	7.0866	3.3464	1.61	0.12	14,080	NS 85	8.4645		2.00	0.12	19,140
NM 90	7.4803	3.5433	1.69	0.12	16,280	NS 90	8.8582		2.12	0.12	23,320
NM 95	7.8740	3.7401	1.77	0.12	17,380	NS 95	9.6456		2.24	0.12	27,280
NM 100	8.4645	3.9370	1.85	0.12	21,120	NS 100	10.4330		2.36	0.12	37,400

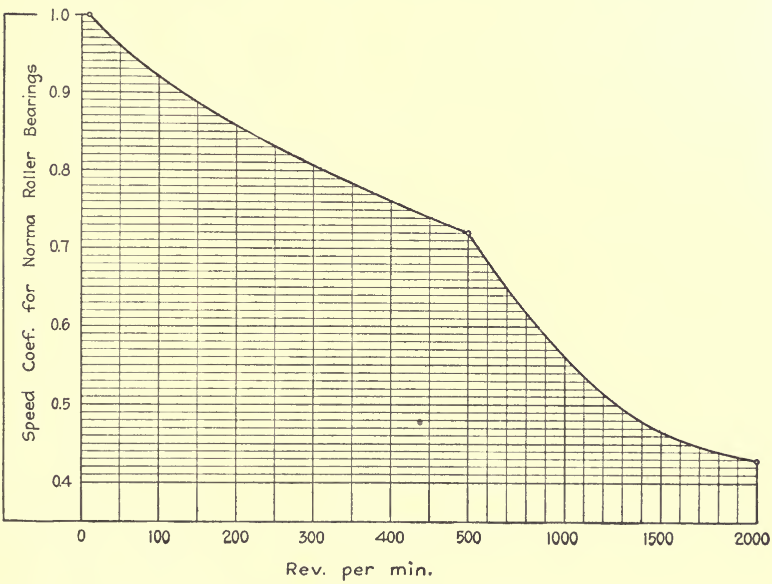

Fig. 323.

light-duty bearing is also manufactured; furthermore, each of the three types is also made in larger sizes than those given.

(*b*) *Hyatt bearings.*—The dimensions and load-carying capacities for the long and short series of the Hyatt high-duty type of roller bearing, similar to that shown in Fig. 320, are given in Table 102. The type of bearing to which the data given in this

Table 102.—Data Pertaining to Hyatt High-duty Bearings

Short series					Long series				
Size	Dimensions			Rating	Size	Dimensions			Rating
	1	2	3			1	2	3	
17,010	1.000	2.249	1.000	460	17,060	2.000	Same as dimension 2 for the short series	Same as dimension 3 for the short series	1,200
17,012	1.000	2.374	1.125	500	17,062	2.000			1,340
17,014	1.125	2.749	1.250	700	17,064	2.250			1,700
17,016	1.125	2.875	1.375	750	17,066	2.250			1,900
17,018	1.250	3.375	1.500	960	17,068	2.500			2,340
17,020	1.250	3.499	1.625	1,040	17,070	2.500			2,530
17,022	1.250	3.625	1.750	1,125	17,072	2.500			2,730
17,024	1.250	3.749	1.875	1,200	17,074	2.500			2,925
17,026	1.375	4.124	2.000	1,470	17,076	2.750			3,490
17,028	1.375	4.249	2.125	1,550	17,078	2.750			3,700
17,030	1.375	4.374	2.250	1,650	17,080	2.750			3,925
17,032	1.500	4.749	2.500	2,060	17,082	3.000			4,820
17,034	1.500	4.999	2.750	2,270	17,084	3.000			5,300
17,036	1.750	5.374	3.000	3,030	17,086	3.500			6,890
17,038	1.750	5.624	3.250	3,400	17,088	3.500			7,600

table applies necessitates the use of a heat-treated or hardened steel shaft, since no inner shell or sleeve is furnished. The load ratings specified in Table 102 represent the load in pounds that any particular bearing is capable of carrying at a speed not to exceed 1,000 revolutions per minute. According to the manufacturer of the Hyatt bearings, the load capacity at 1,500 revolutions per minute should be taken as equivalent to 50 per cent. of that given in the table, and when the speed is 500 revolutions per minute the capacity may be increased 50 per cent. above that given for 1,000 revolutions per minute.

387. Mounting of Roller Bearings.—To obtain satisfactory service, roller bearings of all types must be carefully protected from water, acids, alkalies, dust, and any foreign matter that might ruin them. Protection may be obtained by housing in the bearing and sealing the openings through which the shaft passes with felt packed into grooves provided for that purpose in the end or cover plates. Filling the housing and bearing with a high-grade stiff grease, provided the speed is not too high, will also aid in keeping out foreign matter, and at the same time it will furnish the necessary lubrication.

In roller bearings of the Norma and Hyatt type both the inner and outer races or sleeves are rigidly held in place. Generally the inner race is made a light driving fit on the shaft, and to insure a rigid fastening, the race should be clamped between a suitable shoulder on the shaft and a nut provided with some form of locking device. For various forms of nut locks consult Art. 80. The outer race is usually clamped between a shoulder in the housing and an outside cover plate, or in some cases between two cover plates The shoulders on the shafts or in the housing against

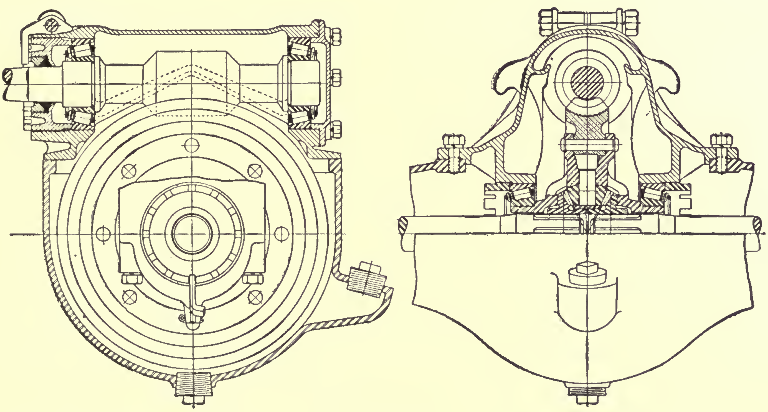

Fig. 324.

which a bearing is clamped should be sufficiently high to provide ample support to the bearing. If, for example, the shoulder on the shaft be made too small, the inner race when pressed into place is liable to slip over this shoulder and cause the race to expand slightly thus producing undue pressure upon the end of the roller. Since Norma and Hyatt bearings cannot take an end thrust, the latter must be taken care of by suitable thrust bearings.

In mounting a conical roller bearing either the cone or the cup must be provided with means for taking up wear. When the inner race or cup is mounted on a non-rotating member, as for example on the front wheel spindle of a motor car, it is considered good practice to fasten the outer race or cup rigidly into the hub casting or forging, and provide the cone with an adjustment for taking up wear by making it an easy sliding fit on the spindle. When the cone is mounted on a rotating member, good practice dictates that the cone be made a tight press fit on the shaft and that the wear be taken up by making the cup adjustable. A good example of an installation of conical roller bearings, in which the cones are mounted on rotating members, is shown in the rear-axle worm-gear transmission of Fig. 324. Attention is directed to the fact that due to the rigid mounting of the outer races against the rim of the end cover plates, the worm shaft is always subjected to a compression, and any expansion of the shaft due to an excessive rise in temperature will cause it to deflect a small amount and necessarily produce undue wear. To obviate such a condition, the bearings may be so arranged that the cups will come against shoulders on the housing or gear case, thus causing the worm shaft to be in tension.

BALL BEARINGS

Formerly ball bearings were used chiefly for light loads, but at the present time they are used in all classes of machinery. In general, a ball bearing consists of a series of balls held by a suitable cage between properly formed hardened steel rings called races. These races may be of such shape that the ball has two points of contact, as shown in Fig. 325, or it may have three or even four points of contact, as shown in Figs. 326 and 327, respectively.

388. Forms of Raceways.—(*a*) *Two-point contact.*—The simplest form of two-point contact is the flat race shown in Fig. 325(*a*). In this construction no provision is made for retaining the balls. To overcome this objection, the races may be curved as shown in Fig. 325(*b*), (*c*) and (*d*), the latter having the greatest carrying capacity. This increase of carrying capacity is no doubt due to the increased area of contact. For a well-designed ball bearing the wear upon the inner and outer races should be the same, which means that the contact pressure upon these races should be the same. The contact pressure depends upon the small contact area, and if these areas are to be equal it is necessary that the radius of curvature of the outer race should be increased. This has been done in the bearing shown in Fig. 325(*d*).

(*b*) *Three-point contact.*—In Figs. 326 and 327 are shown forms of bearing raceways having three and four points of contacts, respectively. To produce true rolling of the ball the races must

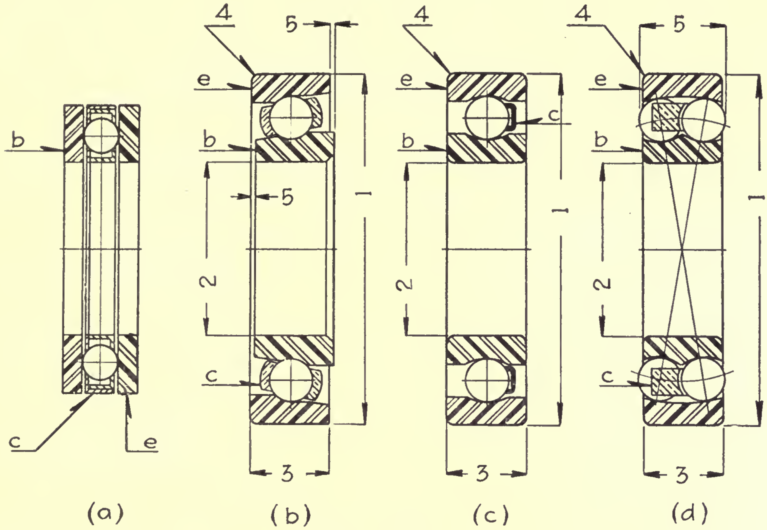

FIG. 325.

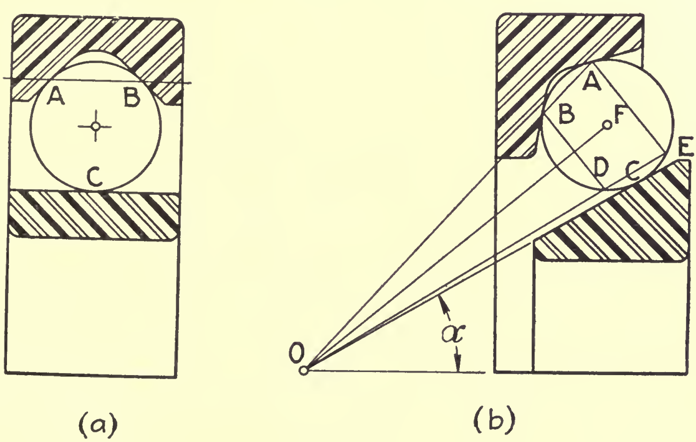

FIG. 326.

be laid out correctly. Referring to Fig. 326(*b*), and letting *A*, *B* and *C* represent the three points of contact, extend the line *AB* until it intersects the center of the shaft at *O*, also draw *OF* through the center of the ball. This latter line represents the axis of rotation of the ball, and the lines AE and BD are projections of two circles of rotation. From similar triangles we have that AE and BD are proportional to OA and OB respectively; therefore it is evident that there is no slipping at the points A and B and that the desired true rolling is obtained. The third point of contact C is determined by drawing OC tangent to the ball. To avoid excessive wedging of the ball, the angle α should be made not less than 30 degrees.

(*c*) *Four-point contact.*—In either of the four-point bearings shown in Fig. 327, the pure rolling of the ball is obtained when

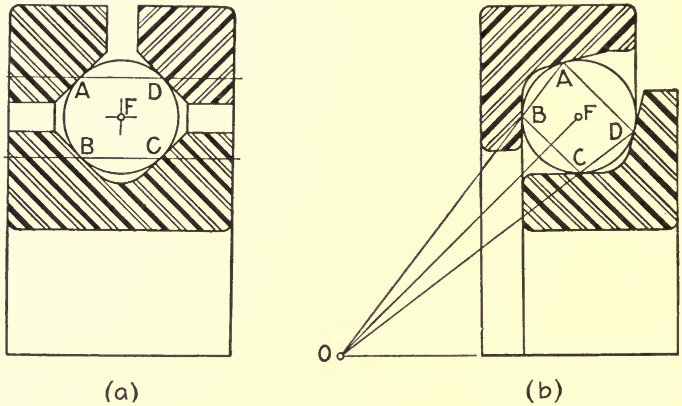

Fig. 327.

$\frac{AD}{BC} = \frac{OA}{OB}$. The various lines required for laying out a bearing of this kind are drawn in a general way, according to the method outlined for the three-point bearing in the preceding paragraph.

389. Experimental Conclusions of Stribeck.—Prof. Stribeck in his investigation of bearings having rolling contact determined how the carrying capacity of a ball was affected by the form of the raceway. For this purpose ball bearings having raceways shown in Figs. 325 to 327 inclusive, except the form shown in Fig. 325(*d*), were used.

Some of the conclusions arrived at were as follows:

(*a*) The form of raceway shown in Fig. 325(*a*) had the least frictional resistance.

(*b*) An increase in the number of points of contact, as shown in Figs. 326 and 327, resulted in higher frictional resistances. It is probable that due to imperfect workmanship the conditions required for pure rolling were not met.

(*c*) The carrying capacities for the forms of raceways shown in Figs. 326 and 327 were practically the same. Theoretically the four-point contact should carry more, but due to difficulties in constructing and adjusting such a bearing it is almost impossible to distribute the load uniformly over the various points of contact.

(*d*) The carrying capacity for the form of raceways shown in Fig. 325(*c*) is considerably greater than for the other forms shown.

(*e*) The frictional resistance for the form indicated by Fig. 325(*c*) is a trifle greater than for the others, but practically it may be considered the same.

390. Radial Ball Bearings.—(*a*) *Single-row bearings.*—A radial ball bearing is used for supporting loads acting at right angles to the axis of rotation. At the present time the two-point contact type having circular raceways is used almost exclusively. Such a bearing consists of an outer and inner race, both provided with curved ball raceways that are uniform and unbroken around the entire circumference. Between these races is located a series of balls separated either by an elastic separator or by a bronze or alloy cage, as shown in Fig. 325. The type of elastic separator mentioned consists of a short helical spring fitted with suitable bearing plates. This separator was formerly used in the Hess-Bright bearings and at the present time is still used under certain special conditions. The majority of the separators or cages now in use are made of brass or bronze and steel and their construction makes them more or less elastic.

(*b*) *S. K. F. bearing.*—Radial bearings having two or more rows of balls, examples of which are shown in Figs. 325(*d*) and 328, have also been devised. In selecting this type of bearing it must not be assumed that doubling the number of balls necessarily doubles the load capacity, for the accuracy of workmanship required for such a condition is not always feasible.

The bearing shown in Fig. 325(*d*) originated in Sweden and is known as the S. K. F. bearing. The outer ball race *e* is a machined and ground spherical surface, the center of which lies on the axis of the bearing. The inner race *b* has two curved ball raceways having a radius slightly larger than the radius of the ball. The balls are staggered and are retained by the phosphor bronze separator or cage *c*. This type of bearing may be dismantled very readily by swinging the race *b* and the balls together within the outer race *e*, and then removing two adjacent balls on either side diametrically opposite to each other. This operation permits the withdrawal of the complete center portion of the bearing. Another important advantage of the S K. F. bearing is its self-aligning feature, which compensates for shaft flexure or deformation.

(*c*) *Norma bearing.*—In Fig. 328 is shown a form of double-row ball bearing manufactured by The Norma Co. of America. It consists of two outer races mounted side by side on a single inner race provided with two raceways. The raceways in the outer rings are ground to the same radius as that used on the inner race, but one-half of the shoulder is ground away to form a cylindrical surface, tangent to the circular raceway, as shown in the figure. It is evident that this form of outer race differs materially from that shown in Fig. 325(*c*). The main advantage of the construction used on the Norma bearing lies in the ease with which that bearing can be assembled and dismantled for inspection. The separator used consists of a light one-piece bronze ring having a channel section. The flanges of this ring separator are provided with spherical seats between which the balls are held with a slight elastic pressure; thus the balls and separator may be removed as a single unit.

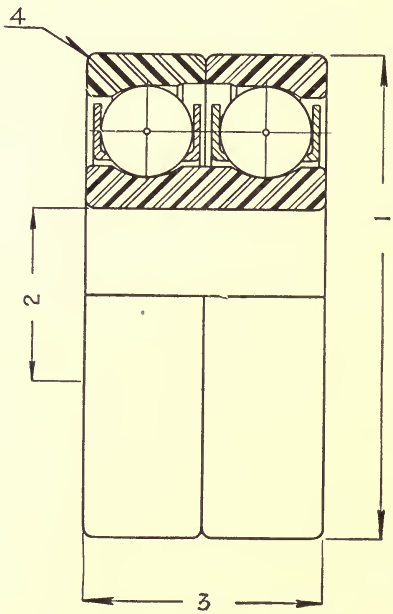

Fig. 328.

391. Thrust Ball Bearings.—(*a*) *Two-point type.*—The modern ball thrust bearing is made with either two- or four-point contact. In Fig. 325(*a*) is shown a two-point contact having flat raceways. It consists of two hardened steel plates or thrust discs *b* and *c* between which is located the cage *c* containing the balls. The cage may be made of one piece by drilling the holes for the balls almost through, then inserting the ball and by means of a special setting tool closing in the upper edge. The cage may also be made in two pieces as shown in the figure.

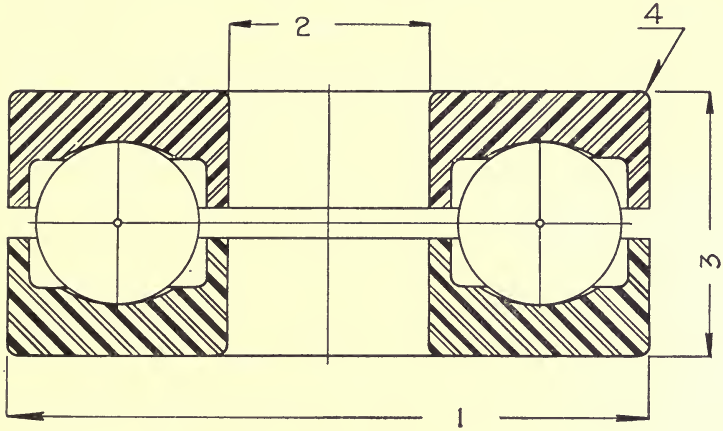

FIG. 329.

The type of thrust bearing having curved or grooved raceways is shown in Figs. 329 and 330(*a*). The constructive features are clearly shown in the figures. These bearings are known as the

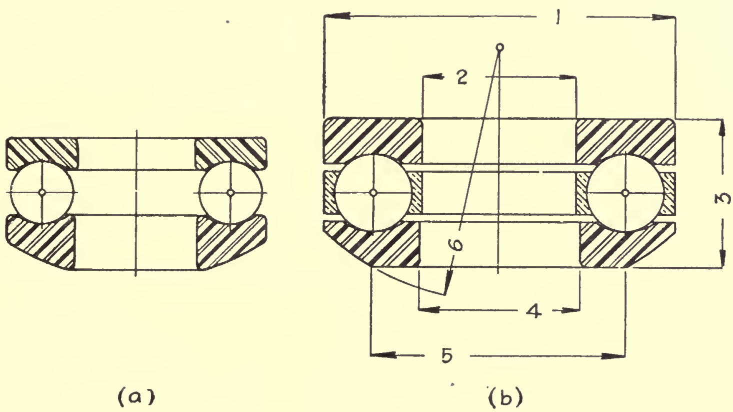

FIG. 330.

full ball or without separator type, and are intended for very heavy service at a slow speed. The type shown in Fig. 330(*a*) being made in small sizes is intended for use on automobile steering pivots, while that illustrated in Fig. 329 is made in the larger sizes and is used on crane hooks. Another type of thrust bearing having curved raceways is shown in Fig. 330(*b*). The balls are separated by a cage made of brass or special alloy.

All thrust bearings thus far shown are intended to take the thrust in one direction only. In cases where the thrust has to be taken care of in both directions, a form known as the double-thrust bearing is used. Such a bearing, shown in Fig. 331, consists of a central grooved disc *f* securely fastened to the shaft, two thrust discs *b* and *e* having grooves to correspond with those on *f*, and two phosphor-bronze cages *c* retaining the balls. The

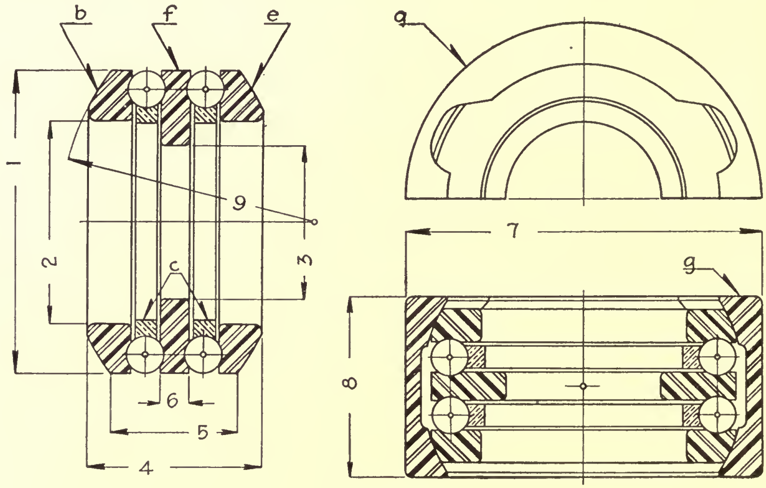

FIG. 331.

form of bearing just described may be so arranged that the combination of balls, cages, and thrust discs form a part of a sphere as shown in Fig. 331. The entire combination is then free to revolve in a specially constructed hardened steel casing *g*. To permit easy assembling two recesses are located in a convenient position on one side of the casing *g*.

(*b*) *Four-point type.*—A four-point bearing made by the Auburn Ball Bearing Co. is shown in Fig. 332. All thrust bearings made by this company are of the four-point type and have no separator for the balls. The condition for pure rolling is fulfilled as may be seen from the geometry of the figure.

(*c*) *Leveling washer.*—In any thrust bearing it is always desirable to distribute the load uniformly over the entire series of balls. This is done by providing one of the thrust discs with a spherical surface thus permitting it to adjust itself. The construction is shown in Figs. 330 and 331. When both discs are flat as is the case of Figs. 329 and 332, a special leveling washer having a spherical seat should be used in connection with the stationary thrust disc.

392. Combined Radial and Thrust Bearing.—A combination radial and thrust bearing is used in places where provision must be made for both radial and axial loads. Some of these bearings are so arranged that, in addition to the radial loads, they will take care of a thrust in only one direction or in both directions.

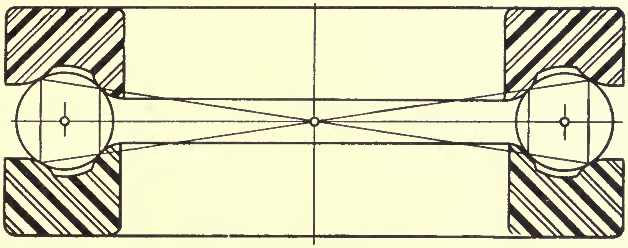

Fig. 332.

(*a*) *Radax bearing.*—A bearing known as *Radax*, manufactured by the New Departure Mfg. Co., is used for both radial and one direction axial loads. The details of this bearing are clearly shown in Fig. 325(*b*). The bearing differs from an ordinary radial bearing in that the ball raceway has a two-point angular contact instead of radial contact. The Radax bearing may readily be assembled and dismantled since the inner race, separator, and balls may easily be withdrawn from the outer race. These bearings are made interchangeable with corresponding sizes of standard radial bearings.

(*b*) *Gurney radio-thrust bearing.*—In Fig. 333 are shown the details of a combined radial and thrust bearing manufactured by the Gurney Ball Bearing Co. The points of contact between the balls and the inner and outer races are not on radial lines, but lie on the lines that intersect the axis of the bearing at the point O, as shown in the figure. The steel separator is made in a single piece having a light but rigid construction. The radio-thrust bearing is well adapted to installations in which there is a combination of radial and axial loads, and where the latter exceeds approximately 25 per cent. of the former. It is generally conceded that ordinary radial bearings should not be subjected to an axial load exceeding 25 per cent. of the radial load. With relation to thrust capacity, the Gurney radio-thrust bearings are made in three types. Each of these types is made in three series, namely, the light, medium, and heavy, and as far as the dimensions are concerned these bearings are interchangeable with the corresponding sizes of standard radial bearings.

(c) *Double-row and duplex bearings.*—The bearings discussed in the preceding paragraphs are used in places where the axial

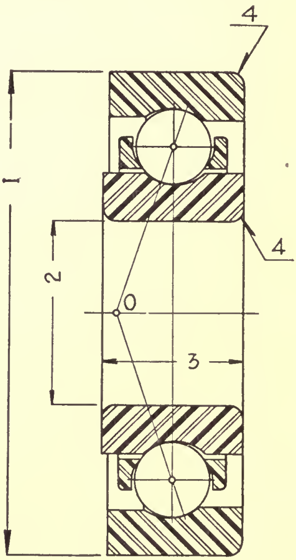

FIG. 333.

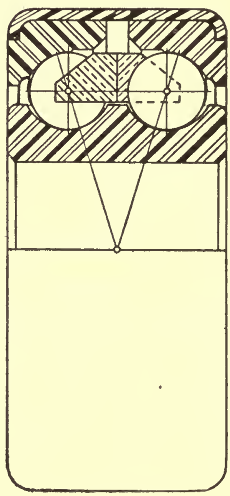

FIG. 334.

loads are always in the same direction. There are, however, many places requiring bearings capable of taking a thrust in either direction. Such a condition can be successfully met by installing *Duplex bearings* which consist of two radio-thrust bearings mounted side by side, or by using a double-row combined radial thrust bearing. A bearing of the latter type, made by the New Departure Mfg. Co., is shown in Fig. 334. It consists of a single inner race containing two raceways, a bronze separator made in two pieces, two rows of balls having two-point angular contact, two outer races, and a thin steel shell which is closed in over the outer races, as shown in the figure, after the bearing is assembled.

393. Allowable Bearing Pressures.—(*a*) *Load per ball.*—According to Stribeck, the carrying capacity in pounds per ball may be determined by the formula

$$w = kd^2 \tag{630}$$

in which d denotes the diameter of the ball in inches, and k a constant depending upon the form and material of the raceway and the speed of the bearing in which the ball is used. The following values of k, due to Stribeck, are based upon a large number of experiments on bearings in which the races were made of a good quality of hardened steel:

1. For a flat or conical raceway having three or four points of contact the value of k varies from 420 to 700.

2. For curved raceways whose radius of curvature equals $\frac{2}{3}d$ and having two-point contact the value of k is 1,400.

3. For special races and balls made of special alloy steel the above values may be increased 50 per cent.

(*b*) *Load per bearing.*—According to Art. 385(*a*), the total load upon a ball bearing may be assumed as being supported by one-fifth of the number of balls in the bearing, hence multiplying the values of w by $\frac{Z}{5}$ we get the total load

$$W = \frac{kZd^2}{5} \tag{631}$$

in which Z denotes the number of balls in the bearing.

(*c*) *Crushing strength of balls.*—In Table 103 are given the approximate crushing strengths of the commercial sizes of regular tool steel balls. These values according to R. H. Grant are considered reliable, and were adopted by the manufacturers

Table 103.—Crushing Strength of Tool-steel Balls

Diam. of ball	Ultimate strength, lb.	Diam. of ball	Ultimate strength, lb.	Diam. of ball	Ultimate strength, lb.
1/16	390	3/8	14,000	15/16	88,000
3/32	875	7/16	19,100	1	100,000
7/64	1,562	1/2	25,000	1 1/8	125,000
1/8	2,450	9/16	31,500	1 1/4	156,000
3/16	3,496	5/8	39,000	1 1/2	225,000
7/32	4,780	3/4	56,250	1 5/8	263,000
1/4	6,215	13/16	66,000	1 3/4	306,000
5/16	9,940	7/8	76,000	2	400,000

after several years of testing. Data pertaining to special alloy steel balls are not available, but it is safe to assume that the crushing loads will exceed those given in Table 103 by 25 to 50 per cent. According to Grant a *factor of safety of ten* should be used in selecting balls for bearings.

394. Coefficient of Friction.—In order to compare ball bearings with ordinary plain bearings, the coefficient of friction is referred to the diameter of the shaft. Stribeck found experimentally that the coefficient of friction of a good radial ball bearing having curved raceways is independent of the speed within wide limits and has an average value of 0.0015. This coefficient will practically be double this value when the load on the bearing is reduced to approximately one-tenth of the maximum load. The magnitude of the coefficient of friction in a radial bearing will also depend upon the axial thrust coming upon it. According to some experimental data published in the *American Machinist* of March, 1909, the coefficient of friction for a radial ball bearing subjected to a constant radial load and a variable axial thrust increased from 0.004 at a speed of 200 revolutions per minute to 0.012 at a speed of 1,200 revolutions per minute.

395. Ball Bearing Data.—Through the efforts of the Committee on Standards appointed by the Society of Automobile Engineers, practically all types of radial ball bearings have been standardized. In a report submitted to the society at the Spring meeting in 1911 were included tables giving standard dimensions of light, medium, and heavy radial bearings. Some manufacturers make a fourth series known as the extra heavy. According to the catalogs of the various prominent manufacturers thrust bearings are made in light, medium, and heavy series. With few exceptions the ball bearing manufacturers have adopted the English unit for the ball dimensions and the metric unit for the remaining dimensions of the bearing.

(*a*) *Hess-Bright radial bearings.*—In Table 104 are given the leading dimensions of the light, medium, and heavy series of the wide-type Hess-Bright radial bearings. The symbols denoting the dimensions refer to the key drawing of Fig. 325(*c*). The load-carrying capacity as given in Table 104 is based on a steady load and a constant speed not exceeding 200 revolutions per minute. The load rating of any size bearing operating at any given speed not exceeding 1,500 revolutions per minute may be determined

Table 104.—Data Pertaining to Hess-Bright Radial Bearings

No. and type of bearing		Dimensions in mm.			Diam. of balls	Capacity at 200 r.p.m.
		1	2	3		
Light series	200	30	10	9	3/16	130
	201	32	12	10	3/16	145
	202	35	15	11	3/16	165
	203	40	17	12	7/32	240
	204	47	20	14	1/4	350
	205	52	25	15	1/4	395
	206	62	30	16	5/16	550
	207	72	35	17	5/16	660
	208	80	40	18	3/8	860
	209	85	45	19	3/8	945
	210	90	50	20	3/8	990
	211	100	55	21	7/16	1,255
	212	110	60	22	1/2	1,630
	213	120	65	23	1/2	1,760
	214	125	70	24	1/2	1,870
	215	130	75	25	9/16	2,200
	216	140	80	26	5/8	2,750
	217	150	85	28	11/16	3,080
	218	160	90	30	3/4	3,630
	219	170	95	32	3/4	3,850
	220	180	100	34	13/16	4,180
	221	190	105	36	7/8	4,840
	222	200	110	38	7/8	5,280
Medium series	300	35	10	11	1/4	220
	301	37	12	12	1/4	265
	302	42	15	13	1/4	285
	303	47	17	14	5/16	395
	304	52	20	15	5/16	440
	305	62	25	17	3/8	660
	306	72	30	19	7/16	880
	307	80	35	21	1/2	1,100
	308	90	40	23	9/16	1,430
	309	100	45	25	5/8	1,760
	310	110	50	27	11/16	2,090
	311	120	55	29	3/4	2,530
	312	130	60	31	13/16	2,970
	313	140	65	33	7/8	3,410
	314	150	70	35	15/16	3,895
	315	160	75	37	1	4,400
	316	170	80	39	1 1/16	4,995
	317	180	85	41	1 1/8	5,500
	318	190	90	43	1 3/16	6,160
	319	200	95	45	1 1/4	6,820
	320	215	100	47	1 5/16	7,435
	321	225	105	49	1 3/8	8,140
	322	240	110	50	1 1/2	9,680
Heavy series	403	62	17	17	1/2	770
	404	72	20	19	9/16	1,145
	405	80	25	21	5/8	1,385
	406	90	30	23	11/16	1,650
	407	100	35	25	3/4	1,980
	408	110	40	27	13/16	2,310
	409	120	45	29	7/8	2,640
	410	130	50	31	15/16	3,080
	411	140	55	33	1	3,465
	412	150	60	35	1 1/16	4,400
	413	160	65	37	1 1/8	4,950
	414	180	70	42	1 1/4	6,095
	415	190	75	45	1 5/16	6,710
	416	200	80	48	1 3/8	7,370
	417	210	85	52	1 7/16	8,030
	418	225	90	54	1 9/16	9,460
	419	250	95	55	1 11/16	10,390
	420	265	100	60	1 13/16	12,650
	421	290	105	65	1 7/8	13,500
	422	320	110	70	2	15,400

by multiplying the capacity given in the table by the speed coefficient obtained from Fig. 335. The graph of Fig. 335 is based upon information published in the trade literature issued by the Hess-Bright Mfg. Co.

(*b*) *S. K. F. radial bearing.*—The leading dimensions and load rating of the light, medium, and heavy series of the narrow type of S.K.F. self-aligning radial bearing, similar to that shown in Fig. 325(*d*), are given in Table 105. The load rating given in

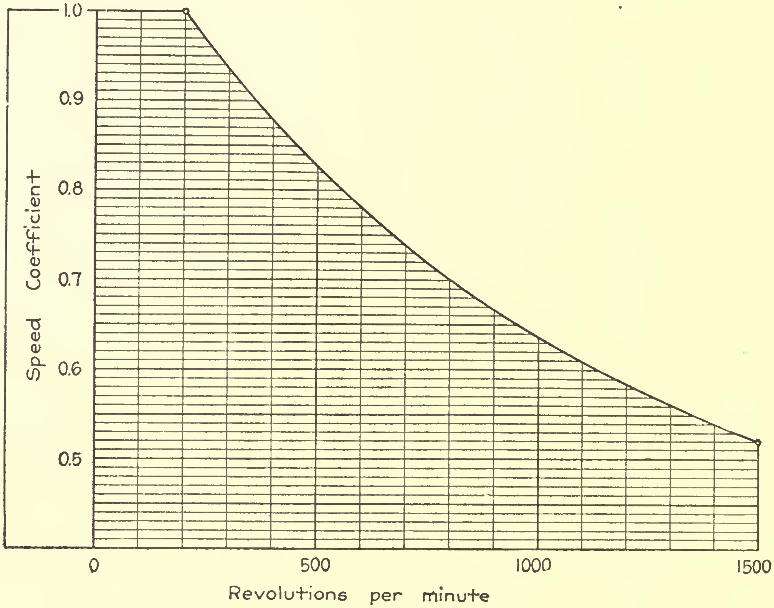

FIG. 335.

this table applies to a steady load and a constant speed not exceeding 300 revolutions per minute. To determine the permissible load capacity for a bearing running at other speeds than 300, the rating given in the table must be multiplied by the speed coefficient obtained from the graph of Fig. 336. This graph was plotted from data deduced from the load-carrying capacities and speeds given in the trade publication issued by the S. K. F. Ball Bearing Co.

(*c*) *F. and S. thrust bearing.*—The dimensions and load ratings given in Table 106 pertain to the light, medium, and heavy series of F. and S. spherical seated type of ball bearing shown in Fig.

TABLE 105.—DATA PERTAINING TO S. K. F. RADIAL BEARINGS

No. and type of bearing		Dimensions in mm.					Capacity at 300 r.p.m.
		1	2	3	4	5	
Light series	1,200	30	10	9	1	....	200
	1,201	32	12	10	1	....	220
	1,202	35	15	11	1	....	285
	1,203	40	17	12	1	....	365
	1,204	47	20	14	1	....	465
	1,205	52	25	15	1	....	630
	1,206	62	30	16	1	....	850
	1,207	72	35	17	2	....	970
	1,208	80	40	18	2	....	1,210
	1,209	85	45	19	2	....	1,400
	1,210	90	50	20	2	....	1,575
	1,211	100	55	21	2	....	1,930
	1,212	110	60	22	2	....	2,205
	1,213	120	65	23	2	....	2,430
	1,214	125	70	24	2	....	2,810
	1,215	130	75	25	2	....	2,980
	1,216	140	80	26	3	....	3,310
	1,217	150	85	28	3	....	4,080
	1,218	160	90	30	3	....	4,630
	1,219	170	95	32	3	....	5,520
	1,220	180	100	34	3	....	6,060
	1,221	190	105	36	3	....	7,060
	1,222	200	110	38	3	....	7,720
Medium series	1,300	35	10	11	1	....	265
	1,301	37	12	12	1	....	350
	1,302	42	15	13	1	....	385
	1,303	47	17	14	1	....	550
	1,304	52	20	15	1	....	575
	1,305	62	25	17	1	....	885
	1,306	72	30	19	2	....	1,100
	1,307	80	35	21	2	....	1,430
	1,308	90	40	23	2	....	1,760
	1,309	100	45	25	2	....	2,200
	1,310	110	50	27	2	....	2,540
	1,311	120	55	29	2	....	3,310
	1,312	130	60	31	2	....	3,860
	1,313	140	65	33	3	....	4,410
	1,314	150	70	35	3	....	5,070
	1,315	160	75	37	3	....	5,850
	1,316	170	80	39	3	....	6,070
	1,317	180	85	41	3	42.0	8,720
	1,318	190	90	43	3	44.0	9,100
	1,319	200	95	45	3	46.0	10,900
	1,320	215	100	47	3	58.0	11,400
	1,321	225	105	49	3	60.0	13,200
	1,322	240	110	50	3	64.0	14,300
Heavy series	402	52	15	15	1	....	630
	403	62	17	17	1	....	885
	404	72	20	19	2	....	1,145
	405	80	25	21	2	....	1,435
	406	90	30	23	2	....	1,765
	407	100	35	25	2	....	2,205
	408	110	40	27	2	....	2,535
	409	120	45	29	2	....	3,300
	410	130	50	31	2	....	3,850
	411	140	55	33	3	....	4,410
	412	150	60	35	3	....	5,070
	413	160	65	37	3	....	6,060
	414	180	70	42	3	....	8,820
	415	190	75	45	3	....	9,080
	416	200	80	48	3	....	11,000
	417	210	85	52	3	54.5	11,000
	418	225	90	54	3	60.0	13,250
	419	250	95	55	3	66.0	15,100
	420	265	100	60	3	68.5	16,550

330(*b*). Using the load capacity as given in the table as a basis, the permissible maximum rating for a bearing running at any constant speed in excess of 50 revolutions per minute may be

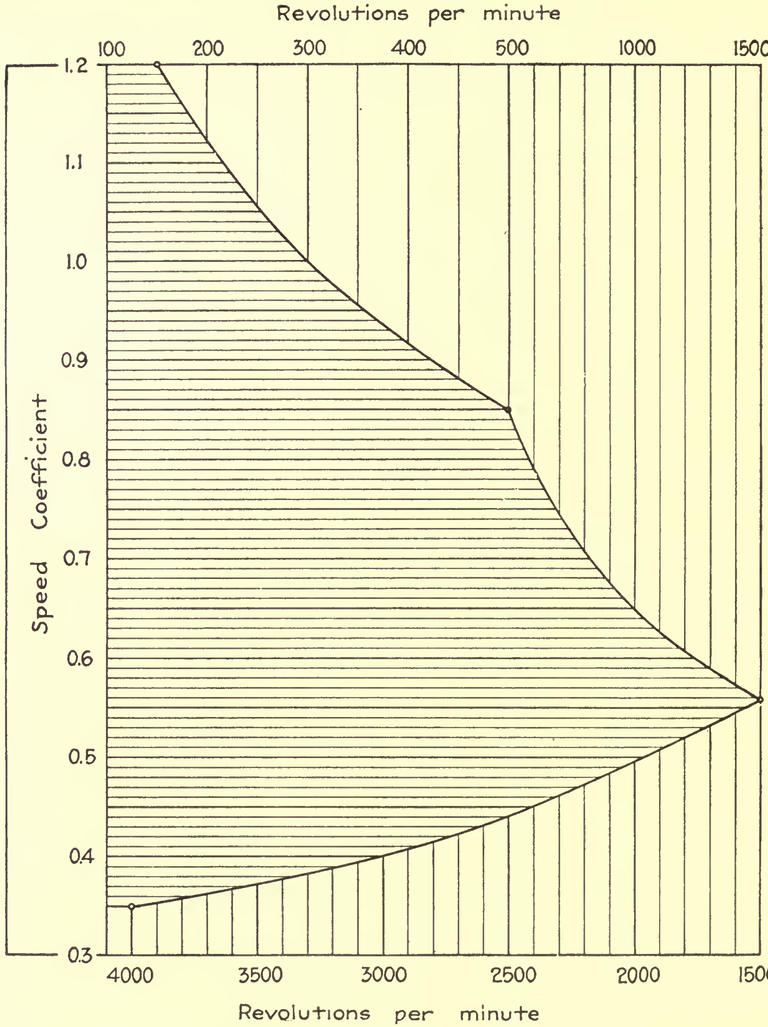

Fig. 336.

determined by multiplying the tabular value by the speed coefficient obtained from Fig. 337. The graph of Fig. 337 is based upon the load capacities given in the trade literature issued by

TABLE 106.—DATA PERTAINING TO F. & S. SPHERICAL SEATED THRUST BEARINGS

	No. and type of bearing	Dimensions in mm.						Balls		Capacity at 50 r.p.m.
		1	2	3	4	5	6	No.	Diam.	
Light series	AJL 10	30	10	14	12	21	21	8	1/4	675
	AJL 15	35	15	16	16	24	25	10	1/4	850
	AJL 20	40	20	16	21	26	30	12	1/4	1,000
	AJL 25	45	25	16	26	33	35	14	1/4	1,200
	AJL 30	53	30	18	32	38	40	16	1/4	1,350
	AJL 35	62	35	21	37	44	50	16	5/16	2,100
	AJL 40	64	40	21	42	49	50	17	5/16	2,250
	AJL 45	73	45	25	47	55	60	16	3/8	3,000
	AJL 50	78	50	25	52	60	65	18	3/8	3,400
	AJL 55	88	55	28	57	65	70	17	7/16	4,400
	AJL 60	90	60	28	62	70	75	18	7/16	4,650
	AJL 65	100	65	32	67	75	80	18	1/2	6,100
	AJL 70	103	70	32	72	80	85	19	1/2	6,400
	AJL 75	110	75	32	77	85	90	20	1/2	6,700
	AJL 80	115	80	35	82	90	95	21	1/2	7,100
	AJL 85	125	85	38	88	97	105	18	5/8	9,500
	AJL 90	132	90	39	93	103	110	19	5/8	10,000
	AJL 95	140	95	41	98	109	115	18	11/16	11,400
	AJL 100	148	100	42	103	118	120	19	11/16	12,000
	AJL 105	155	105	43	110	130	130	18	3/4	13,600
	AJL 110	160	110	43	115	135	135	19	3/4	14,400
Medium series	BJL 25	52	25	19	26	40	40	13	5/16	1,700
	BJL 30	60	30	21	32	45	45	13	3/8	2,450
	BJL 35	68	35	24	37	55	55	13	7/16	3,350
	BJL 40	76	40	27	42	60	60	13	1/2	4,400
	BJL 45	85	45	30	47	65	65	13	9/16	5,500
	BJL 50	92	50	33	52	75	75	13	5/8	6,800
	BJL 55	100	55	35	57	80	80	13	21/32	7,500
	BJL 60	106	60	37	62	85	85	13	11/16	8,250
	BJL 65	112	65	38	67	90	90	14	23/32	9,700
	BJL 70	120	70	40	72	95	95	14	3/4	10,600
	BJL 75	128	75	43	77	105	105	14	13/16	12,400
	BJL 80	136	80	46	82	110	110	14	7/8	14,400
	BJL 85	145	85	49	88	120	120	14	15/16	16,500
	BJL 90	155	90	52	93	125	125	14	1	18,800
	BJL 95	165	95	56	98	130	130	14	1 1/16	21,300
	BJL 100	172	100	59	103	140	140	14	1 1/8	24,000
	BJL 105	180	105	62	110	145	145	14	1 3/16	26,500
	BJL 110	190	110	65	115	155	155	14	1 1/4	29,500
	BJL 115	200	115	68	120	160	160	14	1 5/16	32,500
	BJL 120	210	120	72	125	170	170	14	1 3/8	35,500
Heavy series	CJL 20	50	20	21	21	35	35	10	3/8	1,900
	CJL 25	60	25	25	26	45	45	10	15/32	3,000
	CJL 30	73	30	30	32	50	50	10	9/16	4,300
	CJL 35	80	35	33	37	60	60	10	5/8	5,250
	CJL 40	90	40	38	42	65	65	10	23/32	7,000
	CJL 45	100	45	42	47	75	75	10	13/16	8,900
	CJL 50	110	50	47	52	80	80	10	29/32	11,000
	CJL 55	120	55	52	57	90	90	10	1	13,500
	CJL 60	130	60	56	62	95	95	10	1 1/16	15,200
	CJL 65	140	65	61	67	105	105	10	1 3/16	19,000
	CJL 70	150	70	65	72	110	110	10	1 1/4	21,000
	CJL 75	160	75	70	77	120	120	10	1 3/8	25,500
	CJL 80	170	80	74	82	125	125	10	1 7/16	28,000
	CJL 85	180	85	78	88	135	135	10	1 1/2	30,000
	CJL 90	190	90	83	93	140	140	10	1 5/8	35,500
	CJL 95	195	95	86	98	145	145	10	1 11/16	38,500
	CJL 100	205	100	89	103	155	155	10	1 3/4	41,000
	CJL 115	220	115	93	120	170	170	11	1 13/16	48,500
	CJL 130	240	130	95	135	185	185	12	1 13/16	53,000
	CJL 150	260	150	99	155	205	205	13	1 7/8	61,500

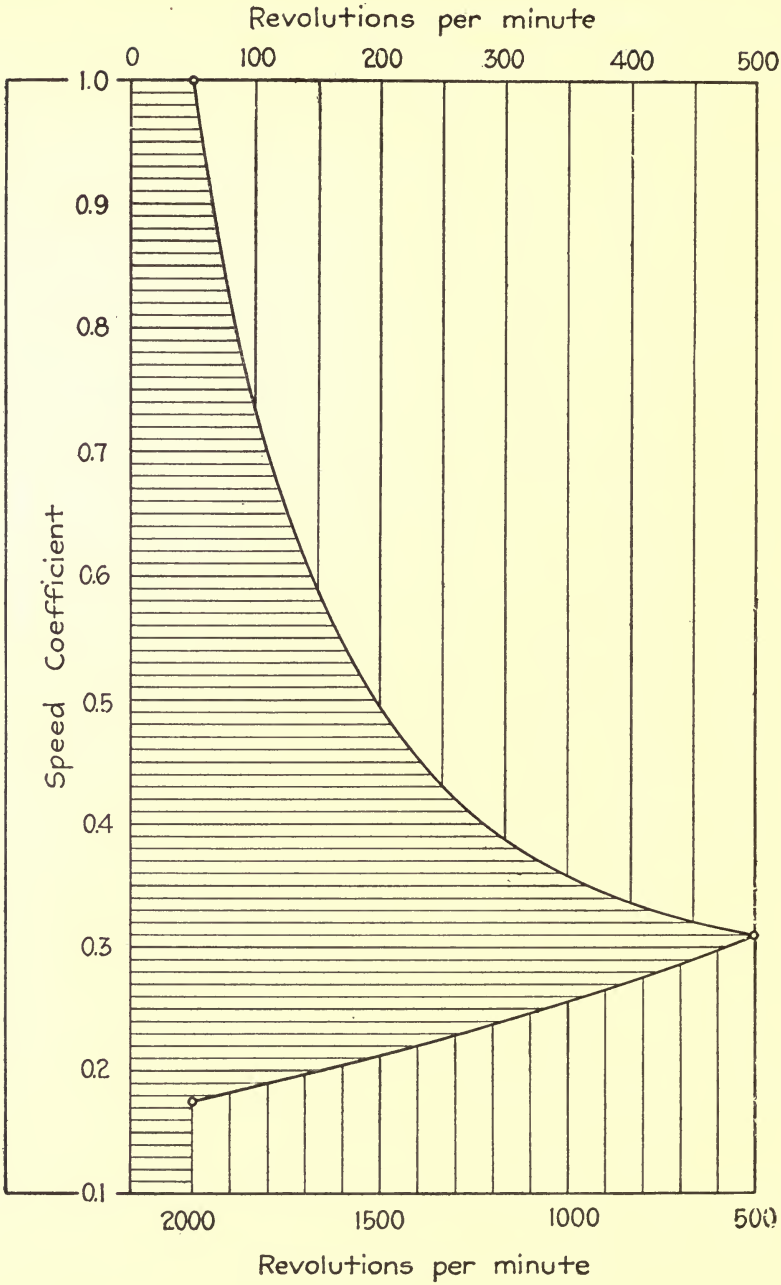
Revolutions per minute
0
100
200
300
400
500
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Speed Coefficient
2000
1500
1000
500
Revolutions per minute

FIG. 337.

The Bearings Co. of America, the distributors of the F. and S. bearings.

(*d*) *Gurney radio-thrust bearing.*—The Gurney radio-thrust bearing is manufactured in the following three standard types:

1. Type *RT* having a thrust capacity equivalent to 100 per cent. of the rated radial load.

2. Type *RT* 150 having a thrust capacity equivalent to 150 per cent. of the rated radial load.

3. Type *RT* 200 having a thrust capacity equivalent to 200 per cent. of the rated radial load.

In Table 107 are given the leading dimensions, load-carrying capacity, and speed rating for the light, medium, and heavy series of the *RT* type Gurney radio-thrust bearing similar to that shown in Fig. 333. It should be understood that the ratings given in Table 107 are not intended for all conditions of operation, but that they apply only to the class of service in which uniform load and constant speed prevail. Furthermore, the speed rating applies to the type of mounting in which the inner race rotates. When the outer race revolves, the permissible speed should be taken as 60 per cent. of that listed in the table. According to information furnished by the manufacturer, the rated capacity of the *RT* 150 type is 95 per cent. and that of the *RT* 200 type is 90 per cent. of the rating given in Table 107. When the speed of the inner race is above or below the value given in the table, the permissible load capacity is obtained by multiplying the rated load by the so-called *load factor.* This factor depends upon the *speed coefficient* and may be determined from the graph of Fig. 338. By the term *speed coefficient* is meant the ratio of the given revolutions per minute to the rated revolutions per minute given in the table.

To determine the capacity of a Gurney radio-thrust bearing having given the radial load coming upon it, the following rule used by the Gurney Ball Bearing Co. is recommended:

Rule I.—"*Subtract the actual radial load from the rated load and multiply the remainder by the thrust percentage of the bearing.*"

If it is required to determine the available radial load capacity of a radio-thrust bearing having given the axial thrust, the following rule should be used:

Rule II.—"*Divide the actual thrust by the thrust percentage of the bearing and subtract the result from the rated load.*"

The use of the various factors and rules just given is shown best by applying them to a problem, as follows:

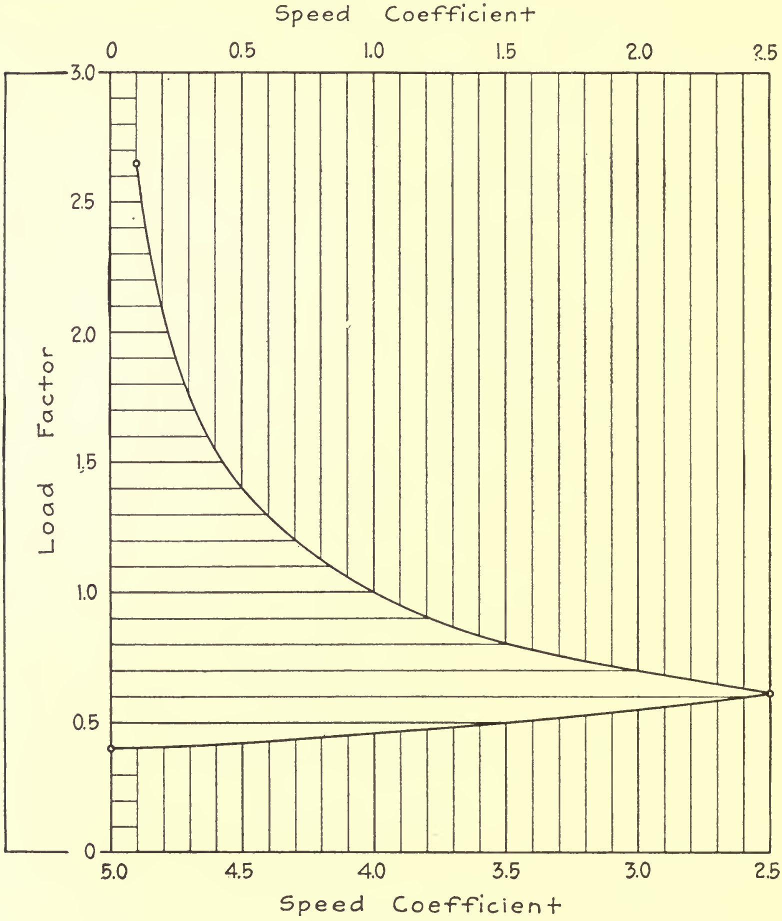

FIG. 338.

Problem.—It is required to determine the thrust capacity of No. 310 RT 150 radio-thrust bearing running at 750 revolutions per minute, assuming that the bearing is carrying a radial load of 1,000 pounds.

Solution.—The load rating of bearing No. 310 RT 150 is 3,000 × 0.95 or 2,850 pounds at 625 revolutions per minute. From Fig. 338 the load factor corresponding to a speed factor of $\frac{750}{625}$, or 1.2, is 0.91, hence the load capacity at 750 revolutions per minute is 2,850 × 0.91 or 2,590 pounds. Applying the first rule given above, the thrust capacity of the given bearing operating under the above conditions is 1.5 (2,590 − 1,000) or 2,385 pounds.

Table 107.—Data Pertaining to Gurney Radio-thrust Bearings

No. and type of bearing		Dimensions				Balls		Load rating	Speed rating
		1 mm.	2 mm.	3 mm.	4 in.	No.	Diam., in.		
Light series	204 RT	47	20	14	3/64	13	9/32	480	1,450
	205 RT	52	25	15	3/64	14	9/32	530	1,250
	206 RT	62	30	16	1/16	15	5/16	720	1,050
	207 RT	72	35	17	1/16	16	3/8	1,100	900
	208 RT	80	40	18	1/16	16	7/16	1,480	800
	209 RT	85	45	19	1/16	17	7/16	1,570	740
	210 RT	90	50	20	5/64	18	7/16	1,670	680
	211 RT	100	55	21	5/64	18	1/2	1 960	600
	212 RT	110	60	22	5/64	17	9/16	2,570	550
	213 RT	120	65	23	5/64	17	5/8	3,230	500
	214 RT	125	70	24	5/64	18	5/8	3,420	475
	215 RT	130	75	25	5/64	19	5/8	3,610	450
	216 RT	140	80	26	3/32	19	11/16	4,370	420
	217 RT	150	85	28	3/32	18	3/4	4,850	390
	218 RT	160	90	30	3/32	18	13/16	5,700	370
	219 RT	170	95	32	1/8	18	7/8	6,650	350
	220 RT	180	100	34	1/8	17	15/16	7,220	330
	221 RT	190	105	36	1/8	17	1	8.080	315
	222 RT	200	110	38	1/8	17	1 1/16	9,310	300
Medium series	304 RT	52	20	15	3/64	13	5/16	620	1.200
	305 RT	62	25	17	1/16	13	3/8	860	1,050
	306 RT	72	30	19	1/16	13	7/16	1,190	950
	307 RT	80	35	21	1/16	13	1/2	1,570	850
	308 RT	90	40	23	5/64	13	9/16	2,000	750
	309 RT	100	45	25	5/64	13	5/8	2,470	675
	310 RT	110	50	27	5/64	13	11/16	3,000	625
	311 RT	120	55	29	3/32	13	3/4	3,570	575
	312 RT	130	60	31	3/32	14	11/16	4,470	525
	313 RT	140	65	33	3/32	14	7/8	5,230	475
	314 RT	150	70	35	3/32	14	15/16	5,990	450
	315 RT	160	75	37	1/8	14	1	6,750	425
	316 RT	170	80	39	1/8	14	1 1/16	7,600	400
	317 RT	180	85	41	1/8	14	1 1/8	8,550	375
	318 RT	190	90	43	1/8	14	1 3/16	9,590	350
	319 RT	200	95	45	1/8	14	1 1/4	10,640	330
	320 RT	215	100	47	1/8	14	1 5/16	11,740	315
	321 RT	225	105	49	1/8	14	1 3/8	12,830	300
	322 RT	240	110	50	1/8	14	1 1/2	15,200	285
Heavy series	404 RT	72	20	19	1/16	9	9/16	1,380	1,250
	405 RT	80	25	21	1/16	9	5/8	1,620	1,075
	406 RT	90	30	23	5/64	10	11/16	2,280	875
	407 RT	100	35	25	5/64	10	3/4	2,660	775
	408 RT	110	40	27	5/64	11	13/16	3,520	700
	409 RT	120	45	29	5/64	11	7/8	4,090	650
	410 RT	130	50	31	3/32	11	15/16	4,700	600
	411 RT	140	55	33	3/32	11	1	5,320	550
	412 RT	150	60	35	3/32	11	1 1/16	5,990	500
	413 RT	160	65	37	3/32	11	1 1/8	6,750	460
	414 RT	180	70	42	3/32	11	1 5/16	9,190	425
	415 RT	190	75	45	1/8	11	1 3/8	10,070	400
	416 RT	200	80	48	1/8	11	1 7/16	11,020	375
	417 RT	210	85	52	1/8	11	1 1/2	11,970	350
	418 RT	225	90	54	1/8	11	1 5/8	14,060	325
	419 RT	250	95	55	1/8	11	1 3/4	16,340	300
	420 RT	265	100	60	1/8	11	1 7/8	18,810	285
	421 RT	290	105	65	1/8	11	2 1/8	23,130	270
	422 RT	320	110	70	1/8	11	2 5/16	28,500	250

Problem.—It is required to determine the radial load capacity of bearing No. 310 RT 150 running at 750 revolutions per minute and carrying an axial load of 2,100 pounds.

Solution.—Applying Rule II, we find that the axial load of 2,100 pounds is equivalent to $\frac{2,100}{1.5}$ or 1,400 pounds. According to the preceding problem the load capacity of the given size of bearing running at 750 revolutions per minute is 2,590 pounds, whence the magnitude of the radial load that may be placed upon the bearing in addition to the 2,100 pounds axial load is 2,590 − 1,400 or 1,190 pounds.

396. Mounting Ball Bearings.—(*a*) *Radial bearings.*—In general, the requirements of a correct mounting for radial ball bearings carrying no axial thrust are as follows.

1. The shaft and sleeves or bosses of pulleys, sprockets, and gears upon which the inner races of radial bearings are to be mounted must be turned and ground accurately, and the housings into which the outer races are fitted must be bored true so as to insure the concentric running of the races.

2. In some installations, as for example on a line- or countershaft, it is impossible to provide the inner race with a driving

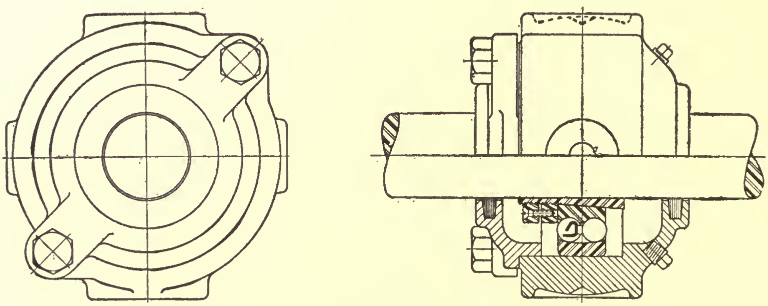

FIG. 339.

fit on the shaft and to clamp it against a shoulder. In such cases a device called an *adapter* is used. The adapter is nothing more than a split conical sleeve fitted over the shaft and provided with a nut and nut-lock as shown in Fig. 339. It is evident that the inner race may be rigidly clamped to the shaft at any desired position.

3. Whenever it is necessary to use a split housing the parts must be fitted together correctly so that the inner race will not become distorted due to any clamping action of the housing.

4. The inner race of the bearing must be retained in a fixed position. To this end it is made with a tight fit on the shaft and is held rigidly in position against a shoulder on the shaft by means of a nut and nut-lock. In Figs. 170 and 171 are shown typical mountings of radial bearings.

5. The outer race should readily take up a free position relative to the balls and inner race, thus insuring a more nearly perfect distribution of the load over the entire outer race. To permit a slight degree of axial movement, the outer race should have a so-called "sucking fit" in the housing and should never be held in place rigidly.

6. The mounting must be so designed that the ball bearing will not be left exposed to the action of water, dust, grit, and other foreign matter. Provision must also be made for retaining the lubricant. Various forms of caps for closing the sides of the housing are used, some of which are shown in Figs. 170, 194, 195, and 339. Experience has demonstrated that one or two cored grooves in the caps packed with a stiff grease or felt are effective in keeping out grit and water, and in preventing the escape of the lubricant. The grooves are generally made from $\frac{1}{4}$ to $\frac{3}{8}$ inch deep, and the bore through which the shaft passes must be made approximately $\frac{1}{64}$ inch larger than the shaft.

7. Whenever a shaft having no thrust bearing is supported by several radial bearings, satisfactory results are assured by securing the outer race of one of these bearings against axial movement, while the outer races of the remaining bearings must be left free to locate themselves.

(*b*) *Single-thrust bearings.*—In mounting one-direction thrust bearings the rotating race must be pressed against a suitable shoulder on the shaft by a light driving fit. The shoulder on the shaft must be of sufficient height so as not to subject the rotating race to an undue bending action. If it is impossible to provide a proper shoulder, a suitable washer or sleeve must be used between the shoulder and the race. The bore of the stationary race is made considerably larger than that of the rotating race. To secure satisfactory service with a thrust bearing, the load upon the balls must be distributed evenly. For this purpose the stationary race is provided with a spherical face, so that the complete thrust bearing may be supported on a spherical seated washer. The bore of the latter must be large and in the majority of installations, this washer should be free to move laterally, thus providing for any shaft deflection that is liable to occur. Whenever possible the center of the spherical seated race and washer should be located at, or near, the center of the radial bearing used in conjunction with the thrust bearing. As in the case of radial bearings, the mountings of all types of thrust bearings must exclude water, grit, and foreign matter and at the same time retain the lubricant. Furthermore, the same degree of workmanship on the various parts of the thrust bearing mounting is required as is necessary for the radial bearing.

(*c*) *Double-thrust bearing.*—In mounting a double-thrust bearing of the type illustrated in Fig. 331, the spherical faced stationary or outer ball races must bear against accurately machined spherical seats in the housing and cap. The central or rotating race must be fastened to the shaft and held against a suitable shoulder by means of a sleeve and nut-lock. The self-contained double-thrust bearing shown in Fig. 331 is mounted by fastening the central race to the shaft in a manner similar to that described for the plain double-thrust bearing. It is unnecessary to have an accurately bored seat for the outer casing, but machined faces must be provided between which the entire bearing may be clamped.

(*d*) *Radio-thrust bearing.*—The single radio-thrust bearings shown in Figs. 325(*b*) and 333 can take a thrust in only one direction and for that reason some care must be exercised in mounting such bearings. The Gurney radio-thrust bearing is made with a certain amount of looseness which must be taken up in the mounting. As in the case of the radial bearings, the inner race is mounted on the shaft with a light press fit and held against a suitable shoulder on the shaft by means of a nut and nut-lock. The outer race is made a push fit in the housing and must be held against a suitable shoulder in the housing or against the end of an adjustable cap. Typical mountings of radio-thrust bearings are shown in Figs. 171 and 195.

The double-row bearing shown in Fig. 334 really consists of two radio-thrust bearings located within an outer casing. Since this type of bearing is capable of supporting a radial load and at the same time take a thrust in either direction, the method of mounting depends upon the nature of the loading. As in the radial bearings, the inner race is fixed to the shaft, and if axial loads from both directions must be taken care of, the outer race must be clamped rigidly between a suitable shoulder on the machine frame and the end of an adjustable cap, as shown in Fig. 194. An installation of an ordinary radial bearing used in conjunction with a double-row radio-thrust bearing, the latter taking a thrust in one direction only, is shown in Fig. 170.

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