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# UNITED STATES
# DEPARTMENT OF THE INTERIOR
## BUREAU OF MINES
## HELIUM ACTIVITY
## HELIUM RESEARCH CENTER

## INTERNAL REPORT

# THERMODYNAMIC PROPERTIES OF A REDLICH-KWONG FLUID IN THE TWO-PHASE REGION

BY

B. J. Dalton
Robert E. Barieau

**BRANCH** Fundamental Research
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# THERMODYNAMIC PROPERTIES OF A REDLICH-KWONG FLUID IN THE TWO-PHASE REGION

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B. J. Dalton[1] and Robert E. Barieau[2]

## ABSTRACT

The Bureau of Mines Helium Research Center has as a long-range objective the development of an equation of state for helium that will allow all of the thermodynamic properties to be calculated within the accuracy with which they are known. If this objective is to be realized, a method of calculation must be known. This report gives the principles of the method we use in such problems.

A computer program (<u>8</u>)[3] was developed for predicting these properties, including those at the critical point, that is perfectly general and applicable to any equation of state. To demonstrate the efficacy of our method, this computer program was used to evaluate thermodynamic properties of a Redlich-Kwong fluid in the two-phase region. Expressions for calculating various thermodynamic functions specific for this particular equation of state are derived. Numerical values are tabulated and graphs are presented for all functions calculated.

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[3] Underlined numbers in parentheses refer to items in the list of references at the end of this report.

It is shown that for a Redlich-Kwong fluid there are finite discontinuities in (1) the second temperature derivative of the Gibbs free energy or chemical potential at the critical point, (2) the second derivative of the vapor pressure curve with regard to the temperature on passing through the critical point, and (3) the measured heat capacity at constant volume on passing through the critical temperature when the calorimeter is filled to the critical density. It is also shown that the heat of vaporization is infinite for a Redlich-Kwong fluid at absolute zero.

## INTRODUCTION

This work is part of a broad program underway at the Helium Research Center for the development of a self-consistent, accurate equation of state for helium and helium mixtures from which all thermodynamic properties can be calculated within the accuracy with which they are known. This includes the calculation of all properties in the two-phase region. Accurate values of the thermodynamic properties of helium and helium mixtures are needed to design improved process and separation cycles for the extraction, concentration, and purification of helium. This information will contribute toward planning and coordinating efficient utilization of the nation's helium resources and for the future development of new uses for helium.

In a previous report (2), formulas were derived for calculating most of the thermodynamic properties of current interest from an empirical equation of state. From this report (2), a computer program

(8) was developed for predicting these properties, including those at the critical point. This computer program is perfectly general and is applicable to any equation of state. This computer program was used in the present work to evaluate various thermodynamic properties of a Redlich-Kwong fluid in the two-phase region.

The notation used for the thermodynamic functions is mainly that of Lewis and Randall (18), an exception being that G is used to represent the molal Gibbs free energy or chemical potential instead of F. The relative molal heat content is defined as Lewis and Randall originally defined this property. Relative molal properties have been extended to include heat capacity, internal energy, entropy, and free energy.

Numerical values are tabulated and illustrations are presented for all functions calculated. The numerical values listed in the tables are in "E format" (i.e., the number to the right of E indicates the exponent of 10 by which the number is to be multiplied). The numerical values listed within the tables and text have been truncated to the number of figures given.

Other than a previous Bureau publication (5) giving results for a van der Waals fluid, the authors are not aware of any previous publication in which these extensive calculations are presented.

## ACKNOWLEDGMENT

The staff of the Helium Activity Branch of Automatic Data Processing wrote the computer program, carried out all calculations using

this program, and prepared the original illustrations using an automatic plotter. The staff of the Helium Activity Branch of Engineering Drafting prepared the illustrative material for publication. The authors thank the staffs of these two groups for this valuable aid.

## MATHEMATICAL NOMENCLATURE AND SYMBOLISM

In this report, all molal thermodynamic functions are assumed to be explicit functions of the molal density, $\rho$, and the absolute thermodynamic temperature, T:

$$W = W(\rho,T) \quad , \tag{1}$$

where W is any molal thermodynamic property of a homogeneous phase. Then for a differential change, we have

$$dW = \left(\frac{\partial W}{\partial \rho}\right)_T d\rho + \left(\frac{\partial W}{\partial T}\right)_\rho dT \quad . \tag{2}$$

For all partial derivatives involving the variables $\rho$ and T, we will use a subscript. Thus, we write

$$W_\rho = \left(\frac{\partial W}{\partial \rho}\right)_T \quad , \tag{3}$$

$$W_T = \left(\frac{\partial W}{\partial T}\right)_\rho \quad . \tag{4}$$

We extend this symbolism to second derivatives by writing

$$W_{\rho\rho} = \left(\frac{\partial W_\rho}{\partial \rho}\right)_T = \left(\frac{\partial^2 W}{\partial \rho^2}\right)_T \quad , \tag{5}$$

$$W_{\rho T} = \left(\frac{\partial W_\rho}{\partial T}\right)_\rho = \left(\frac{\partial W_T}{\partial \rho}\right)_T \quad , \tag{6}$$

$$W_{TT} = \left(\frac{\partial W_T}{\partial T}\right)_\rho = \left(\frac{\partial^2 W}{\partial T^2}\right)_\rho \quad . \tag{7}$$

The extension of this symbolism to higher order derivatives involving the variables $\rho$ and T should be obvious.

Thus, consistent with our symbolism, we write equation 2 as

$$dW = W_\rho d\rho + W_T dT \quad . \tag{8}$$

If we wish to express the change in W with temperature, keeping the pressure, P, constant, we have

$$\left(\frac{\partial W}{\partial T}\right)_P = W_T + W_\rho\left(\frac{\partial \rho}{\partial T}\right)_P \quad . \tag{9}$$

For partial derivatives involving variables other than $\rho$ and T, we will use a small curly d, $\partial$, to indicate a differential as in the left-hand side of equation 9.

Higher order derivatives involving variables other than $\rho$ and T are obtained as follows. Differentiating equation 9 with regard to T, keeping P constant, we have

$$\left(\frac{\partial^2 W}{\partial T^2}\right)_P = \left(\frac{\partial W_T}{\partial T}\right)_P + \left(\frac{\partial W_\rho}{\partial T}\right)_P\left(\frac{\partial \rho}{\partial T}\right)_P + W_\rho\left(\frac{\partial^2 \rho}{\partial T^2}\right)_P \quad . \tag{10}$$

But

$$\left(\frac{\partial W_T}{\partial T}\right)_P = W_{TT} + W_{\rho T}\left(\frac{\partial \rho}{\partial T}\right)_P \tag{11}$$

and

$$\left(\frac{\partial W_\rho}{\partial T}\right)_P = W_{\rho T} + W_{\rho\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \quad . \tag{12}$$

Substituting equations 11 and 12 in equation 10, we find that

$$\left(\frac{\partial^2 W}{\partial T^2}\right)_P = W_{TT} + 2W_{\rho T}\left(\frac{\partial \rho}{\partial T}\right)_P + W_{\rho\rho}\left(\frac{\partial \rho}{\partial T}\right)_P^2 + W_\rho\left(\frac{\partial^2 \rho}{\partial T^2}\right)_P \quad . \tag{13}$$

If we have two phases in equilibrium and we wish to express the change of W with temperature, but maintaining the two phases in equilibrium, we write

$$\frac{dW}{dT} = \left(\frac{\partial W}{\partial \rho}\right)_T \frac{d\rho}{dT} + \left(\frac{\partial W}{\partial T}\right)_\rho ,$$

$$= W_\rho \frac{d\rho}{dT} + W_T \qquad (14)$$

and in this type of derivative, we use a small d to indicate a differential (when two phases remain in equilibrium) as in the left-hand side of equation 14.

## THE REDLICH-KWONG EQUATION OF STATE

The Redlich-Kwong equation of state (21) is

$$P = \frac{RT\rho}{(1-b'\rho)} - \frac{a'\rho^2}{(1+b'\rho)T^{1/2}} , \qquad (15)$$

where R is the gas constant and $a'$ and $b'$ are constants. In terms of the compressibility factor, equation 15 may be written as

$$Z = \frac{P}{\rho RT} = \frac{1}{(1-b'\rho)} - \frac{a'\rho}{RT^{3/2}(1+b'\rho)} , \qquad (16)$$

where Z is the compressibility factor and is defined by equation 16.

We define

$\alpha = \rho/\rho_c$, the reduced molal density, (17)

$\beta = P/P_c$, the reduced pressure, and (18)

$\gamma = T/T_c$, the reduced absolute thermodynamic temperature, (19)

where $\rho_c$, $P_c$, and $T_c$ represent molal density, pressure, and temperature at the critical point.

Substituting equations 17 and 19 in equation 16,

$$Z = \frac{1}{(1-b'\rho_c\alpha)} - \frac{(a'\rho_c/RT_c^{3/2})\alpha}{\gamma^{3/2}(1+b'\rho_c\alpha)} . \qquad (20)$$

The two constants of the Redlich-Kwong equation of state may be determined from the critical conditions (2). The results are:

$$a = (a'\rho_c/RT_c^{3/2}) = [3(2^{1/3}-1)]^{-1} = 1.28244 \quad , \tag{21}$$

$$b = b'\rho_c = (2^{1/3}-1) = 0.259921 \quad , \tag{22}$$

$$Z_c = 1/3 \quad , \tag{23}$$

where $Z_c$ is the compressibility factor at the critical point.

Substituting equations 21 and 22 in equation 20,

$$Z = \frac{1}{(1-b\alpha)} - \frac{a\alpha}{\gamma^{3/2}(1+b\alpha)} \quad . \tag{24}$$

Equation 24 is the expression for the compressibility factor of a Redlich-Kwong fluid in terms of the reduced variables of density and temperature.

The reduced pressure is given by

$$\beta = (P/P_c) = (\rho/\rho_c)(T/T_c)(Z/Z_c) \quad ,$$

$$= \frac{\alpha\gamma Z}{Z_c} \tag{25}$$

or, with equations 23 and 24,

$$\beta = 3\alpha\gamma\left[\frac{1}{(1-b\alpha)} - \frac{a\alpha}{\gamma^{3/2}(1+b\alpha)}\right] \quad , \tag{26}$$

which is the expression for the reduced pressure of a Redlich-Kwong fluid in terms of the reduced variables of density and temperature.

To calculate all of the thermodynamic properties of a pure material, including those in the two-phase region, from a single equation of state, various partial derivatives of the pressure or of the compressibility factor must be evaluated along with the value of the integral

$$X = \int_0^{\alpha} (Z-1)\,\frac{d\alpha}{\alpha} \quad . \tag{27}$$

In addition, various partial derivatives of the above integral must be calculated. Since all functions can be differentiated, if X can be expressed analytically, then all thermodynamic functions can be expressed analytically as explicit functions of the reduced variables of density and temperature.

For a Redlich-Kwong fluid, the integral X as defined above is integrable and differentiable with respect to the reduced variables of density and temperature and can be expressed analytically in terms of these same variables. We show in this report that if the quantities given by equations 28-38 have been calculated, then all thermodynamic properties, including those at the critical, can be determined:

$$Z_{\alpha} = \left(\frac{\partial Z}{\partial \alpha}\right)_{\gamma} = \frac{b}{(1-b\alpha)^2} - \frac{a}{\gamma^{3/2}(1+b\alpha)^2} \quad ; \tag{28}$$

$$Z_{\gamma} = \left(\frac{\partial Z}{\partial \gamma}\right)_{\alpha} = \frac{3a\alpha}{2\gamma^{5/2}(1+b\alpha)} \quad ; \tag{29}$$

$$Z_{\alpha\alpha} = \left(\frac{\partial Z_{\alpha}}{\partial \alpha}\right)_{\gamma} = 2b\left[\frac{b}{(1-b\alpha)^3} + \frac{a}{\gamma^{3/2}(1+b\alpha)^3}\right] \quad ; \tag{30}$$

$$Z_{\alpha\gamma} = \left(\frac{\partial Z_{\alpha}}{\partial \gamma}\right)_{\alpha} = \left(\frac{\partial Z_{\gamma}}{\partial \alpha}\right)_{\gamma} = \frac{3a}{2\gamma^{5/2}(1+b\alpha)^2} \quad ; \tag{31}$$

$$Z_{\gamma\gamma} = \left(\frac{\partial Z_{\gamma}}{\partial \gamma}\right)_{\alpha} = -\frac{15a\alpha}{4\gamma^{7/2}(1+b\alpha)} \quad ; \tag{32}$$

$$Z_{\alpha\alpha\alpha} = \left(\frac{\partial Z_{\alpha\alpha}}{\partial\alpha}\right)_{\gamma} = 6b^2\left[\frac{b}{(1-b\alpha)^4} - \frac{a}{\gamma^{3/2}(1+b\alpha)^4}\right] \quad ; \qquad (33)$$

$$Z_{\alpha\alpha\gamma} = \left(\frac{\partial Z_{\alpha\gamma}}{\partial\alpha}\right)_{\gamma} = \left(\frac{\partial Z_{\alpha\alpha}}{\partial\gamma}\right)_{\alpha} = -\frac{3ab}{\gamma^{5/2}(1+b\alpha)^3} \quad ; \qquad (34)$$

$$Z_{\alpha\alpha\alpha\alpha} = \left(\frac{\partial Z_{\alpha\alpha\alpha}}{\partial\alpha}\right)_{\gamma} = 24b^3\left[\frac{b}{(1-b\alpha)^5} + \frac{a}{\gamma^{3/2}(1+b\alpha)^5}\right] \quad ; \qquad (35)$$

$$X = \int_0^{\alpha}(Z-1)\,\frac{d\alpha}{\alpha} = -\ln(1-b\alpha) - \frac{a}{b\gamma^{3/2}}\ln(1+b\alpha) \quad ; \qquad (36)$$

$$X_{\gamma} = \int_0^{\alpha} Z_{\gamma}\,\frac{d\alpha}{\alpha} = \left(\frac{\partial X}{\partial\gamma}\right)_{\alpha} = \frac{3a}{2b\gamma^{5/2}}\ln(1+b\alpha) \quad ; \qquad (37)$$

$$X_{\gamma\gamma} = \int_0^{\alpha} Z_{\gamma\gamma}\,\frac{d\alpha}{\alpha} = \left(\frac{\partial X_{\gamma}}{\partial\gamma}\right)_{\alpha} = -\frac{15a}{4b\gamma^{7/2}}\ln(1+b\alpha) \quad . \qquad (38)$$

At the critical point, where $\alpha=1$, $\beta=1$, $\gamma=1$:

$$(Z_{\alpha})_{c.p.} = -Z_c = -1/3 \quad ; \qquad (39)$$

$$(Z_{\gamma})_{c.p.} = 1.52681 \quad ; \qquad (40)$$

$$(Z_{\alpha\alpha})_{c.p.} = 2Z_c = 2/3 \quad ; \qquad (41)$$

$$(Z_{\alpha\gamma})_{c.p.} = 1.21183 \quad ; \qquad (42)$$

$$(Z_{\gamma\gamma})_{c.p.} = -3.81702 \quad ; \qquad (43)$$

$$(Z_{\alpha\alpha\alpha})_{c.p.} = 0.144907 \quad ; \qquad (44)$$

$$(Z_{\alpha\alpha\gamma})_{c.p.} = -1/2 \quad ; \qquad (45)$$

$$(Z_{\alpha\alpha\alpha\alpha})_{c.p.} = 0.663623 \quad ; \tag{46}$$

$$(X)_{c.p.} = -0.838988 \quad ; \tag{47}$$

$$(X_{\gamma})_{c.p.} = 1.70998 \quad ; \tag{48}$$

$$(X_{\gamma\gamma})_{c.p.} = -4.27495 \quad . \tag{49}$$

THE PRESSURE-TEMPERATURE COEFFICIENT AT CONSTANT DENSITY

From equations 25 and 26, we find that

$$\beta_{\gamma} = \left(\frac{\partial\beta}{\partial\gamma}\right)_{\alpha} = \frac{\alpha}{Z_c}[Z + \gamma Z_{\gamma}] = 3\alpha\left[\frac{1}{(1-b\alpha)} + \frac{a\alpha}{2\gamma^{3/2}(1+b\alpha)}\right] \quad . \tag{50}$$

At the critical point,

$$(\beta_{\gamma})_{c.p.} = 1 + \frac{(Z_{\gamma})_{c.p.}}{Z_c} = 5.58043 \quad . \tag{51}$$

THE PRESSURE-DENSITY COEFFICIENT AT CONSTANT TEMPERATURE

From equations 25 and 26, we have

$$\beta_{\alpha} = \left(\frac{\partial\beta}{\partial\alpha}\right)_{\gamma} = \frac{\gamma}{Z_c}[Z + \alpha Z_{\alpha}] = 3\gamma\left[\frac{1}{(1-b\alpha)^2} - \frac{a\alpha(2+b\alpha)}{\gamma^{3/2}(1+b\alpha)^2}\right] \quad . \tag{52}$$

At the critical point,

$$(\beta_{\alpha})_{c.p.} = 0 \quad . \tag{53}$$

THE DENSITY-TEMPERATURE COEFFICIENT AT CONSTANT PRESSURE

The density-temperature coefficient at constant pressure may be expressed as

$$\left(\frac{\partial\alpha}{\partial\gamma}\right)_{\beta} = -\frac{(\partial\beta/\partial\gamma)_{\alpha}}{(\partial\beta/\partial\alpha)_{\gamma}} = -\frac{\beta_{\gamma}}{\beta_{\alpha}} \quad . \tag{54}$$

Substituting equations 50 and 52 in equation 54,

$$\left(\frac{\partial\alpha}{\partial\gamma}\right)_\beta = -\frac{\alpha[Z+\gamma Z_\gamma]}{\gamma[Z+\alpha Z_\alpha]} = -\frac{\alpha(1-b\alpha)(1+b\alpha)[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]}{\gamma[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]} \, . \tag{55}$$

At the critical point,

$$\left[\left(\frac{\partial\alpha}{\partial\gamma}\right)_\beta\right]_{c.p.} = \pm\,\infty \quad , \tag{56}$$

where the limiting value of this function depends on the path chosen in approaching the critical point. Obviously, there is only one limiting value when the critical point is approached from physically realizable states. This statement applies to other functions in this report which we indicate are multivalued at the critical point.

## THE SECOND DERIVATIVE OF THE PRESSURE WITH REGARD TO THE TEMPERATURE AT CONSTANT DENSITY

Differentiating equation 50,

$$\beta_{\gamma\gamma} = \left(\frac{\partial\beta_\gamma}{\partial\gamma}\right)_\alpha = \frac{\alpha}{Z_c}\,[2Z_\gamma + \gamma Z_{\gamma\gamma}] = -\frac{9a\alpha^2}{4\gamma^{5/2}(1+b\alpha)} \quad . \tag{57}$$

At the critical point,

$$(\beta_{\gamma\gamma})_{c.p.} = \frac{[2Z_\gamma + Z_{\gamma\gamma}]_{c.p.}}{Z_c} = -2.29021 \quad . \tag{58}$$

## THE COEFFICIENT OF COMPRESSIBILITY AT CONSTANT TEMPERATURE

In a previous Bureau report (5), it is shown that the reduced coefficient of compressibility at constant temperature is expressible as

$$-\frac{P_c}{V}\left(\frac{\partial V}{\partial P}\right)_T = \frac{1}{\alpha}\left(\frac{\partial\alpha}{\partial\beta}\right)_\gamma = \frac{1}{\alpha\beta_\alpha} \quad , \tag{59}$$

where V is the molal volume. Substituting equation 52 in equation 59,

$$\frac{1}{\alpha}\left(\frac{\partial \alpha}{\partial \beta}\right)_{\gamma} = \frac{Z_c}{\alpha\gamma[Z+\alpha Z_{\alpha}]} = \frac{\gamma^{1/2}(1-b\alpha)^2(1+b\alpha)^2}{3\alpha[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]} \quad . \tag{60}$$

At the critical point,

$$\left[\frac{1}{\alpha}\left(\frac{\partial \alpha}{\partial \beta}\right)_{\gamma}\right]_{c.p.} = \pm\infty \quad . \tag{61}$$

## THE COEFFICIENT OF THERMAL EXPANSION AT CONSTANT PRESSURE

The reduced coefficient of thermal expansion at constant pressure in terms of the reduced variables of density and temperature is given by (5)

$$\frac{T_c}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{\alpha}\left(\frac{\partial \alpha}{\partial \gamma}\right)_{\beta} \quad . \tag{62}$$

Substituting equation 55 in equation 62,

$$-\frac{1}{\alpha}\left(\frac{\partial \alpha}{\partial \gamma}\right)_{\beta} = \frac{[Z+\gamma Z_{\gamma}]}{\gamma[Z+\alpha Z_{\alpha}]} = \frac{(1-b\alpha)(1+b\alpha)[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]}{\gamma[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]} \quad . \tag{63}$$

At the critical point,

$$\left[-\frac{1}{\alpha}\left(\frac{\partial \alpha}{\partial \gamma}\right)_{\beta}\right]_{c.p.} = \pm\infty \quad . \tag{64}$$

## PARTIAL DERIVATIVES OF X WHICH CAN BE CALCULATED FROM PREVIOUSLY CALCULATED QUANTITIES

The following partial derivatives of X, which will be used later in this report, can be calculated from previously calculated quantities:

$$X_{\alpha} = \left(\frac{\partial X}{\partial \alpha}\right)_{\gamma} = \frac{Z-1}{\alpha} = \left[\frac{b}{(1-b\alpha)} - \frac{a}{\gamma^{3/2}(1+b\alpha)}\right] \quad ; \qquad (65)$$

$$X_{\alpha\alpha} = \left(\frac{\partial X_{\alpha}}{\partial \alpha}\right)_{\gamma} = \frac{\alpha Z_{\alpha} - (Z-1)}{\alpha^{2}} = b\left[\frac{b}{(1-b\alpha)^{2}} + \frac{a}{\gamma^{3/2}(1+b\alpha)^{2}}\right] \quad ; \qquad (66)$$

$$X_{\alpha\gamma} = \left(\frac{\partial X_{\alpha}}{\partial \gamma}\right)_{\alpha} = \left(\frac{\partial X_{\gamma}}{\partial \alpha}\right)_{\gamma} = \frac{Z_{\gamma}}{\alpha} = \frac{3a}{2\gamma^{5/2}(1+b\alpha)} \quad . \qquad (67)$$

## THE FUGACITY FUNCTION

In a previous report (2), it is shown that

$$\ln \frac{f}{P} = (Z-1) - \ln Z + X \quad , \qquad (68)$$

where f is the fugacity. Substituting equations 24 and 36 in equation 68,

$$\ln \frac{f}{P} = \alpha\left[\frac{b}{(1-b\alpha)} - \frac{a}{\gamma^{3/2}(1+b\alpha)}\right] + \left[1 - \frac{a}{b\gamma^{3/2}}\right] \ln(1+b\alpha)$$

$$- \ln\left[(1+b\alpha) - \frac{a\alpha(1-b\alpha)}{\gamma^{3/2}}\right] \quad . \qquad (69)$$

At the critical point,

$$\ln \frac{f_c}{P_c} = \left[Z_c - 1 - \ln Z_c + (X)_{c.p.}\right] = -0.407043 \quad . \qquad (70)$$

Taking the antilogarithm,

$$\frac{f_c}{P_c} = 0.665615 \quad . \qquad (71)$$

## THE RELATIVE GIBBS FREE ENERGY

From the definition of fugacity (18),

$$(G-G^{0}) = RT \ln f \quad , \qquad (72)$$

where $(G-G^{o})$ is the relative molal Gibbs free energy. In terms of the reduced variables of density and temperature, the relative Gibbs free energy is expressible as

$$\frac{(G-G^{o}) - RT \ln P_c}{RT_c} = \gamma\left[\ln \frac{\alpha\gamma}{Z_c} + (Z-1) + X\right] \quad . \tag{73}$$

Substituting equations 23, 24, and 36 in equation 73,

$$\frac{(G-G^{o}) - RT \ln P_c}{RT_c} = \gamma\left[\ln \frac{3\alpha\gamma}{(1-b\alpha)} + \frac{b\alpha}{(1-b\alpha)} - \frac{a\alpha}{\gamma^{3/2}(1+b\alpha)} - \frac{a}{b\gamma^{3/2}} \ln(1+b\alpha)\right] \quad . \tag{74}$$

At the critical point,

$$\frac{(G_c - G_c^{o}) - RT_c \ln P_c}{RT_c} = [-\ln Z_c + Z_c - 1 + (X)_{c.p.}] = -0.407043 \quad . \tag{75}$$

## THE RELATIVE HELMHOLTZ FREE ENERGY

From thermodynamics (18),

$$(A-A^{o}) = (G-G^{o}) - RT\,(Z-1) \quad , \tag{76}$$

where $(A-A^{o})$ is the relative molal Helmholtz free energy. Substituting equations 68 and 72 in equation 76,

$$\begin{aligned}(A-A^{o}) &= RT\left[\ln \frac{P}{Z} + X\right] \quad , \\ &= RT\left[\ln \rho RT + X\right] \quad .\end{aligned} \tag{77}$$

In terms of the reduced variables of density and temperature, the relative Helmholtz free energy is expressible as

$$\frac{(A-A^{o}) - RT \ln P_c}{RT_c} = \gamma\left[\ln \frac{\alpha\gamma}{Z_c} + X\right] \quad . \tag{78}$$

Substituting equations 23 and 36 in equation 78,

$$\frac{(A-A^{O}) - RT\ \ln P_c}{RT_c} = \gamma\left[\ln \frac{3\alpha\gamma}{(1-b\alpha)} - \frac{a}{b\gamma^{3/2}} \ln(1+b\alpha)\right] . \tag{79}$$

At the critical point,

$$\frac{(A_c - A_c^{O}) - RT_c\ \ln P_c}{RT_c} = \left[-\ln Z_c + (X)_{c.p.}\right] = 0.259623 . \tag{80}$$

## THE RELATIVE ENTROPY

In a previous report (2), it is shown that

$$\frac{(S-S^{O}) + R\ \ln P_c}{R} = -\left[\ln \frac{\alpha\gamma}{Z_c} + X + \gamma X_{\gamma}\right] , \tag{81}$$

where S is the molal entropy at $\alpha$ and $\gamma$, $S^{O}$ is the molal entropy of the hypothetical fluid in the ideal state at unit pressure, and $(S-S^{O})$ is the relative molal entropy.

Substituting equations 23, 36, and 37 in equation 81,

$$\frac{(S-S^{O}) + R\ \ln P_c}{R} = -\left[\ln \frac{3\alpha\gamma}{(1-b\alpha)} + \frac{a}{2b\gamma^{3/2}} \ln(1+b\alpha)\right] . \tag{82}$$

At the critical point,

$$\frac{(S_c - S_c^{O}) + R\ \ln P_c}{R} = \left[\ln Z_c - (X+X_{\gamma})_{c.p.}\right] = -1.96960 . \tag{83}$$

## THE RELATIVE INTERNAL ENERGY

In a previous Bureau report (2), it is shown that

$$\frac{(E-E^{O})}{RT_c} = -\gamma^2 X_{\gamma} , \tag{84}$$

where E is the molal internal energy at $\alpha$ and $\gamma$, $E^{O}$ is the molal internal energy at zero density at the same temperature $\gamma$, and $(E-E^{O})$ is the relative molal internal energy.

Substituting equation 37 in equation 84,

$$\frac{(E-E^{o})}{RT_c} = -\frac{3a}{2b\gamma^{1/2}} \ln(1+b\alpha) \quad . \qquad (85)$$

At the critical point,

$$\frac{(E_c-E_c^{o})}{RT_c} = -(X_\gamma)_{c.p.} = -1.70998 \quad . \qquad (86)$$

THE RELATIVE HEAT CAPACITY AT CONSTANT VOLUME

In a previous report (2), we find

$$\frac{(C_V-C_V^{o})}{R} = -\gamma(2X_\gamma + \gamma X_{\gamma\gamma}) \quad , \qquad (87)$$

where $C_V$ is the molal heat capacity at constant volume at $\alpha$ and $\gamma$, $C_V^{o}$ is the molal heat capacity at constant volume at zero pressure or density, and $(C_V-C_V^{o})$ is the relative molal heat capacity at constant volume.

Substituting equations 37 and 38 in equation 87,

$$\frac{(C_V-C_V^{o})}{R} = \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha) \quad . \qquad (88)$$

At the critical point,

$$\left[\frac{C_V-C_V^{o}}{R}\right]_{c.p.} = -(2X_\gamma + X_{\gamma\gamma})_{c.p.} = 0.854990 \quad . \qquad (89)$$

THE RELATIVE HEAT CONTENT OR THE RELATIVE ENTHALPY

It has been shown in a previous report (2) that

$$\frac{L}{RT_c} = \frac{(H-H^{o})}{RT_c} = \gamma[(Z-1) - \gamma X_\gamma] \quad , \qquad (90)$$

where H is the molal enthalpy at $\alpha$ and $\gamma$, $H^{o}$ is the molal enthalpy

at zero pressure at temperature $\gamma$, and $L = (H-H^{o})$ is the relative molal enthalpy or the relative molal heat content.

Substituting equations 24 and 37 in equation 90,

$$\frac{L}{RT_c} = \frac{b\alpha\gamma}{(1-b\alpha)} - \frac{a[2b\alpha + 3(1+b\alpha)\ ln\ (1+b\alpha)]}{2b\gamma^{1/2}(1+b\alpha)} \quad . \tag{91}$$

At the critical point,

$$\frac{L_c}{RT_c} = [Z_c - 1 - (X_\gamma)_{c.p.}] = -2.37664 \quad . \tag{92}$$

THE RELATIVE HEAT CAPACITY AT CONSTANT PRESSURE

In a previous report (2), it is shown that

$$\frac{J}{R} = \frac{(C_P - C_P^{o})}{R} = \frac{1}{RT_c}\left[L_\gamma + L_\alpha(\partial\alpha/\partial\gamma)_\beta\right] \ ,$$

$$= \frac{1}{RT_c}\left[L_\gamma - L_\alpha \frac{\beta_\gamma}{\beta_\alpha}\right] \ , \tag{93}$$

where $C_P$ is the molal heat capacity at constant pressure at $\alpha$ and $\gamma$, $C_P^{o}$ is the corresponding quantity at the same temperature $\gamma$ but for $\alpha$=0, and $J = (C_P - C_P^{o})$ is the relative molal heat capacity at constant pressure.

From equation 90, one may obtain

$$\frac{L_\gamma}{RT_c} = \frac{1}{RT_c}\left(\frac{\partial L}{\partial\gamma}\right)_\alpha = (Z-1) + \gamma Z_\gamma - \gamma(2X_\gamma + \gamma X_{\gamma\gamma}) \tag{94}$$

or, substituting equation 87 in equation 94,

$$\frac{L_\gamma}{RT_c} = \frac{(C_V - C_V^{o})}{R} + (Z-1) + \gamma Z_\gamma \quad . \tag{95}$$

Also, from equation 90 we find that

$$\frac{L_\alpha}{RT_c} = \frac{1}{RT_c}\left(\frac{\partial L}{\partial \alpha}\right)_\gamma = \gamma(Z_\alpha - \gamma X_{\alpha\gamma}) \tag{96}$$

or, substituting equation 67 in equation 96,

$$\frac{L_\alpha}{RT_c} = \frac{\gamma}{\alpha}\left[\alpha Z_\alpha - \gamma Z_\gamma\right] \ . \tag{97}$$

Substituting equations 95 and 97 in equation 93,

$$\frac{J}{R} = \frac{(C_P - C_P^O)}{R} = \frac{(C_V - C_V^O)}{R} - 1 + (Z + \gamma Z_\gamma) - \frac{\gamma\beta_\gamma}{\alpha\beta_\alpha}(\alpha Z_\alpha - \gamma Z_\gamma) \ . \tag{98}$$

But from equation 50, we find that

$$Z + \gamma Z_\gamma = \frac{Z_c \beta_\gamma}{\alpha} \ . \tag{99}$$

From equation 52, we find that

$$\alpha Z_\alpha = \frac{Z_c \beta_\alpha}{\gamma} - Z \ . \tag{100}$$

From equation 99, it follows that

$$-\gamma Z_\gamma = Z - \frac{Z_c \beta_\gamma}{\alpha} \ . \tag{101}$$

Adding equations 100 and 101,

$$(\alpha Z_\alpha - \gamma Z_\gamma) = \frac{Z_c}{\alpha\gamma}(\alpha\beta_\alpha - \gamma\beta_\gamma) \ . \tag{102}$$

Substituting equations 99 and 102 in equation 98,

$$\frac{J}{R} = \frac{(C_P - C_P^O)}{R} = \frac{(C_V - C_V^O)}{R} - 1 + \frac{Z_c \beta_\gamma}{\alpha}\left\{1 - \frac{(\alpha\beta_\alpha - \gamma\beta_\gamma)}{\alpha\beta_\alpha}\right\} ,$$

$$= \frac{(C_V - C_V^O)}{R} - 1 + \frac{\gamma Z_c \beta_\gamma^2}{\alpha^2 \beta_\alpha} \ . \tag{103}$$

Substituting equations 23, 50, 52, and 88 in equation 103,

$$\frac{J}{R} = \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha) - 1 + \frac{[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]^2}{\gamma^{3/2}[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]} \quad . \qquad (104)$$

At the critical point,

$$\left(\frac{J}{R}\right)_{c.p.} = \left[\frac{C_V - C_V^0}{R}\right]_{c.p.} + \frac{Z_c\,(\beta_\gamma)^2_{c.p.}}{(\beta_\alpha)_{c.p.}} - 1 = \pm\,\infty \quad . \qquad (105)$$

## THE SECOND DERIVATIVE OF THE CHEMICAL POTENTIAL OR GIBBS FREE ENERGY WITH REGARD TO THE TEMPERATURE AT CONSTANT DENSITY

In a previous report (2), we find that

$$\frac{T\left(\frac{\partial^2 G}{\partial T^2}\right)_\rho + C_V^0}{R} = \gamma(2X_\gamma + \gamma X_{\gamma\gamma}) + \gamma(2Z_\gamma + \gamma Z_{\gamma\gamma}) \quad . \qquad (106)$$

From equation 57, it follows that

$$(2Z_\gamma + \gamma Z_{\gamma\gamma}) = \frac{Z_c}{\alpha}\,\beta_{\gamma\gamma} \quad . \qquad (107)$$

Substituting equation 107 in equation 106,

$$\frac{T\left(\frac{\partial^2 G}{\partial T^2}\right)_\rho + C_V^0}{R} = \gamma(2X_\gamma + \gamma X_{\gamma\gamma}) + \frac{Z_c\gamma}{\alpha}\,\beta_{\gamma\gamma} \quad . \qquad (108)$$

Substituting equations 23, 37, 38, and 57 in equation 108,

$$\frac{T\left(\frac{\partial^2 G}{\partial T^2}\right)_\rho + C_V^0}{R} = -\,\frac{3a}{4\gamma^{3/2}}\left[\frac{\alpha}{(1+b\alpha)} + \frac{1}{b}\ln(1+b\alpha)\right] \quad . \qquad (109)$$

At the critical point, from equation 108

$$\left[\frac{T\left(\frac{\partial^2 G}{\partial T^2}\right)_\rho + C_V^0}{R}\right]_{c.p.} = \left[2(X_\gamma)_{c.p.} + (X_{\gamma\gamma})_{c.p.} + Z_c\,(\beta_{\gamma\gamma})_{c.p.}\right] = -1.61839 \quad . \qquad (110)$$

## THE DIFFERENCE BETWEEN THE HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME

In a previous report (2), we find

$$\frac{C_P - C_V}{R} = \frac{(Z + \gamma Z_\gamma)^2}{(Z + \alpha Z_\alpha)} \quad . \tag{111}$$

From equation 52, we find that

$$(Z + \alpha Z_\alpha) = \frac{Z_c \beta_\alpha}{\gamma} \quad . \tag{112}$$

Substituting equations 99 and 112 in equation 111,

$$\frac{C_P - C_V}{R} = \frac{Z_c^2 \beta_\gamma^2 \gamma}{\alpha^2 Z_c \beta_\alpha} = \frac{\gamma Z_c \beta_\gamma^2}{\alpha^2 \beta_\alpha} \quad . \tag{113}$$

Substituting equations 23, 50, and 52 in equation 113,

$$\frac{C_P - C_V}{R} = \frac{[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]^2}{\gamma^{3/2}[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]} \quad . \tag{114}$$

At the critical point,

$$\left[\frac{C_P - C_V}{R}\right]_{c.P.} = \frac{Z_c (\beta_\gamma)^2_{c.P.}}{(\beta_\alpha)_{c.P.}} = \pm\infty \quad . \tag{115}$$

## THE RATIO OF THE HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME

Multiplying equation 113 by $R/C_V$ ,

$$\frac{C_P}{C_V} - 1 = \frac{\gamma Z_c \beta_\gamma^2}{\alpha^2 \beta_\alpha (C_V/R)} \quad , \tag{116}$$

which, from equations 88, 113, and 114, can be written as

$$\frac{C_P}{C_V} = 1 + \frac{[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]^2}{\gamma^{3/2}\left[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)\right]\left[\frac{C_V^0}{R} + \frac{3a}{4b\gamma^{3/2}}\ln(1+b\alpha)\right]} \quad . \tag{117}$$

At the critical point,

$$\left(\frac{C_P}{C_V}\right)_{c.P.} = \frac{Z_c(\beta_\gamma)^2_{c.P.}}{[\beta_\alpha(C_V/R)]_{c.P.}} = \pm\infty \quad . \tag{118}$$

## THE VELOCITY OF SOUND

In a previous Bureau report (5), it is shown that

$$\frac{Ma^2}{RT_c} = \frac{C_P}{C_V} Z_c \beta_\alpha \quad , \tag{119}$$

where M is the molecular weight and $a$ is the velocity of sound.

Substituting equation 116 in equation 119,

$$\frac{Ma^2}{RT_c} = Z_c\beta_\alpha + \frac{\gamma Z_c^2 \beta_\gamma^2}{\alpha^2 (C_V/R)} \quad . \tag{120}$$

Substituting equations 23, 50, 52, and 88 in equation 120,

$$\frac{Ma^2}{RT_c} = \frac{[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]}{\gamma^{1/2}(1-b\alpha)^2(1+b\alpha)^2} + \frac{[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]^2}{\gamma^2(1-b\alpha)^2(1+b\alpha)^2\left[\frac{C_V^0}{R} + \frac{3a}{4b\gamma^{3/2}}\ln(1+b\alpha)\right]} \quad . \tag{121}$$

At the critical point,

$$\frac{Ma_c^2}{RT_c} = \frac{Z_c^2(\beta_\gamma)^2_{c.P.}}{(C_V/R)_{c.P.}} = \left[0.289005(C_V^0/R) + 0.247097\right]^{-1} \quad . \tag{122}$$

## THE TEMPERATURE-PRESSURE COEFFICIENT AT CONSTANT ENTHALPY OR THE JOULE-THOMSON COEFFICIENT

From a previous report (2), it can be shown that the Joule-Thomson coefficient

$$\mu = \left(\frac{\partial T}{\partial P}\right)_H \tag{123}$$

can be expressed, in reduced units, as

$$\frac{\mu P_c}{T_c} = \left(\frac{\partial \gamma}{\partial \beta}\right)_H = \frac{\gamma[\alpha Z_\alpha - \gamma Z_\gamma]}{\gamma[\alpha Z_\alpha - \gamma Z_\gamma]\beta_\gamma - \alpha\left[\frac{C_V}{R} + (Z + \gamma Z_\gamma)\right]\beta_\alpha} \quad . \tag{124}$$

Equation 124 can also be written as

$$\left(\frac{\partial \gamma}{\partial \beta}\right)_H = - \frac{Z_c(\alpha\beta_\alpha - \gamma\beta_\gamma)}{\alpha^2\beta_\alpha(C_P/R)} \quad . \tag{125}$$

Substituting equations 23, 50, and 52 in equation 125,

$$\left(\frac{\partial \gamma}{\partial \beta}\right)_H = - \frac{1}{3(C_P/R)} \frac{[\gamma^{3/2} b(1+b\alpha)^2 - (a/2)(1-b\alpha)^2(5+3b\alpha)]}{[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]} \quad , \tag{126}$$

where, from equations 88, 113, and 114,

$$\frac{C_P}{R} = \frac{C_V}{R} + \frac{\gamma Z_c \beta_\gamma^2}{\alpha^2\beta_\alpha} \quad ,$$

$$= \frac{C_V^0}{R} + \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha) + \frac{[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]^2}{\gamma^{3/2}[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]} \quad . \tag{127}$$

Equation 127 is to be used in equation 126.

At the critical point, we have from equation 124, since $(\beta_\alpha)_{c.p.} = 0$,

$$\left[\left(\frac{\partial \gamma}{\partial \beta}\right)_H\right]_{c.p.} = \frac{1}{(\beta_\gamma)_{c.p.}} = 0.179197 \quad . \tag{128}$$

THE PRESSURE-TEMPERATURE COEFFICIENT AT CONSTANT ENTROPY

In a previous report (2), we find

$$\frac{T_c}{P_c}\left(\frac{\partial P}{\partial T}\right)_S = \left(\frac{\partial \beta}{\partial \gamma}\right)_S = \beta_\gamma + \frac{\alpha^2 C_V \beta_\alpha}{Z_c R \gamma \beta_\gamma} \quad . \tag{129}$$

Substituting equations 23, 50, 52, and 88 in equation 129,

$$\left(\frac{\partial\beta}{\partial\gamma}\right)_S = \frac{3\alpha\left[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)\right]}{\gamma^{3/2}(1-b\alpha)(1+b\alpha)}$$

$$+ \frac{3\alpha\left[\frac{C_V^0}{R} + \frac{3a}{4b\gamma^{3/2}}\ln(1+b\alpha)\right]\left[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)\right]}{(1-b\alpha)(1+b\alpha)\left[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)\right]} \quad . \quad (130)$$

At the critical point, we have from equation 129, since $(\beta_\alpha)_{c.p.} = 0$,

$$\left[\frac{T_c}{P_c}\left(\frac{\partial P}{\partial T}\right)_S\right]_{c.p.} = \left[\left(\frac{\partial\beta}{\partial\gamma}\right)_S\right]_{c.p.} = (\beta_\gamma)_{c.p.} = 5.58043 \quad . \quad (131)$$

THE VOLUME-TEMPERATURE COEFFICIENT AT CONSTANT ENTROPY

From mathematics,

$$\left(\frac{\partial V}{\partial T}\right)_S = -\frac{(\partial S/\partial T)_V}{(\partial S/\partial V)_T} \quad . \quad (132)$$

But (18)

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial P}{\partial T}\right)_\rho = P_T \quad ; \quad (133)$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{C_V}{T} \quad . \quad (134)$$

Substituting equations 133 and 134 in equation 132,

$$\left(\frac{\partial V}{\partial T}\right)_S = -\frac{C_V}{TP_T} \quad (135)$$

or, in reduced units,

$$\rho_c T_c \left(\frac{\partial V}{\partial T}\right)_S = \left(\frac{\partial(1/\alpha)}{\partial\gamma}\right)_S = -\frac{(C_V/R)}{Z_c\gamma\beta_\gamma} \quad . \quad (136)$$

Substituting equations 23, 50, and 88 in equation 136,

$$\rho_c T_c \left(\frac{\partial V}{\partial T}\right)_S = - \frac{\gamma^{1/2}(1-b\alpha)(1+b\alpha)\left[\frac{C_V^0}{R} + \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha)\right]}{\alpha[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]} \quad . \tag{137}$$

At the critical point,

$$\left[\rho_c T_c \left(\frac{\partial V}{\partial T}\right)_S\right]_{c.p.} = - \left[\frac{(C_V/R)}{Z_c \beta_\gamma}\right]_{c.p.} = - \left[0.537592(C_V^0/R) + 0.459636\right] \quad . \tag{138}$$

## THE PRESSURE-VOLUME COEFFICIENT AT CONSTANT ENTROPY

From mathematics,

$$\left(\frac{\partial P}{\partial V}\right)_S = \frac{(\partial P/\partial T)_S}{(\partial V/\partial T)_S} \tag{139}$$

or, in reduced units,

$$\frac{1}{\rho_c P_c}\left(\frac{\partial P}{\partial V}\right)_S = \left(\frac{\partial \beta}{\partial(1/\alpha)}\right)_S = \left(\frac{\partial \beta}{\partial \gamma}\right)_S \Bigg/ \left(\frac{\partial(1/\alpha)}{\partial \gamma}\right)_S \quad . \tag{140}$$

Substituting equations 129 and 136 in equation 140,

$$\frac{1}{\rho_c P_c}\left(\frac{\partial P}{\partial V}\right)_S = - \alpha^2 \beta_\alpha - \frac{Z_c \gamma \beta_\gamma^2}{(C_V/R)} \quad . \tag{141}$$

Substituting equations 23, 50, 52, and 88 in equation 141,

$$\frac{1}{\rho_c P_c}\left(\frac{\partial P}{\partial V}\right)_S = - \frac{3\alpha^2[\gamma^{3/2}(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]}{\gamma^{1/2}(1+b\alpha)^2(1-b\alpha)^2} - \frac{3\alpha^2[\gamma^{3/2}(1+b\alpha) + (a\alpha/2)(1-b\alpha)]^2}{\gamma^2(1-b\alpha)^2(1+b\alpha)^2\left[\frac{C_V^0}{R} + \frac{3a}{4b\gamma^{3/2}}\ln(1+b\alpha)\right]} \quad . \tag{142}$$

At the critical point,

$$\left[\frac{1}{\rho_c P_c}\left(\frac{\partial P}{\partial V}\right)_S\right]_{c.p.} = -\frac{Z_c(\beta_\gamma)^2_{c.p.}}{(C_V/R)_{c.p.}} = -\frac{1}{Z_c}\left(\frac{M a_c^2}{RT_c}\right)$$

$$= -[0.096335(C_V^0/R) + 0.0823657]^{-1} \quad . \quad (143)$$

## THE SECOND VIRIAL COEFFICIENT

In a previous Bureau report (2), the reduced second virial coefficient, $B_r$ , is defined as

$$B_r = B\rho_c = (Z_\alpha)_{\alpha=0} \quad . \quad (144)$$

Thus, from equation 28, evaluated at $\alpha$=0,

$$B_r = (b - a\gamma^{-3/2}) = 0.259921 - 1.282441\ \gamma^{-3/2} \quad . \quad (145)$$

For the critical isotherm,

$$B_r = -1.022519 \quad . \quad (146)$$

## THE BOYLE TEMPERATURE

If we define the Boyle reduced temperature, $\gamma_{Boyle}$, as the reduced temperature where the second virial coefficient is zero, then $\gamma_{Boyle}$ is given by the relation, for a Redlich-Kwong fluid,

$$b - \frac{a}{\gamma_{Boyle}^{3/2}} = 0 \quad (147)$$

or, with equations 21, 22, and 147,

$$\gamma_{Boyle} = (a/b)^{2/3} = 2.89821 \quad . \quad (148)$$

## THE THIRD VIRIAL COEFFICIENT

In a previous Bureau report (2), the reduced third virial coefficient, $C_r$ , is defined as

$$C_r = C\rho_c^2 = (Z_{\alpha\alpha})_{\alpha=0} \quad . \quad (149)$$

Therefore, from equation 30, evaluated at $\alpha=0$, and with a and b defined by equations 21 and 22,

$$C_r = 2b(b + a\gamma^{-3/2}) = 0.135118 + (2/3)\gamma^{-3/2} \quad . \tag{150}$$

For the critical isotherm,

$$C_r = 0.801784 \quad . \tag{151}$$

## THE JOULE-THOMSON INVERSION CURVE

The Joule-Thomson curve is defined as the curve for which $\mu=0$. From equation 124, it can be seen that $\mu$ will equal zero if

$$\alpha Z_\alpha = \gamma Z_\gamma \quad , \tag{152}$$

provided the quantity

$$\alpha\left[\frac{C_V}{R} + (Z + \gamma Z_\gamma)\right]\beta_\alpha \neq 0 \quad . \tag{153}$$

From equations 28 and 29, we see that $\mu=0$ when

$$\frac{b\alpha}{(1-b\alpha)^2} = \frac{a\alpha(5 + 3b\alpha)}{2\gamma^{3/2}(1+b\alpha)^2} \tag{154}$$

or,

$$(\gamma)_{\mu=0} = \left[\frac{a(1-b\alpha)^2(5+3b\alpha)}{2b(1+b\alpha)^2}\right]^{2/3} \quad . \tag{155}$$

Substituting equation 155 in equation 26 and collecting terms, we find that

$$(\beta)_{\mu=0} = \frac{3\alpha[5 - 4b\alpha - 5b^2\alpha^2]}{(1-b\alpha)^2(5+3b\alpha)}\left[\frac{a(1-b\alpha)^2(5+3b\alpha)}{2b(1+b\alpha)^2}\right]^{-2/3} \quad ,$$

$$= 3\alpha[5 - 4b\alpha - 5b^2\alpha^2]\left[\frac{a^2}{4b^2(1+b\alpha)^4(1-b\alpha)^2(5+3b\alpha)}\right]^{1/3} \quad . \tag{156}$$

The $\beta$-$\gamma$ inversion curve has a maximum when $\alpha = 1.40102$, where $\gamma = 2.20101$ and $\beta = 10.8177$.

## CONDITION FOR THERMODYNAMIC EQUILIBRIUM BETWEEN COEXISTING PHASES OF A ONE-COMPONENT SYSTEM

The equations given so far apply to any single phase. We now develop relationships applicable only when two phases are in thermodynamic equilibrium, assuming the effect of a gravitational field on the thermodynamic functions is negligible. The equations to be developed in this section of the report are thus only strictly applicable to the interface region between the two phases, which may be considered at the same level in a gravitational field.

The condition for thermodynamic equilibrium between coexisting phases of a one-component system is that the three quantities pressure, temperature, and chemical potential or Gibbs free energy must be the same in each phase.

In a previous report (2), it is shown that equality of pressure and temperature of the coexisting phases requires that

$$\alpha_1 Z_1 = \alpha_3 Z_3 \quad , \qquad (157)$$

where the subscript 1 refers to the gas phase and the subscript 3 refers to the liquid phase.

Substituting for Z as given by equation 24 in equation 157, we find

$$\alpha_1\left[\frac{1}{(1-b\alpha_1)} - \frac{a\alpha_1}{Y^{3/2}(1+b\alpha_1)}\right] = \alpha_3\left[\frac{1}{(1-b\alpha_3)} - \frac{a\alpha_3}{Y^{3/2}(1+b\alpha_3)}\right] \quad , \qquad (158)$$

where the subscript on $Y$ has been dropped since $Y_1 = Y_3 = Y$.

Equation 158 establishes the reduced temperature as a function of the reduced densities of the coexisting phases and can be solved for $\gamma$ once $\alpha_1$ and $\alpha_3$ are known. Then this value of $\gamma$ can be substituted into either equation 159

$$\beta = \beta_1 = 3\alpha_1 \gamma \left[ \frac{1}{(1-b\alpha_1)} - \frac{a\alpha_1}{\gamma^{3/2}(1+b\alpha_1)} \right] \tag{159}$$

or equation 160

$$\beta = \beta_3 = 3\alpha_3 \gamma \left[ \frac{1}{(1-b\alpha_3)} - \frac{a\alpha_3}{\gamma^{3/2}(1+b\alpha_3)} \right] \tag{160}$$

and the equilibrium reduced pressure $\beta_1 = \beta_3 = \beta$ calculated.

When the coexisting phases of a one-component system are in thermodynamic equilibrium, we are interested in the variables $\alpha_1$, $\alpha_3$, $\beta$, and $\gamma$. We then say we have a four-variable problem or system. Among these variables, we have equation 158, which insures that the reduced pressure and temperature will be the same in each phase, and either equation 159 or equation 160 for the reduced pressure.

With the Gibbs free energy being the same in each phase, we have from equation 73, for each phase

$$Z_1 + X_1 + \ln \alpha_1 = Z_3 + X_3 + \ln \alpha_3 \tag{161}$$

or, substituting equations 24 and 36 in equation 161, we have

$$(\alpha_1 - \alpha_3)\left[ \frac{b}{(1-b\alpha_1)(1-b\alpha_3)} - \frac{a}{\gamma^{3/2}(1+b\alpha_1)(1+b\alpha_3)} \right] = \ln \frac{\alpha_3(1-b\alpha_1)}{\alpha_1(1-b\alpha_3)} + \frac{a}{b\gamma^{3/2}} \ln \frac{(1+b\alpha_1)}{(1+b\alpha_3)} \, . \tag{162}$$

Equations 158, 162, and either equation 159 or equation 160

give a total of three equations between the four quantities $\alpha_1$, $\alpha_3$, $\beta$, and $\gamma$. With two phases in equilibrium, a single component system is said, therefore, to have one degree of freedom.

Equations 158 and 162 are the functional relationships between $\alpha_1$ and $\alpha_3$, when vapor and liquid coexist in equilibrium for a Redlich-Kwong fluid, as a function of $\gamma$. These two equations must be solved for the equilibrium reduced densities as a function of the reduced temperature before any of the other two-phase thermodynamic properties can be calculated.

## METHOD FOR OBTAINING THE SOLUTIONS OF THE FUNCTIONAL RELATIONSHIPS BETWEEN THE EQUILIBRIUM REDUCED DENSITIES AND THE EQUILIBRIUM REDUCED TEMPERATURE OF THE COEXISTING PHASES

Since the relationships given by equations 157 and 161, which assume the form as given by equations 158 and 162 for a Redlich-Kwong fluid, are nonlinear ones, it is necessary to solve them using an iterative technique. We did this in the following way. The functions

$$\theta = \theta(\alpha_1, \alpha_3, \gamma) = \alpha_1 Z_1 - \alpha_3 Z_3 = 0 \tag{163}$$

and

$$\varphi = \varphi(\alpha_1, \alpha_3, \gamma) = Z_1 - Z_3 + X_1 - X_3 + \ln \frac{\alpha_1}{\alpha_3} = 0 \tag{164}$$

were formed, a value for $\gamma$ was specified, and the problem then was to find values for $\alpha_1$ and $\alpha_3$ that reduced $\theta$ and $\varphi$ to zero.

The method of numerical calculation of the reduced densities of the coexisting phases involved expanding $\theta$ and $\varphi$ in a truncated Taylor's expansion about an approximate solution, denoted by an

asterisk, and solving the two equations

$$-\theta^* = \theta^*_{\alpha(1)}\Delta\alpha_1 + \theta^*_{\alpha(3)}\Delta\alpha_3 \qquad (165)$$

and

$$-\varphi^* = \varphi^*_{\alpha(1)}\Delta\alpha_1 + \varphi^*_{\alpha(3)}\Delta\alpha_3 \qquad (166)$$

for the corrections, $\Delta\alpha_1$ and $\Delta\alpha_3$, to be applied to the initial estimates assumed in the beginning. In equations 165 and 166,

$$\theta_{\alpha(1)} = \left(\frac{\partial\theta}{\partial\alpha_1}\right)_{\gamma,\alpha_3} = Z_1 + \alpha_1 Z_{\alpha(1)} = \frac{1}{(1-b\alpha_1)^2} - \frac{a\alpha_1(2+b\alpha_1)}{\gamma^{3/2}(1+b\alpha_1)^2} \ , \qquad (167)$$

$$\theta_{\alpha(3)} = \left(\frac{\partial\theta}{\partial\alpha_3}\right)_{\gamma,\alpha_1} = -Z_3 - \alpha_3 Z_{\alpha(3)} = -\frac{1}{(1-b\alpha_3)^2} + \frac{a\alpha_3(2+b\alpha_3)}{\gamma^{3/2}(1+b\alpha_3)^2} \ , \qquad (168)$$

$$\varphi_{\alpha(1)} = \left(\frac{\partial\varphi}{\partial\alpha_1}\right)_{\gamma,\alpha_3} = Z_{\alpha(1)} + X_{\alpha(1)} + \frac{1}{\alpha_1} \ ,$$

$$= Z_{\alpha(1)} + \frac{Z_1}{\alpha_1} = \frac{1}{\alpha_1(1-b\alpha_1)^2} - \frac{a(2+b\alpha_1)}{\gamma^{3/2}(1+b\alpha_1)^2} \ , \qquad (169)$$

and

$$\varphi_{\alpha(3)} = \left(\frac{\partial\varphi}{\partial\alpha_3}\right)_{\gamma,\alpha_1} = -Z_{\alpha(3)} - X_{\alpha(3)} - \frac{1}{\alpha_3} \ ,$$

$$= -Z_{\alpha(3)} - \frac{Z_3}{\alpha_3} = -\frac{1}{\alpha_3(1-b\alpha_3)^2} + \frac{a(2+b\alpha_3)}{\gamma^{3/2}(1+b\alpha_3)^2} \ . \qquad (170)$$

The corrections $\Delta\alpha_1$ and $\Delta\alpha_3$, are given by

$$\Delta\alpha_1 = \alpha_1 - (\alpha_1)^* \ ; \qquad (171)$$

$$\Delta\alpha_3 = \alpha_3 - (\alpha_3)^* \ . \qquad (172)$$

All calculations were repeated using $[\Delta\alpha_1 + (\alpha_1)^*]$ and $[\Delta\alpha_3 + (\alpha_3)^*]$ for the start of a new iteration. This iterative procedure was repeated until $\theta = \varphi = 0$ to within some predetermined small quantity.

The method outlined in this section was used to solve equations 163 and 164 for the equilibrium reduced densities of the coexisting phases for $0.10 \leq \gamma \leq 0.9999$. A parametric search technique (8) was used to generate initial estimates for $\alpha_1$ and $\alpha_3$.

## COEXISTENCE CURVE OF SATURATED LIQUID AND VAPOR DENSITIES AND VOLUMES AS A FUNCTION OF TEMPERATURE

Table 1 gives values, in reduced variables, of the densities of saturated liquid and vapor and of the rectilinear diameter[4], which is just the arithmetic average density of coexisting liquid and vapor, as a function of temperature. These data, illustrated in figure 1, clearly show that the slope of the rectilinear diameter, for a Redlich-Kwong fluid, is not zero at the critical point.

FIGURE 1. - Redlich-Kwong fluid, saturated densities as a function of temperature.

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[4] A very thorough discussion of rectilinear diameter is given in reference 20.

## TABLE 1. - REDLICH-KWONG FLUID, SATURATED DENSITIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\alpha = \rho/\rho_c$

| $\gamma$ | $\alpha_1$ | $\alpha_3$ | $\frac{\alpha_1 + \alpha_3}{2}$ |
|---|---|---|---|
| 0.00000E-99 | 0.00000E-00 | 3.84732E-00 | 1.92366E-00 |
| 1.00000E-01 | 3.19688E-45 | 3.79768E-00 | 1.89884E-00 |
| 1.50000E-01 | 4.35545E-24 | 3.75561E-00 | 1.87780E-00 |
| 2.00000E-01 | 2.55749E-15 | 3.70515E-00 | 1.85257E-00 |
| 2.50000E-01 | 9.57415E-11 | 3.64702E-00 | 1.82351E-00 |
| 3.00000E-01 | 5.00887E-08 | 3.58155E-00 | 1.79077E-00 |
| 3.50000E-01 | 2.87741E-06 | 3.50882E-00 | 1.75441E-00 |
| 4.00000E-01 | 4.64669E-05 | 3.42870E-00 | 1.71437E-00 |
| 4.50000E-01 | 3.42344E-04 | 3.34087E-00 | 1.67060E-00 |
| 5.00000E-01 | 1.51327E-03 | 3.24479E-00 | 1.62315E-00 |
| 5.50000E-01 | 4.74144E-03 | 3.13971E-00 | 1.57222E-00 |
| 6.00000E-01 | 1.17179E-02 | 3.02460E-00 | 1.51816E-00 |
| 6.50000E-01 | 2.45197E-02 | 2.89809E-00 | 1.46130E-00 |
| 7.00000E-01 | 4.55903E-02 | 2.75834E-00 | 1.40196E-00 |
| 7.50000E-01 | 7.79320E-02 | 2.60278E-00 | 1.34035E-00 |
| 8.00000E-01 | 1.25623E-01 | 2.42756E-00 | 1.27659E-00 |
| 8.50000E-01 | 1.95020E-01 | 2.22642E-00 | 1.21072E-00 |
| 9.00000E-01 | 2.97983E-01 | 1.98745E-00 | 1.14271E-00 |
| 9.50000E-01 | 4.64356E-01 | 1.68067E-00 | 1.07251E-00 |
| 9.52000E-01 | 4.73365E-01 | 1.66595E-00 | 1.06965E-00 |
| 9.54000E-01 | 4.82647E-01 | 1.65095E-00 | 1.06679E-00 |
| 9.56000E-01 | 4.92219E-01 | 1.63565E-00 | 1.06393E-00 |
| 9.58000E-01 | 5.02102E-01 | 1.62003E-00 | 1.06107E-00 |
| 9.60000E-01 | 5.12317E-01 | 1.60408E-00 | 1.05820E-00 |
| 9.62000E-01 | 5.22889E-01 | 1.58776E-00 | 1.05532E-00 |
| 9.64000E-01 | 5.33847E-01 | 1.57104E-00 | 1.05244E-00 |
| 9.66000E-01 | 5.45222E-01 | 1.55391E-00 | 1.04956E-00 |
| 9.68000E-01 | 5.57050E-01 | 1.53631E-00 | 1.04668E-00 |
| 9.70000E-01 | 5.69376E-01 | 1.51820E-00 | 1.04379E-00 |
| 9.72000E-01 | 5.82248E-01 | 1.49955E-00 | 1.04090E-00 |
| 9.74000E-01 | 5.95725E-01 | 1.48028E-00 | 1.03800E-00 |
| 9.76000E-01 | 6.09878E-01 | 1.46032E-00 | 1.03510E-00 |
| 9.78000E-01 | 6.24792E-01 | 1.43960E-00 | 1.03219E-00 |
| 9.80000E-01 | 6.40572E-01 | 1.41801E-00 | 1.02929E-00 |
| 9.82000E-01 | 6.57350E-01 | 1.39540E-00 | 1.02637E-00 |
| 9.84000E-01 | 6.75297E-01 | 1.37163E-00 | 1.02346E-00 |
| 9.86000E-01 | 6.94637E-01 | 1.34645E-00 | 1.02054E-00 |
| 9.88000E-01 | 7.15679E-01 | 1.31956E-00 | 1.01762E-00 |
| 9.90000E-01 | 7.38868E-01 | 1.29051E-00 | 1.01469E-00 |
| 9.92000E-01 | 7.64885E-01 | 1.25863E-00 | 1.01176E-00 |
| 9.94000E-01 | 7.94877E-01 | 1.22277E-00 | 1.00882E-00 |
| 9.96000E-01 | 8.31078E-01 | 1.18069E-00 | 1.00588E-00 |
| 9.98000E-01 | 8.79251E-01 | 1.12664E-00 | 1.00294E-00 |
| 9.99000E-01 | 9.13977E-01 | 1.08896E-00 | 1.00147E-00 |
| 1.00000E-00 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |

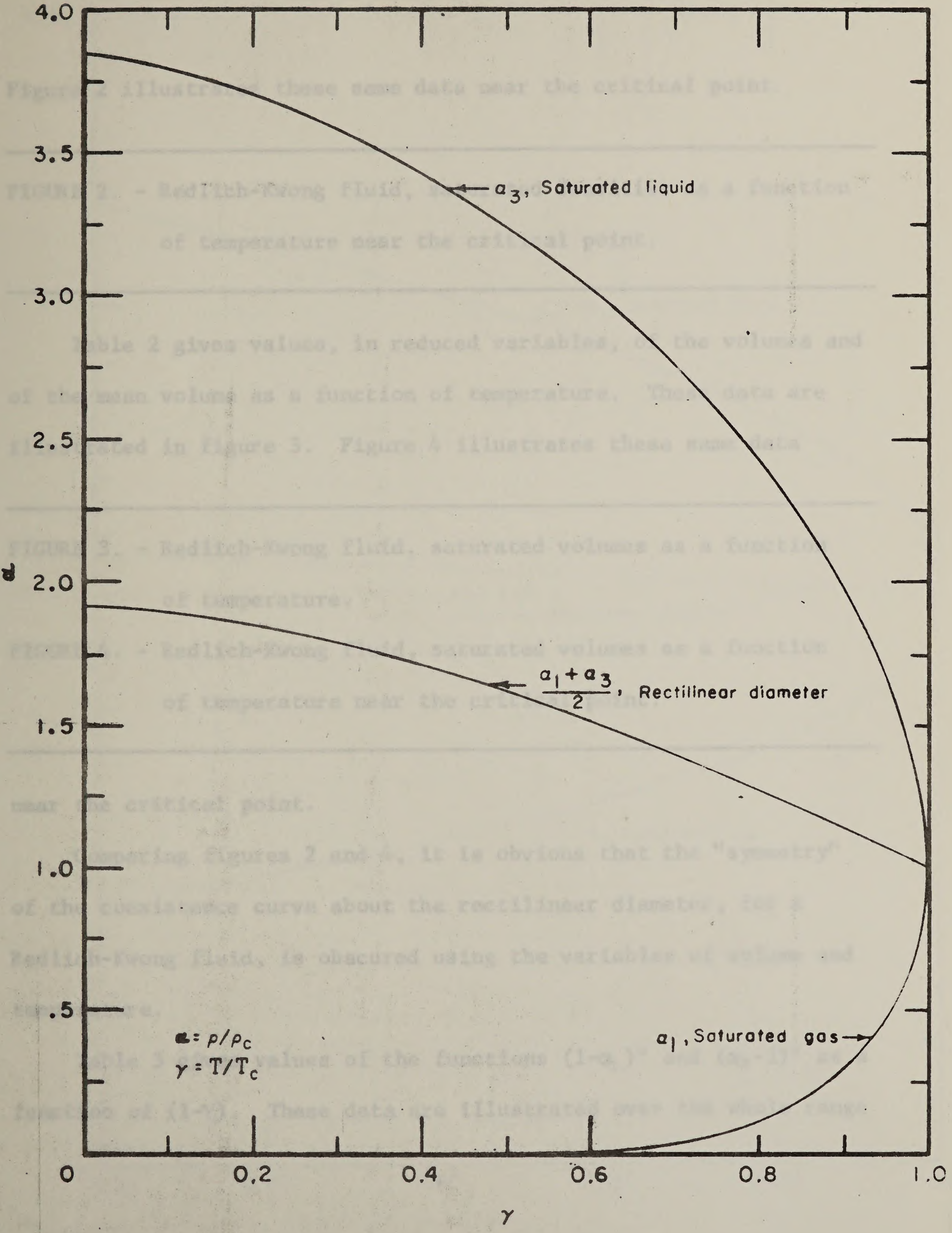

FIGURE I. – Redlich–Kwong Fluid, Saturated Densities as a Function of Temperature.

Figure 2 illustrates these same data near the critical point.

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FIGURE 2. - Redlich-Kwong fluid, saturated densities as a function of temperature near the critical point.

---

Table 2 gives values, in reduced variables, of the volumes and of the mean volume as a function of temperature. These data are illustrated in figure 3. Figure 4 illustrates these same data near the critical point.

---

FIGURE 3. - Redlich-Kwong fluid, saturated volumes as a function of temperature.

FIGURE 4. - Redlich-Kwong fluid, saturated volumes as a function of temperature near the critical point.

---

Comparing figures 2 and 4, it is obvious that the "symmetry" of the coexistence curve about the rectilinear diameter, for a Redlich-Kwong fluid, is obscured using the variables of volume and temperature.

Table 3 gives values of the functions $(1-\alpha_1)^2$ and $(\alpha_3-1)^2$ as a function of $(1-\gamma)$. These data are illustrated over the whole range

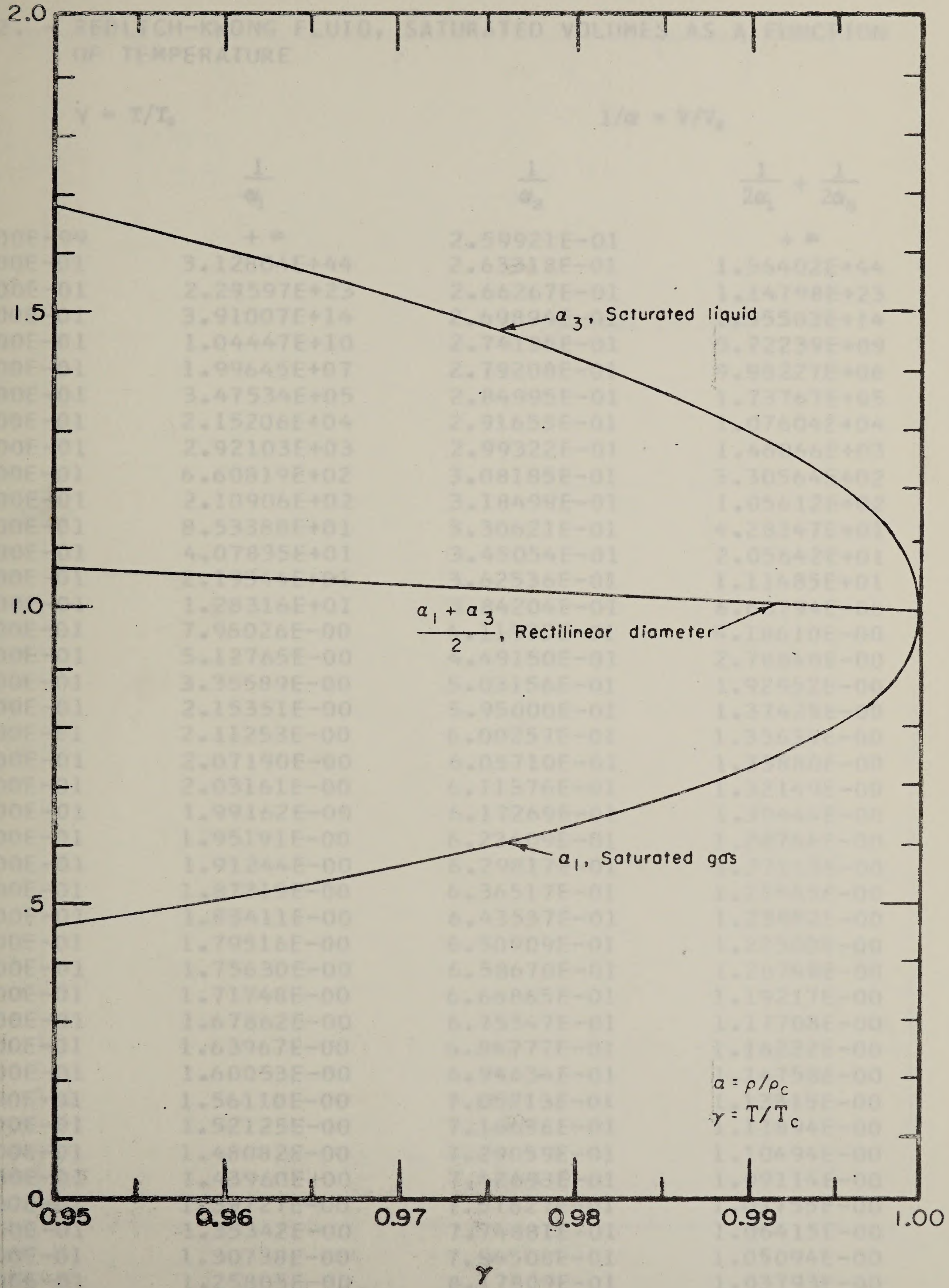

FIGURE 2.– Redlich–Kwong Fluid, Saturated Densities as a Function of Temperature Near the Critical Point.

## TABLE 2. - REDLICH-KWONG FLUID, SATURATED VOLUMES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad$ $1/\alpha = V/V_c$

| $\gamma$ | $\frac{1}{\alpha_1}$ | $\frac{1}{\alpha_3}$ | $\frac{1}{2\alpha_1} + \frac{1}{2\alpha_3}$ |
|---|---|---|---|
| 0.00000E-99 | + ∞ | 2.59921E-01 | + ∞ |
| 1.00000E-01 | 3.12804E+44 | 2.63318E-01 | 1.56402E+44 |
| 1.50000E-01 | 2.29597E+23 | 2.66267E-01 | 1.14798E+23 |
| 2.00000E-01 | 3.91007E+14 | 2.69894E-01 | 1.95503E+14 |
| 2.50000E-01 | 1.04447E+10 | 2.74195E-01 | 5.22239E+09 |
| 3.00000E-01 | 1.99645E+07 | 2.79208E-01 | 9.98227E+06 |
| 3.50000E-01 | 3.47534E+05 | 2.84995E-01 | 1.73767E+05 |
| 4.00000E-01 | 2.15206E+04 | 2.91655E-01 | 1.07604E+04 |
| 4.50000E-01 | 2.92103E+03 | 2.99322E-01 | 1.46066E+03 |
| 5.00000E-01 | 6.60819E+02 | 3.08185E-01 | 3.30564E+02 |
| 5.50000E-01 | 2.10906E+02 | 3.18499E-01 | 1.05612E+02 |
| 6.00000E-01 | 8.53388E+01 | 3.30621E-01 | 4.28347E+01 |
| 6.50000E-01 | 4.07835E+01 | 3.45054E-01 | 2.05642E+01 |
| 7.00000E-01 | 2.19344E+01 | 3.62536E-01 | 1.11485E+01 |
| 7.50000E-01 | 1.28316E+01 | 3.84204E-01 | 6.60794E-00 |
| 8.00000E-01 | 7.96026E-00 | 4.11935E-01 | 4.18610E-00 |
| 8.50000E-01 | 5.12765E-00 | 4.49150E-01 | 2.78840E-00 |
| 9.00000E-01 | 3.35589E-00 | 5.03156E-01 | 1.92952E-00 |
| 9.50000E-01 | 2.15351E-00 | 5.95000E-01 | 1.37425E-00 |
| 9.52000E-01 | 2.11253E-00 | 6.00257E-01 | 1.35639E-00 |
| 9.54000E-01 | 2.07190E-00 | 6.05710E-01 | 1.33880E-00 |
| 9.56000E-01 | 2.03161E-00 | 6.11376E-01 | 1.32149E-00 |
| 9.58000E-01 | 1.99162E-00 | 6.17269E-01 | 1.30444E-00 |
| 9.60000E-01 | 1.95191E-00 | 6.23409E-01 | 1.28766E-00 |
| 9.62000E-01 | 1.91244E-00 | 6.29817E-01 | 1.27113E-00 |
| 9.64000E-01 | 1.87319E-00 | 6.36517E-01 | 1.25485E-00 |
| 9.66000E-01 | 1.83411E-00 | 6.43537E-01 | 1.23882E-00 |
| 9.68000E-01 | 1.79516E-00 | 6.50909E-01 | 1.22303E-00 |
| 9.70000E-01 | 1.75630E-00 | 6.58670E-01 | 1.20748E-00 |
| 9.72000E-01 | 1.71748E-00 | 6.66865E-01 | 1.19217E-00 |
| 9.74000E-01 | 1.67862E-00 | 6.75547E-01 | 1.17708E-00 |
| 9.76000E-01 | 1.63967E-00 | 6.84777E-01 | 1.16222E-00 |
| 9.78000E-01 | 1.60053E-00 | 6.94634E-01 | 1.14758E-00 |
| 9.80000E-01 | 1.56110E-00 | 7.05213E-01 | 1.13315E-00 |
| 9.82000E-01 | 1.52125E-00 | 7.16636E-01 | 1.11894E-00 |
| 9.84000E-01 | 1.48082E-00 | 7.29059E-01 | 1.10494E-00 |
| 9.86000E-01 | 1.43960E-00 | 7.42693E-01 | 1.09114E-00 |
| 9.88000E-01 | 1.39727E-00 | 7.57827E-01 | 1.07755E-00 |
| 9.90000E-01 | 1.35342E-00 | 7.74881E-01 | 1.06415E-00 |
| 9.92000E-01 | 1.30738E-00 | 7.94508E-01 | 1.05094E-00 |
| 9.94000E-01 | 1.25805E-00 | 8.17809E-01 | 1.03793E-00 |
| 9.96000E-01 | 1.20325E-00 | 8.46955E-01 | 1.02510E-00 |
| 9.98000E-01 | 1.13733E-00 | 8.87593E-01 | 1.01246E-00 |
| 9.99000E-01 | 1.09411E-00 | 9.18299E-01 | 1.00620E-00 |
| 1.00000E-00 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |

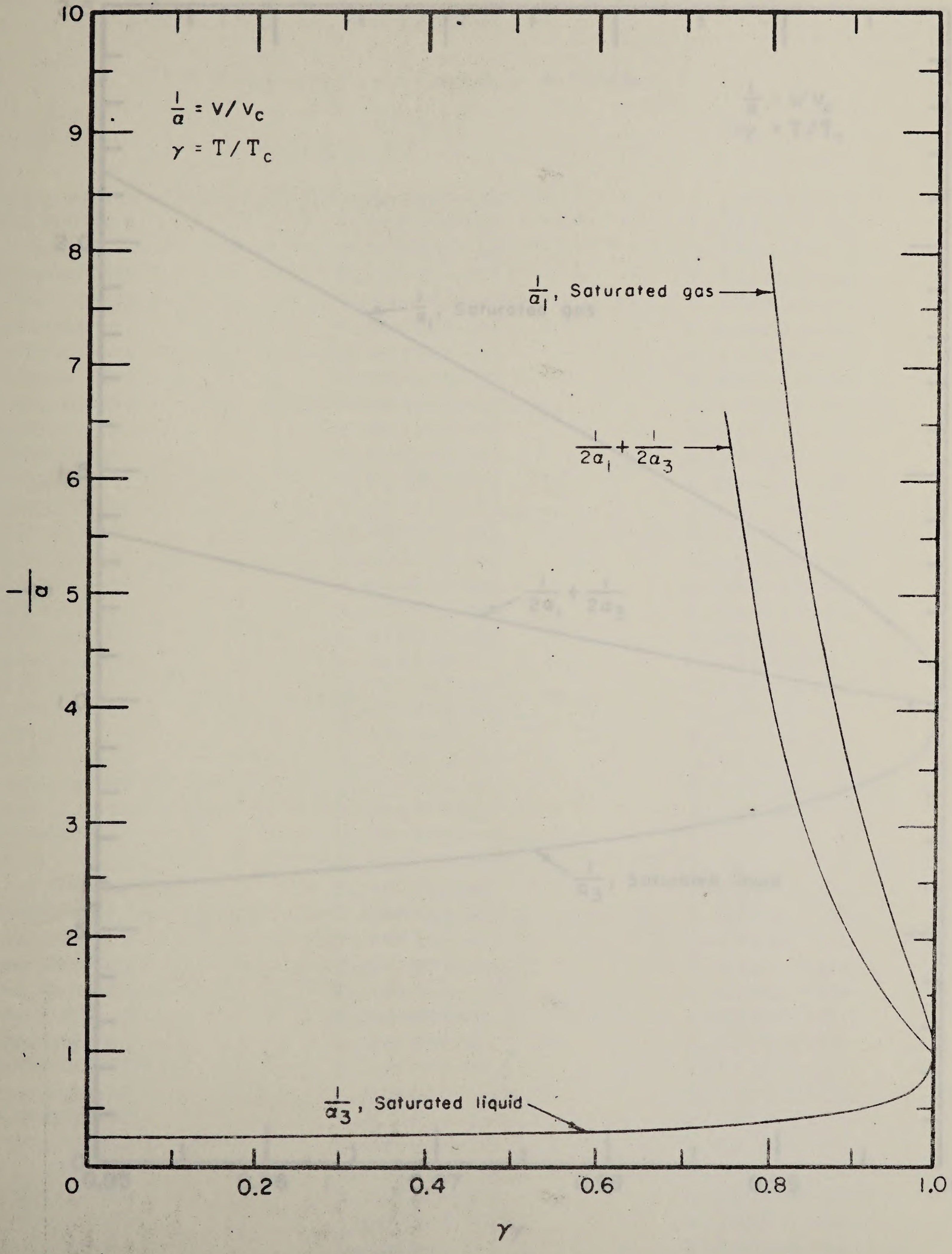

FIGURE 3.- Redlich-Kwong Fluid, Saturated Volumes as a Function of Temperature.

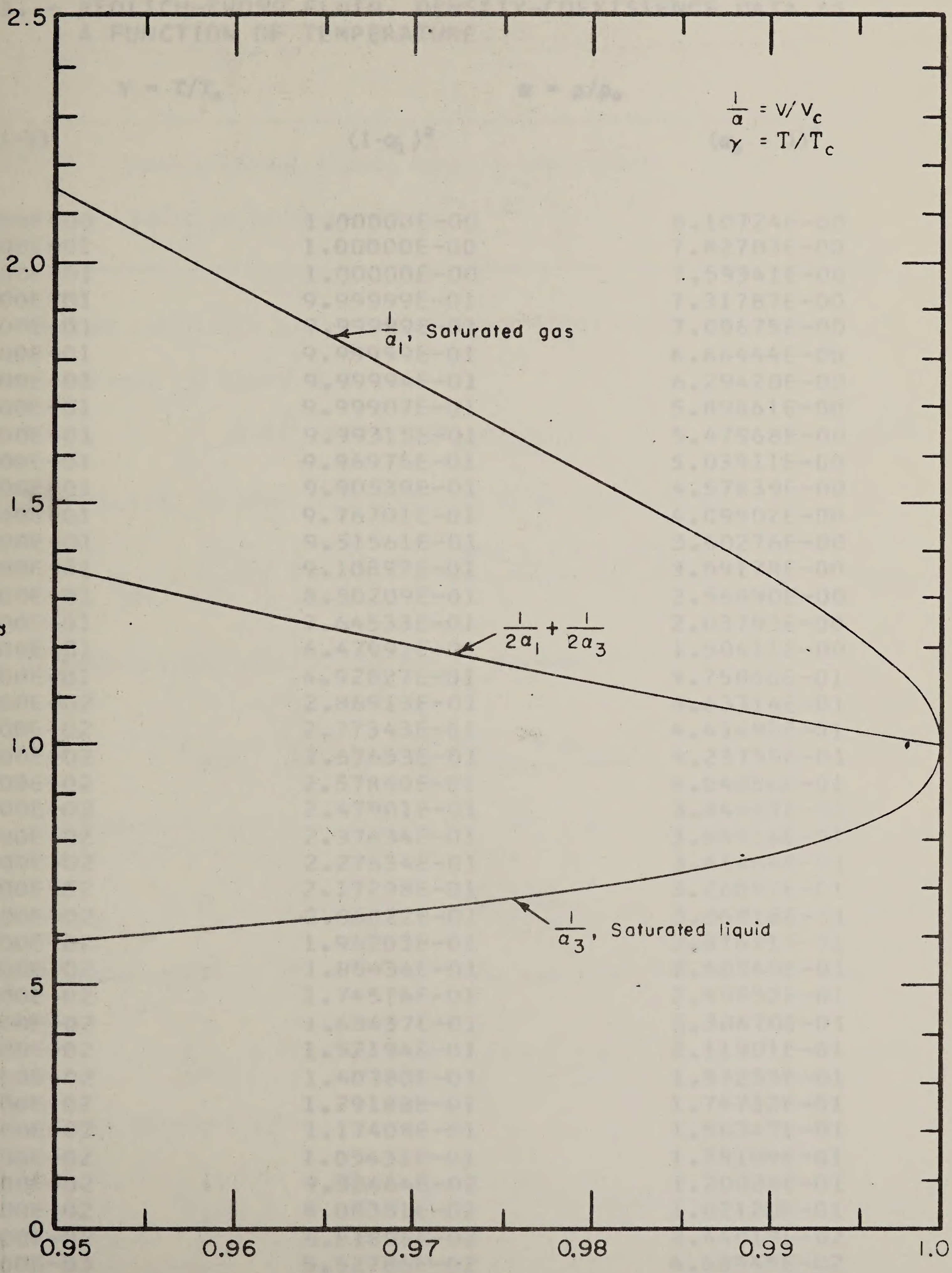

FIGURE 4. - Redlich - Kwong Fluid, Saturated Volumes as a Function of Temperature Near the Critical Point.

TABLE 3. - REDLICH-KWONG FLUID, DENSITY-COEXISTENCE DATA AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\alpha = \rho/\rho_c$

| $(1-\gamma)$ | $(1-\alpha_1)^2$ | $(\alpha_3 - 1)^2$ |
|---|---|---|
| 1.00000E-00 | 1.00000E-00 | 8.10724E-00 |
| 9.00000E-01 | 1.00000E-00 | 7.82703E-00 |
| 8.50000E-01 | 1.00000E-00 | 7.59341E-00 |
| 8.00000E-01 | 9.99999E-01 | 7.31787E-00 |
| 7.50000E-01 | 9.99999E-01 | 7.00675E-00 |
| 7.00000E-01 | 9.99999E-01 | 6.66444E-00 |
| 6.50000E-01 | 9.99994E-01 | 6.29420E-00 |
| 6.00000E-01 | 9.99907E-01 | 5.89861E-00 |
| 5.50000E-01 | 9.99315E-01 | 5.47968E-00 |
| 5.00000E-01 | 9.96975E-01 | 5.03911E-00 |
| 4.50000E-01 | 9.90539E-01 | 4.57839E-00 |
| 4.00000E-01 | 9.76701E-01 | 4.09902E-00 |
| 3.50000E-01 | 9.51561E-01 | 3.60276E-00 |
| 3.00000E-01 | 9.10897E-01 | 3.09178E-00 |
| 2.50000E-01 | 8.50209E-01 | 2.56890E-00 |
| 2.00000E-01 | 7.64533E-01 | 2.03793E-00 |
| 1.50000E-01 | 6.47991E-01 | 1.50411E-00 |
| 1.00000E-01 | 4.92827E-01 | 9.75066E-01 |
| 5.00000E-02 | 2.86913E-01 | 4.63314E-01 |
| 4.80000E-02 | 2.77343E-01 | 4.43492E-01 |
| 4.60000E-02 | 2.67653E-01 | 4.23739E-01 |
| 4.40000E-02 | 2.57840E-01 | 4.04056E-01 |
| 4.20000E-02 | 2.47901E-01 | 3.84447E-01 |
| 4.00000E-02 | 2.37834E-01 | 3.64916E-01 |
| 3.80000E-02 | 2.27634E-01 | 3.45464E-01 |
| 3.60000E-02 | 2.17298E-01 | 3.26097E-01 |
| 3.40000E-02 | 2.06822E-01 | 3.06818E-01 |
| 3.20000E-02 | 1.96203E-01 | 2.87631E-01 |
| 3.00000E-02 | 1.85436E-01 | 2.68540E-01 |
| 2.80000E-02 | 1.74516E-01 | 2.49552E-01 |
| 2.60000E-02 | 1.63437E-01 | 2.30670E-01 |
| 2.40000E-02 | 1.52194E-01 | 2.11901E-01 |
| 2.20000E-02 | 1.40780E-01 | 1.93253E-01 |
| 2.00000E-02 | 1.29188E-01 | 1.74732E-01 |
| 1.80000E-02 | 1.17408E-01 | 1.56347E-01 |
| 1.60000E-02 | 1.05431E-01 | 1.38109E-01 |
| 1.40000E-02 | 9.32464E-02 | 1.20028E-01 |
| 1.20000E-02 | 8.08381E-02 | 1.02120E-01 |
| 1.00000E-02 | 6.81896E-02 | 8.44013E-02 |
| 8.00000E-03 | 5.52786E-02 | 6.68945E-02 |
| 6.00000E-03 | 4.20751E-02 | 4.96300E-02 |
| 4.00000E-03 | 2.85344E-02 | 3.26524E-02 |
| 2.00000E-03 | 1.45802E-02 | 1.60381E-02 |
| 1.00000E-03 | 7.39984E-03 | 7.91564E-03 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

of variables in figure 5.

---

FIGURE 5. - Redlich-Kwong fluid, density-coexistence data as a function of temperature.

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For an analytical fluid sufficiently close to the critical point, Barieau (3) shows that

$$(\alpha-1)^2 = 6(1-\gamma)(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.} \quad . \qquad (173)$$

In equation 173, we have from equation 50 or equation 52

$$\beta_{\alpha\gamma} = \left(\frac{\partial\beta_\alpha}{\partial\gamma}\right)_\alpha = \left(\frac{\partial\beta_\gamma}{\partial\alpha}\right)_\gamma = \frac{(Z+\gamma Z_\gamma) + \alpha(Z_\alpha+\gamma Z_{\alpha\gamma})}{Z_c} \qquad (174)$$

and differentiating equation 52 twice, we have

$$\beta_{\alpha\alpha\alpha} = \left(\frac{\partial^2\beta_\alpha}{\partial\alpha^2}\right)_\gamma = \frac{\gamma}{Z_c}\,[3Z_{\alpha\alpha} + \alpha Z_{\alpha\alpha\alpha}] \quad . \qquad (175)$$

Substituting equations 23, 24, 28, 29, and 31 in equation 174,

$$\beta_{\alpha\gamma} = 3\left[\frac{1}{(1-b\alpha)^2} + \frac{a\alpha(2+b\alpha)}{2\gamma^{3/2}(1+b\alpha)^2}\right] \quad . \qquad (176)$$

Substituting equations 23, 30, and 33 in equation 175, we find that

$$\beta_{\alpha\alpha\alpha} = 18b\gamma\left[\frac{b}{(1-b\alpha)^4} + \frac{a}{\gamma^{3/2}(1+b\alpha)^4}\right] \quad . \qquad (177)$$

At the critical point,

$$(\beta_{\alpha\gamma})_{c.p.} = \frac{(Z_\gamma + Z_{\alpha\gamma})_{c.p.}}{Z_c} = 8.21592 \qquad (178)$$

and

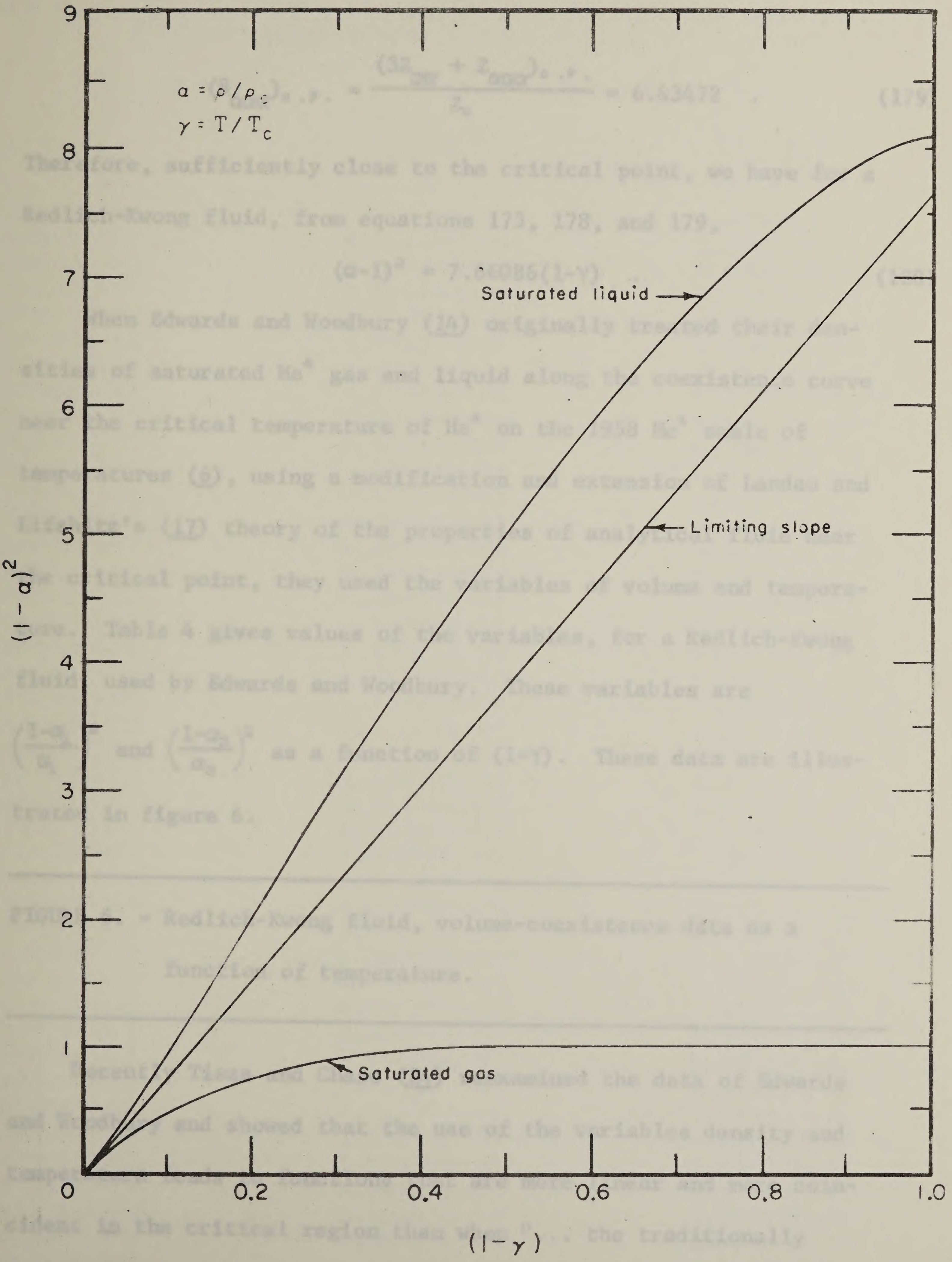

FIGURE 5.- Redlich-Kwong Fluid, Density-Coexistence Data as a Function of Temperature.

$$(\beta_{\alpha\alpha\alpha'})_{c.p.} = \frac{(3Z_{\alpha\alpha} + Z_{\alpha\alpha\alpha'})_{c.p.}}{Z_c} = 6.43472 \quad . \tag{179}$$

Therefore, sufficiently close to the critical point, we have for a Redlich-Kwong fluid, from equations 173, 178, and 179,

$$(\alpha-1)^2 = 7.66086(1-Y) \quad . \tag{180}$$

When Edwards and Woodbury (14) originally treated their densities of saturated $He^4$ gas and liquid along the coexistence curve near the critical temperature of $He^4$ on the 1958 $He^4$ scale of temperatures (6), using a modification and extension of Landau and Lifshitz's (17) theory of the properties of analytical fluid near the critical point, they used the variables of volume and temperature. Table 4 gives values of the variables, for a Redlich-Kwong fluid, used by Edwards and Woodbury. These variables are $\left(\frac{1-\alpha_1}{\alpha_1}\right)^2$ and $\left(\frac{1-\alpha_3}{\alpha_3}\right)^2$ as a function of (1-Y). These data are illustrated in figure 6.

---

FIGURE 6. - Redlich-Kwong fluid, volume-coexistence data as a function of temperature.

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Recently Tisza and Chase (24) reexamined the data of Edwards and Woodbury and showed that the use of the variables density and temperature leads to functions that are more linear and more coincident in the critical region than when "... the traditionally

TABLE 4. - REDLICH-KWONG FLUID, VOLUME-COEXISTENCE DATA AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\quad$ $1/\alpha = V/V_c$

| $(1-\gamma)$ | $\left[\frac{1 - \alpha_1}{\alpha_1}\right]^2$ | $\left[\frac{1 - \alpha_3}{\alpha_3}\right]^2$ |
|---|---|---|
| 1.00000E-00 | $+\infty$ | 5.47716E-01 |
| 9.00000E-01 | 9.78468E+88 | 5.42699E-01 |
| 8.50000E-01 | 5.27149E+46 | 5.38362E-01 |
| 8.00000E-01 | 1.52886E+29 | 5.33054E-01 |
| 7.50000E-01 | 1.09093E+20 | 5.26791E-01 |
| 7.00000E-01 | 3.98583E+14 | 5.19540E-01 |
| 6.50000E-01 | 1.20779E+11 | 5.11231E-01 |
| 6.00000E-01 | 4.63095E+08 | 5.01752E-01 |
| 5.50000E-01 | 8.52658E+06 | 4.90948E-01 |
| 5.00000E-01 | 4.35362E+05 | 4.78607E-01 |
| 4.50000E-01 | 4.40605E+04 | 4.64442E-01 |
| 4.00000E-01 | 7.11304E+03 | 4.48067E-01 |
| 3.50000E-01 | 1.58272E+03 | 4.28954E-01 |
| 3.00000E-01 | 4.38252E+02 | 4.06360E-01 |
| 2.50000E-01 | 1.39988E+02 | 3.79204E-01 |
| 2.00000E-01 | 4.84453E+01 | 3.45819E-01 |
| 1.50000E-01 | 1.70375E+01 | 3.03434E-01 |
| 1.00000E-01 | 5.55021E-00 | 2.46853E-01 |
| 5.00000E-02 | 1.33060E-00 | 1.64024E-01 |
| 4.80000E-02 | 1.23772E-00 | 1.59794E-01 |
| 4.60000E-02 | 1.14898E-00 | 1.55463E-01 |
| 4.40000E-02 | 1.06422E-00 | 1.51028E-01 |
| 4.20000E-02 | 9.83320E-01 | 1.46482E-01 |
| 4.00000E-02 | 9.06140E-01 | 1.41820E-01 |
| 3.80000E-02 | 8.32562E-01 | 1.37035E-01 |
| 3.60000E-02 | 7.62468E-01 | 1.32119E-01 |
| 3.40000E-02 | 6.95747E-01 | 1.27065E-01 |
| 3.20000E-02 | 6.32292E-01 | 1.21864E-01 |
| 3.00000E-02 | 5.72001E-01 | 1.16505E-01 |
| 2.80000E-02 | 5.14778E-01 | 1.10978E-01 |
| 2.60000E-02 | 4.60532E-01 | 1.05269E-01 |
| 2.40000E-02 | 4.09179E-01 | 9.93651E-02 |
| 2.20000E-02 | 3.60639E-01 | 9.32480E-02 |
| 2.00000E-02 | 3.14838E-01 | 8.68990E-02 |
| 1.80000E-02 | 2.71710E-01 | 8.02950E-02 |
| 1.60000E-02 | 2.31197E-01 | 7.34087E-02 |
| 1.40000E-02 | 1.93248E-01 | 6.62068E-02 |
| 1.20000E-02 | 1.57826E-01 | 5.86477E-02 |
| 1.00000E-02 | 1.24906E-01 | 5.06781E-02 |
| 8.00000E-03 | 9.44853E-02 | 4.22267E-02 |
| 6.00000E-03 | 6.65923E-02 | 3.31932E-02 |
| 4.00000E-03 | 4.13128E-02 | 2.34226E-02 |
| 2.00000E-03 | 1.88599E-02 | 1.26352E-02 |
| 1.00000E-03 | 8.85832E-03 | 6.67504E-03 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

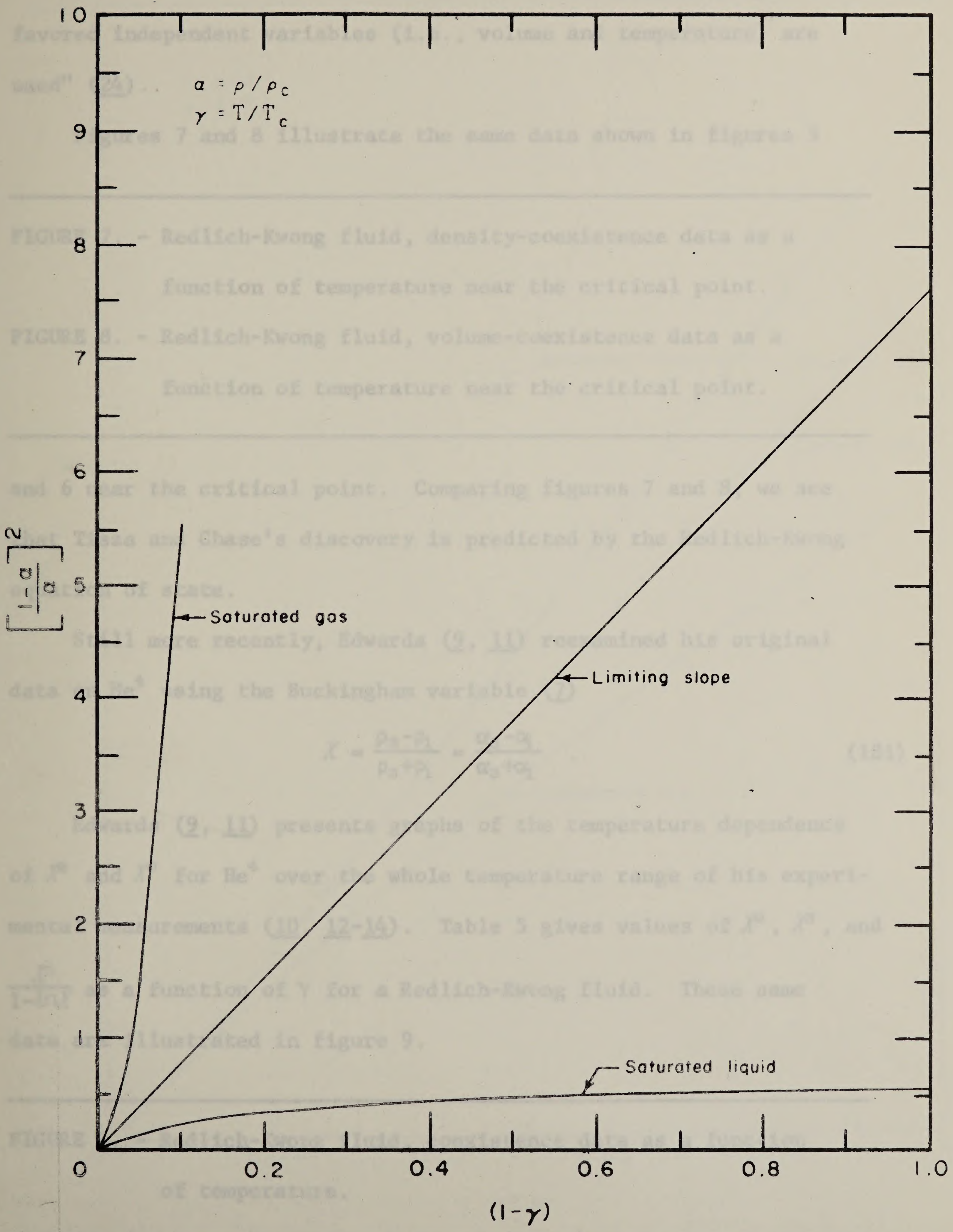

FIGURE 6. — Redlich-Kwong Fluid, Volume-Coexistence Data as a Function of Temperature.

favored independent variables (i.e., volume and temperature) are used" (24).

Figures 7 and 8 illustrate the same data shown in figures 5 and 6 near the critical point. Comparing figures 7 and 8, we see that Tisza and Chase's discovery is predicted by the Redlich-Kwong equation of state.

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FIGURE 7. - Redlich-Kwong fluid, density-coexistence data as a function of temperature near the critical point.

FIGURE 8. - Redlich-Kwong fluid, volume-coexistence data as a function of temperature near the critical point.

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Still more recently, Edwards (9, 11) reexamined his original data on $He^4$ using the Buckingham variable (7)

$$X = \frac{\rho_3-\rho_1}{\rho_3+\rho_1} = \frac{\alpha_3-\alpha_1}{\alpha_3+\alpha_1} \quad . \qquad (181)$$

Edwards (9, 11) presents graphs of the temperature dependence of $X^2$ and $X^3$ for $He^4$ over the whole temperature range of his experimental measurements (10, 12-14). Table 5 gives values of $X^2$, $X^3$, and $\frac{X^2}{1-\ln X}$ as a function of Y for a Redlich-Kwong fluid. These same data are illustrated in figure 9.

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FIGURE 9. - Redlich-Kwong fluid, coexistence data as a function of temperature.

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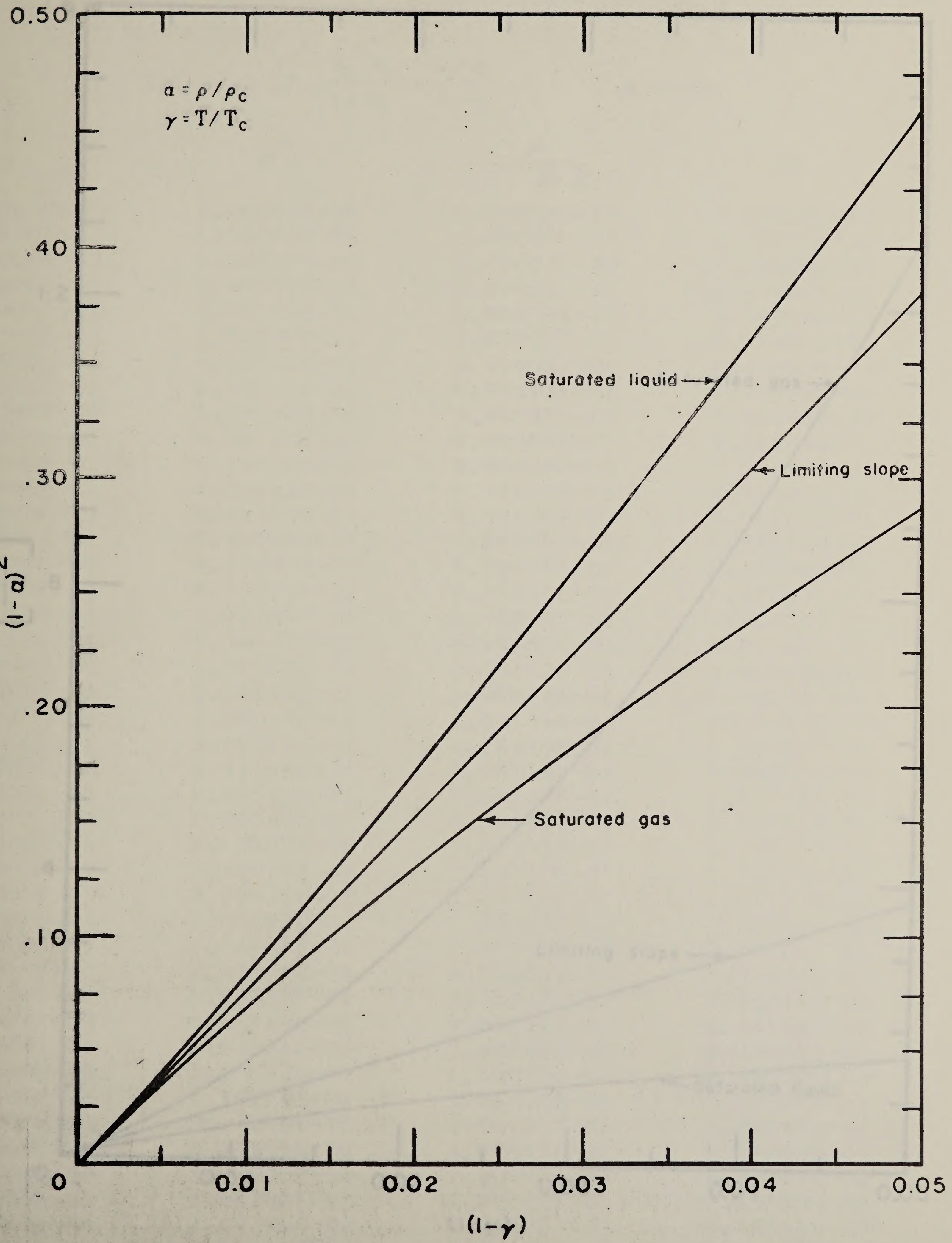

FIGURE 7. — Redlich-Kwong Fluid, Density-Coexistence Data as a Function of Temperature Near the Critical Point.

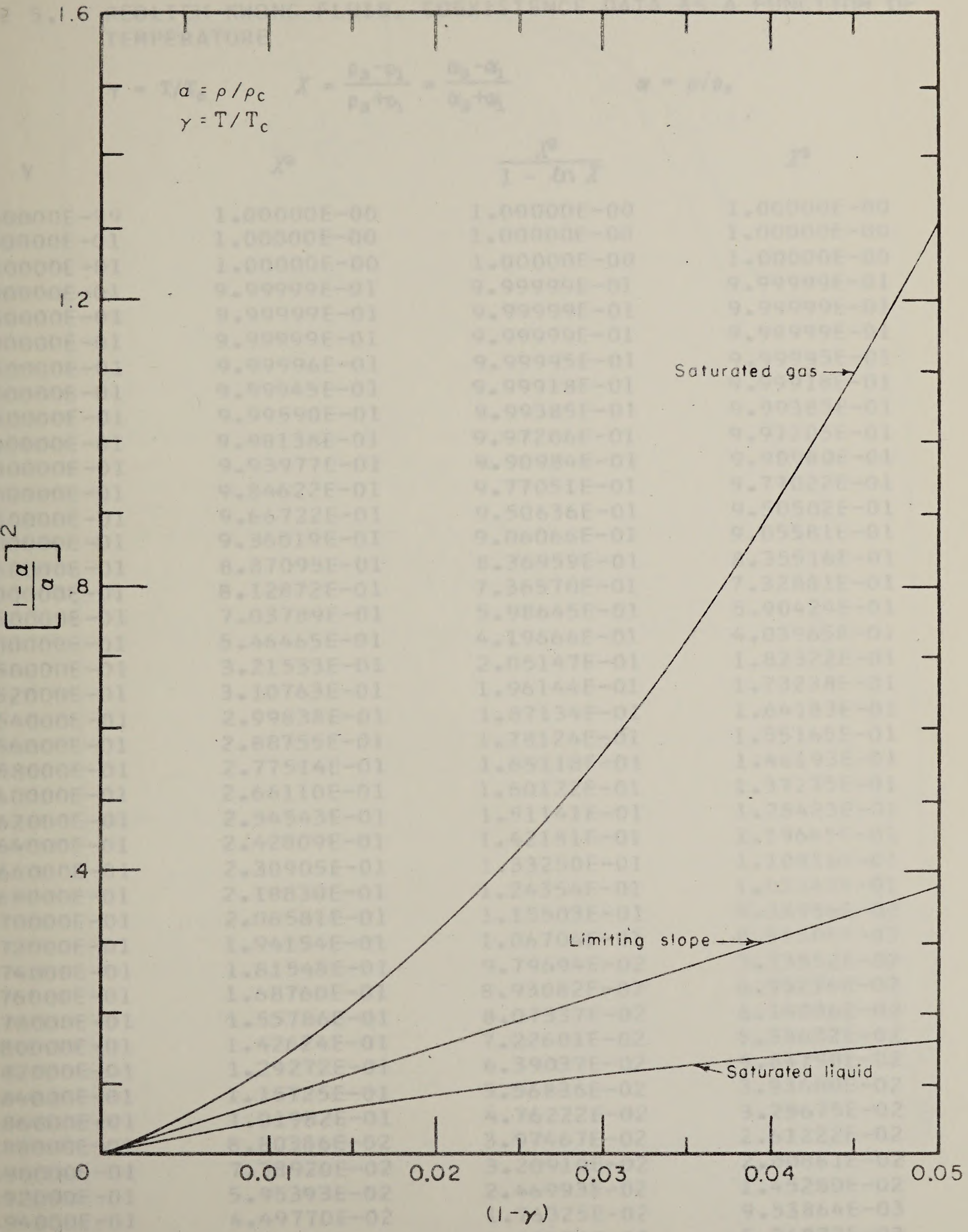

FIGURE 8.- Redlich-Kwong Fluid, Volume-Coexistence Data as a Function of Temperature Near the Critical Point.

TABLE 5. - REDLICH-KWONG FLUID, COEXISTENCE DATA AS A FUNCTION OF TEMPERATURE

$$Y = T/T_c \qquad X = \frac{\rho_3-\rho_1}{\rho_3+\rho_1} = \frac{\alpha_3-\alpha_1}{\alpha_3+\alpha_1} \qquad \alpha = \rho/\rho_c$$

| Y | $X^2$ | $\frac{X^2}{1 - \ln X}$ | $X^3$ |
|---|---|---|---|
| 0.00000E-99 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |
| 1.00000E-01 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |
| 1.50000E-01 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |
| 2.00000E-01 | 9.99999E-01 | 9.99999E-01 | 9.99999E-01 |
| 2.50000E-01 | 9.99999E-01 | 9.99999E-01 | 9.99999E-01 |
| 3.00000E-01 | 9.99999E-01 | 9.99999E-01 | 9.99999E-01 |
| 3.50000E-01 | 9.99996E-01 | 9.99995E-01 | 9.99995E-01 |
| 4.00000E-01 | 9.99945E-01 | 9.99918E-01 | 9.99918E-01 |
| 4.50000E-01 | 9.99590E-01 | 9.99385E-01 | 9.99385E-01 |
| 5.00000E-01 | 9.98136E-01 | 9.97206E-01 | 9.97205E-01 |
| 5.50000E-01 | 9.93977E-01 | 9.90984E-01 | 9.90980E-01 |
| 6.00000E-01 | 9.84622E-01 | 9.77051E-01 | 9.77022E-01 |
| 6.50000E-01 | 9.66722E-01 | 9.50636E-01 | 9.50502E-01 |
| 7.00000E-01 | 9.36019E-01 | 9.06066E-01 | 9.05581E-01 |
| 7.50000E-01 | 8.87095E-01 | 8.36959E-01 | 8.35516E-01 |
| 8.00000E-01 | 8.12872E-01 | 7.36570E-01 | 7.32881E-01 |
| 8.50000E-01 | 7.03789E-01 | 5.98645E-01 | 5.90424E-01 |
| 9.00000E-01 | 5.46465E-01 | 4.19666E-01 | 4.03965E-01 |
| 9.50000E-01 | 3.21533E-01 | 2.05147E-01 | 1.82322E-01 |
| 9.52000E-01 | 3.10763E-01 | 1.96144E-01 | 1.73238E-01 |
| 9.54000E-01 | 2.99838E-01 | 1.87134E-01 | 1.64183E-01 |
| 9.56000E-01 | 2.88755E-01 | 1.78124E-01 | 1.55165E-01 |
| 9.58000E-01 | 2.77514E-01 | 1.69118E-01 | 1.46193E-01 |
| 9.60000E-01 | 2.66110E-01 | 1.60122E-01 | 1.37275E-01 |
| 9.62000E-01 | 2.54543E-01 | 1.51141E-01 | 1.28423E-01 |
| 9.64000E-01 | 2.42809E-01 | 1.42181E-01 | 1.19645E-01 |
| 9.66000E-01 | 2.30905E-01 | 1.33250E-01 | 1.10956E-01 |
| 9.68000E-01 | 2.18830E-01 | 1.24354E-01 | 1.02367E-01 |
| 9.70000E-01 | 2.06581E-01 | 1.15503E-01 | 9.38936E-02 |
| 9.72000E-01 | 1.94154E-01 | 1.06704E-01 | 8.55504E-02 |
| 9.74000E-01 | 1.81548E-01 | 9.79694E-02 | 7.73552E-02 |
| 9.76000E-01 | 1.68760E-01 | 8.93082E-02 | 6.93274E-02 |
| 9.78000E-01 | 1.55786E-01 | 8.07337E-02 | 6.14886E-02 |
| 9.80000E-01 | 1.42624E-01 | 7.22601E-02 | 5.38632E-02 |
| 9.82000E-01 | 1.29272E-01 | 6.39037E-02 | 4.64790E-02 |
| 9.84000E-01 | 1.15725E-01 | 5.56836E-02 | 3.93680E-02 |
| 9.86000E-01 | 1.01982E-01 | 4.76222E-02 | 3.25675E-02 |
| 9.88000E-01 | 8.80386E-02 | 3.97467E-02 | 2.61222E-02 |
| 9.90000E-01 | 7.38920E-02 | 3.20910E-02 | 2.00861E-02 |
| 9.92000E-01 | 5.95393E-02 | 2.46993E-02 | 1.45280E-02 |
| 9.94000E-01 | 4.49770E-02 | 1.76325E-02 | 9.53864E-03 |
| 9.96000E-01 | 3.02019E-02 | 1.09828E-02 | 5.24872E-03 |
| 9.98000E-01 | 1.52107E-02 | 4.91800E-03 | 1.87597E-03 |
| 9.99000E-01 | 7.63305E-03 | 2.22043E-03 | 6.66879E-04 |
| 1.00000E-00 | 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

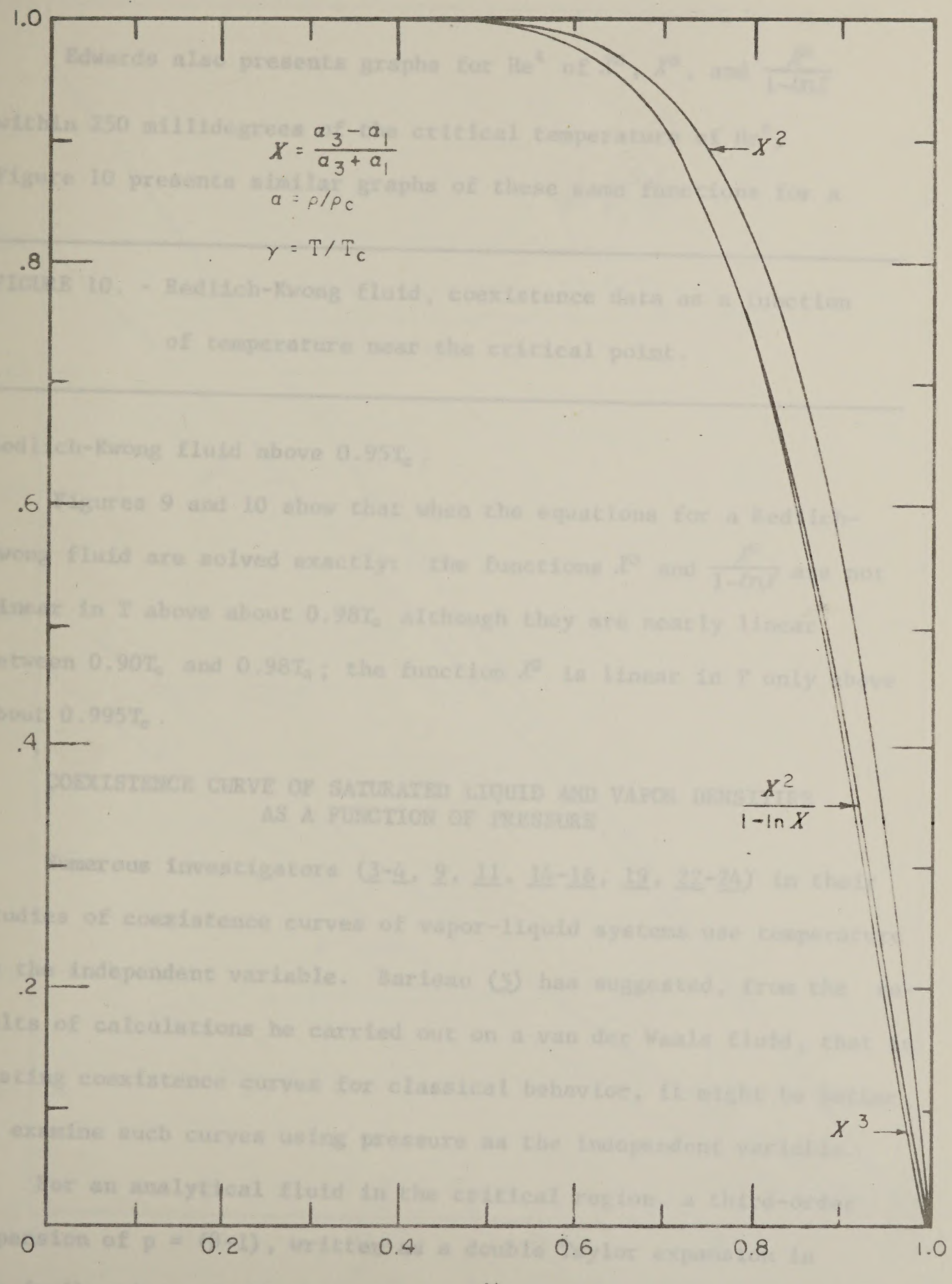

FIGURE 9. - Redlich-Kwong Fluid, Coexistence Data as a Function of Temperature.

Edwards also presents graphs for $He^4$ of $X^2$, $X^3$, and $\frac{X^2}{1-\ln X}$ within 250 millidegrees of the critical temperature of $He^4$. Figure 10 presents similar graphs of these same functions for a Redlich-Kwong fluid above $0.95T_c$.

---

FIGURE 10. - Redlich-Kwong fluid, coexistence data as a function of temperature near the critical point.

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Figures 9 and 10 show that when the equations for a Redlich-Kwong fluid are solved exactly: the functions $X^3$ and $\frac{X^2}{1-\ln X}$ are not linear in T above about $0.98T_c$ although they are nearly linear between $0.90T_c$ and $0.98T_c$; the function $X^2$ is linear in T only above about $0.995T_c$.

## COEXISTENCE CURVE OF SATURATED LIQUID AND VAPOR DENSITIES AS A FUNCTION OF PRESSURE

Numerous investigators (3-4, 9, 11, 14-16, 19, 22-24) in their studies of coexistence curves of vapor-liquid systems use temperature as the independent variable. Barieau (5) has suggested, from the results of calculations he carried out on a van der Waals fluid, that in testing coexistence curves for classical behavior, it might be better to examine such curves using pressure as the independent variable.

For an analytical fluid in the critical region, a third-order expansion of $p = (\beta - 1)$, written as a double Taylor expansion in $a = (\alpha - 1)$ and $t = (\gamma - 1)$ about its value at the critical point,

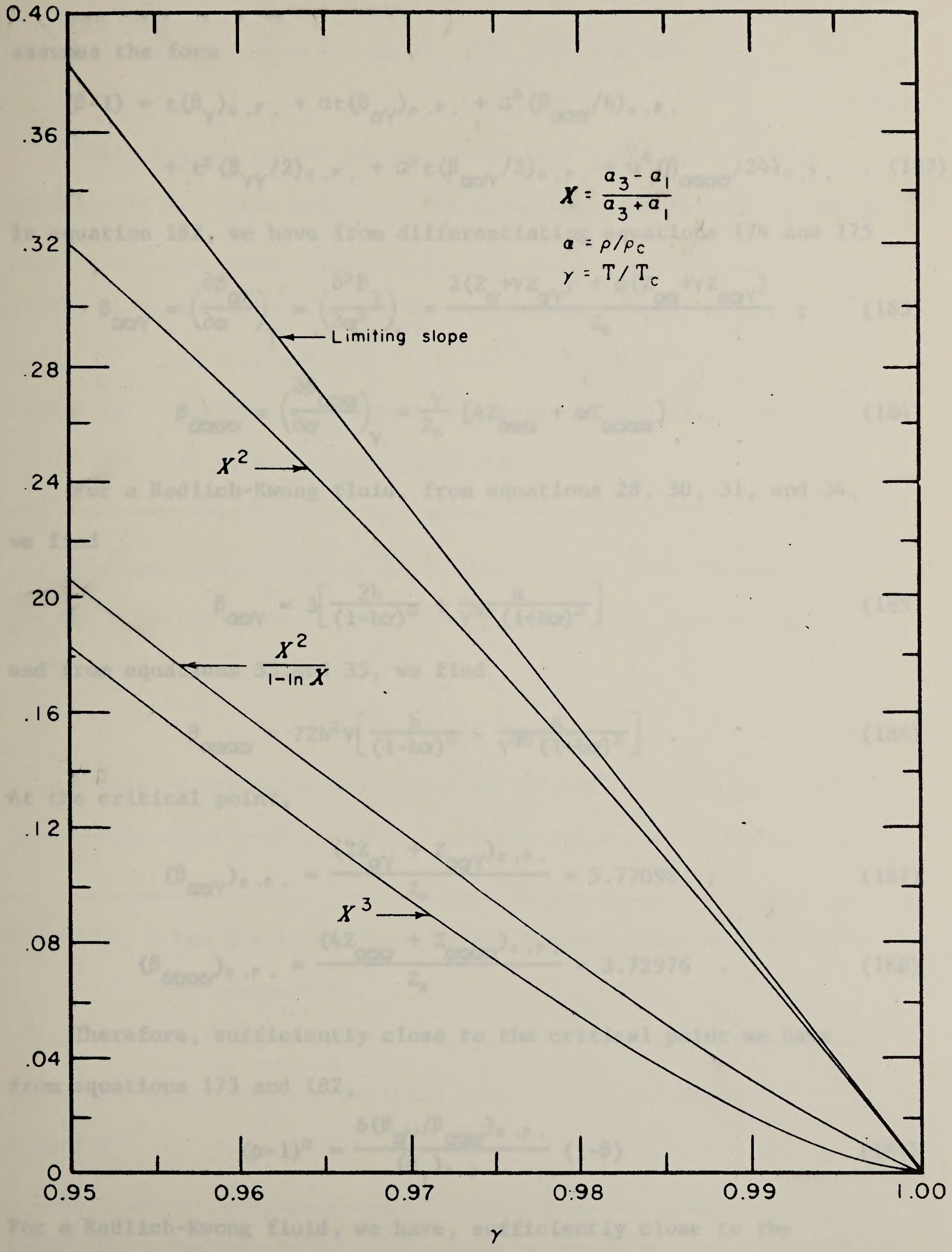

FIGURE 10.—Redlich-Kwong Fluid, Coexistence Data as a Function of Temperature Near the Critical Point.

assumes the form

$$(\beta-1) = t(\beta_{\gamma})_{c.p.} + \alpha t(\beta_{\alpha\gamma})_{c.p.} + \alpha^3(\beta_{\alpha\alpha\alpha}/6)_{c.p.}$$

$$+ t^2(\beta_{\gamma\gamma}/2)_{c.p.} + \alpha^2 t(\beta_{\alpha\alpha\gamma}/2)_{c.p.} + \alpha^4(\beta_{\alpha\alpha\alpha\alpha}/24)_{c.p.} \quad . \qquad (182)$$

In equation 182, we have from differentiating equations 174 and 175

$$\beta_{\alpha\alpha\gamma} = \left(\frac{\partial\beta_{\alpha\gamma}}{\partial\alpha}\right)_{\gamma} = \left(\frac{\partial^2\beta_{\gamma}}{\partial\alpha^2}\right)_{\gamma} = \frac{2(Z_{\alpha}+\gamma Z_{\alpha\gamma}) + \alpha(Z_{\alpha\alpha}+\gamma Z_{\alpha\alpha\gamma})}{Z_c} \quad ; \qquad (183)$$

$$\beta_{\alpha\alpha\alpha\alpha} = \left(\frac{\partial\beta_{\alpha\alpha\alpha}}{\partial\alpha}\right)_{\gamma} = \frac{\gamma}{Z_c}\left[4Z_{\alpha\alpha\alpha} + \alpha Z_{\alpha\alpha\alpha\alpha}\right] \quad . \qquad (184)$$

For a Redlich-Kwong fluid, from equations 28, 30, 31, and 34, we find

$$\beta_{\alpha\alpha\gamma} = 3\left[\frac{2b}{(1-b\alpha)^3} + \frac{a}{\gamma^{3/2}(1+b\alpha)^3}\right] \qquad (185)$$

and from equations 33 and 35, we find

$$\beta_{\alpha\alpha\alpha\alpha} = 72b^2\gamma\left[\frac{b}{(1-b\alpha)^5} - \frac{a}{\gamma^{3/2}(1+b\alpha)^5}\right] \quad . \qquad (186)$$

At the critical point,

$$(\beta_{\alpha\alpha\gamma})_{c.p.} = \frac{(2Z_{\alpha\gamma} + Z_{\alpha\alpha\gamma})_{c.p.}}{Z_c} = 5.77098 \quad ; \qquad (187)$$

$$(\beta_{\alpha\alpha\alpha\alpha})_{c.p.} = \frac{(4Z_{\alpha\alpha\alpha} + Z_{\alpha\alpha\alpha\alpha})_{c.p.}}{Z_c} = 3.72976 \quad . \qquad (188)$$

Therefore, sufficiently close to the critical point we have from equations 173 and 182,

$$(\alpha-1)^2 = \frac{6(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.}}{(\beta_{\gamma})_{c.p.}}(1-\beta) \quad . \qquad (189)$$

For a Redlich-Kwong fluid, we have, sufficiently close to the

critical point, from equations 51, 178, 179, and 189,

$$(\alpha-1)^2 = 1.372808(1-\beta) \quad . \tag{190}$$

Table 6 gives values of the functions $X^2$, $X^3$, and $\frac{X^2}{1-\ln X}$ as a function of $\beta$. These same data are illustrated in figures 11 and 12.

FIGURE 11. - Redlich-Kwong fluid, coexistence data as a function of pressure.

FIGURE 12. - Redlich-Kwong fluid, coexistence data as a function of pressure near the critical point.

A comparison of figures 9-12 clearly shows that the function $X^2$, for a Redlich-Kwong fluid, is more linear when plotted as a function of $\beta$ rather than $\gamma$.

## VAPOR PRESSURE CURVE

Values of the reduced pressure as a function of the reduced temperature are listed in table 7 and are illustrated in figure 13.

FIGURE 13. - Redlich-Kwong fluid, vapor pressure data as a function of temperature.

These same data, near the critical point, are illustrated in figure 14.

FIGURE 14. - Redlich-Kwong fluid, vapor pressure data as a function of temperature near the critical point.

TABLE 6. - REDLICH-KWONG FLUID, COEXISTENCE DATA AS A FUNCTION OF PRESSURE

$$\beta = P/P_c \qquad X = \frac{\rho_3-\rho_1}{\rho_3+\rho_1} = \frac{\alpha_3-\alpha_1}{\alpha_3+\alpha_1} \qquad \alpha = \rho/\rho_c$$

| $\beta$ | $X^2$ | $\frac{X^2}{1 - \ln X}$ | $X^3$ |
|---|---|---|---|
| 0.00000E-99 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |
| 9.59064E-46 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |
| 1.95995E-24 | 1.00000E-00 | 1.00000E-00 | 1.00000E-00 |
| 1.53449E-15 | 9.99999E-01 | 9.99999E-01 | 9.99999E-01 |
| 7.18061E-11 | 9.99999E-01 | 9.99999E-01 | 9.99999E-01 |
| 4.50798E-08 | 9.99999E-01 | 9.99999E-01 | 9.99999E-01 |
| 3.02123E-06 | 9.99996E-01 | 9.99995E-01 | 9.99995E-01 |
| 5.57479E-05 | 9.99945E-01 | 9.99918E-01 | 9.99918E-01 |
| 4.61534E-04 | 9.99590E-01 | 9.99385E-01 | 9.99385E-01 |
| 2.25834E-03 | 9.98136E-01 | 9.97206E-01 | 9.97205E-01 |
| 7.71655E-03 | 9.93977E-01 | 9.90984E-01 | 9.90980E-01 |
| 2.04768E-02 | 9.84622E-01 | 9.77051E-01 | 9.77022E-01 |
| 4.52692E-02 | 9.66722E-01 | 9.50636E-01 | 9.50502E-01 |
| 8.74419E-02 | 9.36019E-01 | 9.06066E-01 | 9.05581E-01 |
| 1.52527E-01 | 8.87095E-01 | 8.36959E-01 | 8.35516E-01 |
| 2.45938E-01 | 8.12872E-01 | 7.36570E-01 | 7.32881E-01 |
| 3.72802E-01 | 7.03789E-01 | 5.98645E-01 | 5.90424E-01 |
| 5.37888E-01 | 5.46465E-01 | 4.19666E-01 | 4.03965E-01 |
| 7.45600E-01 | 3.21533E-01 | 2.05147E-01 | 1.82322E-01 |
| 7.54853E-01 | 3.10763E-01 | 1.96144E-01 | 1.73238E-01 |
| 7.64182E-01 | 2.99838E-01 | 1.87134E-01 | 1.64183E-01 |
| 7.73586E-01 | 2.88755E-01 | 1.78124E-01 | 1.55165E-01 |
| 7.83065E-01 | 2.77514E-01 | 1.69118E-01 | 1.46193E-01 |
| 7.92620E-01 | 2.66110E-01 | 1.60122E-01 | 1.37275E-01 |
| 8.02251E-01 | 2.54543E-01 | 1.51141E-01 | 1.28423E-01 |
| 8.11958E-01 | 2.42809E-01 | 1.42181E-01 | 1.19645E-01 |
| 8.21742E-01 | 2.30905E-01 | 1.33250E-01 | 1.10956E-01 |
| 8.31602E-01 | 2.18830E-01 | 1.24354E-01 | 1.02367E-01 |
| 8.41540E-01 | 2.06581E-01 | 1.15503E-01 | 9.38936E-02 |
| 8.51555E-01 | 1.94154E-01 | 1.06704E-01 | 8.55504E-02 |
| 8.61647E-01 | 1.81548E-01 | 9.79694E-02 | 7.73552E-02 |
| 8.71816E-01 | 1.68760E-01 | 8.93082E-02 | 6.93274E-02 |
| 8.82064E-01 | 1.55786E-01 | 8.07337E-02 | 6.14886E-02 |
| 8.92390E-01 | 1.42624E-01 | 7.22601E-02 | 5.38632E-02 |
| 9.02794E-01 | 1.29272E-01 | 6.39037E-02 | 4.64790E-02 |
| 9.13277E-01 | 1.15725E-01 | 5.56836E-02 | 3.93680E-02 |
| 9.23839E-01 | 1.01982E-01 | 4.76222E-02 | 3.25675E-02 |
| 9.34480E-01 | 8.80386E-02 | 3.97467E-02 | 2.61222E-02 |
| 9.45200E-01 | 7.38920E-02 | 3.20910E-02 | 2.00861E-02 |
| 9.56000E-01 | 5.95393E-02 | 2.46993E-02 | 1.45280E-02 |
| 9.66879E-01 | 4.49770E-02 | 1.76325E-02 | 9.53864E-03 |
| 9.77839E-01 | 3.02019E-02 | 1.09828E-02 | 5.24872E-03 |
| 9.88879E-01 | 1.52107E-02 | 4.91800E-03 | 1.87597E-03 |
| 9.94429E-01 | 7.63305E-03 | 2.22043E-03 | 6.66879E-04 |
| 1.00000E-00 | 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

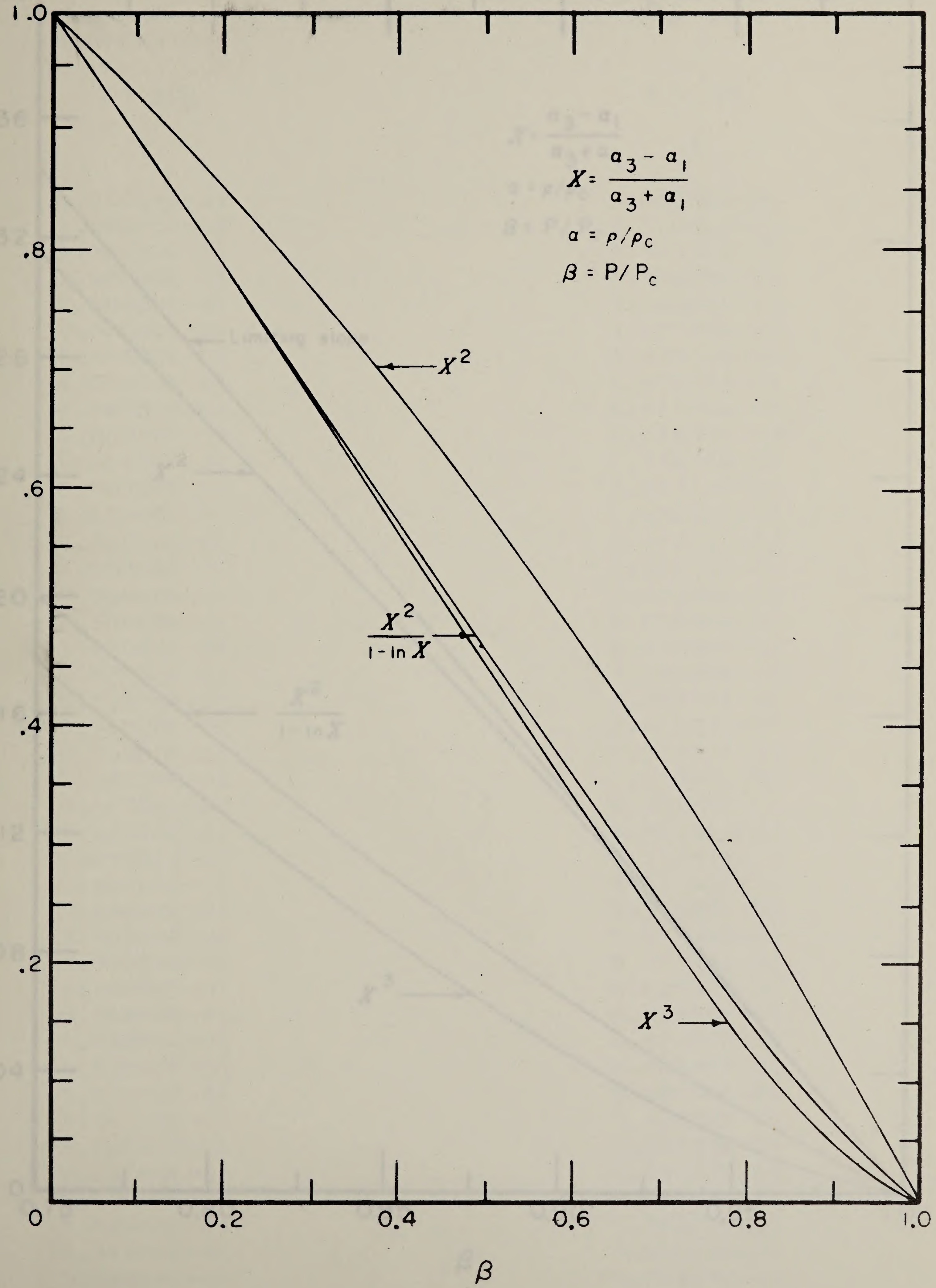

FIGURE 11. - Redlich-Kwong Fluid, Coexistence Data as a Function of Pressure.

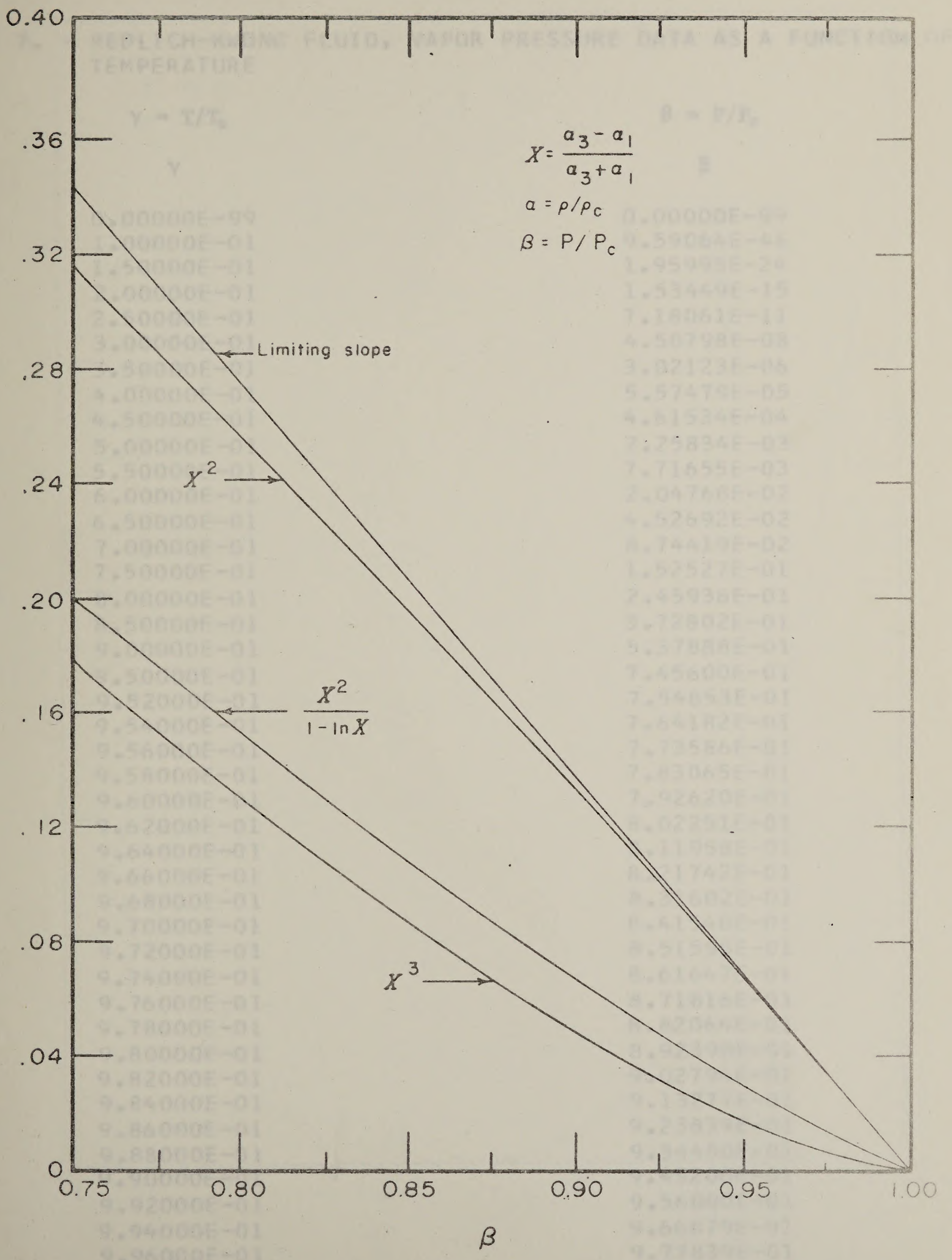

FIGURE 12. - Redlich - Kwong Fluid, Coexistence Data as a Function of Pressure Near the Critical Point.

TABLE 7. - REDLICH-KWONG FLUID, VAPOR PRESSURE DATA AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\beta = P/P_c$ |
|---|---|
| $\gamma$ | $\beta$ |
| 0.00000E-99 | 0.00000E-99 |
| 1.00000E-01 | 9.59064E-46 |
| 1.50000E-01 | 1.95995E-24 |
| 2.00000E-01 | 1.53449E-15 |
| 2.50000E-01 | 7.18061E-11 |
| 3.00000E-01 | 4.50798E-08 |
| 3.50000E-01 | 3.02123E-06 |
| 4.00000E-01 | 5.57479E-05 |
| 4.50000E-01 | 4.61534E-04 |
| 5.00000E-01 | 2.25834E-03 |
| 5.50000E-01 | 7.71655E-03 |
| 6.00000E-01 | 2.04768E-02 |
| 6.50000E-01 | 4.52692E-02 |
| 7.00000E-01 | 8.74419E-02 |
| 7.50000E-01 | 1.52527E-01 |
| 8.00000E-01 | 2.45938E-01 |
| 8.50000E-01 | 3.72802E-01 |
| 9.00000E-01 | 5.37888E-01 |
| 9.50000E-01 | 7.45600E-01 |
| 9.52000E-01 | 7.54853E-01 |
| 9.54000E-01 | 7.64182E-01 |
| 9.56000E-01 | 7.73586E-01 |
| 9.58000E-01 | 7.83065E-01 |
| 9.60000E-01 | 7.92620E-01 |
| 9.62000E-01 | 8.02251E-01 |
| 9.64000E-01 | 8.11958E-01 |
| 9.66000E-01 | 8.21742E-01 |
| 9.68000E-01 | 8.31602E-01 |
| 9.70000E-01 | 8.41540E-01 |
| 9.72000E-01 | 8.51555E-01 |
| 9.74000E-01 | 8.61647E-01 |
| 9.76000E-01 | 8.71816E-01 |
| 9.78000E-01 | 8.82064E-01 |
| 9.80000E-01 | 8.92390E-01 |
| 9.82000E-01 | 9.02794E-01 |
| 9.84000E-01 | 9.13277E-01 |
| 9.86000E-01 | 9.23839E-01 |
| 9.88000E-01 | 9.34480E-01 |
| 9.90000E-01 | 9.45200E-01 |
| 9.92000E-01 | 9.56000E-01 |
| 9.94000E-01 | 9.66879E-01 |
| 9.96000E-01 | 9.77839E-01 |
| 9.98000E-01 | 9.88879E-01 |
| 9.99000E-01 | 9.94429E-01 |
| 1.00000E-00 | 1.00000E-00 |

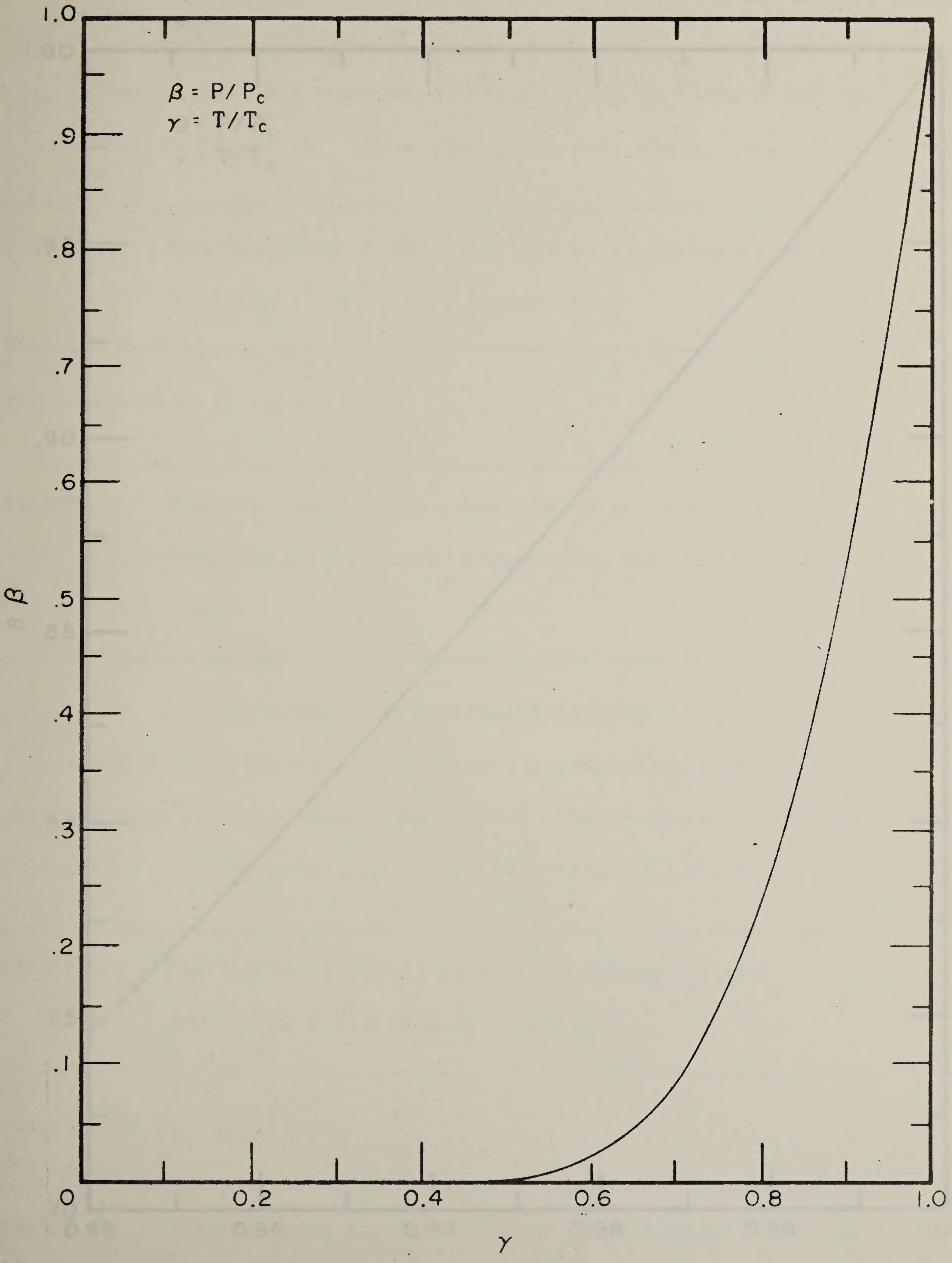

FIGURE 13. - Redlich-Kwong Fluid, Vapor Pressure Data as a Function of Temperature.

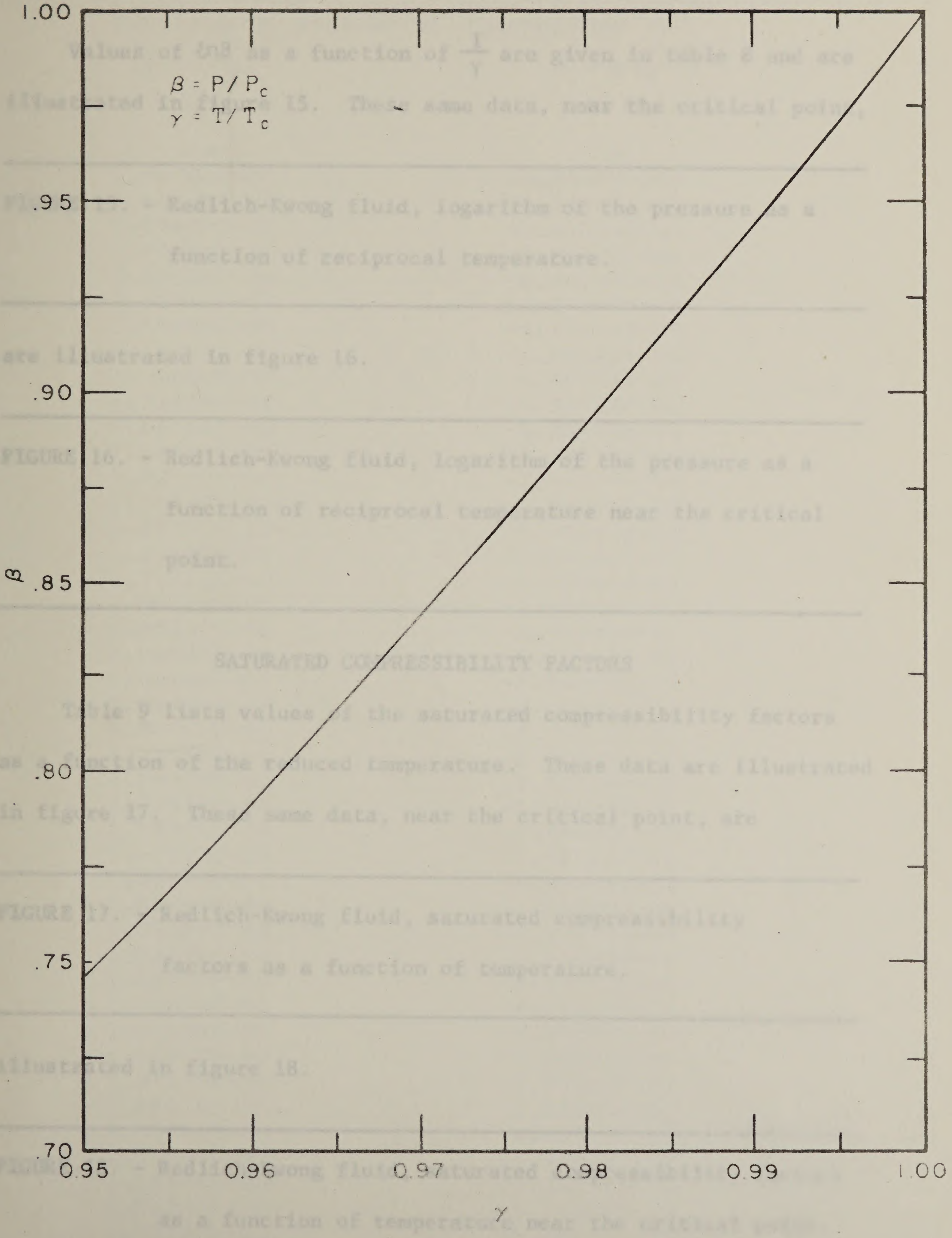

FIGURE 14. - Redlich-Kwong Fluid, Vapor Pressure Data as a Function of Temperature Near the Critical Point.

Values of $\ln\beta$ as a function of $\frac{1}{Y}$ are given in table 8 and are illustrated in figure 15. These same data, near the critical point, are illustrated in figure 16.

FIGURE 15. - Redlich-Kwong fluid, logarithm of the pressure as a function of reciprocal temperature.

FIGURE 16. - Redlich-Kwong fluid, logarithm of the pressure as a function of reciprocal temperature near the critical point.

## SATURATED COMPRESSIBILITY FACTORS

Table 9 lists values of the saturated compressibility factors as a function of the reduced temperature. These data are illustrated in figure 17. These same data, near the critical point, are illustrated in figure 18.

FIGURE 17. - Redlich-Kwong fluid, saturated compressibility factors as a function of temperature.

FIGURE 18. - Redlich-Kwong fluid, saturated compressibility factors as a function of temperature near the critical point.

TABLE 8. - REDLICH-KWONG FLUID, LOGARITHM OF THE PRESSURE AS A FUNCTION OF RECIPROCAL TEMPERATURE

| $\gamma = T/T_c$ | $\beta = P/P_c$ |
|---|---|
| $1/\gamma$ | $\ln \beta$ |
| $+\infty$ | $-\infty$ |
| 1.00000E+01 | -1.03658E+02 |
| 6.66666E-00 | -5.45891E+01 |
| 5.00000E-00 | -3.41105E+01 |
| 4.00000E-00 | -2.33570E+01 |
| 3.33333E-00 | -1.69148E+01 |
| 2.85714E-00 | -1.27098E+01 |
| 2.50000E-00 | -9.79467E-00 |
| 2.22222E-00 | -7.68095E-00 |
| 2.00000E-00 | -6.09312E-00 |
| 1.81818E-00 | -4.86438E-00 |
| 1.66666E-00 | -3.88845E-00 |
| 1.53846E-00 | -3.09512E-00 |
| 1.42857E-00 | -2.43677E-00 |
| 1.33333E-00 | -1.88041E-00 |
| 1.25000E-00 | -1.40267E-00 |
| 1.17647E-00 | -9.86707E-01 |
| 1.11111E-00 | -6.20104E-01 |
| 1.05263E-00 | -2.93566E-01 |
| 1.05042E-00 | -2.81231E-01 |
| 1.04821E-00 | -2.68949E-01 |
| 1.04602E-00 | -2.56718E-01 |
| 1.04384E-00 | -2.44539E-01 |
| 1.04166E-00 | -2.32410E-01 |
| 1.03950E-00 | -2.20333E-01 |
| 1.03734E-00 | -2.08305E-01 |
| 1.03519E-00 | -1.96328E-01 |
| 1.03305E-00 | -1.84400E-01 |
| 1.03092E-00 | -1.72521E-01 |
| 1.02880E-00 | -1.60691E-01 |
| 1.02669E-00 | -1.48909E-01 |
| 1.02459E-00 | -1.37175E-01 |
| 1.02249E-00 | -1.25490E-01 |
| 1.02040E-00 | -1.13851E-01 |
| 1.01832E-00 | -1.02260E-01 |
| 1.01626E-00 | -9.07155E-02 |
| 1.01419E-00 | -7.92172E-02 |
| 1.01214E-00 | -6.77649E-02 |
| 1.01010E-00 | -5.63584E-02 |
| 1.00806E-00 | -4.49972E-02 |
| 1.00603E-00 | -3.36811E-02 |
| 1.00401E-00 | -2.24097E-02 |
| 1.00200E-00 | -1.11828E-02 |
| 1.00100E-00 | -5.58591E-03 |
| 1.00000E-00 | 0.00000E-99 |

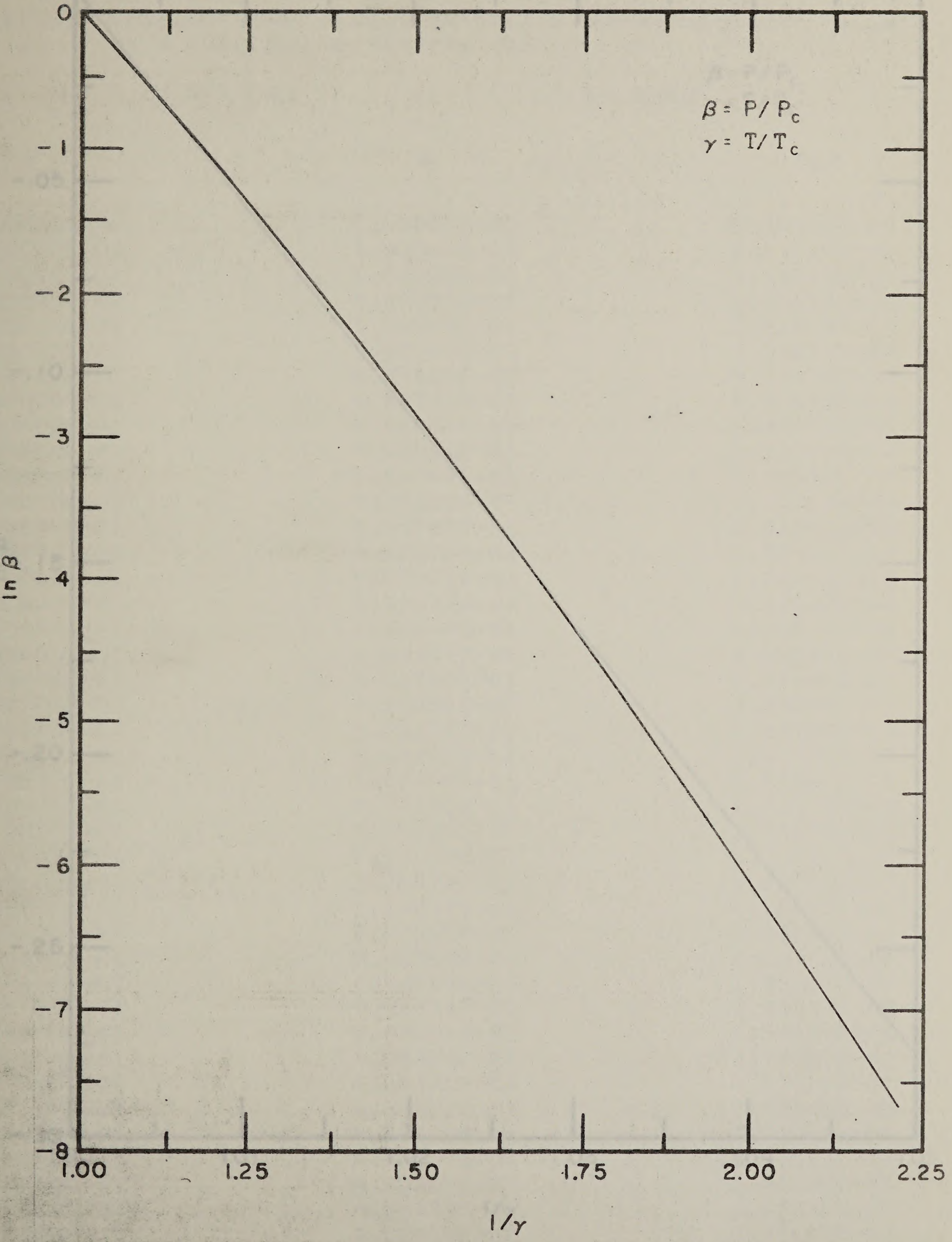

FIGURE 15.- Redlich-Kwong Fluid, Logarithm of the Pressure as a Function of Reciprocal Temperature.

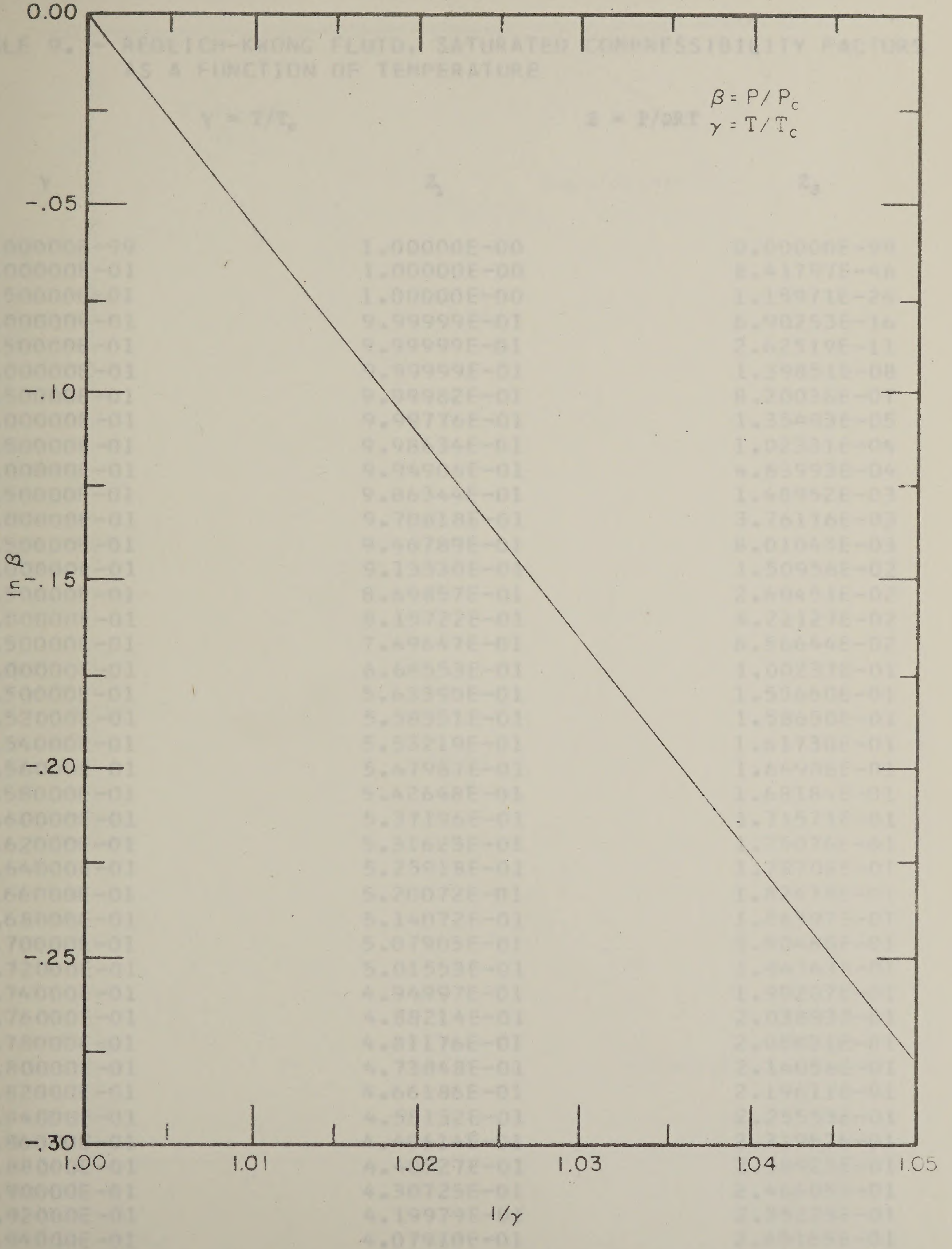

FIGURE 16. - Redlich - Kwong Fluid, Logarithm of the Pressure as a Function of Reciprocal Temperature Near the Critical Point.

### TABLE 9. - REDLICH-KWONG FLUID, SATURATED COMPRESSIBILITY FACTORS AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad$ $Z = P/\rho RT$

| $\gamma$ | $Z_1$ | $Z_3$ |
|---|---|---|
| 0.00000E-99 | 1.00000E-00 | 0.00000E-99 |
| 1.00000E-01 | 1.00000E-00 | 8.41797E-46 |
| 1.50000E-01 | 1.00000E-00 | 1.15971E-24 |
| 2.00000E-01 | 9.99999E-01 | 6.90253E-16 |
| 2.50000E-01 | 9.99999E-01 | 2.62519E-11 |
| 3.00000E-01 | 9.99999E-01 | 1.39851E-08 |
| 3.50000E-01 | 9.99982E-01 | 8.20036E-07 |
| 4.00000E-01 | 9.99776E-01 | 1.35493E-05 |
| 4.50000E-01 | 9.98634E-01 | 1.02331E-04 |
| 5.00000E-01 | 9.94906E-01 | 4.63993E-04 |
| 5.50000E-01 | 9.86344E-01 | 1.48952E-03 |
| 6.00000E-01 | 9.70818E-01 | 3.76116E-03 |
| 6.50000E-01 | 9.46789E-01 | 8.01043E-03 |
| 7.00000E-01 | 9.13330E-01 | 1.50956E-02 |
| 7.50000E-01 | 8.69857E-01 | 2.60451E-02 |
| 8.00000E-01 | 8.15722E-01 | 4.22127E-02 |
| 8.50000E-01 | 7.49647E-01 | 6.56644E-02 |
| 9.00000E-01 | 6.68553E-01 | 1.00237E-01 |
| 9.50000E-01 | 5.63390E-01 | 1.55660E-01 |
| 9.52000E-01 | 5.58351E-01 | 1.58650E-01 |
| 9.54000E-01 | 5.53219E-01 | 1.61730E-01 |
| 9.56000E-01 | 5.47987E-01 | 1.64906E-01 |
| 9.58000E-01 | 5.42648E-01 | 1.68184E-01 |
| 9.60000E-01 | 5.37196E-01 | 1.71571E-01 |
| 9.62000E-01 | 5.31623E-01 | 1.75076E-01 |
| 9.64000E-01 | 5.25918E-01 | 1.78708E-01 |
| 9.66000E-01 | 5.20072E-01 | 1.82478E-01 |
| 9.68000E-01 | 5.14072E-01 | 1.86397E-01 |
| 9.70000E-01 | 5.07905E-01 | 1.90480E-01 |
| 9.72000E-01 | 5.01553E-01 | 1.94743E-01 |
| 9.74000E-01 | 4.94997E-01 | 1.99207E-01 |
| 9.76000E-01 | 4.88214E-01 | 2.03893E-01 |
| 9.78000E-01 | 4.81176E-01 | 2.08831E-01 |
| 9.80000E-01 | 4.73848E-01 | 2.14056E-01 |
| 9.82000E-01 | 4.66186E-01 | 2.19611E-01 |
| 9.84000E-01 | 4.58132E-01 | 2.25553E-01 |
| 9.86000E-01 | 4.49614E-01 | 2.31957E-01 |
| 9.88000E-01 | 4.40527E-01 | 2.38925E-01 |
| 9.90000E-01 | 4.30725E-01 | 2.46605E-01 |
| 9.92000E-01 | 4.19979E-01 | 2.55225E-01 |
| 9.94000E-01 | 4.07910E-01 | 2.65165E-01 |
| 9.96000E-01 | 3.93772E-01 | 2.77170E-01 |
| 9.98000E-01 | 3.75645E-01 | 2.93160E-01 |
| 9.99000E-01 | 3.63037E-01 | 3.04699E-01 |
| 1.00000E-00 | 3.33333E-01 | 3.33333E-01 |

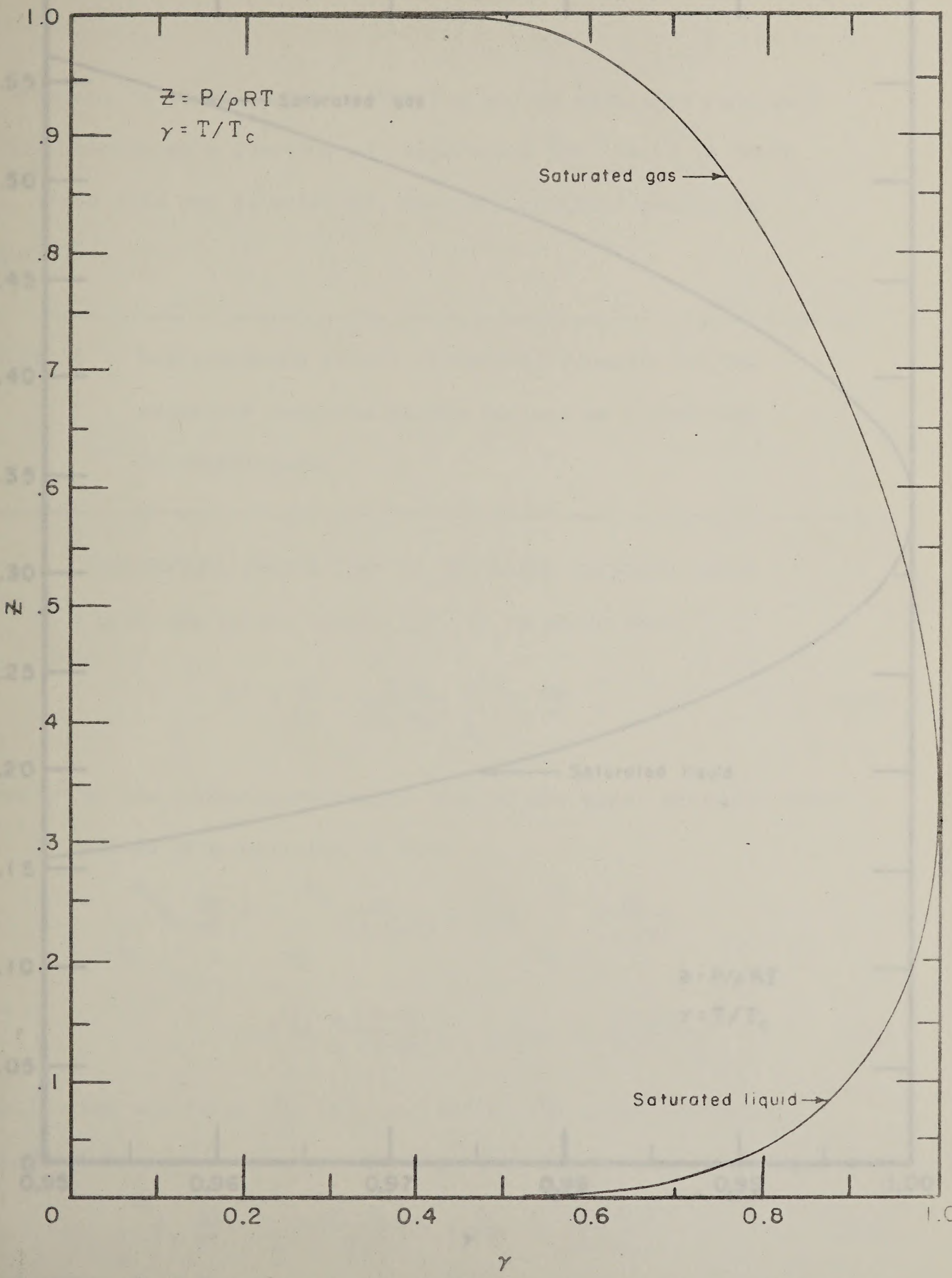

FIGURE 17.- Redlich-Kwong Fluid, Saturated Compressibility Factors as a Function of Temperature.

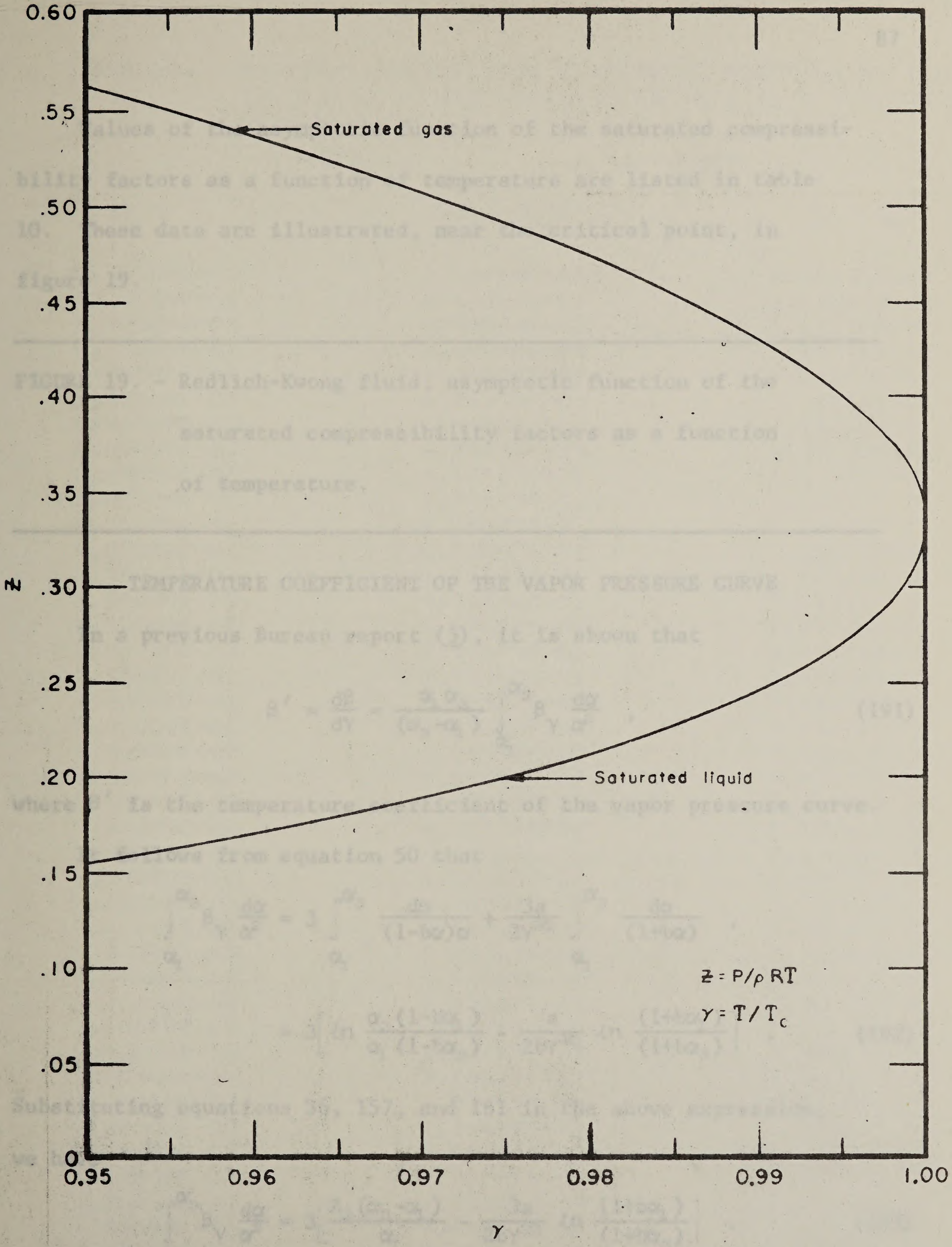

FIGURE 18.— Redlich-Kwong Fluid, Saturated Compressibility Factors as a Function of Temperature Near the Critical Point.

Values of the asymptotic function of the saturated compressibility factors as a function of temperature are listed in table 10. These data are illustrated, near the critical point, in figure 19.

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FIGURE 19. - Redlich-Kwong fluid, asymptotic function of the saturated compressibility factors as a function of temperature.

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## TEMPERATURE COEFFICIENT OF THE VAPOR PRESSURE CURVE

In a previous Bureau report (5), it is shown that

$$\beta' = \frac{d\beta}{d\gamma} = \frac{\alpha_1 \alpha_3}{(\alpha_3 - \alpha_1)} \int_{\alpha_1}^{\alpha_3} \beta_\gamma \frac{d\alpha}{\alpha^2} , \tag{191}$$

where $\beta'$ is the temperature coefficient of the vapor pressure curve.

It follows from equation 50 that

$$\int_{\alpha_1}^{\alpha_3} \beta_\gamma \frac{d\alpha}{\alpha^2} = 3 \int_{\alpha_1}^{\alpha_3} \frac{d\alpha}{(1-b\alpha)\alpha} + \frac{3a}{2\gamma^{3/2}} \int_{\alpha_1}^{\alpha_3} \frac{d\alpha}{(1+b\alpha)} ,$$

$$= 3\left[ \ln \frac{\alpha_3(1-b\alpha_1)}{\alpha_1(1-b\alpha_3)} - \frac{a}{2b\gamma^{3/2}} \ln \frac{(1+b\alpha_1)}{(1+b\alpha_3)} \right] . \tag{192}$$

Substituting equations 36, 157, and 161 in the above expression, we have

$$\int_{\alpha_1}^{\alpha_3} \beta_\gamma \frac{d\alpha}{\alpha^2} = 3\left[ \frac{Z_3(\alpha_3 - \alpha_1)}{\alpha_1} - \frac{3a}{2b\gamma^{3/2}} \ln \frac{(1+b\alpha_1)}{(1+b\alpha_3)} \right] . \tag{193}$$

TABLE 10. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED COMPRESSIBILITY FACTORS AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $Z = P/\rho RT$

| $(1-\gamma)$ | $(Z_1 - Z_c)^2$ | $(Z_3 - Z_c)^2$ |
|---|---|---|
| 1.00000E+00 | 4.44444E-01 | 1.11111E-01 |
| 9.00000E-01 | 4.44444E-01 | 1.11111E-01 |
| 8.50000E-01 | 4.44444E-01 | 1.11111E-01 |
| 8.00000E-01 | 4.44444E-01 | 1.11111E-01 |
| 7.50000E-01 | 4.44444E-01 | 1.11111E-01 |
| 7.00000E-01 | 4.44443E-01 | 1.11111E-01 |
| 6.50000E-01 | 4.44421E-01 | 1.11110E-01 |
| 6.00000E-01 | 4.44146E-01 | 1.11102E-01 |
| 5.50000E-01 | 4.42625E-01 | 1.11042E-01 |
| 5.00000E-01 | 4.37679E-01 | 1.10801E-01 |
| 4.50000E-01 | 4.26423E-01 | 1.10120E-01 |
| 4.00000E-01 | 4.06387E-01 | 1.08617E-01 |
| 3.50000E-01 | 3.76328E-01 | 1.05834E-01 |
| 3.00000E-01 | 3.36396E-01 | 1.01275E-01 |
| 2.50000E-01 | 2.87857E-01 | 9.44260E-02 |
| 2.00000E-01 | 2.32699E-01 | 8.47511E-02 |
| 1.50000E-01 | 1.73317E-01 | 7.16466E-02 |
| 1.00000E-01 | 1.12372E-01 | 5.43335E-02 |
| 5.00000E-02 | 5.29262E-02 | 3.15676E-02 |
| 4.80000E-02 | 5.06333E-02 | 3.05140E-02 |
| 4.60000E-02 | 4.83499E-02 | 2.94474E-02 |
| 4.40000E-02 | 4.60763E-02 | 2.83675E-02 |
| 4.20000E-02 | 4.38129E-02 | 2.72741E-02 |
| 4.00000E-02 | 4.15603E-02 | 2.61667E-02 |
| 3.80000E-02 | 3.93188E-02 | 2.50451E-02 |
| 3.60000E-02 | 3.70891E-02 | 2.39087E-02 |
| 3.40000E-02 | 3.48715E-02 | 2.27572E-02 |
| 3.20000E-02 | 3.26667E-02 | 2.15901E-02 |
| 3.00000E-02 | 3.04753E-02 | 2.04069E-02 |
| 2.80000E-02 | 2.82979E-02 | 1.92070E-02 |
| 2.60000E-02 | 2.61353E-02 | 1.79898E-02 |
| 2.40000E-02 | 2.39883E-02 | 1.67546E-02 |
| 2.20000E-02 | 2.18577E-02 | 1.55006E-02 |
| 2.00000E-02 | 1.97445E-02 | 1.42269E-02 |
| 1.80000E-02 | 1.76498E-02 | 1.29326E-02 |
| 1.60000E-02 | 1.55749E-02 | 1.16165E-02 |
| 1.40000E-02 | 1.35212E-02 | 1.02771E-02 |
| 1.20000E-02 | 1.14906E-02 | 8.91289E-03 |
| 1.00000E-02 | 9.48516E-03 | 7.52169E-03 |
| 8.00000E-03 | 7.50761E-03 | 6.10088E-03 |
| 6.00000E-03 | 5.56169E-03 | 4.64683E-03 |
| 4.00000E-03 | 3.65284E-03 | 3.15423E-03 |
| 2.00000E-03 | 1.79035E-03 | 1.61384E-03 |
| 1.00000E-03 | 8.82350E-04 | 8.19907E-04 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

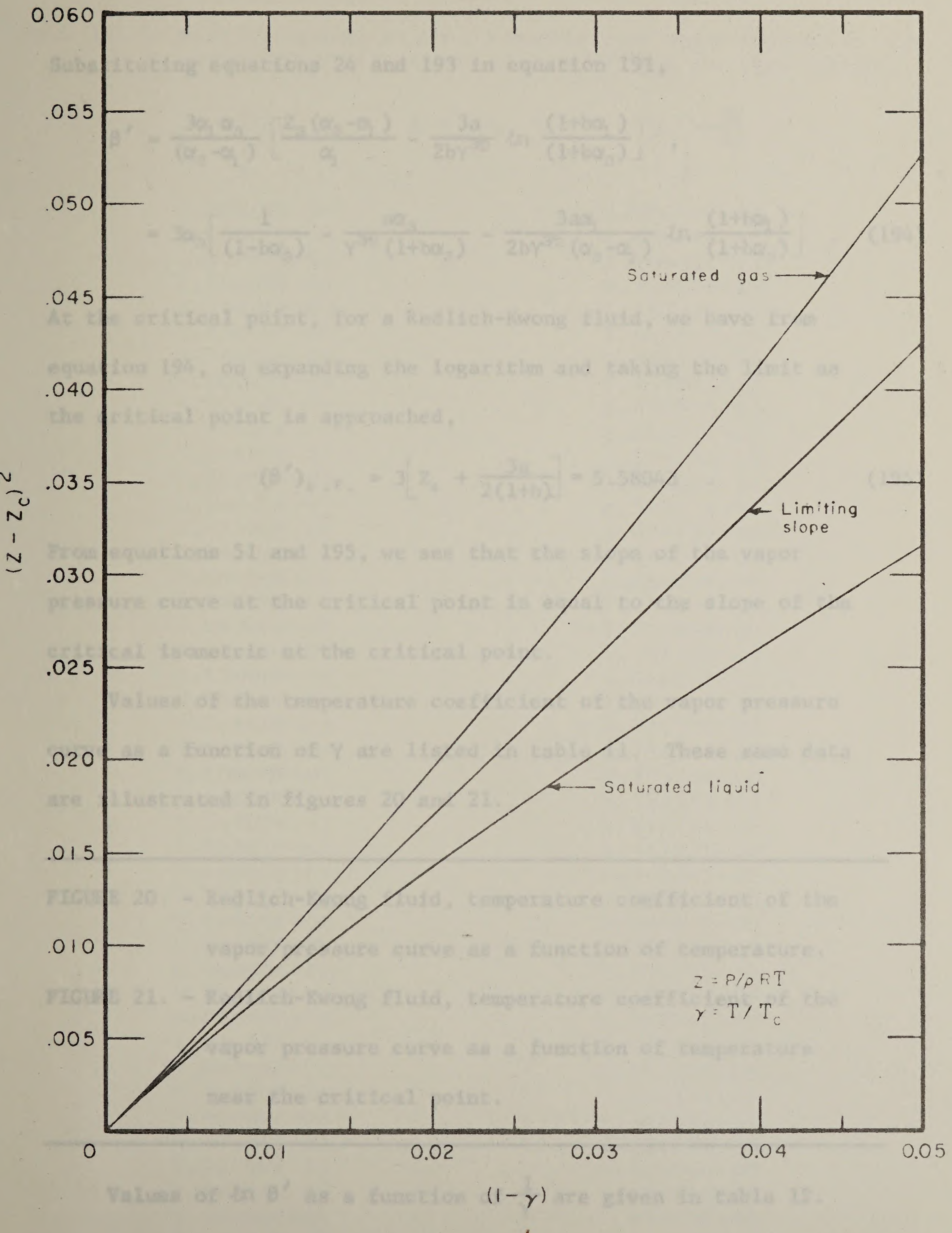

FIGURE 19.—Redlich-Kwong Fluid, Asymptotic Function of the Saturated Compressibility Factors as a Function of Temperature.

Substituting equations 24 and 193 in equation 191,

$$\beta' = \frac{3\alpha_1\alpha_3}{(\alpha_3-\alpha_1)}\left[\frac{Z_3(\alpha_3-\alpha_1)}{\alpha_1} - \frac{3a}{2b\gamma^{3/2}}\ln\frac{(1+b\alpha_1)}{(1+b\alpha_3)}\right] ,$$

$$= 3\alpha_3\left[\frac{1}{(1-b\alpha_3)} - \frac{a\alpha_3}{\gamma^{3/2}(1+b\alpha_3)} - \frac{3a\alpha_1}{2b\gamma^{3/2}(\alpha_3-\alpha_1)}\ln\frac{(1+b\alpha_1)}{(1+b\alpha_3)}\right] . \qquad (194)$$

At the critical point, for a Redlich-Kwong fluid, we have from equation 194, on expanding the logarithm and taking the limit as the critical point is approached,

$$(\beta')_{c.p.} = 3\left[Z_c + \frac{3a}{2(1+b)}\right] = 5.58043 . \qquad (195)$$

From equations 51 and 195, we see that the slope of the vapor pressure curve at the critical point is equal to the slope of the critical isometric at the critical point.

Values of the temperature coefficient of the vapor pressure curve as a function of $\gamma$ are listed in table 11. These same data are illustrated in figures 20 and 21.

---

FIGURE 20. - Redlich-Kwong fluid, temperature coefficient of the vapor pressure curve as a function of temperature.

FIGURE 21. - Redlich-Kwong fluid, temperature coefficient of the vapor pressure curve as a function of temperature near the critical point.

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Values of $\ln \beta'$ as a function of $\frac{1}{\gamma}$ are given in table 12.

TABLE 11. - REDLICH-KWONG FLUID, TEMPERATURE COEFFICIENT OF THE VAPOR PRESSURE CURVE AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\beta = P/P_c$ | $\beta' = \frac{d\beta}{d\gamma}$ |
|---|---|---|
| $\gamma$ | | $\beta'$ |
| 0.00000E-99 | | 0.00000E-99 |
| 1.00000E-01 | | 1.55088E-42 |
| 1.50000E-01 | | 1.14690E-21 |
| 2.00000E-01 | | 4.35885E-13 |
| 2.50000E-01 | | 1.16262E-08 |
| 3.00000E-01 | | 4.60366E-06 |
| 3.50000E-01 | | 2.08613E-04 |
| 4.00000E-01 | | 2.73798E-03 |
| 4.50000E-01 | | 1.67639E-02 |
| 5.00000E-01 | | 6.26287E-02 |
| 5.50000E-01 | | 1.67943E-01 |
| 6.00000E-01 | | 3.58404E-01 |
| 6.50000E-01 | | 6.51267E-01 |
| 7.00000E-01 | | 1.05418E-00 |
| 7.50000E-01 | | 1.56731E-00 |
| 8.00000E-01 | | 2.18623E-00 |
| 8.50000E-01 | | 2.90422E-00 |
| 9.00000E-01 | | 3.71389E-00 |
| 9.50000E-01 | | 4.60810E-00 |
| 9.52000E-01 | | 4.64553E-00 |
| 9.54000E-01 | | 4.68309E-00 |
| 9.56000E-01 | | 4.72077E-00 |
| 9.58000E-01 | | 4.75857E-00 |
| 9.60000E-01 | | 4.79650E-00 |
| 9.62000E-01 | | 4.83455E-00 |
| 9.64000E-01 | | 4.87273E-00 |
| 9.66000E-01 | | 4.91102E-00 |
| 9.68000E-01 | | 4.94944E-00 |
| 9.70000E-01 | | 4.98798E-00 |
| 9.72000E-01 | | 5.02664E-00 |
| 9.74000E-01 | | 5.06543E-00 |
| 9.76000E-01 | | 5.10433E-00 |
| 9.78000E-01 | | 5.14335E-00 |
| 9.80000E-01 | | 5.18249E-00 |
| 9.82000E-01 | | 5.22175E-00 |
| 9.84000E-01 | | 5.26114E-00 |
| 9.86000E-01 | | 5.30063E-00 |
| 9.88000E-01 | | 5.34025E-00 |
| 9.90000E-01 | | 5.37999E-00 |
| 9.92000E-01 | | 5.41984E-00 |
| 9.94000E-01 | | 5.45981E-00 |
| 9.96000E-01 | | 5.49990E-00 |
| 9.98000E-01 | | 5.54011E-00 |
| 9.99000E-01 | | 5.56025E-00 |
| 1.00000E-00 | | 5.58043E-00 |

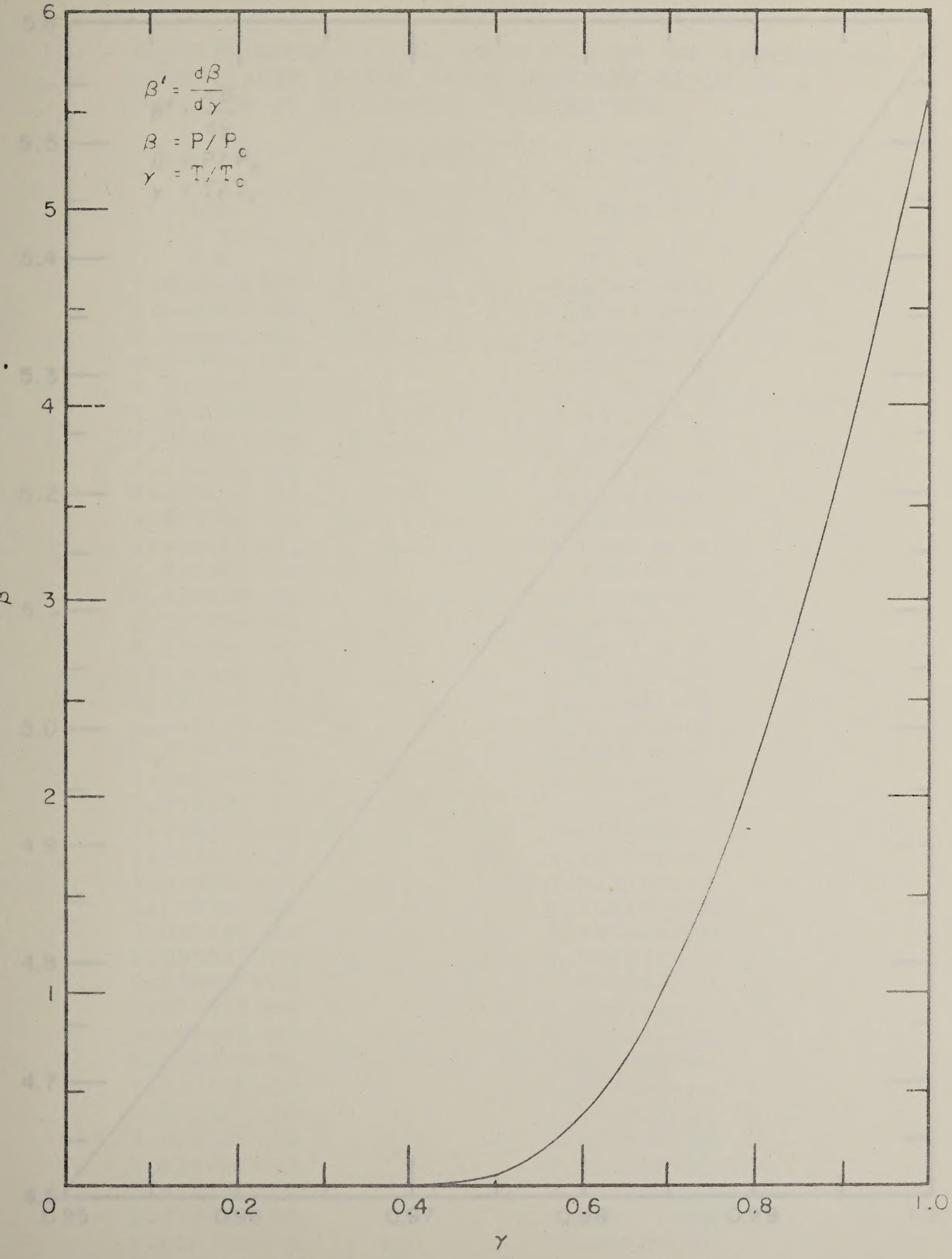

FIGURE 20. - Redlich-Kwong Fluid, Temperature Coefficient of the Vapor Pressure Curve as a Function of Temperature.

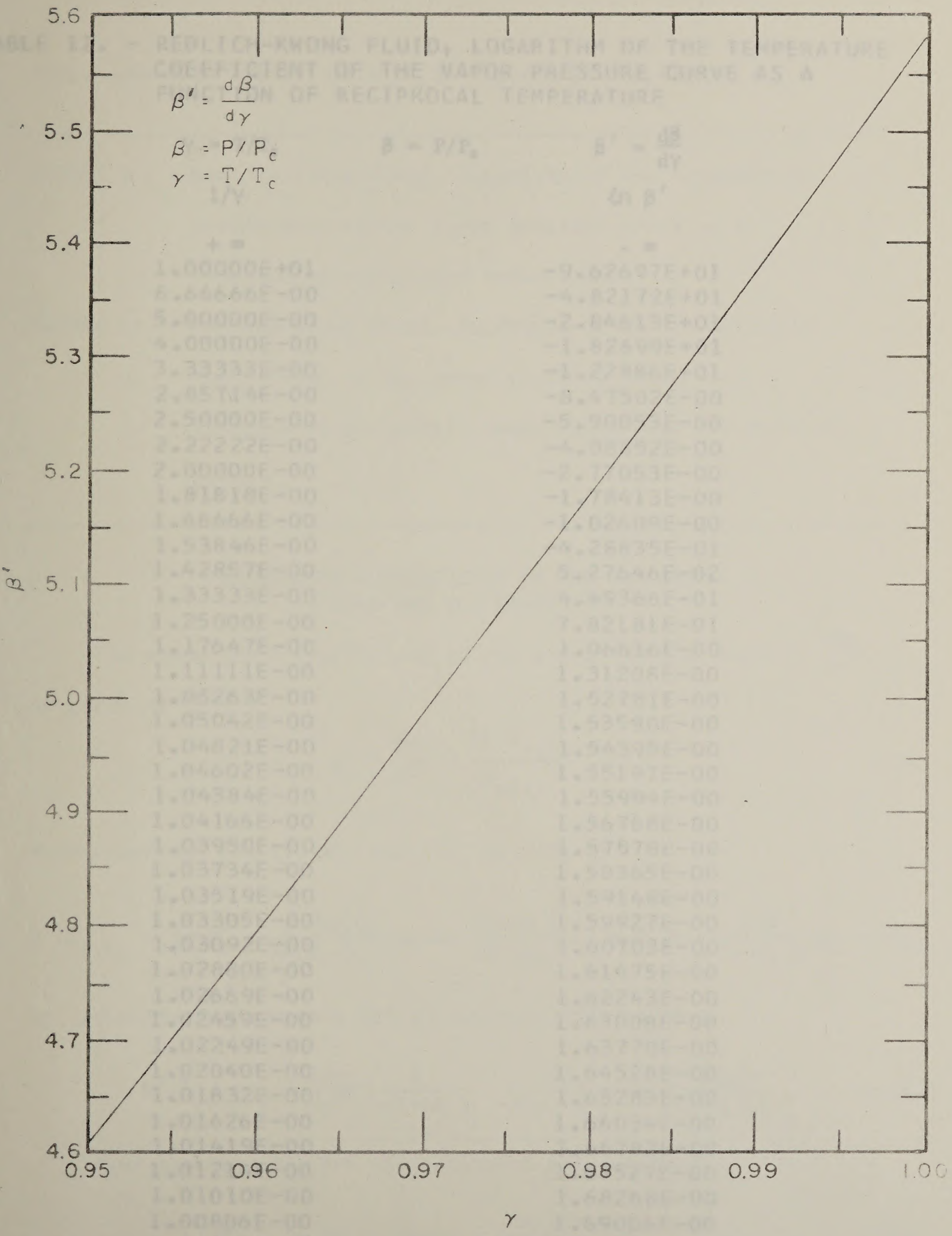

FIGURE 21. - Redlich-Kwong Fluid, Temperature Coefficient of the Vapor Pressure Curve as a Function of Temperature Near the Critical Point.

TABLE 12. - REDLICH-KWONG FLUID, LOGARITHM OF THE TEMPERATURE COEFFICIENT OF THE VAPOR PRESSURE CURVE AS A FUNCTION OF RECIPROCAL TEMPERATURE

$\gamma = T/T_c$ $\beta = P/P_c$ $\beta' = \frac{d\beta}{d\gamma}$

| $1/\gamma$ | $\ln \beta'$ |
|---|---|
| $+\infty$ | $-\infty$ |
| 1.00000E+01 | -9.62697E+01 |
| 6.66666E-00 | -4.82172E+01 |
| 5.00000E-00 | -2.84613E+01 |
| 4.00000E-00 | -1.82699E+01 |
| 3.33333E-00 | -1.22886E+01 |
| 2.85714E-00 | -8.47502E-00 |
| 2.50000E-00 | -5.90053E-00 |
| 2.22222E-00 | -4.08852E-00 |
| 2.00000E-00 | -2.77053E-00 |
| 1.81818E-00 | -1.78413E-00 |
| 1.66666E-00 | -1.02609E-00 |
| 1.53846E-00 | -4.28835E-01 |
| 1.42857E-00 | 5.27646E-02 |
| 1.33333E-00 | 4.49366E-01 |
| 1.25000E-00 | 7.82181E-01 |
| 1.17647E-00 | 1.06616E-00 |
| 1.11111E-00 | 1.31208E-00 |
| 1.05263E-00 | 1.52781E-00 |
| 1.05042E-00 | 1.53590E-00 |
| 1.04821E-00 | 1.54395E-00 |
| 1.04602E-00 | 1.55197E-00 |
| 1.04384E-00 | 1.55994E-00 |
| 1.04166E-00 | 1.56788E-00 |
| 1.03950E-00 | 1.57578E-00 |
| 1.03734E-00 | 1.58365E-00 |
| 1.03519E-00 | 1.59148E-00 |
| 1.03305E-00 | 1.59927E-00 |
| 1.03092E-00 | 1.60703E-00 |
| 1.02880E-00 | 1.61475E-00 |
| 1.02669E-00 | 1.62243E-00 |
| 1.02459E-00 | 1.63008E-00 |
| 1.02249E-00 | 1.63770E-00 |
| 1.02040E-00 | 1.64528E-00 |
| 1.01832E-00 | 1.65283E-00 |
| 1.01626E-00 | 1.66034E-00 |
| 1.01419E-00 | 1.66782E-00 |
| 1.01214E-00 | 1.67527E-00 |
| 1.01010E-00 | 1.68268E-00 |
| 1.00806E-00 | 1.69006E-00 |
| 1.00603E-00 | 1.69741E-00 |
| 1.00401E-00 | 1.70473E-00 |
| 1.00200E-00 | 1.71201E-00 |
| 1.00100E-00 | 1.71564E-00 |
| 1.00000E-00 | 1.71926E-00 |

These same data are illustrated in figures 22 and 23.

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FIGURE 22. - Redlich-Kwong fluid, logarithm of the temperature coefficient of the vapor pressure curve as a function of reciprocal temperature.

FIGURE 23. - Redlich-Kwong fluid, logarithm of the temperature coefficient of the vapor pressure curve as a function of reciprocal temperature near the critical point.

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## TEMPERATURE COEFFICIENTS OF THE SATURATED LIQUID AND GAS DENSITIES

The temperature coefficient of the saturated gas density, $\frac{d\alpha_1}{d\gamma}$, is given as (5),

$$\alpha_1' = \frac{d\alpha_1}{d\gamma} = \frac{\beta' - \beta_{\gamma(1)}}{\beta_{\alpha(1)}} \quad . \tag{196}$$

From equations 23, 29, 50, and 194, we find that

$$\beta' - \beta_{\gamma(1)} = 3\alpha_3 Z_3 - \frac{9a\alpha_1\alpha_3}{2b(\alpha_3-\alpha_1)\gamma^{3/2}} \ln\frac{(1+b\alpha_1)}{(1+b\alpha_3)} - 3\alpha_1 Z_1 - \frac{9a\alpha_1^2}{2(1+b\alpha_1)\gamma^{3/2}} \tag{197}$$

or, substituting equation 157 in equation 197, we have

$$\beta' - \beta_{\gamma(1)} = -\frac{9a\alpha_1}{2\gamma^{3/2}}\left[\frac{\alpha_1}{(1+b\alpha_1)} - \frac{\alpha_3}{b(\alpha_3-\alpha_1)} \ln\frac{(1+b\alpha_3)}{(1+b\alpha_1)}\right] \quad . \tag{198}$$

From equation 52, for the saturated gas, we have

$$\beta_{\alpha(1)} = 3\gamma\left[\frac{1}{(1-b\alpha_1)^2} - \frac{a\alpha_1(2+b\alpha_1)}{\gamma^{3/2}(1+b\alpha_1)^2}\right] \quad . \tag{199}$$

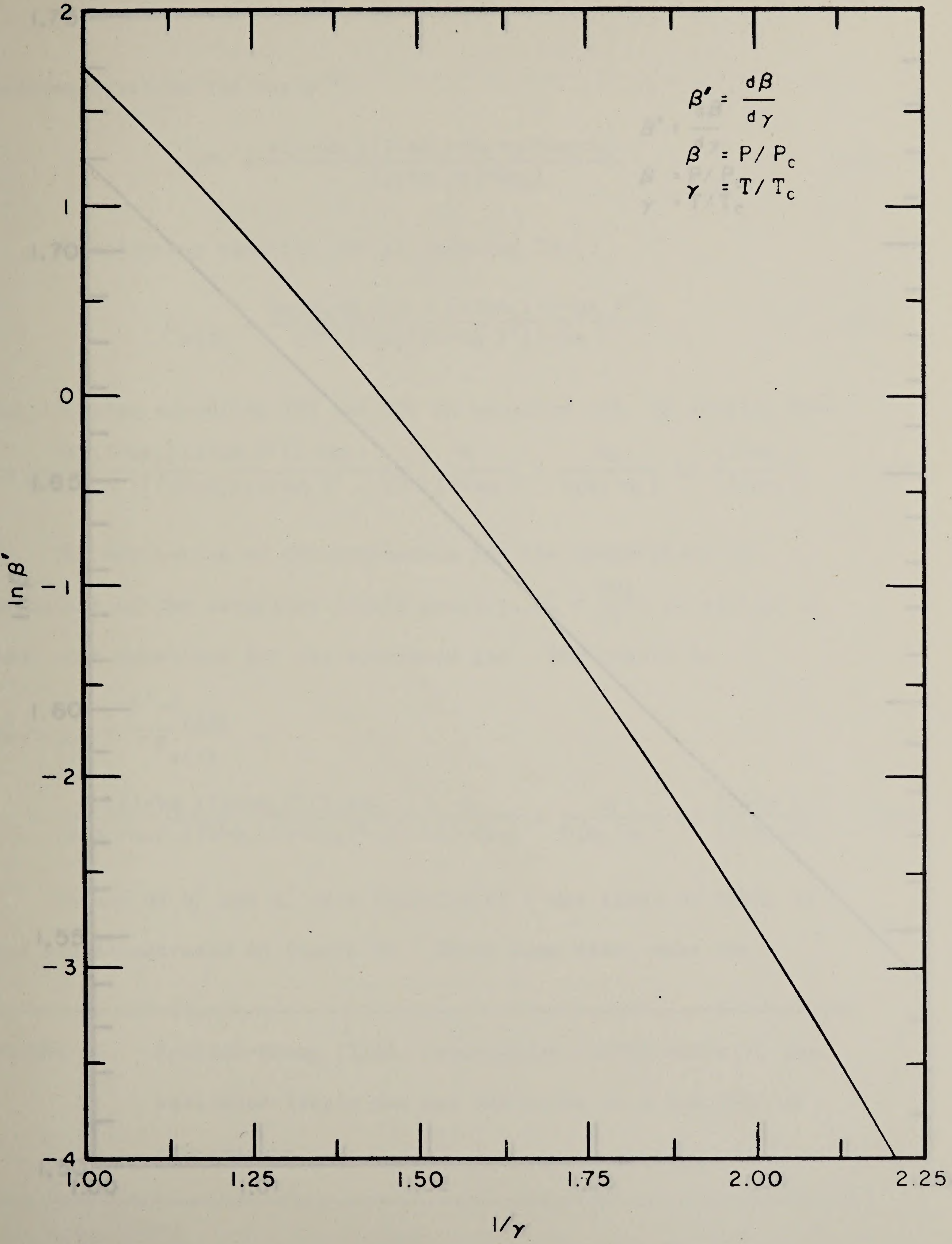

FIGURE 22. - Redlich-Kwong Fluid, Logarithm of the Temperature Coefficient of the Vapor Pressure Curve as a Function of Reciprocal Temperature.

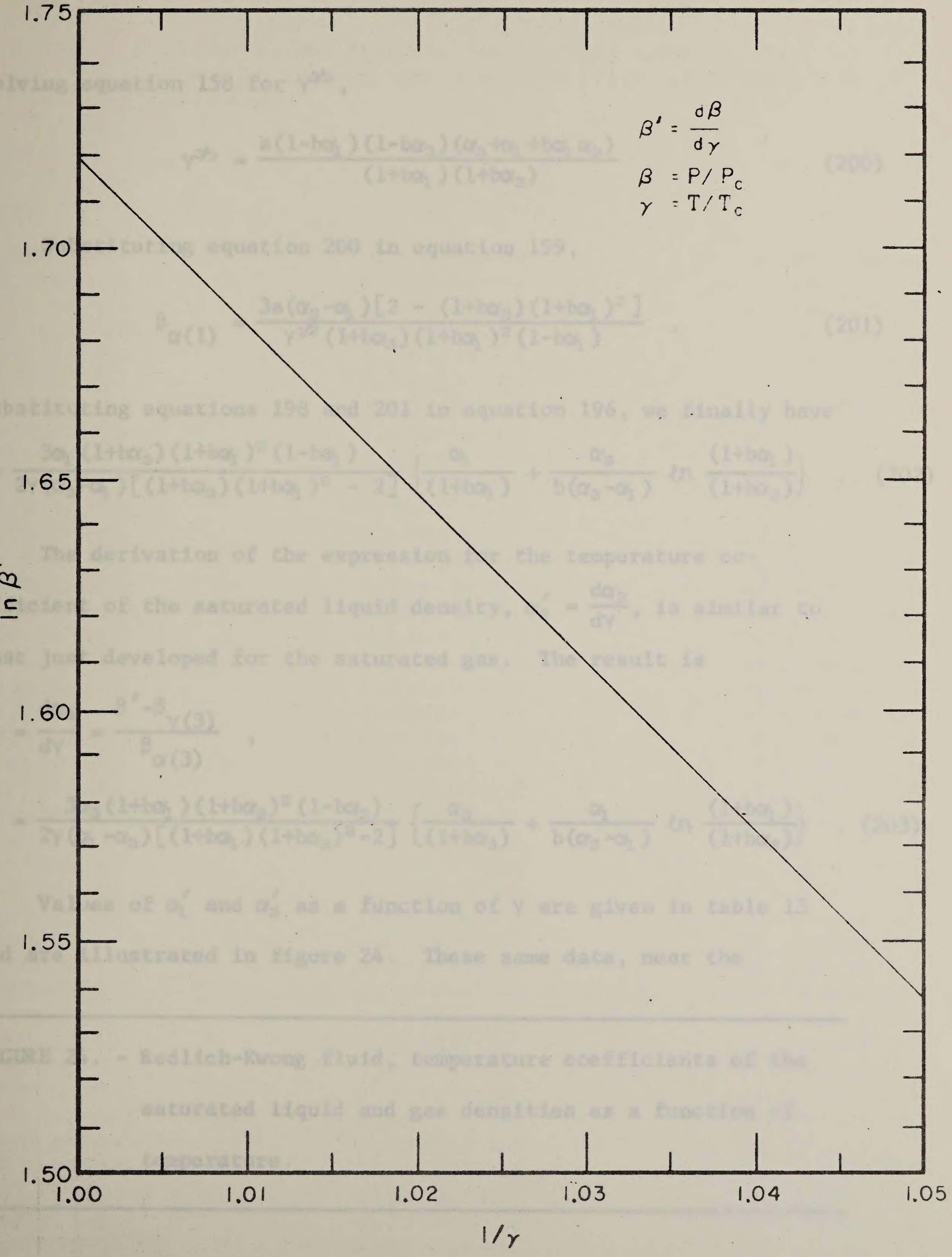

FIGURE 23. - Redlich-Kwong Fluid, Logarithm of the Temperature Coefficient of the Vapor Pressure Curve as a Function of Reciprocal Temperature Near the Critical Point.

Solving equation 158 for $\gamma^{3/2}$,

$$\gamma^{3/2} = \frac{a(1-b\alpha_1)(1-b\alpha_3)(\alpha_3+\alpha_1+b\alpha_1\alpha_3)}{(1+b\alpha_1)(1+b\alpha_3)} \quad . \tag{200}$$

Substituting equation 200 in equation 199,

$$\beta_{\alpha(1)} = \frac{3a(\alpha_3-\alpha_1)[2 - (1+b\alpha_3)(1+b\alpha_1)^2]}{\gamma^{1/2}(1+b\alpha_3)(1+b\alpha_1)^2(1-b\alpha_1)} \quad . \tag{201}$$

Substituting equations 198 and 201 in equation 196, we finally have

$$\alpha_1' = \frac{3\alpha_1(1+b\alpha_3)(1+b\alpha_1)^2(1-b\alpha_1)}{2\gamma(\alpha_3-\alpha_1)[(1+b\alpha_3)(1+b\alpha_1)^2 - 2]}\left\{\frac{\alpha_1}{(1+b\alpha_1)} + \frac{\alpha_3}{b(\alpha_3-\alpha_1)}\ln\frac{(1+b\alpha_1)}{(1+b\alpha_3)}\right\} \quad . \tag{202}$$

The derivation of the expression for the temperature coefficient of the saturated liquid density, $\alpha_3' = \frac{d\alpha_3}{d\gamma}$, is similar to that just developed for the saturated gas. The result is

$$\alpha_3' = \frac{d\alpha_3}{d\gamma} = \frac{\beta' - \beta_{\gamma(3)}}{\beta_{\alpha(3)}} \quad ,$$

$$= \frac{3\alpha_3(1+b\alpha_1)(1+b\alpha_3)^2(1-b\alpha_3)}{2\gamma(\alpha_1-\alpha_3)[(1+b\alpha_1)(1+b\alpha_3)^2-2]}\left\{\frac{\alpha_3}{(1+b\alpha_3)} + \frac{\alpha_1}{b(\alpha_3-\alpha_1)}\ln\frac{(1+b\alpha_1)}{(1+b\alpha_3)}\right\} \quad . \tag{203}$$

Values of $\alpha_1'$ and $\alpha_3'$ as a function of $\gamma$ are given in table 13 and are illustrated in figure 24. These same data, near the

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FIGURE 24. - Redlich-Kwong fluid, temperature coefficients of the saturated liquid and gas densities as a function of temperature.

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TABLE 13. - REDLICH-KWONG FLUID, TEMPERATURE COEFFICIENTS OF THE SATURATED LIQUID AND GAS DENSITIES AS A FUNCTION OF TEMPERATURE

$\Upsilon = T/T_c$ $\alpha = \rho/\rho_c$ $\alpha' = \frac{d\alpha}{d\Upsilon}$

| $\Upsilon$ | $\alpha_1'$ | $\alpha_3'$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |
| 1.00000E-01 | 5.13765E-42 | -7.49513E-01 |
| 1.50000E-01 | 2.51964E-21 | -9.28538E-01 |
| 2.00000E-01 | 7.13688E-13 | -1.08748E-00 |
| 2.50000E-01 | 1.51187E-08 | -1.23662E-00 |
| 3.00000E-01 | 4.94822E-06 | -1.38192E-00 |
| 3.50000E-01 | 1.90465E-04 | -1.52777E-00 |
| 4.00000E-01 | 2.16643E-03 | -1.67811E-00 |
| 4.50000E-01 | 1.16883E-02 | -1.83694E-00 |
| 5.00000E-01 | 3.91147E-02 | -2.00860E-00 |
| 5.50000E-01 | 9.56997E-02 | -2.19798E-00 |
| 6.00000E-01 | 1.90295E-01 | -2.41099E-00 |
| 6.50000E-01 | 3.29859E-01 | -2.65546E-00 |
| 7.00000E-01 | 5.22812E-01 | -2.94293E-00 |
| 7.50000E-01 | 7.84099E-01 | -3.29211E-00 |
| 8.00000E-01 | 1.14387E-00 | -3.73661E-00 |
| 8.50000E-01 | 1.66868E-00 | -4.34586E-00 |
| 9.00000E-01 | 2.53427E-00 | -5.29781E-00 |
| 9.50000E-01 | 4.43884E-00 | -7.29240E-00 |
| 9.52000E-01 | 4.57124E-00 | -7.42850E-00 |
| 9.54000E-01 | 4.71201E-00 | -7.57297E-00 |
| 9.56000E-01 | 4.86208E-00 | -7.72675E-00 |
| 9.58000E-01 | 5.02254E-00 | -7.89092E-00 |
| 9.60000E-01 | 5.19466E-00 | -8.06677E-00 |
| 9.62000E-01 | 5.37995E-00 | -8.25580E-00 |
| 9.64000E-01 | 5.58021E-00 | -8.45980E-00 |
| 9.66000E-01 | 5.79759E-00 | -8.68094E-00 |
| 9.68000E-01 | 6.03472E-00 | -8.92183E-00 |
| 9.70000E-01 | 6.29482E-00 | -9.18571E-00 |
| 9.72000E-01 | 6.58192E-00 | -9.47658E-00 |
| 9.74000E-01 | 6.90108E-00 | -9.79952E-00 |
| 9.76000E-01 | 7.25882E-00 | -1.01610E+01 |
| 9.78000E-01 | 7.66371E-00 | -1.05697E+01 |
| 9.80000E-01 | 8.12722E-00 | -1.10370E+01 |
| 9.82000E-01 | 8.66515E-00 | -1.15788E+01 |
| 9.84000E-01 | 9.30006E-00 | -1.22175E+01 |
| 9.86000E-01 | 1.00653E+01 | -1.29867E+01 |
| 9.88000E-01 | 1.10132E+01 | -1.39384E+01 |
| 9.90000E-01 | 1.22308E+01 | -1.51599E+01 |
| 9.92000E-01 | 1.38776E+01 | -1.68105E+01 |
| 9.94000E-01 | 1.62865E+01 | -1.92234E+01 |
| 9.96000E-01 | 2.03204E+01 | -2.32611E+01 |
| 9.98000E-01 | 2.94087E+01 | -3.23533E+01 |
| 9.99000E-01 | 4.22445E+01 | -4.51910E+01 |
| 1.00000E-00 | $+\infty$ | $-\infty$ |

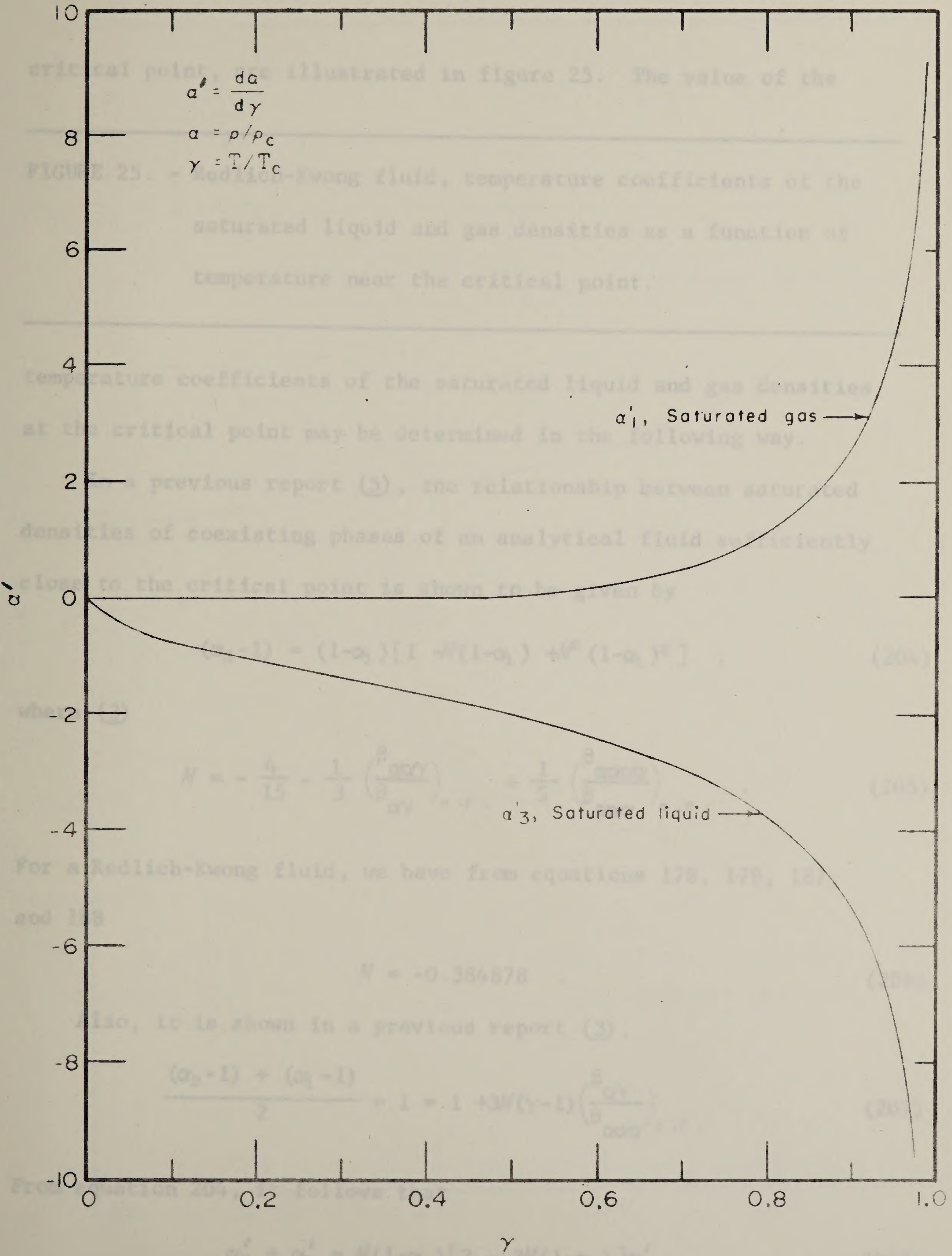

FIGURE 24.- Redlich-Kwong Fluid, Temperature Coefficients of the Saturated Liquid and Gas Densities as a Function of Temperature.

critical point, are illustrated in figure 25. The value of the

FIGURE 25. - Redlich-Kwong fluid, temperature coefficients of the saturated liquid and gas densities as a function of temperature near the critical point.

temperature coefficients of the saturated liquid and gas densities at the critical point may be determined in the following way.

In a previous report (5), the relationship between saturated densities of coexisting phases of an analytical fluid sufficiently close to the critical point is shown to be given by

$$(\alpha_3-1) = (1-\alpha_1)[1 - M(1-\alpha_1) + M^2(1-\alpha_1)^2] \quad , \tag{204}$$

where (3)

$$M = -\frac{4}{15} - \frac{1}{3}\left(\frac{\beta_{\alpha\alpha\gamma}}{\beta_{\alpha\gamma}}\right)_{c.p.} + \frac{1}{5}\left(\frac{\beta_{\alpha\alpha\alpha\alpha}}{\beta_{\alpha\alpha\alpha}}\right)_{c.p.} \quad . \tag{205}$$

For a Redlich-Kwong fluid, we have from equations 178, 179, 187, and 188

$$M = -0.384878 \quad . \tag{206}$$

Also, it is shown in a previous report (3),

$$\frac{(\alpha_3-1) + (\alpha_1-1)}{2} + 1 = 1 + 3M(\gamma-1)\left(\frac{\beta_{\alpha\gamma}}{\beta_{\alpha\alpha\alpha}}\right)_{c.p.} \quad . \tag{207}$$

From equation 204, it follows that

$$\alpha_3' + \alpha_1' = M(1-\alpha_1)[2 - 3M(1-\alpha_1)]\alpha_1' \quad . \tag{208}$$

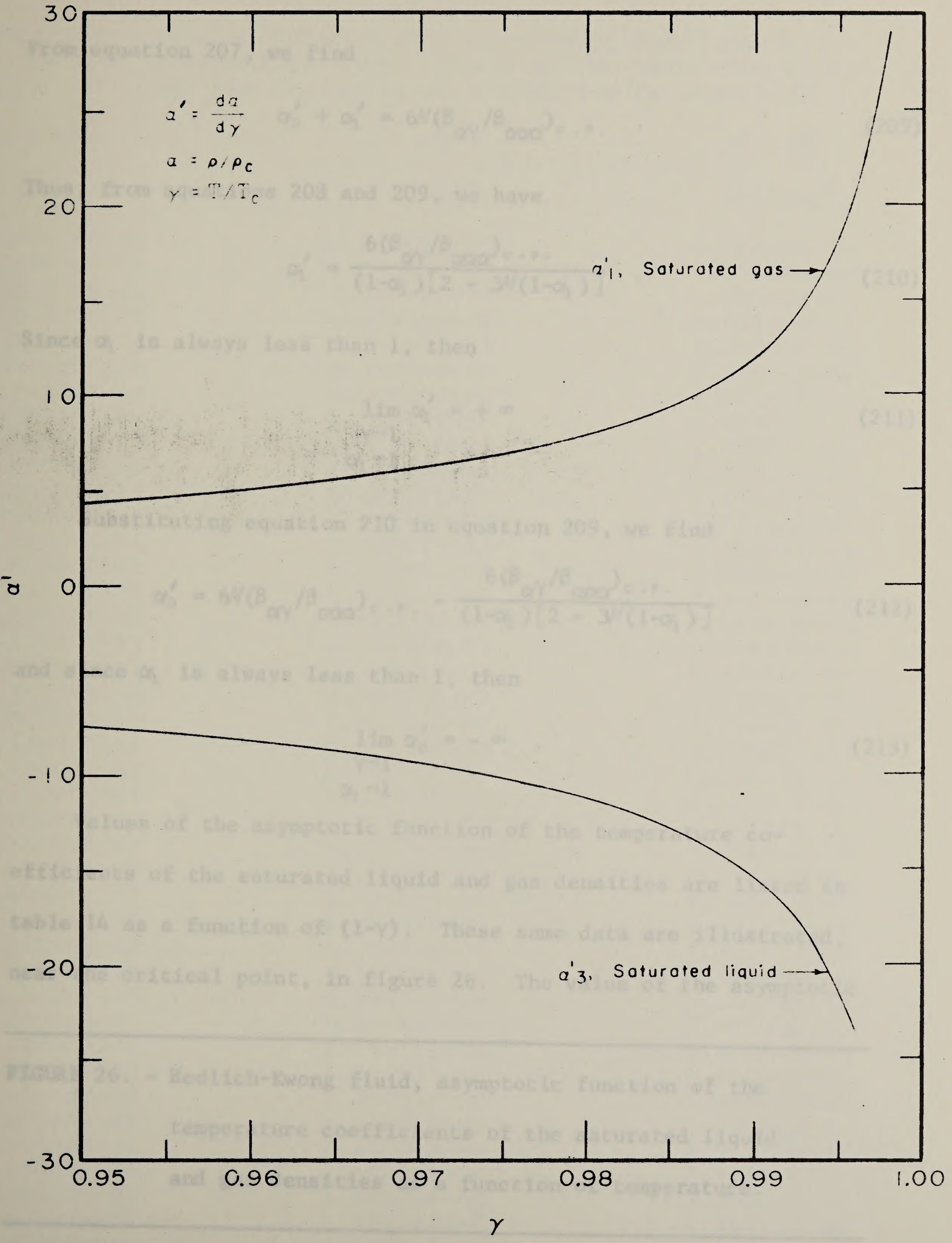

FIGURE 25. - Redlich-Kwong Fluid, Temperature Coefficients of the Saturated Liquid and Gas Densities as a Function of Temperature Near the Critical Point.

From equation 207, we find

$$\alpha_3' + \alpha_1' = 6M(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.} \quad . \tag{209}$$

Thus, from equations 208 and 209, we have

$$\alpha_1' = \frac{6(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.}}{(1-\alpha_1)[2 - 3M(1-\alpha_1)]} \quad . \tag{210}$$

Since $\alpha_1$ is always less than 1, then

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \alpha_1' = +\infty \quad . \tag{211}$$

Substituting equation 210 in equation 209, we find

$$\alpha_3' = 6M(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.} - \frac{6(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.}}{(1-\alpha_1)[2 - 3M(1-\alpha_1)]} \tag{212}$$

and since $\alpha_1$ is always less than 1, then

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \alpha_3' = -\infty \quad . \tag{213}$$

Values of the asymptotic function of the temperature coefficients of the saturated liquid and gas densities are listed in table 14 as a function of (1-$\gamma$). These same data are illustrated, near the critical point, in figure 26. The value of the asymptotic

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FIGURE 26. - Redlich-Kwong fluid, asymptotic function of the temperature coefficients of the saturated liquid and gas densities as a function of temperature.

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function of the temperature coefficients of the saturated liquid

TABLE 14. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE TEMPERATURE COEFFICIENTS OF THE SATURATED LIQUID AND GAS DENSITIES AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\alpha = \rho/\rho_c$ | $\alpha' = \frac{d\alpha}{d\gamma}$ |
|---|---|---|
| $(1-\gamma)$ | $(1-\gamma)(\alpha_1')^2$ | $(1-\gamma)(\alpha_3')^2$ |
| 1.00000E-00 | 0.00000E-99 | 0.00000E-99 |
| 9.00000E-01 | 2.37559E-83 | 5.05592E-01 |
| 8.50000E-01 | 5.39632E-42 | 7.32855E-01 |
| 8.00000E-01 | 4.07480E-25 | 9.46104E-01 |
| 7.50000E-01 | 1.71431E-16 | 1.14693E-00 |
| 7.00000E-01 | 1.71394E-11 | 1.33680E-00 |
| 6.50000E-01 | 2.35800E-08 | 1.51716E-00 |
| 6.00000E-01 | 2.81607E-06 | 1.68964E-00 |
| 5.50000E-01 | 7.51390E-05 | 1.85589E-00 |
| 5.00000E-01 | 7.64983E-04 | 2.01724E-00 |
| 4.50000E-01 | 4.12130E-03 | 2.17402E-00 |
| 4.00000E-01 | 1.44848E-02 | 2.32516E-00 |
| 3.50000E-01 | 3.80825E-02 | 2.46802E-00 |
| 3.00000E-01 | 8.19999E-02 | 2.59826E-00 |
| 2.50000E-01 | 1.53703E-01 | 2.70950E-00 |
| 2.00000E-01 | 2.61689E-01 | 2.79245E-00 |
| 1.50000E-01 | 4.17678E-01 | 2.83297E-00 |
| 1.00000E-01 | 6.42252E-01 | 2.80668E-00 |
| 5.00000E-02 | 9.85167E-01 | 2.65895E-00 |
| 4.80000E-02 | 1.00302E-00 | 2.64876E-00 |
| 4.60000E-02 | 1.02134E-00 | 2.63809E-00 |
| 4.40000E-02 | 1.04015E-00 | 2.62691E-00 |
| 4.20000E-02 | 1.05948E-00 | 2.61520E-00 |
| 4.00000E-02 | 1.07938E-00 | 2.60291E-00 |
| 3.80000E-02 | 1.09986E-00 | 2.59001E-00 |
| 3.60000E-02 | 1.12099E-00 | 2.57646E-00 |
| 3.40000E-02 | 1.14281E-00 | 2.56219E-00 |
| 3.20000E-02 | 1.16537E-00 | 2.54717E-00 |
| 3.00000E-02 | 1.18874E-00 | 2.53131E-00 |
| 2.80000E-02 | 1.21300E-00 | 2.51455E-00 |
| 2.60000E-02 | 1.23824E-00 | 2.49679E-00 |
| 2.40000E-02 | 1.26457E-00 | 2.47793E-00 |
| 2.20000E-02 | 1.29211E-00 | 2.45783E-00 |
| 2.00000E-02 | 1.32103E-00 | 2.43634E-00 |
| 1.80000E-02 | 1.35152E-00 | 2.41325E-00 |
| 1.60000E-02 | 1.38385E-00 | 2.38830E-00 |
| 1.40000E-02 | 1.41836E-00 | 2.36116E-00 |
| 1.20000E-02 | 1.45550E-00 | 2.33136E-00 |
| 1.00000E-02 | 1.49594E-00 | 2.29824E-00 |
| 8.00000E-03 | 1.54071E-00 | 2.26076E-00 |
| 6.00000E-03 | 1.59151E-00 | 2.21723E-00 |
| 4.00000E-03 | 1.65168E-00 | 2.16432E-00 |
| 2.00000E-03 | 1.72975E-00 | 2.09348E-00 |
| 1.00000E-03 | 1.78460E-00 | 2.04223E-00 |
| 0.00000E-99 | 1.91521E-00 | 1.91521E-00 |

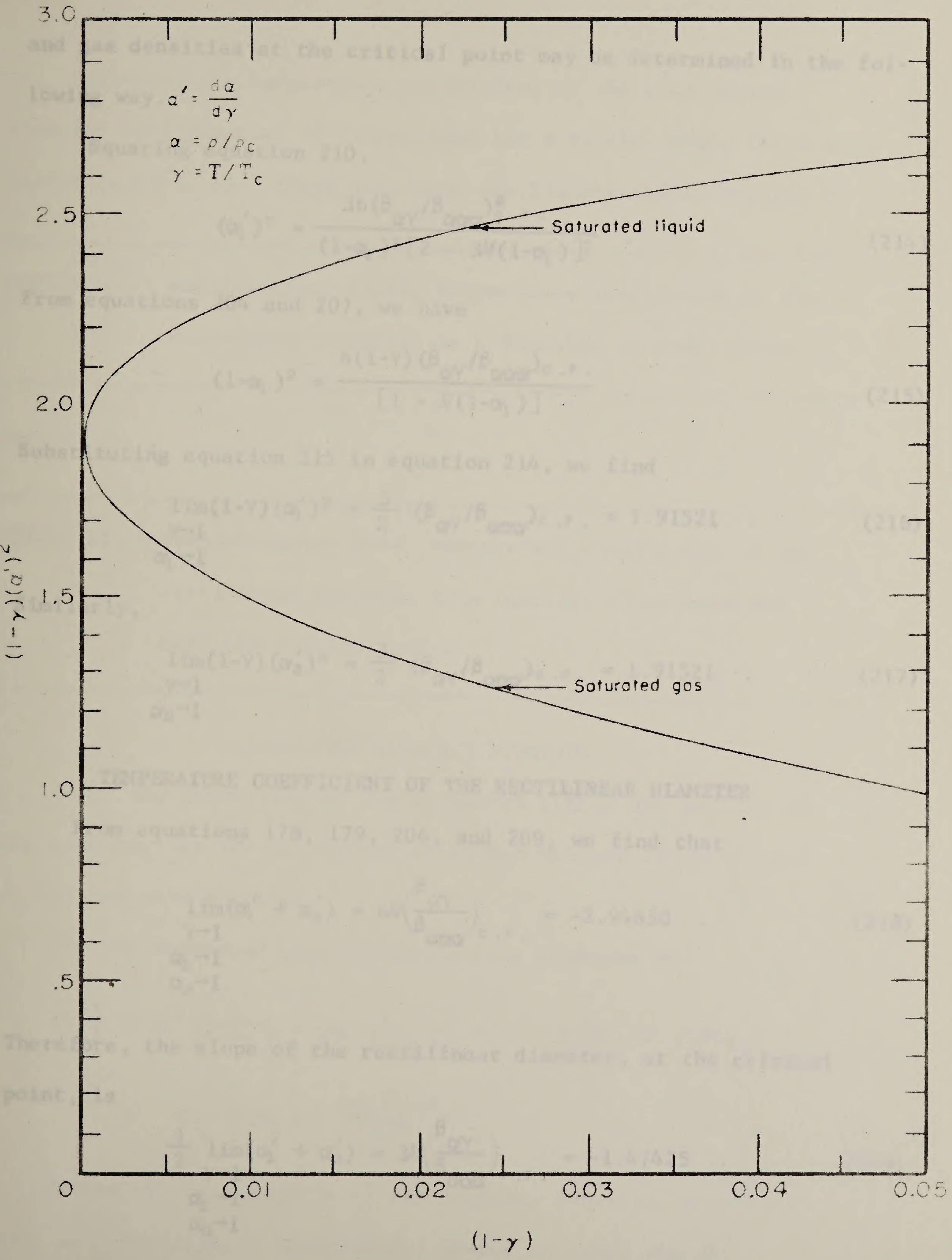

FIGURE 26.- Redlich-Kwong Fluid, Asymptotic Function of the Temperature Coefficients of the Saturated Liquid and Gas Densities as a Function of Temperature.

and gas densities at the critical point may be determined in the following way.

Squaring equation 210,

$$(\alpha_1')^2 = \frac{36(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})^2_{c.p.}}{(1-\alpha_1)^2[2 - 3M(1-\alpha_1)]^2} \quad . \tag{214}$$

From equations 204 and 207, we have

$$(1-\alpha_1)^2 = \frac{6(1-\gamma)(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.}}{[1 - M(1-\alpha_1)]} \quad . \tag{215}$$

Substituting equation 215 in equation 214, we find

$$\lim_{\substack{\gamma\to1\\ \alpha_1\to1}}(1-\gamma)(\alpha_1')^2 = \frac{3}{2}(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.} = 1.91521 \quad . \tag{216}$$

Similarly,

$$\lim_{\substack{\gamma\to1\\ \alpha_3\to1}}(1-\gamma)(\alpha_3')^2 = \frac{3}{2}(\beta_{\alpha\gamma}/\beta_{\alpha\alpha\alpha})_{c.p.} = 1.91521 \quad . \tag{217}$$

## TEMPERATURE COEFFICIENT OF THE RECTILINEAR DIAMETER

From equations 178, 179, 206, and 209, we find that

$$\lim_{\substack{\gamma\to1\\ \alpha_1\to1\\ \alpha_3\to1}}(\alpha_1' + \alpha_3') = 6M\left(\frac{\beta_{\alpha\gamma}}{\beta_{\alpha\alpha\alpha}}\right)_{c.p.} = -2.94850 \quad . \tag{218}$$

Therefore, the slope of the rectilinear diameter, at the critical point, is

$$\frac{1}{2}\lim_{\substack{\gamma\to1\\ \alpha_1\to1\\ \alpha_3\to1}}(\alpha_1' + \alpha_3') = 3M\left(\frac{\beta_{\alpha\gamma}}{\beta_{\alpha\alpha\alpha}}\right)_{c.p.} = -1.47425 \quad . \tag{219}$$

Values of the temperature coefficient of the rectilinear diameter as a function of temperature for a Redlich-Kwong fluid are listed in table 15. These same data are illustrated in figures 27 and 28.

FIGURE 27. - Redlich-Kwong fluid, temperature coefficient of the rectilinear diameter as a function of temperature.

FIGURE 28. - Redlich-Kwong fluid, temperature coefficient of the rectilinear diameter as a function of temperature near the critical point.

## SATURATED FUGACITY FUNCTION

Since the fugacities of the saturated liquid and gas are the same, one may use the properties of either the gas or the liquid to evaluate this function.

For the saturated gas, we obtain from equation 69

$$\ell n \frac{f}{P} = \alpha_1 \left[ \frac{b}{(1-b\alpha_1)} - \frac{a}{\gamma^{3/2}(1+b\alpha_1)} \right] + \left[ 1 - \frac{a}{b\gamma^{3/2}} \right] \ell n\,(1+b\alpha_1)$$

$$- \ell n \left[ (1+b\alpha_1) - \frac{a\alpha_1 (1-b\alpha_1)}{\gamma^{3/2}} \right] . \qquad (220)$$

Values of the logarithm of the fugacity function, at saturation, as a function of temperature, listed in table 16, are

TABLE 15. - REDLICH-KWONG FLUID, TEMPERATURE COEFFICIENT OF THE RECTILINEAR DIAMETER AS A FUNCTION OF TEMPERATURE

$Y = T/T_c$ $\alpha = \rho/\rho_c$ $\alpha' = \frac{d\alpha}{dY}$

| Y | $\frac{\alpha_1' + \alpha_3'}{2}$ |
|---|---|
| 0.00000E-99 | 0.00000E-99 |
| 1.00000E-01 | -3.74756E-01 |
| 1.50000E-01 | -4.64269E-01 |
| 2.00000E-01 | -5.43744E-01 |
| 2.50000E-01 | -6.18314E-01 |
| 3.00000E-01 | -6.90960E-01 |
| 3.50000E-01 | -7.63793E-01 |
| 4.00000E-01 | -8.37974E-01 |
| 4.50000E-01 | -9.12627E-01 |
| 5.00000E-01 | -9.84745E-01 |
| 5.50000E-01 | -1.05114E-00 |
| 6.00000E-01 | -1.11035E-00 |
| 6.50000E-01 | -1.16280E-00 |
| 7.00000E-01 | -1.21006E-00 |
| 7.50000E-01 | -1.25400E-00 |
| 8.00000E-01 | -1.29637E-00 |
| 8.50000E-01 | -1.33858E-00 |
| 9.00000E-01 | -1.38177E-00 |
| 9.50000E-01 | -1.42677E-00 |
| 9.52000E-01 | -1.42862E-00 |
| 9.54000E-01 | -1.43047E-00 |
| 9.56000E-01 | -1.43233E-00 |
| 9.58000E-01 | -1.43419E-00 |
| 9.60000E-01 | -1.43605E-00 |
| 9.62000E-01 | -1.43792E-00 |
| 9.64000E-01 | -1.43979E-00 |
| 9.66000E-01 | -1.44167E-00 |
| 9.68000E-01 | -1.44355E-00 |
| 9.70000E-01 | -1.44544E-00 |
| 9.72000E-01 | -1.44733E-00 |
| 9.74000E-01 | -1.44922E-00 |
| 9.76000E-01 | -1.45112E-00 |
| 9.78000E-01 | -1.45302E-00 |
| 9.80000E-01 | -1.45493E-00 |
| 9.82000E-01 | -1.45684E-00 |
| 9.84000E-01 | -1.45875E-00 |
| 9.86000E-01 | -1.46067E-00 |
| 9.88000E-01 | -1.46260E-00 |
| 9.90000E-01 | -1.46453E-00 |
| 9.92000E-01 | -1.46647E-00 |
| 9.94000E-01 | -1.46840E-00 |
| 9.96000E-01 | -1.47035E-00 |
| 9.98000E-01 | -1.47229E-00 |
| 9.99000E-01 | -1.47326E-00 |
| 1.00000E-00 | -1.47425E-00 |

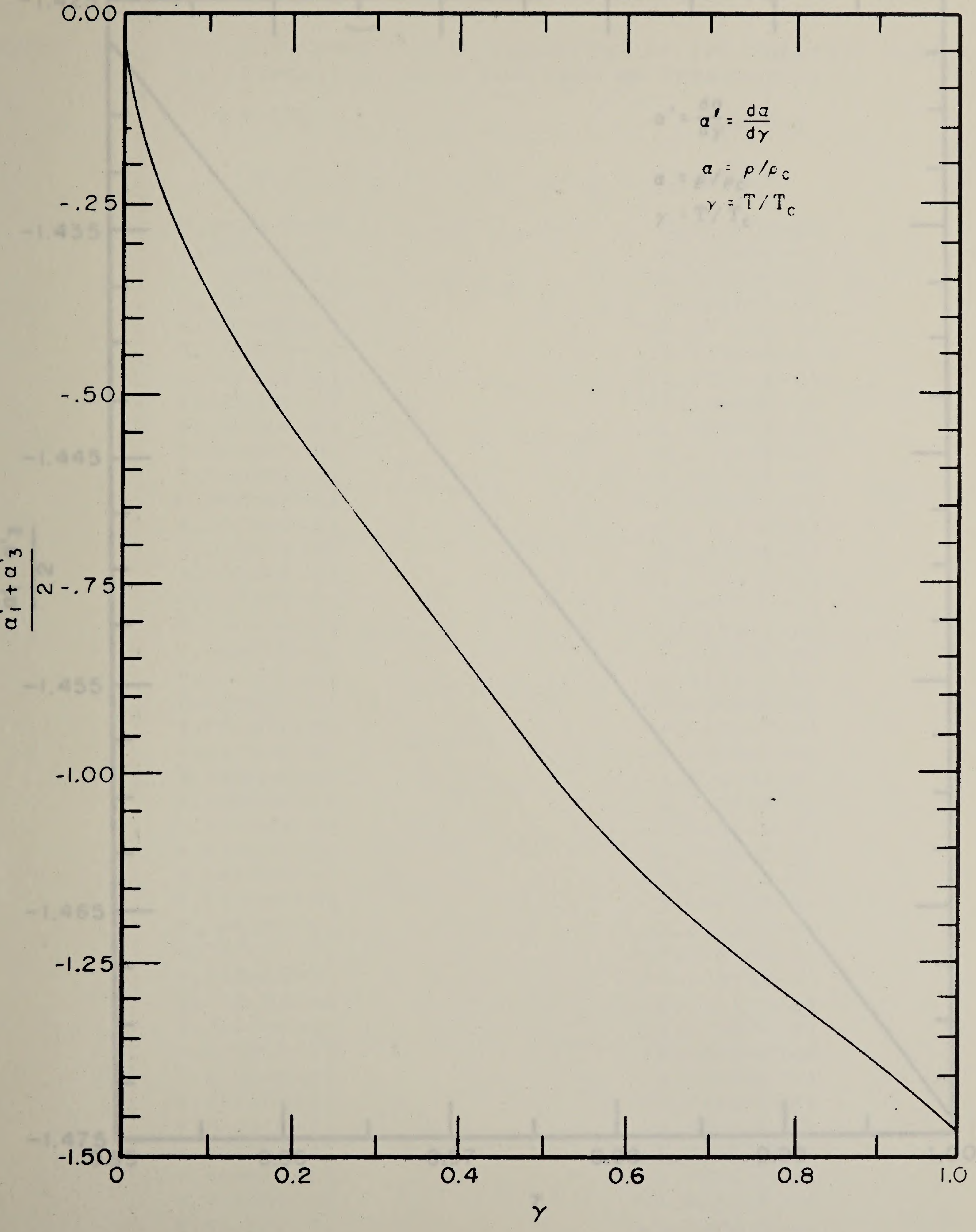

FIGURE 27.-Redlich-Kwong Fluid, Temperature Coefficient of the Rectilinear Diameter as a Function of Temperature.

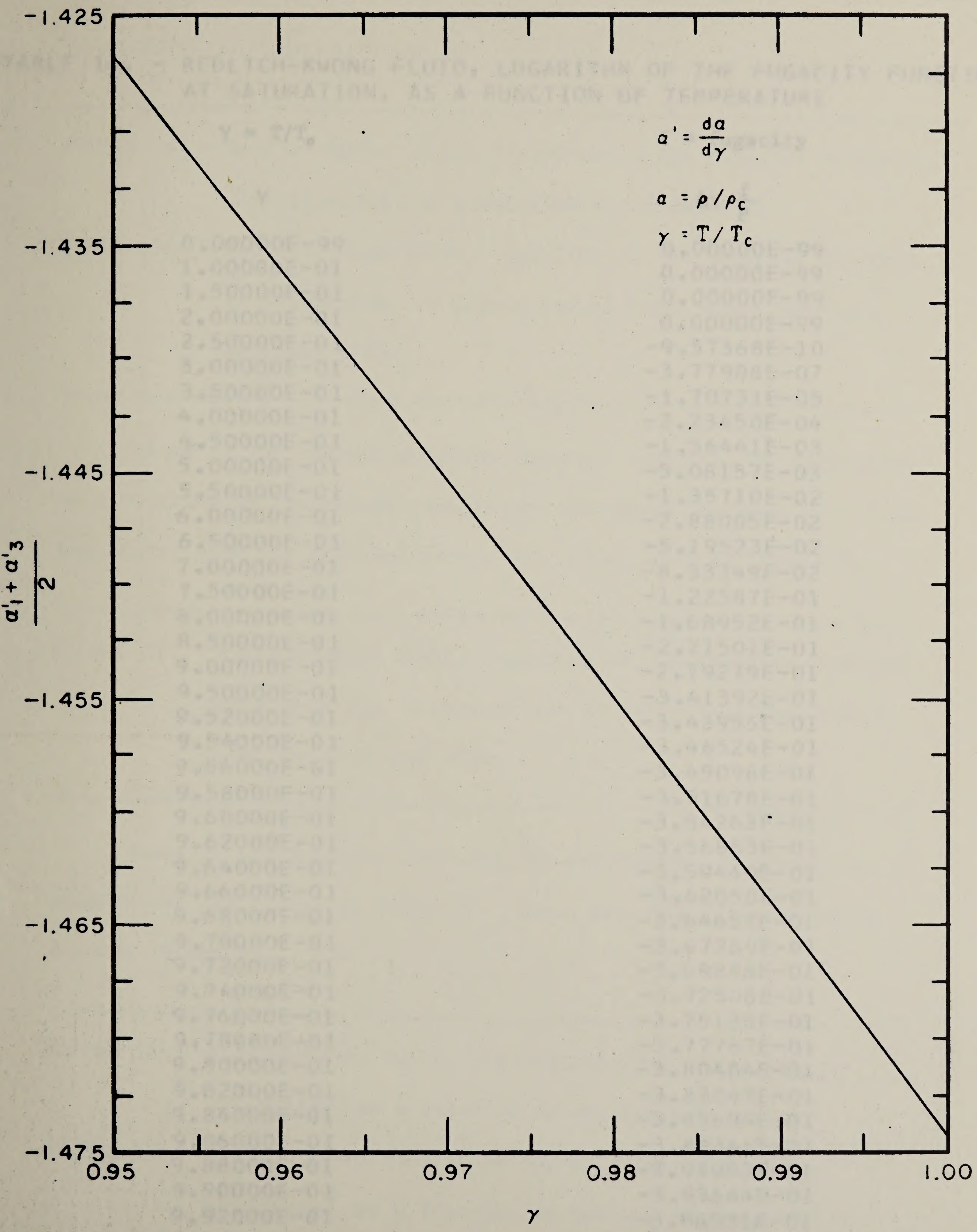

FIGURE 28. - Redlich-Kwong Fluid, Temperature Coefficient of the Rectilinear Diameter as a Function of Temperature Near the Critical Point.

## TABLE 16. - REDLICH-KWONG FLUID, LOGARITHM OF THE FUGACITY FUNCTION, AT SATURATION, AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad$ f = fugacity

| $\gamma$ | $\ln \frac{f}{P}$ |
|---|---|
| 0.00000E-99 | 0.00000E-99 |
| 1.00000E-01 | 0.00000E-99 |
| 1.50000E-01 | 0.00000E-99 |
| 2.00000E-01 | 0.00000E-99 |
| 2.50000E-01 | -9.57368E-10 |
| 3.00000E-01 | -3.77908E-07 |
| 3.50000E-01 | -1.70731E-05 |
| 4.00000E-01 | -2.23450E-04 |
| 4.50000E-01 | -1.36441E-03 |
| 5.00000E-01 | -5.08157E-03 |
| 5.50000E-01 | -1.35710E-02 |
| 6.00000E-01 | -2.88005E-02 |
| 6.50000E-01 | -5.19523E-02 |
| 7.00000E-01 | -8.33349E-02 |
| 7.50000E-01 | -1.22587E-01 |
| 8.00000E-01 | -1.68952E-01 |
| 8.50000E-01 | -2.21501E-01 |
| 9.00000E-01 | -2.79279E-01 |
| 9.50000E-01 | -3.41392E-01 |
| 9.52000E-01 | -3.43955E-01 |
| 9.54000E-01 | -3.46524E-01 |
| 9.56000E-01 | -3.49098E-01 |
| 9.58000E-01 | -3.51678E-01 |
| 9.60000E-01 | -3.54263E-01 |
| 9.62000E-01 | -3.56853E-01 |
| 9.64000E-01 | -3.59449E-01 |
| 9.66000E-01 | -3.62050E-01 |
| 9.68000E-01 | -3.64657E-01 |
| 9.70000E-01 | -3.67269E-01 |
| 9.72000E-01 | -3.69886E-01 |
| 9.74000E-01 | -3.72508E-01 |
| 9.76000E-01 | -3.75135E-01 |
| 9.78000E-01 | -3.77767E-01 |
| 9.80000E-01 | -3.80404E-01 |
| 9.82000E-01 | -3.83047E-01 |
| 9.84000E-01 | -3.85694E-01 |
| 9.86000E-01 | -3.88346E-01 |
| 9.88000E-01 | -3.91003E-01 |
| 9.90000E-01 | -3.93664E-01 |
| 9.92000E-01 | -3.96331E-01 |
| 9.94000E-01 | -3.99002E-01 |
| 9.96000E-01 | -4.01677E-01 |
| 9.98000E-01 | -4.04358E-01 |
| 9.99000E-01 | -4.05700E-01 |
| 1.00000E-00 | -4.07043E-01 |

illustrated in figures 29 and 30.

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FIGURE 29. - Redlich-Kwong fluid, logarithm of the fugacity function, at saturation, as a function of temperature.

FIGURE 30. - Redlich-Kwong fluid, logarithm of the fugacity function, at saturation, as a function of temperature near the critical point.

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Values of the logarithm of the fugacity function, at saturation, as a function of reciprocal temperature are given in table 17. These same data, near the critical point, are illustrated in figure 31.

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FIGURE 31. - Redlich-Kwong fluid, logarithm of the fugacity function, at saturation, as a function of reciprocal temperature near the critical point.

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Values of the fugacity function, at saturation, as a function of temperature are listed in table 18. These data are illustrated in figures 32 and 33.

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FIGURE 32. - Redlich-Kwong fluid, the fugacity function, at saturation, as a function of the temperature.

FIGURE 33. - Redlich-Kwong fluid, the fugacity function, at saturation, as a function of the temperature near the critical point.

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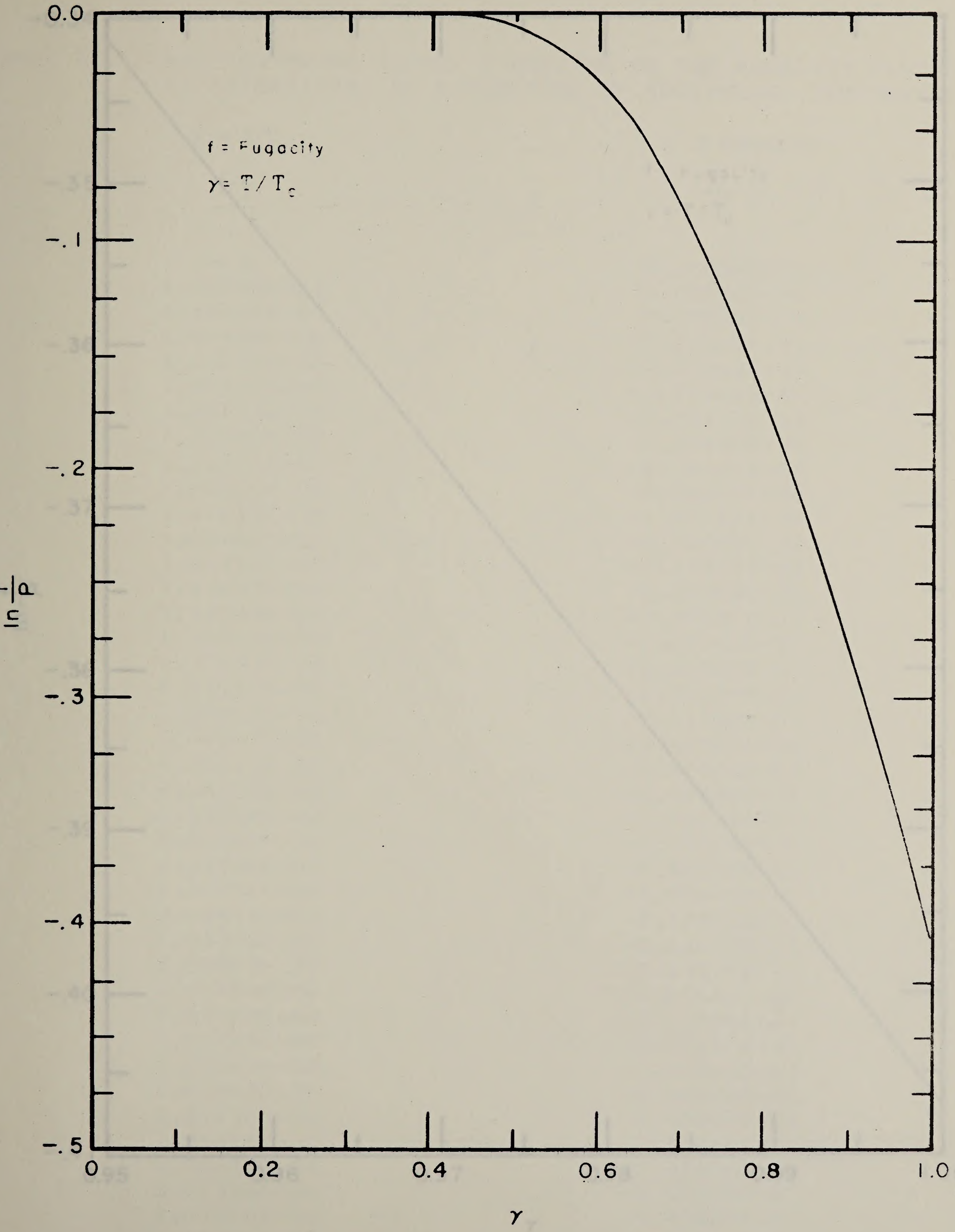

FIGURE 29. - Redlich-Kwong Fluid, Logarithm of the Fugacity Function, at Saturation, as a Function of Temperature.

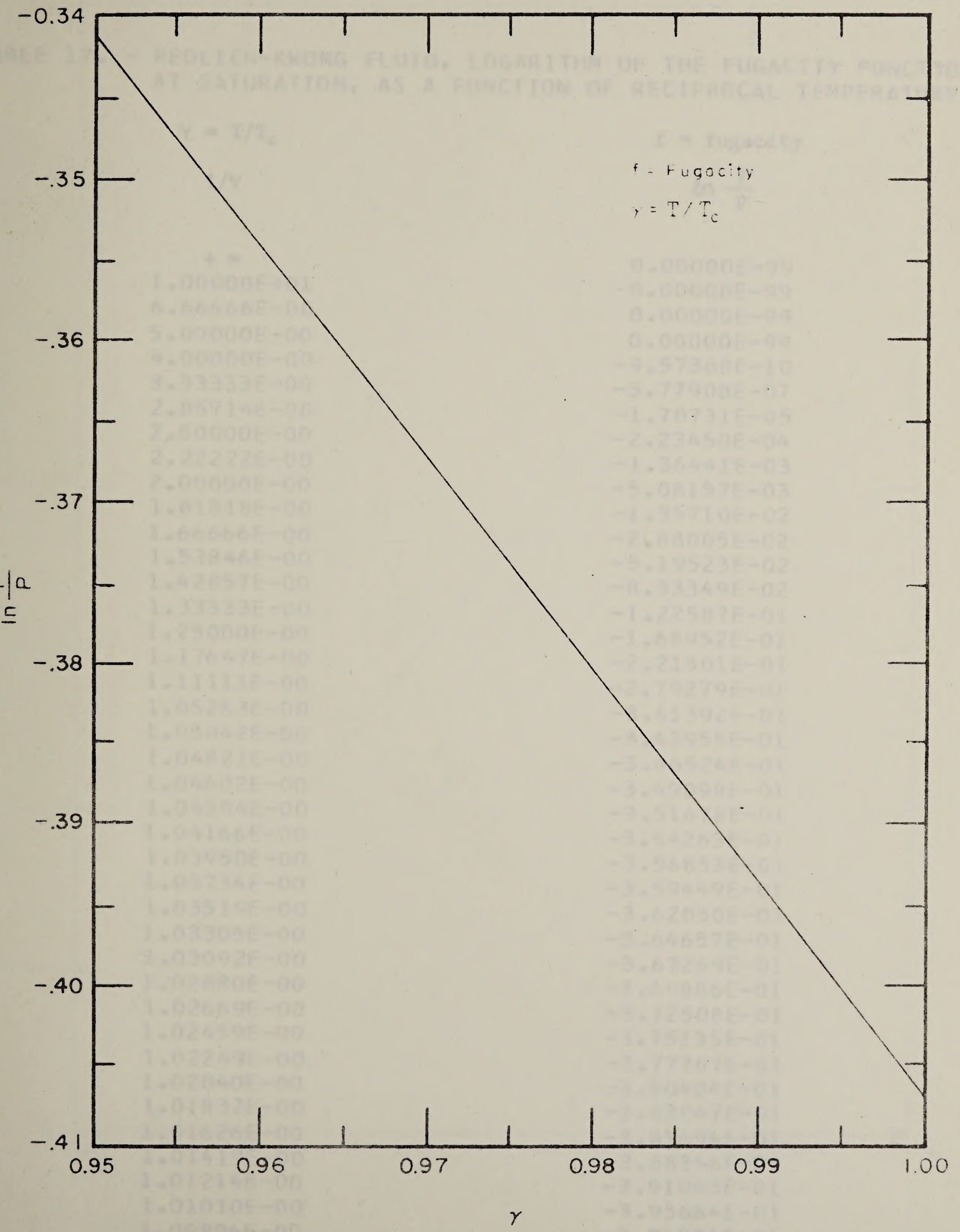

FIGURE 30. - Redlich-Kwong Fluid, Logarithm of the Fugacity Function, at Saturation, as a Function of Temperature Near the Critical Point.

TABLE 17. - REDLICH-KWONG FLUID, LOGARITHM OF THE FUGACITY FUNCTION, AT SATURATION, AS A FUNCTION OF RECIPROCAL TEMPERATURE

| $\gamma = T/T_c$ | f = fugacity |
|---|---|
| $1/\gamma$ | $\ln \frac{f}{P}$ |
| $+\infty$ | 0.00000E-99 |
| 1.00000E+01 | 0.00000E-99 |
| 6.66666E-00 | 0.00000E-99 |
| 5.00000E-00 | 0.00000E-99 |
| 4.00000E-00 | -9.57368E-10 |
| 3.33333E-00 | -3.77908E-07 |
| 2.85714E-00 | -1.70731E-05 |
| 2.50000E-00 | -2.23450E-04 |
| 2.22222E-00 | -1.36441E-03 |
| 2.00000E-00 | -5.08157E-03 |
| 1.81818E-00 | -1.35710E-02 |
| 1.66666E-00 | -2.88005E-02 |
| 1.53846E-00 | -5.19523E-02 |
| 1.42857E-00 | -8.33349E-02 |
| 1.33333E-00 | -1.22587E-01 |
| 1.25000E-00 | -1.68952E-01 |
| 1.17647E-00 | -2.21501E-01 |
| 1.11111E-00 | -2.79279E-01 |
| 1.05263E-00 | -3.41392E-01 |
| 1.05042E-00 | -3.43955E-01 |
| 1.04821E-00 | -3.46524E-01 |
| 1.04602E-00 | -3.49098E-01 |
| 1.04384E-00 | -3.51678E-01 |
| 1.04166E-00 | -3.54263E-01 |
| 1.03950E-00 | -3.56853E-01 |
| 1.03734E-00 | -3.59449E-01 |
| 1.03519E-00 | -3.62050E-01 |
| 1.03305E-00 | -3.64657E-01 |
| 1.03092E-00 | -3.67269E-01 |
| 1.02880E-00 | -3.69886E-01 |
| 1.02669E-00 | -3.72508E-01 |
| 1.02459E-00 | -3.75135E-01 |
| 1.02249E-00 | -3.77767E-01 |
| 1.02040E-00 | -3.80404E-01 |
| 1.01832E-00 | -3.83047E-01 |
| 1.01626E-00 | -3.85694E-01 |
| 1.01419E-00 | -3.88346E-01 |
| 1.01214E-00 | -3.91003E-01 |
| 1.01010E-00 | -3.93664E-01 |
| 1.00806E-00 | -3.96331E-01 |
| 1.00603E-00 | -3.99002E-01 |
| 1.00401E-00 | -4.01677E-01 |
| 1.00200E-00 | -4.04358E-01 |
| 1.00100E-00 | -4.05700E-01 |
| 1.00000E-00 | -4.07043E-01 |

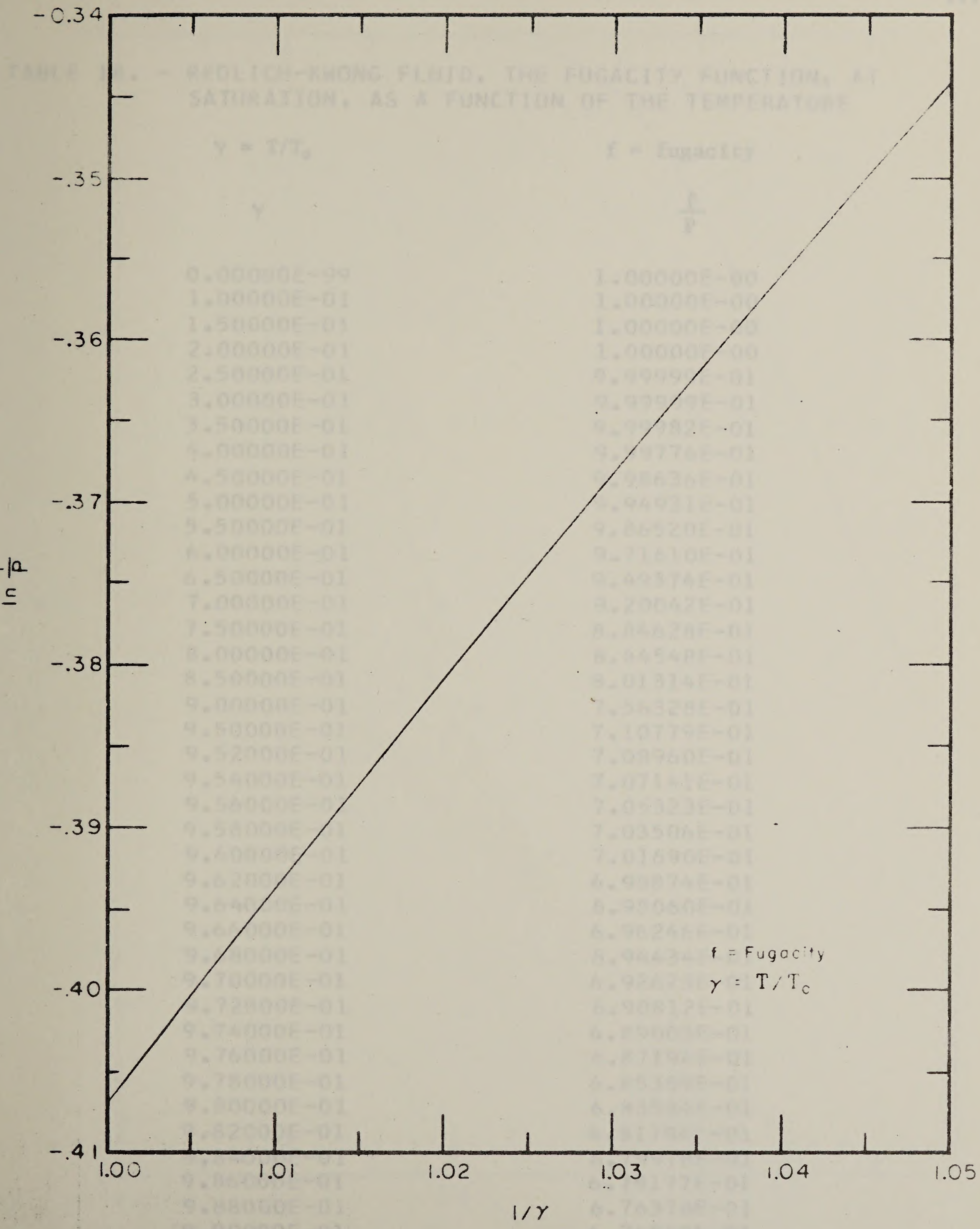

FIGURE 31. - Redlich-Kwong Fluid, Logarithm of the Fugacity Function, at Saturation, as a Function of Reciprocal Temperature Near the Critical Point.

TABLE 18. - REDLICH-KWONG FLUID, THE FUGACITY FUNCTION, AT SATURATION, AS A FUNCTION OF THE TEMPERATURE

| $\gamma = T/T_c$ | f = fugacity |
|---|---|
| $\gamma$ | $\frac{f}{P}$ |
| 0.00000E-99 | 1.00000E-00 |
| 1.00000E-01 | 1.00000E-00 |
| 1.50000E-01 | 1.00000E-00 |
| 2.00000E-01 | 1.00000E-00 |
| 2.50000E-01 | 9.99999E-01 |
| 3.00000E-01 | 9.99999E-01 |
| 3.50000E-01 | 9.99982E-01 |
| 4.00000E-01 | 9.99776E-01 |
| 4.50000E-01 | 9.98636E-01 |
| 5.00000E-01 | 9.94931E-01 |
| 5.50000E-01 | 9.86520E-01 |
| 6.00000E-01 | 9.71610E-01 |
| 6.50000E-01 | 9.49374E-01 |
| 7.00000E-01 | 9.20042E-01 |
| 7.50000E-01 | 8.84628E-01 |
| 8.00000E-01 | 8.44548E-01 |
| 8.50000E-01 | 8.01314E-01 |
| 9.00000E-01 | 7.56328E-01 |
| 9.50000E-01 | 7.10779E-01 |
| 9.52000E-01 | 7.08960E-01 |
| 9.54000E-01 | 7.07141E-01 |
| 9.56000E-01 | 7.05323E-01 |
| 9.58000E-01 | 7.03506E-01 |
| 9.60000E-01 | 7.01690E-01 |
| 9.62000E-01 | 6.99874E-01 |
| 9.64000E-01 | 6.98060E-01 |
| 9.66000E-01 | 6.96246E-01 |
| 9.68000E-01 | 6.94434E-01 |
| 9.70000E-01 | 6.92623E-01 |
| 9.72000E-01 | 6.90812E-01 |
| 9.74000E-01 | 6.89003E-01 |
| 9.76000E-01 | 6.87196E-01 |
| 9.78000E-01 | 6.85389E-01 |
| 9.80000E-01 | 6.83584E-01 |
| 9.82000E-01 | 6.81780E-01 |
| 9.84000E-01 | 6.79978E-01 |
| 9.86000E-01 | 6.78177E-01 |
| 9.88000E-01 | 6.76378E-01 |
| 9.90000E-01 | 6.74580E-01 |
| 9.92000E-01 | 6.72783E-01 |
| 9.94000E-01 | 6.70989E-01 |
| 9.96000E-01 | 6.69196E-01 |
| 9.98000E-01 | 6.67404E-01 |
| 9.99000E-01 | 6.66509E-01 |
| 1.00000E-00 | 6.65615E-01 |

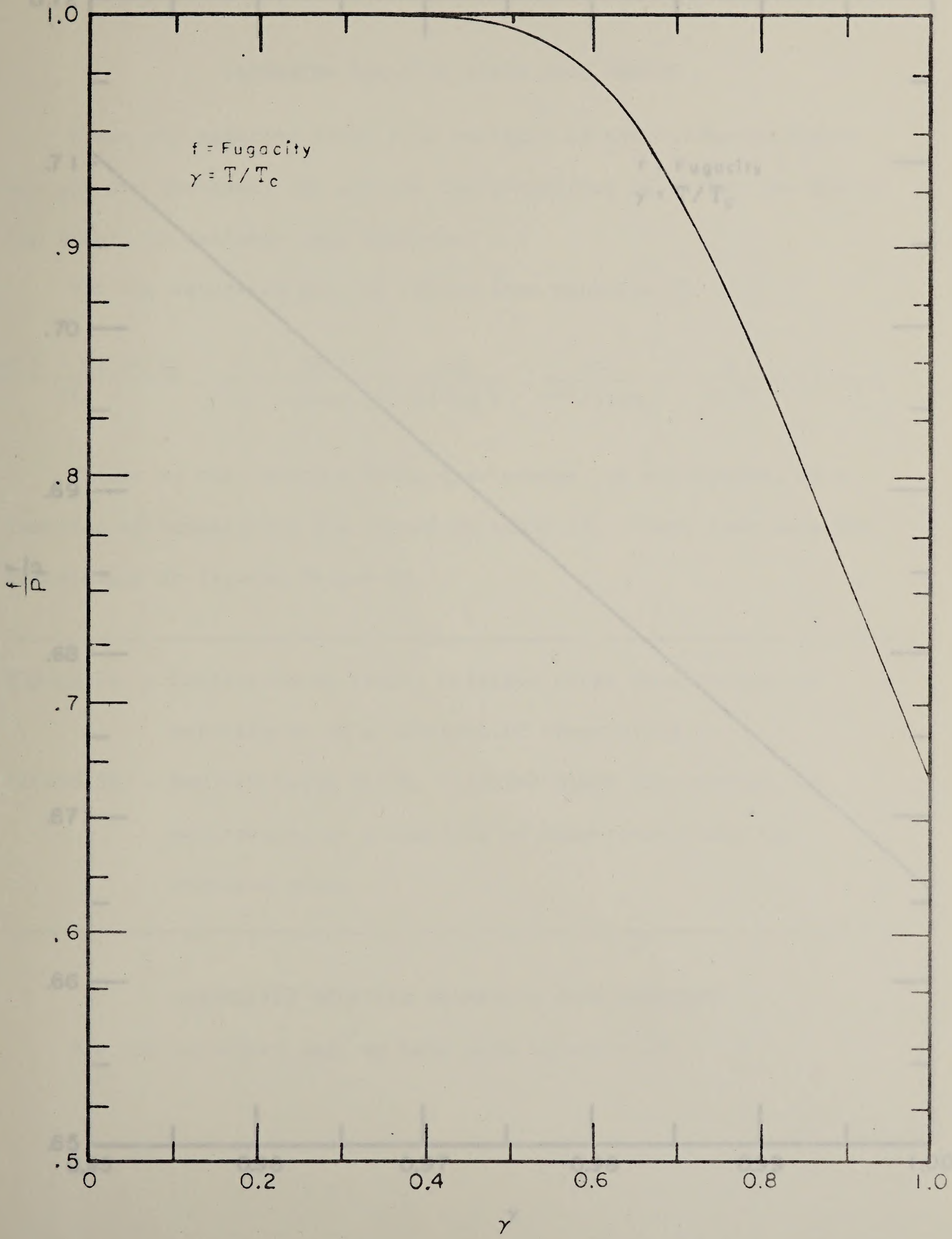

FIGURE 32. — Redlich-Kwong Fluid, the Fugacity Function, at Saturation, as a Function of the Temperature.

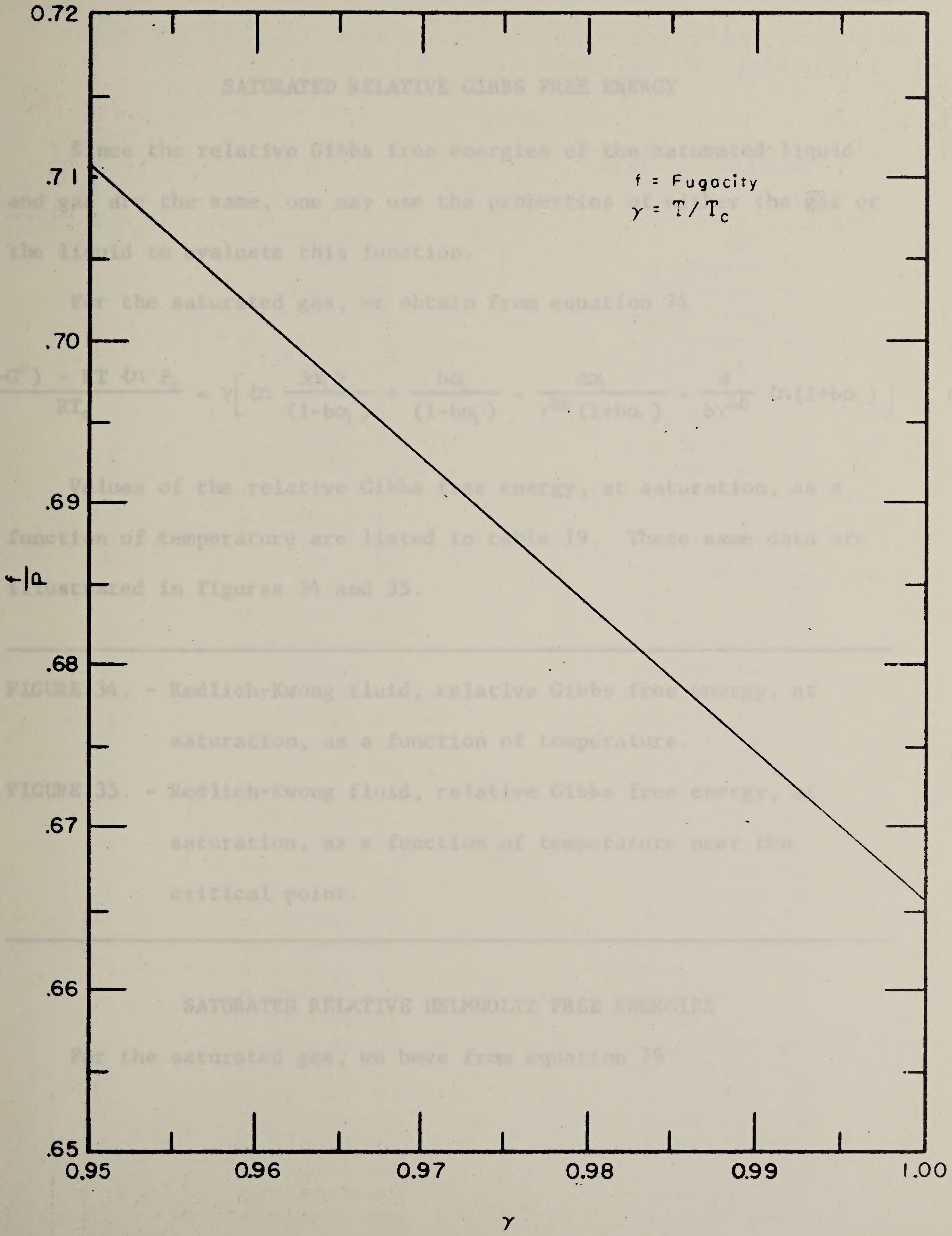

FIGURE 33. - Redlich - Kwong Fluid, the Fugacity Function, at Saturation, as a Function of the Temperature Near the Critical Point.

## SATURATED RELATIVE GIBBS FREE ENERGY

Since the relative Gibbs free energies of the saturated liquid and gas are the same, one may use the properties of either the gas or the liquid to evaluate this function.

For the saturated gas, we obtain from equation 74

$$\frac{(G-G^{o}) - RT \ln P_c}{RT_c} = \gamma\left[\ln \frac{3\alpha_1 \gamma}{(1-b\alpha_1)} + \frac{b\alpha_1}{(1-b\alpha_1)} - \frac{a\alpha_1}{\gamma^{3/2}(1+b\alpha_1)} - \frac{a}{b\gamma^{3/2}} \ln(1+b\alpha_1)\right] \quad . \quad (221)$$

Values of the relative Gibbs free energy, at saturation, as a function of temperature are listed in table 19. These same data are illustrated in figures 34 and 35.

---

FIGURE 34. - Redlich-Kwong fluid, relative Gibbs free energy, at saturation, as a function of temperature.

FIGURE 35. - Redlich-Kwong fluid, relative Gibbs free energy, at saturation, as a function of temperature near the critical point.

---

## SATURATED RELATIVE HELMHOLTZ FREE ENERGIES

For the saturated gas, we have from equation 79

## TABLE 19. - REDLICH-KWONG FLUID, RELATIVE GIBBS FREE ENERGY, AT SATURATION, AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\dfrac{(G-G^\circ) - RT\,\ln P_c}{RT_c}$ |
|---|---|
| 0.00000E-00 | $-\infty$ |
| 1.00000E-01 | -1.03658E+01 |
| 1.50000E-01 | -8.18836E-00 |
| 2.00000E-01 | -6.82211E-00 |
| 2.50000E-01 | -5.83926E-00 |
| 3.00000E-01 | -5.07444E-00 |
| 3.50000E-01 | -4.44845E-00 |
| 4.00000E-01 | -3.91795E-00 |
| 4.50000E-01 | -3.45704E-00 |
| 5.00000E-01 | -3.04910E-00 |
| 5.50000E-01 | -2.68287E-00 |
| 6.00000E-01 | -2.35035E-00 |
| 6.50000E-01 | -2.04560E-00 |
| 7.00000E-01 | -1.76408E-00 |
| 7.50000E-01 | -1.50225E-00 |
| 8.00000E-01 | -1.25730E-00 |
| 8.50000E-01 | -1.02697E-00 |
| 9.00000E-01 | -8.09445E-01 |
| 9.50000E-01 | -6.03210E-01 |
| 9.52000E-01 | -5.95177E-01 |
| 9.54000E-01 | -5.87161E-01 |
| 9.56000E-01 | -5.79160E-01 |
| 9.58000E-01 | -5.71175E-01 |
| 9.60000E-01 | -5.63207E-01 |
| 9.62000E-01 | -5.55253E-01 |
| 9.64000E-01 | -5.47316E-01 |
| 9.66000E-01 | -5.39394E-01 |
| 9.68000E-01 | -5.31487E-01 |
| 9.70000E-01 | -5.23596E-01 |
| 9.72000E-01 | -5.15721E-01 |
| 9.74000E-01 | -5.07860E-01 |
| 9.76000E-01 | -5.00015E-01 |
| 9.78000E-01 | -4.92186E-01 |
| 9.80000E-01 | -4.84371E-01 |
| 9.82000E-01 | -4.76571E-01 |
| 9.84000E-01 | -4.68787E-01 |
| 9.86000E-01 | -4.61017E-01 |
| 9.88000E-01 | -4.53262E-01 |
| 9.90000E-01 | -4.45522E-01 |
| 9.92000E-01 | -4.37797E-01 |
| 9.94000E-01 | -4.30087E-01 |
| 9.96000E-01 | -4.22391E-01 |
| 9.98000E-01 | -4.14710E-01 |
| 9.99000E-01 | -4.10874E-01 |
| 1.00000E-00 | -4.07043E-01 |

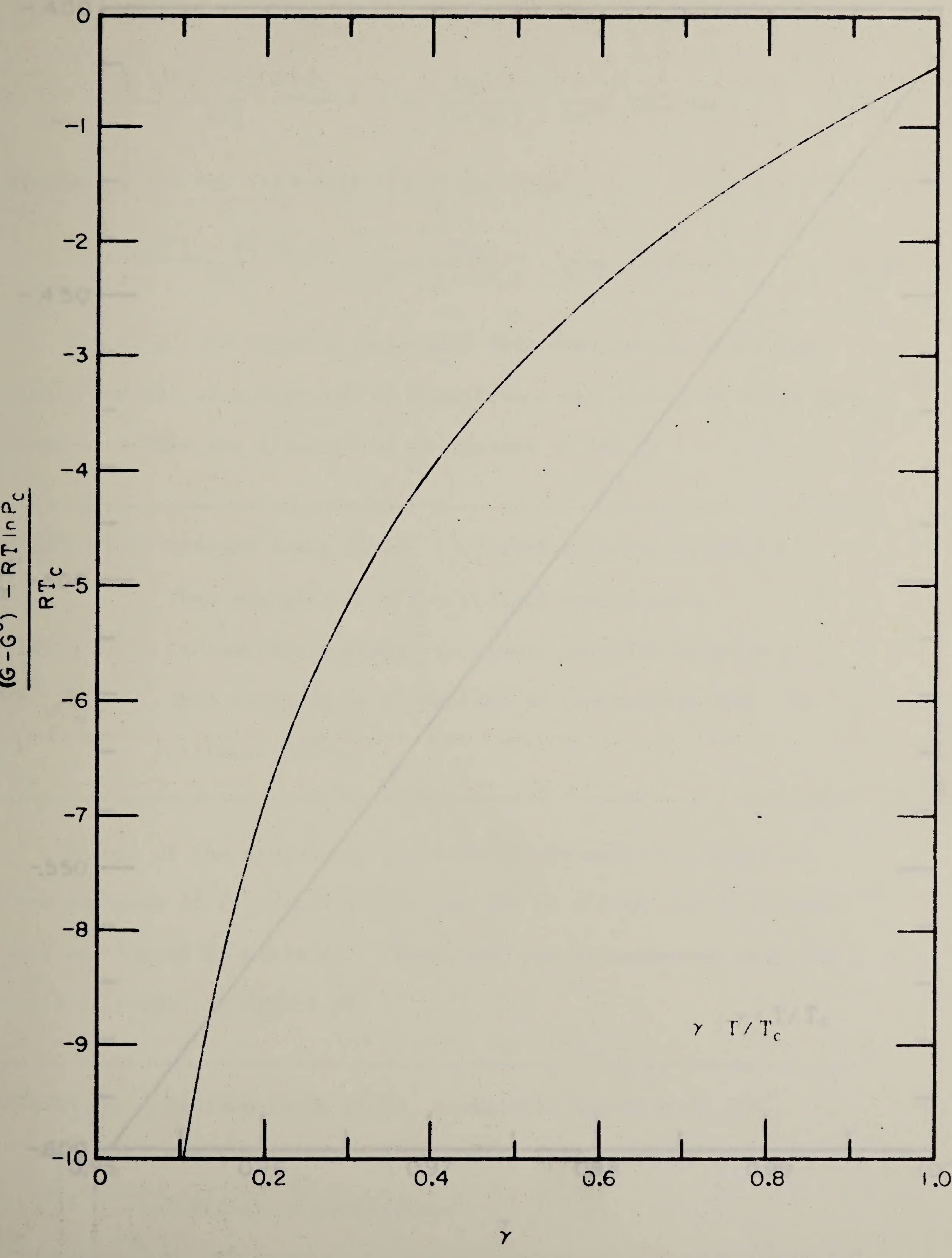

FIGURE 34.- Redlich - Kwong Fluid, Relative Gibbs Free Energy, at Saturation, as a Function of Temperature.

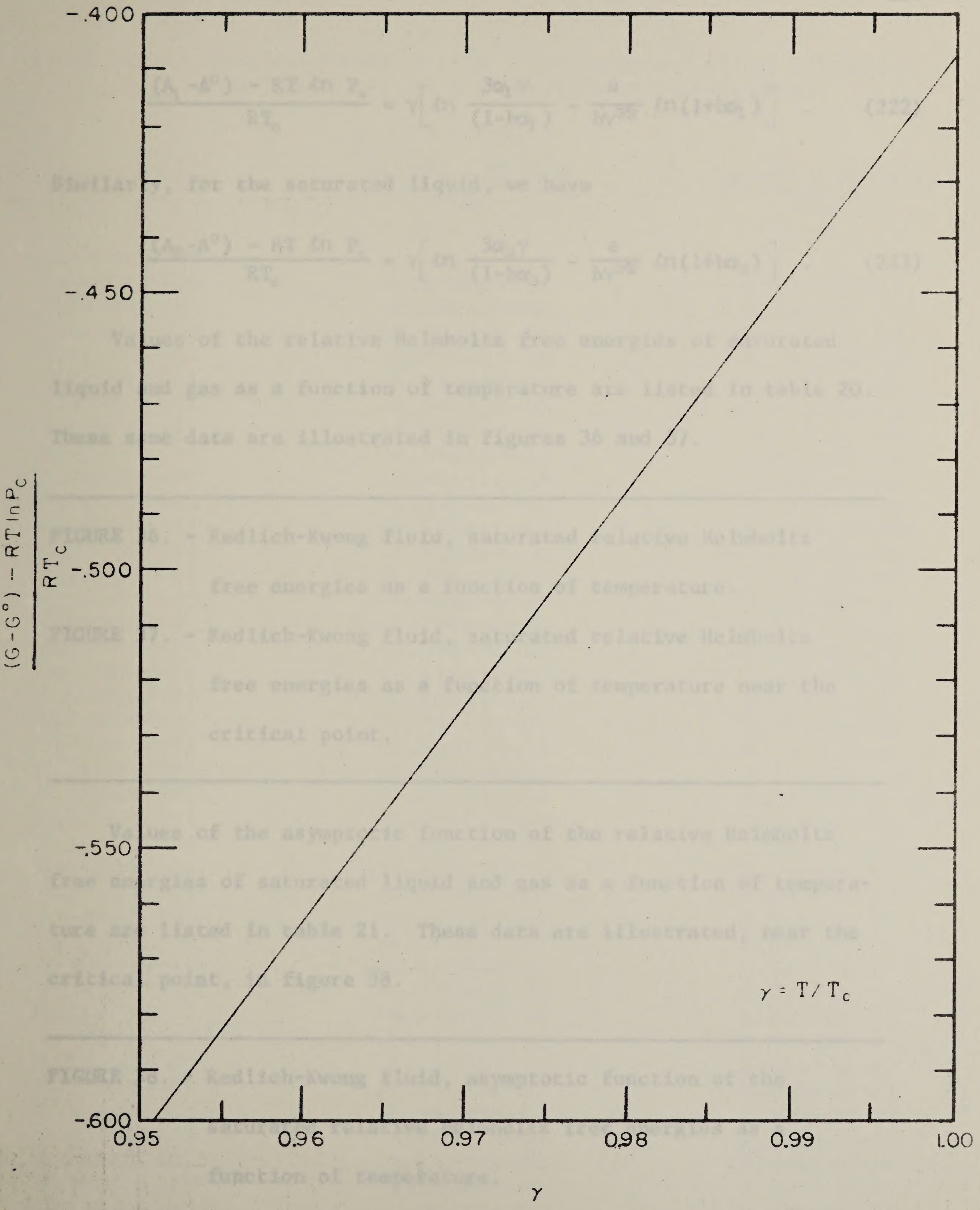

FIGURE 35.—Redlich–Kwong Fluid, Relative Gibbs Free Energy, at Saturation, as a Function of Temperature Near the Critical Point.

$$\frac{(A_1 - A^0) - RT \ln P_c}{RT_c} = \gamma\left[\ln \frac{3\alpha_1\gamma}{(1-b\alpha_1)} - \frac{a}{b\gamma^{3/2}} \ln(1+b\alpha_1)\right] \quad . \qquad (222)$$

Similarly, for the saturated liquid, we have

$$\frac{(A_3 - A^0) - RT \ln P_c}{RT_c} = \gamma\left[\ln \frac{3\alpha_3\gamma}{(1-b\alpha_3)} - \frac{a}{b\gamma^{3/2}} \ln(1+b\alpha_3)\right] \quad . \qquad (223)$$

Values of the relative Helmholtz free energies of saturated liquid and gas as a function of temperature are listed in table 20. These same data are illustrated in figures 36 and 37.

---

FIGURE 36. - Redlich-Kwong fluid, saturated relative Helmholtz free energies as a function of temperature.

FIGURE 37. - Redlich-Kwong fluid, saturated relative Helmholtz free energies as a function of temperature near the critical point.

---

Values of the asymptotic function of the relative Helmholtz free energies of saturated liquid and gas as a function of temperature are listed in table 21. These data are illustrated, near the critical point, in figure 38.

---

FIGURE 38. - Redlich-Kwong fluid, asymptotic function of the saturated relative Helmholtz free energies as a function of temperature.

---

## TABLE 20. - REDLICH-KWONG FLUID, SATURATED RELATIVE HELMHOLTZ FREE ENERGIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{(A_1-A^0) - RT \ln P_c}{RT_c}$ | $\frac{(A_3-A^0) - RT \ln P_c}{RT_c}$ |
|---|---|---|
| 0.00000E-99 | - ∞ | - ∞ |
| 1.00000E-01 | -1.03658E+01 | -1.02658E+01 |
| 1.50000E-01 | -8.18836E-00 | -8.03836E-00 |
| 2.00000E-01 | -6.82211E-00 | -6.62211E-00 |
| 2.50000E-01 | -5.83926E-00 | -5.58926E-00 |
| 3.00000E-01 | -5.07444E-00 | -4.77444E-00 |
| 3.50000E-01 | -4.44844E-00 | -4.09845E-00 |
| 4.00000E-01 | -3.91786E-00 | -3.51796E-00 |
| 4.50000E-01 | -3.45642E-00 | -3.00708E-00 |
| 5.00000E-01 | -3.04655E-00 | -2.54933E-00 |
| 5.50000E-01 | -2.67536E-00 | -2.13369E-00 |
| 6.00000E-01 | -2.33284E-00 | -1.75261E-00 |
| 6.50000E-01 | -2.01101E-00 | -1.40080E-00 |
| 7.00000E-01 | -1.70341E-00 | -1.07464E-00 |
| 7.50000E-01 | -1.40464E-00 | -7.71784E-01 |
| 8.00000E-01 | -1.10988E-00 | -4.91072E-01 |
| 8.50000E-01 | -8.14177E-01 | -2.32792E-01 |
| 9.00000E-01 | -5.11143E-01 | 3.40493E-04 |
| 9.50000E-01 | -1.88431E-01 | 1.98912E-01 |
| 9.52000E-01 | -1.74729E-01 | 2.05786E-01 |
| 9.54000E-01 | -1.60932E-01 | 2.12547E-01 |
| 9.56000E-01 | -1.47036E-01 | 2.19188E-01 |
| 9.58000E-01 | -1.33033E-01 | 2.25703E-01 |
| 9.60000E-01 | -1.18916E-01 | 2.32084E-01 |
| 9.62000E-01 | -1.04675E-01 | 2.38322E-01 |
| 9.64000E-01 | -9.03017E-02 | 2.44408E-01 |
| 9.66000E-01 | -7.57842E-02 | 2.50331E-01 |
| 9.68000E-01 | -6.11101E-02 | 2.56079E-01 |
| 9.70000E-01 | -4.62646E-02 | 2.61637E-01 |
| 9.72000E-01 | -3.12308E-02 | 2.66987E-01 |
| 9.74000E-01 | -1.59885E-02 | 2.72111E-01 |
| 9.76000E-01 | -5.13606E-04 | 2.76983E-01 |
| 9.78000E-01 | 1.52230E-02 | 2.81576E-01 |
| 9.80000E-01 | 3.12570E-02 | 2.85853E-01 |
| 9.82000E-01 | 4.76335E-02 | 2.89769E-01 |
| 9.84000E-01 | 6.44100E-02 | 2.93268E-01 |
| 9.86000E-01 | 8.16627E-02 | 2.96272E-01 |
| 9.88000E-01 | 9.94958E-02 | 2.98679E-01 |
| 9.90000E-01 | 1.18059E-01 | 3.00337E-01 |
| 9.92000E-01 | 1.37582E-01 | 3.01018E-01 |
| 9.94000E-01 | 1.58450E-01 | 3.00338E-01 |
| 9.96000E-01 | 1.81411E-01 | 2.97546E-01 |
| 9.98000E-01 | 2.08395E-01 | 2.90715E-01 |
| 9.99000E-01 | 2.25450E-01 | 2.83730E-01 |
| 1.00000E-00 | 2.59623E-01 | 2.59623E-01 |

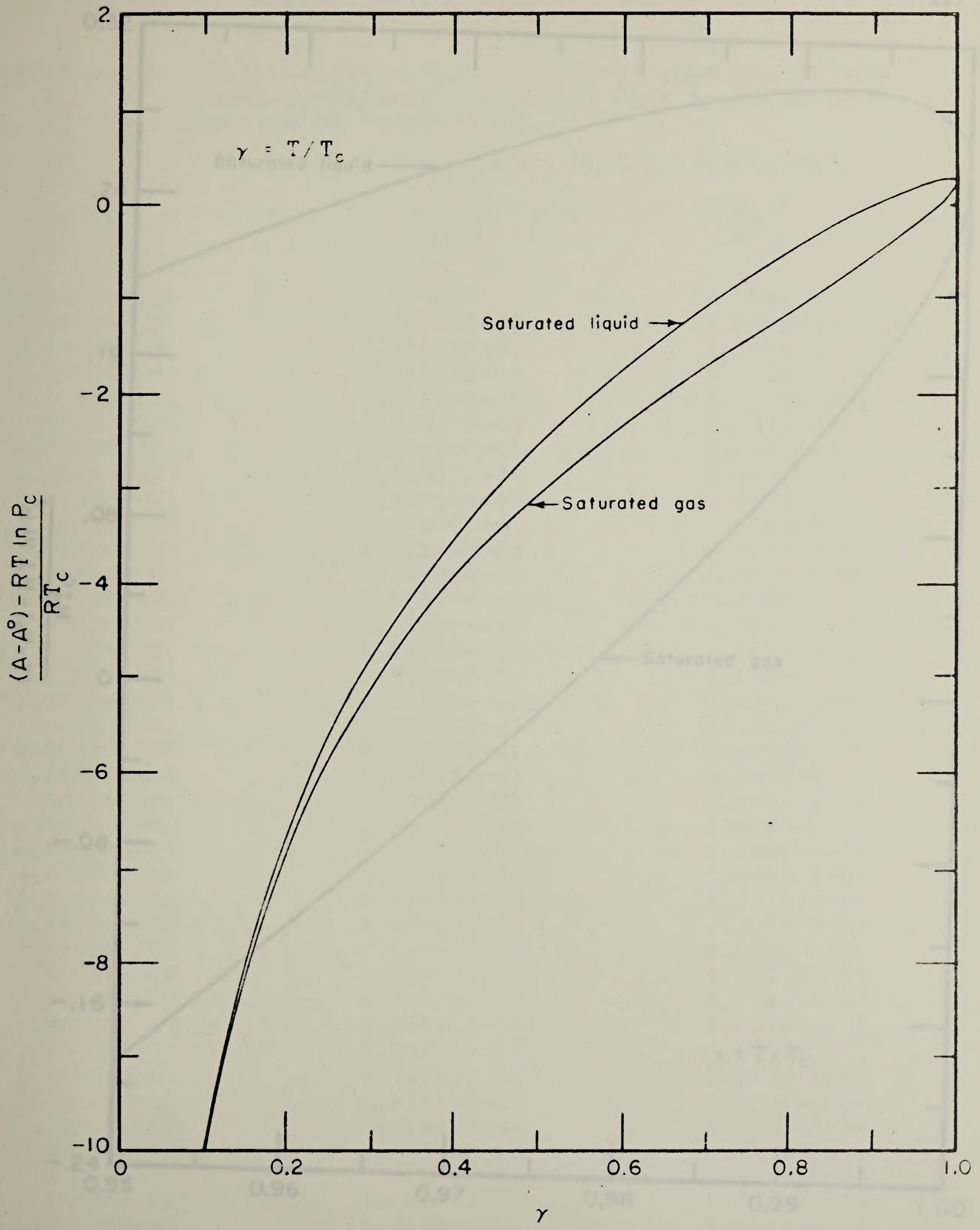

FIGURE 36.- Redlich-Kwong Fluid, Saturated Relative Helmholtz Free Energies as a Function of Temperature.

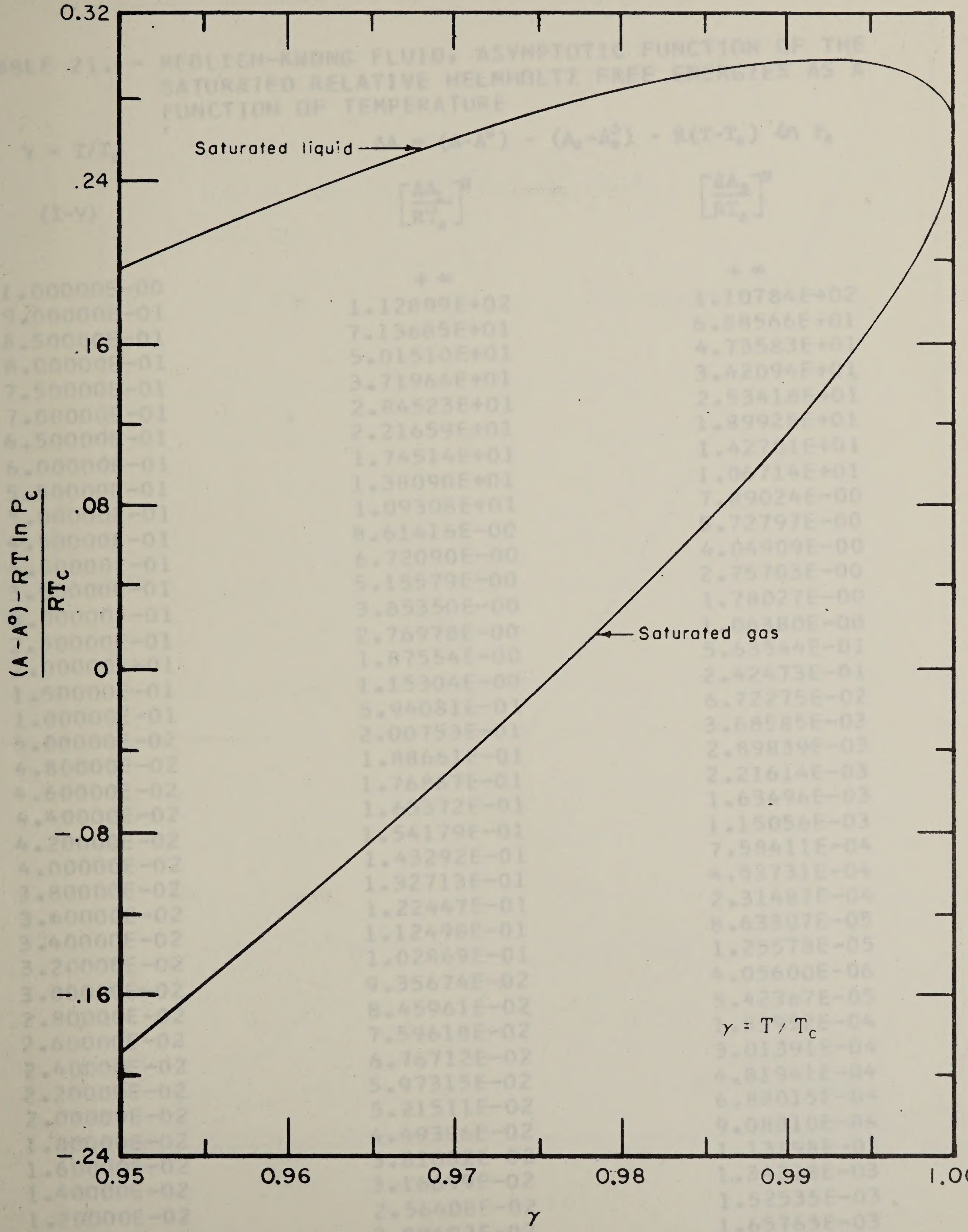

FIGURE 37.– Redlich-Kwong Fluid, Saturated Relative Helmholtz Free Energies as a Function of Temperature Near the Critical Point.

TABLE 21. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED RELATIVE HELMHOLTZ FREE ENERGIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad$ $\Delta A = (A-A^{o}) - (A_c-A_c^{o}) - R(T-T_c)\ \ln P_c$

| $(1-\gamma)$ | $\left[\frac{\Delta A_1}{RT_c}\right]^2$ | $\left[\frac{\Delta A_3}{RT_c}\right]^2$ |
|---|---|---|
| 1.00000E-00 | + ∞ | + ∞ |
| 9.00000E-01 | 1.12899E+02 | 1.10784E+02 |
| 8.50000E-01 | 7.13685E+01 | 6.88566E+01 |
| 8.00000E-01 | 5.01510E+01 | 4.73583E+01 |
| 7.50000E-01 | 3.71964E+01 | 3.42094E+01 |
| 7.00000E-01 | 2.84523E+01 | 2.53418E+01 |
| 6.50000E-01 | 2.21659E+01 | 1.89928E+01 |
| 6.00000E-01 | 1.74514E+01 | 1.42701E+01 |
| 5.50000E-01 | 1.38090E+01 | 1.06714E+01 |
| 5.00000E-01 | 1.09308E+01 | 7.89024E-00 |
| 4.50000E-01 | 8.61416E-00 | 5.72797E-00 |
| 4.00000E-01 | 6.72090E-00 | 4.04909E-00 |
| 3.50000E-01 | 5.15579E-00 | 2.75703E-00 |
| 3.00000E-01 | 3.85350E-00 | 1.78027E-00 |
| 2.50000E-01 | 2.76978E-00 | 1.06380E-00 |
| 2.00000E-01 | 1.87554E-00 | 5.63544E-01 |
| 1.50000E-01 | 1.15304E-00 | 2.42473E-01 |
| 1.00000E-01 | 5.94081E-01 | 6.72275E-02 |
| 5.00000E-02 | 2.00753E-01 | 3.68585E-03 |
| 4.80000E-02 | 1.88661E-01 | 2.89839E-03 |
| 4.60000E-02 | 1.76867E-01 | 2.21614E-03 |
| 4.40000E-02 | 1.65372E-01 | 1.63496E-03 |
| 4.20000E-02 | 1.54179E-01 | 1.15056E-03 |
| 4.00000E-02 | 1.43292E-01 | 7.58411E-04 |
| 3.80000E-02 | 1.32713E-01 | 4.53731E-04 |
| 3.60000E-02 | 1.22447E-01 | 2.31487E-04 |
| 3.40000E-02 | 1.12498E-01 | 8.63307E-05 |
| 3.20000E-02 | 1.02869E-01 | 1.25578E-05 |
| 3.00000E-02 | 9.35674E-02 | 4.05600E-06 |
| 2.80000E-02 | 8.45961E-02 | 5.42367E-05 |
| 2.60000E-02 | 7.59618E-02 | 1.55952E-04 |
| 2.40000E-02 | 6.76712E-02 | 3.01391E-04 |
| 2.20000E-02 | 5.97315E-02 | 4.81941E-04 |
| 2.00000E-02 | 5.21511E-02 | 6.88015E-04 |
| 1.80000E-02 | 4.49396E-02 | 9.08810E-04 |
| 1.60000E-02 | 3.81082E-02 | 1.13198E-03 |
| 1.40000E-02 | 3.16699E-02 | 1.34318E-03 |
| 1.20000E-02 | 2.56408E-02 | 1.52535E-03 |
| 1.00000E-02 | 2.00403E-02 | 1.65765E-03 |
| 8.00000E-03 | 1.48939E-02 | 1.71359E-03 |
| 6.00000E-03 | 1.02360E-02 | 1.65770E-03 |
| 4.00000E-03 | 6.11706E-03 | 1.43817E-03 |
| 2.00000E-03 | 2.62430E-03 | 9.66730E-04 |
| 1.00000E-03 | 1.16778E-03 | 5.81155E-04 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

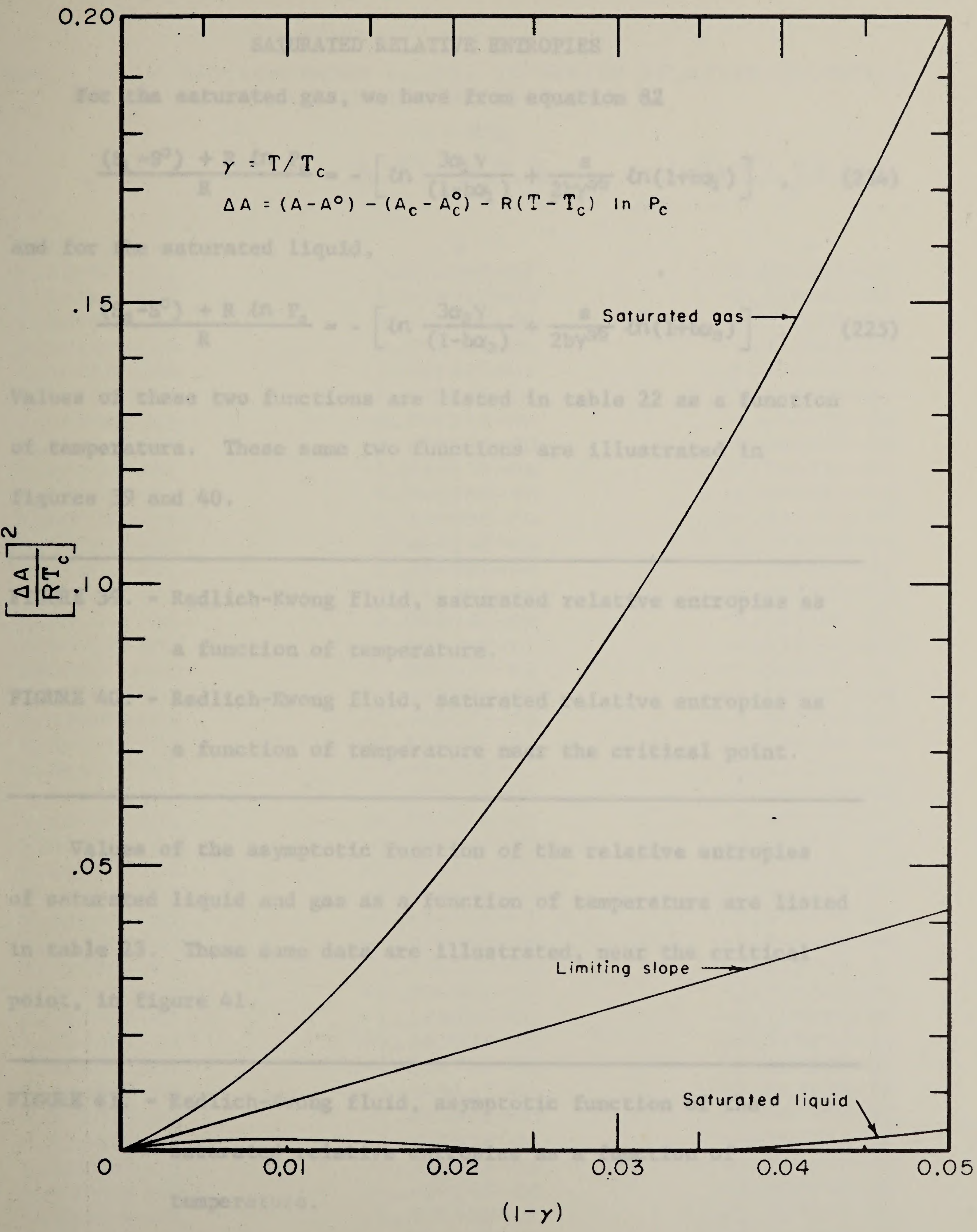

FIGURE 38. - Redlich-Kwong Fluid, Asymptotic Function of the Saturated Relative Helmholtz Free Energies as a Function of Temperature.

## SATURATED RELATIVE ENTROPIES

For the saturated gas, we have from equation 82

$$\frac{(S_1 - S^o) + R \ln P_c}{R} = - \left[ \ln \frac{3\alpha_1 Y}{(1-b\alpha_1)} + \frac{a}{2bY^{3/2}} \ln(1+b\alpha_1) \right] , \qquad (224)$$

and for the saturated liquid,

$$\frac{(S_3 - S^o) + R \ln P_c}{R} = - \left[ \ln \frac{3\alpha_3 Y}{(1-b\alpha_3)} + \frac{a}{2bY^{3/2}} \ln(1+b\alpha_3) \right] . \qquad (225)$$

Values of these two functions are listed in table 22 as a function of temperature. These same two functions are illustrated in figures 39 and 40.

---

FIGURE 39. - Redlich-Kwong fluid, saturated relative entropies as a function of temperature.

FIGURE 40. - Redlich-Kwong fluid, saturated relative entropies as a function of temperature near the critical point.

---

Values of the asymptotic function of the relative entropies of saturated liquid and gas as a function of temperature are listed in table 23. These same data are illustrated, near the critical point, in figure 41.

---

FIGURE 41. - Redlich-Kwong fluid, asymptotic function of the saturated relative entropies as a function of temperature.

---

TABLE 22. - REDLICH-KWONG FLUID, SATURATED RELATIVE ENTROPIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\dfrac{(S_l - S^\circ) + R \ln P_c}{R}$ | $\dfrac{(S_g - S^\circ) + R \ln P_c}{R}$ |
|---|---|---|
| 0.00000E-99 | $+\infty$ | $-\infty$ |
| 1.00000E-01 | 1.03658E+02 | -5.80502E+01 |
| 1.50000E-01 | 5.45891E+01 | -3.31864E+01 |
| 2.00000E-01 | 3.41105E+01 | -2.27008E+01 |
| 2.50000E-01 | 2.33570E+01 | -1.71208E+01 |
| 3.00000E-01 | 1.69148E+01 | -1.37218E+01 |
| 3.50000E-01 | 1.27098E+01 | -1.14569E+01 |
| 4.00000E-01 | 9.79431E-00 | -9.84647E-00 |
| 4.50000E-01 | 7.67877E-00 | -8.64227E-00 |
| 5.00000E-01 | 6.08487E-00 | -7.70411E-00 |
| 5.50000E-01 | 4.84195E-00 | -6.94695E-00 |
| 6.00000E-01 | 3.83965E-00 | -6.31614E-00 |
| 6.50000E-01 | 3.00414E-00 | -5.77459E-00 |
| 7.00000E-01 | 2.28458E-00 | -5.29566E-00 |
| 7.50000E-01 | 1.64435E-00 | -4.85870E-00 |
| 8.00000E-01 | 1.05502E-00 | -4.44579E-00 |
| 8.50000E-01 | 4.90874E-01 | -4.03826E-00 |
| 9.00000E-01 | -7.86947E-02 | -3.61027E-00 |
| 9.50000E-01 | -7.12436E-01 | -3.10637E-00 |
| 9.52000E-01 | -7.41007E-01 | -3.08278E-00 |
| 9.54000E-01 | -7.69992E-01 | -3.05876E-00 |
| 9.56000E-01 | -7.99421E-01 | -3.03429E-00 |
| 9.58000E-01 | -8.29331E-01 | -3.00932E-00 |
| 9.60000E-01 | -8.59760E-01 | -2.98382E-00 |
| 9.62000E-01 | -8.90753E-01 | -2.95773E-00 |
| 9.64000E-01 | -9.22361E-01 | -2.93102E-00 |
| 9.66000E-01 | -9.54642E-01 | -2.90362E-00 |
| 9.68000E-01 | -9.87661E-01 | -2.87547E-00 |
| 9.70000E-01 | -1.02149E-00 | -2.84649E-00 |
| 9.72000E-01 | -1.05623E-00 | -2.81659E-00 |
| 9.74000E-01 | -1.09199E-00 | -2.78566E-00 |
| 9.76000E-01 | -1.12888E-00 | -2.75358E-00 |
| 9.78000E-01 | -1.16707E-00 | -2.72019E-00 |
| 9.80000E-01 | -1.20675E-00 | -2.68530E-00 |
| 9.82000E-01 | -1.24816E-00 | -2.64867E-00 |
| 9.84000E-01 | -1.29160E-00 | -2.60999E-00 |
| 9.86000E-01 | -1.33749E-00 | -2.56884E-00 |
| 9.88000E-01 | -1.38640E-00 | -2.52467E-00 |
| 9.90000E-01 | -1.43914E-00 | -2.47665E-00 |
| 9.92000E-01 | -1.49696E-00 | -2.42353E-00 |
| 9.94000E-01 | -1.56198E-00 | -2.36320E-00 |
| 9.96000E-01 | -1.63833E-00 | -2.29154E-00 |
| 9.98000E-01 | -1.73667E-00 | -2.19787E-00 |
| 9.99000E-01 | -1.80550E-00 | -2.13137E-00 |
| 1.00000E-00 | -1.96960E-00 | -1.96960E-00 |

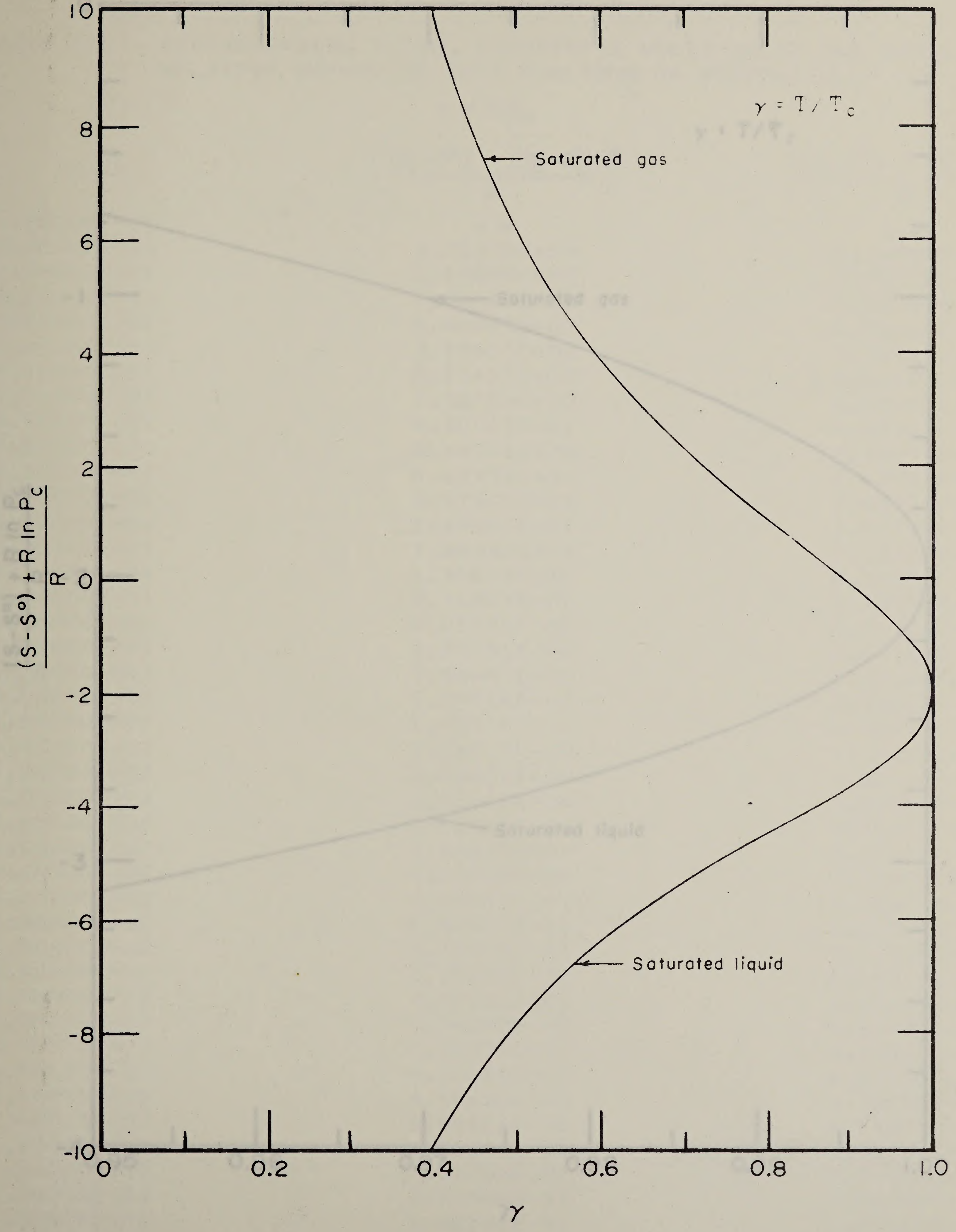

FIGURE 39.- Redlich - Kwong Fluid, Saturated Relative Entropies as a Function of Temperature.

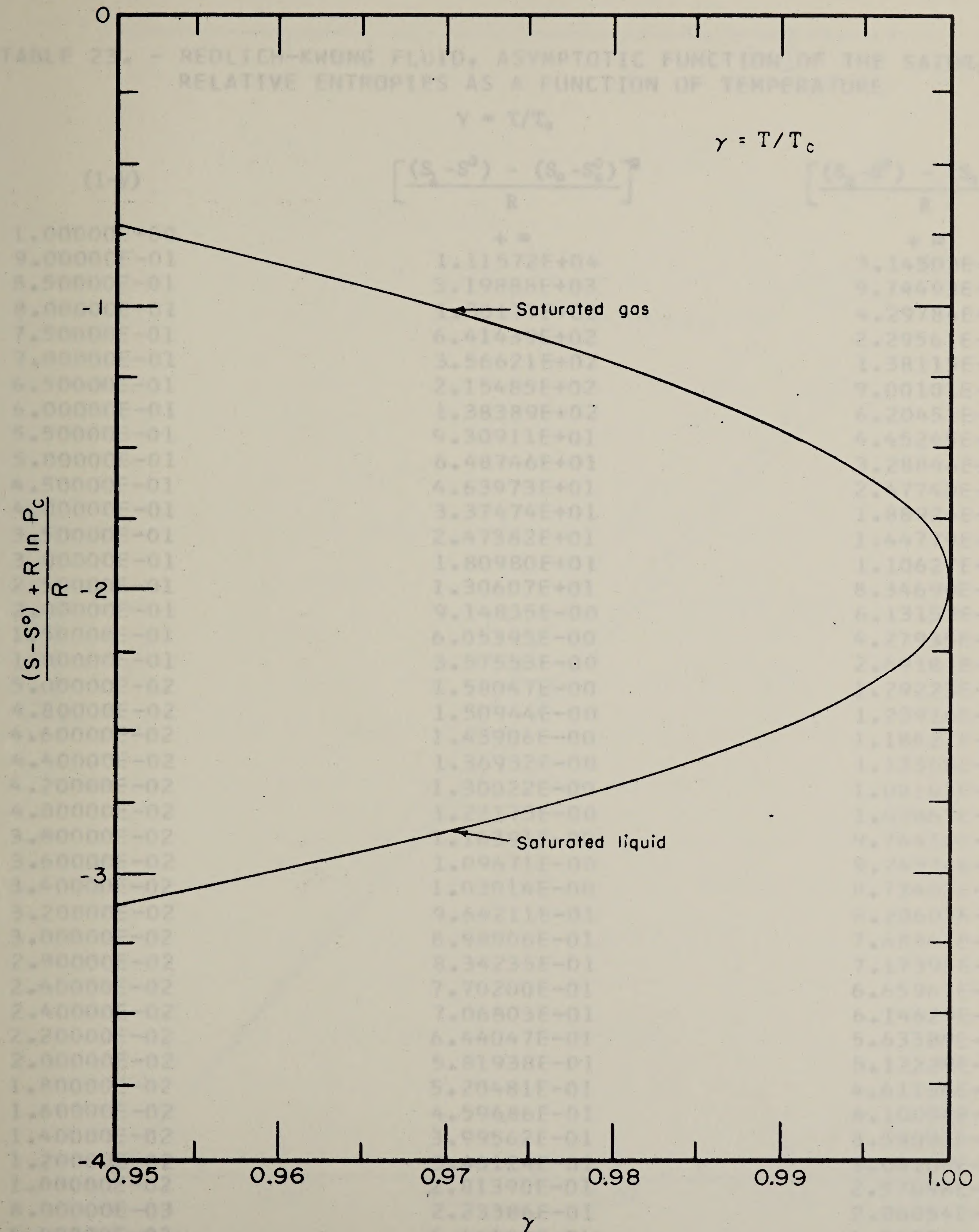

FIGURE 40.—Redlich-Kwong Fluid, Saturated Relative Entropies as a Function of Temperature Near the Critical Point.

### TABLE 23. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED RELATIVE ENTROPIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $(1-\gamma)$ | $\left[\frac{(S_1-S^0) - (S_c-S_c^0)}{R}\right]^2$ | $\left[\frac{(S_3-S^0) - (S_c-S_c^0)}{R}\right]^2$ |
|---|---|---|
| 1.00000E-00 | + ∞ | + ∞ |
| 9.00000E-01 | 1.11572E+04 | 3.14503E+03 |
| 8.50000E-01 | 3.19888E+03 | 9.74493E+02 |
| 8.00000E-01 | 1.30177E+03 | 4.29784E+02 |
| 7.50000E-01 | 6.41439E+02 | 2.29561E+02 |
| 7.00000E-01 | 3.56621E+02 | 1.38115E+02 |
| 6.50000E-01 | 2.15485E+02 | 9.00101E+01 |
| 6.00000E-01 | 1.38389E+02 | 6.20451E+01 |
| 5.50000E-01 | 9.30911E+01 | 4.45245E+01 |
| 5.00000E-01 | 6.48746E+01 | 3.28846E+01 |
| 4.50000E-01 | 4.63973E+01 | 2.47740E+01 |
| 4.00000E-01 | 3.37474E+01 | 1.88924E+01 |
| 3.50000E-01 | 2.47382E+01 | 1.44779E+01 |
| 3.00000E-01 | 1.80980E+01 | 1.10627E+01 |
| 2.50000E-01 | 1.30607E+01 | 8.34690E-00 |
| 2.00000E-01 | 9.14835E-00 | 6.13150E-00 |
| 1.50000E-01 | 6.05395E-00 | 4.27935E-00 |
| 1.00000E-01 | 3.57553E-00 | 2.69181E-00 |
| 5.00000E-02 | 1.58047E-00 | 1.29223E-00 |
| 4.80000E-02 | 1.50944E-00 | 1.23916E-00 |
| 4.60000E-02 | 1.43906E-00 | 1.18627E-00 |
| 4.40000E-02 | 1.36932E-00 | 1.13356E-00 |
| 4.20000E-02 | 1.30022E-00 | 1.08101E-00 |
| 4.00000E-02 | 1.23175E-00 | 1.02863E-00 |
| 3.80000E-02 | 1.16391E-00 | 9.76410E-01 |
| 3.60000E-02 | 1.09671E-00 | 9.24334E-01 |
| 3.40000E-02 | 1.03014E-00 | 8.72403E-01 |
| 3.20000E-02 | 9.64211E-01 | 8.20607E-01 |
| 3.00000E-02 | 8.98906E-01 | 7.68941E-01 |
| 2.80000E-02 | 8.34235E-01 | 7.17395E-01 |
| 2.60000E-02 | 7.70200E-01 | 6.65961E-01 |
| 2.40000E-02 | 7.06803E-01 | 6.14629E-01 |
| 2.20000E-02 | 6.44047E-01 | 5.63389E-01 |
| 2.00000E-02 | 5.81938E-01 | 5.12229E-01 |
| 1.80000E-02 | 5.20481E-01 | 4.61136E-01 |
| 1.60000E-02 | 4.59686E-01 | 4.10096E-01 |
| 1.40000E-02 | 3.99562E-01 | 3.59091E-01 |
| 1.20000E-02 | 3.40124E-01 | 3.08100E-01 |
| 1.00000E-02 | 2.81390E-01 | 2.57098E-01 |
| 8.00000E-03 | 2.23386E-01 | 2.06054E-01 |
| 6.00000E-03 | 1.66149E-01 | 1.54924E-01 |
| 4.00000E-03 | 1.09737E-01 | 1.03645E-01 |
| 2.00000E-03 | 5.42539E-02 | 5.21059E-02 |
| 1.00000E-03 | 2.69279E-02 | 2.61695E-02 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

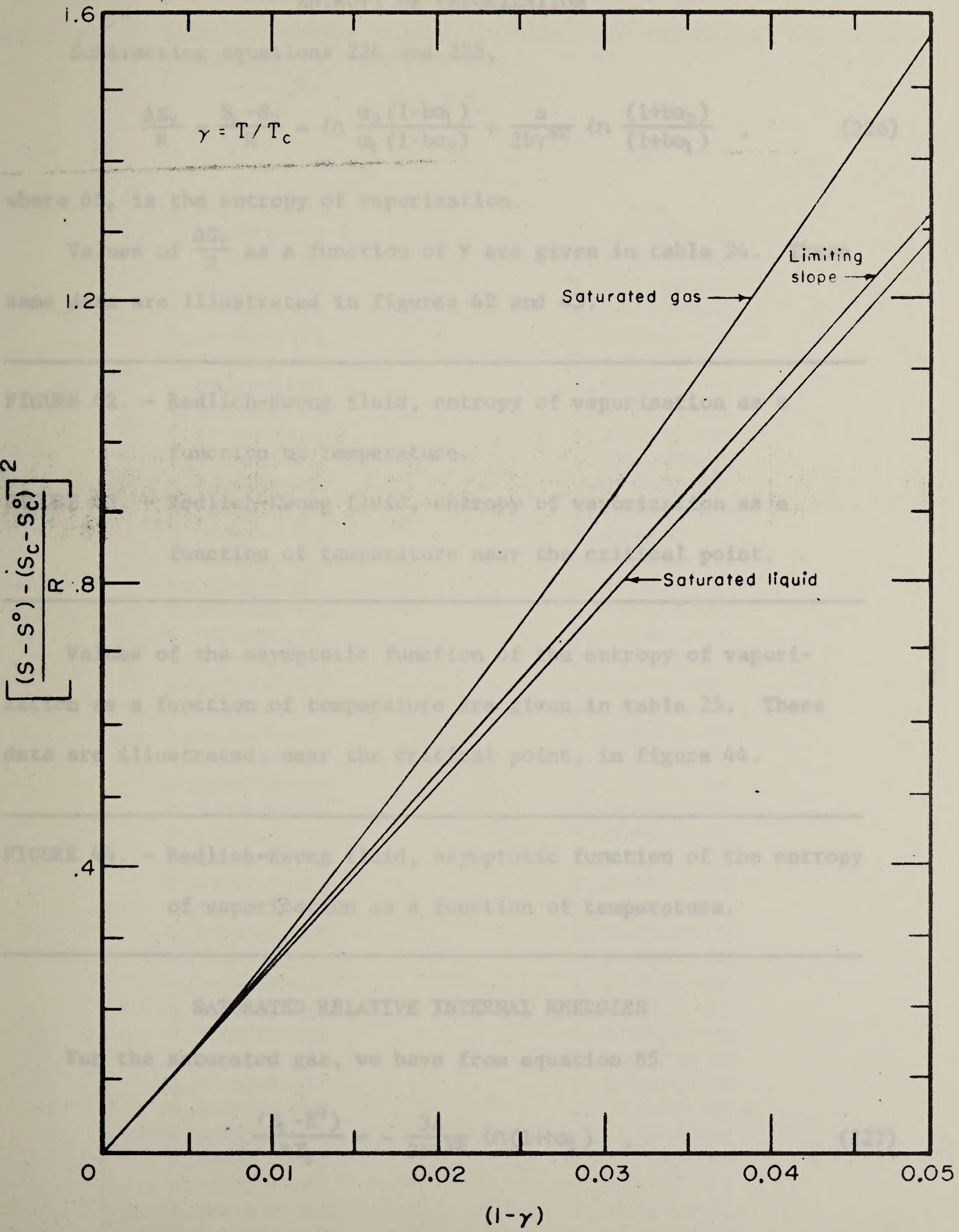

FIGURE 41.—Redlich-Kwong Fluid, Asymptotic Function of the Saturated Relative Entropies as a Function of Temperature.

## ENTROPY OF VAPORIZATION

Subtracting equations 224 and 225,

$$\frac{\Delta S_\gamma}{R} = \frac{S_1 - S_3}{R} = \ln \frac{\alpha_3 (1-b\alpha_1)}{\alpha_1 (1-b\alpha_3)} + \frac{a}{2b\gamma^{3/2}} \ln \frac{(1+b\alpha_3)}{(1+b\alpha_1)} \quad , \tag{226}$$

where $\Delta S_\gamma$ is the entropy of vaporization.

Values of $\frac{\Delta S_\gamma}{R}$ as a function of $\gamma$ are given in table 24. These same data are illustrated in figures 42 and 43.

---

FIGURE 42. - Redlich-Kwong fluid, entropy of vaporization as a function of temperature.

FIGURE 43. - Redlich-Kwong fluid, entropy of vaporization as a function of temperature near the critical point.

---

Values of the asymptotic function of the entropy of vaporization as a function of temperature are given in table 25. These data are illustrated, near the critical point, in figure 44.

---

FIGURE 44. - Redlich-Kwong fluid, asymptotic function of the entropy of vaporization as a function of temperature.

---

## SATURATED RELATIVE INTERNAL ENERGIES

For the saturated gas, we have from equation 85

$$\frac{(E_1 - E^0)}{RT_c} = - \frac{3a}{2b\gamma^{1/2}} \ln (1+b\alpha_1) \quad , \tag{227}$$

TABLE 24. - REDLICH-KWONG FLUID, ENTROPY OF VAPORIZATION AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{\Delta S_v}{R}$ |
|---|---|
| 0.00000E-99 | + ∞ |
| 1.00000E-01 | 1.61708E+02 |
| 1.50000E-01 | 8.77756E+01 |
| 2.00000E-01 | 5.68114E+01 |
| 2.50000E-01 | 4.04779E+01 |
| 3.00000E-01 | 3.06366E+01 |
| 3.50000E-01 | 2.41667E+01 |
| 4.00000E-01 | 1.96407E+01 |
| 4.50000E-01 | 1.63210E+01 |
| 5.00000E-01 | 1.37889E+01 |
| 5.50000E-01 | 1.17889E+01 |
| 6.00000E-01 | 1.01557E+01 |
| 6.50000E-01 | 8.77874E-00 |
| 7.00000E-01 | 7.58024E-00 |
| 7.50000E-01 | 6.50306E-00 |
| 8.00000E-01 | 5.50081E-00 |
| 8.50000E-01 | 4.52914E-00 |
| 9.00000E-01 | 3.53158E-00 |
| 9.50000E-01 | 2.39393E-00 |
| 9.52000E-01 | 2.34177E-00 |
| 9.54000E-01 | 2.28877E-00 |
| 9.56000E-01 | 2.23487E-00 |
| 9.58000E-01 | 2.17999E-00 |
| 9.60000E-01 | 2.12406E-00 |
| 9.62000E-01 | 2.06698E-00 |
| 9.64000E-01 | 2.00866E-00 |
| 9.66000E-01 | 1.94898E-00 |
| 9.68000E-01 | 1.88781E-00 |
| 9.70000E-01 | 1.82499E-00 |
| 9.72000E-01 | 1.76035E-00 |
| 9.74000E-01 | 1.69367E-00 |
| 9.76000E-01 | 1.62469E-00 |
| 9.78000E-01 | 1.55311E-00 |
| 9.80000E-01 | 1.47855E-00 |
| 9.82000E-01 | 1.40051E-00 |
| 9.84000E-01 | 1.31838E-00 |
| 9.86000E-01 | 1.23135E-00 |
| 9.88000E-01 | 1.13826E-00 |
| 9.90000E-01 | 1.03751E-00 |
| 9.92000E-01 | 9.26570E-01 |
| 9.94000E-01 | 8.01219E-01 |
| 9.96000E-01 | 6.53206E-01 |
| 9.98000E-01 | 4.61192E-01 |
| 9.99000E-01 | 3.25867E-01 |
| 1.00000E-00 | 0.00000E-99 |

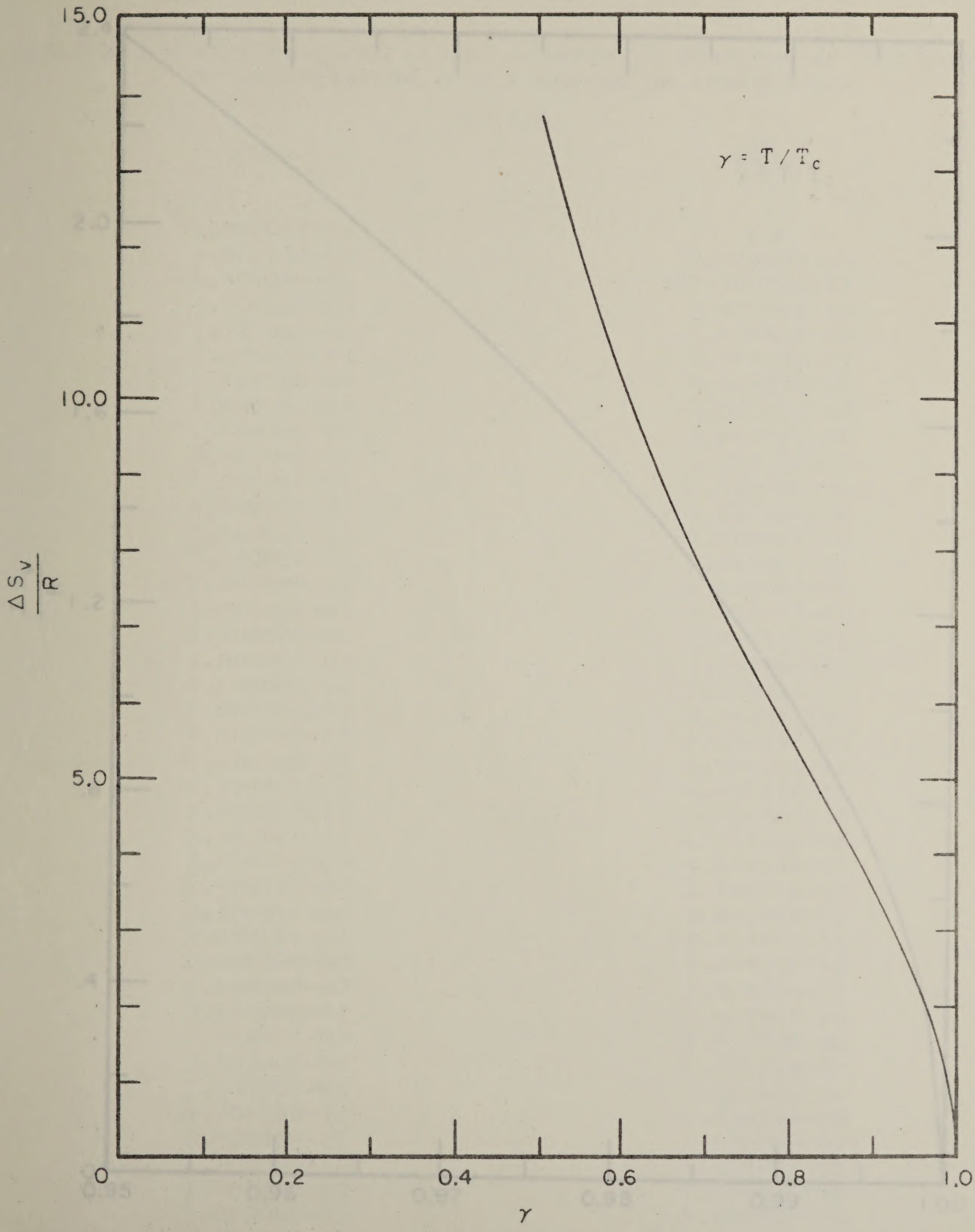

FIGURE 42.-Redlich-Kwong Fluid, Entropy of Vaporization as a Function of Temperature.

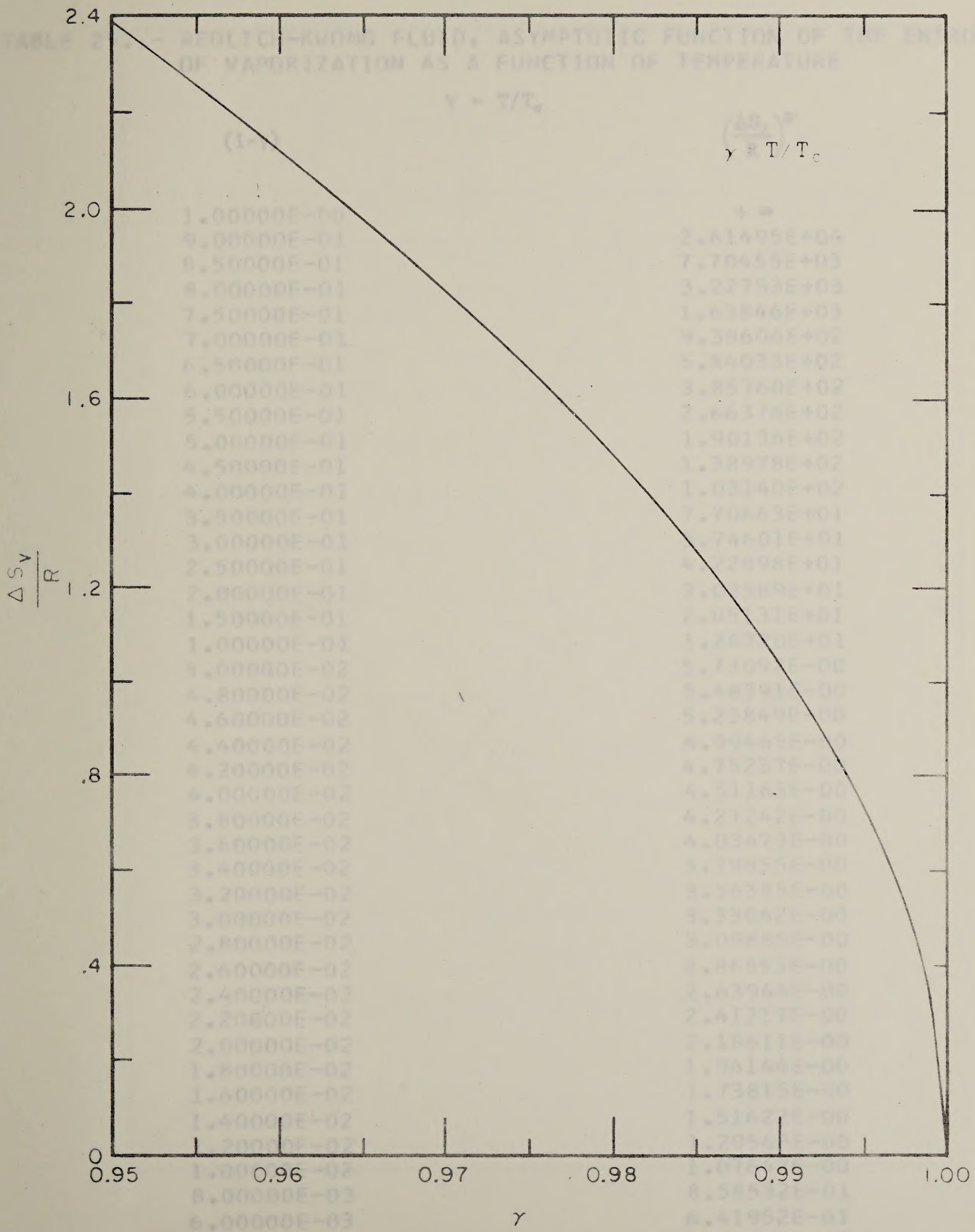

FIGURE 43.- Redlich-Kwong Fluid, Entropy of Vaporization as a Function of Temperature Near the Critical Point.

TABLE 25. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE ENTROPY OF VAPORIZATION AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| (1-γ) | $\left(\frac{\Delta S_v}{R}\right)^2$ |
|---|---|
| 1.00000E-00 | + ∞ |
| 9.00000E-01 | 2.61495E+04 |
| 8.50000E-01 | 7.70455E+03 |
| 8.00000E-01 | 3.22753E+03 |
| 7.50000E-01 | 1.63846E+03 |
| 7.00000E-01 | 9.38606E+02 |
| 6.50000E-01 | 5.84033E+02 |
| 6.00000E-01 | 3.85760E+02 |
| 5.50000E-01 | 2.66376E+02 |
| 5.00000E-01 | 1.90136E+02 |
| 4.50000E-01 | 1.38978E+02 |
| 4.00000E-01 | 1.03140E+02 |
| 3.50000E-01 | 7.70663E+01 |
| 3.00000E-01 | 5.74601E+01 |
| 2.50000E-01 | 4.22898E+01 |
| 2.00000E-01 | 3.02589E+01 |
| 1.50000E-01 | 2.05131E+01 |
| 1.00000E-01 | 1.24720E+01 |
| 5.00000E-02 | 5.73092E-00 |
| 4.80000E-02 | 5.48391E-00 |
| 4.60000E-02 | 5.23849E-00 |
| 4.40000E-02 | 4.99465E-00 |
| 4.20000E-02 | 4.75237E-00 |
| 4.00000E-02 | 4.51163E-00 |
| 3.80000E-02 | 4.27242E-00 |
| 3.60000E-02 | 4.03473E-00 |
| 3.40000E-02 | 3.79855E-00 |
| 3.20000E-02 | 3.56385E-00 |
| 3.00000E-02 | 3.33062E-00 |
| 2.80000E-02 | 3.09885E-00 |
| 2.60000E-02 | 2.86853E-00 |
| 2.40000E-02 | 2.63964E-00 |
| 2.20000E-02 | 2.41217E-00 |
| 2.00000E-02 | 2.18611E-00 |
| 1.80000E-02 | 1.96144E-00 |
| 1.60000E-02 | 1.73815E-00 |
| 1.40000E-02 | 1.51622E-00 |
| 1.20000E-02 | 1.29565E-00 |
| 1.00000E-02 | 1.07642E-00 |
| 8.00000E-03 | 8.58532E-01 |
| 6.00000E-03 | 6.41952E-01 |
| 4.00000E-03 | 4.26678E-01 |
| 2.00000E-03 | 2.12698E-01 |
| 1.00000E-03 | 1.06189E-01 |
| 0.00000E-99 | 0.00000E-99 |

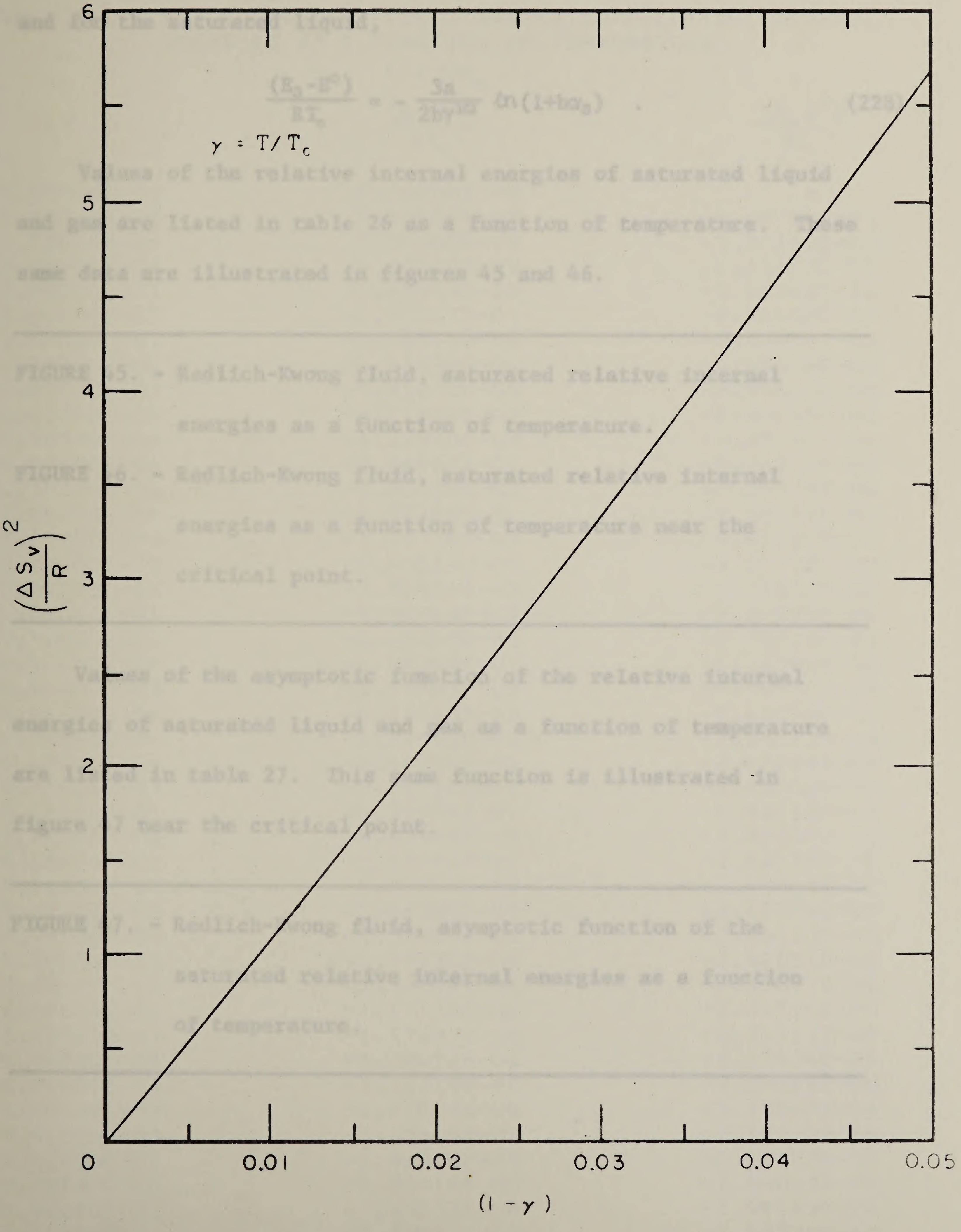

FIGURE 44.-Redlich-Kwong Fluid, Asymptotic Function of the Entropy of Vaporization as a Function of Temperature.

and for the saturated liquid,

$$\frac{(E_3-E^0)}{RT_c} = -\frac{3a}{2b\gamma^{1/2}} \ln(1+b\alpha_3) \quad . \qquad (228)$$

Values of the relative internal energies of saturated liquid and gas are listed in table 26 as a function of temperature. These same data are illustrated in figures 45 and 46.

---

FIGURE 45. - Redlich-Kwong fluid, saturated relative internal energies as a function of temperature.

FIGURE 46. - Redlich-Kwong fluid, saturated relative internal energies as a function of temperature near the critical point.

---

Values of the asymptotic function of the relative internal energies of saturated liquid and gas as a function of temperature are listed in table 27. This same function is illustrated in figure 47 near the critical point.

---

FIGURE 47. - Redlich-Kwong fluid, asymptotic function of the saturated relative internal energies as a function of temperature.

---

TABLE 26. - REDLICH-KWONG FLUID, SATURATED RELATIVE INTERNAL ENERGIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{(E_1 - E^{\circ})}{RT_c}$ | $\frac{(E_3 - E^{\circ})}{RT_c}$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | $-\infty$ |
| 1.00000E-01 | 0.00000E-99 | -1.60708E+01 |
| 1.50000E-01 | 0.00000E-99 | -1.30163E+01 |
| 2.00000E-01 | 0.00000E-99 | -1.11622E+01 |
| 2.50000E-01 | -3.68344E-10 | -9.86948E-00 |
| 3.00000E-01 | -1.75917E-07 | -8.89100E-00 |
| 3.50000E-01 | -9.35614E-06 | -8.10839E-00 |
| 4.00000E-01 | -1.41332E-04 | -7.45655E-00 |
| 4.50000E-01 | -9.81672E-04 | -6.89611E-00 |
| 5.00000E-01 | -4.11599E-03 | -6.40139E-00 |
| 5.50000E-01 | -1.22910E-02 | -5.95452E-00 |
| 6.00000E-01 | -2.90566E-02 | -5.54229E-00 |
| 6.50000E-01 | -5.83185E-02 | -5.15429E-00 |
| 7.00000E-01 | -1.04205E-01 | -4.78161E-00 |
| 7.50000E-01 | -1.71376E-01 | -4.41581E-00 |
| 8.00000E-01 | -2.65864E-01 | -4.04770E-00 |
| 8.50000E-01 | -3.96934E-01 | -3.66531E-00 |
| 9.00000E-01 | -5.81968E-01 | -3.24891E-00 |
| 9.50000E-01 | -8.65245E-01 | -2.75214E-00 |
| 9.52000E-01 | -8.80168E-01 | -2.72902E-00 |
| 9.54000E-01 | -8.95505E-01 | -2.70551E-00 |
| 9.56000E-01 | -9.11283E-01 | -2.68159E-00 |
| 9.58000E-01 | -9.27532E-01 | -2.65722E-00 |
| 9.60000E-01 | -9.44285E-01 | -2.63238E-00 |
| 9.62000E-01 | -9.61580E-01 | -2.60702E-00 |
| 9.64000E-01 | -9.79458E-01 | -2.58110E-00 |
| 9.66000E-01 | -9.97968E-01 | -2.55457E-00 |
| 9.68000E-01 | -1.01716E-00 | -2.52738E-00 |
| 9.70000E-01 | -1.03711E-00 | -2.49946E-00 |
| 9.72000E-01 | -1.05789E-00 | -2.47074E-00 |
| 9.74000E-01 | -1.07959E-00 | -2.44113E-00 |
| 9.76000E-01 | -1.10230E-00 | -2.41051E-00 |
| 9.78000E-01 | -1.12617E-00 | -2.37877E-00 |
| 9.80000E-01 | -1.15136E-00 | -2.34574E-00 |
| 9.82000E-01 | -1.17805E-00 | -2.31122E-00 |
| 9.84000E-01 | -1.20652E-00 | -2.27496E-00 |
| 9.86000E-01 | -1.23710E-00 | -2.23660E-00 |
| 9.88000E-01 | -1.27027E-00 | -2.19569E-00 |
| 9.90000E-01 | -1.30669E-00 | -2.15154E-00 |
| 9.92000E-01 | -1.34740E-00 | -2.10312E-00 |
| 9.94000E-01 | -1.39416E-00 | -2.04869E-00 |
| 9.96000E-01 | -1.45037E-00 | -1.98483E-00 |
| 9.98000E-01 | -1.52481E-00 | -1.90276E-00 |
| 9.99000E-01 | -1.57825E-00 | -1.84551E-00 |
| 1.00000E-00 | -1.70998E-00 | -1.70998E-00 |

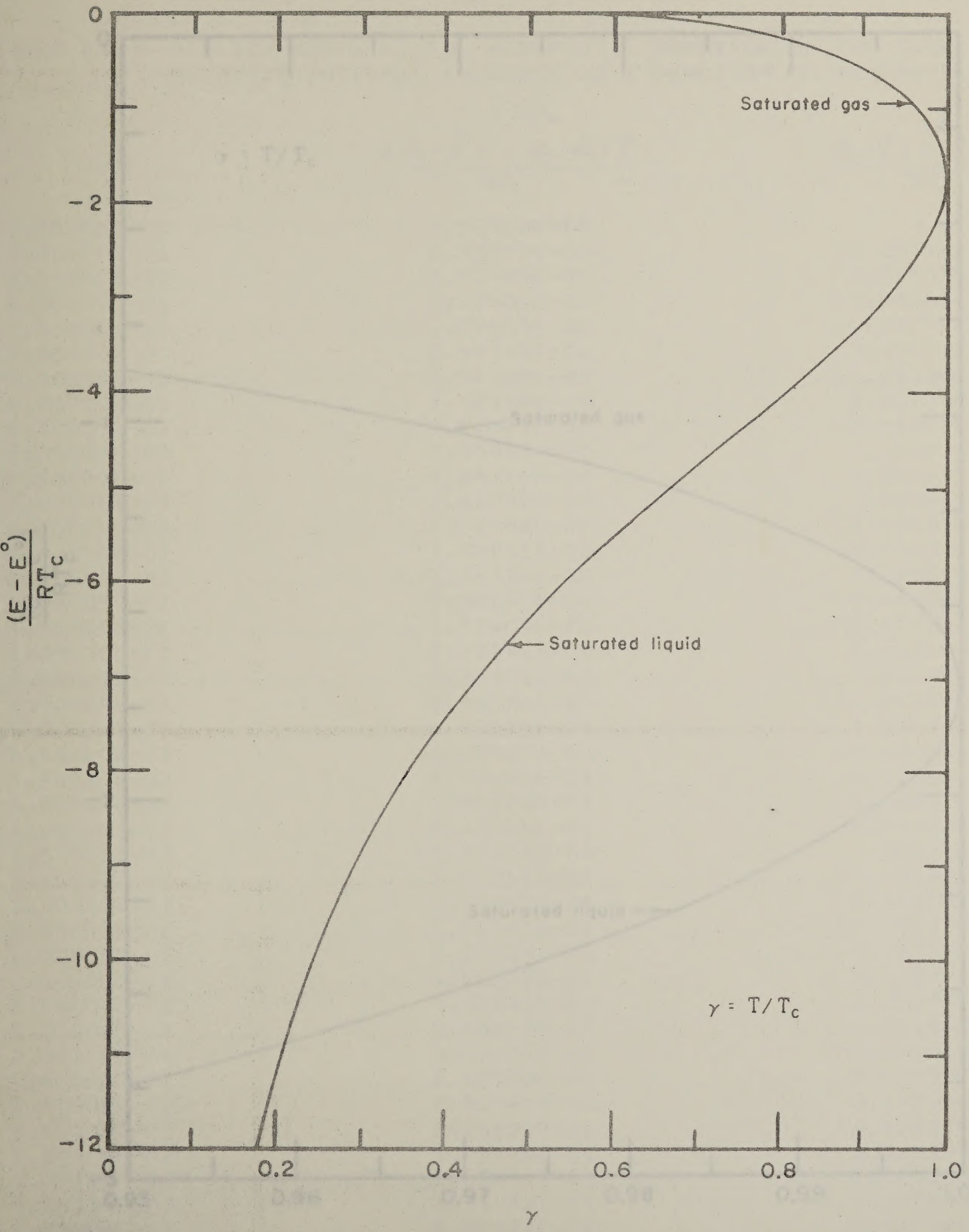

FIGURE 45.- Redlich-Kwong Fluid, Saturated Relative Internal Energies as a Function of Temperature.

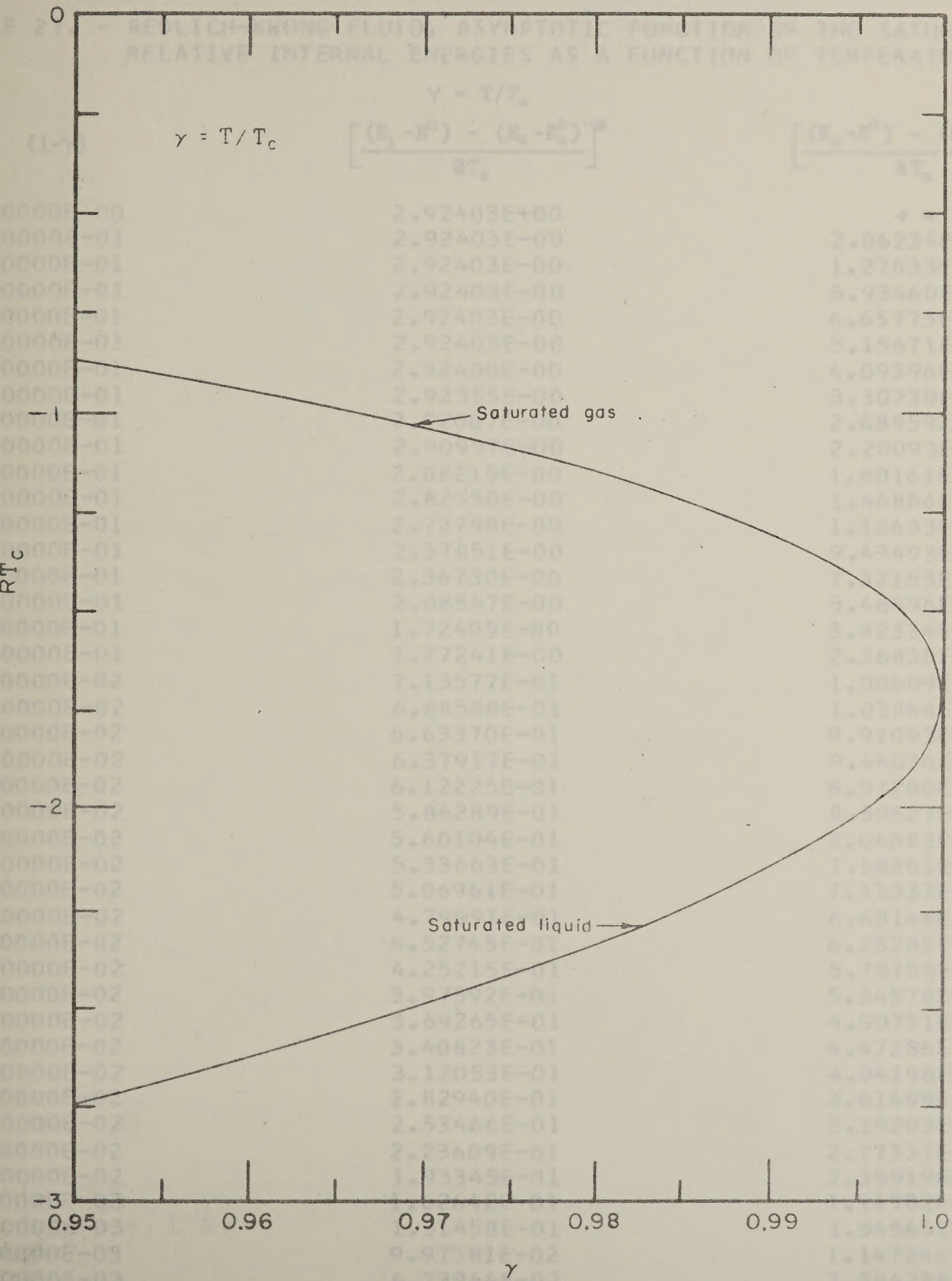

FIGURE 46.- Redlich-Kwong Fluid, Saturated Relative Internal Energies as a Function of Temperature Near the Critical Point.

TABLE 27. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED RELATIVE INTERNAL ENERGIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| (1-$\gamma$) | $\left[\frac{(E_1-E^0) - (E_c-E_c^0)}{RT_c}\right]^2$ | $\left[\frac{(E_3-E^0) - (E_c-E_c^0)}{RT_c}\right]^2$ |
|---|---|---|
| 1.00000E-00 | 2.92403E+00 | + ∞ |
| 9.00000E-01 | 2.92403E-00 | 2.06234E+02 |
| 8.50000E-01 | 2.92403E-00 | 1.27833E+02 |
| 8.00000E-01 | 2.92403E-00 | 8.93460E+01 |
| 7.50000E-01 | 2.92403E-00 | 6.65775E+01 |
| 7.00000E-01 | 2.92403E-00 | 5.15671E+01 |
| 6.50000E-01 | 2.92400E-00 | 4.09396E+01 |
| 6.00000E-01 | 2.92355E-00 | 3.30230E+01 |
| 5.50000E-01 | 2.92067E-00 | 2.68959E+01 |
| 5.00000E-01 | 2.90997E-00 | 2.20093E+01 |
| 4.50000E-01 | 2.88215E-00 | 1.80161E+01 |
| 4.00000E-01 | 2.82550E-00 | 1.46866E+01 |
| 3.50000E-01 | 2.72798E-00 | 1.18633E+01 |
| 3.00000E-01 | 2.57851E-00 | 9.43493E-00 |
| 2.50000E-01 | 2.36730E-00 | 7.32153E-00 |
| 2.00000E-01 | 2.08547E-00 | 5.46496E-00 |
| 1.50000E-01 | 1.72409E-00 | 3.82334E-00 |
| 1.00000E-01 | 1.27241E-00 | 2.36830E-00 |
| 5.00000E-02 | 7.13577E-01 | 1.08609E-00 |
| 4.80000E-02 | 6.88588E-01 | 1.03844E-00 |
| 4.60000E-02 | 6.63370E-01 | 9.91093E-01 |
| 4.40000E-02 | 6.37917E-01 | 9.44036E-01 |
| 4.20000E-02 | 6.12225E-01 | 8.97280E-01 |
| 4.00000E-02 | 5.86289E-01 | 8.50827E-01 |
| 3.80000E-02 | 5.60104E-01 | 8.04683E-01 |
| 3.60000E-02 | 5.33663E-01 | 7.58851E-01 |
| 3.40000E-02 | 5.06961E-01 | 7.13337E-01 |
| 3.20000E-02 | 4.79991E-01 | 6.68146E-01 |
| 3.00000E-02 | 4.52745E-01 | 6.23285E-01 |
| 2.80000E-02 | 4.25215E-01 | 5.78759E-01 |
| 2.60000E-02 | 3.97392E-01 | 5.34578E-01 |
| 2.40000E-02 | 3.69265E-01 | 4.90751E-01 |
| 2.20000E-02 | 3.40823E-01 | 4.47286E-01 |
| 2.00000E-02 | 3.12053E-01 | 4.04198E-01 |
| 1.80000E-02 | 2.82940E-01 | 3.61499E-01 |
| 1.60000E-02 | 2.53466E-01 | 3.19205E-01 |
| 1.40000E-02 | 2.23609E-01 | 2.77337E-01 |
| 1.20000E-02 | 1.93345E-01 | 2.35919E-01 |
| 1.00000E-02 | 1.62642E-01 | 1.94982E-01 |
| 8.00000E-03 | 1.31458E-01 | 1.54565E-01 |
| 6.00000E-03 | 9.97381E-02 | 1.14724E-01 |
| 4.00000E-03 | 6.73966E-02 | 7.55425E-02 |
| 2.00000E-03 | 3.42880E-02 | 3.71638E-02 |
| 1.00000E-03 | 1.73527E-02 | 1.83688E-02 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

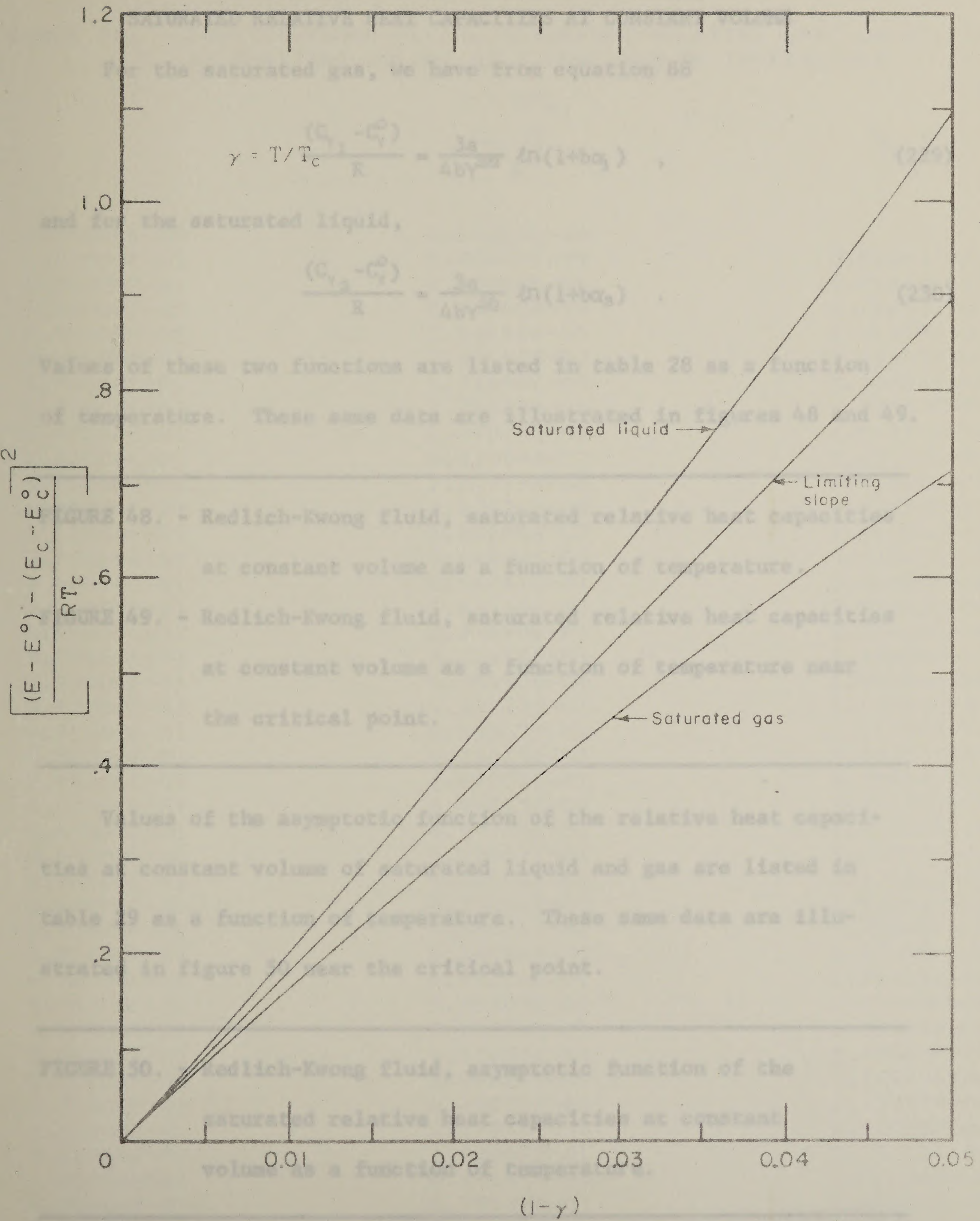

FIGURE 47.- Redlich-Kwong Fluid, Asymptotic Function of the Saturated Relative Internal Energies as a Function of Temperature.

## SATURATED RELATIVE HEAT CAPACITIES AT CONSTANT VOLUME

For the saturated gas, we have from equation 88

$$\frac{(C_{v_1} - C_v^0)}{R} = \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha_1) \quad , \tag{229}$$

and for the saturated liquid,

$$\frac{(C_{v_3} - C_v^0)}{R} = \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha_3) \quad . \tag{230}$$

Values of these two functions are listed in table 28 as a function of temperature. These same data are illustrated in figures 48 and 49.

FIGURE 48. - Redlich-Kwong fluid, saturated relative heat capacities at constant volume as a function of temperature.

FIGURE 49. - Redlich-Kwong fluid, saturated relative heat capacities at constant volume as a function of temperature near the critical point.

Values of the asymptotic function of the relative heat capacities at constant volume of saturated liquid and gas are listed in table 29 as a function of temperature. These same data are illustrated in figure 50 near the critical point.

FIGURE 50. - Redlich-Kwong fluid, asymptotic function of the saturated relative heat capacities at constant volume as a function of temperature.

# TABLE 28. - REDLICH-KWONG FLUID, SATURATED RELATIVE HEAT CAPACITIES AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{C_{V_1} - C_V^0}{R}$ | $\frac{C_{V_3} - C_V^0}{R}$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | + ∞ |
| 1.00000E-01 | 0.00000E-99 | 8.03541E+01 |
| 1.50000E-01 | 0.00000E-99 | 4.33878E+01 |
| 2.00000E-01 | 0.00000E-99 | 2.79057E+01 |
| 2.50000E-01 | 7.36689E-10 | 1.97389E+01 |
| 3.00000E-01 | 2.93195E-07 | 1.48183E+01 |
| 3.50000E-01 | 1.33659E-05 | 1.15834E+01 |
| 4.00000E-01 | 1.76665E-04 | 9.32069E-00 |
| 4.50000E-01 | 1.09074E-03 | 7.66235E-00 |
| 5.00000E-01 | 4.11599E-03 | 6.40139E-00 |
| 5.50000E-01 | 1.11737E-02 | 5.41320E-00 |
| 6.00000E-01 | 2.42138E-02 | 4.61858E-00 |
| 6.50000E-01 | 4.48604E-02 | 3.96484E-00 |
| 7.00000E-01 | 7.44326E-02 | 3.41543E-00 |
| 7.50000E-01 | 1.14251E-01 | 2.94387E-00 |
| 8.00000E-01 | 1.66165E-01 | 2.52981E-00 |
| 8.50000E-01 | 2.33490E-01 | 2.15606E-00 |
| 9.00000E-01 | 3.23316E-01 | 1.80495E-00 |
| 9.50000E-01 | 4.55392E-01 | 1.44849E-00 |
| 9.52000E-01 | 4.62273E-01 | 1.43331E-00 |
| 9.54000E-01 | 4.69342E-01 | 1.41798E-00 |
| 9.56000E-01 | 4.76612E-01 | 1.40250E-00 |
| 9.58000E-01 | 4.84098E-01 | 1.38686E-00 |
| 9.60000E-01 | 4.91815E-01 | 1.37103E-00 |
| 9.62000E-01 | 4.99781E-01 | 1.35500E-00 |
| 9.64000E-01 | 5.08017E-01 | 1.33874E-00 |
| 9.66000E-01 | 5.16546E-01 | 1.32224E-00 |
| 9.68000E-01 | 5.25396E-01 | 1.30546E-00 |
| 9.70000E-01 | 5.34596E-01 | 1.28838E-00 |
| 9.72000E-01 | 5.44184E-01 | 1.27095E-00 |
| 9.74000E-01 | 5.54204E-01 | 1.25314E-00 |
| 9.76000E-01 | 5.64707E-01 | 1.23489E-00 |
| 9.78000E-01 | 5.75756E-01 | 1.21614E-00 |
| 9.80000E-01 | 5.87430E-01 | 1.19680E-00 |
| 9.82000E-01 | 5.99826E-01 | 1.17679E-00 |
| 9.84000E-01 | 6.13072E-01 | 1.15597E-00 |
| 9.86000E-01 | 6.27336E-01 | 1.13418E-00 |
| 9.88000E-01 | 6.42849E-01 | 1.11118E-00 |
| 9.90000E-01 | 6.59945E-01 | 1.08664E-00 |
| 9.92000E-01 | 6.79137E-01 | 1.06004E-00 |
| 9.94000E-01 | 7.01291E-01 | 1.03052E-00 |
| 9.96000E-01 | 7.28098E-01 | 9.96401E-01 |
| 9.98000E-01 | 7.63933E-01 | 9.53286E-01 |
| 9.99000E-01 | 7.89915E-01 | 9.23680E-01 |
| 1.00000E-00 | 8.54990E-01 | 8.54990E-01 |

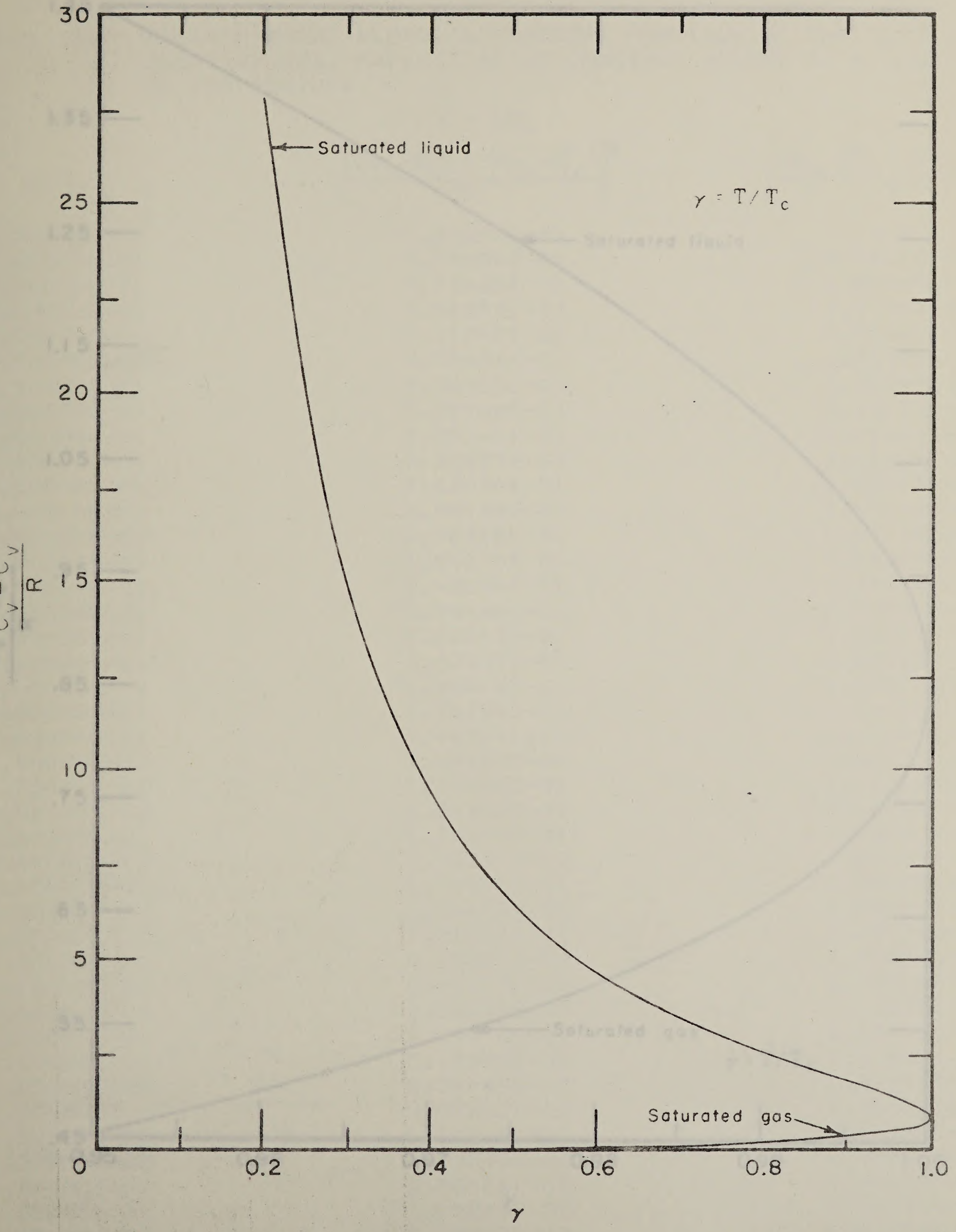

FIGURE 48.- Redlich-Kwong Fluid, Saturated Relative Heat Capacities at Constant Volume as a Function of Temperature.

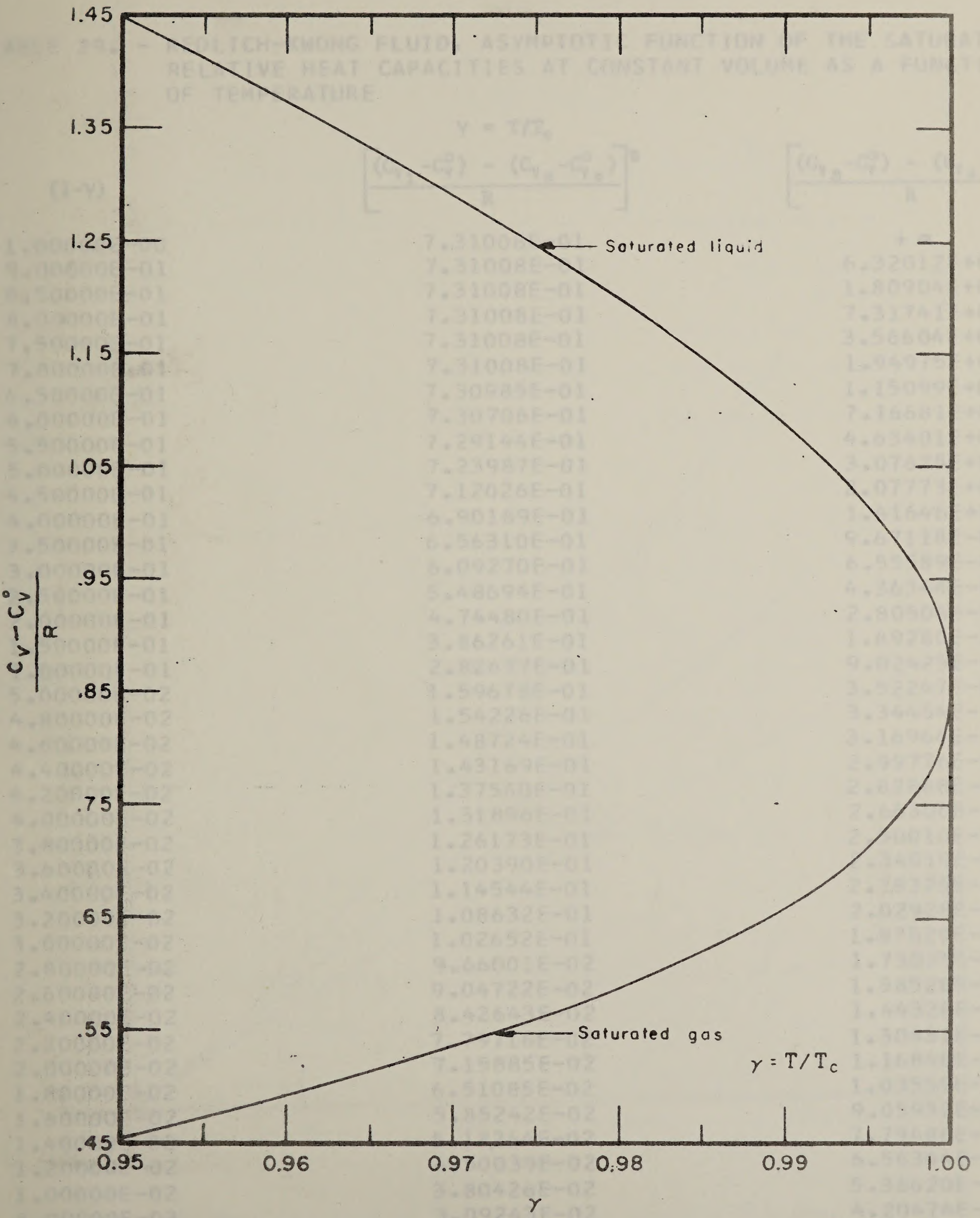

FIGURE 49.—Redlich-Kwong Fluid, Saturated Relative Heat Capacities at Constant Volume as a Function of Temperature Near the Critical Point.

TABLE 29. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED RELATIVE HEAT CAPACITIES AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $(1-\gamma)$ | $\left[\dfrac{(C_{v_1}-C_v^o) - (C_{v_c}-C_{v_c}^o)}{R}\right]^2$ | $\left[\dfrac{(C_{v_3}-C_v^o) - (C_{v_c}-C_{v_c}^o)}{R}\right]^2$ |
|---|---|---|
| 1.00000E-00 | 7.31008E-01 | + ∞ |
| 9.00000E-01 | 7.31008E-01 | 6.32012E+03 |
| 8.50000E-01 | 7.31008E-01 | 1.80904E+03 |
| 8.00000E-01 | 7.31008E-01 | 7.31741E+02 |
| 7.50000E-01 | 7.31008E-01 | 3.56604E+02 |
| 7.00000E-01 | 7.31008E-01 | 1.94975E+02 |
| 6.50000E-01 | 7.30985E-01 | 1.15099E+02 |
| 6.00000E-01 | 7.30706E-01 | 7.16681E+01 |
| 5.50000E-01 | 7.29144E-01 | 4.63401E+01 |
| 5.00000E-01 | 7.23987E-01 | 3.07625E+01 |
| 4.50000E-01 | 7.12026E-01 | 2.07773E+01 |
| 4.00000E-01 | 6.90189E-01 | 1.41646E+01 |
| 3.50000E-01 | 6.56310E-01 | 9.67118E-00 |
| 3.00000E-01 | 6.09270E-01 | 6.55589E-00 |
| 2.50000E-01 | 5.48694E-01 | 4.36344E-00 |
| 2.00000E-01 | 4.74480E-01 | 2.80504E-00 |
| 1.50000E-01 | 3.86261E-01 | 1.69280E-00 |
| 1.00000E-01 | 2.82677E-01 | 9.02423E-01 |
| 5.00000E-02 | 1.59678E-01 | 3.52247E-01 |
| 4.80000E-02 | 1.54226E-01 | 3.34454E-01 |
| 4.60000E-02 | 1.48724E-01 | 3.16964E-01 |
| 4.40000E-02 | 1.43169E-01 | 2.99776E-01 |
| 4.20000E-02 | 1.37560E-01 | 2.82888E-01 |
| 4.00000E-02 | 1.31896E-01 | 2.66300E-01 |
| 3.80000E-02 | 1.26173E-01 | 2.50010E-01 |
| 3.60000E-02 | 1.20390E-01 | 2.34019E-01 |
| 3.40000E-02 | 1.14544E-01 | 2.18325E-01 |
| 3.20000E-02 | 1.08632E-01 | 2.02928E-01 |
| 3.00000E-02 | 1.02652E-01 | 1.87829E-01 |
| 2.80000E-02 | 9.66001E-02 | 1.73029E-01 |
| 2.60000E-02 | 9.04722E-02 | 1.58528E-01 |
| 2.40000E-02 | 8.42643E-02 | 1.44328E-01 |
| 2.20000E-02 | 7.79716E-02 | 1.30431E-01 |
| 2.00000E-02 | 7.15885E-02 | 1.16840E-01 |
| 1.80000E-02 | 6.51085E-02 | 1.03559E-01 |
| 1.60000E-02 | 5.85242E-02 | 9.05932E-02 |
| 1.40000E-02 | 5.18264E-02 | 7.79486E-02 |
| 1.20000E-02 | 4.50039E-02 | 6.56344E-02 |
| 1.00000E-02 | 3.80426E-02 | 5.36620E-02 |
| 8.00000E-03 | 3.09243E-02 | 4.20474E-02 |
| 6.00000E-03 | 2.36233E-02 | 3.08137E-02 |
| 4.00000E-03 | 1.61015E-02 | 1.99969E-02 |
| 2.00000E-03 | 8.29142E-03 | 9.66214E-03 |
| 1.00000E-03 | 4.23476E-03 | 4.71824E-03 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

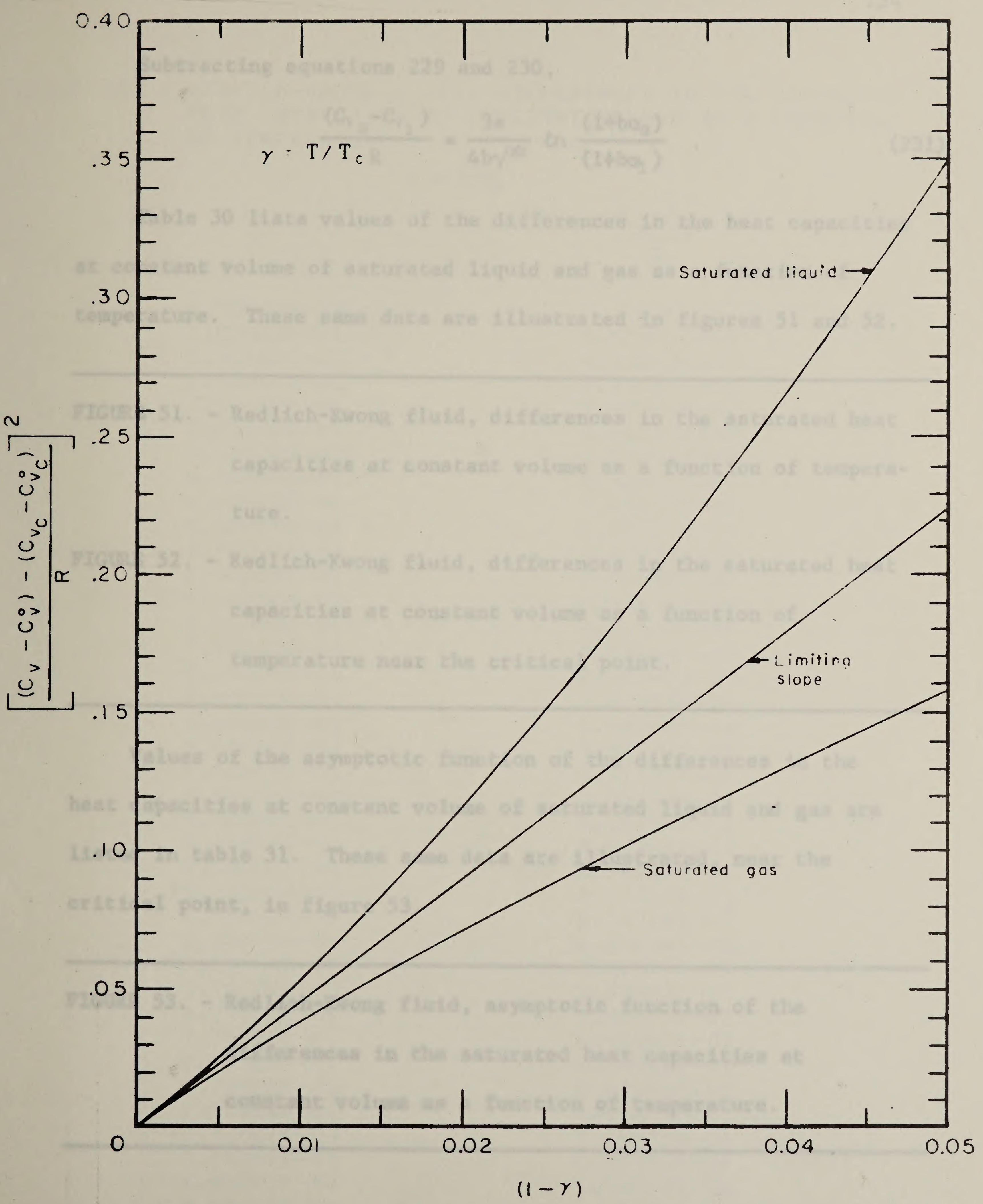

FIGURE 50. - Redlich-Kwong Fluid, Asymptotic Function of the Saturated Relative Heat Capacities at Constant Volume as a Function of Temperature.

Subtracting equations 229 and 230,

$$\frac{(C_{v_3} - C_{v_1})}{R} = \frac{3a}{4b\gamma^{3/2}} \ln \frac{(1+b\alpha_3)}{(1+b\alpha_1)} \quad . \qquad (231)$$

Table 30 lists values of the differences in the heat capacities at constant volume of saturated liquid and gas as a function of temperature. These same data are illustrated in figures 51 and 52.

---

FIGURE 51. - Redlich-Kwong fluid, differences in the saturated heat capacities at constant volume as a function of temperature.

FIGURE 52. - Redlich-Kwong fluid, differences in the saturated heat capacities at constant volume as a function of temperature near the critical point.

---

Values of the asymptotic function of the differences in the heat capacities at constant volume of saturated liquid and gas are listed in table 31. These same data are illustrated, near the critical point, in figure 53.

---

FIGURE 53. - Redlich-Kwong fluid, asymptotic function of the differences in the saturated heat capacities at constant volume as a function of temperature.

---

TABLE 30. - REDLICH-KWONG FLUID, DIFFERENCES IN THE SATURATED HEAT CAPACITIES AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{C_{V_3}-C_{V_1}}{R}$ |
|---|---|
| 0.00000E-99 | $+\infty$ |
| 1.00000E-01 | 8.03541E+01 |
| 1.50000E-01 | 4.33878E+01 |
| 2.00000E-01 | 2.79057E+01 |
| 2.50000E-01 | 1.97389E+01 |
| 3.00000E-01 | 1.48183E+01 |
| 3.50000E-01 | 1.15834E+01 |
| 4.00000E-01 | 9.32051E-00 |
| 4.50000E-01 | 7.66125E-00 |
| 5.00000E-01 | 6.39727E-00 |
| 5.50000E-01 | 5.40202E-00 |
| 6.00000E-01 | 4.59436E-00 |
| 6.50000E-01 | 3.91998E-00 |
| 7.00000E-01 | 3.34100E-00 |
| 7.50000E-01 | 2.82962E-00 |
| 8.00000E-01 | 2.36365E-00 |
| 8.50000E-01 | 1.92257E-00 |
| 9.00000E-01 | 1.48163E-00 |
| 9.50000E-01 | 9.93102E-01 |
| 9.52000E-01 | 9.71037E-01 |
| 9.54000E-01 | 9.48643E-01 |
| 9.56000E-01 | 9.25895E-01 |
| 9.58000E-01 | 9.02764E-01 |
| 9.60000E-01 | 8.79218E-01 |
| 9.62000E-01 | 8.55219E-01 |
| 9.64000E-01 | 8.30728E-01 |
| 9.66000E-01 | 8.05696E-01 |
| 9.68000E-01 | 7.80070E-01 |
| 9.70000E-01 | 7.53787E-01 |
| 9.72000E-01 | 7.26773E-01 |
| 9.74000E-01 | 6.98942E-01 |
| 9.76000E-01 | 6.70188E-01 |
| 9.78000E-01 | 6.40386E-01 |
| 9.80000E-01 | 6.09379E-01 |
| 9.82000E-01 | 5.76970E-01 |
| 9.84000E-01 | 5.42904E-01 |
| 9.86000E-01 | 5.06847E-01 |
| 9.88000E-01 | 4.68333E-01 |
| 9.90000E-01 | 4.26695E-01 |
| 9.92000E-01 | 3.80907E-01 |
| 9.94000E-01 | 3.29237E-01 |
| 9.96000E-01 | 2.68302E-01 |
| 9.98000E-01 | 1.89353E-01 |
| 9.99000E-01 | 1.33764E-01 |
| 1.00000E-00 | 0.00000E-99 |

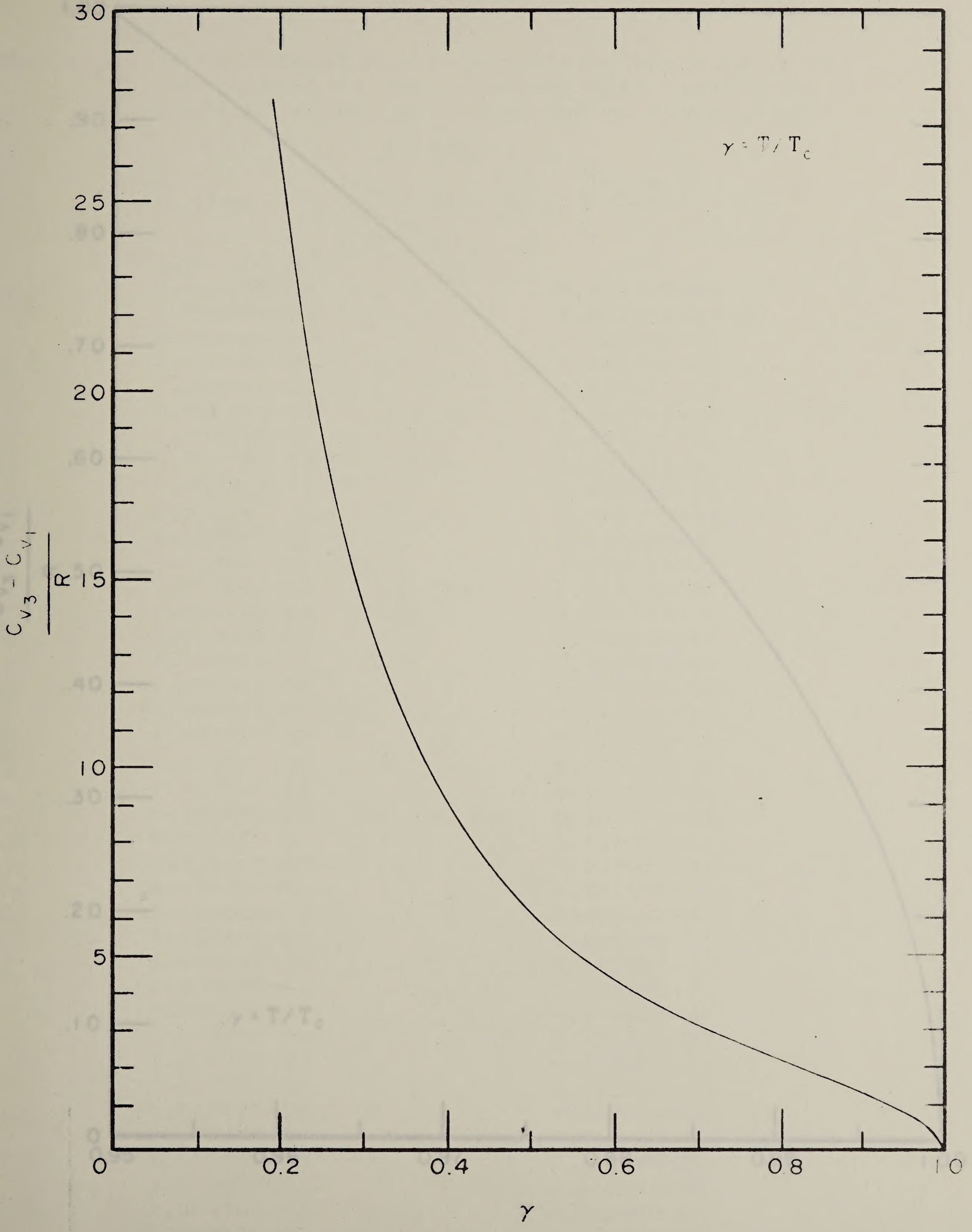

FIGURE 51. —Redlich–Kwong Fluid, Differences in the Saturated Heat Capacities at Constant Volume as a Function of Temperature.

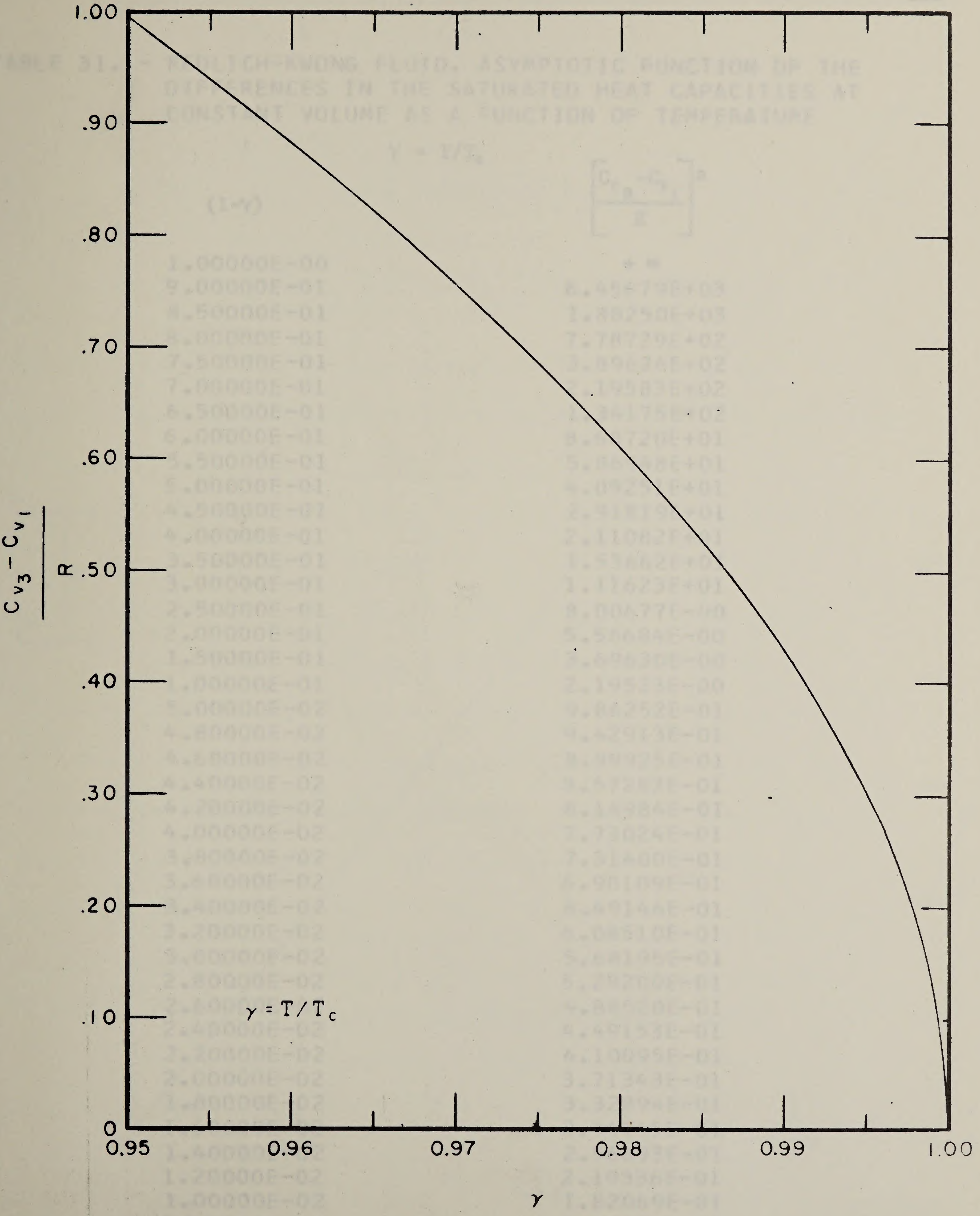

FIGURE 52.—Redlich-Kwong Fluid, Differences in the Saturated Heat Capacities at Constant Volume as a Function of Temperature Near the Critical Point.

TABLE 31. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE DIFFERENCES IN THE SATURATED HEAT CAPACITIES AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $(1-\gamma)$ | $\left[\frac{C_{V_3}-C_{V_1}}{R}\right]^2$ |
|---|---|
| 1.00000E-00 | + ∞ |
| 9.00000E-01 | 6.45679E+03 |
| 8.50000E-01 | 1.88250E+03 |
| 8.00000E-01 | 7.78729E+02 |
| 7.50000E-01 | 3.89626E+02 |
| 7.00000E-01 | 2.19583E+02 |
| 6.50000E-01 | 1.34175E+02 |
| 6.00000E-01 | 8.68720E+01 |
| 5.50000E-01 | 5.86948E+01 |
| 5.00000E-01 | 4.09251E+01 |
| 4.50000E-01 | 2.91819E+01 |
| 4.00000E-01 | 2.11082E+01 |
| 3.50000E-01 | 1.53662E+01 |
| 3.00000E-01 | 1.11623E+01 |
| 2.50000E-01 | 8.00677E-00 |
| 2.00000E-01 | 5.58684E-00 |
| 1.50000E-01 | 3.69630E-00 |
| 1.00000E-01 | 2.19523E-00 |
| 5.00000E-02 | 9.86252E-01 |
| 4.80000E-02 | 9.42913E-01 |
| 4.60000E-02 | 8.99925E-01 |
| 4.40000E-02 | 8.57283E-01 |
| 4.20000E-02 | 8.14984E-01 |
| 4.00000E-02 | 7.73024E-01 |
| 3.80000E-02 | 7.31400E-01 |
| 3.60000E-02 | 6.90109E-01 |
| 3.40000E-02 | 6.49146E-01 |
| 3.20000E-02 | 6.08510E-01 |
| 3.00000E-02 | 5.68195E-01 |
| 2.80000E-02 | 5.28200E-01 |
| 2.60000E-02 | 4.88520E-01 |
| 2.40000E-02 | 4.49153E-01 |
| 2.20000E-02 | 4.10095E-01 |
| 2.00000E-02 | 3.71343E-01 |
| 1.80000E-02 | 3.32894E-01 |
| 1.60000E-02 | 2.94745E-01 |
| 1.40000E-02 | 2.56893E-01 |
| 1.20000E-02 | 2.19336E-01 |
| 1.00000E-02 | 1.82069E-01 |
| 8.00000E-03 | 1.45090E-01 |
| 6.00000E-03 | 1.08397E-01 |
| 4.00000E-03 | 7.19862E-02 |
| 2.00000E-03 | 3.58547E-02 |
| 1.00000E-03 | 1.78929E-02 |
| 0.00000E-99 | 0.00000E-99 |

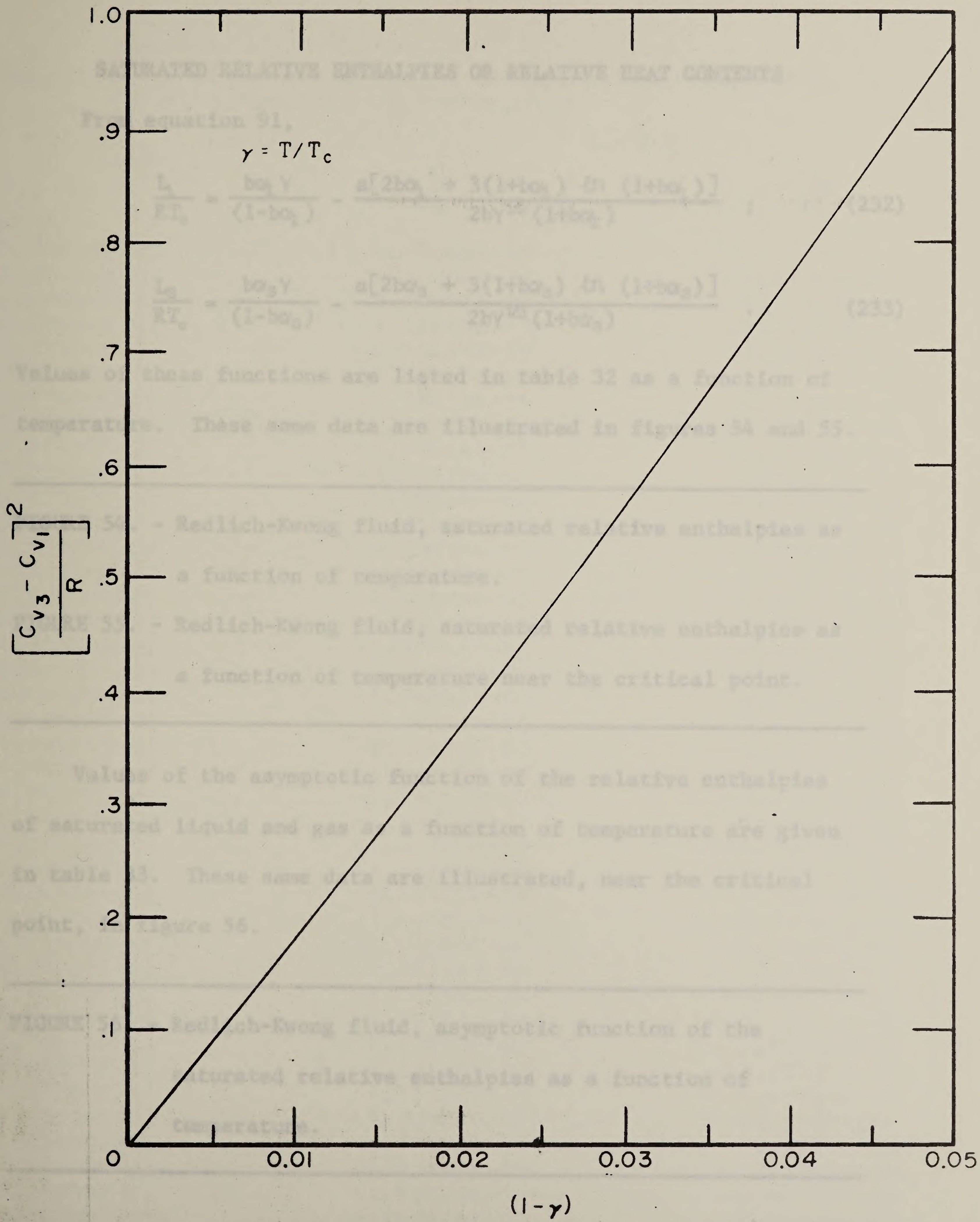

FIGURE 53.–Redlich-Kwong Fluid, Asymptotic Function of the Differences in the Saturated Heat Capacities at Constant Volume as a Function of Temperature.

## SATURATED RELATIVE ENTHALPIES OR RELATIVE HEAT CONTENTS

From equation 91,

$$\frac{L_1}{RT_c} = \frac{b\alpha_1 \gamma}{(1-b\alpha_1)} - \frac{a[2b\alpha_1 + 3(1+b\alpha_1)\, \ln\, (1+b\alpha_1)]}{2b\gamma^{1/2}(1+b\alpha_1)} \quad ; \qquad (232)$$

$$\frac{L_3}{RT_c} = \frac{b\alpha_3 \gamma}{(1-b\alpha_3)} - \frac{a[2b\alpha_3 + 3(1+b\alpha_3)\, \ln\, (1+b\alpha_3)]}{2b\gamma^{1/2}(1+b\alpha_3)} \quad . \qquad (233)$$

Values of these functions are listed in table 32 as a function of temperature. These same data are illustrated in figures 54 and 55.

---

FIGURE 54. - Redlich-Kwong fluid, saturated relative enthalpies as a function of temperature.

FIGURE 55. - Redlich-Kwong fluid, saturated relative enthalpies as a function of temperature near the critical point.

---

Values of the asymptotic function of the relative enthalpies of saturated liquid and gas as a function of temperature are given in table 33. These same data are illustrated, near the critical point, in figure 56.

---

FIGURE 56. - Redlich-Kwong fluid, asymptotic function of the saturated relative enthalpies as a function of temperature.

---

TABLE 32. - REDLICH-KWONG FLUID, SATURATED RELATIVE ENTHALPIES AS A FUNCTION OF TEMPERATURE

$Y = T/T_c$ $L = H\text{-}H^{\circ}$

| Y | $\frac{L_1}{RT_c}$ | $\frac{L_3}{RT_c}$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | $-\infty$ |
| 1.00000E-01 | 0.00000E-99 | -1.61708E+01 |
| 1.50000E-01 | 0.00000E-99 | -1.31663E+01 |
| 2.00000E-01 | -7.20000E-15 | -1.13622E+01 |
| 2.50000E-01 | -6.07689E-10 | -1.01194E+01 |
| 3.00000E-01 | -2.89289E-07 | -9.19100E-00 |
| 3.50000E-01 | -1.53318E-05 | -8.45839E-00 |
| 4.00000E-01 | -2.30721E-04 | -7.85654E-00 |
| 4.50000E-01 | -1.59604E-03 | -7.34606E-00 |
| 5.00000E-01 | -6.66271E-03 | -6.90116E-00 |
| 5.50000E-01 | -1.98014E-02 | -6.50370E-00 |
| 6.00000E-01 | -4.65652E-02 | -6.14004E-00 |
| 6.50000E-01 | -9.29052E-02 | -5.79909E-00 |
| 7.00000E-01 | -1.64874E-01 | -5.47104E-00 |
| 7.50000E-01 | -2.68983E-01 | -5.14628E-00 |
| 8.00000E-01 | -4.13286E-01 | -4.81393E-00 |
| 8.50000E-01 | -6.09734E-01 | -4.45950E-00 |
| 9.00000E-01 | -8.80270E-01 | -4.05869E-00 |
| 9.50000E-01 | -1.28002E-00 | -3.55426E-00 |
| 9.52000E-01 | -1.30061E-00 | -3.52998E-00 |
| 9.54000E-01 | -1.32173E-00 | -3.50522E-00 |
| 9.56000E-01 | -1.34340E-00 | -3.47994E-00 |
| 9.58000E-01 | -1.36567E-00 | -3.45410E-00 |
| 9.60000E-01 | -1.38857E-00 | -3.42767E-00 |
| 9.62000E-01 | -1.41215E-00 | -3.40059E-00 |
| 9.64000E-01 | -1.43647E-00 | -3.37282E-00 |
| 9.66000E-01 | -1.46157E-00 | -3.34430E-00 |
| 9.68000E-01 | -1.48754E-00 | -3.31495E-00 |
| 9.70000E-01 | -1.51444E-00 | -3.28469E-00 |
| 9.72000E-01 | -1.54238E-00 | -3.25345E-00 |
| 9.74000E-01 | -1.57146E-00 | -3.22110E-00 |
| 9.76000E-01 | -1.60181E-00 | -3.18751E-00 |
| 9.78000E-01 | -1.63358E-00 | -3.15253E-00 |
| 9.80000E-01 | -1.66699E-00 | -3.11597E-00 |
| 9.82000E-01 | -1.70226E-00 | -3.07757E-00 |
| 9.84000E-01 | -1.73972E-00 | -3.03701E-00 |
| 9.86000E-01 | -1.77978E-00 | -2.99390E-00 |
| 9.88000E-01 | -1.82302E-00 | -2.94763E-00 |
| 9.90000E-01 | -1.87027E-00 | -2.89740E-00 |
| 9.92000E-01 | -1.92278E-00 | -2.84194E-00 |
| 9.94000E-01 | -1.98270E-00 | -2.77911E-00 |
| 9.96000E-01 | -2.05417E-00 | -2.70476E-00 |
| 9.98000E-01 | -2.14791E-00 | -2.60818E-00 |
| 9.99000E-01 | -2.21457E-00 | -2.54011E-00 |
| 1.00000E-00 | -2.37664E-00 | -2.37664E-00 |

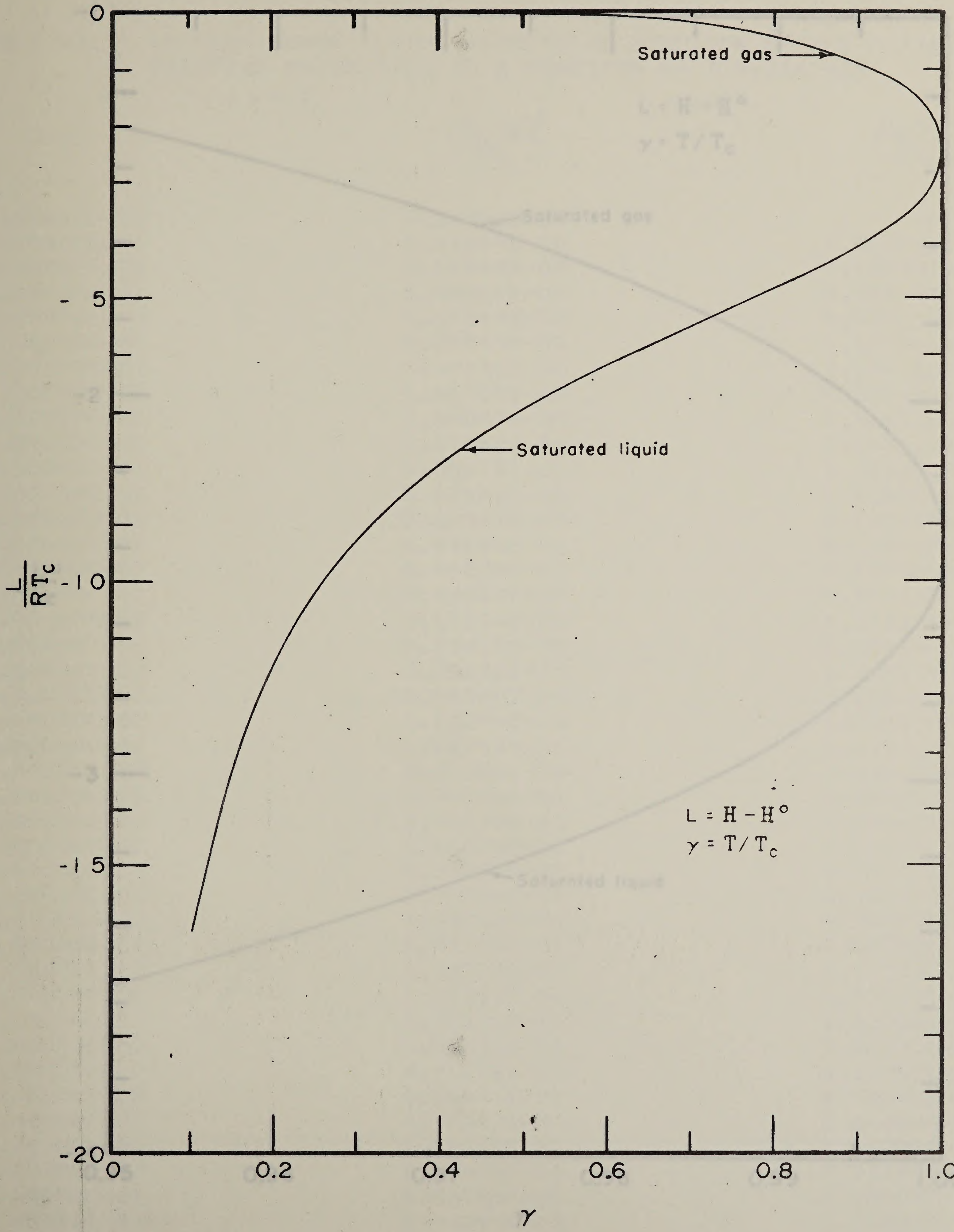

FIGURE 54.– Redlich-Kwong Fluid, Saturated Relative Enthalpies as a Function of Temperature.

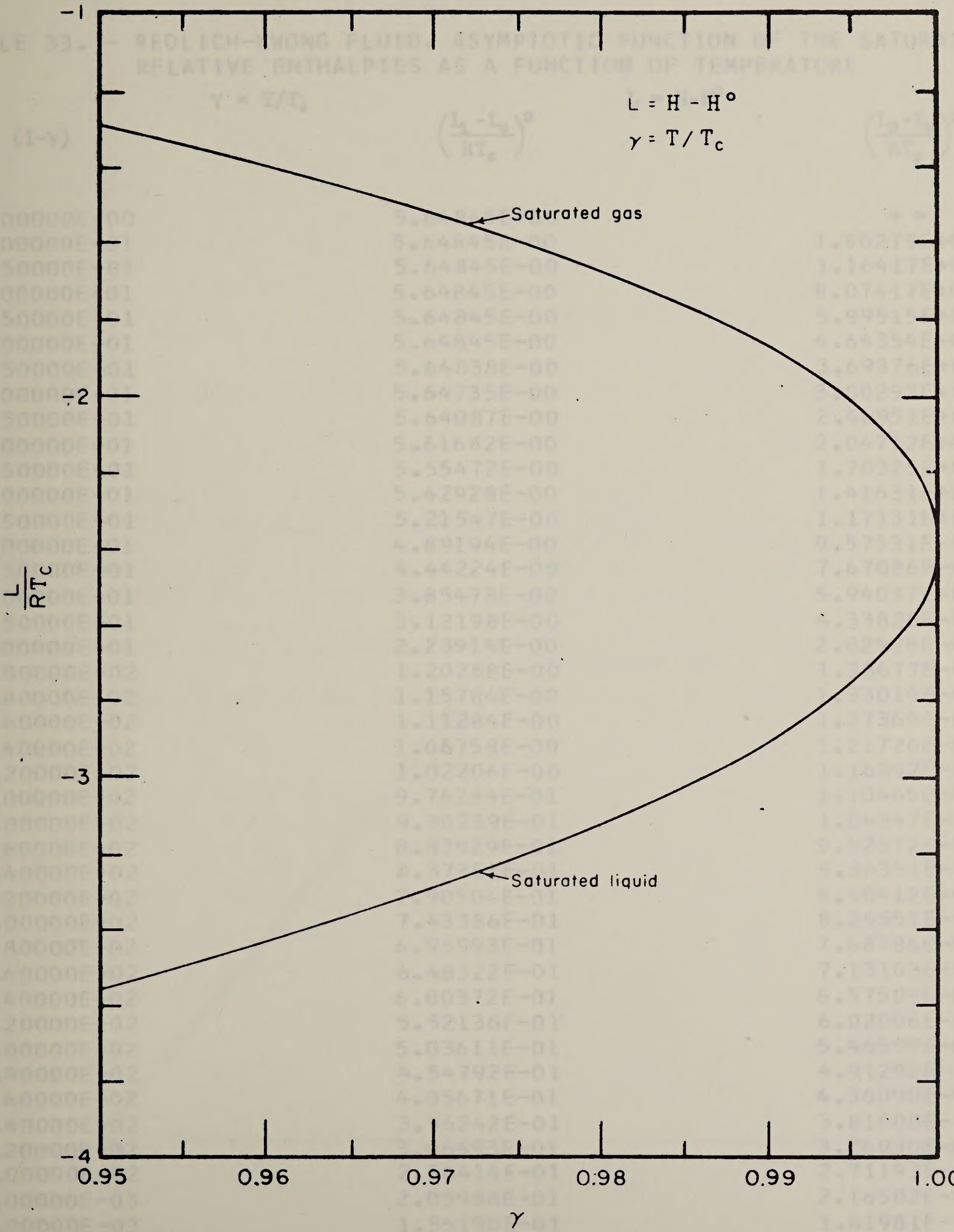

FIGURE 55. – Redlich–Kwong Fluid, Saturated Relative Enthalpies as a Function of Temperature Near the Critical Point.

TABLE 33. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED RELATIVE ENTHALPIES AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $L = H-H^0$

| $(1-\gamma)$ | $\left(\frac{L_1-L_c}{RT_c}\right)^2$ | $\left(\frac{L_3-L_c}{RT_c}\right)^2$ |
|---|---|---|
| 1.00000E-00 | 5.64845E+00 | + ∞ |
| 9.00000E-01 | 5.64845E-00 | 1.90279E+02 |
| 8.50000E-01 | 5.64845E-00 | 1.16417E+02 |
| 8.00000E-01 | 5.64845E-00 | 8.07417E+01 |
| 7.50000E-01 | 5.64845E-00 | 5.99515E+01 |
| 7.00000E-01 | 5.64845E-00 | 4.64354E+01 |
| 6.50000E-01 | 5.64838E-00 | 3.69876E+01 |
| 6.00000E-01 | 5.64735E-00 | 3.00293E+01 |
| 5.50000E-01 | 5.64087E-00 | 2.46951E+01 |
| 5.00000E-01 | 5.61682E-00 | 2.04712E+01 |
| 4.50000E-01 | 5.55472E-00 | 1.70325E+01 |
| 4.00000E-01 | 5.42928E-00 | 1.41631E+01 |
| 3.50000E-01 | 5.21547E-00 | 1.17131E+01 |
| 3.00000E-01 | 4.89194E-00 | 9.57531E-00 |
| 2.50000E-01 | 4.44224E-00 | 7.67086E-00 |
| 2.00000E-01 | 3.85478E-00 | 5.94037E-00 |
| 1.50000E-01 | 3.12198E-00 | 4.33828E-00 |
| 1.00000E-01 | 2.23914E-00 | 2.82928E-00 |
| 5.00000E-02 | 1.20258E-00 | 1.38677E-00 |
| 4.80000E-02 | 1.15784E-00 | 1.33019E-00 |
| 4.60000E-02 | 1.11284E-00 | 1.27369E-00 |
| 4.40000E-02 | 1.06758E-00 | 1.21726E-00 |
| 4.20000E-02 | 1.02206E-00 | 1.16092E-00 |
| 4.00000E-02 | 9.76284E-01 | 1.10465E-00 |
| 3.80000E-02 | 9.30239E-01 | 1.04847E-00 |
| 3.60000E-02 | 8.83929E-01 | 9.92372E-01 |
| 3.40000E-02 | 8.37351E-01 | 9.36351E-01 |
| 3.20000E-02 | 7.90504E-01 | 8.80412E-01 |
| 3.00000E-02 | 7.43386E-01 | 8.24557E-01 |
| 2.80000E-02 | 6.95993E-01 | 7.68786E-01 |
| 2.60000E-02 | 6.48322E-01 | 7.13103E-01 |
| 2.40000E-02 | 6.00372E-01 | 6.57509E-01 |
| 2.20000E-02 | 5.52136E-01 | 6.02006E-01 |
| 2.00000E-02 | 5.03611E-01 | 5.46599E-01 |
| 1.80000E-02 | 4.54792E-01 | 4.91292E-01 |
| 1.60000E-02 | 4.05671E-01 | 4.36090E-01 |
| 1.40000E-02 | 3.56242E-01 | 3.81000E-01 |
| 1.20000E-02 | 3.06493E-01 | 3.26030E-01 |
| 1.00000E-02 | 2.56414E-01 | 2.71193E-01 |
| 8.00000E-03 | 2.05988E-01 | 2.16502E-01 |
| 6.00000E-03 | 1.55190E-01 | 1.61981E-01 |
| 4.00000E-03 | 1.03988E-01 | 1.07663E-01 |
| 2.00000E-03 | 5.23180E-02 | 5.36100E-02 |
| 1.00000E-03 | 2.62670E-02 | 2.67225E-02 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

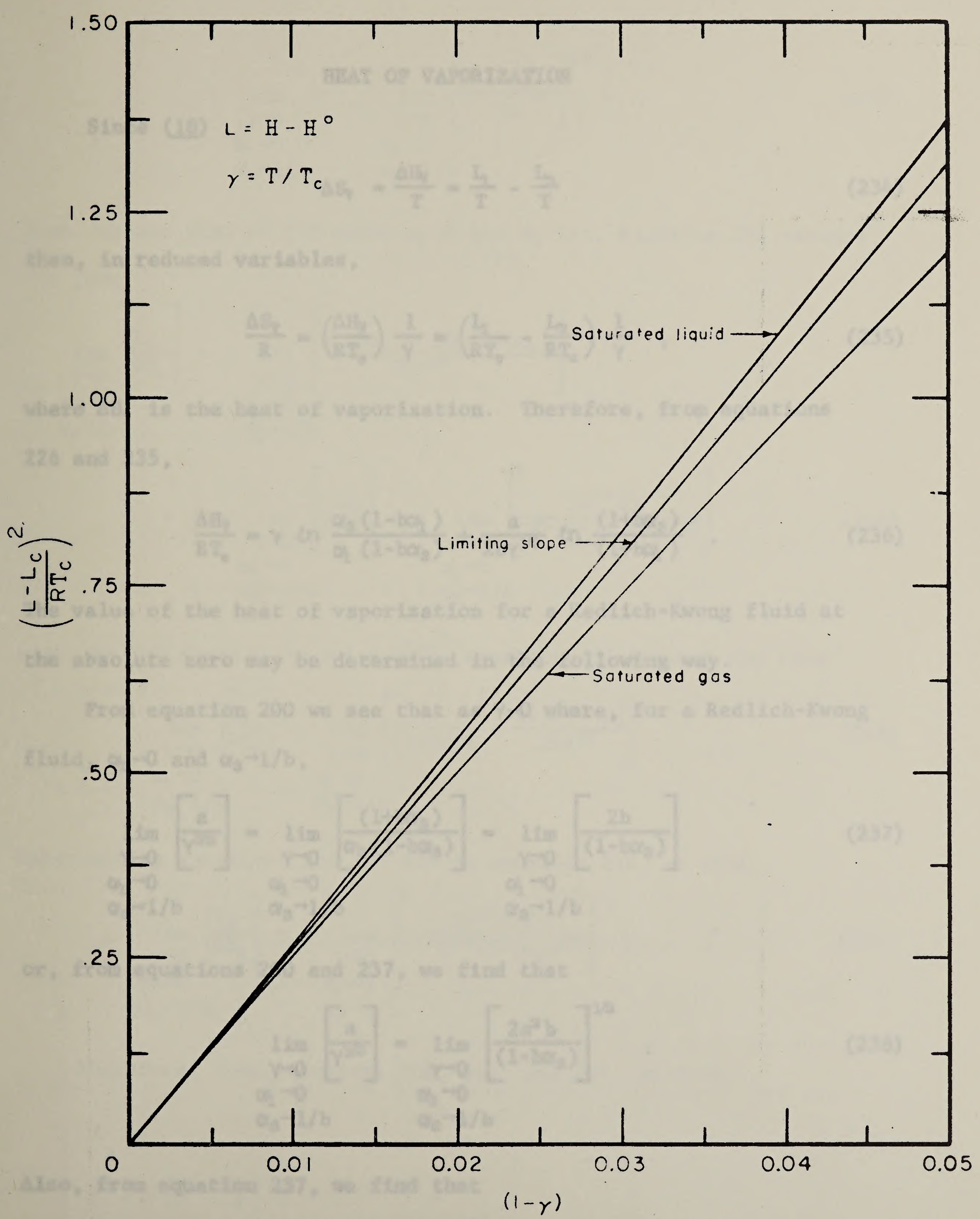

FIGURE 56.-Redlich-Kwong Fluid, Asymptotic Function of the Saturated Relative Enthalpies as a Function of Temperature.

## HEAT OF VAPORIZATION

Since (18)

$$\Delta S_v = \frac{\Delta H_v}{T} = \frac{L_1}{T} - \frac{L_3}{T} \tag{234}$$

then, in reduced variables,

$$\frac{\Delta S_v}{R} = \left(\frac{\Delta H_v}{RT_c}\right)\frac{1}{\gamma} = \left(\frac{L_1}{RT_c} - \frac{L_3}{RT_c}\right)\frac{1}{\gamma} \ , \tag{235}$$

where $\Delta H_v$ is the heat of vaporization. Therefore, from equations 226 and 235,

$$\frac{\Delta H_v}{RT_c} = \gamma \ln \frac{\alpha_3(1-b\alpha_1)}{\alpha_1(1-b\alpha_3)} + \frac{a}{2b\gamma^{1/2}} \ln \frac{(1+b\alpha_3)}{(1+b\alpha_1)} \ . \tag{236}$$

The value of the heat of vaporization for a Redlich-Kwong fluid at the absolute zero may be determined in the following way.

From equation 200 we see that as $\gamma \to 0$ where, for a Redlich-Kwong fluid, $\alpha_1 \to 0$ and $\alpha_3 \to 1/b$,

$$\lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{a}{\gamma^{3/2}}\right] = \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{(1+b\alpha_3)}{\alpha_3(1-b\alpha_3)}\right] = \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{2b}{(1-b\alpha_3)}\right] \tag{237}$$

or, from equations 200 and 237, we find that

$$\lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{a}{\gamma^{1/2}}\right] = \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{2a^2 b}{(1-b\alpha_3)}\right]^{1/3} \ . \tag{238}$$

Also, from equation 237, we find that

$$\lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} [\gamma] = \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{a(1-b\alpha_3)}{2b}\right]^{2/3} . \tag{239}$$

Thus, we see that as $\gamma\to 0$ where $\alpha_1\to 0$ and $\alpha_3\to 1/b$, equation 232 assumes the form

$$\lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{L_1}{RT_c}\right] = - \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{a[2b\alpha_1 + 3\ell n(1+b\alpha_1)]}{2b\gamma^{1/2}}\right] ,$$

$$= - \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{5a\alpha_1}{2\gamma^{1/2}}\right] = - \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{5a^{2/3}b\alpha_1}{2^{2/3}b^{2/3}(1-b\alpha_3)^{1/3}}\right] . \tag{240}$$

Also, as $\gamma\to 0$ where $\alpha_1\to 0$ and $\alpha_3\to 1/b$, equation 233 assumes the form

$$\lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{L_3}{RT_c}\right] = \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{\gamma}{(1-b\alpha_3)} - \frac{a[1+3\ell n2]}{2b\gamma^{1/2}}\right] . \tag{241}$$

Substituting equations 238 and 239 in equation 241, we find

$$\lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{L_3}{RT_c}\right] = - \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{3a^{2/3}\ell n2}{2^{2/3}b^{2/3}(1-b\alpha_3)^{1/3}}\right] . \tag{242}$$

Therefore, from equations 235, 240, and 242, we find

$$\lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{\Delta H_v}{RT_c}\right] = \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{L_1 - L_3}{RT_c}\right] ,$$

$$= \lim_{\substack{\gamma\to 0\\ \alpha_1\to 0\\ \alpha_3\to 1/b}} \left[\frac{a^{2/3}(3\ln 2 - 5b\alpha_1)}{2^{2/3}\, b^{2/3}\,(1-b\alpha_3)^{1/3}}\right] = +\infty \quad . \qquad (243)$$

Thus, the heat of vaporization for a Redlich-Kwong fluid at absolute zero is infinite.

Landau and Lifshitz (17) say that for temperatures appreciably below the critical temperature, $\Delta H_v/RT_c$ is approximately equal to 10; Barieau (5) shows that for a van der Waals fluid $\left.\frac{\Delta H_v}{RT_c}\right|_{\gamma=0} = 3.375$.

Table 34 lists values of the heat of vaporization as a function of temperature. These same data are illustrated in figures 57 and 58.

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FIGURE 57. - Redlich-Kwong fluid, heat of vaporization as a function of temperature.

FIGURE 58. - Redlich-Kwong fluid, heat of vaporization as a function of temperature near the critical point.

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Values of the asymptotic function of the heat of vaporization as a function of $(1-\gamma)$ are listed in table 35. These same data are

TABLE 34. - REDLICH-KWONG FLUID, HEAT OF VAPORIZATION AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{\Delta H_v}{RT_c}$ |
|---|---|
| 0.00000E-99 | $+\infty$ |
| 1.00000E-01 | 1.61708E+01 |
| 1.50000E-01 | 1.31663E+01 |
| 2.00000E-01 | 1.13622E+01 |
| 2.50000E-01 | 1.01194E+01 |
| 3.00000E-01 | 9.19100E-00 |
| 3.50000E-01 | 8.45837E-00 |
| 4.00000E-01 | 7.85631E-00 |
| 4.50000E-01 | 7.34447E-00 |
| 5.00000E-01 | 6.89449E-00 |
| 5.50000E-01 | 6.48390E-00 |
| 6.00000E-01 | 6.09347E-00 |
| 6.50000E-01 | 5.70618E-00 |
| 7.00000E-01 | 5.30617E-00 |
| 7.50000E-01 | 4.87729E-00 |
| 8.00000E-01 | 4.40064E-00 |
| 8.50000E-01 | 3.84976E-00 |
| 9.00000E-01 | 3.17842E-00 |
| 9.50000E-01 | 2.27423E-00 |
| 9.52000E-01 | 2.22937E-00 |
| 9.54000E-01 | 2.18349E-00 |
| 9.56000E-01 | 2.13653E-00 |
| 9.58000E-01 | 2.08843E-00 |
| 9.60000E-01 | 2.03909E-00 |
| 9.62000E-01 | 1.98844E-00 |
| 9.64000E-01 | 1.93635E-00 |
| 9.66000E-01 | 1.88272E-00 |
| 9.68000E-01 | 1.82740E-00 |
| 9.70000E-01 | 1.77024E-00 |
| 9.72000E-01 | 1.71106E-00 |
| 9.74000E-01 | 1.64963E-00 |
| 9.76000E-01 | 1.58570E-00 |
| 9.78000E-01 | 1.51894E-00 |
| 9.80000E-01 | 1.44897E-00 |
| 9.82000E-01 | 1.37530E-00 |
| 9.84000E-01 | 1.29729E-00 |
| 9.86000E-01 | 1.21411E-00 |
| 9.88000E-01 | 1.12461E-00 |
| 9.90000E-01 | 1.02713E-00 |
| 9.92000E-01 | 9.19157E-01 |
| 9.94000E-01 | 7.96412E-01 |
| 9.96000E-01 | 6.50593E-01 |
| 9.98000E-01 | 4.60269E-01 |
| 9.99000E-01 | 3.25541E-01 |
| 1.00000E-00 | 0.00000E-99 |

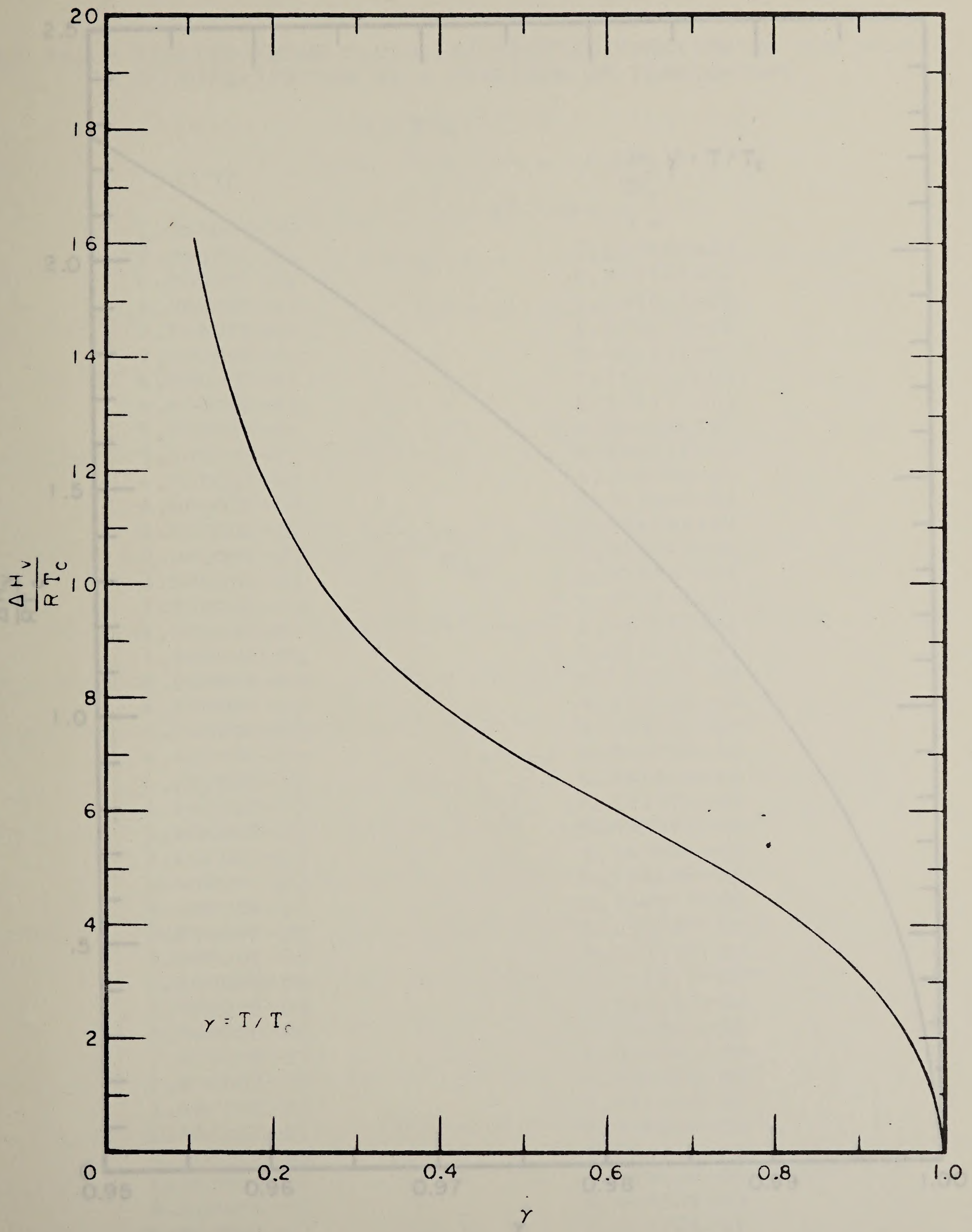

FIGURE 57.–Redlich–Kwong Fluid, Heat of Vaporization as a Function of Temperature.

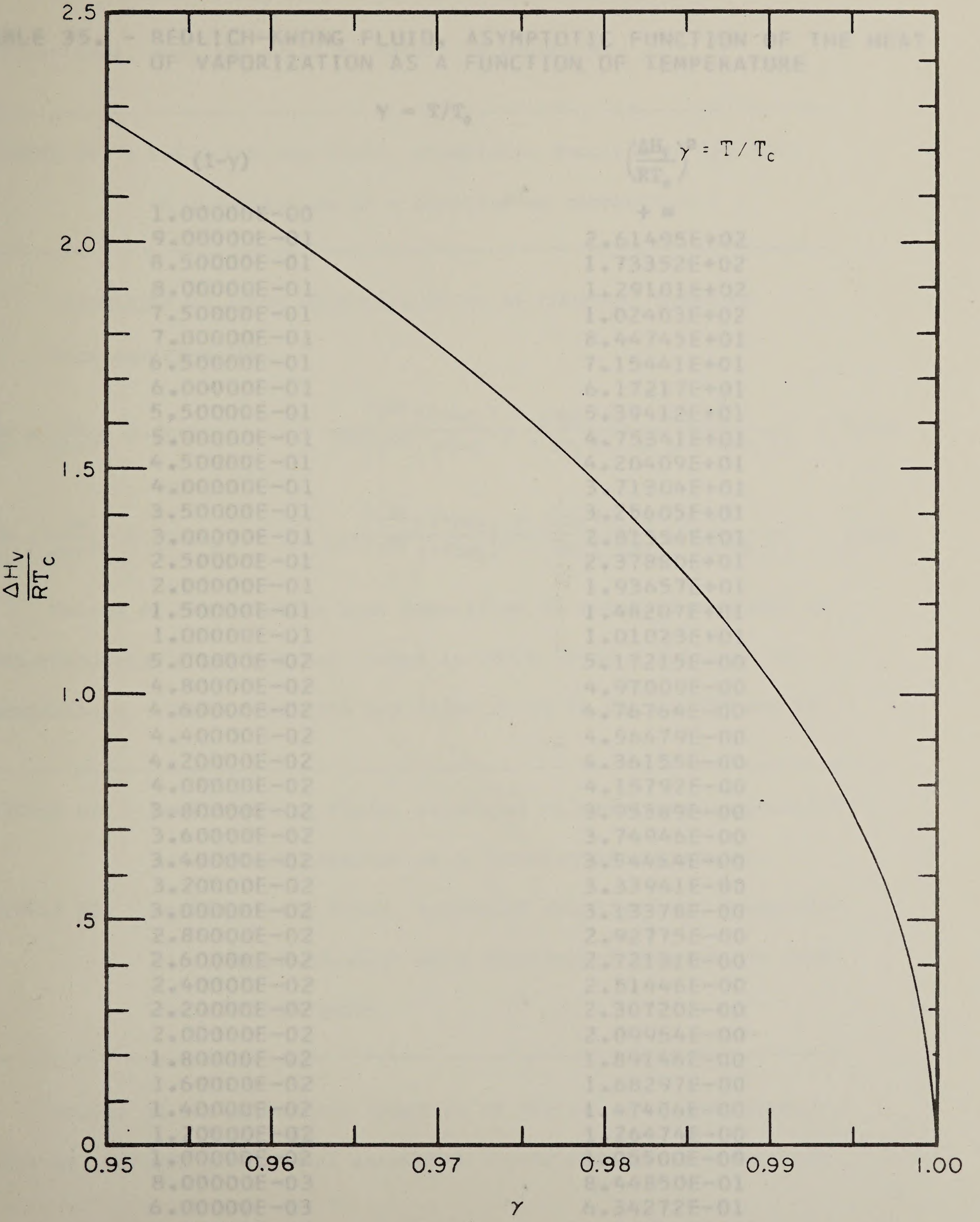

FIGURE 58.— Redlich-Kwong Fluid, Heat of Vaporization as a Function of Temperature Near the Critical Point.

### TABLE 35. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE HEAT OF VAPORIZATION AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $(1-\gamma)$ | $\left(\frac{\Delta H_v}{RT_c}\right)^2$ |
|---|---|
| 1.00000E-00 | $+\infty$ |
| 9.00000E-01 | 2.61495E+02 |
| 8.50000E-01 | 1.73352E+02 |
| 8.00000E-01 | 1.29101E+02 |
| 7.50000E-01 | 1.02403E+02 |
| 7.00000E-01 | 8.44745E+01 |
| 6.50000E-01 | 7.15441E+01 |
| 6.00000E-01 | 6.17217E+01 |
| 5.50000E-01 | 5.39412E+01 |
| 5.00000E-01 | 4.75341E+01 |
| 4.50000E-01 | 4.20409E+01 |
| 4.00000E-01 | 3.71304E+01 |
| 3.50000E-01 | 3.25605E+01 |
| 3.00000E-01 | 2.81554E+01 |
| 2.50000E-01 | 2.37880E+01 |
| 2.00000E-01 | 1.93657E+01 |
| 1.50000E-01 | 1.48207E+01 |
| 1.00000E-01 | 1.01023E+01 |
| 5.00000E-02 | 5.17215E-00 |
| 4.80000E-02 | 4.97009E-00 |
| 4.60000E-02 | 4.76764E-00 |
| 4.40000E-02 | 4.56479E-00 |
| 4.20000E-02 | 4.36155E-00 |
| 4.00000E-02 | 4.15792E-00 |
| 3.80000E-02 | 3.95389E-00 |
| 3.60000E-02 | 3.74946E-00 |
| 3.40000E-02 | 3.54464E-00 |
| 3.20000E-02 | 3.33941E-00 |
| 3.00000E-02 | 3.13378E-00 |
| 2.80000E-02 | 2.92775E-00 |
| 2.60000E-02 | 2.72131E-00 |
| 2.40000E-02 | 2.51446E-00 |
| 2.20000E-02 | 2.30720E-00 |
| 2.00000E-02 | 2.09954E-00 |
| 1.80000E-02 | 1.89146E-00 |
| 1.60000E-02 | 1.68297E-00 |
| 1.40000E-02 | 1.47406E-00 |
| 1.20000E-02 | 1.26474E-00 |
| 1.00000E-02 | 1.05500E-00 |
| 8.00000E-03 | 8.44850E-01 |
| 6.00000E-03 | 6.34272E-01 |
| 4.00000E-03 | 4.23271E-01 |
| 2.00000E-03 | 2.11848E-01 |
| 1.00000E-03 | 1.05977E-01 |
| 0.00000E-99 | 0.00000E-99 |

illustrated, near the critical point, in figure 59.

FIGURE 59. - Redlich-Kwong fluid, asymptotic function of the heat of vaporization as a function of temperature.

SATURATED RELATIVE HEAT CAPACITIES AT CONSTANT PRESSURE

From equation 104,

$$\frac{J_1}{R} = \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha_1) - 1 + \frac{[\gamma^{3/2}(1+b\alpha_1) + (a\alpha_1/2)(1-b\alpha_1)]^2}{\gamma^{3/2}[\gamma^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)]} \quad ; \qquad (244)$$

$$\frac{J_3}{R} = \frac{3a}{4b\gamma^{3/2}} \ln(1+b\alpha_3) - 1 + \frac{[\gamma^{3/2}(1+b\alpha_3) + (a\alpha_3/2)(1-b\alpha_3)]^2}{\gamma^{3/2}[\gamma^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)]} \quad . \qquad (245)$$

Values of the relative heat capacities at constant pressure of saturated liquid and gas are listed in table 36 as a function of temperature. These same data are illustrated in figures 60 and 61.

FIGURE 60. - Redlich-Kwong fluid, saturated relative heat capacities at constant pressure as a function of temperature.

FIGURE 61. - Redlich-Kwong fluid, saturated relative heat capacities at constant pressure as a function of temperature near the critical point.

Values of the asymptotic function of the relative heat capacities at constant pressure of saturated liquid and gas as a function

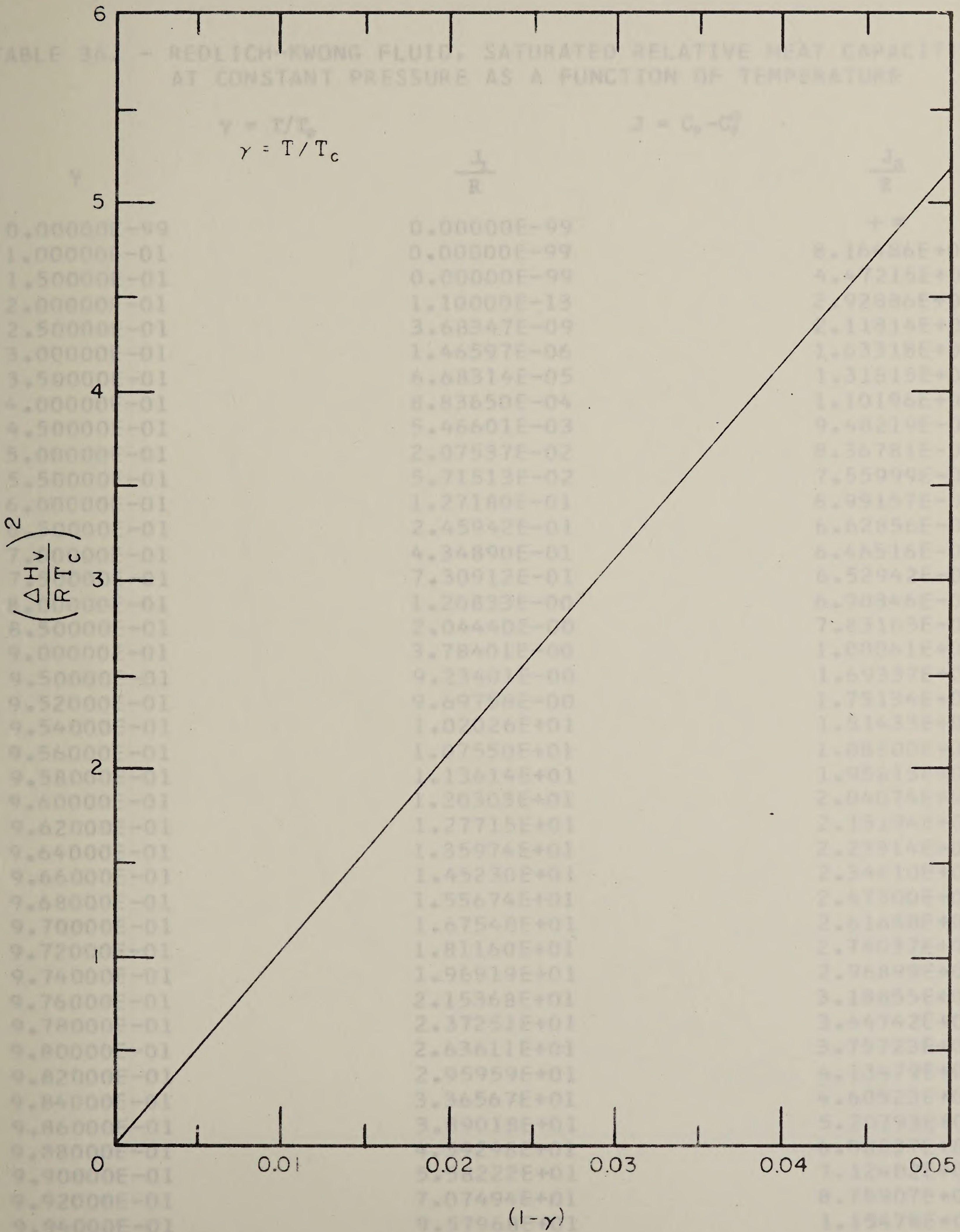

FIGURE 59. - Redlich-Kwong Fluid, Asymptotic Function of the Heat of Vaporization as a Function of Temperature.

## TABLE 36. - REDLICH-KWONG FLUID, SATURATED RELATIVE HEAT CAPACITIES AT CONSTANT PRESSURE AS A FUNCTION OF TEMPERATURE

$\Upsilon = T/T_c$ $\qquad$ $J = C_p - C_p^o$

| $\Upsilon$ | $\frac{J_1}{R}$ | $\frac{J_3}{R}$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | + ∞ |
| 1.00000E-01 | 0.00000E-99 | 8.16486E+01 |
| 1.50000E-01 | 0.00000E-99 | 4.47215E+01 |
| 2.00000E-01 | 1.10000E-13 | 2.92886E+01 |
| 2.50000E-01 | 3.68347E-09 | 2.11814E+01 |
| 3.00000E-01 | 1.46597E-06 | 1.63318E+01 |
| 3.50000E-01 | 6.68314E-05 | 1.31815E+01 |
| 4.00000E-01 | 8.83650E-04 | 1.10196E+01 |
| 4.50000E-01 | 5.46601E-03 | 9.48219E-00 |
| 5.00000E-01 | 2.07537E-02 | 8.36781E-00 |
| 5.50000E-01 | 5.71513E-02 | 7.55999E-00 |
| 6.00000E-01 | 1.27180E-01 | 6.99157E-00 |
| 6.50000E-01 | 2.45942E-01 | 6.62856E-00 |
| 7.00000E-01 | 4.34890E-01 | 6.46516E-00 |
| 7.50000E-01 | 7.30912E-01 | 6.52942E-00 |
| 8.00000E-01 | 1.20833E-00 | 6.90846E-00 |
| 8.50000E-01 | 2.04440E-00 | 7.83163E-00 |
| 9.00000E-01 | 3.78401E-00 | 1.00061E+01 |
| 9.50000E-01 | 9.23401E-00 | 1.69337E+01 |
| 9.52000E-01 | 9.69758E-00 | 1.75134E+01 |
| 9.54000E-01 | 1.02026E+01 | 1.81433E+01 |
| 9.56000E-01 | 1.07550E+01 | 1.88300E+01 |
| 9.58000E-01 | 1.13614E+01 | 1.95815E+01 |
| 9.60000E-01 | 1.20303E+01 | 2.04074E+01 |
| 9.62000E-01 | 1.27715E+01 | 2.13194E+01 |
| 9.64000E-01 | 1.35974E+01 | 2.23314E+01 |
| 9.66000E-01 | 1.45230E+01 | 2.34610E+01 |
| 9.68000E-01 | 1.55674E+01 | 2.47300E+01 |
| 9.70000E-01 | 1.67548E+01 | 2.61658E+01 |
| 9.72000E-01 | 1.81160E+01 | 2.78037E+01 |
| 9.74000E-01 | 1.96919E+01 | 2.96899E+01 |
| 9.76000E-01 | 2.15368E+01 | 3.18855E+01 |
| 9.78000E-01 | 2.37251E+01 | 3.44742E+01 |
| 9.80000E-01 | 2.63611E+01 | 3.75723E+01 |
| 9.82000E-01 | 2.95959E+01 | 4.13479E+01 |
| 9.84000E-01 | 3.36567E+01 | 4.60523E+01 |
| 9.86000E-01 | 3.89018E+01 | 5.20793E+01 |
| 9.88000E-01 | 4.59298E+01 | 6.00837E+01 |
| 9.90000E-01 | 5.58222E+01 | 7.12402E+01 |
| 9.92000E-01 | 7.07494E+01 | 8.78907E+01 |
| 9.94000E-01 | 9.57968E+01 | 1.15478E+02 |
| 9.96000E-01 | 1.46292E+02 | 1.70261E+02 |
| 9.98000E-01 | 2.99350E+02 | 3.33057E+02 |
| 9.99000E-01 | 6.08297E+02 | 6.55831E+02 |
| 1.00000E-00 | + ∞ | + ∞ |

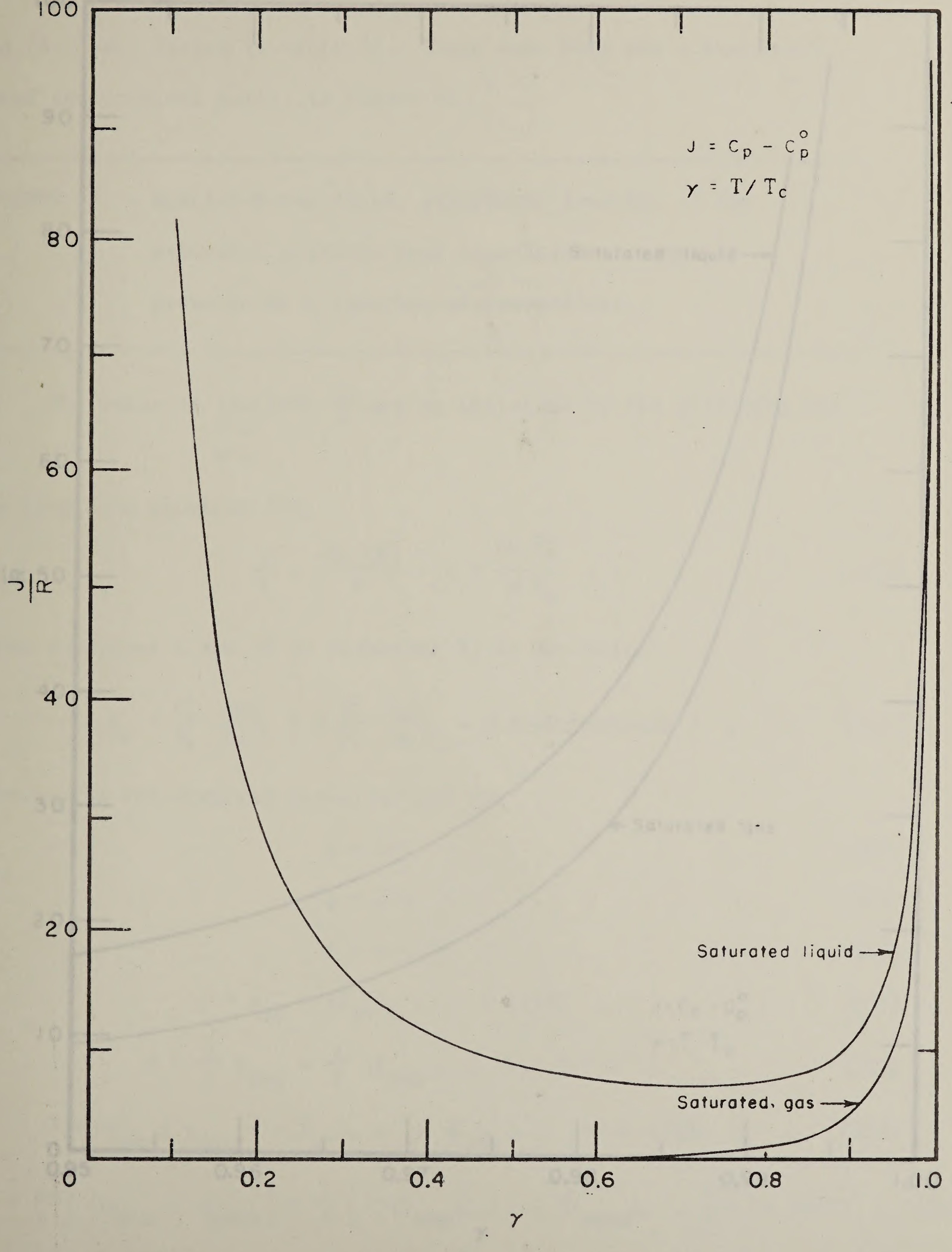

FIGURE 60.- Redlich-Kwong Fluid, Saturated Relative Heat Capacities at Constant Pressure as a Function of Temperature.

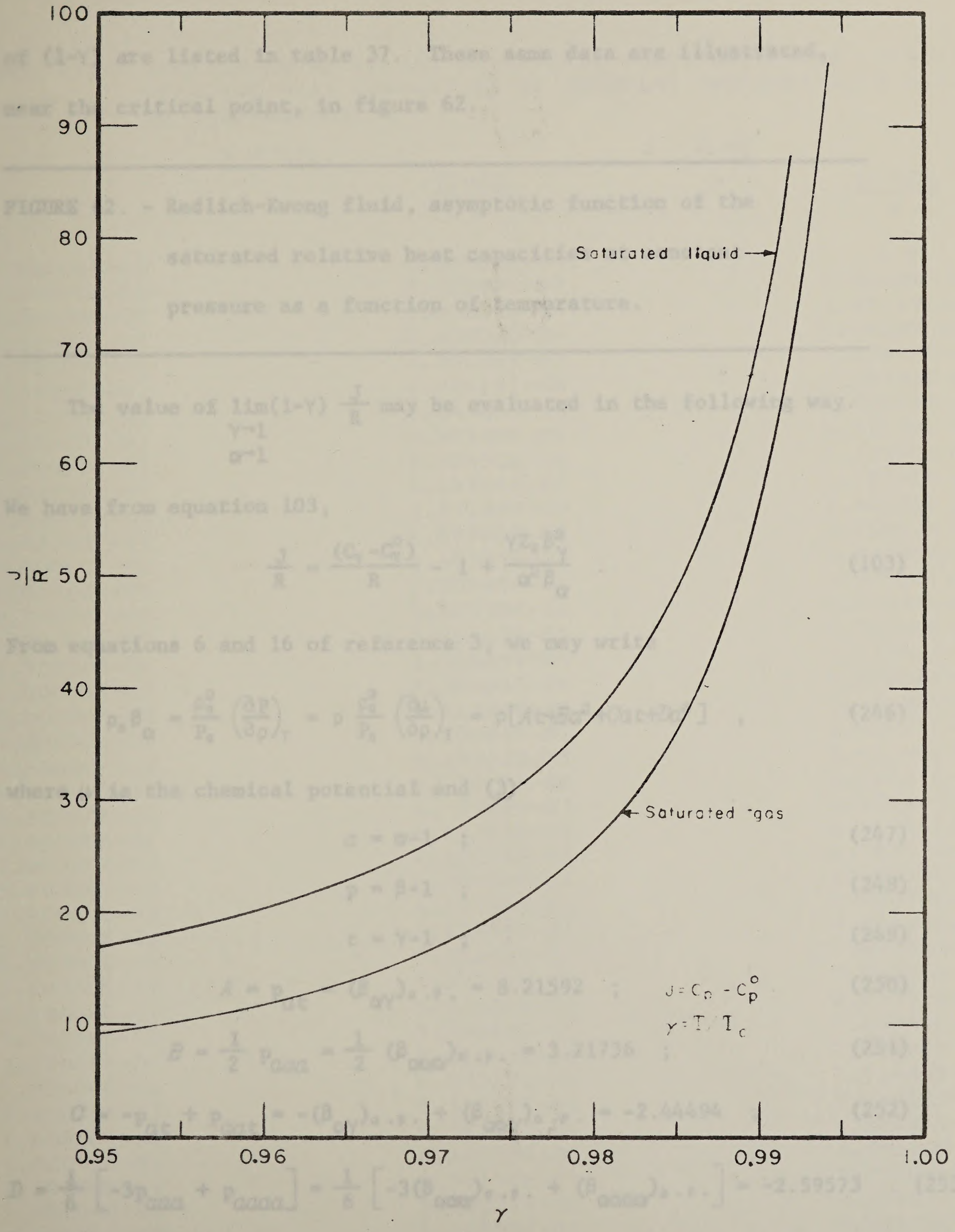

FIGURE 61.—Redlich-Kwong Fluid, Saturated Relative Heat Capacities at Constant Pressure as a Function of Temperature Near the Critical Point.

of (1-γ) are listed in table 37. These same data are illustrated, near the critical point, in figure 62.

---

FIGURE 62. - Redlich-Kwong fluid, asymptotic function of the saturated relative heat capacities at constant pressure as a function of temperature.

---

The value of $\lim\limits_{\substack{\gamma\to 1\\ \alpha\to 1}}(1-\gamma)\,\frac{J}{R}$ may be evaluated in the following way.

We have from equation 103,

$$\frac{J}{R} = \frac{(C_V - C_V^0)}{R} - 1 + \frac{\gamma Z_c \beta_\gamma^2}{\alpha^2 \beta_\alpha} \quad . \qquad (103)$$

From equations 6 and 16 of reference 3, we may write

$$\rho_c \beta_\alpha = \frac{\rho_c^2}{P_c}\left(\frac{\partial P}{\partial \rho}\right)_T = \rho\,\frac{\rho_c^2}{P_c}\left(\frac{\partial \mu}{\partial \rho}\right)_T = \rho[\mathcal{A}t + \mathcal{B}a^2 + \mathcal{C}at + \mathcal{D}a^3] \quad , \qquad (246)$$

where μ is the chemical potential and (3)

$$a = \alpha - 1 \quad ; \qquad (247)$$

$$p = \beta - 1 \quad ; \qquad (248)$$

$$t = \gamma - 1 \quad ; \qquad (249)$$

$$\mathcal{A} = p_{at} = (\beta_{\alpha\gamma})_{c.p.} = 8.21592 \quad ; \qquad (250)$$

$$\mathcal{B} = \frac{1}{2}\,p_{aaa} = \frac{1}{2}\,(\beta_{\alpha\alpha\alpha})_{c.p.} = 3.21736 \quad ; \qquad (251)$$

$$\mathcal{C} = -p_{at} + p_{aat} = -(\beta_{\alpha\gamma})_{c.p.} + (\beta_{\alpha\alpha\gamma})_{c.p.} = -2.44494 \quad ; \qquad (252)$$

$$\mathcal{D} = \frac{1}{6}\left[-3p_{aaa} + p_{aaaa}\right] = \frac{1}{6}\left[-3(\beta_{\alpha\alpha\alpha})_{c.p.} + (\beta_{\alpha\alpha\alpha\alpha})_{c.p.}\right] = -2.59573 \quad . \qquad (253)$$

TABLE 37. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED RELATIVE HEAT CAPACITIES AT CONSTANT PRESSURE AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad$ $J = C_p - C_p^0$

| $(1-\gamma)$ | $\frac{(1-\gamma)J_{\underline{1}}}{R}$ | $\frac{(1-\gamma)J_{\underline{3}}}{R}$ |
|---|---|---|
| 1.00000E-00 | 0.00000E-99 | + ∞ |
| 9.00000E-01 | 0.00000E-99 | 7.34838E+01 |
| 8.50000E-01 | 0.00000E-99 | 3.80133E+01 |
| 8.00000E-01 | 8.80000E-14 | 2.34308E+01 |
| 7.50000E-01 | 2.76260E-09 | 1.58860E+01 |
| 7.00000E-01 | 1.02618E-06 | 1.14323E+01 |
| 6.50000E-01 | 4.34404E-05 | 8.56801E-00 |
| 6.00000E-01 | 5.30190E-04 | 6.61177E-00 |
| 5.50000E-01 | 3.00630E-03 | 5.21520E-00 |
| 5.00000E-01 | 1.03768E-02 | 4.18390E-00 |
| 4.50000E-01 | 2.57180E-02 | 3.40199E-00 |
| 4.00000E-01 | 5.08723E-02 | 2.79662E-00 |
| 3.50000E-01 | 8.60797E-02 | 2.31999E-00 |
| 3.00000E-01 | 1.30467E-01 | 1.93955E-00 |
| 2.50000E-01 | 1.82728E-01 | 1.63235E-00 |
| 2.00000E-01 | 2.41667E-01 | 1.38169E-00 |
| 1.50000E-01 | 3.06660E-01 | 1.17474E-00 |
| 1.00000E-01 | 3.78401E-01 | 1.00061E-00 |
| 5.00000E-02 | 4.61700E-01 | 8.46686E-01 |
| 4.80000E-02 | 4.65484E-01 | 8.40647E-01 |
| 4.60000E-02 | 4.69322E-01 | 8.34594E-01 |
| 4.40000E-02 | 4.73220E-01 | 8.28521E-01 |
| 4.20000E-02 | 4.77182E-01 | 8.22425E-01 |
| 4.00000E-02 | 4.81213E-01 | 8.16299E-01 |
| 3.80000E-02 | 4.85319E-01 | 8.10137E-01 |
| 3.60000E-02 | 4.89507E-01 | 8.03932E-01 |
| 3.40000E-02 | 4.93784E-01 | 7.97677E-01 |
| 3.20000E-02 | 4.98159E-01 | 7.91361E-01 |
| 3.00000E-02 | 5.02644E-01 | 7.84975E-01 |
| 2.80000E-02 | 5.07249E-01 | 7.78505E-01 |
| 2.60000E-02 | 5.11990E-01 | 7.71937E-01 |
| 2.40000E-02 | 5.16884E-01 | 7.65254E-01 |
| 2.20000E-02 | 5.21953E-01 | 7.58432E-01 |
| 2.00000E-02 | 5.27222E-01 | 7.51446E-01 |
| 1.80000E-02 | 5.32726E-01 | 7.44262E-01 |
| 1.60000E-02 | 5.38508E-01 | 7.36836E-01 |
| 1.40000E-02 | 5.44625E-01 | 7.29110E-01 |
| 1.20000E-02 | 5.51158E-01 | 7.21004E-01 |
| 1.00000E-02 | 5.58222E-01 | 7.12402E-01 |
| 8.00000E-03 | 5.65995E-01 | 7.03125E-01 |
| 6.00000E-03 | 5.74781E-01 | 6.92871E-01 |
| 4.00000E-03 | 5.85170E-01 | 6.81047E-01 |
| 2.00000E-03 | 5.98701E-01 | 6.66115E-01 |
| 1.00000E-03 | 6.08297E-01 | 6.55831E-01 |
| 0.00000E-99 | 6.31724E-01 | 6.31724E-01 |

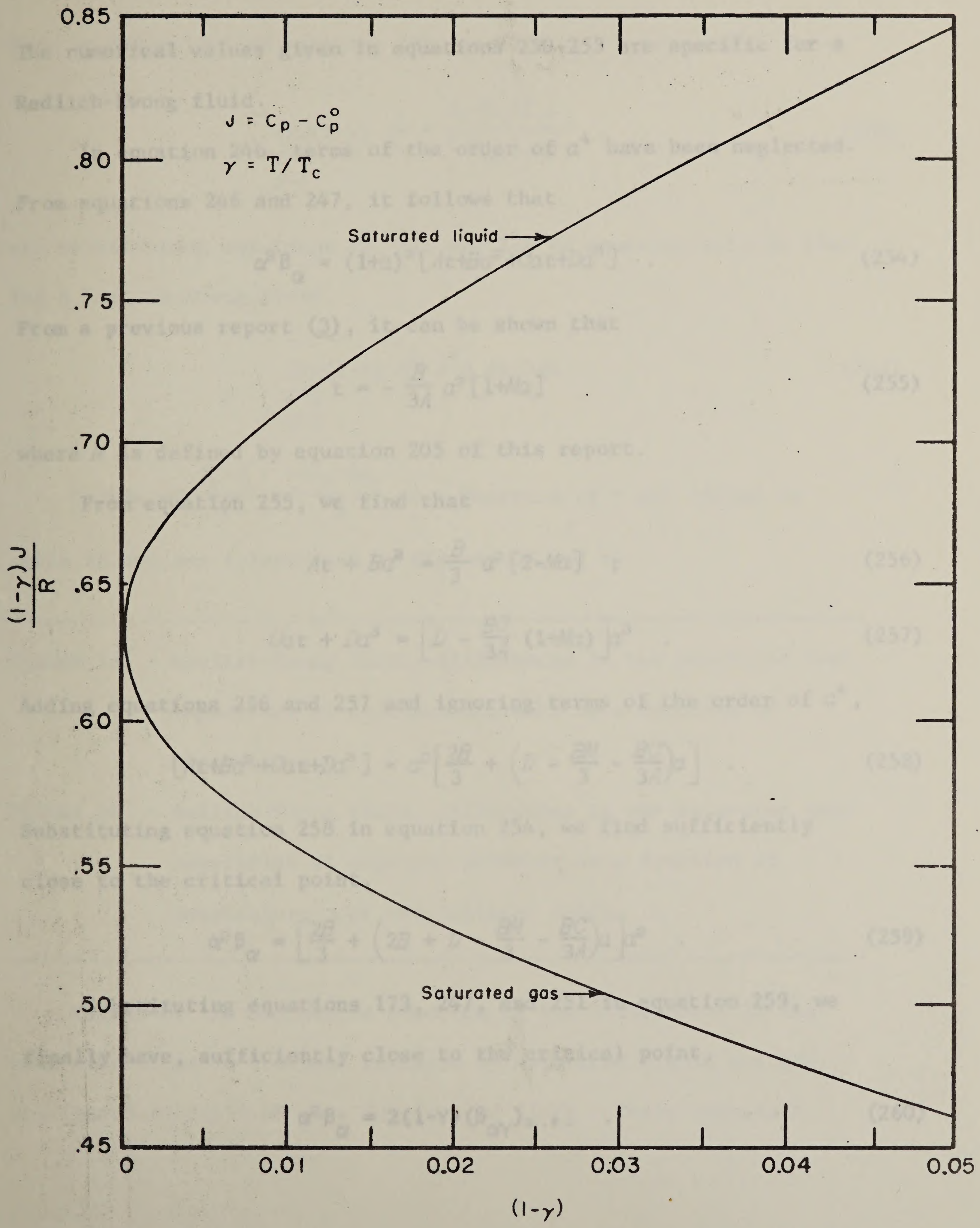

FIGURE 62.- Redlich-Kwong Fluid, Asymptotic Function of the Saturated Relative Heat Capacities at Constant Pressure as a Function of Temperature.

The numerical values given in equations 250-253 are specific for a Redlich-Kwong fluid.

In equation 246, terms of the order of $a^4$ have been neglected. From equations 246 and 247, it follows that

$$\alpha^2\beta_\alpha = (1+a)^3[At+Ba^2+Cat+Da^3] \quad . \tag{254}$$

From a previous report (3), it can be shown that

$$t = -\frac{B}{3A}a^2[1+Ma] \tag{255}$$

where $M$ is defined by equation 205 of this report.

From equation 255, we find that

$$At + Ba^2 = \frac{B}{3}a^2[2-Ma] \quad ; \tag{256}$$

$$Cat + Da^3 = \left[D - \frac{BC}{3A}(1+Ma)\right]a^3 \quad . \tag{257}$$

Adding equations 256 and 257 and ignoring terms of the order of $a^4$,

$$[At+Ba^2+Cat+Da^3] = a^2\left[\frac{2B}{3} + \left(D - \frac{BM}{3} - \frac{BC}{3A}\right)a\right] \quad . \tag{258}$$

Substituting equation 258 in equation 254, we find sufficiently close to the critical point,

$$\alpha^2\beta_\alpha = \left[\frac{2B}{3} + \left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)a\right]a^2 \quad . \tag{259}$$

Substituting equations 173, 247, and 251 in equation 259, we finally have, sufficiently close to the critical point,

$$\alpha^2\beta_\alpha = 2(1-Y)(\beta_{\alpha Y})_{c.p.} \quad . \tag{260}$$

From equation 260 and equation 103, we find

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}}(1-\gamma)\,\frac{J}{R} = \frac{Z_{c}\,(\beta_{\gamma})^{2}_{c.p.}}{2(\beta_{\alpha\gamma})_{c.p.}} \tag{261}$$

or, substituting equations 23, 51, and 178 in equation 261, we find for a Redlich-Kwong fluid,

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}}(1-\gamma)\,\frac{J}{R} = 0.631724\ . \tag{262}$$

Values of $\frac{J_g - J_l}{R} = \frac{C_{P_g} - C_{P_l}}{R}$ as a function of $\gamma$ are listed in table 38 and are illustrated in figures 63 and 64.

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FIGURE 63. - Redlich-Kwong fluid, differences in the saturated heat capacities at constant pressure as a function of temperature.

FIGURE 64. - Redlich-Kwong fluid, differences in the saturated heat capacities at constant pressure as a function of temperature near the critical point.

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Values of the asymptotic function of the differences in the heat capacities at constant pressure of saturated liquid and gas are listed in table 39 as a function of $(1-\gamma)$. These same data,

TABLE 38. - REDLICH-KWONG FLUID, DIFFERENCES IN THE SATURATED HEAT CAPACITIES AT CONSTANT PRESSURE AS A FUNCTION OF TEMPERATURE

| $Y = T/T_c$ | $J = C_p - C_p^0$ |
|---|---|
| $Y$ | $\frac{J_3 - J_1}{R}$ |
| 0.00000E-99 | + ∞ |
| 1.00000E-01 | 8.16486E+01 |
| 1.50000E-01 | 4.47215E+01 |
| 2.00000E-01 | 2.92886E+01 |
| 2.50000E-01 | 2.11814E+01 |
| 3.00000E-01 | 1.63318E+01 |
| 3.50000E-01 | 1.31814E+01 |
| 4.00000E-01 | 1.10187E+01 |
| 4.50000E-01 | 9.47672E-00 |
| 5.00000E-01 | 8.34706E-00 |
| 5.50000E-01 | 7.50284E-00 |
| 6.00000E-01 | 6.86439E-00 |
| 6.50000E-01 | 6.38262E-00 |
| 7.00000E-01 | 6.03027E-00 |
| 7.50000E-01 | 5.79851E-00 |
| 8.00000E-01 | 5.70012E-00 |
| 8.50000E-01 | 5.78722E-00 |
| 9.00000E-01 | 6.22217E-00 |
| 9.50000E-01 | 7.69972E-00 |
| 9.52000E-01 | 7.81591E-00 |
| 9.54000E-01 | 7.94069E-00 |
| 9.56000E-01 | 8.07503E-00 |
| 9.58000E-01 | 8.22007E-00 |
| 9.60000E-01 | 8.37714E-00 |
| 9.62000E-01 | 8.54784E-00 |
| 9.64000E-01 | 8.73404E-00 |
| 9.66000E-01 | 8.93802E-00 |
| 9.68000E-01 | 9.16255E-00 |
| 9.70000E-01 | 9.41103E-00 |
| 9.72000E-01 | 9.68771E-00 |
| 9.74000E-01 | 9.99796E-00 |
| 9.76000E-01 | 1.03487E+01 |
| 9.78000E-01 | 1.07490E+01 |
| 9.80000E-01 | 1.12112E+01 |
| 9.82000E-01 | 1.17520E+01 |
| 9.84000E-01 | 1.23955E+01 |
| 9.86000E-01 | 1.31775E+01 |
| 9.88000E-01 | 1.41538E+01 |
| 9.90000E-01 | 1.54179E+01 |
| 9.92000E-01 | 1.71412E+01 |
| 9.94000E-01 | 1.96817E+01 |
| 9.96000E-01 | 2.39694E+01 |
| 9.98000E-01 | 3.37067E+01 |
| 9.99000E-01 | 4.75335E+01 |
| 1.00000E-00 | + ∞ |

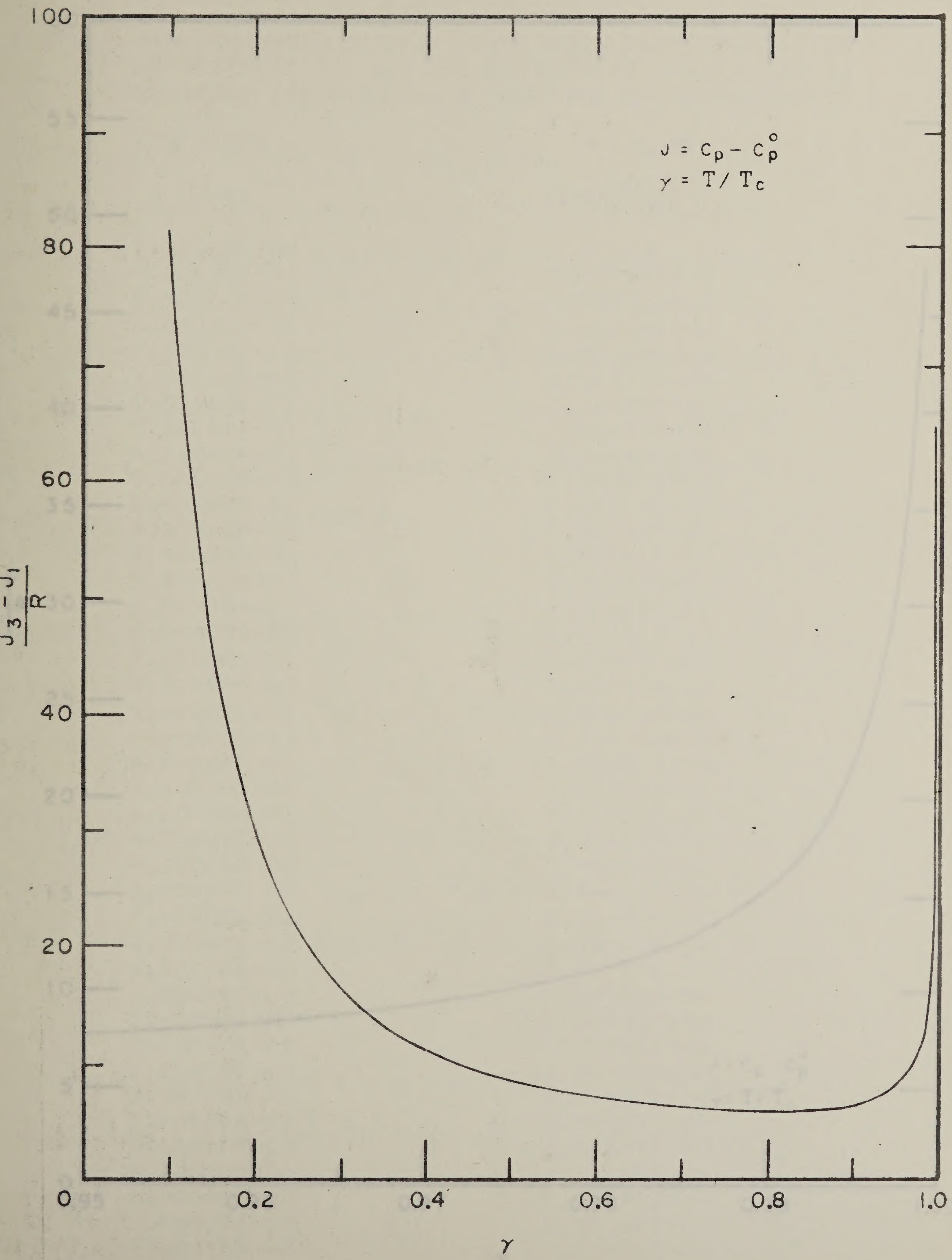

FIGURE 63. – Redlich–Kwong Fluid, Differences in the Saturated Heat Capacities at Constant Pressure as a Function of Temperature.

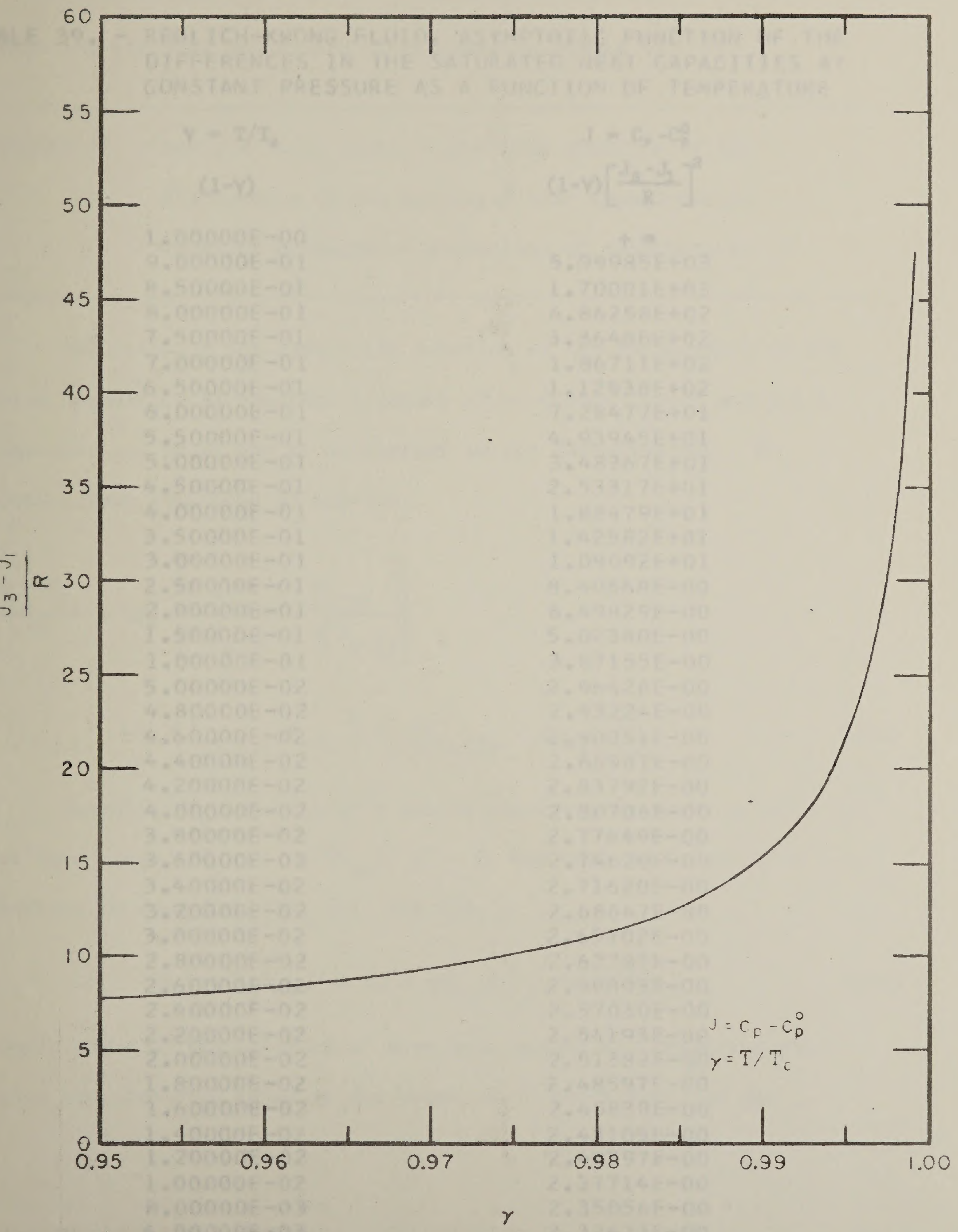

FIGURE 64.—Redlich-Kwong Fluid, Differences in the Saturated Heat Capacities at Constant Pressure as a Function of Temperature Near the Critical Point.

TABLE 39. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE DIFFERENCES IN THE SATURATED HEAT CAPACITIES AT CONSTANT PRESSURE AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$<br>$(1-\gamma)$ | $J = C_p - C_p^0$<br>$(1-\gamma)\left[\frac{J_3-J_1}{R}\right]^2$ |
|---|---|
| 1.00000E-00 | $+\infty$ |
| 9.00000E-01 | 5.99985E+03 |
| 8.50000E-01 | 1.70001E+03 |
| 8.00000E-01 | 6.86258E+02 |
| 7.50000E-01 | 3.36488E+02 |
| 7.00000E-01 | 1.86711E+02 |
| 6.50000E-01 | 1.12938E+02 |
| 6.00000E-01 | 7.28477E+01 |
| 5.50000E-01 | 4.93945E+01 |
| 5.00000E-01 | 3.48367E+01 |
| 4.50000E-01 | 2.53317E+01 |
| 4.00000E-01 | 1.88479E+01 |
| 3.50000E-01 | 1.42582E+01 |
| 3.00000E-01 | 1.09092E+01 |
| 2.50000E-01 | 8.40568E-00 |
| 2.00000E-01 | 6.49829E-00 |
| 1.50000E-01 | 5.02380E-00 |
| 1.00000E-01 | 3.87155E-00 |
| 5.00000E-02 | 2.96428E-00 |
| 4.80000E-02 | 2.93224E-00 |
| 4.60000E-02 | 2.90051E-00 |
| 4.40000E-02 | 2.86907E-00 |
| 4.20000E-02 | 2.83792E-00 |
| 4.00000E-02 | 2.80706E-00 |
| 3.80000E-02 | 2.77649E-00 |
| 3.60000E-02 | 2.74620E-00 |
| 3.40000E-02 | 2.71620E-00 |
| 3.20000E-02 | 2.68647E-00 |
| 3.00000E-02 | 2.65702E-00 |
| 2.80000E-02 | 2.62785E-00 |
| 2.60000E-02 | 2.59893E-00 |
| 2.40000E-02 | 2.57030E-00 |
| 2.20000E-02 | 2.54193E-00 |
| 2.00000E-02 | 2.51382E-00 |
| 1.80000E-02 | 2.48597E-00 |
| 1.60000E-02 | 2.45838E-00 |
| 1.40000E-02 | 2.43105E-00 |
| 1.20000E-02 | 2.40397E-00 |
| 1.00000E-02 | 2.37714E-00 |
| 8.00000E-03 | 2.35056E-00 |
| 6.00000E-03 | 2.32423E-00 |
| 4.00000E-03 | 2.29813E-00 |
| 2.00000E-03 | 2.27228E-00 |
| 1.00000E-03 | 2.25943E-00 |
| 0.00000E-99 | 2.24668E-00 |

near the critical point, are illustrated in figure 65.

---

FIGURE 65. - Redlich-Kwong fluid, asymptotic function of the differences in the saturated heat capacities at constant pressure as a function of temperature.

---

The value of the asymptotic function of the differences in the heat capacities at constant pressure of saturated liquid and gas at the critical point may be determined in the following way. We obtain from equations 87 and 103,

$$\frac{C_{P_3}-C_{P_1}}{R} = \gamma Z_c \left[\frac{\beta^2_{\gamma(3)}}{\alpha_3^2\beta_{\alpha(3)}} - \frac{\beta^2_{\gamma(1)}}{\alpha_1^2\beta_{\alpha(1)}}\right]$$

$$-\gamma\left[2\left(X_{\gamma(3)} - X_{\gamma(1)}\right) + \gamma\left(X_{\gamma\gamma(3)} - X_{\gamma\gamma(1)}\right)\right] \ . \qquad (263)$$

Expanding $\beta_\gamma$ in a Taylor's series about the critical point, we have since, $(\beta_\alpha)_{c.p.} = (\beta_{\alpha\alpha})_{c.p.} = 0$, and with $A$, $B$, and $C$ as defined in equations 250, 251, and 252,

$$\beta_\gamma = (\beta_\gamma)_{c.p.} + A\alpha + t(\beta_{\gamma\gamma})_{c.p.} + \alpha^2\left(\frac{A+C}{2}\right) \ , \qquad (264)$$

where terms of the order of $t^2$ have been neglected. Squaring equation 264, ignoring terms of the order of $\alpha^3$ and higher, we get

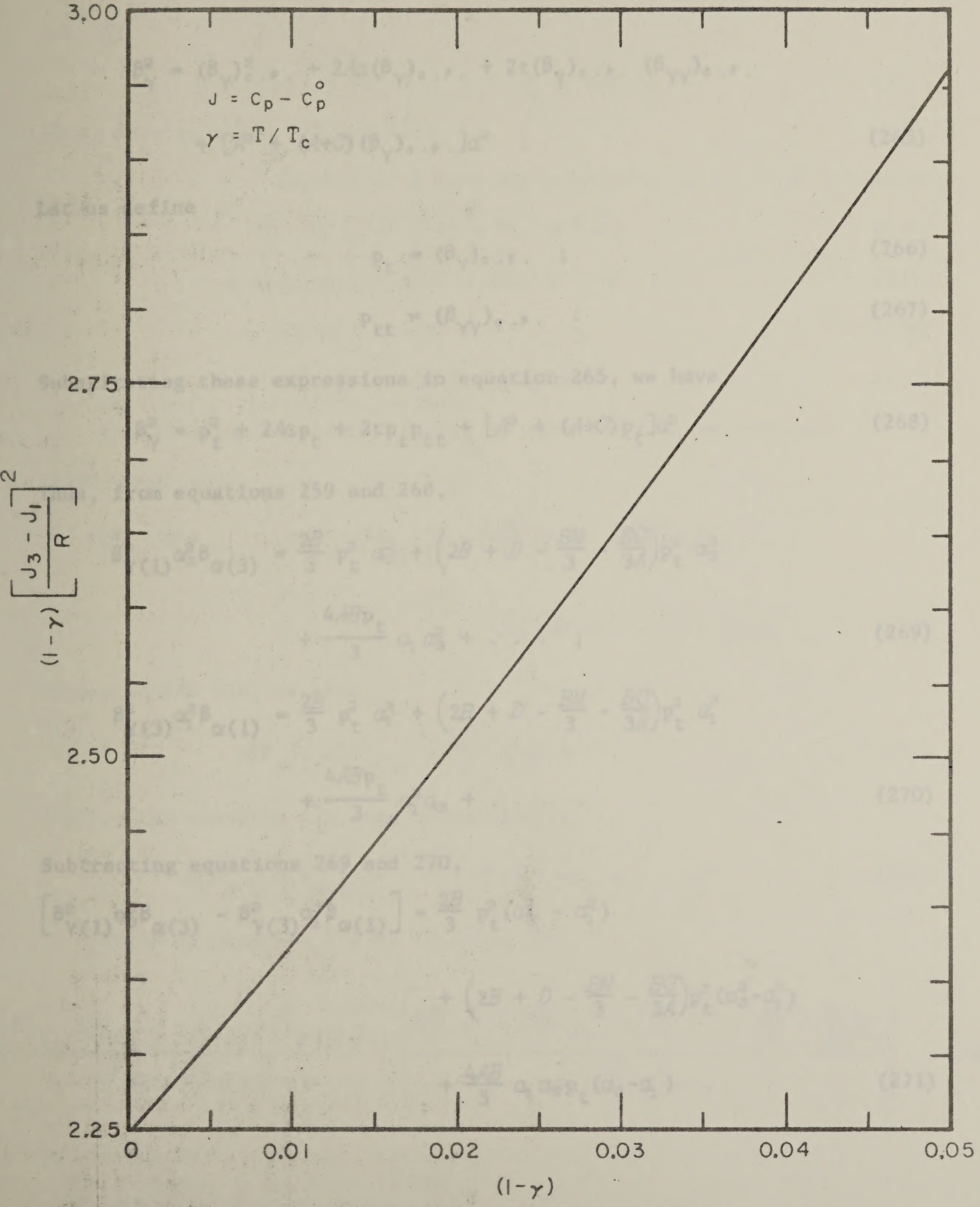

FIGURE 65. — Redlich–Kwong Fluid, Asymptotic Function of the Differences in the Saturated Heat Capacities at Constant Pressure as a Function of Temperature.

$$\beta_\gamma^2 = (\beta_\gamma)^2_{\mathrm{c.p.}} + 2Aa(\beta_\gamma)_{\mathrm{c.p.}} + 2t(\beta_\gamma)_{\mathrm{c.p.}}\ (\beta_{\gamma\gamma})_{\mathrm{c.p.}}$$

$$+ [A^2 + (A+C)(\beta_\gamma)_{\mathrm{c.p.}}]a^2 \quad . \tag{265}$$

Let us define

$$p_t = (\beta_\gamma)_{\mathrm{c.p.}} \quad ; \tag{266}$$

$$p_{tt} = (\beta_{\gamma\gamma})_{\mathrm{c.p.}} \quad . \tag{267}$$

Substituting these expressions in equation 265, we have

$$\beta_\gamma^2 = p_t^2 + 2Aap_t + 2tp_tp_{tt} + [A^2 + (A+C)p_t]a^2 \quad . \tag{268}$$

Thus, from equations 259 and 268,

$$\beta^2_{\gamma(1)}\alpha_3^2\beta_{\alpha(3)} = \frac{2B}{3}\,p_t^2\,\alpha_3^2 + \left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)p_t^2\,\alpha_3^3$$

$$+ \frac{4ABp_t}{3}\,\alpha_1\,\alpha_3^2 + \ldots \quad ; \tag{269}$$

$$\beta^2_{\gamma(3)}\alpha_1^2\beta_{\alpha(1)} = \frac{2B}{3}\,p_t^2\,\alpha_1^2 + \left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)p_t^2\,\alpha_1^3$$

$$+ \frac{4ABp_t}{3}\,\alpha_1^2\alpha_3 + \ldots \quad . \tag{270}$$

Subtracting equations 269 and 270,

$$\left[\beta^2_{\gamma(1)}\alpha_3^2\beta_{\alpha(3)} - \beta^2_{\gamma(3)}\alpha_1^2\beta_{\alpha(1)}\right] = \frac{2B}{3}\,p_t^2(\alpha_3^2 - \alpha_1^2)$$

$$+ \left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)p_t^2(\alpha_3^3 - \alpha_1^3)$$

$$+ \frac{4AB}{3}\,\alpha_1\,\alpha_3\,p_t(\alpha_3 - \alpha_1) \quad . \tag{271}$$

But (3)

$$\alpha_3 = -\alpha_1(1 + M\alpha_1 + \cdots) = -\alpha_1 - M\alpha_1^2 - \cdots \quad . \tag{272}$$

Thus, from equation 272, we find

$$(\alpha_3-\alpha_1) = -\alpha_1(2 + M\alpha_1 + \cdots) \quad ; \tag{273}$$

$$\alpha_1\alpha_3 = -\alpha_1^2(1 + M\alpha_1 + \cdots) \quad ; \tag{274}$$

$$\alpha_1\alpha_3(\alpha_3-\alpha_1) = \alpha_1^3(2 + 3M\alpha_1 + \cdots) \quad ; \tag{275}$$

$$(\alpha_3^2-\alpha_1^2) = 2M\alpha_1^3 + \cdots \quad ; \tag{276}$$

$$(\alpha_3^3-\alpha_1^3) = -\alpha_1^3(2 + 3M\alpha_1 + \cdots) \quad . \tag{277}$$

Substituting equations 275-277 in equation 271, we have to third-order in $\alpha$,

$$[\beta^2_{\gamma(1)}\alpha_3^2\beta_{\alpha(3)} - \beta^2_{\gamma(3)}\alpha_1^2\beta_{\alpha(1)}] = \frac{4BM}{3}\alpha_1^3 p_t^2 + \frac{8AB}{3}\alpha_1^3 p_t - 2\alpha_1^3\left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)p_t^2 \quad . \tag{278}$$

From equation 259, we have

$$\alpha_1^2\beta_{\alpha(1)}\alpha_3^2\beta_{\alpha(3)} = \frac{4B^2}{9}\alpha_1^2\alpha_3^2 + \cdots \tag{279}$$

which, from equation 274, can be written as

$$\alpha_1^2\beta_{\alpha(1)}\alpha_3^2\beta_{\alpha(3)} = -\frac{4B^2}{9}\alpha_1^4(1 + M\alpha_1 + \cdots)^2 \quad . \tag{280}$$

Dividing equation 278 by equation 280,

$$\frac{\left[\beta^2_{\gamma(1)}\alpha_3^2\beta_{\alpha(3)} - \beta^2_{\gamma(3)}\alpha_1^2\beta_{\alpha(1)}\right]}{\alpha_1^2\beta_{\alpha(1)}\alpha_3^2\beta_{\alpha(3)}} = \left[\frac{\frac{4BM}{3}p_t^2 + \frac{8AB}{3}p_t - 2\left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)p_t^2}{\frac{4B^2}{9}(-\alpha_1)}\right] \quad . \tag{281}$$

From equation 255, we have to second order,

$$-\alpha_1 = \left(\frac{-3At}{B}\right)^{1/2}\left[1 - \frac{M}{2}\alpha_1\right] \quad . \tag{282}$$

Substituting equations 23, 249, 281, and 282 in equation 263,

$$\lim_{\substack{\gamma\to1\\ \alpha_1\to1\\ \alpha_3\to1}}\left[(1-\gamma)^{1/2}\left(\frac{C_{p_3}-C_{p_1}}{R}\right)\right] = \frac{3^{3/2}p_tZ_c}{4B^{3/2}A^{1/2}}\left[\frac{8AB}{3} + \frac{4BM}{3}p_t - 2\left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)p_t\right] \quad , \tag{283}$$

or, squaring equation 283, we have

$$\lim_{\substack{\gamma\to1\\ \alpha_1\to1\\ \alpha_3\to1}}\left[(1-\gamma)\left(\frac{C_{p_3}-C_{p_1}}{R}\right)^2\right] = \frac{27p_t^2Z_c^2}{16AB^3}\left[\frac{8AB}{3} + \frac{4BM}{3}p_t - 2\left(2B + D - \frac{BM}{3} - \frac{BC}{3A}\right)p_t\right]^2 \quad ,$$

$$= \frac{27p_t^2Z_c^2}{16AB^3}\left[\frac{8AB}{3} - 2\left(2B + D - BM - \frac{BC}{3A}\right)p_t\right]^2 \quad . \tag{284}$$

The expression given above is true for any analytical fluid.

For a Redlich-Kwong fluid, we then have from equations 23, 51, 206, 250-253, 266, and 284,

$$\lim_{\substack{\gamma\to1\\ \alpha_1\to1\\ \alpha_3\to1}}\left[(1-\gamma)\left(\frac{C_{p_3}-C_{p_1}}{R}\right)^2\right] = 2.24668 \quad . \tag{285}$$

## A FUNCTION RELATED TO THE HEAT OF VAPORIZATION, NAMELY THE HEAT OF VAPORIZATION PER MOLE OF GAS COLLECTED OUTSIDE THE CALORIMETER

In a previous report (5), it is shown that

$$\frac{\Delta H_a}{RT_c} = \frac{Y Z_c}{\alpha_1} \beta' \quad , \tag{286}$$

where $\Delta H_a$ is the heat of vaporization per mole of gas collected outside the calorimeter. Substituting equations 23 and 194 in the above expression,

$$\frac{\Delta H_a}{RT_c} = \frac{\alpha_3 Y}{\alpha_1} \left[ \frac{1}{(1-b\alpha_3)} - \frac{a\alpha_3}{Y^{3/2}(1+b\alpha_3)} - \frac{3a\alpha_1}{2bY^{3/2}(\alpha_3-\alpha_1)} \ln \frac{(1+b\alpha_1)}{(1+b\alpha_3)} \right] \quad . \tag{287}$$

At the critical point,

$$\left[\frac{\Delta H_a}{RT_c}\right]_{c.p.} = Z_c (\beta')_{c.p.} = 1.86014 \quad . \tag{288}$$

Values of $\frac{\Delta H_a}{RT_c}$ as a function of $Y$ are listed in table 40.

These same data are illustrated in figures 66 and 67.

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FIGURE 66. - Redlich-Kwong fluid, heat of vaporization per mole of gas collected outside the calorimeter as a function of temperature.

FIGURE 67. - Redlich-Kwong fluid, heat of vaporization per mole of gas collected outside the calorimeter as a function of temperature near the critical point.

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Values of the asymptotic function of the heat of vaporization per mole of gas collected outside the calorimeter are listed in table 41 as a function of temperature. These data are illustrated,

TABLE 40. - REDLICH-KWONG FLUID, HEAT OF VAPORIZATION PER MOLE OF GAS COLLECTED OUTSIDE THE CALORIMETER AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\alpha = \rho/\rho_c$ $\Delta H_a = \Delta H_V \; \alpha_3/\alpha_3 - \alpha_1$

| $\gamma$ | $\frac{\Delta H_a}{RT_c}$ |
|---|---|
| 0.00000E-99 | $+\infty$ |
| 1.00000E-01 | 1.61708E+01 |
| 1.50000E-01 | 1.31663E+01 |
| 2.00000E-01 | 1.13622E+01 |
| 2.50000E-01 | 1.01194E+01 |
| 3.00000E-01 | 9.19100E-00 |
| 3.50000E-01 | 8.45838E-00 |
| 4.00000E-01 | 7.85642E-00 |
| 4.50000E-01 | 7.34522E-00 |
| 5.00000E-01 | 6.89771E-00 |
| 5.50000E-01 | 6.49370E-00 |
| 6.00000E-01 | 6.11717E-00 |
| 6.50000E-01 | 5.75487E-00 |
| 7.00000E-01 | 5.39534E-00 |
| 7.50000E-01 | 5.02783E-00 |
| 8.00000E-01 | 4.64080E-00 |
| 8.50000E-01 | 4.21935E-00 |
| 9.00000E-01 | 3.73902E-00 |
| 9.50000E-01 | 3.14248E-00 |
| 9.52000E-01 | 3.11425E-00 |
| 9.54000E-01 | 3.08553E-00 |
| 9.56000E-01 | 3.05626E-00 |
| 9.58000E-01 | 3.02641E-00 |
| 9.60000E-01 | 2.99595E-00 |
| 9.62000E-01 | 2.96483E-00 |
| 9.64000E-01 | 2.93299E-00 |
| 9.66000E-01 | 2.90038E-00 |
| 9.68000E-01 | 2.86692E-00 |
| 9.70000E-01 | 2.83254E-00 |
| 9.72000E-01 | 2.79714E-00 |
| 9.74000E-01 | 2.76062E-00 |
| 9.76000E-01 | 2.72285E-00 |
| 9.78000E-01 | 2.68366E-00 |
| 9.80000E-01 | 2.64287E-00 |
| 9.82000E-01 | 2.60022E-00 |
| 9.84000E-01 | 2.55539E-00 |
| 9.86000E-01 | 2.50799E-00 |
| 9.88000E-01 | 2.45741E-00 |
| 9.90000E-01 | 2.40286E-00 |
| 9.92000E-01 | 2.34304E-00 |
| 9.94000E-01 | 2.27584E-00 |
| 9.96000E-01 | 2.19710E-00 |
| 9.98000E-01 | 2.09611E-00 |
| 9.99000E-01 | 2.02583E-00 |
| 1.00000E-00 | 1.86014E-00 |

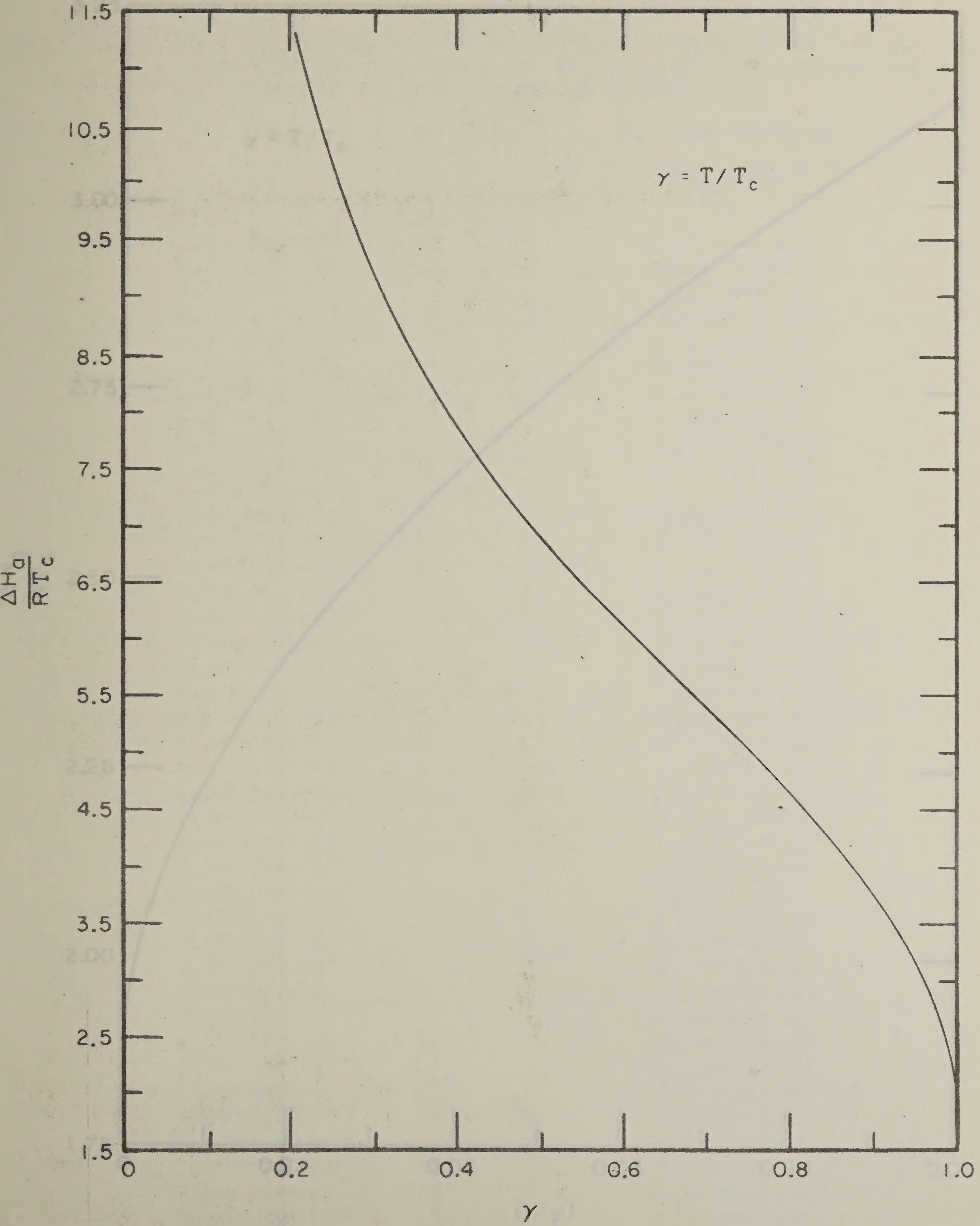

FIGURE 66.– Redlich-Kwong Fluid, Heat of Vaporization Per Mole of Gas Collected Outside the Calorimeter as a Function of Temperature.

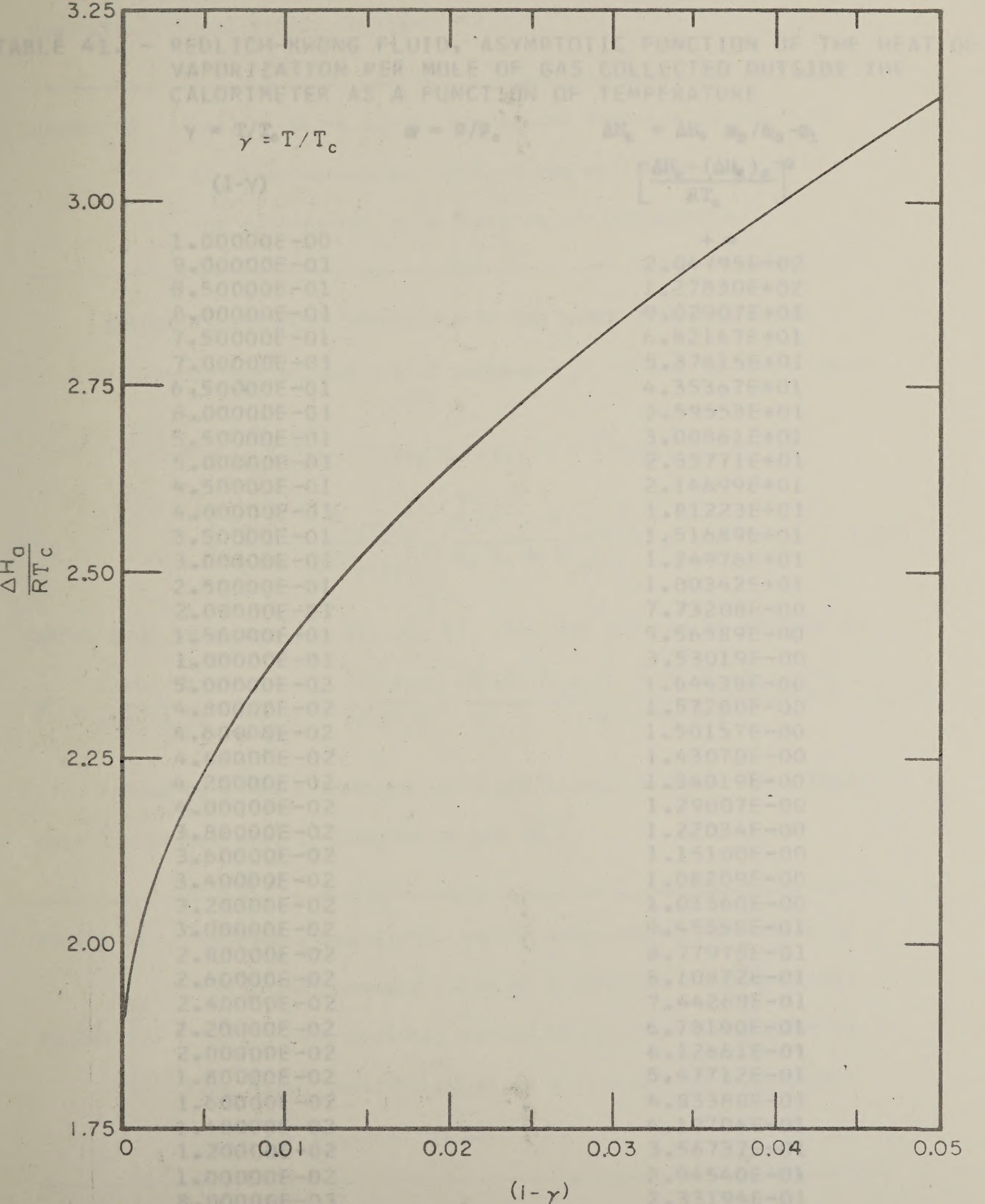

FIGURE 67.—Redlich-Kwong Fluid, Heat of Vaporization Per Mole of Gas Collected Outside the Calorimeter as a Function of Temperature Near the Critical Point.

TABLE 41. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE HEAT OF VAPORIZATION PER MOLE OF GAS COLLECTED OUTSIDE THE CALORIMETER AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\alpha = \rho/\rho_c$ | $\Delta H_a = \Delta H_v \; \alpha_3/\alpha_3 - \alpha_1$ |
|---|---|---|
| $(1-\gamma)$ | | $\left[\frac{\Delta H_a - (\Delta H_a)_c}{RT_c}\right]^2$ |
| 1.00000E-00 | | $+\infty$ |
| 9.00000E-01 | | 2.04795E+02 |
| 8.50000E-01 | | 1.27830E+02 |
| 8.00000E-01 | | 9.02907E+01 |
| 7.50000E-01 | | 6.82167E+01 |
| 7.00000E-01 | | 5.37415E+01 |
| 6.50000E-01 | | 4.35367E+01 |
| 6.00000E-01 | | 3.59553E+01 |
| 5.50000E-01 | | 3.00861E+01 |
| 5.00000E-01 | | 2.53771E+01 |
| 4.50000E-01 | | 2.14699E+01 |
| 4.00000E-01 | | 1.81223E+01 |
| 3.50000E-01 | | 1.51689E+01 |
| 3.00000E-01 | | 1.24976E+01 |
| 2.50000E-01 | | 1.00342E+01 |
| 2.00000E-01 | | 7.73208E-00 |
| 1.50000E-01 | | 5.56589E-00 |
| 1.00000E-01 | | 3.53019E-00 |
| 5.00000E-02 | | 1.64438E-00 |
| 4.80000E-02 | | 1.57280E-00 |
| 4.60000E-02 | | 1.50157E-00 |
| 4.40000E-02 | | 1.43070E-00 |
| 4.20000E-02 | | 1.36019E-00 |
| 4.00000E-02 | | 1.29007E-00 |
| 3.80000E-02 | | 1.22034E-00 |
| 3.60000E-02 | | 1.15100E-00 |
| 3.40000E-02 | | 1.08209E-00 |
| 3.20000E-02 | | 1.01360E-00 |
| 3.00000E-02 | | 9.45558E-01 |
| 2.80000E-02 | | 8.77975E-01 |
| 2.60000E-02 | | 8.10872E-01 |
| 2.40000E-02 | | 7.44269E-01 |
| 2.20000E-02 | | 6.78190E-01 |
| 2.00000E-02 | | 6.12661E-01 |
| 1.80000E-02 | | 5.47712E-01 |
| 1.60000E-02 | | 4.83380E-01 |
| 1.40000E-02 | | 4.19704E-01 |
| 1.20000E-02 | | 3.56737E-01 |
| 1.00000E-02 | | 2.94540E-01 |
| 8.00000E-03 | | 2.33194E-01 |
| 6.00000E-03 | | 1.72808E-01 |
| 4.00000E-03 | | 1.13544E-01 |
| 2.00000E-03 | | 5.56815E-02 |
| 1.00000E-03 | | 2.74526E-02 |
| 0.00000E-99 | | 0.00000E-99 |

near the critical point, in figure 68.

FIGURE 68. - Redlich-Kwong fluid, asymptotic function of the heat of vaporization per mole of gas collected outside the calorimeter as a function of temperature.

SECOND TEMPERATURE DERIVATIVE OF THE VAPOR PRESSURE CURVE

From equations 44 and 251 of reference 3, it can be shown that

$$\beta'' = \frac{d^2\beta}{d\gamma^2} = \frac{\alpha_1\alpha_3}{Z_c(\alpha_3-\alpha_1)}\left[2(X_{\gamma(3)} - X_{\gamma(1)}) + \gamma(X_{\gamma\gamma(3)} - X_{\gamma\gamma(1)})\right] + \frac{\alpha_3[\beta' - \beta_{\gamma(1)}]^2}{\alpha_1(\alpha_3-\alpha_1)\beta_{\alpha(1)}} - \frac{\alpha_1[\beta' - \beta_{\gamma(3)}]^2}{\alpha_3(\alpha_3-\alpha_1)\beta_{\alpha(3)}}, \tag{289}$$

which from equations 23, 37, 38, 57, 196, and 203 can be written as

$$\beta'' = \frac{(1+b\alpha_1)\alpha_3}{b(\alpha_3-\alpha_1)\alpha_1}\beta_{\gamma\gamma(1)}\ln\frac{(1+b\alpha_3)}{(1+b\alpha_1)} + \frac{\alpha_3^2(\beta'-\beta_{\gamma(1)})\alpha_1' - \alpha_1^2(\beta'-\beta_{\gamma(3)})\alpha_3'}{\alpha_1\alpha_3(\alpha_3-\alpha_1)}. \tag{290}$$

Values of $\beta''$ as a function of $\gamma$ are listed in table 42. These data are illustrated in figures 69 and 70.

FIGURE 69. - Redlich-Kwong fluid, second temperature derivative of the vapor pressure curve as a function of temperature.

FIGURE 70. - Redlich-Kwong fluid, second temperature derivative of the vapor pressure curve as a function of temperature near the critical point.

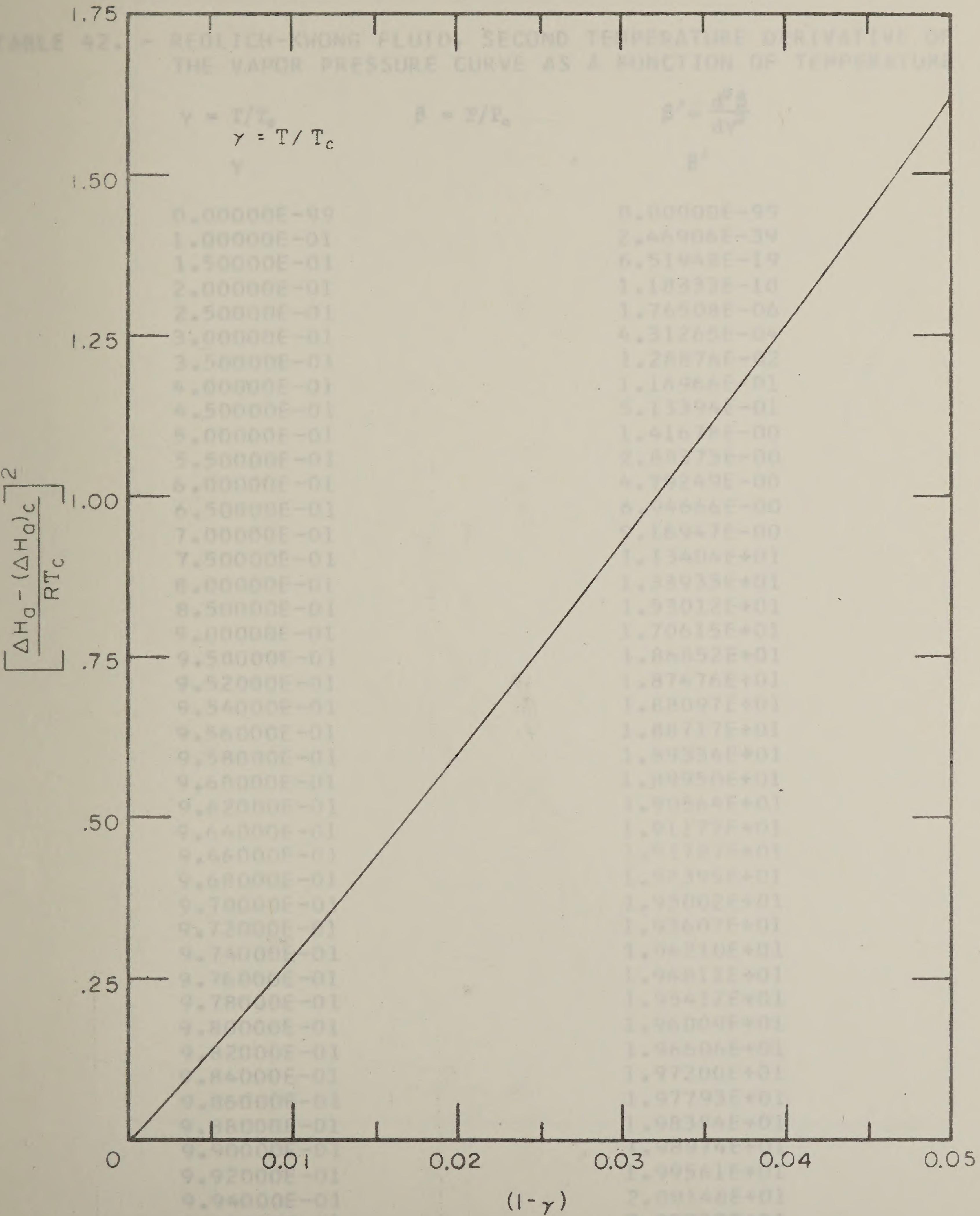

FIGURE 68.- Redlich-Kwong Fluid, Asymptotic Function of the Heat of Vaporization Per Mole of Gas Collected Outside the Calorimeter as a Function of Temperature.

TABLE 42. - REDLICH-KWONG FLUID, SECOND TEMPERATURE DERIVATIVE OF THE VAPOR PRESSURE CURVE AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$     $\beta = P/P_c$     $\beta'' = \dfrac{d^2\beta}{d\gamma^2}$

| $\gamma$ | $\beta''$ |
|---|---|
| 0.00000E-99 | 0.00000E-99 |
| 1.00000E-01 | 2.46906E-39 |
| 1.50000E-01 | 6.51948E-19 |
| 2.00000E-01 | 1.18333E-10 |
| 2.50000E-01 | 1.76508E-06 |
| 3.00000E-01 | 4.31265E-04 |
| 3.50000E-01 | 1.28876E-02 |
| 4.00000E-01 | 1.16966E-01 |
| 4.50000E-01 | 5.13394E-01 |
| 5.00000E-01 | 1.41678E-00 |
| 5.50000E-01 | 2.88373E-00 |
| 6.00000E-01 | 4.79249E-00 |
| 6.50000E-01 | 6.94666E-00 |
| 7.00000E-01 | 9.16947E-00 |
| 7.50000E-01 | 1.13404E+01 |
| 8.00000E-01 | 1.33933E+01 |
| 8.50000E-01 | 1.53012E+01 |
| 9.00000E-01 | 1.70615E+01 |
| 9.50000E-01 | 1.86852E+01 |
| 9.52000E-01 | 1.87476E+01 |
| 9.54000E-01 | 1.88097E+01 |
| 9.56000E-01 | 1.88717E+01 |
| 9.58000E-01 | 1.89334E+01 |
| 9.60000E-01 | 1.89950E+01 |
| 9.62000E-01 | 1.90564E+01 |
| 9.64000E-01 | 1.91177E+01 |
| 9.66000E-01 | 1.91787E+01 |
| 9.68000E-01 | 1.92395E+01 |
| 9.70000E-01 | 1.93002E+01 |
| 9.72000E-01 | 1.93607E+01 |
| 9.74000E-01 | 1.94210E+01 |
| 9.76000E-01 | 1.94812E+01 |
| 9.78000E-01 | 1.95412E+01 |
| 9.80000E-01 | 1.96009E+01 |
| 9.82000E-01 | 1.96606E+01 |
| 9.84000E-01 | 1.97200E+01 |
| 9.86000E-01 | 1.97793E+01 |
| 9.88000E-01 | 1.98384E+01 |
| 9.90000E-01 | 1.98974E+01 |
| 9.92000E-01 | 1.99561E+01 |
| 9.94000E-01 | 2.00148E+01 |
| 9.96000E-01 | 2.00732E+01 |
| 9.98000E-01 | 2.01315E+01 |
| 9.99000E-01 | 2.01607E+01 |
| 1.00000E-00 | 2.01897E+01 |

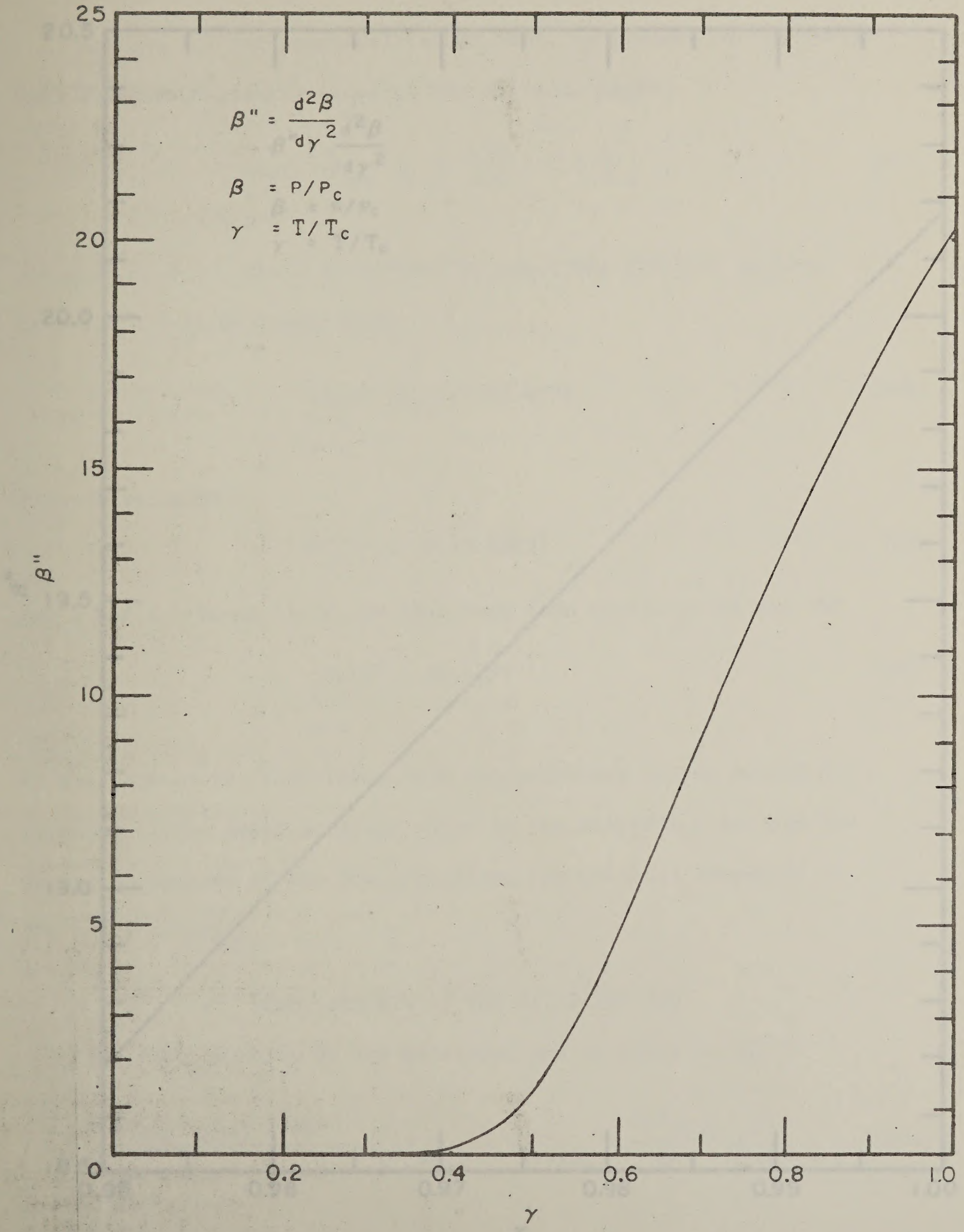

FIGURE 69.– Redlich–Kwong Fluid, Second Temperature Derivative of the Vapor Pressure Curve as a Function of Temperature.

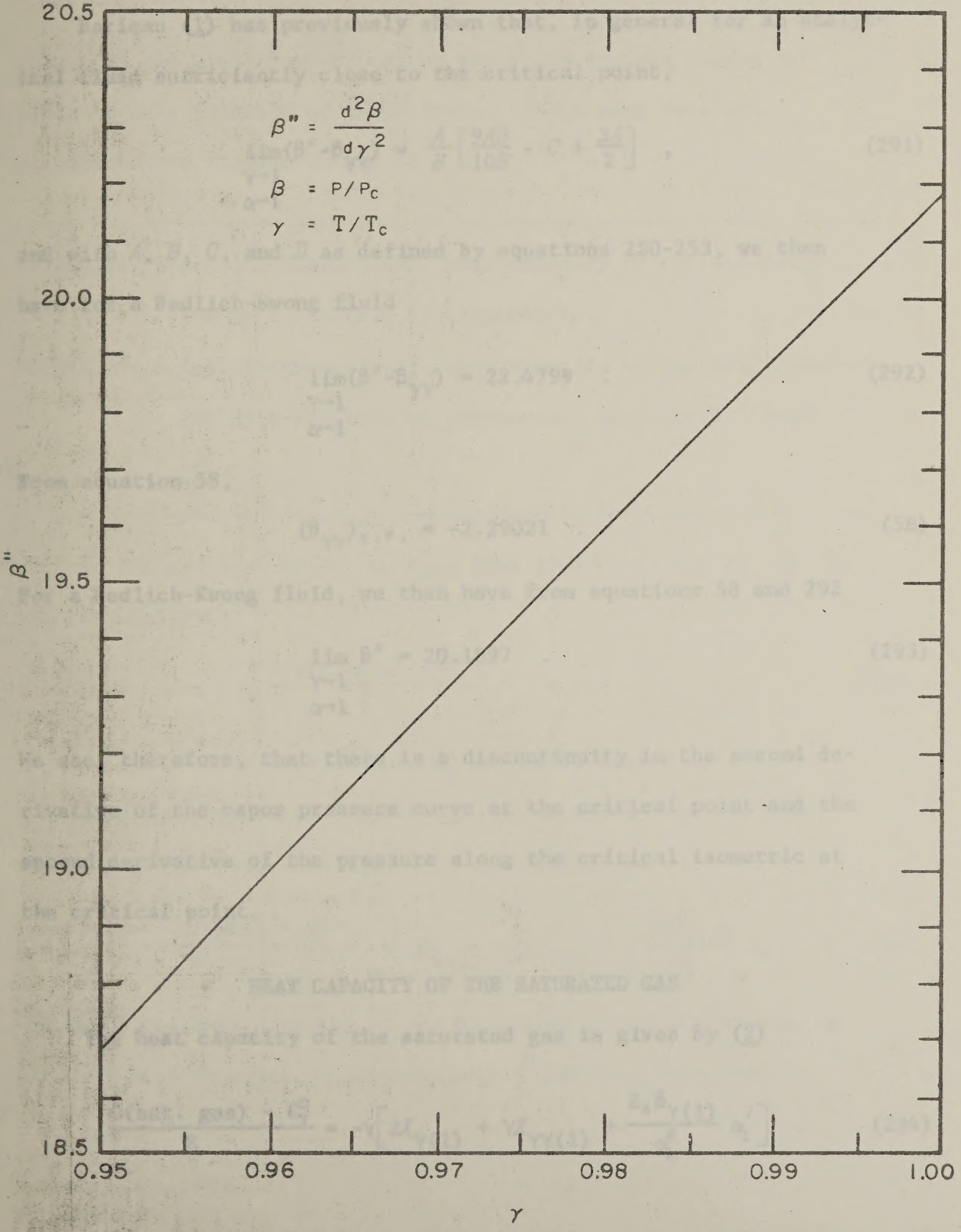

FIGURE 70. — Redlich–Kwong Fluid, Second Temperature Derivative of the Vapor Pressure Curve as a Function of Temperature Near the Critical Point.

Barieau (1) has previously shown that, in general for an analytical fluid sufficiently close to the critical point,

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}}(\beta''-\beta_{\gamma\gamma}) = \frac{A}{B}\left[\frac{9AD}{10B} - C + \frac{3A}{2}\right] \quad , \tag{291}$$

and with $A$, $B$, $C$, and $D$ as defined by equations 250-253, we then have for a Redlich-Kwong fluid

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}}(\beta''-\beta_{\gamma\gamma}) = 22.4799 \quad . \tag{292}$$

From equation 58,

$$(\beta_{\gamma\gamma})_{c.p.} = -2.29021 \quad . \tag{58}$$

For a Redlich-Kwong fluid, we then have from equations 58 and 292

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \beta'' = 20.1897 \quad . \tag{293}$$

We see, therefore, that there is a discontinuity in the second derivative of the vapor pressure curve at the critical point and the second derivative of the pressure along the critical isometric at the critical point.

## HEAT CAPACITY OF THE SATURATED GAS

The heat capacity of the saturated gas is given by (2)

$$\frac{C(\text{sat. gas}) - C_v^o}{R} = -\gamma\left[2X_{\gamma(1)} + \gamma X_{\gamma\gamma(1)} + \frac{Z_c\beta_{\gamma(1)}}{\alpha_1^2}\,\alpha_1'\right] \quad . \tag{294}$$

Values of the heat capacity of the saturated gas are listed in table 43 as a function of temperature. These same data are illustrated in figures 71 and 72.

---

FIGURE 71. - Redlich-Kwong fluid, heat capacity of the saturated gas as a function of temperature.

FIGURE 72. - Redlich-Kwong fluid, heat capacity of the saturated gas as a function of temperature near the critical point.

---

At the critical point, we then have from equations 23, 48, 49, 51, 211, and 294,

$$\left[\frac{C(\text{sat. gas}) - C_V^\circ}{R}\right]_{c.p.} = -\infty \quad . \tag{295}$$

HEAT CAPACITY OF THE SATURATED LIQUID

In a previous Bureau report (2), it is shown that the heat capacity of the saturated liquid is expressible as

$$\frac{C(\text{sat. liquid}) - C_V^\circ}{R} = -\gamma\left[2X_{\gamma(3)} + \gamma X_{\gamma\gamma(3)} + \frac{Z_c\beta_{\gamma(3)}}{\alpha_3^2}\alpha_3'\right] \quad . \tag{296}$$

Values of the heat capacity of the saturated liquid are listed in table 44 as a function of temperature. These same data are

TABLE 43. - REDLICH-KWONG FLUID, HEAT CAPACITY OF THE SATURATED GAS AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad$ $C(\text{sat. gas}) = T\,\frac{dS_1}{dT}$

| $\gamma$ | $\frac{C(\text{sat. gas}) - C_V^\circ}{R}$ |
|---|---|
| 0.00000E-99 | - ∞ |
| 1.00000E-01 | -1.60708E+02 |
| 1.50000E-01 | -8.67756E+01 |
| 2.00000E-01 | -5.58114E+01 |
| 2.50000E-01 | -3.94779E+01 |
| 3.00000E-01 | -2.96367E+01 |
| 3.50000E-01 | -2.31678E+01 |
| 4.00000E-01 | -1.86515E+01 |
| 4.50000E-01 | -1.53753E+01 |
| 5.00000E-01 | -1.29603E+01 |
| 5.50000E-01 | -1.11861E+01 |
| 6.00000E-01 | -9.90634E-00 |
| 6.50000E-01 | -9.01625E-00 |
| 7.00000E-01 | -8.44516E-00 |
| 7.50000E-01 | -8.15678E-00 |
| 8.00000E-01 | -8.15827E-00 |
| 8.50000E-01 | -8.53243E-00 |
| 9.00000E-01 | -9.56335E-00 |
| 9.50000E-01 | -1.24780E+01 |
| 9.52000E-01 | -1.26959E+01 |
| 9.54000E-01 | -1.29290E+01 |
| 9.56000E-01 | -1.31789E+01 |
| 9.58000E-01 | -1.34476E+01 |
| 9.60000E-01 | -1.37375E+01 |
| 9.62000E-01 | -1.40513E+01 |
| 9.64000E-01 | -1.43922E+01 |
| 9.66000E-01 | -1.47643E+01 |
| 9.68000E-01 | -1.51722E+01 |
| 9.70000E-01 | -1.56221E+01 |
| 9.72000E-01 | -1.61211E+01 |
| 9.74000E-01 | -1.66787E+01 |
| 9.76000E-01 | -1.73068E+01 |
| 9.78000E-01 | -1.80211E+01 |
| 9.80000E-01 | -1.88428E+01 |
| 9.82000E-01 | -1.98010E+01 |
| 9.84000E-01 | -2.09374E+01 |
| 9.86000E-01 | -2.23135E+01 |
| 9.88000E-01 | -2.40259E+01 |
| 9.90000E-01 | -2.62357E+01 |
| 9.92000E-01 | -2.92380E+01 |
| 9.94000E-01 | -3.36499E+01 |
| 9.96000E-01 | -4.10715E+01 |
| 9.98000E-01 | -5.78701E+01 |
| 9.99000E-01 | -8.16706E+01 |
| 1.00000E-00 | - ∞ |

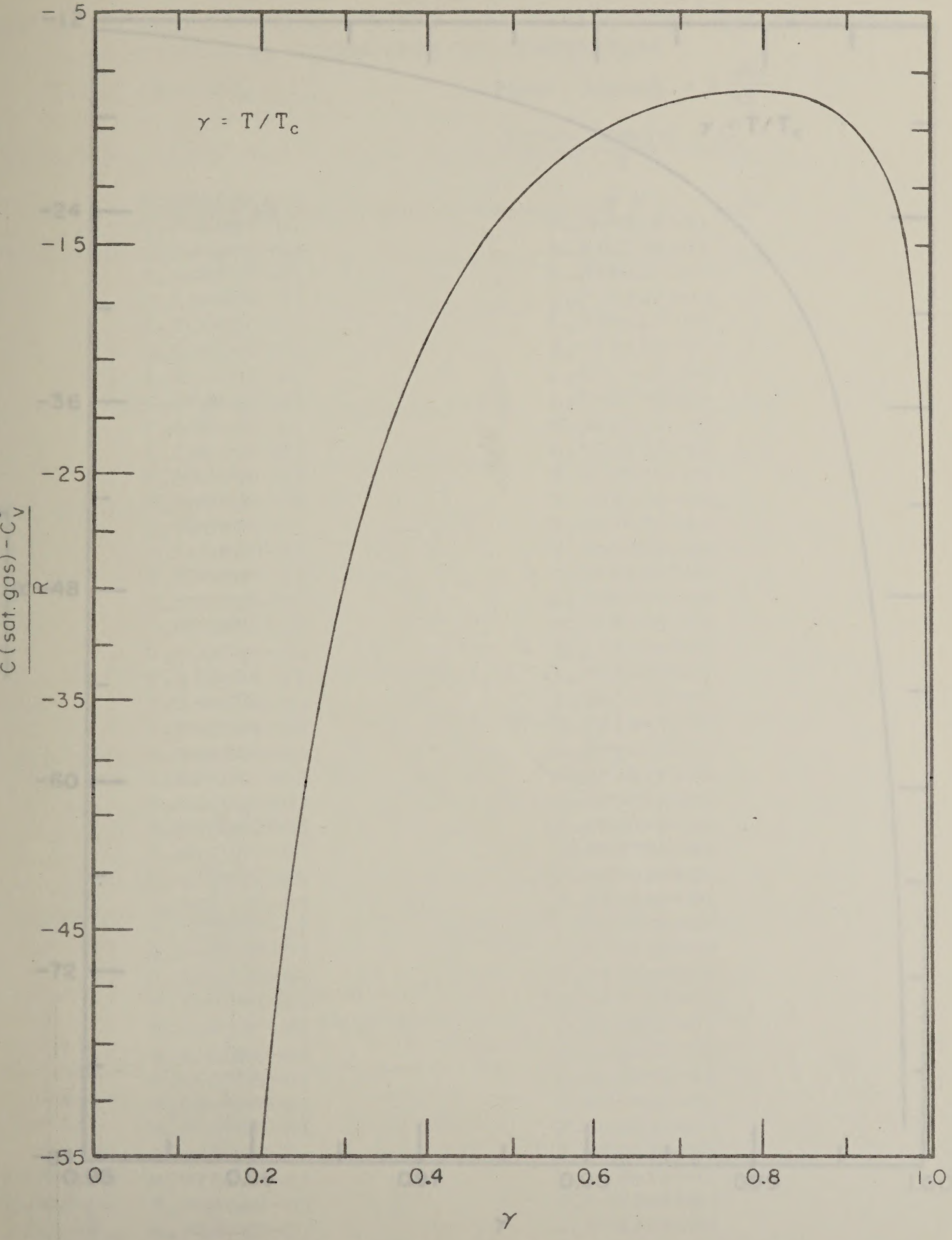

FIGURE 71.- Redlich-Kwong Fluid, Heat Capacity of the Saturated Gas as a Function of Temperature.

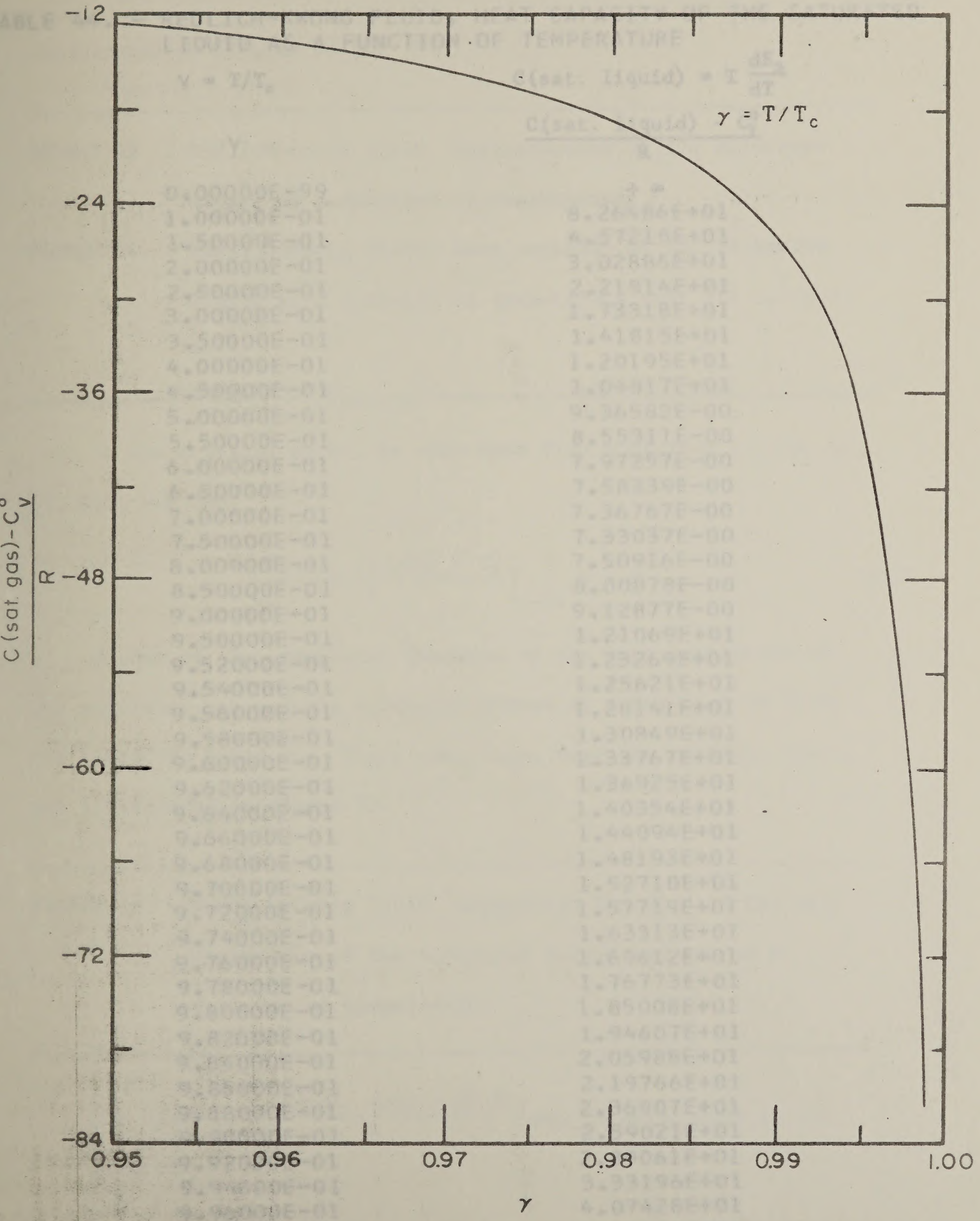

FIGURE 72.— Redlich–Kwong Fluid, Heat Capacity of the Saturated Gas as a Function of Temperature Near the Critical Point.

TABLE 44. - REDLICH-KWONG FLUID, HEAT CAPACITY OF THE SATURATED LIQUID AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad$ $C(\text{sat. liquid}) = T \dfrac{dS_3}{dT}$

| $\gamma$ | $\dfrac{C(\text{sat. liquid}) - C_V^0}{R}$ |
|---|---|
| 0.00000E-99 | + ∞ |
| 1.00000E-01 | 8.26486E+01 |
| 1.50000E-01 | 4.57215E+01 |
| 2.00000E-01 | 3.02886E+01 |
| 2.50000E-01 | 2.21814E+01 |
| 3.00000E-01 | 1.73318E+01 |
| 3.50000E-01 | 1.41815E+01 |
| 4.00000E-01 | 1.20195E+01 |
| 4.50000E-01 | 1.04817E+01 |
| 5.00000E-01 | 9.36582E-00 |
| 5.50000E-01 | 8.55311E-00 |
| 6.00000E-01 | 7.97257E-00 |
| 6.50000E-01 | 7.58339E-00 |
| 7.00000E-01 | 7.36767E-00 |
| 7.50000E-01 | 7.33037E-00 |
| 8.00000E-01 | 7.50916E-00 |
| 8.50000E-01 | 8.00878E-00 |
| 9.00000E-01 | 9.12877E-00 |
| 9.50000E-01 | 1.21069E+01 |
| 9.52000E-01 | 1.23269E+01 |
| 9.54000E-01 | 1.25621E+01 |
| 9.56000E-01 | 1.28141E+01 |
| 9.58000E-01 | 1.30849E+01 |
| 9.60000E-01 | 1.33767E+01 |
| 9.62000E-01 | 1.36925E+01 |
| 9.64000E-01 | 1.40354E+01 |
| 9.66000E-01 | 1.44094E+01 |
| 9.68000E-01 | 1.48193E+01 |
| 9.70000E-01 | 1.52710E+01 |
| 9.72000E-01 | 1.57719E+01 |
| 9.74000E-01 | 1.63313E+01 |
| 9.76000E-01 | 1.69612E+01 |
| 9.78000E-01 | 1.76773E+01 |
| 9.80000E-01 | 1.85008E+01 |
| 9.82000E-01 | 1.94607E+01 |
| 9.84000E-01 | 2.05988E+01 |
| 9.86000E-01 | 2.19766E+01 |
| 9.88000E-01 | 2.36907E+01 |
| 9.90000E-01 | 2.59021E+01 |
| 9.92000E-01 | 2.89061E+01 |
| 9.94000E-01 | 3.33196E+01 |
| 9.96000E-01 | 4.07428E+01 |
| 9.98000E-01 | 5.75430E+01 |
| 9.99000E-01 | 8.13442E+01 |
| 1.00000E-00 | + ∞ |

illustrated in figures 73 and 74.

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FIGURE 73. - Redlich-Kwong fluid, heat capacity of the saturated liquid as a function of temperature.

FIGURE 74. - Redlich-Kwong fluid, heat capacity of the saturated liquid as a function of temperature near the critical point.

---

At the critical point, we then have from equations 23, 48, 49, 51, 213, and 296,

$$\left[\frac{C(\text{sat. liquid}) - C_V^{\circ}}{R}\right]_{c.p.} = +\infty \quad . \qquad (297)$$

Values of the asymptotic function of the heat capacities of the saturated gas and the saturated liquid as a function of $(1-\gamma)$ are listed in table 45. These same data, near the critical point, are illustrated in figure 75.

---

FIGURE 75. - Redlich-Kwong fluid, asymptotic function of the heat capacities of the saturated liquid and gas as a function of temperature.

---

The $\lim\limits_{\substack{\gamma\to 1 \\ \alpha_1 \to 1}} \left[(1-\gamma)\left(\frac{C(\text{sat. gas}) - C_V^{\circ}}{R}\right)^2\right]$ may be determined in the

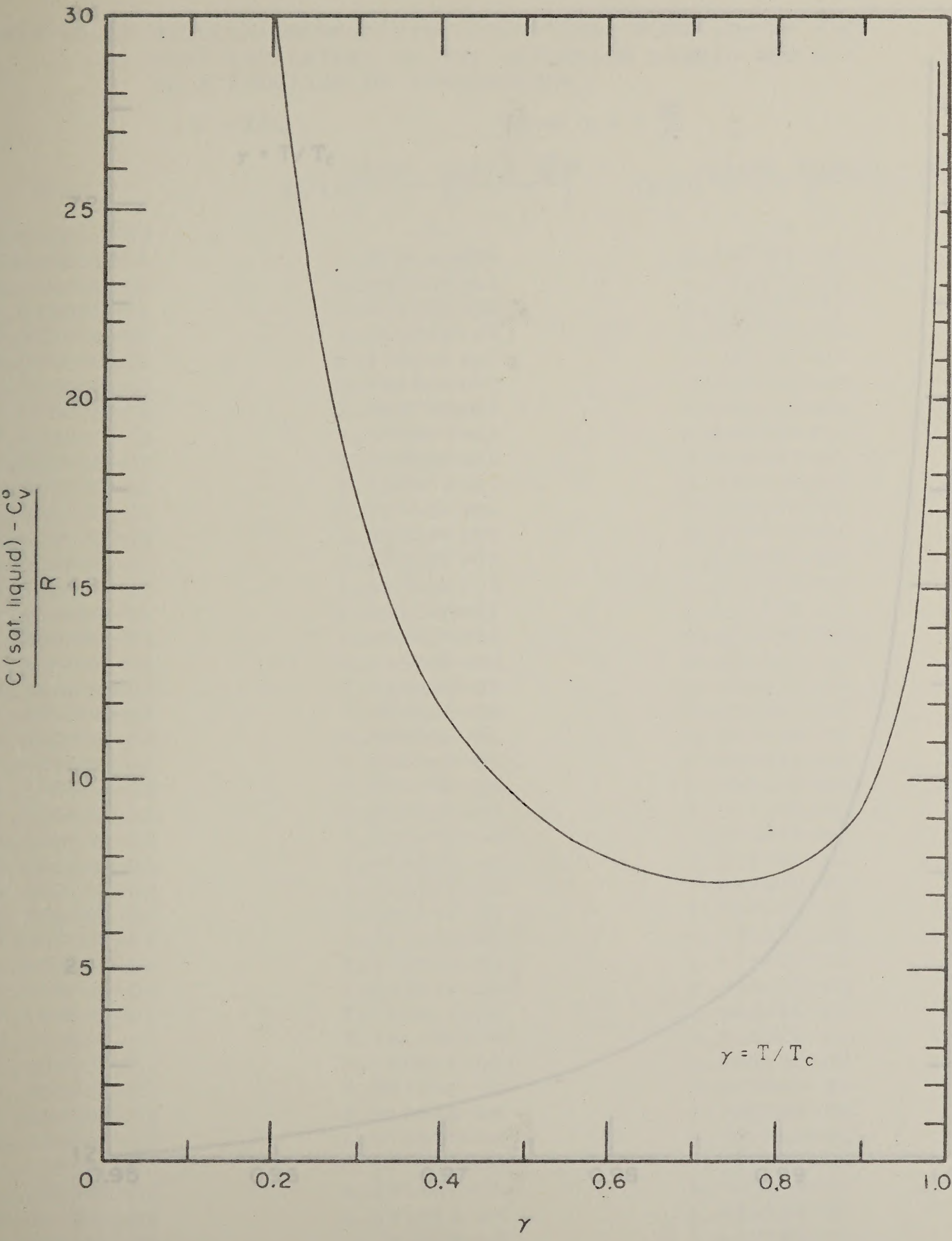

FIGURE 73.— Redlich-Kwong Fluid, Heat Capacity of the Saturated Liquid as a Function of Temperature.

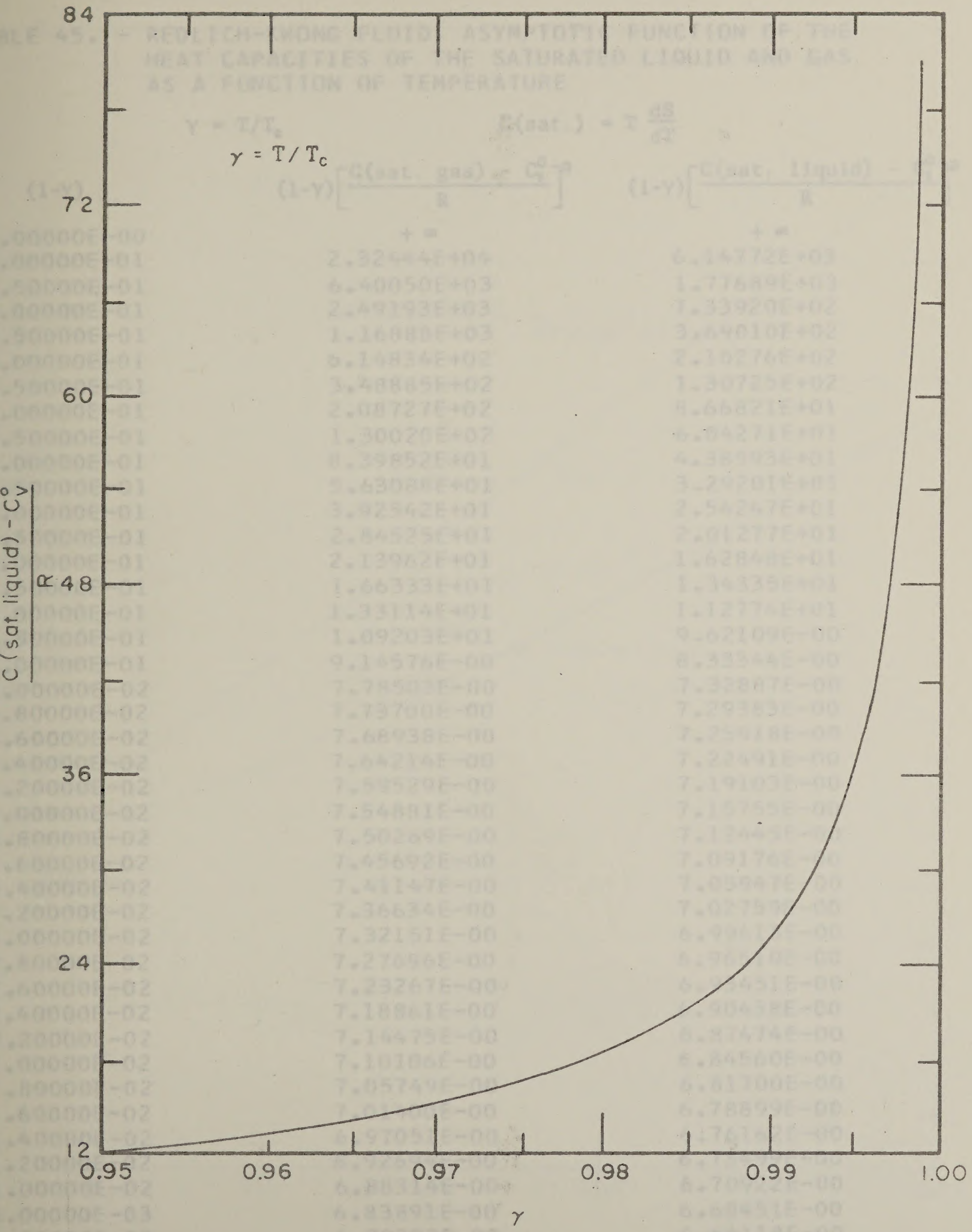

FIGURE 74. - Redlich-Kwong Fluid, Heat Capacity of the Saturated Liquid as a Function of Temperature Near the Critical Point.

TABLE 45. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE HEAT CAPACITIES OF THE SATURATED LIQUID AND GAS AS A FUNCTION OF TEMPERATURE

$Y = T/T_c$ $\qquad$ $C(\text{sat.}) = T\,\dfrac{dS}{dT}$

| $(1-Y)$ | $(1-Y)\left[\dfrac{C(\text{sat. gas}) - C_V^0}{R}\right]^2$ | $(1-Y)\left[\dfrac{C(\text{sat. liquid}) - C_V^0}{R}\right]^2$ |
|---|---|---|
| 1.00000E-00 | + ∞ | + ∞ |
| 9.00000E-01 | 2.32444E+04 | 6.14772E+03 |
| 8.50000E-01 | 6.40050E+03 | 1.77689E+03 |
| 8.00000E-01 | 2.49193E+03 | 7.33920E+02 |
| 7.50000E-01 | 1.16888E+03 | 3.69010E+02 |
| 7.00000E-01 | 6.14834E+02 | 2.10276E+02 |
| 6.50000E-01 | 3.48885E+02 | 1.30725E+02 |
| 6.00000E-01 | 2.08727E+02 | 8.66821E+01 |
| 5.50000E-01 | 1.30020E+02 | 6.04271E+01 |
| 5.00000E-01 | 8.39852E+01 | 4.38593E+01 |
| 4.50000E-01 | 5.63088E+01 | 3.29201E+01 |
| 4.00000E-01 | 3.92542E+01 | 2.54247E+01 |
| 3.50000E-01 | 2.84525E+01 | 2.01277E+01 |
| 3.00000E-01 | 2.13962E+01 | 1.62848E+01 |
| 2.50000E-01 | 1.66333E+01 | 1.34335E+01 |
| 2.00000E-01 | 1.33114E+01 | 1.12774E+01 |
| 1.50000E-01 | 1.09203E+01 | 9.62109E-00 |
| 1.00000E-01 | 9.14576E-00 | 8.33344E-00 |
| 5.00000E-02 | 7.78503E-00 | 7.32887E-00 |
| 4.80000E-02 | 7.73700E-00 | 7.29383E-00 |
| 4.60000E-02 | 7.68938E-00 | 7.25918E-00 |
| 4.40000E-02 | 7.64214E-00 | 7.22491E-00 |
| 4.20000E-02 | 7.59529E-00 | 7.19103E-00 |
| 4.00000E-02 | 7.54881E-00 | 7.15755E-00 |
| 3.80000E-02 | 7.50269E-00 | 7.12445E-00 |
| 3.60000E-02 | 7.45692E-00 | 7.09176E-00 |
| 3.40000E-02 | 7.41147E-00 | 7.05947E-00 |
| 3.20000E-02 | 7.36634E-00 | 7.02759E-00 |
| 3.00000E-02 | 7.32151E-00 | 6.99613E-00 |
| 2.80000E-02 | 7.27696E-00 | 6.96510E-00 |
| 2.60000E-02 | 7.23267E-00 | 6.93451E-00 |
| 2.40000E-02 | 7.18861E-00 | 6.90438E-00 |
| 2.20000E-02 | 7.14475E-00 | 6.87474E-00 |
| 2.00000E-02 | 7.10106E-00 | 6.84560E-00 |
| 1.80000E-02 | 7.05749E-00 | 6.81700E-00 |
| 1.60000E-02 | 7.01400E-00 | 6.78899E-00 |
| 1.40000E-02 | 6.97051E-00 | 6.76162E-00 |
| 1.20000E-02 | 6.92694E-00 | 6.73499E-00 |
| 1.00000E-02 | 6.88314E-00 | 6.70922E-00 |
| 8.00000E-03 | 6.83891E-00 | 6.68451E-00 |
| 6.00000E-03 | 6.79392E-00 | 6.66118E-00 |
| 4.00000E-03 | 6.74749E-00 | 6.63991E-00 |
| 2.00000E-03 | 6.69791E-00 | 6.62239E-00 |
| 1.00000E-03 | 6.67010E-00 | 6.61688E-00 |
| 0.00000E-99 | 6.62690E-00 | 6.62690E-00 |

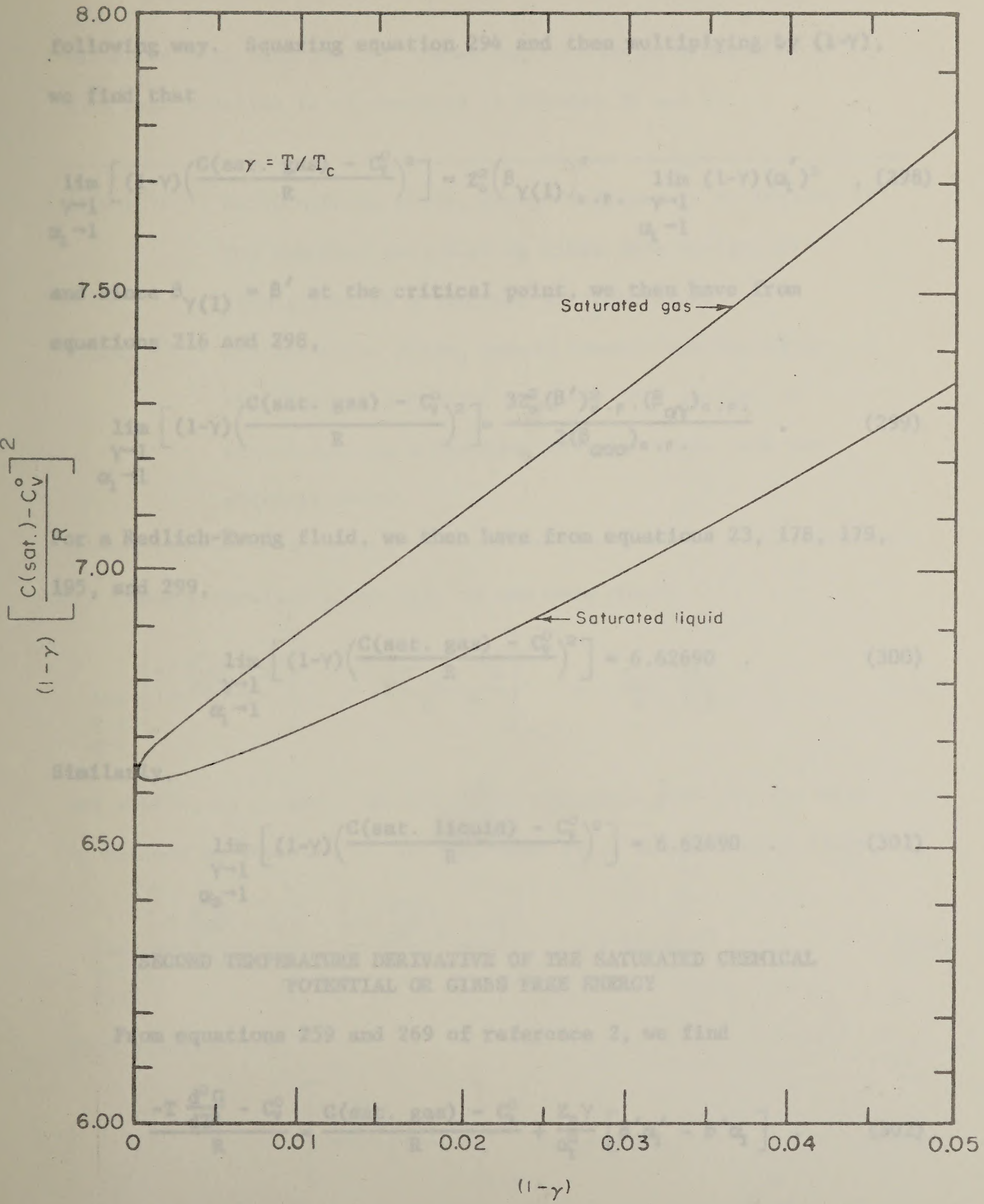

FIGURE 75.- Redlich-Kwong Fluid, Asymptotic Function of the Heat Capacities of the Saturated Liquid and Gas as a Function of Temperature.

following way. Squaring equation 294 and then multiplying by (1-$\gamma$), we find that

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \left[(1-\gamma)\left(\frac{C(\text{sat. gas}) - C_V^0}{R}\right)^2\right] = Z_c^2\left(\beta_{\gamma(1)}\right)^2_{c.p.} \lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} (1-\gamma)(\alpha_1')^2 \quad , \tag{298}$$

and since $\beta_{\gamma(1)} = \beta'$ at the critical point, we then have from equations 216 and 298,

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \left[(1-\gamma)\left(\frac{C(\text{sat. gas}) - C_V^0}{R}\right)^2\right] = \frac{3Z_c^2(\beta')^2_{c.p.}(\beta_{\alpha\gamma})_{c.p.}}{2(\beta_{\alpha\alpha\alpha})_{c.p.}} \quad . \tag{299}$$

For a Redlich-Kwong fluid, we then have from equations 23, 178, 179, 195, and 299,

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \left[(1-\gamma)\left(\frac{C(\text{sat. gas}) - C_V^0}{R}\right)^2\right] = 6.62690 \quad . \tag{300}$$

Similarly,

$$\lim_{\substack{\gamma\to 1\\ \alpha_3\to 1}} \left[(1-\gamma)\left(\frac{C(\text{sat. liquid}) - C_V^0}{R}\right)^2\right] = 6.62690 \quad . \tag{301}$$

## SECOND TEMPERATURE DERIVATIVE OF THE SATURATED CHEMICAL POTENTIAL OR GIBBS FREE ENERGY

From equations 259 and 269 of reference 2, we find

$$\frac{-T\dfrac{d^2G}{dT^2} - C_V^0}{R} = \frac{C(\text{sat. gas}) - C_V^0}{R} + \frac{Z_c\gamma}{\alpha_1^2}\left[\beta'\alpha_1' - \beta''\alpha_1\right] \quad . \tag{302}$$

Values of this function are listed in table 46 as a function of Y. This same function is illustrated in figures 76 and 77.

---

FIGURE 76. - Redlich-Kwong fluid, second temperature derivative of the chemical potential or Gibbs free energy, at saturation, as a function of temperature.

FIGURE 77. - Redlich-Kwong fluid, second temperature derivative of the chemical potential or Gibbs free energy, at saturation, as a function of temperature near the critical point.

---

From a previous paper (1), we can show that

$$\lim_{\substack{Y\to 1\\ \alpha\to 1}} \left[\frac{-T\,\dfrac{d^2G}{dT^2} - C_V^o}{R}\right] - \left[\frac{-T\left(\dfrac{\partial^2 G}{\partial T^2}\right)_\rho - C_V^o}{R}\right]_{c.p.} = -\frac{Z_c A}{B}\left[\frac{9AD}{10B} - C\right], \tag{303}$$

and with $A$, $B$, $C$, and $D$ as defined by equations 250-253, and with $Z_c$ defined by equation 23, we then have for a Redlich-Kwong fluid,

$$\lim_{\substack{Y\to 1\\ \alpha\to 1}} \left[\frac{-T\,\dfrac{d^2G}{dT^2} - C_V^o}{R}\right] - \left[\frac{-T\left(\dfrac{\partial^2 G}{\partial T^2}\right)_\rho - C_V^o}{R}\right]_{c.p.} = 2.99686. \tag{304}$$

From equation 110,

$$\left[\frac{-T\left(\dfrac{\partial^2 G}{\partial T^2}\right)_\rho - C_V^o}{R}\right]_{c.p.} = 1.61839\ . \tag{305}$$

TABLE 46. - REDLICH-KWONG FLUID, SECOND TEMPERATURE DERIVATIVE OF THE CHEMICAL POTENTIAL OR GIBBS FREE ENERGY, AT SATURATION, AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{-T \frac{d^2G}{dT^2} - C_V^\circ}{R}$ |
|---|---|
| 0.00000E-99 | + ∞ |
| 1.00000E-01 | 8.26486E+01 |
| 1.50000E-01 | 4.57215E+01 |
| 2.00000E-01 | 3.02886E+01 |
| 2.50000E-01 | 2.21814E+01 |
| 3.00000E-01 | 1.73318E+01 |
| 3.50000E-01 | 1.41811E+01 |
| 4.00000E-01 | 1.20149E+01 |
| 4.50000E-01 | 1.04583E+01 |
| 5.00000E-01 | 9.29106E-00 |
| 5.50000E-01 | 8.37786E-00 |
| 6.00000E-01 | 7.63678E-00 |
| 6.50000E-01 | 7.01943E-00 |
| 7.00000E-01 | 6.49687E-00 |
| 7.50000E-01 | 6.05069E-00 |
| 8.00000E-01 | 5.66824E-00 |
| 8.50000E-01 | 5.34013E-00 |
| 9.00000E-01 | 5.05902E-00 |
| 9.50000E-01 | 4.81900E-00 |
| 9.52000E-01 | 4.81019E-00 |
| 9.54000E-01 | 4.80143E-00 |
| 9.56000E-01 | 4.79273E-00 |
| 9.58000E-01 | 4.78408E-00 |
| 9.60000E-01 | 4.77549E-00 |
| 9.62000E-01 | 4.76696E-00 |
| 9.64000E-01 | 4.75849E-00 |
| 9.66000E-01 | 4.75007E-00 |
| 9.68000E-01 | 4.74170E-00 |
| 9.70000E-01 | 4.73340E-00 |
| 9.72000E-01 | 4.72514E-00 |
| 9.74000E-01 | 4.71694E-00 |
| 9.76000E-01 | 4.70880E-00 |
| 9.78000E-01 | 4.70071E-00 |
| 9.80000E-01 | 4.69268E-00 |
| 9.82000E-01 | 4.68470E-00 |
| 9.84000E-01 | 4.67677E-00 |
| 9.86000E-01 | 4.66890E-00 |
| 9.88000E-01 | 4.66108E-00 |
| 9.90000E-01 | 4.65331E-00 |
| 9.92000E-01 | 4.64560E-00 |
| 9.94000E-01 | 4.63794E-00 |
| 9.96000E-01 | 4.63033E-00 |
| 9.98000E-01 | 4.62277E-00 |
| 9.99000E-01 | 4.61899E-00 |
| 1.00000E-00 | 4.61526E-00 |

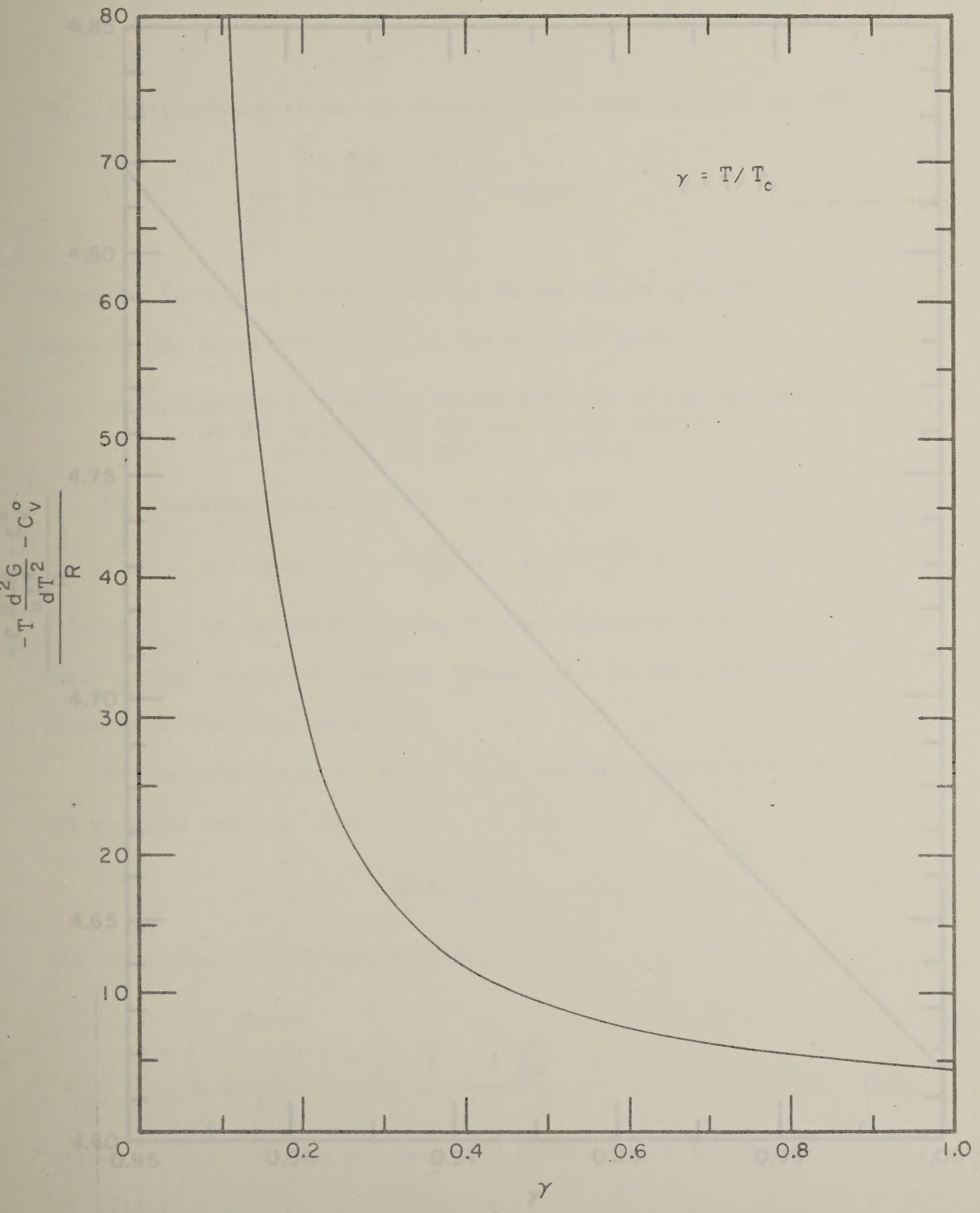

FIGURE 76. — Redlich-Kwong Fluid, Second Temperature Derivative of the Chemical Potential or Gibbs Free Energy, at Saturation, as a Function of Temperature.

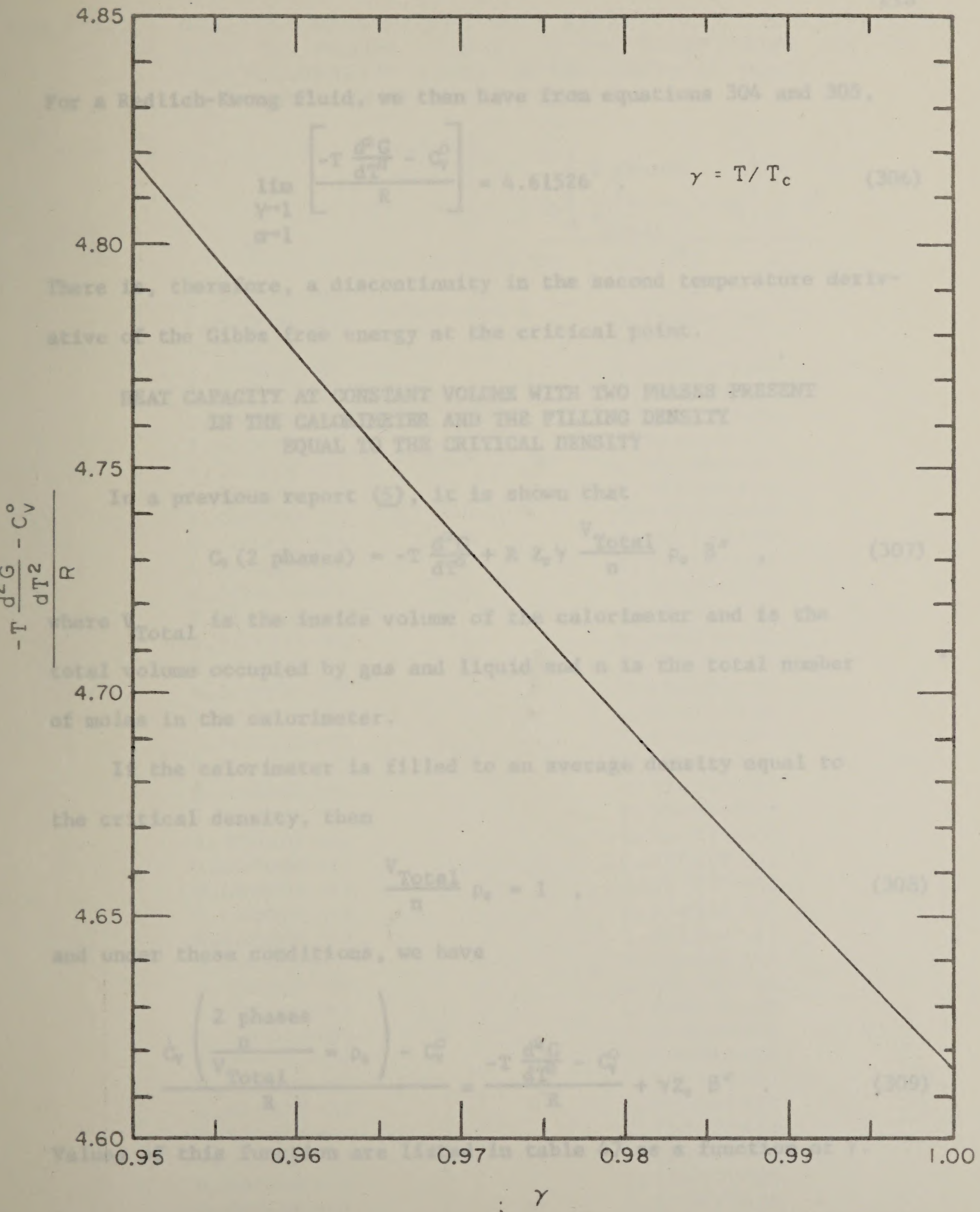

FIGURE 77. – Redlich-Kwong Fluid, Second Temperature Derivative of the Chemical Potential or Gibbs Free Energy, at Saturation, as a Function of Temperature Near the Critical Point.

For a Redlich-Kwong fluid, we then have from equations 304 and 305,

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \left[\frac{-T\,\dfrac{d^2G}{dT^2} - C_V^0}{R}\right] = 4.61526 \quad . \tag{306}$$

There is, therefore, a discontinuity in the second temperature derivative of the Gibbs free energy at the critical point.

## HEAT CAPACITY AT CONSTANT VOLUME WITH TWO PHASES PRESENT IN THE CALORIMETER AND THE FILLING DENSITY EQUAL TO THE CRITICAL DENSITY

In a previous report (5), it is shown that

$$C_V\,(2\ \text{phases}) = -T\,\frac{d^2G}{dT^2} + R\,Z_c\gamma\,\frac{V_{\text{Total}}}{n}\,\rho_c\,\beta'' \quad , \tag{307}$$

where $V_{\text{Total}}$ is the inside volume of the calorimeter and is the total volume occupied by gas and liquid and n is the total number of moles in the calorimeter.

If the calorimeter is filled to an average density equal to the critical density, then

$$\frac{V_{\text{Total}}}{n}\,\rho_c = 1 \quad , \tag{308}$$

and under these conditions, we have

$$\frac{C_V\left(\begin{array}{c}2\ \text{phases}\\ \dfrac{n}{V_{\text{Total}}} = \rho_c\end{array}\right) - C_V^0}{R} = \frac{-T\,\dfrac{d^2G}{dT^2} - C_V^0}{R} + \gamma Z_c\,\beta'' \quad . \tag{309}$$

Values of this function are listed in table 47 as a function of $\gamma$.

TABLE 47. - REDLICH-KWONG FLUID, HEAT CAPACITY AT CONSTANT VOLUME, WITH TWO PHASES PRESENT IN THE CALORIMETER AND THE FILLING DENSITY EQUAL TO THE CRITICAL DENSITY, AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\dfrac{C_V\left(\overset{\text{2 phases}}{\dfrac{n}{V_{\text{Total}}} = \rho_c}\right) - C_V^0}{R}$ |
|---|---|
| 0.00000E-99 | $+\infty$ |
| 1.00000E-01 | 8.26486E+01 |
| 1.50000E-01 | 4.57215E+01 |
| 2.00000E-01 | 3.02886E+01 |
| 2.50000E-01 | 2.21814E+01 |
| 3.00000E-01 | 1.73319E+01 |
| 3.50000E-01 | 1.41826E+01 |
| 4.00000E-01 | 1.20305E+01 |
| 4.50000E-01 | 1.05353E+01 |
| 5.00000E-01 | 9.52719E-00 |
| 5.50000E-01 | 8.90654E-00 |
| 6.00000E-01 | 8.59528E-00 |
| 6.50000E-01 | 8.52455E-00 |
| 7.00000E-01 | 8.63641E-00 |
| 7.50000E-01 | 8.88580E-00 |
| 8.00000E-01 | 9.23980E-00 |
| 8.50000E-01 | 9.67548E-00 |
| 9.00000E-01 | 1.01775E+01 |
| 9.50000E-01 | 1.07360E+01 |
| 9.52000E-01 | 1.07594E+01 |
| 9.54000E-01 | 1.07829E+01 |
| 9.56000E-01 | 1.08065E+01 |
| 9.58000E-01 | 1.08301E+01 |
| 9.60000E-01 | 1.08539E+01 |
| 9.62000E-01 | 1.08777E+01 |
| 9.64000E-01 | 1.09016E+01 |
| 9.66000E-01 | 1.09256E+01 |
| 9.68000E-01 | 1.09496E+01 |
| 9.70000E-01 | 1.09738E+01 |
| 9.72000E-01 | 1.09980E+01 |
| 9.74000E-01 | 1.10223E+01 |
| 9.76000E-01 | 1.10466E+01 |
| 9.78000E-01 | 1.10711E+01 |
| 9.80000E-01 | 1.10956E+01 |
| 9.82000E-01 | 1.11202E+01 |
| 9.84000E-01 | 1.11449E+01 |
| 9.86000E-01 | 1.11697E+01 |
| 9.88000E-01 | 1.11945E+01 |
| 9.90000E-01 | 1.12194E+01 |
| 9.92000E-01 | 1.12444E+01 |
| 9.94000E-01 | 1.12695E+01 |
| 9.96000E-01 | 1.12946E+01 |
| 9.98000E-01 | 1.13198E+01 |
| 9.99000E-01 | 1.13325E+01 |
| 1.00000E-00 | 1.13451E+01 |

This same function is illustrated in figures 78 and 79.

---

FIGURE 78. - Redlich-Kwong fluid, heat capacity at constant volume, with two phases present in the calorimeter and the filling density equal to the critical density, as a function of temperature.

FIGURE 79. - Redlich-Kwong fluid, heat capacity at constant volume, with two phases present in the calorimeter and the filling density equal to the critical density, as a function of temperature near the critical point.

---

From equations 23, 293, 306, and 309, it follows that

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \left[ \frac{C_V\left( \overset{2\ \text{phases}}{\dfrac{n}{V_{\text{Total}}} = \rho_c} \right) - C_V^o}{R} \right] = 11.3451 \quad . \qquad (310)$$

SATURATED PRESSURE-TEMPERATURE COEFFICIENTS
AT CONSTANT DENSITY

From equation 50,

$$\beta_{\gamma(1)} = \left(\frac{\partial\beta}{\partial\gamma}\right)_{\alpha_1} = 3\alpha_1 \left[ \frac{1}{(1-b\alpha_1)} + \frac{a\alpha_1}{2\gamma^{3/2}(1+b\alpha_1)} \right] \quad ; \qquad (311)$$

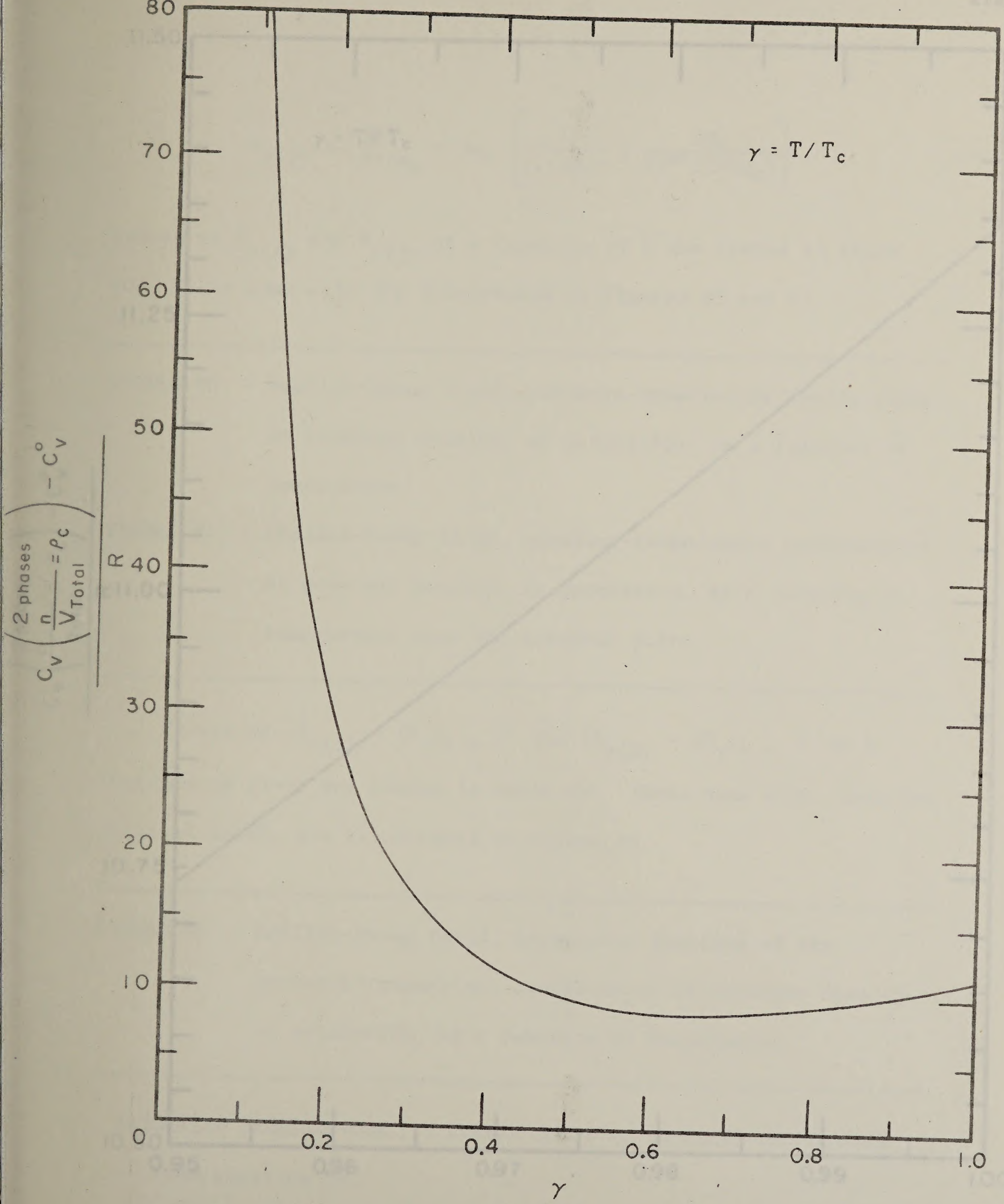

FIGURE 78. – Redlich-Kwong Fluid, Heat Capacity at Constant Volume, With Two Phases Present in the Calorimeter and the Filling Density Equal to the Critical Density, as a Function of Temperature.

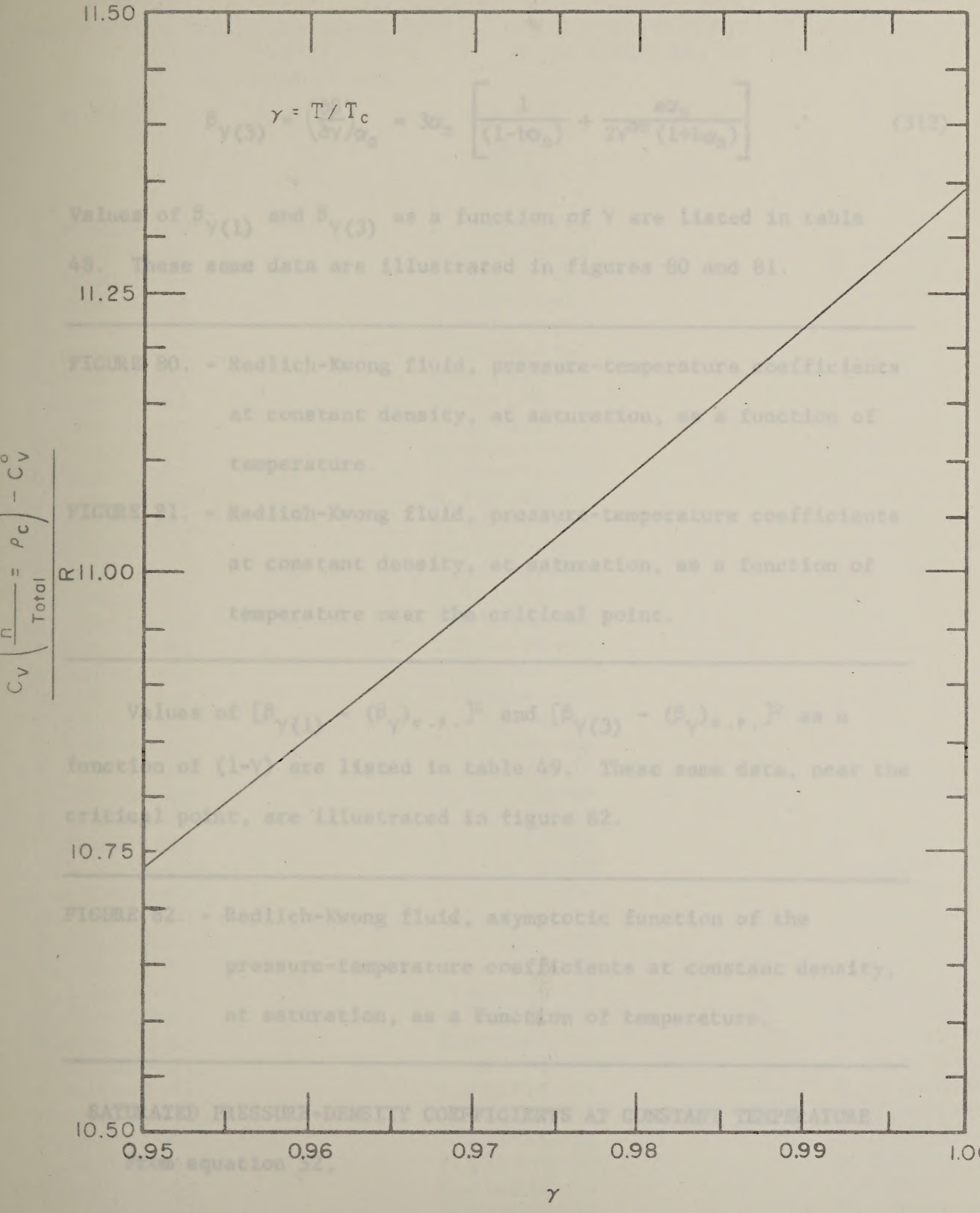

FIGURE 79.-Redlich-Kwong Fluid, Heat Capacity at Constant Volume, With Two Phases Present in the Calorimeter and the Filling Density Equal to the Critical Density, as a Function of Temperature Near the Critical Point.

$$\beta_{\gamma(3)} = \left(\frac{\partial\beta}{\partial\gamma}\right)_{\alpha_3} = 3\alpha_3 \left[\frac{1}{(1-b\alpha_3)} + \frac{a\alpha_3}{2\gamma^{3/2}(1+b\alpha_3)}\right] \quad . \tag{312}$$

Values of $\beta_{\gamma(1)}$ and $\beta_{\gamma(3)}$ as a function of $\gamma$ are listed in table 48. These same data are illustrated in figures 80 and 81.

---

FIGURE 80. - Redlich-Kwong fluid, pressure-temperature coefficients at constant density, at saturation, as a function of temperature.

FIGURE 81. - Redlich-Kwong fluid, pressure-temperature coefficients at constant density, at saturation, as a function of temperature near the critical point.

---

Values of $[\beta_{\gamma(1)} - (\beta_\gamma)_{c.p.}]^2$ and $[\beta_{\gamma(3)} - (\beta_\gamma)_{c.p.}]^2$ as a function of $(1-\gamma)$ are listed in table 49. These same data, near the critical point, are illustrated in figure 82.

---

FIGURE 82. - Redlich-Kwong fluid, asymptotic function of the pressure-temperature coefficients at constant density, at saturation, as a function of temperature.

---

SATURATED PRESSURE-DENSITY COEFFICIENTS AT CONSTANT TEMPERATURE

From equation 52,

TABLE 48. - REDLICH-KWONG FLUID, PRESSURE-TEMPERATURE COEFFICIENTS AT CONSTANT DENSITY, AT SATURATION, AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ | $\alpha = \rho/\rho_c$ | $\beta = P/P_c$

| $\gamma$ | $\left(\frac{\partial\beta}{\partial\gamma}\right)_{\alpha_1}$ | $\left(\frac{\partial\beta}{\partial\gamma}\right)_{\alpha_3}$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | $+\infty$ |
| 1.00000E-01 | 9.59064E-45 | 1.32454E+03 |
| 1.50000E-01 | 1.30663E-23 | 7.09009E+02 |
| 2.00000E-01 | 7.67249E-15 | 4.51218E+02 |
| 2.50000E-01 | 2.87224E-10 | 3.15240E+02 |
| 3.00000E-01 | 1.50266E-07 | 2.33317E+02 |
| 3.50000E-01 | 8.63232E-06 | 1.79464E+02 |
| 4.00000E-01 | 1.39419E-04 | 1.41802E+02 |
| 4.50000E-01 | 1.02787E-03 | 1.14207E+02 |
| 5.00000E-01 | 4.55405E-03 | 9.32338E+01 |
| 5.50000E-01 | 1.43477E-02 | 7.68124E+01 |
| 6.00000E-01 | 3.58279E-02 | 6.36316E+01 |
| 6.50000E-01 | 7.62239E-02 | 5.28235E+01 |
| 7.00000E-01 | 1.45158E-01 | 4.37907E+01 |
| 7.50000E-01 | 2.56260E-01 | 3.61059E+01 |
| 8.00000E-01 | 4.30677E-01 | 2.94487E+01 |
| 8.50000E-01 | 7.05159E-01 | 2.35613E+01 |
| 9.00000E-01 | 1.15467E-00 | 1.82017E+01 |
| 9.50000E-01 | 1.98400E-00 | 1.30372E+01 |
| 9.52000E-01 | 2.03254E-00 | 1.28257E+01 |
| 9.54000E-01 | 2.08294E-00 | 1.26131E+01 |
| 9.56000E-01 | 2.13534E-00 | 1.23993E+01 |
| 9.58000E-01 | 2.18989E-00 | 1.21840E+01 |
| 9.60000E-01 | 2.24676E-00 | 1.19672E+01 |
| 9.62000E-01 | 2.30613E-00 | 1.17486E+01 |
| 9.64000E-01 | 2.36821E-00 | 1.15280E+01 |
| 9.66000E-01 | 2.43327E-00 | 1.13051E+01 |
| 9.68000E-01 | 2.50157E-00 | 1.10797E+01 |
| 9.70000E-01 | 2.57346E-00 | 1.08514E+01 |
| 9.72000E-01 | 2.64930E-00 | 1.06199E+01 |
| 9.74000E-01 | 2.72958E-00 | 1.03847E+01 |
| 9.76000E-01 | 2.81483E-00 | 1.01451E+01 |
| 9.78000E-01 | 2.90572E-00 | 9.90069E-00 |
| 9.80000E-01 | 3.00308E-00 | 9.65041E-00 |
| 9.82000E-01 | 3.10797E-00 | 9.39329E-00 |
| 9.84000E-01 | 3.22171E-00 | 9.12798E-00 |
| 9.86000E-01 | 3.34611E-00 | 8.85269E-00 |
| 9.88000E-01 | 3.48364E-00 | 8.56493E-00 |
| 9.90000E-01 | 3.63785E-00 | 8.26115E-00 |
| 9.92000E-01 | 3.81423E-00 | 7.93585E-00 |
| 9.94000E-01 | 4.02205E-00 | 7.57976E-00 |
| 9.96000E-01 | 4.27942E-00 | 7.17476E-00 |
| 9.98000E-01 | 4.63329E-00 | 6.67391E-00 |
| 9.99000E-01 | 4.89669E-00 | 6.33726E-00 |
| 1.00000E-00 | 5.58043E-00 | 5.58043E-00 |

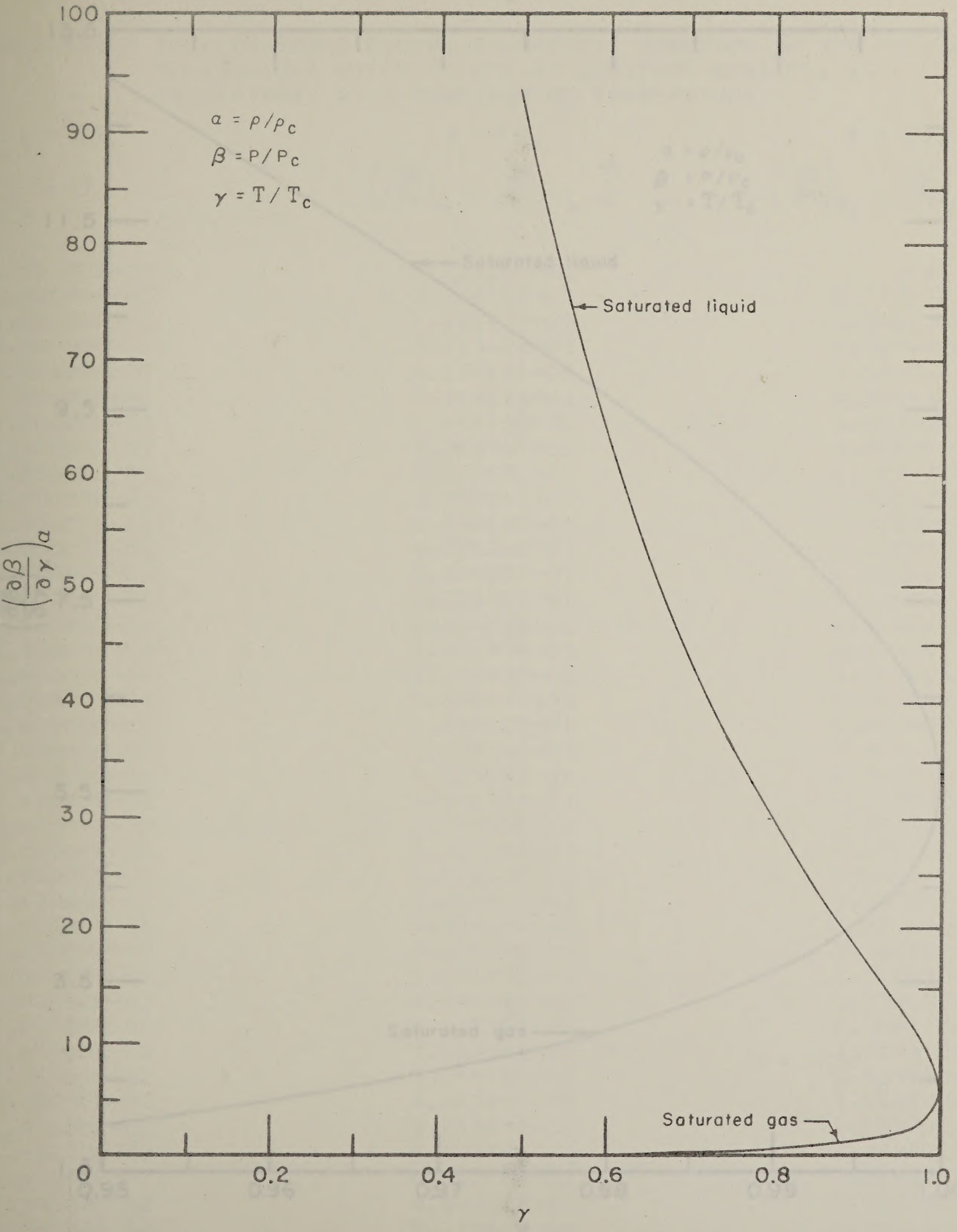

FIGURE 80. - Redlich-Kwong Fluid, Pressure-Temperature Coefficients at Constant Density, at Saturation, as a Function of Temperature.

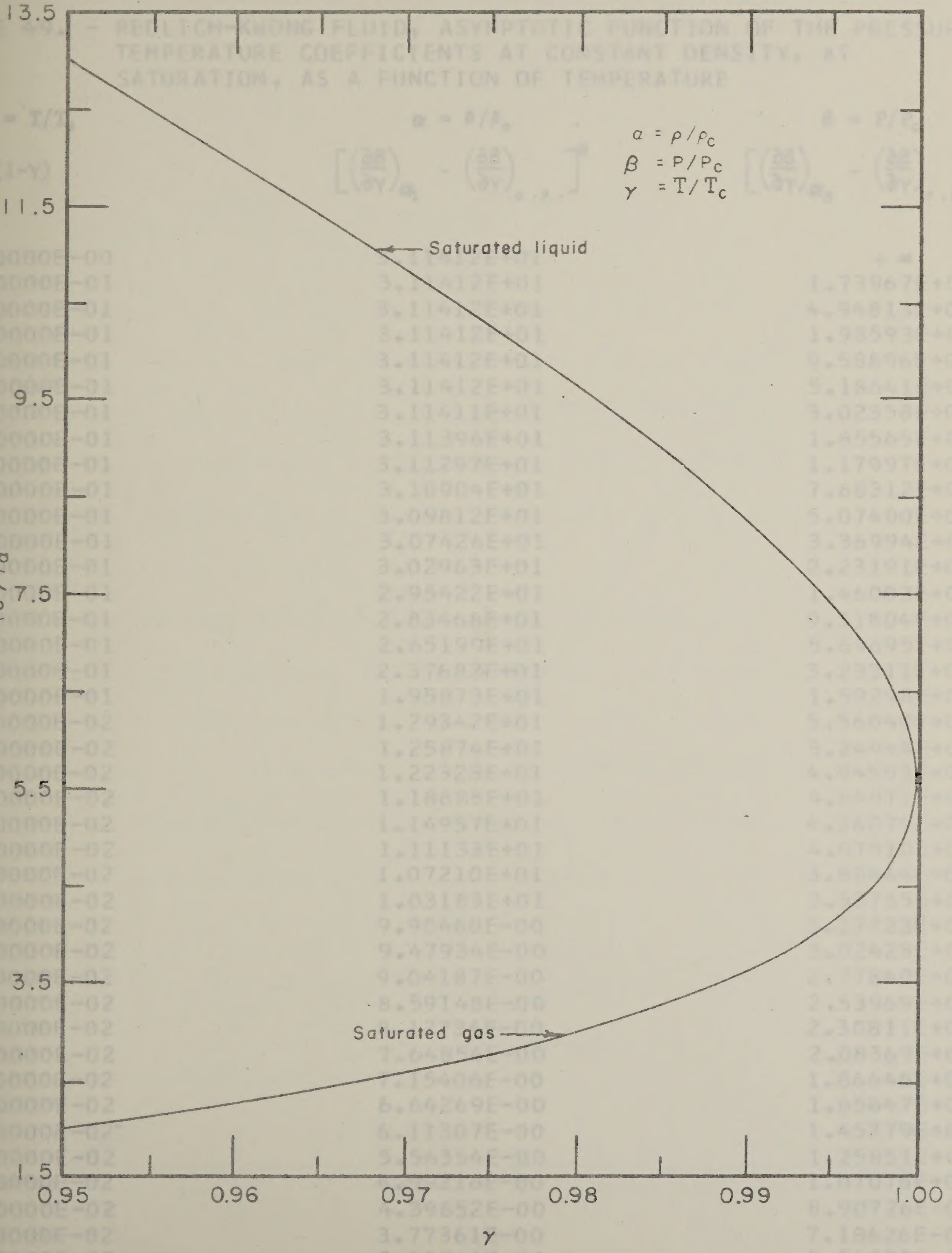

FIGURE 81. - Redlich - Kwong Fluid, Pressure-Temperature Coefficients at Constant Density, at Saturation, as a Function of Temperature Near the Critical Point.

TABLE 49. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE PRESSURE-TEMPERATURE COEFFICIENTS AT CONSTANT DENSITY, AT SATURATION, AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$<br>$(1-\gamma)$ | $\alpha = \rho/\rho_c$<br>$\left[\left(\frac{\partial\beta}{\partial\gamma}\right)_{\alpha_1} - \left(\frac{\partial\beta}{\partial\gamma}\right)_{c.P.}\right]^2$ | $\beta = P/P_c$<br>$\left[\left(\frac{\partial\beta}{\partial\gamma}\right)_{\alpha_3} - \left(\frac{\partial\beta}{\partial\gamma}\right)_{c.P.}\right]^2$ |
|---|---|---|
| 1.00000E-00 | 3.11412E+01 | + ∞ |
| 9.00000E-01 | 3.11412E+01 | 1.73967E+06 |
| 8.50000E-01 | 3.11412E+01 | 4.94813E+05 |
| 8.00000E-01 | 3.11412E+01 | 1.98593E+05 |
| 7.50000E-01 | 3.11412E+01 | 9.58896E+04 |
| 7.00000E-01 | 3.11412E+01 | 5.18641E+04 |
| 6.50000E-01 | 3.11411E+01 | 3.02358E+04 |
| 6.00000E-01 | 3.11396E+01 | 1.85565E+04 |
| 5.50000E-01 | 3.11297E+01 | 1.17997E+04 |
| 5.00000E-01 | 3.10904E+01 | 7.68312E+03 |
| 4.50000E-01 | 3.09812E+01 | 5.07400E+03 |
| 4.00000E-01 | 3.07426E+01 | 3.36994E+03 |
| 3.50000E-01 | 3.02963E+01 | 2.23191E+03 |
| 3.00000E-01 | 2.95422E+01 | 1.46003E+03 |
| 2.50000E-01 | 2.83468E+01 | 9.31804E+02 |
| 2.00000E-01 | 2.65199E+01 | 5.69695E+02 |
| 1.50000E-01 | 2.37682E+01 | 3.23311E+02 |
| 1.00000E-01 | 1.95873E+01 | 1.59298E+02 |
| 5.00000E-02 | 1.29342E+01 | 5.56040E+01 |
| 4.80000E-02 | 1.25874E+01 | 5.24948E+01 |
| 4.60000E-02 | 1.22323E+01 | 4.94593E+01 |
| 4.40000E-02 | 1.18685E+01 | 4.64971E+01 |
| 4.20000E-02 | 1.14957E+01 | 4.36079E+01 |
| 4.00000E-02 | 1.11133E+01 | 4.07910E+01 |
| 3.80000E-02 | 1.07210E+01 | 3.80464E+01 |
| 3.60000E-02 | 1.03183E+01 | 3.53735E+01 |
| 3.40000E-02 | 9.90460E-00 | 3.27723E+01 |
| 3.20000E-02 | 9.47934E-00 | 3.02425E+01 |
| 3.00000E-02 | 9.04187E-00 | 2.77840E+01 |
| 2.80000E-02 | 8.59148E-00 | 2.53969E+01 |
| 2.60000E-02 | 8.12734E-00 | 2.30811E+01 |
| 2.40000E-02 | 7.64854E-00 | 2.08369E+01 |
| 2.20000E-02 | 7.15406E-00 | 1.86646E+01 |
| 2.00000E-02 | 6.64269E-00 | 1.65647E+01 |
| 1.80000E-02 | 6.11307E-00 | 1.45379E+01 |
| 1.60000E-02 | 5.56354E-00 | 1.25851E+01 |
| 1.40000E-02 | 4.99216E-00 | 1.07076E+01 |
| 1.20000E-02 | 4.39652E-00 | 8.90726E-00 |
| 1.00000E-02 | 3.77361E-00 | 7.18626E-00 |
| 8.00000E-03 | 3.11944E-00 | 5.54800E-00 |
| 6.00000E-03 | 2.42854E-00 | 3.99733E-00 |
| 4.00000E-03 | 1.69261E-00 | 2.54190E-00 |
| 2.00000E-03 | 8.97063E-01 | 1.19570E-00 |
| 1.00000E-03 | 4.67499E-01 | 5.72798E-01 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

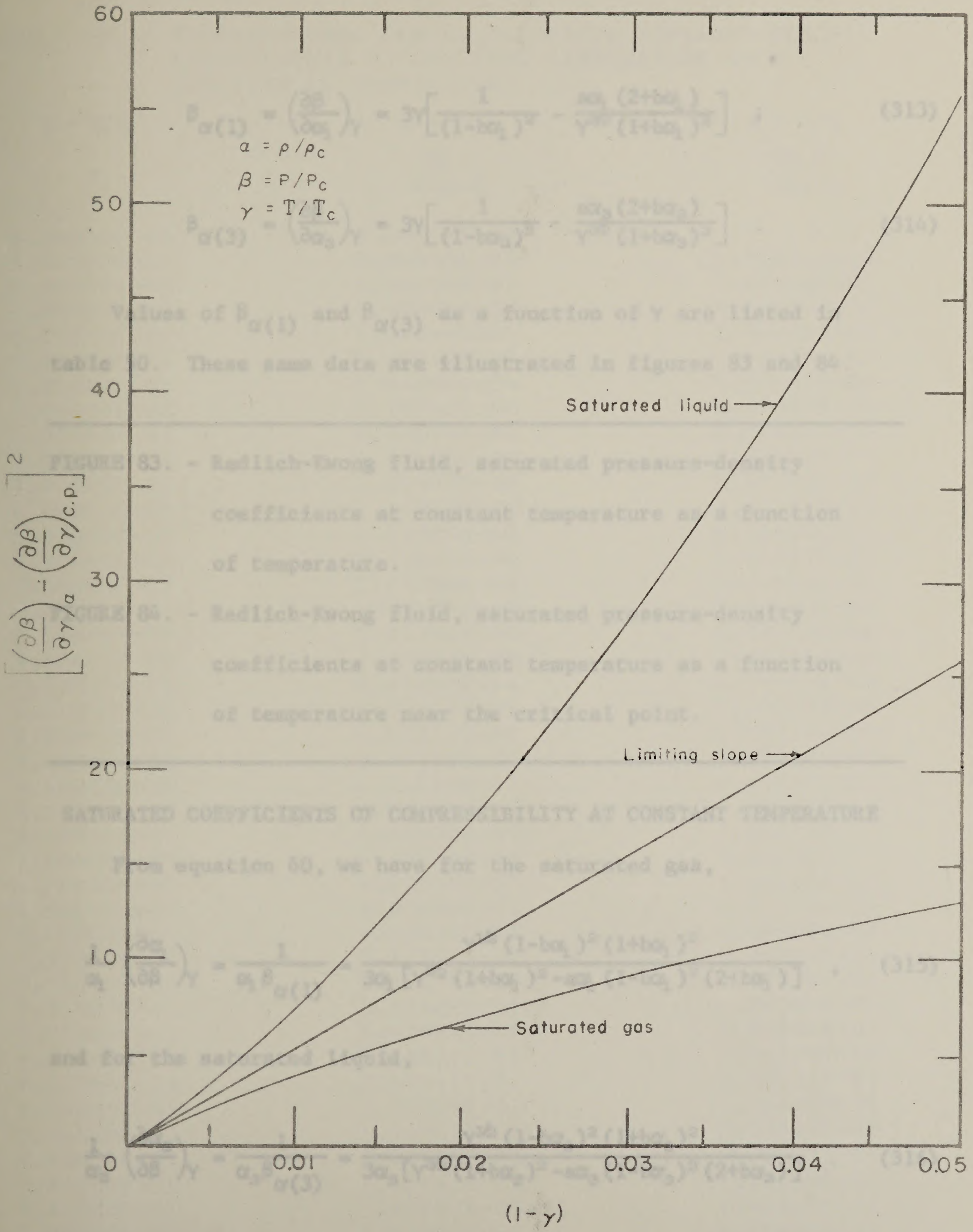

FIGURE 82. - Redlich - Kwong Fluid, Asymptotic Function of the Pressure - Temperature Coefficients at Constant Density, at Saturation, as a Function of Temperature.

$$\beta_{\alpha(1)} = \left(\frac{\partial\beta}{\partial\alpha_1}\right)_\gamma = 3\gamma\left[\frac{1}{(1-b\alpha_1)^2} - \frac{a\alpha_1(2+b\alpha_1)}{\gamma^{3/2}(1+b\alpha_1)^2}\right] ; \tag{313}$$

$$\beta_{\alpha(3)} = \left(\frac{\partial\beta}{\partial\alpha_3}\right)_\gamma = 3\gamma\left[\frac{1}{(1-b\alpha_3)^2} - \frac{a\alpha_3(2+b\alpha_3)}{\gamma^{3/2}(1+b\alpha_3)^2}\right] . \tag{314}$$

Values of $\beta_{\alpha(1)}$ and $\beta_{\alpha(3)}$ as a function of $\gamma$ are listed in table 50. These same data are illustrated in figures 83 and 84.

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FIGURE 83. - Redlich-Kwong fluid, saturated pressure-density coefficients at constant temperature as a function of temperature.

FIGURE 84. - Redlich-Kwong fluid, saturated pressure-density coefficients at constant temperature as a function of temperature near the critical point.

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## SATURATED COEFFICIENTS OF COMPRESSIBILITY AT CONSTANT TEMPERATURE

From equation 60, we have for the saturated gas,

$$\frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma = \frac{1}{\alpha_1\beta_{\alpha(1)}} = \frac{\gamma^{1/2}(1-b\alpha_1)^2(1+b\alpha_1)^2}{3\alpha_1\left[\gamma^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)\right]} , \tag{315}$$

and for the saturated liquid,

$$\frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma = \frac{1}{\alpha_3\beta_{\alpha(3)}} = \frac{\gamma^{1/2}(1-b\alpha_3)^2(1+b\alpha_3)^2}{3\alpha_3\left[\gamma^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)\right]} . \tag{316}$$

TABLE 50. - REDLICH-KWONG FLUID, SATURATED PRESSURE-DENSITY COEFFICIENTS AT CONSTANT TEMPERATURE AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\alpha = \rho/\rho_c$ | $\beta = P/P_c$ |
|---|---|---|
| $\gamma$ | $\left(\frac{\partial\beta}{\partial\alpha_1}\right)_\gamma$ | $\left(\frac{\partial\beta}{\partial\alpha_3}\right)_\gamma$ |
| 0.00000E-99 | 0.00000E-99 | + ∞ |
| 1.00000E-01 | 3.00000E-01 | 1.76721E+03 |
| 1.50000E-01 | 4.50000E-01 | 7.63576E+02 |
| 2.00000E-01 | 5.99999E-01 | 4.14918E+02 |
| 2.50000E-01 | 7.49999E-01 | 2.54919E+02 |
| 3.00000E-01 | 8.99999E-01 | 1.68835E+02 |
| 3.50000E-01 | 1.04996E-00 | 1.17467E+02 |
| 4.00000E-01 | 1.19946E-00 | 8.44995E+01 |
| 4.50000E-01 | 1.34631E-00 | 6.21632E+01 |
| 5.00000E-01 | 1.48472E-00 | 4.63860E+01 |
| 5.50000E-01 | 1.60497E-00 | 3.48702E+01 |
| 6.00000E-01 | 1.69514E-00 | 2.62436E+01 |
| 6.50000E-01 | 1.74329E-00 | 1.96471E+01 |
| 7.00000E-01 | 1.73871E-00 | 1.45217E+01 |
| 7.50000E-01 | 1.67205E-00 | 1.04913E+01 |
| 8.00000E-01 | 1.53474E-00 | 7.29603E-00 |
| 8.50000E-01 | 1.31784E-00 | 4.75327E-00 |
| 9.00000E-01 | 1.00984E-00 | 2.73469E-00 |
| 9.50000E-01 | 5.91165E-01 | 1.15588E-00 |
| 9.52000E-01 | 5.71613E-01 | 1.10119E-00 |
| 9.54000E-01 | 5.51811E-01 | 1.04715E-00 |
| 9.56000E-01 | 5.31752E-01 | 9.93760E-01 |
| 9.58000E-01 | 5.11430E-01 | 9.41014E-01 |
| 9.60000E-01 | 4.90839E-01 | 8.88919E-01 |
| 9.62000E-01 | 4.69972E-01 | 8.37477E-01 |
| 9.64000E-01 | 4.48820E-01 | 7.86692E-01 |
| 9.66000E-01 | 4.27376E-01 | 7.36569E-01 |
| 9.68000E-01 | 4.05630E-01 | 6.87112E-01 |
| 9.70000E-01 | 3.83573E-01 | 6.38328E-01 |
| 9.72000E-01 | 3.61192E-01 | 5.90225E-01 |
| 9.74000E-01 | 3.38475E-01 | 5.42810E-01 |
| 9.76000E-01 | 3.15409E-01 | 4.96094E-01 |
| 9.78000E-01 | 2.91977E-01 | 4.50089E-01 |
| 9.80000E-01 | 2.68161E-01 | 4.04809E-01 |
| 9.82000E-01 | 2.43941E-01 | 3.60272E-01 |
| 9.84000E-01 | 2.19291E-01 | 3.16498E-01 |
| 9.86000E-01 | 1.94183E-01 | 2.73514E-01 |
| 9.88000E-01 | 1.68580E-01 | 2.31351E-01 |
| 9.90000E-01 | 1.42437E-01 | 1.90050E-01 |
| 9.92000E-01 | 1.15697E-01 | 1.49667E-01 |
| 9.94000E-01 | 8.82789E-02 | 1.10279E-01 |
| 9.96000E-01 | 6.00615E-02 | 7.20025E-02 |
| 9.98000E-01 | 3.08347E-02 | 3.50443E-02 |
| 9.99000E-01 | 1.57076E-02 | 1.71938E-02 |
| 1.00000E-00 | 0.00000E-99 | 0.00000E-99 |

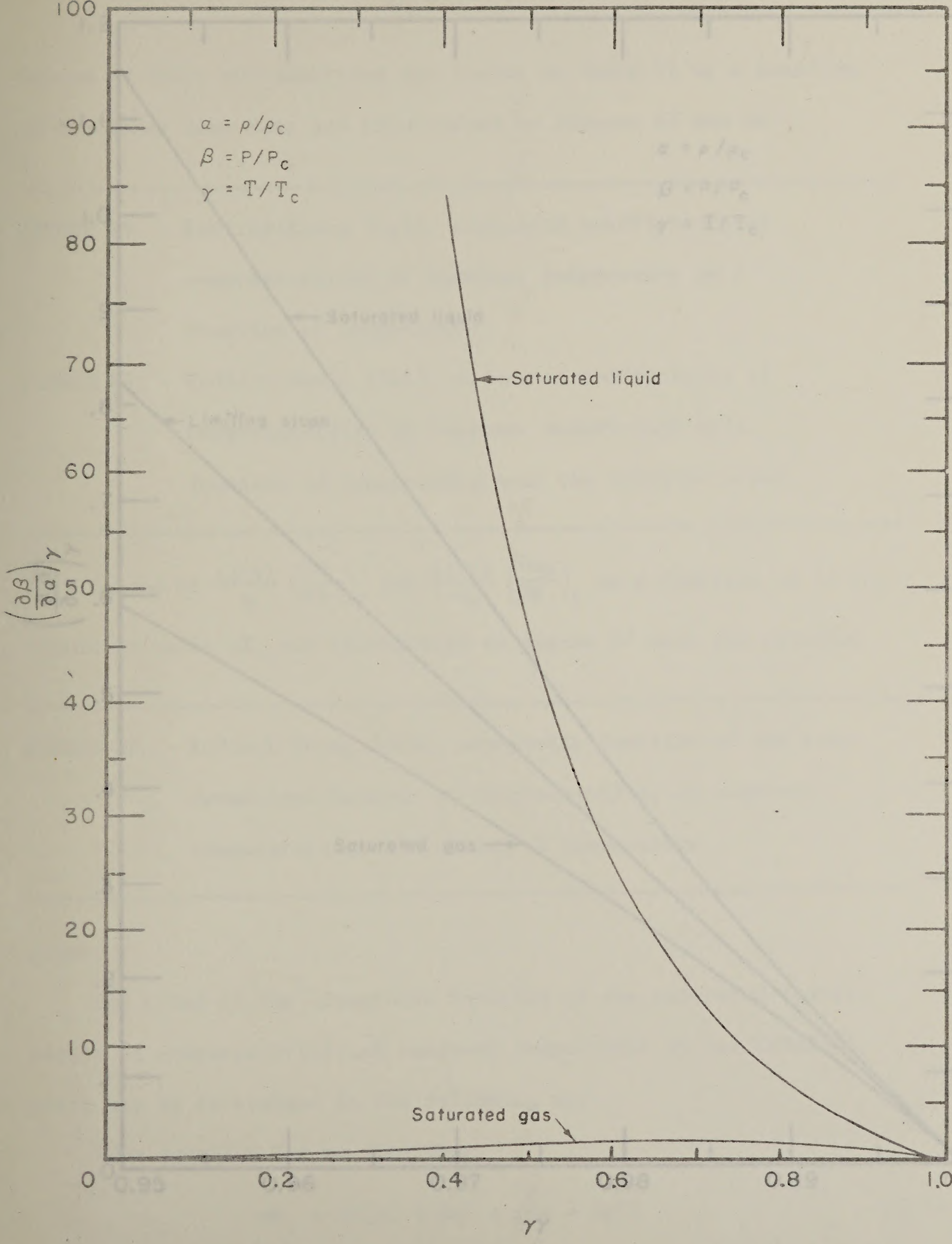

FIGURE 83. - Redlich - Kwong Fluid, Saturated Pressure - Density Coefficients at Constant Temperature as a Function of Temperature.

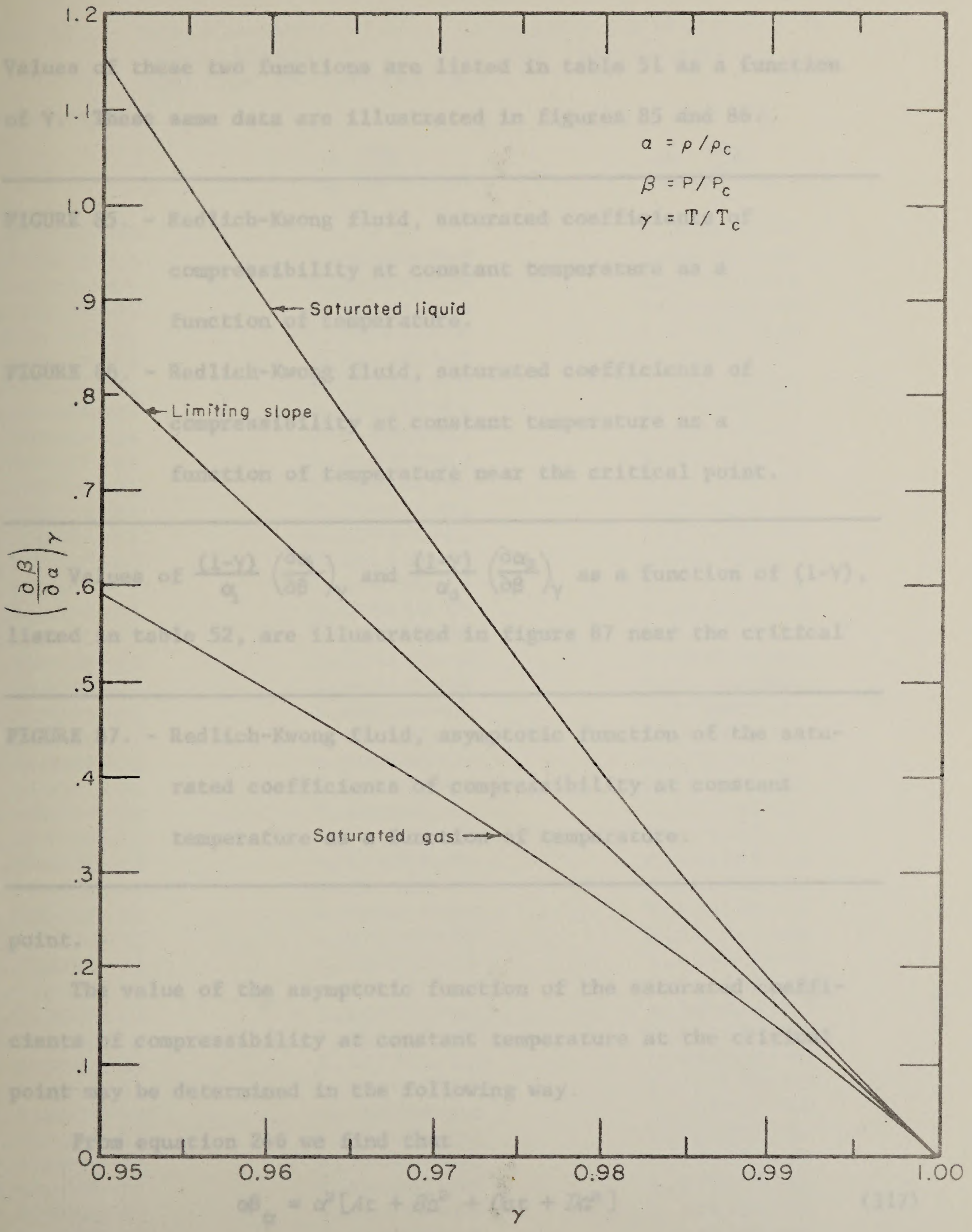

FIGURE 84. - Redlich - Kwong Fluid, Saturated Pressure - Density Coefficients at Constant Temperature as a Function of Temperature Near the Critical Point.

Values of these two functions are listed in table 51 as a function of $\gamma$. These same data are illustrated in figures 85 and 86.

FIGURE 85. - Redlich-Kwong fluid, saturated coefficients of compressibility at constant temperature as a function of temperature.

FIGURE 86. - Redlich-Kwong fluid, saturated coefficients of compressibility at constant temperature as a function of temperature near the critical point.

Values of $\frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma$ and $\frac{(1-\gamma)}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma$ as a function of $(1-\gamma)$, listed in table 52, are illustrated in figure 87 near the critical point.

FIGURE 87. - Redlich-Kwong fluid, asymptotic function of the saturated coefficients of compressibility at constant temperature as a function of temperature.

The value of the asymptotic function of the saturated coefficients of compressibility at constant temperature at the critical point may be determined in the following way.

From equation 246 we find that

$$\alpha\beta_\alpha = \alpha^2[\mathcal{A}t + \mathcal{B}\alpha^2 + \mathcal{C}\alpha t + \mathcal{D}\alpha^3] \tag{317}$$

TABLE 51. - REDLICH-KWONG FLUID, SATURATED COEFFICIENTS OF COMPRESSIBILITY AT CONSTANT TEMPERATURE AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\alpha = \rho/\rho_c$ | $\beta = P/P_c$ |
|---|---|---|
| $\gamma$ | $\frac{1}{\alpha_1}\left(\frac{\partial \alpha_1}{\partial \beta}\right)_\gamma$ | $\frac{1}{\alpha_3}\left(\frac{\partial \alpha_3}{\partial \beta}\right)_\gamma$ |
| 0.00000E-99 | + ∞ | 0.00000E-99 |
| 1.00000E-01 | 1.04268E+45 | 1.49002E-04 |
| 1.50000E-01 | 5.10216E+23 | 3.48711E-04 |
| 2.00000E-01 | 6.51678E+14 | 6.50474E-04 |
| 2.50000E-01 | 1.39263E+10 | 1.07561E-03 |
| 3.00000E-01 | 2.21828E+07 | 1.65373E-03 |
| 3.50000E-01 | 3.30996E+05 | 2.42615E-03 |
| 4.00000E-01 | 1.79418E+04 | 3.45155E-03 |
| 4.50000E-01 | 2.16965E+03 | 4.81510E-03 |
| 5.00000E-01 | 4.45079E+02 | 6.64393E-03 |
| 5.50000E-01 | 1.31408E+02 | 9.13385E-03 |
| 6.00000E-01 | 5.03432E+01 | 1.25981E-02 |
| 6.50000E-01 | 2.33944E+01 | 1.75625E-02 |
| 7.00000E-01 | 1.26153E+01 | 2.49650E-02 |
| 7.50000E-01 | 7.67419E-00 | 3.66211E-02 |
| 8.00000E-01 | 5.18669E-00 | 5.64601E-02 |
| 8.50000E-01 | 3.89095E-00 | 9.44929E-02 |
| 9.00000E-01 | 3.32317E-00 | 1.83989E-01 |
| 9.50000E-01 | 3.64283E-00 | 5.14759E-01 |
| 9.52000E-01 | 3.69573E-00 | 5.45096E-01 |
| 9.54000E-01 | 3.75473E-00 | 5.78434E-01 |
| 9.56000E-01 | 3.82060E-00 | 6.15214E-01 |
| 9.58000E-01 | 3.89422E-00 | 6.55961E-01 |
| 9.60000E-01 | 3.97668E-00 | 7.01311E-01 |
| 9.62000E-01 | 4.06928E-00 | 7.52040E-01 |
| 9.64000E-01 | 4.17359E-00 | 8.09105E-01 |
| 9.66000E-01 | 4.29156E-00 | 8.73695E-01 |
| 9.68000E-01 | 4.42562E-00 | 9.47310E-01 |
| 9.70000E-01 | 4.57880E-00 | 1.03186E-00 |
| 9.72000E-01 | 4.75503E-00 | 1.12984E-00 |
| 9.74000E-01 | 4.95937E-00 | 1.24453E-00 |
| 9.76000E-01 | 5.19855E-00 | 1.38033E-00 |
| 9.78000E-01 | 5.48170E-00 | 1.54332E-00 |
| 9.80000E-01 | 5.82150E-00 | 1.74208E-00 |
| 9.82000E-01 | 6.23616E-00 | 1.98915E-00 |
| 9.84000E-01 | 6.75279E-00 | 2.30351E-00 |
| 9.86000E-01 | 7.41362E-00 | 2.71537E-00 |
| 9.88000E-01 | 8.28848E-00 | 3.27565E-00 |
| 9.90000E-01 | 9.50184E-00 | 4.07723E-00 |
| 9.92000E-01 | 1.13000E+01 | 5.30847E-00 |
| 9.94000E-01 | 1.42509E+01 | 7.41579E-00 |
| 9.96000E-01 | 2.00336E+01 | 1.17628E+01 |
| 9.98000E-01 | 3.68846E+01 | 2.53276E+01 |
| 9.99000E-01 | 6.96549E+01 | 5.34085E+01 |
| 1.00000E-00 | + ∞ | + ∞ |

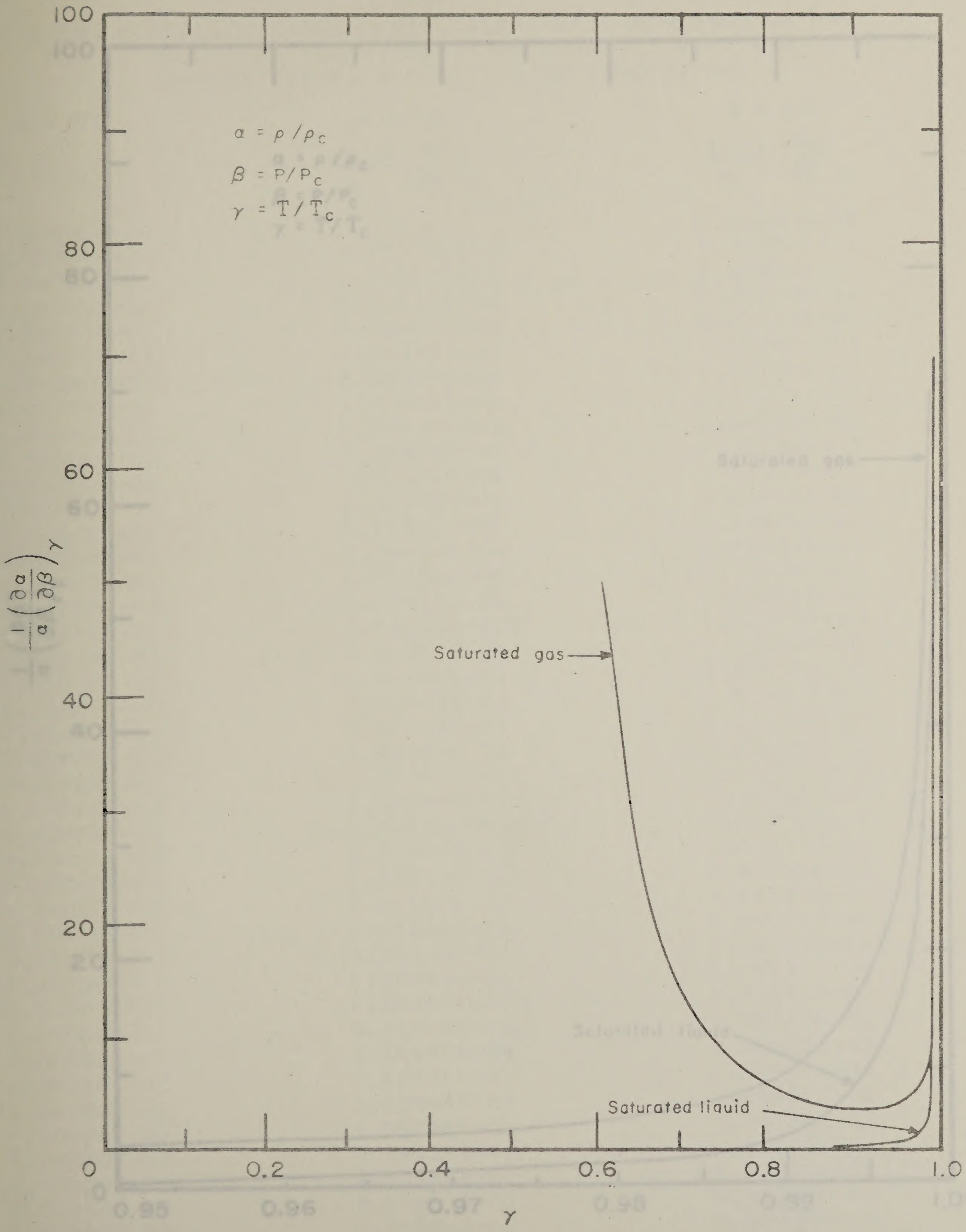

FIGURE 85.—Redlich–Kwong Fluid, Saturated Coefficients of Compressibility at Constant Temperature as a Function of Temperature.

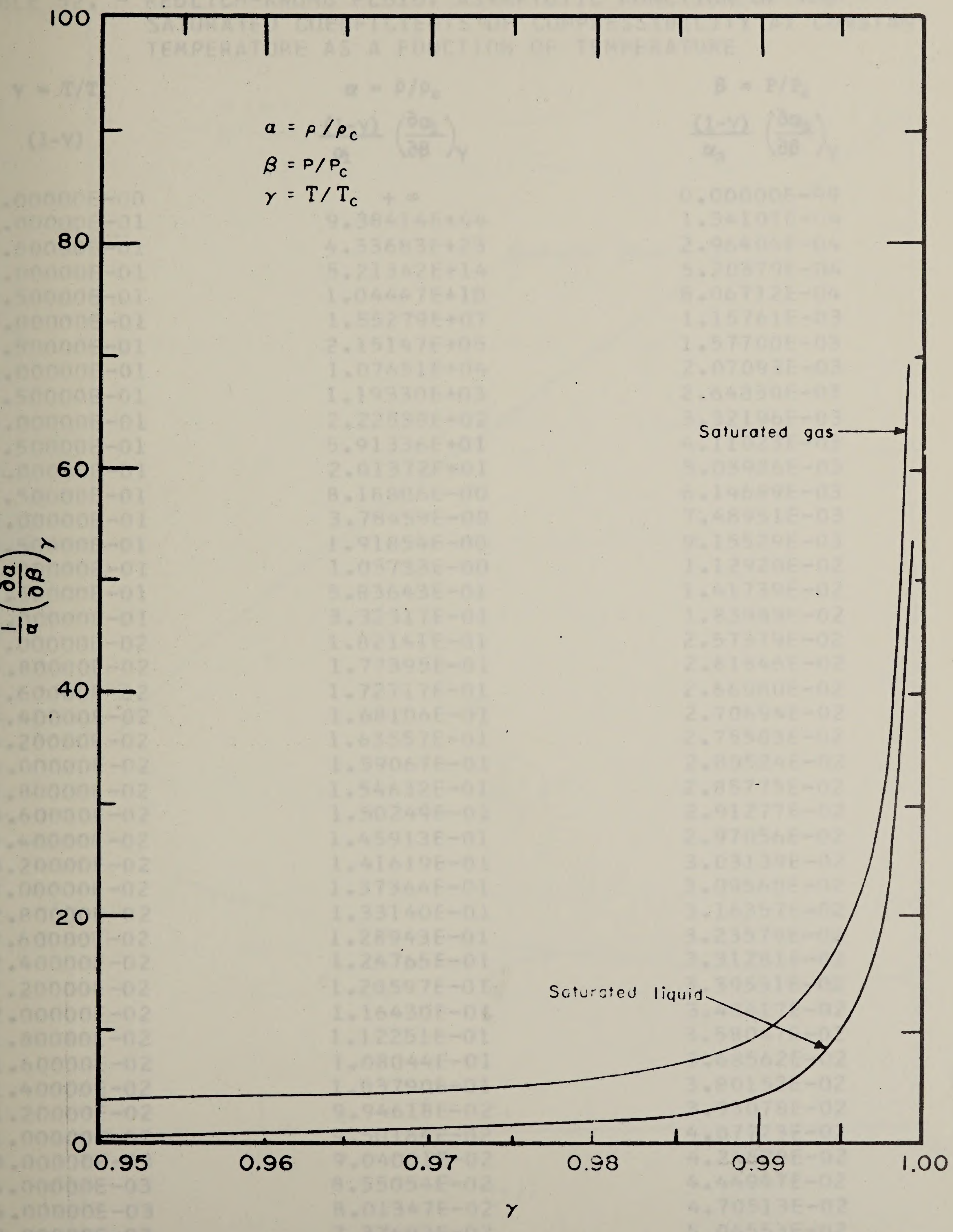

FIGURE 86.— Redlich-Kwong Fluid, Saturated Coefficients of Compressibility at Constant Temperature as a Function of Temperature Near the Critical Point.

TABLE 52. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED COEFFICIENTS OF COMPRESSIBILITY AT CONSTANT TEMPERATURE AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\alpha = \rho/\rho_c$ | $\beta = P/P_c$ |
|---|---|---|
| $(1-\gamma)$ | $\frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma$ | $\frac{(1-\gamma)}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma$ |
| 1.00000E-00 | $+\infty$ | 0.00000E-99 |
| 9.00000E-01 | 9.38414E+44 | 1.34101E-04 |
| 8.50000E-01 | 4.33683E+23 | 2.96404E-04 |
| 8.00000E-01 | 5.21342E+14 | 5.20379E-04 |
| 7.50000E-01 | 1.04447E+10 | 8.06712E-04 |
| 7.00000E-01 | 1.55279E+07 | 1.15761E-03 |
| 6.50000E-01 | 2.15147E+05 | 1.57700E-03 |
| 6.00000E-01 | 1.07651E+04 | 2.07093E-03 |
| 5.50000E-01 | 1.19330E+03 | 2.64830E-03 |
| 5.00000E-01 | 2.22539E+02 | 3.32196E-03 |
| 4.50000E-01 | 5.91336E+01 | 4.11023E-03 |
| 4.00000E-01 | 2.01372E+01 | 5.03926E-03 |
| 3.50000E-01 | 8.18806E-00 | 6.14689E-03 |
| 3.00000E-01 | 3.78459E-00 | 7.48951E-03 |
| 2.50000E-01 | 1.91854E-00 | 9.15529E-03 |
| 2.00000E-01 | 1.03733E-00 | 1.12920E-02 |
| 1.50000E-01 | 5.83643E-01 | 1.41739E-02 |
| 1.00000E-01 | 3.32317E-01 | 1.83989E-02 |
| 5.00000E-02 | 1.82141E-01 | 2.57379E-02 |
| 4.80000E-02 | 1.77395E-01 | 2.61646E-02 |
| 4.60000E-02 | 1.72717E-01 | 2.66080E-02 |
| 4.40000E-02 | 1.68106E-01 | 2.70694E-02 |
| 4.20000E-02 | 1.63557E-01 | 2.75503E-02 |
| 4.00000E-02 | 1.59067E-01 | 2.80524E-02 |
| 3.80000E-02 | 1.54632E-01 | 2.85775E-02 |
| 3.60000E-02 | 1.50249E-01 | 2.91277E-02 |
| 3.40000E-02 | 1.45913E-01 | 2.97056E-02 |
| 3.20000E-02 | 1.41619E-01 | 3.03139E-02 |
| 3.00000E-02 | 1.37364E-01 | 3.09560E-02 |
| 2.80000E-02 | 1.33140E-01 | 3.16357E-02 |
| 2.60000E-02 | 1.28943E-01 | 3.23579E-02 |
| 2.40000E-02 | 1.24765E-01 | 3.31281E-02 |
| 2.20000E-02 | 1.20597E-01 | 3.39531E-02 |
| 2.00000E-02 | 1.16430E-01 | 3.48417E-02 |
| 1.80000E-02 | 1.12251E-01 | 3.58047E-02 |
| 1.60000E-02 | 1.08044E-01 | 3.68562E-02 |
| 1.40000E-02 | 1.03790E-01 | 3.80152E-02 |
| 1.20000E-02 | 9.94618E-02 | 3.93078E-02 |
| 1.00000E-02 | 9.50184E-02 | 4.07723E-02 |
| 8.00000E-03 | 9.04001E-02 | 4.24678E-02 |
| 6.00000E-03 | 8.55054E-02 | 4.44947E-02 |
| 4.00000E-03 | 8.01347E-02 | 4.70513E-02 |
| 2.00000E-03 | 7.37693E-02 | 5.06553E-02 |
| 1.00000E-03 | 6.96549E-02 | 5.34085E-02 |
| 0.00000E-99 | 6.08574E-02 | 6.08574E-02 |

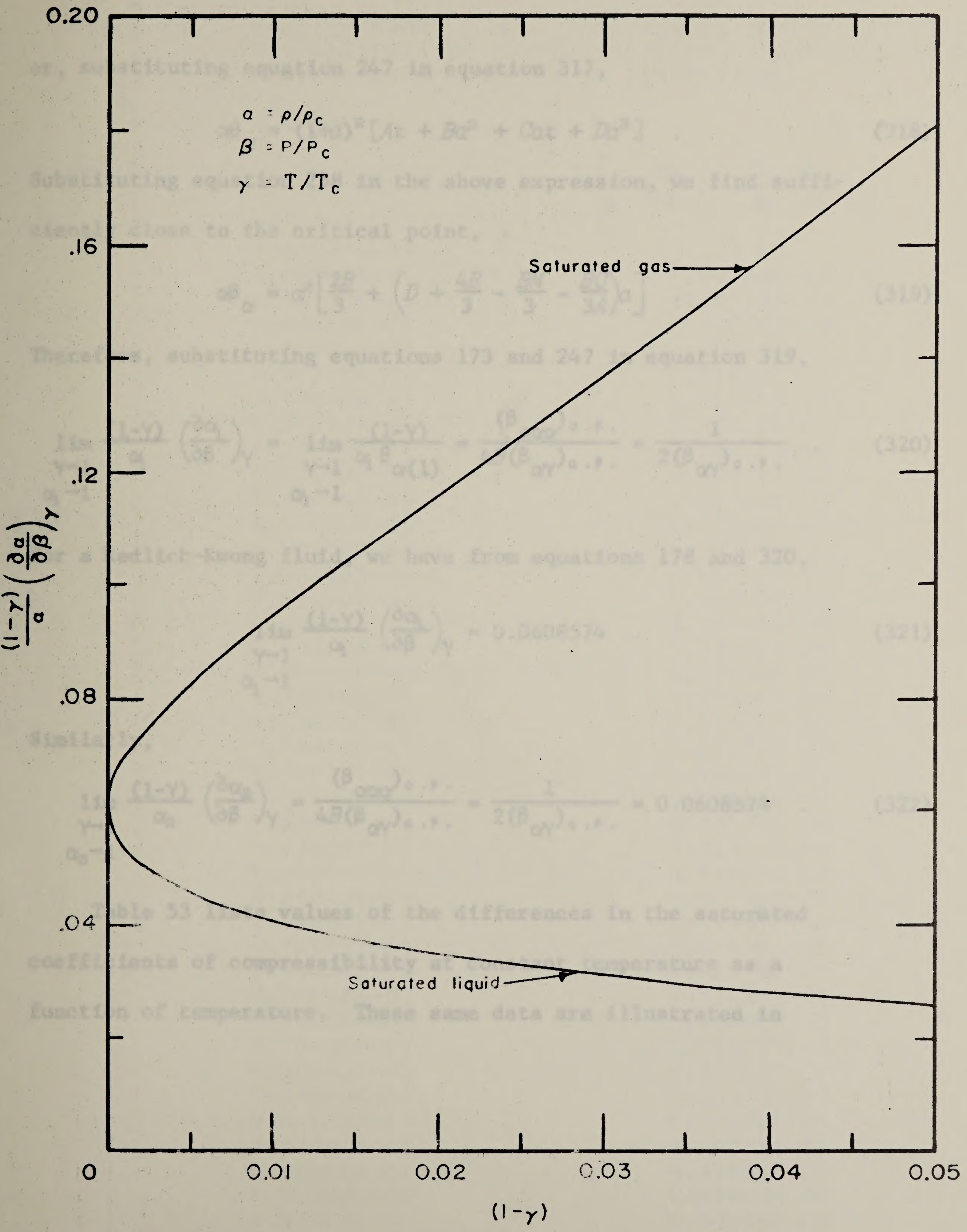

FIGURE 87.— Redlich-Kwong Fluid, Asymptotic Function of the Saturated Coefficients of Compressibility at Constant Temperature as a Function of Temperature.

or, substituting equation 247 in equation 317,

$$\alpha\beta_{\alpha} = (1+a)^2[At + Ba^2 + Cat + Da^3] \quad . \tag{318}$$

Substituting equation 258 in the above expression, we find sufficiently close to the critical point,

$$\alpha\beta_{\alpha} = a^2\left[\frac{2B}{3} + \left(D + \frac{4B}{3} - \frac{BM}{3} - \frac{BC}{3A}\right)a\right] \quad . \tag{319}$$

Therefore, substituting equations 173 and 247 in equation 319,

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_{\gamma} = \lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \frac{(1-\gamma)}{\alpha_1\beta_{\alpha(1)}} = \frac{(\beta_{\alpha\alpha\alpha})_{c.p.}}{4B(\beta_{\alpha\gamma})_{c.p.}} = \frac{1}{2(\beta_{\alpha\gamma})_{c.p.}} \quad . \tag{320}$$

For a Redlich-Kwong fluid, we have from equations 178 and 320,

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}} \frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_{\gamma} = 0.0608574 \quad . \tag{321}$$

Similarly,

$$\lim_{\substack{\gamma\to 1\\ \alpha_3\to 1}} \frac{(1-\gamma)}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_{\gamma} = \frac{(\beta_{\alpha\alpha\alpha})_{c.p.}}{4B(\beta_{\alpha\gamma})_{c.p.}} = \frac{1}{2(\beta_{\alpha\gamma})_{c.p.}} = 0.0608574 \quad . \tag{322}$$

Table 53 lists values of the differences in the saturated coefficients of compressibility at constant temperature as a function of temperature. These same data are illustrated in

TABLE 53. - REDLICH-KWONG FLUID, DIFFERENCES IN THE SATURATED COEFFICIENTS OF COMPRESSIBILITY AT CONSTANT TEMPERATURE AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\qquad \alpha = \rho/\rho_c$ $\qquad \beta = P/P_c$

| $\gamma$ | $\left[\frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma - \frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma\right]$ |
|---|---|
| 0.00000E-99 | + ∞ |
| 1.00000E-01 | 1.04268E+45 |
| 1.50000E-01 | 5.10216E+23 |
| 2.00000E-01 | 6.51678E+14 |
| 2.50000E-01 | 1.39263E+10 |
| 3.00000E-01 | 2.21828E+07 |
| 3.50000E-01 | 3.30996E+05 |
| 4.00000E-01 | 1.79418E+04 |
| 4.50000E-01 | 2.16965E+03 |
| 5.00000E-01 | 4.45072E+02 |
| 5.50000E-01 | 1.31398E+02 |
| 6.00000E-01 | 5.03306E+01 |
| 6.50000E-01 | 2.33769E+01 |
| 7.00000E-01 | 1.25903E+01 |
| 7.50000E-01 | 7.63757E-00 |
| 8.00000E-01 | 5.13023E-00 |
| 8.50000E-01 | 3.79646E-00 |
| 9.00000E-01 | 3.13918E-00 |
| 9.50000E-01 | 3.12807E-00 |
| 9.52000E-01 | 3.15063E-00 |
| 9.54000E-01 | 3.17630E-00 |
| 9.56000E-01 | 3.20538E-00 |
| 9.58000E-01 | 3.23826E-00 |
| 9.60000E-01 | 3.27537E-00 |
| 9.62000E-01 | 3.31724E-00 |
| 9.64000E-01 | 3.36448E-00 |
| 9.66000E-01 | 3.41786E-00 |
| 9.68000E-01 | 3.47830E-00 |
| 9.70000E-01 | 3.54694E-00 |
| 9.72000E-01 | 3.62518E-00 |
| 9.74000E-01 | 3.71483E-00 |
| 9.76000E-01 | 3.81821E-00 |
| 9.78000E-01 | 3.93837E-00 |
| 9.80000E-01 | 4.07941E-00 |
| 9.82000E-01 | 4.24701E-00 |
| 9.84000E-01 | 4.44927E-00 |
| 9.86000E-01 | 4.69825E-00 |
| 9.88000E-01 | 5.01282E-00 |
| 9.90000E-01 | 5.42460E-00 |
| 9.92000E-01 | 5.99154E-00 |
| 9.94000E-01 | 6.83511E-00 |
| 9.96000E-01 | 8.27084E-00 |
| 9.98000E-01 | 1.15569E+01 |
| 9.99000E-01 | 1.62464E+01 |
| 1.00000E-00 | + ∞ |

figures 88 and 89.

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FIGURE 88. - Redlich-Kwong fluid, differences in the saturated coefficients of compressibility at constant temperature as a function of temperature.

FIGURE 89. - Redlich-Kwong fluid, differences in the saturated coefficients of compressibility at constant temperature as a function of temperature near the critical point.

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The limiting value of the differences in the saturated coefficients of compressibility at constant temperature as the critical point is approached may be determined in the following way.

From equation 319, we find

$$\alpha_1 \beta_{\alpha(1)} = \frac{2B}{3} \alpha_1^2 \left[ 1 + \left( \frac{3D}{2B} + 2 - \frac{M}{2} - \frac{C}{2A} \right) \alpha_1 \right] \; ; \tag{323}$$

$$\alpha_3 \beta_{\alpha(3)} = \frac{2B}{3} \alpha_3^2 \left[ 1 + \left( \frac{3D}{2B} + 2 - \frac{M}{2} - \frac{C}{2A} \right) \alpha_3 \right] \; . \tag{324}$$

Substituting equation 272 in equation 324, ignoring terms of the order of $\alpha_1^4$ and higher,

$$\alpha_3 \beta_{\alpha(3)} = \frac{2B}{3} \alpha_1^2 \left[ 1 + \left( \frac{5M}{2} + \frac{C}{2A} - 2 - \frac{3D}{2B} \right) \alpha_1 \right] \; . \tag{325}$$

Subtracting equations 323 and 325,

$$\alpha_3 \beta_{\alpha(3)} - \alpha_1 \beta_{\alpha(1)} = \frac{2B}{3} \alpha_1^3 \left[ 3M + \frac{C}{A} - 4 - \frac{3D}{B} \right] \; . \tag{326}$$

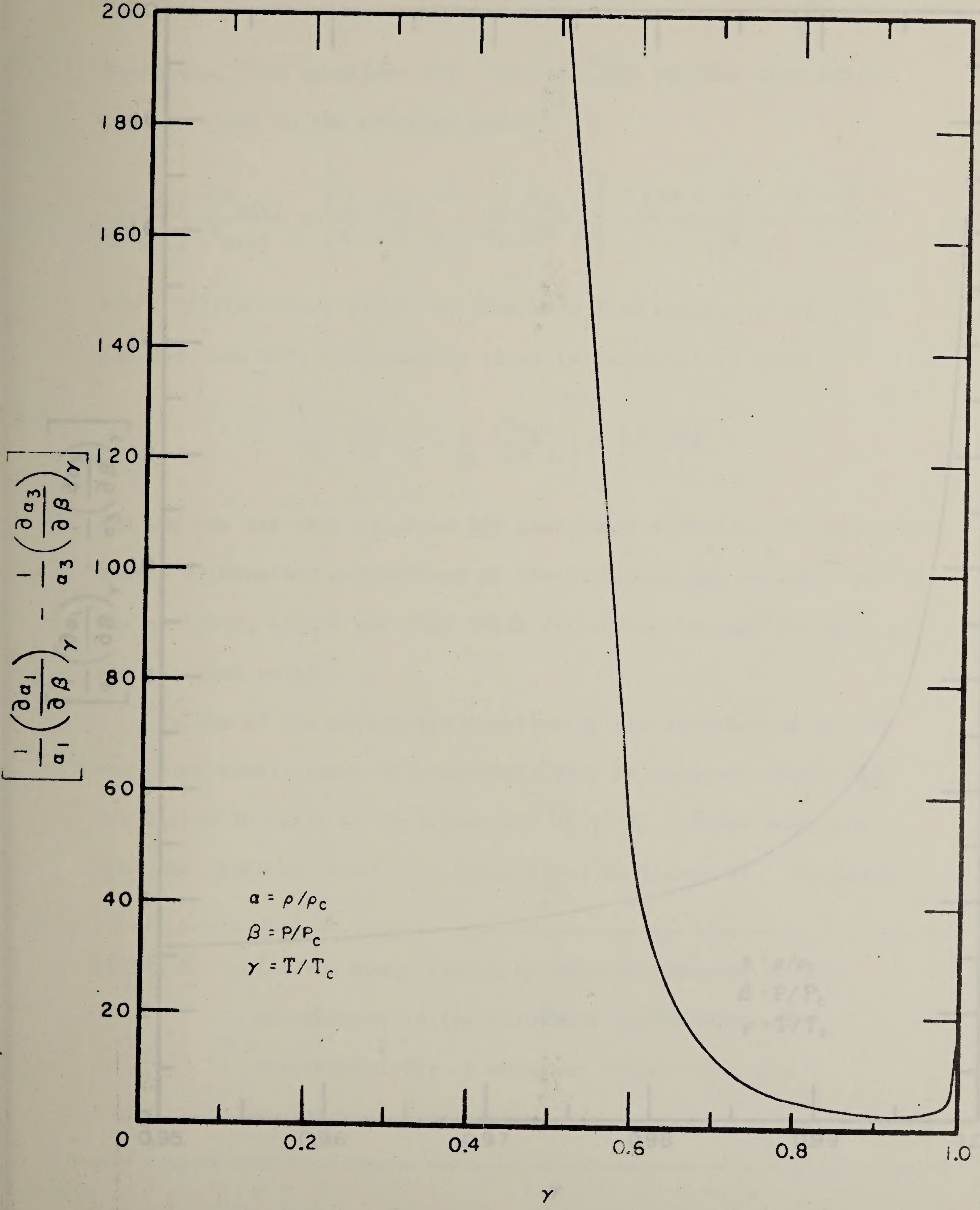

FIGURE 88.-Redlich-Kwong Fluid, Differences in the Saturated Coefficients of Compressibility at Constant Temperature as a Function of Temperature.

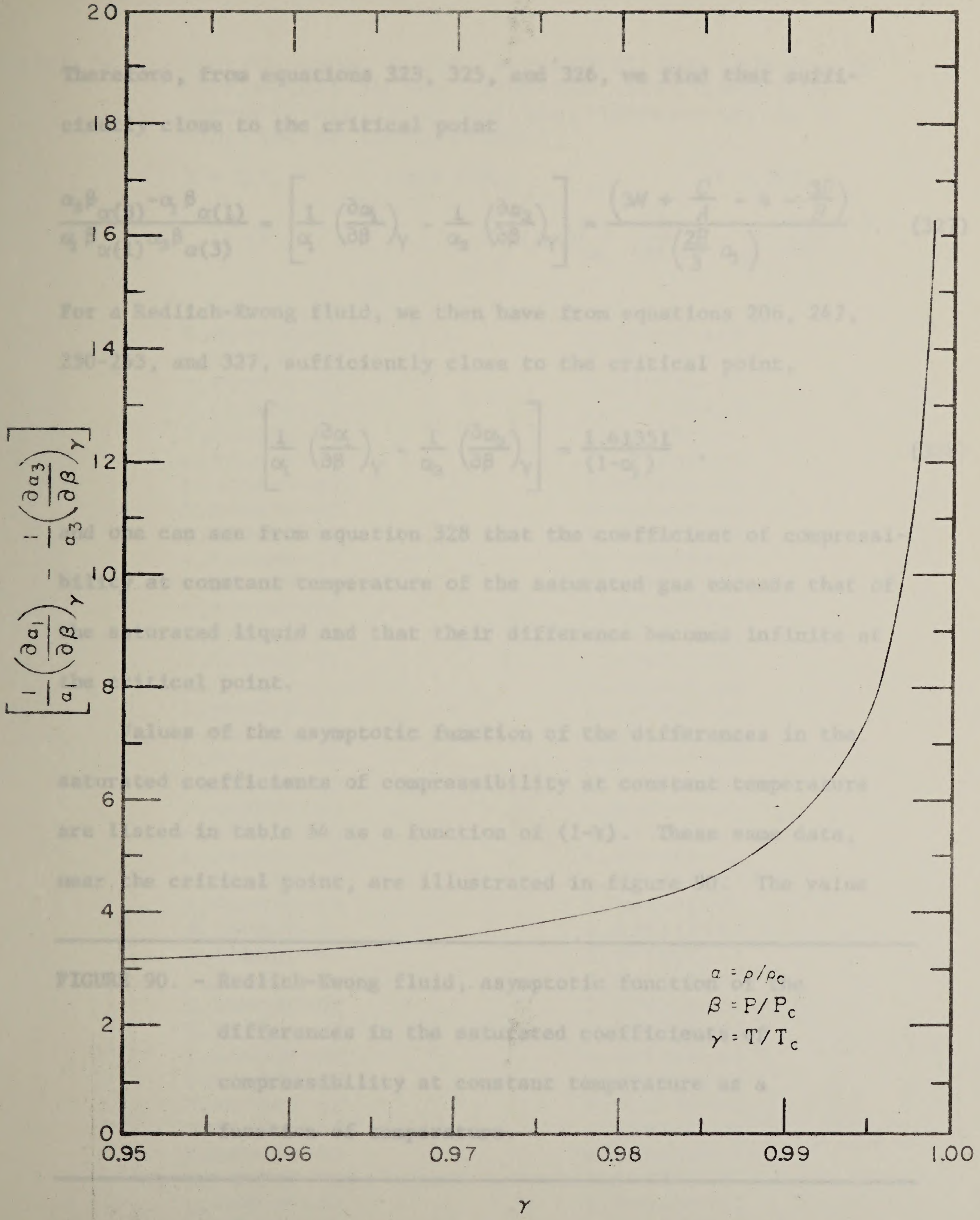

FIGURE 89.- Redlich-Kwong Fluid, Differences in the Saturated Coefficients of Compressibility at Constant Temperature as a Function of Temperature Near the Critical Point.

**Therefore, from equations 323, 325, and 326, we find that sufficiently close to the critical point**

$$\frac{\alpha_3\beta_{\alpha(3)} - \alpha_1\beta_{\alpha(1)}}{\alpha_1\beta_{\alpha(1)}\alpha_3\beta_{\alpha(3)}} = \left[\frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_Y - \frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_Y\right] = \frac{\left(3M + \frac{C}{A} - 4 - \frac{3D}{B}\right)}{\left(\frac{2B}{3}\alpha_1\right)} . \qquad (327)$$

For a Redlich-Kwong fluid, we then have from equations 206, 247, 250-253, and 327, sufficiently close to the critical point,

$$\left[\frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_Y - \frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_Y\right] = \frac{1.41351}{(1-\alpha_1)} , \qquad (328)$$

and one can see from equation 328 that the coefficient of compressibility at constant temperature of the saturated gas exceeds that of the saturated liquid and that their difference becomes infinite at the critical point.

Values of the asymptotic function of the differences in the saturated coefficients of compressibility at constant temperature are listed in table 54 as a function of (1-Y). These same data, near the critical point, are illustrated in figure 90. The value

FIGURE 90. - Redlich-Kwong fluid, asymptotic function of the differences in the saturated coefficients of compressibility at constant temperature as a function of temperature.

TABLE 54. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE DIFFERENCES IN THE SATURATED COEFFICIENTS OF COMPRESSIBILITY AT CONSTANT TEMPERATURE AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\alpha = \rho/\rho_c$ $\beta = P/P_c$

| $(1-\gamma)$ | $(1-\gamma)\left[\frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma - \frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma\right]^3$ |
|---|---|
| 1.00000E-00 | + ∞ |
| 9.00000E-01 | 9.78468E+89 |
| 8.50000E-01 | 2.21272E+47 |
| 8.00000E-01 | 3.39748E+29 |
| 7.50000E-01 | 1.45457E+20 |
| 7.00000E-01 | 3.44455E+14 |
| 6.50000E-01 | 7.12130E+10 |
| 6.00000E-01 | 1.93146E+08 |
| 5.50000E-01 | 2.58905E+06 |
| 5.00000E-01 | 9.90449E+04 |
| 4.50000E-01 | 7.76956E+03 |
| 4.00000E-01 | 1.01326E+03 |
| 3.50000E-01 | 1.91268E+02 |
| 3.00000E-01 | 4.75551E+01 |
| 2.50000E-01 | 1.45831E+01 |
| 2.00000E-01 | 5.26386E-00 |
| 1.50000E-01 | 2.16196E-00 |
| 1.00000E-01 | 9.85449E-01 |
| 5.00000E-02 | 4.89242E-01 |
| 4.80000E-02 | 4.76473E-01 |
| 4.60000E-02 | 4.64089E-01 |
| 4.40000E-02 | 4.52078E-01 |
| 4.20000E-02 | 4.40426E-01 |
| 4.00000E-02 | 4.29123E-01 |
| 3.80000E-02 | 4.18155E-01 |
| 3.60000E-02 | 4.07511E-01 |
| 3.40000E-02 | 3.97182E-01 |
| 3.20000E-02 | 3.87156E-01 |
| 3.00000E-02 | 3.77423E-01 |
| 2.80000E-02 | 3.67974E-01 |
| 2.60000E-02 | 3.58799E-01 |
| 2.40000E-02 | 3.49890E-01 |
| 2.20000E-02 | 3.41237E-01 |
| 2.00000E-02 | 3.32833E-01 |
| 1.80000E-02 | 3.24668E-01 |
| 1.60000E-02 | 3.16737E-01 |
| 1.40000E-02 | 3.09030E-01 |
| 1.20000E-02 | 3.01541E-01 |
| 1.00000E-02 | 2.94263E-01 |
| 8.00000E-03 | 2.87188E-01 |
| 6.00000E-03 | 2.80312E-01 |
| 4.00000E-03 | 2.73627E-01 |
| 2.00000E-03 | 2.67128E-01 |
| 1.00000E-03 | 2.63946E-01 |
| 0.00000E-99 | 2.60808E-01 |

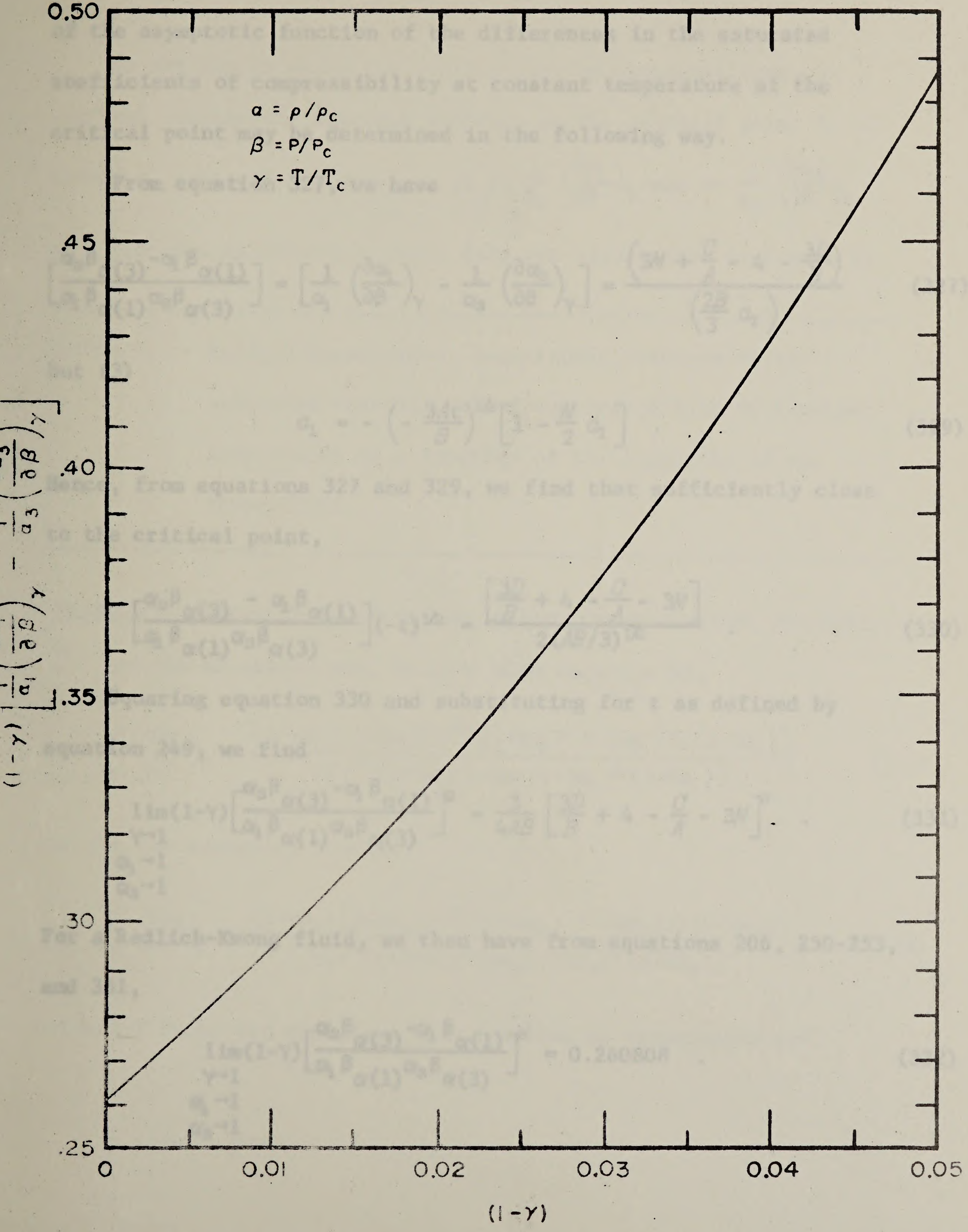

FIGURE 90.— Redlich-Kwong Fluid, Asymptotic Function of the Differences in the Saturated Coefficients of Compressibility at Constant Temperature as a Function of Temperature.

of the asymptotic function of the differences in the saturated coefficients of compressibility at constant temperature at the critical point may be determined in the following way.

From equation 327, we have

$$\left[\frac{\alpha_3\beta_{\alpha(3)} - \alpha_1\beta_{\alpha(1)}}{\alpha_1\beta_{\alpha(1)}\alpha_3\beta_{\alpha(3)}}\right] = \left[\frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma - \frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma\right] = \frac{\left(3M + \frac{C}{A} - 4 - \frac{3D}{B}\right)}{\left(\frac{2B}{3}\,a_1\right)} \quad . \tag{327}$$

But (3)

$$a_1 = -\left(-\frac{3At}{B}\right)^{1/2}\left[1 - \frac{M}{2}\,a_1\right] \quad . \tag{329}$$

Hence, from equations 327 and 329, we find that sufficiently close to the critical point,

$$\left[\frac{\alpha_3\beta_{\alpha(3)} - \alpha_1\beta_{\alpha(1)}}{\alpha_1\beta_{\alpha(1)}\alpha_3\beta_{\alpha(3)}}\right](-t)^{1/2} = \frac{\left[\frac{3D}{B} + 4 - \frac{C}{A} - 3M\right]}{2(AB/3)^{1/2}} \quad . \tag{330}$$

Squaring equation 330 and substituting for t as defined by equation 249, we find

$$\lim_{\substack{\gamma\to1\\ \alpha_1\to1\\ \alpha_3\to1}}(1-\gamma)\left[\frac{\alpha_3\beta_{\alpha(3)} - \alpha_1\beta_{\alpha(1)}}{\alpha_1\beta_{\alpha(1)}\alpha_3\beta_{\alpha(3)}}\right]^2 = \frac{3}{4AB}\left[\frac{3D}{B} + 4 - \frac{C}{A} - 3M\right]^2 . \quad . \tag{331}$$

For a Redlich-Kwong fluid, we then have from equations 206, 250-253, and 331,

$$\lim_{\substack{\gamma\to1\\ \alpha_1\to1\\ \alpha_3\to1}}(1-\gamma)\left[\frac{\alpha_3\beta_{\alpha(3)} - \alpha_1\beta_{\alpha(1)}}{\alpha_1\beta_{\alpha(1)}\alpha_3\beta_{\alpha(3)}}\right]^2 = 0.260808 \quad . \tag{332}$$

As the way in which the coefficients of compressibility at constant temperature approach infinity at the critical point is of interest, table 55 lists values of $\ln \frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma$ and $\ln \frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma$ as a function of $\ln(1-\gamma)$. These data are illustrated in figure 91.

---

FIGURE 91. - Redlich-Kwong fluid, logarithmic function of the saturated coefficients of compressibility at constant temperature as a function of the logarithm of the temperature.

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SATURATED COEFFICIENTS OF THERMAL EXPANSION AT CONSTANT PRESSURE

For the saturated gas, we have from equation 63,

$$-\frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\gamma}\right)_\beta = \frac{(1-b\alpha_1)(1+b\alpha_1)[\gamma^{3/2}(1+b\alpha_1) + (a\alpha_1/2)(1-b\alpha_1)]}{\gamma[\gamma^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)]}, \qquad (333)$$

and for the saturated liquid,

$$-\frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\gamma}\right)_\beta = \frac{(1-b\alpha_3)(1+b\alpha_3)[\gamma^{3/2}(1+b\alpha_3) + (a\alpha_3/2)(1-b\alpha_3)]}{\gamma[\gamma^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)]}. \qquad (334)$$

Values of these two functions as a function of temperature are

TABLE 55. - REDLICH-KWONG FLUID, LOGARITHMIC FUNCTION OF THE SATURATED COEFFICIENTS OF COMPRESSIBILITY AT CONSTANT TEMPERATURE AS A FUNCTION OF THE LOGARITHM OF THE TEMPERATURE

| $\gamma = T/T_c$<br>$\ln(1-\gamma)$ | $\alpha = \rho/\rho_c$<br>$\ln \frac{1}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\beta}\right)_\gamma$ | $\beta = P/P_c$<br>$\ln \frac{1}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\beta}\right)_\gamma$ |
|---|---|---|
| 0.00000E-99 | + ∞ | - ∞ |
| -1.05360E-01 | 1.03658E+02 | -8.81155E-00 |
| -1.62518E-01 | 5.45891E+01 | -7.96126E-00 |
| -2.23143E-01 | 3.41105E+01 | -7.33780E-00 |
| -2.87682E-01 | 2.33570E+01 | -6.83486E-00 |
| -3.56674E-01 | 1.69148E+01 | -6.40472E-00 |
| -4.30782E-01 | 1.27098E+01 | -6.02144E-00 |
| -5.10825E-01 | 9.79489E-00 | -5.66892E-00 |
| -5.97837E-01 | 7.68232E-00 | -5.33599E-00 |
| -6.93147E-01 | 6.09825E-00 | -5.01405E-00 |
| -7.98507E-01 | 4.87830E-00 | -4.69576E-00 |
| -9.16290E-01 | 3.91886E-00 | -4.37420E-00 |
| -1.04982E-00 | 3.15250E-00 | -4.04198E-00 |
| -1.20397E-00 | 2.53491E-00 | -3.69027E-00 |
| -1.38629E-00 | 2.03786E-00 | -3.30712E-00 |
| -1.60943E-00 | 1.64609E-00 | -2.87421E-00 |
| -1.89711E-00 | 1.35865E-00 | -2.35923E-00 |
| -2.30258E-00 | 1.20092E-00 | -1.69287E-00 |
| -2.99573E-00 | 1.29276E-00 | -6.64055E-01 |
| -3.03655E-00 | 1.30717E-00 | -6.06793E-01 |
| -3.07911E-00 | 1.32301E-00 | -5.47429E-01 |
| -3.12356E-00 | 1.34040E-00 | -4.85784E-01 |
| -3.17008E-00 | 1.35949E-00 | -4.21653E-01 |
| -3.21887E-00 | 1.38044E-00 | -3.54803E-01 |
| -3.27016E-00 | 1.40346E-00 | -2.84965E-01 |
| -3.32423E-00 | 1.42877E-00 | -2.11826E-01 |
| -3.38139E-00 | 1.45665E-00 | -1.35023E-01 |
| -3.44201E-00 | 1.48741E-00 | -5.41282E-02 |
| -3.50655E-00 | 1.52143E-00 | 3.13701E-02 |
| -3.57555E-00 | 1.55920E-00 | 1.22084E-01 |
| -3.64965E-00 | 1.60127E-00 | 2.18762E-01 |
| -3.72970E-00 | 1.64838E-00 | 3.22328E-01 |
| -3.81671E-00 | 1.70141E-00 | 4.33940E-01 |
| -3.91202E-00 | 1.76155E-00 | 5.55084E-01 |
| -4.01738E-00 | 1.83036E-00 | 6.87708E-01 |
| -4.13516E-00 | 1.90995E-00 | 8.34436E-01 |
| -4.26869E-00 | 2.00331E-00 | 9.98929E-01 |
| -4.42284E-00 | 2.11486E-00 | 1.18651E-00 |
| -4.60517E-00 | 2.25148E-00 | 1.40541E-00 |
| -4.82831E-00 | 2.42480E-00 | 1.66930E-00 |
| -5.11599E-00 | 2.65682E-00 | 2.00361E-00 |
| -5.52146E-00 | 2.99741E-00 | 2.46494E-00 |
| -6.21460E-00 | 3.60779E-00 | 3.23189E-00 |
| -6.90775E-00 | 4.24355E-00 | 3.97796E-00 |
| - ∞ | + ∞ | + ∞ |

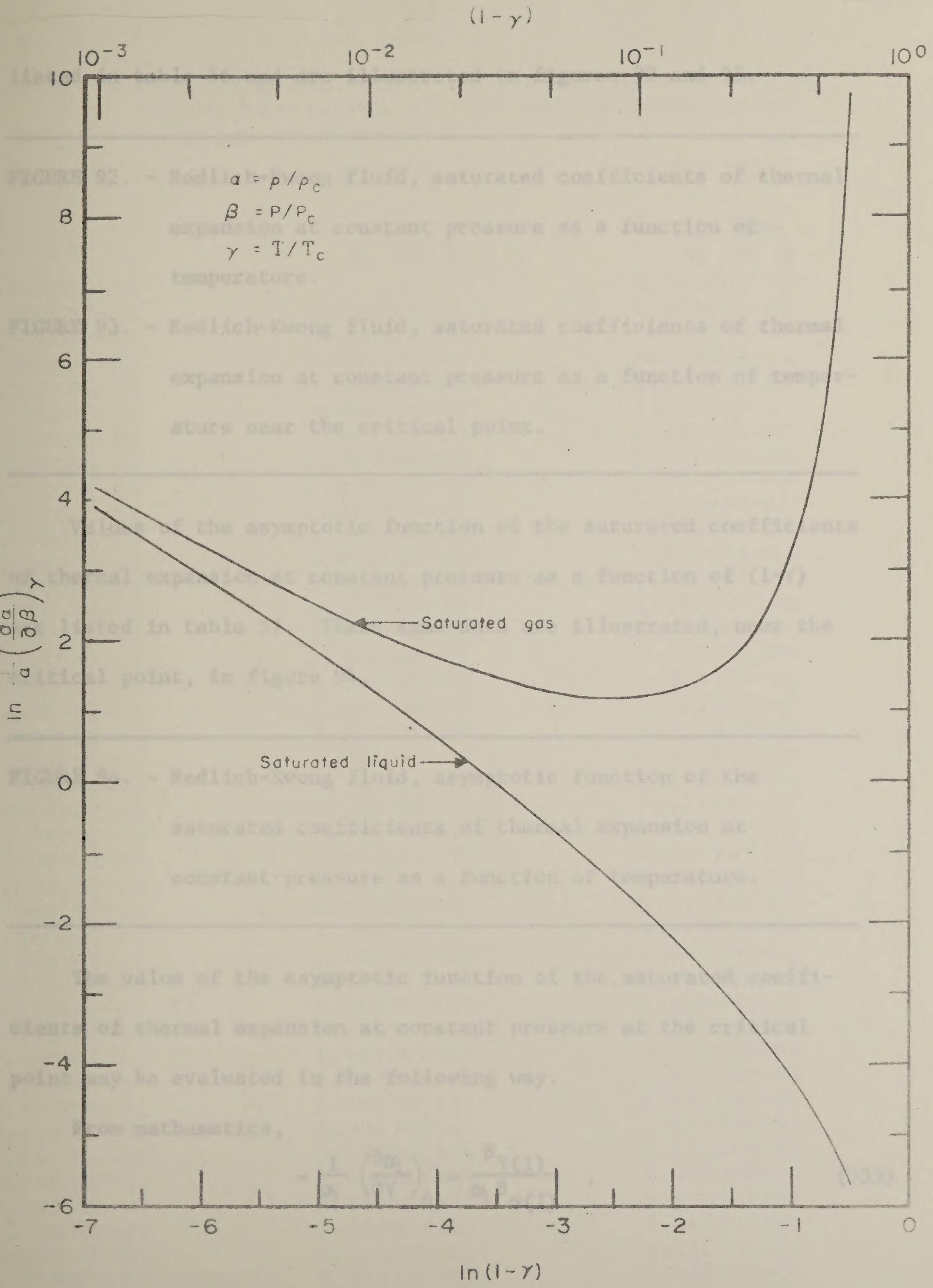

FIGURE 91.— Redlich-Kwong Fluid, Logarithmic Function of the Saturated Coefficients of Compressibility at Constant Temperature as a Function of the Logarithm of the Temperature.

listed in table 56 and are illustrated in figures 92 and 93.

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FIGURE 92. - Redlich-Kwong fluid, saturated coefficients of thermal expansion at constant pressure as a function of temperature.

FIGURE 93. - Redlich-Kwong fluid, saturated coefficients of thermal expansion at constant pressure as a function of temperature near the critical point.

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Values of the asymptotic function of the saturated coefficients of thermal expansion at constant pressure as a function of (1-Y) are listed in table 57. These same data are illustrated, near the critical point, in figure 94.

---

FIGURE 94. - Redlich-Kwong fluid, asymptotic function of the saturated coefficients of thermal expansion at constant pressure as a function of temperature.

---

The value of the asymptotic function of the saturated coefficients of thermal expansion at constant pressure at the critical point may be evaluated in the following way.

From mathematics,

$$-\frac{1}{\alpha_1}\left(\frac{\partial \alpha_1}{\partial Y}\right)_{\beta} = \frac{\beta_{Y(1)}}{\alpha_1 \beta_{\alpha(1)}} . \qquad (335)$$

TABLE 56. - REDLICH-KWONG FLUID, SATURATED COEFFICIENTS OF THERMAL EXPANSION AT CONSTANT PRESSURE AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\alpha = \rho/\rho_c$ | $\beta = P/P_c$ |
|---|---|---|
| $\gamma$ | $-\frac{1}{\alpha_1}\left(\frac{\partial \alpha_1}{\partial \gamma}\right)_\beta$ | $-\frac{1}{\alpha_3}\left(\frac{\partial \alpha_3}{\partial \gamma}\right)_\beta$ |
| 0.00000E-99 | + ∞ | 0.00000E-99 |
| 1.00000E-01 | 1.00000E+01 | 1.97360E-01 |
| 1.50000E-01 | 6.66666E-00 | 2.47239E-01 |
| 2.00000E-01 | 5.00000E-00 | 2.93506E-01 |
| 2.50000E-01 | 4.00000E-00 | 3.39078E-01 |
| 3.00000E-01 | 3.33333E-00 | 3.85844E-01 |
| 3.50000E-01 | 2.85726E-00 | 4.35410E-01 |
| 4.00000E-01 | 2.50144E-00 | 4.89440E-01 |
| 4.50000E-01 | 2.23012E-00 | 5.49920E-01 |
| 5.00000E-01 | 2.02691E-00 | 6.19439E-01 |
| 5.50000E-01 | 1.88541E-00 | 7.01593E-01 |
| 6.00000E-01 | 1.80369E-00 | 8.01642E-01 |
| 6.50000E-01 | 1.78321E-00 | 9.27716E-01 |
| 7.00000E-01 | 1.83121E-00 | 1.09323E-00 |
| 7.50000E-01 | 1.96659E-00 | 1.32224E-00 |
| 8.00000E-01 | 2.23379E-00 | 1.66267E-00 |
| 8.50000E-01 | 2.74374E-00 | 2.22637E-00 |
| 9.00000E-01 | 3.83719E-00 | 3.34894E-00 |
| 9.50000E-01 | 7.22741E-00 | 6.71104E-00 |
| 9.52000E-01 | 7.51175E-00 | 6.99127E-00 |
| 9.54000E-01 | 7.82092E-00 | 7.29589E-00 |
| 9.56000E-01 | 8.15832E-00 | 7.62824E-00 |
| 9.58000E-01 | 8.52795E-00 | 7.99227E-00 |
| 9.60000E-01 | 8.93466E-00 | 8.39274E-00 |
| 9.62000E-01 | 9.38429E-00 | 8.83543E-00 |
| 9.64000E-01 | 9.88398E-00 | 9.32736E-00 |
| 9.66000E-01 | 1.04425E+01 | 9.87725E-00 |
| 9.68000E-01 | 1.10710E+01 | 1.04959E+01 |
| 9.70000E-01 | 1.17833E+01 | 1.11973E+01 |
| 9.72000E-01 | 1.25975E+01 | 1.19989E+01 |
| 9.74000E-01 | 1.35370E+01 | 1.29241E+01 |
| 9.76000E-01 | 1.46330E+01 | 1.40037E+01 |
| 9.78000E-01 | 1.59283E+01 | 1.52800E+01 |
| 9.80000E-01 | 1.74825E+01 | 1.68118E+01 |
| 9.82000E-01 | 1.93818E+01 | 1.86846E+01 |
| 9.84000E-01 | 2.17555E+01 | 2.10264E+01 |
| 9.86000E-01 | 2.48068E+01 | 2.40383E+01 |
| 9.88000E-01 | 2.88741E+01 | 2.80557E+01 |
| 9.90000E-01 | 3.45663E+01 | 3.36826E+01 |
| 9.92000E-01 | 4.31009E+01 | 4.21273E+01 |
| 9.94000E-01 | 5.73179E+01 | 5.62099E+01 |
| 9.96000E-01 | 8.57327E+01 | 8.43957E+01 |
| 9.98000E-01 | 1.70897E+02 | 1.69034E+02 |
| 9.99000E-01 | 3.41078E+02 | 3.38463E+02 |
| 1.00000E-00 | + ∞ | + ∞ |

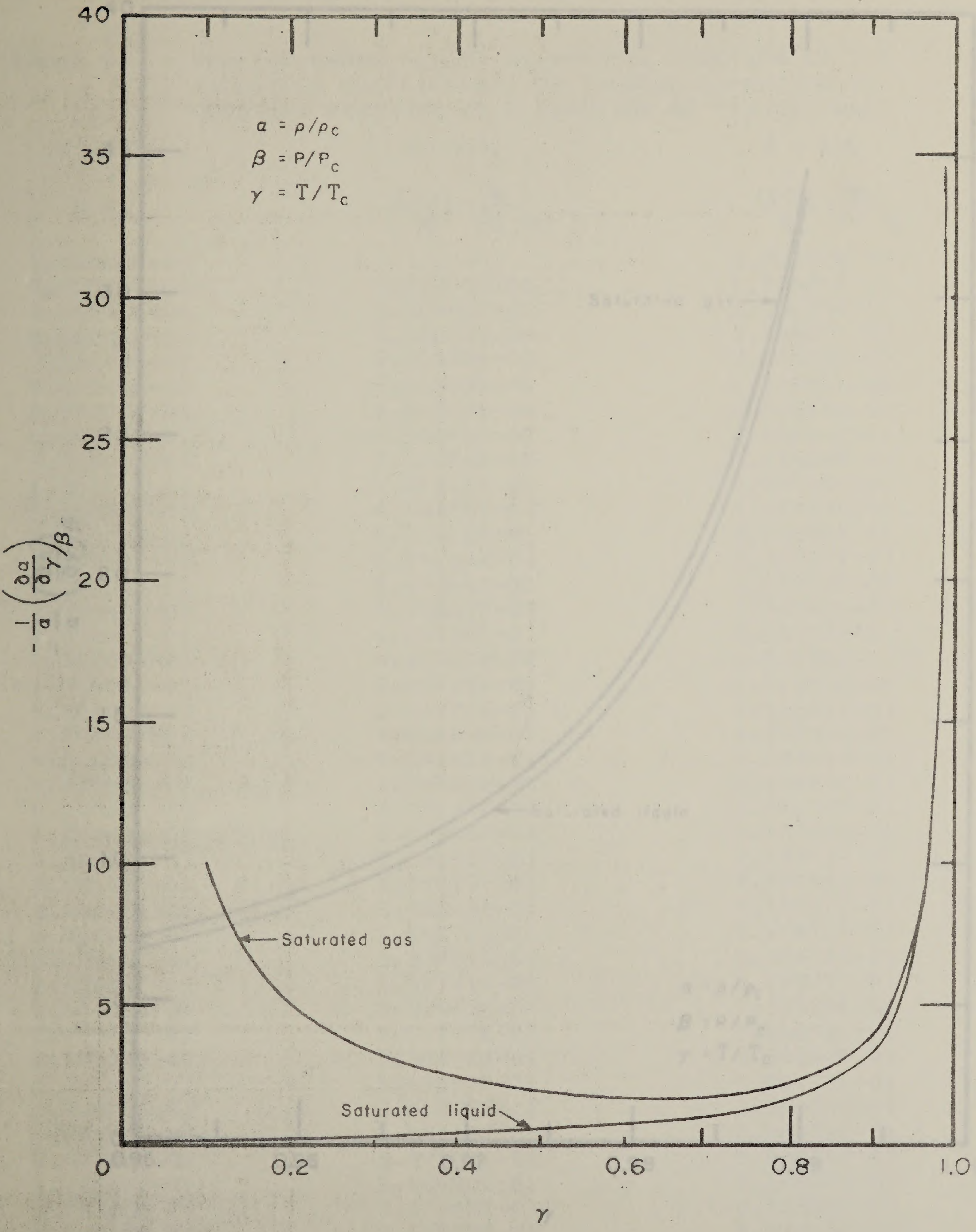

FIGURE 92.— Redlich-Kwong Fluid, Saturated Coefficients of Thermal Expansion at Constant Pressure as a Function of Temperature.

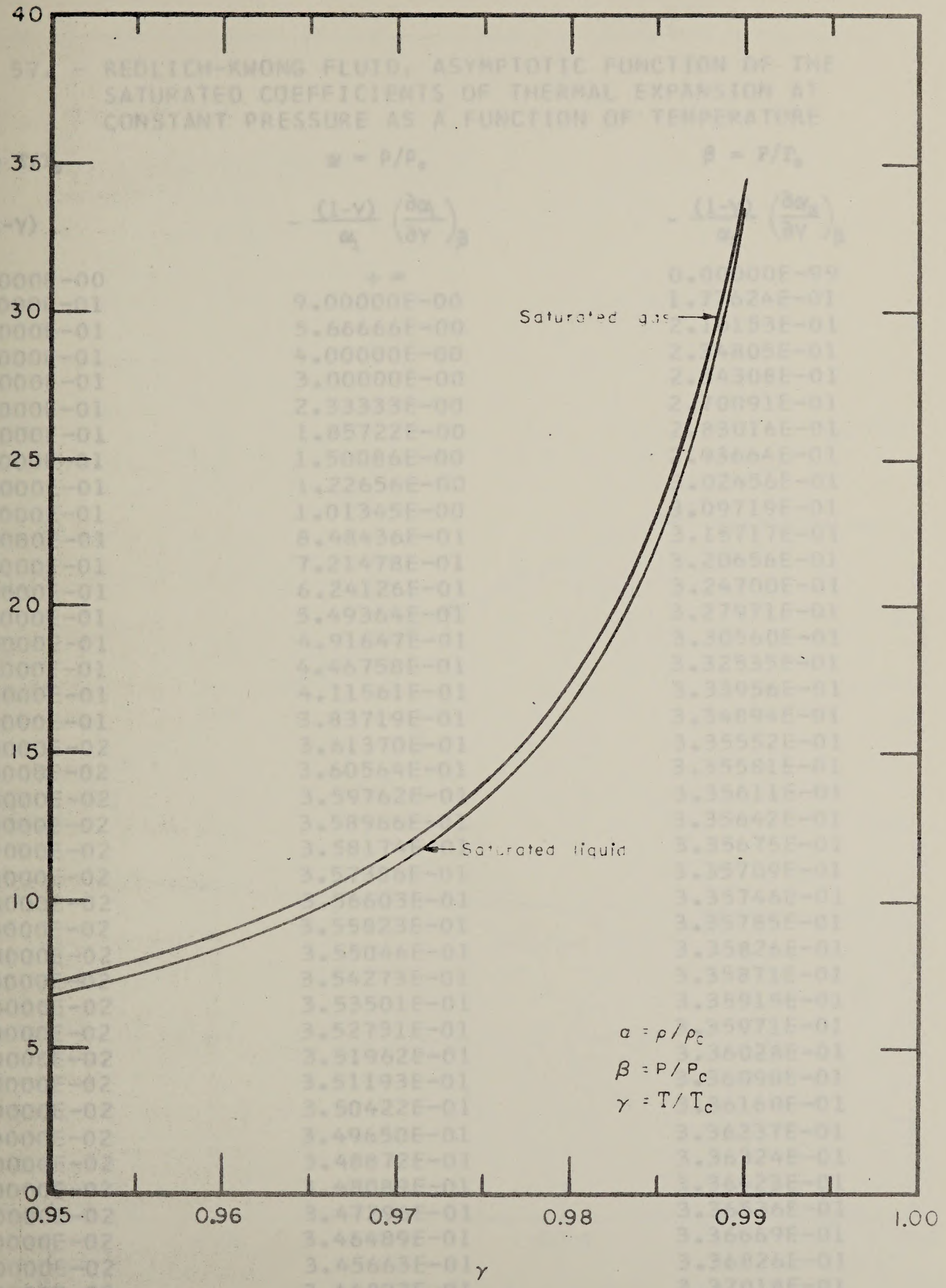

FIGURE 93.–Redlich-Kwong Fluid, Saturated Coefficients of Thermal Expansion at Constant Pressure as a Function of Temperature Near the Critical Point.

TABLE 57. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED COEFFICIENTS OF THERMAL EXPANSION AT CONSTANT PRESSURE AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$<br>$(1-\gamma)$ | $\alpha = \rho/\rho_c$<br>$-\frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\gamma}\right)_\beta$ | $\beta = P/P_c$<br>$-\frac{(1-\gamma)}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\gamma}\right)_\beta$ |
|---|---|---|
| 1.00000E-00 | + ∞ | 0.00000E-99 |
| 9.00000E-01 | 9.00000E-00 | 1.77624E-01 |
| 8.50000E-01 | 5.66666E-00 | 2.10153E-01 |
| 8.00000E-01 | 4.00000E-00 | 2.34805E-01 |
| 7.50000E-01 | 3.00000E-00 | 2.54308E-01 |
| 7.00000E-01 | 2.33333E-00 | 2.70091E-01 |
| 6.50000E-01 | 1.85722E-00 | 2.83016E-01 |
| 6.00000E-01 | 1.50086E-00 | 2.93664E-01 |
| 5.50000E-01 | 1.22656E-00 | 3.02456E-01 |
| 5.00000E-01 | 1.01345E-00 | 3.09719E-01 |
| 4.50000E-01 | 8.48436E-01 | 3.15717E-01 |
| 4.00000E-01 | 7.21478E-01 | 3.20656E-01 |
| 3.50000E-01 | 6.24126E-01 | 3.24700E-01 |
| 3.00000E-01 | 5.49364E-01 | 3.27971E-01 |
| 2.50000E-01 | 4.91647E-01 | 3.30560E-01 |
| 2.00000E-01 | 4.46758E-01 | 3.32535E-01 |
| 1.50000E-01 | 4.11561E-01 | 3.33956E-01 |
| 1.00000E-01 | 3.83719E-01 | 3.34894E-01 |
| 5.00000E-02 | 3.61370E-01 | 3.35552E-01 |
| 4.80000E-02 | 3.60564E-01 | 3.35581E-01 |
| 4.60000E-02 | 3.59762E-01 | 3.35611E-01 |
| 4.40000E-02 | 3.58966E-01 | 3.35642E-01 |
| 4.20000E-02 | 3.58174E-01 | 3.35675E-01 |
| 4.00000E-02 | 3.57386E-01 | 3.35709E-01 |
| 3.80000E-02 | 3.56603E-01 | 3.35746E-01 |
| 3.60000E-02 | 3.55823E-01 | 3.35785E-01 |
| 3.40000E-02 | 3.55046E-01 | 3.35826E-01 |
| 3.20000E-02 | 3.54273E-01 | 3.35871E-01 |
| 3.00000E-02 | 3.53501E-01 | 3.35919E-01 |
| 2.80000E-02 | 3.52731E-01 | 3.35971E-01 |
| 2.60000E-02 | 3.51962E-01 | 3.36028E-01 |
| 2.40000E-02 | 3.51193E-01 | 3.36090E-01 |
| 2.20000E-02 | 3.50422E-01 | 3.36160E-01 |
| 2.00000E-02 | 3.49650E-01 | 3.36237E-01 |
| 1.80000E-02 | 3.48872E-01 | 3.36324E-01 |
| 1.60000E-02 | 3.48089E-01 | 3.36423E-01 |
| 1.40000E-02 | 3.47296E-01 | 3.36536E-01 |
| 1.20000E-02 | 3.46489E-01 | 3.36669E-01 |
| 1.00000E-02 | 3.45663E-01 | 3.36826E-01 |
| 8.00000E-03 | 3.44807E-01 | 3.37018E-01 |
| 6.00000E-03 | 3.43907E-01 | 3.37259E-01 |
| 4.00000E-03 | 3.42931E-01 | 3.37582E-01 |
| 2.00000E-03 | 3.41795E-01 | 3.38069E-01 |
| 1.00000E-03 | 3.41078E-01 | 3.38463E-01 |
| 0.00000E-99 | 3.39610E-01 | 3.39610E-01 |

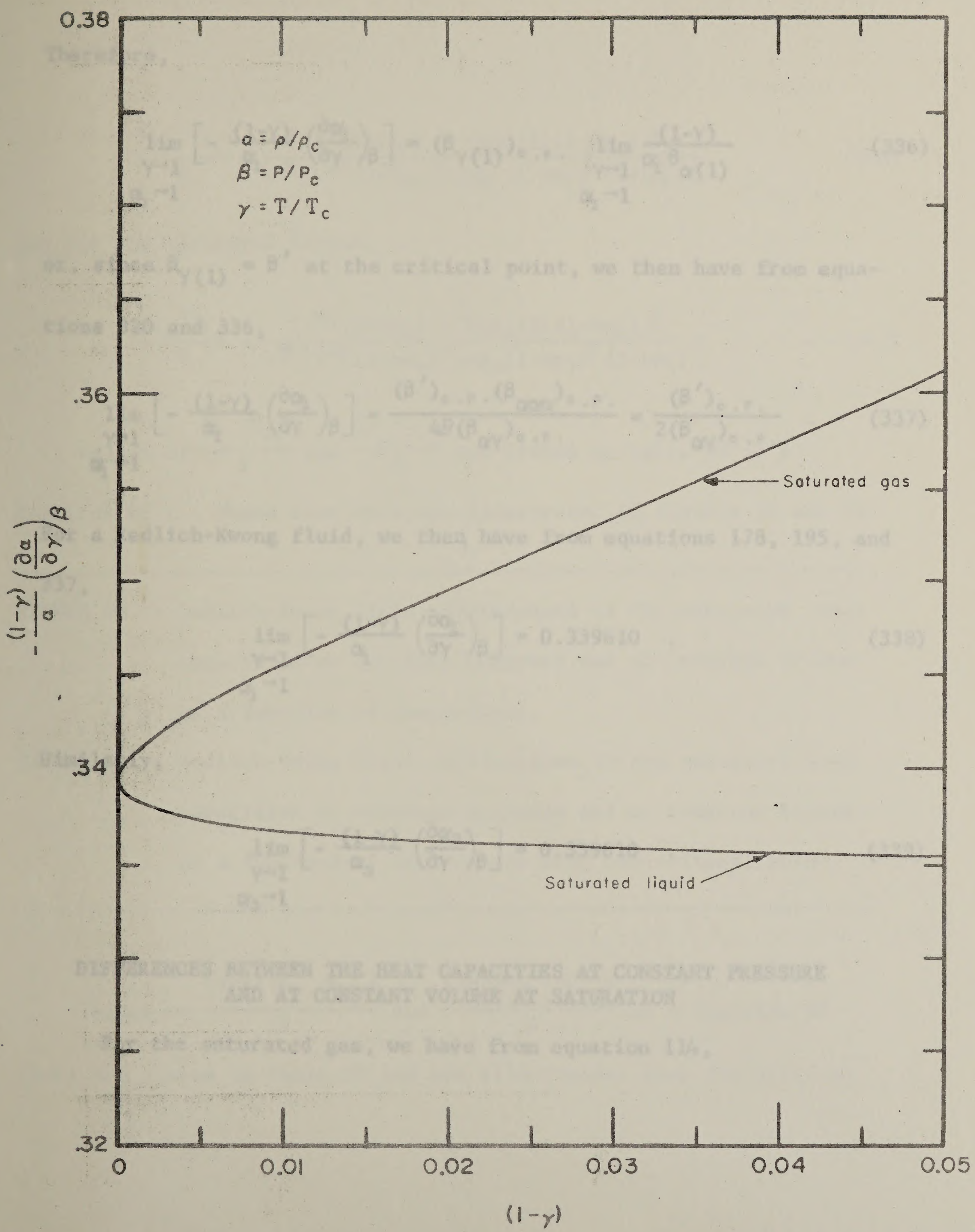

FIGURE 94.—Redlich-Kwong Fluid, Asymptotic Function of the Saturated Coefficients of Thermal Expansion at Constant Pressure as a Function of Temperature.

Therefore,

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}}\left[-\frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\gamma}\right)_\beta\right] = (\beta_{\gamma(1)})_{c.P.}\cdot\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}}\frac{(1-\gamma)}{\alpha_1\beta_{\alpha(1)}} \tag{336}$$

or, since $\beta_{\gamma(1)} = \beta'$ at the critical point, we then have from equations 320 and 336,

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}}\left[-\frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\gamma}\right)_\beta\right] = \frac{(\beta')_{c.P.}\cdot(\beta_{\alpha\alpha\alpha})_{c.P.}}{4B(\beta_{\alpha\gamma})_{c.P.}} = \frac{(\beta')_{c.P.}}{2(\beta_{\alpha\gamma})_{c.P.}}\quad . \tag{337}$$

For a Redlich-Kwong fluid, we then have from equations 178, 195, and 337,

$$\lim_{\substack{\gamma\to 1\\ \alpha_1\to 1}}\left[-\frac{(1-\gamma)}{\alpha_1}\left(\frac{\partial\alpha_1}{\partial\gamma}\right)_\beta\right] = 0.339610\quad . \tag{338}$$

Similarly,

$$\lim_{\substack{\gamma\to 1\\ \alpha_3\to 1}}\left[-\frac{(1-\gamma)}{\alpha_3}\left(\frac{\partial\alpha_3}{\partial\gamma}\right)_\beta\right] = 0.339610\quad . \tag{339}$$

## DIFFERENCES BETWEEN THE HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME AT SATURATION

For the saturated gas, we have from equation 114,

$$\frac{C_{P_1}-C_{V_1}}{R} = \frac{[Y^{3/2}(1+b\alpha_1) + (a\alpha_1/2)(1-b\alpha_1)]^2}{Y^{3/2}[Y^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)]} \tag{340}$$

and for the saturated liquid,

$$\frac{C_{P_3}-C_{V_3}}{R} = \frac{[Y^{3/2}(1+b\alpha_3) + (a\alpha_3/2)(1-b\alpha_3)]^2}{Y^{3/2}[Y^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)]} \; . \tag{341}$$

Values of $\frac{C_{P_1}-C_{V_1}}{R}$ and $\frac{C_{P_3}-C_{V_3}}{R}$ are listed in table 58 as a function of $Y$. These same data are illustrated in figures 95 and 96.

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FIGURE 95. - Redlich-Kwong fluid, differences in the saturated heat capacities at constant pressure and at constant volume as a function of temperature.

FIGURE 96. - Redlich-Kwong fluid, differences in the saturated heat capacities at constant pressure and at constant volume as a function of temperature near the critical point.

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Values of $\frac{(1-Y)(C_{P_1}-C_{V_1})}{R}$ and $\frac{(1-Y)(C_{P_3}-C_{V_3})}{R}$ as a function of $(1-Y)$ are listed in table 59 and are illustrated, near the critical

TABLE 58. - REDLICH-KWONG FLUID, DIFFERENCES IN THE SATURATED HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $\gamma$ | $\frac{C_{P_1} - C_{V_1}}{R}$ | $\frac{C_{P_3} - C_{V_3}}{R}$ |
|---|---|---|
| 0.00000E-99 | 1.00000E+00 | 2.25000E+00 |
| 1.00000E-01 | 1.00000E-00 | 2.29450E-00 |
| 1.50000E-01 | 1.00000E-00 | 2.33377E-00 |
| 2.00000E-01 | 1.00000E-00 | 2.38290E-00 |
| 2.50000E-01 | 1.00000E-00 | 2.44243E-00 |
| 3.00000E-01 | 1.00000E-00 | 2.51355E-00 |
| 3.50000E-01 | 1.00005E-00 | 2.59814E-00 |
| 4.00000E-01 | 1.00070E-00 | 2.69894E-00 |
| 4.50000E-01 | 1.00437E-00 | 2.81984E-00 |
| 5.00000E-01 | 1.01663E-00 | 2.96642E-00 |
| 5.50000E-01 | 1.04597E-00 | 3.14679E-00 |
| 6.00000E-01 | 1.10296E-00 | 3.37299E-00 |
| 6.50000E-01 | 1.20108E-00 | 3.66372E-00 |
| 7.00000E-01 | 1.36045E-00 | 4.04972E-00 |
| 7.50000E-01 | 1.61666E-00 | 4.58554E-00 |
| 8.00000E-01 | 2.04217E-00 | 5.37864E-00 |
| 8.50000E-01 | 2.81091E-00 | 6.67556E-00 |
| 9.00000E-01 | 4.46069E-00 | 9.20124E-00 |
| 9.50000E-01 | 9.77862E-00 | 1.64852E+01 |
| 9.52000E-01 | 1.02353E+01 | 1.70801E+01 |
| 9.54000E-01 | 1.07333E+01 | 1.77253E+01 |
| 9.56000E-01 | 1.12783E+01 | 1.84275E+01 |
| 9.58000E-01 | 1.18773E+01 | 1.91946E+01 |
| 9.60000E-01 | 1.25385E+01 | 2.00364E+01 |
| 9.62000E-01 | 1.32717E+01 | 2.09644E+01 |
| 9.64000E-01 | 1.40894E+01 | 2.19927E+01 |
| 9.66000E-01 | 1.50065E+01 | 2.31388E+01 |
| 9.68000E-01 | 1.60421E+01 | 2.44245E+01 |
| 9.70000E-01 | 1.72202E+01 | 2.58774E+01 |
| 9.72000E-01 | 1.85718E+01 | 2.75328E+01 |
| 9.74000E-01 | 2.01377E+01 | 2.94367E+01 |
| 9.76000E-01 | 2.19721E+01 | 3.16506E+01 |
| 9.78000E-01 | 2.41493E+01 | 3.42580E+01 |
| 9.80000E-01 | 2.67737E+01 | 3.73755E+01 |
| 9.82000E-01 | 2.99961E+01 | 4.11711E+01 |
| 9.84000E-01 | 3.40437E+01 | 4.58963E+01 |
| 9.86000E-01 | 3.92745E+01 | 5.19451E+01 |
| 9.88000E-01 | 4.62870E+01 | 5.99725E+01 |
| 9.90000E-01 | 5.61623E+01 | 7.11536E+01 |
| 9.92000E-01 | 7.10703E+01 | 8.78306E+01 |
| 9.94000E-01 | 9.60955E+01 | 1.15448E+02 |
| 9.96000E-01 | 1.46564E+02 | 1.70265E+02 |
| 9.98000E-01 | 2.99586E+02 | 3.33104E+02 |
| 9.99000E-01 | 6.08507E+02 | 6.55907E+02 |
| 1.00000E-00 | $+\infty$ | $+\infty$ |

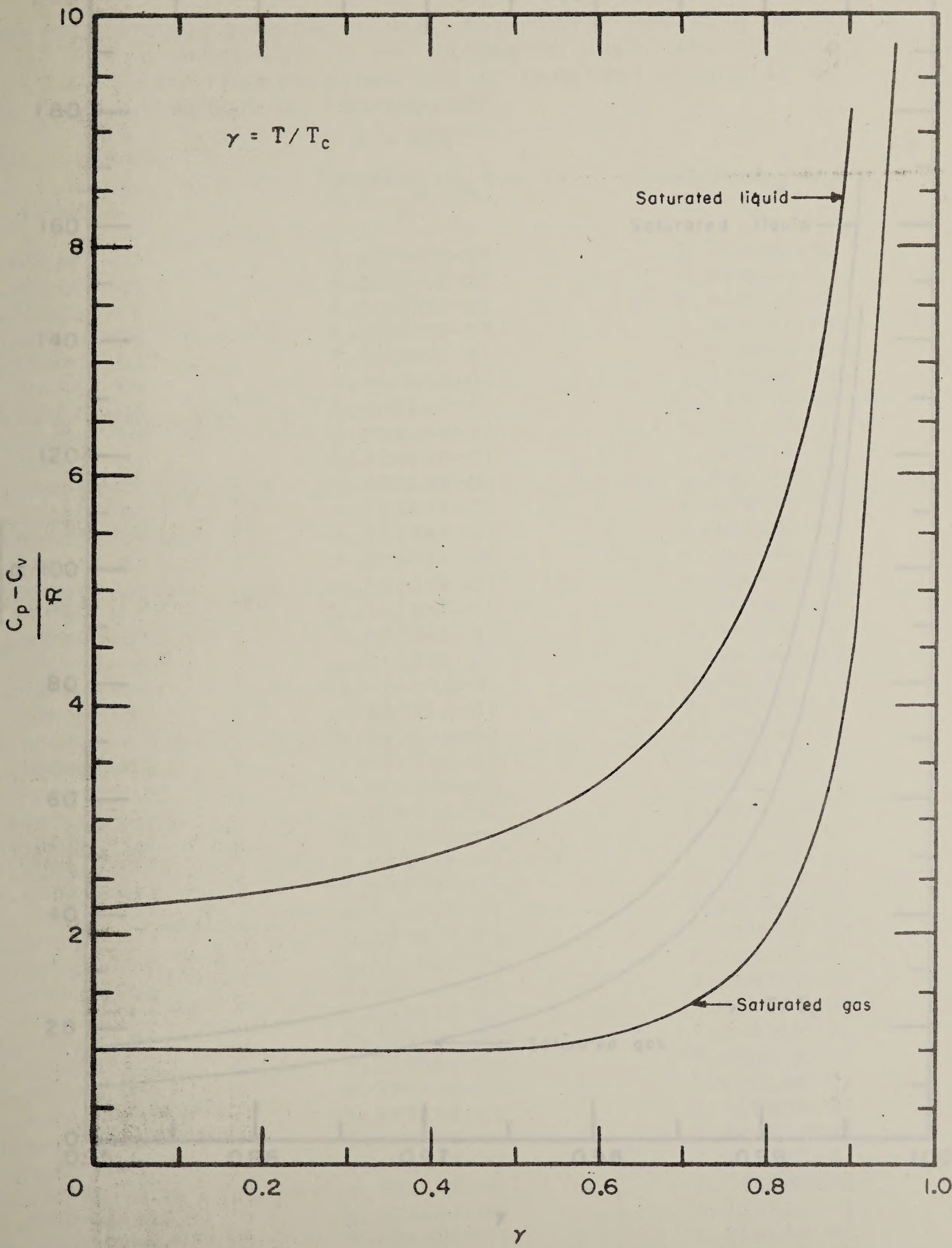

FIGURE 95.– Redlich–Kwong Fluid, Differences in the Saturated Heat Capacities at Constant Pressure and at Constant Volume as a Function of Temperature.

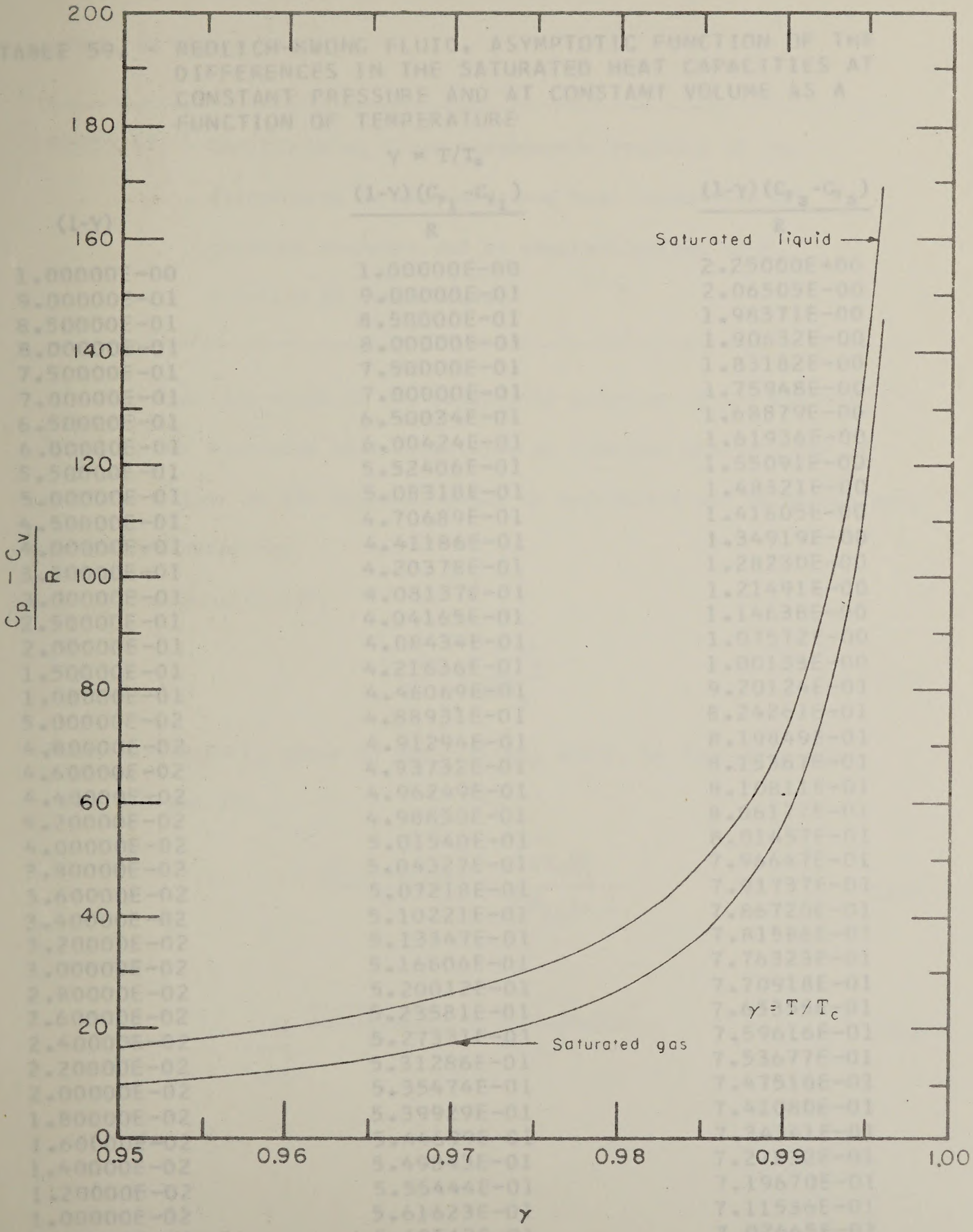

FIGURE 96.-Redlich-Kwong Fluid, Differences in the Saturated Heat Capacities at Constant Pressure and at Constant Volume as a Function of Temperature Near the Critical Point.

TABLE 59. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE DIFFERENCES IN THE SATURATED HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$

| $(1-\gamma)$ | $\frac{(1-\gamma)(C_{P_1}-C_{V_1})}{R}$ | $\frac{(1-\gamma)(C_{P_3}-C_{V_3})}{R}$ |
|---|---|---|
| 1.00000E-00 | 1.00000E-00 | 2.25000E+00 |
| 9.00000E-01 | 9.00000E-01 | 2.06505E-00 |
| 8.50000E-01 | 8.50000E-01 | 1.98371E-00 |
| 8.00000E-01 | 8.00000E-01 | 1.90632E-00 |
| 7.50000E-01 | 7.50000E-01 | 1.83182E-00 |
| 7.00000E-01 | 7.00000E-01 | 1.75948E-00 |
| 6.50000E-01 | 6.50034E-01 | 1.68879E-00 |
| 6.00000E-01 | 6.00424E-01 | 1.61936E-00 |
| 5.50000E-01 | 5.52406E-01 | 1.55091E-00 |
| 5.00000E-01 | 5.08318E-01 | 1.48321E-00 |
| 4.50000E-01 | 4.70689E-01 | 1.41605E-00 |
| 4.00000E-01 | 4.41186E-01 | 1.34919E-00 |
| 3.50000E-01 | 4.20378E-01 | 1.28230E-00 |
| 3.00000E-01 | 4.08137E-01 | 1.21491E-00 |
| 2.50000E-01 | 4.04165E-01 | 1.14638E-00 |
| 2.00000E-01 | 4.08434E-01 | 1.07572E-00 |
| 1.50000E-01 | 4.21636E-01 | 1.00133E-00 |
| 1.00000E-01 | 4.46069E-01 | 9.20124E-01 |
| 5.00000E-02 | 4.88931E-01 | 8.24261E-01 |
| 4.80000E-02 | 4.91294E-01 | 8.19849E-01 |
| 4.60000E-02 | 4.93732E-01 | 8.15367E-01 |
| 4.40000E-02 | 4.96249E-01 | 8.10811E-01 |
| 4.20000E-02 | 4.98850E-01 | 8.06177E-01 |
| 4.00000E-02 | 5.01540E-01 | 8.01457E-01 |
| 3.80000E-02 | 5.04327E-01 | 7.96647E-01 |
| 3.60000E-02 | 5.07218E-01 | 7.91737E-01 |
| 3.40000E-02 | 5.10221E-01 | 7.86720E-01 |
| 3.20000E-02 | 5.13347E-01 | 7.81586E-01 |
| 3.00000E-02 | 5.16606E-01 | 7.76323E-01 |
| 2.80000E-02 | 5.20012E-01 | 7.70918E-01 |
| 2.60000E-02 | 5.23581E-01 | 7.65355E-01 |
| 2.40000E-02 | 5.27331E-01 | 7.59616E-01 |
| 2.20000E-02 | 5.31286E-01 | 7.53677E-01 |
| 2.00000E-02 | 5.35474E-01 | 7.47510E-01 |
| 1.80000E-02 | 5.39929E-01 | 7.41080E-01 |
| 1.60000E-02 | 5.44699E-01 | 7.34341E-01 |
| 1.40000E-02 | 5.49843E-01 | 7.27232E-01 |
| 1.20000E-02 | 5.55444E-01 | 7.19670E-01 |
| 1.00000E-02 | 5.61623E-01 | 7.11536E-01 |
| 8.00000E-03 | 5.68562E-01 | 7.02645E-01 |
| 6.00000E-03 | 5.76573E-01 | 6.92688E-01 |
| 4.00000E-03 | 5.86257E-01 | 6.81062E-01 |
| 2.00000E-03 | 5.99173E-01 | 6.66208E-01 |
| 1.00000E-03 | 6.08507E-01 | 6.55907E-01 |
| 0.00000E-99 | 6.31724E-01 | 6.31724E-01 |

point, in figure 97.

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FIGURE 97. - Redlich-Kwong fluid, asymptotic function of the differences in the saturated heat capacities at constant pressure and at constant volume as a function of temperature.

---

The limiting value of the asymptotic function of the differences in the saturated heat capacities at constant pressure and at constant volume as the critical point is approached may be determined in the following way.

From equation 113,

$$\frac{C_P - C_V}{R} = \frac{\gamma Z_c \beta_\gamma^2}{\alpha^2 \beta_\alpha} \quad . \tag{113}$$

Then, sufficiently close to the critical point, we have from equations 113 and 260,

$$\frac{(1-\gamma)(C_P - C_V)}{R} = \frac{\gamma Z_c \beta_\gamma^2}{2(\beta_{\alpha\gamma})_{c.p.}} \quad . \tag{342}$$

Therefore,

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \frac{(1-\gamma)(C_P - C_V)}{R} = \frac{Z_c (\beta')^2_{c.p.}}{2(\beta_{\alpha\gamma})_{c.p.}} \quad . \tag{343}$$

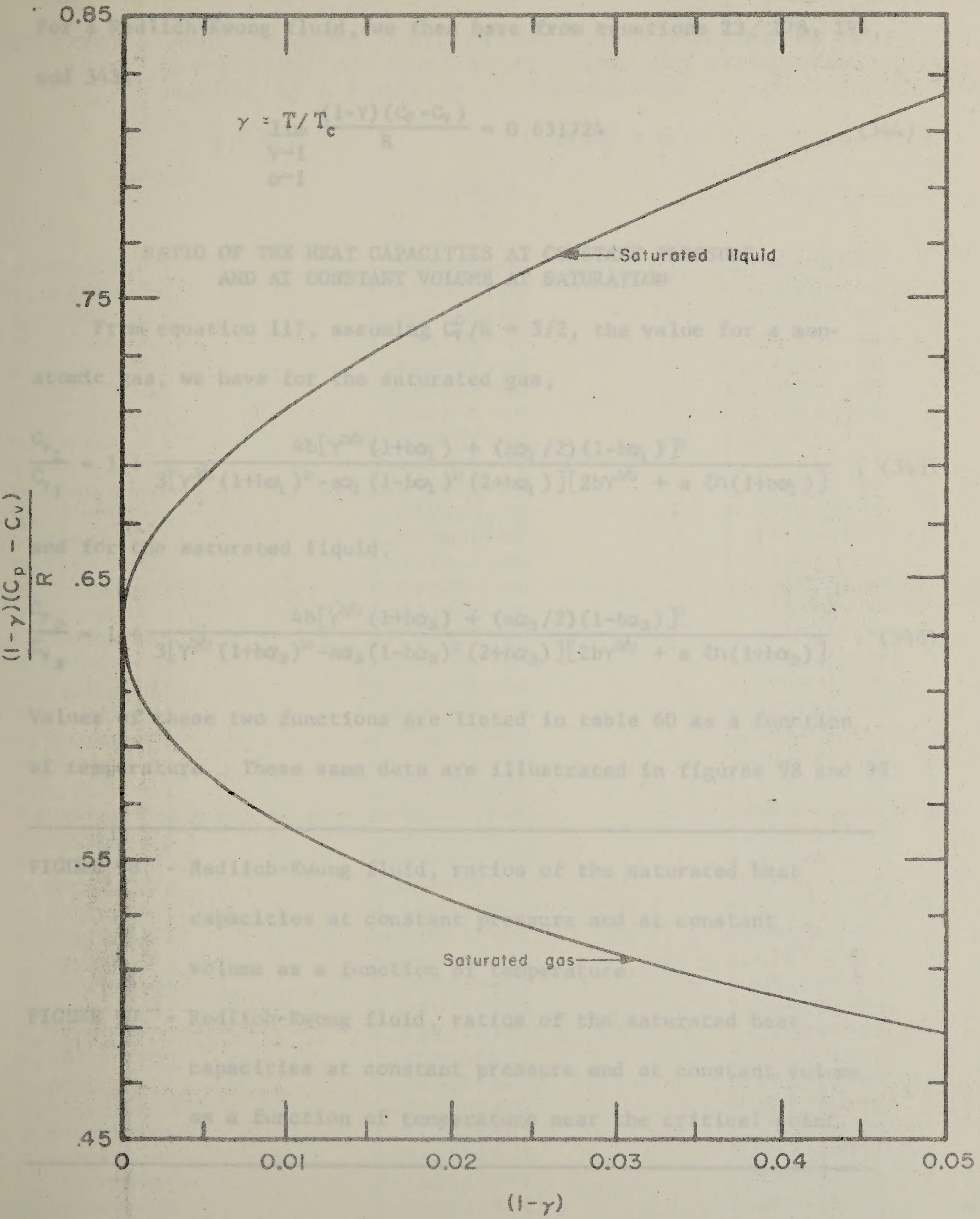

FIGURE 97. - Redlich-Kwong Fluid, Asymptotic Function of the Differences in the Saturated Heat Capacities at Constant Pressure and at Constant Volume as a Function of Temperature.

For a Redlich-Kwong fluid, we then have from equations 23, 178, 195, and 343,

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \frac{(1-\gamma)(C_P - C_V)}{R} = 0.631724 \quad . \tag{344}$$

## RATIO OF THE HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME AT SATURATION

From equation 117, assuming $C_V^0/R = 3/2$, the value for a monatomic gas, we have for the saturated gas,

$$\frac{C_{P_1}}{C_{V_1}} = 1 + \frac{4b[\gamma^{3/2}(1+b\alpha_1) + (a\alpha_1/2)(1-b\alpha_1)]^2}{3[\gamma^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)][2b\gamma^{3/2} + a\,\ln(1+b\alpha_1)]} \quad , \tag{345}$$

and for the saturated liquid,

$$\frac{C_{P_3}}{C_{V_3}} = 1 + \frac{4b[\gamma^{3/2}(1+b\alpha_3) + (a\alpha_3/2)(1-b\alpha_3)]^2}{3[\gamma^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)][2b\gamma^{3/2} + a\,\ln(1+b\alpha_3)]} \quad . \tag{346}$$

Values of these two functions are listed in table 60 as a function of temperature. These same data are illustrated in figures 98 and 99.

---

FIGURE 98. - Redlich-Kwong fluid, ratios of the saturated heat capacities at constant pressure and at constant volume as a function of temperature.

FIGURE 99. - Redlich-Kwong fluid, ratios of the saturated heat capacities at constant pressure and at constant volume as a function of temperature near the critical point.

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TABLE 60. - REDLICH-KWONG FLUID, RATIOS OF THE SATURATED HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$Y = T/T_c$ $C_P^0/C_V^0 = 5/3$

| Y | $(C_{P_1}/C_{V_1})$ | $(C_{P_3}/C_{V_3})$ |
|---|---|---|
| 0.00000E-99 | 1.66666E+00 | 1.00000E+00 |
| 1.00000E-01 | 1.66666E-00 | 1.02803E-00 |
| 1.50000E-01 | 1.66666E-00 | 1.05199E-00 |
| 2.00000E-01 | 1.66666E-00 | 1.08103E-00 |
| 2.50000E-01 | 1.66666E-00 | 1.11499E-00 |
| 3.00000E-01 | 1.66666E-00 | 1.15403E-00 |
| 3.50000E-01 | 1.66669E-00 | 1.19858E-00 |
| 4.00000E-01 | 1.66705E-00 | 1.24942E-00 |
| 4.50000E-01 | 1.66909E-00 | 1.30776E-00 |
| 5.00000E-01 | 1.67590E-00 | 1.37543E-00 |
| 5.50000E-01 | 1.69216E-00 | 1.45518E-00 |
| 6.00000E-01 | 1.72363E-00 | 1.55127E-00 |
| 6.50000E-01 | 1.77746E-00 | 1.67041E-00 |
| 7.00000E-01 | 1.86409E-00 | 1.82387E-00 |
| 7.50000E-01 | 2.00149E-00 | 2.03188E-00 |
| 8.00000E-01 | 2.22567E-00 | 2.33471E-00 |
| 8.50000E-01 | 2.62153E-00 | 2.82588E-00 |
| 9.00000E-01 | 3.44647E-00 | 3.78407E-00 |
| 9.50000E-01 | 6.00084E-00 | 6.59106E-00 |
| 9.52000E-01 | 6.21604E-00 | 6.82283E-00 |
| 9.54000E-01 | 6.45020E-00 | 7.07452E-00 |
| 9.56000E-01 | 6.70592E-00 | 7.34883E-00 |
| 9.58000E-01 | 6.98628E-00 | 7.64897E-00 |
| 9.60000E-01 | 7.29501E-00 | 7.97882E-00 |
| 9.62000E-01 | 7.63661E-00 | 8.34304E-00 |
| 9.64000E-01 | 8.01657E-00 | 8.74733E-00 |
| 9.66000E-01 | 8.44169E-00 | 9.19874E-00 |
| 9.68000E-01 | 8.92047E-00 | 9.70606E-00 |
| 9.70000E-01 | 9.46369E-00 | 1.02804E+01 |
| 9.72000E-01 | 1.00852E+01 | 1.09362E+01 |
| 9.74000E-01 | 1.08031E+01 | 1.16920E+01 |
| 9.76000E-01 | 1.16417E+01 | 1.25729E+01 |
| 9.78000E-01 | 1.26340E+01 | 1.36127E+01 |
| 9.80000E-01 | 1.38261E+01 | 1.48591E+01 |
| 9.82000E-01 | 1.52850E+01 | 1.63807E+01 |
| 9.84000E-01 | 1.71109E+01 | 1.82803E+01 |
| 9.86000E-01 | 1.94618E+01 | 2.07196E+01 |
| 9.88000E-01 | 2.26006E+01 | 2.39675E+01 |
| 9.90000E-01 | 2.70017E+01 | 2.85081E+01 |
| 9.92000E-01 | 3.36139E+01 | 3.53082E+01 |
| 9.94000E-01 | 4.46541E+01 | 4.66221E+01 |
| 9.96000E-01 | 6.67800E+01 | 6.92044E+01 |
| 9.98000E-01 | 1.33330E+02 | 1.36778E+02 |
| 9.99000E-01 | 2.66733E+02 | 2.71624E+02 |
| 1.00000E-00 | + ∞ | + ∞ |

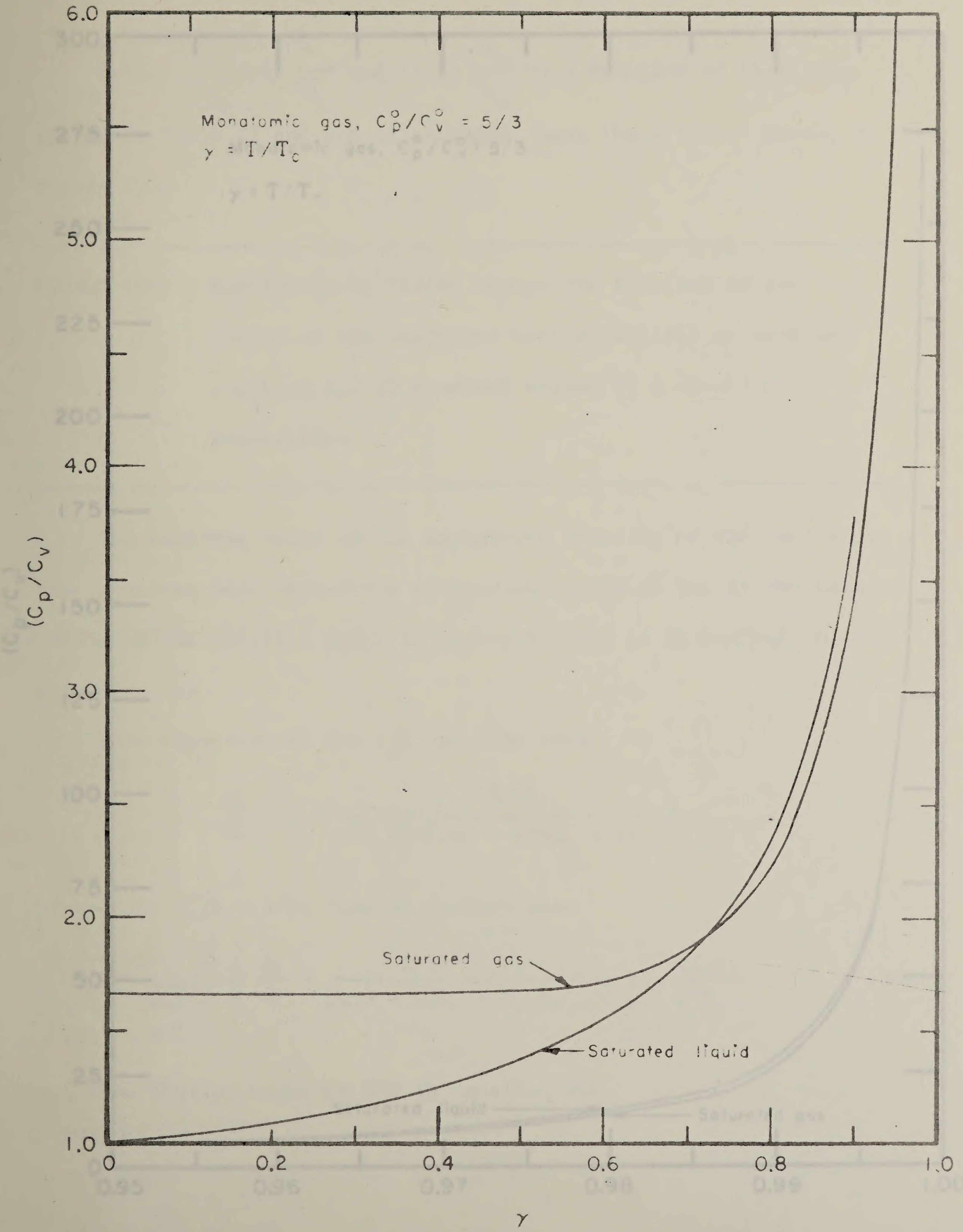

FIGURE 98.–Redlich-Kwong Fluid, Ratios of the Saturated Heat Capacities at Constant Pressure and at Constant Volume as a Function of Temperature.

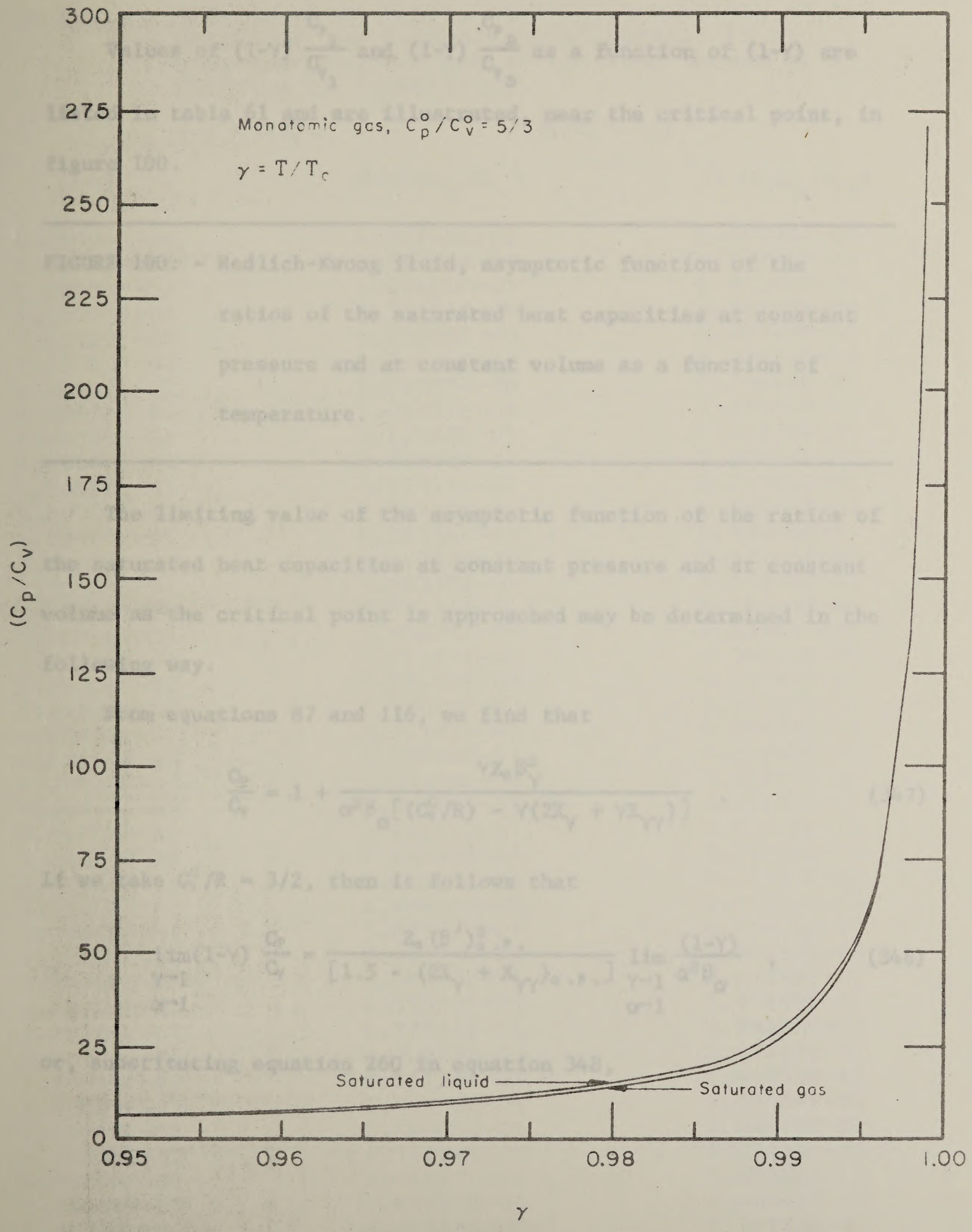

FIGURE 99.- Redlich-Kwong Fluid, Ratios of the Saturated Heat Capacities at Constant Pressure and at Constant Volume as a Function of Temperature Near the Critical Point.

Values of $(1-\gamma)\,\frac{C_{P_1}}{C_{V_1}}$ and $(1-\gamma)\,\frac{C_{P_3}}{C_{V_3}}$ as a function of $(1-\gamma)$ are listed in table 61 and are illustrated, near the critical point, in figure 100.

---

FIGURE 100. - Redlich-Kwong fluid, asymptotic function of the ratios of the saturated heat capacities at constant pressure and at constant volume as a function of temperature.

---

The limiting value of the asymptotic function of the ratios of the saturated heat capacities at constant pressure and at constant volume as the critical point is approached may be determined in the following way.

From equations 87 and 116, we find that

$$\frac{C_P}{C_V} = 1 + \frac{\gamma Z_c \beta_\gamma^2}{\alpha^2 \beta_\alpha [(C_V^0/R) - \gamma(2X_\gamma + \gamma X_{\gamma\gamma})]} \quad . \tag{347}$$

If we take $C_V^0/R = 3/2$, then it follows that

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} (1-\gamma)\,\frac{C_P}{C_V} = \frac{Z_c\,(\beta')^2_{c.p.}}{[1.5 - (2X_\gamma + X_{\gamma\gamma})_{c.p.}]} \lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \frac{(1-\gamma)}{\alpha^2 \beta_\alpha} \quad , \tag{348}$$

or, substituting equation 260 in equation 348,

TABLE 61. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE RATIOS OF THE SATURATED HEAT CAPACITIES AT CONSTANT PRESSURE AND AT CONSTANT VOLUME AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $C_P^o/C_V^o = 5/3$

| $(1-\gamma)$ | $(1-\gamma)(C_{P_1}/C_{V_1})$ | $(1-\gamma)(C_{P_3}/C_{V_3})$ |
|---|---|---|
| 1.00000E-00 | 1.66666E+00 | 1.00000E-00 |
| 9.00000E-01 | 1.49999E-00 | 9.25228E-01 |
| 8.50000E-01 | 1.41666E-00 | 8.94192E-01 |
| 8.00000E-01 | 1.33333E-00 | 8.64828E-01 |
| 7.50000E-01 | 1.25000E-00 | 8.36248E-01 |
| 7.00000E-01 | 1.16666E-00 | 8.07822E-01 |
| 6.50000E-01 | 1.08335E-00 | 7.79078E-01 |
| 6.00000E-01 | 1.00023E-00 | 7.49654E-01 |
| 5.50000E-01 | 9.18003E-01 | 7.19270E-01 |
| 5.00000E-01 | 8.37951E-01 | 6.87715E-01 |
| 4.50000E-01 | 7.61473E-01 | 6.54833E-01 |
| 4.00000E-01 | 6.89452E-01 | 6.20508E-01 |
| 3.50000E-01 | 6.22114E-01 | 5.84645E-01 |
| 3.00000E-01 | 5.59228E-01 | 5.47163E-01 |
| 2.50000E-01 | 5.00373E-01 | 5.07970E-01 |
| 2.00000E-01 | 4.45134E-01 | 4.66942E-01 |
| 1.50000E-01 | 3.93229E-01 | 4.23882E-01 |
| 1.00000E-01 | 3.44647E-01 | 3.78407E-01 |
| 5.00000E-02 | 3.00042E-01 | 3.29553E-01 |
| 4.80000E-02 | 2.98370E-01 | 3.27496E-01 |
| 4.60000E-02 | 2.96709E-01 | 3.25428E-01 |
| 4.40000E-02 | 2.95060E-01 | 3.23348E-01 |
| 4.20000E-02 | 2.93424E-01 | 3.21257E-01 |
| 4.00000E-02 | 2.91800E-01 | 3.19153E-01 |
| 3.80000E-02 | 2.90191E-01 | 3.17035E-01 |
| 3.60000E-02 | 2.88596E-01 | 3.14904E-01 |
| 3.40000E-02 | 2.87017E-01 | 3.12757E-01 |
| 3.20000E-02 | 2.85455E-01 | 3.10594E-01 |
| 3.00000E-02 | 2.83910E-01 | 3.08413E-01 |
| 2.80000E-02 | 2.82386E-01 | 3.06213E-01 |
| 2.60000E-02 | 2.80882E-01 | 3.03993E-01 |
| 2.40000E-02 | 2.79402E-01 | 3.01749E-01 |
| 2.20000E-02 | 2.77948E-01 | 2.99480E-01 |
| 2.00000E-02 | 2.76523E-01 | 2.97183E-01 |
| 1.80000E-02 | 2.75130E-01 | 2.94853E-01 |
| 1.60000E-02 | 2.73775E-01 | 2.92486E-01 |
| 1.40000E-02 | 2.72465E-01 | 2.90075E-01 |
| 1.20000E-02 | 2.71208E-01 | 2.87610E-01 |
| 1.00000E-02 | 2.70017E-01 | 2.85081E-01 |
| 8.00000E-03 | 2.68911E-01 | 2.82465E-01 |
| 6.00000E-03 | 2.67924E-01 | 2.79732E-01 |
| 4.00000E-03 | 2.67120E-01 | 2.76817E-01 |
| 2.00000E-03 | 2.66660E-01 | 2.73557E-01 |
| 1.00000E-03 | 2.66733E-01 | 2.71624E-01 |
| 0.00000E-99 | 2.68249E-01 | 2.68249E-01 |

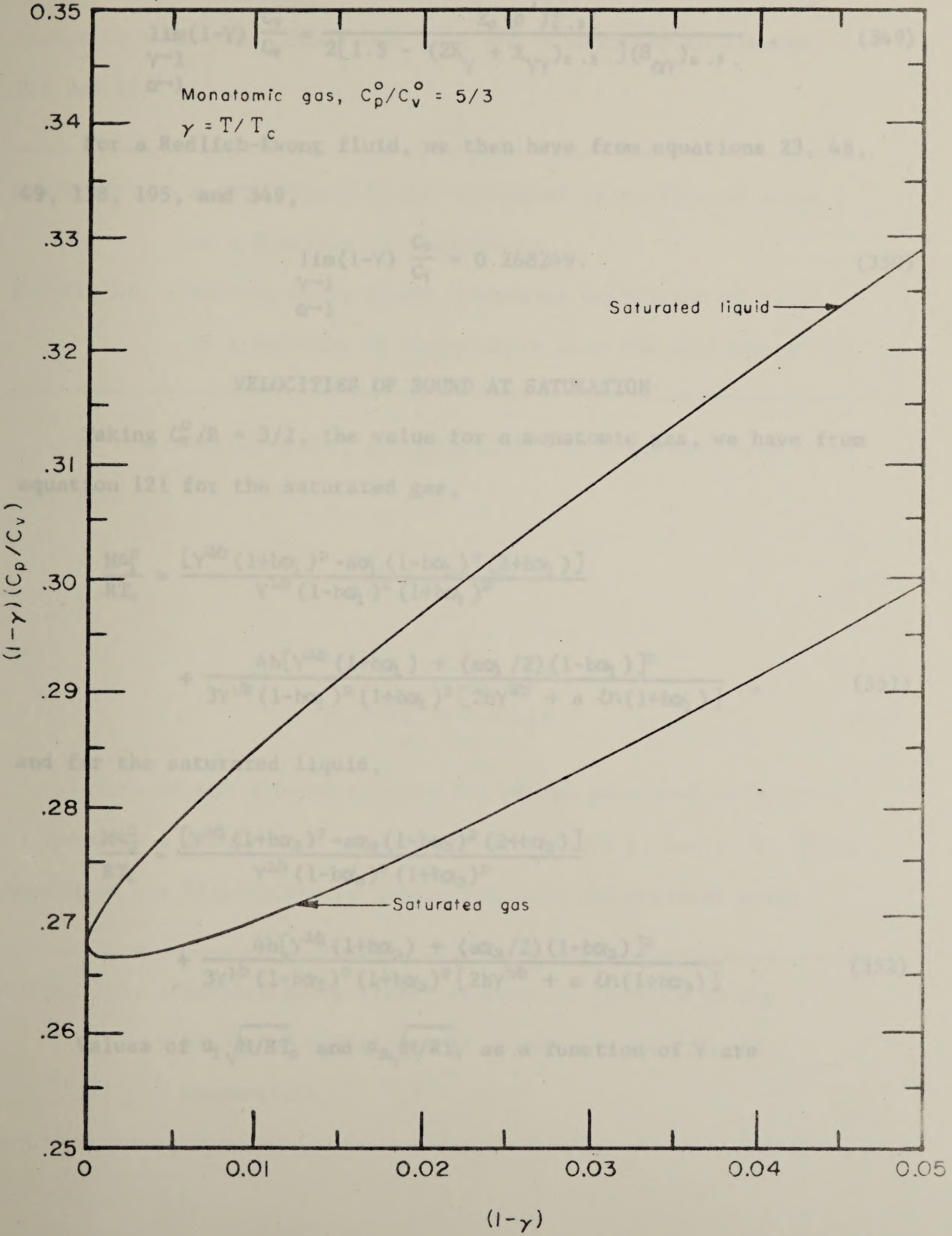

FIGURE 100.–Redlich–Kwong Fluid, Asymptotic Function of the Ratios of the Saturated Heat Capacities at Constant Pressure and at Constant Volume as a Function of Temperature.

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}}(1-\gamma)\,\frac{C_p}{C_v} = \frac{Z_c(\beta')^2_{c.p.}}{2[1.5-(2X_\gamma + X_{\gamma\gamma})_{c.p.}](\beta_{\alpha\gamma})_{c.p.}}\,. \tag{349}$$

For a Redlich-Kwong fluid, we then have from equations 23, 48, 49, 178, 195, and 349,

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}}(1-\gamma)\,\frac{C_p}{C_v} = 0.268249. \tag{350}$$

VELOCITIES OF SOUND AT SATURATION

Taking $C_v^0/R = 3/2$, the value for a monatomic gas, we have from equation 121 for the saturated gas,

$$\frac{Ma_1^2}{RT_c} = \frac{[\gamma^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)]}{\gamma^{1/2}(1-b\alpha_1)^2(1+b\alpha_1)^2}$$

$$+\frac{4b[\gamma^{3/2}(1+b\alpha_1) + (a\alpha_1/2)(1-b\alpha_1)]^2}{3\gamma^{1/2}(1-b\alpha_1)^2(1+b\alpha_1)^2[2b\gamma^{3/2} + a\,\ln(1+b\alpha_1)]}\,, \tag{351}$$

and for the saturated liquid,

$$\frac{Ma_3^2}{RT_c} = \frac{[\gamma^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)]}{\gamma^{1/2}(1-b\alpha_3)^2(1+b\alpha_3)^2}$$

$$+\frac{4b[\gamma^{3/2}(1+b\alpha_3) + (a\alpha_3/2)(1-b\alpha_3)]^2}{3\gamma^{1/2}(1-b\alpha_3)^2(1+b\alpha_3)^2[2b\gamma^{3/2} + a\,\ln(1+b\alpha_3)]}\,. \tag{352}$$

Values of $a_1\sqrt{M/RT_c}$ and $a_3\sqrt{M/RT_c}$ as a function of $\gamma$ are

listed in table 62. These same data are illustrated in figures 101 and 102.

---

FIGURE 101. - Redlich-Kwong fluid, saturated velocities of sound as a function of temperature.

FIGURE 102. - Redlich-Kwong fluid, saturated velocities of sound as a function of temperature near the critical point.

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At the critical point, assuming $C_V^0/R = 3/2$, we have from equation 122,

$$\frac{Ma_c^2}{RT_c} = 1.46927 \tag{353}$$

or,

$$a_c\sqrt{M/RT_c} = 1.21213 \quad . \tag{354}$$

Values of the asymptotic function of the saturated velocities of sound as a function of temperature are listed in table 63. These same data are illustrated in figure 103 near the critical point.

---

FIGURE 103. - Redlich-Kwong fluid, asymptotic function of the saturated velocities of sound as a function of temperature.

---

TABLE 62. - REDLICH-KWONG FLUID, SATURATED VELOCITIES OF SOUND AS A FUNCTION OF TEMPERATURE

$a$ = velocity

M = molecular weight

$Y = T/T_c$

$C_p^0/C_v^0 = 5/3$

| $Y$ | $a_1\sqrt{\frac{M}{RT_c}}$ | $a_3\sqrt{\frac{M}{RT_c}}$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | $+\infty$ |
| 1.00000E-01 | 4.08248E-01 | 2.46086E+01 |
| 1.50000E-01 | 4.99999E-01 | 1.63633E+01 |
| 2.00000E-01 | 5.77350E-01 | 1.22275E+01 |
| 2.50000E-01 | 6.45497E-01 | 9.73370E-00 |
| 3.00000E-01 | 7.07106E-01 | 8.05897E-00 |
| 3.50000E-01 | 7.63756E-01 | 6.85066E-00 |
| 4.00000E-01 | 8.16410E-01 | 5.93227E-00 |
| 4.50000E-01 | 8.65472E-01 | 5.20560E-00 |
| 5.00000E-01 | 9.10723E-01 | 4.61160E-00 |
| 5.50000E-01 | 9.51467E-01 | 4.11269E-00 |
| 6.00000E-01 | 9.86880E-01 | 3.68379E-00 |
| 6.50000E-01 | 1.01630E-00 | 3.30751E-00 |
| 7.00000E-01 | 1.03941E-00 | 2.97130E-00 |
| 7.50000E-01 | 1.05618E-00 | 2.66565E-00 |
| 8.00000E-01 | 1.06705E-00 | 2.38286E-00 |
| 8.50000E-01 | 1.07312E-00 | 2.11598E-00 |
| 9.00000E-01 | 1.07709E-00 | 1.85726E-00 |
| 9.50000E-01 | 1.08742E-00 | 1.59357E-00 |
| 9.52000E-01 | 1.08829E-00 | 1.58253E-00 |
| 9.54000E-01 | 1.08923E-00 | 1.57142E-00 |
| 9.56000E-01 | 1.09024E-00 | 1.56023E-00 |
| 9.58000E-01 | 1.09132E-00 | 1.54895E-00 |
| 9.60000E-01 | 1.09250E-00 | 1.53758E-00 |
| 9.62000E-01 | 1.09376E-00 | 1.52611E-00 |
| 9.64000E-01 | 1.09514E-00 | 1.51453E-00 |
| 9.66000E-01 | 1.09662E-00 | 1.50283E-00 |
| 9.68000E-01 | 1.09824E-00 | 1.49099E-00 |
| 9.70000E-01 | 1.10000E-00 | 1.47899E-00 |
| 9.72000E-01 | 1.10192E-00 | 1.46683E-00 |
| 9.74000E-01 | 1.10402E-00 | 1.45448E-00 |
| 9.76000E-01 | 1.10633E-00 | 1.44191E-00 |
| 9.78000E-01 | 1.10888E-00 | 1.42909E-00 |
| 9.80000E-01 | 1.11170E-00 | 1.41599E-00 |
| 9.82000E-01 | 1.11484E-00 | 1.40256E-00 |
| 9.84000E-01 | 1.11837E-00 | 1.38873E-00 |
| 9.86000E-01 | 1.12237E-00 | 1.37442E-00 |
| 9.88000E-01 | 1.12694E-00 | 1.35952E-00 |
| 9.90000E-01 | 1.13226E-00 | 1.34387E-00 |
| 9.92000E-01 | 1.13857E-00 | 1.32721E-00 |
| 9.94000E-01 | 1.14630E-00 | 1.30912E-00 |
| 9.96000E-01 | 1.15627E-00 | 1.28878E-00 |
| 9.98000E-01 | 1.17064E-00 | 1.26403E-00 |
| 9.99000E-01 | 1.18177E-00 | 1.24770E-00 |
| 1.00000E-00 | 1.21213E-00 | 1.21213E-00 |

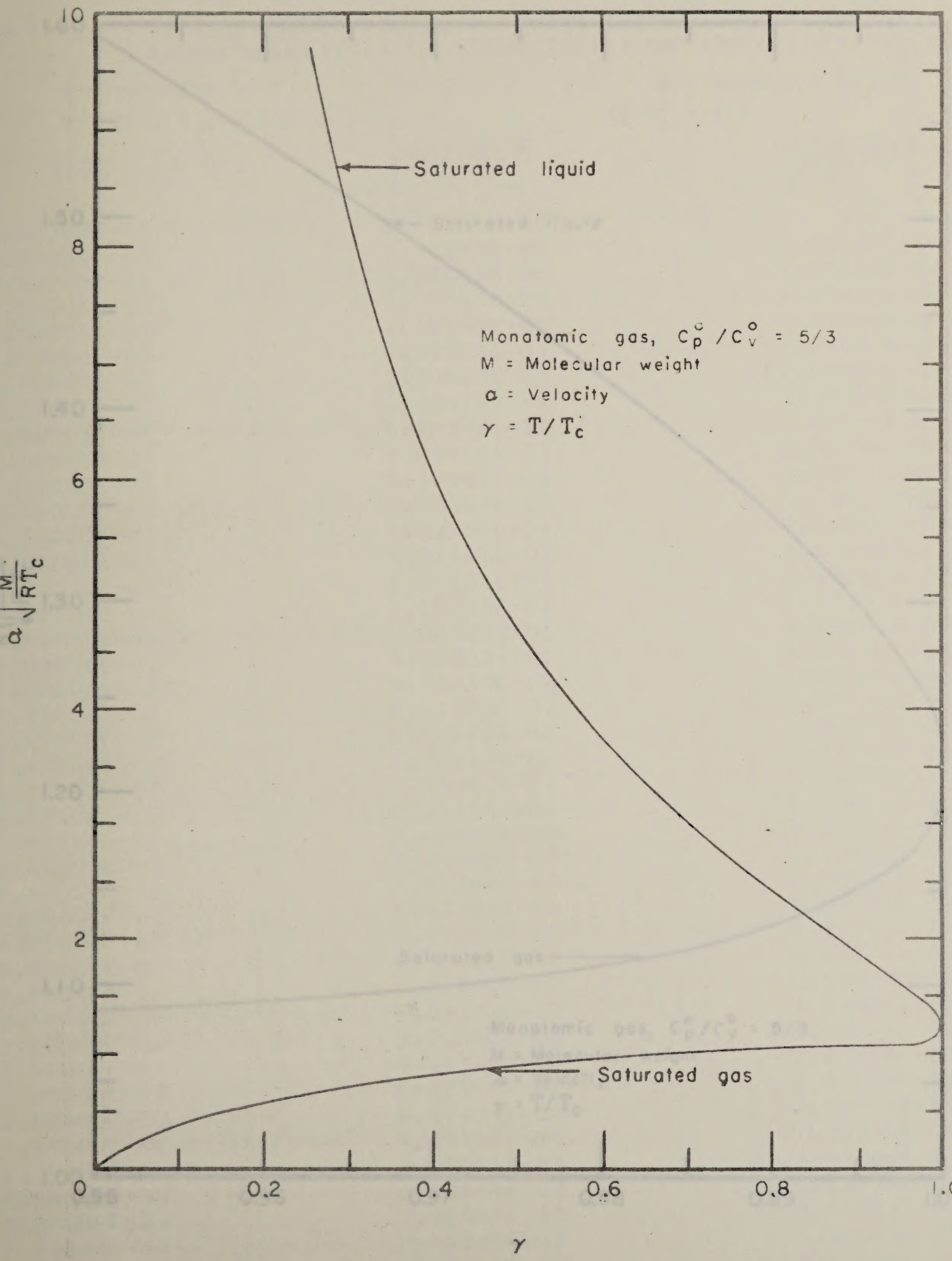

FIGURE 101.—Redlich–Kwong Fluid, Saturated Velocities of Sound as a Function of Temperature.

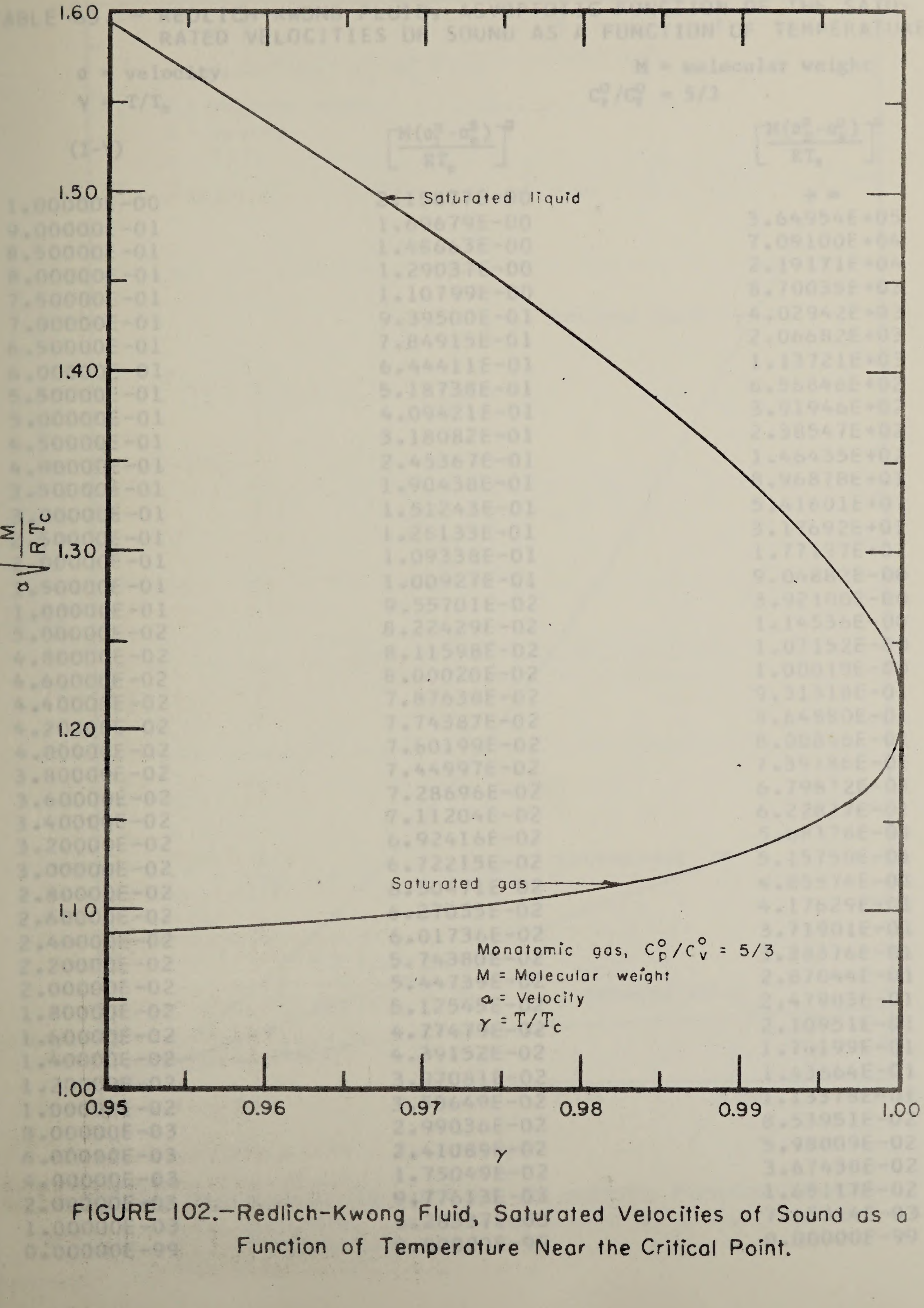

FIGURE 102.—Redlich-Kwong Fluid, Saturated Velocities of Sound as a Function of Temperature Near the Critical Point.

TABLE 63. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED VELOCITIES OF SOUND AS A FUNCTION OF TEMPERATURE

$a$ = velocity

$\gamma = T/T_c$

M = molecular weight

$C_p^o/C_v^o = 5/3$

| $(1-\gamma)$ | $\left[\frac{M(a_1^2-a_c^2)}{RT_c}\right]^2$ | $\left[\frac{M(a_3^2-a_c^2)}{RT_c}\right]^2$ |
|---|---|---|
| 1.00000E-00 | 2.15877E-00 | + ∞ |
| 9.00000E-01 | 1.69679E-00 | 3.64954E+05 |
| 8.50000E-01 | 1.48663E-00 | 7.09100E+04 |
| 8.00000E-01 | 1.29037E-00 | 2.19171E+04 |
| 7.50000E-01 | 1.10799E-00 | 8.70035E+03 |
| 7.00000E-01 | 9.39500E-01 | 4.02942E+03 |
| 6.50000E-01 | 7.84915E-01 | 2.06682E+03 |
| 6.00000E-01 | 6.44411E-01 | 1.13721E+03 |
| 5.50000E-01 | 5.18738E-01 | 6.56846E+02 |
| 5.00000E-01 | 4.09421E-01 | 3.91946E+02 |
| 4.50000E-01 | 3.18082E-01 | 2.38547E+02 |
| 4.00000E-01 | 2.45367E-01 | 1.46435E+02 |
| 3.50000E-01 | 1.90438E-01 | 8.96878E+01 |
| 3.00000E-01 | 1.51243E-01 | 5.41601E+01 |
| 2.50000E-01 | 1.25133E-01 | 3.17692E+01 |
| 2.00000E-01 | 1.09338E-01 | 1.77137E+01 |
| 1.50000E-01 | 1.00927E-01 | 9.04882E-00 |
| 1.00000E-01 | 9.55701E-02 | 3.92100E-00 |
| 5.00000E-02 | 8.22429E-02 | 1.14536E-00 |
| 4.80000E-02 | 8.11598E-02 | 1.07152E-00 |
| 4.60000E-02 | 8.00020E-02 | 1.00019E-00 |
| 4.40000E-02 | 7.87638E-02 | 9.31318E-01 |
| 4.20000E-02 | 7.74387E-02 | 8.64880E-01 |
| 4.00000E-02 | 7.60199E-02 | 8.00846E-01 |
| 3.80000E-02 | 7.44997E-02 | 7.39186E-01 |
| 3.60000E-02 | 7.28696E-02 | 6.79872E-01 |
| 3.40000E-02 | 7.11204E-02 | 6.22877E-01 |
| 3.20000E-02 | 6.92416E-02 | 5.68178E-01 |
| 3.00000E-02 | 6.72215E-02 | 5.15750E-01 |
| 2.80000E-02 | 6.50471E-02 | 4.65574E-01 |
| 2.60000E-02 | 6.27035E-02 | 4.17629E-01 |
| 2.40000E-02 | 6.01736E-02 | 3.71901E-01 |
| 2.20000E-02 | 5.74380E-02 | 3.28376E-01 |
| 2.00000E-02 | 5.44739E-02 | 2.87044E-01 |
| 1.80000E-02 | 5.12545E-02 | 2.47903E-01 |
| 1.60000E-02 | 4.77479E-02 | 2.10951E-01 |
| 1.40000E-02 | 4.39152E-02 | 1.76199E-01 |
| 1.20000E-02 | 3.97081E-02 | 1.43664E-01 |
| 1.00000E-02 | 3.50649E-02 | 1.13378E-01 |
| 8.00000E-03 | 2.99036E-02 | 8.53951E-02 |
| 6.00000E-03 | 2.41089E-02 | 5.98009E-02 |
| 4.00000E-03 | 1.75049E-02 | 3.67438E-02 |
| 2.00000E-03 | 9.77613E-03 | 1.65117E-02 |
| 1.00000E-03 | 5.28337E-03 | 7.65314E-03 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

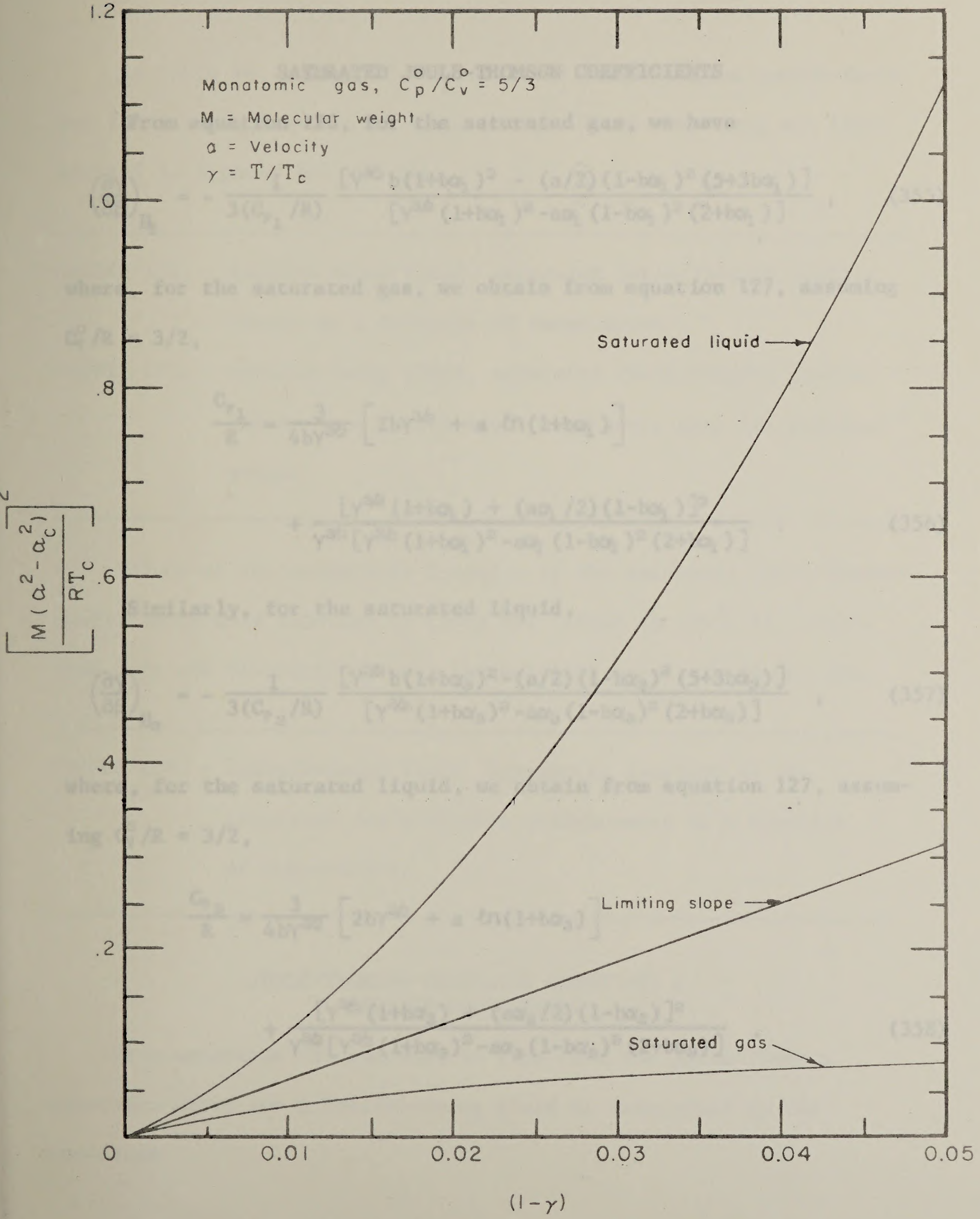

FIGURE 103.-Redlich-Kwong Fluid, Asymptotic Function of the Saturated Velocities of Sound as a Function of Temperature.

## SATURATED JOULE-THOMSON COEFFICIENTS

From equation 126, for the saturated gas, we have

$$\left(\frac{\partial\gamma}{\partial\beta}\right)_{H_1} = -\frac{1}{3(C_{p_1}/R)}\,\frac{[\gamma^{3/2} b(1+b\alpha_1)^2 - (a/2)(1-b\alpha_1)^2(5+3b\alpha_1)]}{[\gamma^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)]}\ , \tag{355}$$

where, for the saturated gas, we obtain from equation 127, assuming $C_V^0/R = 3/2$,

$$\frac{C_{p_1}}{R} = \frac{3}{4b\gamma^{3/2}}\left[2b\gamma^{3/2} + a\ \ln(1+b\alpha_1)\right] + \frac{[\gamma^{3/2}(1+b\alpha_1) + (a\alpha_1/2)(1-b\alpha_1)]^2}{\gamma^{3/2}[\gamma^{3/2}(1+b\alpha_1)^2 - a\alpha_1(1-b\alpha_1)^2(2+b\alpha_1)]}\ . \tag{356}$$

Similarly, for the saturated liquid,

$$\left(\frac{\partial\gamma}{\partial\beta}\right)_{H_3} = -\frac{1}{3(C_{p_3}/R)}\,\frac{[\gamma^{3/2} b(1+b\alpha_3)^2 - (a/2)(1-b\alpha_3)^2(5+3b\alpha_3)]}{[\gamma^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)]}\ , \tag{357}$$

where, for the saturated liquid, we obtain from equation 127, assuming $C_V^0/R = 3/2$,

$$\frac{C_{p_3}}{R} = \frac{3}{4b\gamma^{3/2}}\left[2b\gamma^{3/2} + a\ \ln(1+b\alpha_3)\right] + \frac{[\gamma^{3/2}(1+b\alpha_3) + (a\alpha_3/2)(1-b\alpha_3)]^2}{\gamma^{3/2}[\gamma^{3/2}(1+b\alpha_3)^2 - a\alpha_3(1-b\alpha_3)^2(2+b\alpha_3)]}\ . \tag{358}$$

In table 64, values of the saturated Joule-Thomson coefficients are listed as a function of temperature. These same data are illustrated in figures 104 and 105.

---

FIGURE 104. - Redlich-Kwong fluid, saturated Joule-Thomson coefficients as a function of temperature.

FIGURE 105. - Redlich-Kwong fluid, saturated Joule-Thomson coefficients as a function of temperature near the critical point.

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Values of the asymptotic function of the saturated Joule-Thomson coefficients as a function of (1-Y) are listed in table 65. These same data are illustrated, near the critical point, in figure 106.

---

FIGURE 106. - Redlich-Kwong fluid, asymptotic function of the saturated Joule-Thomson coefficients as a function of temperature.

---

## JOULE-THOMSON INVERSION CURVE FOR A REDLICH-KWONG FLUID

From equations 155 and 156, we find that the Joule-Thomson inversion curve for a Redlich-Kwong fluid is determined by the equations

TABLE 64. - REDLICH-KWONG FLUID, SATURATED JOULE-THOMSON COEFFICIENTS AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\beta = P/P_c$ | $C_p^o/C_v^o = 5/3$ |
|---|---|---|
| $\gamma$ | $\left(\frac{\partial\gamma}{\partial\beta}\right)_{H_1}$ | $\left(\frac{\partial\gamma}{\partial\beta}\right)_{H_3}$ |
| 0.00000E-99 | + ∞ | 0.00000E-99 |
| 1.00000E-01 | 1.34834E+01 | -1.02248E-03 |
| 1.50000E-01 | 7.32367E-00 | -1.80985E-03 |
| 2.00000E-01 | 4.74471E-00 | -2.66396E-03 |
| 2.50000E-01 | 3.38518E-00 | -3.53234E-03 |
| 3.00000E-01 | 2.56690E-00 | -4.37004E-03 |
| 3.50000E-01 | 2.02985E-00 | -5.13477E-03 |
| 4.00000E-01 | 1.65523E-00 | -5.78309E-03 |
| 4.50000E-01 | 1.38203E-00 | -6.26627E-03 |
| 5.00000E-01 | 1.17606E-00 | -6.52490E-03 |
| 5.50000E-01 | 1.01661E-00 | -6.48105E-03 |
| 6.00000E-01 | 8.90226E-01 | -6.02630E-03 |
| 6.50000E-01 | 7.87627E-01 | -5.00191E-03 |
| 7.00000E-01 | 7.02157E-01 | -3.16406E-03 |
| 7.50000E-01 | 6.28751E-01 | -1.17990E-04 |
| 8.00000E-01 | 5.63147E-01 | 4.81828E-03 |
| 8.50000E-01 | 5.01053E-01 | 1.29321E-02 |
| 9.00000E-01 | 4.36747E-01 | 2.70101E-02 |
| 9.50000E-01 | 3.58860E-01 | 5.48602E-02 |
| 9.52000E-01 | 3.55113E-01 | 5.65430E-02 |
| 9.54000E-01 | 3.51289E-01 | 5.82949E-02 |
| 9.56000E-01 | 3.47382E-01 | 6.01208E-02 |
| 9.58000E-01 | 3.43386E-01 | 6.20263E-02 |
| 9.60000E-01 | 3.39294E-01 | 6.40172E-02 |
| 9.62000E-01 | 3.35100E-01 | 6.61005E-02 |
| 9.64000E-01 | 3.30796E-01 | 6.82840E-02 |
| 9.66000E-01 | 3.26371E-01 | 7.05764E-02 |
| 9.68000E-01 | 3.21815E-01 | 7.29878E-02 |
| 9.70000E-01 | 3.17116E-01 | 7.55301E-02 |
| 9.72000E-01 | 3.12260E-01 | 7.82167E-02 |
| 9.74000E-01 | 3.07230E-01 | 8.10639E-02 |
| 9.76000E-01 | 3.02006E-01 | 8.40909E-02 |
| 9.78000E-01 | 2.96565E-01 | 8.73209E-02 |
| 9.80000E-01 | 2.90876E-01 | 9.07826E-02 |
| 9.82000E-01 | 2.84904E-01 | 9.45119E-02 |
| 9.84000E-01 | 2.78601E-01 | 9.85549E-02 |
| 9.86000E-01 | 2.71907E-01 | 1.02972E-01 |
| 9.88000E-01 | 2.64737E-01 | 1.07847E-01 |
| 9.90000E-01 | 2.56971E-01 | 1.13299E-01 |
| 9.92000E-01 | 2.48426E-01 | 1.19511E-01 |
| 9.94000E-01 | 2.38795E-01 | 1.26789E-01 |
| 9.96000E-01 | 2.27481E-01 | 1.35729E-01 |
| 9.98000E-01 | 2.12954E-01 | 1.47859E-01 |
| 9.99000E-01 | 2.02856E-01 | 1.56750E-01 |
| 1.00000E-00 | 1.79197E-01 | 1.79197E-01 |

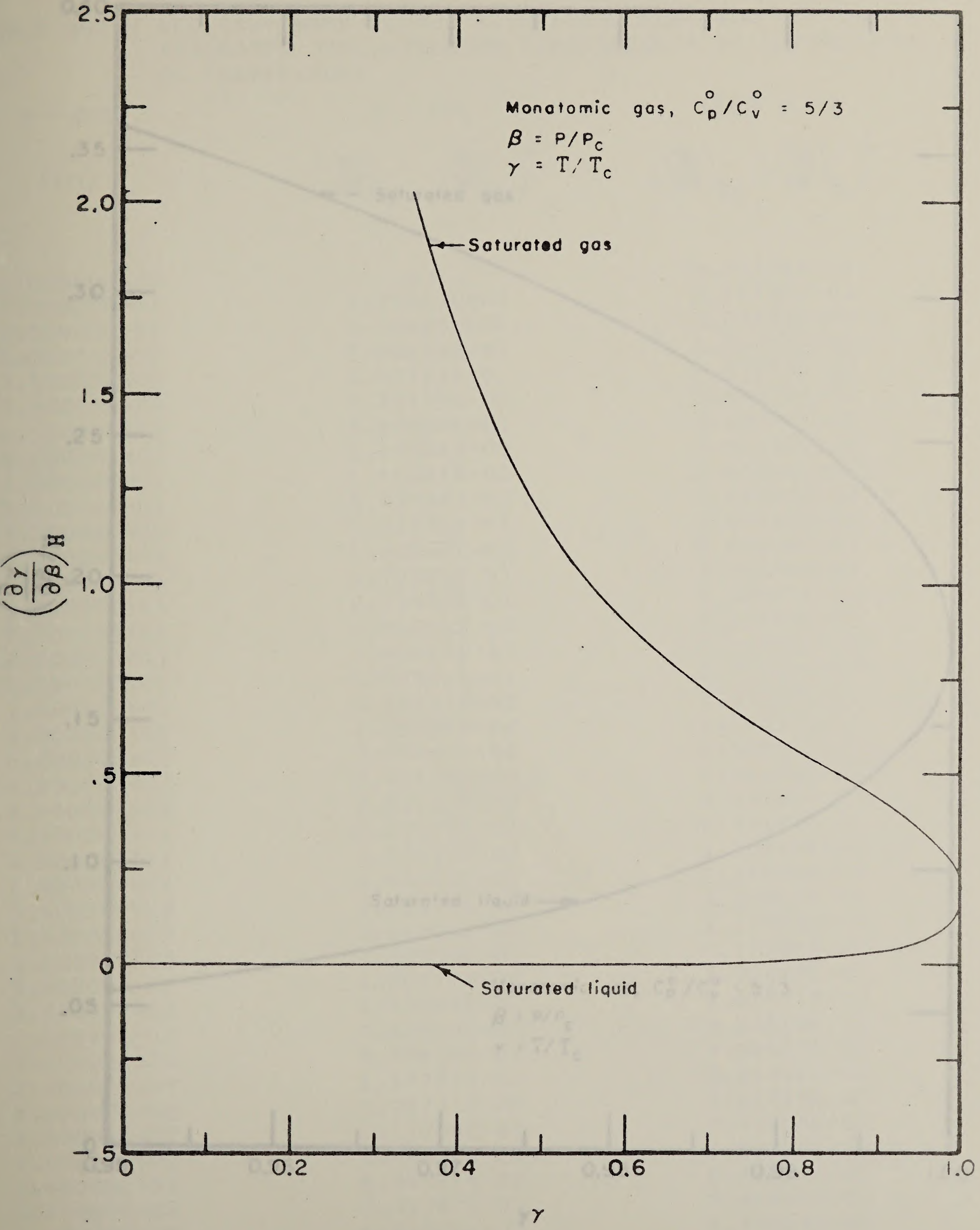

FIGURE 104.-Redlich-Kwong Fluid, Saturated Joule-Thomson Coefficients as a Function of Temperature.

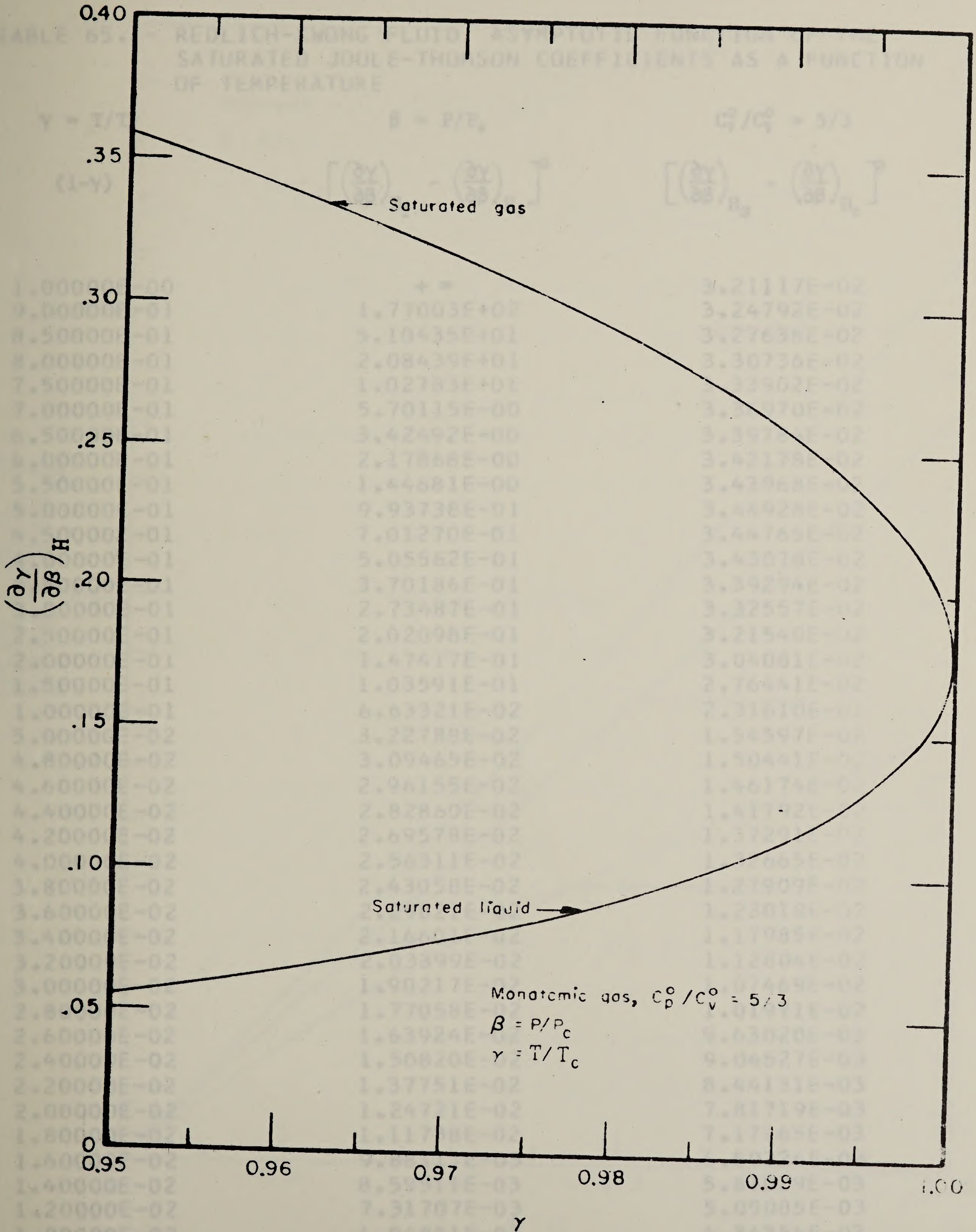

FIGURE 105.—Redlich-Kwong Fluid, Saturated Joule-Thomson Coefficients as a Function of Temperature Near the Critical Point.

TABLE 65. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE SATURATED JOULE-THOMSON COEFFICIENTS AS A FUNCTION OF TEMPERATURE

| $\gamma = T/T_c$ | $\beta = P/P_c$ | $C_p^o/C_v^o = 5/3$ |
|---|---|---|
| $(1-\gamma)$ | $\left[\left(\frac{\partial\gamma}{\partial\beta}\right)_{H_1} - \left(\frac{\partial\gamma}{\partial\beta}\right)_{H_c}\right]^2$ | $\left[\left(\frac{\partial\gamma}{\partial\beta}\right)_{H_3} - \left(\frac{\partial\gamma}{\partial\beta}\right)_{H_c}\right]^2$ |
| 1.00000E-00 | + ∞ | 3.21117E-02 |
| 9.00000E-01 | 1.77003E+02 | 3.24792E-02 |
| 8.50000E-01 | 5.10435E+01 | 3.27636E-02 |
| 8.00000E-01 | 2.08439E+01 | 3.30736E-02 |
| 7.50000E-01 | 1.02783E+01 | 3.33902E-02 |
| 7.00000E-01 | 5.70115E-00 | 3.36970E-02 |
| 6.50000E-01 | 3.42492E-00 | 3.39784E-02 |
| 6.00000E-01 | 2.17868E-00 | 3.42178E-02 |
| 5.50000E-01 | 1.44681E-00 | 3.43968E-02 |
| 5.00000E-01 | 9.93738E-01 | 3.44928E-02 |
| 4.50000E-01 | 7.01270E-01 | 3.44765E-02 |
| 4.00000E-01 | 5.05562E-01 | 3.43078E-02 |
| 3.50000E-01 | 3.70186E-01 | 3.39294E-02 |
| 3.00000E-01 | 2.73487E-01 | 3.32557E-02 |
| 2.50000E-01 | 2.02098E-01 | 3.21540E-02 |
| 2.00000E-01 | 1.47417E-01 | 3.04081E-02 |
| 1.50000E-01 | 1.03591E-01 | 2.76441E-02 |
| 1.00000E-01 | 6.63321E-02 | 2.31610E-02 |
| 5.00000E-02 | 3.22788E-02 | 1.54597E-02 |
| 4.80000E-02 | 3.09465E-02 | 1.50441E-02 |
| 4.60000E-02 | 2.96155E-02 | 1.46174E-02 |
| 4.40000E-02 | 2.82860E-02 | 1.41792E-02 |
| 4.20000E-02 | 2.69578E-02 | 1.37291E-02 |
| 4.00000E-02 | 2.56311E-02 | 1.32665E-02 |
| 3.80000E-02 | 2.43058E-02 | 1.27909E-02 |
| 3.60000E-02 | 2.29821E-02 | 1.23018E-02 |
| 3.40000E-02 | 2.16601E-02 | 1.17985E-02 |
| 3.20000E-02 | 2.03399E-02 | 1.12804E-02 |
| 3.00000E-02 | 1.90217E-02 | 1.07469E-02 |
| 2.80000E-02 | 1.77058E-02 | 1.01971E-02 |
| 2.60000E-02 | 1.63924E-02 | 9.63020E-03 |
| 2.40000E-02 | 1.50820E-02 | 9.04527E-03 |
| 2.20000E-02 | 1.37751E-02 | 8.44131E-03 |
| 2.00000E-02 | 1.24721E-02 | 7.81719E-03 |
| 1.80000E-02 | 1.11738E-02 | 7.17165E-03 |
| 1.60000E-02 | 9.88117E-03 | 6.50324E-03 |
| 1.40000E-02 | 8.59511E-03 | 5.81029E-03 |
| 1.20000E-02 | 7.31707E-03 | 5.09085E-03 |
| 1.00000E-02 | 6.04881E-03 | 4.34254E-03 |
| 8.00000E-03 | 4.79265E-03 | 3.56241E-03 |
| 6.00000E-03 | 3.55186E-03 | 2.74657E-03 |
| 4.00000E-03 | 2.33131E-03 | 1.88950E-03 |
| 2.00000E-03 | 1.13951E-03 | 9.82049E-04 |
| 1.00000E-03 | 5.59755E-04 | 5.03856E-04 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

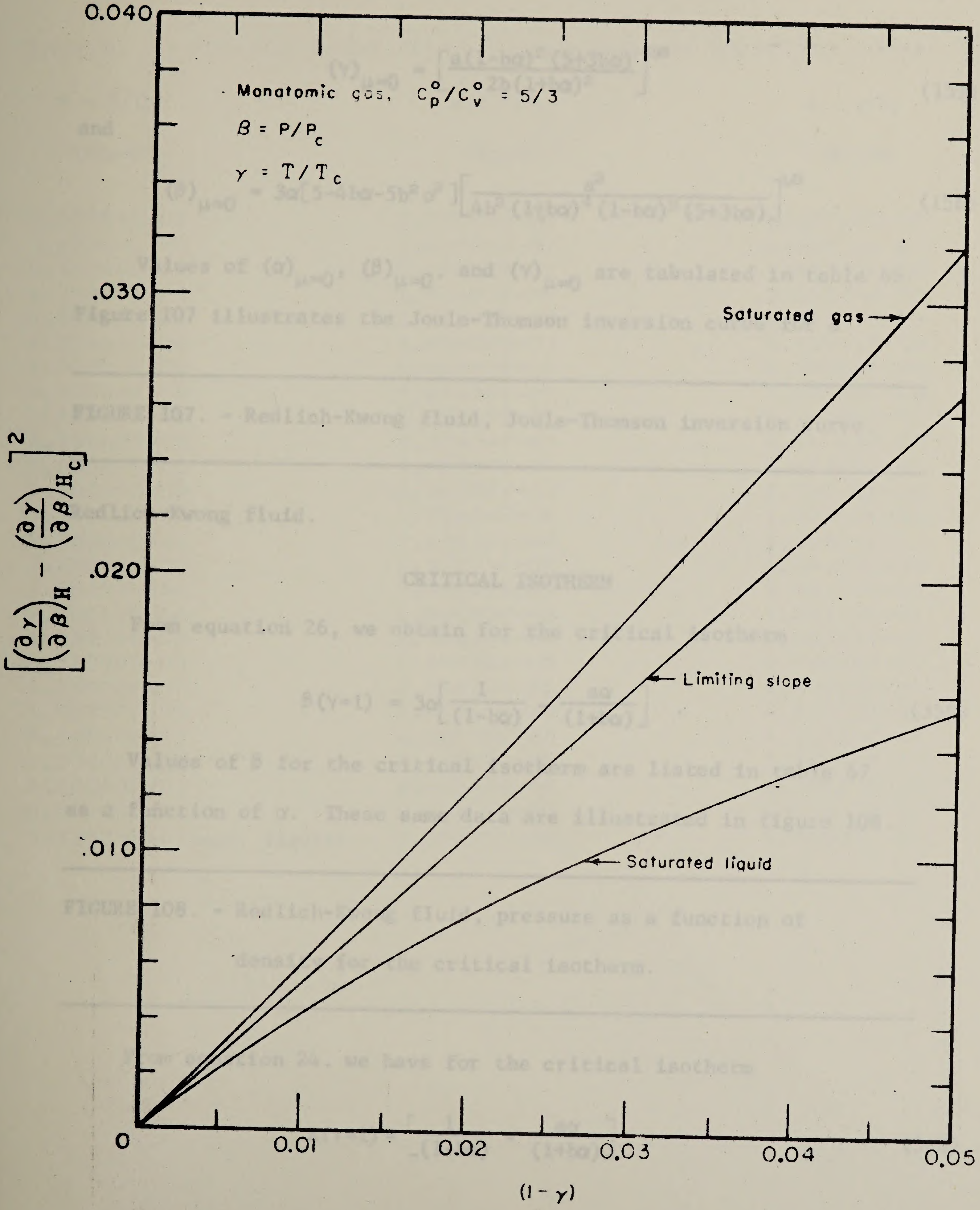

FIGURE 106.–Redlich-Kwong Fluid, Asymptotic Function of the Saturated Joule-Thomson Coefficients as a Function of Temperature.

$$(\gamma)_{\mu=0} = \left[\frac{a(1-b\alpha)^2(5+3b\alpha)}{2b(1+b\alpha)^2}\right]^{2/3} \qquad (155)$$

and

$$(\beta)_{\mu=0} = 3\alpha[5-4b\alpha-5b^2\alpha^2]\left[\frac{a^2}{4b^2(1+b\alpha)^4(1-b\alpha)^2(5+3b\alpha)}\right]^{1/3} . \qquad (156)$$

Values of $(\alpha)_{\mu=0}$, $(\beta)_{\mu=0}$, and $(\gamma)_{\mu=0}$ are tabulated in table 66. Figure 107 illustrates the Joule-Thomson inversion curve for a Redlich-Kwong fluid.

FIGURE 107. - Redlich-Kwong fluid, Joule-Thomson inversion curve.

## CRITICAL ISOTHERM

From equation 26, we obtain for the critical isotherm

$$\beta(\gamma=1) = 3\alpha\left[\frac{1}{(1-b\alpha)} - \frac{a\alpha}{(1+b\alpha)}\right] . \qquad (359)$$

Values of β for the critical isotherm are listed in table 67 as a function of α. These same data are illustrated in figure 108.

FIGURE 108. - Redlich-Kwong fluid, pressure as a function of density for the critical isotherm.

From equation 24, we have for the critical isotherm

$$Z(\gamma=1) = \left[\frac{1}{(1-b\alpha)} - \frac{a\alpha}{(1+b\alpha)}\right] . \qquad (360)$$

TABLE 66. - REDLICH-KWONG FLUID, JOULE-THOMSON INVERSION CURVE

| $Y = T/T_c$<br>$(Y)_{\mu=0}$ | $\alpha = \rho/\rho_c$<br>$(\alpha)_{\mu=0}$ | $\beta = P/P_c$<br>$(\beta)_{\mu=0}$ |
|---|---|---|
| 5.33855E-00 | 0.00000E-99 | 0.00000E-99 |
| 5.25000E-00 | 2.83804E-02 | 4.48972E-01 |
| 5.00000E-00 | 1.11002E-01 | 1.69389E-00 |
| 4.75000E-00 | 1.97537E-01 | 2.90174E-00 |
| 4.50000E-00 | 2.88292E-01 | 4.06697E-00 |
| 4.25000E-00 | 3.83605E-01 | 5.18291E-00 |
| 4.00000E-00 | 4.83860E-01 | 6.24154E-00 |
| 3.75000E-00 | 5.89490E-01 | 7.23308E-00 |
| 3.50000E-00 | 7.00989E-01 | 8.14546E-00 |
| 3.25000E-00 | 8.18924E-01 | 8.96360E-00 |
| 3.00000E-00 | 9.43952E-01 | 9.66838E-00 |
| 2.89265E-00 | 1.00000E-00 | 9.93032E-00 |
| 2.75000E-00 | 1.07684E-00 | 1.02350E+01 |
| 2.50000E-00 | 1.21851E-00 | 1.06310E+01 |
| 2.25000E-00 | 1.37005E-00 | 1.08122E+01 |
| 2.20101E-00 | 1.40102E-00 | 1.08177E+01 |
| 2.00000E-00 | 1.53283E-00 | 1.07171E+01 |
| 1.75000E-00 | 1.70853E-00 | 1.02563E+01 |
| 1.50000E-00 | 1.89936E-00 | 9.29402E-00 |
| 1.25000E-00 | 2.10827E-00 | 7.60985E-00 |
| 1.00000E-00 | 2.33947E-00 | 4.81331E-00 |
| 9.50000E-01 | 2.38890E-00 | 4.06326E-00 |
| 9.00000E-01 | 2.43953E-00 | 3.23096E-00 |
| 8.50000E-01 | 2.49144E-00 | 2.30533E-00 |
| 8.00000E-01 | 2.54471E-00 | 1.27301E-00 |
| 7.51525E-01(sat. liquid) | 2.59775E-00 | 1.54951E-01 |

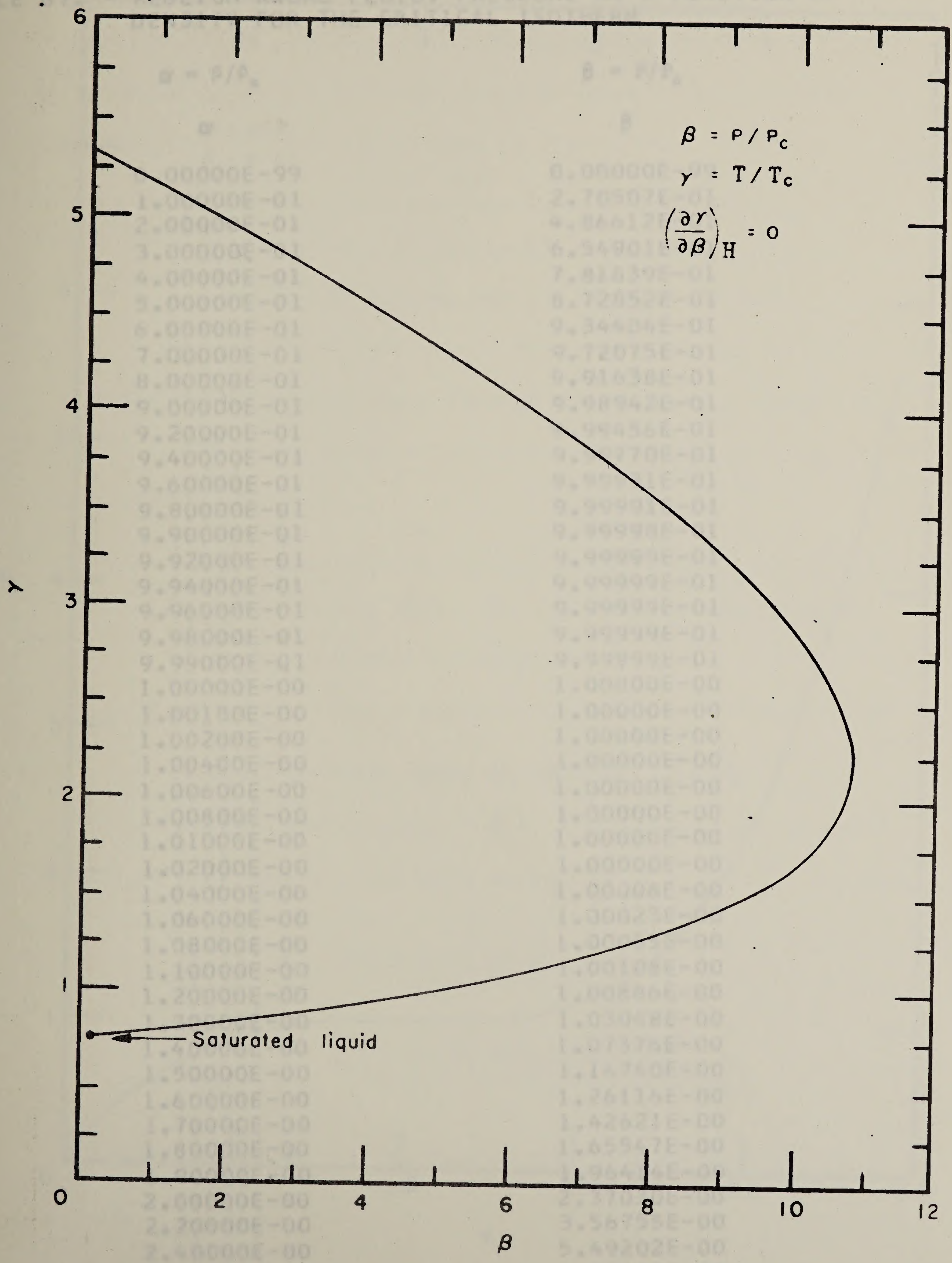

FIGURE 107. – Redlich-Kwong Fluid, Joule-Thomson Inversion Curve.

## TABLE 67. – REDLICH-KWONG FLUID, PRESSURE AS A FUNCTION OF DENSITY FOR THE CRITICAL ISOTHERM

$\alpha = \rho/\rho_c$ $\qquad$ $\beta = P/P_c$

| $\alpha$ | $\beta$ |
|---|---|
| 0.00000E-99 | 0.00000E-99 |
| 1.00000E-01 | 2.70507E-01 |
| 2.00000E-01 | 4.86612E-01 |
| 3.00000E-01 | 6.54901E-01 |
| 4.00000E-01 | 7.81639E-01 |
| 5.00000E-01 | 8.72852E-01 |
| 6.00000E-01 | 9.34404E-01 |
| 7.00000E-01 | 9.72075E-01 |
| 8.00000E-01 | 9.91638E-01 |
| 9.00000E-01 | 9.98942E-01 |
| 9.20000E-01 | 9.99456E-01 |
| 9.40000E-01 | 9.99770E-01 |
| 9.60000E-01 | 9.99931E-01 |
| 9.80000E-01 | 9.99991E-01 |
| 9.90000E-01 | 9.99998E-01 |
| 9.92000E-01 | 9.99999E-01 |
| 9.94000E-01 | 9.99999E-01 |
| 9.96000E-01 | 9.99999E-01 |
| 9.98000E-01 | 9.99999E-01 |
| 9.99000E-01 | 9.99999E-01 |
| 1.00000E-00 | 1.00000E-00 |
| 1.00100E-00 | 1.00000E-00 |
| 1.00200E-00 | 1.00000E-00 |
| 1.00400E-00 | 1.00000E-00 |
| 1.00600E-00 | 1.00000E-00 |
| 1.00800E-00 | 1.00000E-00 |
| 1.01000E-00 | 1.00000E-00 |
| 1.02000E-00 | 1.00000E-00 |
| 1.04000E-00 | 1.00006E-00 |
| 1.06000E-00 | 1.00023E-00 |
| 1.08000E-00 | 1.00055E-00 |
| 1.10000E-00 | 1.00108E-00 |
| 1.20000E-00 | 1.00886E-00 |
| 1.30000E-00 | 1.03048E-00 |
| 1.40000E-00 | 1.07376E-00 |
| 1.50000E-00 | 1.14740E-00 |
| 1.60000E-00 | 1.26116E-00 |
| 1.70000E-00 | 1.42621E-00 |
| 1.80000E-00 | 1.65547E-00 |
| 1.90000E-00 | 1.96414E-00 |
| 2.00000E-00 | 2.37030E-00 |
| 2.20000E-00 | 3.56755E-00 |
| 2.40000E-00 | 5.49202E-00 |
| 2.60000E-00 | 8.53909E-00 |
| 2.80000E-00 | 1.33995E+01 |
| 3.00000E-00 | 2.14097E+01 |
| 3.20000E-00 | 3.55494E+01 |
| 3.40000E-00 | 6.41179E+01 |
| 3.60000E-00 | 1.42245E+02 |
| 3.84732E-00 | + ∞ |

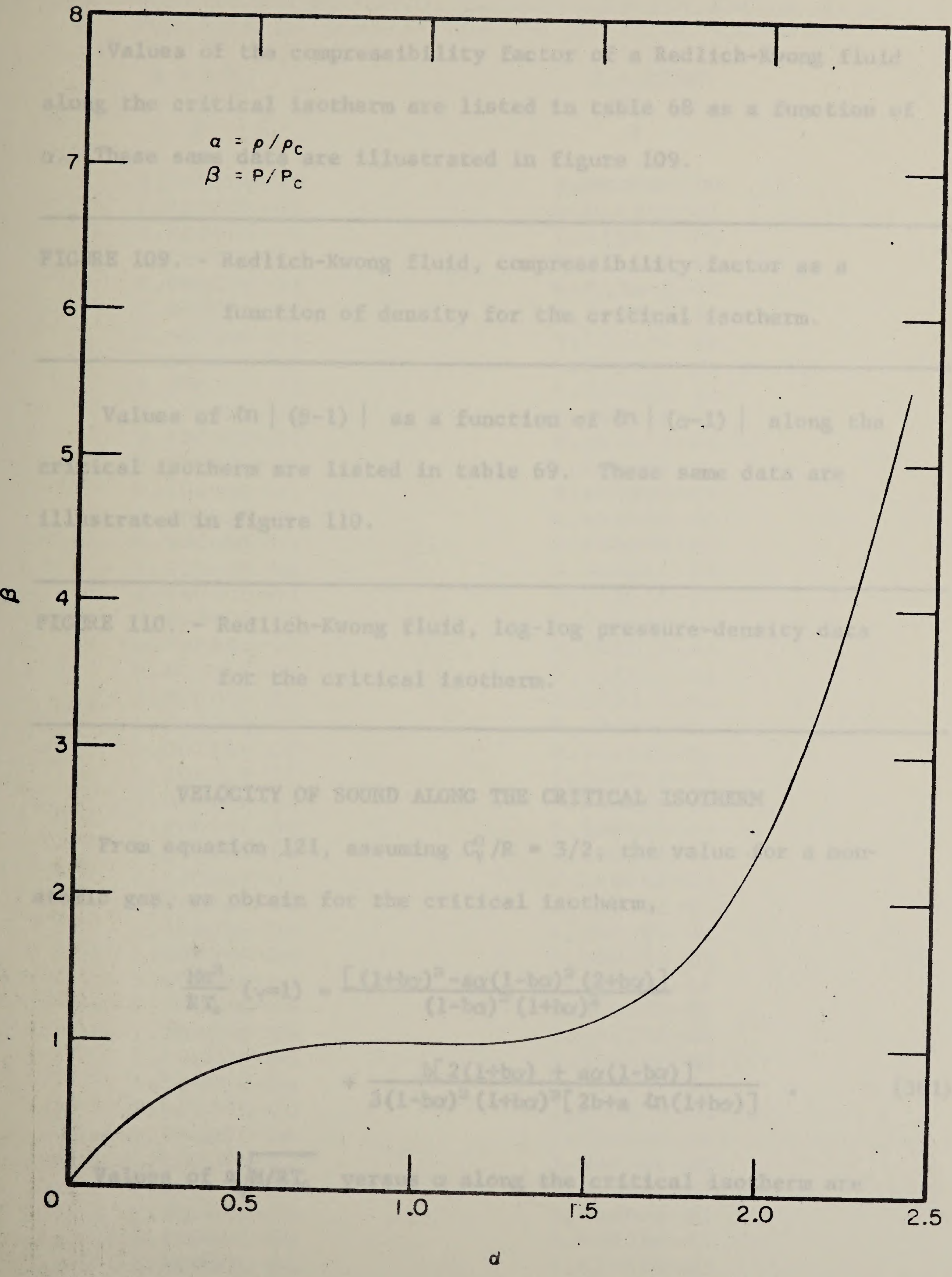

FIGURE 108.—Redlich-Kwong Fluid, Pressure as a Function of Density for the Critical Isotherm.

Values of the compressibility factor of a Redlich-Kwong fluid along the critical isotherm are listed in table 68 as a function of $\alpha$. These same data are illustrated in figure 109.

---

FIGURE 109. - Redlich-Kwong fluid, compressibility factor as a function of density for the critical isotherm.

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Values of $\ln|(\beta-1)|$ as a function of $\ln|(\alpha-1)|$ along the critical isotherm are listed in table 69. These same data are illustrated in figure 110.

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FIGURE 110. - Redlich-Kwong fluid, log-log pressure-density data for the critical isotherm.

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## VELOCITY OF SOUND ALONG THE CRITICAL ISOTHERM

From equation 121, assuming $C_V^0/R = 3/2$, the value for a monatomic gas, we obtain for the critical isotherm,

$$\frac{Ma^2}{RT_c}(\gamma=1) = \frac{[(1+b\alpha)^2 - a\alpha(1-b\alpha)^2(2+b\alpha)]}{(1-b\alpha)^2(1+b\alpha)^2} + \frac{b[2(1+b\alpha) + a\alpha(1-b\alpha)]}{3(1-b\alpha)^2(1+b\alpha)^2[2b+a\ \ln(1+b\alpha)]} \ . \qquad (361)$$

Values of $a\sqrt{M/RT_c}$ versus $\alpha$ along the critical isotherm are

## TABLE 68. – REDLICH-KWONG FLUID, COMPRESSIBILITY FACTOR AS A FUNCTION OF DENSITY FOR THE CRITICAL ISOTHERM

| $\alpha = \rho/\rho_c$ | $Z = P/\rho RT$ |
|---|---|
| $\alpha$ | Z |
| 0.00000E-99 | 1.00000E-00 |
| 1.00000E-01 | 9.01690E-01 |
| 2.00000E-01 | 8.11021E-01 |
| 3.00000E-01 | 7.27668E-01 |
| 4.00000E-01 | 6.51366E-01 |
| 5.00000E-01 | 5.81901E-01 |
| 6.00000E-01 | 5.19113E-01 |
| 7.00000E-01 | 4.62893E-01 |
| 8.00000E-01 | 4.13182E-01 |
| 9.00000E-01 | 3.69978E-01 |
| 9.20000E-01 | 3.62122E-01 |
| 9.40000E-01 | 3.54528E-01 |
| 9.60000E-01 | 3.47198E-01 |
| 9.80000E-01 | 3.40133E-01 |
| 9.90000E-01 | 3.36699E-01 |
| 9.92000E-01 | 3.36021E-01 |
| 9.94000E-01 | 3.35345E-01 |
| 9.96000E-01 | 3.34671E-01 |
| 9.98000E-01 | 3.34001E-01 |
| 9.99000E-01 | 3.33666E-01 |
| 1.00000E-00 | 3.33333E-01 |
| 1.00100E-00 | 3.33000E-01 |
| 1.00200E-00 | 3.32668E-01 |
| 1.00400E-00 | 3.32005E-01 |
| 1.00600E-00 | 3.31345E-01 |
| 1.00800E-00 | 3.30688E-01 |
| 1.01000E-00 | 3.30033E-01 |
| 1.02000E-00 | 3.26800E-01 |
| 1.04000E-00 | 3.20534E-01 |
| 1.06000E-00 | 3.14538E-01 |
| 1.08000E-00 | 3.08813E-01 |
| 1.10000E-00 | 3.03360E-01 |
| 1.20000E-00 | 2.80239E-01 |
| 1.30000E-00 | 2.64225E-01 |
| 1.40000E-00 | 2.55659E-01 |
| 1.50000E-00 | 2.54979E-01 |
| 1.60000E-00 | 2.62743E-01 |
| 1.70000E-00 | 2.79650E-01 |
| 1.80000E-00 | 3.06570E-01 |
| 1.90000E-00 | 3.44586E-01 |
| 2.00000E-00 | 3.95050E-01 |
| 2.20000E-00 | 5.40538E-01 |
| 2.40000E-00 | 7.62781E-01 |
| 2.60000E-00 | 1.09475E-00 |
| 2.80000E-00 | 1.59519E-00 |
| 3.00000E-00 | 2.37886E-00 |
| 3.20000E-00 | 3.70306E-00 |
| 3.40000E-00 | 6.28607E-00 |
| 3.60000E-00 | 1.31708E+01 |
| 3.84732E-00 | + ∞ |

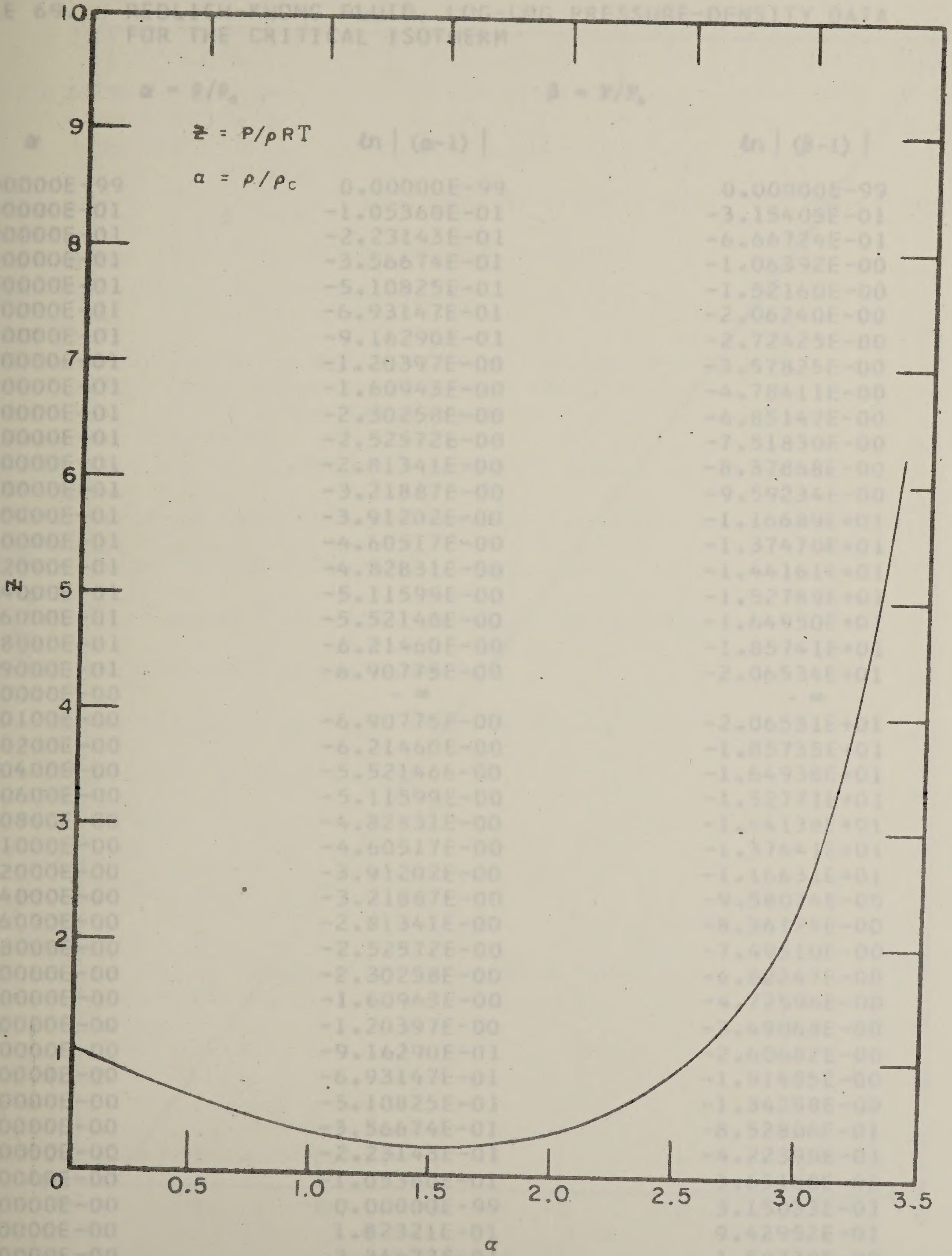

FIGURE 109.–Redlich-Kwong Fluid, Compressibility Factor as a Function of Density for the Critical Isotherm.

# TABLE 69. - REDLICH-KWONG FLUID, LOG-LOG PRESSURE-DENSITY DATA FOR THE CRITICAL ISOTHERM

$\alpha = \rho/\rho_c$ $\beta = P/P_c$

| $\alpha$ | $\ln\|(\alpha-1)\|$ | $\ln\|(\beta-1)\|$ |
|---|---|---|
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |
| 1.00000E-01 | -1.05360E-01 | -3.15405E-01 |
| 2.00000E-01 | -2.23143E-01 | -6.66724E-01 |
| 3.00000E-01 | -3.56674E-01 | -1.06392E-00 |
| 4.00000E-01 | -5.10825E-01 | -1.52160E-00 |
| 5.00000E-01 | -6.93147E-01 | -2.06240E-00 |
| 6.00000E-01 | -9.16290E-01 | -2.72425E-00 |
| 7.00000E-01 | -1.20397E-00 | -3.57825E-00 |
| 8.00000E-01 | -1.60943E-00 | -4.78411E-00 |
| 9.00000E-01 | -2.30258E-00 | -6.85147E-00 |
| 9.20000E-01 | -2.52572E-00 | -7.51830E-00 |
| 9.40000E-01 | -2.81341E-00 | -8.37868E-00 |
| 9.60000E-01 | -3.21887E-00 | -9.59234E-00 |
| 9.80000E-01 | -3.91202E-00 | -1.16689E+01 |
| 9.90000E-01 | -4.60517E-00 | -1.37470E+01 |
| 9.92000E-01 | -4.82831E-00 | -1.44161E+01 |
| 9.94000E-01 | -5.11599E-00 | -1.52789E+01 |
| 9.96000E-01 | -5.52146E-00 | -1.64950E+01 |
| 9.98000E-01 | -6.21460E-00 | -1.85741E+01 |
| 9.99000E-01 | -6.90775E-00 | -2.06534E+01 |
| 1.00000E-00 | $-\infty$ | $-\infty$ |
| 1.00100E-00 | -6.90775E-00 | -2.06531E+01 |
| 1.00200E-00 | -6.21460E-00 | -1.85735E+01 |
| 1.00400E-00 | -5.52146E-00 | -1.64938E+01 |
| 1.00600E-00 | -5.11599E-00 | -1.52771E+01 |
| 1.00800E-00 | -4.82831E-00 | -1.44138E+01 |
| 1.01000E-00 | -4.60517E-00 | -1.37441E+01 |
| 1.02000E-00 | -3.91202E-00 | -1.16631E+01 |
| 1.04000E-00 | -3.21887E-00 | -9.58074E-00 |
| 1.06000E-00 | -2.81341E-00 | -8.36128E-00 |
| 1.08000E-00 | -2.52572E-00 | -7.49510E-00 |
| 1.10000E-00 | -2.30258E-00 | -6.82247E-00 |
| 1.20000E-00 | -1.60943E-00 | -4.72596E-00 |
| 1.30000E-00 | -1.20397E-00 | -3.49068E-00 |
| 1.40000E-00 | -9.16290E-01 | -2.60682E-00 |
| 1.50000E-00 | -6.93147E-01 | -1.91455E-00 |
| 1.60000E-00 | -5.10825E-01 | -1.34258E-00 |
| 1.70000E-00 | -3.56674E-01 | -8.52806E-01 |
| 1.80000E-00 | -2.23143E-01 | -4.22390E-01 |
| 1.90000E-00 | -1.05360E-01 | -3.65160E-02 |
| 2.00000E-00 | 0.00000E-99 | 3.15033E-01 |
| 2.20000E-00 | 1.82321E-01 | 9.42952E-01 |
| 2.40000E-00 | 3.36472E-01 | 1.50230E-00 |
| 2.60000E-00 | 4.70003E-01 | 2.02010E-00 |
| 2.80000E-00 | 5.87786E-01 | 2.51766E-00 |
| 3.00000E-00 | 6.93147E-01 | 3.01601E-00 |
| 3.20000E-00 | 7.88457E-01 | 3.54239E-00 |
| 3.40000E-00 | 8.75468E-01 | 4.14500E-00 |
| 3.60000E-00 | 9.55511E-01 | 4.95049E-00 |
| 3.84732E-00 | 1.04637E-00 | $+\infty$ |

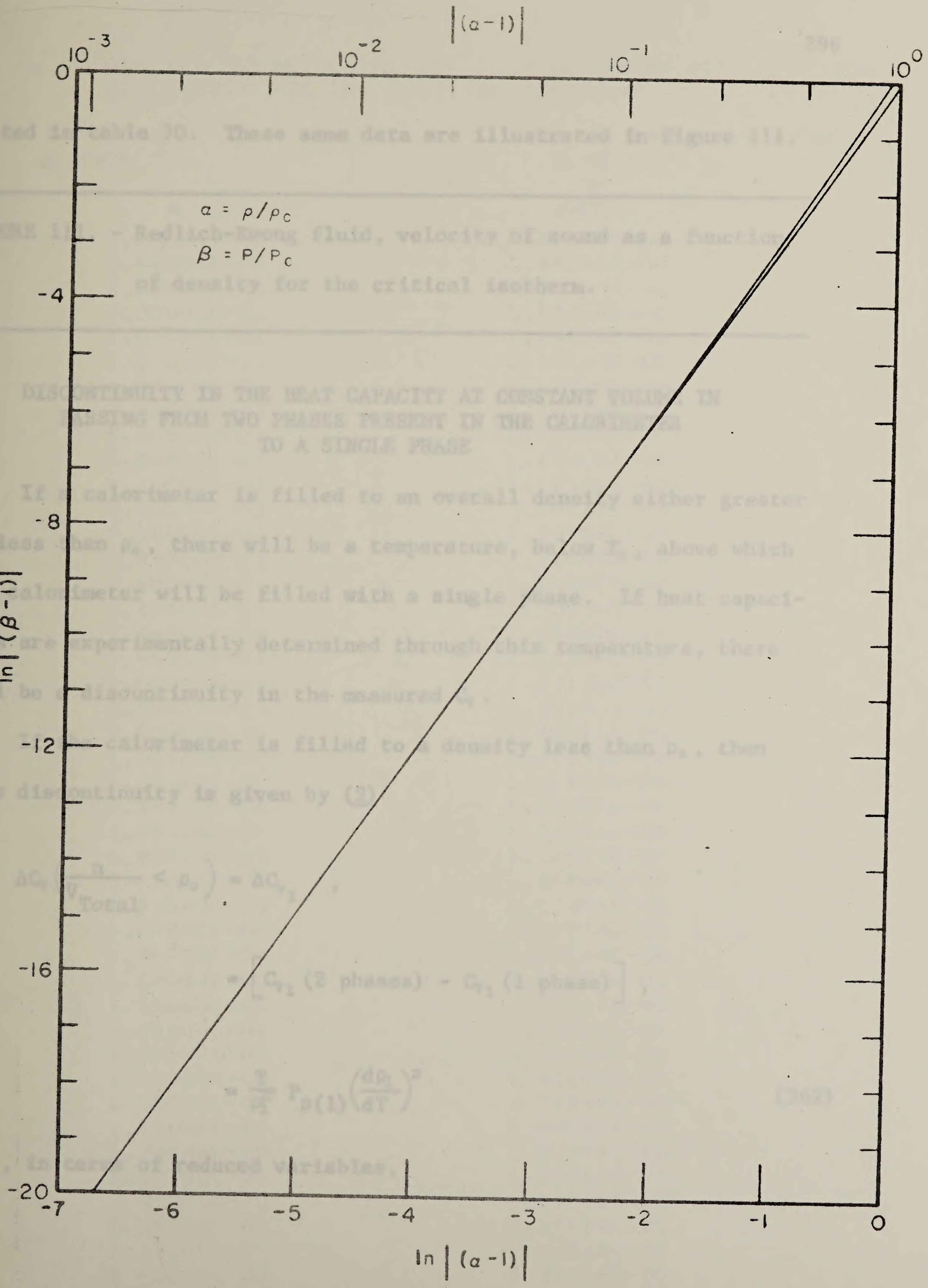

FIGURE 110.— Redlich—Kwong Fluid, Log—Log Pressure—Density Data for the Critical Isotherm.

listed in table 70. These same data are illustrated in figure 111.

---

FIGURE 111. - Redlich-Kwong fluid, velocity of sound as a function of density for the critical isotherm.

---

## DISCONTINUITY IN THE HEAT CAPACITY AT CONSTANT VOLUME IN PASSING FROM TWO PHASES PRESENT IN THE CALORIMETER TO A SINGLE PHASE

If a calorimeter is filled to an overall density either greater or less than $\rho_c$, there will be a temperature, below $T_c$, above which the calorimeter will be filled with a single phase. If heat capacities are experimentally determined through this temperature, there will be a discontinuity in the measured $C_V$.

If the calorimeter is filled to a density less than $\rho_c$, then this discontinuity is given by (2)

$$\Delta C_V\left(\frac{n}{V_{\text{Total}}} < \rho_c\right) = \Delta C_{V_1} \quad ,$$

$$= \left[C_{V_1}\text{ (2 phases)} - C_{V_1}\text{ (1 phase)}\right] \; ,$$

$$= \frac{T}{\rho_1^2} P_{\rho(1)} \left(\frac{d\rho_1}{dT}\right)^2 \qquad (362)$$

or, in terms of reduced variables,

## TABLE 70. - REDLICH-KWONG FLUID, VELOCITY OF SOUND AS A FUNCTION OF DENSITY FOR THE CRITICAL ISOTHERM

$a$ = velocity

$\alpha = \rho/\rho_c$

M = molecular weight

$C_p^o/C_v^o = 5/3$

| $\alpha$ | $a\sqrt{\frac{M}{RT_c}}$ |
|---|---|
| 0.00000E-99 | 1.29099E-00 |
| 1.00000E-01 | 1.24541E-00 |
| 2.00000E-01 | 1.20733E-00 |
| 3.00000E-01 | 1.17710E-00 |
| 4.00000E-01 | 1.15510E-00 |
| 5.00000E-01 | 1.14169E-00 |
| 6.00000E-01 | 1.13718E-00 |
| 7.00000E-01 | 1.14184E-00 |
| 8.00000E-01 | 1.15583E-00 |
| 9.00000E-01 | 1.17926E-00 |
| 9.20000E-01 | 1.18508E-00 |
| 9.40000E-01 | 1.19127E-00 |
| 9.60000E-01 | 1.19785E-00 |
| 9.80000E-01 | 1.20480E-00 |
| 9.90000E-01 | 1.20842E-00 |
| 9.92000E-01 | 1.20916E-00 |
| 9.94000E-01 | 1.20989E-00 |
| 9.96000E-01 | 1.21064E-00 |
| 9.98000E-01 | 1.21138E-00 |
| 9.99000E-01 | 1.21176E-00 |
| 1.00000E-00 | 1.21213E-00 |
| 1.00100E-00 | 1.21251E-00 |
| 1.00200E-00 | 1.21289E-00 |
| 1.00400E-00 | 1.21364E-00 |
| 1.00600E-00 | 1.21441E-00 |
| 1.00800E-00 | 1.21517E-00 |
| 1.01000E-00 | 1.21594E-00 |
| 1.02000E-00 | 1.21984E-00 |
| 1.04000E-00 | 1.22793E-00 |
| 1.06000E-00 | 1.23639E-00 |
| 1.08000E-00 | 1.24523E-00 |
| 1.10000E-00 | 1.25445E-00 |
| 1.20000E-00 | 1.30618E-00 |
| 1.30000E-00 | 1.36734E-00 |
| 1.40000E-00 | 1.43801E-00 |
| 1.50000E-00 | 1.51840E-00 |
| 1.60000E-00 | 1.60883E-00 |
| 1.70000E-00 | 1.70985E-00 |
| 1.80000E-00 | 1.82218E-00 |
| 1.90000E-00 | 1.94686E-00 |
| 2.00000E-00 | 2.08524E-00 |
| 2.20000E-00 | 2.41064E-00 |
| 2.40000E-00 | 2.81976E-00 |
| 2.60000E-00 | 3.34905E-00 |
| 2.80000E-00 | 4.06402E-00 |
| 3.00000E-00 | 5.09293E-00 |
| 3.20000E-00 | 6.72329E-00 |
| 3.40000E-00 | 9.75734E-00 |
| 3.60000E-00 | 1.75923E+01 |
| 3.84732E-00 | + ∞ |

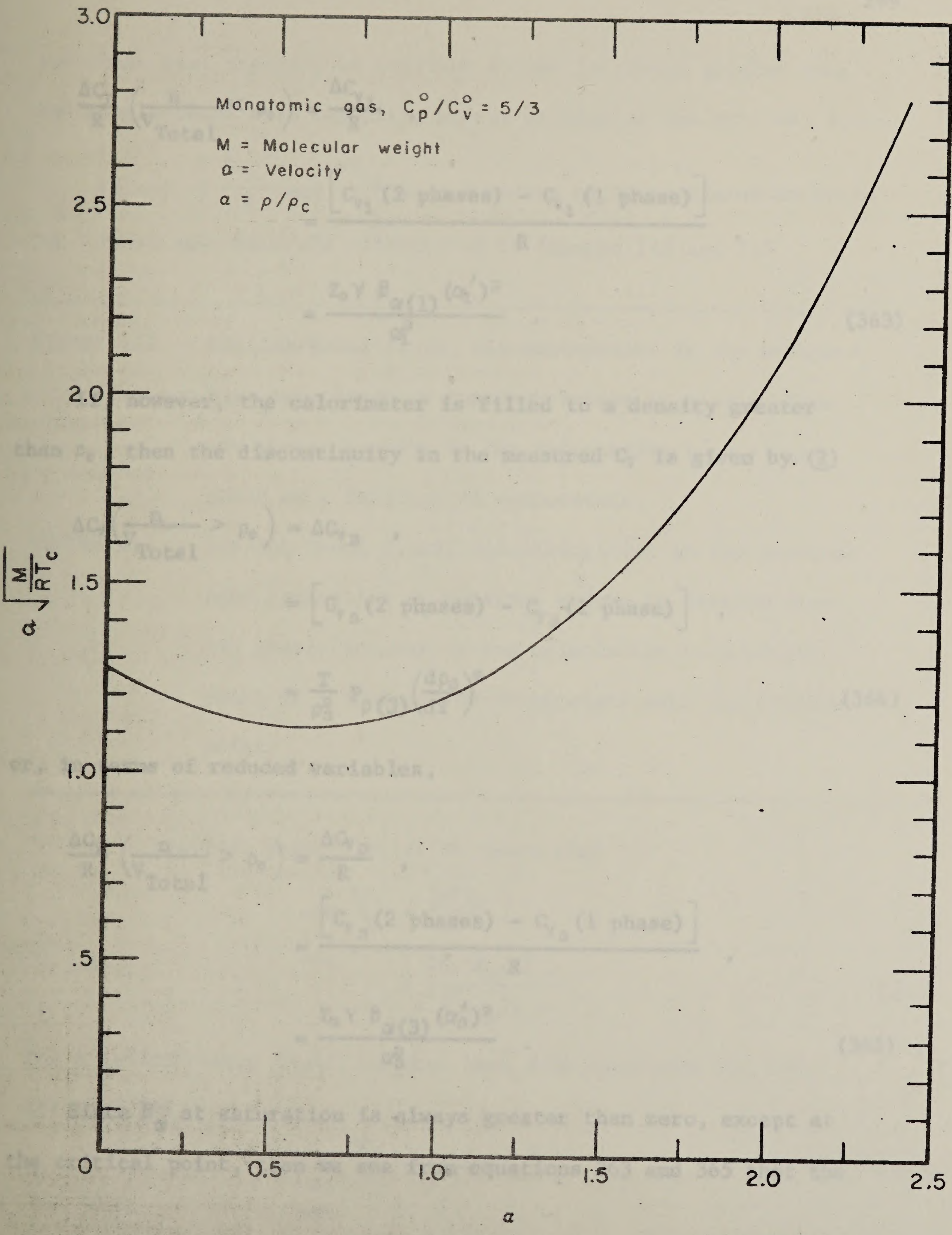

FIGURE 111.— Redlich—Kwong Fluid, Velocity of Sound as a Function of Density for the Critical Isotherm.

$$\frac{\Delta C_V}{R}\left(\frac{n}{V_{\text{Total}}} < \rho_c\right) = \frac{\Delta C_{V_1}}{R} ,$$

$$= \frac{\left[C_{V_1}(2 \text{ phases}) - C_{V_1}(1 \text{ phase})\right]}{R} ,$$

$$= \frac{Z_c \gamma \, \beta_{\alpha(1)} (\alpha_1')^2}{\alpha_1^2} . \tag{363}$$

If, however, the calorimeter is filled to a density greater than $\rho_c$, then the discontinuity in the measured $C_V$ is given by (2)

$$\Delta C_V \left(\frac{n}{V_{\text{Total}}} > \rho_c\right) = \Delta C_{V_3} ,$$

$$= \left[C_{V_3}(2 \text{ phases}) - C_{V_3}(1 \text{ phase})\right] ,$$

$$= \frac{T}{\rho_3^2} P_{\rho(3)} \left(\frac{d\rho_3}{dT}\right)^2 \tag{364}$$

or, in terms of reduced variables,

$$\frac{\Delta C_V}{R}\left(\frac{n}{V_{\text{Total}}} > \rho_c\right) = \frac{\Delta C_{V_3}}{R} ,$$

$$= \frac{\left[C_{V_3}(2 \text{ phases}) - C_{V_3}(1 \text{ phase})\right]}{R} ,$$

$$= \frac{Z_c \gamma \, \beta_{\alpha(3)} (\alpha_3')^2}{\alpha_3^2} . \tag{365}$$

Since $\beta_\alpha$ at saturation is always greater than zero, except at the critical point, then we see from equations 363 and 365 that the

two-phase heat capacity at constant volume is always greater than the single-phase heat capacity, except perhaps at the critical point.

Values of $\frac{\Delta C_{V_1}}{R}$ and $\frac{\Delta C_{V_3}}{R}$ as a function of $\gamma$ are listed in table 71. These same data are illustrated in figures 112 and 113.

---

FIGURE 112. - Redlich-Kwong fluid, discontinuities in the measured heat capacities at constant volume in passing from two phases present in the calorimeter to a single phase as a function of temperature.

FIGURE 113. - Redlich-Kwong fluid, discontinuities in the measured heat capacities at constant volume in passing from two phases present in the calorimeter to a single phase as a function of temperature near the critical point.

---

In a previous paper (1), it is shown that

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \frac{\Delta C_V}{R} = \frac{3\mathcal{A}^2 Z_c}{2\mathcal{B}} ,$$

$$= \frac{3Z_c\,(\beta_{\alpha\gamma})^2_{c.p.}}{(\beta_{\alpha\alpha\alpha})_{c.p.}} . \qquad (366)$$

For a Redlich-Kwong fluid, we then have from equations 23, 178, 179, and 366,

$$\lim_{\substack{\gamma\to 1\\ \alpha\to 1}} \frac{\Delta C_V}{R} = 10.4901 . \qquad (367)$$

TABLE 71. - REDLICH-KWONG FLUID, DISCONTINUITIES IN THE MEASURED HEAT CAPACITIES AT CONSTANT VOLUME IN PASSING FROM TWO PHASES PRESENT IN THE CALORIMETER TO A SINGLE PHASE AS A FUNCTION OF TEMPERATURE

$Y = T/T_c$ $\Delta C_V = C_V\,(2\text{ phases}) - C_V\,(1\text{ phase})$

| Y | $\frac{\Delta C_{V_1}}{R}$ | $\frac{\Delta C_{V_3}}{R}$ |
|---|---|---|
| 0.00000E-99 | + ∞ | 2.25000E+00 |
| 1.00000E-01 | 2.58271E+04 | 2.29450E-00 |
| 1.50000E-01 | 7.53000E+03 | 2.33377E-00 |
| 2.00000E-01 | 3.11491E+03 | 2.38290E-00 |
| 2.50000E-01 | 1.55850E+03 | 2.44243E-00 |
| 3.00000E-01 | 8.78333E+02 | 2.51355E-00 |
| 3.50000E-01 | 5.36718E+02 | 2.59813E-00 |
| 4.00000E-01 | 3.47639E+02 | 2.69883E-00 |
| 4.50000E-01 | 2.35403E+02 | 2.81901E-00 |
| 5.00000E-01 | 1.65326E+02 | 2.96244E-00 |
| 5.50000E-01 | 1.19869E+02 | 3.13304E-00 |
| 6.00000E-01 | 8.94097E+01 | 3.33510E-00 |
| 6.50000E-01 | 6.83582E+01 | 3.57394E-00 |
| 7.00000E-01 | 5.33522E+01 | 3.85709E-00 |
| 7.50000E-01 | 4.23156E+01 | 4.19608E-00 |
| 8.00000E-01 | 3.39326E+01 | 4.60968E-00 |
| 8.50000E-01 | 2.73368E+01 | 5.13129E-00 |
| 9.00000E-01 | 2.19127E+01 | 5.82946E-00 |
| 9.50000E-01 | 1.71059E+01 | 6.89113E-00 |
| 9.52000E-01 | 1.69158E+01 | 6.94795E-00 |
| 9.54000E-01 | 1.67252E+01 | 7.00650E-00 |
| 9.56000E-01 | 1.65338E+01 | 7.06690E-00 |
| 9.58000E-01 | 1.63415E+01 | 7.12929E-00 |
| 9.60000E-01 | 1.61482E+01 | 7.19381E-00 |
| 9.62000E-01 | 1.59537E+01 | 7.26063E-00 |
| 9.64000E-01 | 1.57577E+01 | 7.32996E-00 |
| 9.66000E-01 | 1.55601E+01 | 7.40202E-00 |
| 9.68000E-01 | 1.53606E+01 | 7.47706E-00 |
| 9.70000E-01 | 1.51588E+01 | 7.55539E-00 |
| 9.72000E-01 | 1.49545E+01 | 7.63736E-00 |
| 9.74000E-01 | 1.47471E+01 | 7.72338E-00 |
| 9.76000E-01 | 1.45361E+01 | 7.81395E-00 |
| 9.78000E-01 | 1.43210E+01 | 7.90969E-00 |
| 9.80000E-01 | 1.41009E+01 | 8.01134E-00 |
| 9.82000E-01 | 1.38750E+01 | 8.11987E-00 |
| 9.84000E-01 | 1.36419E+01 | 8.23648E-00 |
| 9.86000E-01 | 1.34001E+01 | 8.36282E-00 |
| 9.88000E-01 | 1.31472E+01 | 8.50113E-00 |
| 9.90000E-01 | 1.28801E+01 | 8.65466E-00 |
| 9.92000E-01 | 1.25936E+01 | 8.82840E-00 |
| 9.94000E-01 | 1.22795E+01 | 9.03078E-00 |
| 9.96000E-01 | 1.19211E+01 | 9.27831E-00 |
| 9.98000E-01 | 1.14756E+01 | 9.61379E-00 |
| 9.99000E-01 | 1.11744E+01 | 9.86033E-00 |
| 1.00000E-00 | 1.04901E+01 | 1.04901E+01 |

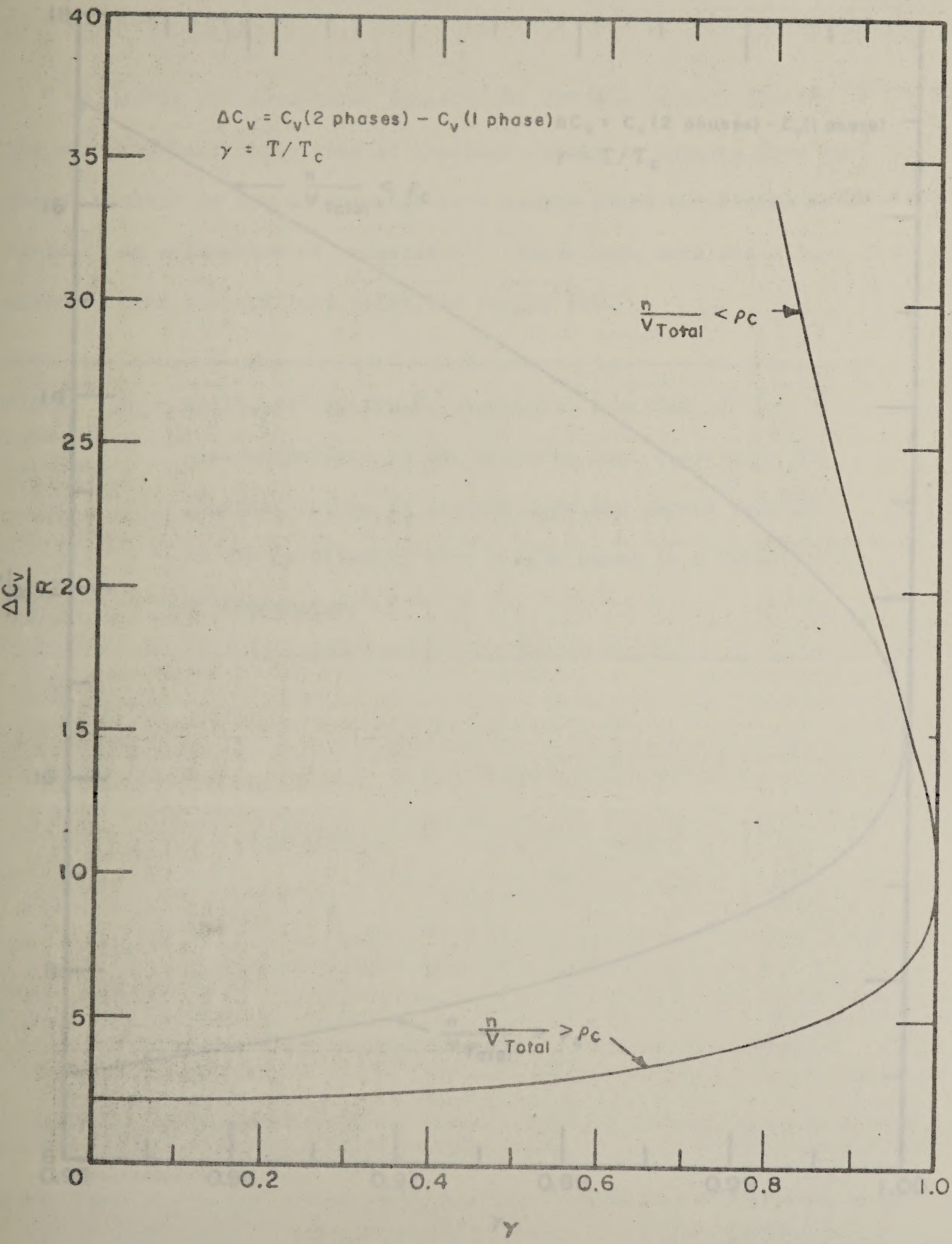

FIGURE 112.— Redlich-Kwong Fluid, Discontinuities in the Measured Heat Capacities at Constant Volume in Passing from Two Phases Present in the Calorimeter to a Single Phase as a Function of Temperature.

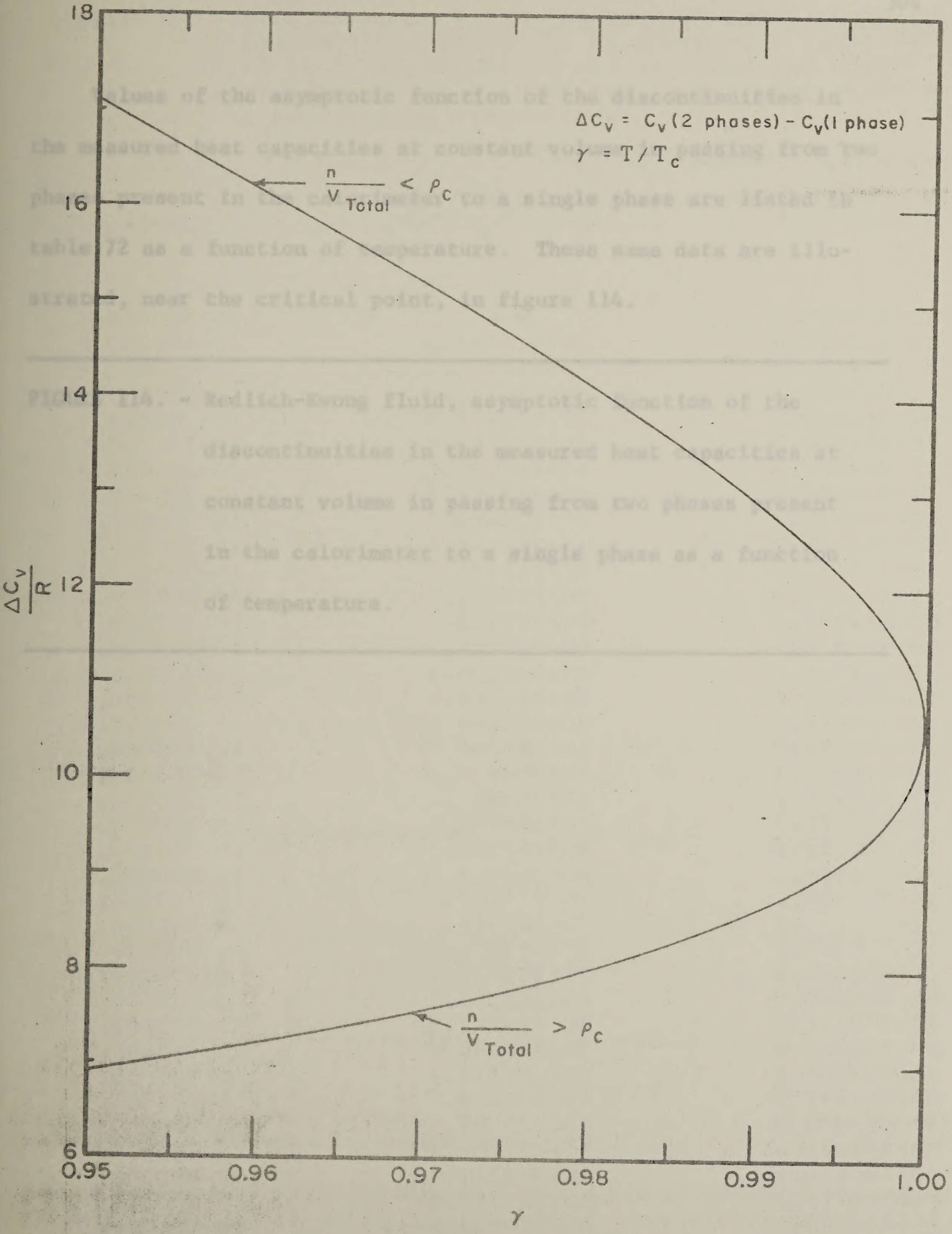

FIGURE 113.—Redlich-Kwong Fluid, Discontinuities in the Measured Heat Capacities at Constant Volume in Passing from Two Phases Present in the Calorimeter to a Single Phase as a Function of Temperature Near the Critical Point.

Values of the asymptotic function of the discontinuities in the measured heat capacities at constant volume in passing from two phases present in the calorimeter to a single phase are listed in table 72 as a function of temperature. These same data are illustrated, near the critical point, in figure 114.

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FIGURE 114. - Redlich-Kwong fluid, asymptotic function of the discontinuities in the measured heat capacities at constant volume in passing from two phases present in the calorimeter to a single phase as a function of temperature.

# TABLE 72. - REDLICH-KWONG FLUID, ASYMPTOTIC FUNCTION OF THE DISCONTINUITIES IN THE MEASURED HEAT CAPACITIES AT CONSTANT VOLUME IN PASSING FROM TWO PHASES PRESENT IN THE CALORIMETER TO A SINGLE PHASE AS A FUNCTION OF TEMPERATURE

$\gamma = T/T_c$ $\Delta C_V = C_V\,(2\text{ phases}) - C_V\,(1\text{ phase})$

| $(1-\gamma)$ | $\left[\frac{\Delta C_{V_1} - (\Delta C_V)_{c.p.}}{R}\right]^2$ | $\left[\frac{\Delta C_{V_3} - (\Delta C_V)_{c.p.}}{R}\right]^2$ |
|---|---|---|
| 1.00000E-00 | + ∞ | 6.79005E+01 |
| 9.00000E-01 | 6.66501E+08 | 6.71691E+01 |
| 8.50000E-01 | 5.65431E+07 | 6.65268E+01 |
| 8.00000E-01 | 9.63746E+06 | 6.57278E+01 |
| 7.50000E-01 | 2.39635E+06 | 6.47662E+01 |
| 7.00000E-01 | 7.53152E+05 | 6.36266E+01 |
| 6.50000E-01 | 2.76916E+05 | 6.22843E+01 |
| 6.00000E-01 | 1.13669E+05 | 6.07050E+01 |
| 5.50000E-01 | 5.05860E+04 | 5.88468E+01 |
| 5.00000E-01 | 2.39743E+04 | 5.66668E+01 |
| 4.50000E-01 | 1.19638E+04 | 5.41274E+01 |
| 4.00000E-01 | 6.22829E+03 | 5.11951E+01 |
| 3.50000E-01 | 3.34871E+03 | 4.78343E+01 |
| 3.00000E-01 | 1.83715E+03 | 4.39978E+01 |
| 2.50000E-01 | 1.01286E+03 | 3.96156E+01 |
| 2.00000E-01 | 5.49548E+02 | 3.45802E+01 |
| 1.50000E-01 | 2.83809E+02 | 2.87176E+01 |
| 1.00000E-01 | 1.30475E+02 | 2.17222E+01 |
| 5.00000E-02 | 4.37688E+01 | 1.29531E+01 |
| 4.80000E-02 | 4.12897E+01 | 1.25473E+01 |
| 4.60000E-02 | 3.88755E+01 | 1.21359E+01 |
| 4.40000E-02 | 3.65254E+01 | 1.17187E+01 |
| 4.20000E-02 | 3.42384E+01 | 1.12955E+01 |
| 4.00000E-02 | 3.20137E+01 | 1.08660E+01 |
| 3.80000E-02 | 2.98504E+01 | 1.04299E+01 |
| 3.60000E-02 | 2.77477E+01 | 9.98694E+00 |
| 3.40000E-02 | 2.57050E+01 | 9.53669E+00 |
| 3.20000E-02 | 2.37216E+01 | 9.07884E+00 |
| 3.00000E-02 | 2.17969E+01 | 8.61295E+00 |
| 2.80000E-02 | 1.99303E+01 | 8.13856E+00 |
| 2.60000E-02 | 1.81215E+01 | 7.65517E+00 |
| 2.40000E-02 | 1.63699E+01 | 7.16217E+00 |
| 2.20000E-02 | 1.46755E+01 | 6.65890E+00 |
| 2.00000E-02 | 1.30379E+01 | 6.14460E+00 |
| 1.80000E-02 | 1.14573E+01 | 5.61836E+00 |
| 1.60000E-02 | 9.93384E+00 | 5.07912E+00 |
| 1.40000E-02 | 8.46787E+00 | 4.52562E+00 |
| 1.20000E-02 | 7.06016E+00 | 3.95628E+00 |
| 1.00000E-02 | 5.71186E+00 | 3.36911E+00 |
| 8.00000E-03 | 4.42479E+00 | 2.76151E+00 |
| 6.00000E-03 | 3.20180E+00 | 2.12984E+00 |
| 4.00000E-03 | 2.04759E+00 | 1.46861E+00 |
| 2.00000E-03 | 9.71184E-01 | 7.68057E-01 |
| 1.00000E-03 | 4.68247E-01 | 3.96707E-01 |
| 0.00000E-99 | 0.00000E-99 | 0.00000E-99 |

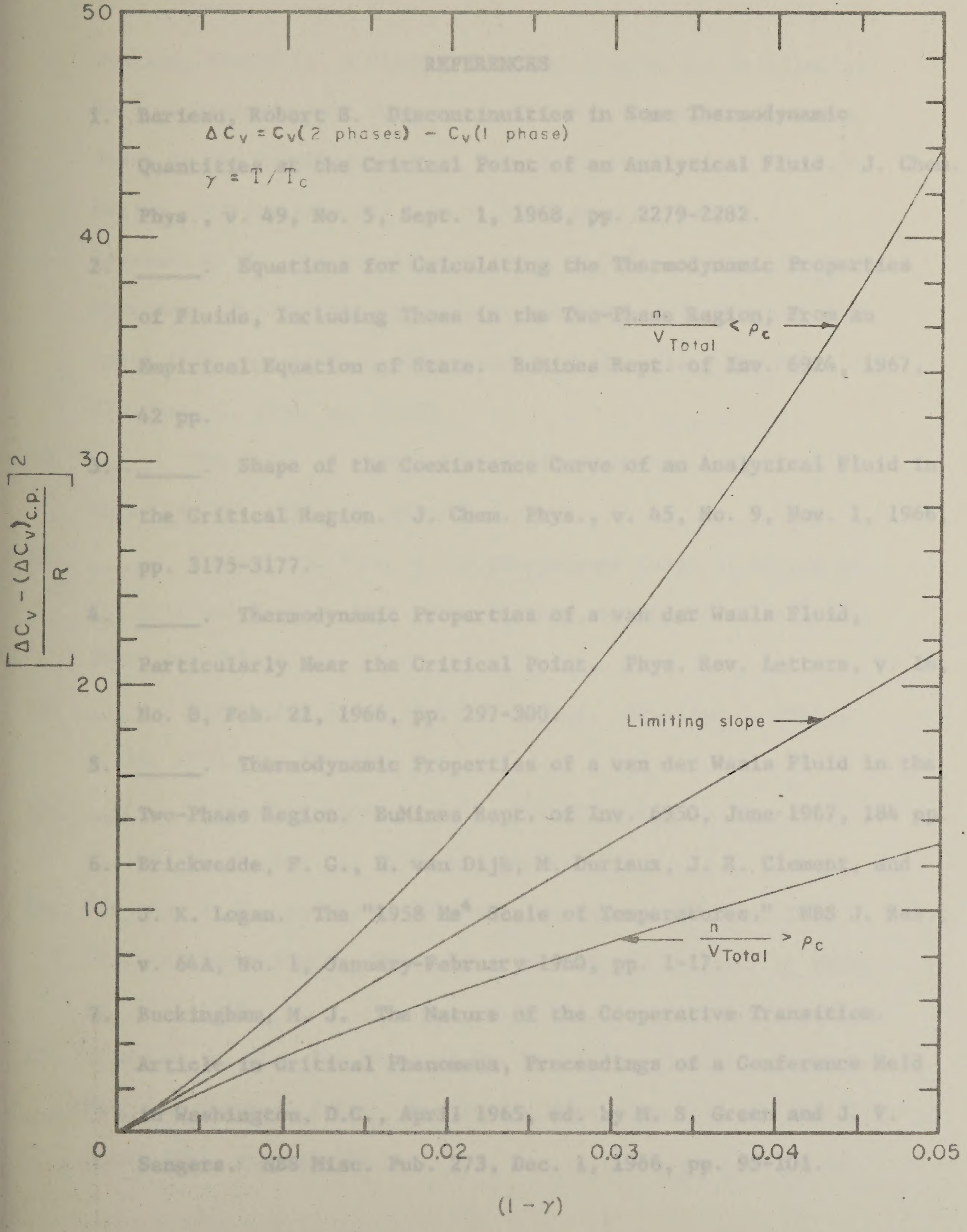

FIGURE 114.—Redlich-Kwong Fluid, Asymptotic Function of the Discontinuities in the Measured Heat Capacities at Constant Volume in Passing from Two Phases Present in the Calorimeter to a Single Phase as a Function of Temperature.

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