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TRANSIENT PHENOMENA OF AN ALTERNATOR UPON CONDENSIVE LOAD

By

Masakazu TAKAHASHI

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ELECTROTECHNICAL LABORATORY,
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SYNOPSIS

The theories on the transient phenomena of an alternator upon condensive load are treated mathematically by the method of symmetrical co-ordinates with the differential equations for a symmetrical three-phase machine and a salient-pole machine.

The author has introduced under some assumptions the general approximate solutions and obtained the approximate magnitudes, angular velocities, and attenuation constants of transient currents with respect to the symmetrical alternator.

The phenomenon of self-excitation is explained as a phenomenon due to the existence of an amplifying free oscillation; and the theoretical range of this free oscillation is discussed for a certain wide range of values of connected capacitance. The angular velocity of this free oscillation is proved to be smaller than the rotational angular velocity in the case of the symmetrical alternator, whereas it is just equal to the rotational angular velocity within a certain range of the connected capacitance in the case of the salient-pole alternator. The theoretical proof is treated by the consideration

of a symmetrical alternator with fictitious negative resistance, and also from the solution of differential equations for the salient-pole alternator.

Examples of numerical calculation with regard to both the symmetrical alternator and the salient-pole alternator are given together with the results of the experiments carried out with the same machines.

In addition, the differential equations and some fundamental characteristics for a general n-phase machine with m-phase rotor winding are described.

May, 1933.

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Masakazu TAKAHASHI

Chapter I. Introduction.

This paper treats theories on transient phenomena of an alternator upon condensive load chiefly from the differential equations for a three-phase symmetrical machine and partly from those for a salient-pole machine.

One of the noticeable phenomena of an alternator upon condensive load is the phenomenon of self-excitation when it is connected to the no-load transmission line. Ordinarily graphical methods are employed for the explanation of this phenomenon using the saturation characteristic excited by armature current and the line charging characteristic. The author has reported in his previous papers that these graphical methods give fair coincidence in their results with the actual test data either for a balanced problem or a certain unbalanced problem.(2) (3)

An explanation of the phenomenon of self-excitation based upon the theories on the transient phenomena, however, has not yet been fully discussed in mathematical form; and, so far as the author is concerned, the general nature of transient currents has not been adequately dealt with. The author believes these may give a fundamental basis on the solution by graphical methods.

The author has introduced under some assumptions the general approximate solutions for these transients, and obtained the approximate magnitudes, angular velocities, and attenuation constants of transient currents from the differential equations for the symmetrical three-phase alternator. From these results, the phenomenon of self-excitation is explained to be caused by the existence of a free oscillation of increasing amplitude, that is, by the existence of the transient current that has a negative attenuation constant. The author calls this oscillation the "amplifying free oscillation." The theoretical range of the amplifying free oscillation against the connected capacitance is discussed and the angular velocity of this oscillation is proved to be generally smaller than the rotational angular velocity in the case of the symmetrical alternator; whereas it is just equal to the rotational angular velocity within a certain range of capacitance in the case of the salient-pole alternator. The proof of this statement has been carried out by considering a symmetrical alternator with a fictitious negative resistance, which is a conventional form of introducing the characteristic of saliency to a symmetrical alternator. Another proof has been given from the solution of differential equations for a salient-pole alternator.

The solution with regard to the symmetrical alternator upon condensive load has been treated in the author's previous paper (published in Japanese in 1928)[4], in which a cubic equation with coefficients of complex quantities was given, and the present report is the extension of the work treated in that paper. Dr. S. Bekku has also treated the problem of self-excitation and proved that an oscillation of ever-increasing amplitude superimposed upon a sustained state occurs within a certain range of static condenser for a symmetrical alternator.[6]

Dr. R. Rüdenberg makes the statement in his book based upon the study of steady state characteristics[8], in the sence that the phenomenon of self-excitation occurs when a capacitance has a value just in resonance with the synchronous reactance, and continues to increase to another value just resonant with the armature leakage reactance. This statement must, however, be altered so far as a symmetrical alternator is concerned, from the study on the transient phenomena. The author has proved that the symmetrical alternator is theoretically self-excited with the upper limit of capacitance much larger than the value just resonant with the total leakage reactance.

The author has deduced the differential equations and some fundamental characteristics for a n-phase machine with m-phase rotor winding expressed in symmetrical co-ordinates. These are described in Appendices.

Chapter II. Mathematical Expressions of a Symmetrical Three-phase Alternator upon Condensive Load.

Let us consider a symmetrical three-phase generator with uniform air gap just as a wound-rotor-type induction generator, having the windings which are of star connection (See Fig. 1.). The fundamental differential equations for this machine

may be written in terms of instantaneous values of symmetrical co-ordinates as follows: (See Appendix II.)

Stator:

$$\left.\begin{aligned} v_{a0} &= -\{R_a + pL_{a0}\} i_{a0} \\ v_{a1} &= -\{R_a + pL_a\} i_{a1} - Mp\varepsilon^{j\theta} i_{u1} \\ v_{a2} &= \overline{v}_{a1} \text{ (conjugate value)} \end{aligned}\right\} \quad (2.1)$$

Rotor:

$$\left.\begin{aligned} v_{u0} &= -\{R_u + pL_{u0}\} i_{u0} \\ v_{u1} &= -\{R_u + pL_u\} i_{u1} - Mp\varepsilon^{-j\theta} i_{a1} \\ v_{u2} &= \overline{v}_{u1} \text{ (conjugate value)} \end{aligned}\right\} \quad (2.2)$$

where

$$v_{a0} = \frac{1}{3}(v_a + v_b + v_c),\ v_{a1} = \frac{1}{3}(v_a + av_b + a^2v_c),\ v_{a2} = \frac{1}{3}(v_a + a^2v_b + av_c)$$

$$i_{a0} = \frac{1}{3}(i_a + i_b + i_c),\ i_{a1} = \frac{1}{3}(i_a + ai_b + a^2i_c),\ i_{a2} = \frac{1}{3}(i_a + a^2i_b + ai_c)$$

$$v_{u0} = \frac{1}{3}(v_u + v_v + v_w),\ v_{u1} = \frac{1}{3}(v_u + av_v + a^2v_w),\ v_{u2} = \frac{1}{3}(v_u + a^2v_v + av_w)$$

$$i_{u0} = \frac{1}{3}(i_u + i_v + i_w),\ i_{u1} = \frac{1}{3}(i_u + ai_v + a^2i_w),\ i_{u2} = \frac{1}{3}(i_u + a^2i_v + ai_w)$$

v_{a0}, v_{a1}, v_{a2}; i_{a0}, i_{a1}, i_{a2} = zero, positive, and negative phase sequence components for instantaneous values of armature voltage and current.

v_{u0}, v_{u1}, v_{u2}; i_{u0}, i_{u1}, i_{u2} = ditto for instantaneous values of rotor voltage and current.

$$a = \varepsilon^{j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

R_a, L_a = resistance and positive phase (synchronous) inductance of armature circuit (per phase).

R_u, L_u = ditto of field circuit.

$M=\frac{3}{2}M'$=mutual inductance between armature and field.

$=\frac{3}{2}\times$ {max. mutual inductance between one phase of the armature (A) and that of the rotor (U)}

$\theta=\omega t+\varphi$; ω=angular velocity of rotor ; φ=constant.

$p=\frac{d}{dt}$=time differential operator.

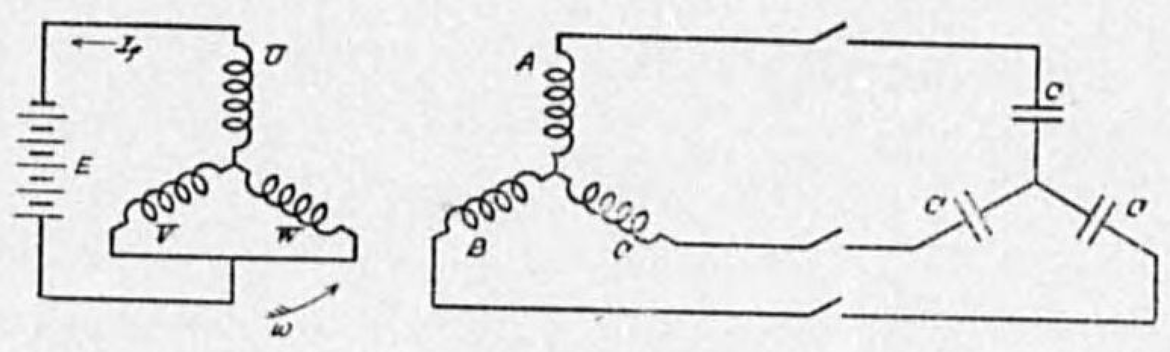

Fig. 1.

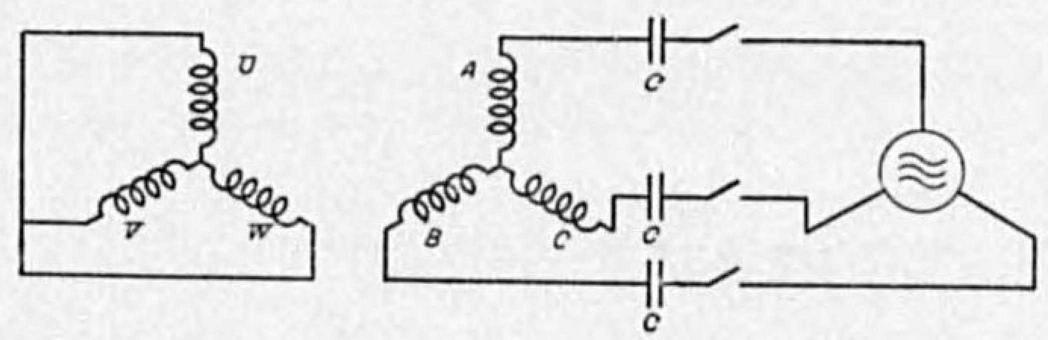

Fig. 2.

To this machine the balanced star-connected capacitance C is connected as shown in Fig. 1. The voltage equations for this condensive load at the terminals of the machine are given as follows:

$$v_{a0}=\frac{i_{a0}}{pC};\quad v_{a1}=\frac{i_{a1}}{pC};\quad v_{a2}=\overline{v}_{a1} \tag{2.3}$$

Now this alternator is assumed to have a no-load induced e.m.f. in the armature circuit, either by a residual magnetism or a field current, the manner of excitation being as shown in Fig. 1. Let the no-load induced voltages be represented by (See Appendix II),

$$v_{a1}=\dot{E}_{a1}\varepsilon^{j\omega t};\quad v_{a2}=\overline{v}_{a1};\quad v_{a0}=0 \tag{2.4}$$

The alternator is then considered to be switched on the above condensive load at this voltage. Using the principle of superposition, the transient current for this case can be obtained by superposing the current distribution before the switching on to that distribution, which will be obtainable by considering the sudden application of the voltage equal in magnitude and opposite in sign to the e.m.f. expressed as (2.4), to the armature circuit, provided that the value of e.m.f. is zero in the armature circuit as well as in the rotor circuit, as indicated in Fig. 2. Under this consideration, we get the following relations:

$$\left.\begin{aligned} v_{a1}&=-\dot{E}_{a1}\varepsilon^{j\omega t}=-\left\{R_a+pL_a+\frac{1}{pC}\right\}i_{a1}-Mp\varepsilon^{j\theta}i_{u1}\\ v_{a0}&=-\left\{R_a+pL_{a0}+\frac{1}{pC}\right\}i_{a0}=0;\quad v_{a2}=\overline{v}_{a1}\end{aligned}\right\} \tag{2.5}$$

$$\left.\begin{aligned} v_{u1}&=-\{R_u+pL_u\}i_{u1}-Mp\varepsilon^{-j\theta}i_{a1}=0\\ v_{u2}&=\overline{v}_{u1};\quad v_{u0}=0\end{aligned}\right\} \tag{2.6}$$

From (2,6), we have by the shift principle*

$$i_{u1}=\frac{-Mp\varepsilon^{-j\theta}i_{a1}}{R_u+pL_u}=-M\varepsilon^{-j\theta}\frac{(p-j\omega)i_{a1}}{R_u+(p-j\omega)L_u} \tag{2.7}$$

Substituting this value into (2.5), we get

$$i_{a1}=\dot{E}_{a1}\frac{p\{R_u+(p-j\omega)L_u\}}{\left(pR_a+p^2L_a+\frac{1}{C}\right)\{R_u+(p-j\omega)L_u\}-M^2p^2(p-j\omega)}\varepsilon^{j\omega t}$$

$$\equiv\dot{E}_{a1}\frac{Y_{a1}}{Z_{a1}}\varepsilon^{j\omega t} \tag{2.8}$$

where

$$Y_{a1}\equiv p\{R_u+(p-j\omega)L_u\}$$

* We use the formula: $\Psi(D)\{\varepsilon^{ax}X\}=\varepsilon^{ax}\Psi(D+a)X$, where $X=f(x)$, $D=\frac{d}{dx}$

$$Z_{a1} \equiv \left(pR_a + p^2 L_a + \frac{1}{C}\right)\{R_u + (p - j\omega)L_u\} - M^2 p^2 (p - j\omega) \tag{2.9}$$

The solution of a symbolic equation (2.8) can be obtained by the aid of Heaviside's expansion theorem in the following form:

$$i_{a1} = \dot{E}_{a1}\left\{\frac{Y_{a1}(j\omega)}{Z_{a1}(j\omega)}\varepsilon^{j\omega t} - \sum_{m=1}^{m=n} \frac{Y_{a1}(p_m)}{(j\omega - p_m)\left.\frac{\partial Z_{a1}}{\partial p}\right|_{p=p_m}}\varepsilon^{p_m t}\right\} \tag{2.10}$$

where p_m = roots of $Z_{a1}(p) = 0$

Now, let us put $p = j\alpha$ and a linear transformation is made by changing the variable p into α; then Heaviside's expansion theorem may be rewritten

$$i_{a1} = \dot{E}_{a1}\left\{\frac{Y(\omega)}{Z(\omega)}\varepsilon^{j\omega t} - \sum_{m=1}^{m=n} \frac{Y(\alpha_m)}{(\omega - \alpha_m)\left.\frac{\partial Z(\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_m}}\varepsilon^{j\alpha_m t}\right\} \tag{2.11}$$

where

$$\left.\begin{aligned} Y(\alpha) &= Y_{a1}(p = j\alpha) = j\alpha\{R_u + j(\alpha - \omega)L_u\} \\ Z(\alpha) &= Z_{a1}(p = j\alpha) = j\left(R_a\alpha + jL_a\alpha^2 + \frac{1}{jC}\right)\{R_u + j(\alpha - \omega)L_u\} \\ &\qquad + jM^2\alpha^2(\alpha - \omega) \\ \alpha_m &= \text{roots of } Z(\alpha) = 0 \end{aligned}\right\} \tag{2.12}$$

From the determinantal equation $Z(\alpha) = 0$, we have to solve the following equation:

$$\begin{aligned} &(L_a L_u - M^2)\alpha^3 + \{-\omega(L_a L_u - M^2) - j(R_a L_u + R_u L_a)\}\alpha^2 \\ &\quad + \left(-R_a R_u - \frac{L_u}{C} + jR_a L_u \omega\right)\alpha + \frac{L_u \omega}{C} + j\frac{R_u}{C} = 0 \end{aligned} \tag{2.13}$$

Next we put,

$$\left.\begin{aligned} \sigma &= \frac{L_a L_u - M^2}{L_a L_u} = 1 - \frac{M^2}{L_a L_u} \\ \rho_a &= \frac{R_a}{\sigma L_a} \\ \rho_u &= \frac{R_u}{\sigma L_u} \\ k &= \omega^2 L_a C \end{aligned}\right\} \tag{2.14}$$

This σ may be called the "total leakage coefficient" of the machine, and ρ_a is the reciprocal of the time constant of armature winding for total leakage field; whereas ρ_u is ditto of field winding. And (2.13) will further be written by (2.14) as

$$\begin{aligned} &\alpha^3 + \alpha^2\{-\omega - j(\rho_a + \rho_u)\} + \alpha\left\{-\sigma\rho_a\rho_u - \frac{1}{\sigma C L_a} + j\omega\rho_a\right\} \\ &\qquad + \frac{\omega}{\sigma C L_a} + j\frac{\rho_u}{C L_a} = 0 \end{aligned} \tag{2.15}$$

Or in another form as,

$$\begin{aligned} &\alpha^3 - \alpha^2\{\omega + j(\rho_a + \rho_u)\} + \alpha\left(-\sigma\rho_a\rho_u - \frac{\omega^2}{\sigma k} + j\omega\rho_a\right) \\ &\qquad + \frac{\omega^3}{\sigma k} + j\frac{\rho_u\omega^2}{k} = 0 \end{aligned} \tag{2.16}$$

This equation (2.15) or (2.16) is a cubic equation of complex quantities with respect to α, and by solving the equation there must be three roots of α in general. Let these roots be represented by α_1, α_2, and α_3. Then the solution for i_{a1} expressed by (2.11) will be written as follows:

$$i_{a1} = \dot{I}_s\varepsilon^{j\omega t} + \dot{I}_1\varepsilon^{j\alpha_1 t} + \dot{I}_2\varepsilon^{j\alpha_2 t} + \dot{I}_3\varepsilon^{j\alpha_3 t} \tag{2.17}$$

where

$$\dot{I}_s = \frac{\dot{E}_{a1}Y(\omega)}{Z(\omega)} = \frac{\dot{E}_{a1}}{R_a + j\left(\omega L_a - \frac{1}{\omega C}\right)} \tag{2.18}$$

$$\begin{aligned}
\dot{I}_1 &= -\frac{\dot{E}_{a1}Y(\alpha_1)}{(\omega-\alpha_1)\dfrac{\partial Z}{\partial \alpha}\alpha=\alpha_1} = \frac{\dot{E}_{a1}\alpha_1\{R_u+j(\alpha_1-\omega)L_u\}}{(\omega-\alpha_1)\sigma L_a L_u(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)} \\
\dot{I}_2 &= -\frac{\dot{E}_{a1}Y(\alpha_2)}{(\omega-\alpha_2)\dfrac{\partial Z}{\partial \alpha}\alpha=\alpha_2} = \frac{\dot{E}_{a1}\alpha_2\{R_u+j(\alpha_2-\omega)L_u\}}{(\omega-\alpha_2)\sigma L_a L_u(\alpha_2-\alpha_3)(\alpha_2-\alpha_1)} \\
\dot{I}_3 &= -\frac{\dot{E}_{a1}Y(\alpha_3)}{(\omega-\alpha_3)\dfrac{\partial Z}{\partial \alpha}\alpha=\alpha_3} = \frac{\dot{E}_{a1}\alpha_3\{R_u+j(\alpha_3-\omega)L_u\}}{(\omega-\alpha_3)\sigma L_a L_u(\alpha_3-\alpha_1)(\alpha_3-\alpha_2)}
\end{aligned} \qquad (2.19)$$

The relations (2.19) can readily be calculated, because

$$Z(\alpha) = -j(L_a L_u - M^2)(\alpha-\alpha_1)(\alpha-\alpha_2)(\alpha-\alpha_3)$$

$$= -j\sigma L_a L_u(\alpha-\alpha_1)(\alpha-\alpha_2)(\alpha-\alpha_3) \qquad (2.20)$$

$$\frac{\partial Z}{\partial \alpha}\alpha=\alpha_1 = -j\sigma L_a L_u(\alpha_1-\alpha_2)(\alpha_1-\alpha_3) \qquad (2.21)$$

We can write i_{a2} directly from (2.17) as the conjugate to i_{a1} as follows:

$$i_{a2} = \overline{i_{a1}} = \bar{I}_s\varepsilon^{-j\omega t} + \bar{I}_1\varepsilon^{-j\bar{\alpha}_1 t} + \bar{I}_2\varepsilon^{-j\bar{\alpha}_2 t} + \bar{I}_3\varepsilon^{-j\bar{\alpha}_3 t} \qquad (2.22)$$

The instantaneous armature currents of the phases i_a, i_b, and i_c will be easily obtained from the symmetrical components as (See Appendix I),

$$\begin{aligned}
i_a &= i_{a1} + i_{a2} = i_{a1} + \overline{i_{a1}} = 2 \times (\text{Real part of } i_{a1}) \\
i_b &= a^2 i_{a1} + a i_{a2} = a^2 i_{a1} + \overline{a^2 i_{a1}} = 2 \times (\text{Real part of } a^2 i_{a1}) \\
i_c &= a i_{a1} + a^2 i_{a2} = a i_{a1} + \overline{a i_{a1}} = 2 \times (\text{Real part of } a i_{a1})
\end{aligned} \qquad (2.23)$$

where

$$a = \varepsilon^{j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = \overline{a^2}$$

The above method of attack will be applicable not only to a symmetrical three-phase machine, but to a general symmetrical n-phase machine(1) (n may be any integer) with a slight modification (See Appendix IV).

Next the instantaneous value of rotor current will be calculated as in the following. From (2.7), we have already

$$i_{u1} = -M\varepsilon^{-j\theta}\frac{(p-j\omega)i_{a1}}{R_u+(p-j\omega)L_u} \qquad (2.24)$$

If we consider only the free oscillation, this i_{u1} can also be obtained from (2.5), by putting $\dot{E}_{a1}$ zero. Thus, we have

$$-\left\{R_a + pL_a + \frac{1}{pC}\right\}i_{a1} = Mp\varepsilon^{j\theta}i_{u1}$$

And therefore,

$$i_{u1} = -\varepsilon^{-j\theta}\frac{\left\{R_a+pL_a+\dfrac{1}{pC}\right\}}{pM}i_{a1} \qquad (2.25)$$

Now if we put, $i_{a1} = \dot{I}\varepsilon^{j\alpha t}$, then from (2.7) and (2.25), we have

$$i_{u1} = \frac{-j\beta M}{R_u+j\beta L_u}(\dot{I}\varepsilon^{-j\gamma})\varepsilon^{j\beta t} = -\frac{\left\{R_a+j(\alpha L_a-\dfrac{1}{\alpha C})\right\}}{j\alpha M}(\dot{I}\varepsilon^{-j\gamma})\varepsilon^{j\beta t} \qquad (2.26)$$

where $\beta \equiv \alpha - \omega$

The instantaneous value of transient rotor current can be written corresponding to (2.17) as follows:

$$i_{u1} = \dot{I}_s' + \dot{I}_1'\varepsilon^{j\beta_1 t} + \dot{I}_2'\varepsilon^{j\beta_2 t} + \dot{I}_3'\varepsilon^{j\beta_3 t} \qquad (2.27)$$

where,

$\dot{I}_s'$ = symmetrical component of initial exciting current

$= \frac{1}{2} \times$ initial exciting current $I_f = \frac{E}{3K_u}$

$$\left.\begin{aligned}
\dot{I}_1' &= -\frac{j\beta_1 M\varepsilon^{-j\varphi}}{R_u+j\beta_1 L_u}\,\dot{I}_1 = -\frac{\left\{R_a+j(\alpha_1 L_a-\frac{1}{\alpha_1 C})\right\}\varepsilon^{-j\varphi}}{j\alpha_1 M}\,\dot{I}_1 \\
\dot{I}_2' &= -\frac{j\beta_2 M\varepsilon^{-j\varphi}}{R_u+j\beta_2 L_u}\,\dot{I}_2 = -\frac{\left\{R_a+j(\alpha_2 L_a-\frac{1}{\alpha_2 C})\right\}\varepsilon^{-j\varphi}}{j\alpha_2 M}\,\dot{I}_2 \\
\dot{I}_3' &= -\frac{j\beta_3 M\varepsilon^{-j\varphi}}{R_u+j\beta_3 L_u}\,\dot{I}_3 = -\frac{\left\{R_a+j(\alpha_3 L_a-\frac{1}{\alpha_3 C})\right\}\varepsilon^{-j\varphi}}{j\alpha_3 M}\,\dot{I}_3 \\
\beta_1 &= \alpha_1-\omega\ ;\ \beta_2=\alpha_2-\omega\ ;\ \beta_3=\alpha_3-\omega
\end{aligned}\right\} \qquad (2.28)$$

As i_{u2} is the conjugate of i_{u1}, we can write

$$i_{u2}=\overline{I}_s'+\overline{I}_1'\varepsilon^{-j\bar{\beta}_1 t}+\overline{I}_2'\varepsilon^{-j\bar{\beta}_2 t}+\overline{I}_3'\varepsilon^{-j\bar{\beta}_3 t} \qquad (2.29)$$

And the instantaneous rotor current of each phase will be expressed by

$$\left.\begin{aligned}
i_u &= i_{u1}+i_{u2} = 2\times(\text{Real part of } i_{u1}) \\
i_v &= a^2 i_{u1}+a i_{u2} = 2\times(\text{Real part of } a^2 i_{u1}) \\
i_w &= a i_{u1}+a^2 i_{u2} = 2\times(\text{Real part of } a i_{u1})
\end{aligned}\right\} \qquad (2.30)$$

Chapter III. Approximate General Solutions.

As already described, the instantaneous armature currents expressed in symmetrical components are as follows:

$$i_{a1}=\dot{I}_s\varepsilon^{j\omega t}+\dot{I}_1\varepsilon^{j\alpha_1 t}+\dot{I}_2\varepsilon^{j\alpha_2 t}+\dot{I}_3\varepsilon^{j\alpha_3 t}$$

$$i_{a2}=\overline{i}_{a1}$$

The first term $\dot{I}_s\,\varepsilon^{j\omega t}$ is a steady-state component and its angular velocity is the same as that of rotation. The other three terms are transient components, and their generalized angular velocities α_1, α_2, and α_3 are of complex quantities, that is, the real part representing an ordinary angular velocity and the imaginary part an attenuation constant or a damping factor.

Let these angular velocities and attenuation constants be represented as,

$$\alpha_1=\omega_1+ja_1\ ;\ \alpha_2=\omega_2+ja_2\ ;\ \alpha_3=\omega_3+ja_3 \qquad (3.1)$$

Then the armature current may be represented in a form,

$$i_{a1}=\dot{I}_s\varepsilon^{j\omega t}+\dot{I}_1\varepsilon^{(j\omega_1-a_1)t}+\dot{I}_2\varepsilon^{(j\omega_2-a_2)t}+\dot{I}_3\varepsilon^{(j\omega_3-a_3)t}$$

$$i_{a2}=\overline{i}_{a1} \qquad (3.2)$$

And the instantaneous value of rotor current may be written as

$$i_{u1}=\dot{I}_s'+\dot{I}_1'\varepsilon^{\{j(\omega_1-\omega)-a_1\}t}+\dot{I}_2'\varepsilon^{\{j(\omega_2-\omega)-a_2\}t}+\dot{I}_3'\varepsilon^{\{j(\omega_3-\omega)-a_3\}t} \qquad (3.3)$$

(1) An approximate solution for α.

We consider first an ideal case when the resistance of stator and rotor is negligibly small. Putting

$R_a=R_u=0$ or $\rho_a=\rho_u=0$ in (2.15), we get

$$\alpha^3-\omega\alpha^2-\frac{1}{\sigma CL_a}\alpha+\frac{\omega}{\sigma CL_a}=0 \qquad (3.4)$$

or

$$(\alpha-\omega)\left(\alpha-\frac{1}{\sqrt{\sigma CL_a}}\right)\left(\alpha+\frac{1}{\sqrt{\sigma CL_a}}\right)=0$$

We have three roots of α from (3.4) as,

$$\alpha_1=\omega_1=\omega\ ;\ \alpha_2=\omega_2=\omega_n=\frac{1}{\sqrt{\sigma CL_a}}\ ;\ \alpha_3=\omega_3=-\omega_n=-\frac{1}{\sqrt{\sigma CL_a}} \qquad (3.5)$$

We can say from this result that one root α_1 has the same angular velocity as that of rotation, whereas the other two roots α_2 and α_3 have an angular velocity corresponding to that obtainable under the resonance condition with the total leakage inductance (σL_a) and the connected capacitance C.

We consider next the approximate values of attenuation constants with an assumption that the angular velocities of α are ω and $\pm\ \omega_n$. And we put,

$$\left.\begin{aligned}\alpha_1&=\omega+ja_1\\ \alpha_2&=\omega_n+ja_2\\ \alpha_3&=-\omega_n+ja_3\end{aligned}\right\}\qquad \omega_n=\frac{1}{\sqrt{\sigma CL_a}} \tag{3.6}$$

From (2.15) we can write the following expressions as the relations between the roots and the coefficients of an cubic equation,

$$\alpha_1+\alpha_2+\alpha_3=\omega+j(\rho_a+\rho_u) \tag{3.7}$$

$$\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1=-\sigma\rho_a\rho_u-\frac{1}{\sigma CL_a}+j\omega\rho_a \tag{3.8}$$

$$\alpha_1\alpha_2\alpha_3=-\frac{\omega}{\sigma CL_a}-j\frac{\rho_u}{CL_a} \tag{3.9}$$

By the relations (3.6), we get

$$a_1+a_2+a_3=\rho_a+\rho_u \tag{3.10}$$

$$j\omega(a_2+a_3)+j\omega_n(a_3-a_2)=j\omega\rho_a-\sigma\rho_a\rho_u+\sum a_1a_2 \tag{3.11}$$

$$j\omega\omega_n(a_3-a_2)-j\omega_n^2a_1=-j\frac{\rho_u}{CL_a}+ja_1a_2a_3+a_2a_3\omega+\omega_na_1(a_3-a_2) \tag{3.12}$$

We assume that σ, ρ_a, ρ_u, a_1, a_2, and a_3 are very small as compared with ω and ω_n, and also $\omega_n>\omega$. Then we can put approximately (3.11) and (3.12) as follows:

$$\omega(a_2+a_3)+\omega_n(a_3-a_2)\doteqdot\omega\rho_a \tag{3.13}$$

$$\omega_n\omega(a_3-a_2)-\omega_n^2a_1\doteqdot-\frac{\rho_u}{CL_a} \tag{3.14}$$

From (3.10), (3.13), and (3.14), we get

$$a_1=\frac{\rho_u\left(\dfrac{1}{CL_a}-\omega^2\right)}{\omega_n^2-\omega^2} \tag{3.15}$$

$$a_2=\frac{1}{2}\left\{\rho_a+\rho_u-a_1+\frac{\omega}{\omega_n}(\rho_u-a_1)\right\} \tag{3.16}$$

$$a_3=\frac{1}{2}\left\{\rho_a+\rho_u-a_1-\frac{\omega}{\omega_n}(\rho_u-a_1)\right\} \tag{3.17}$$

These results indicate us the general approximate nature of transient currents on the foregoing assumptions. We can conclude from (3.15) that an attenuation constant or a decrement factor a_1 becomes negative in sign within a certain range. A negative damping indicates the ever-increasing amplitude of a transient current, and explains the phenomenon of self-excitation of an alternator upon condensive load. For the examination of a_1, we put

$$C_0=\frac{1}{\omega^2L_a};\quad k=\frac{C}{C_0}=\omega^2L_aC \tag{3.18}$$

where C_0 represents a capacitance, the condensive reactance of which is resonant with ωL_a (synchronous reactance), and k means a numerical value of the connected capacitance as compared with C_0. Therefore, (3.15) may be written,

$$a_1=\frac{\rho_u\left(\dfrac{1}{CL_a}-\omega^2\right)}{\dfrac{1}{\sigma CL_a}-\omega^2}=\frac{\sigma\rho_u\left(1-\dfrac{C}{C_0}\right)}{1-\dfrac{\sigma C}{C_0}}=\sigma\rho_u\frac{1-k}{1-\sigma k}$$

$$=\frac{R_u}{L_u}\,\frac{1-k}{1-\sigma k}=\frac{1}{T_0}\,\frac{1-k}{1-\sigma k} \tag{3.19}$$

where $T_0\equiv\dfrac{L_u}{R_u}$ = time constant of field circuit.

The relation (3.19) indicates that if the connected capacitance C is greater than the resonance capacitance C_o for the synchronous reactance, viz. k is greater than unity, the attenuation constant a_1 becomes negative and the self-excitation will occur. The approximate value of a_1 is inversely proportional to the time constant of the field circuit and a certain function of k, which is nearly in linear relation for the small value of σ.

We see from (3.16) and (3.17) that a_2 and a_3 are positive in sign for a negative value of a_1. The precise range for a negative damping is discussed in the later chapter.

(2) The more precise solutions for α.

If numerical values for ω, ρ_a, ρ_u, σ, L_u, and C (or k) are given, an equation (2.15) or (2.16) can be numerically solved and the precise numerical values for α will be calculated. It is, however, difficult from the mathematical point of view to solve this equation in a general form in terms of the constants ω, ρ_a, ρ_u, σ, etc.

Solutions for α_1, α_2, and α_3 may, however, be obtained more precisely as long as C is not very large.

(A) Solution for α_1.

Let one root of α, the angular velocity of which is near ω be represented as α_1; and in a form as,

$$\alpha_1=\omega_1+ja_1=(1-s)\omega+ja_1 \tag{3.20}$$

where s represents a slip. Substituting (3.20) into (2.15) and equating the real and imaginary parts to null respectively, we get

$$(3s-2)a_1^2+\{\rho_a+2\rho_u-2(\rho_a+\rho_u)s\}a_1$$
$$+\left\{-\omega^2s^3+2\omega^2s^2+\left(-\omega^2+\sigma\rho_a\rho_u+\frac{1}{\sigma CL_a}\right)s-\sigma\rho_a\rho_u\right\}=0 \tag{3.21}$$

$$\omega^2(3a_1-\rho_a-\rho_u)s^2+\omega^2(\rho_a+2\rho_u-4a_1)s-a_1^3+(\rho_a+\rho_u)a_1^2$$

$$+\left(\omega^2-\sigma\rho_a\rho_u-\frac{1}{\sigma CL_a}\right)a_1+\frac{\rho_u}{CL_a}-\rho_u\omega^2=0 \tag{3.22}$$

It is very difficult to solve these simultaneous equations regarding s and a_1 and the following equations may be used for an approximate calculation instead of (3.21) and (3.22). Assuming small values of s, σ, and a_1, we have

$$-2a_1^2+(\rho_a+2\rho_u)a_1+\left(\frac{1}{\sigma CL_a}-\omega^2\right)s-\sigma\rho_a\rho_u=0 \tag{3.23}$$

$$\omega^2(\rho_a+2\rho_u-4a_1)s+a_1\left(\omega^2-\frac{1}{\sigma CL_a}\right)+\rho_u\left(\frac{1}{CL_a}-\omega^2\right)=0 \tag{3.24}$$

Solving these, except near the region of $\frac{1}{\sigma CL_a}=\omega^2$, we get

$$a_1=\frac{\rho_u\left(\frac{1}{CL_a}-\omega^2\right)}{\frac{1}{\sigma CL_a}-\omega^2}+\frac{\omega^2(\rho_a+2\rho_u-4a_1)s}{\frac{1}{\sigma CL_a}-\omega^2}\doteqdot\frac{\rho_u\left(\frac{1}{CL_a}-\omega^2\right)}{\frac{1}{\sigma CL_a}-\omega^2} \tag{3.25}$$

$$s=\frac{2a_1^2+\sigma\rho_a\rho_u-(\rho_a+2\rho_u)a_1}{\frac{1}{\sigma CL_a}-\omega^2} \tag{3.26}$$

(B) Solutions for α_2 and α_3.

Next we proceed to obtain the other two roots α_2 and α_3 of α. Now, if a cubic equation in a form of

$$\alpha^3+P\alpha^2+Q\alpha+R=0 \tag{3.27}$$

has a root α_1, then the other two roots can be obtained by solving a quadratic equation

$$\alpha^2+(P+\alpha_1)\alpha+\{\alpha_1(P+\alpha_1)+Q\}=0 \tag{3.28}$$

Now, comparing (3.27) with (2.16), we have

$$\left.\begin{aligned} P &= -\omega - j(\rho_a + \rho_u) \\ Q &= -\sigma\rho_a\rho_u - \frac{1}{\sigma C L_a} + j\omega\rho_a \end{aligned}\right\} \tag{3.29}$$

Substituting (3.29) and (3.20) into (3.28), we have a quadratic equation for α as

$$\alpha^2 + \{-\omega s - j(\rho_a + \rho_u - a_1)\}\alpha - \omega^2 s(1-s) + a_1(\rho_a + \rho_u - a_1)$$

$$-\sigma\rho_a\rho_u - \frac{1}{\sigma C L_a} - j\omega\{\rho_u - a_1 - s(\rho_a + \rho_u) + 2sa_1\} = 0 \tag{3.30}$$

Solving this equation, α may be obtained in the following form,

$$\alpha = \frac{1}{2}\left\{\omega s + j(\rho_a + \rho_u - a_1) \pm (B + jD)\right\} \tag{3.31}$$

where

$$(B + jD)^2 = \{\omega s + j(\rho_a + \rho_u - a_1)\}^2$$

$$+4\left[\omega^2 s(1-s) - a_1(\rho_a + \rho_u - a_1) + \sigma\rho_a\rho_u + \frac{1}{\sigma C L_a}\right.$$

$$\left. + j\omega\{\rho_u - a_1 - s(\rho_a + \rho_u) + 2sa_1\}\right] \tag{3.32}$$

Assuming the small values of s, σ, a_1 and not a very large value of C, we take an approximate relation,

$$(B + jD)^2 = \frac{4}{\sigma C L_a} - (\rho_a + \rho_u - a_1)^2 + j4\omega(\rho_u - a_1) \tag{3.33}$$

B and D will be calculated by using a formula

$$\sqrt{a + jb} = \sqrt{\frac{1}{2}(a + \sqrt{a^2 + b^2})} + j\sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} - a)}, \quad (b > 0) \tag{3.34}$$

And assuming a small value of resistance component and not a very large value of C, we calculated as,

$$B = \sqrt{\frac{1}{2}\left[\frac{4}{\sigma C L_a} - (\rho_a + \rho_u - a_1)^2 + \sqrt{\left\{\frac{4}{\sigma C L_a} - (\rho_a + \rho_u - a_1)^2\right\}^2 + 4^2\omega^2(\rho_u - a_1)^2}\right]}$$

$$\fallingdotseq \sqrt{\frac{4}{\sigma C L_a} - (\rho_a + \rho_u - a_1)^2}$$

$$= 2\sqrt{\frac{1}{\sigma C L_a} - \frac{(\rho_a + \rho_u - a_1)^2}{4}} = 2\omega_n \tag{3.35}$$

$$D = \sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} - a)} = \sqrt{\frac{1}{2}a\left\{\sqrt{1 + \left(\frac{b}{a}\right)^2} - 1\right\}}$$

$$= \sqrt{\frac{1}{2}a\left\{1 + \frac{1}{2}\left(\frac{b}{a}\right)^2 + \cdots\cdots - 1\right.} \fallingdotseq \sqrt{\frac{b^2}{4a}} \qquad \begin{pmatrix}\text{assuming} \\ |a| > |b\end{pmatrix}$$

$$= \sqrt{\frac{4^2\omega^2(\rho_u - a_1)^2}{4 \times 4\omega_n^2}} = \frac{\omega}{\omega_n}(\rho_u - a_1) \tag{3.36}$$

where

$$\omega_n \equiv \sqrt{\frac{1}{\sigma C L_a} - \frac{(\rho_a + \rho_u - a_1)^2}{4}} \tag{3.37}$$

and ω_n is assumed larger than ω.

Substituting (3.35) and (3.36) into (3.31), we get

$$\alpha = \frac{1}{2}\left[\omega s + j(\rho_a + \rho_u - a_1) \pm \left\{2\omega_n \pm \frac{\omega}{\omega_n}(\rho_u - a_1)\right\}\right] \tag{3.38}$$

Hence α_2 and α_3 may be written from (3.38)

$$\alpha_2 = \omega_2 + ja_2 = \omega_n + \frac{\omega s}{2} + j\frac{1}{2}\left\{\rho_a + \rho_u - a_1 + \frac{\omega}{\omega_n}(\rho_u - a_1)\right\} \tag{3.39}$$

$$\alpha_3 = \omega_3 + ja_3 = -\omega_n + \frac{\omega s}{2} + j\frac{1}{2}\left\{\rho_a + \rho_u - a_1 - \frac{\omega}{\omega_n}(\rho_u - a_1)\right\} \tag{3.40}$$

and

$$\omega_n = \sqrt{\frac{1}{\sigma C L_a} - \frac{(\rho_a + \rho_u - a_1)^2}{4}}$$

(3) Some remarks on the angular velocity and the attenuation constant.

From the approximate solution above obtained, we may conclude regarding the angular velocity and the attenuation constant as follows: (Cf. numerical examples given in Chapter V.)

1. In a symmetrical alternator, the frequency ω_1 that is very near the rotational frequency ω has, strictly speaking, a slip in general. In other words, the frequency in the case of self-excitation has a slip and is generally a little smaller than the rotational frequency.

2. The frequency components of α_2 and α_3, or ω_2 and ω_3, are nearly equal to ω_n, and ω_n is nearly equal to the resonant angular velocity between the total leakage inductance σL_a and the connected capacity C.

ω_2 means a revolving field of the same direction to the rotor, while ω_3 has a negative sign and means a revolving field of the opposite direction to the rotor.

3. The algebraic sum of ω_2 and ω_3 is equal to ωs, and ω_2 is larger than $|\omega_3|$ in a case of s being positive.

4. Comparing a_2 and a_3, or the damping components of α_2 and α_3, we see a_2 is larger than a_3.

5. We may say ω_n is a natural frequency for a circuit consisting of the total leakage inductance, capacity, and resistance in series during transients.

The expression of ω_n in (3.37) is to give its approximate value. For a circuit consisting of inductance L, capacity C, and resistance R in series, the usual expression of attenuation constant and angular velocity for transient component is as well known,

$$j\alpha = -\frac{R}{2L} + j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

And we can write from (3.37),

$$\omega_n = \sqrt{\frac{1}{(\sigma L_a)C} - \left\{\frac{1}{2}\left(\frac{R_a}{\sigma L_a} + \frac{R_u}{\sigma L_u} - a_1\right)\right\}^2}$$

Comparing these two expressions, we see the similarity if the attenuation constant a_1 is considered to have a similar dimension of $\frac{R}{L}$.

(4) Approximate solution for the magnitudes of currents.

We can calculate an approximate value for the magnitudes of armature and field currents, in a relatively simple form assuming a negligible resistance and in some parts neglecting the attenuation.

Let the approximate value for α be

$$\left.\begin{aligned} \alpha_1 &= \omega + j\frac{\rho_u\left(\frac{1}{CL_a} - \omega^2\right)}{\omega_n^2 - \omega^2} \\ \alpha_2 &= \omega_n \\ \alpha_3 &= -\omega_n; \quad \omega_n = \frac{1}{\sqrt{\sigma C L_a}} \end{aligned}\right\} \tag{3.41}$$

Then the magnitudes of armature currents $\dot{I}_1$, $\dot{I}_2$, and $\dot{I}_3$ will be calculated from (2.19) by the substitution of (3.41) as,

$$\dot{I}_1 = \frac{\dot{E}_{a1}\alpha_1\{R_u + j(\alpha_1 - \omega)L_u\}}{(\omega - \alpha_1)\sigma L_a L_u(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}$$

$$\doteqdot \frac{\dot{E}_{a1}\omega\left\{R_u + j^2\frac{\rho_u\left(\frac{1}{CL_a} - \omega^2\right)L_u}{\omega_n^2 - \omega^2}\right\}}{-j\frac{\rho_u\left(\frac{1}{CL_a} - \omega^2\right)}{\omega_n^2 - \omega^2}\sigma L_a L_u(\omega - \omega_n)(\omega + \omega_n)}$$

$$= \frac{\dot{E}_{a1}\omega\left\{1 - \frac{1}{\sigma}\frac{\left(\frac{1}{CL_a} - \omega^2\right)}{\omega_n^2 - \omega^2}\right\}}{j\left(\frac{1}{CL_a} - \omega^2\right)L_a}$$

$$=-\frac{\dot{E}_{a1}}{j\left(\omega L_a-\frac{1}{\omega C}\right)}+\frac{\dot{E}_{a1}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)} \tag{3.42}$$

$$\dot{I}_2=\frac{\dot{E}_{a1}\alpha_2\{R_u+j(\alpha_2-\omega)L_u\}}{(\omega-\alpha_2)\sigma L_a L_u(\alpha_2-\alpha_3)(\alpha_2-\alpha_1)}$$

$$\fallingdotseq\frac{\dot{E}_{a1}\omega_n\{j(\omega_n-\omega)L_u\}}{(\omega-\omega_n)\sigma L_a L_u(2\omega_n)(\omega_n-\omega)}=\frac{\dot{E}_{a1}}{j2\sigma L_a(\omega_n-\omega)}$$

$$=\frac{\dot{E}_{a1}}{j2\sigma L_a\frac{\omega}{\omega_n+\omega}\left(\frac{\omega_n^2}{\omega}-\omega\right)}=-\frac{1}{2}\frac{\omega+\omega_n}{\omega}\frac{\dot{E}_{a1}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)} \tag{3.43}$$

$$\dot{I}_3\fallingdotseq-\frac{\dot{E}_{a1}}{j2\sigma L_a(\omega_n+\omega)}=-\frac{1}{2}\frac{\omega-\omega_n}{\omega}\frac{\dot{E}_{a1}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)} \tag{3.44}$$

and

$$\dot{I}_s=\frac{\dot{E}_{a1}}{R_a+j(\omega L_a-\frac{1}{\omega C})}\fallingdotseq\frac{\dot{E}_{a1}}{j\left(\omega L_a-\frac{1}{\omega C}\right)} \tag{3.45}$$

An approximate magnitude of the field current may be calculated by neglecting the resistance component from equations (2.28) and (3.41), and further by the substitution of (3.42), (3.43) and (3.44) as follows:

$$\dot{I}_1'=\frac{-\left\{R_a+j\left(\alpha_1 L_a-\frac{1}{\alpha_1 C}\right)\right\}\varepsilon^{-j\varphi}\dot{I}_1}{j\alpha_1 M}\fallingdotseq\frac{\left(\omega L_a-\frac{1}{\omega C}\right)}{\omega M}\varepsilon^{-j\varphi}\dot{I}_1$$

$$\fallingdotseq\frac{-\left(\omega L_a-\frac{1}{\omega C}\right)}{\omega M}\left\{-\frac{\dot{E}_{a1}\varepsilon^{-j\varphi}}{j\left(\omega L_a-\frac{1}{\omega C}\right)}+\frac{\dot{E}_{a1}\varepsilon^{-j\varphi}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}\right\} \tag{3.46}$$

$$\dot{I}_2'\fallingdotseq-\frac{M}{L_u}\varepsilon^{-j\varphi}\dot{I}_2\fallingdotseq\frac{-\left(\omega_n L_a-\frac{1}{\omega_n C}\right)}{\omega_n M}\varepsilon^{-j\varphi}\dot{I}_2$$

$$\fallingdotseq\frac{\left(\omega_n L_a-\frac{1}{\omega_n C}\right)}{\omega_n M}\frac{1}{2}\frac{\omega+\omega_n}{\omega}\frac{\dot{E}_{a1}\varepsilon^{-j\varphi}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)} \tag{3.47}$$

$$\dot{I}_3'\fallingdotseq-\frac{M}{L_u}\varepsilon^{-j\varphi}\dot{I}_3\fallingdotseq\frac{-\left(-\omega_n L_a+\frac{1}{\omega_n C}\right)}{-\omega_n M}\varepsilon^{-j\varphi}\dot{I}_3$$

$$\fallingdotseq\frac{\left(\omega_n L_a-\frac{1}{\omega_n C}\right)}{\omega_n M}\frac{1}{2}\frac{\omega-\omega_n}{\omega}\frac{\dot{E}_{a1}\varepsilon^{-j\varphi}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)} \tag{3.48}$$

Now, if we assume a field excitation as shown in Fig. 1, the no-load induced voltage can be represented as follows (See Appendix II, (B. 10)):

$$v_{a1}=-Mp\varepsilon^{j\theta}i_{u1}=-j\omega M\frac{I_f}{2}\varepsilon^{j\varphi}\varepsilon^{j\omega t} \tag{3.49}$$

where

$$I_f=\text{initial exciting current}=i_u=\frac{2E}{3R_u}$$

Because the no load voltage is hitherto represented as $v_{a1}=\dot{E}_{a1}\varepsilon^{j\omega t}$, we have the identity

$$\dot{E}_{a1}=-j\omega M\frac{I_f}{2}\varepsilon^{j\varphi} \tag{3.50}$$

By the substitution of (3.50), we get the approximate magnitude for field current in somewhat different form from (3.46) to (3.48) as follows:

$$\left.\begin{aligned}
\dot{I}_1' &= \frac{-\left(\omega L_a-\frac{1}{\omega C}\right)}{\omega M}\left\{-\frac{\varepsilon^{-j\varphi}}{j\left(\omega L_a-\frac{1}{\omega C}\right)}+\frac{\varepsilon^{-j\varphi}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}\right\}\left(-j\omega M\frac{I_f}{2}\varepsilon^{j\varphi}\right)\\
&= -\frac{I_f}{2}+\frac{\omega L_a-\frac{1}{\omega C}}{\sigma\omega L_a-\frac{1}{\omega C}}\,\frac{I_f}{2}=-\frac{I_f}{2}+\frac{k-1}{\sigma k-1}\,\frac{I_f}{2}\\
&= \frac{k(\sigma-1)}{1-\sigma k}\,\frac{I_f}{2}\\
\dot{I}_2' &= -\frac{1}{2}\,\frac{\omega+\omega_n}{\omega}\,\frac{1-\frac{1}{\omega_n^2 L_a C}}{\sigma-\frac{1}{\omega^2 L_a C}}\,\frac{I_f}{2}=-\frac{1}{2}\,\frac{\omega+\omega_n}{\omega}\,\frac{k(\sigma-1)}{1-\sigma k}\,\frac{I_f}{2}\\
\dot{I}_3' &= -\frac{1}{2}\,\frac{\omega-\omega_n}{\omega}\,\frac{1-\frac{1}{\omega_n^2 L_a C}}{\sigma-\frac{1}{\omega^2 L_a C}}\,\frac{I_f}{2}=-\frac{1}{2}\,\frac{\omega-\omega_n}{\omega}\,\frac{k(\sigma-1)}{1-\sigma k}\,\frac{I_f}{2}\\
\text{where}\quad k &= \frac{C}{C_0}=C\omega^2 L_a
\end{aligned}\right\}\quad(3.51)$$

(5) Summary for approximate general solutions.

No-load induced voltage is expressed in symmetrical components:

$$v_{a1}=\dot{E}_{a1}\,\varepsilon^{j\omega t}=-j\omega M\frac{I_f}{2}\varepsilon^{j\varphi}\varepsilon^{j\omega t}$$

$$\dot{E}_{a1}=-j\omega M\frac{I_f}{2}\varepsilon^{j\varphi}=-j\frac{e_0}{2}\varepsilon^{j\varphi}$$

No-load voltage in ordinary phase-component values becomes as follows:

$$\left.\begin{aligned}
v_a &= v_{a1}+v_{a2}=\omega MI_f\sin(\omega t+\varphi)=e_0\sin(\omega t+\varphi)\\
v_b &= a^2v_{a1}+av_{a2}=\omega MI_f\sin\left(\omega t-\frac{2}{3}\pi+\varphi\right)=e_0\sin\left(\omega t-\frac{2}{3}\pi+\varphi\right)\\
v_c &= av_{a1}+a^2v_{a2}=\omega MI_f\sin\left(\omega t-\frac{4}{3}\pi+\varphi\right)=e_0\sin\left(\omega t-\frac{4}{3}\pi+\varphi\right)
\end{aligned}\right\}\quad(3.52)$$

where

$$e_0\equiv\omega MI_f$$

The instantaneous armature current expressed in symmetrical components is as above described in one approximate form:

$$\begin{aligned}
i_{a1} &= -j\omega M\frac{I_f}{2}\varepsilon^{j\varphi}\left[\frac{\varepsilon^{j\omega t}}{j\left(\omega L_a-\frac{1}{\omega C}\right)}+\left\{\frac{\varepsilon^{j(1-s)\omega t}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}-\frac{\varepsilon^{j(1-s)\omega t}}{j\left(\omega L_a-\frac{1}{\omega C}\right)}\right\}\varepsilon^{-a_1t}\right.\\
&\quad\left.-\frac{1}{2}\,\frac{\omega+\omega_n}{\omega}\,\frac{\varepsilon^{j\omega_n t}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}\varepsilon^{-a_2t}-\frac{1}{2}\,\frac{\omega-\omega_n}{\omega}\,\frac{\varepsilon^{-j\omega_n t}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}\varepsilon^{-a_3t}\right]\quad(3.53)
\end{aligned}$$

$$i_{a2}=\overline{i_{a1}}$$

An approximate value of the angular velocities and attenuation constants:

$$a_1=\frac{\rho_u\left(\frac{1}{CL_a}-\omega^2\right)}{\omega_n^2-\omega^2}=\frac{1}{T_0}\,\frac{1-k}{1-\sigma k}$$

$$a_2=\frac{1}{2}\left\{\rho_a+\rho_u-a_1+\frac{\omega}{\omega_n}(\rho_u-a_1)\right\}$$

$$a_3=\frac{1}{2}\left\{\rho_a+\rho_u-a_1-\frac{\omega}{\omega_n}(\rho_u-a_1)\right\}$$

$$s=\frac{2a_1^2+\sigma\rho_a\rho_u-(\rho_a+2\rho_u)a_1}{\omega_n^2-\omega^2}$$

$$\omega_n = \frac{1}{\sqrt{\sigma C L_a}} = \frac{\omega}{\sqrt{\sigma k}}$$

$$\frac{\omega}{\omega_n} = \sqrt{\frac{\sigma C}{C_0}} = \sqrt{\sigma k}$$

The instantaneous field current corresponding to (3.53) in symmetrical components is as follows:

$$i_{u1} = \frac{I_f}{2} + \frac{k(\sigma-1)}{1-\sigma k}\frac{I_f}{2}\varepsilon^{-js\omega t}\varepsilon^{-a_1 t}$$
$$-\frac{1}{2}\frac{\omega+\omega_n}{\omega}\frac{k(\sigma-1)}{1-\sigma k}\frac{I_f}{2}\varepsilon^{j(\omega_n-\omega)t}\varepsilon^{-a_2 t}$$
$$-\frac{1}{2}\frac{\omega-\omega_n}{\omega}\frac{k(\sigma-1)}{1-\sigma k}\frac{I_f}{2}\varepsilon^{-j(\omega_n+\omega)t}\varepsilon^{-a_3 t} \qquad (3.54)$$

$$i_{u2} = \overline{i_{u1}}$$

The instantaneous armature current in ordinary phase components corresponding to (3.53) becomes as:

$$i_a = -\frac{e_0\cos(\omega t+\varphi)}{\omega L_a - \frac{1}{\omega C}} - \left\{\frac{e_0\cos\{(1-s)\omega t+\varphi\}}{\sigma\omega L_a - \frac{1}{\omega C}} - \frac{e_0\cos\{(1-s)\omega t+\varphi\}}{\omega L_a - \frac{1}{\omega C}}\right\}\varepsilon^{-a_1 t}$$
$$+\frac{1}{2}\frac{\omega+\omega_n}{\omega}\frac{e_0\cos(\omega_n t+\varphi)}{\sigma\omega L_a - \frac{1}{\omega C}}\varepsilon^{-a_2 t}$$
$$+\frac{1}{2}\frac{\omega-\omega_n}{\omega}\frac{e_0\cos(-\omega_n t+\varphi)}{\sigma\omega L_a - \frac{1}{\omega C}}\varepsilon^{-a_3 t} \qquad (3.55)$$

i_b, i_c = substitution of $\left(\varphi-\frac{2}{3}\pi\right), \left(\varphi-\frac{4}{3}\pi\right)$ instead of φ in (3.55)

The instantaneous field current corresponding to (3.54) in ordinary phase components is as follows:

$$i_u = I_f + \frac{k(\sigma-1)}{1-\sigma k}I_f\varepsilon^{-a_1 t}\cos s\omega t$$
$$-\frac{1}{2}\frac{\omega+\omega_n}{\omega}\frac{k(\sigma-1)}{1-\sigma k}I_f\varepsilon^{-a_2 t}\cos(\omega_n-\omega)t$$
$$-\frac{1}{2}\frac{\omega-\omega_n}{\omega}\frac{k(\sigma-1)}{1-\sigma k}I_f\varepsilon^{-a_3 t}\cos(\omega_n+\omega)t \qquad (3.56)$$

$$i_v = -\frac{I_f}{2} + \frac{k(\sigma-1)}{1-\sigma k}I_f\varepsilon^{-a_1 t}\cos\left(s\omega t+\frac{2}{3}\pi\right)$$
$$-\frac{1}{2}\frac{\omega+\omega_n}{\omega}\frac{k(\sigma-1)}{1-\sigma k}I_f\varepsilon^{-a_2 t}\cos\left\{(\omega_n-\omega)t-\frac{2}{3}\pi\right\}$$
$$-\frac{1}{2}\frac{\omega-\omega_n}{\omega}\frac{k(\sigma-1)}{1-\sigma k}I_f\varepsilon^{-a_3 t}\cos\left\{(\omega_n+\omega)t+\frac{2}{3}\pi\right\} \qquad (3.57)$$

i_w = Substitution of $\frac{4}{3}\pi$ instead of $\frac{2}{3}\pi$ in (3.57)

Assumptions used in obtaining the above approximate solutions:

1. Small value of resistance. ρ_a and ρ_u are small.
2. Small value of σ.
3. ω is large enough comparing with a_1, a_2, a_3 and s.
4. ω_n is larger than ω or $k < \frac{1}{\sigma}$.

   But for frequency components ω_1 and ω_n; a_1 and $\dot{I}_1$, the above approximate solutions are applicable when $\omega_n < \omega$ or $k > \frac{1}{\sigma}$.

Chapter IV. Theoretical Range of Self-excitation.

In the preceding chapter, we arrived at the conclusion that a decrement factor or an attenuation constant of a free oscillation becomes negative within a certain range of condensive load, provided that other proper conditions are fulfilled. The negative decrement factor means an oscillation of negative damping or an oscillation of ever-increasing amplitude. And this is a mathematical basis of explaining the phenomenon of self-excitation.

In the present chapter a precise mathematical range for this negative damping will be discussed.

Let k be, as already described, represented by

$$k=\frac{C}{C_0}=C\omega^2L_a\,;\ \text{and}\ \frac{1}{\omega C_0}=\omega L_a \tag{4.1}$$

And we start from the equation (2.16), which is rewritten as

$$\alpha^3+\alpha^2\Big\{-\omega-j(\rho_a+\rho_u)\Big\}+\alpha\Big\{-\sigma\rho_a\rho_u-\frac{\omega^2}{\sigma k}+j\omega\rho_a\Big\}+\frac{\omega^3}{\sigma k}+j\frac{\rho_u\omega^2}{k}=0 \tag{4.2}$$

Instead of obtaining the condition that directly expresses a negative attenuation, we proceed to get a relation corresponding to zero attenuation. In other words, the attenuation constant a will change its sign from positive to negative value passing through zero in accordance with the change of the value of k, ρ_a, ρ_u, etc., and therefore the condition $a=0$ will be a boundary relation for the mathematical criterion of self-excitation. (Cf. numerical examples in Chapters VI and IX.)

From (4.2), we have

$$k=\frac{j\rho_u\omega^2+\dfrac{\omega^2}{\sigma}(\omega-\alpha)}{j\{\alpha^2\rho_u+\alpha\rho_a(\alpha-\omega)\}+\alpha^2(\omega-\alpha)+\alpha\sigma\rho_a\rho_u} \tag{4.3}$$

Now, if we assume zero attenuation, α becomes a real value, and owing to the condition that k must be a real value, the ratio of the imaginary part in numerator to the imaginary part in denominator in (4.3) must be identical to the corresponding ratio for the real parts, so that we have

$$k=\frac{\rho_u\omega^2}{\alpha^2\rho_u+\alpha\rho_a(\alpha-\omega)}=\frac{\dfrac{\omega^2}{\sigma}(\omega-\alpha)}{\alpha^2(\omega-\alpha)+\alpha\sigma\rho_a\rho_u} \tag{4.4}$$

Next, we put

$$\alpha=(1-s)\omega \tag{4.5}$$

Because of the real values of α and ω, s takes also a real value representing a slip, and (4.4) can be rewritten,

$$k=\frac{1}{(1-s)\left(1-s-s\dfrac{\rho_a}{\rho_u}\right)}=\frac{1}{(1-s)\sigma\left(1-s+\dfrac{\sigma}{s}\dfrac{\sigma\rho_a\rho_u}{\omega^2}\right)} \tag{4.6}$$

Equation (4.6) can further be rewritten as follows:

$$k=\frac{1}{(1-s)(1-s-s\mu)}=\frac{1}{(1-s)\left(1-s+\dfrac{\sigma}{s}\chi\right)\sigma} \tag{4.7}$$

where

$$\left.\begin{aligned}\mu&=\frac{\rho_a}{\rho_u}=\frac{R_a}{\sigma\omega L_a}\div\frac{R_u}{\sigma\omega L_u}\\ \chi&=\frac{\rho_a\rho_u}{\omega^2}=\frac{R_a}{\sigma\omega L_a}\times\frac{R_u}{\sigma\omega L_u}\end{aligned}\right\} \tag{4.8}$$

From (4.7), we have

$$(1-s)\Big\{1-s-s\mu-\sigma\Big(1-s+\frac{\sigma\chi}{s}\Big)\Big\}=0 \tag{4.9}$$

Solving (4.9) with respect to s, we obtain

$$s=1$$

$$s=\frac{1-\sigma}{2(1-\sigma+\mu)}\left\{1\pm\sqrt{1-\frac{4(1-\sigma+\mu)\sigma^2\chi}{(1-\sigma)^2}}\right\} \tag{4.10}$$

Equation (4.10) is a relation which s fulfills under the condition of zero attenuation, and the value of k for this condition can be obtained by the substitution of (4.10) into (4.7) or (4.6).

We see that the value of k for zero attenuation depends upon values of σ, μ, and χ that are merely dimensionless numeric values.

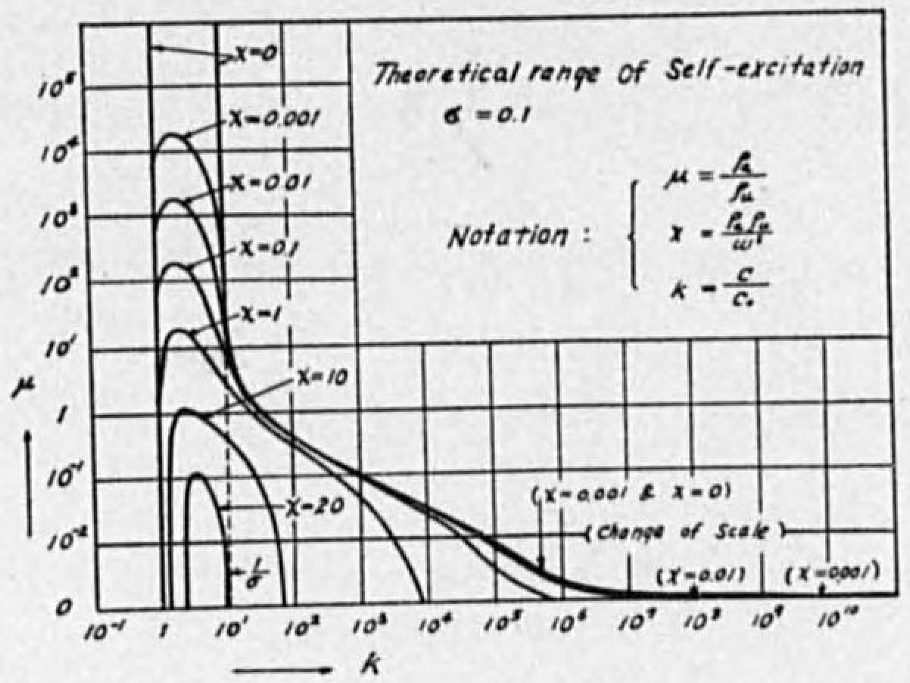

Fig. 3.

Curves of k for zero attenuation corresponding to various values of μ and χ; σ=0.1

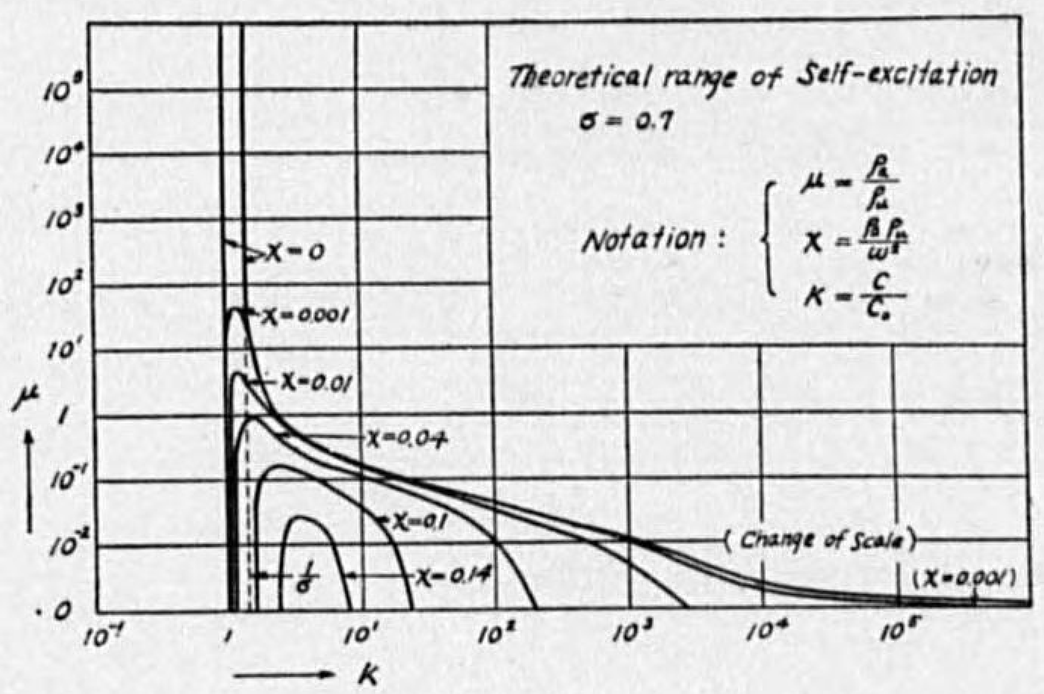

Fig. 4

Curves of k for zero attenuation corresponding to various values of μ and χ; σ=0.7

Examples of curves of k for zero attenuation assuming the constant values of $\sigma=0.1$ and $\sigma=0.7$, corresponding to various μ and χ are given in Figs. 3 and 4.

Each of these represents the $k-\mu$ curve for constants σ and χ. The attenuation constant of one root of equation (4.2) becomes negative in sign for the inside values of

this curve, and becomes positive in sign for the outside values of the curve. In other words, the inside region bounded by these curves indicates the occurrence of a free oscillation of negative damping. The value $\sigma=0.1$ may be said a normal value, while $\sigma=0.7$ is especially a large one, taken merely for the purpose of knowing the effect of σ.

If a machine for consideration is given, σ may be taken as a constant unless an additional inductance is introduced in the circuits and the theoretical region of negative damping for various values of armature circuit resistance, field circuit resistance and connected capacitance will be seen directly from these curves drawn for that value of σ.

Theoretical range of self-excitation.

If we define the term "self-excitation upon condensive load," as the occurrence of a free oscillation of negative damping character from the mathematical point of view, we can conclude from above results that the value k for this range must lie between k_1 and k_2 that are expressed by

$$k_1=\frac{1}{(1-s_1)(1-s_1-s_1\mu)}\ ;\quad k_2=\frac{1}{(1-s_2)(1-s_2-s_2\mu)} \tag{4.11}$$

where

$$\left.\begin{matrix} s_1 \\ s_2 \end{matrix}\right\}=\frac{1-\sigma}{2(1-\sigma+\mu)}\left\{1\pm\sqrt{1-\frac{4(1-\sigma+\mu)\sigma^2\chi}{(1-\sigma)^2}}\right\} \tag{4.12}$$

As an approximate calculation, we may take the square root value in(4.12) is nearly equal to unity for small values of σ and χ. Then the approximate range of self-excitation becomes,

$$k_1=1 \quad \text{to} \quad k_2=\frac{(1-\sigma+\mu)^2}{\mu^2\sigma} \tag{4.13}$$

And the corresponding values of slip are

$$s_1=0 \quad \text{and} \quad s_2=\frac{1-\sigma}{1-\sigma+\mu} \tag{4.14}$$

The condition $k=1$ when $s=0$ is as well known the lower limit of self-excitation, especially for a salient-pole alternator; and under this condition the connected capacity is in resonance with the synchronous reactance of the machine. The condition $k=\frac{(1-\sigma+\mu)^2}{\mu^2\sigma}$ when $s=\frac{1-\sigma}{1-\sigma+\mu}$ is the upper limit and this limit has frequently been supposed to coincide with a condition under which the connected capacitance is in resonance with the leakage reactance. Strictly speaking, this supposition has been found to be theoretically a special case.

We may take the condition $k=\frac{1}{\sigma}$ as an approximate limit only for a small value of χ and a relatively large value of μ as compared with unity.

Dr. R. Rüdenberg makes the following statements in his book* in the sence that the self-excitation of an alternator is only possible within a certain range of connected capacity and this range begins from a synchronous-reactance (un-saturated) resonance condition and continues up to the leakage reactance resonance condition. His conclusion comes, however, from a consideration of the steady state with the rotational frequency and not from attacking the transient phenomena. As above discussed, the statement for this range especially the upper limit must be altered in the point of transient theory for a symmetrical alternator when the resistance component of the machine is taken into consideration. The leakage reactance must naturally be altered to the total leakage reactance which corresponds to the sum of the field leakage and the armature leakage reactance, and is nearly equal to transient reactance as is well known from the recent studies on the sudden short circuit of an alternator. To take the resonance condition for the total leakage reactance as the upper limit is an approximate one, and it is in general a special case. According to the traditional idea, an alternator will be self-excited by an armature reaction of leading current when it is suddenly connected to a condensive load of large value; while if the load becomes larger than that to give the resonance condition for leakage reactance, the armature current will change to lag, hence there will be no self-excitation. This idea is, however, based upon the assumption that the frequency of self-excitation is exactly equal to that of rotation.

As will be seen from (4.10) or (4.14) the theoretical frequency of undamped

* References (8) p. 308 and p. 311.

oscillation is lower than that of rotation. The frequency of the amplifying free oscillation becomes lower and lower as the value of capacity increases.

One numerical example of calculation shows that the frequency of self-excitation decreases down to nearly one half of the rotational frequency (See Chapter VI).

This theoretical result does not coincide with the assumption used in the traditional idea, and we can see that the incorrectness of the conclusion deduced from the old idea is a natural consequence of the unreasonable assumption, although the discussion as to the upper limit of the range of self-excitation is comparatively of academic nature.

We may, however, adopt a conventional explanation that, when the capacity takes a larger value than that corresponding to the resonance condition for the leakage reactance, a free oscillation, the frequency of which nearly equals the rotational frequency, damps away and a free oscillation the frequency of which is nearly equal to the resonance frequency for the total leakage inductance and capacity amplifies in turn and the self-excitation continues up to a larger condensive load.

The theoretical lower limit of self-excitation for a salient-pole alternator is somewhat strictly discussed in Chapter XIII.

Chapter V. Some Physical Explanations of Mathematical Results and Graphical Solutions.

(1) Sudden short circuit of a symmetrical alternator.

Sudden short circuit of an alternator may be dealt with as a special case of the transient phenomena upon condensive load.

We can obtain a mathematical solution by putting $C=\infty$ in the equations previously described. Putting $C=\infty$ or $\omega_n=0$, we can write the following approximate relations from (3.10), (3.11) and (3.12):

$$\left.\begin{aligned} &a_1+a_2+a_3=\rho_a+\rho_u \\ &\omega(a_2+a_3)=\omega\rho_a \\ &a_2a_3(ja_1+\omega)=0 \end{aligned}\right\} \qquad (5.1)$$

From (5.1), we obtain

$$a_1=\rho_u\,;\quad a_2a_2=0\,;\quad a_2+a_3=\rho_a \tag{5.2}$$

And from (5.2) we determine as,

$$a_1=\rho_u\,;\quad a_2=\rho_a\,;\quad a_3=0 \tag{5.3}$$

And we have an approximate relation,

$$\alpha_1=\omega+j\rho_u\,;\quad \alpha_2=0+j\rho_a\,;\quad \alpha_3=0 \tag{5.4}$$

Approximate values of armature currents, $\dot{I}_1$ and $\dot{I}_2$ will be calculated from (2.19) by the substitution of (5.4) in the similar manner as we have calculated in (4) of Chapter III, and we obtain,

$$\left.\begin{aligned}\dot{I}_1&=\left(\frac{1}{j\sigma\omega L_a}-\frac{1}{j\omega L_a}\right)\dot{E}_{a1}\\ \dot{I}_2&=-\frac{\dot{E}_{a1}}{j\sigma\omega L_a},\quad \dot{I}_3=0\end{aligned}\right\} \tag{5.5}$$

This approximate value $\dot{I}_1$ is the same result as obtained by putting $C=\infty$ into the expression of $\dot{I}_1$ in (3.42).

Thus we obtain one form of the instantaneous approximate value of armature current expressed in symmetrical components as follows:

$$\left.\begin{aligned}i_{a1}&=\dot{E}_{a1}\left[\frac{\varepsilon^{j\omega t}}{j\omega L_a}+\left\{\frac{\varepsilon^{j\omega t}}{j\sigma\omega L_a}-\frac{\varepsilon^{j\omega t}}{j\omega L_a}\right\}\varepsilon^{-\rho_u t}-\frac{1}{j\sigma\omega L_a}\varepsilon^{-\rho_a t}\right]\\ i_{a2}&=\overline{i}_{a1}\end{aligned}\right\} \tag{5.6}$$

And the ordinary phase components of armature current corresponding to (5.6) will be obtained in a similar way as described in (5) of Chapter III as follows:

$$\begin{aligned}i_a=&-\frac{e_0\cos(\omega t+\varphi)}{\omega L_a}-\left\{\frac{e_0\cos(\omega t+\varphi)}{\sigma\omega L_a}-\frac{e_0\cos(\omega t+\varphi)}{\omega L_a}\right\}\varepsilon^{-\rho_u t}\\ &-\frac{e_0\cos\varphi}{\sigma\omega L_a}\varepsilon^{-\rho_a t}\end{aligned} \tag{5.7}$$

i_b, i_c = Substitution of $\left(\varphi-\frac{2}{3}\pi\right)$, $\left(\varphi-\frac{4}{3}\pi\right)$ instead of φ in (5.7).

The equations (5.6) and (5.7) are well-known approximate expressions for sudden short-circuit currents.

(2) Comparison between the transients for condensive load and sudden short-circuit.

We can write an approximate expression for a sudden short-circuit armature current as in the following form from (5.6),

$$i_{a1}=\underset{\text{(Rotational freq.)}}{\dot{I}_s\varepsilon^{j\omega t}}+\underset{\text{(Rotational freq.)}}{\dot{I}_1\varepsilon^{j\alpha_1 t}}+\underset{\text{(D.C.)}}{\dot{I}_2\varepsilon^{j\alpha_2 t}} \tag{5.8}$$

And for condensive load, an approximate expression may be written assuming k is not very large,

$$i_{a1}=\underset{\text{(Rotational freq.)}}{\dot{I}_s\varepsilon^{j\omega t}}+\underset{\text{(Rotational freq.)}}{\dot{I}_1\varepsilon^{j\alpha_1 t}}+\underset{(\omega_n)}{\dot{I}_2\varepsilon^{j\alpha_2 t}}+\underset{(-\omega_n)}{\dot{I}_3\varepsilon^{j\alpha_3 t}} \tag{5.9}$$

If we consider the A.C. component of the sudden short-circuit current, it will be the sum of $\dot{I}_s$ and $\dot{I}_1$, which is approximately expressed by

$$\dot{I}_s+\dot{I}_1=\frac{\dot{E}_{a1}}{j\sigma\omega L_a} \tag{5.10}$$

And this is also equal and opposite to $\dot{I}_2$ at the instant $t=0$, and satisfies the condition $i_{a1}=0$. The term $\sigma\omega L_a$ is known as the total leakage reactance or transient reactance which is the sum of field leakage reactance and armature leakage reactance.

In the case of the condensive load, $\dot{I}_s$ and $\dot{I}_1$ may also be considered in the same manner as in the case of the sudden short-circuit and the magnitude of A.C. component of the rotational frequency is represented by

$$\dot{I}_s + \dot{I}_1 \doteqdot \frac{\dot{E}_{a1}}{j\left(\sigma\omega L_a - \frac{1}{\omega C}\right)} \qquad (5.11)$$

And we may use the term "transient reactance" for this case with regard to $\left(\sigma\omega L_a - \frac{1}{\omega C}\right)$. The sum of this A.C. component, $\dot{I}_2$ and $\dot{I}_3$ satisfies of course the boundary condition that the total current is zero at the instant $t=0$.

In the case of the sudden short-circuit, $\dot{I}_2$ is a damped D.C. component and may be considered as a stationary damping vector or a current system in space fixed at a certain axis which is determined at the instant of the occurrence of short circuit. But, in the case of the condensive load, this stationary current may be considered to become an oscillatory current composed of the two revolving current systems that are represented by $\dot{I}_2$ and $\dot{I}_3$. The former revolves in the same direction as the rotor, while the latter in the opposite direction.

(3) Explanation with regard to graphical solution.

An ordinary explanation of the behaviour of an alternator upon condensive load can be made graphically with the aid of two characteristics. One is a charging characteristic and the other is a saturation characteristic excited by armature current. The example of this graphical solution, when the effect of magnetic saturation is taken into account, is given as shown in Fig. 5.

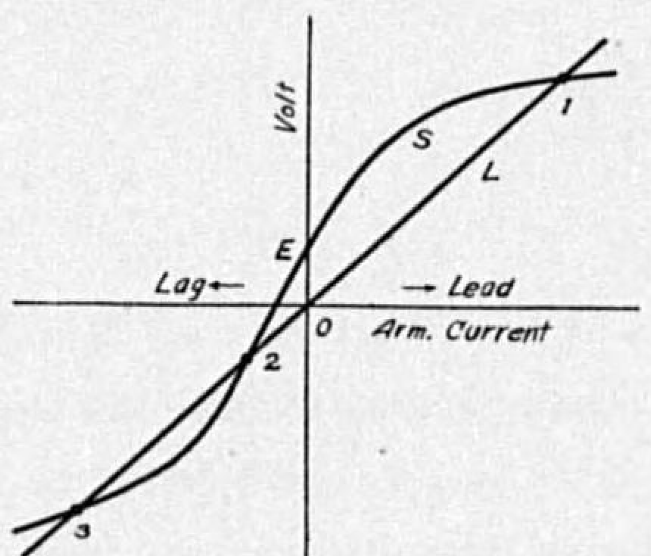

Fig. 5. Graphical solution.

L represents the charging characteristic $\left(\frac{I_{a1}}{\omega C}\right)$ and S the saturation characteristic excited by armature current $(\omega L_a I_{a1})$. The precise method for obtaining these characteristics for a salient-pole machine has been described in the previous report of the author.(2)

When $k<1$ or the initial inclination of S is smaller than that of L, characteristics S and L have only one point of intersection as (1) in the leading-current region, and this point is a stable point which corresponds to $\dot{I}_s$.

When $k>1$ or the initial inclination of S is greater than that of L, there are three points of intersection for S and L as (1), (2), and (3) as shown in the figure. The detailed discussions of the stability characteristic for these points are omitted here and the discussion is chiefly limited to points like (1) and (2) when the machine is suddenly switched.

When the alternator is switched on to the condensive load with the initial armature voltage $\overline{OE}$, whether the phenomenon advances to the point (1) or to the point (2), entirely depends upon the nature of the transient term which is determined by the circuit constants. In other words, this determination as to whether the phenomenon advances to the leading-current region or to the lagging-current region is of a decisive nature and not a matter of chance. And this depends upon the damping or amplifying nature of the transient term $\dot{I}_1 \varepsilon^{j\alpha_1 t}$.

Dr. R. Rüdenberg describes* that this depends upon the phase relation of voltage with which an alternator is switched on to the circuit. The present author, however, believes that Rüdenberg's description must be altered in accordance with the study on the transient phenomena.

We have the following examples which will verify the foregoing statements of the present author.

A symmetrical alternator (wound-rotor-type induction generator) is excited by the direct current in the field circuit through a considerably large resistance and then it is switched upon the condensive load of negligible resistance and in a relation of $1<k<\frac{1}{\sigma}$. And the steady-state characteristics drawn for the rotational frequency are as shown in Fig. 5. Then the sign of the attenuation constant of the transient

* References (8) 1923. p. 306.

term $I_1\varepsilon^{j\alpha_1 t}$ becomes positive, because ρ_n is very large, consequently μ being small, and χ is large, so that the attenuation constant corresponds to the outside region of the curves shown in Fig. 3. And the alternator advances to the point (2) after it is switched on to the load and results in a stable operation in the lagging-current region operating at the rotational frequency.

On the contrary, assume that the field is excited through a comparatively low resistance; the magnitude of the exciting current and that condensive load are kept at the same value as those in the foregoing case. Then the attenuation constant of the transient term becomes negative corresponding to the case of the self-excitation. The alternator advances to the point (1) after it is switched on to the load with the same initial voltage $\overline{OE}$ and results in a stable operation in the leading-current region.

The occurrence of the phenomena corresponding to these two cases have been ascertained experimentally by the present author.

The operating frequency in the latter case is equal to the rotational frequency when the initial excitation is not very small, perhaps due to the effect of magnetic saturation. The alternator operates with a continuous slip after self-excitation, as long as the initial excitation is kept very small. This corresponds to the superposition of the rotational frequency due to the direct-current excitation and the frequency a little lower than the rotational frequency due to the free oscillation.

As will be seen from the above example, we have always two operating points (1) and (2) for the symmetrical alternator when the steady-state characteristics are in the relation as shown in Fig. 5. We can not therefore, judge the actual operating point merely by these steady-state characteristics when the machine is switched on to the load.

We have noticed that there are somewhat different behaviours for a salient pole alternator. In an ordinary state of self-excitation for a salient-pole alternator, we can judge the operating point after it is switched on to the condensive load by the steady state characteristics alone. Because we have generally no actual operating point like (2) within a certain range of the values of connected capacitance [(a certain value) $>k>1$, see Chapter VIII], independent of the field-circuit condition. An alternator gives rise to self-excitation even if the field circuit is open.

The reason for this fact may be explained conventionally that the equivalent field circuit is always closed when the alternator is considered as an equivalent symmertical machine, even if the field circuit is open. In other words, the equivalent resistance component of the field circuit treated as a symmetrical alternator is not so large as to make the attenuation constant of the free oscillation positive. (See examples in Chapter IX.)

The same fact will be strictly explained from the circuit theory that a salient-pole alternator has an amplifying free oscillation, even if the field circuit is open, provided that the value of a connected capacitance is within a certain range.

This result will be obtained from the mathematical attack of the differential equations for a salient-pole alternator. The amplifying free oscillation of the rotational frequency for the salient-pole alternator is discussed in Chapter VIII.

The operating point (1) in Fig. 5 is the well-known point for a salient-pole alternator. The point (2) is obtained when the connected capacitance is extraordinarily large and the alternator is considered to be short-circuited by the capacitance. The author has also experienced that the operating point (2) exists in a special range when the connected capacitance is nearly equal to the resonant value with the quadrature reactance for a salient-pole alternator which has no damper winding.

The point (3) is obtained in the case of the negative field excitation. The author has experienced the stable operation corresponding to the point (3) within a certain range.

(4) Voltage build-up time due to self-excitation.

The instantaneous value of terminal voltage of an alternator due to the transient current upon condensive load is written from (2.3) in terms of symmetrical components as,

$$v_{a1}=\frac{1}{pC}i_{a1}$$

The value i_{a1} in this condition is rewritten from (2.17) as,

$$i_{a1}=\dot{I}_s\varepsilon^{j\omega t}+\dot{I}_1\varepsilon^{j\alpha_1 t}+\dot{I}_2\varepsilon^{j\alpha_2 t}+\dot{I}_3\varepsilon^{j\alpha_3 t}$$

Therefore we have by the substitution of i_{a1},

$$v_{a1}=\frac{\dot{I}_s}{j\omega C}\varepsilon^{j\omega t}+\frac{\dot{I}_1}{j\alpha_1 C}\varepsilon^{j\alpha_1 t}+\frac{\dot{I}_2}{j\alpha_2 C}\varepsilon^{j\alpha_2 t}+\frac{\dot{I}_3}{j\alpha_3 C}\varepsilon^{j\alpha_3 t} \quad (5.12)$$

We may be able to obtain the change of terminal voltage by the calculation of equation (5.12).

Next we consider the approximate value of the envelope of terminal voltage with regard to the lapse of time, chiefly due to the amplifying free oscillation, neglecting the damping and different frequency terms $\dot{I}_2$ and $\dot{I}_3$. We assume the terminal voltage of the machine in a convenient and approximate form as follows:

$$v' = V_s + V_1 \varepsilon^{-a_1 t} \tag{5.13}$$

where v' is the terminal voltage corresponding to the envelope, and a_1 the attenuation constant.

From (5.13), we get

$$\frac{dv'}{dt} = -a_1(v' - V_s)$$

$$t + C' = \frac{1}{-a_1}\int \frac{dv'}{v' - V_s} = \frac{1}{-a_1} \log (v' - V_s)$$

where C' is the integration constant. If we represent the initial voltage when $t=0$ by V_0 (eff. or max. value), then we will obtain,

$$C' = \frac{1}{-a_1} \log (V_0 - V_s)$$

$$\therefore \quad t = \frac{1}{-a_1} \log \frac{v' - V_s}{V_0 - V_s} \tag{5.14}$$

Substituting an approximate value of a_1 from (3.19), we get

$$t = T_0 \frac{1-\sigma k}{k-1} \log \frac{v' - V_s}{V_0 - V_s} \tag{5.15}$$

We may say that the voltage build-up time is approximately proportional to the open-circuit time constant of field winding and $\frac{1-\sigma k}{k-1}$.

When the magnetic saturation is taken into account or k is considered as variable with voltage or current, an approximate build-up time will be calculated from the open-circuit time constant of field winding, saturation characteristic excited by armature current, and charging characteristic. The details has been reported in the previous paper of the author. (5) (9)

Chapter VI. Numerical Examples for a Symmetrical Alternator.

Numerical examples of calculation are given herewith for a symmetrical alternator in order to apprehend the general nature of frequency and attenuation characteristics with the change of connected capacitance in a wide range. These are examples of the numerical general solution for α or the equations of (2.15) and (2.16), and not of approximate solutions.

The machine under test was a wound-rotor-type induction motor used as a generator and had a rating of 7.5 HP., 200V, 50 cycles, and 1,000 R.P.M.

The machine constants have been determined as follows:

$$\left.\begin{array}{ll} L_a = 0.0570\ H & R_a = 0.168\ \Omega \\ L_u = 0.0029\ H & R_u = 0.0127\ \Omega \\ M = 0.0119\ H & \end{array}\right\} \tag{6.1}$$

These are values taken for unsaturated parts, and from (6.1) we calculated:

$$\sigma = 0.143;\quad \rho_a = 20.61;\quad \rho_u = 30.87$$

And we took,

$$\omega = 2\pi f = 100\pi \tag{6.2}$$

The numerical equation (2.15) with the constants expressed in (6.2) is calculated as follows:

$$\alpha^3 - (314.16 + j51.48)\alpha^2 - \left(90.981 + \frac{122.68}{C} - j6474.8\right)\alpha$$

$$+ \frac{1}{C}(38542 + j541.58) = 0 \tag{6.3}$$

The numerical equation of (2.16) corresponding to (6.3) is calculated as,

$$\alpha^3-(314.16+j51.48)\alpha^2-\left(90.981+\frac{690200}{k}-j6474.8\right)\alpha$$

$$+\frac{10^6}{k}(216.84+j3.0469)=0 \qquad (6.4)$$

The machine was driven at rated speed (angular velocity $\omega=2\pi f=314.16$, 50~) and was connected to a condensive load through a bank of step-up transformers as shown in Fig. 6.

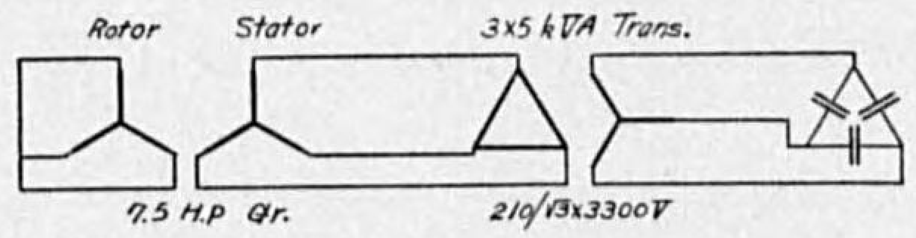

Fig. 6. Test diagram for a symmetrical alternator.

When the connected capacitance was about 0.00035 F (equivalent star value at generator terminal), the machine gave rise to self-excitation with rotor circuit closed and the terminal voltage was built up slowly until about 250V.

When the connected capacitance was about 0.000116F, there was no phenomenon of self-excitation, or any appreciable voltage rise was not perceived.

The numerical solutions of the equation (6.3) or (6.4) against these capacitances are as follows:

When $C=0.00035$ F, or $k=1.969$,

$\alpha_1=\omega_1+ja_1=313.36-j5.84$ (49.87 cycles, builds up)

$\alpha_2=\omega_2+ja_2=592.41+j38.38$ (94.28 cycles, damps)

$\alpha_3=\omega_3+ja_3=-591.61+j18.94$ (94.09 cycles, damps)

When $C=0.000116$ F, or $k=0.6526$,

$\alpha_1=\omega_1+j\imath_1=314.17+j1.67$ (50.0 cycles, damps)

$\alpha_2=\omega_2+ja_2=1025.17+j29.38$ (163.2 cycles, damps)

$\alpha_3=\omega_3+ja_3=-1025.18+j20.43$ (163.2 cycles, damps)

In these calculations losses in the transformers and condensers are neglected. When $C=0.00035$ F or $k=1.969$, we have two damped transient components and one amplifying transient component. When $C=0.000116F$ or $k=0.6526$, all transient components damp away. Thus we have an amplifying transient component corresponding to the case of self-excitation.

How the angular velocity and attenuation constant of the transient components vary with the change of capacitance C or k in this example, are tabulated in Table I. These calculations are all the results of the general solution of (6.3) or (6.4) and not of approximate solutions.

Table I.

Angular velocities and attenuation constants against the variation of connected capacitance. The solution of cubic equation (6.3).

	C (farad)	$k=\frac{C}{C_0}$	$\alpha_1=\omega_1+ja_1$	$\alpha_2=\omega_2+ja_2$	$\alpha_3=\omega_3+ja_3$	Remarks
1	0.00005	0.2813	314.18+j 3.31	1566.21+j26.85	−1565.23+j21.32	
2	0.0001	0.5626	314.17+j 2.10	1107.35+j28.77	−1107.36+j20.61	
3	0.0001777	1.000	314.11+j0.00212	830.48+j31.58	−830.43+j19.90	Near the lower limit of self-excitation.
4	0.00035	1.969	313.36−j 5.84	592.41+j38.38	−591.61+j18.94	Self-excitation.
5	0.0007	3.938	303.74−j22.06	428.5 +j55.69	−418.08+j17.85	〃
6	0.001	5.626	285.27−j29.86	378.51+j64.07	−349.62+j17.26	〃
7	0.00124	6.993	270.15−j30.23	357.44+j64.81	−313.44+j16.90	〃 $k=1/\sigma$
8	0.003	16.88	197.69−j11.33	317.81+j47.35	−201.35+j15.45	〃
9	0.00645	36.30	137.61+j 0.0004	313.34+j37.19	−136.79+j14.28	Near the upper limit of self-excitation.
10	0.0124	69.97	99.46+j 4.17	312.72+j33.88	−98.03+j13.42	
11	0.1	562.6	34.37+j 8.79	312.44+j31.26	−32.66+j11.41	
12	1.0	5626.	5.216+j11.12	312.41+j30.96	−3.47+j 9.39	

The results of calculations are also plotted in curves as shown in Figs. 7 and 8.

In Figs. 7 and 8, we can see the variation of attenuation constants and angular velocities with the change of values of connected capacity or k, provided that the constants of the machine are unchanged.

The attenuation constant a_1 is negative in the range of values of k from nearly $k=1$ and up to $k=36.3$ These regions are hatched in the figures and signify the occur-

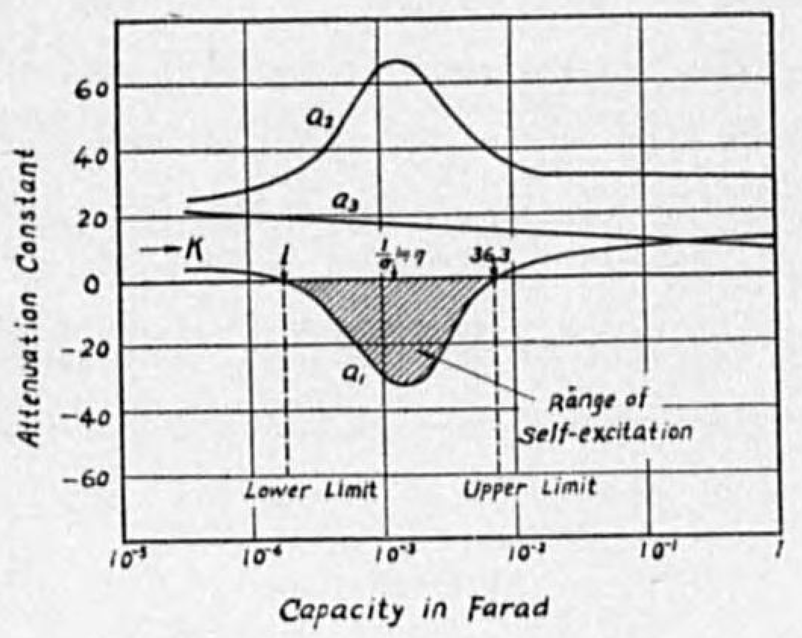

Fig. 7. Attenuation constant curves.

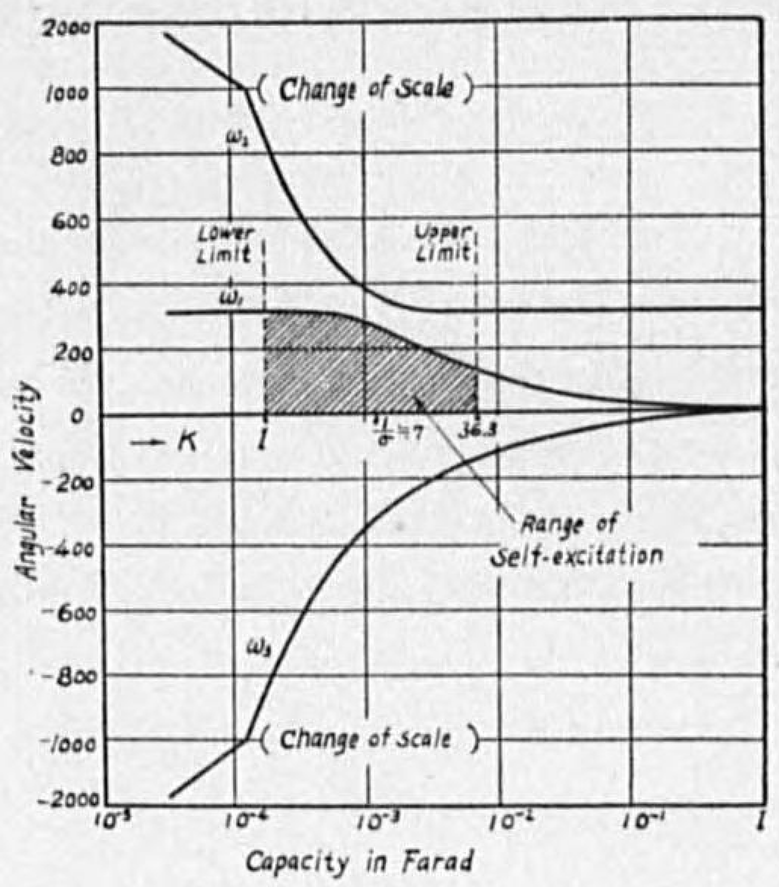

Fig. 8. Angular velocity curves.

rence of the amplifying free oscillation or self-excitation. The attenuation constant a_2 is generally greater than a_3. The angular velocity ω_1 changes its magnitude from the value nearly equal to ω down to a value as low as zero. ω_2 and ω_3 are nearly equal to ω_n at small values of k, and ω_2 takes a magnitude nearly equal to ω when k becomes greater than $\frac{1}{\sigma}$. ω_1 and ω_3 may be considered as nearly equal to ω_n at very large values of k, where ω_n means $\frac{1}{\sqrt{\sigma C L_a}}$.

In the actual experiments with the above machine, the operating point after self-excitation was nearly equal to the point of intersection of the line charging curve and the saturation curve excited by armature current drawn at the rotational frequency when the connected capacity was nearly 0.00035 F. The armature terminal voltage was about 250V, and the frequency of armature voltage and current was about 49.83 cycles, a little lower than 50 cycles.

Chapter VII. Equivalent Symmetrical Alternator with Fictitious Negative Armature Resistance.

In the case of self-excitation in a symmetrical alternator, the voltage builds up

with a slip as has already been described in the foregoing Chapter.

In a salient-pole alternator, however, the voltage builds up without slip in an ordinary case. We have two methods for the explanation of such a difference.

One method is to solve a differential equation for a salient-pole alternator which has a term expressing the nature of saliency as will be discussed in Chapter VIII, and the other is to explain in a conventional form from a equivalent symmetrical alternator which has a conventional character of saliency.

In the present chapter we deal with only the latter method of the two. It is an ordinary way to determine the constants as an equivalent symmetrical alternator for a salient-pole machine that R_a is taken from the ohmic resistance, L_a from synchronous direct reactance, and chiefly R_u, and M are chosen as equivalent values, or in other words, ρ_u and σ will be determined as equivalent values.

The author has an idea to supplement a character of saliency by introducing a negative armature resistance as an effective resistance of armature circuit not only for a steady-state current, but also for a transient current, the frequency of which is equal to that of rotation, with regard to a certain problem. That is to vary the conventional value of ρ_a from positive to negative for a certain range of values of connected capacitance as described below.

(1) Effective resistance of armature circuit in steady state.

First, we will show that the effective armature resistance and reactance of a salient-pole machine vary in accordance with the phase angle of armature current in steady state.

The differential equation for a salient-pole alternator may be written as follows: [See Appendix III. (C.9)]

$$\left.\begin{aligned} v_{a1} &= -\{R_a+pL_a\}i_{a1}-L_r p\varepsilon^{j2\theta}i_{a2}-\frac{M'}{2}p\varepsilon^{j\theta}i_u \\ v_{a2} &= \bar{v}_{a1} \end{aligned}\right\} \qquad (7.1)$$

Now considering a balanced three-phase armature current in steady state, the axis of reaction of which is lagging by an angle ψ behind the axis of field pole, we may put as follows:

$$i_{a_1}=I\varepsilon^{-j\psi}\varepsilon^{j\theta}\,;\quad i_{a_2}=I\varepsilon^{j\psi}\varepsilon^{-j\theta}\,;\quad \theta=\omega t+\varphi \tag{7.2}$$

Substituting (7.2) into (7.1), we get the equation of voltage-drop in the armature circuit due to this armature current as,

$$v_{a1}=-\{R_a+j\omega L_a\}i_{a1}-j\omega L_r\varepsilon^{-j2\psi}i_{a1}$$
$$=-\{R_a-\omega L_r\sin 2\psi+j\omega(L_a+L_r\cos 2\psi)\}i_{a1} \tag{7.3}$$

We have from (7.3) the effective armature-circuit constants due to the variation of phase angle ψ as follows:

$$\left.\begin{aligned}&\text{Effective resistance of armature circuit } =R_a-\omega L_r\sin 2\psi\\&\text{Synchronous reactance of armature circuit } =\omega L_a+\omega L_r\cos 2\psi\end{aligned}\right\} \tag{7.4}$$

When $\psi=0$, or the axis of armature reaction coincides with the axis of field pole, we have a direct synchronous reactance x_d as,

$$x_d=\text{direct (synchronous) reactance}=\omega L_a+\omega L_r \tag{7.5}$$

This is the maximum value and ordinarily taken merely as a synchronous reactance.

When $\psi=90°$ or the axis of reaction is at the midway between poles, we have as a quadrature synchronous reactance x_q

$$x_q=\text{quadrature (synchronous) reactance}=\omega L_a-\omega L_r \tag{7.6}$$

From (7.5) and (7.6), we get the constant to express the saliency as,

$$L_r=\frac{1}{2\omega}(x_d-x_q) \tag{7.7}$$

It is clear from (7.4) that the effective armature resistance varies from positive to negative in value according to the phase angle ψ, when R_a is neglected. This fictitious negative resistance becomes maximum when $\psi=45°$. If we denote the maximum value of this fictitious resistance by R_r, then

$$R_r=\omega L_r=\frac{1}{2}(x_d-x_q) \tag{7.8}$$

This fictitious resistance occurs in accordance with the difference of x_d and x_q or the salient nature of the machine. The salient-pole machine can generate or consume power proportional to the square of armature current without field excitation due to the so-called "reaction torque" and this nature may coventionally be expressed by a fictitious resistance. We can, therefore, express the effective resistance of armature circuit as

$$R=R_a-R_r\sin 2\psi \tag{7.9}$$

If we use the above value of R instead of a mere ohmic resistance R_a, the value ρ_a can fictitiously be considered from zero to a certain negative value for a current of the rotational frequency.

(2) Solution of differential equations for a symmetrical alternator with fictitious negative armature resistance.

Next we will explain the fact that an amplifying free oscillation of the rotational frequency can be obtainable, as a particular solution for transient current of the symmetrical alternator with fictitious negative armature resistance, within a certain range of values of connected capacitance.

Now we assume one root of (2.15) is in a form,

$$\alpha_1=\omega+ja_1\,,\quad a_1<0 \tag{7.10}$$

and will discuss the condition to satisfy the above solution.

First, we put $s=0$ in (3.21) and (3.22) and obtain the following equations.

$$2a_1^2-(\rho_a+2\rho_u)a_1+\sigma\rho_a\rho_u=0 \tag{7.11}$$

$$-a_1^3+(\rho_a+\rho_u)a_1^2+\left(\omega^2-\sigma\rho_a\rho_u-\frac{1}{\sigma CL_a}\right)a_1+\rho_u\left(\frac{1}{CL_a}-\omega^2\right)=0 \tag{7.12}$$

Substituting $k=C\omega^2L_a$ into (7.12) and further eliminating a_1^3, we get

$$\rho_a a_1^2+\left(2\omega^2-\sigma\rho_a\rho_u-\frac{2\omega^2}{\sigma k}\right)a_1+2\rho_u\omega^2\left(\frac{1}{k}-1\right)=0 \tag{7.13}$$

From (7.11), we have

$$\rho_a=\frac{2a_1(\rho_u-a_1)}{\sigma\rho_u-a_1} \tag{7.14}$$

We can say from (7.14),

$$\rho_a<0, \quad \text{if } a_1<0$$

That is, if we want to obtain the negative attenuation constant, ρ_a and consequently the armature resistance must necessarily be negative. And from (7.13), we have

$$k=\frac{2\omega^2\left(\dfrac{a_1}{\sigma}-\rho_u\right)}{\rho_a a_1^2+(2\omega^2-\sigma\rho_a\rho_u)a_1-2\omega^2\rho_u} \tag{7.15}$$

We can say from (7.15),

$$k>0 \text{ and real, if } a_1<0 \text{ and } \rho_a<0$$

That is, if we assume ρ_a is negative and a_1 is the corresponding negative value, we have necessarily a positive value of k. As above discussed, the necessary condition to satisfy the solution (7.10) is $\rho_a<0$ and a certain value of $k>0$. Thus we can say that, if we assume a certain value of ρ_a which is negative, we will obtain, for the corresponding certain positive value of k, the amplifying free oscillation of the rotational frequency. Equation (7.15) will be rewritten as,

$$a_1=\frac{\sigma\rho_a^2\rho_u+4\rho_u\omega^2\left(1-\dfrac{1}{k}\right)}{4\omega^2\left(1-\dfrac{1}{\sigma k}\right)+\rho_a^2+2\rho_a\rho_u-2\sigma\rho_a\rho_u} \tag{7.16}$$

A reasonable approximate range of k to satisfy the condition (7.10) will be taken as,

$$1<k< \text{ a certain value less than } \frac{1}{\sigma}.$$

If we assume $a_1=0$, then $\rho_a=0$ from (7.14) and $k=1$ from (7.16). And the condition $k=1$ may be conventionally considered as the lower limit of self-excitation.

A negative value of ρ_a will conventionally be considered for the salient-pole alternator within a range of the allowable negative resistance as already expressed in (7.9).

Numerical examples for the relation of a_1, ρ_a, and k with given σ and ρ_u are shown in Fig. 9. The portion denoted by full-line in the figure may be considered as the allowable range of negative resistance.

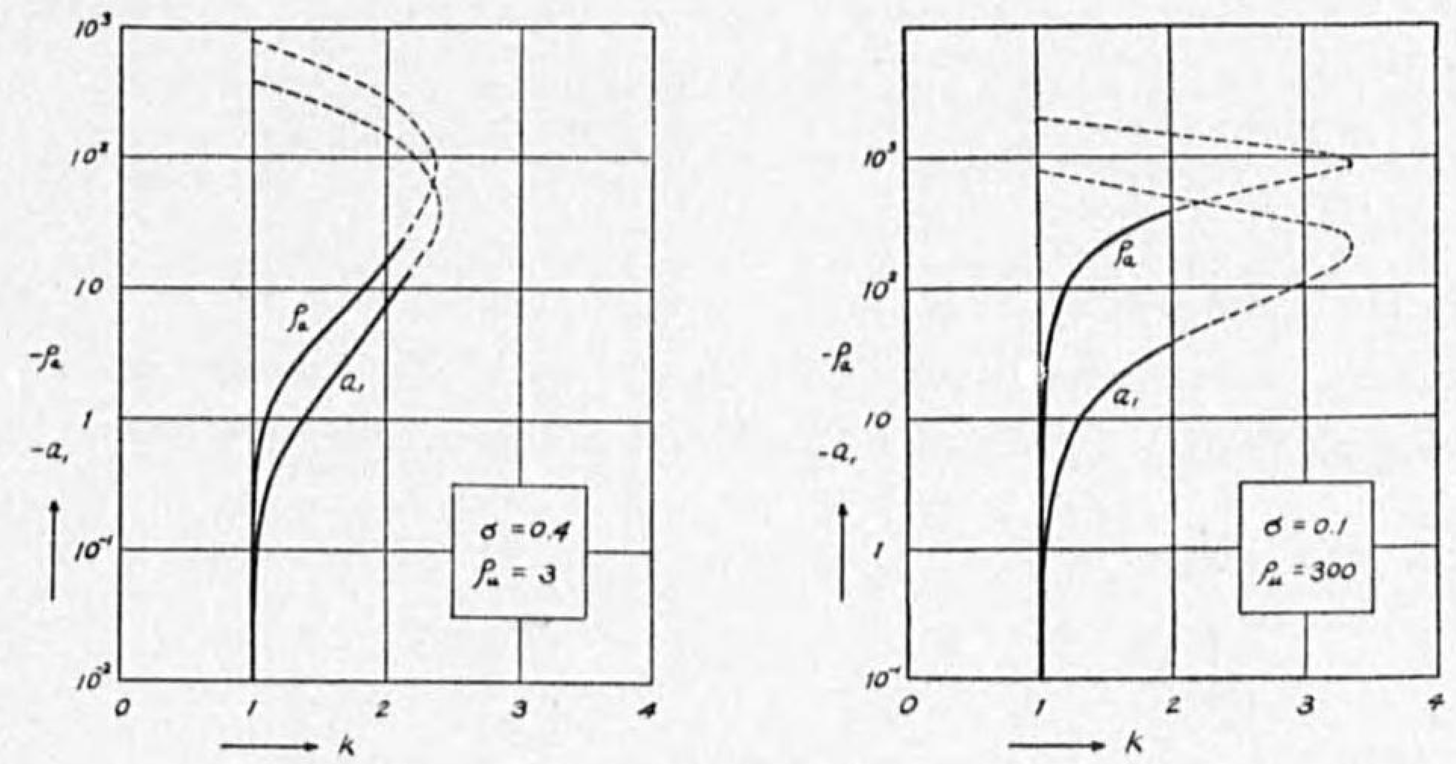

Fig. 9.
Range for negative attenuation constant corresponding to negative armature resistance.

We may conclude from the above discussion that a symmetrical alternator with the proper value of fictitious negative armature resistance may be said,

(i) To have an amplifying free oscillation of rotational frequency, within a certain range of k greater than 1.
Or the frequency with which the voltage builds up during self-excitation is rotational and has no slip within a certain range.

(ii) To operate at the rotational frequency after self-exicitation.

The foregoing statement may conventionally be applicable to a salient-pole alternator as an equivalent symmetrical alternator. It is, however, not preferable to determine by such a conventional method, the strict range of capacitance with which an amplifying free oscillation of the rotational frequency occurs in the actual salient-pole machine, which has a complicated circuit condition, especially in field circuit, such as the existence of damper windings, etc.

Chapter VIII. Theoretical Consideration about the Salient-Pole Alternator.

In this chapter there is given a theoretical consideration of the fact that an ordinary self-excitation of a salient-pole alternator takes place with rotational frequency, or without a slip; this will be discussed from a solution of the differential equation for a salient-pole alternator. That is, we have the amplifying free oscillation of rotational frequency when a salient-pole alternator is suddenly switched on to a capacitance of a certain magnitude, and the discussion given here is chiefly limited to this point.

Instead of employing equation (2.5) we use the differential equations for a salient-pole alternator when the field circuit is considered as a circuit consisting of a single-phase winding, which are given as follows: [See Appendix III. (C.9)]

$$\left.\begin{aligned} v_{a1} &= -\dot{E}_{a1}\varepsilon^{j\theta} \\ &= -\left\{R_a+pL_a+\frac{1}{pC}\right\} i_{a1}-L_r p\varepsilon^{j2\theta} i_{a2}-\frac{M'}{2}p\varepsilon^{j\theta} i_u \\ v_{a2} &= \overline{v}_{a1} \\ v_u &= -\{R_u+pL_u\}i_u-\frac{3}{2}M'p(\varepsilon^{-j\theta}i_{a1}+\varepsilon^{j\theta}i_{a2})=0 \end{aligned}\right\} \qquad (8.1)$$

Considering the free oscillation alone, and putting $\dot{E}_{a1}=0$, we have

$$\left\{R_a+pL_a+\frac{1}{pC}-\frac{M_s^2 p(p-j\omega)}{R_u+(p-j\omega)L_u}\right\}i_{a1}$$

$$+L_r p\varepsilon^{j2\theta}i_{a2}-M_s^2\,\varepsilon^{j2\theta}\frac{(p+j2\omega)(p+j\omega)}{R_u+(p+j\omega)}i_{a2}=0 \qquad (8.2)$$

where $M_s^2\equiv\frac{3}{4}M'^2$

We consider only the free oscillation with rotational frequency and put as follws:

$$\left.\begin{aligned} & i_{a1}=I\varepsilon^{-j\psi}\varepsilon^{j\theta-a_1 t}; \quad i_{a2}=I\varepsilon^{j\psi}\varepsilon^{-j\theta-a_1 t} \\ & \theta=\omega t+\varphi \end{aligned}\right\} \qquad (8.3)$$

where a_1 represents the attenuation constant, and ψ the phase angle of transient current. In the following attack, the author makes it a principal object to obtain the negative value of a_1, that is to get the amplifying free oscillation of rotational frequency. By the substitution of (8.3) into (8.2), we get

$$R_a+(j\omega-a_1)L_a+\frac{1}{(j\omega-a_1)C}+\frac{M_s^2(j\omega-a_1)a_1}{R_u-a_1L_u}$$

$$+\left\{L_r(j\omega-a_1)-\frac{M_s^2(j\omega-a_1)a_1}{R_u-a_1L_u}\right\}\varepsilon^{j2\psi}=0 \qquad (8.4)$$

Putting the real and imaginary parts in (8.4) zero respectively, we have

$$R_a-a_1L_a-\frac{a_1}{(a_1^2+\omega^2)C}-\omega L_r\sin 2\psi-a_1L_r\cos 2\psi$$

$$-\frac{M_s^2a_1^2}{R_u+a_1L_u}-\frac{M_s^2a_1^2\cos 2\psi}{R_u+a_1L_u}-\frac{M_s^2\omega a_1\sin 2\psi}{R_u+a_1L_u}=0 \qquad (8.5)$$

$$\omega L_a+\omega L_r\cos 2\psi-\frac{\omega}{(a_1^2+\omega^2)C}-a_1L_r\sin 2\psi$$

$$+\frac{\omega a_1M_s^2}{R_u+a_1L_u}+\frac{M_s^2\omega a_1\cos 2\psi}{R_u+a_1L_u}-\frac{M_s^2a_1^2\sin 2\psi}{R_u+a_1L_u}=0 \qquad (8.6)$$

Eliminating $\sin 2\psi$ and $\cos 2\psi$ between (8.5) and (8.6), we get

$$\frac{2M_s^2}{R_u-a_1L_u}\left\{L_a+\frac{1}{(a_1^2+\omega^2)C}-L_r\right\}a_1^3$$

$$+\left[\left\{L_a+\frac{1}{(a_1^2+\omega^2)C}\right\}^2-L_r^2-\frac{2R_aM_s^2}{R_u-a_1L_u}\right]a_1^2$$

$$-2\left[R_a\left\{L_a+\frac{1}{(a_1^2+\omega^2)C}\right\}-\frac{\omega M_s^2}{R_u-a_1L_u}\left(\omega L_a-\omega L_r-\frac{\omega}{(a_1^2+\omega^2)C}\right)\right]a_1$$

$$+\left\{\omega L_a-\frac{\omega}{(a_1^2+\omega^2)C}\right\}^2-(\omega^2L_r^2-R_a^2)=0 \qquad (8.7)$$

Equation (8.7) is a polynomial algebraic equation of the 7th degree with respect to a_1. We will treat several special cases as given in the following.

(1) The case $a_1=0$, or the attenuation constant being zero.

Putting $a_1=0$ in (8.7), we get

$$\left.\begin{aligned}&\frac{1}{\omega C}=\omega L_a\pm\sqrt{\omega^2L_r^2-R_a^2}\\&R_a<\omega L_r\end{aligned}\right\}\qquad(8.8)$$

And we put the above relation as,

$$\left.\begin{aligned}&\frac{1}{\omega C_1}=\omega L_a+\sqrt{\omega^2L_r^2-R_a^2}\equiv x_d'\\&\frac{1}{\omega C_2}=\omega L_a-\sqrt{\omega^2L_r^2-R_a^2}\equiv x_q'\end{aligned}\right\}\qquad(8.9)$$

The attenuation constant becomes zero when the connected capacitance C satisfies the above relation, provided that $R_a<\omega L_r$. If R_a, the resistance of armature circuit, is greater than ωL_r or R_r the maximum negative resistance, we have no capacitance which satisfies the condition $a_1=0$. The capacitance C_1 in (8.9) may be taken as the lower limit of self-excitation. x_d' is a reactance very near the direct synchronous reactance in value; and x_q', the quadrature reactance. If $R_a=0$,

$$x_d'=x_d=\omega L_a+\omega L_r=\text{direct reactance, and}$$

$$x_q'=x_q=\omega L_a-\omega L_r=\text{quadrature reactance.}$$

The phase angle ψ for the condition of (8.9) is obtained from $\sin 2\psi=\frac{R_a}{\omega L_r}$ as,

$$\psi=\frac{1}{2}(-1)^n\sin^{-1}\frac{R_a}{\omega L_r}+\frac{n\pi}{2}$$

where n being any integer.

(2) The case $R_u=\infty$, or the field circuit being open.

We consider the case when the field circuit is open, and putting $R_u=\infty$ into (8.7), we get

$$\left[\left\{L_a+\frac{1}{(a_1^2+\omega^2)C}\right\}^2-L_r^2\right]a_1^2-2R_a\left\{L_a+\frac{1}{(a_1^2+\omega^2)C}\right\}$$

$$+\left\{\omega L_a-\frac{\omega}{(a_1^2+\omega^2)C}\right\}^2-(\omega^2L_r^2-R_a^2)=0\qquad(8.10)$$

Assuming a_1 is small as compared with ω, and neglecting a_1^2 for ω^2, we get

$$\left\{\left(L_a+\frac{1}{\omega^2C}\right)^2-L_r^2\right\}a_1^2-2R_a\left(L_a+\frac{1}{\omega^2C}\right)a_1$$

$$+\left(\omega L_a-\frac{1}{\omega C}\right)^2-(\omega^2L_r^2-R_a^2)=0\qquad(8.11)$$

The coefficient of a_1^2 is always positive and that of a_1 is negative, so that the negative value for a_1 can be obtained when C is chosen as,

$$\left(\omega L_a-\frac{1}{\omega C}\right)^2-(\omega^2L_r^2-R_a^2)<0$$

or

$$\left.\begin{aligned}&\omega L_a+\sqrt{\omega^2L_r^2-R_a^2}>\frac{1}{\omega C}>\omega L_a-\sqrt{\omega^2L_r^2-R_a^2}\\&R_a<\omega L_r\end{aligned}\right\}\qquad(8.12)$$

The relation (8.12) may be rewritten by using the notation in (8.9) as,

$$x_d'>\frac{1}{\omega C}>x_q'\quad\text{or}\quad C_1<C<C_2$$

The relation $R_a<\omega L_r$ shows the condition of maximum limit for R_a in obtaining the

the amplifying free oscillation of the rotational frequency and this condition coincides with the maximum allowable equivalent negative resistance ($R_r=\omega L_r$) due to reaction torque in steady state. The numerical calculation for a_1 with ordinary numerical values for L_a, L_r, and R_a in (8.11) shows that the value of a_1^2 is sufficiently small as compared with ω^2. Therefore the relation (8.11) may be said preferable. Thus we have the negative attenuation constant as,

$$a_1=\frac{R_a\left(L_a+\frac{1}{\omega^2 C}\right)-\sqrt{R_a^2\left(L_a+\frac{1}{\omega^2 C}\right)^2-\left\{\left(L_a+\frac{1}{\omega^2 C}\right)^2-L_r^2\right\}\left\{\left(\omega L_a-\frac{1}{\omega C}\right)^2-(\omega^2 L_r^2-R_a^2)\right\}}}{\left(L_a+\frac{1}{\omega^2 C}\right)^2-L_r^2} \tag{8.13}$$

When $R_a\to 0$, we have also a negative attenuation constant approximately as,

$$a_1=-\omega\sqrt{\frac{-\left(x_d-\frac{1}{\omega C}\right)\left(x_q-\frac{1}{\omega C}\right)}{\left(x_d+\frac{1}{\omega C}\right)\left(x_q+\frac{1}{\omega C}\right)}} \tag{8.14}$$

It is a theoretical explanation of an ordinary self-excitation for a salient-pole alternator in view of the circuit theory that the amplifying free oscillation of the rotational frequency is existing even when the field circuit is open.

(3) Approximate solution when the field circuit is closed.

Assuming $a_1^2+\omega^2 \doteqdot \omega^2$ in (8.7), we have

$$\left(L_a+\frac{1}{\omega^2 C}-L_r\right)\left\{2M_s^2-L_u\left(L_a+\frac{1}{\omega^2 C}+L_r\right)\right\}a_1^3$$

$$+\left[R_u\left\{\left(L_a+\frac{1}{\omega^2 C}\right)^2-L_r^2\right\}+2L_uR_a\left(L_a+\frac{1}{\omega^2 C}\right)-2R_aM_s^2\right]a_1^2$$

$$+\left[-R_aR_u\left(L_a+\frac{1}{\omega^2 C}\right)+2\omega M_s^2\left(\omega L_a-\omega L_r-\frac{1}{\omega C}\right)-L_u\left\{\left(\omega L_a-\frac{1}{\omega C}\right)^2\right.\right.$$

$$\left.\left.-\left(\omega^2 L_r^2-R_a^2\right)\right\}\right]a_1+R_u\left[\left(\omega L_a-\frac{1}{\omega C}\right)^2-\left(\omega^2 L_r^2-R_a^2\right)\right]=0 \tag{8.15}$$

(i) When $R_u\to 0$, we have $a_1=0$ from (7.15).

(ii) When $R_u\neq 0$, and $R_a\to 0$, we get from (8.15) neglecting the term a_1^3,

$$R_u\left\{\left(L_a+\frac{1}{\omega^2 C}\right)^2-L_r^2\right\}a_1^2+\left(\omega L_a-\omega L_r-\frac{1}{\omega C}\right)\left\{2\omega M_s^2-L_u\left(\omega L_a+\omega L_r-\frac{1}{\omega C}\right)\right\}a_1$$

$$+R_u\left(\omega L_a+\omega L_r-\frac{1}{\omega C}\right)\left(\omega L_a-\omega L_r-\frac{1}{\omega C}\right)=0 \tag{8.16}$$

or,

$$\frac{R_u}{\omega^2}\left(x_d+\frac{1}{\omega C}\right)\left(x_q+\frac{1}{\omega C}\right)a_1^2+\left(x_q-\frac{1}{\omega C}\right)\left\{2\omega M_s^2-L_u\left(x_d-\frac{1}{\omega C}\right)\right\}a_1$$

$$+R_u\left(x_d-\frac{1}{\omega C}\right)\left(x_q-\frac{1}{\omega C}\right)=0 \tag{8.17}$$

If we take the value of C within the range of

$$x_d>\frac{1}{\omega C}>x_q$$

the coefficient of a_1^2 is positive, that of a_1 is negative, and the constant term is negative; so that, one root of a_1 is negative. This value a_1 is also small, and is negligible for ω.

We have approximate value of a_1 in the neighbourhood of $\frac{1}{\omega C}\doteqdot x_d$ from (8.17) as,

$$a_1=-\frac{R_u\left(x_d-\frac{1}{\omega C}\right)}{2\omega M_s^2} \tag{8.18}$$

(iii) When $R_u\neq 0$, $R_a\neq 0$, and $R_a<\omega L_r$.

If C is chosen in the relation of $x_d > \frac{1}{\omega C} > x_q$, and also approximately assuming $L_d L_u \doteqdot 2M_s^2$ in (8.15), we see that the coefficient of a_1^3 is negative, that of a_1^2 is positive, that of a_1 is negative, and the constant term is negative when we check it with normal numerical values. And from these we may say that we can obtain one root of real and negative value for a_1 which is also negligibly small as compared with ω.

(4) Remarks on the salient-pole machine.

1. We have the amplifying free oscillation of rotational frequency for a salient-pole alternator with the single-phase field winding, when it is suddenly switched on to a capacitance between C_1 and C_2 as given in the relation of

$$\frac{1}{\omega C_1} = \omega L_a + \sqrt{\omega^2 L_r^2 - R_a^2} \doteqdot x_d = \text{direct reactance}$$

$$\frac{1}{\omega C_2} = \omega L_a - \sqrt{\omega^2 L_r^2 - R_a^2} \doteqdot x_q = \text{quadrature reactance}$$

provided that R_a is less than ωL_r.

2. C_1 may be taken as the lower limit of capacitance for self-excitation.

3. If we consider x_q as 55–65% of x_d, then $K = \frac{C_2}{C_1} = \frac{x_d}{x_q} = 1.5 \sim 1.8$. We can say that the phenomena of self-excitation may be considered in rotational frequency without slip for a salient-pole alternator when it is switched on until about 50—80% larger capacitance than the lower limiting value for self-excitation, provided that the resistance of armature circuit is negligible. This fact has also been ascertained by experiments on a salient pole alternator having no damper winding in its rotor.

4. The author has also obtained the amplifying free oscillation of rotational frequency from the theoretical attack on a salient-pole alternator whose field winding being considered as three-phase winding, though the limiting range for the capacitance is somewhat different from the foregoing result. The author has also carried out experiments on the salient-pole generator having in its rotor a damper winding, somewhat similar to these considerations.

Chapter IX. Experiments and Calculations for a Salient-Pole Alternator.

An alternator under experiments was 10 kVA, 100V, 57.5 A, 3ϕ, 1500 R.P.M., 50~, and had a salient-pole revolving field with damper winding.

The following calculations were made as an equivalent three-phase symmetrical alternator.

The no-load saturation curve and three phase short-circuit characteristic are as shown in Fig. 10.

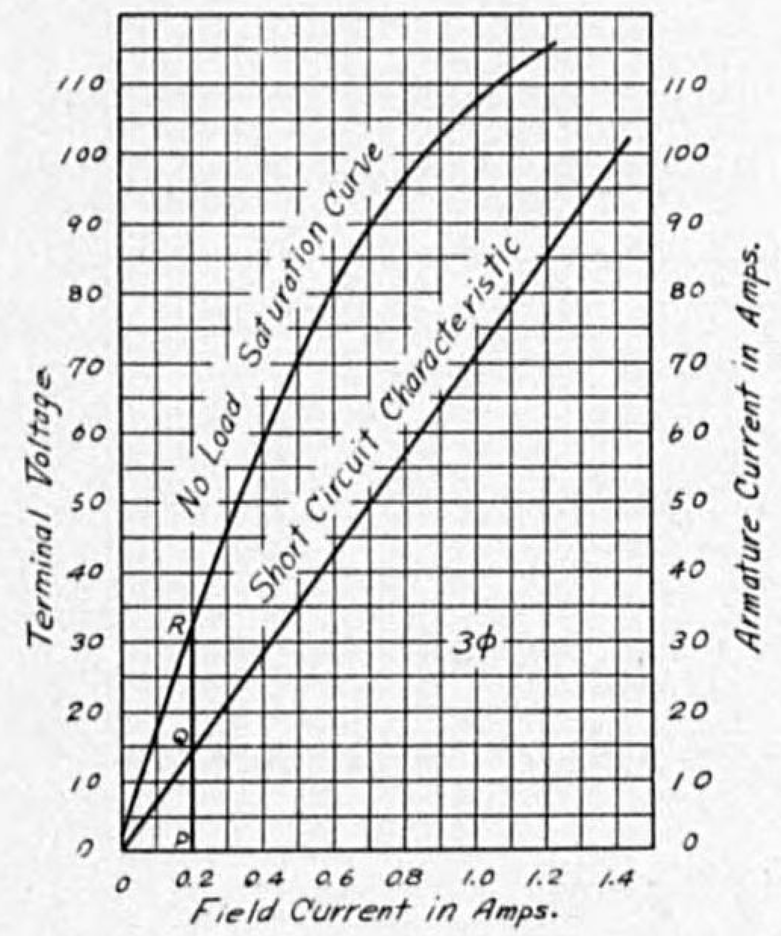

Fig. 10.
No-load saturation and short-circuit characteristics for 10-kVA alternator.

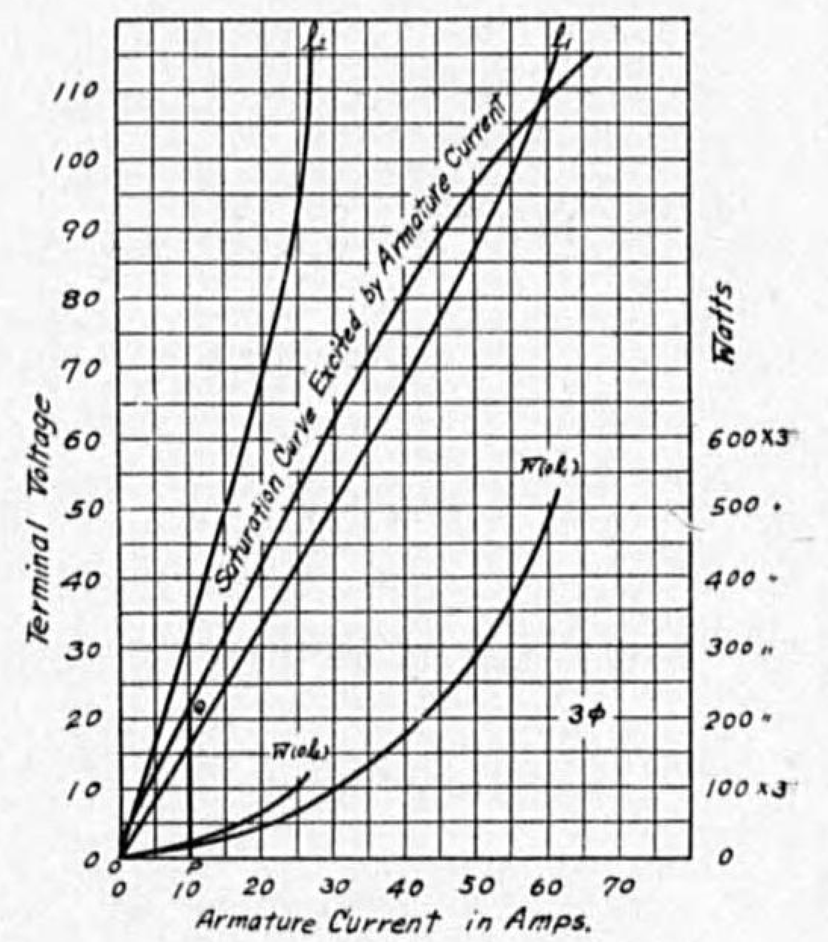

Fig. 11.
Line charging and saturation characteristics for 10- kVA alternator.

Measured armature resistance $R_a = 0.0372\ \Omega$ (per phase)

$$L_a = \frac{1}{2\pi f}\sqrt{\left(\frac{\overline{FR}}{\sqrt{3}\,\overline{FQ}}\right)^2 - R_a^2} = 0.00378\ H \quad \text{no-saturation)}$$

$$M = \frac{\sqrt{2}\,\overline{PR}}{2\pi f\sqrt{3}\,\overline{OP}} = 0.380\ H \quad \text{(no-saturation)}$$

Equivalent field-circuit constants were calculated by using negative phase sequence impedance Z_2. We have the relation

$$Z_2 = R_2 + j\omega L_2 = R_a + j\omega L_a + \frac{2\omega^2 M^2}{R_u + j2\omega L_u}$$

And from this, we have

$$R_u = \frac{2\omega^2 M^2 (R_2 - R_a)}{(R_2 - R_a)^2 + \omega^2 (L_2 - L_a)^2}$$

$$L_u = \frac{\omega^2 M^2 (L_a - L_2)}{(R_2 - R_a)^2 + \omega^2 (L_2 - L_a)^2}$$

We obtained from experiments,

$$Z_2 = 0.136\underline{/\ 57°39'} \quad \text{(field winding open)}$$

$$= 0.115\underline{/\ 61°27'} \quad \text{(field winding short)}$$

And we calculated as constants for unsaturated part,

$$R_u = 883, \quad L_u = 42.3 \quad \text{(field winding open)}$$

$$R_u = 485, \quad L_u = 43.1 \quad \text{(field winding short)}$$

The saturation curve excited by zero p.f. armature leading current was obtained by the method previously reported by the author(3), and it is shown in Fig. 11.

The alternator was connected to a condensive load through a step-up transformer bank. When the charging characteristic was like ol_1 in Fig. 11 ($k \fallingdotseq 1.2$), the voltage was built up without slip and operated stably at about 100V.* When the charging characteristic was like ol_2 ($k \fallingdotseq 0.6$), any appreciable voltage rise was not perceived. The calculations were made for these two cases. The equivalent capacitance C and equivalent series line resistance R_e were measured for the unsaturated part as,

(i) ol_1: $C = 0.003308, \quad k = 1.22, \quad R_e = 0.1067\ \Omega$

(ii) ol_2: $C = 0.001654, \quad k = 0.61, \quad R_e = 0.185\ \Omega$

And we calculated σ, ρ_a, ρ_u as shown in Table II. (R_a in the table includes R_e).

Table II.

C	0.003308		0.001654
	With field open	With field short-circuited	With field short-circuited
L_a	0.00378	0.00378	0.00378
L_u	42.3	43.1	43.1
M	0.38	0.38	0.38
R_a	0.144	0.144	0.222
R_u	883	458	458
σ	0.0969	0.1137	0.1137
ρ_a	293.14	335.1	516.5
ρ_u	215.43	93.46	93.46

With numerical values as obtained in the table, the equation (2.15) was solved and we got three roots of α as shown in Table III.

Table III.

	$C=0.003308,\ k=1.22$		$C=0.001654,\ k=0.61$
	With field open	With field short-circuited	With field short-circuited
α_1	$309.36-j3.774$ (49.24~& amplifies)	$311.74-j2.206$ (49.61~& amplifies)	$313.70+j4.449$ (49.93~& damps)
α_2	$865.59+j345.2$ (137.76~& damps)	$815.23+j233.5$ (129.75~& damps)	$1148.2+j314.9$ (182.75~& damps)
α_3	$-860.79+j267.1$ (137.80~& damps)	$-812.80+j197.2$ (129.36~& damps)	$-1147.8+j290.7$ (182.68~& damps)

* When the same experiment was carried out with a low-loss condenser, the voltage built up was 106 V; this value is in good coincidence with that obtained from the graphical solution.

It is clear from the table that the attenuation constant corresponding to the case of self-excitation becomes negative and shows the occurrence of the amplifying free oscillation. The frequency of this free oscillation is a little lower than 50 cycles, but by introducing the conventional idea for negative resistance as described in the previous chapter, this must be altered to 50 cycles and the actual experiment was, as above described, just in the rotational frequency, and gave rise no slip. This was ascertained by means of the neon stroboscopic method.

Next we calculated that constants corresponding to the voltage of about 110V, a little higher than that given by the point of intersection of ol_1 and the saturation characteristic in Fig. 11. The constants and the calculated α are shown in Tables IV and V.

Table IV.

	$C=0.003007$	
	With field open	With field short-circuited
L_a	0.003312	0.00331
L_u	25.63	26.26
M	0.275	0.275
R_a	0.183	0.183
R_u	619.5	324.7
σ	0.1091	0.1305
ρ_a	506.4	423.46
ρ_u	221.53	94.76

Table V.

	$C=0.003007$	
	With field open	With field short-circuited
α_1	$310.21+j1.849$ (49.37~& damps)	$313.51+j0.423$ (49.90~& damps)
α_2	$897.53+j400.8$ (142.85~& damps)	$839.63+j276.5$ (133.63~& damps)
α_3	$-893.58+j325.3$ (142.22~& damps)	$-838.99+j241.3$ (133.53~& damps)

It will be seen from the table that all the attenuation constants are of positive value, and the transient terms will damp with the lapse of time.

The above calculations are examples of the conventional treatment of a salient-pole alternator considered as an equivalent symmetrical machine and the phenomena of self-excitation may be acknowledged as an amplifying free oscillation as in the case of a symmetrical alternator.

Negative Resistance and Slip.

In order to measure the equivalent negative resistance R_r, a balanced three-phase e.m.f. was applied to the armature terminals and the speed of the prime mover was controlled just to slip a pole.

From the limiting value just to slip a pole, we got

$$R_r \doteqdot 0.24\ \Omega \qquad (100\text{—}110\ \text{V.})$$

The effect of field winding condition either closed or opened was not perceived in this measurement. Next we measured the quadrature reactance by the slip method and obtained $x_q = 0.583\ \Omega$ at approximately 10 V. By equation (7.8), we calculated R_r from x_d and x_q in accordance with terminal voltage, taking x_d from the saturation characteristic curve, and assuming x_q constant. The results are as shown in the following.

V	60	90	100	110	120
x_q	0.583	0.583	0.583	0.583	0.583
x_d	1.210	1.125	1.085	1.040	0.960
R_r	0.313	0.281 [7]	0.251	0.229	0.188
$K=\frac{x_d}{x_q}$	2.07	1.93	1.86	1.78	1.64

We have experienced the following cases with regard to the slip phenomena.

(i) No slip during and after the voltage built up.

One example was the case when the alternator was switched through a transformer bank (105/3000V, ΔY) on a condensive load connected in delta which consisted of $3\times40\times0.01\ \mu F$. The voltage was slowly built up without slip and stably operated at the condition: 100V, 58.5A, 400 watt losses per phase. The equivalent series line resistance is calculated as

$$R_e=\frac{400}{58.5^2}=0.117\ \Omega$$

Adding to this value an armature resistance of 0.037, we obtain 0.154 Ω, which is less than the maximum allowable negative resistance or $R_r \doteqdot 0.24$. The range for no-slip condition during build-up transient was experimentally obtained as $K \doteqdot 1.3$ when the field winding was short-circuited, and $K \doteqdot 2.1$ when the field winding was open. The difference will probably be caused by the effect of the damper winding.

When this alternator was experimented by employing a similar rotor without damper winding, the experimental range for no-slip condition for the connected capacitance during build-up transient was about $K \doteqdot 1.5$, independent of the field condition of opening or closing the exciting winding. The detailed description for a salient-pole machine when the machine is not treated as an equivalent alternator will be left for further consideration.

(ii) With a slip during transients and no slip after the build-up.

When the above alternator was switched on a condensive load of $3\times 55\times 0.01\mu F$ at the high-tension side under similar conditions as (i), with the additional resistance load connected in parallel, the machine slipped once during the build-up transient, and stably operated without slip after self-excitation. The condition of operation was 112.3 V, 89.5 A, 1530 watts per phase. The equivalent series line resistance was calculated, as

$$R_e=\frac{1530}{89.5^2}=0.191\ \Omega$$

Adding an armature resistance of 0.037 Ω, to 0.191 Ω, we obtain 0.228 Ω which is a value close to the maximum allowable negative resistance.

(iii) With a continuous slip after the self-excitation.

An additional resistance load was connected to the machine under the condition of (ii) after the build-up. We experienced the continuous slipping phenomena on the

self-excitation. The additional load was 112.3 V, 3.62 A, 235 watts (per phase). We calculated as an equivalent series resistance,

$$R_e=\frac{1530+235}{90.5^2}=0.216\ \Omega$$

Adding to this value the armature resistance of 0.037 Ω, we obtain 0.253 Ω, which is greater than the maximum allowable negative resistance of 0.24 Ω. And thus, by the slip of pole, the deficient energy is comprehended to be supplied to the armature circuit as in the case of an induction generator.

Chapter X. Summary.

The conclusions obtained in the present report on the transient phenomena of a symmetrical three-phase alternator which is suddenly switched upon a condensive load are briefly summarized as follows:

1. We have three kinds of free oscillation. The one has a frequency nearly equal to the rotational frequency, and the other two have the frequency nearly equal to that with which the total leakage inductance of the machine and the connected capacitance give rise to resonance.

2. One free oscillation has the nature of increasing amplitude within a certain range of values of connected capacitance. Its frequency is generally lower than the rotational frequency.

3. We can explain the phenomena of self-excitation by this amplifying free oscillation or by the existence of a transient current which has a negative attenuation constant.

4. Theoretical range of the amplifying free oscillation against the connected capacitance is limited within the values between k_1 and k_2. The values of k_1 and k_2 are obtained from the relation

$$k=\frac{1}{(1-s)(1-s-s\mu)}$$

by substituting the two roots s_1 and s_2 that are calculated from

$$s=\frac{1-\sigma}{2(1-\sigma+\mu)}\left\{1\pm\sqrt{1-\frac{4(1-\sigma+\mu)\sigma^2\chi}{(1-\sigma)^2}}\right\}$$

5. An approximate range of self-excitation is from

$$k_1=1 \text{ to } k_2=\frac{1}{\sigma}\times\frac{(1-\sigma+\mu)^2}{\mu^2}$$

6. The frequency of self-excitation is a little lower than the rotational frequency between $k\doteqdot 1$ and $k\doteqdot\frac{1}{\sigma}$, whereas it is much lower (for example one half) for a large value of k. It may conveniently be understood that a transient term the frequency of which is nearly rotational, amplifies with the lower value of k; and a transient term which has a resonant frequency between the total leakage inductance and capacitance, amplifies with the upper value of k.

7. When the suddenly connected capacitance is infinitely large, the frequencies of three transient terms are zero, very low frequency approximately zero, and that nearly equal to the rotational frequency.

8. Approximate magnitudes for three transient terms when k is not larger than $\frac{1}{\sigma}$ are,

$$\dot{I}_1=-\frac{\dot{E}_{a1}}{j\left(\omega L_a-\frac{1}{\omega C}\right)}+\frac{\dot{E}_{a1}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}$$

$$\dot{I}_2=-\frac{1}{2}\frac{\omega+\omega_n}{\omega}\cdot\frac{\dot{E}_{a1}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}$$

$$\dot{I}_3=-\frac{1}{2}\frac{\omega-\omega_n}{\omega}\cdot\frac{\dot{E}_{a1}}{j\left(\sigma\omega L_a-\frac{1}{\omega C}\right)}$$

9. We may take as a transient reactance for a.c. component (nearly equal to the rotational frequency) approximately,

$$\text{Transient reactance } =\sigma\omega L_a-\frac{1}{\omega C}=$$

(field leakage + armature leakage + condensive) reactance

10. The time required for the voltage to build-up under the condition of self-excitation is approximately proportional to $T_0\frac{1-\sigma k}{k-1}$.

The conclusions obtained in the present work on the transient phenomena of a salient-pole alternator upon condensive load are briefly given as follows:

1. The transient phenomena of a salient-pole machine may conveniently be dealt with as an equivalent symmetrical alternator. And the theory derived from the symmetrical alternator is applicable in its conventional form.

2. We have the amplifying free oscillation of rotational frequency against a certain range of the connected capacitance from the solution of an equivalent symmetrical alternator with fictitious negative resistance.

3. We have the amplifying free oscillation of rotational frequency from the study of a differential equation for a salient-pole alternator with a single-phase field winding, for the capacitance between C_1 and C_2 that are in the relation of

$$\frac{1}{\omega C_1}=\omega L_a+\sqrt{\omega^2L_r^2-R_a^2}\doteqdot \text{direct reactance } x_d$$

and

$$\frac{1}{\omega C_2}=\omega L_a-\sqrt{\omega^2L_r^2-R_a^2}\doteqdot \text{quadrature reactance } x_q$$

independent of the field-circuit condition, provided that R_a is less than ωL_r.

4. The negative attenuation constant when the field winding is open and the armature resistance is negligibly small, is approximately given by

$$a_1=-\omega\sqrt{\frac{-\left(x_d-\frac{1}{\omega C}\right)\left(x_q-\frac{1}{\omega C}\right)}{\left(x_d+\frac{1}{\omega C}\right)\left(x_q+\frac{1}{\omega C}\right)}}$$

for the free oscillation of the rotational frequency under the condition of self-excitation.

5. The negative attenuation constant when the field winding is closed and the armature resistance is small is approximately given by

$$a_1 = -\frac{R_u\left(x_a - \frac{1}{\omega C}\right)}{2\omega M_s^2}$$

for the free oscillation of rotational frequency under the condition of self-excitation when $\frac{1}{\omega C}$ does not deviate so much from x_a.

6. We have the cases of slipping phenomena during and after the building up of voltage under the condition of self-excitation in actual experiments, when the connected capacitance is comparatively large and is in a certain relation or in cases when the resistance of the armature circuit is large.

Acknowledgment—The author wishes to acknowledge his indebtedness to Messrs. S. Sakurai, T. Maejima, Y. Takeuchi, and late S. Toh for their kind assistance in carrying out the present work.

Appendix I.

Symmetrical Components for Instantaneous Values.

Let v_a, v_b, v_c be the instantaneous values of armature voltages, and i_a, i_b, i_c be those of armature currents. We put as follows:

$$\left.\begin{array}{ll} v_{a0} = \frac{1}{3}(v_a + v_b + v_c) & i_{a0} = \frac{1}{3}(i_a + i_b + i_c) \\ v_{a1} = \frac{1}{3}(v_a + av_b + a^2 v_c) & i_{a1} = \frac{1}{3}(i_a + ai_b + a^2 i_c) \\ v_{a2} = \frac{1}{3}(v_a + a^2 v_b + av_c) & i_{a2} = \frac{1}{3}(i_a + a^2 i_b + ai_c) \end{array}\right\} \quad (A.1)$$

We call these values as zero, positive and negative phase sequence components for instantaneous values.

And then the ordinary phase components are obtained as follows:

$$\left.\begin{array}{ll} v_a = v_{a0} + v_{a1} + v_{a2} & i_a = i_{a0} + i_{a1} + i_{a2} \\ v_b = v_{a0} + a^2 v_{a1} + av_{a2} & i_b = i_{a0} + a^2 i_{a1} + ai_{a2} \\ v_c = v_{a0} + av_{a1} + a^2 v_{a2} & i_c = i_{a0} + ai_{a1} + a^2 i_{a2} \end{array}\right\} \quad (A.2)$$

From the above, we have the following conjugate relations,

$$v_{a2} = \overline{v}_{a1}, \qquad i_{a2} = \overline{i}_{a1} \quad (-\text{means conjugate value}) \quad (A.3)$$

Instantaneous power will be represented as,

$$P = v_a i_a + v_b i_b + v_c i_c = 3(v_{a0} i_{a0} + v_{a1} i_{a2} + v_{a2} i_{a1})$$

$$= 3(v_{a0} i_{a0} + v_{a1} \overline{i}_{a1} + \overline{v}_{a1} i_{a1}) \quad (A.4)$$

We can write the similar relations as to the rotor circuit using subscripts u, v, w instead of a, b, c as follows:

$$\left.\begin{array}{ll} v_{u0} = \frac{1}{3}(v_u + v_v + v_w) & i_{u0} = \frac{1}{3}(i_u + i_v + i_w) \\ v_{u1} = \frac{1}{3}(v_u + av_v + a^2 v_w) & i_{u1} = \frac{1}{3}(i_u + ai_v + a^2 i_w) \\ v_{u2} = \frac{1}{3}(v_u + a^2 v_v + av_w) & i_{u2} = \frac{1}{3}(i_u + a^2 i_v + ai_w) \end{array}\right\} \quad (A.5)$$

And

$$\left.\begin{array}{ll} v_u = v_{u0} + v_{u1} + v_{u2} & i_u = i_{u0} + i_{u1} + i_{u2} \\ v_v = v_{u0} + a^2 v_{u1} + av_{u2} & i_v = i_{u0} + a^2 i_{u1} + ai_{u2} \\ v_w = v_{u0} + av_{u1} + a^2 v_{u2} & i_w = i_{u0} + ai_{u1} + a^2 i_{u2} \end{array}\right\} \quad (A.6)$$

Appendix II.

Differential Equations for the Symmetrical Three-Phase Machine with Uniform Air Gap.

Let us consider a symmetrical three-phase alternator with uniform air gap in which stator and rotor have star-connected windings (See Fig. A).

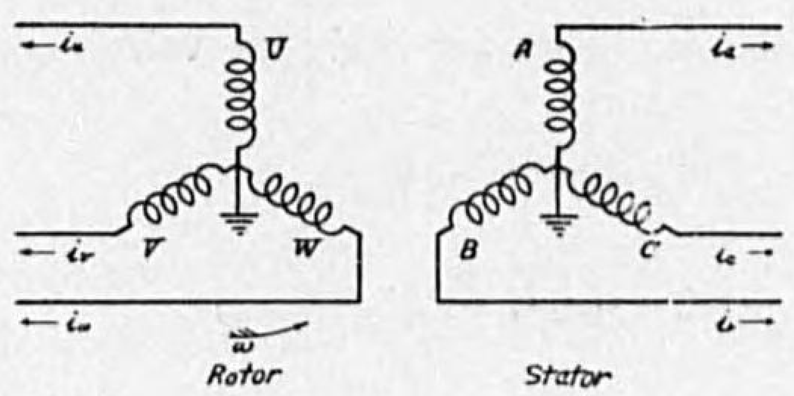

Fig. A.

The following assumptions are adopted:

1. The machine has symmetrical construction.
2. Effects of slot and saturation are neglected.
3. Mutual inductances vary with the cosine function of angle.

Subscripts used have the following meanings:

a ——armature circuit; u——field or rotor circuit.

a, b, c ——each phase of armature circuit.

u, v, w——each phase of field circuit.

0, 1, 2 ——zero, positive, and negative phase sequence.

Notations:

v = instantaneous value of voltage with proper subscripts.

i = instantaneous value of current with proper subscripts.

$p = \frac{d}{dt}$ = time differential operator.

ω = instantaneous value of angular velocity of rotor in the direction of ABC.

R_a = resistance of armature circuit (per phase).

L_1 = self-inductance of armature circuit (per phase).

$M_1 \cos \frac{2}{3}\pi = -\frac{1}{2}M_1$ = mutual inductance of armature circuit.

$$M_{ab} = M_{bc} = M_{ca} = M_{ba} \ldots\ldots = M_1 \cos \frac{2}{3}\pi = -\frac{1}{2}M_1$$

R_u = resistance of field circuit (per phase).

L_2 = self-inductance of field circuit (per phase).

$M_2 \cos \frac{2}{3}\pi = -\frac{1}{2}M_2$ = mutual inductance of field circuit (per phase).

$\theta = \omega t + \varphi$ = angle between stator and rotor.

M_{au} = mutual inductance between U and A.

$$M_{au} = M_{bv} = M_{cw} = M' \cos \theta$$

$$M_{av} = M_{bw} = M_{cu} = M' \cos\left(\theta + \frac{2}{3}\pi\right)$$

$$M_{aw} = M_{bu} = M_{cv} = M' \cos\left(\theta + \frac{4}{3}\pi\right)$$

M' = max. inductance between U and A.

We have differential equations for armature circuit as follows:

$$\left.\begin{aligned} v_a &= -(R_a + pL_1)i_a - p\left(-\frac{1}{2}M_1\right)(i_b + i_c) \\ &\quad - pM'\left\{\cos\theta\, i_u + \cos\left(\theta + \frac{2}{3}\pi\right)i_v + \cos\left(\theta + \frac{4}{3}\pi\right)i_w\right\} \\ v_b &= -(R_a + pL_1)i_b - p\left(-\frac{1}{2}M_1\right)(i_c + i_a) \\ &\quad - pM'\left\{\cos\theta\, i_v + \cos\left(\theta + \frac{2}{3}\pi\right)i_w + \cos\left(\theta + \frac{4}{3}\pi\right)i_u\right\} \end{aligned}\right\} \quad \text{(B.1)}$$

$$v_c=-(R_a+pL_1)i_c-p\left(-\frac{1}{2}M_1\right)(i_a+i_b)$$

$$-pM'\left\{\cos\theta\, i_w+\cos\left(\theta+\frac{2}{3}\pi\right)i_u+\cos\left(\theta+\frac{4}{3}\pi\right)i_v\right\}$$

Similarly we have for rotor circuit,

$$\left.\begin{aligned}
v_u&=-(R_u+pL_2)i_u-p\left(-\frac{1}{2}M_2\right)(i_v+i_w)\\
&\quad-pM'\left\{\cos\theta\, i_a+\cos\left(\theta+\frac{4}{3}\pi\right)i_b+\cos\left(\theta+\frac{2}{3}\pi\right)i_c\right\}\\
v_v&=-(R_u+pL_2)i_v-p\left(-\frac{1}{2}M_2\right)(i_w+i_u)\\
&\quad-pM'\left\{\cos\theta\, i_b+\cos\left(\theta+\frac{4}{3}\pi\right)i_c+\cos\left(\theta+\frac{2}{3}\pi\right)i_a\right\}\\
v_w&=-(R_u+pL_2)i_w-p\left(-\frac{1}{2}M_2\right)(i_u+i_v)\\
&\quad-pM'\left\{\cos\theta\, i_c+\cos\left(\theta+\frac{4}{3}\pi\right)i_a+\cos\left(\theta+\frac{2}{3}\pi\right)i_b\right\}
\end{aligned}\right\}\quad \text{(B.2)}$$

Now representing these values in symmetrical components by using the equations (A. 1) and (A. 5), we get

$$\left.\begin{aligned}
v_{a0}&=-\{R_a+p(L_1-M_1)\}i_{a0}\\
v_{a1}&=-\left\{R_a+p\left(L_1+\frac{1}{2}M_1\right)\right\}i_{a1}-\frac{3}{2}M'p\varepsilon^{j\theta}i_{u1}\\
v_{a2}&=\bar{v}_{a1}
\end{aligned}\right\}\quad \text{(B.3)}$$

$$\left.\begin{aligned}
v_{u0}&=-\{R_u+p(L_2-M_2)\}i_{u0}\\
v_{u1}&=-\left\{R_u+p\left(L_2+\frac{1}{2}M_2\right)\right\}i_{u1}-\frac{3}{2}M'p\varepsilon^{-j\theta}i_{a1}\\
v_{u2}&=\bar{v}_{u1}
\end{aligned}\right\}\quad \text{(B.4)}$$

Next we put:

$L_{a0}=L_1-M_1=$ zero phase sequence inductance of armature.

$L_a=L_1+\frac{1}{2}M_1=$ positive phase sequence inductance of armature.

$L_{u0}=L_2-M_2,\quad L_u=L_2+\frac{1}{2}M_2$

$M=\frac{3}{2}M'=$ mutual inductance between stator and rotor for symmetrical component.

Then (B.3) and (B.4) become

$$\left.\begin{aligned}
v_{a0}&=-(R_a+pL_{a0})i_{a0}\\
v_{a1}&=-(R_a+pL_a)i_{a1}-Mp\varepsilon^{j\theta}i_{u1}\\
v_{a2}&=\bar{v}_{a1}
\end{aligned}\right\}\quad \text{(B.5)}$$

$$\left.\begin{aligned}
v_{u0}&=-(R_u+pL_{u0})i_{u0}\\
v_{u1}&=-(R_u+pL_u)i_{u1}-Mp\varepsilon^{-j\theta}i_{a1}\\
v_{u2}&=\bar{v}_{u1}
\end{aligned}\right\}\quad \text{(B.6)}$$

Equations (B.5) and (B. 6) may be taken as fundamental differential equations for the three-phase symmetrical machine.*

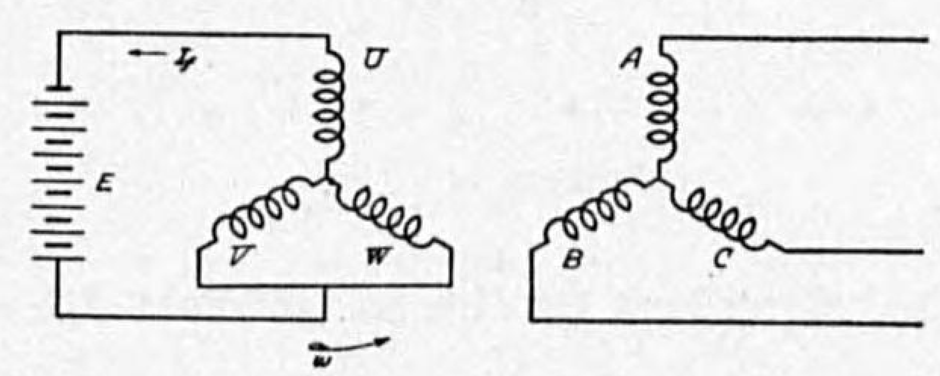

Fig. B.

* These equations have already been obtained by Dr. C. L. Fortescue in somewhat different form (T.A.I.E.E. 1918. p. 1065) and also by Dr. S. Bekku (Researches of the Electrotechical Laboratory, No. 203).

No-load induced voltage.

Let the rotor be excited as shown in Fig. B by *d.c.* voltage E and exciting current I_f at no-load. Then we have,

$$v_v - v_u = v_w - v_u = E;\quad i_a = i_b = i_c = 0 \tag{B.7}$$

From these relations we get by (A 6)

$$\left.\begin{aligned} & v_{u1} = v_{u2} = -\frac{E}{3} \qquad v_{u0} = 0 \\ & i_{a0} = i_{a1} = i_{a2} = 0 \end{aligned}\right\} \tag{B.8}$$

And as $i_{u1} = -v_{u1}/R_u$ for *d.c.* value from (B.6), we get

$$i_{u1} = i_{u2} = \frac{E}{3R_u} = \frac{I_f}{2} = I_s' \tag{B.9}$$

No-load induced voltage is obtained from (B. 5) in symmetrical components as,

$$\begin{aligned} v_{a1} &= -Mp\varepsilon^{j\theta} i_{u1} \\ &= -Mp\varepsilon^{j(\omega t+\varphi)}\frac{E}{3R_u} = -j\omega M\frac{E}{3R_u}\varepsilon^{j\varphi}\varepsilon^{j\omega t} \\ &= -j\omega M\frac{I_f}{2}\varepsilon^{j\varphi}\varepsilon^{j\omega t} \end{aligned} \tag{B.10}$$

Or $$v_{a1} = \dot{E}_{a1}\varepsilon^{j\omega t};\quad \dot{E}_{a1} = -j\omega M\frac{I_f}{2}\varepsilon^{j\varphi} \tag{B.11}$$

Appendix III.

Differential Equations for the Salient-Pole Alternator.

We use almost the same notations and assumptions as used in Appendix II.

Now the author make a further assumption in order to represent the salient nature of the machine as follows: "Self-inductances of armature circuit L_A, L_B, L_C, and

mutual inductances between two phases of armautre circuit M_{ab}, M_{bc}, M_{ca}, pulsate with double frequency of the rotor revolution."

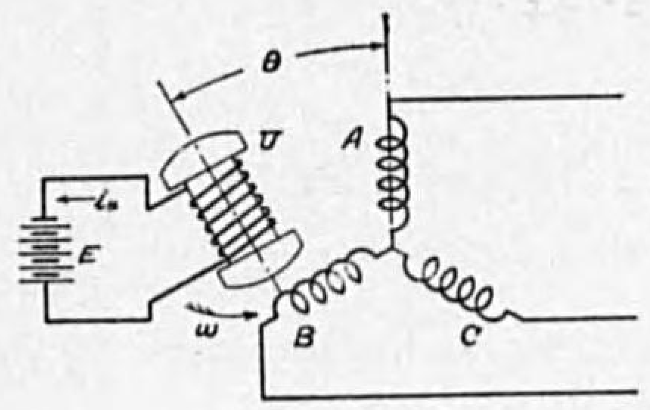

Fig. C.

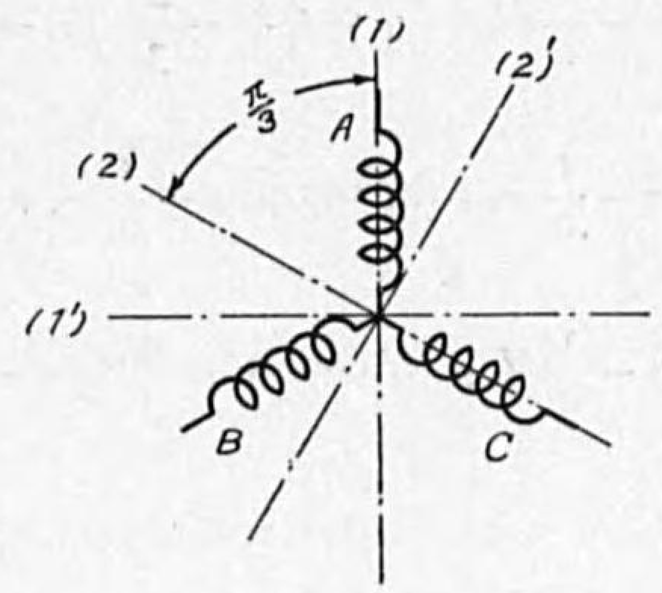

Fig. D.
Axes of symmetry in armature circuit.

We consider the circuit as shown in Fig. C; the armature ABC and the field U, which may be considered as one circuit U among U, V, W in Fig. B.

We may take an axis of symmetry as (1) or (1)′ in Fig. D, considering the pulsation of self-inductance L_A of A-phase winding against the rotor according to the above assumption. Considering axis (1), we put

$$L_A = L_1 + L_1' \cos 2\theta \tag{C.1}$$

where L_1 is the constant term, L_1' the pulsating term, and $\theta = \omega t + \varphi$.

Next we may take an axis of symmetry as (2) or (2)′ in Fig. D for M_{ab}, the mutual inductance between A and B phase windings.

If we choose axis (2), we may write,

$$M_{ab} = M_1 \cos\frac{2}{3}\pi + M_1' \cos 2\left(\theta - \frac{1}{3}\pi\right) \tag{C.2}$$

Thus we can write self-inductances as,

$$\left.\begin{aligned} L_A &= L_1 + L_1'\cos 2\theta = L_1 + \frac{1}{2}L_1'(\varepsilon^{j2\theta} + \varepsilon^{-j2\theta}) \\ L_B &= L_1 + L_1'\cos\left(2\theta - \frac{4}{3}\pi\right) = L_1 + \frac{1}{2}L_1'(a\varepsilon^{j2\theta} + a^2\varepsilon^{-j2\theta}) \\ L_C &= L_1 + L_1'\cos\left(2\theta - \frac{2}{3}\pi\right) = L_1 + \frac{1}{2}L_1'(a^2\varepsilon^{j2\theta} + a\varepsilon^{-j2\theta}) \end{aligned}\right\} \tag{C.3}$$

And mutual inductances as,

$$\left.\begin{aligned} M_{ab} &= -\frac{1}{2}M_1 + M_1' \cos\left(2\theta - \frac{2}{3}\pi\right) = -\frac{1}{2}M_1 + \frac{1}{2}M_1'(a^2\varepsilon^{j2\theta} + a\varepsilon^{-j2\theta}) \\ M_{bc} &= -\frac{1}{2}M_1 + M_1' \cos 2\theta = -\frac{1}{2}M_1 + \frac{1}{2}M_1'(\varepsilon^{j2\theta} + \varepsilon^{-j2\theta}) \\ M_{ca} &= -\frac{1}{2}M_1 + M_1' \cos\left(2\theta - \frac{4}{3}\pi\right) = -\frac{1}{2}M_1 + \frac{1}{2}M_1'(a\varepsilon^{j2\theta} + a^2\varepsilon^{-j2\theta}) \end{aligned}\right\} \quad \text{(C.4)}$$

We can write the differential equations regarding to the armature circuit as already described,

$$\left.\begin{aligned} v_a &= -(R_a + pL_A)i_a - p(M_{ab}i_b + M_{ac}i_c) \\ v_b &= -(R_a + pL_B)i_b - p(M_{bc}i_c + M_{ab}i_a) \\ v_c &= -(R_a + pL_C)i_c - p(M_{ac}i_a + M_{bc}i_b) \end{aligned}\right\} \quad \text{(C.5)}$$

By the substitution of (C. 3) and (C. 4) into (C. 5) and by the further linear transformation into symmetrical components, we get

$$\left.\begin{aligned} v_{a0} &= -\{R_a + p(L_1 - M_1)\}i_{a0} - \frac{1}{2}(L_1' - M_1')p\varepsilon^{j2\theta}i_{a1} \\ &\quad - \frac{1}{2}(L_1' - M_1')p\varepsilon^{-j2\theta}i_{a2} \\ v_{a1} &= -\left\{R_a + p\left(L_1 + \frac{M_1}{2}\right)\right\}i_{a1} - \left(\frac{1}{2}L_1' + M_1'\right)p\varepsilon^{j2\theta}i_{a2} \\ &\quad - \frac{1}{2}(L_1' - M_1')p\varepsilon^{-j2\theta}i_{a0} \\ v_{a2} &= \overline{v}_{a1} \end{aligned}\right\} \quad \text{(C.6)}$$

Here we put,

$$L_{a0} = L_1 - M_1, \quad L'_{a0} = \frac{1}{2}(L_1' - M_1')$$

$$L_a = L_1 + \frac{M_1}{2}, \quad L_r = \frac{1}{2}L_1' + M_1'$$

And (C. 6), the differential equations for armature are written,

$$\left.\begin{aligned} v_{a0} &= -(R_a + pL_{a0})i_{a0} - L'_{a0}(p\varepsilon^{j2\theta}i_{a1} + p\varepsilon^{-j2\theta}i_{a2}) \\ v_{a1} &= -(R_a + pL_a)i_{a1} - L_r p\varepsilon^{j2\theta}i_{a2} - L'_{a0}p\varepsilon^{-j2\theta}i_{a0} \\ v_{a2} &= \overline{v}_{a1} \end{aligned}\right\} \quad \text{(C.7)}$$

We can write fundamental differential equations of a salient-pole machine as (C. 7) plus the field circuit terms as follows:

$$\left.\begin{aligned} v_{a0} &= -(R_a + pL_{a0})i_{a0} - L'_{a0}(p\varepsilon^{j2\theta}i_{a1} + p\varepsilon^{-j2\theta}i_{a2}) \\ v_{a1} &= -(R_a + pL_a)i_{a1} - L_r p\varepsilon^{j2\theta}i_{a2} - L'_{a0}p\varepsilon^{-j2\theta}i_{a0} - \frac{M'}{2}p\varepsilon^{j\theta}i_u \\ v_{a2} &= \overline{v}_{a1} \\ v_u &= -(R_u + pL_u)i_u - \frac{3}{2}M'p(\varepsilon^{-j\theta}i_{a1} + \varepsilon^{j\theta}i_{a2}) \end{aligned}\right\} \quad \text{(C.8)}$$

When the neutral of armature circuit is isolated, or no neutral current flows, $i_a + i_b + i_c = 0$; and putting $i_{a0} = 0$ in (C. 8), we have

$$\left.\begin{aligned} v_{a0} &= -L'_{a0}(p\varepsilon^{j2\theta}i_{a1} + p\varepsilon^{-j2\theta}i_{a2}) \\ v_{a1} &= -(R_a + pL_a)i_{a1} - L_r p\varepsilon^{j2\theta}i_{a2} - \frac{M'}{2}p\varepsilon^{j\theta}i_u \\ v_{a2} &= \overline{v}_{a1} \\ v_u &= -(R_u + pL_u)i_u - \frac{3}{2}M'p(\varepsilon^{-j\theta}i_{a1} + \varepsilon^{j\theta}i_{a2}) \end{aligned}\right\} \quad \text{(C.9)}$$

The constant L_r and L_a may be determined from the relation

$$x_d = \omega L_a + \omega L_r = \text{direct synchronous reactance}$$

$$x_q = \omega L_a - \omega L_r = \text{quadrature synchronous reactance}$$

and

$$\left.\begin{aligned} L_a &= \frac{1}{2\omega}(x_d + x_q) \\ L_r &= \frac{1}{2\omega}(x_d - x_q) \end{aligned}\right\} \qquad \text{(C.10)}$$

The deduction is described in Chapter XIII.

Appendix IV.

Differential Equations and Characteristics for the N-phase Rotary Machine.

(1) E. m. f. and Current relation of n-phase circuit.

Let

n = number of circuits.

1, 2, 3, ... n = terminals.

$i_1, i_2, i_3, \ldots i_n$ = instantaneous value of current flowing out of the terminals 1, 2, 3, ... n.

$v_1, v_2, v_3, \ldots v_n$ = instantaneous value of e.m.f. due to the currents $i_1, i_2, \ldots i_n$ at the terminals 1, 2, 3,....n.

Z = generalized impedance operator with proper subscripts. For example, Z_{lk} means an operator to represent e.m.f. v_l at l terminal due to current i_k at k terminal.

Then we can write the general relation between e.m.fs. and currents of n-circuits as follows:

$$\left.\begin{aligned} v_1 &= z_{11}i_1 + z_{12}i_2 + z_{13}i_3 + \cdots\cdots + z_{1n}i_n \\ v_2 &= z_{21}i_1 + z_{22}i_2 + z_{23}i_3 + \cdots\cdots + z_{2n}i_n \\ &\vdots \\ v_r &= z_{r1}i_1 + z_{r2}i_2 + z_{r3}i_3 + \cdots\cdots + z_{rn}i_n \\ &\vdots \\ v_n &= z_{n1}i_1 + z_{n2}i_2 + z_{n3}i_3 + \cdots\cdots + z_{nn}i_n \end{aligned}\right\} \qquad \text{(D.1)}$$

If we assume the symmetrical construction of n-phase circuit or the symmetry of circuit constants, we can write the identical relation of n operators as follows:

$$\left.\begin{aligned} & z_{11} = z_{22} = z_{33} = \cdots\cdots = z_{nn} \\ & z_{12} = z_{23} = z_{34} = \cdots\cdots = z_{n1} \\ & z_{13} = z_{24} = z_{35} = \cdots\cdots = z_{n2} \\ & \vdots \\ & z_{1r} = z_{2(r+1)} = z_{3(r+2)} = \cdots\cdots = z_{n(r-1)} \\ & \vdots \\ & z_{1n} = z_{21} = z_{32} = \cdots\cdots = z_{n(n-1)} \end{aligned}\right\} \qquad \text{(D.2)}$$

By the substitution of (D. 2) into (D. 1), we get

$$\left.\begin{aligned} v_1 &= z_{11}i_1 + z_{12}i_2 + z_{13}i_3 + \cdots\cdots + z_{1n}i_n \\ v_2 &= z_{1n}i_1 + z_{11}i_2 + z_{12}i_3 + \cdots\cdots + z_{1(n-1)}i_n \\ & \cdots\cdots\cdots\cdots\cdots\cdots \\ v_n &= z_{12}i_1 + z_{13}i_2 + z_{14}i_3 + \cdots\cdots + z_{11}i_n \end{aligned}\right\} \qquad \text{(D.3)}$$

The relation (D. 3) is a general relation of e.m.fs. and currents of n-phase circuit. These relations are definitely determined if the n-impedance operators above described are known.

We carry out the linear transformation of instantaneous values of e.m.fs. and currents into symmetrical components as given in the following.

Symmetrical components for voltages:

$$
\left.\begin{aligned}
v_{a0}&=\frac{1}{n}(v_1+v_2+v_3+\ \cdots\cdots\cdots\cdots\ +v_n)\\
v_{a1}&=\frac{1}{n}(v_1+av_2+a^2v_3+\ \cdots\cdots\ +a^{n-1}v_n)\\
v_{a2}&=\frac{1}{n}(v_1+a^2v_2+a^4v_3+\ \cdots\cdots+a^{2(n-1)}v_n\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
v_{ar}&=\frac{1}{n}(v_1+a^rv_2+a^{2r}v_3+\ \cdots\cdots\ +a^{(n-1)r}v_n)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
v_{a(n-1)}&=\frac{1}{n}(v_1+a^{-1}v_2+a^{-2}v_3+\ \ldots\ a^{-(n-1)}v_n)
\end{aligned}\right\}\qquad \text{(D.4)}
$$

Symmetrical components for currents:

$$
\left.\begin{aligned}
i_{a0}&=\frac{1}{n}(i_1+i_2+i_3+\ \cdots\cdots\cdots\cdots\ +i_n)\\
i_{a1}&=\frac{1}{n}(i_1+ai_2+a^2i_3+\ \cdots\cdots\ +a^{n-1}i_n)\\
i_{a2}&=\frac{1}{n}(i_1+a^2i_2+a^4i_3+\ \cdots\cdots+\ a^{2(n-1)}i_n)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
i_{a(n-1)}&=\frac{1}{n}(i_1+a^{-1}i_2+a^{-2}i_3+\ \cdots\cdots a^{-(n-1)}i_n)
\end{aligned}\right\}\qquad \text{(D.5)}
$$

where,

$$a=\varepsilon^{j\frac{2\pi}{n}}=\cos\frac{2\pi}{n}+j\sin\frac{2\pi}{n}$$

We may call,

v_{a0}, i_{a0} = zero phase sequence component for instantaneous value.

v_{a1}, i_{a1} = positive phase sequence component for instantaneous value.

v_{a2}, i_{a2} = second phase sequence component for instantaneous value.

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$

$v_{a(n-1)}$, $i_{a(n-1)}$ = negative phase sequence component for instantaneous value.

By the substitution of (D. 3) into (D. 4), we get the general relation of e.m.f. and current for n-phase circuit represented by the instantaneous values of symmetrical components as follows:

$$
\left.\begin{aligned}
v_{a0}&=(z_{11}+z_{12}+z_{13}+\ \cdots\cdots\cdots\cdots\ +z_{1n})i_{a0}\\
v_{a1}&=(z_{11}+a^{n-1}z_{12}+a^{n-2}z_{13}+\ \cdots\cdots\ +az_{1n})i_{a1}\\
v_{a2}&=(z_{11}+a^{2(n-1)}z_{12}+a^{2(n-2)}z_{13}+\ \ldots+a^2z_{1n})i_{a2}\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
v_{ar}&=(z_{11}+a^{r(n-1)}z_{12}+a^{r(n-2)}z_{13}+\ldots+a^rz_{1n})i_{ar}\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
v_{a(n-1)}&=(z_{11}+az_{12}+a^2z_{13}+\ldots+a^{n-1}z_{1n})i_{a(n-1)}
\end{aligned}\right\}\qquad \text{(D.6)}
$$

The relation between ordinary phase values of e.m.f. and current, and symmetrical components are as follows:

$$
\left.\begin{aligned}
v_1&=v_{a0}+v_{a1}+v_{a2}+\cdots\cdots\cdots\cdots\cdots\cdots+v_{a(n-1)}\\
v_2&=v_{a0}+a^{-1}v_{a1}+a^{-2}v_{a2}+\cdots\cdots\cdots\cdots+a^{-(n-1)}v_{a(n-1)}\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
v_r&=v_{a0}+a^{-(r-1)}v_{a1}+a^{-2(r-1)}v_{a2}+\cdots\cdots\ a^{-(n-1)(r-1)}v_{a(n-1)}\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
v_n&=v_{a0}+a^{-(n-1)}v_{a1}+a^{-2(n-1)}v_{a2}+\cdots\cdots\cdots\cdots a^{-1}v_{a(n-1)}
\end{aligned}\right\}\qquad \text{(D.7)}
$$

And

$$\left.\begin{array}{l} i_1 = i_{a0} + i_{a1} + \cdots\cdots\cdots\cdots + i_{a(n-1)} \\ i_2 = i_{a0} + a^{-1} i_{a1} + a^{-2} i_{a2} + \cdots\cdots + a^{-(n-1)} i_{a(n-1)} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ i_n = i_{a0} + a^{-(n-1)} i_{a1} + a^{-2(n-1)} i_{a2} + \ldots + a^{-1} i_{a(n-1)} \end{array}\right\} \quad \text{(D.8)}$$

The relation (D. 6) is the general relation of e.m.fs. and currents for one group of n-phase circuits. In the similar manner, we can represent the general relation of e.m.fs. and currents for two groups of symmetrical n-phase circuits. Let one group of n-phase circuits be $A, B, C, \ldots D, E, F$ and the other of n-phase circuits be $U, V, W, \ldots X, Y, Z$. If Z with proper subscripts represents the generalized mutual impedance operator, we may write the general relations in the similar way as follows:

$$\left.\begin{array}{l} v_{a0} = (z_{11} + z_{12} + z_{13} + \cdots\cdots\cdots + z_{1n}) i_{u0} \\ v_{a1} = (z_{11} + a^{n-1} z_{12} + a^{n-2} z_{13} + \cdots\cdots + a z_{1n}) i_{u1} \\ v_{a2} = (z_{11} + a^{2(n-1)} z_{12} + a^{2(n-2)} z_{13} + \ldots + a^2 z_{1n}) i_{u2} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ v_{a(n-1)} = (z_{11} + a z_{12} + a^2 z_{13} + \ldots + a^{n-1} z_{1n}) i_{u(n-1)} \end{array}\right\} \quad \text{(D.9)}$$

The above relations for instantaneous values are also applicable to the vector relations of the general n-systems.

(2) Differential Equations for the Symmetrical N-phase Machine with Uniform Air Gap.

We consider a symmetrical n-phase machine with uniform air gap, in which the windings of both stator and rotor are star-connected.

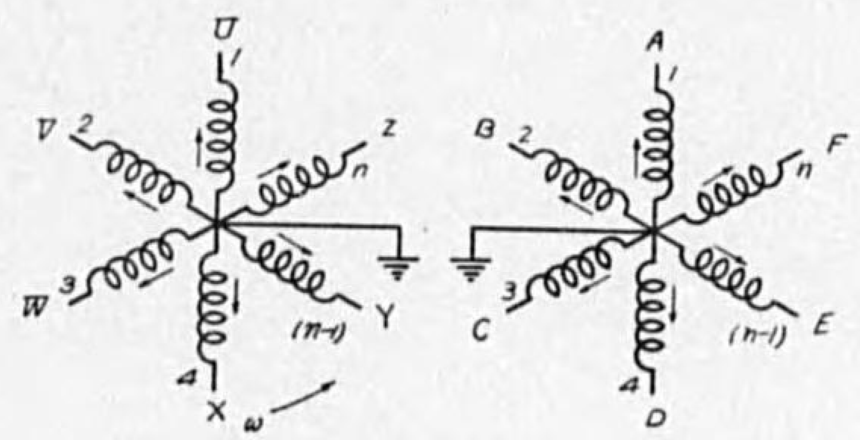

Fig. E.
Schematic diagram of symmetrical n-phase machine.

The effects of slots, iron loss, and magnetic saturation are neglected.

We assume that the mutual inductances vary with the cosine function of angle.

The positive direction of currents is taken as shown in Fig. E and we determine the following notations.

Stator:

R_a = resistance per Phase.

L_1 = self-inductance per Phase.

$$M_{12} = M_{ab}, \text{ etc.} = M_1 \cos \frac{2\pi}{n} = \frac{M_1}{2}(a + a^{-1})$$

= mutual inductance between adjacent windings.

$$M_{13} = M_{ac}, \text{ etc.} = M_1 \cos \frac{4\pi}{n} = \frac{M_1}{2}(a^2 + a^{-2})$$

= mutual inductance between A and C windings, or the first and the third windings.

. . . .

$$M_{1n} = M_{af}, \text{ etc.} = M_1 \cos \frac{2\pi}{n}(n-1) = \frac{M_1}{2}(a^{-1} + a)$$

= mutual inductance between A and F windings, or the first and (n − 1)th windings.

$L_{a0} = L_1 - M_1$ = zero phase sequence inductance.

$L_a = L_1 + \frac{n-2}{2} M_1$ = positive phase sequence or synchronous inductance.

Rotor:

R_u = resistance per phase.

L_2 = self-inductance per phase.

$$M_{12}=M_{uv}=M_2\cos\frac{2\pi}{n}=\frac{M_2}{2}(a+a^{-1})$$

= mutual inductance between adjacent windings.

$$M_{13}=M_{uw}=\frac{M_2}{2}(a^2+a^{-2})$$

= mutual inductance between the first and the third winding.

....

$$M_{1n}=M_{uz}=M_2\cos\frac{2\pi}{n}(n-1)=\frac{M_2}{2}(a^{-1}+a)$$

= mutual inductance between the first and $(n-1)$th windings;

$$L_{u0}=L_2-M_2;\quad L_u=L_2+\frac{n-2}{2}M_2.$$

Mutual inductances between stator and rotor:

Rotor → Stator

$$M_{11}=M_{au}=M'\cos\theta=\frac{M'}{2}(\varepsilon^{j\theta}+\varepsilon^{-j\theta})$$

$$M_{12}=M_{av}=M'\cos\left(\theta+\frac{2\pi}{n}\right)=\frac{M'}{2}(a\varepsilon^{j\theta}+a^{-1}\varepsilon^{-j\theta})$$

$$M_{13}=M_{aw}=M'\cos\left(\theta+\frac{2\pi}{n}\times 2\right)=\frac{M'}{2}(a^2\varepsilon^{j\theta}+a^{-2}\varepsilon^{-j\theta})$$

....

$$M_{1n}=M_{az}=M'\cos\left(\theta+\frac{2\pi}{n}(n-1)\right)=\frac{M'}{2}(a^{-1}\varepsilon^{j\theta}+a\varepsilon^{-j\theta})$$

Stator → Rotor

$$M_{11}=M_{ua}=M'\cos\theta=\frac{M'}{2}(\varepsilon^{j\theta}+\varepsilon^{-j\theta})$$

$$M_{12}=M_{ub}=M'\cos\left(\theta-\frac{2\pi}{n}\right)=\frac{M'}{2}(a^{-1}\varepsilon^{j\theta}+a\varepsilon^{-j\theta})$$

$$M_{13}=M_{uc}=M'\cos\left(\theta-\frac{2\pi}{n}\times 2\right)=\frac{M'}{2}(a^{-2}\varepsilon^{j\theta}+a^2\varepsilon^{-j\theta})$$

....

$$M_{1n}=M_{uf}=M'\cos\left(\theta-\frac{2\pi}{n}(n-1)\right)=\frac{M'}{2}(a\varepsilon^{j\theta}+a^{-1}\varepsilon^{-j\theta})$$

$\theta=\omega t+\varphi$

= Angle measured from A to U in the positive direction. The positive direction is the direction of 1, 2, 3 ... n or counter-clockwise in the figure.

ω = instantaneous angular velocity of rotor.

φ = constant angle.

$$p=\frac{d}{dt}$$

We proceed to obtain the relation of e.m.fs. and currents with the aid of generalized impedance operators as described in the following.

(i) E. m. fs. induced in Stator due to stator current.

We obtain the following impedance operators using the notations explained above.

$$\left.\begin{aligned}
Z_{11}&=-(R_a+pL_1)\\
Z_{12}&=-pM_{12}=pM_1\cos\frac{2\pi}{n}\\
Z_{13}&=-pM_{13}=-pM_1\cos\frac{2\pi}{n}\times 2\\
&\vdots\\
Z_{1n}&=-pM_{1n}=-pM_1\cos\frac{2\pi}{n}(n-1)
\end{aligned}\right\}\qquad \text{(D.10)}$$

By the substitution of (D. 10) into (D.6), we get

$$\left.\begin{aligned} v_{a0}&=-\{R_a+p(L_1-M_1)\}i_{a0}\\ v_{a1}&=-\{R_a+p(L_1+\alpha_1 M_1)\}i_{a1}\\ v_{a2}&=-\{R_a+p(L_1+\alpha_2 M_1)\}i_{a2}\\ &\cdots\cdots\cdots\cdots\cdots\cdots\\ v_{a(n-1)}&=-\{R_n+p(L_1+\alpha_{(n-1)}M_1)\}i_{a(n-1)} \end{aligned}\right\} \quad (D.11)$$

where,

$$\left.\begin{aligned} \alpha_1&=a^{n-1}\cos\frac{2\pi}{n}+a^{n-2}\cos\frac{2\pi}{n}\times 2+\cdots\cdots+a\cos\frac{2\pi(n-1)}{n}\\ \alpha_2&=a^{2(n-1)}\cos\frac{2\pi}{n}+a^{2(n-2)}\cos\frac{2\pi}{n}\times 2+\cdots\cdots+a^2\cos\frac{2\pi(n-1)}{n}\\ &\vdots\\ \alpha_{n-1}&=a\cos\frac{2\pi}{n}+a^2\cos\frac{2\pi}{n}\times 2+\cdots\cdots+a^{n-1}\cos\frac{2\pi(n-1)}{n} \end{aligned}\right\} \quad (D.12)$$

And we calculate, when $n \geqq 3$,

$$\alpha_1=\alpha_{n-1}=\frac{n-2}{2}\,;\quad \alpha_2=\alpha_3=\cdots\cdots=\alpha_{n-2}=1 \quad (D.13)$$

By the substitution of (D.13) and the notation

$$L_{a0}=L_1-M_1\,;\ L_a=L_1+\frac{n-2}{2}M_1$$

into (D.11), we obtain

$$\left.\begin{aligned} v_{a0}&=-\{R_a+pL_{a0}\}i_{a0}\\ v_{a1}&=-\{R_a+pL_a\}i_{a1}\\ v_{a2}&=-\{R_a+pL_{a0}\}i_{a2}\\ &\cdots\cdots\cdots\cdots\\ v_{a(n-1)}&=-\{R_a+pL_a\}i_{a(n-1)} \end{aligned}\right\} \quad (D.14)$$

(ii) E. m. fs. induced in rotor due to rotor current.

In the similar way as in (i), and writing R_u, L_2, M_2, L_{u0}, and L_u instead of R_a, L_1, M_1, L_{a0}, and L_a respectively, we get

$$\left.\begin{aligned} v_{u0}&=-\{R_u+pL_{u0}\}i_{u0}\\ v_{u1}&=-\{R_u+pL_u\}i_{u1}\\ v_{u2}&=-\{R_u+pL_{u0}\}i_{u2}\\ &\cdots\cdots\cdots\cdots\\ v_{u(n-1)}&=-\{R_u+pL_u\}i_{u(n-1)} \end{aligned}\right\} \quad (D.15)$$

(iii) E. m. fs. induced in stator due to rotor current.

$$\left.\begin{aligned} z_{11}&=-pM_{11}=-pM'\cos\theta=-\frac{pM'}{2}(\varepsilon^{j\theta}+\varepsilon^{-j\theta})\\ z_{12}&=-pM_{12}=-pM'\cos\left(\theta+\frac{2\pi}{n}\right)=-\frac{pM'}{2}(a\varepsilon^{j\theta}+a^{-1}\varepsilon^{-j\theta})\\ z_{13}&=-pM_{13}=-pM'\cos\left(\theta+\frac{4\pi}{n}\right)=-\frac{pM'}{2}(a^2\varepsilon^{j\theta}+a^{-2}\varepsilon^{-j\theta})\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ z_{1n}&=-pM_{1n}=-pM'\cos\left(\theta+\frac{2\pi}{n}(n-1)\right)=-\frac{pM'}{2}(a^{-1}\varepsilon^{j\theta}+a\varepsilon^{-j\theta}) \end{aligned}\right\} \quad (D.16)$$

By the substitution of (D.16) into (D.6), we can calculate as follows:

$$\left.\begin{aligned} v_{a0}&=0\\ v_{a1}&=-p\frac{nM'}{2}\varepsilon^{j\theta}i_{u1} \end{aligned}\right\} \quad (D.17)$$

$$\left.\begin{aligned} &v_{a2}=v_{a3}=\cdots\cdots=v_{a(n-2)}=0 \\ &v_{a(n-1)}=-p\frac{nM'}{2}\varepsilon^{-j\theta}i_{u(n-1)} \end{aligned}\right\}$$

(iv) E. m. fs. induced in rotor due to stator current.

By the interchange between a and u; between θ and $-\theta$ in (D.17), we obtain

$$\left.\begin{aligned} &v_{u0}=v_{u2}=v_{u3}=\cdots\cdots=v_{u(n-2)}=0 \\ &v_{u1}=-p\frac{nM'}{2}\varepsilon^{-j\theta}i_{a1} \\ &v_{u(n-1)}=-p\frac{nM'}{2}\varepsilon^{j\theta}i_{a(n-1)} \end{aligned}\right\} \quad \text{(D.18)}$$

(v) Fundamental differential equations of the symmetrical n-phase machine.

By the superposition of (D. 14), (D. 15), (D. 17), and (D. 18), we may write the differential equations for a symmetrical n-phase machine expressed in symmetrical co-ordinates for instantaneous values as follows:

$$\left.\begin{aligned} &v_{a0}=-\{R_a+pL_{a0}\}i_{a0} \\ &v_{a1}=-\{R_a+pL_a\}i_{a1}-p\frac{nM'}{2}\varepsilon^{j\theta}i_{u1} \\ &v_{a2}=-\{R_a+pL_{a0}\}i_{a2} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &v_{a(n-1)}=-\{R_a+pL_a\}i_{a(n-1)}-p\frac{nM'}{2}\varepsilon^{-j\theta}i_{u(n-1)} \end{aligned}\right\} \quad \text{(D.19)}$$

$$\left.\begin{aligned} &v_{u0}=-\{R_u+pL_{u0}\}i_{u0} \\ &v_{u1}=-\{R_u+pL_u\}i_{u1}-p\frac{nM'}{2}\varepsilon^{-j\theta}i_{a1} \\ &v_{u2}=-\{R_u+pL_{u0}\}i_{u2} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &v_{u(n-1)}=-\{R_u+pL_u\}i_{u(n-1)}-p\frac{nM'}{2}\varepsilon^{j\theta}i_{a(n-1)} \end{aligned}\right\} \quad \text{(D.20)}$$

In a special case, the differential equations for a symmetrical three-phase machine may be written by putting $n=3$, and $\frac{3}{2}M'=M$ in (D. 19) and (D. 20) as described in Chapter II and Appendix II as follows:

$$\left.\begin{aligned} &v_{a0}=-\{R_a+pL_{a0}\}i_{a0} \\ &v_{a1}=-\{R_a+pL_a\}i_{a1}-Mp\varepsilon^{j\theta}i_{u1} \\ &v_{a2}=-\{R_a+pL_a\}i_{a2}-Mp\varepsilon^{-j\theta}i_{u2} \end{aligned}\right\} \quad \text{(D.21)}$$

$$\left.\begin{aligned} &v_{u0}=-\{R_u+pL_{u0}\}i_{u0} \\ &v_{u1}=-\{R_u+pL_u\}i_{u1}-Mp\varepsilon^{-j\theta}i_{a1} \\ &v_{u2}=-\{R_u+pL_u\}i_{u2}-Mp\varepsilon^{j\theta}i_{a2} \end{aligned}\right\} \quad \text{(D.22)}$$

(vi) Differential Equations for a n-pease machine with single-phase rotor winding.

We can get the differential equations for a n-phase machine with symmetrical stator construction having a single-phase rotor winding by a slight modification in the equations previously obtained.

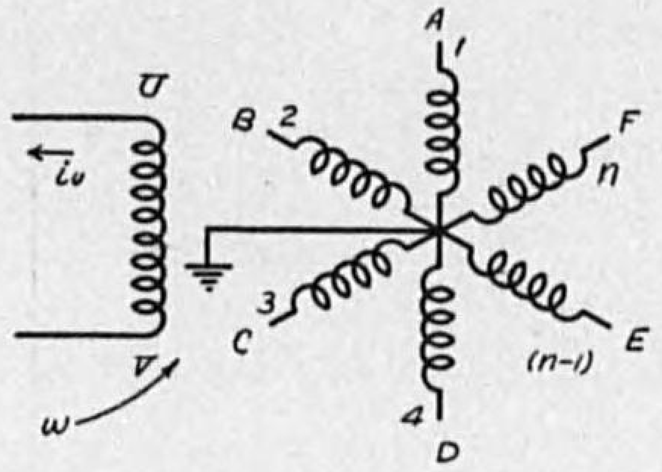

Fig. F. Schematic diagram of n-phase machine with single-phase rotor winding.

We consider to use only a single-winding U (see Fig. F) in the previous symmetrical n-phase machine, and the following equations may be obtained.

$$\left.\begin{aligned}
v_{a0}&=-\{R_a+pL_{a0}\}i_{a0}\\
v_{a1}&=-\{R_a+pL_a\}i_{a1}-p\frac{M'}{2}\varepsilon^{j\theta}i_u\\
v_{a2}&=-\{R_a+pL_a\}i_{a2}\\
&\cdots\cdots\cdots\cdots\\
v_{a(n-1)}&=-\{R_a+pL_a\}i_{a(n-1)}-p\frac{M'}{2}\varepsilon^{-j\theta}i_u
\end{aligned}\right\}\quad (D.23)$$

$$v_u=-\{R_u+pL_u\}i_u-p\frac{nM'}{2}\{\varepsilon^{-j\theta}i_{a1}+\varepsilon^{j\theta}i_{a(n-1)}\}\quad (D.24)$$

In a special case, the equations for a three-phase machine may be written by putting $n=3$ as follows:

$$\left.\begin{aligned}
v_{a0}&=-\{R_a+pL_{a0}\}i_{a0}\\
v_{a1}&=-\{R_a+pL_a\}i_{a1}-p\frac{M'}{2}\varepsilon^{j\theta}i_u\\
v_{a2}&=-\{R_a+pL_a\}i_{a2}-p\frac{M'}{2}\varepsilon^{-j\theta}i_u\\
v_u&=-\{R_u+pL_u\}i_u-p\frac{3}{2}M'\{\varepsilon^{-j\theta}i_{a1}+\varepsilon^{j\theta}i_{a2}\}
\end{aligned}\right\}\quad (D.25)$$

(3) Differential Equations for the n-phase rotary machine with m-phase rotor winding.

We consider a fictitious symmetrical $(m\times n)$ phase machine and use the symmetrical n-phase windings in the stator and the symmetrical m-phase windings in the rotor as shown in Fig. G.

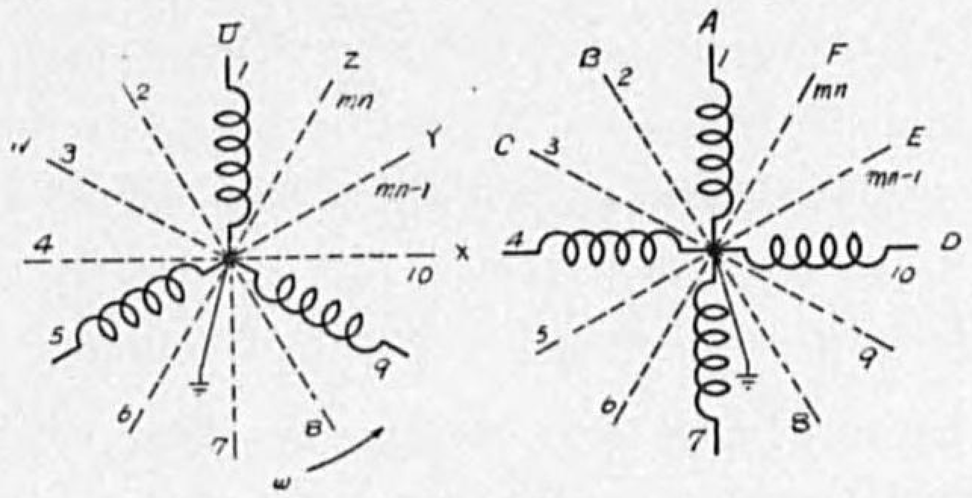

Fig. G.

In these consideration we may finally obtain the differential equations as follows:

$$\left.\begin{aligned}
v_{a0}&=-\{R_a+pL_{a0}\}i_{a0}\\
v_{a1}&=-\{R_a+pL_a\}i_{a1}-p\frac{m}{2}M'\varepsilon^{j\theta}i_{u1}\\
v_{a2}&=-\{R_a+pL_{a0}\}i_{a2}\\
&\cdots\cdots\cdots\cdots\\
v_{a(n-1)}&=-\{R_a+pL_a\}i_{a(n-1)}-p\frac{m}{2}M'\varepsilon^{-j\theta}i_{u(m-1)}
\end{aligned}\right\}\quad (D.26)$$

$$\left.\begin{aligned}
v_{u0}&=-\{R_u+pL_{u0}\}i_{u0}\\
v_{u1}&=-\{R_u+pL_u\}i_{u1}-p\frac{n}{2}M'\varepsilon^{-j\theta}i_{a1}\\
v_{u2}&=-\{R_u+pL_{u0}\}i_{u2}
\end{aligned}\right\}\quad (D.27)$$

....

$$v_{u(m-1)} = -\{R_u + pL_u\} i_{u(m-1)} - p\frac{n}{2} M' \varepsilon^{j\theta} i_{a(n-1)}$$

where,

$$L_{a0} = L_1 - M_1, \qquad L_a = L_1 + \frac{n-2}{2} M_1$$

$$L_{u0} = L_2 - M_2, \qquad L_u = L_2 + \frac{m-2}{2} M_2$$

The reductions are fully treated in the author's previous paper published in Japanese.(1)

(4) Some characteristics for a N-phase machine represented in symmetrical co-ordinates for instantaneous values.

We treat a n-phase machine with m-phase rotor winding.

(i) Conjugate relations.

We have the following conjugate relations as the general nature of symmetrical co-ordinates for instantaneous values as will be seen from (D. 4) and (D. 5):

$$\left.\begin{aligned} v_{a1} &= \bar{v}_{a(n-1)}; \quad i_{a1} = \bar{i}_{a(n-1)} \\ v_{u1} &= \bar{v}_{u(m-1)}; \quad i_{u1} = \bar{i}_{u(m-1)} \end{aligned}\right\} \quad \text{(D.28)}$$

(" – " denotes conjugate value)

And

$$v_{ar} = \bar{v}_{a(n-r)}; \quad i_{ar} = \bar{i}_{a(n-r)} \quad \text{(D.29)}$$

etc.

The positive phase sequence component for instantaneous values is conjugate to

the negative phase sequence component for instantaneous values and the r-th phase sequence component is conjugate to the (n–r)th phase sequence component.

(ii) Impedance drops.

The impedance operators ($R_a + pL_a$, etc.) for positive and negative phase sequence components are in the same form, and they correspond to the synchronous impedances under the steady state condition.

The impedance operators ($R_a + pL_{a0}$, etc.) for other phase sequence components are in the same form and equal to those for zero phase sequence components; they correspond to zero phase sequence impedances in steady state.

The positive and negative phase sequence components have the mutual action between stator and rotor, while other phase sequence components have none.

(iii) Power.

The instantaneous power output P_a from the stator is calculated in the following way.

$$P_a = \sum vi = v_1 i_1 + v_2 i_2 + \cdots\cdots + v_n i_n \quad \text{(D.30)}$$

By the substitution of (D. 7) and (D. 8) into (D. 30), we finally obtain

$$\begin{aligned} P_a &= n\{v_{a0} i_{a0} + v_{a1} i_{a(n-1)} + v_{a2} i_{a(n-2)} + \cdots\cdots + v_{a(n-1)} i_{a1}\} \\ &= n\{v_{a0} i_{a0} + v_{a1} \bar{i}_{a1} + v_{a2} \bar{i}_{a2} + \cdots\cdots + v_{a(n-1)} \bar{i}_{a(n-1)}\} \end{aligned} \quad \text{(D.31)}$$

The instantaneous power output P_u from the rotor is written in the similar way as above,

$$P_u = m\{v_{u0} i_{u0} + v_{u1} \bar{i}_{u1} + v_{u2} \bar{i}_{u2} + \cdots\cdots + v_{u(m-1)} \bar{i}_{u(n-1)}\} \quad \text{(D.32)}$$

The resistance loss $P_{r(a)}$ in the ~~rotor~~ stator is written,

$$P_{r(a)} = \sum_1^n v_n i_n = \sum_1^n i_n^2 R_a$$

$$=nR_a\{i_{a0}^2+i_{a1}\bar{i}_{a1}+i_{a2}\bar{i}_{a2}+\cdots\cdots\cdots+i_{a(n-1)}\bar{i}_{a(n-1)}\}$$

$$=nR_a\{i_{a0}^2+|i_{a1}|^2+|i_{a2}|^2+\cdots\cdots\cdots+|i_{a(n-1)}|^2\} \tag{D.33}$$

The resistance loss $P_{r(u)}$ in the rotor is written.

$$P_{r(u)}=mR_u\{i_{u0}^2+i_{u1}\bar{i}_{u1}+i_{u2}\bar{i}_{u2}+\cdots\cdots\cdots+i_{u(m-1)}\bar{i}_{u(m-1)}\} \tag{D.34}$$

And we have the relation

$$\sum_1^n i_n^2=n\{i_{a0}^2+i_{a1}\bar{i}_{a1}+i_{a2}\bar{i}_{a2}+\cdots\cdots\cdots+i_{a(n-1)}\bar{i}_{a(n-1)}\} \tag{D.35}$$

(iv) Energy.

a) Energy E_l due to the stator self-inductance L_1 is expressed by

$$E_l=\sum_1^n \frac{1}{2}L_1 i_n^2 \tag{D.36}$$

By the substitution of (D. 35) into (D. 36), we get

$$E_l=\frac{1}{2}nL_1\{i_{a0}^2+i_{a1}\bar{i}_{a1}+i_{a2}\bar{i}_{a2}+\cdots\cdots\cdots+i_{a(n-1)}\bar{i}_{a(n-1)}\} \tag{D.37}$$

b) Energy E_m due to the stator mutual-inductance M_1 is expressed by

$$E_m=\sum M_{12}i_1i_2 \tag{D.38}$$

By the substitution of the relations $M_{12}=\frac{M_1}{2}(a+a^{-1})$, etc. and also (D.5), (D.8), and (D. 36) into (D. 38), we finally obtain

$$E_m=\frac{1}{2}M_1n\{ni_{a1}\bar{i}_{a1}-(i_{a0}^2+i_{a1}\bar{i}_{a1}+i_{a2}\bar{i}_{a2}+\cdots\cdots$$

$$\cdots\cdots\cdots+i_{a(n-1)}\bar{i}_{a(n-1)})\} \tag{D.39}$$

c) Total energy E_A due to stator-inductances is expressed by

$$E_A=E_l+E_m \tag{D.40}$$

By the substitution of (D. 37), (D. 39), and also the relations

$$L_{a0}=L_1-M_1;\ L_a=L_1-M_1+\frac{n}{2}M_1,\ \text{into (D.40)},$$

we get

$$E_A=\frac{n}{2}\{i_{a0}^2L_{a0}+i_{a1}\bar{i}_{a1}L_a+i_{a2}\bar{i}_{a2}L_{a0}+\cdots\cdots\cdots+i_{a(n-1)}\bar{i}_{a(n-1)}L_a\} \tag{D.41}$$

d) Energy $E_{(A.U)}$ due to mutual inductances between stator and rotor is expressed by

$$E_{(A.U)}=\sum M_{au}i_ui_a \tag{D.42}$$

By the substitution of the relations $M_{au}=M_{12}=\frac{1}{2}M'(\varepsilon^{j\theta}+\varepsilon^{-j\theta})$, etc., and also (D. 5), (D. 8) into (D. 42), we finally obtain

$$E_{(A.U)}=\frac{1}{2}mnM'\{\bar{i}_{a1}i_{u1}\varepsilon^{j\theta}+i_{a1}\bar{i}_{u1}\varepsilon^{-j\theta}\} \tag{D.43}$$

(v) Torque.

Torque T due to the electro-magnetic energy between stator and rotor may be written as follows:

$$T=\frac{\partial E_{(A.U)}}{\partial\theta} \tag{D.44}$$

where $E_{(A.U)}$ is the total energy due to mutual inductances between stator and rotor, and $\frac{\partial}{\partial\theta}$ is operated with the assumption of constant currents. By the substitution of (D. 43) into (D. 44), we get

$$T=\frac{1}{2}mnM'\{j\varepsilon^{j\theta}i_{u1}\bar{i}_{a1}-j\varepsilon^{-j\theta}\bar{i}_{u1}i_{a1}\} \tag{D.45}$$

The instantaneous mechanical output P_τ at the rotor shaft due to T may be written as follows:

$$P_\tau = \omega T = \frac{1}{2}\omega m n M' \{ j\varepsilon^{j\theta} i_{u1}\bar{i}_{a1} - j\varepsilon^{-j\theta}\bar{i}_{u1} i_{a1} \} \quad \text{(D.46)}$$

As will be seen from (D. 45), the torque for a n-phase machine with m-phase rotor winding depends on the positive and negative phase sequence components, and is independent of other symmetrical components.

(vi) Power relations for a n-phase machine with m-phase rotor winding.

Let

$-P_u$ = input to the rotor circuit (m-phase).

$-P_\tau$ = mechanical input from shaft.

P_a = output from stator circuit (n-phase).

P_r = resistance losses in stator and rotor circuits.

P_e = rate of increase of total electro-magnetic stored energy.

Then we can write from (D. 31), (D. 32), (D. 33), (D. 34), (D. 41), (D. 43), and (D. 45) as follows:

$$-P_u = -m\{ v_{u0} i_{u0} + v_{u1}\bar{i}_{u1} + v_{u2}\bar{i}_{u2} + \cdots\cdots + v_{u(m-1)}\bar{i}_{u(m-1)} \}$$

$$-P_\tau = -\omega T = -\frac{1}{2}\omega m n M' \{ j\varepsilon^{j\theta} i_{u1}\bar{i}_{a1} - j\varepsilon^{-j\theta}\bar{i}_{u1} i_{a1} \}$$

$$P_a = n\{ v_{a0} i_{a0} + v_{a1}\bar{i}_{a1} + v_{a2}\bar{i}_{a2} + \cdots\cdots + v_{a(n-1)}\bar{i}_{a(n-1)} \}$$

$$P_r = nR_a\{ i_{a0}^2 + i_{a1}\bar{i}_{a1} + i_{a2}\bar{i}_{a2} + \cdots\cdots + i_{a(n-1)}\bar{i}_{a(n-1)} \}$$

$$+ mR_u\{ i_{u0}^2 + i_{u1}\bar{i}_{u1} + i_{u2}\bar{i}_{u2} + \cdots\cdots + i_{u(m-1)}\bar{i}_{u(n-1)} \}$$

$$P_e = p\frac{n}{2}\{ 2L_a i_{a1}\bar{i}_{a1} + L_{a0}(i_{u0}^2 + i_{a2}\bar{i}_{a2} + \cdots\cdots + i_{a(n-2)}\bar{i}_{a(n-2)}) \}$$

$$+ p\frac{m}{2}\{ 2L_u i_{u1}\bar{i}_{u1} + L_{u0}(i_{u0}^2 + i_{u2}\bar{i}_{u2} + \cdots\cdots + i_{u(m-2)}\bar{i}_{u(m-2)}) \}$$

$$+ p\frac{mn}{2} M' \{ \varepsilon^{j\theta}\bar{i}_{a1} i_{u1} + \varepsilon^{-j\theta}\bar{i}_{u1} i_{a1} \} \quad \text{(D.47)}$$

By the substitution of (D. 26), (D. 27), and also by the application of the shift principle, we get the following relations.

$$-P_u - P_\tau = P_a + P_r + P_e \quad \text{(D.48)}$$

Or

$$P_u + P_\tau + P_a + P_r + P_e = 0 \quad \text{(D.49)}$$

The relation (D. 49) represents the law of the conservation of energy. The relation (D. 49) is applicable in either case when the machine is considered as a generator or as a motor.

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