



# MFV approach to robust estimate of neutron lifetime

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**Abstract** Aiming at evaluating the lifetime of the neutron, we introduce a novel statistical method to analyse the updated compilation of precise measurements including the 2022 dataset of Particle Data Group (PDG). Based on the minimization for the information loss principle, unlike the median statistics method, we apply the most frequent value (MFV) procedure to estimate the neutron lifetime, irrespective of the Gaussian or non-Gaussian distributions. Providing a more robust way, the calculated result of the MFV is  $\tau_n = 881.16_{-2.35}^{+2.25}$  s with statistical bootstrap errors, while the result of median statistics is  $\tau_n = 881.5_{-3}^{+5.5}$  s according to the binomial distribution. Using the different central estimates, we also construct the error distributions of neutron lifetime measurements and find the non-Gaussianity, which is still meaningful.

## 1 Introduction

It is well-known that the decay of a free neutron related to the weak interaction is the most important  $\beta$ -decay process producing a proton, an electron, and an antineutrino with a lifetime of about 880 seconds. Furthermore, the neutron lifetime plays a fundamental role not only in particle physics but also astrophysics, such as solar physics and cosmology. Obviously, a more accurate value of neutron lifetime will improve our understanding of related fields [1, 2].

On account of the great significance of the  $\beta$ -decay of the free neutron, there are three different experimental methods to measure the free neutron lifetime over the last 60 years [1]. For the first method, the neutron lifetime in 'beam' experiments can be evaluated on the neutron decay rate from the number of decay particles [3]. For the second method,

the neutron lifetime in 'bottle' experiments can be determined by measuring the remaining number of ultra-cold neutrons stored in the container [4–6]. Because the magnetic trap technique can overcome the disadvantages of the traditional material wall, recently several studies have utilized a magneto-gravitational trap technique to determine a neutron lifetime with increasing precision [7, 8]. The third method applies the space-based technique to determine the neutron lifetime using the neutron spectrometer on NASA's lunar prospector mission [9]. It is noteworthy that over the past decades, many more precise measurements of neutron lifetime are obtained, which prompted the research on theoretical physics. However, there is still a significant discrepancy ( $\sim 4\sigma$  [8, 10]) among the above methods, which has long been a puzzling problem involving particle physics, nuclear physics, and astrophysics.

The conventional median technique is one of the most widely used statistical procedures to estimate the characteristics of different observational quantities in many statistical applications because it is not sensitive to outliers [11]. There have been a number of known examples of non-Gaussian data [12] to apply the median statistics, involving Hubble constant [13, 14],  ${}^7\text{Li}$  abundance [15, 16], LMC and SMC distance [17], deuterium abundance and spatial curvature constraints [18], the distance to the Galactic center [19], galactic rotational velocity [20, 21] and Newton's constants [22]. Recently, Rajan and Desai [23] investigated the measurements of neutron lifetime tabulated by Tanabashi et al. from Particle Data Group (PDG, hereafter, the 2019 edition) [24] based on a meta analysis. Furthermore, the results demonstrated that the error distribution of neutron lifetime measurements is not a Gaussian feature. Especially, the contributions of unidentified systematic effects and uncertainties are proposed to explain the anomalous discrepancy between different methods of neutron lifetime, which might imply the existence of new physics.

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Obviously, from the standpoint of robust statistics, it is very important to estimate the true value of neutron lifetime considering the holistic characteristic of all observations. Despite a great deal of research on data analysis and uncertainty in physics and technology over the past several decades, the current status of discrepancy among different measurements still needs a novel statistical method to enhance the robustness of the model, which has motivated us to apply the MFV procedure [25–28] to reanalyze this tension. Besides the conventional statistical algorithms, as a robust and resistant procedure, the MFV statistics have been used widely to seek the robust estimate in different natural science problems [29–33], such as the lithium abundances problem and Hubble constant tension [16,34].

In the next section we describe the latest compilation of neutron lifetime in PDG. Section 3 utilizes the MFV method to estimate the neutron lifetime value. To compare with past traditional results, we also calculate the confidence intervals and illustrate its advantage of MFV. In Sect. 4 we describe our analysis of error distributions around the different central estimates (the weighted mean, median, and MFV). Conclusions are given in Sect. 5.

## 2 Neutron lifetime data

Up to now, many measured values of neutron lifetime using different techniques have been published, e.g., Ezhov et al. [35] with  $\tau_n = 878.3 \pm 1.6_{stat} \pm 1.0_{sys}$  s, Serebrov et al. [6] with  $\tau_n = 881.5 \pm 0.7_{stat} \pm 0.6_{sys}$  s, Pattie et al. [7] with  $\tau_n = 877.7 \pm 0.7_{stat} \pm 0.4 / -0.2_{sys}$  s, Leung et al. [36] with  $\tau_n = 887 \pm 39$  s, Arzumanov et al. [37] with  $\tau_n = 880.2 \pm 1.2$  s, Yue et al. [3] with  $\tau_n = 887.7 \pm 1.2_{stat} \pm 1.9_{sys}$ , Steyerl et al. [38] with  $\tau_n = 882.5 \pm 2.1$  s, Pichlmaier et al. [39] with  $\tau_n = 880.7 \pm 1.3 \pm 1.2$  (also see Rajan and Desai [23] and Tanabashi et al. [24]). In our analysis, we use the updated compilation from Rajan and Desai [23] and the latest results of PDG, including Gonzalez et al. 2021 [8] and Wilson et al. 2021 [9] (see <https://pdglive.lbl.gov/DataBlock.action?node=S017T>, the 2022 edition of PDG [40]). Moreover, we adopt descriptive statistics methods to analyze the used measurements of neutron lifetime and plot the histogram of the number of all data, as shown in Fig. 1. Following graphical depictions of the observed data as a function of publication data [41,42], Fig. 2 illustrates a summary of neutron lifetime experimental results used for analysis since 1972, including measurements listed in/not\_in PDG and results unused in PDG following the taxonomic approach of Rajan and Desai [23]. Moreover, the marginal panels associated with the main panel show the distribution of neutron lifetime contributing to the published years. As summarized in Fig. 3, we give a graphical representation of neutron lifetime data based on beam, bottle, and space-based methods provided by different

authors. Then it becomes apparent that the highly frequent data focus on bottle experiments, and many more space-based measurements are required [9]. Besides, it is possible to constrain neutron lifetime from cosmological  $Y_p$  values and Big Bang nucleosynthesis [43], which is model-dependent. Following Rajan and Desai [23], we also do not consider cosmological parameter estimation in our calculation. Finally, we present all neutron lifetime measurements from PDG and Rajan and Desai in Table 1.

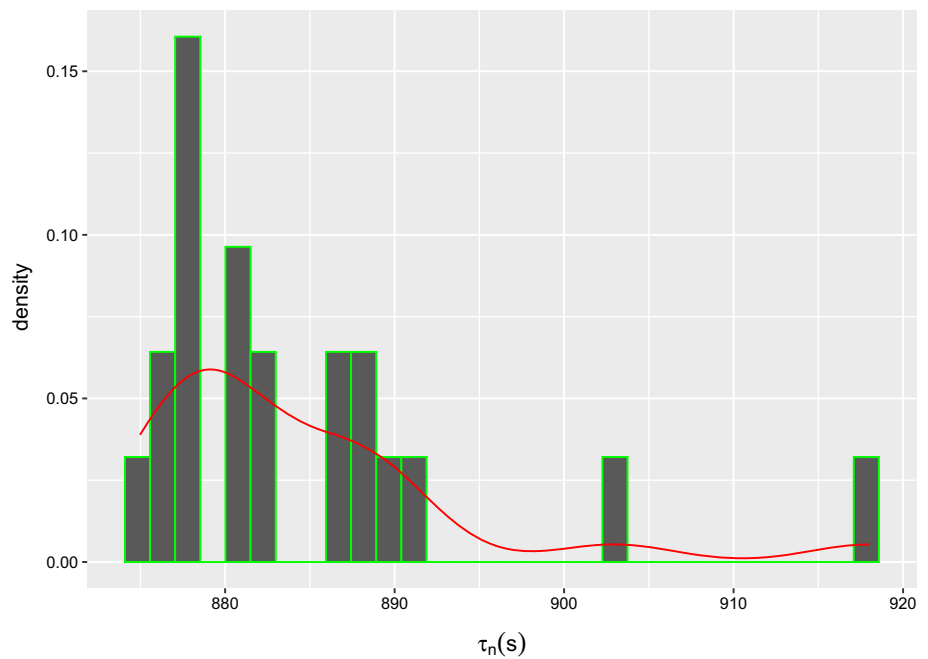
## 3 Analysis

In nuclear physics fields, the estimation of neutron lifetime from the observed measurements is one of the vital problems [23]. For decades, a considerable amount of statistical methods have been widely used, such as median statistics [11,14,55], maximum likelihood estimation, and Bayesian statistics [56]. Naturally, there is no motive for researchers to permanently suppose that the prior distribution of a physical quantity is always normal. In many cases, Gaussian distribution is not the only way even considering the larger sample. For example, it is the most common situation that if the dataset produces from Cauchy distribution, the sampling distribution is non-Gaussian, regardless of the size of the sample space.

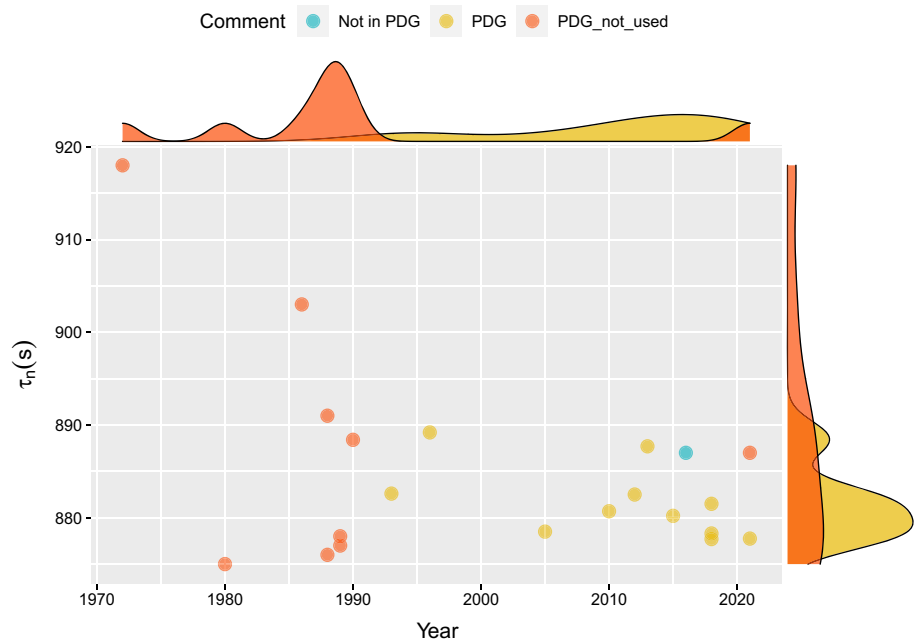
In general,  $\chi^2$  analyses, least square method (LSM) and median statistics are utilized to obtain the key information from the measurement data [11,23]. Moreover, provided that the feature of prior distributions is non-Gaussian, differences in the prior distributions need to be taken into account in the calculation of more precise results. On the other hand, as is known to all that George E. P. Box gave the famous quote “all models are wrong, but some are useful”. Every computational statistical model may have some additional potential hypotheses. Obviously, we should ruminate the normality of distribution, including the error distribution and prior distribution before statistical computation [16,27,34]. In addition, the heteroscedastic analysis is always a big problem for the conventional procedures. Thus, it is necessary to uncover the significant details about distributions of neutron lifetime measurements, shown in Figs. 4 and 5. These figures present the violin plot and kernel density plot for different methods, which is especially useful for analyzing summary and descriptive statistics. Furthermore, we can directly compare the median and MFV values, and intuitively see the distributions of different subgroups implying the non-Gaussian errors in neutron lifetime data of different approaches.

On the base of central limit theorem, the distribution of measurement data should be normal in most cases. Even so, the error distributions of measurement data are still probably non-Gaussian. For instance, it is possible that the measurement data may not originate from a random sample of

**Fig. 1** Histogram and probability density (red line) of neutron lifetime measurements



**Fig. 2** Published neutron lifetime as a function of publication date. The taxonomic approach of Not\_in\_PDG, PDG and PDG\_not\_used for analysis is similar to Rajan and Desai [23]



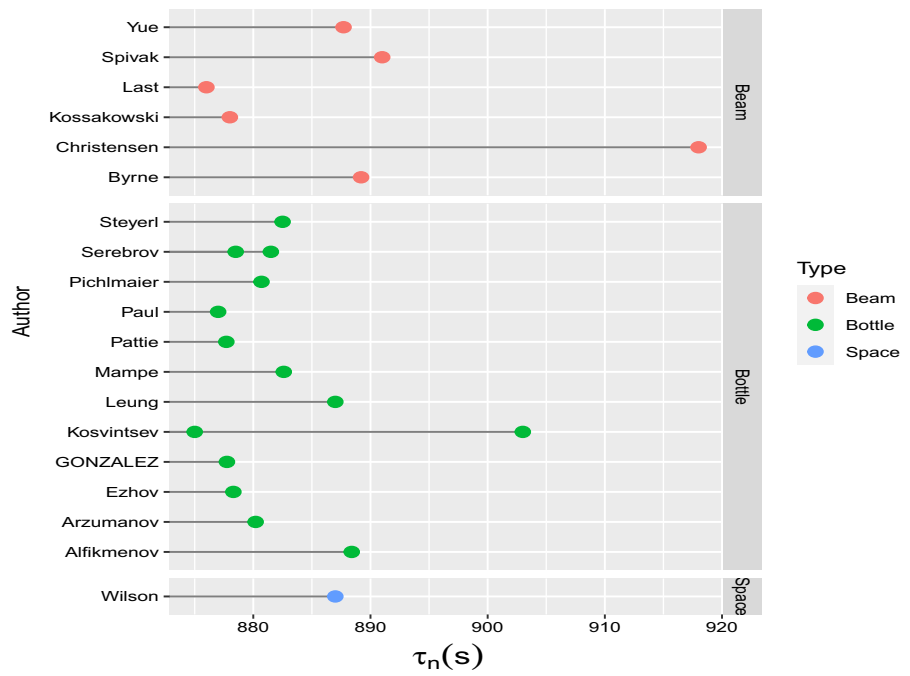
independent, identically distributed random variables, which should be proved. Another non-negligible point is the heavy-tailed problem of the observed distribution. Essentially, no one can be accurately aware whether or not the normality feature is intrinsic for the measurement data [12, 16, 55, 57, 58]. In short, these difficulties also motivate us to apply the novel robust MFV method to evaluate the true value of neutron lifetime.

Because of the ideal condition of pure mathematics, it is on the ground of expediency that some plausible hypotheses have to be accepted temporarily. According to the mea-

surement data of neutron lifetime, physicists expect to estimate accurately the real value from different prior distributions. For the sake of a better implementation of this purpose, Steiner [26, 27] proposed a more robust statistical algorithm — MFV method, based on the minimization for the information loss principle. Nearly regardless of considering the normality of the prior distribution, the MFV is not only highly robust efficiency but also dispose of the deficiency, for example, high sensitivity for outliers of some data [16, 34].

In order to elucidate the effect of prior distribution and error distributions, we utilize the MFV procedure to assess

**Fig. 3** A summary of the experimental results used for the analysis from different authors



**Table 1** 21 neutron lifetime measurements from PDG. The taxonomic approach of Not\_in\_PDG, PDG and PDG\_not\_used for analysis is similar to Rajan and Desai [23]

Neutron lifetime (s)	References	Type	Comment
$878.3 \pm 1.6 \pm 1.0$	Ezhov 2018 [35]	Bottle	PDG
$881.5 \pm 0.7 \pm 0.6$	Serebrov 2018 [6]	Bottle	PDG
$877.7 \pm 0.7 + 0.4/ - 0.2$	Pattie 2018 [7]	Bottle	PDG
$887 \pm 39$	Leung 2016 [36]	Bottle	Not in PDG
$880.2 \pm 1.2$	Arzumanov 2015 [37]	Bottle	PDG
$887.7 \pm 1.2 \pm 1.9$	Yue 2013 [3]	Beam	PDG
$882.5 \pm 1.4 \pm 1.5$	Steyerl 2012 [38]	Bottle	PDG
$880.7 \pm 1.3 \pm 1.2$	Pichlmaier 2010 [39]	Bottle	PDG
$878.5 \pm 0.7 \pm 0.3$	Serebrov 2005 [44]	Bottle	PDG
$889.2 \pm 3.0 \pm 3.8$	Byrne 1996 [45]	Beam	PDG
$882.6 \pm 2.7$	Mampe 1993 [46]	Bottle	PDG
$888.4 \pm 2.9$	Alfikmenov 1990 [47]	Bottle	PDG_not_used
$878 \pm 27 \pm 14$	Kossakowski 1989 [48]	Beam	PDG_not_used
$877 \pm 10$	Paul 1989 [49]	Bottle	PDG_not_used
$876 \pm 10 \pm 19$	Last 1988 [50]	Beam	PDG_not_used
$891 \pm 9$	Spivak 1988 [51]	Beam	PDG_not_used
$903 \pm 13$	Kosvintsev 1986 [52]	Bottle	PDG_not_used
$875 \pm 95$	Kosvintsev 1980 [53]	Bottle	PDG_not_used
$918 \pm 14$	Christensen 1972 [54]	Beam	PDG_not_used
$877.75 \pm 0.28 + 0.22/ - 0.16$	Gonzalez 2021 [8]	Bottle	PDG
$887 \pm 14 + 7/ - 3$	Wilson 2021 [9]	Space	PDG_not_used

the characteristics of datasets of neutron lifetime. Unlike the traditional methods such as Maximum likelihood estimation or LSM, Steiner put forward the maximum reciprocals principle,

$$\sum_i \frac{1}{X_i^2 + S^2} = \max, \tag{1}$$

where  $X_i$  is the residuals or deviations, i.e.,  $X_i = T_i^{measured} - T_i^{computed}$ , and  $S$  is the parameter of scale characterizing the measurement error, denoted by  $\varepsilon$  (called dihesion). According to the minimization of the information divergence (relative entropy) demonstrating the measure of information loss [26,27,59], Steiner suggested the MFV method and the scaling factor  $\varepsilon$ , i.e., dihesion, for the sake of evaluating the parameter of scale to some extent to reduce the information loss. Furthermore, using the iteratively re-weighted least squares procedure, we can calculate the MFV and the dihesion via iterations [27,28,30]. Especially, Steiner had proved the MFV procedure has the advantages of resistance and robustness [26,27]. After the  $(j+1)$ -th step of the MFV procedure, the relative equation of iterations for the most frequent value  $M$  is as follows:

$$M_{j+1} = \frac{\sum_{i=1}^n \frac{\varepsilon_j^2 x_i}{\varepsilon_j^2 + (x_i - M_j)^2}}{\sum_{i=1}^n \frac{\varepsilon_j^2}{\varepsilon_j^2 + (x_i - M_j)^2}}, \tag{2}$$

where  $x_i$  is a series of the measurements and the dihesion  $\varepsilon_j$  can be calculated by

$$\varepsilon_{k+1}^2 = \frac{3 \sum_{i=1}^n \frac{\varepsilon_k^4 (x_i - M_j)^2}{[\varepsilon_k^2 + (x_i - M_j)^2]^2}}{\sum_{i=1}^n \frac{\varepsilon_k^4}{[\varepsilon_k^2 + (x_i - M_j)^2]^2}}. \tag{3}$$

Where the iterative initial value  $M_0$  is chosen as the average value of the measurements, and the initial value of  $\varepsilon$  is obtained as

$$\varepsilon_0 = \frac{\sqrt{3}}{2} (x_{max} - x_{min}). \tag{4}$$

In addition, we take the fixed threshold criterion to restrain the precision in iterations. Through a series of iterations, the most frequent value  $M$  and dihesion  $\varepsilon$  can be obtained when the dihesion is smaller than threshold value (e.g.  $10^{-5}$ ). Apparently, the dihesion  $\varepsilon$  is not like the standard deviation in LSM, which is sensitive to the outliers. Using all data listed in column 1 of Table 1, we obtain the result of MFV calculations is  $\tau_n = 881.16$  s, which agrees with the recent experimental data. Similarly, the calculated MFV using beam and bottle-based measurements are 888.83 s and 879.85 s shown in Fig. 4, respectively.

According to the fact that the experimental data reflect the nature of physical quantities, some statistics can be used to characterize the data, such as the MFV of the sample. To

estimate the uncertainty of physical quantities, the bootstrap method is one of the most effective methods and is significant to assess the rationality of the calculated results [60]. The basic process of the bootstrap method to calculate the confidence interval is as follows. Suppose that the experimental data set of neutron lifetime is  $(t_1, \dots, t_i)$  chosen from the distribution of true values of neutron lifetime with the corresponding statistic  $\theta(t_1, \dots, t_i)$ , i.e. MFV. First, we produce a bootstrap sample  $(t_1^*, \dots, t_i^*)$  from the initial experimental data with replacement. Next, the important statistic, i.e. MFV, is used for the bootstrap sample. Finally, repeating this process  $B$  times (usually 1000-3000 times) generates the distribution of the MFV. Therefore, these distributions can be used to evaluate confidence intervals (usually 68.27% or 95.45%) of the MFV for different measuring methods of neutron lifetime. The 68.27% confidence interval for all measurements is [878.81, 883.41], involving statistical bootstrap errors, while the 95.45% confidence interval for all data is [877.72, 885.61].

The second technique to estimate confidence intervals is the median statistics [11, 19]. Based on the binomial test and estimation in non-parametric statistics [61], the probability of the median between values  $x^{(r)}$  and  $x^{(s)}$  is

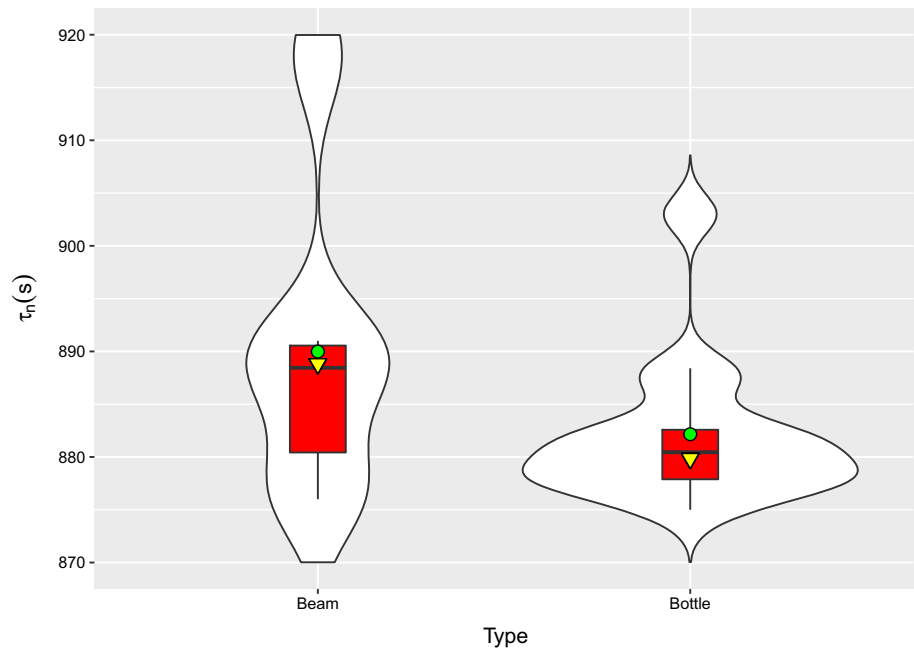
$$p(x^{(r)} \leq med \leq x^{(s)}) = p(med \leq x^{(s)}) - p(med < x^{(r)}) = \sum_{i=r}^s \binom{n}{i} / 2^n, \tag{5}$$

where  $x^{(i)}$  is the order statistic. By application of this formula, the median for all measurements is 881.5 and the calculated 68.27% confidence interval is [878.5, 887.0], while the 95.45% confidence interval for all data is [878.0 887.7].

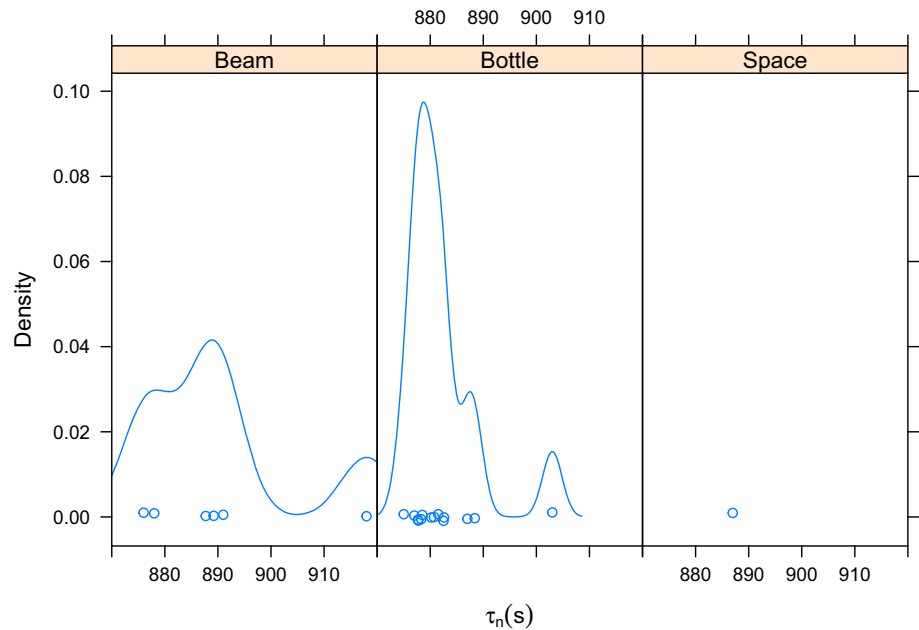
#### 4 Error distribution for PDG dataset

It is worthwhile to investigate the distributions of deviations of the neutron lifetime measurements mentioned in PDG 2022 from the central estimate. Among all the measured data, asymmetric systematic uncertainties should be treated in a clear and consistent manner. Especially, the asymmetric errors of  $877.7 \pm 0.7 + 0.4 / - 0.2$  [7], and  $877.75 \pm 0.28 + 0.22 / - 0.16$  [8] are widely used. Nevertheless, the effect of asymmetric uncertainties are difficult to assess exactly [62,63]. There are several ways to evaluate the asymmetric errors, including maximum absolute error, average error, piecewise linear error, quadratic error, and Fechner distribution methods [23,62–66]. Based on these methods, we have calculated the variance, and find a tiny dispersion using the central value to be in the middle between the upper and lower uncertainties. For example as proposed by Audi et al. [65, Appendix A], we apply the formula  $Var = (1 - 2/\pi)(a - b)^2 + a * b$  to obtain  $\sigma = 0.307$  for

**Fig. 4** Violin plot of neutron lifetime measurements. The MFV and average values from different methods are indicated by the inverted triangles and circles, respectively, while the horizontal solid lines in boxes illustrate the median



**Fig. 5** Density plot of neutron lifetime measurements from different methods. Circles represent the observed values



Pattie et al. [7], while the average value of asymmetric systematic errors is 0.3. Furthermore, for experimental neutron lifetime measurements, the symmetrization of asymmetric uncertainties lead to a tiny bias (generally very small in relative error) in the estimate of combined values, which can be neglected. Therefore, for simplicity, here we choose the measured midpoint and average value of asymmetric uncertainties motivated by Audi et al. [67] and Barlow [68].

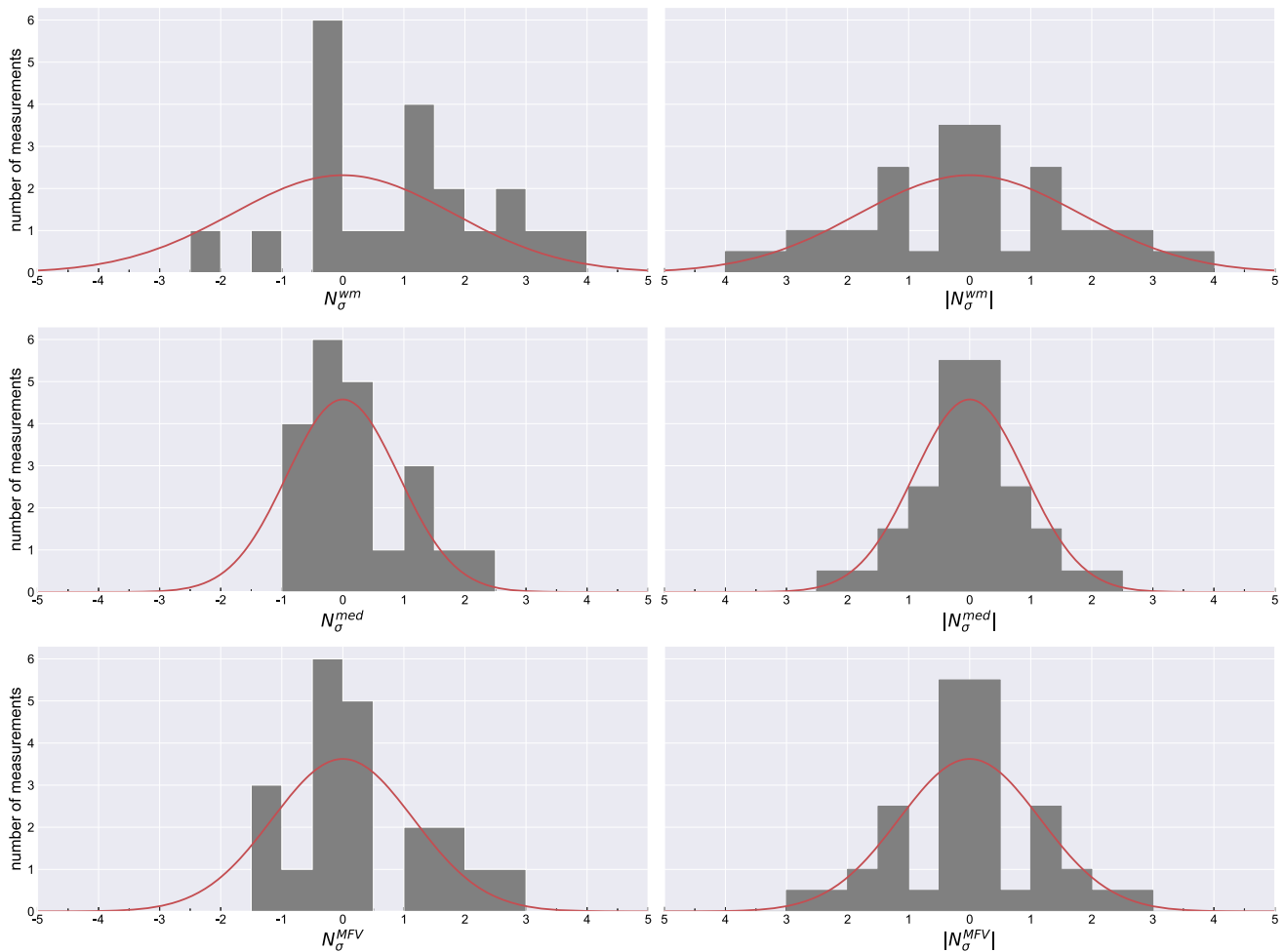
There are three statistical central estimates: weighted mean [69], median, and MFV. The weighted mean is given

by

$$T_{wm} = \frac{\sum_{i=1}^N T_i / \sigma_i^2}{\sum_{i=1}^N 1 / \sigma_i^2}, \tag{6}$$

where  $T_i$  is the measurement of neutron lifetime and  $\sigma_i$  is the one standard deviation error, i.e. the quadrature sum of the statistical and systematic uncertainties. The weighted mean standard deviation is

$$\sigma_{wm} = \frac{1}{\sqrt{\sum_{i=1}^N 1 / \sigma_i^2}}. \tag{7}$$



**Fig. 6** Histograms of the number of standard deviations in half bins away from the weighted mean, median, and MFV listed in the top, middle, and bottom rows. The left (right) column illustrates the signed

(absolute) deviation, where the smooth curves in panels represent the best-fit Gaussian. The  $N_\sigma$  of positive and negative cases indicate greater and less than the weighted mean, median, and MFV

The goodness of fit  $\chi^2$  is

$$\chi^2 = \frac{1}{N-1} \sum_{i=1}^N (T_i - T_{wm})^2 / \sigma_i^2. \tag{8}$$

The number of standard deviations that  $\chi$  deviates from unity [17, 70, 71] is described by

$$N_\sigma = |\chi - 1| \sqrt{2(N-1)}. \tag{9}$$

We can also use the median and MFV statistics approaches to construct the error distributions. Similar to the median statistics to hypothesize statistical independence of all measurements, the MFV statistics does not utilize the individual measurement uncertainties leading to a wider interval of errors on the central value in comparison to the weighted mean technique. Under the conditions of the specified central estimate of all measurements, the error distribution associ-

ated with standard deviations [17, 19] is defined as

$$N_{\sigma_i} = \frac{T_i - T_{CE}}{\sqrt{\sigma_i^2 + \sigma_{CE}^2}}, \tag{10}$$

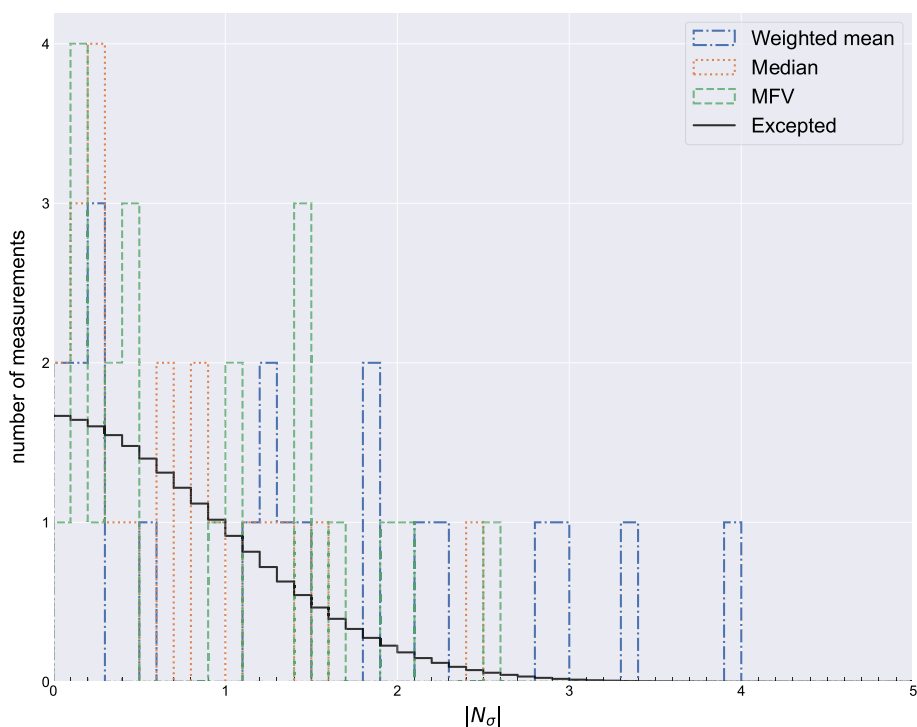
where  $T_{CE}$  is the central estimate of neutron lifetime measurements, either the median  $T_{med}$  or MFV  $T_{MFV}$ , and  $\sigma_{CE}$  is the uncertainty of  $T_{CE}$ , either  $\sigma_{med}$  or  $\sigma_{MFV}$ . These different combinations of central estimates and uncertainties are given by

$$N_{\sigma_i}^{wm} = \frac{T_i - T_{wm}}{\sqrt{\sigma_i^2 + \sigma_{wm}^2}}, \tag{11}$$

$$N_{\sigma_i}^{med} = \frac{T_i - T_{med}}{\sqrt{\sigma_i^2 + \sigma_{med}^2}}, \tag{12}$$

$$N_{\sigma_i}^{MFV} = \frac{T_i - T_{MFV}}{\sqrt{\sigma_i^2 + \sigma_{MFV}^2}}. \tag{13}$$

**Fig. 7** Histogram of the error distribution in  $|N_\sigma| = 0.1$ . The solid black line indicates the expected Gaussian probabilities for all data and the dash-dotted, dash, and dot lines denote the numbers of  $|N_\sigma|$  values for the weighted mean, MFV, and median, respectively



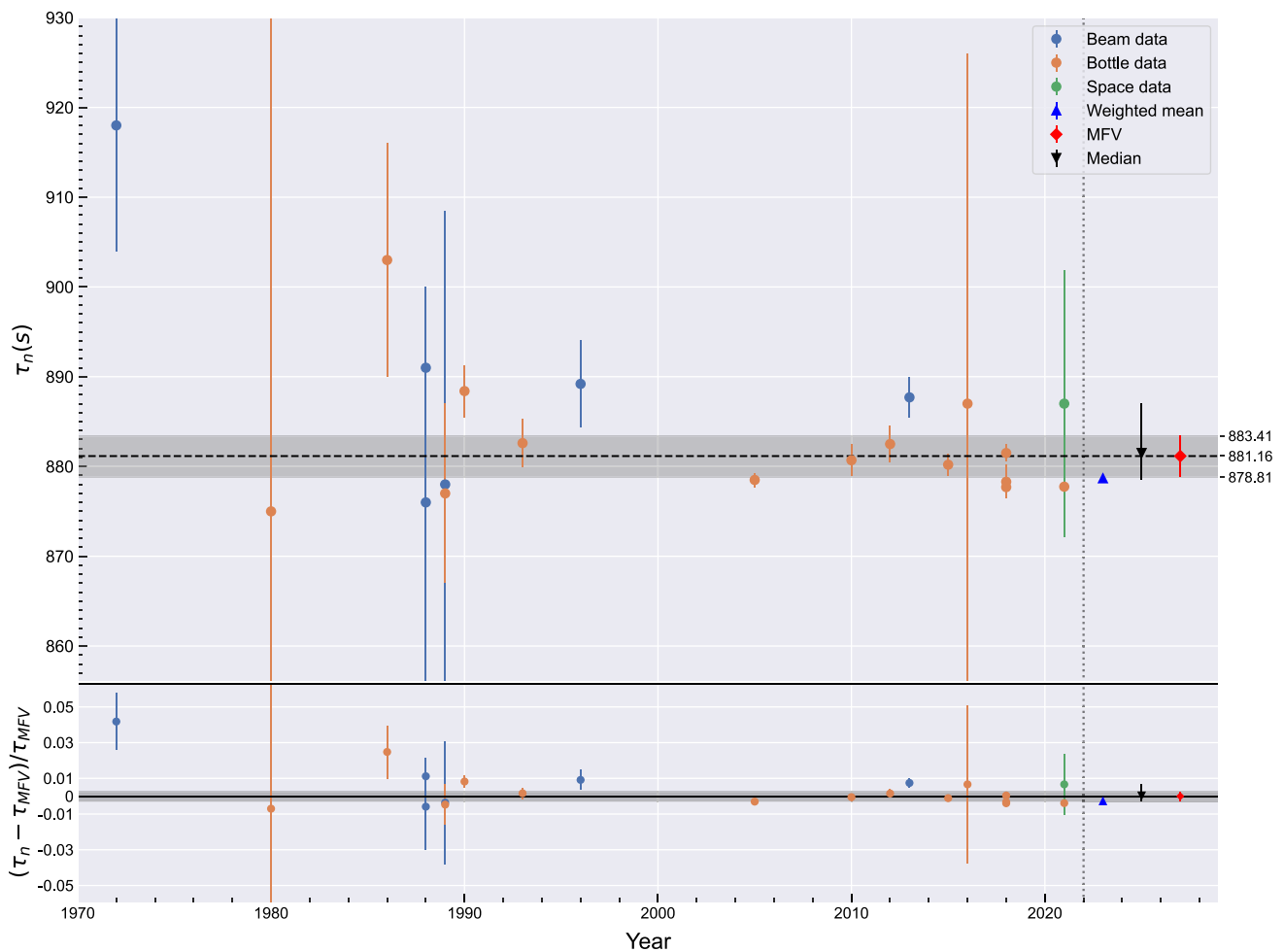
Applying weighted mean statistics, the neutron lifetime measurements produce a central estimate of  $\tau_n = 878.69 \pm 0.25$  s, while the PDG average is  $\tau_n = 878.4 \pm 0.5$  s including scale factor of 1.8 [40]. We also obtain  $\chi^2 = 3.64$  and the number of standard deviations is  $N = 5.75$ . Based on the median statistics, we have obtained a central estimate of  $\tau_n = 881.5^{+5.5}_{-3}$  s with a  $1\sigma$  range of [878.5, 887.0]. The MFV estimate is given by  $\tau_n = 881.16^{+2.25}_{-2.35}$  s with uncertainty corresponding to 68.27% confidence intervals.

We now apply these statistical methods to plot the error distributions of neutron lifetime measurements illustrated as a function of  $N_\sigma$  [17], Eqs. (11)–(13), in Fig. 6, which demonstrates the  $N_\sigma$  and symmetrical  $|N_\sigma|$  histograms using the weighted mean, median, and MFV. The histogram of error distributions of the measurements is shown in Fig. 7 with  $|N_\sigma| = 0.1$  bin size for a more specific viewpoint. As can be seen from these figures, the weighted mean case is wider than the expected Gaussian, which should generate a single value with  $|N_\sigma| \geq 2$  and none value with  $|N_\sigma| \geq 3$ . But there are 6 values with  $|N_\sigma| \geq 2$ , 2 with  $|N_\sigma| \geq 3$ , and none with  $|N_\sigma| \geq 4$ . Remarkably, 68.3% of the  $N_{\sigma_{wm}}$  error distribution lies within  $-1.24 \leq N_\sigma \leq 1.87$  while 95.4% falls within  $-2.22 \leq N_\sigma \leq 3.99$ . The observed  $N_{\sigma_{wm}}$  error distribution has constraints of  $|N_\sigma| \leq 1.87$  and  $|N_\sigma| \leq 3.99$  respectively, and 38.1% and 71.4% of the values lie within  $|N_\sigma| \leq 1$  and  $|N_\sigma| \leq 2$ , respectively. For the median case, the distribution has a narrower tail than the expected Gaussian distribution, with 1 value of  $|N_\sigma| \geq 2$  and none with  $|N_\sigma| \geq 3$ . For signed  $N_\sigma$ , 68.3% of the data

lie within  $-0.69 \leq N_\sigma \leq 1.29$ , while 95.4% fall within  $-0.88 \leq N_\sigma \leq 2.50$ . The absolute  $|N_\sigma|$  error distribution has constraints of  $|N_\sigma| \leq 0.88$  and  $|N_\sigma| \leq 2.50$ , respectively. Moreover, 76.2% and 95.2% of the values lie within  $|N_\sigma| \leq 1$  and  $|N_\sigma| \leq 2$ , respectively.

On the other hand, for the MFV case, we gain a central estimate of  $\tau_n = 881.16$  s, see Fig. 8, and also find a non-Gaussian error distribution with 2 values of  $|N_\sigma| \geq 2$  and none with  $|N_\sigma| \geq 3$ . 68.3% of the data falls within  $-0.97 \leq N_\sigma \leq 1.66$ , while 95.4% lie within  $-1.47 \leq N_\sigma \leq 2.60$ . The  $|N_\sigma|$  error distribution has constraints of  $|N_\sigma| \leq 1.43$  and  $|N_\sigma| \leq 2.60$ , respectively, and 57.1% and 90.5% of the values fall within  $|N_\sigma| \leq 1$  and  $|N_\sigma| \leq 2$ , respectively. These data highlight that the error distribution for the MFV case is broader than that of the median case and narrower than that of the weighted mean case. The incrementally narrowing distributions of the weighted mean, MFV, and median might be related to unaccounted-for systematic uncertainties or correlations among measurements of neutron lifetime [17, 41, 72]. Obviously, these detailed statistical descriptions represent that the error distributions of the weighted mean, median, and MFV are non-Gaussian, and the MFV technique is appropriate for a robust analysis of neutron lifetime measurements. In Fig. 8 we show all neutron lifetime measurements of bottle, beam, and space-based methods in the top panel, while the calculated residuals  $\Delta\tau/\tau_{MFV}$  from the MFV are demonstrated in the bottom panel. This provides a strong evidence for the non-Normality of error distributions, further confirming the effectiveness and rationality of MFV





**Fig. 8** (Top) Neutron lifetime measurements as a function of publication date. The points are the measured data using bottle, beam, and space-based methods, while the triangle, diamond, and inverted triangle

depict the weighted mean, MFV, and median, respectively. (Bottom) We show the residuals of fit or data with respect to the MFV as a function of publication date, i.e.  $\Delta\tau/\tau_{MFV}$ . Data set key: PDG 2022 edition [40]

statistics. The data of neutron lifetime and codes to reproduce above results are available in the following repository: <https://gitee.com/zhangphysics/neutron-lifetime>.

### 5 Conclusion

In summary, the problem of the long-standing tension in neutron lifetime measurements is one of the important challenges of astrophysics and particle physics. In this paper, from the perspective of robust statistics of the observed data, we have applied the MFV statistics technique to explore a detailed statistical analysis for the updated PDG dataset of neutron lifetime measurements that extend from beam to bottle methods. The MFV estimate is given by  $\tau_n = 881.16^{+2.25}_{-2.35}$  s with uncertainty corresponding to 68.27% confidence interval, while the result of median statistics is  $\tau_n = 881.5^{+5.5}_{-3}$  s. Additionally, we have investigated the error distributions of neutron lifetime measurements compiled by PDG. Due to the non-

Gaussian error distributions, the weighted mean technique is more inappropriate than the median and MFV statistics. Similar to median statistics, the MFV statistics error distributions are clearly different from Gaussian, which implies that it might be caused by unaccounted-for systematic uncertainties or correlations among all data. As can be seen, the MFV technique is a powerful tool and appropriate for analysis of non-Gaussian distributions. Using the MFV statistics, we can gain insight into the details of how error distributions construct. Moreover, the consistent results demonstrated the usage and robustness of MFV statistics, which will inspire further uncertainty research into the application of the MFV method in some similar situations.

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