Pl. XIV.

APPLETONS'

CYCLOPÆDIA OF TECHNICAL DRAWING.

EMBRACING

The Principles of Construction

AS APPLIED TO

PRACTICAL DESIGN.

WITH NUMEROUS ILLUSTRATIONS OF

TOPOGRAPHICAL, MECHANICAL, ENGINEERING, ARCHITECTURAL, PERSPECTIVE, AND FREE-HAND DRAWING.

EDITED BY

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NEW YORK:
D. APPLETON AND COMPANY,
1, 3, AND 5 BOND STREET.
1892.

PREFACE.

"At the suggestion of the publishers, this work was undertaken to form one of their series of dictionaries and cyclopædias. In this view, it has been the intention to make it a complete course of instruction and book of reference to the mechanic, architect, and engineer. It has not, therefore, been confined to the explanation and illustration of the methods of projection, and the delineation of objects which might serve as copies to the draughtsman, matters of essential importance for the correct and intelligible representation of every form; but it contains the means of determining the amount and direction of strains to which different parts of a machine or structure may be subjected, and the rules for disposing and proportioning of the material employed, to the safe and permanent resistance of those strains, with practical applications of the same. Thus, while it supplies numerous illustrations in every department for the mere copyist, it also affords suggestions and aids to the mechanic in the execution of new designs. And, although the arranging and properly proportioning alone of material in a suitable direction, and adequately to the resistance of the strains to which it might be exposed, would produce a structure sufficient in point of strength for the purposes for which it is intended, yet, as in many cases the disposition of the material may be applied not only practically, but also artistically, and adapted to the reception of ornament, under the head of Architectural Drawing, the general characteristics of various styles have been treated of and illustrated, with brief remarks on proportion and the application of color." . . . 1857.

Since its first publication, this work has been subjected from time to time to revision. It has now been deemed necessary to almost entirely rearrange and rewrite it; to add largely to the subject-matter and to the illustrations, introducing examples of later practice and experience; to extend the scope of the work, and make it more nearly a cyclopædia of drawing and design. There are no changes in the principles of projection as applied to drawing, and no marked improvement in drawing-instruments; but in the present practice finished drawings in shade and color are exceptional. It is sufficient, for almost every purpose, for the draughtsman to make accurate projections with pencil on paper, and trace them afterward on cloth. The pencil-drawings can be readily altered or amended, and, where there are many repetitions of the same parts, but a single one may be drawn. On the tracing they are made complete, and these are preserved as originals in the office, while sun-prints of them are used for details of construction in the shop, or distributed as circulars among customers.

In the sale of former editions of this work, it has been found that its success has been largely due to its value as a book of design. Great attention has therefore been given to secure practical illustration of constructions and machines of recent date; the nature of materials in common use has been more extensively treated, and the character and effect of stresses and strains, their kind and direction, more fully explained, with as simple rules as possible to determine them for practical application.

Of late years the science of "graphics" has become of great importance, and is here fully illustrated in its varied applications, showing not only this method of recording the facts of the statistician, and affording comparisons of circumstances and times, the growth of population, the quantities and cost of agricultural and mechanical production, and of their transport, movements of trade, fluctuations of value, the atmospheric conditions, death-rates, etc., but also in its application to the plotting of formulæ for their ready solution, by the draughtsman and designer. For many of the rules in this work the results of the formulæ of various authors have been plotted graphically, and the rule given one deemed of the greatest weight, not always by average, but most consistent with our own experience.

In astronomical calculations every decimal may have its importance. It is not so in those of the mechanical or architectural designer; solutions by graphics are sufficient for their purpose, and, simpler than mathematical calculations, they are thus less liable to error; it is very good practice to use one as a check on the other. It is to be remarked that inaccuracy in facts, and carelessness in observation, are not eliminated from an equation by closeness of calculation, and when factors are not established within the limits of units it is useless to extend the results to many places of decimals. It is of the utmost importance to know at first well the object and purposes of the design, the stresses to which its parts are to be subjected, and the strength and endurance of the materials of which it is to be composed. In establishing rules for ourselves, be sure of the facts, and that there are enough of them for a general application. Rules are necessary, but their application and usefulness depend largely on the experience of the user, and life must be a record of applications and effects. It is comparatively easy to make a work strong enough; but to unite economy with proportion is difficult.

To make the work complete in itself, so as to form a sort of single book for most of the purposes of the draughtsman and designer—embracing the profession of surveyor, engineer, and architect—tables of logarithms, latitudes and departures, squares and cubes, and square and cube roots, weights and measures, and weights of material, have been added.

W.

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DESCRIPTION OF PLATES.

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I. Shading of prism and cylinder by flat tints. Referred to on pages 126–7.

II. Shading of cylinder and segment of hexagonal pyramid. Referred to on pages 128–9.

III, IV. Finished shading and shadows of different solids. Referred to on page 131.

V. Shades and shadows on screws. Referred to on page 126.

VI. Example of topographical drawing, done entirely with the pen.

VII. The same, with the brush, in black.

VIII. The same, with the brush, in color. Referred to on page 174.

IX. Contoured map of Staten Island, shaded by superimposed washes, the washes increasing in intensity or strength as required to produce the effect.

X. Geological map of part of New Jersey, colored to show the different formations.

XI. Finished, shaded sectional view, colored to show the different metals, of a balanced leather cup-valve. The body is of cast-iron ; the piston, brass ; packing, leather ; piston-rod, wrought-iron—this last not distinctively colored.

XII. Finished perspective drawing, with shades and shadows, of a large bevel-wheel and two pinions, with shifting clutches.

XIII. Front elevation of a building, in color.

XIV. Perspective view of Gothic church, finished in color.

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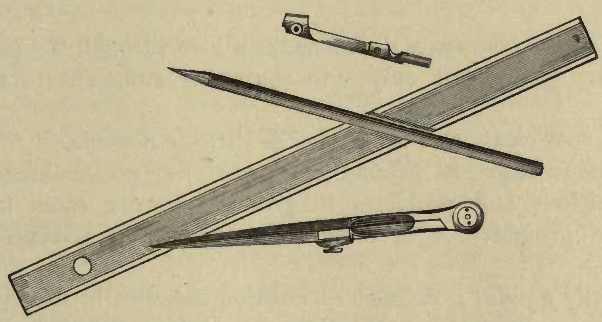
XVI to XX. Details of progressive perspective projections of Plate XV, as shown completed in Plate XII.

APPLETONS'

CYCLOPÆDIA OF DRAWING.

CONSTRUCTION OF GEOMETRICAL PROBLEMS.



Most persons, at some time, have made use of the simple drawing instruments, pencils, straight-edges or rulers, and compasses or dividers with changeable points, and many suppose that there can be no skill in their use; but to one critical in these matters there are great differences to be observed even in common drawings, in the straightness and uniformity of the lines, and in the care of the surface of the paper.

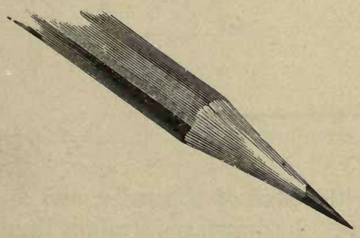


Fig. 1.

Select for the geometrical problems and for usual drawings a No. 3 or H H H pencil. It should be sharpened to a cone-point (Fig. 1). Where a pencil is used for drawing lines only, some draughtsmen sharpen the pencil to a wide edge, like a chisel.

In drawing a straight line, hold the ruler firmly with the left hand; with the right hand hold the pencil lightly but without slackness, and a little inclined in the direction of the line to be drawn, keeping the pencil against the edge of the ruler, and in the same relative position to the edge during the whole operation of drawing the line.

To draw a clean line and preserve the point of the pencil, the part of the cone a little above the point of the pencil should bear against the edge of the ruler, and the pencil should be carried steadily while drawing. Any oscillation will throw the point farther from or nearer the ruler, and the line will not be straight (Fig. 2).

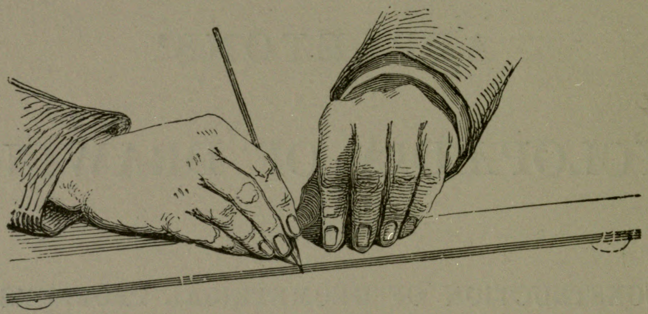


FIG. 2.

In the use of the compasses do not make a hole through the paper with the needle or sharp point, but only into the paper sufficient to maintain the position.

Keep the paper clean, and use rubber as little as possible.

As drawing is based on geometrical principles, we commence with geometrical definitions and problems to give the learner some knowledge of the science of geometry, with a valuable exercise in the use of drawing instruments.

Geometrically a *point* is defined as position merely : in drawing, the positions of points are marked on the paper by prick-marks of a needle or sharp point, and by the dot of a pencil ; sometimes inclosed ⊙, sometimes designated by the intersection of two short lines × >.

Geometrically *lines* have but one dimension, *length*, and the direction of a line is the direction from point to point of the points of which the line is composed : in drawing, lines are visible marks of pencil or pen upon paper.

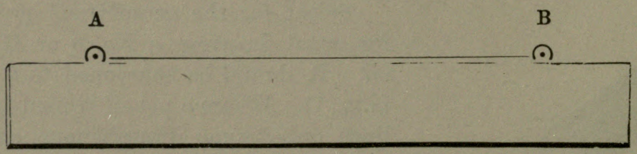


FIG. 3.

A *straight line* is such as can be drawn along the edge of the ruler, and is one in which the direction is the same throughout. In drawing a straight line through two given points, place the edge of the ruler very near to and at equal distances from the points, as the point of the pencil or pen should not be in contact with the edge of the ruler (Fig. 3).

Lines in geometry and drawing are generally of limited extent. A *given* or known line is one established on paper or fixed by dimensions. Lines of the same length are *equal.*

To draw Curved Lines.—Insert the pencil-point in the compasses, and open them to a suitable extent. With the needle or sharp point resting on the paper describe a line with the pencil around this point; the line thus described is usually called a *circle*—more strictly it is the *circumference* of a circle—the circle being the space inclosed. A portion of a circumference is an *arc.* The point around which the circumference is described is the *center* of the circle (Fig. 4).

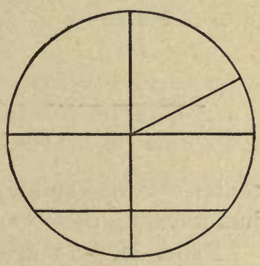


Fig. 4.

If a line be drawn from the center to the circumference it is called a *radius.* As it is the length embraced between the points of the compasses, it is often called by mechanics the sweep.

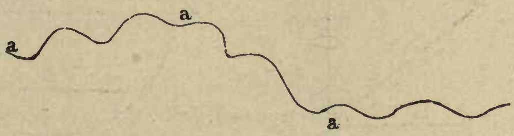
If a line be drawn through the center, and limited by the circumference, it is called the *diameter,* and is equal to two radii.

A radius is a semi-diameter ; a diameter is the longest line that can be contained within a circumference. Lines limited by the circumference, and which are not diameters, are *chords.*

It will be observed that *arcs* are lines which are continually changing the directions, and are called curved lines, but there are other curved lines than those described by compasses, of which the construction will be explained hereafter.

Besides straight and curved lines there are often lines, in drawing, which can neither be drawn by rulers or compasses, as lines representing the directions of brooks and rivers, the margins of lakes and seas, points in which are established by surveys, defined on paper, and connected by hand-drawing. These may be called *irregular* or *crooked* lines.

Where it is necessary to distinguish lines by names, we place at their extremities letters or figures, as A———B, 1———2 ; the line A B, or 1 2. But in lines other than straight, or of considerable extent, it is often necessary to introduce intermediate letters and figures, as *a a a.*



In the following problems, unless otherwise implied or designated, where lines are mentioned, straight lines are intended.

If we conceive a straight line to move sideways in a single direction, it will sweep over a *surface* which is called a *plane.* All drawings are projections on planes of paper or board.

Two lines drawn on paper, and having the same direction, can never come any nearer each other, and must always be at the same distance apart, however far extended. Such lines are called *parallel* lines.

PROB. I.—*To draw a line parallel to a given line, and at a given distance from it* (Fig. 5).

Draw the line A B for the given line, and take in the compasses the distance A C—the distance at which the other line is to be drawn. On A, as a

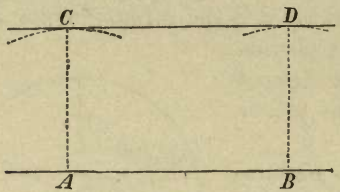


FIG. 5.

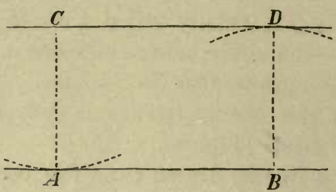


FIG. 6.

center, describe an arc, and on B, as a center, another arc; draw the line C D just touching these two arcs, which will be the parallel line required.

PROB. II.—*To draw a line parallel to a given line through a given point outside this line* (Fig. 6).

Draw the given line A B, and mark the given point C. With C as a center, find an arc that shall only just touch A B; and with B as a center, and the same radius, describe an arc D. Draw through the point C a line just touching this last arc, and the line C D will be the parallel line required.

Two lines in the same plane, not parallel to each other, will come together if extended sufficiently far. The coming together, cutting, or intersection of two lines, is called an *angle* (Fig. 7).

If but two lines come together, the angle may be designated by a single letter at the *vertex*, as the angle E.

But, if three or more lines have a common vertex, the angles are designated by the lines of which they are composed, as the angle D B C of the lines D B

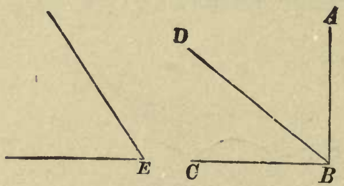


FIG. 7.

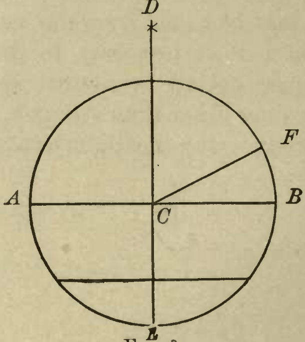


FIG. 8.

and B C; the angle A B C of A B and B C; the angle A B D of A B and B D. The letter at the vertex is not repeated, and must always be the central letter.

Describe a circle (Fig. 8). Draw the diameter A B. From A and B as centers, with any opening of the compasses greater than the radius, describe two arcs cutting each other as at D. Through the intersection of these arcs and the center C, draw the line D E. D E makes, with the diameter A B, four angles, viz., A C D, D C B, B C E, and E C A. Angles are equal whose lines have equal inclination to each other, and whose lines, if placed one on the other, would coincide. By construction, the points C and D have, respectively, equal distances from A and B; the line D C can not, therefore, be inclined more to one side than to the other, and the angle A C D must be equal to the angle B C D. Such angles are called right angles. It can be readily proved that all the four angles, formed by the intersection of D E with A B, are equal, and are right angles.

The angles A C D and D C B, on the same side of A B, are called *adjacent* angles; as also D C B and B C E, on the same side of D E.

When a line, standing on another line, makes the two adjacent angles equal, the angles are right angles, and the first line is perpendicular to the other.

If the second or *base* line be parallel with the surface of still water, it is called an *horizontal* line, and the perpendicular line is called a *vertical* line.

Draw the line C F. It will be observed that the angle F C D is less than a right angle, and it is called an *acute* angle; the angle F C A is greater than a right angle, and it is called an *obtuse* angle. It will be observed that, no matter how many lines be drawn to the center, the sum of all the angles on the one side of A B can only be equal to two right angles, and, on both sides of A B, can only be equal to four right angles. It will be observed that the angles at the center include greater or less arcs between their sides, according to the greater or less inclination of their sides to each other; that the right angles intercept equal arcs, and that, no matter how large the circle, the proportion of the circle intercepted by the sides of an angle is always the same, and that the arcs can therefore be taken as the measures of angles. For this purpose the whole circumference is supposed to be divided into three hundred and sixty degrees (360°), each degree subdivided into sixty minutes (60′), and each minute into sixty seconds (60″). Each right angle has for its measure one quarter of the whole circumference ($\frac{360}{4}$), or 90°, and is called a *quadrant*.

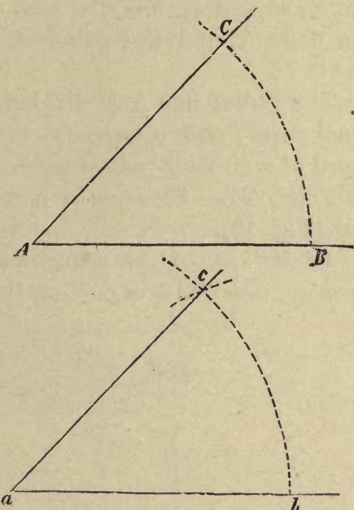


FIG. 9.

PROB. III.—*To construct an angle equal to a given angle* (Fig. 9).

Draw any angle, as C A B, for the given angle, and the line *a b* as the base of the required angle. From A, with any suitable radius, describe the arc B C, and from *a*, with the same radius, describe the arc *b c*. Measure the length of the arc B C, or rather the chord, that is, the distance in a straight line from B to C, and lay off the same distance on the arc *b c*. Draw the line *a c*, and the angle *c a b* will be equal to C A B.

PROB. IV.—*To construct an angle of sixty degrees* (Fig. 10).

Lay off any base-line, and from A, with any radius, describe an arc, and from B, with the same radius, describe another arc cutting the first, as at C. Draw the line C A, and the angle C A B will be an angle of sixty degrees.

The reason of this construction will be readily understood if, on the circumference of any circle, chords equal to the radius are stepped off successively. Six will exactly complete the circle, making 360°, or each 60°, and the angle corresponding will be 60°.

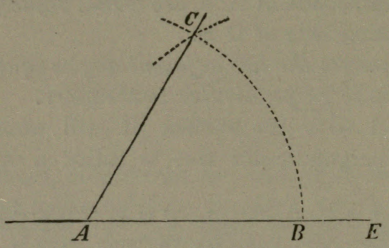


Fig. 10.

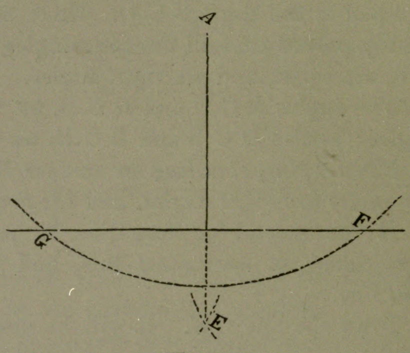


Fig. 11.

Prob. V.—*To draw a perpendicular to a line from a point without the line* (Fig. 11).

Draw a line, and mark the given point outside it, A. From A as a center, with a suitable radius, describe an arc cutting the line at G and F. From G and F, as centers, describe arcs cutting each other. The line drawn through the point A, and the point of intersection E, will be perpendicular to the line G F.

The radial line A E divides the chord G F and the arc G E F into two equal parts; and, conversely, the line perpendicular to the middle point of a chord of a circle is radial—passes through the center of that circle.

Prob. VI.—*To draw a perpendicular to a line from a point within that line* (Fig. 12).

1st Method.—Draw a line, and take the point A in the line. From A, as a center, describe arcs cutting the line on each side at B and C. From B and C, as centers, describe intersecting arcs at D. Draw a line through D and A, and it will be perpendicular to the line B C at A.

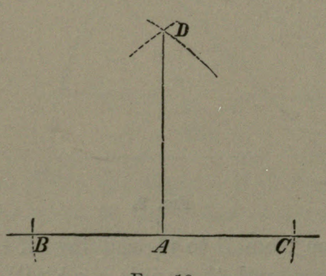


Fig. 12.

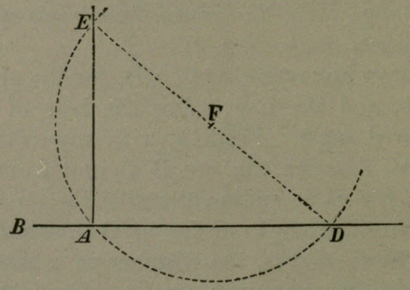


Fig. 13.

2d Method (Fig. 13).—Draw the line, and mark the point A as before. From any center F, without the line, and not directly over A, with a radius equal to F A, describe more than a half-circle cutting the line, as at D. From D, through the center F, draw a line cutting the arc at E. Draw A E, and it will be the perpendicular to the line A D.

It will be observed that the line D E is the diameter of a circle, and that the angle D A E, with its vertex at A in the circumference, would embrace with its sides half a circle, had a full circle been described. It has been shown that angles at the center of a circle have for their measure the arc embraced by their sides. It is easily demonstrable that angles, with their vertices in the circumference, have for their measure half the arc embraced by their sides, and, consequently, angles embracing half a circumference are right angles, and their sides are perpendicular to each other.

PROB. VII.—*To draw a perpendicular to the middle point of a line* (Fig. 14).

From the extremities A and B of the line, as centers, describe intersecting arcs above and below the line. Through these intersections draw the line C D. It will be perpendicular to the line A B, and bisect or divide it into two equal parts.

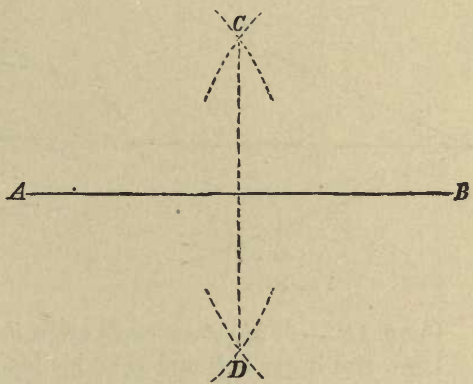


Fig. 14.

If the line A B be considered the chord of a circle, its center would lie in the line C D.

This construction is sometimes used merely to divide a line into two equal parts, or bisect it; but if we have dividers or compasses, with both points sharp, it can be more readily done with them (Fig. 15).

Place one point of the dividers on one end of the line, and open the dividers to a space as near as may be half the line. Turn the dividers on the central point; if the other point then falls exactly on the opposite extremity

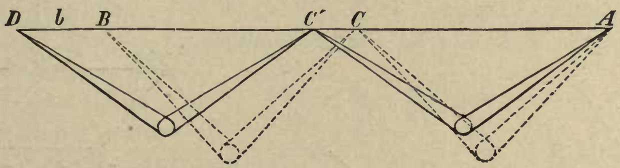


Fig. 15.

of the line, it is properly divided; but, if the point falls either within or without the extremity of the line, divide the deficit or excess by the eye, in halves, and contract or extend the dividers by this measure. Then apply the dividers as before, and divide deficit or excess till a revolution exactly covers the length of the line. By accustoming one's self to this process, the eye is made accurate, and one estimate is sufficient for a correct division of any deficit or excess. By a similar process it is evident that a line can be divided into any number of equal parts, by assuming an opening of the dividers as nearly as possible to that required by the division, and, after spacing the line, dividing the deficit or excess by the required number of parts, contracting or expanding the dividers by one of these parts, and spacing the line again, and so on till it is accurately divided.

Prob. VIII.—*To bisect a given angle* (Fig. 16).

Construct an angle, and from its vertex A, as a center, describe an arc cutting the two sides of the angle at B and C. From B and C, as centers, describe intersecting arcs. Draw a line through A and the point of intersection D, and this line will bisect the angle.

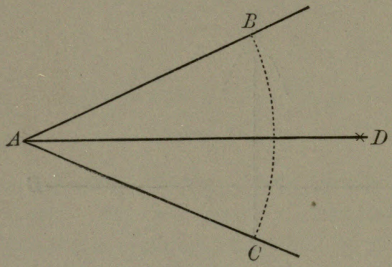


Fig. 16.

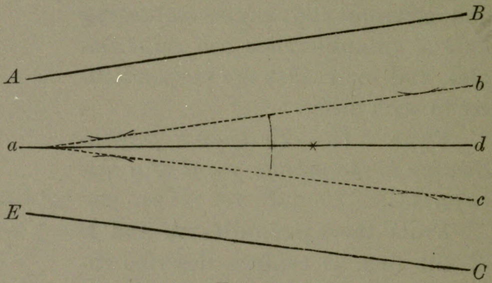


Fig. 17.

Prob. IX.—*To bisect an angle when the vertex is not on the paper* (Fig. 17).

Draw two lines, A B and E C, inclined to each other, but not intersecting. Draw two lines intersecting each other, *a b* and *a c*, inside and parallel to A B and E C. Bisect *b a c* by the line *a d*, and this line will also bisect the angle whose vertex is not on the paper.

Prob. X.—*Through two given points to describe an arc of a circle with a given radius* (Fig. 18).

From B and C, the two given points, with an opening of the dividers equal to the given radius, describe two arcs crossing at A. From A, as a center, with the same radius, describe an arc, and it will be the one required.

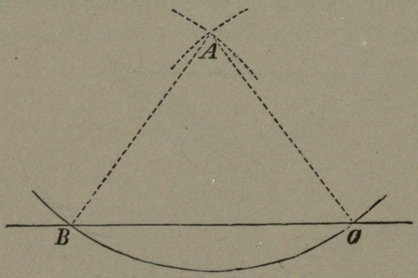


Fig. 18.

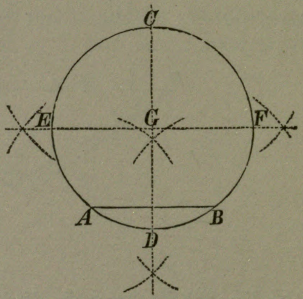


Fig. 19.

Prob. XI.—*To find the center of a given circle, or of an arc of a circle.*

Of a circle (Fig. 19).—Draw the chord A B. Bisect it by the perpendicular C D, whose extremities lie in the circumference, and bisect C D. G, the point of bisection, will be the center of the circle.

Of an arc, or of a circumference (Fig. 20).—Select the points A, B, and C in the circumference, well apart. With the same radius from A and B as centers, and then from B and C as centers, describe arcs cutting each other; draw the two lines D E and F G through their intersections. The point O, where these lines meet, is the center required.

Prob. XII. — *To describe a circle passing through three given points* (Fig. 20).

Proceed, as in the last problem, to find the center O. From O, as a center, with a radius O A, describe a circle, and it will be the one required.

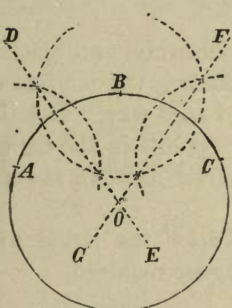


Fig. 20.

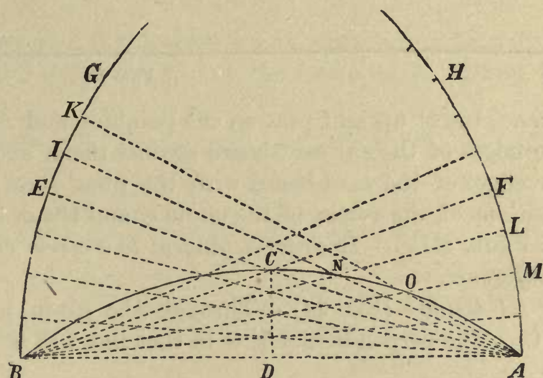


Fig. 21.

Prob. XIII. — *To describe a circle passing through three given points, where the center is not available.*

1st Method (Fig. 21). — From the extreme points A and B, as centers, describe the arcs B G and A H. Through the third point, C, draw A E and B F, cutting the arcs. Divide the arcs A F and B E into any number of equal parts, and set off a series of equal parts of the same length on the upper portions of the arcs beyond E and F. Draw straight lines, B L, B M, etc., to the points of division in A F, and A I, A K, etc., to the points of division in E G; the successive intersections N, O, etc., of these lines are points in the circle required between the given points A and C, which may be filled in accordingly. Similarly, the remaining part of the curve, B C, may be described.

2d Method (Fig. 22).—Let A, D, and B be the given points. Draw A B, A D, and D B. Draw *e f* parallel to A B. Divide D A into a number of equal parts at 1, 2, 3, etc., and from D describe arcs through these points to meet *e f*. Divide the arc A *e* into the same number of equal parts, and draw straight lines from D to the points of division. The intersections of these lines successively with the arcs are points in the circle, which may be filled in as before.

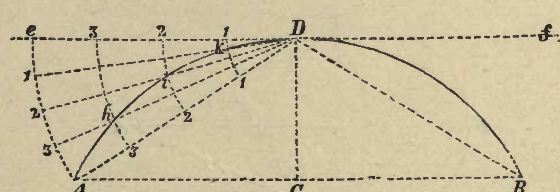


Fig. 22.

Note.—The second method is not perfectly true, but sufficiently so for arcs less than one fourth of a circle.

To describe the arc mechanically.—Let *a, c, b* be the three points of a curve; transfer these points to a piece of stout card-board, and draw the lines *a c* and *c b,* and extend them beyond *a* and *b.* Cut out the card-board along these

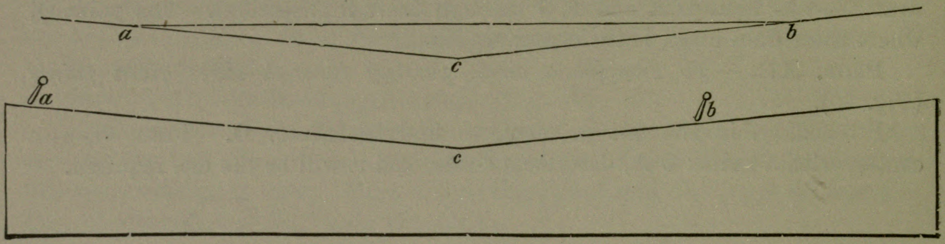


Fig. 23.

lines. Insert upright pins on the points *a* and *b* of the drawing, and placing the edges of the cut card-board against them, and maintaining the contact of the edges of the card-board with the pins, slide the card each way. Dot the positions of the vertex of the angle *c,* and the dots will be points in the curve.

Prob. XIV.—*To draw a tangent to a circle from a given point in the circumference.*

1st Method (Fig. 24).—Through the given point A draw the radial line A C. The perpendicular F G to that line will be the tangent required.

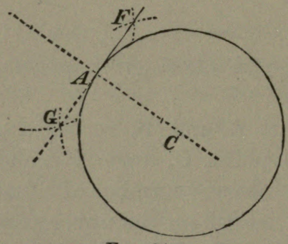


Fig. 24.

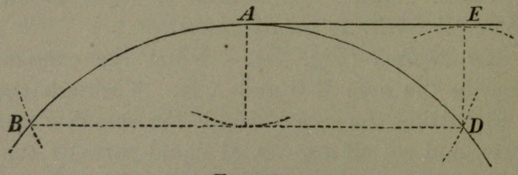


Fig. 25.

2d Method (Fig. 25).—From the given point A set off equal arcs, A B and A D. Join B and D. Through A draw A E parallel to B D, and it will be the tangent required. This method is useful when the center is inaccessible.

Prob. XV.—*To draw tangents to a circle from a point without it* (Fig. 26). From the given point A draw A C to the center of the circle. From D, the

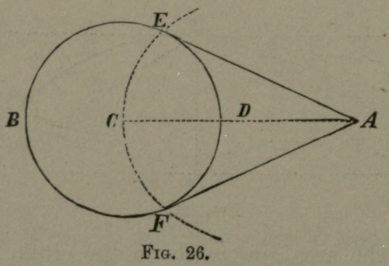


Fig. 26.

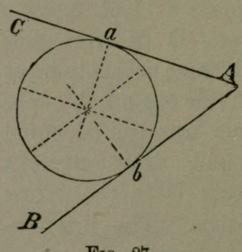


Fig. 27.

intersection of A C with the circle, describe an arc, with a radius D C, cutting the circle at E and F. Draw A E and A F, and they will be the tangents required.

To construct within the sides of an angle a circle tangent to these sides at a given distance from the vertex (Fig. 27).—Let *a* and *b* be the given points equally distant from the vertex A. Draw a perpendicular to A C at *a*, and to A B at *b*. The intersection of these perpendiculars will be the center of the required circle.

In the same figure, *to find the center, the radius being given,* and the points *a* and *b* not known. Draw lines parallel to A C and A B, at a distance equal to the given radius, and their intersection will be the center required.

PROB. XVI.—*To describe a circle from a given point to touch a given circle* (Figs. 28 and 29).

D E being the given circle, and B the given point, draw a line from B to the center C, and produce it, if necessary, to cut the circle at A. From B,

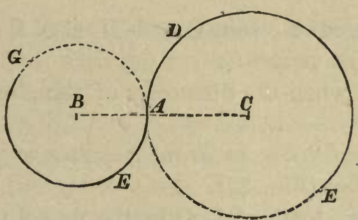


FIG. 28.

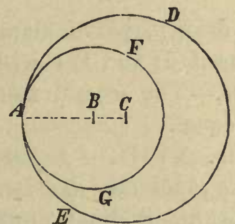


FIG. 29.

as a center, with a radius equal to B A, describe the circle F G, touching the given circle, and it will be the circle required.

The operation is the same whether the point B is within or without the circle.

It will be remarked that, in all cases of circles tangent to each other, their centers and their points of contact must lie in the same straight line.

PROB. XVII.—*To draw tangents to two given circles.*

1st Method (Fig. 30).—Draw the straight line A B C through the centers of the two given circles. From the centers A and B draw parallel radii, A D

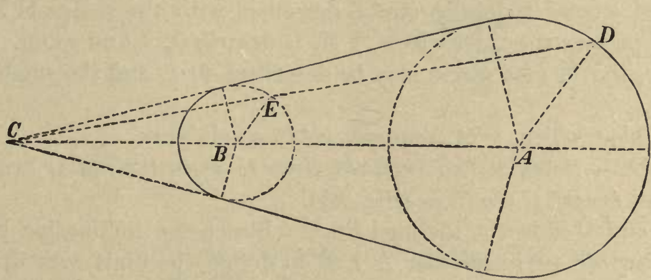


FIG. 30.

and B E, in the same direction. Draw a line from D to E, and produce it to meet the center line at C; and from C draw tangents to one of the circles by Problem XV. Those tangents will touch *both* circles as required.

2d Method (Fig. 31).—Draw the line A B connecting the two centers. Draw in the larger circle any radius, A H, on which set off H G, equal to the radius of the smaller circle. On A describe a circle with the radius A G, and draw tangents, B I and B K, to this circle from the other center, B. From A

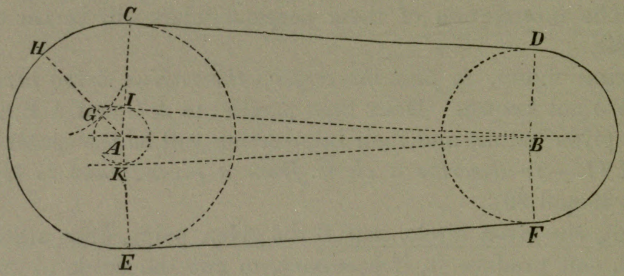


FIG. 31.

and B draw perpendiculars to these tangents. Join C and D, also E and F. The lines C D and E F will be the required tangents.

Note.—The second method is useful when the diameters of the circles are nearly equal.

PROB. XVIII. —*Between two inclined lines to draw a series of circles touching these lines and touching each other* (Fig. 32).

Bisect the inclination of the given lines A B and C D by the line N O; this is the center line of the circles to be inscribed. From a point, P, in this line, draw P B perpendicular to the line A B; and from P describe the circle B D, touching the given lines, and cutting the center line at E. From E draw E F perpendicular to the center line, cutting A B at F; from F describe an arc, with a radius, F E, cutting A B at G; draw G H parallel to B P, giving H the center of the second touching circle, described with the radius H E or H G. By a similar process the third circle, I N, is described. And so on.

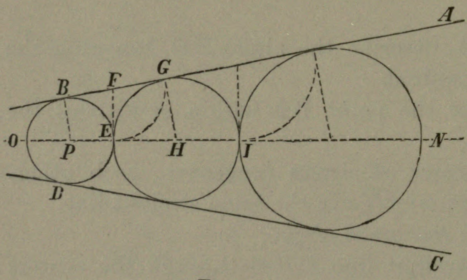


FIG. 32.

Inversely, the largest circle may be described first, and the smaller ones in succession.

Note.—This problem is of frequent use in scroll-work.

PROB. XIX.—*Between two inclined lines to draw a circular arc to fill up the angle, and touching the lines* (Fig. 33).

Let A B and D E be the inclined lines. Bisect the inclination by the line F C, and draw the perpendicular A F D to define the limit within which the circle is to be drawn. Bisect the angles A and D by lines cutting at C, and from C, with radius C F, draw the arc H F G, which will be the arc required.

PROB. XX.—*To fill up the angle of a straight line and a circle, with a circular arc of a given radius* (Fig. 34).

On the center C, of the given circle A D, with a radius C E equal to that of the given circle plus that of the required arc, describe the arc E F. Draw

G F parallel to the given line H I, at the distance G H, equal to the radius of the required arc, and cutting the arc E F at F. Then F is the required

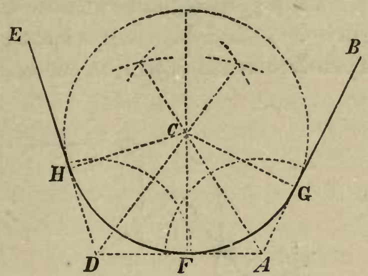


FIG. 33.

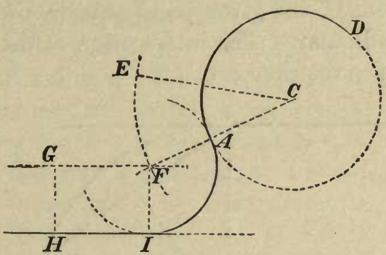


FIG. 34.

center. Draw the perpendicular F I, and the line F C, cutting the circle at A; and, with the radius F A or F I, describe the arc A I, which will be the arc required.

PROB. XXI.—*To fill up the angle of a straight line and a circle, with a circular arc to join the circle at a given point* (Fig. 35).

In the given circle B A draw the radius to A, and extend it. At A draw a tangent, meeting the given line at D. Bisect the angle A D E, so formed, with a line cutting the radius, as extended at F; and, on the center F, with radius F A, describe the arc A E, which will be the arc required.

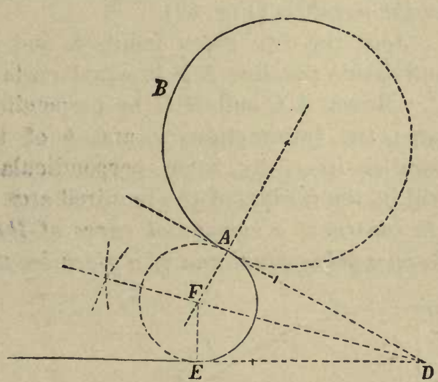


FIG. 35.

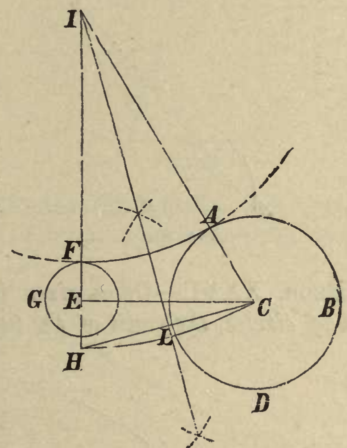


FIG. 36.

PROB. XXII.—*To describe a circular arc joining two circles, and to touch one of them at a given point* (Fig. 36).

Let A B and F G be the given circles to be joined by an arc touching one of them at F.

Draw the radius E F, and produce it both ways; set off F H equal to the radius, A C, of the other circle; join C to H, and bisect it with the perpendicular L I, cutting E F at I. On the center I, with radius I F, describe the arc F A, which will be the arc required.

PROB. XXIII.—*To find the arc which shall be tangent to a given point on a straight line, and pass through a given point outside the line* (Fig. 37).

Erect at A, the given point on the given line, a perpendicular to the line. From C, the given point outside the line, draw C A, and bisect it with a perpendicular. The intersection of the two perpendiculars at *a* will be the center of the required arc.

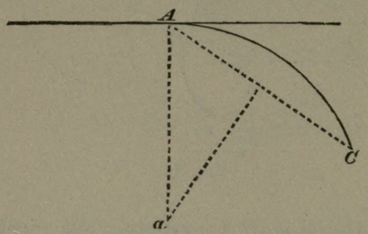


FIG. 37.

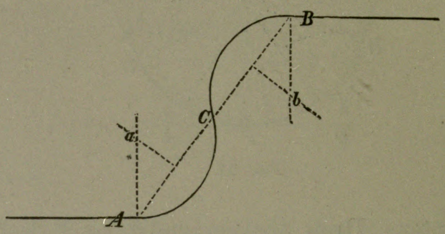


FIG. 38.

PROB. XXIV.—*To connect two parallel lines by a reversed curve composed of two arcs of equal radii, and tangent to the lines at given points* (Fig. 38).

Join the two given points A and B, and divide the line A B into two equal parts at C; bisect C A and C B by perpendiculars; at A and B erect perpendiculars to the given lines, and the intersections *a* and *b* will be the centers of the arcs composing the required curve.

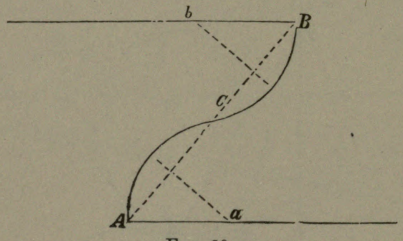


FIG. 39.

PROB. XXV.—*To join two given points in two given parallel lines by a reversed curve of two equal arcs, whose centers lie in the parallels* (Fig. 39).

Join the two given points A and B, and divide the line A B in equal parts at C. Bisect A C and B C by perpendiculars; the intersections *a* and *b* of the parallel lines, by these perpendiculars, will be the centers of the required arcs.

PROB. XXVI.—*On a given line, to construct a compound curve of three arcs of circles, the radii of the two side ones being equal and of a given length,*

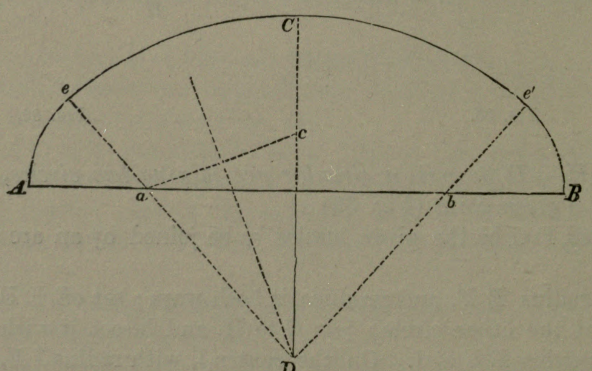


FIG. 40.

and their centers in the given line; the central arc to pass through a given point on the perpendicular, bisecting the given line, and to be tangent to the other two arcs (Fig. 40).

Let A B be the given line, and C the given point. Draw C D perpendicular to A B; lay off A *a*, B *b*, and C *c*, each equal to the given radius of the side arcs; draw *a c*, and bisect it by a perpendicular; the intersection of this line with the perpendicular C D will be the required center of the central arc *e C e'*. Through *a* and *b* draw the lines D *e* and D *e'*; from *a* and *b*, with the given radius equal to *a* A or *b* B, describe the arcs A *e* and B *e'*. From D, as a center, with a radius equal to C D, and, consequently, by construction, equal to D *e* and D *e'*, describe the arc *e* C *e'*. The entire curve A *e* C *e'* B is the compound curve required.

It will be observed in all the preceding problems that, when a line is tangent to a curve, the center of that curve must be in the perpendicular to the line at its tangent point; and that, when two curves are tangent to each other, their centers must be in the same radial line passing through the point of tangency.

PROBLEMS ON POLYGONS AND CIRCLES.

Three lines inclosing a space form a *triangle* (Fig. 41). If two of the sides are of equal length, it is an *isosceles* triangle; if all three are of equal length,

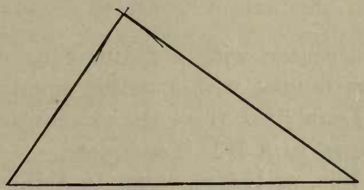


Fig. 41.

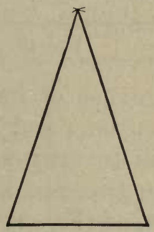


Fig. 42.

it is an *equilateral* triangle. If one of the angles is a right angle, it is a *right-angled* triangle, and if no two of the sides are of equal length, and not one of the angles a right angle, it is a *scalene* triangle.

Prob. XXVII.—*To construct an isosceles triangle* (Fig. 42).

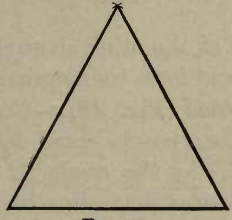


Fig. 43.

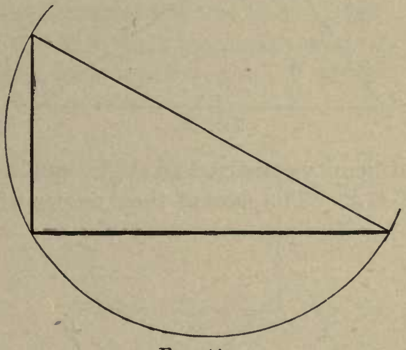


Fig. 44.

Draw any line as a base, and, from each extremity as a center, with equal radius, describe intersecting arcs. Draw a line from each extremity of the base to this point of intersection, and the figure is an isosceles triangle.

Prob. XXVIII.—*To construct an equilateral triangle* (Fig. 43).

Draw a base line, and from each extremity as a center, with a radius equal to the base line, describe intersecting arcs. Draw lines from the extremities of the base to this point of intersection, and the figure is an equilateral triangle.

PROB. XXIX.—*To construct a right-angled triangle* (Fig. 44).

Construct a right angle by any one of the methods before described. Draw a line from the extremity of the one side to the extremity of the other side, and the figure is a right-angled triangle.

It will be evident, in looking at any right-angled triangle, that the side opposite the right angle is longer than either of the other or *adjacent* sides; this side is called the *hypothenuse*.

PROB. XXX.—*To construct a triangle equal to a given triangle.*

Let A B C (Fig. 45) be the given triangle.

1st Method (Fig. 46).—Draw a base line, and lay off its length equal to

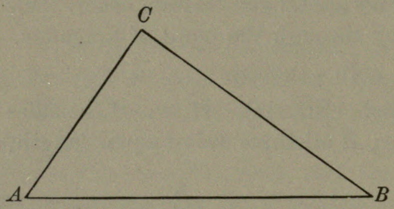


FIG. 45.

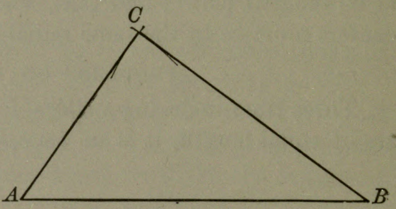


FIG. 46.

A B; from one of its extremities, as a center, with a radius equal to A C, describe an arc; and, from its other extremity, with a radius equal to B C, describe an arc intersecting the first. Draw lines from the extremities to the point of intersection, and the triangle equal to A B C is complete.

2d Method (Fig. 47).—Draw a base line, as before, equal to A B. At one

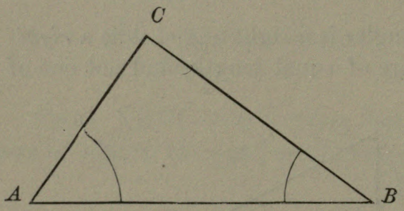


FIG. 47.

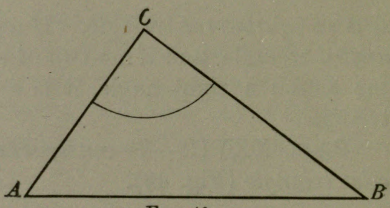


FIG. 48.

extremity construct an angle equal to C A B, and at the other an angle equal to C B A. The sides of these angles will intersect, and form the required triangle.

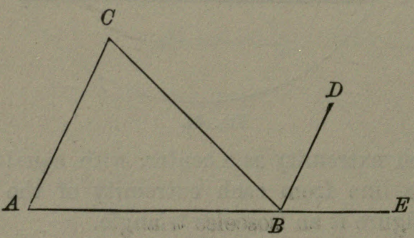


FIG. 49.

3d Method (Fig. 48).—Construct an angle of the triangle equal to any angle of A B C, say the angle A C B. On one of its sides measure a line equal to C A, and on the other side one equal to C B; connect the two extremiities by a line, and the triangle equal to A B C is complete.

From the above constructions it will

be seen that, if the three sides of a triangle, or two sides and the included angle, or one side and the two adjacent angles are known, the triangle can be constructed.

Construct a triangle, A B C (Fig. 49). Extend the base to E, and draw B D parallel to A C. As A C has the same inclination to C B that B D has, the angle C B D is equal to the angle A C B. As A C has the same inclination to A E that B D has, the angle D B E is equal to C A B. That is, the two angles formed outside the triangle are equal to the two inside at A and C; and the three angles at B are equal to the three angles of the triangle, and their sum is equal to two right angles. Therefore, the sum of the three angles of a triangle is equal to two right angles.

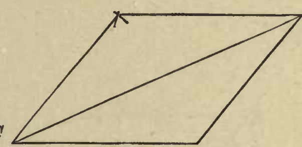


FIG. 50.

On one side of a triangle (Fig. 50) construct a triangle equal to the first, with opposite sides parallel.

The exterior lines of the two triangles form a four-sided or *quadrilateral* figure, of which the opposite sides are equal and parallel, and the opposite angles equal. This figure is called a *parallelogram,* and the line C B, extending between opposite angles, is a *diagonal.*

On the hypothenuse of a right-angled triangle (Fig. 51) construct another equal to it, and the exterior lines form a parallelogram, which, as all the angles are right angles, is called a *rectangle.* If the four sides are all equal, it is called a *square.*

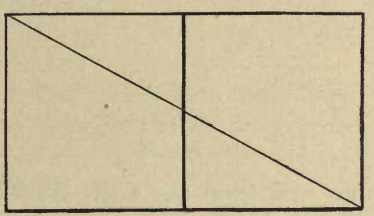


FIG. 51.

A parallelogram in which all the sides are equal, but the angles not right angles, is called a *rhombus* (Fig. 52); if only the opposite sides are equal, it is called a *rhomboid;* if only two sides are parallel, the figure is a *trapezium* (Fig. 53).

Describe a circle (Fig. 54). Draw a diameter, and erect on its center C the perpendicular C F. Draw at any angle with the diameter the line C A. Draw D H and A B perpendicular to the diameter, the first from the intersection of the line C A with the circumference, the other from the extremity B of the diameter. Draw D G and E F perpendicular to the radius C F, one from the point D, the other from the extremity of the radius C F. The angles D C G and D C H are *complements* of each other; that is, together they form a right angle, as it completes with it a right angle. The line D H is the

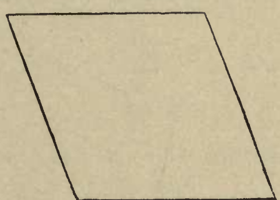


FIG. 52.

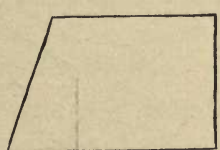


FIG. 53.

sine of the angle D C H and the *cosine* of D C G. D G is the sine of the angle D C G and the cosine of D C H. A B is the *tangent* of the angle D C B and the *cotangent* of D C G. E F is the tangent of the angle D C G and the cotangent of D C H. A C is the *secant* of the angle D C H and the *cosecant* of D C G. C E is the secant of the angle D C G and the cosecant of D C H. H B is the *versed sine* of the angle D C H, and G F of D C G.

It will be observed that the angles of the triangle D C H are equal to those of A C B, and that, if we suppose C A to be the radius of a larger circle, the arcs, and consequently the half-cords or sines D H and A B, will be proportionate to the radii; that is, D H will be to A B as C D is to C A.

Triangles which have equal angles have their sides proportional, and are called similar. This is demonstrable of other triangles than the right-angled ones in the figure.

Take any figure (Fig. 55) of more than three sides bounded by straight lines, and from any angle draw lines to the opposite angles. The figure will be divided into as many triangles as there are sides, less two, and the sum of the angles of the figure will be equal to as many times two right angles as there are sides, less two.

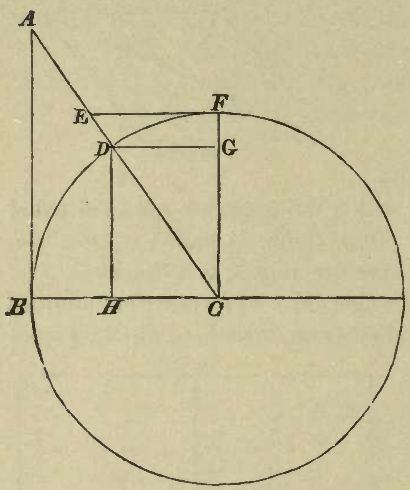


Fig. 54.

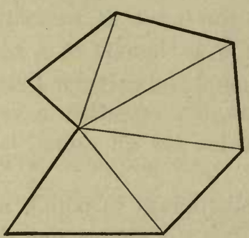


Fig. 55.

If another figure were made with similar triangles, similarly placed, the two figures would be similar.

Polygons, or many-sided figures, are similar when their angles are equal to each other and similarly placed, and their *homologous* sides, or sides including these angles, proportional.

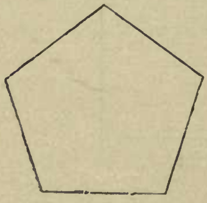


Fig. 56.

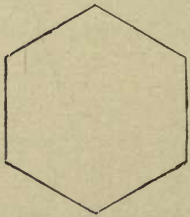


Fig. 57.

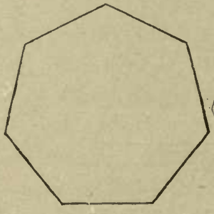


Fig. 58.

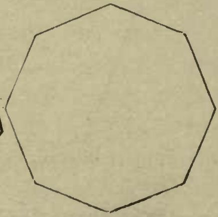


Fig. 59.

On this principle of similarity of figures the science of drawing is based. With a scale of equal parts, one inch on paper, for instance, representing a foot, a yard, or a mile, in nature, the figure drawn on that scale will represent the object accurately in reduced form; and measurements may be made in detail by the scale as well as from the natural object in the shop or on the estate.

Polygons, with their sides and angles equal, are called *regular* polygons (Figs. 56, 57, 58, 59).

Regular polygons are easily constructed by means of circles, whose circumferences are divided into the number of sides required, with chords drawn representing the sides. As the circle is then outside the polygon, the circle is said to be *described* about it, while the polygon is inscribed within the circle. If the polygon is described about the circle, its sides are tangent to it.

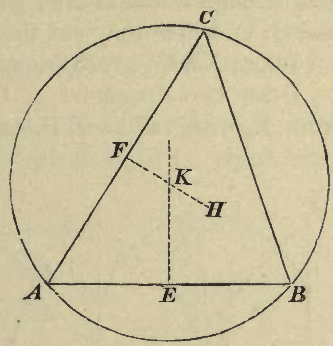


FIG. 60.

PROB. XXXI.—*To describe a circle about a triangle* (Fig. 60).

Bisect two of the sides A B, A C, of the triangle at E, F; from these points draw perpendiculars cutting at K. From the center K, with K A as radius, describe the circle A B C, as required.

PROB. XXXII.—*To inscribe a circle in a triangle* (Fig. 61).

Bisect two of the angles A, C, of the triangle A B C, by lines cutting at D; from D draw a perpendicular D E to any side, as A C; and with D E as radius, from the center D, describe the circle required.

When the triangle is equilateral, the center of the circle is more easily found by bisecting two of the sides, and drawing perpendiculars, as in the previous problem. Or, draw a perpendicular from one of the angles to the opposite side, and from the side set off one third of the length of the perpendicular.

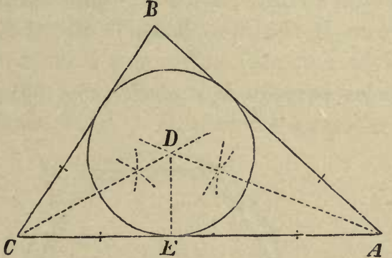


FIG. 61.

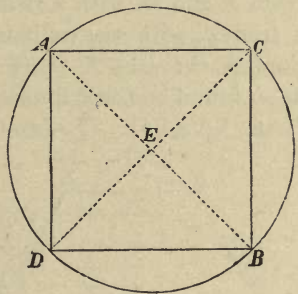


FIG. 62.

PROB. XXXIII.—*To inscribe a square in a circle; and to describe a circle about a square* (Fig. 62).

To inscribe the square. Draw two diameters, A B, C D, at right angles, and join the points A, B, C, D, to form the square as required.

To describe the circle. Draw the diagonals A B, C D, of the given square, cutting at E; on E as a center, with E A as radius, describe the circle as required.

In the same way, a circle may be described about a rectangle.

PROB. XXXIV.—*To inscribe a circle in a square; and to describe a square about a circle* (Fig. 63).

To inscribe the circle. Draw the diagonals A B, C D, of the given square, cutting at E; draw the perpendicular E F to one of the sides, and with the radius E F, on the center E, describe the circle.

To describe the square. Draw two diameters A B, C D, at right angles, and produce them; bisect the angle D E B at the center by the diameter F G, and through F and G draw perpendiculars A C, B D, and join the points A, D, and B, C, where they cut the diagonals, to complete the square.

PROB. XXXV.—*To inscribe a pentagon in a circle* (Fig. 64).

Draw two diameters, A C, B D, at right angles; bisect A O at E, and from E, with radius E B, cut A C at F; from B, with radius B F, cut the

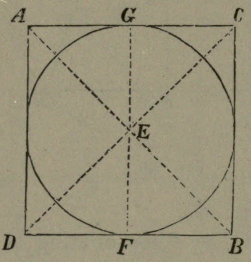


FIG. 63.

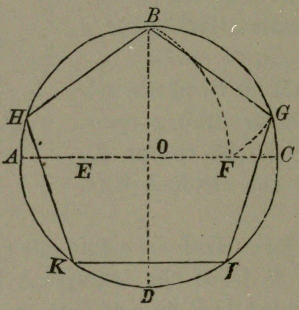


FIG. 64.

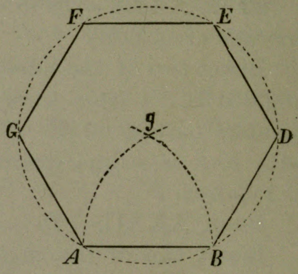


FIG. 65.

circumference at G and H, and with the same radius step round the circle to I and K; join the points so found to form the pentagon.

PROB. XXXVI.—*To construct a regular hexagon upon a given straight line* (Fig. 65).

From A and B, with a radius equal to the given line, describe arcs cutting at *g*; from *g*, with the radius *g* A, describe a circle; with the same radius set off from A the arcs A G, G F, and from B the arcs B D, D E. Join the points so found to form the hexagon.

PROB. XXXVII.—*To inscribe a regular hexagon in a circle* (Fig. 66).

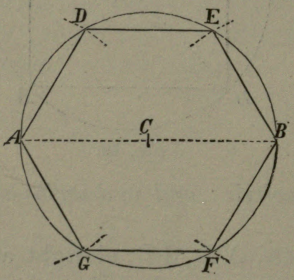


FIG. 66.

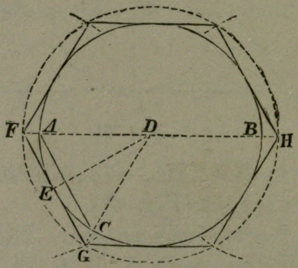


FIG. 67.

Draw a diameter, A B; from A and B as centers, with the radius of the circle A C, cut the circumference at D, E, F, G; draw straight lines A D, D E, etc., to form the hexagon.

The points of contact, D, E, etc., may also be found by setting off the radius six times upon the circumference.

PROB. XXXVIII.—*To describe a regular hexagon about a circle* (Fig. 67).

Draw a diameter, A B, of the given circle. With the radius A D from A as a center, cut the circumference at C; join A C, and bisect it with the radius D E; through E draw F G parallel to A C, cutting the diameter at F, and with the radius D F describe the circle F H. Within this circle describe a regular hexagon by the preceding problem; the figure will touch the given circle as required.

PROB. XXXIX.—*To construct a regular octagon upon a given straight line* (Fig. 68).

Produce the given line A B both ways, and draw perpendiculars A E, B F; bisect the external angles at A and B by the lines A H, B C, which make

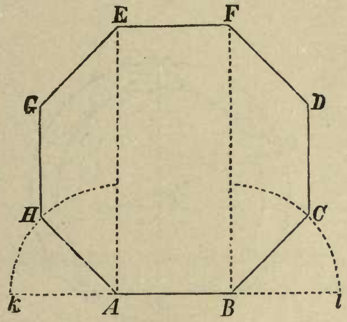


FIG. 68.

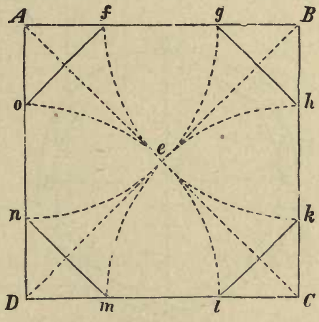


FIG. 69.

equal to A B; draw C D and H G parallel to A E and equal to A B; and from the centers G, D, with the radius A B, cut the perpendiculars at E, F, and draw E F to complete the octagon.

PROB. XL.—*To convert a square into a regular octagon* (Fig. 69).

Draw the diagonals of the square intersecting at *e*; from the corners A, B, C, D, with A *e* as radius, describe arcs cutting the sides at *g h*, etc.; join the points so found to complete the octagon.

PROB. XLI.—*To inscribe a regular octagon in a circle* (Fig. 70).

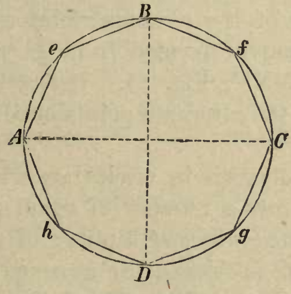


FIG. 70.

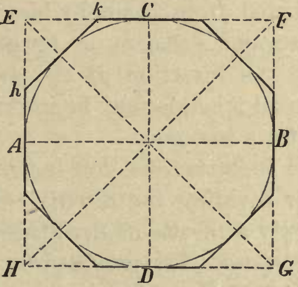


FIG. 71.

Draw two diameters, A C, B D, at right angles; bisect the arcs A B, B C, etc., at *e*, *f*, etc.; and join A *e*, *e* B, etc., for the inscribed figure.

Prob. XLII.—*To describe a regular octagon about a circle* (Fig. 71).

Describe a square about the given circle A B; draw perpendiculars *h k*, etc., to the diagonals, touching the circle. The octagon so formed is the figure required.

Or, to find the points *h*, *k*, etc., cut the sides from the corners of the square, as in Prob. XL.

Prob. XLIII.—*To inscribe a circle within a regular polygon.*

When the polygon has an even number of sides, as in Fig. 72, bisect two opposite sides at A and B, draw A B, and bisect it at C by a diagonal D E drawn between opposite angles; with the radius C A describe the circle as required.

When the number of sides is odd, as in Fig. 73, bisect two of the sides at A and B, and draw lines A E, B D, to the opposite angles, intersecting at C; from C, with C A as radius, describe the circle as required.

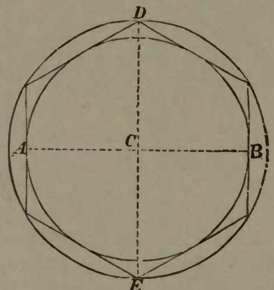


Fig. 72.

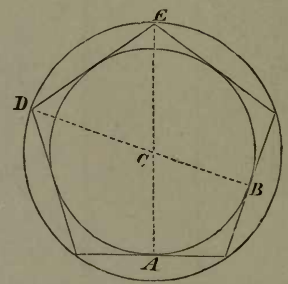


Fig. 73.

Prob. XLIV.—*To describe a circle without a regular polygon.*

When the number of sides is even, draw two diagonals from opposite angles, like D E (Fig. 72), to intersect at C; and from C, with C D as radius, describe the circle required.

When the number of sides is odd, find the center C (Fig. 73) as in last problem, and, with C D as radius, describe the circle.

The foregoing selection of problems on regular figures is the most useful in mechanical practice on that subject. Several other regular figures may be constructed from them by bisection of the arcs of the circumscribing circles. In this way a decagon, or ten-sided polygon, may be formed from the pentagon by the bisection of the arcs in Prob. XXXV, Fig. 64. Inversely, an equilateral triangle may be inscribed by joining the alternate points of division found for a hexagon.

The constructions for inscribing regular polygons in circles are suitable also for dividing the circumference of a circle into a number of equal parts. To supply a means of dividing the circumference into *any* number of parts, including cases not provided for in the foregoing problems, the annexed table of angles relating to polygons, expressed in degrees, will be found of general utility. In this table, the angle at the center is found by dividing 360°, the number of degrees in a circle, by the number of sides in the polygon, and by setting off round the center of the circle a succession of angles by means of the protractor, equal to the angle in the table due to a given number of sides: the radii so drawn will divide the circumference into the same number of parts. The triangles thus formed are termed the elementary triangles of the polygon.

TABLE OF POLYGONAL ANGLES.

Number of Sides of Regular Polygon; or number of equal parts of the circumference.	Angle at Center.	Number of Sides of Regular Polygon.	Angle at Center.
No.	Degrees.	No.	Degrees.
3	120	12	30
4	90	13	$27\frac{9}{13}$
5	72	14	$25\frac{5}{7}$
6	60	15	24
7	$51\frac{3}{7}$	16	$22\frac{1}{2}$
8	45	17	$21\frac{3}{17}$
9	40	18	20
10	36	19	$18\frac{18}{19}$
11	$32\frac{8}{11}$	20	18

CONSTRUCTION OF THE ELLIPSE, PARABOLA, HYPERBOLA, CYCLOID, EPICYCLOID, INVOLUTE, AND SPIRAL.

An *ellipse* is an oval-shaped curve (Fig. 74), in which, if from any point, P, lines be drawn to two fixed points, F and F′, *foci*, their sum will always be the same. The line A B passing through the foci is the *transverse* axis, and the perpendicular C D at the center of it is the *conjugate* axis.

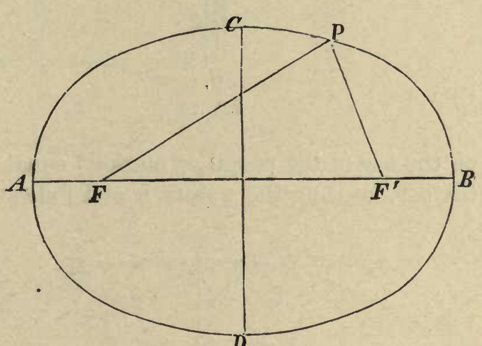


Fig. 74.

Prob. XLV. — *To construct an ellipse, the axes being known* (Fig. 75).

1st Method.—Let the two axes be the lines A B and C D. From C as a center, with a radius equal to E B (half the transverse axis), describe an arc cutting this axis at two points, F and F′, which are the foci. Insert a pin in each of the foci, and loop a thread upon them, so that, when stretched by a pencil inside the loop, the point of the pencil will coincide with C. Move the pencil round, keeping the loop evenly stretched, and it will describe an ellipse. This construction follows the definition above given of an ellipse, that the sum of the distances of every point of the curve from the foci is equal. It is seldom used by the draughtsman, as it is difficult to keep a thread evenly stretched; but for gardeners, laying out beds or plots, it is very convenient and sufficiently accurate.

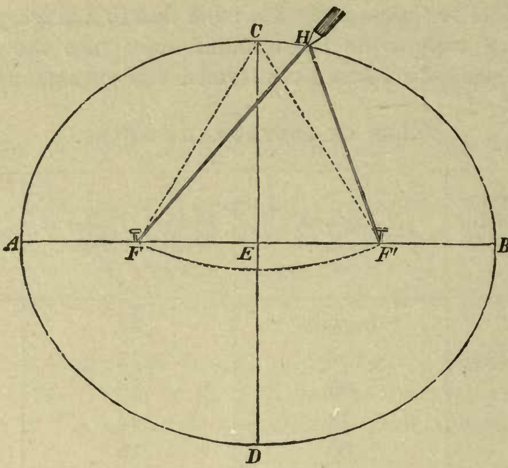


Fig. 75.

2d Method.—Carpenters, almost invariably, lay out an ellipse by means of a *trammel* (Fig. 76), which consists of a rectangular cross, E G F H, with guiding grooves, in which metal rods, attached to slides on a bar, are fitted so as to move easily and uniformly. In describing the ellipse, the trammel is placed with its grooves on the lines of the axes. Adjust the metal points, *k* and *l*, which slide in the grooves, so as to have between them a distance equal to half the conjugate axis, and make the distance from *k* to *m* (the position on the bar of the pencil or marker) equal to half the transverse axis. Revolve the bar, keeping the points *k* and *l* always in the grooves, and the pencil will describe an ellipse. Neat instruments of this sort are made for the use of the draughtsman, but, for offices where curves of this sort are required but little, a substitute for the trammel can be had in a strip of paper (Fig. 77), by marking the straight edge at *a* and *b* and *c*, the distance *c a* being made equal to half the transverse axis, and the distance *c b* to half the conjugate

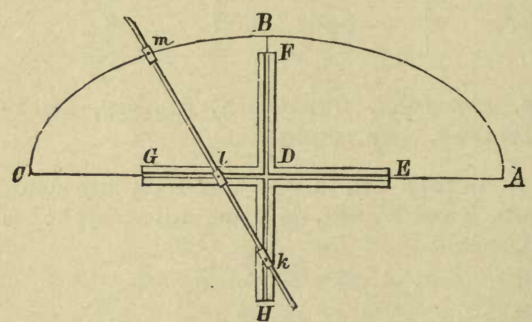


Fig. 76.

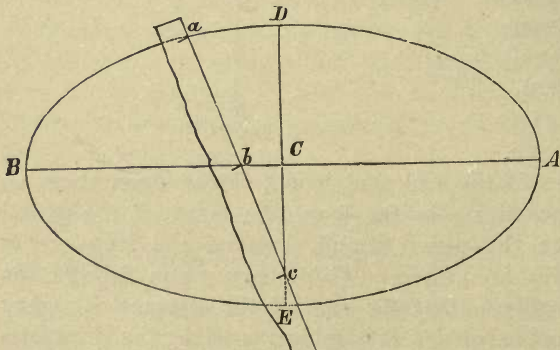


Fig. 77.

axis. Revolving the strip of paper, keeping *b* on the line of the transverse axis, and *c* on the line of the conjugate axis, and dotting the positions of *a* at short intervals, enough points of the curve will be determined through which the ellipse may be drawn readily.

Prob. XLVI. — *To describe an ellipse approximately, by means of circular arcs.*

First, with arcs of two radii (Fig. 78). Take the difference of the transverse and conjugate axes, and set it off from the center O to *a* and *c*, on O A and O C; draw *a c*, and set off half *a c* to *d;* draw *d i* parallel to *a c*, set off O *e* equal to O *d*, join *e i*, and draw *e m*, *d m*, parallels to *d i*, *i e*. On center *m*, with radius *m* C, describe an arc through C, and from center *i* describe an arc through D; on centers *d*, *e*, also, describe arcs through A and B. The four arcs thus described form approximately an ellipse. This method does not apply satisfactorily when the conjugate axis is less than two thirds of the transverse axis.

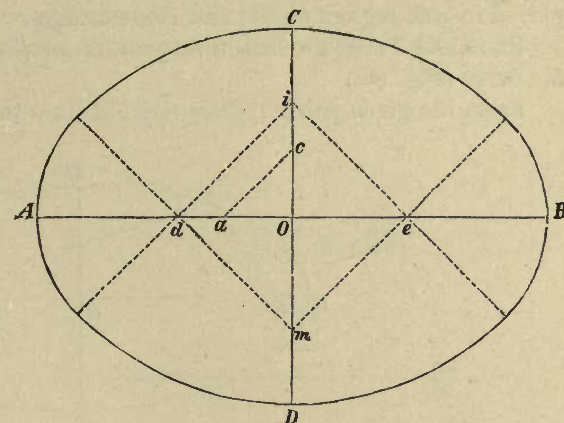


Fig. 78.

Second, with arcs of three radii (Fig. 79). On the transverse axis A B, draw the rectangle B G, equal in height to O C, half the conjugate axis.

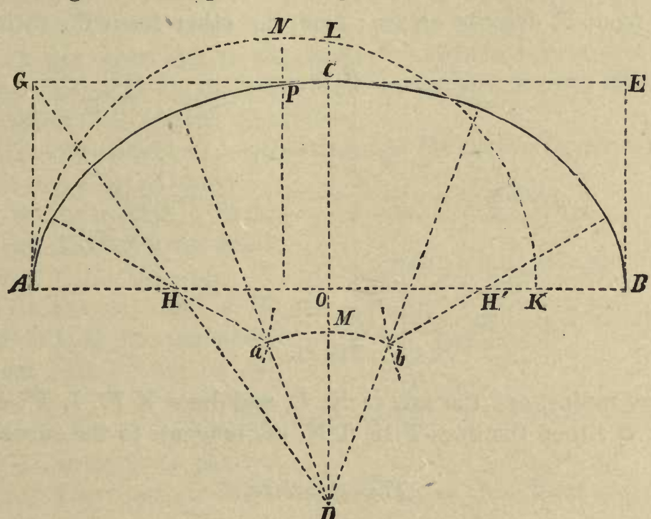


Fig. 79.

Extend O C above and below the rectangle. Draw G D perpendicular to A C, intersecting O C extended at D. Set off O K equal to O C, and on A K as a diameter describe the semicircle A N K; draw a radius parallel to O C, intersecting the semicircle at N and the line G E at P; set off O M equal to P N, and on D as a center, with a radius D M, describe an arc; from A and B as centers, with a radius O L, intersect this arc at *a* and *b*. The points H, *a*, D, *b*, H′, are the centers of the arcs required; produce the lines *a* H, D *a*. D *b*, *b* H′, and the spaces inclosed determine the lengths of each arc.

This process works well for nearly all proportions of ellipses. It is employed in striking out vaults and stone bridges.

PROB. XLVII.—*To draw a tangent to an ellipse through a given point in the curve* (Fig. 80).

From the given point T draw straight lines to the foci F, F′; produce F T

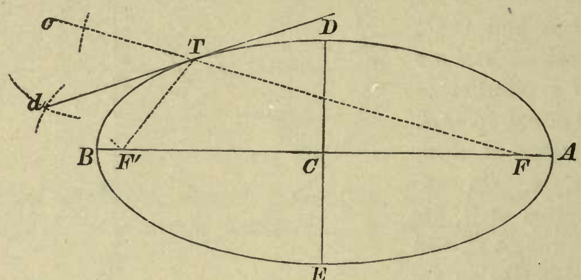


FIG. 80.

beyond the curve to *c*, and bisect the exterior angle *c* T F′ by the line T *d*. This line T *d* is the tangent required.

PROB. XLVIII.—*To draw a tangent to an ellipse from a given point without the curve* (Fig. 81).

From the given point T as center, with a radius equal to its distance from the nearest focus F, describe an arc; from the other focus F′, with the transverse axis as radius, cut the arc at K, L, and draw K F′, L F′, cutting the curve at M, N; then the lines T M, T N, are tangents to the curve.

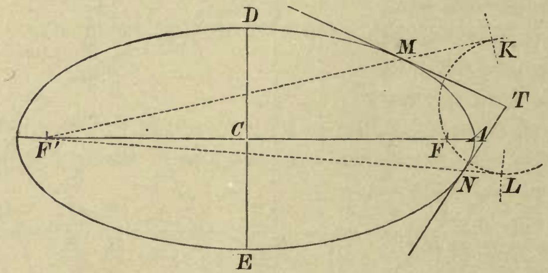


FIG. 81.

The Parabola.

The *parabola* may be defined as an ellipse whose transverse axis is infinite; its characteristic is that every point in the curve is equally distant from the *directrix* E N, and the *focus* F (Fig. 82).

PROB. XLIX.—*To construct a parabola when the focus and directrix are given.*

1st Method (Fig. 82).—Through the focus F draw the axis A B perpendicular to the directrix E N, and bisect A F at *e*, then *e* is the *vertex* of the curve. Through a series of points, C, D, E, on the directrix, draw parallels to A B; connect these points, C, D, E, with the focus F, and bisect by perpendiculars the lines F C, F D, F E. The intersections of these perpendiculars with the parallels will give points, C′, D′, E′, in the curve, through which trace the parabola.

2d Method (Fig. 83).—Place a straight-edge to the directrix E N, and apply to it a square L E G; fasten at G one end of a cord, equal in length

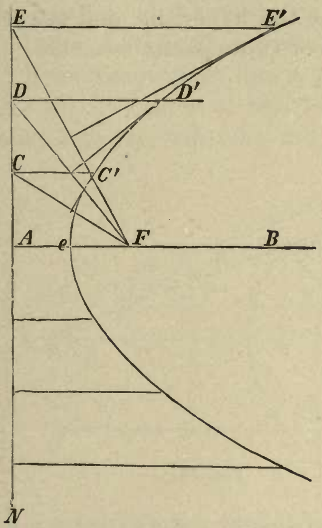


Fig. 82.

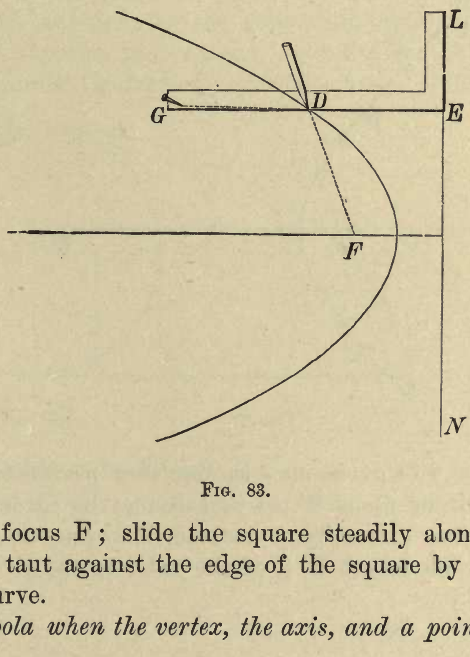


Fig. 83.

to E G; fix the other end to the focus F; slide the square steadily along the straight-edge, holding the cord taut against the edge of the square by a pencil, D, and it will describe the curve.

Prob. L.—*To construct a parabola when the vertex, the axis, and a point of the curve are given* (Fig. 84).

Let A be the vertex, A B be the axis, and M the given point of the curve.

Construct the rectangle A B-M C. Divide M C into any number of equal parts, four, for instance; divide A C in like manner; draw A 1, A 2, A 3; through 1′, 2′, and 3′, draw lines parallel to the axis. The intersections I, II, and III, of these lines are points in the required curve.

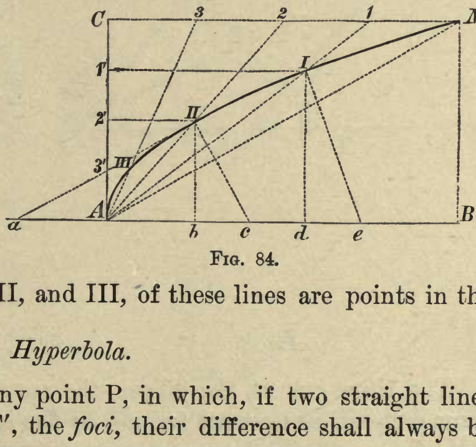


Fig. 84.

The Hyperbola.

An *hyperbola* is a curve from any point P, in which, if two straight lines be drawn to two fixed points, F, F′, the *foci*, their difference shall always be the same.

PROB. LI.—*To describe an hyperbola* (Fig. 85).

From one of the foci F, with an assumed radius, describe an arc, and from the other focus F′, with another radius exceeding the former by the given difference, describe two small arcs, cutting the first as at P and *p*. Let this operation be repeated with two new radii, taking care that the second shall exceed the first by the same difference as before, and two new points will be determined; and this determination of points in the curve may thus be continued till its track is obvious. By making use of the same radii, but transposing, that is, describing with the greater about F, and the less about F′, we have another series of points equally belonging to the hyperbola, and answering the definition; so that the hyperbola consists of two separate branches.

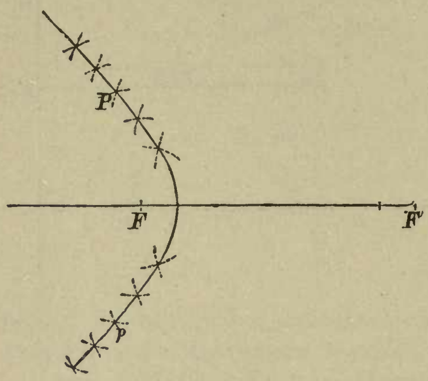


FIG. 85.

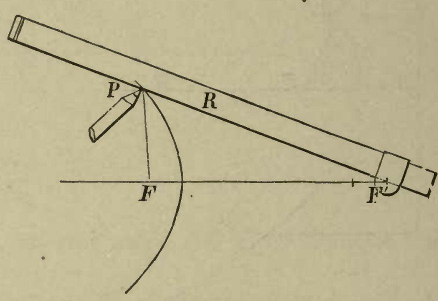


FIG. 86.

The curve may be described mechanically (Fig. 86).—By fixing a ruler to one focus F′, so that it may be turned round on this point, connect the other extremity of the ruler R to the other focus F by a cord shorter than the whole length F′ R of the ruler by the given difference; then a pencil P keeping this cord always stretched, and at the same time pressing against the edge of the ruler, will, as the ruler revolves around F′, describe an hyperbola, of which F F′ are the foci, and the differences of distances from these points to every point in the curve will be the same.

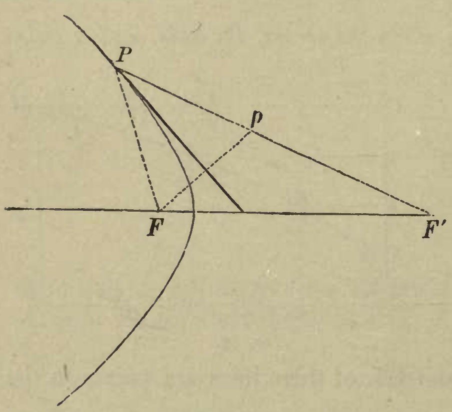


FIG. 87.

PROB. LII.—*To draw a tangent to any point of an hyperbola* (Fig. 87).

Let P be the point. On F′ P lay off P *p*, equal to F P; draw the line F *p*; from P let fall a perpendicular on this line, P *p*, and it will be the tangent required.

The three curves, the ellipse, the parabola, and the hyperbola, are called conic sections, as they are formed by the intersections of a plane with the surface of a cone. See CONSTRUCTION OF THE CONIC SECTIONS.

If the cone be cut through both its sides by a plane not parallel to the base, the section is an ellipse; if the intersecting plane be parallel to the side of the cone, the section is a parabola; if the plane have such a position that, when produced, it meets the opposite cone, the section is an hyperbola. The opposite cone is a reversed cone formed on the apex of the other by the continuation of its sides.

The Cycloid.

The *cycloid* is the curve described by a point in the circumference of a circle rolling on a straight line.

PROB. LIII.—*To describe a cycloid* (Fig. 88).

Draw the straight line A B as the base; describe the generating circle tangent at the center of this line, and through the center C draw the line E E parallel to the base; let fall a perpendicular from C upon the base; divide

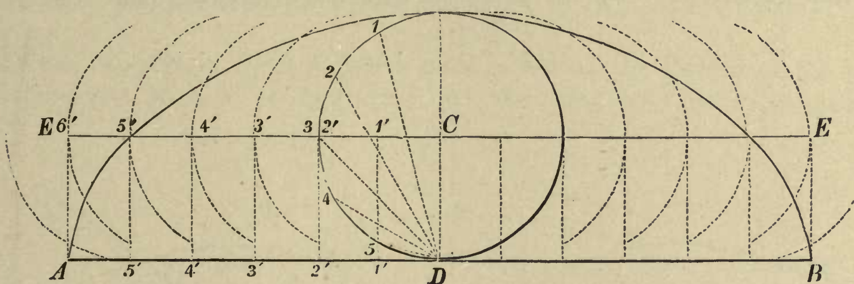


FIG. 88.

the semi-circumference into any number of equal parts, for instance, six; lay off on A B and C E distances C 1,′ 1′ 2′..., equal to the divisions of the circumference; draw the chords D 1, D 2... ; from the points 1′, 2′, 3′...on the line C E, with radii equal to the generating circle, describe arcs; from the points 1′, 2′, 3′, 4′, 5′, on the line B A, and with radii equal successively to the chords D 1, D 2, D 3, D 4, D 5, describe arcs cutting the preceding, and the intersections will be points of the curve required.

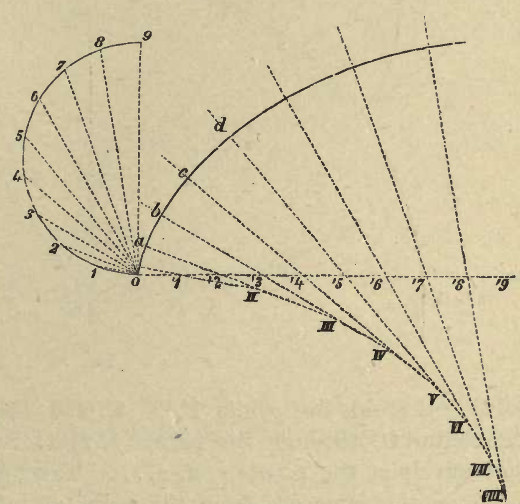


FIG. 89.

2d Method (Fig. 89).—Let 0 9′ be the base-line, 0 4 9 the half of the generating circle; divide the half circle into any number of equal parts, say nine, and draw the chord 0 1, 0 2, 0 3, etc.; lay off on the base 0 1′, 1′ 2′, 2′ 3′...., equal respectively to the length of one of the divisions of the half circle 0 1; draw through the points

1′, 2′, 3′....lines parallel to the chords 0 1, 0 2, 0 3....; the intersections I, II, III....of these lines are centers of the arcs 0 *a, a b, b c....,* of which the cycloid is composed.

The Epicycloid.

The *epicycloid* is formed by a point in the circumference of a circle revolving either externally or internally on the circumference of another circle as a base.

PROB. LIV.—*To describe an epicycloid.*

Let us in the first place take the exterior curve. Divide the circumference A B D (Fig. 90) into a series of equal parts 1, 2, 3...., beginning from the point A; set off in the same manner, upon the circle A M, A N, the divisions 1′, 2′, 3′....equal to the divisions of the circumference A B D. Then, as the circle A B D rolls upon the circle A M A N, the points 1, 2, 3 will coincide successively with the points 1′, 2′, 3′; and, drawing radii from the

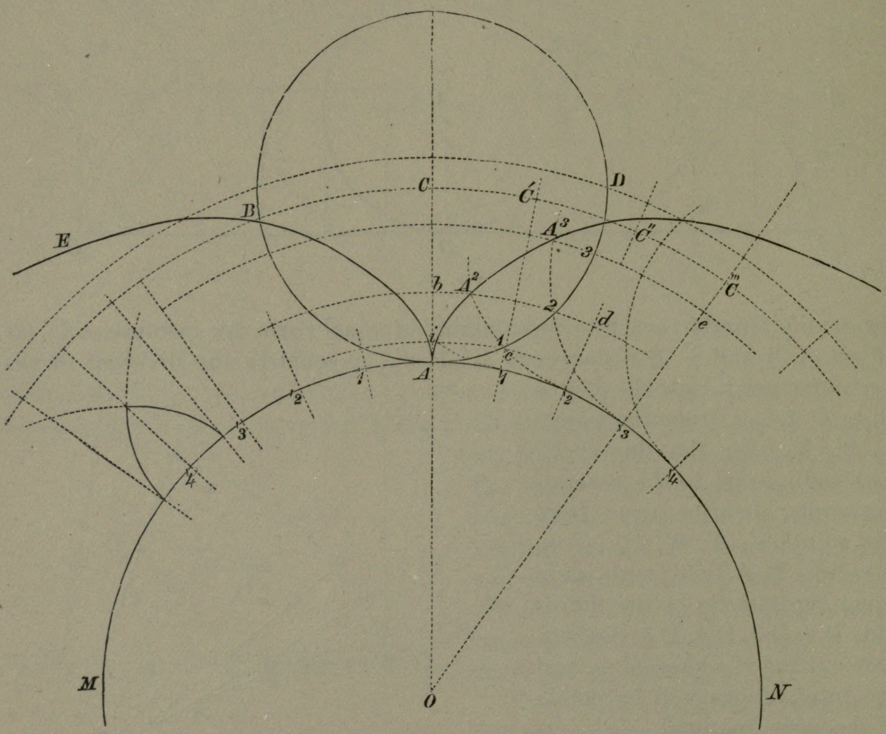


FIG. 90.

point O through the points 1′, 2′, 3′, and also describing arcs of circles from the center O, through the points 1, 2, 3...., they will intersect each other successively at the points *c, d, e....* Take now the distance 1 to *c,* and set it off on the same arc from the point of intersection *i,* of the radius A C; in like manner, set off the distance 2 to *d,* from *b* to A^2, and the distance 3 to *e,* to A^3, and so on. Then the points A^1, A^2, A^3, will be so many points in the epicycloid; and their frequency may be increased at pleasure by shortening the divisions of the circular arcs. Thus the form of the curve may be determined to any amount of accuracy, and completed by tracing a line through the points found.

As the distances 1 to c, ... which are near the commencement of the curve, must be very short, it may, in some instances, be more convenient to set off the whole distance i to 1 from c, and in the same way the distance b to 2 from d to A^2, and so on. In this manner the form of the curve is the more likely to be accurately defined.

2d Method.—To find the points in the curve, find the positions of the center of the rolling circle corresponding to the points of contact 1′, 2′, 3′, etc., which may be readily done by producing the radii from the center O, through the points 1′, 2′, 3′, ... to cut the circle B C. From these centers describe arcs of a circle with the radius of C A, cutting the corresponding arcs described from the center O, and passing through the points 1, 2, 3, ... as before. The intersections of these arcs at A^1, A^2, A^3, ... give points of the curve.

When the moving circle A B D is made to roll on the interior of the circumference A M, A N, as shown (Fig. 91), the curve described by the point

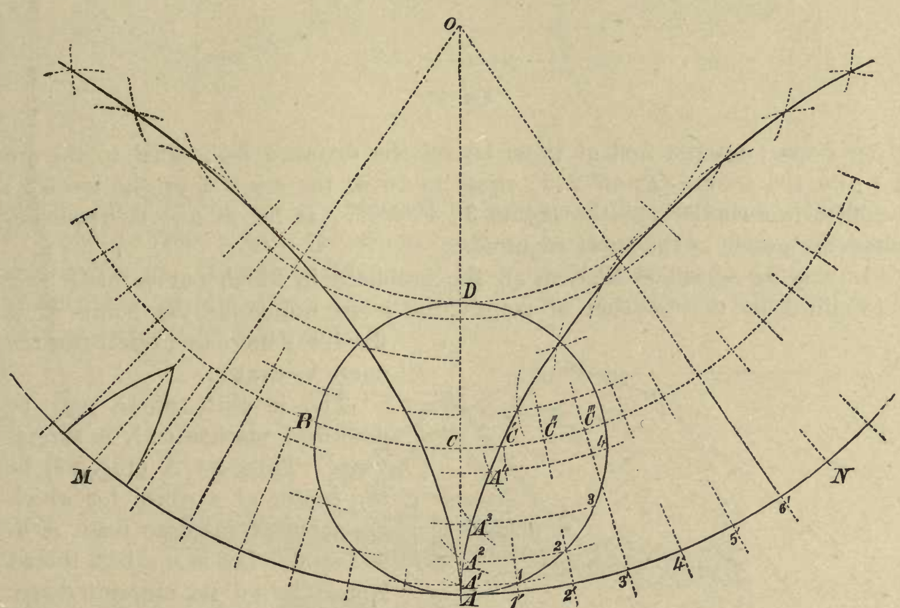


Fig. 91.

A is called an *interior epicycloid*. It may be constructed in the same way as in the preceding case, as may be easily understood, the same figures and letters of reference being used in both figures.

The Involute.

The *involute* is a curve traced by the extremity of a flexible line unwinding from the circumference of a circle.

PROB. LV.—*To describe an involute.*

Divide the circumference of the given circle (Fig. 92) into any number of equal parts, as 0, 1, 2, 3, 4, ... ; at each of these points draw tangents to the

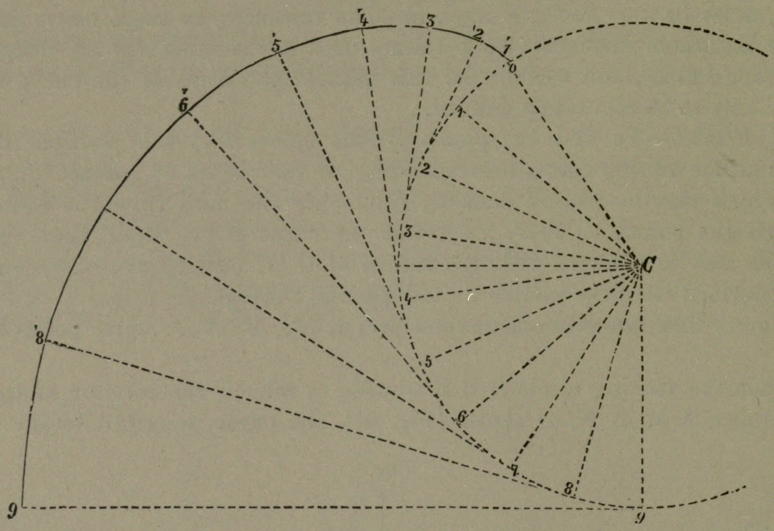


FIG. 92.

given circle; on the first of these lay off the distance 1 1′, equal to the arc 0 1; on the second lay off 2 2′, equal to twice the arc 0 1 or the arc 0 2: establish in a similar way the points 3′, 4′, 5′, ... as far as may be requisite, which are points in the curve required.

It may be remarked that, in all the problems in which curves have been determined by the position of points, the more numerous the points thus fixed, the more accurately can the curve be drawn.

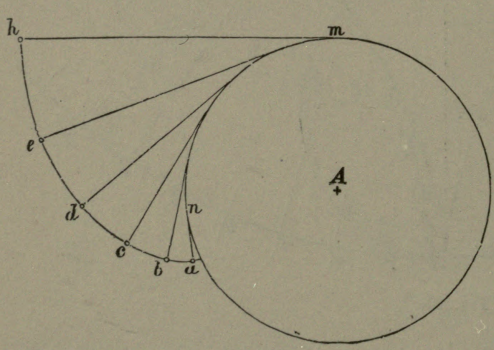


FIG. 93.

The involute curve may be described mechanically in several ways. Thus, let A (Fig. 93) be the center of a wheel for which the form of involute teeth is to be found. Let *m n a* be a thread lapped round its circumference, having a loop-hole at its extremity, *a;* in this fix a pin, with which describe the curve or *involute a b . . . . h,* by unwinding the thread gradually from the circumference, and this curve will be the proper form for the teeth of a wheel of the given diameter.

The Spiral.

The *spiral* is the involute of a circle produced beyond a single revolution.

PROB. LVI.—*To describe a spiral* (Fig. 94, and Fig. 95 of the primary on a larger scale).

Divide the circumference of the primary into any number of equal parts, say not less than eight. To these points of division *o, e, f, i*, etc., draw tangents, and from these points draw a succession of circular arcs; thus, from *o e* lay

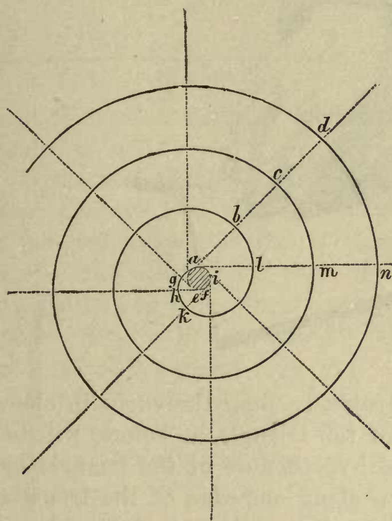


FIG. 94.

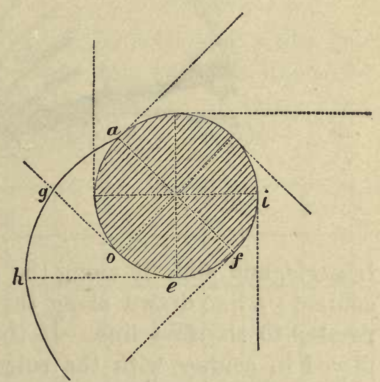
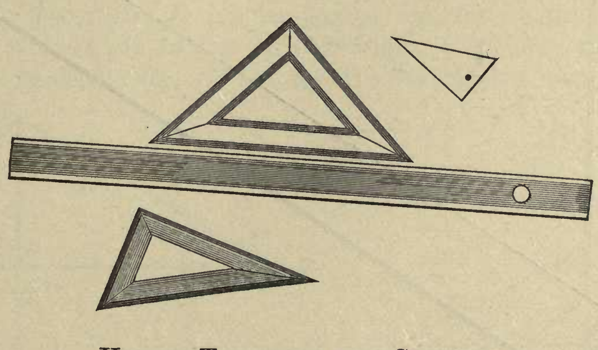


FIG. 95.

off *o g*, equal to the arc *a o* reduced to a straight line, and connect *a* and *g* by a curve; from *e*, with the radius *e g*, describe the arc *g h*; from *f* the next arc, and so on. Continue the use of the centers successively and repeatedly to the extent of the revolutions required. Thus the point *a* in the figure is used as a center for three arcs, *b l, c m, d n*.



USE OF TRIANGLE AND SQUARE.

Right-angled triangles constructed of wood, hard rubber, or metal, are very useful in connection with a straight-edge, or ruler, in drawing lines parallel or perpendicular to other lines.

To draw lines parallel to each other, place any edge of the triangle in close contact with the edge of the ruler. Hold the ruler (Fig. 96) firmly with the thumb and little finger of the left hand, and the triangle with the other three fingers; with a pencil or pen in the right hand, draw a line along one of the free edges of the triangle; withdraw the pressure of the three fingers upon the

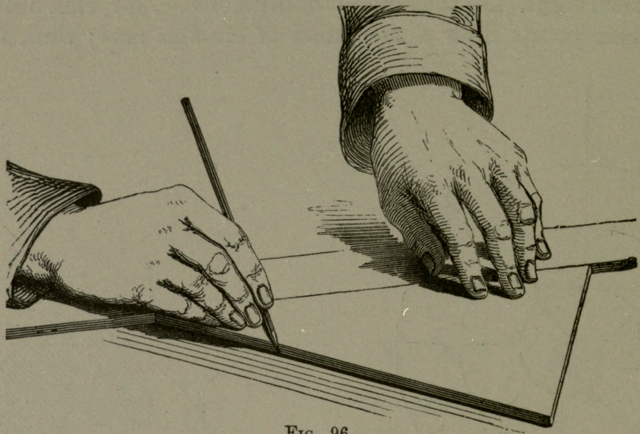


Fig. 96.

triangle, and slide it along the edge of the ruler, keeping the edges in close contact; a line drawn along the same edge of the triangle, as before, will be parallel to the first line. If the edge of the hypothenuse of the triangle be placed in contact with the ruler, lines drawn along one edge of the triangle will be at right angles to those drawn along the other.

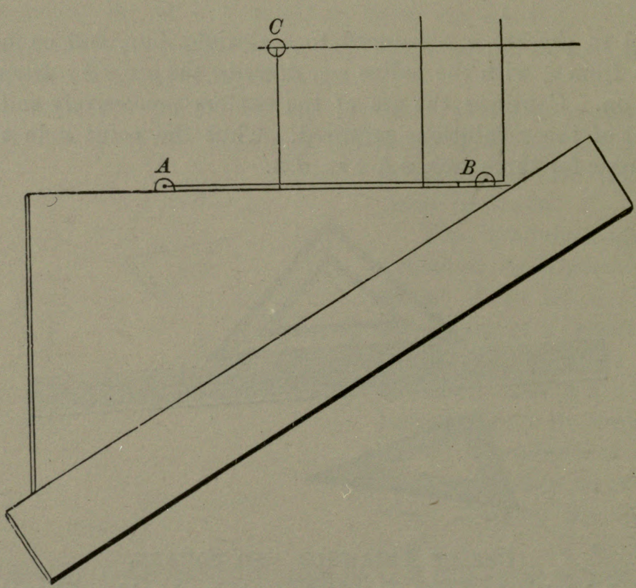


Fig. 97.

Prob. LVII.—*Through a given point to draw a line parallel to a given line* (Fig. 97).

Place one of the shorter edges of the triangle along the given line A B, and bring the ruler against the hypothenuse ; slide the triangle up along the edge of the ruler until the upper edge of the ruler is sufficiently near to the given

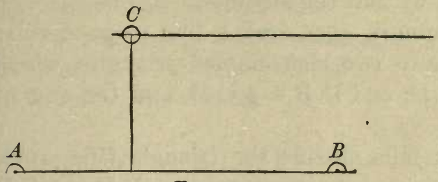


FIG. 98.

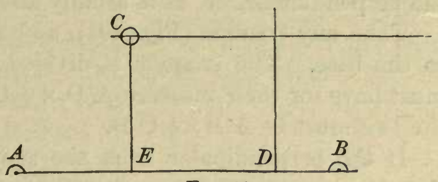


FIG. 99.

point C to allow a line to be drawn through it. Draw the line, and it will be parallel to A B.

If the triangle be slid farther up along the edge of the ruler, and a line be drawn through C along the other edge of the triangle (Fig. 98), this line will be perpendicular to A B. If the triangle be slid still farther up along the edge of the ruler, and a third line be drawn touching A B, the figure constructed will be a rectangle ; and if E D be laid off on A B, equal to C E, the figure inclosed is a square (Fig. 99).

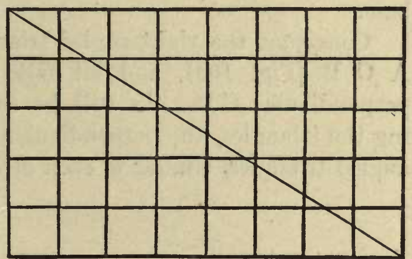


FIG. 100.

It will be seen that the triangle and ruler afford a much readier way of drawing parallel lines, and lines at right angles, than the compasses and ruler, and may be used in solving the following problems :

The area of a figure is the quantity of space inclosed by its lines.

Construct a right angle (Fig. 100). Divide the base and the perpendicular by dividers into any number of equal spaces ; for instance, eight on the one and five on the other. Construct a rectangle with this base and perpendicular, and through the points of division lay off lines parallel to the base and perpendicular. The rectangle will be divided into forty equal squares, and its measure in squares will be the divisions eight in the base, multiplied by the five in the perpendicular. If the division were inches, then the area of

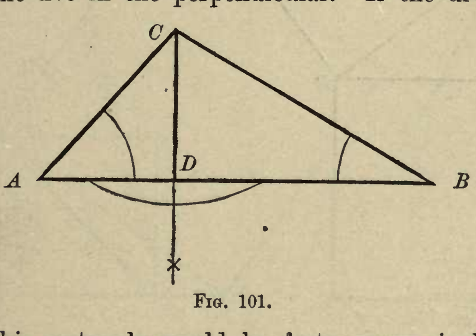


FIG. 101.

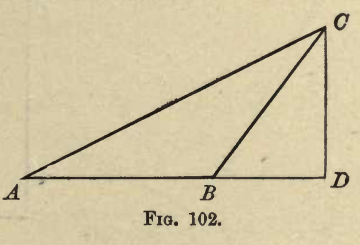


FIG. 102.

this rectangle would be forty square inches ; if feet, then forty square feet. If there were but five divisions in the base and five in the perpendicular, the surface would be twenty-five squares. Therefore, a rectangle has for its measure the base multiplied by its adjacent side or height.

Draw a diagonal, and the rectangle is divided into two equal triangles. Each triangle must therefore have for its measure the base multiplied by half the perpendicular, or, as is usually said, by half the altitude.

Take any triangle (Fig. 101), and from its apex draw a line perpendicular to the base. The triangle is divided into two right-angled triangles, which must have for their measure A D × ½ C D, and D B × ½ C D, and the sum of the two must be A B × ½ C D.

If the perpendicular from the apex falls outside the triangle (Fig. 102), then the triangles B D C and A D C will have for their measure B D × ½ C D and A D × ½ C D; and as the original triangle A B C is the difference of these two triangles, its measure must be A B × ½ C D. Every triangle must have for its measure the base multiplied by half the altitude, and it makes no difference which side is taken as the base.

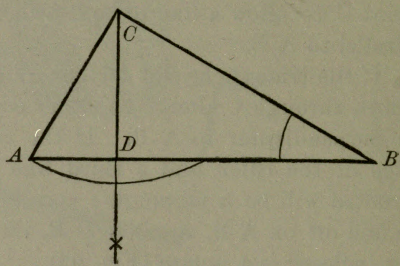


Fig. 103.

Construct the right-angled triangle A C B (Fig. 103), and let fall the perpendicular C D. As will be seen by the equality of the angles composing the triangles, the perpendicular divides the original triangle into two right-angled triangles, similar to each other and to the original triangle. Therefore

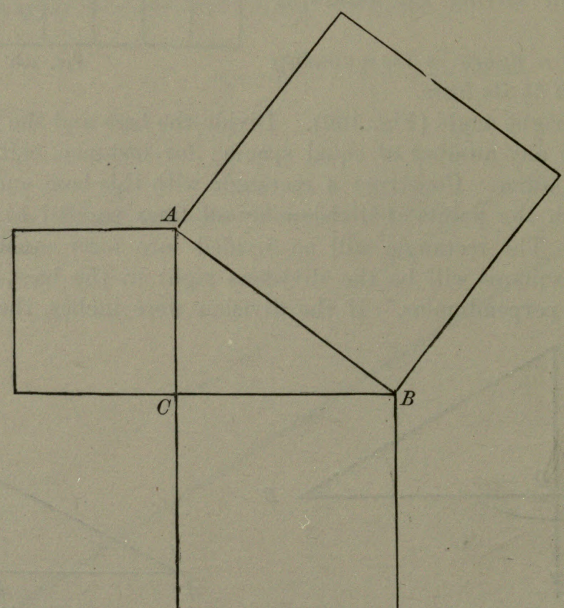


Fig. 104.

A D is to C D as C D is to B D, or, expressed by signs, A D : C D : : C D : B D; therefore, by the Rule of Three, A D × B D = C D^2; that is, C D is a mean proportional between A D and B D. So that the perpendicular let fall from the vertex of a right angle upon the hypothenuse of the triangle, is a mean proportional between the two parts of the hypothenuse into which it is divided by the perpendicular.

In comparing the two triangles with the original triangle, A C is a mean proportional between A D and A B, and B C is a mean proportional between B D and A B; that is, $A\,C^2 = A\,D \times A\,B$

$$B\,C^2 = B\,D \times A\,B$$

adding the two, $A\,C^2 + B\,C^2 = (A\,D + B\,D) \times A\,B$

and as $A\,D + B\,D = A\,B$, we have $A\,C^2 + B\,C^2 = A\,B^2$; that is, the square on the hypothenuse is equal to the sum of the squares on the other two sides.

Construct squares on the three sides of a right-angled triangle (Fig. 104).

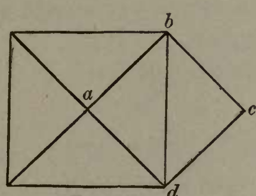


FIG. 105.

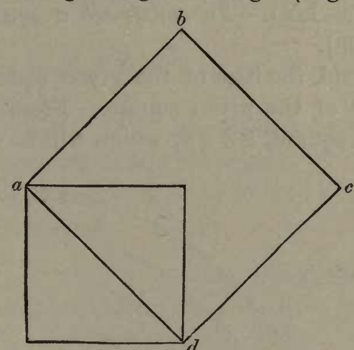


FIG. 106.

PROB. LVIII.—*To construct a square equal to one half of a given square* (Fig. 105).

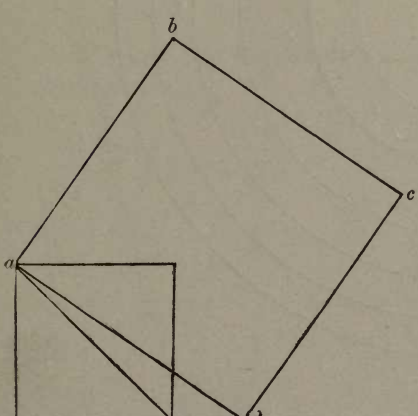


FIG. 107.

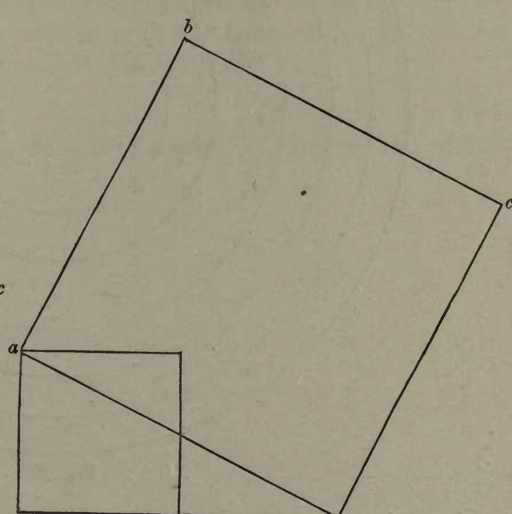


FIG. 108.

Construct the given square, and draw diagonals in it. The square, *a b c d*, constructed on one half of one of these diagonals will be equal to one half the given square.

PROB. LIX. — *To construct a square equal to double a given square* (Fig. 106).

Construct a square on one of the diagonals in the given square, or enclose the square with parallels to the diagonals of the square.

PROB. LX.—*To construct a square equal to three times a given square* (Fig. 107).

Extend the base of the given square, and lay off on it the length of its diagonal. Draw a line from the point at which this diagonal ends to the extreme angle of the square, and upon this line erect a square, which will be the square required.

For a square four times the size of a given square, make the base double that of the given square.

PROB. LXI.—*To construct a square equal to five times a given square* (Fig. 108).

Extend the base of the given square, making the extension to *d* equal to the base of the given square. From *d* draw a line to *a*, and on this line construct a square, *a b c d*, which will be the square required.

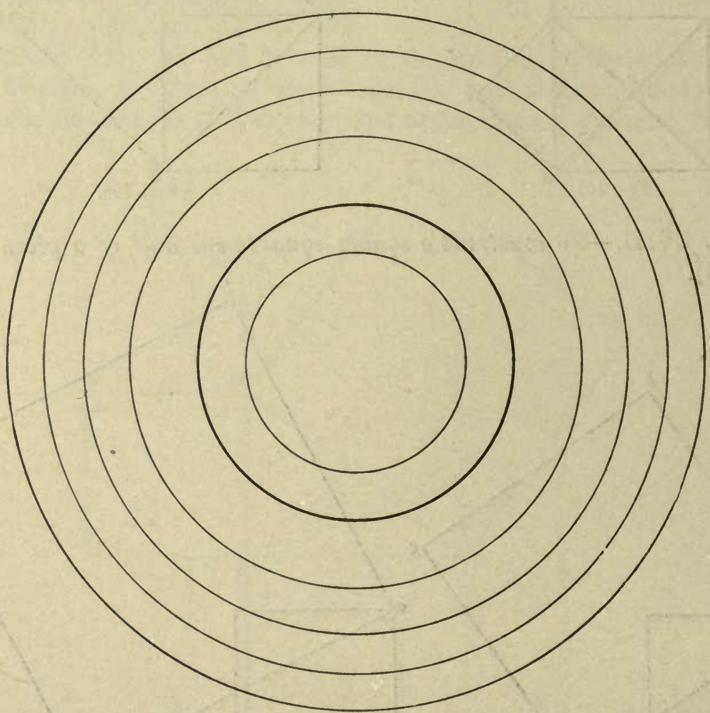


FIG. 109.

Assuming the side of the given square in Figs. 105, 106, 107, and 108 to be the radius (or diameter) (Fig. 109) of a given circle, then the side of the square to be constructed half, twice, three, four, or five times the size of the given square will be the radii (or diameters) of the circles half, twice, three, four, or five times the size of the given circle.

PROB. LXII. — *To determine how much is added to a given square by extending its base and constructing a square thereon* (Fig. 110).

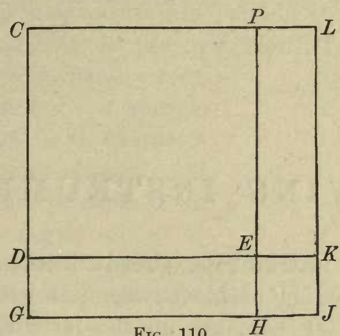


FIG. 110.

Let a represent the length C D of the base of the given square. Its square will be $a \times a$ or a^2.

Extend the base C D by a certain length, D G, represented by b. Then the new square $(a+b) \times (a+b)$ will be made up of the old square, or a^2
and two rectangles, D G E H and P E K L, or $2\ (a \times b)$ or $2\ a\ b$
and one square, E H K J, or $b \times b$ or b^2

$$(a+b)^2 = a^2 + 2\ a\ b + b^2$$

PROB. LXIII.—*To determine how much is taken from the area of a given square, by reducing its base and constructing a square* (Fig. 110).

Let a represent the length C G of the base of the given square. Reduce C G by a certain length, G D, to be represented by b.

Then the new square $(a-b)^2$ will be the old square, or a^2
diminished by two rectangles, D G J K and P L J H, or $-2\ a\ b$
excepting one square, E H J K, or $b \times b$ or $+b^2$

$$(a-b)^2 = a^2 - 2\ a\ b + b^2$$

The last two constructions, in default of a table of squares, may often be found of use.

DRAWING INSTRUMENTS.

THE simple drawing instruments, already illustrated and applied in the construction of the preceding problems, together with scales of equal parts, a protractor and a drawing pen, are all the instruments essential for topographical or mechanical drawing. It is often convenient, for facility in working, to have compasses of varied sizes and modifications, and these, together with an assortment of rulers, triangles, squares, scales, and protractors, adapted to varied work, are included in boxes of drawing instruments as furnished by dealers. The smaller rulers and triangles, as furnished, are generally of hard rubber, and the larger of wood. As it is often inconvenient to carry long rulers, and difficult to procure them ready-made, the draughtsman may have to depend on a carpenter or joiner for them.

Rulers should be of close-grained, thoroughly-seasoned wood, such as mahogany, maple, pear, etc. They should be about $\frac{1}{8}$ of an inch thick in the square or slightly rounded edges, 1 to $2\frac{1}{2}$ inches wide, according to their length. As the accuracy of a drawing depends greatly on the straightness of the lines, the edge of the ruler should be perfectly straight. To test this, place a sheet of paper on a perfectly smooth board; insert two very fine needles in an upright position through the paper into the board, distant from each other nearly the length of the ruler to be tested; bring the edge of the ruler against these needles, and draw a line from one needle to the other; reverse the ruler, bringing the same edge on the opposite side and against the needles, and again draw a line. If the two lines coincide, the edge is straight; but, if they disagree, the ruler is inaccurate, and must be re-jointed. When one ruler has been tested, the other can be examined by placing their edges against the correct one, and holding them between the eye and the light.

Triangles may be made of the same kinds of wood as the ruler, and somewhat thinner, and of various sizes. They should be right-angled, with acute angles of 45°, or of 60° and 30°. The most convenient size for general use measures from 3 to 6 inches on the side. A larger size, from 8 to 10 inches long on the side, is convenient for making drawings to a large scale. Circular openings are made in the body of the triangle for the insertion of the end of the finger to give facility in sliding the triangle on the paper. Triangles are sometimes made as large as 15 to 18 inches on the side; but in this case they are framed in three pieces of about $1\frac{1}{4}$ wide, leaving the center of the triangle open. The value of the triangle in drawing perpendicular lines depends on the accuracy of the right angle. To test this (Fig. 111), draw a line with an accurate ruler on paper. Place the right angle of the triangle near the center of this line, and make one of the adjacent sides to coincide with the line ; now draw a line along the other adjacent side, which, if the angle is strictly a right angle, will be perpendicular to the first line. Turn the triangle on this perpendicular side, bringing it into the position A B C′; if now the sides of the triangle agree with the line B C′ and A B, the angle is a right angle, and the sides straight. If they do not agree, they must be made to do so with a plane, if right angles are to be drawn by the triangle. The straightness of the hypothenuse or longest side can be tested like a common ruler.

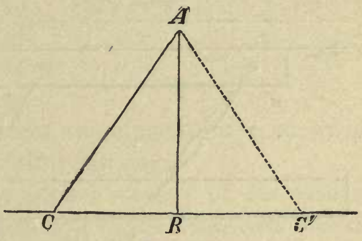


FIG. 111.

The T *square* is a thin "straight edge" or ruler, *a* (Fig. 112), fitted at one end with a stock, *b*, applied transversely at right angles. The stock being so formed as to fit and slide against one edge of the drawing-board, the blade reaches over the surface, and presents an edge of its own at right angles to

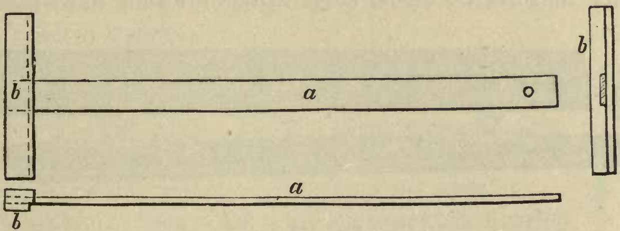


FIG. 112.

that of the board, by which parallel straight lines may be drawn upon the paper. The stock should be long enough to give sufficient bearing on the edge of the board, and heavy enough to act as a balance to the blade, and to relieve the operation of handling the square. The blade should be sunk flush into the upper half of the stock on the inside, and very exactly fitted. It should be inserted full breadth, as shown in the figure ; notching and dovetailing is a mistake, as it weakens the blade, and adds nothing to the security. The upper half of the stock should be about $\frac{1}{4}$ inch broader than the lower half, to rest firmly on the board and secure the blade lying flatly on the paper.

One half of the stock, *c* (Fig. 113), is in some cases made loose, to turn

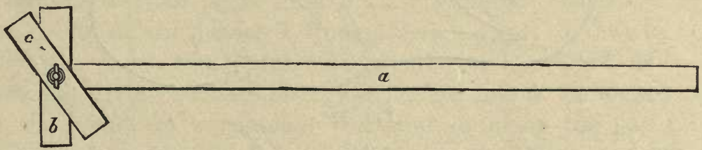


FIG. 113.

upon a brass swivel to any angle with the blade *a*, and to be clenched by a screwed nut and washer. The loose stock is useful for drawing parallel lines obliquely to the edges of the board, such as the threads of screws, oblique columns, and connecting-roads of steam-engines.

In many drawing-cases will be found the parallel ruler (Fig. 114), consisting of two rulers connected by two bars moving on pivots, and so adjusted that the rulers, as they open, form the sides of a parallelogram. The edge of one of the rulers being retained in a position coinciding with, or parallel to, a given line, the other ruler may be moved, and lines drawn along its edge must also be parallel to the given line. This instrument is only useful in drawing small parallels, and in accuracy and convenience does not compare with the triangle and ruler, or T square.

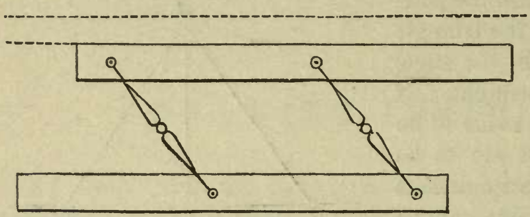


Fig. 114.

An improvement on the above parallel ruler has been patented by Lieutenant-Commander Sigsbee, U. S. N. (Fig. 115), in which the blades are made

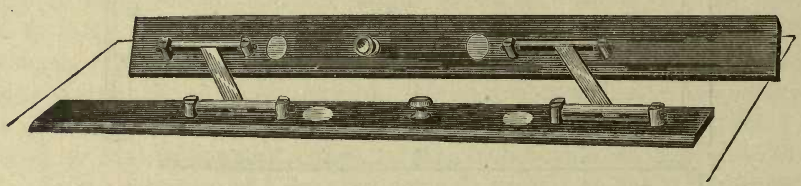


Fig. 115.

with hinges, by which, holding one blade on the paper, the other may be raised over creases or torn edges of the paper, or over thumb-tacks. One blade can be raised, if necessary, at right angles to the other, still preserving the parallelism of lines that may be drawn along these edges. Small cushions of rubber inserted in the blades, pressed by the fingers, prevent the slipping of the blades.

Fig. 116.

SWEEPS AND VARIABLE CURVES.

For drawing arcs of a large radius, beyond the range of ordinary compasses, and lines not circular but varying in curvature, thin slips of wood, termed sweeps (Figs. 116 and 117), are usually employed. These two forms are of very general application, but others of almost every form, and made of hard rubber, can be purchased. Whatever be the nature of the curve, some portion of the sweep will be found to coincide with its commencement, and it can be continued throughout its extent by applying, successively, such parts of the sweep as are suitable, care being taken that the parts are tangent to each other, and that the continuity is not injured by unskillful junction.

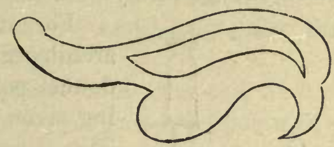


Fig. 117.

No varnish of any description should be applied to any of the wooden instruments used in drawing, as the best varnish will retain dust, and soil the paper. Use the wood in its natural state, keeping it carefully wiped. Various other materials besides wood have been used, as steel for the blades of the T square and the ruler; the objection is the liability to soil the paper. Glass is frequently used for the ruler and the triangle, and retains its correctness of edge and angle, but it is too heavy, and liable, of course, to fracture.

Thin splines are also to be had, which, held in position by leaden weights, serve admirably for a guide to the pen in describing curves (Fig. 118). For the same purpose a thin, hard rubber ruler, with soft rubber backing, answers well, and, as it can be readily rolled up, is extremely portable.

The weights above shown are very convenient in holding the drawing-paper on the board, but the *drawing-pins* (Fig. 119), steel points, or tacks, with large, flat heads, are in general use.

Elliptic and parabolic curves are furnished in sets, but the draughtsman can readily make a model out of thick card-board, with which he can draw a very uniform curve.

For the drawing of ellipses, very neat trammels or compasses, with elliptic guides or patterns, may be purchased.

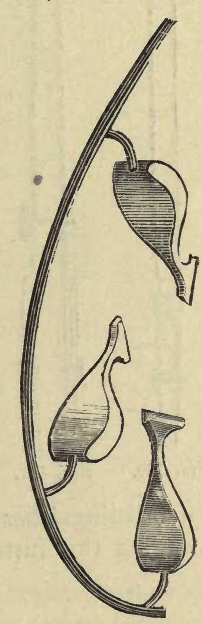


Fig. 118.

The *drawing-pen* (Fig. 120) is used for drawing straight lines. It consists of two blades with steel points fixed to a handle; and they are so bent that a sufficient cavity is left between them for the ink, when the ends of the steel points meet close together, or nearly so. The blades are set with the points more or less open by means of a mill-headed screw, so as to draw lines of any required fineness or thickness. One of the blades is framed with a joint, so that by taking out the screw the blades may be completely opened, and the points effectively cleaned after use. The ink is to be put between the blades by a common pen, and in using the pen it should be slightly inclined in the direction of the line to be drawn, and care should be taken that both points touch the paper; and these observations equally apply to the pen-points of the compasses before described. The drawing-pen should be kept close to the ruler or straight edge, and in the same direction during

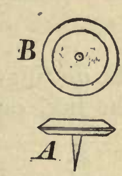


Fig. 119.

the whole operation of drawing the line. Care must be taken in holding the straight edge firmly with the left hand, that it does not change its position.

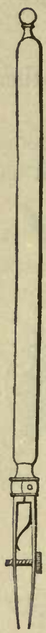


Fig. 120.

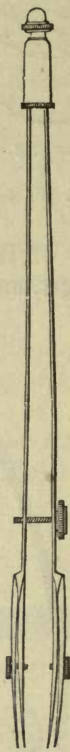


Fig. 121.

For drawing close parallel lines in mechanical and architectural drawings, or to represent canals or roads, a double pen (Fig. 121) is frequently used, with an adjusting screw to set the pens to any required small distance. This is usually called the road-pen.

Border-pens, for drawing broad lines, are double pens with an intermediate blade, and are applicable to the drawing of map-borders. The same work may be done by drawing the outer lines with the common drawing-pen, and filling in with a goose-quill, cut as shown in Fig. 122. In drawing with this pen, incline the drawing-board so that the ink will follow the pen.

The *curve-pen* (Fig. 123) is especially designed for the ready drawing of curved lines.

The *dotting-point* (Fig. 124) resembles a drawing-pen, except that the points are not so sharp. On the back blade, as seen in the engraving, is a pivot, on which may be placed a dotting-wheel, *a*, resembling the rowel of a spur; the screw *b* is for opening the blades to remove the wheel for cleaning after use, or replacing it with one of another character of dot. The cap *c*, at the upper end of the instrument, is a box containing a variety of dotting-wheels, each producing a different-shaped dot. These are used as distinguishing marks for different classes of boundaries on maps; for instance, one kind of dot distinguishes *county* boundaries, another kind *town* boundaries, a third kind distinguishes that which is both a county and a town boundary, etc., etc. In using this instrument, the ink must be inserted between the blades above

Fig. 122.

the dotting-wheel, so that, as the wheel revolves, the points shall pass through the ink, each carrying with it a drop, and marking the paper as it passes.

Fig. 123.

It sometimes happens that the wheel will revolve many times before it begins to deposit its ink on the drawing, thereby leaving the first part of the line altogether blank, and, in attempting to go over it again, the first-made dots are liable to get blotted. This evil may be mostly remedied by placing a piece of blank paper over the drawing to the very point the dotted line is to commence at, then begin with drawing the wheel over the blank paper first, so that, by the time it will have arrived at the proper point of commencement, the ink may be expected to flow over the points of the wheel, and make the dotted line perfect as required.

The best *pricking-point* (Fig. 125) is a fine needle held in a pair of forceps, and is used to transfer drawings by pricking through at the points of a drawing into the paper placed beneath. When drawings are transferred by

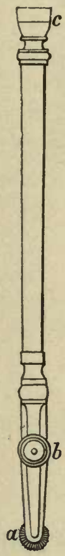


FIG. 124.

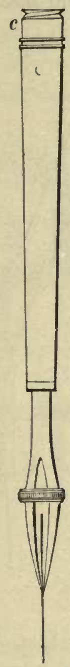


FIG. 125.

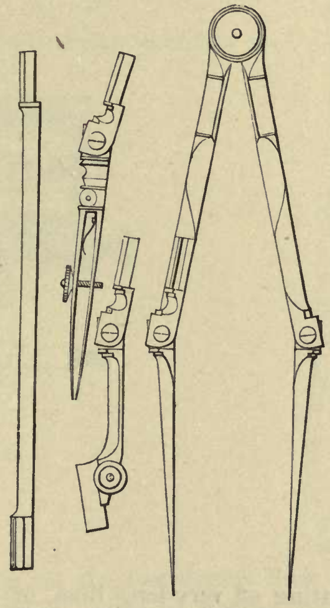


FIG. 126.

tracing—a prepared black sheet being placed between the drawing and the paper to receive the tracing—the eye-end of the needle forms a good tracing-point.

Compasses, in addition to pencil-points, as before shown, are fitted with movable ink-points and lengthening bars, so that larger circles may be drawn. Compasses should have joints in the legs, so that the points, pencil, and pen may be set perpendicular to the planes in which the circles are described (Fig. 126). Compasses of this general form may be had in sizes of 3½ to 7 inches.

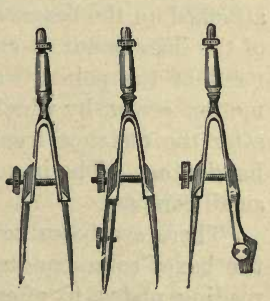


FIG. 127.

For the measurement and laying off of small spaces, and the describing of small circles, there are small *bow-compasses* (Fig. 127). These are sometimes made with jointed legs.

For the measurement or laying off of distances the plain dividers are convenient, but for ready and close adjustment the *hair dividers* (Fig. 128) are most suitable. The only difference is that, in the hair dividers, one of the points is attached to the body by a spring, and by means of the screw *b* it can be moved toward or from the fixed point a very small amount more accurately than by closing or opening the dividers. In dividing a line into equal parts especially, it enables one to divide the excess or deficit readily.

Large screw dividers (Fig. 129) are used for the same purpose, but they belong rather to the mechanic than to the draughtsman.

For convenience of carrying in the pocket, there are *portable* or turn-in *compasses* (Fig. 130).

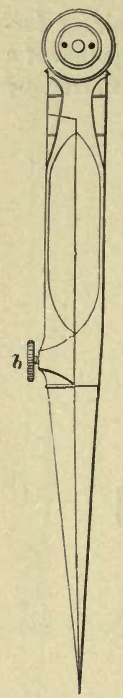


Fig. 128.

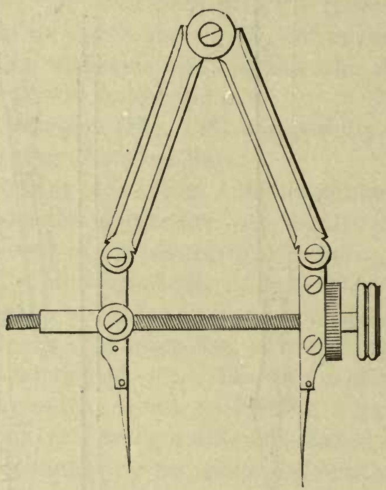


Fig. 129.

For setting off very long lines, or describing circles of large radius, *beam compasses* are used (Fig. 131). These consist of a mere slip of wood, A

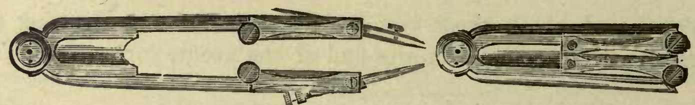


Fig. 130.

which is readily procured; two brass boxes, B and C, which can easily be attached to the beam, and connected with the brass boxes are the two points of the instrument, G and H. The object of this instrument is the nice adjustment of the points G and H at any definite distance apart; at F is a slow-motion screw, by which the joint G may be moved any very minute quantity after the distance from F to G has been adjusted as nicely as possible by the hand alone. The important parts of this instrument can be carried in a very small compass.

There are beam compasses in which the beam is graduated, and in which the boxes corresponding to B and C, in Fig. 131, are fitted with vernier or reading plates, to afford the means of minutely subdividing the divisions on the beam.

Proportional dividers (Fig. 132), for copying and reducing drawings, are found in most cases of instruments.

Closing the dividers and loosening the screw C, the slide may be moved up in the groove until the mark on the slide or index corresponds with the required number; then clamping the screw, the space inclosed between the long points, A B, will be as many times that between the short points, E D, as is shown by the number opposite the index. If the lines are to be reduced, the distances are measured with the long points, and set off by the short ones; if the lines are to be enlarged, then *vice versa*.

It often happens that the length of the points becomes reduced by use or accident, and the division on the instrument then becomes useless, but the purpose may be served by trial on paper, moving the slide up or down until a measured line is reduced or enlarged, as required.

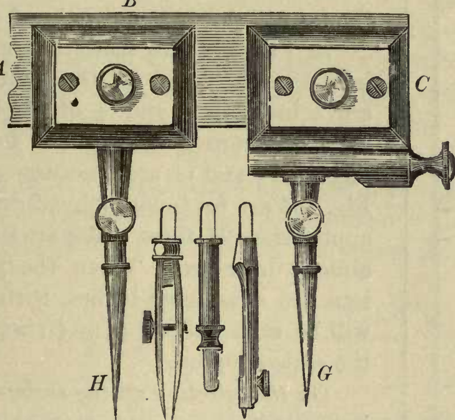


Fig. 131.

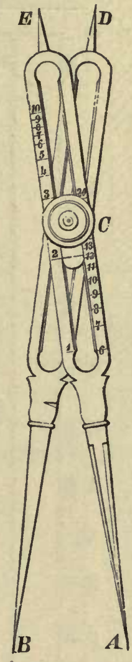


Fig. 132.

SCALES.

Practically, a two-foot rule, with its division into inches, half inch, quarter inch, eighth inch, and sixteenth inch, may be made use of as a scale of equal parts, the inch or any of its parts being taken as the unit to represent a foot, a yard, or a mile; but among drawing instruments, scales especially adapted to the purpose are found in great varieties of form, division, and material.

Fig. 133 represents the usual scale to be found in the common boxes of drawing instruments. It contains, on its two sides, simply divided scales—a *diagonal scale* on the reverse side and a *protractor* along the edges. The simply divided scales consist of a series of equal divisions of an inch, which are numbered 1, 2, 3, etc., beginning from the second division on the left hand; the upper part of the left division in each is subdivided into 12 equal parts, and the lower part into 10 equal parts. In Fig. 134 the scales are marked 30, 35, 40, etc., and the subdivisions of tenths can be considered as units, one mile, or one chain, or one foot, then each primary division will represent ten units, ten miles, ten chains, or ten feet, and the scale is said to be 30, 35, 40 (according to the scale selected) miles, chains, or feet to the inch. Thus, suppose that it were required, on a scale of 30 feet to the inch, to lay off 47 feet. On the scale marked 30, place one point of the compasses or dividers at 4, and bring the other point to the seventh lower subdivisions, counting from the right, and we have the distance required. Each of the primary divisions may be regarded as unit, one foot for instance; then the upper subdivisions are twelfths of a foot or inches, and the lower subdivisions tenths of an inch.

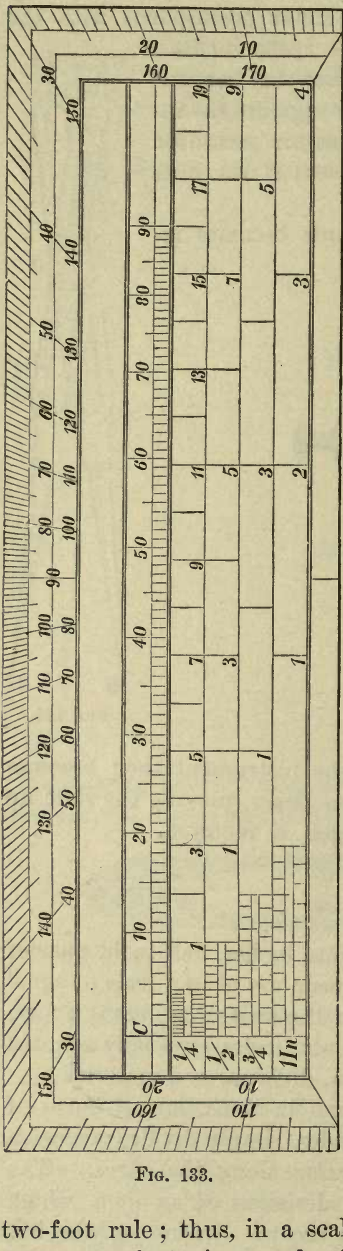


Fig. 133.

In Fig. 133 the scales are marked at the left, 1 inch, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$; the primary divisions are 1 inch, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ of an inch. These scales are more generally used for drawings of machinery and of architecture, while those of Fig. 134 are for topographical drawings. The applications of these scales are similar to those already described. When the primary divisions are considered inches, then the drawings will be each full, $\frac{3}{4}$, $\frac{1}{2}$, or $\frac{1}{4}$ size, according to the scale adopted.

On the selection of the scale.—In all working architectural and mechanical drawings, use as large a scale as possible; neither depend, even in that case, that the mechanics employed in the construction will measure correctly, but *write in* the dimensions as far as practicable. For architectural plans, the scale of $\frac{1}{4}$ an inch to the foot is one of very general use, and convenient for the mechanic, as the common two-foot rule carried by all mechanics is subdivided into $\frac{1}{4}$ths, $\frac{1}{8}$ths, and sometimes sixteenths of an inch, and the distances on a drawing to this scale can therefore be easily measured by them. This fact should not be lost sight of in working drawings. When the dimensions are not written, make use of such scales that the distances may be measured by the subdivisions of the common two-foot rule; thus, in a scale of $\frac{1}{2}$ or $\frac{1}{4}$ full size, 6 inches or 3 inches represent one foot; in a scale of an inch to the foot or twelfth full size, each $\frac{1}{2}$ an inch represents 6 inches, $\frac{1}{4}$ of an inch, 3 inches; but when $\frac{1}{8}$ or $\frac{1}{10}$ an inch to the foot, or any similar scale, is adopted, it is evident that these divisions can not be taken by the two-foot rule. The scale should be written on every drawing, or the scale itself should be drawn on the margin. In topographical and geodesic drawings the latter is essential, as the scale adopted frequently has to be drawn for the specific purpose, and the paper

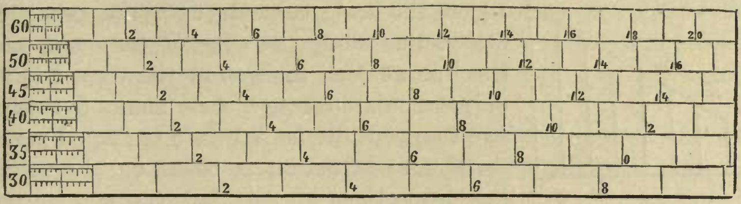


Fig. 134.

itself contracts or expands with every atmospheric change, and the measurements will therefore not agree at all times with a detached scale; and, moreover, a drawing laid down from such a detached scale, of wood or ivory, will not be uniform throughout, for on a damp day the measurements will be too short, and on a dry day too long. Mr. Holtzapffel has sought to remedy this inconvenience by the introduction of *paper* scales; but all kinds of paper do not contract and expand equally, and the error is therefore only partially corrected by his ingenious substitution of one material for another.

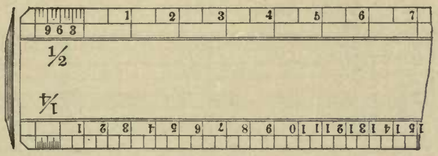


Fig. 135.

Plotting scales (Fig. 135) are scales of equal parts, with the divisions on a fiducial edge, by which any length may be marked off on the paper without using dividers. There are also small *offset* scales, for use of which see "Topographical Drawing."

Sometimes these scales are made with edges chamfered on both sides, and graduated to four different scales. Sometimes the section of the scale is triangular (Fig. 136), with six scales on the different edges. Both of these scales are convenient as portable instruments. To avoid the objection that having

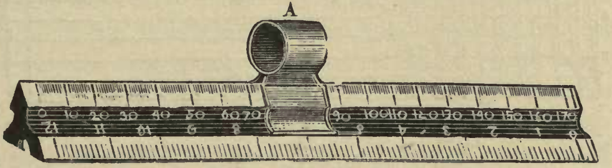


Fig. 136.

many scales on one ruler leads the draughtsman into error by the confusion of the scales, the triangular has a small slip of metal, A, readily put on, which covers partially the scales not in use.

To divide a given line into any number of equal parts (Fig. 137).

Let A B be the line, and the number of parts be ten. Draw a perpendicular at one extremity, A, of the line; with a plotting scale place the zero at the other extremity, B, of the line; make the mark 10 on the scale coincide with the perpendicular; draw a line along the edge of the scale, and mark the line at each division of the scale 1 to 9; draw perpendiculars through these marks to the line A B, and they will divide A B into ten equal parts.

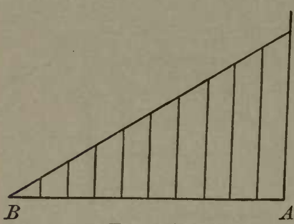


Fig. 137.

The construction is based on the principle of the proportions of parts between similar triangles, and it is evident that if the perpendicular at 1 be taken as a unit, that at 2 will be two units, and so on. This way of dividing a line will often be found convenient in practice. The lines may be at any angle to each other, and the lines connecting the divisions must be parallel to the line completing the triangle. The above figure illustrates the construction of *diagonal scales.* The simply divided scales give only two denominations, primaries and tenths, or twelfths; but more minute subdivision is attained by the diagonal scale, which consists of a number of primary divisions, one of which is divided into tenths, and subdivided into hundredths by diagonal lines (Fig. 138). This scale is constructed in the following manner: Eleven

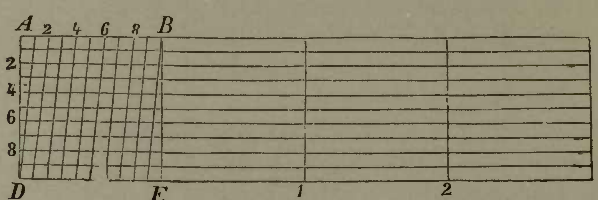


Fig. 138.

parallel lines are ruled, inclosing ten equal spaces; the length is set off into equal primary divisions, as D E, E 1, etc.; the first D E is subdivided, and diagonals are then drawn from the subdivisions between A and B, to those

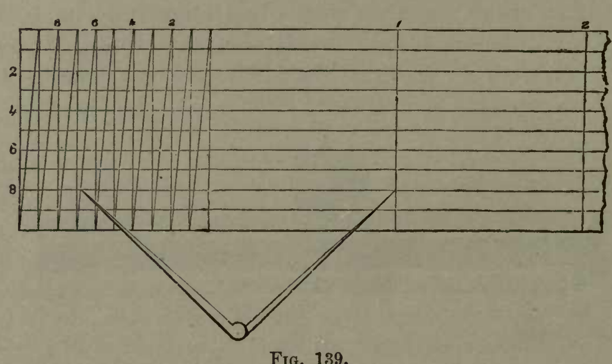


Fig. 139.

between D and E, as shown in the diagram. Hence it is evident that at every parallel we get an additional tenth of the subdivisions, or a hundredth of the primaries, and can therefore obtain a measurement with great exactness to three places of figures. To take a measurement of (say) 168, we place one foot of the dividers on the primary 1, and carry it down to the ninth parallel, and then extend the other foot to the intersection of the diagonal, which falls from the subdivision 6, with the parallel that measures the eight-hundredth part (Fig. 139). The primaries may, of course, be considered as yards, feet, or inches; and the subdivisions as tenths and hundredths of these respective denominations.

The diagonals may be applied to a scale where only one subdivision is required. Thus, if seven lines be (Fig. 140) ruled, inclosing six equal spaces,

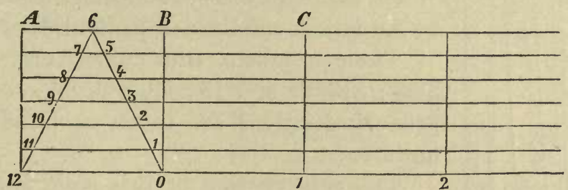


Fig. 140.

and the length be divided into primaries, as A B, B C, etc., the first primary, A B, may be subdivided into twelfths by two diagonals running from 6, the middle of A B, to 12 and 0. We have here a very convenient scale of feet and inches. From C to 6 is 1 foot 6 inches; and from C on the several parallels to the various intersections of the diagonals we obtain 1 foot and any number of inches from 1 to 12.

Vernier scales are preferred by some to the diagonal scale already described. To construct a vernier scale (Fig. 141) by which a number to three places may be taken, divide all the primary divisions into tenths, and number

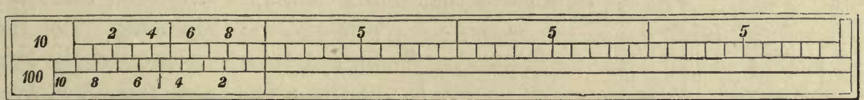


Fig. 141.

these subdivisions 1, 2, 3, from left to right. Take off now with the compasses eleven of these subdivisions, set the extent off backward from the end of the first primary division, and it will reach beyond the beginning of this division, or zero point, a distance equal to one of the subdivisions. Now divide the extent thus set off into ten equal parts, marking the divisions on the opposite side of the divided line to the lines marking the primary divisions and the subdivisions, and number them 1, 2, 3, etc., backward from right to left. Then, since the extent of eleven subdivisions has been divided into ten equal parts, so that these ten parts exceed by one subdivision the extent of ten subdivisions, each one of these equal parts, or, as it may be called, one division of the vernier scale, exceeds one of the subdivisions by a tenth part of a subdivision, or a hundredth part of a primary division; thus, if the subdivision be considered 10, then from 0 to the first division of the vernier will be 11; to the second, 22; to the third, 33; to the fourth, 44; to the fifth, 55, and so on, 66, 77, 88, 99.

To take off the number 253 from this scale, place one point of the dividers at the third division of the vernier; if the other point be brought to the primary division 2, the distance embraced by the dividers will be 233, and the dividers must be extended to the second subdivision of tenths to the right of 2. If the number were 213, then the dividers would have to be closed to the second subdivision of tenths to the left of 2. To take off the number 59 from the scale, place one point of the dividers at the ninth division of the vernier; if the other point be extended to the 0 mark, the dividers will embrace 99, and must therefore be closed to the fourth subdivision to the left of 0.

These numbers, thus taken, may be 253, 25·3, 2·53; 213, 21·3, 2·13; 59, 5·9, .59, according as the primary divisions are taken as hundreds, tens, or units.

The construction of this scale is similar to that of the verniers of theodolites and surveying instruments; but, in its application to drawing, is not as simple as the diagonal scales (Figs. 138, 140).

The *sector* (Fig. 142), now seldom used, consists of two flat rulers united by a central joint, and opening like a pair of compasses. It carries several plain scales on its faces, but its most important lines are in the pairs or double scales, running accurately to the central joint.

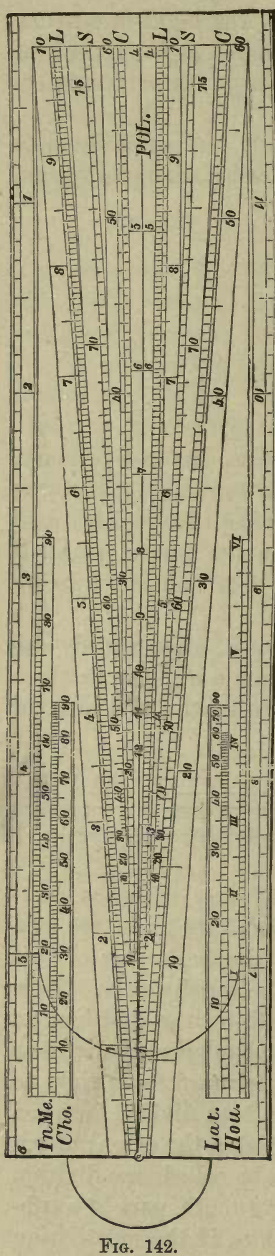


Fig. 142.

The principle on which the double scales are constructed is that similar triangles have their like sides proportional (Fig. 143). Let the lines A B, A C, represent the legs of the sector, and A D, A E, two equal sections from the center; then, if the points B C and D E be connected, the lines B C and D E will be parallel; therefore, the triangles A B C, A D E, will be similar, and, consequently, the sides A B, B C, A D, D E, proportional—that is, as A B : B C : : A D : D E; so that if A D be the half, third, or fourth part of A B, then D E will be a half, third, or fourth part of B C; and the same holds of all the rest. Hence, if D E be the chord, sine, or tangent of any arc, or of any number of degrees to the radius A D, then B C will be the same to the radius A B. Thus, at every opening of the sector, the *trans-*

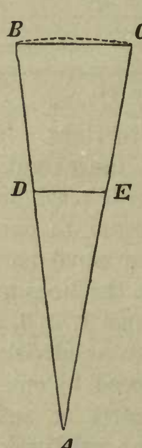


Fig. 143.

verse distances D E and C B from one ruler to another are proportional to the *lateral* distances, measured on the lines A B, A C. It is to be observed that all measures are to be taken from the inner lines, since these only run accurately to the center.

On the scale in common boxes of drawing instruments, the edge of one side is divided as a *protractor,* for the laying out of angles, whose use will be readily understood from the description of the instrument, when by itself. It consists of a semicircle of thin metal or horn (Fig. 144), whose circumference is divided into 180 equal parts or degrees (180°). In the larger protractors each of these divisions is subdivided.

Application of the protractor (Fig. 144).—To lay off a given angle from a given point on a straight line, let the straight line *a b* of the protractor coin-

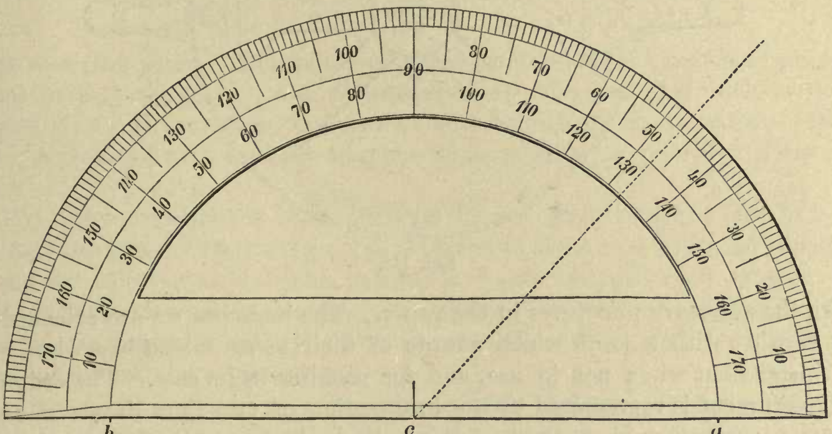


Fig. 144.

cide with the given line, and the point *c* with the given point; now mark on the paper against the division on the periphery coinciding with the angle required; remove the protractor, and draw a line through the given point and the mark.

For plotting field-notes expeditiously, drawing paper can be obtained with large, full circular protractors printed thereon, on which the courses can be readily marked, and thus transferred to the part of the paper required by a parallel ruler, or by triangle and ruler. These sheets are of especial use in plotting at night the day's work, as, on account of the large size of protractor, angles can be laid off with greater accuracy than by the usual protractor of a drawing-instrument case, with less confusion of courses, and more expeditiously.

For accurate plotting of angles, the circular protractor (Fig. 145) is one of the best. It is a complete circle, A A, connected with its center by four radii, *a a a a*. The center is left open, and surrounded by a concentric ring or collar, *b*, which carries two radial bars, *c c*. To the extremity of one bar is a pinion, *d*, working in a toothed rack quite round the outer circumference of the protractor. To the opposite extremity of the other bar, *c*, is fixed a vernier, which subdivides the primary divisions on the protractor to single minutes, and by estimation to 30 seconds. This vernier is carried round the protractor by turning the pinion *d*. Upon each radial bar, *c c*, is placed a branch, *e e*, carrying at their extremities a fine steel pricker, whose points are kept above the surface of the paper by springs placed under their supports, which give way when the branches are pressed downward, and allow the points to

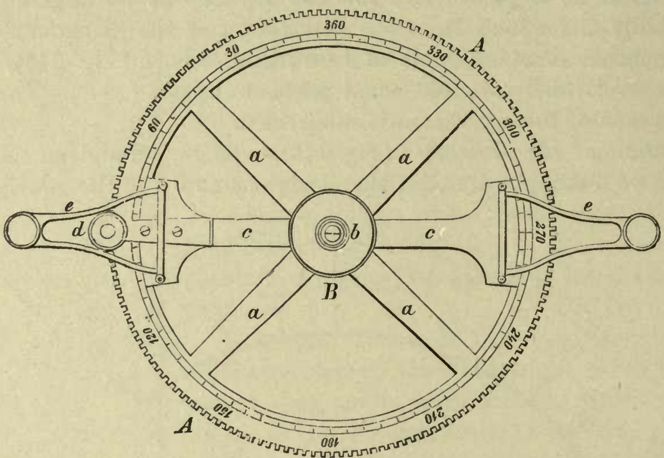


Fig. 145.

make the necessary punctures in the paper. The branches *e e* are attached to the bars *c c* with a joint which admits of their being folded backward over the instrument when not in use, and for packing in its case. The center of the instrument is represented by the intersection of two lines drawn at right angles to each other on a piece of plate glass, which enables the person using it to place it so that the center or intersection of the cross-lines may coincide with any given point on the plan. If the instrument is in correct order, a line connecting the fine pricking points with each other would pass through the center of the instrument, as denoted by the before-mentioned intersection of the cross-lines upon the glass. In using this instrument, the vernier should first be set to zero (or the division marked 360) on the divided limb, and then placed on the paper, so that the two fine steel points may be on the given line (from whence other and angular lines are to be drawn), and the center of the instrument coincides with the given angular point on such line. This done, press the protractor gently down, which will fix it in position by means of very fine points on the under side. It is now ready to lay off the given angle, or any number of angles that may be required, which is done by turning the pinion *d* till the opposite vernier reads the required angle. Then press downward the branches *e e*, which will cause the points to make punctures in the paper at opposite sides of the circle; which being afterward connected, the line will pass through the given angular point, if the instrument was first correctly set. In this manner, at one setting of the instrument, a great number of angles may be laid off from the same point.

The *pantagraphs* are used for the copying of drawings either on the same scale, on a reduced scale, or on an enlarged scale, as may be required. The form of pantagraph as shown in Fig. 146 consists of a set of jointed rulers, A, B, and another, C, D, about one half the length of the former. The free ends of the smaller set are jointed to the larger at about the center. Casters are placed at *a a*, etc., to support the instrument and to allow an easy move-

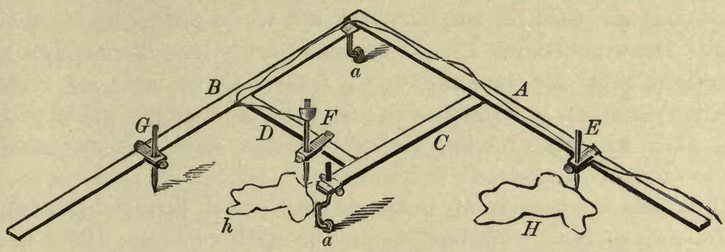


Fig. 146.

ment over the paper. The rulers A and C are divided with a scale of proportional parts, marked $\frac{1}{4}$, $\frac{1}{2}$, etc. These arms are also provided with movable indices, E, F, which can be fastened at any division by clamp screws. Each index is provided with a socket adapted to carry either a pencil or a tracing point.

Fig. 146 represents the instrument in the act of reducing the plan H to *h*, one half the size. The tracing point is placed in the socket at E, the pencil at F, and the fulcrum at G. The indices, E, F, are clamped each at $\frac{1}{2}$ on the scales. If the instrument is correct, the points E, F, G, are in a straight line. Pass the tracing point delicately over the plan H, and the pencil point F will trace *h*, one half the original size.

If the object had been to enlarge the drawing to double its scale, then the tracer must have been placed at F, and the pencil at E. And if a copy be required, retaining the scale of the original, then the slides E and F must be placed at the divisions marked 1. The fulcrum must take the middle station, and the pencil and tracer those on the exterior rules A and B of the instrument. Another form of this instrument is shown in Fig. 147.

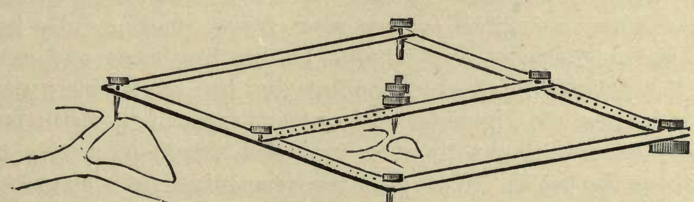


Fig. 147.

The *camera lucida* is sometimes used for copying and reducing topographical drawings. A description of the use of this instrument will be found under the head of topographical drawing.

The drawing table and drawing board.—The usual size of the drawing table should be from 5 to 6 feet long and 3 feet wide, of $1\frac{1}{2}$- or 2-inch white pine plank well seasoned, without any knots, closely joined, glued, doweled, and clamped. It should be fixed on a strong, firm frame and legs, and of such a height that the draughtsman, as he stands up, may not have to stoop to his work. The table is usually provided with a shallow drawer to hold paper or drawings. Drawing tables are made portable by having two horses for their supports, and a movable drawing board for the top ; this board is made similar to the top of the drawing table, but of inch boards, and barred at the ends. Various woods are used for the purposes, but white pine is by far the cheapest and best. Drawing boards should be made truly rectangular, and with perfectly straight sides for the use of the T square. Two sizes are sufficient for common purposes, 41 × 30 inches to carry double elephant paper with a margin, and 31 × 24 inches for imperial and smaller sizes. Boards smaller than this are too light and unsteady in handling.

Small boards are occasionally made, as loose panels fitting into a frame, flush on the drawing surface, with buttons on the back to secure them in position. The panel is mostly of white pine, with a hard-wood frame.

DRAWING PAPER.

Hand-made drawing paper is usually made to certain standard sizes about as follows :

Demy...........	20 inches by	15¼ inches.		Columbier.......	35 inches by	23½ inches.
Medium.........	22¾ "	17½ "		Atlas...........	34 "	26 "
Royal...........	24 "	19¼ "		Double Elephant.	40 "	27 "
Super Royal.....	27¼ "	19¼ "		Antiquarian.....	53 "	31 "
Imperial.........	30 "	22 "		Emperor.........	68 "	48 "
Elephant........	28 "	23 "				

But of late machine-made papers are the most used, and are furnished in rolls of widths up to 58 inches, and wider can be obtained by order.

Whatman's white paper is the quality most usually employed for finished drawings ; it will bear wetting and stretching without injury, and, when so treated, receives color readily. For ordinary working drawings, where damp-stretching is dispensed with, cartridge paper, in rolls of a coarser, harder, and tougher quality, is preferable. It bears the use of India-rubber better, receives ink on the original undamped surface more freely, shows a fully better line, and, as it does not absorb very rapidly, tinting lies better and more evenly upon it. For delicate small-scale line-drawing, the thick blue paper, such as is used for ledgers, etc., imperial size, answers exceedingly well ; but it does not bear damp-stretching without injury, and should be merely pinned or waxed down to the board. With good management, there is no ground to fear the shifting of the paper. Good letter paper receives light drawing very well ; of course, it does not bear much fatigue.

Drawings destined for rough usage and frequent reference should be on sheet or roll drawing paper, backed with cotton cloth, which can be purchased at the stationer's.

Tracing paper is a preparation of tissue paper, transparent and qualified to receive ink lines and tinting without spreading. When placed over a drawing already executed, the drawing is distinctly visible through the paper, and may be copied or *traced* directly by the ink instruments ; thus an accurate copy may be made with great expedition. Tracings may be folded and stowed away very conveniently ; but, for good service, they should be mounted on cloth, or on paper and cloth, with paste.

Tracing paper may be prepared from thick tissue paper by sponging over one surface with a mixture of one part raw linseed oil and five spirits of turpentine ; five gills of turpentine and one of oil will go over from forty to fifty sheets of paper.

Tracing cloth is a similar preparation of linen, and is preferable for its toughness and durability. Tracing paper and cloth are usually to be had in rolls, and tracings on cloth are now preserved as originals, and copies are made from them by some sun process.

Mouth Glue, for the sticking of the edges of drawing paper to the board, is made of glue and sugar or molasses ; it melts at the temperature of the mouth, and is convenient for the draughtsman.

Drawing paper may be fixed down on the drawing board by the pins at the corners, by weights, or by gluing the edges. The first is sufficient when no shading or coloring is to be applied, and if the sheet is not to be a very long time on the board ; and it has the advantage of preserving the paper in its natural state. For shaded or tinted drawings, the paper must be damped and glued at the edges, as the partial wetting of paper, loose or fixed at the corners merely, by the water-colors, distorts the surface.

Damp-stretching is done as follows : The edges of the paper should first be cut straight, and, as near as possible, at right angles with each other ; also, the sheet should be so much larger than the intended drawing and its margin as to admit of being afterward cut from the board, leaving the border by which it is attached thereto by glue or paste, as we shall next explain.

The paper must first be thoroughly and equally damped with a sponge and clean water, on the opposite side from that on which the drawing is to be made. When the paper absorbs the water, which may be seen by the wetted side becoming dim, as its surface is viewed slantwise against the light, it is to be laid on the drawing board with the wetted side downward, and placed so that its edges may be nearly parallel with those of the board ; otherwise, in using a T square, an inconvenience may be experienced. This done, lay a straight flat ruler on the paper, with its edge parallel to, and about half an inch from, one of its edges. The ruler must now be held firm, while the said projecting half-inch of paper be turned up along its edge ; then a piece of solid or mouth glue, having its edge partially dissolved by holding it in boiling or warm water for a few seconds, must be passed once or twice along the turned-up edge of the paper, after which, by sliding the ruler over the glued border, it will be again laid flat, and, the ruler being pressed down upon it, that edge of the paper will adhere to the board. If sufficient glue has been applied, the ruler may be removed directly, and the edge finally rubbed down by an ivory book-knife, or by the bows of a common key, by rubbing on a slip of paper placed on the drawing paper, so that the surface of the latter may not be soiled, which will then firmly cement the paper to the board. This done, another but *adjoining* edge of the paper must be acted upon in like manner, and then the remaining edges in succession ; we say the adjoining edges, because we have occasionally observed that, when the opposite and parallel edges have been laid down first, without continuing the process progressively round the board, a greater degree of care is required to prevent undulations in the paper as it dries.

Sometimes strong paste is used instead of glue; but, as this takes a longer time to set, it is usual to wet the paper also on the upper surface to within an inch of the paste mark, care being taken not to rub or injure the surface in the process. The wetting of the paper in either case is done for the purpose of expanding it; and the edges, being fixed to the board in its enlarged state, act as stretchers upon the paper, while it contracts in drying, which it should be allowed to do gradually. All creases or undulations by this means disappear from the surface, and it forms a smooth plane to receive the drawing.

To remove the paper after the drawing is finished, cut off inside the pasted edge, and remove the edge by warm water and the knife.

With paneled boards, the panel is taken out, and the frame inverted; the paper, being first damped on the back with a sponge slightly charged with water, is applied equally over the opening to leave equal margins, and is pressed and secured into its seat by the panel and bars.

MOUNTING PAPER AND DRAWINGS, VARNISHING, ETC.

When paper of the requisite quality or dimension can not be purchased already backed, it may be mounted on cloth. The cloth should be well stretched upon a smooth flat surface, being damped for that purpose, and its edges glued down, as was recommended in stretching drawing paper. Then with a brush spread strong paste upon the canvas, beating it in till the grain of the canvas be all filled up; for this, when dry, will prevent the canvas from shrinking when subsequently removed; then, having cut the edges of the paper straight, paste one side of every sheet, and lay them upon the canvas sheet by sheet, overlapping each other a small quantity. If the drawing paper is strong, it is best to let every sheet lie five or six minutes after the paste is put on it, for, as the paste soaks in, the paper will stretch, and may be better spread smooth upon the canvas; whereas, if it be laid on before the paste has moistened the paper, it will stretch afterward and rise in blisters when laid upon the canvas. The paper should not be cut off from its extended position till thoroughly dry, which should not be hastened, but left in a dry room to do so gradually, if time permit; if not, it may be exposed to the sun, unless in the winter season, when the help of a fire is necessary, provided it is not placed too near a scorching heat.

In joining two sheets of paper together by overlapping, it is necessary, in order to make a neat joint, to feather-edge each sheet; this is done by carefully cutting with a knife half way through the paper near the edges, and on the sides which are to overlap each other; then strip off a feather-edged slip from each, which, if done dexterously, will form a very neat and efficient joint when put together.

For mounting and varnishing drawings or prints, stretch a piece of linen on a frame, to which give a coat of isinglass or common size, paste the back of drawing, which leave to soak, and then lay it on the linen. When dry, give it at least four coats of well-made isinglass size, allowing it to dry between each coat. Take Canada balsam diluted with the best oil of turpentine, and with a clean brush give it a full flowing coat.

MANAGEMENT OF THE INSTRUMENTS.

In constructing preparatory pencil-drawings, it is advisable, as a rule of general application, to make no more lines upon the paper than are necessary to the completion of the drawing in ink ; and also to make these lines just so dark as is consistent with the distinctness of the work. With respect to the first idea, it is of frequent application : in the case, for example, of the teeth of spur wheels, where, in many instances, all that is necessary to the drawing of their end view in ink are three circles, one of them for the pitch line, and the two others for the tops and bottoms of the teeth ; and again, to draw the face view of the teeth—that is, in the edge view of the wheel—we have only to mark off by dividers the positions of the lines which compose the teeth, and draw four pencil lines for the two sides, and the top and bottom of the elevation. And here we may remark the inconvenience of that arbitrary rule, by which it is by some insisted that the pupil should lay down in pencil every line that is to be drawn before finishing it in ink. It is often beneficial to ink in one part of a drawing before touching other parts at all ; it prevents confusion, makes the first part of easy reference, and allows of its being better done, as the surface of the paper inevitably contracts dust and becomes otherwise soiled in the course of time, and therefore the sooner it is done with the better.

Circles and circular arcs should, in general, be inked in before straight lines, as the latter may be more readily drawn to join the former than the former the latter. When a number of circles are to be described from one center, the smaller should be inked first, while the center is in better condition. When a center is required to bear some fatigue, it should be protected with a thickness of stout card glued or pasted over it, to receive the compass-leg.

India-rubber is the ordinary medium for cleaning a drawing, and for correcting errors in the pencil. For slight work it is quite suitable ; that substance, however, operates to destroy the surface of the paper ; and, by repeated application, it so ruffles the surface, and imparts an unctuosity to it, as to spoil it for fine drawing, especially if ink shading or coloring is to be applied. It is much better to leave trivial errors alone, if corrections by the pencil may be made alongside without confusion, as it is, in such a case, time enough to clear away superfluous lines when the inking is finished.

For cleaning a drawing, a piece of bread two days old is preferable to India-rubber, as it cleans the surface well and does not injure it. When ink lines to any considerable extent have to be erased, a small piece of damped soft sponge may be rubbed over them till they disappear. As, however, this process is apt to discolor the paper, the sponge must be passed through clean water, and applied again to take up the straggling ink. For ordinary small erasures of ink lines, a sharp rounded pen-blade, applied lightly and rapidly, does well, and the surface may be smoothed down by the thumb-nail. In ordinary working drawings, a line may readily be taken out by damping it with a hair-pencil and quickly applying the India-rubber ; and to smooth the surface so roughened, a light application of the knife is expedient. In drawings intended to be highly finished, particular pains should be taken to avoid the necessity for corrections, as everything of this kind detracts from the appearance.

In using the square, the more convenient way is to draw the lines off the left edge with the right hand, holding the stock steadily but not very tightly against the edge of the board with the left hand. The convenience of the left edge for drawing by is obvious, as we are able to use the arms more freely, and we see exactly what we are doing.

To draw lines in ink with the least amount of trouble to himself, the mechanical draughtsman ought to take the greater amount of trouble with his tools. If they be well made, and of good stuff originally, they ought to last through three generations of draughtsmen ; their working parts should be carefully preserved from injury, they should be kept well set, and, above all, scrupulously clean. The setting of instruments is a matter of some nicety, for which purpose a small oil-stone is convenient. To dress up the tips of the blades of the pen or of the bows, as they are usually worn unequally by the customary usage, they may be screwed up into contact in the first place, and passed along the stone, turning upon the point in a directly perpendicular plane, till they acquire an identical profile. Being next unscrewed and examined to ascertain the parts of unequal thickness round the nib, the blades are laid separately upon their backs on the stone, and rubbed down at the points, till they be brought up to an edge of uniform fineness. It is well to screw them together again, and to pass them over the stone once or twice more, to bring up any fault ; to retouch them also on the outer and inner side of each blade, to remove barbs or fraying ; and, finally, to draw them across the palm of the hand.

The China ink which is commonly used for line-drawing ought to be rubbed down in water to a certain degree, avoiding the sloppy aspect of light lining in drawings, and making the ink just so thick as to run freely from the pen. This medium degree may be judged of after a little practice by the appearance of the ink on the palette. The best quality of ink has a soft feel when wetted and smoothed ; free from grit or sediment, and musky. The rubbing of China ink in water tends to crack and break away the surface at the point ; this may be prevented by shifting at intervals the position of the stick in the hand while being rubbed, and thus rounding the surface. Nor is it advisable, for the same reason, to bear very hard, as the mixture is otherwise more evenly made, and the enamel of the palette is less rapidly worn off. When the ink, on being rubbed down, is likely to be for some time required, a considerable quantity of it should be prepared, as the water continually vaporizes ; it will thus continue for a longer time in a condition fit for application. The pen should be leveled in the ink, to take up a sufficient charge ; and, to induce the ink to enter the pen freely, the blades should be lightly breathed upon before immersion. After each application of ink, the outsides of the blades should be cleaned, to prevent any deposit of ink upon the edge of the squares.

To keep the blades of his *inkers* clean is the first duty of a draughtsman who is to make a good piece of work. Pieces of blotting or unsized paper and cotton velvet, wash-leather, or even the sleeve of a coat, should always be at hand while a drawing is being inked. When a small piece of blotting paper is folded twice so as to present a corner, it may usefully be passed between the blades of the pen now and then, as the ink is liable to deposit at the point and obstruct the passage, particularly in fine lining ; and for this purpose the pen must be unscrewed to admit the paper. But this process may be delayed by drawing the point of the pen over a piece of velvet, or even over the surface of thick blotting-paper ; either method clears the point for a time. As soon as any obstruction takes place, the pen should be immediately cleaned, as the trouble thus taken will always improve and expedite the work. If the pen should be laid down for a short time with the ink in it, it should be unscrewed to keep the points apart, and so prevent deposit ; and, when done with altogether for the occasion, it ought to be thoroughly cleaned at the nibs. This will preserve its edges and prevent rusting.

For the designing of machinery, it is very convenient to have some scale of reference by which to proportion the parts ; for this purpose a vertical and horizontal scale may be drawn on the walls of the room.

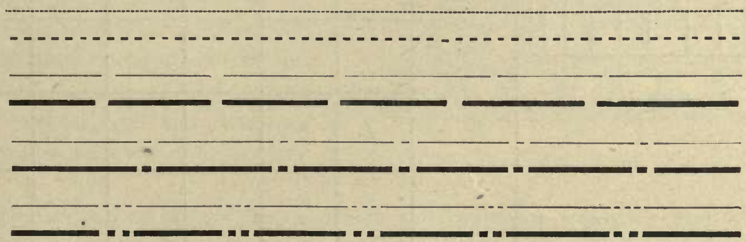
Exercises with the Drawing-Pen.

Before proceeding to the construction of finished drawings, skill should be acquired in the use of the drawing-pen, supplemented often by the steel pen. Beginning with lines, outlines of figures, alphabets, and the like, the draughtsman should strive to acquire the habit of readily drawing clean, uniform lines, without abruptness or breaks, where straight lines connect with curved ones. Draw straight lines of different grades :

as, fine
medium
coarse

at first, lines of indefinite length, taking care that they are drawn perfectly straight and of uniform width or grade ; then draw lines of definite length between assumed points, taking care to terminate the lines exactly at these points. Lines as above are *full* lines, the grades depending on the effect which the draughtsman wishes to give.

Draw *dotted lines, broken lines*, and *broken* and *dotted* lines, of different grades :



Draw fine lines at uniform distances from each other :

To give uniform appearance, the lines must be of uniform grade and equally spaced. Practice in lines of this sort is important, as they are much used in drawing to represent sections, shades, and conditions, as soundings on charts, density or characteristics of population, areas of rain, temperature, and the like. Draw lines as in Fig. 148. These lines are diagonal with the border-lines, and

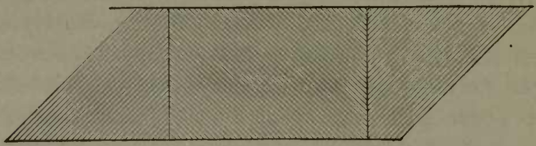


Fig. 148.

are used to represent *sections* of materials. In the figure, lines differently inclined represent different pieces of the same material.

Sections of different materials may be represented in different kinds of lines, as in Figs. 149, 150, 151.

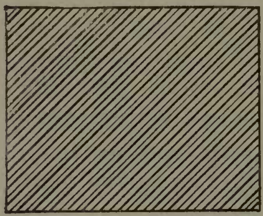


Fig. 149.

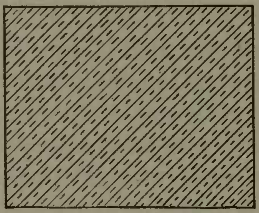


Fig. 150.

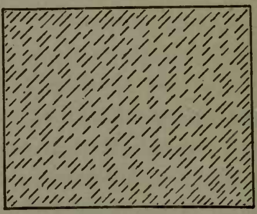


Fig. 151.

These particular ones are used to represent sections of wrought-iron, steel, and cast-iron; but they may be used to represent different colors, the location of different mineral or agricultural products, etc.

To represent cylindrical surfaces (Fig. 152).

Draw a semi-circumference, and mark on it a number of points, at equal distances apart, and through these points draw lines perpendicular to the

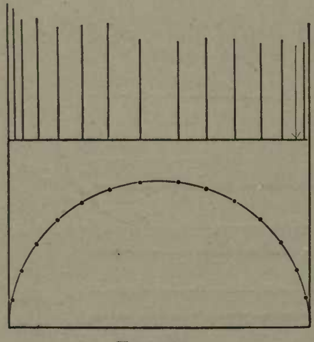


Fig. 152.

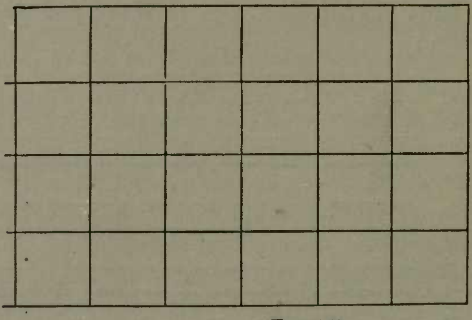


Fig. 153.

diameter across the surface to be represented. It is not absolutely necessary that the central space should be equal to the others; it will be more effective to leave out two of the lines, and make it to this extent wider.

To construct a mass of equal squares (Fig. 153).

Lay off a right angle, and on its sides mark as many points, at equal distances apart, as may be necessary ; through these points draw lines parallel to the sides.

Or, construct a rectangle ; mark on its sides as many points, at equal distances apart, as may be necessary ; through these points draw the lines.

To construct the squares diagonally to the base (Fig. 154).

Mark on the sides of the right angle as many points, at distances apart equal to the diagonal of the required squares, as may be necessary. Connect these points by lines as shown, and through the same points draw lines at right angles to the others.

Or, as above, construct a rectangle, and mark on its sides points at distances apart equal to the diagonal of the required squares.

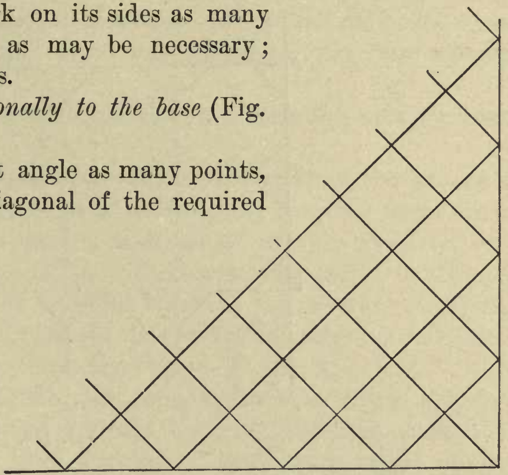


FIG. 154.

To cover a surface with equilateral triangles (Fig. 155).

Construct an angle of 60°, and mark on its sides points at distances apart equal to the side of the triangle. Connect these points ; and through these points draw lines parallel to the sides of the angle.

Figures composed of two triangles, with the same base, are called *lozenges.* Six triangles may be arranged as a hexagon. The whole surface may be arranged in lozenges or hexagons.

To cover a surface with octagons and squares (Fig. 156).

Lay off the surface in squares having sides equal to the width of the octagons. Corner the outer squares to form octagons, as by Prob. XL., page 21. Extend the sides of these octagons across the other squares, and similar corners will be cut off, and the octagons and squares required will be complete.

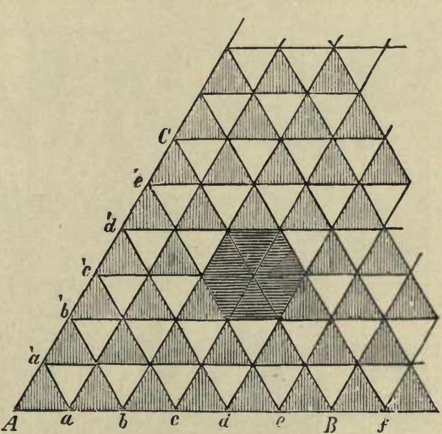


FIG. 155.

With the aid of paper thus covered with squares, triangles, and lozenges, various geometrical designs may be readily constructed, pleasing in their effect, and affording good practice to young draughtsmen.

In the examples given of designs constructed on squares or triangles, if it is desired to increase or diminish the size of the original designs, it is only necessary to make the sides of the squares or triangles larger or smaller, and taking relatively the same squares for the construction of the figures. In transferring designs and drawings from books or plates, on which squares can not be drawn, it is very convenient to have a square of glass, with squares upon it, which may be laid on the drawing, and thus serve the same purpose as if squares had been

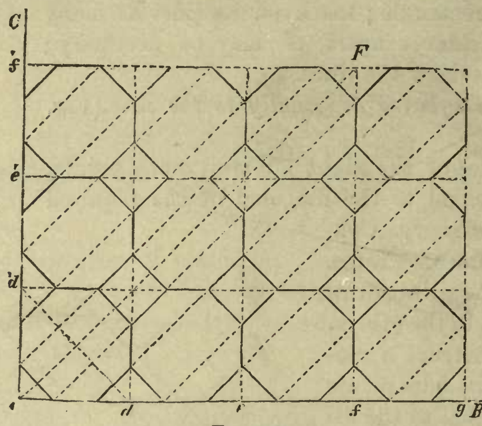


Fig. 156.

drawn. The glass may be readily prepared by painting one of its surfaces with a thin coat of gum, and drawing squares upon it with the drawing-pen ; if every fifth or tenth line be made fuller or in a different color, it will be still more convenient for reference.

Fig. 157 is the front view and side of an acanthus-leaf, of which the surfaces are covered with squares, somewhat larger than would be recommended

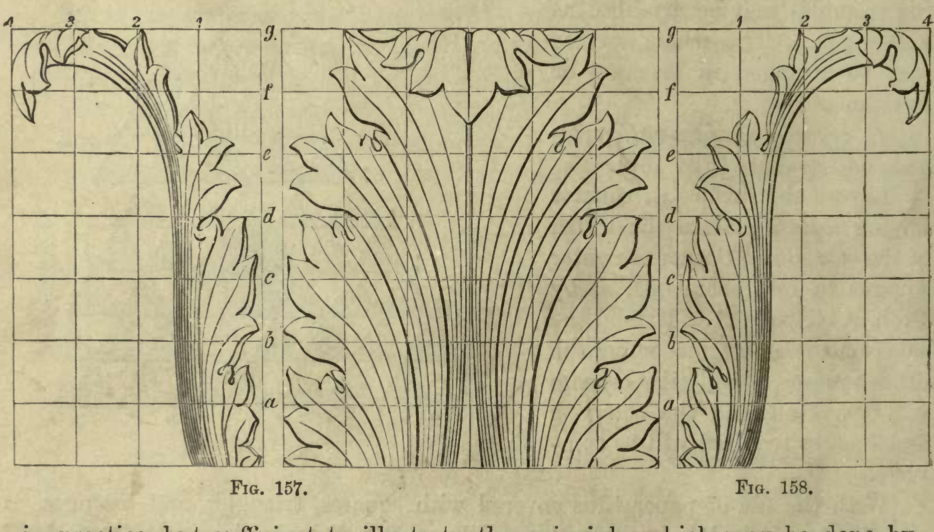


Fig. 157. Fig. 158.

in practice, but sufficient to illustrate the principle, which may be done by the learner on the same or other sized squares. If the same size, the intersections of the lines of the figure with those of the squares are easiest transferred by a straight-edged slip of paper, placed along a line, and making all the intersections at once, and then transferring the marks to the copy.

Fig. 158 is the side-view of the acanthus-leaf, in a reversed position from the original (Fig. 157) ; that is, right-handed, while the original is left-handed. It will readily be understood how this may be done by observing the letters on the side and the numerals at the top of the squares.

Fig. 159 represents the construction of *Gothic* letters and numerals on a system of squares. These letters are formed mechanically by the drawing-pen and dividers.

Fig. 160 are *Italic* letters, drawn on rhombs, in which the upright lines are inclined to horizontal.

On pages 66, 67, 68, 69, are specimens of type taken from the printer's font, which can be readily transferred to a drawing, by covering them with a bit of glass or horn, laid off in squares, as described above. Printers' letters are in general well proportioned, but it is customary often to distort letters, to call attention to them, or to adapt them to the position in which they are to be placed. Spaces between the letters are in printing uniform, but in drawing, when such letters come together as F and A, L and T, one wide at top and the other at bottom, the spacing between them may be reduced a little. The acquisition of a ready hand in lettering enables a draughtsman to give a finish to a good drawing or map which might otherwise be spoiled by poor lettering.

Fig. 159.

Fig. 160.

LARGE ROMAN.

ABC DE FGH IJ
KLMN OP QRST
UV WX YZ

SMALL ROMAN.

abc de fgh ij klmn op
qrst uv wx yz
1234567890

ENGLISH GOTHIC.

ABC DE FGH IJ KLMN OP
QRST UV WX YZ
1234567890

ITALIC.

ABC DE FGH IJ KLMN OP QRST
UV WX YZ

abc de fgh ij klmn op qrst uv wx yz

TUSCAN.

ABC DE FGH IJ KLMN OP QRST
UV WX YZ
1 2 3 4 5 6 7 8 9 0

ABC DE FGH IJ
KLMN OPQRST
UV WX YZ
abc de fgh ij klmn op
qrst uv wx yz

TELEGRAPH.

ABC DE FGH IJ KLMN
OP QRST UV WX YZ

abc de fgh ij klmn op qrst
uv wx yz

1234567890

ORNAMENTED.

ABC DE FGH IJ KLMN
OP QRST UV WX YZ

abc de fgh ij klmn op qrst
uv wx yz

OLD ENGLISH.

ABC DE FGH IJ KLMN
OP QRST UV WX YZ

abc de fgh ij klmn op qrst uv wx yz

ENGLISH CHURCH TEXT.

ABC DE FGH IJ KLMN
OP QRST UV WX YZ

abc de fgh ij klmn op qrst uv wx yz

MEDIÆVAL.

ABC DE FGH IJ KLMN OP QRST
UV WX YZ

abc de fgh ij klmn op qrst uv wx yz

ABC DE FGH IJ KLMN OP QRST
UV WX YZ

abc de fgh ij klmn op qrst uv wx yz

Paper printed in squares is used by designers of figures for calicoes, silks, and woolens. For the engineer, there is a class of papers called cross-section papers, sold in sheets or rolls, and of various scales, originally intended, as the name implies, for cross-sections of railway or canal cuts, but now extensively employed by the architectural and mechanical designer for the rough sketches of works either executed or to be executed ; by the sanitarian for the plotting of death-rates ; for thermometric and hygrometric readings ; by the broker and merchant for the graphic representation of the prices of gold, stocks, or articles of merchandise, during a term of years ; by the railway superintendent for the movement of trains ; and for multitudes of other uses. These may hardly be considered in the light of drawings ; but, as they involve the drawing of lines, shading of spaces, and lettering, and as there is no head of drawing under which this use of cross-section paper can be classed, it seems proper to give here a few illustrations, which will show its general application.

Fig. 161 shows a graphical method of determining the equivalent values of the metric system of measurements in United States units, or *vice versa.* The vertical scale represents the metric units, and the horizontal the common or

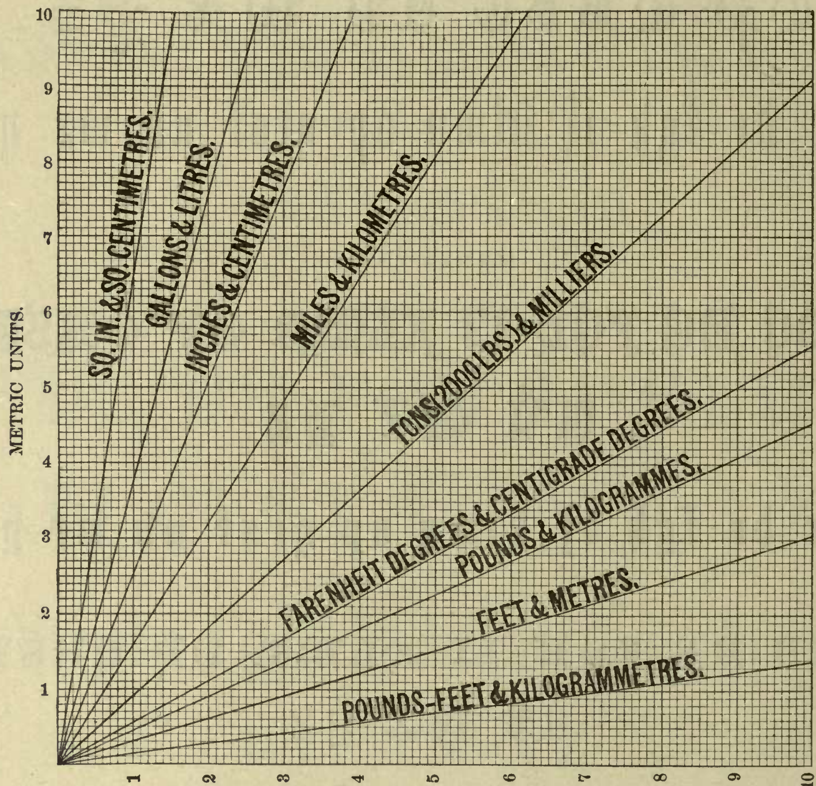


Fig. 161.

United States units. The method of using the diagram can be best shown by taking one or two examples.

What is the equivalent value of seven kilometres in miles ? Read upward on the metric scale to 7, then read on that horizontal line to the point of intersection with the line designated "MILES & KILOMETRES," that is, at the point on the United States scale of units representing 4·35 ; therefore, seven kilometres are equal to 4·35 miles.

What is the value of five pounds in kilogrammes ? The process is the same as the foregoing, except that, to change United States units into the metric units, first read horizontally, then upward. The result will be in this case that five pounds is found equal to 2·25 kilogrammes. The divisions may represent single units, ten units, one hundred units, etc. ; that is, if we had wished to find the equivalent of 500 pounds, it would have been 225 kilogrammes.

Fig. 162 is a diagram illustrating graphically the difference charged on a ton of merchandise per mile, on the New York Central and Hudson River Railroad and the Erie Canal, for every year between 1857 and 1880; the values being

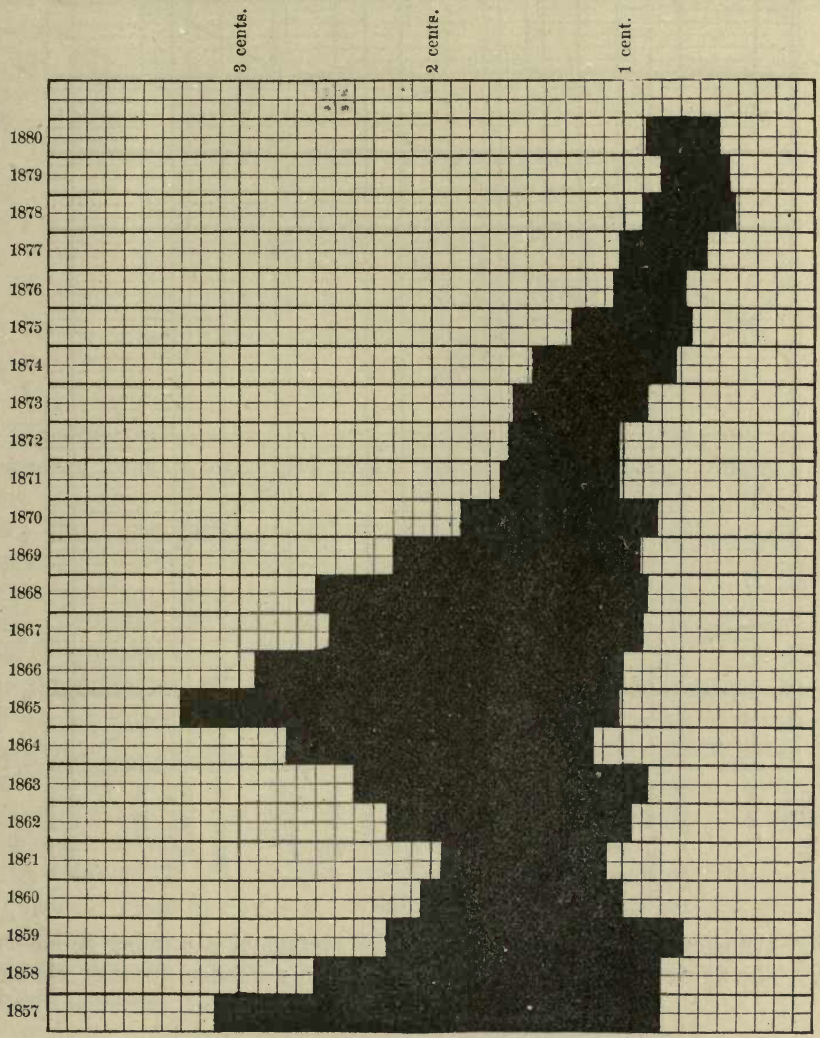


Fig. 162.

published in the Report of the United States Bureau of Statistics for 1880. The higher values in every case represent the railroad rates and the lower the canal rates. The black band shows the difference between these values. In 1865, for instance, the railroad rates were 3·30 cents, and the canal 1·02 cents, the difference being 2·28 cents.

Fig. 163 is made up from the time-table of the New York, New Haven, and Hartford Railroad, showing the movement of trains, two from New York and two from New Haven, the abscissas (horizontal lines) being cut off on a scale of miles for each station, the ordinates (vertical lines) being a scale of hours.

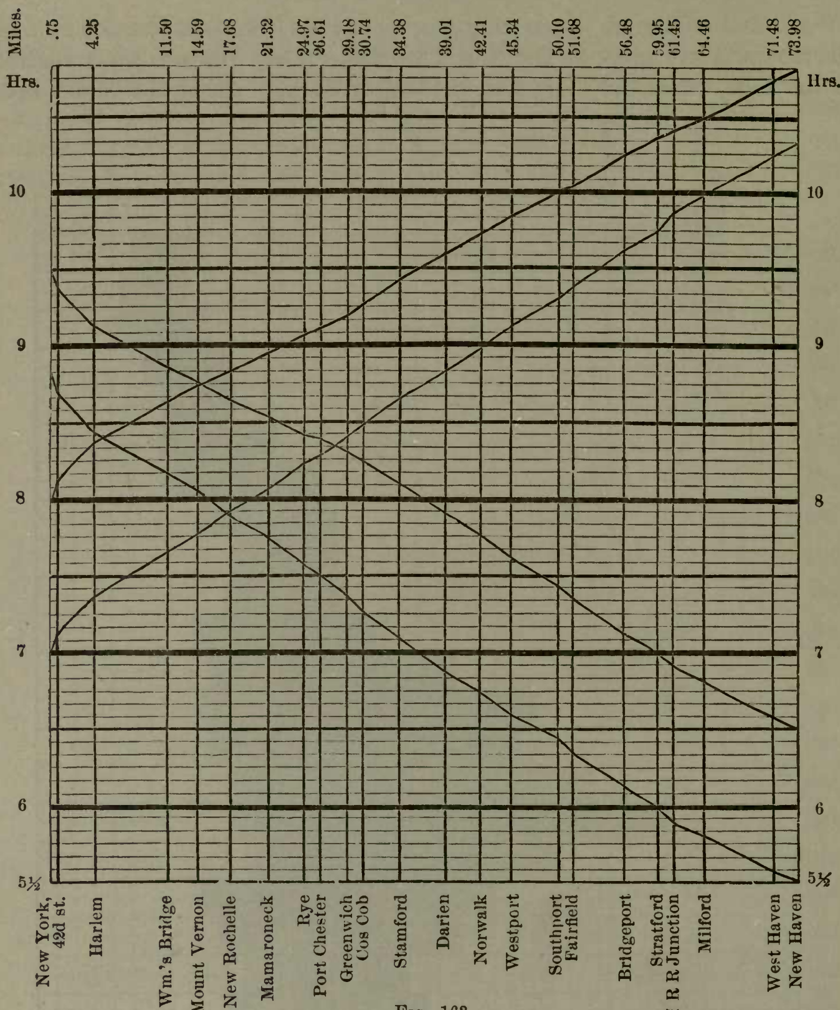


Fig. 163.

Fig. 164 shows the method of finding the average of a number of observations. The figure represents the path of a float in a wooden flume or channel,

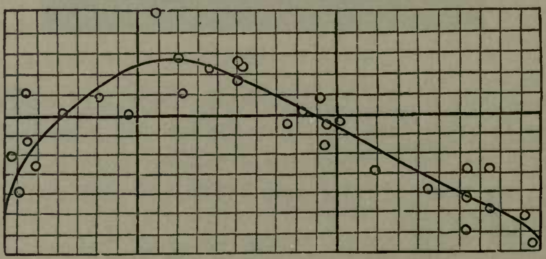


Fig. 164.

taken from the last edition of Francis's "Lowell Hydraulic Experiments." The cut was copied directly on the wood, and is therefore reversed. The width of the cut represents the width of the flume, each abscissa being one foot; the ordinates are the speeds of float in divisions of 0·1 foot per second; the o o on the cut are meant to represent the floats in their observed path and speed; and the curved line the average velocity in the different threads of the stream.

Fig. 165 is from Clarke's "Railway Machinery." The abscissas represent the speed in miles per hour; the ordinates the pounds per ton resistance of a 100-ton train.

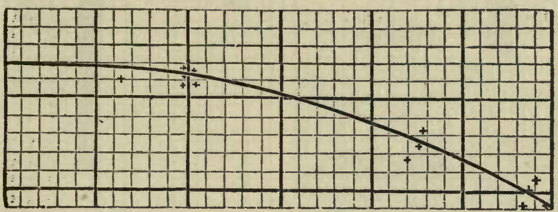


Fig. 165.

Fig. 166 is a diagram illustrating the daily mortality during the month of November, 1873, in New York City. The figure is a copy of a portion of the chart published in the Report of the Metropolitan Board of Health for that year. The lower irregular line shows the daily mortality. The upper single irregular line shows the daily average temperature. The terminal cross-lines at the ends of perpendicular bars show the daily range of temperature. The double irregular line shows the daily humidity, saturation being 100 on the scale of temperatures. The black bands in the upper portion of the diagram give the daily rain-fall in inches. This method of representing the rain-fall will do for this chart, but, for most meteorological purposes, is insufficient. The time of the commencement and end of the rain-fall should be given where any effect due to the rain is to be detected. These few diagrams illustrate the method of graphical representation, so that any one should with little difficulty be able now to make them for such cases as he may see fit.

On pages 75, 76, 77, are some designs, showing other uses to which squared or quadrille paper can be put. The execution of such ornamental designs is greatly facilitated by the use of this paper. The figure on page 77 illustrates how color may be represented in a design, by different grades and directions, of black lines and white spaces.

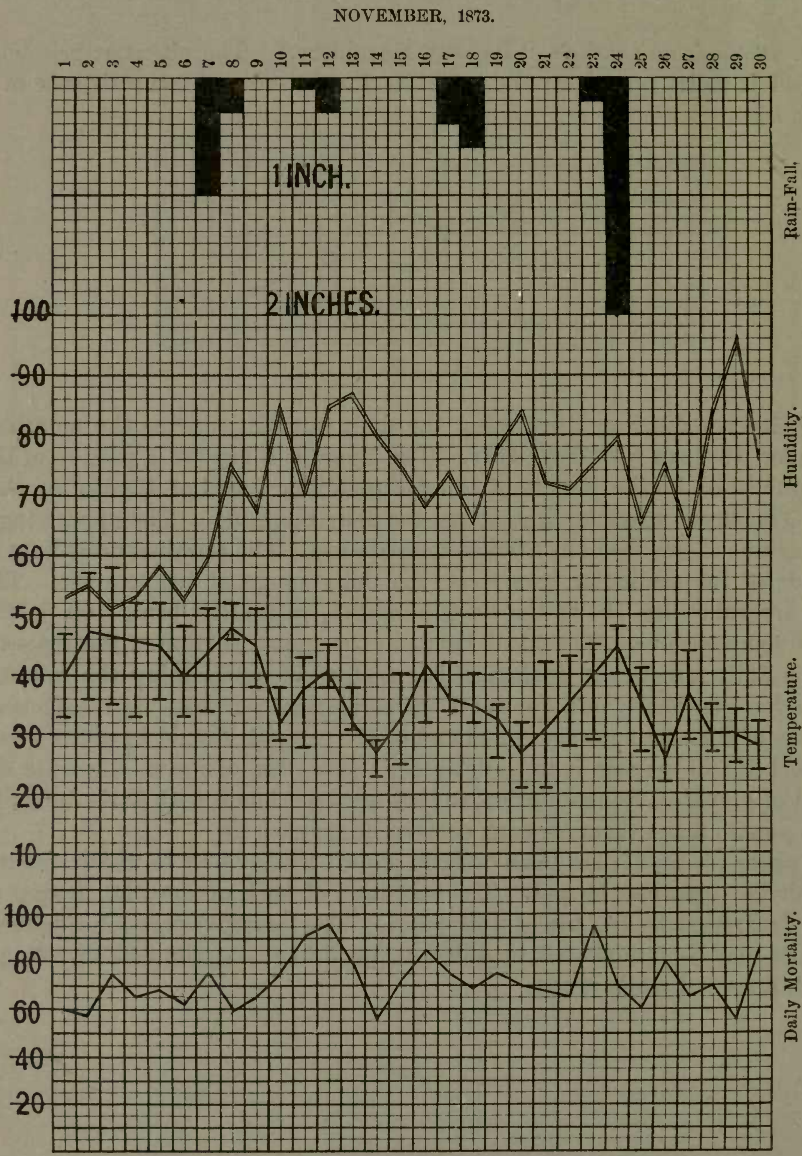
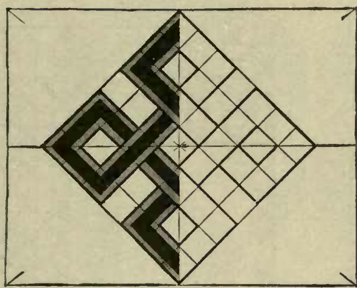
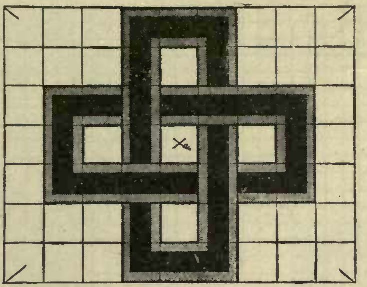
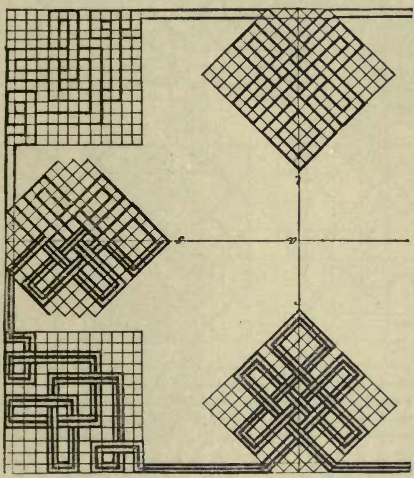
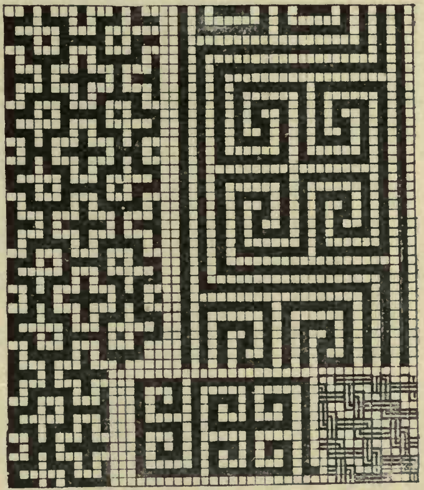

NOVEMBER, 1873.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
1 INCH.
2 INCHES.
100
90
80
70
60
50
40
30
20
10
100
80
60
40
20
Rain-Fall.
Humidity.
Temperature.
Daily Mortality.

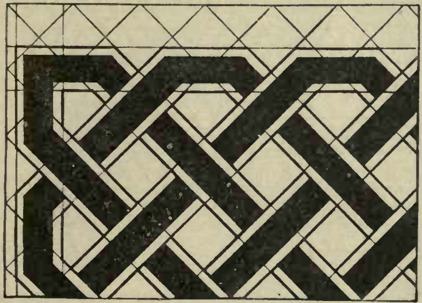
Fig. 166.

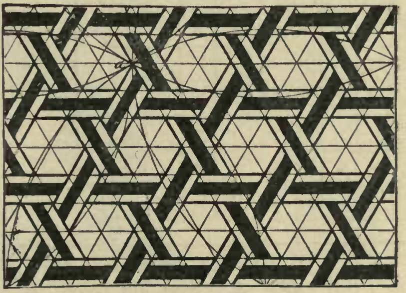


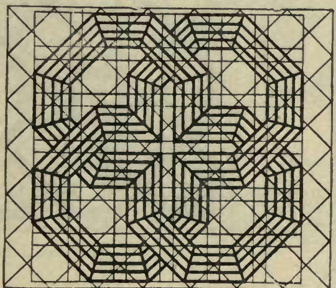


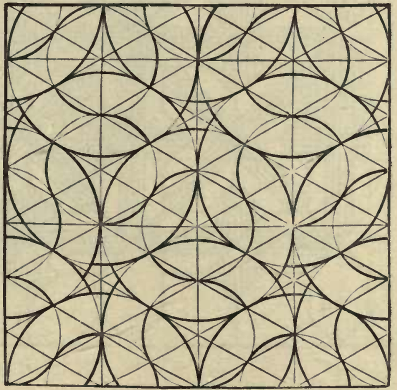


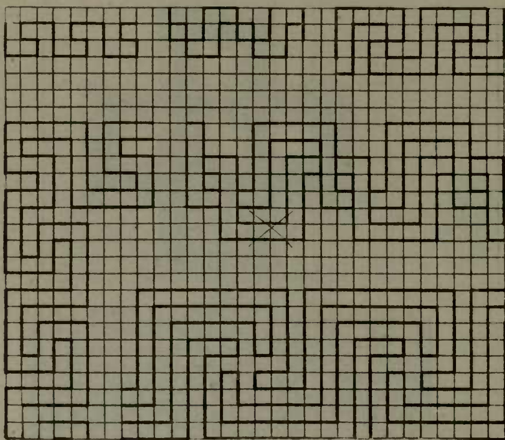


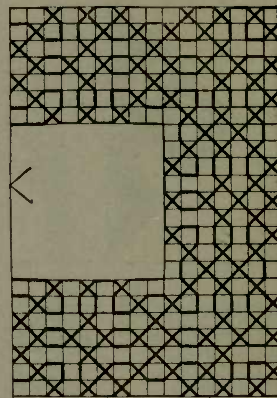


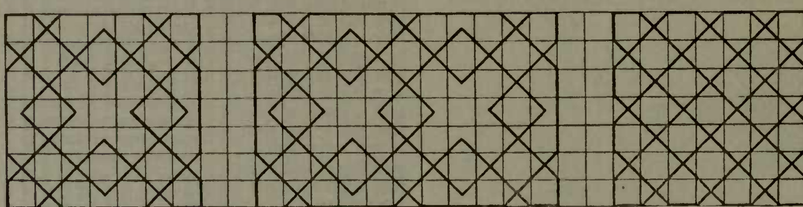


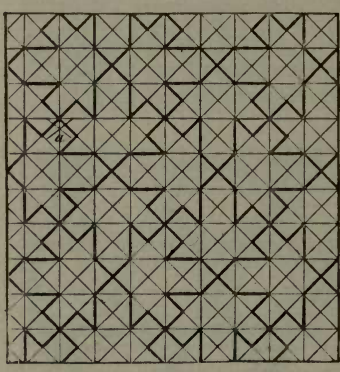


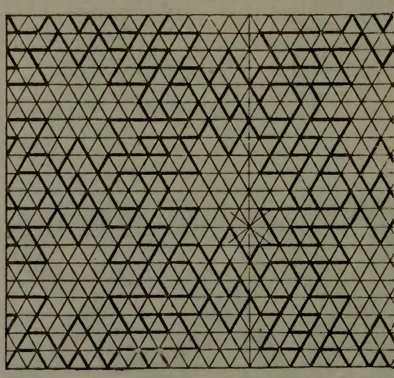


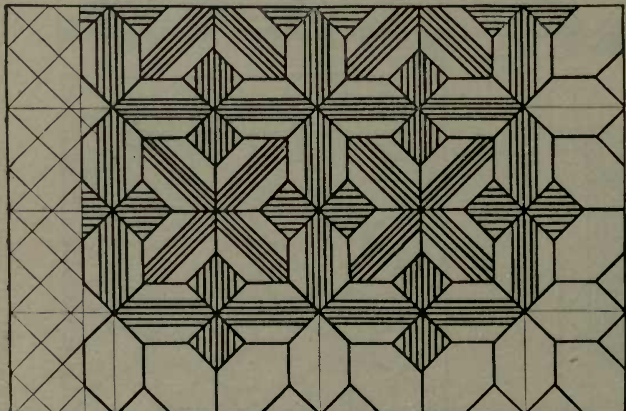


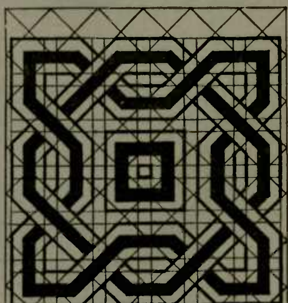


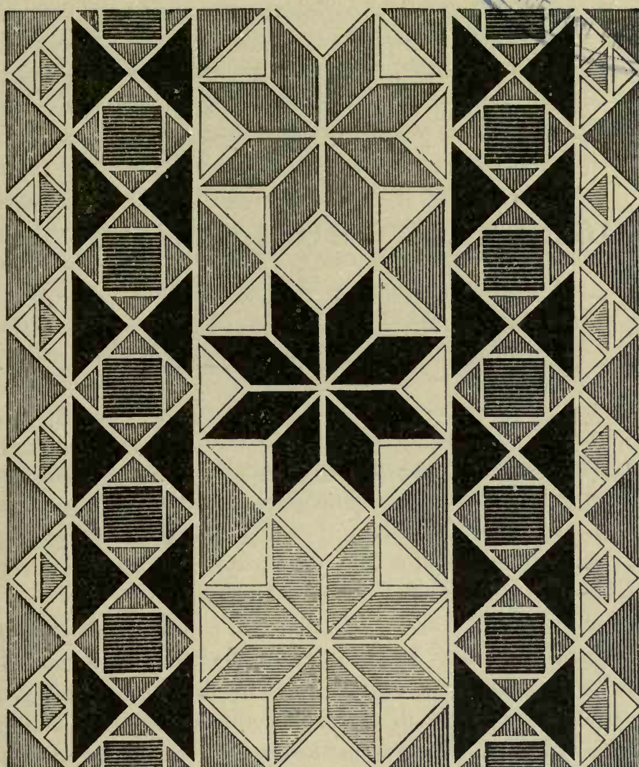


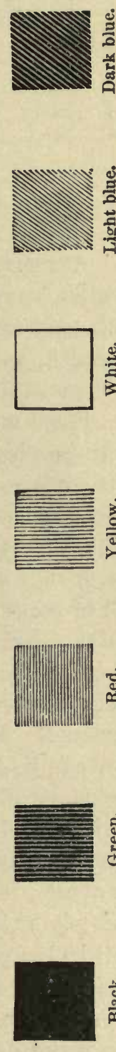


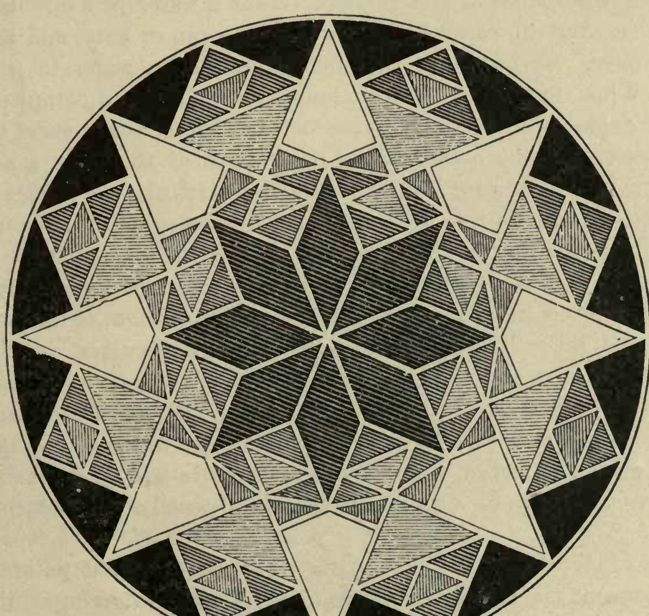




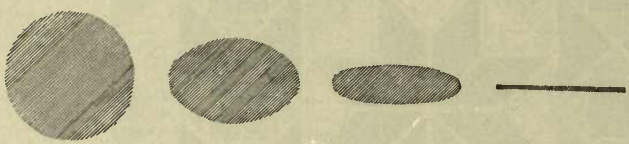





Dark blue.
Light blue.
White.
Yellow.
Red.
Green.
Black.



ORTHOGRAPHIC PROJECTION.



ARCHITECTURAL and mechanical drawings are usually the delineation of bodies by *orthographic projection*, the representation on a sheet of paper having only two dimensions, length and breadth, of solids having three, length, breadth, and thickness; and on such scales that dimensions can be taken from the parts, and structures and machines constructed therefrom.

Place any surface—for instance, a sheet of paper or a drawing-board—at right angles to the sun's rays. This may be readily done by inserting a pin into the surface, and making it vertical to the surface in every direction by a right-angled triangle; then place the surface in the direct rays of the sun, and in such a position that there will be no shadow on the surface from the pin; the sun's rays are then perpendicular to the surface. Take a wafer or a circular bit of paper, and hold it over the paper by means of a long pin or wire, and we obtain shadows, as above, varying with the inclination of the wafer to the plane of the paper. When parallel with the plane, the shadow is a complete circle; when at right angles, a line; and varying between them as the wafer is inclined. These shadows are the *orthographic* projections of the wafer; no line can be longer than it is naturally, but, if inclined or vertical, it is reduced in length till it becomes a point only. The orthographic projection of the pin which has determined the position of the surface is merely the shadow of the head. If the pin be inclined at all, the body of the pin is projected as a shadow by a line; if the pin be laid on the surface, its shadow, or projection, is that of the whole length of the pin. The sun's rays act as perpendiculars, which will be hereafter spoken of as projecting the points of an object upon a surface which will represent the object itself in drawing; and, should any confusion occur to the draughtsman of how an object is to be projected or drawn, if he can make the outline of the object on any convenient scale in wire and get its shadows by the sun's vertical rays on a plane, he can readily see how the object should be drawn.

Since the surfaces of all bodies may be considered as composed of points, the first step is to represent the position in space of a point, by referring it to planes whose position is established. The projection of a point upon a plane is the foot of the perpendicular let fall from the point on the plane. If, therefore, on two planes not parallel to each other, whose positions are known, we have the projections of a point, the position of this point is completely determined by erecting perpendiculars from each plane at the projected points : their intersection will be the point.

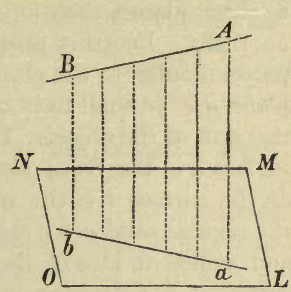


Fig. 167.

If from every point of an indefinite straight line, A B (Fig. 167), placed in any manner in space, perpendiculars be let fall on a plane, L M N O, whose position is given, then all the points in which these perpendiculars meet the plane will form another indefinite straight line, *a b:* this line is called the projection of the line A B on this plane. Since two points are sufficient to determine a straight line, it is only necessary to project two points of the line, and the straight line drawn through the two projected points will be the projection of the given line. The projection of a straight line, itself perpendicular to the plane, is the point in which this perpendicular meets the plane.

If the projections *a b* and *a′ b′* of a straight line on the two planes L M N O and L M P Q (Fig. 168) are known, this line A B is determined ; for if,

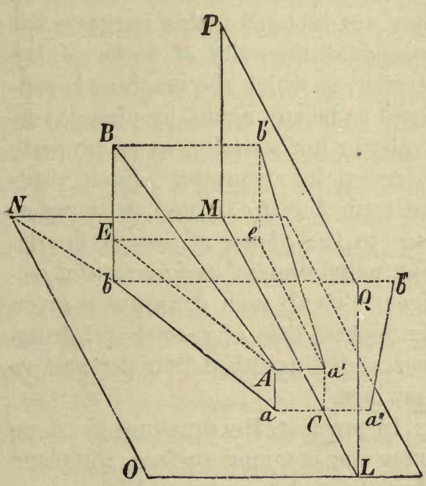


Fig. 168.

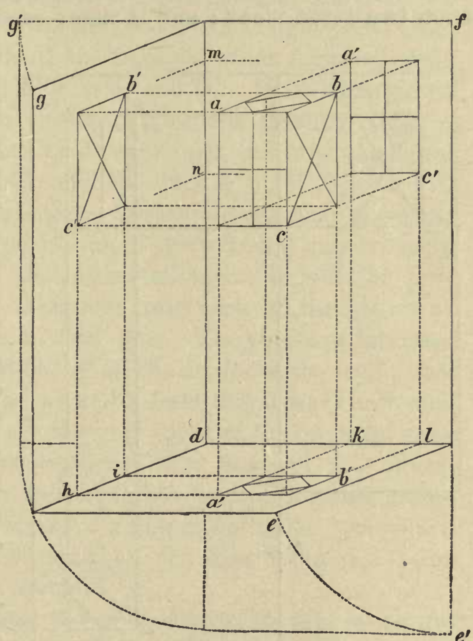


Fig. 169.

through one of its projections, *a b,* we suppose a plane drawn perpendicularly to L M N O, and if through *a′ b′* another plane be drawn perpendicular to L M P Q, the intersection of the two planes will be the line A B.

To delineate a solid, as the form of a machine, for instance, it must be referred to three series of dimensions, each of them at right angles to the plane of the other.

Thus, let *a b c* (Fig. 169) be a parallelopiped in an upright position, of which the plane *a b* is horizontal, and the planes *a c* and *c b* vertical. Let *d e*, *d f*, and *d g*, be the planes of projection. The sides of the body being parallel to these planes, each to each, let the figure of the parallelopiped be projected on them. For this purpose draw parallel lines from the angles of the body perpendicular to the planes, as indicated by the dotted lines; then upon the plane *d e* we shall have *a′ b′*, the projection of the surface *a b*: this is called the *plan* of the object. Upon the plane *d f* we have *a′ c′*, the projection of the surface *a c*, the *front elevation*; and upon the plane *d g*, the projection *b′ c′* of the surface *b c*, the *side elevation*. We have then three distinct views of the regular solid *a b c* delineated on plane surfaces, which convey an accurate and sufficient idea of its form. Indeed, any two of these representations are sufficient as a description of the object. From the two figures *a′ c′*, *b′ c′*, for example, the third figure *a′ b′* may be compounded, by merely drawing the vertical lines *c′ h b′ i*, and *a′ k*, *c′ l*, to meet the plane *d e*, and by producing them horizontally till they meet and form the figure *a′ b′*. Similarly, the figure *b′ c′* may be deduced from the other two by the aid of the lines *h*, *i*, from *a′ b′*, and the lines *m*, *n*, from *a′ c′*.

It is in this way that a third view of any piece of machinery is to be found from two given views; and in many cases two elevations, or one elevation and a plan, may afford a sufficiently complete idea of the construction of a machine. In other cases, many parts may be concealed by others in which they are inclosed; this suggests the occasional necessity of views of the interior, in which the machine is supposed to be cut across by planes, vertically or horizontally, so as properly to reveal its structure. Such views are termed *sections*, and, with reference to the *planes of section*, are denominated *vertical and horizontal sections*. To all such drawings is given the general title of *geometrical drawings*, as distinguished from perspective drawings.

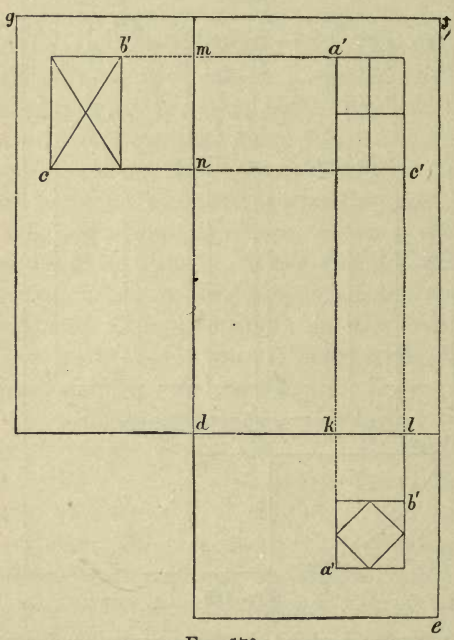


Fig. 170.

In practice, the drawings are done upon one common surface, the plane of paper, and we may readily suppose the plane *d g* (Fig. 169) revolved back into the position *d g′*, and *d e* also moved to *d e′*, both of these positions being in the plane of *d f*. This being done, we have the three views depicted on one plane surface (Fig. 170). In this figure, the same letters of reference are employed as in Fig. 169; *d l* and *d m* are the ground and vertical lines. It is evident that the positions of the same points in *a′ c′* and *a′ b′* are in the same perpendicular from the ground-line: that, in short, the position of a point in the plane may be found by applying the edge of the square to the same point as represented in the elevation. The same remark is applicable as between the two elevations. Hence the method of drawing several views of one machine upon the same surface of paper in strict agreement with each other.

PROJECTIONS OF SIMPLE BODIES.

In most of the following examples, the projections of the bodies are given both with and without the construction lines.

Right projections of a regular hexagonal pyramid (Fig. 171).—It is evident that two distinct geometrical views are necessary to convey a complete idea of the form of the object : an elevation to represent the sides of the body, and to express its height ; and a plan to express the form horizontally.

Draw a horizontal straight line L T through the center of the sheet to represent the ground-line. Then draw a perpendicular S S′ to the ground-line to represent the axis of the pyramid. For the sake of preserving the symmetry of the drawing, the centers of the horizontal projections of Figs. 171 and 172 are in the same straight line A′ S′, drawn parallel to the ground-line.

In delineating the pyramid, it is necessary, in the first place, to construct the plan. Take any point, S′, on the line S S′ as the center of the figure, and from this point, with a radius equal to the side of the hexagon which forms the base of the pyramid, describe a circle, cutting A′ S′ at A′ and D′. From these points with the same radius, draw four arcs of circles, cutting the primary circle in four points. These six points being joined by straight lines, will form the figure A′ B′ C′ D′ E′ F′, the base of the pyramid ; and the lines A′ S′, B′ S′, etc., will represent the projections of its edges shortened as they would appear in the plan.

By the help of this plan the vertical projection of the pyramid may be easily constructed. Since its base rests upon the horizontal plane, it must be projected vertically upon the ground-line ; therefore, from each of the angles at A′, B′, C′, and D′, erect perpendiculars to that line. The points of intersection, A, B, C, and D, are the true positions of all the angles of the base ; and it only remains to lay off the height of the pyramid, from the point G to S, and to draw S A, S B, S C, and S D, which are the only edges of the pyramid visible in the elevation. Of these it is to be remarked that S A and S D alone, being parallel to the vertical plane, are seen in their true length ; and, moreover, that from the assumed position of the solid under examination, the points F′ and E′ being situated in the lines B B′ and C C′, the lines S B and S C are each the projections of two edges of the pyramid.

To construct the projections of the same pyramid, having its base set in an inclined position, but with its edges S A *and* S D *still parallel to the vertical plane* (Fig. 172).

It is evident that, with the exception of the inclination, the vertical projection of this solid is precisely the same as in the preceding example, and it is only necessary to copy that elevation. To do this, fix the position of the point D upon the ground-line, through which draw D A, making with L T the desired inclination of the base of the pyramid. Make D A equal to the A D of the preceding figure, and on this erect the vertical projection S A D of that figure.

Since the edges S A and S D are still parallel to the vertical plane, and

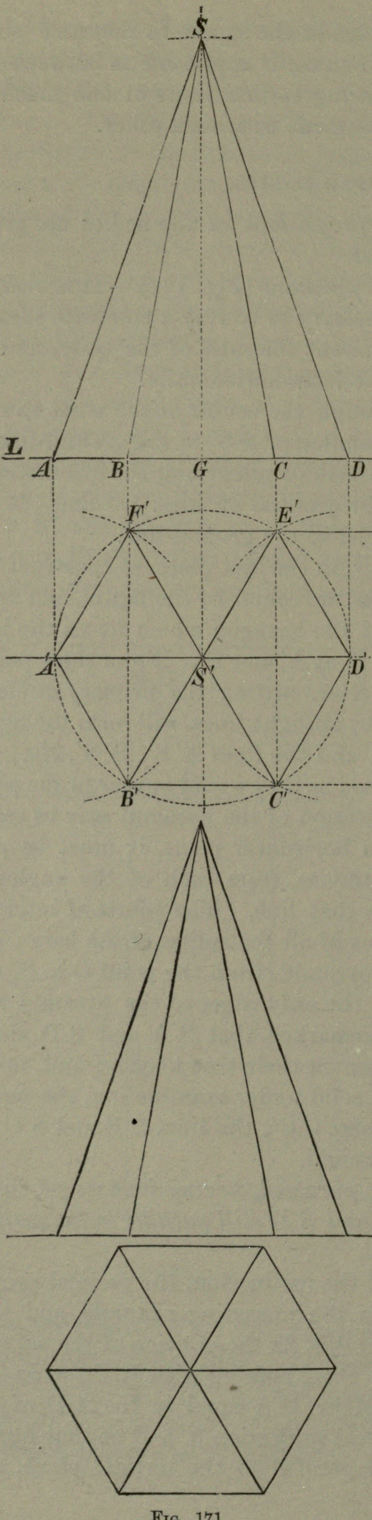


Fig. 171.

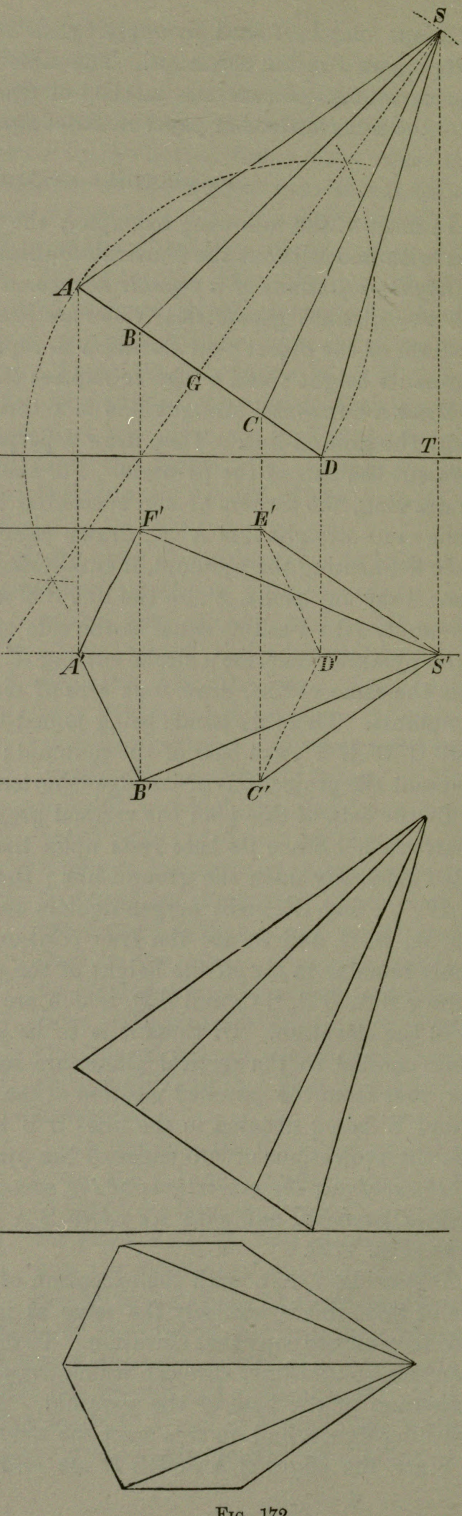


Fig. 172.

the point D remains unaltered, the projection A′ of the point A will still be in the line M N. The remaining points B′, C′, etc., in the projection of the base, are found by the intersections of perpendiculars let fall from the corresponding points in the elevation, with lines drawn parallel to M N, at a distance equal to the width of the base. By joining all the contiguous points, we obtain A′ B′ C′ D′ E′ F′, the horizontal projection of the base, two of its sides, however, are dotted, being concealed by the body of the pyramid. The vertex S having been similarly projected to S′, and joined by straight lines to the several angles of the base, the projection of the solid is completed.

To find the horizontal projection of a transverse section of the same pyramid, made by a plane perpendicular to the vertical, but inclined at an angle to the horizontal plane of projection; and let all the sides of the base be inclined to the ground-line (Fig. 173).

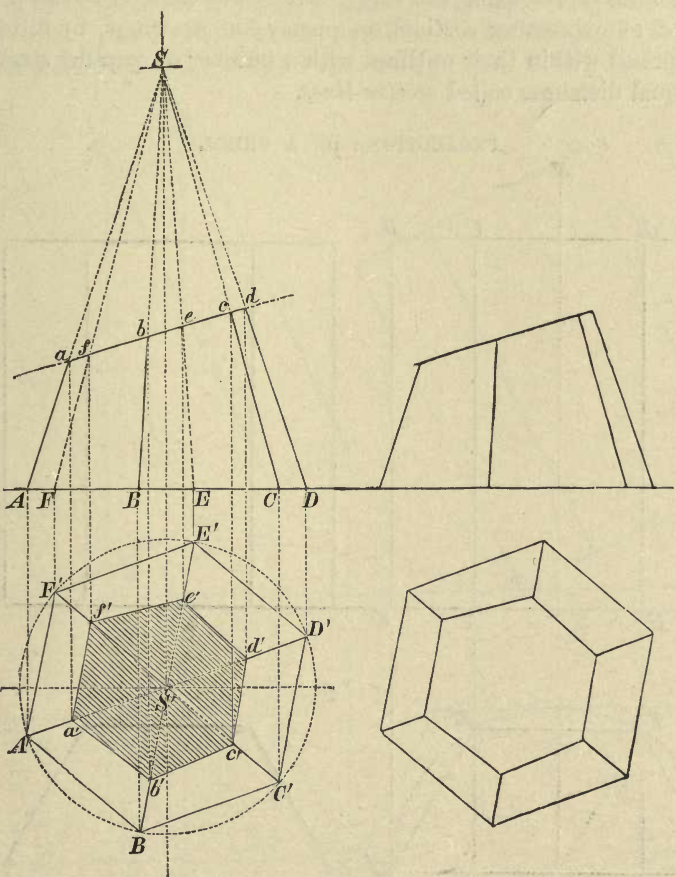


Fig. 173.

Since none of the sides of the base are to be parallel with the ground-line, draw a diameter A′ D′ making the required angle with that line, and from the points A′ and D′ proceed to set out the angular points of the hexagon as in the figure. Then, in order to obtain the projections of the edges of the pyramid, join the angular points which are diametrically opposite; and, following the method pointed out in reference to Fig. 171, project the figure thus obtained upon the vertical plane, as shown in the elevation.

Now, if the cutting plane be represented by the line *a d* in the elevation, it is obvious that it will expose, as the section of the pyramid, a polygon whose angular points, being the intersections of the various edges with the cutting plane, will be projected in perpendiculars drawn from the points where it meets these edges respectively. If, therefore, from the points *a*, *f*, *b*, etc., we let fall the perpendiculars *a a′*, *f f′*, *b b′*, etc., and join their contiguous points of intersection with the lines A′ D′, F′ C′, B′ E′, etc., we shall form a six-sided figure, which will represent the section required. The edges F S and E S, being concealed in the elevation, but necessary for the construction of the plan, have been expressed in dotted lines, as also the portion of the pyramid situated above the cutting plane, which, though supposed to be removed, is necessary in order to draw the lines representing the edges. We have here introduced the ordinary method of expressing sections in purely line-drawings, by filling up the spaces comprised within their outlines with a number of parallel straight lines drawn at equal distances called *section-lines.*

PROJECTIONS OF A PRISM.

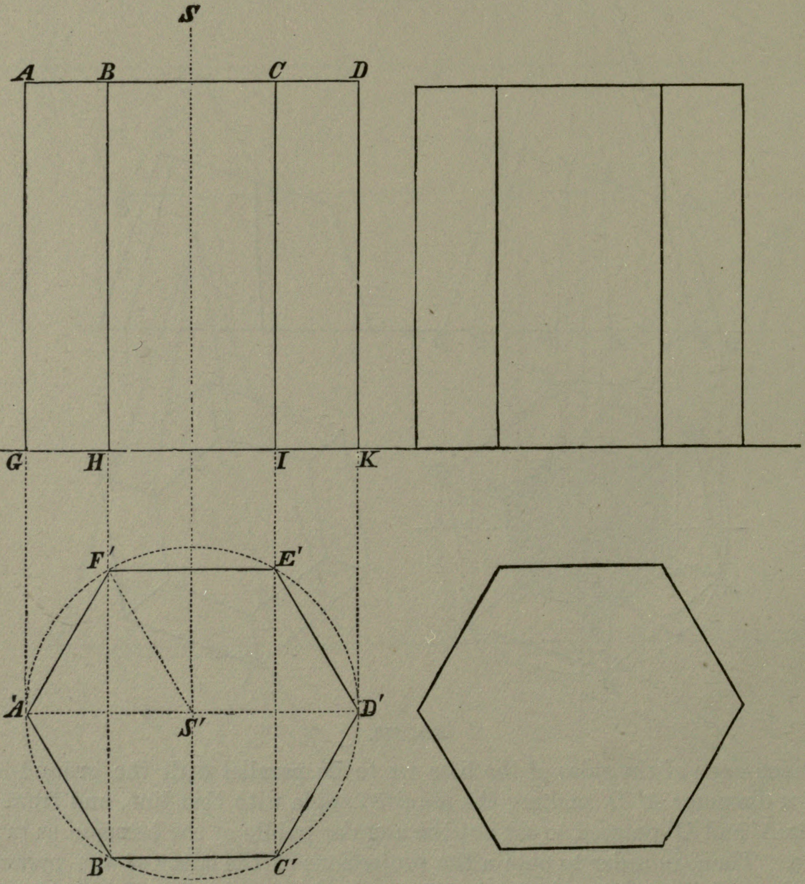


Fig. 174.

Required to represent in plan and elevation a regular six-sided prism in an upright position (Fig. 174).

Lay down the ground-line G K and draw the axis of the prism S S′. Describe the hexagonal plan A′ B′ C′ D′ E′ F′, as in the previous example. From each of the angular points, A′, B′, etc., erect perpendiculars to the ground-line, and on one of these perpendiculars set off A G, the height of the prism, and draw a parallel A D to the ground-line, which completes the vertical projection. The face, B C H I, being parallel to the vertical plane, is seen in its true size. B′ C′ being equal to one half of A′ D′, therefore H I is equal to one half of G K. We have then G H and I K equal each to one half of H I. This enables us to draw the elevation of such a prism situated as is this one without constructing the plan. This fact should be remembered in the drawing of nuts, bolt-heads, etc., in machine-drawing, where it is of frequent application.

To form the projections of the same prism, supposing it to have been moved round the point G, *in a plane parallel to the vertical plane* (Fig. 175).

Copy the elevation (Fig. 174) on the inclined base G K. Let fall perpen-

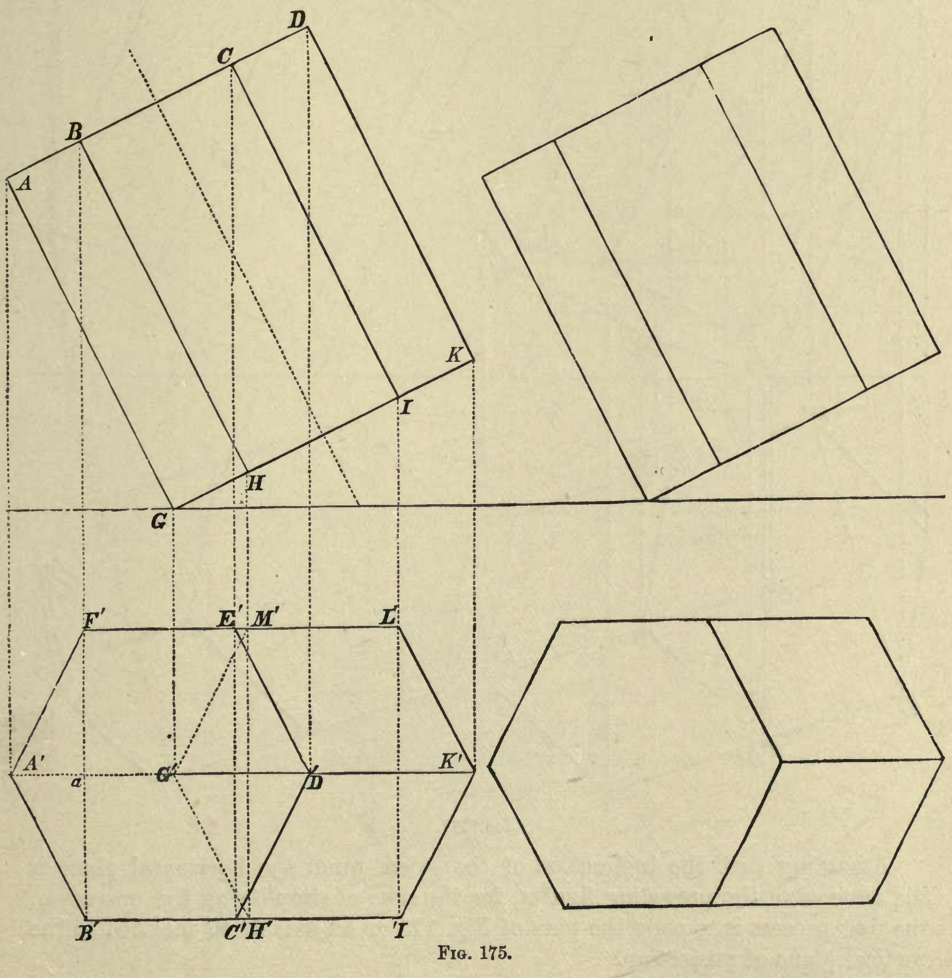


Fig. 175.

diculars from all the angles in the elevation, and, joining the contiguous points of intersection with the horizontal lines appropriate to these points respectively, the plan of course remaining the same width as before, we obtain the polygon A′ B′ C′ D′ E′ F′ as the projection of the upper surface, and G′ H′ I′ K′ L′ M′ as that of the base of the prism. Finally, it will be observed that all the edges are represented, in the horizontal projection, by equal straight lines, as D′ K′, A′ G′, etc., and that the sides A′ B′, G′ H′, etc., remain still parallel to each other, which will afford the means of verifying the accuracy of the drawings. As the upper surface and the base are seen obliquely in this projection, of course they do not appear as true hexagons in the plan.

Required the projections of the same prism set into a position inclined to both planes of projection (Fig. 176).

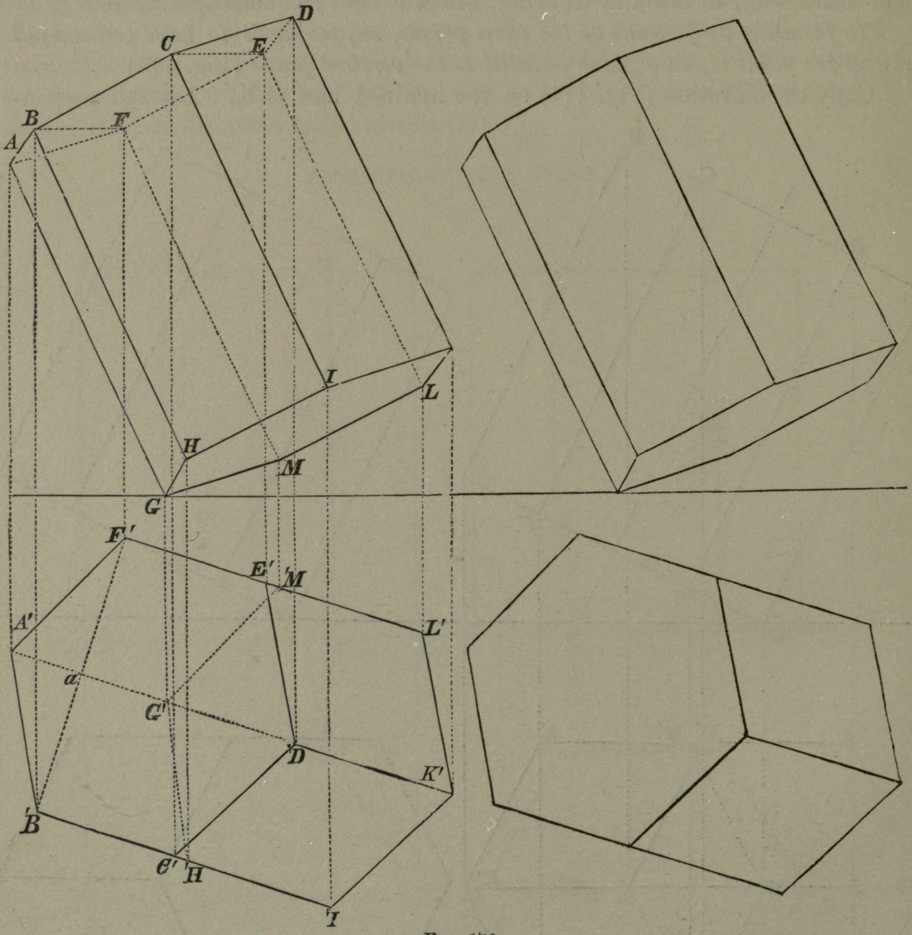


Fig. 176.

Assuming that the inclination of the prism upon the horizontal plane is the same as in the preceding figures, for the sake of simplifying the operation, the first process is to copy the plan of Fig. 175 on an axis A′ K′ inclined to the vertical plane of projection.

Now, since the prism has been supposed to have preserved its former inclination to the horizontal plane, it is obvious that every point in it, such as A, has, in assuming its new position, simply moved in a horizontal plane, and will therefore be at the same distance above the ground-line that it was in the elevation (Fig. 175), and it will also be in the perpendicular A′ A ; the point of intersection A is, therefore, its projection in the elevation. The remaining angular points in this view are all determined in the same manner, and, having joined the contiguous points, and the corresponding angles of the upper and lower surface, we obtain the complete vertical projection of the prism in its doubly-inclined position.

CONSTRUCTION OF THE CONIC SECTIONS.

The *plan* of the cone (Fig. 177) is simply a circle, described from the center S′, with a diameter equal to that of the base. Its *elevation* is an isosceles triangle, obtained by drawing tangents A′ A, B′ B, perpendicular to and inter-

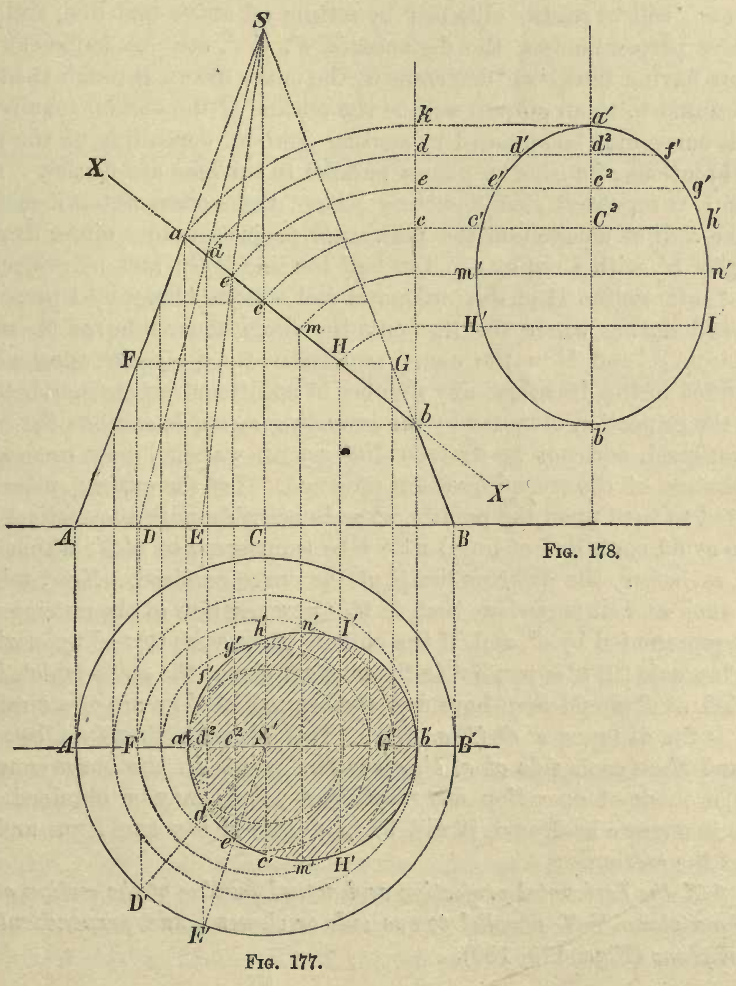


Fig. 178.

Fig. 177.

secting the ground-line; then set off upon the center line the height C S, and join S A, S B. These lines are called the exterior *elements* of the cone.

Given the projections of a cone, and the direction of a plane X X, *cutting it perpendicularly to the vertical, and obliquely to the horizontal plane; required to find, first, the horizontal projection of this section; and, secondly, the outline of the ellipse thus formed* (Figs. 177, 178).

Through the vertex of the cone draw a line S E to any point within the base A B; let fall a perpendicular from E, cutting the circumference of the base in E′, and join E′ S′; then another perpendicular let fall from *e* will intersect E′ S′ in a point *e′*, which will be the horizontal projection of a point in the curve required; and so on for any required number of points.

The exterior generatrices A S and B S being both projected upon the line A′ B′, the extreme limits of the curve sought will be at the points *a′* and *b′* on that line, which are the projections of the points of intersection *a* and *b* of the cutting plane with the outlines of the cone. And since the line *a′ b′* will obviously divide the curve symmetrically into two equal parts, the points *f′*, *g′*, *h′*, etc., will be readily obtained by setting off above that line, and on their respective perpendiculars, the distances $d' d^2$, $e' e^2$, etc. A sufficient number of points having thus been determined, the curve drawn through them (which will be found to be an ellipse) will be the outline of the section required.

This curve may be obtained by another method, depending on the principle that all sections of a cone by planes parallel to the base are circles. Thus, let the line F G represent such a cutting plane; the section which it makes with the cone will be denoted on the horizontal projection by a circle drawn from the center S′, with a radius equal to half the line F G; and by projecting the point of intersection H of the horizontal and oblique planes by a perpendicular H H′, and noting where this line cuts the circle above referred to, we obtain two points H′ and I′ in the curve required. By a similar construction, as exemplified in the drawings, any number of additional points may be found.

As the projection obtained by the preceding methods exhibits the section as fore-shortened, and not in its true dimensions, we shall now proceed to the consideration of the second question proposed. Let the cutting plane X X be conceived to turn upon the point *b*, so as to coincide with the vertical line *b k*, and (to avoid confusion of lines) let *b k* be transferred to *a′ b′*, which will represent, as before, the extreme limits of the curve required. Now, taking any point, such as *d*, it is obvious that, in this new position of the cutting plane, it will be represented by d^2, and, if the cutting plane were turned upon *a′ b′* (Fig. 178) as an axis till it is parallel to the vertical plane, the point which had been projected at d^2 would then have described round *a′ b′* an arc of a circle, whose radius is the distance $d' d^2$ (Fig. 177). This distance, therefore, being set off at *d′* and *f′* on each side of *a′ b′*, gives two points in the curve sought. By a similar mode of operation any number of points may be obtained, through which, if a curve be drawn, it will be an ellipse of the true form and dimensions of the section.

To find the horizontal projection and actual outline of the section of a cone, made by a plane Y Y *parallel to one side or element, and perpendicular to the vertical plane* (Figs. 179, 180).

Determine by the second method laid down in the preceding problem any number of points, as F′, G′, J′, K′, etc., in the curve representing the horizontal projection of the section specified. The horizontal plane passing through M gives only one point M′, which is the vertex of the curve sought.

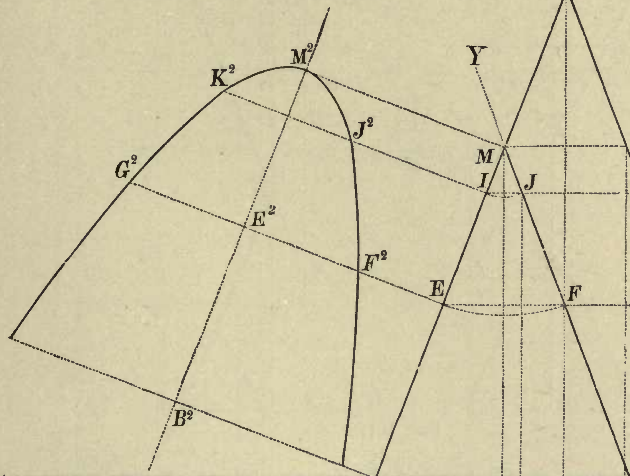


Fig. 180.

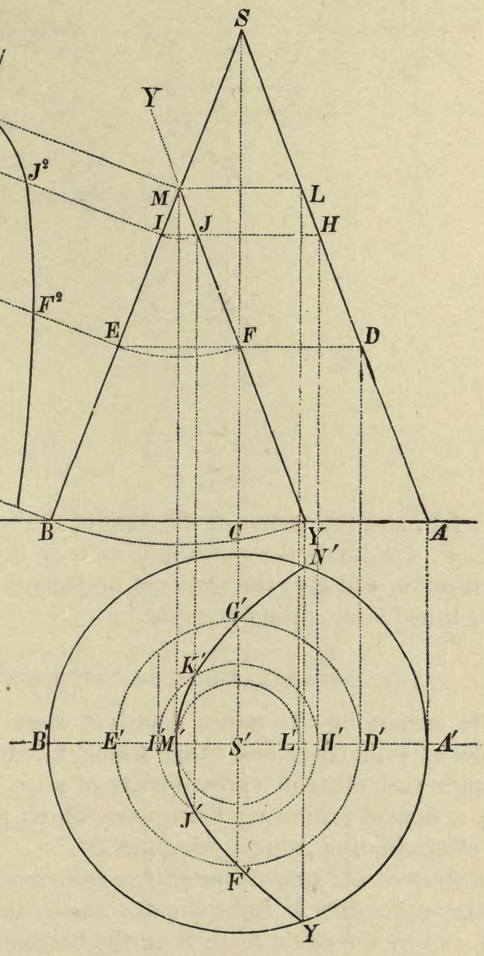


Fig. 179.

In order to determine the actual outline of this curve, suppose the plane Y Y to turn as upon a pivot at M, until it has assumed the position M B, and transfer M B parallel to itself to M^2 B^2 (Fig. 180). The point F will thus have first described the arc F E till it reaches the point E, which is then projected to E^2; suppose the given plane, now represented by M^2 B^2, to turn upon that line as an axis, until it assumes a position parallel to the vertical plane, the point E^2, which is distant from the axis M′ B′ by the distance F′ S′ (Fig. 179), will now be projected to F^2 and G^2, two points in the curve required, which is a *parabola*.

To draw the vertical projection of the sections of two opposite cones made by a plane parallel to their axis (Fig. 181).

Let C E D and C B A be the two cones, and X X the position of the cutting plane. Project in plan either of the cones, as *b* E′ D′; from its center, with a radius equal to L H, describe a circle, and draw the tangent *b a*; *b a* will be the horizontal projection of the cutting plane. Draw the line H′ M′ parallel to the cutting plane; H′, M′ corresponding in position to the intersections H, M, of the plane with the cones. From H′ and M′ lay off distances equal to L K, K I, and the length of the cone, and through these points draw perpendiculars, as *f′ e′*, *d′ c′*, *b′ a′*, etc., which must be made equal to the chords *f e*, *d c*, *b a*, made by the cutting plane *a b*, with circles whose radii are

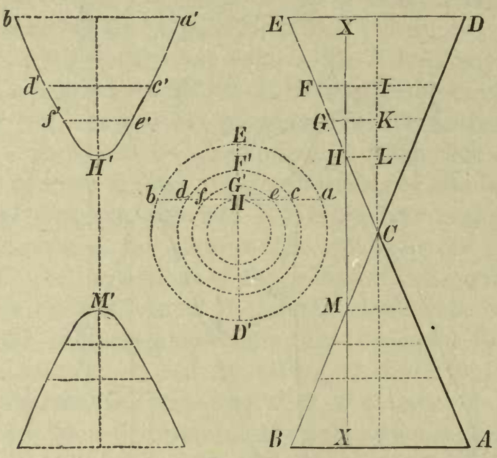


Fig. 181.

G K, I F, and the radius of the base of the cone. Through the points *a′*, *c′*, *e*, H′, *f′*, *d′*, *b′*, draw the curve, and we have the projection required. A similar construction will give the sectional projection of the opposite cone at M′. The curve thus found is the *hyperbola*.

PENETRATIONS OR INTERSECTIONS OF SOLIDS.

On examining the minor details of most machines, we find numerous examples of cylindrical and other forms, fitted to, and even appearing to pass through, each other in a great variety of ways. The examples given are selected with a view of exhibiting those cases which are of most frequent occurrence, and of elucidating general principles.

Represent the projections of two cylinders of unequal diameters (Fig. 182) meeting each other at right angles ; one by the rectangle A B E D in the vertical, and by the circle A′ H′ B′ in the horizontal projections ; the other, which is supposed to be horizontal, is indicated in the former by the circle L P I N, and in the latter by the rectangle L′ I′ K′ M′. From the position of these two solids it is evident that the curves formed by their junction will be projected horizontally in the curves O′ H′ P′, R′ S′ T′, and vertically by L P I N.

But, if the position of these bodies be changed into that represented by Fig. 183, the lines of their intersection will assume in the vertical projection a totally different aspect, and may be accurately determined as follows :

Through any point taken upon the plan of Fig. 183 draw a horizontal line *a′ b′*, which is to be considered as indicating a plane cutting both cylinders parallel to their axes ; this plane would cut the vertical cylinder in lines drawn perpendicularly through the points *c′* and *d′*. To find the vertical projection of its intersection with the other cylinder, conceive its base I′ L′, after being

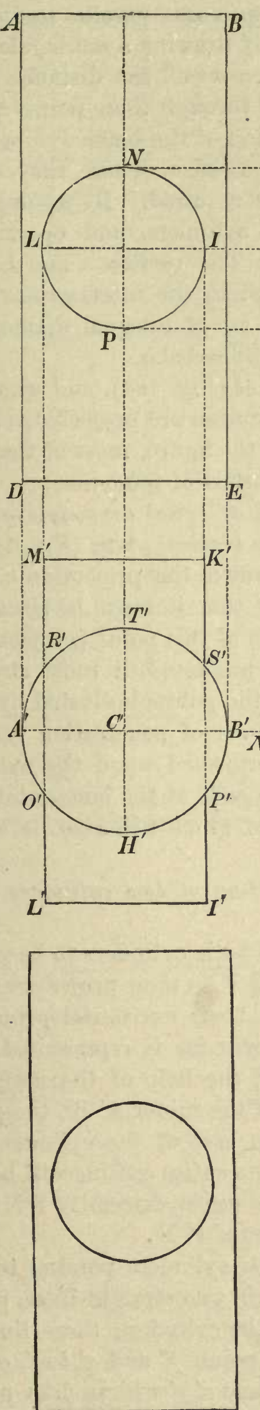


Fig. 182.

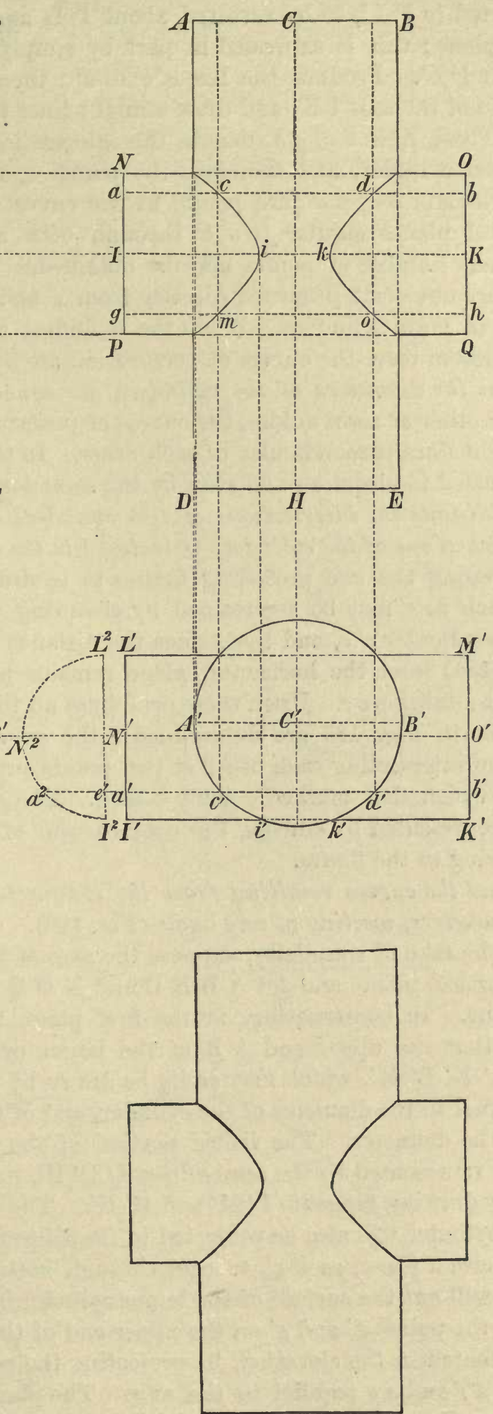


Fig. 183.

transferred to $I^2 L^2$, to be revolved about $I^2 L^2$ as an axis parallel to the horizontal plane; this is expressed in part by simply drawing a semicircle of the diameter $I^2 L^2$. Produce the line $a' b'$ to a^2; then set off the distance $a^2 e'$ on each side of the axis I K, and draw straight lines through these points parallel to it. These lines *a b, g h,* denote the intersection of the plane $a' b'$ with the horizontal cylinder, and therefore the points *c, d, m, o,* where they cut the perpendiculars $c\ c'$, $d\ d'$, are points in the curve required. By passing other horizontal planes similar to $a' b'$ through both cylinders, and operating as before, any number of points may be obtained. The vertices *i* and *k* of the curves are obviously projected directly from i' and k', the intersections of the outlines of both cylinders. When the cylinders are of unequal diameters, as in the present case, the curves of penetration are hyperbolas.

When the diameters of the cylinders are equal (Fig. 184), and when they cut each other at right angles, the curves of penetration are projected vertically in straight lines perpendicular to each other. In the figure, most of the points are indicated in elevation and plan by the same letters of reference.

To delineate the intersections of two cylinders of equal diameters at right angles, when one of the cylinders is inclined to the vertical plane (Fig. 185).

Supposing the two preceding figures to be drawn, the projection *c* of any point such as c' may be ascertained by observing that it must be situated in the perpendicular $c'\ c$, and that, since the distance of this point (projected at *c* in Fig. 184) from the horizontal plane remains unaltered, it must also be in the horizontal line *c c.* Upon these principles all the points indicated by literal references in Fig. 185 are determined; the curves of penetration resulting therefrom intersecting each other at two points projected upon the axial line L K, of which that marked *q* alone is seen. The ends of the horizontal cylinder are represented by ellipses, the construction of which will also be obvious on referring to the figure.

To find the curves resulting from the intersection of two cylinders of unequal diameters, meeting at any angle (Fig. 186).

For the sake of simplicity, suppose the axes of both cylinders to be parallel to the vertical plane, and let A B E D and N O Q P be their projections upon that plane. In constructing, in the first place, their horizontal projection, observe that the upper end A B of the larger cylinder is represented by an ellipse A′ K′ B′ M′, which may easily be drawn by the help of the major axis K′ M′ equal to the diameter of the cylinder, and of the minor A′ B′, the projection of the diameter. The visible portion of the base of the cylinder being similarly represented by the semi-ellipse L′ D′ H′, its entire outline will be completed by drawing tangents L′ M′ and H′ K′. The upper extremity P N of the smaller cylinder will also be projected in the ellipse $p'\ i'$ N′.

Conceive a plane, as $a'\ g'$, to pass through both cylinders parallel to their axes; it will cut the surface of the larger cylinder in two straight lines, passing through the points f' and g' on the upper end of the cylinder; these lines will be represented in the elevation, by projecting the points f' and g' to *f, g*; and drawing *a f* and *c g* parallel to the axis. The plane $a'\ g'$ will in like manner cut the smaller cylinder in two straight lines, which will be represented in the vertical projection by *d h* and *e i,* and the intersections of these lines with *a f*

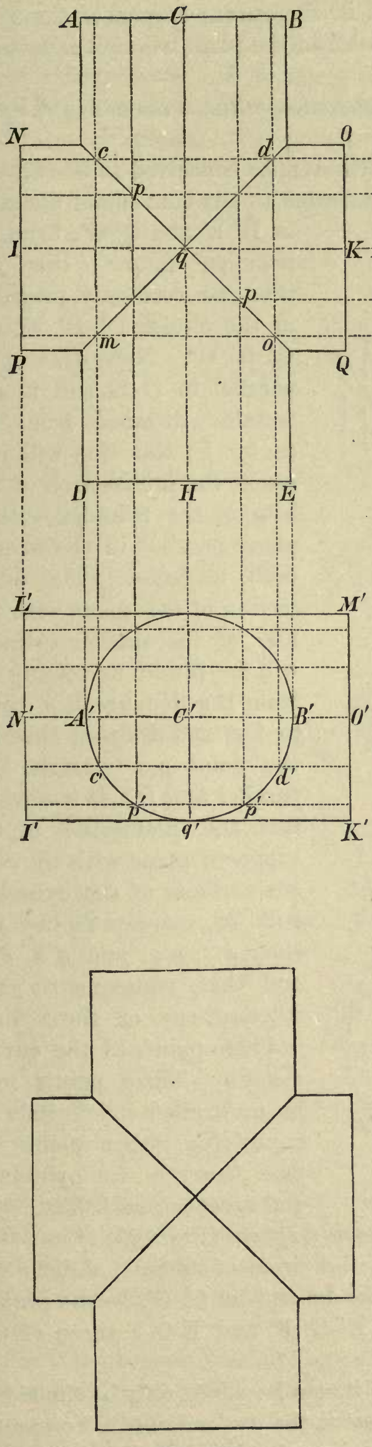


Fig. 184.

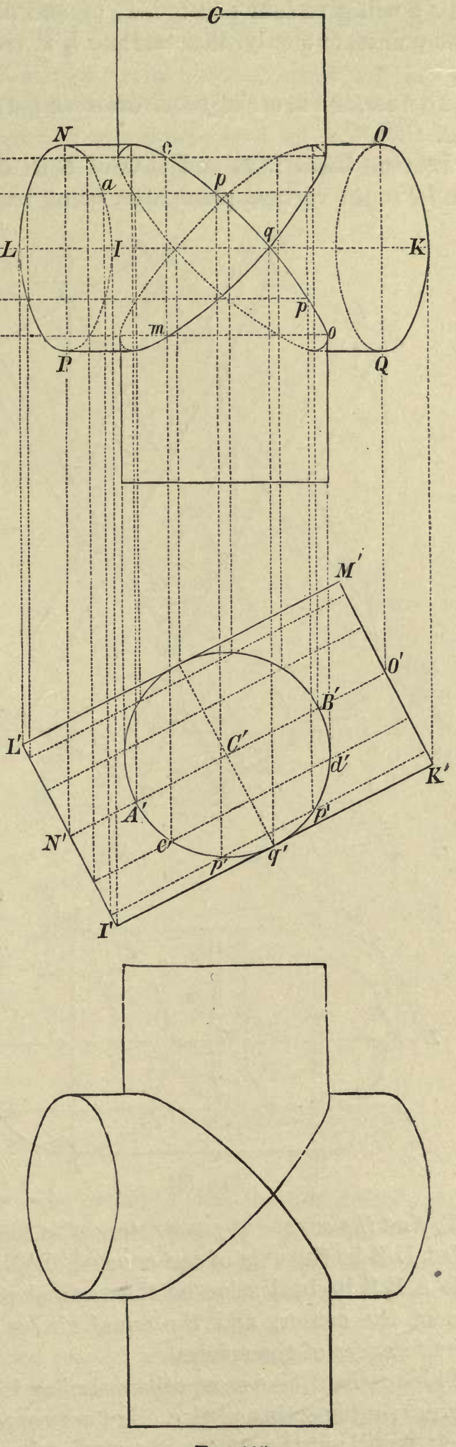


Fig. 185.

and *c g* will give four points *l, k, m,* and *n,* in the curves of penetration. Of these points, one only, that marked *l,* is visible in the plan, where it is denoted by *l′*.

To find the curves of penetration in the elevation without the aid of the plan (Fig. 186).

Let the bases D E and Q O of both cylinders be conceived to be revolved parallel to the vertical plane after being transferred to any convenient distance, as D^2 E^2 and Q^2 O^2, from the principal figure; they will then be vertically projected in the circles D^2 H^2 E^2 and Q^2 G′ O^2. Now draw a^2 c^2 parallel to D E, and at any suitable distance from the center I; this line will represent the intersection of the base of the cylinder with a plane parallel to the axes of both, as before. The intersection of this plane with the base of the smaller cylinder will be found by setting off from R a distance R *p*, equal to I *o*, and drawing through the point *p* a straight line parallel to Q O. It is obvious that the intersection of the supposed plane with the convex surfaces of the cylinders will be represented by the lines *a f, c g,* and *d h, e i*; and that, consequently, the intersections of these lines indicate points in the curves sought. These points may be multiplied indefinitely by conceiving other planes to pass through the cylinders, and operating as before.

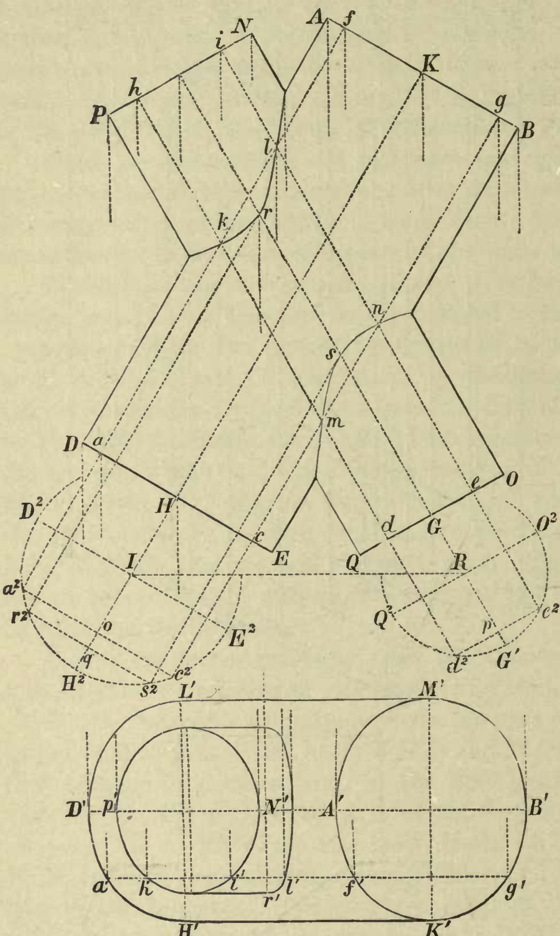


Fig. 186.

To find the curves of penetration of a cone and sphere (Fig. 187).

Let D S be the axis of the cone, A′ L′ B′ the circle of its base, and the triangle A B S its projection on the vertical plane; and let C, C′, be the projections of the center, and the equal circles E′ K′ F′ and E G F those of the circumferences of the sphere.

This problem, like most others similar to it, can be solved only by the aid of imaginary intersecting planes. Let *a b* represent the projection of a horizontal plane; it will cut the sphere in a circle whose diameter is *a b,* and which is partially drawn from the center C′ in the plan, as *a′ f′ b′*. Its intersection with the cone is also a circle described from the center S′ with the diameter *c d* as *c′ f′ d′* ; the points *e′* and *f′*, where these two circles cut each other, are the horizontal projections of two points in the lower curve, which is evidently entirely hidden by the sphere. The points referred to are projected vertically upon the line *a b* at *e* and *f*. The upper curve, which is seen in both projections, is ob-

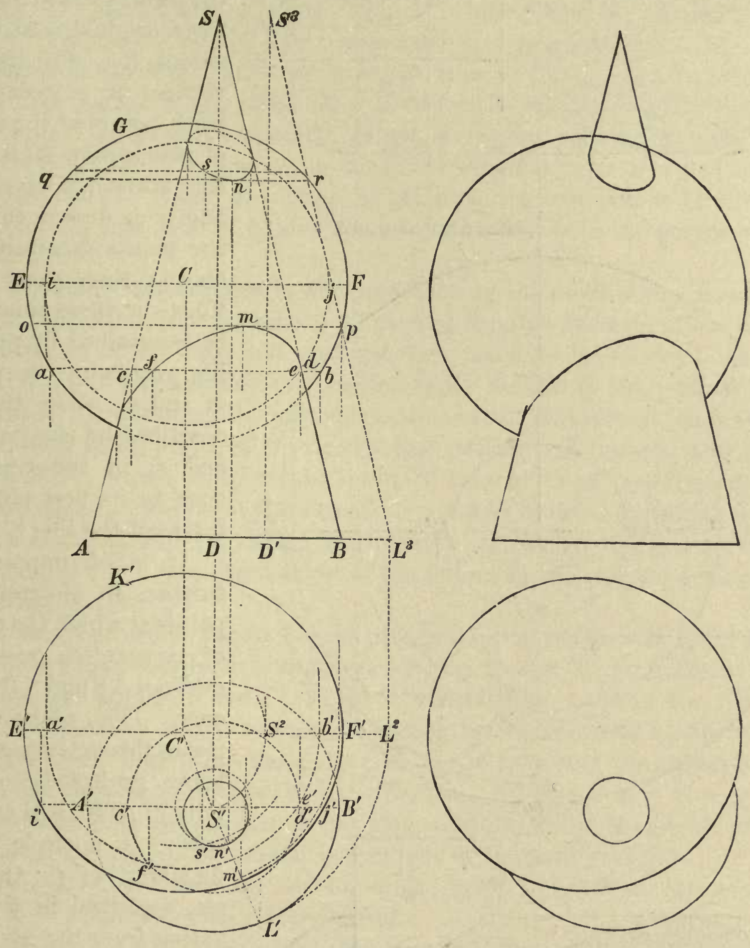


FIG. 187.

tained by a similar process ; but it is to be observed that the horizontal cutting planes must be taken in such positions as to pass through both solids in circles which shall intersect each other. For our guidance in this respect it will be necessary, first, to determine the vertices *m* and *n* of the curves of penetration.

For this purpose, conceive a vertical plane passing through the axis of the cone and the center of the sphere ; its horizontal projection will be the straight line C′ L′ joining the centers of the two bodies. Let us also make the supposition that this plane is turned upon the line C C′ as on an axis, until it becomes parallel to the vertical plane ; the points S′ and L′ will now have assumed the positions S^2 and L^2, and consequently the axis of the cone will be projected vertically in the line D′ S^3, and its side in S^3 L^3, cutting the sphere at the points *p* and *r*. Conceive the solids to have resumed their original relative positions, it is clear that the vertices or adjacent limiting points of the curves of penetration must be in the horizontal lines *p o* and *r q*, drawn through the points determined as above ; their exact positions on these lines may be ascertained by projecting vertically the points *m′* and *n′*, where the arcs described by the points *p* and *r*, in restoring the cone to its first position, intersect the line S′ L′.

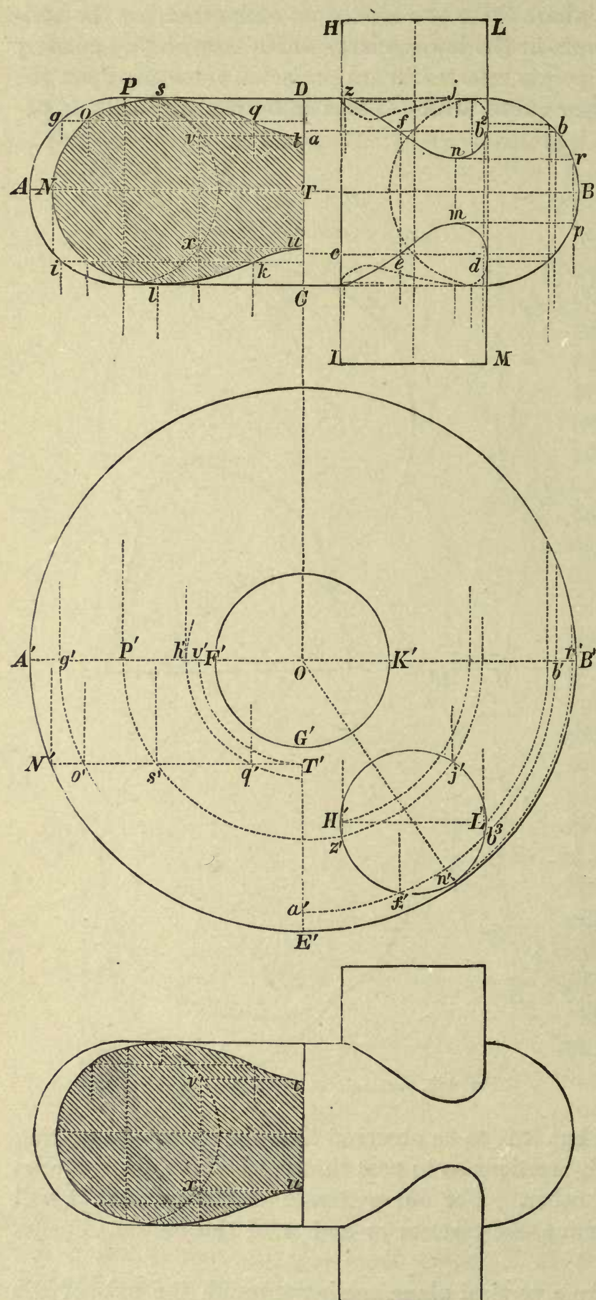


Fig. 188.

It is of importance, further, to ascertain the points at which the curves of penetration meet the outlines A S and S B of the cone. The plane which passes through these lines, being projected horizontally in A′ B′, will cut the sphere in a circle whose diameter is *i′ j′* ; this circle, described in the elevation from the center C, will cut the sides A S and S B in four points, at which the curves of penetration are tangent to the outlines of the cone.

To find the lines of penetration of a cylinder and a cylindrical ring or torus (Fig. 188).

Let the circles A′ E′ B′, F′ G′ K′, represent the horizontal, and the figure

A C B D the vertical projection of the torus, and let the circle H′ f′ L′, and the rectangle H I M L be the analogous projections of the cylinder, which passes perpendicularly through it. Conceive, as before, a plane, *a b*, to pass horizontally through both solids ; it will obviously cut the cylinder in a circle which will be projected in the base H′ f′ L′ itself, and the ring in two other circles, of which one only, part of which is represented by the arc f' b^3 b', will intersect the cylinder at the points f' and b^3, which, being projected vertically, will give two points *f* and b^2 in the upper curve of penetration.

Another horizontal plane, taken at the same distance *below* the center line A B as that marked *a b* is above it, will evidently cut the ring in circles coinciding with those already obtained ; consequently the points f' and b^3 indicate points in the lower as well as in the upper curves of penetration, and are projected vertically at *d* and *e*. Thus, by laying down two planes at equal distances on each side of A B, by one operation four points in the curves required are determined.

To determine the vertices *m* and *n*, following the method explained in the preceding problem, draw a plane O *n′*, passing through the axis of the cylinder and the center of the ring, and conceive this plane to be revolved about the point O until it has assumed the position O B′, parallel to the vertical plane ; the point *n′*, representing the extreme outline of the cylinder in plan, will now be at *r′*, and, being projected vertically, that outline will cut the ring in two points *p* and *r*, which would be the limits of the curves of penetration in the supposed relative position of the two solids ; and by drawing the two horizontal lines *r n* and *p m*, and projecting the point *n′* vertically, the intersections of these lines, *m* and *n*, are the vertices of the curves in the actual position of the penetrating bodies.

The points at which the curves are tangents to the outlines H I and L M of the cylinder, may readily be found by describing arcs of circles from the center O through the points H′ and L′, which represent these lines in the plan, and then proceeding, as above, to project the points thus obtained upon the elevation. Lastly, to determine the points, as *j*, *z*, etc., where the curves are tangents to the horizontal outlines of the ring, draw a circle P′ *s′ j′* with a radius equal to that of the center line of the ring, namely, P D ; the points of intersection *z′* and *j′* are the horizontal projections of the points sought.

Required to represent the section which would be made in this ring by a plane, N′ T′, *parallel to the vertical plane.*

Such a section will be represented in its actual form and dimensions in the elevation. To determine its outlines, let two horizontal planes, *g q* and *i k*, equidistant from the center line A B, be supposed to cut the ring ; their lines of intersection with it will have their horizontal projections in the two circles *g′ o′* and *h′ q′*, which cut the given plane N′ T′ in *o′* and *q′*. These points being projected vertically to *o*, *q*, *k*, etc., give four points in the curve required. The line N′ T′ cutting the circle A′ E′ B′ at N′, the projection N of this point is the extreme limit of the curve.

The circle P′ *s′ j′*, the center line of the rim of the torus, is cut by the planes N′ T′ at the point *s′*, which, being projected vertically upon the lines D P and C *l*, determines *s* and *l*, the points of contact of the curve with the horizontal

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outlines of the ring. Finally, the points t and u are obtained by drawing from the center O a circle, T′ v', tangent to the given plane, and projecting the point of intersection v' to the points v and x, which are then to be replaced upon C D by drawing the horizontals v t and x u.

Required to delineate the lines of penetration of a sphere and a regular hexagonal prism whose axis passes through the center of the sphere (Fig. 189).

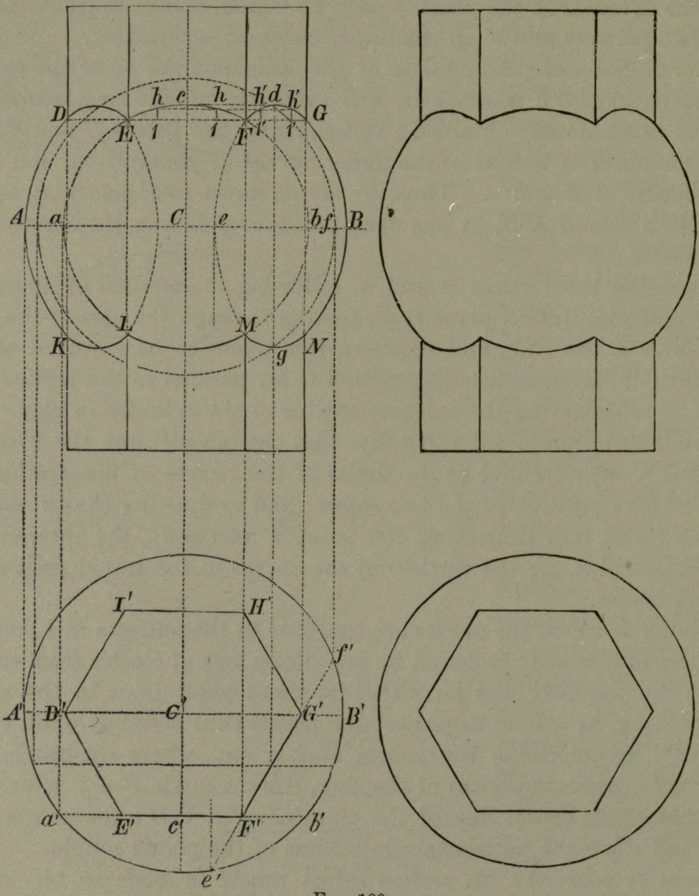


Fig. 189.

The centers of the circles forming the two projections of the sphere are, according to the terms of the problem, upon the axis C C′ of the upright prism, which is projected horizontally in the regular hexagon D′ E′ F′ G′ H′ I′. Hence it follows that, as all the lateral faces of the prism are equidistant from the center of the sphere, their lines of intersection with it will necessarily be circles of equal diameters. The perpendicular face, represented by the line E′ F′ in the plan, will meet the surface of the sphere in two circular arcs, E F and L M, described from the center C, with a radius equal to c' b' or a' c'. And the intersections of the two oblique faces D′ E′ and F′ G′ will obviously be each projected in two arcs of an ellipse, whose major axis d g is equal to e' f', and the minor axis is the vertical projection of e' f'. But, as it is necessary to draw small portions only of these curves, the following method may be employed :

Draw D G through the points E, F ; divide the portions E F and F G respectively into the same number of equal parts, and, drawing perpendiculars through the points of division, set off from F G the distances from the corresponding points in E F to the circular arc E C F, as points in the elliptical arc required. The remaining elliptical arcs can be traced by the same method.

Required to draw the lines of penetration of a cylinder and a sphere, the center of the sphere being without the axis of the cylinder (Fig. 190).

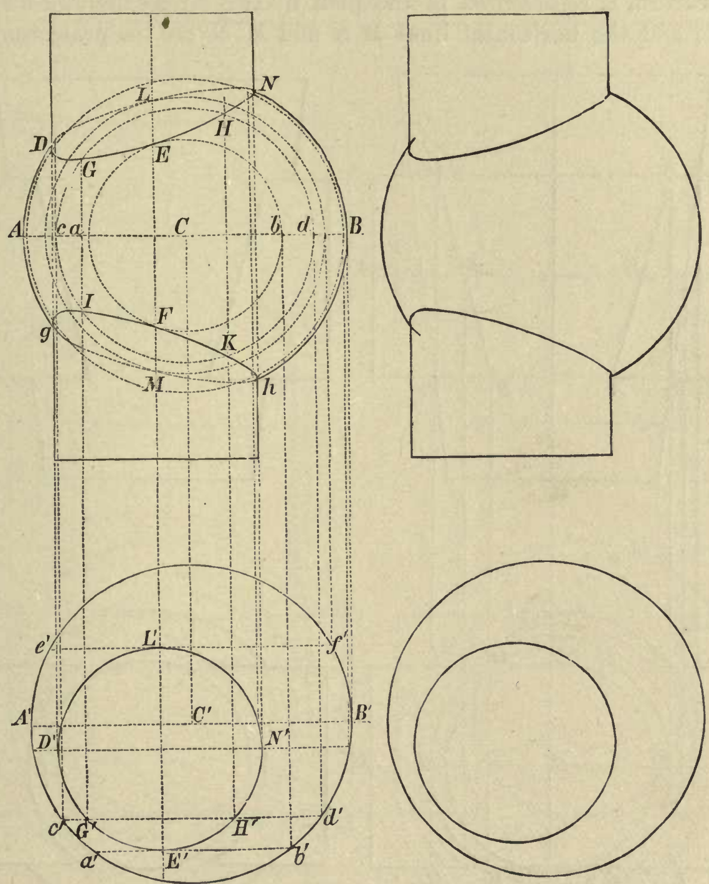


FIG. 190.

Let the circle D′ E′ L′ be the projection of the base of the given cylinder, and let A B be the diameter of the given sphere. If a plane, as *c′ d′*, be drawn parallel to the vertical plane, it will evidently cut the cylinder in two straight lines, G G′, H H′. This plane will also cut the sphere in a circle described from the center C with a radius of half the line *c′ d′* ; its intersection with the lines G G′ and H H′ will give so many points in the curves sought, viz., G, H, I, K.

The planes *a′ b′* and *e′ f′*, which are tangents to the cylinder, furnish respectively only two points in the curves ; of these points, E and F alone are visible, the other two, L and M, being concealed by the solid ; therefore, the planes drawn for the construction of the curves must be all taken between *a′ b′* and *e′ f′*. The plane which passes through the axis of the cylinder cuts the sphere in a circle whose projection upon the vertical plane will meet at the points D, N, and *g, h,* the outlines of the cylinder, to which the curves of penetration are tangents.

To find the lines of penetration of a frustum of a cone and a prism (Fig. 191).

The frustum is represented in the plan by two circles described from the center C′ ; and the horizontal lines M N and M′ N′ are the projections of the

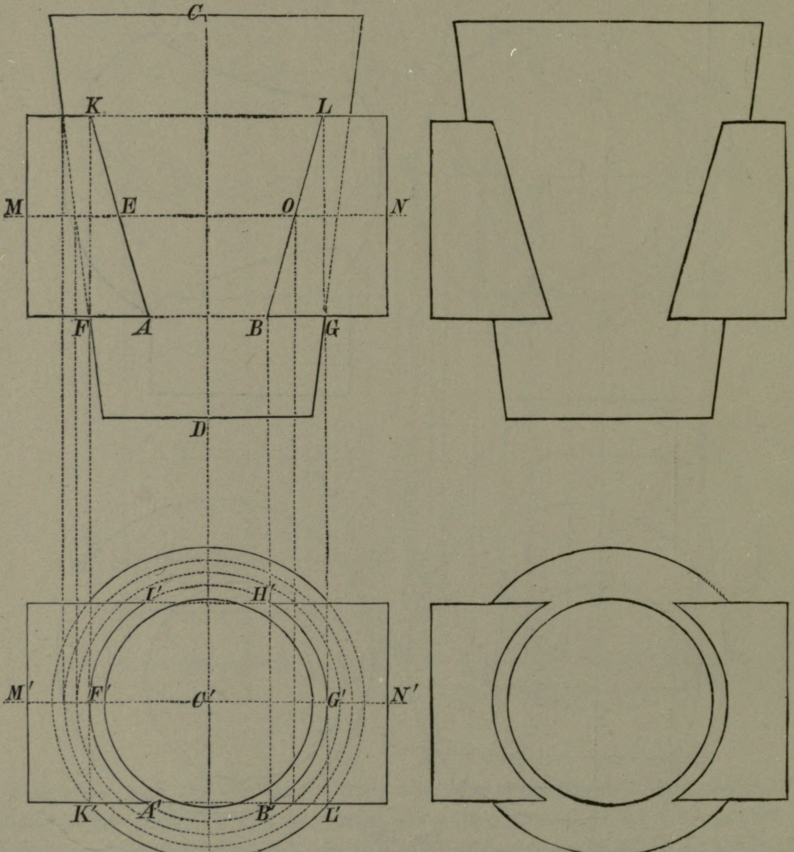


Fig. 191.

axis of a prism of which the base is square, and the faces respectively parallel and perpendicular to the planes of projection.

In laying down the plan of this solid, it is supposed to be inverted, in order that the smaller end of the cone and the lines of intersection of the lower surface, F G, of the prism may be exhibited. According to this arrangement, the letters A′ and B′ ought, strictly speaking, to be marked at the points I′ and H′, and conversely ; but, as it is quite obvious that the part above M′ N′ is exactly symmetrical with that below it, the distribution of the letters of reference adopted in our figures can lead to no confusion.

The intersection of the plane F G with the cone is projected horizontally in a circle described from the center C′, with the diameter F′ G′. The arcs I′ F′ A′ and H′ G′ B′ are the only parts of this circle which require to be drawn. In the vertical projection the extreme points K, L, A, B need only be found, for the lines of intersection are here projected straight.

To describe the curves formed by the intersection of a cylinder with the frustum of a cone, the axes of the two solids cutting each other at right angles (Fig. 192).

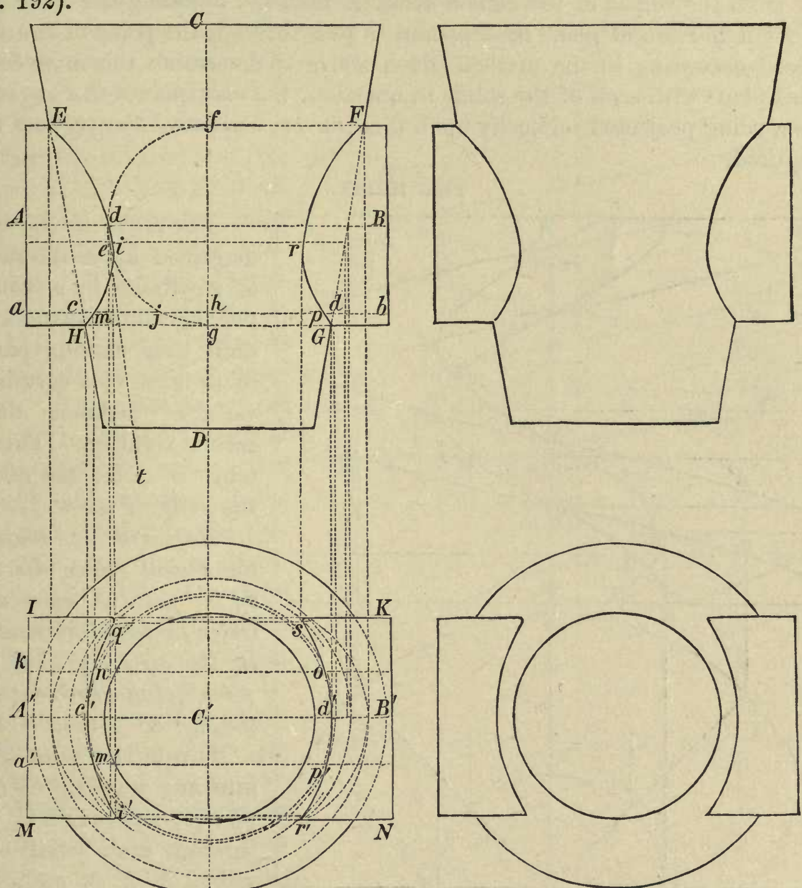


Fig. 192.

The projections of the solids are laid down in the figure precisely as in the preceding example. The intersections of the outlines in elevation furnish, obviously, four points in the curves of penetration ; these points are all projected horizontally upon the line A′ B′. Now pass a plane, as *a b*, horizontally through both solids ; its intersection with the cone will be a circle of the diameter *c d*, while the cylinder will be cut in two parallel straight lines, represented in the elevation by *a b*, and whose horizontal projection may be determined in the following manner : Conceive a vertical plane, *f g*, cutting the cylinder at right angles to its axis, and let the circle *g e f* thereby formed be described from the intersection of the axes of the two solids ; the line *j h* will now represent, in this position of the section, the distance of one of the lines sought from the axis of the cylinder. Set off this distance on both sides of the point A′, and through the points *k* and *a′* thus obtained, draw straight lines parallel to A′ B′ ; the intersections of these lines with the circle drawn from the center C′ of the diameter *c d* will give four points *m′*, *p′*, *n*, and *o*, which, being projected vertically upon *a b*, determine two points, *m* and *p*, in the curves required.

In order to obtain the vertices or adjacent limiting points of the curves, draw from the vertex of the cone a straight line, *t e*, touching the circle *g e f*, and let a horizontal plane be supposed to pass through the point of contact *e*. Proceed according to the method given above to determine the intersections of this plane with each of the solids in question, the four points *i′*, *r′*, *q*, and *s*, which, being projected vertically upon the line *e r*, determine the vertices *i* and *r* required.

THE HELIX.

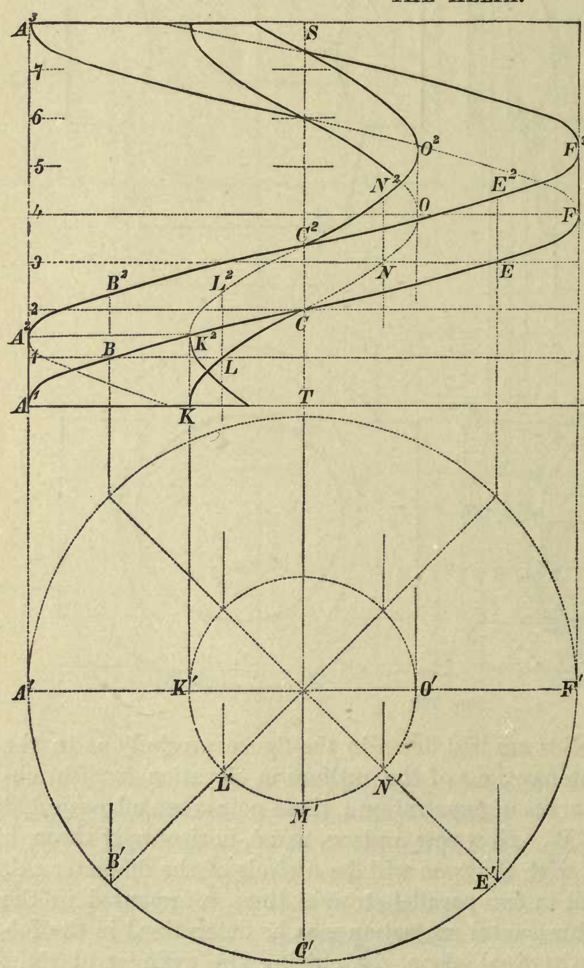


Fig. 193.

The Helix is the curve described upon the surface of a cylinder by a point revolving round it, and at the same time moving parallel to its axis by a certain invariable distance during each revolution. This distance is called the *pitch* of the helix or screw.

Required to construct the helical curve described by the point A^1 *upon a cylinder projected horizontally in the circle* A′ C′ F′, *the pitch being represented by the line* A^1 A^3 (Fig. 193).

Divide the pitch A^1 A^3 into any number of equal parts, say eight; and through each point of division, 1, 2, 3, etc., draw straight lines parallel to the ground-line. Then divide the circumference A′ C′ F′ into the same number of equal parts; the points of division, B′, C′, E′, F′, etc., will be the horizontal projections of the different positions of the given point during its motion round the cylinder. Thus, when the point is at B′ in the plan, its vertical projection will be the point of intersection B of the perpendicular drawn through B′ and the horizontal drawn through the first point of division. Also, when the point arrives at C′ in the plan, its vertical projection is the point C, where the perpendicular drawn from C′ cuts the horizontal passing through the second point of division, and so on for all the remaining points. The curve A^1 B C F A^3, drawn through all the points thus obtained, is the helix required.

To draw the vertical elevation of the solid contained between two helical surfaces and two concentric cylinders (Fig. 193).

A *helical surface* is generated by the revolution of a straight line round the axis of a cylinder, its outer end moving in a helix, and the line itself forming with the axis a constant and invariable angle.

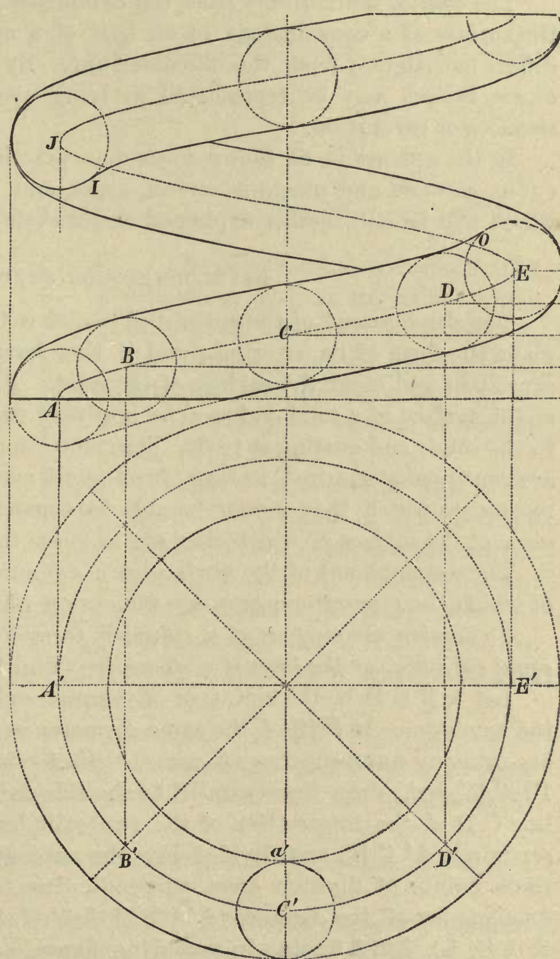


Fig. 194.

Let A′ C′ F′ and K′ M′ O′ represent the concentric bases of the cylinders, whose common axis S T is vertical; the curve of the exterior helix A^1 C F A^3 is the first to be drawn according to the method above shown. Then, having set off from A^1 to A^2 the thickness of the required solid, draw through A^2 another helix equal and similar to the former. Now construct, as above, similar helices, K C O and K^2 C^2 O^2, of the same pitch as the last, but on the interior cylinder. The lines A′ K′, B′ L′, C′ M′, etc., represent the horizontal projections of the various positions of the generating straight line, which, in the present example, has been supposed to be horizontal; and these lines are projected vertically at A^1 K, B L, etc.

It will be observed that in the position A^1 K the generating line is projected in its actual length, and that at the position C′ M′ its vertical projection is the point C. The same remark applies to the generatrix of the second helix. The parts of both curves which are visible in the elevation may be easily determined by inspection.

To determine the vertical projection of the solid formed by a sphere moving in a helical curve (Fig. 194).

Let A′ C′ E′ be the base of a cylinder, upon which the center point C′ of a sphere whose radius is *a*′ C′ describes a helix, which is projected on the vertical plane in the curve A C E J, determined as before. From the various points A, B, C, D . . . . . , in this curve, as centers, describe circles with the radius *a*′ C′ ; these denote the various positions of the sphere during its helical motion ; and, if lines be drawn touching them, the curves thereby formed will constitute the figure required. One of these curves disappears at O, but reappears again at I. The exterior and interior circles of the plan represent the horizontal projection of the solid in question.

The *conical* helix differs from the cylindrical one in that it is described on the surface of a cone instead of on that of a cylinder ; but the construction differs but slightly from the one described. By following out the same principles, helices may be represented as lying upon spheres or upon any other surfaces of revolution.

In the arts are to be found numerous practical applications of the helical curve, as wood and machine screws, gears, and staircases, the construction of which will be still further explained under their appropriate heads.

DEVELOPMENT OF SURFACES.

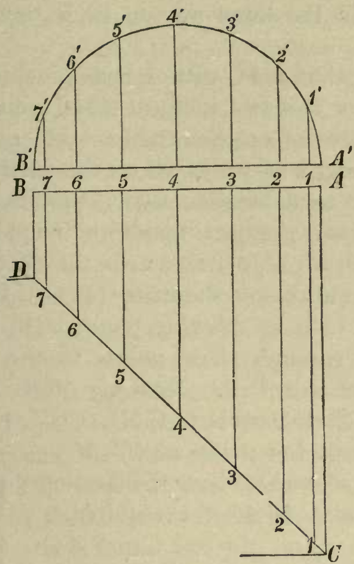
The *development* of the surface of a solid is the drawing or unrolling on a plane the form of its covering ; and if that form be cut out of paper, it would exactly fit and cover the surface of the solid. Frequently in practice, the form of the surface of a solid is found by applying paper or thin sheet-brass directly to the solid, and cutting it to fit. Tin and copper smiths, boiler-makers, etc., are continually required to form from sheet-metal forms analogous to solids; to execute which they should be able to construct geometrically the development of the surface of which they are to make the form.

The development of the surface of a cylinder is evidently but a flat sheet, of which the circumference is one dimension while its length is the other.

To develop the surface of a cylinder formed by the intersection of another equal cylinder, as the knee of a stove-pipe (Fig. 195).

Let A B C D be the elevation of the pipe or cylinder. Above A B describe the semicircle A′ 4′ B′ of the same diameter as the pipe ; divide this semicircle into any number of equal parts, eight for instance ; through these points, 1′, 2′, 3′, etc., draw lines parallel to the side A C of the pipe, and cutting the line C D of the intersection of the two cylinders. Lay off A″ B″ equal to the semicircle A′ 4′ B′, and divided into the same number of equal parts ; through these points of division erect perpendiculars to A″ B″, and on these perpendiculars lay off the distances A″ C″, 1″ 1″, 2″ 2″, 3″ 3″, and so on, corresponding to A C, 1 1, 2 2, 3 3, etc., in preceding figure. Through the points C″, 1″, 2″, ———D″, draw connecting lines, and we have the developed surface required. It is to be remarked that this gives but one half of the surface of the pipe, the other being exactly similar to it.

To develop the surface of a cylinder intersected by another cylinder, as in the formation of a T*-pipe* (Fig. 196).



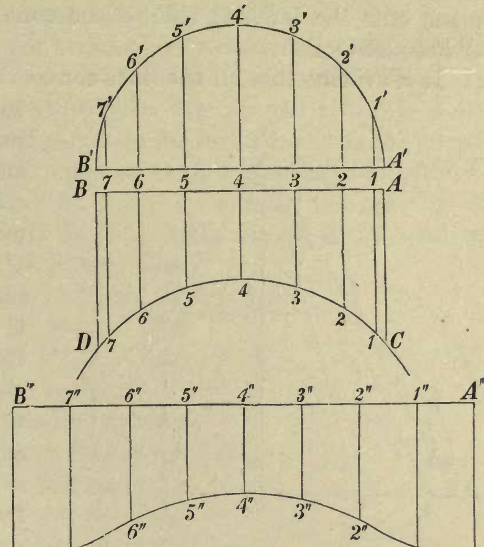


Fig. 196.

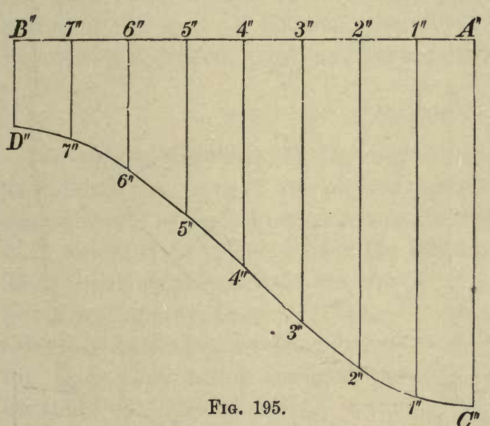


Fig. 195.

The construction is similar to the preceding, and, as the same letters and figures are preserved relatively, the demonstration will be easily understood from the foregoing.

To develop the surface of a right cone (Fig. 197).

From C′ as a center, with a radius, C′ A′, equal to the inclined side A C of the cone, describe an arc of a circle A′ B′ A″ ; on this arc lay off the distance A′ B′ A″ equal to the circumference of the base of the cone; connect A′ C′ and C′ A″, and A′ B′ A″ C′ is the developed surface required.

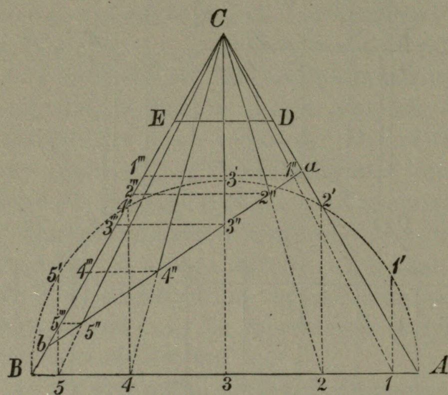
To develop the surface of the frustum of a cone, D A B E (Fig. 197).

D′ E′ D″ is the development of the cut-off cone C D E as shown by the preceding construction, and we therefore have A′ B′ A″ D″ E′ D as the developed surface of the frustum.

To develop the surface of a frustum of a cone, when the cutting plane a b is inclined to the base (Fig. 197).

On A B, the base, describe the semicircle A 3′ B ; divide the semicircle into any number of equal parts, six for instance ; from each point of division, 1′, 2′, 3′, 4′, 5′, let fall perpendiculars to the base ; at 1, 2, 3, 4, 5, connect each of these last points with the apex C. Divide now the arc A′ B′ A″, equal to the base A 3 B, into twelve equal parts ; each of these parts by the construction is equal to the arc A 1′, 1′ 2′ ; connect these points of division with the point C′ ; on C′ A′ take C′ *a*′ equal to C *a, a* being the point at which the plane cuts the inclined side of the cone; in the same way on C′ B′, lay off C′ *b*′ equal to C *b*.

It is evident that all the lines connecting the apex C with the base, included within the two inclined sides, are represented as less than their actual length, and must be projected on the inclined sides to determine their absolute dimensions; project, therefore, the points 1″, 2″, 3″, 4″, 5″, at which the cutting plane intersects the lines C 1, C 2, C 3, C 4, C 5, by drawing parallels to the base through these points to the inclined side C B. Now lay off C′ 1‴, C′ 2‴, etc., equal to C 1′″, C 2′″, etc.; connect the points *a*′, 1‴, 2‴, —— *b*′, —— *a*″, and we have the developed surface *a*′ A′ B′ A″ *a*″ *b*′ required.



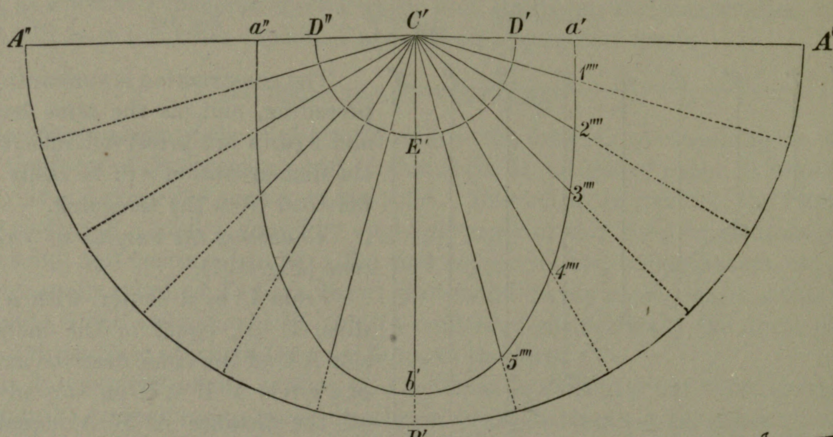


Fig. 197.

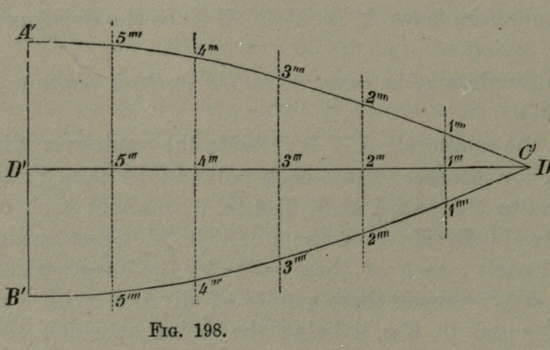


Fig. 198.

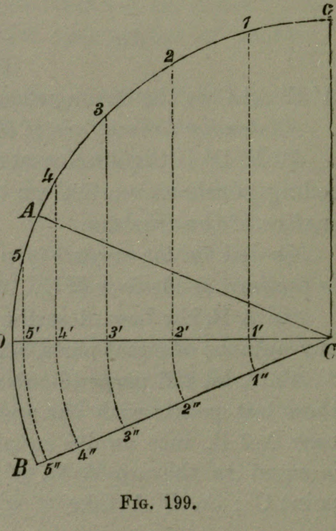


Fig. 199.

To develop the surface of a sphere or ball (Figs. 198, 199).

It is evident that the surface can not be accurately represented on a plane. It may be done approximately by a number of gores. Let C A B (Fig. 199) be the eighth of a hemisphere ; on C D describe the quarter circle D A *c* ; divide this arc into any number of equal parts, six for instance ; from the points of division 1, 2, 3, . . . let fall perpendiculars on C D, and from the intersections with this line describe arcs 1′ 1″, 2′ 2″, 3′ 3″, . . . cutting the line C B at 1″, 2″, 3″, ... ; on the straight line C′ D′ (Fig. 198), lay off C′ D′ equal to the arc D A *c*, with as many equal divisions ; then from either side of this line lay off 1‴ 1⁗, 2‴ 2⁗ . . . . D′ B′ equal to the arcs 1′ 1″, 2′ 2″ . . . . D B (Fig. 199). Connect the points C′, 1⁗, 2⁗, ... and C′ A′ B′ is approximately the developed surface.

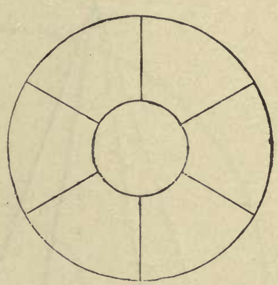


Fig. 200.

It is to be remarked that, in the preceding demonstrations, the forms are described to cover the surface only ; in construction, allowance is to be made for lap by the addition of margins on each side as necessary. It is found difficult, in the formation of hemispherical ends of boilers, to bring all the gores together at the apex ; it is usual, therefore, to make them, as shown (Fig. 200), by cutting short the gores, and surmounting the center with a cap-piece.

SHADE-LINES.

In *outline drawings*, or drawings which consist simply of the lines employed to indicate the form of the object represented, the roundness, the flatness, or the obliquity of individual surfaces, is not indicated by the lines, although it may generally be inferred from the relation of different views of the same part. The direct significance of an outline drawing may, however, be considerably increased, by strengthening those lines which indicate the contours of surfaces resting in the shadow ; and this distinction also improves the general appearance of the drawing. The strong lines, to produce the best effect, ought to be laid upon the sharp edges at the summits of salient angles ; but bounding lines for curve surfaces should be drawn finely, and should be but slightly, if at all, strengthened on the shade side. This distinction assists in contrasting flat and curve surfaces. To understand and apply the shade-lines, however, we must know the direction in which the light is supposed to fall upon the object, and thence the locality of the shadows.

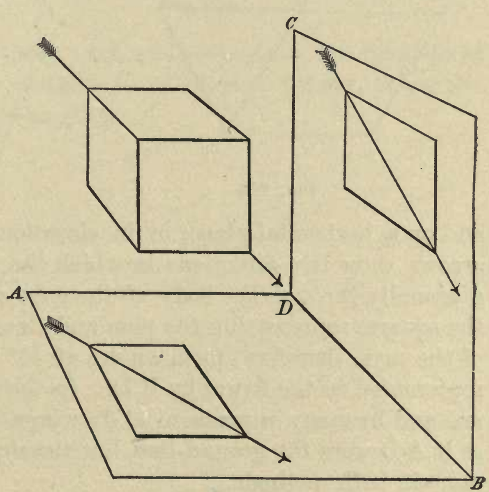


Fig. 201.

It is necessary, for the explicitness of the drawing, that, firstly, the light be supposed to fall upon the object in parallel lines, that all the parts may be shade-lined according to one uniform rule; secondly, that the light should be supposed to fall upon the object obliquely, as in this way both the horizontal and vertical lines may be relieved by shading.

Fig. 201 represents the drawing of a cube, with its projections on a vertical

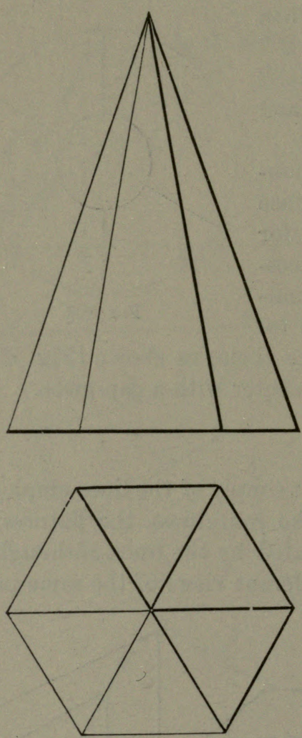


Fig. 202.

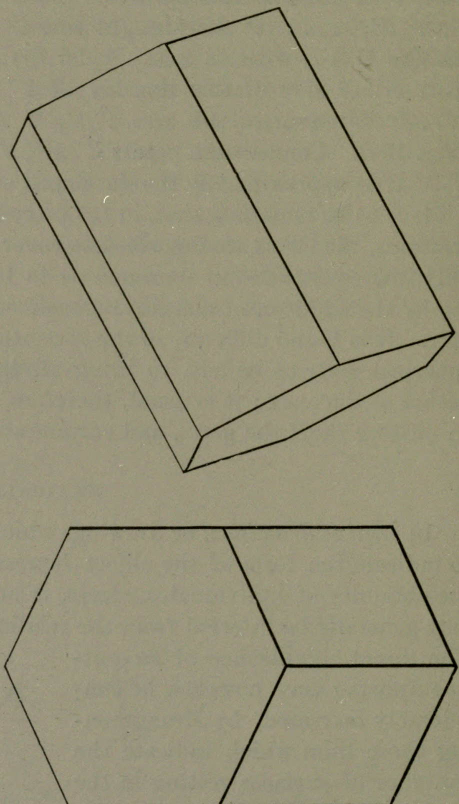


Fig. 203.

and on a horizontal plane, or in elevation and plan, all in perspective. The arrows show the directions in which the light is supposed to fall: in space diagonally through the body of the cube and in projection diagonally through the squares representing the plan and elevation of the cube. The projections of the rays, therefore, form angles of 45° with the ground-line, which line is represented in the figure by B D. In the old method, still used in topographical and by many in mechanical drawings, the light is supposed to fall *in space*, as if A D were the ground-line, but the shade-lines in the vertical plane are the same in both methods.

Copies of a few of the preceding projections are here given, with the proper shade-lines, according to the first or French method (Fig. 201). The outlines to be shaded can be determined, ordinarily, by mere inspection and by using a 45° triangle. Such a triangle gives the direction of the projected rays, and determines the surfaces in shadow. Fig. 202 is a reproduction of Fig. 171; Fig. 203 is a reproduction of Fig. 176; Fig. 204 is a reproduction of Fig. 184; Fig. 205 is a reproduction of the plan of Fig. 188. The outlines on which the light falls are represented by fine lines, the others by coarse lines. In general, it is not customary to use more than two grades of lines, one for the outlines in light, and the other for those in shade; but, for lines parallel with the

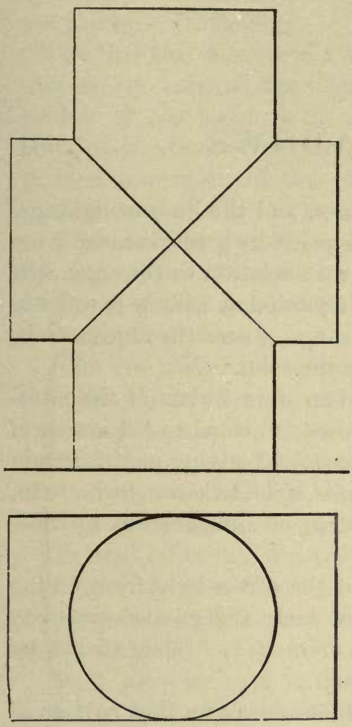


FIG. 204.

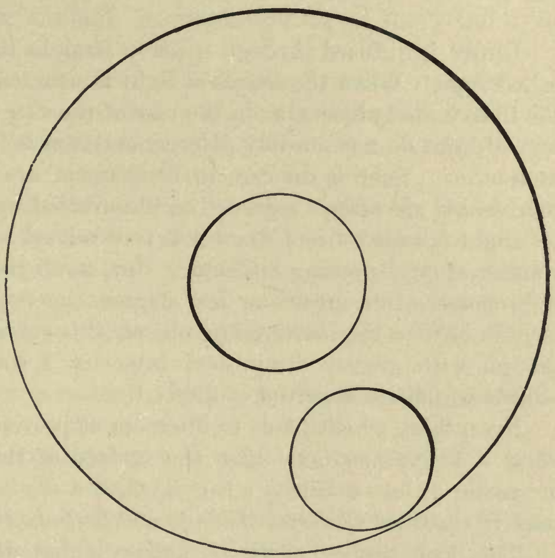


FIG. 205.

rays of light, medium lines are sometimes used, and sometimes the shade-lines are proportioned to the depth of the surfaces to which they belong, below the original surfaces from which the shadows arise.

SHADES AND SHADOWS.

Light is diffused through space in straight lines, and the lines of light are called *rays*. When the source of light is situated at a very great distance from the illuminated objects, as in the case of the sun with relation to the earth, the rays of light do not sensibly diverge, and may be regarded as *exactly parallel* to each other. Such is the case in mechanical drawings, where the objects to be represented are always regarded as illuminated by the solar light.

Light is called *direct* when it is transmitted to an object without the intervention of any opposing medium. But, as all bodies subjected to the action of light possess, in a greater or less degree, the property of giving out a certain portion of it to the surrounding objects,this *reflected light* becomes in its turn, though with greatly diminished intensity, a source of illumination to those objects which are deprived of direct light.

Everything which tends to intercept or prevent the direct light from falling upon a body, produces upon the surface of that body a degree of obscurity of greater or less intensity ; this is called a *shade* or *shadow*. Such effects are usually classified as *direct shadows* and *cast shadows*.

The shade proper, or *direct shadow*, is that which occurs on that portion of the surface of a body which is situated opposite to the enlightened part, and is the natural result of the form of the body itself, and of its position with regard to the rays of light. The cast shadow, on the other hand, is that which is produced upon the surface of one body by the interposition of another between the former and the source of light ; thus intercepting the rays which would otherwise illuminate that surface. Cast shadows may also obviously be produced upon the surface of a body by the form of the body itself ; as, for example, if it contain projecting or concave parts.

The limit of the direct shadow on any body, whatever may be its form or position, is a line of greater or less distinctness, termed *the line of shade ;* this line is, of course, determined by the contact of the luminous rays with the surface of the body ; and, if these rays be prolonged till they meet a given surface, by joining all the points of intersection with that surface, we obtain the *outline of the shadow cast* upon it by the part of the body which is deprived of light.

The rays of light being regarded as parallel to each other, it is obvious that, in the delineation of shadows, it is only necessary to know the direction of one of them ; and, as that direction is arbitrary, we have adopted the usual and confessedly the most convenient mode of regarding the rays as in all cases falling in the direction of the diagonal of a cube, of which the sides are parallel to the planes of projection. This is graphically shown in Fig. 201 of the preceding chapter. The projections of the ray form each an angle of 45° with the ground-line. This is not true of the ray itself in space, for that forms an angle of 54° 44′ with the ground-line, and an angle of 35° 16′ with each of the planes of projection.

To find the shadow of a point, as A, A′ (Fig. 206), on either plane of projection, the vertical, for instance, we draw a line through the horizontal projection of the point A′ at an angle of 45° with the ground-line, and at the point of intersection of those lines, *a′*, erect a perpendicular to intersect the vertical projection of the ray through A, which will be at the point *a*, the shadow in question.

This, as may readily be seen, is simply finding the point of intersection of the ray passing through the point and the vertical plane of projection. The converse of this method will as easily determine the shadow of the point on the horizontal plane.

The line A *a* in the elevation being equal in every case to the line A′ *a′* in the plan, it will in some cases be found more convenient to use the compasses instead of a geometrical construction; for example, in place of projecting the point *a′* by a perpendicular to the ground-line, in order to obtain the position of the required shadow *a*, that point may be found by simply setting off upon the line A *a* a distance equal to A′ *a′*.

In the following illustrations the same letter accented is employed in the plan as in the elevation to refer to the same object or point.

Required to determine the shadow cast upon the vertical wall X Y *by the straight line* A B (Fig. 206).

It is obvious that in this case the shadow itself will be a straight line; hence, to solve the problem, it is only necessary to find two points in that line. We have seen that the position of the shadow thrown by the point A is at *a*;

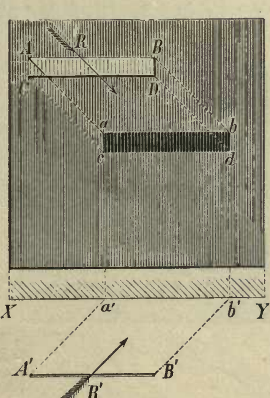


Fig. 206.

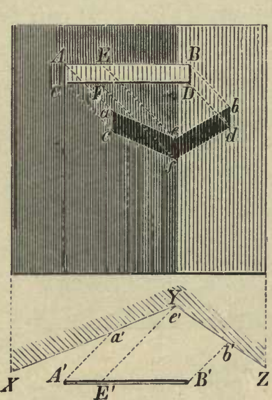


Fig. 207.

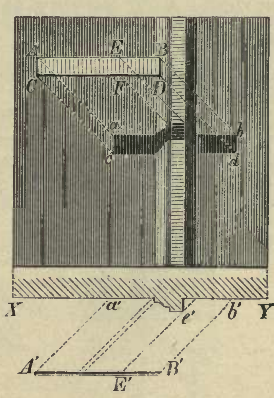


Fig. 208.

by a similar process we can easily determine the point *b*, the position of the shadow thrown by the opposite extremity B of the given line; the straight line *a b*, which joins these two points, is the shadow required.

It is evident, from the construction of this figure, that the line *a b* is equal and parallel to the given line A B; this results from the circumstance that the latter is parallel to the vertical plane X Y. Hence, *when a line is parallel to a plane, its shadow upon that plane is a line which is equal and parallel to it.*

Suppose now that, instead of a mere line, a parallel slip of wood or paper, A B C D, be taken, which, for the sake of greater simplicity, we shall conceive as having no thickness. The shadow cast by this object upon the same vertical plane X Y is a rectangle *a b c d*, equal to that which represents the projection of the slip, because all the edges of the latter are parallel to the plane upon which the shadow is thrown. Hence, in general, *when any surface,* whatever may be its form, *is parallel to a plane, its shadow thrown upon that plane is a figure similar to it, and similarly situated.* This principle facilitates the delineation of shadows in many cases. In the present example, an idea may be formed of its utility; for, after having determined the position of any one of the points *a, b, c, d,* the figure may be completed by drawing lines equal and parallel to the sides of the slip, without requiring to go through the operations in detail.

When the object is not parallel to the given plane, the shadow cast is no longer a figure equal and similarly placed; the method of determining it remains, however, unchanged; thus (Fig. 207), take the portion A E of the slip A B, which throws its shadow on the plane X Y; draw the projections of the rays of light A *a*, E *e*, C *c*, F *f*, and A′ *a′*. E′ *e′*, and project *a′* vertically to *a*, *c*, and *e′* to *e*, *f*; connect *a*, *e*, *f*, *c*, and we have the outline of the shadow of the slip A E.

By an exactly similar construction we have the shadow of the portion E B on the plane Y Z, which, being inclined to the plane of projection in a direction contrary to X Y, necessarily causes the shadow to be broken, and the part *e d* to lie in a contrary direction to *a f*.

The determination of the shadow of the slip upon a molding placed on the plane X Y parallel to the slip (Fig. 208) can be readily determined by an inspection of the figure.

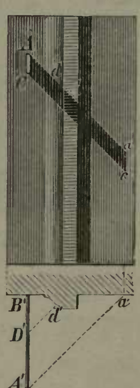


Fig. 209.

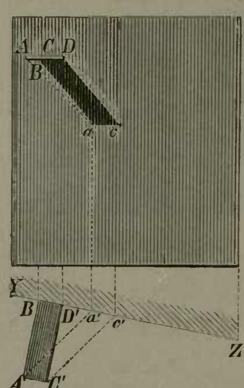


Fig. 210.

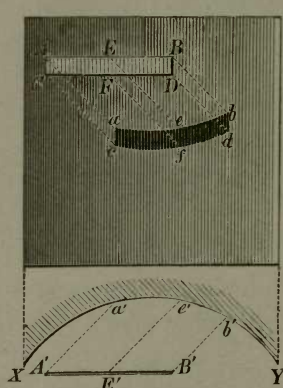


Fig. 211.

When the slip is placed perpendicularly to a given plane, X Y (Fig. 209), on which a projecting molding, of any form whatever, is situated, the shadow of the upper side A′ B′, which is projected vertically in A, will be simply a line, A *a*, at an angle of 45°, traversing the entire surface of the molding, and prolonged unbroken beyond it. This may easily be demonstrated by finding the position of the shadow of any number of points such as D′, taken at pleasure upon the straight line A′ B′. The shadow of the opposite side, projected in C, will follow the same rule, and be denoted by the line C *c*, parallel to the former. Hence, as a useful general rule : *in all cases where a straight line is perpendicular to a plane of projection, it throws a shadow upon that plane in a straight line, forming an angle of* 45° *with the ground-line.*

When the slip is set horizontally in reference to its own surface, and perpendicularly to the given plane X Y (Fig. 210), the shadow commences from the side D B, which is in contact with this plane, and terminates in the horizontal line *a c*, which corresponds to the opposite side A C of the slip.

To find the shadow cast by a slip, A B C D, *upon a curved surface, either convex or concave, whose horizontal projection is represented by the line* X *e*′ Y (Fig. 211).

This construction is similar to the foregoing illustrations, and requires no explanation more than the figure.

Required to find the shadow cast upon a vertical plane, X Y, *by a given circle parallel to it* (Fig. 212).

Let C, C′, be the projections of the center of the circle, and R, R′, those of the rays of light.

The position of the shadow of the center C, according to the rules already fully developed, is easily fixed at *c* ; from which point, if a circle equal to the given circle be described, it will represent the outline of the required shadow, according to the principle previously enunciated on page 112.

When the circle is perpendicular to both planes of projection (Fig. 213), its

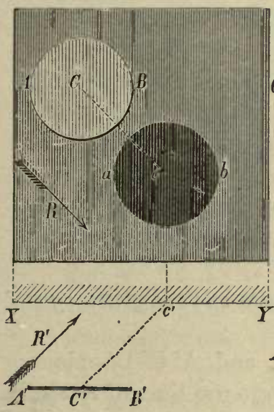


Fig. 212.

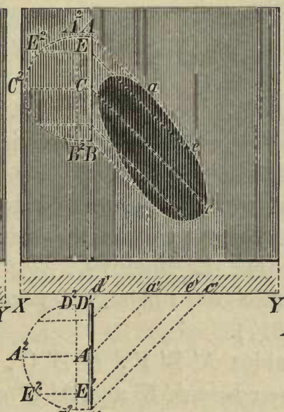


Fig. 213.

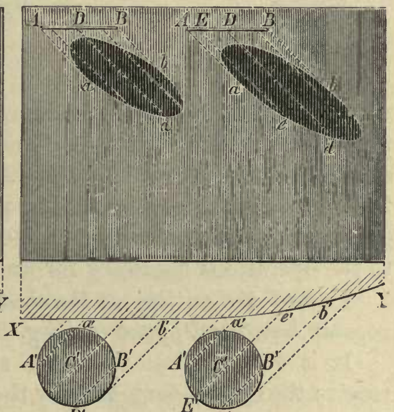


Fig. 214.

projection upon each will obviously be represented by the equal diameters A B and C′ D′, perpendicular each to the ground-line. To determine the cast shadow, describe the given circle upon both planes, as indicated in the figures, and divide the circumference of each into any number of equal parts ; then, having projected the points of division, as A^2, E^2, C^2, etc., to their respective diameters A B and C′ D′, draw from them lines parallel to the rays of light, which, by their intersection with the given plane, will indicate so many points in the outline of the cast shadow.

If the given circle be horizontal (Fig. 214), its shadows cast upon the straight and curved portions of the vertical plane X Y become ellipses, which must be constructed by means of points, as indicated in the figure.

If the plane of the circle is situated perpendicularly to the vertical projection of the luminous rays (Fig. 215), the method of constructing the cast shadow does not differ from that pointed out in reference to Fig. 214. It is obvious that, instead of laying down the entire horizontal projection of this circle, all that is necessary is to set off the diameter D′ E′ equal to A B, because the shadow of this diameter, transferred in the usual way, gives the major axis of the ellipse which constitutes the outline of the shadow sought, while its minor axis is at once determined by *a b*, equal and parallel to A B.

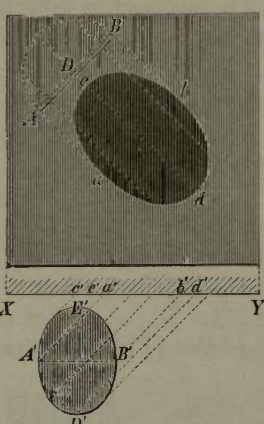


Fig. 215.

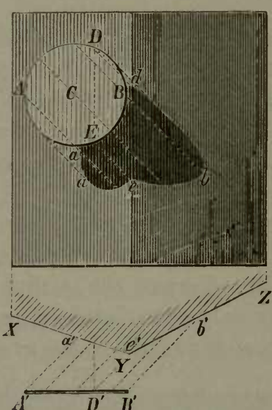


Fig. 216.

To delineate the shadow of a circle parallel to the vertical plane of projection, throwing its shadow at once upon two plane surfaces inclined to each other (Fig. 216), all that it is necessary specially to point out is, that the points *d* and *e* are found by drawing from Y a line Y D′, parallel to the rays of light, and projecting the point D′ to D and E.

We may here remark that, in every drawing where the shadows are to be inserted, it is of the utmost importance that the projections which represent the object whose shadow is required should be exactly defined, as well as the surface upon which this shadow is cast; it is therefore advisable, in order to prevent mistakes and to insure accuracy, to draw the figures in India ink, and to erase all pencil-marks before proceeding to the operations necessary for finding the shadows.

To find the outline of the shadow cast upon both planes of projection by a regular hexagonal pyramid (Fig. 217).

It is obvious that the three sides A′ B′ F′, A′ B′ C′, and A′ C′ D′, alone receive the light; consequently the edges A′ F′ and A′ D′ are the lines of shade. To solve this problem, then, we have only to determine the shadow cast by these two lines, which is accomplished by drawing from the projections of the vertex of the pyramid the lines A *b*′ and A′ *a*′ parallel to the ray of light, then raising from the point *b*′ a perpendicular to the ground-line, which gives at *a*′ the shadow of the vertex on the horizontal plane (on the other side of the ground-line), and finally by joining this last point *a*′ with the points D′ and F′; the lines D′ *a*′ and F′ *a*′ are the outlines of the required shadow on the horizontal plane. But, as the pyramid happens to be situated sufficiently near the vertical plane to throw a portion of its shadow toward the vertex upon it, this portion may be found by raising from the point *c*, where the line A′ *a*′ cuts the ground-line, a perpendicular *c a*, intersecting the line A *b*′ in *a*; the lines *a d* and *a e* joining this point with those where the horizontal part of the shadow meets the ground-line, will be its outline upon the vertical plane.

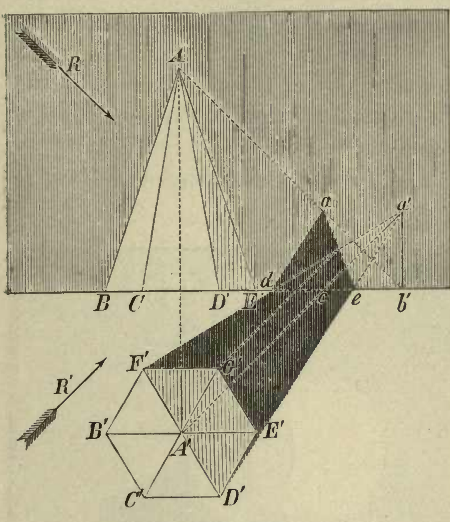


Fig. 217.

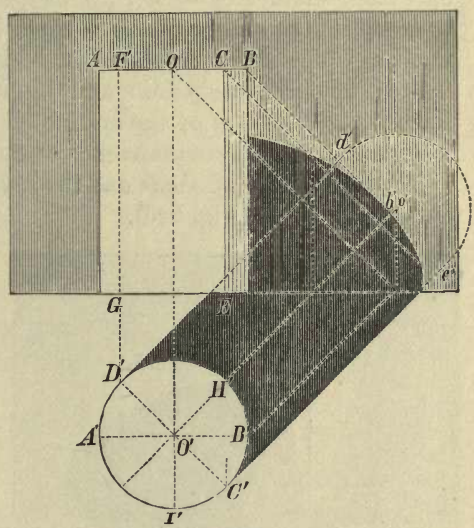


Fig. 218.

To determine the limit of shade on a cylinder placed vertically, and likewise its shadow cast upon the two planes of projection (Fig. 218).

The lines of shade on a cylinder situated as indicated are at once found by drawing two tangents to its base, parallel to the ray of light, and vertically projecting through the points of contact lines parallel to the axis of the cylinder.

Draw the tangents D′ *d*′ and C′ *c*′ parallel to the rays of light; these are the outlines of the shadow cast upon the horizontal plane. Through the point of contact C′ draw the vertical line C C′; this line denotes the line of shade upon the surface of the cylinder. It is obviously unnecessary to draw the perpendicular from the opposite point D′, because it is altogether concealed in the vertical elevation of the solid. In order to ascertain the points C′ and D′ with accuracy, draw through the center O′ a diameter perpendicular to the rays of light.

Had this cylinder been placed at a somewhat greater distance from the vertical plane of projection, its shadow would have been entirely cast upon the horizontal plane, in which case it would have terminated in a semicircle drawn from the center *o*′, with a radius equal to that of the base. But, as a portion of the shadow of the upper part is thrown upon the vertical plane, its outline will be defined by an ellipse drawn in the manner indicated in Fig. 214.

To find the line of shade in a reversed cone, and its shadow cast upon the two planes of projection (Fig. 219).

From the center A′ of the base draw a line parallel to the ray of light; from the point *a′* where it intersects the perpendicular, describe a circle equal to the base, and from the point A′ draw the lines A′ *b′* and A′ *c′*, tangent to this circle; these are the outlines of the shadow cast upon the horizontal plane. Then from the center A′ draw the radii A′ B′ and A′ C′ parallel to *a′ b′* and *a′ c′*; these radii are the horizontal projections of the lines of shade, the former of which, transferred to B D, is alone visible in the elevation. But, in order to complete the outline of the shadow, it is necessary to project the point C′ to C, from which, by a construction which will be manifest by inspecting the figures, we derive the point *c* and the line *c d* as part of the cast shadow of the line C′ A′. The rest of the outline of the vertical portion of the cast shadow is derived from the circumference of the base, as in Fig. 218.

To find the line of shade and the shadow of a horizontal cylinder inclined to the vertical plane (Fig. 220).

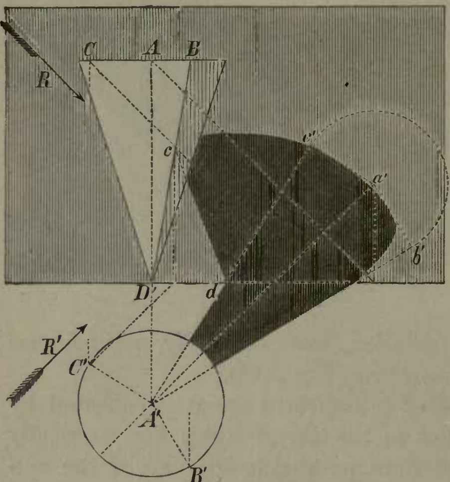


FIG. 219.

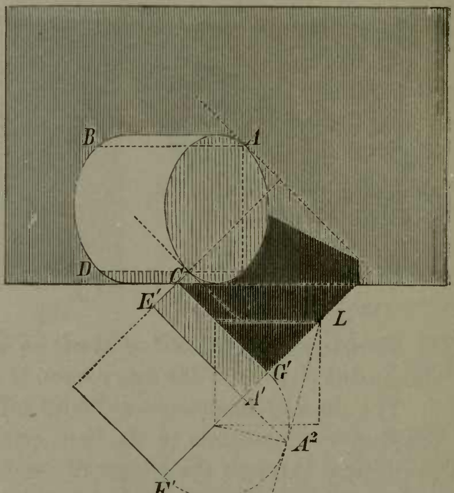


FIG. 220.

The construction in this case is the same as that explained by Fig. 218. Of the horizontal lines of shade A B and C D, the latter alone is visible in the elevation, while, on the other hand, the former, A B alone, is seen in the plan, where it may be found by drawing a perpendicular from A meeting the base F′ G′ in A′. The line A′ E′, drawn parallel to the axis of the cylinder, is the line of shade required. Project the shadow of the line A B on the vertical plane as in previous examples, and the construction will define the outline of the shadow of the cylinder.

The example here given presents the particular case in which the bases of the cylinder are parallel to the direction of the rays of light. In this case, to determine the line A′ E′, lay off the angle A′ L A^2 equal to 35° 16′, which the ray of light makes with the horizontal plane, so that the side A^2 L shall be tangent to the circle F′ A^2 G′ (which represents the base of the cylinder laid down on the horizontal plane); through the point of tangency A^2, draw a line, A′ E′, parallel to the axis of the cylinder, which will be the line of shade, as before.

To determine the shadows cast upon a cylinder by various shaped caps.

Fig. 221 represents a cylinder upon which a shadow is thrown by a rectangular prism, of which the sides are parallel to the planes of projection. The shadow in this case is derived from the edges A′ D′ and A′ E′, the first of which, being perpendicular to the plane of projection, gives, according to principles already laid down, a straight line at an angle of 45° for the outline of its shadow, whereas the side A′ E′ being parallel to that plane, its shadow is determined by a portion of a circle, *a b c*, described from the center *o*.

If the prism be hexagonal (Fig. 222), or a cylinder be substituted for it

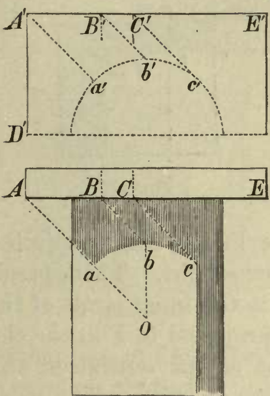


Fig. 221.

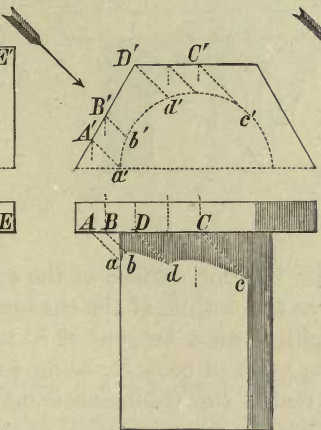


Fig. 222.

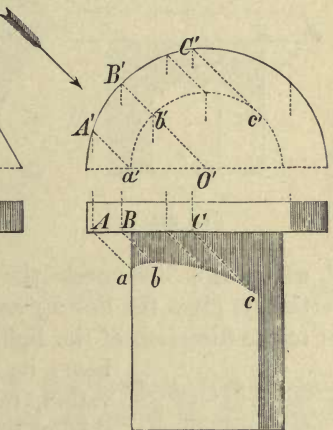


Fig. 223.

(Fig. 223), the mode of construction remains the same. But it should be observed that it is best in all such cases to commence by finding the points which indicate the main direction of the outline. To ascertain the point *a* at which the shadow commences, draw from *a*′ the line *a*′ A′ at an angle of 45°, which is then to be projected vertically to *a* A. Then the highest point *b* (Fig. 223) should be determined by the intersection of the radius O′ B′ (drawn parallel to the ray) with the circumference of the base of the cylinder on which the required shadow is cast ; and, finally, the point *c*, where the outline of the cast shadow intersects the line of shade, should be determined by a similar process.

To determine the shadows cast upon a hexagonal prism by the same caps.

Fig. 224 represents a hexagonal prism upon which a shadow is thrown by a rectangular prism.

Fig. 225 represents a hexagonal prism upon which a shadow is cast by another hexagonal prism.

Fig. 226 represents a hexagonal prism upon which a shadow is cast by a cylinder. These three cases do not materially differ from the preceding three, and can easily be understood from an examination of the figures.

To define the shadows cast upon the interior of a hollow cylinder, in section, by itself, and by a circular piston fitted into it (Fig. 227).

The figure shows the section of a steam-cylinder, by a plane passing through its axis, with its piston and rod in full.

Conceive, in the first instance, the piston P to be removed ; the shadow cast into the interior of the cylinder will then consist, obviously, of that projected by the vertical edge B C, and by a portion of the horizontal edge B A. To find the first, draw through B′ a line, B′ *b*′, at an angle of 45° with B′ A′; the point

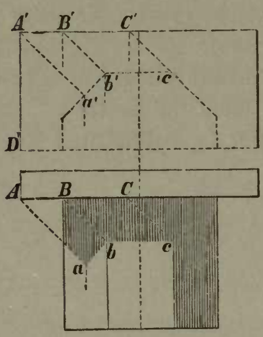


Fig. 224.

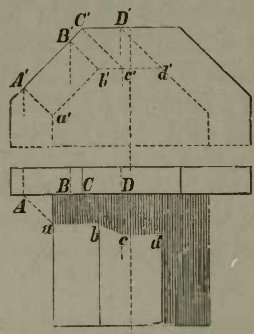


Fig. 225.

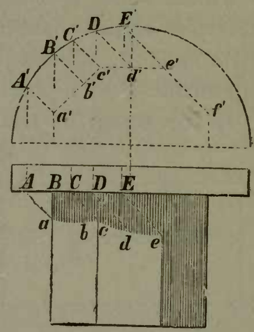


Fig. 226.

b′, where this line meets the interior surface of the cylinder, being projected vertically, gives the line *b f* as the outline of the shadow sought. Then, parallel to the direction of the light, draw a tangent at F′ to the inner circle of the base; its point of contact, being projected to F in the elevation, marks the commencement of the outline of the shadow cast by the upper edge of the cylinder. The point *b*, where it terminates, will obviously be the intersection of the straight line *f b* already determined, with a ray, B *b*, from the upper extremity of the edge B C; and any intermediate point in the curve, as *e*, may be found in precisely the same way. The outline of the shadow required will then be the curve F *e b* and the straight line *b f*. Suppose, now, the piston P and its rod T to be inserted into the cylinder, as shown. The lower surface of the piston will then cast a shadow upon the interior surface of the cylinder, of which the outline D *d h o* may be formed in the same way, as will be obvious from inspection of the figures and comparison of the letters of reference. The piston-rod T being cylindrical and vertical, it casts also its shadow into the interior of the cylinder; it will obviously consist of the rectangle *i j l k* drawn parallel to the axis.

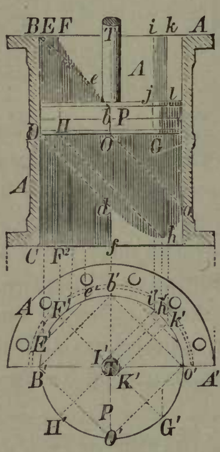


Fig. 227.

To find the shadow cast in the interior of a hollow cylinder, surmounted by a circular disk or cover, sectioned through the center, where it is also penetrated by a cylindrical aperture (Fig. 228).

The construction necessary for finding the outlines of the cast shadow will obviously be the same as already laid down. To know beforehand what parts of the upper and lower edges of the central aperture cast their shadows into the interior of the cylinder, in order to avoid unnecessary work, we should first determine the position of the point of intersection, *c*, of the two curves *b c f* and *a c e*, shadows of these edges, which is the cast shadow of the lowest point, C, in the curve D C, previously laid down in the circular opening of the cover, in the manner indicated in the previous example.

To find the shadow cast in the interior of a cylinder, in section, inclined to the horizontal plane (Fig. 229).

In any convenient part of the paper, draw the diagonal *m o* parallel to the line of light A′ E, and construct a square *m n o p* (Fig. 230) ; from one of the

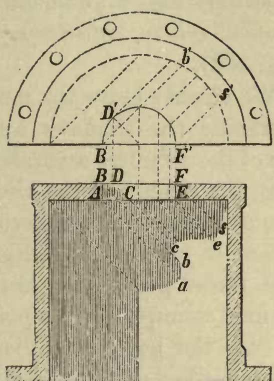


Fig. 228.

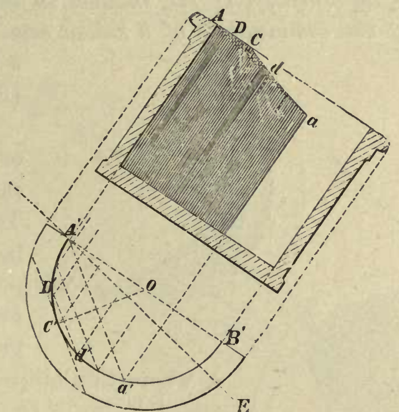


Fig. 229.

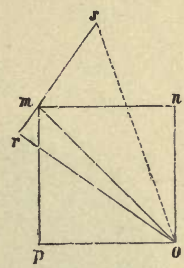


Fig. 230.

extremities, *o*, draw the line *o r* parallel to A′ B′, and through the opposite extremity, *m*, draw a perpendicular, *r s*, to this line, and set off on the perpendicular the distance *r s* equal to the side of the square, and join *s o*. Now, draw through the point A′, in the original figure, a line, A′ *a*′, parallel to *s o*, intersecting the circle A′ *a*′ B′ in the point *a*′, which, being projected by a line parallel to the axis of the cylinder, and meeting the line A *a*, drawn at an angle of 45°, gives the first point *a* in the curve C *d a*. The other points will be obtained in like manner, by drawing at pleasure other lines, such as D′ *d*′, parallel to A′ *a*′.

To find the outline of the shadow cast into the interior of a hollow hemisphere (Fig. 231).

Let A B C D represent the horizontal projection of a concave hemisphere. Here it is sufficiently obvious that, if we draw through the center of the sphere a line perpendicular to the ray of light A C, the points B and D will at once give

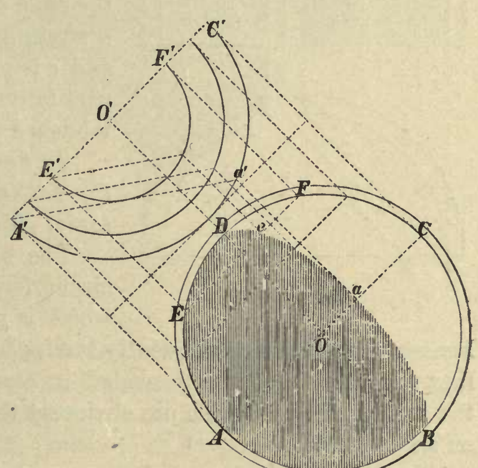


Fig. 231.

the extremities of the curves sought. On any point of B D produced as O′, construct the semicircle A′ *a*′ C′ with a radius, A′ O′, equal to A O. At A′ draw the line A′ *a*′, making an angle of 35° 16′ with A′ C′. This angle, as has been said before, is equal to that made by the ray of light in space with the planes of projection. The point of intersection of the line with the semicircle at a' projected to a, gives a point of the outline of the shadow. Similar sections, as E F parallel to A C, will give other points. But, as this outline cover of the shadow is an ellipse whose axes are B D and twice O a, it may be constructed, when the point a is determined, by the ordinary methods for ellipses.

To construct the outlines of the shadow in the interior of a concave surface, formed by the combination of a hollow semi-cylinder and a quadrant of a hollow sphere, called a niche (Fig. 232).

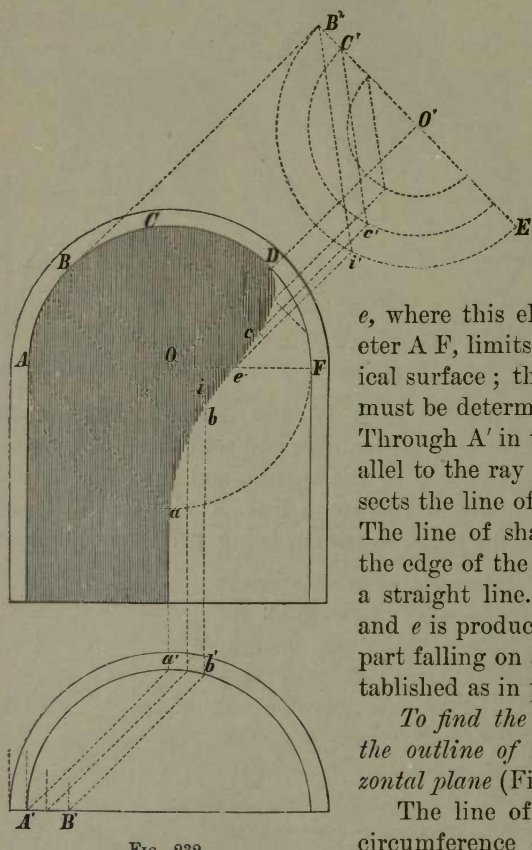


FIG. 232.

We already know the mode of tracing the shadows upon each of these figures separately. Thus, the shadow of the circular outline upon the spherical portion is part of an ellipse, i c D, found precisely as in the previous example. The point e, where this ellipse cuts the horizontal diameter A F, limits the cast shadow upon the spherical surface ; therefore, all the points beneath it must be determined upon the cylindrical part. Through A′ in the plan draw the line A′ a' parallel to the ray of light ; project a' till it intersects the line of light A a in the elevation at a. The line of shadow below a is the shadow of the edge of the cylinder, and must therefore be a straight line. The line of shadow between a and e is produced by the outline of the circular part falling on a cylindrical surface, and is established as in previous constructions.

To find the line of shade in a sphere, and the outline of its shadow cast upon the horizontal plane (Fig. 233).

The line of shade in a sphere is simply the circumference of a great circle, the plane of which is perpendicular to the direction of the luminous rays, and consequently inclined to the two planes of projection. This line will therefore be represented in elevation and plan by two equal ellipses, the major axes of which are obviously the diameters C D and C′ D′, drawn at an angle of 45°.

To find the minor axes of these curves, assume any point, O^2, upon the prolongation of the diameter of the perpendicular C′ D′, draw through this point the straight line O^2 o', inclined at an angle of 35° 16′, to A′ B′ or its parallel, and erect upon it the perpendicular E^2 F^2. The projection of the two extremities E^2 and F^2 upon the line A′ B′ will give in the plan the line E′ F′ for the length of the required minor axis of the ellipse, i. e., of the line of shade in the plan ; and this line, being again transferred to the elevation, determines the minor axis E F of the line of shade in the elevation.

Supposing it were required to draw these ellipses, not by means of their axes, but by points, any number of these may be obtained by making horizontal sections of the sphere. Thus, for example, if we draw the chord G H parallel to A′ B′, to represent one of these sections, and from the point *a*, where it cuts the diameter E^2 F^2, if we draw a perpendicular to A′ B′, the points *a′ a′*, where

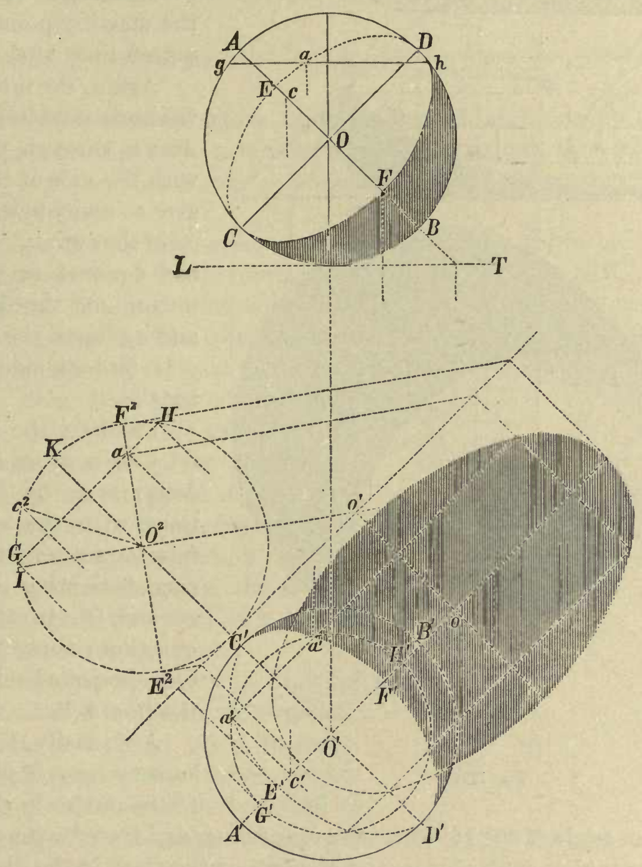


Fig. 233.

it intersects the circumference of the circle G′ *a′* H′, representing the section G H in plan, will be two points in the line of shade required. These points may be transferred, by supposing a section, *g h*, to be made in the elevation corresponding to G H, and projecting the points *a′ a′* by perpendiculars to *g h*, the line representing the cutting plane.

The outline of the shadow cast by the sphere upon the horizontal plane is also obviously an ellipse ; it may be constructed either by means of its two axes or by the help of points, in the manner indicated in the figure.

To draw the line of shade on the surface of a ring of circular section, in vertical section, elevation, and plan (Fig. 234).

We shall first point out the mode of obtaining those primary points in the curve which are most easily found, and then proceed to the general case of determining any point whatever.

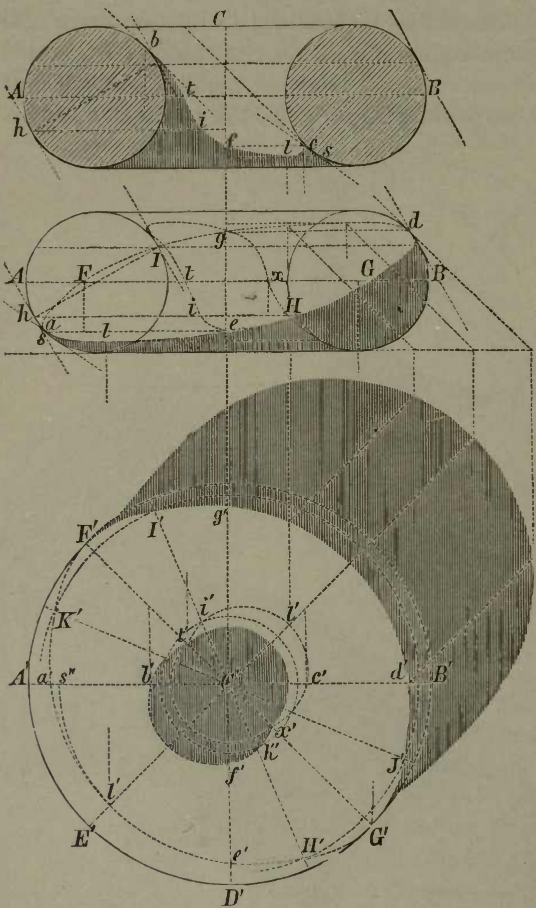


Fig. 234.

If tangents be drawn to the circles represented in both elevation and section, parallel to the rays of light, their points of contact, *a*, *b*, *c*, *d*, will be the starting-points of the required lines of shade.

Again, the intersections of the horizontal lines *a e*, *d g*, *c f*, drawn through these points, with the axis of the ring, will give so many new points, *e*, *g*, *f*, in the curve. These points are denoted in the plan by setting off the distances *a e* and *c f* upon the vertical line *g′* D, on both sides of the center C′.

Further, the diameter F′ G′, drawn at an angle of 45°, determines, by its intersections with the exterior and interior circumferences of the ring, four other points, F′, *t′*, *x′*, and G′, in the curve in question ; these points are all to be projected vertically upon the line A B.

And, lastly, to obtain the lowest points, *l l*, draw tangents to the circles in elevation and section at an angle of 35° 16′ with the ground-line, and transfer the distances between the points of contact, *s*, *s*, and the axis of the ring, to the diameter E′ J′, where they are denoted by *l′ l′* ; these points are then projected to *l*, *l*, upon the horizontal lines drawn through the same points *s*, *s*.

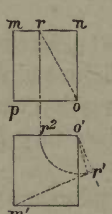


Fig. 235.

To determine any other points, draw through the center C′ a diameter, I′ H′, in any direction. Draw through *o′*, one of the angular points of the horizontal projection of the cube, made at any convenient size (Fig. 235), a straight line, *o′ r′*, parallel to I′ H′, and from the opposite point *m′* draw a perpendicular, *m′ r′*, to *o′ r′*. Then, having revolved the point *r′* to r^2, and projected r^2 to *r*, join *o* and *r*.

Applying this construction to the figures before us, we now draw tangents to the circles represented in elevation and section parallel to the line *o r*, and, taking as radii the distances from their respective points of contact, h and I, to the axis of the ring, we describe corresponding circles about the center C′ of the plan. We thus obtain four other points in the curves required, namely, I′, i', h'', and H′, which may also be projected upon the horizontal lines drawn through the points h or I.

By drawing the straight line J′ K′ so as to form with F′ G′ the same angle which the latter makes with the line H′ I′, we obtain, by the intersection of that line with the circles last named, four other points of the curves in question.

To determine the shadows cast upon the surfaces of grooved pulleys (Fig. 236).

The construction of cast shadows upon surfaces of the kind now under consideration is founded upon the principle, already announced, that *when a circle is parallel to a plane, its shadow, cast upon that plane, is another circle equal to the original circle.*

Take the case of a circular-grooved pulley; the cast shadow on its surface is obviously derived from the circumference of the upper edge A B. To determine its outline, take any horizontal line D E in upper fig. and describe from the center C′ a circle with a radius equal to the half of that line; then draw through the same center a line parallel to the ray of light, which will intersect the plane D E in c; lastly, describe from the point c', as a center, an arc of a circle with a radius equal to A C; the point of intersection, a', of this arc, with the circumference of the section D E, will give, when projected to a, one of the points in the curve required.

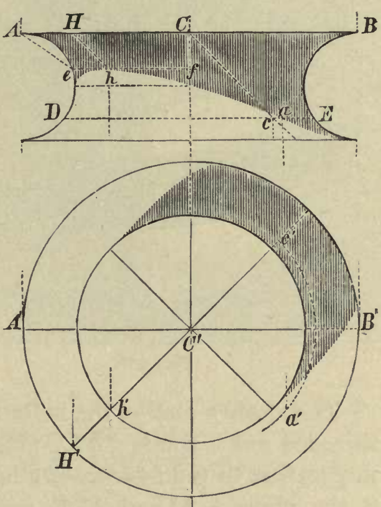


Fig. 236.

To avoid unnecessary labor in drawing more lines parallel to D E than are required, it is important, in the first place, to ascertain the highest point in the curve sought. This point is the shadow of that marked H on the upper edge of the pulley, and which is determined by the intersection of the ray C′ H′ with the circumference of that edge in the plan; and it is obtained by drawing through the point A a straight line at an angle of 35° 16′ with the line A B, and through the point e, striking a horizontal line ef, which by its intersection with the line H h, drawn at an angle of 45°, will give the point sought.

In the plan, the pulley is supposed to be divided horizontally in the center, and the shadow represented is derived from the smaller circle, and is easily constructed by methods already described.

To trace the outlines of the shadows cast upon the surfaces of a square-threaded nut and screw (Figs. 237, 238).

Fig. 238 represents the projections of a screw with a single square thread, and placed in a horizontal position, A′ a' being the direction of the ray of light.

In this example, the shadow to be determined is simply that cast by the outer edge, A B, of the thread upon the surface of the inner cylinder ; therefore, its outline is to be delineated in the same manner as we have already pointed out, in treating of a cylinder surmounting another of smaller diameter (Fig. 223).

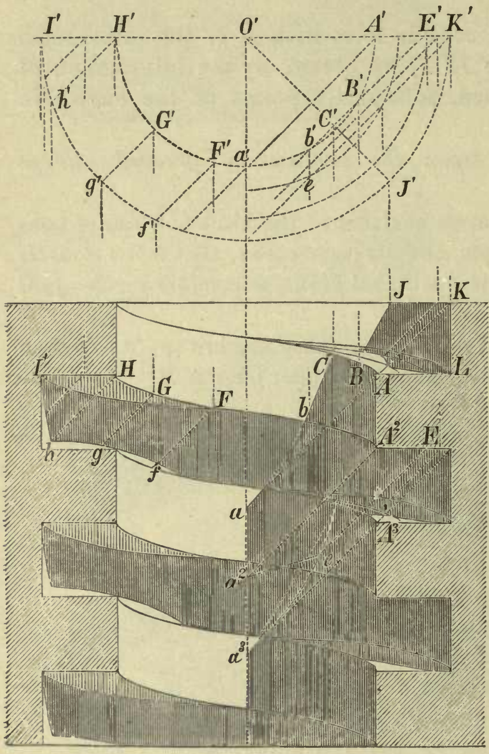


Fig. 237.

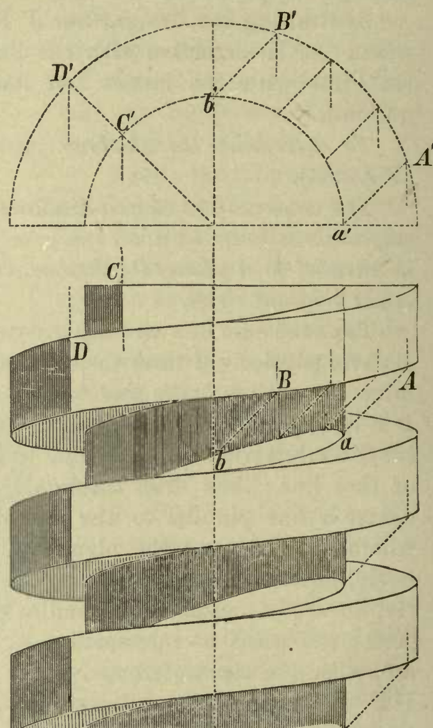


Fig. 238.

The shadow cast by the helix A B C upon the concave surface of the square-threaded nut is a curve, *a b* C (Fig. 237), which is to be determined in the same way as that in the interior of a hollow cylinder. The same observation applies to the edges A A^2 and A^2 E, as well as to those of the helix F G H and the edge H I. With regard to the shadow of the two edges J K and K L, they will follow the rules laid down in reference to the following figures, seeing that they are thrown upon an inclined helical surface, of which A L is the generatrix.

To determine the outlines of the shadows cast upon the surfaces of a triangular-threaded nut and screw (Figs. 239, 240).

Fig. 240 represents the case of a triangular-threaded screw, and does not admit of so easy a solution as the square-threaded, because the outer edge, A C D, of the thread, in place of throwing its shadow upon a cylinder, projects it upon a helical surface inclining to the left, of which the generatrix is known. Describe from the center O a number of circles, representing the bases of so many cylinders, on the surfaces of which we must suppose helical lines to be traced, of the same pitch as those which form the exterior edges of the screw. We must now draw any line, such as B′ E′, parallel to the ray of light, and cutting all the circles described in the plan in the points B′, F′, G′, E′, which are then to be successively projected to their corresponding helical lines in the elevation, where they are denoted by B^2, F, G, and E. Then, transferring the point B′ to its appropriate position, B, on the edge A C D, and

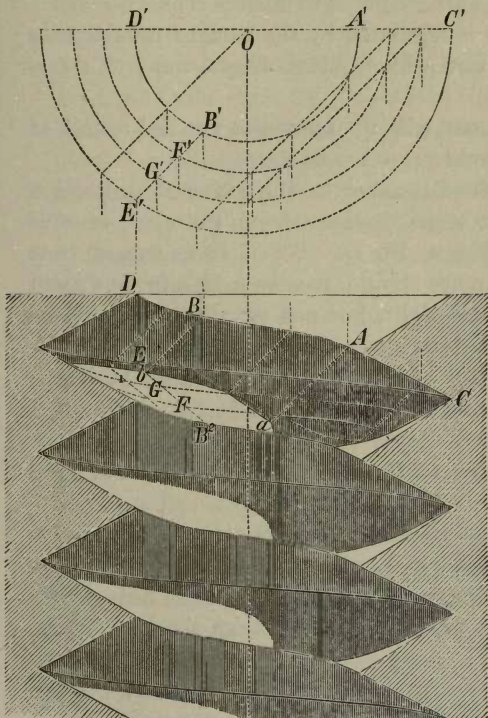


Fig. 239.

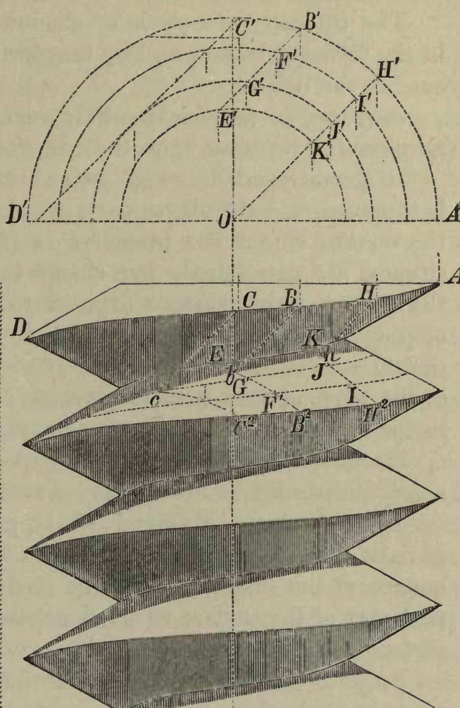


Fig. 240.

drawing through the latter a line, B *b*, at an angle of 45°, its intersection with the curve B^2 G E will give one point in the curve of the shadow required. In the same manner, by constructing other curves, such as H^2 J K, the remaining points, as *h*, in the curve may be found.

The same processes are requisite in order to determine the outlines of the shadows cast into the interior surfaces of the corresponding nut, as will be evident from an inspection of Fig. 239. These shadows are derived not only from the helical edge A B D, but also from that of the generatrix A C.

The principles so fully laid down and illustrated in the preceding pages will be found to admit of a ready and simple application to the delineation of the shadows of all the ordinary forms and combinations of machinery and architecture, however varied or complicated ; and the student should exercise himself, at this stage of his progress, in tracing, according to the methods above explained, the outlines of the cast shadows of pulleys, spur-wheels, and such simple and elementary pieces of machinery. It must be observed that the student should never copy the figures as here represented, but should adopt some convenient scale somewhat larger than our figures, and construct his drawings according to the description, looking to the figures as mere illustrations; in this way, the principles of the construction will be more surely understood, and more firmly fixed in his mind.

MANIPULATION OF SHADING AND SHADOWS.—METHODS OF TINTING.

The intensity of a shade or shadow is regulated by the various peculiarities in the forms of bodies, and by the position which objects may occupy in reference to the light.

Surfaces in the Light.—Flat surfaces wholly exposed to the light, and at all points equidistant from the eye, should receive a uniform tint.

In geometrical drawings, every surface parallel to the plane of projection is supposed to have all its parts at the same distance from the eye; such is the vertical side of the prism *a b c d* (Fig. 4, Pl. I). When two surfaces thus situated are parallel, the one nearer the eye should receive a lighter tint than the other. Every surface exposed to the light, but not parallel to the plane of projection, and therefore having no two points equally distant from the eye, should receive an unequal tint. The tint should gradually increase in depth as the parts of such a surface recede from the eye. This effect is represented in the same figure on the inclined surface, *a d f e*.

If two surfaces are unequally exposed to the light, the one which is more nearly perpendicular to the rays should receive the fainter tint.

Thus, the face *e′ a′* (Fig. 1, Pl. I), presenting itself more directly to the rays of light than the face *a′ b′*, receives a tint which, although graduated in consequence of the inclination of this face to the plane of projection, becomes at that part of the surface situated nearest to the eye fainter than the tint on the surface *a′ b′*.

Surfaces in Shade.—When a surface entirely in the shade is parallel to the plane of projection, it should receive a uniform dark tint.

When two objects parallel to each other are in the shade, the one nearer the eye should receive the darker tint.

When a surface in the shade is inclined to the plane of projection, those parts which are nearest to the eye should receive the deepest tint. This can be seen on the face *b g h c* (Fig. 4), where the tint is much darker toward the line *b c*, than where it approaches the line *g h*.

If two surfaces exposed to the light, but unequally inclined to its rays, have a shadow cast upon them, that part of it which falls upon the surface more directly influenced by the light should be darker than where it falls upon the other surface.

Exemplifications of the foregoing rules may be seen on the various figures of Plates I to V.

In order that these rules may be practiced with proper effect, we shall give some directions for using the brush or hair-pencil, and explain the usual methods employed for tinting and shading.

The methods of shading most generally adopted are either by the superposition of any number of flat tints, or by tints softened off at their edges. The former method is the more simple of the two, and should be the first attempted.

To shade a prism by means of flat tints (Pl. I).

According to the position of the prism, as shown by its plan, the face *a b c d* (Fig. 4) being parallel to the plane of projection, should receive a uniform tint either of India ink or sepia. When the surface to be tinted happens to be very large, it is advisable to put on a very light tint first, and then to go over the surface a second time with a tint sufficiently dark to give the desired tone to the surface.

The face *b g h c* being inclined to the plane of projection, should receive a graduated tint from the line *b c* to the line *g h*. This gradation is obtained by laying on a succession of flat tints in the following manner: First, divide the plan $b' g'$ into equal parts at the points 1′, 2′, and from these points project lines upon, and parallel to, the sides of the face *b g h c*. These lines should be drawn very lightly in pencil, as they merely serve to circumscribe the tints. A grayish tint is then spread over that portion of the face *b g h c* (Fig. 2), between the lines *b c* and 1, 1. When this is dry, a similar tint is to be laid on, extending over the space comprised within the lines *b c* and 2, 2 (Fig. 3). Lastly, a third tint, covering the whole surface *b c h g* (Fig. 4) imparts the desired graduated shade to that side of the prism. The number of tints designed to express such a graduated shade depends upon the size of the surface to be shaded; and the depth of tint must vary according to this number.

As the number of these washes is increased, the whole shade gradually presents a softer appearance, and the lines which border the different tints become less harsh and perceptible. For this reason the foregoing method of representing a shade or graduated tint by washes successively passing over each other is preferable to that sometimes employed, of first covering the whole surface *b g h c* with a faint tint, then putting on a second tint *b* 2 2 *c*, followed, lastly, by a narrow wash *b* 1 1 *c*; because, in following this process, the outline of each wash remains untouched, and presents, unavoidably, a prominence and harshness which, by the former method, are in a great measure subdued.

The face *a d f e* being also inclined to the plane of projection, should, as it is entirely in the light, be covered by a series of much fainter tints than the surface *b g h c*, which is in the shade, darkening, however, toward the line *e f*. The gradation of tint is effected in the same way as on the face *b g h c*.

To shade a cylinder by means of flat tints (Pl. I).

In shading a cylinder, it will be necessary to consider the difference in the tone proper to be maintained between the part in the light and that in the shade. It should be remembered that the line of separation between the light and shade *a b* (Fig. 6) is determined by the radius O a' (Fig. 5), drawn perpendicular to the rays of light R O. That part, therefore, of the elevation of the cylinder which is in the shade is comprised between the lines *a b* and *c d*. This portion, then, should be shaded conformably to the rule previously laid down for treating surfaces in the shade inclined to the plane of projection. All the remaining part of the cylinder which is visible presents itself to the light; but, in consequence of its circular figure, the rays of light form angles varying at every part of its surface, and consequently this surface should receive a graduated tint. In order to represent with effect the rotundity, it will be necessary to determine with precision the part of the surface which is most directly affected by the light. This part, then, is situated about the line *e i* (Fig. 12). As the visual rays, however, are perpendicular to the vertical plane, and therefore parallel to V O, it follows that the part which appears clearest to the eye will be near this line V O, and may be limited by the line T O, which bisects the angle V O R. By projecting the points *e′* and *m′*, and drawing the lines *e i* and *m n* (Fig. 12), the surface comprised between these lines will represent the lightest part of the cylinder.

This part should have no tint upon it whatever if the cylinder happen to be polished—a turned iron shaft or a marble column, for instance; but if the surface of the cylinder be rough, as in the case of a cast-iron pipe, then a very light tint—considerably lighter than on any other part—may be given it.

Again, let us suppose the half-plan of the cylinder *f′ m′ a′ c′* to be divided into any number of equal parts. Indicate these divisions upon the surface of the cylinder by faint pencil-lines, and begin the shading by laying a tint over all that part of the cylinder in the shade *a c d b* (Fig. 6). This will at once render evident the light and dark parts of the cylinder. When this is dry, put on a second tint covering the line *a b* of separation of light and shade, and extending over one division, as shown in Fig. 7. Proceed in this way until the whole of that part of the cylinder which is in the shade is covered. The successive stages of this process may be seen in Figs. 6 to 12.

Treat in a similar manner the part *f e i g* (Fig. 12), and complete the operation by covering the whole surface of the cylinder—excepting only the division *e m n i*—with a very light tint; the cylinder will then assume the appearance presented by Fig. 12.

To shade the segment of an hexagonal pyramid by means of softened tints (Pl. II).

The plan of this figure is similar to that of the prism (Pl. I). Its position in reference to the light is also the same. Thus, the face *a b c d* should receive a uniform flat tint. If, however, it be desired to adhere rigorously to the preceding rules, the tint may be slightly deepened as it approaches the top of the pyramid, seeing that the surface is not quite parallel to the vertical plane.

The face *b g h c* being inclined and in the shade, should receive a dark tint. The darkest part of this tint is where it meets the line *b c*, and gradually becomes lighter as it approaches the line *g h*. To produce this effect, apply a narrow strip of tint to the side *b c* (Fig. 6), and then, qualifying the tint in the brush with a little water, join another strip to this, and finally, by means of another brush moistened with water, soften off this second strip toward the line 1, 1, which may be taken as the limit of the first tint.

When the first tint is dry, cover it with a second, which must be similarly treated, and should extend beyond the first up to the line 2, 2 (Fig. 7). Proceed in this manner with other tints, until the whole face *b g h c* is shaded, as presented in Fig. 8.

In the same way the face *e a d f* is to be covered, though with a considerably lighter tint, for the rays of light happen to fall upon it almost perpendicularly.

It may be observed that, consistently to carry out the rules we have laid down, the tint on these two faces should be slightly graduated from *e a* to *f d*, and from *c h* to *b g*. But this exactitude may be disregarded until some proficiency in shading has been acquired.

To shade a cylinder by means of softened tints (Pl. II).

The boundary of each tint being indicated in a manner precisely similar to that shown in Pl. I, the first strip of tint must cover the line of extreme shade *a b*, and then be softened off on each side. Other and successively wider strips of tint are to follow, and receive the same treatment as the one first put on. The results of this process are shown in the figures.

As this method requires considerable practice before it can be performed with much nicety, the learner need not be discouraged at the failure of his first attempts, but persevere in practicing on simple figures of different sizes.

If, after shading a figure by the foregoing method, any very apparent inequalities present themselves in the shades, such defects may be remedied, in some measure, by washing off excesses of tint with a brush or a damp sponge, and by supplying a little color to those parts which are too light.

Dexterity in shading figures by softened tints will be facilitated in practicing upon *large* surfaces ; this will be the surest way of overcoming that timidity and hesitation which usually accompany all first attempts, but which must be laid aside before much proficiency in shading can be acquired.

ELABORATION OF SHADING AND SHADOWS.

Thus far the simplest primary rules for shading isolated objects have been laid down, and the easiest methods of carrying them into operation explained. It is now proposed to exemplify these rules upon more complex forms, to show where the shading may be modified or exaggerated, to introduce additional rules more especially adapted for mechanical coloring, and to offer some observations and directions for effectively shading the drawing of machines in their entity.

Whatman's best rough-grained drawing-paper is better adapted for receiving color than any other. Of this paper, the *double-elephant* size is preferable, as it possesses a peculiar consistency and grain. The face of the paper to be used is the one on which the *water-mark* is read correctly.

The paper for a colored drawing ought always to be strained upon a board with glue or strong gum. Before doing this, care must be taken to dampen the *face* of the paper with a sponge well charged with water, in order to remove any impurities from its surface, and as a necessary preparation for the better reception of the color. The sponge should merely touch the paper lightly, and not rub it. The whole of the surface is to be dampened, that the paper may be subjected to a uniform degree of expansion, thereby insuring, as it dries, a uniform degree of contraction. Submitted to this treatment, the sheet of paper will present, when thoroughly dry, a clean, smooth surface, agreeable to work upon.

The size of the brushes to be used will, of course, depend upon the scale to which the drawing is made. Long, thin brushes, however, should be avoided. Those possessing corpulent bodies and fine points are to be preferred, as they retain a greater quantity of color, and are more manageable.

During the process of laying on a flat tint, if the surface be large—though this is seldom the case except in topographical drawings—the drawing may be slightly inclined, and the brush well charged with color, so that the edge of the tint may be kept in a moist state until the whole surface is covered. In tinting a small surface, the brush should never have much color in it, for, if it have, the surface will unavoidably present coarse, rugged edges, and a coarse, uneven appearance throughout.

In the examples of shading which are given in this work, it may be observed that all objects with curved outlines have a certain amount of reflected light imparted to them. It is true that all bodies, whatever may be their form, are affected by reflected light ; but, with a few exceptions, this light is only appreciable on curved surfaces.

All bodies in the light reflect on those objects which surround them more or less light according to the situation. Wherever light extends, reflection follows. If an object be isolated, it is still reached, by reflected light, from the ground on which it rests, or from the air which surrounds it.

In proportion to the degree of polish or brightness in the color of a body, is the amount of reflected light which it spreads over adjacent objects, and also its own susceptibility of illumination under the reflection from other bodies. A polished steam-cylinder or a white porcelain vase receives and imparts more reflected light than a rough casting or a stone pitcher.

Shade, even the most inconsiderable, ought never to extend to the outline of any smooth circular body. On a polished sphere, for instance, the shade should be delicately softened off just before it meets the circumference, and, when the shading is completed, the body-color intended for the sphere may be carried on to its outline. This will give a transparency to that part of the sphere influenced by reflected light, which it could not have possessed if the shade-tint had been extended to its circumference. Very little shade should be suffered to reach the outlines even of rough circular bodies, lest the coloring look harsh, and present a coarse appearance quite at variance with its natural aspect. Shadows also become lighter as they recede from the bodies which cast them, owing to the increasing amount of reflection which falls on them from surrounding objects.

Shadows appear to increase in depth as their distance from the spectator diminishes. In nature this increase is only appreciable at considerable distances. Even on extensive buildings, inequalities in the depth of the shadows are hardly perceptible ; much less, then, can any natural gradation present itself in the shadows on a machine, which, supposing it to be of the largest construction, is confined to a comparatively small space. It is most important, however, for the effective representation of machinery, that the variation in the distance of each part of a machine from the spectator should at once strike the eye ; and an exaggeration in expressing the varying depths of the shadows is one means of effecting that object. The shadows on the nearest and most prominent parts of a machine should be made as dark as color can render them, the colorist being thus enabled to exhibit a marked difference in the shadows on the other parts of the machine as they recede from the eye. The same direction is applicable in reference to shades. The shade on a cylinder, for instance, situated near the spectator, ought to be darker than on one more remote; in fact, the gradation of depth for the shades follows that which depicts the shadows. As a general rule, the color on a machine, no matter what it may be intended to represent, should become lighter as the parts on which it is placed recede from the eye.

Plates III and IV present some very good examples of finished shading.

Plate III represents, both in elevation and plan, different solids variously penetrated and intersected. The rules for the projection of these solids have been given under the head of *Orthographic Projection.* They are selected with a view of exhibiting those cases which are of most frequent occurrence, and at the same time elucidating the general principles of shading.

Plate IV presents examples of shading and shadow.

Fig. 1 presents a hexagonal prism surmounted by a fillet. The most noticeable part of this figure is the shadow of the prism in the plan view. It presents a good example of the graduated expression which should be given to all shadows cast upon plain surfaces. Its two extremities are remarkably different in their tone. As the shadow nears the prism, it increases rapidly in depth; on the contrary, as it approaches the other end, it assumes a comparatively light appearance. This difference is doubtlessly a great exaggeration upon what it would naturally display. Any modification of it, however, in the representation would destroy the best effect of the shadow.

The direction which the shades and shadows take, in all the plans of the figures in this plate, is from the left-hand lower corner. This is rigorously correct, supposing the objects to remain stationary, while the spectator views them in both a vertical and a horizontal position. Nevertheless, to many, this upward direction given to the shadows has an awkward appearance, and, perhaps, in the plan of an entire machine, the shadows may *look* better if their direction coincide with that which is given to them in the elevation. If, however, the shadows be correctly projected, their direction is an arbitrary matter, and may be left to the taste of the draughtsman.

Figs. 2, 3, and 6 exemplify the complex appearance of shade and shadow presented on concave surfaces. It is worthy of particular notice that the shadow on a concave surface is darkest toward its outline, and becomes lighter as it nears the edge of the object. Reflection from that part of the surface on which the light falls most powerfully causes this gradual diminution in the depth of the shadow, the greatest amount of reflection being opposite the greatest amount of light.

It may be as well to remark here that no *brilliant or extreme* light should be left on concave surfaces, as such lights would tend to render it doubtful at first sight whether the objects represented were concave or convex. After the body-color—which shall be treated in a subsequent section—has been put on, a faint wash should be passed very lightly over the whole concavity. This will not only modify and subdue the light, but tend to soften any asperities in the tinting, which are more unsightly on a concave surface than on any other.

The lightest part of a sphere (Fig. 4) is confined to a mere point, around which the shade commences and gradually increases as it recedes. This point is not indicated on the figure referred to, because the shade-tint on a sphere ought not to be spread over a greater portion of its surface than is shown there. The very delicate and hardly perceptible progression of the shade in the immediate vicinity of the light-point should be effected by means of the body-color of the sphere. If, for instance, the material of which the sphere is composed be brass, the body-color itself should be lightened as it nears the light point. In like manner all polished or light-colored curved surfaces should be treated; the part bordering upon the extreme light being covered with a tint of body-color somewhat fainter than that used for the flat surfaces. Again, if the sphere be of cast-iron, then the ordinary body-color should be deepened from the light point until it meets the shade-tint, over which it is to be spread uniformly. Any curved unpolished surface is to be thus treated; the body-color should be gradually deepened as it recedes from that part of the surface most exposed to the light. Considerable management is necessary in order to shade a sphere effectively. The best way is to put on two or three softened-off tints in the form of crescents converging toward the light-point, the first one being carried over the point of deepest shade.

A ring (Fig. 5) is a difficult object to shade. To change with accurate and effective gradation the shade from the inside to the outside of the ring, to leave with regularity a line of light upon its surface, and to project its shadow with precision, require a degree of attention and care in their execution greater, perhaps, than the shade and shadow of any other simple figure. The learner, therefore, should practice the shading of this figure, as he will seldom meet with one presenting greater difficulties.

Figs. 7 and 8 show the peculiarities of the shadows cast by a conical form on a sphere or cylinder. The following fact should be well noted in the memory: That the depth of a shadow on any object is in proportion to the degree of light which it encounters on the surface of that object. In these figures very apt illustrations of this fact may be remarked. It will be seen, by referring to the plan (Fig. 7), that the shadow of the apex of the cone happens to fall upon the lightest point of the sphere, and is, therefore, the darkest part of the shadow. So also the deepest portion of the shadow of the cone on the cylinder in the plan (Fig. 8) is exactly where it coincides with the line of extreme light. *Flat* surfaces are similarly affected, the shadows thrown on them being less darkly expressed, according as their inclination to the plane of projection increases. The body-color on a flat surface should, on the contrary, increase in depth as the surface becomes more inclined to this plane.

Another notable fact is exemplified by these figures—that reflected light is incident to shadows as well as to shades. This is very observable where the shadow of the cone falls upon the cylinder. It may likewise be remarked, though to a less extent, on other parts of these figures. The reflected light on the cone from the sphere or cylinder is also worthy of observation. This light adds greatly to the effect of the shadows, and, indeed, to the appearance of the objects themselves. Altogether, these figures offer admirable scope for study and practice.

The concentration within a small space of nearly all the peculiarities and effects of light, shade, and shadow, may be seen on Plate V in the examples of screws there given.

Under the head of Topographical, Mechanical, and Architectural Drawing, will be given examples of drawings in shade and shadow, and in varied colors expressing conditions of surfaces or materials of composition. In the topographical and architectural examples, often a certain amount of artistic effect can be introduced, but, in the mechanical, distinctness of outline and accuracy of expression are essential; but, to maintain harmony in the coloring, and to equalize the appearance of the drawing, large shades should be colored less darkly than small, as they may be situated at the same distance from the eye, and no very dark shading is permissible.

In preparing colors for tints, great care should be used in grinding. The end of the cake should be slightly wetted and rubbed on a porcelain palette, and then transferred by a wet brush to another saucer, and water added to bring to the required tint. Mixed colors should be intimately blended by the brush. Grind in excess enough of all the tint required, and let it stand in the saucer till the grosser particles have settled and the liquid is of clear and uniform tint. It is very common to make little boxes or bag-like receptacles of waste drawing-paper to hold the colors instead of saucers; the gross matters, settling on the bottom, are not then so readily disturbed.

Instead of hard cakes of color, moist colors are used, either in cakes or collapsible tubes, which preclude the necessity of grinding. For flat tints or washes, aniline colors, dissolved in water and kept bottled, afford the readiest means of coloring, but are not applicable to finished work.

Sometimes the surface of the paper is, as it were, greasy, and resists colors; in that case, dissolve a piece of ox-gall, the size of a pea, in a tumbler of water, and use this solution with the colors instead of plain water.

When the brush is too full, as it comes toward the limit of the tint, take up the surplus moisture on a wet sponge or piece of cloth or blotting-paper.

An expeditious way of shading a cylinder or expressing the shores of a stream or lake, is by drawing with a brush full of the darkest tint along the sides of cylinder or shores of water, and then, with a wet brush, modifying this tint toward the light from the sides, so as to give a shaded appearance. For this purpose, two brushes will be necessary, one with color, the other with water; also, a tumbler of water, and a piece of blotting-paper, to take up the excess of moisture from paper or brush. Often a single line of dark color blended this way will express all that is necessary, but the effect may be improved by a sort of stippling with the color-brush and extending the line of shade.

The same effect is obtained better by drawing two faint pencil-lines on the elevation of the cylinder, for instance, to indicate the extremes of light and shade on its surface. Pass the brush, moderately full of the *darkest* tint, down the line of deepest shade, spreading the color more or less on either side, according to the diameter of the cylinder; then, if possible, before this layer of tint is dry, toward the line of extreme light, beginning at the top, and encroaching slightly over the edge of the first tint, lay on another not quite so dark, but about double its width. It may be observed that it is not very essential to put on the second tint befor the *first* is dry, for the latter should be so dark and thick that its edges may be easily softened at any time. While this second tint is still wet, with a much lighter color in the brush, proceed in the same manner with a third tint, and so on until the line of extreme light is nearly attained. Repeat this process on the other side of the first tint, approaching the outline of the cylinder with a very faint wash, so as to represent the reflected light which progressively modifies the shade as it nears that line. Then let a darkish narrow strip of tint meet and pass along the outline of the cylinder on the other side of the extreme line of light, after which gradually fainter tints should follow, treated in a manner similar to that which has been already described, and becoming almost imperceptible just before arriving at the line of light.

This is a very expeditious way of shading a cylinder ; but even to the most experienced colorist it is not possible, by the above-described means alone, to impart a sufficient degree of well-regulated rotundity to the appearance of such an object. Superfluities and deficiencies of color will appear here and there. It will be necessary, therefore, to equalize to some extent, by a species of gross stippling, the disparities which present themselves. This is done by spreading a little color over the parts where it is deficient, and then passing very lightly over nearly the whole width of the shade, with the brush supplied with a very light wash. This process may be repeated to suit the degree of *finish* which it is desired to give the drawing. In the same manner the shading of all curved surfaces is to be treated.

The shades being put in, that of the shadows follows. The outline of any shadow being drawn in pencil, along its inner line—the line which forms a portion of the figure of the object whose shadow is to be represented—lay on a strip of the darkest tint, wide or narrow, according to the width of the shadow, and then, before it is dry, soften off its outer edge. This may be repeated as often as the taste of the colorist may dictate, but the color should not spread itself over much more than half the space occupied by the shadow. These preliminary touches will add to the intensity of the proposed shadow, and neutralize a certain harshness of appearance inevitable to all shadows made equally dark throughout.

The finish is made by a light wash or two of the body-color, and in passing over the shades and shadows care must be taken to manœuver the brush at such parts quickly and lightly.

The shades and shadows of a machine are modified in intensity as their distance from the eye increases. Its body-color should be treated in a similar manner, becoming lighter and less bright as the parts of the machine which it covers recede from the spectator.

When the large circular members of a machine have been shaded, the shadows, and even the body-color on those parts farthest removed from the eye, are to follow, and the proportion of India ink in the tint used should increase as the part to be colored becomes more remote. A little washing, moreover, of the most distant parts is allowable, as it gives a pleasing appearance of atmospheric remoteness, or depth, to the color thus treated.

The amount of light and reflection on the members of a machine should diminish in intensity as the distance of such objects from the spectator increases. As it is necessary, for effect, to render, on those parts of a machine nearest the eye, the contrast of light and shade as intense as possible, so, for the same object, the light and shade on the remotest parts should be subdued and blended according to the extent or size of the machine.

A means of adding considerably to the definiteness of a colored mechanical drawing, and of promoting, in a remarkable degree, its effective appearance, is obtained by leaving a very narrow margin of light on the edges of all surfaces, no matter what may be the angles which they may form with the surfaces that join them. This should be done invariably ; but the margin of those edges which happen to have shadows falling on them, instead of being left quite white, may be slightly subdued.

To effect this, suppose the object about to receive the color to be the elevation of a long, flat rod or lever, on the edge of which a line of light is to be left. Fill the drawing-pen as full as it will conveniently hold with tint destined to cover the rod or lever, and draw a broad line just within, but not touching, the edge of the lever exposed to the light. As it is essential for the successful accomplishment of the desired effect that this line of color should not dry, even partially, until the tint on the whole side of the lever has been put on, it will be as well to draw the pen again very lightly over the same part, so that the line may retain as much tint as possible. Immediately this has been done, the brush, properly filled with the same tint, is to pass along and join the inner edge of this narrow strip of color, and the whole surface of the lever filled in. Thus a distinct and regular line of light is obtained, and, in fact, the lever, or whatever else the object may be, covered in a shorter time than usual. A still more expeditious way of coloring such surfaces is to draw a second line of color along and joining the opposite edge of the lever or other object, and then expeditiously to fill in the intermediate space between the two wet lines by means of the brush. By similar means the line of light on a cylinder, shaft, or other circular body, may be beautifully expressed. To indicate this light with perfect regularity is highly important, for, if a strict uniformity be not maintained throughout its whole length, the object will look crooked or distorted. After having marked in pencil, or guessed the position of the extreme light, take the drawing-pen, well filled with a just perceptible tint, and draw a line of color on one side the line of light, and almost touching it ; then with the brush, filled with similar light tint, join this line of color while still wet, and fill up the space unoccupied by the shade-tint, within which the very light color in the brush will disappear. Let that part of the object on the other side of the line of light be treated in the same way, and the desired effect of a stream of light clear and mathematically regular will be obtained. The extreme depth of shade, as well as the line of light in such rods, may, with great effect, be indicated by filling the pen with dark shade-tint, and drawing it exactly over the line representing the deepest part of the shade. On either side and joining this strip of dark color, another, composed of lighter tint, is to be drawn. Others successively lighter are to follow, until, on one side, the line of the rod is joined, and on the other the lightest part of the rod is nearly reached. The line of light is then to be shown, and the faint tint used on this occasion spread with the brush lightly over the whole of that part of the rod situated on either side of this line, thus blending into smooth rotundity the graduated strips of tint drawn by the pen.

In all tinted drawings the more important parts, whether the machinery or the structure, should be more conspicuously expressed than those parts which are mere adjuncts. Thus, if the drawing be to explain the construction of the machine, the tint of edifice and foundations may be kept lighter and more subdued than those of the machine ; and if the machine, on the contrary, be unimportant, it may be represented quite light, or in mere outline, while the edifice is brought out conspicuously.

With regard to washings, the soft sponge is an implement not to be neglected by the draughtsman ; it is an excellent means of correcting great errors in drawing, better than rubber or an eraser, but care of course must be taken to wash and not to rub off the surface, and for errors in coloring washing is almost the only corrector. In removing or softening color on large surfaces, the sponge is to be used, and for small spots the brush. While coloring, keep a clean, moist brush by you : it will be extremely useful in removing or modifying a color.

The immediate effect of washing is to soften a drawing, an effect often very desirable in architectural and mechanical drawings, and the process is simple and easily acquired ; keep the sponge or brush and water used clean ; after the washing is complete, take up the excess of moisture by the sponge or brush, or by a piece of clean blotting-paper. Where great vigor is required, let the borders of the different tints be distinct.

There are no conventional tints that draughtsmen have agreed upon to be uniformly used, to represent different materials. India ink is not a black, but a brown, making with a blue a greenish cast, and with gamboge a smear. A colored drawing is better without the use of India ink at all ; any depth of color may be as well obtained with blue as with black ; there is also an objection to gamboge, that it is gummy, and does not wash well, and the effect is better obtained with yellow ochre. For the reds, the madder colors are the best, as they stand washing ; for the shade-tint of almost every substance a neutral tint, Payne's gray, or madder brown subdued with indigo.

PLOTTING.

PLOTTING is the laying out on paper in plan or in horizontal projection the boundaries of lots, estates, farms, etc., portions of the earth's surface of greater or less extent, from the notes of surveys or other records. When the extents are large, beyond the usual limits of personal property, and embracing degrees of latitude and longitude, the plots are designated as maps ; but if of small extent—as lots, estates, and farms—they are usually designated as plans or plots. After completing the outlines, it is usual to fill up the plot, with the characteristic features, geographical, geological, agricultural, industrial, and domestic, which are expressed more or less conventionally, as will be shown under the head of "Topographical Drawing."

Scales.—The choice of the scale for the plot depends in a great measure on the purpose for which the plan is intended. It should be large enough to express all the details desirable, modified by the circumstances whether the map is to be portable or whether space can be afforded for the exhibition of a large plan. We must adapt our plan for the purposes which it is intended to illustrate, and the place it is to occupy.

Plans of house-lots are usually named as being so many feet to the inch ; plots of farm-surveys, as so many chains to the inch ; maps of surveys of States, as so many miles to the inch ; and maps of railway-surveys, as so many feet to the inch, or so many inches to the mile.

Formerly the lines of farms were measured by the four-rod chain ; latterly the 100-foot chain is more usually adopted. Two to three chains to the inch was then a very common scale.

State surveys are of course plotted on a smaller scale than those of farms. On the United States Coast Survey all the scales are expressed fractionally and decimally. The original surveys are generally on a scale of one to ten or twenty thousand, but in some instances the scale is larger or smaller. The public surveys embrace three general classes : 1. Small harbor-charts. 2. Charts of bays, sounds, etc. 3. General coast-charts.

The scales of the first class vary from 1 : 5,000 to 1 : 60,000, according to the nature of the harbor and the different objects to be represented.

The scale of the second class is usually fixed at 1 : 80,000. Preliminary charts, are, however, issued of various scales, from 1 : 80,000 to 1 : 200,000.

Of the third class the scale is fixed at 1 : 400,000 for the general chart of the coast from Gay Head to Cape Henlopen, although considerations of the proximity and importance of points on the coast may change the scales of charts of other portions of our extended coast.

On all plots of large surveys, it is very desirable that the scales adopted should bear a definite numerical proportion to the linear measurement of the ground to be mapped, and that this proportion should be expressed fractionally on the plan, even if the scale be drawn or expressed some other way, as chains to the inch. The decimal system has the most to recommend it, and is generally adopted in government surveys.

For railroad-surveys, the New York general railroad law directs the scale of map which is to be filed in the State Engineer's office, to be 500 feet to one tenth of a foot, 1 : 5,000.

For the canal-maps, a scale of two chains to the inch, 1 : 1,584 is employed. In England, plans and sections for projected lines of inland communication, or generally for public works requiring the sanction of the Legislature, are required, by the "standing orders," to be drawn to scales not less than four inches to the mile, 1 : 15,840, for the plan, and 100 feet to the inch, 1 : 1,200, for the profiles.

In the United States engineer service the following scales are prescribed :

General plans of buildings	10 feet	to the	inch,	1 : 120
Maps of ground with horizontal curves 1 foot apart	50 "		"	1 : 600
Topographical maps 1½ mile square	1 mile	to	2 feet,	1 : 2,640
Topographical maps comprising 3 miles square	1 "		1 foot,	1 : 5,280
Topographical maps comprising between 4 and 8 miles	1 "		6 in.,	1 : 10,560
Topographical maps comprising 9 miles square	1 "		4 "	1 : 15,840
Maps not exceeding 24 miles square	1 "		2 "	1 : 31,680
Maps comprising 50 miles square	1 "		1 inch,	1 : 63,360
Maps comprising 100 miles square	1 "		½ "	1 : 126,720
Surveys of roads and canals	50 feet	to	1 "	1 : 600

In cities and towns, lots and squares are generally rectangular, and they can be readily plotted on any convenient scale.

Fig. 241 is a plan of the usual New York city lot, 25 × 100, on a scale of 20 feet to the inch, or $\frac{1}{240}$ full size.

Fig. 242 is a city square containing thirty-two of these lots, on a scale of 100 feet to the inch, or $\frac{1}{1200}$. The most accurate way is to plot the large rectangle 400 × 200 feet, and then subdivide it.

Fig. 243 is a plan of the same city squares, with the inclosing streets, on a scale 200 feet to the inch, or $\frac{1}{2400}$.

But there are many lots, and most estates, which are not rectangular, the angles of which are recorded, which must be plotted by the aid of a protractor.

If the survey has been made by triangles, the principal triangles are first laid down in pencil by the intersection of their sides, the length being taken from the scale and described with compasses. In general, when the surveys have been conducted without instruments to measure the angles, as the compass or theodolite, the position of the points on paper are determined by the intersection and construction of the same lines as has been done in the field.

Surveys are mostly conducted by measuring the inclination of lines to a meridian or to each other by the compass or by the theodolite. In the surveys of farms, where great accuracy is not required, the compass is most used. The compass gives the direction of a line in reference to the magnetic meridian. The variation from the true meridian, or a direct north-and-south line, varies considerably in different parts of the country. In 1875 the line of variation in which the needle pointed directly north, passed in a nearly straight direction from Wilmington, North

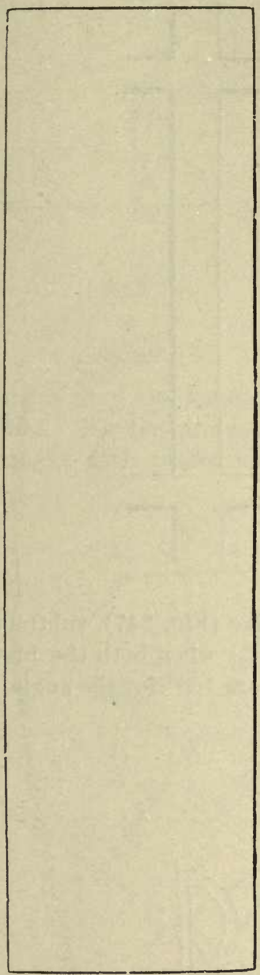


FIG. 241.

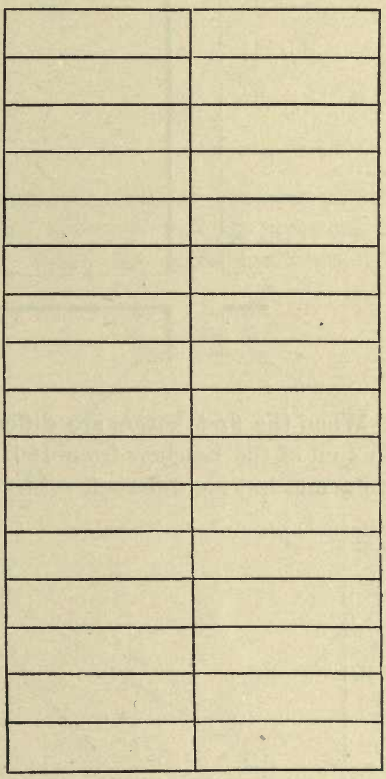


FIG. 242.

Carolina, to Cleveland, Ohio. At all places east of this line the variation is westerly, that is, the needle points west of the line north. West of this line the variation is easterly.

Fig. 244 represents the plot of a compass survey, with the positions of the protractor in laying off the angles. To the left of the figure are given the field-notes. In this way of plotting, a meridian is laid off at the intersection of each set of lines. Sometimes the angles are plotted directly from the determination of the angle of deflection of two courses meeting at any point, without laying down more than one meridian (Fig. 245). When the first letters of the bearing are alike, that is, both N. or both S., and the last letters also alike, both E. or both W., the angle of deflection C B B′ will be the difference of the bearings, or, in this instance, 20°.

When the first letters are alike and the last different (Fig. 246), the angle C B B′ will be the sum of the two bearings.

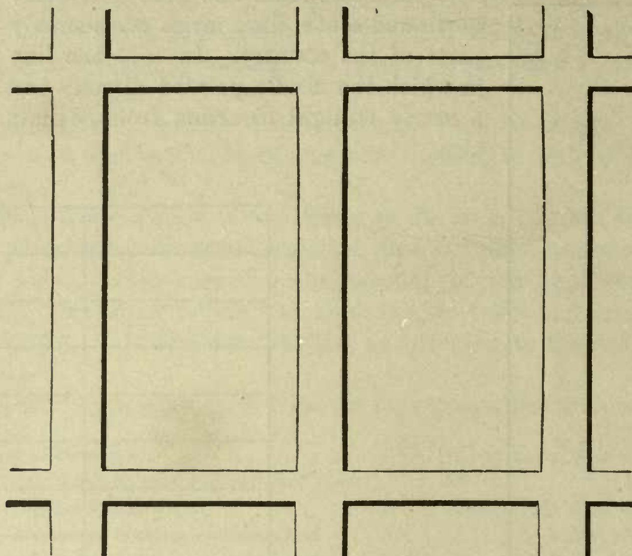
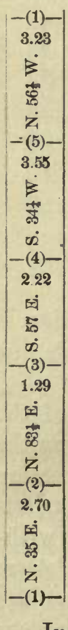


FIG. 243.

When the first letters are different and the last alike (Fig. 247), subtract the sum of the bearings from 180° for the angle C B B′; when both the first letters and last are different, subtract their difference from 180° for the angle.



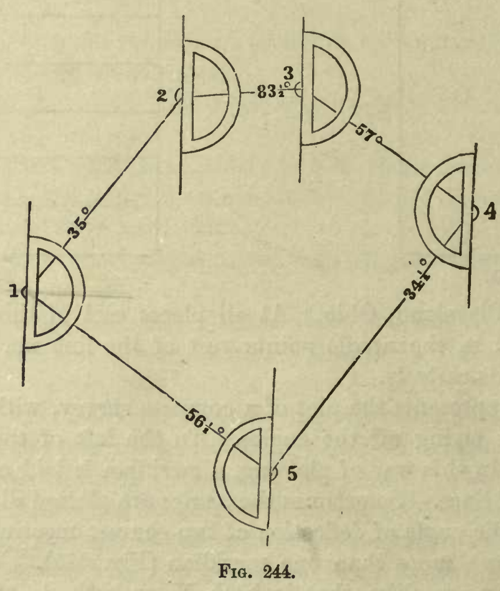


FIG. 244.

Instead of drawing a meridian through each station, or laying off the angle of deflection, by far the easiest way is to lay off but a single meridian near the middle of the sheet; lay off all the bearings of the survey from some one point of it, as shown in Fig. 248, and number to correspond with the stations from which the bearings are taken, and then transfer them to the places

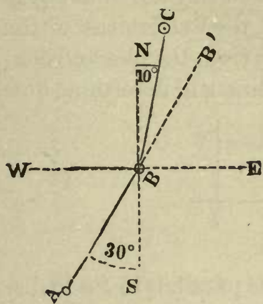


FIG. 245.

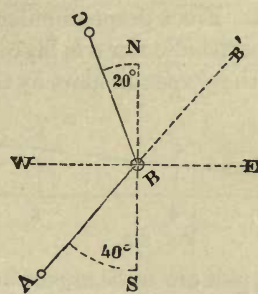


FIG. 246.

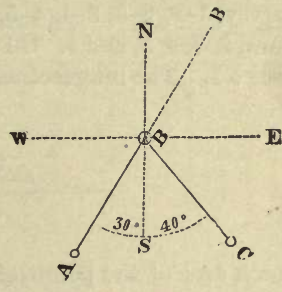


FIG. 247.

where they are wanted by any of the instruments used for drawing parallel lines. For the protracting of the rough plan, sheets of drawing-paper can be bought with protractors printed on them. When the plans are large, it is

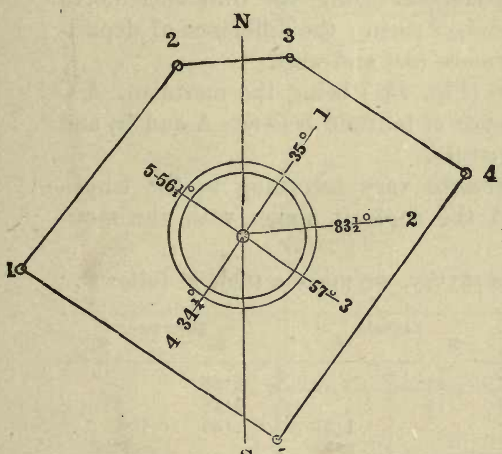


FIG. 248.

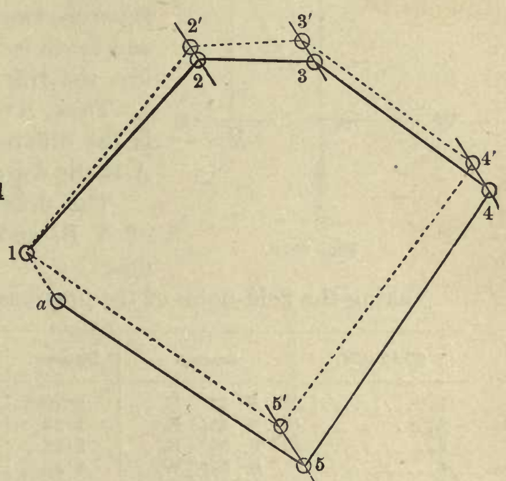


FIG. 249.

often convenient to lay out two or three meridians on different parts of the sheet, and lay off the bearings of lines adjacent to each meridian upon them.

In plotting from a survey by a theodolite or transit, it is generally usual to lay off the angles of deflection of the different lines as taken in the field, plotting all the tie-lines as corrections.

When the plot of a survey does not close—that is, come together, or return to the point of commencement, as it seldom does exactly—it may be corrected or *forced;* but first be sure that the bearings and distances as recorded are laid down accurately, and then proceed to correct as follows:

If the plot of the last line does not close up the outline of the figure exactly, by its extremity falling upon the point of beginning of the plot, as upon the point *a* (Fig. 249), instead of upon 1, either the survey or the plotting is incorrect.

If the latter be correct, the error of the survey must be *balanced*, or distributed through the lines and angles of the plot. Connect 1 with *a*, and draw lines parallel to 1 *a* through 2, 3, 4, 5, of the plot. Draw an indefinite line, 1 *b* (Fig. 250), and on this, with any convenient scale, lay off consecutively the lines of the survey, 1–2, 2–3, 3–4, 4–5, 5–*a*. Erect perpendiculars at the extremities of the lines, 2, 3, 4, 5, and *b*. On the perpendicular *a b*, lay off 1 *a* from the plot and connect *b* 1. The intersections of the perpendiculars by this line will determine how

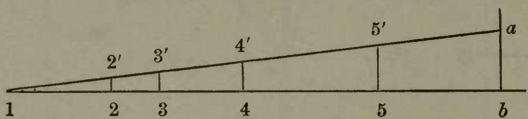


FIG. 250.

much each of the points of the plot are to be moved on the parallels to 1 *a* to distribute the error. The dotted lines on the figure show the corrected outline.

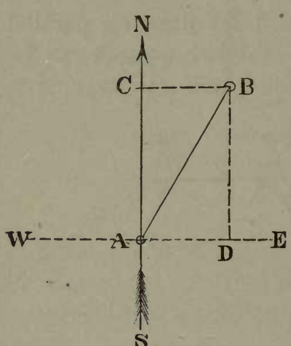


FIG. 251.

By the aid of the Traverse Table a survey may be balanced and accurately plotted. The Traverse Table (see appendix) is a table of differences of latitudes and departures, the difference of latitude between two stations being the difference north and south between them ; the difference of departure, the difference east and west.

Thus, N S (Fig. 251) being the meridian, A C is the difference of latitude between A and B, and A D the departure.

The differences vary according to the length of A B, and the angle it makes with the meridian.

Taking the field-notes of the previous survey, we make a table as follows :

STATION.	Bearing.	Distance.	Latitude. N.	Latitude. S.	Departure. E.	Departure. W.
1.............	N. 35° E.	2·70	2·21		1·55	
2.............	N. 83½° E.	1·29	·15		1·28	
3.............	S. 57° E.	2·22		1·21	1·86	
4.............	S. 34¼° W.	3·55		2·93		2·00
5.............	N. 56½° W.	3·23	1·78			2·69
			4·14	4·14	4·69	4·69

In the Traverse Table, on the line with 35°, and in

column 2, latitude = 1·638 departure = 1·147
" 7, " = ·573 " = ·401

2·211 1·548

Again, on the line with 83½°, in

column 1, latitude = ·113 departure = ·994
" = ·0226 " = ·1987
" = ·01019 " = ·08942

·14579 1·28212

And in the same manner the table is completed.

The table following is constructed by adding up the northings and subtracting the southings for the latitude, and by adding up the eastings and subtracting the westings for the departures.

STATION.	Total latitude from station.	Total departure from station 1 —.
1..	0·00	0·00
2..	+ 2·21 N.	+ 1·55 E.
3..	+ 2·36 N.	+ 2·83 E.
4..	+ 1·15 N.	+ 4·69 E.
5..	— 1·78 S.	+ 2·69 E.
1..	0·00	0·00

From this table the survey can be readily plotted (Fig. 252). Draw the meridian through the point taken for station 1; measure to the north 2·21 chains to A; draw an easterly line, or one perpendicular to the meridian at A, and lay off on it 1·55 chains, and we have station 2; measure again from 1

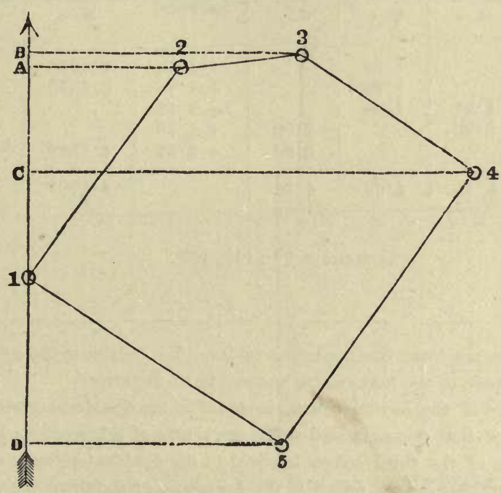


Fig. 252.

northerly 2·36 chains to B, and lay off from B due easterly 2·83 chains for station 3; measure again from 1 northerly 1·15 chains to C, and lay off from C due easterly 4·69 chains for station 4; measure again from 1 southerly 1·78 chains to D, and lay off from D easterly 2·69 chains for station 5. Connect 1, 2, 3, 4, 5, and 1 for the complete plot.

In this survey the latitudes balance, 4·14 to 4·14, and the departures balance, 4·69 to 4·69, but this seldom happens. Generally there is a difference which must be balanced before plotting. For instance, in this survey, had the

northings been $\left\{\begin{array}{r} 2{\cdot}20 \\ {\cdot}15 \\ 1{\cdot}70 \\ \hline 4{\cdot}05 \end{array}\right.$ and had the southings been $\left\{\begin{array}{r} 1{\cdot}24 \\ 2{\cdot}95 \\ \hline 4{\cdot}19 \end{array}\right.$ the difference would have been ·14 to be divided, in proportion to their lengths, between the northings and the southings, adding to the former and deducting from the latter. The total northings and southings is 4·05 + 4·19 = 8·24 chains, in which an error of 14 links is to be balanced, or about ·017 chain to each chain. In the 2·20 N. the correction will be 2·20 × ·017 = ·0374, or about ·04, and without much calculation we can see that

the corrected northings will be { 2·24, ·15, 1·73; total 4·12 } and the corrected southings will be { 1·22, 2·90; total 4·12 }

The same calculation is applied to the departures when there is a difference in the total eastings and westings.

The errors are to be balanced before the survey is plotted.

When a field has been plotted, it can be divided into triangles, and its area can be calculated; but, having the latitudes and departures balanced and tabulated, the area can be calculated as follows:

STATION.	Latitude.		Departure.		Double longitude.	Double area.	
	N +	S. −	E. +	W. −		N. +	S. −
1.......	2·21		1·55		+ 1·55	3·4255	
2.......	·15		1·28		+ 4·38	0·6570	
3.......		1·21	1·86		+ 7·52		9·0992
4.......		2·93		2·00	+ 7·38		21·6234
5.......	1·78			2·69	+ 2·69	4·7882	
	4·14	4·14	4·69	4·69		8·8707	30·7226
							8·8707
			Content = 1A., 0R., 15P.				2)21·8519
							10·9259 = 1·09259 acres.

The first five columns are from the preceding tables. To construct the column of double longitudes: the double longitude of the first course is equal to its departure.

The double longitude of the second course is equal to the double longitude of the first course, added to the departure of that course, added to the departure of the second course.

The double longitude of the third course is equal to the double longitude of the second course, added to the departure of that course, added to the departure of the third course.

The double longitude of any course is equal to the double longitude of the preceding course added to the departure of that course, added to the departure of the course itself; the double longitude of the last course is equal to its departure.

Thus, the double longitude of first course is its departure............................ = 1·55
add the departure of first course.. 1·55
add the departure of second course.. 1·28
and we have the double longitude of second course.............................. = 4·38
add the departure of second course.. 1·28
add the departure of third course.. 1·86
and we have the double longitude of third course.............................. = 7.52
add the departure of third course.. 1·86
subtract the departure of fourth course.............................. 2.00
and we have the double longitude of fourth course.............................. = 7·38
subtract the departure of fourth course.............................. 2·00
subtract the departure of fifth course.............................. 2·69 4·69
and we have the double longitude of fifth course.............................. = 2·69

Multiply the double longitude of each course by the latitude of that course, placing the north products in one column and the south products in another; subtract the lesser total of the one column from the greater total of the other, and divide the difference by two. The product will be in square chains, which, divided by ten, will give the result in acres and decimals.

The area of an irregular figure can be calculated most conveniently, and with sufficient accuracy, by dividing it into triangles, measuring the height and base of each, calculating the area of each, and adding the areas together.

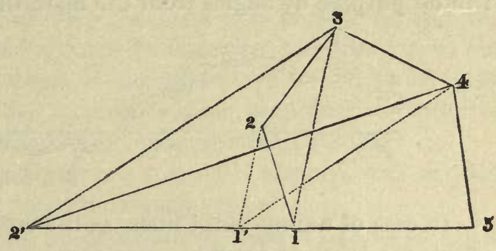


Fig. 253.

Or, the polygon may be resolved readily into a single triangle, and its area calculated. For instance, take the five-sided polygon, 1, 2, 3, 4, 5 (Fig. 253). Call the side 5 1 the base, and extend it. Join 1 and 3. Draw 2 1′ parallel to 1 3. Join 1′ and 4. Draw 2′ 3 parallel to 1′ 4. Join 2′ and 4. The triangle 2′ 4 5 will be a triangle equal to the polygon.

The same construction will apply to a figure of a greater number of sides.

The area of a triangle can be calculated graphically (Fig. 254). Let the scale be two chains to the inch. Prepare a strip of drawing-paper one inch wide, and divide it by perpendicular lines in

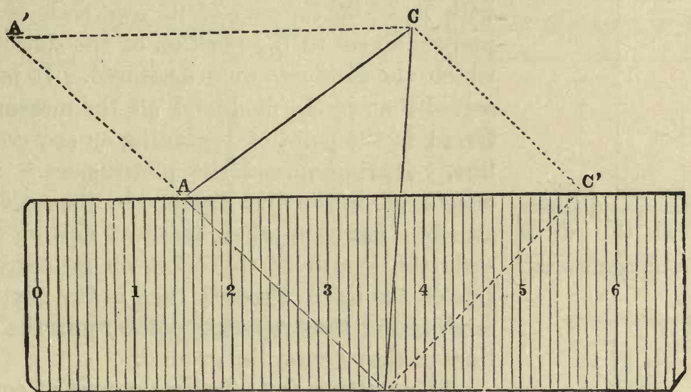


Fig. 254.

20ths of an inch. Apply it to the triangle A B C so that one edge will fall upon A, and the other at B. Keeping the same points on the extended line A′ B, slide the scale up till its upper edge arrives at the point C. The line A′ C in divisions of the scale is the area of the triangle in square chains.

If the scale had been three chains to the inch, the strip should have been $\frac{2}{3}$ of an inch in width; if four chains to the inch, then $\frac{2}{4}$ of an inch in width, and so on.

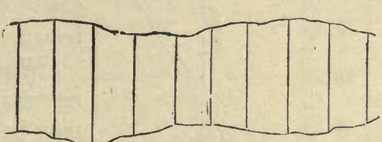


Fig. 255.

When the lines of a plot are irregular, as in Fig. 255, draw across it a number of equidistant parallel lines, and with a strip of paper measure these lines, one after another, till the sum of their lengths is marked on the edge of the strip. Cut the strip at the last mark, and fold it in two. This measure (half the length of the strip), multiplied by the uniform width between the parallel lines, will give very nearly the area.

Having completed the plot—that is, the main lines of the survey—the filling of other points may in general be done on paper, the same way that they have been established in the field. Intersections of the main lines by roads, streams, fences, and the like, are measured off; other points not intersecting, are usually fixed by triangles or by offsets from the main lines, or lines run on purpose by angles from the main lines.

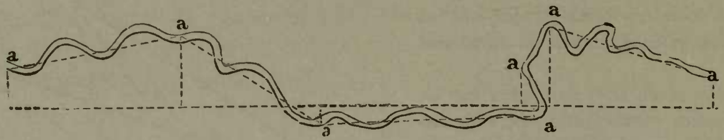


Fig. 256.

In case of unimportant lines, as the crooked brook, for instance (Fig. 256), offsets are taken to the most prominent angles, as, *a*, *a*, *a*, and the intermediate bends are sketched by eye into the field-book. In copying them on the plan a similar construction is adopted.

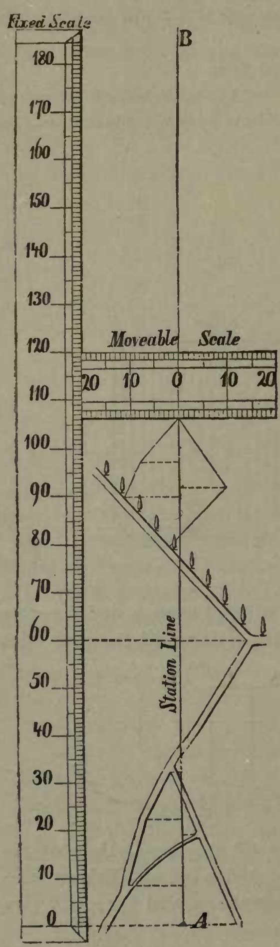


Fig. 257.

The most rapid way of plotting the offsets is by the use of a plotting and offset scale (Fig. 257), the one being fixed parallel to the line A B from which the offsets are to be laid off, at such a distance from it, that the zero-line on the movable scale coincides with it, while the zero of its own scale is on a line perpendicular to the position of the station A from which the distances were measured. It is to be observed that in the field-book all the measures are referred to the point of beginning on any one straight line. Having placed the plotting-scale, move the offset-scale to the first distance by the scale at which an offset has been taken, mark off now on the offset-scale the length of the offset on its corresponding side of the line. Proceed then to the next distance, establishing thus repeated points, join the points by lines as they are on the ground.

The plotting and offset scale must of course be of the same scale as the rest of the drawing, on which account it may not always be possible to obtain such scales adapted to those of the plan; but they may be easily constructed of thick drawing-paper or pasteboard.

When a great deal of plotting to one scale is necessary, as in government surveys, the offset-scale may be made to slide in a groove upon the plotting-scale.

In protracting the triangles of an extended trigonometrical survey in which the sides have been calculated or measured, it is better to lay down the triangles from the length of their sides rather than by measuring the angles, because measures of length can be taken with more accuracy from a scale, and transferred to the plan with more exactness than angles can be pricked off from a protractor; but, for ordinary surveys, the triangulation is most frequently and expeditiously plotted by the means of a protractor.

The outlines of the survey having been balanced and plotted in, and the subsidiary points, as established by offsets and by triangles, the filling in of the interior detail, with the natural features of the ground, from the skeleton or suggestions in the field-book or other records, is done according to imitative and conventional signs, to be shown under "Topographical Drawing."

The public lands of the United States are surveyed, mapped, and divided into nearly square tracts, according to the following system :

Ranges.—Standard lines must first be determined, from which to measure. Accordingly, in each land-district some meridian-line is run due north and south ; this is called the Principal Meridian. From some point of the Principal Meridian is also run a line due east and west, called the Base-Line.

Other lines are then run in the same direction as the Principal Meridian, at distances of six miles (measured on the Base-Line) on each side of it. The strip between the Principal Meridian and the first line thus run east of it is known as *Range 1 East ;* the second strip is *Range 2 East,* etc. And so on the west ; the successive strips running north and south, six miles wide, are called *Range 1 West, Range 2 West,* etc. This division is shown in Fig. 258.

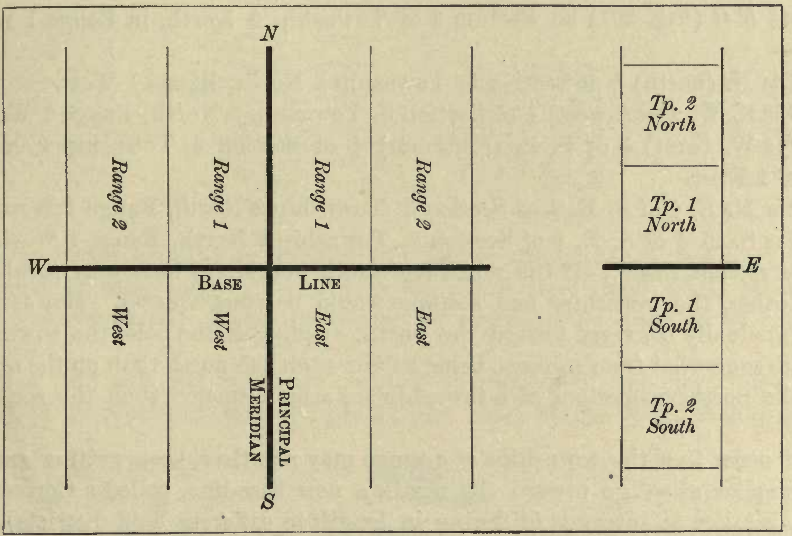


FIG. 258. FIG. 259.

Townships.—In like manner, lines are run north and south of the Base-Line at intervals of six miles. These lines cut at right angles those which separate the ranges, and with them form squares six miles on each side, called townships. Each township contains thirty-six square miles.

The township nearest the Base-Line on the north is known as *Township 1 North,* of whatever range it may be in ; the next farther north is *Township 2 North,* of that range—and so on. In like manner, going south from the Base-Line, we have in succession *Township 1 South, Township 2 South,* etc. (Fig. 259).

Sections.—Each township is divided into thirty-six squares, called Sections, each one mile long and one mile wide, and therefore having an area of one square mile. The sections of a township are numbered 1, 2, 3, etc., up to 36, beginning at the northeast, and running alternately from right to left and from left to right, as shown in Fig. 260.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 260.

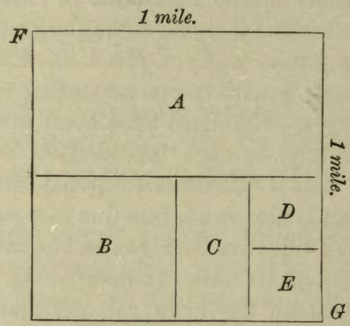


FIG. 261.

A section may be subdivided into half-sections, quarter-sections, eighths, and sixteenths, designated as in the example that follows:

Let *F G* (Fig. 261) be Section 3 of Township 2 North, in Range 1 West; then—

A is N. (north) ½ of Section 3, Township 2 North, Range 1 West.

B is S. W. (southwest) ¼ of Section 3, Township 2 North, Range 1 West.

C is W. (west) ½ of S. E. (southeast) ¼ of Section 3, Township 2 North, Range 1 West.

D is N. E. ¼ of S. E. ¼ of Section 3, Township 2 North, Range 1 West.

E is S. E. ¼ of S. E. ¼ of Section 3, Township 2 North, Range 1 West.

Correction-Lines.—If the north-and-south (meridian) lines were parallel to each other, the townships and sections would be exact squares. But as these lines gradually converge toward the north, meeting at the pole, the townships deviate somewhat from squares, being narrower on the north than on the south; and the northern sections of a township are a little smaller than the southern ones.

In order that the townships of a range may not thus keep getting smaller and smaller as we go toward the north, a new base-line, called a Correction-Line, is taken at intervals (differing in length in different land-districts), and new north-and-south lines are run at distances of six miles measured on the Correction-Lines.

The system of survey described above is not used in Texas, the public lands there being State property.

TOPOGRAPHICAL DRAWING.

Topographical Drawing is the delineation of the surface of a locality, with the natural and artificial objects, as houses, roads, rivers, hills, etc., upon it in their relative dimensions and positions, giving, as it were, a miniature copy of the farm, field, district, etc., as it would be seen by the eye moving over it. Many of the objects thus to be represented can be defined by regular and mathematical lines, but many other objects, from their irregularity of outline, it would be very difficult thus to distinguish ; nor are the particular irregularities necessary for the expression. Certain conventional signs have


Fig. 262.


Fig. 263.


Fig. 264.


Fig. 265.

therefore been adopted in general use among draughtsmen, some of which resemble, in some degree, the objects for which they stand, while others are purely conventional. These signs may be expressed by lines, or by tints, or by both.

Figs. 262 and 263 represent meadow or grass land, the short lines being supposed to represent tufts of grass ; the bases of the tufts should always


Fig. 266.


Fig. 267.

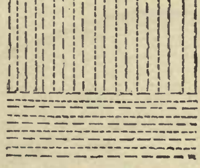


Fig. 268.

be parallel to the base of the drawing, whatever may be the shape of the inclosure.

Figs. 264, 265, 266, 267, give various methods of representing trees. Figs. 264 and 265 represent in plan a forest and an orchard, while Figs. 266 and 267 show the same in elevation. The latter method of representing trees is not consonant with the projection of the plan, but to many is more expressive and intelligible.

Fig. 268 represents cultivated land. The lines are supposed to represent plow-furrows, and adjacent fields should be distinguished from each other by different inclinations of lines.

Figs. 269 and 270 represent marsh or bog land. Fig. 269 is the more ordinary mode of representing fresh-water bog, and Fig. 270 of salt-marsh.

FIG. 269.

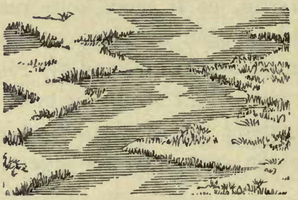


FIG. 270.

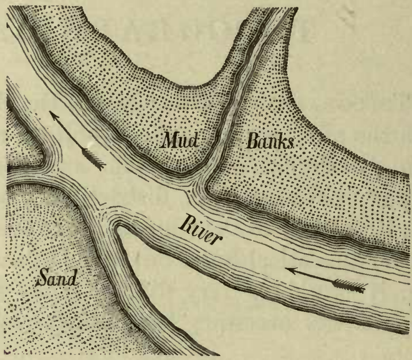


FIG. 271.

Fig. 271 represents a river, with mud and sand banks. Sand is designated by fine dots, made with the point of the pen; mud in a similar way, but the dots should be much closer together. Gravel is represented by coarser dots, and stones by irregular angular forms.

Water is almost invariably represented in the same way, except in connection with bogs, by drawing a line parallel to the shore, following its windings and indentations closely; then another parallel a little more distant; a

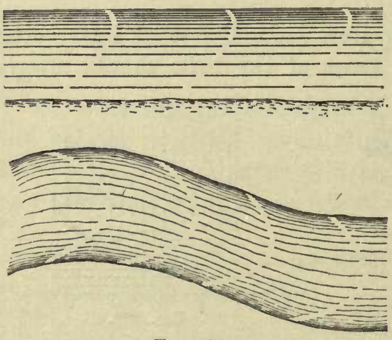


FIG. 272.

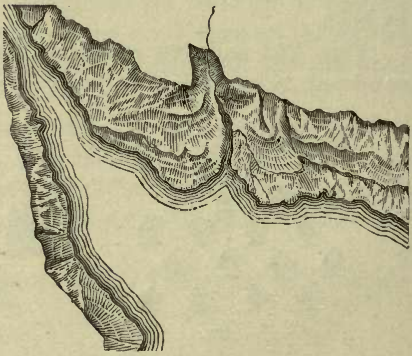


FIG. 273.

third still more so; and so on. Brooks, and even rivers, when the scale is small, are represented by one or two lines. Fig. 272 gives a plan and sectional view of water, in which the white curves represent the character and direction of the flow of streams, retarded at bottom and sides, and more rapid near the surface and at center, therefore convex down stream. The direction of the current may also be shown by arrows, as in Fig. 271.

Fig. 273 represents a bold shore bounded by cliffs.

Fig. 274 represents a turnpike. If the toll-bar and marks for a gate be omitted, it is a common highway. Fig. 275 represents a road as sunk or cut

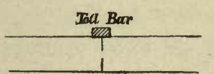


Fig. 274.

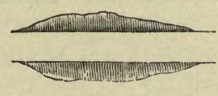


Fig. 275.

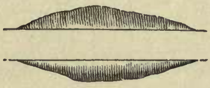


Fig. 276.

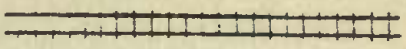


Fig. 277.

through a hill. Fig. 276, one raised upon an embankment. Fig. 277 is a railroad, often represented without the cross-ties by two heavy parallel lines, sometimes by but one.

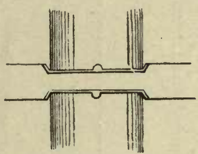


Fig. 278.

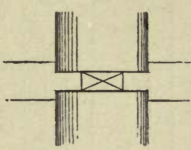


Fig. 279.

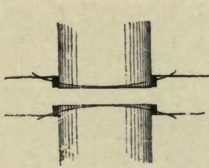


Fig. 280.

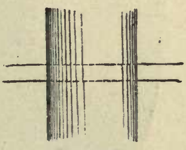


Fig. 281.

Fig. 278 represents a bridge with a single pier. Fig. 279, a swing or draw bridge. Fig. 280, a suspension bridge, and Fig. 281 a ford. Fig. 282, a lock of a canal. Canals are represented like roads, except that in the latter the side from the light is the shaded line ; in the former, the side to the light.

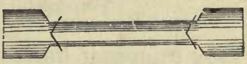
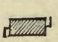
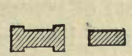
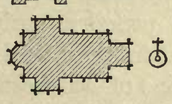


Fig. 282.

The more important objects that are likely to need representation on a map have conventional signs, as follows :

Signal of Survey,		*Saw-mill,*	
Telegraph,		*Wind-mill,*	
Court-house,		*Steam-mill,*	
Post-office,		*Furnace,*	
Tavern,		*Woolen-factory,*	
Blacksmith's shop,		*Cotton-factory,*	
Guide-board,		*Dwellings,*	
Quarry,		*Churches,*	
Grist-mill,		*Grave-yards,*	

The localities of mines may be represented by the signs of the planets, which were anciently associated with the various metals, and a black circle for coal. Thus, ☿ Mercury, ♀ Copper, ♄ Lead, ☽ Silver, ⊙ Gold, ♂ Iron, ♃ Tin, ● Coal.

The Representation of Hills.—The two methods in general use for representing with a pen or pencil the slopes of ground, are known as the vertical and horizontal. In the first (Fig. 283), the strokes of the pen follow the course that water would take in running down these slopes. In the second (Fig. 284),

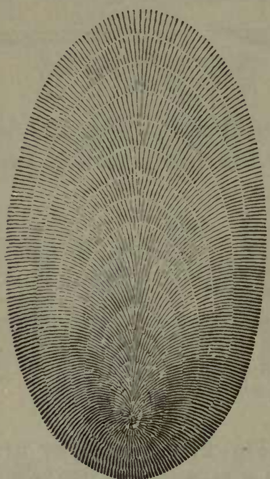


Fig. 283.

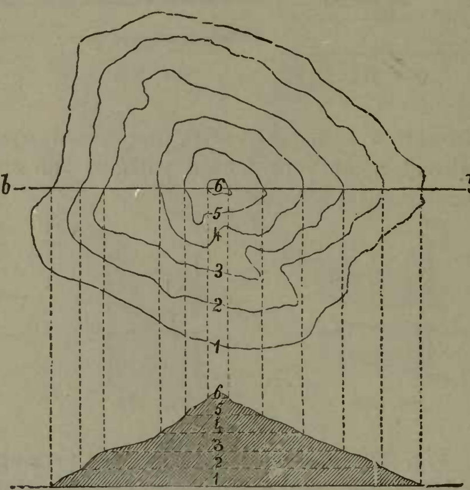


Fig. 284.

they represent horizontal lines traced round them, such as would be shown on the ground by water rising progressively by stages, 1, 2, 3, 4, 5, 6, up the hill. The last is the more correct representation of the general character and features of the ground, and, when vertical levels or contours have been traced by level at equal vertical distances over the surface of the ground, they should be so represented ; or when, by any lines of levels, these contours can be traced on the plans with accuracy, the horizontal system should be adopted : but where, as in most plans, the hills are but sketched in by the eye, the vertical system should be adopted ; it affords but proximate data to judge of the slope, whereas, by the contour system, the slope may be measured exactly. It is a good maxim in topographical drawing not to represent as accurate anything which has not been rigorously established by surveys. On this account, for general plans, when the surface of the ground has not been leveled, nor is required to be determined with mathematical precision, we prefer the vertical to the horizontal system of representing slopes.

On drawing hills on the vertical system, it is very common to draw contour-lines in pencil as guides for the vertical strokes. If the horizontal lines be traced at fixed vertical intervals, and vertical strokes be drawn between them in the line of quickest descent, they supply a sufficiently accurate representation of the face of the country for ordinary purposes. It is usual to make the vertical strokes heavier the steeper the inclination, and systems have been proposed and used, by which the inclination is defined by the comparative thickness of the line and the intervening spaces.

In describing ground with the pen, the light is generally supposed to descend in vertical rays, and the illumination received by each slope is diminished in proportion to its divergence from the plane of the horizon. Thus, in Fig. 285, it will be seen that a horizontal surface receives an equal portion of light with the inclined surface resting upon it, and, as the inclined surface is of greater extent, it will be darker than the horizontal in proportion to the inclination and consequent increase of the surface, and on this principle varied forms of ground are represented by proportioning the thickness of stroke to the steepness of the slope.

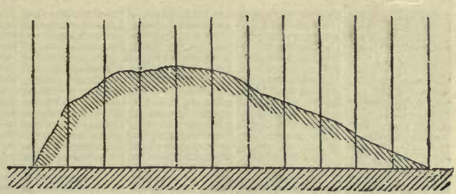


FIG. 285.

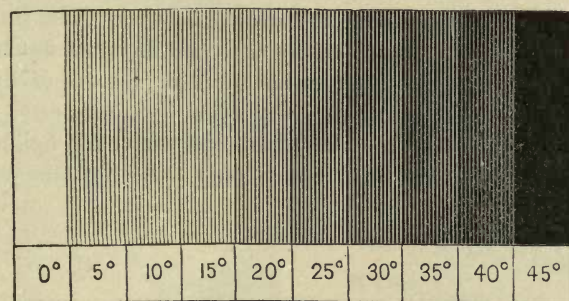


FIG. 286.

In the German system, as proposed by Major Lehmann, of representing the slopes of ground by a scale of shade, the slope at an angle of 45°, as reflecting its light horizontally, is supposed to be the greatest ever required to be shown, and is represented by black, while the horizontal plane reflecting all rays upward is represented by white. Fig. 286 gives the intervening proportions of black and white.

Slope.	Proportion of	
	Black.	White.
2½° or 2¾°	1	10
5° or 6°	2	9
10° or 11°	3	8
15° or 16°	4	7
25° or 26°	5	6
35°	6	5
45°	7	4
60°	8	3
75°	9	2

A modification of Lehmann's method, proposed by the United States Coast Survey, has the advantage of discriminating between slopes of greater inclination than 45°. The table gives the proportions of black and white for different inclinations, and the construction may easily be understood from Fig. 287.

Contour-Lines.—Conceive a hill to be completely covered with water. Then suppose the water to be drawn down, say five feet at a time. Each line of contact of the hill and the water will be a *contour-line*, or a line every point of which is at the same height or level above a fixed horizontal plane, called the *datum-plane.* For a small hill, stake out the ground in squares of say fifty feet to the side, and take levels at each point of these squares, and as many intermediates as the change of slope makes necessary. To draw the map, lay off these squares to a scale, and mark the elevation of each point and the intermediates in pencil. Then by the eye draw in the contours at such vertical distances apart as the requirements of the map call forth. For a large survey, say of a mountain, such a method is impracticable. In this case, the surveyor

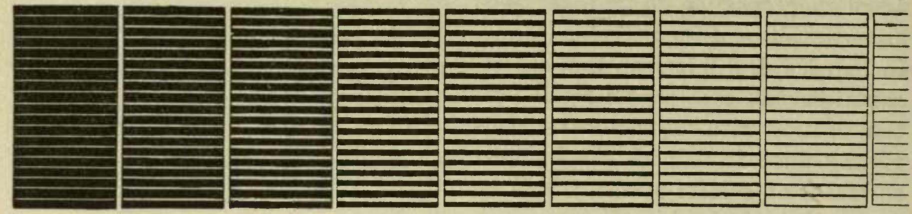


FIG. 287.

fixes a number of points at the same level, the points being absolutely established by the transit or compass so that they can be plotted accurately. Connect all points at the same level, and fill in the distances between by the eye, on the supposition that the slope is uniform between these lines. The lines absolutely established and those merely sketched in must not be confounded, and should be distinguished apart either by color, by size of lines, or by dotting. The contour-lines denoting every even five, ten, etc., feet above the datum or plane of reference may be numbered with such height. This is an effective way of representing hills, but is only to be recommended when lines

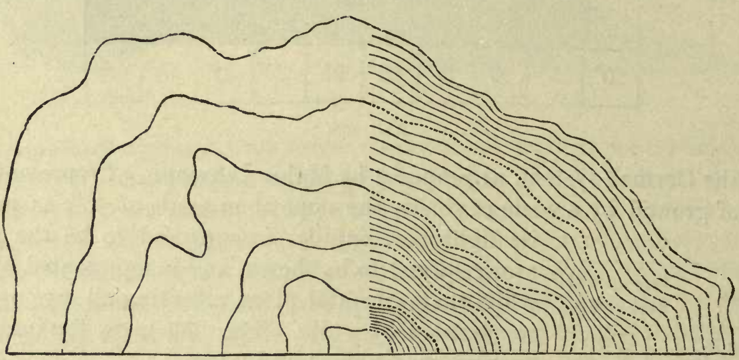


FIG. 288.

have been traced and it becomes a record of facts. Fig. 288 represents, on double the scale, the half of the hill, Fig. 284, with one half completed by drawing the intermediate contour lines.

The objection to the drawing of hills by any system is that the depths of shade representing different slopes conflict with the lights and shades of the drawing, and are therefore confusing. The plan adopted by Von Eggloffstein in his maps was to form a model and then put in the hills as they appeared, with the rays of light inclined 45° to the plan of the drawing. He adopted a ready way of forming his model. The contours were cut out of sheet-wax under the needle of a sewing-machine, then properly superimposed on one another. A mold was then taken from them in plaster. A model from the mold, also in plaster, was then taken. This was watered while fresh by a vertical rain from a water-pot, which broke down the vertical edge of the contours, and gave natural lines of water shed. This model would then be photographed

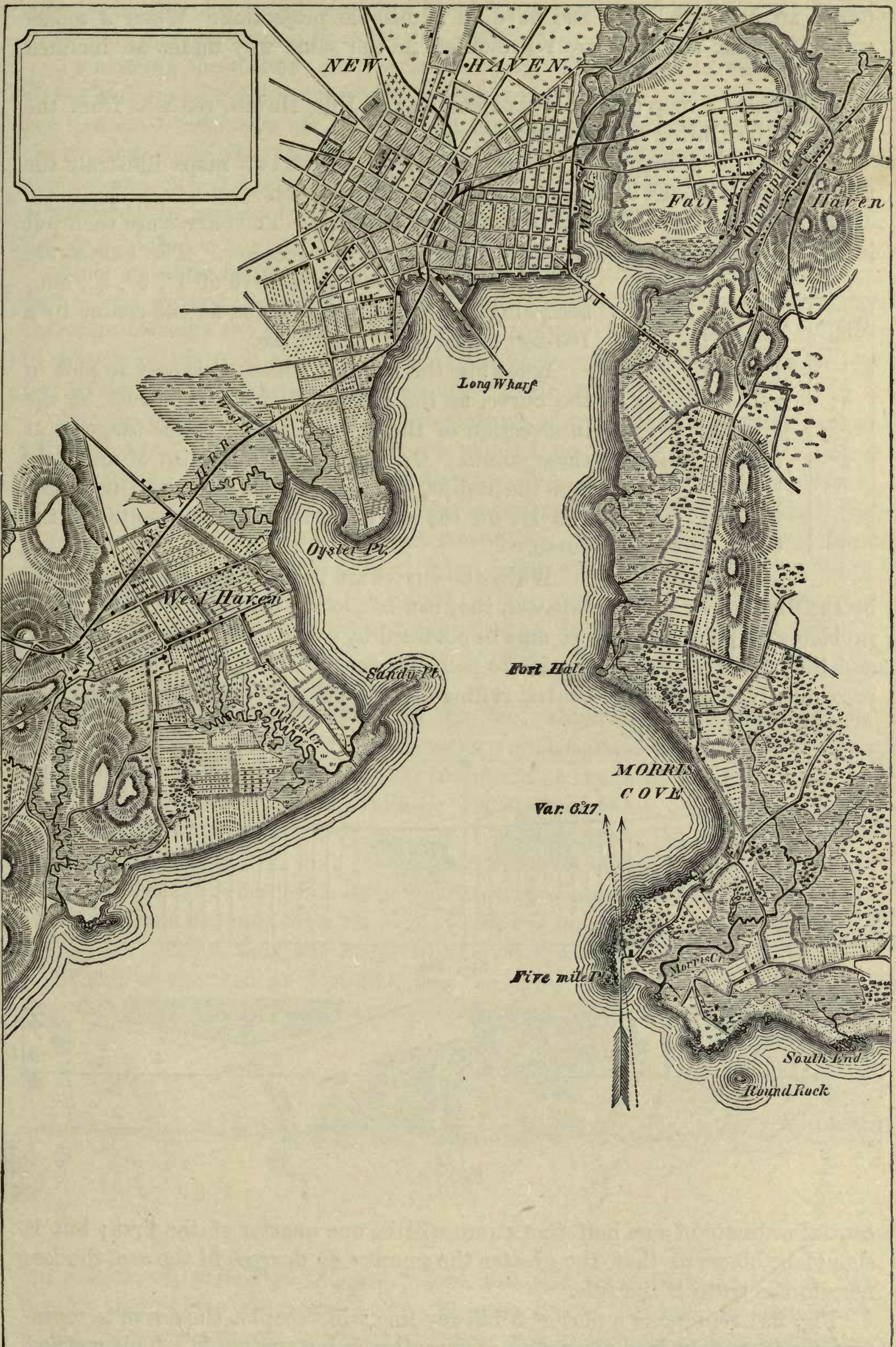

NEW HAVEN
Fair Haven
Mill R.
Quinnipiac R.
Long Wharf
West R.
N.Y. & N.H. R.R.
Oyster Pt.
West Haven
Sandy Pt.
Fort Hale
MORRIS COVE
Var. 6°17.
Five mile Pt.
Morris Cr.
South End
Round Rock

FIG. 289.

under an inclined light, and gave an admirable projection. When a model was not made, the hills are represented in the same way under an inclined light of 45°.

Fig. 289 is a map of the harbor and city of New Haven, reduced from the charts of the United States Coast Survey.

Plate VI is a map of a farming country. These two maps illustrate the practical applications of topographical conventionalities.

Railway surveys are usually plotted by tangents. The curves are then put in, and the topographical features for the width necessary. The curves are designated by degrees, as a curve of 1°, 2°, 3°, etc., according as the angle subtended at the center by a 100-feet chord is 1°, 2°, 3°, etc.

Degree.	Radii, ft.	Central Ordinate.
1°	5729·65	0·218
2°	2864·93	0·436
3°	1910·08	0·655
4°	1432·69	0·873
5°	1146·28	1·091
6°	955·37	1·309
7°	819·02	1·528
8°	716·78	1·746
9°	637·27	1·965
10°	573·69	2·183

Knowing the tangent points, it is easy to plot in the curve, as the center of the curve must be the intersection of the perpendiculars to the tangents at these points. Or, if we know one point of tangency and the radius, erect a perpendicular at this point, and lay off the radius on it to get the center of the curve.

When the curves are larger than can be described by the dividers or beam compasses, they can be plotted as shown in geometrical problems, or points of a curve may be obtained by calculation of their ordinates, and the curves drawn from point to point by sweeps and variable curves. Approximately, knowing the central ordinate of the curve between two points, the

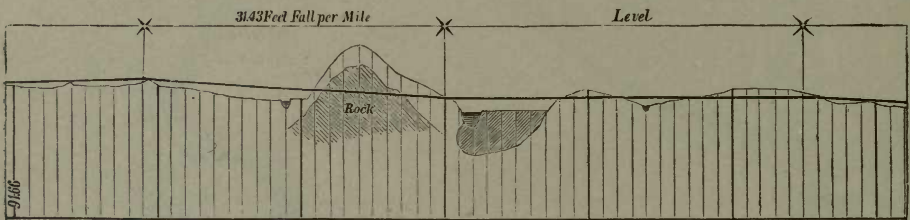


FIG. 290.

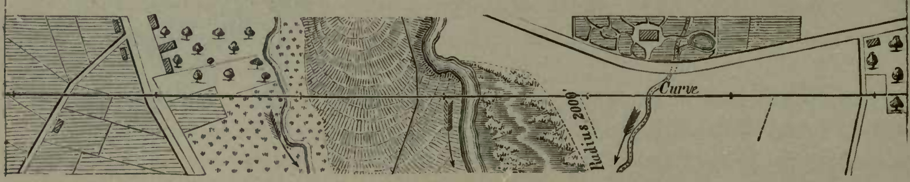


FIG. 291.

central ordinate of one half that curve will be one quarter of the first; but it should be observed that, the greater the number of degrees in the arc, the less near to the truth is the rule.

Fig. 291 represents a plot of a railway line; in this plot the curve is represented as a straight line, the radius of curvature being written in. This method is sometimes adopted when it is desirable to confine the plot within a limited space upon the sheet, and it is convenient when plotted thus directly beneath the profile or longitudinal section (Fig. 290).

In plotting the section, a horizontal or base line is drawn on which are laid off the stations or distances at which levels have been taken ; at these points perpendiculars or ordinates are erected, and upon them are marked the heights of the ground above the base, and the marks are joined by straight lines. To express rock in a cut, it is generally represented by diagonal lines ; rivers are represented in section by cross-lines or colored in blue ; a mud-bottom by masses of dots.

Since it would be in general impossible to express the variations of the surface of the ground in the same scale as that adopted for the plan, it is usual therefore to make the vertical scale larger than that of the horizontal, usually in proportion of 10 or 20 to 1. Thus, if the horizontal scale of the plan be 400 feet to the inch, the vertical scale would be 40 or 20 feet to the inch.

For the purpose of facilitating the plotting of profiles, profile-paper can be obtained from stationers, on which are printed horizontal and vertical lines ; the horizontal lines being ruled at a distance of $\frac{1}{30}$ of an inch from each other, every fifth line being coarser, and every twenty-fifth still heavier than the others. Each of the spaces is usually considered one foot. The vertical lines are one quarter of an inch distant from each other, every tenth line being made more prominent than the others ; these spaces in general represent a distance of 100 feet, the usual distance between stations on a railroad. Much time is saved by the use of this paper, both in plotting, and in reading the measurements after they are plotted.

In the plotting of sections across the line, which are extended but little beyond the line of the cut or embankment, equal vertical and horizontal scales are adopted ; these plots are mostly to determine the position of the slope, or to assist in calculating the excavation. To facilitate these, cross-section paper is sold, ruled with vertical and horizontal lines, forming squares of $\frac{1}{10}$ of an inch each. Every fifth line in each direction is made prominent. When cross-sections are extended to show the grade of cross-road, or changes of level at considerable distance from the line of rail, the same scales, vertical and horizontal, are adopted as in the longitudinal section or profile.

It will be observed, in Fig. 290, that the upper or heavy line represents the line of the rail, the grades being written above; this is the more usual way,

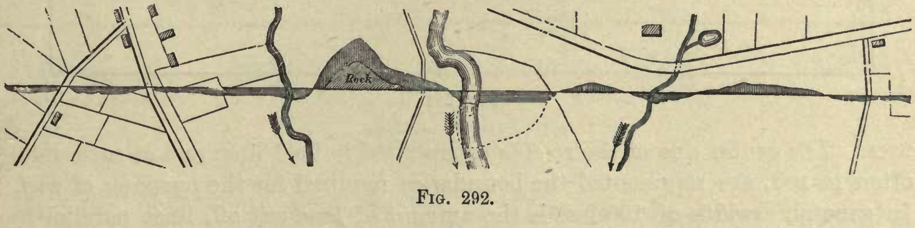


FIG. 292.

but sometimes, as in Fig. 292, the profile and plan are combined ; that is, the heights and depths above and below the grade-line of the road are transferred to the plan, and referred to the line in plan, which becomes thus a representation both in plan and elevation.

Cross-sections, for grades of cross-roads, etc., are usually plotted beneath or above the profile ; they may, if necessary, be plotted across the line when plan and profile are combined.

Besides the complete plans as above, giving the details of the location, land plans, so called, are required, showing the position and direction of all lines of fences and boundaries of estates, with but very few of the topographical feat-

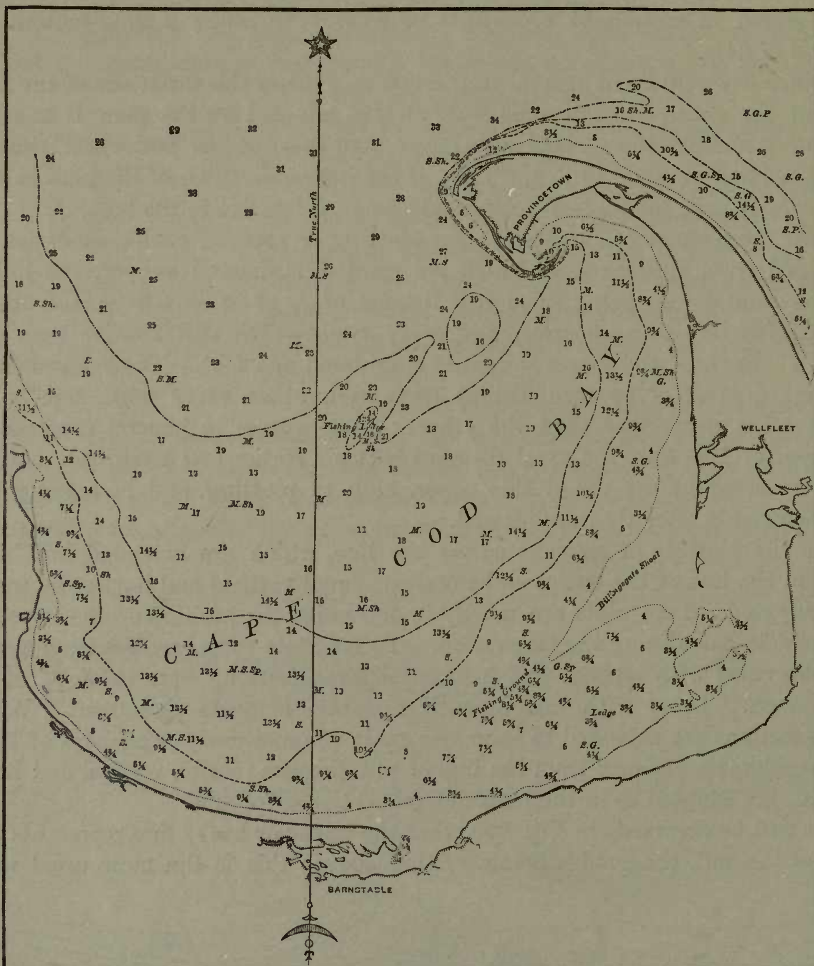


FIG. 293.

ures. The center line of the road is represented in bold line, and at each side, often in red, are represented the boundaries required for the purposes of way. In general, a width of 100 feet is the amount of land set off, lines parallel to the central line being at a distance of 50 feet on each side ; but when, owing to the depth of the cut or embankment, the slopes run out beyond this limit, the extent is determined by plotting a cross-section and transferring the distances thus found to the plan, and inclosing all such points somewhat within the limits as set off for railway purposes. These plans are generally filed in the register's office for the county through which the line passes.

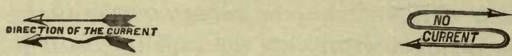
Hydrometrical or Marine Surveys.—In plotting hydrometrical or marine surveys, the depths of soundings are seldom expressed by sections, but by figures written on the plan, expressing the sounding or depth below a datum-line, generally that of high water. The low-water line is usually represented by a single continued line. The soundings are generally expressed in fathoms, sometimes in feet.

Fig. 293 is a map of Cape Cod Bay plotted by this method. The depths are expressed in fathoms (six feet), and the dotted lines inclose depths between certain fixed limits so as to plainly indicate a channel or bar, as the case may be.

Another and an exceedingly effective way of making a marine chart is to express the different depths by lines varying in direction, distance apart, width,

Fig. 294.

etc. Fig. 294 is a chart of the Isle of Wight and the surrounding water, with the depths expressed as shown at the bottom of the cut. Sections are often used for rivers, especially for those like our Western ones, that have a very changeable bottom. By plotting sections, taken at different times, over one another, distinguishing them apart by a difference in color and variety of line, the changes that take place in the bottom of the river, and the erosion of the banks, are more boldly shown than by the use of any other method. The ordinary marine conventionalities are as follows :



Anchorage for ships, *Buoys,* *Light-house,*

Anchorage for coasters, *Wrecks,* *Signal-house,*

Rocks always covered, *Harbors,* *Channel-marks,*

Representation of Geological and Statistical Features.—The geological features of a country may be readily expressed on a map by the use of lines as in

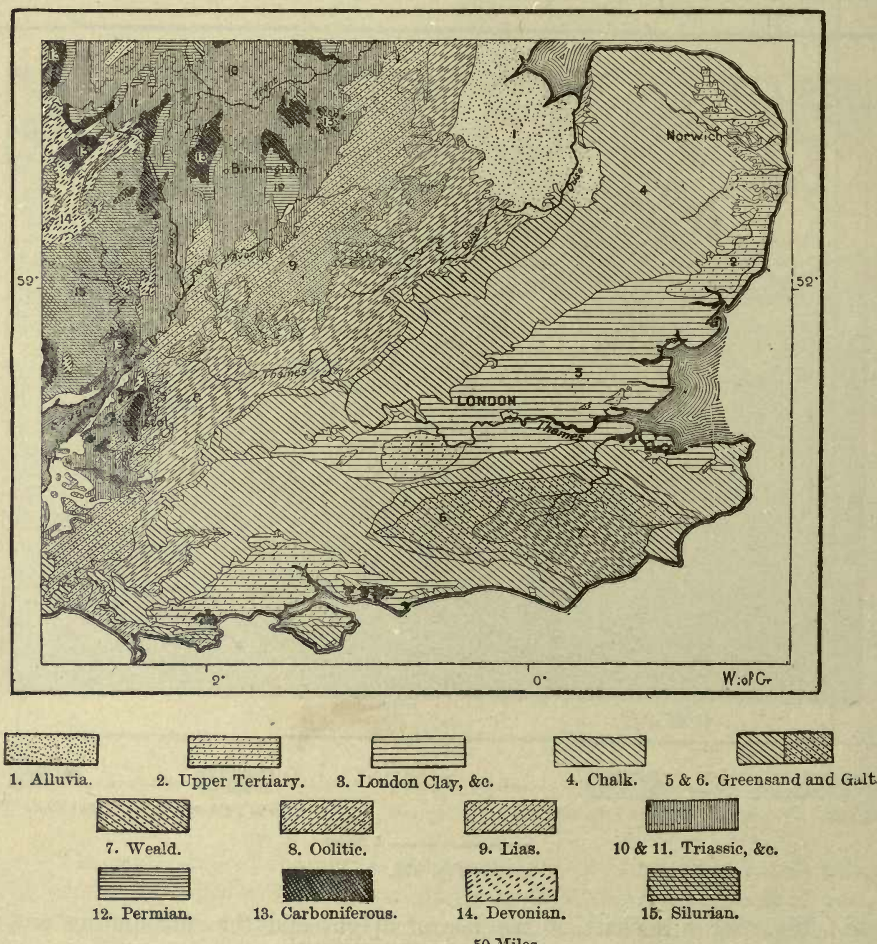


FIG. 295.

marine charts. Fig. 295 is a geological map of Southeastern England, and will be easily understood by inspection.

A geological profile may be represented in the same way. The different rocks or formations are usually distinguished by color and explained by marginal notes and squares, but more often by marks, dots, or cross-hatchings, as

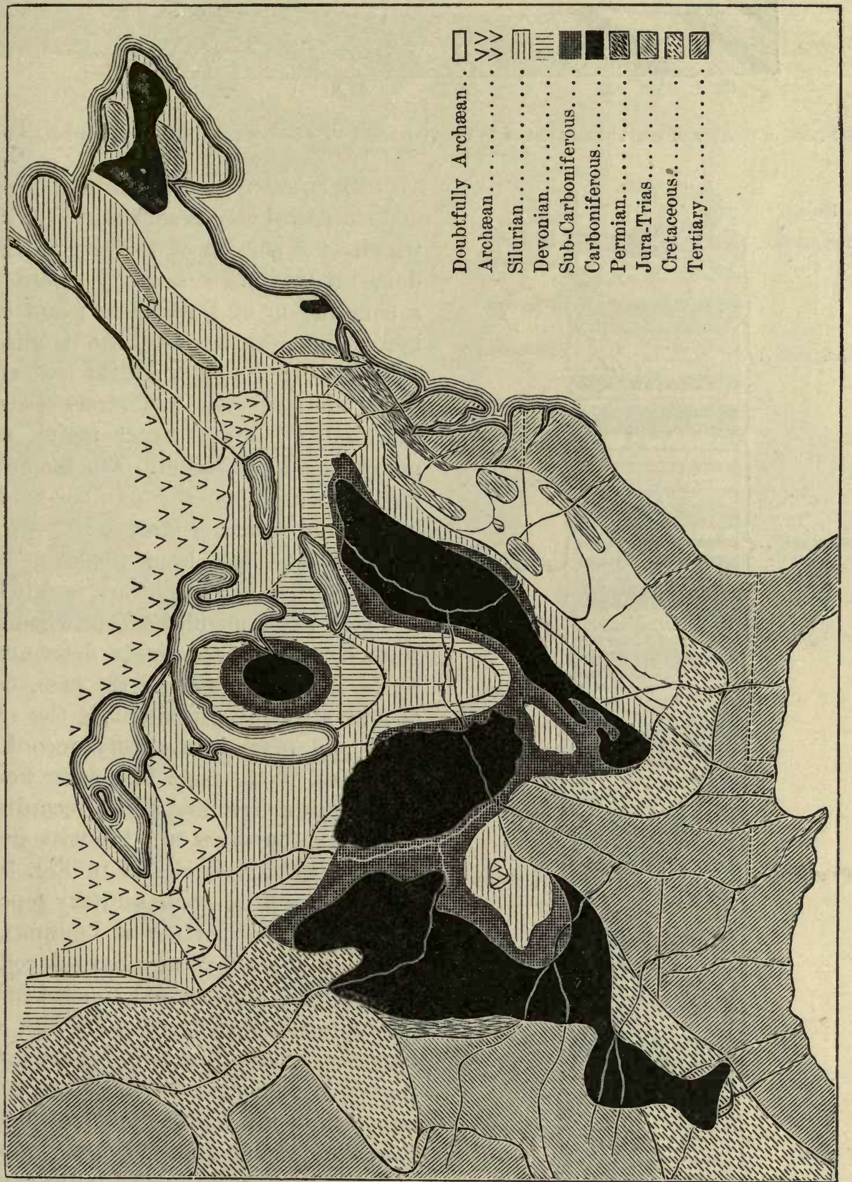


Fig. 296.

in Fig. 296, which exhibits the geological features of the United States east of the Rocky Mountains and Canada to the south of the St. Lawrence.

Fig. 297 is a section from Pennsylvania to Canada, showing the relations of the subdivisions to each other.

Fig. 298 represents an ideal diagram of the principal groups in American geology, in the order of their superposition.

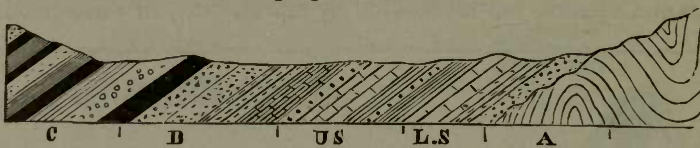


Fig. 297.
Ideal Section north and south from Canada to Pennsylvania: *A*, Archæan; *L S* and *U S*, Silurian; *D*, Devonian; *C*, Carboniferous.

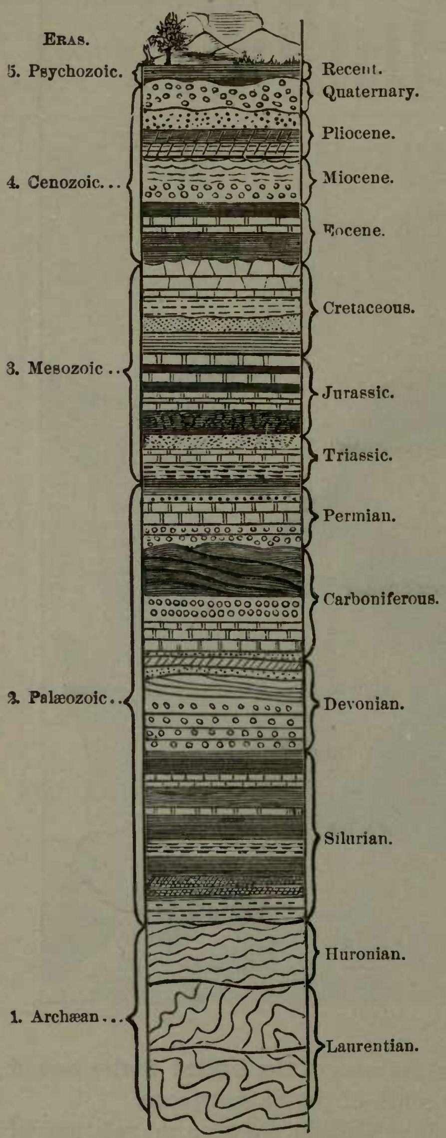


Fig. 298.
Ideal General Section of the Whole Series of Strata, showing the Principal Divisions and Subdivisions.

Still another form of a topographical and statistical map is shown in Fig. 299, which is a portion of the city of London, taken from a sanitary report by a commission of Parliament; and embodies, in a graphic way, the details in regard to drainages, natural and artificial, contour-lines and street-sewers; position of gas and water mains, and occupancy of buildings. On the original are also given the number of the houses and names of streets.

Reference has been made to the drawing of hills by contours, and it has not been recommended except when the lines have been accurately determined by level. When this is the case, they should always be used; it is the simplest and most explanatory record of facts, and if the facts have been worth determining they are worth recording. When contour-lines are brought more closely together (as shown in Fig. 300, which is from the same sanitary report, and of a larger portion of London), it produces the effect of physical relief, and shows at a glance the lines of natural drainage, and from it profiles can be made, in any direction, for the grading of streets or sewers. Were town and county maps thus drawn with contour-lines, much time and money would be saved in the location of highways and railways.

Transferring.—It is usual, in plotting from a field-book, to make first but a rough draft, and then make a finished copy on another sheet. In the first, many lines of construction, balances of survey, and trial lines are drawn, which are unnecessary in the copy; outlines of natural features are sketched roughly, but the plotting of surveys, and such lines and points as are to be preserved in the copy, must be done with accuracy.

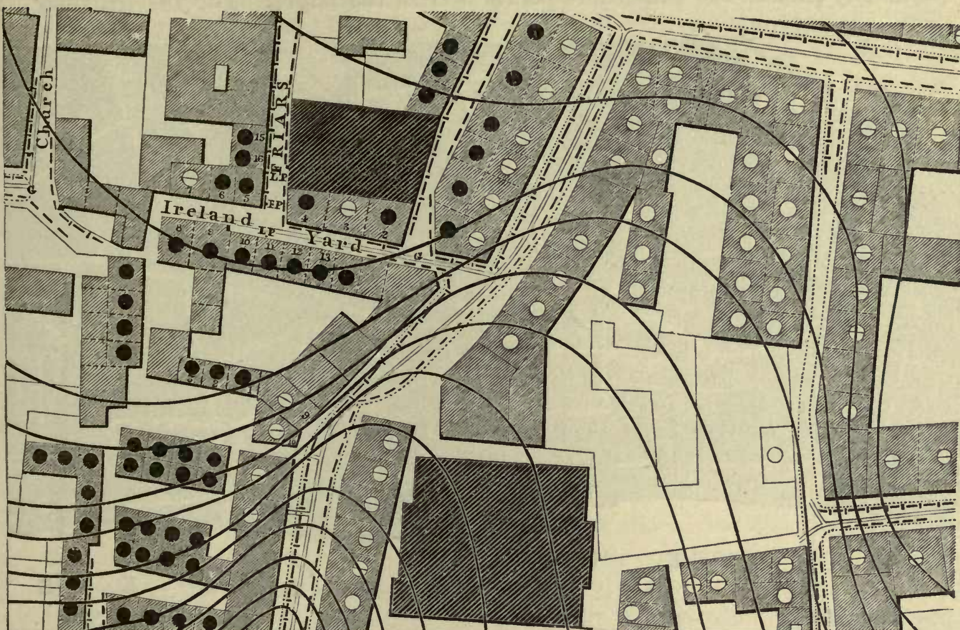


Fig. 299.

Private houses (occupied by persons not in receipt of wages).

Offices and shops.

Houses occupied by persons in receipt of wages.

Warehouses.

Stables and outhouses.

Public buildings.

Contours; vertical distances between lines, two feet.

Sewers.

Gas-pipes.

Water-pipes.

The most common way of transferring, for a fair copy, is by superposition of the plan above the sheet intended for the copy, and pricking through every intersection of lines on the plan, and all such points as may be necessary to preserve. The clean paper should be laid and fastened smoothly on the drawing-board; the rough draft should be laid on smoothly, and retained in its position by weights, glue, or tacks. The needle must be held perpendicular to the surface of the plan, and pressed through both sheets ; begin at one side and work with system, so as not to prick through each point but once, nor omit any ; make the important points a trifle the larger. For the irregular

Fig. 300.

curves, as of rivers, make frequent points, but very small ones. On removing the plan, select the important points, those defining leading lines ; draw in these, and the other points will be easily recognized from their relative position to these lines. When any point has not been pricked through, its place may be determined by taking any two established points adjacent to the one required, and with radii equal to their distance, on the plan, from the point required, describing arcs, on the copy, on the same side of the two points ; their intersection will be the point desired. In this way, as in a trigonometrical survey, having established the two extremes of a base, a whole plan may be copied. In extensive drawings it is very common to prick off but a few of the salient points, and fill in by intersections, as above, or by copying detached portions on tracing-paper, and transferring them to the copy ; the position of each sketch being determined by the points pricked off, the transfer is made by pricking through as above, or by transfer-paper placed between the tracing and the copy.

If tracing paper or cloth (pages 56, 57) be placed above the drawing, every line will show through, and can be traced directly with the pen, in India ink. These tracings are used mostly to preserve duplicates of finished drawings.

Duplicates of drawings, contracts, estimates, etc., on paper allowing the light to pass through are readily made by the use of ferro-prussiate paper, or the *blue-print* process. Paper can be prepared by washing it with a mixture of $1\frac{1}{8}$ ounces of citrate of iron and ammonia with 8 ounces of water, and $1\frac{1}{4}$ ounces of red prussiate of potash and 8 ounces of water, dissolved separately and mixed. The mixture and prepared paper should be kept from the light. The prepared paper in close rolls can be readily purchased. For the manipulation there is needed plate-glass, and a blanket a little larger than the drawing, a shallow tin dish, that the drawing can be placed in flat for washing. Lay down the blanket on a drawing-board, above that the ferro-prussiate paper, next the drawing, and then the glass. Expose to the sunlight for about ten minutes if the drawing is on tracing-paper or cloth, and longer for thicker paper; when done, the background should be a metallic gray. Now lay the ferro-prussiate paper in the tin dish, cover with water, and leave it for five to ten minutes; wash thoroughly and dry. The lines will be white on a blue ground. The negative of ferro-prussiate paper gives blue lines on a white ground, and other processes black lines on a buff ground.

An accurate and rapid way of tracing, on drawing-paper, plans of small extent, is by means of an instrument called a copying-glass. It consists of a large piece of plate-glass set in a frame of wood, which can be inclined at any angle. On this glass is first laid the original plan, and above, the fair sheet, and the frame being raised to a suitable angle, a strong light is thrown by reflectors or otherwise on the under side of the glass, whereby every line in the original plan is seen distinctly through the fair sheet, and the copy is made at once, as on tracing-paper. This same process, on a small scale, is adopted by putting the plans upon a pane of glass in a window.

Plans mounted on cloth, or on opaque paper, do not admit of being traced in this way. In such cases the copy may be made by means of transfer-paper. The plan is first traced on tracing-paper or cloth, black-leaded or transfer paper is then placed on the fair sheet, and the tracing-paper copy is placed above. All is steadied by numerous weights along the edges, or by drawing-pins fixed into the drawing-board. A fine and smooth point is then passed over each boundary or mark on the tracing with a pressure of the hand sufficient to cause a clear, penciled mark to be left on the fair sheet by the black-leaded or transfer paper. The whole outline is thus obtained, and afterward drawn in ink. The copyist should be careful in his manipulations, so as not to transfer any other lines than those required, nor leave smutches on the fair sheet.

Plans may be copied, on a reduced or enlarged scale, by means of the pantagraph (Figs. 146, 147), or by the method of squares (pages 63, 64).

Map Projections.—For a farm or other small survey, the surface of the earth can be conceived to be *flat*, and the map a horizontal projection of the plane surface on a reduced scale; the error being practically insignificant, while the labor is greatly reduced by making this assumption. But, for large maps of countries, States, rivers, etc., where the meridians and parallels of latitude are represented, such a system would be so erroneous as to be impracticable. The surface of the earth being a sphere, it is incapable of development on a plane, so that it becomes necessary to make the best approximation possible in form, relation, and proportional area of the portions to be represented on a map or chart. There are many different kinds of projection, all more or less imperfect, but most of which possess advantages for some descriptions of maps or charts. They may be divided into four classes, as follows :

Class I. Perspective projection on planes.
" II. Developed perspective projections.
" III. Projections by developing elements.
" IV. Projections conformed to some arbitrary condition.

In *Class I,* the more important kinds are the globular or equidistant, and the stereographic.

Globular or Equidistant Projection of the Sphere.—According to this method the eye is placed at a distance from the center of the earth, equal to 1.707 × radius. The plane of projection passes through the center perpendicular to the central ray. This method is quite common in school maps. The following is the construction :

Draw two lines (Fig. 301), at right angles to and intersecting each other ; from the point C of their intersection as a center, with a radius equal to that

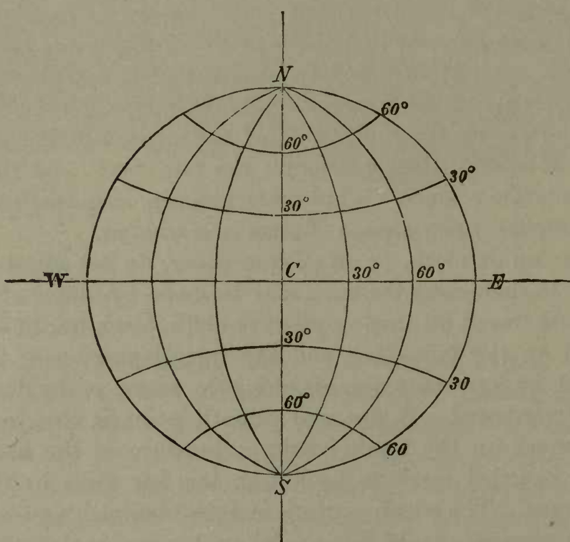


FIG. 301.

intended for the hemisphere, describe a circle, and mark the points N, S, W, E. N and S will be the poles, the line N S the central meridian, and W E the equator. Divide N S and W E into as many equal parts as there are degrees or numbers of degrees to be represented—in the figure in divisions of 30°—and meridian and equator into six equal parts, as the hemisphere embraces 180°. Commence at C, and divide the half-lines into three equal parts. Divide the arcs N W, N E, S W, and S E, each into three equal parts. There will be now determined three points in two parallels of north and south latitude, 30° and 60°, through which to describe the arcs representing the parallels. The center of these arcs will be in the line N S ; describe the arc, and with the same radius from a center on the line N S below the S pole, describe a similar arc passing through the S 30° point on the meridian. Therefore, keeping the steel point of the dividers on the line N S, by trial radii may be found of arcs which shall pass through the points on the central meridian and on the circle. With the radii describe arcs for the parallels in north and south latitude. All the meridians pass through the N and S poles, and through the divisions of degrees on the equator. There are three points, therefore, determined in the arc of each meridian which may be described from centers found by trial on the line E W.

Stereographic Projection.—In this method the eye is taken at the center of the earth, at the pole of the great circle used as a plane of projection. Circles are stereographically projected into circles. An increasing exaggeration outward from the center is its principal defect. To project stereographically the hemisphere on the plane of the meridian, draw the central meridian, equator,

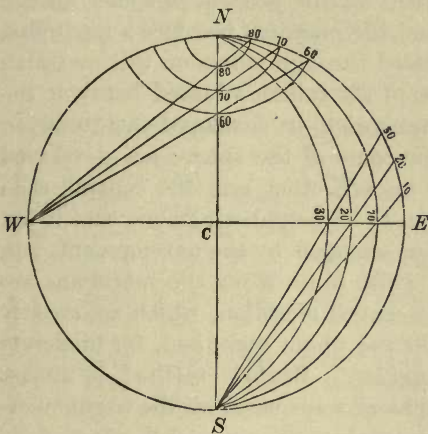


FIG. 302.

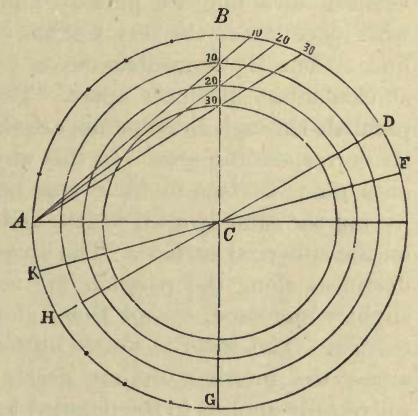


FIG. 303.

and circle (Fig. 302), as in the preceding problem. To project the other meridians (say every 10°), divide the quadrant N E into nine equal parts; from S to these points of division, 10, 20, 30, draw lines intersecting C E in 10, 20, 30. These latter points are in the meridians through which N and S arcs are to be described from centers on the line E W.

To find in like manner the three points in the parallels of latitude, divide the quadrants into nine parts, 80, 70, 60, and through these points draw lines to W; the intersections with the central meridian 80, 70, 60, will with the points of the quadrant furnish three points through which to describe arcs of parallels of latitude.

To project the hemisphere on the plane of the equator (Fig. 303). Draw two lines at right angles to each other; describe the circle and divide the circumference as before. The center C will be the projection of N or S pole, the lines at right angles to each other will be meridians, as well as any other diameters, as D H, F K, drawn through some division of the circumference.

To project the parallels of latitude. The circle represents the projection of the equator, and the other parallels must be arcs on the same center C, of which the radii are to be determined by the intersections of the line C B by lines drawn from A to the divisions of the circle 10, 20, 30.

In *Class II*, instead of projecting directly on planes, an intermediate cone or cylinder is employed to receive the projection, which is then developed on a tangent plane. The cylinder or cone must always be employed, because they are the only surfaces that can be developed on a plane. The eye is always conceived to be at the center of the earth in all the projections of this class.

In *Class III* the portions of the earth's surface are mapped by being divided into small or differential elements which are successively developed. This method admits of greater accuracy than any of the four classes. The two most important subdivisions are Bonne's and the Polyconic.

In Bonne's projection, assume a central meridian, and a central parallel with a cone tangent along the latter. The central meridian is then developed on that element of this cone to which it is tangent, and the cone is then developed on a tangent plane. The parallel, by this process, becomes an arc with its center at the vertex of the cone, and the meridian becomes a graduated line. Conceive concentric circles to be traced through points on this meridian at elementary distances apart. The zones of the sphere situated between the parallels through these points are then conceived to be developed each between its corresponding arcs. In this way all the zones of the sphere are developed on a plane surface in their true relation to each other and the central, each having the same length, width, and relation to its neighboring zone that it did on the spherical surface. The areas are not changed by the development, and distances along the parallels are correct, while those along the meridians are slightly increased, except those along the central meridian, which are strictly correct. The scale is nearly uniform over the whole map, and, for moderate areas, the intersections are nearly rectangular. Bonne's method is almost universally applied to the detailed topographical maps based on the trigonometrical surveys of the different states of Europe.

The Polyconic has been adopted by the United States Coast Survey, and all their maps are projected by this method. Each parallel is supposed to be represented on a plane by the development of a cone having the parallel for its base, and its vertex at the point of intersection of a tangent to the parallel and the earth's axis. The map thus becomes the development of the surface of successive cones, and the degrees of the parallel preserve their true length. The following tables are given for use in projecting large maps. Their use will be explained in an example. For making small maps, with a great degree of accuracy, tables are published by the United States Coast Survey.

Co-ordinates of Curvature in Miles for Maps of Large Extent.

LONGITUDE.	Latitude 20°.		Latitude 24°.		Latitude 28°.		Latitude 32°.	
	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.
°								
2....	130·0	0·8	126·4	0·9	122·2	1·0	117·4	1·1
4....	260·0	3·1	252·8	3·6	244·4	4·0	234·8	4·3
6....	390·0	6·9	379·2	8·1	366·5	9·0	352·0	9·8
8....	520·0	12·4	505·5	14·4	488·6	16·0	469·3	17·3
10....	649·8	19·4	631·7	22·4	610.4	25·0	586·3	27·1
12....	779·7	27·8	757·9	32·2	732·4	36·0	703·5	39·1
14 ...	909·2	38·0	883·6	43·9	853·7	49·0	819·6	53·1
16....	1039·2	49·6	1009·9	57·4	975·7	64·1	936·8	69·5
18....	1168·1	62·8	1134·8	72·6	1096·0	80·9	1051·9	87.8
20....	1298·0	77·6	1261·2	89·7	1218·8	100·1	1169·2	108·6
R.	10892		8905		7458		6348	

Co-ordinates of Curvature in Miles for Maps of Large Extent.—(Continued.)

LONGITUDE.	Latitude 36°.		Latitude 40°.		Latitude 44°.		Latitude 48°.	
	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.	D. M.	D. P.
°								
2 ...	112·0	1·2	106·1	1·2	99·7	1·2	92·7	1·2
4....	224·0	4·6	212·2	4·8	198·9	4·8	185·4	4·8
6....	335·9	10·3	318·1	10·7	298·7	10·9	277·9	10·8
8....	447·7	18·4	423·9	18·9	398·0	19·3	370·3	19·2
10....	559·2	28·7	529·4	29·7	497·1	30·2	462·3	30·0
12....	670·5	41·3	634·7	42·8	595·9	43·4	554·1	43·2
14....	781·6	56·2	739·7	58·2	694·3	59·1	645·6	58·8
16....	892·3	73·4	844·3	76·0	792·3	77·1	736·5	76·7
18....	1002·6	92·8	948·5	96·1	889·9	97·5	827·0	97·0
20....	1112·5	114·5	1052·3	118·5	986·9	120·2	916·9	119.6
R.	5461		4729		4110		3575	

Length of a Degree of Longitude at Different Latitudes, and at Sea-Level.

Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.	Deg. of Lat.	Miles.
0	69·16	14	67·12	28	61·11	42	51·47	56	38·76	70	23·72
2	69·12	16	66·50	30	59·94	44	49·83	58	36·74	72	21·43
4	68·99	18	65·80	32	58·70	46	48·12	60	34·67	74	19·12
6	68·78	20	65·02	34	57·39	48	46·36	62	32·55	76	16·78
8	68·49	22	64·15	36	56·01	50	44·54	64	30·40	78	14·42
10	68·12	24	63·21	38	54·56	52	42·67	66	28·21	80	12·05
12	67·66	26	62·20	40	53·05	54	40·74	68	25·98	82	9·66

Lengths for intermediate degrees can be found accurately by proportion. At the equator, 1° of latitude = 68·70 miles; at latitude 20° = 68·78; at 40° = 69·00; at 60° = 69·23; at 80° = 69·39; at 90° = 69·41.

To draw a map according to the tables, we lay off on the straight line (Fig. 304) N S, representing the middle meridian, the lengths representing the ten degrees of latitude between 20° and 30°, 30° and 40°, etc. Through these points draw circular arcs with the radii designated by R in the preceding tables. On these arcs lay off the lengths of ten degrees of longitude for each corresponding latitude on each side of the center meridian. Through the points thus formed draw the meridians, which will be found slightly concave toward the middle one. If the scale is so large that it is impossible to draw the circular arcs with beam-compasses, erect perpendiculars at the points 20°, 30°, 40°, and 50°, and on them lay off the values *d m* from the tables. At each of the points so found erect perpendiculars, and set off on them the corresponding values of *d p*. Through the points thus found draw the parallels and meridians. The principal advantages of this projection are—a minimum amount of distortion at any portion of the map; a scale of degrees and minutes of the parallels and meridians, by means of which, positions, determined by their latitudes and longitudes, may be readily inserted on the maps; the use of a linear scale in any portion or direction of the map; and the intersection of parallels and meridians at nearly right angles.

In *Class IV* some arbitrary mathematical condition is imposed, for some practical purpose, usually giving rise to distorted maps.

For polar projections, *De Lorgne's* has much merit. Calculate first a circle with an area equivalent to that of the hemisphere to be projected. Draw

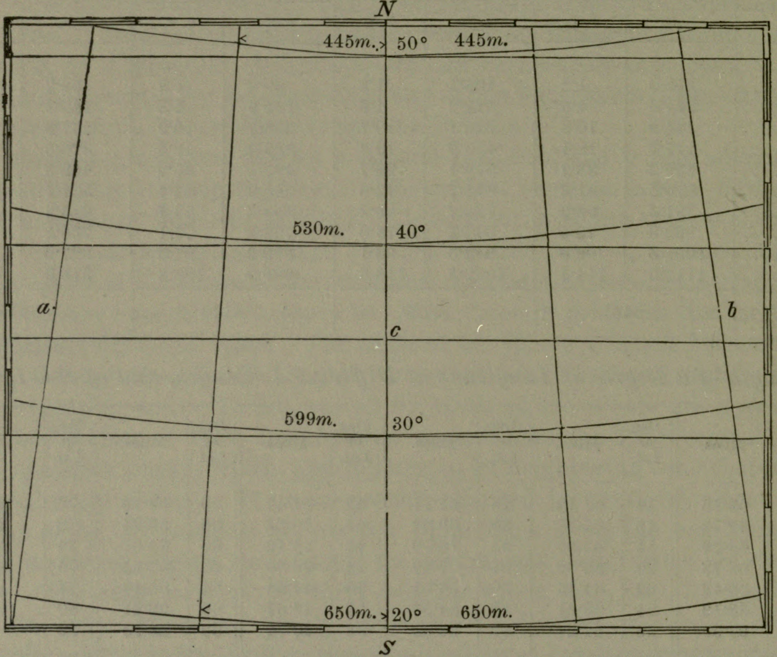


Fig. 304.

such a circle and connect the graduations of the circumference with the center. These represent the meridians. The radius can be divided into ninety equal

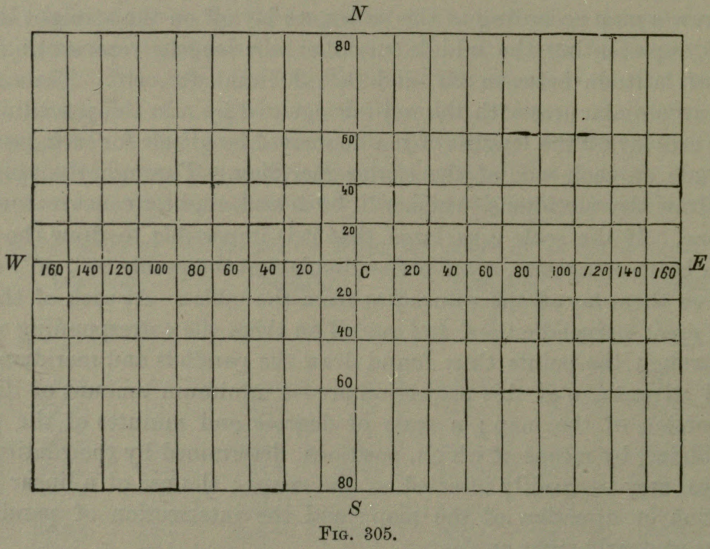


Fig. 305.

parts ; but, where it is possible, the chords of the polar distances of the parallels should be used for determining the parallels.

Mercator's chart is especially valuable to the navigator. By it he can lay off his course accurately on the chart in a straight line. It has little value for the other purposes of a chart. Meridians are represented by equidistant parallel straight lines, and the parallels by a perpendicular set of parallel straight lines, whose distances from each other increase from the equator outward in the same ratio as the corresponding longitudinal degrees diminish. By this means the relation between the latitude and longitude measurements on the chart is preserved uniformly as on the earth's surface.

To construct a Mercator's Chart (Fig. 305). Draw two straight lines, W E and N S, intersecting each other at right angles at C. W E is the equator, N S the meridian passing through the middle of the chart. From C set off equal parts on the equator both ways, to represent degrees of longitude, subdivided into minutes if the size of the chart will admit of it. Assuming the equator as a scale of minutes, set off from C, toward N and S, the number of minutes in the enlarged meridian corresponding to each degree of latitude, as shown by the table of meridional parts. Draw lines parallel to N S through the divisions of the equator for meridians, and parallels to W E through the divisions of N S for parallels of latitude.

To find the bearing of any one place from another, it is only necessary to draw a straight line between the two points, and observe the angle it makes with the meridians.

Table of Meridional Parts.

Latitude.	Meridional parts.	Latitude.	Meridional parts.	Latitude.	Meridional parts.
°		°		°	
0	0·00	35	2244·29	70	5965·92
5	300·38	40	2629·69	75	6970·34
10	603·07	45	3029·94	80	8375·20
15	910·46	50	3474·47	85	10764·62
20	1225·14	55	3967·97	90	Infinite.
25	1549·99	60	4527·37		
30	1888·38	65	5178·81		

COLORED TOPOGRAPHY.

Topographical features may be represented effectively and expeditiously by means of the brush and water-colors, either by India ink alone, or by various tints, or by the union of both.

The most important colors for conventional tints are (besides India ink), indigo (blue), carmine (or crimson lake), and gamboge (yellow), used separately or compounded. Besides these, burnt sienna, yellow ochre, and vermilion are sometimes used, although the first three are susceptible of the best combinations, and the others are generally used alone.

The following conventional colors are used by the French military engineers in their colored topography: Woods, *yellow;* using gamboge and a very little indigo. Grass-land, *green;* made of gamboge and indigo. Cultivated land, *brown;* lake, gamboge, and a little India ink; "burnt sienna" will answer. Adjoining fields should be slightly varied in tint. Sometimes furrows are indicated by strips of various colors. Gardens are represented by small rectangular patches of brighter *green* and *brown*. Uncultivated land, marbled *green* and light *brown*. Brush, brambles, etc., marbled green and yellow. Heath, furze, etc., marbled *green* and *pink*. Vineyards, *purple;* lake and indigo. Sands, a light *brown;* gamboge and lake; "yellow ochre" will do. Lakes and rivers, light *blue*, with a darker tint on their upper and left-hand sides. Seas, dark *blue*, with a little yellow added. Marshes, the *blue* of water, with spots of grass *green*, the touches all lying horizontally. Roads, *brown;* between the tints for sand and cultivated ground, with more India ink. Hills, *greenish* brown; gamboge, indigo, lake, and India ink. Woods may be finished up by drawing the trees and coloring them green, with touches of gamboge toward the light (the upper and left-hand side), and of indigo on the opposite side.

In addition to the conventional colors, a sort of imitation of the conventional signs is introduced in color with the brush, and shadows are almost invariably introduced. The light is supposed to come from the upper left-hand corner, and to fall nearly vertical, but sufficiently oblique to allow of a decided light and shade to the slopes of hills, trees, etc. After the shadow has been painted, the outline of the object is strengthened by a heavy black line on the side opposite the light. The flat tints are first laid on as above, and then the conventional signs are drawn in with a pencil and colored in with appropriate and more intense tints; the shadows are generally represented in India ink.

Hills are usually shaded, not as they would appear in nature, but on the conventional system of making the slopes darker in proportion to their steepness; the summits of the highest ranges being left white—an arrangement incorrect in theory, but generally understood by those not accustomed to plan-drawing, and is easy of execution. Wash the surface first with the proper flat tint, trace in with a pencil outlines, then lay on in India ink tints proportioned in intensity to the height of the hills and steepness of the slopes. To soften the tints, two brushes are used, one as a color-brush, the other as a water-brush: the tints are laid on with the first, and softened by passing the water-brush rapidly along the edges. The water-brush must not have too much water, as it would in that case lighten the tint to a greater extent than is intended, and leave a ragged, harsh edge. Tints may be applied in very light shades, one tint over another, with the boundary of the upper tint not reaching the extreme limit of the tint below it. When depth of shade is required, it is best produced by application of several light tints in succession; no tint is to be laid over the other until the first is dry.

When woods have to be represented, the shading used for the trees, instead of interfering with the shadows due to the slopes, may be made to harmonize with them, and contribute to the general effect by presenting greater or less depth, according to the position of the woods on the sides, or summits of the hills.

An expeditious and effective way of representing hills with a brush, a species of imitation of hills drawn with a pen on the vertical system, is effected by pressing out flat the brush to a sort of comb-like edge; drawing this over a nearly dry surface of India ink, and then brushing lightly or more heavily between the contours, according to the steepness of the slope, each of the comb-like teeth making its mark.

Rivers and masses of water may be shaded in with a color and water brush as above, or, by superposition of light tints, a shadow may be thrown from the bank toward the light, and the outline of this bank strengthened with a heavy black line. The tints are to be in indigo, the shadows in India ink.

Topographical drawings may be made in water-color with but one tint, as India ink, or ink mixed with a little sepia. The conventional signs are in imitation of pen-drawings, the hills in softened tint, or drawn with the comb-edged brush, and the rivers shaded with superposed tints.

Most artistic and effective drawings are made of hills as they would appear in nature, under an oblique light ; the sides of the hills next the light receiving it more or less brilliantly, according as they are inclined more or less at right angles with its rays, and the shades on the sides removed from the light, increasing in intensity as the slopes increase in steepness.

Having damp-stretched the paper upon the drawing-board, first draw in the lines in pencil, and afterward repeat them with a very light ink-line ; a soft sponge, well saturated, should then be passed quickly over the surface of the drawing, in order to remove any portions of the ink which would be liable to mix with the tint and mar its uniformity.

The moistened surface will prevent the tint from drying too rapidly at the edges. In tinting, never allow the edge to dry until the whole surface is covered ; leave a little superfluous color along the edge while filling the brush. In applying a flat tint to large surfaces, let the drawing-board be inclined upward at an angle of five or six degrees, so as to allow the color to flow downward over the surface. With a moderately full brush, commence at the upper outline, and carry the color along uniformly from left to right and from right to left in horizontal bands, taking care not to overrun the outlines, in approaching which the point of the brush should be used, and at the lower outline let there be only sufficient color in the brush to complete the tinting.

No color should be allowed to accumulate in inequalities of the paper, but should be evenly distributed over the whole surface.

Too much care can not be given to the first application of color ; as any attempt to remedy a defect by washing or applying fresh tints will be found extremely difficult, and to generally make bad worse.

Erasers should never be used on a tinted drawing, as the paper, when scratched, receives the tint more readily, and retains a larger portion of color than other parts, thereby causing a darker tint.

Marbling is done by using two separate tints, and blending them at their edges. A separate brush is required for each tint ; before the edge of the first is dry, pass the second tint along the edge, blending one tint into the other, and continue with each tint alternately.

In reference to the general effect to be produced in tinted topographical drawings, as to intensity, everything should be subordinate to clearness ; no tint should be prominent or obtrusive. Tints that are of small extent must be a little more intense than large surfaces, or they will appear lighter in shade. Keep a general tone throughout the whole drawing. Beginners will find it best to keep rather low in tone, strengthening their tints as they acquire boldness of touch.

Plate VIII gives an example of colored topography.

In lettering tinted drawings, let the letters harmonize with the rest of the plan ; let them be in tint more intense than the topography, prominent but not obtrusive.

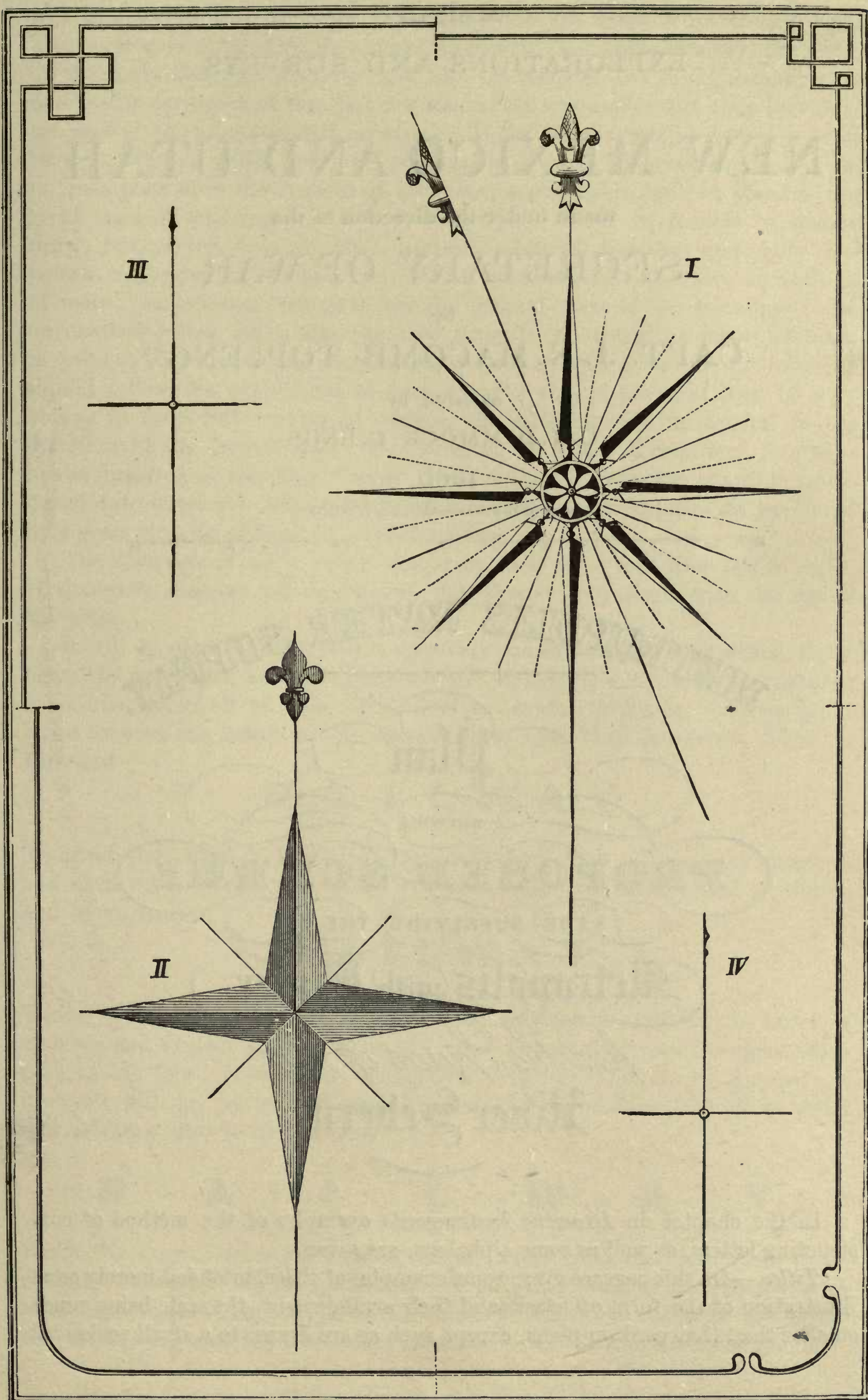
Finishing the Plan or Map.—In general, in topographical drawings, the light is supposed to fall upon the surface in a diagonal direction from the upper left-hand corner. This rule is not uniform ; by some draughtsmen the light is introduced at the lower left, and hills are mostly represented under a vertical light, although the oblique adds more to the picturesque effect. The plan is usually so drawn that the top may represent the north, and the upper left-hand corner is then the northwest.

In inking in, commence first with the light lines, since a mistake in these lines may be covered by the shade-lines. Describe all curves which are to be drawn with compasses or sweeps before the straight lines, for it is easier to join neatly a straight line to a curve than the opposite. Ink in with system, commencing say at the top ; ink in all light lines running easterly and westerly, then all light lines running northerly and southerly, then commence in the same way and draw in the shade-lines. It will of course be understood that elevated objects have their southern and eastern outline shaded, while depressions have the northern and western ; thus, in conventional signs, roads are shaded the opposite to canals. Having inked in all lines that are drawn with a ruler or described with compasses, commence again at one corner to fill in the detail, keeping all the rest of the plan except what you are actually at work upon covered with paper, to protect it from being soiled. The curved lines of brooks, fences, etc., are sometimes drawn with a drawing-pen, sometimes with a steel pen or goose-quill. The latter are generally used in drawing the vertical lines of hills.

Boundary-lines of private properties, of townships, of counties, of States, etc., are indicated by various combinations of short lines and dots, thus :

All plans should have meridian lines drawn on them ; also scales, and the dates on which the plans were finished. Page 175 gives several designs for meridians and borders. In these diagrams it will be observed that both true and magnetic meridians are drawn ; this is desirable when the variation is known, but in many surveys merely the magnetic meridian is taken ; in these cases this line is simply represented with half of the barb of the arrow at the north point, and on the opposite side of the line from the true meridian. Scales are drawn or represented in various forms, or the proportion of the plan to the ground is expressed decimally, as the number of feet, chains, etc., to the inch.

Lettering.—The style in which this is done very much affects the general appearance of the plan. Great care must be taken in the selection and character of the type, and in the execution.


I
II
III
IV

MAP OF

EXPLORATIONS AND SURVEYS

IN

NEW MEXICO AND UTAH

made under the direction of the

SECRETARY OF WAR

by

CAPT. J. N. MACOMB TOP^{L}. ENG^{RS}.

assisted by

C.H. DIMMOCK, C. ENG^{R}

1860

Scale of 12 Miles to one Inch or 1:760320

METROPOLIS WATER SUPPLY.

Plan

shewing

PROPOSED SCHEME

FOR SUPPLYING THE

Metropolis with Water.

from the Sources of the

River Severn.

In the chapter on *Drawing Instruments* examples of the method of constructing letters, as well as some alphabets, are given.

Titles.—On this pageare given some examples of titles, intended merely as an illustration of the form of letters and their arrangement, the scale being much smaller than that used on plans, except such as are drawn to a small scale. It will be observed that the more important words are made in prominent type. The lower part of the title should always contain, in small character, the name of the party making the survey, and also the name of the draughtsman, with date of the execution of the plan : if the survey was made some time previous, the date of the survey should be given. If the plan is compiled from several surveys, the authorities should, if possible, be given. The lettering of the title in lines parallel to the bottom of the plan is preferable, and, in general, the great mass of lettering in the body of the plan should be formed in similar lines ; but curved lines are often not only essential, but they materially contribute to the beauty of the plan. Thus, on crooked boundaries, on outlines of maps, the lettering should follow the general curve of the boundary ; also on crooked rivers, lakes, seas, etc. ; on irregular or straggling pieces of land, in order to show the extent, connection, or proprietorship thereof, the lettering should follow the central line of such a tract ; and, if pieces of land be very oblong in form but regular in outline, the lettering will be central in the direction of the longest side. The lettering of roads, streets, etc., is always in the direction of the line of road. Curved lines of lettering are often introduced into extended titles to take off the monotonous appearance presented by a great number of straight lines of writing.

The direction of all lettering should be so as to be read from left to right. If shades or shadows are introduced, they should be uniform with the rest of the plan.

It will be observed that letters vary very considerably in their width, the *I* being the narrowest, and the *W* the widest ; if, therefore, the letters composing a word be spaced off at equal distances from center to center, the interval or space between the letters will be more in some cases than in others. Thus, in the word

R A I L W A Y

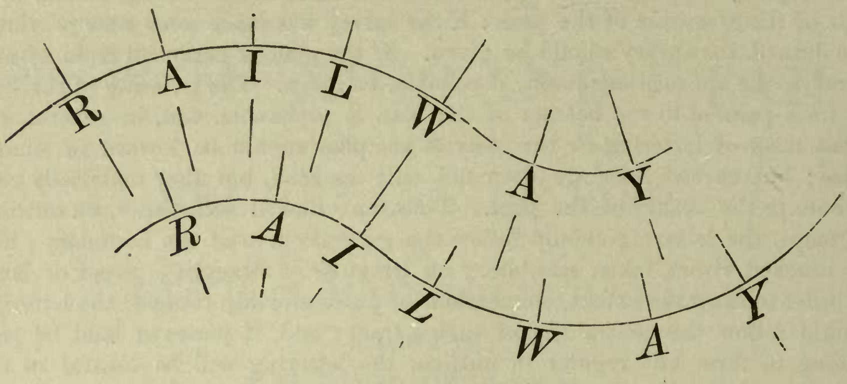
To avoid this, write in first one letter, and then space off a proper interval, and then write in the next letter, and then space off the interval as before, and so on, thus :

R A I L W A Y

When, as frequently happens, the words are very much extended, in order to embrace and explain a large extent of surface or boundary, and the space occupied by the letter is small in comparison with the interval, the disparity of intervals will not be noticed, and the letters may be then laid off at equal spaces from center to center, thus :

R A I L W A Y

When the lines of lettering are curved, the same rules for spacing are to be observed as above. If the letters are upright, as Roman or Gothic, the sides of each letter are to be parallel to the radius drawn to the center of the letter, and the bottom and top lines at right angles to it. If the letters be inclined, as Italic letters, then the side-lines of the letters must be inclined to the central radial line, as on a horizontal line they are inclined to the perpendicular.



In laying off letters by equal intervals, it is usual to count the number of letters in the word, and fix the position on the plan of the central one, and then space off on each side; this is particularly important in titles, when it is necessary that many lines should have their extremities at uniform distances from the center line. In laying off the title, we determine what is necessary to be included in the title, the space it must occupy, the number of lines necessary, and the style and arrangement of characters to be used. Thus, if the title were, *Plan of the Proposed Terminus of the Harlem Railroad at New York, 1857,* knowing the space to be occupied, we can write the title thus:

Plan

of the

Proposed Terminus

of the

Harlem Railroad,

at

New York,

1857.

We now draw parallel lines at intervals suited to the character of the type we intend to employ for the different words. *Harlem Railroad* is the line to be made most prominent; this, calling the interval between the words one letter, includes 15 letters; or, if we consider *I,* with its proper interval, but half a letter (which will be found a very good rule in spacing), 14½; hence the center of the line will be 7¼ letters from the beginning, or ½ of the space occupied by the letter *R* and its interval. Draw a perpendicular line at the center, and write in *R* in such a character as may suit the position to be filled, and lay off by letters and spaces the other letters. The line *Harlem Railroad* is intended to occupy the whole length of space; that is, it must be the longest line in the title, and the lines above and below must gradually diminish, forming a sort of double pyramid. *Proposed Terminus* includes 16½ letters; the *I* and interval between the words being rated as above, we find the center to be nearly midway between the words. These words, including more letters, and being confined within less space, must be in smaller character than the preceding; and, as a further distinction, a different style should be adopted. Having determined this, we proceed to write in the letters as before, and in the same way with the other lines; the prepositions, as unimportant, are always written in small type.

Plan

of the

PROPOSED TERMINUS

of the

HARLEM RAILROAD

at

NEW YORK

1857

In general, it is better that letters should be first written on a piece of paper, distinct from the plan, as repeated trials may be necessary before one is arranged to suit the draughtsman. Having formed a model title, it may be copied in the plan by measures or by tracing and transfer paper. There are some words, such as plan, map, section, scale, elevation, etc., which, as they are of constant occurrence, may be cut in stencil; sometimes whole alphabets are thus cut and words compounded. It will be found very convenient for a draughtsman if he makes tracing or copies of such titles as he meets with, and preserves them as models; for there is no manipulation on a plan that contributes more to the effect than good lettering and arrangement of titles, and considerable practice should be expended in acquiring a facility in lettering, and, for the first start, perhaps nothing will be found more valuable than tracing good examples.

We have treated of mechanical methods by which most persons can learn to form letters and words; but it must be borne in mind that the distances between letters on the plan are only intended to suit the eye; if, therefore, a person accustom himself to spacing, so that his eye is correct, there will be no necessity of laying off by dividers ; in this mode, such letters as A and V, L and T, are brought nearer each other than the regular interval. In general, it may be observed, in reference to the lettering of topographical drawings, stiff letters like those of stencil should not be introduced, but there should be such variety, incident on construction by the pen, as may be consonant with the rest of the drawing. Of late, rubber type have been introduced, of fair forms, much used on common drawings, by which lettering is very rapidly executed, and is an improvement on that of most draughtsmen.

MATERIALS.

VARIED materials enter into the composition of structures and machines, or form their supports, which are not only to be represented by the draughtsman, but he should also understand the composition and properties of these materials, that he may use them appropriately in his designs, and devise proper forms to resist adequately and economically the strains to which they are to be subjected. The earths and rocks, in their natural position, serve as the supports of structures and machines ; they may be represented as shown under the head of "Topographical Drawing," or by a closer imitation of nature, with or without color.

Fig. 306 represents a plan and section of an earth-bank of a canal, with a paved rock-slope. A breakwater, of which the base A is a mass of loose stone, is represented by Fig. 307. A base of rock may be represented by a stratification (Fig. 308). For the foundation of a structure, nothing is better than solid rock, but the rock should either have a horizontal bed or be cut in horizontal steps, so that the walls resting on it may not slide. The base of the wall need not be widened. Sand and gravel are also very good foundations, but the base resting on the earth should in general be about double the width or thickness of the wall resting on it. For extensive buildings it is important that the areas of the bases of its different parts should be proportioned to the weights upon them, and it is also important that soundings should be made to determine whether

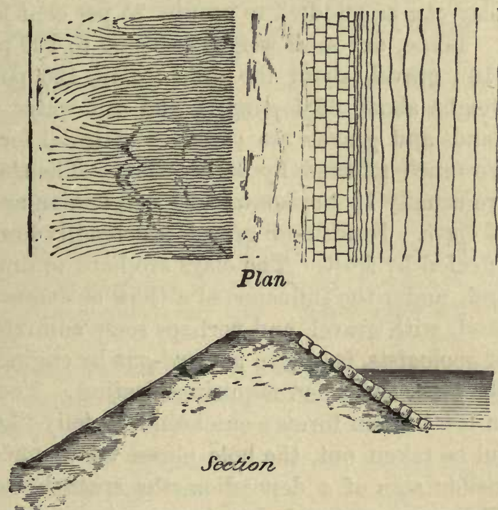


FIG. 306.

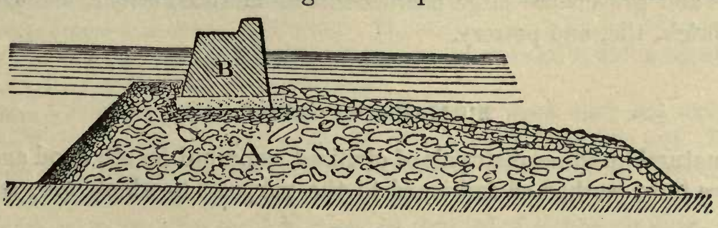


FIG. 307.

there are any compressible or sliding strata below. A stratum of 3 to 5 feet of gravel upon a stony stratum is sufficient foundation to support 1 to 1½ tons per square foot; but, if the sand rests upon rock, even at a very great depth, it is not unusual to load it with 2 to 5 tons per square foot. On sand and gravel, the building may settle somewhat, but with proper bases uniformly; on wet clay, it is more uncertain; the building may settle by displacement, as on a fluid; and, if the stratum is inclined, it is extremely apt to slide under its load. There are others still more fluid, as quicksand and marshy deposits, where support must be obtained by extending the bases. On water itself, it is obtained by means of a scow or tight box, the displacement being equal to the weight of box and structure.

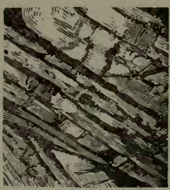


Fig. 308.

Earth, when first dug, occupies more space than when in its natural condition, but, after a time, it shrinks and becomes more compact. The earth dug out of a hole, when settled, will not fill the hole. Sand, gravel, loam, and clay, will occupy from 8 to 12 per cent less space than when in the natural cut. Clay can be puddled to occupy 25 per cent less.

Loose, dry sand weighs from 90 to 100 pounds per cubic foot; compacted, 110; gravel, about the same; clay, 120 pounds. Fresh water, at 60° Fahr., weighs about 62·4 pounds, and salt water about 64·1 pounds per cubic foot. Sands and gravels are excellent material for embankments and fills, but clays are much affected by the weather. The slopes of the former in cuts and fills are usually 1½ horizontal to 1 perpendicular; no fixed slope can be predicated of clays. Sands and gravels are readily drained, and, when dry, are but little affected by frost. The clays are hard to drain, heave with the frost when wet, and, under the influence of a thaw or excess of water, become fluid. Very fine sand, with gravel, and perhaps some admixture of clay, forming the *glacier till* of geologists, is known as *hard-pan* by engineers, very difficult to be moved with the pick, and often requiring blasting. The same material without the gravel in low bottom forms a quicksand—a jelly-like material—from which, if a spadeful be taken out, the hole closes up at once, and excavation shows but little visible sign of a depression, the space being made good from the entire mass. This same material, dry, is a species of hard-pan. There is another material, called quicksand, which is rather a running sand—even when not wet, it rests with a very flat slope; the particles are very fine, and flow like the sands in an hour-glass.

Sands and gravels are large components of mortars, *bétons*, and concrete; clay, of brick, tile, and pottery.

BUILDING MATERIALS.

The natural building materials of civilized communities are wood and stone, which are to be worked or fashioned to the purposes to which they are to be applied.

Figs. 309, 310, 311, are drawings of wood, longitudinal and sectional, in which the grain of the wood is imitated, but wood is more often represented in plain outline, and the cross-section of a timber thus (Fig. 312), or by mere hatching. When distinguished by color, burnt sienna is used commonly for wood, but sometimes the color of the wood is imitated.

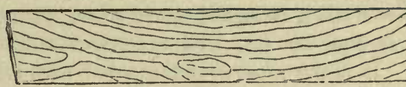


FIG. 309.

FIG. 310.

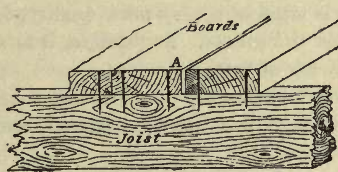


FIG. 311.

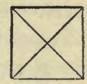


FIG. 312.

The draughtsman, for his designs, will probably have to confine himself to the timber within his reach. But he should know what is best for his purpose, reference being had to economy in cost and maintenance. For most purposes, wood should be seasoned, so that joints may not open under this operation after the material is in the structure. But, for work under water, wood should be but slightly seasoned, as a swelling of the wood may be disastrous. Seasoning of timber may be done by exposure for a time to outer air-currents; if in a kiln, it can be done speedily with heated air, or by steam. For beams, girders, and the like, there should be few knots, especially on the outer edges—for posts, small ones are not objectionable; while for sidings and under-floors, firm, large knots do not impair the work; but no smooth work can be made with knotty lumber.

The trunk of the tree is composed of sap-wood and heart-wood: the one soft, readily rotting; the other more dense and durable. In most specifications, lumber is "to be square-edged, without sap, and large or loose knots."

In selecting lumber for a permanent structure, the life and endurance of the material are to be considered. Most of the woods, sheltered from the wet and exposed to air-currents, will last for a very long time; but many will check and warp and become distorted. All lumber in earth beneath the level of water will last indefinitely. In salt water, above the earth, all are subject to the attacks of the worm—the *Teredo* and *Limnoria*—and, where the water is pure, the destruction is very rapid. Sewer-water and fresh water are both destructive to the worm.

The life of lumber, in situations exposed to wet and dry, can be prolonged by impregnating it with creosote or with various metallic salts, as the chlorides of zinc, mercury, pyrolignite of iron, and others.

CHARACTERISTICS AND USE.

White Pine.—A wood of the most general application in the market; is light, stiff, easily worked, nails are easily driven into it, and takes paint well, warps and checks but little in seasoning, endures well in exposed situations; clear stuff, of best quality, useful for patterns and models, for interior finish of houses, doors, window-sashes, furniture. It forms base or inner core of the best veneered work, holds glue well, and the composite structure is better than single solid wood. The cheaper kinds of pine are used for frames of buildings, posts, girders, and beams. Even with large knots is well adapted for boardings, and is extensively used for goods-boxes.

Southern Pine.—A heavy, strong, resinous, lasting wood, clear and mostly without knots, hard to be worked by hand-tools, and when seasoned difficult to nail. The surfaces, from their resinous character, do not hold paint well. It is used very largely for girders, beams, and posts of mills and warehouses, and for floors of the same, when exposed to heavy work or travel. For the first, it can be obtained of almost any dimension to suit; for floors, it is sold in long strips, from two to six inches wide, of varied lengths, tongued and grooved, and when laid is *blind-nailed, toeing* the nail through the tongue, so that the nail-head does not show.

Canadian Red, Norway, and Silver Pines.—Are resinous woods, like the Southern pine, and are used for similar purposes, but are not as valuable—woods less straight in the grain, and with more knots.

Spruce.—A light, straight-grained wood, with but few knots, which are small and often decayed. It does not last well exposed to the weather, and checks and warps badly in seasoning. It is the most common wood here for floor-beams and common floors, but it must be well braced and nailed, and is not fitted for joiner-work.

Hemlock is similar to the spruce, and, when selected, is less liable to check and twist in seasoning. It is often of a very poor quality, *brash* and shaky. Exposed, it is but little better, if any, than the spruce. For stables, it is well adapted for grain-boxes, as the fiber prevents the gnawing of rats.

Ash.—Some of the ashes are of exceeding toughness. A straight, close-grained wood. It is used for carriage and machine frames, and for interiors, doors, wainscot, floors, when no paint is used.

Chestnut.—Somewhat like the ash in appearance, but coarser-grained, and very enduring in exposed positions. It is most largely used for cross-ties of railways. As a roof-frame exposed in the inside, and in general interior finish without paint, the effect is very good. The closer-grained woods are very often thus used.

Black Walnut.—Is, in the trunk, a straight-grained, gummy wood, clogging the plane a little in its working; the knots are useful for veneer. Were the wood cheap enough, it would undoubtedly make a good frame. It is used here for desks and counters, for furniture and interior finish, as an ornamental wood.

Butternut.—Similar to the black walnut, less commonly used, but fully equal as an ornamental wood.

Hickory.—A strong, tough wood; is used for cogs of mortise-wheels, handspikes, axe-helves, and wheelwrights' work.

Beech.—A close-grained wood, but of little application in this market. Sometimes used for cogs of wheels, for small tool-handles, and in marquetry.

Oak, Live.—A very strong, tough, enduring wood, used industrially almost entirely for ship-building. Ornamentally, in marquetry and panels.

Oak, White.—A very valuable, strong, tough wood, with great endurance. It is heavy, and hard to work, and was formerly used largely for the frames of houses, but has been superseded by the white pine. It is used in ship-yards and in water-works—for the frames of flumes, penstocks, and dams, and for the planking of the latter, for dock-buffers and piles, and for railway and warehouse platforms. The red and black oaks may in general be considered a cheaper and poorer quality of the white oak. All have a handsome grain, that adapts them to ornamental work.

Bass, Poplar, White-wood, are light woods, mostly used in the manufacture of furniture, for drawer-bottoms, cabinet-backs, panels; they are very clear stock, easily worked, and can be readily obtained in thin, wide boards.

Cedar.—A straight-grained, light wood, of great endurance, valuable for posts, sills, shingles; used for pails and domestic utensils. The red variety, from its odor, is admirable for drawers and chests, preserving their contents from moths.

Locust is in the market only in small sticks; is of extreme endurance. It is used almost invariably here for the sills of the lowest floors of buildings, where there can be no ventilation, and for treenails of ship-planks.

Elm.—Although a tree of wide diffusion, is but little used as lumber. It is kept for an ornamental tree, beyond its usefulness for any other purpose but fuel. Well selected, it is said to be an enduring timber, useful for piles and places exposed to wet.

Maples are tough, close-grained woods. rather to be considered among the ornamental woods, for furniture and interior finish. The same may be said of the cherry, plum, and apple tree, of which the denser woods are admirably adapted for the handles of small tools, for bushings of spools and bobbins.

The list of imported woods is extremely large, mostly for ornamental purposes; but the mahogany is one of the very best of woods for patterns and small models, as it changes but little in seasoning; and the lignum-vitæ, a very hard and heavy wood, is used for pulley-sheaves, packing-rings of pumps, water-wheel steps, and shaft-bushings.

The weight and strength of the several woods are usually given in tables, but specimens of the same wood differ essentially in both particulars. For all practical purposes, the weight per cubic foot of white-pine, spruce, hemlock, poplar, bass, and cedar, may be taken at from 23 to 30 pounds. Ash, cherry, chestnut, black-gum, black-walnut, and butternut, from 32 to 45 pounds. Birch, beech, and elm, from 40 to 50 pounds. The oaks, except live, from 40 to 55 pounds. Locust and hickory, from 50 to 60 pounds. Live-oak and pitch-pine, from 60 to 70 pounds. Lignum-vitæ, from 75 to 80 pounds. Their resistance to crushing varies from 4,000 to 11,000 pounds per square inch, and to tension, from 1,100 to 4,000 pounds, but their practical use will be given in future illustrations.

STONES.

In selecting the form of construction, and the stones of which it is to be composed, the draughtsman must be governed by the fitness for the purpose and the cost. He must select from what he can readily get, and arrange the form to suit the material. He must know what is to be the exposure, and what the effect will be on the stones. Almost any stone will stand in a protected wall, but many of the sandstones and slates disintegrate and exfoliate under the influence of the weather, heat, cold, frost, and moisture. Even the granites are liable to serious decomposition when the feldspars are alkaline; and the limestones (dolomites), of which the English Houses of Parliament are composed, have failed in the sulphurous air of London smoke, while at Southwell Minster they have stood for over 800 years. Chemical tests of stone to determine endurance are deceptive. The safe way is to see how the material has stood in like situations to the one in which it is to be employed, and, if this is not possible, go to the quarry, and see how the stones have weathered there.

The strength of stones to resist crushing, as determined by experimental cubes, is even in the weaker stones much in excess of what would be required in structures, but most stones are weak under cross-strains, and failures in construction are more likely to occur by faulty workmanship or design, by which the stones are subjected to unequal strains, and for which they are not adapted. The weight should not be brought on the outer edges or *arrises*, as the faces will chip readily; nor should most stones be used for wide-span lintels, unless they form a part of the masonry above the opening, so that the whole is a beam.

TECHNICAL TERMS OF MASONRY.

We follow the nomenclature recommended in "Transactions of the American Society of Civil Engineers," November, 1877:

Rubble masonry includes all stones which are used as they come from the quarry, prepared at the work by roughly knocking off their corners. It is called *uncoursed rubble* (Fig. 313) when it is laid without any attempt at regular courses; *coursed rubble*, when leveled off at specified heights to a horizontal surface (Fig. 314).

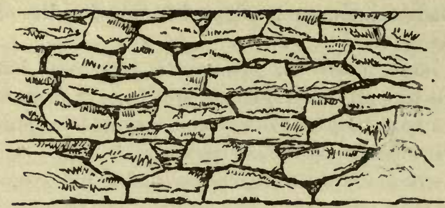


Fig. 313.

Fig. 314.

Square-stoned Masonry.—Square stones cover all stones that are roughly squared and roughly dressed on bed and joints, and when the joints, when laid, are one half inch or more.

Quarry-faced stones are those which are left untouched as they come from the quarry.

Pitch-faced stones are those on which the *arris* is clearly defined beyond which the rock is cutting away by pitching-tool.

Drafted stones are those in which the face is surrounded by a chisel-draft.

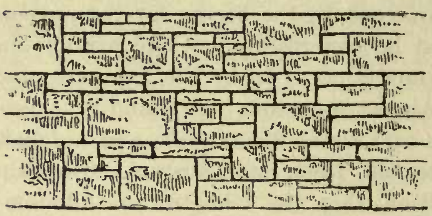


Fig. 315.

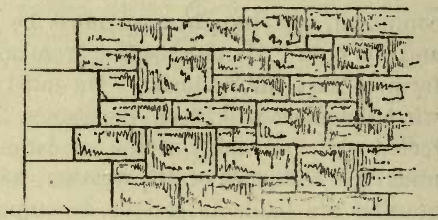


Fig. 316.

If laid in regular courses of about the same rise throughout, it is *range-work* (Fig. 315). If laid in courses that are not continuous, it is *broken range* (Fig. 316).

Cut stones or ashlar covers all squared stones with smoothly-dressed bed and joints. Generally, all the edges of cut stone are drafted, if the face is not entirely fine cut, but they may be quarry-faced or pitch-faced; as a rule, the

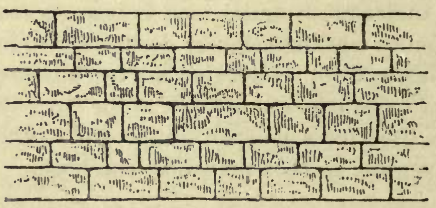


Fig. 317.

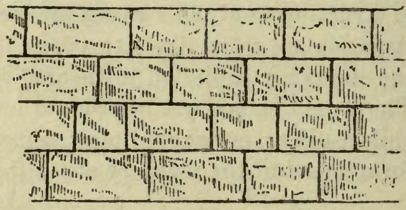


Fig. 318.

courses are continuous (Figs. 317, 318), but, if broken by the introduction of smaller stones of the same kind, it is called *broken ashlar* (Fig. 316). If the courses are less than one foot in height, it is *small ashlar* (Fig. 317).

Square-stoned masonry is usually backed up with rubble masonry. Any of this masonry may be laid dry, or with mortar or cement, which is to be specified.

The joints in one course should not come directly over those of another;

there should be a lap or *bond*, and, in connecting the front or face with the backing, headers must be introduced for bond. Headers are stones extending into the wall, stretchers running with the face.

In addition to the classes of stone-work, there is an old form lately come into use called *hock and ham* by old English builders ; it is a species of rubble, in which there are no courses. The stones are very carefully selected in size and shape, so as to make an ornamental work ; the joints are close, but have no uniformity of direction.

For rubble-work, all varieties of sound stone are used, and of almost any size. In dry work, for foundations and for heavy revetment-walls, the stones are laid with derricks, but they must have fair beds and builds. If bowlders, they must be split, and cobbles in the filling are worse than useless.

For rubble laid in mortar, the usual size is such as can be laid by hand.

GRANITIC STONES.

Granite and syenite are by builders classed as granites. The granite in general rifts in any direction, and works well under the hammer and points. From these circumstances it is more desirable than the syenites, which are much harder to be worked. Both are admirable stones for heavy dock-walls, bridge-abutments, river-walls, either as rubble-squared stones or cut work, and are very enduring. They are also used for the faces of important buildings, either as fine-cut, quarry, or pitched-face. Ornamental work of the simpler kind is readily produced ; more elaborate is expensive, but it is about the only stone in this climate in which foliage and sharp undercut work will stand the weather without exfoliating. These stones, especially the syenites, admit of a high polish, and are used considerably for columns and panels in buildings, and in monumental work. Gneiss is of the granitic order, but a cheaper, poorer stone. It splits with difficulty, except parallel with line of bed. It has a foliated structure, and is not adapted for ashlar, but is very good for squared-stone masonry and rubble-work, and often used for sidewalk-covers of vaults.

ARGILLACEOUS STONES.

The slates or stones thus designated by builders were formerly in very common use as roofing material, and were almost entirely from Wales, but latterly they are taken from Vermont and Pennsylvania, and other parts of the United States. They are also used, in thicknesses of one inch and above, for floors, platforms, facing of walls, mantels, and for wash-tubs by plumbers. Soap-stone may be classed under the clay stones; also, used for tubs, for stoves, and for the lining of grates and furnaces.

The Ulster, or North River blue stone of this market, is a coarser slate, a very strong and enduring stone; it can be quarried of varying thickness up to twelve inches, and of any dimension that can be transported. It can be readily cut, hammer-dressed, axed, planed, and rubbed. Is generally used for sidewalks under these various forms. It is used as bond-stones in brick piers, for caps, sills, and string-courses.

THE SANDSTONES.

Sandstones, called also freestones, from the ease with which they are worked; and from their colors, are very popular for the fronts of edifices. In general, they are not very enduring stones, and when laid must be set parallel to their natural beds, as otherwise they flake off under the influence of the weather. The sandstones are not all of the same quality; those in which the cementing material is nearly pure silex, are strong, enduring stones, but not those in which the cementing material is alumina, or lime. By examining a fresh fracture, the character of the stone can generally be detected. A clear, shining surface with sharp grains indicate a good stone; while rounded grains, a dull, mealy surface, indicate a soft, perishable stone. None of the sandstones in this locality are used for heavy pier or abutment work and the like, but there are sandstones in other localities adapted to it.

LIMESTONE.

The coarser calcareous stones are of great variety; some are well adapted for building stones, being hard and compact, while others are soft and friable. They are more easily worked than granite, but are not considered as enduring. They are well adapted to the same class of heavy work, and the locks of the Erie and Northern Canals and the dam across the Mohawk, at Cohoes, are built from limestone on the line of the canals.

The finer kinds of limestones are classed under the head of marbles. They are easily worked, sawed, turned, rubbed, and polished. Marble is not popular as a building material, although more enduring than most sandstones, but is susceptible to the action of sulphurous gases in the smoky air of cities; and it is said that the Capitol at Washington, D. C., built of marble, is suffering from disintegration. But, for interior finish, as tiles, wainscots, architraves, mantels, linings of walls, it is admirably adapted, and from its richness, cleanliness, and variety of color, it is very ornamental and effective.

ARTIFICIAL BUILDING MATERIAL.

The most common and useful are bricks. They are generally made of clay, with an admixture of sand, well incorporated together, and mixed with water to the consistency of a smooth, strong, viscous mud, pressed into molds, dried, and burned, the best quality being those in the interior of the kiln. The exteriors are light, friable bricks adapted to walls supporting but little weight and not exposed to wet. The brick forming the arches are very hard-burned, dark in color, often swelled and cracked; but by proper selection, they can be used for foot-walks. A good brick is well burned throughout; when struck, it gives a ringing sound, and is of uniform shape.

Bricks vary somewhat in size and weight in different localities—from 8 to 8½ inches long × 3½ to 4 inches broad × 2 to 2½ inches thick; in general, the thickness of a wall with the joints is called some multiple of 4″, as 8″, 12″, 16″. Here an 8-inch wall by 1 foot face contains 14 bricks; 12-inch, 21; 16-inch, 28 bricks—9 courses high are equal to 2 feet. In the Eastern States the brick is somewhat thinner—5 courses to the foot. The best front brick are pressed, and are a little larger than the common brick. Philadelphia and Baltimore pressed brick are distinguished by their clear, cherry-red color; Milwaukee are of a pale-straw color.

Bricks are laid in mortar, of lime, lime and cement, or cement only—all with an admixture of sand; in common walls, in lime; in walls of heavy buildings, above-ground, in lime and cement; beneath, and in wet, exposed positions, in cement only. The common bond of the different courses of brick is by header-courses every fifth or seventh course. When bricks are laid in arches they are set on edge, and turned in 4-inch rings, sometimes without any bond between the different rings; sometimes with a bond of brick lengthways, when two courses come on the same line.

Bricks set on edge, as in arches or in a level course, are here termed *rollocks*.

Arch brick, between iron beams, to reduce the weight, are often made hollow, and laid in flat arches; that is, the joints are radial, but the upper and lower surfaces are level. Hollow brick are also used for walls and partitions.

Fire-brick can be made of any size and pattern, but are usually 9 × 4½ × 2¾. They are used for the lining of furnaces, flues, and chimneys, exposed to the action of flame or great heat. Fire-clay, with an admixture of sawdust, which is burned out in the firing, leaves a light, porous, spongy mass, which can be sawed in sheets or strips, and is well adapted for covering the exposed parts of iron beams and girders, and, as it admits of nailing, is convenient for partitions.

Enameled Brick.—The English size is that of fire-brick—the American is that of common brick. The brick, on the faces to be exposed, are covered with glaze of varied colors and designs, and fired. They make a handsome ornamental face for walls, do not absorb moisture, and can be washed.

Tile are a species of brick, with or without enamel. The latter were originally used for roof-covering, but now are used in flooring walks and the like. The enameled or encaustic tile are generally in squares, 4″ × 4″, 6″ × 6″, 8″ × 8″, but there are smaller ones for tessellation, and rectangular strips for borders. They can be obtained of any color or design, forming beautifully ornamented floors and wall-panels.

Terra-cotta, a kind of brick, is now largely used for exterior decoration. It is molded in every variety of capitals, cornices, caps, friezes, and panels. It is a good, strong brick, with all the good qualities of such a material.

Mortars.—Brick are never laid dry, except in the under part of drains, to admit of the removal of ground-water. Stone-work, except in rough, heavy, rubble-work, is also generally laid in mortar. Where cut-work is backed with rubble, the joints in the latter should be as close as possible, and full of mortar, that the settling of the wall in itself may not be more in the backing than in the face. Some lay the rubble dry, and fill in with cement grout, or cement mortar made liquid to flow into the interstices, but the sand is apt to separate and get to the bottom of the course.

By mortar, is usually understood a mixture of quicklime and sand, but mortar may have an addition of cement to the lime, or it may be cement only with sand.

Lime, or properly quicklime, is made by the calcination of limestone, shells, and substances composed largely of carbonate of lime, carbonic-acid gas, water of crystallization, and organic coloring-matter. Quicklime, brought in contact with water, rapidly absorbs it, with a great elevation of temperature, and bursting of the lime into pieces, reducing it to a fine powder, of from two to three and a half times the volume of the original lime. This is *slaked* lime. It may be slaked slowly by exposure to the air, from which it will take the moisture. This is *air-slaked* lime. Barrels of lime exposed to rain often take fire from the heat caused by slaking. The paste of slaked lime may be kept uninjured for a considerable time, if protected from the air, and this may readily be done by a covering of sand, and it is customary, in some places, to hold it over one season, as an improvement to the uniformity of quality in the paste. But, in general, the lime is used soon after slaking, and is thoroughly mixed with sand, in various proportions, generally about two of sand to one of lime. The theory of the mixture is, that the lime should fill the void spaces in the sand, and the space occupied by the mortar is a little in excess of that occupied by the sand alone.

The sand should be sharp, clean, silicious grains, from one twelfth to one sixtieth of an inch in diameter. Close brick-joints do not admit of as coarse sand as those of cut-stone work, and, in rubble-work, sand coarser than the above can be used, and there will be considerable saving of lime in using a mixture of coarse and fine sand.

The hydraulic limes contain a small proportion of silica, alumina, and magnesia; slake with but little heat, and small increase of volume; are more or less valuable, according to the property which they have for hardening under water; but, in this particular, are not equal to the hydraulic cements, which contain a larger proportion of silica, alumina, and magnesia. They are made by calcining natural rocks, or by the combination of clay and soft carbonate of lime, or chalk, calcining and grinding. The cements make a paste with water, with little or no heat on slaking, and *set*, in open air or under water, with more or less rapidity; but this is not a sure criterion of the value of the cement, when time comes in as an element before the work is subject to stress.

Cement is mixed with sand in varied proportions—from 1 to 1 to 1 to 3—it is stronger without any admixture of sand, but is seldom used neat, except in pointing, and for very close joints. By experiments of Mr. F. O. Norton (Trans. Am. Soc. of C. E's.) it was found that Portland cement, with two volumes of sand, was equal to that of Rosendale (or native cement) with one part of sand. In purchasing cement, it is usual that it should be required to be up to a certain standard; that is, made up into a ball with water at 65° temperature; it should set in water to withstand a pressure of say one pound on a one-quarter inch wire within so many minutes. By many, cements are required to be of a certain degree of fineness—that only a very small per cent should be left on the screen, say of one sixtieth-inch mesh, and that it should weigh about 80 pounds to the bushel, and that it should have a certain tensile strength after so long a set.

Cement is used in all masonry in exposed and wet situations. With a small admixture of lime, it works better under the trowel, and for brick-work it does not sensibly impair its value. Cement adds to the strength of lime-mortar, and gives it an amount of hydraulicity. To increase the quick-setting of cement, it may sometimes be necessary to add a little plaster-of-Paris, but it is preferable to get a quick-setting cement.

Concrete or *béton* are terms now used for the same material. It consists of cement, sand, and gravel, or broken stone, which may be intimately mixed, in varied proportions, according to the quality of the cement and the character of the inert materials. For the blocks of the New York city docks, the proportions were:

Portland cement..................................	3	volumes.
Sand, damp......................................	5	"
Broken stone....................................	10	"

This is a strong mixture. It is not uncommon to make a cement of Rosendale cement 1, sand 2 to 3, and broken stone or clean gravel as much as can be well covered by the mixture, but it should have time to set.

Concrete is used for the base course or foundations of walls, and is formed *in situ*, that is, depositing and ramming it in the trench where it is to be left; or by forming in molds, in immense blocks, for docks or break-water, or in the smallest forms of brick and moldings.

The bituminous cements are formed of natural bitumens, or artificial from coal-tar mixed with various proportions of gravel and inert material. The mixture is usually heated, put down in layers, and rolled or rammed. It is used for roads and sidewalks, and for water-proof covering of vaults. For the covering of roofs, coarse paper, saturated with bitumen, is put on in layers, one over the other, breaking joints, cemented with the bitumen, the last coat being of bitumen, in which gravel is imbedded. For an anti-damp course in a wall, or for the joints in the bricks of a wet cellar-floor, or on top of a roof, bitumen is used as a cementing material—the bricks must be dry, bitumen hot.

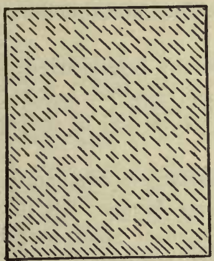
Plastering.—Coarse-stuff is nothing more than common brick-mortar, with an admixture of bullock's hair. When time can not be given for the setting it is *gauged*, that is, mixed with some plaster-of-Paris. *Fine-stuff* is made of pure lump-lime with an admixture of fine sand, and perhaps plaster-of-Paris. *Hard-finish* is composed of fine-stuff and plaster-of-Paris. *One-coat work* is of coarse-stuff, which may be *rendered*, that is, put on masonry, or *laid* on laths. *Two-coat work* is a coat of coarse-stuff, or scratch-coat; that is, after the coat is partially dry it is scored or scratched for a back for the next or fine coat. In three coats, the first coat is a scratch-coat, the second the brown-coat, and the third is hard-finish, or stucco. Keene's cement, for the last finish, gives a very hard surface, which admits of washing.

A single brick weighs between 4 and 5 pounds; but a cubic foot, well laid in cement, with full joints, will weigh about 112 pounds. They have resisted, in an experimental test, as high as 13,000 pounds to the square inch, but 12 tons should be the limit to the load per square foot; and the brick should be uniform, well burned, and closely laid in cement, and without cross-strain. In lime mortar, the load should not exceed 3 tons per square foot.

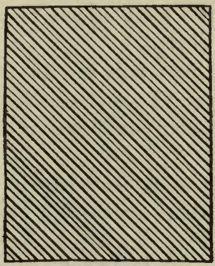
The granites weigh from 160 to 180 pounds per cubic foot; the limestones from 150 to 175; the sandstones from 130 to 170; the slates from 160 to 180; mortar, set, about 100 pounds; masonry, laid full in mortar, according to the quality of the stone and the percentage of mortar, from 150 to 170 pounds. Some of the granites have withstood a crushing strain of 15,000 pounds per square inch, and, when structures are important, and subject to great strains, specimens of the stones to be employed should be tested; but, for practical purposes, common mortar-rubble is not considered equal in strength to a brick wall, as it is seldom laid with equal care, and the joints are not as likely to be well filled, and the load as evenly distributed; but cut stones will sustain more, and ashlar, up to 50 tons per square foot for sound, strong stones.

METALS.

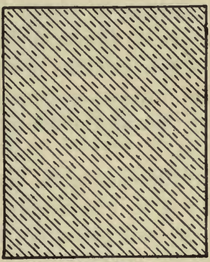
Metals are often to be shown distinctively by the draughtsman. If he can use color, he will in a measure imitate that of the material. For cast-iron, India-ink, with indigo, and a slight admixture of lake; for wrought-iron, the same colors, with stronger predominance of the blue; steel, in Prussian-blue; brass, in a mixture of gamboge and burnt sienna; copper, gamboge and crimson lake. But it is often requisite to express distinctive metals in drawings where no color is admissible. When the drawings may be required for photographing, or reproduced in printing, some conventional hatchings are used to represent sections of metals, but none have been so established as to have a universal application. The following are submitted to represent the most common industrial metals:



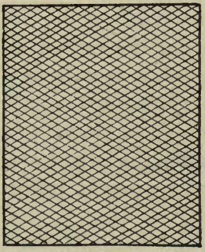
Cast Iron.
FIG. 319.



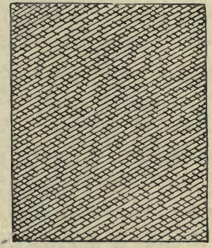
Wrought Iron.
FIG. 320.



Steel.
FIG. 321.



Brass.
FIG. 322.



Lead.
FIG. 323.

Under the term iron may be included cast-iron, wrought-iron, and steel, differing from each other in the percentage of carbon contained, and in the uses to which they are applied. Cast-iron contains more carbon than the others, say from two to five per cent. It can be cast in varied forms in molds, but can not be welded or tempered. The usual molds are in sand or loam, in which the pattern is imbedded, and when drawn out the space is filled with molten metal. The drawing of patterns for molding involves a knowledge of the art of founding. The shrinkage of the metal, usually about one per cent, for which provision must be made in increased size of pattern, is provided for by the pattern-maker, the draughtsman giving finished sizes, but the draughtsman must know whether the pattern can be drawn from the sand, and by what system of cores voids can be left; or it may often happen that castings, designed as a whole, will have to be made in a number of pieces, involving flanges and bolts. In cooling, the shrinkage takes place the soonest in the thinnest parts, and, if great care be not taken by the molder in exposing the thicker parts to the air first, the parts will shrink unequally, and there will be a strain induced which will materially weaken the casting, and it may even break in the mold. The draughtsman, in his design, should make the parts of as uniform thickness as possible.

Castings cool from the outside inward, in annular crystals perpendicular to the face, as in Figs. 324 and 325. Now, if the casting consist of a right angle (Fig. 326), there will evidently be a weak place along the line A B, but, if the angle be eased by a curve, the

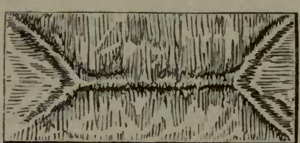


Fig. 324.

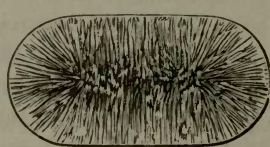


Fig. 325.

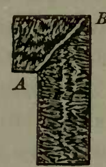


Fig. 326.

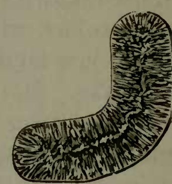


Fig. 327.

crystallization takes place as in Fig. 327, and the line of weakness is avoided. This is effected by a very small easement of the angle, and a cove is almost invariably introduced. In castings, in almost all metals, the same effects result from cooling, and therefore the changes of direction should not be abrupt.

When castings are ordered for important structures, iron of certain tensile strength is called for, and specimens of the metal, in small rectangular bars, are required, cast at the same time and under as nearly the same conditions as the casting which may be subjected to test.

If the casting be made in dry sand, it cools slowly, and the surface is comparatively soft; if in greensand—sand somewhat moist—the surface becomes harder; but if cast on an iron plate, or *chill*, some irons become as hard as the hardest steel, useful in surfaces exposed to heavy wear, as the treads of railway-wheels. Cast-iron, in general, is brittle under the blows of a hammer, but some mixtures, under a process of annealing, become *malleable iron*, used largely for steam-fittings, parts of agricultural machines, forms requiring the toughness of wrought-iron, but difficult to forge.

Wrought-iron is produced from cast-iron by removing the carbon and impurities by puddling, squeezing, heating, and rolling. As a material, it is sold in all sizes of wire, rods, shafts, bars, plates, shapes—girders and beams, chains and anchors. Its application industrially is well known. When hot, it can be welded, forged, drawn, and swaged into almost any required shape. Under the steam-hammer, the largest shafts, anchors, and cranks can be built, or by hand or by machinery it can be wrought into tacks, nuts, bolts, nails, or drawn into the finest wire.

For shafts of mills it is generally turned in a lathe and polished, but of late it can be bought, up to four inches diameter, *cold-rolled*, which adds very considerably to the strength, and is ready for use.

Bessemer and Siemens-Martin metals are made by burning out the carbon from a melted iron, and then reintroducing a known quantity, say from 0·03 to 0·6 per cent of carbon. There are other patents covering somewhat different irons, but the above are the best known. All are commonly classed as steel, but by many are called homogeneous metal;

first-class iron, of very uniform texture and great strength, but not equal to that of the best steel.

Steel is produced from pure wrought-iron by what is called cementation—heating the bars in contact with charcoal, by which a certain amount of carbon is taken up. The bars, when taken out, are covered with blisters, apparently from the expansion of minute bubbles within; hence called *blistered steel.* From this *shear-steel* can be produced by piling, heating, and hammering, or *cast-steel* from melting in a crucible.

Steel, when broken, does not show the fibrous character of wrought-iron. The fracture of shear-steel is fine, with a crystalline appearance. The fracture of cast-steel is very fine, requiring very close inspection to show the crystals or granulations; its appearance is that of a fine, light, slaty-gray tint, almost without luster. Steel is stronger than any of the other iron products, and especially applicable for the piston-rods of steam-engines, and positions requiring great strength and stiffness, with the minimum of space. But it is the way in which steel can be hardened and tempered which adapts it to its peculiar applications.

When the malleable metals are hammered or rolled, they generally increase in hardness, elasticity, and denseness, and some kinds of steel springs are made by the process of hammer-hardening; but the usual process of hardening and tempering is by heating the steel to a degree required by the use to which it is to be applied, and cooling it more or less suddenly by immersing in water or oil. The greater the difference between the heated steel and the cooling medium, the greater the hardness, but too much heat may burn the steel, and too sudden cooling make it too brittle. Steel, in tempering, is heated from 430° Fahr. to 630°. The temperature is shown by the color—from a pale yellow to deeper yellow, light purple to a dark purple, dark blue to a light blue, with a greenish tinge.

Steel is used for the edges of all cutting-tools, faces of hammers and anvils, and is generally welded to bodies of wrought-iron, but often composing the entire tool; for saws, springs, railway tires, pins, and can be bought in the form of wire, rods, bars, sheets, and plates, in varied forgings and castings.

All irons are very liable to rust, and must be protected where exposed to moisture. Polished surfaces are kept wiped and oiled, others painted, others galvanized or plated with some less oxidizable metal, generally tin, zinc, or nickel. Of late, a process has been introduced of coating them with black oxide, but is yet of no general application.

Antimony, *bismuth*, *copper*, *lead*, *tin*, *and zinc*, are used more or less industrially, and alloys of them are extremely useful. They may be hardened somewhat by the process of rolling and hammering, but can not be welded. Joinings are made by soldering or brazing or burning—that is, melting together.

Antimony expands by cooling. With tin, in equal proportions, it makes speculum-metal, and is used, with lead, to make type. Type metal makes a very good bearing for shafts and axles.

Bismuth is chiefly used as a constituent of fusible metal: 3 bismuth, 5 lead, and 3 tin, is an alloy which melts at 212°. Other mixtures are made, increasing the melting-point to adapt the metal for fusible plugs in boilers, or lowering the melting-point, so that, in case of fire in a building, a heat of say 140° melts the joint made by the metal, and lets water through sprinklers, to automatically put out the fire.

Copper is very malleable and ductile. In sheets, it is used for the cover of roofs, gutters, leaders, lining of bath-tubs, kettles, stills, and kitchen utensils. It is worked more easily than iron, and is stronger than lead or zinc, but it is much more costly than either of these metals, and its oxide is so poisonous that, without great care and cleaning, it can not be used to transmit or contain anything that may be used as food, without a cover of tin. It oxidizes slowly, and is used extensively for ships' fastenings and for bottom-sheathing. It is the most important element in all the brass and bronze alloys.

Brass, in common use, covers most of the copper alloys, no matter what the other components are, whether zinc, tin, or lead, or all three.

Copper and zinc will mix in almost any proportions. The ordinary range of good yellow brass is from 4½ to 9 ounces of zinc to the pound of copper. With more zinc it becomes more crystalline in its structure, but, as zinc is very much cheaper than copper, the founder is apt to increase the percentage of zinc, with the addition of a small percentage of lead. Muntz metal, in its best proportion, contains 10⅔ ounces of zinc to the pound of copper.

Copper and tin mix in almost any proportion. The composition of ancient bronzes is from 1 to 3 ounces of tin to the pound of copper. Ten parts of tin to 90 of copper is the usual mixture for field-pieces, and this is used in steam-engine work, often under the name of composition. Bell-metal is from 4 to 5 ounces of tin to the pound of copper; Babbit-metal, for journal-boxes, 90 of tin to 10 of copper.

Copper and lead mix in any proportion up to nearly one half lead, when they separate in cooling.

An addition of from one quarter to one half ounce of tin to the pound of yellow brass renders it sensibly harder. A quarter to one half ounce of lead makes it more malleable.

German-silver is 50 copper, 25 zinc, and 25 nickel.

Holzapfel gives the following alloys:

1½ ounce tin, ½ ounce zinc, to 16 ounces copper, for works requiring great tenacity.

1½ to 1¾ ounces tin, 2 ounces brass, to 16 ounces copper, for cut wheels.

2 ounces tin, 1½ ounce brass, to 16 ounces copper, for turning-work.

2½ ounces tin, 1½ ounce brass, to 16 ounces copper, for coarse-threaded nuts and bearings.

2½ ounces tin, 2½ ounces zinc, to 16 ounces copper, Sir F. Chantry's mixture, from which a razor was made, nearly as hard as tempered steel.

Professor R. H. Thurston, of Stevens Technological Institute, has tested various alloys of copper, tin, and zinc, and, by a graphic method, determines the best alloy for toughness as well as strength to be—copper 55, tin 2·5, zinc 44·5.

There are various other alloys, as phosphate bronze, aluminium bronze, Sterro-metal, of which the strength will be given hereafter in a table.

Lead is a very soft metal, that can be readily rolled into sheets and drawn into pipes, and is so flexible that it can be readily fitted in almost any position. It is, therefore, especially adapted to the use of plumbers, for the lining of cisterns and tanks, and for pipes for the conveyance of water and waste. For pipes for conveying pure water for drinking purposes, or for cisterns containing it, it is objectionable, as it oxidizes, and the oxide is a dangerous and a cumulative poison, but, in common waters which are more or less hard, the insides of the pipes become covered with a deposit which protects them. It is well, before drinking from a lead pipe in which the water has stood for a time, to draw off all the water, and, in lead-lined cisterns exposed more or less to the air, to protect them by a coating of asphalt varnish. Lead expands readily, and has so little tenacity that, in many positions, if heated, it has not strength in cooling to bring it back to its original position. It remains in wrinkles on roofs, and, for pipes conveying hot water, unless continuously supported, it will hang down in loops, continuously increasing under variations of temperature, to rupture. But it makes a very good plating for sheet-iron for roofs, and its oxides are the most valuable of all pigments.

Tin, in a pure state, is used for domestic utensils, as block-tin, and has also been used for pipes in the conveyance of water by parties who feared the poisonous qualities of lead pipe. But its chief use is for the covering of sheet-iron, which is sold under the name of tin or tin-plate, and is of universal application for architectural, industrial, and domestic purposes. Its oxide is not injurious, and it is so little affected by air and moisture that roofs, in many places, covered with it, need no painting, and oxidization takes place in the iron beneath only from deficiency in plating, or from the abrasion or breaks in it.

Zinc, in the pure form of spelter, is crystalline and brittle, but, at a temperature between 210° and 300°, it is so ductile and malleable that it can be readily rolled into sheets, and of late has been used as a cheap substitute for sheet-copper; but, under considerable variations of temperature, as for lining of bath-tubs, it takes permanent wrinkles, and, for coverings of roofs, suitable provision must be made for its expansion. But as a plating of iron, under the name of galvanizing, it affords an admirable protection, cheaply, and extends the use of iron in sheets, bolts, and castings, where it would not otherwise be applicable. Zinc, as a pigment, does not discolor, like lead, under the action of sulphureted hydrogen, but is objected to by painters for its want of body or cover.

METALS.

METALS AND ALLOYS.	Specific gravity.	Weight per c. ft.	Melting-point.	Resistance in pounds per square inch.	
				To crushing.	To tension.
			° Fahr.		
Aluminum-bronze	.....	....	....	....	73,000–96,000
Antimony, cast	4·500	280	932	....	1,060
Bismuth	9·900	617	476	....	3,250
Brass	8·500	530	1,873	50,000–160,000	18,000
Copper	8·726	537	4,587	117,000	22,000
Gold	19·238	1,200	5,237	....	....
Iron	7·20	450	18,000	82,000–145,000	13,000–25,000
Lead	11·479	716	594	7,000	1,800
Phosphor-bronze	.....	....	....	....	22,000–50,000
Platinum, cast	21·500	1,340	3,080	....	55,000
Silver, "	10·480	654	3,677	....	40,000
Steel, "	7·800	486	....	125,000–295,000	85,000–145,000
Tin, "	7·250	450	442	15,500	4,600
Zinc, "	7·215	450	700	....	2,500
Iron, forged	7·77	485	2,822	40,000– 65,000	40,000– 60,000
Steel, "	7·85	490	2,462	100,000–180,000	70,000–120,000
Iron wire (unannealed)	....	....	....	....	50,000–100,000
Steel wire "	....	....	....	....	120,000–200,000
Sterro-metal	7	....	....	....	50,000– 85,000

Fig. 328 is an admirable illustration of the graphic representation of facts adopted by Professor R. H. Thurston of exhibiting the results of his tests on the strength of alloys—which not only exhibits the results, but enables others to judge the probable strength of other mixtures. The apices of the triangle marked copper, tin, and zinc, represent the points of pure metal, 100 per cent. The lines opposite the apex of any metal represent the 0 of such metal—thus the base opposite copper represents an alloy of tin and zinc only, without any copper, and every line drawn above this base, and parallel to it, will contain a percentage of copper increasing by regular scale, from the base to the apex, and so with lines opposite tin and zinc ; the first contains only copper and zinc, the latter tin and copper, and the percentages of tin and zinc increase with the distance from their opposite lines to their vertices. It will be seen that the intersections of these percentage parallels define the percentages of each metal, their sum always making 100 per cent. If, then, the strength of such alloy, as obtained by test, be supposed to represent an ordinate or elevation, on any convenient scale, and be represented by this height at its opposite intersection of percentage, a contour map, as in the figure, may be formed—which the professor has not only done, but made a model from it. The summit, 65,000 on the figure, represents the position of the strongest alloy found: if through the scales marked copper on each side, we find the parallel to the base, which passes through this summit, it will be found to be about 55, that is, 55 per cent copper. In like manner, the parallel to the *o* zinc base, intersecting this summit, will be about 43 per cent zinc; and, in the same way, tin is 2 per cent. If we

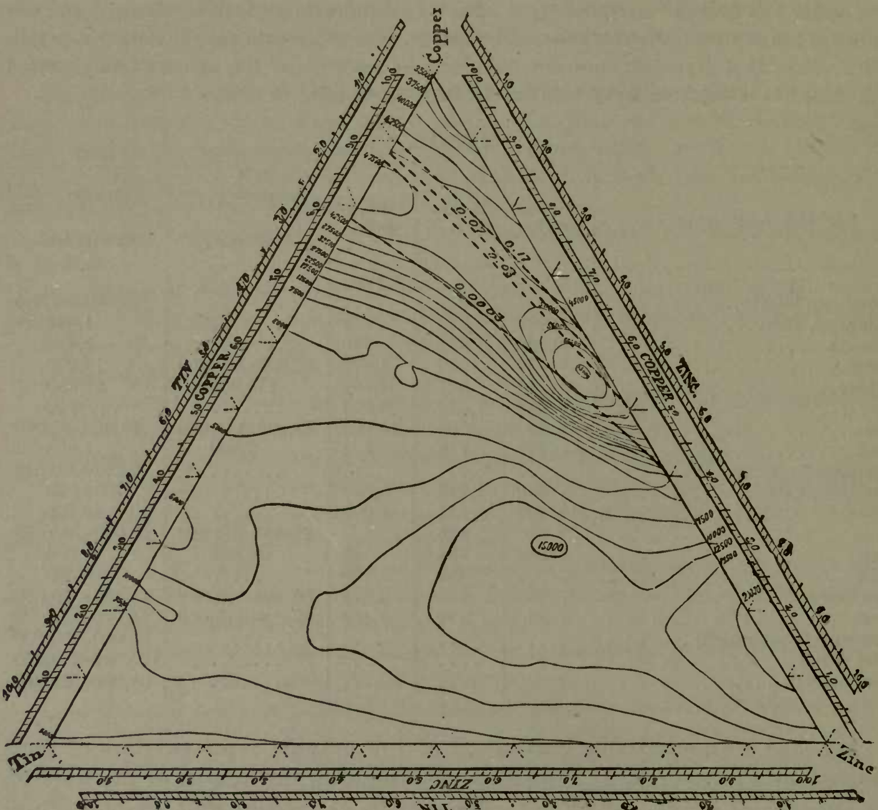


FIG. 328.

wish to find the probable strength of any mixture, it is only necessary to find the contour intersected by the triple parallels representing the percentages which we are investigating. It is said *probable* strength, because the care and manipulation of the founder are such important factors in the result.

Sulphur, when used in sufficiently large masses as to show on a drawing, may be represented by a reddish-yellow tint, or some distinctive hatching. It melts at 248° Fahr., and, from its fluidity, answers admirably for the filling of joints between stones, beneath the balls of iron columns, between wood and stone, and around anchor-bolts in stone, forming, when cold, a strong, uniform bearing, and adapting itself to the roughness of the material, and is detached with difficulty. It is used largely for the bases of engines, and for the joints of the cap-stones of dams. On the dam across the Mohawk, at Cohoes, many tons were used in these joints, the depth of sulphur being about 6 inches, and now, after seventeen years' use, but few of the joints are little worn, and there has been no injurious effect from the sulphur on the limestone, of which the apron or capping is composed. It is better for most of the above purposes than lead, being cheaper, more fluid when molten, shrinks less in cooling, is less affected by temperature, and its crushing strength is adequate to any of the positions of use above, but it is brittle under blows. It sometimes rusts the bolts or iron with which it is brought in contact, but this is prevented by an addition of about 20 per cent of coal tar. This mixture is used as a cement to fasten lights in illuminated tile and vault covers.

When heated to about 300°, sulphur begins to grow viscid, and at 428° it has the consistency of thick molasses. Above this, it begins to grow thin again. Heated to 518°, and thrown into cold water, it becomes for a time plastic, and is used for taking molds or casts.

Sulphur, in powder, mixed in proportions of one sal-ammoniac, two sulphur, and fifty of iron-filings, makes a mastic which is used for calking the joints of iron pipes, especially gas-pipes. The joint is called a rust-joint.

Glass, in drawing, is represented by a bluish tint or by different shades or hatchings, expressive of the effect of light upon it, whether the light is reflected or transmitted.

Fig. 329 represents a portion of a mirror when the light is reflected. The exterior of windows is often represented in the same way, but with deeper shades, and often with a piece of curtain behind in white with dim outline. A window viewed from inside is represented in shades less than in the figure, or as transparent, which is conveyed by the dimness of outline of figures or skies seen beyond.

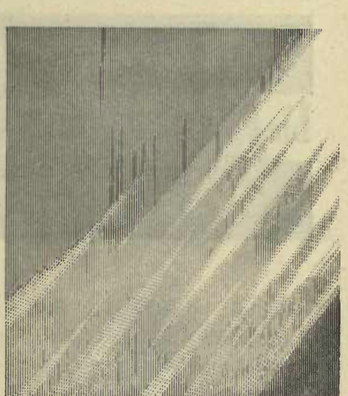


Fig. 329.

Fig. 330 represents a glass flask.

Fig. 331 represents a glass box with glass sides.

Fig. 332 represents a glass jar containing fluids of different densities.

Figs. 333 and 334 represent spars, which may be taken for any transparent substances, as glass, ice, and the like.

Fig. 330.

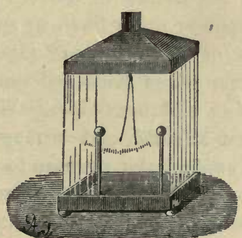


Fig. 331.

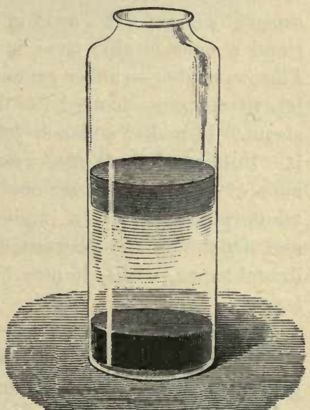


Fig. 332.

Common window-glass is blown in the form of cylinders (hence called cylinder-glass), flatted out, and cut in lights of varying dimensions, from 6 × 8 up to 30 × 30 inches, and put up in boxes containing about fifty square feet. It is classed as single-thick (about $\frac{1}{16}$ inch) and double-thick ($\frac{1}{8}$ inch). When the squares are large, or used for sky-lights, they should be the latter. Plate-glass—polished plate is used for windows of stores and first-class buildings. It can be got of almost any dimensions, and of a thickness from $\frac{3}{16}$ to $\frac{3}{4}$ of an inch. Rough plate is largely used for floor-lights and sky-lights. It is cut to required sizes, and of a thickness from $\frac{3}{8}$ to one inch.

Single thick cylinder-glass cuts off from about 8 to 15 per cent of the light.

Double-cylinder, from 12 to 20 per cent of the light.

Polished plate, three sixteenth inch thick, from 5 to 7 per cent of the light.

Rough plate, one half inch thick, from 20 to 30 per cent of the light.

Rough plate, one inch thick, from 30 to 40 per cent of the light.

This is when the glass is clean; but there is always a film of moisture on its surface, which attracts dust, and impairs very much the transmitted light. Rough plate more readily retains the dirt, and, when it is used as floor-lights, becomes scratched. It is therefore usual, in the better class of buildings, to use a cast white glass, set in iron frames. In outer, or platform lights, these lights are in the form of lenses, set in cast-iron frames with an asphalt putty, or resting on iron frames and imbedded in Portland cement.

FIG. 333.

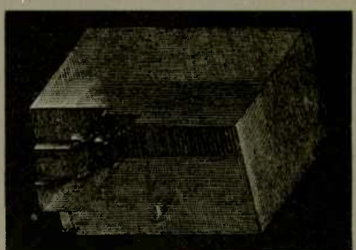


FIG. 334.

Rubber, mixed and ground with sulphur, subjected to heat, becomes *vulcanized*, and is not affected by moderate variations in temperature. Soft rubber, most extensively used for industrial purposes, is subjected to a heat of from 265° to 300°, and for a time can withstand a temperature a little below this without losing its elasticity; after a time it will harden. Soft rubber is classed as pure rubber, and fibrous rubber, or rubber with cloth. Pure rubber contains about fifty per cent of rubber and fifty per cent of compound, white lead and sulphur. It is used for the buffers and springs of railway-carriages, and for the faces of valves and seats of water-pumps, but it is not well suited for the pumping of hot water, especially above 212°, as it is liable to lose its elasticity; and, although some valves may stand a considerable time, it is almost impossible to secure uniformity in the rubber. Fibrous rubber—rubber ground with cotton or other fiber, or spread on cloth, on more or less thicknesses—is used for the packing of faced joints of pipes and gaskets for water or steam. It makes a stanch joint, and, even when hardened under heat, it still preserves it. Rubber cloth is also used for belting and hose-pipes. When used for the conveyance of steam, the inner coat is the first affected, and it may be some time before the whole pipe suffers. In buying rubber, explain the purpose to which it is to be applied, and depend on the guarantee of the vender. Rubber is often to be designated by the draughtsman, which it may be by a bluish-black tint, or by lines across it parallel to its length.

Paints are used for a twofold purpose—for covering and preserving the material to which they are applied, and for ornamentation. The best and the most general is white-lead ground with linseed-oil, either used by itself or mixed with various other pigments, as ochre, chrome, lamp-black, etc. It is often adulterated with barytes. For the covering of iron, or for the packing of close joints in it, nothing is better than pure red-lead, but many of the oxides of iron, red or yellow, form good covers of iron, and, as cheap and good paints, are used on tin roofs. All the leads and pigments are ground in oil: if the oil is raw, it dries slowly; driers, as litharge, are added to hurry the process, but, with boiled oil, no drier is necessary. Almost any inert substance, as cement, chalk, or sand, if fine enough, can be ground with oil for a paint, and make a good cover, and for these fish-oil will answer. The general specification for painting is " paint with — good coats of white-lead, of such color as may be directed." The priming-coat of new wood-work requires more oil than paint. For the next coats, one-half pound of paint to the square yard would be considered a good coat. If the paint is on old work, or that which has been already painted, there will be a little less lead required. Wood should be fairly dry before the application of paint, so that it may properly adhere and not inclose moisture that may rot the wood. The knots should be *killed*, that is, covered with shellac varnish or similar preparation, to prevent the exuding of the resin. The heads of nails should be sunk, and the holes and cracks filled with putty, and the surface of the wood smoothed.

Coals and other minerals are represented like rocks or stones, in varied shades of tones and colors. Fig. 334*a* represents the fire-box of a locomotive, with coal in the state of ignition in its usual type. In color, flame is represented in streaks of red-yellow, with dark tints for smoke. Water occupies the lower half of the boiler: but, as steam under

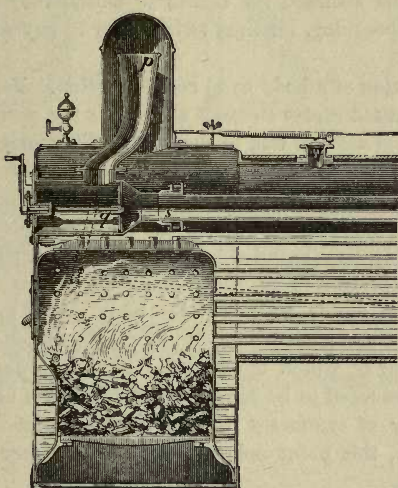


Fig. 334*a*.

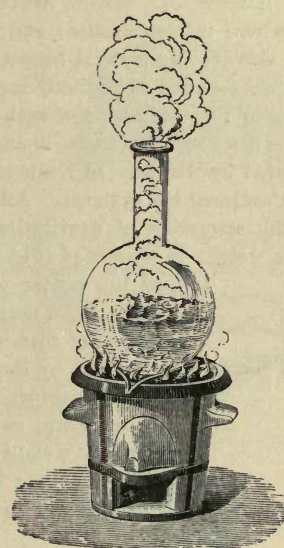


Fig. 334*b*.

pressure is invisible like gas, the space occupied by it is shown as empty. If the direction of its movement is desired, it is indicated by arrows. Steam issuing into atmosphere, or boiling in an open kettle, has the appearance of a very light smoke or cloud (Fig. 334*b*).

There are many substances used in such masses in construction, or to be shown in the processes of manufacture, that must be graphically represented by the draughtsman by a general imitation of their natural appearance, or conventionally with explanatory marginal blocks and legends.

MECHANICS.

The draughtsman, in designing a structure, should be conversant not only with the nature of the material, but also with the forces to which it is to be subjected—their magnitude, direction, and points of application, and their effects; that is, he should know the first principles of mechanics, the science of rest, motion, and force—to wit, Statics, Dynamics, and Kinematics. Statics treats of balanced forces, or rest; dynamics, of unbalanced forces, where motion ensues; and kinematics, of the comparison of motions with each other. Considering statical forces simply in the abstract, the bodies to which they are applied are assumed as perfectly rigid, without breaking, binding, twisting, or in any wise changing by the application of such forces.

Force is a cause tending to change the condition of a body as to rest or motion. Force is measured by weight. In England and the United States the unit of force is the pound, on the Continent the gramme. All bodies fall, or tend to fall, to the earth. This force is called the attraction of gravitation. Its direction is shown by that of a string from which a weight is suspended (Fig. 335). It is called a vertical line, and its direction is toward the center of the earth. Practically, these lines are considered parallels. Let a mass, P (Fig. 336), be suspended by a cord. Each particle is acted on by *gravity*, and the *resultant* of all these parallel forces is the force resisted by the cord, or the entire weight of the body. If a mass (Fig. 337) be suspended from two different points, P and Q, the directions of the string will meet at a point C, which is called the *center of gravity*, where all the weight may be considered to be concentrated. When a body of uniform density has a center of symmetry (a point which bisects all straight lines drawn through it), this point coincides with the center of gravity, as the middle of a straight line, the center of a circle, the intersection of the diagonals of a parallelogram, the intersection of lines drawn from any two angles of a triangle to the centers of the opposite sides; in solids, the center of a sphere, the middle point of the axis of a cylinder, and the intersection of the diagonals of a parallelopiped.

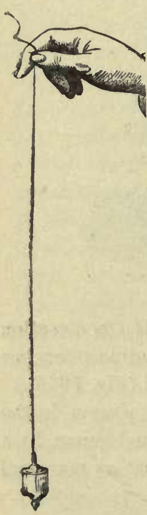


FIG. 335.

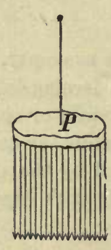


FIG. 336.

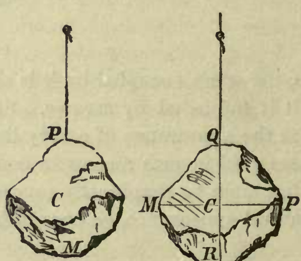


FIG. 337.

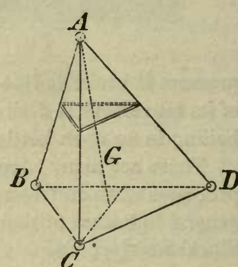


FIG. 338.

The center of gravity of the triangular pyramid, Fig. 338, is in the straight line A E, connecting the apex A with the center of gravity of the base triangle B C D, and distant $\frac{1}{4}$ of the length of the line A E from E.

The center of gravity of solids, which may be divided into symmetrical figures and pyramids, as for all practical purposes most may be, can be found by determining the center of gravity of each of the solids of which it is compounded, and then compounding them, observing that each center of gravity represents the solid contents of its own mass or masses of which it may be composed. The center of gravity of bodies enclosed by more or less regular contours, as a ship for instance, is determined by dividing it into parallel and equidistant sections, finding the center of gravity of each, and compounding them into a single one.

The center of gravity of a body may be determined practically, as shown above, by its suspension from different points. It can be done generally more readily by balancing the body in horizontal positions on different lines of support; the center of gravity will lie in the intersection of planes perpendicular to these lines. A body placed in a horizontal position will fall over, unless the vertical line from the center of gravity falls within the

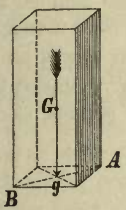


FIG. 339.

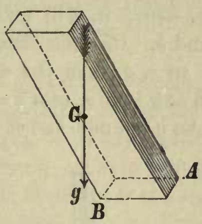


FIG. 340.

FIG. 341.

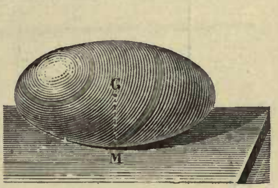


FIG. 342.

base of support; as Fig. 339 will stand, while Fig. 340 will fall over. A person carrying a weight insensibly throws a portion of the body forward, backward, or laterally, to balance the load. Thus, in Fig. 341, the body is thrown back, so that the vertical from the center of gravity *g*, compounded of the center of gravity G of the woman and of the load H, falls within the base of the feet.

When a figure rests in such a position that its center of gravity is in its lowest position, it is said to be in *stable* equilibrium. It may, like a ball, rest in any position, as the center of gravity is neither depressed nor raised by movement; but, in the ellipsoidal form (Fig.


FIG. 343.

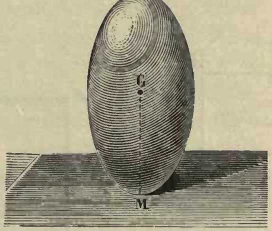


FIG. 344.

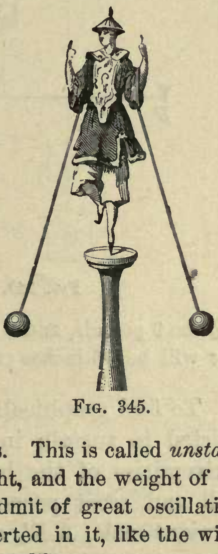


FIG. 345.

342) or in the toy (Fig. 343), any movement tends to raise the center of gravity, and, on the cessation of the force, the body returns to its original position. The ellipsoidal form (Fig. 344), placed on its pointed end, is balanced, but the slightest movement lowers the center of gravity, and, without the application of an outside force, it can not be raised, and therefore falls. This is called *unstable* equilibrium. In the toy (Fig. 345), the body of the figure is light, and the weight of the balls brings the center below the point of support. This will admit of great oscillation, and return to its original position. A cork with two forks inserted in it, like the wires of the balls, and resting on the top of a glass, will illustrate this readily.

When two parallel forces, F F′, are applied at the extremities of a straight line (Fig. 346), they have a resultant, R, equal to their sum, and acting at a point, C, which divides the line inversely proportional to the forces. If the forces are equal, the point C will be at the center of the line; if the force F is double that of F′, C A will be equal to one half C B. This is called the principle of the *lever*.

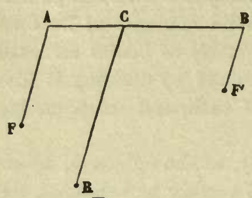


Fig. 346.

Levers, in practice, are called of the first (Fig. 347), second (Fig. 348), and third class (Fig. 349), according to the position, weight, W, power applied, P, and *fulcrum*, support or turning-point, C, of the lever. They are all forces, and only vary in name. The two extreme forces must always act in the same direction; the middle one must act in the opposite direction, and be equal to the sum of the other two; and the magnitude of the extreme forces be inversely proportional to their distances from the middle one. Let the middle force C be measured by a spring-balance (Fig. 350); it will mark the sum of the weights *a* and *b*. Call the distance from *a* to *c*, *x*, and from *b* to *c*, *y*, then the weight *a* will be to the weight *b* as *y* is to *x*, or $a\,x = b\,y$. Suppose the weight *a* to be 6 pounds and at *b* 3 pounds, at *c* it will be 9 pounds, and *a c* or *x* will be to *b c* or *y* as 6 to 3, or, if the lever is 48 inches, *b c* will be 16 inches and *a c* 32 inches.

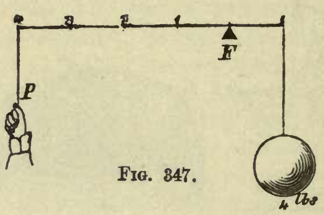


Fig. 347.

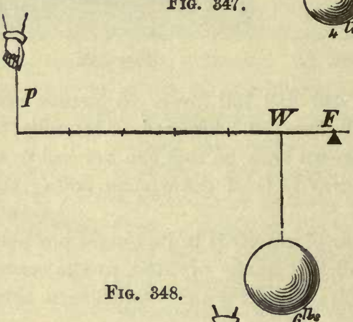


Fig. 348.

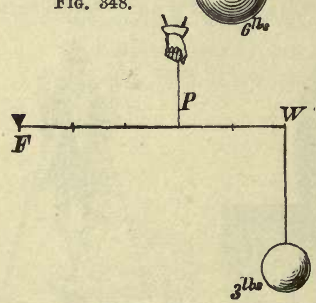


Fig. 349.

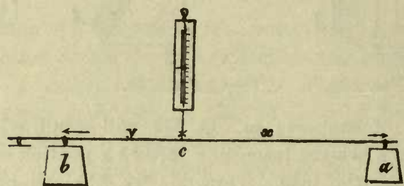


Fig. 350.

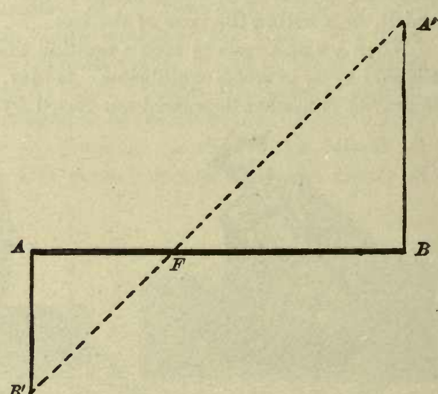


Fig. 351.

To find graphically the fulcrum, or point, at which a lever should be supported to sustain in equilibrium weights, or equivalent forces, acting at the extremities of the lever. Let A B (Fig. 351) be the lever. At A and B let fall and erect perpendiculars to the lever. Lay off from A, on any convenient scale, A B′, corresponding to the weight applied at B; and at B, on the same scale, B A′, the weight applied at A; draw the line A′ B′; its intersection, F, with the lever will be the position of the fulcrum. This is on the hypothesis that there is no weight to the lever, or that, after determining the position of the fulcrum, the lever itself is balanced on the point by the addition of weight on the short arm F A, or the reduction of weight on the long one F B. If the lever is of uniform weight, on perpendiculars to C, the center of the lever (Fig. 352), and to F, the fulcrum, as before determined, lay off F C′, the weight of the lever, and C F′, the sum of the weights applied at A and B; draw C′ F′. Its intersection, F″, will be the actual fulcrum, taking into consideration the weight of the lever in addition to the weights suspended at the extremities.

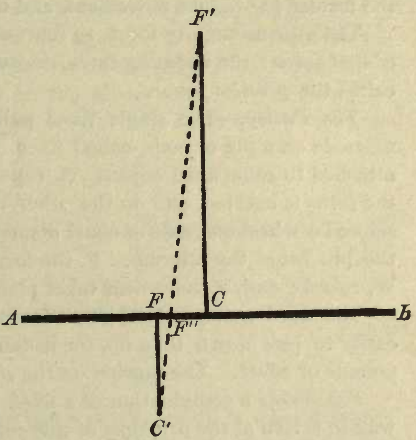


Fig. 352.

The Wheel and Axle.—If a weight, P, be suspended from the periphery of a wheel (Fig. 353), while another weight, W, is suspended on the opposite side of a barrel or axle attached to the wheel, the principle of action is the same as that of the lever. P multiplied by its length of lever or radius *ca* of the wheel is equal to W multiplied by its length of lever or radius of the axle *cb*; the axis *c* is the fulcrum. If a movement downward be communicated to P, as shown by the dotted line, a rotary motion is given to the wheel and axle; the cord of P is unwound while that of W is wound up, but P is still suspended from *a* and W from *b*; the leverage, or distance from the fulcrum, of each is the same as at first. The wheel and axle is a lever of continuous and uniform action. Since the wheel has a larger circumference than the axle, by their revolution more cord will be unwound from the former than is wound up on the latter, P will descend faster than W is raised, in the proportion of the circumference of the wheel to that of the axle, or of their radii *c a* to *c b*. When P has reached the position P′, W will have reached W′. If *c a* be four times *c b*, then P will have moved four times the distance that W has. The movement is directly as the length of the levers, or the radii of the points of suspension. It will be perceived, therefore, to move a large weight by the means of a smaller one, that the smaller must move through the most space, and that the spaces described are as the opposite ends of the lever, or inversely as the weights.

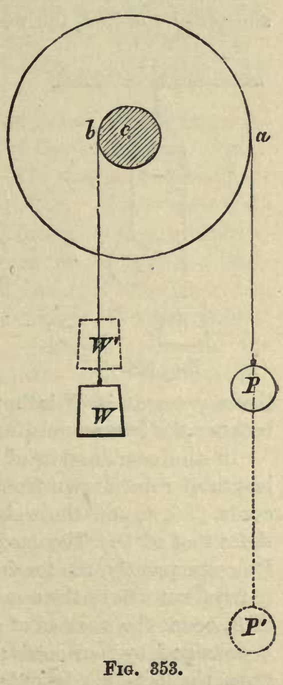


Fig. 353.

It is the fundamental principle of the action of all mechanical powers, that whatever is "gained in power," as it is said, is lost in space traveled; that, if a weight is to be raised a certain number of feet, the force exerted to do this must always be equal to the product of the weight by the height to which it is to be raised; thus, if 200 pounds are to be raised 50 feet, the force exerted to do this must be equal to a weight, which, if multiplied by its fall, will be equal to the product 200 × 50, or 10,000; and it is immaterial whether the force be a weight of 10,000 pounds falling 1 foot, or 1 pound 10,000 feet.

It is now common to refer all forces exerted to a unit of *pounds-feet*, that is, 1 pound falling 1 foot; and the effect to the same unit of pounds-feet, 1 pound raised 1 foot. Thus, in the example above, the force exerted or power is 10,000 pounds-feet falling; the effect 10,000 pounds-feet raised. In practice, the pounds-feet of force exerted must always be more than the pounds-feet of effect produced; that is, there must be some excess of the former to produce movement, and to overcome resistance and friction of parts.

The measure of any force, as represented by falling weight, is termed the *absolute power* of that force; the resulting force, or useful effect for the purposes for which it is applied, is called the *effective power.*

The Pulley.—The single fixed pulley (Fig. 354) consists of a single grooved wheel movable on a pin or axis, called fixed, because the strap through which the pin passes is attached to some fixed object. A rope passes over the wheel in the groove; on one side the force is exerted, and on the other the weight is attached and raised. It may be considered a wheel and axle of equal diameters, or as a lever in which the two sides are equal, the pin being the fulcrum. P, the force exerted, must therefore be equal to the weight W, raised; and, if movement takes place, W will rise as much as P descends.

The fixed pulley is used for its convenience in the application of the force; it may be easier to pull down than up, for instance; but the pounds of force must be equal to the pounds of effect. The tension on the rope is equal to either the force or weight.

Fig. 355 is a combination of a fixed pulley, A, and a movable pulley, B. The simplest way to arrive at the principle of this combination is to consider its action. Let P be pulled down, say two feet; the length of rope drawn to this side of the pulley must be furnished from the opposite side. On that side there is a loop, in which the movable pulley, with the weight W attached, is suspended. Each side of this loop, 2 and 3, must go to make up the two feet for the side or end 1. Cords 2 and 3 will therefore furnish each one foot. As these cords are shortened one foot, the weight W is raised one foot, and, as

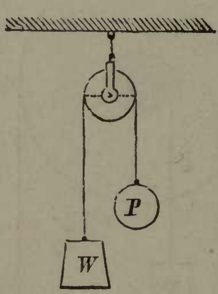


Fig. 354.

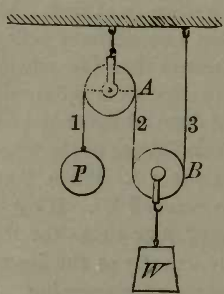


Fig. 355.

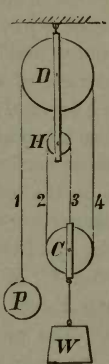


Fig. 356.

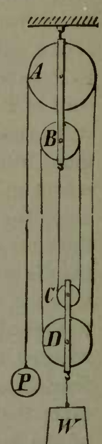


Fig. 357.

the movement of W is but one foot for the two feet of P, W must be twice that of P, because the two pounds-feet of P must equal the pounds-feet of W.

In the combination of pulleys (Fig. 356), let P be pulled, say three feet; then this length of rope, drawn from the opposite side of the pulley, is distributed over the three cords, 2, 3, 4, and the weight W is raised one foot; consequently, the weight W is three times that of P. The cord 1 supports P, the cords 2, 3, 4, the weight W, or three times P; consequently, the tension on every cord is alike. The same rope passing freely around pulleys must have the same tension throughout; so that, to determine the relation of W to P, count the number of cords which sustain the weight. Thus, in Fig. 357, the weight is sustained by four cords; consequently, it is four times the tension of the cord, or four times the force P. In order not to confuse the cords, the pulleys are represented as in the figures; but, in construction, the pulleys, or sheaves, are usually of the same diameter, and those in connection, as A and B, and C and D, run on the same pin.

The Inclined Plane.—To support a weight by means of a single fixed pulley, the force must be equal to the weight. Suppose the weight, instead of hanging freely, to rest upon an inclined plane *b d* (Fig. 358); if motion ensue, to raise the weight W the height *a b*, the rope transferred from the weight side of the pulley will be equal to *b d*, and P will have, consequently, fallen this amount; thus, if *b d* be six feet, and *a b* one foot, while W is raised one foot, P has descended six feet, and, as pounds-feet of power must equal pounds-feet of effect, P will be one sixth of W; and, by reference to the figure, P is to W as *a b* is to *b d*, or as the height of the incline is to its length. If the end of the plane *d* be raised, till it becomes horizontal, the whole weight would rest on the plane, and no force would be necessary at P to keep it in position; if the plane be revolved on *b*, till it becomes per-

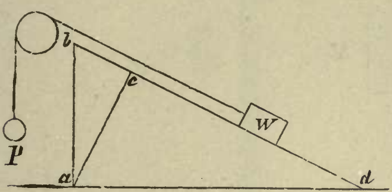


Fig. 358.

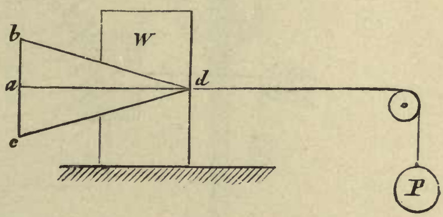


Fig. 359.

pendicular, then the weight is not supported by the plane at all, but it is wholly dependent on the force P, and is equal to it. Between the limits, therefore, of a level and a perpendicular plane, to support a given weight W, the force P varies from nothing to an equality with the weight.

The construction (Fig. 359) illustrates the principle of the *wedge*, which is but a movable inclined plane; if the wedge be drawn forward by the weight P, and the weight W be kept from sliding laterally, the fall of P a distance equal to *a d* will raise the weight W a height *c b*. P will therefore be to W as *c b* is to *a d*. For example, if the length of the wedge *a d* be ten feet, and the back *c b* two feet, then P will be to W as two to ten, or one fifth of it.

Let the inclined plane *a b d* (Fig. 359) be bent round, and attached to the drum A (Fig. 360), to which motion of revolution on its axis is given, by the unwinding of the turns of a cord from around its periphery, through the action of a weight P suspended from a cord passing over a pulley. If the weight W be retained in its vertical position, by the revolution of the drum, it will be forced up the incline, and when the cord has unwound one half turn from the drum, and consequently the weight P descended a distance, *c e*, equal to one half the circumference of the drum, the weight W has been raised to the height *a b* by the half revolution of the plane; P must therefore be to W as *a b* is to one half the circumference. Extend the inclined plane so as to encircle the drum (Fig. 361). The figure illustrates the mechanism of the screw, which may be considered as formed by wrapping a fillet-band or thread around a cylinder at a uniform inclination to the axis. In practice, the screw or nut, as the case may be, is moved by means of a force applied at the extremity of a lever, a complete revolution raises the

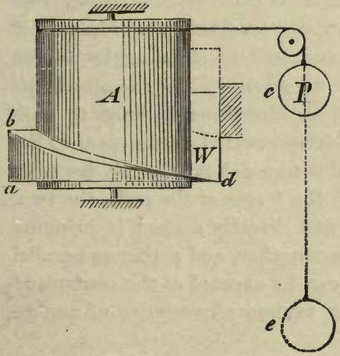


Fig. 360.

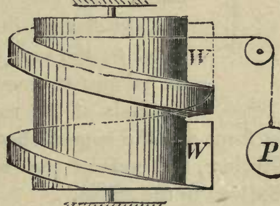


Fig. 361.

weight the distance from the top of one thread to the top of the one above, or *the pitch.* If the force be always exerted at right angles to the lever (Fig. 362), the lever may be considered the radius of a wheel, at the circumference of which the force is applied. Thus, if the lever be three feet long, the diameter of the circle would be six feet, and the cir-

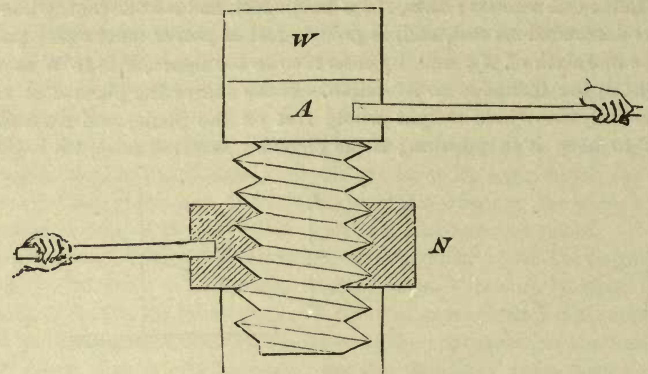


Fig. 362.

cumference 6 × 3·1416, or $18\frac{85}{100}$ feet; if the pitch be one inch, or one twelfth of a foot, then the force would be to the weight as one twelfth is to 18·85; and if the force be one pound, the weight would be 226·20 pounds.

The resultant of two forces of exertion, as has been seen, is their sum, and counterbalances the force of resistance, which must be applied at a point intermediate between, and distant from each of them, inversely as the forces exerted.

The resultant of any number of parallel forces acting in one direction is equal to their sum acting in the same direction at some intermediate point; that is, the effect of all these forces is just the same as if there were but one force, equal to their sum, acting at this point, and is balanced by an equal force acting in the opposite direction. This central point may be determined by finding the resultant, i. e., the sum, and the point of application for any two of the forces, as shown graphically in Figs. 351, 352, and then of other two, the resultants thus determined being again added together like simple forces.

Inclined Forces are those whose directions are inclined to each other. When two men of equal strength pull directly opposite to each other, the resultant is nothing. Let a third take hold of the center of the rope (Fig. 363), and pull at right angles to the rope; he will make an angle in the rope, and the other two now pull in directions inclined to each other. The less the force exerted at the center, the less the flexure in the rope; but when it becomes equal to the sum of the forces at the ends, the two, to balance it, must pull directly against it, bringing the ends of the rope together, and acting as parallel forces. Between the smallest force and the largest that can be exerted at the center and maintain a balance or equilibrium, the ends of the rope assume all varieties of angles, which angles bear definite relations to the forces.

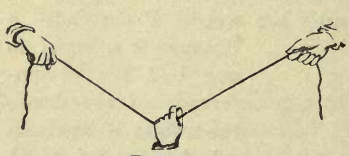


Fig. 363.

Represent these forces by weights (Fig. 364). Let P and P′ be the extreme forces acting over the pulleys M and N, and tending to draw the rope straight, which the weight P″ prevents. Lay off the weight of P (90 pounds) along A B, and the weight of P′ (60 pounds) along A C. Draw Bn parallel to A C, and Cn parallel to A B. Connect n with A. If this is measured with the same scale that A B and A C were laid off with, it will be found that it equals 120 pounds, which will be found to be the same as the weight P″. An, therefore, gives the amount and direction of the resultant of the two forces P and P′, which resultant is balanced by P″. In the same way the resultant of any number of inclined

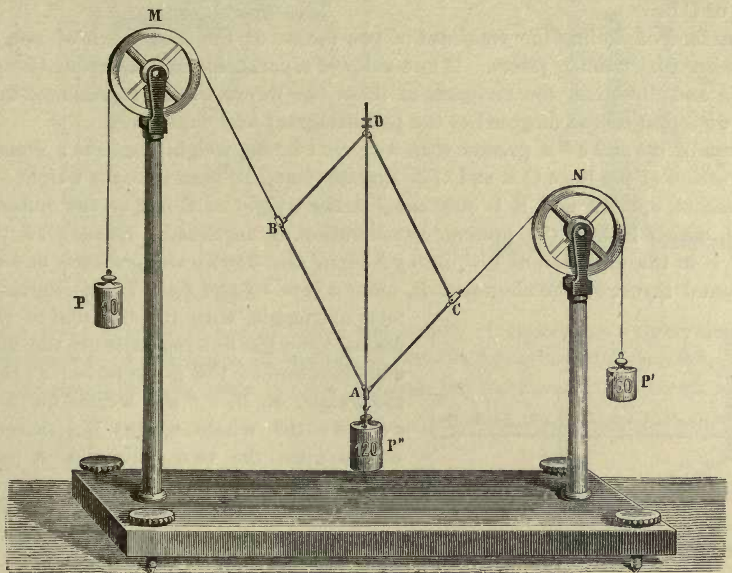


Fig. 364.

forces (Fig. 365) may be found by compounding the resultant of any two forces with a third, and so on.

As two forces may be compounded into a single resultant, so conversely one force may be resolved into two components; thus, let the weight P (Fig. 366) be supported by two inclined rafters, C A and C B. Each resists a part of the force exerted by the weight P. To find the force exerted against the abutments A and B, in the direction of C A and C B, draw *c* A′ (Fig. 367) parallel to C A, *c* B′ to C B, and *c d*, a parallel to the line C P, the direction in which the weight P acts; lay off *c d* from a scale of equal parts, a length which

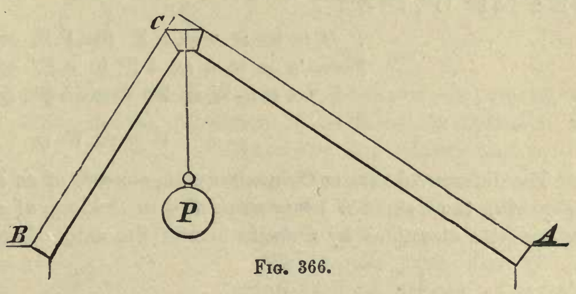


Fig. 366.

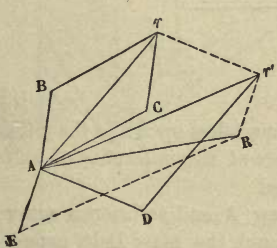


Fig. 365.

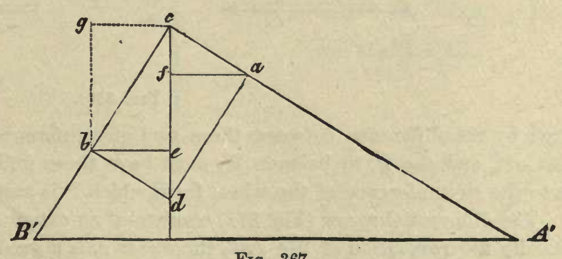


Fig. 367.

will represent the number of pounds, or whatever unit of weight there may be in the weight P; draw *d a* parallel to *c* B′, and *d b* parallel to *c* A′; *c a*, measured on the scale of equal parts adopted, will represent the pounds or units of weight exerted against A in the direction of C A, and *c b* the pounds or units of weight exerted against B in the direction of C B.

This method of finding the *resultant* of two forces, or the *components* of one force, is called the *parallelogram of forces.* If two sides of a parallelogram represent two forces in magnitude and direction, the resultant of these two forces will be represented in magnitude and direction by the diagonal of the parallelogram and conversely.

The sum of *a c* and *c b* is greater than *c d*; that is, the weight P exerts a greater force in the direction of the lines C A and C B, against A and B, than its own weight; but the down pressure upon A and B is only equal to the weight of P and of the rafters which support it, which last, in the present consideration, is neglected. Resolve *c b*, the force acting on B in the direction of *c* B′, into *g b* or *c e* the downward pressure, and *c g* or *e b* the horizontal thrust on the abutment B, and *c a* into *c f* and *f a*. To decompose a force, form a triangle, with the direction of the other forces, upon the line representing the magnitude and direction of the given force; *c e* represents the weight on B, *c f* the weight on A; *c d*, or *c e* + *d e*, the whole weight P; therefore, the weight upon the two abutments A and B is equal to the whole weight of P.

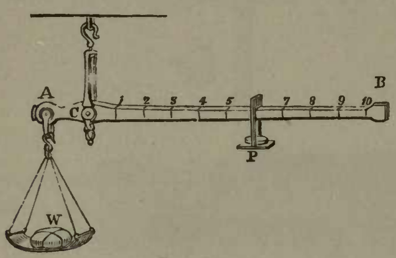


Fig. 368.

The steelyard (Fig. 368) is a lever, from the short arm of which a dependent hook or scale supports the article to be weighed; while, on the long arm, a fixed weight, P, is slid in or out from the fulcrum till it balances the article; the distance as marked on a scale on the long arm determines the weight. In platform-scales, when very heavy weights are balanced by small weights on a graduated arm, combinations of levers are used, the principle of which can be understood from Fig. 369. Thus, suppose P F to be 7″, *a* F 2″, *a* F′ 9″, *b* F′ 2″, *b* F″ 11″, F″ W 3″.

P is to force *a* as *a* F to P F,	or as	2 to 7
Force *a* is to *b* as *b* F′ to *a* F,	or as	2 to 9
b is to W as F″ W to *b* F″,	or as	3 to 11
P is to W as		12 to 693

The differential axle, or Chinese capstan, consists of an axle with two different diameters (Fig. 370), the weight W being suspended in the loop of a cord wound around these axles in opposite directions by a single turn of the axle. The weight is only raised or low-

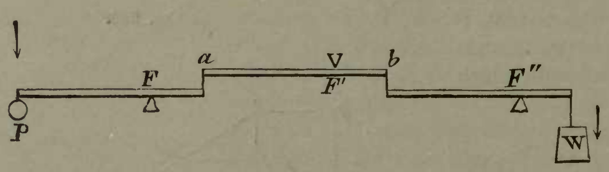


Fig. 369.

ered by the difference between these two circumferences; one takes up while the other lets out, and the P, to balance W, must be as these differences of circumference of axles is to the circumference of the wheel from which P is suspended.

The differential screw (Fig. 371) consists of an exterior screw, A, and an interior screw, B. By the revolution of the arm, the screw A is moved through the plate D in proportion to its pitch, but the interior screw B moves inward its pitch, and the movement of W is only the pitch of A less that of B, and the power applied is to the weight moved as the difference of these pitches is to the circumference described by the power.

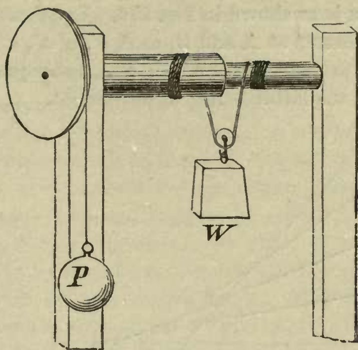


Fig. 370.

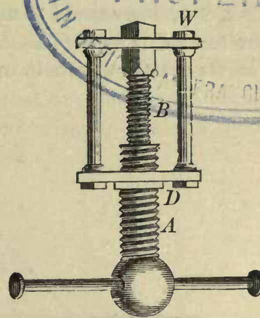


Fig. 371.

As the lever (Fig. 372) moves under the action of power or weight, the lever becomes inclined to the direction of the forces, but the forces are still parallel. The relations of the forces to each other are not changed, but the absolute action of each is only

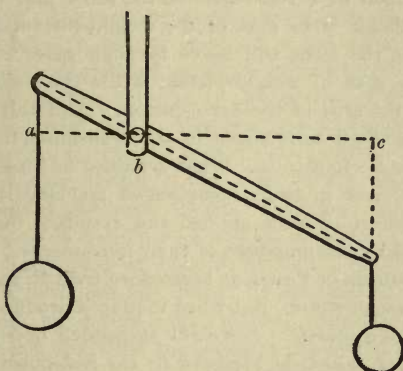


Fig. 372.

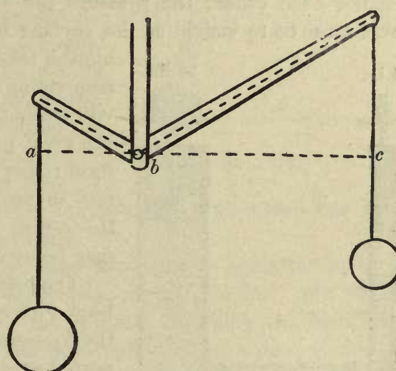


Fig. 373.

that due to the length *a b* and *b c*, to which the directions of the forces are perpendicular. In the bent levers (Figs. 373 and 374) the action of the forces is estimated on lengths of arms, determined by the perpendiculars *a b* and *b c* let fall from the fulcrum on the directions of the forces.

The toggle-joint (Fig. 375) is much used for presses. The force is exerted in the direction of the arrow at C, and the effective force

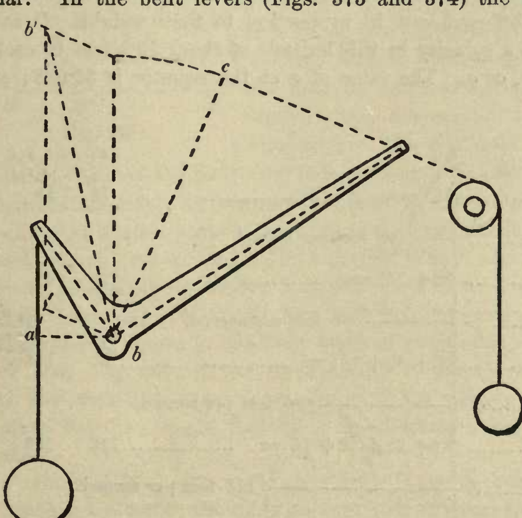


Fig. 374.

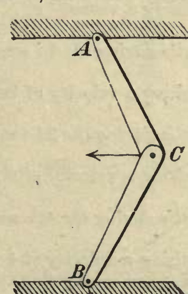


Fig. 375.

is to separate the plates A and B. The action is as shown in Fig. 376. Equal movements, as C–1, 1–2, 2–3, correspond to unequal movements at A and B, as A a', a' a^2, a^2 a^3. The nearer the force C is to the line A B, the less the movement a^2 a^3; and, consequently, the force C exerts greater effects in intensity, but the latter is less in movement.

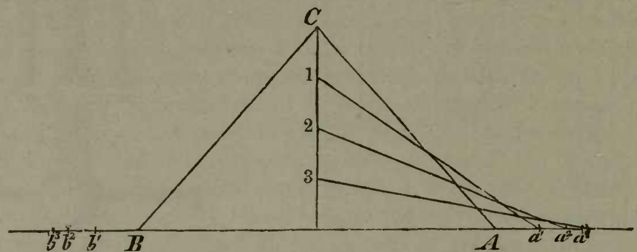


Fig. 376.

Fig. 377 exhibits the principle of the hydraulic press. The small plunger or piston may be considered the application of the force, and the large one the weight to be raised to balance each other; the pressure per square inch of surface must be the same, and the force must be to weight as the surface of its piston is to that of the weight-piston. If motion takes place, the force will move through space corresponding to the area of weight-piston, while the weight will move that of the area of the force-piston. And this is the great principle of all mechanism in the transmission of force: there can be no total gain. What is gained in force is lost in movement, and in many complicated machines the theoretical comparison of force applied and resultant force may be ascertained by the measures of their movements.

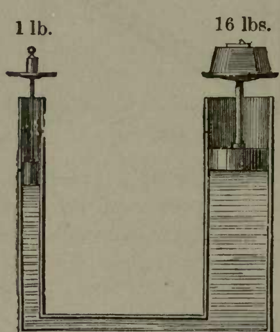


Fig. 377.

The resultant effects of forces, as heretofore treated, have been without motion, or static. But when motion is produced, the forces are called *dynamic*. A weight suspended or supported exerts a force, which is balanced by the resistance of the suspending or supporting medium; but a falling weight acquires an increasing velocity with every unit of time or space passed. All bodies would fall with the same velocities were it not for the different resistances from the air due to their different bulk in proportion to their weight. Dense articles, as stones and metals, acquire a velocity in this latitude of about 32·2 feet in each second, called the intensity of gravity, or g. The value of g at the equator is 32·088; at the poles, 32·253. A body

		Ft.	Tot. Fall.
Starting with a velocity	0		
Falls during the 1st second	16 =	16	16
Acquiring a velocity of	32 feet per second.		
Falls during the 2d second	32 + 16 =	48	64
Acquiring a velocity of twice 32, or	64 feet per second.		
Falls during the 3d second	32 + 32 + 16 =	80	144
Acquiring a velocity of 3 × 32 =	96 feet per second.		
Falls during the 4th second	32 + 32 + 32 + 16 =	112	256
Acquiring a velocity of 4 × 32 =	128 feet per second.		
Falls during the 5th second	32 + 32 + 32 + 32 + 16 =	144	400
Acquiring a velocity of 5 × 32 =	160 feet per second.		

Calling *s* the space passed over, *v* the terminal velocity in feet, *t* the time in seconds of falling, $s=\frac{1}{2}gt^2$, $v=gt$ or $=\sqrt{64.4s}$. In determining the velocity of issuing water under a head *h*, corresponding to *s* in the equation, it is generally near enough to reckon *v* as eight times the square root of the head ($\sqrt{h}$).

The motion of falling bodies is a uniformly accelerated one, but there are also uniformly retarded motions in which the velocity is decreased by equal losses in equal times. There are also uniform motions when bodies are impelled by a constant force and opposed by constant resistances.

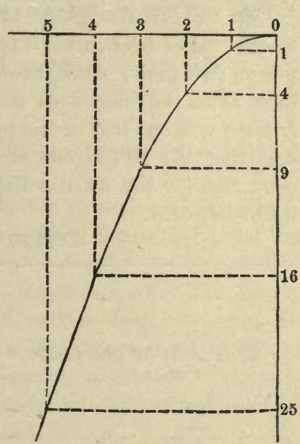


Fig. 378.

In Fig. 378, *o s* represents the trace of a body impelled horizontally by a uniform, but falling through the action of gravity with an accelerated, force. This curve, a parabola, represents approximately the curve of the thread of stream issuing from an orifice, or flowing.

It will be seen that to produce twice the velocity the body must fall through four times the space; that there is four times the force stored in the body. But to maintain this velocity uniformly, only twice the force is necessary. The *momentum* of a body is its mass multiplied by its velocity, but its *inertia* is as the square of the velocity. It is an established principle of mechanics that the results must be proportional to the causes: if a body has to be raised four feet to obtain a double velocity in falling, the destructive result of that fall must also be four times.

Under statics, it has been shown that forces may be resolved and compounded. The same may be done dynamically—that which has been treated as weight must now be considered as momentum.

In treating of dynamic forces the resultants have been considered as equal to the exertion, without any losses by resistances. This never happens in practice; the resistances are a very large element. Resistances from the medium in which the bodies are moved are from the surfaces on which the bodies are supported; resistances due to the displacement of the fluid in which the bodies move, and frictional resistances, or what is termed skin-resistances, of bodies moving through air or water; and the surface-resistance of bodies sliding or rolling on each other. Suppose a weight to rest on a horizontal surface—it will take a certain force to move the insistent weight depending on the amount of this weight and the kind of surfaces in contact, and the force that will just cause motion overcomes the friction, or frictional force, and is equal to it. The frictional force is only a percentage of the insistent force of the body, and this percentage is called the *co-efficient of friction*. If the horizontal surface of support be raised at one end, so as to make the surface inclined, it will after a time become so steep that the insistent body will slide down the surface. Thus, in Fig. 379, if the body Q is ready to slip on the surface A B, the angle B A C, which represents the angle of the surface with the horizontal, is called the *angle of repose, or limiting angle of frictional resistance;* or thus (Fig. 380), if the force acting in the direction P″ M is just sufficient to produce motion of the mass M along the plane F Q, the angle P M P″ is the limiting angle of resistance.

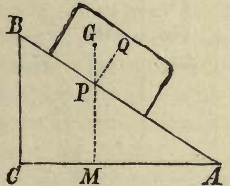


Fig. 379.

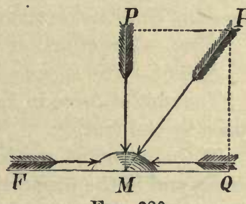


Fig. 380.

General Morin has made an elaborate course of experiments on friction, the results of which are given in the table on page 212. It was formerly held that friction was directly as the weight, without regard to the amount of surface or velocity of movements. And M. Morin's experiments, as rather applicable to the friction of quiescence and slow movements, come within this rule. But in practice it has been found that the co-efficient of friction with unguents is reduced by increase of velocity and temperature; that extent of surface may be prejudicial; and that careful selection of unguents, according to the work to be done, must be made to economize power by the reduction of friction.

Mr. C. I. H. Woodbery, in his experiments on the driving of cotton-spindles, found the co-efficient of friction to be from 7 to 20 per cent, the load being from one to five pounds per square inch; while Professor Thurston, with heavy loads of 1,000 pounds per square inch, as on the crank-pins of the North River steamboat-engines, found the co-efficient of friction was one half of one per cent, the unguent being sperm-oil. Practically it may be said that the co-efficient of friction for light-running spindles should not exceed 10 per cent, and for the usual work in shops, of say 100 to 200 pounds, should not exceed from 2 to 3 per cent.

EXPERIMENTS ON FRICTION, BY M. MORIN.

SURFACES OF CONTACT.	WITHOUT UNGUENTS.				UNCTUOUS SURFACES.			
	FRICTION OF MOTION.		FRICTION OF QUIESCENCE.		FRICTION OF MOTION.		FRICTION OF QUIESCENCE.	
	Co-efficient of friction.	Limiting angle of resistance.	Co-efficient of friction.	Limiting angle of resistance.	Co-efficient of friction.	Limiting angle of resistance.	Co-efficient of friction.	Limiting angle of resistance.
Oak upon oak, fibers parallel to the motion	0·478	25° 33′	0·625	32° 1′	0·108	6° 10′	0 390	21° 19′
Oak upon oak, fibers of the moving body, perpendicular to the motion	0·324	17·58	0·540	28·23	0·143	8° 9′	0·314	17° 26′
Oak upon elm, fibers parallel	0·246	13·50	0·376	20·37	0·136	7·45	....	....
Elm upon elm, " "	....	....	....	....	0·140	7·59	....	....
Wrought-iron upon oak	0·619	31° 47′	0·619	31° 47′	....	...	....	....
" " wrought-iron	0·138	7·52	0·137	7·49	0·177	10·3	....	...
" " cast-iron	0·194	10° 59′	0·194	10·59	....	....	0·118	6·44
" " brass	0·172	9·46	....	....	0·160	9·6	...	....
Cast-iron on elm	0·195	11·3	....	....	0·125	7·8	....	....
" " cast-iron	0·152	8·39	0·162	9·13	0·144	8·12	....	....
" " wrought-iron		....	....	....	0·143	8·9	....	....
" " brass	0·147	8·22	....	...	0·132	7·32	....	....
Brass upon cast-iron	0·217	12·15	....	....	0·107	6·7	....	....
" " wrought-iron	0·161	9·9	....	....	....	....	....	....
" " brass	0 201	11·22	....	....	0·134	7·38	0·164	9·19
Leather ox-hide, well tanned, on oak	0·296	16·30	....	....	....	....	....	....
" " on cast-iron, wetted	....	....	....	....	0·229	12·54	2·67	14·57
" belts on oaken drums	0·27	....	0·47	....	....	....	....	....
" " cast-iron pulleys	0·28	....	....	....	....	....	....	....
Common building-stones upon the same	0·38 to 0·65	20·49—33·2	0·65—0·75	33·2—36·53	....	....	....	...

MECHANICAL WORK OR EFFECT.

Mechanical work is the effect of the simple action of a force upon a resistance which is directly opposed to it, and which it continnously destroys, giving motion in that direction to the point of application of the resistance; it is, therefore, the product of two indispensable qualities or terms:

First.—The effort, or pressure exerted.

Second.—The space passed through in a given time, or the velocity.

The unit of force in England and here is represented by the pound, and the unit of space by the foot.

The amount of mechanical work increases directly as the increase of either of these terms, and in the proportion compounded of the two when both increase. If, for example, the pressure exerted be equal to 4 pounds, and the velocity one foot per second, the amount of work will be expressed by $4 \times 1 = 4$. If the velocity be double, the work becomes $4 \times 2 = 8$, or double also; and if, with the velocity double, or 2 feet per second, the pressure be doubled as well—that is, raised to 8 pounds—the work will be $8 \times 2 = 16$ pounds feet. It is more usual to write *foot-pounds*, but we invariably use the former, following the Continental idiom of *kilogrammetre*, in which the unit of force, kilogramme, precedes that of space, the metre.

In comparison of motors with each other, it is usual to speak of them as so many horse-power equivalent to 550 pounds feet per second, or 33,000 pounds feet per minute. The Continental horse-power is equal to 75· kilogrammetres or 542·48 pounds feet per second.

It is very common to use other units of force and space, as tons-miles; and train-miles, in railway practice.

The *time* must also be expressed or understood. It is impossible to express or state intelligibly an amount of mechanical effect, without indicating all the three terms—force, space, and time.

The motors generally employed in manufactures and industrial arts are of two kinds—living, as men and animals; and inanimate, as water and steam.

What may be termed the amount of a day's work, producible by men and animals, is the product of the force exerted, multiplied into the distance or space passed over, and the time during which the action is sustained. There will, however, in all cases be a certain proportion of effort, in relation to the velocity and duration, which will yield the largest possible product or day's work for any one individual, and this product may be termed the maximum effect. In other words, a man will produce a greater mechanical effect by exerting a certain effort at a certain velocity, than he will by exerting a greater effort at a less velocity, or a less effort at a greater velocity, and the proportion of effort and velocity which will yield the maximum effect is different in different individuals.

In the manner and means in which the strength of men and animals is applied, there are three circumstances which demand attention:

1. The power, when the strength of the animal is exerted against a resistance that is at rest.

2. The power, when the stationary resistance is overcome, and the animal is in motion. And,

3. The power, when the animal has attained the highest amount of its speed.

In the first case, the animal exerts not only its muscular force or strength, but at the same time a very considerable portion of its weight or gravity. The power, therefore, from these causes must be the greatest possible. In the second case, some portion of the power of the animal is withdrawn to maintain its own progressive motion; consequently, the amount of useful labor varies with the variations of speed. In the third case, the power of the animal is wholly expended in maintaining its locomotion; it therefore can carry no weight.

Weisbach calls the mean effort of an animal one fifth its weight, which may serve as a general rule, but, in practice, will be considerably modified, when applied to the individual, depending upon the exertions, and the conditions and circumstances under which it is made. A man-power is usually estimated at one sixth of a horse-power (H. P.); yet, if the muscular force of a man be required for an effort of short duration, it will exceed one horse-power. Thus, a horse-power is equal to 33,000 pounds feet per minute, or 550 pounds feet per second; and, if a man weighing 150 pounds move up-stairs at the rate of four feet per second, he exerts a force of 600 pounds feet, which he can readily double for a few seconds.

The force of a man is utilized mechanically through levers, as in pumping or rowing, or at a vertical capstan, or at a crank, carrying or dragging loads, shoveling, etc. In continuous work at the lever he will exert from 25 to 30 pounds; at the crank, from 15 to 20 pounds.

The muscular force of horses is utilized in the draft of carriages, in hoisting, and in horse-powers, either moving in a circle round a central shaft or on a revolving platform, or on an endless belt. The draught of a horse varies with the speed of movement and its duration. Trautwine gives the draught of a horse at two and a half miles per hour for 10 hours per day, 100 pounds; 8 hours, 125 pounds; 6 hours, 166⅔ pounds; 5 hours, 200 pounds. The omnibus-horses here average nearly six miles per hour, and make 16 to 24 miles per day; the average will not exceed 16 miles. At the Manhattan Gas Works, a span of horses hoist from the lighter 200 tons gross in 10 hours to the height of say 25 feet, with charges of 6 to the ton, in a bucket weighing 150 pounds, the rope passing over a single block and through a snatch-block. On a horse-power, the force exerted by a single horse is from 125 to 175 pounds, at an average speed of about three miles per hour, and for eight hours per day. Beyond a speed of four miles per hour, the pounds foot of work of a horse will decrease in an increasing ratio up to the limits of his speed, when the whole work done will be used up in locomotion. In proportioning levers, cranks, traces, chains, through which animal force is transmitted to machines, or for mechanical purposes, it is not safe to estimate the stress as the average force; there are impulses and stresses in action which will exceed the weight of the animal.

Water-Power.—Water, used for the purposes of power, moves machinery either by its weight, by pressure, by impact, or by reaction, and is applied through various forms of wheels. However used, the mechanical effect inherent in water is the product of its weight into the height from which it falls; but there are many losses incurred in its application, so that only a portion of the mechanical effect becomes available; and the comparative efficiency of any water-wheel or motor is represented by this percentage of the absolute effect of the water applicable to power.

The quantity of water supplied to the mills at Lowell, permanently, for the working hours per day is about 4,000 cubic feet per second, and the entire fall 33 feet. In the distribution of the water by the canals about two feet of fall is lost, and the mill-powers, as leased to the mills, would be about 4,000 cubic feet per second, on a 31-foot fall. In the passage of the water through the trunks or pent-stocks to the wheels, and from the wheels to the river or other canals, there is still another loss of head, which may be considered about one foot, so that the net fall is only 30 feet.

$$\frac{4{,}000 \text{ cu. ft.} \times 62{\cdot}33 \text{ weight of water per cu. ft.} \times 30 \text{ ft. fall}}{550 \text{ lbs. ft. per H. P. per sec.}}$$

= 13,600 horse-power.

But only a percentage of this power is available for mechanical power. The efficiency of the turbines, the wheels now generally in use here, may be taken at 80 per cent of the gross horse-power. The net horse-power will then be 13,600 × ·80 = 10·880 horse-power.

Wind is applied for the purposes of power; but, as there is no constancy in its action, its use is mostly confined to the purpose of raising water by means of pumps into cisterns or reservoirs.

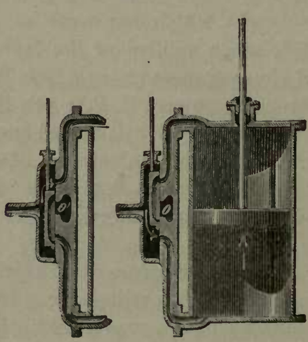


Fig. 381.

Steam is the elastic fluid into which water is converted by a continuous application of heat. It is used to produce mechanical action almost invariably by means of a piston movable in a cylinder. Thus, in Fig. 381, the steam entering through the lower channel-way, or *port*, presses against the under side of the piston in the direction of the arrow, the piston is forced upward, the steam above the piston escaping through the exhaust-channel *o*. When the piston reaches the top of the cylinder, the valve is changed by mechanism, the steam enters above the piston, and the steam below it escapes through the exhaust. In this way a reciprocating movement is established. To determine the horse-power of a steam-engine, multiply the area of the piston in square inches by the effective pressure in pounds on each square inch of piston, and the product by the travel in feet through which the piston moves per minute, and divide this last product by 33,000. The travel is the length of stroke multiplied by the number of strokes, or double the number of revolutions per minute.

Example.—Let the diameter of the piston be 18 inches, the effective pressure 45 pounds per square inch, the stroke 30″, the revolutions 60, or 300 feet travel per minute, what will be the horse-power of the engine ?

Area of piston, 254·46 square inches.

$$\frac{254{\cdot}46 \times 45 \times 300}{33{,}000} = 104{\cdot} \text{ horse-power.}$$

As steam in its passage through channels and in the cylinder is subject to various losses of pressure, and as the steam is worked under more or less expansion, and as the exhaust steam is discharged under more or less pressure, whether into the air or into a condenser, it is impossible to determine the effective pressure except by the means of an indicator.

The principle of working steam expansively is as follows: If a cubic foot of air of the atmospheric density be compressed into the compass of half a cubic foot, its elasticity will be increased from 15 pounds on the square inch to 30 pounds; if the volume be enlarged

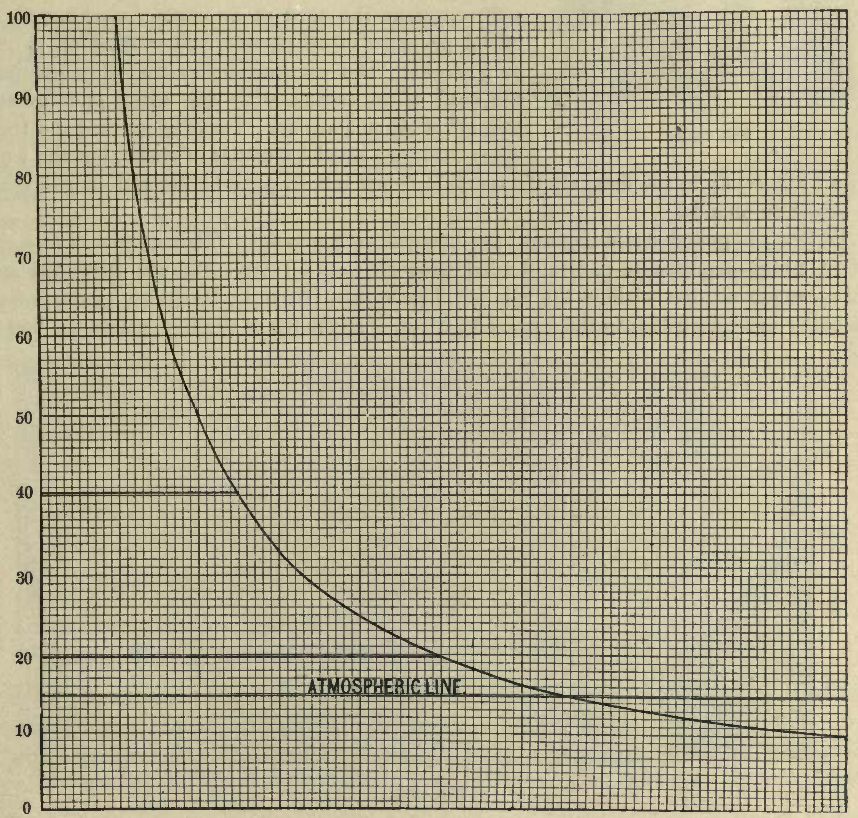


FIG. 382.

to two cubic feet, the pressure will be one half, or 7½ pounds. The same law holds in all other proportions for gases and vapors, provided their temperature is unchanged.

Fig. 382 illustrates this graphically. Suppose the piston in the cylinder to have made one tenth of its stroke, and to be at .1, and the pressure at 100 pounds above the absolute 0 (or vacuum) to which expansion is referred, and not to the atmospheric line representing nearly 15 pounds pressure: if the steam-valve be now closed, and the piston be moved to the position .2, the space occupied by the steam will be double what it was at first, and the pressure one half, $\frac{100}{2}$, or 50 pounds. If the piston be moved to .3, the pressure will be $\frac{1}{3}$, or $33\frac{1}{3}$ pounds; to .4, $\frac{1}{4}$, or 25 pounds; and so on to .5, .6, .7, .8, .9, .1.0, the pressure will be $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$; and, at the end, the expansion will be said to be ten times, and the cut-off (or where the steam was shut off from the cylinder), at $\frac{1}{10}$ of the stroke.

When the steam is cut off, if there be no leak through the valves or by the piston, this quantity may be considered constant, although there are losses by condensation from the surfaces of cylinder and piston, and the conversion of heat into work. But it will generally be found that the weight of steam, as represented by the volumes, will be greater at the end of the stroke than at the cut-off, owing to re-evaporation of condensed or conveyed water by the cylinder surfaces.

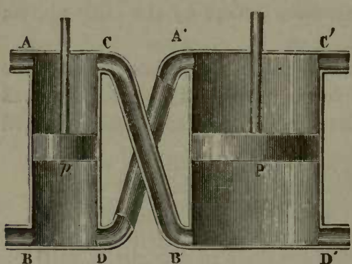


Fig. 383.

To illustrate the theoretical advantages of a cut-off, draw lines across the card (Fig. 382) at 40 and 20 pounds. The portions of the card below these lines will represent the card of an engine, working at an initial pressure of 40 pounds, and cutting off at .25, or $\frac{1}{4}$ stroke. The portions below the 20-pound line, the card of an engine, with this initial pressure cutting off at .5, or $\frac{1}{2}$ stroke. The original card and these other cards use equal quantities of steam, but the work is very different; in the first, all the work is below the 40-pound line, and in the other all below 20 pounds.

Of late compound steam-engines have become very popular. They consist of two cylinders, a high-pressure (h. p. c.) and a low-pressure (l. p. c.) one. Fig. 383 shows the general arrangement, but without the valves. The h. p. c. (A, B, C, D) draws its steam from the boiler and exhausts into the l. p. c. (A′, B′, C′, D′); the top of the h. p. c. into the bottom

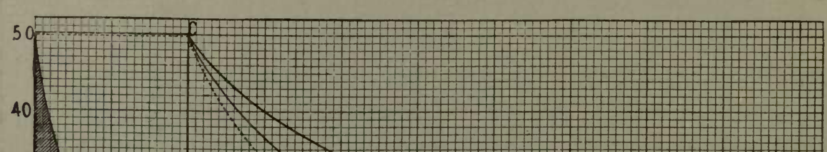


Fig. 384.

of the l. p. c., and *vice versa*, so that the pressure on the pistons of the two cylinders is in the same direction.

For the comparison of the theoretical effect of the single cylinder and compound engines, construct the card of a single cylinder, Fig. 384, shown in dotted line, in which 0·8 is the length of stroke, 50 pounds the initial pressure, and .2 the point of cut-off. If 50, C, .2, .0 represent the h. p. c. of a compound, and the cylinder be filled at the pressure of 50 pounds, the quantity of steam used at each stroke will be the same as in the single cylinder, and, to expand equally with this, the stroke of the l. p. c. is represented by .2, .1, .0. When the piston of the h. p. c. is about to commence its stroke downward, for instance, the cylinder beneath it is full of steam at 50 pounds. As the steam rushes in above the piston from the boiler, the steam below the piston begins to exhaust into the upper part of the l. p. c., and consequently falls off, while the pressure above the h. p. c. piston in connection with the boiler maintains its 50 pounds.

When expansion commences in the l. p. c., the pressure is the same as in the h. p. c., 50 pounds; but the expansion takes place differently from that in a single cylinder. At the end of the first eighth of the stroke, the space in the l. p. c. is equal to ½ that of the h. p. c., but its space has been reduced by the movement of the piston ⅛; therefore, the space now occupied by the steam is ½ + ⅞, or $\frac{11}{8}$ of what it was before expansion, and the 50 pounds becomes $\frac{8 \times 50}{11}$ = 36·4 nearly. At ¼ stroke, the space in the l. p. c. is equal to that of the h. p. c., and in the h. p. c. it is reduced to ¾; the total space is now 1 + ¾ = 7/4, and the expansion is $\frac{50 \times 4}{7}$, or 28·6 pounds nearly. At ⅜ stroke the space is 1½ × ⅝, and the pressure, consequently, 23·5 nearly. At the end of the stroke the space of the h. p. c. is entirely shut off, and that of the l. p. c. filled with expanded steam, at 12½ pounds, ¼ of the initial pressure. The full line, C T′, represents the expansion as it has taken place in the l. p. c.; but, as said above, the pressure below the piston in the h. p. c. falls off as expansion goes on in the l. p. c. The pressure in the l. p. c. at the top is the same as the h. p. c. at the bottom, and, if these pressures be transferred to the h. p. c. card, there will be a curve, 50 T″, which will represent the back pressure beneath the h. p. c. piston. The back pressure is shown in the shaded portion, above which is the net pressure on the piston; if these net pressures be divided by 4, and plotted, as shown, above the l. p. c. expansion, curve C T′, then the curve C H will represent the curves of pressures of the united h. p. c. and l. p. c. on the same scale as that of the single cylinder.

Figs. 385, 386 represent these cards, both on the same scale, and it will be observed that, theoretically, there is no difference in effect between steam used in a single cylin-

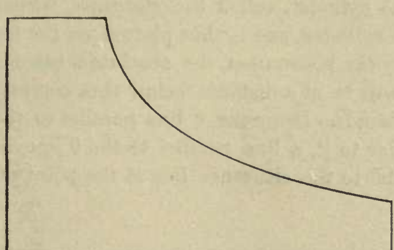


Fig. 385.

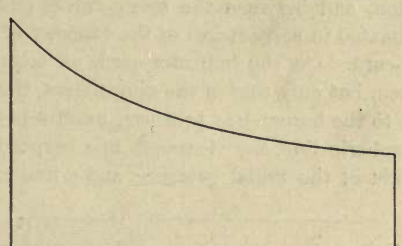


Fig. 386.

der or in a compound. But, practically, the compound is, for many purposes, found the most economical, due in part to the smaller condensation, since the surfaces in the h. p. c. are never cooled below the limit of expansion, in example 12½ pounds (204°), while the l. p. c. and the single cylinder are cooled to the limit of condensation, or probably about 126°.

In addition, comparing the two cards (Figs. 385, 386), it will be observed that the forces in the compound cylinders are less irregular than in the single cylinder, and the necessities of a fly-wheel, to equalize forces and resistances, are less.

The cards of the compound engines above drawn do not take into consideration the loss of pressure in the channels between the h. p. c. and l. p. c., and there is a class of compound engines in which the h. p. c. exhausts into an intermediate chamber, between it and the l. p. c., to which the construction of cards given is not applicable. They can best be determined from practical examples.

The above illustrations represent purely the theoretical card. The vacuum is perfect, and the steam in the cylinders at full pressure, both in introduction and at relief, without any wire-drawing, reduction, or rounding, incident on actual practice.

Fig. 387 represents a real card taken from a steam-cylinder of a condensing engine. To determine the mean effective pressure, divide the atmospheric line, embraced in the card, into 20 equal parts, and draw ordinates through the .1, .3, .5, .7, .9, .11, .13, .15, .17, .19th divisions. The lines embraced between the card outlines represent the pressure at different parts of the stroke—.05, .15, and so on, on the scale of the indicator-spring; these, added together and divided by 10, give the mean effective pressure (m. e. p.) on this card, 43·4 pounds.

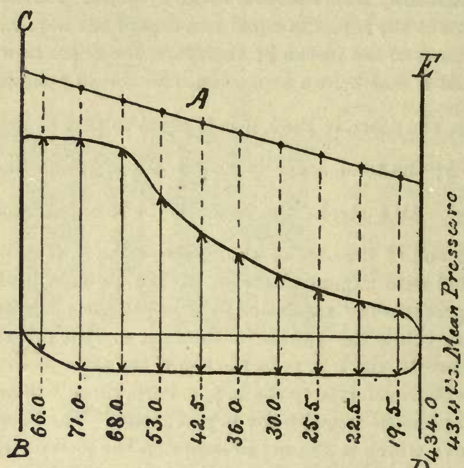


Fig. 387.

The mean effective pressure multiplied by the area of piston, in square inches, by the length of stroke, and number of strokes per minute, gives the pounds-feet of work per minute, which, divided by 33,000, will give the indicated horse-power (i. h. p.) of the engine.

To determine whether a steam-engine is working properly, it is necessary to compare the absolute card with the theoretical one.

Fig. 388 represents an indicator-card, as taken from a steam-cylinder in which there is no condensation; the exhaust is directly into the air. On this is shown the construction of the isothermal curve. It will be observed that there is a line, A B, to the top of the card. The space between this and the card represents the spaces between the cylinder-head and piston, and between the steam-valves and the cylinder, called the *clearance*, which are estimated in percentages of the capacity of the cylinder, and is thus plotted on the indicator-card. On the indicator-card, as taken by the instrument, the absolute 0 can not be taken, but only that of the atmosphere, the 0 will be at a distance below this, corresponding to the barometric pressure, usually 14·8 pounds. Draw the 0 line parallel to the atmospheric line, the clearance line perpendicular to it, a line parallel to the 0 line, at the height of the initial pressure, and a line parallel to the clearance line at the point of cut-

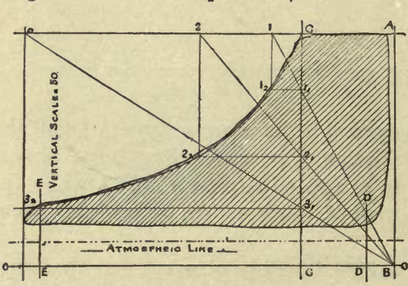


Fig. 388.

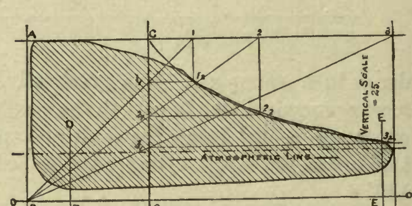


Fig. 389.

off on the initial pressure line. Any point on the expansion line, as 1_2, 2_2, 3_2, may be determined by drawing lines B 1, B 2, B 3, and then horizontal lines 1_2 1_1, 2_2 2_1, 3_2 3_1 from their intersections 1_1, 2_1, 3_1. With the cut-off line, parallel to the 0 line, and perpendiculars from 1, 2, 3, the intersections of these two lines, 1_2, 2_2, 3_2, will be the points in the curve. The curves in the outline of the cards, at the times of admission, cut-off, and exhaust, show the action of the valves and time occupied in change of condition. The stroke commences at A, cuts off at C, commences to exhaust at E; about D the exhaust-valve closes, and the steam between the piston and the ends of the cylinder begins to be compressed, and the curve developed is called the *curve of compression.*

In expanding, steam does not maintain the same temperature; there is a fall of temperature, and consequently less space occupied than shown by the isothermal curve; the curve thus developed is called the adiabatic curve. In Fig. 389 the construction of this curve, the line C c, through the point of cut-off, is inclined to the line A B, 1° 43′, to which the lines 1 1_2, 2 2_2 are drawn parallel, but otherwise the same as in the preceding figure; practically, the isothermal curve corresponds more nearly with that formed by the cards, as, especially near the end of the stroke, there is considerable transmission of heat from the cylinder surfaces to the steam, more than that lost by mere expansion.

The indicator-cards show very fairly the amount of power exerted on the piston, but they do not show the economy of the whole machine including the boilers. The boilers may be faulty, in that they do not evaporate sufficient water for the coal consumed, or that the ebullition is too local and violent, without sufficient steam-space, so that water is taken off with the steam; or the steam-cylinder and its working may be faulty, in that the steam is condensed therein without doing any work. The economic value of the boiler may be determined by the measure of the quantity of water pumped into the boilers, and the quality of the steam.

MACHINE DESIGN AND MECHANICAL CONSTRUCTIONS.

In the designing of new machines and mechanical constructions, the draughtsman must draw from his knowledge of well-known forms and parts, and combine them; but, to proportion them properly, and adapt them to the purposes required, he must understand the stresses to which they are to be subjected, and the action and endurance of the material to be used, to withstand these stresses.

In the present technical application of the term, ***stress*** is confined to a force exerted between two bodies or parts of a body, such as a pull, push, or twist. Strain is the alteration produced by a stress. Stress is the cause, strain the effect; the first is measured by the load, the latter by the deformation of the body produced by the first. A stress, not greater than the elastic limit of the material acted upon, produces a strain which disappears as soon as the load is removed: up to this limit the strain is proportional to the stress; beyond, the strain increases faster than the stress, up to the point of rupture. The elastic limit is a percentage of the breaking strain, varying with the kind of material and application of stress. Stress is usually designated as load, meaning thereby the sum of all the external forces acting on the member or structure, together with its weight.

Dead load, or weight, is a steady, unchangeable load. *Live loads* are variable, alternately imposed and removed, or varying in intensity or direction. It is usual, in designing constructions, to proportion the parts to resist a much greater load than will be brought on them in the structure; the load is multiplied by a factor termed *factor of safety*, as a security against imperfections in material and workmanship, contingencies of settlement, and other incidental stresses. But it must be observed that these imperfections are such as can not be seen and met; there can be no factor of safety to provide for poor and unknown material and defective workmanship.

The factor of safety adopted for dead loads varies but little with the same kind of material; but, for live loads, the factor varies not only with the material, but with the character of the stresses, whether they are applied and relieved gradually or suddenly; whether they only vary in intensity, or also in direction, alternately compressive or tensile. In this latter case the load should never be considered less than the sum of the stresses. with a large factor of safety. Vibrations, shocks, and changes in the direction of stresses, concentrate the strains at the weakest point of the construction, and rupture takes place at these points, which would be adequate to the strain if the form throughout were uniform with that at these points. Thus, boiler-plates show wear just at the edge of the lap of the sheets, and car-axles (Fig. 390), with sharp angles at the journals, are known to break after a time, while under the same stresses an axle of uniform size with the journal would not break; nor if a slight curve or rim ¼ inch radius (Fig. 391) be made in the angle to distribute stress.

Besides provisions for strength, the draughtsman should understand the necessities of the construction, and the character of the material to be used. He should know what parts of the design are to be forged, cast, framed, and how it is to be done. He should know what wear is to be met, and what waste, as rust or rot, to be provided for. This knowledge can only be arrived at by reference to examples of practice and by observation of results under similar conditions of use and time.

The stresses to which constructions and parts of constructions are subjected are the *tensile* or stretching stress, tending to lengthen a body in the direction of the stress; the *compressive* or crushing stress, tending to shorten a body in the direction of the stress; the *shearing* or cutting stress, tending to elongate, compress, and deflect; the *torsional* or

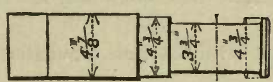


Fig. 390.

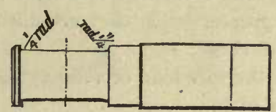


Fig. 391.

twisting stress, the effect being an angular deflection of the parts of the body; and the *transverse* or lateral stress, tending to bend the body or break it across.

At page 195 is given a table of the strength of various metals to resist compression and tensional stresses, and examples will hereafter be given of varied constructions, with their usual or required factors of safety; but, for a practical rule for the common necessities of the above stresses, under dead loads, 10,000 pounds per square inch for wrought-iron may be considered perfectly safe.

Posts in structures are subjected to compressive stresses; but, as the action is modified somewhat by a tendency to bend, depending on the proportion of the length to the diameter, and the material of which they are composed, the usual tables of crushing strength are not generally applicable, and the formulæ to be depended on are those deduced from practical tests. The best tests of wooden posts are those made by Professor Lanza, for the Boston Manufacturers' Mutual Fire-Insurance Company, and the following are the results:

"That the strength of a column of hard pine or oak, with flat ends, the load being uniformly distributed over the ends, is practically independent of the length, such columns giving way by direct crushing, the deflection, if any, being very small. Tests were on columns 6″ to 10″ diameter × 12 feet. The average crushing strength of very highly-seasoned, hard pine was 7,386 pounds per square inch. Some very slow-growth and highly-seasoned, 9,339 pounds; very wet and green, 3,015 pounds; seasoned about three months, 3,400 pounds; not very well seasoned and not very green, 4,400 to 4,700 pounds. The average of two specimens of thoroughly-seasoned white-oak, 7,150 pounds; for green and knotty, average, 3,200 pounds. Spruce, nearly 5,000 pounds. Whitewood, 3,000 pounds.

"That it is a mistake to turn columns, taper, or even turn them at all, square columns being much stronger, cheaper, and better, and that accuracy of fitting is of great consequence, that the stress may be directly vertical." The professor recommends that longitudinal holes be bored through the center of columns to allow of the circulation of air (in the experiments the holes were 1·7″ diameter), and that iron caps be used instead of wooden bolsters, as the wooden bolster will fail at a pressure far below that which the column is capable of resisting, and the unevenness of pressure brought about by the bolster is sometimes so great as to crack the column. He also recommends horizontal holes in the iron caps to connect the longitudinal ones in the column with the outer air.

From the whole of the experiments, we estimate the safe load, for fair-grained, well-seasoned oak or yellow-pine columns to be from 1,000 to 1,500 pounds per square inch; for the more imperfect and green specimens, from 300 to 500 pounds; for good specimens of whitewood, about 300 pounds; and of spruce, about 500 pounds.

Cast-Iron.—For the columns of buildings where the load is dead, cast-iron is very generally used. They are, in interiors, mostly of circular section, but for outer columns forms are used suited to the necessities of their position or style of architecture. They admit of considerable ornamentation and finish direct from the mold; but, as they are liable to defects not readily detected in the process of casting, the factor of safety is usually taken as high as 5. To protect them against the effects of fire and water in conflagrations, they are often covered with an outer shell of cast-iron or plaster, or of both.

The experiments of Hodgkinson are the usual basis of all formulæ on the strength of circular cast-iron columns, and the ends of all columns are now required to be faced by architects and by the rules of building departments, since Mr. Hodgkinson states this rule, that "in all long columns, of the same dimensions, the resistance to fracture by flexion is three times greater when they are flat and firmly bedded than when they are rounded and capable of moving."

Table of the safe load of solid cylindrical columns, with flat ends calculated with a factor of safety of 5.

TABLE OF SAFE LOADS FOR SOLID CAST-IRON COLUMNS, WITH FLAT ENDS.

Diam.	8′ 1,000 lbs.	9′ 1,000 lbs.	10′ 1,000 lbs.	11′ 1,000 lbs.	12′ 1,000 lbs.	13′ 1,000 lbs.	14′ 1,000 lbs.	15′ 1,000 lbs.	16′ 1,000 lbs.	17′ 1,000 lbs.	18′ 1,000 lbs.	19′ 1,000 lbs.	20′ 1,000 lbs.	21′ 1,000 lbs.	22′ 1,000 lbs.	23′ 1,000 lbs.	24′ 1,000 lbs.
3″	29·	23·	20·	17·	14·	13·	11·	10·	9·	8·	7·	7·	6·	6·	5·	5·	4·
3¼″	40·	31·	26·	22·	19·	17·	15·	13·	12·	11·	10·	9·	8·	7·	7	6·	6·
3½″	50·	41·	34·	29·	25·	22·	19·	17·	15·	14·	12·	11·	10·	10·	9·	8·	8·
3¾″	63·	54·	43·	37·	32·	28·	24·	22·	19·	18·	16·	15·	13·	12·	11·	11·	10·
4″	77·	66·	54·	46	40·	35·	31·	27·	24·	22·	20·	18·	17·	15·	14·	13·	12·
4¼″	92·	80·	70·	57·	49·	43·	38·	34·	30·	27·	25	23·	21	19·	18·	16·	15·
4½″	110·	96·	84·	74·	61·	53·	47·	41·	37·	33·	30·	28·	25·	23·	22·	20·	19·
4¾′	130·	113·	99·	88·	73·	64·	56·	50·	45·	41·	37·	34·	31·	28·	26·	24·	23·
5	152·	133·	117·	103·	92·	77·	68·	60·	54·	49·	44·	40·	37·	34·	31·	29·	27·
5¼″	176·	154·	136·	121·	108·	97·	81·	72·	64·	58·	53·	48·	44·	40·	37·	35·	32·
5½″	201·	177·	157·	140·	125·	113·	95·	85·	76·	68·	62·	57·	52·	48·	44·	41·	38·
5¾″	230·	203·	180·	161·	144·	130·	118·	99·	89·	80·	73·	66·	61·	56·	52·	48·	45·
6′	260·	230·	205·	183·	165·	149·	135·	115·	103·	93·	84·	77·	71·	65·	60·	56·	52·
6¼″	292·	260·	232·	208·	187·	169·	154·	140·	119·	108·	98·	89·	82·	75·	69·	64·	60·
6½″	327·	292·	261·	234·	212·	192·	174·	159·	146·	124·	112·	102·	94·	86·	80·	74·	69·
6¾″	364·	326·	292·	263·	238·	216·	197·	180·	165·	141·	128·	117·	107·	99·	91·	85·	79·
7′	404·	362·	325·	293·	266·	242·	221·	202·	186·	171·	146·	133·	122·	112·	104·	96·	90·
7¼′	445·	400·	361·	326·	296·	269·	246·	226·	208·	192·	177·	151·	138·	127·	118·	109·	101·
7½′	489·	441·	398·	361·	328·	299·	274·	251·	231·	214·	198·	170·	156·	143·	133·	123·	114·
7¾′	536·	484·	438·	398·	362·	331·	303·	278·	257·	237·	220·	204·	175·	161·	149·	138·	128·
8″	584·	529·	480·	436·	398·	364·	334·	308·	284·	263·	244·	227·	196·	180·	167·	155·	144·
8½′	689·	626·	571·	521·	477·	437·	402·	371·	343·	318·	296·	275·	257·	241·	207·	192·	178·
9′	802·	733·	670·	614·	564·	519·	479	442·	410·	381·	354·	331·	309·	290·	272·	235·	218·
9½″	926·	849·	780·	717·	660·	609·	563·	522·	484·	451·	420·	393·	367·	345·	324·	305·	265·
10′	1058·	975·	898·	829·	765·	708·	656·	609·	566·	528·	493·	461·	432·	406·	382·	360·	340·
10½″	1195·	1108·	1026·	957·	892·	823·	779·	740·	698·	658·	610·	580·	546·	511·	485·	459·	433·
11″	1359·	1264·	1159·	1083·	1017·	950·	889·	846·	798·	751·	703·	665·	627·	589·	561·	542·	513·
11½″	1517·	1413·	1319·	1226·	1147·	1080·	1018·	956·	904·	852·	810·	758·	727·	691·	655·	613·	587·
12″	1674·	1583·	1470·	1380·	1289·	1221·	1142·	1074·	1018·	973·	916·	871·	746·	701·	667·	645·	611·

Solid columns are very seldom used in constructions; they are almost invariably made hollow, the shell being ½″ to 2″ thick. To determine the safe load of a hollow column, it will be sufficiently accurate to take from the table the safe load of a column equal to that of the exterior diameter, and subtract from this the safe load of a column of a diameter equal to the core.

Example.—To find the safe load of a column 12 feet long, 8″ exterior diameter, shell ¾″.

Safe load of 8″ column..................................	398,000 lbs.
" " " 6½″ "	212,000 "
" " required column	186,000 "

For square box-columns, it will be safe to estimate that a square column will support as much as a round one, the side of the one being equal to the diameter of the other, and the thickness of shell the same.

For a star-column (Fig. 392), the load should be about ½ less than on a cylindrical column of same diameter and same area of section.

Wrought-Iron Columns.—With the decrease in the cost of the manufacture of shapes in wrought-iron, columns of this material have largely superseded those of cast-iron in constructions liable to varying loads and shocks. Fig. 393 shows the section of a Phœnix column, Fig. 394 of the Piper, Fig. 395 of the Keystone.

The Phœnix columns vary in the number of segments, and in the thickness of shells.

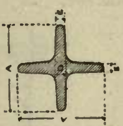


Fig. 392.

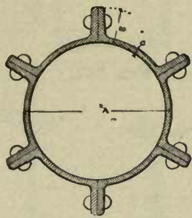


Fig. 393.

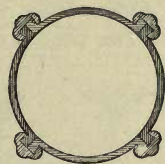


Fig. 394.

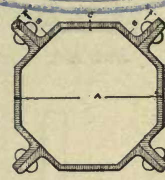


Fig. 395.

TABLE OF PHŒNIX COLUMNS.

MARK OF COLUMN.	Thickness in inches.	Area in square inches.	Weight in pounds per foot.	Internal diameter.
A	$\frac{1}{8}$	2·8	9·3	$3\frac{5}{8}$
4 segments	$\frac{5}{16}$	5·8	19·4	
B	$\frac{3}{16}$	5·0	16·7	$4\frac{13}{16}$
4 segments	$\frac{5}{8}$	14·8	51.	
B²	$\frac{3}{16}$	5·8	19·4	$5\frac{15}{16}$
4 segments	$\frac{5}{8}$	17·	58·6	
C	$\frac{3}{16}$	8·8	30·3	$7\frac{3}{16}$
4 segments	$1\frac{3}{16}$	40·	138.	
D	$\frac{1}{4}$	14·0	48·2	$9\frac{1}{8}$
5 segments	$\frac{5}{8}$	26·	89·7	
E	$\frac{1}{4}$	16·	55·2	11
6 segments	$1\frac{3}{16}$	60·	207·	
F	$\frac{3}{8}$	24·5	84·5	13
7 segments	$\frac{5}{8}$	36·4	125·6	
G	$\frac{5}{16}$	24·	82·8	$14\frac{5}{8}$
8 segments	$1\frac{5}{16}$	80·	276·	

TABLE OF PIPER AND KEYSTONE COLUMNS.

	4-inch Column.				6-inch Column.				8-inch Column.				10-inch Column.			
	Piper.		Keystone.		Piper.		Keystone.		Piper.		Keystone.		Piper.		Keystone.	
	Area, sq. in.	Weight per ft.	Area, sq. in.	W'ight per ft.	Area, sq. in.	Weight per ft.	Area, sq. in.	W'ight per ft.	Area, sq. in.	Weight per ft.	Area, sq. in.	W'ight per ft.	Area, sq. in.	Weight per ft.	Area, sq. in.	Weight per ft.
$\frac{3}{16}$	5·2	17·4					5·6	18·7								
$\frac{1}{4}$	6·	20·			7·3	24·3	7·1	23·8	11·	36·6	9·8	32·6				
$\frac{5}{16}$	6·8	22·7			8·4	28·1	8·7	28·9	12·5	41·7	11·8	39·3	16·	53·3	14·2	47·4
$\frac{3}{8}$	7·6	25·3	7·1	23·7	9·0	31·8	10·2	34·	14·	46·8	13·8	46·	17·9	59·7	16·6	55·3
$\frac{7}{16}$	8·4	28·	8·2	27·3	10·7	35·6	11·7	39·1	15·6	51·8	15·8	52·8	19·8	66·	18·9	63·1
$\frac{1}{2}$			9·3	30·9	11·8	39·4	13·3	44·2	17·1	56·9	17·9	59·5	21·7	72·3	23·7	78·9
$\frac{9}{16}$							14·8	49·3	18·6	62·	19·9	66·2	23·6	78·7	26·	86·7
$\frac{5}{8}$							16·3	54·4	20·1	67·1	21·9	72·9	25·5	85·	28·4	94·6
$\frac{11}{16}$											23·9	79·6	27·4	91·3	30·7	102·4
$\frac{3}{4}$											25·9	86·4	29·3	97·7	33·1	110·3
$\frac{13}{16}$															35·5	118·2
$\frac{7}{8}$																

Figs. 396–399 are sections of box-columns; the covers of 398 and 399 must be made in short pieces, to admit of the inside riveting, and with close butt-joints to preserve the strength. The thickness of the webs should exceed $\frac{1}{30}$ of the width, to prevent buckling under stress.

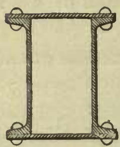


FIG. 396.

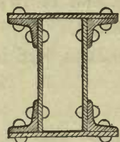
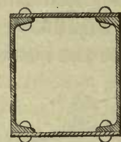


FIG. 397.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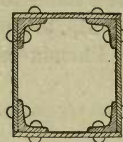
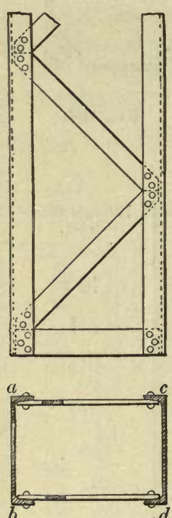


FIG. 399.



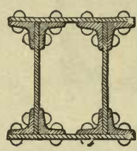


FIG. 401.

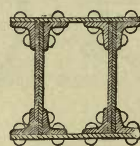


FIG. 402.

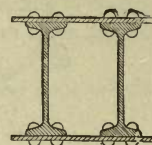


FIG. 403.

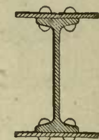


FIG. 404.

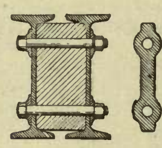


FIG. 405.

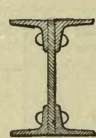


FIG. 406.

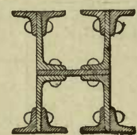


FIG. 407.

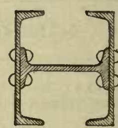


FIG. 408.

FIG. 400.

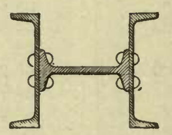


FIG. 409.

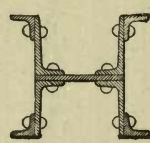


FIG. 410.

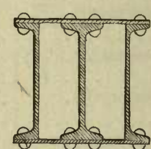


FIG. 411.

Fig. 400 shows the elevation and section of an open or lattice column, common in bridge and railway work. In estimating strength by area of section, in lattice-columns, the areas of continuous support, as of the *channel-irons*, *a b* and *c d* in the figure, are only considered.

Figs. 401–403 are sections of other open columns.

Figs. 404–411 are sections of various forms of made-up columns.

The caps and bases are usually of cast-iron and molded to the requirements of position.

On the Strength of Wrought-Iron Columns.—The upper curve, Fig. 412, represents graphically the average breaking load, taken from experiments on the Phœnix, Keystone, Piper, and open columns, with flat ends. Horizontal distances give the proportions of lengths of columns to diameters, or $\frac{\text{length}}{\text{diameter}}$, the vertical distances the loads in pounds. The lower curves represent the safe loads, under factors of safety of 3, 4, and 5. In looking at these curves, it will be observed that, within the common limits of practice, of 15 to 35 $\frac{\text{length}}{\text{diameter}}$, these lines may be considered straight; that with iron of a breaking strength of 52,000 pounds per square inch, and within the above limits, and a factor of safety of 3, the safe load may be taken at 11,000 per square inch; with a factor of safety of 4, at 8,000 pounds; with a factor of safety of 5, at 6,500 pounds; and that for common and usual purposes 10,000 pounds per square inch is a safe load.

It has generally been considered that columns with pin or cylindrical ends had about $\frac{2}{3}$ of the resisting strength of flat ends, but if the pin-ends are closely fitted, so that the strains are uniformly in the direction of the length of the column, the difference is but little between the two kinds of ends.

The sectional areas of I, channel, and angle irons, of which the above posts are composed, will be given hereafter.

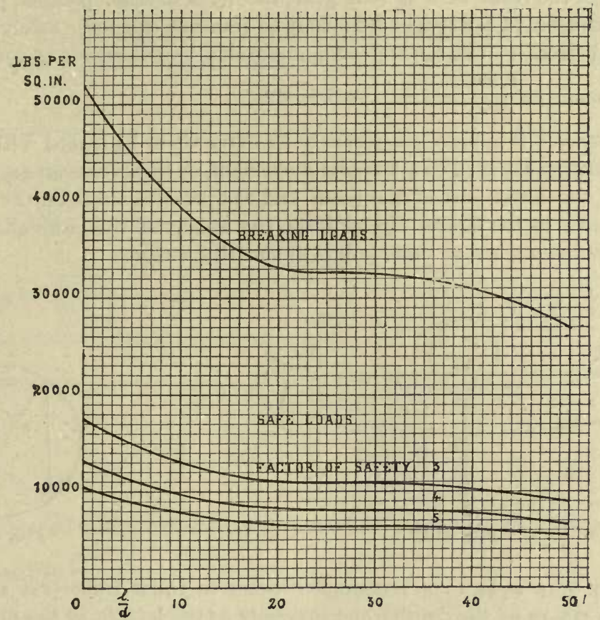


Fig. 412.

Shearing Stresses.—Parts of machines and of constructions subjected to these stresses have often the resistances modified by friction, combined with other stresses. The sizes of parts necessary to resist such stresses practically, as in the cases of bolts, rivets, and the like, will be hereafter illustrated by examples and determined by particular rules. In general, the strength to resist shearing stress is, in wrought-iron and steel, from 70 to 80 per cent of its tensile strength; in cast-iron, about 40 per cent of its crushing strength. The softer woods, as spruce, white pine, hemlock, resting on walls or girders, will safely sustain a load of 200 to 300 pounds per square inch of bearing surface, and the harder woods, as oak and Southern pine, 300 to 500 pounds. By experiment, oak treenails, 1″ to $1\frac{3}{4}''$ diameter, were found to have an ultimate shearing strength of about two tons per square inch of section; but, according to Rankine, the planks thus connected together should have a thickness of at least three times the diameter of the treenails. In 3″ planks, $1\frac{3}{4}''$ treenails bore only 1·43 tons per square inch of section; in 6″ plank, 1·73 tons.

Torsional Stress.—Every shaft through which power is transmitted, whether through gears, cranks, or pulleys, is subjected to a torsional stress, of which the power acting tangentially to the shaft in one direction is resisted by the load in an opposite direction. When this stress exceeds a certain limit depending on the material, the fibers are twisted asunder, but much below this limit the elasticity of the shaft may be too great to transmit power uniformly.

The *length* of the axle subjected to torsion does not affect the actual amount of pressure required to produce rupture, but only the angle of torsion which precedes rupture, and therefore the space through which the pressure must be made to act.

A torsional deflection of 1° in a length equal to twenty diameters of the shaft, is a good working limit of deflection—that is, $\frac{1}{360}$ part of a full turn. D. V. Clark gives the following rule: "To find the diameter of a shaft capable of transmitting a given torsional stress within good working limits. Divide the torsional stress in foot-pounds by 18·5 for cast-iron; 27·7 for wrought-iron; and 57·2 for steel. The cube root of the quotient is the diameter of the shaft in inches.

Example.—On the teeth of a 4½-foot gear, the force exerted is 2,800 pounds. What should be the diameter of a wrought-iron shaft to transmit this force safely?

The torsional stress will be 2,800 pounds multiplied by the radius of leverage, 2¼ feet, or 6,300 foot-pounds $= \frac{6,300}{27·7} = 228$, $\sqrt[3]{228} = 6·1$.

Transverse Stress.—If a beam supported at its extremities be loaded with a weight, W, Fig. 413, the beam is subjected to a bending movement, or transverse stress, composed of a tensile stress on the lower part of the beam and compressive stress on the upper part, as will be readily seen by the figure. In addition, the weight of the beam and its load, supported on the abutments, act at these points as shearing stresses.

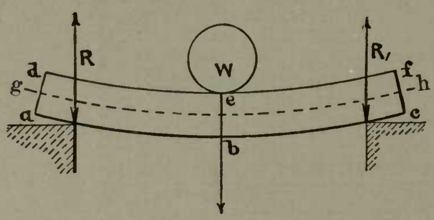


Fig. 413.

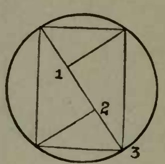


Fig. 414.

The strength of a square or rectangular beam to resist transverse stress is as the breadth and the square of the depth; and inversely as the length, or the distance from or between the points of support. Thus a beam twice the breadth of another, other proportions being alike, has twice the strength; or twice the depth, four times the strength; but twice the length, only half the strength.

It is evident, therefore, that, with the same area of section, the deeper a beam the stronger it will be, if the breadth is sufficient to prevent lateral buckling.

To cut the best beam from a log, Fig. 414, the section of which is a circle: draw a diameter, divide it into three equal parts, erect perpendiculars at the points of division 1, 2, and they will intersect the circumference at the corners of the beam, of which the extremities of the diameter are the other two.

For the transverse strength of rectangular beams the general formula is $W = \frac{S b d^2}{l}$, in which W is the breaking weight; S, a number determined by experiment on different materials; b, the breadth, and d, the depth in inches; and l, the length in feet.

Figs. 415 to 422 represent the usual methods of loading beams, and the loads as drawn represent the comparative strength of beams under these different conditions. Thus, in

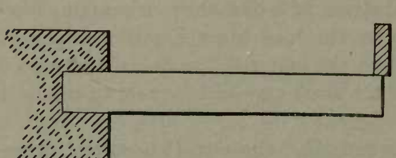


Fig. 415.

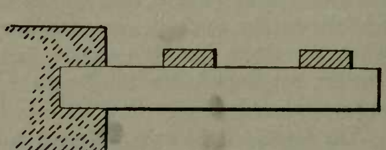


Fig. 416.

Fig. 415, the beam supports but one unit of load, while Fig. 416 supports twice as much. The formulæ given represent the safe dead loads with a factor of safety of 6, deduced from experiments of Mr. C. J. H. Woodbury on Southern pine. For spruce the co-efficient would be about ⅕ less, and for live loads the factor of safety should be 12.

Beams fixed at one end and loaded at the other (Fig. 415).

Safe load $= 30 \frac{b\,d^2}{l}$.

Beams fixed at one end and load distributed uniformly, not as represented in the figure, as the two units of weight would be spread over the whole length of the beam (Fig. 416).

Safe load $= 60 \frac{b\,d^2}{l}$.

Beams supported at the extremities and loaded at the middle (Fig. 417).

Safe load $= 120 \frac{b\,d^2}{l}$.

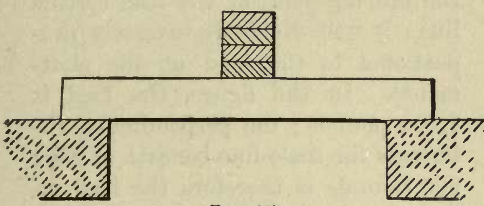


FIG. 417.

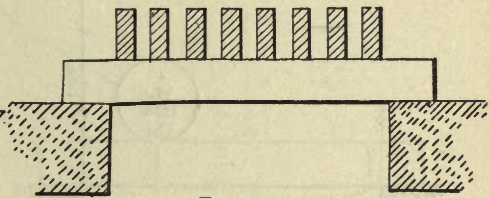


FIG. 418.

Beams supported at the extremities and the load uniformly distributed (Fig. 418).

Safe load $= 240 \frac{b\,d^2}{l}$.

Beams, one end firmly fixed, the other supported, and loaded at the middle (Fig. 419).

Safe load $= 160 \frac{b\,d^2}{l}$.

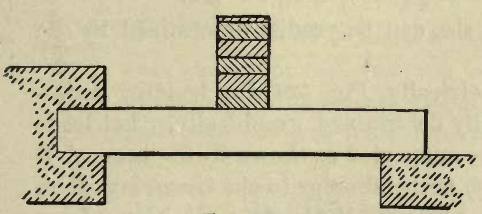


FIG. 419.

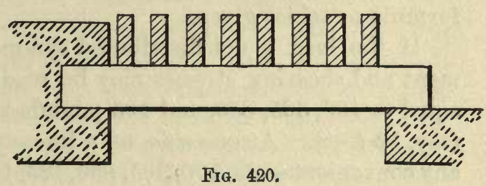


FIG. 420.

Beams with one end fixed, the other supported, and load uniformly distributed (Fig. 420).

Safe load $= 240 \frac{b\,d^2}{l}$.

This formula, although given by good authorities, is evidently too small; it should be probably about $300 \frac{b\,d^2}{l}$.

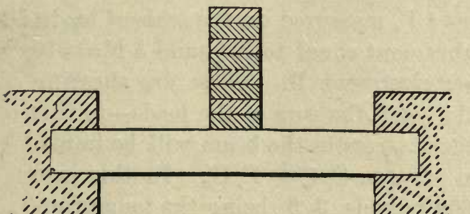


FIG. 421.

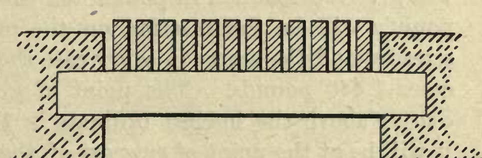


FIG. 422.

Beams with both ends fixed, and loaded at center (Fig. 421).

Safe load $240 = \frac{b\,d^2}{l}$.

Beams with both ends fixed, and load uniformly distributed (Fig. 422).

Safe load $= 360\,\frac{b\,d^2}{l}$.

If the load on the beam be neither at its center nor distributed as in Fig. 423, lay off on any convenient scale an inclined line, A C, between the abutments, equal to the weight of the load. Let fall a perpendicular from the bearing-point of the load to this line; it will divide it inversely proportional to the load on the abutments. In the figure, the load is 1,200 pounds; the perpendicular intersects the scale-line beneath at 900; 900 pounds is therefore the load on the abutment at B, and the balance of the weight, or 300 pounds, on the abutment A. To determine the size of beam of uniform section to resist the bending movements of the loads, multiply the loads on the abutments together, and divide by one quarter of the sum of the two loads. Thus, in the figure, $\frac{900 \times 300}{\frac{1}{4}\,(900 + 300)} = \frac{270,000}{300} = 900$, the load at the center of the beam, and the size can be readily determined by the formula or tables given.

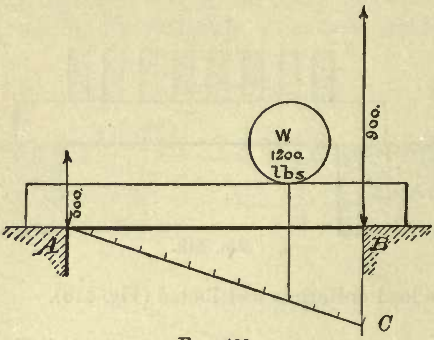


Fig. 423.

If the load is not distributed symmetrically, Fig. 424, the bending movement and shearing stresses may be readily determined graphically. Let loads equal to 100, 365, 850, and 125 pounds be supported as shown by the beam A B (say, 12 feet). At one side, on a line *a b*, perpendicular to the beam, lay off on any convenient scale, 100, 365, 850, 125, to represent the loads on the beam; from 1, 2, 3, 4, 5 draw lines meeting at some point, C. The point C can be chosen anywhere, but, for reasons that will be hereafter self-evident, it will be better to take C at a horizontal distance C D of either 10, 100, 1,000, etc., measured on the same scale as the loads on the line *a b*. From the points of support of the loads on the beam A B, let fall perpendiculars; from any point $C_,$ on line A $C_,$ draw the line $C_,\ 1_,$ parallel to C 1, $1_,\ 2_,$ parallel to C 2, $2_,\ 3_,$ parallel to C 3, $3_,\ 4_,$ parallel to C 4, and $4_,\ F_,$ parallel to C 5. Connect $C_,$ and $F_,$ and draw the line C F parallel to this. The distance 1 F, measured on the scale of loads, will give the reaction in pounds on the abutment equal to 530, and 5 F = 910 pounds will be the reaction on the other abutment B. These are shearing stresses, and their sum in every case should equal the sum of the loads—in this case, 1,440 pounds. The point of greatest stress in the beam will be immediately above the longest ordinate in the polygon $C_,\ 1_,$ — $F_,\ C_,$. In this case it will be at the point of support of the 850 pounds, $3_,\ 3_{,,}$ being the longest ordinate in the polygon. This ordinate, 2·7, measured on the scale of the beam

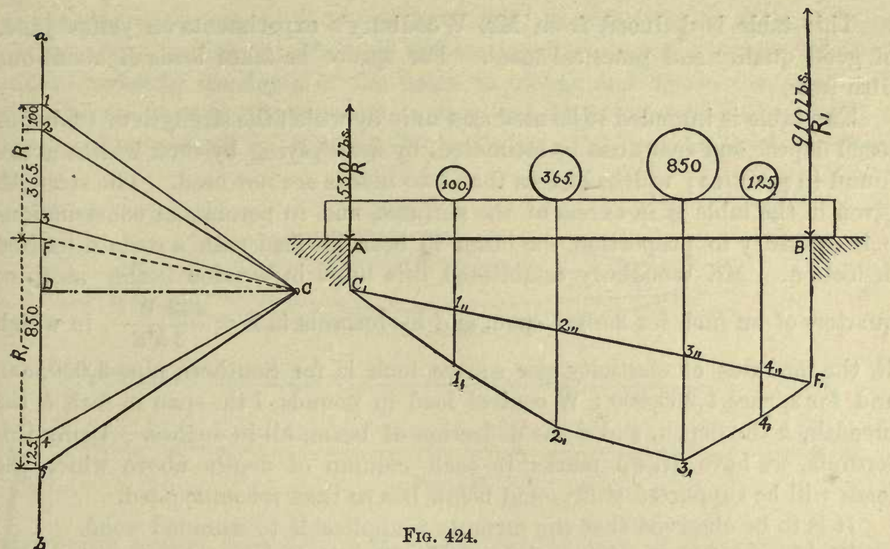


FIG. 424.

multiplied by the horizontal ordinate C D (taken here at 1,000), will give 2,700. This number, divided by 3, one quarter of the span A B of the beam, will give the center load, equal to 900, for which the size can be determined by the formula or tables as before.

TABLE OF THE SAFE CENTRAL LOAD OF YELLOW-PINE BEAMS, CALCULATED FROM THE FORMULA $-120 \frac{b d^2}{l}$.

Span in feet.	DEPTH IN INCHES OF YELLOW-PINE BEAMS, ONE INCH WIDE.													
	3 ins.	4 ins.	5 ins.	6 ins.	7 ins.	8 ins.	9 ins.	10 ins.	11 ins.	12 ins.	13 ins.	14 ins.	15 ins.	16 ins.
4	270·	480·	750·	1080·	1470·	1920·	2430·	3000·	3630·	4320·	5070·	5880·	6750·	7680·
5	216·	384·	600·	864·	1176·	1536·	1944·	2400·	2904·	3456·	4056·	4704·	5400·	6144·
6	180·	320·	500·	720·	980·	1280·	1620·	2000·	2420·	2880	3380·	3920·	4500·	5120·
7	154·	274·	430·	616·	840·	1097·	1389·	1714·	2074·	2469·	2897·	3360·	3857·	4388·
8	135·	240·	375·	540·	735·	960·	1215·	1500·	1815·	2160·	2535·	2940·	3375·	3840·
9	120·	213·	333·	480·	653·	853·	1080·	1333·	1613.	1920·	2253·	2613·	3000·	3413·
10	108·	192·	300·	432·	588·	768·	972·	1200·	1452.	1728·	2028·	2352·	2700·	3072·
11		175·	273·	392·	535·	700·	882·	1092·	1320·	1571·	1844.	2140·	2457·	2793·
12		160·	250·	360·	490·	640·	810·	1000·	1210·	1440·	1690·	1960·	2250·	2560·
13			230·	332·	452·	592·	747·	923·	1117·	1328·	1560·	1808·	2070·	2363·
14			215·	308·	420·	548·	693·	860·	1037·	1234·	1448·	1680·	1928·	2192·
15				288·	392·	512·	648·	800·	968·	1155·	1352·	1568·	1800·	2048·
16				270·	368·	480·	607·	748·	907·	1080·	1267·	1470·	1688·	1920·
17				254·	346·	452·	566·	704·	854·	1016·	1193·	1384·	1588·	1808·
18					327·	427·	540·	668·	806·	960·	1126·	1307·	1500·	1707·
19						404·	512·	632·	764·	909·	1067·	1238·	1422·	1616·
20						384·	486·	600·	726·	864·	1014·	1176·	1350·	1536·
21							463·	572·	691·	823·	966·	1120·	1287·	1463·
22							442·	546·	660·	785·	922·	1070·	1228·	1395·
23								522·	631·	752·	882·	1023·	1178·	1329·
24								500·	605·	720·	845·	980·	1125·	1280·
25									581·	691·	811·	940·	1080·	1230·
26									558·	665·	780·	904·	1035·	1182·

This table is deduced from Mr. Woodbury's experiments on yellow pine, of good quality and practical sizes. For spruce he takes loads of about one fifth less.

The table is intended to be used as a unit by which the strength of timber of usual depths and spans can be estimated, by multiplying by such widths as are found in practice; widths of less than two inches are not used. The strength given in the table is in excess of the stiffness, and in permanent constructions it is necessary to proportion the beam to bear its load with a certain limited deflection. Mr. Woodbury established this limit in wooden beams at three quarters of an inch for 25-feet span, and his formula is $E = \frac{432\ W\ l^3}{b\ h^3 d}$, in which E, the modulus of elasticity per square inch is for Southern pine 2,000,000, and for spruce 1,200,000: W central load in pounds, l the span in feet, b the breadth, h the depth, and d the deflection of beam, all in inches. Using this formula, we have drawn marks in each column of depth, above which the loads will be supported stiffly, and below less so than recommended.

It is to be observed that the formula is applicable to seasoned wood.

Wooden and wrought-iron beams are of uniform section for their entire span, but cast-iron can be readily adapted in form to the load to be sustained.

The forms of beams which afford equal strength throughout are parabolic (Figs. 425, 426, 427), of which the axis A B and the vertex A are given, and

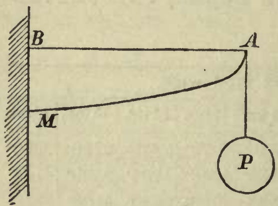


Fig. 425.

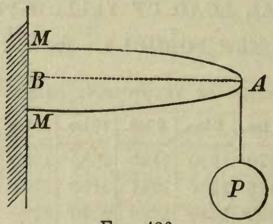


Fig. 426.

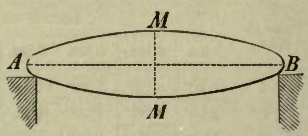


Fig. 427.

the points M determined by calculations. Figs. 426, 427 are oftener used when the force is applied on alternate sides of A B.

A beam subjected to a transverse stress, as shown in Fig. 413, one side is compressed, while the other side is extended; and therefore, where extension terminates and compression begins, there is a lamina or surface, $g\ h$, which is neither extended nor compressed, called the *neutral surface.* As the strains are proportional to the distance from this surface, the material of which the beam is composed should be concentrated as much as possible at the outer surfaces, as can readily be done in beams of cast and wrought iron. Acting on these principles, Mr. Hodgkinson has determined the most economical form for cast-iron beams or girders, of which the section is given (Fig. 428); it has been found that the strength of cast-iron to resist compression is about six times that to resist extension; the top web is therefore made only one sixth the area of the lower one. The depth of the beam is generally about one sixteenth of its length, the deeper of course the stronger; the thickness of the stem or the upright part should be from ½ an inch to 1½ inch, according to the size of the beam. The rule for finding the ultimate strength of beams of the above section is: Multiply the sectional area of the bottom flange in square inches by the depth of the beam in inches, and divide the product by the distance between the supports in feet, and 2·42 times the quotient will be the breaking weight in tons (2,000 pounds). As has already been shown

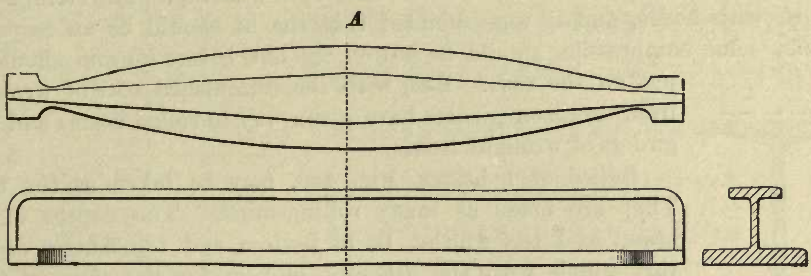


Fig. 428.

above, the section thus determined need only be that of the greatest strain, and can be reduced toward the points of support, either by reducing the width of the flanges to a parabolic form (Fig. 428), or by reducing the *thickness* of the bottom flange; the reduction of the girder in *depth* is not in general as economical or convenient.

For railway structures subject to an impulsive force, Mr. Joseph Cubitt, C. E., recommends that the section of the upper flange should be one third that of the lower.

Fig. 429 is side elevation, plan, and section of cast-iron girder, adopted by

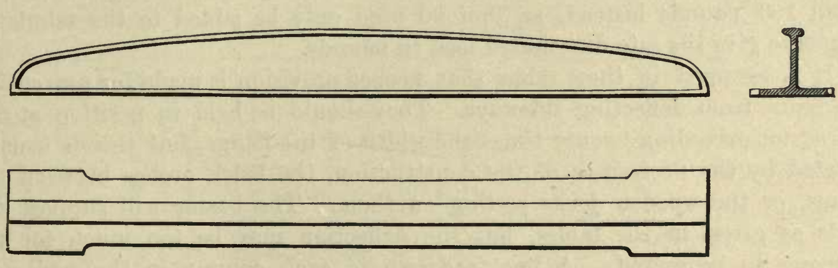


Fig. 429.

him for railway purposes, a pair of girders for each track, the rails being supported on wooden cross-beams.

DIMENSIONS FOR DIFFERENT SPANS.

Opening.	Bearing on abutment.	Height of girder at center.	Top flange.	Bottom flange at center.	At end.	Thickness of middle web.
12 ft.	1′·6″	1′·3″	3″ × 1½″	1′·4″ × 1½″	1′·8″ × 1½″	1½″
30 ft.	2′·6″	3′·	5″ × 2″	1′·6″ × 2″	1′·10″ × 2″	2″
45 ft.	2′·9″	3′·9″	7″ × 2½″	2′· × 2½″	2′· × 2½″	2″

Some years since the bow-string girder was in very common use in this city for span openings of from fifteen to twenty-five feet in the fronts and rears of stores and warehouses. The bow was made of cast-iron, in a ⊥-form, and the strings, or tension-rods, were of wrought-iron. In this composite structure it was impossible to calculate the strength of the girder, to decide how much was borne by the bow and how much by the string. The strings were forged with heads, and it was intended that the fit should be an easy one, so that some compression should be put on the bow before tension should be put on the rods. But, with the diminished cost of wrought-iron, cast-iron girders have given way to rolled beams and box-girders of wrought iron.

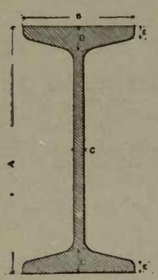


Fig. 430.

Rolled or I beams, Fig. 430, may be taken as the type. They are made at many rolling-mills. The depths of the beams and the widths, B, of bottom and top flanges do not vary much with the different makers for the same class of beams; the thickness of the stems varies somewhat more proportionally. For each depth there are usually two weights—the *light* and *heavy*—and are thus classed in the trade, as *light twelves* and *heavy twelves,* and lighter or heavier weights may be made to order.

There is considerable difference in the strengths of these beams as given in the tables of the different makers: in the table on page 233 we have tried to modify these discrepancies as far as possible, adopting that of no single maker; and to give dimensions such as will suffice for the purpose of the draughtsman in illustration, with tables of strength which can be relied on as practical. We have discarded the usual practice of stating strength in tons, and have taken 100 pounds instead, so that 00 need only be added to the tabulated figures to give the safe distributed load in pounds.

It is assumed in these tables that proper provision is made for preventing the beam from deflecting sideways. They should be held in position at distances not exceeding twenty times the width of the flange, but this is usually effected by the necessities of the construction, the brick arches between the beams, or the wooden joists resting on them. The beams will support the loads as given in the tables, but the deflection may be too much for the purposes to be served. A line is drawn in each column in the tables, at which the deflection is $\frac{1}{360}$, or one inch for every thirty feet of span, beyond which, if the beams carry plastered ceilings, the deflection is apt to crack the plastering.

A common formula for determining the strength of a wrought-iron beam is $W = \frac{8\,D\,(a + \frac{a'}{6})\,S}{L}$, in which W is the load in pounds, equally distributed on the beam, D the effective depth between the centers of gravity of the flanges, and L the clear span, both in the same unit, feet or inches; a the area of the top or bottom flange in square inches; a' the area of the stem.

To find the sectional area of a beam-plate or rod from its weight, divide the weight per yard by 10; and, conversely, to determine the weight per

TABLE OF THE COMPARATIVE STRENGTH AND STIFFNESS OF DIFFERENT SECTIONS OF WROUGHT-IRON I-BEAMS.

Depth of Beam.		3		4			5		6		7	
Width of Flange.	INCHES.	2·32	2·52	2·48	2·75	3·	2·75	3·	3·	3·46	3·5	3·91
Thickness of Stem.		0·19	0·39	0·23	0·25	0·313	0·25	0·313	0·25	0·46	0·35	0·53
Weight per foot.	LBS.	7·	9·	8·	10·	12⅜	10·	13¼	13⅜	18·	18	25·
DISTANCE BETWEEN SUPPORTS, IN FEET.	5	33·	38·	50·	60·	73·	78·	97·	131·	152·	210·	248·
	6	27·	31·	41·	50·	60·	65·	82·	109·	126·	175·	207·
	7	24·	27·	35·	42·	52·	56·	69·	93·	108·	150·	177·
	8	21·	24·	31·	37·	45·	49·	60·	82·	95·	131·	155·
	9	18·	21·	28·	33·	40·	43·	53·	73·	84·	116·	138·
	10	16·	19·	25·	29·	36·	39·	48·	65·	76·	105·	124·
	11	15·	17·	23·	26·	32·	36·	43·	59·	69·	95·	113·
	12	14·	16·	21·	24·	29·	33·	40·	54·	63·	87·	103·
	13			19·	22·	27·	30·	36·	50·	58·	81·	95·
	14			18·	21	25·	28·	33·	47·	54·	75·	89·
	15			17·	20·	23·	26·	31·	44·	51·	70·	83·
	16						25·	29·	41·	47·	66·	78·
	17						23·	27·	38·	45·	62·	73·
	18						22·	25·	36·	42·	58·	69·
	19								34·	40·	55·	65·
	20								33·	38·	52·	62·
	21								31·	36·	50·	59·
	22										48·	56·
	23										46·	54·
	24										44·	52·

Depth of Beam.		8		9		10		10¼		12		15		
Width of Flange.	INCHES.	4·	4·29	4·	5·375	4·32	4·77	4·54	5·	4·64	5·09	5·	5.4	5·81
Thickness of Stem.		0·3	0·79	0·26	0·6	0·32	0·77	0·41	0·47	0·51	0·96	0·5	0·7	0·93
Weight per foot.	LBS.	22·	35·	23½	50·	30·	45·	31½	45·	42·	60·	50·	67·	80·
DISTANCE BETWEEN SUPPORTS, IN FEET.	5	280·	362·											
	6	233·	301·											
	7	200·	258·											
	8	175·	226·											
	9	156·	201·											
	10	140·	181·	174·	394·	240·	300·	251·	—	367·	454·	565·	772·	800·
	11	127·	164·	158·	356·	218·	273·	228·	322·	334·	412·	513·	700·	727·
	12	117·	151·	145·	328·	200·	250·	209·	295·	306·	378·	471·	643·	667·
	13	108·	139·	134·	304·	185·	231·	193·	271·	282·	349·	434·	594·	615·
	14	100·	129·	124·	282·	171·	214·	179·	251·	262·	324·	404·	551·	571·
	15	93·	121·	116·	264·	160·	200·	167·	233·	245·	302·	377·	518·	533·
	16	88·	113·	109·	246·	150·	188·	157·	218·	230·	284·	353·	482	500·
	17	82·	106·	102·	232·	141·	176·	148·	204·	216·	267·	332·	453·	471·
	18	78·	100·	96·	218·	133·	167·	140·	192·	204·	252·	314·	429·	444·
	19	74·	95·	91·	206	126·	158·	132·	181·	193·	239·	297·	406·	421·
	20	70·	90·	87·	196·	120·	150·	126·	171·	184·	227·	282·	386·	400·
	21	67·	86·	83·	186·	114·	143·	120·	162·	175·	216·	269·	367·	381·
	22	64·	82·	79·	178·	109·	136·	114·	154·	167·	206·	257·	350·	364·
	23	61·	79·	75·	170·	104·	130·	109·	146·	160·	197·	245·	335·	348·
	24	58·	75·	72·	164·	100·	125·	105·	139·	153·	189·	235·	322·	333·
	25	56·	72·	69·	158·	96·	120·	100·	133·	147·	181·	226·	309·	320·
	26	54·	70·	67·	152·	92·	115·	97·	127·	141·	174·	217·	297·	308·
	27	52·	67·	64·	146·	89·	111·	93·	121·	136·	168·	209·	286·	296·
	28			62·	140·	86·	107·	90·	116·	131·	162·	202·	275·	286·
	29			60·	136·	83·	103·	87·	111·	126·	156·	195·	266·	276·
	30			58·	132·	80·	100·	84·	106·	122·	151·	188·	257·	267·
	31					77·	97·	81·	102·	118·	146·	182·	249·	258·
	32					75·	94·	79·	98·	114·	142·	177·	241·	250·
	33					73·	91·	76·	94·	111·	137·	171·	233·	242·
	34							74·	91·	108·	133·	166·	225·	235·
	35							72·	87·	105·	130·	161·	218·	229·
	36									102·	126·	157·	211·	222·
	37									99·	123·	153·	204·	216·
	38											149·	197·	211·
	39											145·		205·

linear foot from the sectional area, multiply the area by 10 and divide the product by 3.

Thus, if a bar a yard long weigh 40 pounds, its sectional area will be 4 square inches; and a bar of 9 square inches section will weigh $\frac{9 \times 10}{3} = 30$ pounds per foot.

For naval constructions, *deck-beams*, Fig. 431, are rolled at different mills, from 3″ to 12″ deep, and varied widths of flanges and thicknesses of stem; in general, not quite up to the grades of heavy and light I-beams in weight, but they can be rolled to order to any desirable dimensions within the limits of depth given. Properly proportioned, they should be equal in strength to the I-beams.

Coupled I-Beams.—When the load is beyond the strength of a single I-beam, two or more may be united, as shown in Fig. 432. A cast-iron block, or

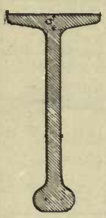


FIG. 431.

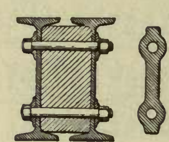


FIG. 432.

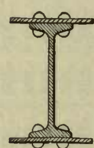


FIG. 433.

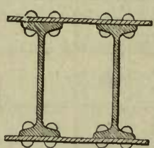


FIG. 434.

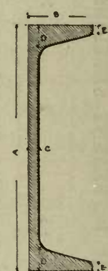


FIG. 435.

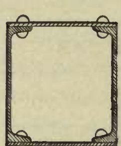


FIG. 436.

FIG. 437.

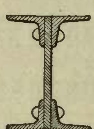


FIG. 438.

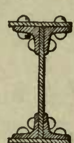


FIG. 439.

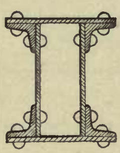


FIG. 440.

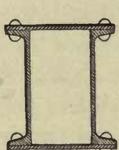


FIG. 441.

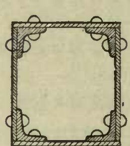


FIG. 442.

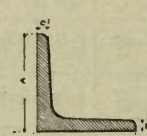


FIG. 443.

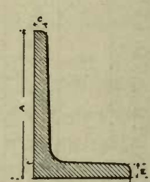


FIG. 444.

separator, is inserted between the beams, and two bolts, passing through them and the block, add lateral strength. The bolt-holes, if placed at some distance from the center of the span, do not reduce the transverse strength.

It is not unusual to strengthen an I-beam by the riveting of a plate on top (Fig. 433). It adds to the areas of the flanges by the area of the plate, less that of the rivet-holes in both plate and flange.

Box-girders are sometimes made up in the same way by two I's and plates across top and bottom (Fig. 434); but, as the access to the inside for holding the rivets is usually impossible, channel-beams (Fig. 435) are preferred for these forms, within the limits to which these beams are rolled.

Channel-beams can be furnished of depths the same as I-beams, from three to fifteen inches, of varied grades of light and heavy, and within any desirable limits of weight.

TABLE OF DIMENSIONS OF CHANNEL-BEAMS IN INCHES.

DEPTH.	Web. Thickness.	Flange.	
		Width.	Thickness.
3	·2 to ·3	1·51 to 1·61	$\frac{1}{4}$ to $\frac{7}{16}$
4	·24 " ·39	1·74 " 1·89	$\frac{1}{4}$ " $\frac{1}{2}$
5	·25 " ·55	1·93 " 2·23	$\frac{5}{16}$ " $\frac{9}{16}$
6	·23 " ·53	1·98 " 2·28	$\frac{5}{16}$ " $\frac{5}{8}$
7	·30 " ·55	2·30 " 2·55	$\frac{5}{16}$ " $\frac{3}{4}$
8	·30 " ·75	2·30 " 2·75	$\frac{3}{8}$ " $\frac{13}{16}$
9	·31 " ·71	2·43 " 2·83	$\frac{3}{8}$ " $\frac{7}{8}$
10	·31 " ·76	2·56 " 3·01	$\frac{3}{8}$ " $\frac{15}{16}$
12	·46 " ·96	2·71 " 3·21	$\frac{9}{16}$ " 1
15	·53 " ·93	3·53 " 3·93	$\frac{1}{2}$ " $\frac{7}{8}$

It may be desirable, on account of position, to finish a box-girder as in Fig. 436; in this case the dimensions must be such as to admit of a helper inside to hold the rivets. Fig. 437 shows a closed box-beam made of channel-bars and plates. The lower channel is first riveted, and the upper one afterward. This form gives a *clean* surface below, but the lower channel-bar can be reversed and riveted the same as the upper.

Where the purpose can be served by I-beams, either single, or coupled, as in Fig. 432, or in numbers, they afford the best and cheapest construction. But, where the spans are large and loads heavy, it is often economical to obtain greater depth by means of plate-girders, as in Figs. 438, 439, 440, 441, or perhaps from requirements of position, as in Fig. 442, subject as above to the necessities of large inside dimensions. These girders are made up of plates of uniform thickness, and angle-irons riveted together.

Angle-irons are made of varied dimensions, and are classed as *equal-legged* (Fig. 443), *unequal-legged* (Fig. 444), and *square-root angles* when the thickness of the iron is uniform throughout, and consequently the interior angle a complete right angle without rounding. The following table gives the dimensions and weights of the angles to be found at different mills, but weights can be increased to order:

ANGLE-IRON.—WEIGHT IN POUNDS PER FOOT.

SIZE, INCHES.	Average Thickness.												
	$\frac{1}{8}''$	$\frac{3}{16}''$	$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{7}{16}''$	$\frac{1}{2}''$	$\frac{9}{16}''$	$\frac{5}{8}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$\frac{7}{8}''$
Equal Legs.													
6 × 6							19·2	21·7	24·2	26·7	29·2	31·7	34·2
4 × 4					9·5	11·2	12·9	14·5	16·2	17·9	19·5		
$3\frac{1}{2} \times 3\frac{1}{2}$					8·3	9·7	11·2	12·7	14·1	15·6	17·0		
$3\frac{1}{4} \times 3\frac{1}{4}$					7·7	9·0	10·4	11·7	13·1	14·4	15·8		
3 × 3				5·9	7·2	8·4	9·7	10·9	12·2				
$2\frac{3}{4} \times 2\frac{3}{4}$				5·4	6·5	7·7	8·8						
$2\frac{1}{2} \times 2\frac{1}{2}$				4·9	5·9	7·0	8·0						
$2\frac{1}{4} \times 2\frac{1}{4}$			3·5	4·5	5·4	6·4	7·3						
2 × 2			3·1	4·0	4·8	5·6							
$1\frac{3}{4} \times 1\frac{3}{4}$		2·1	2·8	3·5	4·3	5·0							
$1\frac{1}{2} \times 1\frac{1}{2}$		1·8	2·4	3·0	3·6								
$1\frac{1}{4} \times 1\frac{1}{4}$	1·0	1·5	2·0										
$1\frac{1}{8} \times 1\frac{1}{8}$	0·9	1·4	1·8										
1 × 1	0·8	1·2	1·6										
$\frac{3}{4} \times \frac{3}{4}$	0·6	0·9											

ANGLE-IRON.—WEIGHT IN POUNDS PER FOOT.

SIZE, INCHES.	AVERAGE THICKNESS.												
	$\frac{1}{8}''$	$\frac{3}{16}''$	$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{7}{16}''$	$\frac{1}{2}''$	$\frac{9}{16}''$	$\frac{5}{8}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$\frac{7}{8}''$
UNEQUAL LEGS.													
6½ × 4						14·6	16·6	18·6	20·6	22·7	24·7	26·7	28·7
6 × 4						13·9	16·0	18·1	20·2	22·3	24·4	26·4	28·4
6 × 3½					11·3	13·2	15·0	16·6	18·4	20·2	22·1		
5 × 4					10·8	12·7	14·5	16·4	18·3	20·2	22·0		
5 × 3½					10·2	11·9	13·7	15·5	17·2	19·0	20·8		
5 × 3					9·5	11·2	12·9	14·5	16·2	17·9	19·5		
4 × 3½					8·9	10·5	12·0	13·6	15·2	16·7	18·3		
4 × 3					8·3	9·7	11·2	12·7	14·1	15·6	17·0		
3½ × 3					7·7	9·0	10·4	11·7	13·1	14·4	15·8		
3¼ × 2			4·2	5·3	6·4	7·4	8·5						
3 × 2½			4·4	5·5	6·7	7·8	9·0						
3 × 2			4·0	5·0	6·0	7·1	8·1						
2½ × 2			3·5	4·5	5·4	6·4	7·3						
2¼ × 1½		2·5	3·0	3·8	4·5								
2 × 1⅝		2·0	2·6	3·3	4·0								

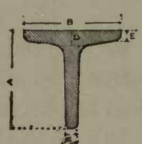


FIG. 445.

T-irons (Fig. 445) may be used for top and bottom flanges in the manufacture of plate-girders, by riveting a web on one side of the T, or on both sides, with a separator between of the thickness of the stem E; but, as the areas of section of T-irons to be had are small, the flanges will be too slight in proportion to the webs at depths above that of rolled beams. Angle-irons are then to be preferred for flanges. The T-irons are well adapted in many positions as struts or braces, and can be bought of varied dimensions and weights, from widths, B, of from 2 to 5 inches, and equal or less depths, A, and thicknesses from $\frac{3}{16}''$ to $\frac{5}{8}''$.

Rivets for plate-girders are usually from $\frac{3}{8}''$ to $\frac{7}{8}''$ diameter, and *pitched* or spaced not more than 6″ nor less than 3″ between centers. The number of rivets through flange and stem are the same, but alternating. Usually angle irons and plates can be had of the full length of girder, but, where joints are necessary, they should be butt, with a splicing-piece to make the strength as nearly as possible uniform. Stiffeners are often necessary for the webs, which may be of band, angle, or T iron, and one should always be placed at each end, where the shearing stress is the greatest.

To construct a diagram from the formula, $W = \frac{8\,D\,(a + \frac{a'}{6})\,S}{L}$, in which the relation of the factors may be shown. Let S be 10,000, on account of loss of strength by rivet-holes, then $W = \frac{D}{L} \times (a + \frac{a'}{6})\,80{,}000$. On a sheet of cross-section paper, from a corner, O, lay off on the line of ordinates, 5, 10, 15, 20, 25, representing the factor $a + \frac{a'}{6}$. From the same O, on the line of abscissas, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{30}$, $\frac{1}{35}$, $\frac{1}{40}$, representing $\frac{D}{L}$. Suppose $\frac{D}{L} = \frac{1}{40}$, then $W = (a + \frac{a'}{6})\,2{,}000$. If $a + \frac{a'}{6}$ be $= 10$, then $W = 20{,}000$. From the intersection of ordinate on line of $\frac{1}{40}$, and abscissa line of 10, draw a line to the point O. This line will represent the safe distributed load W, and its intersections of the ordinates and abscissas will represent the relative proportions of the two factors $\frac{D}{L}$ and $a + \frac{a'}{6}$ under this load. On the abscissa line 15, and ordinate $\frac{1}{40}$, W $= 30,000$, on line 20, 40,000, and so on, and lines drawn from these intersections to O will represent W.

Fig. 446 is thus constructed, but lines below 5 and above 30 on line of ordinates are erased, as within these limits may be found most of the proportions required in practice.

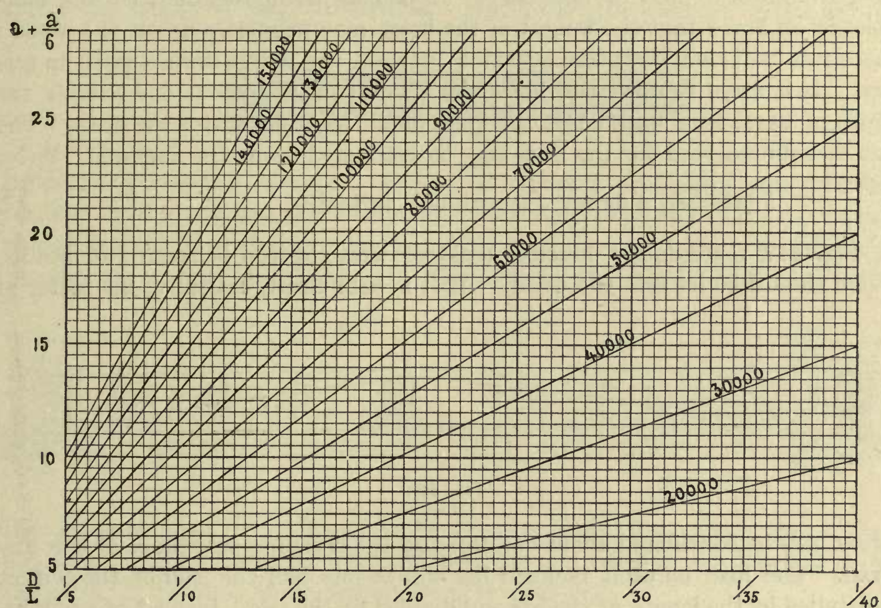


FIG. 446.

We should recommend to every draughtsman who needed this sort of table to construct one for himself on cross-section paper.

Application of the Diagram.—What will be the area of section $a + \frac{a'}{6}$ of a girder, 40-foot span, depth 32″, distributed load 90,000 pounds?

D in the formula represents the distance between the centers of gravity of the flanges, which will be somewhat less than the depth of beam. Approximately we assume it at 30″, $\frac{D}{L} = \frac{480''}{30''} = \frac{1}{16}$, and the intersection of the line of load, 90,000, with the ordinate $\frac{1}{16}$, will be 18, on the line of $a + \frac{a'}{6}$. A fair proportion of a to a' is 5 to 6, therefore $\frac{5a'}{6} + \frac{a'}{6} = 18$ or $a' = 18$. $\frac{18''}{30} = 0.6'' =$ thickness of web, and $a = \frac{5}{6}$ of $18 = 15$, or weight per foot of one flange $= \frac{15 \times 10}{3} = 50$ pounds, which is slightly in excess of the weight of two angle-irons $6 \times 4 \times \frac{3}{4}$, compensated by thickness of web outside centers of gravity.

This calculation is sufficiently near for all practical purposes, but D can be found more accurately by plotting the angle-irons as above, on thick card-board, cutting out and then balancing for the center of gravity.

Composite Beams.—Often, in constructions where the beams or girders are of wood, and on account of extent of spans and loads, the stress is beyond the strength and stiffness of beams of this material, of readily available dimensions, it is usual to supplement by some application of iron. A simple form, in which the iron is not exposed to view, is by bolting a plate or *flitch* of wrought-iron between two beams, of the full length and depth of the beams, and of such thickness as may be necessary. In bolting them together, let the bolt-holes be so bored that the weight of the beam may primarily be on the wood; the stress will then be better adjusted between the two materials when in service. It is usual to make the holes zigzag, in two lines about one quarter the depth of beam from each edge, the holes closer together nearer the ends. The safe-distributed load for the iron may be estimated from the formula: $W. = \frac{15000\,b\,h^2}{l}$, b breadth, h depth, l length—all in inches.

Fig. 447 represents a bracing truss of wrought-iron between two beams, which should be let into the wood. As it is held firmly laterally, the factor of

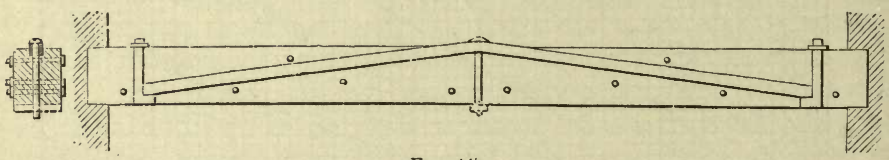


FIG. 447.

safety may be considered about one third of the crushing resistance of the material. The load on each inclined bar will be one half the load on the center, multiplied by the length of the bar and divided by the rise. Instead of wrought-iron, cast-iron or wood is used.

In Fig. 448 the beams are strengthened by a tension-rod, of which the strength may be determined by that of the material; allowing the usual factor

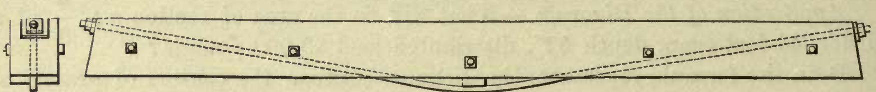


FIG. 448.

of safety, the load is obtained as in the example above. The deeper the block beneath the center of the beam, the less the stress on the rods for the same load. In construction, the beam should not be cambered by the screwing up of the rod; but, if the beams are crowning, the convex side should be placed upward, the nut turned by hand just to a bearing, and the tension put on by the settlement of the beams under the load.

Fig. 449 represents the trussing of a beam by two struts and a tension-rod. The stress on the tension-rod is the load on c, multiplied by the length $a\,d$, divided by $c\,d$.

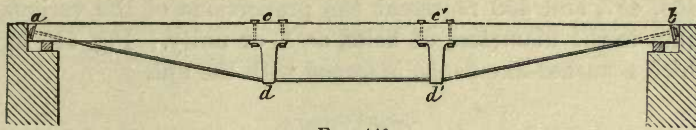


Fig. 449.

The theory of trusses will be treated and illustrated under "Bridges" and "Roofs," and the proportions of rivets and forms of plate-iron joints under "Boiler Construction."

Bolts and *nuts* are of such universal application that their manufacture forms the center of large industries. Much thought has been given to their

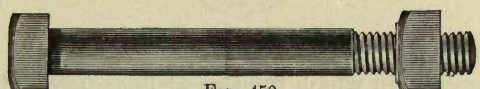


Fig. 450.

proportions and the forms of thread, but without producing complete uniformity in the practice of different countries and makers. The old form of thread was the A or sharp pitch (Fig. 451), still used by some, especially when the threads are cut in a lathe. In this country the standard U. S. thread is

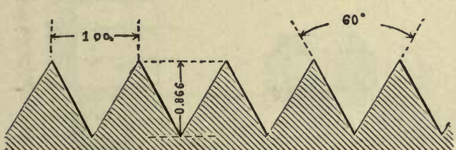


Fig. 451.

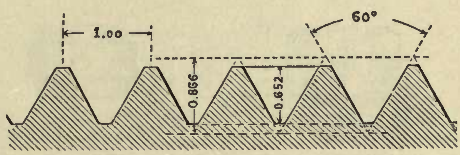


Fig. 452.

that recommended by the Franklin Institute in 1864 (Fig. 452). The angle is 60°, with straight sides and flat surface at top and bottom, equal to one eighth the *pitch*, or distance from center to center of threads.

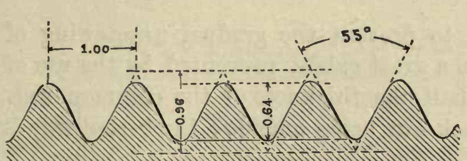


Fig. 453.

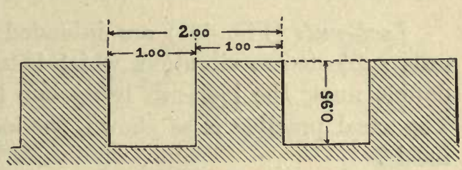


Fig. 454.

In England, the standard thread for bolts and nuts is the Whitworth (Fig. 453); the angle is 55°, with top and bottom rounded.

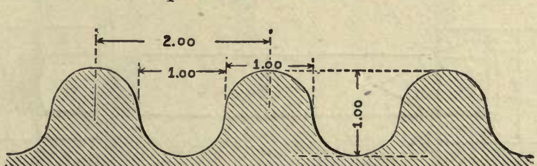


Fig. 455.

The square and rounded threads (Figs. 454 and 455) are only made to order and used in presses and the like as parts of machines.

Figs. 456, 457, and 458 represent the proportions of the various parts of English nuts to the diameters of bolts, as 1, or unity. Fig. 457 is a flange-nut, in which a washer-like flange is forged with the nut.

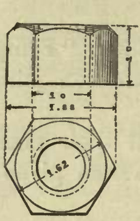


FIG. 456.

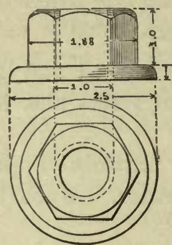


FIG. 457.

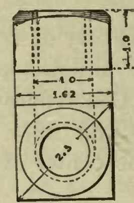


FIG. 458.

Fig. 459 is a cap-nut, in which the thread does not go through the nut, to prevent leaking along the thread, and a soft copper washer is introduced to prevent leakage below the nut.

Figs. 460 and 461 are circular nuts, in one of which holes are drilled to insert a rod for turning, and in the other grooves for a *spanner*.

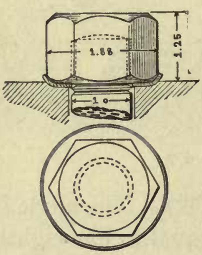


FIG. 459.

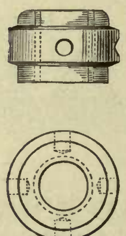


FIG. 460.

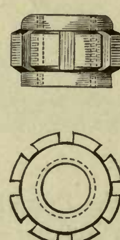


FIG. 461.

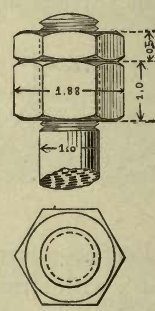


FIG. 462.

Lock-nuts (Fig. 462) are intended to prevent the gradual unscrewing of nuts subjected to vibration, which is to a great extent prevented by the use of double nuts, the lock-nut being one half the thickness of the common nut. The usual practice is as shown, the lock-nut being outside; the better way is inside.

The following figures are from trade circulars; the limits of sizes given are such as can usually be found in stock.

Figs. 463, 464, and 465 are machine-bolts, from $\frac{1}{4}''$ to $\frac{3}{4}''$ diameter, and 1″

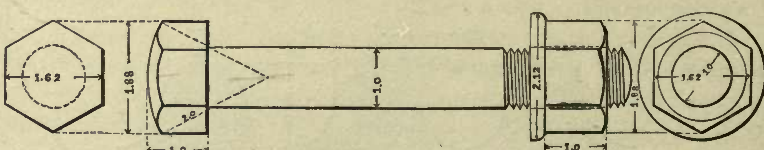


FIG. 463.

to 4″ long, but not flanged, as in Fig. 463, unless expressly ordered; the dotted line shows the radius of curvature of a finished head. The diagonal lines beneath the head (Figs. 464 and 465) represent square bolts tapering into round bolts, as shown by the curved lines.

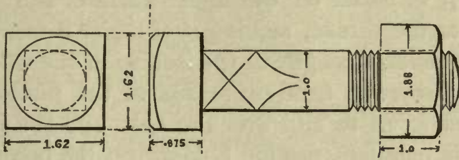


FIG. 464.

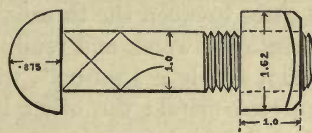


FIG. 465.

Figs. 466 and 467 are tap-bolts and set screws, from $\frac{1}{4}''$ to $\frac{3}{4}''$ diameter, and from 1″ to 3″ long.

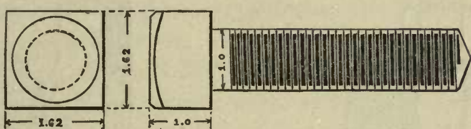


FIG. 466.

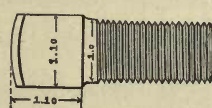


FIG. 467.

Fig. 468 is a carriage-bolt, from $\frac{1}{4}''$ to $\frac{3}{4}''$ diameter, and from 1″ to 16″ long. Fig. 469 is a plow-bolt, from $\frac{3}{8}''$ to $\frac{1}{2}''$ diameter, and from 1″ to 4″ long.

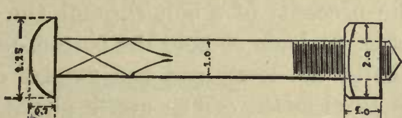


FIG. 468.

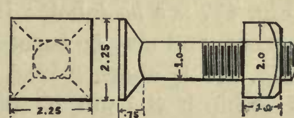


FIG. 469.

Fig. 470 is a stove-bolt, from $\frac{1}{4}''$ diameter and from $\frac{3}{4}''$ to 3″ long.

Figs. 471 and 472 are machine-screws without nuts ; the holes in the metals are tapped to receive them ; Fig. 471 is button-headed ; Fig. 472 a countersunk head—both slotted to admit of driving by a screw-driver. They are

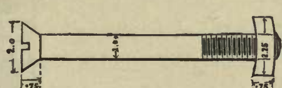


FIG. 470.

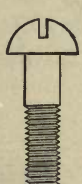


FIG. 471.

FIG. 472.

FIG. 473.

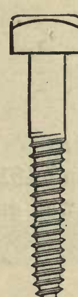


FIG. 474.

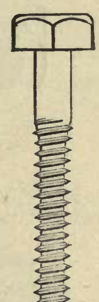


FIG. 475.

made of various sized wire and lengths, and sold by the gross like the common wood-screw (Fig. 473). The wood-screw is for connecting pieces of wood together, or metal to wood. They are of very great variety, usually with a gimlet-point, so that they can be driven into the wood, without any holes being previously made. When made of rods, with a square or hexagonal head (Figs.

474 and 475) to admit of the use of a wrench, they are called *lag-screws.* It will be seen that wood-screws differ in their thread from bolts and machine-screws. The thread is a very sharp V, flatter on the upper surface, and the flat space between the threads wide as the thread, making it of easier introduction into the wood, and retaining as much strength in the iron as in the wood.

Fig. 476 is a *stud*-bolt, which is screwed firmly into one of the pieces of connected metal; the other is bored so as to slip over the bolt, and the nut then brought down upon it. It is in common use for holding on the bonnets of steam-chests and water-chambers, the bolt remaining permanent.

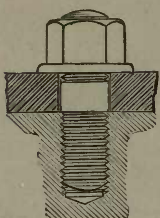


Fig. 476.

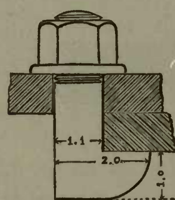


Fig. 477.

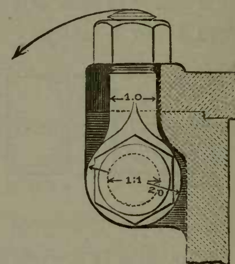


Fig. 478.

Fig. 477 is a *hook*-bolt; it relieves the necessity of a bolt through the bottom-piece, and may be turned like a button, to loose or hold the bottom-plate.

Fig. 478 is another kind of button-bolt; the lower end can revolve on a stud or pin if the nut be raised enough to clear the cap or upper plate. By this arrangement there is no necessity of taking off the nut entirely; the bolt lies in a slot in the cap, and the nut bears on three sides.

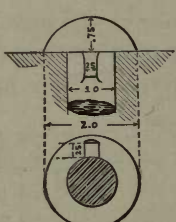


Fig. 479.

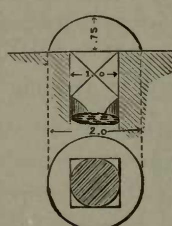


Fig. 480.

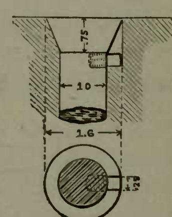


Fig. 481.

Figs. 479, 480, and 481 show expedients to prevent the bolt from turning when the nut is screwed on or off.

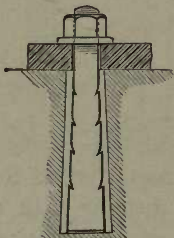


Fig. 482.

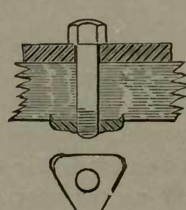


Fig. 483.

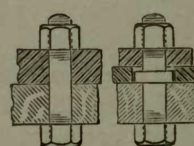


Fig. 484.

Fig. 482 is an *anchor*-bolt, flattened and jagged, introduced into a hole in masonry, and then leaded or sulphured in; but the more common way is to split the lower end of the bolt, insert a wedge into the cleft, place the bolt in the hole, and drive the wedge in against the bottom of the hole, thus keying the bolt in the hole.

Fig. 483 is a bolt with a *fang*-nut or corner turned down and driven into the wood to prevent turning; the screwing to be done at the head.

It is often convenient to use bolts with two nuts, as in Fig. 484, or *collar*-bolts, which are readily made to order, and of any dimensions.

Fig. 485 is a *hanger*-bolt; the lag-screw part is screwed into the wooden beam, the hanger then put over the bolt, and the nut put on.

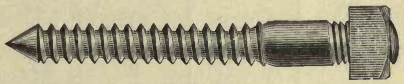


Fig. 485.

Figs. 486 and 487 represent forms of *turn-buckles*, and the *swivel* and *pipe*, sometimes designated as *swivels*. Turn-buckles are very useful in straining tierods, where neither end of the bolts can be got at. By turning the buckle, the rod can readily be made longer or shorter. In the pipe-swivel, *right* and *left* threads are cut on the bolts, so that each turn of the pipe shortens or lengthens the tie by double the pitch of the screw. The turn-buckle is also made in the same way, with two screws instead of a head at one end.

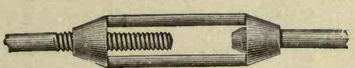


Fig. 486.

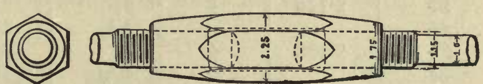


Fig. 487.

Screws, unless otherwise ordered, are made right-handed; that is, turning the nut to screw up, the hand moves from left to right, the apparent motion of the sun.

On the Strength of Bolts.—The strength of a bolt depends on its smallest section—that is, between the bottom of the threads. It is very common, therefore, especially in long bolts, to *upset* the screw-end, so that the screw may be cut entirely from this extra boss, or re-enforce. Bolt-ends (Fig. 488) are sold either with or without re-enforce, to be welded to bolts. It will be observed that the ends of the pipe-swivel bolts (Fig. 487) are thus upset.

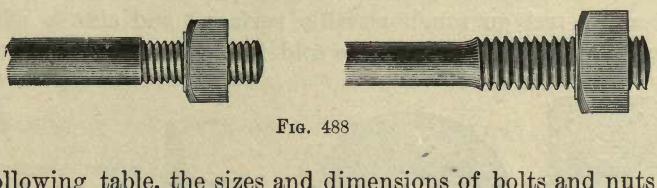


Fig. 488

In the following table, the sizes and dimensions of bolts and nuts are from the United States standard, and the strength, or safe-load of the bolts, is computed from the report of the committee on the test of wrought-iron and chain-cables to the United States Government in 1879. Nuts and heads as furnished are either hexagonal or square. Columns 4, 5, and 6 apply equally to either.

Diameter of screw in inches.	Diameter at root of thread.	Thread per inch of length.	Short diameter of nut and head.	Thickness of nut.	Thickness of head.	Safe-load of upset bolts.	Safe-load of plain bolts.
1/8			5/16	1/8	5/32		
3/16			13/32	3/16	13/64		
1/4	·185	20	1/2	1/4	1/4		
5/16	·240	18	19/32	5/16	19/64		
3/8	·294	16	11/16	3/8	11/32		1,700
7/16	·344	14	25/32	7/16	25/64		1,900
1/2	·400	13	7/8	1/2	7/16		2,200
9/16	·494	12	31/32	9/16	31/64		2,500
5/8	·507	11	1 1/16	5/8	17/32		2,800
11/16			37/32	11/16	37/64		3,200
3/4	·620	10	1 1/4	3/4	5/8	6.000	3,600
13/16			1 11/32	13/16	43/64	7,000	4,300
7/8	·731	9	1 7/16	7/8	23/32	8,000	5,100
1	·837	8	1 5/8	1	13/16	10,000	7,000
1 1/8	·940	7	1 13/16	1 1/8	29/32	12,000	9,000
1 1/4	1·065	7	2	1 1/4	1	15,000	11,000
1 3/8	1·160	6	2 3/16	1 3/8	1 3/32	18,000	13,500
1 1/2	1·284	6	2 3/8	1 1/2	1 3/16	21,000	16,000
1 5/8	1·389	5 1/2	2 9/16	1 5/8	1 9/32	24,000	19,000
1 3/4	1·490	5	2 3/4	1 3/4	1 3/8	28,000	22,300
1 7/8	1·615	5	2 15/16	1 7/8	1 15/32	32,000	25,500
2	1·712	4 1/2	3 1/8	2	1 9/16	36,000	29,300
2 1/8			3 5/16	2 1/8	1 21/32	40,000	33,000
2 1/4	1·962	4 1/2	3 1/2	2 1/4	1 3/4	45,000	37,000
2 3/8			3 11/16	2 3/8	1 27/32	50,000	41,500
2 1/2	2·175	4	3 7/8	2 1/2	1 15/16	55,000	46.000
2 5/8			4 1/16	2 5/8	2 1/32		
2 3/4	2·425	4	4 1/4	2 3/4	2 1/8		
2 7/8			4 7/16	2 7/8	2 7/32		
3	2·629	3 1/2	4 3/8	3	2 3/16		
3 1/4	2·879	3 1/2	5	3 1/4	2 1/2		
3 1/2	3·100	3 1/4	5 3/8	3 1/2	2 11/16		
3	3·317	3	5 3/4	3 3/4	2 7/8		
4	3·567	3	6 1/8	4	3 1/16		
4 1/4	3·798	2 7/8	6 1/2	4 1/4	3 1/4		
4 1/2	4·028	2 3/4	6 7/8	4 1/2	3 7/16		
4 3/4	4·255	2 5/8	7 1/4	4 3/4	3 5/8		
5	4·480	2 1/2	7 5/8	5	3 13/16		
5 1/4	4·730	2 1/2	8	5 1/4	4		
5 1/2	5·053	2 3/8	8 3/8	5 1/2	4 3/16		
5 3/4	5·203	2 3/8	8 3/4	5 3/4	4 3/8		
6	5·423	2 1/4	9 1/8	6	4 9/16		

Washers (Fig. 489)—in common use to provide seatings for nuts which would otherwise rest on rough metallic surfaces, and also to adapt bolts to shorter spaces than their lengths—are sold for bolts up to 2″ diameter. Cir-

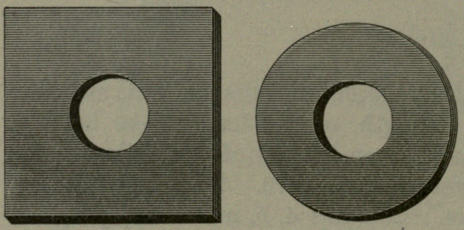


Fig. 489.

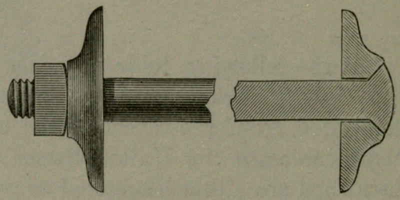


Fig. 490.

cular in form, their diameter is slightly in excess of that of the largest diameter of the nut, and the hole that of the bolt, and thickness from $\frac{1}{20}''$ to $\frac{1}{8}''$, according to the diameter of the bolt. The square washer is used under both head and nut on surfaces of wood, and of dimensions suited to the stress. That they may neither sink into the wood, nor bend or break, cast-iron is frequently used, and often, as shown in Fig. 490, for roof-frames.

Shafts and Axles.—Short shafts, revolving in bearings or boxes, or fastened with pulleys, drum, or wheels revolving on them, are called axles ; but shafts of large dimension or extent, and revolving, are usually termed shafts, as water-wheel shafts and fly-wheel shafts. These may be independent ; that is, a single shaft, revolving in its bearings, or they may be coupled together, forming what is termed a *line of shafting.* The small shafts, as in clock-work and spinning-machinery, are termed *pins* and *spindles.*

Shafts and axles are made of wood and metal, and of varied sections and form.

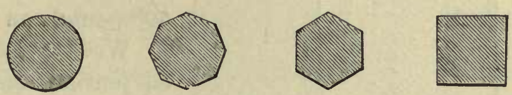


Fig. 491.

Wooden shafts are polygonal, circular, or square section (Fig. 491).

Wrought metal, iron, or steel shafts, are almost invariably circular in section, but sometimes square.

Cast-iron is used in great variety of section and form for shafts (Fig. 492) ; without uniformity longitudinally, but adapted to their position and load.

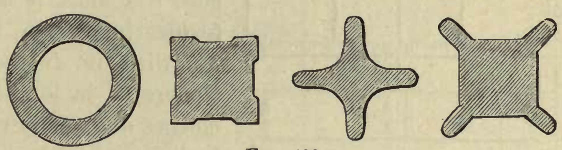


Fig. 492.

Formerly, either wood or cast-iron was invariably used for water-wheel shafts ; but a change of motors, from the breast, over-shot and under-shot wheels to reactors or turbines, has involved an entire change of construction, and now only wrought-iron is used. Still, wooden shafts are often used in machines subject to wet or shock, and often from greater convenience in procuring the material ; and, from the same cause, the bearings or bushings on which the shafts revolve are of the same material, and serve a good purpose where the movements are not continuous or rapid. But it is usual to make metal boxes, in which the rounded ends of shafts revolve ; these ends are called *journals* or *gudgeons.* The diameters and lengths of journals depend on the weight to be supported, the material of shafts and bearings, and the velocity at which the shafts are run.

TABLE OF DIAMETER OF JOURNALS FOR HEAVY WORK.

Total load in pounds.	Diameter in inches.		Total load in pounds.	Diameter in inches.		Total load in pounds.	Diameter in inches.	
	Cast-iron.	Wrought-iron.		Cast-iron.	Wrought-iron.		Cast-iron.	Wrought-iron.
1,100	2	1·7	30,000	6	5·1	137,000	10	8·6
3,700	3	2·5	44,000	7	6·0	183,000	11	9·4
8,800	4	3·4	70,000	8	6·9	237,000	12	10·3
17,000	5	4·3	100,000	9	7·7	312,000	13	11·2

The usual length of journals is from once to twice their diameters, but this is to be modified by the speed at which the shafts are run; if slow-moving, one diameter in length is ample, but at very high speed, and small shafts, like those of circular saws, from 4 to 6 diameters is not uncommon. If the boxes are of cast-iron, they will sustain the load given in the above table for large journals; but when the boxes are lined with brass and composition, or Babbit-metal, the first should not be loaded beyond 500 pounds per square inch, on half-circumferential section, or 750 pounds on the axial section. Babbit-metal should have a somewhat less load, say 500 pounds on the axial section.

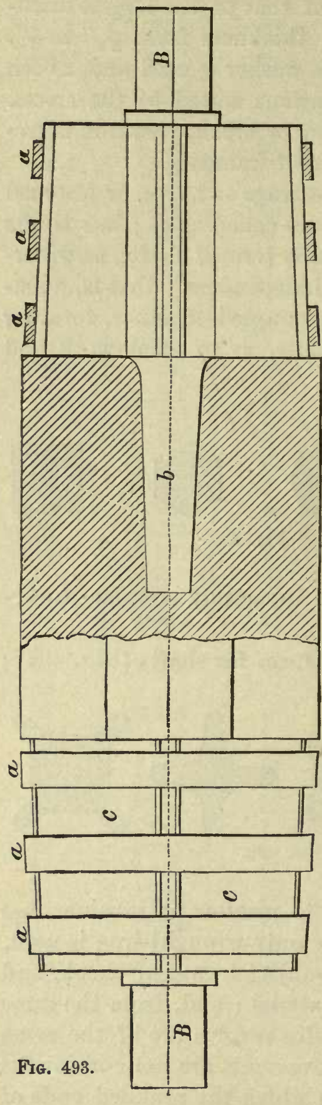


Fig. 493.

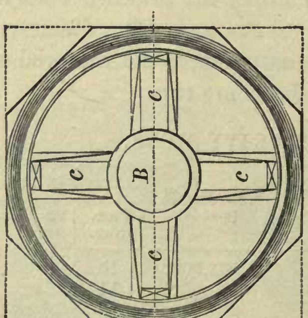


Fig. 494.

Wooden shafts are sometimes fitted with wooden journals and boxes, but the usual practice is to insert cast-iron journals.

Figs. 493 and 494 represent different views of a wooden shaft. Fig. 493 shows at one end the side elevation of the shaft, furnished with its iron ferules or collars and its gudgeon; at the other end, the shaft is shown in sections, giving the ferules in section, but showing the central spindle with its feathers in an external elevation. Generally, in longitudinal sections of objects inclosing one or more pieces, the innermost or central piece is not sectioned unless it has some internal peculiarity, the object of a section being to show and explain peculiarities, and being therefore unnecessary when the object is solid; on this account, bolts, nuts, and solid cylindrical shafts are seldom drawn in section. Fig. 494 is an end view of the shaft, showing the fitting of the spindle B and its feathers into the end of the shaft, and the binding of the whole by ferules or hoops, *a a*. The spindles B, which are let into the ends, are cast with four feathers or wings, *c*. The tail-piece *b* is by most millwrights omitted. The ends of the beam are bored for the spindle, and grooved to receive the feathers; the casting is then driven into its place, hooped with hot ferules, and after this hard-wood wedges are driven in on each side of the feathers, and iron spikes are sometimes driven into the end of the wood.

Figs. 495, 496, and 497 represent different views of a cast-iron shaft of a water-wheel. Fig. 495 is an elevation of the shaft, with one half in section to show the form of the core ; Fig. 496, an end elevation ; Fig. 497, a section on the line *c c* across the center. The body is cylindrical and hollow, and is cast with four feathers, *c c*, disposed at right angles to each other, and of an external parabolic outline. Near the extremities of these feathers four projections are cast, for the attachment of the bosses of the water-wheel. These projections are made with facets, so as to form the corners of a circumscribing square, as shown in Fig. 496, and they are planed to receive the keys by which they are fixed to the naves which are grooved to receive them. The shaft is cast in one entire piece, and the journals are turned.

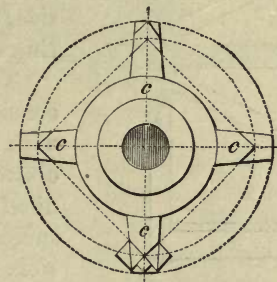


Fig. 496.

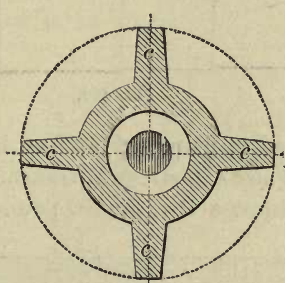


Fig. 497.

It will be observed that although no weight is supported at the center, yet there is an increase in the diameter of the feathers at this line ; the weight of the shaft itself is a considerable factor.

Fig. 498 represents the section of a portion of a breast water-wheel, with a cast-iron shaft, formerly much in use in this country, in which stiffness was given to the wheel and shaft by wooden trusses. These shafts are cast circular, in two lengths, connected at the center, with circular bosses on which the naves of the wheel are keyed.

Journals of independent shafts are always of less diameters than those of the rest of the shafts, and if the load on each is nearly equal the diameters of the two journals are equal ; but, if the load is not central, the diameters are proportioned to their several loads, as shown on pages 228, 229.

Shafts of wrought-iron, of less diameter than six inches, are fitted by turning down the journals

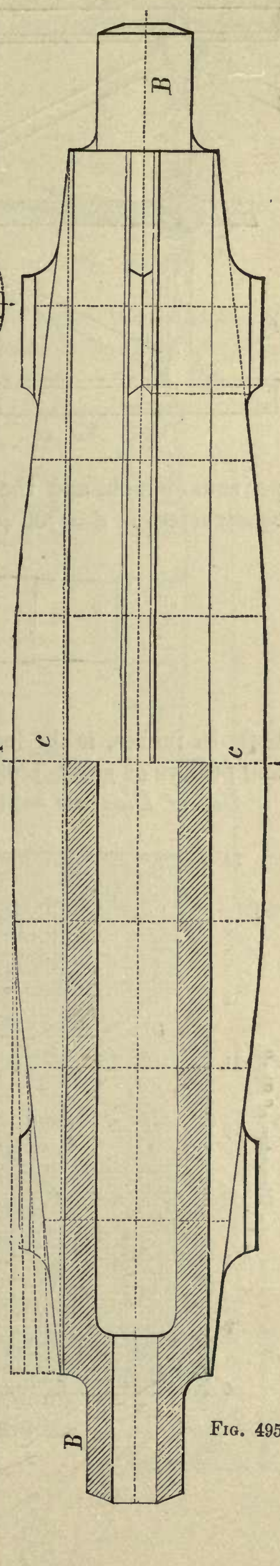


Fig. 495.

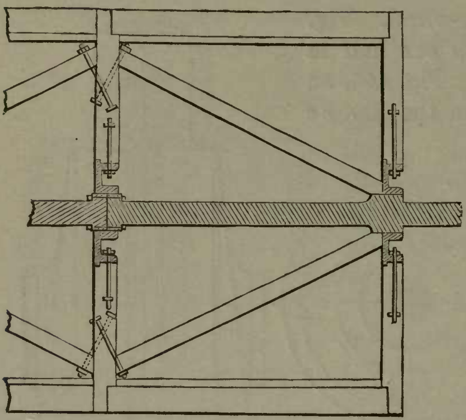


FIG. 498.

only. Large shafts are generally forged, mostly in steps, as in Fig. 499, with the largest boss beneath the gear or pulley-hub, and sufficiently above the next boss, on each side, to admit of the planing or cutting of the key-seats.

To determine the size of a shaft, considered as a beam merely, but with a shafting load—as by the revolution of the shaft—each longitudinal line of surface has to undergo successively tension and compression. The safe load of wrought-iron is estimated at 6,000 pounds per square inch, and the formula on which the graphic diagram (Fig. 500) is constructed is $d = \cdot06 \sqrt[3]{w\,l}$, d being diameter, l = length between bearings,

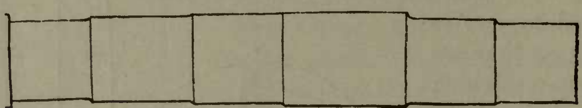


FIG. 499.

both in inches, w the load in pounds; the load is not only the weight of shaft and pulleys or gears, but also the stress in transmitting the power.

Use of Diagram.—Suppose w = 50,000 pounds, and l = 6 feet = 72″, then

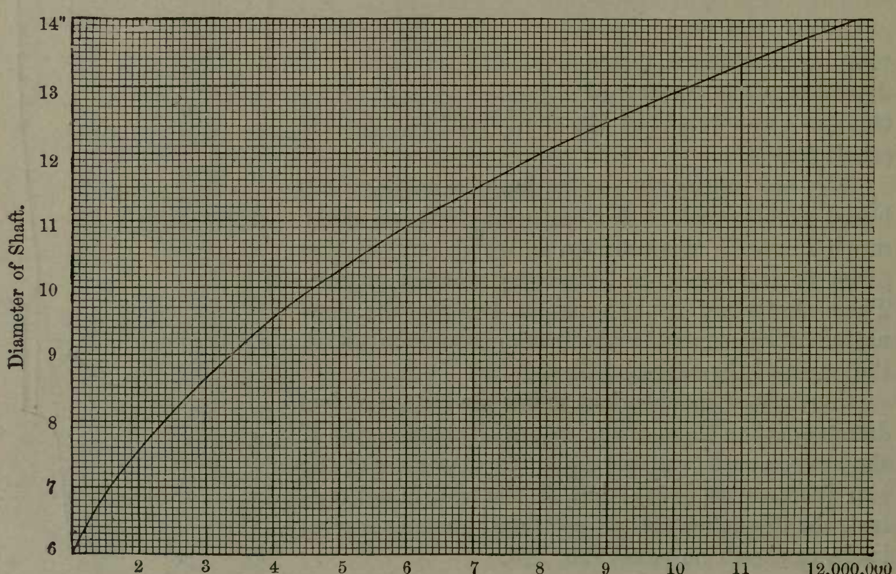


FIG. 500.

$w\,l = 3,600,000$, the ordinate of 3·6 cuts the curve on the abscissa 9·2, which is the required diameter of the shaft in inches.

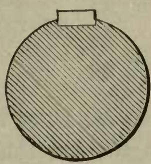


Fig. 501.

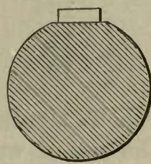


Fig. 502.

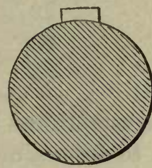


Fig. 503.

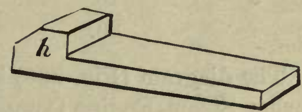


Fig. 504.

Keys are pieces of metal, usually steel, employed to secure the hubs of pulleys, gears, and couplings to shafts. They may be *sunk* keys (Fig. 501), *flat* keys (Fig. 502), and *hollow* keys (Fig. 503). The shaded circle represents the shaft. The breadth of the key (Fig. 504) is uniform, but the thickness is tapered about one eighth of an inch per foot. The shoulder h is for the purpose of drawing out the key. Sunk keys are not necessarily taper. Some prefer them of uniform section, and to force the hub on over the key.

PROPORTIONS OF SUNK KEYS.

DIAMETER OF SHAFT, IN INCHES.	1	2	3	4	5	6
Breadth of key	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{5}{8}$
Thickness of key	·25	·34	·43	·52	·61	·71
Depth sunk in shaft	·10	·125	·15	·175	·20	·225
Depth sunk in wheel	·15	·215	·28	·345	·41	·485

DIAMETER OF SHAFT, IN INCHES.	7	8	9	10	11	12
Breadth of key	$1\frac{7}{8}$	$2\frac{1}{8}$	$2\frac{3}{8}$	$2\frac{5}{8}$	$2\frac{7}{8}$	$3\frac{1}{8}$
Thickness of key	·80	·89	·98	1·07	1·16	1·25
Depth sunk in shaft	·25	·275	·30	·325	·35	·375
Depth sunk in wheel	·55	·615	·68	·745	·81	·875

Car-Axles.—Fig. 505 is the form and dimensions of axle adopted as standard by the American Master Car-Builders' Association for wrought-iron and steel.

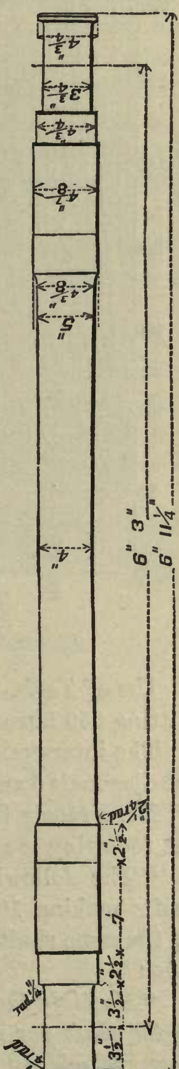


Fig. 505.

Shafting.—Thus far, independent shafts or axles have been treated of, and the dimensions have been established mostly by the load acting transversely; but, in transferring power to machines, lines of shafting are necessary, almost invariably of wrought-iron or steel bars, which are subject not only to transverse but also torsional stress. When there are no pulleys or gears on the shafts between the bearings, and the couplings are close to the bearings, there is still an amount of deflection due to the weight of the shaft. James B. Francis, C. E., puts the maximum distances between bearings for shafts of wrought-iron or steel, under these conditions, as follows:

Diameter of shaft.	Distance between bearings.	Diameter of shaft.	Distance between bearings.	Diameter of shaft.	Distance between bearings.
1″	12 ft.	5″	21 ft.	9″	26 ft.
2	15	6	22	10	27
3	18	7	24	11	28
4	20	8	25	12	28

The diagram (Fig. 506) is one established by J. T. Henthorn, M. E. of the Corliss Steam-Engine Company, to determine the size of wrought-iron shafting, to transmit a fixed amount of horse-power.

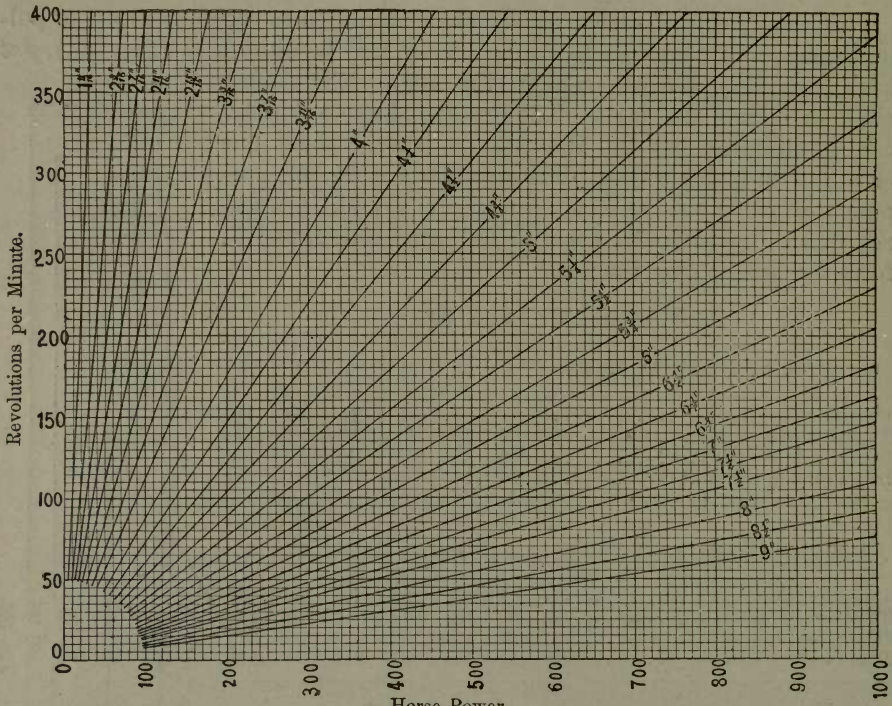


FIG. 506.

Use of Table.—To find the size of a shaft making 150 revolutions, and transmitting 350 horse-power.

The intersection of the ordinate of 350 with the abscissa of 150 is between the diagonals 5 and 5¼, and the diameter of the shaft may be taken safely at 5¼″.

Mr. Francis has constructed a table from his own experiments, of which the following is a synopsis :

"The following table gives the power which can be safely carried by shafts making 100 revolutions per minute. The power which can be carried by the same shafts at any other velocity may be found by the following simple rule :

"*Multiply the power given in the table by the number of revolutions made by the shaft per minute ; divide the product by 100 ; the quotient will be the power which can be safely carried.*"

Horse-power which can be safely transmitted by shafts making 100 revolutions per minute, in which the transverse strain, if any, need not be considered; if of

Diameter in inches.	Wrought-iron.	Steel.	Diameter in inches.	Wrought-iron.	Steel.	Diameter in inches.	Wrought-iron.	Steel.
1·	2·0	3·2	4·5	182·	291·	7·5	843·	1350·
1·5	6·7	10·7	5·	250·	400·	8·	1024·	1638·
2·	16·0	25·6	5·5	332·	532·	8·5	1228·	1965·
2·5	31·2	50·	6·	432·	691·	9·	1458·	2332·
3·	54·0	86·4	6·5	549·	878·	9·5	1714·	2743·
3·5	85·7	137·	7·	686·	1097·	10·	2000·	3200·
4·	128·	204·						

The diagram and table given are applicable to shafts which are called second movers, subject to no sudden shock. For first movers, Mr. Francis takes but one half the horse-power given in the table for any diameter of shafts. Of late, cold-rolled shafts can be procured in the market, which are much stiffer than turned shafts, but not equal to that given for steel in the table.

It is usual to make the shafts of second and third movers throughout manufactories and shops of uniform diameter, without reduction at the journals, the end-slip being prevented by collars keyed or fastened by set-screws. The usual length between bearings is from 7 to 10 feet; but that they may run smooth, and not spring intermediately, it is desirable that they should never be less than 2 inches diameter, and that the pulleys or gears through which the power is transmitted to the next mover or to the machine should be as near as possible to the bearing.

Fig. 507 represents a line of shafting. A is an upright shaft; *a a*, bevel-gears; *b b*, bearings for the shafts; *c*, coupling or connection of the several pieces of shafting. These shafts are intended to be of wrought-iron. No reduction is made for the journal, no bosses for pulleys or gears. As the power is distributed from this line of shafting, the torsional strain diminishes with

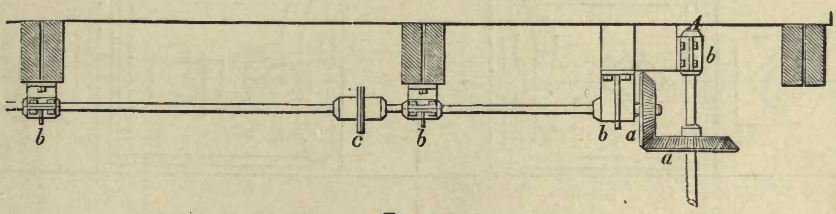


Fig. 507.

the distance from the bevel-gears or first movers, and the diameter of each piece of shafting may be reduced consecutively, if necessary; but uniformity will generally be found to be of more importance than a small saving of iron. The drawing given is of a scale large enough to order shafting by, but the dimensions should be written in.

In laying out lines of shafting, the position of the bearings is usually fixed, and the lengths of shafts must be determined thereby, with as few couplings as possible. When there is no necking or reduction of the shafts, which is usually the case, the orders given for shafting will be so many lengths and of such diameters, and so many couplings and hangers. When there is to be a necking, the sketch for the order may be very simple, showing length and diameter of shaft, and position, length, and diameter of bearing.

The joints or couplings are generally made near the bearings, and it is also usual to bring the pulleys as near the bearings as possible. It frequently happens, therefore, that the coupling and pulley are needed at the same point; to remedy this, as the position of the pulley depends on the machine which it is required to drive, it frequently can not be moved without considerable inconvenience or loss of room; the shaft will have, therefore, to be lengthened or shortened, to change position of coupling; or, if the couplings are plate couplings, the coupling and pulley may be made together.

When a horizontal shaft is supported from beneath, its bearing is usually called a *pillow-* or *plumber-block,* or *standard;* if suspended, the supports are called *hangers.*

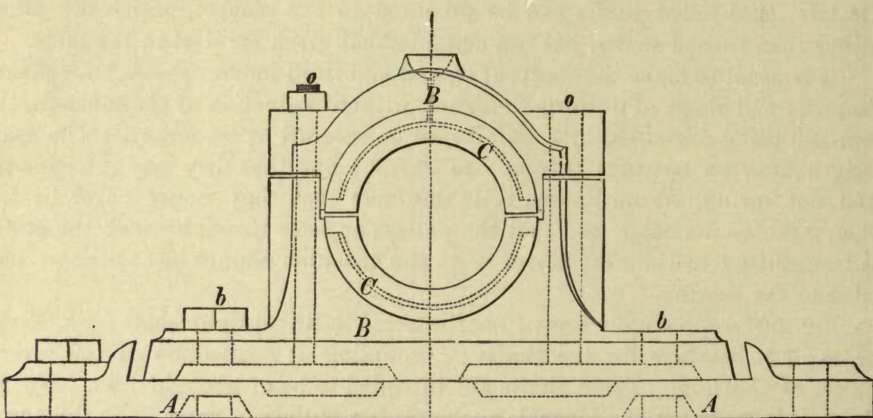


Fig. 508.

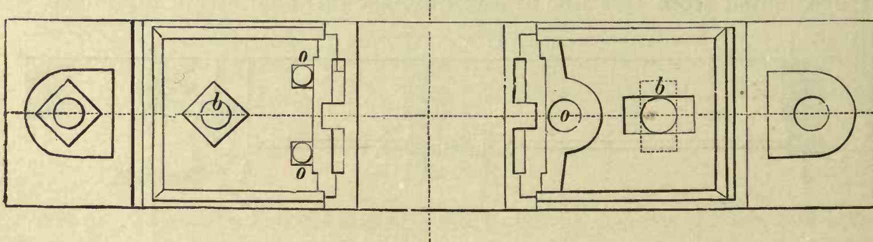


Fig. 509.

Figs. 508 and 509 are the elevation and plan of a pillow-block. It consists of a base plate, A, the body of the block B, and the box C. The plate, as in the step, is bolted securely to its base, the surface on which the block B rests being horizontal. A and B are connected by bolts passing through oblong holes, so as to adjust the position in either direction laterally. The box or bush C is of brass, in two parts or halves, extending through the block, and forming a collar by which it is retained in its place. The cap of the block is retained by the screws *o o o;* in the figure there are two screws on one side and one on the other; often four are used, two on each side, but most frequently but one on each side.

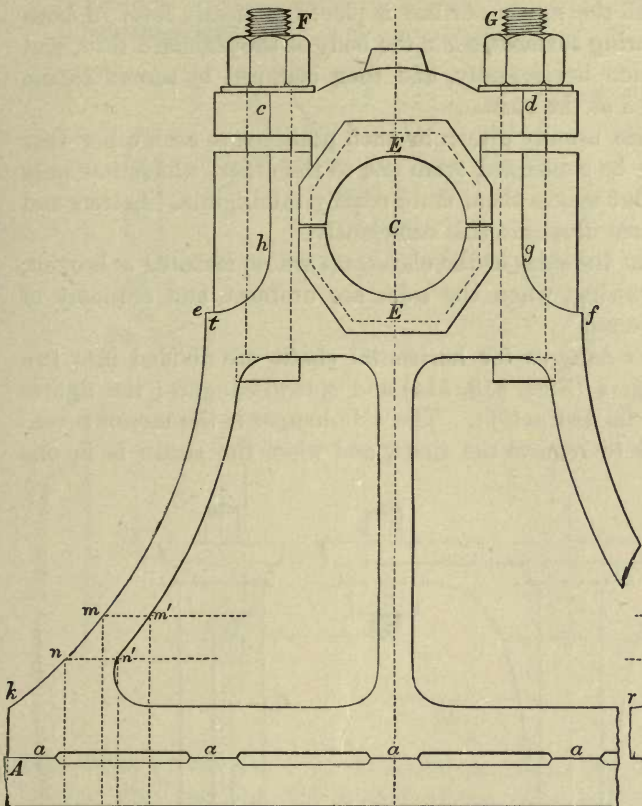


FIG. 510.

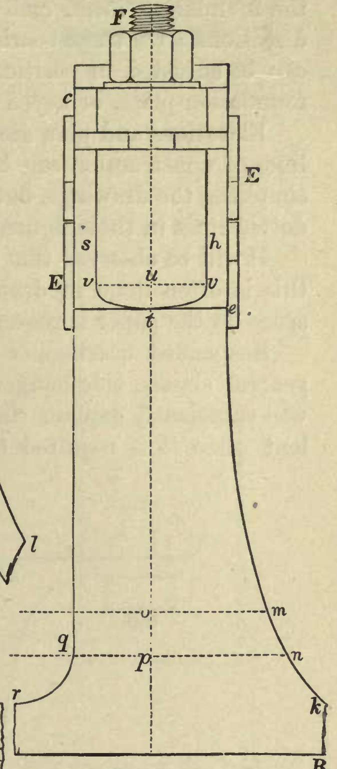


FIG. 512.

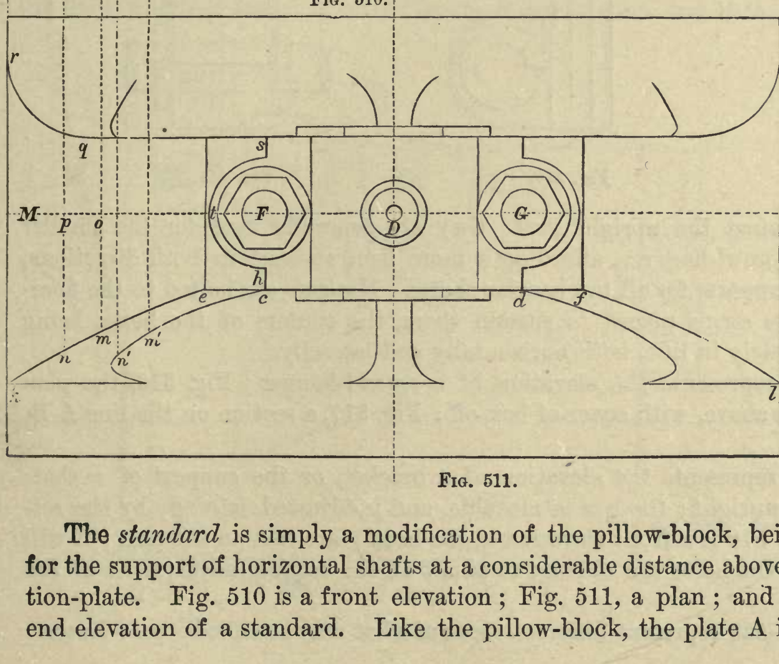


FIG. 511.

The *standard* is simply a modification of the pillow-block, being employed for the support of horizontal shafts at a considerable distance above the foundation-plate. Fig. 510 is a front elevation; Fig. 511, a plan; and Fig. 512, an end elevation of a standard. Like the pillow-block, the plate A is fastened to the foundation itself, and the upper surface is placed perfectly level in both directions. On these bearing surfaces, *a a a*, the body of the standard rests, and can be adjusted in position horizontally, and then clamped by screws to the foundation-plate, or keyed at the ends.

Elevations and plan are usually drawn in such positions to each other that lines of construction can be continued from one to the other, which not only simplifies the drawings, but makes them more readily intelligible. Letters and dotted lines in these figures illustrate this sufficiently.

It will be observed that the sides of the elevations are represented as broken; this is often done in drawing, when the sides are uniform, and economy of space on the paper is required.

Suspended bearings or *hangers* for horizontal shafts are divided into two general classes, side-hangers (Figs. 513, 514) and sprawl-hangers; the figures will sufficiently explain the distinction. The side-hanger is the more convenient when it is required to remove the shaft, and when the strain is in one

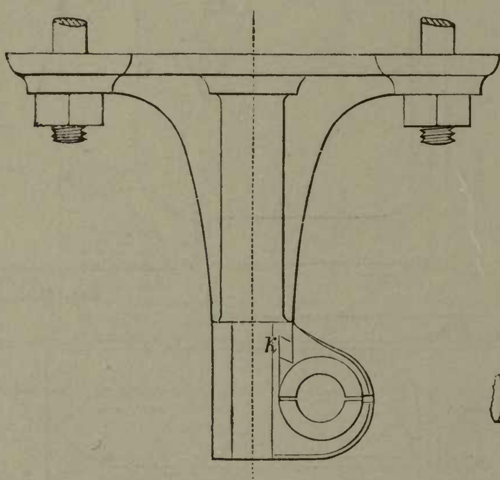


FIG. 513.

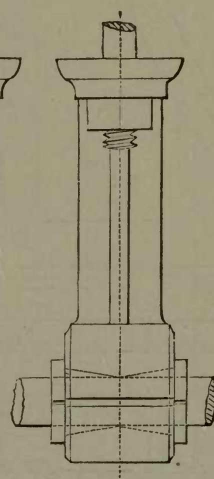


FIG. 514.

direction, against the upright part; they are generally used for the smaller shafts, but sprawl-hangers, affording a more firm support in both directions, are used as supports for all the heavier shafts. Hangers are bolted to the floor-timbers, or to strips placed to sustain them, the centers of the boxes being placed accurately in line, both horizontally and laterally.

Fig. 515 represents the elevation of a sprawl-hanger; Fig. 516, the plan looking from above, with cover of box off; Fig. 517, a section on the line A B, Fig. 515.

Fig. 518 represents the elevation of a bracket, or the support of a shaft bolted to an upright; the box is movable, and is adjusted laterally by the set-screws. Fig. 519 is a front elevation of the back plate cast on the post; it will be seen that the holes are oblong, to admit of the vertical adjustment of the bracket.

Figs. 520–523 represent different views of what may be called a *yoke-hanger*.

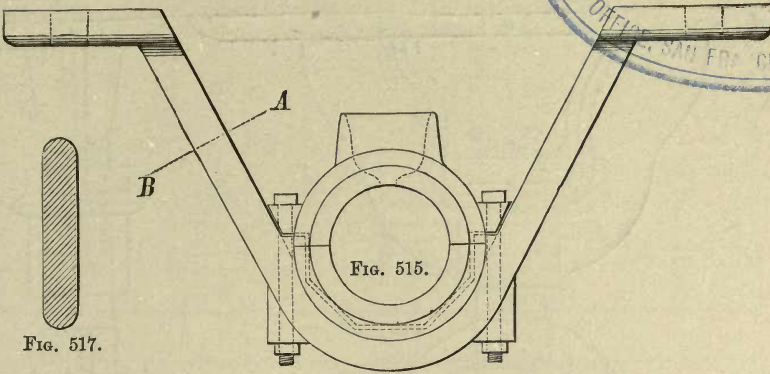


Fig. 515.

Fig. 517.

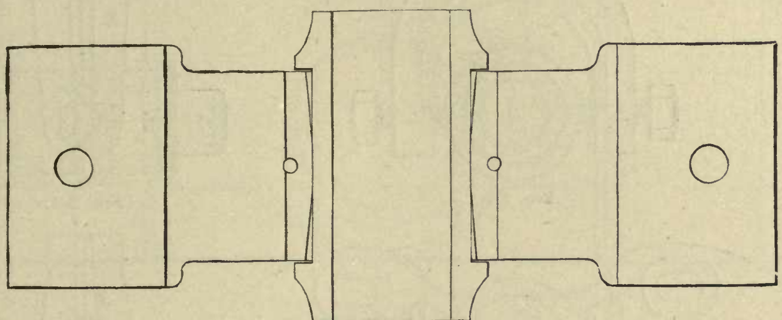


Fig. 516.

Fig. 520 is a front and Fig. 521 a side elevation; Fig. 522 a plan of the hanger, looking up; and Fig. 523 a plan of the yoke, looking down upon it. A is the plate which is fastened to the beam, E is the yoke, and B the stem of the

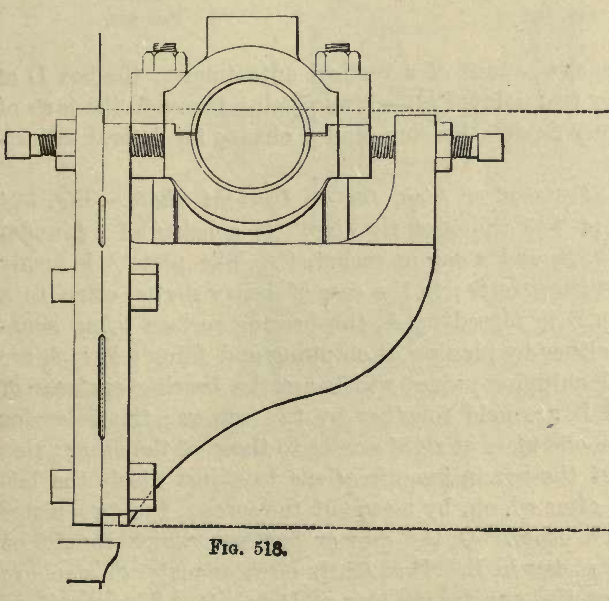


Fig. 518.

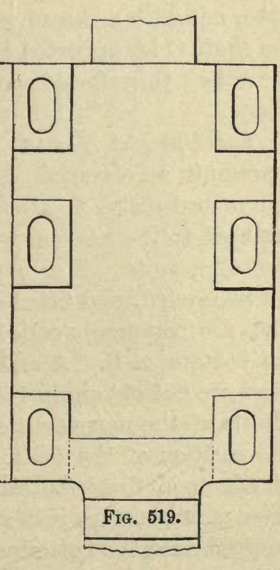


Fig. 519.

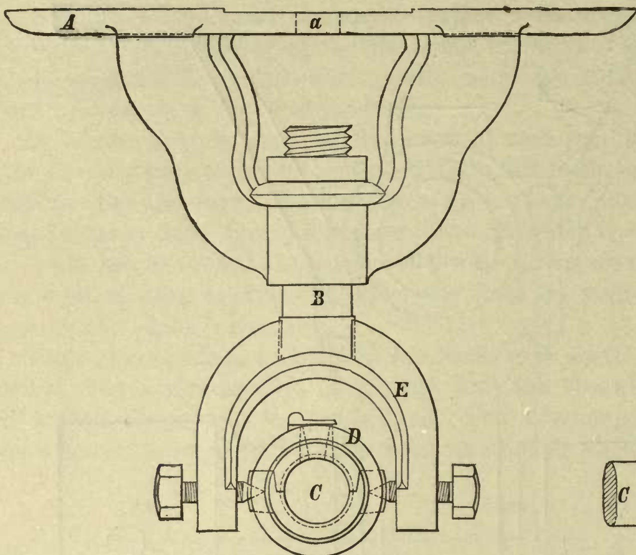


Fig. 520.

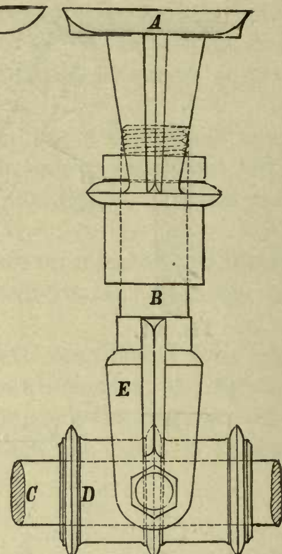


Fig. 521.

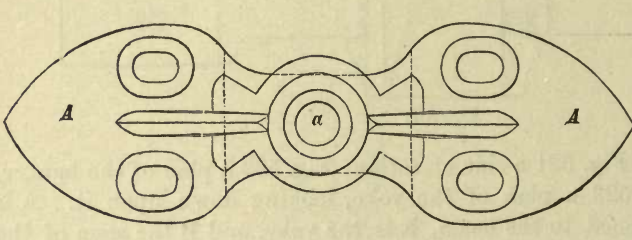


Fig. 522.

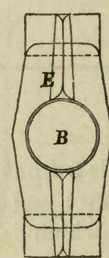


Fig. 523.

yoke, cut with a thread so as to admit of a vertical adjustment ; the box D of the shaft C is supported by two pointed set-screws passing through the jaws of the yoke ; this affords a very flexible bearing, and a chance for lateral adjustment.

For Upright Shafts.—Footstep, or Step, for an Upright Shaft.—Fig. 524 represents an elevation, Fig. 525 a plan of the step. It consists of a foundation or bed-plate, A, a box, B, and a cap or socket, C. The plate A is firmly fastened to the base on which it rests ; in the case of heavy shafts, often to a base of granite. The box B is placed on A, the bearing surface being accurately leveled, and fitted either by planing or chipping and filing ; *b, b, b,* are what are commonly called chipping-pieces, which are the bearing surfaces of the bottom of B. A and B are held together by two screws ; the holes for these are cut oblong in the one plate at right angles to those of the other ; this admits of the movement of the box in two directions to adjust nicely the lateral position of the shaft, after which, by means of the screws, the two plates are clamped firmly to each other. C, the cup or bushing, which should be made of brass, slips into a socket in B. Frequently circular plates of steel are dropped into the bottom of this cup for the step of the shaft. The cup C, in

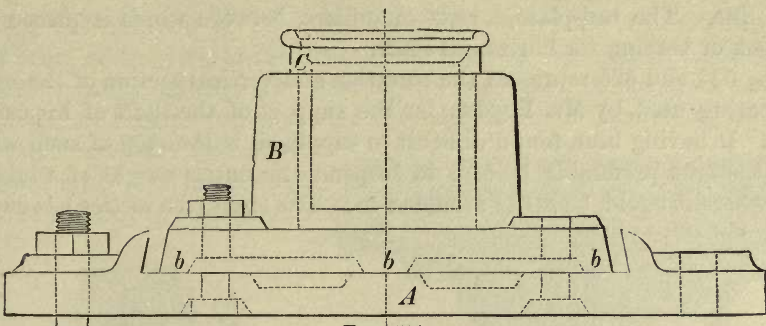


Fig. 524.

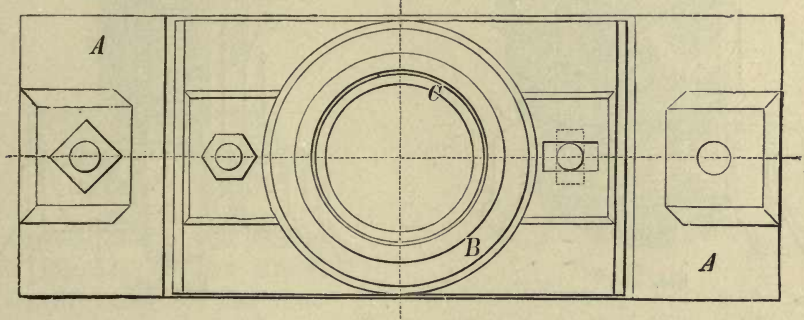


Fig. 525.

case of its sticking to the shaft, will revolve with the shaft in the box B; if plates are used, these also admit of movement in the cup.

Fig. 526 represents the elevation of a bearing for an upright shaft, in which

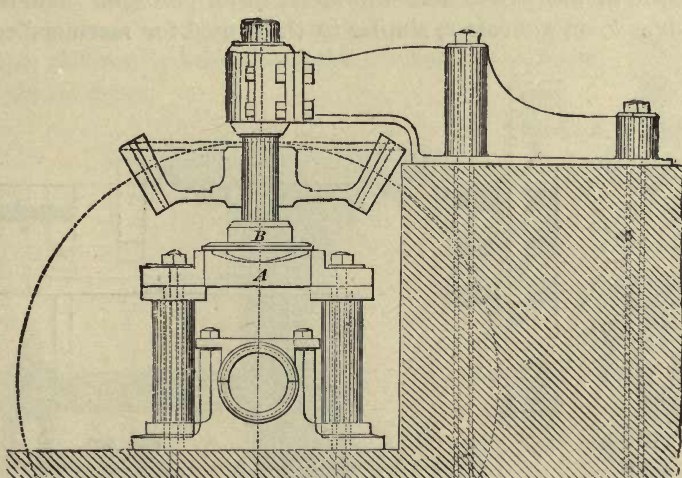


Fig. 526.

the shaft is held laterally by a box and bracket above the step. The step B is made larger than the shaft, so as to reduce the amount of wear incident to a heavy shaft. The end of the shaft and the cup containing oil are shown in dotted line. The bed-plate A rests on pillars, between which is placed a pillow-block or bearing for horizontal shaft.

Figs. 527 and 528 represent the elevation and vertical section of the suspension bearing used by Mr. Boyden for the support of the shaft of his turbine-wheels. It having been found difficult to supply oil to the step of such wheels, it was thought preferable by him to suspend the entire weight of wheel and shaft, where it could be easily attended to. The shaft (see section) is cut into

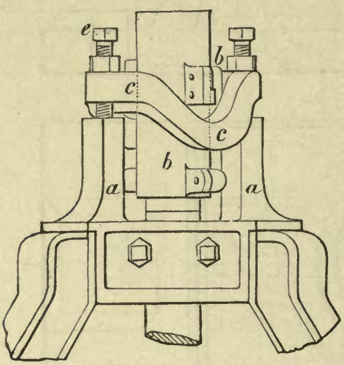


FIG. 527.

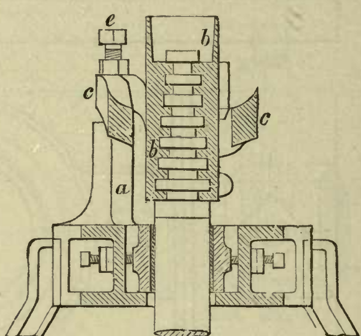


FIG. 528.

necks, which rest on corresponding projections cast in the box *b*; the spaces in the box are made somewhat larger than the necks of the shaft, to admit of Babbitting, as it is termed, the box; that is, the shaft being placed in its position in the box, Babbitt, or some other soft metal melted, is poured in round the shaft, and in this way accurate bearing surfaces are obtained; projections or holes are made in the box to hold the metal in its position. The box is suspended by lugs *b*, on gimbals *c*, similar to those used for mariners' compasses,

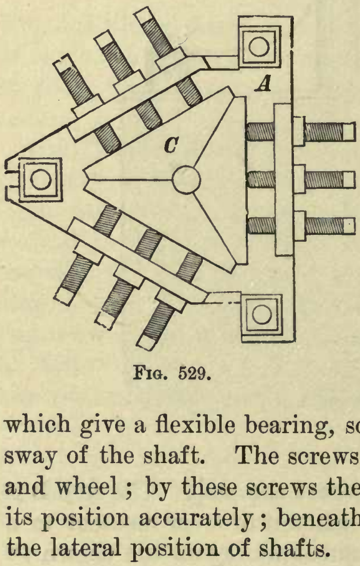


FIG. 529.

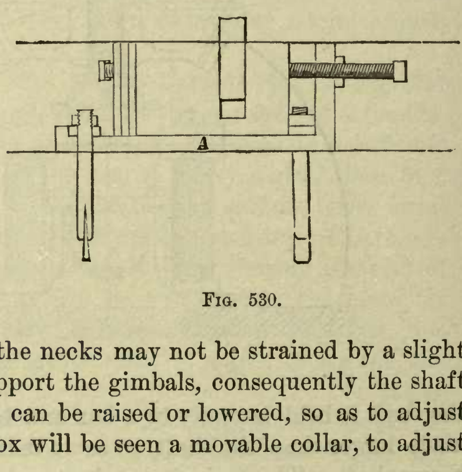


FIG. 530.

which give a flexible bearing, so that the necks may not be strained by a slight sway of the shaft. The screws *e e* support the gimbals, consequently the shaft and wheel; by these screws the wheel can be raised or lowered, so as to adjust its position accurately; beneath the box will be seen a movable collar, to adjust the lateral position of shafts.

Figs. 529 and 530 are the plan and elevation for the *step*, or rather *guide* (as it bears no weight), of the foot of the shaft of these same turbines. The plate A is firmly bolted to the floor of the wheel-pit; the cushions C, holding the shaft, are either wooden or cast-iron, and admit of lateral adjustment by the three rows of set-screws. In construction, the hanger and guide of Mr. Boyden were found

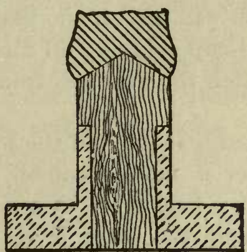


FIG. 531.

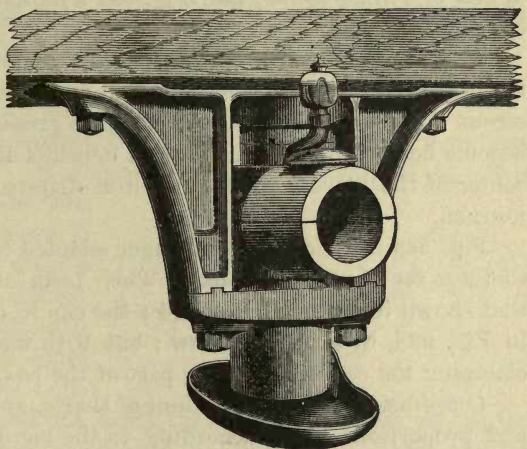


FIG. 532.

to be too expensive, and wooden steps (Fig. 531) are now almost universally used for turbines. They are made either conical or a portion of a sphere, of various woods, usually lignum-vitæ, but oak and poplar are preferred by some. The load is from fifty to seventy-five pounds per square inch. The fibers of the wood are placed vertically, and afford a very excellent bearing surface. Water is sometimes introduced into the center of the wood, or into a box around it, from the upper level of water. When cast-iron or steel is used for the step, it is usual to incase the box and supply oil by leading a pipe, sufficiently high above the surface of the water, to force the oil down.

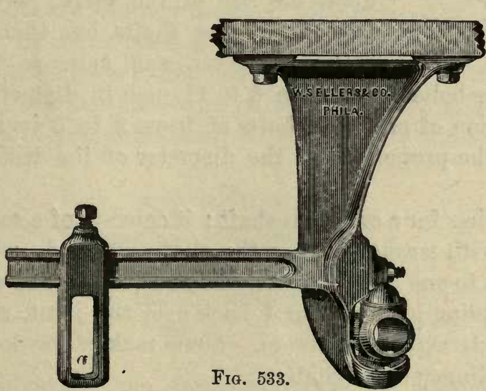


FIG. 533.

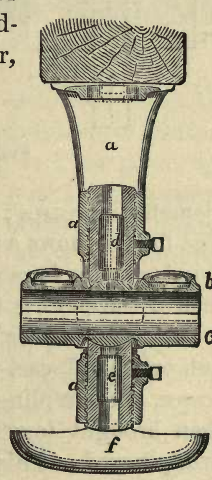


FIG. 534.

For long, upright shafts, it is very usual to suspend the upper portion by a suspension-box, and to run the lower on a step, connecting the two portions by a loose sleeve or expansion coupling, to prevent the unequal meshing of the bevel-wheels, incident to an alteration of the length of shaft by variations of temperature. The suspension is frequently made by a single collar at the top of the shaft.

Figs. 532 and 533 are perspectives of the hangers made by William Sellers & Co., of Philadelphia; and Fig. 534 a section showing the adjustment of the boxes in Fig. 532.

The boxes are of cast-iron, long in proportion to the diameters of shafts; the center bearings are spherical, and are adjusted in position vertically by the screws *d* and *e*; *b b* are cups, to contain grease, which will melt if the bearings become heated, but the lubrication depends on an oil-cup dripping oil into the center of the bearing; *f* is a cast-iron drip-pan to catch the waste oil from the journal.

Fig. 533 is a view of side hanger adapted to a counter shaft, and the square slot *a* is for the shipping-bar. This form of hanger is more common than that shown in Figs. 513 and 514; the cap in this last is held down by a wedge, in Fig. 533, by a lateral screw; but with most makers the screw is vertical, clamping the cap to the lower part of the box.

Couplings are the connections of shafts, and are varied in their construction and proportions often according to the mere whim of the mechanic making them.

The *Face Coupling* (Fig. 535) is the one in most general use for the connecting of wrought-iron shafts; it consists of two plates or disks with long, strong hubs, through the center of which holes are accurately drilled to fit the shaft; one half is now drawn on to the shaft, and tightly keyed; the plates are faced square with the shaft, and the two faces are brought together by bolts. The number and size of the bolts depend upon the size of the shaft; never less than 4 for shafts less than 3 inches diameter, and more as the diameter increases; the size of the bolts varies from $\frac{5}{8}$ to $1\frac{1}{4}$ inch in diameter. The figure shows a usual proportion of parts for shafts of from 2 to 5 inches diameter; for larger than these, the proportion of the diameter of the disk to that of the shaft is too large.

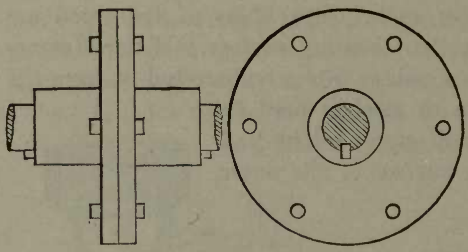


Fig. 535.

Fig. 536 is a rigid sleeve coupling for a cast-iron shaft; it consists of a solid hub or ring of cast-iron hooped with wrought-iron; the shafts are made with bosses, the coupling is slipped on to one of the shafts, the ends of the two are then brought together; the coupling is now slipped back over the joint, and firmly keyed. This is an extremely rigid connection. Some makers use keys without taper, and force the couplings on the shafts.

Fig. 537 is a screw coupling for the connecting of the lighter kinds of shafts. It will be observed that this coupling admits of rotation but in one direction, the one tending to bring the ends of the shafts toward each other; the reverse motion tends to unscrew and throw them apart, and uncouple them.

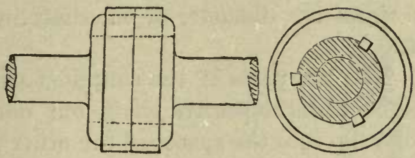


FIG. 536.

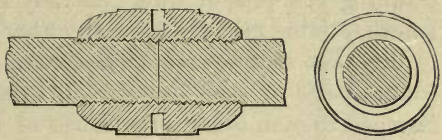


FIG. 537.

Fig. 538 is a clamp coupling for a square shaft.

William Sellers & Co., Philadelphia, make a double-cone vice coupling, which is largely used (Fig. 539). It is shown complete on shaft at A. B is the outer shell or sleeve, C the two cones, and D the bolts. The sleeve is cylindrical outside, but bored with a double taper inside, smallest at center. The cones are bored to fit the shaft, and turned outside to fit the interior cones of the sleeve. There are three bolt-grooves in the cones and sleeve, and one is cut through to give elasticity to the cones. The sleeve and cones are adjusted over the joint of the shafts, leaving at an easy fit some ⅜″ between the ends of the cones; if now the bolts be intro-

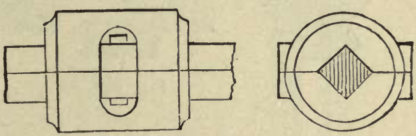


FIG. 538.

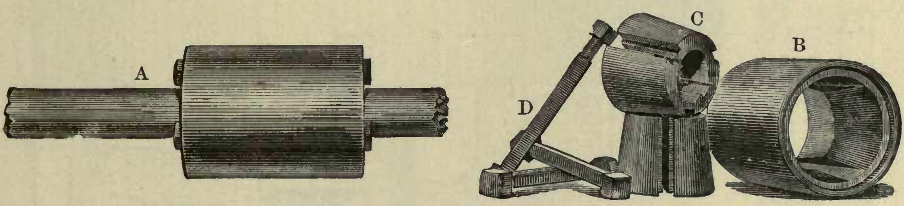


FIG. 539.

duced and screwed up, the cones are brought nearer to each other, and the shafts are securely clamped together. Fig. 540 shows the coupling in section.

In many cases it occurs that rigid couplings, such as we have given, are objectionable; they necessarily imply that, to run with the least strain possible, the bearings should be in accurate line; *any* displacement involves the springing of the shaft, and, if considerably moved, fracture of shaft or coupling.

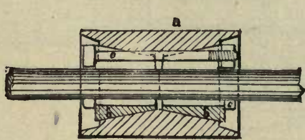


FIG. 540.

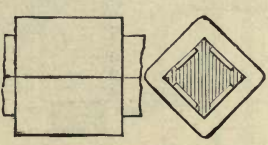


FIG. 541.

Wherever, then, from any cause the alignment can not be very nearly accurate, some coupling that admits of lateral movement should be adopted. The simplest of these is the *box* or *sleeve* coupling (Fig. 541), sliding over the end of two square shafts, keyed to neither, but often held in place by a pin passing through the coupling into one of the shafts. For round shafts, the loose sleeve coupling is a pipe or hub, generally 4 to 6 times the diameter of the shaft in length, sliding on keys fixed on either shaft.

Fig. 542 represents a *horned* coupling. The two parts of the coupling are counterparts of each other, each firmly keyed to its respective shaft, but not fastened to each other ; the horns of the one slip into the spaces of the other ; if the faces of the horns are accurately fitted, it affords an excellent coupling, and is not perfectly rigid.

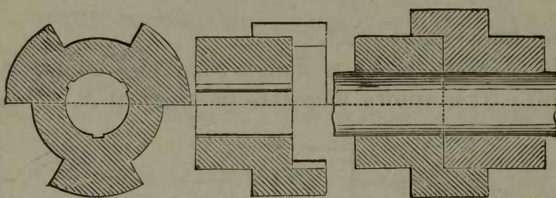


Fig. 542.

It often happens that some portion of a shaft or machine is required to be stopped while the rest of the machinery continues in motion. It is evident that, if one half of a horned coupling be not keyed to the shaft, but permitted to slide lengthways on the key—the key being fixed in the shaft, forming in this case what is more usually called a feather—by sliding back the half till the horns are entirely out of the spaces of the other half, communication of motion will cease from one shaft to the other.

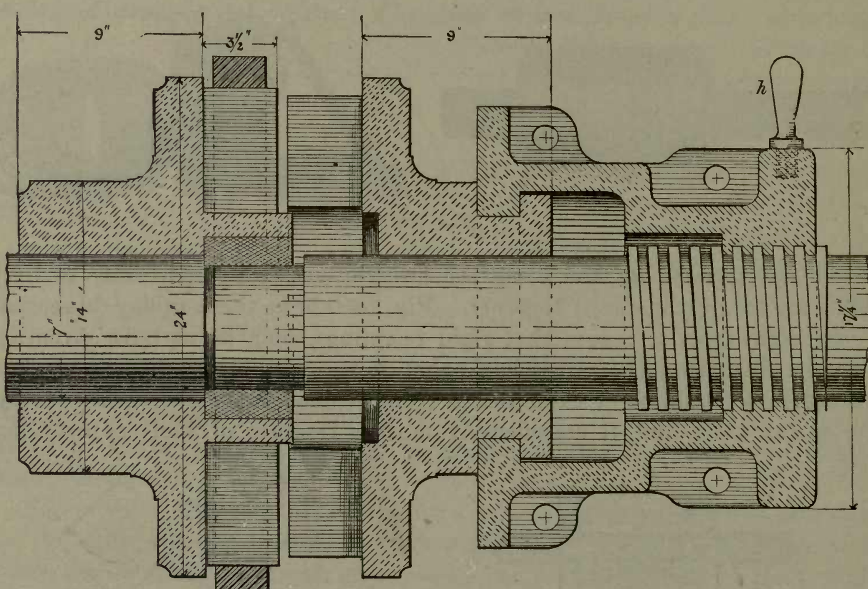


Fig. 543.

Fig. 543 represents a coupling of this sort for a large shaft, from the Corliss Steam-Engine Company. The horns are 8 in number on each part, and are thrown readily in or out of action by the handle *h* turning the loose part of the clutch on the screw cut on the shafts.

Fig. 544 is another form of disengaging a large pulley from a main shaft, from the Corliss Steam-Engine Company. The pulley is fastened to a cast-

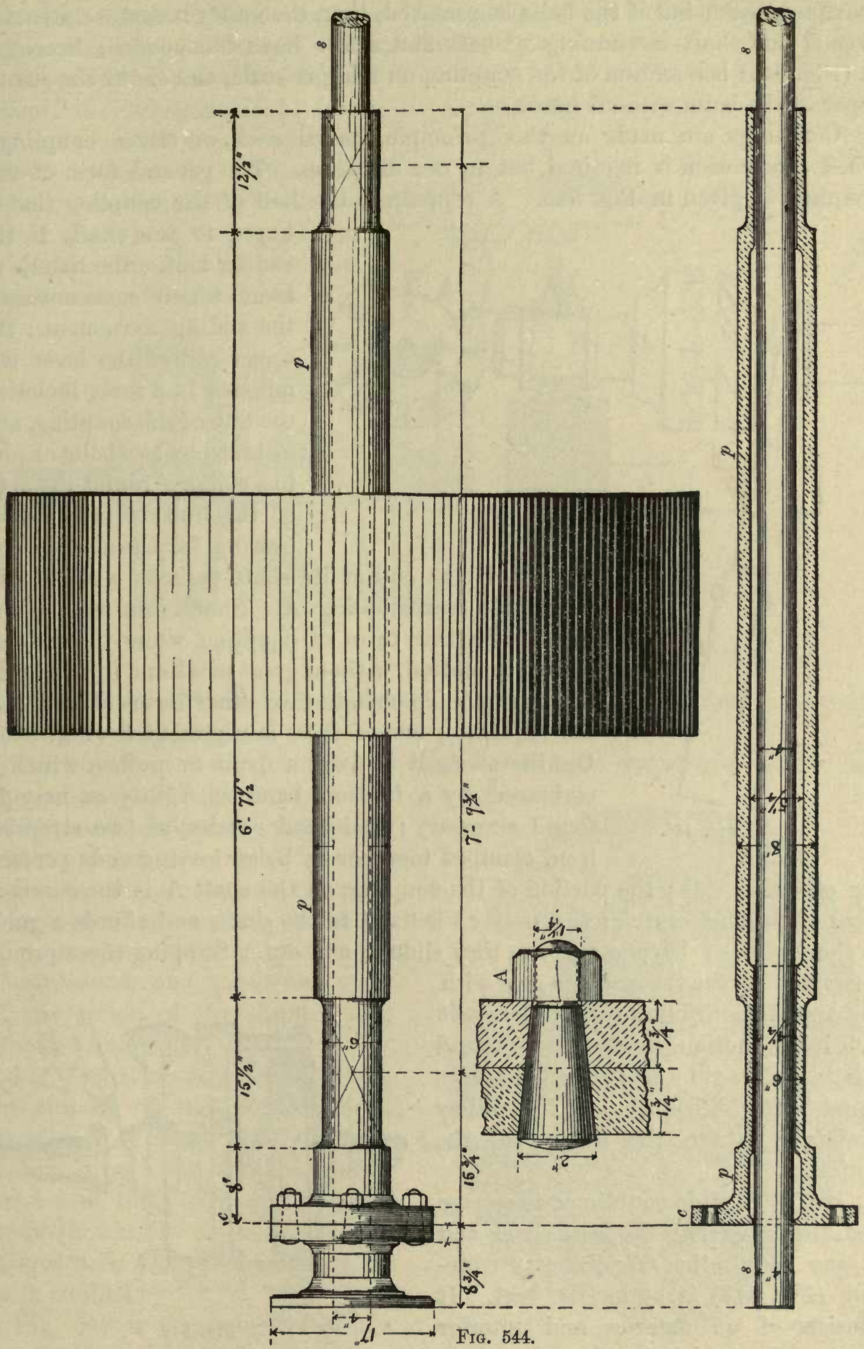


FIG. 544.

iron pipe or sleeve *p* through which the main shaft *s* passes. The two are attached by means of the coupling *c*, one half of which is attached to the shaft and the other to the sleeve. When bolted together, the pulley and main shaft move together; but if the bolts be removed, then the pulley becomes stationary even if the shaft is running. Shaft and sleeve have independent bearings. A (Fig. 544) is a section of the coupling on a larger scale, and shows the strong taper of the bolts without head.

Couplings are made on this principle, called *slide* or *clutch* couplings, when the motion is required but in one direction. The general form of this coupling is given in Fig. 545. A represents the half of the coupling that is keyed to the shaft, B the sliding half, *c* the handle or lever which communicates the sliding movement; the upper end of the lever terminates in a fork, inclosing the hub of the coupling, and fastened by two bolts or pins to a collar *c′* round the neck of the hub; *b* is a box or bearing for the shaft A; to support B the end of its shaft extends a slight distance into the coupling A. Shafts can not be engaged with this form of coupling while the driving shaft is in motion, without great shock and injury to the machinery. To obviate this, other forms of coupling are requisite; one of these is represented (Fig. 546). On the shaft B is fixed a drum or pulley, which is embraced by a friction band as tightly as may be found necessary; this band consists of two straps of iron, clamped together by bolts, leaving ends projecting on either side; the portion of the coupling on the shaft A is the common form of *bayonet* clutch; the part *c c* is fixed to the shaft, and affords a guide to the prongs or bayonets *b b*, as they slide in and out. Slipping these prongs forward, they are thrown into gear with the ears of the friction band; the shaft A being in motion, the band slips round on its pulley till the friction becomes equal to the resistance, and the pulley gradually attains the motion of the clutch.

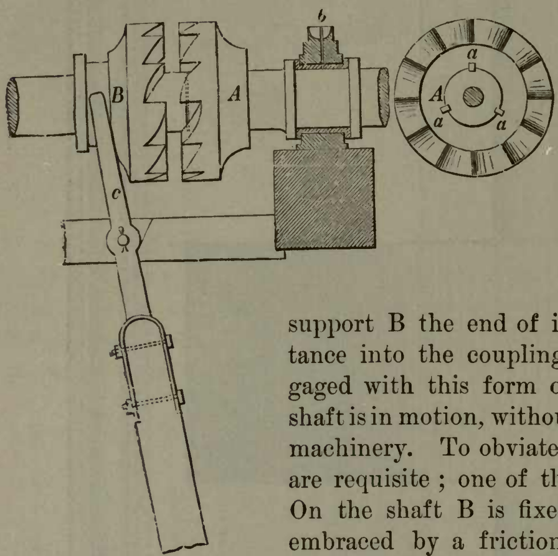


Fig. 545.

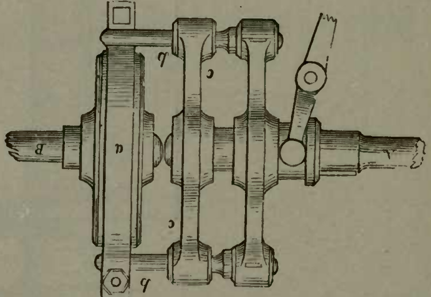


Fig. 546.

But of all slide couplings, to engage and disengage with the least shock and at any speed, the *friction cone* coupling (Fig. 547) is by far the best. It consists of an exterior and interior cone, *a*, *b*; *a* is fastened to the shaft A, while *b* slides in the usual way on the feather *f* of the shaft B; pressing *b* forward, its exterior surface is brought in contact with the interior conical surface of *a*; this should be done gradually; the surfaces of the two cones slip on each other till the friction overcomes the resistance, and motion is transmitted comparatively gradually, and without danger to the machinery. The longer the taper of the cones, the more difficult the disengagement; but the more blunt the cones, the more difficult to keep the surfaces in contact. An angle of 8° with the line of shaft is a very good one for surfaces of cones of cast-iron on cast-iron. When thrown into

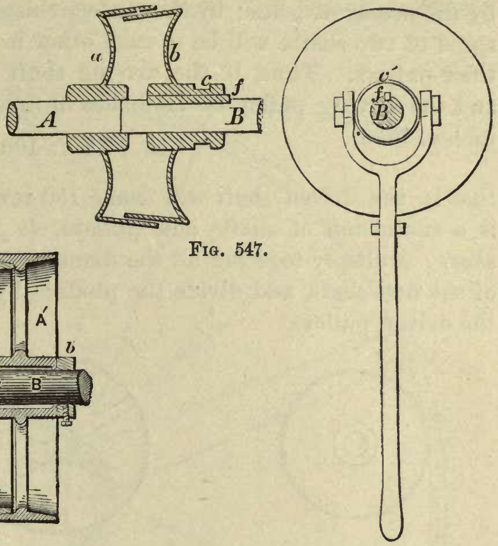


Fig. 547.

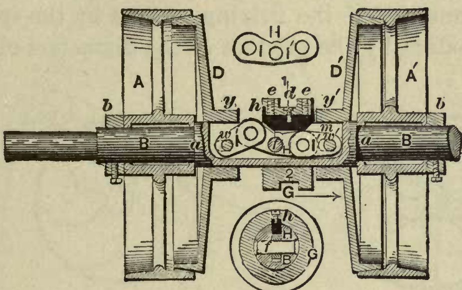


Fig. 548.

gear, the handle of the lever or *shipper* is slipped into a notch, that it may not be thrown out by accident.

The objection to this coupling is that it will work out of gear unless the shipper-handle is held firmly in its position, and producing considerable friction against the collar. To obviate this the shipper is made to act on a toggle-joint fastened to the shaft, and, once thrown, the pressure is self-continued, and preserved without any action of the shipper, and without friction.

Fig. 548 represents a double-friction clutch, of the Weston-Capen patent. The clutch G is slid over the toggle, and the friction cone is forced into the pulley and engaged therewith. In the figure, D′ is thus engaged with A′, while D and A are not in contact.

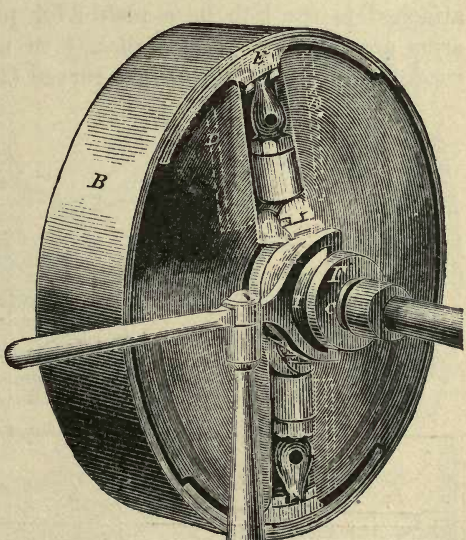


Fig. 549.

Fig. 549 is a perspective of the Mason clutch, in which two toggles are attached to the sliding hub F. By the action of the shipper moving the hub inward the two toggles force the two segments E E against the inner periphery of the pulley, which is turned parallel with the axis. The toggles are so adjusted that when forced in they are a trifle within the straight line, so that there is no tendency for them to fly out.

Pulleys are used for the transmission of motion from one shaft to another by the means of belts ; by them every change of velocity may be effected. The speed of two shafts will be to each other in the inverse ratio of the diameter of their pulleys. Thus, if the driving shaft make 100 revolutions per minute, and the driving pulley be 18 inches in diameter, while the driven pulley is 12 inches, then,

$$12 : 18 :: 100 : 150 ;$$

that is, the driven shaft will make 150 revolutions per minute. Where there is a succession of shafts and pulleys, *to find the velocity of the last driven shaft:* Multiply together all the diameters of the driving pulleys by the speed of the first shaft, and divide the product by the product of the diameters of all the driven pulleys.

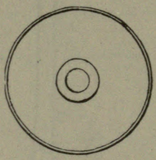


Fig. 550.

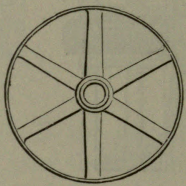


Fig. 551.

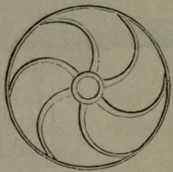


Fig. 552.

Pulleys are made of cast-iron and of every diameter, from 2 inches up to 20 feet. The number of arms vary according to the diameter ; for less than 8 inches diameter the *plate* pulley is preferable (Fig. 550) ; that is, the rim is attached to the hub by a plate ; for pulleys of larger diameters, those with arms are used, never less than 4 in number. The arms are made usually straight (Fig. 551), sometimes curved (Fig. 552).

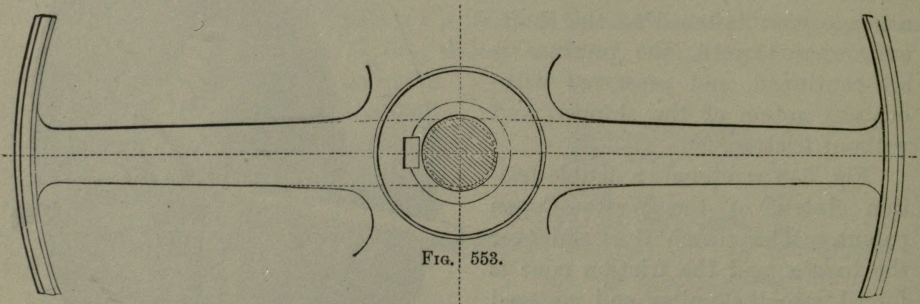


Fig. 553.

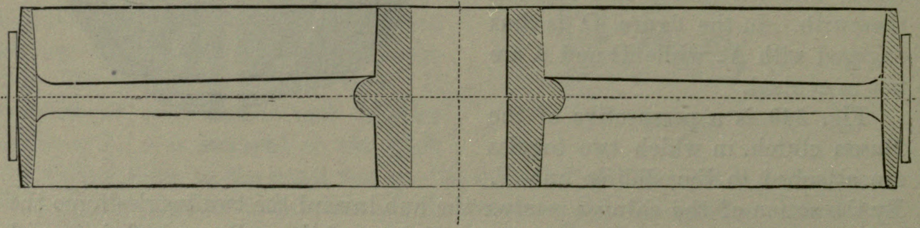


Fig. 554.

Fig. 553 represents a portion of the elevation of a pulley sufficient to show the proportion of the several parts, and Fig. 554 a section of the same. The parts may be compared proportionately with the diameter of shaft; thus the thickness of the hub is about $\frac{1}{2}$ the diameter of the shaft; this proportion is also

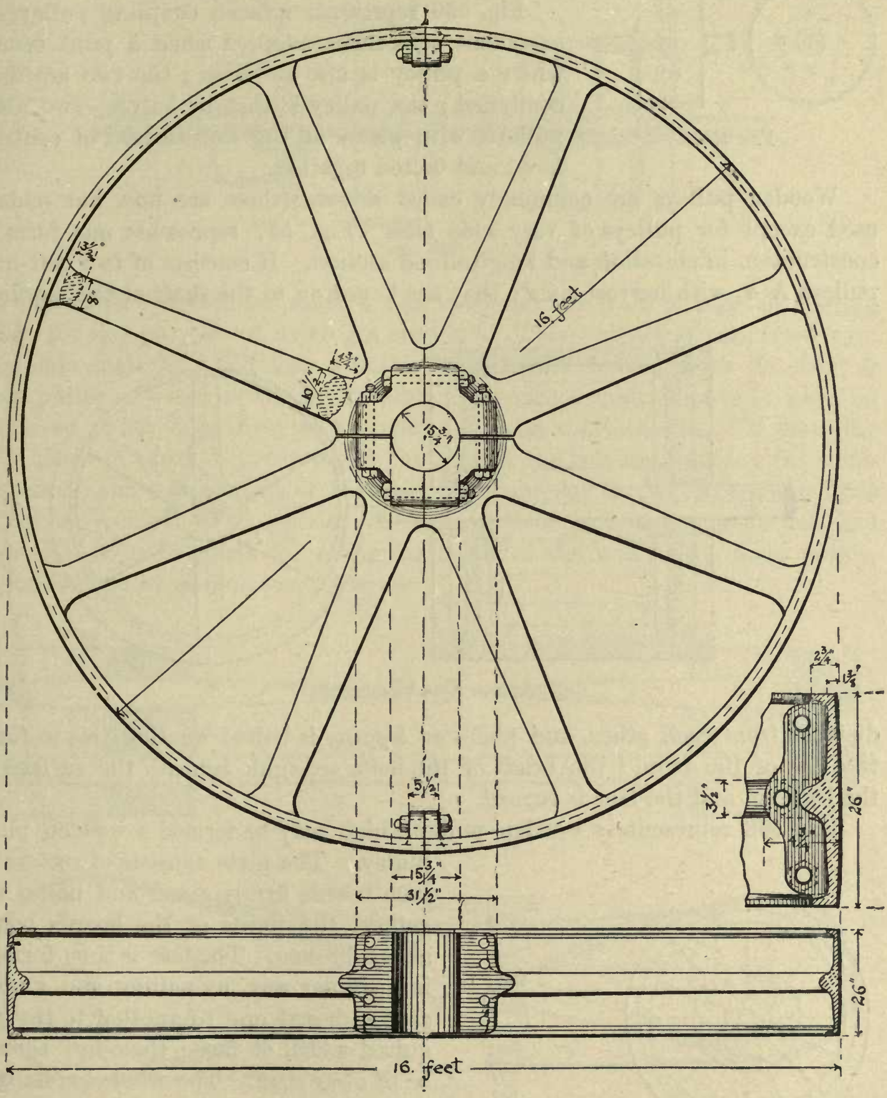


FIG. 555.

used for the hubs of couplings; the width of the arms from $\frac{3}{4}$ to full diameter; the thickness half the width; the thickness of the rim from $\frac{1}{5}$ to $\frac{1}{6}$ the diameter; the length of hub the same as the width of face.

Fig. 555 is a large pulley of the Southwark Foundry pattern. The hub is cast with four divisions, to admit of contraction in cooling, and the rim is in halves, to admit of the pulley being put on the shaft without removing it from its bearings. This is now very common practice with large pulleys. Wrought-iron rim-pulleys have lately been introduced in which the spider—that is, the hub and arms—are of cast-iron, and a wrought-iron plate-rim is bolted to flanges on the extremities of the arms.

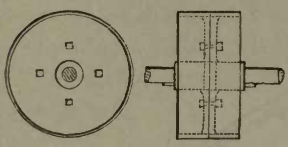


Fig. 556.

Fig. 556 represents a faced coupling pulley, an expedient sometimes adopted when a joint occurs where a pulley is also required; the two are then combined; the pulley is cast in halves—two plate pulleys, with plates at the side instead of central, faced and bolted together.

Wooden pulleys are commonly called *drums*; these are now but seldom used except for pulleys of very wide face. Fig. 557 represents one form of construction in elevation and longitudinal section. It consists of two cast-iron pulleys A A, with narrow rims; they are keyed on to the shaft at the required

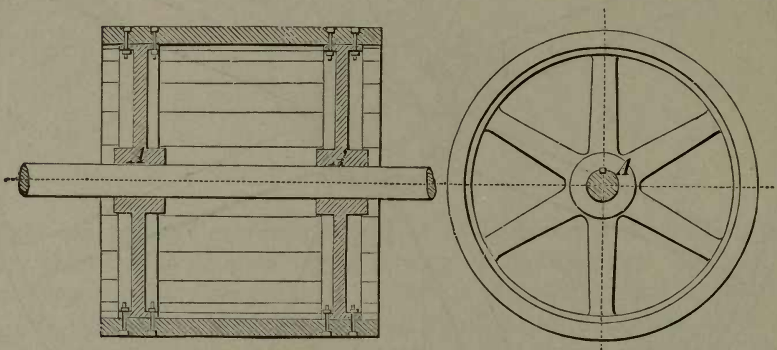


Fig. 557.

distance from each other, and plank or *lagging* is bolted on the rims to form the face of the drum; the heads of the bolts are sunk beneath the surface of the lagging, and the face is turned.

Fig. 558 represents a wooden pulley which may be termed a wooden plate pulley. The plate consists of sectors of inch boards firmly glued and nailed together, the joints of the boards being always broken. The face is then formed in a similar way, by nailing and gluing arcs of board one to another to the required width of face; these last should be of clear stuff. The whole is retained on the shaft by an iron hub, cast with a plate on one side, and another separate plate sliding on to the hub; the hub is placed in the center of the pulley, the two plates are brought in contact with the sides of the pulley, and bolted through; the face of the pulley is now turned in the lathe. A similar arrangement of hub is used for the hanging of grindstones.

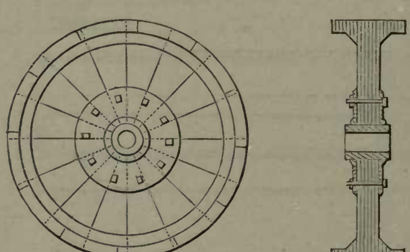


Fig. 558.

Cone pulleys are used to change the speed of the driven shaft. Fig. 559 represents a cone pulley *c* on a shaft with a fast pulley *d*, or one attached to the shaft, while the other, *e*, is loose and revolves on it. The cone pulleys are for changes of speed on the machine, which has upon it another set of cone pulleys, but in reverse position, the small one being opposite the large one on the

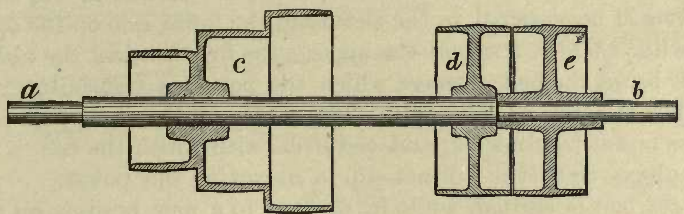


Fig. 559.

counter shaft. A counter shaft is one disconnected from a main or leading shaft, for the purpose of driving a machine. This counter is connected with the main shaft by a belt from a pulley on this shaft passing over the fixed or loose pulley. When on the fixed pulley, the counter shaft is moved; when on the loose pulley it revolves on the shaft, and the shaft is still. To move the belt, there is a fork between which the sides of the belt approaching the counter passes, and a movement of this fork by a shipper throws the counter in or out of movement. The faces of the fast or loose pulleys are made flat, and provision is to be made for oiling the inside of the hub of the loose pulley, which is done by oil-holes and grooves.

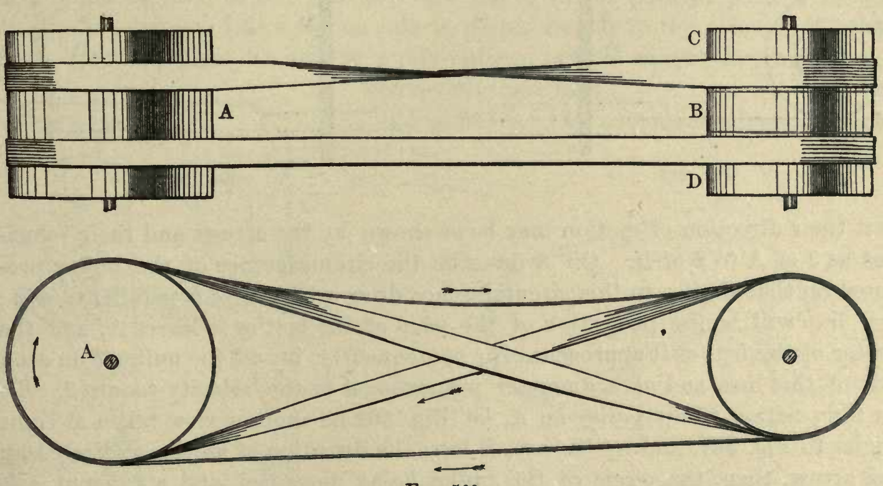


Fig. 560.

It is often necessary to reverse the motion of a machine. This is readily done by a system of fast and loose pulleys, as shown in the plan and elevation, Fig. 560, in which A is a drum or wide-faced pulley on the driving-shaft, B a fast pulley on the driven shaft, and C and D loose pulleys on the same. The action will be understood from the direction of the arrows. The driving-shaft revolves always in the same direction, but on the driven shaft the loose pulley of the straight belt is drawn from the bottom, and partakes of the same motion as the driving-pulley ; while by the cross-belt the draft is at the top of its pulley, and the motion reversed. If the straight or *open* belt be shipped on to the fast pulley B, the motion given to the shaft is like that of the driving-shaft ; if the cross-belt be shipped on to the fast pulley, the motion of the shaft is reversed. It will be observed in the elevation, the lower side of the open belt is straight, while there is a sag in the upper ; the first is called the *tight* or *leading* belt, it being the belt through which the power is transmitted, while the upper side is the *loose* or *slack* belt. The stress on the tight belt is equal to that of the power transmitted, and the stress with which the belt is stretched over the pulleys, so that it will not slip in conveying this power.

When the belt is shifted, while in motion, to a new position on a drum or pulley, or from fast to loose pulley, or *vice versa,* the lateral pressure must be applied on the *advancing* side of the belt, on the side on which the belt is approaching the pulley, and not on the side on which it is running off. It is only necessary that a belt, to maintain its position, should have its advancing side in the plane of rotation of that section of the pulley on which it is required to remain, without regard to the retiring side. On this principle, motion may be conveyed by belts to shafts at any angle to each other. Let A and B (Fig. 561) be two shafts at right angles to each other, A vertical, B horizontal, so that the line run perpendicular to the direction of one axis is also perpendicular to the other, and let it be required to connect them by pulleys and a belt,

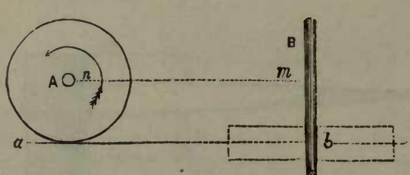


Fig. 561.

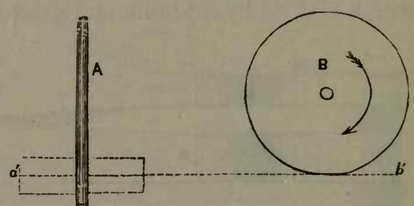


Fig. 562.

that their direction of motion may be as shown by the arrows and their velocities as 3 of A to 2 of B. On A describe the circumference of the pulley proposed on that shaft ; to this circumference draw a tangent *a b* parallel to *m n* ; this line will be the projection of the edge of the belt as it leaves A, and the center of the belt as it approaches B ; consequently, lay off the pulley *b* on each side of this line, and of a diameter proportional to the velocity required. To fix the position of the pulley on A, let Fig. 562 be another view taken at right angles to Fig. 561, and let the axis B have the direction of motion indicated by the arrow, then, the circle of the pulley being described, and a tangent *a' b'* drawn to it perpendicular to the axis B as before determined, the position of the pulley on the shaft A is likewise fixed.

The positions of the two pulleys are thus fixed in such a way that the belt is always delivered by the pulley it is receding from into the plane of rotation of the pulley toward which it is approaching. If the motion be reversed, the belt will run off.

Figs. 563 and 564 are the plan and elevation, on a large scale, of a similar arrangement of pulleys and belts.

It is not an essential condition that the shafts should be at right angles to each other to have motion transferred by a belt. They may be placed at

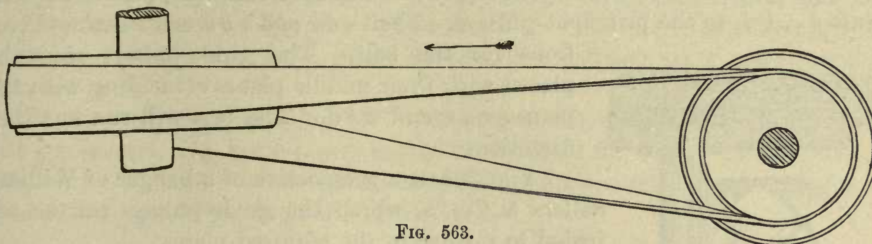


Fig. 563.

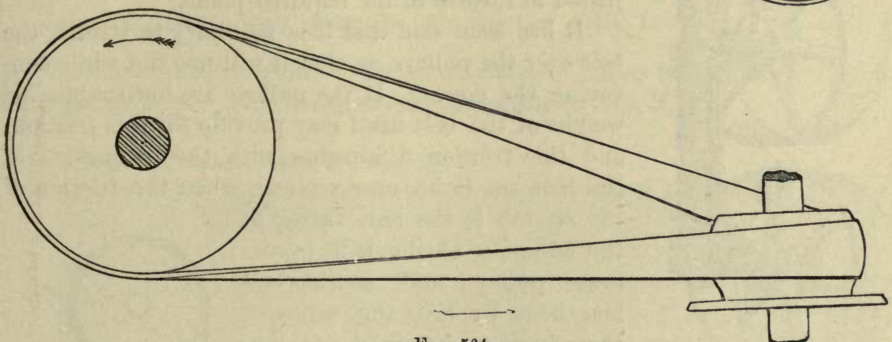


Fig. 564.

any angle to each other, provided the shafts lie in parallel planes, so that the perpendicular drawn to one axis is perpendicular to the other. If otherwise, recourse must be had to guide-pulleys, which should be considerably convex on their face.

Fig. 565 is an arrangement adopted in port-

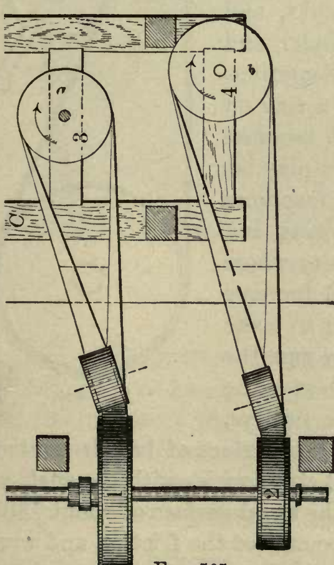


Fig. 565.

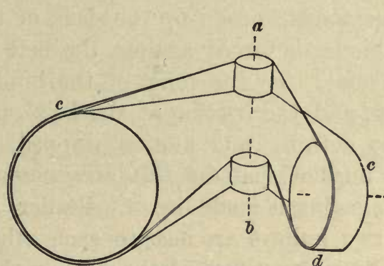


Fig. 566.

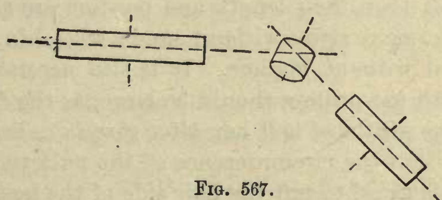


Fig. 567.

able grist-mills for driving the vertical shafts *a*, *b*, of mill-stones, from pulleys on a horizontal shaft. Here it is thought necessary to use guide-pulleys.

Figs. 566 and 567 are the elevation and plan of another arrangement of pulleys and guide-pulleys ; *a b* is the intersection of the middle plane of the principal pulleys. Select any two points *a* and *b* on this line, and draw tangents *a c*, *b d*, to the principal pulleys. Then *c a c* and *a b d* are suitable directions for the belt. The guide-pulleys must be placed with their middle planes coinciding with the planes *c a c* and *a b d*. The belt will run in either direction.

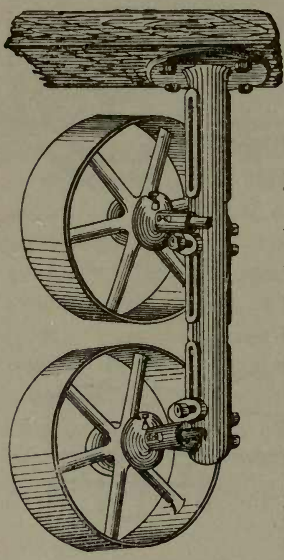


Fig. 568.

Fig. 568 is a perspective of a hanger of William Sellers & Co., in which the guide-pulleys can be adjusted to revolve in the required plane.

It has been said that it is necessary to stretch the belt over the pulleys, so that it will not slip while conveying the power. If the pulleys are horizontal, the weight of the belt itself may provide for this friction, and this friction diminishes with the inclination of the belt till it becomes vertical, when the friction of the stretch is the only factor of the adhesion of the belt to the lower pulley ; and, as the belt lengthens by use, the value of this friction becomes nothing. This position of pulleys should not obtain if it can be avoided ; but if not, the friction-stress should be by means of a binder on the loose belt. The binder (Fig. 569) hangs in a loose frame or links, and rests on the belt, so that the weight of the binder and frame tends to take up the slack of the belt. Sometimes the binder is forced against the belt by a screw acting on its frame. By the relief of the binder the belt becomes slack, and the friction of the belt on the pulleys may become nothing, and motion stopped. On many machines and lines of shafting this arrangement for engaging and disengaging is made use of. Binders are a necessity where the two pulleys are near to each other, either to increase the bearing surface of the belt on the pulleys or to make up for the slight weight of a short belt. Belts run the best when their length and position are such as to give the frictional stress without much stretching on the pulleys, and without binders. It is also necessary that the surface of belt in contact with the pulleys should be large, as the frictional stress varies with the surface. The widths of belt hereafter given are based on the usual surface of about 180°, or half the circumference of the pulleys. On account of the friction and wear it is usual to put the hair side of the belt next the pulley.

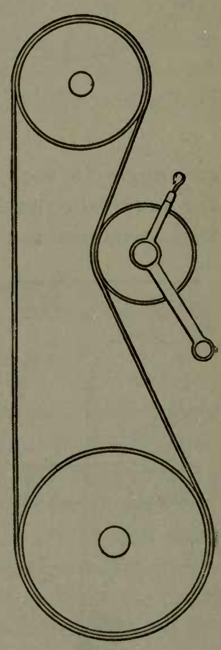


Fig. 569.

In determining the necessary length for any position, the simplest way is to measure it, if the construction is complete ; if not, to make a drawing of the pulleys in position to a scale, and measure on the drawing.

The width of the belts should always be a little less than the face of the pulley ; both are to be determined by the power to be transmitted and the velocity of movement. For the lighter stress of belt a single thickness is only necessary, but for belts from prime movers, transmitting great power, double belts are used.

For single belts, embracing 180° of the circumference, with a velocity of 10 feet per second, one horse-power can be transmitted for each inch in width of belt, with a maximum stress on the belt of 50 pounds, and pressure on journals of about 85 pounds per inch of width of belt.

John T. Henthorn's formula for double belts is $\frac{D \times \pi \times R}{450} = \text{H. P.}$ per inch in width, in which D is the diameter of pulley in feet, R the revolutions per minute. This is expressed graphically in Fig. 570.

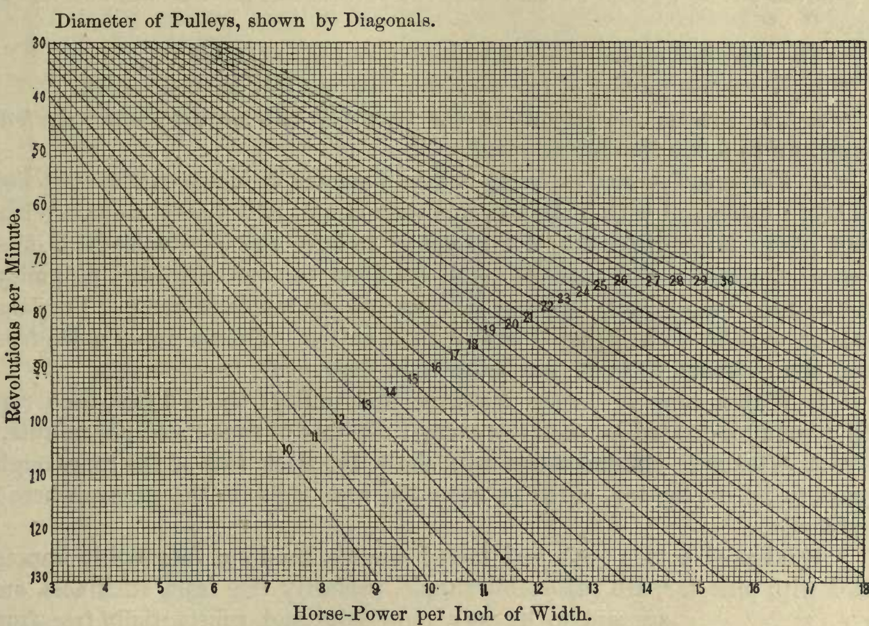


Fig. 570.

Use of Diagram.—To find the horse-power that can be transmitted by a 24″ belt on a 20-foot pulley making 100 revolutions per minute : The abscissa line 100 intersects the diagonal 20 on the ordinate line 14 ; $14 \times 24 = 336 =$ horse-power transmissible.

To find the belt necessary to transmit 100 horse-power through a 10-foot pulley and 120 revolutions per minute of shaft : The abscissa 120 cuts the diagonal 10 on the ordinate line $8\frac{1}{3}$; $\frac{100}{8\frac{1}{3}} = 12''$ width of belt. If the pulley were 12-foot instead of 10, it will be seen by the diagram, the intersection of diagonal would be at 10, and the width of belt $\frac{100}{10} = 10''$.

The above rules are applicable to leather belts, but belts of India-rubber and canvas are largely used, and can be procured of any desirable dimensions, and the strength and adhesion are generally considered greater than those of leather belts. They are especially valuable in situations exposed to wet, where leather is not admissible.

It is the present practice to run belts at high speed ; 5,000 to 6,000 feet is admissible with suitable pulleys and position.

The use of ropes instead of belts has not obtained largely in this country, but in a late report of Mr. Edward Atkinson on English practice, he says that in first-class mills ropes instead of leather belts have taken the place of the upright shafts and gears which were formerly used. He instances in one mill,

"The main wheel on the 2d-motion shaft from the engine or driving-pulley is grooved ; it is 12 feet in diameter, 104 revolutions per minute, and has 20 ropes."

"The rules for rope-driving have been given me as follows :

"1. Never use pulleys of less diameter than six feet for main work.

"2. The greater the velocity of the rope per minute the greater the efficiency, up to 5,000 feet per minute.

"3. For great power, ropes $2\frac{1}{4}$ inches diameter, 2 inches when stretched, are best ; cable-laid with 3 strands, and each strand of 3 finer strands. Where small power is required it is not necessary to have the rope cable-laid, or so great in diameter. For ropes of small diameter smaller diameters of pulleys may be used than 6 feet, and cotton ropes are preferable to hemp. For large ropes or outside work, hemp is better than cotton. Cotton ropes made from yarn, counts about 20 to 30, are better than those made from rovings.

"4. With a rope $2\frac{1}{4}$ inches diameter, and pulleys above 6 feet, each rope will drive 10 indicated horse-power every 1,000 feet of rope-speed per minute.

"5. Whenever circumstances will allow, the slack side of the ropes ought always to be on the top, so as to keep the rope tight in the groove where it stretches.

"The tarred cotton rope and tarred spindle banding, thoroughly impregnated with pine tar, are reasonably supple, perfectly free from stickiness, and are said to be very non-elastic and substantially free from the effects of humidity."

Fig. 571.

Fig. 572.

Fig. 571 represents a cross-section of single-grooved rim for a cotton or hemp rope as used in this country, the groove being simply turned and polished.

Fig. 572 is a cross-section of the rim of wheel for wire rope, showing the rubber lining contained in a dovetailed recess at the bottom of the groove.

From the circular of Messrs. Roebling & Sons we make the following table of transmission of power by wire, the number of revolutions per minute being 100 :

Diameter of wheel in feet.	Diameter of rope.	Horse-power.	Diameter of wheel in feet.	Diameter of rope.	Horse-power.
4	3/8	3·3	10	5/8 11/16	68·7 73·
4	3/8	4·1			
5	7/16	8·6	11	5/8 11/16	81·1 94·4
6	1/2	13·4	12	11/16 3/4	116·7 124·1
7	9/16	21·1			
8	5/8	27·5	13	11/16 3/4	140· 153·2
9	9/16 5/8	50· 51·9	14	3/4 7/8	185· 176·
10	5/8 11/16	55· 58·4	15	3/4 7/8	259· 259·

Gearing.—The term gearing, in general sense, is applied to all arrangements for the transmission of power; it is also used in a particular sense, as toothed gearing.

Toothed gearing may be divided into two great classes—*spur* and *bevel* wheels. In the former, the axes of the driving and driven wheels are parallel to each other; in the latter they may be situated at any angle; if of equal size and at right angles, they are called *miter*-gears.

Spur-wheels, strictly so called, consist of wheels of which the teeth are disposed at the outer periphery of the wheel (Fig. 583), in direction of radii from their centers.

Internal gearing, in which the teeth are disposed in the interior periphery of the wheel, in direction of radii from their centers (Fig. 596).

Rack-gear and pinion are employed to convert a rotatory into a rectilinear motion, or *vice versa.* In this arrangement the pinion is a spur-wheel, acting on teeth placed along a straight bar (Fig. 595).

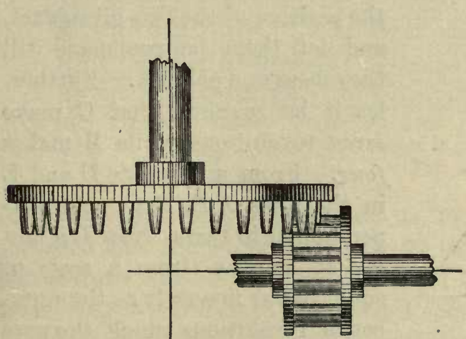


FIG. 573.

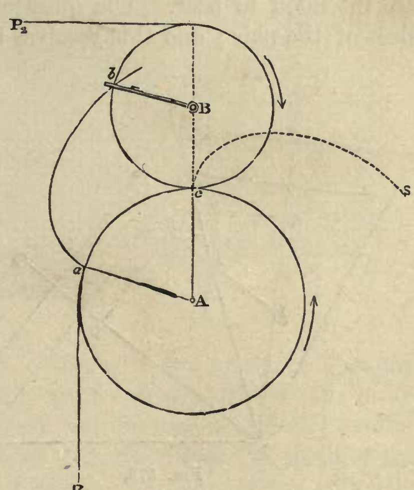


FIG. 574.

Bevel-gearing, strictly so called, consists of toothed wheels formed to work together in different planes, their teeth being disposed at an angle to the plane of their faces (Fig. 591).

Trundle-pins or *wheels* (Fig. 578) are constructed with cylindrical pieces, called *staves* or *pins,* instead of teeth. Fig. 573 is an illustration of trundle-gears with wooden pins; the pinion with double plates is called a *lantern.* This construction is very useful when iron gears can not be easily got or repaired. The trundle may be used either with a spur-wheel to transmit motion to parallel shafts, or with *face* or *crown* wheels.

The primary object of toothed gears is the uniform transmission of power supposed to be constant and equal; the one wheel conducts the other, and they are designated severally as *driver* and *driven,* or *leader* and *follower.* There must be a central line of contact of the teeth, when the surfaces move with the same velocity. In spur-wheels this line of contact is represented by circles, as A and B (Fig. 574). These circles are called pitch-circles—they must have the same angular velocity, and the number of revolutions of each wheel in a given time must be inversely as their diameters.

To find the relative radii of two wheels whose number of revolutions are known: Divide the distances between their centers into parts inversely proportional to the number of revolutions which the wheels are to make in the same unit of time. Thus, let A and B (Fig. 574) be the given centers, the ratio of their velocities being respectively *two* and *three;* if the line joining the centers A and B be divided into $2 + 3 = 5$ equal parts, that is, into as many equal parts as there are units in the terms of the given ratio, the radius of the wheel upon A will contain three of these parts, and the radius of the pinion on B will contain the remaining two parts.

The sizes of a pair of bevels are, however, limited to no particular diameters as when the axes are parallel; the wheels may be made of any convenient sizes, and the teeth consequently of any breadth, according to the stress they are intended to bear. The question is the mode of determining the *relative* sizes of the pair; and this resolves itself into a division of the angle included between the two axes inversely as the ratio of their angular velocities. Let B and C (Fig. 575) be the position of the two given axes, and let them be prolonged till they meet in a point A. Further, let it be required that C make *seven* revolutions while B makes *four.* From any points D and E in the lines A B, A C, and perpendicular to them, draw D *d* and E *e* of lengths (from a scale of equal parts) inversely as the number of revolutions which the axes are severally required to make in the same unit of time. Thus, the angular velocity of axis B being 4 (Fig. 575), and that of the axis C being 7, the line D *d* must be drawn = 7, and the line E *e* = 4. Then through *d* and *e* parallel with the axes A B and A C draw *d c* and *e c* till they meet in *c*. A straight line drawn from A through *c* will

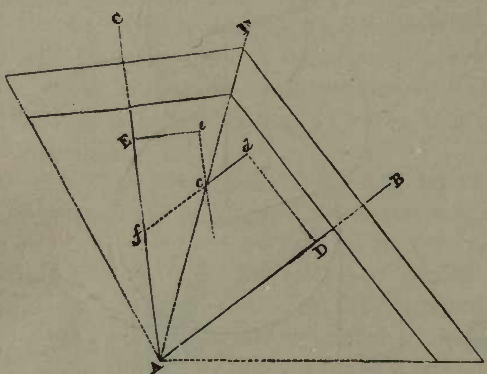


Fig. 575.

then make the required division of the angle B A C, and define the line of contact of the two cones, by means of which the two rolling frusta may be projected at any convenient distance from A.

Otherwise, having determined the relative perimeters, diameters, or radii, of the pair, then the lines D *d* and E *e* are to each other directly as these quantities. B F and C F are radii of the pitch-circle.

The case in which the axes are neither parallel nor intersecting admits of solution by means of a pair of bevels upon an intermediate axis, so situated as to meet the others in any convenient points.

When the contiguity of the shafts is such as to permit of their being connected by a single pair, skewed bevels are sometimes employed.

When the axes are at right angles to each other, and do not intersect, the wheel and screw may be employed to connect them. The velocity of motion is in this arrangement immediately deduced from that of the screw, its number of threads, and the number of teeth in its gearing-wheel. Thus, if it be required to transmit the motion of one shaft to another, contiguous and at right angles

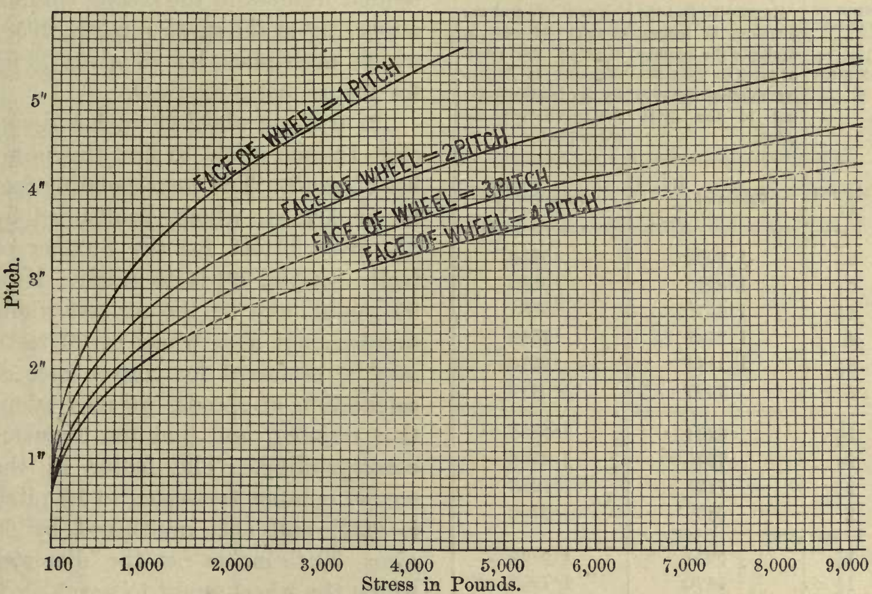


FIG. 576.

to it—the angular motions being as 20 to 1—then, if the screw be a single-threaded one, the wheel must have 20 teeth ; but if double-threaded, the number of teeth will be increased to 40, for 2 teeth will be passed at every revolution. If the screw have few threads compared with the number of teeth of the wheel, it must always assume the position of driver on account of the obliquity of the thread to the axis ; and in this respect its action is analogous to that of a traveling rack, moving endwise one tooth, while the screw makes one revolution on its axis.

If the pitch-circle be divided into as many equal parts as there are teeth to be given to the wheel, the length of one of these parts is termed the *pitch of* *the teeth.* One of these arcs comprehends a complete *tooth and space,* meaning by *space* the hollow opening between two contiguous teeth.

The pitch depends on the power to be transmitted or the stress on each tooth. The diagram (Fig. 576) is by John T. Henthorn, M. E., in which pitch and face, represented by multiples of the pitch, are proportioned to the stress in pounds.

If the pitch be known, the number of teeth in a wheel can be determined approximately by dividing the circumference of the wheel by the pitch, but there must be no remainder in the quotient—there can be no fraction of a pitch—either the pitch or diameter of wheel must be changed if necessary to produce this result; generally the latter, as gears are usually made of determinate inches and fractions, as given in the table, by which also calculation for diameters and number of teeth is much simplified.

PITCH IN INCHES AND PARTS OF AN INCH.	$D = \frac{P}{\pi} \times N.$ Rule.—To find the diameter in inches, multiply the number of teeth by the tabular number answering to the given pitch.	$N = \frac{\pi}{P} \times D.$ Rule.—To find the number of teeth, multiply the given diameter in inches by the tabular number answering to the given pitch.
Values of P.	Values of $\frac{P}{\pi}$	Values of $\frac{\pi}{P}$
6	1·9095	·5236
5	1·5915	·6283
4½	1·4270	·6981
4	1·2732	·7854
3½	1·1141	·8976
3	·9547	1·0472
2¾	·8754	1·1333
2½	·7958	1·2566
2¼	·7135	1·3963
2	·6366	1·5708
1⅞	·5937	1·6755
1¾	·5570	1·7952
1⅝	·5141	1·9264
1½	·4774	2·0944
1⅜	·4377	2·2848
1¼	·3979	2·5132
1⅛	·3568	2·7926
1	·3183	3·1416
⅞	·2785	3·5904
¾	·2387	4·1888
⅝	·1989	5·0266
½	·1592	6·2832
⅜	·1194	8·3776
¼	·0796	12·5664

Example 1.—Given a wheel of 88 teeth, 2½-inch pitch, to find the diameter of the pitch-circle. Here the tabular number in the second column answering to the given pitch is ·7958, which multiplied by 88 gives 70·03 for the diameter required.

2. Given a wheel 33 inches diameter, 1¾-inch pitch, to find the number of teeth. The corresponding factor is 1·7952, which, multiplied by 33, gives 59·242 for the number of teeth—that is, 59¼ teeth nearly. Now 59 would here be the nearest whole number, but as a wheel of 60 teeth may be preferred for convenience of calculation of speeds, we may adopt that number, and find the diameter corresponding. The factor in the second column answering to 1¾ pitch is ·557, and this multiplied by 60 gives 33·4 inches as the diameter which the wheel ought to have.

Another mode of sizing wheels in relation to their pitches, diameters, and number of teeth, is adopted in some machine shops, by dividing the diameter of the pitch-circle into as many equal parts as there are teeth to be given to the wheel. To illustrate this by an arithmetical example, let it be assumed that a wheel of 20 inches diameter is required to have 40 teeth; then the *diametral* pitch,

$$\frac{20}{40} = \frac{1}{m} = \tfrac{1}{2} \text{ inch };$$

that is, the diameter being divided into equal parts corresponding in number to the number of teeth in the circumference of the wheel, the length of each of these parts is $\frac{1}{2}$ an inch, consequently $m = 2$; and according to the phraseology of the workshop, the wheel is said to be one of *two pitch.*

In this mode of sizing wheels, a few determined values are given to m, as 20, 16, 14, 12, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, which includes a variety of pitches from $\frac{1}{8}$-inch up to 3 inches, according to the following table, which shows the value of the circular pitches corresponding to the assigned values of m.

VALUES OF m.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	12.	14.	16.	20.
Corresponding circular pitch in decimals of an inch.	3·142	1·571	1·047	·785	·628	·524	·449	·393	·349	·314	·262	·224	·196	·157

Fundamental principle.—In order that two circles A and B (Fig. 577) may be made to revolve by the contact of the surfaces of the curves *m m* and *n n* of their teeth precisely as they would by the friction of their circumferences, it is necessary and sufficient that a line drawn from the point of contact *t* of the teeth to the point of contact *c* of the circumferences (pitch-circles) should, in every position of the point *t*, be perpendicular to the surfaces of contact at that point ; that is, in the language of mathematicians, that the straight line be a

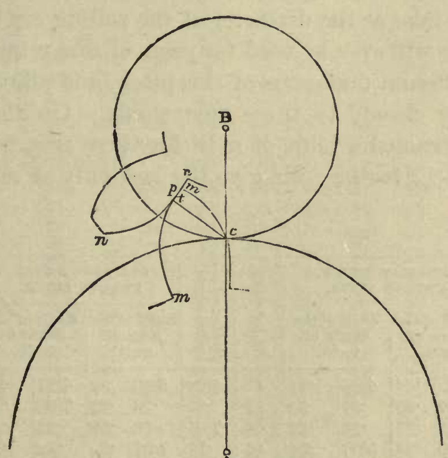


Fig. 577.

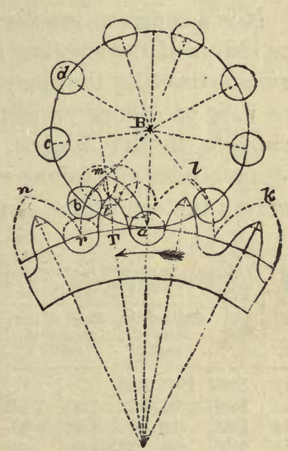


Fig. 578.

normal to both the curves *m m* and *n n*. The principle here announced exhibits a special application of one particular property of that curve known to mathematicians as the *epicycloid* (see page 30).

Of epicycloidal teeth.—The simplest illustration of the action of epicycloidal teeth is when they are employed to drive a trundle, as represented in Fig. 578.

Let it be assumed that the staves of the trundle have no sensible thickness; that the distance of their centers apart, that is their *pitch*, and also their distance from the center of the trundle, that is their pitch-circle, are known. The pitch-circles of the trundle and wheel being then drawn from their respective centers B and A, set off the pitches upon these circumferences, corresponding to the number of teeth in the wheel and number of staves in the trundle; let five pins, *a b c*, etc., be fixed into the pitch-circle of the trundle to represent the staves, and let a series of epicycloidal arcs be traced with a describing circle, equal in diameter to the radius of the pitch-circle of the trundle, and meeting in the points *k l m n*, etc., alternately from right and left. If, now, motion be given to the wheel in the direction of the arrow, then the curved face *m r* will press against the pin *b*, and move it in the same direction; but as the motion continues, the pin will slide upward till it reaches *m*, when the tooth and pin will quit contact. Before this happens, the next pin *a* will have come into contact with the face *a l* of the next tooth, which repeating the same action, will bring the succeeding pair into contact; and so on continually.

To allow of the required thickness of staves, it is sufficient to diminish the size of the teeth of the wheel by a quantity equal to the radius of the staves (sometimes increased by a certain fraction of the pitch for clearance), by drawing within the primary epicycloids, at the required distance, another series of curves parallel to these. In practice, a portion must be cut from the points of the teeth, and also a space must be cut out within the pitch-circle of the driver, to allow the staves to pass; but no particular form is requisite, the condition to be attended to is simply to allow of sufficient space for the staves to pass without contact.

It is a common practice of shops to take as the diameter of the rolling circle the radius of the smallest pinion which will ever be used for gears of this pitch, and constructing the epicycloids for different diameters of this pitch, and allowing arcs of circles corresponding very closely to these epicycloids. On this principle, Robert Adcock, C. E., constructed a table of radii for these arcs, for rolling circles of pinions of 8, 10, and 12 teeth. We give the last only as answering the conditions of practice:

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH.			
		Radil of the faces of teeth.		Radil of the flanks of teeth.	
12	1·93	1·88	0·75		
13	2·09	2·04	0·76	7·45	7·14
14	2·25	·19	·77	4·86	4·27
15	2·40	·35	·78	3·92	3·04
16	2·56	·50	·78	·62	3·53
17	2·72	·66	·79	·58	2·22
18	2·88	·82	·80	·59	2·02
19	3·04	·97	·81	·63	1·87
20	3·20	3·13	·81	·73	·76
21	3·35	·29	·82	·83	·68
22	3·51	·44	·82	·95	·61
23	3·67	·60	·83	4·07	·56
24	3·83	·76	·83	·21	·51
25	3·99	·91	·84	·34	·47
26	4·15	4·07	·84	·48	·44

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH.			
		Radil of the faces of teeth.		Radii of the flanks of teeth.	
27	4·31	·23	·84	4·63	·41
28	·46	·39	·85	·37	·38
29	·62	·55	·85	·92	·36
30	·78	·70	·86	5·07	·34
31	·94	·86	·86	·21	·32
32	5·10	5·02	·86	·37	·30
33	·26	·18	·86	·52	·29
34	·42	·34	·87	·67	·28
35	·58	·49	·87	·82	·26
36	·74	·65	·87	·97	·25
37	·90	·81	·88	6·13	·24
38	6·05	·97	·88	·29	·23
39	·21	6·13	·88	·44	·23
40	·37	·28	·88	·60	·22
41	·53	·44	·89	·75	·21

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH.			
		Radii of the faces of teeth.		Radii of the flanks of teeth.	
42	6·69	6·60	·89	6·91	·20
43	·85	·76	·89	7·06	·20
44	7·01	·92	·89	·22	·19
45	·17	7·07	·89	·38	·18
46	·33	·23	·90	·53	·18
47	·49	·39	·90	·09	·17
48	·64	·55	·90	·84	·16
49	·80	·71	·90	8·00	·16
50	·96	·86	·90	·96	·16
51	8·12	8·02	·91	·31	·16
52	·28	·18	·91	·47	·15
53	·44	·34	·91	·63	·15
54	·60	·50	·91	·79	·14
55	·76	·66	·91	·95	·14
56	·92	·81	·91	9·10	·14

No. of teeth.	Radius of pitch-circle.	Smallest pinion, twelve teeth.				No. of teeth.	Radius of pitch-circle.	Smallest pinion, twelve teeth.				No. of teeth.	Radius of pitch-circle.	Smallest pinion, twelve teeth.			
		Radii of the faces of teeth.		Radii of the flanks of teeth.				Radii of the faces of teeth.		Radii of the flanks of teeth.				Radii of the faces of teeth.		Radii of the flanks of teeth.	
57	9·08	8·97	·91	9·26	·13	120	19·10	18·99		19·10		183	29 13	29·00	·97	29·27	
58	·23	9·13	·91	·42	·13	121	·26	19·15		·42	·06	184	·28	·16		·43	1·03
59	·37	·29	·92	·57	·13	122	·42	·30	·95	·57		185	·44	·32		·59	
60	·55	·45	·92	·73	·13	123	·58	·46		·73	·06	186	·60	·48	·97	·74	1·03
61	·71	·61	·92	·89	·12	124	·74	·62		·89		187	·76	·64		·90	
62	·87	·77	·92	10·05	·12	125	·90	·78	·95	20·05	·06	188	·92	·80		30·06	
63	10·03	·92	·92	·20	·12	126	20·05	·94		·21		189	30·08	·96	·98	·22	1·03
64	·19	10·08	·92	·36	·12	127	·21	20·10		·37	·05	190	·24	30·12		·38	
65	·35	·24	·92	·52	·12	128	·37	·26	·96	·53		191	·40	·28	·98	·54	1·02
66	·51	·40	·92	·68	·11	129	·53	·42		·69	·05	192	·55	·43		·70	
67	·67	·56	·92	·84	·11	130	·68	·58		·84		193	·71	·59		·86	
68	·83	·72	·92	·99	·11	131	·84	·74	·96	21·00	·05	194	·87	·75	·98	31·02	1·02
69	·98	·88	·93	11·15	·11	132	21·00	·89		·16		195	31·03	·91		·18	
70	11·14	11·04	·93	·31	·11	133	·17	21·05		·32	·05	196	·19	31·07		·33	1·02
71	·30	11·20	·93	·47	1·10	134	·33	·21	·96	·48		197	·35	·23	·98	·49	
72	·46	·35	·93	·63	·10	135	·49	·37		·64	·05	198	·51	·39		·65	1·02
73	·62	·51	·93	·79	·10	136	·65	·53	·96	·80		199	·67	·55		·81	
74	·78	·67	·93	·95	·10	137	·81	·69		·96	·05	200	·83	·71	·98	·97	
75	·94	·83	·93	12·10	·10	138	·96	·85		22·11		201	·99	·87		32·13	
76	12·10	11·99	·93	·26	·09	139	22·12	22·01	·96	·27	·05	202	32·15	32·02		·29	
77	·26	12·15		·42	·09	140	·28	·17		·43	·05	203	·30	·18		·45	
78	·42	·30	·93	·58	·09	141	·44	·33		·59		204	·46	·34		·61	
79	·58	·47		·74	·09	142	·60	·48	·96	·75	·05	205	·62	·50		·77	
80	·73	·63	·93	·90	·09	143	·76	·64		·91		206	·78	·66		·92	
81	·89	·79		13·06	·09	144	·92	·80		23·07	·05	207	·94	·82		33·08	
82	13·05	·94	·93	·22	·09	145	23·08	·96	·96	·23		208	33·10	·98		·24	
83	·21	13·10		·38	·09	146	·24	23·12		·38	1·04	209	·26	33·14		·40	
84	·37	·26	·94	·53	·08	147	·40	·28		·54		210	·42	·30		·56	
85	·53	·42	·94	·69	·08	148	·56	·44		·70	·04	211	·58	·46		·72	
86	·69	·58		·85	·08	149	·72	·60	·96	·86		212	·74	·61		·88	
87	·85	·74	·94	14·01	·08	150	·87	·76		24·02	·04	213	·90	·77		34·04	
88	14·01	·90	·94	·17	·08	151	24·03	·92		·18		214	34·06	·93		·20	
89	·17	14·06		·33	·08	152	·19	24·07	·96	·34		215	·21	34·09		·36	
90	·33	·22	·94	·49	·08	153	·35	·23		·50	·04	216	·37	·25		·51	
91	·49	·38	·94	·65	·08	154	·51	·39		·65		217	·53	·41		·67	
92	·64	·53		·81	·08	155	·67	·55	·96	·81	·04	218	·69	·57		·83	
93	·80	·69	·94	·97	·08	156	·83	·71		·98		219	·85	·73		·99	
94	·96	·85	·94	15·12	·07	157	·99	·87		25·13	·04	220	35·01	·89		35·15	
95	15·14	15·01		·30	·07	158	25·15	25·03	·97	·29		221	·17	35·05		·31	
96	·28	·17	·94	·44	·07	159	·31	·19		·45	·04	222	·33	·20		·47	
97	·44	·33		·60	·07	160	·47	·35		·61		223	·49	·36		·63	
98	·60	·49	·94	·76	·07	161	·62	·51	·97	·77		224	·65	·52		·79	
99	·76	·65		·92	·07	162	·78	·66		·93	·04	225	·80	·68		·95	
100	·92	·81	·95	16·08	·07	163	·94	·82		26·09		226	·96	·84		36·10	
101	16·08	·97		·24	·07	164	26·10	·98	·97	·25	·04	227	36·12	36·00		·26	
102	·24	16·13		·40		165	·26	26·14		·42		228	·28	·16		·42	
103	·39	·28	·95	·56	·07	166	·42	·30		·56	·04	229	·44	·32		·58	
104	·55	·44		·72		167	·58	·46		·72		230	·59	·48		·74	
105	·71	·60		·87	·07	168	·74	·62	·97	·88	·03	231	·75	·64		·90	
106	·87	·76	·95	17·03		169	·90	·78		27·04		232	·91	·79		37·06	
107	17·03	·92		·19	·06	170	27·06	·94		·22	·03	233	37·08	·95		·22	
108	·19	17·08		·35		171	·22	27·10	·97	·36		234	·24	37·11		·58	
109	·35	·24	·95	·51	·06	172	·38	·25		·52	·03	235	·40	·27		·54	
110	·51	·40		·67		173	·53	·41		·68		236	·56	·43		·69	
111	·67	·56		·83	·06	174	·69	·57	·97	·84		237	·72	·59		·85	
112	·83	·71	·95	·99		175	·85	·73		1·00	·03	238	·87	·75		38·01	
113	·99	·87		18·15	·06	176	28·01	·89		28·16		239	38·03	·91		·17	
114	18·15	18·03	·95	·30		177	·17	28·05	·97	·31	·03	240	·19	38·07		·33	
115	·30	·19		·46	·06	178	·33	·21		·47		241	·35	·23		·69	
116	·46	·35		·26		179	·48	·37		·63		242	·51	·38		·85	
117	·62	·51	·95	·62	·06	180	·64	·53	·97	·79	·03	243	·67	·54		39·01	
118	·78	·67		·78		181	·80	·69		·95		244	·83	·70		·17	
119	·94	·83	·95	·94	1·06	182	·97	·84		29·11	1·03	245	·99	·86		·33	

No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.	Radii of the flanks of teeth.	No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.	Radii of the flanks of teeth.	No. of teeth.	Radius of pitch-circle.	SMALLEST PINION, TWELVE TEETH. Radii of the faces of teeth.	Radii of the flanks of teeth.
246	39·15	39 02	39·28	265	42·17	42·04	42·31	284	45·19	45·06	45·33
247	·31	·18	·44	266	·33	·20	·46	285	·35	.22	·49
248	·47	·34	·60	267	·49	·36	·62	286	·51	·38	·64
249	·64	·50	·76	268	·64	·52	·78	287	·67	·54	·80
250	·78	·86	·92	269	·80	·68	·94	288	·83	·70	·96
251	·94	·82	40·08	270	·97	·84	43·10	289	·99	·86	46 12
252	40·10	·97	·24	271	43·13	1·00	·26	290	46·15	46·02	·28
253	·26	40·13	·40	272	·28	43·15	·42	291	·30	·17	·44
254	·42	·30	·56	273	·44	·31	·58	292	·46	·33	·60
255	·59	·45	·72	274	·60	·47	·74	293	·62	·49	·76
256	·74	·61	·87	275	·76	·63	·90	294	·78	·65	·82
257	·90	·77	41·03	276	·92	·79	44·05	295	·94	·81	·98
258	41·06	·93	·20	277	44·08	·96	·21	296	47·10	·97	47·13
259	·22	41·09	·36	278	·24	44·11	·37	297	·25	47·13	·29
260	·38	·25	·51	279	·40	·27	·53	298	·42	·29	·45
261	·53	·41	·67	280	·55	·43	·69	299	·58	·45	·61
262	·69	·56	·83	281	·71	·59	·85	300	·74	·61	·77
263	·85	·72	·99	282	·87	·74	45·01	Rak		·129 1·00	0·129 1·00
264	42·01	·89	42·15	283	45·03	·90	·17				

Rule.—Seek in the first column of the table for the number of teeth it is proposed that the wheel shall contain. In a line with such number of teeth take from columns 2, 3, 4, 5, and 6 the numbers that are in them ; and in every case multiply such numbers by the pitch. The products will be the number of inches and parts of inches to which the compasses must be opened to describe the circles and parts of circles that are required.

Example.—Suppose that a wheel is to be made to contain thirty teeth, and that the pitch of the teeth is to be $2\frac{1}{2}$ inches, proceed as follows : Seek in column 1 for 30, the number of proposed teeth, and take from column 2 the numbers 4·783, which multiply by $2\frac{1}{2}$ inches, the product will be 11″·957. Open the compasses, therefore, to this radius and describe a circle, which will be the "pitch-circle." On an arc of this circle lay off $2·5 \times ·48 = 1·2''$ for the thickness of a tooth, and $2·5 \times 5·2 = 1·3''$ for the space. Having determined the number of teeth and pitch, next, in column 3, and in the same line with 30 teeth, will be found the numbers 4·704, which multiply by $2\frac{1}{2}$ inches—the product will be 11·75. With the compasses opened to this distance, and from the same center as the last, describe another circle, which will be the paths of centers for the curves of the faces of the teeth. From column 4 similarly take the numbers 0·865 and multiply by $2\frac{1}{2}$ inches. The product is 2·15, to which distance the compasses must be opened to describe the faces of the teeth.

Again, in column 5, multiply $5·07 \times 2·5 = 12''·675$, and from the center, with this radius, describe another circle for the paths of centers of flanks of the teeth, from column 6, $1·34 \times 2·5 = 3·35$, the radius of the flanks of the teeth.

For the height of a tooth a common proportion is $\frac{3}{10}$ of pitch outside of pitch-circle, and $\frac{4}{10}$ of pitch within, which leaves $\frac{1}{10}$ pitch for clearance at the bottom, where usually small arcs are described to connect the teeth with the wheel.

Having described a few teeth of any gear to its full size, the rest may be laid off from a templet, or cutters made by which the teeth may be accurately formed. In the illustration (Fig. 579) the teeth and spaces are proportioned to a common form, but there is considerable variation in proportion, as—

Thickness of teeth, from ·45 to ·48 pitch.

Space between teeth, from ·55 to ·52 pitch.

Height of teeth outside of pitch-circle, from ·2 to ·3 pitch.

Depth of teeth inside of pitch-circle, from ·3 to ·4 pitch.

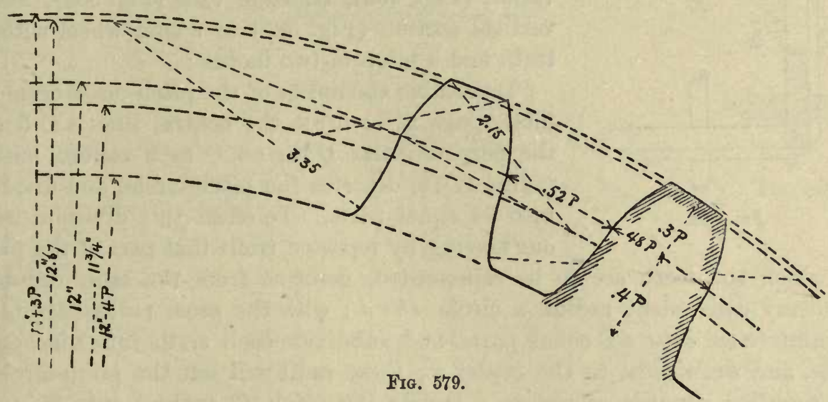


Fig. 579.

It is not uncommon to make one of the set of gears with wooden teeth, mortices being cast in the periphery of the wheel for the insertion of these teeth—hence called mortise wheels—the elasticity of the wood diminishes the effect of shocks, and they run with less noise.

The usual proportions and construction of mortise wheels are shown in Fig. 580, a section across and with the rim of the wheel. The figures represent the proportions to pitch as unity; *b* is from 2 to 3 *p*. The teeth are held in position by wooden dovetailed keys.

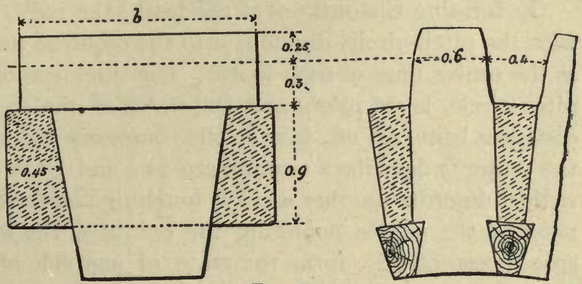


Fig. 580.

Fig. 581 is a section across the rim of mortised bevel-gear; the figures are as before in ratios to *p*. In this illustration the teeth are held in by pins, not unusual also in spur-mortise-gears.

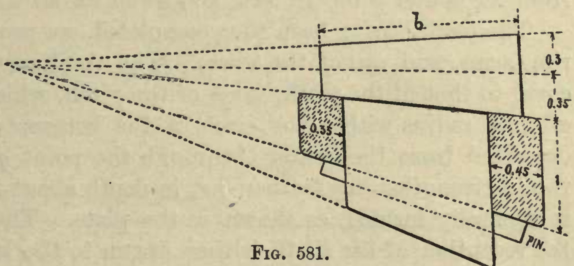


Fig. 581.

It is unusual in drawings to complete gears with teeth according to the examples given ; it is sufficient for the purposes of pattern-making that the pitch-circle, pitch and form of one tooth be given. For lines of shafting, spur-gears may be represented, like plain pulleys, of the diameters of the pitch-circle, with the pitch and number of teeth written in: bevel-gears, as in Fig. 582. But, as in finished drawings all the detail is necessary, we proceed to give the simplest forms of describing spur- and bevel-gears with sufficient accuracy for all practical purposes.

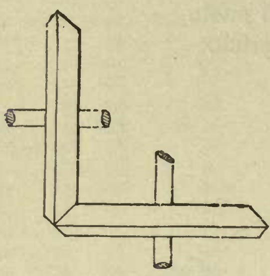


Fig. 582.

Projections of a Spur-Wheel.—To draw side elevation (Fig. 583), an edge view (Fig. 584), and a vertical section (Fig. 585) of a spur-wheel with 34 teeth and a pitch of two inches :

Determine the radius of the pitch-circle from the table, page 278 ; draw the central line A C B and the perpendicular D E ; on C as a center, with a radius 17·19, describe the pitch-circle, and divide it into 54 equal parts. To effect this division, without fraying by repeated trials that part of the paper on which the teeth are to be represented, describe from the same center *c*, with any convenient radius, a circle *a b c d* ; with the same radius divide its circumference into six equal parts, and subdivide each sixth into nine equal parts, and draw radii to the center *c* ; these radii will cut the pitch-circle at the required number of points. Divide the pitch (2 inches) into 10 equal parts ; mark off beyond the pitch-circle a distance equal to 3 of these parts, and within it a distance equal to 4 parts, and from the center C describe circles passing through these points ; these circles are projections of the cylinders bounding the points of the teeth and the roots of the spaces respectively.

In forming the outlines of the teeth, the radii, which, by their intersections with the pitch-circle, divide it into the required number of parts, may be taken as the center lines of each tooth. The thickness of the tooth, measured on the pitch-circle, is ·46 pitch, and the width of the space is equal to ·54 *p*. These distances being set off, take in the compasses the length of the pitch, and from the center *g* describe a circular arc *h i* ; and from the center *j*, with the same radius, describe another arc *h k* touching the former ; these arcs, being terminated at the circles bounding the points of the teeth and the bottoms of the spaces respectively, form the curve of one side of a tooth. The other side is formed in a similar manner, by drawing from the center *l* the arc *m n*, and from the center *p* the arc *m o*, and so on for all the rest of the teeth.

The teeth having been thus completed, we proceed to the delineation of the rim, arms, and eye of the wheel. The thickness of the rim is usually made equal to that of the teeth, say $\frac{1}{2}$ of the pitch, which distance is accordingly set off on a radius within the circle of the bottoms of the spaces, and a circle is described from the center C through the point *q* thus obtained. Within the rim, a strengthening feather *q r*, in depth about $\frac{3}{4}$ of the thickness of the rim, is generally formed, as shown in the plate. The *eye*, or central aperture for the reception of the shaft, is then drawn to the specified diameter, as also the circle representing the thickness of metal round the eye, which is usually made equal to the pitch of the wheel.

To draw the arms, from the center C, with the radius C *u* equal to the pitch, describe a circle ; draw all the radii, as C L, which are to form the cen-

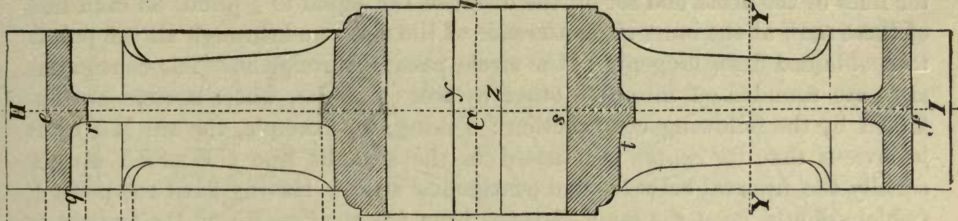


Fig. 585.

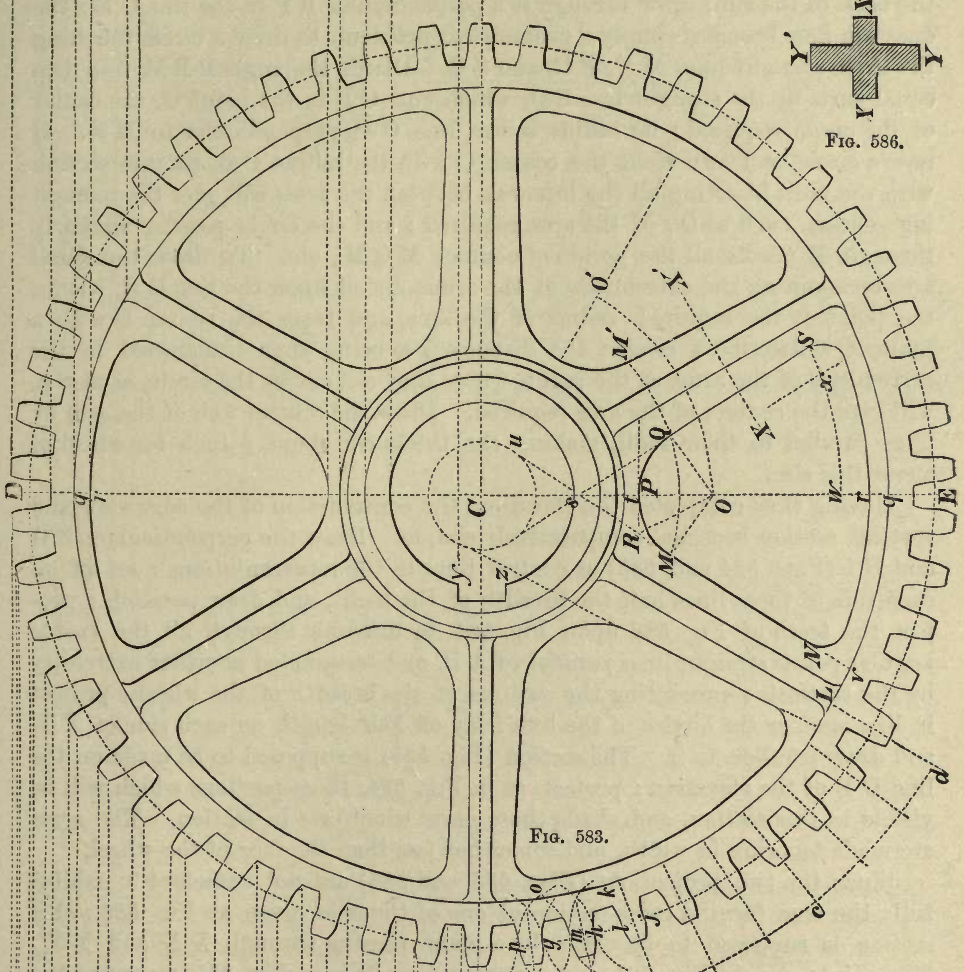


Fig. 586.

Fig. 583.

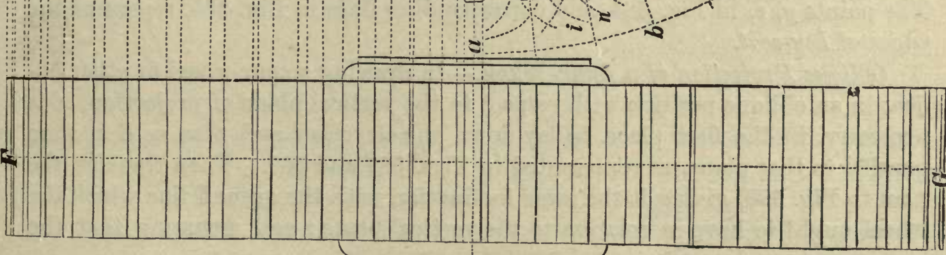


Fig. 584.

ter lines of the arms, and set off the distance L v, equal to $\frac{1}{8}$ pitch, on each side of these radii at the inner circumference of the rim; and through all the points thus obtained draw tangents to the circle passing through u. The contiguous arms are rounded off into each other by arcs of circles, whose centers are obtained by the following construction: Taking, for example, the arc M P Q, it is obvious that its center is situated in the straight line C E which divides equally the interval between two contiguous arms. Having fixed the point P (which should be at the same distance from t as the breadth of the feather at the back of the rim) draw through it a perpendicular R P to the line C E; the question now becomes simply a geometrical problem, to draw a circle touching the three straight lines M N, P R, and S Q. Divide the angle P R M into two equal parts by the straight line R O, which cuts C E in the point O, the center of the circle required; its radius is the line O M perpendicular to M N. If now a circle be drawn from the center C with the radius C O, its intersection with the radii bisecting all the intervals between the arms will give the remaining centers, such as O′, of the arcs required; and the circle passing similarly through M marks all the points of contact M Q M′, etc. To draw the small arcs terminating the extremities of the arms, set off upon the line C E, within the point r, the required radius of the arcs, and from the center C with a radius C w describe a circle; the distance $r\,w$ being then transferred to the extremities of the arms at the points where they are cut by the circle, as at S x, will give the centers of the arcs required. Draw the central web of the arm by lines parallel to their radii, making the thickness about $\frac{3}{4}$ inch for wheel of about this size.

Having thus completed the elevation, the construction of the edge view and vertical section becomes comparatively simple. Draw the perpendiculars F G and H I (Figs. 584 and 585) as central lines in the representations; set off on each side of these lines half the breadth of the teeth, and draw parallels; project the teeth of Fig. 583 upon Fig. 584, by drawing through all the visible angular points straight lines parallel to A B, and terminated at either extremity by the verticals representing the outlines of the breadth of the wheel; project in like manner the circles of the hub; lay off half length on each side of F G, and draw parallels to it. The section (Fig. 585) is supposed to be made on the line D E of the elevation; project, as in Fig. 584, those portions which will be visible in this section, and shade those parts which are in section. The arms are made tapering in width, and somewhat less than the face of the wheel.

Since the two projections (Figs. 583 and 585) are not sufficient to exhibit fully the true form, a cross-section of one of them is given at Fig. 586; this section is supposed to be made by a plane passing through X X′ and Y Y′. The points y, z, in Fig. 583, and corresponding lines in Fig. 585, represent the edges of *key-seat.*

Oblique Projection of a Spur-Wheel.—In drawing a spur-wheel or other object in an oblique position with respect to the vertical plane of projection, it is necessary, in the first place, to lay down the elevation and plan as if it were parallel to that plane, as represented in Figs. 587 and 589. Then transfer the plan to Fig. 590, giving it the same inclination with the ground line which the wheel ought to have in relation to the vertical plane; and assuming that the

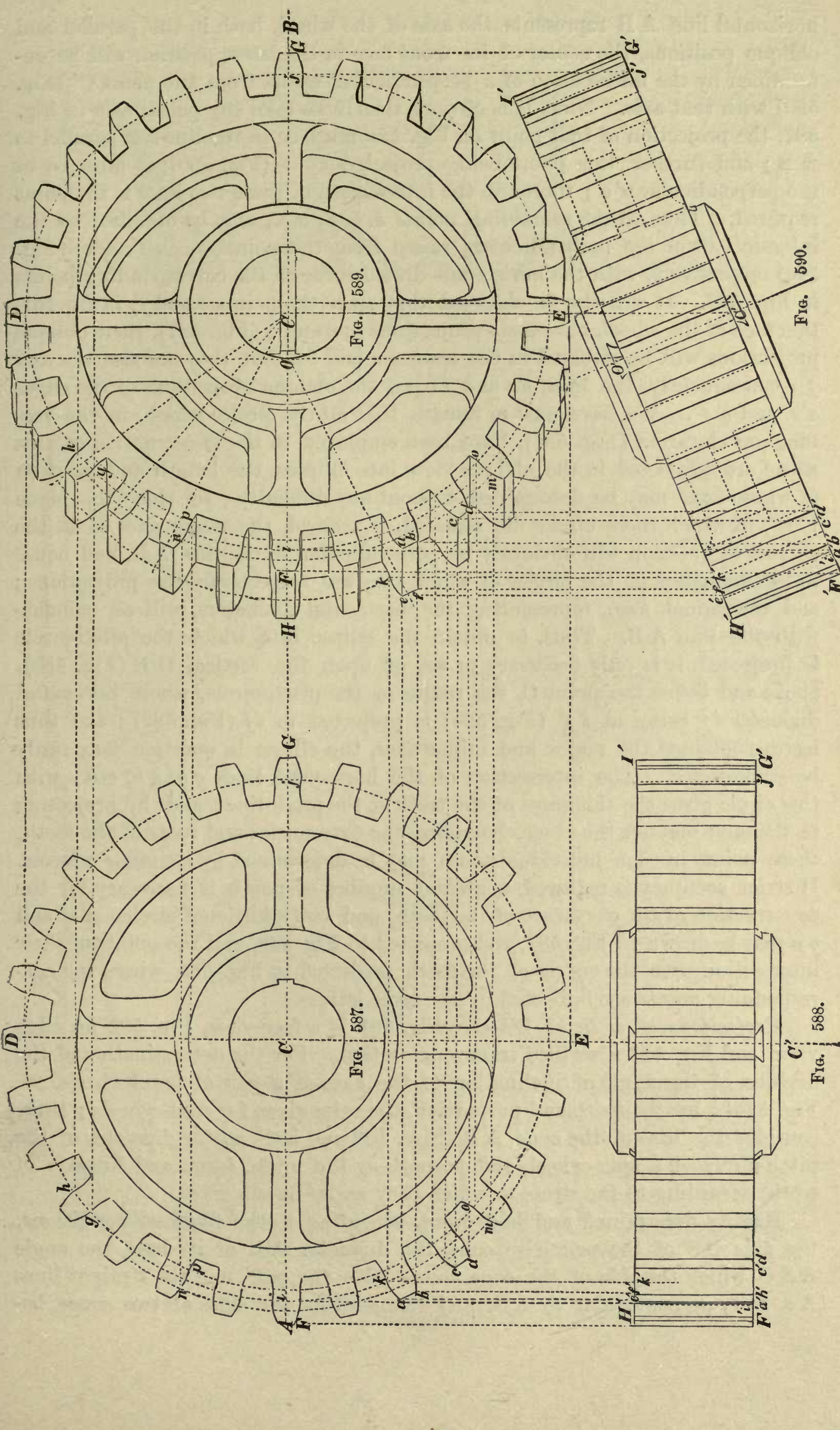


Fig. 589.

Fig. 590.

Fig. 587.

Fig. 588.

horizontal line A B represents the axis of the wheel, both in the parallel and oblique positions, the center of its front face in the latter position will be determined by the intersection of a perpendicular raised from the point C′ (Fig. 590) with that axis. Now, it is obvious that if we take any point, as *a* in Fig. 587, the projection of that point on Fig. 589 must be in the line *a a*, parallel to A B; and further, this point being projected at *a*′ (Fig. 590), it must be in the perpendicular *a*′ *a*; therefore the intersection of these two lines is the point required. Thus all the remaining points *b*, *c*, *d*, etc., may be obtained by the intersections of the perpendiculars raised from the points *b*′, *c*′, *d*′, etc. (Fig. 590) respectively, with the horizontals drawn through the corresponding points in Fig. 587. It will also be observed that since the points *e* and *f*, in the further face of the wheel, have their projections in *a* and *b* (Fig. 587), their oblique projections will be situated in the lines *a a* and *b b*, but they are also at *e* and *f*; consequently, the lines *e a* and *f b* are the oblique projections of the edges *a*′ *e*′ and *b*′ *f*′. We have now to remark that all the circles which, in the rectangular elevation (Fig. 587), have been employed in the construction of this wheel are projected in the oblique view into ellipses, the length and position of whose axes may be determined without any difficulty; for since the plane F′ G′, in which these circles are situated, is vertical, the major axes of all the ellipses in question will obviously be perpendicular to the line A B, and equal to the diameters of the circles of which they are respectively the projections; and the minor axes, representing the horizontal diameters, will all coincide with the line A B. Thus, to obtain the ellipse into which the pitch-circle is projected, it is only necessary to set off upon the vertical D E (Fig. 589), above and below the point C, the radius of the pitch-circle, whose horizontal diameter *i j* being at *i*′ *j*′ (Fig. 590) is projected to *i j* (Fig. 589); and thus having obtained the major and minor axes, the ellipse in question may easily be constructed. The intersection of the horizontal lines *g g*, *h h*, etc., with this circle gives the thickness of the teeth at the pitch-line; and, by projecting in the same manner the circles bounding the extremities and roots of the teeth, these points in each individual tooth may be determined by a similar process. If strict accuracy is required, a greater number of points is necessary for the construction of the curvature of the teeth, and two additional circles *m n* and *o p* may be drawn on Fig. 587, and projected to Fig. 589, and the points of their intersection with the curves of the teeth projected to Fig. 589, where the corresponding points are indicated by the same letters.

Projections of a Bevel-Wheel.—Fig. 591 is a face view, Fig. 592 an edge view, and Fig. 593 a vertical transverse section. For the determination of the division of the angle of inclination of the axes of a pair of bevel-wheels, see Fig. 575); for their size and proportion, the rules given for spur-wheels; thus, consider the base of the cone A B (Figs. 592 and 593) as the diameter of the pitch-circle of a spur-wheel, and proportion the pitch, form, and breadth of teeth, according to the stress to which they are to be subjected.

Having determined and laid down, according to the required conditions, the axis O S of the primitive cone, the diameter A B of its base, the angle A S O which the side of the cone makes with the axis, and the straight lines A *o*, D *o*′, perpendicular to A S, and representing the sides of two cones, be-

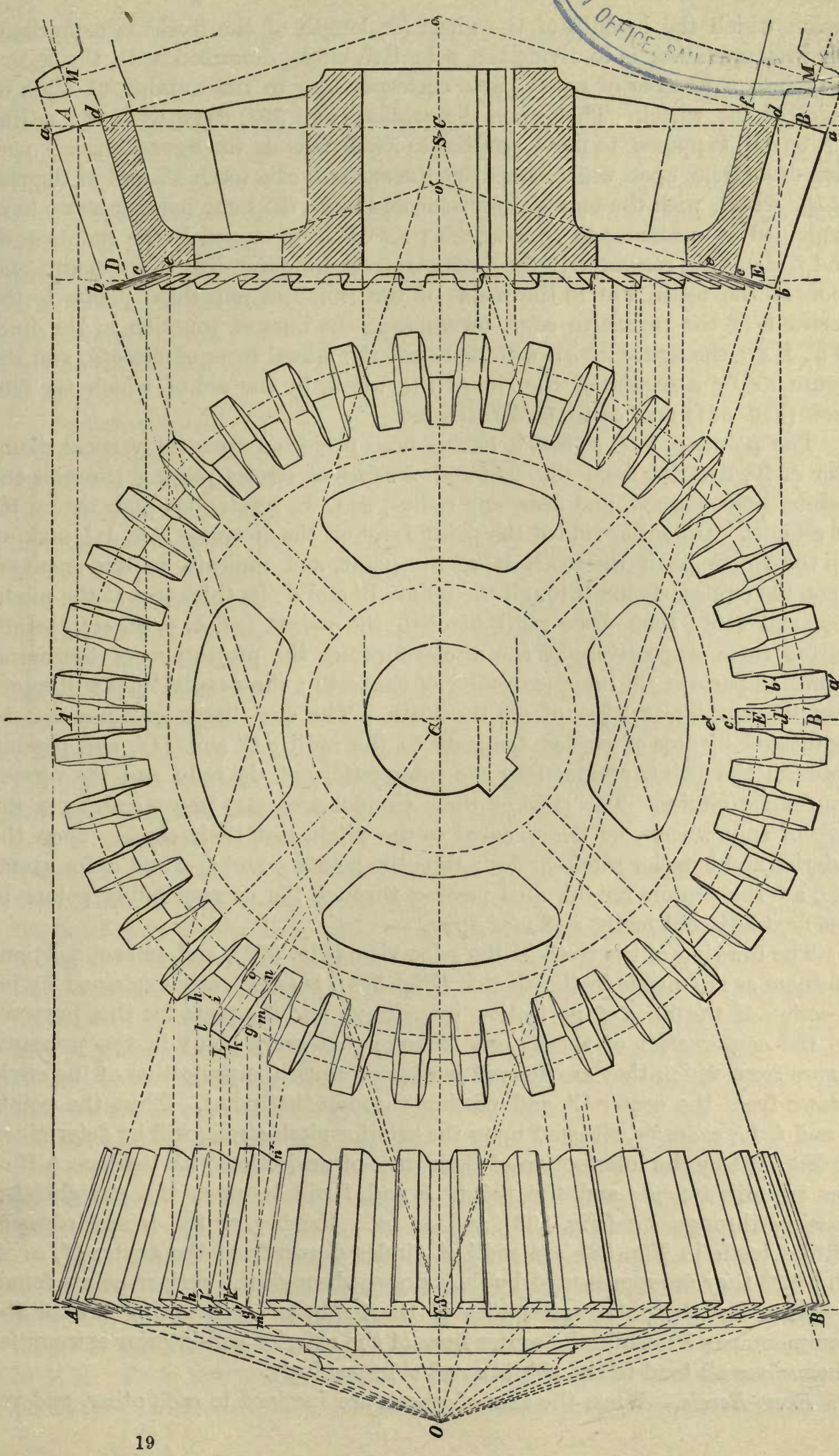


Fig. 593.

Fig. 592.

Fig. 591.

tween which the breadth of the wheel (or length of the teeth) is comprised, the first operation is to divide the primitive circle, described with the radius A C, into a number of equal parts corresponding to the number of teeth or *pitch* of the wheel. Then upon the section (Fig. 593) draw with the radius *o* A or *o* B, supposed to move parallel to itself, outside the figure, a small portion of a circle, upon which construct the outlines of a tooth M, and of the rim of the wheel, with the same proportions and after the same manner as we have explained in reference to spur-wheels; set off from A and B the points *a, d,* and *f,* denoting respectively the distances from the pitch-line to the points and roots of the teeth, and to the inside of the rim, and join these points to the vertex S of the primitive cone, terminating the lines of junction at the lines D *o'*, E *o'*; the figure *a b c d* will represent the lateral form of a tooth, and the figure *c d f e* a section of the rim of the wheel, by the aid of which the face view (Fig. 591) may easily be constructed.

The points *a, b, c, d,* and *e,* having been projected upon the vertical diameter A' B', describe from the center C' a series of circles passing through the points thus obtained, and draw any radius, as C' L, passing through the center of a tooth. On either side of the point L set off the distances L *k,* L *l,* making up the thickness of the tooth M at the point, and indicate, in like manner, upon the circles passing through the points B' and *d'*, its thickness at the pitch-line and root; then draw radii through the points *i, l, k, g, m,* etc., terminating them respectively at the circles forming the projections of the corresponding parts at the inner extremity of the teeth; these radial lines will represent the rectilinear edges of all the teeth. The curvilinear outlines may be delineated by arcs of circles, tangents to the radii *g* C' and *i* C', and passing through the points obtained by the intersections of the radii and the various concentric circles. The radii of these circular arcs may in general, as in the case of spur-wheels, be taken equal to the pitch, and their centers upon the interior and exterior pitch-circles; thus the points *g* and *i, n* and *o,* for example, are the centers for the arcs passing through the corresponding points in the next adjacent teeth, and *vice versa.*

The drawing of the teeth in the edge view (Fig. 592), and of such portions of them as are visible in the section (Fig. 593), is sufficiently explained by inspection of the lines of projection introduced into the plate for this purpose. In the construction of these views, observe that every point in the principal figure from which they are derived is situated upon the projection of the circle drawn from the center C', and passing through that point. Thus the points *g* and *i,* for example, situated upon the exterior pitch-circle, will be determined in Fig. 592 by the intersection of their lines of projection with the base A B of the primitive cone; and the points *k* and *l* will be upon the straight line passing through *a a* (Fig. 593), and so on. Farther, as the lateral edges of all the teeth in Fig. 591 are radii of circles drawn from the center C', so in Fig. 592 they are represented by lines drawn through the various points found as above for the outer extremities of the teeth, and converging toward the common apex S; while the center lines of the exterior and interior extremities themselves all tend to the points *o* and *o'* respectively.

Skew-Bevels.—When the axes of wheels are inclined to each other, and yet do not meet in direction, and it is proposed to connect them by a single pair of bevels, the teeth must be inclined to the base of the frusta to allow them to come into contact. Set off *a e* (Fig. 594) equal to the shortest distance between the axes (called the *eccentricity*), and divide it in *c*, so that *a c* is to *e c* as the mean radius of the frustum to the mean radius of that with which it is to work; draw *c m d* perpendicular to *a e*. The line *c m d* gives the direction of the teeth; and, if from the center *a*, with radius *a c*, a circle be described, the direction of any tooth of the wheel will be a tangent to it, as at *c*. Draw the line *d e* perpendicular to *c m d*, and with a radius *d e* equal to *c e* describe a circle; the direction of the teeth of the second wheel will be tangents to this last, as at *d*.

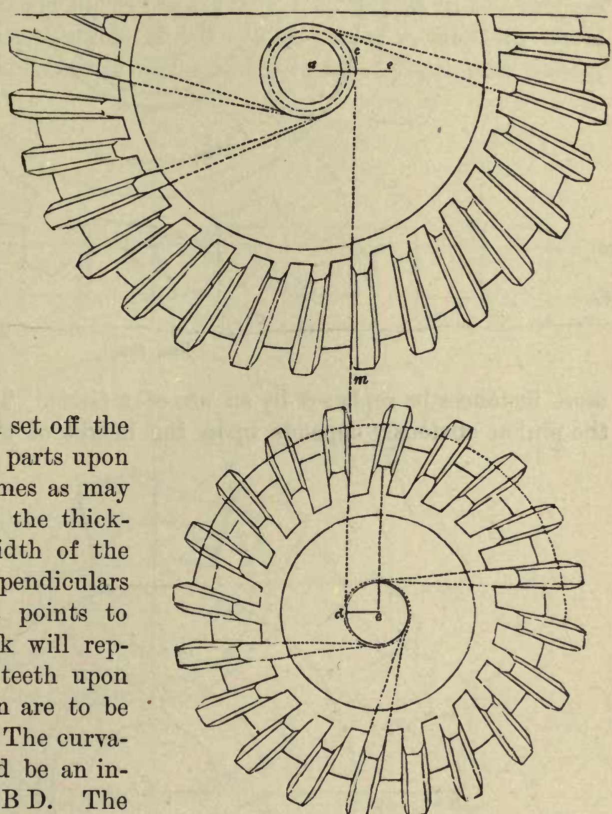


Fig. 594.

System composed of a Pinion driving a Rack (Fig. 595).—The pitch-line M N of the rack and the pitch-circle A B D of the pinion being laid down touching one another, divide the latter into twice the number of equal parts that it is to have of teeth, and set off the common distance of these parts upon the line M N, as many times as may be required; this marks the thickness of the teeth and width of the spaces in the rack. Perpendiculars drawn through all these points to the solid part of the rack will represent the flanks of the teeth upon which those of the pinion are to be developed in succession. The curvature of these latter should be an involute A *c* of the circle A B D. The teeth might be cut off at the point of contact *d* upon the line M N, for at this position the tooth A begins its action upon that of the rack E; but it is better to allow a little more length; in other words, to describe the circle bounding the points of the teeth with a radius somewhat greater than C *d*.

With regard to the form of the spaces in the rack, all that is required is to set off from M N, as at the point *e*, a distance slightly greater than the difference A *a* of the radius of the pitch-circle, and that of the circle limiting the points of the teeth, and through this point to draw a straight line F G parallel to M N. From this line the flanks of all the teeth of the rack spring, and their points are terminated by a portion of a cycloid A *b*, which, however, may in

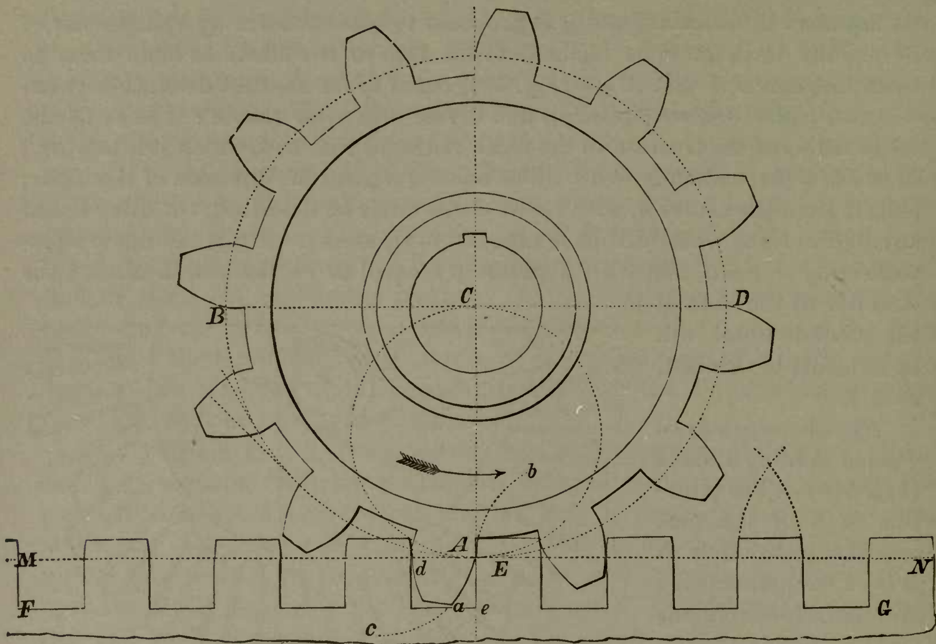


Fig. 595.

most instances be replaced by an arc of a circle. The depth of the spaces in the pinion obviously depends upon the height of this curved portion of the

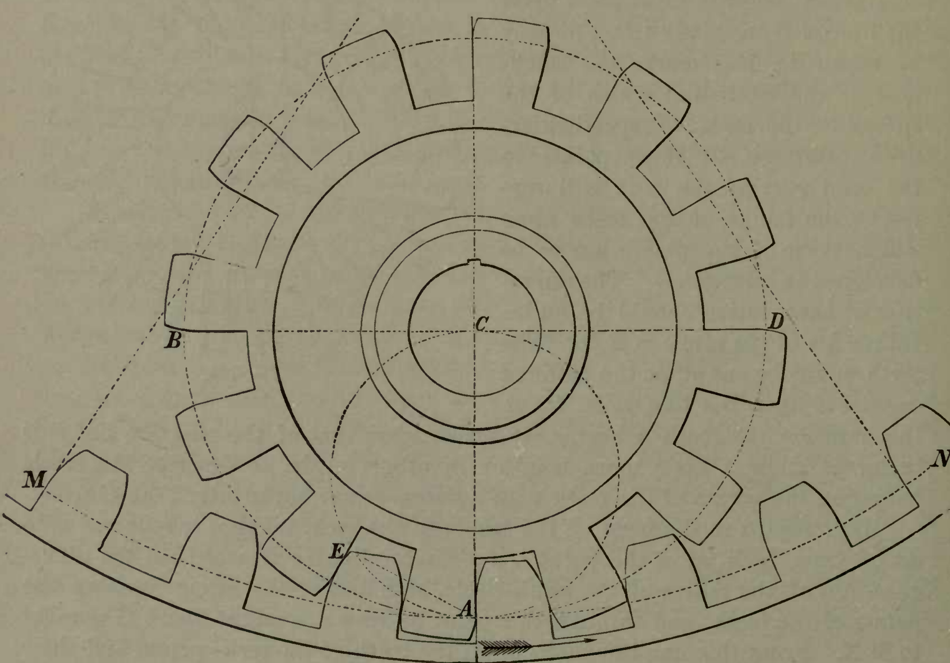


Fig. 596.

teeth; their outline is formed by a circle drawn from the center C, with a radius a little less than the distance from this point to the straight line bounding the upper surface of the teeth of the rack.

System composed of a Rack driving a Pinion.—In this case the construction is in all respects identical with that of the preceding example, with this exception, that the form proper to be given to the teeth of the rack is a cycloid generated by a point A in the circumference of the circle A E C rolling on the line M N. The curvature of the teeth is an involute as before.

System composed of an Internal Spur-Wheel driving a Pinion (Fig. 596).—The form of the teeth of the driving-wheel is in this instance determined by the epicycloid described by a point in the circle A E C, rolling on the concave circumference of the primitive circle M A N. The points of the teeth are to be cut off by a circle drawn from the center of the internal wheel, and passing

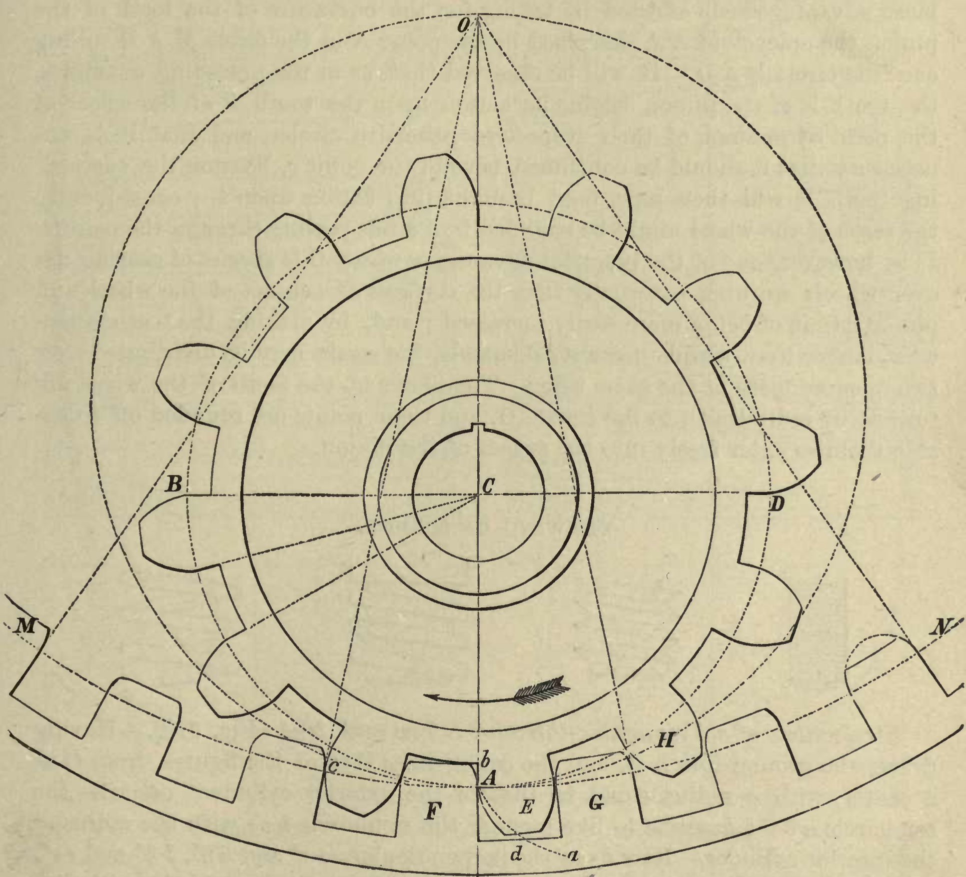


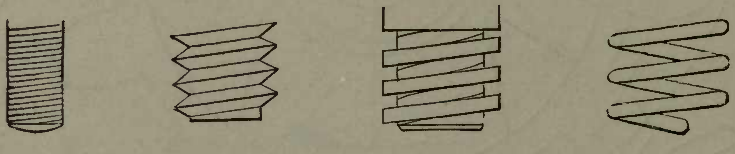
FIG. 597.

through the point E, which is indicated, as before, by the contact of the curve with the flank of the driven tooth.

The wheel being supposed to be invariably the driver, the curved portion of the teeth of the pinion may be very small. This curvature is a part of an epicycloid generated by a point in the circle M A N rolling upon B A D.

System composed of an Internal Wheel driven by a Pinion (Fig. 597).—This problem involves a different mode of treatment from that employed in the preceding cases. The epicycloidal curve A *a*, generated by a point in the circle having the diameter A O, the radius of the circle M A N, and which rolls upon the circle B A D, can not be developed upon the flank A *b*, the line described by the same point in the same circle in rolling upon the concave circumference M A N; and for this obvious reason, that that curve is situated *without* the circle B A D, while the flank, on the contrary, is *within* it. It becomes necessary, therefore, in order that the pinion may drive the wheel uniformly according to the required conditions, to form the teeth so that they shall act always upon one single point in those of the wheel. This may be most advantageously effected by taking for the curvature of the teeth of the pinion the epicycloid A *d*, described by the point A in the circle M A N rolling over the circle B A D. It will be observed that, as in the preceding examples, the tooth E of the pinion begins its action upon the tooth F of the wheel at the point of contact of their respective primitive circles, and that it is unnecessary that it should be continued beyond the point *c*, because the succeeding tooth H will then have been brought into action upon G; consequently the teeth of the wheel might be bounded by a circle passing through the point *c*. It is, however, one of the practical advantages which this species of gearing has over wheels working externally that the surfaces of contact of the wheel and pinion admit of being more easily increased; and, by making the teeth somewhat longer than simple necessity demands, the strain may be distributed over two or more teeth at the same time. The flanks of the teeth of the wheel are formed by radii drawn to the centre O, and their points are rounded off to enable them to enter freely into the spaces of the pinion.

DRAWING OF SCREWS.



Projections of a Triangular-threaded Screw and Nut (Fig. 598).—Having drawn the ground line A B, and the center lines C C′ of the figures, from O as a center, with a radius equal to that of the exterior cylinders, describe the semicircle *a* 3 6; describe in like manner the semicircle *b c e* with the radius of the interior cylinder. Now draw the perpendiculars *a a*″ and 6 6″, *b b*″ and *e e*″, which will represent the vertical projections of the exterior and interior cylinders. Then divide the semicircle *a* 3 6 first described into any number of equal parts, say 6, and through each part draw radii, which will divide the interior semicircle similarly. On the line *a*′ *a*″ set off the length of the pitch as many times as may be required; and through the points of division draw straight lines parallel to the ground line A B. Then divide each distance or pitch into twice the number of equal parts that the semicircles have been divided into, and, following instructions already laid down (page 102), construct the helix *a'* 3' 6 both in the screw and nut.

Having obtained the point *b'*, by the intersection of the horizontal line passing through the middle division of *a' a* with the perpendicular *b b''*, describe the helix *b' c' e'*, which will represent the bottom of the groove. The apparent out-

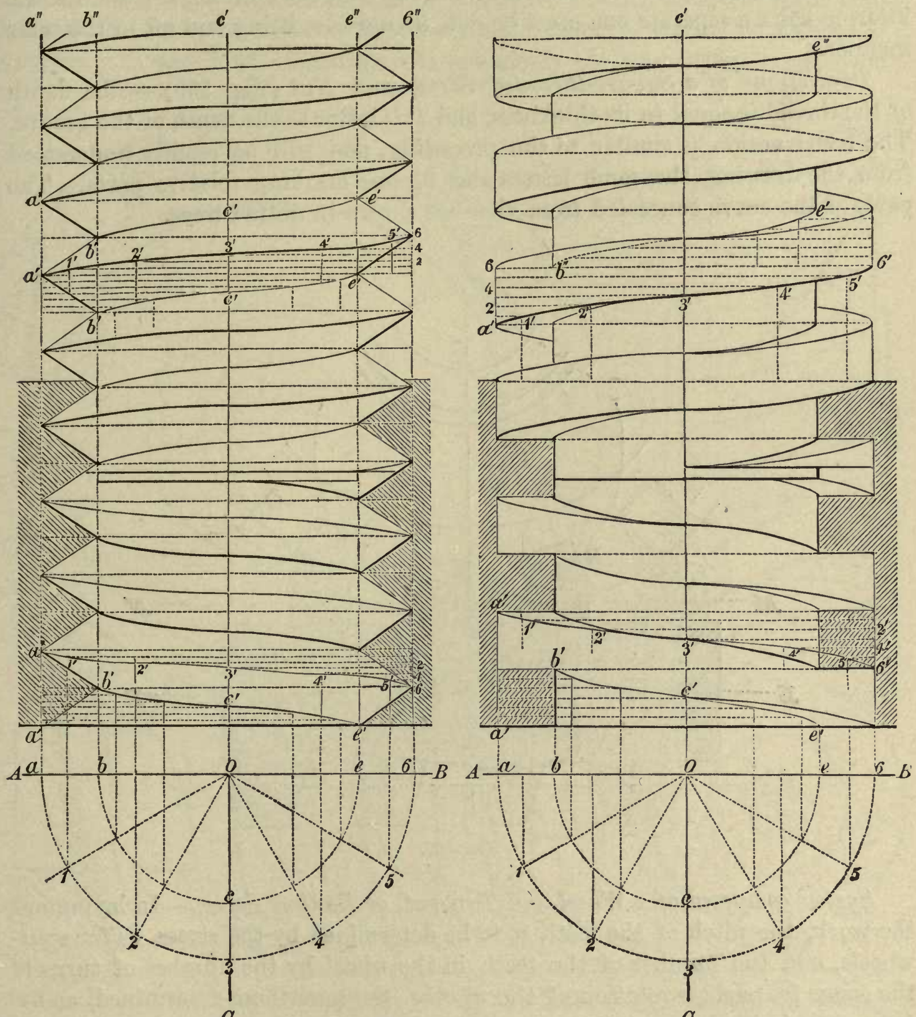


FIG. 598. FIG. 599.

lines of the screw and its nut will then be completed by drawing the lines *b' a'*, *a' b'*, etc., to the curves of the helices ; these are not, strictly speaking, straight lines, but their deviation from the straight line is, in most instances, so small as to be imperceptible, and it is therefore unnecessary to complicate the drawing.

When a long series of threads have to be delineated, they should be drawn mechanically, by means of a *mold* or *templet* constructed in the following manner: Take a small slip of thin wood or pasteboard, and draw upon it the helix a' 3′ 6 to the same scale as the drawing, and pare the slip carefully and accurately to this line. By applying this templet upon Fig. 598, so that the points a' and 6 on the plate shall coincide with a' and 6 on the drawing, the curve a' 3′ 6 can be drawn mechanically, and so on for the remaining curves of the outer helix. The same templet may be employed to draw the corresponding curves in the screw-nut by simply inverting it; but for the interior helix a separate one must be cut, its outlines being laid off in the same manner.

Projections of a Square-threaded Screw and Nut (Fig. 599).—The depth of the thread is equal to its thickness, and this latter to the depth of the groove. The construction is similar to the preceding, and will be readily understood from the drawing, the same letters and figures marking relative parts. The parts of the curve concealed from view are shown in dotted lines.

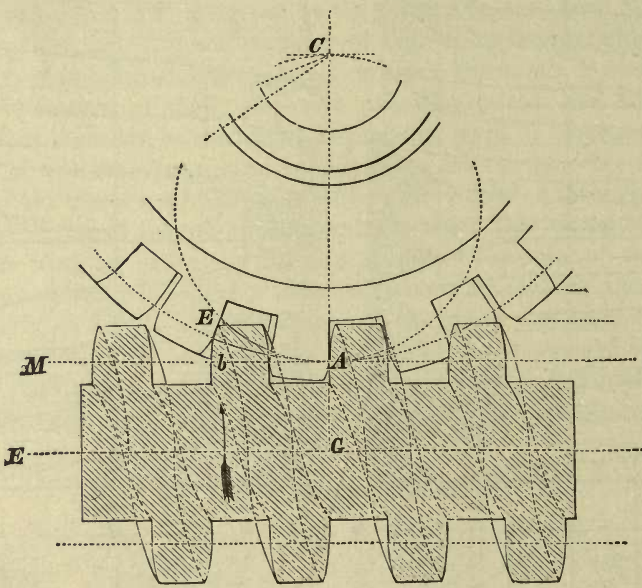


FIG. 600.

System composed of a Wheel and Tangent, or Endless Screw.—In laying out the work, the pitch of the teeth is to be determined by the stress, as for spur-wheels, and the number of the teeth in the wheel by the number of turns of the screw for each revolution of the wheel. Suppose these determined, and C (Fig. 600) to be the center of the wheel, E F the axis of the screw, C A the radius of the pitch-circle of the wheel, and G A that of the pitch-cylinder of the screw; the line M N drawn through A, parallel to E F, will be the generatrix of that cylinder, which will serve the purpose of determining the form of the teeth. The section is made through the axis, and is obviously the case of a rack driving a pinion; consequently the curve of the teeth, or rather *thread*, of the screw should be simply a cycloid generated by a point in the circle A E C, described upon A C as a diameter, and rolling upon the straight line

M N. The outlines of the teeth are helical surfaces described about the cylinder forming the screw, with the pitch A *b* equal to the distance, measured upon the primitive scale, between the corresponding points of two contiguous teeth. These curves are expressed by dotted lines. The teeth of the wheel are set at angle to the plane of its face, and with surfaces corresponding to the inclination and helical form of the thread of the screw. Usually the points of the teeth and bottoms of the spaces are formed of a concave outline, adapted to the convexity of the screw, in order to present as much bearing surface as possible to its action. In this kind of gearing it is invariably the screw that imparts the

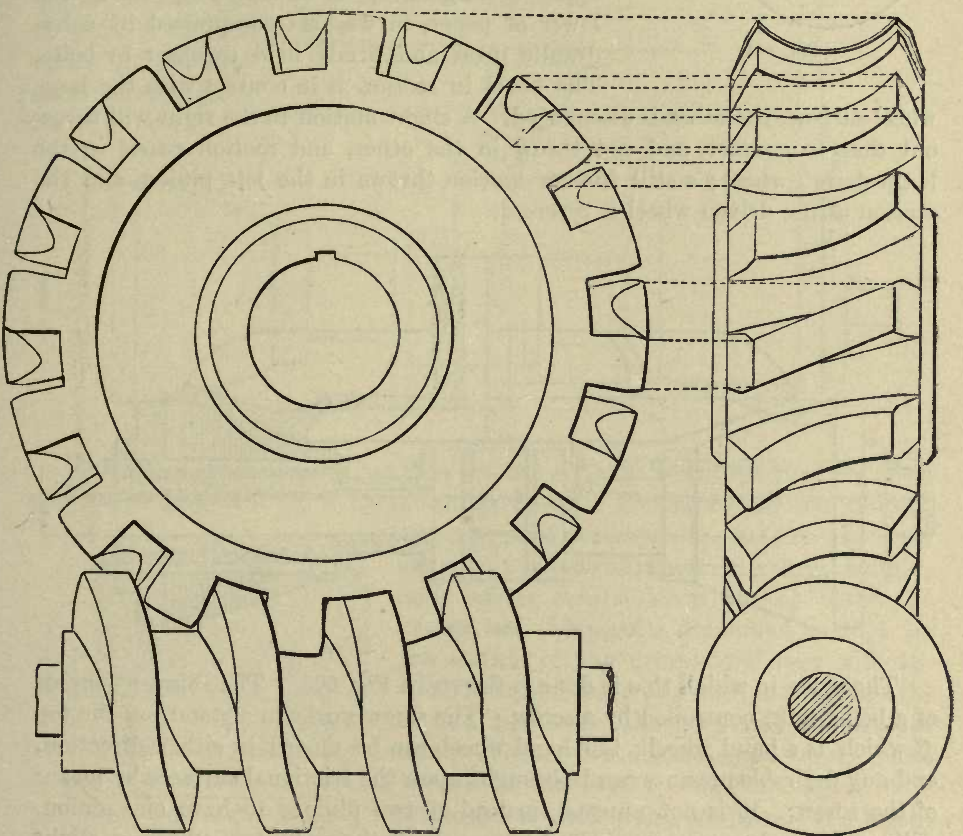


Fig. 601. Fig. 602.

motion ; but in the proportions adopted by the Yale & Towne Manufacturing Co. for worm gearing, the wheel under the weight will revolve the screw slowly. This angle of the teeth is found to be the best adapted for economy of power. In the wheel the teeth in section are those of a spur-wheel, cut with a chasing cutter, and in the screw turned in a lathe.

Figs. 601 and 602 are two views, worm and wheel, with such lines of construction dotted as will explain the manner of drawing.

Frictional Gearing.—When motion is not continuous for a long time, either having frequently to be stopped and started or reversed, frictional gearing is very often used. The starting is with as little shock as with belting, and under the proper conditions of pressure it is fully as positive, and by the usual appliances this pressure may be applied gradually. The simplest form of frictional gearing is that in which the surfaces in contact correspond to that of the pitch-circles (Fig. 603).

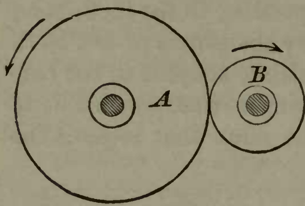


Fig. 603.

Fig. 604 is a bevel frictional gear, such as is used in Dow's grain stores, Brooklyn, N. Y. One half is shown in section. The surface of the upper or larger gear is of cast-iron, that of the lower of paper, in washers compressed by a hydraulic press and firmly held together by bolts. The bevel in section is in contact with the large wheel-surface, the other is disengaged. A slight motion to the right will throw out that in contact, and not throw in the other, and motion ceases in the large driven wheel; a still further motion throws in the left pinion, and the motion of the driven wheel is reversed.

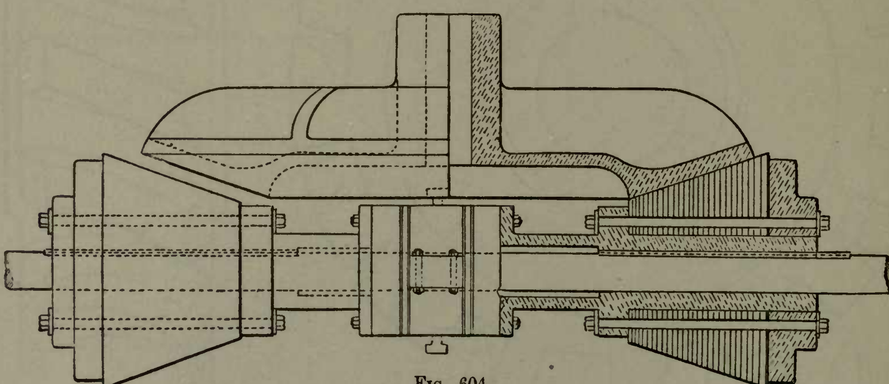


Fig. 604.

The mode in which this is done is shown in Fig. 605. The shipper consists of a bell-crank, controlled by a screw. The screw works in a stand, on the top of which is a hand wheel; the hand wheel can be moved in either direction, and any desirable pressure can be brought upon the frictional surfaces by means of the screw. It is not unusual, instead of two pinions to have one pinion, with a little clearance on each side, revolving between two wheels, a slight lateral motion, in either direction, bringing it in contact with one or the other of the wheels. Some provision, by a loose coupling or otherwise, must be made to admit of this lateral movement in the pinion shaft. Straight pulleys, or what would correspond to spur-gears without teeth, are constructed, as in the example given, and are thrown in or out of gear by a lateral motion of the pinion.

In proportioning the face of the pulleys it has been found safe to consider it the same as belts, given in the table (page 273). The pressure can be applied according to the requirements of driving, and there is no falling off in the friction. The frictional surfaces are not always paper; wood, leather, and prepared rubber are frequently used.

Wedge Gearing, or Robertson Grooved-Surface Frictional Gearing.—Fig. 606 is the cross-section of the rims of two wheels of this gearing. The angle recommended by Robertson is 50° (usually not over 30° in our practice), and the pitch to vary somewhat with the velocity and power to be transmitted. The adhesion, under a pressure equal to that of the tension of a belt, is proved to be greater, and it would be safe to make the horizontal face equal to that of a belt under the same circumstances of transfer of power.

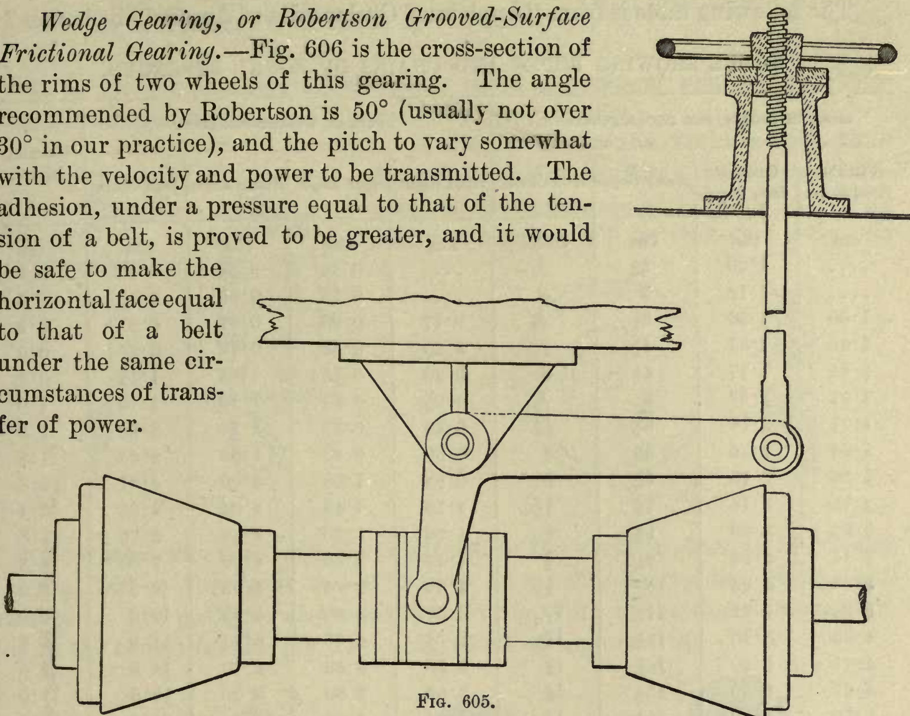


Fig. 605.

The use of ropes as belts has been treated of (page 274), but they are often used, as in Fig. 607, for a reciprocating power. The ropes are not endless, but consist of two ropes, the ends of which are attached to two drums parallel with each other, each having several turns on the barrels or drums, but in opposite directions, so that, by the motion of the drums, one rope will unwind from one drum and wind up on the other, and *vice versa*, the length of the reciprocatory movement being measured by the turns on one of the drums. This arrangement is sometimes applied to run the barrels of a hoist; the barrels being attached to one drum and the power applied at the other, and in this form the application may be at considerable distance apart.

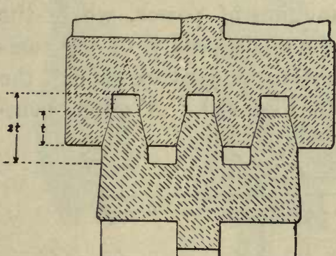


Fig. 606.

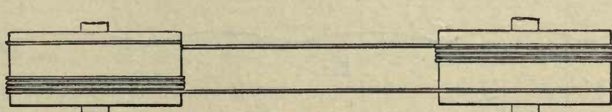


Fig. 607.

A similar arrangement with chains, instead of ropes, was much used for the reciprocating motion of the bed in the older type of planers.

The following table is from "Appletons' Cyclopædia of Applied Mechanics":

TABLE SHOWING ROPES AND CHAINS OF EQUAL STRENGTH.

SIZES, IN INCHES, FOR EQUAL STRENGTH.				AVERAGE WEIGHT PER FOOT.				Working Strain.
Crucible Steel Rope.	Charcoal Iron Rope.	Hemp Rope.	Iron Chain.	Steel Rope.	Iron Rope.	Hemp Rope.	Iron Chain.	
Cir.	Cir.	Cir.	Diam.	Lbs.	Lbs.	Lbs.	Lbs.	Tons.
....	1·00	2¾	7/32	...	0 14	0·34	0·50	0·3
....	1·18	3	¼	....	0·21	0·46	0·65	0·4
1·00	1·39	3½	9/32	0·17	0·28	0·67	0·81	0·5
1·26	1·57	4¼	5/16	0·25	0·33	0·75	0·96	0·6
1·45	1·77	4½	⅜	0·30	0 45	0·83	1·38	0·8
1·57	1·97	5	7/16	0·35	0·57	1·16	1·76	1·0
1·77	2·19	5½	15/32	0·45	0·70	1·20	2·20	1 3
1·96	2·36	5¾	½	0·59	0·83	1·60	2·63	1·5
2·36	2·75	6¾	⅝	0 85	1·08	2·00	4·21	2·3
2·75	3·14	7¾	11/16	1·10	1·43	2·65	4·83	3·1
2·95	3·53	8¾	¾	1·28	1·80	3·35	5·75	3·8
3·14	3·93	9¾	⅞	1·45	2·30	4·00	7·50	4·8
3·53	4·32	10½	15/16	1·83	2·94	4 92	9·33	5·9
3·93	4·71	11¾	1 1/16	2·33	3·56	5·83	10·6	7·0
4·32	5·10	12¾	1⅛	2 98	4·00	6 20	11·9	8·2
4·71	5·50	14¾	1¼	3·58	4·80	8·70	14·5	9·5
4·81	5·89	15¼	1⅜	3·65	5 60	9·00	17·6	11·0
5·10	6·28	15¾	1½	4·04	6·30	10·1	20·0	12·5
5·89	7·07	17¾	1⅝	5·65	7·95	13 7	22·3	15·9
6·35	7·85	19½	1¾	6 50	9·81	16·4	24·3	19·6

Endless chains are often used for the transmission of power, where the stress is great and the movement slow. When the chain used is of the common form, the wheels must be fitted with depressions or caps to receive the flat links, with a slot for the vertical links, as in Fig. 617. A chain com-

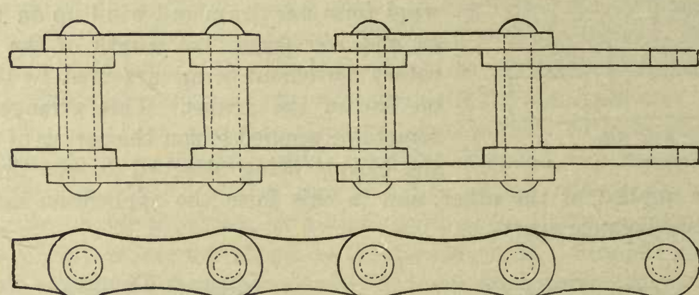


FIG. 608.

posed of punched links, as in Fig. 608, admits of a tooth between the links, and the wheels on which these run have therefore teeth adapted to the chain, which is composed of links of uniform length.

But the chief application of ropes and chains is for the purpose of hoisting or lowering heavy weights or loads, by the means of pulleys and blocks, or barrels and capstans.

Rope for running-rigging is usually made of hemp or manilla, and wire-rope for this purpose is mostly made with hemp centers.

A simple rule for the working-strength of these ropes is to multiply the square of the *girth* or circumference of the rope by 100 for hemp or manilla, 600 for iron-wire rope, and 1,000 for steel-wire rope, and the result will be the working-strength in pounds.

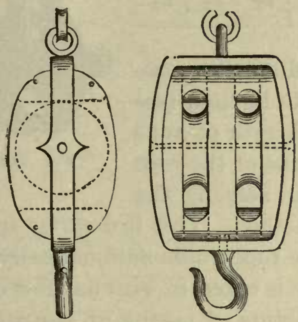


FIG. 609.

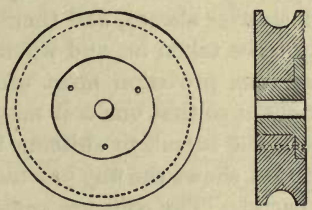


FIG. 610.

Fig. 609 is the front and side view of a common wooden block, iron-strapped. The pulley or *sheave* is shown in Fig. 610 ; the section shows a bushing at the center for the pin ; the sheaves are of lignum vitæ.

FIG. 611.

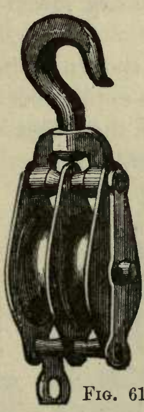


FIG. 612.

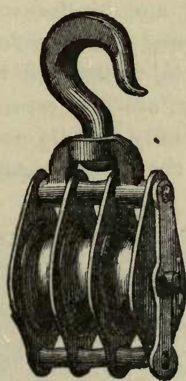


FIG. 613.

FIG. 614.

Figs. 611, 612, 613, 614 are wrought-iron tackle-blocks of the Yale & Towne Manufacturing Company's pattern. The lower block of every set is always sent with a becket attached, as shown in Fig. 612.

	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
Diameter of sheave.............	2½	3½	4	4¾	5	6½	7	8	9	10	11
Will take rope, diameter........	½	⅝	¾	⅞	1	1¼	1½	1¾	2	2¼	2½
Will take chain, diameter.... ...				3/16	¼	5/16	⅜	7/16	9/16	⅝	¾

Gin-blocks (Fig. 615).—These blocks are made with wrought- and malleable-iron frames and wrought swivel-hook.

	In.	In.	In.	In.	In.	In.	In.
Diameter of wheel................	10	12	14	16	18	20	22
Will take rope, diameter..........	1	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.
Will carry about..................	1	$1\frac{1}{2}$	$1\frac{1}{2}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{1}{2}$

Fig. 615.

Winding-drums or barrels must have their diameters proportioned to the diameters of the rope or chain to be used (see table of sheaves above), and their length to the length of rope or chain to be taken in, and when the coils or turns of the rope are numerous provision must often be made for keeping the rope or chain so that one coil may not ride on another. This is done by spiral grooves in the barrel, or shifting the barrel or the rope-guide automatically.

Fig. 1504 shows the way in which a chain cable is taken in with but few coils on the barrel. The coils are sufficient for the friction of taking up the cable ; the tight cable is wound on the larger part of the barrel, and as the coils are unwound on the slack side the tight coil slips down to a smaller diameter ; the weight of the chain on the slack side, as it drops into the locker, is sufficient to preserve the friction ; but with a rope, a man takes in the rope and exerts at the same time a little strain. The application of a barrel of this form for hoisting is very common ; by exerting a slight stress the man can hoist a weight on a revolving barrel, and by slacking he can lower without changing the direction of motion or speed of the barrel.

Chain-wheels with pockets, which have been spoken of in their application to the transmission of power, are also especially applicable to the purpose of hoisting, requiring a width only slightly greater than that of the chain, and a diameter sufficient to give the proper engagement with it.

Fig. 618.

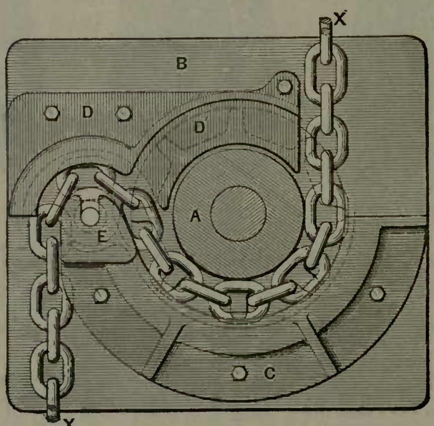


Fig. 617.

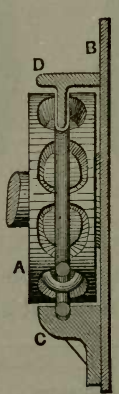


Fig. 616.

Flat punched links are of uniform length, and can be purchased of any desirable sizes, and put together in multiples ; common chain has not that uniformity in length to adapt it nicely to the pockets of the wheel. The Yale

& Towne Manufacturing Company have made a spiral chain, of common form but of uniform length, especially adapted to hoists, and Figs. 616, 617, and 618 illustrate the construction of their chain-wheel. A is a pocketed chain-wheel, made of soft cast-iron, mounted on a frame B. C is the chain-guide enveloping the lower half of the chain-wheel. The inner curved surface of the

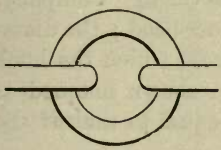


Fig. 619.

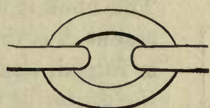


Fig. 620.

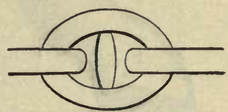


Fig. 621.

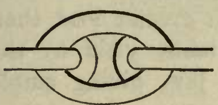


Fig. 622.

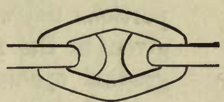


Fig. 623.

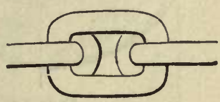


Fig. 624.

chain-guide is grooved, and is of such a shape as to leave a space between it and the periphery of the chain-wheel merely sufficient to admit the chain ; it must then enter properly and continue engaged with the chain-wheel. E is a chain-guide roller, that delivers the slack chain into the box or locker. D is the chain-stripper, bolted also to the plate B, with a tongue or rib projecting into the center groove of the wheel which disengages the chain.

The usual forms of chain-cables are represented by the open circular link (Fig. 619), the open oval (Fig. 620), oval with pointed stud (Fig. 621), oval with broad-headed stud (Fig. 622), an obtuse angled stud-link (Fig. 623), and the parallel-sided stud-link (Fig. 624). The usual proportions of chain-links are 6 diameters of the iron in length by 3½ in width. The end links, which terminate each 15 fathoms of chain, are 6·5 in length to 4·1 in breadth, and the iron about 1·2 the diameter of the rest of the chain.

Hooks.—Figs. 625 and 626 (from Redtenbacher) represent two wrought-iron hooks, in which the material is distributed according to the strain to which the parts may be subjected. The following are the proportions on which Fig. 625 is constructed: Assuming the neck of the hook as the modulus or 1, the diameter of journals of the traverse are 1·1 ; width of traverse at center, 2 ; distance from the center of the hook to the center of the traverse, 7·5 ; interior circle of the hook, 3·4 ; greatest

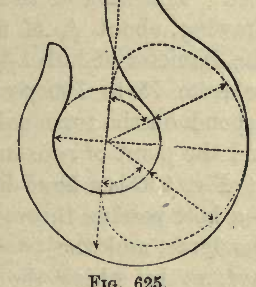


Fig. 625.

Fig. 626.

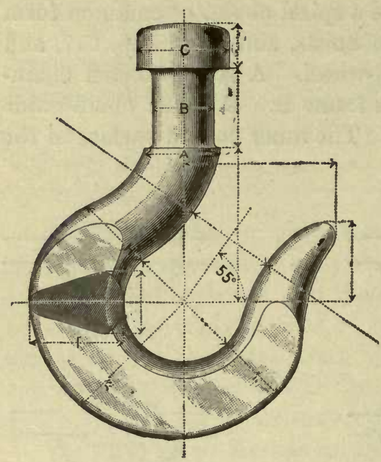


FIG. 627.

thickness of the hook, 2·8. Assuming (Fig. 626) the diameter of the wire of the chain as 1 : interior circle of hook is 3·2, and greatest thickness of hook 3·5.

Fig. 627 represents a hook as made by the Yale & Towne Manufacturing Company. This hook is fitted in a cross-head ; the diameter at A is that of iron from which the hook is forged, and the section shown hatched at the center of the hook is equal to that of the round iron.

It has been shown that hooks, of the proportions but with a much greater load than given in the following table, yield by the gradual opening of the jaw, giving ample notice before rupture.

	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.	Ton.
Capacity of hook..........	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4	5	6	8	10
	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
Dimensions of A...........	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{7}{8}$	$3\frac{1}{4}$
Dimensions of D...........	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{1}{2}$	$4\frac{1}{4}$	$5\frac{1}{4}$	$6\frac{1}{4}$

All parts of the hook are expressed in parts of A, and can readily be determined from the scale above.

Figs. 628 and 629 are side and front elevations of an ordinary straight lever on a shaft ; both are shown broken, either because the length is indefinite, or because it is inconvenient to put on the paper. The handle should be from 5 to 6 inches long, and $1\frac{1}{4}$ diameter. The bar beneath the handle to be square, and of uniform width on one side of the lever and a taper on the other, as shown, of about $\frac{1}{2}''$ in 4 feet on each side. The sides of the square at the handle to be $\frac{1}{4}\sqrt[3]{\text{length}}$ in inches, or say $\frac{3}{4}''$ for 30″ lever, $\frac{7}{8}''$ for 4 feet, and 1″ for 5 feet. The neck of the shaft to be, as proportioned in the drawing, about $\frac{8}{10}$ of the greatest width of the lever, and the diameter of hub $1\frac{3}{10}$. The stress exerted by a man may be from 75 to 100 pounds, and the size of the shaft will depend on the torsional stress between the hub of the lever and the point of resistance.

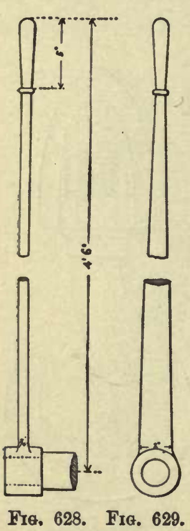


FIG. 628. FIG. 629.

Fig. 630 is a hand-lever forming one arm of a bell-crank —a bolt passing through a slot in the frame and the arm of the lever, and the two are clamped together by a *thumb-nut*, *n*, by which the lever can be held in any position. The same purpose is often effected by notches in the frame, into which the arm of the lever is caught, or by spring latches, as in Fig. 631.

Figs. 632 and 633 are side view and plan of a foot-lever. The foot-plate is 8″ × 5″ × $\frac{5}{8}$″, and as the lever is subject to double the stress of the hand-lever above, the dimension should be somewhat increased. The side of square next

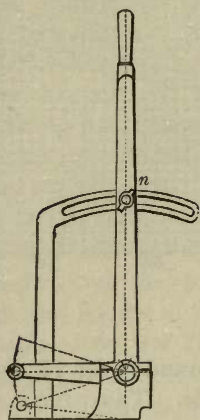


Fig. 630.

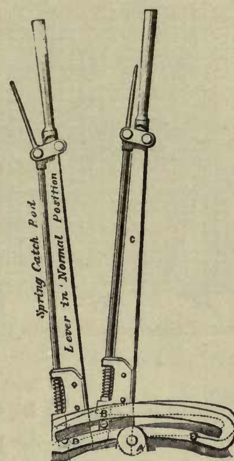


Fig. 631.

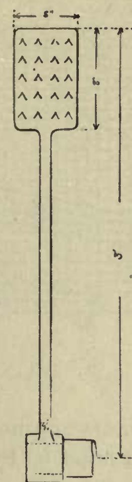


Fig. 632.

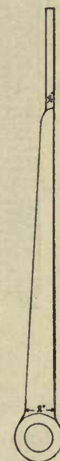


Fig. 633.

the foot-plate should be, say for a lever of 30″, 1″ ; of 4 feet, 1$\frac{1}{8}$; of 5 feet, 1$\frac{1}{4}$; the form and taper as in the hand-lever.

Figs. 634 and 635 are views of a hand-crank. The diameter of the handles, for convenience in grasping, should not be less than 1$\frac{1}{4}$″ ; if for the force of two men, 1$\frac{1}{2}$″, and from the diameter of the handle the rest may be proportioned as in the figure. The length of handle for a single man should be from 10″ to 12″ ; for two men, from 20 to 24 ; the crank from 15″ to 18″, and the height of shaft above the foot support for the men from 2′10″ to 3′2″.

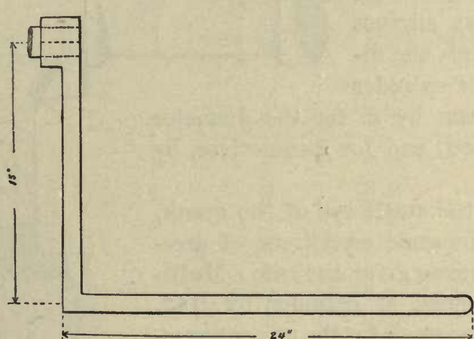


Fig. 634.

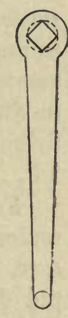


Fig. 635.

Engine-Cranks.—Fig. 636 is a graphic representation made from a table from Bourne's "Handbook of the Steam-Engine," for determining "the diameters of wrought crank-shaft journals"—i. e., of the large eye of the crank. The ordinates are diameters in inches of the steam cylinder, the inclined lines the stroke in feet, and the abscissas the diameters of the eye in inches.

Use of the Table.—To find the diameter of large eye of crank of a steam-engine 40″ cylinder and 4-foot stroke. Find on what line of abscissa is the intersection of the ordinate 40″ with the diagonal 4′ of stroke, which will be about 8$\frac{1}{8}$″, the diameter of crank-eye.

The table is calculated on a steam pressure in the cylinder of 25 pounds; not the average pressure, but the maximum. This pressure is much less than present practice, but the table can be readily adapted to any pressure. For

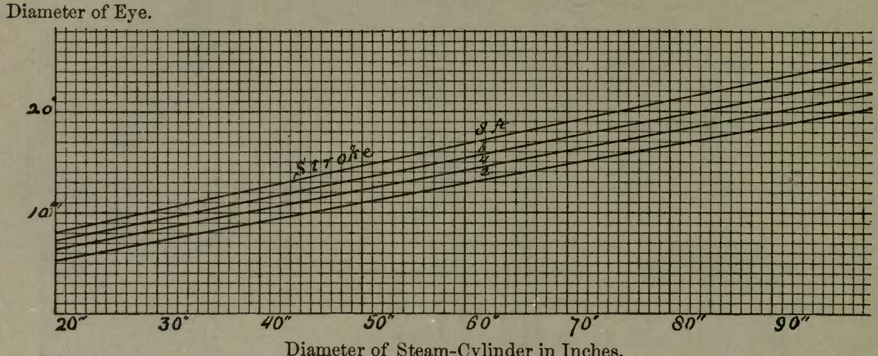


Fig. 636.

most stationary engines the pressure is from 75 to 100 pounds; for 75, the area on diagram must be three times what it is for 25 pounds. Thus, for a steam-cylinder of 30″ diameter and under 75 pounds pressure, multiply the area of $30''\text{D.} \times \frac{75}{25} = 706{\cdot}9 \times 3 = 2120{\cdot}7$ = area of 52″ diameter, which use for determining the diameter of eye instead of 30″. It will agree very nearly with common practice for stationary engines to multiply the diameter of cylinder in diagram by 2, for the diameter to be used, and for locomotives, by 2·5.

For the small eye of the crank, under the same conditions of pressure, Bourne gives the rule: Multiply diameter of cylinder by ·142. This is too small for the present practice, which is from ·17 to ·25 or $\frac{1}{8}$ to $\frac{1}{4}$ the diameter of the cylinder. The crank-pins are made of steel or iron case-hardened. Eyes are bored by hydraulic or screw press to a very tight fit, and forced on to the shaft or pin, or heated and shrunk on.

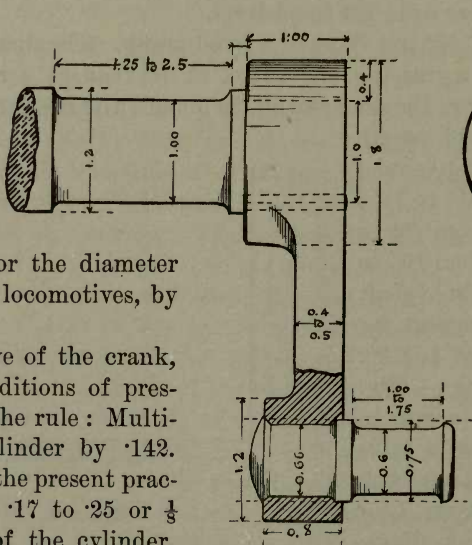


Fig. 637.

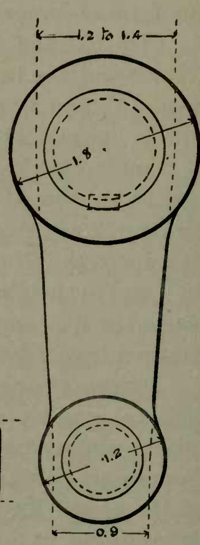


Fig. 638.

Figs. 637 and 638 are two views of a wrought-iron crank, and Figs. 639 and 640 of a cast-iron crank,* both proportioned in their parts to the diameter

* "Elements of Machine Design," Unwin.

of the large eye as unity, but, as shown by the diagram and rule following, these figures can only apply to a single throw of crank, as the diameters of the two eyes vary as their distances apart.

Taking the diameter of the large eye of the crank, Redtenbacher gives in the table the relative sizes of central and end eyes of cranks, depending on the proportion between the length of crank and the diameter of central eye. The first column exhibits the number of times the diameter of eye is contained in the length of crank; the second and third columns give the suitable diameters of crank-pins.

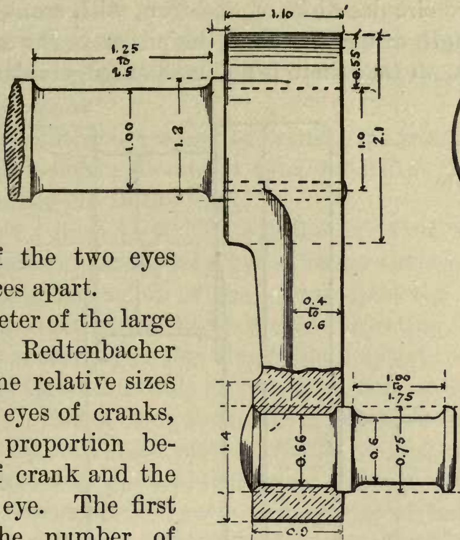


Fig. 639.

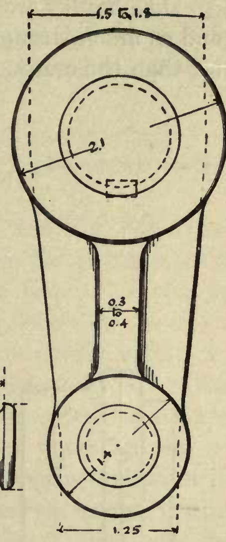


Fig. 640.

Figs. 641 and 642 represent a side and front elevation of a crank, such as is used on engines of American river boats. The main body of the crank is of cast-iron, with two horns *a a* projecting from the central hub, and the whole is bound with a strap of wrought-iron.

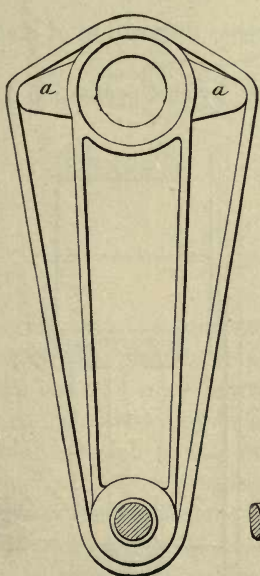


Fig. 641.

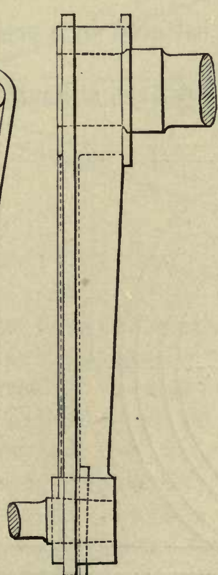


Fig. 642.

DIAMETER OF EYE, BEING UNITS.

	For wrought-iron shafts.	Cast-iron shafts
2	0·85	0·62
3	0·69	0·51
4	0·60	0·44
5	0·54	0·39
6	0·49	0·36
7	0·45	0·33
8	0·42	0·31
9	0·40	0·29
10	0·38	0·28
11	0·36	0 26
12	0·34	0·25
13	0·33	0·24

The diameters of crank-pins as above given are on the basis of a length of from 1 to 1¼ of the diameter; if the length be increased beyond this the diameter should be increased in the ratio of 1 to the square root of the diameter.

Disk-cranks are circular disks of cast-iron, with crank-pins of iron or steel, and as much strength of metal around the pin as in the crank. They are better than the crank, in that there is no unbalanced crank and pin, and part of

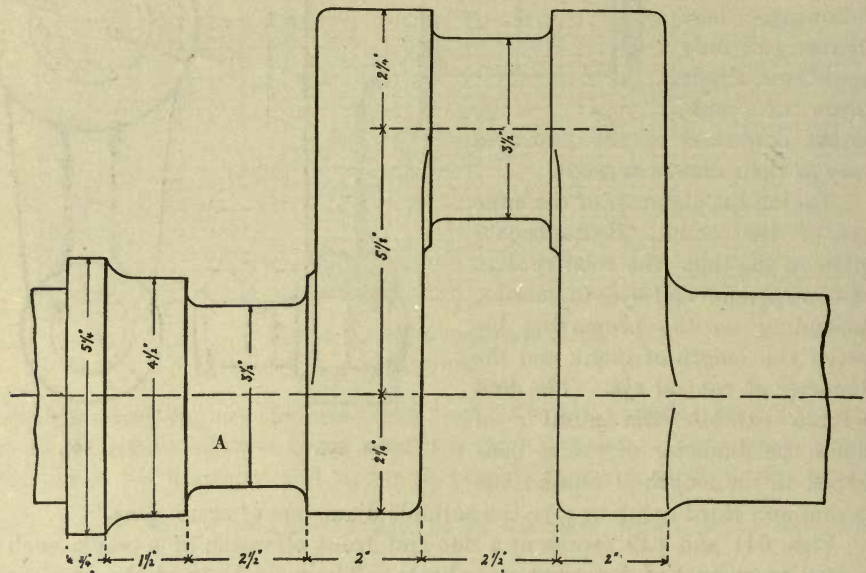


FIG. 643

the weight of the connection can be balanced by a proper disposition of metal within the area of the disk.

Fig. 643 is a plan of a double crank-axle, although by the projection the

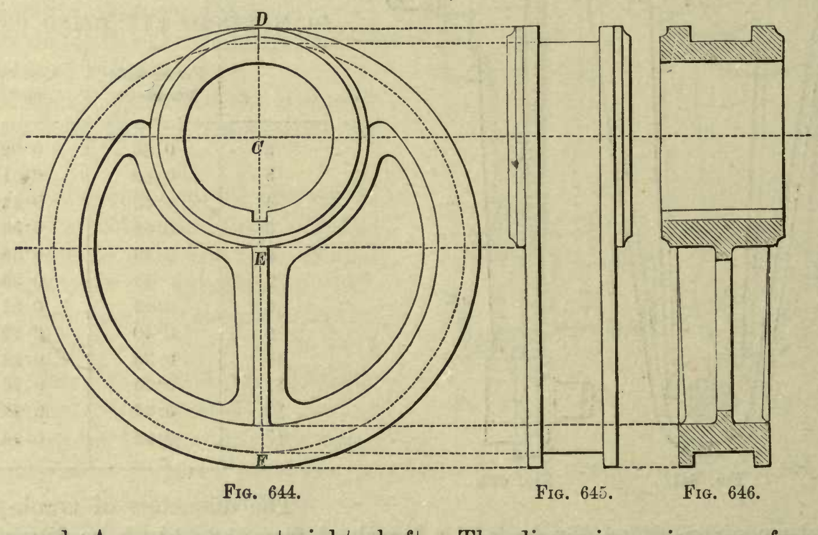


FIG. 644. FIG. 645. FIG. 646.

lower axle A appears as a straight shaft. The dimensions given are from an axle in use. In construction the cranks are rectangular in section, of which the width is $\frac{7}{10}$ the depth, and the depth 1·5 the diameter of crank-journal.

Cranks are usually forged solid, and the slot for the crank cut out; that shown in the figure was cast in steel for a double compound engine, 7 × 15 × 15, and although there is often great condensation in the cylinders, it has worked satisfactorily for many years.

Eccentrics.—An eccentric is a modified crank; the crank-pin is enlarged so as to include the crank-shaft; motion is conveyed through the crank to the pin, and not through the pin to the shaft.

Fig. 644 represents a front view, Fig. 645 the side view, and Fig. 646 a section, of a form of eccentric usually adopted in steam-engines for giving motion to the valves regulating the action of the steam upon the piston. A ring or hoop, eccentric strap, is accurately fitted within projecting ledges on the outer circumference of the eccentric, so that the latter may revolve freely within it; this ring is connected by an inflexible rod with a system of levers, by which the valve is moved. It is evident, that as the shaft to which the eccentric is fixed revolves, an alternating rectilinear motion will be impressed upon the rod, its amount being determined by the eccentricity, or distance between the center of the shaft and that of the exterior circle. The *throw* of the eccentric is twice the eccentricity C E; or it may be expressed as the diameter of the circle described by the point E. The nature of the alternating motion generated by the circular eccentric is identical with that of the crank.

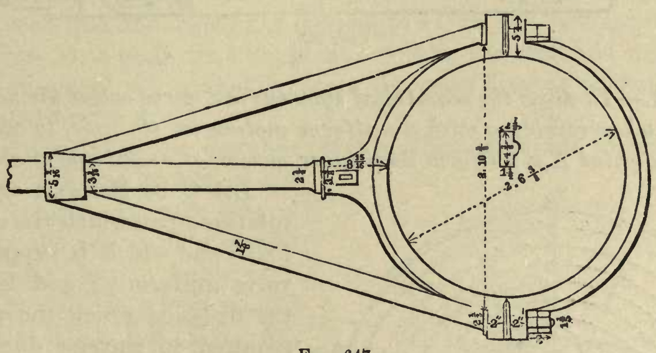


Fig. 647.

Fig. 647 is a common form of eccentric strap and rod adapted to the drawing of the eccentric given; it is usually fitted with a composition bush, and a pan must be provided beneath to catch any oil that may drip from the eccentric. This last may be avoided by the use of an eccentric strap, Figs. 648, 649, 650, in which it will be seen that the strap forms a cup-section (Fig. 650) which secures a projecting ring on the eccentric, and retains the oil. These figures represent the eccentric strap of a locomotive, and are made entirely of cast-iron; the bolts are very long, and the strap exceedingly rigid.

In practice, the term *eccentric* is generally confined to the *circular* eccentric; all others, with exception of that last described, being called *cams* or *wipers.*

Projections of Eccentrics.—The term *eccentric* is often applied in general to all such curves as are composed of points situated at unequal distances from a central point or axis.

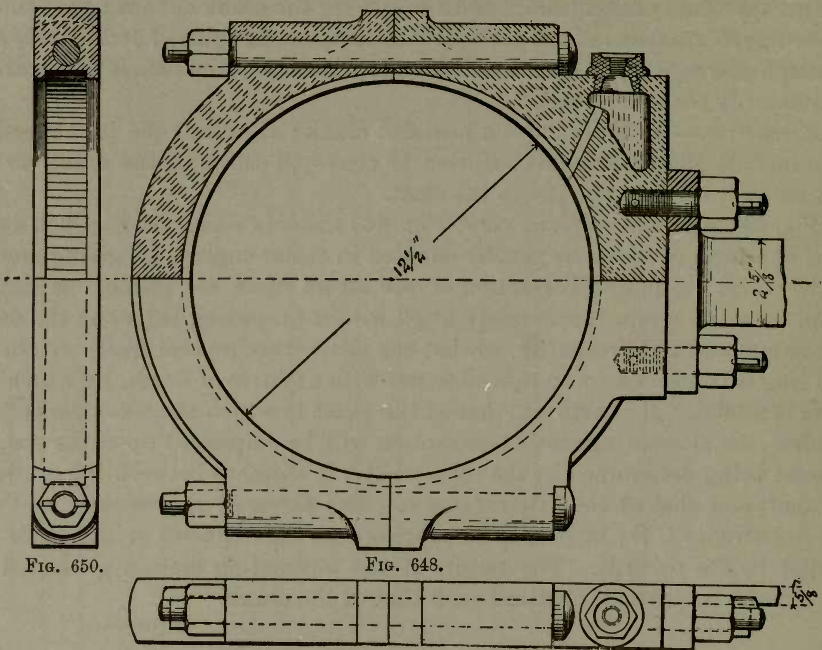


FIG. 650. FIG. 648.

FIG. 649.

Fig. 651.—*To draw the eccentrical symmetrical curve called the heart, which is such as, when revolving with a uniform motion on its axis, to communicate to a movable point A, a uniform rectilinear motion of ascent and descent.*

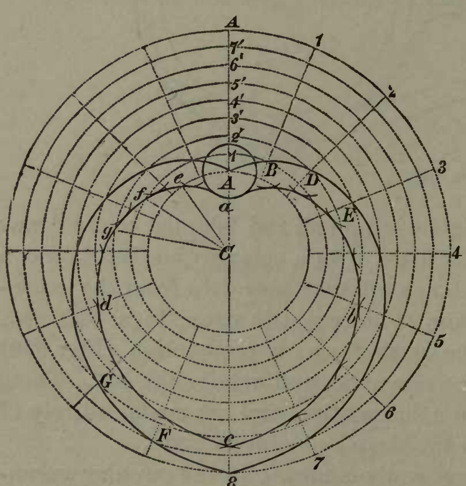


FIG. 651.

Let C be the axis or center of rotation upon which the eccentric is fixed, and which is supposed to revolve uniformly; and let A A′ be the distance which the point A is required to traverse during a half revolution of the eccentric. From the center C, with radii respectively equal to C A and C A′, describe two circles; divide the greatest into any number of equal parts (say 16), and draw through these points of division the radii C 1, C 2, C 3, etc. Then divide the line A A′ into the same number of equal parts as are contained in the semicircle (that is, into 8 in the example now before us), and through all the points 1′, 2′, 3′, etc., draw circles concentric with the former; the points of their intersection B, D, E, etc., with the respective radii C 1, C 2, C 3, etc., are points in the curve required, its vertex being at the point 8.

It will now be obvious that when the axis, in its angular motion, shall have passed through one division; in other words, when the radius C 1 coincides with C A′, the point A, being urged upward by the curvature of the revolving body on which it rests, will have taken the position indicated by 1′; and further, when the succeeding radius C 2 shall have assumed the same position, the point A will have been raised to 2′, and so on till it arrives at A′, after a half revolution of the eccentric. The remaining half, A G F 8, of the eccentric, being exactly symmetrical with the other, will enable the point A to descend in precisely the same manner as it is elevated. It is thus manifest that this curve is fitted to impress a uniform motion upon the point A itself, but in practice a small friction roller is usually interposed between the surface of the eccentric and the piece which is to be actuated by it. Accordingly, the point A is to be taken as the center of this roller, and the curve whose construction we have just explained is replaced by another, similar to and equidistant from it, which is drawn tangentially to arcs of circles described from the various points in the primary curve with the radius of the roller. This second curve is manifestly endowed with the same properties as the other; for, supposing the point *e*, for example, to coincide with A, if we cause the axis to revolve through a distance equal to one of the divisions the point *f*, which is the intersection of the curve with the circle whose radius is C 1′, will then obviously have assumed the position 1′; at the next portion of the revolution, the point *g* (which is such that the angle *f* C *g* is equal to *e* C *f*) will have arrived at 2′, and so on. Thus it is plain that the point *a* will be elevated and depressed uniformly by means of the second curve, in the same manner as that denoted by A is actuated by the first.

It is obvious that the movable point *a* must, in actual working, be held in contact with the surface of the eccentric; this is generally accomplished by the action of a weight or of a spring; but in forms similar to Fig. 651, in which all the diameters, as A A 8, B F, D G, etc., are equal, two frictions connected and placed diametrically opposite each other may be used, which will be thus alternately and similarly impelled; in many cases an eccentric groove is cut, and the friction roll or point *a* is made to slide in this groove.

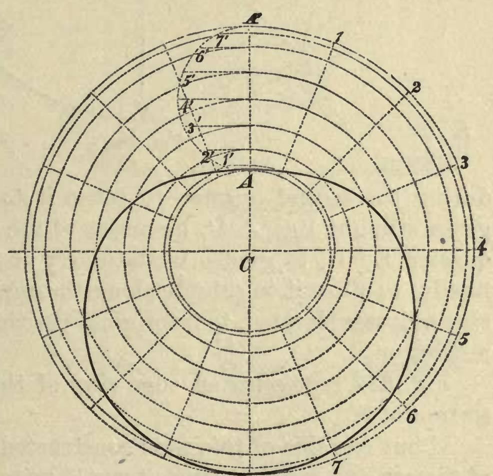


Fig. 652.

Fig. 652.—*To draw a double and symmetrical eccentric curve, such as to cause the point* A *to move in a straight line, and with an unequal motion; the velocity of ascent being accelerated in a given ratio from the starting-point to the vertex of the curve, and the velocity of descent being retarded in the same ratio.*

Upon A A′ as a diameter describe a semicircle, and divide it into any number of equal parts; draw from each point of division 1, 2, 3, etc., perpendiculars upon C A′; and through the points of intersection 1′, 2′, 3′, etc., draw circles having for their common center the point C, which is to be joined, as before, to all the points of division on the circle (A′ 4 8). The points of intersection of the concentric circles with the radii C 1, C 2, C 3, etc., are points in the curve required.

Fig. 653.—*To construct a double and symmetrical eccentric, which shall produce a uniform rectilinear motion, with periods of rest at the points nearest to, and farthest from, the axis of rotation.*

The lines in the figure above referred to indicate sufficiently plainly, without the aid of further description, the construction of the curve in question, which is simply a modification of the eccentric represented at Fig. 651. In the present example, the eccentric is adapted to allow the movable point A to remain in a state of rest during the first quarter of a revolution B D; then,

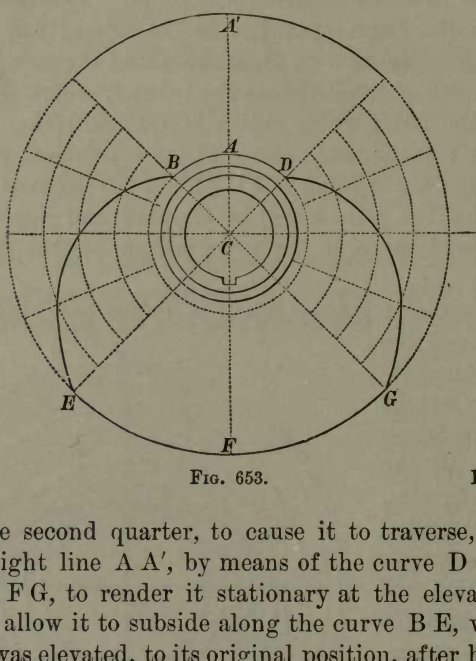


Fig. 653.

Fig. 654.

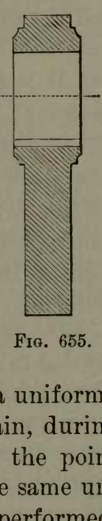


Fig. 655.

during the second quarter, to cause it to traverse, with a uniform motion, a given straight line A A′, by means of the curve D G; again, during the next quarter E F G, to render it stationary at the elevation of the point A′; and finally, to allow it to subside along the curve B E, with the same uniform motion as it was elevated, to its original position, after having performed the entire revolution.

Fig. 654 represents an edge view of this eccentric, and Fig. 655 a vertical section of it.

If but one side of this were constructed, and the motion only equal to that of the arc and reciprocating, it would raise and lower every point resting on it, and would be called a wiper. The wiped surface is generally flat, an arm extending out from the rod to be raised, and a curve D G may be formed adapted to any height of lift, and action during the lift.

Connections.—Figs. 656 and 657 are sections of cottered joints of wrought-iron bars, the first made with a socket and the end of one of the bars; the latter by a sleeve connecting the two bars. The bars in the socket and sleeve are upset, to give more section than the bars themselves, so that the slots cut for the cotters *c c* will not reduce the strength below that of the bars. The cotters must have sufficient shearing strength and bearing surface, and at the

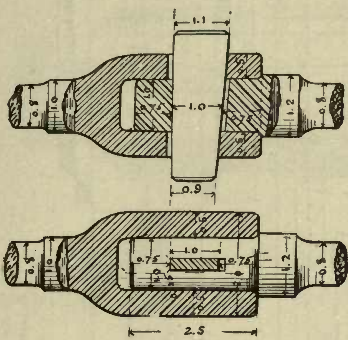


Fig. 656.

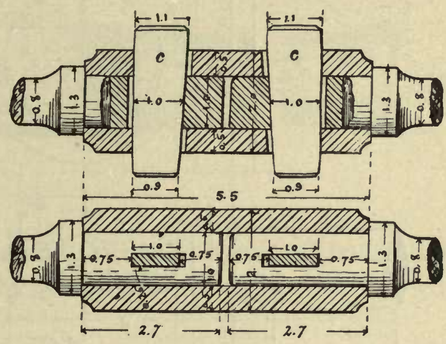


Fig. 657.

same time diminish as little as possible the section of the parts connected. The proportions given in the figures are drawn to a scale of the diameter of the enlarged part as the unit, and the proportions given in figures are such as obtain in practice for wrought-iron. If the cotter be of steel, its breadth may be $\frac{3}{4}$ of that given, preserving the other dimensions the same; the thickness is ·25 of the unit.

The knuckle-joint (Fig. 658) is given in dimensions of the bar as a unit, and adapted to usual work. If there is much motion at the joint, the wearing-surface should be larger, by increasing the width of the eyes and the length of the pin. The pin in the drawing is through the collar; usually the pin is extended, and the pin passes through the bolt outside the collar.

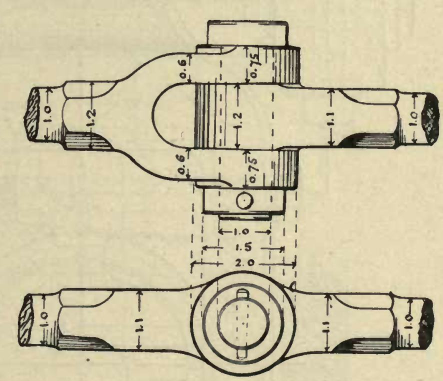


Fig. 658.

Connecting-rods, in their application to steam-engines, are the rods connecting the piston through the cross-head to the crank. When two cranks are connected it is called a *coupling-rod*.

Figs. 659, 660, and 661 are side plan and end views of a connecting-rod, as made by the Southwark Foundry, of Philadelphia, and used on their fast-running Porter-Allen engines.

The cross-head end is a *strap-end*, while that of the crank is a *box-end*, and the latter is made of larger diameter than the former on account of the application of the stress to the crank-pin, and the wear, this pin is made larger than the pin of the cross-head. The length of the page does not admit of the representation of the full length of the connecting-rod on the scale; it is

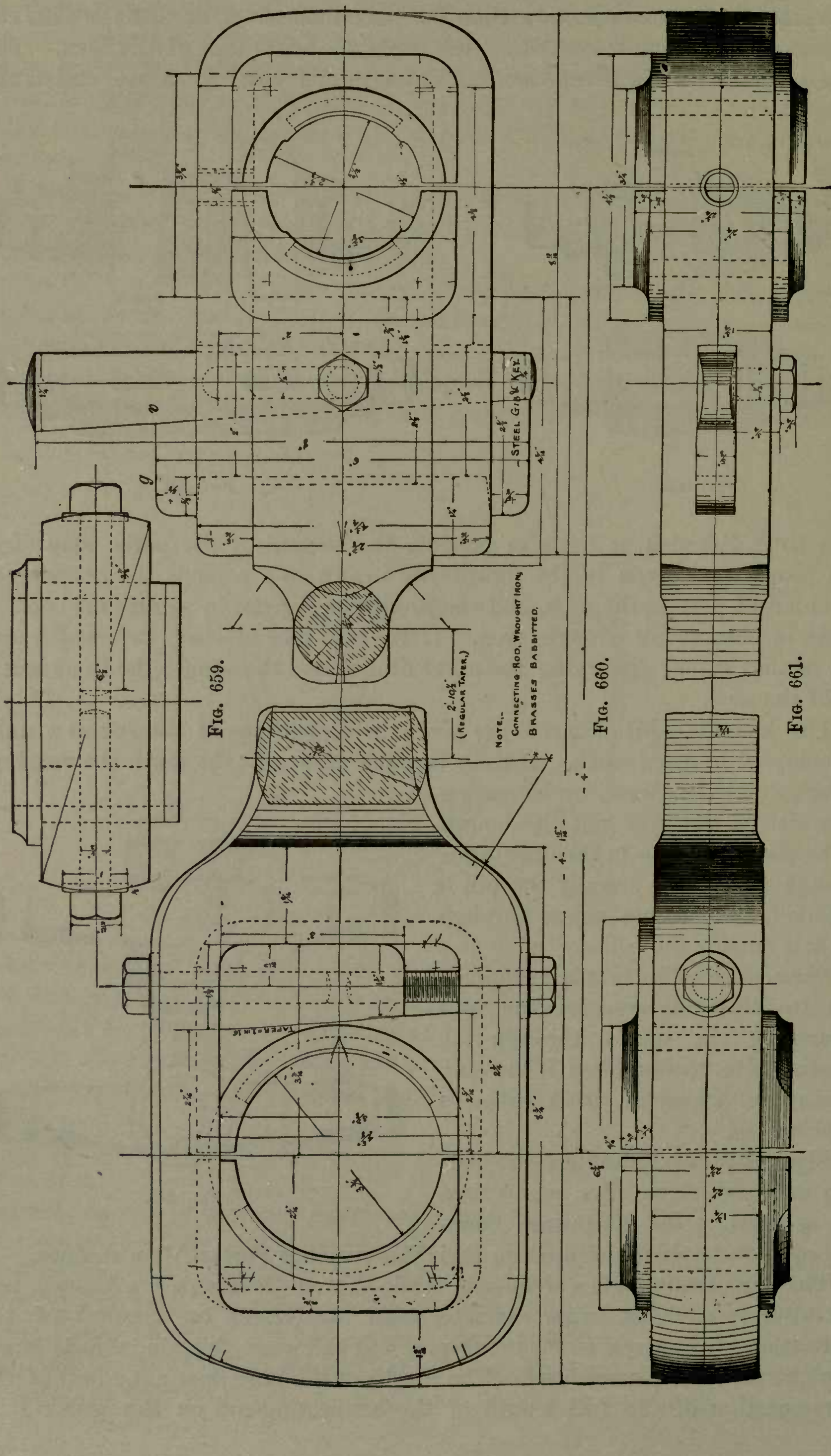


FIG. 659.

FIG. 660.

FIG. 661.

therefore shown broken, with the dimensions figured in. The sections of the two ends are drawn in on the rods; the circular section A is the same as that

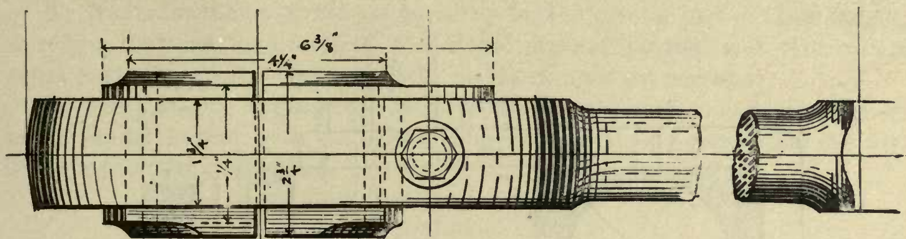


FIG. 662.

of the piston-rod, and both are represented in the conventional hatching of cast-iron. This is of wrought-iron. The gib *g* and key or cotter *v* at the strap-

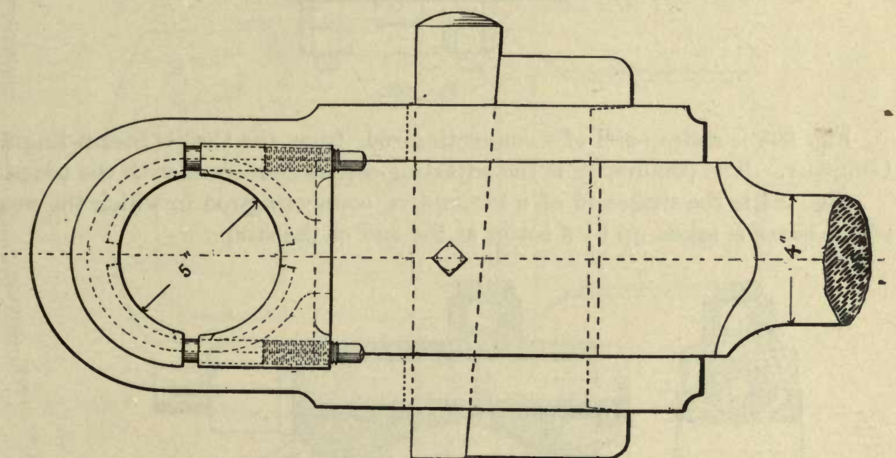


FIG. 663.

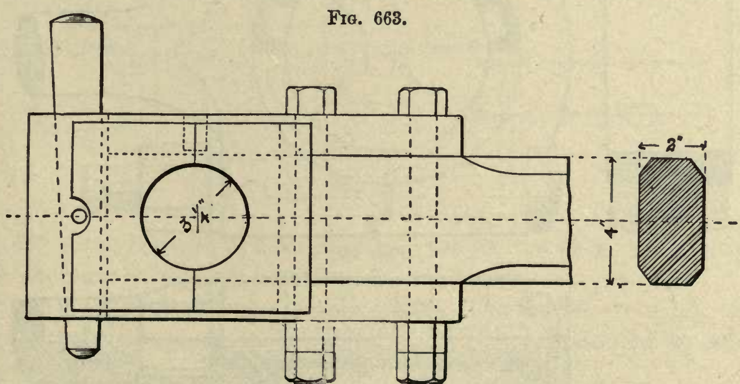


FIG. 664.

end are of steel, and the key is fastened when in position by a set-screw through the head. At the box-end, a wedge and screw forces the box into position.

It will be observed on the plan that this rod is drawn as though it were flat on top; but as the tops are curved, it is more accurately represented in Fig. 662.

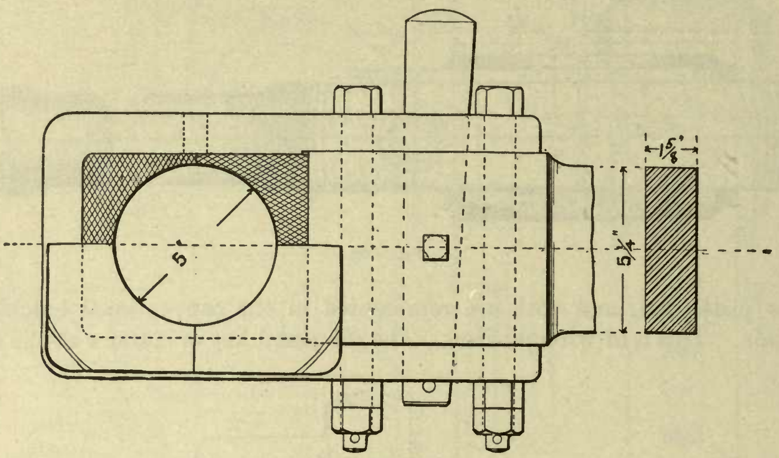


Fig. 665.

Fig. 663 is a strap-end of a connecting-rod, from the Corliss Steam-Engine Company. The peculiarity is the adjusting-screws connected with the boxes.

Fig. 664 is the strap-end of a locomotive connecting-rod in which the wear of the boxes is taken up by a cotter at the end of the strap.

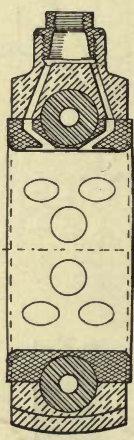


Fig. 668.

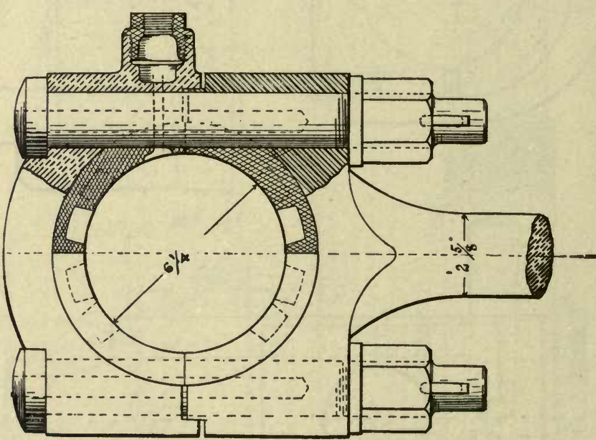


Fig. 666.

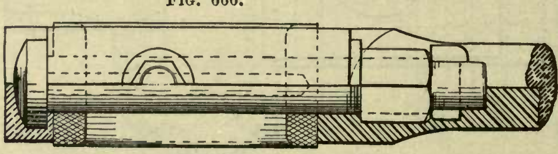


Fig. 667.

In Fig. 665 the key is between the bolts; the weakness from bolt-holes or cotter-slot is compensated by the width of the strap.

Fig. 666 is a cast-iron eccentric strap; the bolts are very long and the connection very rigid. The box is fitted with metalline, which is put in small disks; oiling is thereby avoided.

The bolts for the large end are bored up for the greater part of their length, to reduce their sectional area to that of the screwed portion and thus secure equal elasticity; with these long bolts no check-nuts are necessary.

In many marine engines the boxes of both crank and cross-head pins are made similar to this, with the bolts strong and heavy, and connecting the two boxes without any other rod.

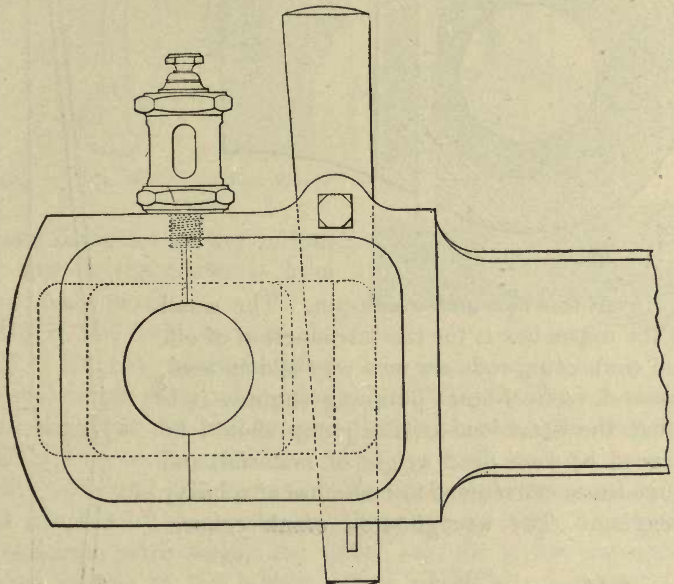


Fig. 669.

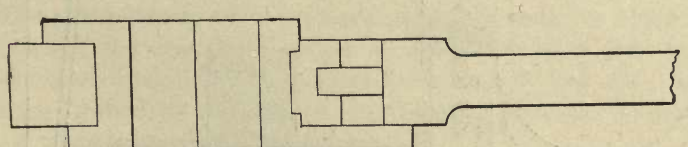


Fig. 670.

Fig. 669 is the box-end of a locomotive; the section (Fig. 670) is expressed in shade line merely, without hatching.

Fig. 671 is the stub end of a coupling-rod. The bushes are solid, of brass, and kept from turning round by taper-pins, which are secured by set-screws pressing on the larger end; taper, $\frac{1}{16}$ in 3 inches.

Fig. 672 represents the forked end of a cast-iron connecting-rod of an English type, the end of the working-beam coming within the forks. Wrought-iron connecting-rods of this kind are most generally used. One side of the fork is shown in section, with its bosses, *a b*, and the cotters, *c c*.

Fig. 673 is a section of the lower end of the same beam. The lower box *n* is held in position by a spherical boss, filling a recess in the rod, the upper brass by the cotter; there

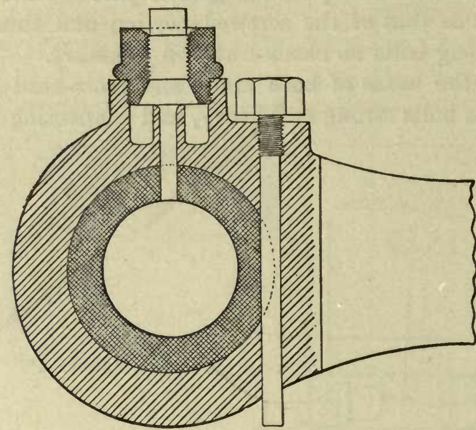


FIG. 671.

is a cover *c* over the box and crank-pin. The small channel in the upper box is for the introduction of oil.

Cast-iron connecting-rods are now very seldom used. In some cases of vertical-beam pumping engines, it is necessary that the water-load of the pump should be counterbalanced by some dead weight of material, and it is then sometimes convenient to make use of a heavy pump-connection. The wrought-iron crank connec-

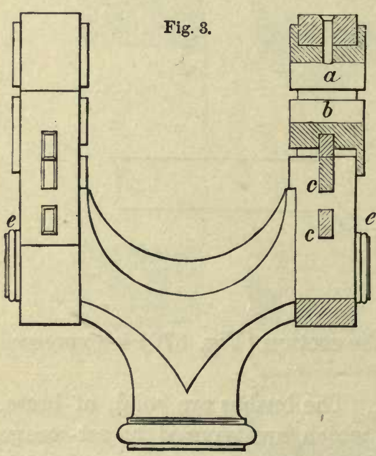


FIG. 672.

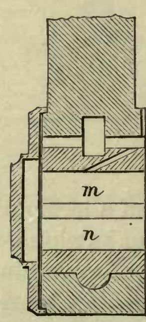


FIG. 673.

tions of American river-boat engines are peculiar in their construction. They are made as light as possible, with very great stiffness. Fig. 674 represents the side ele-

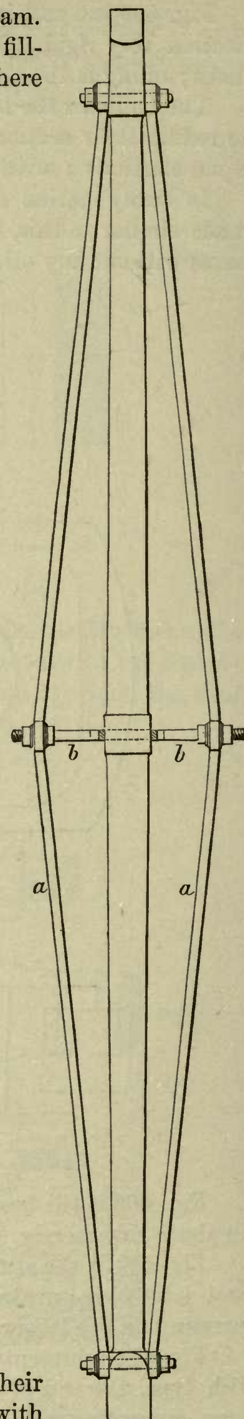


FIG. 674.

vation of such a connecting-rod. The means adopted to give the required stiffness consist of a double-truss brace, *a a*, of round iron, which is fastened by bolts to the rod near each end ; struts *b b*, cut with a screw, and furnished with nuts, pass through the center of the brace, by which means the braces are tightened. The connecting-rod at its smallest part near the extremities is

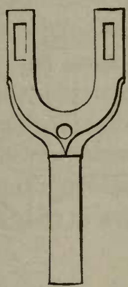


Fig. 675.

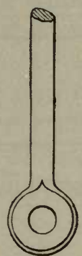


Fig. 676.

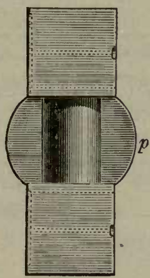


Fig. 678.

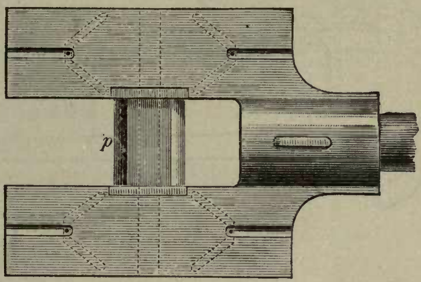


Fig. 677.

of the same diameter as the piston-rod ; the boss in the center is from one to two inches more.

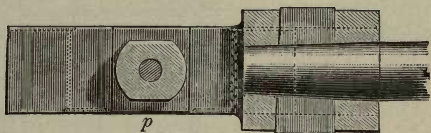


Fig. 679.

Fig. 675 is the front view of the forked end of the rod, which is fitted with the usual straps, gibs, and cotters. Fig. 676 is the side view of the brace-rod.

The *cross-head* is the link between the piston-rod of the steam-engine and the connecting-rod to the crank.

Figs. 677, 678 and 679 represent the plan, end view, and section of the cross-head adopted by the Southwark Foundry for their high-speed engines. It is of cast-iron, with large, flat faces, the pin *p* for the connecting-rod being in the middle of the length. This pin is of wrought-iron, large and flattened on top and bottom, so that the boxes of the rod can never bind on the pin at the extreme of the vibrations of the rod ; usually these pins are round. The pin is formed with large squares at the ends, by which it is fitted into the jaws of the cross-head, where it is secured by a steel pin passing through the cross-head. The bearing surfaces of the head and those of the guide-bars are finished by scraping to true planes ; there are no means of adjustment, as there is no wear if kept clean.

It is to be understood that the piston-rod moves in a straight line, and that the stress on the connecting-rod pin is mostly oblique. Guides are to be provided, between which the cross-head slides, to take the oblique stress off the piston-rod.

Figs. 680 and 681 are elevation and plan of *guide-bars* which are in common use for both vertical and horizontal engines. Lugs or ears are cast on the steam-cylinders, and on the frames to which the bars are bolted, and between which the cross-head slides. The grooves or notches across the guide-bars, at the ends of the stroke, are to throw off any grease or dirt that may be carried along by the head and prevent their accumulation. The stress on the guide-bars is due to the pressure of the steam on the piston acting obliquely on the crank through the connecting-rod, and is the greatest when the crank is at right angles to the piston. It can be determined by multiplying the pressure on the piston by the length of the crank, and dividing the product by the length of the connecting-rod, which will be the stress tending to separate the guides. If the

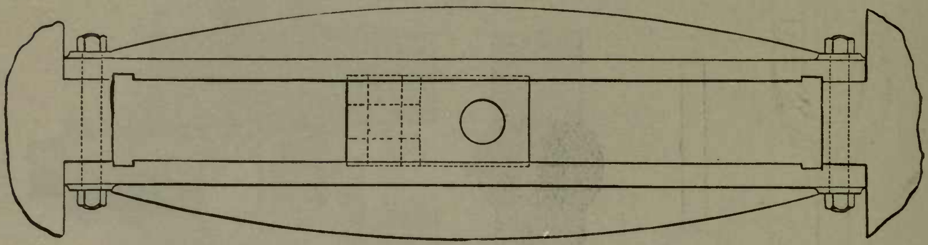


Fig. 680.

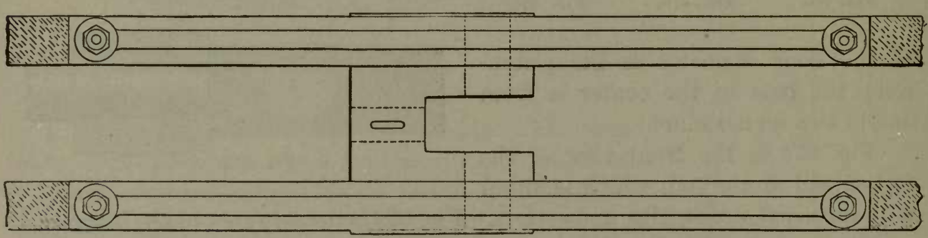


Fig. 681.

connecting-rod be 3 times the stroke, or 6 times that of the crank, which is the usual proportion, then the stress is $\frac{1}{6}$ the pressure on the piston. Sometimes the proportion of connecting-rod to stroke is $2\frac{1}{2}$ to 1. When a portion of the force of the steam is opposed directly to the resistance, as in direct-acting pumps, and only the irregularities in the steam-pressure are transmitted through the connecting-rods, the proportion of rod to stroke may be still smaller. In this case the force transmitted to the fly-wheel is retransmitted to the cross-

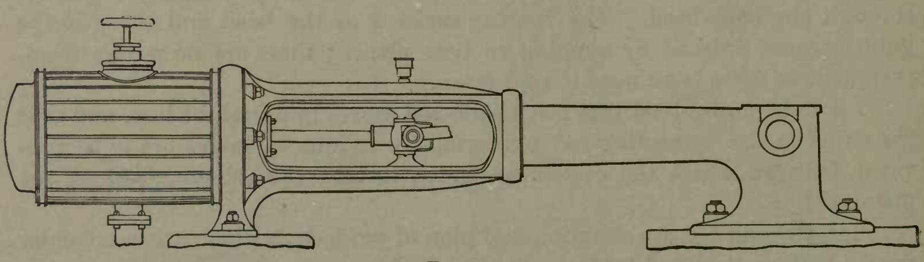


Fig. 682.

head, whenever the resistance in the pumps exceeds the pressure of the steam, thus utilizing the expansive properties of the steam by a cut-off.

When the top of the engine-frame is horizontal it may form the lower guide of the cross-head. In many engines the guides are formed in the frame itself (Fig. 682), in which the bearing surfaces of the guides are arcs of circles within a pipe, open on the face, which forms a part of the frame and is bored at the same time with the cylinder, and consequently in true line. On locomotives it is not unusual to have the guide on one side, as in Fig. 683, where the slide-bars are of wrought-iron and the slide-block is fastened between the two plates of the cross-head by bolts. It is the most common practice in this country to use guides with vertical engines, even when the connection is with working-beams, but abroad the parallel motion is more popular. The working-beam is seldom applied to stationary engines, but only to marine and pumping engines.

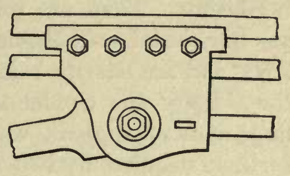


Fig. 683.

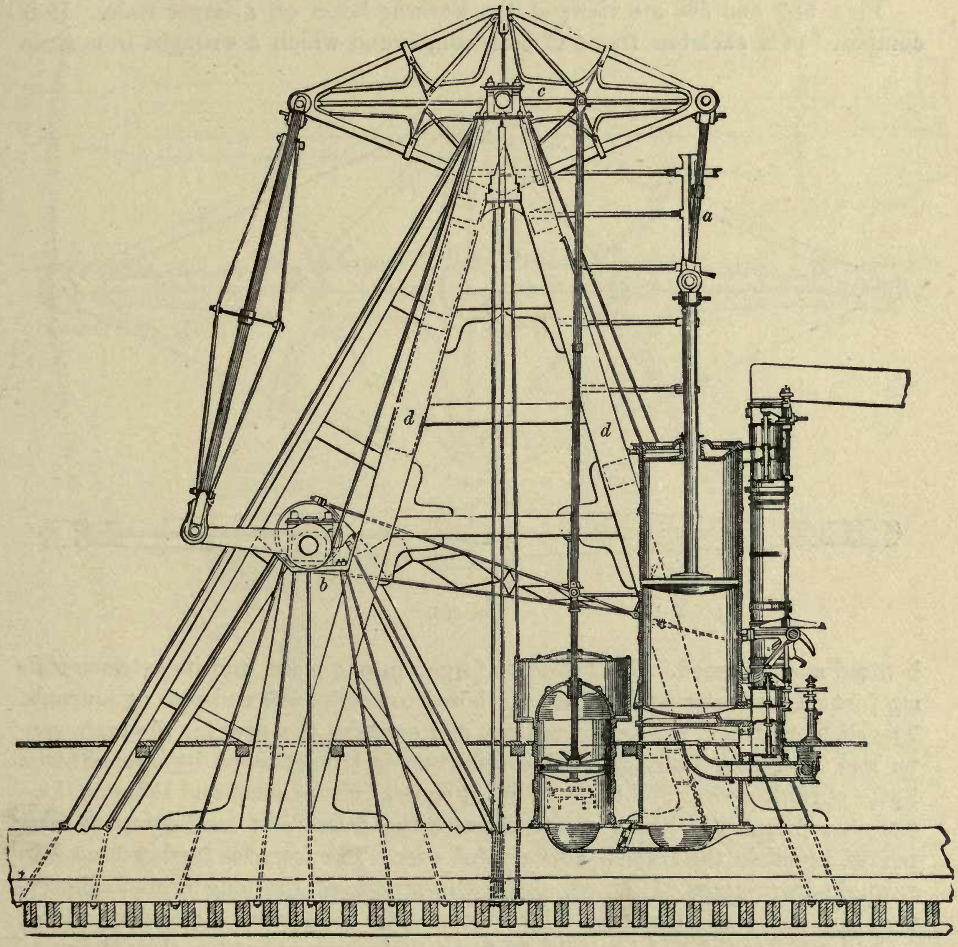


Fig. 684.

Fig. 684, elevation of engine of the "New World," may be taken as the type of a North River steamboat engine. The frame-work is composed of four pieces of heavy pine timber, *d d*, which are formed into two triangles, and inclined slightly laterally to each other; their lower ends rest on the keelsons, and upon their upper extremities are placed the pillow-block *c* of the working-beam. They are solidly fastened together and to the boat by numerous horizontal and diagonal timbers, which are secured by wooden knees and keys, and are heavily bolted. The two front legs are bolted to flanges cast on the sides of the condenser, and the other end of the framing is attached to a large mass of timbers, which support the shaft pillow-block *b*; the framing is further steadied by two additional timbers, and rods running from the beam pillow-blocks outside the shaft to the keelsons of the boat. The guides *a* are bolted at the bottom to the cylinder-flange, and retained in their vertical position by wrought-iron braces connected with the framing. The height of the frame is 46 feet, width at bottom 31 feet.

Figs. 685 and 686 are views of the working-beam on a larger scale. It is composed of a skeleton frame of cast-iron, round which a wrought-iron strap

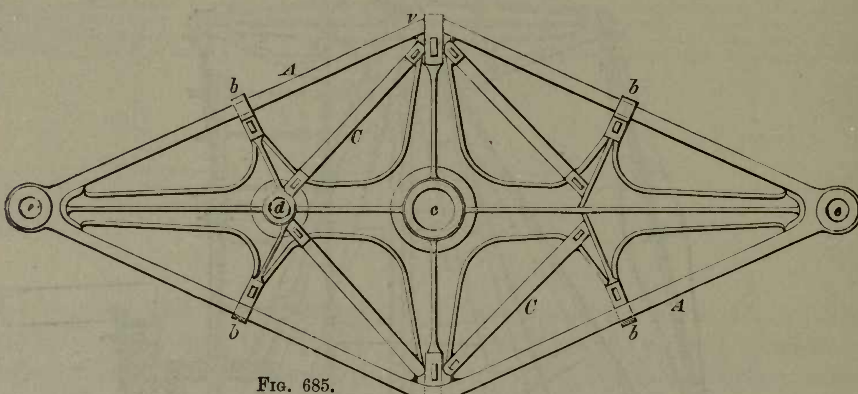


Fig. 685.

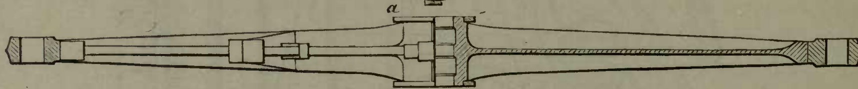


Fig. 686.

is fitted and fastened. This strap is forged in one piece, and its extreme ends are formed into large eyes, which are bored to receive the end-pins or journals. The skeleton frame is a single casting, and contains the eyes for the main center and air-pump journals; the center hub is strengthened by wrought-iron hoops shrunk upon it. At the points of contact of the strap and skeleton, key-beds are prepared. Small straps connect the frame and main-strap at these points, keyed to the frame—keys riveted over. The frame is further braced by wrought-iron straps, C C, which tie the middle of the long arms to the extremities of the shorter ones. The following are the general dimensions: From center to center of end-journals, 26 feet; this is somewhat less than the usual proportion to length of stroke, being slightly less than double the stroke; length of center hub, 26″, *a a*; diameter of main center eye *c*, 15⅝″; of air-pump journal-eye *d*, 6¾″; of end-journals *e e*, 8½.

Fig. 687 is the side elevation, Fig. 688 a plan, and Fig. 689 a section through the hub of a cast-iron working-beam. The proportions are as in practice, but the end as shown is not usual. Fig. 690 shows the way in which the

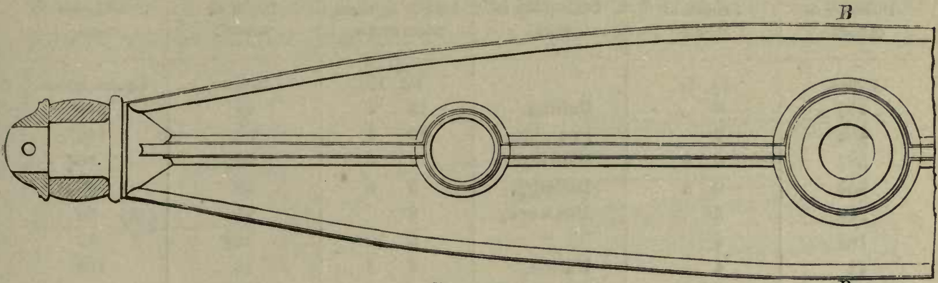


Fig. 687.

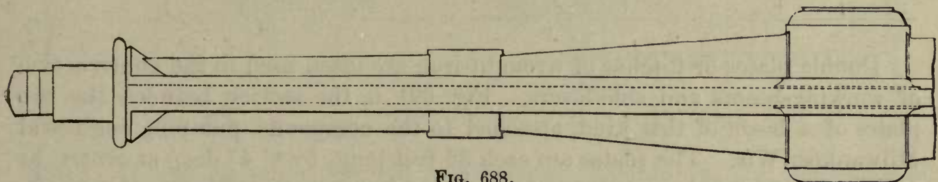


Fig. 688.

connection-rod is attached, the dotted lines showing the head, which passes over the end pivot. The common form of the end is like that of the working-beam (Fig. 685).

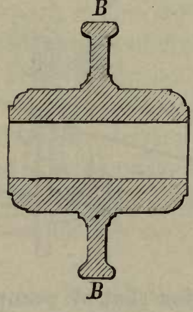


Fig. 689.

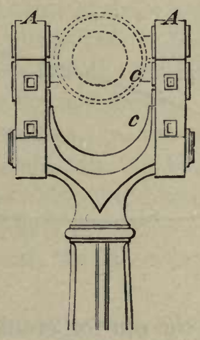


Fig. 690.

From the following table of practical examples from "Architecture of Machinery," it is safe to assume as a rule for the working-beams of land engines, that the depth at center should be the diameter of the cylinder, and the length of beam three times the length of stroke. The outline is parabolic, having for the vertex the extremity of the beam and the point B in the curve at the center. The sectional area may be estimated from rules already given, knowing the load at the extremity, that is, the pressure on the piston, the weight of the same and its connections, and also the force required to drive the air-pump, estimated at the extremity of the lever. As an engine is subject to shocks, the load should be estimated at six times the absolute load. Five per cent of the nominal power of the engine may be considered the maximum of power required to drive the air-pump.

Diameter of cylinder.	Length of stroke.	Description of work.	Length of beam from center.	Depth at center.	Sectional area.
Inches.	Ft. In.		Ft. In.	Inches.	Square inches.
47¼	8	Rolling,	12 4	48	240
40¾	7	Pumping,	10 4	36	162
39½	6 9	Blowing,	9 6	38½	96¼
36⅝	6 3	Rolling,	9 3	30	60
24¾	5	Mill-work,	8	25	50
18¼	4	"	6 10	22¼	50
42	4	Marine,	6 3	23	138
42	4 2	"	6 6	27	216
32	3	"	5	22	132

Double plates or flitches of wrought-iron are often used in the construction of working-beams and side-levers. Fig. 691 is the section between the two plates of a beam of this kind, attached to the compound pumping engines at Milwaukee, Wis. The plates are each 30 feet long, by 6′ 4″ deep at center, by 1¾″ thick. The connections between the two, shown in section in the figure, are cast-iron pipes with wide flanges at each end riveted or bolted to the plates. The main center and other small journal-pins are rods of wrought-iron, passing through the pipes, and extending outside the plates to form the journals; *c* is

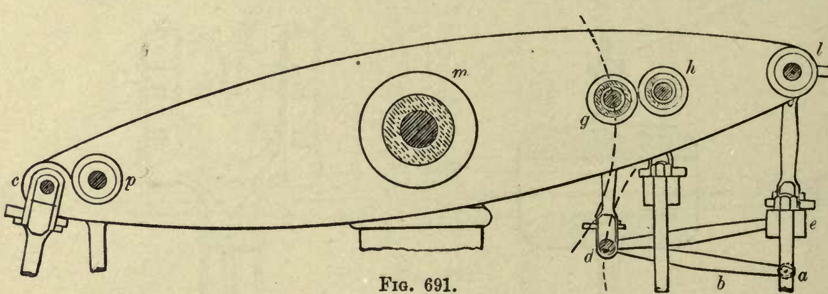


FIG. 691.

the section of the pin for crank connection, *p* for that of pump, *h* for that of high-pressure cylinder, *l* for that of low-pressure cylinder, *m* for main center-pin, and *g* for the parallel-motion links. This last is usually the position of the air-pump center, but in this engine the air-pump is below the high-pressure cylinder, and its piston-rod is extended to the air-pump piston. The dimensions are—H. P., 36″ × 62″; L. P., 58″ × 8 feet. The action of the parallel motion, in keeping the cross-head of the low-pressure cylinder in a vertical line, will be understood by the arcs described from the main center *m* and from the fixed point *a*, or the journal of the radius bar *b*. The point *e*, the angle of a parallelogram formed of rods and links, must partake of the motions of these two arcs, and for a portion of movement it is in a straight line parallel to that of the motion of the piston-rod cross-head. It is usual to make the radii of these arcs equal.

Fig. 692 is a general mode of finding the length of the radius rod *g c*. F is the main center of the beam, *a c* is a strap or link attached to the beam at *a*, the piston-rod to be attached to some point nearly central on the link, which must move in a straight line. Moving the beam up and down, keep the point *b* on the vertical line, and mark the positions of the lower end of the

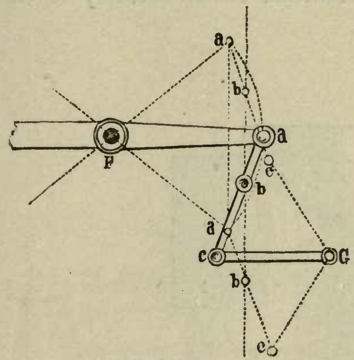


Fig. 692.

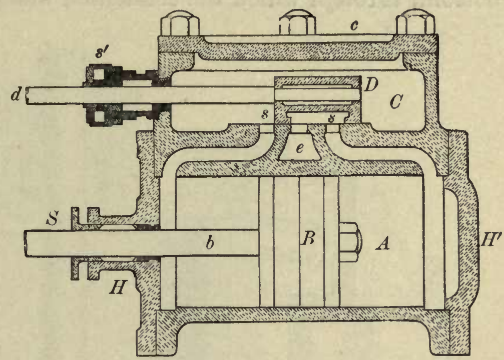


Fig. 693.

link *c c c*; find an arc which will pass through these points, and the center of this arc will be the fixed center *g* of the radius bar, and the radius that of this bar.

Steam-Cylinders.—Fig. 693 is a sectional plan of a common form of small steam-cylinder. A is the cylinder, B the piston, *b* the piston-rod, D the slide-valves, *d* the valve-rod, C the valve-chest, *c* the chest-cover, *s s* the steam-ports, *e* the exhaust-port, S the stuffing-box of the piston-rod, *s′* that of the valve-rod. H is the front head and H′ the back head of the cylinder. The bolts attaching the heads to the body of the cylinder are not shown.

Length of Cylinder.—It is the present practice, in the construction of stationary engines for driving machinery, to make the stroke not over twice the diameter of the cylinder, and for diameters above 24″ about $1\frac{1}{2}$ times the diameter of the cylinder, and invariably to place the cylinders horizontally with a direct connection with the crank, without the intervention of a working-beam.

Fig. 694 is the longitudinal section of a Corliss steam-cylinder which has two steam-valves, *s s*, and two exhaust-valves, *e e*. The steam-pipe S is attached to the top of the steam-chest, and the exhaust E to the bottom of the exhaust-channel; the bolts on cylinder-heads or stuffing-box are not shown. The thickness of shell, Mr. Hawthorne finds by many examples in Corliss's large practice, to conform to the formula $t = \cdot 268\ \sqrt{d}$; t and d being in inches. Thus the thickness of the shell of 16″ cylinder will be $\sqrt{16} \times \cdot 268 = 4 \times \cdot 268 = 1 \cdot 072$, a little more than 1″. The thickness of flanges should exceed that of the shell by $\frac{1}{8}$ to $\frac{1}{4}$ its thickness. The bolts should not be less than $\frac{5}{8}$″ and seldom more than 1″. It is better to increase the number of bolts than their diameter, the breadth of flange about 3 times the diameter of the bolts, and the pitch of the bolts, or the distance between centers, about 6 times the diameter of the bolts.

Fig. 695 is a sectional elevation of a Cornish pumping engine's steam-cylinder. The valves are in pipes outside the cylinder, as in most of our North River boats, and there is what is called a *steam-jacket*—that is, a shell, *j j*, outside the shell of the main cylinder, inclosing a narrow space filled with steam by a pipe connection directly from the boiler, and with a pipe at the bottom, through which the condensed water is returned either directly to the

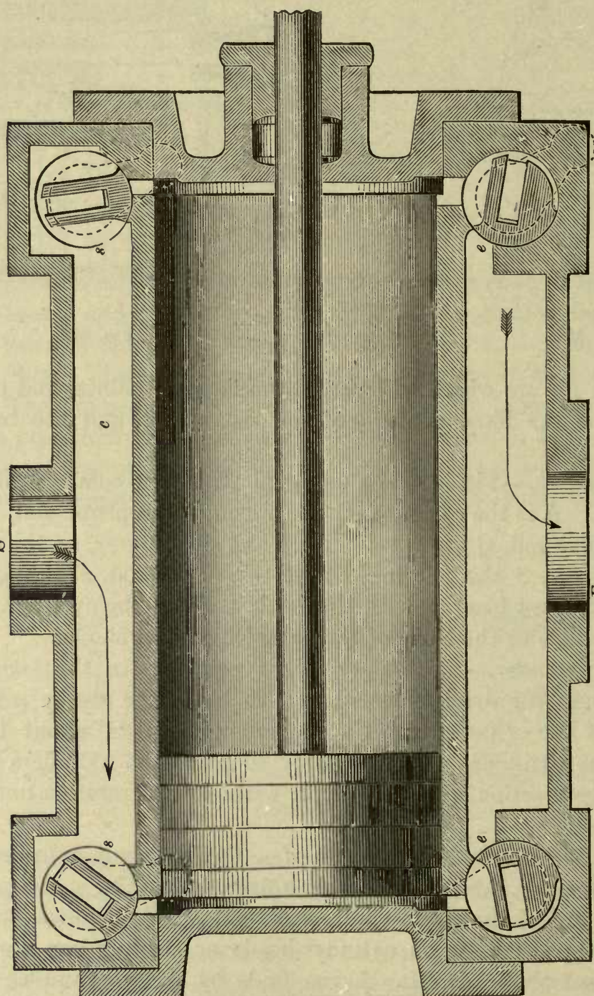


Fig. 694.

boiler or discharged into the hot well. All steam-cylinders, whether with or without jackets, should be clothed—that is, covered with some preparation to prevent the escape of heat from contact with air. The usual clothing is hair-felt, with a *lagging*, *b b*—that is, an exterior shell of some wood, usually black-walnut.

Figs. 696 and 697 represent sections of two types of water-cylinders. In Fig. 696 the pump-barrel is long and the piston short; in Fig. 697 the pump-barrel is only about equal to the diameter of the piston in length, but the length of the piston is equal to that of the stroke of the pump and that of the pump-barrel. The figures are taken from the Worthington pump, and represent his arrangements of valves and passage-ways. I I are

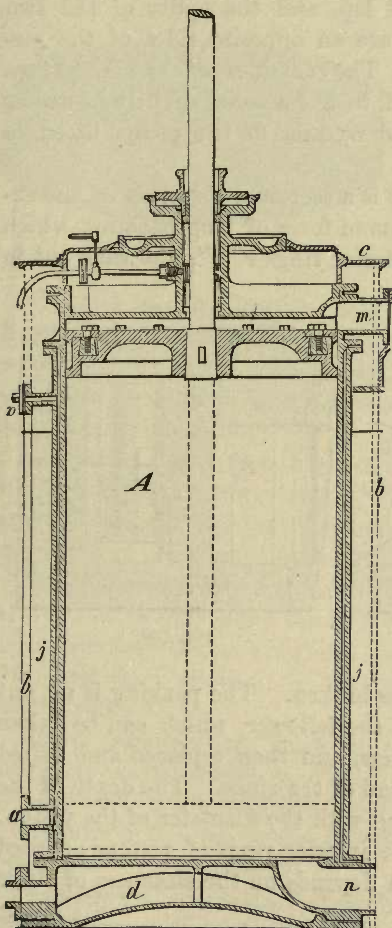


FIG. 695.

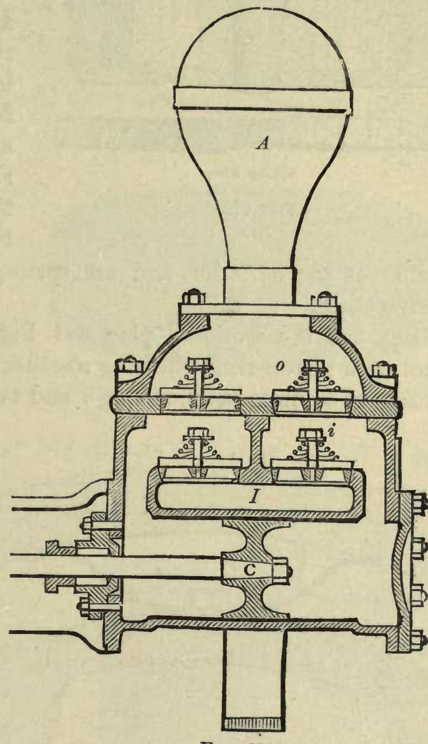


FIG. 696.

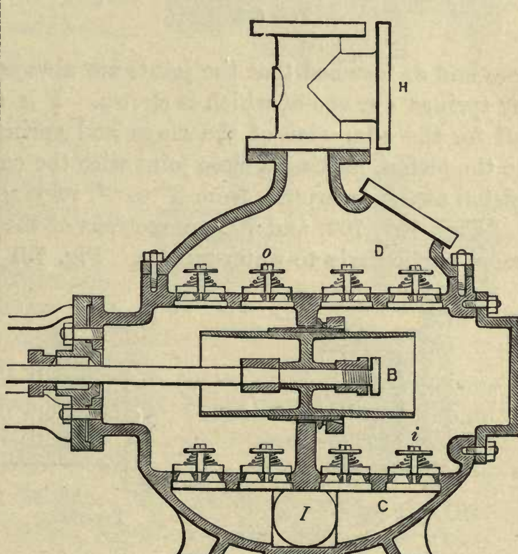


FIG. 697.

the inlet chambers, *i i* the lower valves, and *o o* the upper ones. A is the air-chamber.

Pistons are of great variety and of different proportions, according to the work to be done, the medium in which they move, and the friction

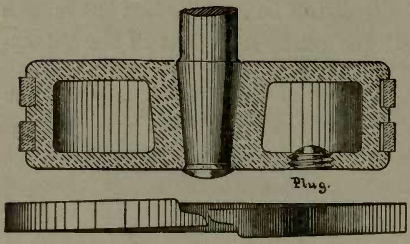


FIG. 698.

due to their weight on the sides of the cylinders.

Fig. 698 is the cast-iron piston of a locomotive. The spring or snap-rings forming the packing are of cast-iron, $1\frac{1}{2}''$ wide by $\frac{1}{2}''$ thick, of uniform section. The split is made with a half lap, and the splits of the two rings are on opposite sides of the piston. The outsides of the rings are turned to a diameter slightly in excess of that of the cylinder, and are sprung into recesses of the piston fitted to receive them.

Fig. 699 is a sectional plan and Fig. 700 is a sectional elevation of the exterior of a piston-ring, showing another common form of ring packing, which consists of a single exterior ring *r* and two exterior rings r'' r'', and each cut in

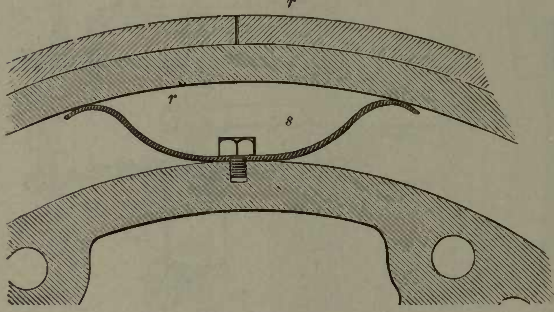


FIG. 699.

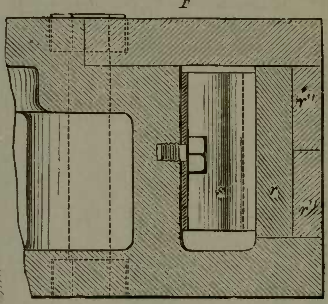


FIG. 700.

two and so fastened that the joints are always broken. The packing is set out by springs *s s*, one of which is shown. F is the follower, which can be taken off for the admission of the rings and springs, and then replaced and bolted to the piston, making a close joint with the end of the rings. The depth of the piston at the exterior is from $3''$ to $9''$, varying with the diameter of the piston.

Figs. 701, 702, and 703 are sections of the exterior rings of pistons adapted more particularly to water-pumps. Fig. 701 depends on the closeness of fit of

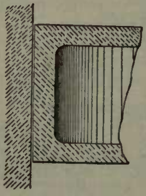


FIG. 701.

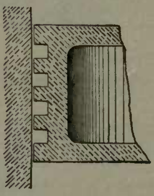


FIG. 702.

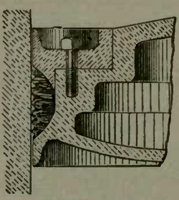


FIG. 703.

the exterior of the piston with the inner surface of the cylinder, and when accurately turned and fitted the leak is very inconsiderable. By the use of grooves in the piston (Fig. 702) this leak is still further reduced, as the thread of the water in passing through the joint is broken by the grooves.

In Fig. 703 the joint between the piston and the cylinder is made tight by a gasket, usually of hemp, compressed by a joint ring or follower, *a*, in the pocket between piston and cylinder.

When the water-pressure is very great, as in hydraulic presses, peculiar packing-rings of leather are used.

Fig. 704.

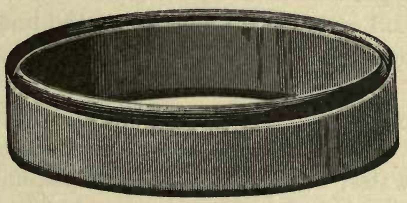


Fig. 705.

Fig. 704 is a cup leather packing, and Fig. 705 is a U-packing. The application of the first will be understood from Fig. 706, in which the piston is packed with two cup leathers, in this case to withstand pressure in both directions. Were the piston single-acting, but one cup would be necessary—and if from beneath the piston, this would be the lower cup. The flexible flange is pressed against the inside of the cylinder, and the joint is perfectly stanch.

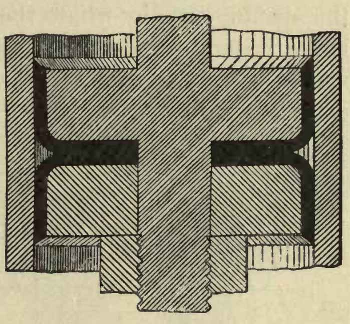


Fig. 706.

Fig. 707 shows the application of the U-packing; it is put into a recess in the cylinder by bending the packing into a saddle-bag form, and then allowing it to spring back into the recess.

Hemp packings are made to serve the same purpose, as shown in Fig. 703. They are more easily made than the U-packing, but they require a follower or cap to retain them in position.

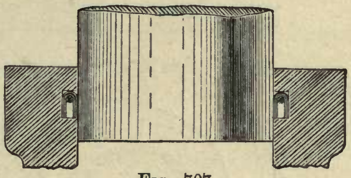


Fig. 707.

Packings can be obtained from hydraulic-pump and press manufacturers, and are kept in stock of all the usual sizes. Their depths are from 1″ to $1\frac{1}{2}$″ for diameters varying from 4″ to 14″, and the space occupied by the thickness in the U from $\frac{1}{4}$″ to $\frac{3}{8}$″. A filling of flat braided hemp is placed inside the U to keep it tight when not under pressure. The packings are made by steeping the leather in warm water, forcing them into a mold, and leaving them to dry and harden. The molds are made of either metal or wood; frequently the rings are of metal, and the piston over which the cup is formed, of wood.

Clearances in cylinders include, in general signification, not only the spaces between the piston and cylinder-heads at the ends of the stroke, but also the spaces between the cylinder and the valves; and as those spaces are voided in a steam-cylinder at each stroke for which adequate work from the steam is not obtained, they are usually made as small as possible. If the steam is fairly dry, from $\frac{1}{2}''$ to $1''$ will be sufficient for end-clearances—that is, minimum distance between piston and cylinder-head.

Piston-rods are proportioned to the stress on them, usually one square inch of section to each 5,000 pounds of stress. In Fig. 698 the tapered end fits a taper hole in the piston, and is riveted over. It is more usually held by a nut, and some use a shoulder on the inner end of the piston-rod instead of a taper, and the nut brings the piston strongly up against this shoulder.

Piston-rods are made either of steel or hammered iron, some makers of engines preferring one and some the other material.

Stuffing-boxes are the mechanisms to prevent the leakage of steam, air, or water, in the passage of the piston or other rod out of the cylinder or chest. They consist of an annular chamber around the rod, filled most generally with gaskets of hemp, which is forced down by a ring or *gland* into close contact with the rod and the sides of the box. In Fig. 693 there are two stuffing-boxes shown, one for the main piston-rod, the other for the valve-rod. In the latter the cap of the gland is fitted with a screw to connect it with the side of the stuffing-box, by which the gasket may be more or less compressed. This is the general form of stuffing-box for small stems or pistons used on steam-valves, but sometimes with a ring or follower on the top of the gasket, which is forced down by the gland without turning the ring or gasket. In the figure the stuffing-box is made of brass, and screwed into the end of the steam or valve-chest.

The stuffing-box to the piston is cast with the head of the cylinder, and is bored

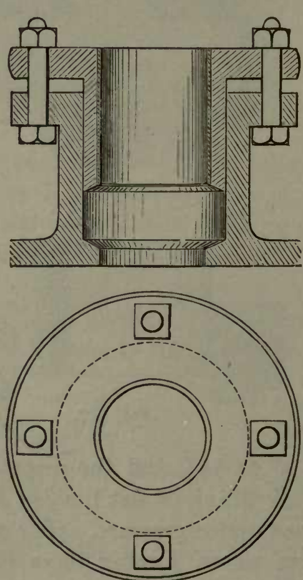


Fig. 708.

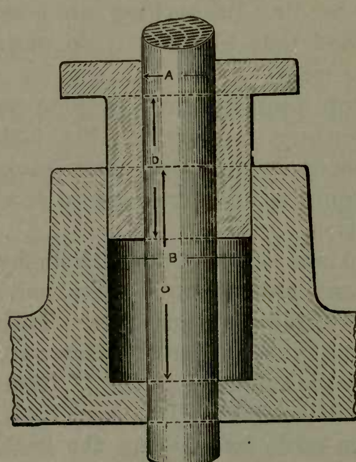


Fig. 709.

out, and a brass bushing fitted and driven into the end of the box. The hole through the bushing in most boxes fits the piston-rod accurately. The gland is of cast-iron, turned to fit the stuffing-box, and bored to fit the piston-rod ; after packing the box the gland is forced in and retained by screws.

Fig. 708 is the plan and section of a common stuffing-box, in which the thickness of packing is from ½″ to 1½″, and the depth from 1½ to 2 times the diameter of the piston-rod. The number of bolts vary with the diameter of the piston—seldom more than four, and, for the size of engines mostly in use, but two.

Fig. 709 is the section of a stuffing-box of the proportions adopted by the Southwark Foundry. Taking the diameter of piston-rod A as the unit, B is 2, C 3, D 2, all scant up to a 3″ rod, or 22″ cylinder. For a 28″ × 42″, A = 4, with an allowance of $\frac{1}{32}$″ for clearance, B 6¾, C 9, D 6¼″.

It has been said that hemp gaskets were in most common use for the packing of stuffing-boxes, and they can be procured readily ; but there are a very great variety of packings, patented or otherwise, which are very good, adapted to common stuffing-boxes ; and there are also metallic packings which have given great satisfaction, and can be easily procured.

Valves—Steam-Cylinder Valves.—The simplest and most common is the slide D, shown in Fig. 693. The function of the valve is to admit the steam alternately into the ends of the steam-cylinder, and, while steam is being admitted through one port to one end of the cylinder, the other end is being exhausted or the steam discharged through the other port. It is absolutely necessary (Figs. 710, 711, 712, 713) that one port should be closed before the other is opened, that the steam may not be admitted to both ends of the cylinder at the same time, nor that it may flow through from either end into the exhaust. The simplest form of valve is shown in different positions in the sections. The face of the valve-seat is shown in Fig. 713 ; *s* and *s′* are the steam-ports, and *e* the exhaust-port. The valve only just covers the ports, so that there is no leak, and in Fig. 712 it is in the position in which the steam can neither enter nor escape from the cylinder. Suppose there be a movement of the valve to the left, the steam will be admitted through the steam-port *s′*, and the steam can escape through the other port *s* into the exhaust ; at the end of the movement of the valve it will be as shown in Fig. 711, with full opening of steam into *s′*, and full exhaust through *s*. If the motion be now alternated the ports will be gradually closed till the valve returns to its first position (Fig. 712), and then, as the valve continues its movement, the port *s* begins to take steam, and the port *s′* to connect with the exhaust, till at the end of the motion in this direction the valve will be in the position shown in Fig. 710. With this valve there can be no economical use of steam ; it follows to the end of the stroke without cut-off, without benefit of expansion, except that which may come from *throttling*, that is, impeding the flow resulting from the gradual contraction of the steam openings.

FIG. 710.

FIG. 711.

FIG. 712.

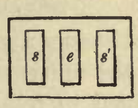


FIG. 713.

Of the Size of Ports or Openings.—Under "Steam-pipes" will be given the formula for the flow of steam, but the general rule of proportioning the ports of a cylinder is to consider the velocity of steam 100 feet per second, and of the exhaust 80 feet per second. It will be seen from the movement of the slide-valve that the opening is made gradually, and closed in the same way, thereby throttling the flow of the steam. To avoid this, Mr. Corliss, in his engine (Fig. 694), has made his ports long and narrow; the steam-valves open quickly and close at once by a drop. It will be seen that the valves have cylindrical faces and seats, and are moved by a central rocking-bar.

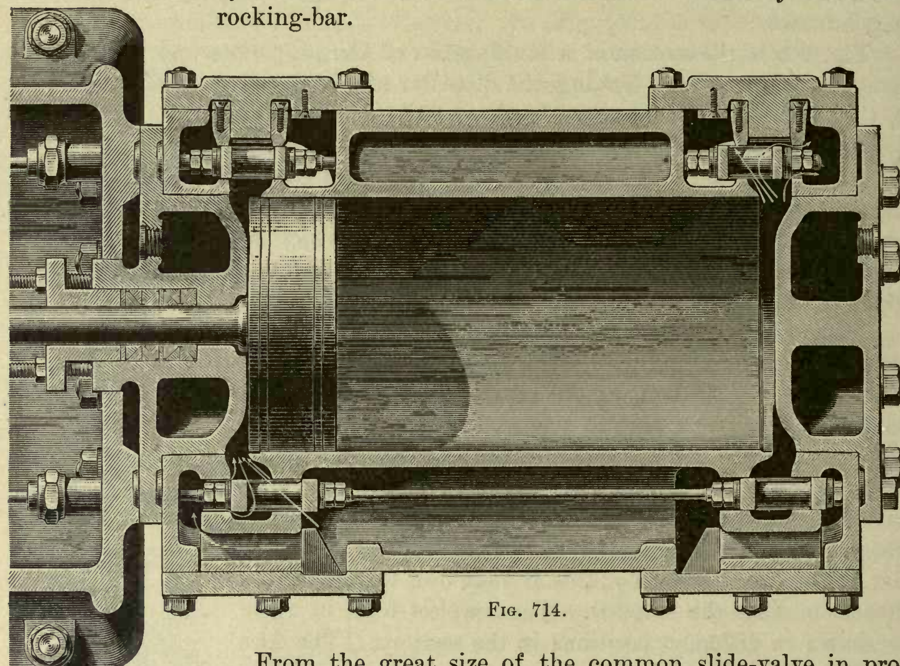


Fig. 714.

From the great size of the common slide-valve in proportion to its port, the bearing-surface extending all round, there ensues a great pressure on the surface, tending to wear it, and also making the movement of the valve more difficult. Various expedients have been adopted to relieve this pressure, which is especially desirable in quick-running engines.

Fig. 714 is a horizontal section of cylinder, through steam and exhaust-valves, of a Porter-Allen engine, and Fig. 715 a vertical cross-section through cylinder and valves. The valves are four in number, one for admission and one for exhaust, at each end of the cylinder, and on opposite sides. They stand vertically so as to drain the cylinder. The valves work between opposite parallel seats; the exhaust-valves nearly and the admission-valves wholly in equilibrium. The action of the back plate, and how the wear is taken up, will be understood from the section (Fig. 715), which passes through the middle of one pressure-plate. It is made hollow, and most of the steam supplied to two of the openings passes through it. It is arched to resist the pressure of the steam without deflection. It rests on two inclined supports, one above and the other below the valve. These inclines are so steep that the plate will move down under steam pressure; and also that it may be closed up to the valve with only a small vertical movement, the pressure-plate is held in its correct position by projections in the chest on one side and tongues projecting from the cover in the other, which bear against it at the near end, as shown. Between these

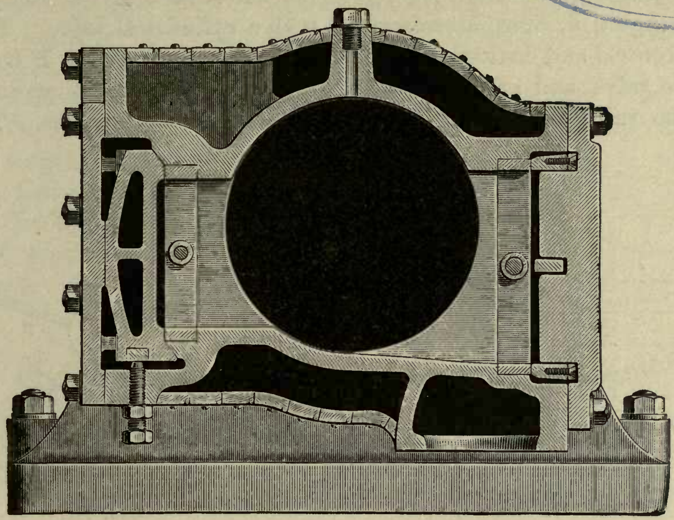


FIG. 715.

guides it is capable of motion up and down and back and forth from $\frac{1}{16}''$ to $\frac{1}{8}''$. The pressure of the steam on this plate tends to force it down the inclines to

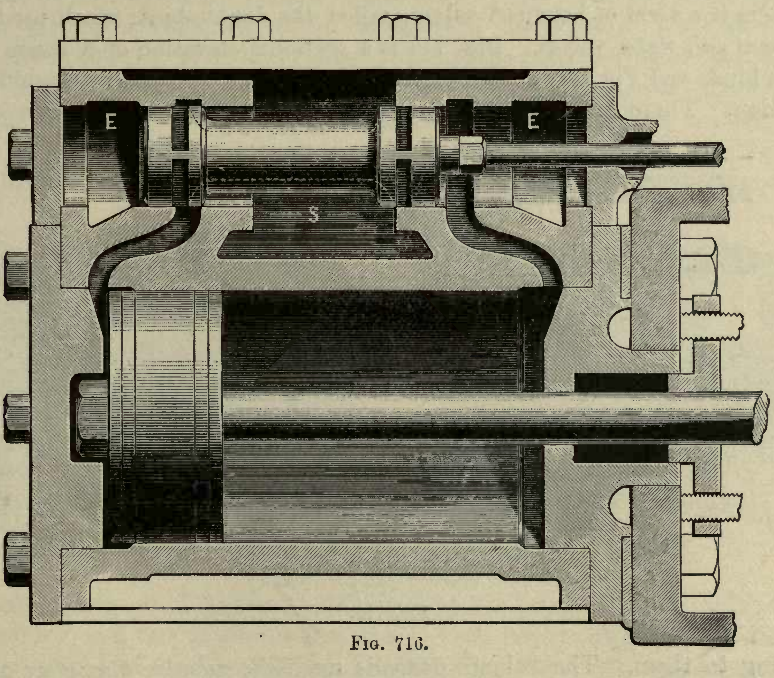


FIG. 716.

rest on the valve. By the means of the screw the plate is forced up and away from the valve, and can be so nicely adjusted that the valve works freely and perfectly steam-tight. When the pressure is greater in the cylinder than in the chest, the pressure-plate is forced back, to the instant relief of the cylinder.

Cylindrical Valves.—Fig. 716 represents the section of the steam-cylinder of an Armington & Sims' steam-engine with a cylindrical valve. The steam-chest S is central and incloses the valve; the exhaust chambers E E are at the ends of the valve, and are connected through the hollow stem or body of the valve. The valve depends on its accuracy of fit for its tightness. The valve-

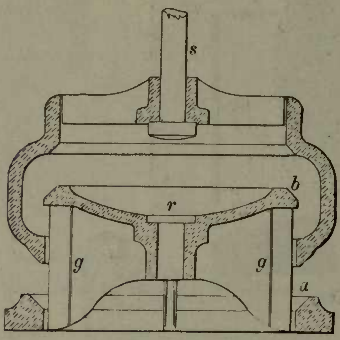


Fig. 717.

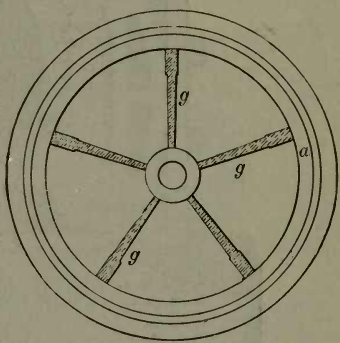


Fig. 718.

chamber is bored out and ground, the valve is turned, ground, and carefully worked by hand, to so close a fit that there is no loss of steam in action, and the valve is completely balanced.

There is a form of balanced valves, called the double-beat, much used both for steam and water valves. Fig. 717 is a sectional elevation of a steam valve of this kind, and Fig. 718 a plan of the lower seat *a*, with the valve-guides *g g* in section. There are two seats, *a* and *b*, and two faces on the valve corre-

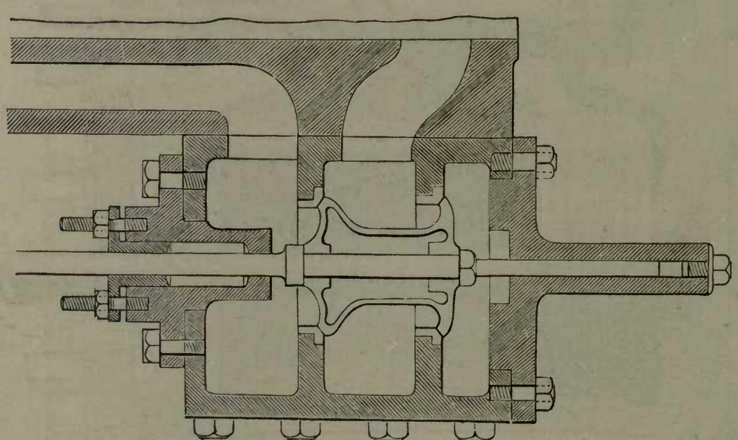


Fig. 719.

sponding to them. The balance depends upon the relative diameters of the bearing-lines of the two faces. In the figure, if the exterior of the bearing at *b* and the interior at *a* are both tight, the valve is balanced under any pressure, except as to its own weight; s is the valve-stem, and the hole r is for a bolt to fasten the valve-seat to the casting of the steam-chest. The scale is $\frac{1}{8}$ full size.

Fig. 719 is another form of balance, consisting of two equal poppet-valves connected together—the steam passage to the cylinder being central, and the steam-chest at each end, and connected.

Automatic valves, that are moved by the action of the fluid in which they are placed.—Figs. 720 and 721 are the plan and section of a disk valve for the air-pump of a condensing steam-engine. The valve consists of a disk of rubber lying on a flat grating or perforated plate of brass, held in position between the grating and a spherical guard by a central bolt. The shape of the guard gives a uniform flexure to the rubber in lifting, and an easy flow to the current of air and water. The rubber is not closely clamped between the guard and plate, as will be seen in the figure. The lower nut, after being screwed home, is riveted, and the upper nut usually pinned to prevent turning. The size of the apertures in the grating are adapted to the thickness of the rubber. With an external diameter of opening of 6″, and rubber $\frac{1}{2}''$ thick, the exterior ring of openings may be $\frac{3}{4}''$ by $\frac{3}{4}''$, the *lands* or spaces between openings $\frac{1}{4}''$ wide, and exterior lap of the rubber $\frac{1}{2}$ inch. With larger diameters and larger openings thicker rubber must be used. This valve is often made of a long strip or flap of rubber, on a suitable grating, with a curved guard attached on one side. For the common

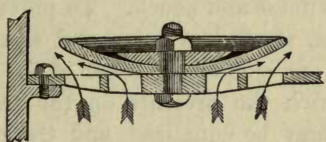


Fig. 720.

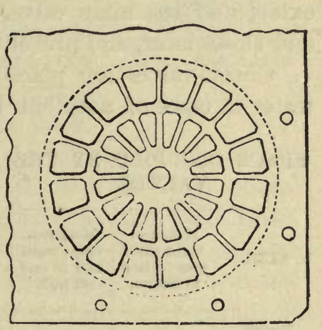


Fig. 721.

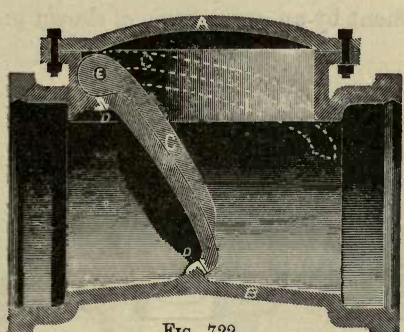


Fig. 722.

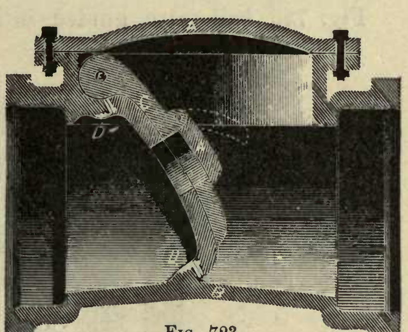


Fig. 723.

air-pump pressure, $\frac{3}{4}''$ rubber is sufficient for apertures $1'' \times 4''$. With the use of backing and face plates on the rubber flaps, the gratings may be dispensed with. Thimbles are inserted in the rubber, and the rivets connecting the two plates pass through these thimbles. The valves to the Boston sewage pumping-engines are of this description. Clear opening in seats $13\frac{1}{2}'' \times 4\frac{1}{2}''$, rubber $\frac{3}{4}''$

thick, toe of guards curved to a $2\frac{1}{2}''$ radius for the hinge of the rubber; the guards have leather pads for the valves to cushion on in their lift.

Fig. 722 is the section of a metallic flap-valve or *check-valve* of the Ludlow Valve Manufacturing Company pattern. Body and valve are of cast-iron, with valve faces and seats of bronze. The bottom of the case B is flattened and raised toward the seat, so that gravel and stones may not lodge against it.

In Fig. 723, section of a like valve, there is a small secondary valve on the exterior of the main valve, which, being lighter than the latter, opens earlier and closes later, and prevents shock to the main and to the valve.

Check-valves are placed outside of large pumps to prevent the return of water in cases of accident to the pumps, and for facility of their examination. Valves of this kind open from the pressure of water beneath, and, from a state of rest, with some suddenness and shock. To prevent this in large valves, there is a valve and small by-pass pipe, from one side to the other of the valves, by opening which the pressure on the two sides of the valve may be equalized, and the excess due to the starting of the pump distributed. At many pumping works the by-pass is kept open except when necessary to get at the pumps. In case of accident to the pumps the flow through the by-pass would be comparatively small, and readily shut off.

SPACE OCCUPIED BY THE VALVES.

SIZE.	Measurement from face to face of flange.	Measurement from end to end of hub.
Inches.		
4	$11\frac{3}{4}$	$13\frac{3}{8}$
5	$14\frac{1}{2}$	16
6	$14\frac{1}{2}$	16
8	$17\frac{3}{4}$	19
10	$21\frac{1}{2}$	24
12	$24\frac{1}{2}$	$26\frac{1}{2}$
16	29	31
18	33	35
20	$35\frac{1}{4}$	38
24	$39\frac{3}{4}$	39

Fig. 724 is a section of a poppet-valve; the body is of cast-iron, but the valve and seat are of brass. The valve is guided in rising and falling by three feathers on the valve. The lift of the valve is controlled by the projection on the cover; a screw is often substituted for this, as it admits of adjustment to varied lifts. Poppets are often guided by stems.

Fig. 725, ball-valve, guided in its movement by an open cage, *c*, shown in

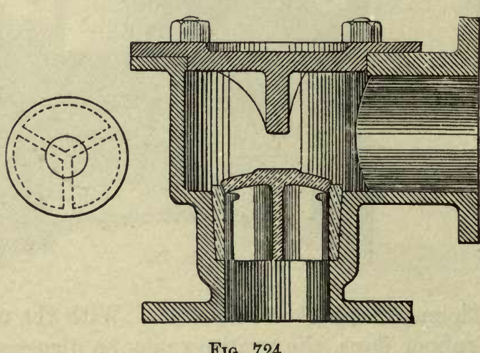


Fig. 724.

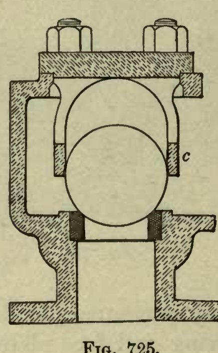


Fig. 725.

section and attached to the cover. Ball-valves are usually small metallic balls on metallic or wooden seats, or rubber balls on metallic seats; and cylindrical valves have been made of the same section as in the figure; the body of the valves of brass pipes with rubber jackets.

Fig. 726 is a section of a rubber disk-valve in very common use in direct-acting pumps and small pumping-engines; sometimes with a thimble in the rubber as a guide; usually with a metallic plate on top of the rubber for the bearing of the spring; valve-seat generally of composition, with spindle riveted or screwed into it. Sometimes the rubber is held in a metallic plate or cup.

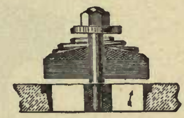


FIG. 726.

Large valves, either poppets or disks, are objectionable from the great lift required for an outlet, proportionate to the area of opening in the seat, making shocks both in the lifting and seating. Consequently, these kinds of valves are made small, the requisite area of outlet being made up by the number of the valves.

The balance-valve (Fig. 717) is commonly used in Cornish and large pumping-engines. From its two beats, the lift is about one half that of a plain valve. There must be difference enough in the faces to admit of the lift of the valve by the pressure of water acting on this difference. The seats of the valves are often made of wood, set endways. Automatic valves should have springs at their backs to cushion the blow on the lift, and to start the valve downward promptly on the check of the water-flow at the end of the stroke. The great desideratum of water-valves is that there should be little lift but ample water-way.

Valves controlled by Hand.—Fig. 727 represents a side view of a water *bib-cock,* called a *hose*-bib, because the outlet end is fitted with a screw to adapt it to a hose. Without this screw it is a *plain bib.* If both ends of the cock are in the same line, it is

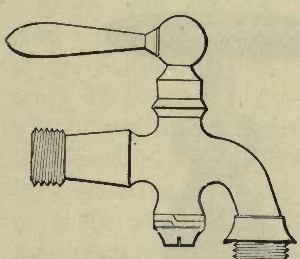


FIG. 727.

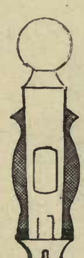


FIG. 728.

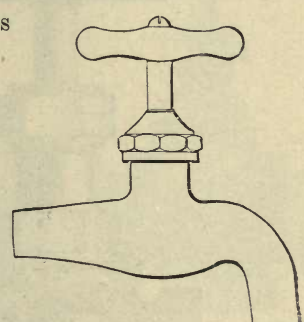


FIG. 729.

called a *stop-cock.* The ends may not be fitted with screws, as in the figure; the screws are sometimes female screws, and often with taper ends, to solder lead pipe to, or to drive into a cask. These cocks come under the common designation of plug-cocks, from their interior construction, which will be readily understood from the section given in Fig. 728. They are used in both steam and water pipes, but not in the former when the use is frequent and daily, and then usually not over 2″ in diameter of passage.

Fig. 729 is the side view of a compression water-bib, used when the pressure of the water is great. The section is somewhat similar to that of Fig. 732, in which a rubber disk is forced against a metallic seat to shut off the flow.

Figs. 730 and 731 are side views of common air-cocks for boilers and steam work; they are plugs in their construction, as are the cocks used in gas-fitting; size of air-cocks to $\frac{3}{8}''$ diameter.

Fig. 732 is the section in part of a globe steam or water valve with a rubber disk; soft rubber for cold water, hard rubber for hot water or steam. The

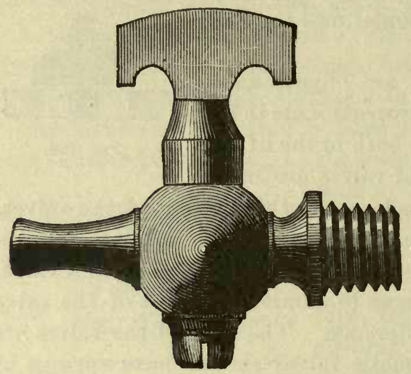


Fig. 730.

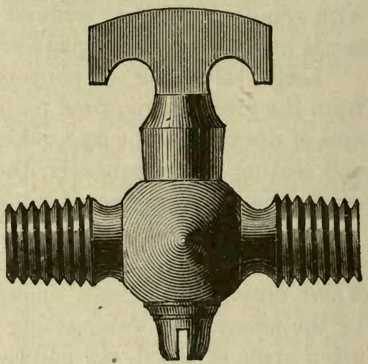


Fig. 731.

fluid enters below the diaphragm and passes up through the aperture in it, which is controlled by the valve; a screw in the stem, below the stuffing-box, bringing it in close contact with the face, or raising it to any height required.

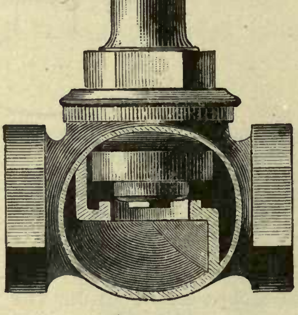


Fig. 732.

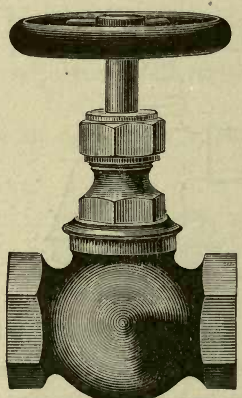


Fig. 733.

They are called globe-valves from the shape inclosing the valve (Fig. 733). They are not necessarily rubber disks; the smaller sizes are metallic poppet-valves.

The dimensions of straightway globe-valves in common use are as follows, from "Warming Buildings by Steam" (Briggs):

Diameter of opening in seat.	Body—gun-metal or cast-iron.	Nozzles—tipped or flanged.	Length over all.	Diameter of flanges.	Number of bolts each flange.
			Inches.	Inches.	
¼	Gun-metal.	Tipped.	1·45		
⅜	"	"	1·78		
½	"	"	2·20		
¾	"	"	2·65		
1	"	"	3·30		
1¼	"	"	3·85		
1½	"	"	4·35		
	Cast-iron.	"	5·10		
2	Gun-metal.	"	5·30		
	Cast-iron.	"	5·90		
	"	Flanged.	5·75	6	4
2½	Gun-metal.	Tipped.	6·75		
	Cast-iron.	"	7·30		
	"	Flanged.	7·25	7	4
3	Gun-metal.	Tipped.	7·75		
	Cast-iron.	"	9·25		
	"	Flanged.	9·25	7½	4
3½	"	Tipped.	10·25		
	"	Flanged.	10·25	8	5
4	"	"	11·25	9	5
5	"	"	13·25	10	6
6	"	"	15·25	11	6
8	"	"	19·	13½	8
10	"	"	23·	16	10
12	"	"	27·	19	10

Figs. 734 and 735 are elevations of valves of the same type as the last, but from their form are called *angle* and *cross* valves.

Figs. 736 and 737 are the plan and section of a steam valve of the Southwark Foundry pattern. Its construction and action will be readily understood from the drawing. The valve is with inclined faces, and seat ground to a fit, and is guided in its movement by three wings, *w*, *w*. This is a common type of throttle-valve for steam use.

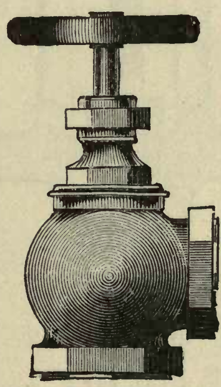


Fig. 734.

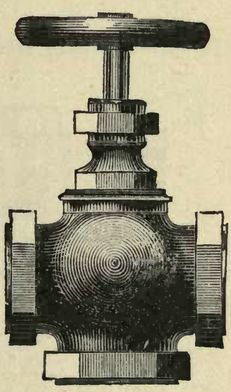


Fig. 735.

It will be observed that in the section (Fig. 737), and especially in that of the globe-valve (Fig. 732), the flow of the fluid passing through them is very disturbed and impeded ; to avoid this, straightway gates are almost invariably used on water mains, in which the gate is raised entirely out of the line of pipe, so as to leave the flow unobstructed.

Fig. 738 is a section of one of the oldest types of this kind of valve, the Coffin valve, with double disks, *d*, *d*, self-adjusting on their seats. The screw works within a long pipe or nut, and when raised the disk-valves are above the line of pipe within the large circular chest.

In "Scraps" is a perspective view of a similar valve of another maker.

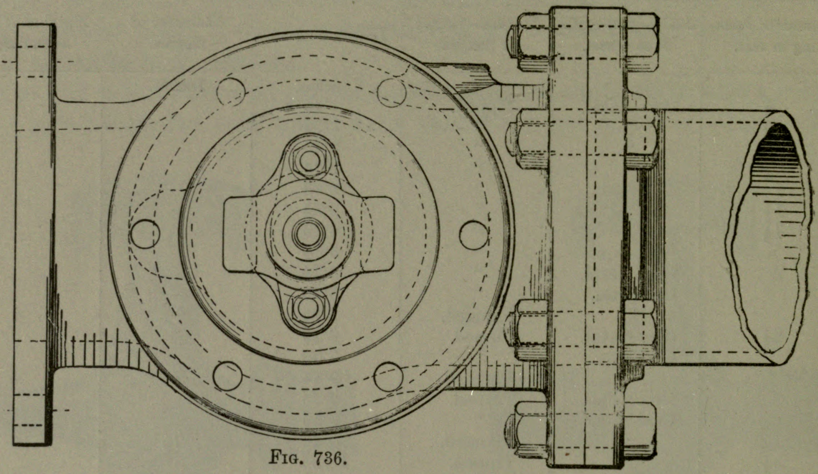


Fig. 736.

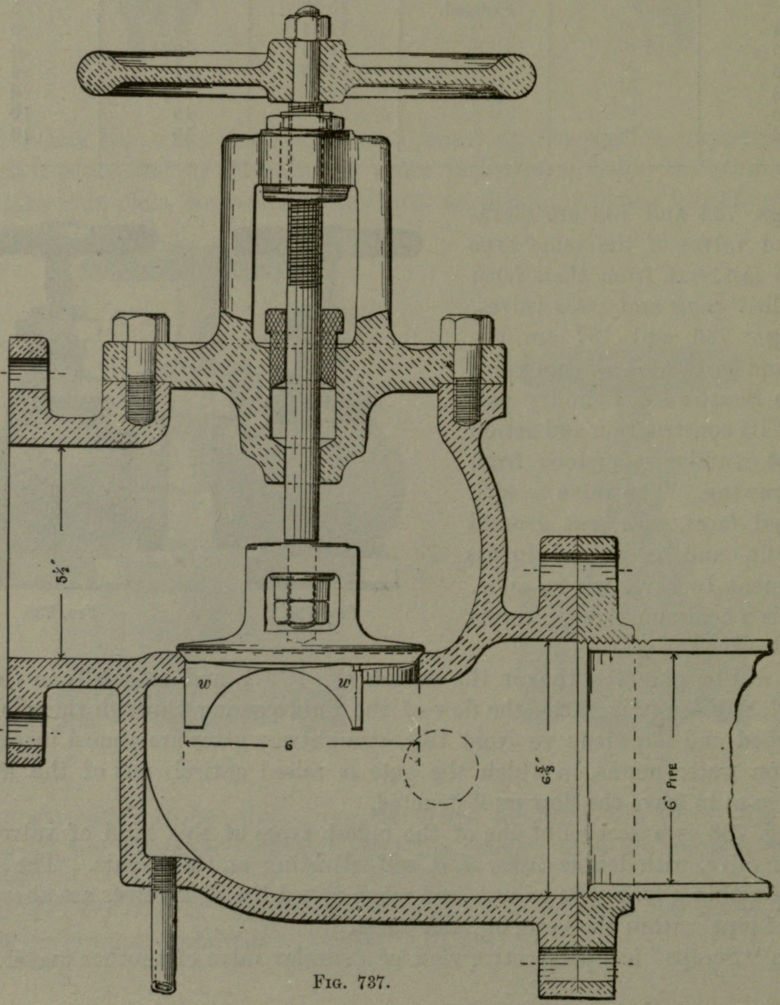


Fig. 737.

Fig. 740, the *Safety-Valve.*—The illustration is of the common type; a poppet-valve, with a stem bearing on the top, and this weighted by a scale-beam, by which any desirable pressure can be put on the valve. To every boiler it is absolutely indispensable that there should be such a valve attached directly, without any means of shutting it off, as in Fig. 740, where B is the boiler, S the steam-pipe, and *b* the blow-off from safety-valve. The United States rules require for the safety-valves of this pattern,

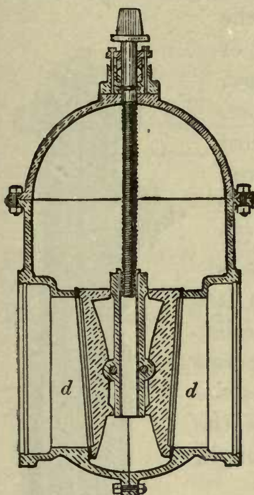
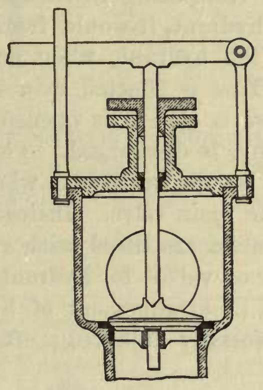


FIG. 738.



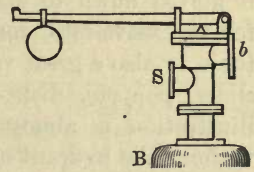


FIG. 740.

for ocean and river service, that they "shall have an area of not less than one square inch for every two square feet of grate-surface."

"But when safety-valves are used, the lift of which will give an effective area of one half of that due the diameter of the valve, the area required shall not be less than one half of one square inch to two feet of grate-surface."

Fig. 741 is what is termed a pop safety-valve; the steam issuing as the valve rises, impinges on a cup surface to force the valve further open. The valve is held down by a spring, but the valve can be raised by the lever *l*. Valves of this kind are often inclosed in a locked box, that they may not be tampered with.

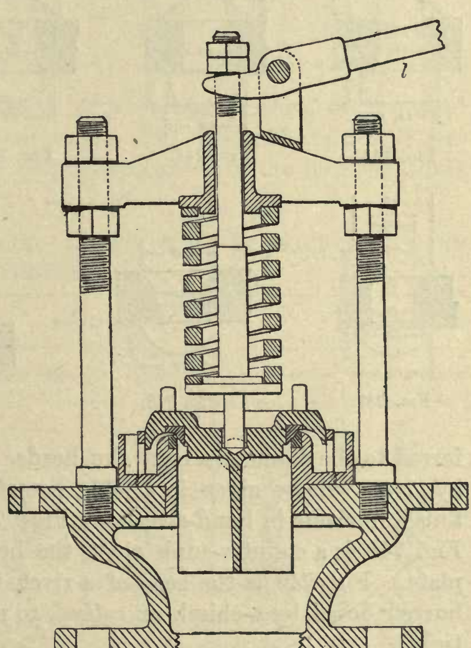


FIG. 741.

Hydrants.—For water-service in connection with high-pressure mains.

Fig. 742 is a section of the Matthews post-hydrant, one of the best known of the type. The valve *v* consists of a series of leather disks bolted together and turned conical, which is brought in contact with a corresponding seat by the valve-rod and its screw at the top of the hydrant. The valve is opened by being forced down into the cavity of a branch of the pipe-main ; *n* is the nozzle for the coupling of the hose ; outside the main pipe of the hydrant there is a case, extending from near the line of valve to the ground line, called the hydrant or frost case, which prevents the hydrant from being lifted by the frost. Were the water left in the hydrant, it would freeze in most exposures during winter ; the hydrant, when not in use, is therefore kept empty. This is effected by a small hole at *v*, which, when the valve is closed, is opened, and the water in the hydrant, if any, is discharged. This vent is closed by a slide attached to the valve-rod, when this last is moved down to open the main valve. Instead of leather for the valve-face, many valves are fitted with rubber ; and there is also a great variety of valves for hydrant purposes—slides, poppets, disks—but in arrangement of hydrants the illustration is almost universally followed ; often, though, without the hydrant case.

Riveted Joints, as used in the construction of Boilers.—Figs. 743–749 are forms of rivets with their proportions re-

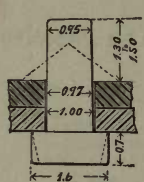


FIG. 743.

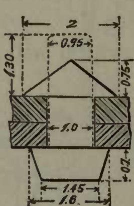


FIG. 744.

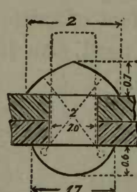


FIG. 745.

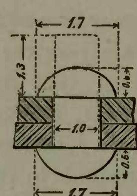


FIG. 746.

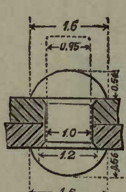


FIG. 747.

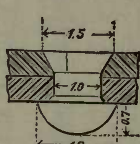


FIG. 748.

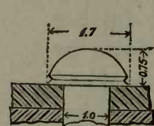


FIG. 749.

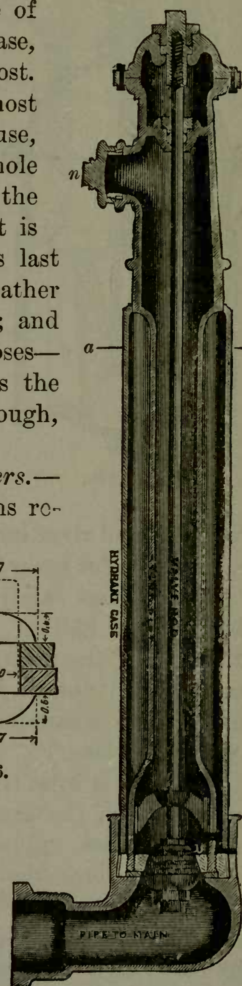


FIG. 742.

ferred to the diameters next the heads. The thickness of the plate connected by rivets will be given in a table hereafter. Figs. 744 and 745 are the usual finish of rivets in hand-riveting ; Figs. 746 and 747, when done by machines. Fig. 748 is a counter-sunk rivet, the head being flush with the outside of the plate. Fig. 749 is the head of a rivet, in which a narrow strip at the edge is burred down by a chisel, or *calked,* to make the joint between rivet and plate tight.

Fig. 750 is a plan and section of a single riveted lap-joint. Joints of this kind fail from the tear of the plate on the line of rivets if the rivets are too close; by the shear of the rivets if too few; or by the bursting of the plate from the rivet to the outside if the space is too small. The great difference in the quality of boiler-plates and rivets, and the uncertainty as to the effect of

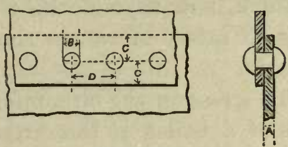


FIG. 750.

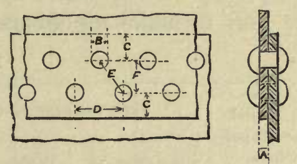


FIG. 751.

punching plates, prevent any accurate determination of the exact proportion of riveted joints. We insert the tables from a practical "Treatise on High-Pressure Steam-Boilers" by William M. Barr. Dimensions in inches:

TABLE SHOWING DIAMETER AND SPACING OF RIVETS IN SINGLE-RIVETED LAP-JOINTS.

Thickness of plate.	Diameter of rivet.	Length of rivet.	Center of rivet to edge of plate.	Center to center of rivets or pitch.	Thickness of plate.	Diameter of rivet.	Length of rivet.	Center of rivet to edge of plate.	Center to center of rivets or pitch.
A	B		C	D	A	B		C	D
$\frac{3}{16}$	$\frac{1}{2}$	1	$1\frac{13}{16}$	$1\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{8}$	$2\frac{1}{4}$	$1\frac{3}{8}$	$2\frac{3}{8}$
$\frac{1}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	1	$1\frac{3}{4}$	$\frac{9}{16}$	$\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{3}{8}$	$2\frac{1}{2}$
$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{1}{2}$	1	$1\frac{7}{8}$	$\frac{5}{8}$	1	$2\frac{3}{4}$	$1\frac{9}{16}$	$2\frac{5}{8}$
$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{3}{16}$	2	$\frac{11}{16}$	1	3	$1\frac{9}{16}$	$2\frac{7}{8}$
$\frac{7}{16}$	$\frac{3}{4}$	2	$1\frac{3}{16}$	$2\frac{1}{8}$	$\frac{3}{4}$	$1\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{3}{4}$	3

Single-riveted joints have the strength of about 56 per cent of the solid plate; double-riveted joints about 70 per cent. Fig. 751 is the plan and section of a double riveted joint, and the proportions given in the table are those recommended by Barr:

TABLE SHOWING DIAMETER AND SPACING OF RIVETING IN DOUBLE-RIVETED LAP-JOINTS.

Thickness of plate.	RIVETS.					
	Diameter.	Length.	Center to edge.	Pitch.	Center to center.	Center to center
			C	D	E	F
$\frac{1}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	1	2	$1\frac{7}{8}$	$1\frac{9}{16}$
$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{1}{2}$	1	$2\frac{1}{4}$	2	$1\frac{21}{32}$
$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{3}{16}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{23}{32}$
$\frac{7}{16}$	$\frac{3}{4}$	2	$1\frac{3}{16}$	$2\frac{3}{4}$	$2\frac{1}{4}$	$1\frac{3}{4}$
$\frac{1}{2}$	$\frac{7}{8}$	$2\frac{1}{4}$	$1\frac{3}{8}$	3	$2\frac{7}{16}$	$1\frac{29}{32}$
$\frac{9}{16}$	$\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{3}{8}$	$3\frac{1}{4}$	$2\frac{9}{16}$	2
$\frac{5}{8}$	1	$2\frac{3}{4}$	$1\frac{9}{16}$	$3\frac{1}{2}$	$2\frac{3}{4}$	$2\frac{1}{8}$
$\frac{11}{16}$	1	3	$1\frac{9}{16}$	$3\frac{3}{4}$	$2\frac{7}{8}$	$2\frac{3}{16}$
$\frac{3}{4}$	$1\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{3}{4}$	4	3	$2\frac{1}{4}$

For the most part in this country, rivet-holes are punched; some drill them. By punching first a small central hole, and then using a *pin* or *teat-drill*, an annular washer is taken out, leaving a clean hole, and a ready means of testing the quality of the material by bursting the washer by a drift. It is the practice here to make no boilers less than ¼″ thick, and beyond this to use a factor of safety of six, as shown in the table.

Strength of solid plate—pounds per square inch.	SAFE WORKING LOAD.	
	Single-riveted.	Double-riveted.
50,000	4,700	5,800
60,000	5,600	7,000
70,000	6,500	8,200

The stress on the circumferential seams of a boiler is the circular or end area in square inches multiplied by the pressure per square inch, and this is to be met by the circumferential section of the shell. The longitudinal stress can be estimated by multiplying the diameter of the boiler in inches by the pressure per square inch, and this stress is to be resisted by one inch in length on each side of the boiler, or by a section of plate 2″ wide by its thickness, and with a proper factor for riveted joints.

Fig. 752 is the plan and section of a single-riveted butt-joint, and Fig. 753 the same of a double-riveted one. The two plates are brought close to each other, and the joint is made by a cover, proportioned in the pitch of the rivets

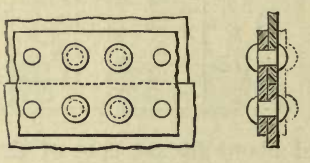


Fig. 752.

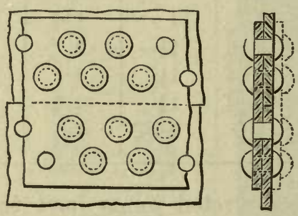


Fig. 753.

and distances of centers from edges of plates, as in rules above given; and although this form of joint in some cases is convenient, it has not been found practically stronger than the lap-joint.

But butt-joints with double covers, one on each side of the plates, increase the shearing resistance of the rivets, so that rupture always takes place in the

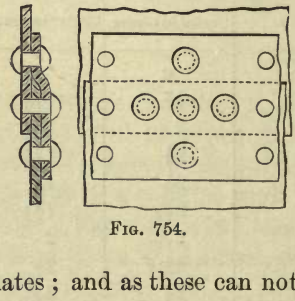


Fig. 754.

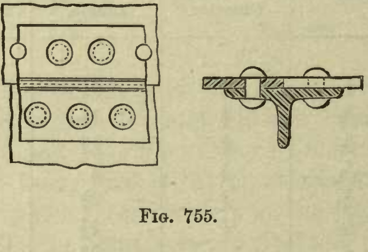


Fig. 755.

plates; and as these can not bend, and there is considerable frictional resistance between the plates, the strength of the joint has been found to be more than that due to the net section of the plates between the rivets.

Fig. 754 is a plan and section of a combined lap and butt joint. The pitch of the exterior rows is double that of the central one ; for a $\frac{5}{8}''$ plate, 4″ for the former and 2″ for the latter.

Fig. 755 is the plan and section of a butt-joint when the cover is of T-iron—a not uncommon form of strengthening flues to resist collapse.

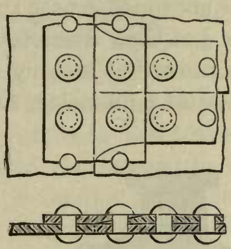


Fig. 756.

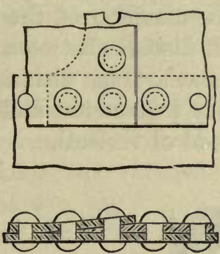


Fig. 757.

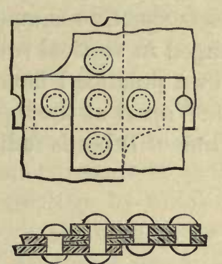


Fig. 758.

Junction of more than two plates, shown in plans and sections (Figs. 756, 757, and 758).—These become necessary when cross-joints intersect longitudinal ones. At these joints one or more of the plates are thinned or drawn out by forging.

Fig. 759 is the plan and section of an angular connection of plates by the means of angle-iron ; this should be a little thicker than the plates, and its width four times the diameter of the rivets.

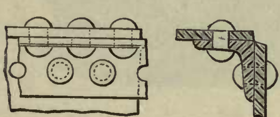


Fig. 759.

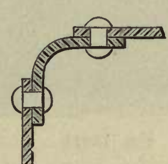


Fig. 760.

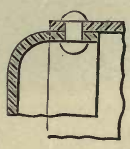


Fig. 761.

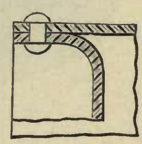


Fig. 762.

Figs. 760, 761, and 762 are sections of angular connections by flanging the plates. The iron should be good and the curvature easy ; inside radius at least four times the thickness of the plates.

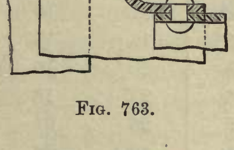


Fig. 763.

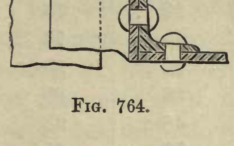


Fig. 764.

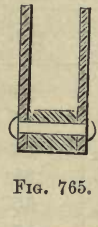


Fig. 765.

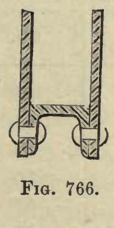


Fig. 766.

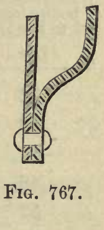


Fig. 767.

Figs. 763 and 764 are sections of joints of cylinders of unequal diameters, or surfaces not in line with each other.

Figs. 765, 766, and 767 are sections of fire-box legs.

In all connections provisions are to be made for the means of holding the head of the rivet, and for riveting and for calking the joints.

Fig. 768 is the perspective view of a boiler of the type most commonly used when the fuel is anthracite, and often also when bituminous, called the *horizontal tubular.* The proportions of the boiler vary with the requirements of their position, and with the views of the mechanical engineer or maker constructing them. Those in most extensive use are with shells of 4 to 5 feet inside diameter and 3″ to 3½″ tubes, 14 to 16 feet long. The line of the top of the upper tubes is usually about $\frac{1}{10}$ of the diameter of the boiler above its center ; tubes arranged in vertical rows, with distance between tubes ⅓ of their diameter. In my own practice I have kept the average distance the same, but making them farther apart at the top row, say ½ diameter, and the lowest ¼ diameter, so that the line of tubes is radial instead of vertical.

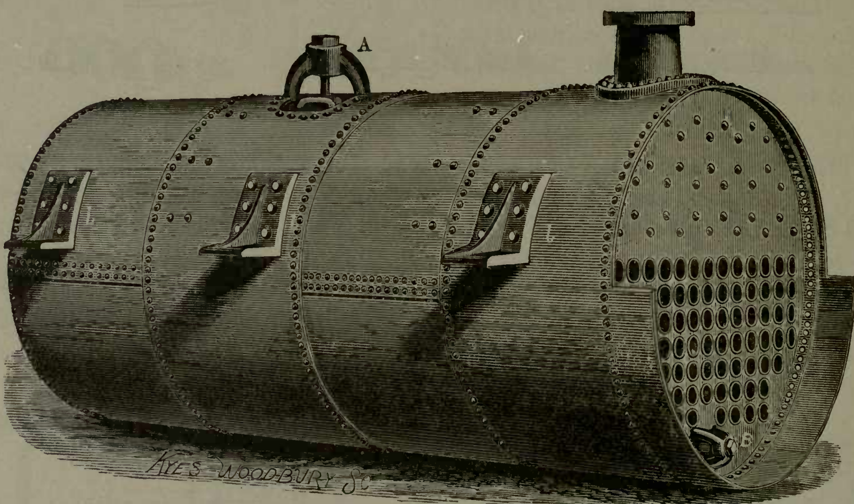


FIG. 768.

The following table is from Barr, showing the greatest number of tubes which should be put in a given head, no tube to come nearer to the shell than 2″ for boilers of small diameter, 2½″ for medium, and 3″ for the larger series :

Diameters of bodies inside, in inches.	NUMBER OF TUBES (outside diameter).						
	3 in.	3¼ in.	3½ in.	3¾ in.	4 in.	4½ in.	5 in.
36	26	23	20	19	16	12	10
40	34	34	25	23	20	14	14
44	48	36	32	25	25	20	16
48	50	38	36	30	26	21	18
52	57	50	48	38	32	26	21
56	72	57	55	48	41	32	23
60	80	68	62	55	46	36	30

A (Fig. 768) is the *man-hole,* to enable the mechanic to get into the boiler to examine it. It consists of a cast-iron frame, bolted to the shell of the boiler, with an elliptical opening usually 9″ × 15″ in the clear; the valve laps about 1″ on each side. In closing the opening the valve is passed down into the boiler, and is brought up against the valve-seat, where it is held by its stem passing up through a movable yoke, and brought up tight by a nut and screw. The joint is made with a gasket or with sheet-rubber. The man-hole is often placed in one of the boiler heads. B is the hand-hole, of the same general construction as the man-hole, but smaller, to enable the fireman to clean the boiler. Formerly this hand-hole was quite small, but of late the practice is to make them 6″ × 8″, or even as large as 8″ × 12″ for large boilers—in fact, a man-hole. There should be a hand-hole in the other end of the boiler, so that by taking off both hand-holes one can look directly through the boiler. As this hand-hole is exposed to the flame and products of combustion, it is well to make it smaller than the front one, say 3″ × 5″; *l l l* are lugs by which the boiler is supported on brick-work. It will be observed that in the head above the tubes there are rivet-heads, and also in the sides back of the first seams at each end. These are for the attachment of diagonal stays. The tubes themselves serve as stays in the lower part of the boiler, but above the flat surface needs something to prevent the head from moving out under pressure. The stays are made of round or flat iron, bolted directly to the shell, the round part being flattened, and connected by a yoke and pin to a crow-foot or piece of angle-iron attached to the head. The stays are from $\frac{3}{4}$″ to $1\frac{1}{4}$″ diameter or equivalent sections.

BARR'S PROPORTIONS FOR STAY-BOLTS FOR FLAT SURFACES.

Pressure per square inch.	CENTER TO CENTER OF STAY-BOLTS IN SQUARE INCHES.				
	$\frac{1}{4}$″ plate.	$\frac{5}{16}$″ plate.	$\frac{3}{8}$″ plate.	$\frac{7}{16}$″ plate.	$\frac{1}{2}$″ plate.
60	$5\frac{3}{8}$	$6\frac{3}{8}$	$7\frac{1}{4}$	$8\frac{1}{8}$	9
80	$4\frac{5}{8}$	$5\frac{1}{2}$	$6\frac{1}{4}$	$7\frac{1}{8}$	$7\frac{7}{8}$
100	$4\frac{1}{4}$	$4\frac{3}{4}$	$5\frac{1}{2}$	$6\frac{1}{4}$	7
120	$3\frac{7}{8}$	$4\frac{1}{4}$	5	$5\frac{3}{4}$	$6\frac{3}{8}$
140	$3\frac{5}{8}$	$4\frac{1}{8}$	$4\frac{5}{8}$	$5\frac{1}{4}$	6

Figs. 769 and 770 are a longitudinal and a half transverse section of an anthracite-burning locomotive from the New Jersey Railroad, which illustrates the stays used in such forms of boilers. Water-spaces are 4″ wide in front, 3″ at sides, and 6″ at rear; stay-bolts in water-spaces $\frac{7}{8}$″ diameter, 4″ centers. The crown-sheet of fire-box is supported by double cast-iron girders, extending across the boiler, ends resting on the inner plates of fire-box, and also supported by hangers *h h* from the outer shell, and the inside of the steam-drum. These hangers have a fork at one end, through which a pin is passed to connect it with the foot riveted to the boiler; the other end passes into the space between the double girder, and they are pinioned together. The crown-sheet is held by bolts passing down through the double girder. The bottom of the water-space is made with a wrought-iron ring. The opening for the door is made by turning a flange on the inside plate, to which a plate ring is riveted,

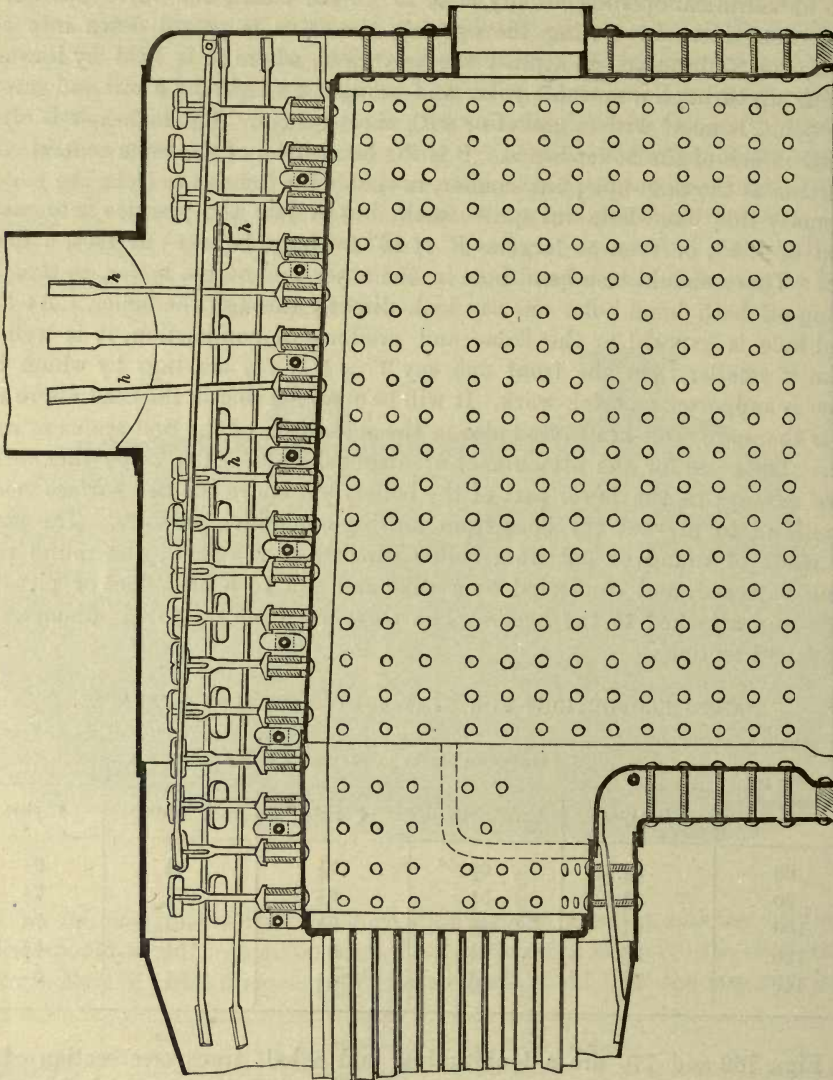


Fig. 769.

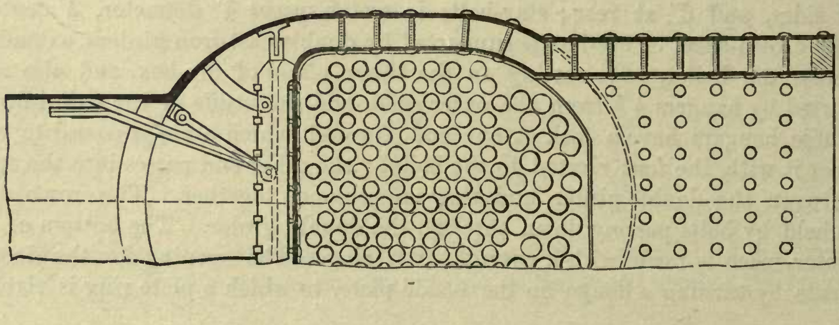


Fig. 770.

and the joint with the outside plate is made by a ring of angle-iron. The boiler has 130 2″ tubes, and 26 $2\frac{1}{4}$″ tubes.

Fig. 771 is a *gusset* stay, used in angles, consisting of a triangular plate with the edges flanged and riveted to the shell.

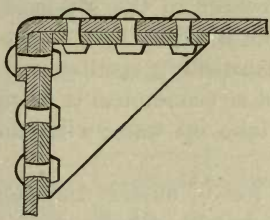


Fig. 771.

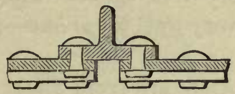


Fig. 772.

Fig. 773.

Flue Boilers.—Where bituminous coal is used, small tubes become clogged with soot; it was therefore customary to construct boilers with larger tubes or flues of boiler-iron riveted together, which sometimes failed from collapse, their resistance being uncertain, due largely to the defect of an accurate circular section. Mr. Fairbairn made experiments on the resistance of tubes to collapse, but it has been demonstrated that the rule does not apply within the limits of length adapted for boiler-flues, and it may be considered ample to make the tubes subject to outside stress fifty per cent thicker than for bursting, especially for the large drawn tubes now made. From Mr. Fairbairn's experiments it was considered necessary to make the joints of tubes subject to collapse as in Figs. 772 and 773, which may be useful against deterioration of force in riveted boiler-flues, and might in long mains be of importance, especially if of the form of Fig. 773, which, besides strengthening the tube, provides for expansion.

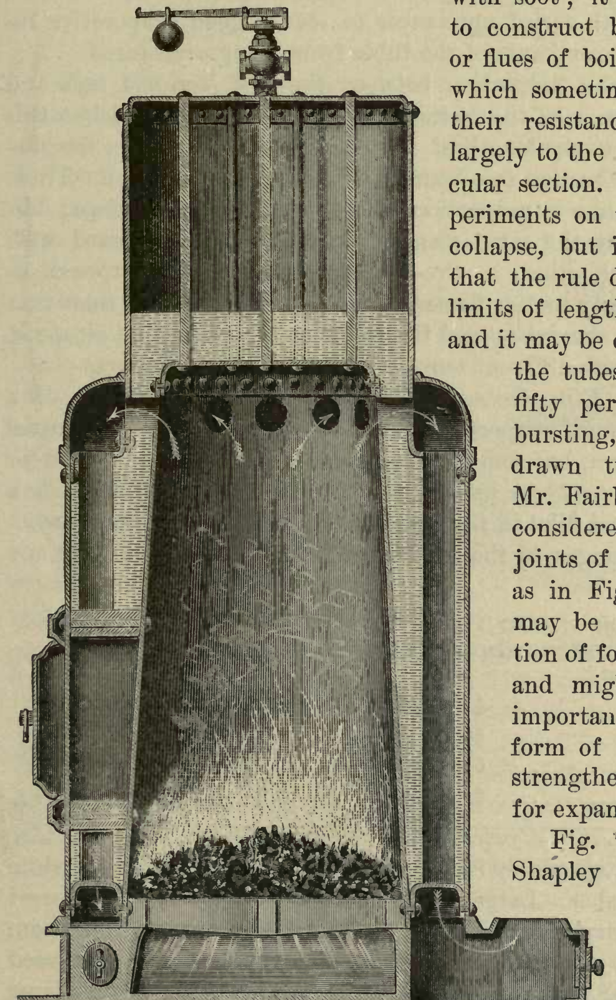


Fig. 774.

Fig. 774 is a section of the Shapley boiler, as made by the Knowles Steam-Pump Works — a good form of upright boiler, with the crown-sheet simply stayed and well covered by water. It is an admirable illustration for the draughtsman of how a boiler in action may be represented.

The usual form of upright boiler consists of a fire-box, extending a little above the door, and tubes extending from the crown-sheet to the top-head, over which there is a bonnet to secure the smoke, which is led off by a smoke-pipe. These are very convenient forms of boilers for furnishing small power, as they occupy comparatively little space. They are not as economical in their combustion, and they are very apt to *prime*—that is, take up water with the steam.

The common vertical boilers are from 2 feet 6″ to 4 feet 6″ outside diameter of shell, with water space in legs of $2\frac{1}{2}''$ to 3″; extreme height of boiler from 2 to $2\frac{1}{2}$ times the outside diameter of fire-box; tubes from 2″ to $2\frac{1}{2}''$ diameter, and spaced from 1″ to $1\frac{1}{2}''$ apart. Water-line from 10″ to 15″ above crown-sheet.

On account of small ground-space, vertical boilers are popular with some makers, and are made with varied appliances to secure good evaporative results and to protect the upper joints of the tubes from being overheated.

There is supposed to be a proportion between the tube sectional area and the grate-surface, say from $\frac{1}{7}$ in the horizontal to $\frac{1}{8}$ in the vertical; but this rule is entirely empirical, as the length of the tube is a large factor in the discharge of products of combustion (see Sturtevant tables in appendix). There is also a proportion of grate to heating surface; but only the same class of boilers can be compared with each other, as fire-box surface—and that exposed directly to the flame—is much more effective than that of the tubes, and the products of combustion escape at much different temperatures in different boilers.

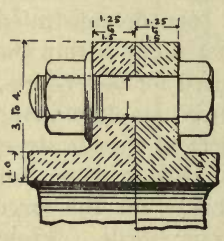


Fig. 775.

Pipe Connections.—Fig. 775 is the section of a flanged connection of a cast-iron pipe of the most usual form, but some thicken or reinforce the pipe a little for 1″ to 2″ in length next the flange; but if there is a good fillet in the angle of the flange it is unnecessary.

The proportions of flanges to the thickness of the pipe at the joint are given below:

DIMENSIONS OF CAST-IRON FLANGED PIPE TO WITHSTAND SAFELY A PRESSURE OF ONE HUNDRED POUNDS PER SQUARE INCH.

Diameter of pipe	4	6	8	10	12	16	20
Thickness of pipe	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
Number of bolts	5	6	8	10	10	14	18
Diameter of bolts	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$

The flanges are almost invariably faced, and joints made with red and white lead, or a sheet-rubber washer. Large cast-iron flanged pipe is but little used for street mains; water-service socket-pipes are invariably used, and for steam connections wrought-iron pipe is to be preferred, and it can now be purchased of any necessary diameter; and when steam-drums are requisite, or very large connections, they are made of riveted plate.

Fig. 776 is a section of the joint used by Sir William Armstrong for the pipes of his accumulator. For a working pressure of 800 pounds per square inch, pipes of 5″ diameter are made 1″ thick and tested to 3,000 pounds per square inch. The flange is elliptical, and there are but two bolts ; one pipe slightly enters the other, forming a dovetailed recess in which is placed a gutta-percha ring ¼″ diameter.

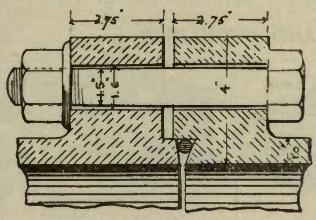


Fig. 776.

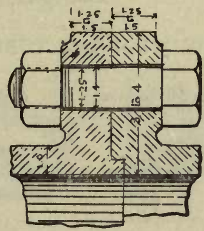


Fig. 777.

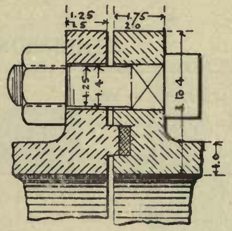
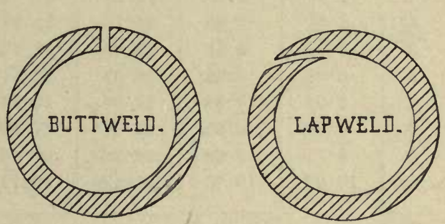


Fig. 778.

Figs. 777 and 778 are sections of two other forms of cast-iron flanged pipes, both with projections fitting into grooves. The packing in Fig. 778 is a ring of lead. In Siemens's air reservoirs, where the pressure sustained by steel rings is 1,000 pounds per square inch, the joint is made by turning a V groove in the face of the rings, and placing in it a ring of annealed copper $\frac{5}{16}''$ diameter. This form is adopted by many mechanics for forming flanged joints even for steam purposes.

Wrought-Iron Pipe Connections.—With the present cost of wrought-iron pipes they are almost invariably used for the conveyance of steam, but are more liable to rust for water purposes than cast-iron. Wrought-iron pipes are either butt-welded or lap-welded. It is a mere question of manufacture. It is difficult to make a lap-welded tube less than 1½″ diameter, and, therefore, below this size they are usually butt-welded ; but this size and above, lap-welded.



Wrought-iron pipes in continuous length are connected by socket or sleeve couplings, shown partly in section (Fig. 779), which are almost invariably of wrought-iron. A thread is cut on each end of the pipes, and internal threads in the coupling. The coupling is screwed on to the end of one pipe and the other pipe screwed into the coupling. The screw in the coupling is tapped parallel usually, but the ends of the tubes are cut with a taper thread, uniform with all makers, of 1 in 32 to the axis. The length of the screwed portion varies with the diameter.

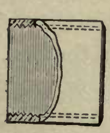


Fig. 779.

Fig. 780 is the longitudinal section of tapering tube-end with the screw-thread as actually formed, and considered standard by the late Robert Briggs, C. E., in his "Treatise on Warming Buildings by Steam." It is shown in the figure double full size for a nominal 2½″ tube.

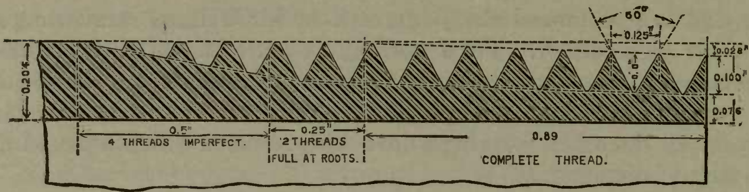


Fig. 780.

DIMENSIONS OF WROUGHT TUBES AND COUPLINGS.

DIAMETER OF TUBE.			CIRCUMFERENCE.		SCREWED ENDS.		Weight per foot in length.	COUPLINGS.	
Nominal inside.	Actual inside.	Actual outside.	Inside.	Outside.	No. of threads per end.	Length of screw.		Outside diameter.	Length.
In.	In.	In.	In.	In.	In.	In.	Lbs.	In.	In.
⅛	0·27	0·41	0·85	1·27	27	0 19	0·21	0·55	⅞
¼	0·36	0·54	1·14	1·70	18	0·29	0·42	0·70	1
⅜	0·49	0·67	1·55	2·12	18	0 30	0·56	0·83	1
½	0·62	0·84	1·96	2·65	14	0·39	0·84	1·01	1$\frac{5}{16}$
¾	0·82	1·05	2·59	3·30	14	0·40	1·13	1·24	1⅜
1	1·05	1·31	3·29	4·13	11½	0·51	1·67	1·53	1¾
1¼	1·38	1·66	4 33	5·21	11½	0·54	2·26	1·89	1¾
1½	1 61	1·90	5·06	5·97	11½	0·55	2·69	2·17	2
2	2·07	2·37	6·49	7·46	11½	0 58	3·67	2 68	2¼
2½	2·47	2·87	7·75	9·03	8	0·89	5·77	3·19	2¾
3	3·07	3·50	9·64	11·00	8	0·95	7·55	3·87	3
3½	3·55	4·00	11·15	12·57	8	1·00	9·06	4 40	3⅛
4	4·03	4·50	12·65	14·14	8	1·05	10·73	4 99	3¼
4½	4·51	5·00	14·15	15·71	8	1·10	12·49	5·49	3⅜
5	5·04	5·56	15·85	17·47	8	1·16	14·56	6·19	3½
6	6·06	6·62	19·05	20·81	8	1·26	18·77	7·24	3¾
7	7·02	7·62	22·06	23·95	8	1·36	23·41	8·36	4
8	7·98	8·62	25·08	27·10	8	1·46	28 35	9·49	4¼
9	9·	9·69	28·28	30·43	8	1·57	34·08	10·54	4½
10	10·02	10·75	31·47	33·77	8	1·68	40·64	11·72	5

When pipes are thus put together in lengths, with couplings, it is frequently impossible to take out a length of pipe for repairs or alterations without breaking a coupling or fitting; provision is made for disconnections by the insertion of a *union* or unions in the line.

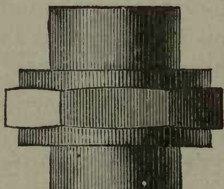


Fig. 781.

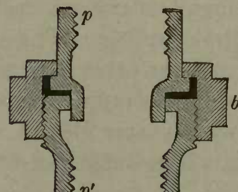


Fig. 782.

Fig. 781 is an exterior view, and Fig. 782 a section, of the common malleable-iron union; *p* and *p′* are the halves into which the tube is screwed, and the joint is made by a male and female coupling. The male, *b*, turning on a flange on the tube *p*, is screwed to the other half of the coupling, and the joint is made tight by a rubber washer, shown in black. These unions are used only in the smaller sizes of pipes. The flange coupling (Fig. 783) is preferred by most fitters, and they are made of diameters up to 14″; the thickness is about one half that of the length of a coupling of the same diameter. The bolts are from $\frac{3}{8}$″ to $\frac{3}{4}$″, and spaced somewhat larger than that given for cast-iron flanges. The width of flange is such as to admit of the head and nut of the bolt without projection beyond the edge of the flange.

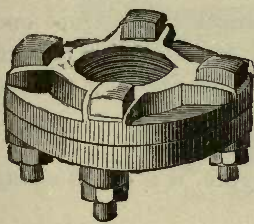


Fig. 783.

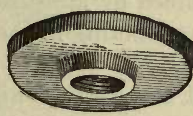


Fig. 784.

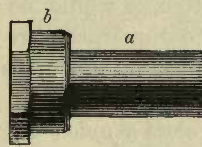


Fig. 785.

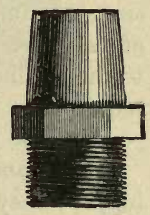


Fig. 786.

Fig. 784 is a common cast-iron flange, and with about the same proportions as in Fig. 783. When the lines are long, and provision can not be made by bends for the expansion and contraction of pipes under changes of temperature, a fitting like a stuffing-box is often used, the end of one of the tubes being attached to the box, and the other sliding in and out like a piston-rod; sometimes expansion is permitted by two flexible flanges, admitting of a sort of bellows-like movement; sometimes by a connection between pipes of a ring, as in Fig. 773, or a succession of corrugations.

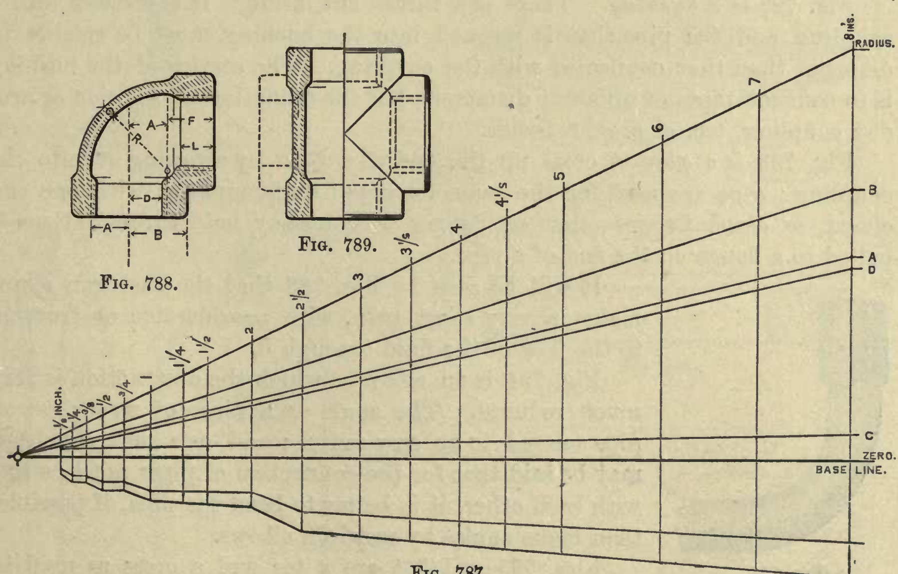


Fig. 788.

Fig. 789.

Fig. 787.

Fig. 785 is a *soldering union;* the ring *b* is like that of the male coupling (Fig. 782), which is screwed directly to the wrought-iron pipe, while *a* is a brass tube, with a shoulder on the bottom on which the coupling *a* turns, and a lead pipe is soldered to the tube. If it is not necessary to break the joints, a *soldering nipple* (Fig. 786) only is necessary, one end of which is screwed into the wrought-iron pipe, and the other soldered to the lead pipe.

Figs. 787, 788, 789 are taken from Briggs's treatise, and give the dimensions of the parts of elbow, tees, crosses, and branches. Fig. 788 shows the parts of an elbow designated by letters in Fig. 787, and Fig. 789 shows the applicability of the same to tees and crosses. The scale is one quarter full size; if much used, it would be better for the draughtsman to construct one of full size. The dimensions are obtained by measuring from the base or zero to the inclined lines, on ordinates corresponding to the inside diameter of pipe required.

Fig. 790 is a *close nipple;* Fig. 791 is a *shoulder nipple.*

If the uncut part of the tube is longer than in the figure, it is called a *long* nipple; they serve the purpose of short pipes.

Fig. 790.

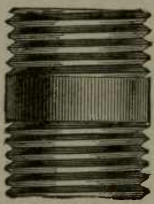


Fig. 791.

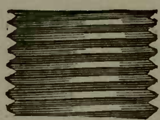


Fig. 792.

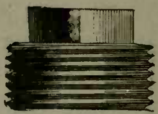


Fig. 793.

Fig. 792 is a *bushing.* There is a thread cut inside. It is screwed into a coupling, and the pipe that is screwed into the bushing must be smaller in diameter than that connected with the coupling. The service of the bushing is to connect pipes of different diameters, but the reduction of one side or arm of a coupling, tee, or cross is better.

Fig. 793 is a *plug* to close up the end of a pipe by screwing it into the coupling; *caps* are used for the same purpose; half-couplings with one end closed, or *blank* flanges—that is, flanges without any hole through them—bolted to a flange on the end of a pipe.

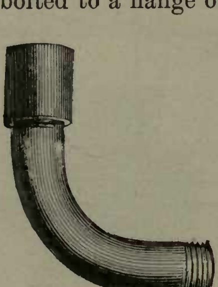


Fig. 794.

It will be seen by Fig. 788 that the cast-iron elbow makes a very short turn, with considerable obstruction to the flow of the fluid through it.

Fig. 794 is an elbow in which the obstruction is very much reduced. The angle is a piece of wrought-iron pipe curved to an easy radius; and, as a general rule, it may be said that for the connection of pipes not in a line with each other, it is better to bend the pipe, if possible, than make angles by cast-iron elbows.

Figs. 795 and 796 are a tee and a cross as used in connections of hydraulic presses, made of composition. The tubes are of wrought-iron, extra thick. The usual dimensions for such are as follows:

Outside diameter........................	$1\frac{3}{16}''$	$\frac{7}{8}''$	$1''$
Inside diameter........................	$\frac{3}{8}''$	$\frac{3}{8}''$	$\frac{1}{2}''$

The joints are made by leather washers, square ends on square seats.

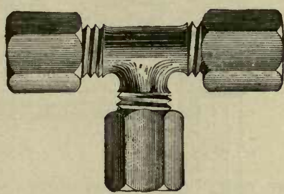


FIG. 795.

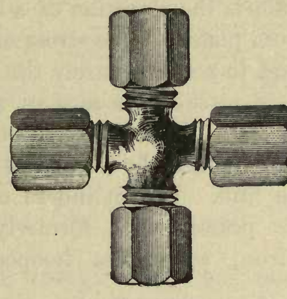


FIG. 796.

FRAMES.

Fig. 797 is the section of a common jack-screw, in which the pressure is vertical; the base is made extended to give it stability.

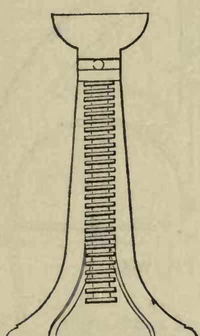


FIG. 797.

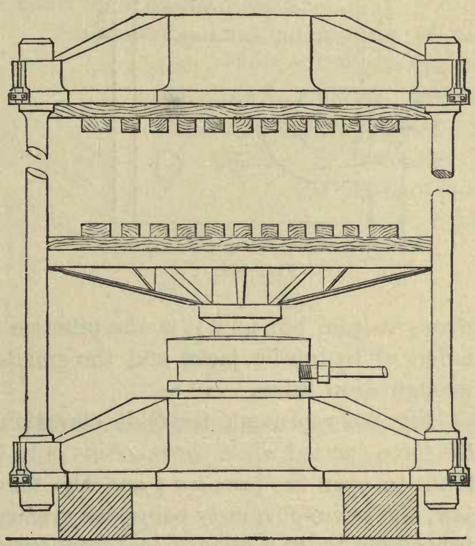


FIG. 800.

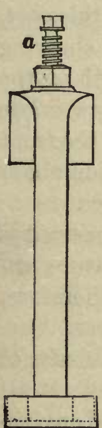


FIG. 798.

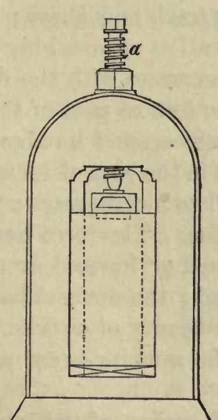


FIG. 799.

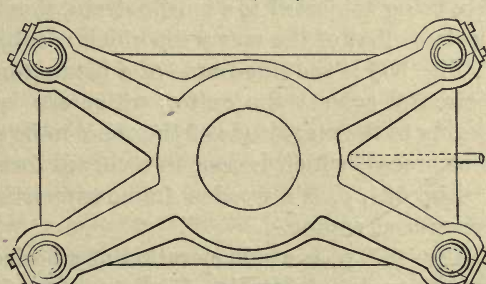


FIG. 801.

Figs. 798 and 799 are side and end views of a cast-iron *housing* for rolls. The screw exerts the pressure on the box of the roll-journal, and the reaction is a tensile pressure on the sides of the frame; but there is in addition much percussion and intermittent stress that is to be provided against.

Fig. 800 is the elevation of a hydraulic press, and Fig. 801 the plan of top and bottom plates. The stress on the rods is tensile, and they must be of sectional area to resist securely the power exerted on the press. The plates are beams held at the four corners, and the stress central. The platen *p* attached to the ram is braced by triangular flanges from the hub. The cylinders of large hydraulic presses were formerly made of cast-iron, sometimes hooped with wrought-iron, but now it is the practice to make them of cast-steel. The cylinders of hydraulic jacks and the smaller presses are made of drawn steel or wrought-iron tubes.

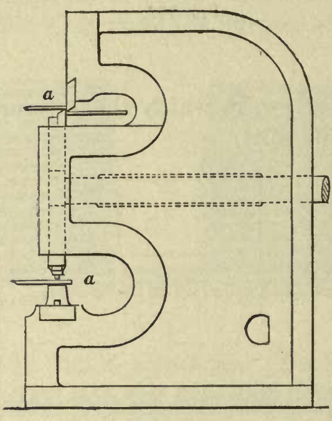


Fig. 802.

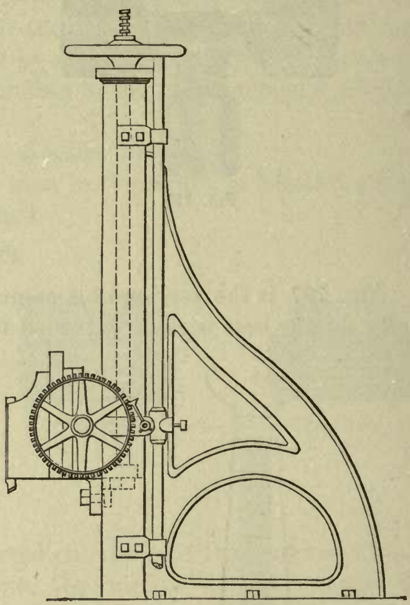


Fig. 803.

Fig. 802 represents the (side elevation) cam-punch and shear; in this case, the force exerted while the machine is in the operation of punching or shearing tends to open the jaws *a a*; and the tendency increases with the depth of the jaw, the stress obviously being the greatest at the inmost part of the jaw. The frame consists of a plate of cast-iron, with two webs around its edges; the front web, being subjected to a tensile strain, should be in the area of its section about six times that of the rear web, which is subjected to a compressive force.

Fig. 803 is the side-frame of a planing-machine. The force here exerted is horizontal against the cutter, which can be raised or lowered at pleasure, according to the magnitude of the work to be planed; the upright has, therefore, to be braced, which is done in a curved form for beauty of outline.

Fig. 684, *p*, is a wooden frame supporting the working-beam and shafts of a river-boat engine.

Fig. 682, *p*, is a side elevation of a horizontal engine, of the type of engine-frame introduced by Mr. Corliss; (Fig. 804) is a plan of the same. The old type of steam-engine frame was a rectangular cast-iron frame; the steam-cylinder resting on the top side flanges, the pillow-block being bolted on the top of one side flange, and the crank and connecting-rod working centrally between the sides.

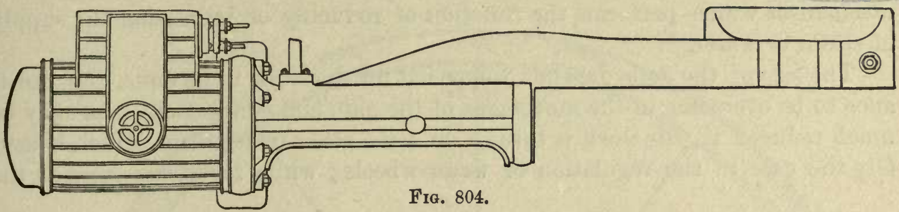


Fig. 804.

Fig. 805 is a side view of the inclined wrought-iron box-frame of the war steamer Susquehanna. The steam-cylinder rests between the frames, and is bolted to them. The two frames are securely stayed to each other, and bolted to the keelson and the bottom of the ship. For small inclined engines, as used on ferry-boats, the frames are of wood, as also in many of the horizontal engines of boats on Western waters.

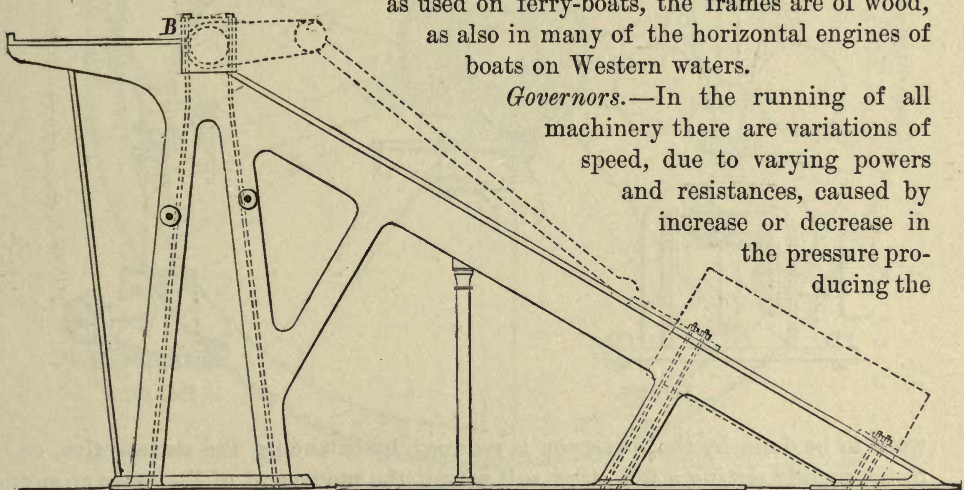


Fig. 805.

Governors.—In the running of all machinery there are variations of speed, due to varying powers and resistances, caused by increase or decrease in the pressure producing the power, as of steam or water, or in the resistances of the machinery, from more or less being brought into action, or through inequalities of work done. To maintain the speeds at as much uniformity as possible, governors are used, which, applied to steam-engines or water-wheels, open or close valves or gates, and increase or reduce the supply of steam or water to the cylinders or wheels, according to the varying necessities. The ordinary governor (Fig. 806) consists of two heavy balls, suspended by links from a spindle, and caused to revolve by some connection with the shaft of the motor. In the figure the governor is driven by a belt-connection to the pulley, *p*, bevel-geared to the governor. When at rest, the balls hang close to the spindle, but when in motion the balls rise by the centrifugal force. When the motor is running at its established speed, the links assume a position nearly at 45° with the spindle. If the speed falls off, the balls fall, and, acting on the lever, as shown in side view, open the valve or gate controlling the passage of steam to the cylinder or water to the wheel; if the speed rises, the balls rise and close the valve or gate. The lever does not always connect directly with the gate, nor is there always a lever, but the rise or fall of the balls acts on some mechanism which performs the function of reducing or increasing the supply of steam or water.

The size of the balls depends somewhat on the work to be done, the resistance to be overcome in the movement of the gate and connections, and may be much reduced if this work is thrown on some other mechanism, which is usually the case in the regulation of water-wheels; while for steam-engines the

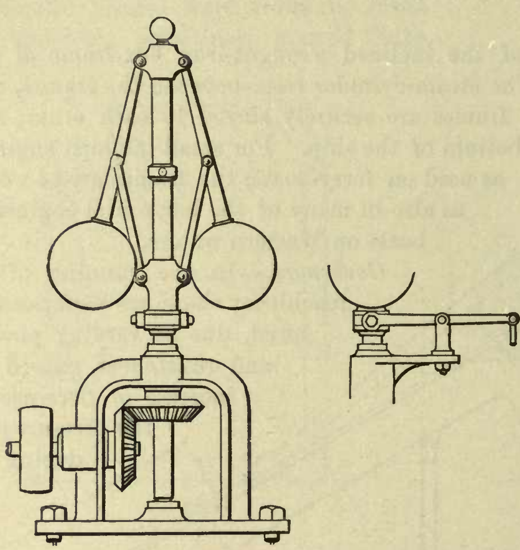


FIG. 806.

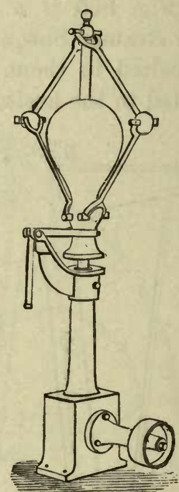


FIG. 807.

work to be done by the governor is reduced by balancing the steam-valve, or to the merely setting a trip, that will permit the movement of the valve at any point of cut-off.

In the Porter governor (Fig. 807) the balls of the governor are comparatively light, but they are connected to a heavy central weight by levers, the same as those connecting the balls with the spindle.

Fly-Wheels.—In most machinery there would be great inequality of movements, from the great difference in power exerted or resistances to be overcome, and in the application of the force, as through cranks. To obviate this, fly-wheels are used, which absorb energy in one part of their revolution and give it out at another, or by their mass in movement overcome resistances, as in the punching, shearing, and rolling of metal, which comes only periodically, and is much in excess of that usually required. In addition, fly-wheels give governors time to act, and consequently the motion is more uniform and constant. For the speed and weight of fly-wheels the conditions vary so much at different times, even with the same engines, that it is impossible to get data for an estimate by any mathematical formula embracing the conditions. From the experience of the best mechanical engineers, and from published examples of constructions, are deduced the following rules, applicable to common practice for the fly-wheels of steam-engines : The diameter of fly-wheel to be 4 times

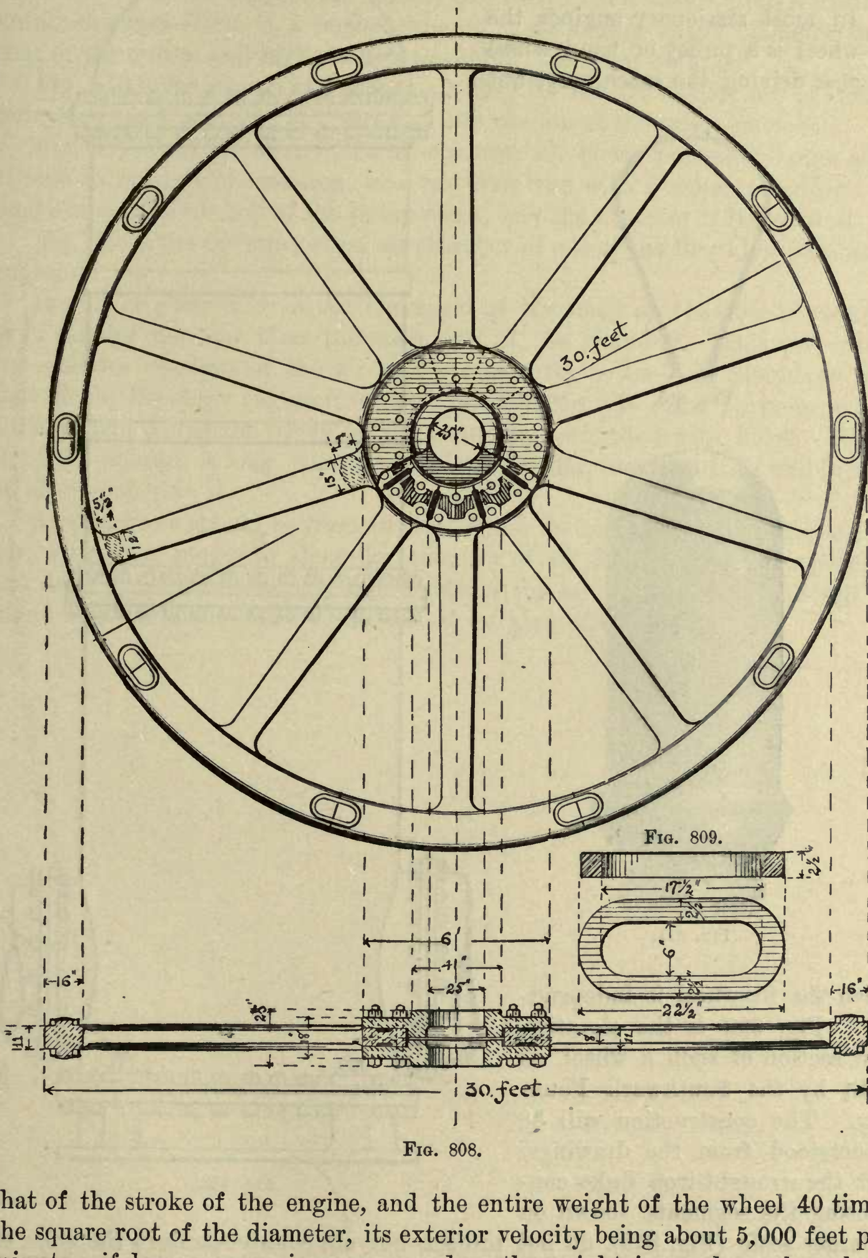


Fig. 809.

Fig. 808.

that of the stroke of the engine, and the entire weight of the wheel 40 times the square root of the diameter, its exterior velocity being about 5,000 feet per minute ; if less or more, increase or reduce the weight inversely as the velocity. The rim is generally a little less than ¾ of the whole weight. For rolling-mill engines, Mr. C. B. Richards takes the weight of the fly-wheel at 60 times the square of the diameter of the cylinder, and the diameter of the wheel 5 times that of the stroke, and rim velocity not to exceed 125 feet per second.

In most stationary engines the fly-wheel is a pulley or band wheel or gear driving the machinery, but

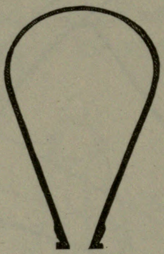


Fig. 810.

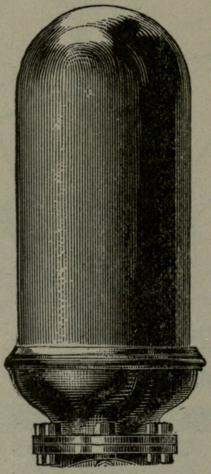


Fig. 811.

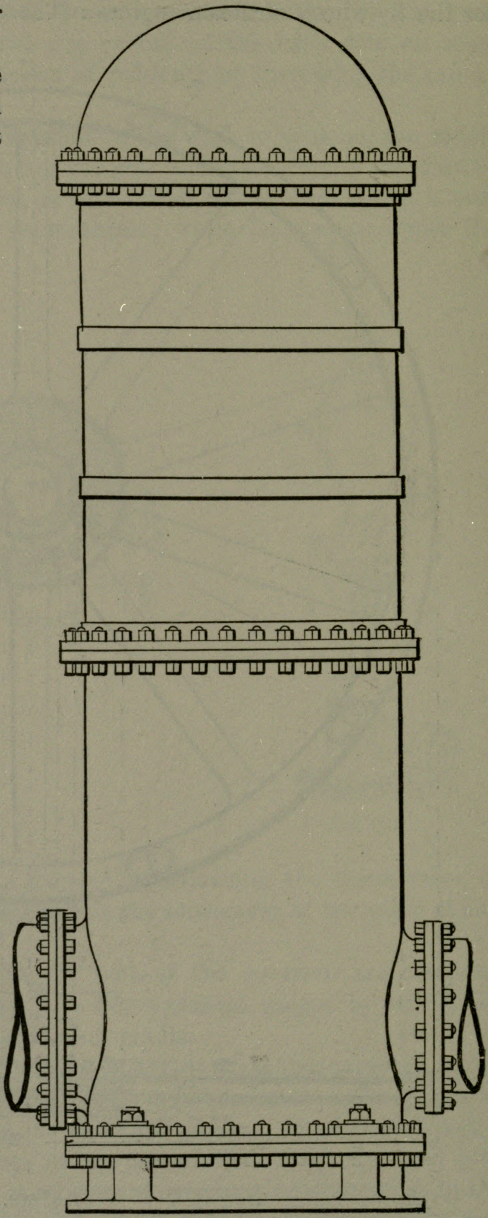


Fig. 812.

often the fly-wheel is independent. Fig. 808 is the elevation and section of such a wheel, as built by the Southwark Foundry. The construction will be understood from the drawings, but the wrought-iron links connecting the segments, shown on a larger scale (Fig. 809), do not project, but are counter-sunk in the sides of the rim.

Air-Chambers.—The action of the air-chamber is very similar to that of a fly-wheel; it tends to make the outflowing or inflowing pressure of the fluid uniform, and cushions or prevents the reaction that takes place from the fluid in reciprocating pumps, especially crank-pumps; but pumps in which the pistons or plungers start very slowly and stop equally so, require but little air-chamber. Cornish engines are usually provided with a stand-pump instead of an air-chamber—that is, a vertical pipe of considerably larger diameter than that of the pump, and high enough to contain the water-column.

Fig. 810 is the section of a copper air-chamber for the smaller size of steam-pumps or hand-pumps. It is screwed into the top of the pump-chamber.

Fig. 811 is the elevation of an air-chamber for power pumps of larger size. It may be entirely of cast-iron, or a cast-iron base with a copper chamber. A flange is cast on the top of the pump-chest, and the chamber is bolted to it.

Fig. 812 is the elevation of an air-chamber of one of the Brooklyn pumping-engines.

It will be observed that the lower end of the small air-chamber is necked, or of smaller diameter than the main part of the chamber. This prevents a too sensitive reaction of the air and prevents its escape. In chambers like that of the Brooklyn engine it is good practice, for the same purpose, to put a diaphragm across the inside of the chamber, perforated with holes. When the inlet column is long, whether suction or under pressure, it is well to put an air-chamber on it.

Air-chambers should be from 10 to 15 times the capacity of the pump-cylinder, with glass gauges to show the quantity of air in them for large pumps, and some provision to supply and maintain the air at such levels as will be found by experiment suited to the easiest working of the pump.

ENGINEERING DRAWING.

There is no part of engineering more important than that of securing a good foundation for the structure. Where likely to be disturbed by frost, the structure should start below it, unless, as in the extreme northern regions where frost is permanent at certain depths, the support should be in it. In preparing the foundation for any structure, there are two sources of failure which must be carefully guarded against : viz., inequality of settlement, and lateral escape of the supporting material ; and, if these radical defects can be guarded against, there is scarcely any situation in which a good foundation may not be obtained. It is therefore important that, previous to the commencement of the work, soundings should be taken to ascertain the nature of the soil and the lay of the strata, to determine the kind of foundation ; and, the more important and weighty the superstructure, the more careful and deeper the examination. But it must be understood that in general it is not an *unyielding* but a *uniformly yielding* stratum that is required, and that a moderate settlement is not objectionable, but an inequality of settlement.

In good sand or gravel, the common load per square foot is from three to five tons. Many soils are very compressible, not supporting one ton per square foot ; if the structure is important, the bearing resistance of the strata should be tested by experiment. The base of the wall is extended to secure the requi-

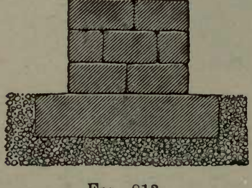


Fig. 813.

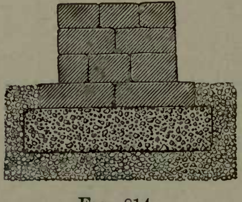


Fig. 814.

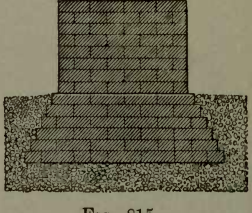


Fig. 815.

site area of bearing-surface, either by a base-stone (Fig. 813), by a bed of concrete (Fig. 814), or by extending the wall by steps (Fig. 815), with or without concrete base, or the weight may be distributed by inverted arches between walls and piers. The walls themselves should sustain from three to ten tons per square foot.

When the foundation is beneath water, the base may be made of plank, or a grillage of plank and timber (Fig. 816). But the character of the soil must be well understood. There are positions, as in the foundation of the Custom-

House and other public buildings at New Orleans, in which it would appear that there could be no practicable area of surface that would secure a permanent foundation for an extensive building. A pile foundation in such earth is more satisfactory, but all timber should be covered with water to prevent rot.

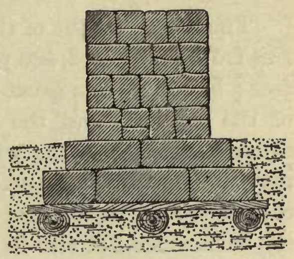


Fig. 816.

Figs. 817 and 818 represent plan and elevation of a pile foundation ; the piles are usually from 10″ to 14″ diameter, and driven at about 3 feet between centers. The tops are cut off square, and capped with timber ; the caps treenailed or rag-bolted to the piles, and plank spiked to the timber. In the figure a sheet-piling, *s s*, is shown, inclosing the piles ; the spaces between piles and timbers are often filled with concrete, small stone, or closely packed earth.

Piles are used either as posts or columns driven through soft earth to a hard bottom, or depending on their exterior frictional surface to give the necessary support, either in earth naturally compact or made so by the driving of the piles. In the first case, care must be taken that the piles be driven sufficiently deep into the lower strata to secure their ends from slipping laterally, and soundings should be made carefully to ascertain the dip and character of these strata. In many places, from the hardness and the inclined position of the lower strata, this kind of foundation is inapplicable and unsafe.

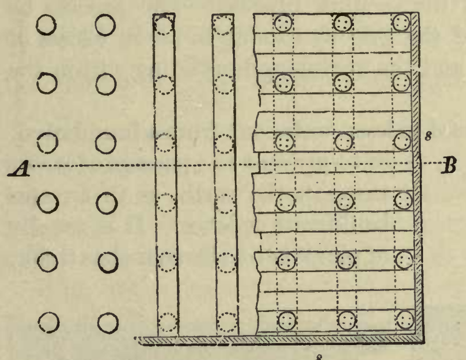


Fig. 817.

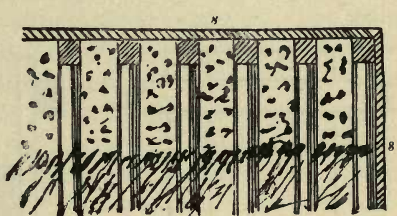


Fig. 818.

Where a firm foundation is required to be formed in a situation where no firm bottom can be found within an available depth, piles are driven, to consolidate the mass, a few feet apart over the whole area of the foundation, which is surrounded by a row of sheet-piling to prevent the escape of the soil ; the space between the pile-heads is then filled to the depth of several feet with stones or concrete, and the whole is covered with a timber platform on which to commence the solid work.

In the case in which the support from the piles depends on the exterior frictional resistance, the rule most generally adopted by engineers is that of Major Saunders, published in the "Journal of the Franklin Institute" for 1851:

Multiply the weight of the ram by the distance which the ram falls, in inches, at last blow, divided by 8 times the depth driven or set at that blow.

Thus, suppose the ram to be 1,600 pounds weight, the fall 20 feet, or 240″, and the set 1 inch, then the safe load would be $\frac{240 \times 1,600}{1 \times 8} = 48,000$ pounds.

The usual weight of the ram or hammer employed on our public works varies from 1,400 to 2,400 pounds, and the height of leaders or fall from 20 to 35 feet; but there is a great advantage in reducing the fall, increasing the weight of the hammer, and the frequency of the blows. As generally driven, and of average size, when the whole weight is to be supported by the pile, ten tons may be considered a usual load, but when additional support is received from compacted earth, broken stone, or concrete between piles and caps, this bearing-surface should also be taken into consideration. In some loose, sandy soils piles are set, not by driving, but by the water-jet; a 1″ or 1½″ pipe is lashed to the whole length of the pipe, and a force of water through this pipe clears out a hole for the settlement of the pile. When in position, the pile is held, the pipe is withdrawn, and the sand settles around the pile.

Iron pipes with a cast-iron foot are sunk also by a water-jet, the water being forced into the pile and out beneath the foot.

Hollow cast-iron piles have been driven by exhausting the air from the inside; then the weight of the pile, and sometimes an added load, cause the pile to settle into the earth; this is called the vacuum process. The process by plenum is by expelling the water out of the pile by forcing in air in excess of the pressure of the surrounding water, and the workmen descending within the pile and excavating the material.

Sheet-piling (Figs. 819 and 820) is used to keep water out from a foundation, or to prevent the passage of water through the earth, as in an embankment or levee. It is usually of plank two to three inches thick, set or driven. For driving, the bottom of the plank should be sharpened to a chisel-edge, a little out of center toward the timber side, and cornered slightly at the outer edge, that it may hug the timber and the plank while being driven.

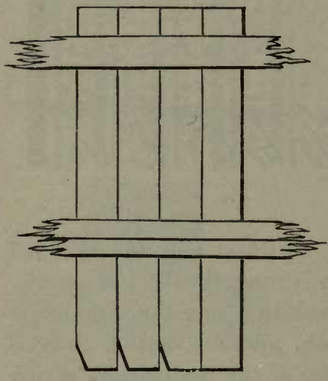


Fig. 819.

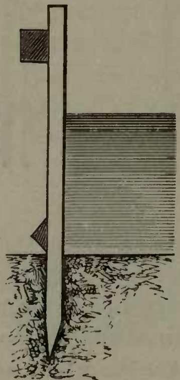


Fig. 820.

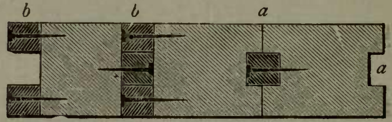


Fig. 821.

Fig. 821 is the section of a timber sheet-piling, in which a tongue and groove forms the guide, the grooves being either made in the timber, as shown at *a a*, or planted on, *b b*. The pile should be of uniform thickness, but the widths may be random; six inches thick is a good practical thickness, driving well under short and frequent blows; the tongue should be of hard, straight-grained wood, 2 inches by 2 inches, and well spiked to the pile.

Frequently, to secure the foundation from water, a wall is constructed of two rows of sheet-piling, driven one within the other, and the space between the two filled with clay or some compact earth. This is called a *coffer-dam;* the two pilings are stayed to each other by bolts, and if the wall is wide enough no other stays or braces will be necessary.

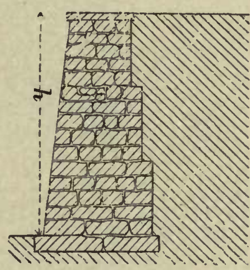


Fig. 822.

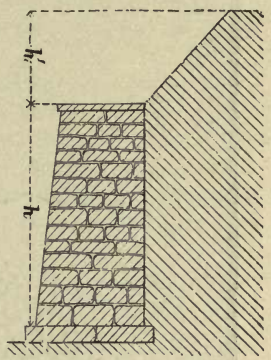


Fig. 823.

Retaining-walls are such as sustain a lateral pressure from an embankment or head of water (Figs. 822 and 823). The width of a retaining-wall depends upon the height of the embankment which it may have to sustain, the kind of earth of which it is composed (the steeper the natural slope at which the earth would stand, the less the thrust against the wall), and the comparative weight of the earth and of the masonry. The formula given by Morin for ordinary earths and masonry is $b = 0{\cdot}285\,h + h'$; that is, to find the breadth of a wall laid in mortar, multiply the whole height of the embankment above the footing by $\frac{285}{1000}$; for dry walls make the thickness one fourth more.

Most retaining-walls have an inclination or *batter* to the face, sometimes also the same in the back, but *offsets* (Fig. 822) are more common. The usual batter is from one to three inches horizontal for each foot vertical. To determine the thickness of a wall having a batter, "determine the width by the rule above, and make this width at one ninth of the height above the base."

Fig. 824 is one section of the bulk-head wall, as constructed by the Department of Docks of the City of New York, on the North River side, and in positions where the mud is deep.

The site of the wall is first dredged to hard mud compacted with sand. The vertical piles are then driven, and small cobble-stones mixed with coarse gravel put around and among the piles to the height of the under side of the binding frames, and rip-rap stone placed outside the piles, in front and rear.

The binding frames are then slid down to their places. These binding frames were made of two pieces of spruce plank 5 × 10 inches, placed edgewise one over the other, and running from front to rear of the piles between the rows. An oak beam 8 × 8 inches is let through these planks in front of the front row and in rear of the rear row of piles, and an oak wedge block fitted and placed by the divers between the oak beam and each pile nearest it. The duty of these frames is to hold the front rows of piles firmly, in case there should be any tendency in them to tilt outward.

More cobble-stone is then put in to the height of the bottom of the base blocks of the wall, weighting the binding frames and preventing any tendency to floating.

The bracing piles are then driven on a slope of six inches horizontal to twelve inches vertical, between the rows of vertical piles, and spaced three feet from center to center longitudinally and transversely. All the piles are staylathed and adjusted in position as soon as they are driven.

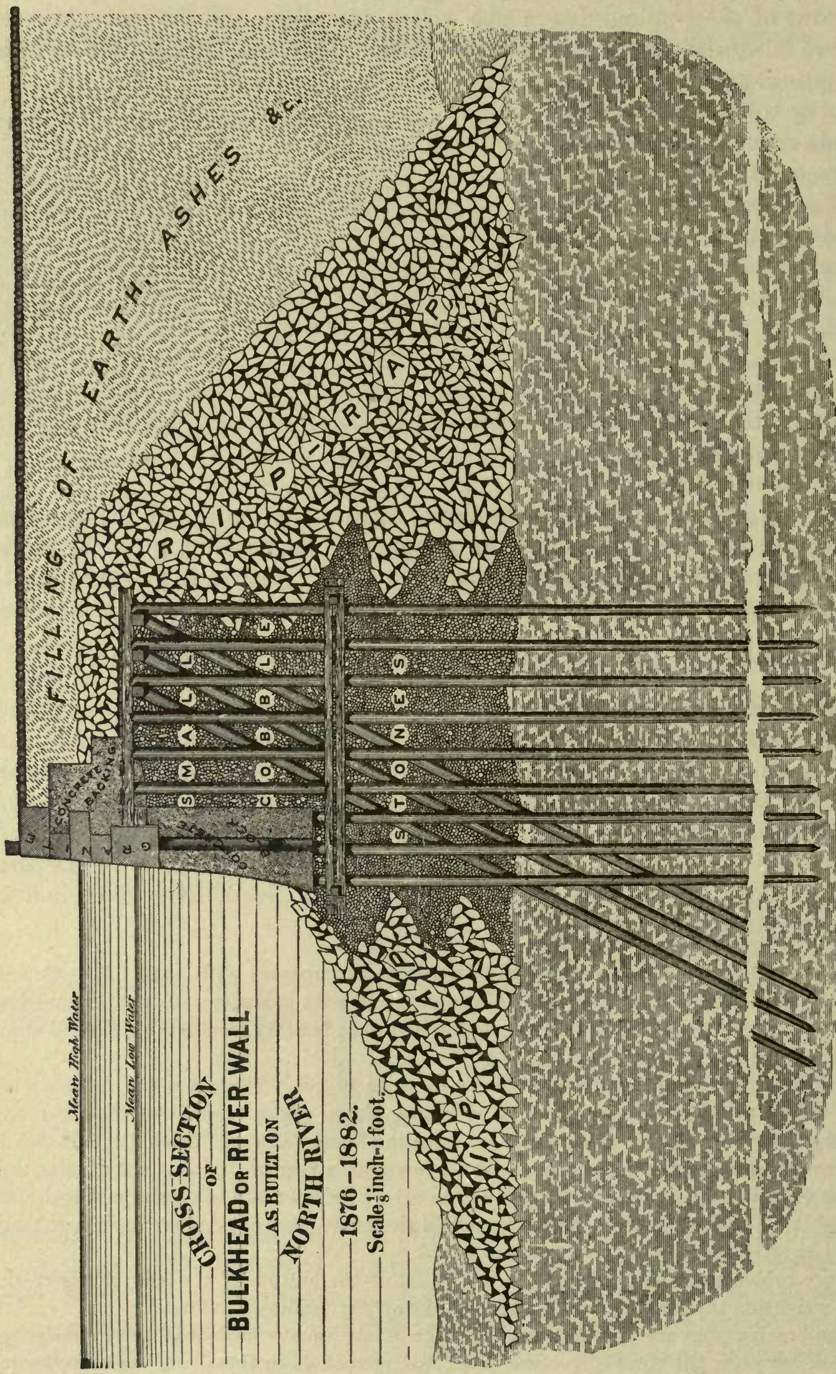

FILLING OF EARTH, ASHES &c.
RIPRAP
CONCRETE BACKING
SMALL COBBLE STONES
RIPRAP
Mean High Water
Mean Low Water
CROSS SECTION OF BULKHEAD OR RIVER WALL AS BUILT ON NORTH RIVER
1876-1882.
Scale 1/8 inch-1 foot.

Fig. 824.

The bracing piles are cut off at right angles to their axis, about one foot below mean low water, and capped with twelve inch square timber, running longitudinally. The sides of the caps are kept horizontal and vertical, and a sloping recess or notch made to receive the head of each bracing pile, and give it a good bearing.

The six rear rows of vertical piles are cut off at two inches above mean low water, and notched front and rear to give an eight inch wide bearing across their tops for the transverse caps.

The three front rows of vertical piles are cut off by a circular saw, suspended in the ways of a pile-driver, at 15·3 feet below mean low water mark, to receive the concrete base-blocks of the wall. It being impossible to cut off piles at this distance below the surface of the water to exactly the same height, and as the bottom of the concrete base-blocks would rest only upon the highest piles of those under them, a mattress of burlap, containing freshly mixed soft mortar, in a layer about two inches thick, placed on a network

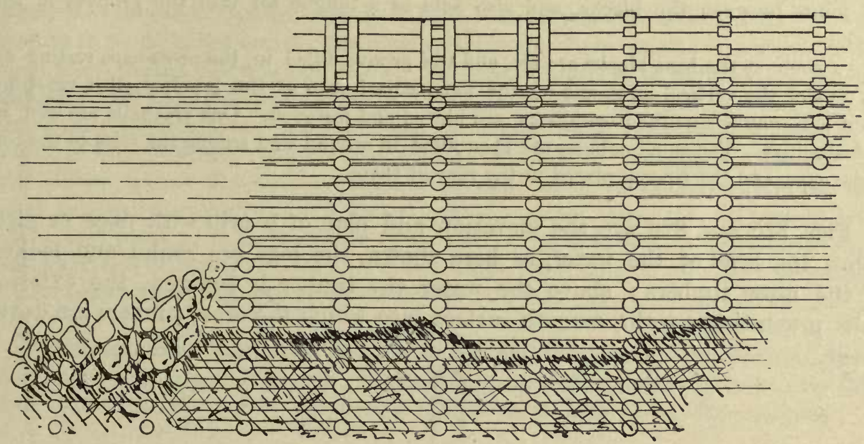


FIG. 825.

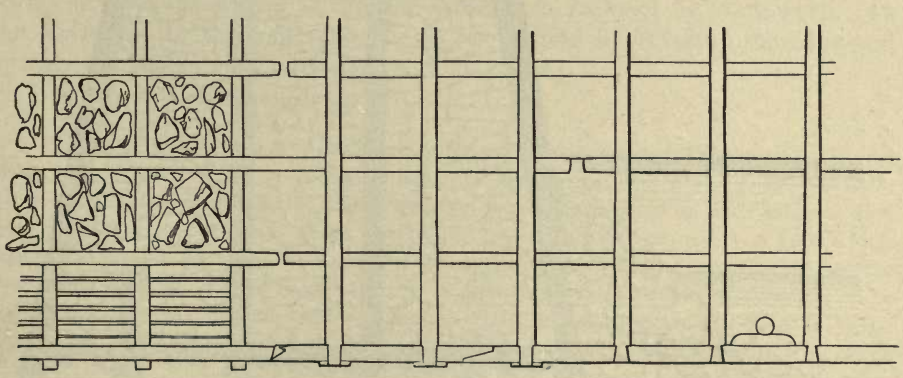


FIG. 826.

of marline stuff, supported by a plank frame about its edges, is lowered upon the tops of these piles immediately before setting the base-blocks upon them. The diver then cuts the netting between the edge of the mattress and the plank frame, and the frame floats to the surface of the water.

The base-block is then immediately placed in position upon the mattress of mortar resting on the piles, and the excess of mortar is pressed out from between the head of the pile and the bottom of the base-block, until each pile has a well and evenly distributed portion of the load to carry.

The concrete base-blocks for this section are 7 feet wide at the bottom and 5 feet wide at the top; on the front the vertical height is 13 feet, and on the rear 14 feet. The top has a step on the rear of 1 foot height and 1½ foot wide, extending the entire length of the block, for the purpose of giving the mass concrete backing of the granite superstructure a good hold upon the block. For handling, grooves for chains are molded in the end, and a longitudinal hole, 2 feet in the clear above the bottom, connects them, with the corners rounded, to enable the chain to render easily. The face is curved inward, to save material while giving a broad base; their length is 12 feet.

After the blocks are set, the vertical chain-grooves in each block, coming opposite to each other, are filled in with concrete in bags, well rammed into place. This closes the joints between the blocks, and also acts as a tongue set into the grooves in the blocks.

As soon as the base-blocks are set, and the groove filled in, the cross-caps resting on the tops of the vertical piles, and on the longitudical caps of the bracing piles, reaching about half way across the base-blocks, are placed and fastened. Oak treenails are used in all fastenings. The small cobbles are then filled in around and among the piles to the top of the caps, and the rip-rap placed in the rear of them.

Figs. 825 and 826 are the elevation and plan of a crib with dock or pier. Below the level of the water, as here shown, the logs are round and locked to the cross timbers; above the water the timber is squared, the exterior walls presenting a tight, smooth surface into which the cross timbers are dovetailed.

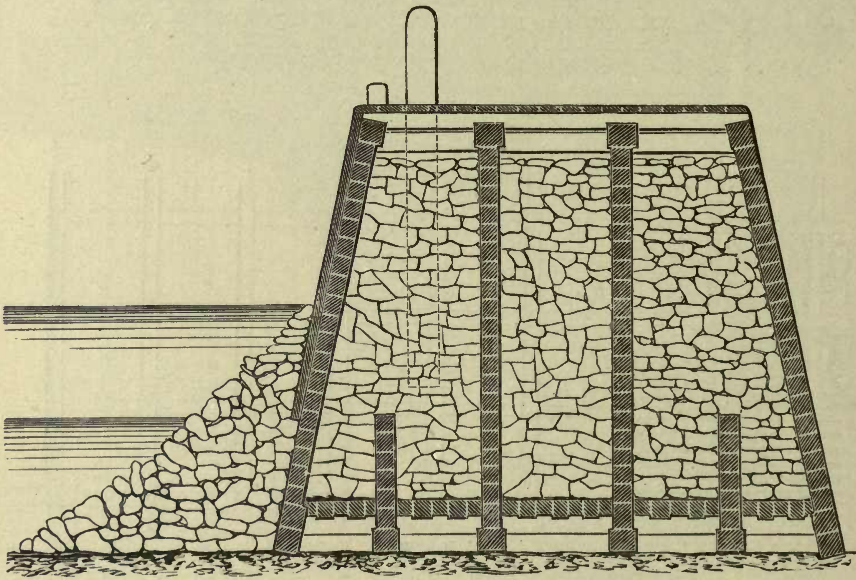


Fig. 827.

Fig. 827 is a section of the outer wall of the crib-pier erected on the West Bank for the Quarantine Department of the Port of New York by Mr. J. W.

Ritch. The structure consists of an outer wall of crib-work, with an interior filling of sand, 228 feet wide by 488 feet long. The interties occur at intervals of 6 feet spaces, or 7 feet centers.

Extracts from specifications :

"The exterior wall to be built in blocks up to low water, of about 80 feet in length, sunk to a line, and to be filled up to low water with stone-filling. From the low water the construction of the exterior wall will be continuous, breaking the joints of the logs throughout the entire length. The base of the blocks will be formed with timbers 14 inches square; two rows on the outside, held together with interties of timber 12 inches square, each end dovetailed into the outside, and shiplapped to the other timbers, secured at each end and intersection with iron bolts, 1 inch square, 14 inches long, well driven home.

"The cribs of the entire exterior wall to be built with sound timber 12 inches square, laid so that they touch each other, secured at every crossing or intersection, and in the center between each crossing, with iron bolts, 1 inch square, 20 inches long. The cross timbers to be all in one length; the ranging timbers to be in lengths of not less than 46 feet; joints broken over the logs below. The cross-timbers to be dovetailed at the ends, and shiplapped at intersections. The under tier of timbers to be secured to the logs below, the ranging timbers to the under tier, and the upper tier to the ranging timbers, as follows: at each end and every crossing with an iron bolt, 1 inch square, 21 inches long, well driven home. The entire exterior to be close fendered. extending from the deck-plank to low water, with sawn white-oak plank, 5 inches thick, and not over 12 inches wide; each plank to be secured with 7 iron bolts, 3 inches square, 15 inches long. The 6 corners of this fendering to have each 3 iron bands, 5 feet long on each limb, 3½ inches by 1 inch counter-sunk holes to receive 5 iron bolts. ¾ inch square, 15 inches long, in each limb.

"Each crib to be filled, from the floor-logs to within 6 inches of the deck-plank, with stone, granite, gneiss, or trap-rock; none of the stone to be more than 2 feet in any direction. The entire exterior to be protected with stone, in large pieces, done in riprap."

Fig. 828 is a transverse section of the river-wall Thames embankment, Middlesex side. It may be said to be a wall of concrete, etc., faced with granite, with a sewer and subway within the same, both inclosed by brick-work. In the drawings the different material is represented by different shadings and letters: *g*, granite; *b b*, brickwork; *c c*, concrete.

Extracts from specifications :

"The embankment-wall is to be formed within iron caissons or coffer-dams, as the engineer may direct. As soon as the excavations shall have been made to the requisite depths, and the works cleared of water, the trenches shall be filled up with concrete to a level of 12½ feet below datum, and a bed dressed to the proper slope and level for the footings of the brick wall. This wall shall be formed thereon (when the concrete has become thoroughly hard and consolidated) at a true slope in sets-off, as shown on drawing. The brick-work generally shall be laid in courses at right angles to the face of the wall. The low level sewer is to be formed on concrete foundation carried down as shown. The sewer shall be 7 feet 9 inches in the clear diameter for a length of 1,820 feet, and 8 feet 3 inches in diameter for the remainder of its length, the whole to be formed in brick-work 1 foot 1½ inch thick. The subway shall be formed 7 feet 6 inches high by 9 feet wide in the clear, generally; the side-walls to be 18 inches, the arch 1 foot 1½ inch thick. The subway sewer and river-wall shall be tied into each other, at intervals of 6 feet, by cross or counterfort walls 18 inches thick, extending from the brickwork of the wall to a vertical line 9 inches beyond the side of the sewer farthest from the said wall, and from footings 9 feet below datum, which are to be bedded on a concrete foundation 12 inches thick, up to the under side of the subway. The upper arch of the subway, and all other similar arches, shall be coated on their outside circumference with a layer of Claridge's patent Seyssel asphalt, 1 inch thick, laid on hot, and returned up all spandrel walls rising above the arch to a height of 9 inches. The river-wall shall be faced with granite, generally to a level of 8 feet below datum, and shall be surmounted with a molded parapet of solid granite; the stones to be laid in courses, in alternate headers and stretchers.

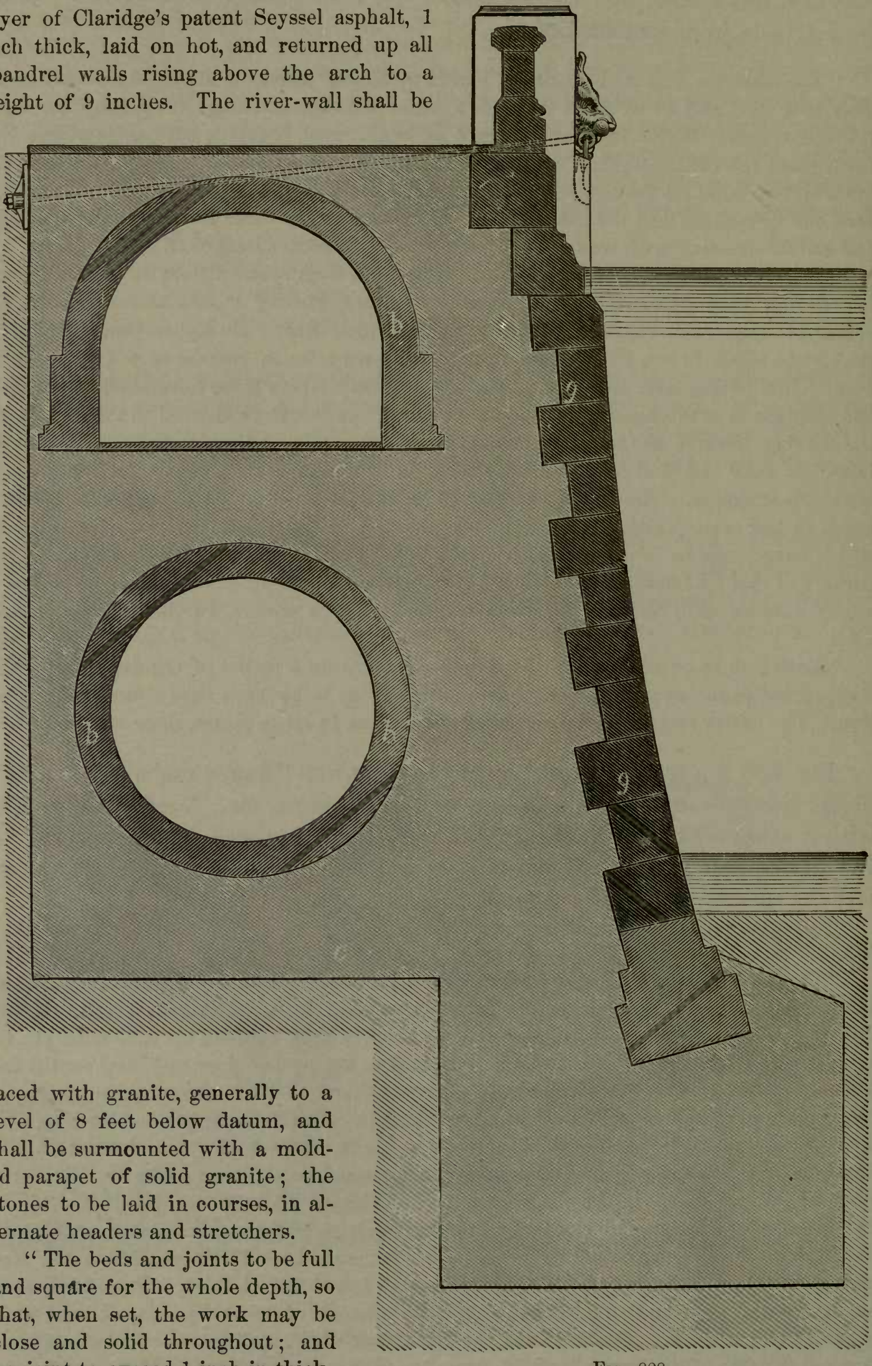


FIG. 828.

" The beds and joints to be full and squáre for the whole depth, so that, when set, the work may be close and solid throughout; and no joint to exceed ⅛ inch in thickness. The whole of the stones above the given level (11½ feet above datum) to be doweled together in bed and joints with slate-dowels, not less than 5 for every foot run of wall; each 2½ inches square at least, let fully 2½ inches into each stone, very accurately fitted, and run in with neat cement; the stones to be bedded and jointed in cement, and the joints struck with neat cement.

"The whole iron-work to be delivered on the works perfectly free from paint or other coatings."

Fig. 829 is an isometrical view of the overflow and outlet of the Victoria and Regent Street sewers in the Thames embankment. S is the main sewer, and W the subway shown in Fig. 828; *s s s* the street-sewers, discharging into the overflow basin O; *w w* the weirs over which the water is discharged into the weir-chamber *c c*; *p* is the penstock-chamber, which is but a continuation of the weir-chamber. It has been attempted in the drawing, by breaks, to explain, as far as possible, the whole construction. Whenever, from storms, the discharge from the street-sewers (*s s s*) is greater than can be carried off by the main sewer (S), the water rises in the overflow-chamber (O), passes over the weirs (*w w*) down into the weir-chamber (*c*), then into the penstock-chamber, and through the flap-gates (*g*) into the river.

Extracts from the specifications:

"The foundation to be of concrete, not less than 2 feet in thickness; upon this brick-work shall be built for the flooring of the chambers, and for the side-end and weir-walls. The weir-chamber shall be divided in the direction of its length, by a brick wall, into two rectangular overflow-channels, covered with cast-iron plates, 6 feet 8½ inches long, 3 feet wide by ⅞ inch general thickness, with strong ribs and flanges on the under side, properly bolted together and jointed with iron cement, and bolted down to stones which are to be built into the under side of the brick-work of the basement-chamber. Arches on either side, running parallel thereto, and communicating with this chamber and with the weirs which are to be formed, upon which weir-walls, divided so as to correspond with these arches, are to be built in brick-work, capped with granite blocks, 4 feet long, 2 feet deep, and 2 feet 3 inches in the bed. The floor of the penstock-chamber to be formed with York landings, 6 inches thick, having a fall of 3 inches to the river. The outlets for the penstock-chamber through the river-wall shall be formed by an arch-recess in granite. and fixed with two tidal flaps, well hung, and firmly secured to the masonry by strong bolts and screws.

"The subway is to be continued over the low-level sewer, and across the overflow chamber, by cast-iron plates, curved to the form of the arch, ⅞ inch general thickness, with strong ribs and flanges on the upper side, properly bolted together, and strongly bolted down to the brick-work; jointed with iron cement, and covered with brick-work, to form the floor of the subway. From a point of 10 feet 3 inches on either side of the central longitudinal line of the chamber, where the sewer and subway are farthest from the river-wall, these are again to be brought into their general position by two curves, each not less than 80 feet in length.

"The whole of the cast-iron shall receive one coat priming of red lead and linseed oil, and three coats best coal-tar, before fixing; and the accessible surfaces one further coat best coal-tar, when fixed."

Foundations for piers and abutments of bridges beneath the surface of water are formed by piles, by throwing down masses of stone or beton until the mass reaches the surface of the water, by open caisson or by inclosing the space within a coffer-dam, and proceeding as in common foundations, or by an inverted caisson and air-lock.

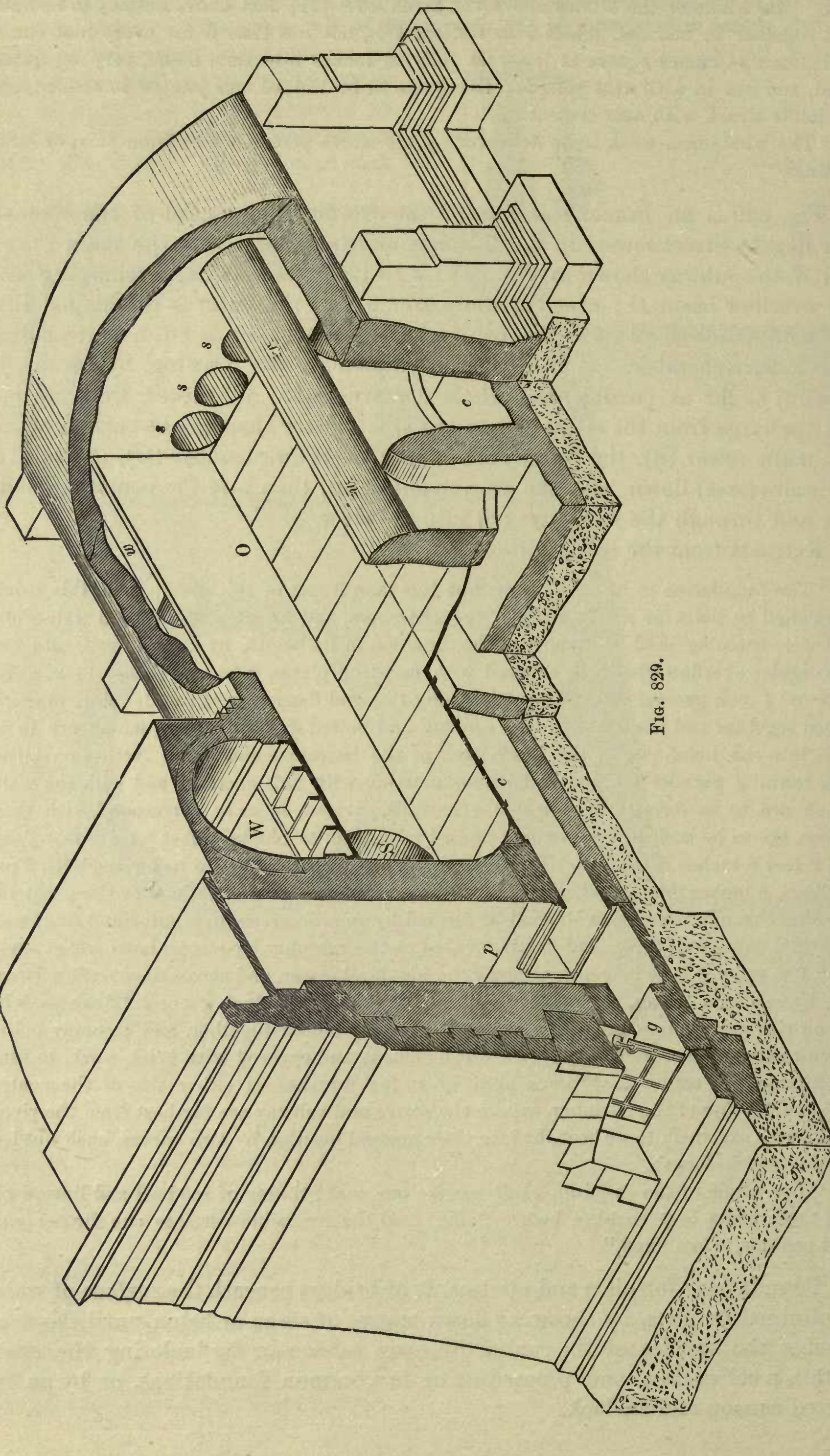


FIG. 829.

An open caisson is a chest of timber which is floated over the site of the work, and, being kept in its place, is loaded with stone until it rests firmly on the ground. In some cases the stone is merely thrown in, the regular masonry commencing with the top of the caisson, which is sunk a little below the level of low water, so that the whole wood-work may be always covered, and the caisson remains as part of the structure. In others the masonry is built on the bottom of the caisson, and when the work reaches the level of the water the sides of the caisson are removed.

The general plan adopted by G. A. Parker, C. E., in the erection of the piers of the Susquehanna bridge, was :

First to dredge away as much as possible of the material in the bed of the river at the pier site. A ⅜-inch thick boiler-iron curb was then sunk and secured in its place. The curb was about 30 feet wide and 50 to 60 feet long, and of sufficient height to reach above the bed of the river. The material was then pumped by sand-pumps out of the curb, which gradually undermined, and settled down to the required depth, or on to the bed-rock. When stumps, logs, or bowlders were met with, they were removed by divers working in a bell. After the rock had been thoroughly cleaned off, it was brought to a uniform level by a solid bed of concrete extending over a greater space than the size of the bottom of the pier, using the diving-bell for this purpose.

Three guide-piles on each side, and one at each end, were fixed firmly in position. A strong platform of solid timber, the size of the bottom of the pier, was then placed in position over the curb, and at the surface of the water. On this was placed a caisson of iron large enough to contain the pier, and with sides and ends high enough to reach to the level of high water after the caisson is landed on the bottom. The caisson was then made water-tight. The bottom was then floored over with masonry and stone, and laid in mortar up the sides of the caisson to the top, thus constituting a stone caisson inside of an iron one. This was secured to the guide piles, and the masonry of the pier proper was laid up, the caisson sinking as the weight of masonry inside increased, until it finally settled upon the bottom which had been prepared for it, as already described. At some of the piers

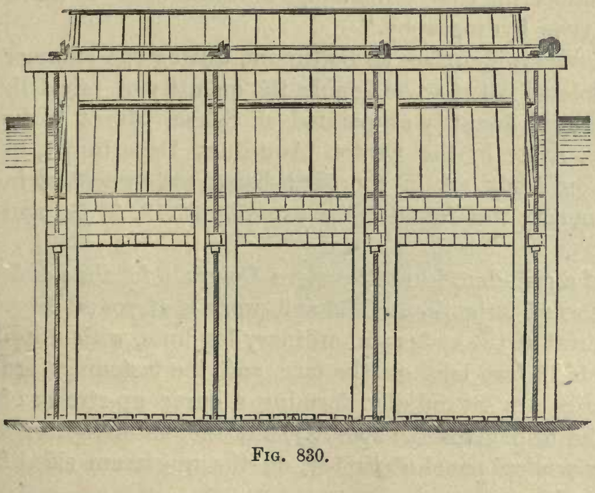


FIG. 830.

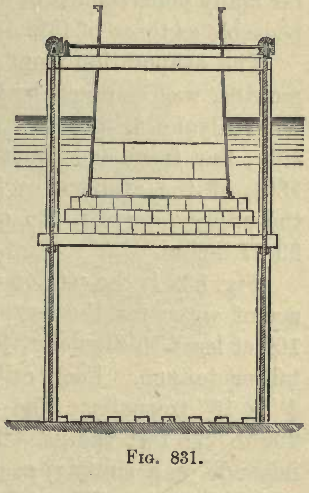


FIG. 831.

(Figs. 830 and 831) screw-rods were used to suspend the pier and gearing attached, governed by one man, who at pleasure could raise or lower without assistance the whole pier. Wooden piles were driven and cut off by machinery just above the ground, and the platform, with its incumbent pier, lowered upon them; at other piers the foundation was on rock.

Piers are sometimes made by sinking a wrought-iron curb, extending from the bottom to above the level of the water, driving within it the usual proportion of piles, and then filling the spaces entirely with concrete.

Dams are constructed to pond water for the supply of cities and towns; for inland navigation, by deepening the water over shoals, and the feeding of canals; for power in its application to mills and workshops; and for irrigation. To whatever purpose the water is to be applied, there are two questions to be settled: Whether the level will be raised high enough by the construction, and whether the flow of the stream be sufficient for the purpose required; and further, it may often be important to know how large a pond will be thus formed, how ample a reservoir for unequal flow, or intermittent use. If the pond be small, so that the water can not be retained, and the supply is only the natural run of the stream at a high level, then the minimum flow of the stream is the measure of its capacity.

The rule that obtains on the Merrimack River, at Lowell, and Lawrence, where the pondage is more than the average, is that 1 cubic foot per second per day of 12 hours per square mile of water-shed can be depended on for permanent mill-power. On very small streams it may often happen that pondage may be secured, and the supply be equal to one half the rain-fall.

Blodgett, in his "Climatology of the United States," says that "in this sense of permanence as a physical fact, we may consider the quantity of rain for a year as a surface-stratum, on the Atlantic slope and in the central States of 3½ feet, which may be diminished to half this quantity, or increased to twice as great a depth in the extreme years. But, with such an average and such a known range, we may deal with the quantity as definitely as with a stream of which we know the mean volume and the extremes to which it is liable, and for many departments of engineering these climatological measures are as indispensable as those of tide or river hydrography."

The evaporation from a reservoir-surface at Baltimore, during the summer months, was assumed by Colonel Abert to be double the quantity of rain-fall. Dr. Holyoke assigns the annual quantity evaporated at Salem, Mass., to be 56″; but from experiments made by the Croton Aqueduct Department, in 1864, of the evaporation from a box set in the earth-bank, and two afloat in the upper reservoir, the quantity was found to be severally 37·12, 37·53, and 39·97 inches.

Fig. 832 is the section of a crib-dam in northeastern Colorado for the pondage of water for the purposes of irrigation. The crib-work is of round logs, 10″ at least in diameter, joined at the ends as in ordinary log huts, with dovetail or tongue. Each crib is 18 feet long on the face, and the fastenings are 2″ × 18″ treenails. The cribs are set radially, forming a curve up-stream of 200 to 238 feet radius. The crib gives the stability, but the water-tightness depends on a shutter, *p*, or vertical panel of timber, on the up-stream side of which there is a filling of earth.

Crib or wooden dams, when the timber is not kept covered with water, fail from the decay or rot of the timber.

Fig. 833 is a section of the dam across the Croton River, constructed under the direction of Mr. John B. Jervis, for the supply of the aqueduct for the city of New York. This dam was built on an earth foundation, with curved roll in cut stone, extended by a timber-apron some 50 feet, supported by strong

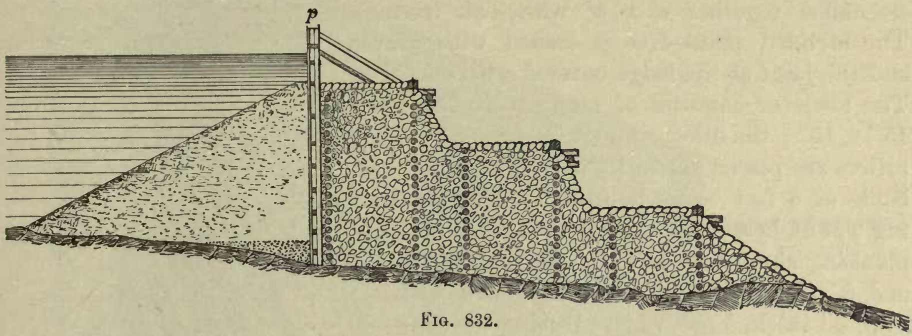


FIG. 832.

crib-work. Originally there was a secondary dam still farther down, to throw back-water on this apron. In the erection of this dam, excavation was made of all loose material ; the cribs C and D were built up, and the tops were planked ; on this planking were carried up the cribs F and G. Between these piers the space E, as well as *e* below and on the cribs, was filled in with concrete ; on this the body of the dam was erected in stone-masonry, laid in cement. The face-work of granite is cut to admit of a joint, not exceeding $\frac{3}{16}$ of an inch.

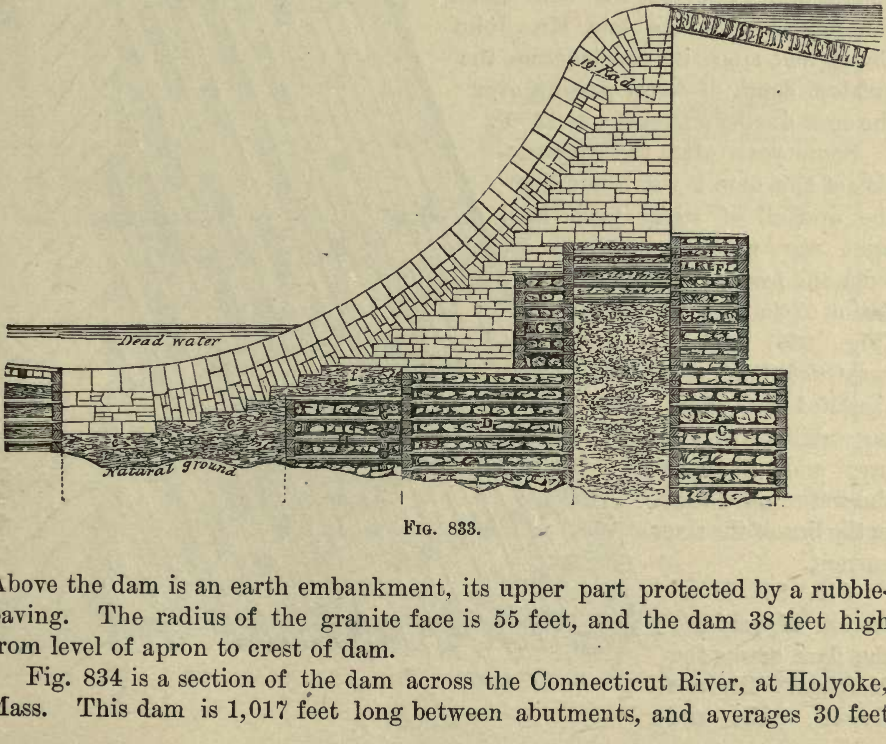


FIG. 833.

Above the dam is an earth embankment, its upper part protected by a rubble-paving. The radius of the granite face is 55 feet, and the dam 38 feet high from level of apron to crest of dam.

Fig. 834 is a section of the dam across the Connecticut River, at Holyoke, Mass. This dam is 1,017 feet long between abutments, and averages 30 feet high by a base of 80 feet. It is constructed of timber crib-work, loaded in with stone for about ⅓ its height. The foot of each rafter is bolted to the ledge, and all timbers at their intersections are treenailed together with 2″ white-oak treenails. The inclined plank-face is loaded with gravel, and the joint at the ledge covered with concrete. The lower or base-tier of ranging timbers were 15″ × 15″; the other timbers, 12″ × 12″. The rafters are placed vertically over each other, in bents of 6 feet between centers. The planking was of hemlock, 6″ thick, with oak cross-planking at crest of dam, 4″ thick at bottom and 8″ at top. The crest was plated with iron, ¼″ thick, 5 feet wide. During the construction the dam was planked first about 30 feet on the incline; a space was then left of about 16 feet width by sufficient length, through which the water flowed; and the balance of the dam was then completed. A plank-flap was then made for the opening, and when every thing was ready, it was shut down, and the pond filled. The dam was built under the direction of the late Mr. John Chase, and since its construction the greatest depth of water passing over the crest during a freshet was 12′ 6″.

Some years after the construction of this dam it was found that the overfall of water from its crest was wearing away the ledge and jeopardized the foundation of the dam. An apron (Fig. 835) was therefore constructed of crib-work, sheathed with plank, adding stability to the structure, and discharging the water more nearly in the line of the river current.

Fig. 836 is a section of part of the dam across the Merrimack River, at Lowell, built under

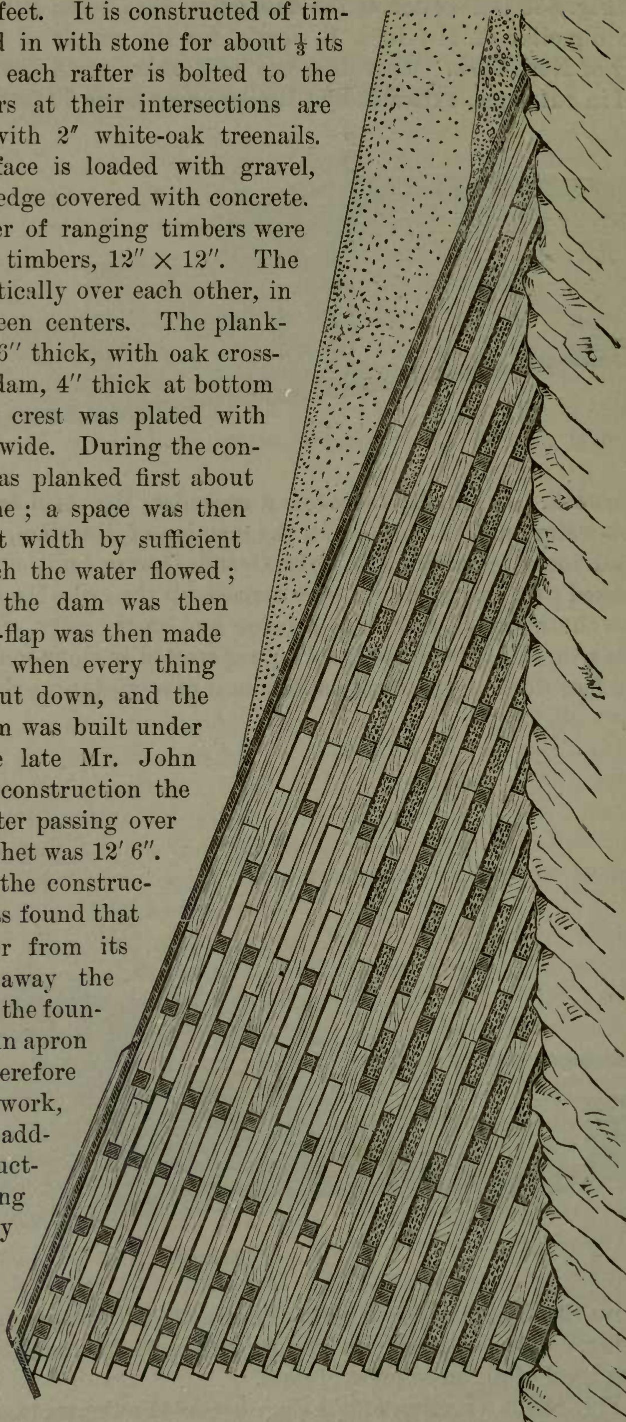


Fig. 834.

Scale: 1 inch = 12 feet.

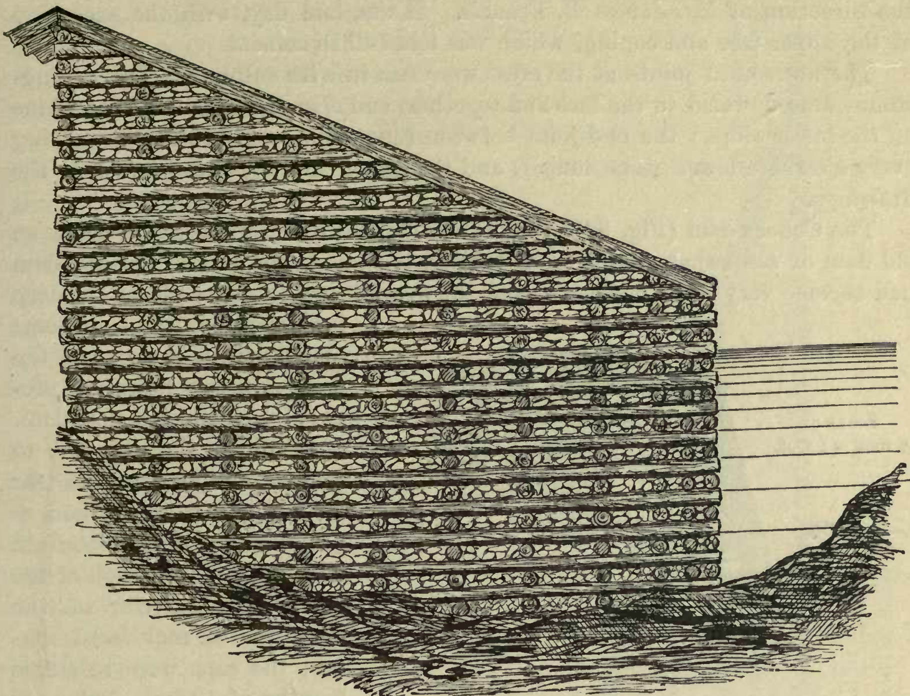


Fig. 835.

Scale: ¼ inch = 1 foot.

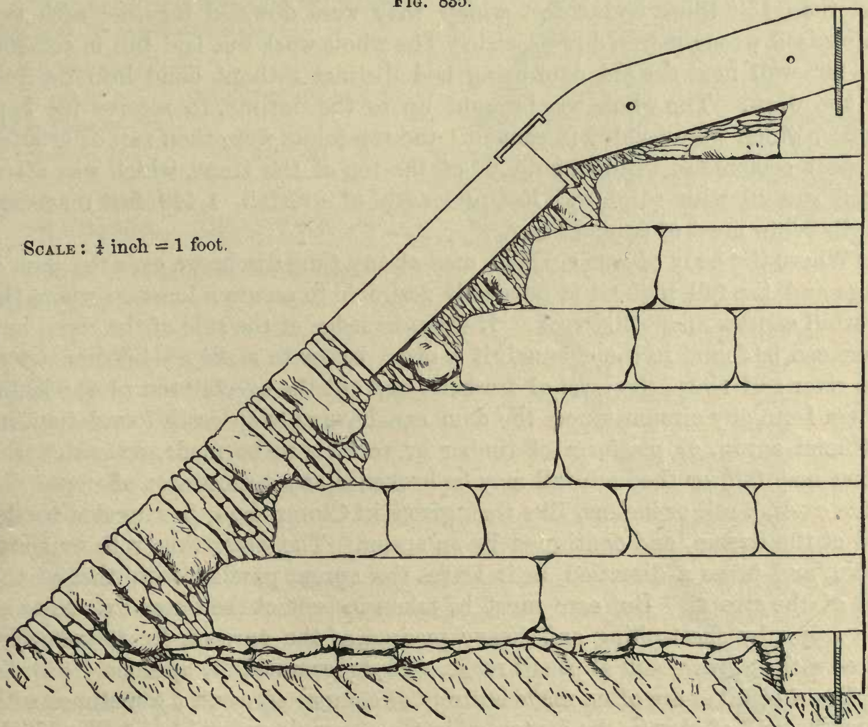


Fig. 836.

the direction of Mr. James B. Francis. It was laid dry, with the exception of the upper face and coping, which was laid full in cement.

The horizontal joints at the crest were run in with sulphur. The coping-stones were doweled to the face and together, and clamped to an inclined stone on the lower slope ; the end-joint between these stones was broken by making every alternate lower stone longer, and the upper shorter, than shown in the drawings.

The Cohoes dam (Fig. 837) was built under my direction, directly below an old dam of somewhat similar construction to that of Holyoke. The old dam had become very leaky and worn, and the overfall had in many places cut deep into the rock, and in some places within the line of the dam. It was therefore proposed to make the new dam, as a roll to the old one, to discharge the water as far from the foot of the dam as possible, and to keep the old dam for the protection of the new. The exterior of the dam was of rock-faced ashlar ; the caps were in single lengths of 10 feet, and none less than 15″ thick and 2 feet wide ; they were doweled together with two galvanized wrought-iron dowels each. The whole work was laid full in cement, the 20″ wall next the old dam being laid distinct without bond into the rest of the work. The whole was brought up to the outline, to receive the cap-stones, which were bedded in cement ; the top-joints were then run or grouted in neat cement, to within about 6″ of the top of the stone, which was afterward run in with sulphur. Entire length of overfall, 1,443 feet ; average depth below crest of dam, 12 feet.

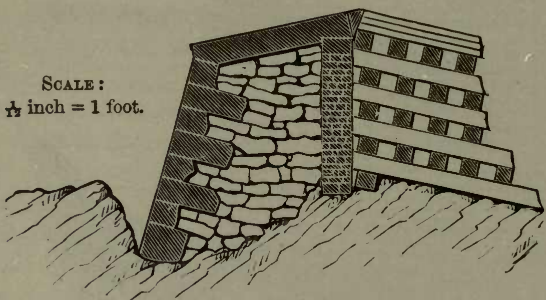


FIG. 837.

Where the body of water which may at any time discharge over the dam is large and the fall high, it is especially desirable to secure a location where the overfall can be upon solid rock. If there be ledge at the side of the river, and none can be found in the channel, it is often better to make a solid dike across the river and above the level of freshets, and cut the overfall out of the bank. When from any circumstances the dam can have only an earth foundation, an artificial apron, or platform of timber or rock, is to be made, on which the water may fall, or the high fall may be broken up by a succession of steps. In some cases, a roll or incline, like that given in Croton dam, is extended to the bed of the stream, and continued by an apron. The water thus rolls or slides down, and takes a direction, as it leaves the apron, parallel with that of the bed of the stream. But care must be taken to protect the outer extremity of the apron by sheet-piling and heavy paving, as the current, by its velocity, takes with it gravel and all small rocks, and undermines the apron.

Dams or dikes are often made entirely of compacted earth ; sometimes with a puddle-wall of clay in the center, as in the reservoir embankment (Fig. 860), or a sheet-piling. Dikes across salt marshes are made of material taken from the marsh at some distance from the site of the dike, well packed in thin layers on a base prepared on the soil without excavation. Sand and gravel, being heavier than the moist material, break through it and settle to the bottom, involving often the construction of a large embankment, while, by the use of a homogeneous material, the foundation is not displaced but compressed.

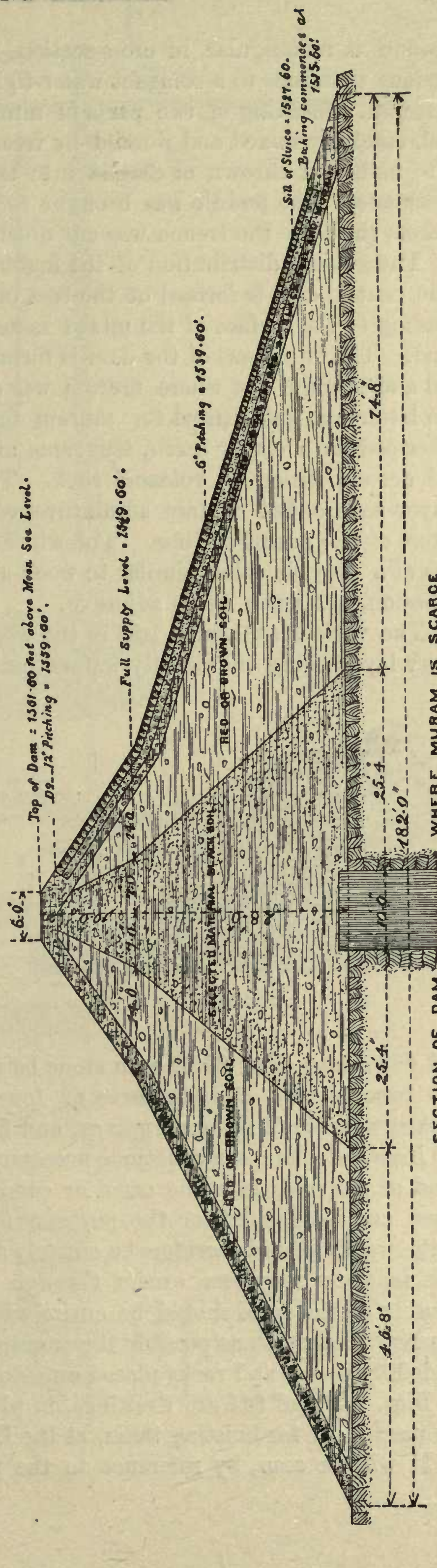
Fig. 838 is a section of the dike or embankment for the Ashti Tank or Reservoir, constructed for retaining water for irrigation purposes in India. The following is an abstract of the description of the work given in the "Minutes of the Proceedings of the Institute of Civil Engineers," vol. lxxvi :

"The net supply available for irrigation may be calculated thus:

Available capacity of tank........	1,348,192,450 cub. ft.
Deduct loss by evaporation, etc....	233,220,240 "
Net supply available for irrigation...	1,114,972,210 "

"Area of catchment basin nearly 92 square miles."

The total length of the dam is 12,709 feet; the breadth at the top, which is uniform throughout, six feet; breadth at full supply-level, 42 feet; height of the top of the dam above full supply-level, 12 feet; greatest height of dam, 58 feet. The seat of the dam throughout was cleared of vegetable mold, stones, and loose material, all trees and shrubs with their roots being completely grubbed or dug out. The puddle-trench laid in the natural



SECTION OF DAM WHERE MURAM IS SCARCE

Fig. 838.

ground is rectangular in cross-section, 10 feet in width, excavated through various materials to a compact water-tight bed, and then filled in with puddle material, consisting of two parts of muram or sand, and three parts of black soil, carefully mixed and worked by treading with the feet, and then kneaded into balls and thrown or dashed into the trench in layers up to 12 inches in thickness. The puddle was brought to a level of one foot above the ground. Across the river the trench was cut down to the rock and filled with concrete.

The general distribution of the material of the dam is shown in the figure. The central core is formed of the best black soil attainable; on each side, extending to the surface of the mixed material, brown, reddish, or white earth is used. The outer part of the dam is formed of a mixture of equal parts of black soil and muram, but where muram was difficult to obtain, and sand plentiful, the latter was substituted for muram in the mixture. The black soil may be described as a clayey earth, tenacious and adhesive when wet—a product of the decomposition of volcanic rock. The brown and reddish soils are of a clayey nature, but contain admixtures of fine sand, kunkun nodules, and thin layers of fine grains of lime. The white soil consists of finely powdered particles of a grayish color, similar to wood-ashes, which when dry possesses little adhesion, but when wet is adhesive.

The various soils were laid in the work in layers eight inches in thickness, every layer being thoroughly watered and rolled with iron rollers. The outer slope was protected by a mixture of equal parts of soil and reddish muram, and with sods of grass, laid about three feet apart, which in time extended over the whole slope.

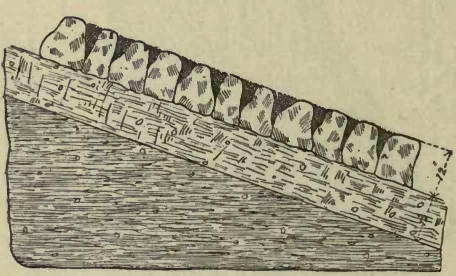


Fig. 839.

The inner slope is protected from the action of the waves by being pitched or faced with dry stone, set by hand, and laid on a layer of coarse muram. The stones of the pitching were bedded on the slope, and were laid with their broadest end downward (Fig. 839), each stone being roughly squared with the hammer, and touching for at least three or four inches. The interstices were then packed with small stone-chippings, and finished off with muram.

Head-gates are constructions necessary to control the flow from the river-pond or reservoir into the canal or conduit by which the water is to be conveyed and distributed for the purposes to which it is to be applied. The top of the works should therefore be entirely above the level of the highest freshets, that no water may pass, except through the gates; and it is better that the opening of the gates should be entirely below the level of the top of the dam, to prevent as much as possible the passage of drift and ice, which are often excluded by booms and racks placed outside the gates.

Figs. 840 and 841 are drawings, in plan and detail, of the head-gates, and the machinery for hoisting them, at the Cohoes Company's dam.

It will be seen, by reference to the plan, that there are ten gates. The

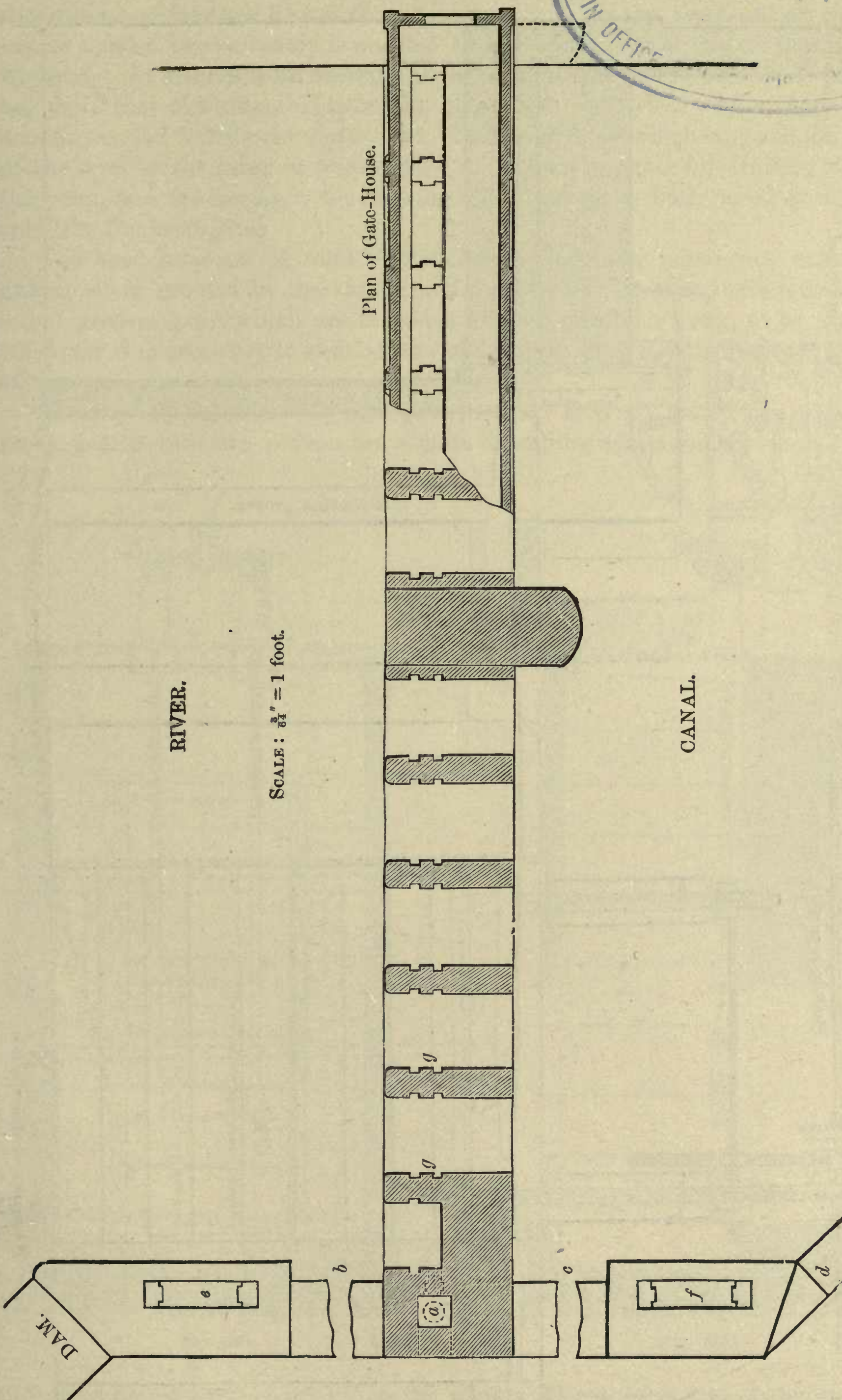


Fig. 840.

Erratum.—This plate is wrongly cut—should be reversed.

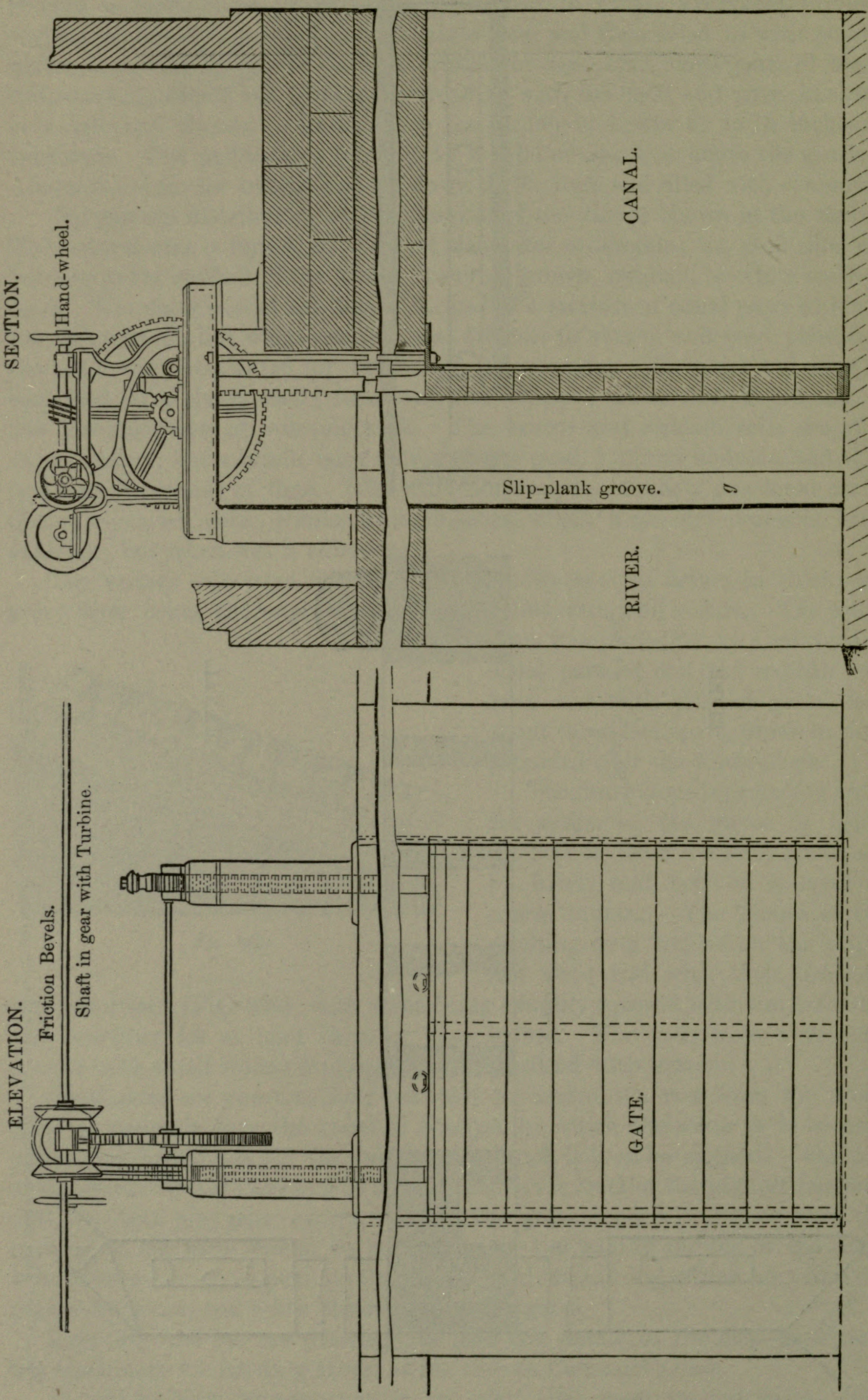


Fig. 841.

Scale: $\frac{1}{4}$ inch = 1 foot.

dimensions of four are 8′ × 6′ 6″; and six, 8′ × 9′, in the clear—all of which can be hoisted by machinery connected with a turbine-wheel at *a*, or separately by hand. At *b* there is an overfall, at the same height as the dam, over which any drift that is brought against the gate-house is carried. At *c* there is a similar overfall within the gates, and another at *d*, by which any sudden rise of the level of the canal is prevented. At *e* there is a gate for drawing down the pond, and another at *f*, for drawing off by the canal, both raised and lowered like the head-gates.

The head-gates are of solid timber bolted together, moving in cast-iron guides set in grooves in the stone; in front of these grooves there is another set of grooves (*g g*), which are intended for slip-planks or gates, to be put in whenever it is necessary to shut off the water from the gates themselves in case of repairs.

Hoisting Apparatus.—To each gate there are strongly bolted two cast-iron racks, geared into two pinions on a shaft extending across the gate-space, and

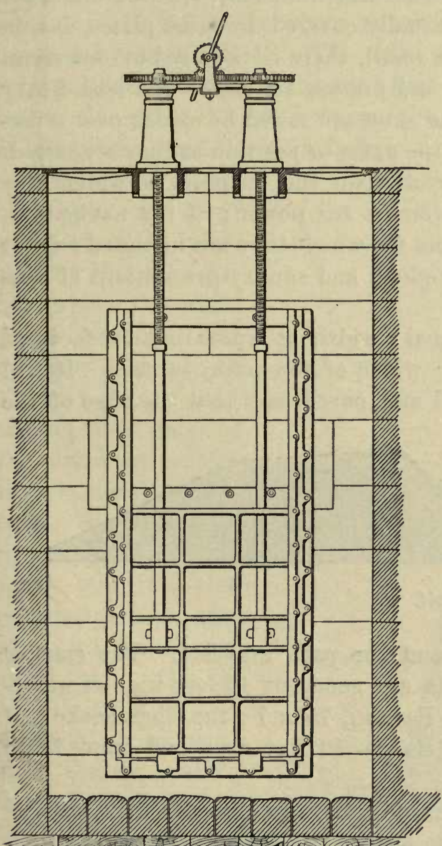


Fig. 842.

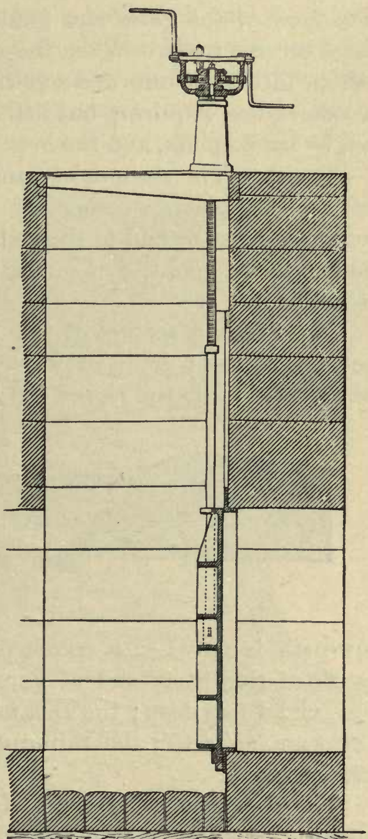


Fig. 843.

supported on cast-iron standards on the piers. At one extremity of this shaft, there is a worm-wheel, driven by a worm or screw on a shaft perpendicular to the pinion-shaft. The worm-shaft can be driven either by a hand-wheel at one end, or by the friction-bevel at the other. The friction-bevel can be driven in either direction by being brought in contact with one or other of the friction-bevels on a shaft extending the whole length of the gate-house, and in gear directly with the small turbine at *a*. The small turbine draws its supply through a pipe, built in the walls, and opening into the space between the gates and the slip-plank groove.

Figs. 842 and 843 are the front elevation and section of the gates of Farm Pond, Sudbury River Conduit, Boston Water-Works. The main web or plate of the gate is $1\frac{1}{4}''$ thick, the ribs $6''$ deep, the gate-stems $2\frac{1}{2}''$ diameter. The nuts by which the gates are raised are geared together, and actuated by a double crank. For smaller gates it is usual to have but a single stem, and the nut in a hand-wheel on top of the standard. The gates and guides are faced with brass, about $\frac{3}{16}''$ thick.

Gates of this form are very common, consisting of plates of cast-iron strengthened by ribs; the guides are also of cast-iron, bolted to the masonry. The faces of the gates and guides are usually covered by brass plates, as iron faces become rusty. When the gates are small, there is usually but one stem. Often, instead of nuts and screws, racks and pinions are used; and with heavy wooden gates, requiring but little use, the gates are raised by chains over a barrel, by hand-spikes, and ratchets to hold the gates in position as they are raised.

Canals.—The sections of canals depend upon the purposes to which they are to be applied, whether for navigation or for power; if for navigation, reference must be had to the class of boats for which they are intended; if for power, to the quantity of water to be supplied, and sundry precautions of construction.

Fig. 844 is a section of the Erie Canal: width at water-line, 70 feet; at bottom, 28 feet; depth of water, 7 feet; width of tow-path, 14 feet. It will be observed that the slopes are graveled and paved, and that the edge of the

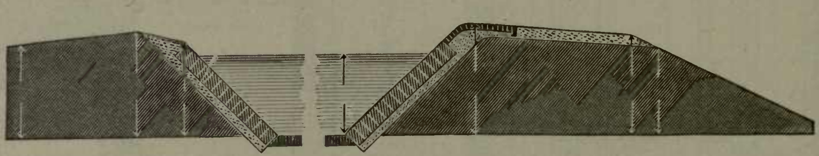


Fig. 844.

tow-path is paved with cobble-paving, and the path graveled. The smaller canals of this State and of Pennsylvania are generally 40 feet wide at water-line, and 4 feet deep; the Delaware and Raritan, $75' \times 7'$; the Chesapeake and Delaware, $66' \times 10'$; the ship-canals of Canada, 10 feet deep and from 70 to 190 feet wide.

The dimensions for canals for the supply of mills depend—first, on the quantity of water to be delivered. Their area of cross-section should be such that the average velocity of flow should not exceed two feet per second, and in northern climates less velocity than this would be still better; it should always be such that during the winter the canals may be frozen over, and remain so, to prevent the obstruction from drift and anchor-ice in the water-wheels. The usual depths of the larger canals are from 10 to 15 feet; with such depths the cover of ice which reduces the section by the amount of its thickness does not materially increase the velocity of flow, nor diminish, consequently, very perceptibly the available head.

Fig. 845 is a section of the Northern Canal, at Lowell, Mass., which may be considered a model for large works. The width at water-line is 103 feet,

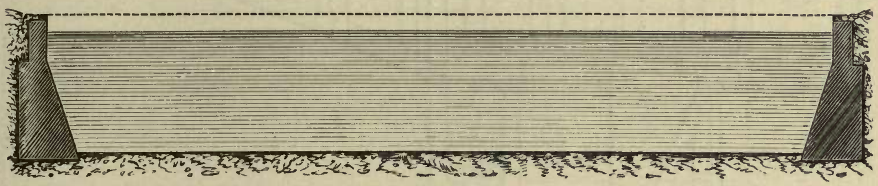


Fig. 845.

and the depth 16′, and is intended for an average flow of 2,700 cubic feet per second. The fall in the whole length of 4,300 feet is between 2″ and 3″; when covered by ice, about 4″. The sides are walled in dry rubble, and coped by split granite. It will be observed that the portion above, and about three feet below, the water-line, or between the limits of extreme fluctuations of level, is laid plumb, that the ice may have as free a movement as possible vertically.

Fig. 846 is a section, on a scale of $\frac{1}{8}''$ = 1 foot, of the river-wall of this same canal, where the canal passes out into and occupies a portion of the river-channel, and the depth of water in the canal is greater than in the above section. The main wall is in dry masonry, faced on river-side with rough-faced ashlar, pointed beds and end-joints. The inside lining is of two courses of cement-wall, the dry rubble backing being first laid, then pointed with cement, against which is laid the first cement lining, which is plastered on the inside, and the interior wall is then laid; the granite inside wall, above lining, is also laid in cement.

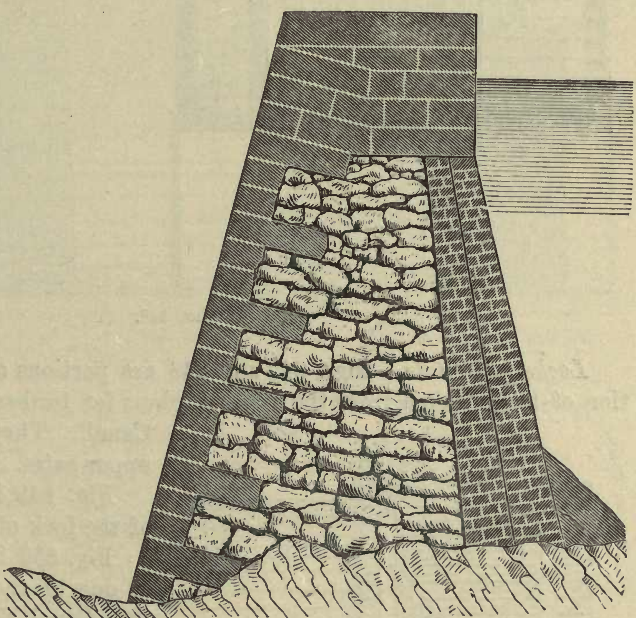


Fig. 846.

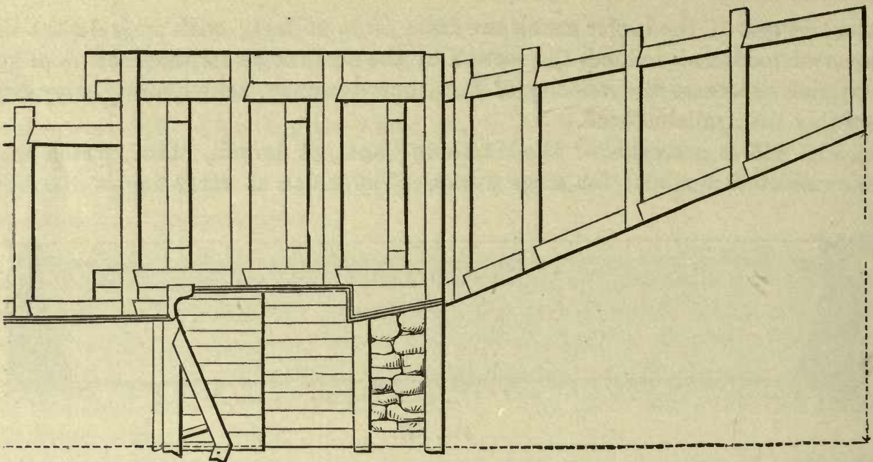
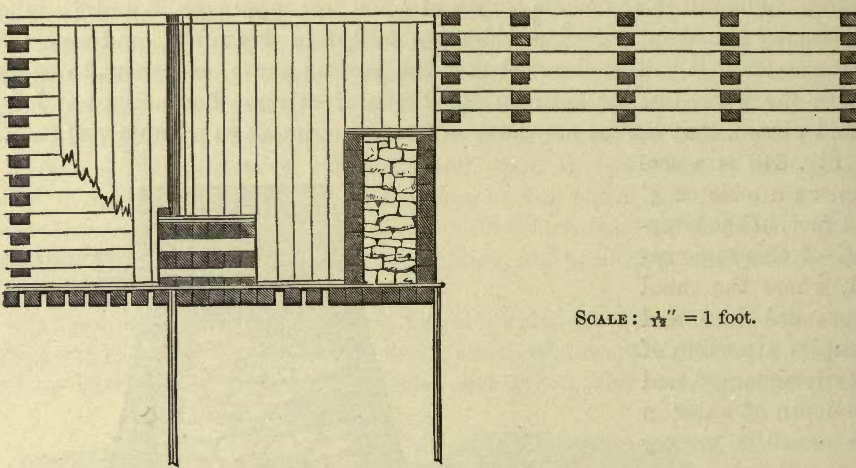


FIG. 847.



SCALE: $\frac{1}{12}'' = 1$ foot.

FIG. 848.

Locks of Canals.—Figs. 847 and 848 are portions of plan and vertical section of locks, taken from the general plans for timber locks on the Chemung Canal. They represent the half of upper gates.

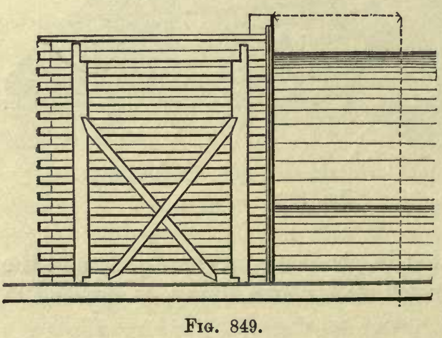


FIG. 849.

Fig. 849 is a section of one side of the lock of the same.

Fig. 850 is the plan of a portion of one of the enlarged locks of the Erie Canal, showing one of the upper gates and the side-walls.

Fig. 851 is a cross-section of one of the same locks, showing the culvert in the center between the locks, used for the supply of the waste of the lower level, to preserve the proper height of water in this level controlled by gates in the upper level.

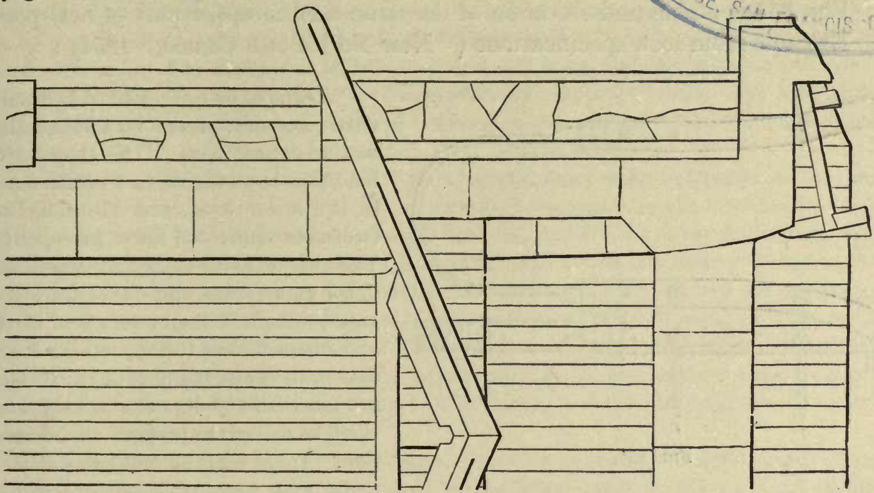


Fig. 850.

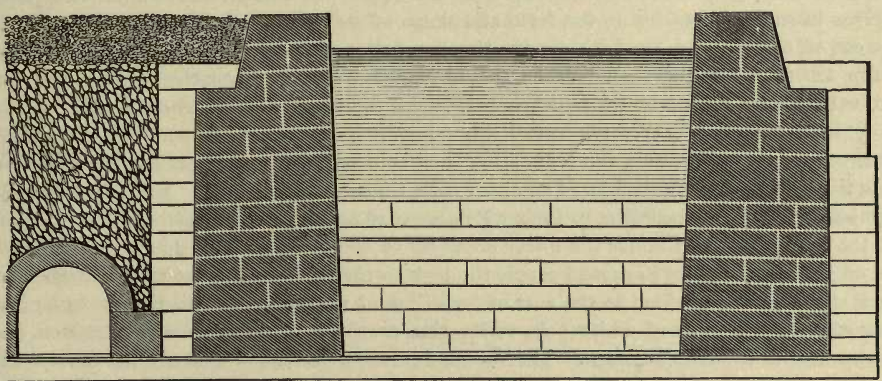


Fig. 851. Scale: $\frac{1}{16}'' = 1$ foot.

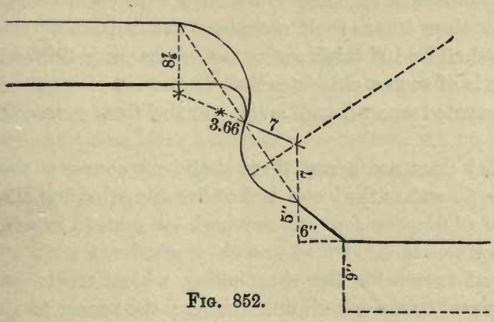
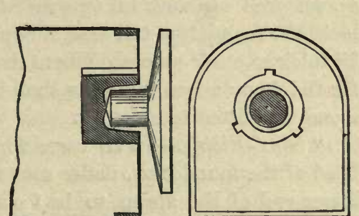


Fig. 852.



$\frac{1}{8}$ full size.

Fig. 853.

Fig. 852 is a drawing, in outline, of the hollow quoin of the lock-gate, on a scale of $\frac{1}{24}$ full size (Chemung Canal).

Fig. 853 is a plan and elevation of pintal for heel-post of lock, with a section of the bottom of the post. The pintal is imbedded in bottom timber or stone, as the case may be.

Fig. 854 is a plan and elevation of the strap for the upper part of heel-post.

Extracts from lock specifications ("New York State Canals," 1854):

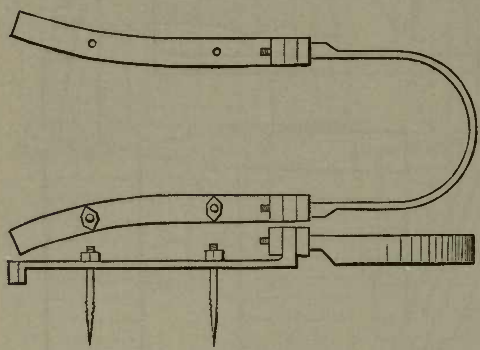


Fig. 854.

"Locks to be composed of hydraulic stone masonry, placed on a foundation of timber and plank. The chamber to be 18′ wide at the surface of the water in the lower level, and 110′ long between the upper and lower gate-quoins. The side-walls to extend 21′ above the upper gate-quoins, and 14′ below lower gate-quoins. If the bottom is of earth, and not sufficient to support the foundation, then bearing-piles of hard wood, not less than 10″ diameter at small end, shall be driven, to support the foundation. There shall be four rows of piles under each main wall, and one row in center of lock; the piles shall be driven in rows, at 3′ from center to center. The piles to support the wing and breast-walls and wing buttresses, and also under the miter-sills, to be driven in rows to conform to the form and shape of the same. The heads of the piles to be cut off smooth and level, to receive the foundation timbers. The foundation timbers to be 12″ × 12″, and of such lengths as will extend from and cover the outside piles, and to be treenailed with a 2″ white-oak or white-elm treenail, 24″ long, to each pile.

"If the bottom is of earth sufficiently compact and firm to support the foundation without bearing-piles, then the foundation shall be composed of timber, 12″ thick and not less than 10″ wide, counterhewed on upper side, timbers to average 12″ wide, to be placed at uniform distance, according to their width, so as to occupy or cover at least ⅔ of the area of the foundation, and under the lower miter-sill to be placed side by side: in all cases to be of sufficient length to extend across the lock to the back line of the center buttresses, and at the head and foot to the rear or back line of wing-walls. The timber under the lower miter-sill to be of white oak, white elm, or red beach, the other foundation and apron timber to be of hemlock. The foundation to be extended 3′ above the face of the main wall at the head of the lock, and at the foot from 25′ to 30′ below the exterior wing—that portion of the spaces between the timbers in all cases to be filled with clean coarse gravel, well rammed in, or concrete. In cases where rock composes the bottom of the lock, the foundation timbers, if required, shall be 10″ thick under the lower miter-sill, and 8″ thick at other places. Where the rock is of such a character that timber is not required for the foundation, the same shall be excavated smooth and level, and the first course of stone well fitted to the rock.

"*Sheet-Piling.*—In all cases where rock does not occur, there shall be a course at the head of the foundation, under each miter-sill, and at the lower end of the wings, and at the lower end of the apron, to be from 4′ to 6′ deep as may be required—in each to extend across the whole foundation. The sheet-piling to be of 2″ hemlock plank, lined with 1″ pine boards. Ditches are to be excavated to receive the sheet-piling, which are to be placed edge to edge, and the top well secured to the foundation timber; the spaces to be filled up with fine hard gravel, well puddled in, or with concrete.

"*Flooring.*—A course of 2½″ pine or hemlock plank to be laid over the whole of the foundation timbers, except a space, 3′ wide, under the face-line of each wall to be 2½″ white oak: the whole to be well jointed, and every plank to be treenailed with two white-oak treenails at each end, and at every 3′ in length, to enter the timber at least 5″, or with wrought-iron spikes, treenails to fill 1¼″ bore. *Platform for the upper miter-sill* to be 5′ 10″ wide, and 6′ high above foundation, and to extend across from side-wall to side-wall, to be composed of masonry, coped with white-oak timbers, which are to extend 6″ into each side-wall. The timbers to be 12″ deep and 14″ wide, covered with two courses of 1½″ white-oak plank. *Miter-sills* to be of best white-oak timber, 9″ thick, to be well jointed, and bolted to the foundation or platform timbers, as the case may be, with bolts of iron, 20″ long, 1″ × 1″, well ragged and headed, eight bolts to each side.

"*Masonry.*—The main walls, for 21′ 6″ in length, from wing-buttresses at the head, and 32′ at lower end, to be 9′ 8¾″ thick, including recesses, and for the intermediate space, 7′ 8¾″ thick, with three buttresses projecting back 2½′, and 9′ long at equal distances apart. The *quoin-stones*, in which the heel-post is to turn, shall not be less than 4′ 6″ in length in line of the chamber, to be alternately header and stretcher. The *recesses* for the gates to be 20″ wide at top of wall, 12′ long, with sub-recesses, 9″ wide, 6′ high, 10′ long, for the valve-gates. *Breast-wall* to commence 5′ below upper end of foundation, 5′ wide, 8′ high, finished with a coping of cut stone. The interior *wing-walls*, and exterior wing from main walls to the termination of first curve, to be 7′ 6″ thick, and the running curve of exterior wing to be 6′ thick on the foundation.

"*Culvert between Locks.*—In such cases as may be required, a culvert shall be constructed, to pass the water from the upper to the lower level, as follows: A foundation of suitable timber and plank, as for lock-walls, and covering all the space between the lock-foundations, shall be put down. Three apertures for the sluice-way shall be made in the head-wall with cut-stone jambs, grooves to be cut in the jambs for the sluice-gates, and the coping to form a recess, corresponding with the grooves in the jambs; grooves to be cut on the top and bottom coping, 1″ deep, to secure the jambs. The bottom of the aperture to be of cut stone, with lower corner beveled off, over which the water will fall into the well, the bottom of which shall be covered with a sheeting of cut stone, 6″ thick. The apertures to be 3′ 6″ deep, placed immediately below the coping-stone, and 4′ long. Suitable gates of plank, for regulating the water in passing the sluice, to be prepared; the well to commence on the foundation, to be made of substantial hydraulic masonry.

"*Second flooring* of seasoned 2″ first-quality white-pine plank, to be well jointed, and laid on the foundation between the walls, from the breast-wall to lower end of main wall, and also on the floor of the well, to be close and firmly jointed to miter-sills and walls, so as to make a water-tight flooring. The plank to butt, or the end-joints to come to the center of a foundation timber, and each plank to be treenailed with two treenails at end and two at every 3′ intermediate: treenails 10″ long, to fill 1¼″ bore.

"*Gates.*—The framing to be made of best quality white-oak timber; the cross-bar to be framed into heel and toe posts with double tenons, each tenon to be 7″ long, and thickness equal to the thickness of the bar, and secured with wrought-iron Ts, well bolted. The heel and the posts to be framed to the balance-beam by double tenons, and secured by a wrought-iron strap and balance-rod, from the top of the beam to the under side of the upper bar. The lower ends of the heel-posts to be banded with wrought-iron bars; the collar and other hangings to be of wrought-iron, secured together with a double nut and screw, and to the coping by bedding the depth of the iron in, and by screw-bolts fastened with sulphur and sand-cement. The pivots and sockets which support the heel-posts to be of best cast-iron; a chilled cast-iron elliptical ball, 2½″ horizontal, and 1″ vertical diameter, to be placed on the pivot and in the socket of each heel-post, to facilitate the movement of the gate. The gates to be planked with seasoned first-quality 2″ white-pine plank, jointed, grooved, and tongued—tongues of white oak—the plank to be secured by 6″ pressed spike. On the chamber-side of the gates, fenders of white-oak plank, to be put on with pressed spike."

Water, ponded by dams, and conveyed by canals for use as mill-power, is carried within the workshops or manufactories, to be applied on water-wheels, by some covered channels. These channels, although of various forms, are usually designated as *flumes*. The common form of a flume for the conveyance of water to breast, overshot, or undershot wheels, is of a rectangular section, framed with sills, side-posts, and cap, and, if large section is required, intermediate posts are set in. The sills are set, and earth well rammed in the spaces between them ; the bottom plank is then laid, posts and cap framed with tenon and mortice, set and pinned, and the plank is then firmly spiked on the outside of posts and caps. The planks are usually nearly green, jointed, and brought to close joints ; the size of timbers will depend on the depth beneath the soil, or the insistent load. Within the mill, and just above the wheel, the flume is framed without a cover, and the posts and side-planks are brought above the level of the water. This open flume is termed the *penstock*, especially necessary, in the class of wheel above referred to, to secure the full head of water.

Many flumes are made of a circular section, pipes of iron, or wood. For the conveyance of water to turbine-wheels, wrought-iron pipes are almost invariably used. Cast-iron is also sometimes used, with flange, or hub and spigot-joints. Plank-pipes are also used, made with continuous staves, and hooped with wrought-iron ; these constructions are much cheaper, and serve a very good purpose. The head-gates of flumes are placed at the head of the flumes, in a recess back from the face of the canal, with racks in front to prevent the passage of any drift that might obstruct or injure the wheel. The total area of passages through the racks should liberally exceed the area of cross-section of the flume, not only on account of the extra lateral friction of the rack-bars, but also on account of their liability to become obstructed. Sometimes two sets of racks are placed in front of the flumes, especially for turbines and reacting wheels : a coarse rack with wide passages, say 2″ spaces outside, and a finer one inside, say of $\frac{5}{8}''$ to $\frac{3}{4}''$ spaces. The head-gates to the flume, directly back of the racks, in their function are like the head-gates at the dam, and are similar in construction—strong plank gates, moving in slides, vertically or horizontally, with a paddle-gate in them, to fill the flume when empty, so that the gates themselves may be opened without any pressure due to a difference of head outside and inside of the gates, and also to prevent any damage to the flume by the water-ram, which might result from a too sudden filling of the flume by the opening of a large gate suddenly.

Fig. 855 is the elevation and section of the head-gates manufactured at Holyoke, Massachusetts. G G are plank gates, sliding laterally, moved by two pinions, working into racks on top and bottom of gates, turned by a hand-spike. P is the paddle-gate ; R, the rack ; F, the flume, or plank-pipe ; A, air-pipe, for the escape of air from the flume while being filled.

Conduits for the supply of water to cities and towns are of masonry, or cast or wrought iron pipes. Their capacity to deliver the required quantity depends upon the area and form of cross section, and the velocity of flow due to the loss of head or of pressure permissible ; this velocity being due primarily to gravity, but largely modified by conditions of structure, as the kind and amount of wetted surface, and length and directness of line.

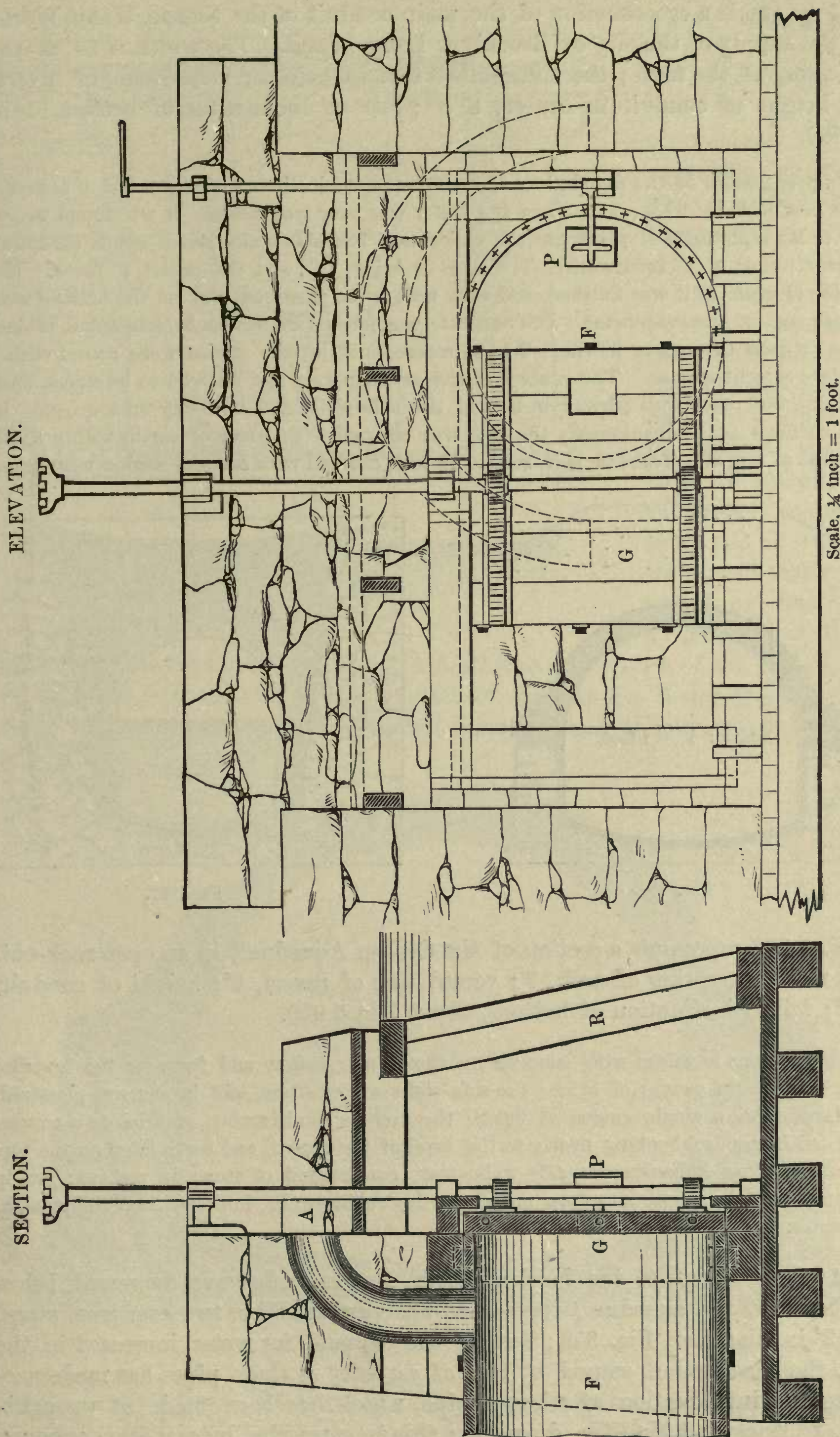


Fig. 855.

Fig. 856 is a cross-section of the main conduit of the Nassau Water-Works for the supply of the city of Brooklyn, Long Island. The width is 10′ at the springing of the arch; the side-walls 3 feet in height; versed sine of invert, 8″; height of conduit in center, 8′ 8″; fall or inclination of bottom, 1 in 10,000.

In preparation of the foundations the contract specifications required a bed of concrete to be first laid, 15′ wide; but, when the water was very troublesome, it was found necessary to lay a platform of plank for the concrete. The side-walls are of stone, except an interior lining of 4″ brickwork. The arch is brick, 12″, and the invert 4″ thick. The outside of arch, as it was finished, and each wall, were plastered over on the outside with a thick coat of cement-mortar. The concrete was formed from clean broken stone, broken so as to pass through a 2″ ring; 2 to 2½ measures of broken stones were mixed with 1 measure cement-mortar. The centers of the arching were not allowed to be struck until the earth had been well packed in behind the side-walls and half-way up the arch. In both cuttings and embankments the arch was covered with 4 feet of earth, with a width of 8 feet at top, and slopes on each side of 1½ to 1, covered with soil and seeded with grass.

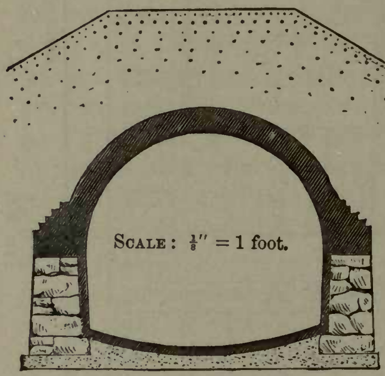


FIG. 856.

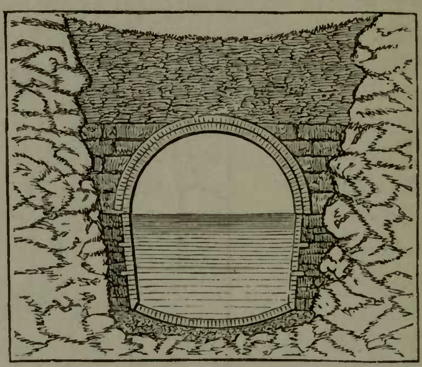


FIG. 857.

Fig. 857 represents a section of the Croton Aqueduct, in an open rock-cut. The width at spring of arch, 7′; versed sine of invert, 6″; height of conduit, 8′ 6″; fall or inclination of bottom, about 1 in 5,000.

The bottom is raised with concrete to the proper height and form for the inverted arch, of a single course of brick; the side-walls are of stone, laid in cement, plastered, and faced with a single course of brick; the arch is semicircular, of brick two courses thick, with spandrel backing nearly to the level of the crown, and earth filled on the top. In earth-cuts or embankments, side-walls were constructed of stone, in cement; and in embankments the whole structure rested on dry rubble-walls, built up from solid earth-foundations.

At the crossing of the Harlem River, as the bridge was depressed below the level of the aqueduct, the water was conveyed by two cast-iron pipes, a a, 3′ in diameter, Fig. 858; but, as the demand for water increased in the city, the obstruction caused by lack of capacity in these pipes has made necessary the introduction of a larger pipe, which has been made of wrought-iron, ½″ thick and 7′ 6½″ in diameter; this is supported by cast-iron columns

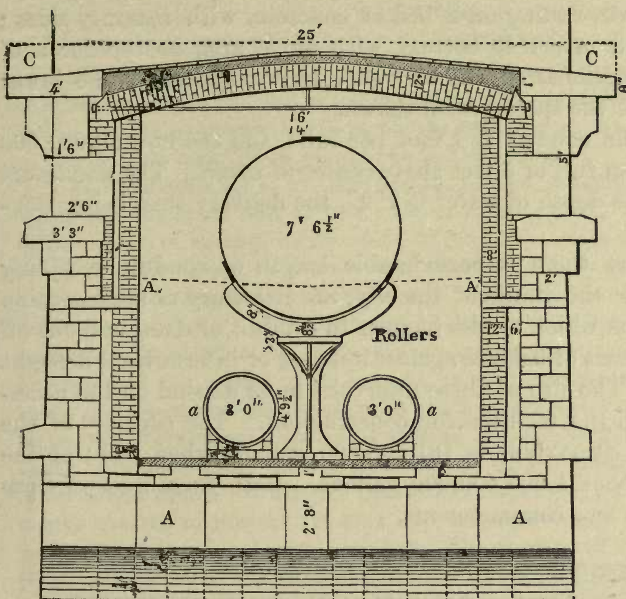


Fig. 858.

which admit of a rocking movement, and slip-joints are also made in the pipe to compensate for any expansion or contraction by changes of temperature. The pipes are inclosed in a long chamber or passage, extending the whole length of the bridge, covered by a brick arch, laid in cement with a cover of asphalt, and a brick pavement over all, affording a wide promenade protected on each side by cast-iron railings, fastened to the coping-stones, C C. A A are the arch-stones of the bridge.

Fig. 859 is a section of the conduit of the Boston Water-Works. The inside section is equal to a circle $8\frac{1}{2}$ feet diameter, and is uniform throughout

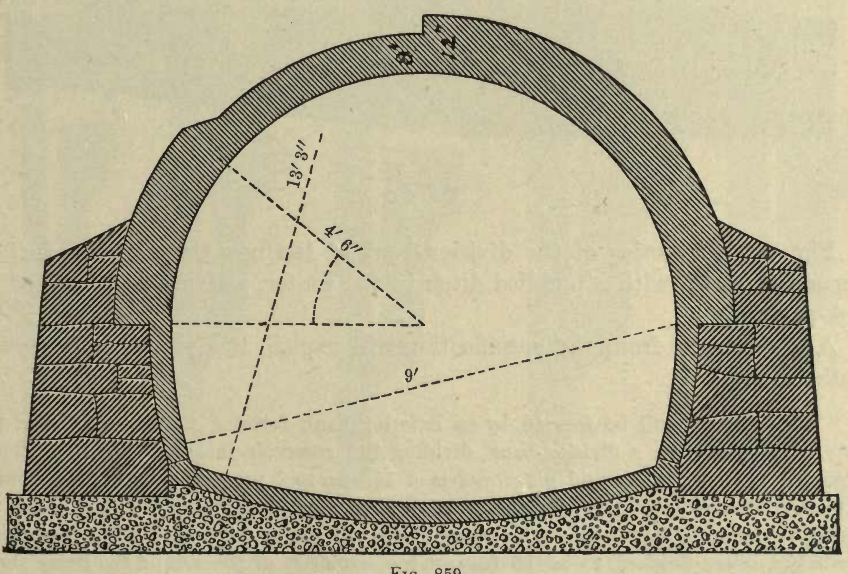


Fig. 859.

except in tunnels. The exterior lines vary according to the material on which it is built and the cover or load on the top. The section given may be considered the general one, resting on a bed of concrete, with masonry sides; brick lining at sides and invert at bottom, with an 8″ arch at top for a 4′ cover, and 12″ for exceptional depths or under railway-tracks. The lower corners were of brick, of the special form shown.

The inclination of the conduit is 1 foot per mile, and the flow 80,000,000 gallons per 24 hours when full or 5 feet above center of invert. The maximum flow takes place when the depth of water is 7′ 2″, the delivery then being 109,000,000 gallons.

In large works, where there is considerable length of conduit, receiving reservoirs, within or near the limits of the city, are necessary as a precaution to guard against accidents which might happen to conduit or dam, and cut off the supply, and also as a sort of balance against unequal or intermittent draught among the consumers. The size of these reservoirs must depend on the necessities of the case, and on the facilities for construction. The capacity of the Ridgewood reservoir, at Brooklyn, is 161,000,000 gallons when full; of the new Croton reservoir, about 1,000,000,000 gallons. Both these reservoirs are made double—that is, in two compartments.

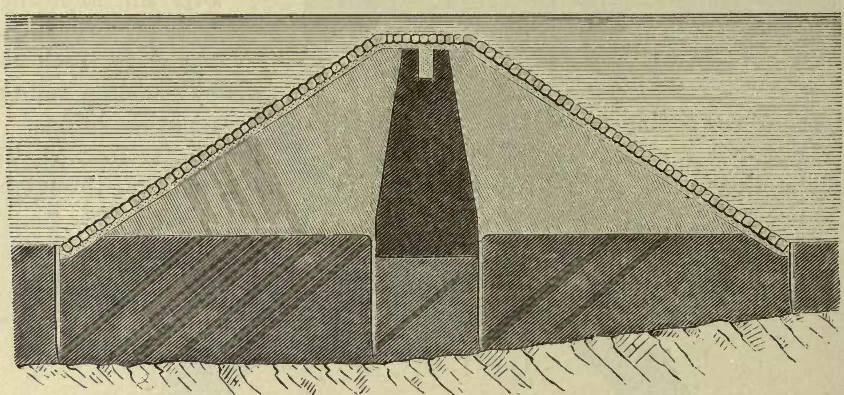


Fig. 860.

Fig. 860 is a section of the division-bank of the new Croton reservoir. It is made of earth, with a puddled ditch in the center, and slopes protected by rock-paving.

A few extracts from the specification will explain the general construction of the reservoir:

"The reservoir will be formed by an exterior bank forming the outer sides of the basin. There will be a division-bank, dividing the reservoir into two basins. All the banks will have the inner and outer slopes of 1½ base to 1 perpendicular. All the inner or water-slopes will be covered with 8″ of broken stone, on which will be placed the stone pavement, 1½ feet thick. The outer slopes will be covered with soil 1 foot thick. The banks, when finished, to be 15 feet on top, exclusive of the soil on the outer slope. The top of the outer bank to be 4 feet above water-line; the top of the division-bank to be 3 feet below water-line. In the center of all the banks a puddle-bank will be built, extending from the rock to the paving in the division-bank, and to within 2 feet of the top of the outer bank. It will be 6′ 2″ wide at top in division-bank, and 14′ wide at top in exterior bank, and 16′ wide at a plane 38′ below top of exterior bank. In the middle of the division-bank there will be built a brick wall,* laid in cement-mortar, 4′ high, 20″ wide, the top of the wall to be connected with the bottom of the stone pavement; 8″ thickness of concrete is to be laid on the top of the bank, on each side of, and connected with, this wall. On the pavement 18″ thick will be laid in concrete. The slope-wall on each side of the division-bank, 10′ in width, to be laid in cement.

"Puddle-ditches are to be excavated to the rock under the center of all embankments where the rock is not over 46′ below top of exterior bank. Where the rock is more than 46′, two rows of sheet-piling are to be driven to the rock, 16′ apart, and the material between them excavated, so as to remove all soil, muck, or vegetable matter. Sheet-piling will be formed of spruce or pine plank, 6″ thick, tongued and grooved; the tongue and groove to be 1½″ × 1″. The earth within the working-lines of interior slopes will be excavated to the depth of 40′ below top of exterior bank, rock 36′. The puddle-ditch will be formed of clay, gravel, sand, or earth, or such admixture of these materials, or any of them, as the engineer may direct, to be laid in layers of not more than 6″, well mixed with water, and worked with spades by cutting through vertically, in two courses at right angles with each other; the courses to be 1″ apart, and each spading to extend 2″ into the lower course or bed. Whenever the work is suspended, the puddle must be covered with boards or earth to prevent cracking, and, whenever cracks do occur in the puddle, those parts must be removed and reworked. The puddle will extend to all the masonry and pipes, and along and around it and them as the engineer may direct.

"The embankments will be formed in layers of not more than 6″, well packed by carting and rolling, and, in such places as the rollers can not be effectually used, by ramming. The embankments will be worked to their full width as they rise in height, and not more than 2′ in advance of the puddle. The interior slopes of all the banks will be covered with 8″ thickness of stone, broken to pass through a 2″ ring. On this will be laid the paving, 18″ in thickness, of a single course of stones set on edge at right angles with the slope, laid dry, and well wedged with pinners."

Fig. 861.

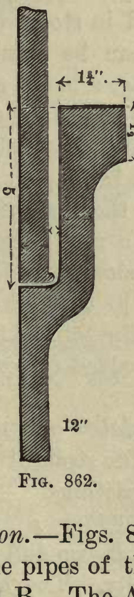


Fig. 862.

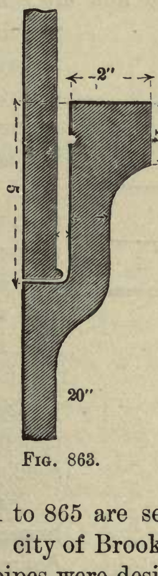


Fig. 863.

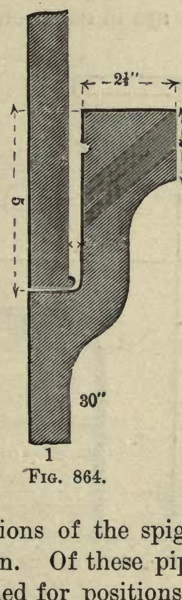


Fig. 864.

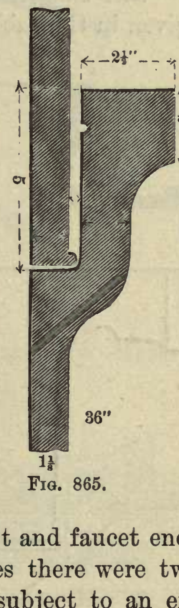


Fig. 865.

Distribution.—Figs. 861 to 865 are sections of the spigot and faucet ends of some of the pipes of the city of Brooklyn. Of these pipes there were two classes, A and B. The A pipes were designed for positions subject to an ex-

* This wall was formed of concrete.

treme head of 120′, the B pipes for positions below this level, subject to a head of from 120 to 170 feet.

The thicknesses of these pipes is greater than those which now obtain in practice. The following table, made from the average of formulas and of the dimensions in use in different cities, may be considered safe for a static pressure of 100 lbs., or 231 feet. But pipes should be tested at the manufactories to three times this pressure. The weights given are the pipes as delivered in lengths of 12′ or 12′ 5″; as laid, the laps are 5″, and for running feet about 4 per cent should be added to the table-weights:

Diameter.	Thickness.	Weight.	LEAD JOINT.	
			Depth.	Weight.
	In.	Lbs.		
4	·42	18	1⅛	4¼
6	·47	30	1⅛	6¼
8	·52	44	1⅛	8¼
10	·58	60	1⅛	10¼
12	·63	78	2	13
16	·73	120	2⅛	24¼
20	·83	170	2¼	31
24	·94	228	2⅜	38
30	1·10	330	2½	57
36	1·24	450	2½	30
48	1·44	700	2⅝	111

The smallest water-pipe laid in large cities now is the 6″; the other sizes given in the table are in common use and are found in stock, except the 10″, which can be obtained by order. In laying, a hemp gasket is forced down to the lower end of the bell to prevent the molten lead escaping into the pipe. The end of the pipe is then stopped by the *clay roll*, or a rope covered with clay, or clay alone, and the melted lead poured in through an aperture or *gate* at the top. After cooling, the lead is calked or compacted in the joint.

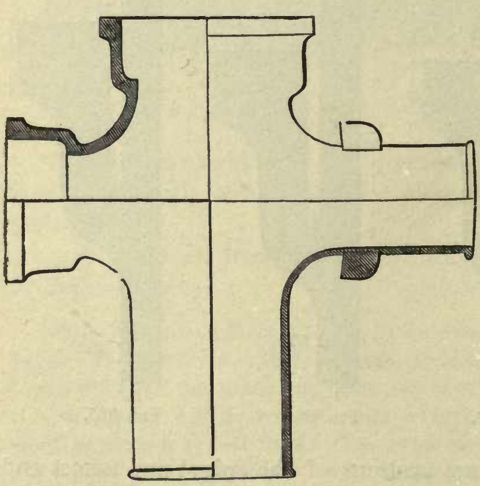


Fig. 866.

Specials.—All parts of a main except the straight pipes are called special castings.

Fig. 866 is a 12″ × 8″ 4-way branch, shown full and in section, diagonally. The horns on the 4″ branch are for the straps which hold in the plug, or cap, or a connected short or curved pipe. The 4-way branches are often called crosses, and the 3-way

T's, or single branches. The branches may be of any appropriate size. In ordering, designate diameter of main pipe first, and then that of the branches. It is very common in these pipes to make all the ends bell ends—it saves sleeves when pipes are cut, as they usually are at street intersections.

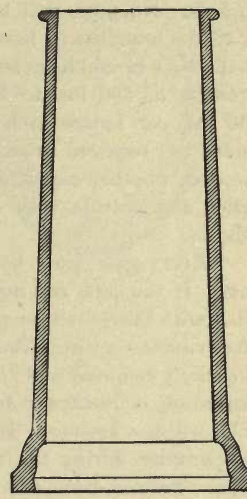


Fig. 868.

Fig. 867 is a section of a sleeve for uniting cut pipes or uncut spigot-ends; a kind of double hub is often used for the former. Sometimes sleeves are made in halves, and bolted together.

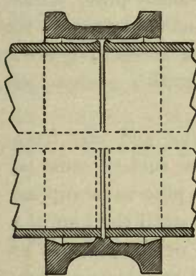


Fig. 867.

Fig. 868 is the section of a *reducer* for the connection of pipes of unequal diameters.

Fig. 869 is the section of a *bend;* the horns on the outer circle are for straps between the pipes, as the pressure is unbalanced.

Fig. 870 is a section of the connection of two wrought-iron pipes by a bell riveted to the end of one, and a fillet or ring to the end of the other.

House-services are usually through lead pipes; the taps allowed on the mains for house-connections being usually from $\frac{1}{2}''$ to $\frac{5}{8}''$.

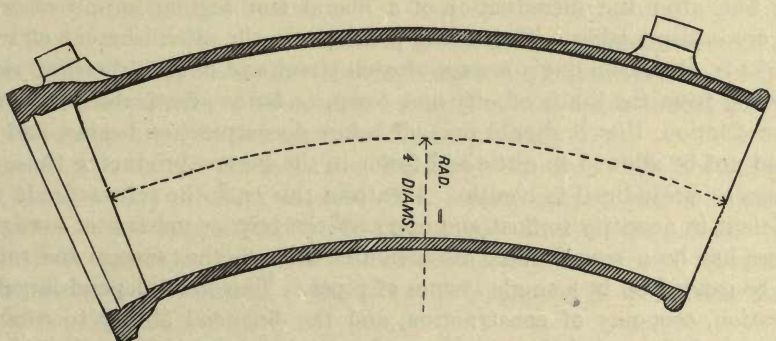


Fig. 869.

From the specifications of "Cast-Iron Distribution-Pipes and Pipe-Mains, with their Branches," etc., Brooklyn, L. I.:

Fig. 870.

"All pipes of 20″ diameter and upward to be formed so as to give a lead joint of not less than $\frac{3}{8}''$ in thickness all round, and not more than $\frac{7}{16}''$; those of 12″ diameter and under, not exceeding $\frac{3}{8}''$, and not less than $\frac{5}{16}''$. The straight pipes of 12″ diameter and upward shall be cast in dry sand molds, vertically. The smaller pipes may be cast at an angle with the horizon of not less than 12°. The pipes shall be free from scoria, sand-holes, air-bubbles, cold-short cracks, and other defects or imperfections; they shall be truly cylindrical in the bore, straight in the axes of the straight pipes, and true to the required curvature or form in the axes of the other pipes; they shall be internally of the full specified diameters, and have their inner and outer surfaces concentric. No plugging or filling will be allowed. They shall be perfectly fettled and cleaned; no lumps or rough places shall be left in the barrels or sockets. No pipes will be received which are defective in joint-room. The spigot ends of all the branches to have lugs or horns cast on each. Every pipe-branch and casting shall pass a careful hammer-inspection, and shall be subject thereafter to a proof by water-pressure of 300 lbs. to the square inch for all pipes 30″ in diameter and under, and 250 lbs. per square inch for all pipe-mains exceeding 30″ diameter. Each pipe, while under the required pressure, shall be rapped with a hand-hammer from end to end, to discover whether any defects have been overlooked. The pipes shall be carefully coated inside and outside with coal-pitch and oil, according to Dr. R. A. Smith's process, as follows:

"Every pipe must be thoroughly dressed and made clean from sand and free from rust. If the pipe can not be dipped presently after being cleansed, the surface must be oiled with linseed-oil to preserve it until it is ready to be dipped; no pipe to be dipped after rust has set in. The coal-tar pitch is made from coal-tar, distilled until the naphtha is entirely removed and the material deodorized. The mixture of five or six per cent of linseed-oil is recommended by Dr. Smith. Pitch, which becomes hard and brittle when cold, will not answer. The pitch must be heated to 300° Fahr., and maintained at this temperature during the time of dipping. Every pipe to attain this temperature before being removed from the vessel of hot pitch. It may then be slowly removed and laid upon skids to drip."

Sewers.—For the removal of waste water from houses and rainfall, sewers are very convenient in towns and cities, even before the construction of water-works; but, after the introduction of a liberal and regular supply of water, sewers are indispensable. The ruling principle in the establishment of sewerage-works is, that each day's sewage of each street and of each dwelling should be removed from the limits of city and town, as far as practicable, on the day of its production, that it should pass off before decomposition begins, and that it should not be allowed to settle and fester in the sewers, producing those noxious gases so prejudicial to health. To attain this end, the refuse fluids must be sufficient in quantity to float and carry off the heavier matters of sewage.

There has been considerable discussion of late whether sewage and rainfall should be carried off by a single system of pipes. This must depend largely on the location, economy of construction, and the financial ability to carry out the design. If the rainfall can be provided for by street gutters, the pipes for the conveyance of house-waste may be very small.

If the rate of inclination of a sewer be not less than 1 in 440, the experience of Brooklyn, and other cities equally well supplied with water, shows that the fluid of domestic sewage is sufficient to carry off all the heavier matters, and keep the sewers free and clean, provided the form is such as to concentrate as much as possible the sewage waters. Less inclination than 1 in 440 will require some means of flushing. In the Brooklyn system of sewers, adopted on the report and plans of Colonel J. W. Adams, the principle of construction has been, to make the sewers as small as the service required of them will admit, to maintain as much velocity of flow as possible, so that nothing may be deposited; and without any provision for a man entering and passing through the sewer, which has been found by experience unnecessary.

The value of sewers depends on the correctness of their lines, uniformity of descent, and smoothness of interior surface. The pipes used in Brooklyn have generally been strong glazed earthenware pipes, of 12″, 15″, and 18″ diameter. Many cement-pipes have also been used, and, in such situations as required great capacity, brick sewers were used, the leading forms of which are egg-shaped, as in Fig. 871, of which the dimensions are as follow: R (as in table) the longest diameter D, and the longest radius R′, each 3 times R, and R″ ½ R.

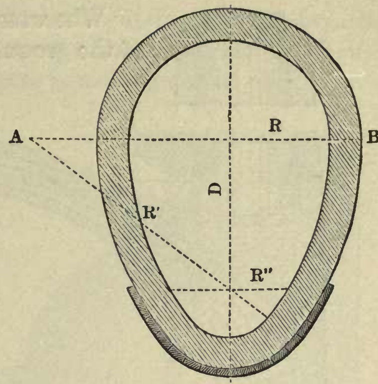


Fig. 871.

AREA.	R.	D.
	″	″
60″ diameter circular..	24·8	74·4
48″ " "	19·8	59·5
36″ " "	14·9	44·7
24″ " "	9·9	29·8

Thickness of brickwork, 8″; boards shown at bottom only used in cases of soft earth for convenience of construction. For area of egg-shaped sewer of above section, multiply R^2 by 4·6.

In some locations the depth did not admit of the egg-shaped section. A circular form of 6 feet diameter was adopted for the Union Avenue sewer, and one of a section similar to the main conduit of the water-works, 10 feet in width and 9 feet high, in the clear, for Kent Avenue.

Fig. 872 is a section of the largest Washington sewer. The bottom course of the large sewers, or where exposed to a strong current, are of stone; the ring-courses, of brick, are 3 for the 13-foot sewers and 2 for the 7-foot.

Man-holes are built along the line of sewers, at a distance of from 100 to 150 feet apart, to give access to the sewers for purposes of inspection and removal of deposit.

Figs. 873 and 874 are section and plan of the man-hole at present used by the Croton Sewer Department. It consists of a funnel-shaped brick well, oval at the bottom, 4′ × 3′, circular at top, 2′ diameter, curbed with cast-iron frame and covered by cast-iron plate. Side-walls, 8″ thick, through which the pipe-sewers pass at the bottom of the well. Across the open space the sewer is formed in brick, whose bottom section corresponds to that of pipe, side-walls carried up perpendicular to top of sewer; the flat spaces at the sides of sewer are flagged. The top of the sewer is a heavy cast-iron frame, fitted with a strong cover, which may or may not be perforated, for ventilation. In the figure the main sewer is 12″ pipe, with a 12″ branch entering at an acute angle, as all branches and connections with a sewer should. The short lines on the left vertical wall represent sections of ∩ staples, built in to serve for a ladder.

Wherever necessary, from the slope and conformation of the ground, to remove the surface or rain water direct-

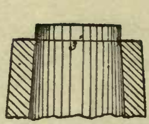
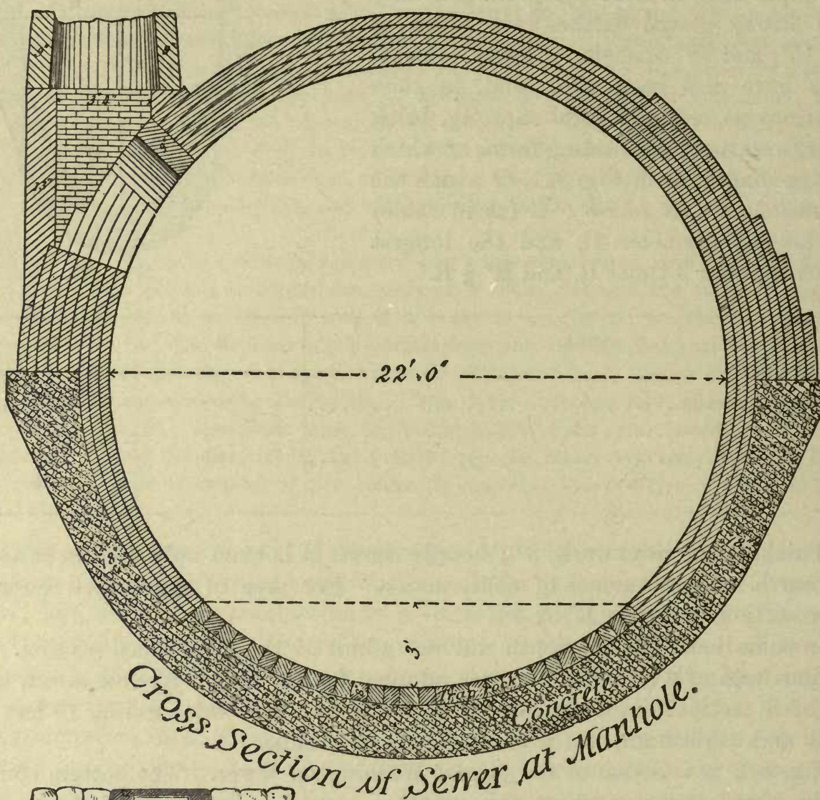



FIG. 872.

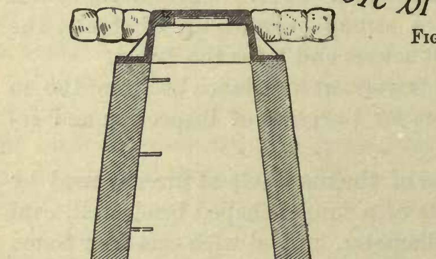
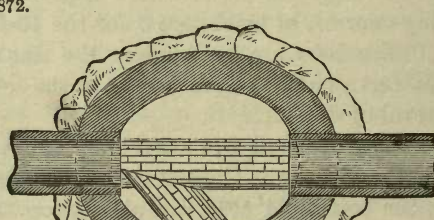


FIG. 873.



SCALE: $\frac{1}{4}''$ = 1 foot.

FIG. 874.

ly from the street-gutters into the sewers, *catch-basins* are placed generally at the corners of streets.

Figs. 875 and 876 are section and plan of the Croton sewer catch-basins, on a scale of $\frac{1}{4}'' = 1$ foot. The intention of the catch-basin is to receive the street washings, retain the heaviest portion in the basin, and let the liquid escape into the sewer. The basin in the figure is rectangular in plan, with a semicircular end, 3′ 8″ in width by 5′ 1″ long; bottom of flag and side-walls of brick, 12″ thick. It will be observed that a piece of flag is built into the side-walls from the top, extending about half-way to the bottom; this divides the upper part of the basin into two parts; the sewer enters the basin three feet above bottom flag; the dividing flag comes to within 2′ 6″; before any water can flow out through the sewer-pipe this flag must be submerged 6″; a trap is thus formed, which cuts off any smell from the sewer escaping into the street. This trap is sometimes made of a cast-iron elbow, turned down and bolted to the sewer-pipe in the wall. The water is received into the basin through the channel C, which is curbed with granite, and protected by a grating. The coping (*b*) is of granite, and forms a portion of the sidewalk; through this there is a man-hole cut, 16″ diameter, for access to the basin, for removal of the deposit; it is covered by a strong cast-iron plate.

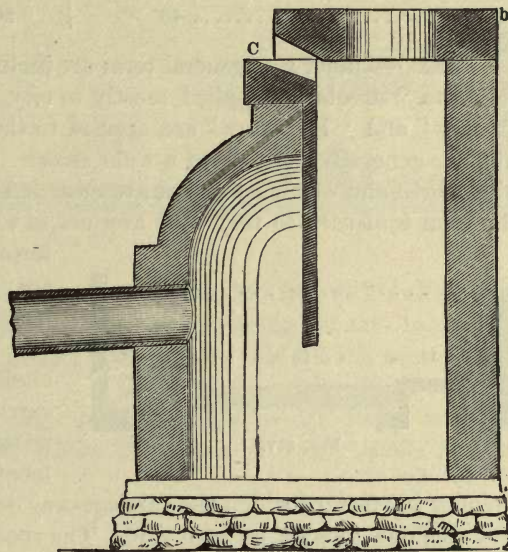


Fig. 875.

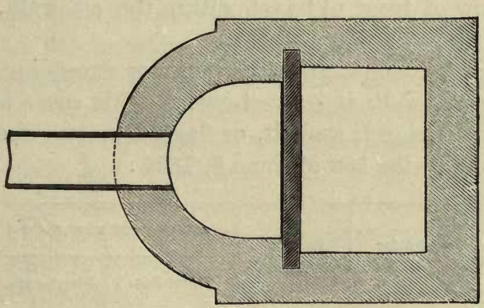


Fig. 876.

Gas-Supply.—Next in importance to the necessities of a city or town for water-supply and sewerage, is the luxury of gas-supply. The gas-works should always be placed remote from the thickly-populated part of a city, for under the best regulations some gas will escape in the manufacture, offensive and deleterious. They should be placed at the lowest level, for, gas being light, readily rises, and the portions of the city below the works are supplied at less pressure than those above. Gas-mains, like those for water, are of cast-iron, and put together in the same way; but, as they have to resist no pressure beyond that of the earth in which they are buried, they are never made of as great thickness as those of water-pipes, but drips must be provided, and the pipe laid with such inclination to them that the condensed tar may be received in them and pumped out.

WEIGHT OF GAS-PIPES PER RUNNING FOOT.

3″	12 lbs.	10″	50 lbs.
4″	16 "	12″	62 "
6″	27 "	16″	103 "
8″	40 "	20″	150 "

Roads.—Under this general term are included all routes of land-travel ; but the term "streets" is applied mostly to city, town, and suburban roads, while "roads" and "highways" are applied to those of the country. By an "avenue" is generally understood a wide street. In New York all the streets running north and south are called avenues, and those at right angles, streets, and the term *boulevard* to very wide avenues in which there are rows of trees. The terms street and avenue, as laid out, are the established bounds within which no buildings may be erected. The street, therefore, technically includes the street or traveled way for carriages, and the sidewalks and front areas. New York streets above Fourteenth Street are 60 and 100 feet wide, avenues 100 feet, of which the carriage-way occupies one half, and the sidewalks and area one quarter on each side. The space occupied by areas, is from 5 to 8 feet, which may be inclosed by iron fence, but can not be included within the building above the level of sidewalk. The stoop-line extends into the sidewalk beyond the area-line some 1′ to 18″, fixing the limit for the first step and newel to a high stoop or platform. The boulevard in the old line of upper Broadway and the Bloomingdale Road is 150 feet wide, of which 100 feet are to be carriage-way, and 25 feet on each side for sidewalk and area, the latter not to exceed 7 feet; one row of trees to be set within the sidewalk, about 2 feet from the curb.

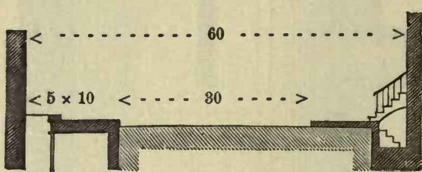


Fig. 877.

In Paris, there is no area; the sidewalk comes up to the house or street-line, and there is a space for trees between sidewalk and street-curb. This space is available for pedestrians, a part being a gravel, asphalt, or flagged walk. The following are the dimensions according to the law of June 5, 1856 :

Entire width of boulevard and avenues.	Width of carriage-way.	Width of sidewalk.	Width for trees.	Rows of trees.	DISTANCE OF ROW FROM	
					Street-line.	Street-curb.
Metres.	Metres.	Metres.	Metres.		Metres.	Metres.
26 to 28	12			1	5·5 to 6·5	1·5
30 " 34	14			1	6·5 " 8·5	1·5
36 " 38	12 to 13	3·5	8· to 8·5	2	5· " 5·5	1·5
40	14	3·5	9·5	2	6·5	1·5

1 metre = 3·281 feet.

The foot-walks in this city and vicinity are generally formed of flags, or what is here termed blue-stone, laid on a bed of sand or cement-mortar. The flags are from 2″ to 4″ thick. In the more important streets the upper surface is axed, the quality of the stone selected, and the sidewalk often covered by a single width of stone. Brick are often used in towns, or places where good flag can not be readily obtained, usually laid flatways on a sand-bed. Granite

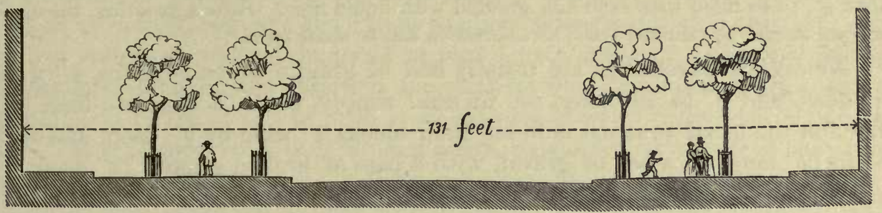


FIG. 878.

is very often employed in business streets, in lengths the full width of the sidewalk, and about 1′ in thickness, the inner ends resting on an iron girder, and the outer on the vault-wall, forming in this way a roof for the vault and the outer ends a curb for the street.

Curbs here are generally of flag, about 4″ thick by 20″ deep, extending 10″ above the gutter-stone; but, where the street is nearly level, and the gutter-stones have to be raised to give sufficient descent for the flow of the water, the curbs, in extreme cases, are not more than 4″ exposed. When sidewalks are stone of large dimensions, they extend over the curbs. The *gutter-stones* are from 12″ to 15″ in width, and from 3″ to 5″ in thickness, laid close, and bedded in cement. The *bridge* or crossing-*stones* are of blue-stone or granite, from 2′ to 15″ wide, and not less than 5″ thick, laid in double rows.

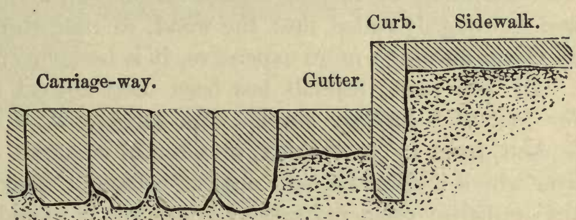


FIG. 879.

Carriage-way.—Streets and avenues were formerly paved entirely of cobblestone, and, if selected so as to be of a uniform size and shape, and properly bedded in sand and well rammed, they formed a cheap and very fair roadway; but the cubical block-stone pavement of trap or granite, often called the Belgian, is in every way to be preferred. Mr. Kneass, the engineer of the city of Philadelphia, recommends:

"That the blocks should not exceed 3″ in width, 6″ in depth, nor 8″ in length; that, as to depth, they should be uniform within ½″, and in length be not less than 6″. For foundations the material should be taken out to a depth of 20″ below the proposed surface of paving, and to be made to accurately conform to shape of finished road. After being compactly rolled with a heavy roller, it should have a covering of clean anthracite coal-ashes placed upon it to a depth of 10″, laid on in two layers, each well rolled; the ashes to be scrupulously clean—i. e., free from any organic matter. Upon this should be laid a bed of clean gravel, 4″ in depth, and rolled; upon which again should be a layer of sand, clean and sharp, or fine-screened gravel, in which to set and bed the stone blocks. Each layer of ashes and gravel should in surface conform to the outline intended for the surface of the stone. The stone should be carefully assorted, so that, when laid across the street, the joint-lines may be straight; and each stone should be *set on its bed* fair and square, so that no edge shall extend above the general level of the surface, and the surface of each stone shall be an extension of that lying next to it. The joints I would not make smaller than ½″, to be filled with sand and grouted with liquid lime. Before grouting, the entire surface should be rammed *until no impression can be made on it.*"

New York pavements are usually laid of granite or trap-blocks, 4″ wide, 6″ deep, and 8″ to 12″ long, set in sand simply, or on a concrete base. In London the usual practice is to set their blocks 3″ wide by 9″ deep, and from 6″ to 12″ long, on a bed of gravel, with a base of broken granite 12″ deep.

Wooden pavements of various kinds have been tried. The "Nicholson" consists of pieces of 3″ plank, 6″ long, set on a board base supported by a sand-bed. The plank is set on end in lines perpendicular to line of street, with a strip of board 1″ wide between the rows, nailed to the blocks; the top of strip being some 2″ to 3″ below top of plank. Boiled coal-tar is used freely while setting the blocks, and is poured into the interstices; the 1″ joint is filled with gravel, wet with tar, and well rammed. Instead of plank, blocks of wood, sawed from trunks or limbs, from 4″ to 9″ diameter, with the bark removed, are set on a board or plank base, with the interstices filled with gravel, into which coal-tar or melted asphalt is poured, and the top covered with gravel and thoroughly rolled into the wood, so that the wear is on the gravel. In all cases, although more expensive, it is better to make a concrete base.

Of late years, asphalt has been used abroad to a very great extent, both for foot and carriage ways. The carriage-ways are composed of a layer of asphalt, from 1½″ to 2″ thick, on a bed of concrete, or on a worn Macadam road, over which is spread a thin coat of cement. The cement having become dry, the asphaltic rock, reduced to a powder, is spread over the surface to a depth of about ·40 per cent more than the finished thickness; it is then rammed with rammers warmed by portable furnaces, beginning gradually, and increasing the force of the blows as the work approaches completion. For a footway the same concrete bed is used, and the layer of asphalt is about ⅝″. Walks and roads have been constructed in this country with an artificial asphalt, prepared from coal-tar mixed with gravel.

Pavements of mineral asphalt have also been laid in many of our cities. In Washington, the asphalt pavement consists of 6″ of hydraulic cement concrete and a wearing surface of bituminous mastic laid in two coats respectively ½″ and 2″ thick when compressed. The mastic is composed of the following parts by weight:

Asphaltic cement (refined Trinidad asphalt) 100 parts, petroleum oil 20 parts.	15 to 18
Limestone powder..	15 to 17
Sand..	70 to 65
	100 to 100

Roads and Highways.—Macadam was the first to reduce the construction of broken-stone roads to a science, and has given his name, in this country, to all this class of roads. He says that "the whole science of artificial road-making consists in making a dry, solid path on the natural soil, and then keeping it dry by a durable water-proof covering." The road-bed, having been thoroughly drained, must be properly shaped, and sloped each way from the center, to discharge any water that may penetrate to it. Upon this bed a coating of 3″ of clean broken stone, free from earth, is to be spread on a dry day. This is then to be rolled, or worked by travel till it becomes almost consolidated ; a second 3″-layer is then added, wet down so as to unite more readily with the first; this is then rolled, or worked, and a third and fourth layer, if necessary, added. Macadam's standard for stone was 6 ounces for the maximum weight, corresponding to a cube of 1½″, or such as would pass in any direction through a 2½″ ring. The Telford road is of broken stone, supported on a bottom course or layer of stone set by hand in the form of a close, firm pavement.

At the New York Central Park, after trials of the Macadam and Telford roads, the gravel-road (of which Fig. 880 is a cross-section of one half) was adopted, as being, according to the statement of their engineer, Mr. Grant, "the easiest and

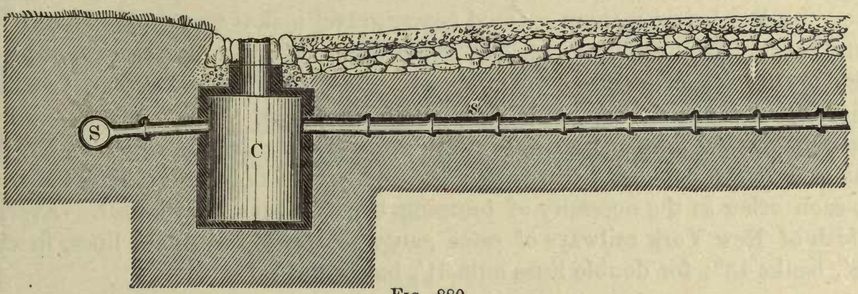


FIG. 880.

most agreeable kind of road for both carriages and horses ; that it is the cheapest at first cost, and can be kept in repair at an equal if not less cost than any other equally satisfactory road." This road consists of a layer of rubble-stones, about 7″ thick, on a well-rolled or packed bed, with a covering of 5″ of clean gravel. C is the catch-basin for the reception of water and deposit of silt from the gutters ; S is the main sewer or drain, and *s* a sewer-pipe leading to catch-basin on opposite side of the road. In wider roads each side has its own main drain, and there is no cross-pipe *s*. The road-bed was drained by drain-tiles of from 1½″ to 4″-bore, at a depth of 3′ to 3½ below the surface. The maximum grade of the Park roads is 1 in 20. The grades of the streets of Paris vary from 1 in 20 to 1 in 200. The best grade is from 1 in 50 to 1 in 100 ; this gives ample descent for the flow of water in the gutters. Many of our street-gutters have a pitch not exceeding 1 foot in the width of a block, or 200 feet.

The grade of a road is described as 1 in so many ; so many feet to the mile, or such an angle with the horizon.

Inclination.	Feet per mile.	Angle.	Inclination.	Feet per mile.	Angle.
1 in 10	528	5° 43′	1 in 30	176	1° 55′
1 " 11	462	5°	1 " 40	132	1° 26′
1 " 14	369	4°	1 " 50	106	1° 9′
1 " 20	264	2° 52′	1 " 57	92	1°
1 " 29	184	2°	1 " 100	53	35″

The best transverse profile for a road on nearly level ground is that formed by two inclined planes, meeting in the center, and having the angle rounded. The degree of inclination depends somewhat on the surface of the road. A medium for broken-stone roads is about $\frac{1}{2}''$ in $1'$, or 1 in 24; but Telford, on the Holyhead Road, adopted 1 in 30; and Macadam, 1 in 36, and even 1 in 60. For paved streets in Paris, a crown of $\frac{1}{50}$ of the width is adopted, and for Macadamized, $\frac{1}{100}$. The inclination of sidewalks should not exceed $\frac{3}{4}''$ in 1 foot, and, when composed of granite, the surface should be roughened.

The necessity of a well-drained road-bed is as important beneath rails as on a highway. The cuts should be excavated to a depth of at least 2 feet below grade, with ditches at the sides still deeper, for the discharge of water. The embankments should not be brought within 2 feet of grade; this depth to be left in cut and on embankment for the reception of ballast. The best ballast is Macadam stone, on which the cross-ties are to be bedded, and finer broken stone packed between them. Good coarse gravel makes very good ballast; but sand, although affording filtration for the water, is easily disturbed by the passage of the trains, raising a dust, an annoyance to travelers, and an injury to the rolling-stock by getting into boxes and bearings. The average length of sleepers on the 4.8½ gauge railways is about 8 feet; bearing surface, 7″; distance between centers, 2 feet to 2′ 6″, except at joints, where they are as close to each other as the necessity of tamping beneath them will admit. Average width of New York railways of same gauge as above, for single lines, in cuts 18′, banks 13′; for double lines cuts 31′, banks 26½′.

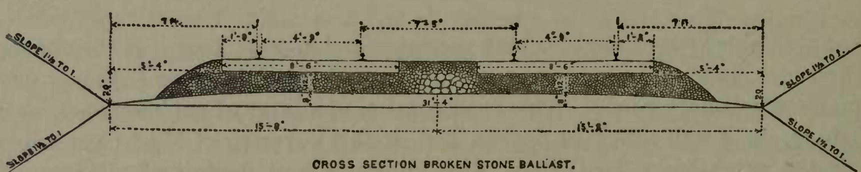


FIG. 881.

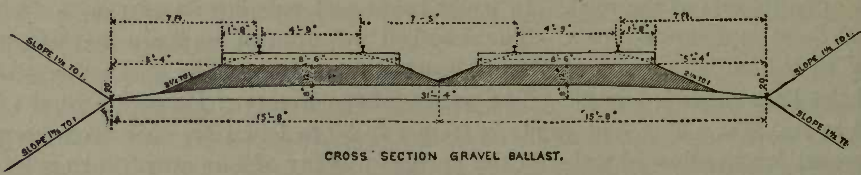


FIG. 882.

Figs. 881 and 882 are two standard sections of the permanent way of the Pennsylvania Railroad, in which the width of cuts and top of embankments are the same, 31′ 4″, and other dimensions equally ample.

Sections of rail are of infinite variety and weights, adapted to the class of railroads on which they are to be used, and the loads and speed of trains to which they are to be subjected. For roads of the common gauge, the weight of rails is from 56 to 70 lbs. per yard. The joints are made with a fish-plate.

Figs. 883, 884, and 885 are the elevation, section, and plan of the standard rail-joint of the New York, West Shore and Buffalo Railroad.

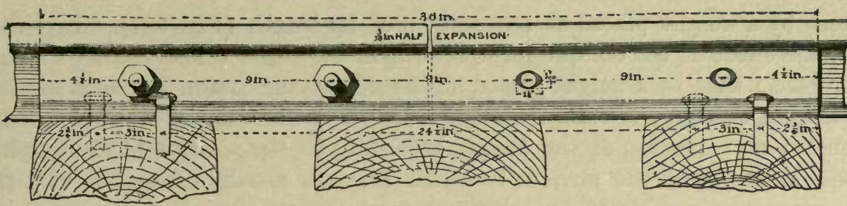


FIG. 883.

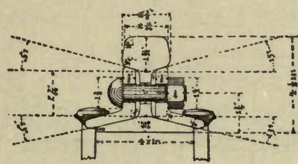


FIG. 884.

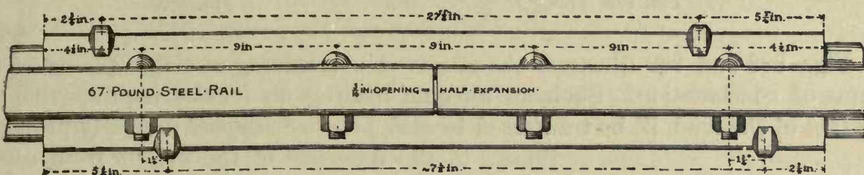


FIG. 885.

ROOFS AND BRIDGES.

At pages 238 and 239 will be found illustrations of the trussing of wooden beams. These are simple forms, which may be used in roofs or bridges, and rules are given for the proportion of parts. Rolled I-beams or plate-girders will serve also for floor-beams and moderate spans, but with modern necessities much more complicated structures are required.

On the General Principles of Bracing.—Let Fig. 886 be the elevation of a common roof-truss, and let a weight, W, be placed at the foot of one of the suspension-rods. Now, if the construction consisted merely of the rafter C′ B, and the collar-beam C′ C, resting against some fixed point, then the point B would support the whole downward pressure of the weight; but in consequence of the connection of the parts of the frame, the pressure must be resolved into components in the direction C′ A and C′ B; C′ *b* will represent the pressure in the direction C′ B, C′ *w* the portion of the weight supported at B, C′ *a* the pressure in the direction C′ A, and *w* W the portion of the weight supported on A. The same resolution obtains to determine the direction and amount of force exerted on a bridge-truss of any number of

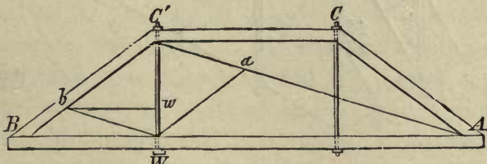


FIG. 886.

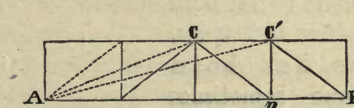


FIG. 887.

panels, by a weight placed at any point *p* of its length (Fig. 887). In either case, the effect of the oblique form C′ A upon the angle C is evidently to force it upward ; that is, a weight placed at one side of the frame has, as in case of the arch, a tendency to raise the other side. The effect of this upward force is a tension on a portion of the braces, according to the position of the weight; but as braces, from the manner in which they are usually connected with the frame, are not capable of opposing any force of extension, it follows that the only resistance is that which is due to the weight of a part of the structure.

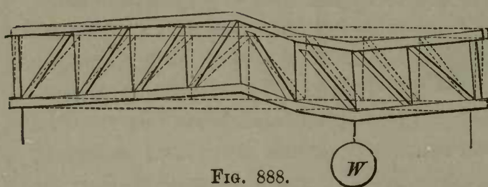


Fig. 888.

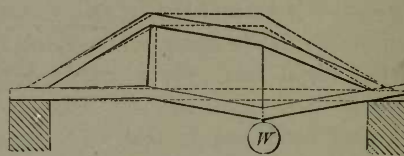


Fig. 889.

Figs. 888 and 889 illustrate the effects of overloading at single points such forms of construction. Such an unequal loading on trusses requires that a *portion* of the load *W* be tranferred to *each point of support* inversely proportionate to the distances of the weight from each support. The above trusses are not prepared to transfer this weight to but one support. To remedy the difficulty, it will be necessary to add braces running in the opposite direction, as shown by dotted lines (Fig. 890), at every point subject to the above distortion. These are called *counter-braces.*

Fig. 890.

To prevent the *braces* from becoming loose when the counter-braces are in action, it is always customary to give the braces and counter-braces an initial compression, by putting a moderate tension on the suspension-rods. In this case, therefore, the passage of a load would produce no additional strain upon any of the timbers, but would tend to relieve the counters. The counter-braces do not, of course, assist in sustaining the weight of the structure ; on the contrary, the greater the weight of the structure itself, the more will the counter-braces be relieved.

If, instead of the counter-braces, the braces themselves are made to act both as ties and struts, as has been done sometimes in iron bridges and trusses, then the upward force will be counteracted by the tension of the brace.

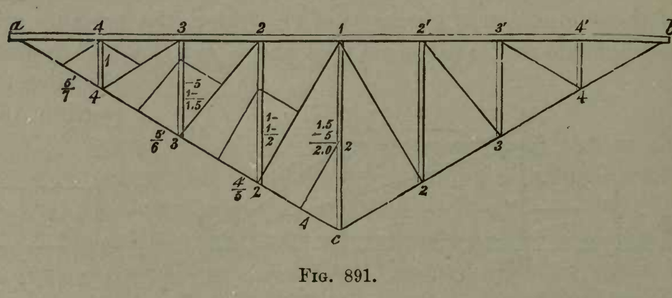


Fig. 891.

Suppose a system to be composed of a series of suspension-trusses, as in Fig. 891, in which the load is uniformly distributed. If we represent the load at each of the points, 4, 3, 2, 1, 2′, etc., by 1, the load at 4 will be supported $\frac{1}{2}$ upon *a* and $\frac{1}{2}$ upon 3 ; hence the strut 3 will have to support a load of $1 + {\cdot}5 = 1{\cdot}5$; of this, $\frac{2}{3}$ will be supported by 2 and $\frac{1}{3}$ by *a* ; $\frac{2}{3}$ of $1{\cdot}5 = 1$, $1 + 1 = 2$, load on strut 2 ; $\frac{3}{4}$ of this load, or $1{\cdot}5$, will be supported at 1, and since from the opposite side there is an equal force exerted at 1, therefore the strut 1 supports $1 + 1{\cdot}5 + 1{\cdot}5 = 4$.

The tension on the rod $c\text{–}2 = 2\ \dfrac{c\,a}{c\,1}$.

"	"	$2\text{–}3 = 2\frac{1}{2}$	"
"	"	$3\text{–}4 = 3$	"
"	"	$4\text{–}a = 3\frac{1}{2}$	"

If this construction be reversed, the parts which now act as ties must be made as braces, and braces, ties ; then we have a roof-truss, and the force exerted on the several parts may be estimated in a similar way as for the suspension-truss.

It is evident that neither of these constructions would serve for a bridge-truss, subject to the passage of heavy loads, but is only fit to support uniform and equally distributed loads.

To frame a construction so that it may be completely braced—that is, under the action of any arrangement of forces—the angles must not admit of alteration, and consequently the shape can not. The form should be resolvable into either of the following elements (Figs. 892, 893, and 894) :

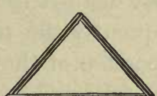


FIG. 892.

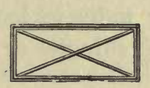


FIG. 893.

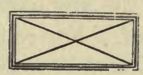


FIG. 894.

In these figures, lines ═══════ represent parts required to resist compression ; lines ——————— parts to resist tension only ; lines ≡≡≡≡≡≡≡ parts to resist both tension and compression.

It is evident that, in a triangle (Fig. 892), an angle can not increase or diminish, without the opposite angles also increasing or diminishing. In the form Fig. 893, a diagonal must diminish ; in Fig. 894 a diagonal must extend, in order that any change of form may take place. Consequently, all these forms are completely braced, as each does not permit of an effect taking place, which would necessarily result from a change of figure. Hence, also, any system composed of these forms, properly connected, breaking joint as it were into each other, must be braced to resist the action of forces in any direction ; but as in general all bridge-trusses are formed merely to resist a downward pressure, the action on the top chord being always compression, it is not necessary that these chords should act in both capacities.

Roofs.—The roofs of city dwellings and stores are generally flat, that is, with but very little inclination, from half an inch to two inches per foot, merely sufficient to discharge the water. The beams are laid from wall to wall, the same as floor-timbers, but usually of less depth, or at greater distances between centers, and with one or two rows of bridging.

Figs. 895, 896, and 897 represent side or portions of side elevations of the usual form of framed roofs. The same letters refer to the same parts in all the figures. T T are the *tie*-beams, R R the *main rafters, rr* the *jack-rafters,* P P the *plates, p p* the purlines, K K the *king-posts, k k king-bolts, q q queen-bolts*—both are called *suspension*-bolts—C C the *collar* or *straining* beams, B B *braces* or *struts, b b* ridge-boards, *e* corbels.

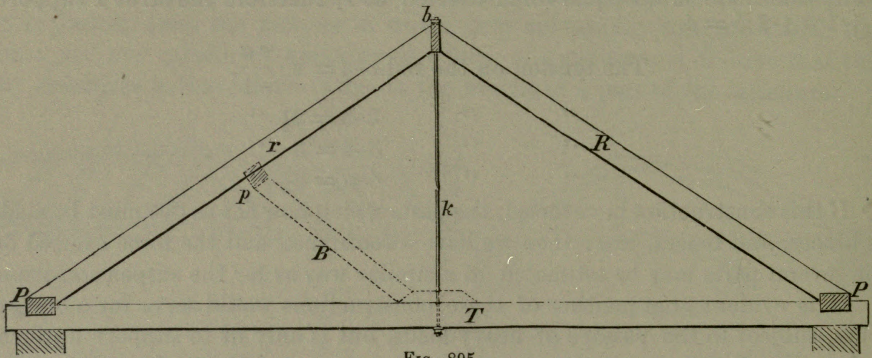


Fig. 895.

The *pitch* of the roof is the inclination of the rafters, and is usually designated in reference to the span, as $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{8}$, etc., pitch ; that is, the height of the ridge above the plate is $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{8}$, etc., of the span of the roof at the level of the plate. The steeper the pitch of the roof, the less the thrust against the side-walls, the less likely the snow or water to lodge, and consequently the tighter the roof. For roofs covered with shingles or slate, in this portion of the coun-

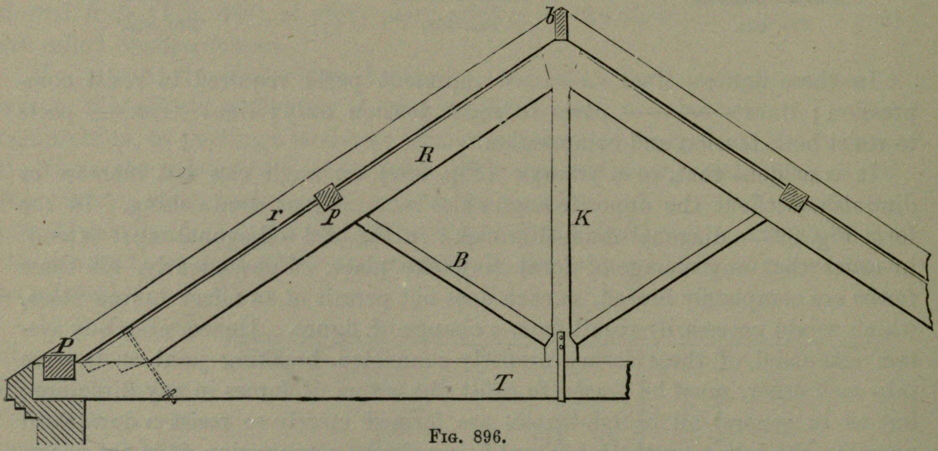


Fig. 896.

try, it is not advisable to use less than $\frac{1}{4}$ pitch ; above that, the pitch should be adapted to the style of architecture adopted. The pitch in most common use is $\frac{1}{3}$ the span.

Fig. 895 represents the simplest framed roof : it consists of rafters, resting upon a plate framed into the ceiling-beam ; this beam is supported by a suspension-rod, *k*, from the ridge, but, if supported from below, this rod may be omitted. As shown, the rafters are to be spaced from 1 to 2 feet centers, and the tie-beams at intervals of from 6 to 8 feet ; the roof cover to be of boards nailed directly to the rafters. This form of construction is sufficient for any roof of less than 25 feet span, and of the usual pitch, and may be used for a 40-foot span by increasing the depth of the rafters ; deep rafters should always be bridged. By the introduction of a purline extending beneath the center of the

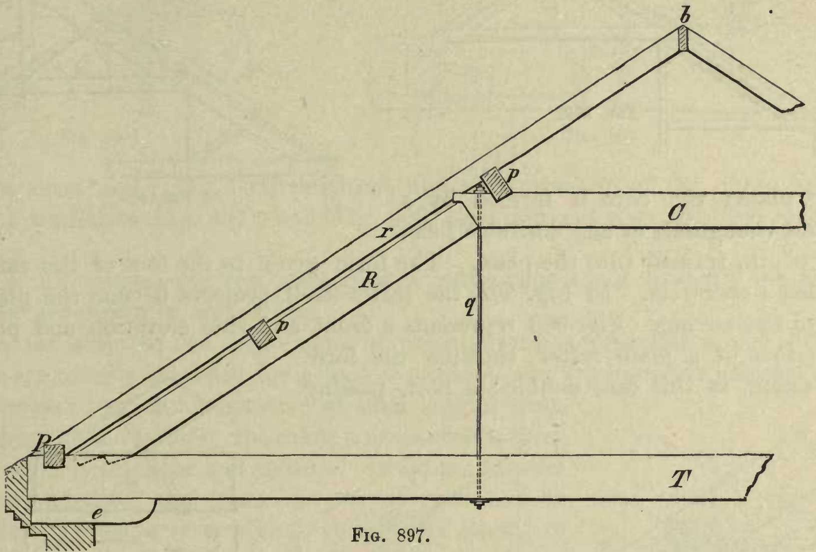


FIG. 897.

rafter, supported by a brace to the foot of the suspension-rod, as shown in dotted line, the depth of the rafters may obviously be reduced. It often happens that the king-bolt may interfere with the occupancy of the attic ; in that case the beam is otherwise supported. Again, it may be necessary that the tie-beam, which is also a ceiling and floor beam, should be below the plate some 2 to 4 feet ; in that case, the thrust of the roof is resisted (Fig. 898) by bolts, *b b*, passing through the plate and the beam, and by a collar-plank, C, spiked on the sides of the rafters, high enough above the beam to afford good head-room. For roofs $\frac{3}{8}$ pitch and under 20 feet span, the bolts are unnecessary, the collar alone being sufficient.

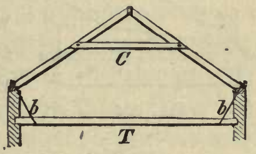


FIG. 898.

Fig. 896 represents a roof, a larger span than Fig. 895 ; the frame may be made very strong and safe for roofs of 60 feet span. King-bolts or suspension-rods are now oftener used than posts, with a small triangular block of hard wood or iron, at the foot of the bolts, for the support of the braces. The objection to this form of roof is that the framing occupies all the space in the attic ; on this account the form, Fig. 897, is preferred for roofs of the same span, and is also applicable to roofs of at least 75 feet span, by the addition of a brace to the rafter from the foot of the queen-bolt. The collar-beam (Fig. 900) is also trussed by the framing similar to Fig. 896.

In many church and barn roofs the tie-beam is cut off (Fig. 899) ; the queen-post being supported on a post, or itself extending to the base, with a short tie-rod framed into it from the plate.

Figs. 901 and 902 are representations of the feet of rafters on an enlarged scale. In Fig. 901, the end of the rafter does not project beyond the face of

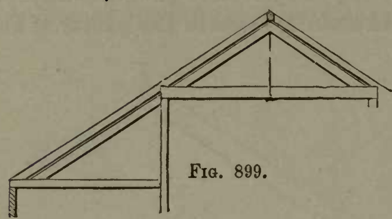


Fig. 899.

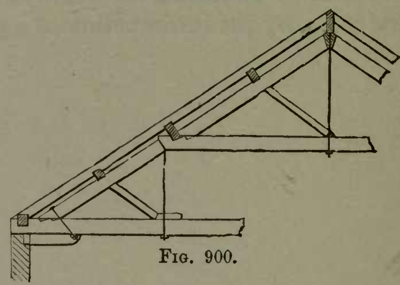


Fig. 900.

the plate; the cove is formed by a small triangular, or any desirable form of plank, framed into the plate. The form given to the foot of the rafter is called a *crow-foot.* In Fig. 902, the rafter itself projects beyond the plate to form the coving. Fig. 903 represents a front and side elevation and plan of the foot of a main rafter, showing the form of tenon, in this case double; a bolt, passing

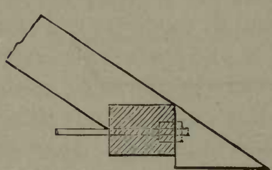


Fig. 901.

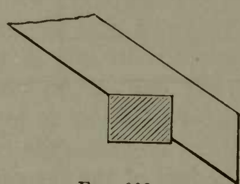


Fig. 902.

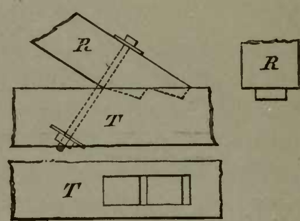


Fig. 903.

through the rafter and beam, retains the foot of the former in its place. Fig. 904 represents the foot of a main rafter, with a wooden shoe too short at *a*, outside of the rafter; it should be framed as in Fig. 903. In Fig. 901, of a similar construction to Fig. 895, the tie-rod passes directly through the plate. In general, when neither ceiling nor flooring is supported by the tie-beam, a rod is preferable.

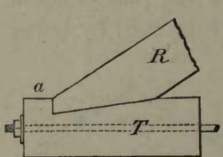


Fig. 904.

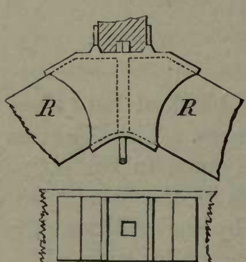


Fig. 905.

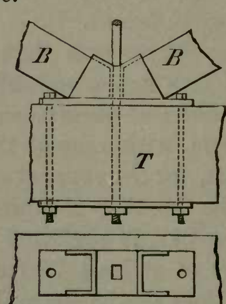


Fig. 906.

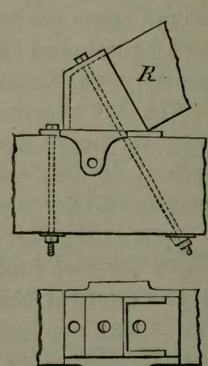


Fig. 907.

Roofs are now very neatly and strongly framed by the introduction of cast-iron shoes and abutting plates for the ends of the braces and rafters. Fig. 905

represents the elevation and plan of a cast-iron king-head for a roof similar to Fig. 896; Fig. 906, that of the brace-shoe; Fig. 907, that of the rafter-shoe

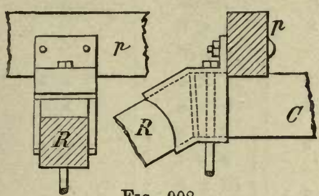


FIG. 908.

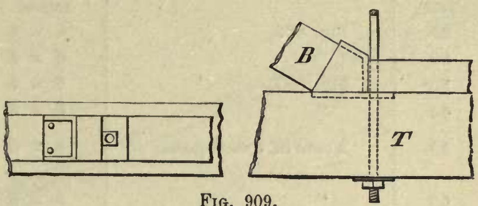


FIG. 909.

for the same roof; Fig. 908, the front and side elevation of the queen-head of roof similar to Fig. 897; and Fig. 909, elevation and plan of queen brace-shoe.

Fig. 910 represents the section of a rafter-shoe for a tie-rod; the side flanches are shown in dotted line.

On *the size and the proportions* of the different members of a roof: Tie-beams are usually intended for a double purpose, and are therefore affected by two strains: one in the direction of their length, from the thrust of the rafters; the other a cross-strain, from the weight of the floor and ceiling. In estimating the size necessary for the beam the thrust need not be considered, because it is always abundantly strong to resist this strain, and the dimensions are to be determined as for a floor-beam merely, each point of suspension being a support. When tie-rods are used, the strain is in the direction of their length only, and their dimensions can be calculated, knowing the pitch, span, and weight of the roof per square foot, and the distance apart of the ties, or the amount of surface retained by each tie.

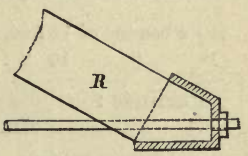


FIG. 910.

The weight of the wood-work of the roof may be estimated at 40 pounds per cubic foot; slate at 7 to 9 pounds, shingles at 1½ to 2 pounds per square foot. The force of the wind may be assumed at 15 pounds per square foot. The excess of strength in the timbers of the roof, as allowed in all calculations, will be sufficient for any accidental and transient force beyond this. Knowing the weights, pressures, and their directions on parts of a roof, their stresses may be determined by the parallelograms of forces and dimensions proportioned to the strength of the materials of which the roof is composed. It will generally be sufficient for the draughtsman to have practical examples of construction to draw from. Dimensions are therefore given of the parts of wooden roofs already illustrated. With further examples of actual constructions, the beams are usually proportioned to the weight that they are to sustain in floors and load, but where tie-rods are used, the stress upon them may be determined by the following rule:

Rule.—Multiply one half the weight of the roof and load by one half the span, and divide the product by the rise or height of ridge above eaves.

Gwilt, in his "Architecture," recommends the following dimensions for portions of a roof:

Span.	Form of Roof.	Rafters.	Braces.	Posts.	Collar-beams.
Feet.		Inches.	Inches.	Inches.	Inches.
25	Fig. 896,	5 × 4	5 × 3	5 × 5	
30	"	6 × 4	6 × 3	6 × 6	
35	Fig. 897,	5 × 4	4 × 2	4 × 4	7 × 4
45	"	6 × 5	5 × 3	6 × 6	7 × 6
50	2 sets of queen-posts,	8 × 6	5 × 3	{ 8 × 8 8 × 4 }	9 × 6
60	" "	8 × 8	6 × 3	{ 10 × 8 10 × 4 }	11 × 6

These dimensions, for rafters, are somewhat less than the usual practice in this country ; no calculations seem to have been made for using the attic. An average of common roofs here would give the following dimensions nearly : 30 feet span, 8 × 5 inches ; 40 feet, 9 × 6 ; 50 feet, 10 × 7 ; 60 feet, 11 × 8 ; collar-beams the same size as main rafters. Roof-frames from 8 to 12 feet from center to center.

Dimensions for jack-rafters, 15 to 18 inches apart :

For a bearing of 12 feet.... 6 × 3 inches.
" " 10 " .... 9 × 3 "

For a bearing of 8 feet.... 4 × 3 inches.
" " 20 " ....10 × 3 "

Purlines :

Length of Bearing.	Distances apart in Feet.			
Feet.	6	8	10	12
8	7 × 5	8 × 5	9 × 5	9 × 6
10	9 × 5	10 × 5	10 × 6	11 × 6
12	10 × 6	11 × 6	12 × 7	13 × 8

The pressure on the plates is transverse from the thrust of the rafters, but in all forms except Fig. 895, owing to the notching of the rafters on the purlines, this pressure is inconsiderable. The usual size of plates for Figs. 895 and 896 is 6 × 6 inches.

In the framing of roofs, it is now customary, for roofs of mills, to omit purlines, jack-rafters, and plates, and make the roof-boards of plank stiff enough to supply their places, from 2″ to 3″ thick (according to the space between the frames), tongued and grooved, and strongly spiked to the main rafters.

The dimensions of rafters depend on the distances between their supports and between centers. The depth in all such cases to be greater than the width ; 2 to 6 inches may be taken as the width, 8 to 12 for the depth.

When there are no purlines, and the roof is covered with plank, there is no need of plates ; the plank forms a deep beam, and, if the ends of the frame are secured, there may be no need of intermediate ties.

Iron Roofs.—Roofs of less than 30 feet span are often made of corrugated iron alone, curved into a suitable arc, and tied by bolts passing through the iron about 2 to 4 feet above the eaves.

Fig. 911 represents the half elevation of an iron roof of a forge at Paris; Figs. 912, 913, and 914, details on a larger scale. This is a common type of

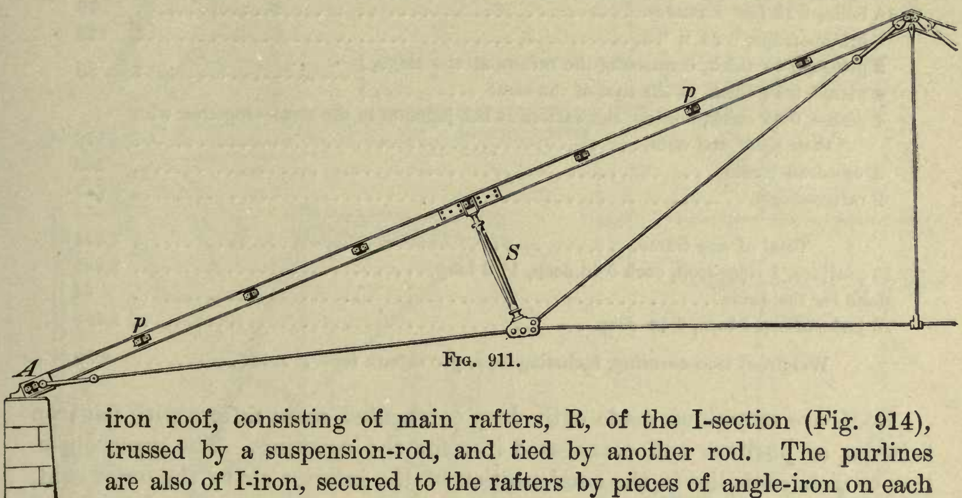


FIG. 911.

iron roof, consisting of main rafters, R, of the I-section (Fig. 914), trussed by a suspension-rod, and tied by another rod. The purlines are also of I-iron, secured to the rafters by pieces of angle-iron on each side; and the roof is covered with either sheet-iron resting on jack-rafters, or corrugated iron extending from purline to purline. The rafter-shoe, A, and the strut,

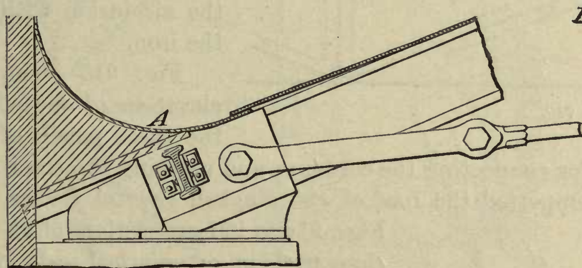


FIG. 912.

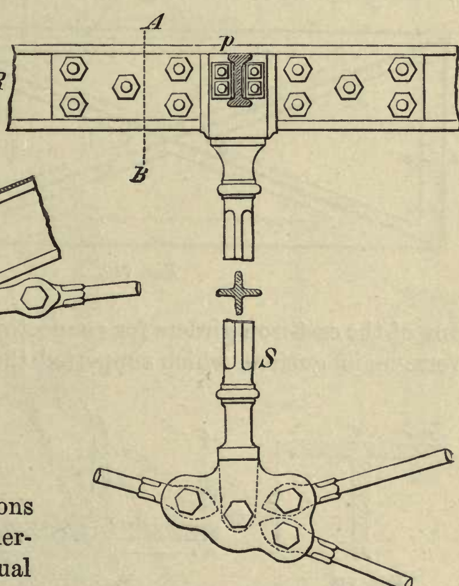


FIG. 913.

S, are of cast-iron; all the other portions of the roof are of wrought-iron. In American practice it is usual to make the strut of wrought-iron, with a single pin connection at its foot, instead of as in the figure.

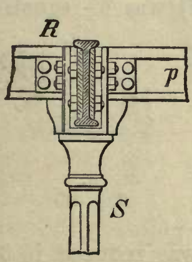


FIG. 914.

The surface covered by this particular roof is 53 metres (164 feet) long and 30 metres (98½ feet) wide. There are eleven frames, including the two at the ends, which form the gables.

The following are the details of the dimensions and weights of the different parts:

	Pounds.
2 rafters, 0·72 feet deep; length together, 99·1 feet	1,751
5 rods, 0·13 feet diameter; length together, 131·4 feet	882
16 bolts, 0·13 feet diameter	79
8 bridle-straps, 0·24 × ·05	123
2 pieces, 0·46 thick, connecting the rafters at the ridge, 4 pieces, 0·46 thick, at the foot of the strut	88
4 pieces, 0·36 thick, uniting the rafters at the junction in the strut—together with their bolts and nuts	176
2 cast-iron struts	308
2 rafter-shoes	287
Total of one frame	3,694
16 purlines, 1 ridge-iron, each 0·46 deep, 17·2 long	2,985
Bolts for the same	64
16 jack-rafters, I-iron, 0·16 deep	2,489
Weight of iron covering, including laps, per square foot	2·88

Roofs are sometimes made with deep corrugated main rafters with flat iron between, or purlines and corrugated iron for the covering. The great objection to iron roofs lies in the condensation of the interior air by the outer cold, or, as it is termed, sweating; on this account, they are seldom used for other buildings than boiler-houses or depots, except a ceiling be made below to prevent the contact of the air inside with the iron.

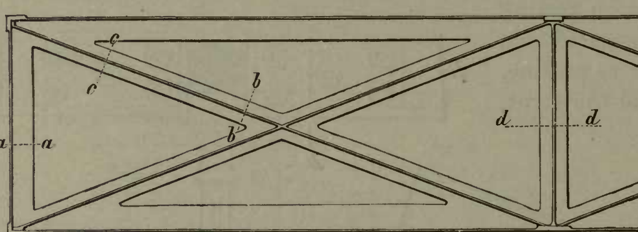


Fig. 915.

Fig. 915 is an elevation of one of the three panels of one of the cast-iron girders for connecting the columns, and carrying the transverse main gutters, which supported the roof of the English Crystal Palace. Figs. 916 to 921 are sections of various parts on an enlarged scale.

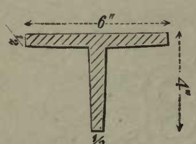


Fig. 916.

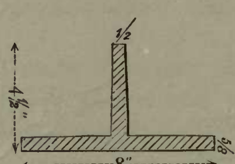


Fig. 917.

The depth of the girder was 3 feet, and its length was 23 feet $3\frac{3}{4}$ inches. The sectional area of the bottom rail and flange in the center (Fig. 917) was $6\frac{1}{4}$ square

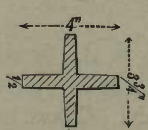


Fig. 918.

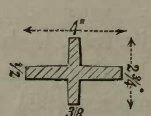


Fig. 919.

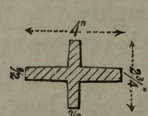


Fig. 920.

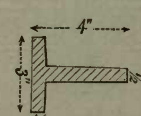


Fig. 921.

inches; the width of both bottom and top rail (Fig. 916) was reduced to 3 inches at their extremities.

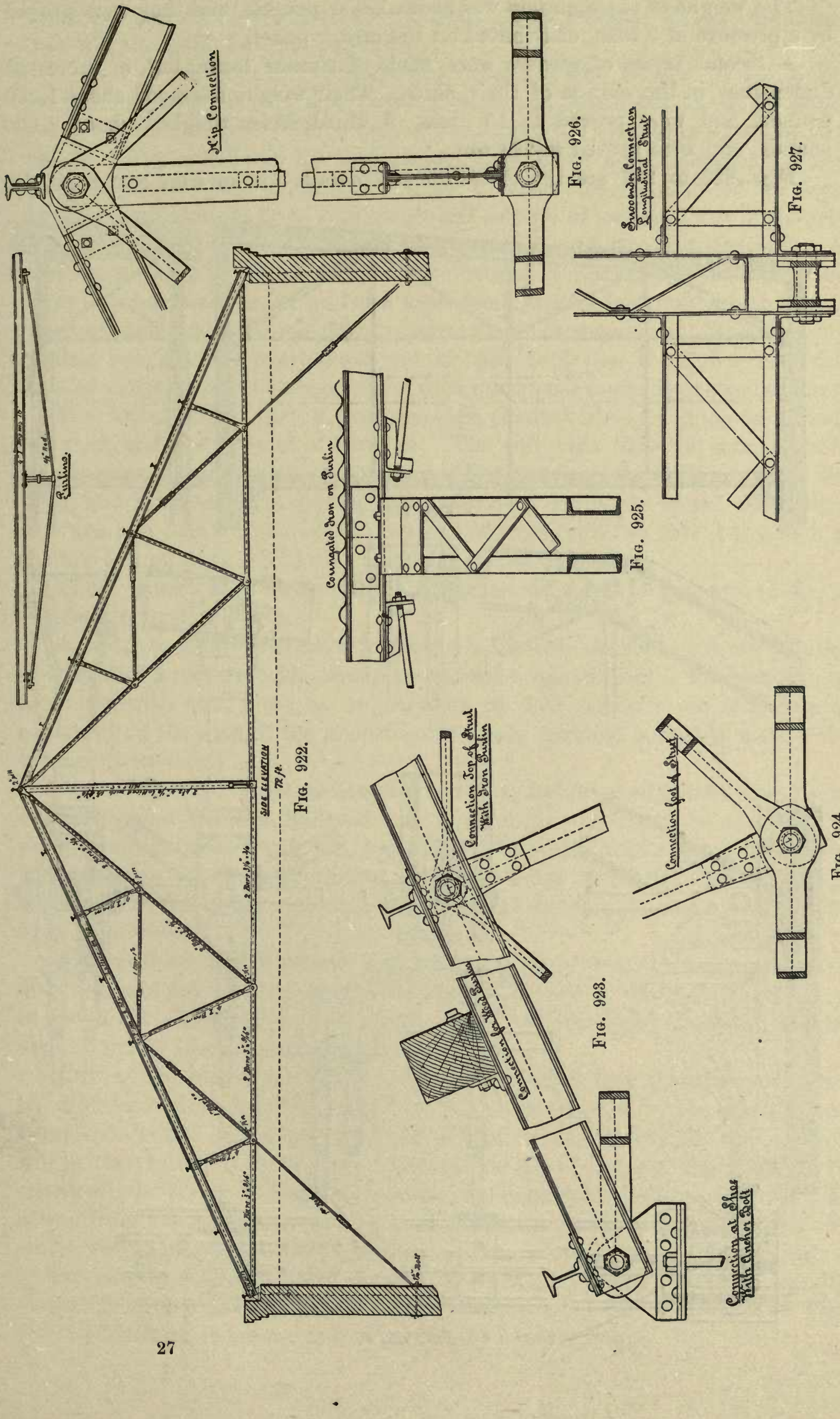


FIG. 922.

FIG. 923.

FIG. 924.

FIG. 925.

FIG. 926.

FIG. 927.

The weight of these girders was about 1,000 pounds, and they were proved by a pressure of 9 tons, distributed on the center panel.

A second series of girders were made of similar form, but of increased dimensions in the section of their parts. Their weight averaged about 1,350 pounds, and were proved to 15 tons. A third series weighed about 2,000 pounds, and were proved to 22½ tons.

Figs. 922 to 927 are the elevation and details for an iron roof-truss, for

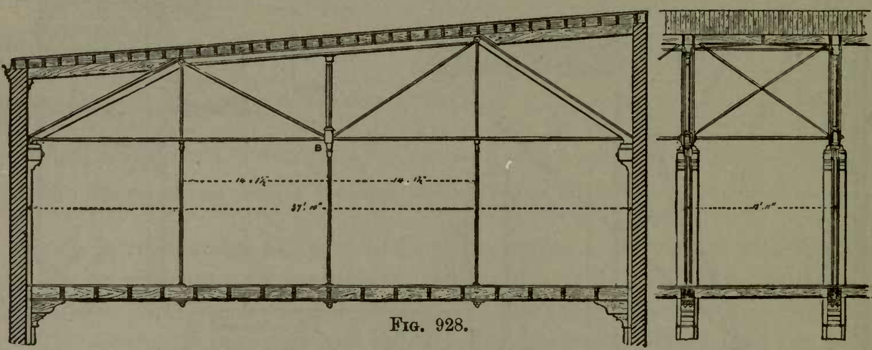


FIG. 928.

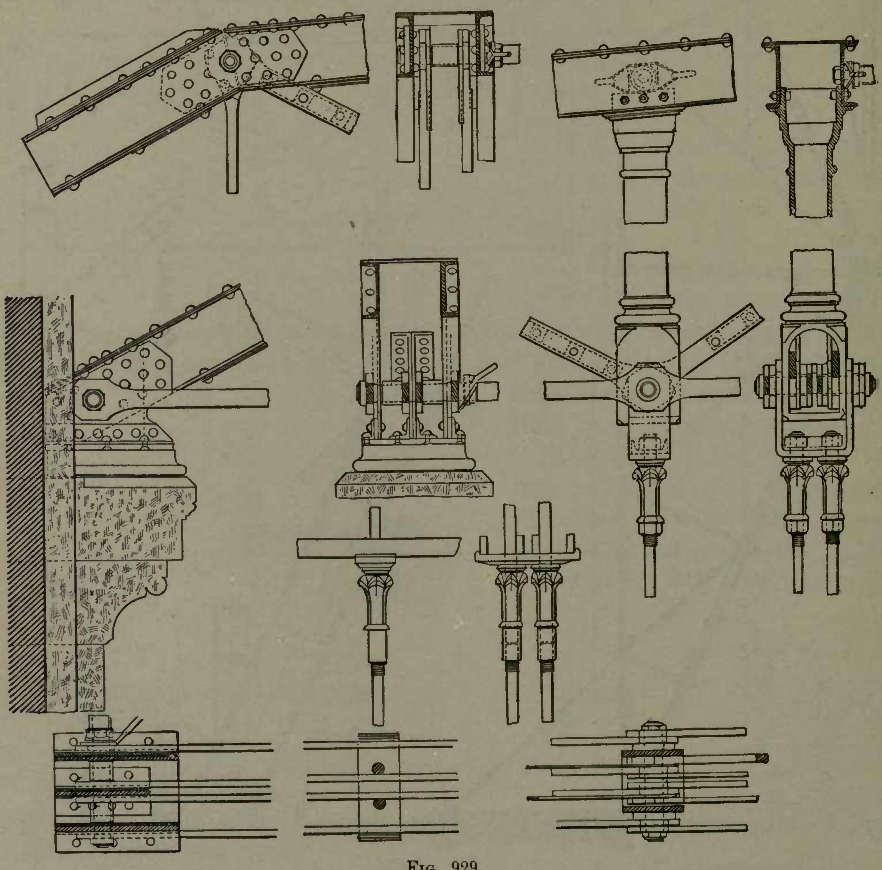


FIG. 929.

wood, slate, or corrugated iron covers, built by the Missouri Valley Bridge and Iron Works, A. S. Tulloch, engineer.

Figs. 928 and 929 are sections and details of the trusses for sustaining the roof and floor of the new English High and Latin School Gymnasium, Boston, Massachusetts. The object of sustaining the gymnasium-floor by rods was to secure a drill-hall for the military exercises of the school, and trusses were designed to have sufficient strength to resist the vibration of the floor. As the trusses were to be in sight, a central column of cast-iron was introduced to sustain the center of the top chord, instead of some wrought-iron construction less pleasing to the eye, with lattice between the main diagonals to enable them to act as counters, instead of a more complicated construction introducing counters, and a 3½-inch gas-pipe for horizontal bracing-struts. The floor-sustaining rods all have upset ends, and at their tops pass through ornamental foliated castings, but their connection with the trusses is wholly of wrought-iron.

The top chords consist of two nine-inch channel-irons weighing 50 pounds per yard, and one plate 12 × ⅝ inches. The end-posts have the same section. The bottom chord consists of four bars 2½ × 1 inch at the shallow end of the truss, and four bars 2½ × ¾ of an inch at the deep end of the truss. The diagonals are two bars 3 × 1 inch at deep end of truss, and two bars 3 × ½ inch at shallow end of truss. The pins are all 2½ inches diameter.

These trusses were designed and constructed by D. H. Andrews, C. E., of the Boston Bridge Works.

In order to secure free space in the room beneath the roof, it is my practice to construct a roof or bridge truss above, and suspend from it the roof framed as a floor, with such pitch as is requisite to shed rainfall. In this form of construction the span of the unobstructed space required is readily met by the truss construction.

Fig. 930 is a half cross-section of a two-story freight-shed for the New York, Lake Erie and Western Railroad. It is a simple and cheap construction of wood, readily framed and put together. The shed rests upon a pile-dock. The platform for the reception of freight is 4 feet above the dock-planking, and about 26 feet wide, with occasional inclined runs for the transfer of freight to or from vessels.

Bridge-Trusses.—Whatever may be the form of truss or arrangement of the framing, provided its weight is supported only at the ends, the tension of the lower chord, or the compression of the upper chord at center, may be determined by this common rule :

Rule.—The sum of the total weight of the truss, and the maximum distributed load which it will be called on to bear, multiplied by the length of the span, and divided by 8 times the depth of the truss in the middle, the quotient will be the tension of lower chord and compression of upper at the middle. In nearly all the forms of diagonal bracing, if the uniform load be considered as acting from the center toward each abutment, each tie or brace sustains the whole weight between it and the center, and the strain is this weight multiplied by the length of tie or brace, divided by its height. Any diagonals, equally distant from the center, sustain all the intermediate load : if rods, as in Fig. 932, by tension ; if braces, as in Fig. 931, by compression.

CROSS-SECTION OF ONE HALF OF A FREIGHT-SHED, NEW YORK, LAKE ERIE AND WESTERN RAILROAD.

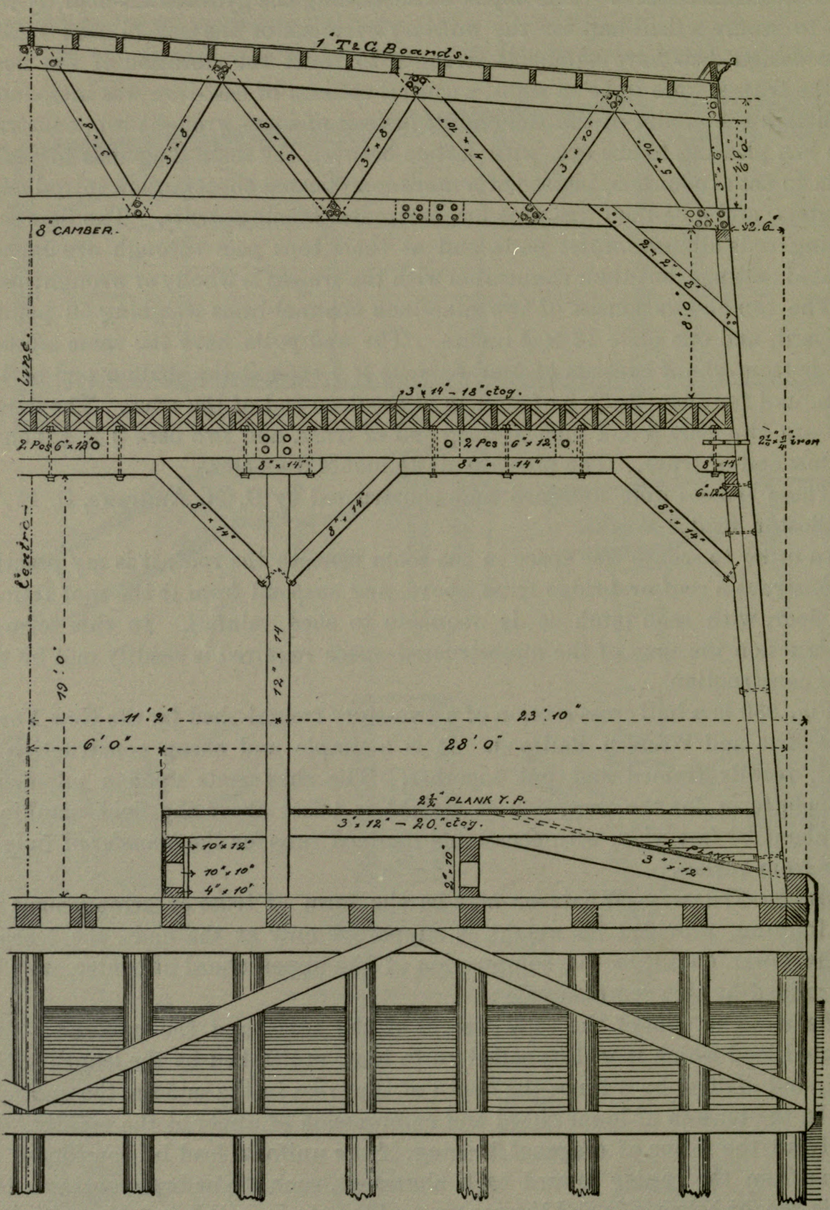


Fig. 930.

It follows, therefore, that in all these trusses the upper and lower chords should be stronger at the center than at the ends, while diagonals should be largest at the abutments. Unless the weight of the bridge is great compared with the moving loads, counter-braces become necessary.

The general rule adopted in the construction of the Howe truss is, to make the height of the truss $\frac{1}{8}$ of the length up to 60 feet span ; above this span the trusses are 21 feet high, to admit of a system of lateral bracing, with plenty of head clearance for a person standing on the top of a freight-car. From 175 feet to 250 feet span, height of truss gradually increased up to 25 feet. Moving load for railroad-bridge calculated at 1 ton per running foot. Center to center of panels not exceeding 11 feet.

Wooden Truss-Bridges.—Fig. 931 is the elevation of a few panels of a Howe truss, and Fig. 932 of a Pratt truss. The Howe truss is by far the most popu-

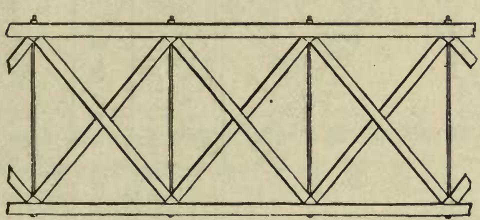


FIG. 931.

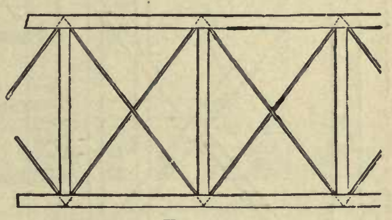


FIG. 932.

lar of all wooden trusses, being readily framed and put together, uniting great strength with simplicity of construction.

Fig. 933 is the side elevation of three of the five panels of a Howe truss highway-bridge of the New York, Lake Erie and Western Railroad. Fig. 934 is a cross-section. It will be observed that there is a section of 3″ plank laid close, and another beneath, laid with spaces; these planks are laid diagonally across the floor-beams, and at right angles to each other, and are made to act as lateral bracing. Fig. 935 are the details of the abutment end of bridge; the foot of the brace rests on a cast-iron shoe. The length over all—that is, including the portions on the abutment—is 81′ 2″, or 75 feet between abutments, usually designated as the span.

Figs. 936, 937, and 938 are the side elevation, floor cross-section, and plan of floor and bottom chord of three of the twelve panels of a single-track railway Howe truss. Their length is each 10′ $10\frac{5}{16}$″. The center braces are two, 7″ × 10″ ; the center rods three, $1\frac{1}{2}$″ diameter. The counters, each one 6″ × 8″ ; lateral brace top and bottom, 6″ × 6″; rods $1\frac{1}{4}$ inch; top chord, four pieces, 7″ × 12″; bottom chord, four pieces, 7″ × 15″; floor-beams, 7″ × 16″. The shoes, splices, and blocks between chord-timber are of cast-iron.

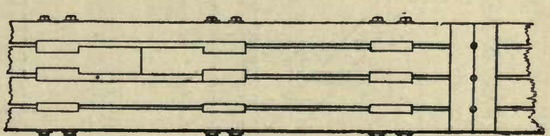


FIG. 939.

In the earlier practice the angle-blocks were of oak, and the splices made as in Fig. 939. Both of these were satisfactory.

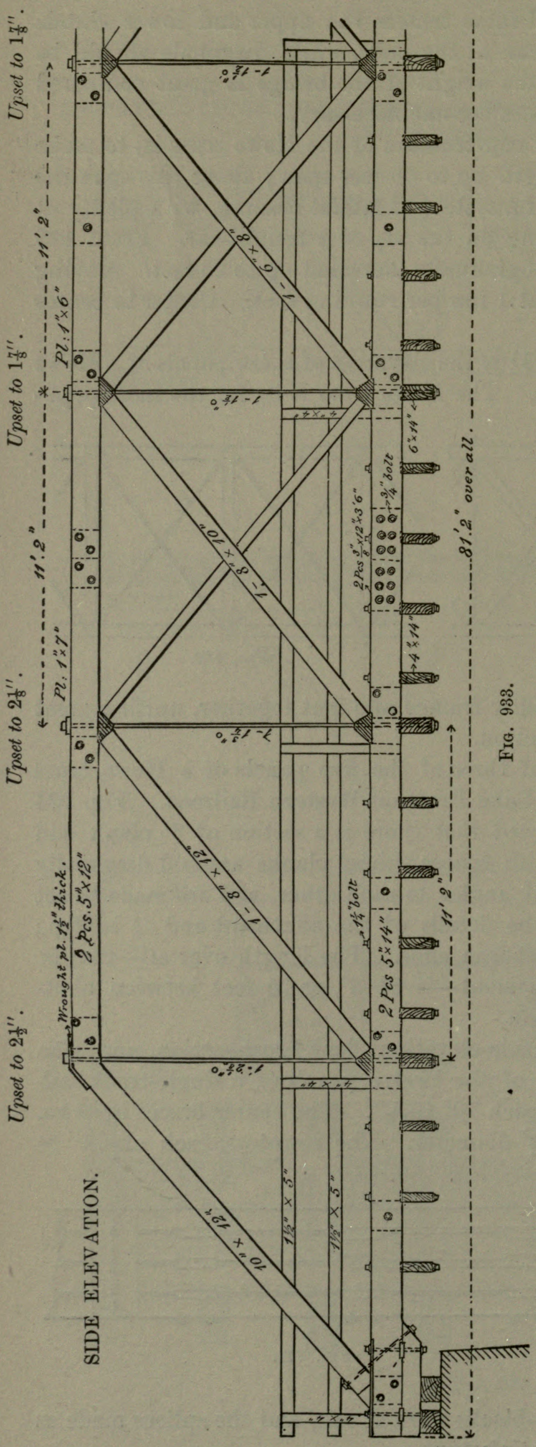


FIG. 933.

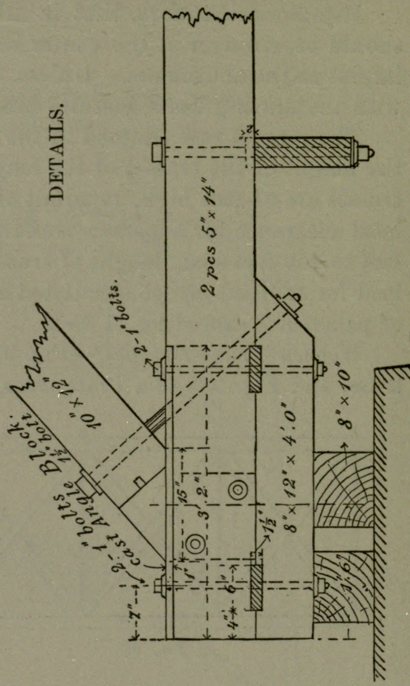


FIG. 935.

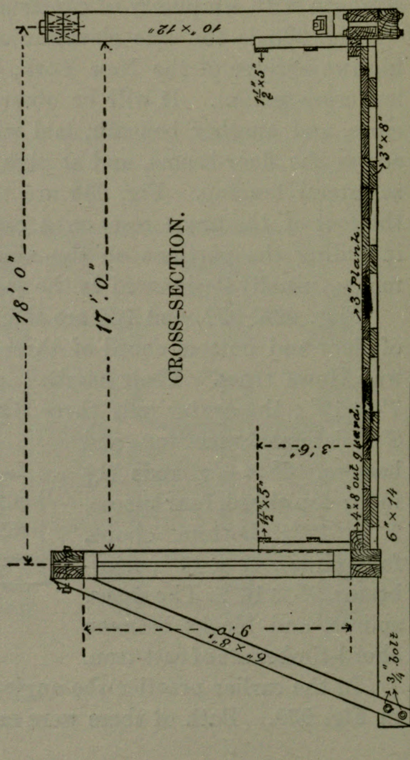


FIG. 934.

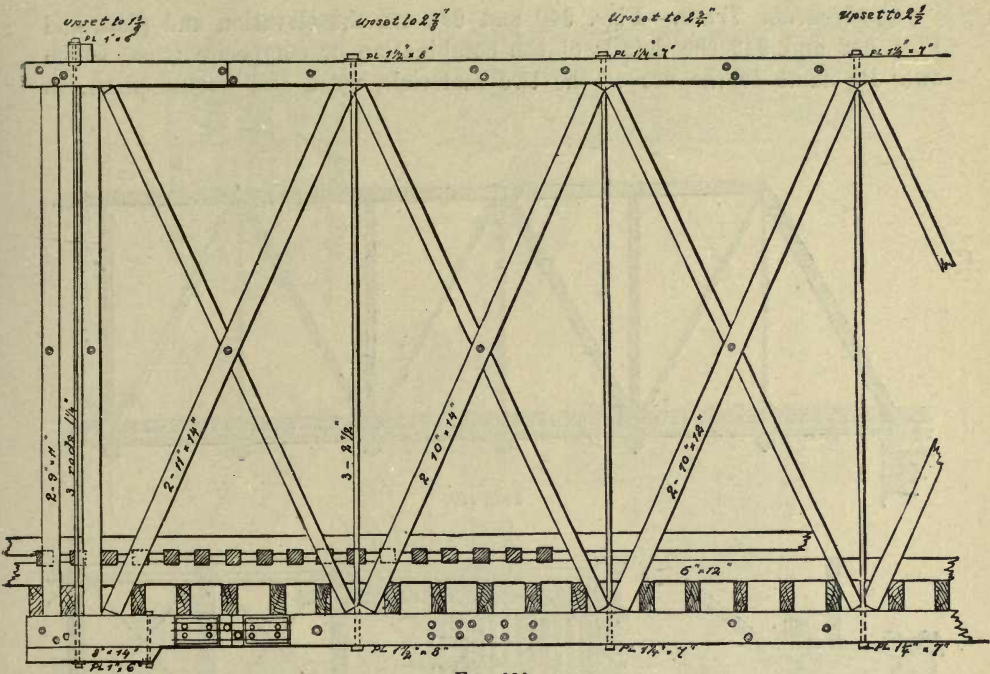


Fig. 936.

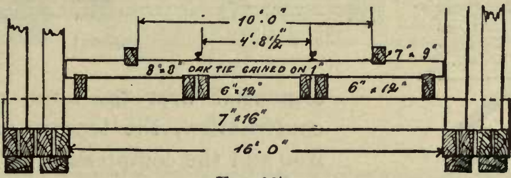


Fig. 937.

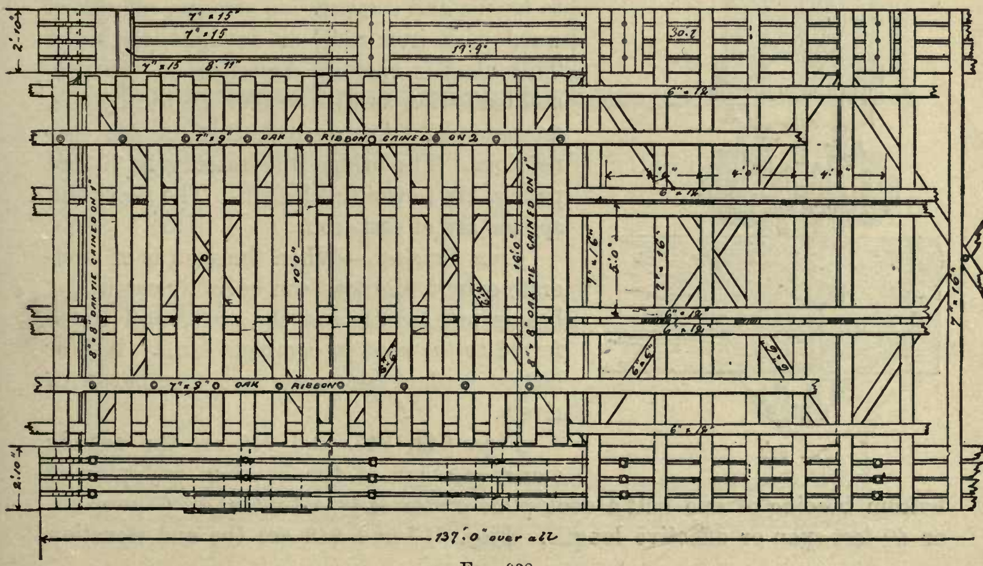


Fig. 938.

Combination Truss.—Figs. 940 and 941 are the elevation and plan, and Figs. 942 and 943 the details of the combination or composite truss, which owes its name to the use of the two materials, wood and wrought-iron, in

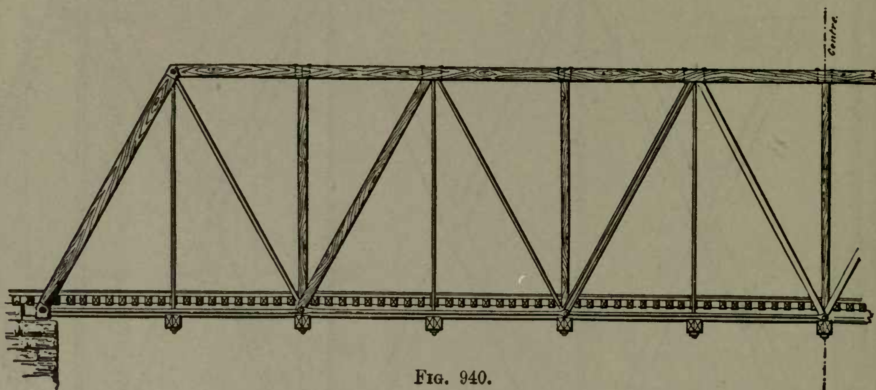
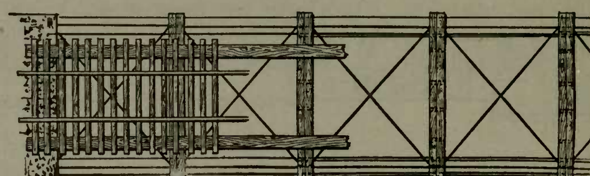


Fig. 940.



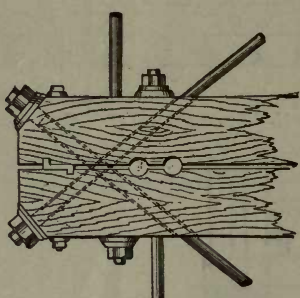


Fig. 941.

somewhat near the same proportion in its construction, the tension members being of iron and the compression of wood. The central braces, which are subjected alternately to tensile and compression stresses, may be of wood with iron rods, or wrought-iron only. This class is entirely American in practice, and embodies, as will be seen in the details, an essentially American feature, of *pin* connections. The bridge illustrated is in 30-foot panels, six to the full length. The shoes and splices are of cast-iron.

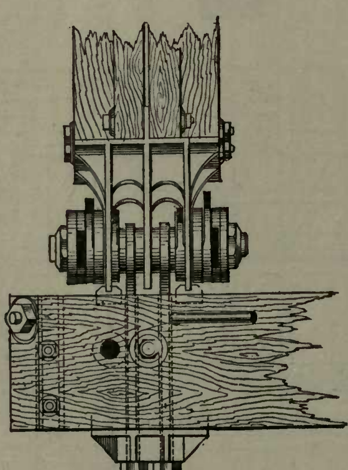


Fig. 942.

Iron Bridges.—When the span is of moderate extent, the load can be safely carried by beams put together at the works and transferred to the road in complete form. Web or lattice girders are used, put together with rivets.

Figs. 944, 945, and 946 are the outside elevation, plan of top bracing, and plan of bottom bracing of one half a deck plate-girder railway-bridge, 42′ 6″ over all, or 40 feet span or effective length. Figs. 947 and 948 are the end-elevation

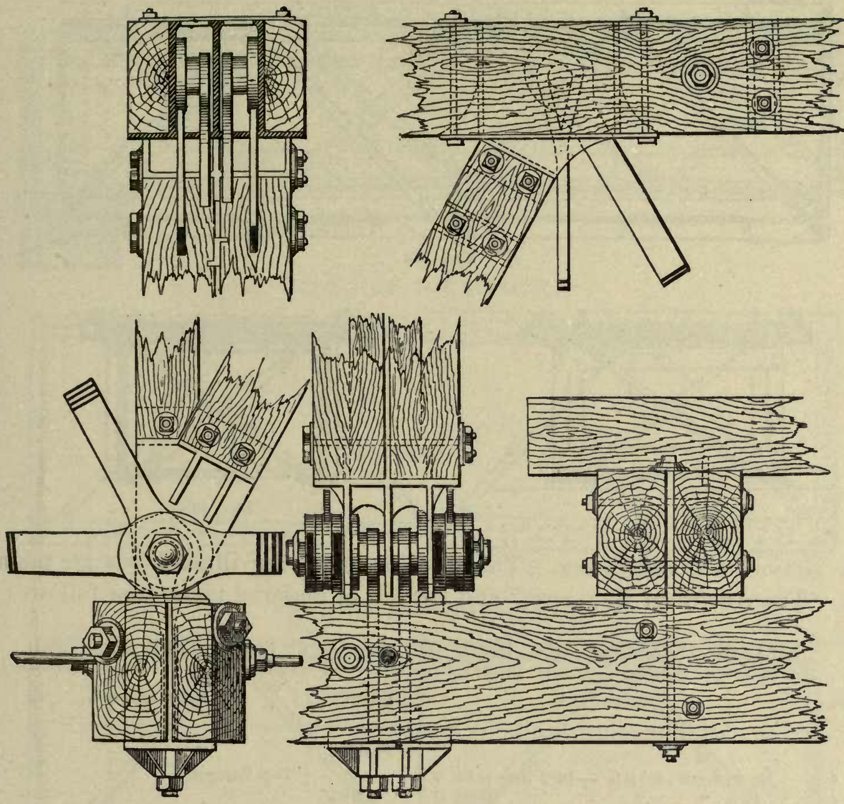


FIG. 943.

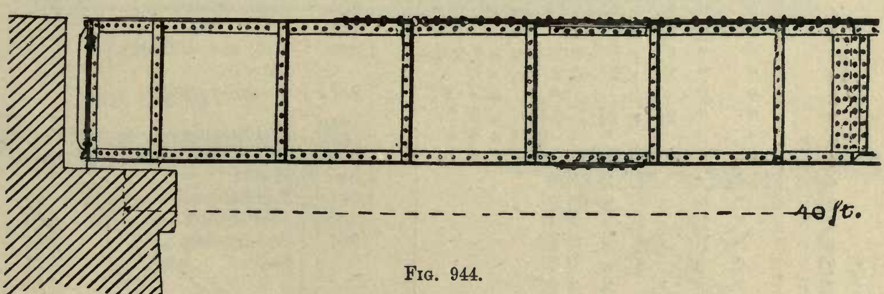


FIG. 944.

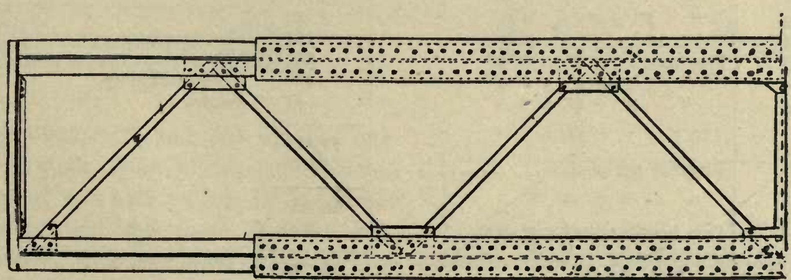


FIG. 945.

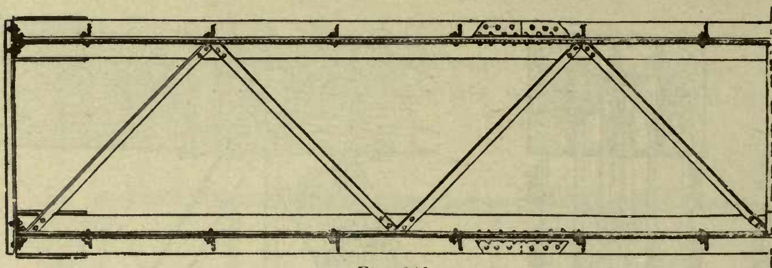


FIG. 946.

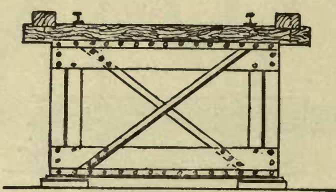


FIG. 947.

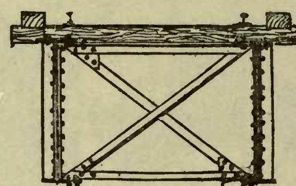


FIG. 948.

and a section near the center. This and the following illustrations are taken from "The American Engineer," and the bill of material given is as follows :

BILL OF MATERIAL FOR DECK GIRDER, 42′ 6″ LONG OVER ALL.

No.	DIMENSIONS.	Weight.	For what used.
		Pounds.	
4	Bars, angle, 4″ × 5″—14·2 lbs. × 14′ 0″		Top flanges.
4	" " " " × 28′ 0″	2,386	" "
4	" " 4″ × 6″—24½ lbs. × 14′ 0″		Bottom flanges.
4	" " " " × 28′ 0″	4,116	" "
4	" " 3½″ × 5″—20·8 lbs. × 2′ 8″	222	Angle-covers.
4	" " 3½″ × 4″—12 lbs. × 2′ 8″	128	" "
32	" " 3″ × 4″—8·3 lbs. × 3′ 10½″	1,029	Ends and stiffeners.
16	" " 3″ × 3″—7·2 lbs. × 7′ 5″		Lateral.
2	" " " " × 5′ 4″	931	Center-bracing.
2	" " 2½″ × 2½″—5 lbs. × 5′ 9″		" "
4	" " " " " × 6′ 5″		End-bracing.
4	" " " " " × 5′ 10″	303	" "
4	Plates, 48″ × ⅜″ × 21′ 0″	5,040	Webs.
2	" 14″ × ⅝″ × 29′ 0″	1,691	Top flanges.
4	" 12″ × ⅜″ × 3′ 4″	200	Joint-covers.
4	" 10″ × 5/16″ × 6′ 5″	267	End-bracing.
7	" 9″ × ⅜″ × 1′ 9″		Lugs.
1	" " × 2′ 0″		"
1	" " × 1′ 0″	172	"
2	" 8″ × ⅜″ × 1′ 0″	20	"
4	" 14″ × ½″ × 2′ 0″	187	Bearing-plates.
32	Flat bars, 3″ × ⅝″ × 3′ 4″	667	Fillers.
24	" 3″ × ½″ × 3′ 4″	400	Inside stiffeners.
4	" 6″ × ½″ × 2′ 5″	97	Joints.
		17,856	
	Rivets 6 per cent....................	1,070	
		18,926	
4	Cast bearing-blocks, @ 200..........	800	
	Total weight................ ...	19,726	

OUTSIDE ELEVATION.

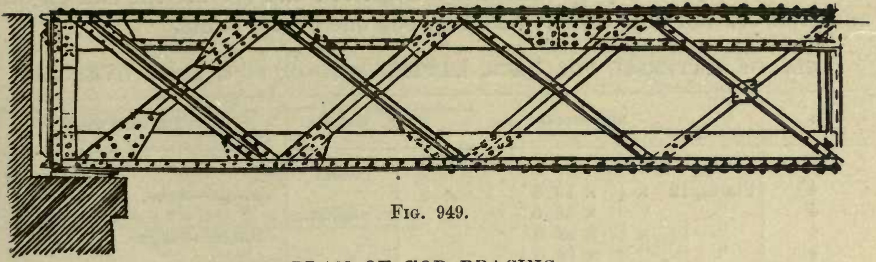


Fig. 949.

PLAN OF TOP BRACING.

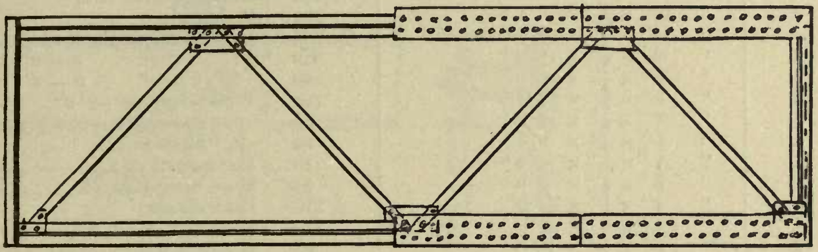


Fig. 950.

PLAN OF BOTTOM BRACING.

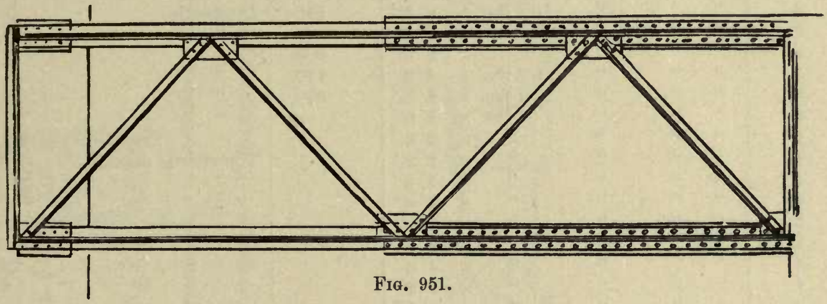


Fig. 951.

END ELEVATION.

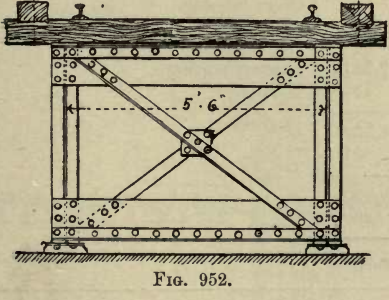


Fig. 952.

CROSS-SECTION NEAR CENTER.

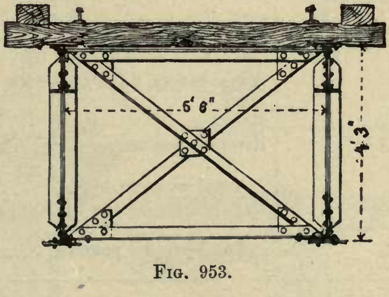


Fig. 953.

JOINT ⅞ RIVETS.

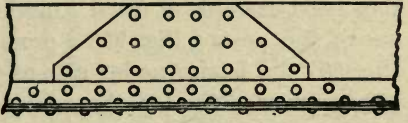


Fig. 954.

Figs. 949, 950, and 951 are the outside elevation, plan of top and bottom bracing of one half a deck lattice-girder railway-bridge, the same span as Fig. 944 above, and intended to carry the same load—rolling 4,000 pounds, and dead load 900 pounds per lineal foot. Figs. 952 and 953 are the end elevation and cross-section near center, and Fig. 954 one of the joints.

BILL OF MATERIAL FOR DECK LATTICE-BRIDGE, 42′ 6″ LONG OVER ALL.

No.	DIMENSIONS.	Weight.	For what used.
		Pounds.	
4	Plates, 12″ × ½″ × 13′ 6″		Chords—webs.
4	" " × 28′ 6″	3,360	" "
4	" 10″ × ⅝″ × 22′ 0″		Chords—flanges.
2	" " × 12′ 0″		" "
1	" " × 6′ 0″	1,475	Bearing-plates = 4.
4	" 10″ × $\frac{5}{16}$″ × 6′ 1″	254	End-bracing.
1	" 18″ × ½″ × 9′ 6″	285	Lugs on diagonals 8—12″ × ½″
1	" 9″ × ½″ × 18′ 8″	280	" " " 8— 9″ × ½″
1	" 6″ × ½″ × 16′ 0″	160	" " " 8— 6″ × ½″
1	" 9″ × ⅝″ × 16′ 4″	184	Web-covers—8—2′ 6″
1	" 7″ × ½″ × 2′ 8″	53	" gussets—4—0′ 8″
1	" 6″ × ½″ × 9′ 7″	96	" fillers = 12.
1	" 7″ × $\frac{5}{16}$″ × 1′ 4″	10	End-gussets = 2.
1	" 8″ × ⅝″ × 2′ 9″	28	Sway brace-lugs, etc.
1	" 7″ × ⅝″ × 24′ 0″	210	Lateral lugs.
1	" 8″ × ½″ × 9′ 0″	120	End-posts, 4—2′ 3″
2	" 5″ × ½″ × 10′ 0″	83	End-post splices, 16—1′ 3″
1	Flat bar, 3″ × ½″ × 12′ 0″	60	Fillers.
8	Bars, angle, 3″ × 3″—9·7 lbs. × 13′ 6″		Chords.
8	" " " " × 28′ 6″	3,249	"
8	" " " 10·8 lbs. × 6′ 3″	540	Diagonals.
8	" " " 8·2 lbs. × 6′ 0″		"
8	" " " " × 6′ 3″	804	"
4	" " " 6·8 lbs. × 6′ 3″	170	"
8	" " " 7·4 lbs. × 6′ 3″	370	"
8	" " " 6 lbs. × 6′ 0″		"
4	" " " " × 6′ 3″		"
4	" " " " × 2′ 2″		Top chords outside.
4	" " " " × 2′ 6″		" " "
4	" " " " × 2′ 9″		" " "
2	" " " " × 8′ 0″		" " "
4	" " " " × 4′ 9″		" " inside.
4	" " " " × 4′ 6″		" " "
4	" " " " × 2′ 3″		" " "
4	" " " " × 2′ 4″		" " "
4	" " " " × 6′ 1″		End-bracing.
2	" " " " × 6′ 0″		Cross-bracing.
2	" " " " × 5′ 5″		" "
8	" " " " × 4′ 3″		End posts.
16	" " " " × 7′ 6″	2,251	Lateral bracing.
8	Angle-covers, 2½ × 2½—8 lbs. × 1′ 10″	117	Chords.
		14,159	
	Rivets 6 per cent..	841	
	Total wrought-iron................	15,000	
4	Cast bearing-blocks @ 100 lbs.......	400	
	Total iron in pounds..............	15,400	

Figs. 955 to 959 are details of portions of a wrought-iron truss-bridge, a very good example of usual American practice. Fig. 955 is a side elevation of one of the posts ; Fig. 956 a cross-section as far as the first rail of the road ; Fig. 957 the lattice under side of the top chord—the top is a plate. Fig. 958 is a top view of the top chord, showing the lateral bracing, consisting of a lattice box-girder and diagonal rods. As the bridge is a skew, this box-girder is

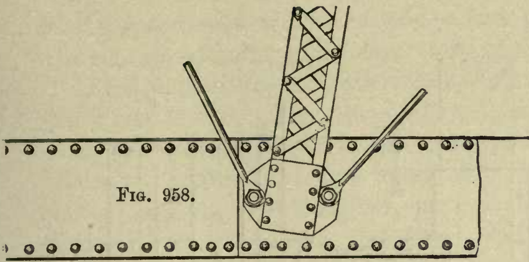


Fig. 958.

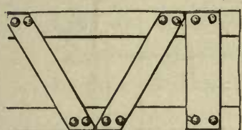


Fig. 957.

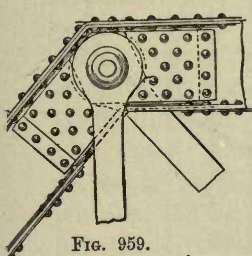


Fig. 959.

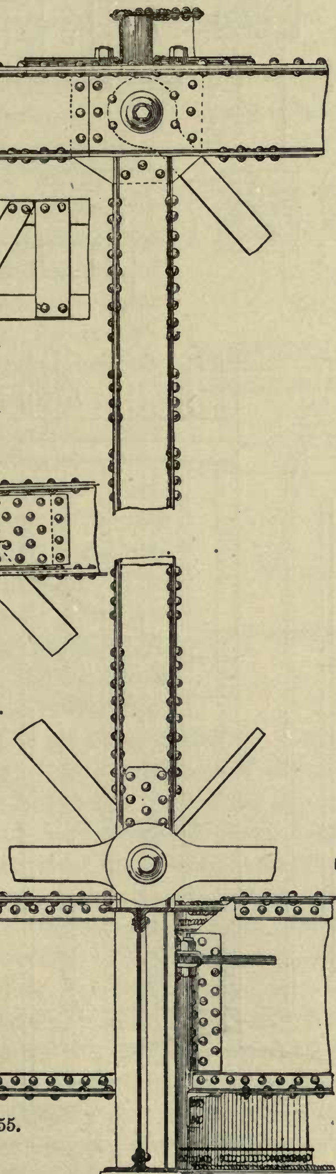


Fig. 955.

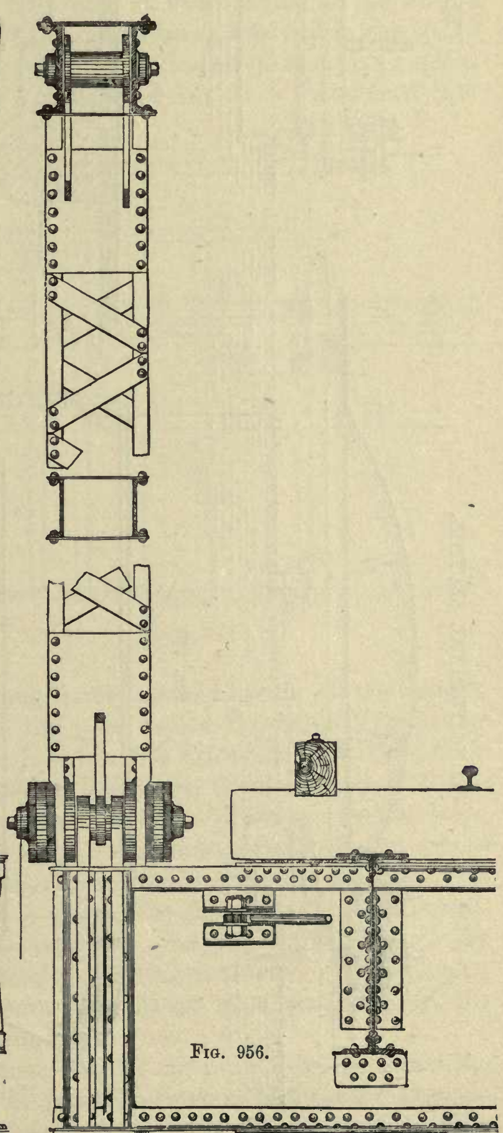


Fig. 956.

ELEVATION.

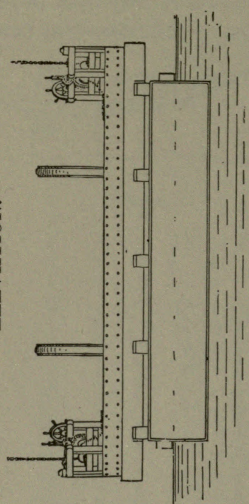


Fig. 961.

LONGITUDINAL SECTION.

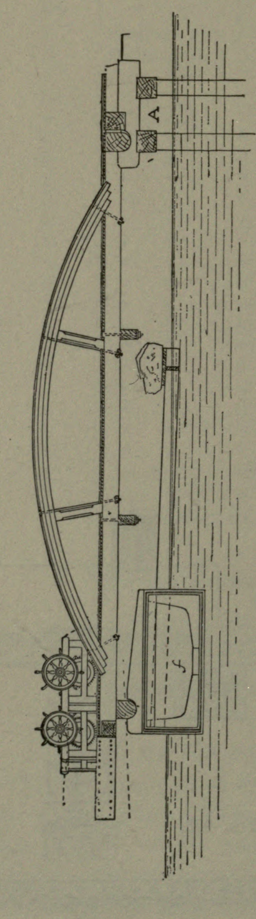


Fig. 960.

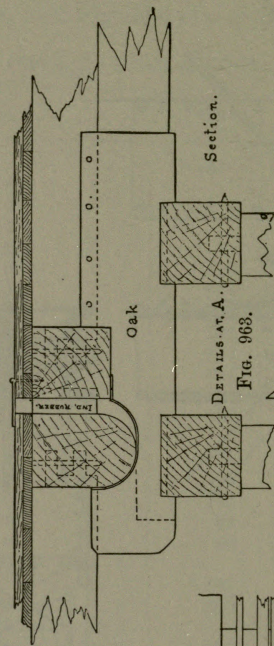


Fig. 963.

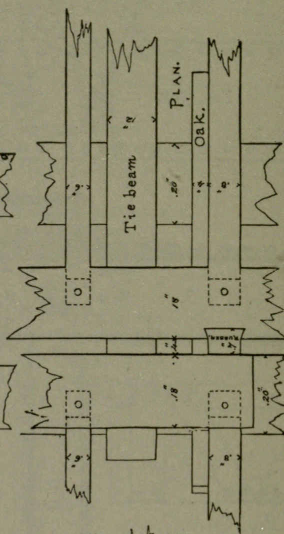


Fig. 964.

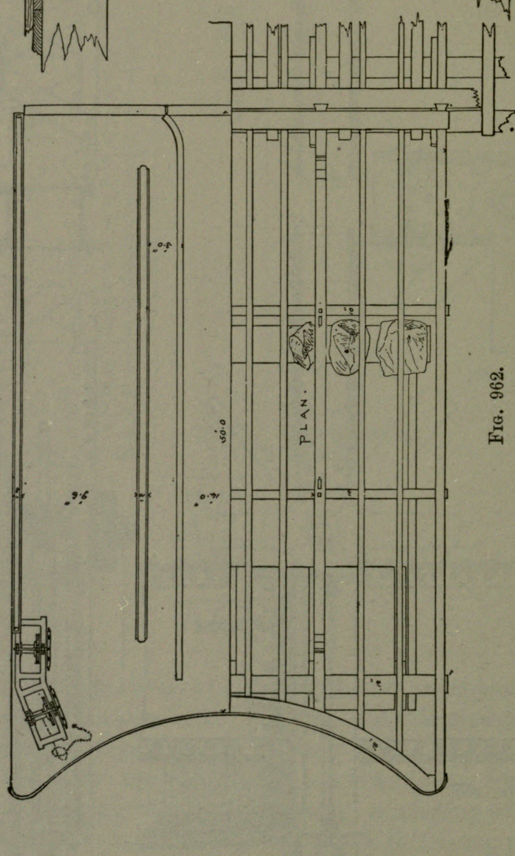


Fig. 962.

not perpendicular to line of bridge, but parallel with abutment. Fig. 959 is the side elevation of angle connection of end-brace and top chord.

Figs. 960 to 964 are illustrations of the landing-bridge common at New York city ferries. Fig. 960 is a longitudinal section, showing a section of the float, *f*, with its lever and stone counterpoise to balance the weight of the bridge, the end of which is thrown to one side of the float. Fig. 961 is the front elevation, and Fig. 962 the plan, one half being planked, and one half showing framing. It will be seen that there are two chain-barrels, on each side of the bridge, worked by hand-wheels; on the outer ones are the chains by which the boat is drawn up to the bridge; on the inner ones the chains by which the bridge is adjusted to the load on the boat, and by which a part of the weight of the bridge is held, the upper ends of the chains being attached to the frame of the overhead. The details (Figs. 963 and 964) in section and plan explain the construction of the land-hinge; a cushion of rubber is introduced into the joint to modify the shock caused by the boat striking the bridge, and a flap of wrought-iron to cover the joint, for protection to travel, and security from dirt.

Piers.—Fig. 965 is an elevation of a pile-pier for a bridge. Tenons are cut on the top of the piles, and a cap (*a*) mortised on. The two outer piles are driven in an inclined position, and the heads bolted to the piles adjacent. The piles are made into a strong frame laterally by the planks *b* and *c*, and plank-braces *d d* on each side of the piles, bolted through. The string-pieces of the bridge rest on the cap. Longitudinal braces are often used, their lower ends resting on the plank *b*—which should be, then, notched on to the piles—and their upper ends coming together, or with a straining-piece between, beneath the string-pieces, acting not only as supports to the load, but also as braces to prevent a movement forward of the frames. As the tendency of a moving train is to push the structure on which it is supported forward, in railway-bridges especially, great care is taken to brace the structure in every way—vertically and horizontally, laterally and longitudinally. If the plank *c* be a timber-sill, and the piles beneath be replaced by a masonry-pier, the structure will represent a common form of trestle.

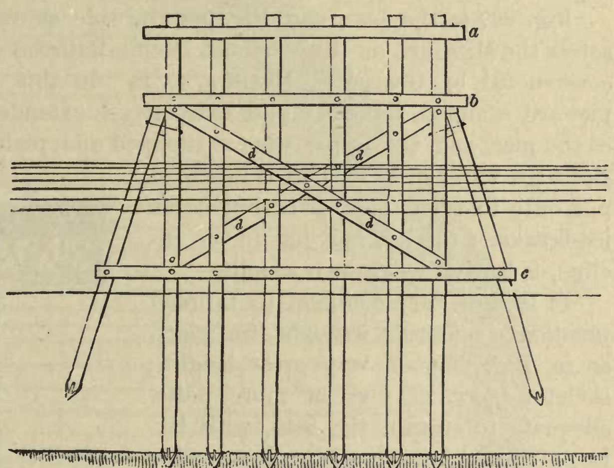


Fig. 965.

Fig. 966 is a plan of one of the stone piers of the railway-bridge across the Susquehanna, at Havre de Grace. To lessen as much as possible the obstruction to the flow of the stream, it is usual to make both extremities of the piers pointed or rounded. Sometimes the points are right angles ; sometimes, angles of 60° ; often, a semicircle, the width of the pier being the diameter ; occasionally, pointed arches, of which the radii are the width of the pier, the centers being alternately in one side, and their arcs tangent to the opposite side. It will be observed (Fig. 966) that none of the stones break joint at the angle—this is important in opposing resistance to drift-wood and ice. It is not unusual, in very exposed places, to make distinct ice-breakers above each pier, usually of strong crib-work, with a plank-slope like a dam, of 45°, and with a width somewhat greater than that of the pier—a cheap structure as a protection to an expensive bridge.

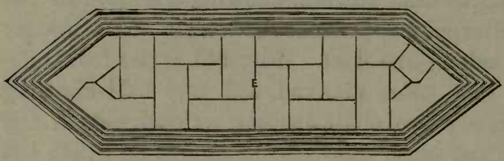


Fig. 966.

Fig. 967 is the plan and Fig. 968 the side elevation of a pier of a bridge across the Missouri, on the Northern Pacific Railroad at Bismarck, designed and constructed by George S. Morison, C. E. In this design both ends of the pier are rounded, but the upper extremity is extended beyond the main body of the pier, and the upper edge is inclined and plated with iron between low and high water mark. This is intended not only to turn aside drift, but as an ice-breaker ; the ice, moving up the incline, is broken by its own weight.

It is now very common in railroad practice to construct wrought-iron piers, as in Fig. 969, of very great height ; skeleton-piers, of four or more posts, adequate to sustain the load, with lattice girts and lateral rod-bracing.

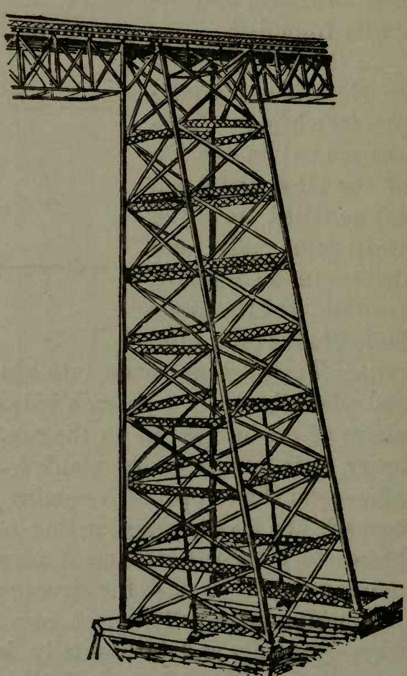


Fig. 969.

Fig. 970 is a section of the foundation of the Bismarck bridge, showing the construction of the inverted caisson, similar to that used for the Brooklyn bridge pier and others. The caissons are 74′ long, 26′ wide, and 17′ high outside ; the working-chamber 7 feet high. The caissons are built of pine, sheathed with two thicknesses of 3″ oak-plank. Above this is crib-work filled in with Portland cement concrete ; *a a* are the air-locks. The sand was removed from the caissons by water-ejectors.

Arch bridges are of stone, brick, or metal ; the parts of the arch exert a direct thrust upon the abutments, resisted by the inherent weight of the latter, or its absolute fixed mass, as in the case of natural rock abutments.

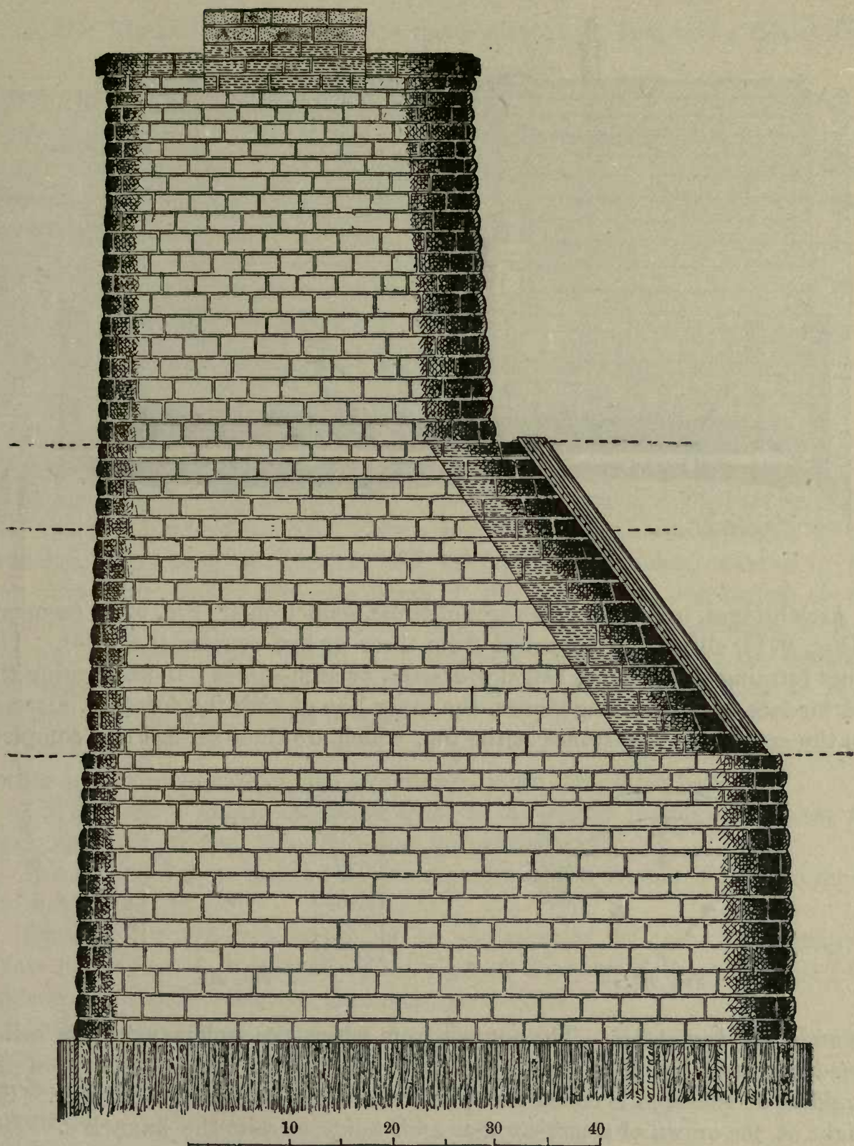


Fig. 967.

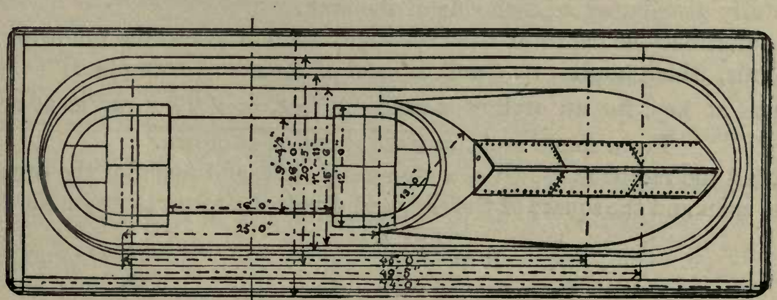


Fig. 968.

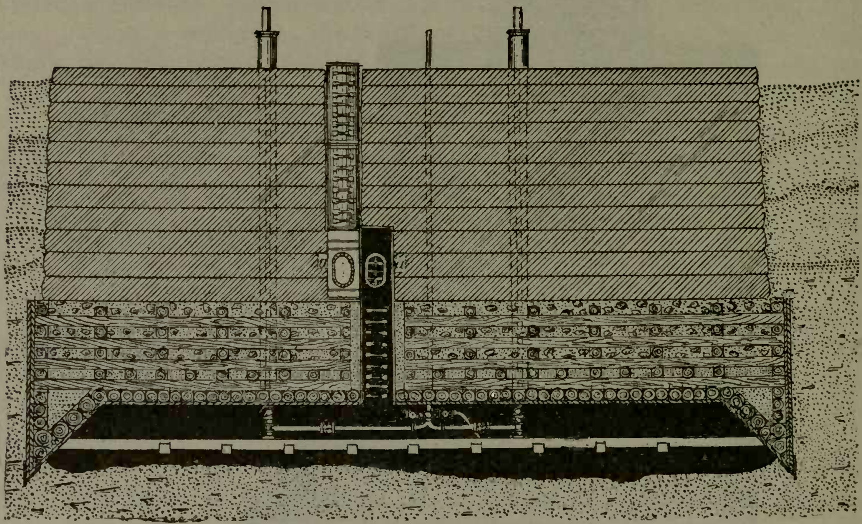


Fig. 970.

Arch bridges, in masonry, are arcs of circle, semicircular (Fig. 972), segmental (Fig. 971), elliptic, or described from three or five centers (page 25). The stones forming the arch are called *voussoirs*, or arch-stones ; those forming the exterior face are called ring-stones, the inner line of arch the *intrados*, exterior line the *extrados*. The stones at the top, which are those set last and complete

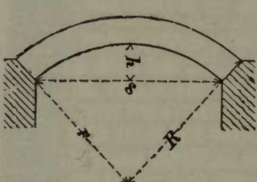


Fig. 971.

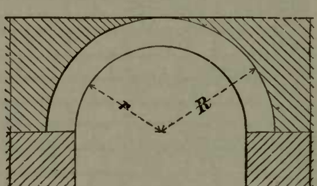


Fig. 972.

the arch, are *key-stones*. The courses from which the arches spring are called *skew-backs*, and the first course the *springing-course*. The masonry on the shoulders of the arch is called the *spandrel-courses*, or *spandrel-backing*. The weight at the crown of a semicircular arch tends to raise the haunches. This is counteracted by the spandrel-backing, and by the earth-load, which should be carefully distributed on each side of the arch.

To determine the depth of the key-stone, Rankin gives the following empirical rule, which applies very well to most of the above examples :

Depth at key, for an arch of a series, in feet, $= \sqrt{\cdot 17 \times \text{radius at crown.}}$

For a single arch, $= \sqrt{\cdot 12 \times \text{radius at crown.}}$

To find the radius at crown of a segmental arch, add together the square of half the span and the square of the rise, and divide their sum by twice the rise—

$$R = \frac{\overline{\tfrac{1}{2}S}^2 + r^2}{2r}$$

Thus, the Blackwall Railway-bridge has a span of 87 feet, and a rise of 16—

$$\frac{\overline{43{\cdot}5}^2 + 16^2}{2 \times 16} = \frac{1892{\cdot}25 + 256}{32} = 67{\cdot}1.$$

To find the radius of an elliptical arch, on the hypothesis that it is an arch of five centers (Fig. 79, page 25), the half-span is a mean proportional between the rise and the radius. Thus, for example, the Great Western Railway-bridge is 128′ span, and 24·25′ rise—

$$\overline{64}^2 = 24{\cdot}25 \times R$$

$$R = \frac{4096}{24{\cdot}25} = 169 \text{ feet.}$$

To find the depth of key-stone, by rule above, as in one of a series—

$$d = \sqrt{17 \times 169} = \sqrt{2873} = 5{\cdot}33.$$

The depth of the voussoir is increased in most bridges from the key-stone to the springing-course, but not always ; it is safer to increase the depth.

If an arch be loaded too heavily at the crown, the lines of pressure pass above the extrados of the crown, and below the line of intrados at the haunches, depressing the crown and raising the haunches, separating the arch into four pieces, and *vice versa* if the arches are overloaded at the haunches. To prevent such effects, especially from moving loads, in construction the arches are loaded with masonry and earth, that the constant load may be in such excess that there will be no dangerous loss of equilibrium by accidental changes of load.

The horizontal thrust may be determined, according to Rankin, by the following approximate rule, which seldom errs more than 5 per cent :

The horizontal thrust is nearly equal to the weight supported between the crown and that part of the soffit whose inclination is 45°.

This thrust is to be resisted by the masonry of the abutment and the earth-load behind it.

Thus, if Fig. 973 be a section of an abutment of an arch, the horizontal thrust exerted at T is resisted by the mass of masonry of the abutment; the tendency is to slide back the abutment on its base A C, or turn it over on the point A. The sliding motion is resisted by friction, being a percentage, say from ½ to ⅔ of the weight of the abutment and of half the arch which is supported by this base ; but, in turning over the abutment on the point A, the action may be considered that of a lever, the force T acting with a lever T C to raise the weight of the abutment on a lever A B (G being the center of gravity, and G B the perpendicular let fall on the base), and the weight of half of the arch on the lever A C. That is, to be in equilibrium, the horizontal thrust T × T C must be less than the sum of the weights of the abutment multiplied by A B, and the weight of the arch multiplied by A C.

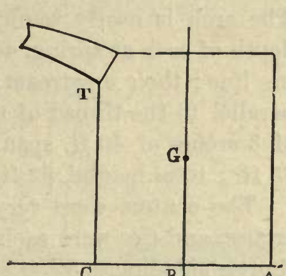


Fig. 973.

Skew bridges are those in which the abutments are parallel, but not at right angles to the center line, and the arches oblique. To construct these in cut stone requires intelligence and education both in the designer and stone-cutter; but, when the work is laid full in cement, so that the joints are as strong as the material itself, this refinement of stone-cutting is not necessary. The arch may safely be constructed as a regular cylinder of a diameter equal to the rectangular distance between the abutments, with its extremity cut off parallel to the upper line of road. For such an arch hard-burned brick is the best material, the outer voussoirs being cut stone.

In the rules above given no consideration is paid to the strength of the cement in which the stones are bedded. When the cement is thoroughly set, the structure is in a measure monolithic, and the thrust is inconsiderable.

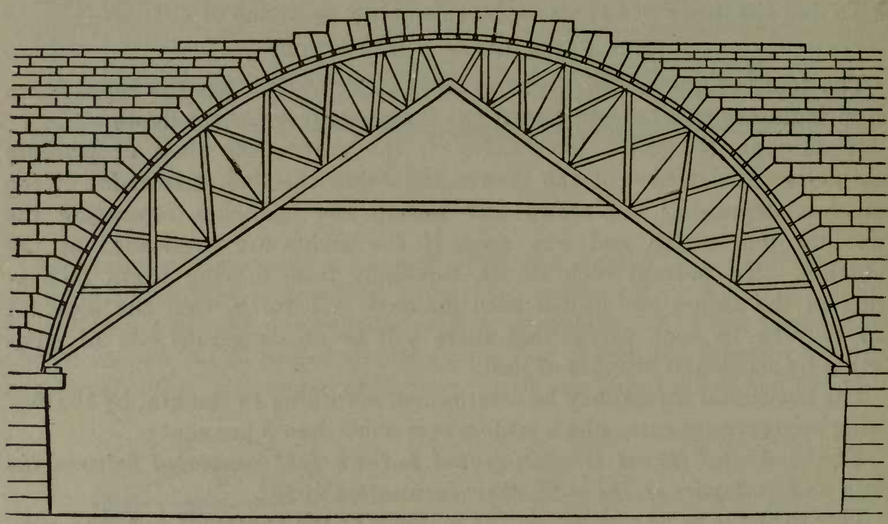


Fig. 974.

Fig. 974 is the elevation of one of the stone arches of the Minneapolis Union Railway Viaduct, with the timber centers on which the arch was turned. The arch is nearly semicircular, 97·82 ft. span, 50 ft. rise; width, 28 ft.; depth of arch at spring, 40″; at key, 36″. The piers are 10 ft. thick at springing line; their up-stream ends are at angles to the main body of the piers, and parallel to the thread of the stream. The whole length is 2,100 ft., composed of 3 arches of 40 ft. span, 16 of 80 ft., and 4 of 100 ft. Height above water, 65 ft.; total height, 82 ft.

The centers were very light frames, 5 to each arch; the chords, timber arches, and ties were each 12″ × 12″, the central braces 10″ × 10″, and the shorter side-braces 8″ × 8″; the bolts, single, 1½″ diameter.

The bridge was constructed after the designs and under the direction of Charles C. Smith, C. E., Chief Engineer of the St. Paul, Minneapolis and Manitoba Railway, and is an example of a very economical and stable construction. The piers are of Minnesota granite, but above springing line the masonry is of magnesian limestone. It was commenced in February, 1882, and completed in November, 1883.

LOCATION.	Material.	Form of arch.	Span.	Rise.	Depth at crown.	Depth at spring.
Manchester and Birmingham Railroad....	Brick.	Semicircular.	18	9	1·6	Unif'rm
" " " "	"	"	63	31·6	3	"
London and Brighton Railroad...........	"	"	30	15	1·6	2·3
" " Blackwall "	"	Segmental.	87	16	4·1½	Unif'rm
Great Western Railroad.................	"	Elliptical.	128	24·3	5	7·1½
Chestnut Street (Philadelphia) Railroad...	"	Segmental.	60	18	2·6	
High Bridge, Harlem River, New York...	Stone.	Semicircular.	80	40	2·8	
St. Paul, Minneapolis & Manitoba Railroad (largest arch), at Minneapolis	"	Segmental.	97·8	50	3	3·4
Cabin John Washington Aqueduct.......	"	Elliptical.	220	57·3	4·2	
Licking Aqueduct and Ohio Canal........	"	"	90	15	2·10	
Monocacy "	"	"	54	9	2·6	
Hutcheson "	"	Segmental.	79	13·6	3·6	4·6
Chemin du Fer du Nord, sur l'Oise.......	"	"	82·5	13·5	4·6	
D'Enghien Railroad du Nord............	"	Semicircular.	24·4	12·2	1·4	
Du Crochet Railroad...................	"	"	13·2	6·6	1·7½	
Experimental arch, designed and built by M. Vaudray, Paris....................		Segmental.	124	6·11	2·8	3·7

The arch last in the list was a very bold specimen of engineering, built as an experiment, preliminary to the construction of a bridge over the Seine. It was made of cut stone, laid in Portland cement, with joints of $\frac{3}{8}''$, and left to set four months; the arch was 12′ wide; the centers rested on posts in wrought-iron boxes filled with sand, and, as the centering was eased by the running out of the sand, the crown came down $\frac{6}{10}''$; the joints of one of the skew-backs opening $\frac{7}{1000}''$ during the first day, it came down $\frac{7}{100}''$. It was then loaded with a distributed weight of 300 tons; under this load the crown settled $\frac{3}{10}''$ more. Since then nothing has stirred, although it was afterward tested by allowing five tons to fall vertically 1′ 6″ on the roadway over the key-stone. This bridge will not come within any of the rules laid down for other constructions. It will be observed that the rise is about $\frac{1}{18}$ the span, although the usual practice for segmental and elliptical arches is more than $\frac{1}{6}$, or within the limits of $\frac{1}{4}$ and $\frac{1}{8}$.

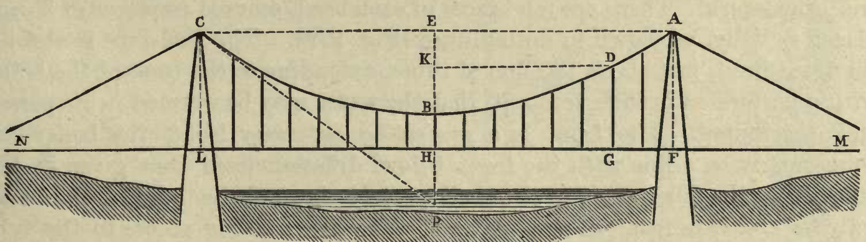


FIG. 976.

In suspension-bridges the platform of the bridge is suspended from cables, or chains, the ends of which are securely anchored within the natural or artificial abutments.

The curve of a suspended chain is that known as the catenary, and, if the whole weight of the structure were in the chain itself, this would be the curve of the chains of a suspension-bridge ; but, as a large part of the weight and the whole of the loading lies in the platform, the curve assimilates to that of a parabola, and in all calculations it is so regarded.

Let Fig. 976 represent a suspension-bridge, in which A, B, C, are points in a parabolic curve.

Rule.—Add together four times the square of the deflection $(E\,B)^2$ and the square of half-span $(A\,E)^2$, and take the square root of this sum ; multiply this result by the total weight of *one* chain and all that is suspended from it, including the distributed load, and divide this product by four times the deflection (E B) of the cable at the center, and the result will be the tension on one chain, at each point of support, A and C. The angle made by the chain at the point of support, viz., angle P C L and the angle of the back-stays, or continuation of the chain (angle L C N) should be equal to each other, in order that there be no tendency to overset the tower C L and A F.

BRIDGES.	Main spans.	Deflection of chain or cable.	No. of chains and cables.	Total effective section of cable in square inches.	Mean weight of cable per foot of span (pounds).	Fixed load per foot of span (lbs.).	Breadth of platform in feet.
Menai........	570	43	16	260	880		28
Chelsea......	348	29	4	230	767		47
Pesth	666	47·6	4	507	1,690	9,892	46
Bamberg.....	211	14·1	4	40·2	137	1,581	30·5
Freyburg.....	870	63	4	49	167	760	21·25
Niagara Falls.	821	54 and 64	4	241·6	820	2,032	24
Cincinnati ...	1,057	89	2	172·6	516	2,580	36
Brooklyn	1,595						

BOILER-SETTING.

Fig. 977 is a longitudinal section, Fig. 978 a plan with section of wall, and Fig. 979 an elevation half-front and half-sectional of a boiler and setting as recommended by the Hartford branch of the Hartford Steam-Boiler and Inspection Company, showing the interior bracing, steam and water connections, and brick-work. There are ten braces in each head, secured to pieces of T-iron, placed radially, as shown in dotted lines (Fig. 979). The feed-pipe is through the front-head, just above the line of tubes, extending to the back of the boiler, with a perforated branch across it, that the water may be warmed in its passage and distributed. The front is a *projecting cut-away* front, the boiler-head being nearly on a line with the front below, different from that given in Fig. 768, where the lower part of the shell projects beyond the head of the boiler, and the cast-iron front covers the end. The doors giving access to the tubes are usually semicircular, and hung on the top diameter, but it will be found more convenient to form them in two quadrants, and hung so as to move horizontally. The boilers are to be protected against radiation by a covering of ashes, or a brick arch, resting on the side-walls. My own practice is to make the boiler without lugs to support it on the side-walls, but to hang the boiler

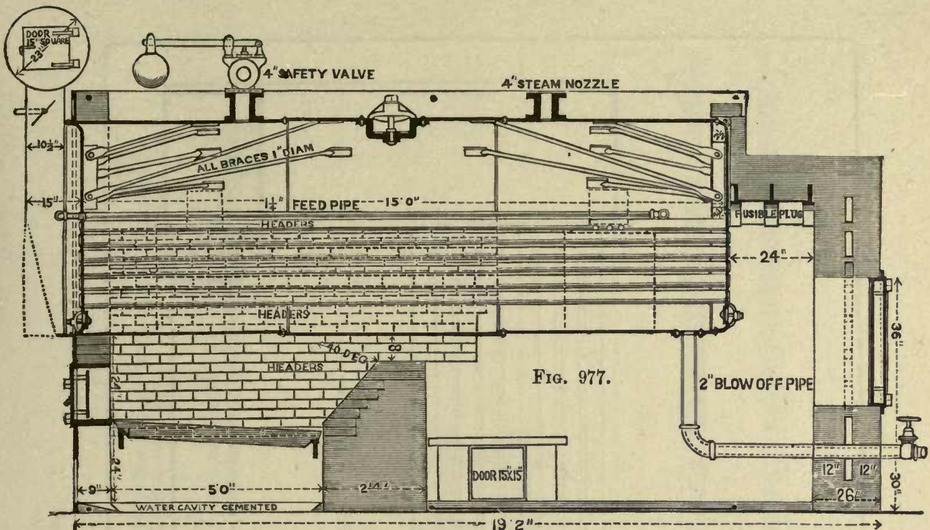


Fig. 977.

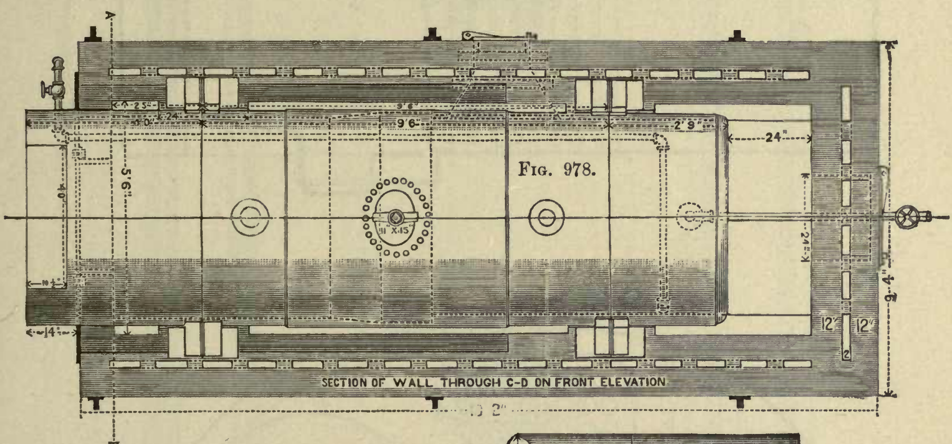


Fig. 978.

from wrought-iron cross-bars, resting on the top of the side-walls, and putting small bars across just above the top of the boiler, to roof over with sheet-iron and fill above with ashes, leaving the spandrels as hot-air spaces.

It will be observed (Fig. 978) that the manhole-frame is riveted to the inside of the boiler; frequently it is on the outside. For most positions I prefer that the manhole should be placed in the back-head, as easier of access, and in my form of cover there is no disturbance of ashes for access to the manhole. It is often well to make the blow-off pipe a circulating pipe by

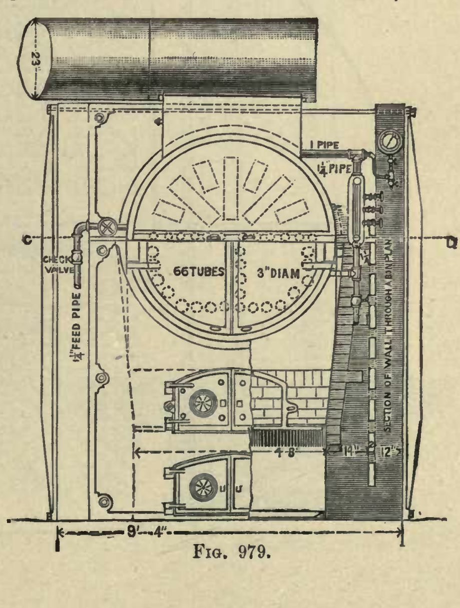


Fig. 979.

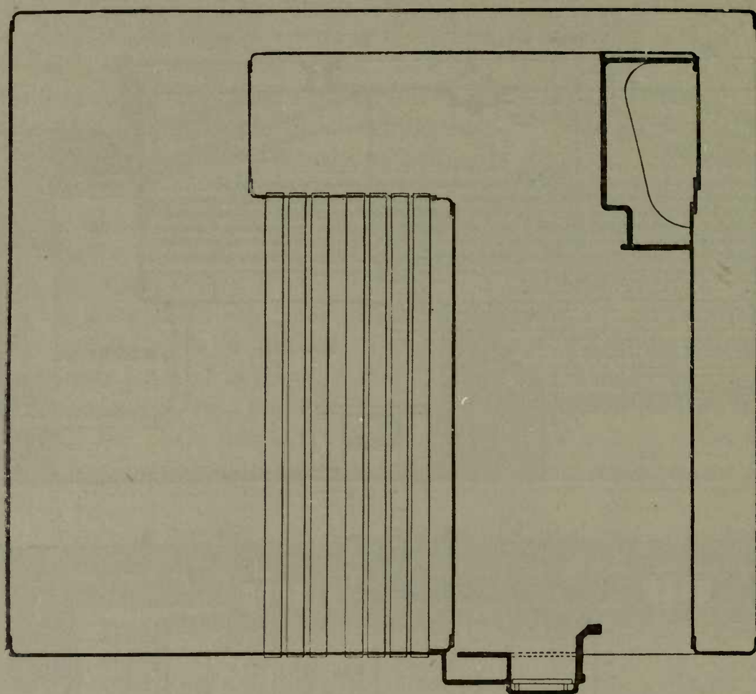


FIG. 981.

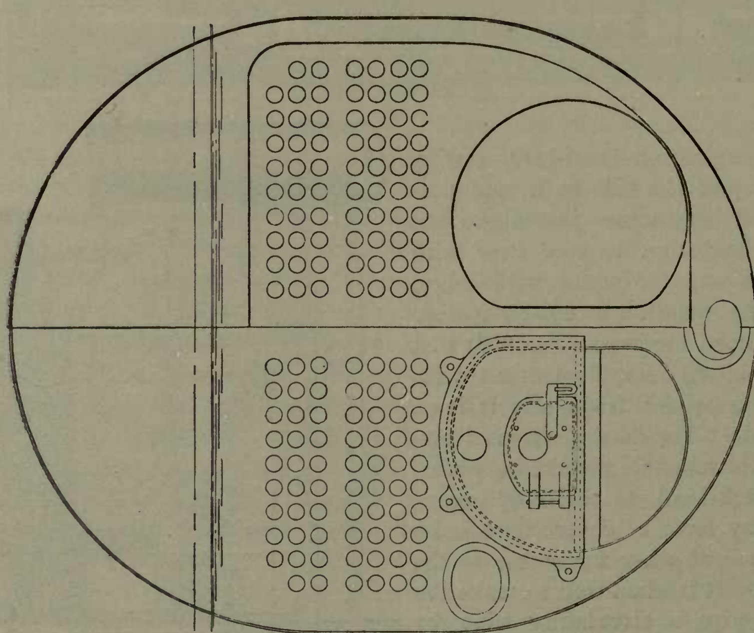


FIG. 980.

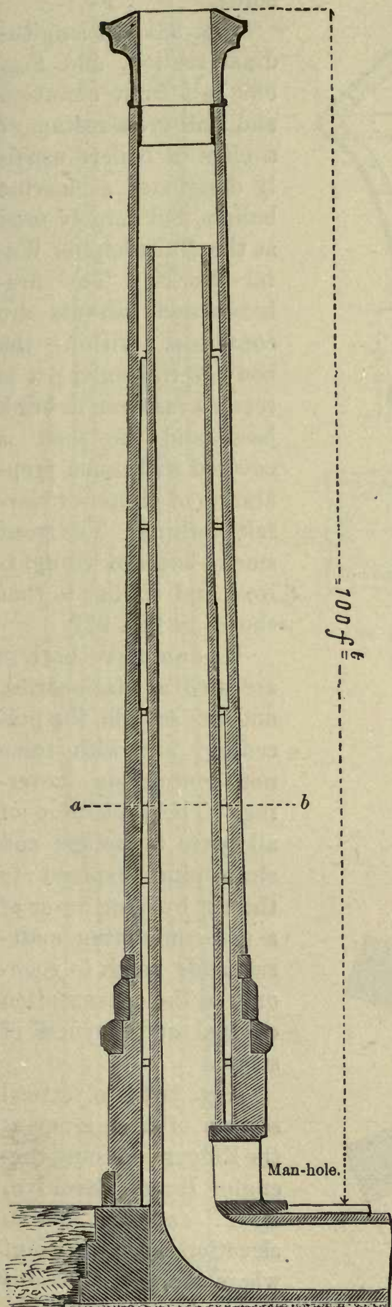


FIG. 982.

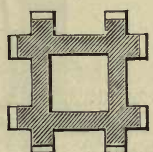


FIG. 986.

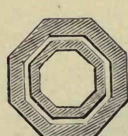


FIG. 984.

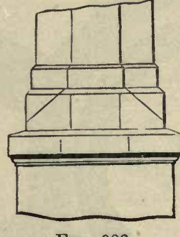


FIG. 983.

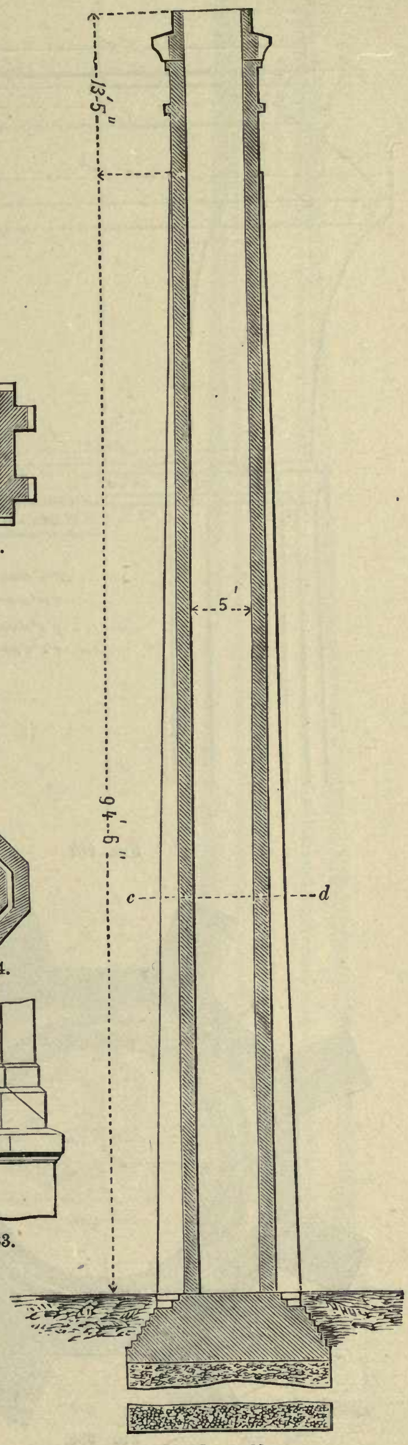


FIG. 985.

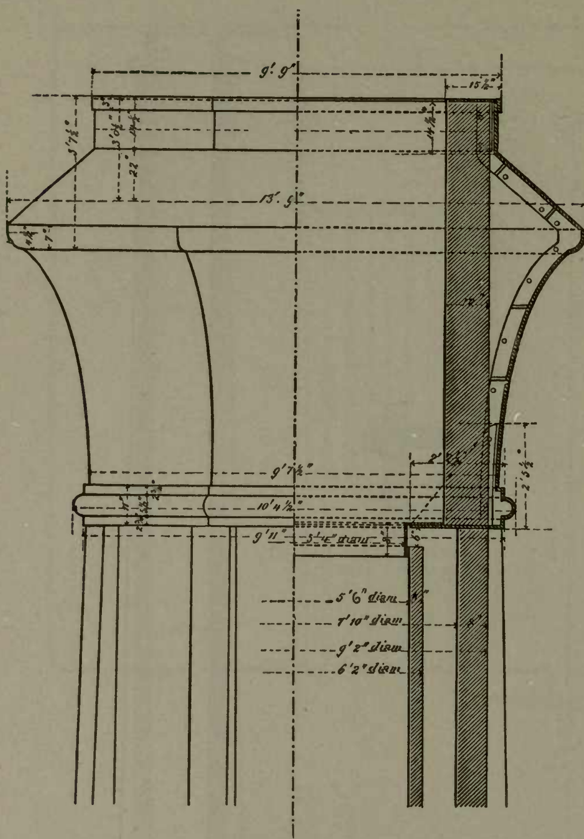


Fig. 987.

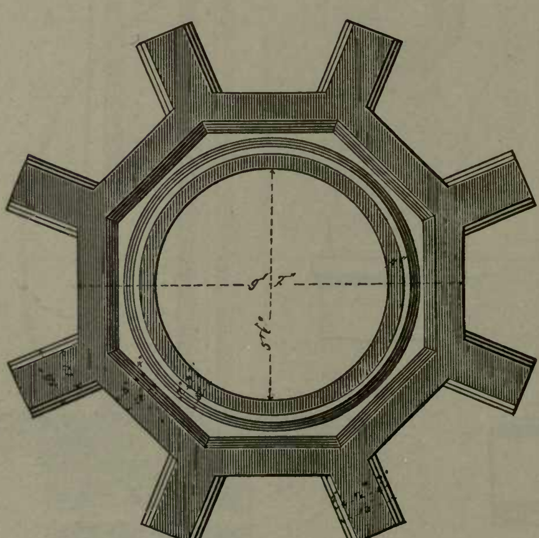


Fig. 988.

connecting an inch pipe inside the valve with the upper water-space of the boiler.

Fig. 981 is a longitudinal section, and Fig. 980 half front elevation and half cross-section of a class of boilers usually designated as marine boilers, but largely used at the Philadelphia Water-Works. The fire-boxes and ash-pits are contained within the body of the boiler; it is set on a cast-iron or brick base, and the shell is covered with some preparation of plaster or hair-felt clothing. The front smoke-box is of wrought-iron, and similar to that shown in Fig. 977.

Locomotive-boilers are used as stationaries, and are set like the preceding, but with some non-conducting covering. The protection of all parts of boilers and steam-pipes exposed to the air by some cover of a non-conducting material adds much to economy in the consumption of coal and dryness of steam.

Fig. 982 is a vertical section of a chimney at the Ridgewood Pumping-engine House, Brooklyn, N. Y., and Fig. 983 an elevation at the point where the square base is changed into an octagonal.

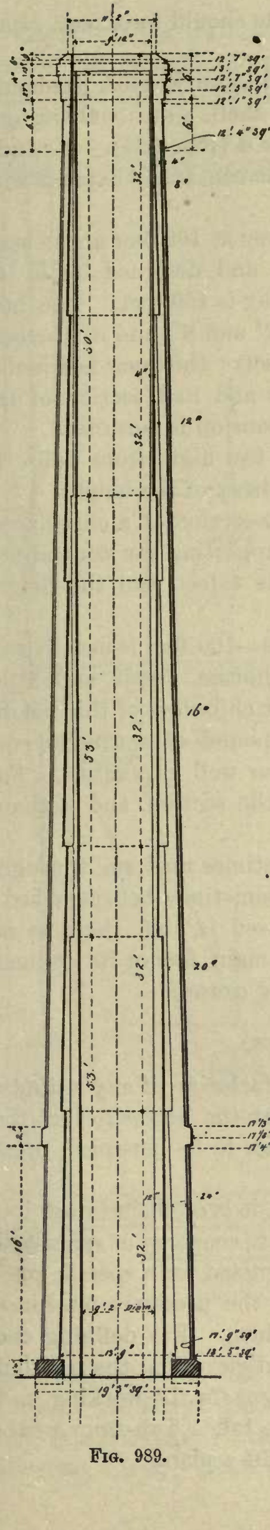


FIG. 989.

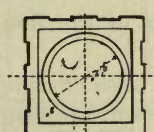


FIG. 990.

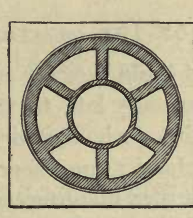


FIG. 992.

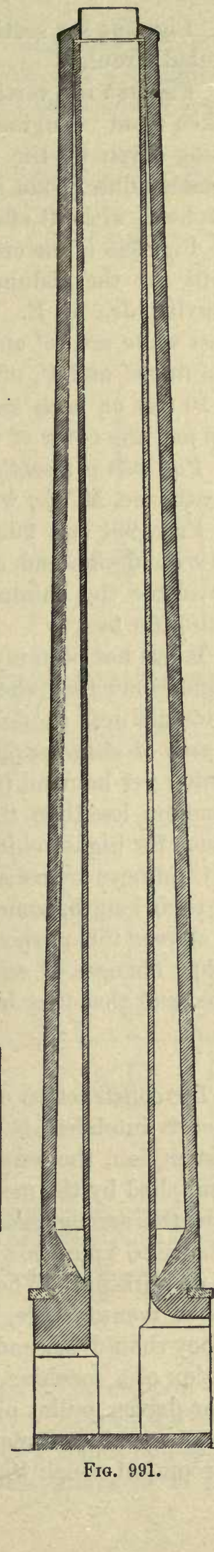


FIG. 991.

Fig. 984 is a section of the shaft at *a b*, but the flue should have been represented circular.

Fig. 985 is a vertical section of a chimney attached to an English gas-house, taken from "Engineering," with a uniform flue and shell, additional strength being given by the buttresses shown in section at *c d* (Fig. 986). No independent flue inside is shown, but it is desirable, as it can freely expand with the heat, without affecting the outer shell.

Fig. 988 is the cross-section of a buttressed chimney at 100 feet above base, built for the Calumet & Hecla Mining Company, and designed by E. D. Leavitt, Jr., M. E. The whole height of the chimney is 150 feet. The buttress walls are 16″ and 12″ thick, that of the body 12″ and 8″, and of the central flue 8″ and 4″, offsetting into each other by 1″ offsets ; the taper is 4 inches to 10 feet on each side. Fig. 987 is a half elevation and half section of the cap and the cover of the interior flue by which its expansion is permitted.

Fig. 989 is a sectional elevation of a chimney 160 feet high, from John T. Henthorne, M. E., with a cross-section (Fig. 990) midway of the height.

Figs. 991 and 992 are sectional elevation and cross-section of a chimney of my own design and construction. The buttresses supporting the central flue are inside the chimney. The diameter of the flue is 4 feet, and the height about 100 feet.

It has not been my practice to build high chimneys—100 feet is usually sufficient—but they should extend above surrounding houses, woods, and hills, which are near enough to influence the draught. For chimneys of this height an area of chimney-flue of one square inch for every pound of anthracite coal burned per hour on the grate has been found to answer well in practice. For chimneys less than this height, it is well to increase the section, and perhaps reduce for higher chimneys.

Chimneys are constructed of various sections, sometimes uniform throughout their length, sometimes tapering at the top, and sometimes bell-mouthed ; all answer the purpose. The great point to be observed is, that there be no abrupt changes of section or direction, either in the main flue or its connections, and that they be carried well above all disturbing causes.

ON THE LOCATION OF MACHINES.

The construction of buildings for mills and manufactories (if any æsthetic effect is intended) is usually left to the architect, but the necessities of the construction, the weights to be supported, and the space to be occupied, must be supplied by the mechanical engineer or millwright.

In the arrangement of a manufactory or workshop, it is of the utmost importance to know how to place the machinery, both as to economy of space and also of working. Where a new building is to be constructed for a specific purpose of manufacture, it will be found best to arrange the necessary machines as they should be, and then build the edifice to suit them. For defining the position of a machine, the space it occupies in plan and elevation, the position of the driven pulley or gear, of the operative, and spaces for the working and access to parts, are required. To illustrate this subject, take a two-story weaving-room, of which Fig. 993 is an elevation, and Fig. 994 a plan.

Lay down the outlines of an interior angle of the building, and dot in, or draw in red or blue, the position and width of beams. This last is of importance, as it will be observed (Fig. 993) that no driving-pulley can come beneath

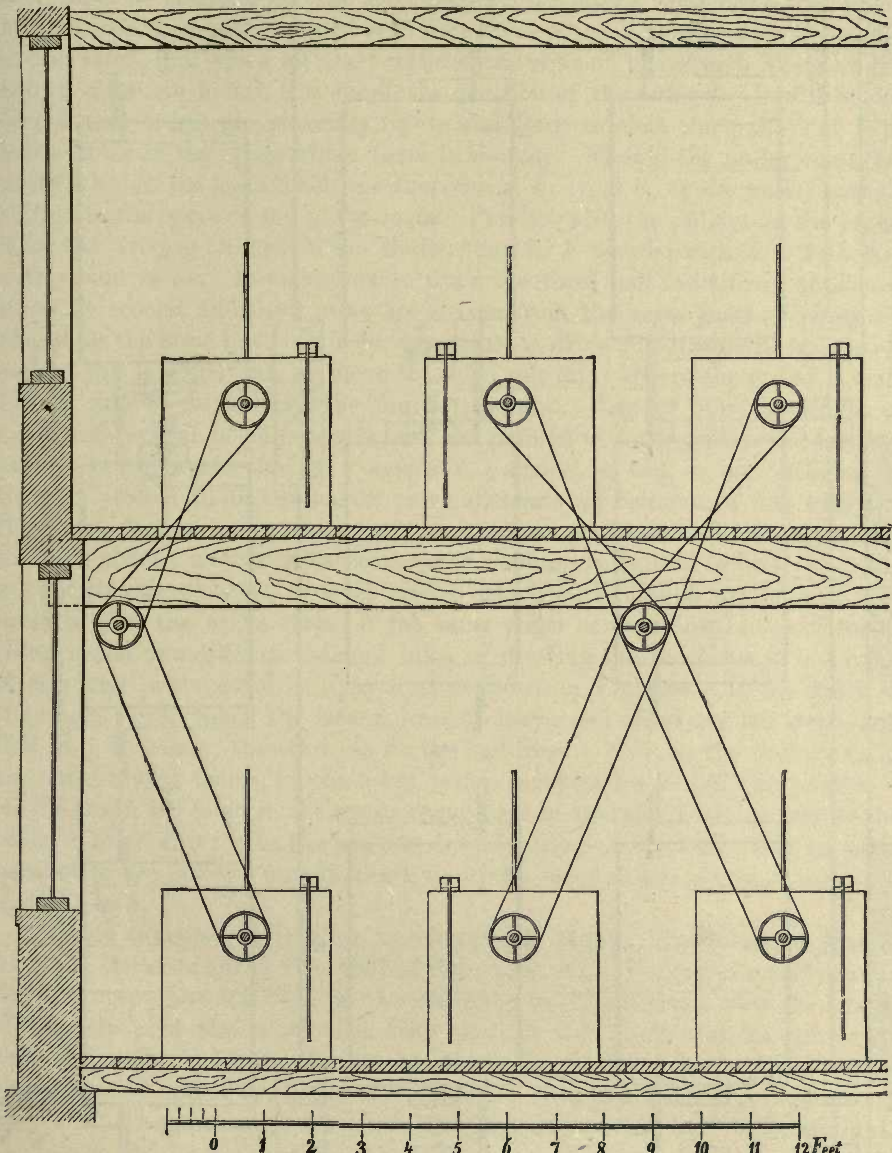


FIG. 993.

the beam, and also that this is the position for the hanger. Lay off now the width of the alleys and of the machines. The first alley, or nearest the side-wall, is a back alley; that is, where the operative does not stand, and so on alternate alleys. Draw the lines of shafting central to the alleys, as in this

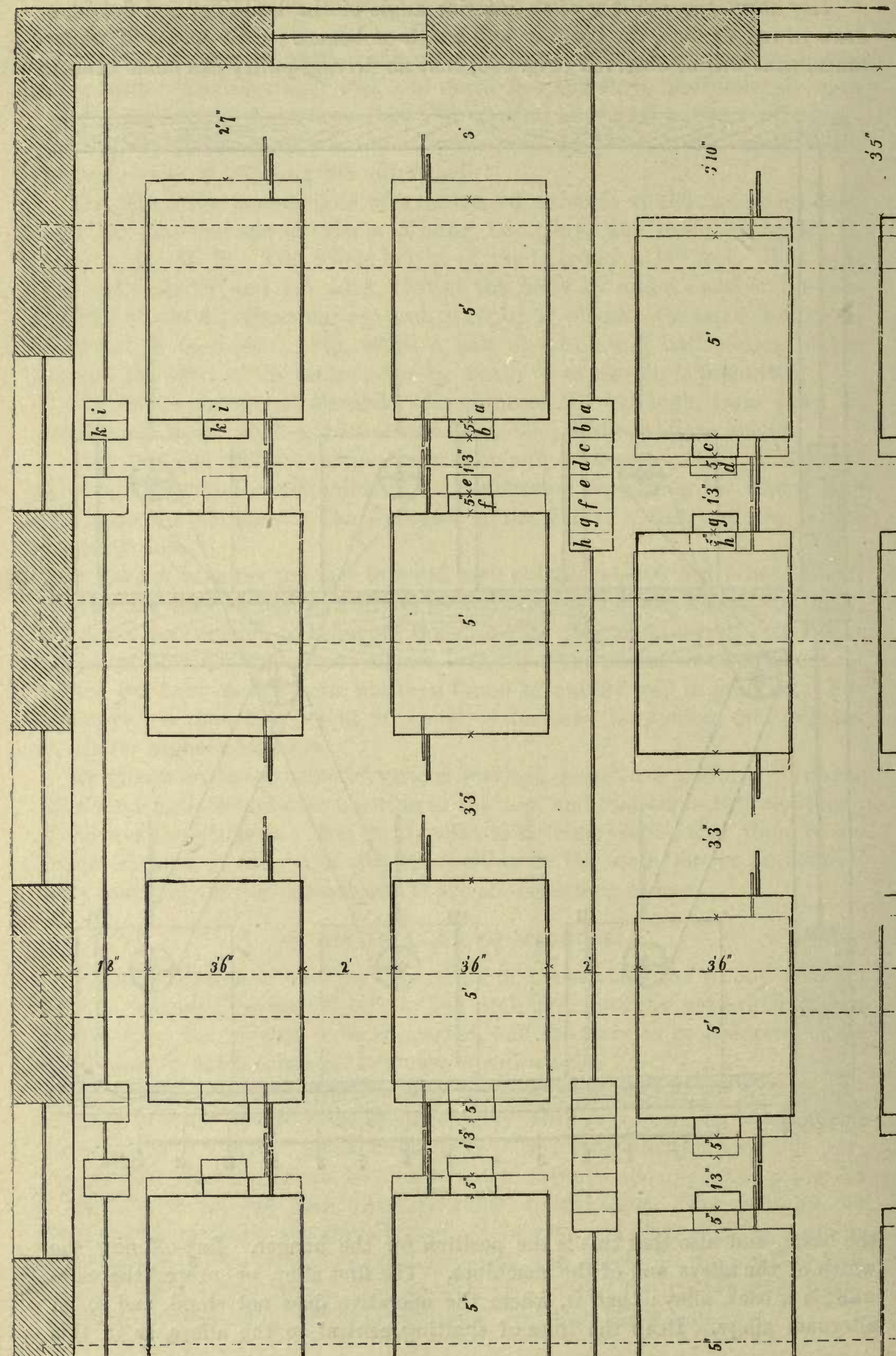


Fig. 994.

position the belts are least in the way. One operative usually tends four looms; they are therefore generally arranged in sets of four, two on each side of the main alley, where the operative stands; the twos are placed as close to each other as possible, say one inch between the lays, a small cross-alley being left between them and the next set. Lay off now the alley necessary at the end of the room, and space off the length of two rows of looms with alleys at the end of alternate looms, and mark the position of the pulleys. It will be observed that looms are generally rights and lefts, so that the pulleys of both looms come in the space where there is no alley. Should the pulley come beneath a beam, the loom must be either moved to avoid it, or the pulley may be shifted to the opposite end of the loom. Parallel with the pulleys on the looms draw the driving-pulleys on the shafts, that is, *k* parallel with *k*, *b* with *b*, *f* with *f*, and so on. Proceed now to draw the third and fourth row of looms, since the second and third rows are driven from the same shaft; if they are placed on the same line, it will be impossible to drive both from the same end, and, as this is important, we move the third row the width of the pulley *b*, and, for the sake of uniformity, the fourth row also. Lay off now the length of looms and position of pulleys as before, and parallel with the pulleys the driving-pulleys on the shaft, that is, *c* against *c*, *g* against *g*, and so on. Having in this way plotted in all the looms, every alternate set being on a line with the third and fourth row, proceed now to lay down the position of the looms in the floor above; and since for economy of shafting it is usual to drive from the lines in the lower rooms, to avoid errors, interference of belts and pulleys, it is usual to plot the upper room on the same paper or board as the lower room, using either two different colored inks, or drawing the machines in one room in deep and in the other in light line, as shown in Fig. 994. If the width of the rooms is the same, the lateral lines of looms and alleys are the same, and it is only necessary, therefore, to fix the end lines. Now, as the first loom in the outer row of looms, in the lower room, occupies for its belt the position *k* on the shaft, the loom in the upper room must be moved either one way or the other to avoid this; thus the position *i* of the pulley on the loom must be made parallel to the pulley *i* on the shaft, so in the other looms *a* to *a*, *e* to *e*, *d* to *d*, and *h* to *h*.

Besides the plan, it is often necessary, and always convenient, to draw a sectional elevation (as in Fig. 993) of the rooms, with the relative positions of the driving-pulleys and those on the machines, to determine suitably the length of the belts, and also to see that their position is in every way the most convenient possible. In the figure, one of the lower belts should have been a cross-belt, and one of the upper ones straight: now, had the belts to the second row of looms in the upper story been drawn as they should have been, straight, the belt would have interfered a little with the alley, and it would have been better to have moved the driving-shaft a trifle toward the wall.

From this illustration of the location of machines, knowing all the requirements, in a similar way any machinery may be arranged with economy of space, materials, power, and attendance. These last two items are of the more importance as they involve a daily expense, where the others are almost entirely in the first outlay.

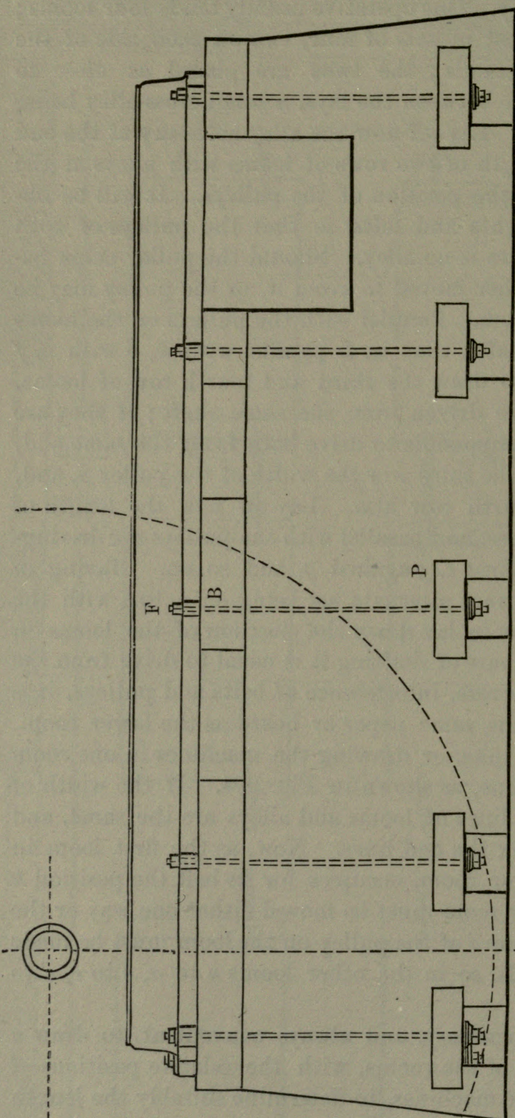


FIG. 995.

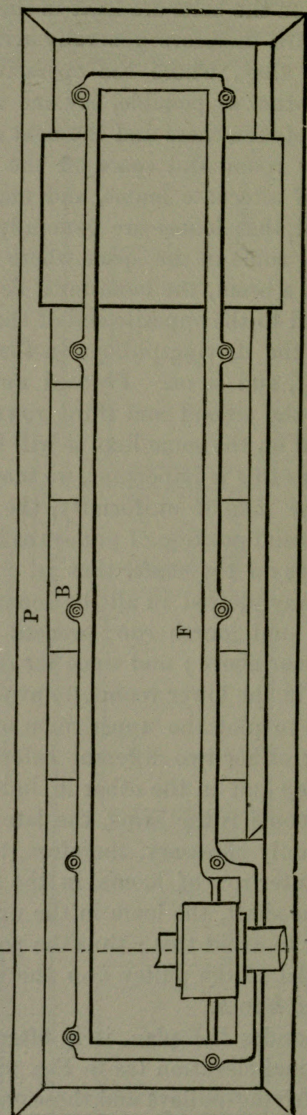


FIG. 997.

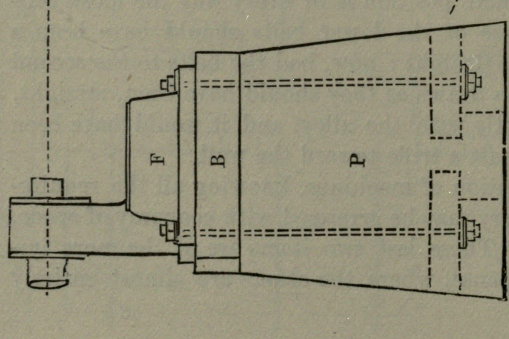


FIG. 996.

Machine Foundations.—Figs. 995, 996, and 997 are side and end elevation, and plan, of the foundation of the stationary steam-engine. F is the cast-iron frame or bed-plate of the engine; B the granite bed of engine, or coping of foundation; P the stone or brick pier, laid full in cement. The sides and surfaces of granite exposed are usually fine-hammered, the upper bed or build to receive the engine-frame, hammer-dressed and set level. Strong wrought-iron bolts pass through frame, bed, and pier, with nuts at each end, and the whole is strongly bolted together. Pockets are left in the pier near the bottom for access to nuts, and these pockets are covered by granite caps or iron plates.

Few stationary steam-engines are now built with bed-plates extending the whole length of the engine, but the illustration is applicable to the partial plates supporting the cylinder and pillow-block, and to engines and machines for which heavy foundations are necessary. It is not an uncommon practice now, instead of granite caps, to use timber, as cushioning the shocks and blows incident to most machinery.

Tunnels.—Figs. 998 to 1007 are illustrations, with description, taken from "Tunneling," a standard work on this subject by H. S. Drinker.

Figs. 998 to 1003 illustrate the principles of timbering applied to driving a gallery through running material. Figs. 998 and 999 are parts of the construc-

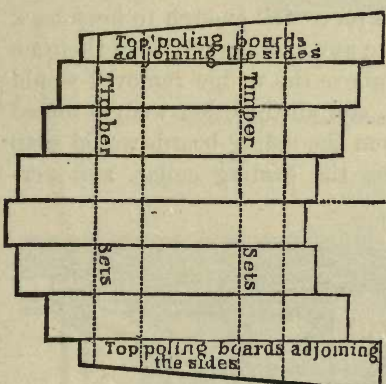


FIG. 998.

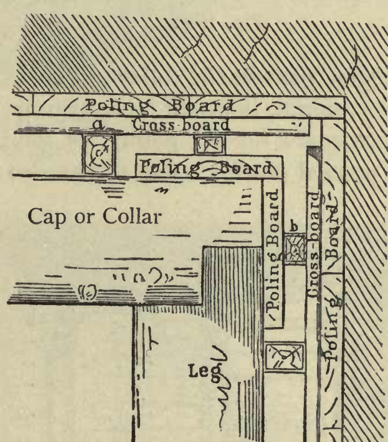


FIG. 999.

tion on a large scale, with the technical names of the parts. Each frame is called a *timber-set.* Suppose a leading set (Figs. 1000 and 1001) is in place, close to the face, and that the leading ends of the poling-boards resting above this leading set are held up from the collar by wedges sufficiently high to allow the insertion of the new poling-boards. In Fig. 1001 the sets *e e*, standing midway between the front and the hind ends of the poling-boards, serve as middle sets between the main sets *d d.* By turning to the plan (Fig. 1003) of a gallery thus timbered it will be seen that, owing to the fact that the side-*poling* has also to be wedged *out* at its leading end, just as the roof-poling is wedged *up*, therefore the space to be filled across the top by the roof-poling is wider over a front main-set than over a back one. Owing to this fact, the two outer top poling-boards, as shown in Fig. 998, are made wider at their leading ends than at their back ends. Now, to begin inserting the roof-poling, the miners, at either corner of the face, remove the extreme end-wedges between the collars and the poling, and into this space the new poling-boards (i. e., the ones shown in Fig. 998) that are wider at their leading ends are driven. But, though the

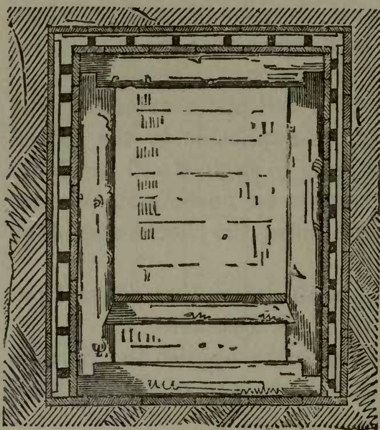


Fig. 1000.

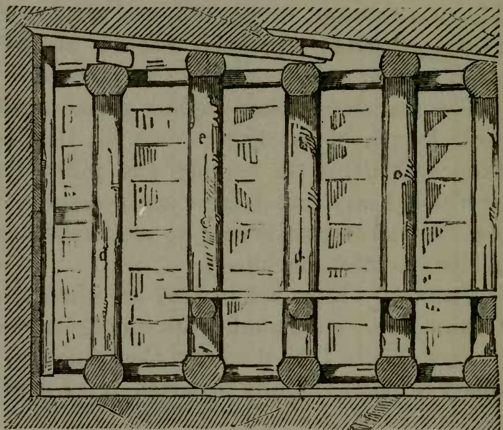


Fig. 1001.

wedges between the collar and the poling-boards serve well enough to keep back the material, it would be dangerous thus to take any of them out were there no other guard for the poling, as the board just above the wedge removed would be pressed down ; a run might also be started, and all the other wedges forced out, when the poling-boards would snap down on the leading collar, and per-

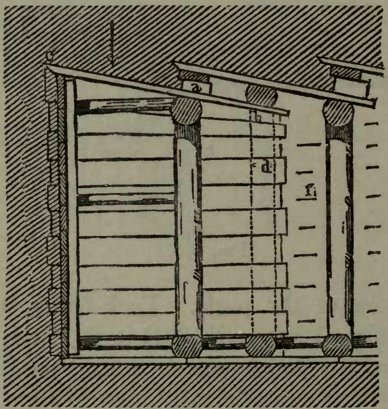


Fig. 1002.

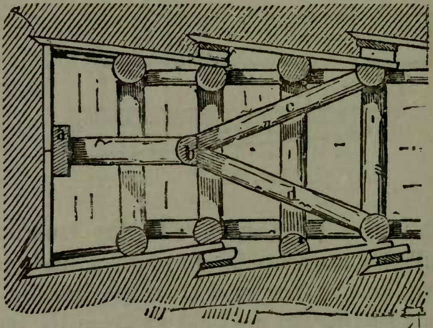


Fig. 1003.

haps break off ; in any event, it would be a matter of great trouble to get them wedged up again. In order to guard against this trouble, a cross-board or plank *a* (Fig. 999) is placed just under the poling-boards, and over the wedges. Then, when one wedge is removed, this cross-connection holds in place the poling-board that is immediately above the wedge removed, until the new board

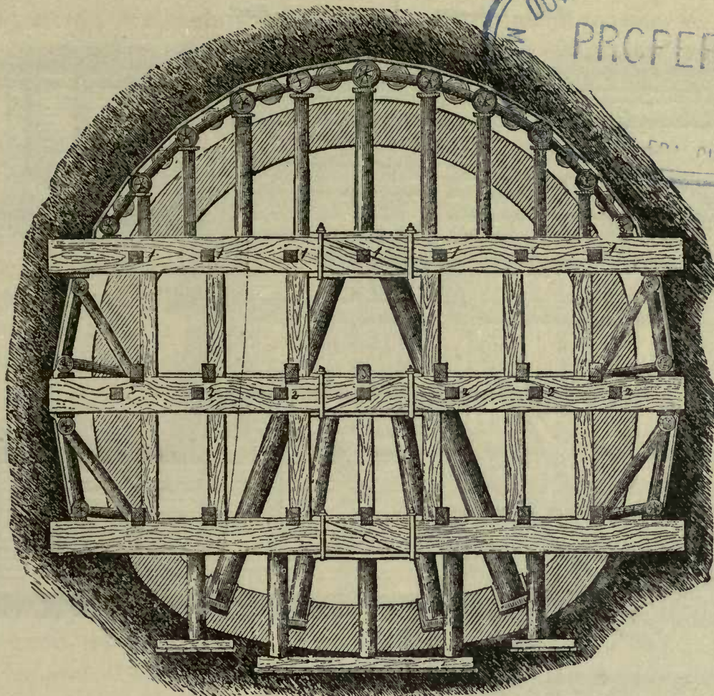


Fig. 1004.

(Section of Fig. 1006, through A B, looking west.)

HOOSAC TUNNEL.

Timbering and arching through soft ground at the West End. Scale, 11′ = 1″.

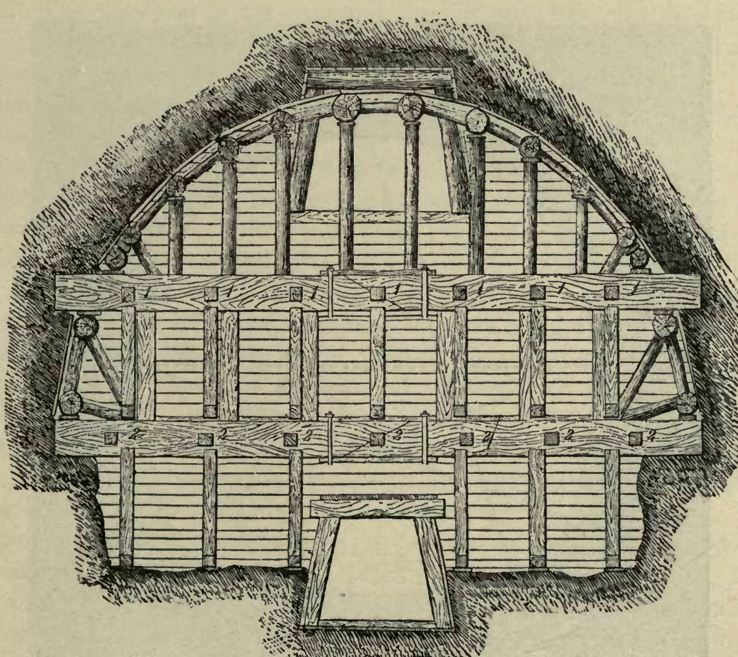


Fig. 1005.

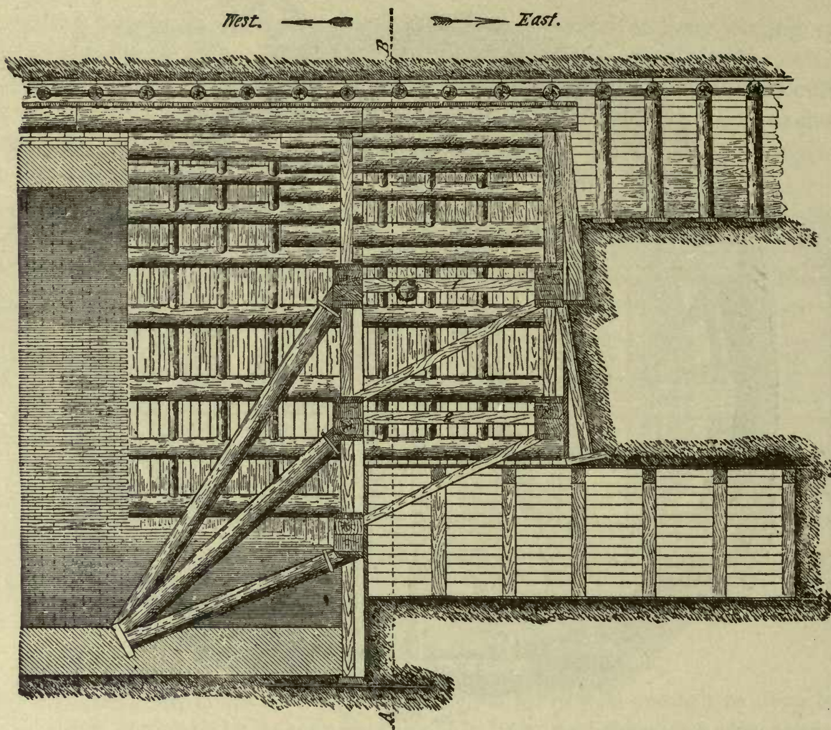


Fig. 1006.

HOOSAC TUNNEL.

West End. Scale, 11′ = 1.

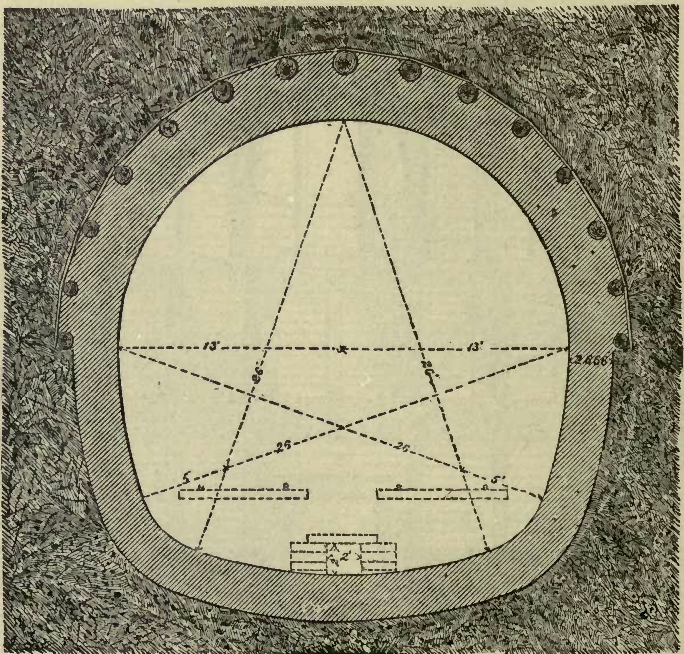


Fig. 1007.

can be put in ; it also stays the tendency to any general movement. The new poling-board being inserted, it is now driven ahead six or twelve inches, and then temporarily stayed by wedges, *b* (Fig. 1001). The corner roof-polings being thus in place, the middle ones (Fig. 998) are similarly inserted. Then the top retaining-board in the face is cut out, and the material allowed to flow into the heading through the space. As room is thus given ahead, the poling-boards are gradually driven forward, say 24 or 30 inches, or about half the length of a board, supposing they are 5 feet long. Whenever they are thus tapped, the wedges *b* (Fig. 999) must be loosened, and then tightened again after the driving. The side-poling is similarly thus advanced ; and we must bear in mind that, as space is gained ahead, it must be protected by new face-boarding, stayed by stretchers. Thus the work can be gradually carried down to the floor of the heading, by successively taking out the face-boards. Often the floor of the gallery also has to be planked, and, in very extreme cases, to be poled similarly to the roof and sides.

We now have reached the point, shown in Fig. 1002, where the new poling-board has been inserted for its half-length. During this operation the boards have been held in place by the double support offered by *a* and *b* (Fig. 1002). The face retaining-boards are kept back by a vertical plank laid across them, and stayed by stretchers. On this newly-excavated chamber the outside pressure will be great, especially acting, as it does, on the front half length of the poling-board *c a*, and, if the remaining work is not rapidly executed, the front ends of the boards may be snapped beyond *a* ; then, if it were attempted to drive the remaining portion of the board on, as soon as its back end left *b* it would snap between *a* and *b*. A middle set is therefore required at once. The middle set being in position, the work of excavating the face can be proceeded with as before. The face-boards are removed, one by one, from top to bottom, and the polings are driven in to their full length ; then in the new length ahead the next main set is erected.

Such are the general principles of heading-driving through running ground, or *sheet-piling* in tunneling.

Figs. 1004 to 1007 show the English system of bar-timbering, as used at the Hoosac Tunnel for the soft ground at the west end. The material was of the worst character, and was exceedingly difficult to drive through. Figs. 1004 and 1005 are cross-sections, the one looking west from A B, the other east. Fig. 1006 is a longitudinal section. Fig. 1007 is a cross-section of the tunnel as completed with an invert, and the bars not drawn but bricked in.

Railway Stock.—Figs. 1008 and 1009 are the elevation and plan of a standard box-car of the New York Central and Hudson River Railroad.

Figs. 1010 to 1013 are the plan and elevations of the truck for the same car.

Figs. 1014, 1015, and 1016 are end-elevations and cross-sections, Figs. 1017 and 1019 longitudinal sections, and Fig. 1018 plan of a standard passenger-car of the Pennsylvania Railroad.

Figs. 1020 to 1023 are elevations, in full and parts, and Fig. 1024 a plan of the trucks of the above car.

In the figures, both of standard box and passenger cars, the elevations and plans are usually broken, to show the construction. When the two sides or

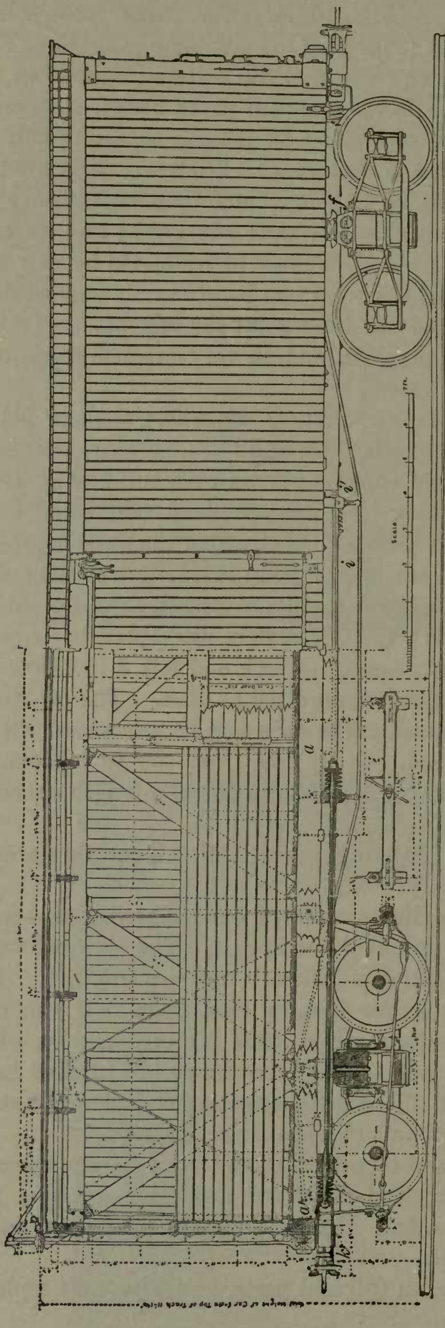


Fig. 1008.

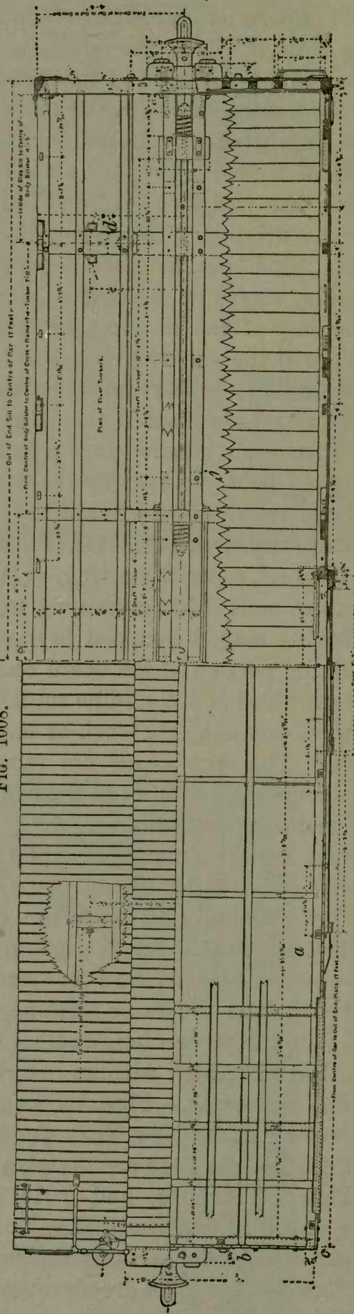


Fig. 1009.

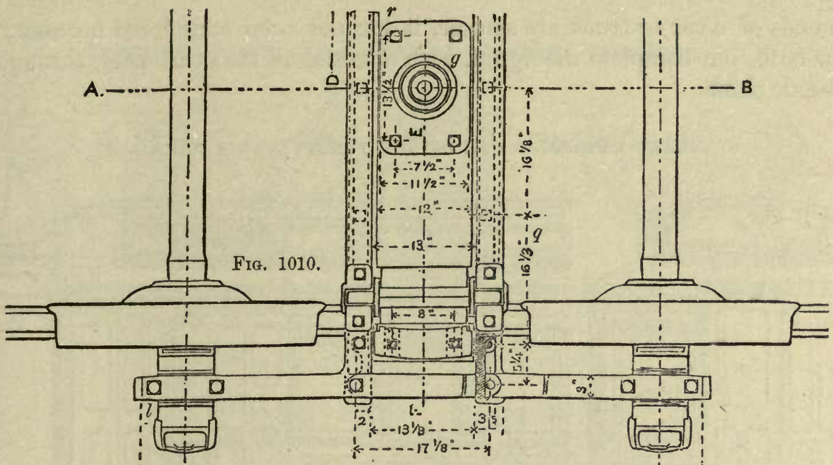


FIG. 1010.

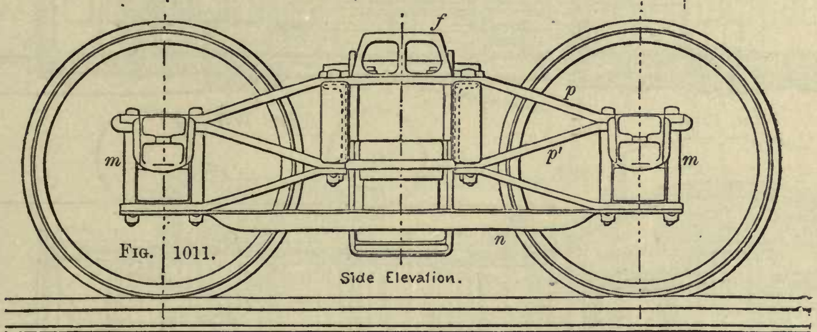


FIG. 1011.

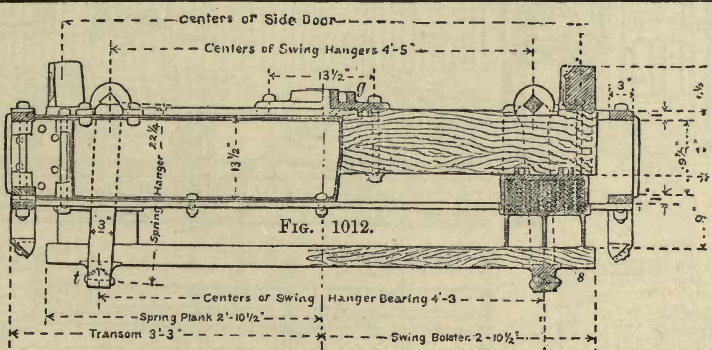


FIG. 1012.

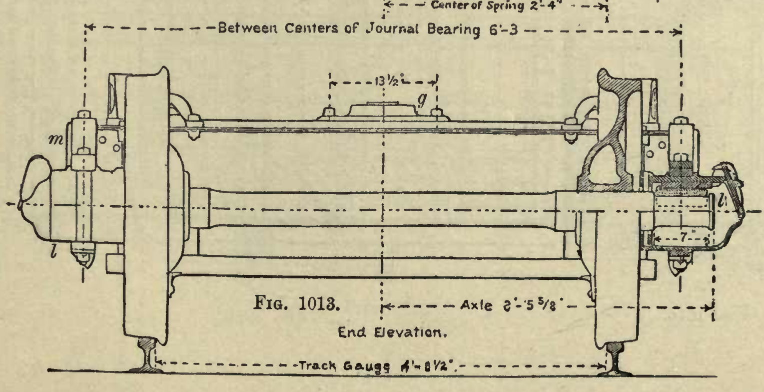


FIG. 1013.

two ends of a car or truck are similar, it has not been considered necessary to show both, but complete the figure, with a section of the other part, through a different plane.

STANDARD PASSENGER CAR OF THE PENNSYLVANIA RAILROAD.

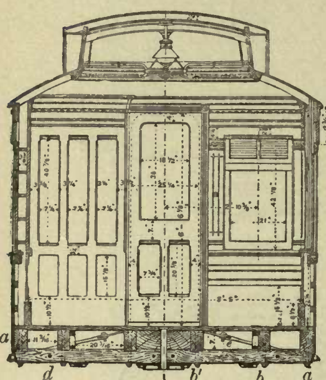


Fig. 1014.

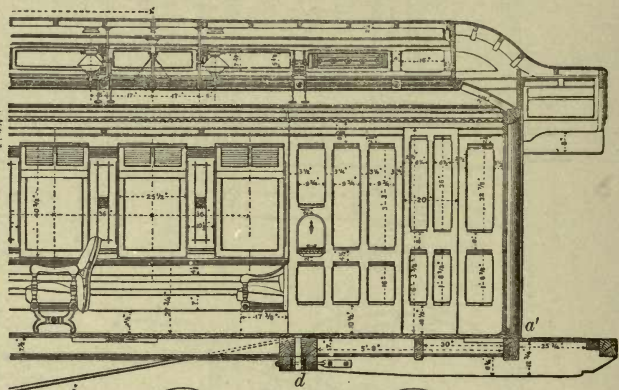


Fig. 1017.

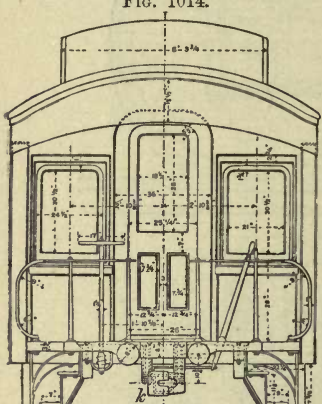


Fig. 1015.

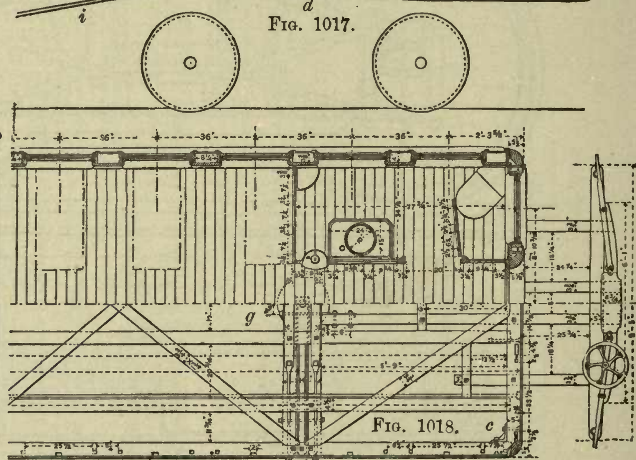


Fig. 1018.

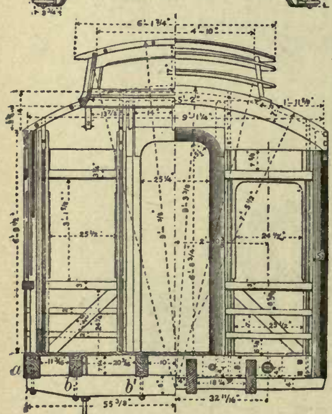


Fig. 1016.

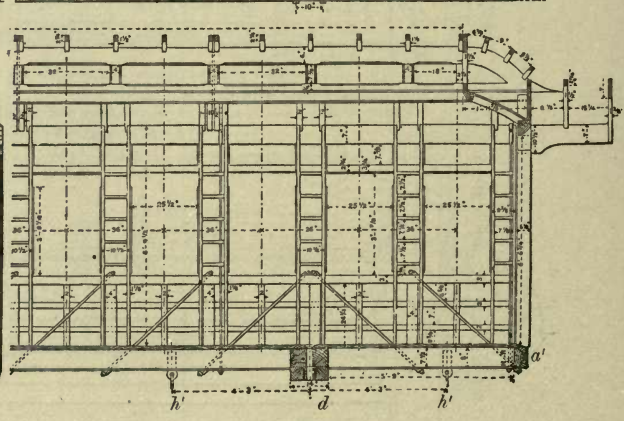


Fig. 1019.

TRUCK OF PENNSYLVANIA RAILROAD STANDARD PASSENGER CAR.

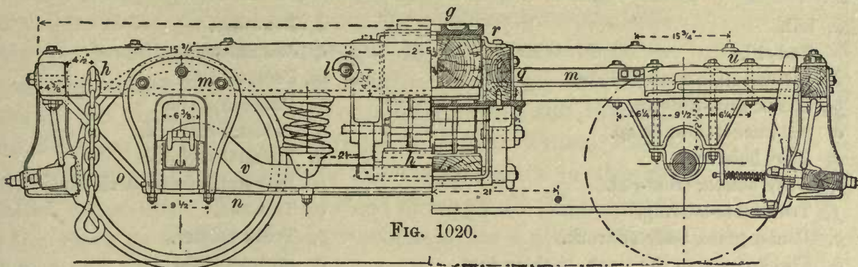


Fig. 1020.

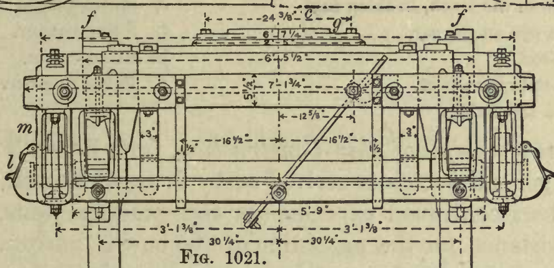


Fig. 1021.

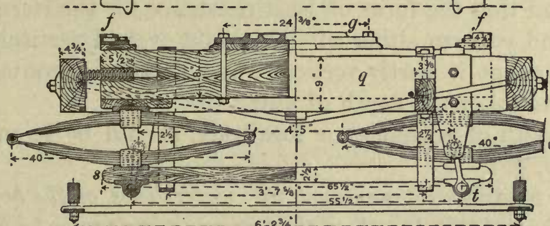


Fig. 1022.

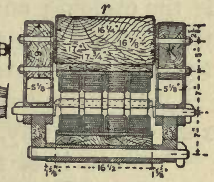


Fig. 1023.

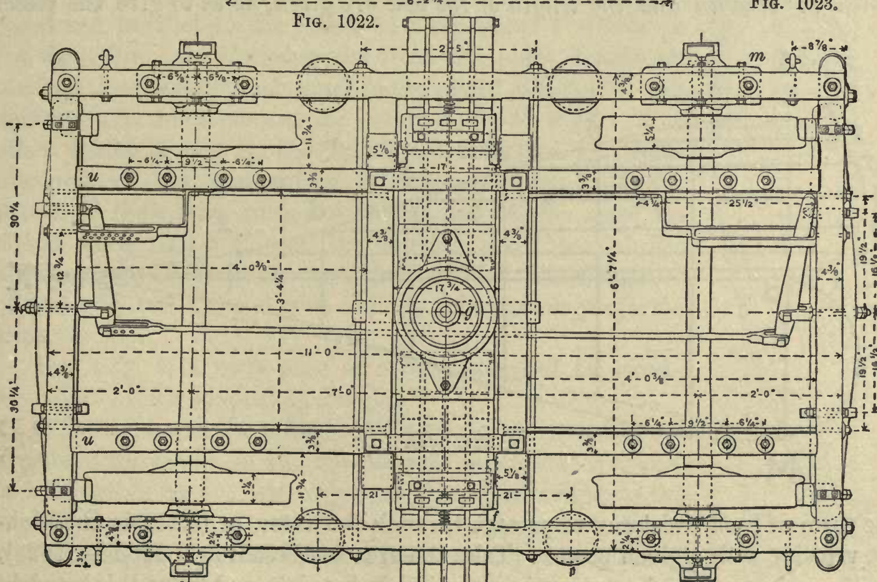


Fig. 1024.

The following letters of reference and technical names of similar parts apply equally to all the figures :

a, Sill.
a′, End-sill.
b, Intermediate floor-timbers.
b′, Center floor-timbers.
c, Sill knee-iron or strap.
d, Body bolster.
e, Body bolster truss-rod.
f, Truck side-bearing.
g, Center plate, body or truck.
h, Check-chain on the truck, hooking into
h′, Check-chain eye on the car.
i, Body truss-rod.
i′, Body truss-rod queen-post.
j, Cross-frame tie-timber.
k, Draw-bar.
l, Journal-box.
m, Pedestal.
n, Pedestal tie-bar.
o, Pedestal stay-rod.
p, Pedestal arch-bar.
p′, Pedestal inverted arch-bar.
q, Transom.
r, Truck bolster.
s, Spring-plank.
t, Swing-hanger.
u, Safety-beam.
v, Equalizing-bar.

The Wave-line Principle of Ship-Construction, from Russell's "Naval Architecture."—The general doctrines arrived at by J. Scott Russell, F. R. S., from numerous and long-continued experiments and practical tests, is "that the form of least resistance for the water-line of the bow is horizontally the curve of versed sines, and that the form of least resistance for the stern of the vessel is the cycloid ; and you can either adopt the said cycloid vertically or horizontally, or you can adopt it partly vertically and partly horizontally, according to the use of the vessel or the depth of water."

"That the length of entrance, or fore body, should be $\frac{3}{5}$, and that of the run, or after body, $\frac{2}{5}$."

"When it is required *to construct the water-lines of the bow of a ship of which the breadth and the length of the bow are given,* so as to give the vessel

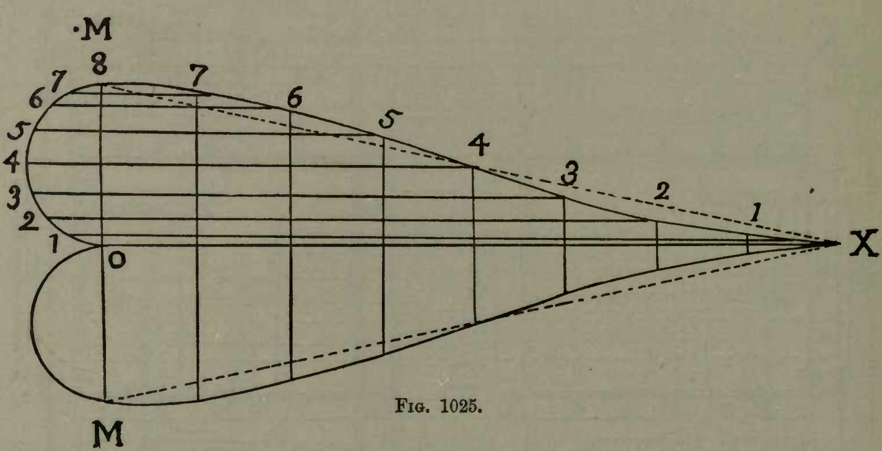


Fig. 1025.

the form of least resistance to passage through the water, or to obtain the highest velocity with a given power : Take the greatest breadth, M M (Fig. 1025), on the main section of construction at midship-breadth, and halve this breadth, M O ; at right angles to M M at O draw the center line of the length of the bow, O X ; on each half-breadth describe a half-circle, dividing its circumference into, say, eight equal parts. Divide the length O X into the same number of equal parts. The divisions of the circle, reckoned successively from the extreme breadth, indicate the breadths of the water-line at the successive corresponding points of the line of length. Through the divisions of the circles draw lines parallel to O X, and through the divisions of O X lines parallel to M M. These, intersecting one another, show the successive points in the required water-line. The line traced through all these points is the *wave water-line of least resistance* for a given length of bow and breadth of body."

To construct the water-lines of the after body or run of a ship (Fig. 1027), the mid-section (Fig. 1026) being given: The bow is constructed as in Fig. 1025, but the divisions are 12 on the center line; for the run lay off 8 divisions, each

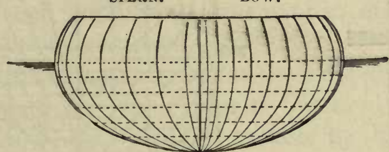


Fig. 1026.

equal to those of the bow; divide the half circle into 8 equal parts, and draw chords to these divisions from 0 to 1, 2, 3, 4. From the point 1 on the center line lay off an inclined line equal and parallel to the chord 0 1; the point 1′ will be in the water-line. In the same way from the point 2 draw an inclined line parallel and equal to the chord 0 2, for 2′, and determine in the same way the points 3′, 4′, 5′, 6′, 7′. The other circles drawn in the figure are described on semi-diameters of the mid-section at different levels, and the points of their wave-lines are determined on the same inclined lines 1 1′, 2 2′, but the lengths are those of the chords of the different circles. In Fig. 1026, the elevations of the mid body, the curved lines of sections are projected from the plan.

Fig. 1028 is a body plan of a vessel adapted to speed; Fig. 1029 of one adapted to freight.

"To determine the after body it is expedient to construct a vertical wave-line on the run as well as a horizontal one, and in designing shallow vessels to give more weight to the vertical wave-line."

"The wave system destroys all idea of any proportion of breadth to length being required for speed. An absolute length is required for entrance and run, but, these being formed in accordance with the wave principle for any given

Fig. 1027.

speed, the breadth may have any proportion to that which the uses of the ship and the intentions of the constructor require."

"The wave system allows us to give the vessel as much length as we please. It is by this means that we can give to a vessel of the wave form the capacity we may require, but which the ends may not admit. Thus, the Great Eastern, which is a pure example of the wave form, has an entrance or fore body of 330′, a run or after body of 220′, and a middle body of 120′, which was made of this length merely to obtain the capacity required. The lengths of the fore and after body are indicated by the required speed, and if the beam is fixed, it is only by means of a due length of middle body that the required capacity, stability, and such other qualities are to be given as will make a ship, as a whole, suit its use."

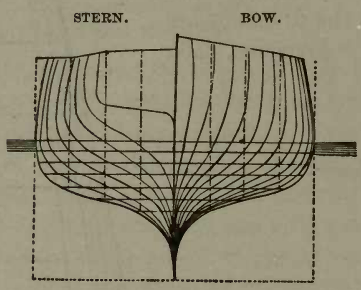


FIG. 1028.

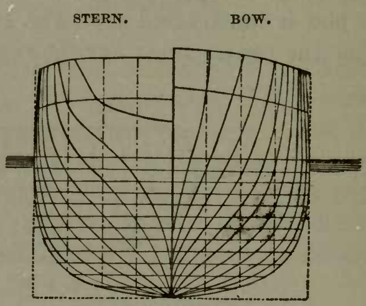


FIG. 1029.

Length of entrance of a vessel for a 10-mile speed should be 42 feet, of run 30 feet; for a 20-mile speed, 168 and 120 feet; that is, the lengths increase as the squares of the speed.

Under Isometrical Drawing are given illustrations of vessels constructed on wave-lines.

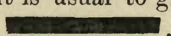
ARCHITECTURAL DRAWING.

It is the duty of an architect to design a building to be suitable and convenient for the purposes for which it is intended ; to select and dispose of the materials of which it is composed to withstand securely and permanently the stresses and wear to which they may be subjected ; to arrange the parts to produce the artistic effects consistent with the use of the building and its location, and to apply such appropriate ornament as may express the purpose and harmonize with the construction.

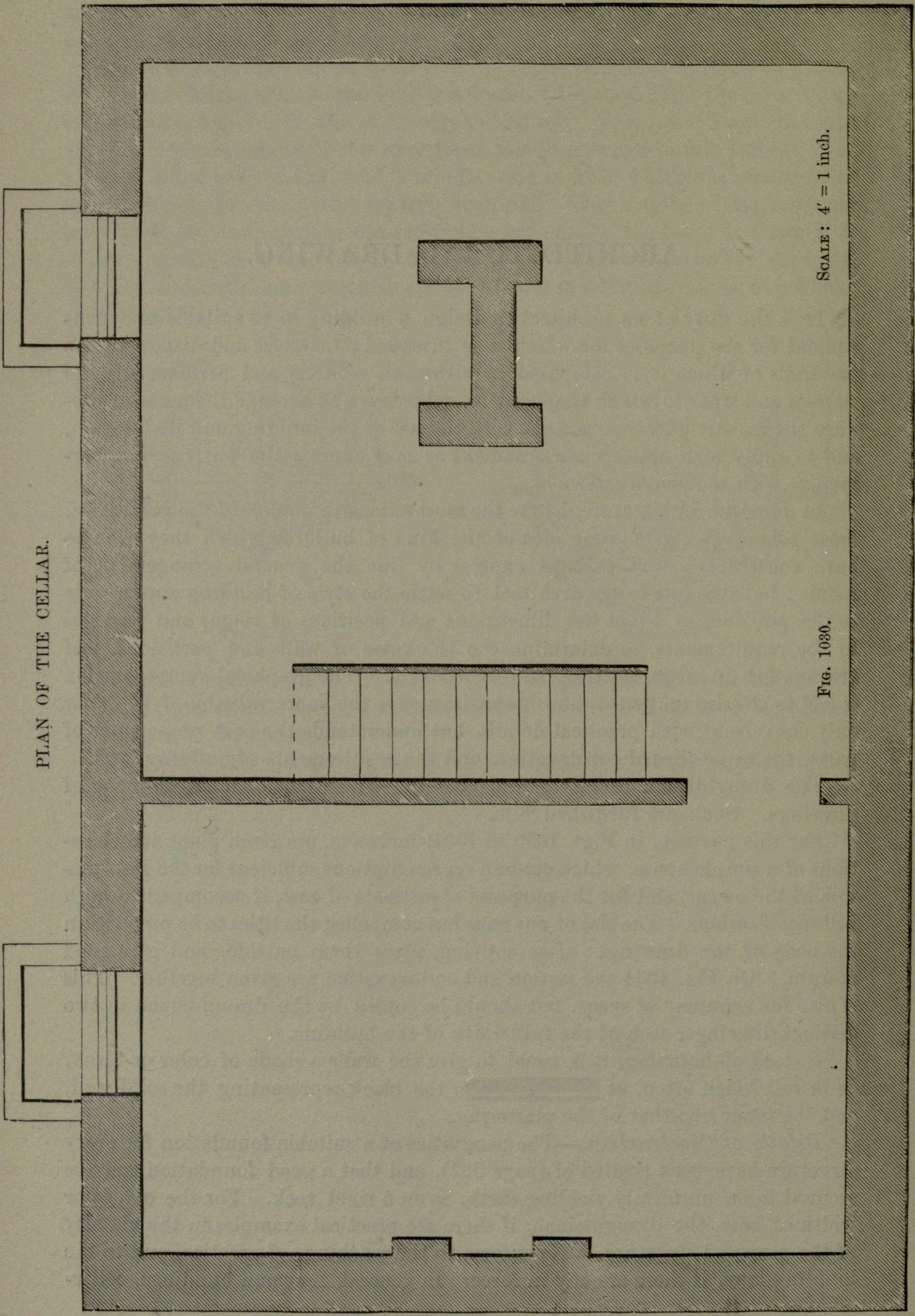
In domestic architecture, by far the most extensive branch of the profession, most persons can give some idea of the kind of building which they wish to have constructed, and perhaps express by line the general arrangement of rooms ; but it is left to the architect to settle the style of building appropriate to the position, to adapt the dimensions and positions of rooms and passages to the requirements, to determine the thickness of walls and partitions, and arrange for drainage, heating, and ventilating. The graphical representation is left to the draughtsman, and his assistance is the more valuable if he is not only conversant with practical details, but understands the best proportions of parts, the necessities of construction, and the requirements of building laws.

The draughtsman usually commences his education with the copying of drawings. Such are furnished him.

For this purpose, in Figs. 1030 to 1034, inclusive, are given plans and elevations of a simple house, which contain representations sufficient for the information of the owner, and for the purposes of estimate of cost, if accompanied with full specifications. The size of our page has compelled the titles to be put within the body of the drawings ; after copying, place them outside, and give good margin. On Fig. 1034 the section and end-elevation are given together. This is also for economy of space, but should be copied by the draughtsman in two distinct drawings, each of the full width of the building.

Instead of hatching, it is usual to give the walls a shade of color or black, or in full black often, as , the black representing the solid wall, and the inner line that of the plastering.

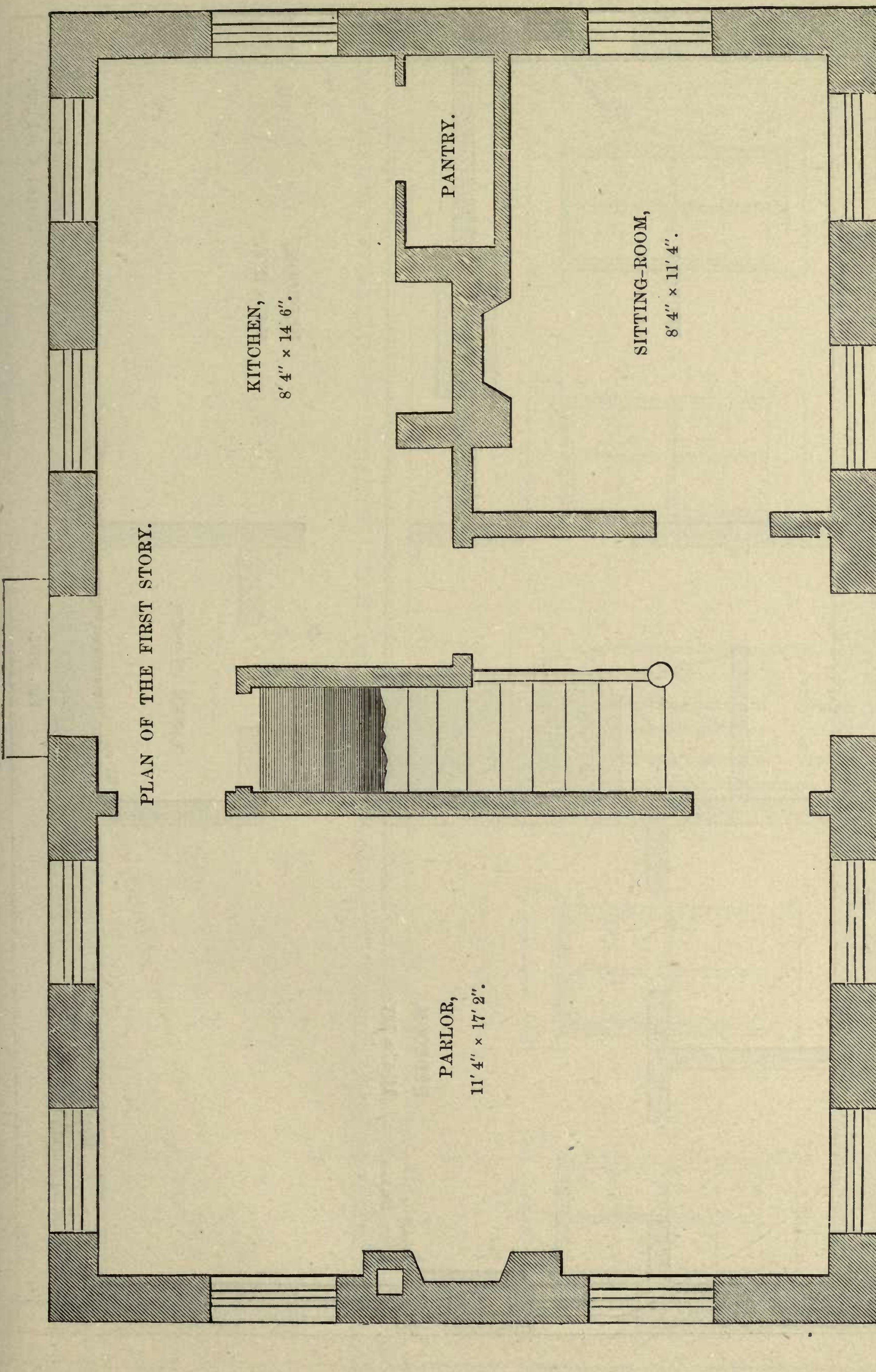
Details of Construction.—The necessities of a suitable foundation for every structure have been treated of (page 362), and that a good foundation may be secured in an uniformly yielding earth, as on a rigid rock. For the extent or width of base, the draughtsman, if there are practical examples in the vicinity of the proposed structure, will conform to the teachings of practice, and to the building laws, if there are any in force. In general, for small buildings, cellar-



PLAN OF THE CELLAR.

Fig. 1030.

Scale: 4′ = 1 inch.



SCALE: 4′ = 1 inch.

FIG. 1031.

PLAN OF THE ATTIC.

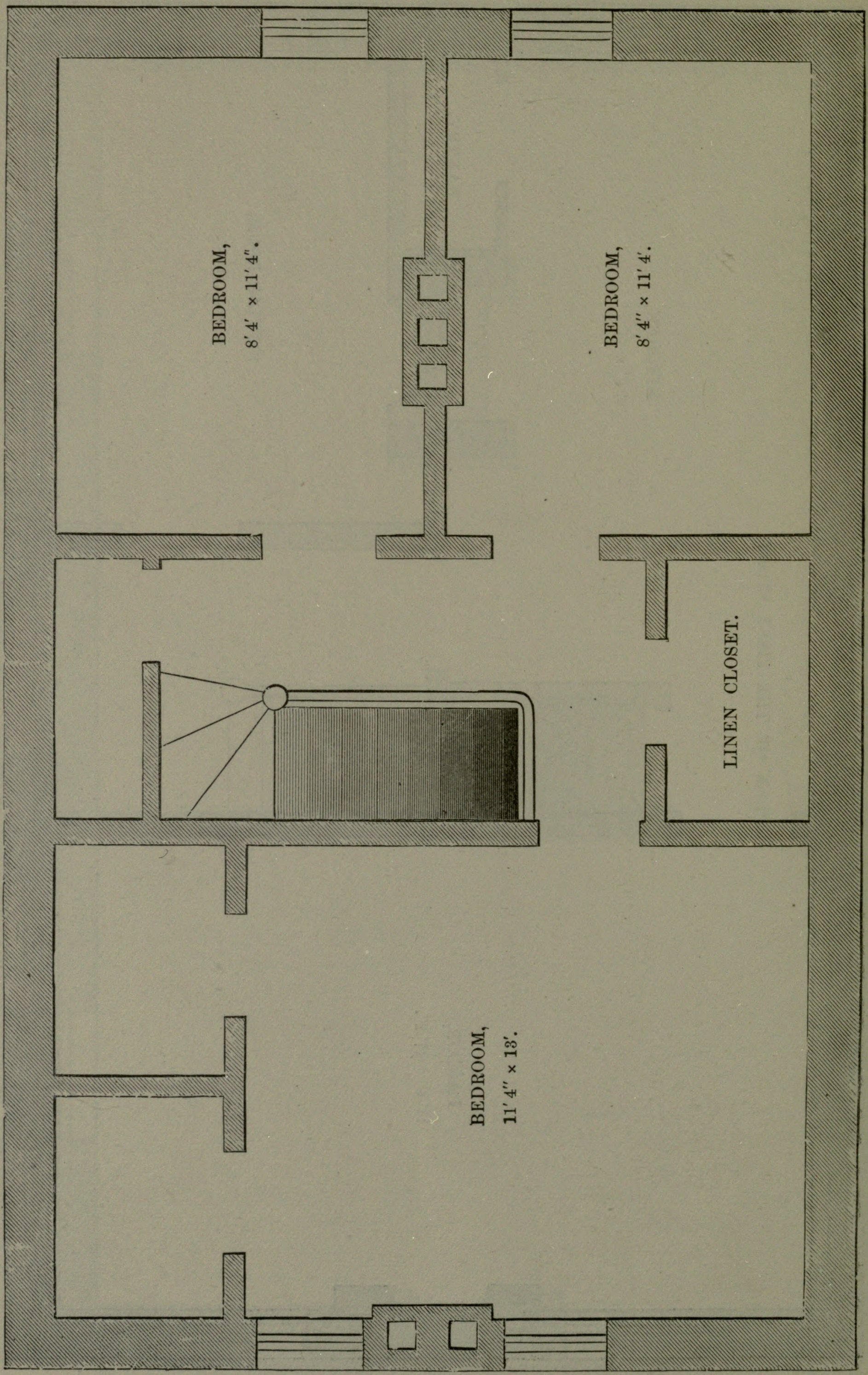


Fig. 1032.

Scale: 4′ = 1 inch.

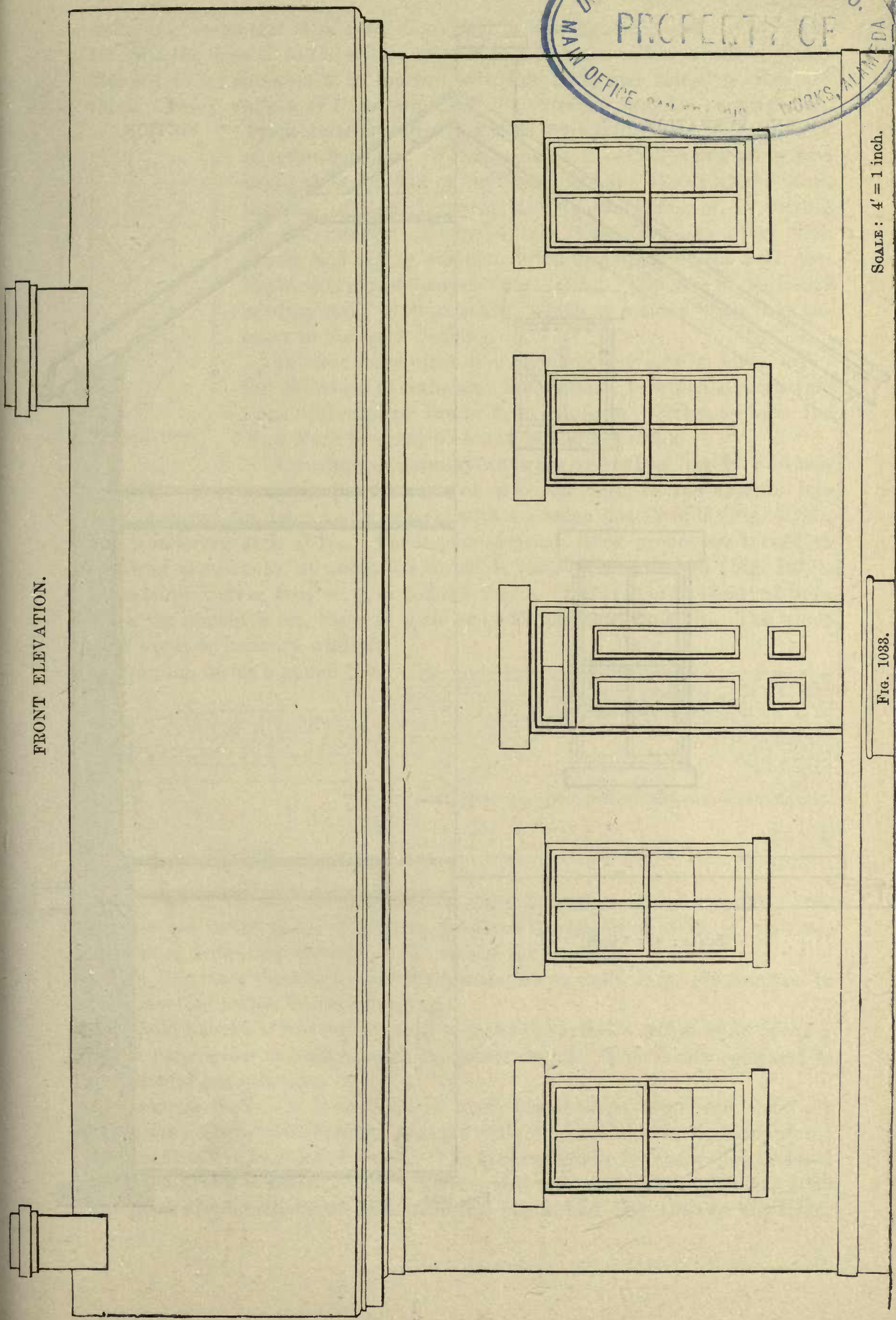


Fig. 1033.

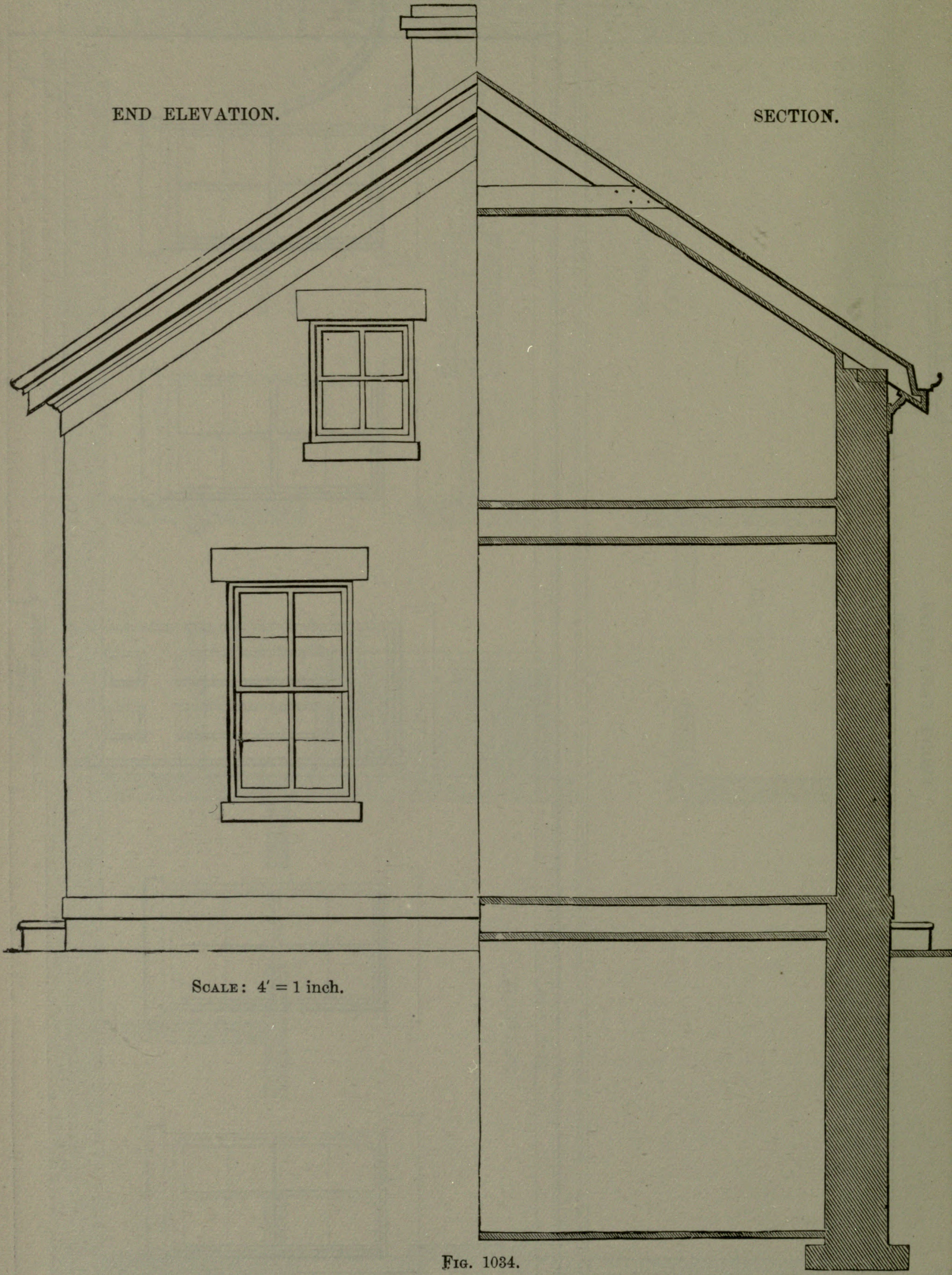


FIG. 1034.

walls, if of stone laid in mortar, should not be less than 18″ thick; if of brick, 16″, and the base 6″ to 12″ wider. For walls above the cellar, it will be found difficult to lay stone walls in mortar, with fair bond and face, less than 16″ thick. Brick walls may be as thin as 8″ for exteriors, and for partitions 4″.

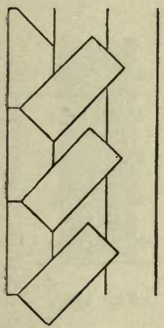


FIG. 1035.

Brick walls are usually bonded by heading-courses every fifth to seventh course. Where the outside course is pressed or face brick, these are laid on stretchers, and the bond with the backing may be thin strap-iron, laid in the joints, or, by cutting off the interior corners of the face-course, say every fifth course, and laying common brick diagonally of the wall resting in this clipped corner (Fig. 1035). The face of buildings is often built of thin ashlar, which is secured with iron anchors to the brick backing.

In most large cities there are building acts in force, defining thickness of walls and foundations, to which all constructions within their limits must conform. Extracts from the New York law may be found in the Appendix.

Openings in masonry-walls are covered by lintels or arches, or both. It is usual to place a stone or cast-iron lintel in the exterior face over openings for doors and windows, with a wooden lintel inside (Fig. 1036), and a relieving arch above. For larger openings, brick arches are turned in cast-iron skew-backs, of which the thrust is resisted by a tie-bolt (Fig. 1037), or cast-iron lintels, box, or ⊥, or roller I-beams. But it is to be observed that, when the cement is set, there is little or no thrust from the arch. The whole *dead work*, or masonry without an opening, forms a monolithic

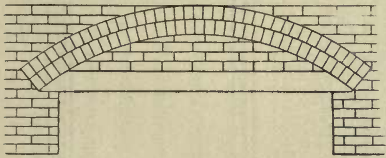


FIG. 1036.

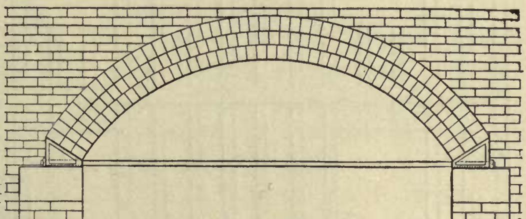


FIG. 1037.

beam, and, if there is depth enough of this, the arch is of no account. It is the custom in the north of Italy to construct flat lintels of brick, of considerable span, depending entirely on the mortar for strength.

To distribute the weight over the foundation or walls, it is very common to turn inverted arches beneath openings.

In old houses, it was not unusual to make the exterior arches of an opening flat or rectangular in outline, with the joints radial. This is now relegated to ornamental construction.

Concrete Walls.—It is common in many places where brick and stone are expensive and gravel is abundant to make walls of concrete, in proportions of one of cement to five to seven of gravel. The space requisite for the wall is inclosed with plank, and is filled in with concrete, well rammed. Figs. 1038 and 1039 are plans of concrete walls with inclosing plank, and Fig. 1040 an elevation.

The planks are held by bolts passing through wall and plank, all of which are removed after the wall is set, and the bolt-holes are then filled with cement. The thickness of walls should be a little in excess of those of brick.

Wooden walls are framed. Fig. 1041 represents the frame

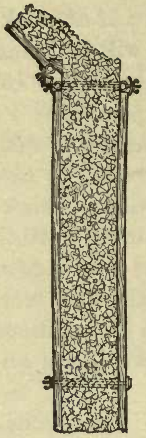


Fig. 1038.

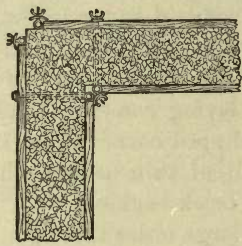


Fig. 1039.

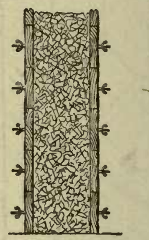


Fig. 1040.

of the side of a wooden house, in which A A are the *posts,* B the *plate,* C C *girts* or *interties,* D D *braces,* E *sill,* F *window-posts* or *studs,* G G *studs.*

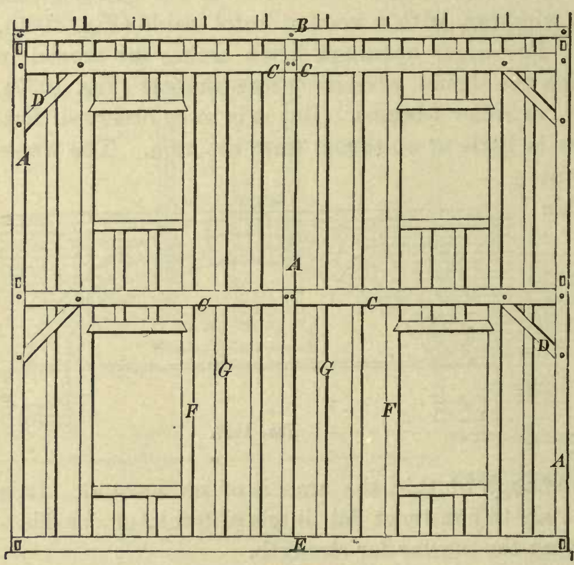


Fig. 1041.

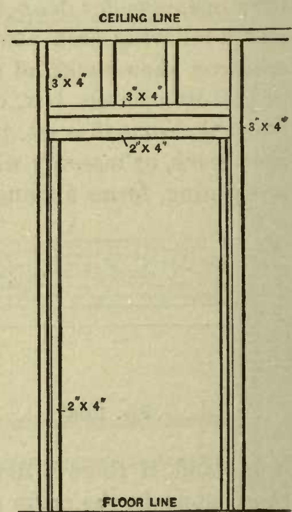


Fig. 1042.

The studs at all door-openings should be set at least 2″ wider, and 3″ higher than the size of the finished opening. It is not unusual to have double studs (2″ × 4″) to inclose these openings (Fig. 1042). This leaves the doorway more or less independent of the partition.

Usual dimensions of timber for frame of common dwelling-houses: sills 6″ × 8″, posts 4″ × 8″, studs 2″ × 4″ or 3″ × 4″, girts 6″ × the depth of floor-joists, plates 4″ × 6″; the floor-joists (J, Fig. 1043) are notched into the girts. The posts and studs are *tenoned* into the sills and girts. Fig. 1045 represents a *tenon, b c,* in side and end elevation, and *mortice, a*; the portions of the end of the stud resting on the beam are called the *shoulders* of the tenon. In the balloon-frame the girts are omitted; the studs are of the same length as the posts, and the floor-joists are supported by a board, *a*, 3″ or 4″ × 1″, let into the studs (Fig. 1044), and firmly nailed; the joists are also nailed strongly to the studs.

The frame is covered with boards usually 1″ thick, laid either horizon-

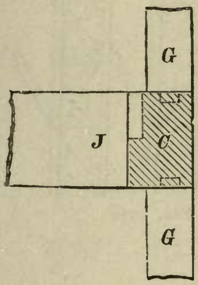


Fig. 1043.

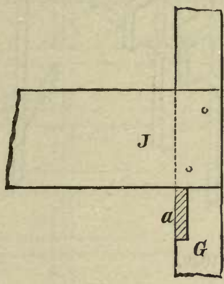


Fig. 1044.

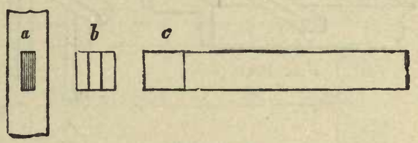


Fig. 1045.

tally or diagonally, and nailed strongly to the posts or studs. Fig. 1046 is the elevation of the end frame of a house, showing by breaks the diagonal cover of boards and the inner lathing. The lower story is sheathed or ceiled with narrow boards, the upper shingled. With balloon frames, the bracing depends largely on the diagonal boarding.

Partitions are usually simply studs set at intervals of 12 or 16 inches, these spaces being adapted to the length of the lath (48 inches). The sizes of the studs are generally 2 × 4, 3 × 5, or 3 × 6 inches, according to the height of the partition; for very high partitions, greater depth may be required for the studs, but three inches will be sufficient width.

Partitions are usually cut in between sills placed on the floor-beams (Fig. 1047), and similar caps above, beneath the beams. Where partitions of the second story are directly above those on the first story it is better to foot the studs on the caps of the latter, and not on the beams (Fig. 1048). Where there are double floors, the sills are placed on the bot-

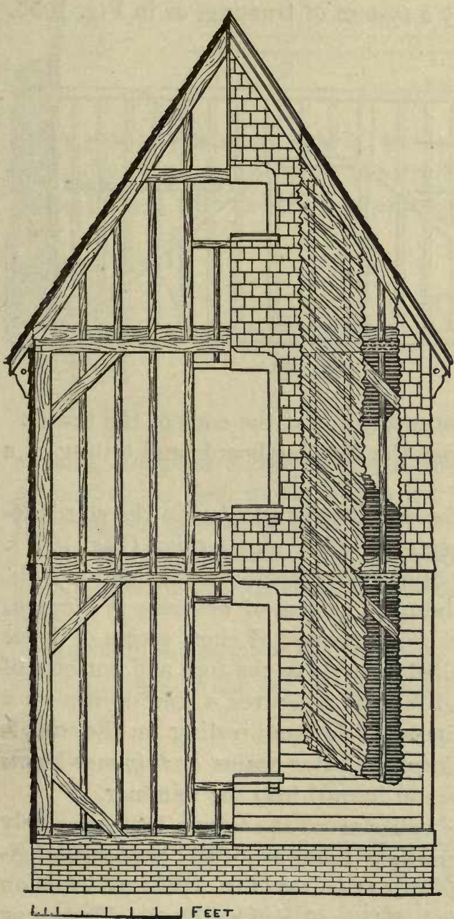


Fig. 1046.

tom floor, or on the floor without a sill. It may be important that the partitions should be self-sustaining. This is effected by simple bridging, well

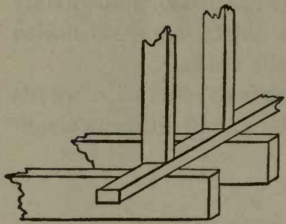


Fig. 1047.

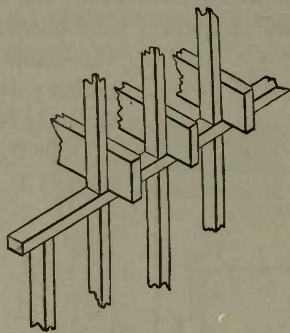


Fig. 1048.

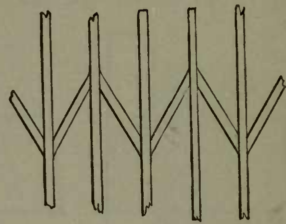


Fig. 1049

nailed to the studs, as shown in Fig. 1049, or by herring-bone bridge, as shown in plan of floor (Fig. 1051), or by a system of trussing, as in Fig. 1050. This method of trussing must vary with the position of opening. The foot of the braces should rest on a positive support.

The bridging should be accurately cut and firmly nailed. Bridging distributes the weight of the partition, but trussing concentrates it at the ends of the braces.

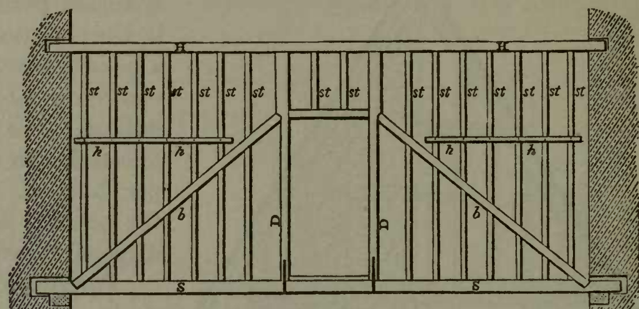


Fig. 1050.

Flooring.—The timbers which support the flooring-boards and ceiling of a room are called the naked flooring.

The simplest form of flooring, and the one usually adopted in the construction of city houses and stores, is represented in plan and section (Fig. 1051). It consists of a single series of beams or deep joists, reaching from wall to wall. As a lateral brace between each set of beams a system of *bridging* is adopted, of which the best is the *herring-bone* bridging, formed of short pieces of joists about 2 × 3, crossing each other, and nailed securely to the tops and bottoms of the several beams, represented by *a* and *b*; and wherever a flue occurs, or a stairway or well-hole prevents one or more joists from resting on the wall, a *header,* H, is framed across the space into the outer beams or *trimmer-beams* T T, and the beams cut off or *tail-beams* are framed into the trimmer.

Whenever the distances between the walls exceed the length that can safely be given to joists in one piece, an intermediate beam or girder, running longitudinally, is introduced, on which the joist may be set (Fig. 1052), notched on (Fig. 1053), or boxed in (Fig. 1054), or both boxed and notched. They may also be framed in with tenon and mortice; the best form is the *tusk*-tenon

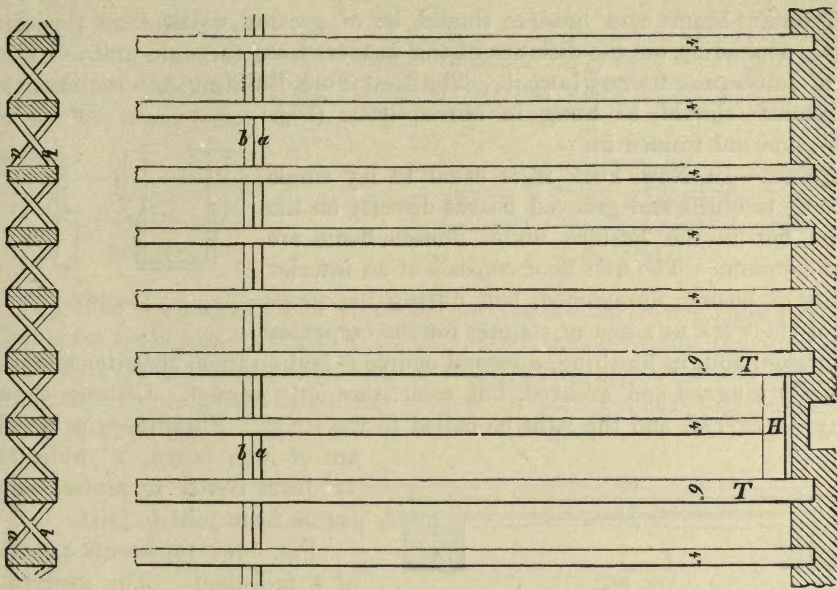


FIG. 1051.

(Fig. 1055). Flooring is still further varied, by framing with girders longitudinally; beams crosswise, and framed into or resting on the girders; and joists framed into the beams, running the same direction as the girders. It is

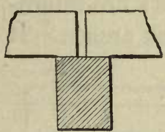


FIG. 1052.

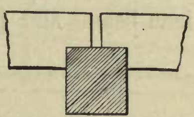


FIG. 1053.

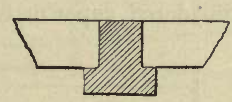


FIG. 1054.

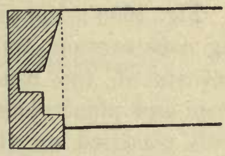


FIG. 1055.

evident, when the joists are not flush or level with the bottom of the beams or girders, either that in the finish the beams will show, or that ceiling-joists or furrings will have to be introduced.

On the Size of Joists.—The following dimensions may be considered as safe sizes for ordinary constructions, the distances from center to center being one foot.

Joists in floors, clear bearing—

Exceeding	7	feet,	and not	exceeding	10	feet,	to be	not less than	6 × 2	inches.
"	10	"	"	"	12	"	"	"	6 × 3	"
"	12	"	"	"	14	"	"	"	7 × 3	"
"	14	"	"	"	16	"	"	"	9 × 3	"
"	16	"	"	"	18	"	"	"	9 × 3	"
"	18	"	"	"	20	"	"	"	10 × 3	'
"	20	"	"	"	22	"	"	"	11 × 3	"
"	22	"	"	"	24	"	"	"	12 × 3	"

It is to be observed that lumber is seldom sawed to dimensions of fractions of an inch.

Trimmer-beams and headers should be of greater width than the other beams, depending on the distance of the headers from the wall, and the number of tail-beams framed into it. The New York Building Act requires that all headers should be hung in stirrup-irons (Fig. 1056), and not framed in.

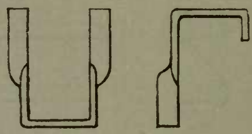


FIG. 1056.

Floors.—In New York it is usual to lay single floors of tongued and grooved boards directly on the beams, but in the Eastern States double floors are more common. The first floor consists of an inferior quality of boards, unmatched, laid during the progress of the work as a sort of staging for the carpenter and mason, and, in finishing, a second course is laid on them of better material, generally tongued and grooved, but sometimes only jointed. Ceilings should always be *furred*, and the laths be nailed to the strips. Furring-strips usually are of inch board, 2″ wide, and 12″ from center to center, nailed across from joist to joist.

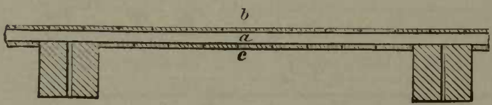


FIG. 1057.

Fig. 1057 represents a section of a mill-floor. The girders or beams, generally in pairs, with a space of about an inch between them, are placed at a distance of from seven to nine feet from center to center, and are of from twelve to sixteen inches in depth. On these, a tongued and grooved plank floor of from 3″ to 4″ thick is laid.

Fig. 1058 is the section of a beam and mill-floor now adopted as a fire-retarding construction, and considered superior to iron beams and brick arches. It consists of the usual beam and plank floor; both plastered on the under side and on the lateral surfaces. The lathing consists of wire cloth stapled through furring strips $\frac{3}{8}''$ to $\frac{1}{2}''$, and then the usual three-coat plaster. In addition, it is common to lay roofing-felt on the upper surface of the plank, with 1″ to $1\frac{1}{2}''$ of cement mortar; with the usual floor on the top of this, the floor being nailed to strips attached to the plank, and serving as guides to surface the cement mortar. Various methods are given (page 238) of trussing beams when the spans or loads are in excess of the strength of lumber of the usual dimensions.

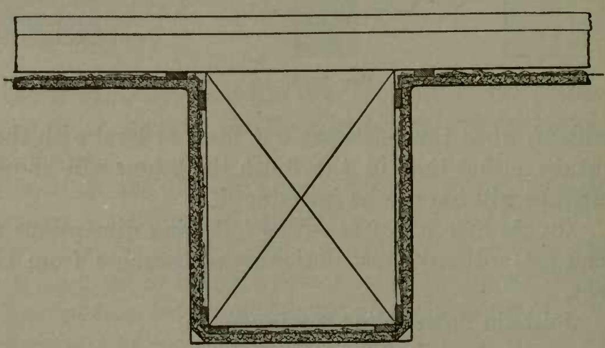


FIG. 1058.

Joinings.—As timber can not always be obtained of sufficient lengths for the different portions of a frame, or to tie the walls of a building, it is often necessary to unite two or more pieces together by the ends, called scarfing or lapping. Fig. 1059 is a most common means of lapping or halving employed when there is not much longitudinal stress, and when a post is to be placed beneath the lower joint.

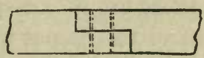


Fig. 1059.

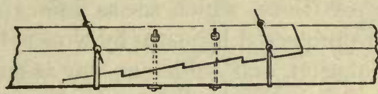


Fig. 1060.

Fig. 1060 is a long scarf, in which the parts are bolted through and strapped, suitable for tie-beams. Joints (Figs. 1061, 1062, and 1063) are also often made by abutting the pieces together, and bolting splicing-pieces on each side ; still further security is given by cutting grooves in both timbers and pieces, and driving in keys, *k k*.

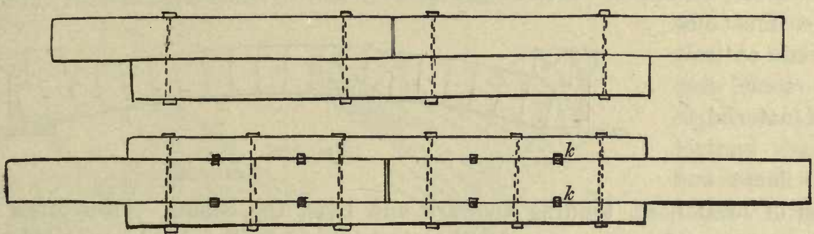


Fig. 1061.

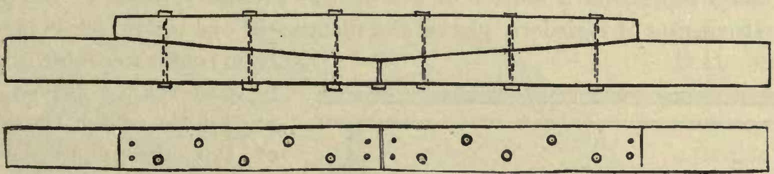


Fig. 1062.

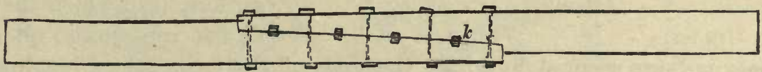


Fig. 1063.

Floor-beams in a building acting as ties are usually strapped, or anchored together by iron bars, spiked to the top or bottom of the beams, often sunk into the beam.

Fig. 1064.

Fig. 1065.

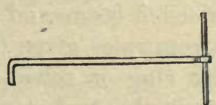


Fig. 1066.

Figs. 1064, 1065, and 1066 are common forms of anchors. The first two for connecting beams, the last for beams and walls. In warehouses, it is usual to carry the anchors entirely through the wall, with a washer and nut outside. The beams are often joint-bolted together like stair-rails.

Fire-resisting Floors.—Flames spread through buildings by means of the spaces left between floors and ceilings, and between walls and furrings and hollows in partitions, which act as flues. But when the wooden beams and plank floors are protected beneath by wire netting and plaster there are no air-spaces for circulation, and sufficient stay is made in the progress of the flames to admit of the application of means for extinguishment. And if the beams are placed close together, and the joints filled with cement, there is still greater security. Experiments were made in Paris on asphalt floors laid on plank, and they resisted for a very long time the spread of flames, both when fires were kindled beneath the floors, and directly on top of the asphalt. In the latter case, a thin layer carbonized, and afforded a good fire-proof material.

Iron beams and brick arches, as in Fig. 1067, are the usual form of fire-proof floors, but when efficient protection against fire is desired the bottom flange must be covered entirely with some fire-proof material, to prevent contact with flame and excess of local heat, tending to warp and twist the beams. Iron often becomes necessary for spans greater than can be met by wooden beams, and they should be protected by some fire-proof covering.

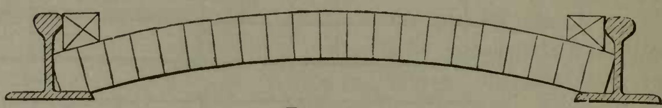


Fig. 1067.

Fig. 1068 represents a section of one of the French systems of fire-proof floors. It consists of I-girders, placed at a distance of one metre (39·38 inches) from center to center, slightly *cambered* or curved upward in the center, the depth of the girders to depend upon the span. Stirrups of cast-iron are slid upon the girders, into which the ends of flat iron joists, set edgeways, pass and are secured by pins; the ends of the joists take a bearing also on the bottom flanges of the girders. The joists are placed at a distance of one metre from center to center. Upon the joists rest rods of square iron, which in this way form a *grillage* for the support of a species of rough-cast and the ceiling. By this and other very similar systems, the French have succeeded in reducing the cost of such floors to that of wooden ones.

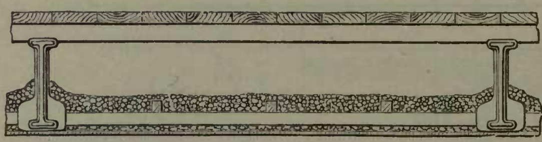


Fig. 1068.

The dimensions of beams and girders for the above constructions can readily be determined from rules given (page 233). The brick arches (Fig. 1067) are usually in single ring or rolock courses, and beams spaced from 3′ to 6′ centers. Strips of plank are fastened on the top or at the side of the beam to receive the floor, and the spandrels leveled up with concrete.

Floors constructed of concrete, in plain cylindrical or groined arches (Fig. 1069), are cheap and efficient constructions. One of the warehouses of the publishers is covered by arches of this last form. Posts of brick, 2 feet square, 13 feet centers, arches arcs of circles, depth of concrete at spring 21″, at key 9″ to a level floor, supporting presses.

In Italy ceilings are made in single courses of brick, and groined, laid without centers, the arcs being described on the side-walls, and the bricks laid to a line in plaster. The spandrels may be leveled up with concrete, when rooms above are to be occupied, but often there is only the brick arch forming the ceiling of the principal rooms, with a light wooden roof above.

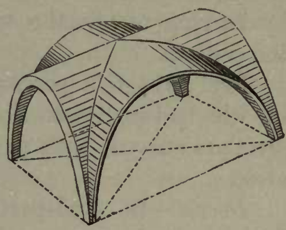


Fig. 1069.

Figs. 1070 to 1073 are illustrations of Roman constructions in masonry, from "Dictionnaire Raisonné de l'Architecture," par M. Viollet Le Duc.

Fig. 1070 is a perspective view of a cylindrical arch in process of construction. The centers A and lagging B are quite light, as the full load of the arch is never borne by them. On the lagging, B, a cover of flat tile, C, is laid in cement, and above ribs, D D, and girts, E E, in brick masonry, shown on a larger scale in Fig. 1071, with the plank P used for the support of the girt bricks E, which is removed after the mortar is set. The panels are now filled with concrete.

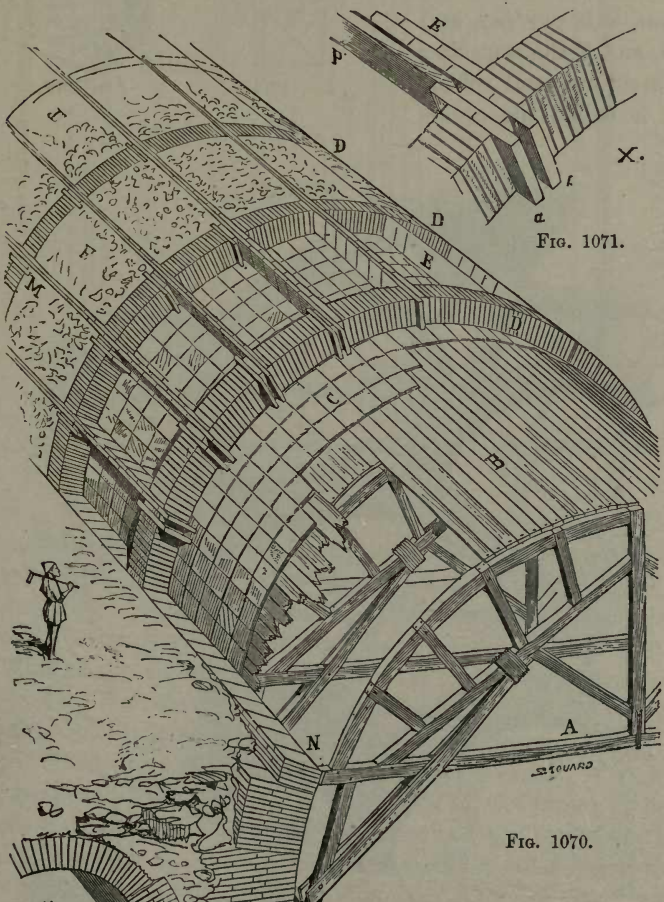


Fig. 1071.

Fig. 1070.

Fig. 1072 represents rib and portions of girts of a groin shown in plan, Fig. 1073, *e f g h* being that of the rib; *K*, a timber of the center.

A similar construction also obtained for domes, the girts being of the same width as the ribs, and sunk panels formed by furring up on the wooden lagging of the centers.

Fig. 1074 is a perspective of a dome, in which the brick skeleton, ribs, and girts are curved, with panels, B B, of concrete.

Doors.—In stud-partitions, the openings for doors are framed as in Fig. 1042, the door-frame being independent of the studs.

Fig. 1075 represents the elevation and Fig. 1076 the horizontal section of a common inside-door. A A are the *stiles*, B, C, H, D, the *bottom, lock, parting*, and *top rail*, E the *panels*, and F the *muntin*; the combination of moldings and offsets around the door, G, is called the *architrave*; in the section, *a a* are the partition-studs, *b b* the door-*jambs*.

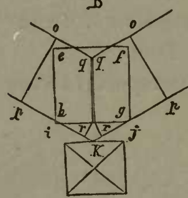


Fig. 1073.

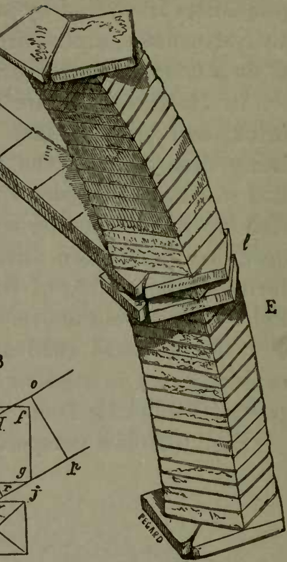


Fig. 1072.

Fig. 1077 represents the forms of the parts of a door, and the way in which they are put together. When the tenons are to be slipped into the mortises, they are covered with glue, and, after being closed up, keys are driven in.

With regard to the proportions of internal doors, they should depend in some degree on the size of the apartments; in a small room a large door always

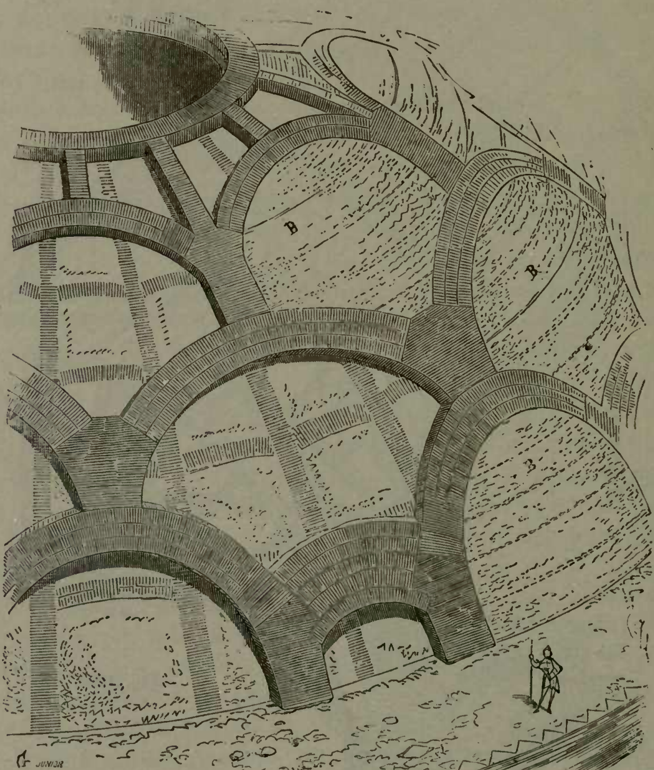


Fig. 1074.

gives it a diminutive appearance, but doors leading from the same room or passage, which are brought into the same view, should be of uniform height. The smaller doors which are found on sale are 2 feet 4 inches × 6 feet; for water-closets, or very small pantries, they are sometimes made as narrow as 15 inches, but any less height than 6 feet will not afford requisite head-room; 2 feet 9 inches × 7 feet, 3 feet × 7 feet 6 inches, or 3 feet 6 inches × 8 feet, are well-proportioned, six-paneled doors. But the apparent proportions of a door may be varied by the omission of the parting-rail, making the door four-paneled, or narrowed still more by the omission of the lock-rail, making a two-paneled door. Sometimes the muntin is omitted, making but one panel; but this, of course, will not add to the appearance of width, but the reverse. Wide panels are objectionable, as they are apt to shrink from the moldings and crack. The moldings are generally planted on, and nailed to the stiles and rails, but sometimes formed on them.

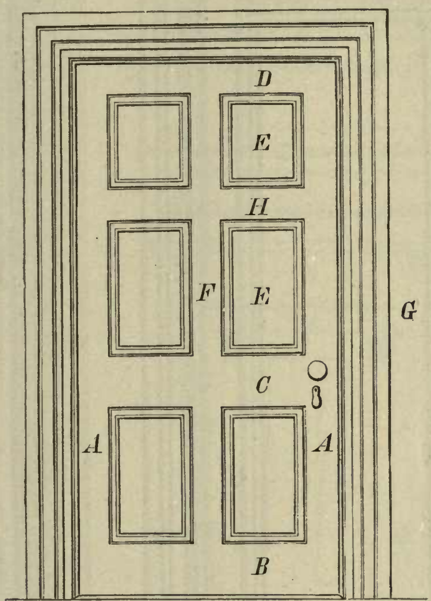


Fig. 1075.

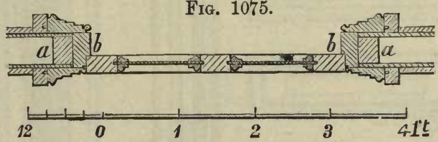


Fig. 1076.

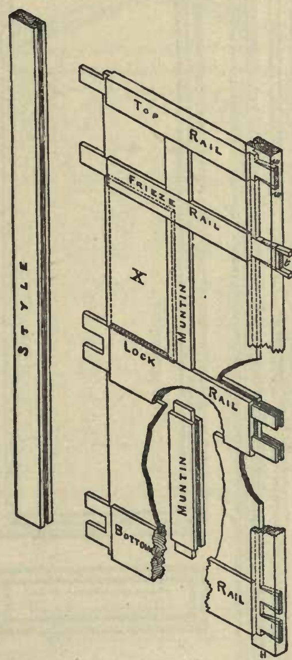


Fig. 1077.

When the width of the door exceeds five feet, it is generally made in two parts, each part being hung to its side of the frame, or one part hung to the other, so as to fold back like a shutter; or the parts may be made to slide back into pockets or grooves in the partition. The doors may be supported on wheels, and run on tracks at the floor-level; or the tracks may be above the doors, and the doors suspended; or they may be supported by levers, and be moved parallel without rollers.

Figs. 1078, 1079, and 1080 are the elevation, vertical and horizontal sections of a pair of sliding-doors. There are no knobs, but countersunk pulls to the

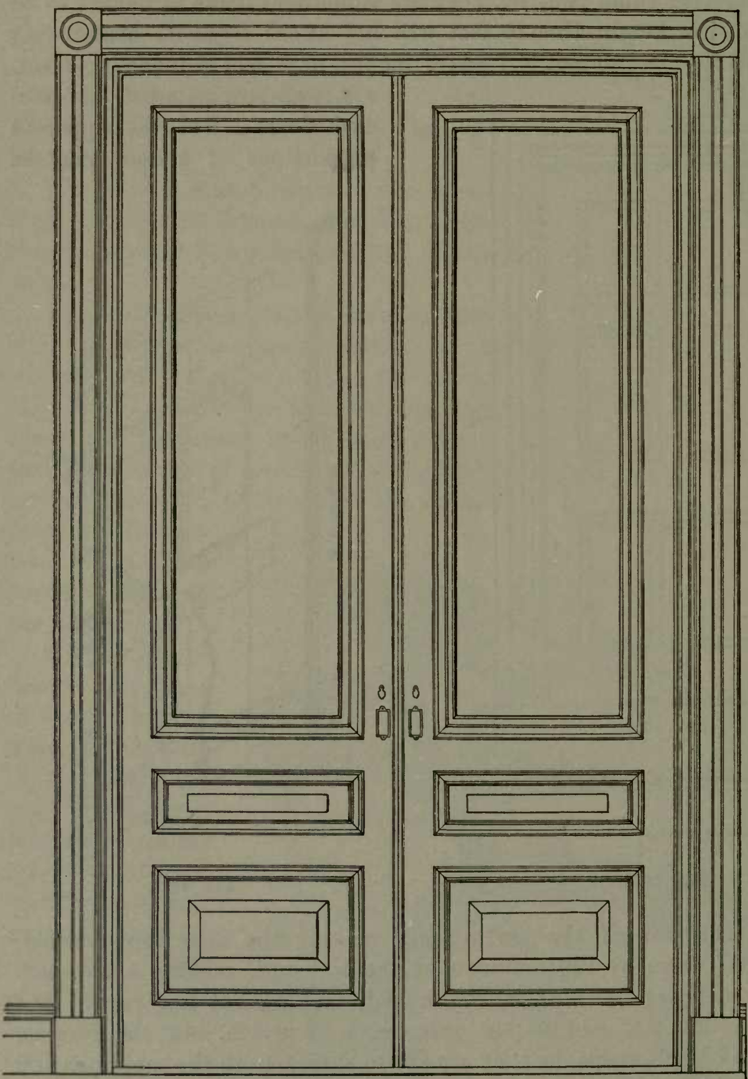


Fig. 1078.

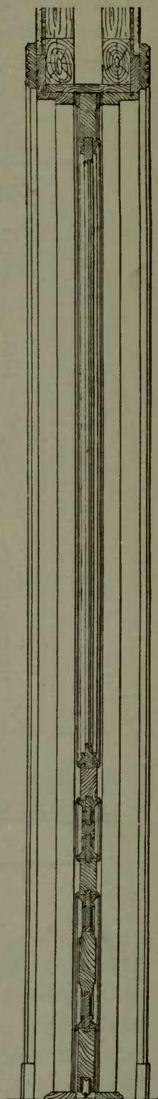


Fig. 1079.

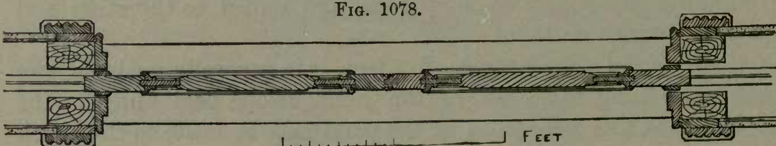


Fig. 1080.

doors, that they may be slid entirely within the pockets, with a special handle in the locks at the edges of the doors for withdrawing them.

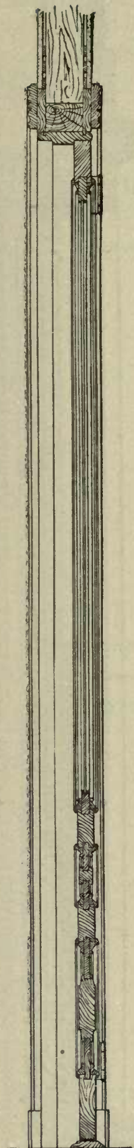


Fig. 1081.

Fig. 1082.

Figs. 1081 and 1082 are vertical and horizontal sections of the same doors hung on butts or hinges.

Figs. 1083 and 1084 are the elevation and horizontal section of an antæ-finished outside-door, with the *side-lights* C C, and a *top, fan,* or *transom light* B. The

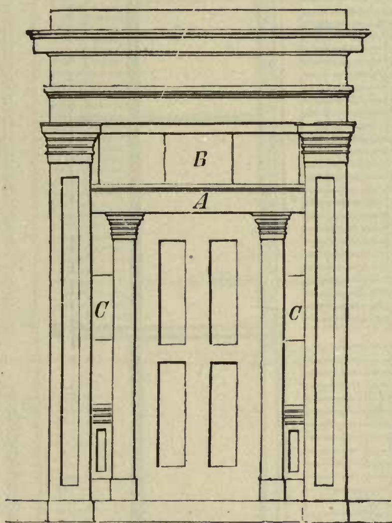


Fig. 1083.

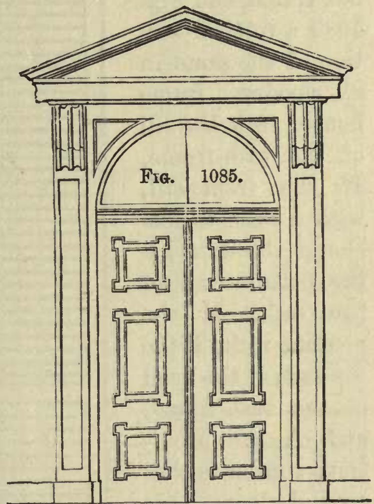


Fig. 1085.

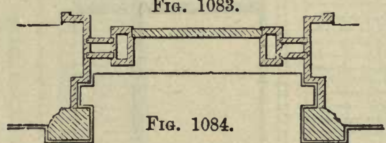


Fig. 1084.

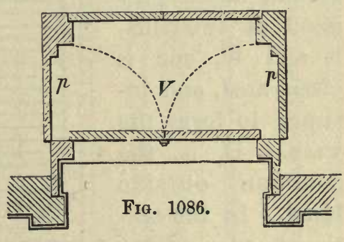


Fig. 1086.

bar A is called a transom, and this term is applied generally to horizontal bars extending across openings, or even across rooms.

Fig. 1085 is the elevation of an outside folding-door. The plan (Fig. 1086) shows a vestibule, V, and an interior door. The outer doors open, as shown by the arcs, and fold back into the pockets or recesses, *p p,* in the wall. This is a very common form of doors for first-class houses in this city. The fan-lights are made semicircular, and also the head of the upper panels of the door; these panels in the interior or vestibule door are of glass.

Windows are usually understood to be glazed apertures. The sashes may be stationary, but for most positions they are made to open either by sliding vertically, or laterally, or like doors. The first is the common form of window, and the sashes are generally balanced by weights; the second, except in a cheap form in mechanics' shops, are seldom used; the third, often used for access to balconies or between rooms, are called casements, or French windows.

Figs. 1087 and 1088 are the outside elevation and horizontal section of one half of a common box-frame, and Fig. 1089 a vertical section of the same in a wooden frame house. S is the sill of the sash-frame, W the frame-sill, with a wash to discharge the water, B the bottom rail of the sash, M the meeting rails, T the top rail, H the head of the sash-frame, and A the architrave similar to that around doors. Instead of two sills, S and W, one is often used, and inclined to form the wash. D is the common outside blind. In the sectional plan (Fig. 1094), C C′ are the *window-stiles*, F the *pulley-stile*, *w w* the sash-weights, *p* the parting strip, and D D double-fold shutters.

Figs. 1090 and 1091 are the interior elevation and vertical section of a box-frame window in a masonry wall;

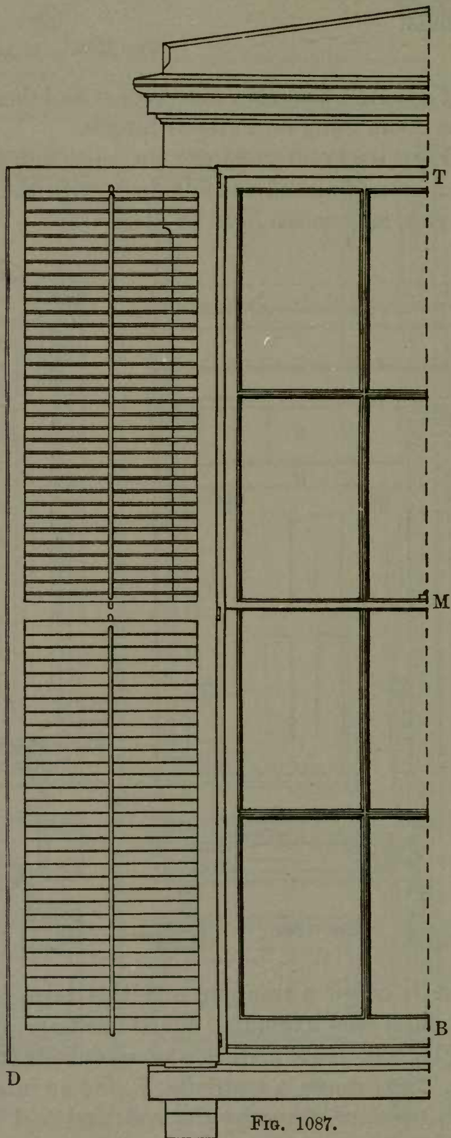


Fig. 1087.

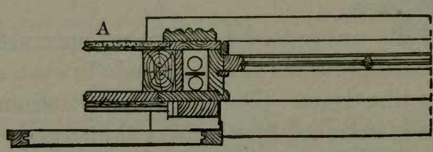


Fig. 1088.

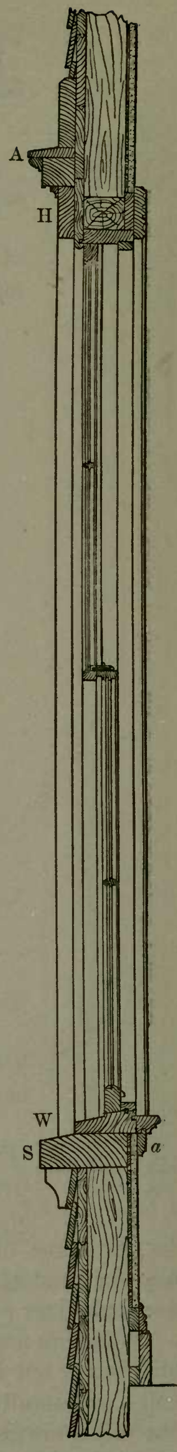


Fig. 1089.

Fig. 1092 is an exterior view of the same window, and Fig. 1093 a horizontal section.

Unless the windows begin from, or nearly from, the floor, the point *a* (Fig. 1089) may be fixed at a

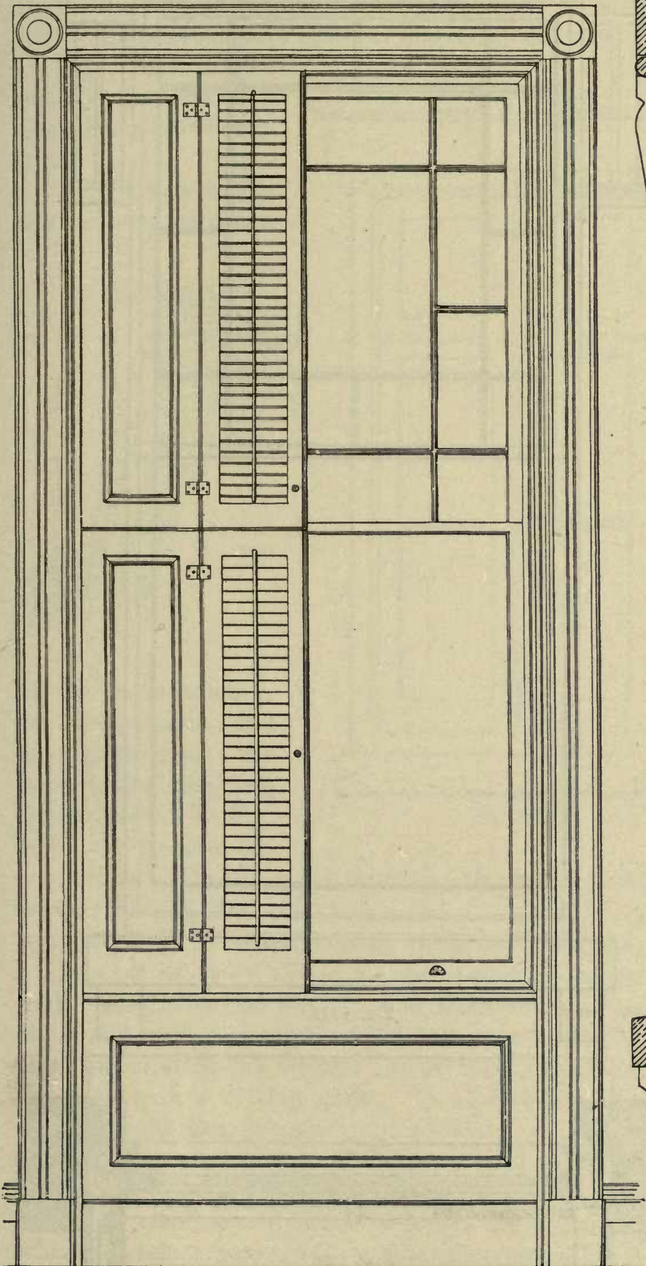


Fig. 1090.

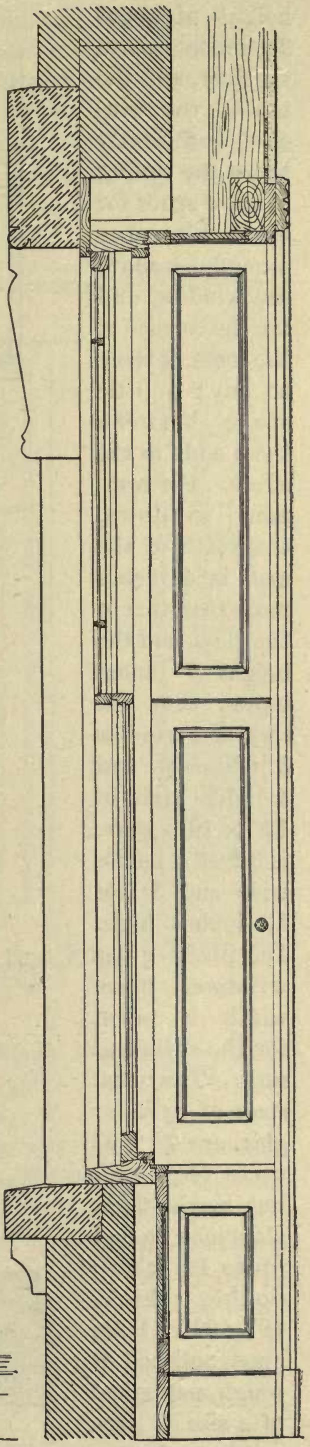


Fig. 1091.

height of about 30 inches above the floor, and the top of the window sufficiently below the ceiling to allow space for the architrave or other finish above the window, and for the cornice of the room, if there be any; a little space between these adds to the effect. For common windows, the width of the sash is 4 inches more than that of the glass, and the height 6 inches more; thus the sash of a window 3 lights wide and 4 lights high, of 12″ × 16″ glass, is 3 feet 4 inches wide and 5 feet 10 inches high. In plate-glass windows more width is taken for the stiles and rails. The usual sizes of cylinder glass are 7″ × 9″ up to 24″ × 36″, but single thick glass may be had up to 40″ × 60″; double thick, 48″ × 62″. Plate glass, polished or rough, may be had of a size as large as 14 × 8 feet.

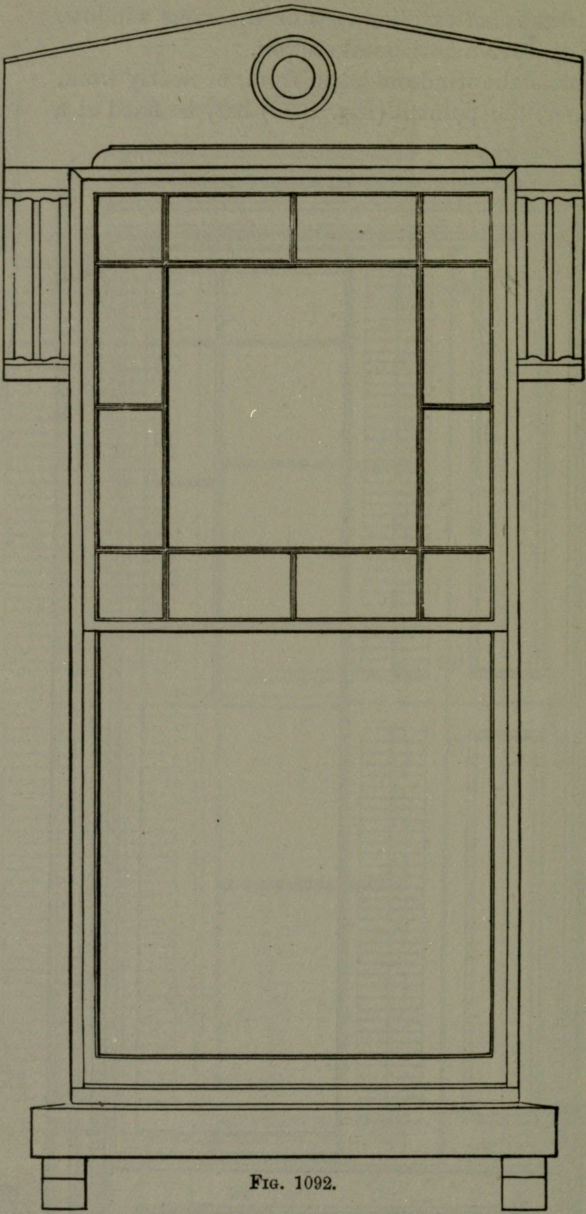


Fig. 1092.

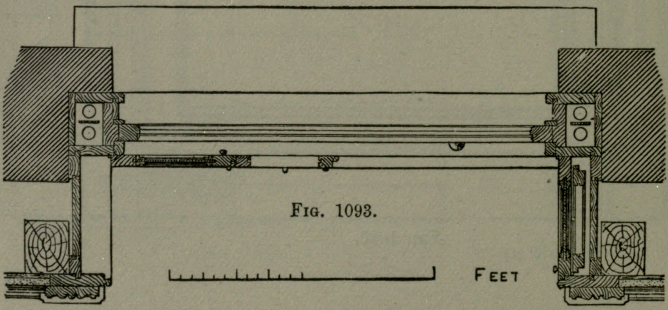


Fig. 1093.

In Fig. 1087 the blind D is hinged to the hanging stile, and folds within the opening in the masonry. The slats are movable on pin tenons, and those of each half, upper and lower, are connected by a central bar, so that they are moved together, and adjusted at any angle to the light. In Fig. 1093 the blinds are inside, 4-fold, and folding back into pockets. It is more usual to make the pockets for the blinds inclined to the window, as in Fig. 1094, giving to the interior more light, or ampler space for curtains.

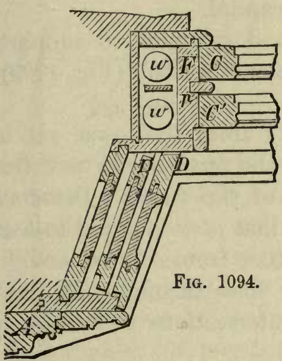


FIG. 1094.

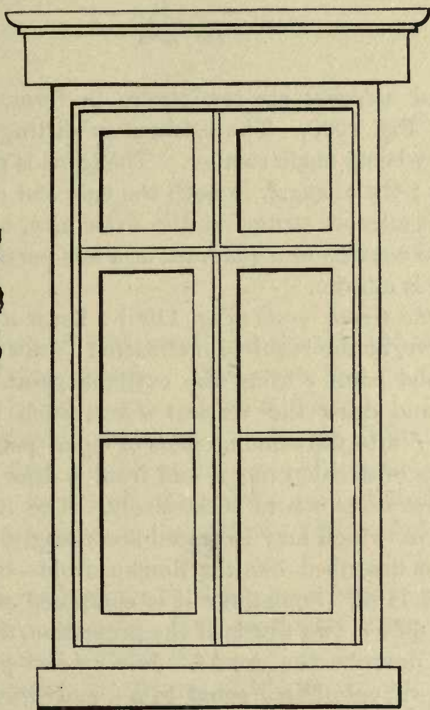


FIG. 1095.

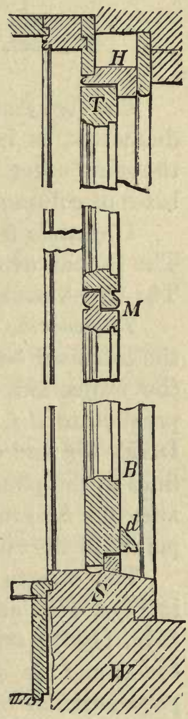


FIG. 1096.

Fig. 1095 is the outside elevation of a French window or casement.

Fig. 1096 represents the sectional elevation of the same window, in broken lines, and on a larger scale; the same letters designate similar parts as in Fig. 1089. A transom-bar is often framed between the meeting-rails, and in this case the upper sash may be movable; in Fig. 1096 it is fixed. An upright, called a *mullion,* is often introduced in the center, against which the sash shuts.

For use as doors, the lower sashes should not be less than 5 feet 6 inches high. It will be seen that in these forms of sash the rails and stiles are wide, and that for the same aperture the French window admits the least light. The chief objection to this window lies in the difficulty of keeping out the rain at the bottom in a driving storm. To obviate this, the small molding *d,* with a drip or undercut, is nailed to the bottom rail; but the more effectual means is the patent weather-strip, the same as used on outside doors.

Dormer or attic windows are framed and set as in an upright stud-partition.

In all architectural finish *moldings* are a necessity, the simpler forms of which are taken from Greek or Roman examples.

Greek and Roman Moldings.—The regular Greek moldings are eight in number: the Fillet or Band, Torus, Astragal or Bead, Ovolo, Cavetto, Cyma Recta or Ogee, Cyma Reversa or Talon, and Scotia.

The *fillet* (*a*, Fig. 1097) is a small rectangular member, on a flat surface, whose projection is usually made equal to its height.

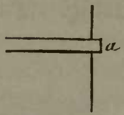


Fig. 1097.

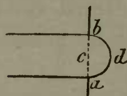


Fig. 1098.

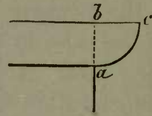


Fig. 1099.

The *torus* and *astragal* are semicircles in form, projecting from vertical diameters, as in Fig. 1098. The astragal is distinguished from the torus in the same order by being made smaller. The torus is generally employed in the bases of columns; the astragal, in both the base and capital.

The *ovolo* is a member strong at the extremity, and intended to support. The Roman ovolo consists of a quadrant or a less portion of a circle (Fig. 1099). The Greek ovolo is elliptic.

To describe the Greek ovolo (Fig. 1100): Draw *d f* from the lower end of the proposed curve, at the required inclination; draw the vertical *g e f* to define the projection, the point *e* being the extreme point of the curve. Draw *e h* parallel to *d f*, and draw the vertical *d h k*, such that *d h* is equal to *h k*. Divide *e h* and *e f* into the same number of equal parts; from *d* draw straight lines to the points of division in *e f*, and from *k* draw lines through the divisions in *e h* to meet those others successively. The intersections so found are points in the curve, which may be traced accordingly.

The *cavetto* is described like the Roman ovolo—by circular arcs, as shown in Figs. 1101 and 1102. Sometimes it is composed of two circular arcs united (Fig. 1103); set off *b e*, two thirds of the projection, draw the vertical *b d* equal to *b e*, and on *d* describe the arc *b i*. Join *e d* and produce it to *p*; draw *i n* perpendicular to *e d*, set off *n o* equal to *n i*, and draw the horizontal line *o p* meeting *e p*; on *p* describe the arc *i o* to complete the curve.

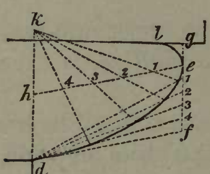


Fig. 1100.

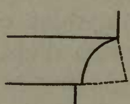


Fig. 1101.

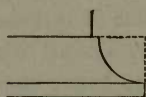


Fig. 1102.

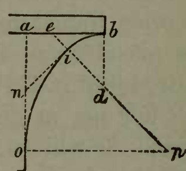


Fig. 1103.

The *ogee*, or cyma recta (Fig. 1104), is compounded of a concave and a convex surface. Join *a* and *b*, the extremities of the curve, and bisect *a b* at *c*; on *a*, *c*, as centers, with the radius *a c*, describe arcs cutting at *d*; and on *b*, *c*, describe arcs cutting at *e*. On *d* and *e*, as centers, describe the arcs *a c*, *c b*, composing the molding.

The cyma reversa, or talon (Fig. 1105), is a compound curve, distinguished from the ogee by having the convex part uppermost.

If the curve be required to be made quicker, a shorter radius than *a c* must be employed. The projection of the molding *n b* (Fig. 1104) is usually equal to the height *a n*.

To describe the Greek talon: Join the extreme points *a, b* (Fig. 1106); bisect *a b* at *c*, and on *a c, c b,* describe the semicircles *b d c* and *c a*. Draw perpendiculars *d o*, etc., from a number of points in *a c, c b,* meeting the circumferences;

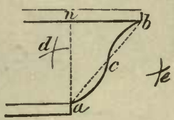


Fig. 1104.

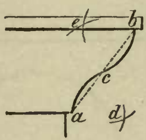


Fig. 1105.

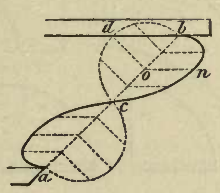


Fig. 1106.

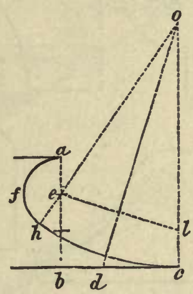


Fig. 1107.

and from the same points set off horizontal lines equal to the respective perpendiculars: *o n* equal to *o d*, for example. The curve line *b n a*, traced through the ends of the lines, will be the contour of the molding.

To describe a scotia: Divide the perpendicular *a b* (Fig. 1107) into three equal parts, and with the first, *a e*, for radius, on *e* as a center, describe the arc *a f h*, in the perpendicular *c o* set off *c l* equal *a e*, join *e l*, and bisect it by the perpendicular *o d*, meeting *c o* at *o*, on the center *o*, with *o c* for radius, complete the figure by the arc *c h*.

These moldings, and combinations of them, are *stuck* in wood, and are to be purchased in every variety. Fig. 1108 represents some of the common forms always to be had, and of suitable sizes.

Stairs consist of the *tread* or *step* on which we set our feet, and *risers*, upright pieces supporting the treads—each tread and riser forms a stair. If the treads are parallel they are called *fliers;* if less at one end than the other, they are called *winders*, *f* and *w* (Fig. 1115). The top step, or any intermediate wide step, for the purpose of resting, is called a *landing*. The height

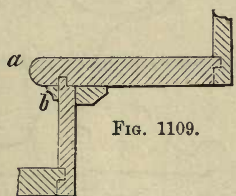


Fig. 1109.

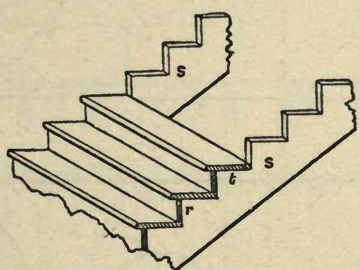


Fig. 1110.

from the top of the nearest step to the ceiling above is called the *headway*. The rounded edge of the step is called a *nosing* (*a*, Fig. 1109); if a small hollow (*b*) be glued in the angle of the nosing and riser, it is called a *molded*

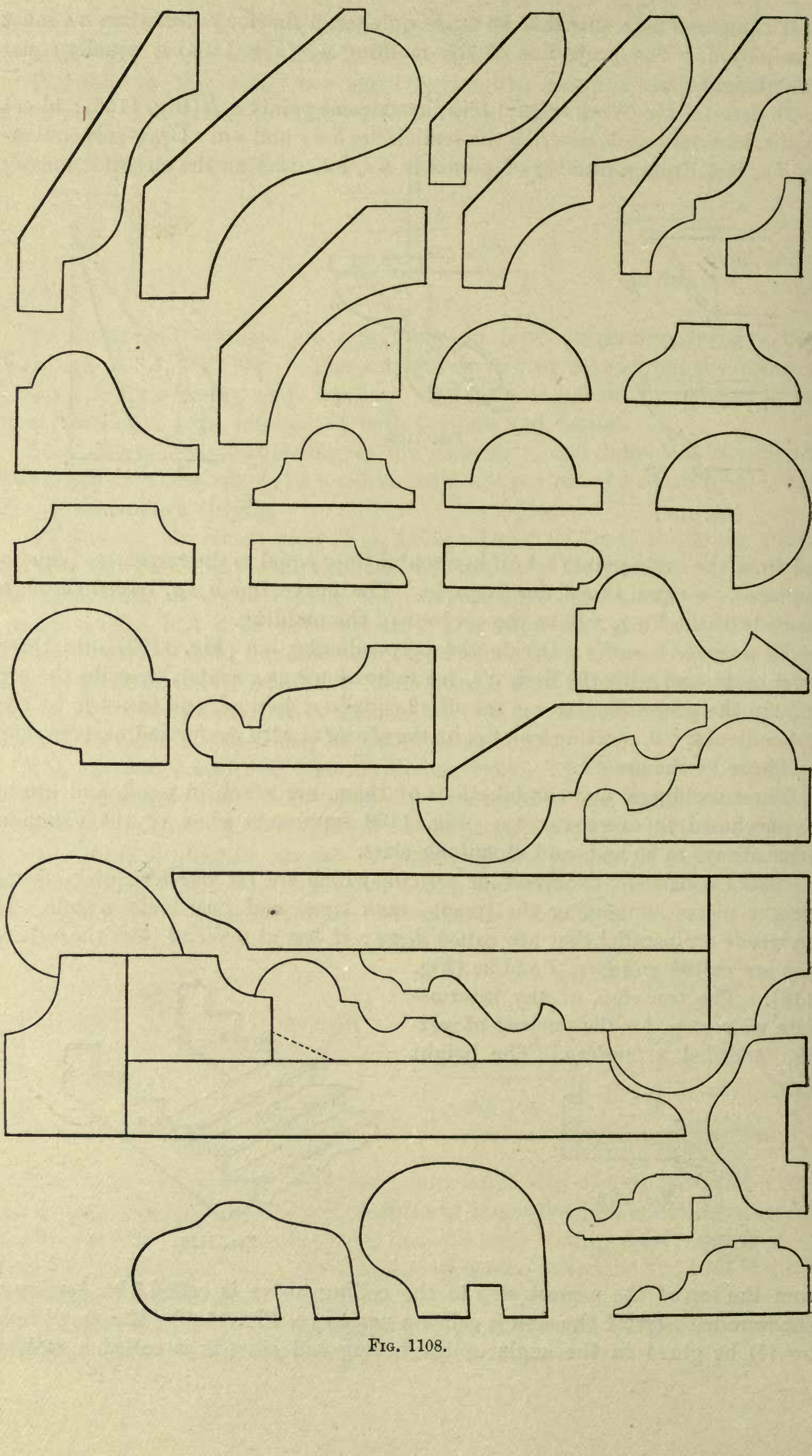


FIG. 1108.

nosing. The pieces which support the ends of the stairs are called strings (Fig. 1110); that against the wall the *wall-string,* the other the *outer string.* Besides these strings, pieces of timber are framed and placed beneath

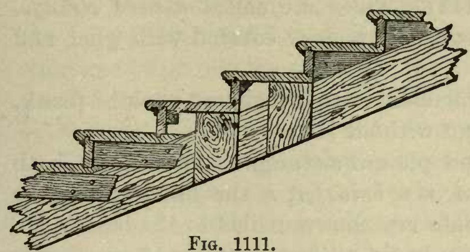


Fig. 1111.

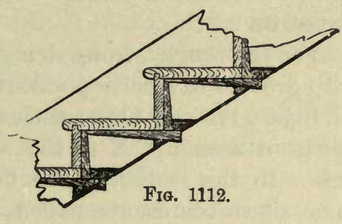


Fig. 1112.

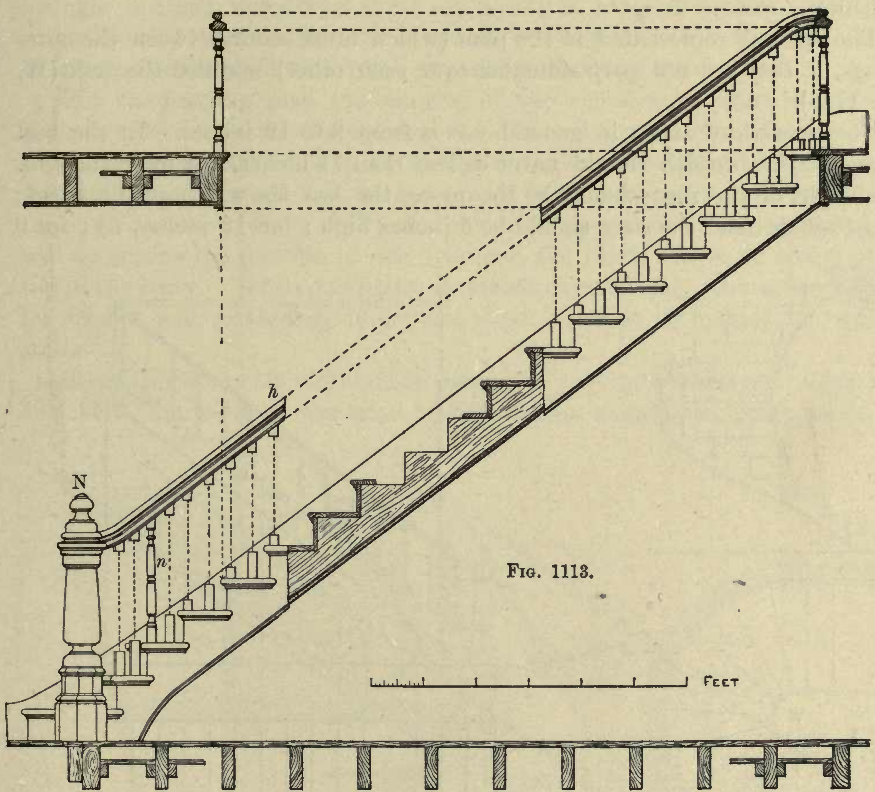


Fig. 1113.

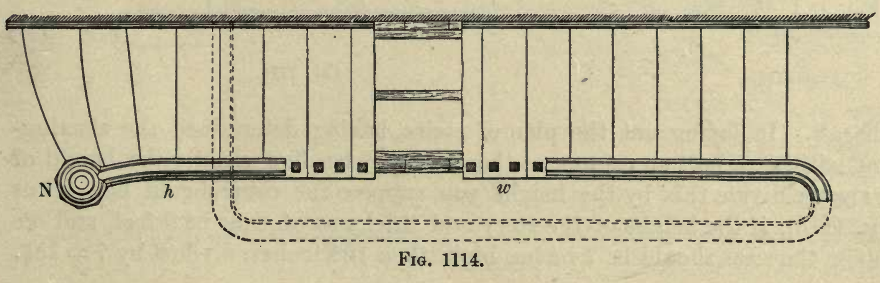


Fig. 1114.

the fliers, when the stairs are wide (Fig. 1111), called *carriages*. Sometimes the strings, instead of being notched out to receive the steps, have the upper and lower edges parallel, with grooves cut in their inner faces to receive the ends of the steps and risers (Fig. 1112). These are called *housed strings*. Steps and risers are secured in the grooves by wedges covered with glue, and driven in.

For the rough, strong strings of warehouses the carriages are made of plank, with grooves to receive plank-treads, and without risers.

Figs. 1113 and 1114 are elevation and plan of a straight run of stairs, both partly in section. N is the *newel-post*, *n* a *baluster*, *h* the hand-rail, *w* the well. In the section of the floors, cleats are shown nailed to the beams; on these short boards are nailed to form a box for the reception of mortar for deafening.

The opening represented in the plan (which must occur between the outer strings, if they are not perpendicular over each other) is called the *well* (W, Fig. 1115).

The breadth of stairs in general use is from 9 to 12 inches. In the best staircases, the breadth should never be less than 11 inches, nor more than 15. The height of the riser should be the more, the less the width of the tread; for a 15-inch tread the riser should be 5 inches high; for 12 inches, $6\frac{1}{2}$; for 9

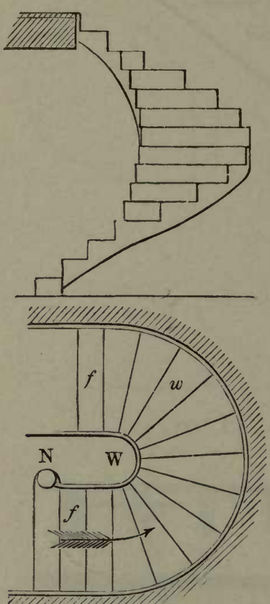


Fig. 1115.

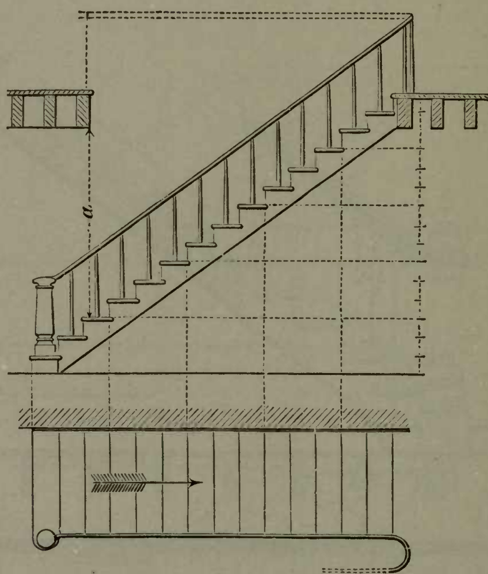


Fig. 1116.

inches, 8. In laying out the plan of stairs, having determined the starting-point, either at bottom or top, as the case may be, find exactly the height of the story; divide this by the height you suppose the riser should be. Thus (Fig. 1116), if the height of the story and thickness of floor be 9 feet, and we suppose the riser should be 7 inches high, then 108 inches, divided by $7 = 15\frac{3}{7}$.

It is clear that there must be an even number of steps, either 16 or 15 ; to be near the supposed height of the riser, adopt 15, then—

$$\frac{108}{15} = 7\tfrac{3}{15} \text{ inches, height of riser.}$$

For this particular case, assume the breadth of the step as 10 inches, and the length at 3 feet, a very usual length, seldom exceeding 4 feet in the best staircases of private houses. For the plan—lay off the outside of the stairs, two parallel lines 3 feet apart, and space off from the point of beginning 14 treads of 10 inches each, and draw the cross-parallel lines.

To construct the elevation, project the lines of the steps in plan, and divide the height, either on a perpendicular or by an inclined line, into the number of risers (15), and draw cross-parallels through these points ; or the same points may be determined by intersection of the projections of the plan with a single inclined line drawn along the nosing of top and bottom steps. It is to be observed that the number of treads is always one less than the number of risers, the reason of which will appear by observing the elevation.

For the framing plan the drawing of the elevation of stairs is in general necessary, to determine the opening to be framed in the upper floor, to secure proper headway. Thus (Fig. 1116), the distance between the nearest stair and the ceiling at *a* should not be less than 6 feet 6 inches ; a more ample space improves the look of the stairway ; but if we are confined in our limits, this will determine the position of one trimmer, the other will be of course at the top of the stairs. When one flight is placed over another, the space required for timber and plastering, under the steps, is about 6 inches for ordinary stairs.

When the stairs are circular, or consist in part of winders and fliers, as in Fig. 1115, the width of the tread of the winders should be measured on the

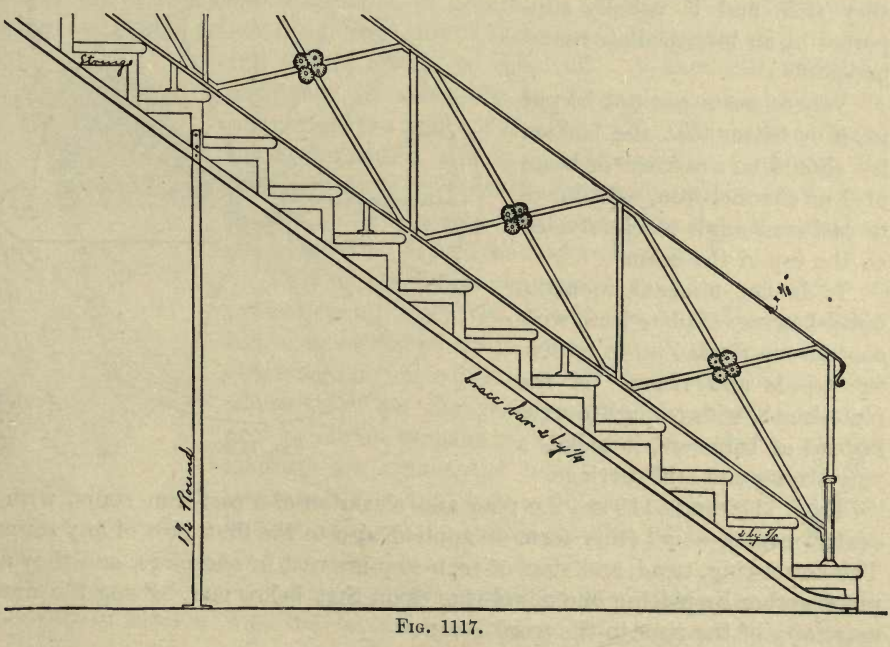


Fig. 1117.

central line. The construction of the elevation is similar to that of the straight run (Fig. 1116), by dividing the space between the stories by a number of parallel lines equal to the number of risers, and intersecting the parallels by projections from the plan.

The objection to all circular stairs of this form, or with a small well-hole or central shaft, is that there is too much difference between the width of the tread, but a small portion being of a suitable size. The handsomest and easiest stairs are straight *runs*, divided into landings, intermediate of the stories, and either continuing then in the same line, or turning at right angles, or making a full return.

Fig. 1117 is the side elevation of a stairs with wrought-iron string and rail. The string is made of wrought-iron knees, welded together continuously, with a flat bottom-bar riveted across the lower angle of the knees. The construction is not very stiff, and is usually supported by an intermediate round bar-post.

Where posts can not be put in, it is better that the bottom bar should be a carriage or beam of I or channel-iron, with knees or cast-iron angle-blocks riveted on the top of the beam.

It is not unusual to make housed strings of plate-iron, with angle-irons riveted on to receive the treads and risers. If the plate-iron be wide enough to serve instead of balusters, it makes a very strong and stiff carriage.

ELEVATION.

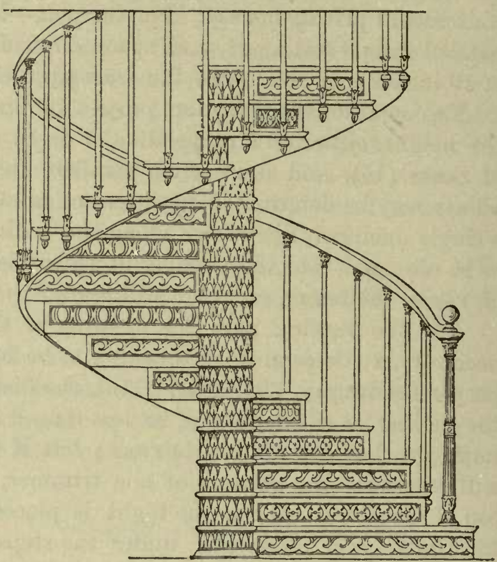


Fig. 1118.

PLAN.

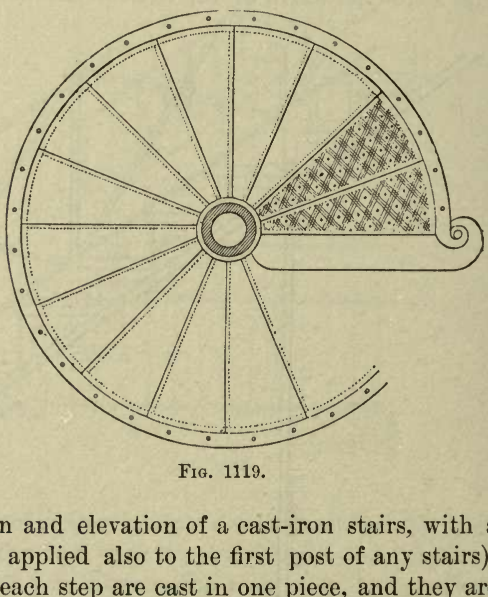


Fig. 1119.

Figs. 1118 and 1119 are the plan and elevation of a cast-iron stairs, with a central post or *newel* (this term is applied also to the first post of any stairs). The newel-ring, tread, and riser of each step are cast in one piece, and they are put together by placing one newel-ring upon that below and bolting the outer extremity of the riser to the tread below.

Fig. 1120 is a form of cast-iron stairs with a well instead of a newel; the step and riser are bolted together by the flanges. It will be seen that one tread is wider than the others; this is a landing.

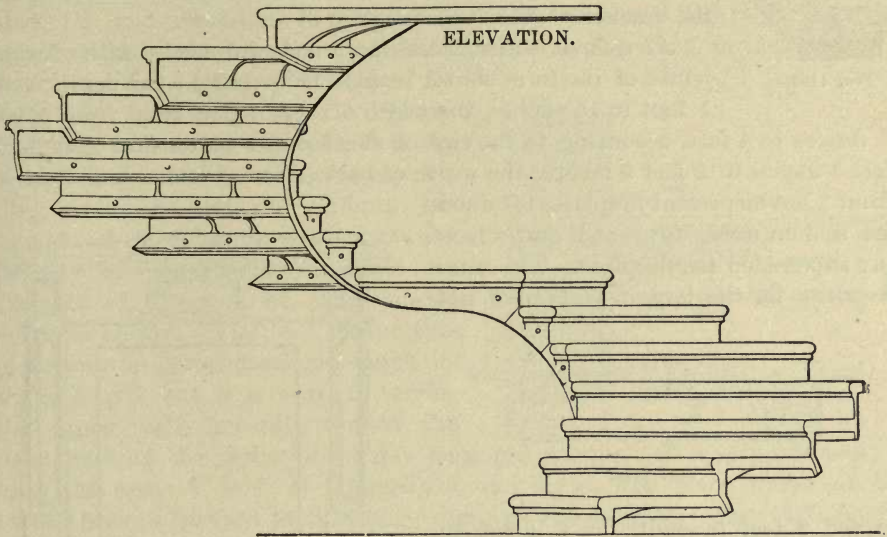


Fig. 1120.

It is at times fashionable to make the *newel* a prominent feature in the hall, often occupying valuable space. It is sufficient that it be large and stiff enough for a support to the hand-rail.

The top of the hand-rail should, in general, be about 2′ 8″ to 3′ above the nosing, and should follow the general line of the steps. The angles of the hand-rail should always be eased off. A hand-rail, affording assistance in ascending or descending, should not be wider than the grasp of the hand (Fig. 1121); but where, for architectural effect, a more massive form may be necessary, it is very convenient, and may be very ornamental, to have a sort of double form, that is, a smaller one planted on top of the larger (Fig. 1122).

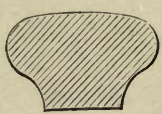


Fig. 1121.

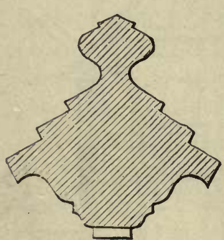


Fig. 1122.

To a draughtsman conversant with the principles of projection already given, it will not be difficult to draw in the hand-rail of stairs, or to lay off the mold for its construction. It will follow the line of stair-nosing, and where there are changes of pitch they are made to connect by curves tangent to these pitches, except where the landings are square, and newels set at the head of the landings, the rail is made to bolt into the newel. At the bottom the rail is curved to the horizontal, when it comes into or upon top of the newel.

Balusters are of great variety—usually turned forms—attached to the treads by dovetails, covered with the returned nosing, or with pin-ends and holes in

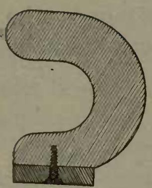


Fig. 1123.

treads and under side of caps. Sometimes (especially in iron-work) the baluster is set in a bracket from the face of the string, as in Fig. 1123. These brackets are often very ornamental, and the balusters may be cast on the same piece with the bracket.

Fireplaces.—Fireplaces for wood are made with flaring jambs of the form shown in plan (Fig. 1124); the depth from 1 foot to 15 inches, the width of opening in front from 2 feet 6 inches to 4 feet, according to the size of the room to be warmed; height 2 feet 3 inches to 2 feet 9 inches, the width of back about 8 inches less than in front; but at present fireplaces for wood are seldom used, stoves and grates having superseded the fireplace. The space requisite for the largest grate need not exceed 2 feet in width by 8 inches in depth. The requisite depth is given by the projection of the grate, and the mantel-piece. Ranges require from 4 feet 4 inches to 6 feet 4 inches wide × 12 inches to 20 inches deep; jambs 8 inches to 12 inches.

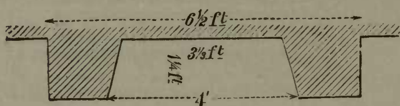


Fig. 1124.

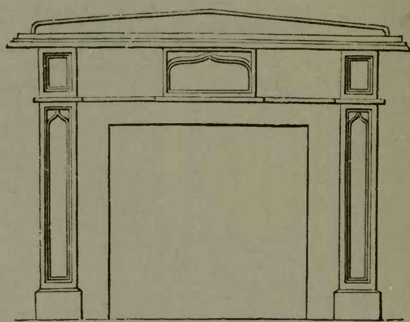


Fig. 1125.

Fig. 1125 represents the elevation of a mantel-piece of very usual proportions. The length of the mantel is 5 feet 5 inches, the width at base 4 feet 6 inches, the height of opening 2 feet 7 inches, and width 2 feet 9 inches. A portion of this opening is covered by the iron sides or architrave of the grate, and the actual open space would not probably exceed 18 inches in width by 2 feet in height. In brick or stone houses the flues are formed in the thickness of the wall, but when distinct they have an outside shell of a half-brick or 4 inches, and sometimes 8″ (Fig. 1126); the *withs* or division-walls always 4″.

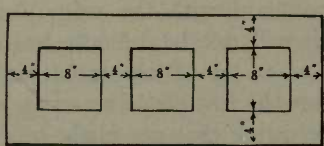


Fig. 1126.

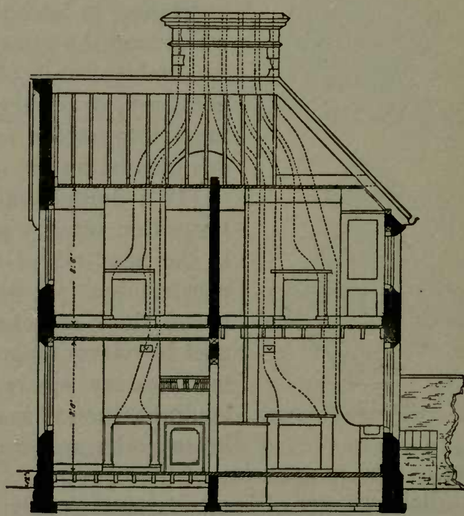


Fig. 1127.

The size of house flues is usually 8″ × 8″, but some are 4″ × 8″, 4″ × 12″, and 8″ × 12″. The flues of different fireplaces should be distinct. Those from the lower stories pass up through the jambs of the upper fireplaces, and, keeping side by side with but 4-inch brick-work between them, are topped out above the roof, sometimes in a double and often in a single line 16 inches wide by a breadth required by the number of flues, as in Fig. 1126, or in Fig. 1127. The latter is an illustration of how far flues may be diverted from a vertical line, but it is to be observed that the construction must be stable, as any settling or cracks not only injures the draught of the chimney, but impairs the security of the building against fire. Changes of direction of flues should never be abrupt. The back of the fireplace may be perpendicular through its whole height, but it is usual to incline the upper half inwardly toward the room, making the throat to the flue long and narrow. It is very common to form the upper 3″ to 4″ of the inclined back by an iron plate, which can be turned back or forward to increase or diminish the draught. Fig. 1128 represents the arrangement of frame and brick arch for the support of the hearth. The chimney is generally capped with stone, sometimes with tile or cement pots. As an architectural feature, the chimney is often made to add considerably to the effect of a design.

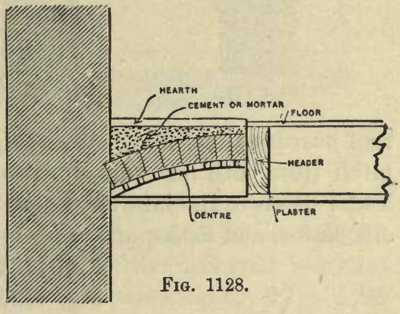


FIG. 1128.

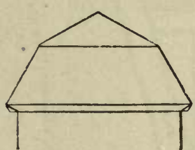


FIG. 1129.

Roofs.—Framed roofs have been illustrated (page 410). City roofs are usually flat, and timbered similarly to floors, but not so strongly, with a slight pitch to discharge rainfall. Roofs of country dwellings are usually framed like stud-partitions, with inclined studs somewhat deeper than if they were vertical, depending on the inclination from the vertical; if flat, depth like that of a floor. The theory of the construction of the gambrel or Mansard roof (Fig. 1129) is a roof with two kinds of pitch; it is that of the polygon of rods, and self-supporting; but, in general, they have central support from partitions, and their outlines are much varied by curves in the lower rafters cut from plank.

Fig. 1130 is the plan of a roof as usually drawn, shaded strongly at the ridges. The transept roof is *hipped* at A and B.

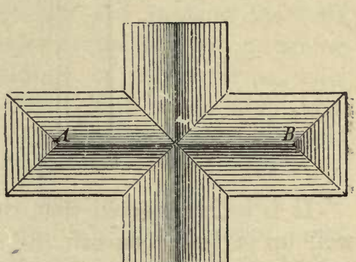


FIG. 1130.

Gutters are generally formed in the cornice (Fig. 1131); sometimes on the roof (Fig. 1132), and sometimes by raising a parapet (Fig. 1133) and forming a *valley*. The intersection of two roofs forms a valley.

Fig. 1131 represents a form of gutter very common to city buildings, the

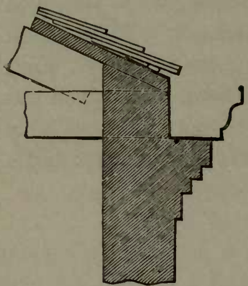


Fig. 1131.

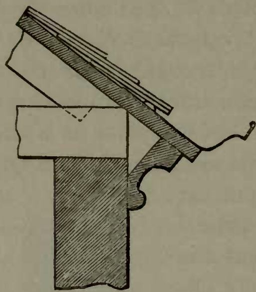


Fig. 1132.

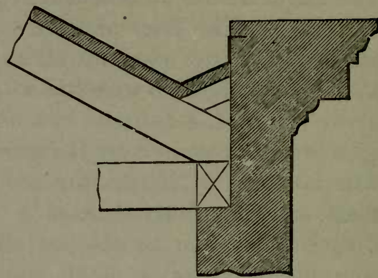


Fig. 1133.

roof boarding extending over the gutter ; but it is preferable to make the roof pitch from both rear and front to the center of the building, and to carry the leader down in the interior, where it may serve as a soil-pipe for the water-closets, basins, and baths, affording ventilation in fair weather and a scour in rains.

Fig. 1134 is a gutter of a cottage roof.

Fig. 1135 is the section of a Mansard roof, so called, showing the side elevation of a dormer-window, with the gutter below its sill.

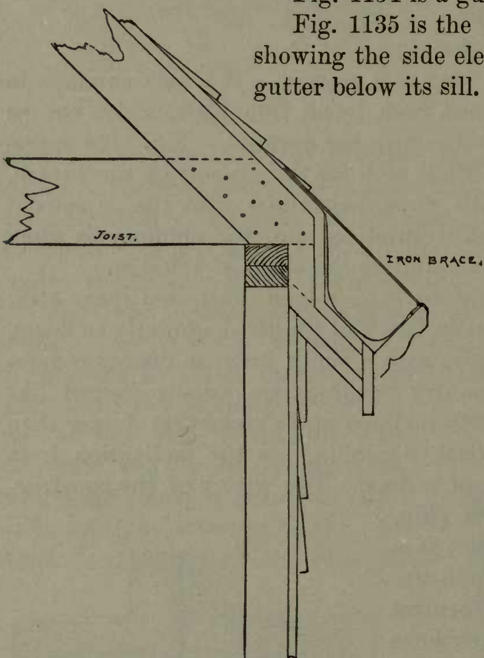


Fig. 1134.

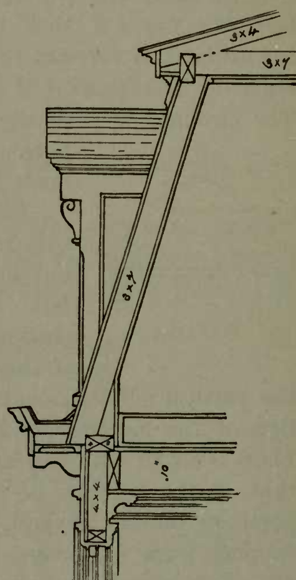


Fig. 1135.

It is to be observed that the sheet-metal forming the gutter must extend well up or back beneath the shingles or felt, or be soldered to the tin of the roof, to prevent water finding its way into the interior ; and at the sides *flashings* of tin must extend on the walls above the roof and into the joints of the brick.

Plastering.—To prevent damp striking through the plastering of outer walls, and cracks in ceilings, it is usual to *fur* walls and beams ; that is, to nail vertical strips of wood to the walls, and across from beam to beam. Furring-strips are from $1\frac{1}{2}''$ to $2''$ wide, and about $\frac{7}{8}''$ thick, nailed at distances of $12''$ or $16''$ centers (usually the former), adapted to the length of the laths, which are 4 feet long, and about $1\frac{1}{4}'' \times \frac{1}{4}''$ = spaces between laths $\frac{1}{4}''$ to $\frac{3}{8}''$. The first coat of mortar is the scratch-coat, which is forced through the interstices between

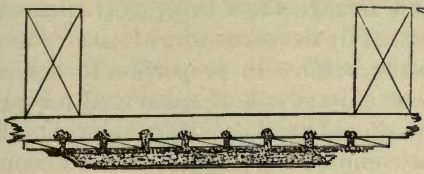


FIG. 1136.

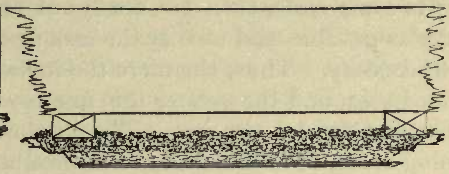


FIG. 1137.

the laths, to make a lock to retain it. This coat is about $\frac{1}{4}''$ thick. The next or brown coat is about $\frac{1}{2}''$ thick, and if the last coat is a sand-finish, it will be less than $\frac{1}{8}''$ thick; while, if the last coat is a hard finish, its thickness will be almost imperceptible. Figs. 1136 and 1137 are sections of furring and plastering.

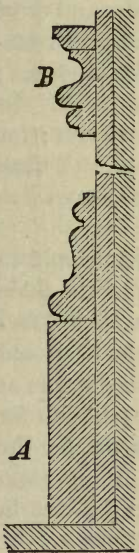


FIG. 1138.

The brown coat is usually carried down to the floor. Over this is nailed the base-board, A (Fig. 1138), for the finish around the bottom of the walls of the room. Above the base is a molding forming a part of the base; above this, there may be a molded

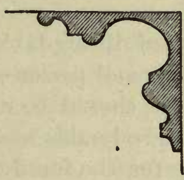


FIG. 1139.

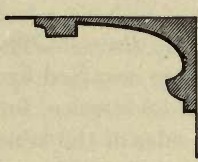


FIG. 1140.

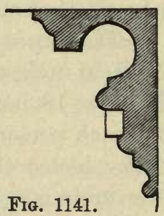


FIG. 1141.

rail, B, called the chair-rail, or surbase, and between a panel, termed a *dado*. The walls of stores are generally ceiled up as high as the surbase. For the finish of the angle of the wall and ceiling, it is usual in the better rooms to form a cornice in plaster. The cornices are moldings of varied forms, with or without *enrichments*—that is, plaster ornaments. Figs. 1139, 1140, and 1141 are sections of cornices. If the rooms are low, the cornice should extend but little on the wall, but well out on the ceiling.

Proportions and Distribution of Rooms and Passages.—Rooms of dwelling-houses are to be proportioned and arranged according to the necessities of position and use, the space that can be occupied, the financial means available, and often to suit the peculiar wishes of owners or occupants. In cities, the limits of the lot restrict the arrangements to a small ground-space, and require an increase in the number of stories. Use has established certain forms often peculiar to different cities, beyond which there is little change; but in the country, where there is plenty of ground-space, and where many stories are usually injurious to the æsthetic effect, and where there are few canons in architecture to be observed, there is little limit to the variety of forms and arrangements of country-houses.

In designing a country-house, where one is not restricted to room, it is often convenient to mark out the rooms of the desired size on slips of paper, according to some scale, then cut them out and arrange them in as convenient an order as possible, and modify the arrangement by the necessities of construction and economy. Thus, the more the inclosing surface in proportion to the included area, and the greater the number of chimneys and space used for passages, the greater the cost. The kitchen should be of convenient access to the dining-room, both should have large and commodious pantries, and all rooms should have an access from an entry, without being compelled to pass through other rooms ; this is particularly applicable to the communication of the kitchen with the front door. Outside doors for common and indiscriminate access should not open into important rooms.

As to the size of the different rooms, they must of course depend on the purposes to which they are to be applied, the class of house, and the number of occupants. The *kitchen* for the poorer class of houses is also used as an eating-room, and should therefore be of considerable size to answer both purposes ; for the richer houses, size is necessary for the convenience of the work. In New York city houses the average will be found to be about 16 × 20 feet ; for medium houses in the country they are in general less, say 12 × 16. A back kitchen, scullery, or laundry, should be attached to the kitchen, and may serve as a passage-way out.

The Dining or Eating Rooms.—The width of dining-tables varies from 3 to 5 feet 6 inches ; the space occupied by the chair and person sitting at the table is about 18 inches ; the table-space, for comfort, should be not less than 2 feet for each person at the sides of the table, and considerable more at the head and foot ; hence the space that will be necessary for the family and its visitors at the table may be calculated. Allow a further space of 2 feet at each side for passages, and some 3 to 5 at the head for the extra tables or chairs, for the minimum of space required ; but, if possible, do not confine the dining-room to meager limits, unless for very small families ; let not the parties be lost in the extent of space, nor let them appear crowded.

The show-room parlors, if there are any intended for such in the house, should be made according to the rules given below, not square, but the length about once and a half the width ; if much longer than this, break up the walls by transoms or projections. As to the particular dimensions, no rules can be given ; they must depend on every person's taste and means ; 20 × 16 may be considered a fair medium size for a regular living-room parlor, not a drawing-room. The same size will answer very well for a *sleeping-room.* The usual width of single beds is 2 feet 8 inches ; of three-quarter, 3 feet 6 inches ; of whole, 4 feet 6 inches ; the length, 6 feet 6 inches ; and as the other furniture may be made to consist of but very few pieces, if adequate means of ventilation are provided, it is easy to see into how small quarters persons may be thrust. The bed should not stand too near the fire, nor between two windows ; its most convenient position is head against an interior wall, with a space on each side of at least 2 feet. To the important bedrooms of first-class houses, dressing-rooms should be attached, and, if there is water and sewer service, fitted with set bowls and baths and water-closets. If possible, there should be windows opening to the outer air, but always with flue-ventilation.

Pantries.—Closets for crockery should not be less than 14 inches in depth in the clear; for the hanging up of clothes, not less than 18 inches, and should be attached to every bedroom. For medium houses, the closets of large sleeping-rooms should be at least 3 feet wide, with hanging-room, and drawers and shelves. There should also be blanket-closets, for the storing of blankets and linen; these should be accessible from the entries, and may be in the attic. Store-closets should also be arranged for groceries and sweetmeats.

Passages.—Front entries are usually 6 feet wide in the clear; common passage-ways, 3 feet; these are what are required, but ample passages give an important effect to the appearance of the house. The width of principal stairs should be not less than 3 feet, and all first-class houses, especially those not provided with water-closets and slop-sinks on the chamber-floor, should have two pairs of stairs, a front and a back pair; the back stairs need not necessarily be over 2 feet 6 inches in width.

The Height of Stories.—It is usual to make the height of all the rooms on each floor equal; it can be avoided by furring down, or by the breaking up of the stories, by the introduction of a mezzonine or intermediate story over the smaller rooms. Both remedies are objectionable.

The average height of the stories for common city dwellings is: Cellar, 6 feet 6 inches; common basement, 8 to 9 feet; English basement, 9 to 10 feet; principal story, 12 to 15 feet; first chamber floor, 10 to 12 feet; other chamber-floors, 8 to 10 feet—all in the clear. For country-houses, the smaller of the dimensions are more commonly used. Attic stories are sometimes but a trifle over 6 feet in height, but are, of course, objectionable.

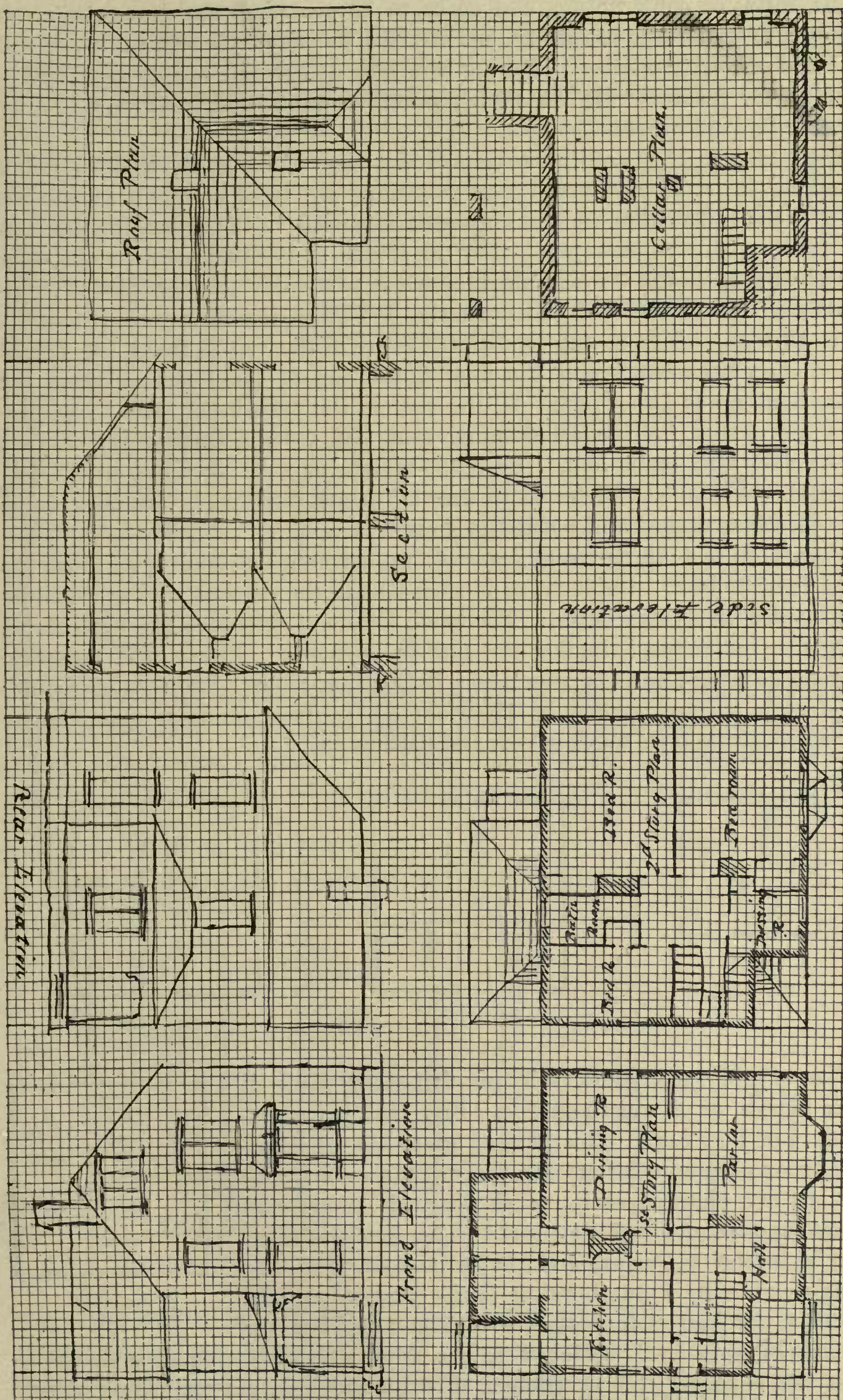
Privies, Water-Closets, and Out-Houses.—The size of privies must depend greatly on the uses of the building to which they are to be attached, its position, and the character of its occupants. Allowing nothing for evaporation and absorption, the entire space necessary for the excrementitious deposits of each individual, on an average, will be about seven cubic feet for six months, of which three quarters is fluid. In the country, vaults are usually constructed of dry rubble-stone, and the fluid matters are expected to be filtered through the earth, the same as in cesspool-waste; but great care must be taken that they neither vitiate the water-supply nor the air of the house. A brick and cement vault, air and water tight, with a ventilating-pipe into a hot chimney-flue, is the best preventive, and may even be built within the house. In all other cases there should be free air-space between the house and privy. In the city, where there is adequate water-supply and sewerage, the water-closet should be adopted, except in houses occupied by many ignorant and irresponsible tenants, who throw extraneous matters into the hoppers, and obstruct the sewer-pipes. In these, tight privy-vaults, with trapped sewer connections, and with all the house-waste and roof-water discharging into them, are the easiest kept in order. The water-closet, or privy, with a single seat, should occupy a space not less than 4′ × 2′ 6″. The rise of seat should be about 17″ high; and the hole egg-shaped, 11″ × 8″. The earth-closet, when properly taken care of, is an extremely useful appendage to a country-house, and the space requisite for it is the same as that of a water-closet. It is the most common practice to place the water-closet in the bath-room. A common bath-tub will occupy a floor-space of 6′ × 2′, and 18″ deep; the French tub, so called, is much shorter, often not over 4′ 6″, but deeper. The water-closet seat will occupy about 2 feet in width × 20 inches in depth.

The forms of modern water appliances, and the means to get rid of house-waste, will be illustrated hereafter, under the heads of Ventilation and Plumbing.

For Wood or Coal Sheds or Bins.—In estimating the size of these accessories, it may only be necessary to state that a cord of wood contains 128 cubic feet, and a ton of coal occupies a space of about 40 cubic feet.

On the Size and Proportion of Rooms in general.—"Proportion and ornament," according to Ferguson, "are the two most important resources at the command of the architect, the former enabling him to construct ornamentally, the latter to ornament his construction." A proportion to be good must be modified by every varying exigence of a design; it is of course impossible to lay down any general rules which shall hold good in all cases; but a few of its principles are obvious enough. To take first the simplest form of the proposition, let us suppose a room built, which shall be an exact cube—of say 20 feet each way—such a proportion must be bad and inartistic; and, besides, the height is too great for the other dimensions. As a general rule, a square in plan is least pleasing. It is always better that one side should be longer than the other, so as to give a little variety to the design. Once and a half the width has been often recommended, and with every increase of length an increase of height is not only allowable, but indispensable. Some such rule as the following meets most cases: "The height of the room ought to be equal to half its width plus the square root of its length"; but if the height exceed the width the effect is to make the room look narrow. Again, by increasing the length we diminish, apparently, the other two dimensions. This, however, is merely speaking of plain rooms with plain walls; it is evident that it will be impossible, in any house, to construct all the rooms and passages to conform to any one rule of proportion, nor is it necessary, for in many rooms it would not add to their convenience, which is often the most desirable end; and, if required, the unpleasing dimensions may be counteracted by the art of the architect, for it is easy to increase the apparent height by strongly marked vertical lines, or bring it down by horizontal ones. Thus, if the walls of two rooms of the same dimensions be covered with the same strongly marked striped paper, in one case the stripes being vertical and in the other horizontal, the apparent dimensions will be altered very considerably. So also a deep, bold cornice diminishes the apparent height of a room. If the room is too long for its other dimensions, this can be remedied by breaks in the walls, by the introduction of pilasters, etc. So also, as to the external dimensions of a wall, if the length is too great it is to be remedied by projections, or by breaking up the lengths into divisions.

Understanding the general necessities of a dwelling, the proportions of



Figs. 1142 to 1149.

rooms, forms of construction, and space to be occupied, the draughtsman is prepared to undertake designing, and for this purpose cross-section paper will be found of very great use. Taking the side of a small square as a unit—one foot, for instance—he can readily pencil in rooms and passages, and alter and modify at pleasure.

Figs. 1142 to 1149 are illustrations of this form of designing, making rough sketches. It is to be observed that partitions are to be as much as possible one over the other, and the posts or walls arranged in the cellar, for the support of these lines of partitions. For the sketch, it is sufficient to make door and window openings 3 feet, unless for some particular purpose bow or mullioned windows are required. In arranging the stairs, the clear space is roughly about 12 feet, and from the foot of the stairs to the top $1\frac{1}{2}$ times the height of the story from the top of the floor to the top of the floor, counting the square landings as 1 foot each. In the sketch, the stair-head room to be provided for is that for the cellar-stairs, that lead from a small entry between the kitchen and main hall. Chimney-breasts may be sketched as $4' \times 2'$. When the sketch is transferred to drawing-paper, the spaces are then to be more exactly arranged and plotted to a scale.

Figs. 1150 to 1165 represent plans of familiar forms of houses, all drawn to the scale of 32 feet to the inch, as illustrations to the student, and as examples to be copied on a larger scale. The same letters of reference are used on all the plans, for rooms intended for similar purposes. Thus, K K designate kitchens, cooking-rooms, or laundries; D D eating-rooms; S S sleeping-rooms; P P drawing-rooms, parlors, or libraries; *p p* pantries, china or store closets, or clothes-presses; *c c* water-closets and bath-rooms.

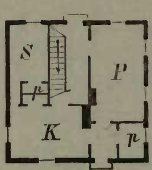


FIG. 1150.

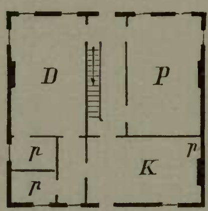


FIG. 1151.

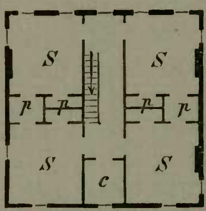


FIG. 1152.

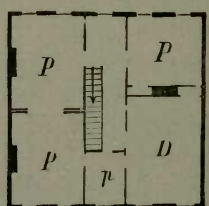


FIG. 1153.

Figs. 1150, 1151, and 1153 are first-story plans of square houses, or of square outline. Fig. 1152 is the second story of Fig. 1151. This form of house has the greatest interior accommodations for the outside cover, and, although not picturesque in its elevation, is a very convenient and economical structure. The kitchen (Fig. 1153) is in the basement, and the connection with the dining-room is by a dumb-waiter in the pantry (*p*). In Fig. 1154 the plan is the same as in Fig. 1153, but the kitchen (*k*) is in an L attached to the house; there is a small opening between the pantry (*p'*) and kitchen, through which dishes are passed to and from the dining-room.

Fig. 1155 is the plan of a very small but convenient floor, of prettier outline than the square; *v* is a portico or veranda. No chimney is shown in the sleeping-room S; there should be one either against the stairs or the back wall.

Figs. 1156 and 1157 are first-story plans of houses still more extensive. All of the above are adapted to the country, dependent on lights on all sides, and ample spaces.

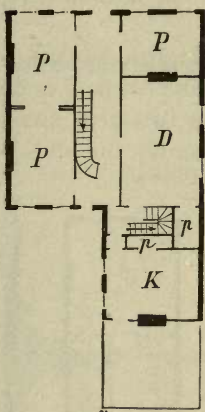


FIG. 1154.

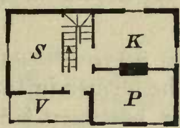


FIG. 1155.

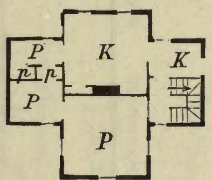


FIG. 1156.

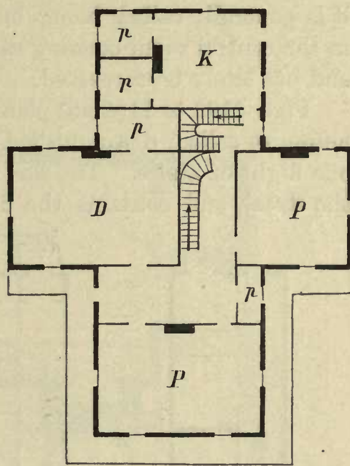


FIG. 1157.

In the cities, houses are mostly confined to one form in their general outline—a rectangle. Figs. 1158 and 1162 may be taken as the usual type of New York city houses. Figs. 1158, 1159, and 1160 are the basement, first and second floor plans of a three-rooms-deep, high-stoop house, as the first floor is

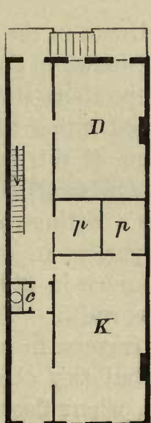


FIG. 1158.

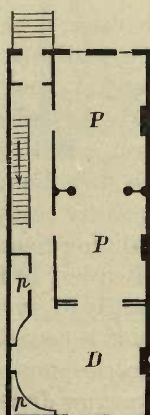


FIG. 1159.

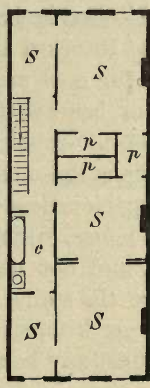


FIG. 1160.

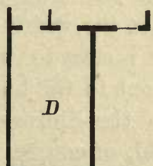


FIG. 1161.

reached by an outside flight of steps about 6 feet high. There is usually a cellar beneath the basement, but in some cases there are front vaults, entered beneath the steps to the front door; the entrance to the basement itself is also beneath the steps. The front room of the basement may be used as an eating-room, for the servants' sleeping-room, billiards, or library. The usual dining-room is on the first floor; a dumb-waiter being placed in the butler's pantry, *p*, for convenience in transporting dishes to and from the kitchen. The objection to three-rooms-deep houses is that the central room is too dark, being lighted by sash folding-doors between that and the front or rear rooms, or both. Fig. 1161 is a modification to avoid this objection, the dining-room, or tea-room, as it is generally called, being built as an L, so that there is at least one window in the central room opening directly out-doors. This was an old fashion here, and has lately been revived.

Figs. 1162 to 1165 are plans of the several floors of an English basement-house, so called, distinguished from the former in that the principal floor is up one flight of stairs. The first story or basement is but one or two steps above the street, and contains the dining-room, with its butler's pantry and dumb-

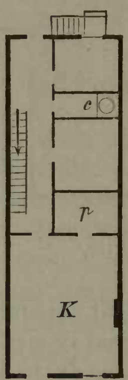


Fig. 1162.

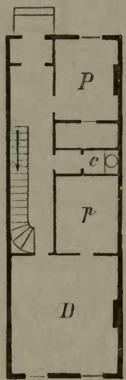


Fig. 1163.

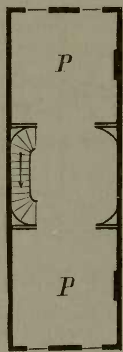


Fig. 1164.

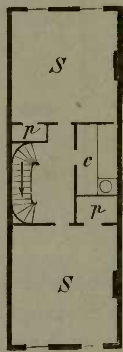


Fig. 1165.

waiter, a small sitting-room, with, in some cases, a small bedroom in the space in the rear of it. The kitchen is situated beneath the dining-room, in the sub-basement. The grade of the yard is in general some few steps above the floor of the kitchen. Vaults for coal and provisions are excavated either beneath the pavement in front or beneath the yard. The advantages of this form of house are the small reception-room on the first floor, which in small families and in the winter months is the most frequently occupied as a sitting-room of any in the house; the spaciousness of its dining-room and parlors in proportion to the width of the house, which is often but 16 feet 8 inches in width, or three houses to two lots, and not unfrequently of even a less width. The objections to the house are the stairs, which it is necessary to traverse in passing from the dining-rooms or kitchen to the sleeping-rooms, but this objection would, of course, lie against any house of narrow dimensions, where floor-space is supplied by height.

In New York, outside access to the kitchen is from the front, as there is no back street or alley. In Philadelphia, where the lots are deeper, and there is a street in the rear, the kitchen is usually in a rear L, on the level of the first floor, with the dining-room above it on a mezzonine or half-story between the first and second floors.

Figs. 1166 to 1171 are plans and elevations of a country-house in the Flemish or Queen Anne style.

PLAN OF FIRST FLOOR.

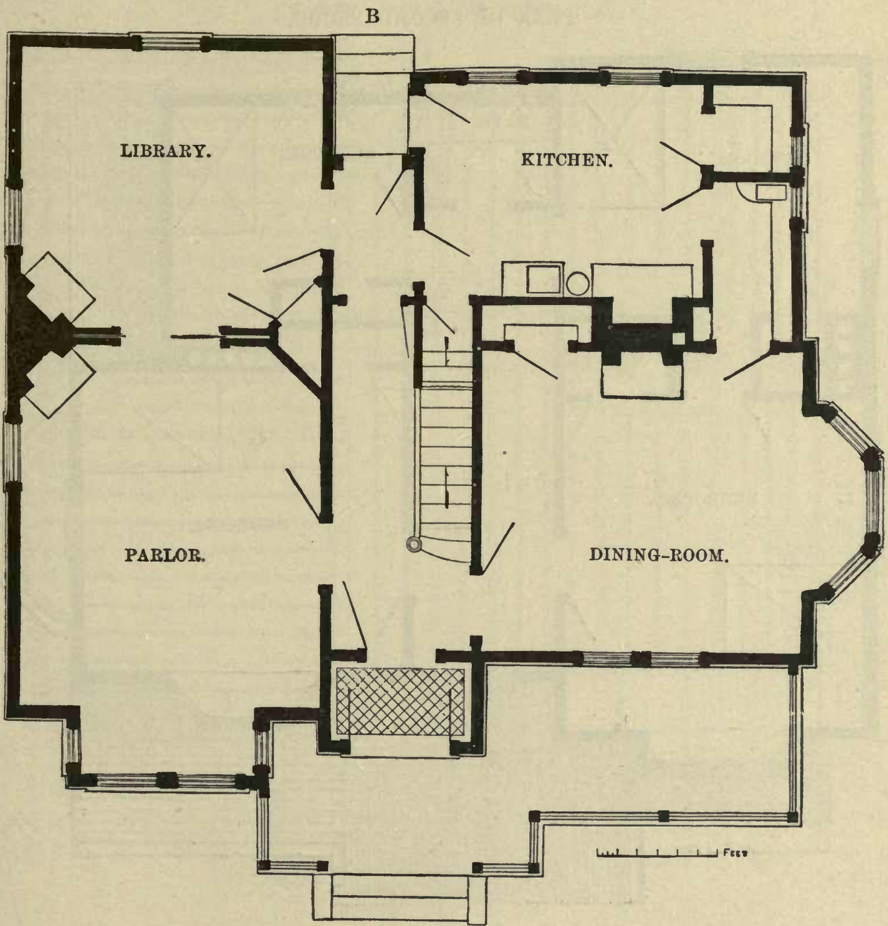


Fig. 1166.

PLAN OF SECOND FLOOR.

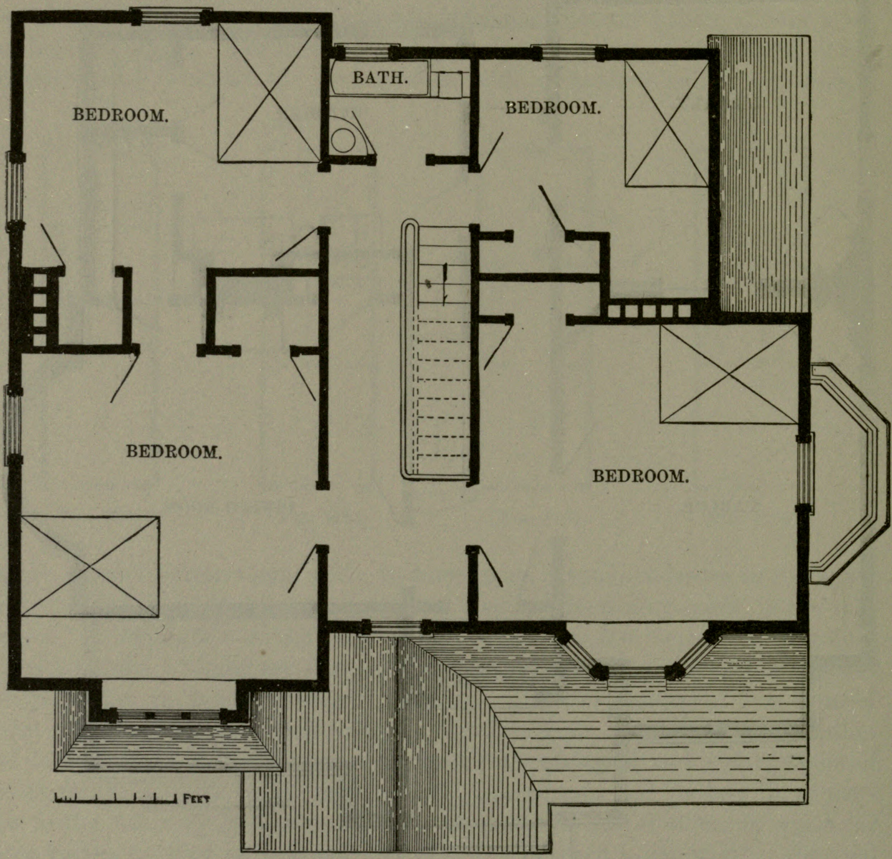


FIG. 1167.

FRAMING-PLAN OF FIRST FLOOR.

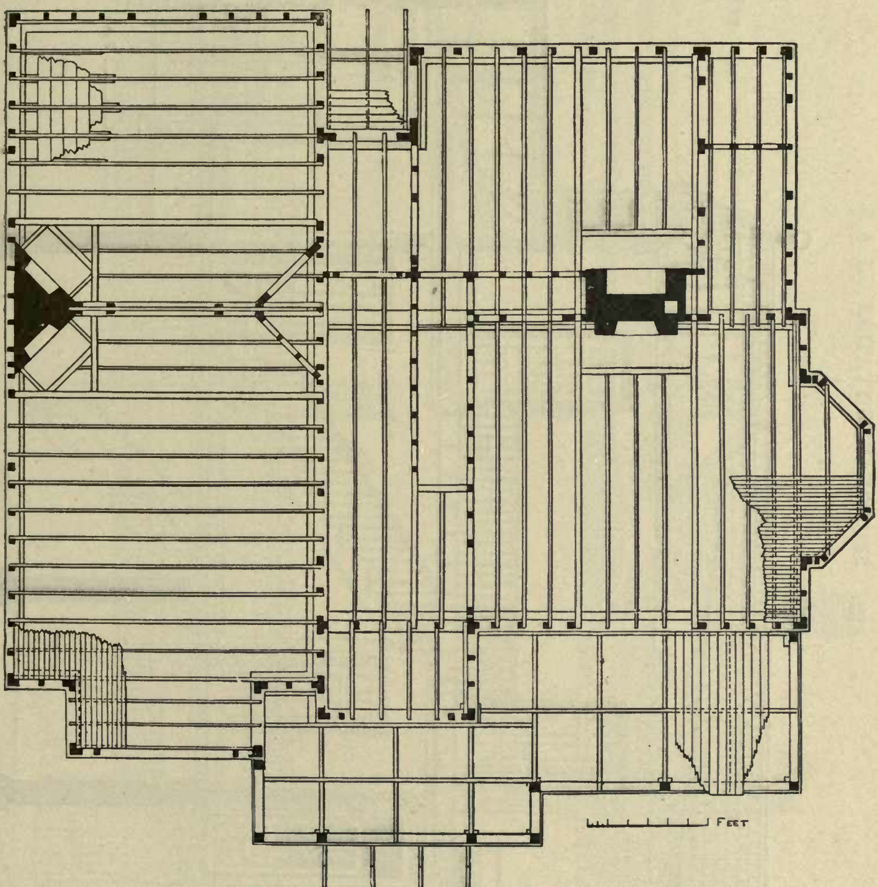


FIG. 1168.

SECTIONAL ELEVATION ON A B.

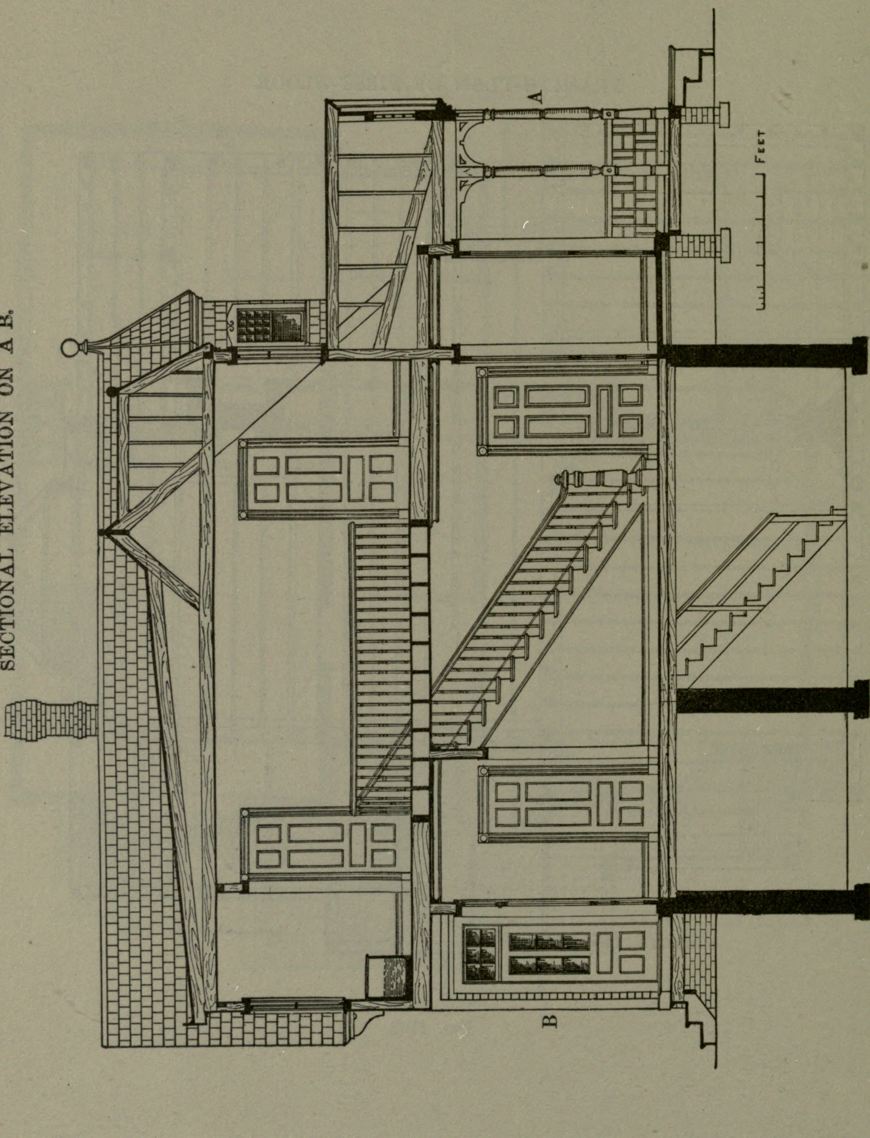


Fig. 1169.

ELEVATION OF CHIMNEY OF DINING-ROOM. SECTION.

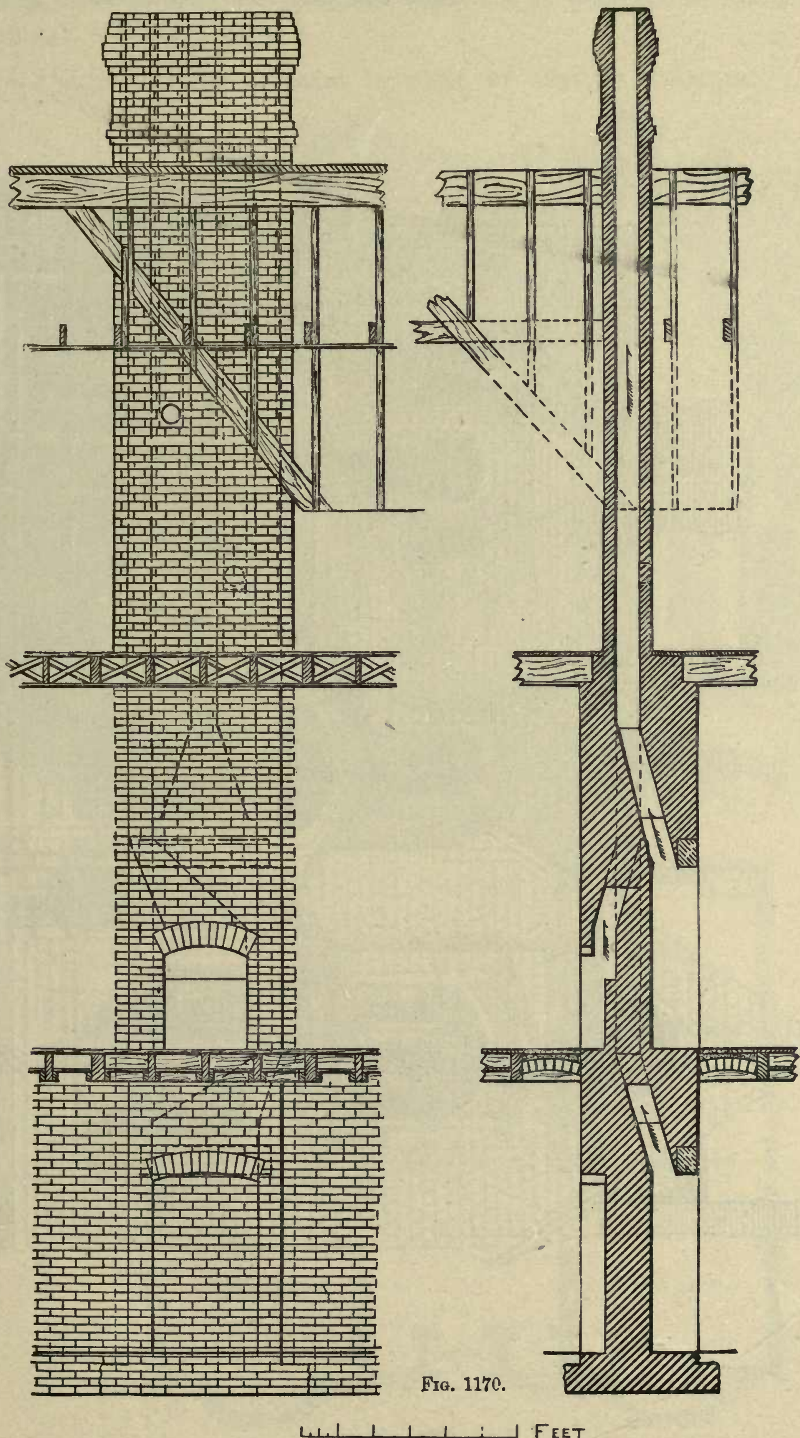


FIG. 1170.

FRONT ELEVATION.

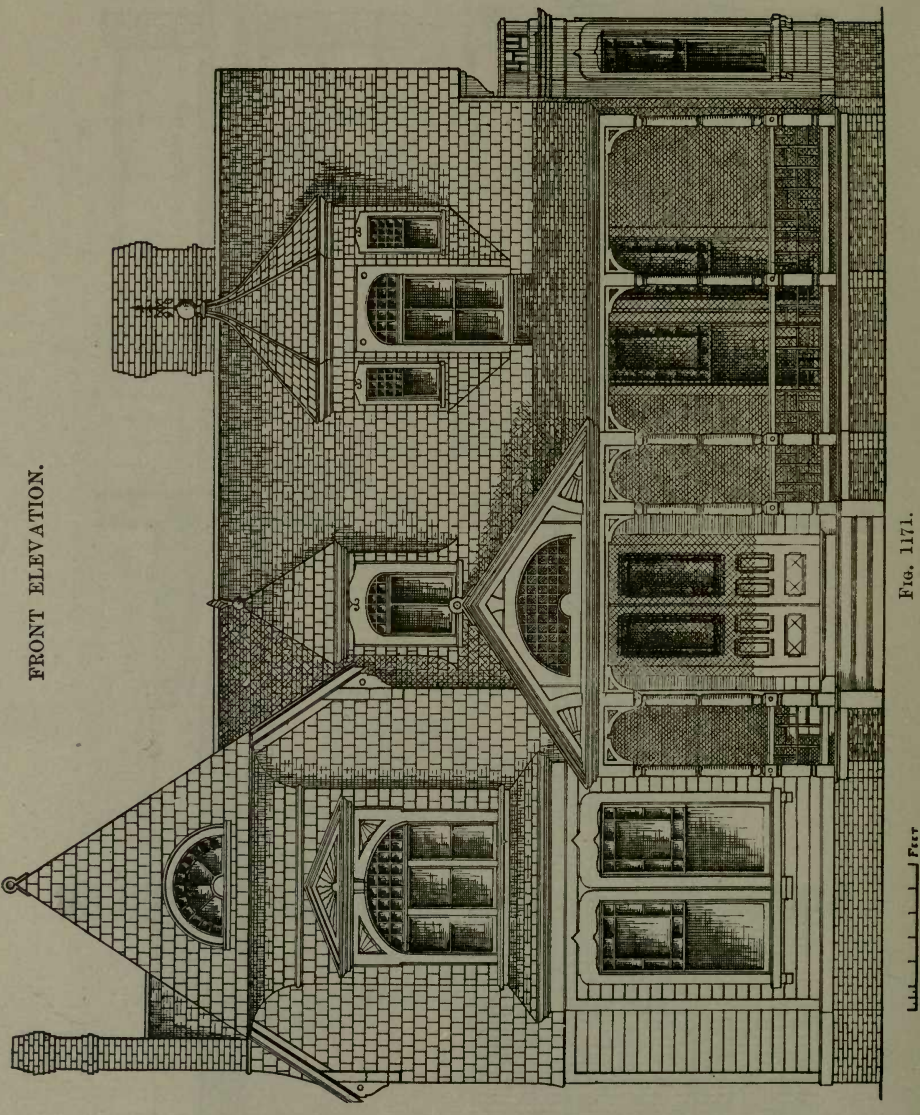
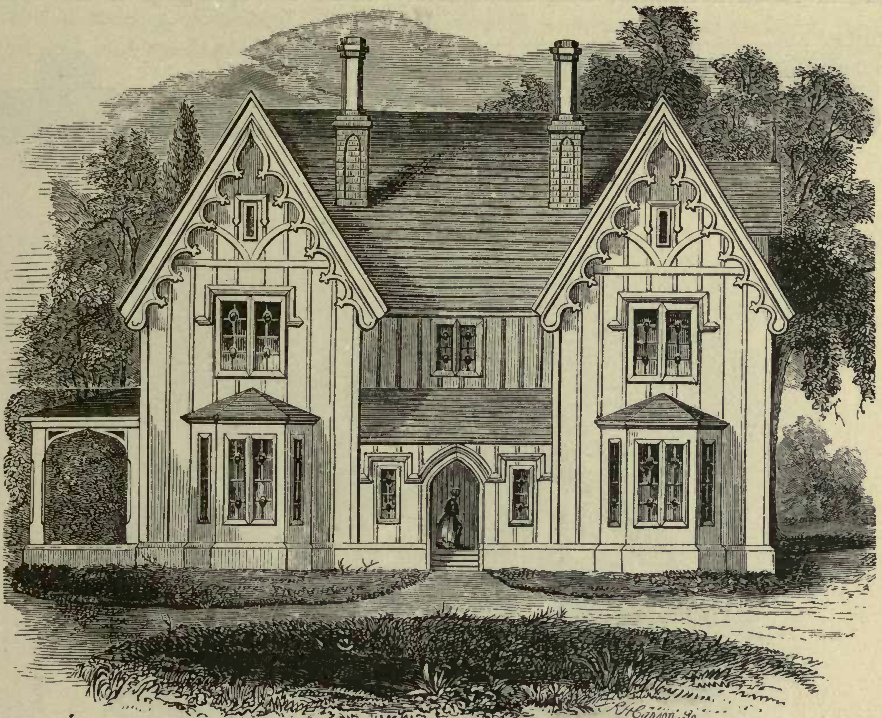


Fig. 1171.

Figs. 1172 to 1177 are plans and elevations of country residences, from Downing's "Cottage Houses."

ELEVATION OF A TIMBER COTTAGE, BY GERVASE WHEELER.



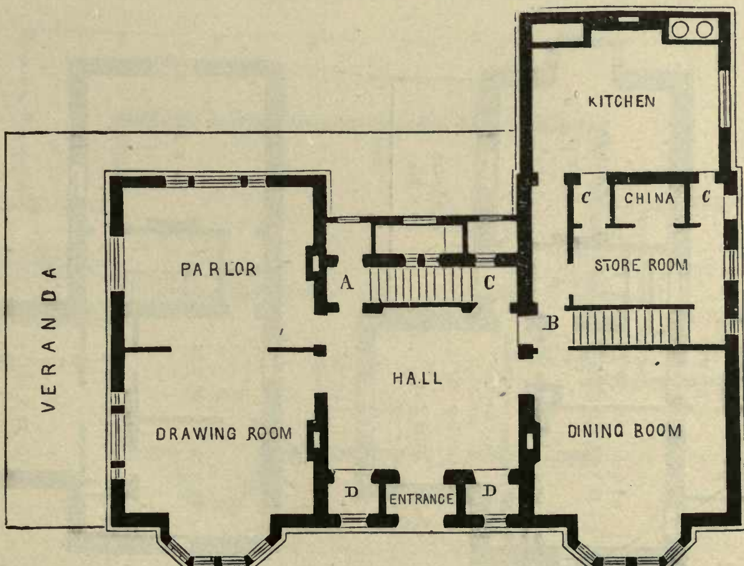


Fig. 1172.

The construction of Fig. 1172, though simple, is somewhat peculiar. It is framed in such a manner that the construction is manifest on the exterior. At the corners are heavy posts, roughly dressed and chamfered, and into them are mortised horizontal ties, immediately under the springing of the roof;

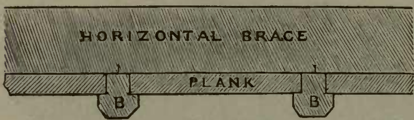


Fig. 1173.

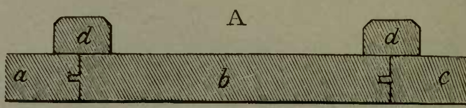


Fig. 1174.

ENGLISH RURAL STYLE.

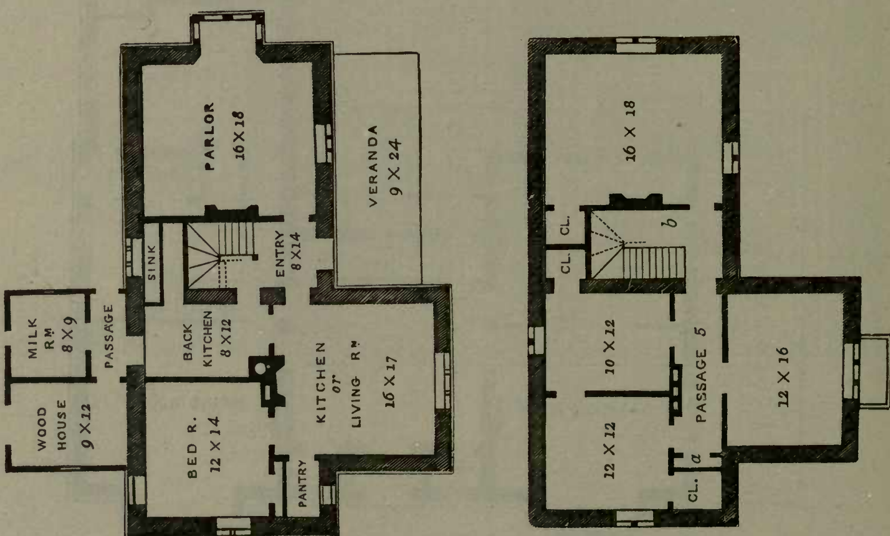


Fig. 1175.

these, with the posts and the studs, and the framing of the roof, show externally. Internally are nailed horizontal braces at equal distances apart, stopping on the posts and studs of the frame, and across these the furring and lathing cross diagonally in different directions. On these horizontal braces, the sheathing, composed of plank placed in a perpendicular position, is supported and retained in its place by battens two and a half inches thick, and

"RURAL GOTHIC STYLE."

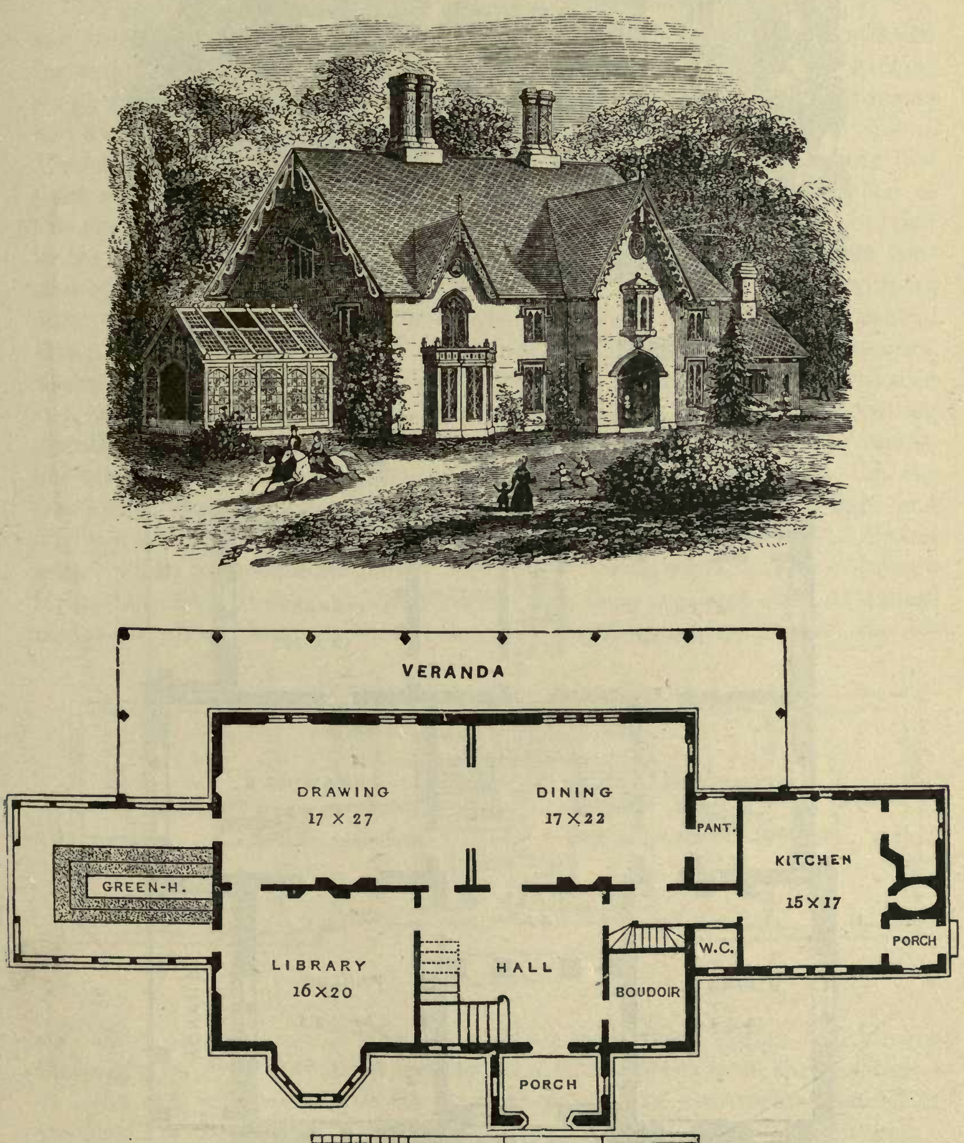
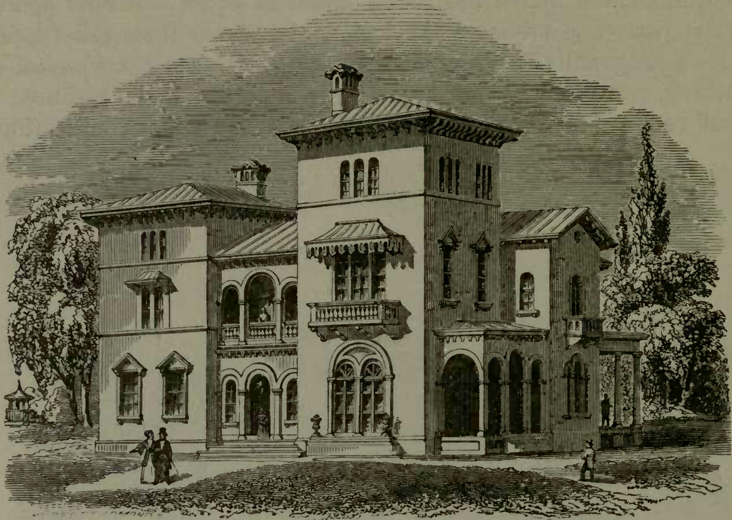


Fig. 1176.

ITALIAN VILLA, BY UPJOHN.



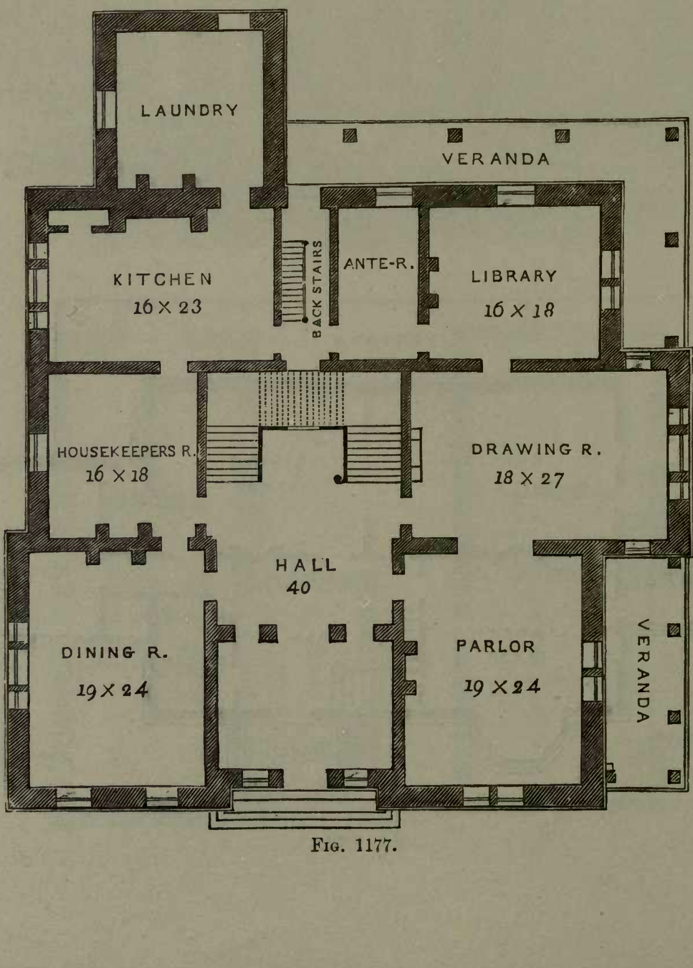


Fig. 1177.

made with a broad shoulder. These battens are pinned to the horizontal braces, confining the planks, but leaving spaces for shrinking and swelling, thus preventing the necessity of a single nail being driven through the planks. Fig. 1173 represents the batten, B, and the mode of framing.

Fig. 1174 represents the usual form of vertical boarding, which is less expensive than the first illustration, and, in general, will be found sufficiently secured for the class of buildings to which it is applied.

Fig. 1178 represents the front elevation of a high-stoop house of T. Thomas design, New York city.

To accommodate the poor and people of small means in all cities, it was, and to some extent still is, the custom to divide houses which were intended for single occupation into small apartments for many families, or to let rooms singly for this purpose. This was found to be objectionable to both occupants and owners, and houses have been constructed especially for the poorer classes. Virtually, they are now nearly all apartment-houses, each family having distinct rooms or suites to itself. But the term tenement-houses is applied to the cheaper kind of apartments, occupied by the poorer class, and situated in the least expensive localities. The common form of tenement-house consists of two buildings, one in the front and one in the rear of the lot, with an outer or air space between. A hall leads through the first story to the central area; on each side of this hall there may be small stores and apartments. Stairs from the hall lead to the apartments above. The 25 feet is divided in two, making two living-rooms on each front; these are the only rooms opening directly into the outer air. Bedrooms are attached to each of these rooms, but take their light and air from the staircases, or small light-wells. In the rear houses there are two tenements to each story; they take their light and air from the central and back areas. Water-closets or privies are in the central area. These tenements are mostly occupied by work-people, largely of foreign birth, dependent directly on small wages. But there is a large class, of limited means, to whom these accommodations are insufficient; parties who can not

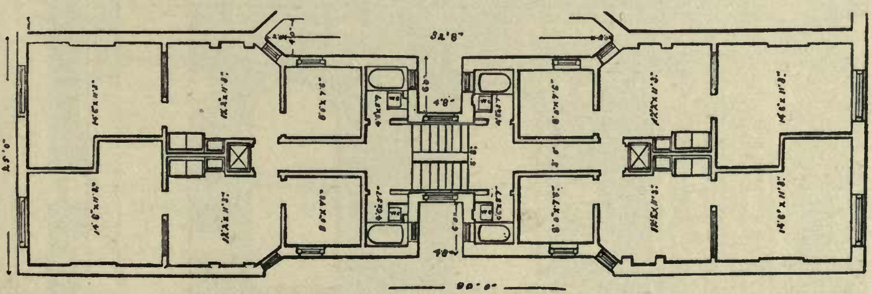


Fig. 1179.

well afford an entire house, but still wish for the privacy of one. Within the limits of a lot 25′ × 100′ it has been found difficult to secure all the necessaries of light and ventilation, with the number of suites of apartments adapted to the means of the occupants, and satisfactory as an investment to the owners.

Fig. 1179 is a plan of one of the best of these designs. It provides for

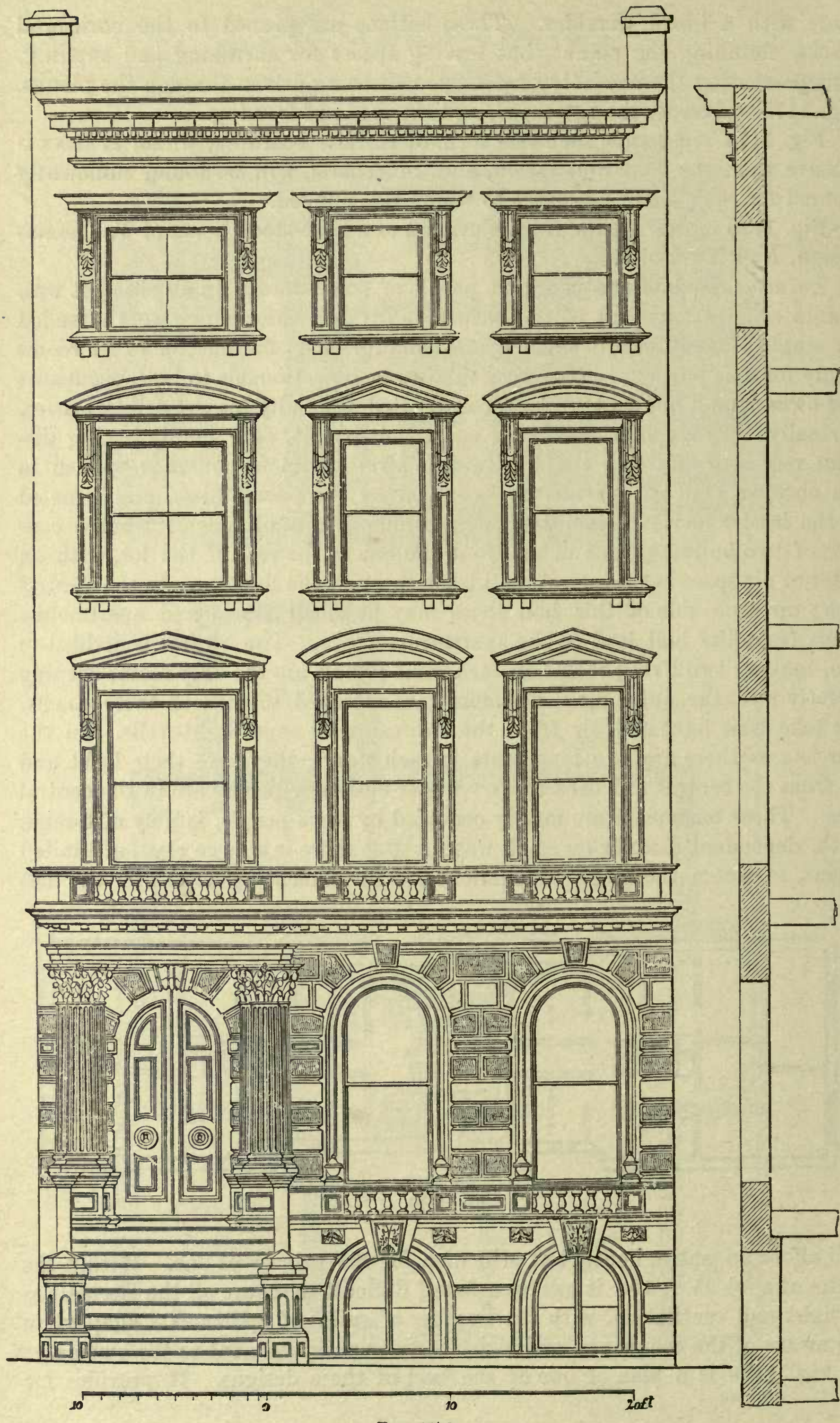


Fig. 1178.

four families on each story, although it will be observed by the plan of the stairs that the front and rear tenements are not on the same flat; they are separated by the half flight of stairs. By means of the cross-shaped court be-

PLAN.

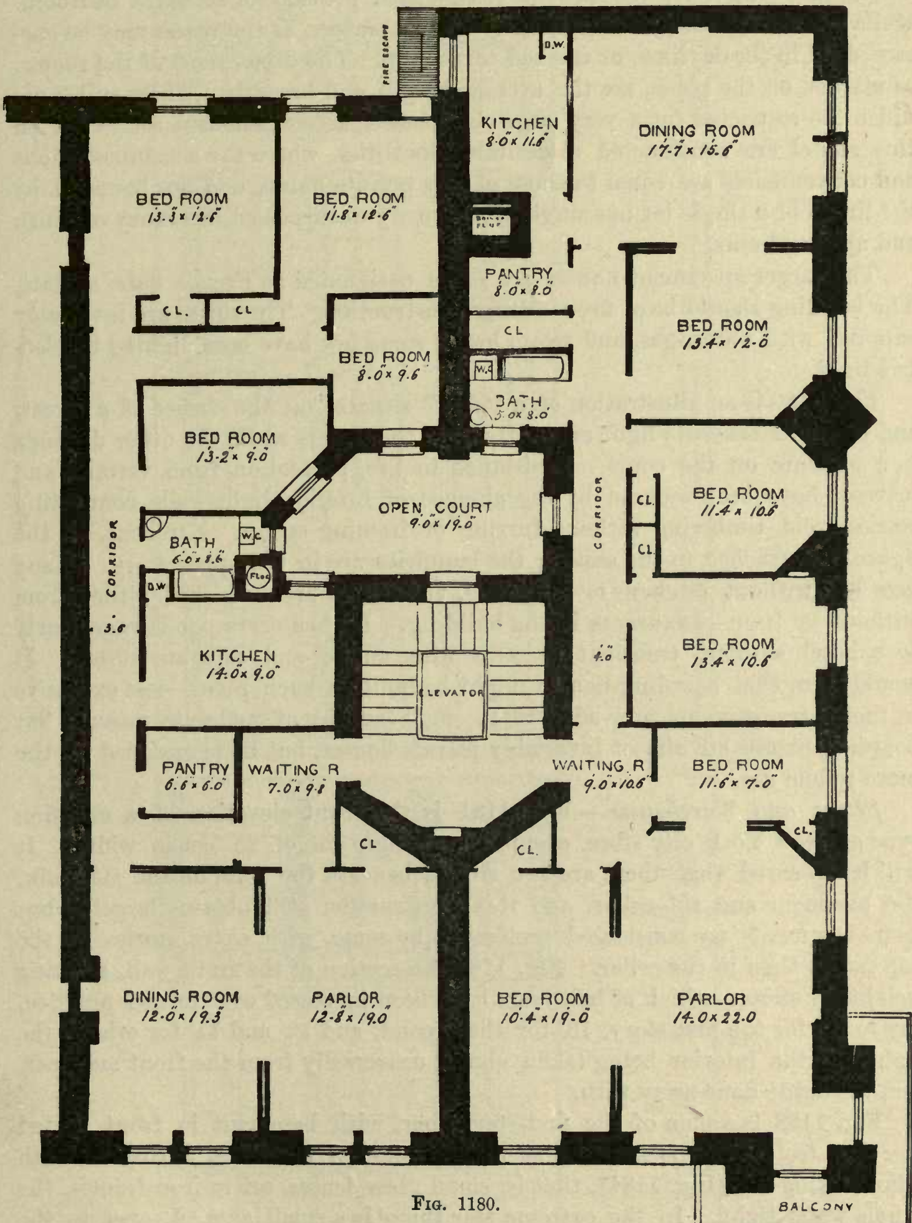


Fig. 1180.

tween the adjacent houses, every room, including the bath-room, has a window to the open air. This is the most commendable feature of the plan. It is remarkable, also, however, for providing more conveniences than have been customary in dwellings of this class, as, for instance, a small bath-tub as well as a water-closet for each family, and two wash-tubs as well as a sink ; also, a dumb-waiter (common to two families) for bringing up fuel, provisions, etc. The large rooms have recesses for beds, which provide for an extra bedroom, while detracting but little from their value as parlors, as the recess may be curtained off in the daytime, or the bed turned up. The dimensions of the rooms, as marked on the plans, are the average length and breadth. These suites are much too restricted for a very large class, but apartment-houses somewhat on this model are constructed in desirable localities, where the accommodations and conveniences are equal to those of any private house, and not bounded by the limits of a single lot nor single story, many unsurpassed in luxury of finish and appointments.

The larger apartment-houses are often designated as French flats, or flats. The building should be of fire-resisting construction. The suites are invariably supplied with water, gas, and steam heat ; some few have been lighted by electric light.

Fig. 1180 is an illustration of a "flat" situated on the corner of a street, and one suite takes its light exteriorly from the streets while the other depends in a measure on the court. Resistance to fire, protection from vermin, and privacy, have been secured by the absence of interior light-wells connecting stories, solid timbering without furring or framing spaces. Kitchens, in the figure, are attached to the suites ; the laundries are in the upper story. Many flats are without kitchens or laundries, and meals are furnished either from without or from restaurants in the building. It then corresponds very nearly to a hotel without transient custom, with ample and separate suites. It would seem that boarding-houses might be built on such plans—less extensive in their arrangements and adapted to small families of moderate means ; but boarding-houses are almost invariably private houses, but little modified for the more public use.

Stores and Warehouses.—Fig. 1181 is the front elevation of a common type of New York city store, occupying a single lot of 25 feet in width. It will be observed that there are two stories beneath the level of the sidewalk, the basement and sub-cellar, and this construction still obtains largely ; but deep basements are considered preferable by some, with extra stories at the top rather than in the cellar. Fig. 1182 is a section of the front wall, showing heights of stories, which of late years have been increased over former practice, say to 16′ for the first story, 13′ for the second, and 12′ and 11′ for others, the light for the interior being taken almost universally from the front and rear, and skylights done away with.

Fig. 1183 is a plan of the first-story floor, with basement in front dotted in ; five feet of this space, or that usually allotted for areas, is covered with illuminating tile (Fig. 1184), that is, small glass lenses, set in iron frames, the whole water-tight. In the extreme rear there is a small area, A, open to the air, of about 5 feet, for light and air to the basement and cellar. The offices of the first story are situated at B, over which there is usually a curved lean-to of illuminating tile. The main wall above this story is on the line *a b*—plain

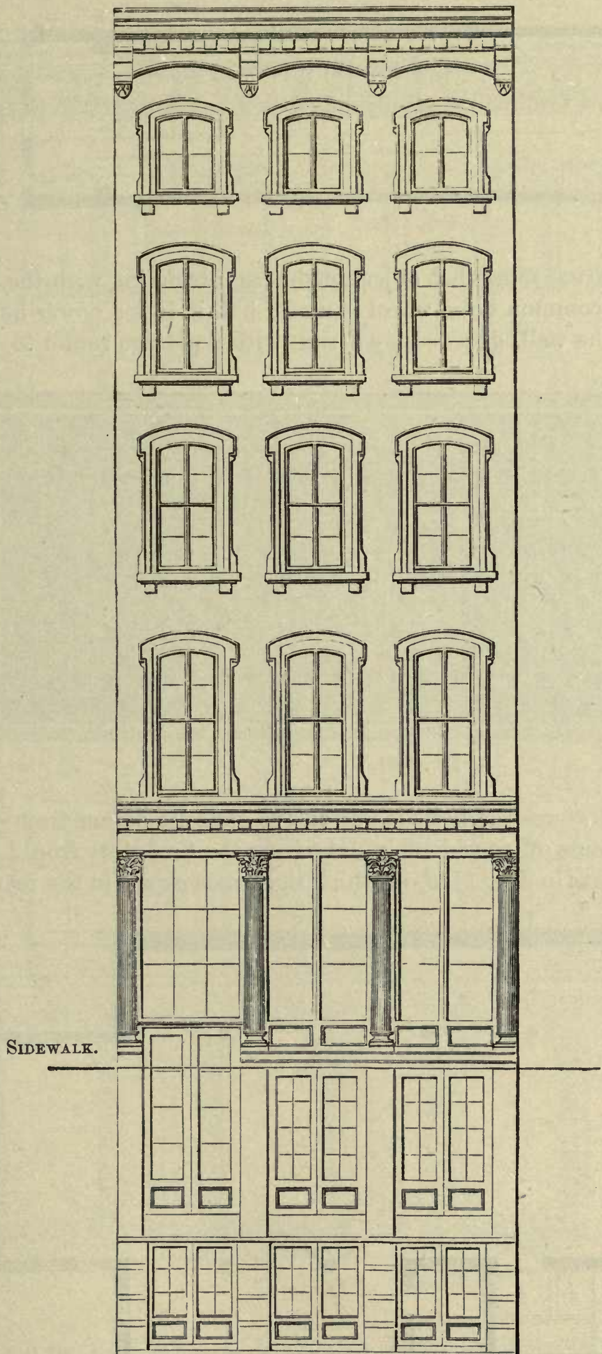


FIG. 1181.

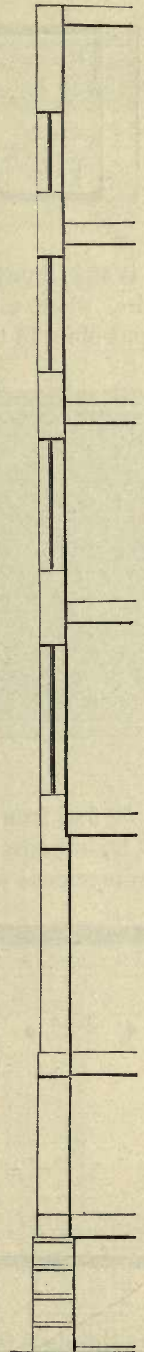


FIG. 1182.

brick—with iron shutters. When shutters are used to close the first-story front they are mostly rolling shutters of sheet-steel. The hoist-way to the up-

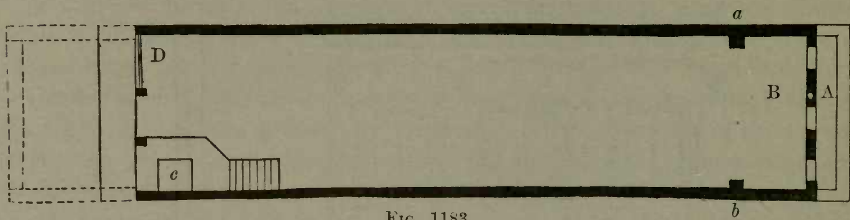


Fig. 1183.

per stories is at *c*, a position somewhat objectionable as interfering with the use of the stairs, when a common hoist-wheel is used; but if it is a power-hoist, then it is put close to the wall, guarded by a rail, with a passage round to the

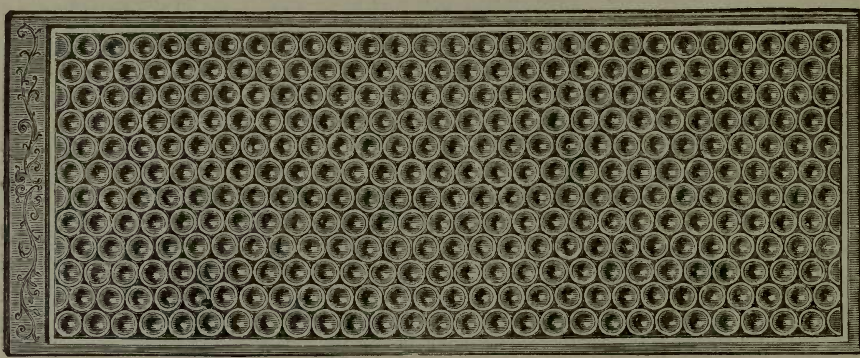


Fig. 1184.

stairs. In 50 feet front stores the hoist is put on the opposite corner from the stairs, as at D, but this cuts off considerable light from the first-story front. In some the arrangement is as in Fig. 1185, in which the hoists *c c* are in the rear of

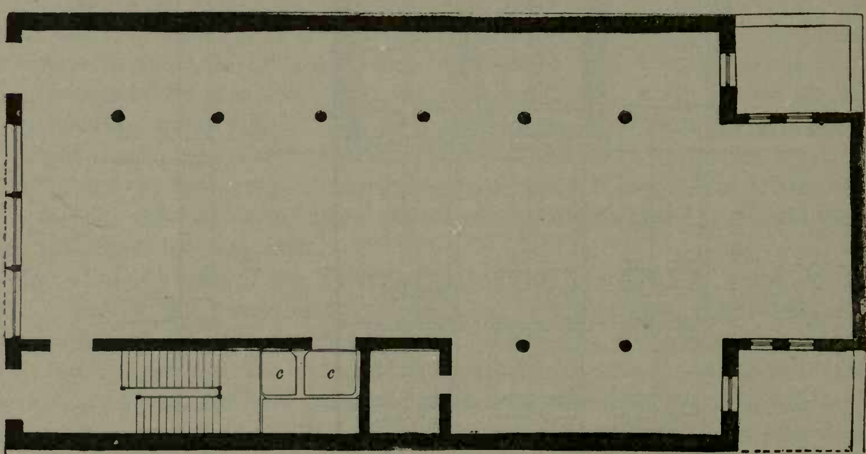


Fig. 1185.

the stairs. The arrangement for offices in the rear of the first story is in a T, with spaces at the sides for the ventilation and light of the lower stories. It

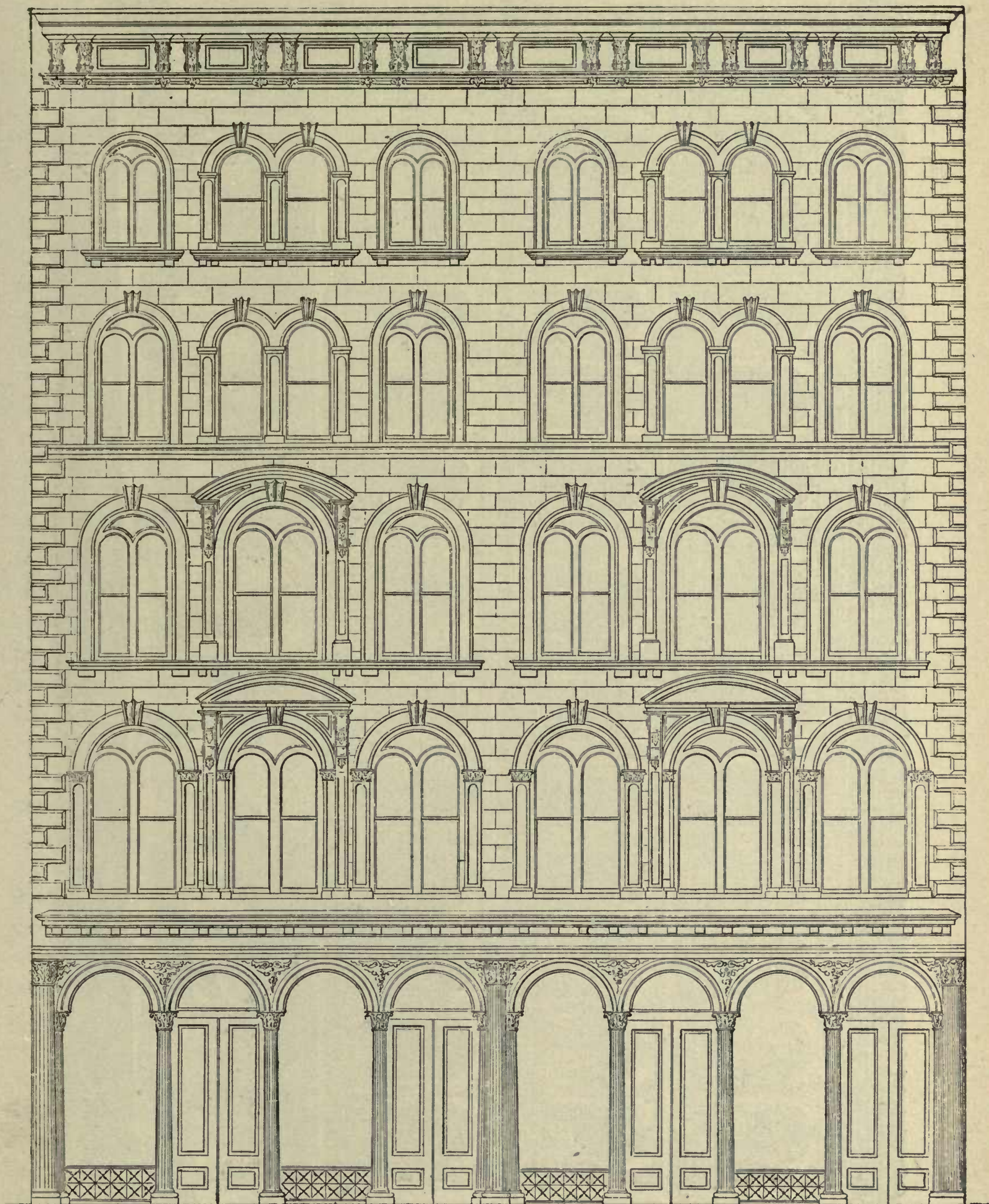


Fig. 1186.

will be observed that there is no central door, as in the elevation (Fig. 1181), which last most usually obtains for wholesale stores. For retail stores, there are usually four openings in the 25 feet, as shown in the double stores (Fig. 1186), a design of J. B. Snook.

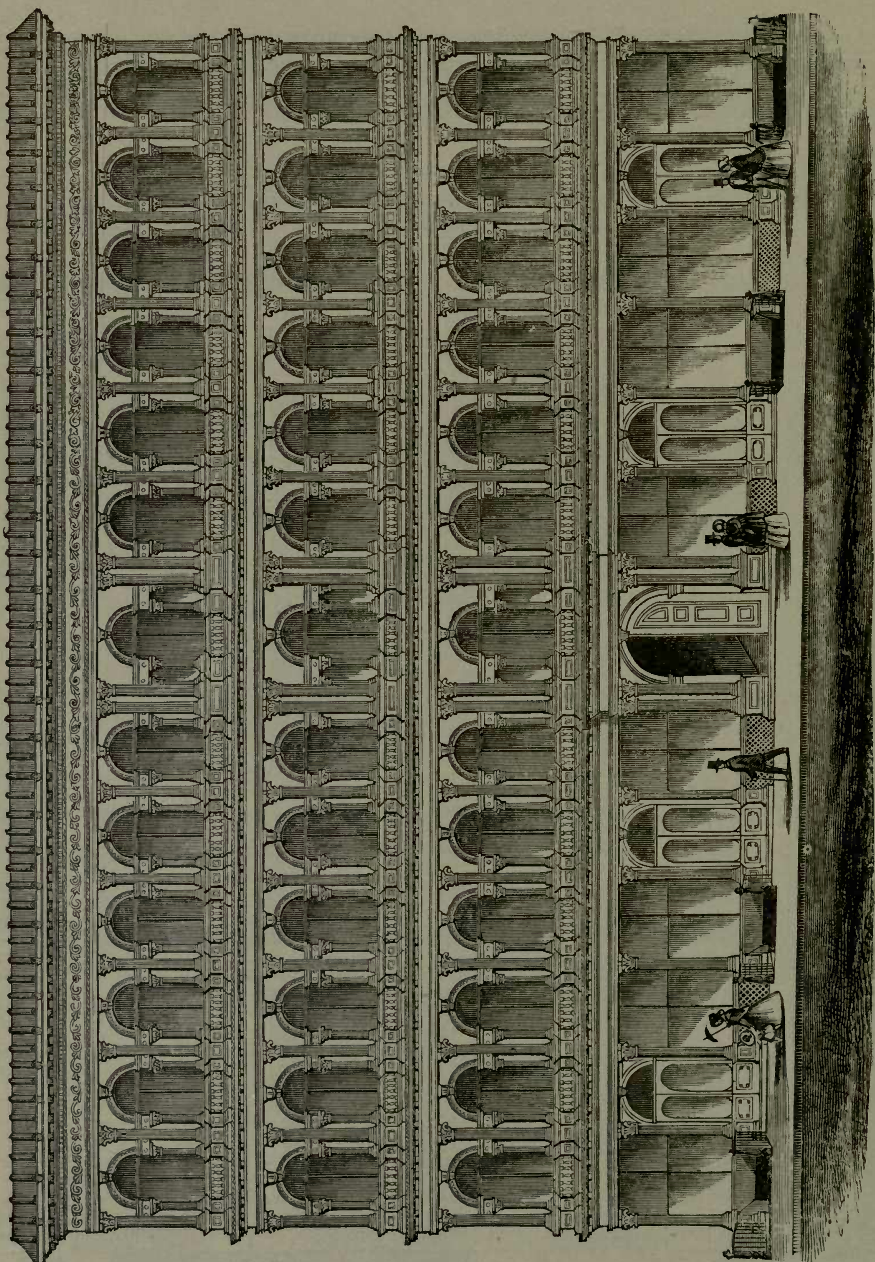


Fig. 1187.

When lots are only 100 feet in depth, 85 feet can be utilized by the building with sufficient light from the ends, but very often the stores run through from street to street, or 200 feet. Formerly the central portion was lighted by sky-lights, but this was found very objectionable, and it is now usual to leave an open-air shaft on one side, inclosed by brick walls, and the windows protected by iron shutters. The space should be 30 to 40 feet long and 6 feet wide, which may be covered in the first story with glass. If this recess is on the side occupied by the staircases, it does not detract from the inside finish of the stores.

Hoists now in large stores are power-hoists—that is, worked by either steam or water. The platform of a freight-hoist is usually 5 feet square ; for passenger-hoists, in wholesale stores, somewhat less—4′ × 5′. For the raising of goods from the basement or sub-cellar to the sidewalk there is a hatch in the front light platform, opposite some window, and the space is like that of freight-hoists, 5′ × 5′ ; these may be power or hand hoists. For the delivery of goods into these stores there is often a slide or incline, iron-plated, ending at the bottom with an easy curve to the horizontal, down which boxes and bales are slid.

Fig. 1187 is the elevation of an iron-front store 100 feet in width, among the earliest built in New York city, and in its effect is as satisfactory as any since constructed.

Fig. 1188 is a perspective view of a machine and blacksmith shop, built by the author many years since. It was built for a purpose, and to express the purpose constructionally and economically. As regards convenience and strength, it was found to be, on occupation, all that could be wished. Some allowance should be made for absence of color in the sketch, which contributed much to architectural effect. Posts, lintels, window-frames, sashes, and ornamental letters, were of iron, and painted a very deep green ; the structure was of brick, with sills and bands of rubbed Ulster bluestone, roof of Welsh slate. The building occupied one corner of Greene and Houston Streets, in this city, but was burned, and can not, therefore, be referred to practically. The chimneys shown in front, although not dummies, were never used. Power and heat were supplied by steam-boilers in the front vault, with a long, slightly inclined flue leading to a chimney at the center of the side blank wall. On each side of this chimney, and separated by a thin *with*, there were flues. Forges occupied all the exterior walls of the basement, front and side areas, and the draught was upward and then down into the nearly horizontal flues connected with the central flues, and the draught was invariably good. Care was taken that all angles, horizontal and vertical, should be rounded.

School-Houses.—Figs. 1189 and 1190 are an elevation and plan of a country district school-house, with seats for forty-eight scholars. There are two entrances, one for each sex, with ample accommodations of entry or lobby-room for the hanging up of hats, bonnets, and cloaks. A side door leads from each entry into distinct yards, and an inside door opens into the school-room. The desk, T, of the teacher, is central between the doors, on a platform, P, raised some 6″ or 8″ above the floor. In the rear of the teacher's desk is a closet or small room, for the use of the teacher. The seats are arranged two to each desk, with two alleys of 18″ and a central one of 2′. The passages around the room are 3′.

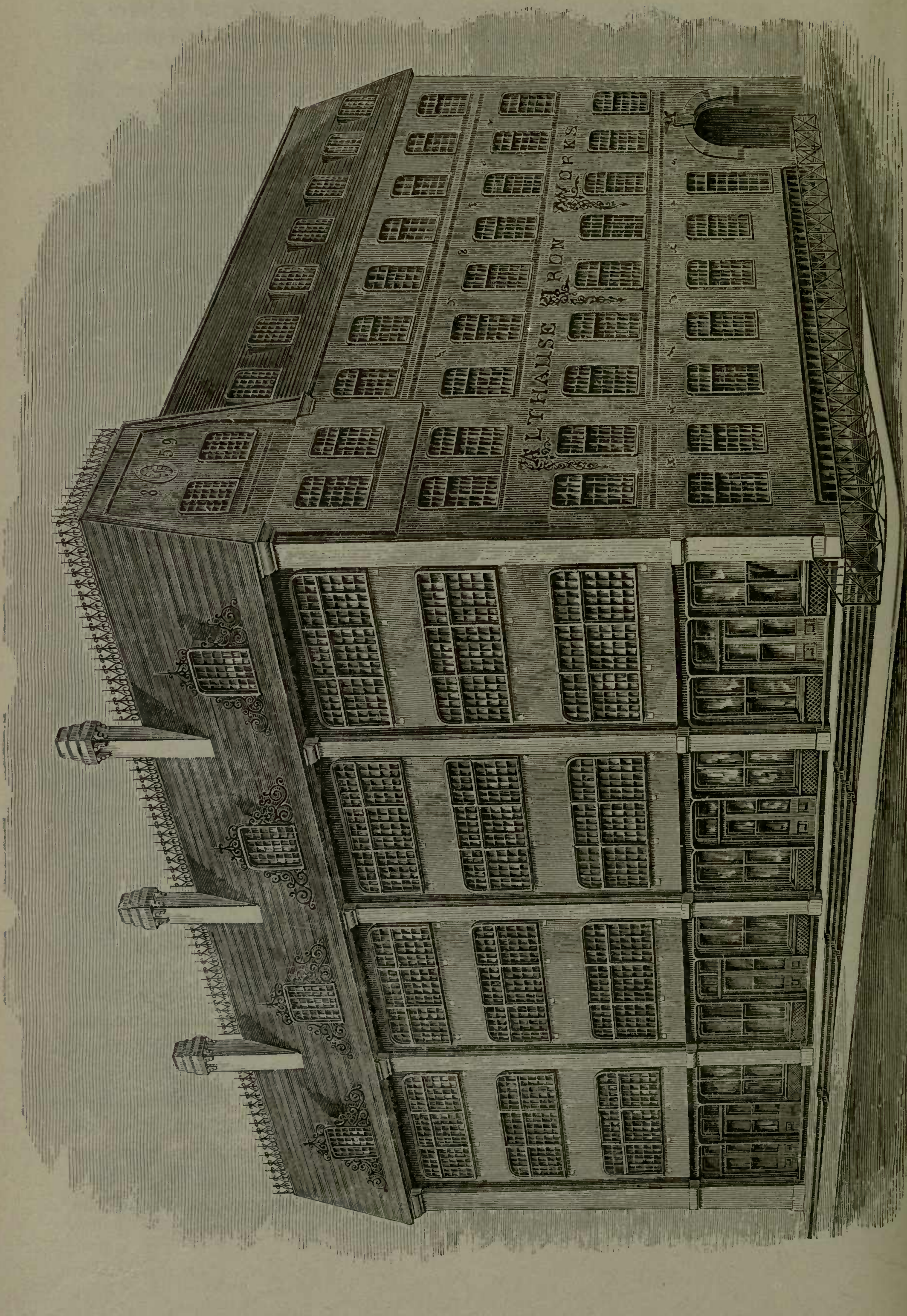


Fig. 1188.

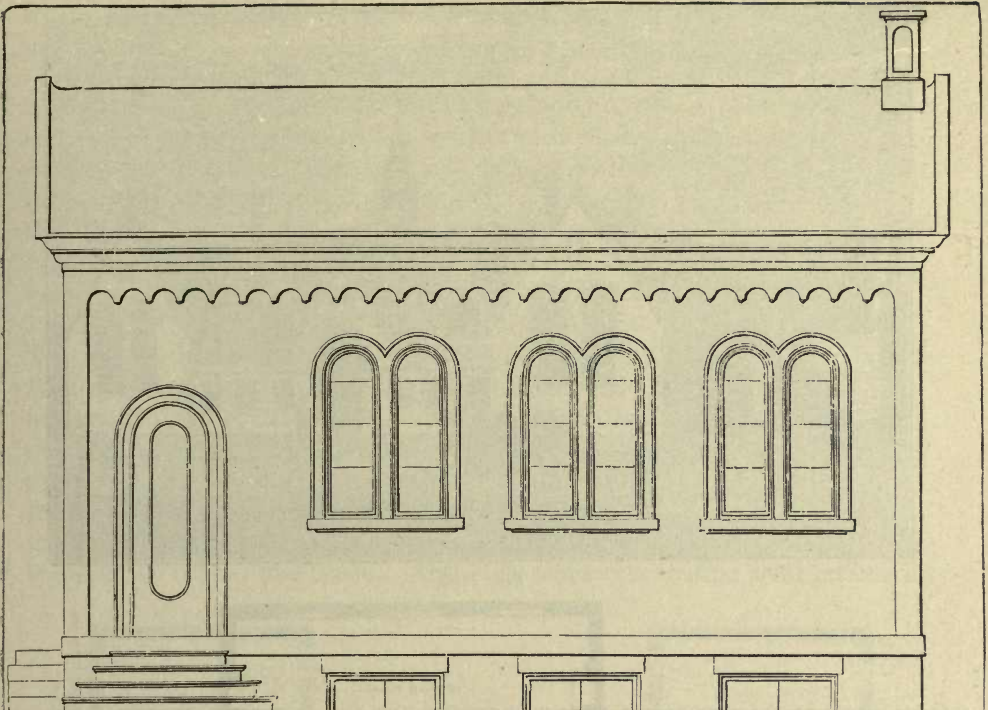


Fig. 1189.

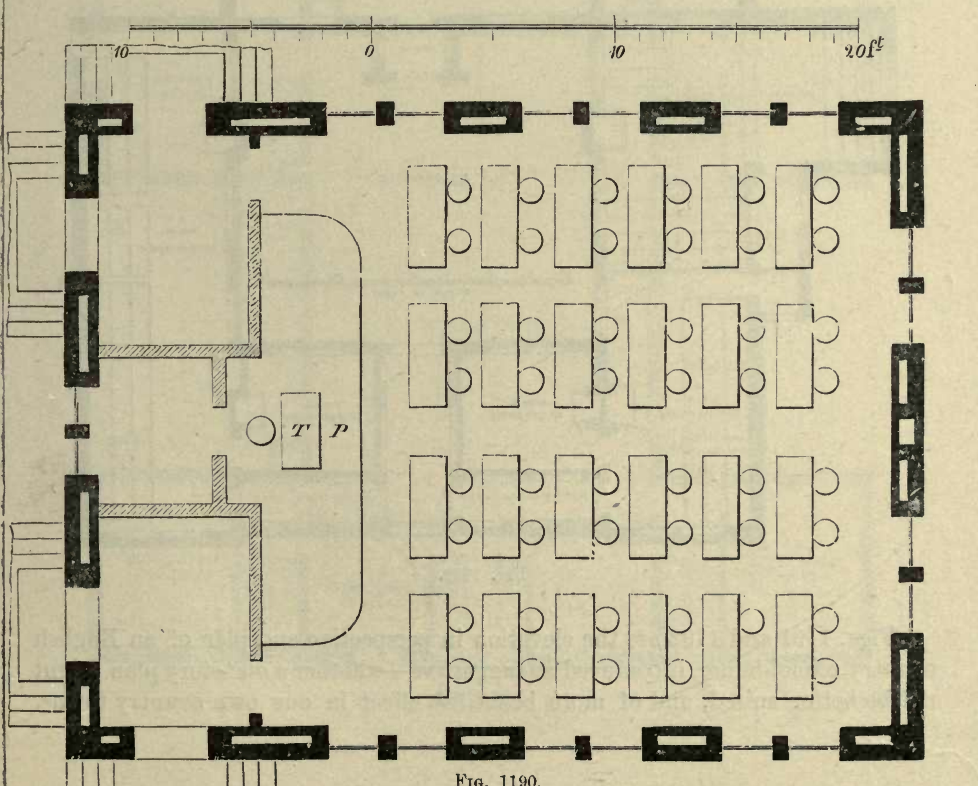


Fig. 1190.

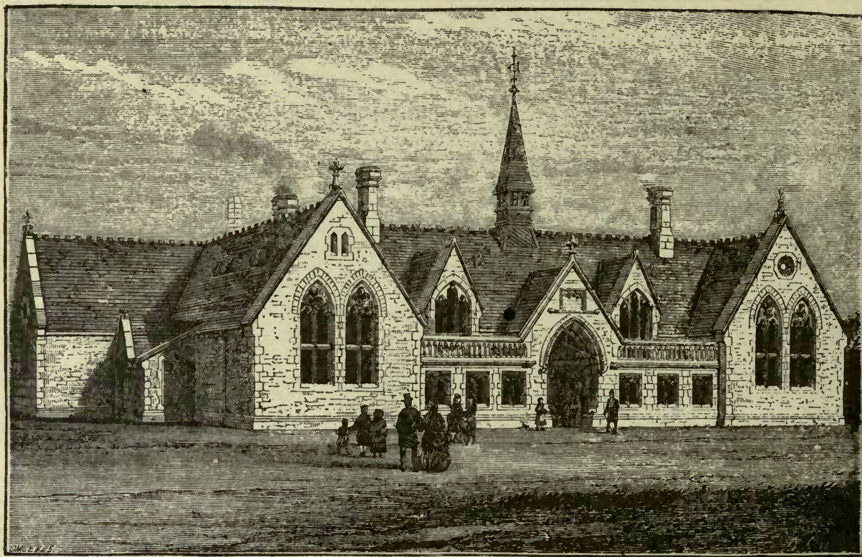


Fig. 1191.

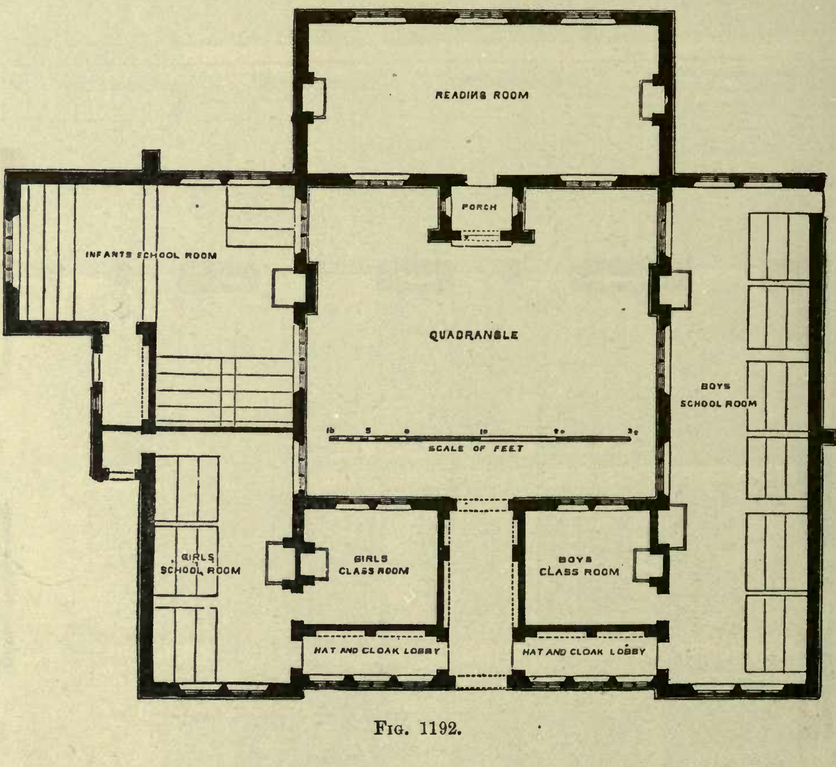


Fig. 1192.

Figs. 1191 and 1192 are the elevation in perspective and plan of an English country school-house, introduced as suggestive—whether a one-story plan might not be better suited, and of more beautiful effect in our own country towns, where there is plenty of ground space, than the imitation of city edifices of many stories.

On the Requirements of a School-House.—Every scholar should have room enough to sit at ease, his seat should be of easy access, so that he may go to and fro, or be approached by the teacher without disturbing any one else. The seat and desk should be properly proportioned to each other and to the size of the scholar for whom it is intended. The seats, as furnished by the different makers of school furniture, vary from 9″ to 14″ in height; and the benches from 17″ to 28″; measuring on the side next the scholar. The average width of the desk is about 18″, and it is formed with a slope of from $1\frac{1}{2}$″ to $2\frac{1}{2}$″, with a small horizontal piece of from 2″ to 3″ at top. There is a shelf beneath for books, but it should not come within about 3″ of the front. The width of the seat varies from 10″ to 14″, with a sloping back, like that of a chair; it should, in fact, be a comfortable chair. It will be observed that, in the figure, two scholars occupy one bench. Fig. 1193 represents another arrangement, in

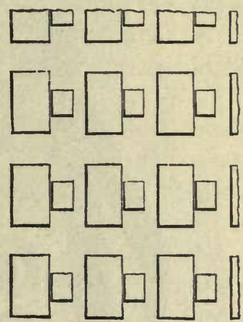


Fig. 1193.

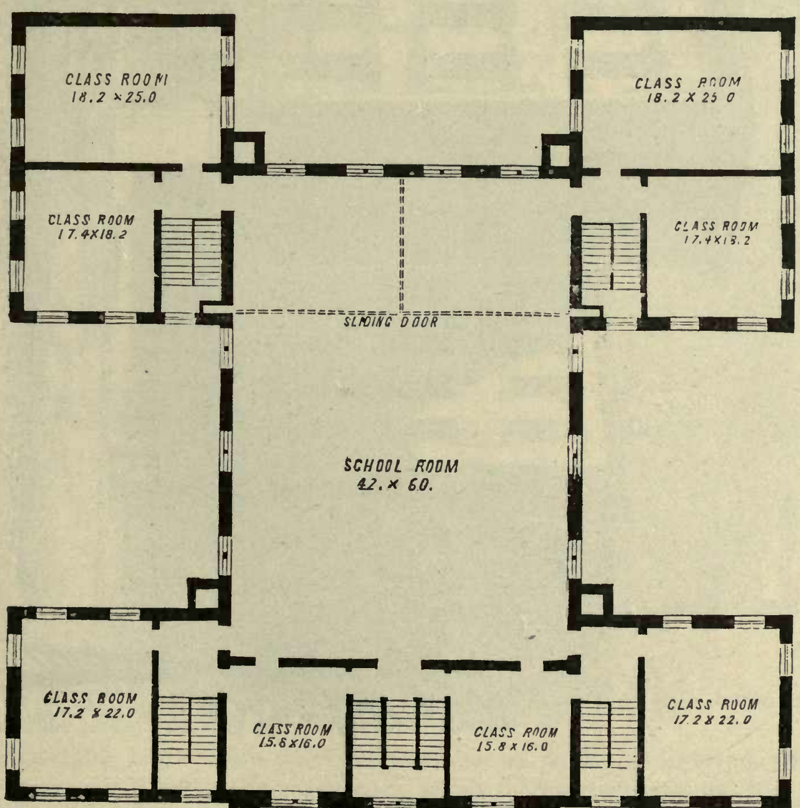


Fig. 1195.

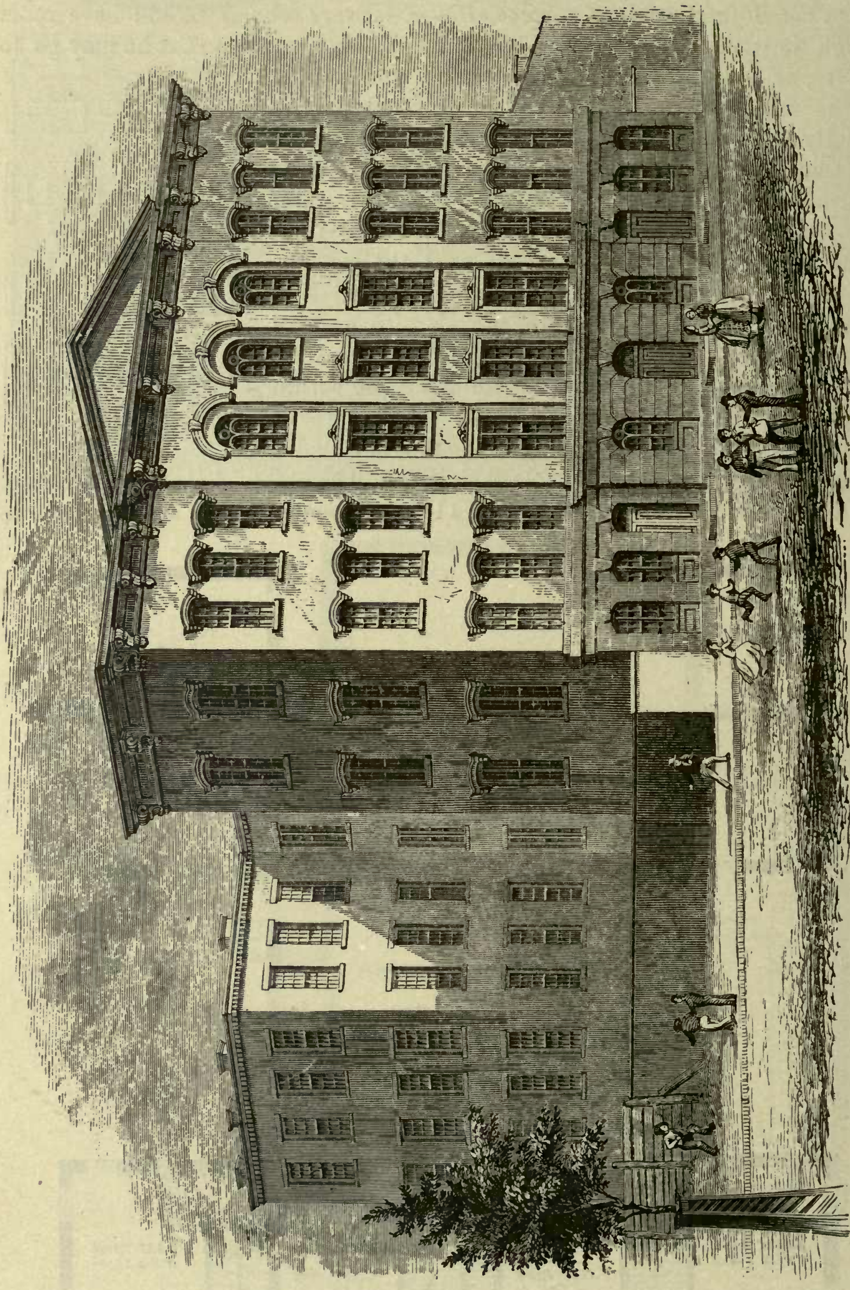


Fig. 1194.

which each scholar has a distinct bench ; this is more desirable, but not quite so economical in room. In primary schools, desks are not necessary ; and in many of the intermediate schools the seat of one bench is formed against the back of the next bench ; but seats distinct are preferable. The teacher's seat is invariably on a raised platform, and had better be against a dead wall than where there are windows. Blackboards and maps should be placed along the walls. Care should be taken in the warming and ventilation ; warm air should be introduced in proportion to the number of scholars, and ventiducts should be formed to carry off the impure air.

In cities and large towns it is almost indispensable to build school-houses many stories in height, dividing the rooms in each story according to the necessities of their occupancy. The management of schools differs in different localities. This will be seen in the illustrations given below, showing the arrangements of school-houses in the city of New York and of Cleveland, Ohio.

Fig. 1194 is an elevation in perspective of one of the largest of the New York city schools, showing the yards around it. Fig. 1195 is the plan of the gram-

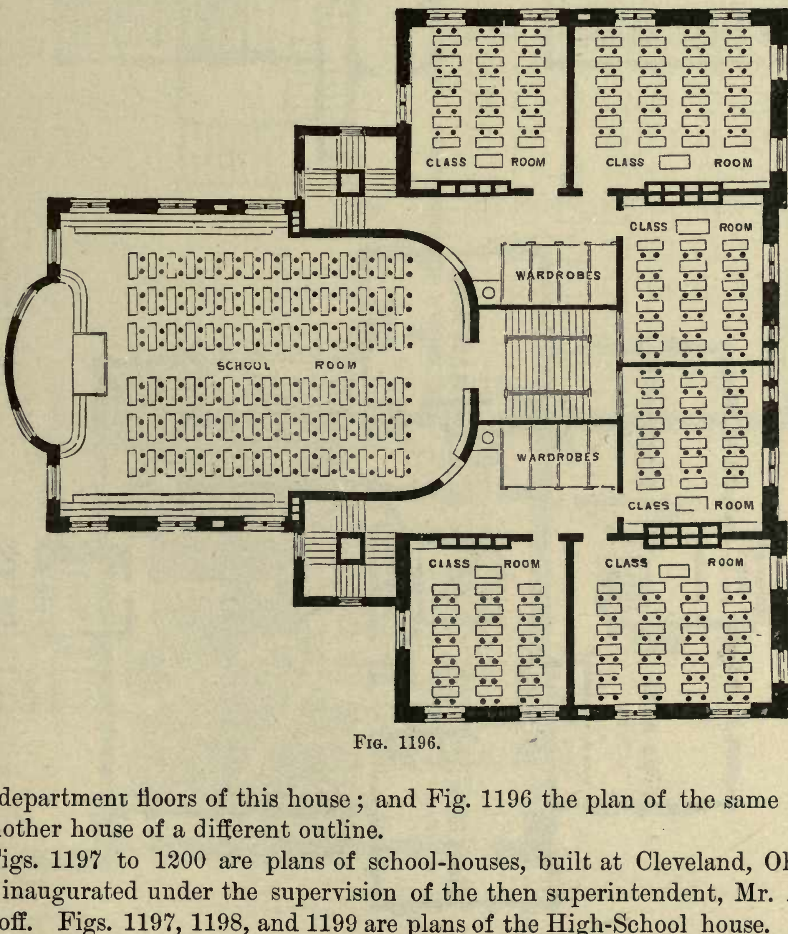


Fig. 1196.

mar-department floors of this house ; and Fig. 1196 the plan of the same floors of another house of a different outline.

Figs. 1197 to 1200 are plans of school-houses, built at Cleveland, Ohio, a type inaugurated under the supervision of the then superintendent, Mr. A. J. Rickoff. Figs. 1197, 1198, and 1199 are plans of the High-School house. Fig.

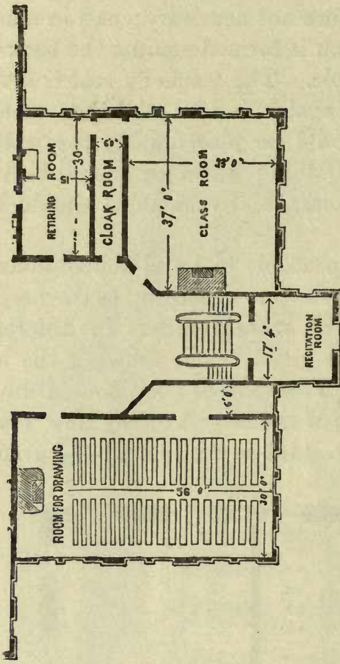


Fig. 1198.

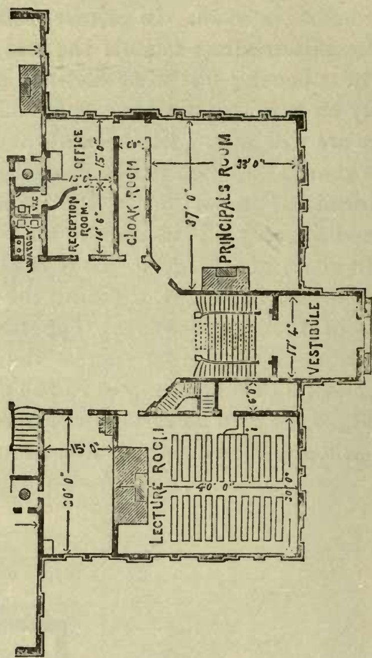


Fig. 1199.

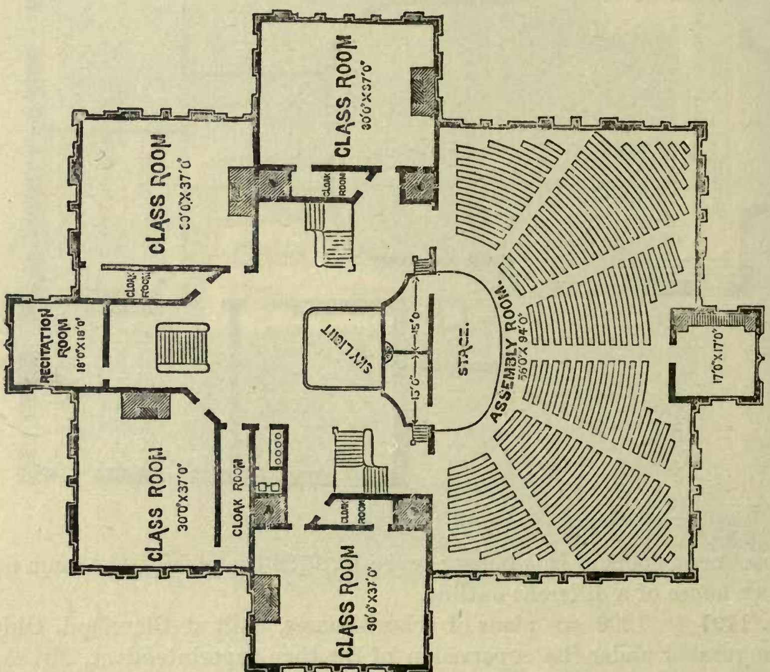
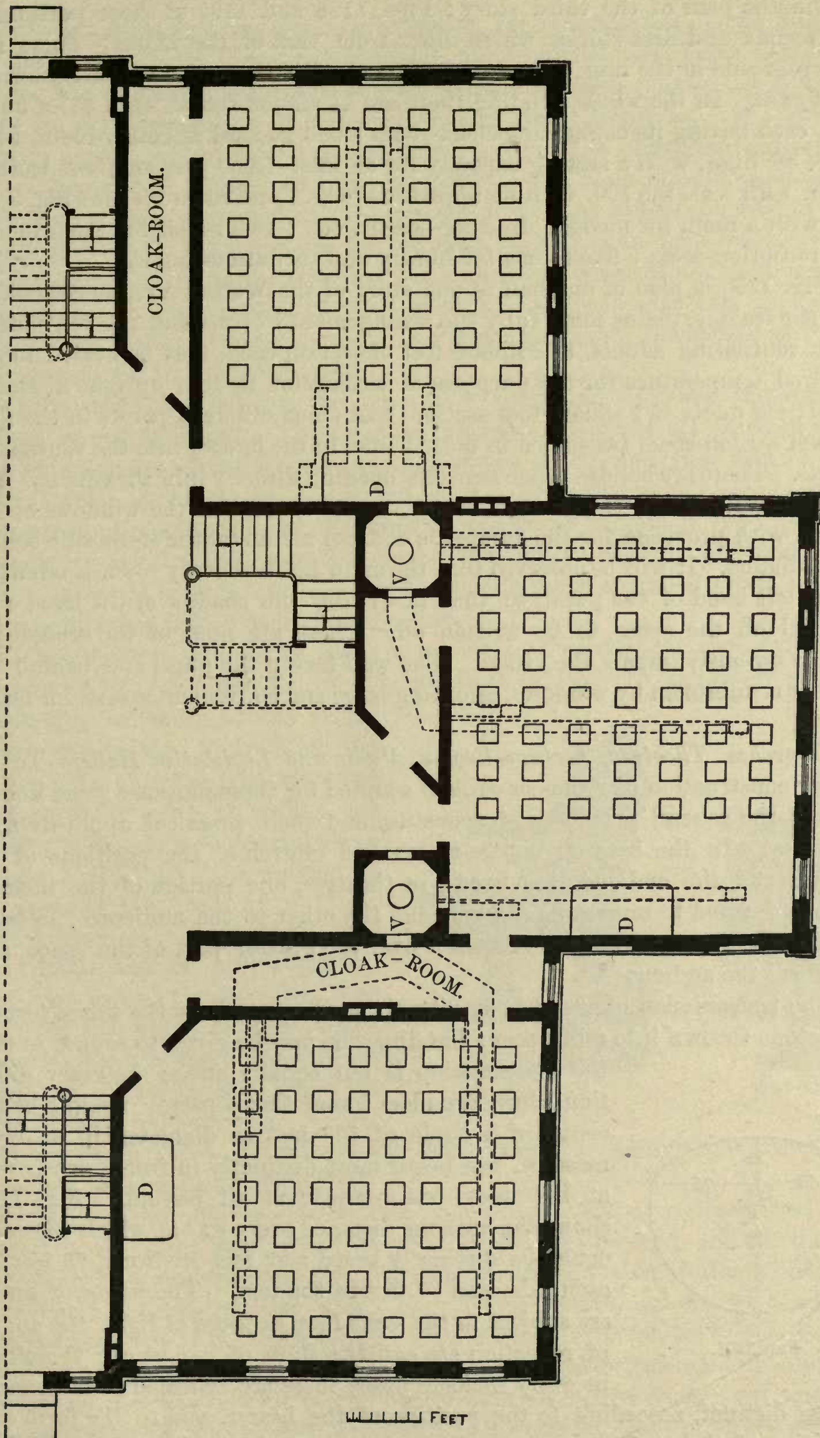


Fig. 1197.



PLAN OF HALF OF ONE STORY OF WALTON AVENUE SCHOOL, CLEVELAND, OHIO.

Fig. 1200.

1197 is the plan of the third story; Figs. 1198 and 1199 of those portions of the second and first stories which differ from that of the third. There is a rear vestibule in the first story to correspond with the one in front, shown in the figure. In the whole building there are 14 session-rooms, each 37′ × 30′ × 16′; each having its connecting cloak-room; one general assembly-room, 94′ × 56′ × 38′ high, with a seating capacity for at least 1,000 persons; one lecture-room, with seats for 100, with an apparatus-room; one room for drawing, 30′ × 55′, with a room for models, drawing-boards, etc.; two rooms for the principal and reception-room; five rooms for library and recitation-rooms.

Fig. 1200, a plan of one half of one story of the Walton Avenue School, on a larger scale, explains more fully the arrangement of seats and the ventilation. Four ventilating educts, of 8 square feet of section each, may be heated to any required temperature for the purposes of circulation by four upright 2″ steam-pipes; six ducts of 1 square foot section lead from different points in the floor of each session-room (as shown in dotted lines in the figure) into the ventilating educts. There are besides other registers opening directly into the educts. The building is heated by steam coils or radiators placed under the windows of the rooms, with provision for the admission of fresh air under the stone sills behind the radiators. It will be observed that the main light of every room is admitted at the left hand of the pupil, so that in writing the shadow of the hand does not fall on the space to be written on. There are none of the cross-lights that so seriously impair the vision. The wall facing the pupil and behind the teacher is unbroken by windows, affording large and convenient spaces for black-boards.

Churches, Theatres, Lecture-Rooms, Music and Legislative Halls.—To the proper construction of rooms or edifices adapted for these purposes some knowledge of the general principles of acoustics, and their practical application, is necessary. In the case of lecture-rooms and churches, the positions of the speaker and the audience are fixed; in theatres, one portion of the inclosed space is devoted to numerous speakers and the other to the audience; in legislative halls, the speakers are scattered over the greater part of the space, and also form the audience.

The transmission of sound is by vibrations, illustrated by the waves formed by a stone thrown into still water; but direction may be given to sound, so that the transmission is not equally strong in every direction; thus, Saunders found that a person reading at the center of a circle of 100 feet in diameter, in an open meadow, was heard most distinctly in front, not as well at the sides, but scarcely at all behind. Fig. 1201 shows the extreme distance every way at which the voice could be distinctly heard: 92 feet in front, 75 feet on each side, and 31 feet in the rear. The waves of sound are subject to the same laws as those of light, the angles of reflection are equal to those of incidence; therefore, in every inclosed space there are reflected sounds, more or less distinct, according to the position of the hearer, and to the form and condition of the surfaces against which the waves of sound impinge. Thus,

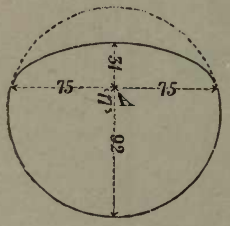


Fig. 1201.

of all the sounds entering a parabolic sphere, the reflected sounds are collected at the focus. Solid bodies reflect sound, but draperies absorb it. As, in all rooms, the audience can never be concentrated at focal points, nor is it possible in any construction to make calculation for all positions, it is in general best to depend on nothing but the direct force of the voice, and not to construct larger than can be heard directly without aids from reflected sounds.

There is great difference in the strength of voice of different speakers ; the limits as given in the figure are for ordinary reading in an open space. In inclosed spaces, owing to the reflected sounds or some other cause, there are certain pitches or keys peculiar to every room, and to speak with ease the speaker must adapt his tone to those keys. The larger the room, the slower and more distinct should be the articulation.

It has been observed that the direction of the sound influences the extent to which it may be heard. The direction of the currents of air through which the sound passes affects the transmission of the sound, and this may be made useful when the rooms are heated by hot air, by introducing the air near the speaker, and placing the ventilators or educts at the outside of the rooms, and by placing their apertures rather nearer the bottom of the room than at the top. It would seem much better and easier to make a current of air a vehicle of sound rather than depend on reflection.

On the Space occupied by Seats in general.—A convenient arm-chair occupies about 20″ × 20″, the seat itself being about 18″ in depth, and the slope of the back 2″ ; 18″ more affords ample space for passage in front of the sitter. In churches the seats are arranged by pews or stalls ; the width of each pew in general being about 2′ 10″. In the arrangement of seats at the Academy of Music the bottom turns up (Figs. 1202 and 1203), and 29″ only is allowed for both seat and passage-way, and 18″ for the width of seat, which may be taken as the average allowance in width to each sitter in comfortable public rooms. In lecture-rooms, benches and settees are often used, the space there occupied by seat and passage being about 2′ 6″.

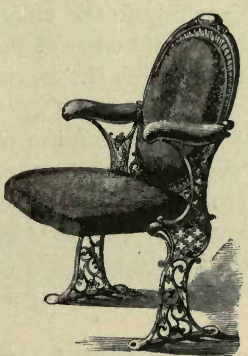


Fig. 1202.

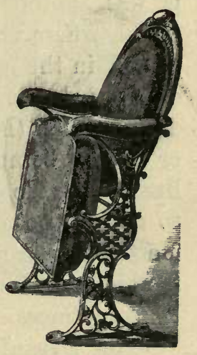


Fig. 1203.

In the earlier churches, ceremonies and rites formed a very large part of the worship, the sight was rather appealed to than the hearing, and for this purpose churches were constructed of immense size, and with all the appliances of ornament and construction, with pillars, vaults, groins, and traceried windows. In the churches of this country, the great controlling principle in the construction of a church is its adaptation to the comfortable hearing and seeing the preacher. In this view alone, the church is but a lecture-room ; but since even the character of the building may tend to devotional feelings in the audience, and since certain styles and forms of architecture have long been used for church edifices, and seem particularly adapted for this purpose, it has been the custom to follow these time-honored examples, adapting them to the modern requirements of church worship.

Fig. 1205 is a plan of an ancient basilicon or Romanesque church. Fig. 1204 is a sectional elevation of the same. Fig. 1206 is a plan of a Gothic church, in which C is the chancel, usually at the eastern extremity, T T the transept, and N the nave. In general elevation the Gothic and Romanesque agree : a high central nave and low side aisles. In the later Romanesque the transept is also added.

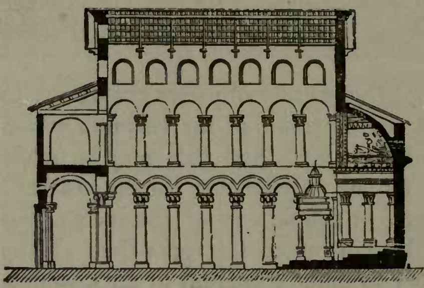


FIG. 1204.

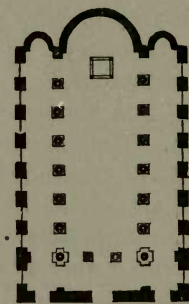


FIG. 1205.

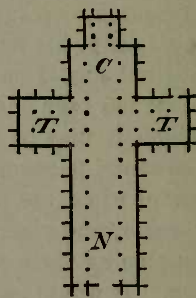


FIG. 1206.

The basilicas aggregated within themselves all the offices of the Romish church. The circular end or apse, and the raised platform, or dais, in front of it, was appropriated entirely to the clergy ; beneath was the crypt or confessional, where were placed the bodies of the saints and martyrs, and pulpits were placed in the nave, from which the services were said or sung by the inferior order of clergy.

The plan (Fig. 1206) is that of the original Latin cross, the eastern limb or chancel being the shortest, and the nave the longest. Sometimes the eastern limb was made equal to that of the transept, sometimes even longer, but never to exceed that of the nave. In the Greek cross all the limbs are equal. In most of the French Gothic churches the eastern end is made semicircular, often inclosed by three or more apsidal chapels, that is, semi-cylinders, surmounted by semi-domes.

The Byzantine church consisted internally of a large square or rectangular chamber, surmounted in the center by a dome, which rested upon massive piers; an apse was formed at the eastern end. Circular churches were built in the earlier ages for baptisteries, and for the tombs of saints and emperors.

The Greek, Roman, and English churches conform in their cathedrals and larger edifices nearly to the Romanesque or Gothic models. But as the general requirements for church services now are those of a lecture-room—comfortable seats, convenient for hearing and seeing the preacher, with adequate means of heating and ventilation, for which the older forms are not suited—modern churches are constructed adapted to these purposes, and, in cities, to the size and form of the lots, with some ecclesiastical accessories of towers and steeples, windows and doors and interior finish.

Fig. 1207.

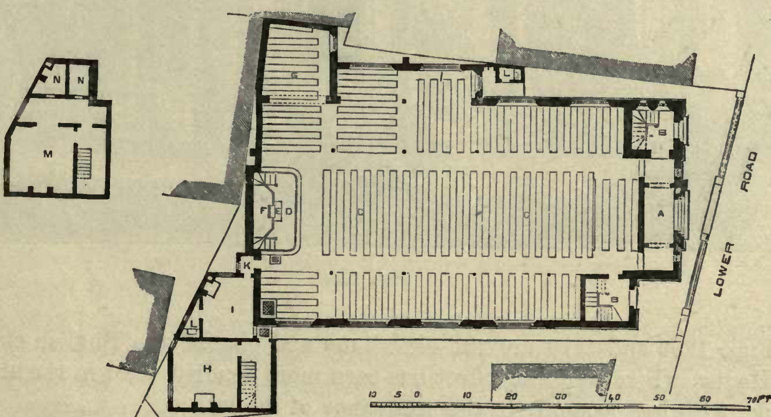


Fig. 1208.

Figs. 1207 and 1208 are the elevation and plan of a London Wesleyan chapel characteristic of the above.

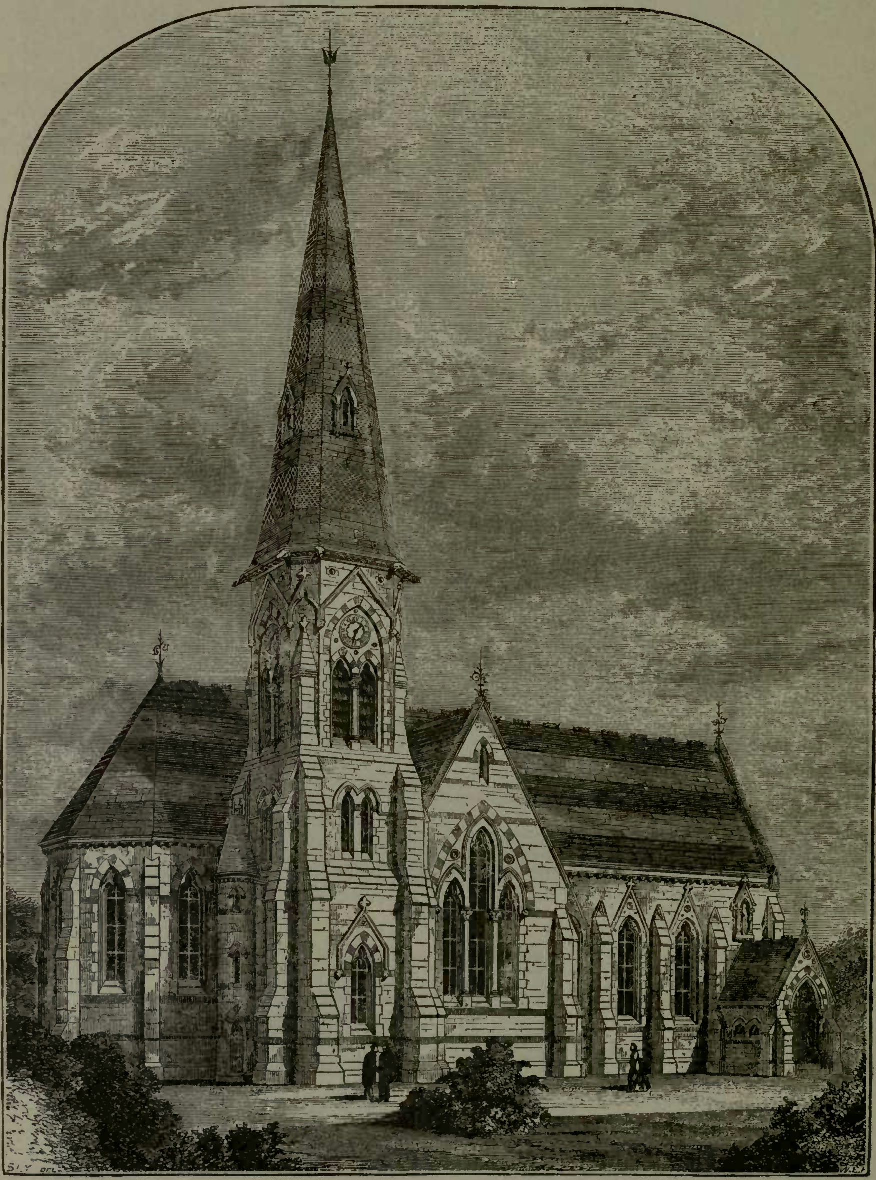


Fig. 1209.

Figs. 1209 and 1210 are the elevations and plan of the English church at the Hague, where æsthetic effect has been more studied than in the above example, with less economy in the occupancy of the lot.

The length of pews is various, being generally of two sizes, adapted to either small or large families, say from 7′ 6″ to 11′ 6″, 18″ being allowed for each sitter. In arrangement it is always considered desirable that there should be a

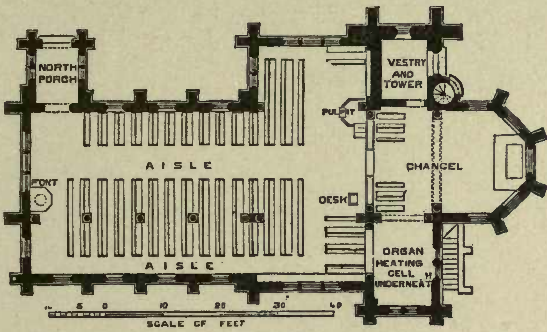


Fig. 1210.

central aisle, and if but four rows of pews, two aisles against the wall; if six rows, one row on each side will be wall-pews. Formerly it was the universal practice to construct pews with doors, but of late it is more customary to omit the doors, making the pews open stalls.

Few churches are now without an organ ; its dimensions should of course depend on the size of the church. In form it may be adapted somewhat to the place which may be appropriated to it—either in a gallery over the main entrance, or at the side of the chancel, as in Fig. 1210. In general, it is oblong in form, the longer side being with the keys. The dimensions suited to a medium-sized church are about 9′ × 15′, and 12′ in height.

The vestry-room, if used for the purposes of its meetings, should be adapted in size to the purpose ; but if only for a withdrawing or robing room for the clergyman, it may be of very small dimensions, and should be accessible from without. The Sunday-school room, in general, requires in plan about half the area of the church. From motives of economy it is usually placed in the basement of the church ; but, in the country especially, it is better that it should be a separate building, and form one of the group of church, parsonage, and Sunday-school house.

In elevation, city churches are Greek with porticoes in front, Romanesque, and Gothic, occasionally Byzantine. The Greek have no tower, but often a spire above the portico ; the Romanesque and Gothic generally one tower, over the central door of entrance, or at one corner ; sometimes two, one at each side of the principal door, almost invariably surmounted by spires, high and tapering, usually of wood, but in some instances of stone.

Fig. 1211 is the front elevation of the Roman Catholic cathedral in Fifth avenue, New York city, from designs by James Renwick, architect. The style is the French Decorated Gothic.

Fig. 1212 is a perspective view of the Episcopal church of St. Bartholomew, corner of Forty-fourth Street and Madison Avenue, New York ; Renwick and Sands, architects. The style is Romanesque ; the vestry and parsonage are connected with the church.

Fig. 1211.

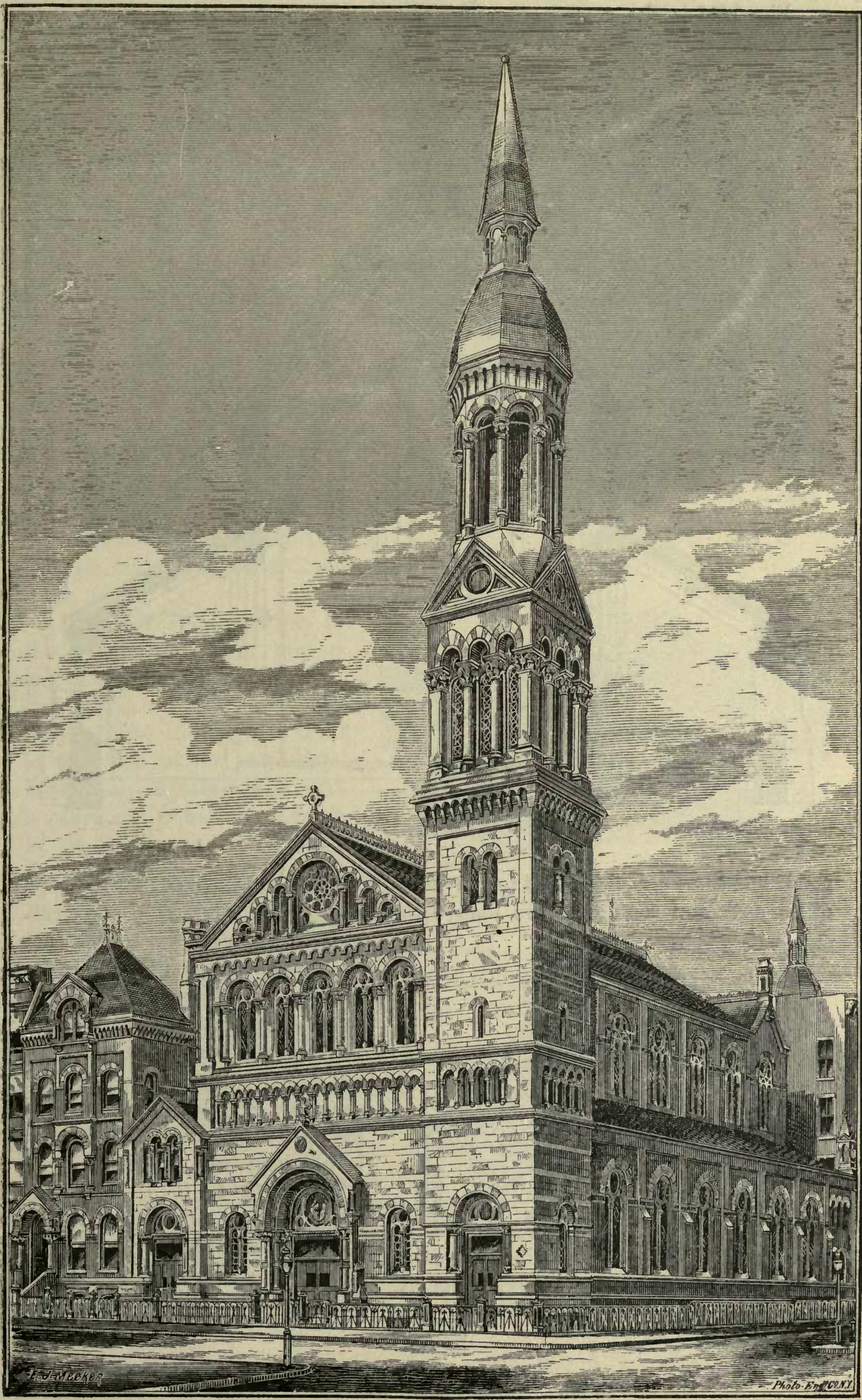


FIG. 1212.

Fig. 1213 is the cross-section of a common form of small country church, with nave *n*, aisles *a a*, and clear-story *c*. The effect, both inside and out, is

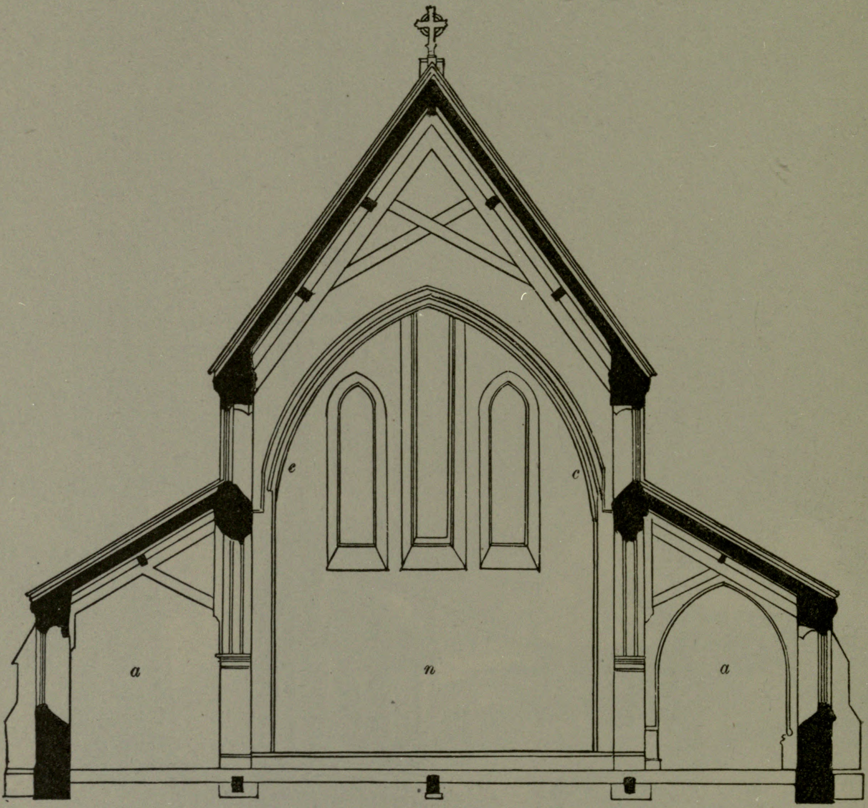


Fig. 1213.

good, but there are objections to the masonry-columns, which cut off the view of the desk and the altar from many sitters, and to the windows of the clear-story, that in the winter they act as coolers to the air which descends in draughts upon the heads of the congregation beneath them. Neither columns nor clear-story are constructively necessary; the span can readily be met by a single roof, and sufficient light can be obtained from the sides.

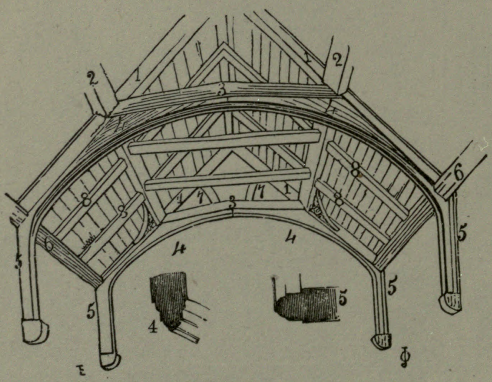


Fig. 1214.

Figs. 1214, 1215, and 1216 are examples of open-timbered Gothic roofs of churches.

The technical names (Fig. 1214) are: 1, Principals; 2, Purlines; 3, Collars; 4, Braces; 5, Wall-pieces; 6, Wall-plates; 7, Struts; 8, Rafters. 4 and 5 are shown in section.

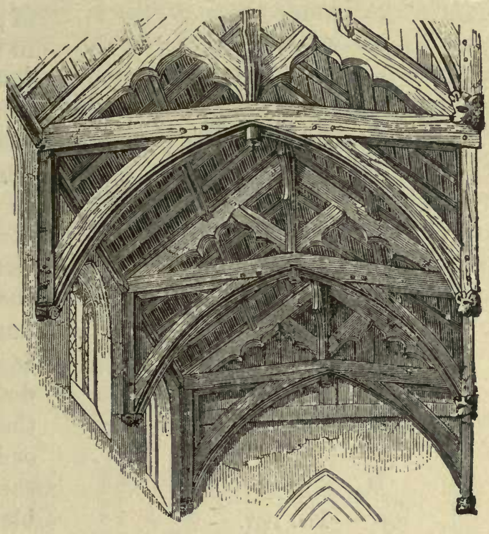


Fig. 1215.

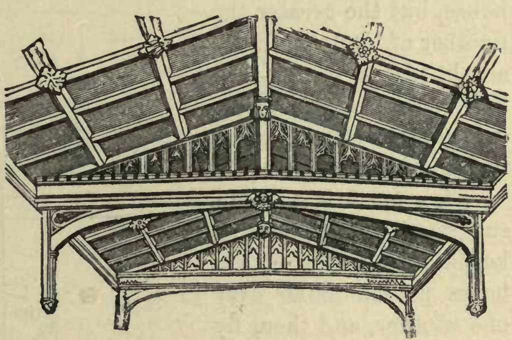


Fig. 1216.

Theatres.—In theatres and opera-houses it is not only necessary that the audience should have a good position for hearing and seeing the performance upon the stage, but also to see each other. The most approved form, now, for the body of a dramatic theatre is a circular plan, the opening for the stage occupying from one fourth to one fifth of the circumference, the sides of the proscenium being short tangents; but for a lyric theatre, where music only is performed, and where, consequently, hearing is easier, the curve is elongated into an ellipse, with its major axis toward the stage.

In the general position of the stage, proscenium, orchestra, orchestra-seats, parquette, and boxes, but one plan is followed. The line of the front of the stage, at the foot-lights, is generally slightly curved, with a sweep, say, equal to the depth of the stage, and the orchestra and parquette seats are arranged in circles concentric with it: of the space occupied by seats we have already spoken. The entrance to the parquette may be through the boxes, near the proscenium, and centrally, but better at the sides, dividing the boxes into three equal benches; the seats in the boxes are usually concentric with the walls, and more roomy than those of the parquette. The orchestra seats are of a height to bring the shoulders of the sitter level with the floor of the stage, and the floor of the parquette rises to the outside, 1 in 15 to 18. The floor of the first row of boxes is some 2 to 3 feet above the floor of the parquette at the front center, and rises, by steps at each row, some 4 inches; in the next tier of boxes the steps are considerably more in height, and so on in the boxes above. In general, three rows of boxes are all that is necessary; in front, above the second, the view of the stage is almost a bird's-eye view. The floor of the stage descends to the foot-lights at the rate of about 1 in 50. In large theatres it is of the utmost importance that all the lobbies or entries should be spacious, and the means of exit numerous and ample—the staircases broad, in short flights and square landings, and not circular, as, in case of fright, the pressure of persons behind may precipitate those in front the whole length of the flight. Ladies' drawing-rooms should be placed convenient to the lobbies, of a size adapted to that of the theatre, arranged with water-closets ; there should also be provided rooms for the reception of gentlemen's canes and umbrellas, with water-closets attached. The box-office should be, of course, near the entrance, but so arranged as to interfere as little as possible with the approach to the doors of the house. At the entrance there should be a very spacious lobby, or hall, so that the audience may wait sheltered from the weather ; if possible, there should be a long portico over the sidewalk, to cover the approach to the carriages. Only single entrances are necessary to distinct parts of the house, but the greater the number of, and the more ample places for exit at the conclusion of the piece, or for the contingency of fire, the better.

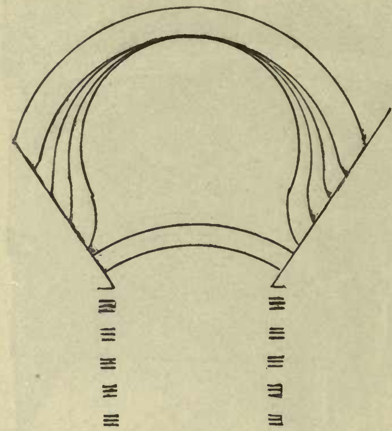


Fig. 1217.

Fig. 1217 is a plan suggested by Ferguson of keeping the center of the boxes perpendicular over one another, and then, by throwing back each tier of side-boxes till the last is a semicircle, the whole audience would sit more directly facing the stage, would look at it at a better angle, and the volume of sound be considerably increased by its freer expansion immediately on leaving the stage.

Fig. 1218 and 1219 are a plan and section of Wagner's theatre.

PLAN.

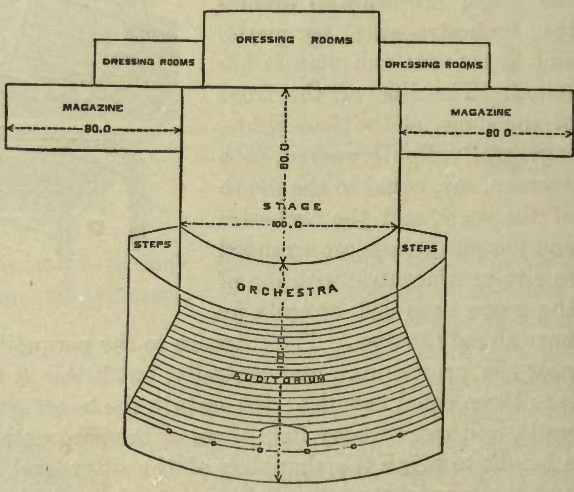


Fig. 1218.

SECTION.

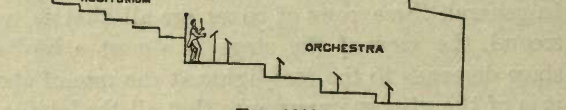


Fig. 1219.

In cities, the auditoria

of dramatic theatres conforming to the shape of the lots are rectangular in their outline, and seldom exceed a seating capacity of 1,000. Lyric theatres are much larger, seating often as many as 2,000, and conforming in their interior outline to the art requirements. Lecture-rooms are usually arranged with the audience-floor flat, room rectangular, with reading-desk or platform raised, and with or without galleries. The same form usually obtains for music-halls, only they are much greater in extent; the first being capable of containing from 500 to 800 persons; whereas some music-halls will contain 2,000, and Ferguson thinks that a music-hall might be arranged so that even 10,000 might hear as well as in those of present construction. The lecture and music halls are seldom devoted to a single purpose, but are used for political meetings, for fairs, and dances, and the construction must be such as to serve these other purposes.

COMPARATIVE TABLE OF THE DIMENSIONS OF A FEW THEATRES.

NAME AND LOCATION.	DISTANCE, IN FEET.						HEIGHT, IN FEET.	
	Between boxes and footlights.	Between footlights and curtain.	Between curtain and back of stage.	Greatest breadth of pit.	Breadth of curtain.	Breadth of stage between side-walls.	From floor of pit to cornice.	From floor of pit to center of ceiling.
Alexandre, St. Petersburg	65	11	84	58	56	75	53	58
———, Berlin	62	16	76	51	41	92	43	47
La Scala, Milan	77	18	78	71	49	86	60	64
San Carlo, Naples	77	18	74	74	52	66	81	83
Grand Théâtre, Bordeaux	46	10	69	47	37	80	50	57
Salle Lepelletier, Paris	67	9	82	66	43	78	52	66
Covent Garden, London	66*		55	51	32	86	54	
Drury Lane, London	64*		80	56	32	48	60	
Boston, Boston	53	18	68		46	87	55½	58
Academy of Music, New York	74	13	71	62	48	83	74	
Grand Opera-House, New York	54	8½	63½	48	44	76	52	67
Opera-House, Philadelphia	61	17	72	66	48	90	64½	74

* These dimensions include the distance between the footlights and curtain.

Legislative Halls.—Although much has been written about their construction in relation to acoustic principles, there yet seems to be great disagreement in practical examples, and in the deductions of scientific men. The Chamber of French Deputies was constructed after a report of most celebrated architects, in a semicircular form, surmounted by a flat dome, but as the member invariably addresses the house from the tribune, at the center, in its requirements it is but a lecture-room. Mr. Mills, architect, of Philadelphia, recommends for legislative or forensic debate, a room circular in its plan, with a very slightly concave ceiling. Dr. Reid, on the contrary, in reference to the Houses of Parliament, gave preference to the square form, with a low, arched ceiling. The Hall of Representatives, at Washington, is 139 feet long by 93 feet wide, and about 36 feet high, with a spacious retiring gallery on three sides, and a reporters' gallery behind the Speaker's chair. The members' desks are arranged in a semicircular form. The ceiling is flat, with deep-sunk panels, openings for ventilation, and glazed apertures for the admission of light. The ventilation is intended, in a measure, to assist the phonetic capacity of the hall, the air being forced in at the ceiling and drawn out at the bottom.

In reviewing the general principles of acoustics, it will be found that those rooms are the best for hearing in which the sound arrives directly to the ear, without reflection; that the sides of the room should neither be reflectors nor sounding-boards, and that surfaces absorbing sound are less injurious than those that reflect. Slight projections, such as ornaments of the cornices and shallow pilasters, tend to destroy sound, but deep alcoves and recessed rooms produce echoes. Let the ceiling be as low as possible, and slightly arched or domed; all large external openings should be closed; as M. Meynedier expresses it, in his description of an opera-house, "Let the hall devour the sound; as it is born there, let it die there."

Hospitals.—In large cities, hospitals, by necessity, are confined to narrow spaces, but they should be placed, if possible, on river fronts or on open parks, to secure as much open-air ventilation as possible. They are usually many stories in height, with large wards one above the other. Sir J. T. Simpson alleges a very high rate of mortality in hospitals after surgical operations as compared with the mortality after the same operations when performed at the homes of the patients, and asserts that the mortality after operations performed in hospitals containing more than 300 beds is in excess of that in hospitals containing less; that great hospitals are great evils in exact proportion to their magnitude, and suggests the construction of smaller hospitals.

Figs. 1220 and 1221 are an elevation and plan of an English country hospital.

Stables.—Under this general name are included the barn, or the receptacle of hay and fodder, the carriage-house, and the stable proper, or lodging-house for horses and cows. The first two may be included under one roof, the carriages on the first floor, and hay in the loft; but the lodging-place should be distinct, in a wing attached to the barn, that the odors from the animals may not impregnate their food, or the cloth-work of the carriages, or the ammonia tarnish their mountings.

Hay in bulk, in the mow, occupies about 340 cubic feet per ton; bales average 2′ 4″ × 2′ 6″ × 4′, and weigh from 220 to 320 pounds. The door-space for a load of hay in the bulk should be from 12 to 13 feet high and 12 feet wide. The floor beneath the hay should be tight, so that dust and seed may not drop on the carriage. A door for carriages should be 10 feet 6 inches high by 9 feet wide.

The horse is to be treated with greater care than any other domestic animal. His stable is to be carefully ventilated, that he may have fresh air without being subject to cross-draughts. Preferably, the floor should be on the ground, that there may be no cold from beneath. He should stand as near as possible level; and for this purpose a grated removable floor, with small interstices, should be laid over a concrete bottom, with a drip toward the rear of the stall, and the urine should be collected in a drain, and discharged into a trapped manure-tank outside the stable. In Fig. 1222 the pitch of bottom of stalls is

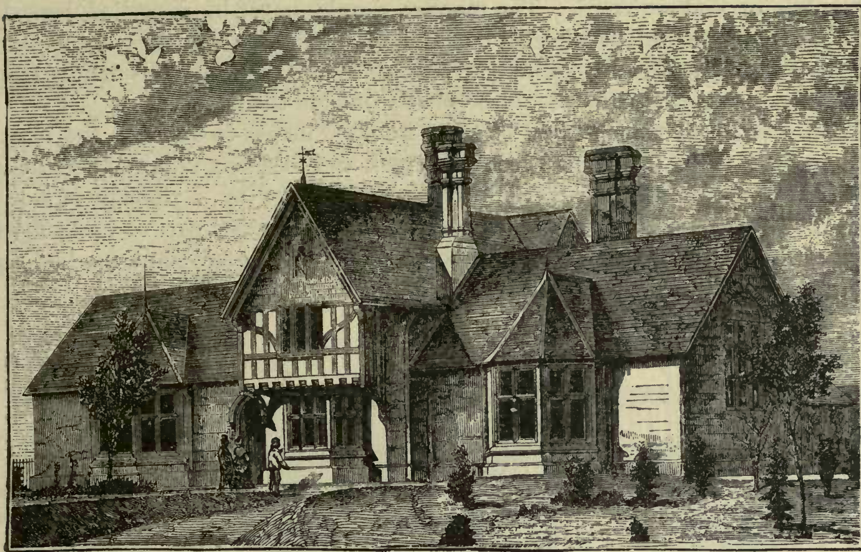


Fig. 1220.

GROUND PLAN.

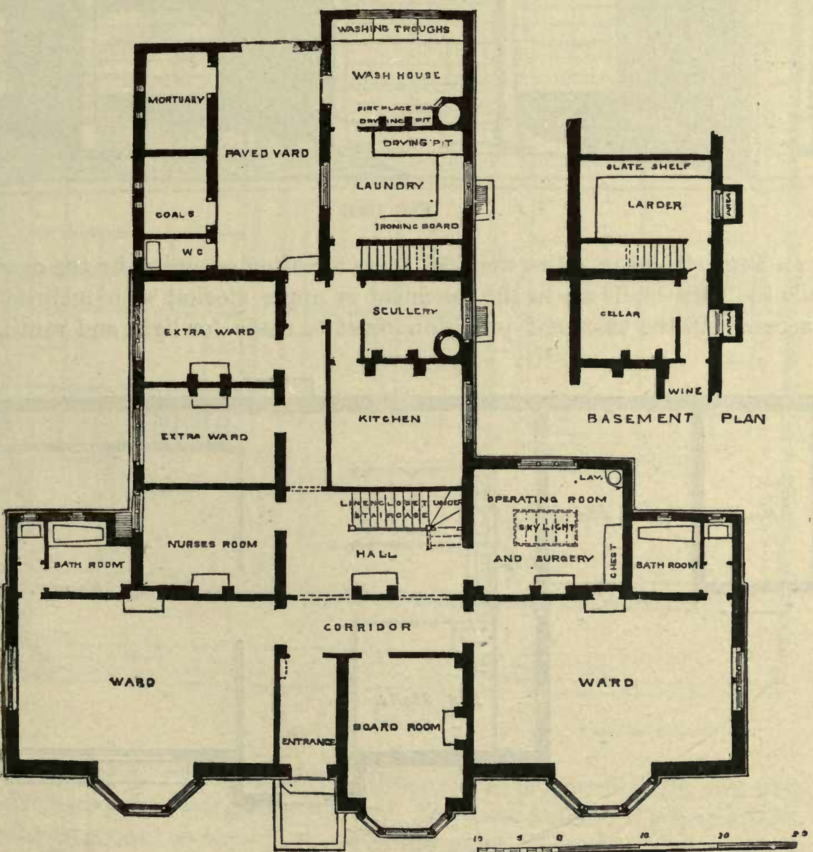


Fig. 1221.

to the center and outward. The manure should never be deposited beneath the stable, but should be wheeled out and deposited in a manure-yard or tank daily. It is as essential that all excrements should be removed entirely from the stable as that the privy should be placed outside the house.

The breadth of stalls should be from 4 feet 6 inches to 5 feet in the clear; the length, 7 feet 6 inches to 8 feet; the rack and feed-box require two feet in addition, to which access is given in the best stables by a passage in front. Rack and feed-boxes are often made of iron, and the upper part of stalls fitted with wrought-iron guards. Box-stalls, in which horses are shut up but not tied in cases of sickness or foaling, are about 10 feet square.

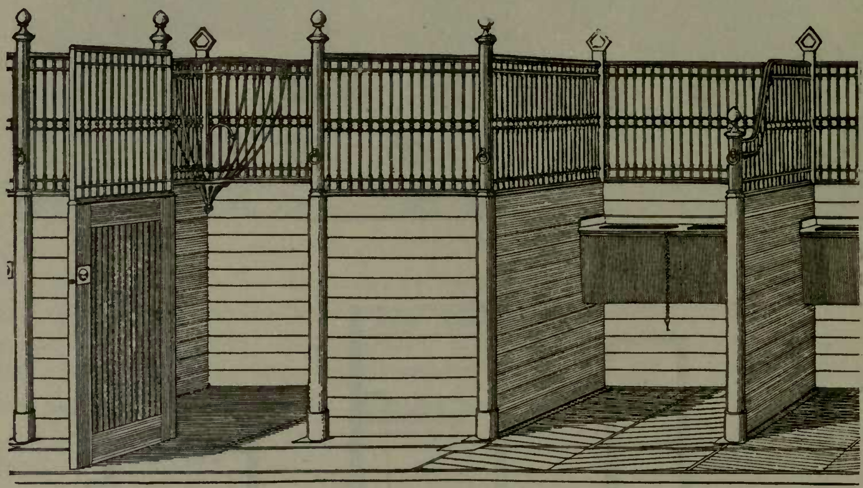


Fig. 1222.

In large stables in cities the first floors are often occupied by the carriages, while the horse-stalls are in the basement or upper stories, with inclined ways of access. In the basement provision must be made for light and ventilation.

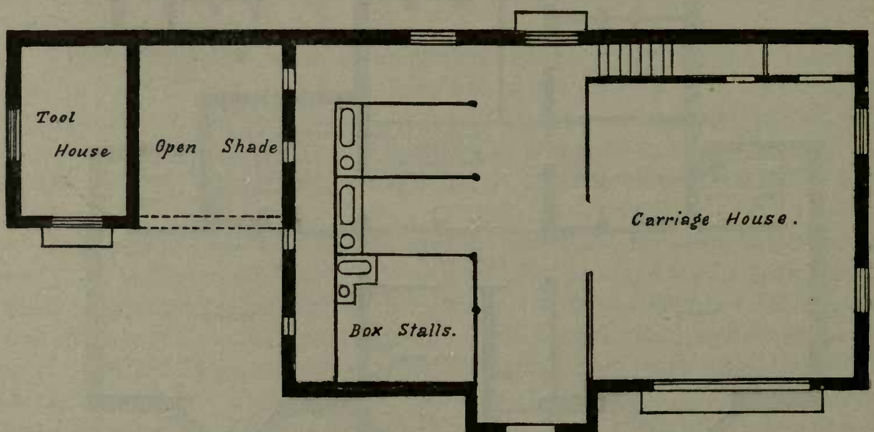


Fig. 1223.

In the upper stories these may be secured more readily, but the floors must be made tight and deafened, that the urine may not leak through, nor the cold come through from below to make too cool a bed for the horse.

Fig. 1222 is an elevation in perspective of two first-class stalls, a box shown with the door open, and a single stall. The lower part of the inclosures is of plank, with wrought-iron guards and ramp above. The posts are of oak, and the hay-boxes or mangers of cast-iron ; the hay-rack in the box-stall is of wrought-iron. These are of common manufacture, and are of varied patterns ; but in the country they are usually made of wood, and connected with the stall.

Fig. 1223 is the plan of a small country stable, showing the desirable passages around the stalls and exterior windows in front of each stall, that the horses may not only have light and air, but can see out.

Cow-houses, for cows giving milk, should be constructed with care

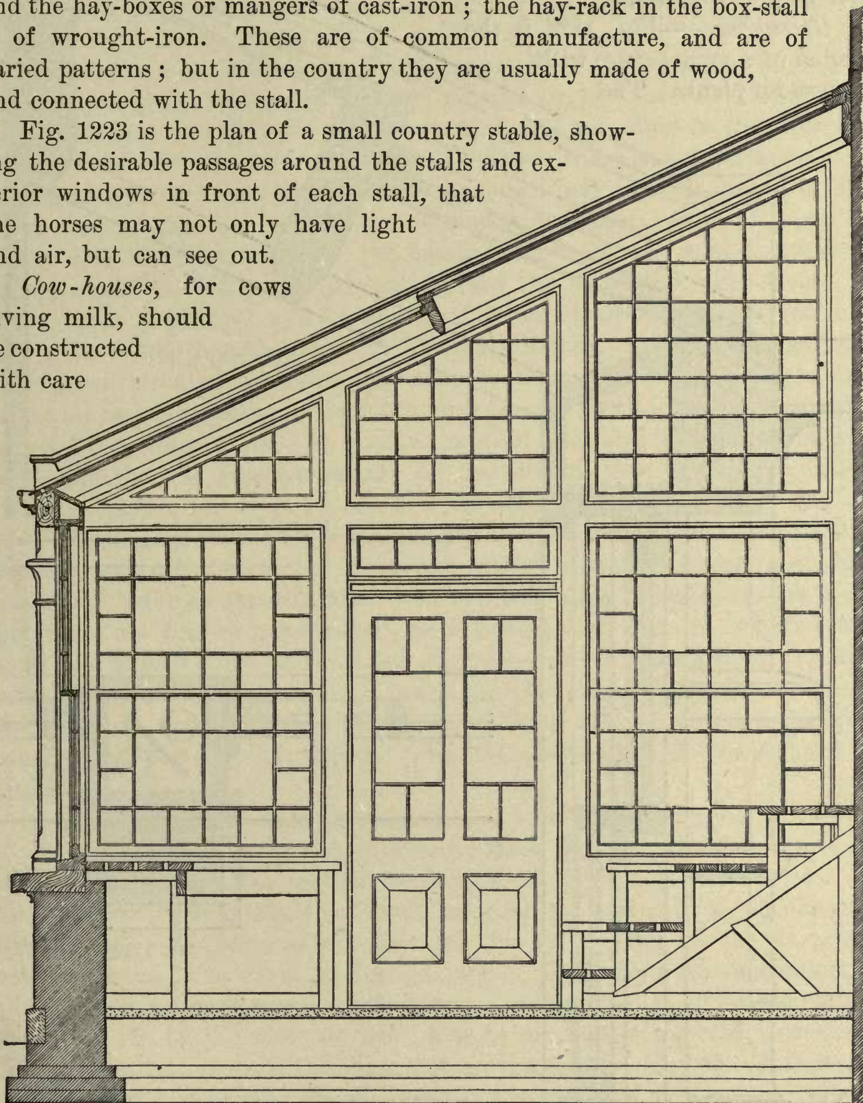


Fig. 1224.

for ventilation, light, and cleanliness. Other cattle are usually left out, with sheds under which they can go for shelter. For those housed, the spaces occupied should be about the same per head as the single horse-stall. The manger should be on the floor, 12″ to 18″ high, and about 18″ wide. It is not usual to have partitions, but there ought to be between every pair, reaching from the manger half-way to the gutter behind. The floor should be level, grated, with a drip beneath, and cleansed by washing out. In England the partition and mangers are often of cast-iron, and are on sale, but here they are of wood.

Greenhouses.—Fig. 1224 is the section of a greenhouse, with shelves for plants. The

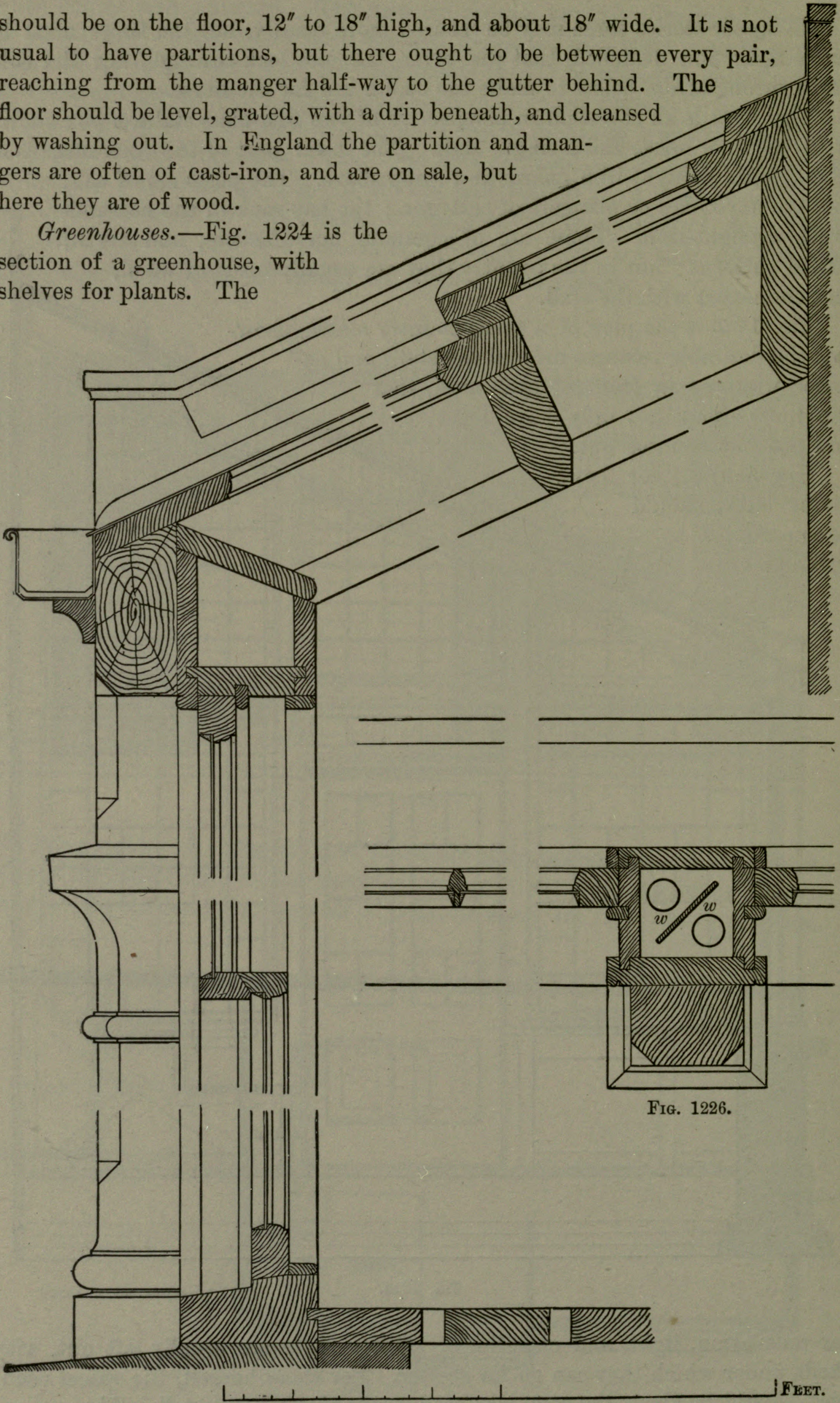


Fig. 1226.

Fig. 1225.

floor is of concrete and the walls are of masonry ; the northern exposure is a blank wall.

Fig. 1225 are the details of windows. The sides are box-sash, hung with weights (*w*, *w*, Fig. 1226). The lower roof sash is firmly fixed, but the upper one can be slid down ; it is usually retained in place by a cord attached to the lower part of the sash, passing over a pulley on the upper bar of the frame, with the loose end within reach of the gardener, who can fasten it to a cleat.

Ventilation and Warming.—The purposes of ventilation are not changes of air merely, but the removal of foul and vitiated air, and the substitution therefor of pure air ; and this air may be warm or cool according to the necessities of the season and personal requirements. Open space is not necessarily well ventilated ; there must be circulation, outward and inward, the latter from purer sources than the former, or the change is useless. With an equal discharge and supply of pure air, the smaller the room, the more frequent the change of air, the better its distribution, and the better the ventilation. But if the means of removal, supply, and distribution of air be proportioned to the size of the room, then the larger the room the better. Apertures do not necessarily mean circulation ; a flue may draw or it may not draw, it may be inert, or the air may come down ; a window may be open, with little or no inward or outward movement of air. In a house exposed to a fresh breeze, on the windward side there is an air-pressure ; on the leeward side there is an eddy or vacuum. Air is forced in on the first through every crack of door and window—often down chimney-flues—and drawn out on the other side. This often happens even with fires in the chimneys, and with heat in ventilating educts. If one will make an experiment in cold weather, when the windows are closed, and there are fires in some rooms, he will find that there is cold air coming down the unused flues, and will feel the cold current flowing down the stairs, and along the floors to the fires. Architects have placed kitchens in the basement, and in the attic, and the smell of cooking will rise through the house, usually from the one, but descend from the other when the air is light and muggy.

Every room should have its separate flue ; for if the current is not upward it will probably be downward, affording a fresh supply if there is an exit elsewhere. A chimney-flue may be too large for the purposes of a fire ; for most fires a flue $8'' \times 8''$ is amply sufficient, and, for the purposes of ventilation in the common occupation of a house, this flue will answer all the purposes in cold weather. It is usual to depend largely on windows for ventilation, but the space on which they open may be too circumscribed to afford the requisite change of air, or the outer air itself may be too hot, or too cold, or too malarial or offensive, to make the change of air sanitary or pleasant. In tenement or apartment houses care should especially be taken that the inner windows on different flats open into as large air-shafts as possible, and that these shafts should have free opening to the outer air without sky-lights ; and that the floors should be tight, so that the smells may not pass from one flat to another. Nothing more surely shows faults in ventilation than the diffusion of kitchen-smells or tobacco-smoke. For the separation of apartments, let every room have its own flue, and this flue extending independently well above the roof, and not into an attic with a ventilating louver. In this case the air may ascend one flue and descend another, and not out of the louver.

The quantity of air taken into and expired from the lungs by a single individual is quite small, probably about 13 cubic feet on an average per hour. The usual gas-burner delivers from 4 to 6 cubic feet per hour, under a pressure of 1″ and 2″ of water. It will be seen, therefore, how small apertures are necessary to supply the lungs of a person, if it could be provided directly to him and taken away without vitiating other air. But, in addition, air is vitiated by personal emanations, and consumed by lights. These last can readily be made, not only to remove all their products of combustion, but also increase the circulation in flues for the ventilation of the room.

All systems of ventilation are based on the idea that so many individuals within a room and so many lights burning vitiate so much air, and that consequently a very large quantity of outer air must be introduced to reduce the percentage of vitiation, and generally with very little consideration as to the distribution of this air, although it is in every one's experience that the air in some portions may be fresh, in others stifling ; that in hospital wards there are often dead ends where the air does not circulate, and where patients do not as a rule recover. The system is to provide somewhere in a room air enough, and trust to chance for its distribution.

Some architects make the educts at the ceiling, some at the floor, some at both, with registers to control the openings. For sleeping-apartments, if there is a fireplace, this is all that will be necessary ; if the air goes up or comes down, it does not make draughts about the heads of the occupants.

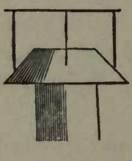


Fig. 1227.

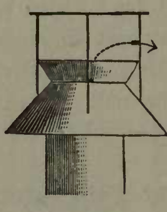


Fig. 1228.

To make flues draw, various forms of chimney-tops or cowls are adopted. The best and simplest are the Emerson (Fig. 1227), and a modification of the same (Fig. 1228) ; there are also various forms of self-acting flaps, turn-cowls, etc., the principle being to take advantage of the wind to make a draught. With the wind blowing across the top of a chimney, a bit of square-ended iron pipe extending above the chimney will answer as an expirator, but without a wind the draught must depend on circumstances within the dwelling and artificial draught. When sufficient circulation can not be obtained from natural differences of temperature in the atmosphere, or from winds, it is usual to have recourse to fans, to force air into or draw it from a building, or by heat applied to the air in flues, ducts, or chambers in the hot-air furnaces. Both the air and the heat are necessary. When heat is applied for ventilation only, as in mines, a fire is built in a flue near the top, and the air necessary for combustion is drawn from the mines ; the flue extends from the bottom of the mine, with a chimney above the surface of the ground, and ducts are led from the bottom of the flue to the face of the workings, the cold air for ventilation being drawn down through the working-shafts and drifts. In buildings, steam-pipes and gas-burners are put in flues.

Methods of Heating.—The open fireplace grate heats by radiation, communicating heat to objects, which by contact transfer it to the air. Persons coming in contact with rays are themselves heated, while the air around them is cool and invigorating for breathing; the bright glow has a cheering and animating effect upon the system, somewhat like that of sunlight. As a ventilator, an open fire is one of the most important, drawing in air not only for the support of combustion, but also, by the heat of the fire and flue, making a very considerable current through the throat of the chimney above the fire. From this cause, although there is a constant change of air, yet there arises one great inconvenience of disagreeable draughts, especially along the floor, if the air-supply be drawn directly from the outer cold air; but in connection with properly regulated furnaces or stoves, the open fireplace becomes the most perfect means of heating and ventilation. As a heater merely, the open grate, in very cold weather, is not satisfactory; its influence is only felt in its immediate vicinity, and but from 10 to 15 per cent of the heat of the fuel is rendered available.

Fig. 1229 represents an old form of open fire used in a tavern bar-room and office, which answered admirably for heating and ventilation, and admitted of access to many persons. It consisted of a circular grate at the level of the floor in the center of the room. In the cellar beneath was an ash-pit, *a*, in brick-work, with an opening, *o*, to supply air for the combustion of the fuel. Above the grate was a counter-weighted sheet-iron hood, *h*, connected by a pipe with the chimney, which could be raised or lowered, to suit the required draught. Around the grate was a ring-guard to rest the feet on, and the customers ranged themselves in a circle round the fire.

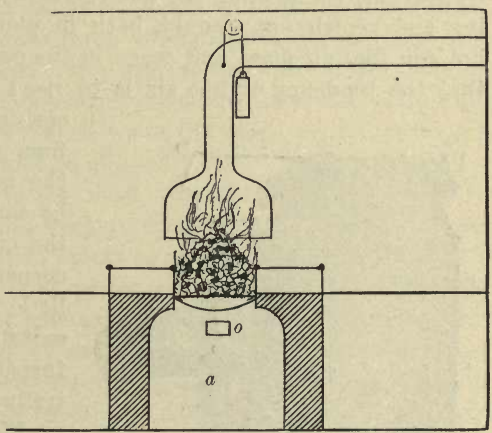


Fig. 1229.

Stoves.—Open stoves heat by direct radiation, and by heating the air in contact with them, and close stoves by the latter way only; as economical means of heating, the latter are the best, and, when properly arranged, give both a comfortable and wholesome atmosphere. There should be some dish of water upon them to supply a constant evaporation, sufficient to compensate for increased capacity of the air for moisture due to its increased heat. In the hall there will be no objection to a close stove, letting it draw its supply of air as it best can; but in close rooms the open stove is best, on the plan of the old Franklin stove, or, if a close stove, somewhat on the plan of a furnace, with an outer air-supply for combustion and ventilation.

Hot-air furnaces are close cast-iron stoves, inclosed in air-chambers of brick or metal, into which external air is introduced, heated, and distributed by metal pipes to the different rooms of a house. Furnaces have been, of late, very much decried, but under proper regulation they are very cheap, economical, and even healthful means of ventilation and warming. The heating-surface should be very large, the pot thick, or even incased with fire-brick, that it may not become too hot ; there should be a plentiful supply of water in the chamber for evaporation, perhaps also beneath the opening of each register ; the air-supply should always be drawn from the outer air and unobjectionable sources, through ample and tight ducts, without any chance of draught from the cellar ; the pot, and all joints in the radiator, should be perfectly gas-tight, so that nothing may escape from the combustion into the air-chamber. With these provisions on a sufficient scale, and proper means for distribution of the heated air and escape of foul air, almost any edifice may be very well heated and ventilated. The air should be delivered through the floor or the base-board of the room, and at the opposite side from the flue for the escape of foul air, making as thorough a current as possible across the room, and putting the whole air in motion. In dwelling-houses the fireplace will serve the best means of exit ; in public rooms distinct flues will have to be made for this purpose, and they should be of ample dimensions and well distributed, with openings at the floor and ceiling with registers, and means should be provided for heating the flues. An architect, in laying out flues for heating and ventilation, should, both in plan and elevation, fix the position of hot and foul air flues, and trace in the current of air, always keeping in mind that the tendency of hot air is to rise ; he will then see that, if the exit-opening be directly above the entrance-flue, the hot air will pass out, warming the room but little ; if the exit-opening be across the room and near the ceiling, the current will be diagonal, with a cold corner beneath, where there will be very little circulation or warmth. To heat the exit-flue, a very simple way is to make the furnace-flue of iron, and let it pass up centrally through the exit-flue.

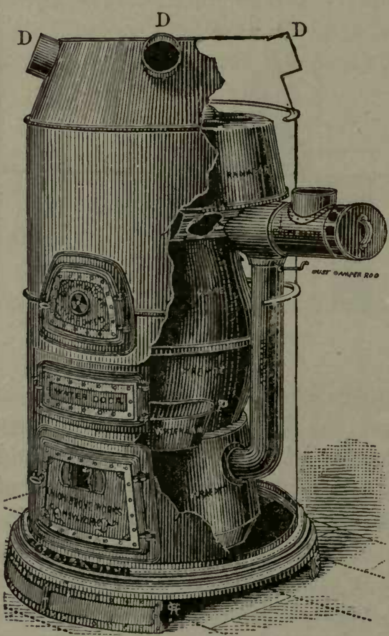


Fig. 1230.

Fig. 1230 may be taken as a type of a portable (so named on account of its small size and metallic case) hot-air furnace. The air is introduced at the bottom of the case, passes up and around the stove, and out through the ducts D, D, D to different parts of the building. The water-pan *p* is indispensable to the hot-air furnace, and should be of capacity enough for a day's supply, or have automatic means of keeping up the supply.

Air in winter is very dry, but as its volume is enlarged by heat, it draws a supply of moisture from everything with which it comes in contact—from the skin and lungs, creating that parched and feverish condition experienced in many furnace-heated houses ; from furniture and wood-work, snapping joints and making unseemly cracks.

Thus, taking the air at 10°, and heating it to 70°, the ordinary temperature of our rooms requires about nine times the moisture contained in the original external atmosphere, and, if heated to 100°, as most of our hot-air furnaces heat the air, it would require about 23 times.

The portable furnace is not so economical as the furnace set in brick-work, as more heat escapes through the metallic case. The former are usually made from 12″ to 24″ diameter of pot, from 2′ to 4′ outside diameter, and 5′ to 6′ height of case.

The brick-set furnaces are from 20″ to 28″ pot, outside brick-work from 5′ to 6′ square, walls 4″ thick, height 6′ to 7′. The size of air-ducts is proportioned to size of furnace. The inlet should be, say, equal to that of the grate, and the sum of the outlets but little in excess of this area. It is difficult to give any rule for the heating capacity. A 22″ pot should be adequate for the heating of a common 25′ × 60′ city house, and the higher the air-duct the less its diameter.

Steam and hot-water circulation are applied to the heating of buildings by means of wrought or cast iron pipes connected with boilers. In the simplest form, as common in workshops and factories, steam is made to give warmth without ventilation by direct radiation from wrought-iron pipes. The general arrangement is by rows of 1″ pipe hung against the walls of the room, or suspended from the ceilings, 3′ of 1″ pipe being considered adequate to heat 200 cubic feet of space ; if there are many windows in the room, or the building is very much exposed, more length should be allowed.

Steam, as a means of heating, is the most convenient and surest in its application to extensive buildings and works. From boilers, located at some central point, steam can be conveyed to points so remote that in many cities it is matter of sale, both for heating and power purposes. The limits of the extension of steam-pipes economically have not yet been determined, but within the range of the buildings occupied by any single textile manufacturing industry steam-heating has proved satisfactory, and is of almost universal adoption. For stores, warehouses, large buildings of all sorts, where there are extensive or numerous rooms to be heated, steam has been long used, and the appliances for its use can be as readily obtained in all our cities and large towns as stoves or grates. Steam is used for heating at either high or low pressures ; under 5 or 6 pounds would be considered low pressure. A low-pressure apparatus may draw direct from a boiler, or be supplied from the exhaust of a steam-engine ; if from the latter, a certain amount of back pressure must be put on the engine to establish a circulation in the steam-heating pipes.

In the operation of heating by steam, the steam, in giving off its latent heat through the pipes to the air of the room, returns to water ; the apparatus would then be nothing but pipes to convey the steam to radiators to condense it, and pipes to return the water to the boiler, were it not for air invariably in water and steam. This necessitates a more complicated circulation ; there should be a regular flow outward of steam from the boiler, and inward of water and steam to it, both as far as possible together, and in the same direction. When hot water is used for heating, there must be circulation throughout the system; the water flows out from the top of the boiler, gives out its heat, and returns, practically of the same bulk, cold to the bottom of the boiler, and any radiator out of the line of this current is of no use. A single valve shuts off the circulation in the hot-water apparatus, while two are necessary with a steam apparatus, for the steam cut off on the direct pipes may back up through the return-pipe.

Steam is used for heating rooms either *directly* or *indirectly*. Direct steam-heating is like that of common stoves, without any considerations for ventilation. Indirect steam-heating is like that of hot-air furnaces. Steam radiators are inclosed in a box or chamber, into which air is drawn or forced, and then distributed by ducts to the rooms to be warmed and ventilated. Thus, when ventilation is combined with steam or hot-water heating, the metallic surfaces brought in contact with the air usually range from 212° to 250°, while the pot

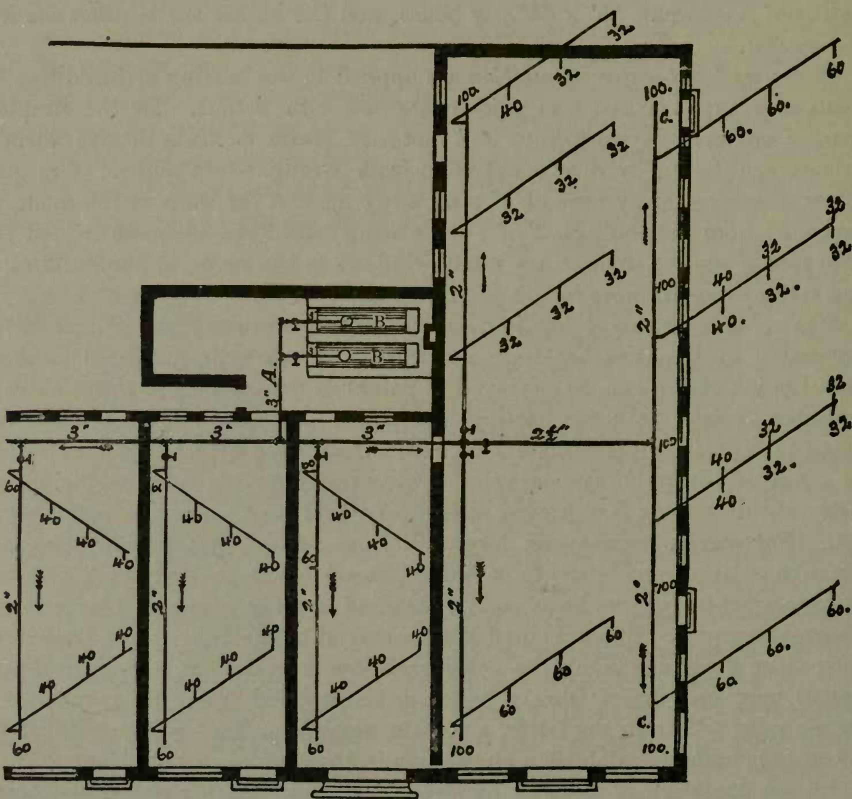


Fig. 1231.

of the air-furnace may be near a white heat. In a sanitary point of view, hot-water or low-steam coils in air-chambers are a more surely healthy means of warming and ventilation; the greatest objection is their expense, the care requisite in attending them, and the danger of freezing and bursting the pipes if worked intermittently in winter. In the arrangement it is usual, in dwelling-houses, to place the coils at different points in the cellar, as near as possible beneath the rooms to be heated. In public buildings frequently a very large space in the cellar is occupied by the coils, into which the air is forced by a fan, and then distributed by flues or ducts throughout the building.

All inlet or outlet ventilating flues should be provided with dampers or registers, to control the supply or discharge of air, cutting it off when sufficient heat is secured, or retaining the warmth when ventilation is not required.

Fig. 1231 (an illustration from "The Sanitary Engineer") is the plan of a portion of a large building heated by steam. B B are two boilers, either of which would be sufficient for the purpose ; the steam mains are shown by black lines following those of the building, with the sizes marked upon them ; the risers by inclined lines, with the square foot of radiating surface on each story, marked. This is a very convenient form of drawing, explanatory of the system. It is usual to draw the steam mains and risers in red, and the returns in black, with the diameters on each.

Fig. 1232 is the elevation of a small steam-heating apparatus, illustrating the general action. B is the boiler, and R and R′ radiators on different stories ; *s* is the steam-pipe, and *r r′* return or drip pipes. The steam is drawn from the top of the boiler, and the returns must be below the surface, W. L., of the water in the boiler. The circulation is simple and intelligible, and applicable to a hot-water apparatus ; as a steam apparatus, if it is required to shut off the lower radiator, R, both the inlet and outlet valves on the radiator must be shut. If only the top valve be shut, the steam in the radiator will be condensed, and the pressure from the boiler will fill it with water. If the lower valve only be shut, the radiator will still act as a condenser till it is filled with water. In the upper radiator, R′, there is no outlet-valve, as the radiator is supposed to be set at a level above the height to which the water would be raised by the pressure of the steam in the boiler. This arrangement of separate returns for each radiator is sometimes used, but the usual practice is to have single returns, into which there are branches from each radiator, controlled by valves. In low-steam apparatus,

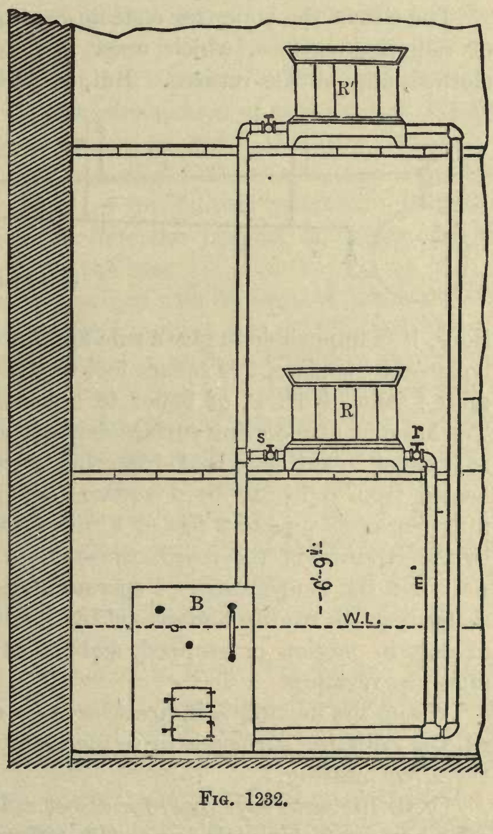


FIG. 1232.

the steam is introduced and the water removed by the same pipe, and controlled by a single valve.

Fig. 1233 is an elevation, showing the usual arrangement of mains, *s s*, and returns, *r r*, when the horizontal distance from the boiler is small and the risers few. The inclination of the mains is toward the boiler, and their condensed water returns by them to the boiler.

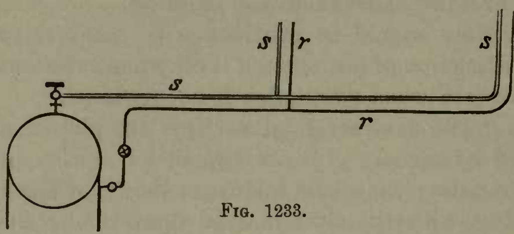


Fig. 1233.

Fig. 1234 is the better practice, and necessary if the steam is high pressure, the mains extended, and the branches numerous. The inclination of the mains, *s s*, is from the boiler, and the condensed water flows down to the lowest angle, where it is connected with the return, *r*, and is by this brought back to the boiler.

The size of the boiler for a steam-heating apparatus is based on the amount of radiating surface, which must include that of the steam-mains, if not clothed, and of the returns. But, as boilers vary so much in their propor-

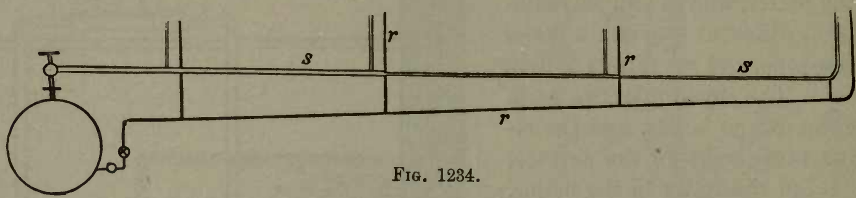


Fig. 1234.

tions, it is impossible to give a rule applicable to all of them. Some estimate by boiler-grate surface, 500 square feet of radiating surface to each square foot of grate; some, 1 H. P. of boiler to each 200 square feet of radiating surface. The amount of radiating surface depends on the cubic feet of air to be heated. It is usual to estimate that from 150 to 200 cubic feet of room-space can be heated from 0 to 70° by 1 square foot of radiating surface; or, say, 4 running feet of ¾″ pipe or 3 feet of 1″ pipe. But this is to be modified very much by the exposure of the room, the amount of glass surface, the thickness of wall, and the temperature of surroundings. The effect of glass as a cooling surface can be readily understood by the difference one experiences in the heat of cars in motion or stopped, and the advantages of double windows in the same conveyances.

Where the heating is indirect, as there are more cubic feet of air to be heated, the radiating surface is to be increased, usually to about three times that of the direct heating.

C. B. Richards says that, for direct radiators, 1 square foot of surface gives off 3 heat-units for each degree (1°) difference of temperature between the air of the room and that of the steam in the radiator.

As the boiler must be proportioned to the requirements of heating, as determined by the square feet of radiators, the sizes of mains and returns are also measured by the same standard. A common rule is 1″ diameter of main for each 100 square feet of radiating surface, varying with the squares of the diameters—2″, 400 square feet; 3″, 900; 4″, 1,600. In fact, the larger sizes will be sufficient for a much larger radiating surface than given by this rule.

For the returns, one size less than that of the steam mains is the rule; thus, a ¾″ return for a 1″ pipe, but no pipe of less diameter than ¾″ is used; for a 2½″ steam a 2″ return, and a larger than 2″ is seldom used. It may not be always practicable to return the condensed water, as shown in the figures above, by gravitation, but there are various forms of receivers or traps in which the water is collected and returned, automatically as in the Albany trap, or by pumping to the boiler.

Figs. 1235 to 1241 are common forms of radiators. Fig. 1235 is a bench coil, often called a mitre coil, from the vertical or horizontal angle made at the end of the pipes to admit of their unequal expansion. In the circulating coil (Fig. 1236), often called the trombone, the circulation is alternately forward and back; when placed in rows, as in Fig. 1237, it is a box coil; the ends of the pipes at both top and bottom are connected in heads. Fig. 1238 is a horizontal radiator, similar in its action. Figs. 1239 and 1240 are vertical radiators, the first composed of wrought-iron pipes inserted in a hollow cast-iron base, circulation being obtained by a sheet-iron division in the pipe as in the Nason radiator, by an inside pipe, or by the connection of two pipes at top by a return bend. Fig. 1240 is a Bundy radiator, in which there are twin cast-iron pipes connected at the top and bottom. Fig. 1241 are cast-iron pin radiators, so called from the projections, effective for indirect radiators. In measuring the surface of circulating coils, include the lengths of angles and all fittings; in the vertical radiators, include the base.

On the radiator (Fig. 1240) a small pipe (*p*) will be seen, which is an air-vent, often automatic, but indispensable for a prompt start of the circulation.

Plumbing.—The conveniences for comfort in modern buildings require the introduction of water and its removal. Most cities have water-supplies and a system of sewers, and the plumber makes the connections with both. In the country, for the better class of houses there are private expedients to supply their places, largely by wells and pumping, and connections to cesspools. The quantity used in each household varies with the wants and habits of the occupants. An average bath will take 25 gallons; each use of a water-closet from 2 to 3 gallons. A wash-tub will hold from 10 to 20 gallons. If the water is to be pumped by hand, from 7 to 10 gallons will be reckoned as the use by each person; if from aqueduct, 30 to 50 gallons is ample.

The regulation size of taps for city mains is from ½″ to ⅝″, and the pipes leading into the house from ¾″ to 1″ diameter. The pipes are usually of lead, as most waters are not affected sensibly by lead, if the pipes are always kept full, and there is fair circulation. In some cases block-tin pipes are used; or iron, galvanized, or coated with some preparation of asphalt, or glass-lined.

The soil or house-sewer pipe connections with the main sewer or cesspool are usually vitrified stone-ware pipe, from 4″ to 6″ diameter, as they are not only for the discharge of the sewage, but also for the rainfall from the roof. Within the house the pipe is either of stone-ware or cast-iron; invariably of the latter if the pipe is exposed. The rising pipe to the roof is here, also,

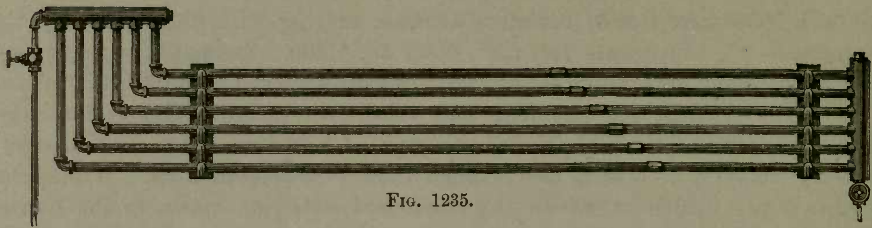


Fig. 1235.

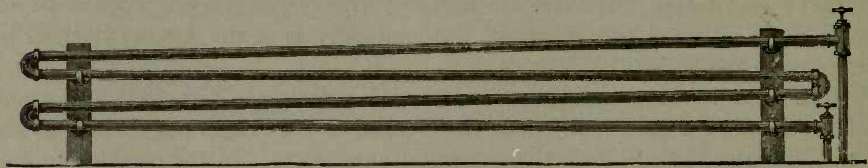


Fig. 1236.

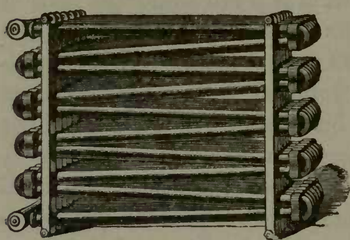


Fig. 1237.

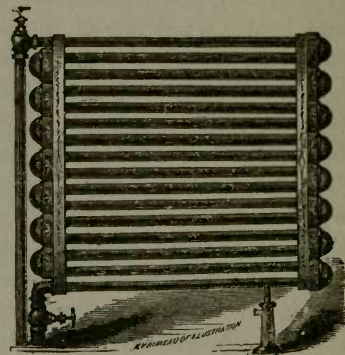


Fig. 1238.

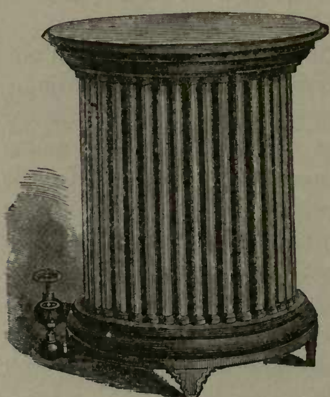


Fig. 1239.

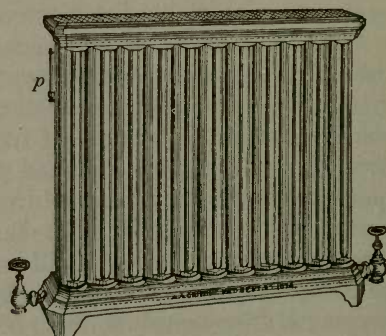


Fig. 1240.

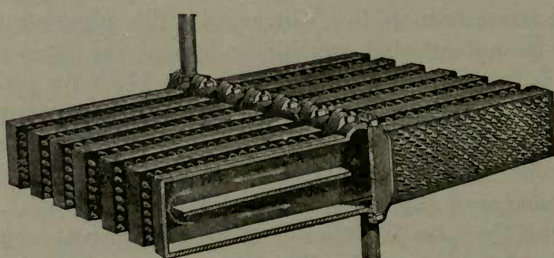


Fig. 1241.

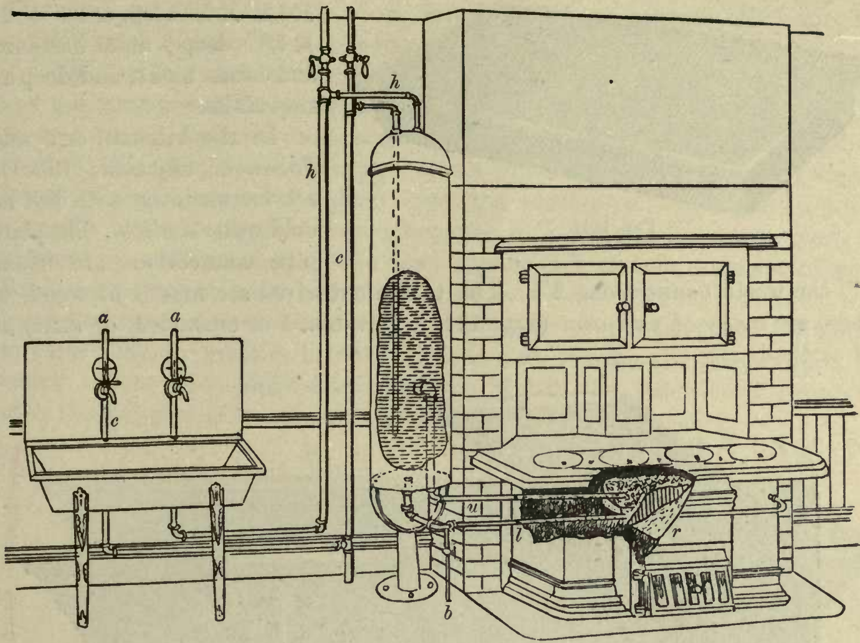
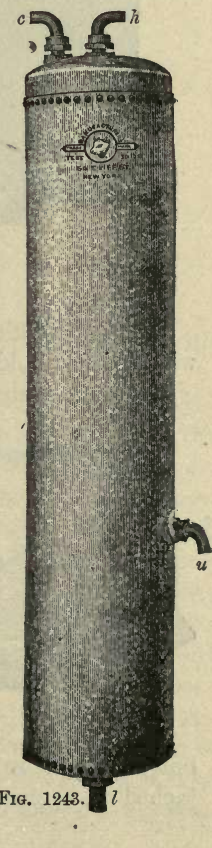


Fig. 1242.



usually of cast-iron, and 4″ diameter may be considered ample for a common house; the smaller branches may also be of iron, but when as small as 2″ are usually of lead.

Fig. 1242 is the perspective of a kitchen-range boiler and sink: *c* is the cold-water pipe leading to the sink and to the boiler; it enters the top of the boiler, and is led down nearly to the bottom. The hot water is drawn from the top, through the pipe *h*, is led down to the sink and up for distribution through the house. The water is heated in the boiler by the connection with the water-back of the range, *r*; the water flows through the pipe, *l*, is connected with the lower part of the water-back, and returns by the pipe, *u*, from the top of the water-back to a higher point in the boiler; *b* is the blow-off pipe.

It will be observed that at the draw-cocks over the sink there are pipes, *a a*, turned up; these are air-chambers, to cushion the blow of the water-hammer when the cocks are shut quickly. Beneath the sink there is a trapped connection with the sewer-pipe.

Fig. 1243 is the elevation of a galvanized-iron boiler, but those in general use here are of copper.

Fig. 1244 is the perspective drawing of a cast-iron sink, of the usual form and material. They are to be obtained of all suitable dimensions, rectangular, from 16″

Fig. 1243.

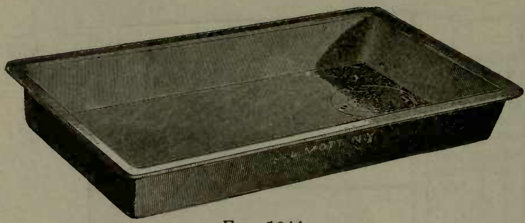


Fig. 1244.

× 12″ × 5″ deep, to 96″ × 24″ × 10″ deep; also, half-circle and corner sinks, and deep and slop sinks.

In the kitchen, or a laundry-room adjacent, tubs are set for washing, with hot and cold water service. The water-pipe connections are usually ¾″, the waste connections 2″. The tubs themselves are mostly of wood, but there are many of cast-iron (Fig. 1245), galvanized or enameled, of slate, and of earthenware.

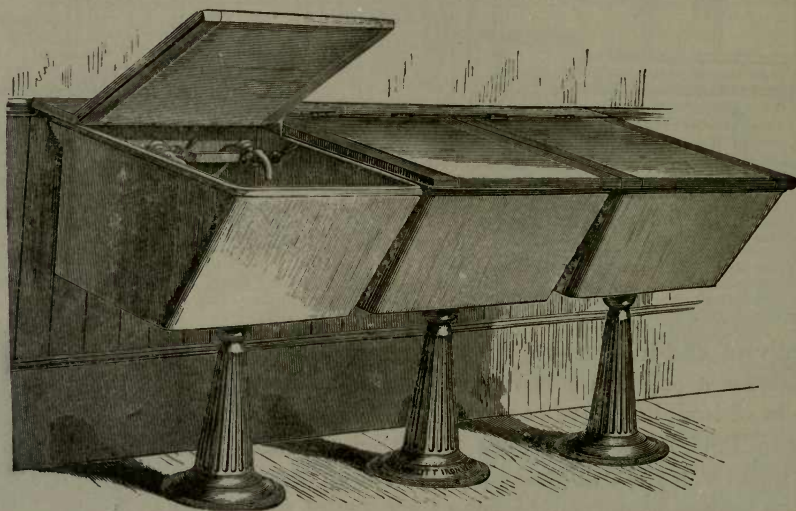


Fig. 1245.

In the butler's pantry there is usually a sink set of planished tinned-copper, with hot and cold water connections.

In the chambers and dressing-rooms, bowls of earthenware are set, with like connections. The sizes of basins vary from 12″ to 18″ outside diameters.

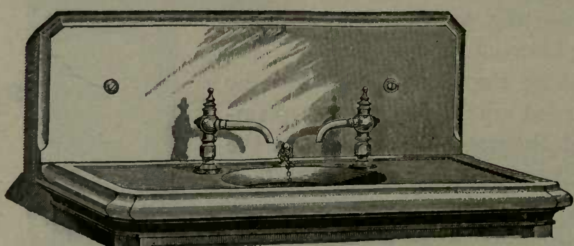


Fig. 1246.

Fig. 1246 shows the usual form of setting of a wash-basin in a countersunk marble slab, with a back of the same material; these are the common ground key swinging faucets for the supply of hot and cold water, and the waste is closed by a metal or rubber plug, attached to a chain, with the other end fastened to a pin in the marble slab.

The sides are inclosed with wood, forming a closet beneath the basin, with usually small drawers for towels at each side of the closet.

Fig. 1247 is a cast-iron bath-tub, of a simple pattern, with an overflow, *o*, and apertures *c* and *h* near the bottom for hot and cold water connections ; the waste is closed, as in the basin above, by a plug. When there is no overflow to the tub, this plug is a hollow pipe, down which there is an overflow when its lower extremity or annular plug closes the waste-pipe. Bath-tubs are more generally made of planished tinned-copper in a wooden box or support, and inclosed by wooden panels. The more expensive bath-tubs are made of porcelain, and may or may not be inclosed. In most bath-rooms there is a set wash-hand basin and a water-closet—often a foot-bath and bidet-pan. Formerly it was the common practice to have but one trap beneath the water-closet, into which all the waste-pipes discharged, but of late the water-closet connection with the soil-pipe is independent of the others.

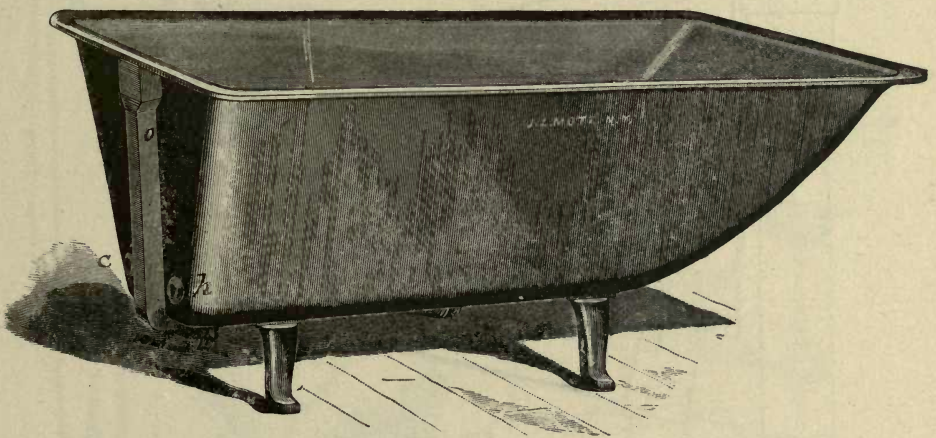


Fig. 1247.

It is preferable to make the water-closet in a separate room, distinct, with its own water and sewer-service and means of ventilation.

The construction of one form of water-closet, with all the modern appliances for the removal of soil and for ventilation, will be understood from the section (Fig. 1248). The seat is not shown, but is just above the basin, B, which contains some water to receive the defecations, to prevent the soil attaching to the side of the basin, and in a measure to check its offensive smell. T is the trap or water-seal which prevents the smell from the soil-pipe S passing up through the basin. The water-discharge from the pipe W is through a rim-flush around the edge of the basin. The sudden discharge washes out the basin B into the trap T, which is also cleaned by the rush of water. The soil-pipe S extends up through the roof, and may or may not also serve as a rain-leader. A sudden flow of water down the soil-pipe often acts as an ejector to draw the water out of the trap T, and break the water-seal ; to prevent this, there is an air connection, A, leading also to the top of the house. But as the offense of a water-closet is largely due to its recent use, and as smell once getting into the room is with difficulty and slowly removed, there is a ventilating-pipe, V, connecting the basin B with a ventilating-flue. It will be observed that this is the most important part of the apparatus; connected with a chamber commode, it would remove all smell, and if there were no trap to the soil-pipe, or were

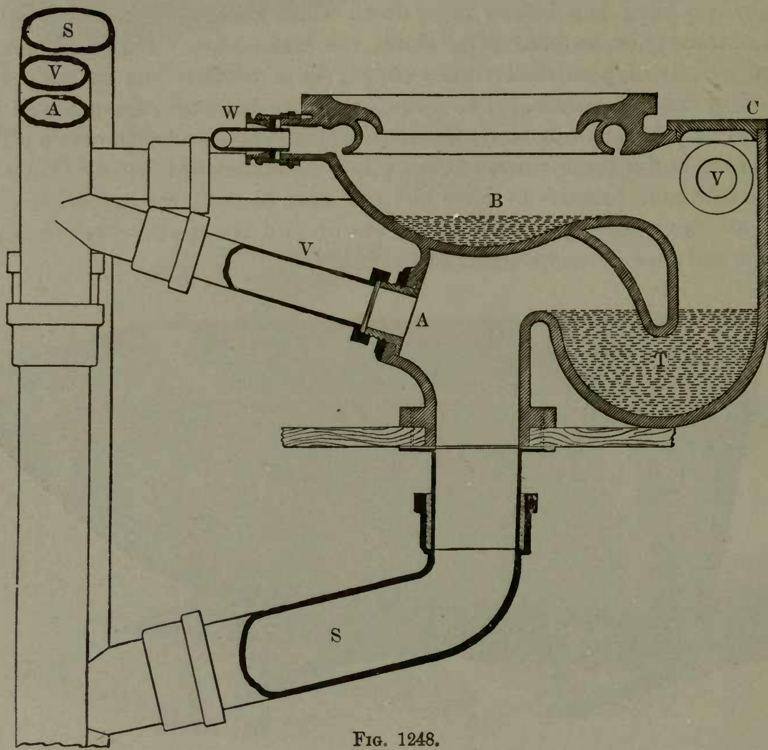


Fig. 1248.

the water-seal broken, it would still prevent any offensive smell from penetrating the house. If the soil-pipe be made also a ventilating-pipe, as is frequently done by its connection with the hot-air flue, then the trap and pipes A and V are unnecessary.

Fig. 1249.

Fig. 1249 is an elevation of the simplest form of closet—the *hopper*-closet—and in many respects the best. It is shown in section (Fig. 1250), with its water, soil-pipe, and water connection. By pulling up the handle, *h*, the disk-valve in the cistern is raised, and water supplied to the closet-basin.

Fig. 1251 is the section of a *pan*-closet, for many years the most popular closet. The copper pan, when shut, cuts off the view of the trap below and any odor from it; with a small flow of water the basin is readily kept clean, but soil is apt to lodge in the iron receiver, and the odor to arise from it when the pan is down. There is an annular ventilating-tube beneath the seat, with an air-shaft attached, but of altogether inadequate dimension for the purpose, as may be said of all such vents attached to water-closets. There is also the air-vent to prevent the water being drawn from the trap. No water connections are shown in the figure.

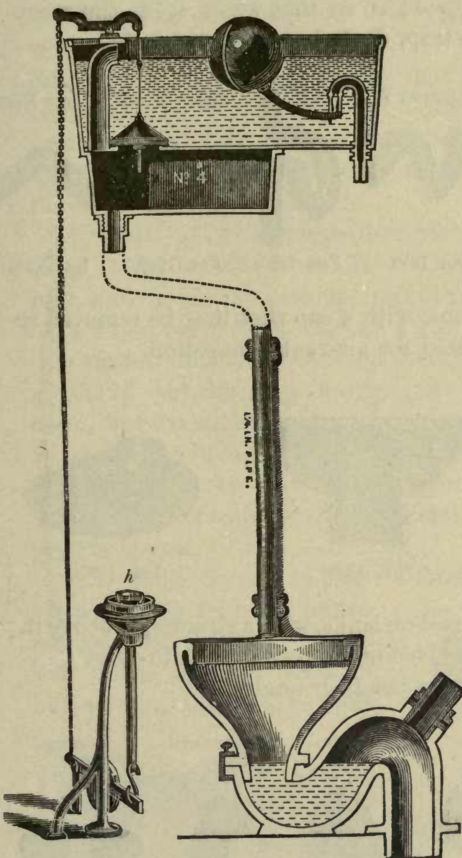


Fig. 1250.

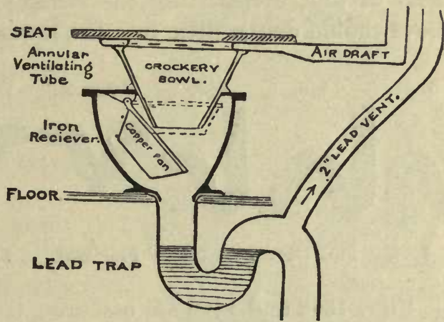


Fig. 1251.

Fig. 1252 is the section of a *flap*-closet, in which a flap-valve supplies the place of a pan.

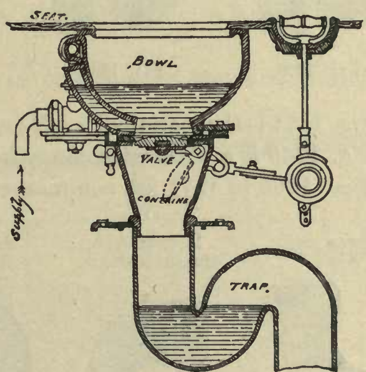


Fig. 1252.

Fig. 1253 is the section of a siphon-jet closet. In addition to the *fan* flush, *f*, into the basin, it has a jet-pipe *j* at its bottom, inducing a current in the direction of the inclined leg of the trap, and by flush and jet the water is siphoned from the basin.

The use of traps has already been explained, but they are varied in their form, all answering the same purpose, to cut off the air-connection of the soil-pipe with the room in which the appliance is placed. The smaller traps are invariably in lead.

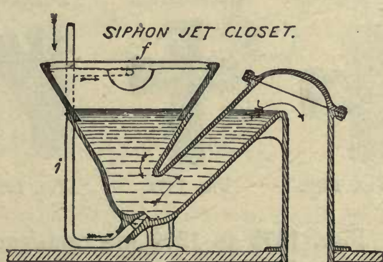


Fig. 1253.

Figs. 1254 to 1261 represent the usual forms of lead traps. It will be observed that there are screw-plugs at the bottom of the traps, which can be taken out to remove any obstruction. As the water may be drawn out of any trap by the passage of water down the pipe with which it is connected, an air-vent, as already described, in the water-closet trap, is put on these small traps. Instead of this, by inserting the rising pipe at *a*, so that water from the waste above should drip a little into the lower trap, draft from it is prevented.

S.

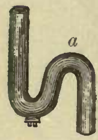


Fig. 1254.

¾S.

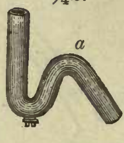


Fig. 1255.

½S.

Fig. 1256.

Running.

Fig. 1257.

Running Y.

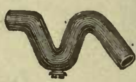


Fig. 1258.

Bag.

Fig. 1259.

Short Bend.

Fig. 1260.

Long Bend.

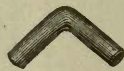


Fig. 1261.

Figs. 1262 and 1263 are cast-iron traps, with a cap that may be removed to clean the trap, or the aperture may be used for air-vent connection.

S-Trap.

Fig. 1262.

Trap with Side Outlet.

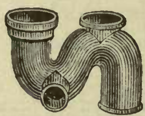


Fig. 1263.

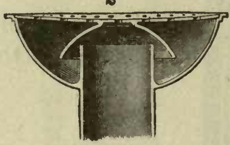


Fig. 1264.

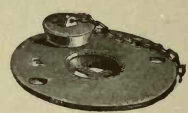


Fig. 1265.

Fig. 1264 is the section of a *bell*-trap, used on sinks, with a strainer, S, above it.

Fig. 1265 is a plate with plug, for the bottom of sinks and bath-tubs.

Figs. 1266 to 1271 are common cast-iron bends or angles.

Quarter Bend.

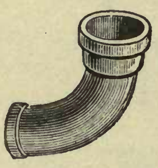


Fig. 1266

Double Hub, Quarter Bend.

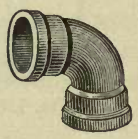


Fig. 1267.

Eighth Bend.

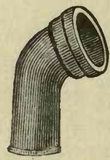


Fig. 1268.

Sixth Bend.

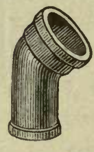


Fig. 1269.

Sixteenth Bend.

Fig. 1270.

Return Bend.

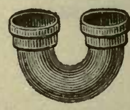


Fig. 1271.

T-Branch.

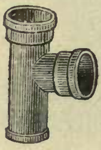


Fig. 1272.

Cross-Head.

Fig. 1273.

Y-Branch.

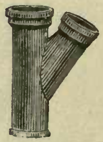


Fig. 1274.

Half Y-Branch.

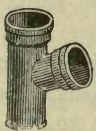


Fig. 1275.

Double Y-Branch.

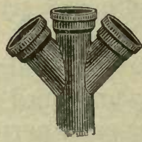


Fig. 1276.

Double Half Y-Branch.

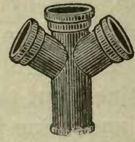


Fig. 1277.

Figs. 1272 to 1277 are cast-iron branches. The T branch and cross-head are objectionable, as the flows from the branches and mains are at right angles, and mutually obstructive; whereas in the Y, especially the full Y, the flows are at acute angles with each other, and the currents converge. Similar fittings are used for water, but they are much heavier.

Fig. 1278.

Most water-closet basins are inclosed by a lidded seat and riser, but the less woodwork about a basin the better. The seat is generally hung with hinges, so that it can be raised, and the basin used as a urinal for men; the upper edge of the basin being extended or covered with an earthenware tray, sloping toward the basin.

Urinals, of which one form is shown (Fig. 1278), are often used in public buildings, and in airy situations; although they have water connection, *w*, and a rim flush, it is almost impossible to keep them sweet; a cake of carbolic soap is often put in the basin, but the most effectual means adopted on many railway-cars is a piece of ice. As the raising of the seat of the water-closet makes this convenience a good urinal, the distinctive one is but little used in private houses.

The Water-Service to Water-Closets.—In the cheaper hoppers the supply is often directly from the house-service. In these the trap is well down, and the flush, if not certain, there is nothing objectionable to sight. The water may be let on by hand, or by an automatic valve connected with sitting down on the seat, or by opening or shutting the closet-door.

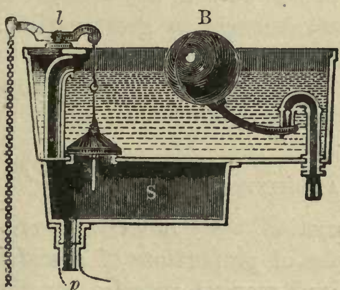


Fig. 1279.

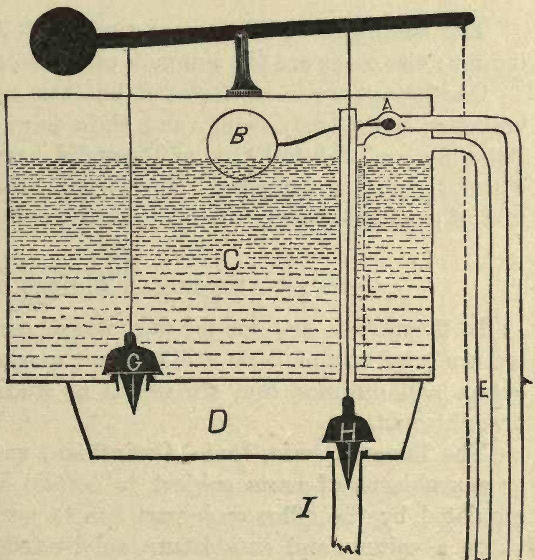


Fig. 1280.

As the supply from the service is uncertain if there is a draught in another quarter, it is now more common to have a cistern-supply, shown in section, Fig. 1279. B is a ball-cock, operating a valve in the water-pipe, by which the water is admitted to the cistern whenever the water is below a certain level; *l* is a lever, by which the discharge-valve is raised or lowered; the valve-opening is large and the water flows into a service-box, S, filling it, and at the same time discharging through the *p* into the closet-basin. When the valve is closed, the water still continues to flow from the service-box, vent being given through the air-pipe, which in this case serves also as an overflow.

Fig. 1280 is the section of another cistern, in which the ball, B, or float, operates a common plug-valve, A. The service-box, D, acts as a sort of a measure of the quantity of water used. When it is filled by means of the valve G, the valve H is closed, and then, when the valve H is raised for the flush of the closet, G is closed. There is an air-vent around the chain or rod of the valve H, and the overflow E is independent. The supply-pipe is extended nearly to the bottom of the cistern by a short, loose pipe, as shown at L, to avoid noise from falling water.

Lighting.—It may be needless to say that the light in a building should be as much as possible from natural sources, as it conduces to health and cleanliness, and economy in conducting any industrial pursuits. But, for artificial lighting, the present permanent fixtures are usually for the use of gas. In the distribution of gas through the building, wrought-iron pipes are invariably used. The old English rule for the sizes of these pipes:

¼″......	6 feet long......	1 outlet.	1″	70 feet long......	35 outlets.
⅜″......	20 "	...... 3 outlets.	1¼″.....	100 "	...... 60 "
½″......	30 "	...... 6 "	1½″.....	150 "	......100 "
¾″......	90 "	......20 "	2″	200 "	......200 "

The couplings to elbows are similar to those used in steam-fitting, but lighter; the cocks are the common plug-cocks.

Gas fittings are in all forms of brackets and pendants, with any number of branches, with fixed, swing, and slide joints, and burners in great variety. Bat-wing and fish-tail tips and Argand burners are the most used, with or without globes and shades. The Argand must have a chimney. Consumption of gas is commonly from 3 to 6 ft. per hour per burner.

GREEK AND ROMAN ORDERS OF ARCHITECTURE.

In themselves, and for the purposes of construction, the "orders of architecture" are now of little utility; but, as examples of proportions of graceful curves and outlines, they are useful as studies and manual practice for the draughtsman.

The Tuscan, Doric, Ionic, Corinthian, and Composite orders, are systems or assemblages of parts subject to certain uniform established proportions, regulated by the office each part has to perform, consisting of two essential parts, a column and entablature, subdivided into three parts each: the first into the base, the shaft, and the capital; the second into the architrave, or chief beam, C, Fig. 1281, which stands immediately on the column; the frieze, B, which lies on the architrave; and the cornice, A, which is the crowning or uppermost member of an order. In the subdivisions certain

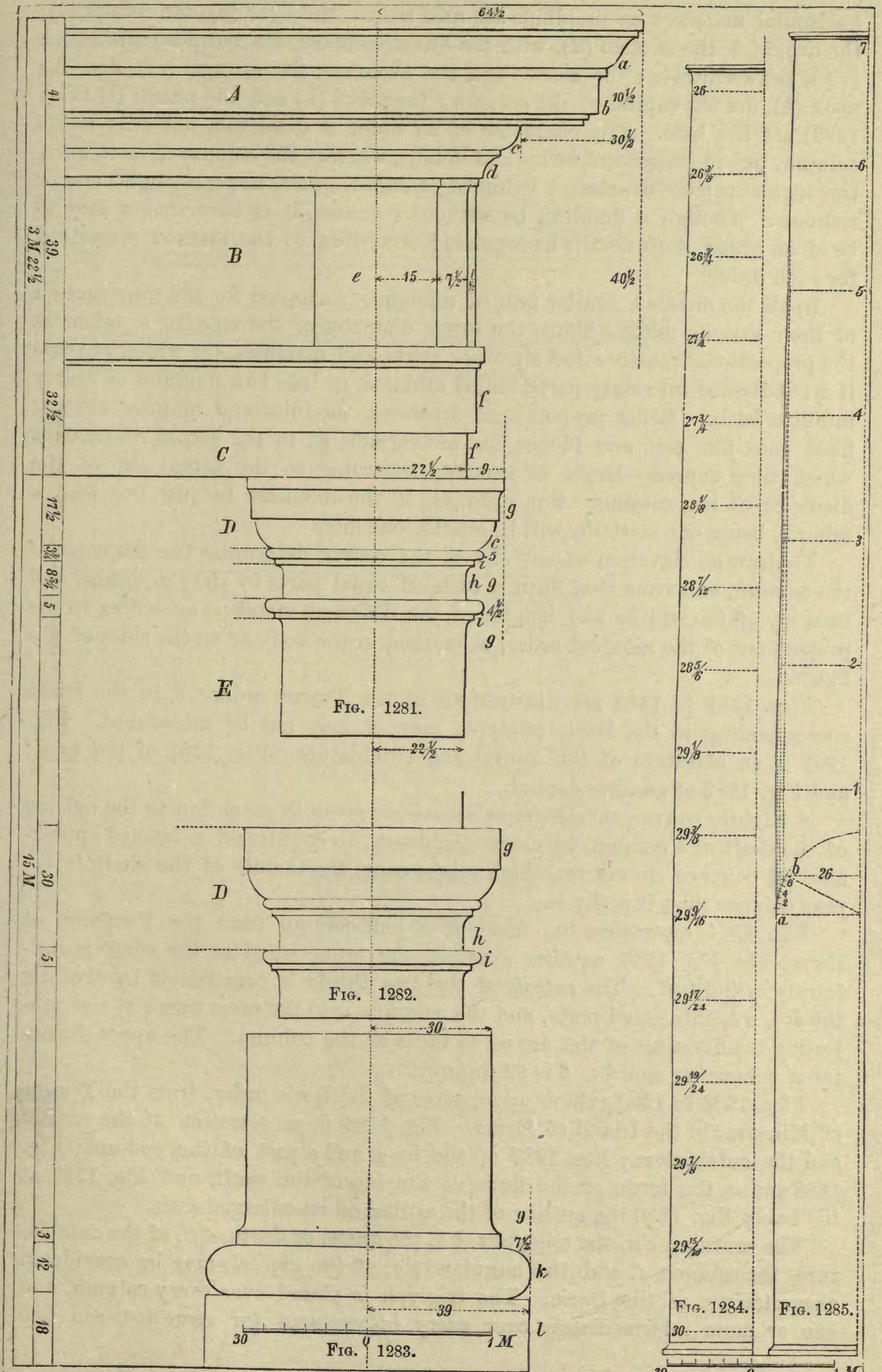


Fig. 1281.

Fig. 1282.

Fig. 1283.

Fig. 1284.

Fig. 1285.

horizontal members or moldings are used: thus, the ogee (*a*), the corona (*b*), the ovolo (*c*), the cavetto (*d*), with the fillets, compose the cornice; the fasciæ (*f f*), the architrave; the abacus (*g*), the ovolo (*c*), the astragal (*i i*), and the neck (*h*), are the capital of the column; the torus (*k*) and the plinth (*l*) (Fig. 1283) are the base. The character of an order is displayed, not only in its column, but in its general forms and details, whereof the column is, as it were, the regulator; the expression being of strength, grace, elegance, lightness, or richness. Though a building be without columns, it is nevertheless said to be of an order, if its details be regulated according to the method prescribed for such order.

In all the orders a similar unit of reference is adopted for the construction of their various parts. Thus, the lower diameter of the column is taken as the proportional measure for all other parts and members, for which purpose it is subdivided into sixty parts, called minutes, or into two modules of thirty minutes each. Being proportional measures, modules and minutes are not fixed ones like feet and inches, but are variable as to the actual dimensions which they express—larger or smaller, according to the actual size of the diameter of the column. For instance, if the diameter be just five feet, a minute, being one sixtieth, will be exactly one inch.

To draw an elevation of any one of the orders, determine the diameter of the column, and from that form a scale of equal parts by sixty divisions, and then lay off the widths and heights of the different members according to the proportions of the required order, as marked in the body or on the sides of the figures.

Figs. 1281 to 1285 are illustrations of the Tuscan order: *e*, in the frieze corresponding to the Doric triglyph, may or may not be introduced. Fig. 1281 is an elevation of the capital and entablature; Fig. 1283 of the base; and Fig. 1282 of another capital.

A slightly convex curvature, or *entasis*, is given in execution to the outline of the shaft of a column, by classic architects, to counteract a fancied appearance of concave curvature, which might cause the middle of the shaft to appear thinner than it really is.

Fig. 1284 represents the form of a half-column from the Pantheon at Rome. In Fig. 1285, another example, the lower third of the shaft is uniformly cylindrical. The *entasis* of the two thirds is constructed by dividing the arc, *a b*, into equal parts, and the columns into the same number, and projecting the divisions of the arc on to those of the column. The upper diameter of column or chord at *b* is 52 minutes.

Figs. 1286 to 1290 exhibit an example of the Doric order, from the Temple of Minerva, in the Island of Egina. Fig. 1286 is an elevation of the capital and the entablature; Fig. 1287 of the base, and a part of the podium; Fig. 1288 shows the forms of the flutes at the top of the shaft, and Fig. 1289 at the base; Fig. 1290 the outline of the capital on an enlarged scale.

The mutules, *a a*, the triglyphs, *b b*, the guttæ or drops, *d d*, of the entablature, the echinus, *f*, and the annulets, *g g*, of the capital, may be considered characteristic of the Doric. The triglyph is placed over every column, and one or more intermediately over every *intercolumn* (or span between two

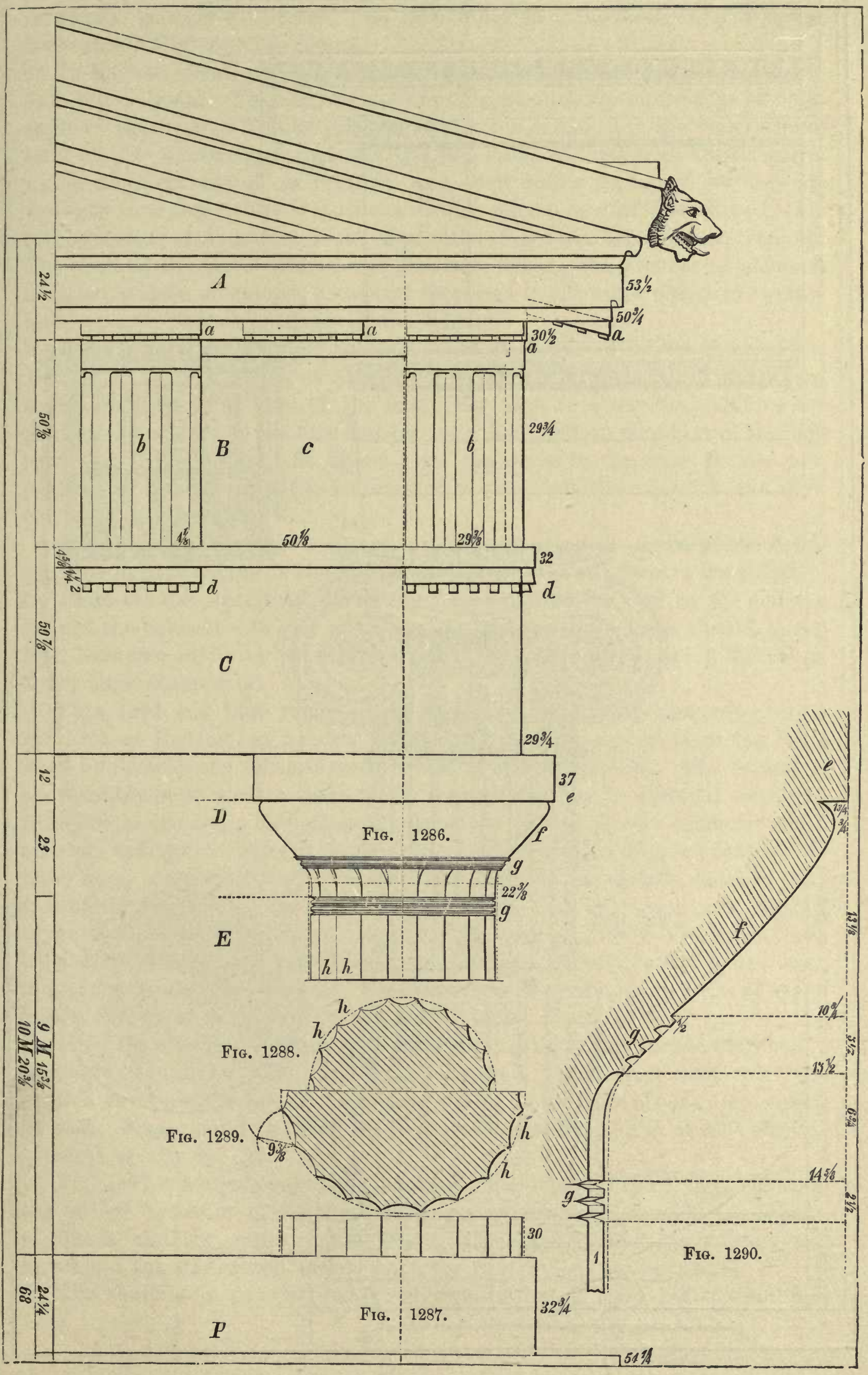


FIG. 1286.

FIG. 1287.

FIG. 1288.

FIG. 1289.

FIG. 1290.

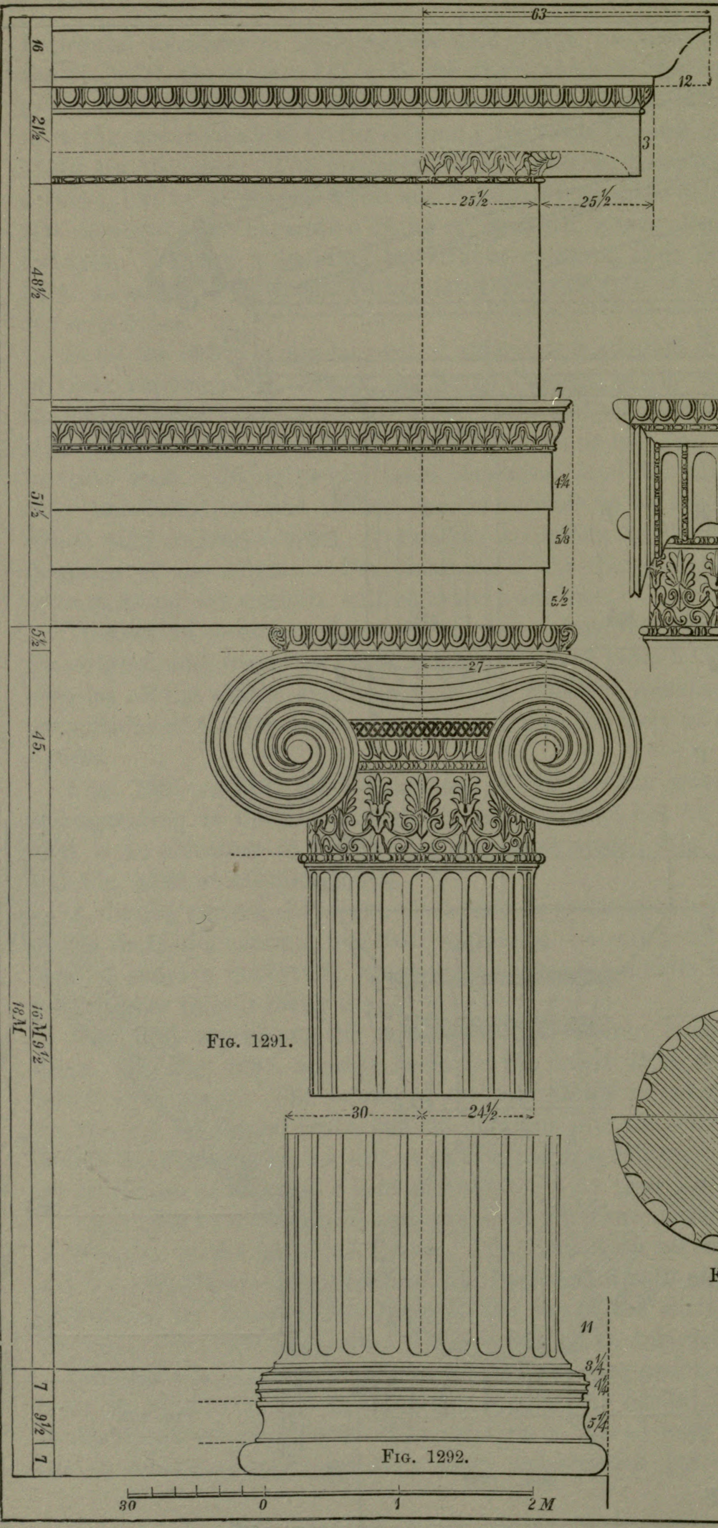


Fig. 1291.

Fig. 1292.

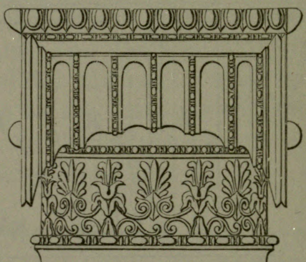


Fig. 1294.

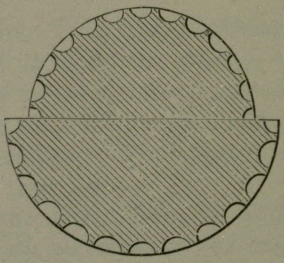


Fig. 1293.

columns), at such a distance from each other that the metopes, *c*, or spaces between the triglyphs, are square.

In the best Greek examples of the order, there is only a single triglyph over each intercolumn. The end triglyphs are placed quite up to the edge or outer angle of the frieze. The mutules are thin plates attached to the under side or soffit of the corona, over each triglyph and each metope, with the former of which they correspond in breadth, and their soffits or under surfaces are wrought into three rows of guttæ or drops, conical or otherwise shaped, each row consisting of six guttæ, or the same number as those beneath each triglyph. The shaft of the Doric column was generally fluted; the number of channels is either sixteen or twenty, afterward increased in the other orders to twenty-four, a center flute on each side of the column.

Figs. 1291 to 1294 exhibit an example of the Ionic order, taken from the Temple of Minerva Polias, at Athens. Fig. 1291 is an elevation of the capital and entablature; Fig. 1292, of the base; Fig. 1293 is a sectional half of the plan of the column at the base and the top; Fig. 1294 an elevation of the baluster side of the capital. It differs from the Doric in the more slender proportions of its shaft, and the addition of a base; but the capital is the indicial mark of the order.

When a colonnade was continued in front and along the flanks of the building, this form of capital in the end column occasioned an offensive irregularity; for while all the other columns on the flanks showed the volutes, the end one showed the baluster side. It was necessary that the end column should, therefore, have two adjoining volute faces, which was effected by placing the volute at the angle diagonally.

Figs. 1295 and 1296 represent an example of the Corinthian order, from the Arch of Hadrian, at Athens. This order is distinguished from the Ionic more by its deep and foliaged capital than by its proportions. The capital is considerably more than a diameter in height, varying in different examples from one to one and a half diameter, upon the average about a diameter and a quarter, and has two rows of leaves, eight in each row, so disposed that of the taller ones, composing the upper row, one comes in the middle, beneath each face of the abacus, and the lower leaves *alternate* with the upper ones, coming between the stems of the latter; so that in the first or lower tier of leaves there is in the middle of each face a space between two leaves occupied by the stem of the central leaf above them. Over these two rows is a third series of eight leaves, turned so as to support the small volutes which, in turn, support the angles of the abacus. Besides these outer volutes, invariably turned diagonally, there are two other smaller ones, termed *caulicoli*, which meet each other beneath a flower on the face of the abacus. The sides of the abacus are concave in plan, being curved outward so as to produce a sharp point at each corner, which is usually cut off.

Fig. 1297 represents one of the capitals of the Tower of the Winds, showing the earliest formation of the Corinthian capital. In this example the abacus is square, and the upper row of leaves, of the kind called *water-leaves*, are broad and flat, and merely carved upon the vase or body of the capital.

The shaft is, in general, fluted, similarly to that of the Ionic column, but

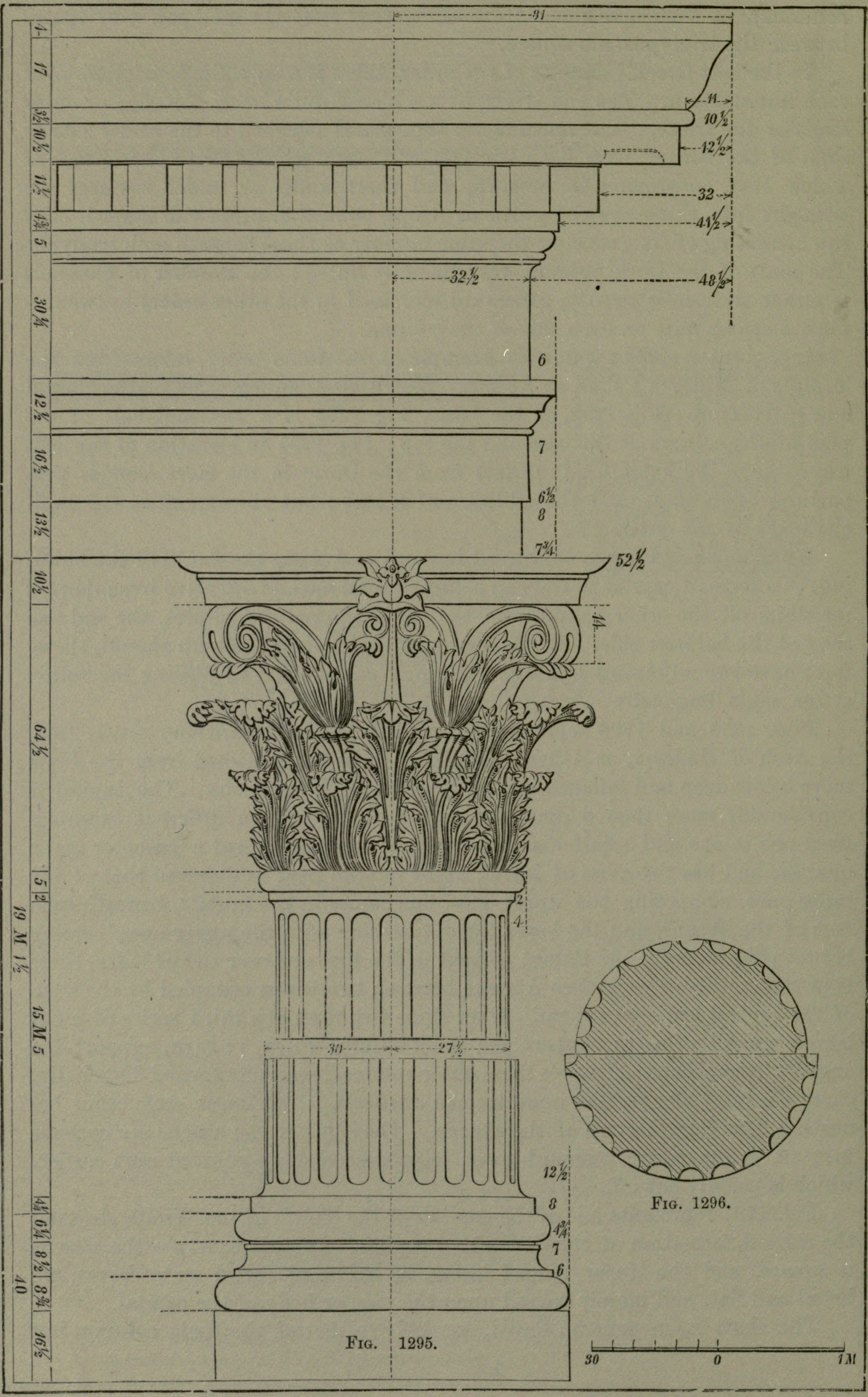


FIG. 1296.

FIG. 1295.

sometimes the flutes are *cabled;* that is, the channels are hollowed out for only about two thirds of the upper part of the shaft, and the remainder cut so that each channel has the appearance of being partly filled up by a round staff or piece of rope.

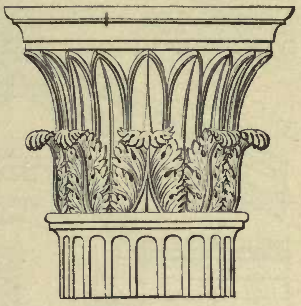


Fig. 1297.

The cornice is very much larger than in the other orders, in height and in projection, consisting of a greater number of moldings beneath the corona, for that and the cymatium over it are invariably the crowning members. In Fig. 1295 square blocks or *dentels* are introduced, but often to the dentels is added a row of *modillions* (Fig. 1418), immediately beneath, and supporting the corona; and between them and the dentels, and also below the latter, are other moldings, sometimes cut, at others left plain.

The Composite Order is a union of the Ionic and Corinthian orders. Its capital consists of a Roman Ionic one, superimposed upon a Corinthian foliaged base, in which the leaves are without stalks, placed directly upon the body of the vase.

The spacing between the columns, or intercolumn, is from one to one and one half diameters, but modern architects have coupled the columns, making a wide intercolumn between every pair of columns, so that as regards the average proportion between solids and voids, that disposition does not differ from what it would be were the columns placed singly. Supercolumniation, or the system of piling up orders, or different stages of columns one above another, was employed for such structures merely as were upon too large a scale to admit of the application of columns at all as their decoration, otherwise than by disposing them in tiers.

The Greeks seldom employed human figures to support entablatures or beams; the female figures, or Caryatides, are almost uniformly represented in an erect attitude, without any apparent effort to sustain any load; while the male figures, Telamones or Atlantes, display strength and muscular action. Besides entire figures, either Hermes' pillars or Termini are occasionally used as substitutes for columns of the usual form, on a moderate scale. The first mentioned consist of a square shaft with a bust or human head for its capital; the latter of a half-length figure rising out of, or terminating in, a square shaft tapering downward. Hermes' pillars are frequently employed by modern architects for the decoration of window architraves.

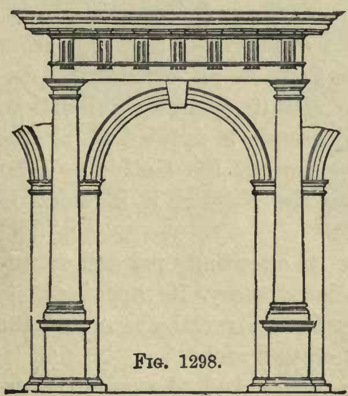


Fig. 1298.

The Romans introduced circular forms and curves, not only in elevation and section, but in plan. The true Roman order consists, not in any of the columnar ordinances, but in an arrangement of two pillars (Fig. 1298) placed at a distance from one another nearly equal to their own height, and having a very long entablature, which, in consequence, required to be supported in the center by an arch springing from piers.

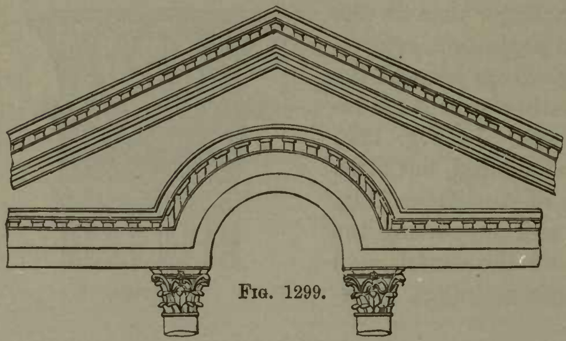


Fig. 1299.

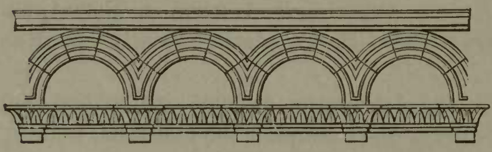


Fig. 1300.

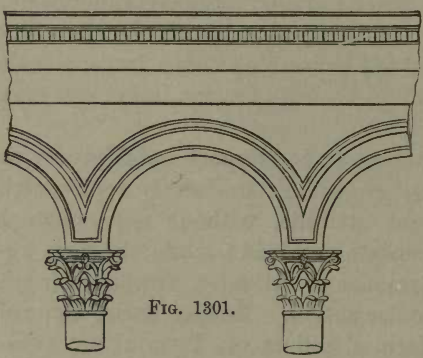


Fig. 1301.

Figs. 1299, 1300, and 1301, from the Palace of Diocletian at Spalatro, are illustrations of the different modes of treatment of the arch and entablature.

Perhaps the most satisfactory works of the Romans are those which we consider as belonging to civil engineering rather than to architecture — their aqueducts and viaducts, all of which, admirably conceived and executed, have furnished practical examples for modern constructions, of which the High Bridge across Harlem River may be taken as an illustration.

The history of Roman architecture is that of a style in course of transition, beginning with purely pagan or Grecian, and passing into a style almost wholly Christian. The first form of Christian art was the Romanesque, which afterward branched off into the Byzantine and the Gothic.

The Romanesque and Byzantine, as far as regards the architectural features, are almost synonymous; in the earlier centuries there is an ornamental distinction. In its widest signification, the Romanesque is applied to all the earlier round-arch developments, in contradistinction to the Gothic or later pointed arch varieties of the North. In this view the Norman is included in the Romanesque.

The general characteristics of the Gothic are its essentially pointed or vertical tendency, its geometrical details, its window-tracery, its openings, its cluster of shafts and bases, its suits of moldings, the universal absence of the dome, and the substitution of the pointed for the round arch.

The Romanesque pillars are mostly round or square, and, if square, generally set evenly, while the Gothic square pillar is set diagonally.

Figs. 1302 to 1306 represent sections of Gothic pillars. Fig. 1307 is half of one of the great western piers of the Cathedral of Bourges, measuring 8 feet

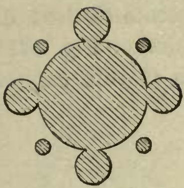


Fig. 1302.

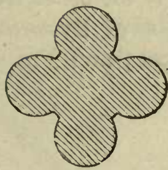


Fig. 1303.

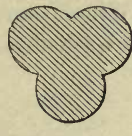


Fig. 1304.

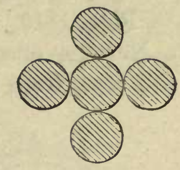


Fig. 1305.

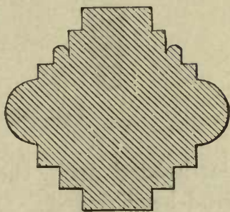


Fig. 1306.

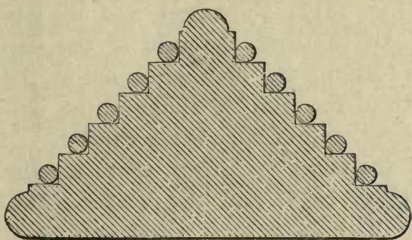


Fig. 1307.

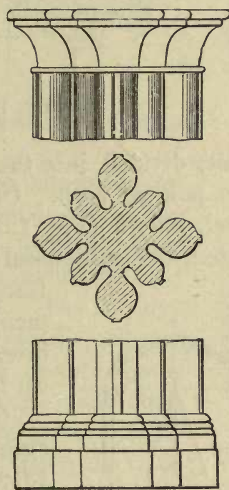


Fig. 1308.

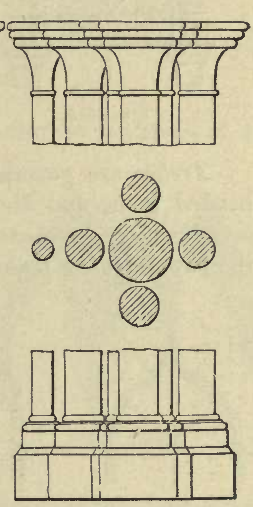


Fig. 1309.

Fig. 1310.

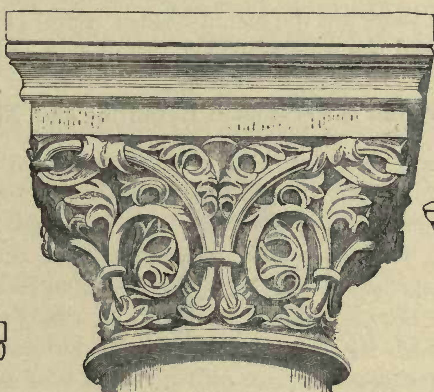


Fig. 1312.

Fig. 1311.

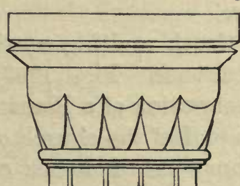


Fig. 1313.

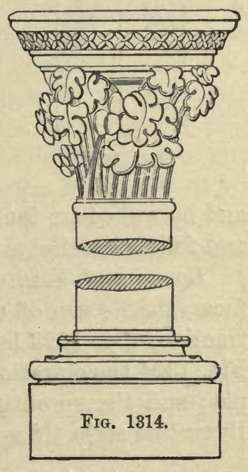


Fig. 1314.

on each side. Figs. 1308 and 1309 are elevations of capitals and bases, and sections of Gothic pillars; one from Salisbury, the other from Lincoln Cathedral.

Figs. 1310, 1311, and 1312 are examples of Byzantine capitals; Fig. 1313 a Norman one, from Winchester Cathedral; and Fig. 1314 a Gothic capital and base, from Lincoln Cathedral.

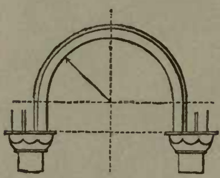


Fig. 1315.

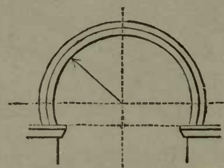


Fig. 1316.

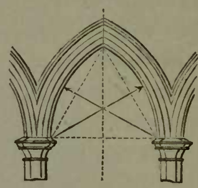


Fig. 1317.

Arches are generally divided into the triangular-headed arch, the round-headed arch, and the pointed arch. Of the round-headed arch, there are semicircular, segmental, stilted (Fig. 1315), and horseshoe (Fig. 1316). Of the two-centered pointed, the equilateral (Fig. 1317), the lancet, and the obtuse. Of the first, the radii of the segments forming the arch are equal to the breadth of the arch, of those of the lancet longer, and of the obtuse shorter.

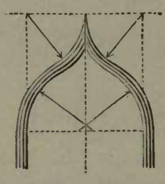


Fig. 1318.

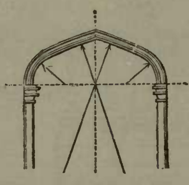


Fig. 1319.

Of the complex arches, there are the ogee (Fig. 1318) and the Tudor (Fig. 1319). The Tudor arch is described from four centers, two on a level with the spring and two below it.

Of foiled arches, there are the round-headed trefoil (Fig. 1320), the pointed trefoil (Fig. 1321), and the square-headed trefoil arch (Fig. 1322). The points *c* are termed cusps.

The semicircular arch is the Roman Byzantine and Norman arch; the ogee

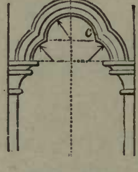


Fig. 1320.

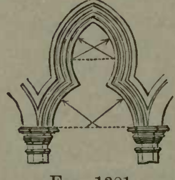


Fig. 1321.

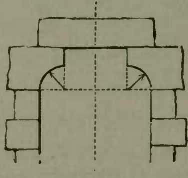


Fig. 1322.

and horseshoe are the profiles of many Turkish and Moorish domes; the pointed and foliated arches are Gothic.

Domes and Vaults.—The Greek vaulting consisted wholly of spherical surfaces, the Roman of cylindrical ones. Figs. 1323 and 1324 illustrate this distinction, Fig. 1323 being the elevation of a Roman cylindrical cross-vault, and Fig. 1324 the elevation of the roof of the church of St. Sophia at Constantinople; and the sprouting of domes out of domes continues to characterize the Byzantine style. As a constructive expedient the cross-vault is to be preferred, as the whole pressure and thrust are collected in four definite resultants, applied at the angles only, so that it might be supported by four flying buttresses, placed in the direction of these resultants, and strong enough not to be crushed by the pressure.

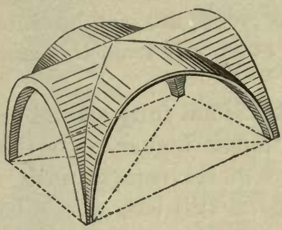


Fig. 1323.

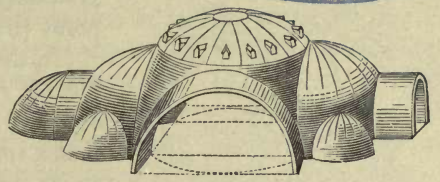


Fig. 1324.

Fig. 1325 represents a compartment of the simplest Gothic vaulting—*a, a,* groin ribs ; *b, b, b,* side ribs.

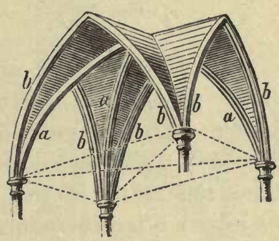


Fig. 1325.

The Romans introduced side ribs, appearing on the inside as flat bands, and harmonizing with the similar form of pilasters in the walls, but they never used groin ribs ; the Gothic builders introduced these, and deepened the Roman ribs. The impenetration of vaults, either round or pointed, produces elliptical groin lines, or else lines of double curvature ; yet the early Gothic architects made their groin ribs usually simple pointed arches of circular curvature, thrown diagonally across the space to be groined, and the four side arches were equally simple, the only care being that all the arches should have their vertices at the same level. The strength depended on the ribs, and the shell was made quite light, often not more than six inches, while Roman vaults of the same span would have been three or four feet. The Romans made their vault *surfaces* geometrically regular, and left the groins to take their chance ; while the early Gothic architects made their groins geometrically regular, and let the intermediate surfaces take their chance.

In the next step the groin ribs were elliptical, and when intermediate ribs or *tiercerons* were inserted, these ribs had also elliptical or cylindrical curvatures, different from the groins, and the ribs were placed near each other, in order that the portion of the vault between each pair might practically be almost cylindrical. In the formation of the compound circular ribs three conditions were to be observed : 1. That the two arcs should have a common tangent at the point of meeting. 2. That the feet of all the ribs should have the same radius, up to the level at which they completely separate from each other. 3. That from this point upward their curvature should be so adjusted as to make them all meet their fellows on the same horizontal plane, so that all the ridges of the vaults may be on one level.

The geometrical difficulty of such works led to what is called *fan-tracery vaulting.* If similar arches spring from each side of the pillars (Fig. 1325), the portion of vault springing from each pillar would have the form of an inverted concave-sided pyramid, its horizontal section at every level being square. The later architects, by converting this section into a circle, the four-sided pyramid became a conoid, and all the ribs forming the conoidal surface became alike in curvature, so that they all might be made simple circular arcs; these ribs are continued with unaltered curvature till they meet and form the ridge; but in this case the ridges are not level, but gradually descend every way from the center point (Fig. 1326).

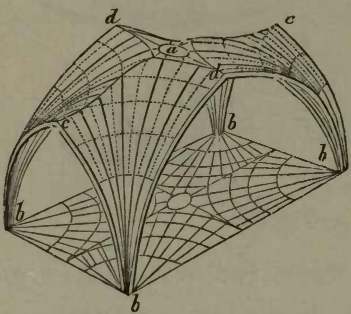


Fig. 1326.

In the figure this is not fully carried out, for no rib is continued higher than those over the longer sides of the compartment, so that a small lozenge is still left, with a boss at its center. When the span of the main arch *b a* was large in proportion to that of *b c*, the arch *b c* became a very acute lancet arch, scarcely admitting windows of an elegant or sufficient size. To obviate this, the compound curve was again introduced.

The four-centered arch is not necessarily flat or depressed; it can be made of any proportion, high or low, and always with a decided angle at the vertex. In general, the angular extent of the lower curve is not more than 65°, nor less than 45°. The radius of the upper curve varies from twice to more than six times the radius of the lower. The projecting points of the trefoil arch, or *cusps*, are often introduced for ornament merely, but serve constructively, both in vaults and arches, as a load for the sides, to prevent them rising from the pressure on the crown.

As vaultings, in general, were contrived to collect the whole pressure of each compartment into four single resultants, at the points of springing, leaving the walls so completely unloaded that they are required only as inclosures or screens, they might be entirely omitted or replaced by windows. Indeed, the real supporting walls are broken into narrow strips, placed at right angles to the outline of the building, and called *buttresses*, and the *inclosing* walls may be placed either at the outer or inner edge of the buttresses. The first, that adopted by the French architects, gave deep recesses to the interiors, while the other, or English method, served to produce external play of light and shade.

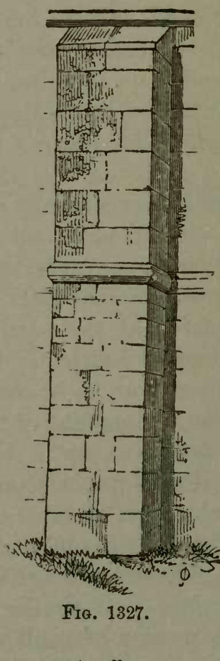


Fig. 1327.

The Norman buttress (Fig. 1327) resembles a flat pilaster, being a mass of masonry with a broad face, slightly projecting from the wall. They are, generally, of but one stage, rising no higher than the cornice, under which they often, but not always, finish with a slope. Sometimes they are carried up to, and terminate in, the corbel table.

Fig. 1328 represents a buttress in two stages, with slopes as set-offs.

Fig. 1329 is a buttress of the Early English style, having a plain triangular

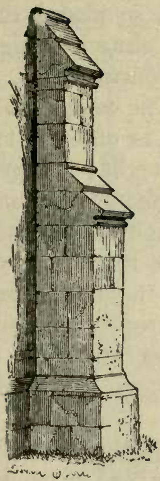


FIG. 1328.

or pedimental head. The angles were sometimes chamfered off, and sometimes ornamented with slender shafts. In buttresses of different stages, the triangular head or gable is used as a finish for the intermediate stages.

In the Decorated style, the outer surfaces of the buttresses are ornamented with niches, as in Fig. 1330. In the Perpendicular style the outer surface is often partially or wholly covered with panel-work tracery (Fig. 1331).

The buttress was a constructive expedient to resist the thrust of vaulting; but to resist the thrust of the principal vault, or that over the nave or central part of the church, buttresses of the requisite depth would have filled up the side

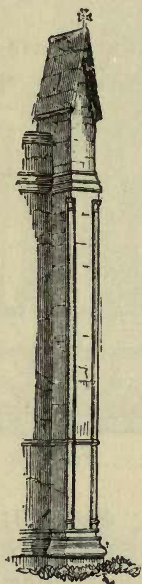


FIG. 1329.

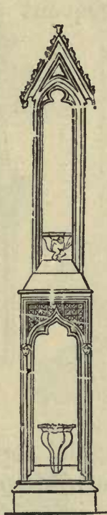


FIG. 1330.

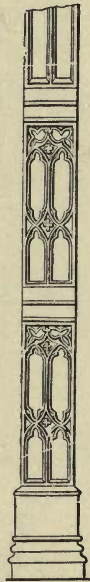


FIG. 1331.

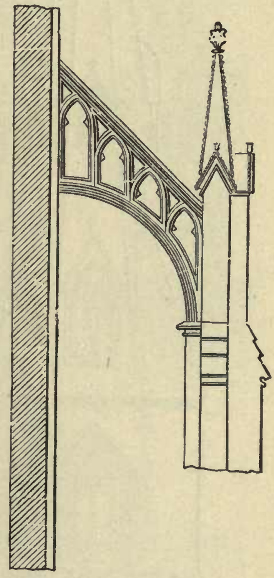


FIG. 1332.

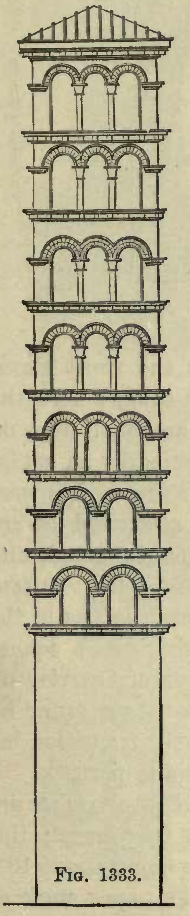


FIG. 1333.

aisles entirely. To obviate this, the system of flying buttresses was adopted; that is, the connection of the interior with the outer buttress, by an arch or system of arches, as shown in Fig. 1332. The outer piers were surmounted by pinnacles, to render them a sufficiently steady abutment to the flying arches.

The earlier towers of the Romanesque style were constructed without spires. All are square in plan, and extremely similar in design. Fig. 1333 is an elevation of the tower attached to the church of Sta. Maria, in Cosmedin, and is one of the best and most complete examples of this style. It is 15 feet broad and 110 feet high. These towers are the types of the later Italian campaniles, generally attached to some angle of churches; if detached, so placed that they still form a part of the church design. Sometimes they are but civic constructions, as belfries, or towers of defense. The campanile is square, carried up without break or offset to two thirds, at least, of its intended height; it is generally solid to a considerable height, or with only such openings as serve to admit light to the staircases. Above this solid part one round window is introduced in each face; in the next story, two; in the one above this, three; then four, and lastly five; the lights being separated by slight piers, so that the upper story is virtually an open loggia.

The Gothic towers have projecting buttresses, frequent offsets, lofty spires, and a general

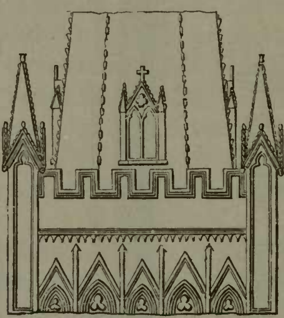


Fig. 1335.

pyramidal form. Fig. 1334 is the front elevation of a simple English Gothic tower; here the plain pyramidal roof, rising at an equal slope on each of the four sides, is intersected by an octagonal spire of steep pitch. The first spires were simple quadrangular pyramids; afterward the angles were cut off, and they became octagonal, and this is the general Gothic form of spire. Often, instead of intersecting the square roof, as in the figure, the octagonal spire rests upon a square base, and the angles of the tower are carried up by pinnacles, or the sides by battlements, or by both, as in Fig. 1335, to soften the transition between the perpendicular and sloping part.

In general the spires of English churches are more lofty than those on the Continent; the angle at the apex in the former being about 10°, and in the latter about 15°. The apex angle of

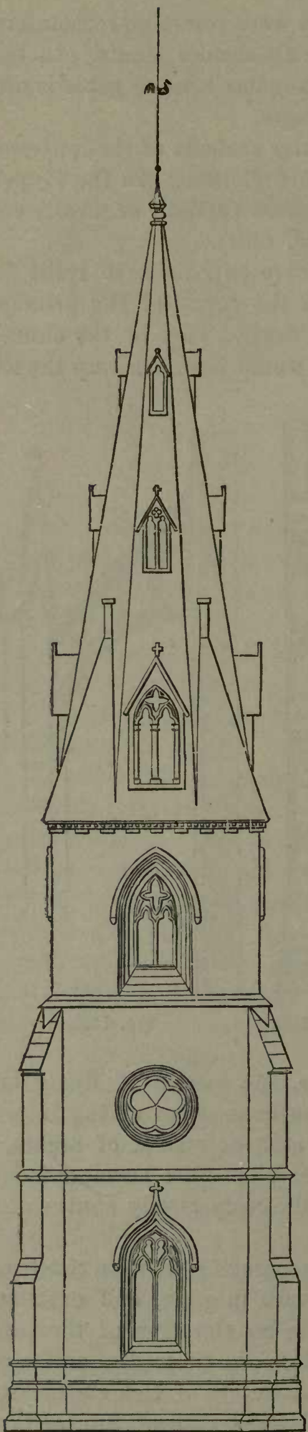


Fig. 1334.

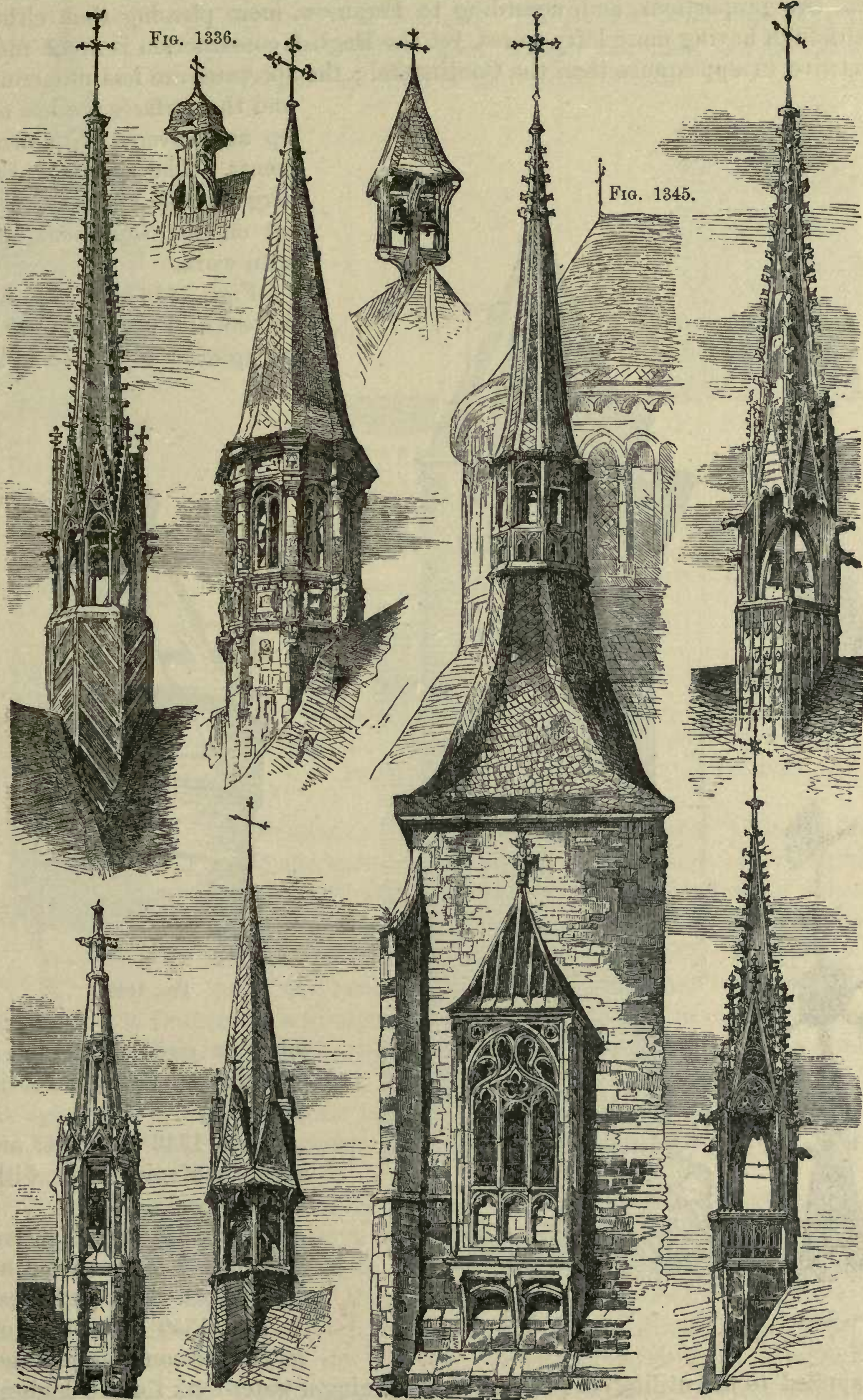


Fig. 1338. Fig. 1336. Fig. 1339. Fig. 1337. Fig. 1341. Fig. 1345.

Fig. 1342. Fig. 1343. Fig. 1340. Fig. 1344.

the spires of Chichester and Lichfield are from 12° to 13°, or a mean between the two proportions, and, according to Ferguson, more pleasing than either. Although having more lofty spires, yet the English construction is much more massive in appearance than the Continental; the apertures are less numerous, and the surfaces are less cut up and covered with ornaments. The spires of Friberg Church, and many others on the Continent, are open work.

Figs. 1336 and 1337 are bell-cots. Figs. 1338 to 1344 are spires. Fig. 1345 is an

Fig. 1346.

Fig. 1347.

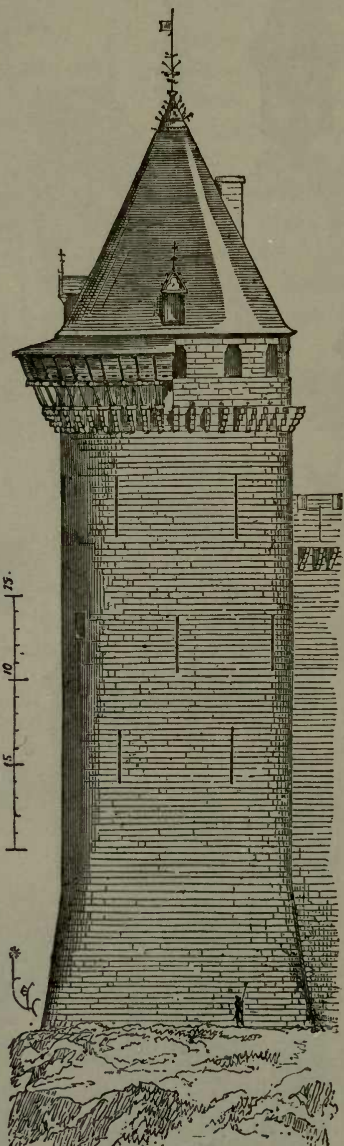


Fig. 1348.

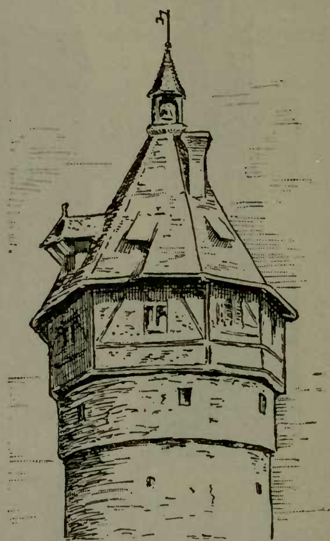


Fig. 1349.

apse, or circular end of a church, from German Gothic examples.

Figs. 1346 and 1347 are examples of spire finials, with weather-cocks.

Figs. 1348 and 1349 are examples of towers not connected with church edifices.

Fig. 1350 is a tower of very recent construction, and is applied to the utilitarian purpose of sustaining a water-tank for the highest service of the Croton in New York city.

Fig. 1351 represents the upper portion of the tower of Ivan Veliki, at Moscow. The Russian towers are generally constructed independent of their churches, and are intended for the reception of their massive bells.

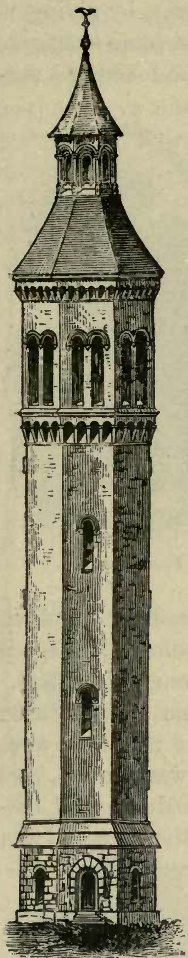


Fig. 1350.

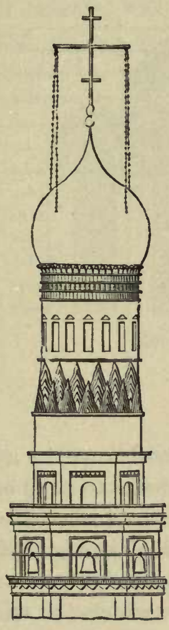


Fig. 1351.

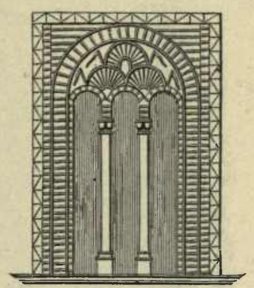


Fig. 1352.

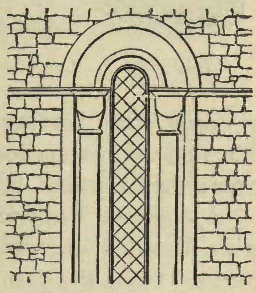


Fig. 1353.

Windows.—Before the use of painted glass, as very small apertures sufficed for the introduction of the required quantity of light into a church, the windows of the Romanesque churches were generally small, and devoid of tracery; and as the Byzantine architects, adorning the walls with paintings, could not use stained glass, they followed in general form the Romanesque window, apertures with circular heads, either single or in groups (Fig. 1353 or Fig. 1352). The Norman windows were also small, each consisting of a single light, semicircular in the head, and placed as high as possible above the ground; at first splayed on the inside only, afterward the windows began to be recessed with moldings and jamb-shafts in the angles, as in Fig. 1353.

The Lancet, in general use in the early Gothic period, was of the simplest arrangement: in these windows the glass was brought within three or four inches of the outside of the wall, and the openings were widely splayed in the interior. The proportions of these windows vary considerably; in some the height being but five times the width, in others as much as eleven; eight or nine times may be taken as the average. Lancet windows occur singly (Fig. 1354), or in groups of two, three, five, and seven, rarely of four and six. The triplet (Fig. 1355) is the most beautiful arrangement of lancet windows. It was customary to mark with greater importance the central light, by giving it additional height, and in most cases increased width also. In some examples the windows of a lancet triplet are placed within one drip-stone forming a sin-

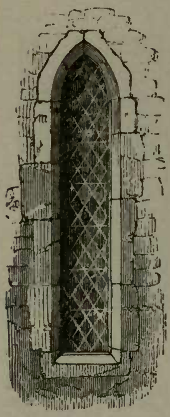


Fig. 1354.

Fig. 1355.

Fig. 1356.

gle arch, thus bearing a strong resemblance to a single three-light window. The first approximation to tracery appears to have been the piercing of the space over a double lancet window comprised within a single drip-stone (Fig. 1356).

A traceried window is a distinctive characteristic of Gothic architecture; with the establishment of the principle of window tracery the mullions were recessed from the face of the wall in which the window arch was pierced, and the fine effect thus produced was speedily enhanced by the introduction of distinct orders of mullions, and by recessing certain portions of the tracery from the face of the primary mullions and their corresponding tracery bars.

Decorated window tracery is divided into two chief varie-

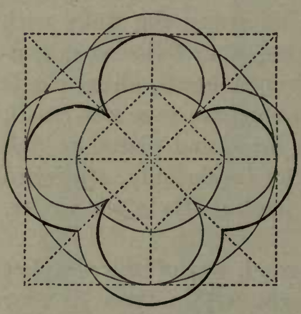


Fig. 1357.

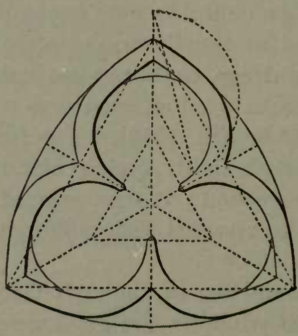


Fig. 1358.

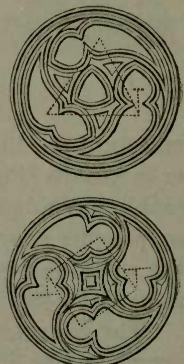


Fig. 1359.

ties, Geometrical and Flowing; the former consisting of geometrical figures, as circles, trefoils, quatrefoils, curvilinear triangles, lozenges, etc.; while in flowing tracery these figures, though still existing, are gracefully blended together in one design.

Fig. 1357 represents a quatrefoil window, Fig. 1358 a pointed trefoil in outline with the centers of the different circles and such constructive lines indicated as may be necessary. Fig. 1359 represents two forms of circular windows, or *roses tournantes.*

Fig. 1360 represents an example of the earlier decorated tracery window-head, consisting of two foiled lancets, with a pointed quatrefoil in the spandrel between them. One half of the windows in this, as in some of the following figures, is drawn in skeleton, to explain their construction. Fig. 1361 is another example of Decorated tracery.

Fig. 1360.

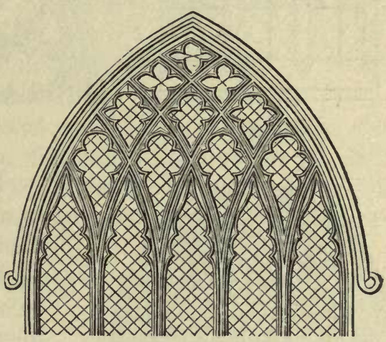


Fig. 1361.

Fig. 1362 is an example of the English leaf tracery; Fig. 1363, of the French flamboyant. The difference between the two styles is, that while the upper ends of the English loops or leaves are round, or simply pointed; the upper ends of the latter terminate, like their lower ones, in angles of contact, giving a flame-like form to the tracery bars and form pieces.

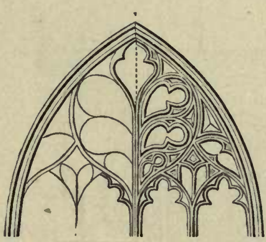


Fig. 1362.

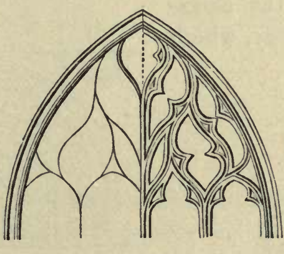


Fig. 1363.

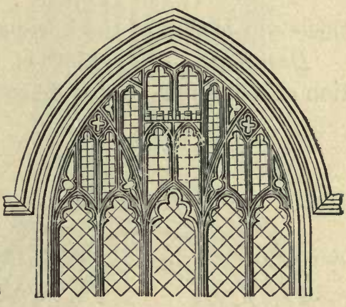


Fig. 1364.

In England the Perpendicular style succeeded the Decorated; the mullions, instead of diverging in flowing or curvilinear lines, are carried up straight through the head of the windows; smaller mullions spring from the head of the principal lights, and thus the upper portion of the window is filled with panel-like compartments. The principal as well as the subordinate lights are foliated in their heads, and large windows are often divided horizontally by transoms. The forms of the window arches vary from simple pointed to the complex four-centered, more or less depressed.

Fig. 1364 is an example of a Perpendicular window.

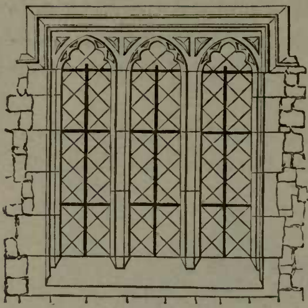


Fig. 1365.

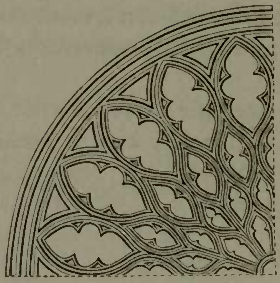


Fig. 1366.

Fig. 1367.

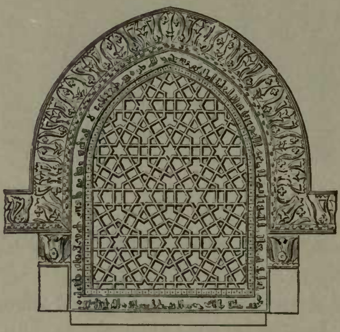


Fig. 1368.

Fig. 1365 is a square-headed window, such as were usual in the clear-stories of Perpendicular architecture.

Figs. 1366 and 1367 are quadrants of circular windows, used more especially in France, for the adornment of the west ends and transepts of the cathedrals.

Besides the tracery characteristic of Gothic architecture, there is a tracery peculiar to the Saracenic and Moorish style, of which Fig. 1368 may be taken as an example—it being a window of one of the earliest mosques. The general form of the window and door-heads of this style is that of the horse-shoe, either circular or pointed.

Doorways.—Fig. 1369 is the elevation of a circular-headed doorway, which

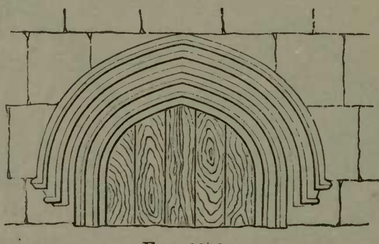


Fig. 1370.

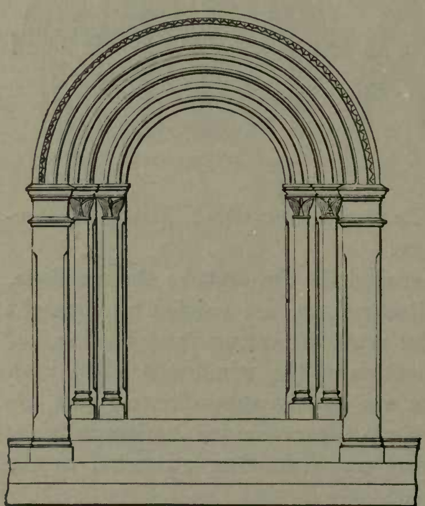


Fig. 1369.

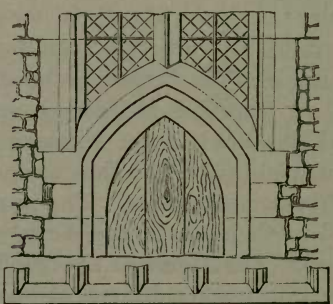


Fig. 1371.

may be considered the type of many entrances both in Romanesque, Gothic, and later styles. It consists of two or more recessed arches, with shafts or moldings in the jambs. In the earlier styles the arches were circular, in the later Gothic, generally pointed, but sometimes circular; in the earlier, the angles in which the shafts are placed are rectangular; in the later, the shaft is often molded on a *chamfer* plane, that is, a plane inclined to the face of the wall, generally at an angle of 45°; often the chamfer and rectangular planes are used in connection.

Fig. 1370 is a simple head of a depressed four-centered or Tudor-arched doorway, with a hood molding.

Fig. 1371 represents the incorporation of a window and doorway. Sometimes the doorway pierces a buttress; in that case, the buttress expands on either side, forming a sort of porch. The Gothic architects placed doors where they were necessary, and made them subservient to the beauty of the design.

Fig. 1372 is an example of a gabled doorway with crockets and finial.

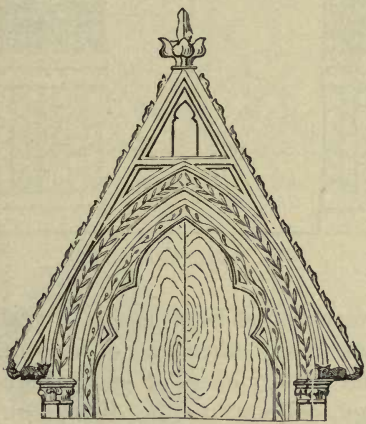


Fig. 1372.

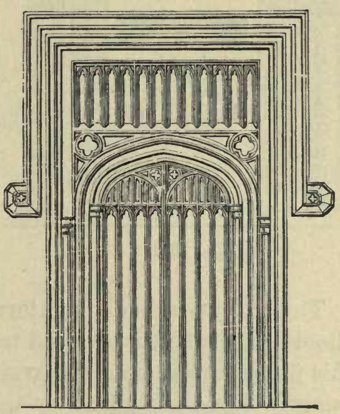


Fig. 1373.

Fig. 1373 is an example of a perpendicular doorway, with a label or hood molding above, and ornamented spandrels.

Fig. 1374 is an example of a Byzantine, and Fig. 1375 of a Saracenic doorway.

The Renaissance style was, originally, but the revival or a fair rendering of the classical orders of architecture, with ornaments from the Byzantine and Saracenic styles.

Garbett divides this style into three Italian schools, the Florentine, Venetian, and Roman. The Florentine admits of little apparent ornament, but any degree of real richness, preserving in its principal forms severe *contrast;* powerful masses self-poised without corbeling, without arching; *breadth* of everything, of light, of shade, of ornament, of plain wall; *depth* of recess in the openings, of perspective in the whole mass, of projection in the cornice. Absence of features useless to convenience or stability, admitting of great plainness, or of very florid enrichment.

The aim of the Venetian school was splendor, variety, show, and ornament; not so much real as effective ornament. Thus, it rarely contains as much carving or minute enrichment as the Florentine admits; but it has larger ornaments, constructed (or built) ornaments, great features useless except for ornament, such as inaccessible porticoes, detached columns, and architraves supporting no ceiling, towers built only for breaking an outline.

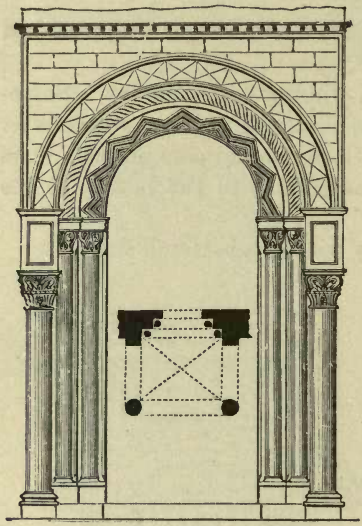


FIG. 1374.

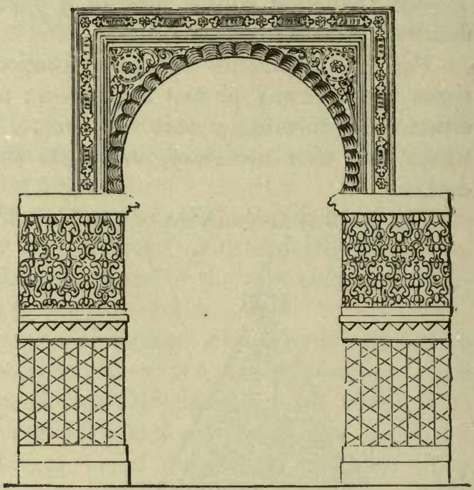


FIG. 1375.

The Roman school is intermediate in every respect between the two other schools. It is better adapted to churches than to any other class of buildings. This fitness arises from the grand, simple, and unitary effect of one tall order, generally commencing at or near the ground, obliterating the distinction of two or three stories, making a high building appear a single story.

Moldings.—"All classical and Romanesque architecture is composed of bold independent shafts, plain or fluted, with bold detached capitals forming arcades or colonnades where they are needed, and of walls whose apertures are surrounded by courses of parallel lines called moldings, and have neither shafts nor capitals. The shaft system and molding system are entirely separate; the Gothic architects confounded the two; they clustered the shafts till they looked like a group of moldings, they shod and capitaled the moldings till they looked like a group of shafts." The moldings appear in almost every conceivable position; from the bases of piers and piers themselves, to the ribs of the fretted vaults which they sustain.

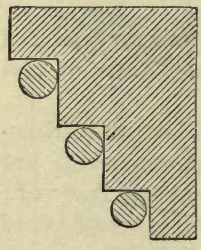


FIG. 1376.

In the earliest examples of Norman doorways the jambs are mostly simply squared back from the walls; recessed jambs succeeded, and are common in both Norman and Gothic architecture; and when thus recessed, detached shafts were placed in each angle (Fig. 1376).

In the later styles the shafts were almost invariably attached to the structure. The angles themselves were often cut or chamfered off, and the moldings attached to the chamfer-plane. The arrangement of window jambs, during the successive periods, was in close accordance with that of doorways.

In the richer examples small shafts were introduced, which, rising up to the springing of the window, carried one or several of the arch moldings. Yet

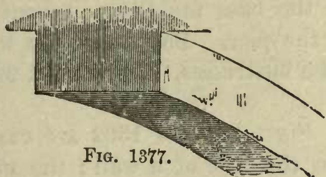


Fig. 1377.

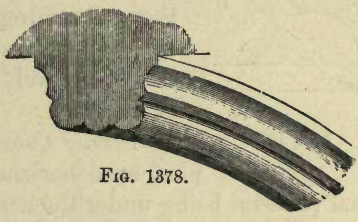


Fig. 1378.

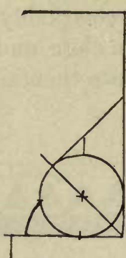


Fig. 1379.

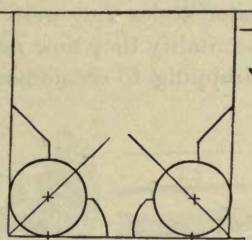


Fig. 1380.

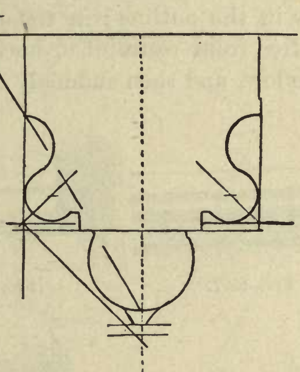


Fig. 1381.

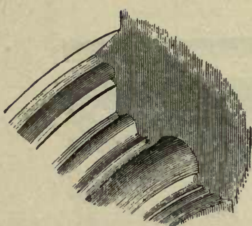


Fig. 1382.

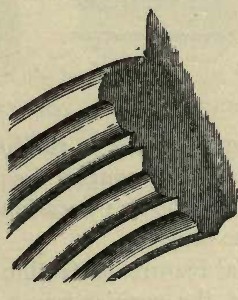


Fig. 1383.

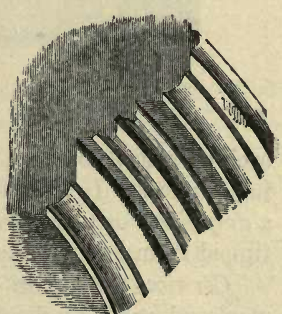


Fig. 1384.

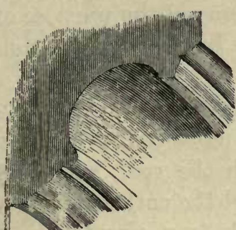


Fig. 1385.

moldings are nevertheless not essential accessories; many windows of the richest tracery have their mullions and jambs composed of simple chamfers.

Figs. 1377 to 1385 are examples of *arch and architrave moldings*, which, even when not continuous, partook of the same general arrangement as those in the jambs, with greater richness of detail. When shafts were employed, they carried groups of moldings more elaborate than those of the jambs, though still falling on the same planes.

Capitals were either molded or carved with foliage, animals, etc. ; they always consisted of three distinct parts (Fig. 1386)—the head mold (A), the bell (B), and the neck mold (C). In Norman examples the head mold was almost invariably square ; in the later styles it is circular, or corresponding to the form of the pillar.

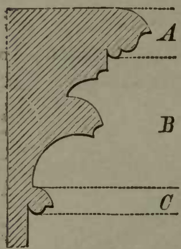


Fig. 1386.

Bases consist of the plinth and the base moldings. The plinth was square in the Norman style, afterward octagonal ; then, assuming the form of the base moldings, it bent in and out with the outline of the pier. Base moldings were also extensively used round the buttresses, towers, and walls of churches.

String Courses, of which Figs. 1387 to 1392 are examples, were horizontal courses in the face of a wall ; the most usual position being under the windows. In the Norman styles they were usually heavy in the outline ; in the later styles they were remarkably light and elegant ; free from restraint or horizontality they now rose close under the sill of the window, and then suddenly dropping to accommodate themselves to the

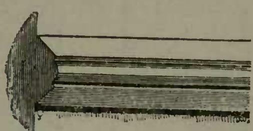


Fig. 1387.

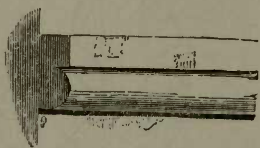


Fig. 1388.

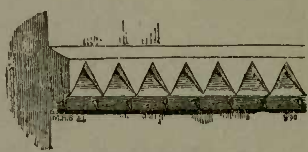


Fig. 1389.

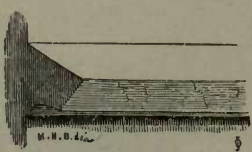


Fig. 1390.

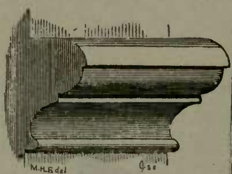


Fig. 1391.

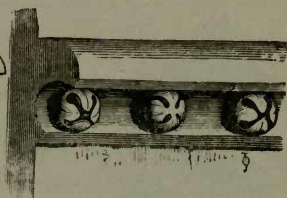


Fig. 1392.

arch of a low doorway, and again rising to run immediately under the adjoining window. In this way the string courses frequently served the purpose of a *drip-stone* or hood molding over doors ; occasionally the hood mold was continued from one window to the other.

Cornices are not an essential feature in Gothic architecture. In the Norman and early English styles, the cornice was a sort of enlarged, projecting string course, forming a drip-stone beneath the roof, which, if supported on brackets or *corbels,* was termed the corbel table.

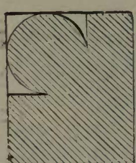


Fig. 1393.

The earliest molding in Norman work is a circular bead strip, worked out of the edges of a recessed arch, called a circular *bowtel* (Fig. 1393). From a circular form the bowtel soon became pointed, and, by an easy transition, into the bowtel of one, two, or three fillets.

Figs. 1394 to 1399 are sections of Romanesque drip- or cap stones, adapted to different pitches of roof.

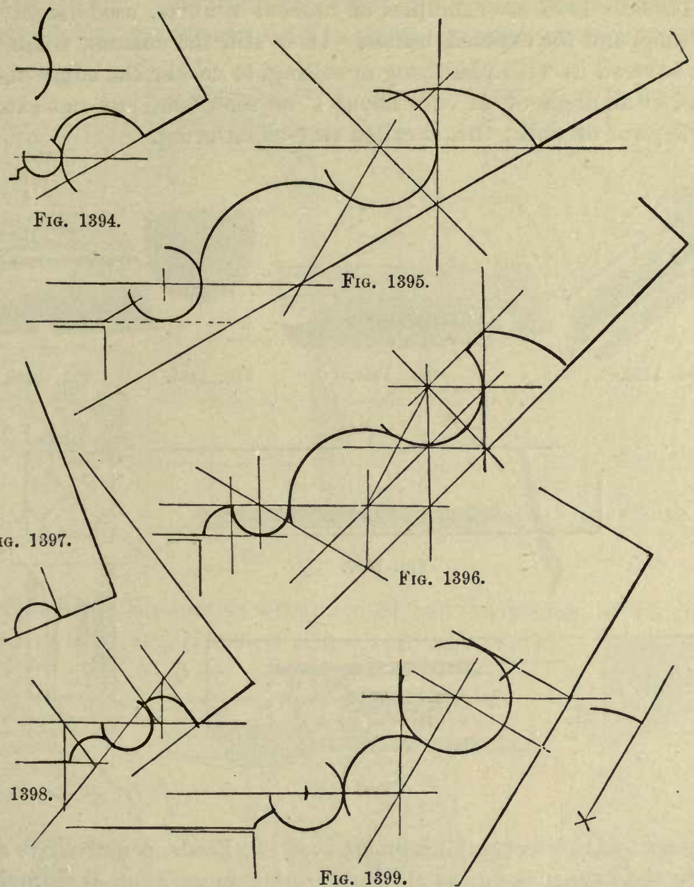


FIG. 1394. FIG. 1395. FIG. 1396. FIG. 1397. FIG. 1398. FIG. 1399.

Fig. 1400 is the *scroll* molding; a simple filleted bowtel, with the fillet undeveloped on one side, as shown by the dotted lines. If this molding be cut in half, through the center of the fillet, we have on the developed side the molding now termed by carpenters the rule joint, which, by rounding off the corners by reverse curves, becomes the wave molding.

FIG. 1400.

Fig. 1401 is a Gothic example of the filleted bowtel with prominent alternate hollows.

FIG. 1401.

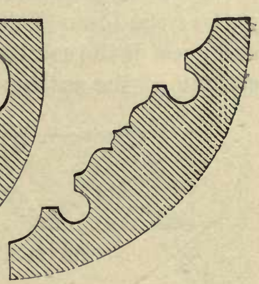


FIG. 1402.

Fig. 1402 is an example of the perpendicular style, an insignificant hollow separating groups of moldings.

Figs. 1403 to 1408 are examples of molded timbers, used largely in open-timbered roofs and for exposed beams. It is still the custom, when the framing is not covered in with plastering or ceiling, to corner the edges of the joists and beams, at an angle of 45°, for about 1″ on each face, but not extending it close to the joint or wall ; this is called stop-chamfering.

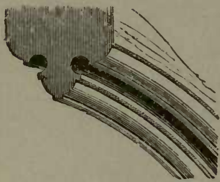


Fig. 1403.

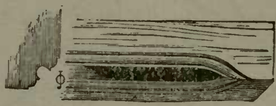


Fig. 1404.

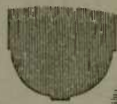


Fig. 1405.

Fig. 1406.

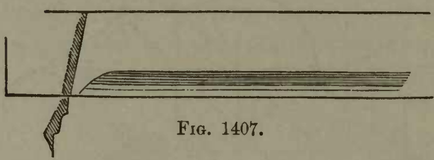
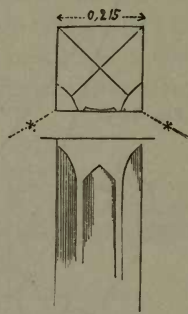


Fig. 1407.



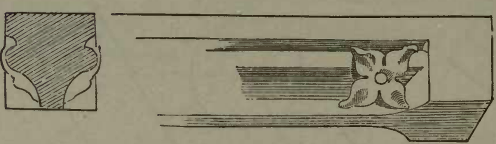


Fig. 1408.

Ornament.—Architectural ornament is of two kinds, constructive and decorative. By the former is meant all those contrivances, such as capitals, brackets, vaulting-shafts, and the like, which serve to explain or give expression to the construction ; by the latter, such as moldings, frets, foliage, etc., which give grace and life, either to the actual constructive form, or to the constructive decoration. Moldings of the different styles have been already treated of ; it is proposed to give now what are even more purely decorations of a style.

In the Grecian orders the Doric (Fig. 1286) has the triglyph mutules and guttæ ; the Ionic (Fig. 1291) has various moldings of the cornice, frieze, abacus, and neck of the column enriched. The principal ornament of the neck of the column is the anthemion, commonly known, in its most simple form, as the

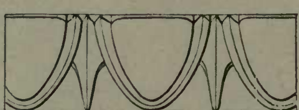


Fig. 1409.

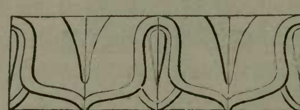


Fig. 1410.

honeysuckle or palmetto ; in the anthemion, as represented in the figure, the palmetto alternates with the lily or some analogous form. The ornament of the abacus is the egg and dart (Fig. 1409) ; the ornament of the frieze and cornice (Fig. 1410). The fret (Fig. 1411) and the guilloche (Fig. 1412) are also common Greek ornaments, used to adorn the soffits of beams and ceilings.

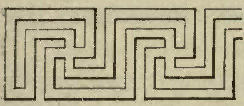


Fig. 1411.

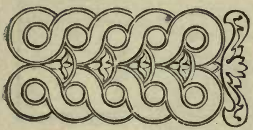


Fig. 1412.

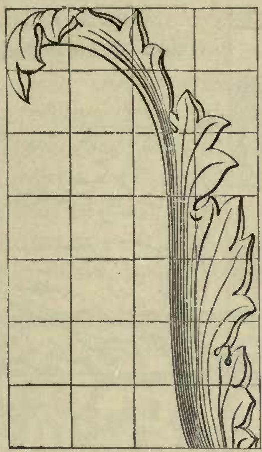


Fig. 1413.

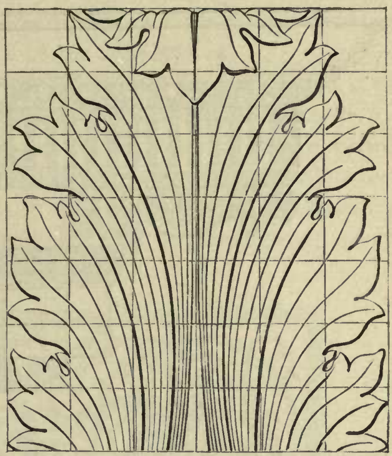


Fig. 1414.

The acanthus is the distinctive ornament of the Corinthian, of which a leaf is represented in front and side view (Figs. 1413 and 1414).

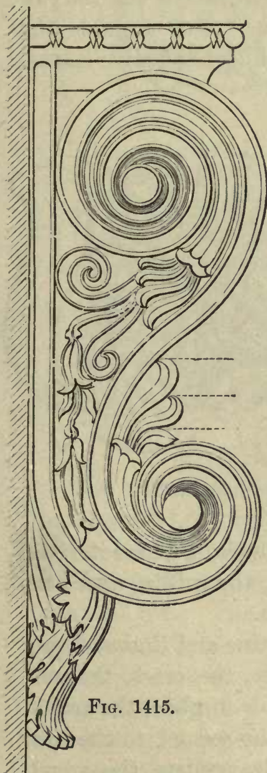


Fig. 1415.

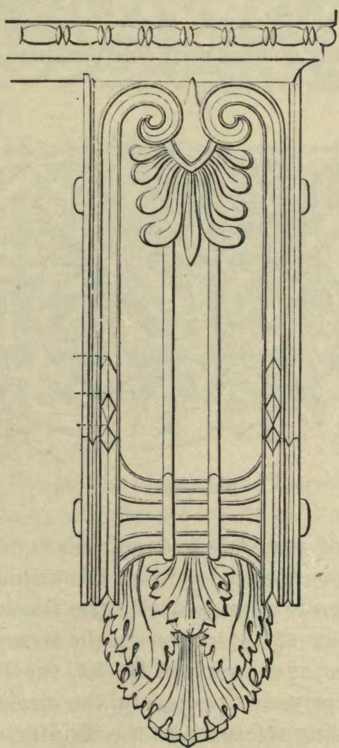


Fig. 1416.

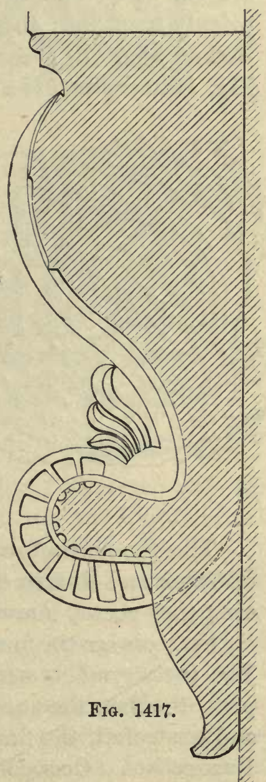


Fig. 1417.

Figs. 1415, 1416, and 1417 are the side elevation, front elevation, and section of a Greek bracket, the principal ornaments of which are taken from the anthemion and acanthus.

Fig. 1418 is an elevation of a portion of an enriched cornice from the

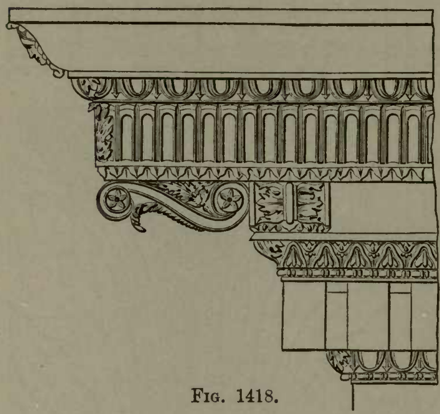


Fig. 1418.

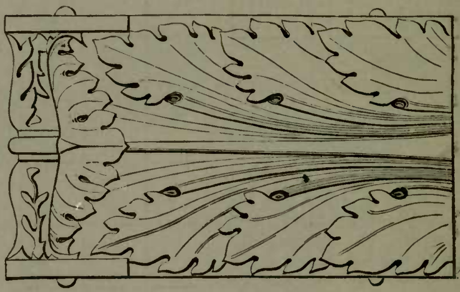


Fig. 1419.

temple of Jupiter Stator, at Rome, of the Corinthian order of architecture. Fig. 1419 is the under side of the modillion, on a larger scale.

The chief characteristic of Roman ornament is its uniform magnificence, an enrichment of the Greek. The most used elements of the Roman decorations are the scroll and the acanthus. The acanthus of the Greeks is the narrow prickly acanthus; that of the Roman, the soft acanthus. For capitals the Roman acanthus is commonly composed of conventional clusters of olive-leaves. Fig. 1420 represents a Roman acanthus scroll.

Fig. 1420.

The free introduction of monsters and animals is likewise a characteristic of Greek and Roman ornament, as the sphinx, the triton, the griffin, and others ; they occur, however, more abundantly in the Roman.

Symbols are the foundation of decorations in the Byzantine and Romanesque. The early symbols were the monogram of Christ, the lily, the cross, the serpent, the fish, the aureole, or vesica piscis, and the circle or nimbus, the trefoil and quatrefoil, the first having reference to the Trinity, the second to the four Evangelists. Occasionally the symbolic images of the Evangelists, the angel, the lion, the ox, and the eagle, are represented within these circles. The hand in the attitude of benediction, and the lily (the fleur-de-lis), the emblem of the virgin and purity, are common ; also a peculiarly formed leaf, somewhat resembling the leaf of the ordinary thistle. The serpent figures largely in Byzantine art as the instrument of the fall, and one type of the redemption.

Pagan ornaments, under certain symbolic modifications, were admitted into Christian decorations. Thus the foliations of the scroll were terminated by lilies, or by leaves of three, four, and five blades, the number of blades being significant ; and in a similar way the anthemion and every other ancient ornament. In the Byzantine, all their imitations of natural forms were invariably conventional ; it is the same even with animals and the human figure ; every saint had his prescribed colors, proportions, and symbols.

Fig. 1421.

Fig. 1422.

The Saracenic was the period of gorgeous diapers (Figs. 1421 and 1422), for their habit of decorating the entire surfaces of their apartments was highly favorable to the development of this class of design. The Alhambra displays almost endless specimens, and all are in relief and enriched with gold and color, chiefly blue and red. The religious cycles and symbolic figures of the Byzantine are excluded. Mere curves and angles or interlacings were now to bear the chief burden of a design, but distinguished by a variety of color. The curves, however, very naturally fell into standard forms and floral shapes, and the lines and angles were soon developed into a very characteristic species of tracery, or interlaid strap-work, very agreeably diversified by the ornamental introduction of the inscriptions, which last custom of elaborating inscriptions with their designs was peculiarly Saracenic. Although flowers were not palpably admitted, yet the great mass of the minor details of Saracenic designs are composed of flower forms disguised—the very inscriptions are sometimes thus grouped as flowers; still, no actual flower ever occurs, as the exclusion of all natural images is fundamental to the style in its purity.

All the symbolic elements of the Byzantine are continued in the Gothic. Ornamentally, the Gothic is the geometrical and pointed element elaborated to the utmost; its only peculiarities are its combinations of details; at first the conventional and geometrical prevailing, and afterward these combined with the elaboration of natural objects in its decoration. The most striking feature of all Gothic work is the wonderful elaboration of its geometric tracery; vesicas, trefoils, quatrefoils, cinquefoils, and an infinity of geometric varieties besides. The tracery is so paramount a characteristic that the three English varieties, the early English, the decorated, and the perpendicular, and the French flamboyant, are distinguished almost exclusively by this feature. (See Figs. 1360 to 1364.)

The ornamental moldings used in the decorative details are numerous, among which the more common is the chevron or zigzag (Fig. 1423), simple

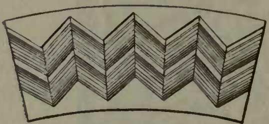


Fig. 1423.

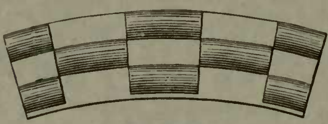


Fig. 1424.

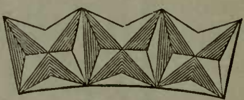


Fig. 1425.


Fig. 1426.

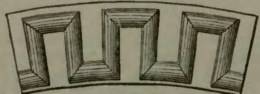


Fig. 1427.


Fig. 1428.


Fig. 1429.

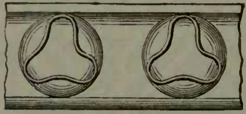


Fig. 1430.

as the indented, or duplicated, triplicated, or quadrupled; the billet, the prismatic billet, the square billet, and the alternate billet (Fig. 1424); the star (Fig. 1425), the fir-cone; the cable (Fig. 1426); the embattled (Fig. 1427); the nail-head (Fig. 1428), the dog-tooth (Fig. 1429); the ball-flower (Fig. 1430), and the serpentine vine-scroll.

The crocket, in its earliest form, was the simple arrow-head of the episcopal pastoral staff; subsequently finished with a trefoil, and afterward still further enriched. Figs. 1431 and 1432 are early English crockets; Fig. 1433 a decorated one. Fig. 1434 is a finial of the same style. Both finials and crockets in detail display a variety of forms.

The parapets of the early English style are often a simple horizontal course, supported by a corbel table, sometimes relieved by a series of sunk blank trefoil-headed panels; sometimes a low embattled parapet crowns the wall. In the decorated style the horizontal parapet is sometimes pierced with trefoils, sometimes with wavy, flowing tracery (Fig. 1435). Grotesque spouts or gargoyles discharge the water from the gutters. The parapets of the perpendicular style

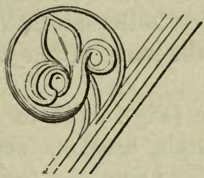


Fig. 1431.

Fig. 1433.

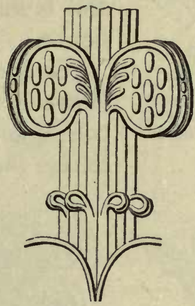


Fig. 1432.

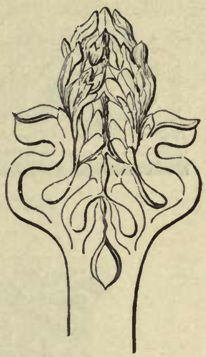


Fig. 1434.

are frequently embattled (Fig. 1436), covered with sunk or pierced paneling, and ornamented with quatrefoil, or small trefoil-headed arches ; sometimes

Fig. 1435.

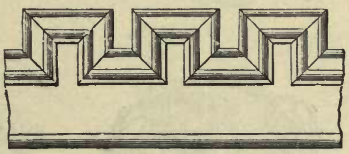


Fig. 1436.

not embattled but covered with sunk or pierced quatrefoils in circles, or with trefoils in triangular spaces, as in Fig. 1437.

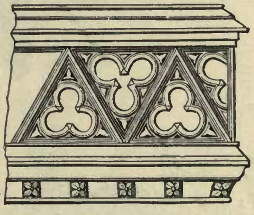


Fig. 1437.

Among the varieties of ornamental work, the mode of covering small plain surfaces with diapering (Fig. 1438) was sometimes used ; the design being in exact accordance with the architectural

Fig. 1438.

Fig. 1439.

features and details of the style. The rose (Fig. 1439), the badge of the houses of York and Lancaster, is often met with in the perpendicular style ; and tendrils, leaves, and fruit of the vine are carved in great profusion in the hollows of rich cornice moldings, especially on screen-work in the interior of a church. Fig. 1440, in its original type a Byzantine ornament, an alternate lily and cross, is a common finish to the cornice of rich screen-work in the latest Gothic, and is known under the name of the Tudor flower.

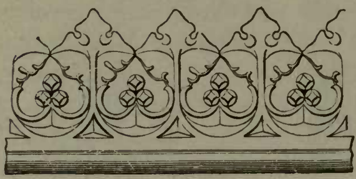


Fig. 1440

Sculptured foliage (Figs. 1441 to 1446) is much used in capitals, brackets, corbels, bosses, and crockets. Among the forms of foliage the trefoil is most predominant.

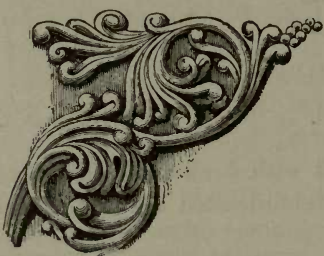


Fig. 1441.

Fig. 1442.

Fig. 1443.

Fig. 1444.

Fig. 1445.

Fig. 1446.

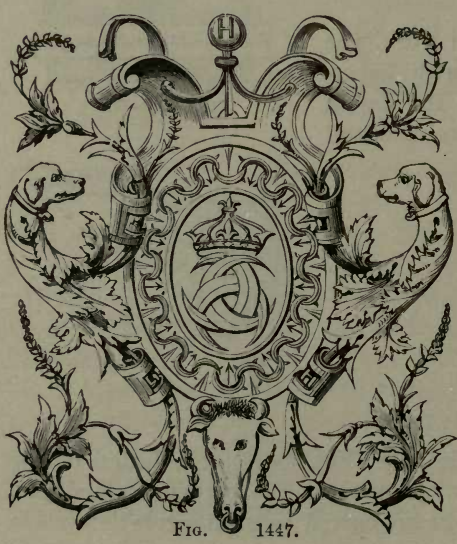


Fig. 1447.

The Ornaments of the Renaissance.—The term Renaissance is used in a double sense; in a general sense implying the revival of art, and specially signifying a peculiar style of ornament. It is also sometimes, in a very confined sense, applied in reference to ornament of the style of Benvenuto Cellini; or, as it is sometimes designated, the Henry II (of France) style.

The mixture of various elements is one of the essentials of this style. These elements are the classical ornaments; unnatural and natural flowers and foliage; men and animals, natural and grotesque; cartouches, or pierced and scrolled shields, in great prominence; tracery independent, and developed from the scrolls of the cartouches; and jewel forms (Fig. 1447 and 1448).

The Elizabethan is a partial elaboration of the same style; the present Elizabethan exhibits a very striking preponderance of strap and shield work; but

Fig. 1448.

the earlier is much nearer allied to the Continental styles of the time, classical ornaments but rude in detail, occasional scroll and arabesque work, and strapwork, holding a much more prominent place than the pierced or scrolled shields. Fig. 1449 is an example of the style from the old guard chamber, Westminster.

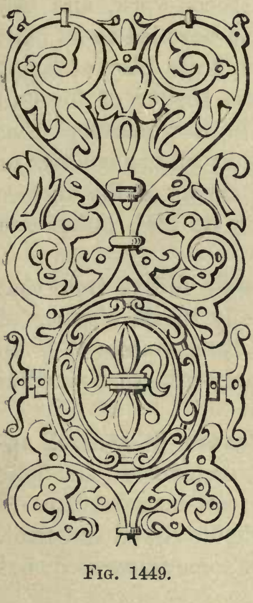


Fig. 1449.

Fig. 1450.

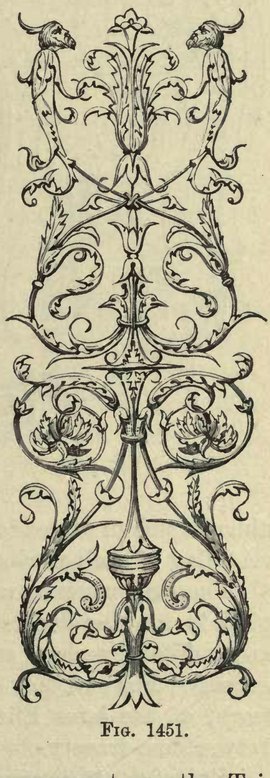


Fig. 1451.

Of the earliest and transition styles of Renaissance ornament are the Tricento and the Quatrecento; the great features of the first are its intricate tracery and delicate scroll-work of conventional foliage, the style being but a slight remove from the Byzantine and Saracenic; of the second, elaborate natural imitations of fruit, flowers, birds, or animals (Fig. 1450), all disposed simply with a view to the ornamental; also occasional cartouches, or scrolled shield-work.

The Renaissance is something more approximative to a combination of previous styles than a revival of any in particular, developed solely on æsthetic principles, from a love of the forms and harmonies themselves, as varieties of effect and arrangements of beauty, not because they had any particular signification, or from any superstitious attachment to them as heirlooms.

Fig. 1451 is an example of ornament in the Cinquecento style. The arabesque scroll-work is the most prominent feature of the Cinquecento, and with this in its elements, it combines every other feature of classical art, with the unlimited choice of natural and conventional imitations from the entire animal and vegetable kingdom, both arbitrarily disposed and combined. Absolute works of art, such as vases and implements, and instruments of all kinds, are prominent elements of the Cinquecento arabesque, but cartouches and strap-work wholly disappear from the best examples. Another chief feature of the Cinquecento is the admirable play of color in its arabesques and scrolls; and it is worthy of note that the three secondary colors, orange, green, and purple, perform the chief parts in all the colored decorations.

Fig. 1452.

Fig. 1452 is an example of the Louis Quatorze style of ornament. The great medium of this style was gilt stucco-work, and this absence of color seems to have led to its most striking characteristic, infinite play of light, of shade; color, or mere beauty of form in detail, having no part in it whatever. Flat surfaces are not admitted; all are concave or convex: this constant varying of the surface gives every point of view its high lights and brilliant contrasts.

The Louis Quinze style differs from that of Louis Quatorze chiefly in its absence of symmetry; in many of its examples it is an almost random dispersion of the scroll and shell, mixed only with that peculiar crimping of shell-work, the coquillage.

The ornaments of which we have thus given examples are, in general, applied to interior decorations, to friezes, pilasters, panels, architraves, the faces and soffits of arches, ceilings, etc., to furniture, and to art-manufactures in general. For exteriors these ornaments are sparingly applied; shield and scroll work, of the later Elizabethan or Renaissance style, is sometimes used, but very seldom tracery.

Principles of Design.—Professedly treating of architecture only in its most mechanical phase of drawing, the history of it as an art, and the distinctions of styles, have been but briefly treated. To one anxious to acquire knowledge in this department we refer, as the very best compendium within our knowledge, to Ferguson's "Hand-Book of Architecture." The study of this work will give direction to a person's observation, but, without referring to actual examples, mere reading will be of little use. Drawings give general ideas of the character of buildings, but no idea of size or of the surroundings of a building. Many a weak design, especially in cast-iron buildings, acquires a sort of strength by the number of its repetitions, giving an idea of extent ; and many a beautiful design on paper has failed in its execution, being dwarfed by its surroundings. With regard to the style of a building, there are none of the ancient styles in their purity adapted to present requirements ; our churches and theatres are more for the gratification of the ear than the eye, and the comforts of our domestic architecture, and the requirements of our stores and warehouses, are almost the growth of the present century. For a design, look first to the requirements of the structure, the purposes to which it is to be applied ; sketch the plan first, arrange the divisions of rooms, the openings for doors and windows, construct the sections, and then the elevations, first in plain outline ; modify each by the exigencies of construction.

"Construction, including in the term the disposition of a building in reference to its uses, is by some supposed to be the common part of the art of architecture, but it is really the bone, muscle, and nerve of architecture, and the arts of construction are those to which the true architect will look, rather than to rules and examples, for the means of producing two at least of the three essential conditions of building well, *commodity*, *firmness*, and *delight*, which conditions have been aptly said to be the end of architecture as of all creative arts.

"The two great principles of the art are : First, that there should be no features about a building which are not necessary for convenience, construction, or propriety ; second, that all ornament should consist of enrichment of the essential construction of the building.

"The neglect of these two rules is the cause of all the bad architecture of the present time. Architectural features are continually tacked on buildings with which they have no connection, merely for the sake of what is termed effect, and ornaments are continually constructed instead of forming the decoration of construction to which in good taste they should always be subservient. The taste of the artist ought to be held merely ancillary to truthful disposition for structure and service. The soundest construction is the most apt in the production or the reproduction, it may be, of real art. The Eddystone Lighthouse is well adapted to its uses ; it is commodious, firm and stable almost to a miracle, and its form is as beautiful in outline to the delight of the eye, as it is well adapted to break and mitigate the force of the sea in defense of its own structure. The Great Exhibition Building of 1851 was most commodious for the purposes of an exhibition, firm enough for the temporary purpose required of it, and there was delight in the simplicity and truth of its combinations ; and all this may be said to have grown out of propriety of construction, as applied to the material, cast-iron. The use of unfitting material, or fitting material inappropriately, leads almost entirely to incommodiousness, infirmity, and offense, or some of them.

"Out of truth in structure, and that structure of a very inartificial sort, grow the beautiful forms of the admirable proportions found in the works of the Greeks; and out of truth in structure, with the strictest regard to the necessities of the composition and of the material employed, and that structure as full of artifice as the artifice employed is of truth and simplicity, grew the classical works vulgarly called Gothic, but now characteristically designated as Pointed, from the arch which is the basis of the style. Structural untruth is not to be justified by authority; neither Sir Christopher Wren, nor the Athenian exemplars of Doric or Ionic in the Propylæum and in the Minerva Polias, with their irregular and inordinately wide intercolumniation, can persuade even the untutored eye to accept weakness for strength, or what is false for truth.

"The Greek examples offer the most beautiful forms for moldings, and the Grecian mode of enriching them is unsurpassed. It should be borne in mind that the object in architectural enrichment is not to show ornament, but to enrich the surface by producing an effective and pleasing variety of light and shade; but still, although ornament should be a secondary consideration, it will develop itself, and therefore should be of elegant form and composition."

We have quoted thus at some length from the article "Architecture," "Encyclopædia Britannica," because with many authority is necessary, and they distrust their own powers of observation and analysis; all must feel the truth of the above, but in practice it is very little appreciated or carried out. The present taste in architecture, as in the theatre, is for the spectacular; breadth or dignity of effect is not popular; edifices are not only covered with, but built up in ornament; and construction is but secondary. The French, having a building-stone that is very easily worked, cut merely the joints, leaving the rough outer surface to be worked after it is laid; chopping out moldings and ornaments almost as readily as though it were in plaster, and the surface when finished is covered with enrichments in low relief. The fashion thus set is imitated in this country at immense cost, in the most unfitting materials, marble and granite. Our architectural buildings express fitly our condition—a rich country, recent and easily acquired wealth, and a desire and rivalry to exhibit it, or a display as a means of advertising, and in this truth of expression will have an archæological interest; although it does not contribute much to present excellence in construction, it still has this value, that the architect or constructor need be governed by no rules or principles—he can make experiments on a pretty extensive scale, and out of much bad construction even forms and ornament may spring up which will stand the test of time, and form a nucleus of a new style adapted to the present wants.

Cast-iron as a building material, with the exception of exhibition-buildings, has seldom been treated distinctively; buildings erected with it have been copies of those in stone, and have been even imitated in color. For the first story of stores, where space is necessary for light and the exhibition of wares, cast-iron columns are almost invariably used, but are objected to architecturally, that they look too weak for the support of the piles of brick and stone above them. The objection should not be to the use, but that the truth of the adequate strength of the cast-iron is not conveyed by the form or color. No one objects that the ankles of Atlas look too light to support the massive figure and globe, or wishes him seated to give the idea of stability; so if the columns and lintels were some other form than Greek or Roman with immense inter-columniations, and colored fitly, the appearance of weakness would be entirely lost sight of.

In conclusion, the draughtsman should be conversant with classic and later styles, still, as he must design to suit the necessities of the times, and the requirements of present tastes and fashions of buildings, he should keep himself posted on what is being done, and he will find it very convenient to have a scrap-book of cuts from which to draw parts of a design, and afford him ready means of combinations. He will find much in illustrated magazines and newspapers, many cuts unpromising as a whole, yet fruitful in suggestions of parts; many an agreeable outline illy filled up; many that are only valuable as showing dimensions requisite for certain uses. But the larger the collection the better for the draughtsman; it will save time to know, as far as possible, what has been done, that he may judge what forms and proportions it will be best for him to use, and what to avoid.

It has been our practice to select, from papers and magazines, cuts which we considered of value, and arrange them in scrap-books with appropriate headings. In the Appendix a few pages of "scraps" are given as illustrations.

PERSPECTIVE DRAWING.

The science of Perspective is the representation by geometrical rules, upon a plane surface, of objects as they appear to the eye, from any point of view.

All the points of the surface of a body are visible by means of luminous rays proceeding from these points to the eye. Thus, let the line A B (Fig. 1453) be placed before the eye, C, the lines drawn from the different points 1, 2, 3, 4, etc., represent the visual rays emanating from each of these points. It is easy to understand that, if in the place of a line a surface is substituted, the result will be a pyramid of rays.

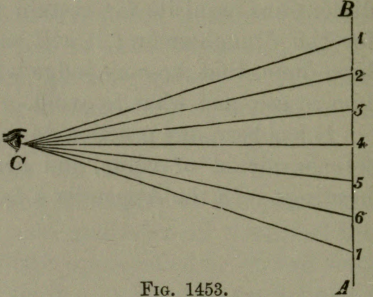


FIG. 1453.

Let A B (Fig. 1454) be a straight line, and let the globe of the eye be represented by a circle, and its pupil by the point C. The ray emanating from A, entering through C, will proceed to the retina of the eye, and be depicted at *a*. And as it follows that all the points of A B will send rays, entering the eye through C, the whole image of A B will be depicted on the retina of the eye in a curved line *a* 3 *b*. Conceive the line A B moved to a greater distance from the eye, and placed at A′ B′, then the optic angle will be reduced, and the image *a*′ 3 *b*′ will be less than before ; and as our visual sensations are in proportion to the magnitude of the image painted on the retina, it may be concluded that the more distant an object is from the eye the smaller the angle under which it is seen becomes, and, consequently, the less it appears.

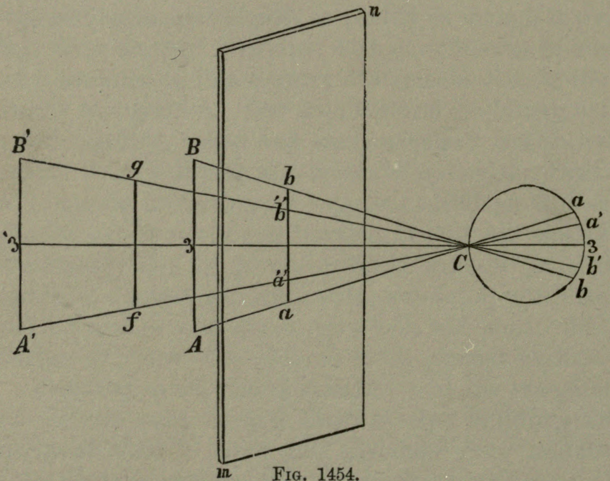


FIG. 1454.

Observation has rendered it evident that the greatest angle under which one or more objects can be distinctly seen is one of 90°. If between the object and the eye there be interposed a transparent plane (such as one of glass, *m n*), the intersections of this plane with the visual rays are termed perspectives of the points from which the rays emanate. Thus *a* is the perspective of A, *b* of B, and so on of all the intermediate points; but, as two points determine the length of a straight line, it follows that *a b* is the perspective of A B, and *a″ b″* the perspective of A′ B′.

It is evident from the figure that objects appear larger or smaller according to the angle under which they are viewed; and further, that objects of unequal size may appear equal if seen under the same angle. For, draw *f g*, and its perspective will be found to be the same as that of A′ B′.

It follows also that a line near the eye may be viewed under an angle much greater than a line of greater dimensions but more distant, and hence a little object may appear to be much greater than a similar object of larger dimensions. Since, therefore, unequally sized objects may appear equal in size, and equally sized objects unequal, and since objects are not seen as they are in reality, but as they appear under certain conditions, perspective may be defined to be a science which affords the means of representing, on any surface whatever, objects such as they appear when seen from a given point of view. It is divided into two branches, the one called linear perspective, occupying itself with the delineation of the contours of bodies, the other called aërial perspective, with the gradations of colors produced by distance. It is the former of these only that is proposed here to be discussed.

The perspective of objects, then, is obtained by the intersection of the rays which emanate from them to the eye, by a plane or other surface (which is called the picture), situated between the eye and the objects.

From the explanation and definition just given, it is easy to conceive that linear perspective is in reality the problem of constructing the section, by a surface of some kind, of a pyramid of rays of which the summit and the base are given. The eye is the summit, the base may be regarded as the whole visible extent of the object or objects to be represented, and the intersecting surface is the picture.

A good idea of this will be obtained by supposing the picture to be a transparent plane, through which the object may be viewed, and on which it may be depicted.

In addition to the vertical and horizontal planes with which we are familiar in the operations of projection, several auxiliary planes are employed in perspective, and particularly the four following:

1. The horizontal plane A B (Fig. 1455), on which the spectator and the object viewed are supposed to stand, for convenience supposed perfectly level, is termed the *ground plane*.

2. The plane M N, which has been considered as a transparent plane placed in front of the spectator, on which the objects are delineated, is called the *plane of projection* or the *plane of the picture*. The intersection M M of the first and second planes is called the line of projection, the *ground*, or *base line of the picture*.

3. The plane E F passing horizontally through the eye of the spectator, and cutting the plane of the picture at right angles, is called the horizontal plane, and its intersection at D D with the plane of the picture is called the *horizon line,* the *horizon* of the picture, or simply the *horizon.*

4. The plane S T passing vertically through the eye of the spectator, and cutting each of the other planes at a right angle, is called the *central plane.*

Point of view, or *point of sight,* is the point where the eye is supposed to be placed to view the object, as at C, and is the vertex of the optical pyramid. Its projection on the ground plane S is termed the *station point.*

The projection of any point on the ground plane is called the seat of that point.

Center of view (commonly, though erroneously, called the point of sight), is the point V where the central vertical line intersects the horizon line; a line drawn from this point to the eye would be in every way perpendicular to the plane of the picture.

Points of distance are points on the horizontal line as remote from the centre of view as the eye.

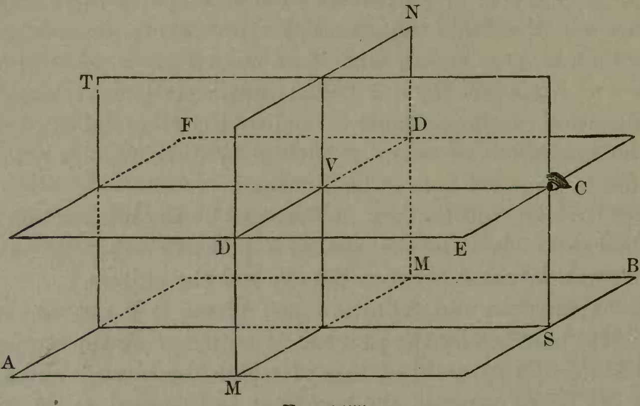


Fig. 1455.

Vanishing points are points in a picture to which all lines converge that in the original object are parallel to each other.

Parallel Perspective.—An object is said to be seen in parallel perspective when one of its sides is parallel to the plane of the picture.

Angular Perspective.—An object is said to be seen in angular perspective when none of its sides are parallel to the picture.

To find the perspective of points, as the points *m, s* (Fig. 1456), in the ground plane, the same letters designating similar planes and points as in Fig. 1455. From the point *m* draw a line to the point of sight C, and also to the station point S; at the intersection of the line *m* S with the base line M S′, erect a perpendicular cutting the line *m* C, the intersection *m′* will be the perspective projection of the point *m,* on the plane of the picture M V. The point *s* being in the central plane, its projection must be in the intersection of that plane by the plane of the picture, at the point *s′* the intersection of the central vertical line by the line *s* C.

In the same way find the perspective $h' m'$ of the line $h m$, and we find that *when an original line is parallel or perpendicular to the base of the picture, the perspective of that line* will also be parallel or perpendicular to it.

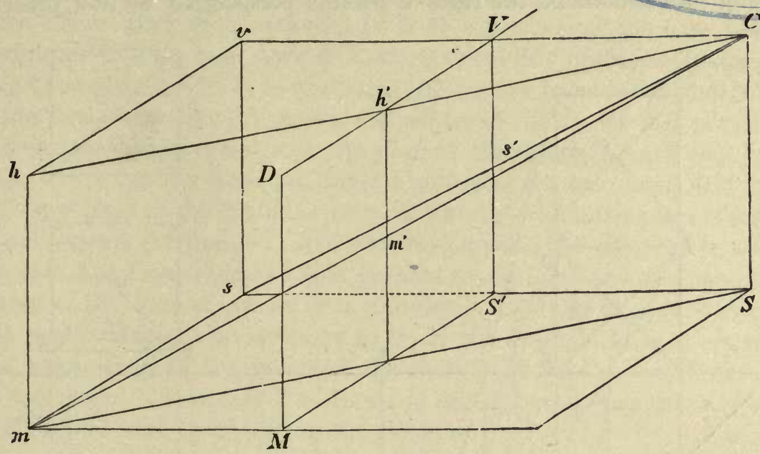


Fig. 1456.

Fig. 1457. Draw the diagonals M s' and m S′, project as in the preceding figure the points m and s into the plane of the picture, draw M m' M S′, and S′ m' ; now, since m and M are the extremities of a line perpendicular to the plane of the picture, the line m' M must be the projection of this line on the plane of the picture, and if this line be extended it will pass through V, which may be demonstrated of all lines perpendicular to the plane of the picture ; *hence the perspective direction of lines perpendicular to the picture is to the center of view.*

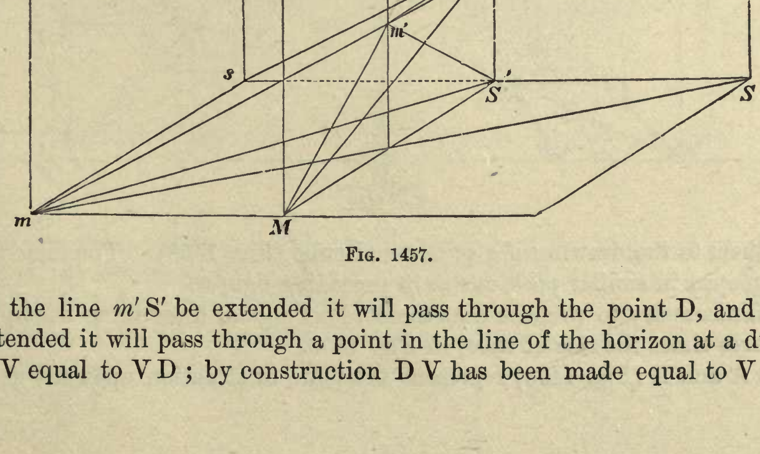


Fig. 1457.

If the line m' S′ be extended it will pass through the point D, and if M s' be extended it will pass through a point in the line of the horizon at a distance from V equal to V D ; by construction D V has been made equal to V C, and as this demonstration is applicable to other similar lines, and since M *m* *s* S′ is a square ; hence *the perspective direction of all lines, making an angle of* 45° *with the plane of the picture, is toward the point of distance.*

Having thus illustrated the rules of parallel perspective, we now proceed to

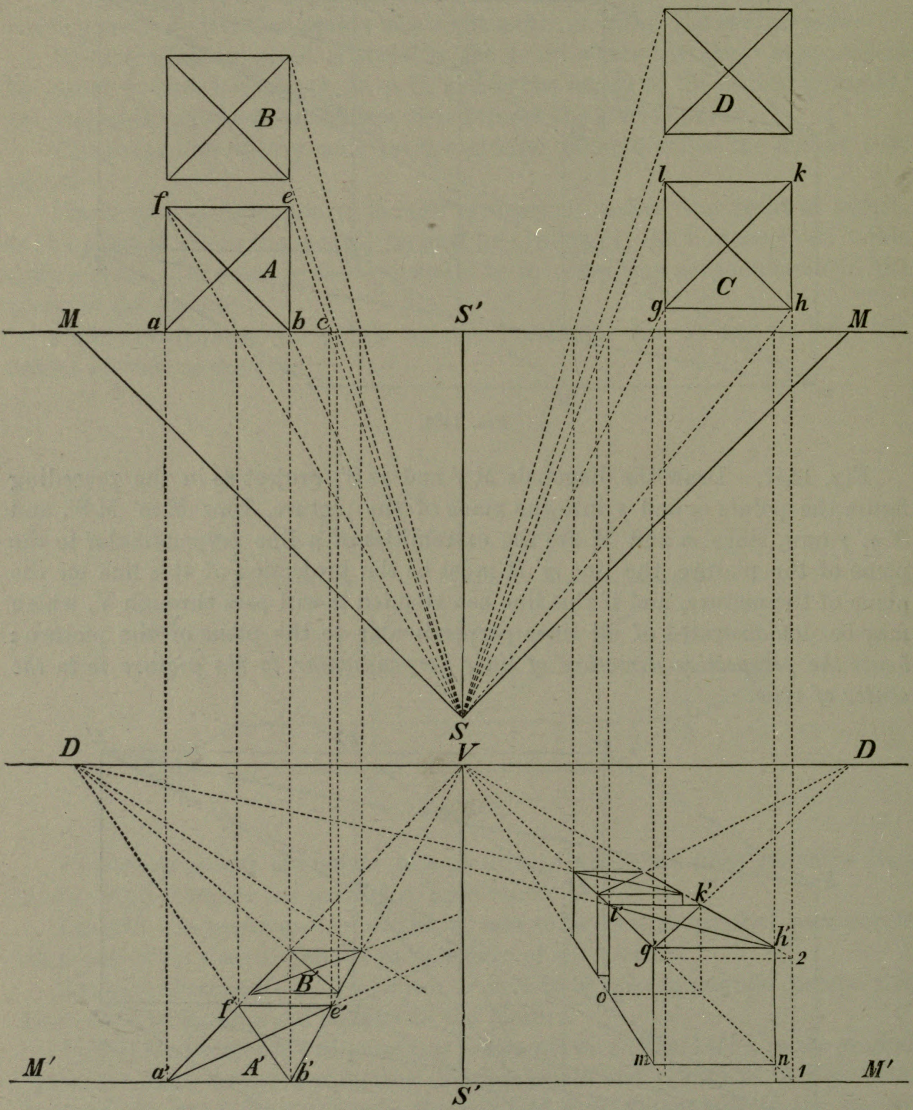


Fig. 1458.

apply them to the drawing of a square and cube (Fig. 1458). The same letters are employed in similar positions as in preceding figures.

It is necessary to premise that the student should draw these examples at least three times the size of those in Fig. 1458.

Let A and B (Fig. 1458) represent the plan, or situation upon the ground, of two squares, of which a perspective representation is required. First draw the line M M, which represents the base line of the picture; make S the station point or place of the observer, and draw lines or rays from all visible angles of the squares, to S; then draw the lines S M, parallel to the diagonal lines of the squares. Now draw M′ M′ parallel to M M representing the base line of the picture in elevation; then draw S′ V, the vertical line immediately opposite the eye; let the distance, S′ V, be the height of the eye from the ground, and draw D D the horizontal line; V being the center of view; let fall perpendicular lines from the angles *a* and *b* of the plan of the square A, and also from the point *c*, where the ray from the angle *e* intersects the base line, M M; from *a′* and *b′* draw lines to the center of view, V; and *e′* where the perpendicular line from *c* intersects the line *b′* V, will give the apparent or perspective width *b′ e′* of the side *b e*; from *e′* draw a line parallel to *a′ b′*, and the perspective representation of the nearest square A is complete. In order to prove the accuracy of this performance, it is necessary to try if the diagonal lines, *a′ e′*, and *b′ f′*, incline respectively to the points of distance, D D, on the horizontal line: if so, it is correct. The square B is drawn in precisely the same manner, and will be easily understood by observing the example.

The plans of the two cubes C and D are the same as the plans of the squares A and B. As neither of these cubes appears to touch the plane of the picture M M, it will be necessary to imagine the sides *l g*, and *k h*, to be continued until they do so; now draw down perpendicular lines from where the continuations of these sides intersect the base line, and set off on them from the line M′ M′, the height of the cube, as 1—2 which is the same as the width, and complete the square shown by the dotted lines; from all four angles of this square draw lines to the center of view—this will give the representation of four lines at right angles with the picture carried on as far as it would be possible to see them; then it only remains to cut off the required perspective widths of the cubes, by the perpendicular lines, from the intersection of the visual rays with the plane of the picture: the completion of this problem will be very easy, if the drawing of the squares is well understood.

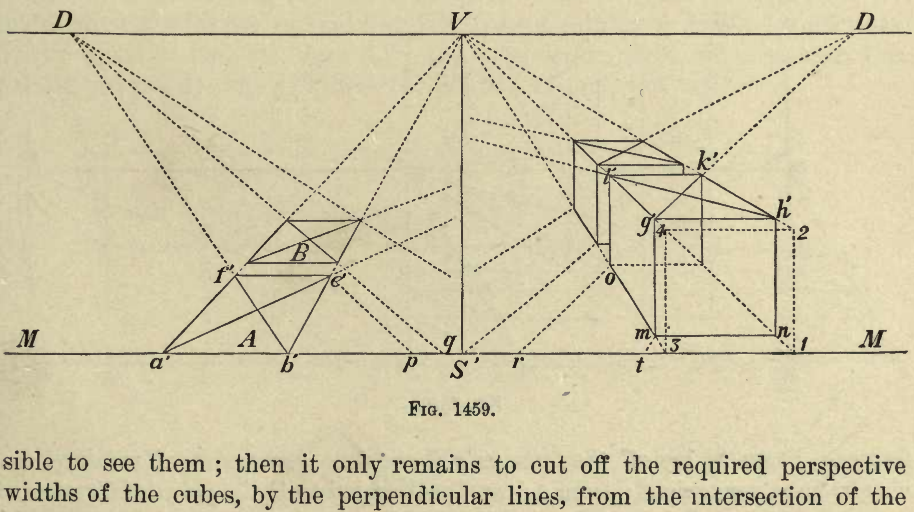


Fig. 1459.

In such simple objects as these it will not be necessary to draw a plan when one side is parallel to the picture, and dimensions are known. In Fig. 1459, the same objects as those in Fig. 1458 are drawn without a plan thus :

Draw the ground line M M, then the vertical line S′ V, and the horizontal line D D, at the height of the eye ; making D D the same distance on each side of V that the eye is from the transparent plane ; for drawing the squares, mark off from S′ to *b′*, on the ground line, the distance that the square is on one side of the observer ; let *b′ a′* be the length of one side of the square ; from *b′* and *a′* draw lines to V, which represent the sides of the square carried on indefinitely ; to cut off the required perspective width of the side *b′ e′* of the square, lay off the width, *a′ b′*, from *b′* to *p*, then draw from *p* to D on the left and the point *e′* where the line D *p* intersects *b′* V will give the apparent width required ; then draw *f′ e′* parallel to *a′ b′*, and the square is complete : this may be proved in the same way as in Fig. 1458. The further square may be obtained in a similar manner, setting off the distance between the squares from *p* to *q*, and the width of the square beyond that, and drawing lines to D as before : some of the lines in this plate are not continued to the ground line, in order to avoid confusion. Proceed with the cubes by the same rule. Let 1, 2, 3, 4, be the size of one side of the cube if continued until touching the picture ; from these points draw rays to V ; from 3 to *t* set off the distance the cube is from the picture, and from *t* to *r*, the width of the cube ; draw from these points to D on the right, and their intersections of the line 3 V in *m*, *o*, will give the perspective width and position of that side of the cube ; then finish the cube as in the figure. The operation of drawing the other cube is similar, and easy to be understood.

From the drawing of a square in parallel perspective, we deduce rules for the construction of a scale in perspective. Let D M M D (Fig. 1460) be the plane of the picture, the same letters of reference being used as in the preceding

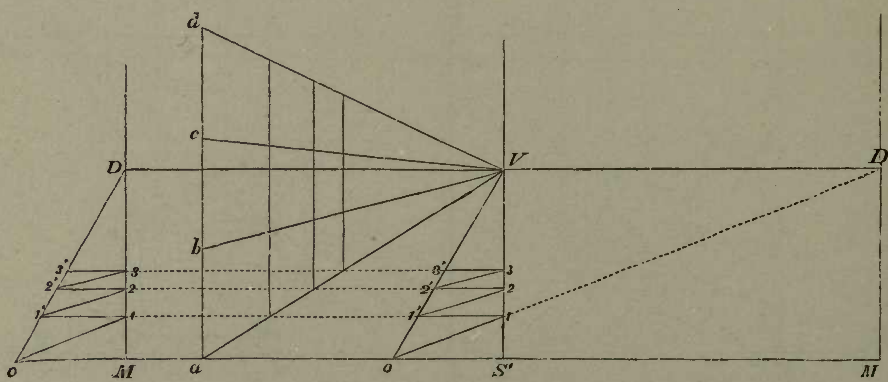


Fig. 1460.

figures. From S′ lay off the distance *o* S′ equal to some unit of measure, as may be most convenient ; from *o* draw the diagonal to D the point of distance ; now draw 1 1′ parallel to the ground line M M, again draw from 1′ the diagonal 1′ D, and lay off the parallel 2 2′, proceed in the same way with the diagonal 2′ D and the parallel 3 3′, and extend the construction as far as may be necessary. It is evident o S′ 1 1′, 1′ 1 2 2′, 2′ 2 3 3′ are the perspective projections of equal squares, and therefore o S′, 1 1′, 2 2′ 3 3′, etc., and S′ 1, 1 2, 2 3, etc., are equal to each other, and that if o S′ is set off to represent any unit of measure, as one foot, one yard, or ten feet, etc., each of these lines represents the same distance, the one being measured parallel to the base line, the others perpendicular to it. In making a perspective drawing a scale thus drawn will be found very convenient; but as in the center of the picture it might interfere with the construction lines of the object to be put in perspective, it is better that the scale be transferred to the side of the picture a M o, the diagonals to be laid off to a point to the right of D equal to the point of distance.

The scales thus projected are for lines in the base or ground plane; for lines perpendicular to this plane the following construction is to be adopted: Upon any point of the base line removed from S′, as a for instance, erect a perpendicular, $a\,d$; on this line, lay off as many of the units o S′ as may be necessary; in this example three have been laid off, that is, $a\,d = 3\,o$ S′. From a and d draw lines to the center of view, and extend the parallels 1 1′, 2 2′, 3 3′; at the intersection of these lines with a V erect perpendiculars. The portions comprehended between the lines a V and d V will be the perspective representations of the line $a\,d$, in planes at distances of 1, 2, 3, o S′ from the base line, and as b, c, d are laid off at intervals equal to o S′, by drawing the lines c V and b V nine equal squares are constructed, of which the sides correspond to the unit of measure o S′

To determine the Perspective Position of any point in the Ground Plane.—Thus (Fig. 1461), to determine the position of the point p, which in plane would be six feet distant from the plane of the picture, M D, and ten feet from the central plane, to the left.

Lay off from S′, to the left, the distance a S′, equal to six feet on the scale adopted; draw the diagonal to the point of distance D on the right; at its intersection f with the vertical line V S′ draw a parallel to M M; lay off from S′, S′ b equal to ten feet, draw b V; the intersection of this line p, with the parallel previously drawn, will be the position of the point required.

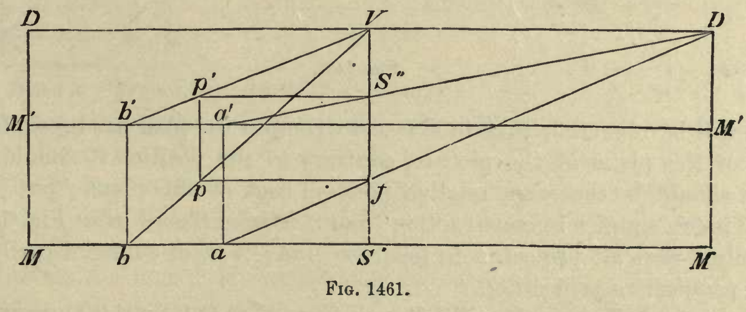


FIG. 1461.

By a similar construction the position of any point in the ground plan may be determined. It is not necessary that the distances should be expressed numerically; they may be shown on the plan and thence be transferred to the base line, and thrown into perspective by the diagonals and parallels. As the intersections of the various lines of the outlines of objects are points, by projecting perspectively these points, and afterward connecting by lines, the perspective of any plane surface on the ground plane may be shown.

If the point *p* were not in the ground plane, but in a position directly above the ground plane, say five feet, then at *b* erect a perpendicular, and lay off *b b′* equal to five feet, connect *b′* V, at *p* erect another perpendicular, and its intersection *p′* with the line *b′* V will be the position of the point required.

To draw an Octagon in Parallel Perspective.—Let A (Fig. 1462) represent the plan of an octagon. Draw M M, S′ V, and D D, as before ; from the points M, *a, b, c,* draw rays to V. Set off on M M from *c* to the right the distances *c e, c d, c f,* from which draw diagonals to D on the left, and at their intersection with the ray *c* V, draw parallels *e′ g′, d′ h′, k′ l′,* to the base line ; these points will correspond to the angles on the plan. Now connect the angles on the perspective view, in the proper succession, and the perspective projection is complete.

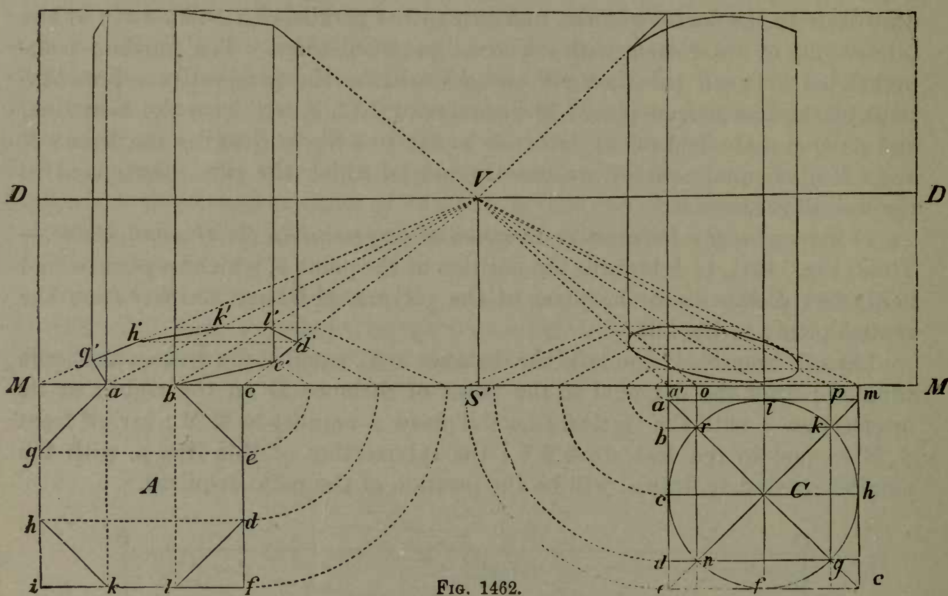


Fig. 1462.

It will be observed, that in this construction the plan has been placed forward of the plane of the picture, contrary to the position it should occupy, which should be the same relative position back of this plane ; but it will be found much simpler in construction than if it were placed as in Fig. 1458, and the points were all projected to the base line ; it is, of course, equally correct in its perspective projection.

To draw a Circle in Parallel Perspective.—Let C (Fig. 1462) represent the plan of a circle, round which let the square *a e c m* be described, two of its sides being parallel to the base line M M ; draw diagonals across the square, and where these intersect the circumference of the circle draw the lines *b k* and *d g* parallel to the base line, and the lines *o n* and *p g* at right angles thereto. Draw also the lines *f l* and *c h* at right angles to each other through the center of the circle, project the points *a, o, l, p, m,* to the base and draw rays to V; set off from *a'* to the left the distances *a' a, a' b, a' c, a' d, a' e,* and draw diagonals to the point of distance D on the right; at their intersection with the line *a'* V draw horizontal lines, or parallels to the base, and there will be projected in perspective the square *a e c m,* with all the lines of parallels and perpendiculars; connect the intersections corresponding to the points *c, n, f, g, h, k, l, r,* and we have the perspective projection of the required circle, which will be an ellipse.

To erect upon the octagonal base A an octagonal pillar or tower. This construction resolves itself into simply constructing another octagon on an upper plane, and connecting the visible angles by perpendiculars; or perpendiculars may be erected at the points M, *a, b, c,* and the heights of the tower laid off upon them, and from these extremities rays drawn to the center of view; the intersection of these rays by perpendiculars from the angles of the octagon beneath will determine the projection of the upper surface of the pillar; represent in full lines all visible outlines, and the projection is complete.

In the same manner a pillar may be erected on the circular base. If the pillars be inclined, the first method of projecting the upper outline on a plane assumed at the height of the pillar must be adopted.

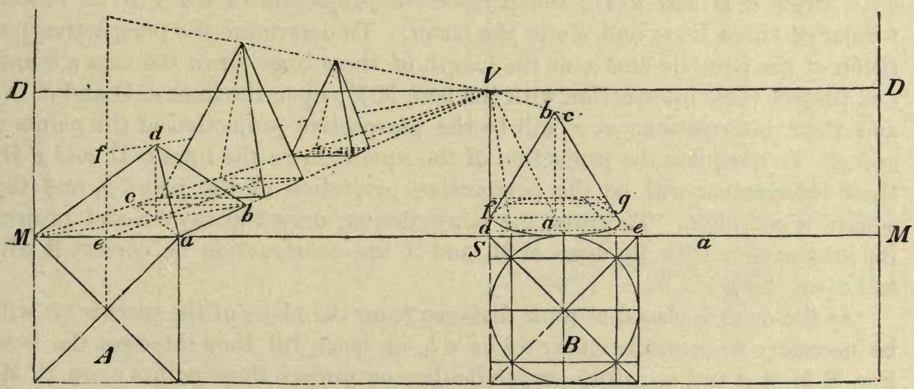


Fig. 1463.

To draw a Pyramid in Parallel Perspective.—Let A (Fig. 1463) be the plan of a pyramid, the diagonal lines represent the angles, and their intersection the vertex; project the plan as in previous examples of squares. Draw diagonal lines from M to *b,* and *a* to *c,* their intersection gives the perspective center of the square; upon this point raise a perpendicular line which is the axis of the pyramid; draw a perpendicular line *e f,* in the center of the line M *a,* upon which set up the height of the pyramid *e f*; from *f* draw a line to V, and its intersection of the axis of the pyramid at *d* will give the perspective height; complete the figure by drawing lines from *d,* the apex, to M, *a, b,* the three visible angles. The other two pyramids are drawn in a similar manner, by setting their distances from the plane of the picture off from *a,* on the ground line to the right, and drawing diagonals to the point of distance on the left.

To draw a Cone in Parallel Perspective.—Let B (Fig. 1463) represent the plan of a cone, apply the same lines of construction as to C (Fig. 1462); and draw the perspective view of the circle, lay off the height and finish precisely as in the preceding case.

To draw a Square and Cube in Angular Perspective.—Let A (Fig. 1464) be the plan of the square, and B the plan of the cube, M M the base or ground line, and S the station point. Draw M′ M′, and D D′ parallel to M M, the one being the ground line and the other the horizon of the plane of the picture; project the point *d* on M M, to *d′* on M′ M′. It has been shown in parallel perspective that the vanishing points of diagonals of squares lie in the points of distance; if through the station point S, in any of the preceding figures, lines be drawn parallel to the diagonals, they will intersect the base lines at distances from the central plane equal to the points of distance. In like manner to find the vanishing points of lines in the ground planes, or in planes parallel to the ground plane, inclined to the plane of the picture, through the station point S draw lines parallel to the inclined lines, and project their intersection with the base line to the horizon of the picture; thus, in the present example, draw S M, S M parallel to *a d*, *e h*, and to *d c*, *h g*; project their intersections M, M, with the base line to D, D′, the horizon of the picture, and D, D′, will be the vanishing points of all lines parallel to *a d* and *d c*. Draw *d′* D and *d′* D′, the perspective projection of *d a* will lie in the former of these lines and *d c* in the latter. To determine the perspective position of the points *a* and *c*, or the length of these lines, draw the rays *a* S and *c* S, project their intersection with the base M M, upon the lines *d′* D and *d′* D′, and their intersections *a′*, *c′* will be the perspective projection of the points *a* and *c*. To complete the projection of the square, draw the lines *a′* D′ and *c′* D, their intersection will be the perspective projection of the point *b*, and the square is complete. To prove the construction, draw the ray *b* S and project its intersection with the base M M, and if the construction be correct it will fall upon the point *b′*.

As the cube is placed at some distance from the plane of the picture, it will be necessary to continue either *e h* or *g h*, or both, till they intersect the base line M M at *n* and *m*; drop perpendiculars or project these points upon M′ M′ at *n′* and *m′*; on these perpendiculars set up the height of the cube *m′ o* and *n′ s*, draw the lines *m′* D′, *o* D′, and *n′* D, *s* D; connect the intersections *h′* and *h″*; draw the rays S *e* and S *g*, and project their intersections with M M, to *g′ e′*; draw the lines *e″* D′ and *g″* D; if the construction be correct, the projection of the intersection of the ray S *f* with the base will fall upon *f″*, and of the ray S *h* will fall upon *h″* and *h′*.

To solve the Same Problem by a Different Construction.—Let A B (Fig. 1464) be as before the plans of the square and of the cube; to project them perspectively on the plane of the picture M D D′ M (Fig. 1465).

From the point M and M (Fig. 1464), set off distances equal to M S, M S, to *p* and *p′*; project these points upon D D′ (Fig. 1465), the point *p′* (Fig. 1465) will be that from which any number of parts may be laid off on lines vanishing in D′; the point *p* will be the corresponding point for lines vanishing in D. These points may be called the points of division. In parallel perspective the points of distance were the points of division, the one for the other. To illus-

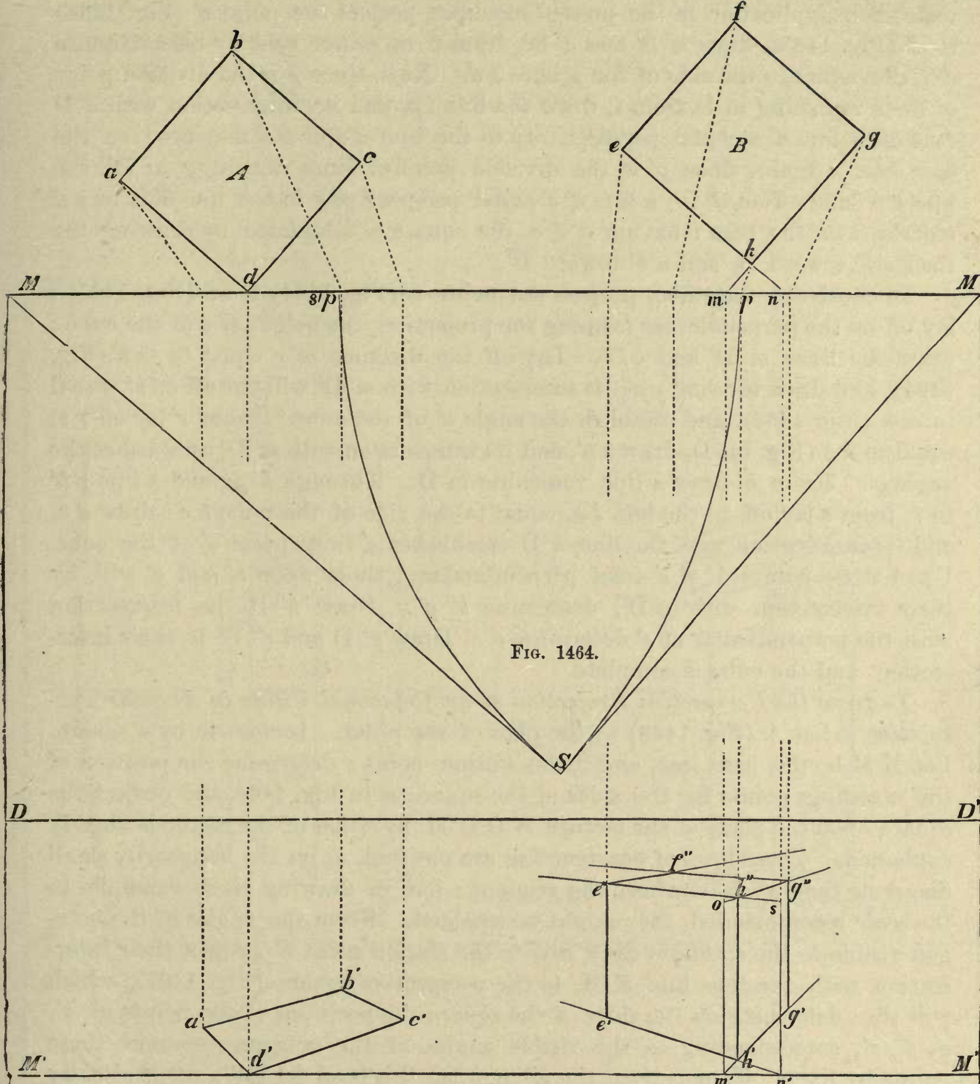


Fig. 1464.

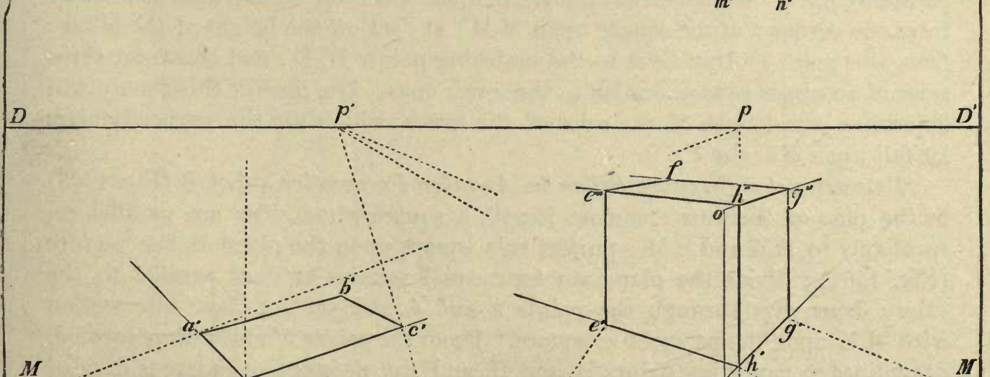


Fig. 1465.

trate their application in the present example, project the point d (Fig. 1464) to d' (Fig. 1465), draw d' D and d' D′, from d' on either side lay off a distance $d'\,i$, $d'\,k$ equal to the side of the square $a\,d$. Now, since p is the division point of lines vanishing in D, from i, draw the line $i\,p$, and its intersection with d' D cuts off a line $d'\,a'$ equal perspectively to the line $d'\,i$ or $a\,d$ measured on the base line. Again, since p' is the division point of lines vanishing in D′, the line $k\,p'$ cuts off on d' D′, a line $d'\,c'$ equal perspectively to the line $d'\,k$, or $a\,d$ measured on the base : having $a'\,d'\,c$, the square is completed by drawing the lines $c'\,b'$ toward D, and $a'\,b'$ toward D′.

To construct the cube, project the point m (Fig. 1464) to m' (Fig. 1465) ; lay off on the perpendicular forming the projection, the height $m'\,o$ of the cube ; draw the lines m' D′ and o D′. Lay off the distance $m'\,r$ equal to $m\,h$ (Fig. 1464), and draw the line $r\,p'$, its intersection with m' D′ will cut off $m'\,h'$, equal to $m\,h$ (Fig. 1464), and establish the angle h' of the cube. From r lay off $r\,s$, equal to $h\,g$ (Fig. 1464), draw $s\,p'$, and its intersection with m' D′ establishes the angle g'. From h' draw a line vanishing in D. Through h' extend a line $p\,h'$ to t, from t lay off to the left $t\,a$, equal to the side of the cube $h\,e$; draw $a\,p$, and its intersection with the line h' D establishes a third point e' of the cube. Upon these points $h'\,g'\,e'$ erect perpendiculars ; those upon h' and g' will, by their intersection with o D′, determine $h''\,g''$. Draw h'' D, its intersection with the perpendicular at e' determine e''. Draw g'' D and e'' D′ to their intersection, and the cube is complete.

To draw the Perspective Projection of an Octagonal Pillar in Angular Perspective.—Let A (Fig. 1466) be the plan of the pillar. Inclose it by a square. Let M M be the base line, and S the station point ; determine the position of the vanishing points for the sides of the square as in Fig. 1464, and project the square upon the plane of the picture M D D′ M′ by either of the methods already explained. These lines of construction are omitted, as on the necessarily small diagrams they would confuse the student ; but in drawing these examples to the scale recommended, they might be retained. From the angles of the octagon visible to the spectator draw rays to the station point S, project their intersection with the base line M M, to the perspective square (Fig. 1467), which will thus determine on the sides of the square the positions of the points a', b', c', d', e', corresponding to the visible angles of the octagon ; connect these points by lines. To construct the pillar upon this base, let fall a perpendicular from the corner f of the square upon M M′, at f set off the height of the pillar ; from this point f' draw lines to the vanishing points D, D′, and construct three sides of an upper square similar to the lower one. The lines of this square will determine the length of the sides of the tower, which are the perpendiculars let fall upon $a'\,b'\,c'\,d'\,e'$.

To construct a Circular Pillar in Angular Perspective.—Let B (Fig. 1466) be the plan of the base ; enclose it with a square whose sides are parallel respectively to S M and S M ; project this square upon the plane of the picture (Fig. 1467) ; divide the plan into four equal squares by lines parallel to the sides ; draw rays through the points h and i, and project their intersection with M M upon the perspective square. From the points h' and i' thus formed, draw lines to vanishing points D′ and D, and the perspective square is divided

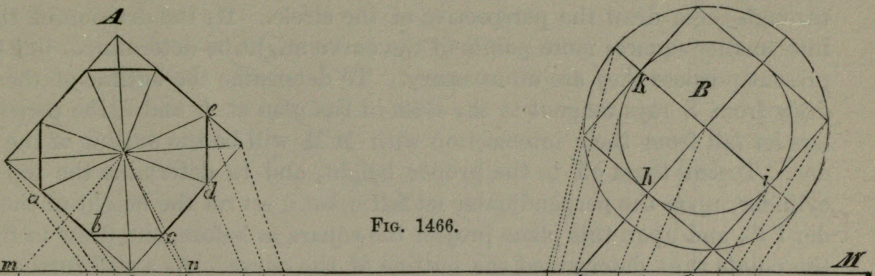


Fig. 1466.

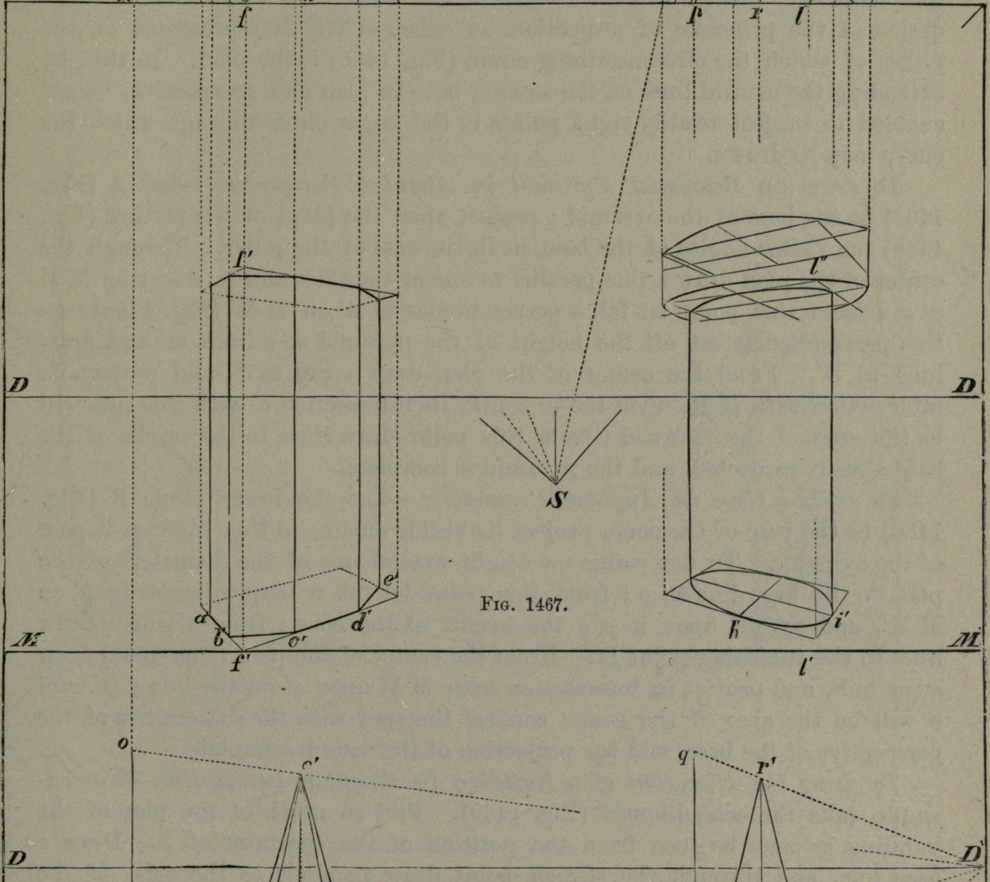


Fig. 1467.

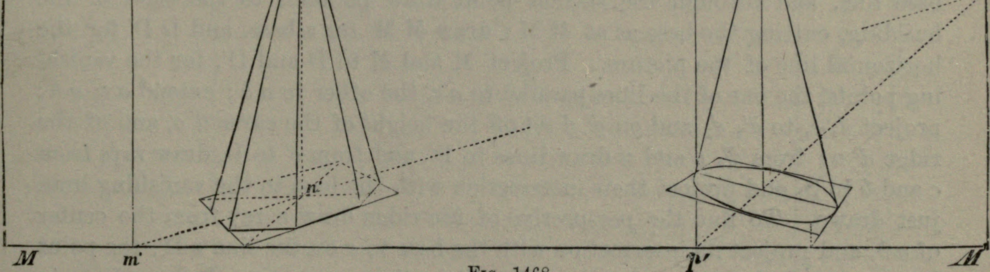


Fig. 1468.

similarly to the original, and there are four points of the circle established: through these draw the perspective of the circle. By the division of the base into smaller squares more points of the curve might be determined, but for the present purpose they are unnecessary. To determine the outline of the pillar, draw from S rays tangent to the sides of the plan at *k* and *i*, the perpendiculars let fall from their intersection with M M will be the outline of the cylinder. To cut them off to the proper height, and to determine the top of the cylinder, upon the perpendicular let fall upon *i*, set off the height of the cylinder *l′ l″*, and upon this plane project the square as before, and draw in through the points thus determined the outline of the curve. As a still further elucidation of the principle of projection, an enlarged cap is represented on the pillar, of which the circumscribing circle (Fig. 1466) is the plan. In this, by extending the central lines of the square, both in plan and perspective, we are enabled to project readily eight points in the larger circle through which the curve may be drawn.

To draw an Octagonal Pyramid in Angular Perspective.—Let A (Fig. 1466) be the base of the pyramid; project upon the plane of the picture (Fig. 1468) the visible angles of the base, as in the case of the pillar. Through the center of the plan draw a line parallel to one of the sides and intersecting M M at *m*; from this point let fall a perpendicular to *m′* on M M′ (Fig. 1468); on this perpendicular set off the height of the pyramid *m′ o* from *m′* and draw lines to D′. From the center of the plan draw a ray to S, and project its intersection with M M, upon the line *o* D′, its intersection *o′* with this line will be the apex of the pyramid: from this point draw lines to the angles of the base already projected, and the pyramid is complete.

To draw a Cone in Angular Perspective.—Let the inner circle B (Fig. 1466) be the base of the cone, project its visible outline to Fig. 1468, as in case of the cylinder. To determine its height extend one of the diameters of the plan to the base line at *p*; from this point let fall a perpendicular to *p′* on M M′, and set off upon it *p′ q* the height of the cone; from *p′* and *q* draw lines to the vanishing point D′. From the center of the plan (Fig. 1466) draw a ray to S, and project its intersection with M M upon *r′* on the line *q* D′, and *r′* will be the apex of the cone: connect the apex with the extremities of the perspective of the base, and the projection of the cone is complete.

To draw the Elevation of a Building in Angular Perspective.—For example, take the school-house (Fig. 1469). Plot so much of the plan of the building as may be seen from the position of the spectator at S. Draw a base line, and through the station point draw parallels to the sides of the building, cutting the base as at M M; draw M M′ for a base, and D D′ for the horizontal line of the picture. Project M and M to D and D′, for the vanishing points, the one of the lines parallel to *a c*, the other to *a b*; extend *a c*, *a b*; project *d*, *e*, to *d′*, *e′*, and on *d′ d* set off the height of the eaves *d′ o*, and of the ridge *d′ n*; from *d′*, *o* and *n* draw lines to D′, and from *e′* to D, draw rays from *c* and *b* to S, and project their intersection with the base to the vanishing lines just drawn. To find the perspective of the ridge draw a ray from the center of *a b*, and project its intersection with the base to *r* on the line *n* D′, the point is the apex of the gable, the line *r* D will be the perspective of the ridge; to

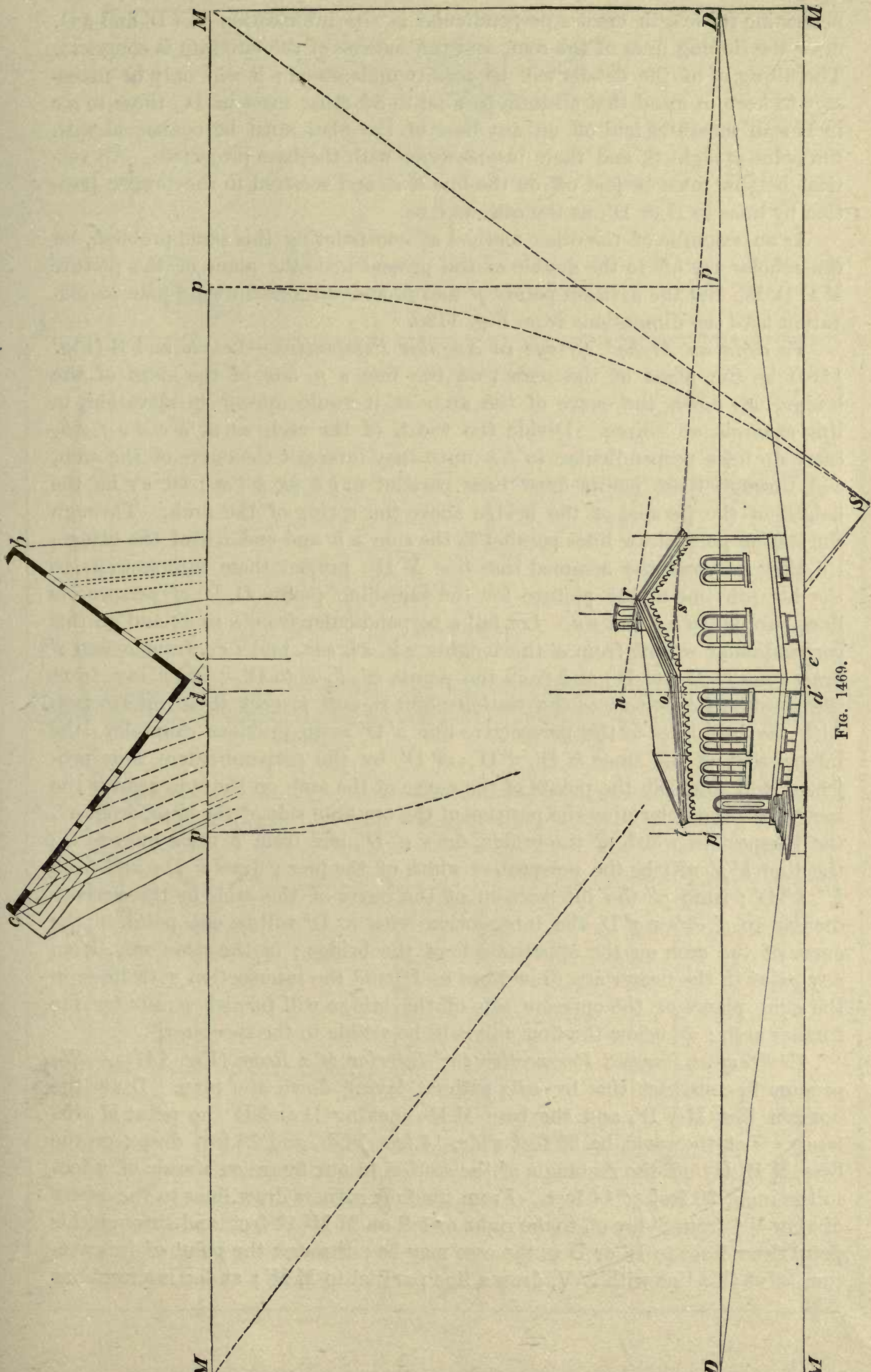


FIG. 1469.

determine its length erect a perpendicular at the intersection of t D′ and s D, draw the sloping lines of the roof, and the outline of the building is complete. The filling in of the details will be readily understood ; it will only be necessary to keep in mind that all lines parallel to $a\,b$ must meet in D′, those to $a\,c$ in D : all measures laid off on any lines of the plan must be connected with the point of sight S, and their intersections with the base projected. All vertical heights must be laid off on the line $d'\,d$, and referred to the proper position by lines to D or D′, as the case may be.

As an example of the other method of constructing this same problem, let the scholar lay off to the double of the present scale the plane of the picture M D D′ M′, and the division points p' and p, and without drawing plan or elevation take the dimensions from Fig. 1190.

To draw an Arched Bridge in Angular Perspective.—Let A and B (Fig. 1470) be the plans of the piers ; on the line $a\ p$, one of the sides of the bridge, lay down the curve of the arch as it would appear in elevation, in this example an ellipse. Divide the width of the arch as at $b\ c\ d\ e\ f\ g\ h$, carry up lines perpendicular to $b\,h$ until they intersect the curve of the arch, and through these points draw lines parallel to $b\,h$ as $k\ l\ m$; let $o\ r$ be the height of the parapet of the bridge above the spring of the arch. Through the station point draw lines parallel to the side $a\,h$ and end $a\,a$ of the bridge, till they intersect the assumed base line M M : project these intersections to the horizon line of the picture for the vanishing points D, D′ of perspective lines parallel to $a\,h$ and $a\,a$. Let fall a perpendicular from a to a', and on this perpendicular set off from a' the heights $s\,k$, $s\,l$, $s\,m$, and $s\,r$; from a' and r' draw lines to D and D′, and from the points m', l', k' to D′. Draw rays from the points $a\,b\,c\,d\,e\,f\,g\,h$ to the station point S, and project their intersection with the base lines to the perspective line a' D′ as in previous examples : the intersection of the lines k' D′, l' D′, m' D′ by the perpendiculars thus projected will establish the points of the curve of the arch on the side nearest the spectator. To determine the position of the opposite side of the arch, from a'', the perspective width of the bridge, draw a'' D′, and from h' draw lines to D ; the line $h'\,p'$ will be the perspective width of the pier ; draw k' D ; and from k'', k'' D′ ; from g'' the intersection of the curve of the arch by the perpendicular to g', draw g'' D, the intersection with k'' D′ will be one point in the curve of the arch on the opposite side of the bridge ; in the same way, from any point in the nearer arc draw lines to D, and the intersection with lines in the same planes on the opposite side of the bridge will furnish points for the further arch ; all below the first only will be visible to the spectator.

To draw in Parallel Perspective the Interior of a Room (Fig. 1471).—We propose to construct this by scale without laying down the plan. Draw the horizon line D V D′, and the base M M′, making D and D′ the point of distance. Let the room be 20 feet wide, 14 feet high, and 12 feet deep ; on the base M M′ lay off the rectangle of the section in our figure on a scale of 8 feet to the inch, 20 feet × 14 feet. From the four corners draw lines to the center of view V ; from S′ lay off to the right or left on M M′ 12 feet, and through this point draw lines to D′ or D as the case may be ; through the point of intersection, a' of this line with S′ V, draw a line parallel to M M′ ; at the intersections

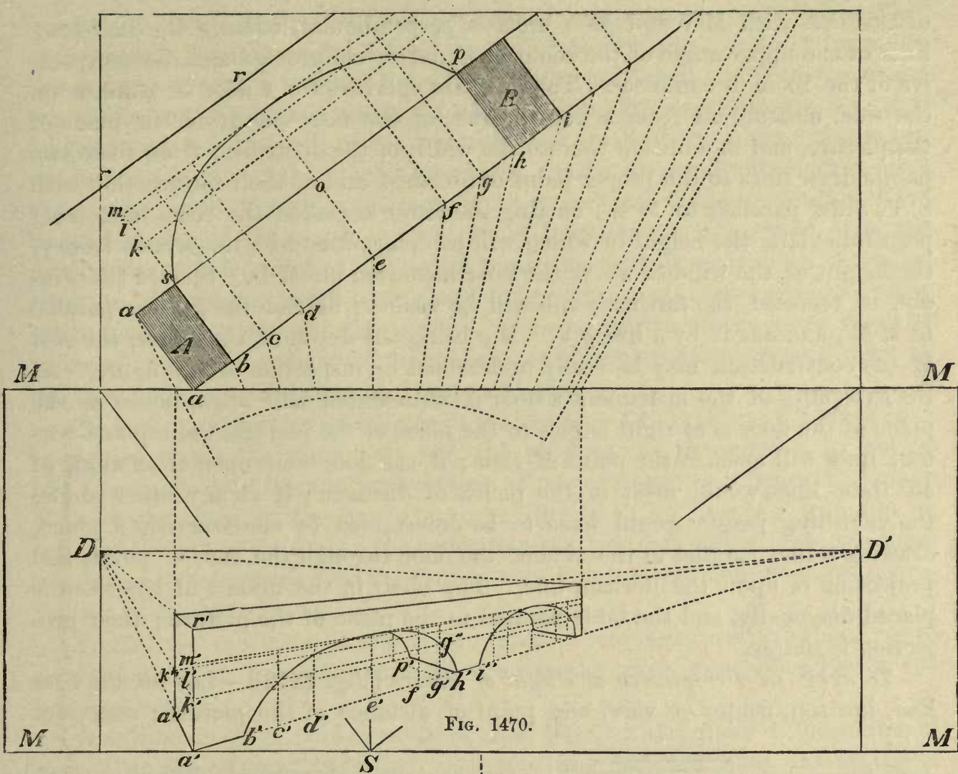


Fig. 1470.

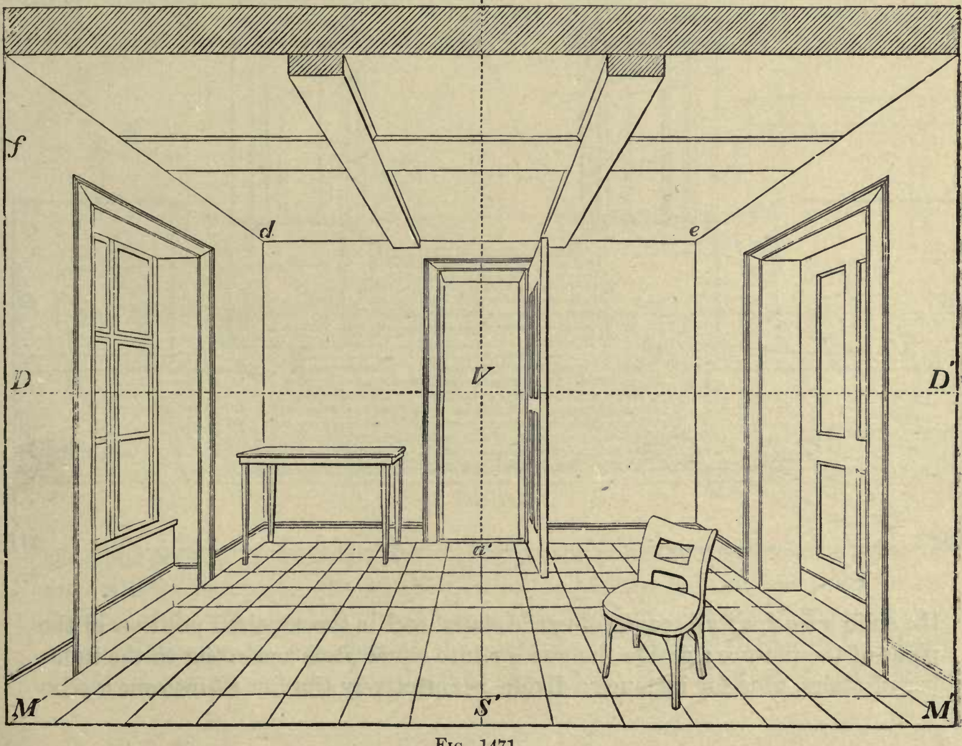


Fig. 1471.

of this line with M V and M′ V erect a perpendicular, cutting the vanishing lines of the upper angle of the room at *d* and *e*; connect *d e* and the perspective of the room is complete. To draw the aperture for a door or window on the side, measure off from S′ the distance of the near side from the plane of the picture, and in addition thereto the width of the aperture; from these two points draw lines to the proper point or distance, and at their intersection with S′ V, draw parallels to M M′, cutting the lower angles of the room, and erect perpendiculars, the height of which will be determined by a line drawn from *f*, the height of the window above the floor measured on M D. Should the window be recessed, the farther jamb will be visible; extend the farther parallel to M M′, and cut it by a line *g* V. M *g* being the depth of the recess, the rest of the construction may be easily understood by inspection of the figure. At the extremity of the apartment a door is represented half open, hence as the plane of the door is at right angles to the plane of the picture, the top and bottom lines will meet in the point of view; if the door were open at an angle of 45° these lines would meet in the points of distance; if at any other angle, the vanishing points would have to be determined by constructing a plane, drawing a line parallel to the side of the door through the station point, and projecting it upon the horizon line. The chair in the middle of the room is placed diagonally, and the table parallel to the plane of the picture; their projection is simple.

To draw in Perspective a Flight of Stairs (Fig. 1472).—Lay off the base line, horizon, center of view, and point of distance of the picture; construct

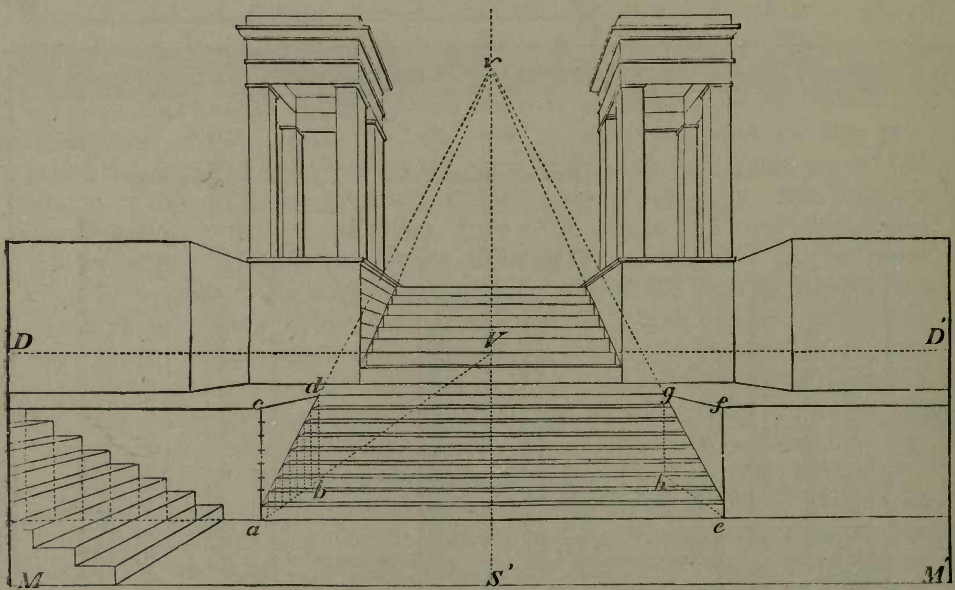


Fig. 1472.

the solid *a b c d*, *e f g h*, containing the stairs, and in the required position in the plane of the picture; divide the rise *a c* into equal parts according to the number of stairs, nine for instance; divide perspectively the line *a b* into one less (8) number of parts ; at the points of division of this latter erect perpendiculars, and through the former draw lines to the center of view ; one will form the rise and the other the tread of the steps. From the top of the first step to the top of the upper continue a line *a d*, till it meets the perpendicular S′ V prolonged in *v* ; this line will be the inclination or pitch of the stairs ; if through the top of the step at the other extremity a similar line be drawn, it will meet the central perpendicular at the same point *v*, and will define the length of the lines of nosing of the steps, and the other lines may be completed. As the pitch lines of both sides of the stairs meet the central vertical in the same point, in like manner *v* will be the vanishing point of all lines having a similar inclination to the plane of the picture. The projection of the other flight of stairs will be easily understood from the lines of construction perpendicular to the base line or parallel thereto, lying in planes.

To find the Reflection of Objects in the Water.—Let B (Fig. 1473) be a cube suspended above the water ; we find the reflection of the point *a*, by letting fall a perpendicular from it, and setting off the distance *a′ w* below the plane of the water equal to the line *a w* above this line, the line *w f′* will also be equal to the line *w f* ; find in the same way the points *b′* and *e′*, through these points construct perspectively a cube in this lower plane, and we have the reflection of the cube above.

To find the reflection of the square pillar D removed from the shore : suppose the plane of the water extended beneath the pillar, and proceed as in the previous example.

It will be observed that those lines of an object which meet in the center of view V, in the original, have their corresponding reflected lines converging to

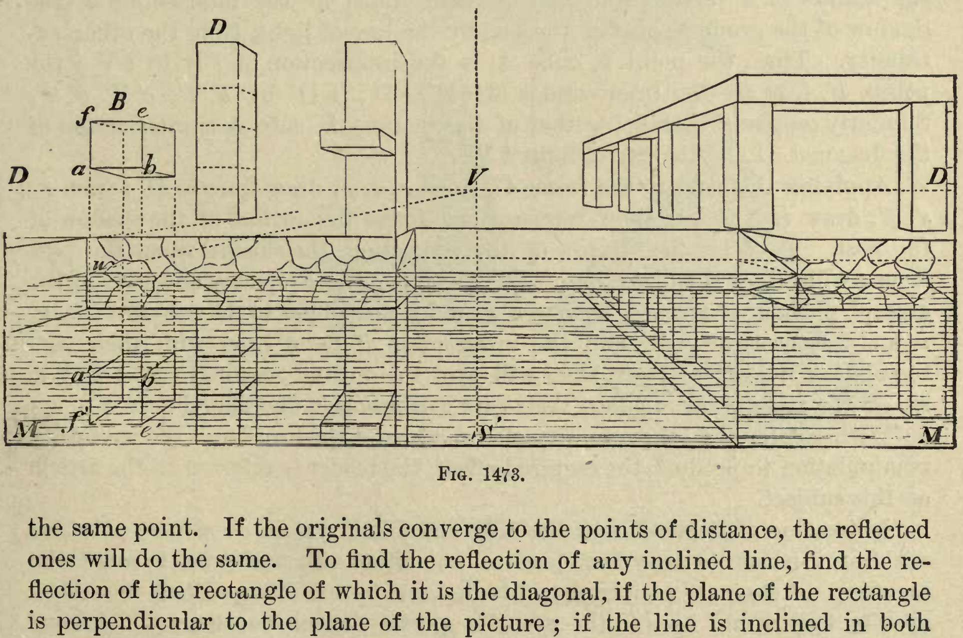


Fig. 1473.

the same point. If the originals converge to the points of distance, the reflected ones will do the same. To find the reflection of any inclined line, find the reflection of the rectangle of which it is the diagonal, if the plane of the rectangle is perpendicular to the plane of the picture ; if the line is inclined in both directions inclose it in a parallelopiped and project the reflection of the solid.

To find the Perspective Projection of Shadows (Fig. 1474).—Let the construction points and lines of the picture be plotted. Let A be the perspective projection of a cube placed against another block, of which the face is parallel to the plane of the picture ; to find the shadow upon the block and upon the ground plane, supposing the light to come into the picture from the upper left-hand corner and at an angle of 45°. Since the angle of light is the diagonal of a cube, construct another cube similar to A, and adjacent to the face *d c g* ; draw the diagonal *b k*, it will be the direction of the rays of light, and *k* will be the shadow of *b* ; connect *f k* and *c k*, *f k* must be the shadow of the line *b f*, and *c k* of *b c* ; the one upon the horizontal plane and the other in a vertical one : the former will have its direction, being a diagonal, toward the point of distance D′, the other being a diagonal in a plane parallel to that of the picture, will be always projected upon this plane in a parallel direction.

Let B be a cube similar to A ; to find its projection upon a horizontal plane, the shadow of the point *b′* may be determined as in the preceding example, but the shadow of the point *c′*, instead of falling upon a plane parallel to the picture, falls upon a horizontal one ; its position must be determined as we did before by *b*. Construct the cube and draw the diagonal *c′ l* ; in the same way determine the point *m* the shadow of *d′* ; connect *c k′ l m n*, and we have the shadow of the cube in perspective on a horizontal plane.

On examination of these projected shadows, it will be found that as the rays of light fall in a parallel direction to the diagonal of the cube, the vanishing point of these rays will be in one point V′ on the line D′ M′ prolonged, at a distance below D′ equal V D′ ; and since the shadows of vertical lines upon a horizontal plane are always directed toward the point of sight, the extent of the shadow of a vertical line may be determined by the intersection of the shadow of the ground point of the line by the line of light, from the other extremity. Thus, the point *k*, cube A, is the intersection of *f* D′ by *b* V′ ; the points *k′*, *l*, *m* are the intersections of *c* D′, *o* D′, *n* D′ by *b′* V′, *c′* V′ *d′* V′. Similarly on planes parallel to that of the picture, *k*, cube A is intersection of the diagonal *c k*, by the ray of light *b* V′.

Applying this rule to the frame C, from *r*, *s*, *p*, draw lines to D′ ; from *r′*, *s′*, *p′*, draw rays to V′ ; their intersections define the outline of the shadow of the post. To draw the shadow of the projection, the shadow upon the post from *t* will follow the direction of the diagonal *c k*. Project *u* and *v* upon the ground plane at *u′* and *v′* ; from *t u′ v′* and *p* draw lines to D′ ; from *t′*, *u*, *v*, *w* and *x* draw rays to V′, and the intersection of these lines with their corresponding lines from their bases will give the outline required ; as *v* and *w* are on the same perpendicular, their rays will intersect the same line *v′* V′.

With reference to the intensity of "shade and shadow," and the necessary manipulation to produce the required effect, the reader is referred to the article on this subject.

In treating of Perspective it has been considered not in an artistic point, as enabling a person to draw from nature, but rather as a useful art to assist the architect or engineer to complete his designs, by exhibiting them in a view such as they would have to the eye of a spectator when constructed. In our examples, owing to size of the page, we have been limited in the scale of the

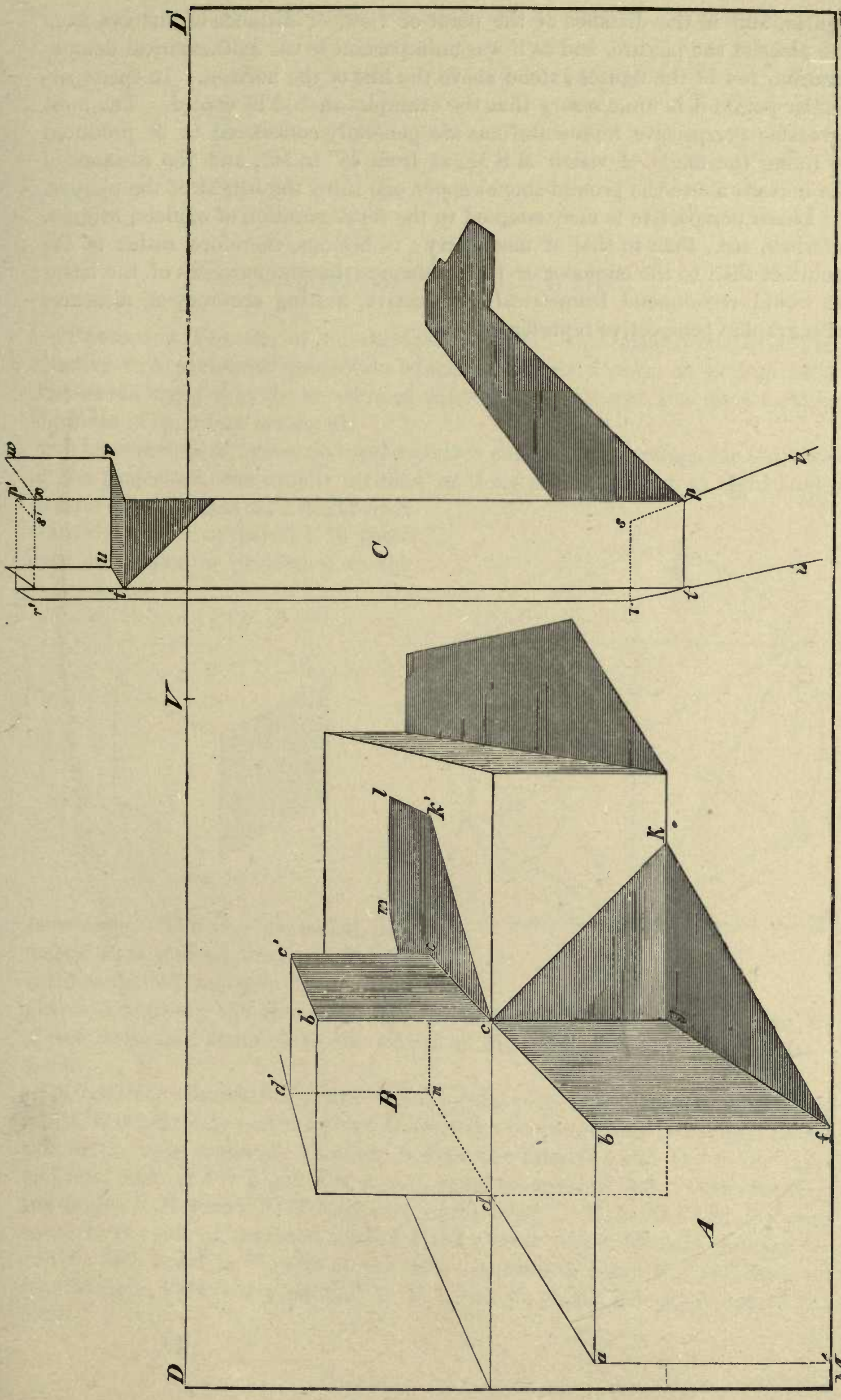


FIG. 1474.

figures, and in the distance of the point of view, or distance of the eye from the plane of the picture, and as it was unimportant to the mathematical demonstration, few of the figures extend above the line of the horizon. In these particular points it is unnecessary that the examples should be copied. The most agreeable perspective representations are generally considered to be produced by fixing the angle of vision M S M′, at from 45° to 50°, and the distance of the horizon above the ground-line at about one third the height of the picture.

Linear perspective is more adapted to the representation of edifices, bridges, interiors, etc., than to that of machinery ; it belongs, therefore, rather to the architect than to the engineer or the mechanic ; for the purposes of the latter we would recommend Isometrical Perspective, uniting accuracy of measures with graphic perspective representation.

ISOMETRICAL DRAWING.

Professor Farish, of Cambridge, has given the term Isometrical Perspective to a particular projection which represents a cube, as in Fig. 1474. The words imply that the measure of the representations of the lines forming the sides of each face are equal.

The principle of isometric representation consists in selecting, for the plane of the projection, one equally inclined to three principal axes, at right angles to each other, so that all straight lines coincident with or parallel to these axes are drawn in projection to the same scale. The axes are called isometric axes, and all lines parallel to them are called isometric lines. The planes containing the isometric axes are isometric planes; the point in the object projected, assumed as the origin of the axes, is called the *regulating-point.*

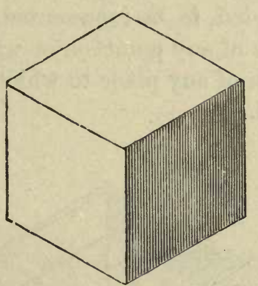


Fig. 1474.

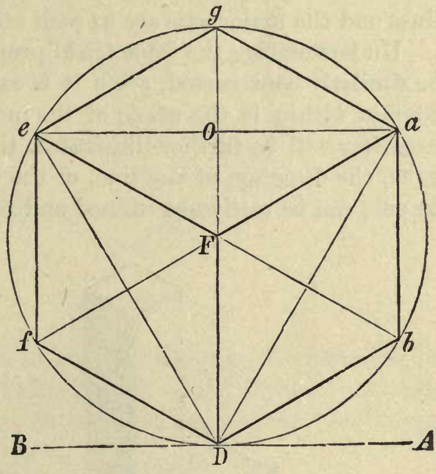


Fig. 1475.

To draw the isometrical projection of a cube (Fig. 1475), draw the horizontal line A B indefinitely; at the point D erect the perpendicular D F, equal to one side of the cube required; through D draw the lines D *b* and D *f* to the right and left, making *f* D B and *b* D A each equal an angle of 30°. Consequently, the angles F D *f* and F D *b* are each equal to 60°. Make D *b* and D *f* each equal to the side of the cube, and at *b* and *f* erect perpendiculars, making *b a* and *f e* each equal to the side of the cube; connect F *a* and F *e*, and draw *e g* parallel to *a* F, and *a g* parallel to F *e*, and we obtain the projection of the cube.

If from the point F, with a radius F D, a circle be described, and commencing at the point D radii be laid off around the circumference, forming a regular inscribed hexagon, and the points D *a* *e* be connected with the center of the circle F, we have an isometrical representation of a cube. The point D is called the *regulating-point.*

If a cube be projected according to the principles of isometrical perspective, in a similar manner as we have constructed one according to the rules of linear perspective, the length of the isometrical lines would be to the original lines as ·8164 to 1, but, since the value of isometrical perspective as a practical art lies in the applicability of common and known scales to the isometric lines, in our constructions we have not thought it necessary to exemplify the principles of the projection, but have drawn our figures without any reference to what would be the comparative size of the original and of the projection, transferring measures directly from plans and elevations in orthographic projections to those in isometry. It will be observed that the isometric scale adopted applies only to isometric lines, as F D, F *a*, and F *e*, or lines parallel thereto; the diagonals which are absolutely equal to each other, and longer than the sides of the cube, are the one less, the other greater; the minor axis being unity, the isometrical lines and the major axis are to each other as, 1. $\sqrt{2}$. $\sqrt{3}$.

Understanding the isometrical projection of a cube, any surface or solid may be similarly constructed, since it is easy to suppose a cube sufficiently large to contain within it the whole of the model intended to be represented, and, as hereafter will be further illustrated, the position of any point on or within the cube, the direction of any line, or the inclination of any plane to which it may be cut, can be easily ascertained and represented.

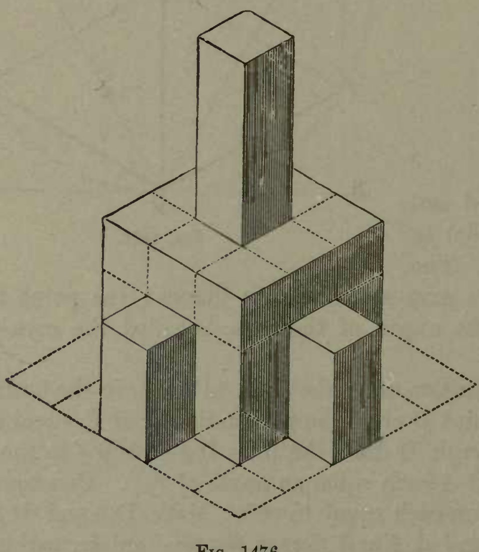


Fig. 1476.

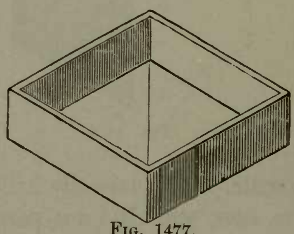


Fig. 1477.

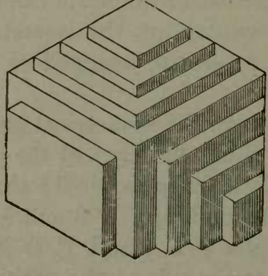


Fig. 1478.

In Figs. 1474 and 1475 one face of the cube appears horizontal, and the other two faces appear vertical. If now the figures be inverted, that which before appeared to be the top of the object will now appear to be its under side.

The angle of the cube formed by the three radii meeting in the center of the hexagon may be made to appear either an internal or external angle ; in the one case the faces representing the interior, and in the other the exterior of a cube.

Figs. 1476, 1477, 1478, illustrate the application of isometrical drawing to simple combinations of the cube and parallelopipedon. The mode of construction of these figures will be easily understood by inspection, as they contain no lines except isometrical ones.

To draw Angles to the Boundary Lines of an Isometrical Cube.—Draw a square (Fig. 1479) whose sides are equal to those of the isometrical cube A (Fig. 1480), and from any of its angles describe a quadrant, which divide

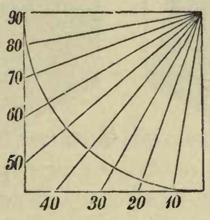


FIG. 1479.

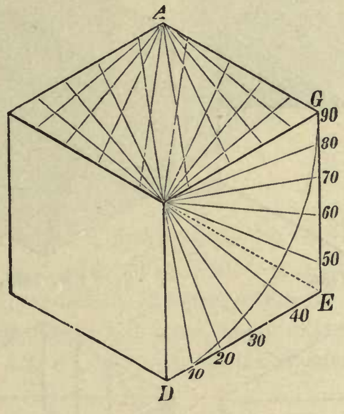


FIG. 1480.

into 90°, and draw radii through the divisions meeting the sides of the square. These will then form a scale to be applied to the faces of the cube ; thus, on D E, or any other, by making the same divisions along their respective edges.

As the figure is bounded by twelve isometrical lines, and the scale of tangents may be applied two ways to each, it can be applied therefore twenty-four ways in all, affording a simple means of drawing, on the isometrical faces of the cube, lines at any angles with their boundaries.

Figs. 1481 to 1486 show the section of the cube by single planes, at various inclinations to the faces of the cubes. Figs. 1487 and 1488 are the same cube, but turned round, with pieces cut out of it. Fig. 1489 is a cube cut by two planes forming the projection of a roof. Fig. 1490 is a cube with all of the angles cut off by planes, so as to leave each face an octagon. Fig. 1491 represents the angles cut off by planes perpendicular to the base of the cube, forming thereby a regular octagonal prism. By drawing lines from each of the angles of an octagonal base to the center point of the upper face of the cube, we have the isometrical representation of an octagonal pyramid.

As the lines of construction have all been retained in these figures, they will

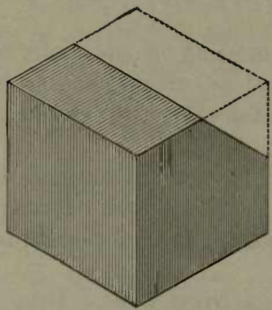


Fig. 1481.

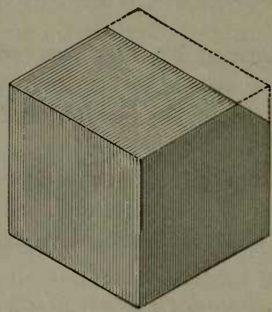


Fig. 1482.

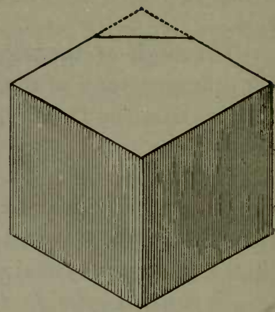


Fig. 1483.

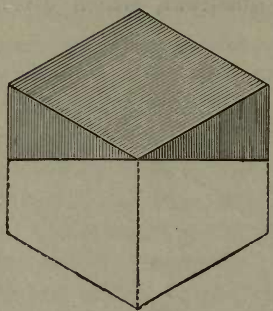


Fig. 1484.

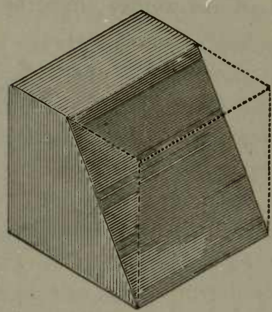


Fig. 1485.

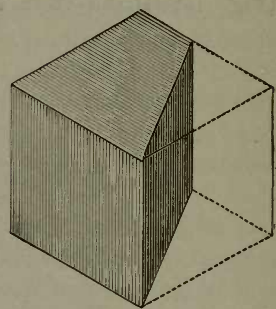


Fig. 1486.

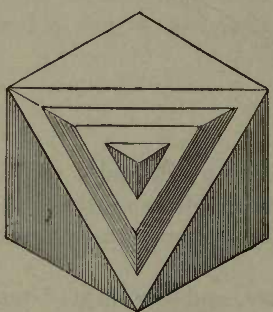


Fig. 1487.

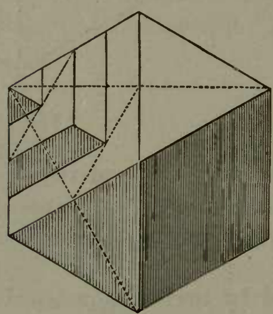


Fig. 1488.

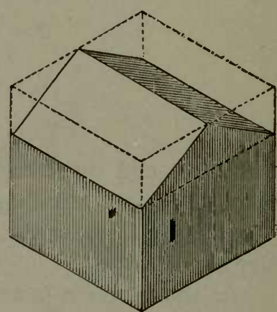


Fig. 1489.

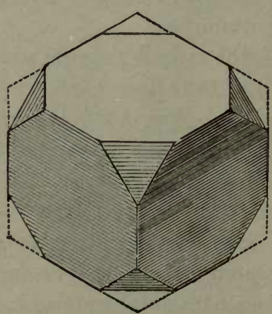


Fig. 1490.

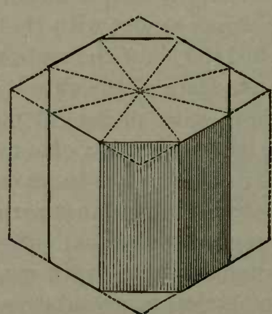


Fig. 1491.

be easily understood and copied, and are sufficient illustrations of the method of representing any solid by inclosing it in a cube.

In the application of this species of projection to curved lines, let A B (Fig. 1492) be the side of a cube with a circle inscribed; and that all the faces of a cube are to have similarly inscribed circles. Draw the diagonals A B, C D, and

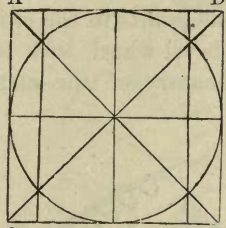


FIG. 1492.

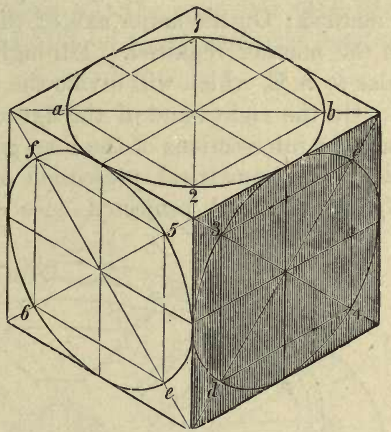


FIG. 1493.

at their intersection with the circumference, lines parallel to A C, B D. Now draw the isometrical projection of the cube (Fig. 1493), and lay out on the several faces the diagonals and the parallels; the projection of the circle will be an ellipse, of which the diagonals being the axes, their extremities are defined by their intersections *f* 6, *e* 5, *a* 2, *b* 1, *d* 3, *c* 4, with the parallels; having

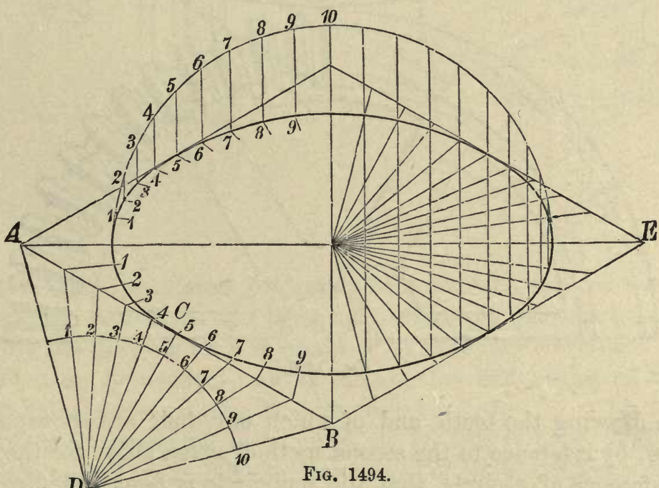


FIG. 1494.

thus the major and minor axis, construct the ellipse by the trammel, or, since the curve is tangent at the center of the sides, we have eight points in the curve; it may be put in by sweeps or by the hand.

To divide the Circumference of a Circle.—First method : On the center of the line A B (Fig. 1494) erect a perpendicular, C D, making it equal to C A or C B ; then from D, with any radius, describe an arc and divide it in the ratio required, and draw through the divisions radii from D meeting A B ; then from the isometric center of the circle draw radii from the divisions on A B, cutting the circumference in the points required.

Second method : On the major axis of the ellipse describe a semicircle, and divide it in the manner required. Through the points of division draw lines perpendicular to A E, which will divide the circumference of the ellipse in the same ratio. On the right hand of the figure both methods are shown in combination, and the intersections of the lines give the points in the ellipse.

Fig. 1495 is an isometrical projection of a bevel-wheel, with a half-plan (Fig. 1496) beneath, and projected lines explanatory of the method to be

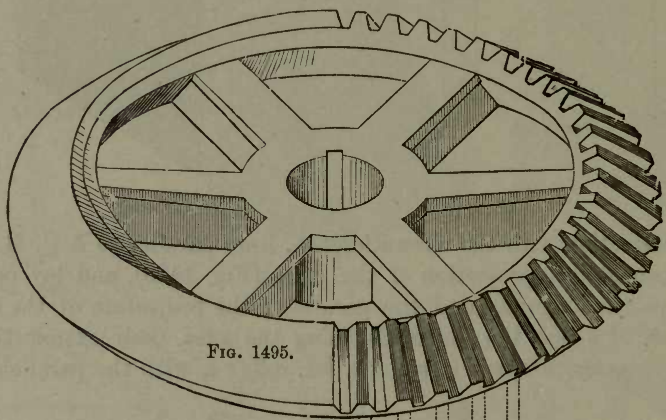


Fig. 1495.

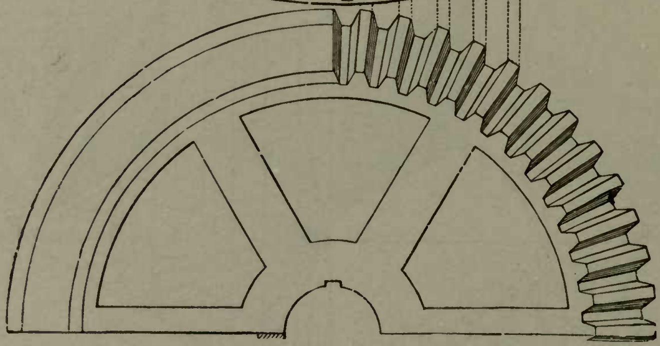


Fig. 1496.

adopted in drawing the teeth, and of which only half are shown as cut. It will be seen, by reference to the second method given above for the division of the circumference of a circle, that the semicircle is described directly on the major axis of the ellipse. In practice it will be found more convenient, when a full drawing is to be made, to draw the semicircle on a line parallel to the major axis, and entirely without the lines of the main drawing ; and also, as in the example of the bevel-gear, complete on the semicircle, or half-plan, the drawings of all lines, the intersections of which with circles it will be necessary to project on the isometrical drawing.

Fig. 1497 is an isometrical projection of a complete pillow-block, with its hold-down bolts. By reference to Fig. 592, and Figs. 508 and 509, it will be seen how much more graphically these forms of gearing are given by isometry than by the usual projection. As an exercise for the learner, it will be very good practice to project isometrically the spur-gear (Fig. 583), and the standard and hanger (Figs. 510 and 515), of which sufficient details are given.

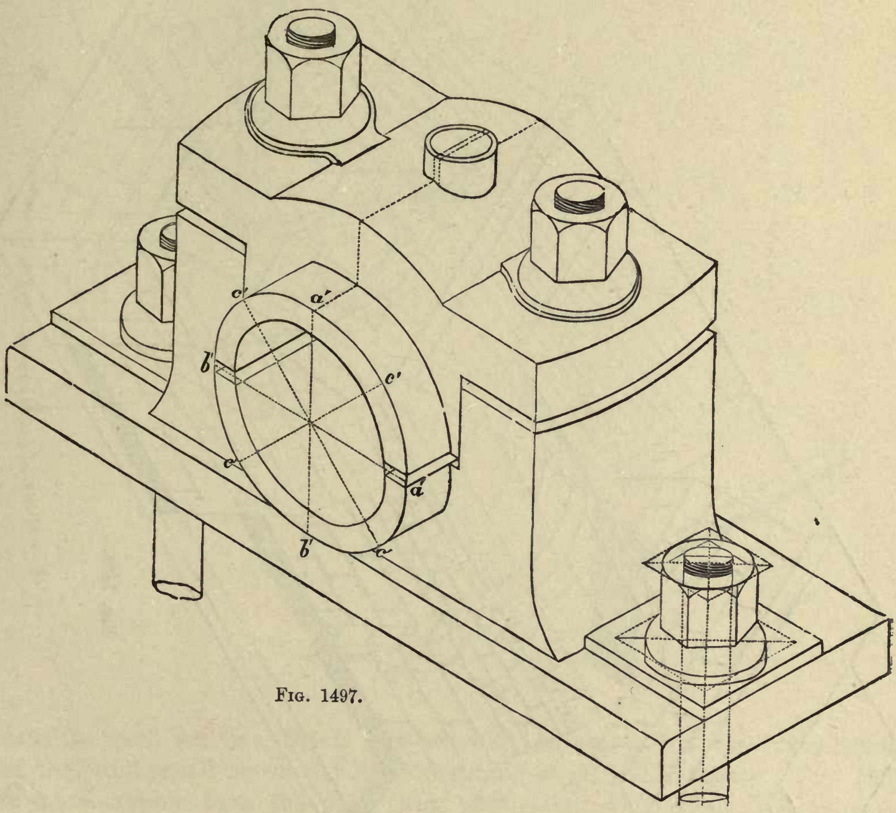


Fig. 1497.

Fig. 1498 is an isometrical projection of a culvert, such as were built beneath the Croton Aqueduct, and is a good example of construction, and better illustrated by the drawing than it would be by plan and elevations.

Fig. 829 is an isometrical view of the overflow and outlet of the Victoria and Regent Street sewers in the Thames embankment.

Fig. 1499 is an isometric elevation of the roof-truss (Fig. 896). No side-view is shown on the plate, but the dimensions of timber and spaces are drawn as usual in practice.

Figs. 1500 and 1501 are the elevation and section in isometry of the district school-house given in Figs. 1189 and 1190. To bring the drawing within the limits of the page, the scale has been necessarily reduced, but it is given in

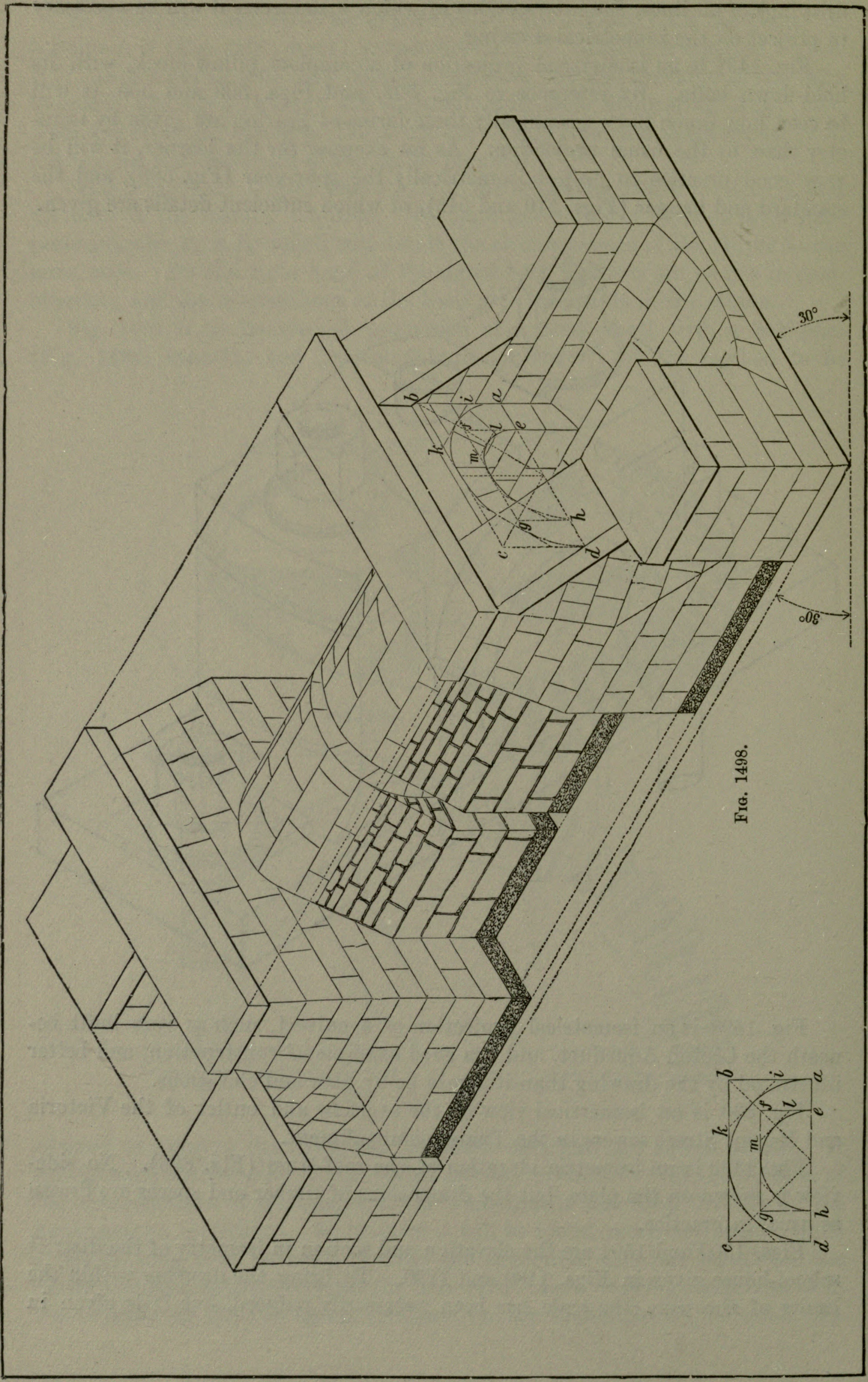


FIG. 1498.

the figure as it should always be, either drawn or written, on all drawings to a scale, not intended for mere pictures or illustrations. The section is drawn at the height of 8 feet above the base course, and higher than is

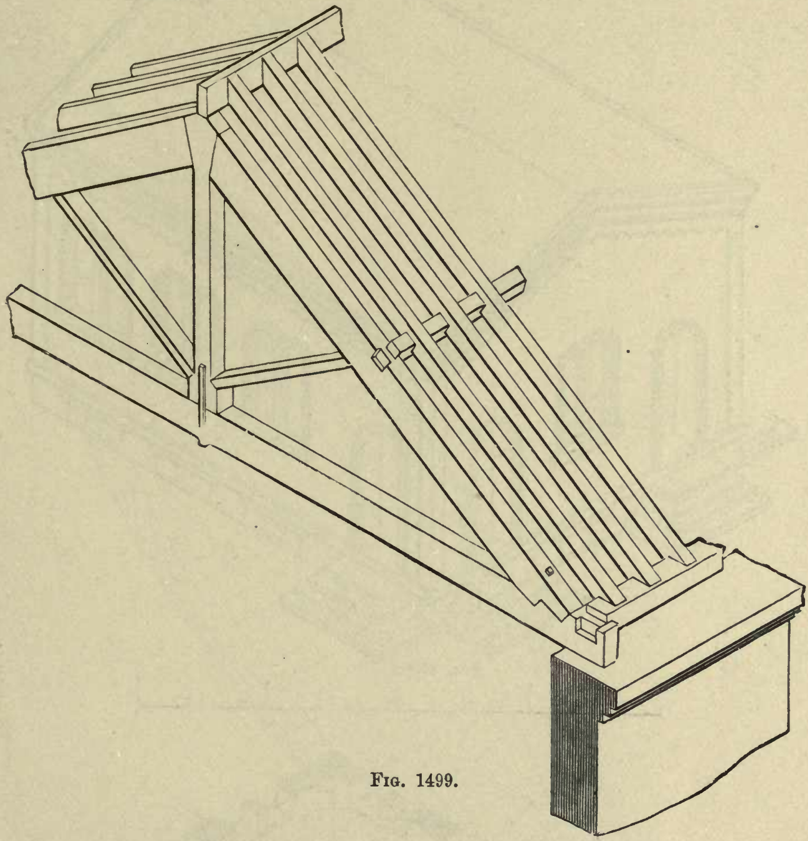


Fig. 1499.

usual in such sections, but it was necessary on account of the extra height of the window-sill above the floor, desirable in all school-rooms. Fig. 1501 is more graphic than the plan (Fig. 1190), and, when there are staircases one above the other in the drawing, they are more intelligibly expressed ; but there is nothing in the present drawing that can not be nearly as well shown by the plan, and to a mechanic, for the purposes of construction, the plan is the simpler.

By comparing the elevation (Fig. 1500) with the perspective (Fig. 1469), the former appears distorted, and out of drawing, but it is much more readily drawn, and has this great convenience, that it is drawn to and can be measured by a scale, but only on the isometric lines : all others are distorted, too long or too short, as may be seen in the major and minor axes of the bevel-gear (Fig. 1496), or the rake-lines of the roof (Fig. 1499).

Fig. 1502 is the isometrical projection, on the wave-line principle, of ship construction, from Russell's "Naval Architecture"—as explained and illus-

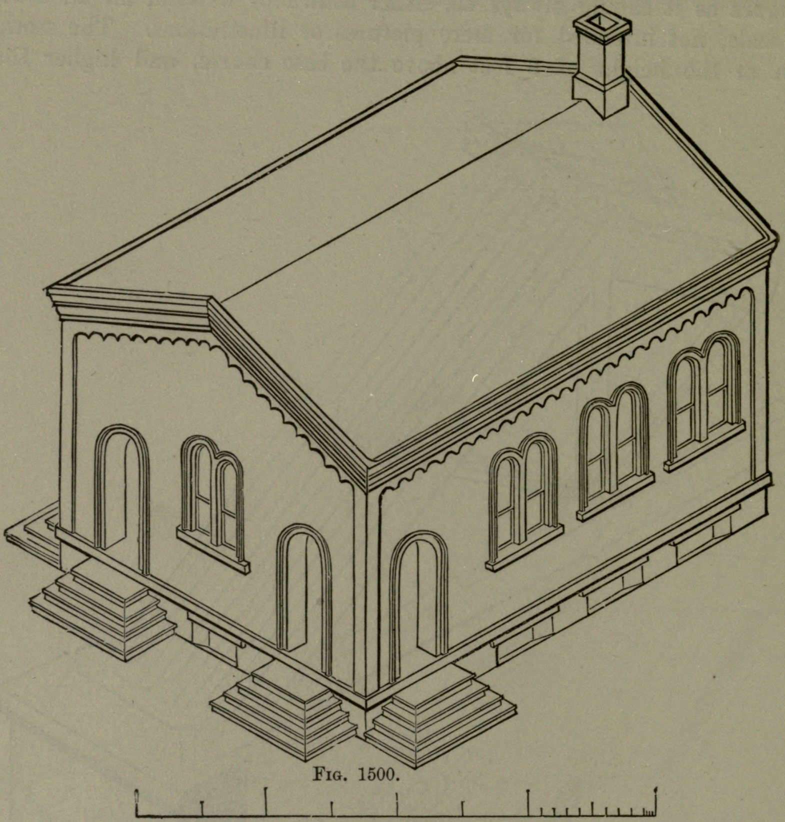


FIG. 1500.

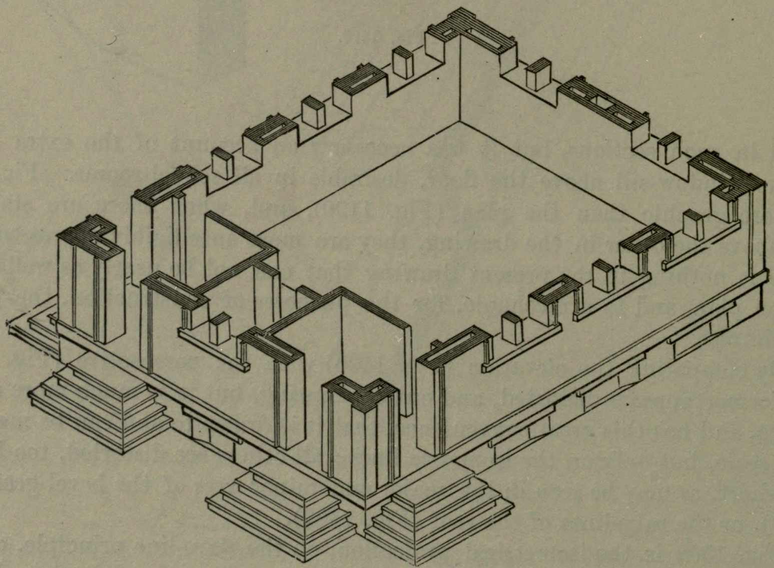


FIG. 1501.

trated on pages 458 and 459—and Fig. 1503, another isometrical drawing from the same work.

We have multiplied examples of isometrical drawing, to show its applica-

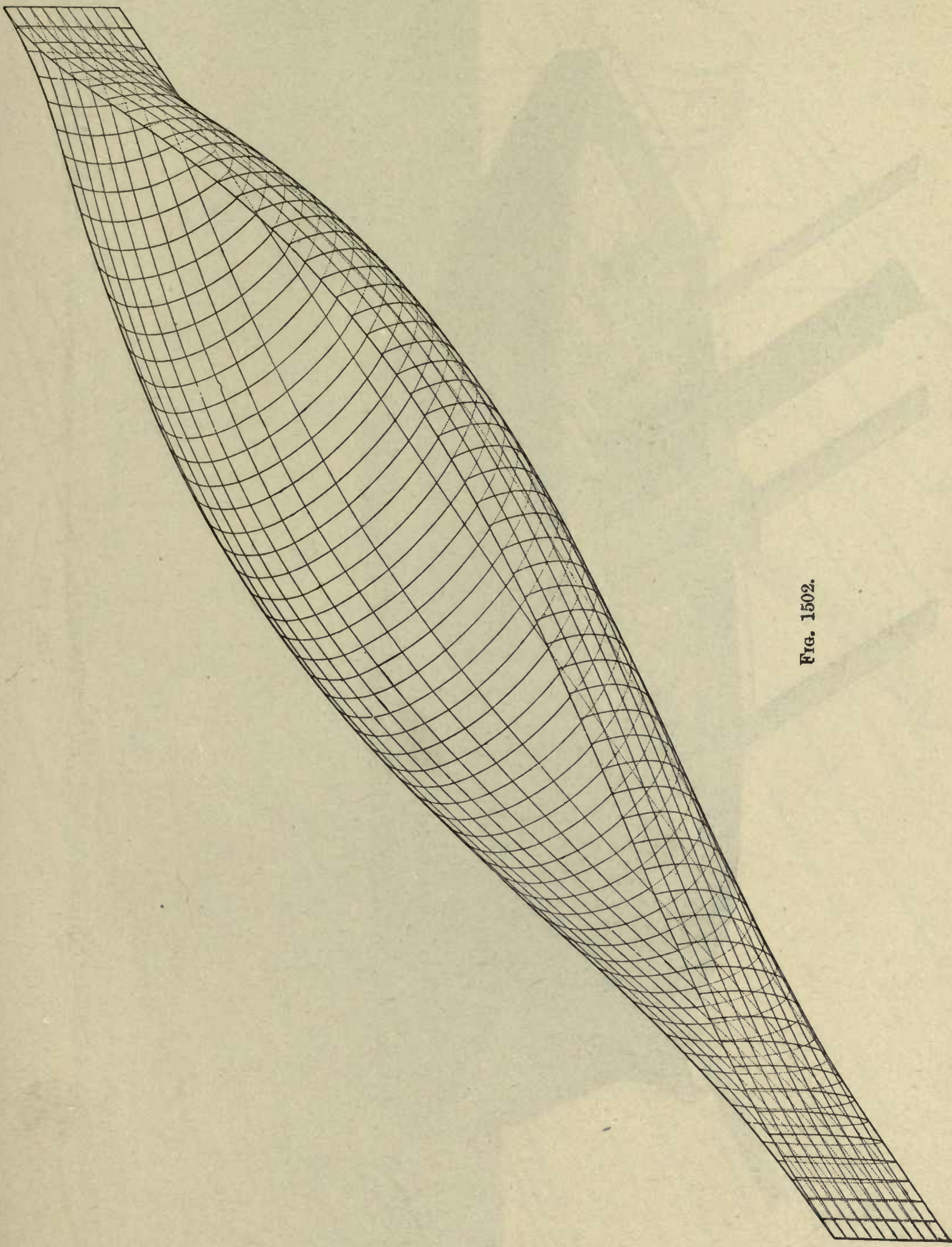


Fig. 1502.

bility to varied forms of construction, mechanical, architectural, and naval. The principles of this projection are easy and intelligible, and their use should

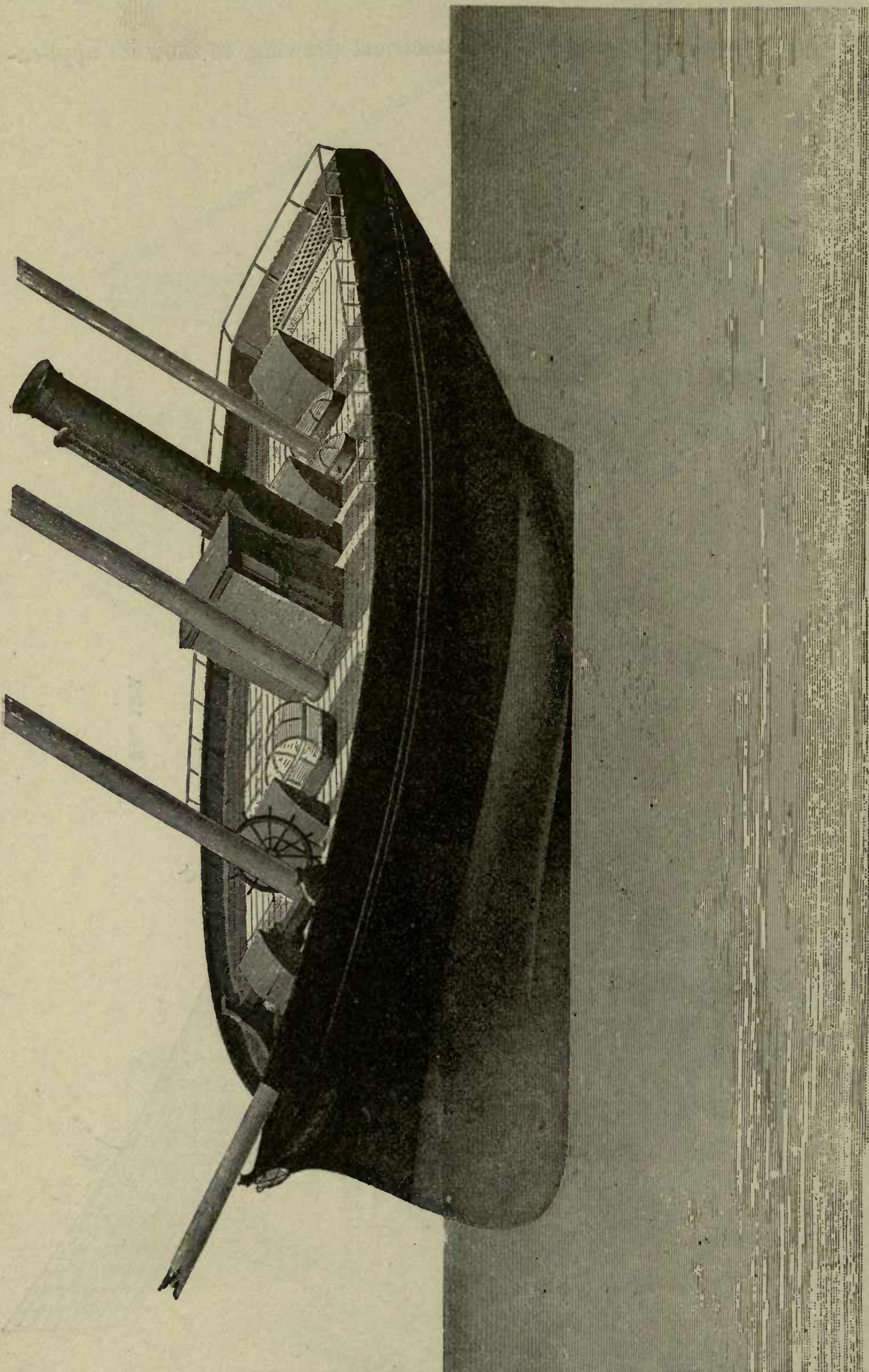


Fig. 1503.

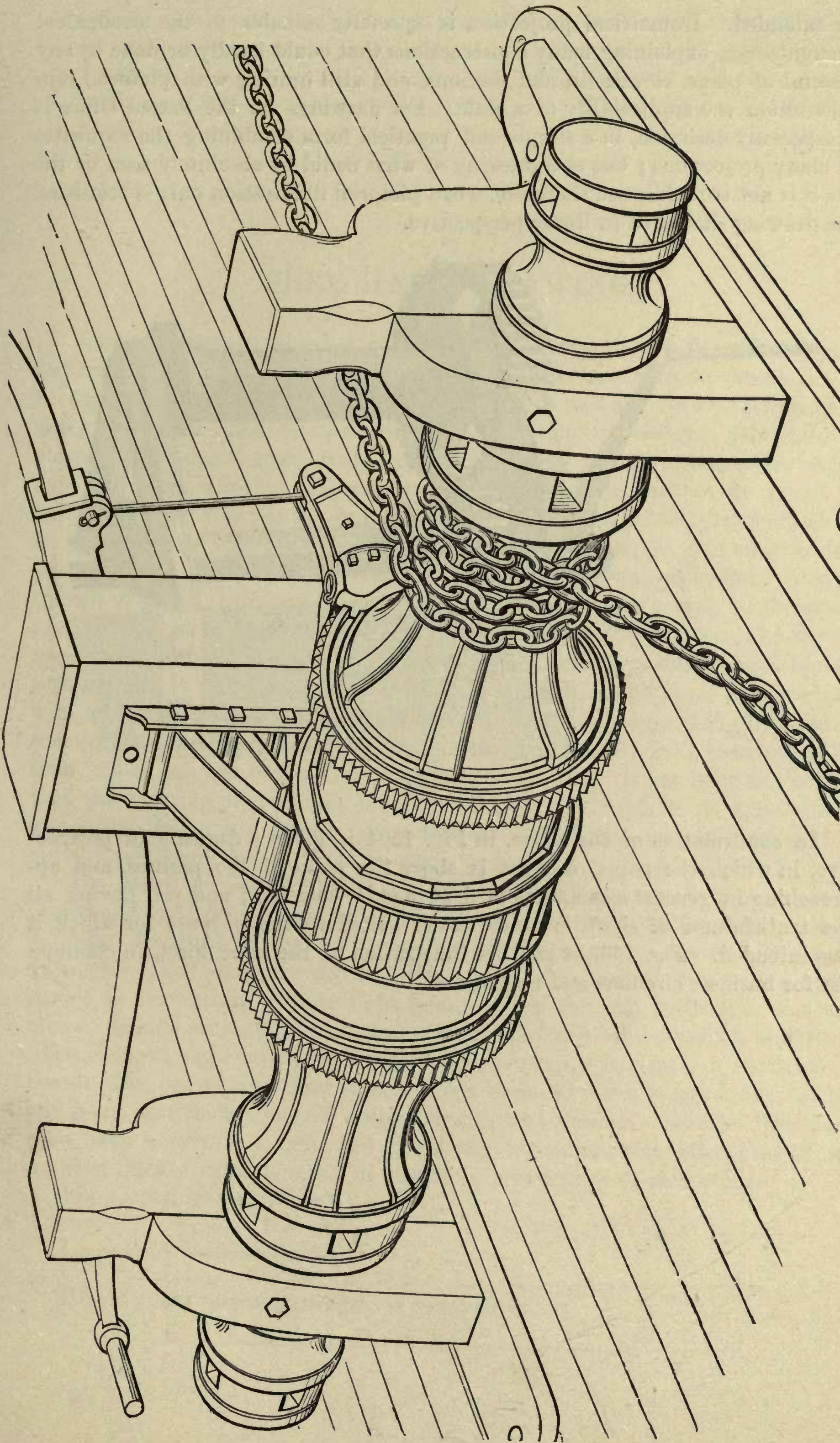


Fig. 1504.

be extended. Isometrical projection is especially valuable to the mechanical draughtsman, explaining many constructions that could hardly be done by any amount of plans, elevations, and sections, and still uniting with pictorial representation the applicability of a scale. For drawings for the Patent Office it is especially desirable, in a simple and practical form combining the requisites of many projections ; but as a drawing of what could be absolutely seen by the eye it is not truthful, and therefore, when pictorial illustration only is requisite, the drawing should be in linear perspective.

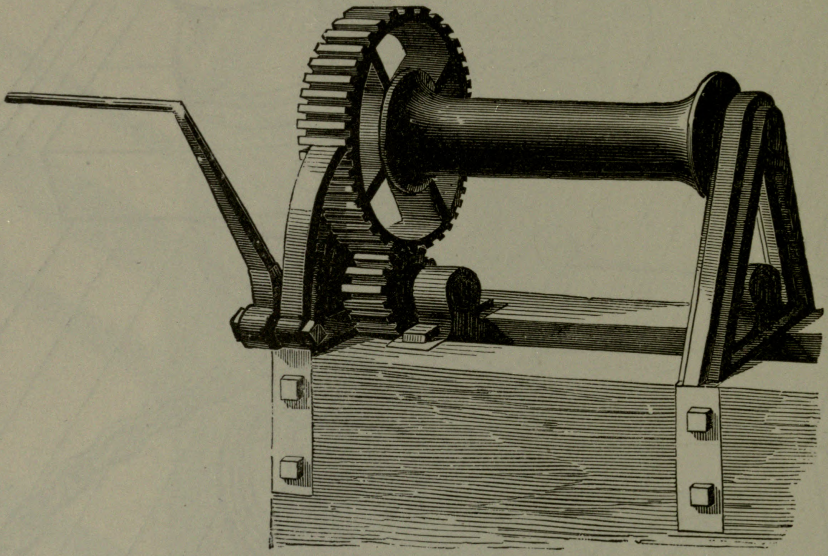


FIG. 1505.

In confirmation of the above, in Fig. 1504 is given a drawing in perspective, in which the point of sight is above the plane of the picture, and approaching in general appearance to drawings in isometry ; and yet, having all the truthfulness of sight, is much better suited to the purpose for which it was intended. Fig. 1505 is another illustration of the same kind, in common use for business circulars and catalogues.

FREE-HAND DRAWING.

A DRAUGHTSMAN, who has made himself conversant with the rules of projection as laid down in this book, and has applied these rules to practice, will be capable of representing correctly such objects as have been illustrated, or make up similar combinations of his own invention and design. But natural objects, as animals, trees, rocks, clouds, etc., can not be imitated on paper with the aid of drawing instruments ; outlines so varied can not be copied in this mechanical way ; it can only be done by what is called free-hand drawing, an educated eye that can recognize proportion and position, and an educated hand that can execute and portray naturally things recognized by the eye, with the aid of pencil, pen, crayon, or brush. A free hand adds largely to the effect of drawings, where close measures are not requisite, giving grace and beauty to mechanical designs, and is especially applicable to architectural ornaments and accessories. It will be found impossible to draw many of these in any other way, and there are few drawings that do not require some patching by hand—short curves, which can be thus done much more readily, and connections of lines, which can not be done by drawing instruments. It has been said before that the lettering of a plan or map contributes very much to its appearance, and as the Italian and Roman characters are now almost universally used it is only by free hand that they can be made ornamental or graceful.

The pencil or pen should be held by the thumb and first finger, and supported and guided by the second. The two fingers touching the pencil should be placed firmly on it, and be perfectly straight, the end of the middle finger at least one inch above the point of the pencil. In drawing, it is well to commence, as in writing, with straight lines. Lines vertical, horizontal, and inclined, parallel to each other and at angles, light and strong—short and long lines, straight and curved, with pen, pencil, or crayon on paper, or chalk on a board. Dot points, and draw lines between them, at a single movement, without going over them a second time, and without patching. Besides direction, lines have a definite length, and the draughtsman must practice himself in drawing lines of equal lengths, or in certain proportions to each other.

Lines equal to each other :

Lines twice another line :

Divide a line into any number of equal parts :

The accuracy of these divisions may be tested by a strip of paper applied along the line, marking off the divisions upon it, and then slipping it along one division, and noting if the divisions on the paper and line still agree. By practice, the eye will be able to make these divisions almost accurately. Having acquired this skill, copy the triangles in the Geometrical Problems, in their proper proportions, and afterwards squares and rectangles.

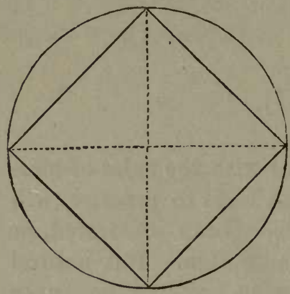


Fig. 1506.

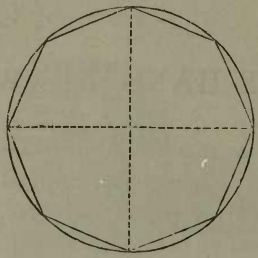


Fig. 1507.

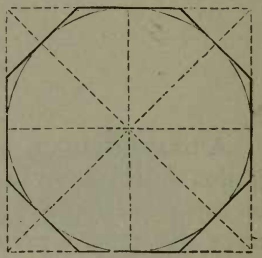


Fig. 1508.

Draw two lines (Fig. 1506) at right angles to each other, and mark equal distances on each one. Through these points draw a circle and a square.

Draw a circle and divide each quadrant into two equal arcs, and connect the chords to form an octagon (Fig. 1507). Or, draw a square, and cut off the corners (Fig. 1508).

Divide a circle into six equal arcs, and connect the chords for a hexagon.

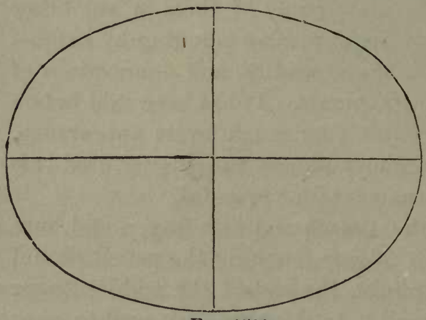


Fig. 1509.

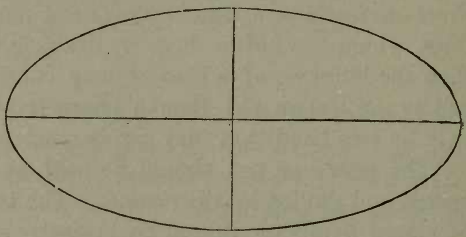


Fig. 1510.

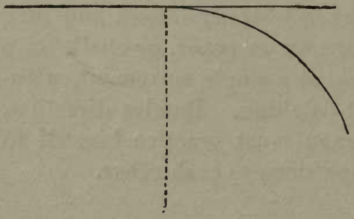


Fig. 1511.

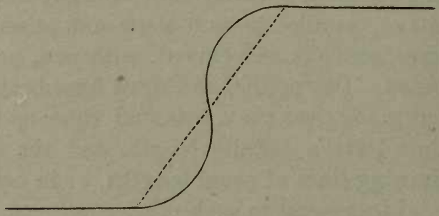


Fig. 1512.

Draw lines at right angles to each other, with only the opposite arms equal, and construct the ellipses (Figs. 1509 and 1510).

Draw an arc tangent to a straight line (Fig. 1511).

Draw two parallel lines (Fig. 1512), and connect them by two equal and reversed arcs, tangent to each other, and to the parallel lines.

Draw a similar curve, with arcs perpendicular to the parallels (Fig. 1513).

Although it will be observed that in all these problems guide or construction lines are used, it is not the intention that any use should be made of drawing instruments, but the construction should be dependent entirely on eye and hand; still it will be found, whether the draughtsman draws from copy or nature, that it is almost impossible to get along well without defining positions by some points in the pictures, and sketching in some defined lines which may serve as guides. All the above examples are from "Geometrical Problems," and it will be found good practice to copy others.

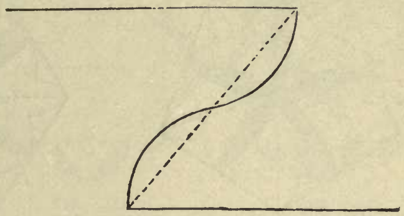
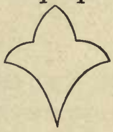


Fig. 1513.

Following this practice of guide lines, it will be well to copy the outlines of architectural moldings, of which most of the ornaments are conventional representations of natural objects.

In design, "a true artistic end has been accomplished when well-observed features of natural objects have been chronicled within the conventionalized limits of a few geometric rules that include proportion, symmetry, and a proper subordination of one part to another."



The following example is from the "Art Journal" (trefoil design):

"In the equilateral triangle (Fig. 1514), each side is divided by a dot, and from the center of the triangle lines are drawn to each angle, and from the dot in the middle of each side to the opposite sides of the figure. The geometrical plan of the design is thus laid out, and the figure is easily filled in by drawing simple curves from the center of the form to the dot on each side of it, and, lastly, filling in the form of the trefoil a little below the point of each corner of the triangle.

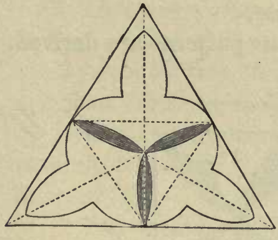


Fig. 1514.

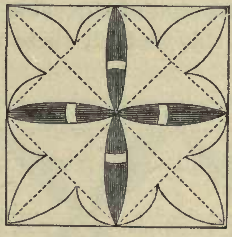


Fig. 1515.

"The square (Fig. 1515), which is the next form, is developed in much the same manner. The sides are bisected, and from a point in the center lines are carried to each angle, and to all the dots on the sides. As in the preceding figure, slight curves are made on either of the side-lines, and the trefoil is added to each angle, with the base of the middle leaf touching the transverse working-lines between the sides. It will be seen that the pentagon (Fig. 1516) and the hexagon (Fig. 1517) also are formed in the same general manner, but the proportion of the top of the trefoil varies from its sides.

"In drawing the circular rosette (Fig. 1518), the circumference should be constructed on a vertical and a horizontal diameter, with two other diameters bisecting it at equal angles, which divide it into eight sections, the half diameters, upon all of which curved lines and the top of the trefoil are made. A

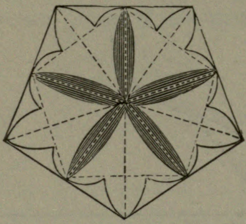


Fig. 1516.

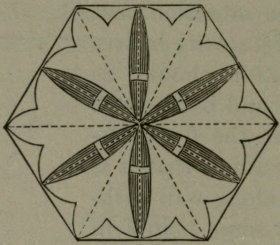


Fig. 1517.

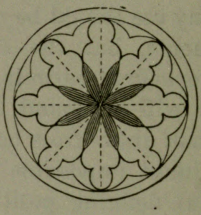


Fig. 1518.

series of arcs may be added at the pleasure of the designer. In the two pieces of molding (Figs. 1519 and 1520), the trefoil is inserted vertically to the sides in one and horizontally in the other. In the latter, a half of the trefoil is added upon the sides to enrich the elementary figure ; and the double line and

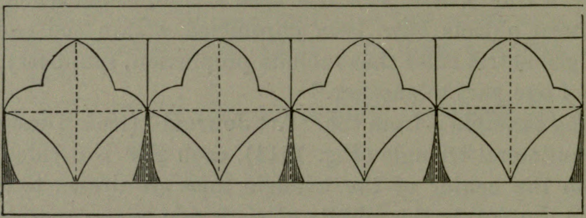


Fig. 1519.

the transverse lines which form the squares are repeated for the sake of symmetry, and as affording an impression of agreeable repose.

"It is from such a basis as this that all these various patterns are derived,

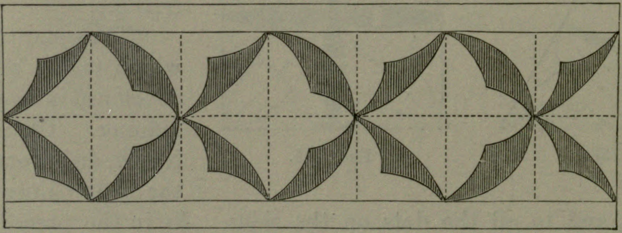


Fig. 1520.

and they produce a result which an inexperienced eye, unaccustomed to analyze designs, could scarcely resolve into its elements."

Figs. 1521–1524 are other illustrations of the same principle, of varieties of rosettes constructed on a similar plan.

All of these designs can be constructed mechanically, but more grace is given to the design by the filling in with free hand, and it is an excellent practice in the execution of the more elaborate Saracenic and Moorish diaper ; but

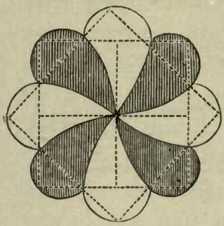


Fig. 1521.

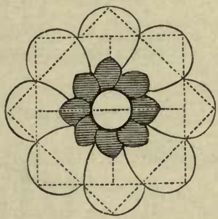


Fig. 1522.

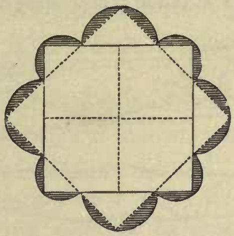


Fig. 1523.

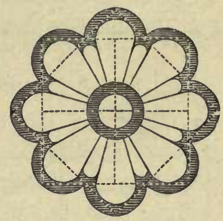


Fig. 1524.

in all of these where there are repetitions of the same figures it is usual to draw but one, and then transfer this, but the finish must be in crayon or pencil.

"Proportions of the Human Frame." By Joseph Bonomi.

The following, with the illustrations, are taken from the above work :

"The human frame is (Figs. 1525 and 1526) divided into four equal measures, by very distinctly marked divisions on its structure and outward form :

"1. From the crown of the head to a line drawn across the nipples.

"2. From the nipples to the pubes.

"3. From the pubes to the bottom of the patella (knee-pan).

"4. From the bottom of the patella to the sole of the foot.

"Again, four measures, equal in themselves, and equal to those just described, and as well marked in the structure of the human body, are seen when the arms are extended horizontally. They are the following :

"From the tip of the middle or longest finger to the bend of the arm is one fourth of the height of the person.

"From the bend of the arm to the pit of the neck is another fourth.

"These two measures, taken together, make the half of the man's height, and with those of the opposite side equal the entire height.

"In the figures, the differences in width between the male and female figures are given from the tables of the Count de Clarac of the Apollino and the Venus de Medici. The male figure is in thicker line than the female, and the measurements referring to it are on your right hand, and those referring to the female on your left.

"The measurements of length, according to Vitruvius and Leonardo da Vinci, are the same in both sexes, and expressed in long horizontal lines running through both the front and profile figures.

"Almost innumerable are the varieties of character to be obtained by the alterations of widths, without making any change in the measurements of length ; nevertheless, some ancient statues differ slightly in these measurements of length.

"No measurement is given in the figure of the width of the foot ; its normal proportion should be one sixteenth of the height. The views of the foot (Fig. 1527) are those of the female.

"The scale, V, used is 8 heads to the height; parts, $\frac{1}{4}$ of a head; and minutes, $\frac{1}{12}$ of a part.

"The whole height is usually taken at 8 heads, but there are slight differences in the classic statues; the height of the Venus de Medici is equal to 7

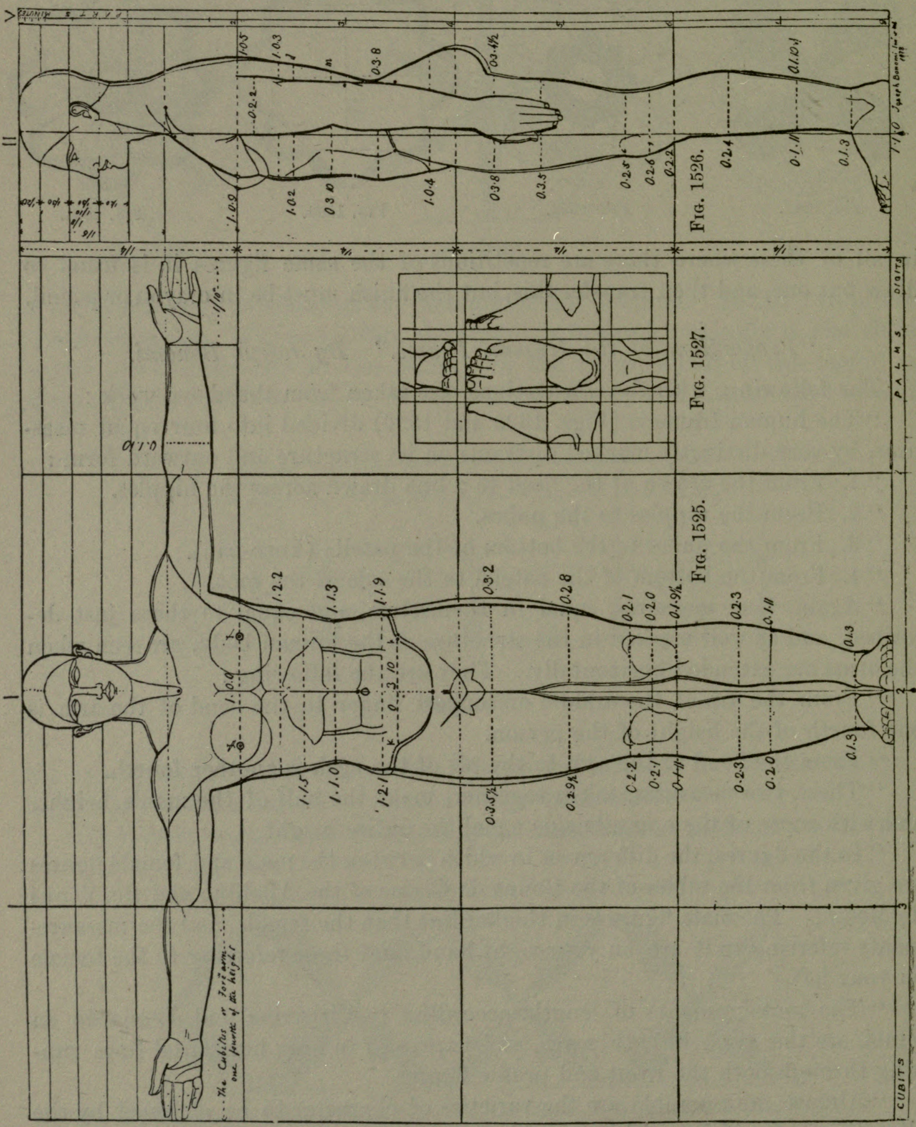


Fig. 1526.

Fig. 1527.

Fig. 1525.

heads, 3 parts, 10 minutes, that of the Apollino of Florence, 7 heads, 3 parts, 6 minutes.

"When the student is acquainted with the forms of the body and limbs in two aspects—viz., the front and side views—and the normal proportions they bear to each other, then will follow the study of the characteristic features of childhood, youth, and mature age, and those niceties of character that the ancients invariably observed in the statues of their divinities, so that in most cases a mere fragment of a statue could be identified as belonging to this or that divinity—as, for instance, the almost feminine roundness of the limbs of the youthful Bacchus, the less round and distinctly marked muscles of the Mercury, and of the statues of the Athletæ."

Figure Drawing.—In the album of Villard de Hennecourt, which dates from the middle of the thirteenth century, certain mechanical processes are given to facilitate the composition and design of figures. According to these sketches, geometry is the generator of movements of the human body, and that of animals, and serves to establish certain relative proportions of the figures. From the time of Villard sculptors have had these practical methods, which, if they could not inspire the artisan with genius, yet prevented him from falling into gross faults. The pen sketch (Fig. 1528) is an example of this prac-

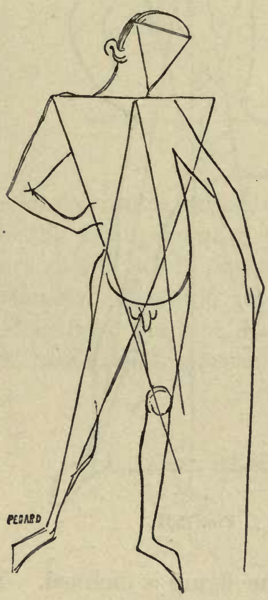


Fig. 1528.

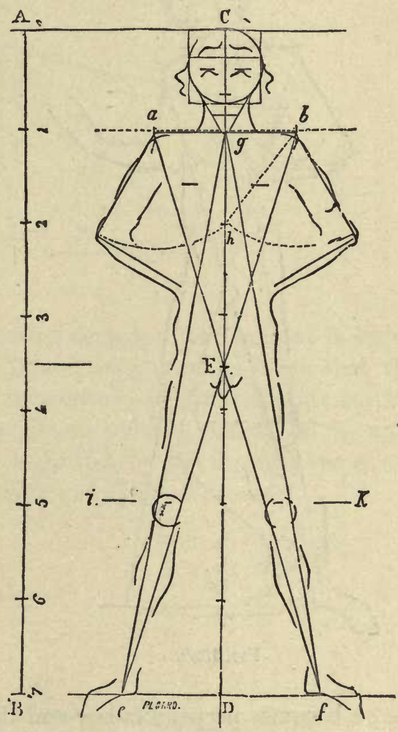


Fig. 1529.

tical process. In comparing this mode of drawing with figures in the vignettes of manuscripts, with designs on glass, and even with statues and bas-reliefs, we must recognize the general employment in the thirteenth and fourteenth centuries of these geometrical means, suited to give figures not only their proportions but also the justness of their movement and bearing. Rectifying the canon of Villard in its proportions by comparison with the best statues, notably those in the interior of the western façade of the Cathedral of Reims, we obtain the Fig. 1529. The line A B, the height of the human figure, is divided into seven equal parts. The upper division is from the top of the head to the shoulders. Let C D be the axis of the figure, the line at the breadth of the shoulders is $\frac{2}{7}$ of the whole height A B. The point E is the center of the line C D; draw through this point two lines, *a f* and *b e*, and from the point *g* two other lines, *g e* and *g f*. The line *b h* is the length of the humerus, and the line of the knee-pan is on *i k*. The length of the foot is $\frac{4}{5}$ of a division, A 1. Having established these proportions, it will be seen by the following cuts how the artisan gave movements to these figures when the movements were not in absolute profile.

Suppose the weight of the figure to be borne upon one leg (Fig. 1530), the

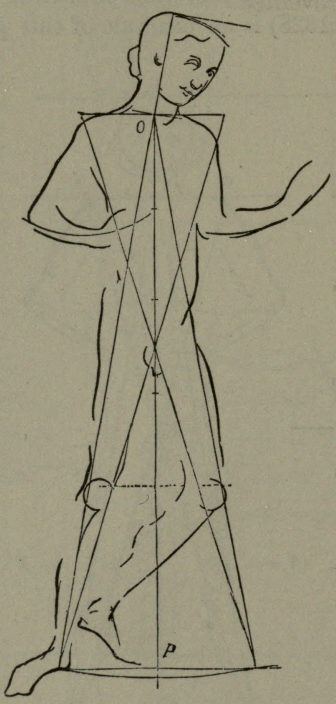


Fig. 1530.

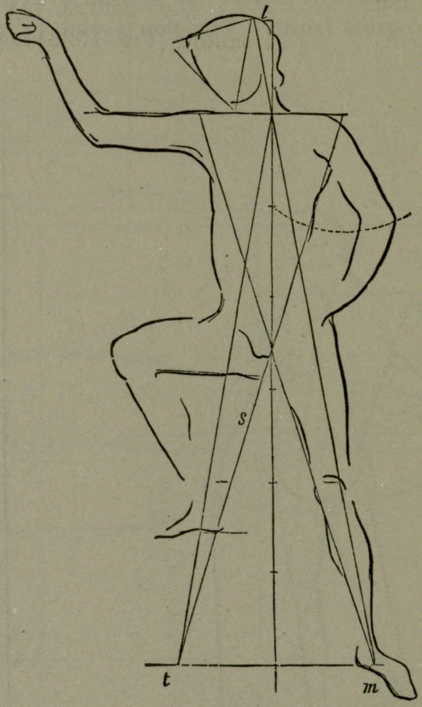


Fig. 1531.

line *g e* becomes perpendicular, and the axis *o p* of the figure is inclined. The movement of the shoulders and trunk follow this inflection; the axis of the head and the right heel are in the same vertical line.

In stepping up (Fig. 1531) the axis of the figure is vertical, and the right heel raised is on the inclined line *s t*, while the line of the neck is on the line *l m*, and the trunk is vertical.

In Fig. 1532 it will be seen how a figure can be submitted to a violent movement and yet preserve the same geometrical trace. The figure is fallen, supported on one knee and one arm, while the other wards off a blow; the head is vertical.

In Fig. 1533, the left thigh being in the line *af*, to determine the position of the heel *c* on the ground, supposed to be level, an arc is to be described from the knee-pan ; the line *ef* is horizontal.

It is clear that, in adopting these practical methods, all the limbs can be developed geometrically without shortening.

The above is from the "Dictionnaire raisonné de l'Architecture" of Viollet Le Duc, and will supply to many a ready means of sketching the human figure

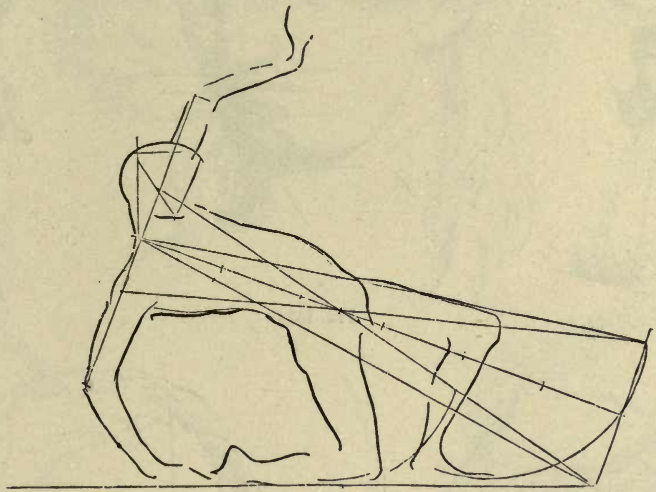


Fig. 1532.

in various attitudes, naked, or in the close-fitting dresses of the present fashion ; but in the arrangement of drapery upon a figure, care must be taken that the drapery should fall in graceful folds. "It is necessary to give the body certain inflections which would be ridiculous in a person walking naked. The walk should be from the hips, with wide-spread legs, and, by the movements of the trunk, make the drapery cling on certain parts and float on others."

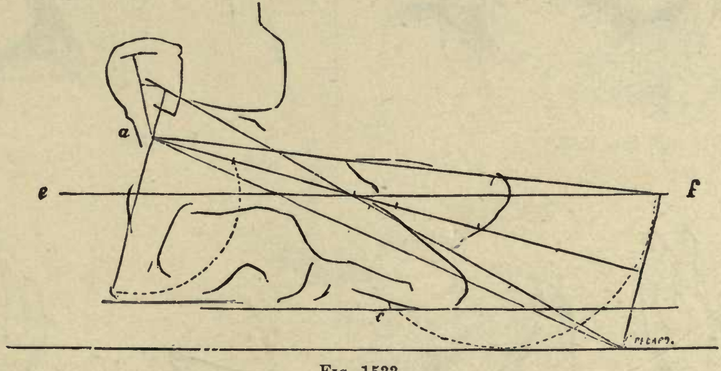


Fig. 1533.

In figures in repose, their centers of gravity must fall within the points of support, but the body can be sustained by muscular exertion, and this should

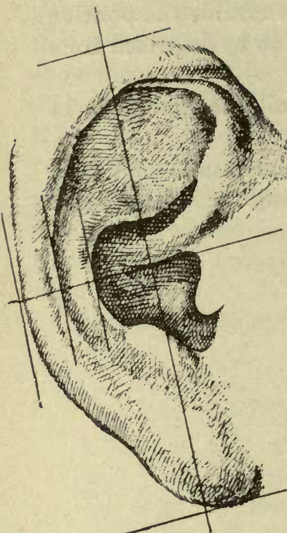


Fig. 1537.

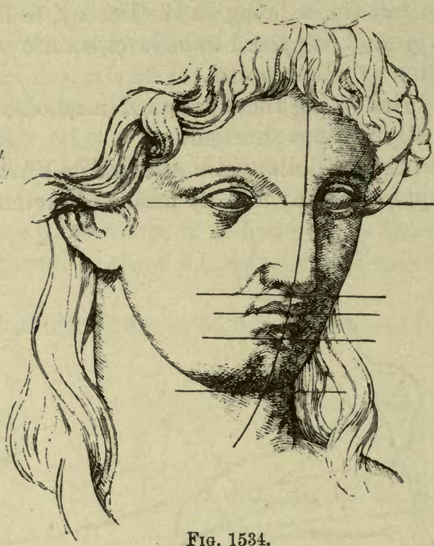


Fig. 1534.

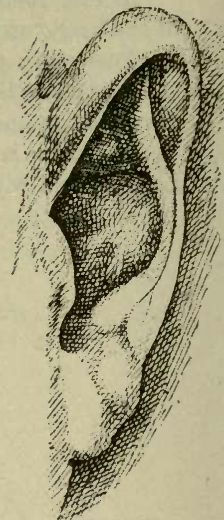


Fig. 1538.

Fig. 1535.

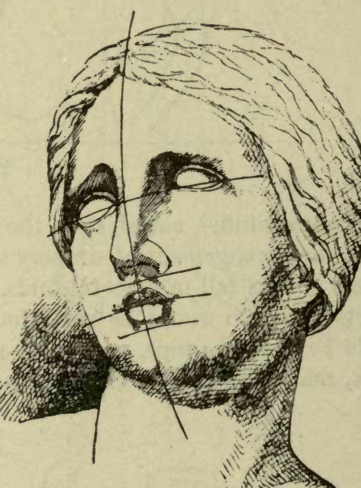


Fig. 1536.

Fig. 1540.

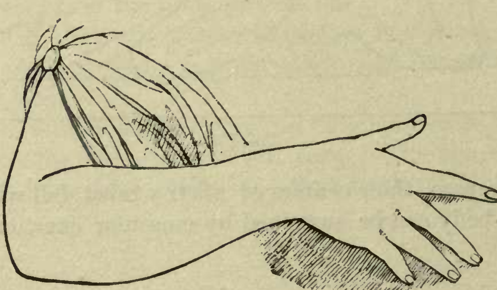


Fig. 1539.

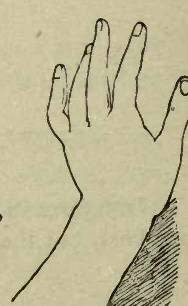


Fig. 1541.

Fig. 1542.

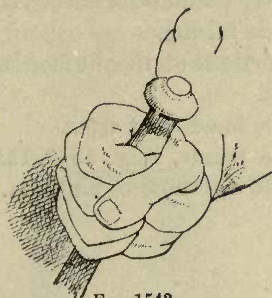


Fig. 1543.

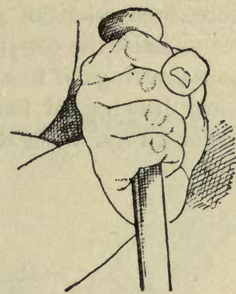


Fig. 1544.

Fig. 1545.

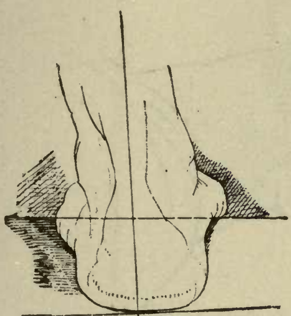


Fig. 1546.

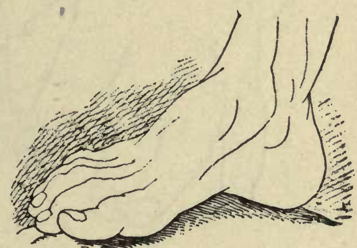


Fig. 1548.

Fig. 1547.

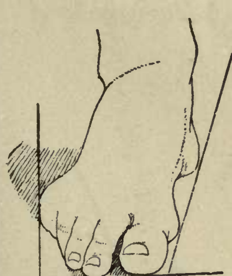


Fig. 1549.

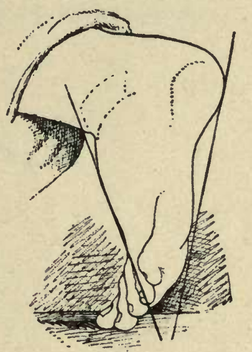


Fig. 1550.

Fig. 1551.

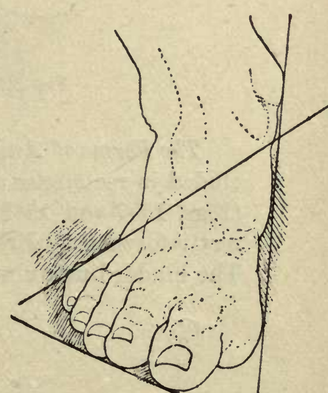


Fig. 1552.

be expressed in such cases by the tension of the muscles on which the position depends. In the act of running, the body inclines forward, its weight assists the movement, and the motions prevent its falling.

Figs. 1534–1538 are illustrations of portions of the human head and face, with some guide-lines to assist the copyist.

Figs. 1539–1541 are drawings of female hands and arms.

Figs. 1542–1545 are drawings of male hands, Figs. 1546–1552 of legs and feet, with guide-lines, and Figs. 1553–1556 are those of children.

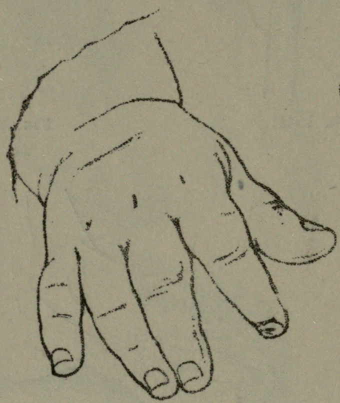


Fig. 1553.

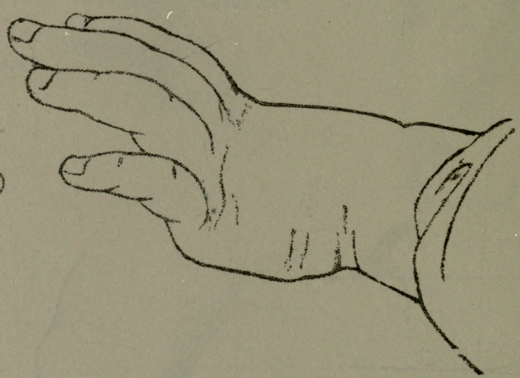


Fig. 1554.

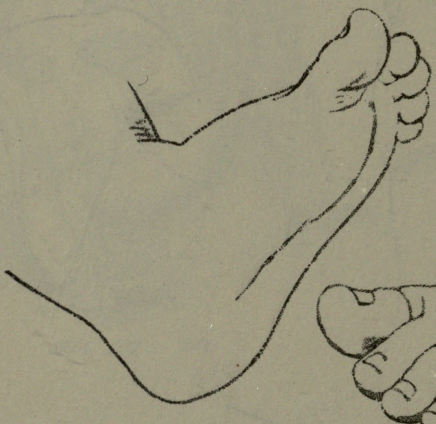


Fig. 1555.

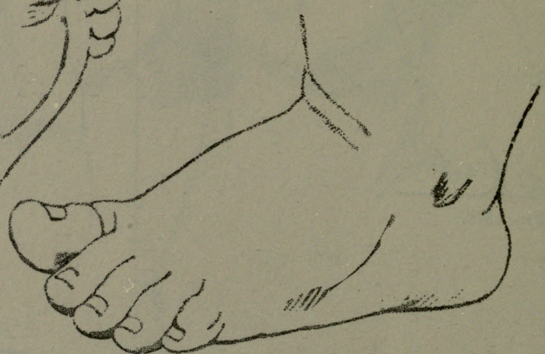


Fig. 1556.

The Forms of Animals.—The bodies of most quadrupeds standing can be included in rectangles as guide-lines ; that of the ox and horse in that of a square (Figs. 1557 and 1558). The action of the limbs of quadrupeds is chiefly directly forward or directly backward, the power of lateral motion being limited. The hinder limbs always commence progressive motion, as in the first position of the walk (Fig. 1559), the fore foot of the same side advances next, then the hind foot of the opposite side, and lastly the fore foot on that side, and so on. In the trot, the hinder leg of one side and the fore leg of the other are raised together (Fig. 1560). In the canter or gallop, both fore legs and one hind

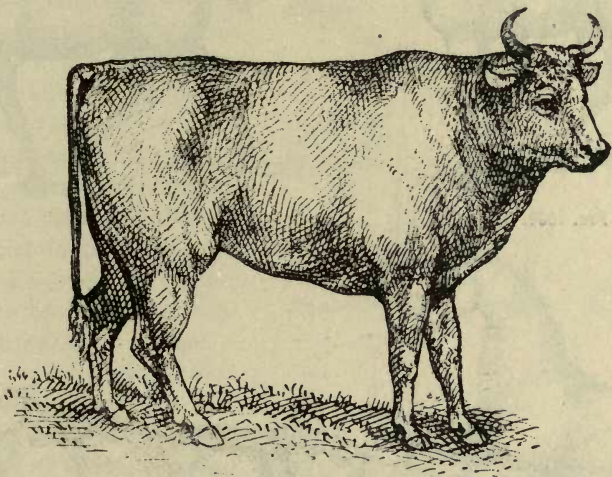


Fig. 1557.

leg are raised together (Fig. 1561) ; when rapidly moving, the two fore legs and two hind legs appear to advance together (Fig. 1562). In fact, all the movements are rather resultants, as they appear to us, but when instantaneously photographed the legs are wonderfully mixed.

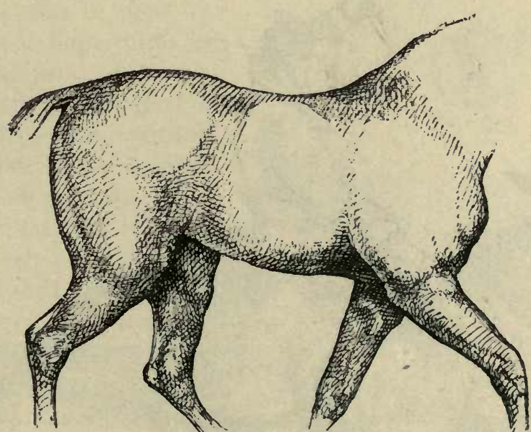


Fig. 1558.

The forms of feet range under two great divisions—hoofs (Fig. 1564) and paws (Fig. 1565). All hoofs, whether whole or cloven, approximate to a right-angled triangle, and all paws to a rhomboid.

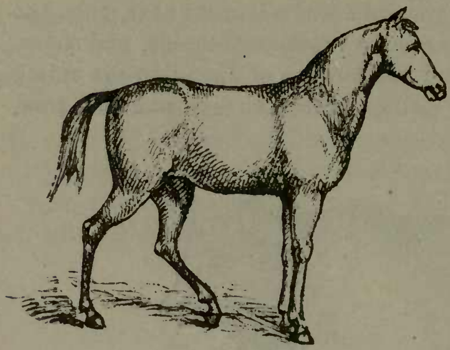


Fig. 1559.

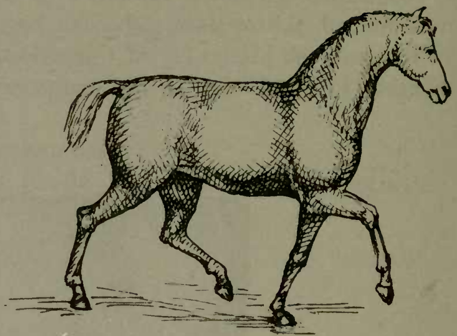


Fig. 1560.

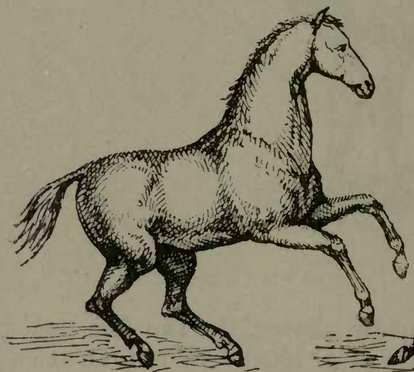


Fig. 1561.

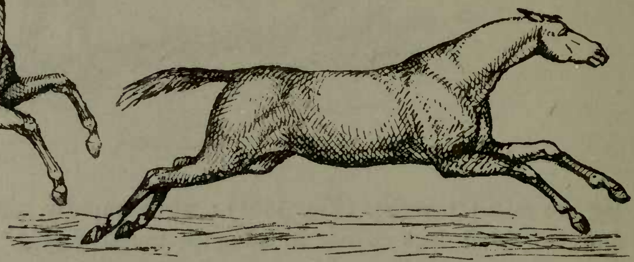


Fig. 1562.

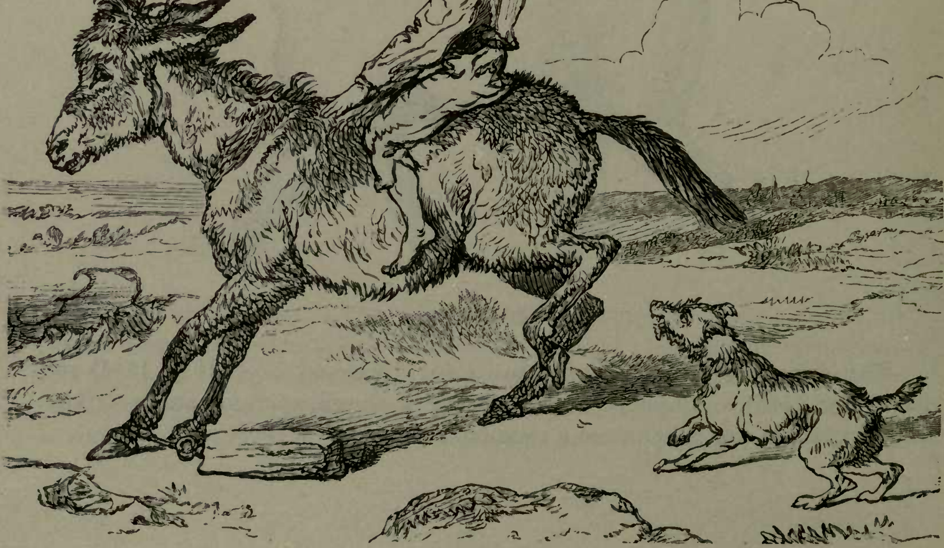


Fig. 1563.

The Noses of Animals.—Fig. 1566 represents that of the horse ; Fig. 1567, that of the ox and deer tribe ; Fig. 1568, those of the carnivori ; Fig. 1569, those of the camel, sheep, and goat tribes ; and Fig. 1570, those of the hog tribes. The muzzles of nearly all quadrupeds will be found to range under one or other of these classes, with minute variations to characterize the different species and individuals.

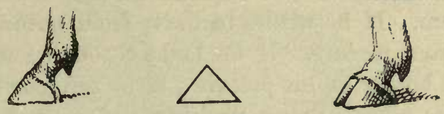


Fig. 1564.

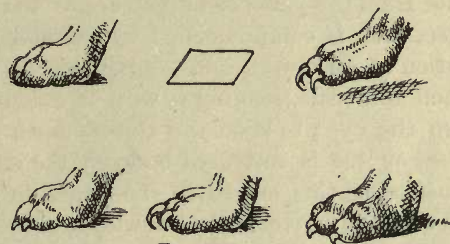


Fig. 1565.

Fig. 1566.

Fig. 1567.

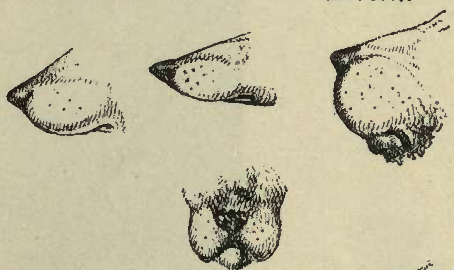


Fig. 1568.

Fig. 1569.

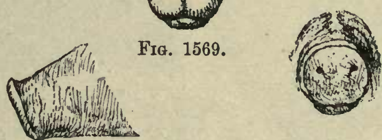
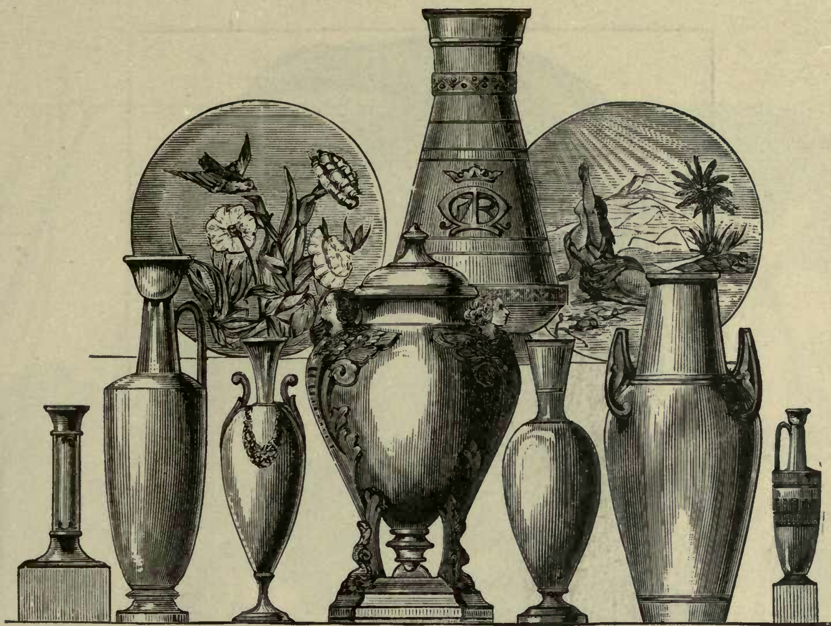


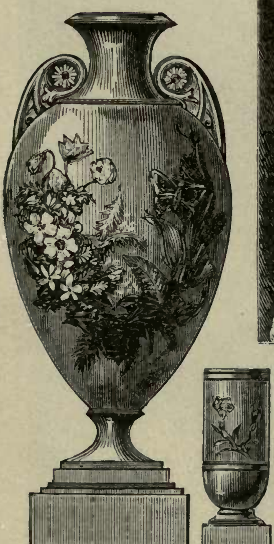
Fig. 1570.

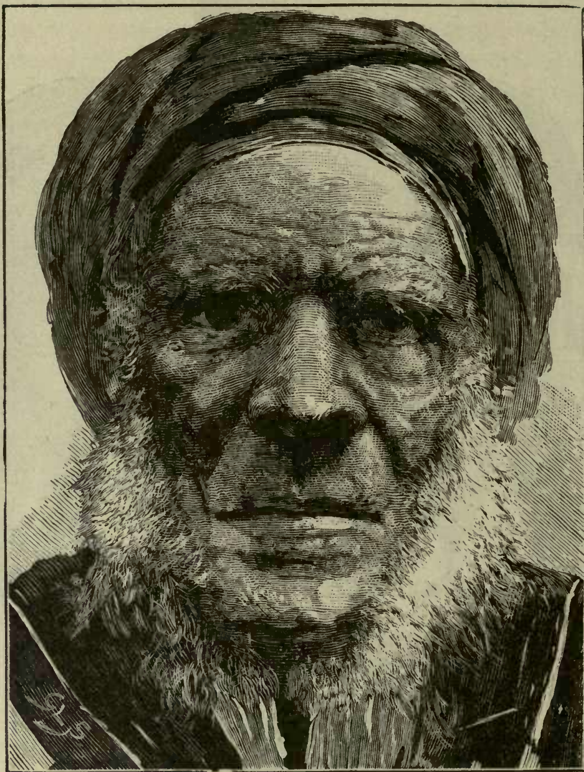
In looking over the varied sketches and engravings of Landseer which have been published, it will be noticed in how varied a manner they are executed. Sometimes in mere outline with lead-pencil, sometimes with a camel's-hair pencil charged with Indian ink or sepia for the outlines, giving effect to the subject by slight tints or washes of the same color ; in others, pen and ink have been alone employed. Some are in oils, others in water-colors ; frequently chalks, both black and colored, were the vehicles used. "As we look at some of these, we are tempted to believe that, of all the instruments that can be used by the artist, there is none quite so wonderful as the pen. A simple sketch with a pen or lead-pencil is naked, unadorned truth, bearing witness to the skill or its opposite of the hand which produced it."

The above quotation is given to show the value of accurate drawing—the skeleton, as it were, may be more suggestive, and convey more skillfully effective truth than the finished drawing, and the first necessity is truth in drawing. Nothing has yet been said of drawing from nature. The copies given are intended as rudiments, and the following illustrations from the "Art Journal" of objects in art, and sketches and pictures of different painters, will serve to show their varied treatment of subjects.

The illustrations given are for the education of the eye of the draughtsman, in showing him the varied appearance of different subjects by different artists, and their modes of expression; and he can acquire facility of hand in copying them. If he wishes to draw from nature, let him look at objects as if they were a picture. If he looks through a window, the frame may be considered the border of his picture; if he can portray what he sees through a square of glass truthfully, in position and proportion, with pencil, chalk, or brush, he has made a picture. He must keep his eyes in one position, or at such a distance from the plane of his picture or the glass that he can not see more of an object than is comprehended by one look. To enable one to judge of the proportion of an object, and its position, it is very common to make use of the pencil as a scale, holding it with an extended arm always at the same distance from the eye; to slide the thumb down on the pencil till the length of the object or line is embraced between the end of the pencil and the thumb, and transferring this length to the paper in its proper position. Practically, in this way, one arrives at the knowledge of perspective, of which the principles have been given in "Perspective Drawing." Aërial perspective, or the tones of lights and shadows according to their distances from the observer and the sources of the light, he will acquire by studies of pictures and observations of nature. The rule in drawing from nature is to draw only what you see, and express it in the most truthful form.

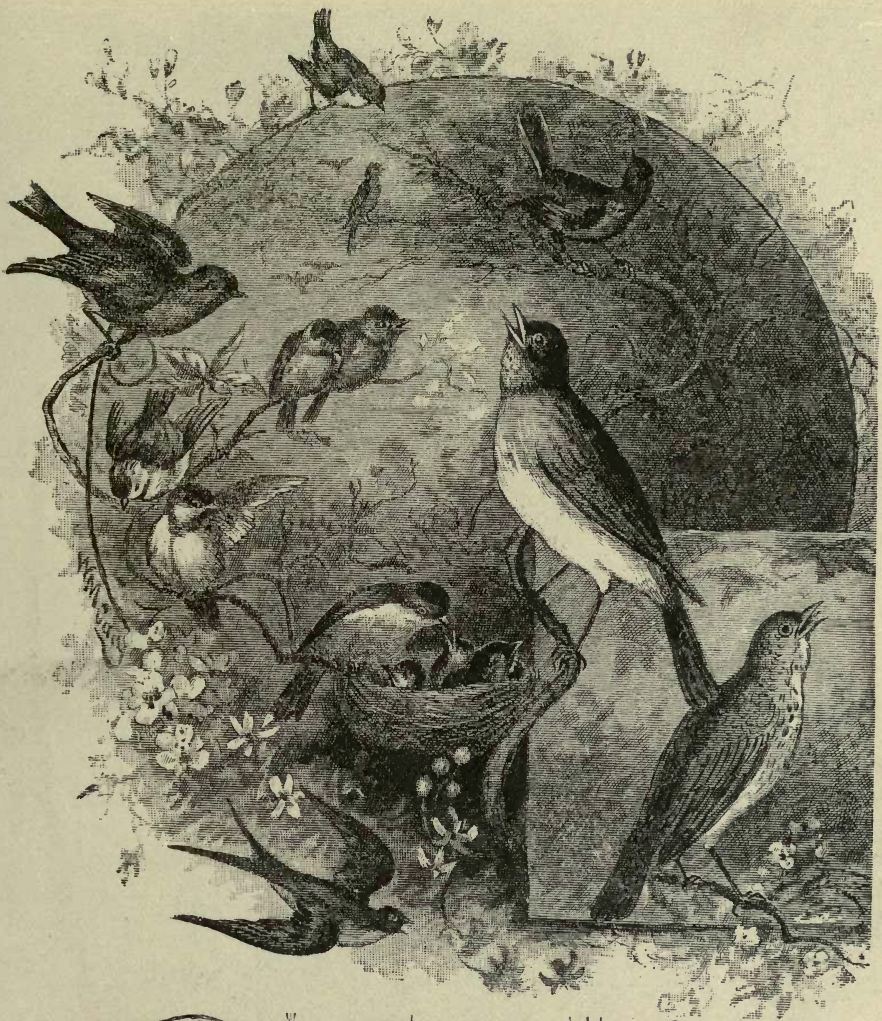


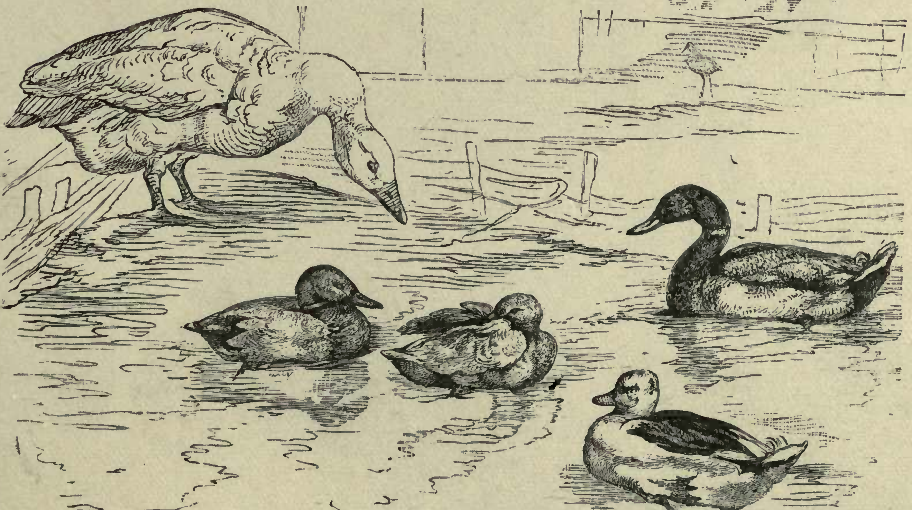




Bacchus and the Water-Thieves. JOHN PENNIEL.

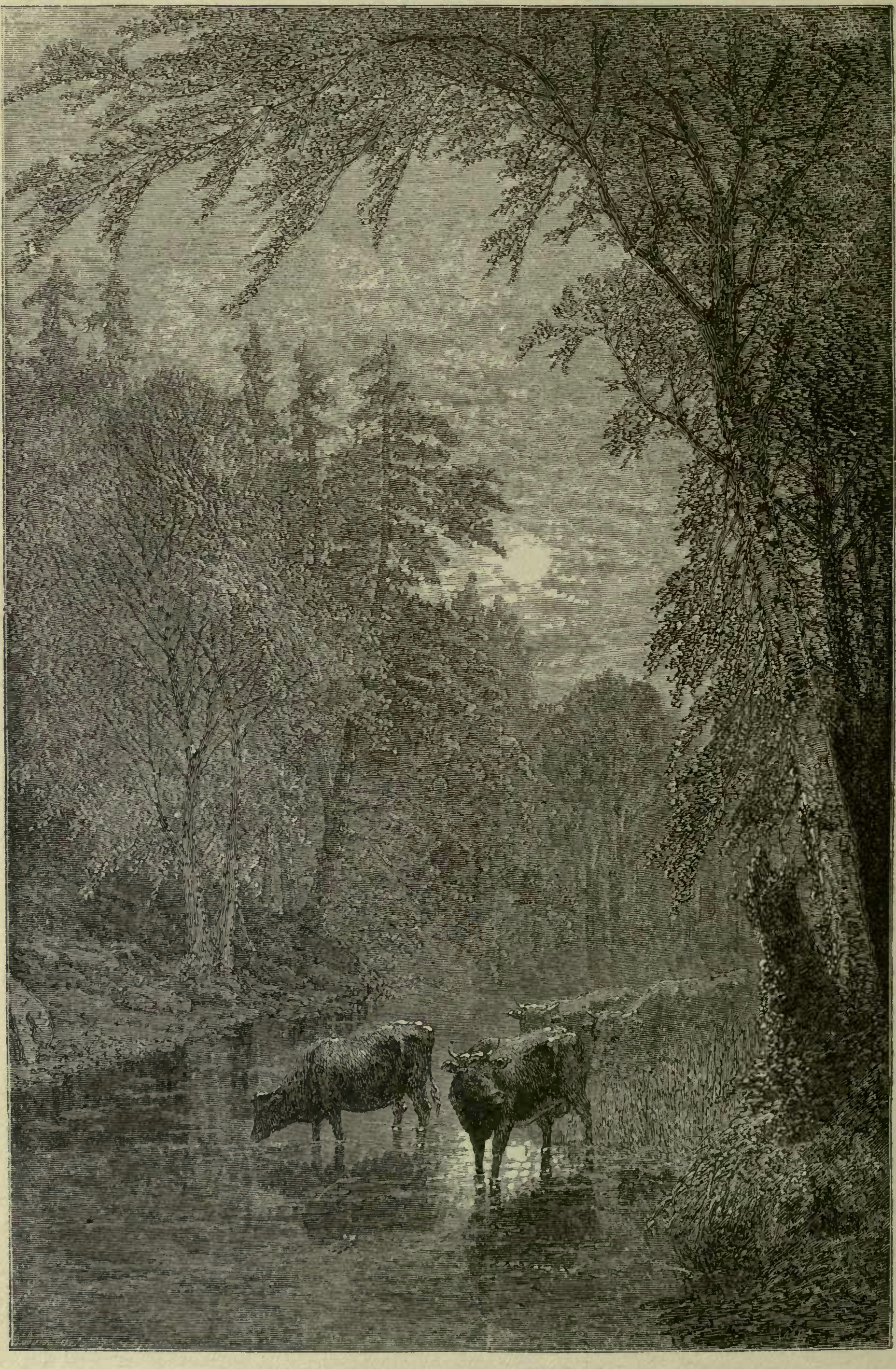
After a Pen-and-ink Design, by FORTUNY.





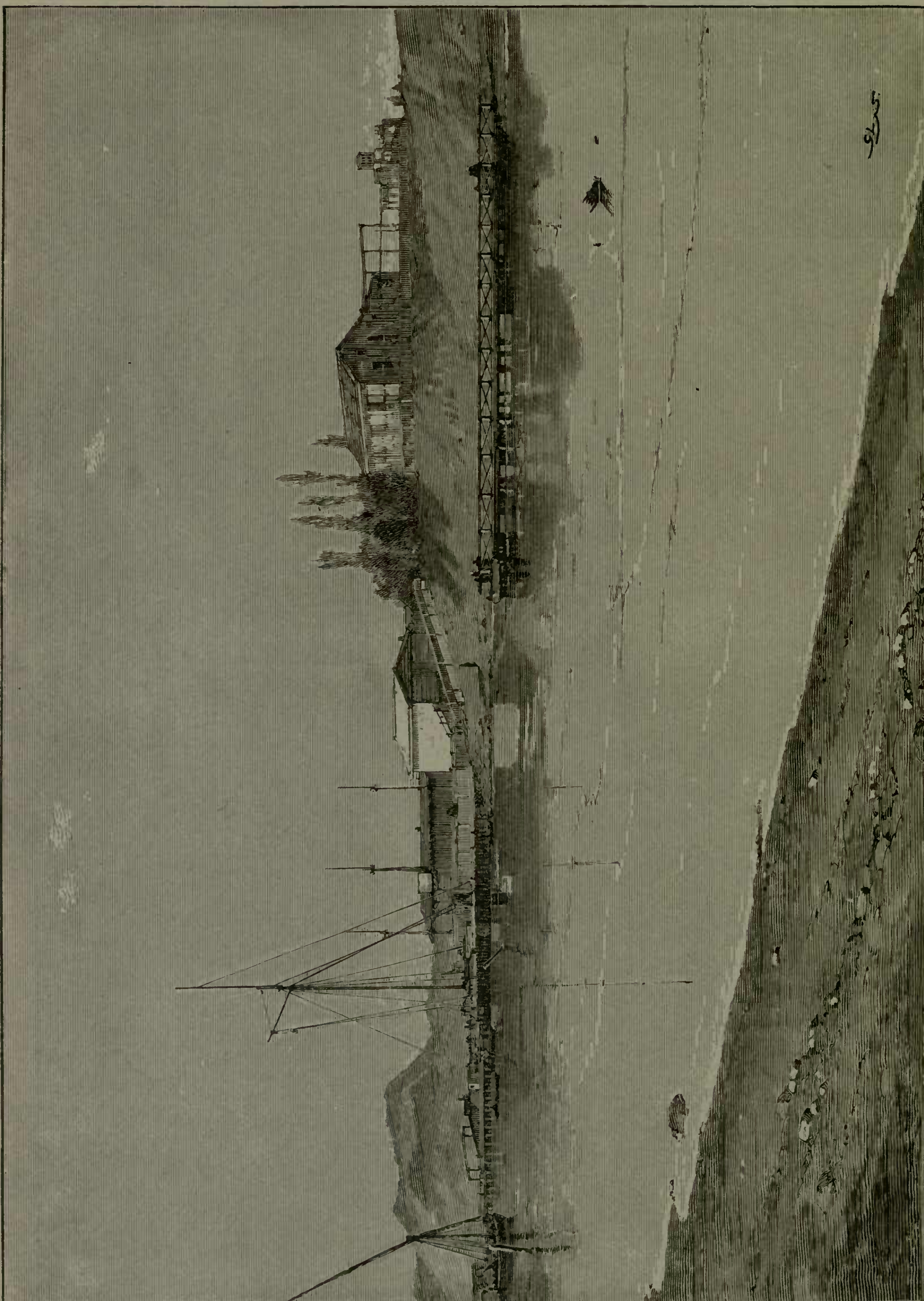
Study of Oak-Trees. E. LANDSEER.

Apple-Blossoms. A. T. BRICHER.



Cattle going Home. James M. Hart.

Morning. H. W. Robbins.



The Station at Kantarah, on the Suez Canal.

APPENDIX.

Extracts from the Acts relating to Buildings in the City of New York.

§ 3. All foundation walls shall be laid not less than 4′ below the surface of the earth, on a good solid bottom, and, in case the nature of the earth should require it, a bottom of driven piles, or laid timbers, of sufficient size and thickness, shall be laid to prevent the walls from settling, the top of such pile or timber bottom to be driven or laid below the water line; and all piers, columns, posts, or pillars resting on the earth, shall be set upon a bottom in the same manner as the foundation walls. Whenever in any case the foundation wall or walls of any building that may hereafter be erected shall be placed on a rock bottom, the said rock shall be graded off level to receive the same. . . .

§ 4. The footing, or base course, under all foundation walls, and under all piers, columns, posts, or pillars resting on the earth, shall be of stone or concrete; and if under a foundation wall shall be at least 12″ wider than the bottom width of the said wall; and if under piers, columns, posts, or pillars, shall be at least 12″ wider on all sides than the bottom width of the said piers, columns, posts, or pillars, and not less than 18″ in thickness; and if built of stone, the stones thereof shall not be less than 2′ × 3′, and at least 8″ in thickness; and all base stones shall be well bedded and laid edge to edge; and if the walls be built of isolated piers, then there must be inverted arches, at least 12″ thick, turned under and between the piers, or two footing courses of large stone at least 10″ thick in each course. All foundation walls shall be built of stone or brick, and shall be laid in cement mortar, and, if constructed of stone, shall be at least 8″ thicker than the wall next above them, to a depth of 16′ below the curb level, and shall be increased 4″ in thickness for every additional 5′ in depth below the said 16′; and if built of brick, shall be at least 4″ thicker than the wall next above them to a depth of 16′ below the curb level, and shall be increased 4″ in thickness for every additional 5′ in depth below the said 16′.

§ 5. In all dwelling-houses that may hereafter be erected not more than 55′ in height, the walls shall not be less than 12″ thick, and if above 55′ in height, and not more than 80′ in height, the outside walls shall not be less than 16″ thick to the top of second story floor-beams; provided the same is 20′ above the curb level, and if not, then to under side of the third story beams, and also provided that portion of the wall that is 12″ thick shall not exceed 40′ above the said 16″ wall; and in every dwelling-house hereafter erected more than 80′ in height, 4″ shall be added to the thickness of the wall for every 15′ or part thereof that is added to the height of the building. All party-walls in dwellings over 55′ in height shall not be less than 16″ in thickness.

§ 6. In all buildings other than dwellings hereafter erected, the bearing walls shall not be less than 12″ thick to the height of 40′ above the curb level; if above 40′ in height and not more than 55′ feet in height, the bearing walls shall not be less than 16″ thick; if above 55′ and not more than 70′ in height, the bearing walls shall not be less than 20″

thick, to the height of 20′ above the curb level or to the next tier of floor-beams above, and not less than 16″ from thence to the height of 55′ above the curb level or to the next tier of floor-beams, and not less than 12″ thick from thence to the top; and if above 70′ and not more than 85′ in height, the bearing walls shall not be less than 24″ thick to the height of 12′ above the curb level or the second story floor-beams, and from thence to the height of 60′ above the curb level, the said walls shall not be less than 20″ thick, and from thence to the top not less than 16″ thick; and if above the height of 85′, the bearing walls shall be increased 4″ in thickness for every 15′, or part thereof, that shall be added to the height of said wall above the 85′. In all buildings over 25′ in width, and not having either brick partition walls or girders supported by columns running from front to rear, the wall shall be increased an additional 4″ in thickness, to the same relative thickness in height as required under this section for every additional 10′ in width of said building, or any portion thereof. It is understood that the amount of materials specified may be used either in piers or buttresses, provided the outside walls between the same shall in no case be less than 12″ in thickness to the height of 40′, and if over that height then 16″ thick; but in no case shall a party wall between the piers or buttresses of a building be less than 16″ in thickness. In all buildings hereafter erected, situated on the street corner, the bearing wall thereof (that is, the wall on the street upon which the beams rest) shall be 4″ thicker in all cases than is otherwise provided for by this act. All walls other than bearing walls may be 4″ less in thickness than required in the clauses and provisions of this section above set forth, provided no wall is less than 12″ in thickness.

§ 7. Every building hereafter erected more than 30′ in width, except churches, theatres, school-houses, car-stables, and other public buildings, shall have one or more stone or brick partition walls running from front to rear, or iron or wooden girders supported on iron or wooden columns; these walls shall be so located that the space between any two of the bearing walls shall not be over 25′. In case iron or wooden girders, supported on iron or wooden columns, are substituted in place of the partition walls, the building may be 75′ in width, but not more; and if there should be substituted iron or wooden girders, supported on iron or wooden columns, in place of partition walls, they shall be made of sufficient strength to bear safely the weight of 250 lbs. for every square foot of the floor or floors that rest upon them, exclusive of the weight of material employed in their construction, and shall have a footing course and foundation wall not less than 16″ in thickness, with inverted arches under and between the columns, or two footing courses of large, well-shaped stone, laid crosswise, edge to edge, and at least 10″ thick in each course, the lower footing course to be not less than 2′ greater in area than the size of the column; and under every column, as above set forth, a cap of cut granite, at least 12″ thick, and of a diameter 12″ greater each way than that of the column, and must be laid solid and level to receive the column. Any building that may hereafter be erected in an isolated position, and more than 100′ in depth, and which shall not be provided with cross walls, shall be securely braced, both inside and out, during the whole time of its erection, if it can be done; but in case the same can not be so braced from the outside, then it shall be properly braced from the inside, and the braces shall be continued from the foundation upward to at least one third the height of the building from the curb level.

§ 8. . . . Every temporary support placed under any structure, wall, girder, or beam during the erection, finishing, alteration, or repairing of any building, or part thereof, shall be equal in strength to the permanent support required for such structure, wall, girder, or beam. And the walls of every building shall be strongly braced from the beams of each story until the building is topped out, and the roof tier of beams shall be strongly braced to the beams of the story below until all the floors in the said building are laid.

§ 9. All stone walls less than 24″ thick shall have at least one header, extending through the walls, in every 3′ in height from the bottom of the wall, and in every 4′ in length; and, if over 24″ in thickness, shall have one header for every six superficial feet on both sides of the wall, and running into the wall at least 2′; all headers shall be at least 18″ in width and 8″ in thickness, and shall consist of a good flat stone, dressed on all sides. In every brick wall every sixth course of brick shall be a heading course, except where walls are faced with brick, in which case every fifth course shall be bonded into the backing by cutting the course of the faced brick, and putting in diagonal headers behind the same, or by splitting face-brick in half, and backing the same by a continuous row of headers. In all walls which are faced with thin ashlar, anchored to the backing, or in which the ashlar has not either alternate headers and stretchers in each course, or alternate heading and stretching courses, the backing of brick shall not be less than 12″ thick, and all 12″ backing shall be laid up in cement mortar, and shall not be built to a greater height than prescribed for 12″ walls. All heading courses shall be good, hard, perfect brick. The backing in all walls, of whatever material it may be composed, shall be of such thickness as to make the walls, independent of the facing, conform as to thickness with the requirements of sections five and six of this act.

§ 10. Every isolated pier less than ten superficial feet at the base, and all piers supporting a wall built of rubble-stone or brick, or under any iron beam or arch girder, or arch on which a wall rests, or lintel supporting a wall, shall, at intervals of not less than 30″ in height, have built into it a bond stone not less than 4″ thick, of a diameter each way equal to the diameter of the pier, except that in piers on the street front, above the curb, the bond stone may be 4″ less than the pier in diameter; and all piers shall be built of good, hard, well-burned bricks and laid in cement mortar, and all bricks used in piers shall be of the hardest quality, and be well wet when laid; and the walls and piers under all compound, cast-iron, or wooden girders, iron or other columns, shall have a bond stone at least 4″ in thickness, and if in a wall at least 2′ in length, running through the wall, and if in a pier, the full size of the thickness thereof, every 30″ in height from the bottom, whether said pier is in the wall or not, and shall have a cap stone of cut granite, at least 12″ in thickness, by the whole size of the pier, if in a pier, and if in a wall it shall be at least 2′ in length, by the thickness of the wall, and at least 12″ in thickness. In any case where any iron or other column rests on any wall or pier built entirely of stone or brick, the said column shall be set on a base stone of cut granite, not less than 8″ in thickness by the full size of the bearing of the pier, if on a pier, and if on a wall the full thickness of the wall. In all buildings where the walls are built hollow, the same amount of stone or brick shall be used in their construction as if they were solid, as above set forth; and no hollow walls shall be built unless the two walls forming the same shall be connected by continuous vertical ties of the same materials as the walls, and not over 24″ apart. The height of all walls shall be computed from the curb level. No swelled or refuse brick shall be allowed in any wall or pier; and all brick used in the construction, alteration, or repair of any building, or part thereof, shall be good, hard, well-burned brick; and if used during the months from April to November, inclusive, shall be well wet at the time they are laid.

§ 12. In no case shall the side, end, or party wall of any building be carried up more than two stories in advance of the front and rear walls. The front, rear, side, end, and party walls of any building hereafter to be erected shall be anchored to each other every 6′ in their height by tie-anchors, made of one and a quarter inch by three eighths of an inch of wrought-iron. The said anchors shall be built into the side or party walls not less than 16″, and into the front and rear walls at least one half the thickness of the front and rear walls, so as to secure the front and rear walls to the side, end, or party walls; and all stone used for the facing of any building, except where built with alternate headers and stretchers, as hereinbefore set forth, shall be strongly anchored with iron anchors in each stone, and all such anchors shall be let into the stone at least 1″. The side, end, or party walls shall be anchored at each tier of beams, at intervals of not more than eight feet apart, with good, strong, wrought-iron anchors, one half inch by one inch, well built into the side walls, and well fastened to the side of the beams by two nails, made of wrought-iron, at least one fourth of an inch in diameter; and where the beams are supported by girders, the ends of the beams resting on the girder shall be butted together end to end, and strapped by wrought-iron straps of the same size, and at the same distance apart, and in the same beam as the wall-anchors, and shall be well fastened.

§ 13. All walls of any buildings over fifteen feet high shall be built up and extended at least 24″ above the roof, and shall be coped with stone or iron. . . .

§ 14. All iron beams or girders used to span openings over 6′ in width, and not more than 12′ in width, upon which a wall rests, shall have a bearing of at least 12″ at each end by the thickness of the wall to be supported; and for every additional foot of span over and above the said 12′, if the supports are iron or solid cut stone, the bearing shall be increased half an inch at each end; but if supported on the ends by walls or piers built of brick or stone, if the opening is over 12′ and not more than 18′, the bearing shall be increased 4″ at each end by the thickness of the wall to be supported; and if the space is over 18′ and not more than 25′ then the bearing shall be at least 20″ at each end by the thickness of the wall to be supported; and for every additional 5′ or part thereof that the space shall be increased, the bearing shall be increased an additional 4″ at each end by the thickness of the wall to be supported. And on the front of any building where the supports are of iron or solid cut stone, they shall be at least 16″ on the face and the width of the thickness of the wall to be supported, and shall, when supported at the ends by brick walls or piers, rest upon a cut granite base block, at least 12″ thick by the full size of the bearing; and in case the opening is less than 12′, the granite block may be 6″ in thickness by the whole size of the bearing; and all iron beams or girders used in any buildings shall be, throughout, of a thickness not less than the thickness of the wall to be supported. All iron beams or girders used to span openings more than 8′ in width, and upon which a wall rests, shall have wrought iron tie-rods of sufficient strength, well fastened at each end of the beam or girder, and shall have cast-iron shoes on the upper side, to answer for the skew-back of a brick or cut-stone arch, which said arch shall always be turned over the same, and the arch shall in no case be less than 12″ in height by the width of the wall to be supported, and the shoes shall be made strong enough to resist the pressure of the arch in all cases. Cut-stone or hard-brick arches, with two wrought-iron tie-rods of sufficient strength, may be turned over any opening less than 30′, provided they have skew-backs of cut stone or cast or wrought-iron, with which the bars or tension-rods shall be properly secured by heavy wrought iron washers, necks, and heads of wrought-iron, properly secured to the skew-backs. The above clause is intended to meet cases where the arch has not abutments of sufficient size to resist its thrust. All lintels hereafter placed over openings in the front, rear, or side of a building, or returned over a corner opening, when supported by brick piers or iron or stone columns, shall be of iron, and of the full breadth of the wall to be supported, and shall have a brick arch of sufficient thickness, with skew-backs and tie-rods of sufficient strength to support the superincumbent lateral weight, independent of the cast-iron lintel. . . .

§ 15. All openings for doors and windows in all buildings, except as otherwise provided, shall have a good and sufficient arch of stone or brick, well built and keyed, and with good and sufficient abutments, or a lintel of stone or iron, as follows: . . . For an opening exceeding 6′ in width, and not more than 8′ in width, the lintel shall be of iron or stone, and of the full thickness of the wall to be supported; and every such opening 6′ or less in width in all walls shall be at least one third the thickness of the walls on which it rests, and shall have a bearing at each end not less than 4″ on the walls; and on the inside of all openings, in which the lintel shall be less than the thickness of the wall to be supported, there shall be a good timber lintel on the inside of the other lintels, which shall rest at each end not more than 4″ on any wall, and shall be chamfered at each end, and shall have a double rolock arch turned over said timber lintel; arches built of stone or brick may be turned over openings on a center, which may be struck after the arch is turned, provided the arch has a good and sufficient rise, and that the piers or abutments are of sufficient strength to bear the thrust of the arch. . . .

§ 17. All chimneys, and all flues in stone or brick walls, in any building hereafter erected, altered, or repaired, without reference to the purpose for which they may be used, shall have the joints struck smooth on the inside, and no parging mortar shall be used on the inside; and the fire-backs of all chimneys hereafter erected shall not be less than 8″ in thickness; . . . no wooden furring or lath shall be placed against any flue, metal pipe, or pipes used to convey heated air or steam in any building; and when any wall shall hereafter be furred or lathed with wood, the space between the lathing and wall shall be filled with plaster between the top and underside of the floor-beams of each story, so as to prevent fire from extending from one floor to another. And no air-flue shall be used at any time as a smoke-flue. No steam-pipe shall be placed within 2″ of any timber or wood-work as aforesaid; when the said space of 2″ around the steam-pipe is objectionable, it shall be protected by a soap-stone or an earthen ring or tube. No base, or flooring, or roofing, or any other wood-work shall be placed against any brick or other flue until the same shall be well plastered with plaster-of-Paris behind such wood-work. . . .

§ 18. No smoke-pipe, in any building with wooden or combustible floors and ceilings, shall hereafter enter any flue unless the said pipe shall be at least 18″ from either the floors or ceilings; and in all cases where smoke-pipes pass through stud or wooden partitions of any kind, whether the same be plastered or not, they shall be guarded by either a double collar of metal, with at least 4″ air space and holes for ventilation, or by a soap-stone ring, not less than 3″ in thickness and extending through the partition, or by a solid coating of plaster-of-Paris, 3″ thick, or by an earthenware ring 3″ from the pipe. . . .

§ 19. In no building, whether the same be a frame building or otherwise, shall any wooden girders, beams, or timbers be placed within 12″ of the inside of any flue, whether the same be a smoke, air, or any other flue. All wooden beams and other timbers in the party wall of every building hereafter to be erected or built, of stone, brick, or iron, shall be separated from the beam or timber entering in the opposite side of the wall by at least 8″ of solid mason-work. No floor-beam shall be supported wholly upon any wood partition, but every beam, except headers and tail-beams, shall rest, at one end, not less than 4″ in the wall, or upon a girder, as authorized by this act. And every trimmer or header more than 4′ long, used in any building except a dwelling, shall be hung in stirrup-irons of suitable thickness for the size of the timbers. . . .

§ 20. In all buildings, every floor shall be of sufficient strength in all its parts to bear safely upon every superficial foot of its surface 75 lbs.; and if used as a place of public assembly, 120 lbs.; and if used as a store, factory, warehouse, or for any other manufacturing or commercial purposes, from 150 to 500 lbs. and upward; and every floor shall be of sufficient strength to bear safely the weights aforesaid, in addition to the weight of the materials of which the floor is composed; and every column, post, or other vertical support shall be of sufficient strength to bear safely the weight of the portion of each and every floor depending upon it for support, in addition to the weight required as above to be supported safely upon said portions of said floors. In all calculations for the strength of materials to be used in any building, the proportion between the safe weight and the breaking weight shall be as one to three for all beams, girders, and other pieces subjected to a cross-strain, and shall be as one to six for all posts, columns, and other vertical supports, and for all tie-rods, tie-beams, and other pieces subjected to a tensile strain. And the requisite dimensions of each piece of material is to be ascertained by computation by the rules given by Tredgold, Hodgkinson, Barlow, or the treatises of other authors now or hereafter used at the United States Military Academy of West Point on the strength of materials, using for constants in the rules only such numbers as have been deduced from experiments on materials of like kind with that proposed to be used. . . .

§ 21. In all fire-proof buildings hereafter to be constructed, where brick walls, with wrought-iron beams or cast or wrought iron columns with wrought-iron beams, are used in the interior, the following rules must be observed:

1. All metal columns shall be planed true and smooth at both ends, and shall rest on cast-iron bed-plates, and have cast-iron caps, which shall also be planed true. If brick arches are used between the beams, the arches shall have a rise of at least an inch and a quarter to each foot of space between the beams.

2. Under the ends of all the iron beams, where they rest on the walls, a stone template must be built into the walls; said templates to be 8″ wide in 12″ walls, and in all walls of greater thickness to be in width not less than 4″ less than the width of said walls, and not to be, in any case, less than 4″ in thickness and 18″ long. . . .

§ 22. All exterior cornices and gutters of all buildings, hereafter to be erected or built, shall be of some fire-proof material. . . .

§ 23. The planking and sheathing of the roof of every building, erected or built as aforesaid, shall in no case be extended across the front, rear, side, end, or party wall thereof, and every such building, and the tops and sides of every dormer-window thereon, shall be covered and roofed with slate, tin, zinc, copper, or iron, or such other equally fire-proof roofing. . . .

PATENT-OFFICE DRAWINGS

must be made upon pure white paper, of a thickness corresponding to three-sheet Bristol board, with surface calendered and smooth. Indian ink alone must be used.

The size of the sheet must be exactly 10 by 15 inches. 1″ from its edges single marginal lines are to be drawn, leaving the "sight" precisely 8″ by 13″. Within this margin all work must be included. Measuring downward from the marginal line of one of the shorter sides, a space of not less than 1¼ inch is to be left blank for the heading of title, name, number, and date.

All drawings must be made with the pen only. All lines and letters must be absolutely black, clean, sharp, and solid, and not too fine or crowded. Surface shading should be open, and used only on convex and concave surfaces sparingly. Sectional shading should be made by oblique parallel lines, which may be about $\frac{1}{20}$″ apart.

Drawings should be made with the fewest lines possible consistent with clearness. The plane upon sectional views should be indicated on the general view by broken or dotted lines. Heavy lines on the shade sides of objects should be used, except where they tend to thicken the work and obscure letters of reference; light to come from the upper left-hand corner, at an angle of 45°.

The scale of the drawing to be large enough to show the mechanism without crowding; but the number of sheets must never be increased unless it is absolutely necessary.

Letters and figures of reference must be carefully formed, and, if possible, measure at least ⅛″ in height, and so placed as not to interfere with a thorough comprehension of the drawing, and therefore should rarely cross the lines. Upon shaded surfaces a blank space must be left in the shading for the letter. The same part of an invention must always be represented by the same character, and the same character must never be used to designate different parts.

The signature of the inventor, by himself or by his attorney, is to be placed at the lower right-hand corner of the sheet, and the signature of two witnesses at the lower left-hand corner, all within the marginal line. The title is to be written with pencil on the back of the sheet. The permanent names and title will be supplied subsequently by the office in uniform style.

Drawings should be rolled for transmission to the office, not folded.

MENSURATION.

Properties of Triangles.—It has been already shown in "Geometrical Problems" that to construct a triangle three dimensions must be known—the three sides, or two sides and the included angle, or one side and the two adjacent angles. If only the three angles are known, triangles of varied sizes may be constructed, but all similar to each other. To determine the length of the side of a right-angled triangle by calculation, the other two sides being known, use these formulæ:

$A^2 = B^2 + C^2$, or $A = \sqrt{B^2 + C^2}$

$B = \sqrt{A^2 + C^2}$, or $\sqrt{\overline{A + C} \times \overline{A - C}}$

$C = \sqrt{A^2 - B^2}$, or $\sqrt{\overline{A + B} \times \overline{A - B}}$.

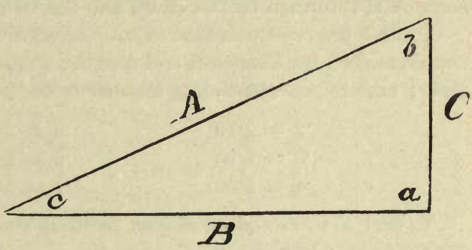


Fig. 1.

The side of any triangle (Figs. 1, 2, or 3) can be found by the following formulæ:

$$A = \frac{B \sin. a}{\sin. b}, \text{ and consequently}$$

$$\sin. b = \frac{B \sin. a}{A}.$$

$$\sin. a = \frac{A \sin. b}{B}.$$

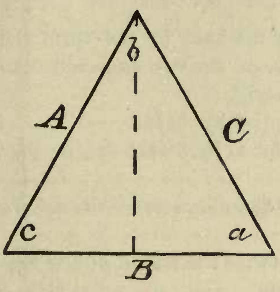


Fig. 2.

The *area* of a triangle is equal to half the product of the base by the height. Taking any side as the base, say B, the height is readily obtained by multiplying the length of the adjacent side A by the natural sine of *c*. All figures bounded by straight lines can be divided into triangles, and their dimensions readily calculated.

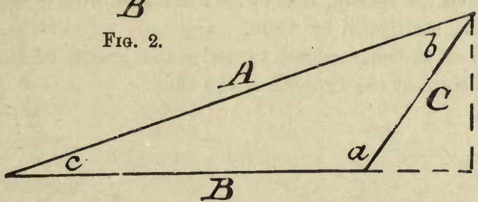


Fig. 3.

Properties of circles.—The circumference of a circle is equal to the diameter multiplied by 3·1416, or π (pi), or approximately $3\frac{1}{7}$.

The area of a circle is equal to the square of the radius multiplied by 3·1416 (π), or the square of the diameter multiplied by ·7854.

The chord A C (Fig. 4) forms, with the chords of half the arc and the three radii, right-angled triangles whose dimensions may be calculated as given above. But the solution by table of natural sines is extremely simple; thus the chord is twice the sine of half the angle A E C at the center made by the radii to the extremities of the chord. D E is the cosine of the angle D E C or D E A, and the versed sine B D is equal to radius less the cosine.

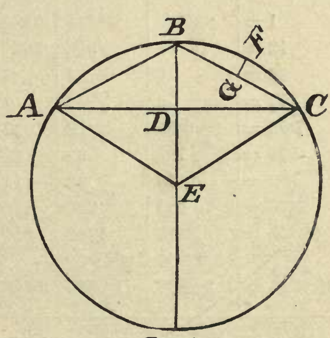


Fig. 4.

The versed sine F G of the half chord is equal to about one quarter of the versed sine D B of the whole chord.

The area of a sector A B C E is to that of the whole circle as the angle at the center A E C is to 360°; or the radius, multiplied by half the length of the arc A B C, will give the area.

The length of an arc of one degree = radius × ·017453.
" " " " " " " minute = " × ·000291.
" " " " " " " second = " × ·000005.

The area of a segment A B C D is equal to that of the sector less the area of the triangle A E C formed by the chord and the two radii.

To find the circumference of an *ellipse*, divide the conjugate or short diameter by the transverse or long diameter, and find the quotient in the first column in the accompanying table; take the corresponding number from the table, and multiply it by the long span.

·2 = 2·10	·5 = 2·43	·8 = 2·84
·3 = 2·20	·6 = 2 56	·9 = 2·99
·4 = 2·30	·7 = 2·69	1·0 = 3·14

To find the *area of an ellipse*, multiply the conjugate by the transverse diameter, and the result by ·7854.

The *area of a parabola* is the product of the base by two thirds the height.

Mensuration of Solids.—The solidity of *parallelopipeds*, *cylinders*, and *prisms* is found by multiplying the base by the altitude.

The solidity of cones or pyramids is found by multiplying the base by one third the vertical height; of frustums of pyramids, the sum of the areas of the two ends added to the square root of their product multiplied by one third the height.

The *solidity of the sphere* is the cube of the diameter multiplied by ·5236.

The area of the surface is the square of the diameter multiplied by 3·1416 (π), or four times the area of the great circle passing through the center.

The curved surface of a *spherical segment* is the product of the diameter of the sphere by the height of the segment by 3·1416.

The solidity is three times the diameter of the sphere, less twice the height of the segment, multiplied by the square of the height, multiplied by ·5236.

The solidity of the *wedge* is the length of the edge added to twice the length of the back, multiplied by the height and by one sixth of the breadth of the back.

LINEAL MEASURE.

Inches.	Feet.	Yards.	Fathoms.	Links.	Rods.	Chains.	Furlongs.	Statute miles.	Nautical miles.	Metres.
1 =	·08333	·02778	·0139	·126	·005	·00126	·000126	·000016	....	·0254
12 =	1	·333	·1667	1·515	·0606	·0151	·00151	·00019	....	0·3048
36 =	3	1	·5	4·545	·182	·0454	·00454	·00057	....	0·9144
72 =	6	2	1	9·1	·364	·091	·0091	·00114	....	1·8289
7·92 =	0·66	·22	·11	1	·04	·01	·001	·000125	....	·2012
198 =	16½	5½	2¾	25	1	·25	·025	·003125	....	5·0294
792 =	66	22	11	100	4	1	·10	·0125	....	20·118
7920 =	660	220	110	1000	40	10	1	·125	....	201·18
63360 =	5280	1760	880	8000	320	80	8	1	0·86755	1609·41
	6086·07	2028·69		....	....	....	..	1·1527	1	1855·11
39·3685 =	3·2807	1·0936	·5468	....	....	....	..	0·000621	....	1

Latin prefixes, as milli-, centi-, deci-, to the French units of length (metre), surface (are), weight (gramme), or volume (litre), signify $^1/_{1000}$, $^1/_{100}$, or $^1/_{10}$ of the unit; as, millimetre, $^1/_{1000}$ of a metre, decigramme, $^1/_{10}$ of a gramme. Greek prefixes, as kilo, hekto, deka, multiples of the unit by 1,000, 100, or 10, as kilometre = 1000 metres.

TABLE OF INCHES AND SIXTEENTHS IN DECIMALS OF A FOOT.

Inches.		$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$	$\frac{8}{16}$	$\frac{9}{16}$	$\frac{10}{16}$	$\frac{11}{16}$	$\frac{12}{16}$	$\frac{13}{16}$	$\frac{14}{16}$	$\frac{15}{16}$
0....	·000	·005	·010	·016	·021	·026	·031	·036	·042	·047	·052	·057	·062	·068	·073	·078
1....	·083	·089	·094	·099	·104	·109	·115	·120	·125	·130	·135	·141	·146	·151	·156	·161
2....	·167	·172	·177	·182	·187	·193	·198	·203	·208	·214	·219	·224	·229	·234	·240	·245
3....	·250	·255	·260	·266	·271	·276	·281	·286	·292	·297	·302	·307	·312	·318	·323	·328
4....	·333	·339	·344	·349	·354	·359	·365	·370	·375	·380	·385	·391	·396	·401	·406	·411
5....	·417	·422	·427	·432	·437	·443	·448	·453	·458	·464	·469	·474	·479	·484	·490	·495
6....	·500	·505	·510	·516	·521	·526	·531	·536	·542	·547	·552	·557	·562	·568	·573	·578
7....	·583	·589	·594	·599	·604	·609	·615	·620	·625	·630	·635	·641	·646	·651	·656	·661
8....	·667	·672	·677	·682	·687	·693	·698	·703	·708	·714	·719	·724	·729	·734	·740	·745
9....	·750	·755	·760	·766	·771	·776	·781	·786	·792	·797	·802	·807	·812	·818	·823	·828
10....	·833	·839	·844	·849	·854	·859	·865	·870	·875	·880	·885	·891	·896	·901	·906	·911
11....	·917	·922	·927	·932	·937	·943	·948	·953	·958	·964	·969	·974	·979	·984	·990	·995

MEASURES OF SURFACE.

Sq. inches.	Sq. feet.	Sq. yards.	Sq. rods.	Roods.	Acres.	Sq. miles.	Sq. metres.	Ares.
1 =	·00694	....	....	....	....	....	....	....
144 =	1	·111	·0037	....	....	....	·0929	·0009
1296 =	9	1	·033	....	....	....	·8361	·0084
....	272¼	30¼	1	·025	·00625	....	25·293	0·253
....	10890	1210	40	1	·25	....	....	....
....	43560	4840	160	4	1	·00156	4046·86	40·47
....	27878400	3097600	....	....	640	1	....	25899
1549·8 =	10·763	1·196	·0395	·0009	·000247	....	1	·01
....	1076·31	119·60	....	....	·02471	....	100	1

MEASURES OF CAPACITY.

LIQUID MEASURE.

Gills.	Pints.	Quarts.	Gallons.	Imp. gallons.	Litres.	Cubic feet.	Cubic in.	Lbs. water at 62°.
1 =	0·25	0·125	·03125	·026	·1183	·0042	7·219	·26
4 =	1	0·5	0·125	·1041	·4731	·01671	28·875	1·0412
8 =	2	1	0·25	·2083	0·9463	·03342	57·75	2·0825
32 =	8	4	1	0·8331	3·7852	0·1337	231	8·33
38·4096 =	9·6024	4·8012	1·2003	1	4·5435	0·1605	277·27	10·00
8·4534 =	2·1133	1·0567	0·26417	0·2201	1	0·0353	61·0279	2·2007
239·36 =	59·84	29·92	7·48	6·232	28·320	1	1728	62·321
·138528 =	·034632	·017316	·004329	·0036	0·01639	0·000579	1	·03606
....	....	......	......	....	......	·01604	27·727	1

DRY MEASURE.

Pints.	Quarts.	Gallons.	Pecks.	Bushels.
1 =	0·50	0·125	·0625	0·01562
2 =	1	0·25	0·125	0·0312
8 =	4	1	0·50	0·125
16 =	8	2	1	0·25
64 =	32	8	4	1

The standard bushel contains 2150·42 cubic inches.

WEIGHTS.

APOTHECARIES'.

Grains.	Scruples.	Drachms.	Ounces.	Pounds.
1 =	·05	·0167	·0021	·00018
20 =	1	·333	·042	·0035
60 =	3	1	·125	·0104
480 =	24	8	1	·083
5760 =	288	96	12	1

TROY.

Grains.	Pennyweights.	Ounces.	Pounds.
1 =	·042	·0021	·00018
24 =	1	·05	·0042
480 =	20	1	·083
5760 =	240	12	1

AVOIRDUPOIS.

Drachms.	Ounces.	Pounds.	Hundred-weights.	Tons.	French grammes.
1 =	·0625	·0039	·000035	·00000174	1·771836
16 =	1	·0625	·000558	·000028	28·34938
256 =	16	1	·00893	·000446	453·59
28672 =	1792	112	1	·05	50802·
573440 =	35840	2240	20	1	1016041·6

It is common usage here to omit hundred-weights (cwt.) and rate tons at 2,000 pounds as net, and 2240 lbs. as gross.

COMPARISON OF WEIGHT.

Pounds apothecaries'.	Pounds Troy.	Pounds avoirdupois.	Kilo-gramme.
1 =	1	0·8229	0·37324
1 =	1	0·8229	0·37324
1·2153 =	1·2153	1	0·4536
2·6792 =	2·6792	2·2046	1

DYNAMIC TABLE.

Pounds, feet.	Kilogramme-metre.	Horse-power.	French horse-power.
1 =	0·13825	·00003	·000031
7·2331 =	1	·000219	·000222
Per min. 33·000 =	4562·3	1	1·01386
32548·9 =	4500	0·98633	1

CUBIC OR SOLID MEASURE.

Cubic inches.	Cubic feet.	Cubic yards.	Cubic metres.	United States gallon.
1 =	·00058	·000021	·000016	·004329
1728 =	1	0·037	0·0283	7·48
46656 =	27	1	0·7646	201·97
61016 =	35·31	1·3078	1	264·141
231 =	0·1337	·00495	·00379	1

TABLE OF WEIGHT OF ONE FOOT IN LENGTH OF ROLLED IRON.

Breadth or diameter in inches.	Round Iron.	Thickness in Inches.																													
		⅛	¼	⅜	½	⅝	¾	⅞	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4½	5	5½	6	7	8	9	10	11	12
⅛	0·04	·05																													
¼	0·17	·10	·21																												
⅜	0·37	·15	·31	·47																											
½	0·66	·21	·42	·63	·84																										
⅝	1·03	·26	·53	·79	1·05	1·31																									
¾	1·49	·31	·63	·95	1·26	1·58	1·90																								
⅞	2·02	·36	·73	1·10	1·47	1·84	2·21	2·58																							
1	2·65	·42	·84	1·26	1·68	2·11	2·53	2·95	3·37																						
1¼	4·13	·53	1·05	1·58	2·11	2·63	3·16	3·68	4·21	5·26																					
1½	5·95	·63	1·26	1·90	2·53	3·16	3·79	4·42	5·05	6·32	7·58																				
1¾	8·10	·74	1·47	2·21	2·95	3·68	4·42	5·16	5·89	7·37	8·84	10·31																			
2	10·58	·84	1·68	2·53	3·37	4·21	5·05	5·89	6·74	8·42	10·10	11·79	13·47																		
2¼	13·39	·95	1·90	2·84	3·79	4·74	5·68	6·63	7·58	9·47	11·37	13·26	15·16	17·05																	
2½	16·53	1·05	2·11	3·16	4·21	5·26	6·32	7·37	8·42	10·53	12·63	14·74	16·84	18·95	21·05																
2¾	20·01	1·16	2·32	3·47	4·63	5·79	6·95	8·10	9·26	11·58	13·89	16·21	18·52	20·84	23·16	25·47															
3	23·81	1·26	2·53	3·79	5·05	6·32	7·58	8·84	10·10	12·63	15·16	17·68	20·21	22·74	25·26	27·79	30·31														
3¼	27·94	1·37	2·74	4·11	5·47	6·84	8·21	9·58	10·95	13·69	16·43	19·17	21·90	24·63	27·37	30·11	32·84	35·57													
3½	32·41	1·47	2·95	4·42	5·89	7·37	8·84	10·32	11·79	14·74	17·68	20·64	23·58	26·53	29·48	32·42	35·37	38·32	41·26												
3¾	37·20	1·58	3·16	4·74	6·32	7·89	9·47	11·05	12·63	15·79	18·95	22·11	25·26	28·42	31·58	34·74	37·89	41·05	44·21	47·37											
4	42·33	1·68	3·37	5·05	6·74	8·42	10·10	11·79	13·47	16·84	20·20	23·58	26·95	30·32	33·69	37·06	40·42	43·79	47·16	50·53	53·89										
4½	53·57	1·90	3·79	5·68	7·58	9·47	11·37	13·26	15·16	18·95	22·75	26·52	30·31	34·10	37·89	41·68	45·47	49·26	53·05	56·84	60·63	68·20									
5	66·18	2·11	4·21	6·32	8·42	10·53	12·63	14·74	16·84	21·05	25·26	29·48	33·68	37·89	42·10	46·31	50·52	54·73	58·94	63·15	67·36	75·78	84·20								
5½	80·02	2·32	4·63	6·95	9·26	11·58	13·89	16·21	18·52	23·16	27·78	32·42	37·04	41·67	46·31	50·95	55·58	60·21	64·84	69·47	74·10	83·36	92·63	101·90							
6	95·23	2·53	5·05	7·58	10·10	12·63	15·16	17·68	20·21	25·26	30·31	35·37	40·42	45·47	50·52	55·57	60·63	65·68	70·73	75·78	80·84	90·95	101·05	111·15	121·25						
7	129·60	2·94	5·90	8·84	11·79	14·74	17·68	20·64	23·58	29·47	35·36	41·25	47·14	53·03	58·92	64·81	70·70	76·60	82·50	88·39	94·28	106·07	117·86	129·65	141·48	165·00					
8	169·30	3·36	6·74	10·10	13·48	16·84	20·21	23·58	26·95	33·69	40·43	47·17	53·90	60·64	67·38	74·12	80·86	87·60	94·33	101·07	107·80	121·28	134·76	148·24	161·71	188·66	215·60				
9	214·30	3·79	7·58	11·36	15·16	18·95	22·75	26·52	30·32	37·89	45·47	53·05	60·63	68·20	75·78	83·36	90·94	98·52	106·10	113·67	121·25	136·40	151·56	166·72	181·88	212·20	242·50	272·80			
10	264·50	4·21	8·42	12·64	16·84	21·05	25·26	29·48	33·68	42·10	50·52	58·94	67·36	75·78	84·20	92·62	101·04	109·46	117·88	126·30	134·72	151·56	168·40	185·24	202·08	235·76	269·44	303·12	336·80		
11	320·10	4·64	9·26	13·90	18·52	23·16	27·78	32·42	37·04	46·30	55·56	64·82	74·08	83·34	92·60	101·87	111·13	120·39	129·65	138·91	148·17	166·70	185·22	203·75	222·27	259·31	296·85	333·40	370·45	407·50	
12	380·90	5·05	10·10	15·16	20·21	25·26	30·31	35·37	40·42	50·52	60·63	70·73	80·83	90·94	101·04	111·14	121·25	131·35	141·45	151·55	161·65	181·86	202·07	222·28	242·50	282·92	323·34	363·76	404·17	444·59	485·00

WEIGHTS OF WROUGHT-IRON AND BRASS PLATES AND WIRE, SOFT ROLLED.

BIRMINGHAM GAUGE.		No. of gauge.	AMERICAN GAUGE.				
				PLATES PER SQUARE FOOT.		WIRE PER LINEAL FOOT.	
Plate iron.	Thickness of each number.		Thickness of each number.	Wrought iron.	Brass.	Wrought iron.	Brass.
Lbs.	Inch.		Inch.	Lbs.	Lbs.	Lbs.	Lbs.
17·025	·454	0000	·46	17·25	19·68	·5607	·6051
15·9375	·425	000	·4096	15·361	17·53	·4447	·4799
14·25	·38	00	·3648	13·68	15·61	·3527	·3806
12·75	·34	0	·3248	12·182	13·90	·2797	·3018
11·25	·3	1	·2893	10·848	12·38	·2218	·2393
10·65	·284	2	·2576	9·661	11·02	·1759	·1898
9·7125	·259	3	·2294	8·603	9·81	·1395	·1505
8·925	·238	4	·2043	7·661	8·74	·1106	·1193
8·25	·22	5	·1819	6·822	7·78	·0877	·0946
7·6125	·203	6	·1620	6·075	6·93	·0695	·0750
6·75	·18	7	·1442	5·410	6·17	·0551	·0595
6·1875	·165	8	·1284	4·818	5·49	·0437	·0472
5·55	·148	9	·1144	4·291	4·89	·0347	·0374
5·025	·134	10	·1018	3·820	4·36	·0275	·0296
4·5	·12	11	·0907	3·402	3·88	·0218	·0235
4·0875	·109	12	·0808	3·030	3·45	·0173	·0186
3·5625	·095	13	·0719	2·698	3·07	·0137	·0148
3·1125	·083	14	·0640	2·403	2·74	·0109	·0117
2·7	·072	15	·0570	2·140	2·44	·00863	·00931
2·4375	·065	16	·0508	1·905	2·17	·00684	·00758
2·175	·058	17	·0452	1·697	1·93	·00542	·00585
1·8375	·049	18	·0403	1·511	1·72	·00430	·00464
1·575	·042	19	·0358	1·345	1·53	·00341	·00368
1·3125	·035	20	·0319	1·198	1·36	·00271	·00292
1·2	·032	21	·0284	1·067	1·21	·00215	·00231
1·05	·028	22	·0253	·9505	1·08	·00170	·00183
·9375	·025	23	·0225	·8464	·9660	·00135	·00145
·825	·022	24	·0201	·7537	·8602	·00107	·00115
·75	·02	25	·0179	·6712	·7661	·00085	·000916
·675	·018	26	·0159	·5977	·6822	·000673	·000726
·6	·016	27	·0141	·5323	·6075	·000534	·000576
·525	·014	28	·0126	·4740	·5410	·000423	·000457
·4875	·013	29	·0112	·4221	·4818	·000336	·000362
·45	·012	30	·0100	·3759	·4290	·000266	·000287
·375	·01	31	·0089	·3348	·3821	·000211	·000228
·3375	·009	32	·0079	·2981	·3402	·000167	·000180
·3	·008	33	·00708	·2655	·3030	·000132	·000143
·2625	·007	34	·00630	·2364	·2698	·000105	·000113
·1875	·005	35	·00561	·2105	·2402	·0000836	·00009015
·15	·004	36	·005	·1875	·214	·0000662	·0000715
		37	·00445	·1669	·1905	·0000525	·00005671
		38	·00396	·1486	·1697	·0000416	·00004496
		39	·00353	·1324	·1511	·0000330	·00003566
		40	·00314	·1179	·1345	·0000262	·00002827

Copper is about 5 per cent heavier than brass. Lead is about 47 per cent heavier than wrought iron. Zinc is about 7 per cent lighter than wrought iron. Sheet copper is rated by weight at so many ounces per square foot, and sheet lead at so many pounds per square foot.

TABLE OF DIMENSIONS AND WEIGHT OF WROUGHT-IRON WELDED TUBES.

Nominal diameter.	External diameter.	Thickness.	Internal diameter.	Internal circumference.	External circumference.	Length of pipe per square foot of internal surface.	Length of pipe per square foot of external surface.	Internal area.	Weight per foot.	No. of threads per inch of screw.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Feet.	Feet.	Inches.	Lbs.	
1/8	·40	·068	·27	·85	1·27	14·15	9 44	·057	·24	27
1/4	·54	·088	·36	1·14	1·7	10·5	7·075	·104	·42	18
3/8	·67	·091	·49	1·55	2·12	7·67	5·657	·192	·56	18
1/2	·84	.109	·62	1·96	2·65	6·13	4·502	·305	·84	14
3/4	1·05	.113	·82	2·59	3·3	4·64	3·637	·533	1·13	14
1	1·31	·134	1·05	3·29	4·13	3·66	2·903	·863	1·67	11 1/2
1 1/4	1·66	·14	1·38	4·33	5·21	2·77	2·301	1·496	2·26	11 1/2
1 1/2	1·9	·145	1·61	5·06	5·97	2·37	2·01	2·038	2·69	11 1/2
2	2·37	·154	2·07	6·49	7·46	1·85	1·611	3·355	3·67	11 1/2
2 1/2	2·87	·204	2·47	7·75	9·03	1·55	1·328	4·783	5·77	8
3	3·5	·217	3·07	9·64	11·	1·24	1·091	7·388	7·55	8
3 1/2	4·	·226	3·55	11·15	12·57	1·08	0·955	9·887	9·05	8
4	4·5	·237	4·07	12·69	14·14	·95	0·849	12·73	10·73	8
4 1/2	5·	·247	4·51	14·15	15·71	·85	0·765	15·939	12·49	8
5	5·56	·259	5·04	15·85	17·47	·78	0·629	19·99	14·56	8
6	6·62	·28	6·06	19·05	20·81	·63	0·577	28·889	18·77	8
7	7·62	·301	7·02	22·06	23·95	·54	0·505	38·737	23·41	8
8	8·62	·322	7·98	25·08	27·1	·48	0·444	50·039	28·35	8
9	9·69	·344	9·	28·28	30·43	·42	0·394	63·633	34·08	8
10	10·75	·366	10·02	31·47	33·77	·38	0·355	78·838	40·64	8

Nominal diameter.	Thickness, extra strong.	Thickness, double extra strong.	Actual inside diameter. Extra strong.	Actual inside diameter. Double extra strong.
Inches.	Inches.	Inches.	Inches.	Inches.
1/8	0·100	.....	0·205	.....
1/4	0·123	.....	0·294	.....
3/8	0·127	.....	0·421	....
1/2	0·149	0·298	0·542	0·244
3/4	0·157	0·314	0·736	0·422
1	0·182	0·364	0·951	0·587
1 1/4	0·194	0·388	1·272	0·884
1 1/2	0·203	0·406	1·494	1·088
2	0·221	0·442	1·933	1·491
2 1/2	0·280	0·560	2·315	1·755
3	0·304	0·608	2·892	2·284
3 1/2	0·321	0·642	3·358	2·716
4	0·341	0·682	3·818	3·136

BOILER TUBES.

External diameter.	Thickness, wire gauge.	Average weight.	External diameter.	Thickness, wire gauge.	Average Weight.
Inches.	No.	Lbs. per foot.	Inches.	No.	Lbs. per foot.
1 1/4	16	1·	3	11	3·5
1 1/2	15	1·16	3 1/4	11	4·
1 3/4	14	1·63	4	8	6·4
2	13	2·	5	7	9·1
2 1/4	12	2·16	6	6	12·3
2 1/2	12	2·56	7	6	15·2
2 11/16	11	2·2	8	6	16·

HEAVY PIPE FOR DRIVEN WELLS.

Tested at 1200 pounds hydraulic pressure. Furnished in five-foot lengths.

Size (inches)	1¼	1½	2	2½	3	3½	4
Weight per foot, lbs	3·62	2·75	3·75	6·00	7·75	9·25	11·00

HEAVY WROUGHT GALVANIZED IRON SPIRAL RIVETED PIPES,

With Flanged Connections.

Tested at 150 pounds hydraulic pressure. Regalvanized after riveting.

Inside diameter (inches)	3	4	5	6	7	8	9	10	11	12
Wire gauge, Nos.	20	20	20	18	18	18	18	16	16	16
Nominal weight per foot, lbs.	2½	4	5	6	7	8	9	12	13	14

Manufactured lengths, 20 feet or less. Elbows and other fittings, cast iron.

Light Pipe, suitable for House Leaders, Ventilating, Air, and Blower Pipes, etc.

Inside diameter (inches)	2	2½	3	3½	4	4½	5	5½	6
Nominal weight per foot, lbs.	⅝	¾	⅞	1	1¼	1⅜	1½	1⅝	1¾

TABLE OF COPPER AND BRASS RODS ONE FOOT IN LENGTH.

To find the weight of copper or brass pipe, take the weight of the exterior diameter from the table, and subtract from it the weight of a rod equal to that of the interior diameter, or bore.

Diamet'r in inches.	Copper.	Brass.	Diamet'r in inches.	Copper.	Brass.	Diamet'r in inches.	Copper.	Brass.
1/8	·047	·045	1 5/8	7·993	7·593	4 1/4	55·62	52·27
3/16	·106	·101	1 11/16	8·630	8·198	4 3/8	58·94	55·39
1/4	·189	·179	1 3/4	9·270	8·806	4 1/2	62·36	58·60
5/16	·296	·281	1 13/16	9·950	9·452	4 5/8	65·87	61·90
3/8	·426	·405	1 7/8	10·642	10·110	4 3/4	69·48	6
7/16	·579	·550	1 15/16	11·370	10·801	4 7/8	73·19	68·77
1/2	·757	·719	2	12·108	11·503	5	77·43	72·76
9/16	·958	·910	2 1/8	13·668	12·985	5 1/8	80·89	76·00
5/8	1·182	1·123	2 1/4	15·325	14·559	5 1/4	84·88	79·76
11/16	1·431	1·360	2 3/8	17·075	16·221	5 3/8	88·97	83·60
3/4	1·703	1·618	2 1/2	18·916	17·970	5 1/2	93·15	87·53
13/16	1·998	1·898	2 5/8	20·856	19·808	5 5/8	97·44	91·56
7/8	2·318	2·202	2 3/4	22·891	21·746	5 3/4	101·81	95·68
15/16	2·660	2·527	2 7/8	25·019	23·768	5 7/8	106·29	99·88
1	3·027	2·876	3	27·243	25·881	6	110·85	104·15
1 1/16	3·417	3·246	3 1/8	29·559	28·081	6 1/4	120 30	113·04
1 1/8	3·831	3·639	3 1/4	31·972	30·373	6 1/2	130·10	122·26
1 3/16	4·269	4·056	3 3/8	34·481	32·757	6 3/4	140·32	131·85
1 1/4	4·723	4·487	3 1/2	37·081	35·227	7	150·86	141·76
1 5/16	5·214	4·953	3 5/8	39·777	37·788	7 1/4	161·87	152·10
1 3/8	5·723	5·437	3 3/4	42·568	40·440	7 1/2	173·22	162·77
1 7/16	6·255	5·943	3 7/8	45·455	43·182	7 3/4	184·97	173·81
1 1/2	6·811	6·470	4	48·433	46·000	8	197·03	185·14
1 9/16	7·390	7·020	4 1/8	52·40	49·24			

NUMBER OF BURDEN'S RIVETS IN ONE HUNDRED POUNDS.

Lengths.	Diameter.				Lengths.	$\frac{7}{8}$ S. B.
	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$		
$\frac{3}{4}$	1,092	665	....	....	5	90
$\frac{7}{8}$	1,027	597	....	....	$5\frac{1}{2}$	85
1	940	538	450	....	6	80
$1\frac{1}{8}$	840	512	415	....	$6\frac{1}{2}$	75
$1\frac{1}{4}$	797	487	389	356	7	70
$1\frac{3}{8}$	760	460	370	329	$7\frac{1}{2}$	67
$1\frac{1}{2}$	730	440	357	280	8	65
$1\frac{5}{8}$	711	420	340	271	$8\frac{1}{2}$	61
$1\frac{3}{4}$	693	390	325	262	9	57
$1\frac{7}{8}$	648	375	312	257	$9\frac{1}{2}$	54
2	608	360	297	243	10	51
$2\frac{1}{8}$	573	354	....	....	$10\frac{1}{2}$	47
$2\frac{1}{4}$	555	347	280	232		
$2\frac{1}{2}$	525	335	260	220		
$2\frac{3}{4}$	500	312	242	208		
3	460	290	224	197		
$3\frac{1}{4}$	433	267	212	180		
$3\frac{1}{2}$	413	248	201	169		
$3\frac{3}{4}$	395	241	192	160		
4	....	230	184	158		
$4\frac{1}{4}$	....	220	177	150		
$4\frac{1}{2}$	....	210	171	146		
$4\frac{3}{4}$	....	200	166	138		
5	....	190	161	135		
$5\frac{1}{4}$	....	180	156	130		
$5\frac{1}{2}$	....	172	151	124		
$5\frac{3}{4}$	....	164	145	120		
6	....	157	140	115		
$6\frac{1}{4}$	....	150	138	111		
$6\frac{1}{2}$	....	146	134	107		
$6\frac{3}{4}$	....	143	129	104		
7	....	140	125	100		

WROUGHT SPIKES—NUMBER TO A KEG OF ONE HUNDRED AND FIFTY POUNDS.

Length.	$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{7}{16}''$	$\frac{1}{2}''$
Inches.					
3	2,250	....	....	....	....
$3\frac{1}{2}$	1,890	1,208	....	....	....
4	1,650	1,135	....	....	....
$4\frac{1}{2}$	1,464	1,064	....	....	....
5	1,380	930	742	....	....
6	1,292	868	570	....	....
7	1,161	662	482	445	306
8	....	635	455	384	256
9	....	573	424	300	240
10	....	....	391	270	222
11	....	....	....	249	203
12	....	....	....	236	180

LENGTHS OF CUT NAILS AND SPIKES, AND NUMBER IN A POUND.

Size.	Length.	No.	Size.	Length.	No.	Size.	Length.	No.
	Inches.			Inches.			Inches.	
3*d.*	$1\frac{1}{4}$	420	8*d.*	$2\frac{1}{2}$	100	30*d.*	4	24
4	$1\frac{1}{2}$	270	10	3	65	40	$4\frac{1}{2}$	20
5	$1\frac{3}{4}$	220	12	$3\frac{1}{4}$	52	60		
6	2	175	20	$3\frac{1}{2}$	28			

WEIGHTS OF LEAD PIPE PER FOOT IN LENGTH.

Caliber.	Mark.							
	AAA	AA	A	B	C	D	E	
	Lbs. oz.	Lbs. oz.	Lbs. oz.	Lbs oz.	Lbs. oz.	Lbs. oz.	Lbs. oz.	Lbs. oz.
$\frac{1}{8}$	.. ..	.. ..	.. ..	.. ..	.. ..	.. .	.. ..	0 2
$\frac{1}{4}$	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	0 2	.. ..
$\frac{3}{8}$	1 12	1 5	1 2	1 0	0 14	0 7	.. ..	0 10
$\frac{7}{16}$	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	0 $9\frac{1}{2}$
$\frac{1}{2}$	3 ..	2 0	1 10	1 3	1 0	0 10	.. ..	.. ..
	2 8	.. ..	.. ..	.. ..	.. ..	0 12	.. ..	.. ..
$\frac{5}{8}$	3 8	2 12	2 8	2 0	1 7	1 4	0 12	.. ..
$\frac{3}{4}$	4 14	3 3	3 0	2 3	1 12	1 3	1 0	.. ..
1	6 ..	4 8	4 0	3 4	2 8	2 4	2 0	1 8
$1\frac{1}{4}$	6 12	5 12	4 11	3 11	3 0	2 8	2 0	.. ..
$1\frac{1}{2}$	8 0	7 0	6 4	5 0	4 4	3 8	2 0	.. ..
	.. ..	.. ..	.. ..	.. ..	.. ..	3 0	.. ..	.. ..
$1\frac{3}{4}$	.. ..	8 8	6 7	5 0	4 0	3 10	.. ..	.. ..
2	10 11	8 14	7 0	6 0	5 0	4 0	.. ..	.. ..
	.. ..	.. ..	.. ..	.. ..	.. ..	3 0	.. ..	.. ..
	Thickness.				Waste.			
	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{3}{16}$				
$2\frac{1}{2}$	16 11	13 10	10 10	7 3	6 0	4 0	.. ..	.. ..
3	19 9	16 0	12 9	9 4	5 0	3 8	.. ..	.. ..
$3\frac{1}{2}$	22 8	18 7	14 8	10 12	.. ..	.. ..	.. ..	.. ..
4	25 6	20 14	16 7	12 2	8 0	6 0	.. ..	.. ..
$4\frac{1}{2}$	.. ..	.. ..	18 6	13 9	.. ..	.. ..	10 8	7 6
5	31 3	.. ..	20 5	15 0	.. ..	.. ..	10 8	.. ..
6	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	12 0	.. ..

TABLE OF THE WEIGHT OF A CUBIC FOOT OF WATER AT DIFFERENT TEMPERATURES.

Fahrenheit.	Centigrade.	Weight in pounds.	Fahrenheit.	Centigrade.	Weight in pounds.	Fahrenheit.	Centigrade.	Weight in pounds.
Degrees.	Degrees.		Degrees.	Degrees.		Degrees.	Degrees.	
32	0	62·42	95	35	62·06	167	75	60·87
39	4	62·42	104	40	61·95	176	80	60·68
41	5	62·42	113	45	61·83	185	85	60·48
50	10	62·41	122	50	61·69	194	90	60·27
59	15	62·37	131	55	61·55	203	95	60·04
68	20	62·32	140	60	61·39	212	100	59·83
77	25	62·25	149	65	61·23			
86	30	62·16	158	70	61·06			

PROPERTIES OF SATURATED STEAM,

From "Richards's Steam-Engine Indicator," by Chas. T. Porter.

Elastic Force.		Heat, in Degrees Fahrenheit.			Weight of one cubic foot, in decimals of a pound.	Specific gravity, the atmosphere at 32° being 1.	Elastic Force.		Heat, in Degrees Fahrenheit.			Weight of one cubic foot, in decimals of a pound.	Specific gravity, the atmosphere at 32° being 1.
In lbs. on the sq. in. from 0.	In inches of mercury at 32°.	Sensible heat.	Latent heat.	Total heat.			In lbs. on the sq. in. from 0.	In inches of mercury at 32°.	Sensible heat.	Latent heat.	Total heat.		
1	2·04	102·	1043·0	1145·0	·0029	·037	64	130·40	296·9	907·6	1204·5	·1416	1·754
2	4·08	126·3	1026·1	1152·4	·0057	·071	65	132·44	298·0	906·8	1204·8	·1436	1·779
3	6·11	141·6	1015·4	1157·1	·0084	·104	66	134·48	299·0	906·1	1205·1	·1456	1·804
4	8·15	153·1	1007·5	1160·6	·0110	·136	67	136·51	300·0	905·4	1205·4	·1476	1·829
5	10·19	162·3	1001·0	1163·4	·0135	·167	68	138·55	300·9	904·8	1205·7	·1496	1·854
6	12·22	170·1	995·6	1165·8	·0160	·198	69	140·59	301·9	904·1	1206·0	·1516	1·879
7	14·26	176·9	990·9	1167·9	·0185	·228	70	142·63	302·9	903·4	1206·3	·1536	1·904
8	16·30	182·9	986·7	1169·7	·0209	·258	71	144·66	303·9	902·7	1206·6	·1556	1·929
9	18·34	188·3	983·0	1171·3	·0233	·288	72	146·70	304 8	902·1	1206·9	·1576	1·954
10	20·38	193·2	979·6	1172·8	·0257	·318	73	148·74	305·7	901·5	1207·2	·1596	1·979
11	22·41	197·8	976·4	1174·2	·0281	·348	74	150·78	306·6	900·9	1207·5	·1616	2·004
12	24·45	201·0	973·5	1175·5	·0304	·377	75	152·81	307·5	900·3	1207·8	·1636	2·029
13	26·48	205·9	970·8	1176·7	·0327	·406	76	154·85	308·4	899·6	1208·0	·1656	2·054
14	28·53	209·6	968·2	1177·8	·0350	·435	77	156·89	309·3	899·0	1208·3	·1676	2·079
14·7	atmos.						78	158·93	310·2	898·4	1208·6	·1696	2·103
15	30·56	213·0	965·8	1178·9	·0373	·463	79	160·96	311·1	897·8	1208·9	·1716	2·127
16	32·60	216·3	963·6	1179·9	·0396	·492	80	163·00	312·0	897·1	1209·1	·1736	2·151
17	34·64	219·4	961·5	1180·9	·0419	·520	81	165·04	312·8	896·6	1209·4	·1756	2·175
18	36·68	222·4	959·4	1181·8	·0442	·548	82	167·08	313·6	896·1	1209·7	·1776	2·199
19	38·71	225·2	957·5	1182·7	·0465	·576	83	169·11	314·5	895·4	1209·9	·1795	2·223
20	40·75	228·0	955·5	1183·5	·0487	·604	84	171·15	315·3	894·8	1210·1	·1814	2·247
21	42·79	230·6	953·7	1184·3	·0510	·632	85	173·19	316·1	894·3	1210·4	·1833	2·271
22	44·83	233·1	951·9	1185·0	·0532	·660	86	175·23	316·9	893·8	1210·7	·1852	2·295
23	46·86	235·5	950·2	1185·7	·0554	·688	87	177·26	317·8	893·1	1210·9	·1871	2·319
24	48·90	237·9	948·6	1186·5	·0576	·715	88	179·30	318·6	892·5	1211·1	·1891	2·343
25	50·94	240·2	947·0	1187·2	·0598	·742	89	181·34	319·4	892·0	1211·4	·1910	2·367
26	52·98	242·3	945·6	1187·9	·0620	·769	90	183·38	320·2	891·4	1211·6	·1930	2·391
27	55·01	244·4	944·1	1188·5	·0642	·796	91	185·41	321·0	890·8	1211·8	·1950	2·415
28	57·05	246·4	942·7	1189·1	·0664	·823	92	187·45	321·7	890·3	1212·0	·1970	2·439
29	59·09	248·4	941·3	1189·7	·0686	·850	93	189·49	322·5	889·8	1212·3	·1990	2·463
30	61·13	250·4	939·9	1190·3	·0707	·877	94	191·53	323·3	889·2	1212·5	·2010	2·487
31	63·16	252·3	938·5	1190·8	·0729	·904	95	193·56	324·1	888·7	1212·8	·2030	2·511
32	65·20	254·1	937·3	1191·4	·0751	·931	96	195·60	324·8	888·2	1213·0	·2050	2·535
33	67·24	255·9	936·1	1192·0	·0772	·958	97	197·64	325·6	887·7	1213·3	·2070	2·559
34	69·28	257·6	934·9	1192·5	·0794	·985	98	199·68	326·3	887·2	1213·5	·2089	2·583
35	71·31	259·3	933·7	1193·0	·0815	1·012	99	201·71	327·1	886·6	1213·7	·2108	2·607
36	73·35	260·9	932·6	1193·5	·0837	1·038	100	203·75	327·8	886·1	1213·9	·2127	2·631
37	75·39	262·6	931·4	1194·0	·0858	1·064	101	205·79	328·5	885·7	1214·2	·2147	2·655
38	77·43	264·2	930·3	1194·5	·0879	1·090	102	207·83	329·2	885·2	1214·4	·2167	2·679
39	79·46	265·8	929·2	1195·0	·0900	1·116	103	209·86	329·9	884·7	1214·6	·2186	2·703
40	81·50	267·3	928·1	1195·4	·0921	1·142	104	211·90	330·6	884·2	1214·8	·2205	2·727
41	83·54	268·7	927·2	1195·9	·0942	1·168	105	213·94	331·3	883·7	1215·0	·2224	2·751
42	85·58	270·2	926·1	1196·3	·0963	1·194	106	215·98	331·9	883·3	1215·2	·2243	2·775
43	87·61	271·6	925·2	1196·8	·0983	1·220	107	218·01	332·6	882·8	1215·4	·2262	2·799
44	89·65	273·0	924·2	1197·2	·1004	1·246	108	220·05	333·3	882·3	1215·6	·2281	2·823
45	91·69	274·4	923·2	1197·6	·1025	1·272	109	222·09	334·0	881·8	1215·8	·2300	2·847
46	93·73	275·8	922·2	1198·0	·1046	1·298	110	224·13	334·6	881·4	1216·0	·2319	2·871
47	95·76	277·1	921·3	1198·4	·1067	1·324	111	226·16	335·3	880·9	1216·2	·2337	2·895
48	97·80	278·4	920·4	1198·8	·1087	1·350	112	228·20	336·0	880·4	1216·4	·2355	2·919
49	99·84	279·7	919·5	1199·2	·1108	1·376	113	230·24	336·7	879·9	1216·6	·2374	2·943
50	101·88	281·0	918·6	1199·6	·1129	1·402	114	232·28	337·4	879·4	1216·8	·2392	2·967
51	103·91	282·3	917·7	1200·0	·1150	1·428	115	234·31	338·0	879·0	1217·0	·2410	2·990
52	105·95	283·5	916·9	1200·4	·1171	1·454	116	236·35	338·6	878·6	1217·2	·2428	3·013
53	107·99	284·7	916·1	1200·8	·1192	1·479	117	238·39	339·3	878·1	1217·4	·2446	3·036
54	110·03	285·9	915·2	1201·1	·1212	1·504	118	240·43	339·9	877·7	1217·6	·2465	3·059
55	112·06	287·1	914·4	1201·5	·1232	1·529	119	242·46	340·5	877·3	1217·8	·2484	3·082
56	114·10	288·2	913·6	1201·8	·1252	1·554	120	244·50	341·1	876·9	1218·0	·2503	3·105
57	116·14	289·3	912·9	1202·2	·1272	1·579	121	246·54	341·8	876·4	1218·2	·2522	3·130
58	118·18	290·4	912·1	1202 5	·1293	1·604	122	248·58	342·4	876·0	1218·4	·2541	3·155
59	120·21	291·6	911·3	1202·9	·1314	1·629	123	250·61	343·0	875·6	1218·6	·2560	3·179
60	122·25	292·7	910·5	1203·2	·1335	1·654	124	252·65	343·6	875·1	1218·7	·2579	3·203
61	124·29	293·8	909·8	1203·6	·1356	1·679	125	254·69	344·2	874·7	1218·9	·2598	3·227
62	126·33	294·8	909·1	1203·9	·1376	1·704	126	256·73	344·8	874·3	1219·1	·2617	3·251
63	128·36	295·9	908·3	1204·2	·1396	1·729	127	258·76	345·4	873·9	1219·3	·2636	3·275

PROPERTIES OF SATURATED STEAM—(*Continued.*)

Elastic force.		Heat, in degrees Fahrenheit.			Weight of one cubic foot, in decimals of a pound.	Specific gravity, the atmosphere at 32° being 1.	Elastic force.		Heat, in degrees Fahrenheit.			Weight of one cubic foot, in decimals of a pound.	Specific gravity, the atmosphere at 32° being 1.
In lbs. on the sq. in. from 0.	In inches of mercury at 32°.	Sensible heat.	Latent heat.	Total heat.			In lbs. on the sq. in. from 0.	In inches of mercury at 32°.	Sensible heat.	Latent heat.	Total heat.		
128	260·80	346·0	873·4	1219·4	·2655	3·299	140	285·25	352·9	868·6	1221·5	·2883	3·582
129	262·84	346·6	873·0	1219·6	·2674	3·323	141	287·29	353·4	868·3	1221·7	·2902	3·605
130	264·88	347·2	872·6	1219·8	·2693	3·347	142	289·33	354·0	867·9	1221·9	·2921	3·628
131	266·91	347·8	872·2	1220·0	·2712	3·371	143	291·36	354·5	867·5	1222·0	·2940	3·651
132	268·95	348·3	871·9	1220·2	·2731	3·395	144	293·40	355·0	867·2	1222·2	·2959	3·674
133	270·99	348·9	871·5	1220·4	·2750	3·419	145	295·44	355·6	866·8	1222·4	·2978	3·697
134	273·03	349·5	871·1	1220·6	·2769	3·443	146	297·48	356·1	866·4	1222·5	·2997	3·720
135	275·06	350·0	870·7	1220·7	·2788	3·467	147	299·51	356·7	866·0	1222·7	·3016	3·743
136	277·10	350·6	870·3	1220·9	·2807	3·490	148	301·55	357·2	865·7	1222·9	·3035	3·765
137	279·14	351·2	869·8	1221·0	·2826	3·513	149	303·59	357·8	865·2	1223·0	·3054	3·787
138	281·18	351·8	869·4	1221·2	·2845	3·536	150	305·63	358·3	864·9	1223·2	·3073	3·809
139	283·21	352·3	869·1	1221·4	·2864	3·559							

TABLE OF MEAN PRESSURES IN STEAM CYLINDERS AT DIFFERENT RATES OF EXPANSION.

Portion of stroke during which steam is admitted.	Mean pressure during whole of stroke.	Portion of stroke during which steam is admitted.	Mean pressure during whole of stroke.	Portion of stroke during which steam is admitted.	Mean pressure during whole of stroke.	Portion of stroke during which steam is admitted.	Mean pressure during whole of stroke.
·80	·98	·56	·88	·40	·77	·24	·58
·77	·97	·54	·87	·38	·75	·22	·55
·74	·96	·52	·86	·36	·73	·20	·52
·70	·95	·50	·85	·34	·71	·18	·49
·68	·94	·48	·83	·32	·68	·16	·45
·66	·93	·46	·82	·30	·66	·14	·42
·62	·92	·44	·80	·28	·64	·12	·37
·60	·90	·42	·78	·26	·61	·10	·33
·58	·89						

Examples of Application of above Table.—To find the mean pressure in a condensing engine with an initial pressure, as shown by the gauge, of 75 pounds, and a cut-off at ·20, or $\frac{1}{5}$ stroke.

The actual initial pressure above 0 is 75 + 15, or 90 pounds. Mean pressure at ·20 cut-off in table ·52 for each pound of initial pressure, 90 × ·52 = 46·8 mean pressure above 0 in cylinder; but as the vacuum in the cylinder can never be perfect, an allowance of two to three pounds is to be made; 46·8 — 2·8 = 44, which may be taken as the probable actual mean pressure to be used in estimating the H. P. or lbs. ft. of work of the engine—made up thus: Mean pressure × area of steam piston in square inches, less $\frac{1}{2}$ that of the piston-rod × length of stroke in feet × number of strokes per minute = lbs. ft. of work per minute, and divided by 33,000 = H. P.

If the engine is non-condensing, then the deduction from the mean pressure would be the whole atmospheric pressure, 14·7, and probably about 1·3 back pressure, or say, 16 pounds, and the mean effective pressure in the cylinder would be for the cut-off and initial power as above, 46·8 — 16, or 30·8 pounds.

In estimating for the per cent of cut-off or steam follow, the clearances are to be estimated with the stroke and cut-off.

It may often be convenient to estimate the amount of water and coal necessary for an engine, which can be done approximately by taking the tension or pressure of the steam at any part of the stroke after the cut-off, finding in table the weight of one cubic foot of steam corresponding to this pressure, and multiplying it by the number of cubic feet in the cylinder at the point taken, which will be the weight of steam used per stroke. Multiplying this product by the number of strokes per working day, will give the total weight of water used as steam ; and if 8 pounds of steam be allowed for each pound of coal, it will give a fair average of the coal consumption during working hours. There will be additional coal used for getting up steam or for banking, and more water will be used than shown by the steam in the cylinder, as there will be water entrained with the steam, and condensed in passages and cylinder, equal to 25 per cent more, say, in total, 10 pounds of water for each pound of coal fed on the grates.

THE FLOW OF WATER.

The velocity of water in a stream or channel is often taken approximately by floats along different threads of the current. If the channel be an artificial one of rectangular section, the average velocity may be determined very nearly by a number of such experiments, with a tube float, extending nearly to the bottom of the channel; but in the rivers and streams, if surface floats be used, allowance is to be made for the friction of water on the bed of the stream, and want of uniformity in the flow. There are a variety of tachometers to determine the velocities beneath the surface, and to afford data for averages.

In the flow of water through apertures the theoretic velocity in feet per second is $8{\cdot}04\sqrt{h}$, h being the head or height of surface of water in feet above the center of the aperture. But in all apertures the discharge is less than the product of the area of their section by the theoretic velocity. There are contractions which reduce the effective section. If the discharge be through a thin plate into air, in which the contractions are around the entire periphery, the discharge is $\frac{6}{10}$ of that due to the section and theoretic velocity. If the edges are rounded, or the discharge be through a short pipe or ajutage, or beneath the surface of the water, the loss is less, and by suitable ajutages it may be almost entirely eliminated.

For the common purpose of gauging or determining the discharge of large pumps or small streams, the most accurate measure is by weirs, on which many experiments have been made, but those of Mr. James B. Francis, C. E., which are embodied in "Lowell Hydraulic Experiments," embrace a more practical range than any other, and are considered standard.

His general formula, on which the following table is calculated, is $Q = 3{\cdot}33\,(l - {\cdot}2h)\,h^{\frac{3}{2}}$, in which Q is the discharge in cubic feet per second, l the length of the weir, and h the height of water above the crest of the weir, both in feet; h is taken either at the side of the weir or a slight distance up stream; usually, a pipe with small perforations is laid parallel with the weir, on the bottom, and connected with a tight vertical box, in which the oscillations of the water surface are reduced to a mean.

In the table, the discharge is given for one foot in length; but as in weirs there are usually two end contractions, virtually reducing the length, and met in the formula above by $-{\cdot}2h$, a column of correction has been added, which is to be subtracted from the product of discharge, as given in the other columns of the table, by the length in feet.

Example.—Let the weir, with end contractions, be 5·3 feet long, and depth of water, or $h = 0{\cdot}612$.

By table the discharge for one foot in length is........................	1·594
	5·3
	8·4482
Correction...	·196
Discharge in cubic feet per second..................................	8·252

DISCHARGE, IN CUBIC FEET PER SECOND, OF A WEIR ONE FOOT LONG, WITHOUT CONTRACTION AT THE ENDS; FOR DEPTHS FROM 0·500 TO 0·999 FEET.

Correction for contractions.	Depth.	0	1	2	3	4	5	6	7	8	9
·012	0·20	0·298	0·300	0·302	0·305	0·307	0·309	0·311	0·314	0·316	0·318
·013	·21	0·320	0·323	0·325	0·327	0·330	0·332	0·334	0·337	0·339	0·341
·015	·22	0·344	0·346	0·348	0·351	0·353	0·355	0·358	0·360	0·362	0·365
·017	·23	0·367	0·370	0·372	0·374	0·377	0·379	0·382	0·384	0·387	0·389
·019	·24	0·391	0·394	0·396	0·399	0·401	0·404	0·406	0·409	0·411	0·414
·021	·25	0·416	0·419	0·421	0·424	0·426	0·429	0·431	0·434	0·436	0·439
·023	·26	0·441	0·444	0·447	0·449	0·452	0·454	0·457	0·459	0·462	0·465
·025	·27	0·467	0·470	0·472	0·475	0·478	0·480	0·483	0·485	0·488	0·491
·028	·28	0·493	0·496	0·499	0·501	0·504	0·507	0·509	0·512	0·515	0·517
·030	·29	0·520	0·523	0·525	0·528	0·531	0·534	0·536	0·539	0·542	0·544
·033	0·30	0·547	0·550	0·553	0·555	0·558	0·561	0·564	0·566	0·569	0·572
·036	·31	0·575	0·577	0·580	0·583	0·586	0·589	0·591	0·594	0·597	0·600
·039	·32	0·603	0·606	0·608	0·611	0·614	0·617	0·620	0·623	0·625	0·628
·042	·33	0·631	0·634	0·637	0·640	0·643	0·646	0·649	0·651	0·654	0·657
·045	·34	0·660	0·663	0·666	0·669	0·672	0·675	0·678	0·681	0·684	0·687
·048	·35	0·689	0·692	0·695	0·698	0·701	0·704	0·707	0·710	0·713	0·716
·052	·36	0·719	0·722	0·725	0·728	0·731	0·734	0·737	0·740	0·743	0·746
·056	·37	0·749	0·752	0·755	0·759	0·762	0·765	0·768	0·771	0·774	0·777
·059	·38	0·780	0·783	0·786	0·789	0·792	0·795	0·799	0·802	0·805	0·808
·063	·39	0·811	0·814	0·817	0·820	0·823	0·827	0·830	0·833	0·836	0·839
·067	0·40	0·842	0·846	0·849	0·852	0·855	0·858	0·861	0·865	0·868	0·871
·072	·41	0·874	0·877	0·881	0·884	0·887	0·890	0·893	0·897	0·900	0·903
·076	·42	0·906	0·910	0·913	0·916	0·919	0·923	0·926	0·929	0·932	0·936
·081	·43	0·939	0·942	0·945	0·949	0·952	0·955	0·959	0·962	0·965	0·969
·085	·44	0·972	0·975	0·978	0·982	0·985	0·988	0·992	0·995	0·998	1·002
·090	·45	1·005	1·009	1·012	1·015	1·019	1·022	1·025	1·029	1·032	1·035
·095	·46	1·039	1·042	1·046	1·049	1·052	1·056	1·059	1·063	1·066	1·070
·100	·47	1·073	1·076	1·080	1·083	1·087	1·090	1·094	1·097	1·100	1·104
·106	·48	1·107	1·111	1·114	1·118	1·121	1·125	1·128	1·132	1·135	1·139
·111	·49	1·142	1·146	1·149	1·153	1·156	1·160	1·163	1·167	1·170	1·174
·118	0·50	1·177	1·181	1·184	1·188	1·191	1·195	1·199	1·202	1·206	1·209
·124	·51	1·213	1·216	1·220	1·223	1·227	1·231	1·234	1·238	1·241	1·245
·130	·52	1·249	1·252	1·256	1·259	1·263	1·267	1·270	1·274	1·278	1·281
·136	·53	1·285	1·288	1·292	1·296	1·299	1·303	1·307	1·310	1·314	1·318
·143	·54	1·321	1·325	1·329	1·332	1·336	1·340	1·343	1·347	1·351	1·355
·150	·55	1·358	1·362	1·366	1·369	1·373	1·377	1·381	1·384	1·388	1·392
·157	·56	1·395	1·399	1·403	1·407	1·410	1·414	1·418	1·422	1·425	1·429
·164	·57	1·433	1·437	1·441	1·444	1·448	1·452	1·456	1·459	1·463	1·467
·171	·58	1·471	1·475	1·478	1·482	1·486	1·490	1·494	1·498	1·501	1·505
·178	·59	1·509	1·513	1·517	1·521	1·524	1·528	1·532	1·536	1·540	1·544
·186	0·60	1·548	1·551	1·555	1·559	1·563	1·567	1·571	1·575	1·579	1·583
·194	·61	1·586	1·590	1·594	1·598	1·602	1·606	1·610	1·614	1·618	1·622
·202	·62	1·626	1·630	1·633	1·637	1·641	1·645	1·649	1·653	1·657	1·661
·210	·63	1·665	1·669	1·673	1·677	1·681	1·685	1·689	1·693	1·697	1·701
·218	·64	1·705	1·709	1·713	1·717	1·721	1·725	1·729	1·733	1·737	1·741
·227	·65	1·745	1·749	1·753	1·757	1·761	1·765	1·769	1·773	1·777	1·781
·236	·66	1·785	1·790	1·794	1·798	1·802	1·806	1·810	1·814	1·818	1·822
·245	·67	1·826	1·830	1·834	1·838	1·843	1·847	1·851	1·855	1·859	1·863
·254	·68	1·867	1·871	1·875	1·880	1·884	1·888	1·892	1·896	1·900	1·904
·263	·69	1·909	1·913	1·917	1·921	1·925	1·929	1·934	1·938	1·942	1·946
·273	0·70	1·950	1·954	1·959	1·963	1·967	1·971	1·975	1·980	1·984	1·988
·283	·71	1·992	1·996	2·001	2·005	2·009	2·013	2·017	2·022	2·026	2·030
·293	·72	2·034	2·039	2·043	2·047	2·051	2·056	2·060	2·064	2·068	2·073
·303	·73	2·077	2·081	2·085	2·090	2·094	2·098	2·103	2·107	2·111	2·115
·314	·74	2·120	2·124	2·128	2·133	2·137	2·141	2·146	2·150	2·154	2·159
·324	·75	2·163	2·167	2·172	2·176	2·180	2·185	2·189	2·193	2·198	2·202

DISCHARGE, IN CUBIC FEET PER SECOND, OF A WEIR ONE FOOT LONG, WITHOUT CONTRACTION AT THE ENDS; FOR DEPTHS FROM 0·500 TO 0·999 FEET.

(*Continued.*)

Correction for contractions.	Depth.	0	1	2	3	4	5	6	7	8	9
·335	·76	2·206	2·211	2·215	2·219	2·224	2·228	2·232	2·237	2·241	2·246
·346	·77	2·250	2·254	2·259	2·263	3·267	2·272	2·276	2·281	2·285	2·290
·358	·78	2·294	2·298	2·303	2·307	2·312	2·316	2·320	2·325	2·329	2·334
·369	·79	2·238	2·343	2·347	2·351	2·356	2·360	2·365	2·369	2·374	2·378
·381	0·80	2·383	2·387	2·392	2·396	2·401	2·405	2·410	2·414	2·419	2·423
·393	·81	2·428	2·432	2·437	2·441	2·446	2·450	2·455	2·459	2·464	2·468
·406	·82	2·473	2·477	2·482	2·486	2·491	2·495	2·500	2·504	2·509	2·513
·413	·83	2·518	2·523	4·527	2·532	2·536	2·541	2·545	2·550	2·554	2·559
·431	·84	2·564	2·568	2·573	2·577	2·582	2·587	2·591	2·596	2·600	2·605
·444	·85	2·610	2·614	2·619	2·623	2·628	2·633	2·637	2·642	2·646	2·651
·457	·86	2·656	2·660	2·665	2·670	2·674	2·679	2·684	2·688	2·693	2·698
·470	·87	7·702	2·707	2·712	2·716	2·721	2·726	2·730	2·735	2·740	2·744
·484	·88	2·749	2·754	2·758	2·763	2·768	2·772	2·777	2·782	2·786	2·791
·498	·89	2·796	2·801	2·805	2·810	2·815	2·819	2·824	2·829	2·834	2·838
·512	0·90	2·843	2·848	2·853	2·857	2·862	2·867	2·872	2·876	2·881	2·886
·526	·91	2·891	2·895	2·900	2·905	2·910	2·915	2·919	2·924	2·929	2·934
·541	·92	2·938	2·943	2·948	2·953	2·958	2·963	2·967	2·972	2·977	2·982
·555	·93	2·986	2·991	2·996	3·001	3·006	3·011	3·015	3·020	3·025	3·030
·570	·94	3·035	3·040	3·044	3·049	3·054	3·059	3·064	3·069	3·074	3·078
·586	·95	3·083	3·088	3·093	3·098	3·103	3·108	3·113	3·117	3·122	3·127
·601	·96	3·132	3·137	3·142	3·147	3·152	3·157	3·162	3·166	3·171	3·176
·677	·97	3·181	3·186	3·191	3·196	3·201	3·206	3·211	3·216	3·221	3·226
·632	·98	3·231	3·235	3·240	3·245	3·250	3·255	3·260	3·265	3·270	3·275
·648	·99	3·280	3·285	3·290	3·295	3·300	3·305	3·310	3·315	3·320	3·325

Flow of Water through Pipes.—Figs. 5 and 6 are diagrams showing, by inspection, the million gallons delivered in 24 hours under varying resistance-heads or sines of slopes $\left(\frac{n}{l}\right)$ of clean cast-iron pipes of diameters from 6″ to 36″. They are calculated from the table of velocities in J. F. Fanning's "Practical Treatise on Hydraulics and Water-Supply Engineering."

Illustration of the Application of Diagram.—To determine the million gallons discharged per 24 hours through a 12″ pipe with ·02 sine of slope. The intersection of the horizontal of ·02 by the curve of 12″ is on the ordinate 4 millions, which will be discharge to be determined.

Again, to determine the loss of head per foot in length of a 30″ pipe in delivering 25 million gallons per 24 hours. The intersection of the ordinate of 25 millions with the 30″ curve is in the horizontal ·0067, the loss of head to be determined.

It will be seen that a 36″ pipe would deliver the same quantity with a loss of but ·0025 feet per foot in length.

These diagrams are applicable to long mains with a uniform current.

Flow through Sewers.—Fig. 7 is a diagram similar to the preceding, by which may be readily determined the cubic feet per second that would flow through circular sewers from 12″ to 72″ diameter, with various falls of from $\frac{1}{1000}$ to $\frac{1}{100}$ of a foot per foot of length.

It is calculated by the formula given by A. Fteley, M. A. S. C. E., in the description of the "Additional Supply from Sudbury River," for the Boston Water-Works, and deduced from experiments made by him on those works.

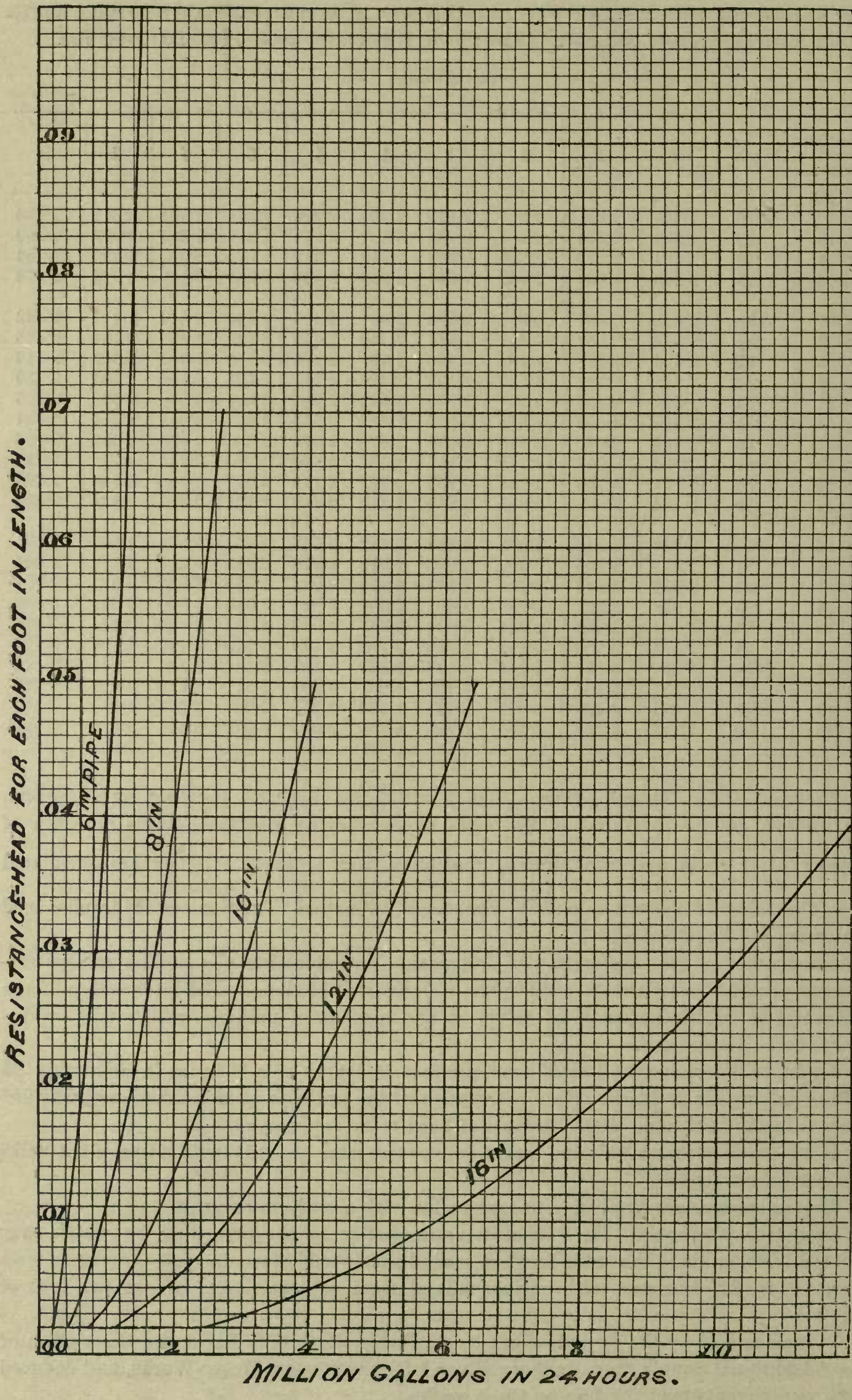

.09
.08
.07
.06
.05
.04
.03
.02
.01
.00
2
4
6
8
10
6 IN PIPE
8 IN
10 IN
12 IN
16 IN
RESISTANCE-HEAD FOR EACH FOOT IN LENGTH.
MILLION GALLONS IN 24 HOURS.

Fig. 5.

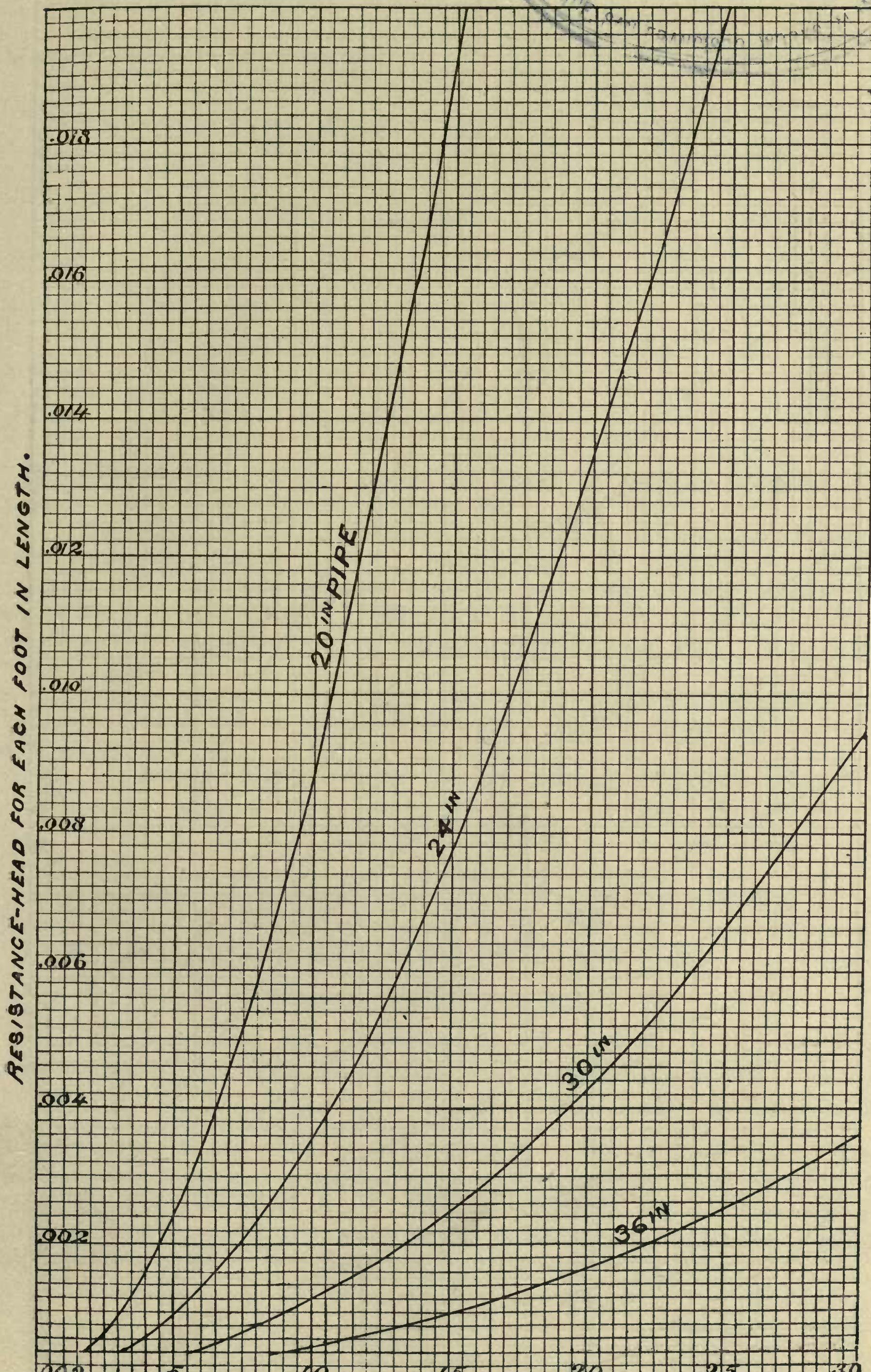

.018
.016
.014
.012
.010
.008
.006
.004
.002
.000
5
10
15
20
25
30
20 IN PIPE
24 IN
30 IN
36 IN
RESISTANCE-HEAD FOR EACH FOOT IN LENGTH.
MILLION GALLONS IN 24 HOURS.

FIG. 6.

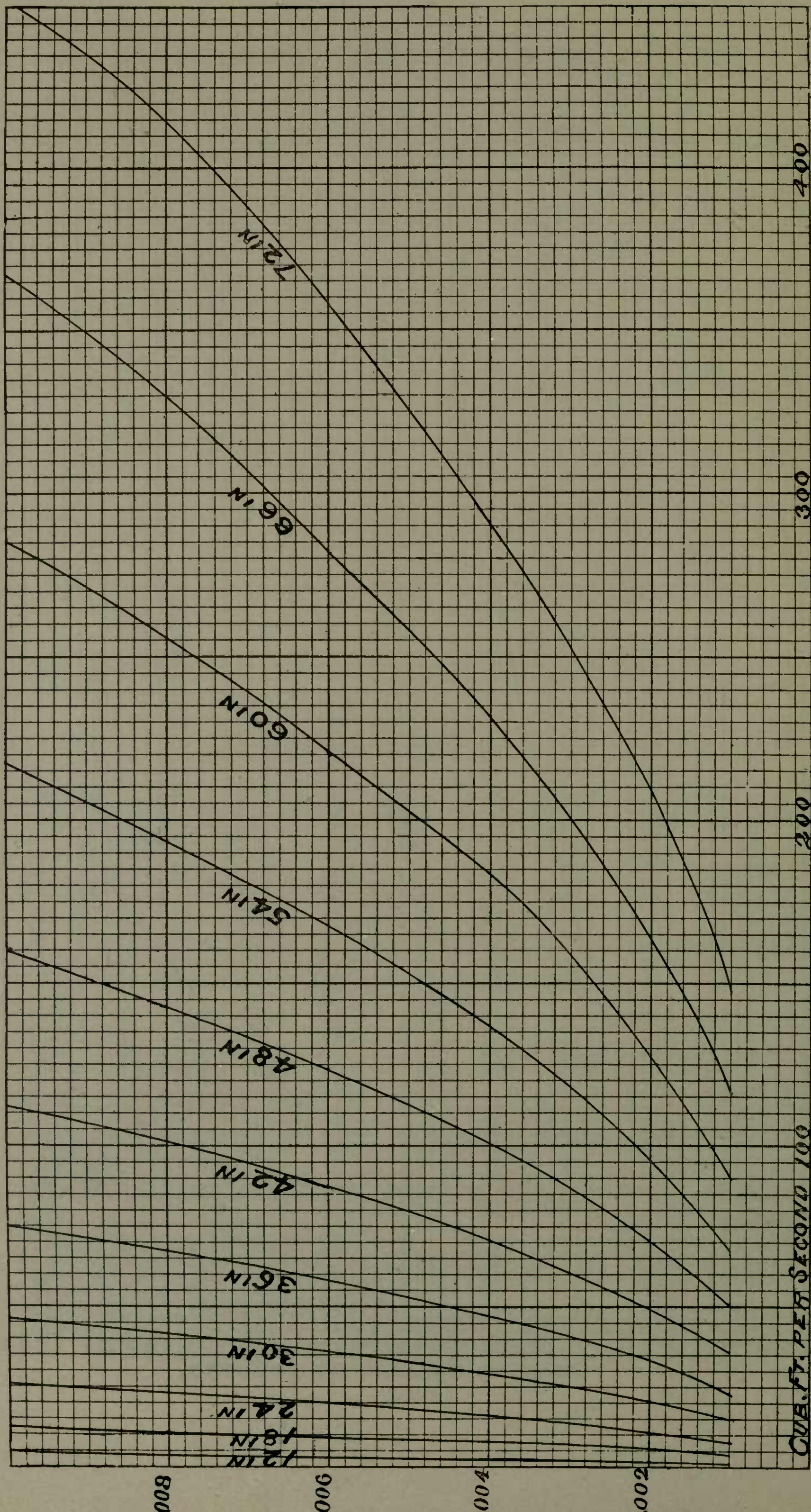

72 IN
66 IN
60 IN
54 IN
48 IN
42 IN
36 IN
30 IN
24 IN
18 IN
12 IN
400
300
200
CUB. FT. PER SECOND 100
.008
.006
.004
.002

Fig. 7.

The formula is—

$$V = C\sqrt{R\,I},$$

in which V = velocity in feet per second,

C = coefficient varying with R, as given in the following table,

R = hydraulic mean radius $= \dfrac{\text{area}}{\text{wetted perimeter}}$,

which in circular sewers is = $\frac{1}{4}$ of the diameter.

I = sine of inclination $= \dfrac{\text{total fall}}{\text{total length}}$.

R	C	R	C	R	C
0·1	96·3	0·6	119·4	1·1	128·5
0·2	104·7	0·7	121·7	1·2	129·8
0·3	109·9	0·8	123·6	1·3	131·1
0·4	113·8	0·9	125·4	1·4	132·2
0·5	116·9	1·0	127·0	1·5	133·3

Example.—To determine the cubic feet per second that would be discharged by a sewer 4′ or 48″ diameter with a fall per foot of ·006.

The intersection of the horizontal ·006 with the 48 in. curve is on the ordinate 124, which is the quantity per second which would be discharged under the conditions of the example.

On the other hand, to determine the fall per foot necessary to give a 60″ sewer to discharge 200 cubic feet per second. Following up the ordinate 200 to its intersection of the 60″ curve, its intersection will be found on the ·0049 horizontal, which will be the fall required.

For the same cubic feet of discharge per second, it will be seen by the diagram that a 72″ sewer would require but ·0018 fall per foot, and a 54″ sewer, for the same discharge, a fall of ·0086 feet.

Flow of Gas through Cast-iron Mains.—The usual formula found in hand-books is

$$Q = 1350\,D^2\sqrt{\frac{H\,D}{G\,L}},$$

in which Q = cubic feet per hour, D diameter, and H head of water-pressure, both in inches, L length of pipe in yards, and G specific gravity of the gas; if the last be taken at ·42, L at 1 mile or 1,760 yards, and H one inch, then $Q = 1200,\ D^{\frac{5}{2}}$, and

$$D = \sqrt[5]{1{,}440{,}000\,Q^2} = 17{\cdot}25\,Q^{\frac{5}{2}}.$$

It will be observed that, in the flow through the pipes, equivalent sections do not imply equal discharges; that, by the formula above, the flow through 4 pipes under the same head is not equal to that of one pipe of double the diameter, but that the flow is as the square root of the 5th power of the diameter ($D^{\frac{5}{2}}$).

Flow of Air through Pipes.—B. F. Sturtevant & Co., in the appplication of their fans and connections, found it very convenient to have tables of the value of pipes of different diameters in conveying air under different pressures, and the practical economy in this application in the matter of power for the transmission of air. On the following page are the tables published by them of the results of their calculations.

TABLE FOR EQUALIZING THE DIAMETER OF PIPES.

Parties putting up blast pipes are very liable to think, because the combined area of four 6-inch pipes is the same as one 12-inch pipe, that the four pipes will convey the same quantity of air with the same ease and freedom that the 12-inch will, whereas it actually does take 5·7—almost six 6-inch pipes. Again, 16 3-inch pipes have the combined area of one 12-inch pipe, but in actual practice it takes just 32 3-inch pipes to do the work of one 12-inch.

This is due to the excess of friction for every cubic foot of air in the small pipes over that in the large.

The large figures at the top of each column give the diameters in inches of the branch pipes.

The figures at the intersection of the horizontal line with the vertical give the number of pipes, of the diameter given at the top of the column, that will be equal in capacity for conveying air to one given opposite in the first column.

Diameter of main Blast Pipe, in Inches.																														
1	1																													
2	5.7	2																												
3	16	2.7	3																											
4	32	5.7	2.	4																										
5	56	9.8	3.6	1.8	5																									
6	88	16	5.7	2.8	1.6	6																								
7	129	23	8.3	4.1	2.3	1.5	7																							
8	180	32	12	5.7	3.2	2.1	1.4	8																						
9	244	42	16	7.6	4.3	2.8	1.9	1.3	9																					
10	317	56	20	9.9	5.7	3.6	2.4	1.7	1.3	10																				
11	402	71	26	12	7.0	4.5	3.1	2.2	1.7	1.3	11																			
12	501	88	32	16	9.0	5.7	3.8	2.8	2.0	1.6	1.2	12																		
13	613	107	39	19	11	6.9	4.7	3.4	2.5	1.9	1.5	1.2	13																	
14	737	129	47	23	13	8.3	5.7	4.1	3.0	2.3	1.8	1.5	1.2	14																
15	876	152	56	27	16	9.9	6.7	4.8	3.6	2.8	2.2	1.8	1.4	1.2	15															
16	1026	180	65	32	18	11	7.9	5.7	4.2	3.2	2.6	2.1	1.7	1.4	1.2	16														
17	1197	208	76	37	21	13	9.2	6.6	4.9	3.8	2.9	2.4	2.0	1.6	1.4	1.2	17													
18	1375	239	88	43	24	16	10	7.7	5.7	4.3	3.4	2.8	2.3	1.9	1.6	1.3	1.2	18												
19	1580	275	100	49	28	18	12	8.8	6.5	5.	3.9	3.2	2.6	2.2	1.8	1.5	1.3	1.2	19											
20	1797	313	114	56	32	20	14	9.9	7.4	5.7	4.5	3.6	2.9	2.5	2.1	1.7	1.5	1.3	1.1	20										
22	2284	398	145	71	41	26	18	13	9.3	7.2	5.7	4.5	3.7	3.1	2.6	2.2	1:9	1.7	1.4	1.3	22									
24	2834	493	180	88	50	32	22	16	12	8.9	7.6	5.7	4.6	3.8	3.2	2.9	2.4	2.1	1.8	1.6	1.2	24								
26	3474	605	219	108	62	39	27	19	14	11	8.6	6.9	5.7	4.7	4.0	3.4	2.9	2.5	2.2	1.9	1.5	1.2	26							
28	4165	725	265	129	74	48	32	23	17	13	10	8.3	6:8	5.7	4.8	4.1	3.5	3.0	2.6	2.3	1.8	1.5	1.2	28						
30	4963	864	315	154	88	56	38	28	20	16	12	9.9	8.0	6.7	5.7	4.7	4.1	3.6	3.0	2.6	2.2	1.7	1.4	1.2	30					
36	7818	1361	497	243	139	88	60	43	32	25	19	16	13	11	8.9	7.6	6.5	5.7	5.0	4.3	3.4	2.7	2.2	1.9	1.6	36				
42	11488	2000	730	358	205	129	88	63	47	36	29	23	19	16	13	11	9.6	8.5	7.3	6.4	5.0	4.1	3.3	2.8	2.3	1.5	42			
48	15989	2792	1081	492	282	180	123	88	66	50	39	32	26	22	18	16	13	12	10	8.9	7.0	5.7	4.7	3.8	3.2	2.1	1.4	48		
54	21560	3753	1368	671	384	244	166	119	88	68	53	43	35	29	24	21	18	16	15	12	9.4	7.6	6.2	5.2	4.3	2.8	1.9	1.3	54	
60	27913	4879	1781	872	499	314	215	154	115	88	69	56	46	38	32	27	23	20	18	16	12	9.9	8.1	6.7	5.7	3.6	2.4	1.8	1.3	

DIAMETER OF PIPES IN INCHES.

Losses of Pressure per 100 Feet must be provided for by Extra Speed and Power on the Blower.

Velocity of air in feet per min.	LOSS OF PRESSURE IN OUNCES PER SQUARE INCH.												
	1 inch.	2 inch.	3 inch.	4 inch.	6 inch.	8 inch.	10 inch.	12 inch.	14 inch	16 inch	18 inch	20 inch	22 inch
100	·011	·006	·004	·003	·002	·001	·001	·001	·001	·001	·001	·001	·001
200	·044	·022	·015	·011	·007	·006	·004	·004	·003	·003	·002	·002	·002
400	·178	·088	·059	·044	·030	·022	·018	·015	·013	·011	·010	·009	·008
600	·400	·200	·133	·100	·067	·050	·040	·033	·029	·025	·022	·020	·018
800	·711	·356	·237	·178	·119	·089	·071	·059	·051	·044	·040	·036	·032
1000	1·111	·556	·370	·278	·185	·139	·111	·092	·079	·069	·062	·056	·051
1200	1·600	·800	·533	·400	·267	·200	·160	·133	·114	·100	·089	·080	·073
1400	2·178	1·089	·726	·544	·363	·282	·218	·181	·156	·136	·121	·109	·099
1600	2·844	1·422	·948	·711	·474	·356	·284	·237	·203	·178	·158	·142	·129
1800	3·600	1·800	1·200	·900	·600	·450	·360	·300	·257	·225	·200	·180	·164
2000	4·444	2·222	1·481	1·111	·741	·556	·444	·370	·317	·278	·247	·222	·202
2200	5·378	2·689	1·793	1·344	·896	·672	·538	·448	·384	·336	·299	·269	·244
2400	6·400	3·200	2·133	1·600	1·067	·800	·640	·533	·457	·400	·356	·320	·291
2600	7·511	3·756	2·504	1·877	1·252	·939	·751	·626	·537	·468	·417	·376	·341
2800	8·711	4·356	2·904	2·178	1·452	1·089	·871	·726	·622	·544	·484	·436	·396
3000	10·000	5·000	3·333	2·500	1·667	1·250	1·000	·833	·714	·625	·556	·500	·455
3200	11·378	5·689	3·792	2·844	1·896	1·422	1·138	·948	·813	·711	·632	·569	·517
3400	12·844	6·422	4·281	3·211	2·141	1·606	1·284	1·070	·917	·827	·714	·642	·584
3600	14·400	7·200	4·800	3·600	2·400	1·800	1·440	1·200	1·029	·900	·800	·720	·655
3800	16·044	8·022	5·349	4·011	2·674	2·006	1·604	1·337	1·146	1·003	·891	·802	·729
4000	17·778	8·889	5·926	4·444	2·963	2·222	1·778	1·481	1·270	1·111	·988	·889	·808
4400		10·705	7·175	5·353	3·569	2·676	2·141	1·784	1·537	1·344	1·189	1·071	·973
4800		12·800	8·533	6·400	4·267	3·200	2·560	2·133	1·829	1·600	1·422	1·280	1·164
5200		15·022	10·015	7·511	5·007	3·756	3·004	2·504	2·146	1·871	1·670	1·502	1·366
5600		17·422	11·615	8·711	5·807	4·356	3·484	2·904	2·489	2·178	1·936	1·742	1·584
6000		20·000	13·333	10·000	6·667	5·000	4·000	3·333	2·857	2·500	2·222	2·000	1·818

TABLES OF THE CIRCUMFERENCES OF CIRCLES TO THE NEAREST FRACTION OF PRACTICAL MEASUREMENT ; ALSO, THE AREAS OF CIRCLES, IN INCHES AND DECIMAL PARTS, LIKEWISE OF FEET AND DECIMAL PARTS.

Circumference in feet and inches.	Diameter in inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in inches.	Area in square inches.	Area in square feet.
				1 6 7/8	6	28·27	·196
·20	1/16	·003		1 7 1/4	6 1/8	29·46	·204
·39	1/8	·012		1 7 5/8	6 1/4	30·68	·212
·59	3/16	·028		1 8	6 3/8	31·92	·220
·78	1/4	·049		1 8 3/8	6 1/2	33·18	·228
·98	5/16	·077		1 8 3/4	6 5/8	34·47	·237
1·18	3/8	·110		1 9 1/4	6 3/4	35·78	·246
1·37	7/16	·150		1 9 1/2	6 7/8	37·12	·256
1·57	1/2	·196		1 10	7	38·48	·267
1·77	9/16	·248		1 10 3/8	7 1/8	39·87	·277
1·96	5/8	·307		1 10 3/4	7 1/4	41·28	·287
2·16	11/16	·371		1 11 1/8	7 3/8	42·72	·297
2·36	3/4	·442		1 11 1/2	7 1/2	44·18	·307
2·55	13/16	·518		1 11 7/8	7 5/8	45·66	·318
2·75	7/8	·601		2 0 3/8	7 3/4	47·17	·328
2·94	15/16	·690		2 0 3/4	7 7/8	48·71	·338
3 1/8	1	·785	·0054	2 1 1/8	8	50·26	·349
3 1/2	1 1/8	·994	·0069	2 1 1/2	8 1/8	51·85	·360
3 7/8	1 1/4	1·23	·0085	2 1 7/8	8 1/4	53·46	·371
4 1/4	1 3/8	1·48	·0103	2 2 1/4	8 3/8	55·09	·383
4 5/8	1 1/2	1·77	·0123	2 2 5/8	8 1/2	56·74	·394
5 1/8	1 5/8	2·07	·0144	2 3	8 5/8	58·43	·406
5 1/2	1 3/4	2·40	·0167	2 3 3/8	8 3/4	60·13	·428
5 7/8	1 7/8	2·76	·0192	2 3 7/8	8 7/8	61·86	·430
6 1/4	2	3·14	·0218	2 4 1/4	9	63·62	·442
6 5/8	2 1/8	3·55	·0246	2 4 5/8	9 1/8	65·40	·455
7	2 1/4	3·98	·0276	2 5	9 1/4	67·20	·467
7 3/8	2 3/8	4·43	·0307	2 5 3/8	9 3/8	69·03	·480
7 7/8	2 1/2	4·91	·0341	2 5 3/4	9 1/2	70·88	·493
8 1/4	2 5/8	5·41	·0376	2 6 1/4	9 5/8	72·76	·506
8 5/8	2 3/4	5·94	·0412	2 6 5/8	9 3/4	74·66	·519
9	2 7/8	6·49	·0450	2 7	9 7/8	76·59	·532
9 3/8	3	7·07	·0490	2 7 3/8	10	78·54	·545
9 3/4	3 1/8	7·67	·0532	2 7 3/4	10 1/8	80·51	·559
10 1/4	3 1/4	8·29	·0576	2 8 1/8	10 1/4	82·52	·573
10 5/8	3 3/8	8·95	·0621	2 8 1/2	10 3/8	84·54	·587
11	3 1/2	9·62	·0668	2 9	10 1/2	86·59	·601
11 3/8	3 5/8	10·32	·0716	2 9 3/8	10 5/8	88·66	·615
11 3/4	3 3/4	11·04	·0766	2 9 3/4	10 3/4	90·76	·630
12 1/8	3 7/8	11·79	·0818	2 10 1/8	10 7/8	92·88	·645
1 0 1/2	4	12·57	·087	2 10 1/2	11	95·03	·660
1 1	4 1/8	13·36	·093	2 10 7/8	11 1/8	97·21	·675
1 1 3/8	4 1/4	14·19	·099	2 11 1/4	11 1/4	99·40	·690
1 1 3/4	4 3/8	15·03	·105	2 11 3/4	11 3/8	101·62	·705
1 2 1/8	4 1/2	15·90	·111	3 0 1/8	11 1/2	103·87	·720
1 2 1/2	4 5/8	16·80	·118	3 0 1/2	11 5/8	106·14	·736
1 2 7/8	4 3/4	17·72	·124	3 0 7/8	11 3/4	108·43	·752
1 3 1/4	4 7/8	18·66	·130	3 1 1/4	11 7/8	110·75	·768
1 3 5/8	5	19·63	·136	3 1 5/8	12	113·10	·785
1 4 1/8	5 1/8	20·63	·143	3 2	12 1/8	115·47	·802
1 4 1/2	5 1/4	21·65	·150	3 2 1/2	12 1/4	117·86	·819
1 4 7/8	5 3/8	22·69	·157	3 2 7/8	12 3/8	120·28	·836
1 5 1/4	5 1/2	23·76	·165	3 3 1/4	12 1/2	122·72	·853
1 5 5/8	5 5/8	24·85	·173	3 3 5/8	12 5/8	125·19	·870
1 6	5 3/4	25·97	181	3 4	12 3/4	127·68	·887
1 6 3/8	5 7/8	27·11	·189	3 4 3/8	12 7/8	130·19	·904

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
3 4¾	13	132·73	·922	5 2⅞	20	314·16	2·182
3 5⅛	13⅛	135·30	·939	5 3¼	20⅛	318·10	2·209
3 5⅝	13¼	137·89	·956	5 3⅝	20¼	322·06	2·237
3 6	13⅜	140·50	·974	5 4	20⅜	326·05	2·265
3 6⅜	13½	143·14	·992	5 4⅜	20½	330·06	2·293
3 6¾	13⅝	145·80	1·011	5 4¾	20⅝	334·10	2·321
3 7⅛	13¾	148·49	1·030	5 5⅛	20¾	338·16	2·349
3 7⅝	13⅞	151·20	1·050	5 5½	20⅞	342·25	2·377
3 8	14	153·94	1·069	5 6	21	346·36	2·405
3 8⅜	14⅛	156·70	1·088	5 6⅜	21⅛	350·50	2·434
3 8¾	14¼	159·49	1·107	5 6¾	21¼	354·66	2·463
3 9⅛	14⅜	162·30	1·126	5 7⅛	21⅜	358·84	2·492
3 9½	14½	165·13	1·146	5 7½	21½	363·05	2·521
3 9⅞	14⅝	167·99	1·166	5 7⅞	21⅝	367·28	2·550
3 10¼	14¾	170·87	1·186	5 8¼	21¾	371·54	2·580
3 10¾	14⅞	173·78	1·206	5 8¾	21⅞	375·83	2·610
3 11⅛	15	176·71	1·227	5 9⅛	22	380·13	2·640
3 11½	15⅛	179·67	1·247	5 9½	22⅛	384·46	2·670
3 11⅞	15¼	182·65	1·267	5 9⅞	22¼	388·82	2·700
4 0¼	15⅜	185·66	1·288	5 10¼	22⅜	393·20	2·730
4 0⅝	15½	188·69	1·309	5 10⅝	22½	397·61	2·761
4 1	15⅝	191·75	1·330	5 11	22⅝	402·04	2·792
4 1½	15¾	194·83	1·352	5 11½	22¾	406·49	2·823
4 1⅞	15⅞	197·93	1·374	5 11⅞	22⅞	410·97	2·854
4 2¼	16	201·06	1·396	6 0¼	23	415·48	2·885
4 2⅝	16⅛	204·22	1·418	6 0⅝	23⅛	420·00	2·917
4 3	16¼	207·39	1·440	6 1	23¼	424·56	2·949
4 3⅜	16⅜	210·60	1·462	6 1⅜	23⅜	429·13	2·981
4 3¾	16½	213·82	1·484	6 1¾	23½	433·74	3·013
4 4¼	16⅝	217·08	1·507	6 2¼	23⅝	438·36	3·045
4 4⅝	16¾	220·35	1·530	6 2⅝	23¾	443·01	3·077
4 5	16⅞	223·65	1·553	6 3	23⅞	447·69	3·109
4 5⅜	17	226·98	1·576	6 3⅜	2 0	452·39	3·142
4 5¾	17⅛	230·33	1·599	6 4⅛	2 0¼	461·86	3·207
4 6⅛	17¼	233·70	1·622	6 4⅞	2 0½	471·44	3·273
4 6½	17⅜	237·10	1·645	6 5¾	2 0¾	481·11	3·341
4 6⅞	17½	240·53	1·669	6 6½	2 1	490·87	3·408
4 7⅜	17⅝	243·98	1·693	6 7¼	2 1¼	500·74	3·477
4 7¾	17¾	247·45	1·718	6 8⅛	2 1½	510·71	3·547
4 8⅛	17⅞	250·95	1·743	6 8⅞	2 1¾	520·77	3·617
4 8½	18	254·47	1·767	6 9⅝	2 2	530·93	3·687
4 8⅞	18⅛	258·02	1·792	6 10½	2 2¼	541·19	3·758
4 9¼	18¼	261·59	1·817	6 11¼	2 2½	551·55	3·830
4 9¾	18⅜	265·18	1·842	7 0	2 2¾	562·00	3·904
4 10⅛	18½	268·80	1·868	7 0¾	2 3	572·56	3·976
4 10½	18⅝	272·45	1·893	7 1⅝	2 3¼	583·21	4·050
4 10⅞	18¾	276·12	1·918	7 2⅜	2 3½	593·96	4·124
4 11¼	18⅞	279·81	1·943	7 3⅛	2 3¾	604·81	4·200
4 11⅝	19	283·53	1·969	7 3⅞	2 4	615·75	4·276
5 0	19⅛	287·27	1·995	7 4¾	2 4¼	626·80	4·352
5 0⅜	19¼	291·04	2·021	7 5½	2 4½	637·94	4·430
5 0⅞	19⅜	294·83	2·047	7 6¼	2 4¾	649·18	4·508
5 1¼	19½	298·65	2·074	7 7	2 5	660·52	4·586
5 1⅝	19⅝	302·49	2·101	7 7⅞	2 5¼	671·96	4·666
5 2	19¾	306·36	2·128	7 8⅝	2 5½	683·49	4·747
5 2⅜	19⅞	310·25	2·155	7 9½	2 5¾	695·13	4·827

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
7 10¼	2 6	706·86	4·908	11 6¼	3 8	1520·5	10·56
7 11	2 6¼	718·69	4·990	11 7	3 8¼	1537·9	10·68
7 11¾	2 6½	730·62	5·073	11 7¾	3 8½	1555·3	10·80
8 0⅝	2 6¾	742·64	5·157	11 8½	3 8¾	1572·8	10·92
8 1⅜	2 7	754·77	5·241	11 9⅜	3 9	1590·4	11·04
8 2⅛	2 7¼	766·99	5·326	11 10⅛	3 9¼	1608·1	11·17
8 2⅞	2 7½	779·31	5·411	11 10⅞	3 9½	1626·0	11·29
8 3¾	2 7¾	791·73	5·498	11 11¾	3 9¾	1643·9	11·41
8 4½	2 8	804·25	5·585	12 0½	3 10	1661·9	11·54
8 5⅜	2 8¼	816·86	5·673	12 1¼	3 10¼	1680·0	11·67
8 6⅛	2 8½	829·58	5·761	12 2	3 10½	1698·2	11·79
8 6⅞	2 8¾	842·39	5·849	12 2⅞	3 10¾	1716·5	11·92
8 7⅝	2 9	855·30	5·939	12 3⅝	3 11	1734·9	12·05
8 8½	2 9¼	868·31	6·029	12 4⅜	3 11¼	1753·4	12·18
8 9¼	2 9½	881·41	6·120	12 5¼	3 11½	1772·0	12·30
8 10	2 9¾	894·62	6·212	12 6	3 11¾	1790·8	12·43
8 10¾	2 10	907·92	6·305	12 6¾	4 0	1809·6	12·57
8 11½	2 10¼	921·32	6·398	12 7½	4 0¼	1828·5	12·70
9 0⅜	2 10½	934·82	6·491	12 8⅜	4 0½	1847·4	12·83
9 1⅛	2 10¾	948·42	6·586	12 9⅛	4 0¾	1866·5	12·96
9 1⅞	2 11	962·11	6·681	12 9⅞	4 1	1885·7	13·09
9 2¾	2 11¼	975·91	6·777	12 10¾	4 1¼	1905·0	13·23
9 3½	2 11½	989·80	6·874	12 11½	4 1½	1924·4	13·36
9 4¼	2 11¾	1003·8	6·970	13 0¼	4 1¾	1943·9	13·50
9 5	3 0	1017·9	7·069	13 1	4 2	1963·5	13·63
9 5⅞	3 0¼	1032·1	7·167	13 1⅞	4 2¼	1983·2	13·77
9 6⅝	3 0½	1046·3	7·266	13 2⅝	4 2½	2003·0	13·91
9 7½	3 0¾	1060·7	7·366	13 3⅜	4 2¾	2022·8	14·05
9 8¼	3 1	1075·2	7·466	13 4¼	4 3	2042·8	14·19
9 9	3 1¼	1089·8	7·567	13 5	4 3¼	2062·9	14·32
9 9⅞	3 1½	1104·5	7·669	13 5¾	4 3½	2083·1	14·46
9 10⅝	3 1¾	1119·2	7·772	13 6½	4 3¾	2103·3	14·61
9 11⅜	3 2	1134·1	7·876	13 7⅜	4 4	2123·7	14·75
10 0⅛	3 2¼	1149·1	7·979	13 8⅛	4 4¼	2144·2	14·89
10 0⅞	3 2½	1164·2	8·085	13 8⅞	4 4½	2164·7	15·03
10 1¾	3 2¾	1179·3	8·189	13 9¾	4 4¾	2185·4	15·18
10 2½	3 3	1194·6	8·295	13 10½	4 5	2206·2	15·32
10 3¼	3 3¼	1209·9	8·403	13 11¼	4 5¼	2227·0	15·46
10 4	3 3½	1225·4	8·509	14 0	4 5½	2248·0	15·61
10 4⅞	3 3¾	1241·0	8·617	14 0⅞	4 5¾	2269·1	15·76
10 5⅝	3 4	1256·6	8·727	14 1⅝	4 6	2290·2	15·90
10 6⅜	3 4¼	1272·4	8·836	14 2⅜	4 6¼	2311·5	16·05
10 7¼	3 4½	1288·2	8·946	14 3¼	4 6½	2332·8	16·20
10 8	3 4¾	1304·2	9·056	14 4	4 6¾	2354·3	16·35
10 8¾	3 5	1320·2	9·169	14 4¾	4 7	2375·8	16·50
10 9½	3 5¼	1336·4	9·211	14 5½	4 7¼	2397·5	16·65
10 10⅜	3 5½	1352·6	9·394	14 6⅜	4 7½	2419·2	16·80
10 11⅛	3 5¾	1369·0	9·506	14 7⅛	4 7¾	2441·1	16·95
10 11⅞	3 6	1385·4	9·62	14 7⅞	4 8	2463·0	17·10
11 0¾	3 6¼	1402·0	9·73	14 8¾	4 8¼	2485·0	17·26
11 1½	3 6½	1418·6	9·84	14 9½	4 8½	2507·2	17·41
11 2¼	3 6¾	1435·4	9·96	14 10¼	4 8¾	2529·4	17·56
11 3	3 7	1452·2	10·08	14 11	4 9	2551·8	17·72
11 3⅞	3 7¼	1469·1	10·20	14 11⅞	4 9¼	2574·2	17·88
11 4⅝	3 7½	1486·2	10·32	15 0⅝	4 9½	2596·7	18·03
11 5½	3 7¾	1503·3	10·44	15 1⅜	4 9¾	2619·3	18·19

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.	Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
15 2¼	4 10	2642·1	18·35	18 10⅛	6 0	4071·5	28·27
15 3	4 10¼	2664·9	18·51	18 10⅞	6 0¼	4099·8	28·47
15 3¾	4 10½	2687·8	18·66	18 11¾	6 0½	4128·2	28·67
15 4½	4 10¾	2710·8	18·82	19 0½	6 0¾	4156·8	28·87
15 5¼	4 11	2734·0	18·98	19 1¼	6 1	4185·4	29·07
15 6⅛	4 11¼	2757·2	19·15	19 2⅛	6 1¼	4214·1	29·27
15 6⅞	4 11½	2780·5	19·31	19 2⅞	6 1½	4242·9	29·47
15 7¾	4 11¾	2803·9	19·47	19 3⅝	6 1¾	4271·8	29·67
15 8½	5 0	2827·4	19·63	19 4½	6 2	4300·8	29·87
15 9¼	5 0¼	2851·0	19·80	19 5¼	6 2¼	4329·9	30·07
15 10	5 0½	2874·8	19·96	19 6	6 2½	4359·2	30·27
15 10¾	5 0¾	2898·6	20·13	19 6¾	6 2¼	4388·5	30·47
15 11⅝	5 1	2922·5	20·29	19 7½	6 3	4417·9	30·68
16 0⅜	5 1¼	2946·5	20·46	19 8⅜	6 3¼	4447·4	30·88
16 1¼	5 1½	2970·6	20·63	19 9⅛	6 3½	4477·0	31·09
16 2	5 1¾	2994·8	20·80	19 9⅞	6 3¾	4506·7	31·30
16 2¾	5 2	3019·1	20·96	19 10¾	6 4	4536·5	31·50
16 3½	5 2¼	3043·5	21·13	19 11½	6 4¼	4566·4	31·71
16 4¼	5 2½	3068·0	21·30	20 0¼	6 4½	4596·3	31·92
16 5⅛	5 2¾	3092·6	21·48	20 1⅛	6 4¾	4626·4	32·13
16 5⅞	5 3	3117·2	21·65	20 1⅞	6 5	4656·6	32·34
16 6⅝	5 3¼	3142·0	21·82	20 2⅝	6 5¼	4686·9	32·55
16 7½	5 3½	3166·9	21·99	20 3½	6 5½	4717·3	32·76
16 8¼	5 3¾	3191·9	22·17	20 4¼	6 5¾	4747·8	32·97
16 9	5 4	3217·0	22·34	20 5	6 6	4778·3	33·18
16 9¾	5 4¼	3242·2	22·51	20 5¾	6 6¼	4809·0	33·40
16 10⅝	5 4½	3267·5	22·69	20 6½	6 6½	4839·8	33·61
16 11⅜	5 4¾	3292·8	22·87	20 7⅜	6 6¾	4870·7	33·82
17 0⅛	5 5	3318·3	23·04	20 8⅛	6 7	4901·6	34·04
17 1	5 5¼	3343·9	23·22	20 8⅞	6 7¼	4932·7	34·25
17 1¾	5 5½	3369·6	23·40	20 9⅝	6 7½	4963·9	34·47
17 2½	5 5¾	3395·3	23·58	20 10½	6 7¾	4995·1	34·69
17 3⅜	5 6	3421·2	23·76	20 11¼	6 8	5026·5	34·91
17 4⅛	5 6¼	3447·2	23·94	21 0⅛	6 8¼	5058·0	35·12
17 4⅞	5 6½	3473·2	24·12	21 0⅞	6 8½	5089·5	35·34
17 5⅝	5 6¾	3499·4	24·30	21 1⅝	6 8¾	5121·2	35·56
17 6½	5 7	3525·1	24·48	21 2⅜	6 9	5153·0	35·78
17 7¼	5 7¼	3552·0	24·67	21 3¼	6 9¼	5184·8	36·01
17 8	5 7½	3578·5	24·85	21 4	6 9½	5216·8	36·23
17 8¾	5 7¾	3605·0	25·03	21 4¾	6 9¾	5248·8	36·45
17 9⅝	5 8	3631·7	25·22	21 5⅝	6 10	5281·0	36·67
17 10⅜	5 8¼	3658·4	25·40	21 6⅜	6 10¼	5313·2	36·89
17 11⅛	5 8½	3685·3	25·59	21 7⅛	6 10½	5345·6	37·12
17 11⅞	5 8¾	3712·2	25·78	21 7⅞	6 10¾	5378·0	37·35
18 0¾	5 9	3739·3	25·96	21 8¾	6 11	5410·6	37·57
18 1½	5 9¼	3766·4	26·15	21 9½	6 11¼	5443·2	37·80
18 2¼	5 9½	3793·7	26·34	21 10¼	6 11½	5476·0	38·03
18 3⅛	5 9¾	3821·0	26·53	21 11⅛	6 11¾	5508·8	38·26
18 3⅞	5 10	3848·5	26·72	21 11⅞	7 0	5541·7	38·48
18 4⅝	5 10¼	3876·0	26·92	22 0⅝	7 0¼	5574·8	38·71
18 5½	5 10½	3903·6	27·11	22 1⅜	7 0½	5607·9	38·94
18 6¼	5 10¾	3931·4	27·30	22 2¼	7 9¾	5641·1	39·17
18 7	5 11	3959·2	27·49	22 3	7 1	5674·5	39·41
18 7¾	5 11¼	3987·1	27·69	22 3¾	7 1¼	5707·9	39·64
18 8⅝	5 11½	4015·2	27·88	22 4½	7 1½	5741·4	39·87
18 9⅜	5 11¾	4043·3	28·08	22 5¼	7 1¾	5775·0	40·10

TABLES OF THE CIRCUMFERENCES OF CIRCLES, ETC.—(*Continued.*)

Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
22 6⅛	7 2	5808·8	40·34
22 6⅞	7 2¼	5842·6	40·57
22 7¾	7 2½	5876·5	40·80
22 8½	7 2¾	5910·5	41·04
22 9¼	7 3	5944·6	41·28
22 10⅛	7 3¼	5978·9	41·52
22 10⅞	7 3½	6013·2	41·76
22 11⅝	7 3¾	6047·6	42·00
23 0⅜	7 4	6082·1	42·24
23 1¼	7 4¼	6116·7	42·48
23 2	7 4½	6151·4	42·72
23 2¾	7 4¾	6186·2	42·96
23 3⅝	7 5	6221·1	43·20
23 4⅜	7 5¼	6256·1	43·44
23 5⅛	7 5½	6291·2	43·68
23 6	7 5¾	6326·4	43·93
23 6¾	7 6	6361·7	44·18
23 7½	7 6¼	6397·1	44·43
23 8¼	7 6½	6432·6	44·67
23 9⅛	7 6¾	6468·2	44·92
23 9⅞	7 7	6503·8	45·17
23 10⅝	7 7¼	6539·6	45·41
23 11⅜	7 7½	6575·5	45·66
24 0¼	7 7¾	6611·5	45·91
24 1	7 8	6647·6	46·16
24 1¾	7 8¼	6683·8	46·42
24 2½	7 8½	6720·0	46·67
24 3¼	7 8¾	6756·4	46·92
24 4⅛	7 9	6792·9	47·17
24 4⅞	7 9¼	6829·4	47·43
24 5¾	7 9½	6866·1	47·68
24 6½	7 9¾	6902·9	47·94
24 7¼	7 10	6939·7	48·19
24 8	7 10¼	6976·7	48·45
24 8¾	7 10½	7013·8	48·71
24 9⅝	7 10¾	7050·9	48·96
24 10⅜	7 11	7088·2	49·22
24 11¼	7 11¼	7125·5	49·48
25 0	7 11½	7163·0	49·74
25 0¾	7 11¾	7200·5	50·00
25 1½	8 0	7238·2	50·26
25 2⅜	8 0¼	7275·9	50·53
25 3⅛	8 0½	7313·8	50·79
25 3⅞	8 0¾	7351·7	51·05
25 4¾	8 1	7389·8	51·32
25 5½	8 1¼	7427·9	51·58
25 6¼	8 1½	7466·2	51·85
25 7	8 1¾	7504·5	52·11
25 7⅞	8 2	7542·9	52·38
25 8⅝	8 2¼	7581·5	52·65
25 9⅜	8 2½	7620·1	52·92
25 10¼	8 2¾	7658·8	53·19
25 11	8 3	7697·7	53·46
25 11¾	8 3¼	7736·6	53·73
26 0½	8 3½	7775·6	54·00
26 1¼	8 3¾	7814·7	54·27

Circumference in feet and inches.	Diameter in feet and inches.	Area in square inches.	Area in square feet.
26 2⅛	8 4	7853·9	54·54
26 5¼	8 5	8011·9	55·64
26 8⅜	8 6	8171·3	56·75
26 11½	8 7	8332·3	57·86
27 2¾	8 8	8494·9	58·99
27 5¾	8 9	8659·0	60·13
27 9	8 10	8824·7	61·28
28 0⅛	8 11	8892·0	62·44
28 3¼	9	9160·9	63·62
28 6⅜	9 1	9331·3	64·80
28 9½	9 2	9503·3	66·00
29 0⅝	9 3	9676·9	67·20
29 3¾	9 4	9852·1	68·42
29 7	9 5	10028·8	69·64
29 10⅛	9 6	10207·1	70·88
30 1¼	9 7	10386·9	72·13
30 4⅜	9 8	10568·3	73·39
30 7½	9 9	10751·3	74·66
30 10¾	9 10	10935·9	75·94
31 1¾	9 11	11122·0	77·24
31 5	10	11309·8	78·54
31 8⅛	10 1	11499·0	79·85
31 11¼	10 2	11689·9	81·18
32 2⅜	10 3	11882·3	82·52
32 5½	10 4	12076·3	83·86
32 8⅝	10 5	12271·9	85·22
32 11¾	10 6	12469·0	86·59
33 2⅞	10 7	12667·7	87·97
33 6⅛	10 8	12868·0	89·36
33 9¼	10 9	13069·8	90·76
34 0⅜	10 10	13273·3	92·17
34 3½	10 11	13478·2	93·60
34 6⅝	11	13684·8	95·03
34 9¾	11 1	13892·9	96·48
35 0⅞	11 2	14142·6	97·93
35 4⅛	11 3	14313·9	99·40
35 7¼	11 4	14526·8	100·88
35 10⅜	11 5	14741·2	102·37
36 1½	11 6	14957·2	103·87
36 4⅝	11 7	15174·7	105·38
36 7¾	11 8	15393·8	106·90
36 10⅞	11 9	15614·5	108·43
37 2	11 10	15836·8	109·98
37 5¼	11 11	16060·6	111·53
37 8⅜	12	16286·0	113·10
37 11½	12 1	16513·0	114·67
38 2⅝	12 2	16741·6	116·26
38 5¾	12 3	16971·7	117·86
38 8⅞	12 4	17203·4	119·47
39 0	12 5	17436·7	121·09
39 3¼	12 6	17671·5	122·72
39 6⅜	12 7	17907·9	124·36
39 9½	12 8	18145·9	126·01
40 0⅝	12 9	18385·4	127·68
40 3¾	12 10	18626·6	129·35
40 6⅞	12 11	18869·2	131·04

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS.

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
1	1	1	1·000	1·000	4096	262144	64	8·000	4·000
4	8	2	1·414	1·259	4225	274625	65	8·062	4·020
9	27	3	1·732	1·442	4356	287496	66	8·124	4·041
16	64	4	2·000	1·587	4489	300763	67	8·185	4·061
25	125	5	2·236	1·709	4624	314432	68	8·246	4·081
36	216	6	2·449	1·817	4761	328509	69	8·306	4·101
49	343	7	2·645	1·912	4900	343000	70	8·366	4·121
64	512	8	2·828	2·000	5041	357911	71	8·426	4·140
81	729	9	3·000	2·080	5184	373248	72	8·485	4·160
100	1000	10	3·162	2·154	5329	389017	73	8·544	4·179
121	1331	11	3·316	2·223	5476	405224	74	8·602	4·198
144	1728	12	3·464	2·289	5625	421875	75	8·660	4·217
169	2197	13	3·605	2·351	5776	438976	76	8·717	4·235
196	2744	14	3·741	2·410	5929	456533	77	8·774	4·254
225	3375	15	3·872	2·466	6084	474552	78	8·831	4·272
256	4096	16	4·000	2·519	6241	493039	79	8·888	4·290
289	4913	17	4·123	2·571	6400	512000	80	8·944	4·308
324	5832	18	4·242	2·620	6561	531441	81	9·000	4·326
361	6859	19	4·358	2·668	6724	551368	82	9·055	4·344
400	8000	20	4·472	2·714	6889	571787	83	9·110	4·362
441	9261	21	4·582	2·758	7056	592704	84	9·165	4·379
484	10648	22	4·690	2·802	7225	614125	85	9·219	4·396
529	12167	23	4·795	2·843	7396	636056	86	9·273	4·414
576	13824	24	4·898	2·884	7569	658503	87	9·327	4·431
625	15625	25	5·000	2·924	7744	681472	88	9·380	4·447
676	17576	26	5·099	2·962	7921	704969	89	9·433	4·464
729	19683	27	5·196	3·000	8100	729000	90	9·486	4·481
784	21952	28	5·291	3·036	8281	753571	91	9·539	4 497
841	24389	29	5·385	3·072	8464	778688	92	9·591	4·514
900	27000	30	5·477	3·107	8649	804357	93	9·643	4·530
961	29791	31	5·567	3·141	8836	830584	94	9·695	4·546
1024	32768	32	5·656	3·174	9025	857374	95	9·746	4·562
1089	35937	33	5·744	3·207	9216	884736	96	9·797	4·578
1156	39304	34	5·830	3·239	9409	912673	97	9·848	4·594
1225	42875	35	5·916	3·271	9604	941192	98	9·899	4·610
1296	46656	36	6·000	3·301	9801	970299	99	9·949	4·626
1369	50653	37	6·082	3·332	10000	1000000	100	10·000	4·641
1444	54872	38	6·164	3·361	10201	1030301	101	10·049	4·657
1521	59319	39	6·244	3·391	10404	1061208	102	10·099	4·672
1600	64000	40	6·324	3·419	10609	1092727	103	10·148	4·687
1681	68921	41	6·403	3·448	10816	1124864	104	10·198	4·702
1764	74088	42	6·480	3·476	11025	1157625	105	10·246	4·717
1849	79507	43	6·557	3·503	11236	1191016	106	10·295	4·732
1936	85184	44	6·633	3·530	11449	1225043	107	10·344	4·747
2025	91125	45	6·708	3·556	11664	1259712	108	10·392	4·762
2116	97336	46	6·782	3·583	11881	1295029	109	10·440	4·776
2209	103823	47	6·855	3·608	12100	1331000	110	10·488	4·791
2304	110592	48	6·928	3·634	12321	1367631	111	10·535	4·805
2401	117649	49	7·000	3·659	12544	1404928	112	10·583	4·820
2500	125000	50	7·071	3·684	12769	1442897	113	10·630	4·834
2601	132651	51	7·141	3·708	12996	1481544	114	10·677	4·848
2704	140608	52	7·211	3·732	13225	1520875	115	10·723	4·862
2809	148877	53	7·280	3·756	13456	1560896	116	10·770	4·876
2916	157464	54	7·348	3·779	13689	1601613	117	10·816	4·890
3025	166375	55	7·416	3·802	13924	1643032	118	10·862	4·904
3136	175616	56	7·4S3	3·825	14161	1685159	119	10·908	4·918
3249	185193	57	7·549	3·848	14400	1728000	120	10·954	4·932
3364	195112	58	7·615	3·870	14641	1771561	121	11·000	4·946
3481	205379	59	7·681	3·892	14834	1815848	122	11·045	4·959
3600	216000	60	7·745	3·914	15129	1860867	123	11·090	4·973
3721	226981	61	7·810	3·930	15376	1906624	124	11·135	4·986
3844	238328	62	7·874	3·957	15625	1953125	125	11·180	5·000
3969	250047	63	7·937	3·979	15876	2000376	126	11·224	5·013

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
16129	2048383	127	11·269	5·026	36100	6859000	190	13·784	5·748
16384	2097152	128	11·313	5·039	36481	6967871	191	13·820	5·758
16641	2146689	129	11·357	5·052	36864	7077888	192	13·856	5·768
16900	2197000	130	11·401	5·065	37249	7189517	193	13·892	5·778
17161	2248091	131	11·445	5·078	37636	7301384	194	13·928	5·788
17424	2299968	132	11·489	5·091	38025	7414875	195	13·964	5·798
17689	2352637	133	11·532	5·104	38416	7529536	196	14·000	5·808
17956	2406104	134	11·575	5·117	38809	7645373	197	14·035	5·818
18225	2460375	135	11·618	5·129	39204	7762392	198	14·071	5·828
18496	2515456	136	11·661	5·142	39601	7880599	199	14·106	5·838
18769	2571353	137	11·704	5·155	40000	8000000	200	14·142	5·848
19044	2628072	138	11·747	5·167	40401	8120601	201	14·177	5·857
19321	2685619	139	11·789	5·180	40804	8242408	202	14·212	5·867
19600	2744000	140	11·832	5·192	41209	8365427	203	14·247	5·877
19881	2803221	141	11·874	5·204	41616	8489664	204	14·282	5·886
20164	2863288	142	11·916	5·217	42025	8615125	205	14·317	5·896
20449	2924207	143	11·958	5·229	42436	8741816	206	14·352	5·905
20736	2985984	144	12·000	5·241	42849	8869743	207	14·387	5·915
21025	3048625	145	12·041	5·253	43264	8998912	208	14·422	5·924
21316	3112136	146	12·083	5·265	43681	9129329	209	14·456	5·934
21609	3176523	147	12·124	5·277	44100	9261000	210	14·491	5·943
21904	3241792	148	12·165	5·289	44521	9393931	211	14·525	5·953
22201	3307949	149	12·206	5·301	44944	9528128	212	14·560	5·962
22500	3375000	150	12·247	5·313	45369	9663597	213	14·594	5·972
22801	3442951	151	12·288	5·325	45796	9800344	214	14·628	5·981
23104	3511008	152	12·328	5·336	46225	9938375	215	14·662	5·990
23409	3581577	153	12·369	5·348	46656	10077696	216	14·696	6·000
23716	3652264	154	12·409	5·360	47089	10218312	217	14·730	6·009
24025	3723875	155	12·449	5·371	47524	10360232	218	14·764	6·018
24336	3796416	156	12·489	5·383	47961	10503459	219	14·798	6·027
24649	3869893	157	12·529	5·394	48400	10648000	220	14·832	6·036
24964	3944312	158	12·569	5·406	48841	10793861	221	14·866	6·045
25281	4019679	159	12·609	5·417	49284	10941048	222	14·899	6·055
25600	4096000	160	12·649	5·428	49729	11089567	223	14·933	6·064
25921	4173281	161	12·688	5·440	50176	11239424	224	14·966	6·073
26244	4251528	162	12·727	5·451	50625	11390625	225	15·000	6·082
26569	4330747	163	12·767	5·462	51076	11543176	226	15·033	6·099
26896	4410944	164	12·806	5·473	51529	11697083	227	15·066	6·100
27225	4492125	165	12·845	5·484	51984	11852352	228	15·099	6·109
27556	4574296	166	12·884	5·495	52441	12008989	229	15·132	6·118
27889	4657463	167	12·922	5·506	52900	12167000	230	15·165	6·126
28224	4741632	168	12·961	5·517	53361	12326391	231	15·198	6·135
28561	4826809	169	13·000	5·528	53824	12487168	232	15·231	6·144
28900	4913000	170	13·938	5·539	54289	12649337	233	15·264	6·153
29241	5000211	171	13·076	5·550	54756	12812904	234	15·297	6·162
29584	5088448	172	13·114	5·561	55225	12977875	235	15·329	6·171
29929	5177717	173	13·152	5·572	55696	13144256	236	15·362	6·179
30276	5268024	174	13·190	5·582	56169	13312053	237	15·394	6·188
30625	5359375	175	13·228	5·593	56644	13481272	238	15·427	6·197
30976	5451776	176	13·266	5·604	57121	13651919	239	15·459	6·205
31329	5545233	177	13·304	5·614	57600	13824000	240	15·491	6·214
31684	5639752	178	13·341	5·625	58081	13997521	241	15·524	6·223
32041	5735339	179	13·379	5·635	58564	14172488	242	15·556	6·231
32400	5832000	180	13·416	5·646	59049	14348907	243	15·588	6·240
32761	5929741	181	13·453	5·656	59536	14526784	244	15·620	6·248
33124	6028568	182	13·490	5·667	60025	14706125	245	15·652	6·257
33489	6128487	183	13·527	5·677	60516	14886936	246	15·684	6·265
33856	6229504	184	13·664	5·687	61009	15069223	247	15·716	6·274
34225	6331625	185	13·601	5·698	61504	15252992	248	15·748	6·282
34596	6434856	186	13·638	5·708	62001	15438249	249	15·779	6·291
34969	6539203	187	13·674	5·718	62500	15625000	250	15·811	6·299
35344	6644672	188	13·711	5·728	63001	15813251	251	15·842	6·307
35721	6751269	189	13·747	5·738	63504	16003008	252	15·874	6·316

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
64009	16194277	253	15·905	6·324	99856	31554496	316	17·776	6·811
64516	16387064	254	15·937	6·333	100489	31855013	317	17·804	6·818
65025	16581375	255	15·968	6·341	101124	32157432	318	17·832	6·825
65536	16777216	256	16·000	6·349	101761	32461759	319	17·860	6·832
66049	16974593	257	16·031	6·357	102400	32768000	320	17·888	6·839
66564	17173512	258	16·062	6·366	103041	33076161	321	17·916	6·847
67081	17373979	259	16 093	6·374	103684	33386248	322	17·944	6·854
67600	17576000	260	16·124	6·382	104329	33698267	323	17·972	6·861
68121	17779581	261	16·155	6·390	104976	34012224	324	18·000	6·868
68644	17984728	262	16·186	6·398	105625	34328125	325	18·027	6·875
69169	18191447	263	16·217	6·406	106276	34645976	326	18·055	6·882
69696	18399744	264	16·248	6·415	106929	34965783	327	18·083	6·889
70225	18609625	265	16·278	6·423	107584	35287552	328	18·110	6·896
70756	18821096	266	16·309	6·431	108241	35611289	329	18·138	6·903
71289	19034163	267	16·340	6·439	108900	35937000	330	18·165	6·910
71824	19248832	268	16·370	6·447	109561	36264691	331	18·193	6·917
72361	19465109	269	16·401	6·455	110224	36594368	332	18·220	6·924
72900	19683000	270	16·431	6·463	110889	36926037	333	18·248	6·931
73441	19902511	271	16·462	6·471	111556	37259704	334	18·275	6·938
73984	20123648	272	16·492	6·479	112225	37595375	335	18·303	6·945
74529	20346417	273	16·522	6·487	112896	37933056	336	18·330	6·952
75076	20570824	274	16·552	6·495	113569	38272753	337	18·357	6·958
75625	20796875	275	16·583	6·502	114244	38614472	338	18·384	6·965
76176	21024576	276	16·613	6·510	114921	38958219	339	18·411	6·972
76729	21253933	277	16·643	6·518	115600	39304000	340	18·439	6·979
77284	21484952	278	16·678	6·526	116281	39651821	341	18·466	6·986
77841	21717639	279	16·703	6·534	116964	40001688	342	18·493	6·993
78400	21952000	280	16·733	6·542	117649	40353607	343	18·520	7·000
78961	22188041	281	16·763	6·549	118336	40707584	344	18·547	7·006
79524	22425768	282	16·792	6·557	119025	41063625	345	18·574	7·013
80089	22665187	283	16·822	6·565	119716	41421736	346	18·601	7·020
80656	22906304	284	16·852	6·573	120409	41781923	347	18·627	7·027
81225	23149125	285	16·881	6·580	121104	42144192	348	18·654	7·033
81796	23393656	286	16·911	6·588	121801	42508549	349	18·681	7·040
82369	23639903	287	16·941	6·596	122500	42875000	350	18·708	7·047
82944	23887872	288	16·970	6·603	123201	43243551	351	18·734	7·054
83521	24137569	289	17·000	6·611	123904	43614208	352	18·761	7·060
84100	24389000	290	17·029	6·619	124609	43986977	353	18·788	7·067
84681	24642171	291	17·058	6·626	125316	44361864	354	18·814	7·074
85264	24897088	292	17·088	6·634	126025	44738875	355	18·841	7·080
85849	25153757	293	17·117	6·641	126736	45118016	356	18·867	7·087
86436	25412184	294	17·146	6·649	127449	45499293	357	18·894	7·093
87025	25672375	295	17·175	6·656	128164	45882712	358	18·920	7·100
87616	25934836	296	17·204	6·664	128881	46268279	359	18·947	7·107
88209	26198073	297	17·233	6·671	129600	46656000	360	18·973	7·113
88804	26463592	298	17·262	6·679	130321	47045831	361	19·000	7·120
89401	26730899	299	17·291	6·686	131044	47437928	362	19·026	7·126
90000	27000000	300	17·320	6·694	131769	47832147	363	19·052	7·133
90601	27270901	301	17·349	6·701	132496	48228544	364	19·078	7·140
91204	27543608	302	17·378	6·709	133225	48627125	365	19·104	7·146
91809	27818127	303	17·406	6·716	133956	49027896	366	19·131	7·153
92416	28094464	304	17·435	6·723	134689	49430863	367	19·157	7·159
93025	28372625	305	17·464	6·731	135424	49836032	368	19·183	7·166
93636	28652616	306	17·492	6·738	136161	50243409	369	19·209	7·172
94249	28934443	307	17·521	6·745	136900	50653000	370	19·235	7·179
94864	29218112	308	17·549	6·753	137641	51064811	371	19·261	7·185
95481	29503609	309	17·578	6·760	138384	51478848	372	19·287	7·191
96100	29791000	310	17·606	6·767	139129	51895117	373	19·313	7·198
96721	30080231	311	17·635	6·775	139876	52313624	374	19·339	7·204
97344	30371328	312	17·663	6·782	140625	52734375	375	19·364	7·211
97969	30664297	313	17·691	6·789	141376	53157376	376	19·390	7·217
98596	30959144	314	17·720	6·796	142129	53582633	377	19·416	7·224
99225	31255875	315	17·748	6·804	142884	54010152	378	19·442	7·230

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
143641	54439939	379	19·467	7·236	195364	86350888	442	21·023	7·617
144400	54872000	380	19·493	7·243	196249	86938307	443	21·047	7·623
145161	55306341	381	19·519	7·249	197136	87528384	444	21·071	7·628
145924	55742968	382	19·544	7·255	198025	88121125	445	21·095	7·634
146689	56181887	383	19·570	7·262	198916	88716536	446	21·118	7·640
147456	56623104	384	19·595	7·268	199809	89314623	447	21·142	7·646
148225	57066625	385	19·621	7·274	200704	89915392	448	21·166	7·651
148996	57512456	386	19·646	7·281	201601	90518849	449	21·189	7·657
149769	57960603	387	19·672	7·287	202500	91125000	450	21·213	7·663
150544	58411072	388	19·697	7·293	203401	91733851	451	21·236	7·668
151321	58863869	389	19·723	7·299	204304	92345408	452	21·260	7·674
152100	59319000	390	19·748	7·306	205209	92959677	453	21·283	7·680
152881	59776471	391	19·773	7·312	206116	93576664	454	21·307	7·685
153664	60236288	392	19·798	7·318	207025	94196375	455	21·330	7·691
154449	60698457	393	19·824	7·324	207936	94818816	456	21·354	7·697
155236	61162984	394	19·849	7·331	208849	95443993	457	21·377	7·702
156025	61629875	395	19·874	7·337	209764	96071912	458	21·400	7·708
156816	62099136	396	19·899	7·343	210681	96702579	459	21·424	7·713
157609	62570773	397	19·924	7·349	211600	97336000	460	21·447	7·719
158404	63044792	398	19·949	7·355	212521	97972181	461	21·470	7·725
159201	63521199	399	19·974	7·361	213444	98611128	462	21·494	7·730
160000	64000000	400	20·000	7·368	214369	99252847	463	21·517	7·736
160801	64481201	401	20·024	7·374	215296	99897344	464	21·540	7·741
161604	64964808	402	20·049	7·380	216225	100544625	465	21·563	7·747
162409	65450827	403	20·074	7·386	217156	101194696	466	21·587	7·752
163216	65939264	404	20·099	7·392	218089	101847563	467	21·610	7·758
164025	66430125	405	20·124	7·398	219024	102503232	468	21·633	7·763
164836	66923416	406	20·149	7·404	219961	103161709	469	21·656	7·769
165649	67419143	407	20·174	7·410	220900	103823000	470	21·679	7·774
166464	67917312	408	20·199	7·416	221841	104487111	471	21·702	7·780
167281	68417929	409	20·223	7·422	222784	105154048	472	21·725	7·785
168100	68921000	410	20·248	7·428	223729	105823817	473	21·748	7·791
168921	69426531	411	20·273	7·434	224676	106496424	474	21·771	7·796
169744	69934528	412	20·297	7·441	225625	107171875	475	21·794	7·802
170569	70444997	413	20·322	7·447	226576	107850176	476	21·817	7·807
171396	70957944	414	20·346	7·453	227529	108531333	477	21·840	7·813
172225	71473375	415	20·371	7·459	228484	109215352	478	21·863	7·818
173056	71991296	416	20·396	7·465	229441	109902239	479	21·886	7·824
173889	72511713	417	20·420	7·470	230400	110592000	480	21·908	7·829
174724	73034632	418	20·445	7·476	231361	111284641	481	21·931	7·835
175561	73560059	419	20·469	7·482	232324	111980168	482	21·954	7·840
176400	74088000	420	20·493	7·488	233289	112678587	483	21·977	7·846
177241	74618461	421	20·518	7·494	234256	113379904	484	22·000	7·851
178084	75151448	422	20·542	7·500	235225	114084125	485	22·022	7·856
178929	75686967	423	20·566	7·506	236196	114791256	486	22·045	7·862
179776	76225024	424	20·591	7·512	237169	115501303	487	22·068	7·867
180625	76765625	425	20·615	7·518	238144	116214272	488	22·090	7·872
181476	77308776	426	20·639	7·524	239121	116930169	489	22·113	7·878
182329	77854483	427	20·663	7·530	240100	117649000	490	22·135	7·883
183184	78402752	428	20·688	7·536	241081	118370771	491	22·158	7·889
184041	78953589	429	20·712	7·541	242064	119095488	492	22·181	7·894
184900	79507000	430	20·736	7·547	243049	119823157	493	22·203	7·899
185761	80062991	431	20·760	7·553	244036	120553784	494	22·226	7·905
186624	80621568	432	20·784	7·559	245025	121287375	495	22·248	7·910
187489	81182737	433	20·808	7·565	246016	122023936	496	22·271	7·915
188356	81746504	434	20·832	7·571	247009	122763473	497	22·293	7·921
189225	82312875	435	20·856	7·576	248004	123505992	498	22·315	7·926
190096	82881856	436	20·880	7·582	249001	124251499	499	22·338	7·931
190969	83453453	437	20·904	7·588	250000	125000000	500	22·360	7·937
191844	84027672	438	20·928	7·594	251001	125751501	501	22·383	7·942
192721	84604519	439	20·952	7·600	252004	126506008	502	22·405	7·947
193600	85184000	440	20·976	7·605	253009	127263527	503	22·427	7·952
194481	85766121	441	21·000	7·611	254016	128024064	504	22·449	7·958

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
255025	128787625	505	22·472	7·963	322624	183250432	568	23·832	8·281
256036	129554216	506	22·494	7·968	323761	184220009	569	23·853	8·286
257049	130323843	507	22·516	7·973	324900	185193000	570	23·874	8·291
258064	131096512	508	22·538	7·979	326041	186169411	571	23·895	8·296
259081	131872229	509	22·561	7·984	327184	187149248	572	23·916	8·301
260100	132651000	510	22·583	7·989	328329	188132517	573	23·937	8·305
261121	133432831	511	22·605	7·994	329476	189119224	574	23·958	8·310
262144	134217728	512	22·627	8·000	330625	190109375	575	23·979	8·315
263169	135005697	513	22·649	8·005	331776	191102976	576	24·000	8·320
264196	135796744	514	22·671	8·010	332929	192100033	577	24·020	8·325
265225	136590875	515	22·693	8·015	334084	193100552	578	24·041	8·329
266256	137388096	516	22·715	8·020	335241	194104539	579	24·062	8·334
267289	138188413	517	22·737	8·025	336400	195112000	580	24·083	8·339
268324	138991832	518	22·759	8·031	337561	196122941	581	24·103	8·344
269361	139798359	519	22·781	8·036	338724	197137368	582	24·124	8·349
270400	140608000	520	22·803	8·041	339889	198155287	583	24·145	8·353
271441	141420761	521	22·825	8·046	341056	199176704	584	24·166	8·358
272484	142236648	522	22·847	8·051	342225	200201625	585	24·186	8·363
273529	143055667	523	22·869	8·056	343396	201230056	586	24·207	8·368
274576	143877824	524	22·891	8·062	344569	202262003	587	24·228	8·372
275625	144703125	525	22·912	8·067	345744	203297472	588	24·248	8·377
276676	145531576	526	22·934	8·072	346921	204336469	589	24·269	8·382
277729	146363183	527	22·956	8·077	348100	205379000	590	24·289	8·387
278784	147197952	528	22·978	8·082	349281	206425071	591	24·310	8·391
279841	148035889	529	23·000	8·087	350464	207474688	592	24·331	8·396
280900	148877000	530	23·021	8·092	351649	208527857	593	24·351	8·401
281961	149721291	531	23·043	8·097	352836	209584584	594	24·372	8·406
283024	150568768	532	23·065	8·102	354025	210644875	595	24·392	8·410
284089	151419437	533	23·086	8·107	355216	211708736	596	24·413	8·415
285156	152273304	534	23·108	8·112	356409	212776173	597	24·433	8·420
286225	153130375	535	23·130	8·118	357604	213847192	598	24·454	8·424
287296	153990656	536	23·151	8·123	358801	214921799	599	24·474	8·429
288369	154854153	537	23·173	8·128	360000	216000000	600	24·494	8·434
289444	155720872	538	23·194	8·133	361201	217081801	601	24·515	8·439
290521	156590819	539	23·216	8·138	362404	218167208	602	24·535	8·443
291600	157464000	540	23·237	8·143	363609	219256227	603	24·556	8·448
292681	158340421	541	23·259	8·148	364816	220348864	604	24·576	8·453
293764	159220088	542	23·280	8·153	366025	221445125	605	24·596	8·457
294849	160103007	543	23·302	8·158	367236	222545016	606	24·617	8·462
295936	160989184	544	23·323	8·163	368449	223648543	607	24·637	8·467
297025	161878625	545	23·345	8·168	369664	224755712	608	24·657	8·471
298116	162771336	546	23·366	8·173	370881	225866529	609	24·677	8·476
299209	163667323	547	23·388	8·178	372100	226981000	610	24·698	8·480
300304	164566592	548	23·409	8·183	373321	228099131	611	24·718	8·485
301401	165469149	549	23·430	8·188	374544	229220928	612	24·738	8·490
302500	166375000	550	23·452	8·193	375769	230346397	613	24·758	8·494
303601	167284151	551	23·473	8·198	376996	231475544	614	24·779	8·499
304704	168196608	552	23·494	8·203	378225	232608375	615	24·799	8·504
305809	169112377	553	23·515	8·208	379456	233744896	616	24·819	8·508
306916	170031464	554	23·537	8·213	380689	234885113	617	24·839	8·513
308025	170953875	555	23·558	8·217	381924	236029032	618	24·859	8·517
309136	171879616	556	23·579	8·222	383161	237176659	619	24·879	8·522
310249	172808693	557	23·600	8·227	384400	238328000	620	24·899	8·527
311364	173741112	558	23·622	8·232	385641	239483061	621	24·919	8·531
312481	174676879	559	23·643	8·237	386884	240641848	622	24·939	8·536
313600	175616000	560	23·664	8·242	388129	241804367	623	24·959	8·540
314721	176558481	561	23·685	8·247	389376	242970624	624	24·979	8·545
315844	177504328	562	23·706	8·252	390625	244140625	625	25·000	8·549
316969	178453547	563	23·727	8·257	391876	245314376	626	25·019	8·554
318096	179406144	564	23·748	8·262	393129	246491883	627	25·039	8·558
319225	180362125	565	23·769	8·267	394384	247673152	628	25·059	8·563
320356	181321496	566	23·790	8·271	395641	248858189	629	25·079	8·568
321489	182284263	567	23·811	8·276	396900	250047000	630	25·099	8·572

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
398161	251239591	631	25·119	8·577	481636	334255384	694	26·343	8·853
399424	252435968	632	25·139	8·581	483025	335702375	695	26·362	8·857
400689	253636137	633	25·159	8·586	484416	337153536	696	26·381	8·862
401956	254840104	634	25·179	8·590	485809	338608873	697	26·400	8·866
403225	256047875	635	25·199	8·595	487204	340068392	698	26·419	8·870
404496	257259456	636	25·219	8·599	488601	341532099	699	26·438	8·874
405769	258474853	637	25·238	8·604	490000	343000000	700	26·457	8·879
407044	259694072	638	25·258	8·608	491401	344472101	701	26·476	8·883
408321	260917119	639	25·278	8·613	492804	345948408	702	26·495	6·887
409600	262144000	640	25·298	8·617	494209	347428927	703	26·514	8·891
410881	263374721	641	25·317	8·622	495616	348913664	704	26·532	8·895
412164	264609288	642	25·337	8·626	497025	350402625	705	26·551	8·900
413449	265847707	643	25·357	8·631	498436	351895816	706	26·570	8·904
414736	267089984	644	25·377	3·635	499849	353393243	707	26·589	8·908
416025	268336125	645	25·396	8·640	501264	354894912	708	26·608	8·912
417316	269586136	646	25·416	8·644	502681	356400829	709	26·627	8·916
418609	270840023	647	25·436	8·649	504100	357911000	710	26·645	8·921
419904	272097792	648	25·455	8·653	505521	359425431	711	26·664	8·925
421201	273359449	649	25·475	8·657	506944	360944128	712	26·683	8·929
422500	274625000	650	25·495	8·662	508369	362467097	713	26·702	8·933
423801	275894451	651	25·514	8·666	509796	363994344	714	26·720	8·937
425104	277167808	652	25·534	8·671	511225	365525875	715	26·739	8·942
426409	278445077	653	25·553	8·675	512656	367061696	716	26·758	8·946
427716	279726264	654	25·573	8·680	514089	368601813	717	26·776	8·950
429025	281011375	655	25·592	8·684	515524	370146232	718	26·795	8·954
430336	282300416	656	25·612	8·688	516961	371694959	719	26·814	8·958
431649	283593393	657	25·632	8·693	518400	373248000	720	26·832	8·962
432964	284890312	658	25·651	8·697	519841	374805361	721	26·851	8·966
434281	286191179	659	25·670	8·702	521284	376367048	722	26·870	8·971
435600	287496000	660	25·690	8·706	522729	377933067	723	26·888	8·975
436921	288804781	661	25·709	8·710	524176	379503424	724	26·907	8·979
438244	290117528	662	25·729	8·715	525625	381078125	725	26·925	8·983
439569	291434247	663	25·748	8·719	527076	382657176	726	26·944	8·987
440896	292754944	664	25·768	8·724	528529	384240583	727	26·962	8·991
442225	294079625	665	25·787	8·728	529984	385828352	728	26·981	8·995
443556	295408296	666	25·806	8·732	531441	387420489	729	27·000	9·000
444889	296740963	667	25·826	8·737	532900	389017000	730	27·018	9·004
446224	298077632	668	25·845	8·741	534361	390617891	731	27·037	9·008
447561	299418309	669	25·865	8·745	535824	392223168	732	27·055	9·012
448900	300763000	670	25·884	8·750	537289	393832837	733	27·073	9·016
450241	302111711	671	25·903	8·754	538756	395446904	734	27·092	9·020
451584	303464448	672	25·922	8·759	540225	397065375	735	27·110	9·024
452929	304821217	673	25·942	8·763	541696	398688256	736	27·129	9·028
454276	306182024	674	25·961	8·767	543169	400315553	737	27·147	9·032
455625	307546875	675	25·980	8·772	544644	401947272	738	27·166	9·036
456976	308915776	676	26·000	8·776	546121	403583419	739	27·184	9·040
458329	310288733	677	26·019	8·780	547600	405224000	740	27·202	9·045
459684	311665752	678	26·038	8·785	549081	406869021	741	27·221	9·049
461041	313046839	679	26·057	8·789	550564	408518488	742	27·239	9·053
462400	314432000	680	26·076	8·793	552049	410172407	743	27·258	9·057
463761	315821241	681	26·095	8·797	553536	411830784	744	27·276	9·061
465124	317214568	682	26·115	8·802	555025	413493625	745	27·294	9·065
466489	318611987	683	26·134	8·806	556516	415160936	746	27·313	9·069
467856	320013504	684	26·153	8·810	558009	416832723	747	27·331	9·073
469225	321419125	685	26·172	8·815	559504	418508992	748	27·349	9·077
470596	322828856	686	26·191	8·819	561001	420189749	749	27·367	9·081
471969	324242703	687	26·210	8·823	562500	421875000	750	27·386	9·085
473344	325660672	688	26·229	8·828	564001	423564751	751	27·404	9·089
474721	327082769	689	26·248	8·832	565504	425259008	752	27·422	9·093
476100	328509000	690	26·267	8·836	567009	426957777	753	27·440	9·097
477481	329939371	691	26·286	8·840	568516	428661064	754	27·459	9·101
478864	331373888	692	26·305	8·845	570025	430368875	755	27·477	9·105
480249	332812557	693	26·324	8·849	571536	432081216	756	27·495	9·109

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
573049	433798093	757	27·513	9·113	672400	551368000	820	28·635	9·359
574564	435519512	758	27·531	9·117	674041	553387661	821	28·653	9·363
576081	437245479	759	27·549	9·121	675684	555412248	822	28·670	9·367
577600	438976000	760	27·568	9·125	677329	557441767	823	28·687	9·371
579121	440711081	761	27·586	9·129	678976	559476224	824	28·705	9·375
580644	442450728	762	27·604	9·133	680625	561515625	825	28·722	9·378
582169	444194947	763	27·622	9·137	682276	563559976	826	28·740	9·382
583696	445943744	764	27·640	9·141	683929	565609283	827	28·757	9·386
585225	447697125	765	27·658	9·145	685584	567663552	828	28·774	9·390
586756	449455096	766	27·676	9·149	687241	569722789	829	28·792	9·394
588289	451217663	767	27·694	9·153	688900	571787000	830	28·809	9·397
589824	452984832	768	27·712	9·157	690561	573856191	831	28·827	9·401
591361	454756609	769	27·730	9·161	692224	575930368	832	28·844	9·405
592900	456533000	770	27·748	9·165	693889	578009537	833	28·861	9·409
594441	458314011	771	27·766	9·169	695556	580093704	834	28·879	9·412
595984	460099648	772	27·784	9·173	697225	582182875	835	28·896	9·416
597529	461889917	773	27·802	9·177	698896	584277056	836	28·913	9·420
599076	463684824	774	27·820	9·181	700569	586376253	837	28·930	9·424
600625	465484375	775	27·838	9·185	702244	588480472	838	28·948	9·427
602176	467288576	776	27·856	9·189	703921	590589719	839	28·965	9·431
603729	469097433	777	27·874	9·193	705600	592704000	840	28·982	9·435
605284	470910952	778	27·892	9·197	707281	594823321	841	29·000	9·439
606841	472729139	779	27·910	9·201	708964	596947688	842	29·017	9·442
608400	474552000	780	27·928	9·205	710649	599077107	843	29·034	9·446
609961	476379541	781	27·946	9·209	712336	601211584	844	29·051	9·450
611524	478211768	782	27·964	9·213	714025	603351125	845	29·068	9·454
613089	480048687	783	27·982	9·216	715716	605495736	846	29·086	9·457
614656	481890304	784	28·000	9·220	717409	607645423	847	29·103	9·461
616225	483736625	785	28·017	9·224	719104	609800192	848	29·120	9·465
617796	485587656	786	28·035	9·228	720801	611960049	849	29·137	9·468
619369	487443403	787	28·053	9·232	722500	614125000	850	29·154	9·472
620944	489303872	788	28·071	9·236	724201	616295051	851	29·171	9·476
622521	491169069	789	28·089	9·240	725904	618470208	852	29·189	9·480
624100	493039000	790	28·106	9·244	727609	620650477	853	29·206	9·483
625681	494913671	791	28·124	9·248	729316	622835864	854	29·223	9·487
627264	496793088	792	28·142	9·252	731025	625026375	855	29·240	9·491
628849	498677257	793	28·160	9·256	732736	627222016	856	29·257	9 494
630436	500566184	794	28·178	9·259	734449	629422793	857	29·274	9·498
632025	502459875	795	28·195	9·263	736164	631628712	858	29·291	9·502
633616	504358336	796	28·213	9·267	737881	633839779	859	29·308	9·505
635209	506261573	797	28 231	9·271	739600	636056000	860	29·325	9·509
636804	508169592	798	28·248	9·275	741321	638277381	861	29·342	9·513
638401	510082399	799	28·266	9·279	743044	640503928	862	29·359	9·517
640000	512000000	800	28·284	9·283	744769	642735647	863	29·376	9·520
641601	513922401	801	28·301	9·287	746496	644972544	864	29·393	9·524
643204	515849608	802	28·319	9·290	748225	647214625	865	29·410	9·528
644809	517781627	803	28·337	9·294	749956	649461896	866	29·427	9·531
646416	519718464	804	28·354	9·298	751689	651714363	867	29·444	9·535
648025	521660125	805	28·372	9·302	753424	653972032	868	29·461	9·539
649636	523606616	806	28·390	9·306	755161	656234909	869	29·478	9·542
651249	525557943	807	28·407	9·310	756900	658503000	870	29·495	9·546
652864	527514112	808	28·425	9·314	758641	660776311	871	29·512	9·550
654481	529475129	809	28·442	9·317	760384	663054848	872	29·529	9·553
656100	531441000	810	28·460	9·321	762129	665338617	873	29·546	9·557
657721	533411731	811	28·478	9·325	763876	667627624	874	29·563	9·561
659344	535387328	812	28·495	9·329	765625	669921875	875	29·580	9·564
660969	537367797	813	28·513	9·333	767376	672221376	876	29·597	9·568
662596	539353144	814	28·530	9·337	769129	674526133	877	29·614	9·571
664225	541343375	815	28·548	9·340	770884	676836152	878	29·631	9·575
665856	543338496	816	28·565	9·344	772641	679151439	879	29·647	9·579
667489	545338513	817	28·583	9·348	774400	681472000	880	29·664	9·582
669124	547343432	818	28·600	9·352	776161	683797841	881	29·681	9·586
670761	549353259	819	28·618	9·356	777924	686128968	882	29·698	9·590

TABLE OF SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS—(*Continued*).

Squares.	Cubes.	No.	Square roots.	Cube roots.	Squares.	Cubes.	No.	Square roots.	Cube roots.
779689	688465387	883	29·715	9·593	894916	846590536	946	30·757	9·816
781456	690807104	884	29·732	9·597	896808	849278123	947	30·773	9·820
783225	693154125	885	29·748	9·600	898704	851971392	948	30·789	9·823
784996	695506456	886	29·765	9·604	900601	854670349	949	30·805	9·827
786769	697864103	887	29·782	9·608	902500	857375000	950	30·822	9·830
788544	700227072	888	29·799	9·611	904401	860085351	951	30·838	9·833
790321	702595369	889	29·816	9·615	906304	862801408	952	30·854	9·837
792100	704969000	890	29·832	9·619	908209	865523177	953	30·870	9·840
793881	707347971	891	29·849	9·622	910116	868250664	954	30·886	9·844
795664	709732288	892	29·866	9·626	912025	870983875	955	30·903	9·847
797449	712121957	893	29·883	9·629	913936	873722816	956	30·919	9·851
799236	714516984	894	29·899	9·633	915849	876467493	957	30·935	9·854
801025	716917375	895	29·916	9·636	917764	879217912	958	30·951	9·857
802816	719323136	896	29·933	9·640	919681	881974079	959	30·967	9·861
804609	721734273	897	29·949	9·644	921600	884736000	960	30·983	9·864
806404	724150792	898	29·966	9·647	923521	887503681	961	31·000	9·868
808201	726572699	899	29·983	9·651	925444	890277128	962	31·016	9·871
810000	729000000	900	30·000	9·654	927369	893056347	963	31·032	9·875
811801	731432701	901	30·016	9·658	929296	895841344	964	31·048	9·878
813604	733870808	902	30·033	9·662	931225	898632125	965	31·064	9·881
815409	736314327	903	30·049	9·665	933156	901428696	966	31·080	9·885
817216	738763264	904	30·066	9·669	935089	904231063	967	31·096	9·888
819025	741217625	905	30·083	9·672	937024	907039232	968	31·112	9·892
820836	743677416	906	30·099	9·676	938961	909853209	969	31·128	9·895
822649	746142643	907	30·116	9·679	940900	912673000	970	31·144	9·898
824464	748613312	908	30·133	9·683	942841	915498611	971	31·160	9·902
826281	751089429	909	30·149	9·686	944784	918330048	972	31·176	9·905
828100	753571000	910	30·166	9·690	946729	921167317	973	31·192	9·909
829921	756058031	911	30·182	9·694	948676	924010424	974	31·208	9·912
831744	758550528	912	30·199	9·697	950625	926859375	975	31·224	9·915
833569	761048497	913	30·215	9·701	952576	929714176	976	31·240	9·919
835396	763551944	914	30·232	9·704	954529	932574833	977	31·256	9·922
837225	766060875	915	30·248	9·708	956484	935441352	978	31·272	9·926
839056	768575296	916	30·265	9·711	958441	938313739	979	31·288	9·929
840889	771095213	917	30·282	9·715	960400	941192000	980	31·304	9·932
842724	773620632	918	30·298	9·718	962361	944076141	981	31·320	9·936
844561	776151559	919	30·315	9·722	964324	946966168	982	31·336	9·939
846400	778688000	920	30·331	9·725	966289	949862087	983	31·352	9·943
848241	781229961	921	30·347	9·729	968256	952763904	984	31·368	9·946
850084	783777448	922	30·364	9·732	970225	955671625	985	31·384	9·949
851929	786330467	923	30·380	9·736	972196	958585256	986	31·400	9·953
853776	788889024	924	30·397	9·739	974169	961504803	987	31·416	9·956
855625	791453125	925	30·413	9·743	976144	964430272	988	31·432	9·959
857476	794022776	926	30·430	9·746	978121	967361669	989	31·448	9·963
859329	796597983	927	30·446	9·750	980100	970299000	990	31·464	9·966
861184	799178752	928	30·463	9·753	982081	973242271	991	31·480	9·969
863041	801765089	929	30·479	9·757	984064	976191488	992	31·496	9·973
864900	804357000	930	30·495	9·761	986049	979146657	993	31·511	9·976
866761	806954491	931	30·512	9·764	988036	982107784	994	31·527	9·979
868624	809557568	932	30·528	9·767	990025	985074875	995	31·543	9·983
870489	812166237	933	30·545	9·771	992016	988047936	996	31·559	9·986
872356	814780504	934	30·561	9·774	994009	991026973	997	31·575	9·989
874225	817400375	935	30·577	9·778	996004	994011992	998	31·591	9·993
876096	820025856	936	30·594	9·782	998001	997002999	999	31·606	9·996
877969	822656953	937	30·610	9·785	1000000	1000000000	1000	31·622	10·000
879844	825293672	938	30·626	9·788	1000201	1003003001	1001	31·638	10·003
881721	827936019	939	30·643	9·792	1004004	1006012008	1002	31·654	10·006
883600	830584000	940	30·659	9·795	1006009	1009027027	1003	31·670	10·009
885481	833237621	941	30·675	9·799	1008016	1012048064	1004	31·685	10·013
887364	835896888	942	30·692	9·802	1010025	1015075125	1005	31·701	10·016
889249	838561807	943	30·708	9·806	1012036	1018108216	1006	31·717	10·019
891136	841232384	944	30·724	9·809	1014049	1021147343	1007	31·733	10·023
893025	843908625	945	30·740	9·813	1016064	1024192512	1008	31·749	10·026

LATITUDES AND DEPARTURES.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
0°	1·000	0·000	2·000	0·000	3·000	0·000	4·000	0·000	5·000	90°
0¼	1·000	0·004	2·000	0·009	3·000	0·013	4·000	0·017	5·000	89¾
0½	1·000	0·009	2·000	0·017	3·000	0·026	4·000	0·035	5·000	89½
0¾	1·000	0·013	2·000	0·026	3·000	0·039	4·000	0·052	5·000	89¼
1°	1·000	0·017	2·000	0·035	3·000	0·052	3·999	0·070	4·999	89°
1¼	1·000	0·022	2·000	0·044	2·999	0·065	3·999	0·087	4·999	88¾
1½	1·000	0·026	1·999	0·052	2·999	0·079	3·999	0·105	4·998	88½
1¾	1·000	0·031	1·999	0·061	2·999	0·092	3·998	0·122	4·998	88¼
2°	0·999	0·035	1·999	0·070	2·998	0·105	3·998	0·140	4·997	88°
2¼	0·999	0·039	1·998	0·079	2·998	0·118	3·997	0·157	4·996	87¾
2½	0·999	0·044	1·998	0·087	2·997	0·131	3·996	0·174	4·995	87½
2¾	0·999	0·048	1·998	0·096	2·997	0·144	3·995	0·192	4·994	87¼
3°	0·999	0·052	1·997	0·105	2·996	0·157	3·995	0·209	4·993	87°
3¼	0·998	0·057	1·997	0·113	2·995	0·170	3·994	0·227	4·992	86¾
3½	0·998	0·061	1·996	0·122	2·994	0·183	3·993	0·244	4·991	86½
3¾	0·998	0·065	1·996	0·131	2·994	0·196	3·991	0·262	4·989	86¼
4°	0·998	0·070	1·995	0·140	2·993	0·209	3·990	0·279	4·988	86°
4¼	0·997	0·074	1·995	0·148	2·992	0·222	3·989	0·296	4·986	85¾
4½	0·997	0·078	1·994	0·157	2·991	0·235	3·988	0·314	4·985	85½
4¾	0·997	0·083	1·993	0·166	2·990	0·248	3·986	0·331	4·983	85¼
5°	0·996	0·087	1·992	0·174	2·989	0·261	3·985	0·349	4·981	85°
5¼	0·996	0·092	1·992	0·183	2·987	0·275	3·983	0·366	4·979	84¾
5½	0·995	0·096	1·991	0·192	2·986	0·288	3·982	0·383	4·977	84½
5¾	0·995	0·100	1·990	0·200	2·985	0·301	3·980	0·401	4·975	84¼
6°	0·995	0·105	1·989	0·209	2·984	0·314	3·978	0·418	4·973	84°
6¼	0·994	0·109	1·988	0·218	2·982	0·327	3·976	0·435	4·970	83¾
6½	0·994	0·113	1·987	0·226	2·981	0·340	3·974	0·453	4·968	83½
6¾	0·993	0·118	1·986	0·235	2·979	0·353	3·972	0·470	4·965	83¼
7°	0·993	0·122	1·985	0·244	2·978	0·366	3·970	0·487	4·963	83°
7¼	0·992	0·126	1·984	0·252	2·976	0·379	3·968	0·505	4·960	82¾
7½	0·991	0·131	1·983	0·261	2·974	0·392	3·966	0·522	4·957	82½
7¾	0·991	0·135	1·982	0·270	2·973	0·405	3·963	0·539	4·954	82¼
8°	0·990	0·139	1·981	0·278	2·971	0·418	3·961	0·557	4·951	82°
8¼	0·990	0·143	1·979	0·287	2·969	0·430	3·959	0·574	4·948	81¾
8½	0·989	0·148	1·978	0·296	2·967	0·443	3·956	0·591	4·945	81½
8¾	0·988	0·152	1·977	0·304	2·965	0·456	3·953	0·608	4·942	81¼
9°	0·988	0·156	1·975	0·313	2·963	0·469	3·951	0·626	4·938	81°
9¼	0·987	0·161	1·974	0·321	2·961	0·482	3·948	0·643	4·935	80¾
9½	0·986	0·165	1·973	0·330	2·959	0·495	3·945	0·660	4·931	80½
9¾	0·986	0·169	1·971	0·339	2·957	0·508	3·942	0·677	4·928	80¼
10°	0·985	0·174	1·970	0·347	2·954	0·521	3·939	0·695	4·924	80°
10¼	0·984	0·178	1·968	0·356	2·952	0·534	3·936	0·712	4·920	79¾
10½	0·983	0·182	1·967	0·364	2·950	0·547	3·933	0·729	4·916	79½
10¾	0·982	0·187	1·965	0·373	2·947	0·560	3·930	0·746	4·912	79¼
11°	0·982	0·191	1·963	0·382	2·945	0·572	3·927	0·763	4·908	79°
11¼	0·981	0·195	1·962	0·390	2·942	0·585	3·923	0·780	4·904	78¾
11½	0·980	0·199	1·960	0·399	2·940	0·598	3·920	0·797	4·900	78½
11¾	0·979	0·204	1·958	0·407	2·937	0·611	3·916	0·815	4·895	78¼
12°	0·978	0·208	1·956	0·416	2·934	0·624	3·913	0·832	4·891	78°
12¼	0·977	0·212	1·954	0·424	2·932	0·637	3·909	0·849	4·886	77¾
12½	0·976	0·216	1·953	0·433	2·929	0·649	3·905	0·866	4·881	77½
12¾	0·975	0·221	1·951	0·441	2·926	0·662	3·901	0·883	4·877	77¼
13°	0·974	0·225	1·949	0·450	2·923	0·675	3·897	0·900	4·872	77°
13¼	0·973	0·229	1·947	0·458	2·920	0·688	3·894	0·917	4·867	76¾
13½	0·972	0·233	1·945	0·467	2·917	0·700	3·889	0·934	4·862	76½
13¾	0·971	0·238	1·943	0·475	2·914	0·713	3·885	0·951	4·857	76¼
14°	0·970	0·242	1·941	0·484	2·911	0·726	3·881	0·968	4·851	76°
14¼	0·969	0·246	1·938	0·492	2·908	0·738	3·877	0·985	4·846	75¾
14½	0·968	0·250	1·936	0·501	2·904	0·751	3·873	1·002	4·841	75½
14¾	0·967	0·255	1·934	0·509	2·901	0·764	3·868	1·018	4·835	75¼
15°	0·966	0·259	1·932	0·518	2·898	0·776	3·864	1·035	4·830	75°
Bearing.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Bearing.
	1		2		3		4		5	

LATITUDES AND DEPARTURES.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
0°	0·000	6·000	0·000	7·000	0·000	8·000	0·000	9·000	0·000	90°
0¼	0·022	6·000	0·026	7·000	0·031	8·000	0·035	9·000	0·039	89¾
0½	0·044	6·000	0·052	7·000	0·061	8·000	0·070	9·000	0·079	89½
0¾	0·065	5·999	0·079	6·999	0·092	7·999	0·105	8·999	0·118	89¼
1°	0·087	5·999	0·105	6·999	0·122	7·999	0·140	8·999	0·157	89°
1¼	0·109	5·999	0·131	6·998	0·153	7·998	0·175	8·998	0·196	88¾
1½	0·131	5·998	0·157	6·998	0·183	7·997	0·209	8·997	0·236	88½
1¾	0·153	5·997	0·183	6·997	0·214	7·996	0·244	8·996	0·275	88¼
2°	0·174	5·996	0·209	6·996	0·244	7·995	0·279	8·995	0·314	88°
2¼	0·196	5·995	0·236	6·995	0·275	7·994	0·314	8·993	0·353	87¾
2½	0·218	5·994	0·262	6·993	0·305	7·992	0·349	8·991	0·393	87½
2¾	0·240	5·993	0·288	6·992	0·336	7·991	0·384	8·990	0·432	87¼
3°	0·262	5·992	0·314	6·990	0·366	7·989	0·419	8·988	0·471	87°
3¼	0·283	5·990	0·340	6·989	0·397	7·987	0·454	8·986	0·510	86¾
3½	0·305	5·989	0·366	6·987	0·427	7·985	0·488	8·983	0·549	86½
3¾	0·327	5·987	0·392	6·985	0·458	7·983	0·523	8·981	0·589	86¼
4°	0·349	5·985	0·419	6·983	0·488	7·981	0·558	8·978	0·628	86°
4¼	0·371	5·984	0·445	6·981	0·519	7·978	0·593	8·975	0·667	85¾
4½	0·392	5·982	0·471	6·978	0·549	7·975	0·628	8·972	0·706	85½
4¾	0·414	5·979	0·497	6·976	0·580	7·973	0·662	8·969	0·745	85¼
5°	0·436	5·977	0·523	6·973	0·610	7·970	0·697	8·966	0·784	85°
5¼	0·458	5·975	0·549	6·971	0·641	7·966	0·732	8·962	0·824	84¾
5½	0·479	5·972	0·575	6·968	0·671	7·963	0·767	8·959	0·863	84½
5¾	0·501	5·970	0·601	6·965	0·701	7·960	0·802	8·955	0·902	84¼
6°	0·523	5·967	0·627	6·962	0·732	7·956	0·836	8·951	0·941	84°
6¼	0·544	5·964	0·653	6·958	0·762	7·952	0·871	8·947	0·980	83¾
6½	0·566	5·961	0·679	6·955	0·792	7·949	0·906	8·942	1·019	83½
6¾	0·588	5·958	0·705	6·951	0·823	7·945	0·940	8·938	1·058	83¼
7°	0·609	5·955	0·731	6·948	0·853	7·940	0·975	8·933	1·097	83°
7¼	0·631	5·952	0·757	6·944	0·883	7·936	1·010	8·928	1·136	82¾
7½	0·653	5·949	0·783	6·940	0·914	7·932	1·044	8·923	1·175	82½
7¾	0·674	5·945	0·809	6·936	0·944	7·927	1·079	8·918	1·214	82¼
8°	0·696	5·942	0·835	6·932	0·974	7·922	1·113	8·912	1·253	82°
8¼	0·717	5·938	0·861	6·928	1·004	7·917	1·148	8·907	1·291	81¾
8½	0·739	5·934	0·887	6·923	1·035	7·912	1·182	8·901	1·330	81½
8¾	0·761	5·930	0·913	6·919	1·065	7·907	1·217	8·895	1·369	81¼
9°	0·782	5·926	0·939	6·914	1·095	7·902	1·251	8·889	1·408	81°
9¼	0·804	5·922	0·964	6·909	1·125	7·896	1·286	8·883	1·447	80¾
9½	0·825	5·918	0·990	6·904	1·155	7·890	1·320	8·877	1·485	80½
9¾	0·847	5·913	1·016	6·899	1·185	7·884	1·355	8·870	1·524	80¼
10°	0·868	5·909	1·042	6·894	1·216	7·878	1·389	8·863	1·563	80°
10¼	0·890	5·904	1·068	6·888	1·246	7·872	1·424	8·856	1·601	79¾
10½	0·911	5·900	1·093	6·883	1·276	7·866	1·458	8·849	1·640	79½
10¾	0·933	5·895	1·119	6·877	1·306	7·860	1·492	8·842	1·679	79¼
11°	0·954	5·890	1·145	6·871	1·336	7·853	1·526	8·835	1·717	79°
11¼	0·975	5·885	1·171	6·866	1·366	7·846	1·561	8·827	1·756	78¾
11½	0·997	5·880	1·196	6·859	1·396	7·839	1·595	8·819	1·794	78½
11¾	1·018	5·874	1·222	6·853	1·425	7·832	1·629	8·811	1·833	78¼
12°	1·040	5·869	1·247	6·847	1·455	7·825	1·663	8·803	1·871	78°
12¼	1·061	5·863	1·273	6·841	1·485	7·818	1·697	8·795	1·910	77¾
12½	1·082	5·858	1·299	6·834	1·515	7·810	1·732	8·787	1·948	77½
12¾	1·103	5·852	1·324	6·827	1·545	7·803	1·766	8·778	1·986	77¼
13°	1·125	5·846	1·350	6·821	1·575	7·795	1·800	8·769	2·025	77°
13¼	1·146	5·840	1·375	6·814	1·604	7·787	1·834	8·760	2·063	76¾
13½	1·167	5·834	1·401	6·807	1·634	7·779	1·868	8·751	2·101	76½
13¾	1·188	5·828	1·426	6·799	1·664	7·771	1·902	8·742	2·139	76¼
14°	1·210	5·822	1·452	6·792	1·693	7·762	1·935	8·733	2·177	76°
14¼	1·231	5·815	1·477	6·785	1·723	7·754	1·969	8·723	2·215	75¾
14½	1·252	5·809	1·502	6·777	1·753	7·745	2·003	8·713	2·253	75½
14¾	1·273	5·802	1·528	6·769	1·782	7·736	2·037	8·703	2·291	75¼
15°	1·294	5·796	1·553	6·761	1·812	7·727	2·071	8·693	2·329	75°
Bearing.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Bearing.
	5	6		7		8		9		

LATITUDES AND DEPARTURES.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
15°	0·966	0·259	1·932	0·518	2·898	0·776	3·864	1·035	4·830	75°
15¼	0·965	0·263	1·930	0·526	2·894	0·789	3·859	1·052	4·824	74¾
15½	0·964	0·267	1·927	0·534	2·891	0·802	3·855	1·069	4·818	74½
15¾	0·962	0·271	1·925	0·543	2·887	0·814	3·850	1·086	4·812	74¼
16°	0·961	0·276	1·923	0·551	2·884	0·827	3·845	1·103	4·806	74°
16¼	0·960	0·280	1·920	0·560	2·880	0·839	3·840	1·119	4·800	73¾
16½	0·959	0·284	1·918	0·568	2·876	0·852	3·835	1·136	4·794	73½
16¾	0·958	0·288	1·915	0·576	2·873	0·865	3·830	1·153	4·788	73¼
17°	0·956	0·292	1·913	0·585	2·869	0·877	3·825	1·169	4·782	73°
17¼	0·955	0·297	1·910	0·593	2·865	0·890	3·820	1·186	4·775	72¾
17½	0·954	0·301	1·907	0·601	2·861	0·902	3·815	1·203	4·769	72½
17¾	0·952	0·305	1·905	0·610	2·857	0·915	3·810	1·220	4·762	72¼
18°	0·951	0·309	1·902	0·618	2·853	0·927	3·804	1·236	4·755	72°
18¼	0·950	0·313	1·899	0·626	2·849	0·939	3·799	1·253	4·748	71¾
18½	0·948	0·317	1·897	0·635	2·845	0·952	3·793	1·269	4·742	71½
18¾	0·947	0·321	1·894	0·643	2·841	0·964	3·788	1·286	4·735	71¼
19°	0·946	0·326	1·891	0·651	2·837	0·977	3·782	1·302	4·728	71°
19¼	0·944	0·330	1·888	0·659	2·832	0·989	3·776	1·319	4·720	70¾
19½	0·943	0·334	1·885	0·668	2·828	1·001	3·771	1·335	4·713	70½
19¾	0·941	0·338	1·882	0·676	2·824	1·014	3·765	1·352	4·706	70¼
20°	0·940	0·342	1·879	0·684	2·819	1·026	3·759	1·368	4·698	70°
20¼	0·938	0·346	1·876	0·692	2·815	1·038	3·753	1·384	4·691	69¾
20½	0·937	0·350	1·873	0·700	2·810	1·051	3·747	1·401	4·683	69½
20¾	0·935	0·354	1·870	0·709	2·805	1·063	3·741	1·417	4·676	69¼
21°	0·934	0·358	1·867	0·717	2·801	1·075	3·734	1·433	4·668	69°
21¼	0·932	0·362	1·864	0·725	2·796	1·087	3·728	1·450	4·660	68¾
21½	0·930	0·367	1·861	0·733	2·791	1·100	3·722	1·466	4·652	68½
21¾	0·929	0·371	1·858	0·741	2·786	1·112	3·715	1·482	4·644	68¼
22°	0·927	0·375	1·854	0·749	2·782	1·124	3·709	1·498	4·636	68°
22¼	0·926	0·379	1·851	0·757	2·777	1·136	3·702	1·515	4·628	67¾
22½	0·924	0·383	1·848	0·765	2·772	1·148	3·696	1·531	4·619	67½
22¾	0·922	0·387	1·844	0·773	2·767	1·160	3·689	1·547	4·611	67¼
23°	0·921	0·391	1·841	0·781	2·762	1·172	3·682	1·563	4·603	67°
23¼	0·919	0·395	1·838	0·789	2·756	1·184	3·675	1·579	4·594	66¾
23½	0·917	0·399	1·834	0·797	2·751	1·196	3·668	1·595	4·585	66½
23¾	0·915	0·403	1·831	0·805	2·746	1·208	3·661	1·611	4·577	66¼
24°	0·914	0·407	1·827	0·813	2·741	1·220	3·654	1·627	4·568	66°
24¼	0·912	0·411	1·824	0·821	2·735	1·232	3·647	1·643	4·559	65¾
24½	0·910	0·415	1·820	0·829	2·730	1·244	3·640	1·659	4·550	65½
24¾	0·908	0·419	1·816	0·837	2·724	1·256	3·633	1·675	4·541	65¼
25°	0·906	0·423	1·813	0·845	2·719	1·268	3·625	1·690	4·532	65°
25¼	0·904	0·427	1·809	0·853	2·713	1·280	3·618	1·706	4·522	64¾
25½	0·903	0·431	1·805	0·861	2·708	1·292	3·610	1·722	4·513	64½
25¾	0·901	0·434	1·801	0·869	2·702	1·303	3·603	1·738	4·503	64¼
26°	0·899	0·438	1·798	0·877	2·696	1·315	3·595	1·753	4·494	64°
26¼	0·897	0·442	1·794	0·885	2·691	1·327	3·587	1·769	4·484	63¾
26½	0·895	0·446	1·790	0·892	2·685	1·339	3·580	1·785	4·475	63½
26¾	0·893	0·450	1·786	0·900	2·679	1·350	3·572	1·800	4·465	63¼
27°	0·891	0·454	1·782	0·908	2·673	1·362	3·564	1·816	4·455	63°
27¼	0·889	0·458	1·778	0·916	2·667	1·374	3·556	1·831	4·445	62¾
27½	0·887	0·462	1·774	0·923	2·661	1·385	3·548	1·847	4·435	62½
27¾	0·885	0·466	1·770	0·931	2·655	1·397	3·540	1·862	4·425	62¼
28°	0·883	0·469	1·766	0·939	2·649	1·408	3·532	1·878	4·415	62°
28¼	0·881	0·473	1·762	0·947	2·643	1·420	3·524	1·893	4·404	61¾
28½	0·879	0·477	1·758	0·954	2·636	1·431	3·515	1·909	4·394	61½
28¾	0·877	0·481	1·753	0·962	2·630	1·443	3·507	1·924	4·384	61¼
29°	0·875	0·485	1·749	0·970	2·624	1·454	3·498	1·939	4·373	61°
29¼	0·872	0·489	1·745	0·977	2·617	1·466	3·490	1·954	4·362	60¾
29½	0·870	0·492	1·741	0·985	2·611	1·477	3·481	1·970	4·352	60½
29¾	0·868	0·496	1·736	0·992	2·605	1·489	3·473	1·985	4·341	60¼
30°	0·866	0·500	1·732	1·000	2·598	1·500	3·464	2·000	4·330	60°
Bearing.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Bearing.
	1		2		3		4		5	

LATITUDES AND DEPARTURES.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
15°	1 294	5·796	1·553	6·761	1·812	7·727	2·071	8·693	2·329	75°
15¼	1·315	5·789	1·578	6·754	1·841	7·718	2·104	8·683	2 367	74¾
15½	1·336	5·782	1·603	6·745	1·871	7·709	2·138	8·673	2·405	74½
15¾	1·357	5·775	1·629	6·737	1·900	7·700	2·172	8·662	2·443	74¼
16°	1·378	5·768	1·654	6·729	1·929	7·690	2·205	8·651	2·481	74°
16¼	1·399	5·760	1 679	6·720	1·959	7·680	2·239	8·640	2·518	73¾
16½	1·420	5·753	1·704	6·712	1·988	7·671	2·272	8·629	2·556	73½
16¾	1·441	5·745	1·729	6·703	2·017	7·661	2·306	8·618	2·594	73¼
17°	1·462	5·738	1·754	6·694	2·047	7·650	2·339	8·607	2·631	73°
17¼	1·483	5·730	1·779	6·685	2·076	7·640	2·372	8·595	2·669	72¾
17½	1·504	5·722	1·804	6·676	2·105	7·630	2·406	8·583	2·706	72½
17¾	1·524	5·714	1·829	6·667	2·134	7·619	2·439	8·572	2·744	72¼
18°	1·545	5·706	1·854	6·657	2·163	7·608	2·472	8·560	2·781	72°
18¼	1·566	5·698	1·879	6·648	2·192	7 598	2·505	8·547	2·818	71¾
18½	1·587	5·690	1·904	6·638	2·221	7·587	2·538	8·535	2·856	71½
18¾	1·607	5·682	1·929	6·629	2·250	7·575	2·572	8·522	2·893	71¼
19°	1·628	5·673	1·953	6·619	2·279	7·564	2·605	8·510	2·930	71°
19¼	1·648	5·665	1·978	6·609	2·308	7·553	2·638	8·497	2·967	70¾
19½	1·669	5·656	2·003	6·598	2·337	7·541	2·670	8·484	3·004	70½
19¾	1·690	5·647	2·028	6·588	2·365	7·529	2·703	8·471	3·041	70¼
20°	1·710	5·638	2·052	6·578	2·394	7·518	2·736	8·457	3·078	70°
20¼	1·731	5·629	2·077	6·567	2·423	7·506	2·769	8·444	3·115	69¾
20½	1·751	5·620	2·101	6·557	2·451	7·493	2·802	8·430	3·152	69½
20¾	1·771	5·611	2·126	6·546	2·480	7·481	2·834	8·416	3·189	69¼
21°	1·792	5·601	2·150	6·535	2·509	7·469	2·867	8·402	3·225	69°
21¼	1·812	5·592	2·175	6·524	2·537	7·456	2·900	8·388	3·262	68¾
21½	1·833	5·582	2·199	6·513	2·566	7·443	2·932	8·374	3·299	68½
21¾	1·853	5·573	2·223	6·502	2·594	7·430	2·964	8·359	3·335	68¼
22°	1·873	5·563	2·248	6·490	2·622	7·417	2·997	8·345	3·371	68°
22¼	1·893	5·553	2·272	6·479	2·651	7·404	3·029	8·330	3·408	67¾
22½	1·913	5·543	2·296	6·467	2·679	7·391	3·061	8·315	3·444	67½
22¾	1·934	5·533	2·320	6·455	2·707	7·378	3·094	8·300	3·480	67¼
23°	1·954	5·523	2·344	6·444	2·735	7·364	3·126	8·285	3·517	67°
23¼	1·974	5·513	2·368	6·432	2·763	7·350	3·158	8·269	3·553	66¾
23½	1·994	5·502	2·392	6·419	2·791	7·336	3·190	8·254	3·589	66½
23¾	2·014	5·492	2·416	6·407	2·819	7·322	3·222	8·238	3·625	66¼
24°	2·034	5·481	2·440	6·395	2·847	7·308	3·254	8·222	3·661	66°
24¼	2·054	5·471	2·464	6·382	2·875	7·294	3·286	8·206	3·696	65¾
24½	2·073	5·460	2·488	6·370	2·903	7·280	3·318	8·190	3·732	65½
24¾	2·093	5·449	2·512	6·357	2·931	7·265	3·349	8·173	3·768	65¼
25°	2·113	5·438	2·536	6·344	2·958	7·250	3·381	8·157	3·804	65°
25¼	2·133	5·427	2·559	6·331	2·986	7·236	3·413	8·140	3·839	64¾
25½	2·153	5·416	2·583	6·318	3·014	7·221	3·444	8·123	3·875	64½
25¾	2·172	5·404	2·607	6·305	3·041	7·206	3·476	8·106	3·910	64¼
26°	2·192	5·393	2·630	6·292	3·069	7·190	3·507	8·089	3·945	64°
26¼	2·211	5·381	2·654	6·278	3·096	7·175	3·538	8·072	3·981	63¾
26½	2·231	5·370	2·677	6·265	3·123	7·160	3·570	8·054	4·016	63½
26¾	2·250	5·358	2·701	6·251	3·151	7·144	3·601	8·037	4·051	63¼
27°	2·270	5·346	2·724	6·237	3·178	7·128	3·632	8·019	4·086	63°
27¼	2·289	5·334	2·747	6·223	3·205	7·112	3·663	8·001	4·121	62¾
27½	2·309	5·322	2·770	6·209	3·232	7·096	3·694	7·983	4·156	62½
27¾	2·328	5·310	2·794	6·195	3·259	7·080	3·725	7·965	4·190	62¼
28°	2·347	5·298	2·817	6·181	3·286	7·064	3·756	7·947	4·225	62°
28¼	2·367	5·285	2·840	6·166	3·313	7·047	3·787	7·928	4·260	61¾
28½	2·386	5·273	2·863	6·152	3·340	7·031	3·817	7·909	4 294	61½
28¾	2·405	5·260	2·886	6·137	3·367	7·014	3·848	7·891	4·329	61¼
29°	2·424	5·248	2·909	6·122	3·394	6·997	3·878	7·872	4·363	61°
29¼	2·443	5·235	2·932	6·107	3·420	6·980	3·909	7·852	4·398	60¾
29½	2·462	5·222	2·955	6·093	3·447	6·963	3·939	7·833	4·432	60½
29¾	2·481	5·209	2·977	6·077	3·474	6·946	3·970	7·814	4·466	60¼
30°	2·500	5·196	3·000	6·062	3·500	6·928	4·000	7·794	4·500	60°
Bearing.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Bearing.
	5	6		7		8		9		

LATITUDES AND DEPARTURES.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
30°	0·866	0·500	1·732	1·000	2·598	1·500	3·464	2·000	4·330	60°
30¼	0·864	0·504	1·728	1·008	2·592	1·511	3·455	2·015	4·319	59¾
30½	0·862	0·508	1·723	1·015	2·585	1·523	3·447	2·030	4·308	59½
30¾	0·859	0·511	1·719	1·023	2·578	1·534	3·438	2·045	4·297	59¼
31°	0·857	0·515	1·714	1·030	2·572	1·545	3·429	2·060	4·286	59°
31¼	0·855	0·519	1·710	1·038	2·565	1·556	3·420	2·075	4·275	58¾
31½	0·853	0·522	1·705	1·045	2·558	1·567	3·411	2·090	4·263	58½
31¾	0·850	0·526	1·701	1·052	2·551	1·579	3·401	2·105	4·252	58¼
32°	0·848	0·530	1·696	1·060	2·544	1·590	3·392	2·120	4·240	58°
32¼	0·846	0·534	1·691	1·067	2·537	1·601	3·383	2·134	4·229	57¾
32½	0·843	0·537	1·687	1·075	2·530	1·612	3·374	2·149	4·217	57½
32¾	0·841	0·541	1·682	1·082	2·523	1·623	3·364	2·164	4·205	57¼
33°	0·839	0·545	1·677	1·089	2·516	1·634	3·355	2·179	4·193	57°
33¼	0·836	0·548	1·673	1·097	2·509	1·645	3·345	2·193	4·181	56¾
33½	0·834	0·552	1·668	1·104	2·502	1·656	3·336	2·208	4·169	56½
33¾	0·831	0·556	1·663	1·111	2·494	1·667	3·326	2·222	4·157	56¼
34°	0·829	0·559	1·658	1·118	2·487	1·678	3·316	2·237	4·145	56°
34¼	0·827	0·563	1·653	1·126	2·480	1·688	3·306	2·251	4·133	55¾
34½	0·824	0·566	1·648	1·133	2·472	1·699	3·297	2·266	4·121	55½
34¾	0·822	0·570	1·643	1·140	2·465	1·710	3·287	2·280	4·108	55¼
35°	0·819	0·574	1·638	1·147	2·457	1·721	3·277	2·294	4·096	55°
35¼	0·817	0·577	1·633	1·154	2·450	1·731	3·267	2·309	4·083	54¾
35½	0·814	0·581	1·628	1·161	2·442	1·742	3·257	2·323	4·071	54½
35¾	0·812	0·584	1·623	1·168	2·435	1·753	3·246	2·337	4·058	54¼
36°	0·809	0·588	1·618	1·176	2·427	1·763	3·236	2·351	4·045	54°
36¼	0·806	0·591	1·613	1·183	2·419	1·774	3·226	2·365	4·032	53¾
36½	0·804	0·595	1·608	1·190	2·412	1·784	3·215	2·379	4·019	53½
36¾	0·801	0·598	1·603	1·197	2·404	1·795	3·205	2·393	4·006	53¼
37°	0·799	0·602	1·597	1·204	2·396	1·805	3·195	2·407	3·993	53°
37¼	0·796	0·605	1·592	1·211	2·388	1·816	3·184	2·421	3·980	52¾
37½	0·793	0·609	1·587	1·218	2·380	1·826	3·173	2·435	3·967	52½
37¾	0·791	0·612	1·581	1·224	2·372	1·837	3·163	2·449	3·953	52¼
38°	0·788	0·616	1·576	1·231	2·364	1·847	3·152	2·463	3·940	52°
38¼	0·785	0·619	1·571	1·238	2·356	1·857	3·141	2·476	3·927	51¾
38½	0·783	0·623	1·565	1·245	2·348	1·868	3·130	2·490	3·913	51½
38¾	0·780	0·626	1·560	1·252	2·340	1·878	3·120	2·504	3·899	51¼
39°	0·777	0·629	1·554	1·259	2·331	1·888	3·109	2·517	3·886	51°
39¼	0·774	0·633	1·549	1·265	2·323	1·898	3·098	2·531	3·872	50¾
39½	0·772	0·636	1·543	1·272	2·315	1·908	3·086	2·544	3·858	50½
39¾	0·769	0·639	1·538	1·279	2·307	1·918	3·075	2·558	3·844	50¼
40°	0·766	0·643	1·532	1·286	2·298	1·928	3·064	2·571	3·830	50°
40¼	0·763	0·646	1·526	1·292	2·290	1·938	3·053	2·584	3·816	49¾
40½	0·760	0·649	1·521	1·299	2·281	1·948	3·042	2·598	3·802	49½
40¾	0·758	0·653	1·515	1·306	2·273	1·958	3·030	2·611	3·788	49¼
41°	0·755	0·656	1·509	1·312	2·264	1·968	3·019	2·624	3·774	49°
41¼	0·752	0·659	1·504	1·319	2·256	1·978	3·007	2·637	3·759	48¾
41½	0·749	0·663	1·498	1·325	2·247	1·988	2·996	2·650	3·745	48½
41¾	0·746	0·666	1·492	1·332	2·238	1·998	2·984	2·664	3·730	48¼
42°	0·743	0·669	1·486	1·338	2·229	2·007	2·973	2·677	3·716	48°
42¼	0·740	0·672	1·480	1·345	2·221	2·017	2·961	2·689	3·701	47¾
42½	0·737	0·676	1·475	1·351	2·212	2·027	2·949	2·702	3·686	47½
42¾	0·734	0·679	1·469	1·358	2·203	2·036	2·937	2·715	3·672	47¼
43°	0·731	0·682	1·463	1·364	2·194	2·046	2·925	2·728	3·657	47°
43¼	0·728	0·685	1·457	1·370	2·185	2·056	2·913	2·741	3·642	46¾
43½	0·725	0·688	1·451	1·377	2·176	2·065	2·901	2·753	3·627	46½
43¾	0·722	0·692	1·445	1·383	2·167	2·075	2·889	2·766	3·612	46¼
44°	0·719	0·695	1·439	1·389	2·158	2·084	2·877	2·779	3·597	46°
44¼	0·716	0·698	1·433	1·396	2·149	2·093	2·865	2·791	3·582	45¾
44½	0·713	0·701	1·427	1·402	2·140	2·103	2·853	2·804	3·566	45½
44¾	0·710	0·704	1·420	1·408	2·131	2·112	2·841	2·816	3·551	45¼
45°	0·707	1·707	1·414	1·414	2·121	2·121	2·828	2·828	3·536	45°
Bearing.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Bearing.
	1		2		3		4		5	

LATITUDES AND DEPARTURES.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
30°	2·500	5·196	3·000	6·062	3·500	6·928	4·000	7·794	4·500	60°
30¼	2·519	5·183	3·023	6·047	3·526	6·911	4·030	7·775	4·534	59¾
30½	2·538	5·170	3·045	6·031	3·553	6·893	4·060	7·755	4·568	59½
30¾	2·556	5·156	3·068	6·016	3·579	6·875	4·090	7·735	4·602	59¼
31°	2·575	5·143	3·090	6·000	3·605	6·857	4·120	7·715	4·635	59°
31¼	2·594	5·129	3·113	5·984	3·631	6·839	4·150	7·694	4·669	58¾
31½	2·612	5·116	3·135	5·968	3·657	6·821	4·180	7·674	4·702	58½
31¾	2·631	5·102	3·157	5·952	3·683	6·803	4·210	7·653	4·736	58¼
32°	2·650	5·088	3·180	5·936	3·709	6·784	4·239	7·632	4·769	58°
32¼	2·668	5·074	3·202	5·920	3·735	6·766	4·269	7·612	4·802	57¾
32½	2·686	5·060	3·224	5·904	3·761	6·747	4·298	7·591	4·836	57½
32¾	2·705	5·046	3·246	5·887	3·787	6·728	4·328	7·569	4·869	57¼
33°	2·723	5·032	3·268	5·871	3·812	6·709	4·357	7·548	4·902	57°
33¼	2·741	5·018	3·290	5·854	3·838	6·690	4·386	7·527	4·935	56¾
33½	2·760	5·003	3·312	5·837	3·864	6·671	4·416	7·505	4·967	56½
33¾	2·778	4·989	3·333	5·820	3·889	6·652	4·445	7·483	5·000	56¼
34°	2·796	4·974	3·355	5·803	3·914	6·632	4·474	7·461	5·033	56°
34¼	2·814	4·960	3·377	5·786	3·940	6·613	4·502	7·439	5·065	55¾
34½	2·832	4·945	3·398	5·769	3·965	6·593	4·531	7·417	5·098	55½
34¾	2·850	4·930	3·420	5·752	3·990	6·573	4·560	7·395	5·130	55¼
35°	2·868	4·915	3·441	5·734	4·015	6·553	4·589	7·372	5·162	55°
35¼	2·886	4·900	3·463	5·716	4·040	6·533	4·617	7·350	5·194	54¾
35½	2·904	4·885	3·484	5·699	4·065	6·513	4·646	7·327	5·226	54½
35¾	2·921	4·869	3·505	5·681	4·090	6·493	4·674	7·304	5·258	54¼
36°	2·939	4·854	3·527	5·663	4·115	6·472	4·702	7·281	5·290	54°
36¼	2·957	4·839	3·548	5·645	4·139	6·452	4·730	7·258	5·322	53¾
36½	2·974	4·823	3·569	5·627	4·164	6·431	4·759	7·235	5·353	53½
36¾	2·992	4·808	3·590	5·609	4·188	6·410	4·787	7·211	5·385	53¼
37°	3·009	4·792	3·611	5·590	4·213	6·389	4·815	7·188	5·416	53°
37¼	3·026	4·776	3·632	5·572	4·237	6·368	4·842	7·164	5·448	52¾
37½	3·044	4·760	3·653	5·554	4·261	6·347	4·870	7·140	5·479	52½
37¾	3·061	4·744	3·673	5·535	4·286	6·326	4·898	7·116	5·510	52¼
38°	3·078	4·728	3·694	5·516	4·310	6·304	4·925	7·092	5·541	52°
38¼	3·095	4·712	3·715	5·497	4·334	6·283	4·953	7·068	5·572	51¾
38½	3·113	4·696	3·735	5·478	4·358	6·261	4·980	7·043	5·603	51½
38¾	3·130	4·679	3·756	5·459	4·381	6·239	5·007	7·019	5·633	51¼
39°	3·147	4·663	3·776	5·440	4·405	6·217	5·035	6·994	5·664	51°
39¼	3·164	4·646	3·796	5·421	4·429	6·195	5·062	6·970	5·694	50¾
39½	3·180	4·630	3·816	5·401	4·453	6·173	5·089	6·945	5·725	50½
39¾	3·197	4·613	3·837	5·382	4·476	6·151	5·116	6·920	5·755	50¼
40°	3·214	4·596	3·857	5·362	4·500	6·128	5·142	6·894	5·785	50°
40¼	3·231	4·579	3·877	5·343	4·523	6·106	5·169	6·869	5·815	49¾
40½	3·247	4·562	3·897	5·323	4·546	6·083	5·196	6·844	5·845	49½
40¾	3·264	4·545	3·917	5·303	4·569	6·061	5·222	6·818	5·875	49¼
41°	3·280	4·528	3·936	5·283	4·592	6·038	5·248	6·792	5·905	49°
41¼	3·297	4·511	3·956	5·263	4·615	6·015	5·275	6·767	5·934	48¾
41½	3·313	4·494	3·976	5·243	4·638	5·992	5·301	6·741	5·964	48½
41¾	3·329	4·476	3·995	5·222	4·661	5·968	5·327	6·715	5·993	48¼
42°	3·346	4·459	4·015	5·202	4·684	5·945	5·353	6·688	6·022	48°
42¼	3·362	4·441	4·034	5·182	4·707	5·922	5·379	6·662	6·051	47¾
42½	3·378	4·424	4·054	5·161	4·729	5·898	5·405	6·635	6·080	47½
42¾	3·394	4·406	4·073	5·140	4·752	5·875	5·430	6·609	6·109	47¼
43°	3·410	4·388	4·092	5·119	4·774	5·851	5·456	6·582	6·138	47°
43¼	3·426	4·370	4·111	5·099	4·796	5·827	5·481	6·555	6·167	46¾
43½	3·442	4·352	4·130	5·078	4·818	5·803	5·507	6·528	6·195	46½
43¾	3·458	4·334	4·149	5·057	4·841	5·779	5·532	6·501	6·224	46¼
44°	3·473	4·316	4·168	5·035	4·863	5·755	5·557	6·474	6·252	46°
44¼	3·489	4·298	4·187	5·014	4·885	5·730	5·582	6·447	6·280	45¾
44½	3·505	4·280	4·206	4·993	4·906	5·706	5·607	6·419	6·308	45½
44¾	3·520	4·261	4·224	4·971	4·928	5·681	5·632	6·392	6·336	45¼
45°	3·536	4·243	4·243	4.950	4·950	5·657	5·657	6·364	6·364	45°
Bearing.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Bearing.
	5	6		7		8		9		

NATURAL SINES AND COSINES.

′	0°		1°		2°		3°		4°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	00000	Unit.	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	Unit.	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	Unit.	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	Unit.	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	Unit.	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	Unit.	01891	99982	03635	99934	05379	99855	07121	99746	55
6	00175	Unit.	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	Unit.	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	Unit.	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	Unit.	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	Unit.	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99841	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	04100	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
60	01745	99985	03490	99939	05234	99863	06976	99756	08716	99619	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	89°		88°		87°		86°		85°		′

NATURAL SINES AND COSINES.

′	5° Sine.	5° Cosine.	6° Sine.	6° Cosine.	7° Sine.	7° Cosine.	8° Sine.	8° Cosine.	9° Sine.	9° Cosine.	′
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
′	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	′
	84°		83°		82°		81°		80°		

NATURAL SINES AND COSINES.

′	10°		11°		12°		13°		14°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	79°		78°		77°		76°		75°		′

NATURAL SINES AND COSINES.

′	15° Sine.	15° Cosine.	16° Sine.	16° Cosine.	17° Sine.	17° Cosine.	18° Sine.	18° Cosine.	19° Sine.	19° Cosine.	′
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
′	74° Cosine.	74° Sine.	73° Cosine.	73° Sine.	72° Cosine.	72° Sine.	71° Cosine.	71° Sine.	70° Cosine.	70° Sine.	′

NATURAL SINES AND COSINES.

′	20°		21°		22°		23°		24°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	69°		68°		67°		66°		65°		′

NATURAL SINES AND COSINES.

′	25°		26°		27°		28°		29°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	64°		63°		62°		61°		60°		′

NATURAL SINES AND COSINES.

′	30°		31°		32°		33°		34°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	59°		58°		57°		56°		55°		′

NATURAL SINES AND COSINES.

′	35°		36°		37°		38°		39°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	54°		53°		52°		51°		50°		′

NATURAL SINES AND COSINES.

′	40°		41°		42°		43°		44°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	49°		48°		47°		46°		45°		′

LOGARITHMS OF NUMBERS.

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100	00 0000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
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102	* 8600	9026	9451	9876	•300	0724	1147	1570	1993	2415	424
103	01 2837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	* 7033	7451	7868	8284	8700	9116	9532	9947	•361	0775	416
105	02 1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	* 9384	9789	•195	0600	1004	1408	1812	2216	2619	3021	404
108	03 3424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	* 7426	7825	8223	8620	9017	9414	9811	•207	0602	0998	396
110	04 1393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	* 9218	9606	9993	•380	0766	1153	1538	1924	2309	2694	386
113	05 3078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	* 6905	7286	7666	8046	8426	8805	9185	9563	9942	•320	379
115	06 0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	* 8186	8557	8928	9298	9668	•038	0407	0776	1145	1514	369
118	07 1882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	* 9181	9543	9904	•266	0626	0987	1347	1707	2067	2426	360
121	08 2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	* 9905	•258	0611	0963	1315	1667	2018	2370	2721	3071	351
124	09 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	* 6910	7257	7604	7951	8298	8644	8990	9335	9681	•026	346
126	10 0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	* 7210	7549	7888	8227	8565	8903	9241	9579	9916	•253	338
129	11 0590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	* 7271	7603	7934	8265	8595	8926	9256	9586	9915	•245	330
132	12 0574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	* 7105	7429	7753	8076	8399	8722	9045	9368	9690	•012	323
135	13 0334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	* 9879	•194	0508	0822	1136	1450	1763	2076	2389	2702	314
139	14 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	* 9219	9527	9835	•142	0449	0756	1063	1370	1676	1982	307
142	15 2288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	* 8362	8664	8965	9266	9567	9868	•168	0469	0769	1068	301
145	16 1368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	17 0262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	* 8977	9264	9552	9839	•126	0413	0699	0985	1272	1558	287
152	18 1844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	* 7521	7803	8084	8366	8647	8928	9209	9490	9771	•051	281
155	19 0332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	* 8657	8932	9206	9481	9755	•029	0303	0577	0850	1124	274
159	20 1397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
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LOGARITHMS OF NUMBERS.

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160	20 4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	* 9515	9783	•051	0319	0586	0853	1121	1388	1654	1921	267
163	21 2188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	22 0108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	* 7887	8144	8400	8657	8913	9170	9426	9682	9938	•193	256
170	23 0449	0704	0960	1215	1470	1724	1979	2234	2488	2742	254
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	* 8046	8297	8548	8799	9049	9299	9550	9800	•050	0300	250
174	24 0549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	* 7973	8219	8464	8709	8954	9198	9443	9687	9932	•176	245
178	25 0420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26 0071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	* 9513	9746	9980	•213	0446	0679	0912	1144	1377	1609	233
187	27 1842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	* 8754	8982	9211	9439	9667	9895	•123	0351	0578	0806	228
191	28 1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29 0035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	* 8853	9071	9289	9507	9725	9943	•161	0378	0595	0813	218
200	30 1030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	* 9630	9843	•056	0268	0481	0693	0906	1118	1330	1542	212
205	31 1754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	32 0146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	* 8380	8583	8787	8991	9194	9398	9601	9805	•008	0211	203
214	33 0414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	* 8456	8656	8855	9054	9253	9451	9650	9849	•047	0246	199
219	34 0444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
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221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	* 8305	8500	8694	8889	9083	9278	9472	9666	9860	⋄054	194
224	35 0248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	* 9835	⋄025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	36 1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	* 9216	9401	9587	9772	9958	⋄143	0328	0513	0698	0883	185
235	37 1068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	* 8398	8580	8761	8943	9124	9306	9487	9668	9849	⋄030	181
240	38 0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	* 9166	9343	9520	9698	9875	⋄051	0228	0405	0582	0759	177
246	39 0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	* 9674	9847	⋄020	0192	0365	0538	0711	0883	1056	1228	173
252	40 1401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	* 9933	⋄102	0271	0440	0609	0777	0946	1114	1283	1451	169
258	41 1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	* 9956	⋄121	0286	0451	0616	0781	0945	1110	1275	1439	165
264	42 1604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	* 9752	9914	⋄075	0236	0398	0559	0720	0881	1042	1203	161
270	43 1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	* 9333	9491	9648	9806	9964	⋄122	0279	0437	0594	0752	158
276	44 0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
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281	* 8706	8861	9015	9170	9324	9478	9633	9787	9941	⬩095	154
282	45 0249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
286	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
288	* 9392	9543	9694	9845	9995	⬩146	0296	0447	0597	0748	151
289	46 0898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	* 9822	9969	⬩116	0263	0410	0557	0704	0851	0998	1145	147
296	47 1292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	7121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	48 0007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	* 9958	⬩099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	49 1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	* 9687	9824	9962	⬩099	0236	0374	0511	0648	0785	0922	137
317	50 1059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
322	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
323	* 9203	9337	9471	9606	9740	9874	⬩009	0143	0277	0411	134
324	51 0545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4414	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	* 9828	9959	⬩090	0221	0353	0484	0615	0745	0876	1007	131
332	52 1138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	* 8917	9045	9174	9302	9430	9559	9687	9815	9943	⬩072	128
339	53 0200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128
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341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
343	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	* 9076	9202	9327	9452	9578	9703	9829	9954	⋄079	0204	125
347	54 0329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	* 9003	9126	9249	9371	9494	9616	9739	9861	9984	⋄106	123
355	55 0228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	* 9907	⋄026	0146	0265	0385	0504	0624	0743	0863	0982	119
364	56 1101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	* 9374	9491	9608	9725	9842	9959	⋄076	0193	0309	0426	117
372	57 0543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	* 9784	9898	⋄012	0126	0241	0355	0469	0583	0697	0811	114
381	58 0925	1039	1153	1267	1381	1495	1608	1722	1836	1950	114
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	* 9950	⋄061	0173	0284	0396	0507	0619	0730	0842	0953	112
390	59 1065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	* 9883	9992	⋄101	0210	0319	0428	0537	0646	0755	0864	109
399	60 0973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
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401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	* 9594	9701	9808	9914	◆021	0128	0234	0341	0447	0554	107
408	61 0660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
416	* 9093	9198	9302	9406	9511	9615	9719	9824	9928	◆032	104
417	62 0136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	* 9410	9512	9613	9715	9817	9919	◆021	0123	0224	0326	102
427	63 0428	0530	0631	0733	0835	0936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	100
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	100
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	99
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	99
436	* 9486	9586	9686	9785	9885	9984	◆084	0183	0283	0382	99
437	64 0481	0581	0680	0779	0879	0978	1077	1177	1276	1375	99
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	98
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	* 9335	9432	9530	9627	9724	9821	9919	◆016	0113	0210	97
447	65 0308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	* 9916	◆011	0106	0201	0296	0391	0486	0581	0676	0771	95
458	66 0865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
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462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	* 9317	9410	9503	9596	9689	9782	9875	9967	•060	0153	93
468	67 0246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	* 9428	9519	9610	9700	9791	9882	9973	•063	0154	0245	91
479	68 0336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	5726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	* 9309	9398	9486	9575	9664	9753	9841	9930	•019	0107	89
490	69 0196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	8970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	* 9838	9924	•011	0098	0184	0271	0358	0444	0531	0617	87
502	70 0704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3895	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	* 9270	9355	9440	9524	9609	9694	9779	9863	9948	•033	85
513	71 0117	0202	0287	0371	0456	0540	0625	0710	0794	0879	85
514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3650	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
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521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	* 9331	9414	9497	9580	9663	9745	9828	9911	9994	⋆077	83
525	72 0159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	* 9974	⋆055	0136	0217	0298	0378	0459	0540	0621	0702	81
538	73 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
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544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	* 9572	9651	9731	9810	9889	9968	⋆047	0126	0205	0284	79
550	74 0363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2646	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	* 9736	9814	9891	9968	⋆045	0123	0200	0277	0354	0431	77
563	75 0508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
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567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	* 9668	9743	9819	9894	9970	⋆045	0121	0196	0272	0347	75
576	76 0422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	* 9377	9451	9525	9599	9673	9746	9820	9894	9968	◆042	74
589	77 0115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	* 9596	9669	9741	9813	9885	9957	◆029	0101	0173	0245	72
603	78 0317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
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607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	* 9581	9651	9722	9792	9863	9933	◆004	0074	0144	0215	70
617	79 0285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
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624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	9341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	80 0029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	.0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
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641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	* 9560	9627	9694	9762	9829	9896	9964	◆031	0098	0165	67
646	81 0233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	* 9544	9610	9676	9741	9807	9873	9939	◆004	0070	0136	66
661	82 0201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	* 9947	◆011	0075	0139	0204	0268	0332	0396	0460	0525	64
677	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	64
681	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	* 9478	9541	9604	9667	9729	9792	9855	9918	9981	◆043	63
692	84 0106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
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701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85 0033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	* 9739	9799	9859	9918	9978	⋄038	0098	0158	0218	0278	60
725	86 0338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	59
741	* 9818	9877	9935	9994	⋄053	0111	0170	0228	0287	0345	59
742	87 0404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	* 9669	9726	9784	9841	9898	9956	⋄013	0070	0127	0185	57
759	88 0242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
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761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	56
771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
773	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
776	* 9862	9918	9974	*030	0086	0141	0197	0253	0309	0365	56
777	89 0421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	* 9821	9875	9930	9985	*039	0094	0149	0203	0258	0312	55
795	90 0367	0422	0476	0531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
801	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	8485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	* 9556	9610	9663	9716	9770	9823	9877	9930	9984	*037	53
813	91 0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
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821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	* 9601	9653	9706	9758	9810	9862	9914	9967	♦019	0071	52
832	92 0123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	* 9930	9981	♦032	0083	0134	0185	0236	0287	0338	0389	51
852	93 0440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	94 0018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
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882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
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944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
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951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
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992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
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997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

The Application of Logarithms.—The logarithm of a number is set down as a decimal, and addition of ciphers to numbers does not change the logarithm; it is the same for 11, 110, 1100, but the value of the number is established by figures to the left of the decimal point; thus, if the number is among the units, the characteristic is 0; if in the tens, 1; in the hundreds, 2; thousands, 3; tens of thousands, 4, and so on; if the number is a decimal fraction and the first figure a tenth, the characteristic is $\bar{1}$, if hundredths $\bar{2}$, thousandths $\bar{3}$.

Multiplication of two numbers is performed by the addition of their logarithms and characteristics, and finding the number corresponding to their sum; thus, to multiply 119 by 2760.

Characteristic of 119 2, logarithm.		2·075547
" 2760 3, "		3·440909
		5·516456
3284		403
401	D =	132)53(401
328440·1		528
		200
		132
		68

As the characteristic is 5, the result is 6 figures of whole numbers.

Division is performed by subtracting the logarithm of the divisor from that of the dividend, and finding the logarithm of the remainder for the quotient. But if the divisor is the larger, then the characteristic of the remainder is —.

Thus, to divide 500 by 63008.

Logarithm of 500		2·698970
Logarithm of 63000 = 4·799341		
D = 69 $\frac{8 \times 69}{10}$ =	55·2	
Logarithm of 63008		4.799396
Corresponding number ·007935 =		$\bar{3}$·899574

Numbers are raised to any power by multiplying their logarithm by the exponents, and roots are extracted by dividing the logarithm. Thus, to get the square of any number, its logarithm is multiplied by 2, for the cube by 3, for the 4th power by 4; in like manner, to obtain the square root of the number, divide the logarithm by 2; by 3 for $\sqrt[3]{}$; by 4 for $\sqrt[4]{}$.

The roots of numbers are better expressed by fractional exponents, thus: $\sqrt{a}$ by $a^{1/2}$, $\sqrt[3]{a}$ by $a^{1/3}$.

The raising of numbers to different powers is extremely simple, by logarithms, when the numbers are whole numbers, but becomes somewhat more complicated when the numbers are decimals.

Thus, to find the 4th power of ·07.

Logarithm ·07		$\bar{2}$·845098
		4
	$\bar{8}$	3·380392
Number ·00002401		$\bar{5}$·380392

To extract the 4th root of ·07—

Logarithm ·07	$\bar{2}$·845098
Add $\bar{2}$ to the characteristic to make it divisible by 4, and a positive 2 to the logarithm to balance it.	$\bar{2}$·2·845098
	4)$\bar{4}$·2·845098
Number ·5143	$\bar{1}$· 711274

The exponent of a root is often a decimal; thus the $\sqrt[4]{\cdot 07}$ may be expressed by $\cdot 07^{\cdot 25}$.

Logarithm ·07	$\bar{2}$·845098
	·25
	4225490
	1690196
	·$\bar{5}$·21127450
	·$\bar{5}$·5
Number ·5143	$\bar{1}$·71127450

Note.—In this example, ·$\bar{5}$ is added to the resultant characteristic to bring it to an integer, and an equal positive amount to the logarithm to balance it.

The same logarithm as by dividing by 4· and corresponding to the number ·5143. The rule is to consider the logarithm as a plus quantity, and multiply by the exponent and the characteristic as minus, and, after similar multiplication, subtract it from the first product. When a characteristic has a minus sign ($\bar{3}$), and it is to be subtracted, the sign is changed and added.

Thus, to divide 10· by $\frac{1}{10}$.

Logarithm 10·	1·00000
$\frac{1}{10}$	$\bar{1}$
Logarithm of 100·	2·000
To divide $\frac{1}{10}$ Logarithm	$\bar{1}$·00000
by $\frac{1}{100}$	$\bar{2}$·
Logarithm of 10·	1·0000
To divide $\frac{1}{1000}$ Logarithm	$\bar{3}$·00000
by 100	2
Logarithm of ·00001	$\bar{5}$

INDEX.

PL. I.

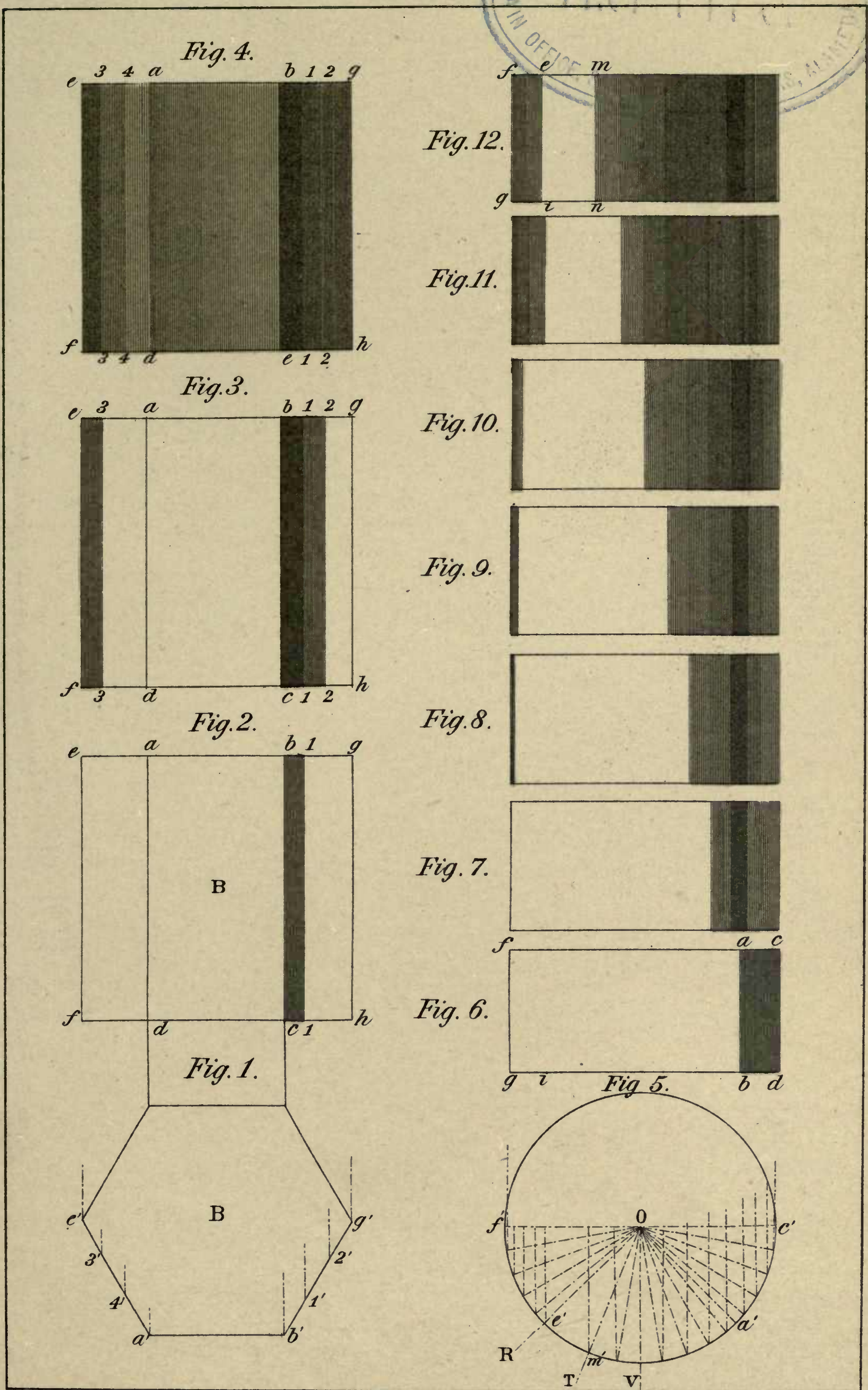
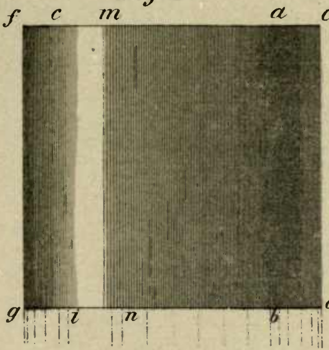


Fig. 4.
Fig. 3.
Fig. 2.
Fig. 1.
Fig. 12.
Fig. 11.
Fig. 10.
Fig. 9.
Fig. 8.
Fig. 7.
Fig. 6.
Fig 5.
B
B
0
R
T
V

PL. II.

Fig. 4.



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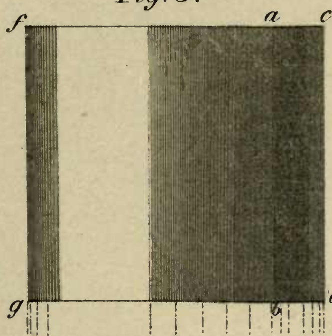


Fig. 2.

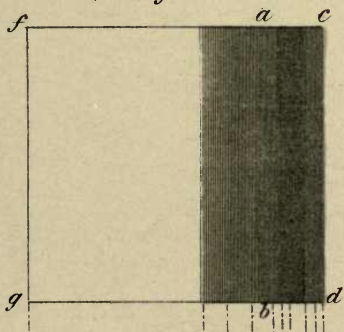
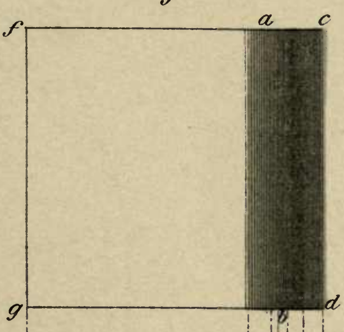


Fig. 1.



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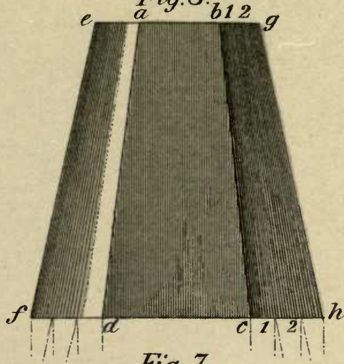

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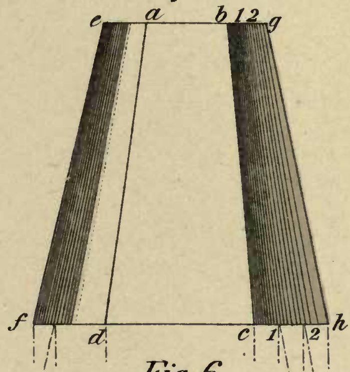

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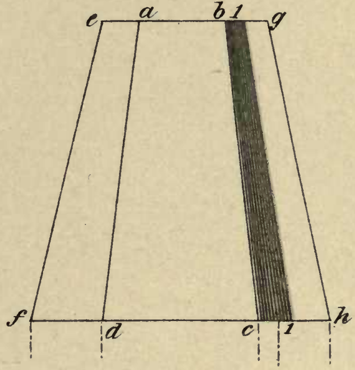

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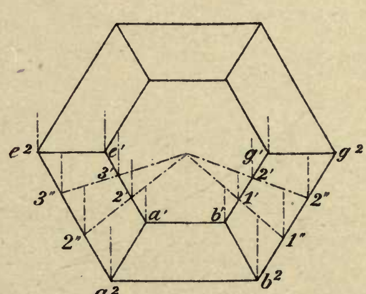

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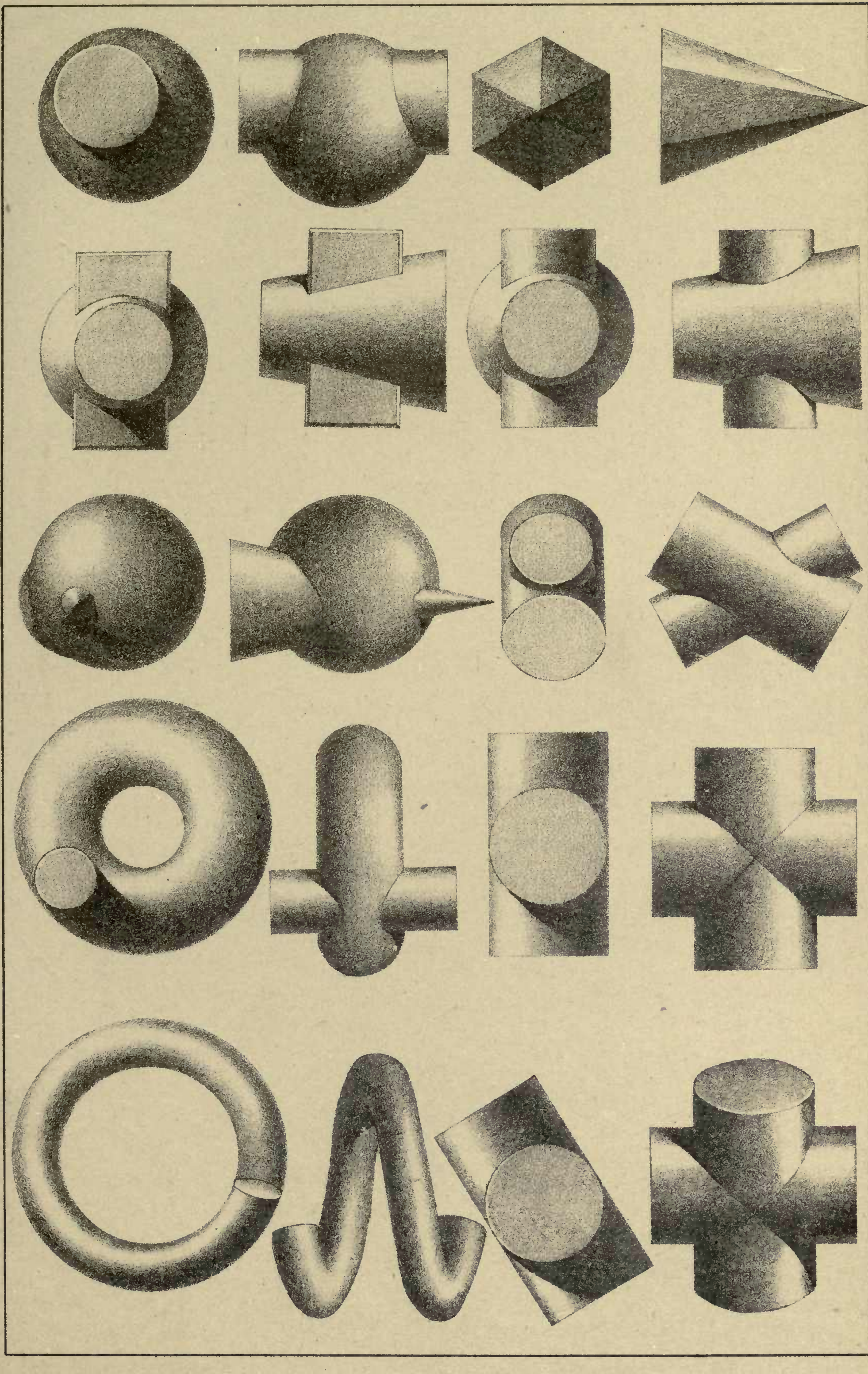

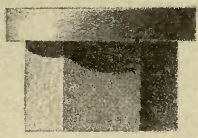

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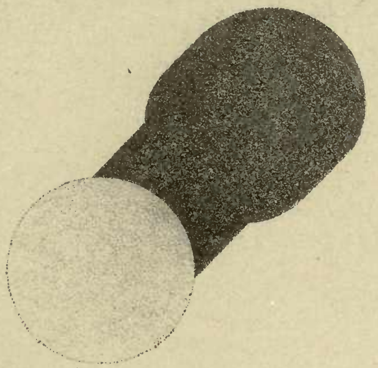

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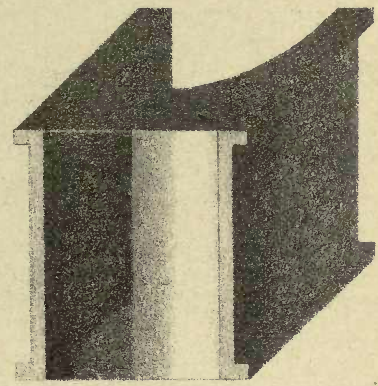


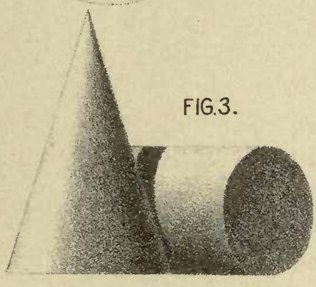
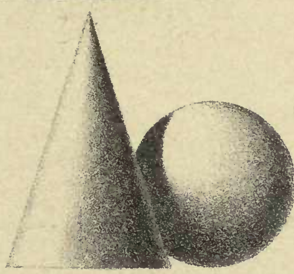


FIG. 3.



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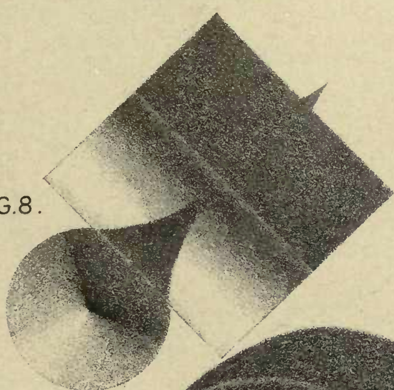
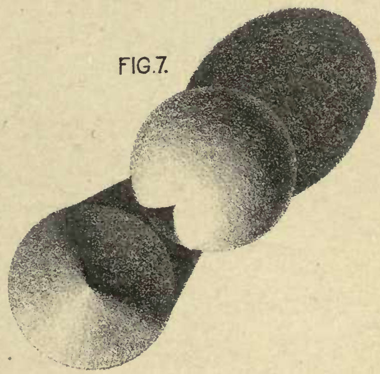


FIG. 8.



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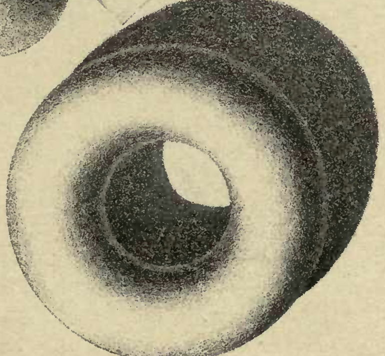

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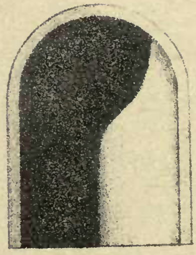

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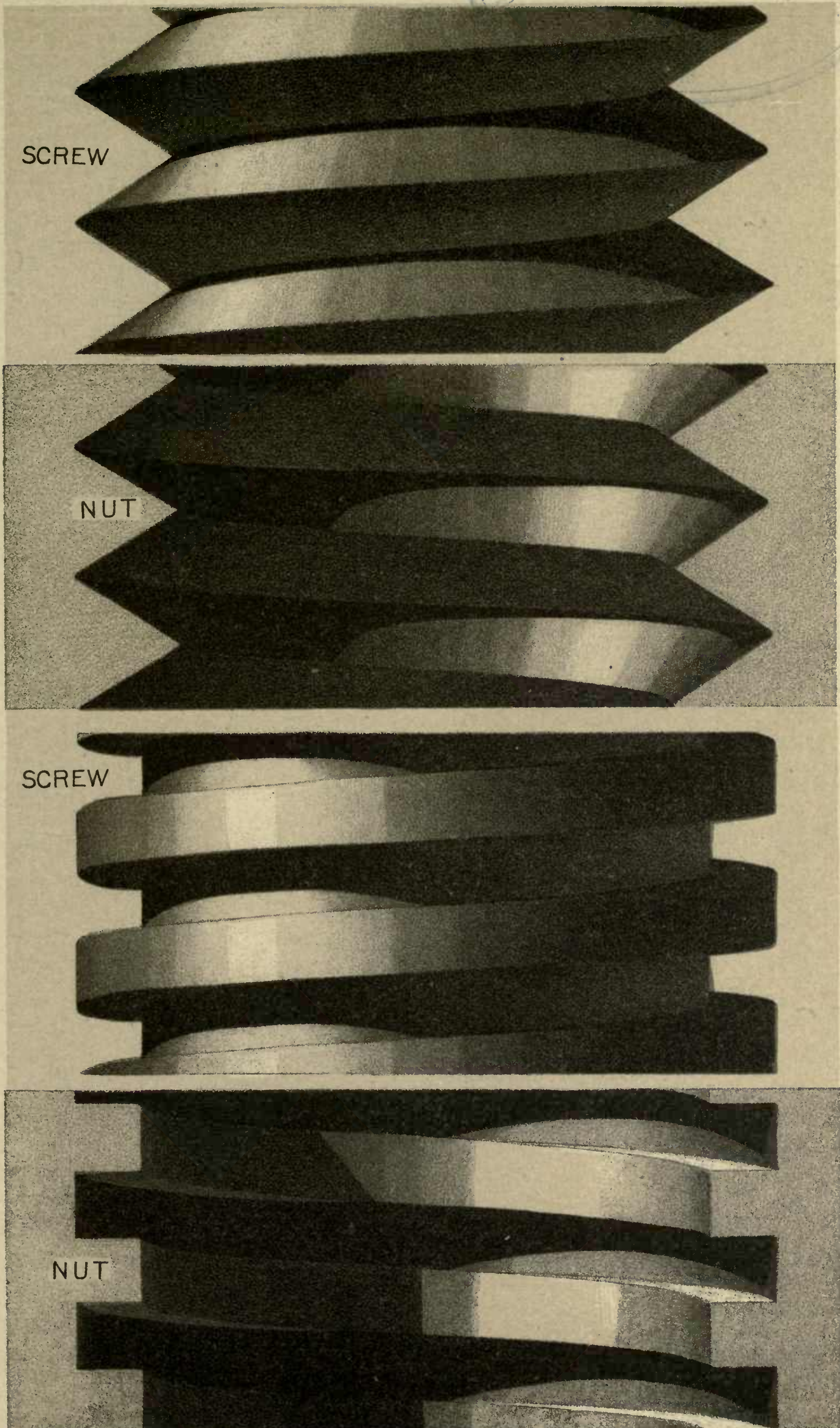
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NUT
SCREW
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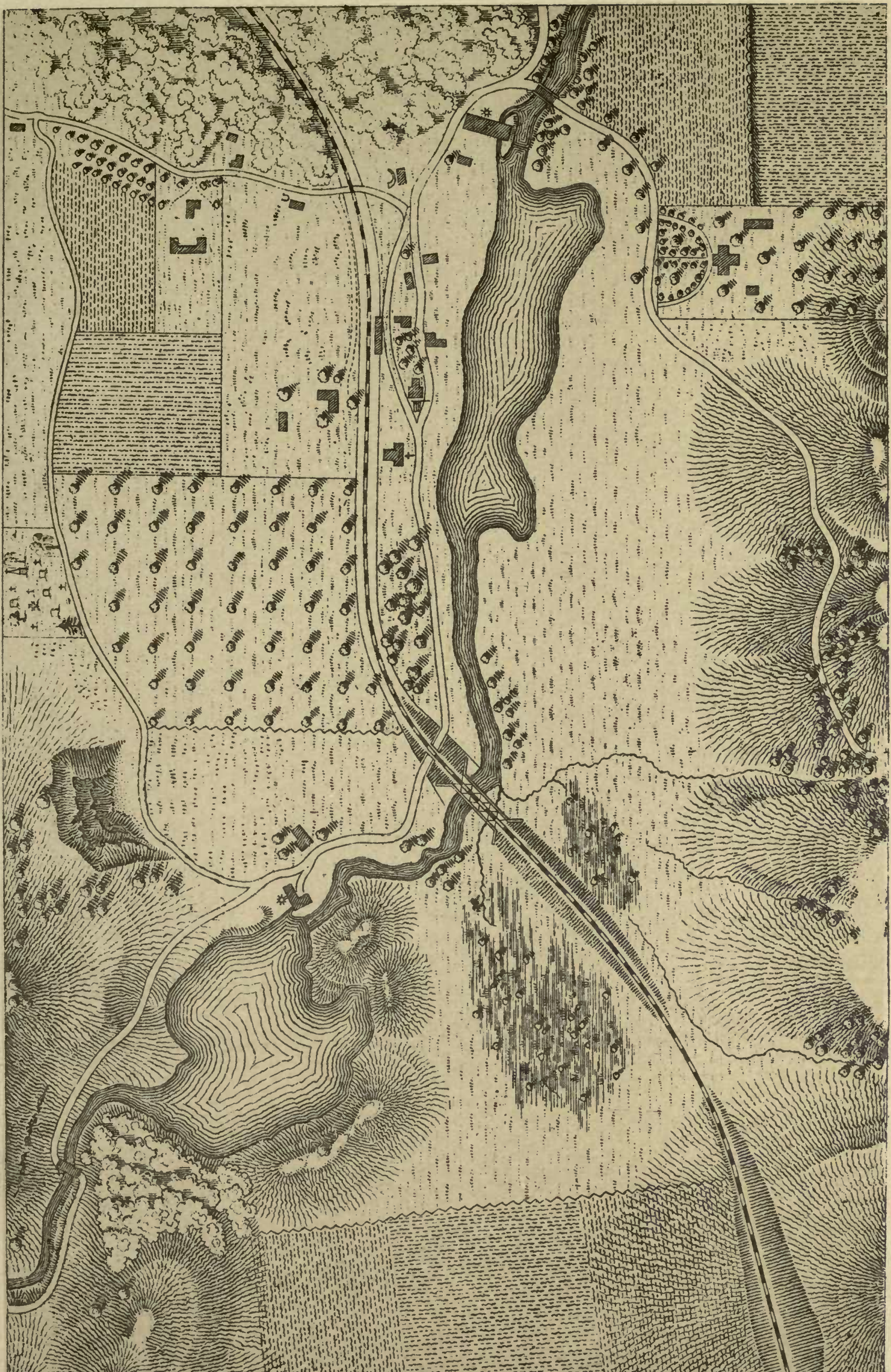
PL.VI.



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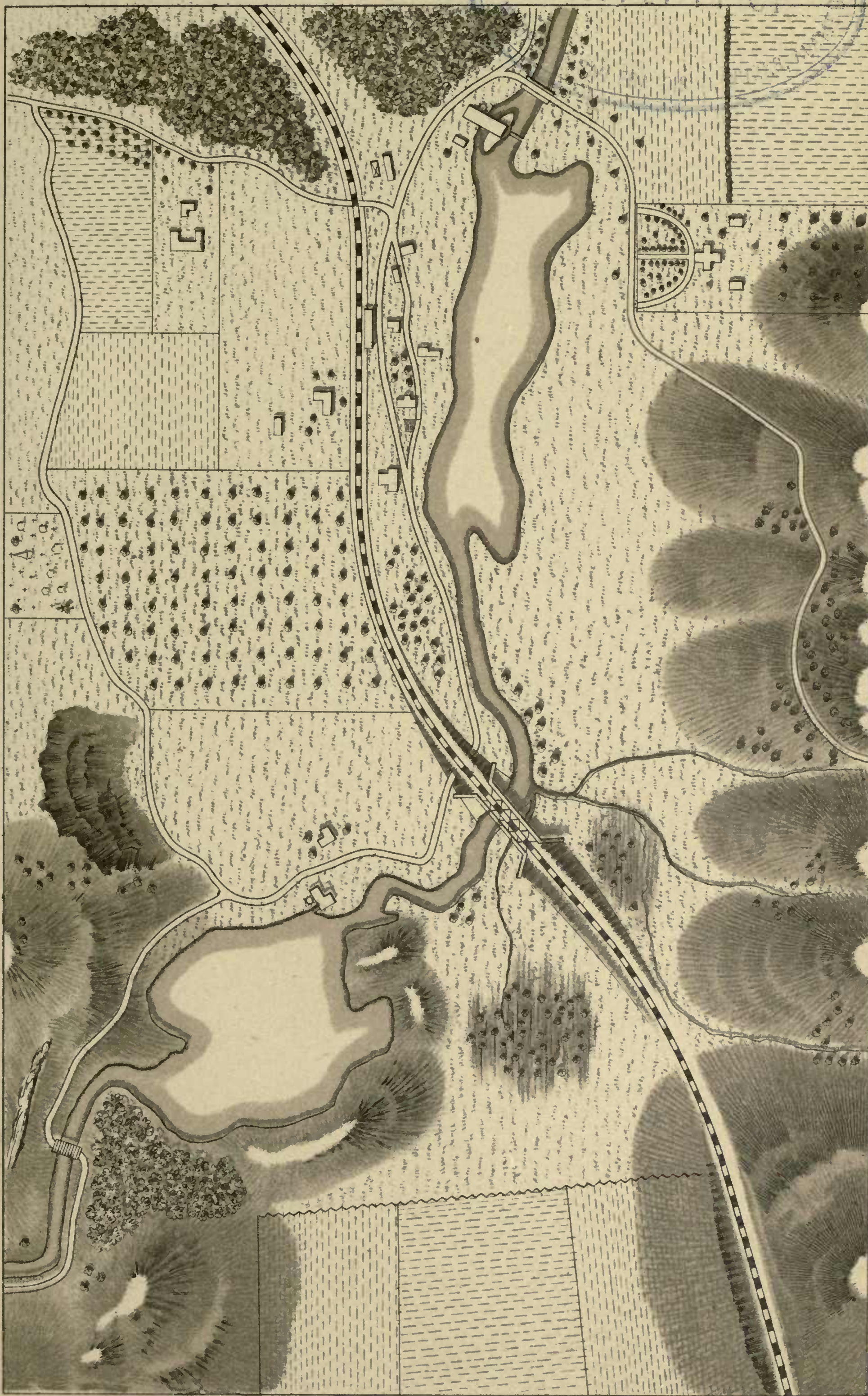
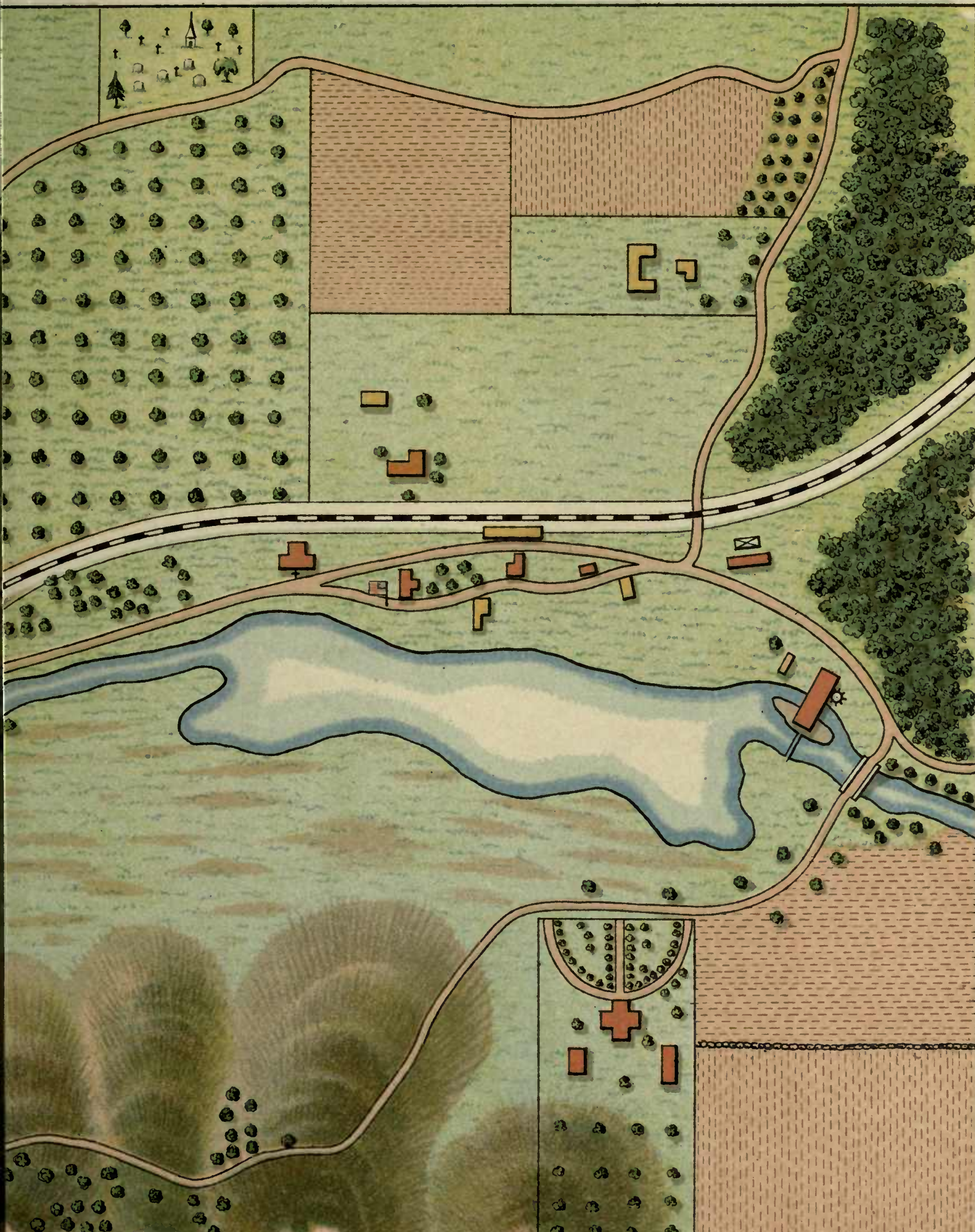
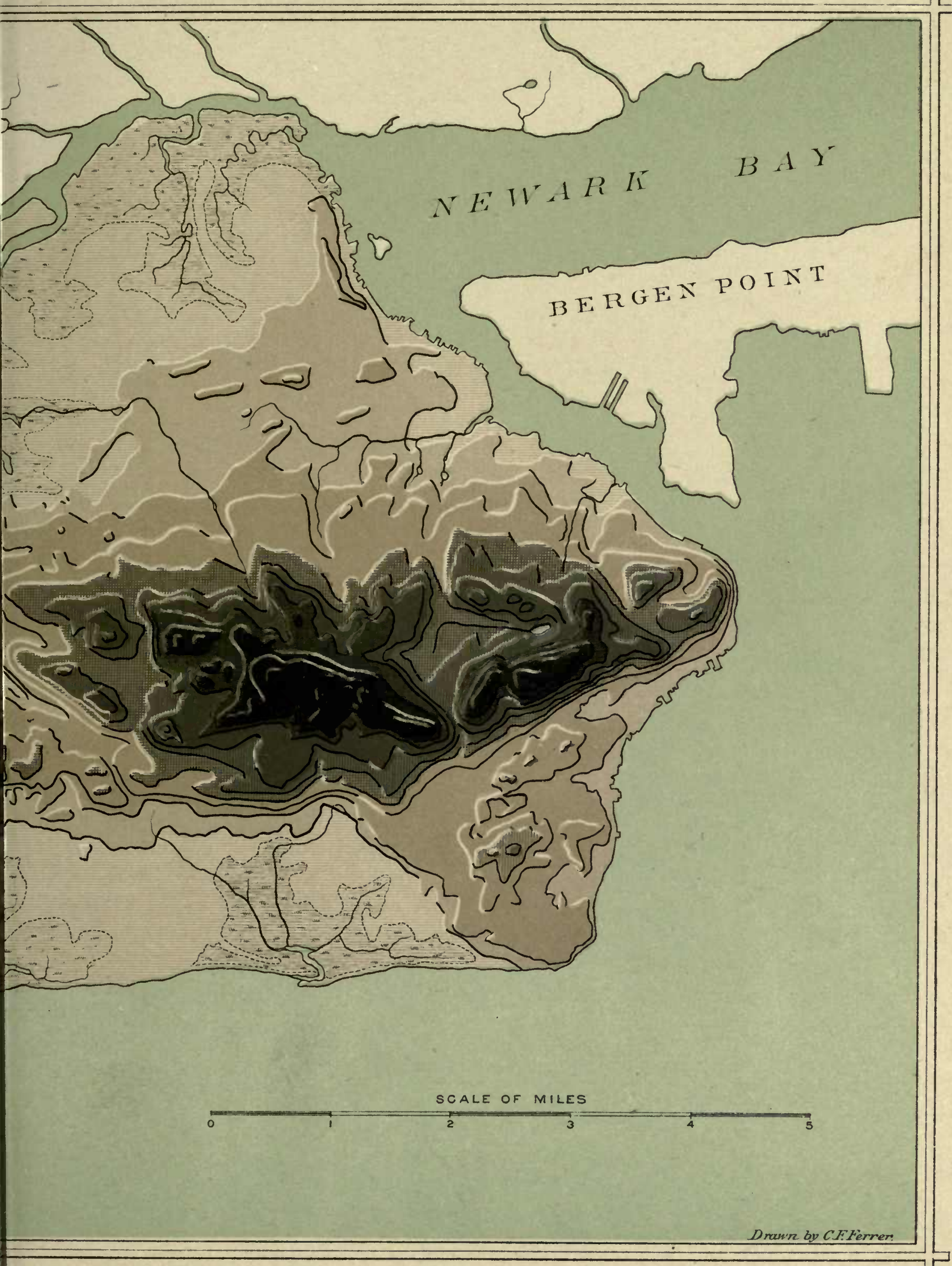


I.




ARTHUR
KILL
CONTOURED MAP
OF
STATEN ISLAND.
CONTOURS 20 FT. APART.

IX.

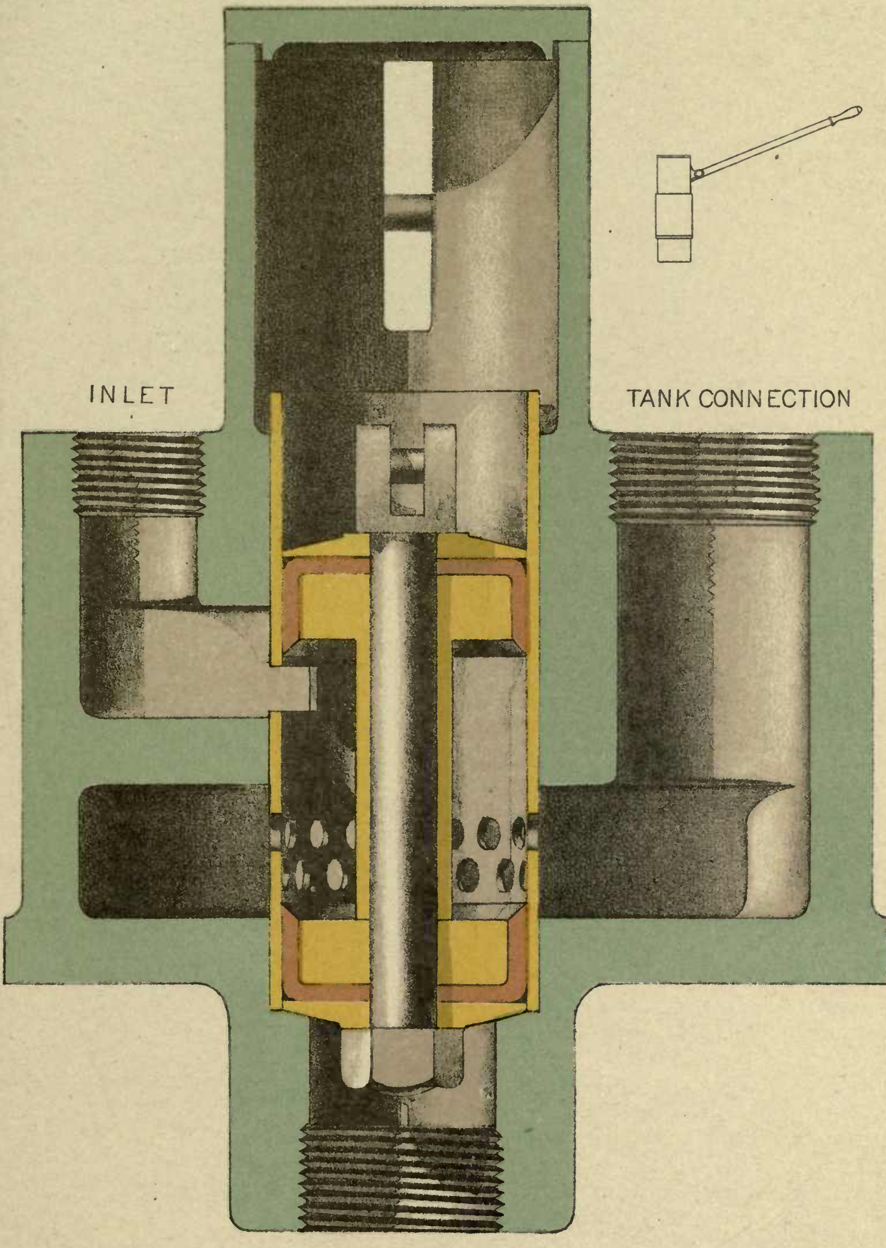

NEWARK BAY
BERGEN POINT
SCALE OF MILES
0 1 2 3 4 5
Drawn by C.F. Ferrer.

PL.X.

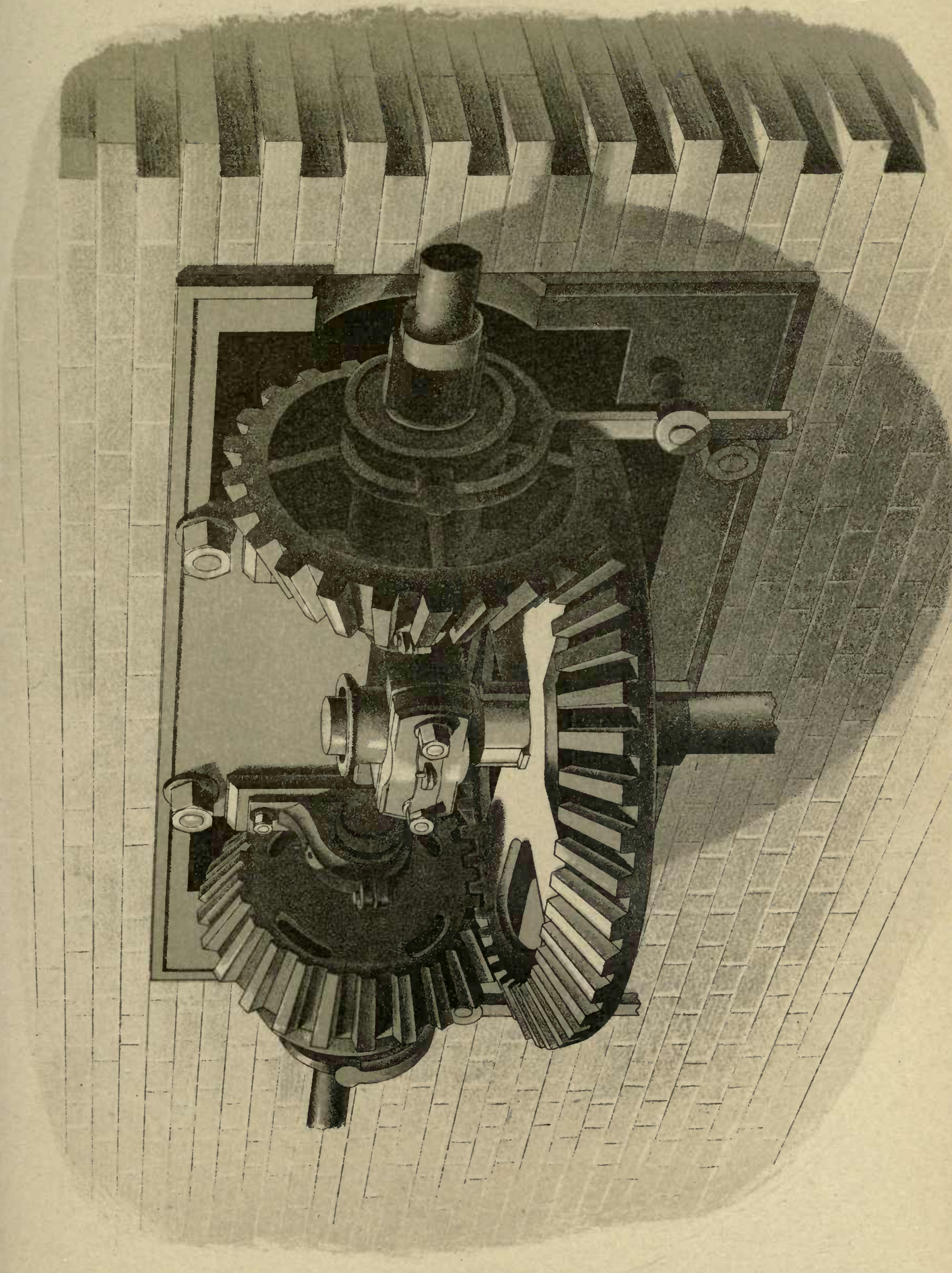
GEOLOGICAL MAP OF NEW JERSEY

GEORGE. H. COOK, STATE GEOLOGIST.

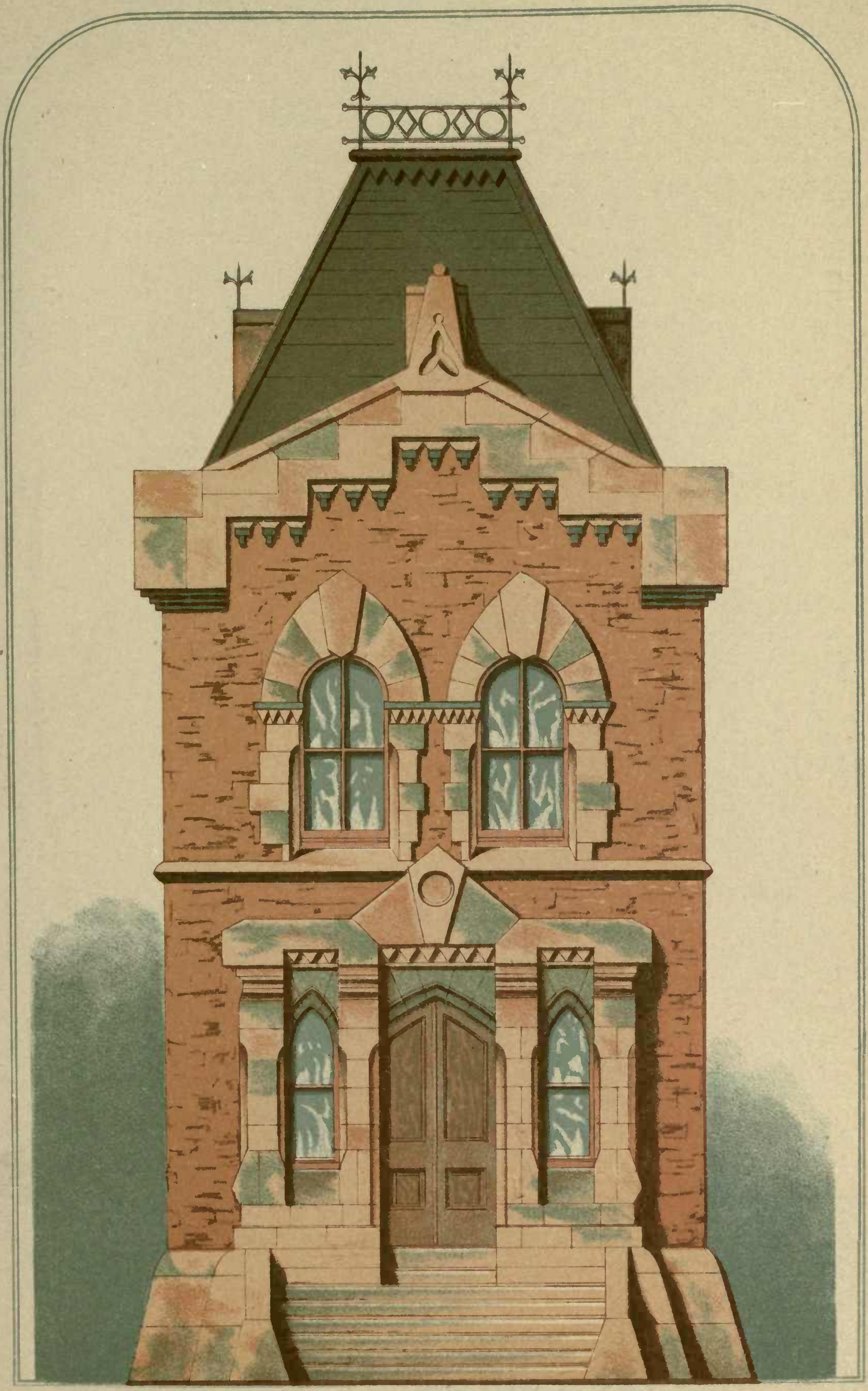
PL.XL



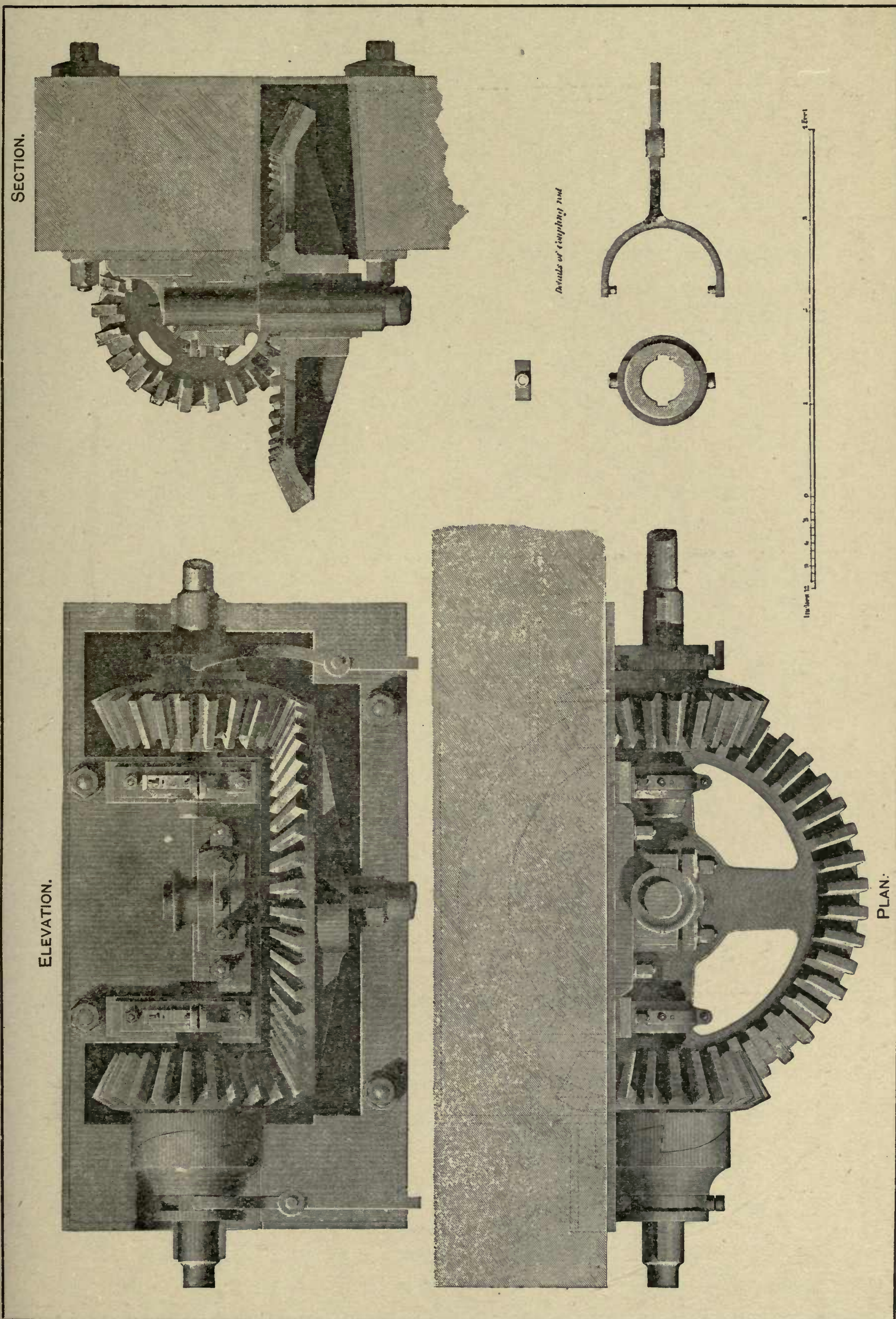
PL. XII.



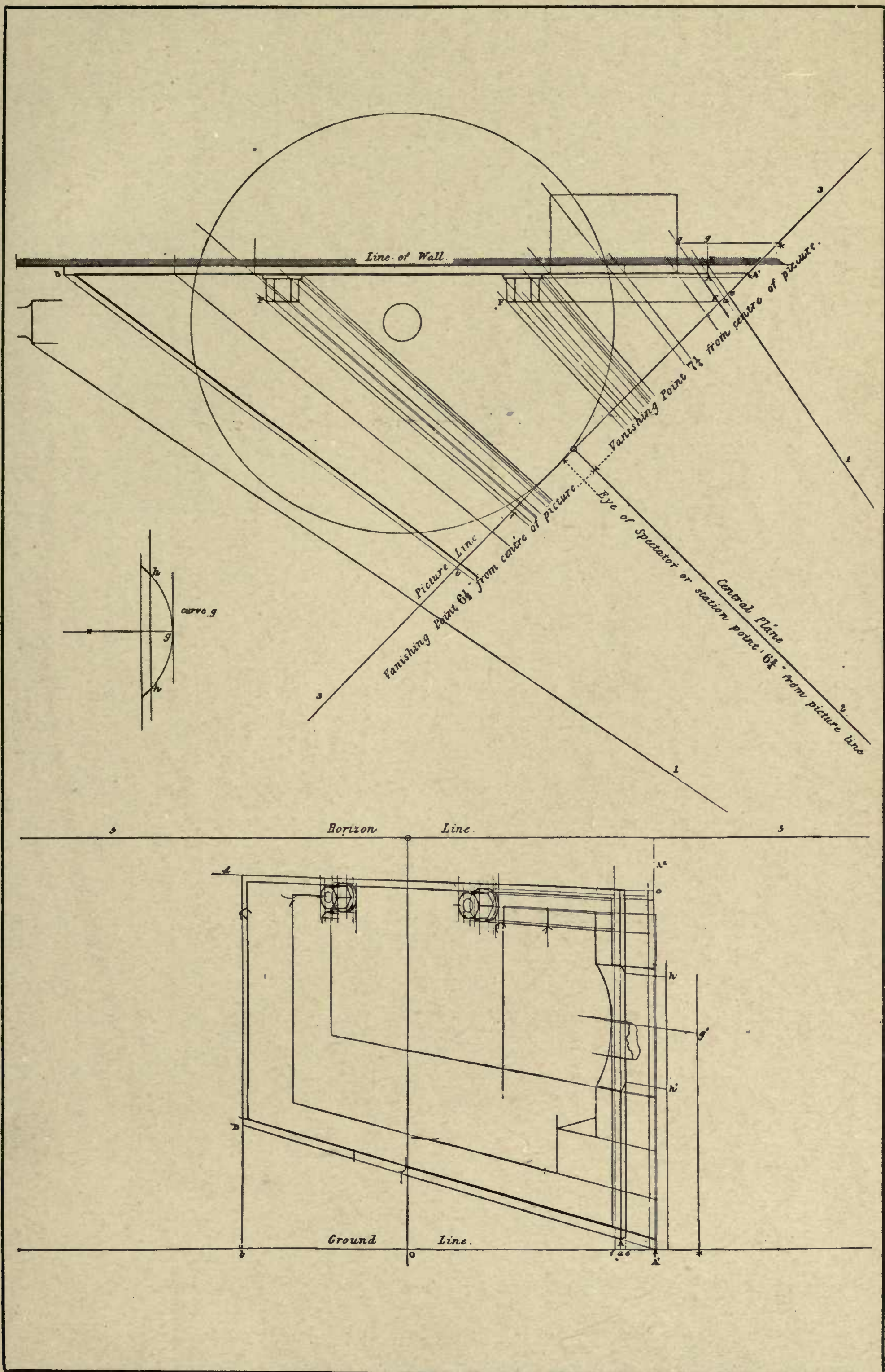
PL. XIII.



PL. XV.

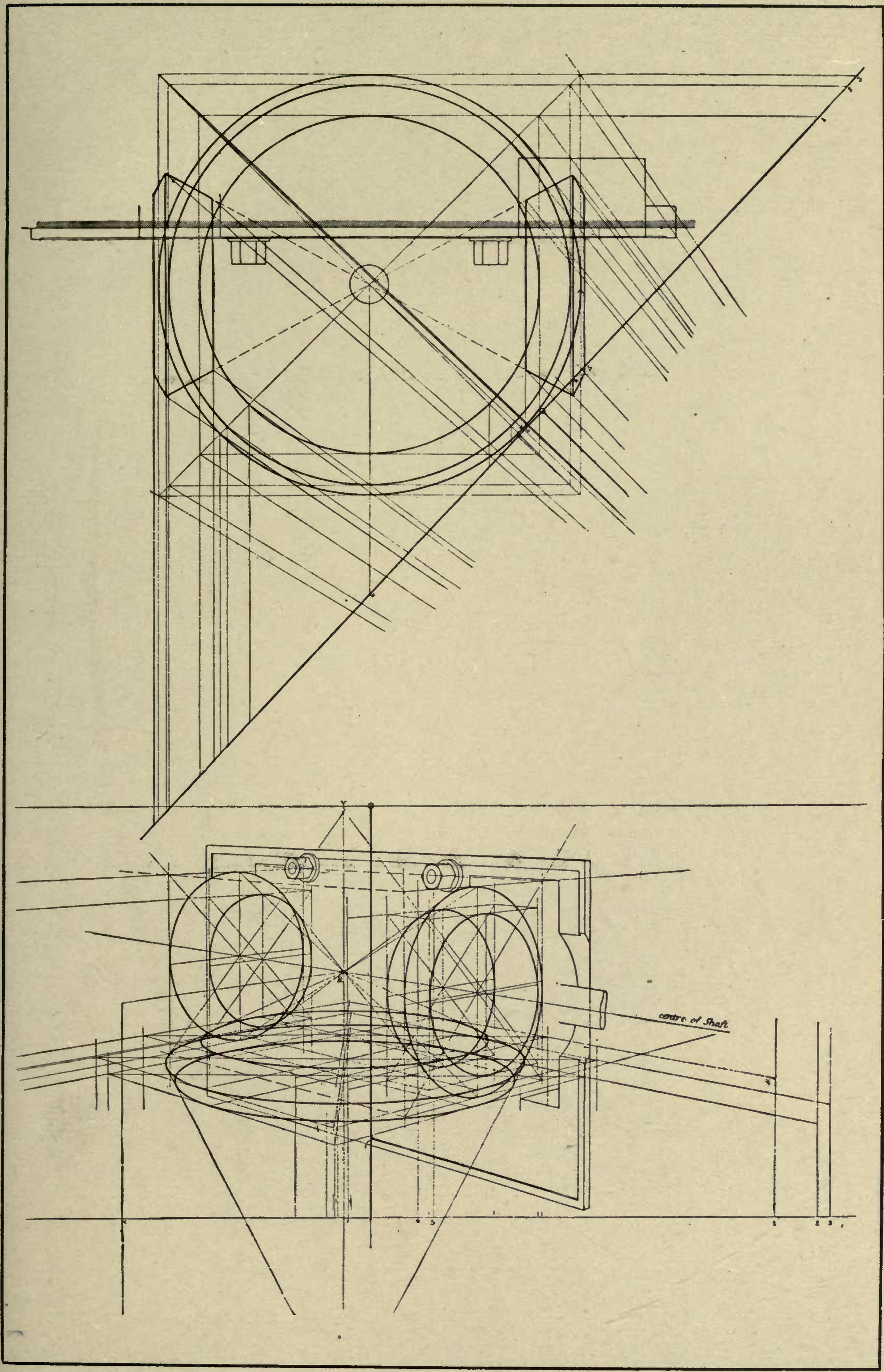


PL. XVI.

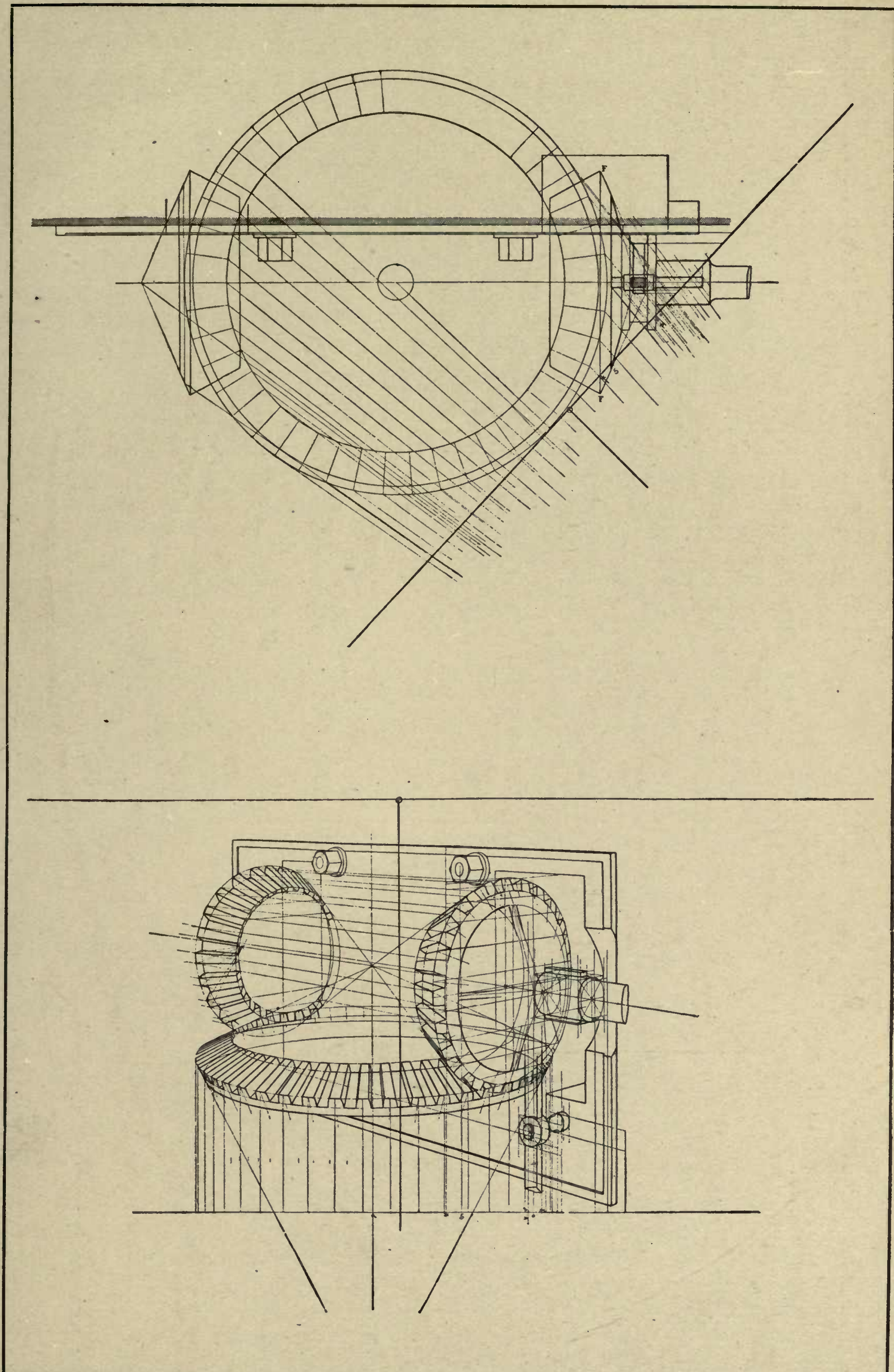


Line of Wall.
curve g
Picture Line
Vanishing Point 7¼ from centre of picture.
Vanishing Point 6¼ from centre of picture
Eye of Spectator or station point 6¼ from picture line
Central Plane
Horizon Line.
Ground Line.

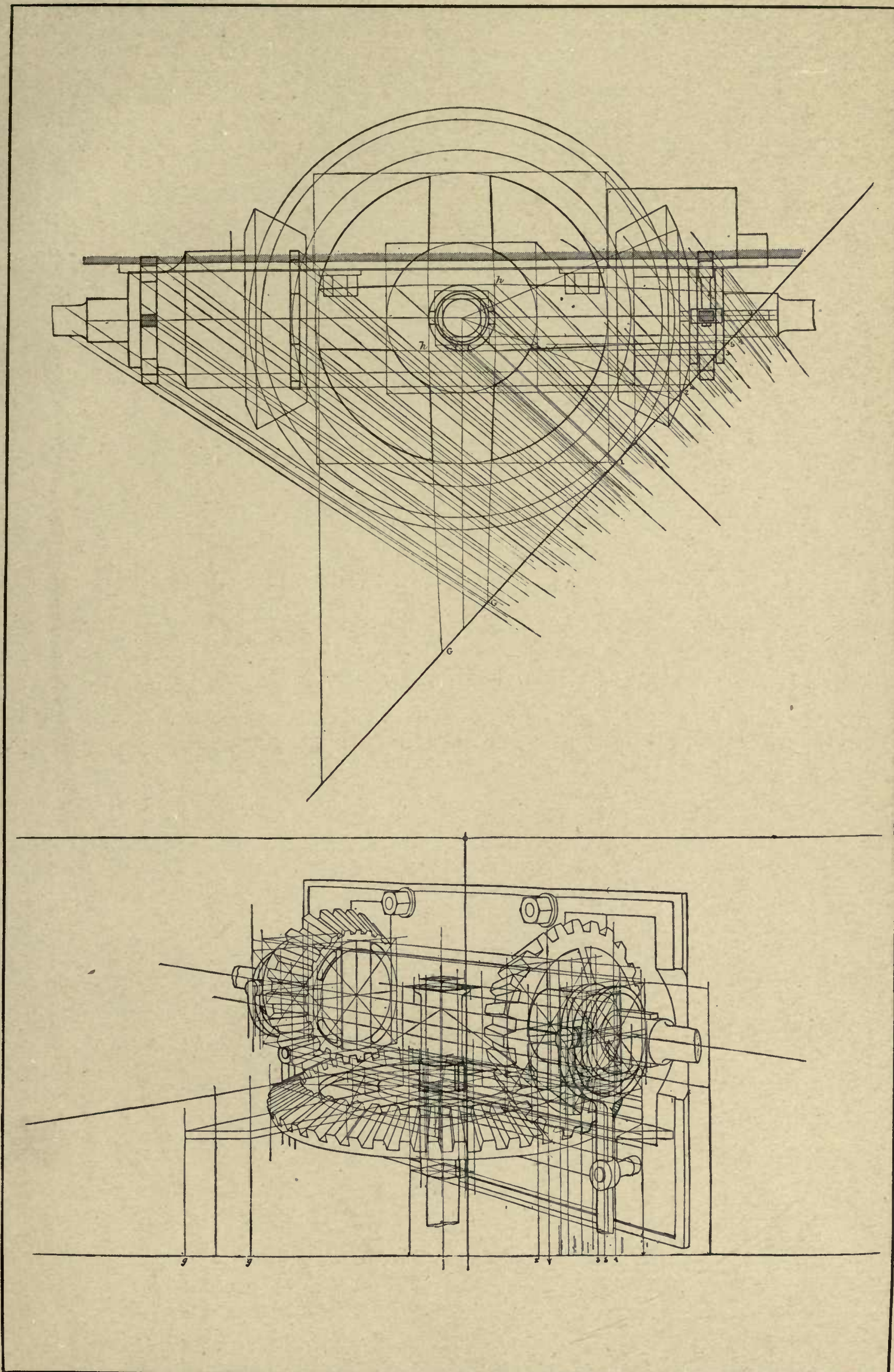
PL. XVII.



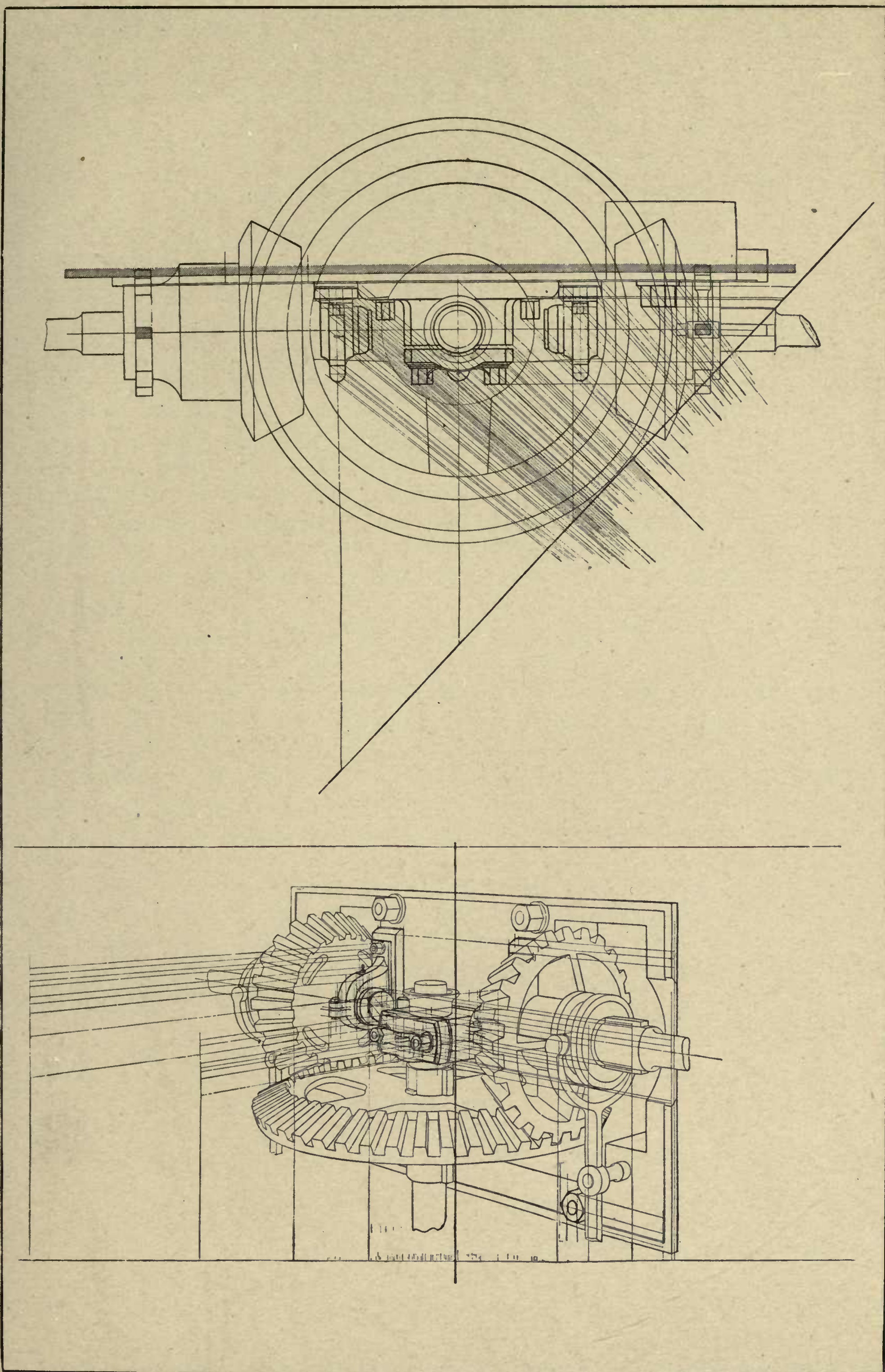
PL. XVIII.



PL. XIX.



PL. XX.

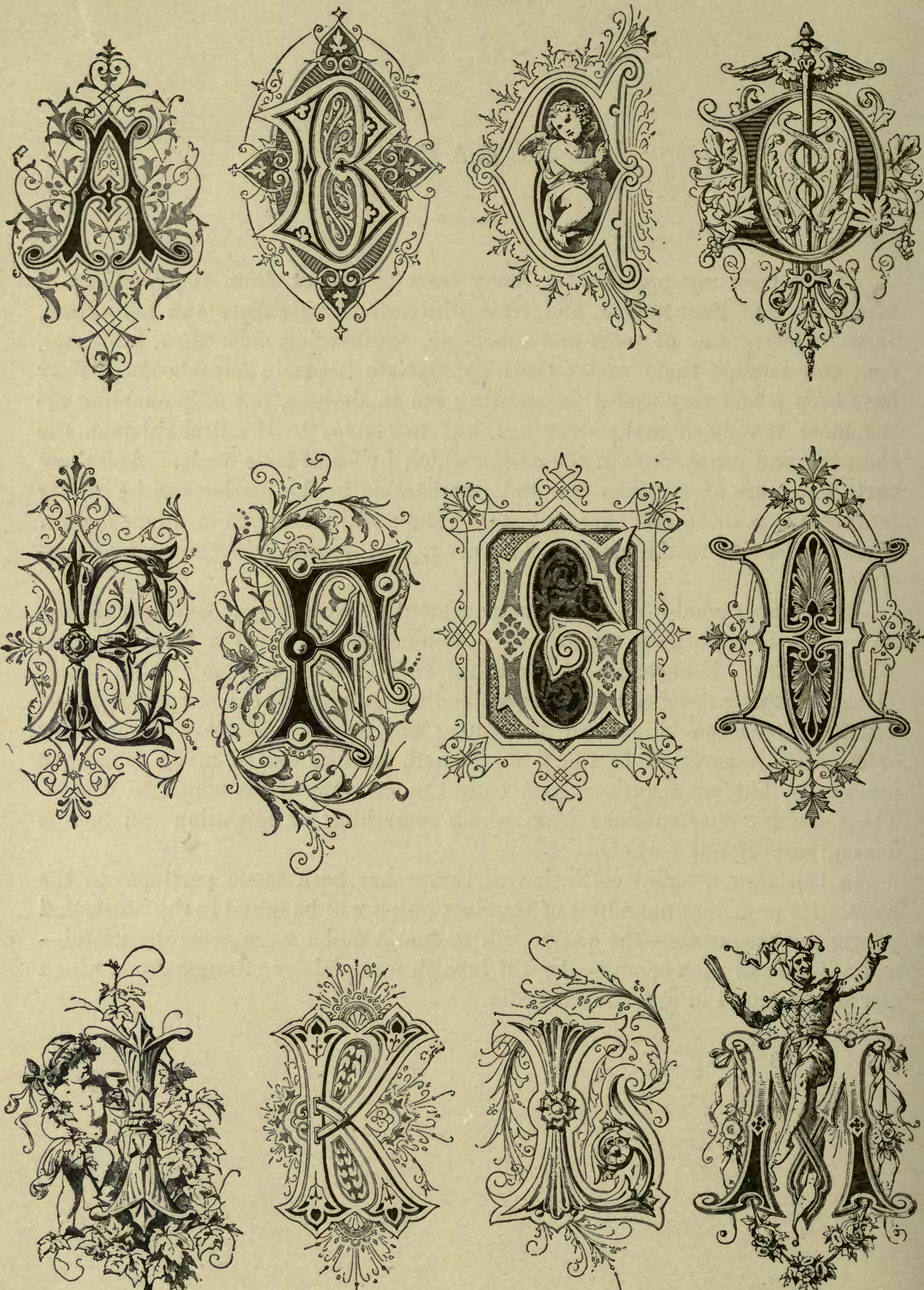


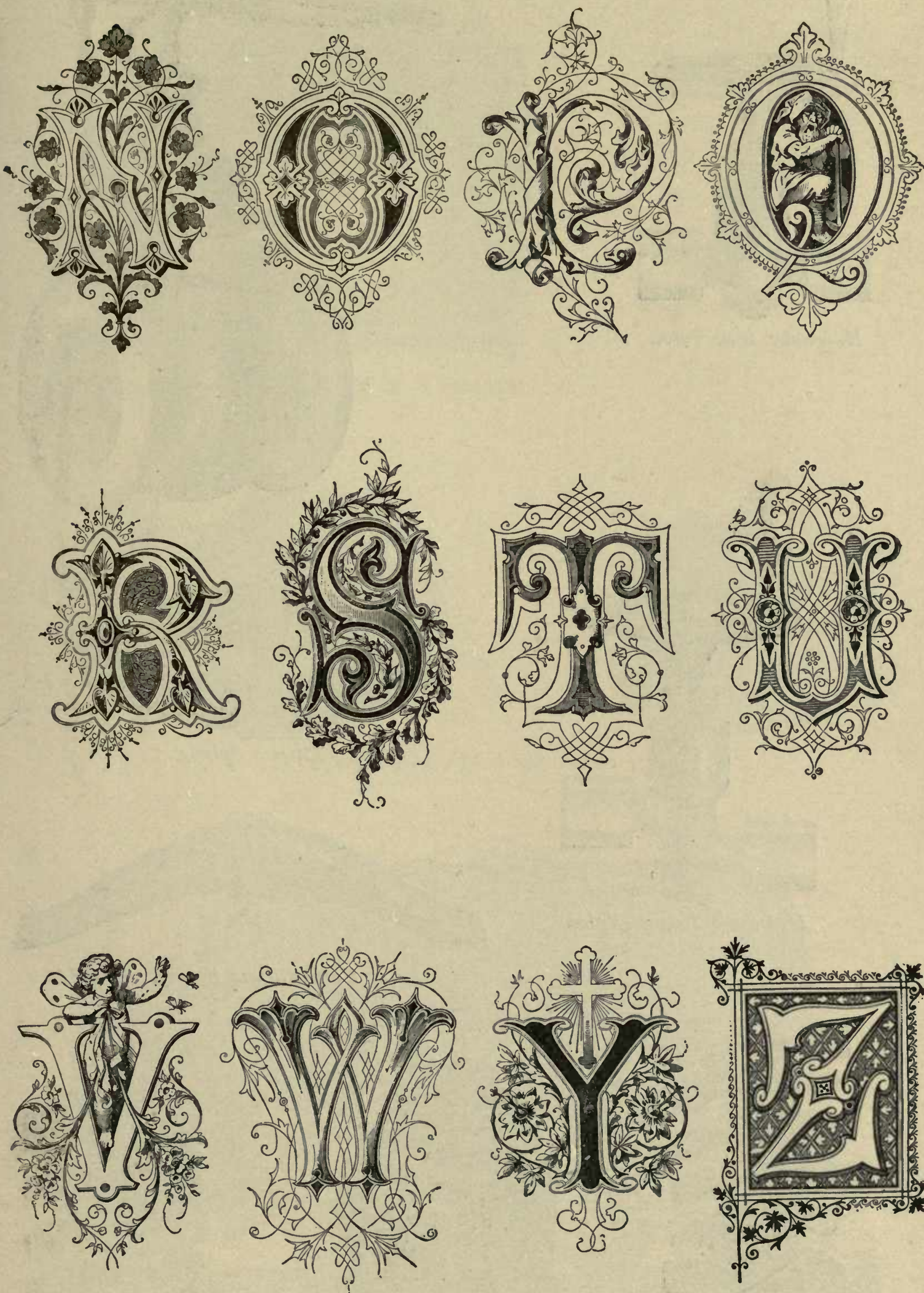
SCRAPS.

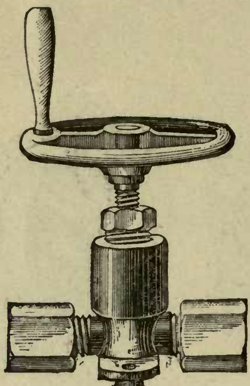
It has been my practice for many years to collect, from the circulars of mechanics and their agents, and from illustrated newspapers and magazines, varied illustrations of tools and machines, engineering structures, buildings, etc., and arrange them under their appropriate heads in scrap-books. They have been found very useful in assisting me in designs, not only enabling me the more readily to make drawings, but to convey to the draughtsman the character and proportions of the design which I wish to have made. And those parts which are of common use and purchasable in the market can be readily arranged in position and executed more economically than from a new design. There is a saving in the matter of drawing, and a saving in the cost of construction.

By a proper combination and arrangement of parts which have practically served a purpose, a more satisfactory design can be made than from attempts at originality. Knowledge of what has been done is economy in all labor. When the thing itself can not be seen bodily, its picture can supply its place, and its details can be studied at leisure; and, as the education of the eye is of essential importance to the draughtsman, let him see as much as he can practically, but yet acquire a good collection of scraps from which to design. There are few constructions from which something of education can not be drawn, parts if not a whole.

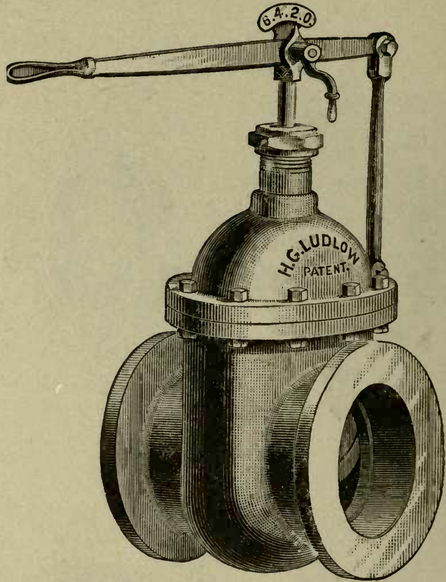
In this view a small collection of scraps has been made pertinent to the book. Its page does not admit of the sizes which will be found in the illustrated papers and magazines—the quarto will be found much more generally useful—and a library of such scrap-books will furnish material for a draughtsman which can not be found in any encyclopædia.



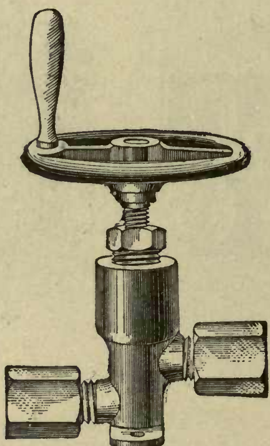




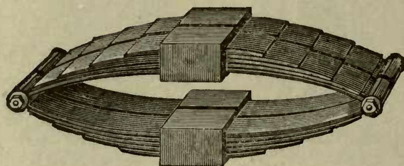
Hydraulic Stop-Valve.



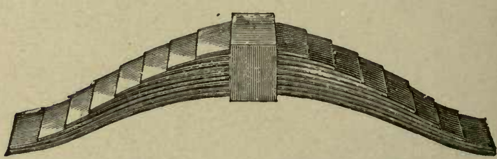
Lever-Gate.



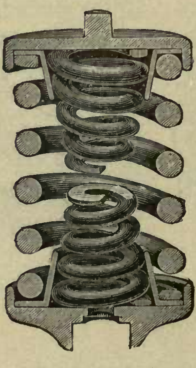
Hydraulic Release-Valve.



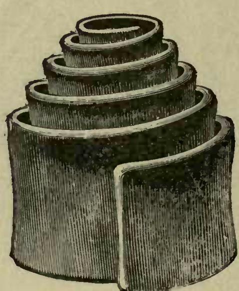
Elliptic Spring.



Half Elliptic Spring.

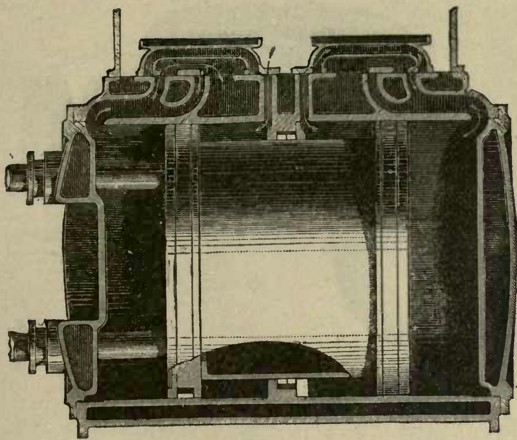


Vose Graduated Spring.

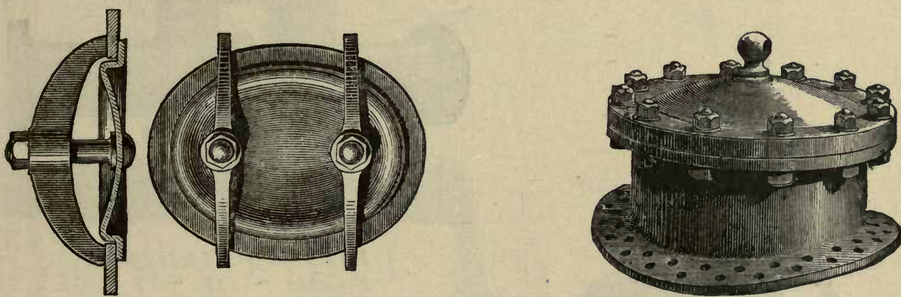


Volute Spring.

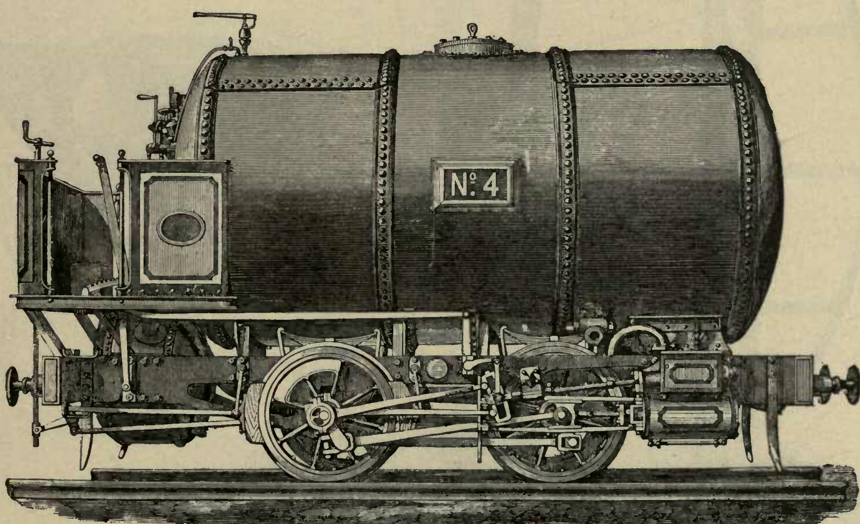
Oval Bar Spring.



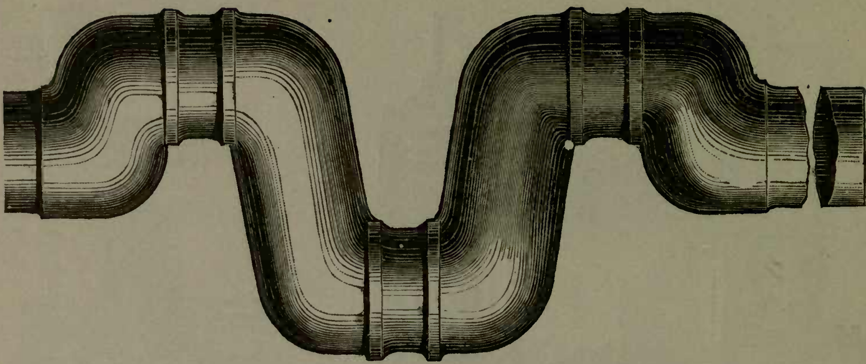
Compound Steam Cylinders. H. M. S. Spartan.



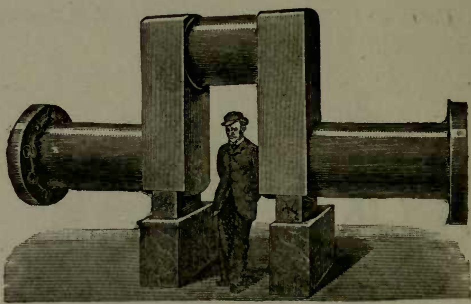
Wrought-Iron Plates and Covers.



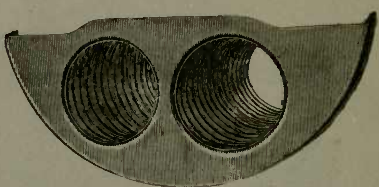
Compressed-Air Locomotive, St. Gothard Tunnel.



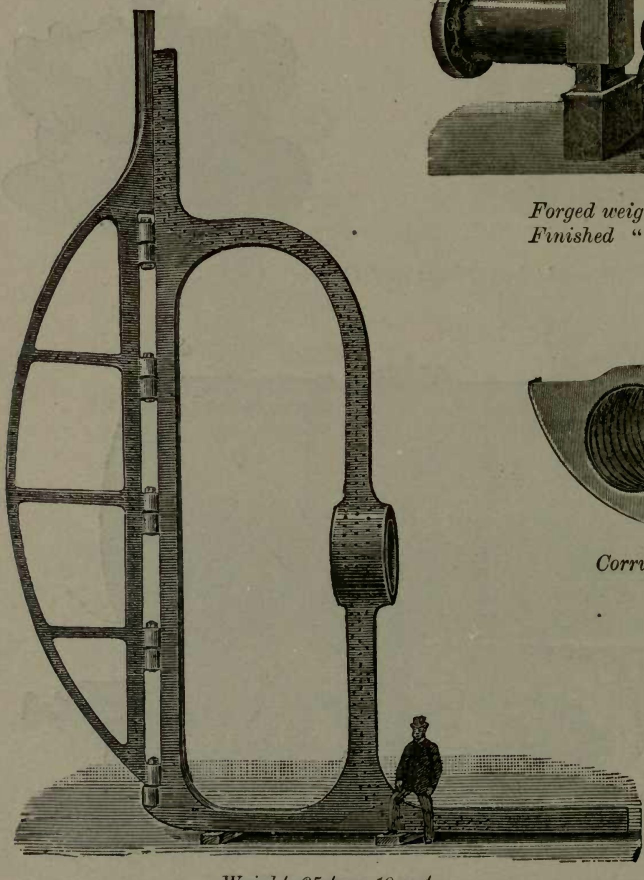
Three-Throw Crank.



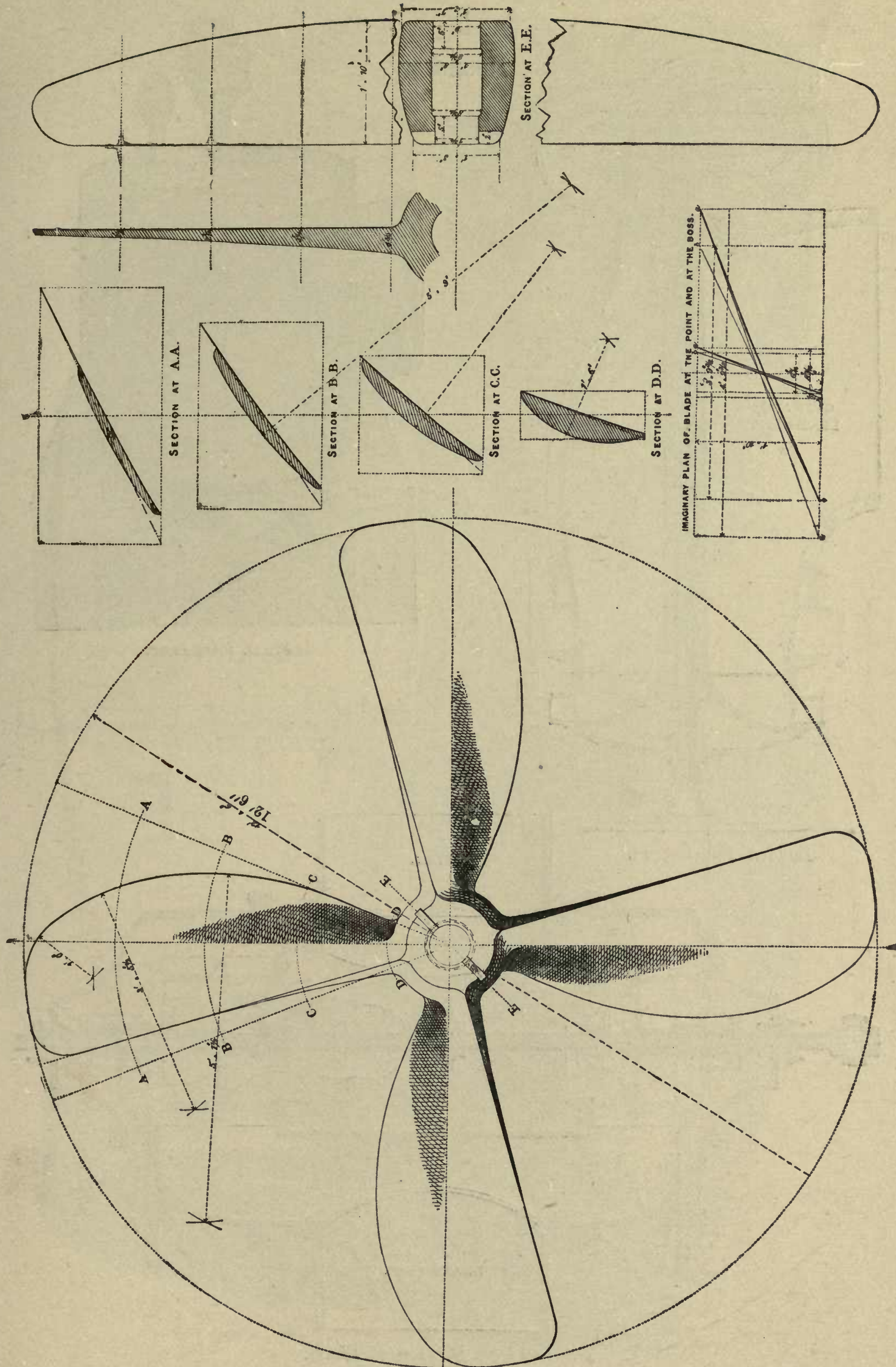
Forged weight, 24 tons 11 cwt.
Finished " 15 " 8 "



Corrugated Boiler-Flues.



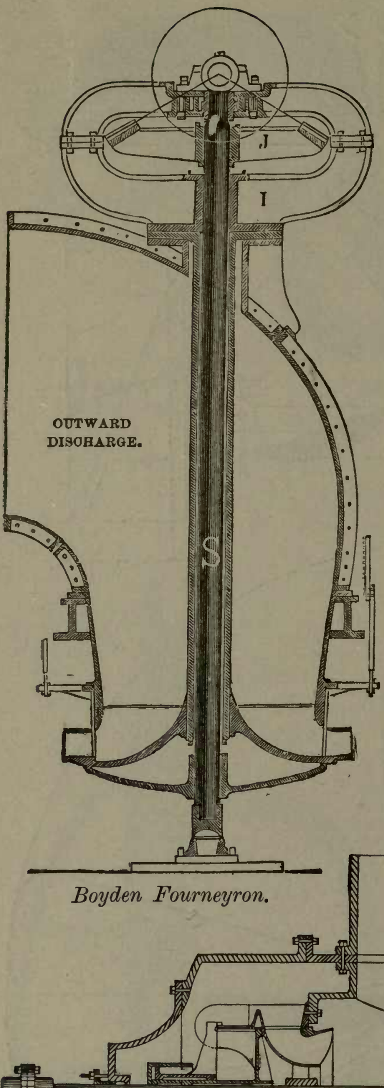
Weight, 25 tons 10 cwt.



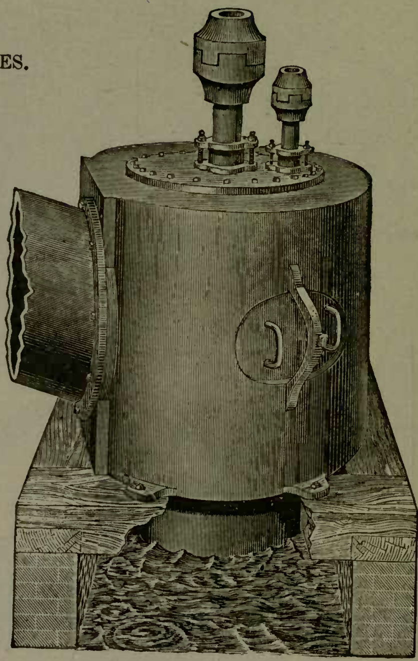
Screw Propeller.

Vessel, 1400 gross tons. Engines, 130 nominal English horse-power.

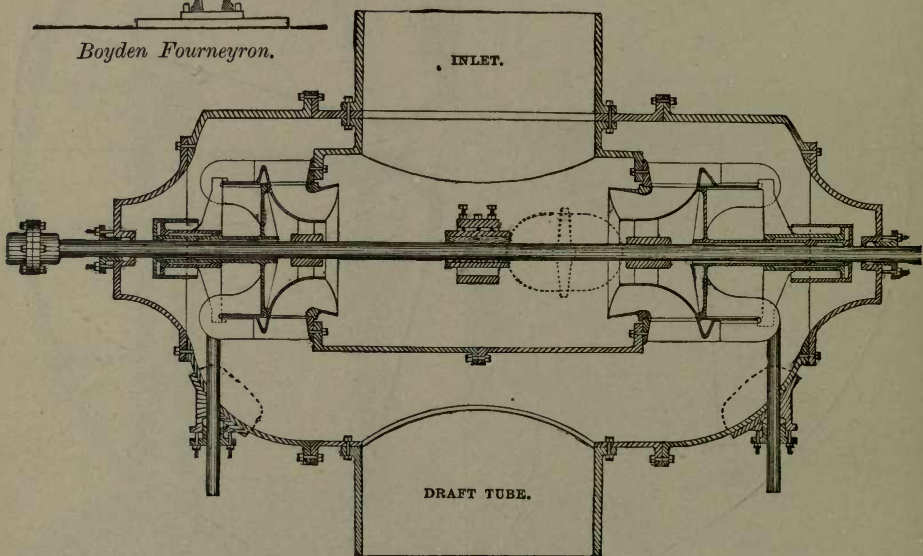
TURBINES.



Boyden Fourneyron.

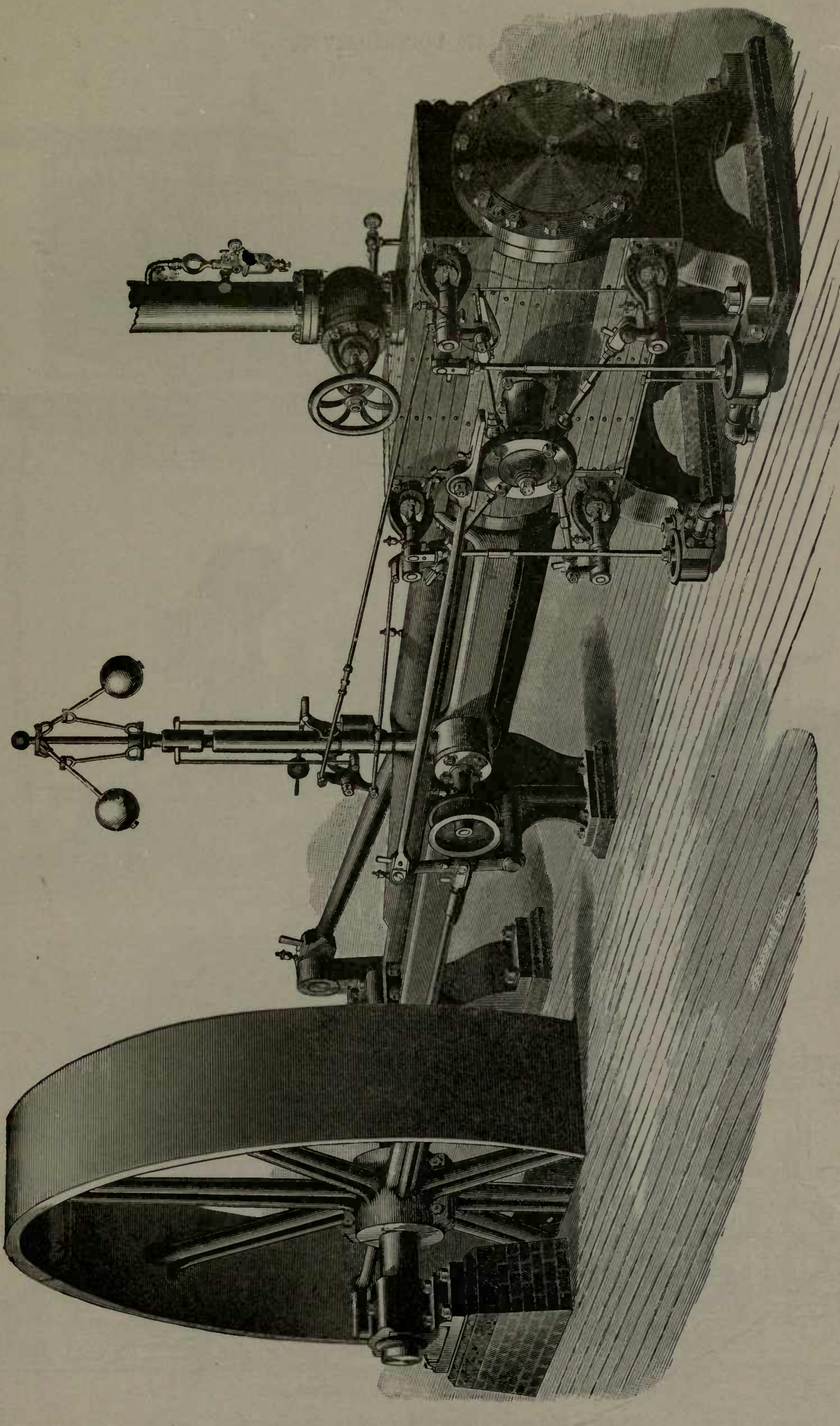


CENTRAL DISCHARGE.



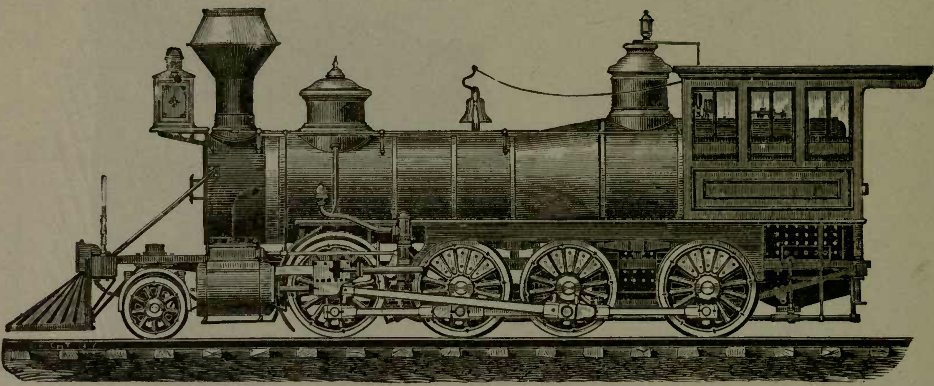
Turbine with Horizontal Shaft.

Longest draft tube, at Manchester, N. H. 26 feet from center of shaft to tail water. Fall, 40 feet.

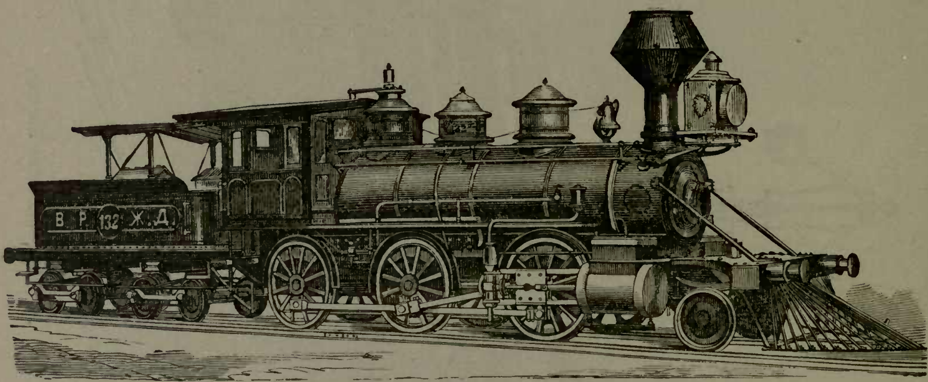


Corliss Steam-Engine.

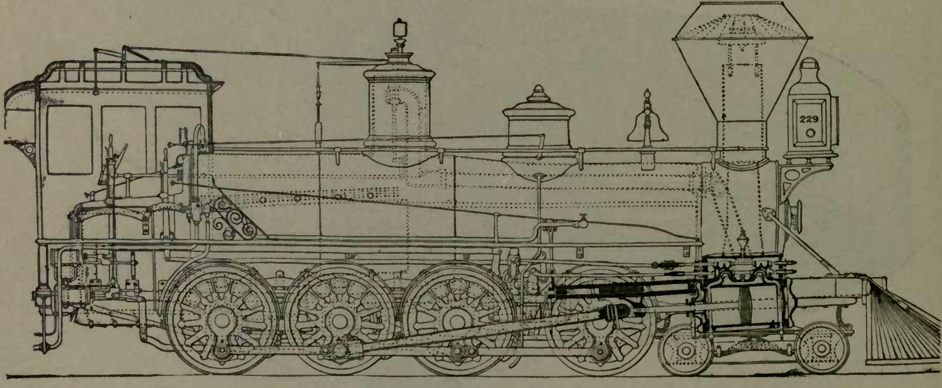
AMERICAN LOCOMOTIVES.



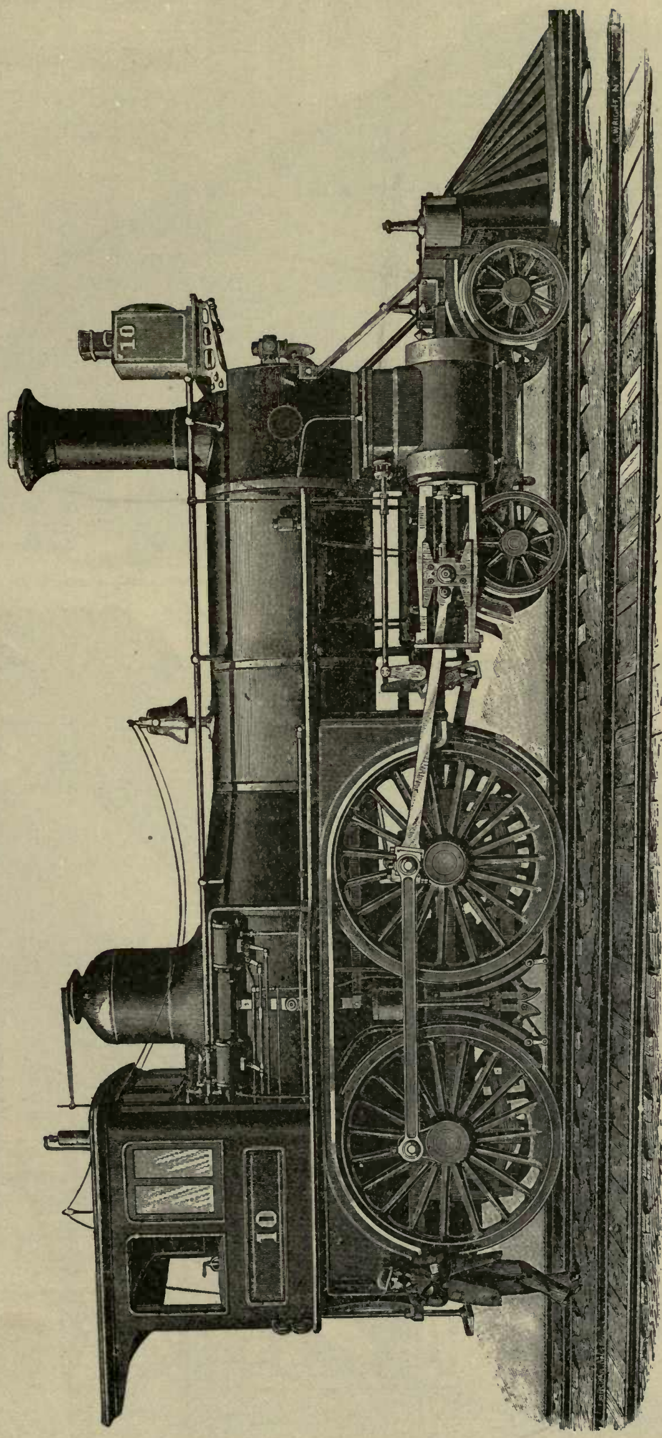
The Consolidation.



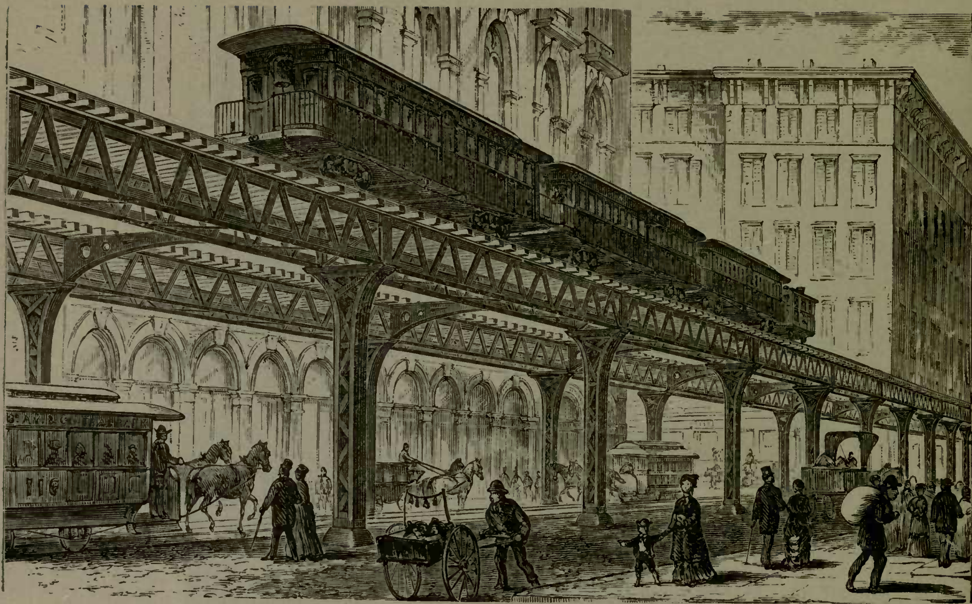
The Mogul.



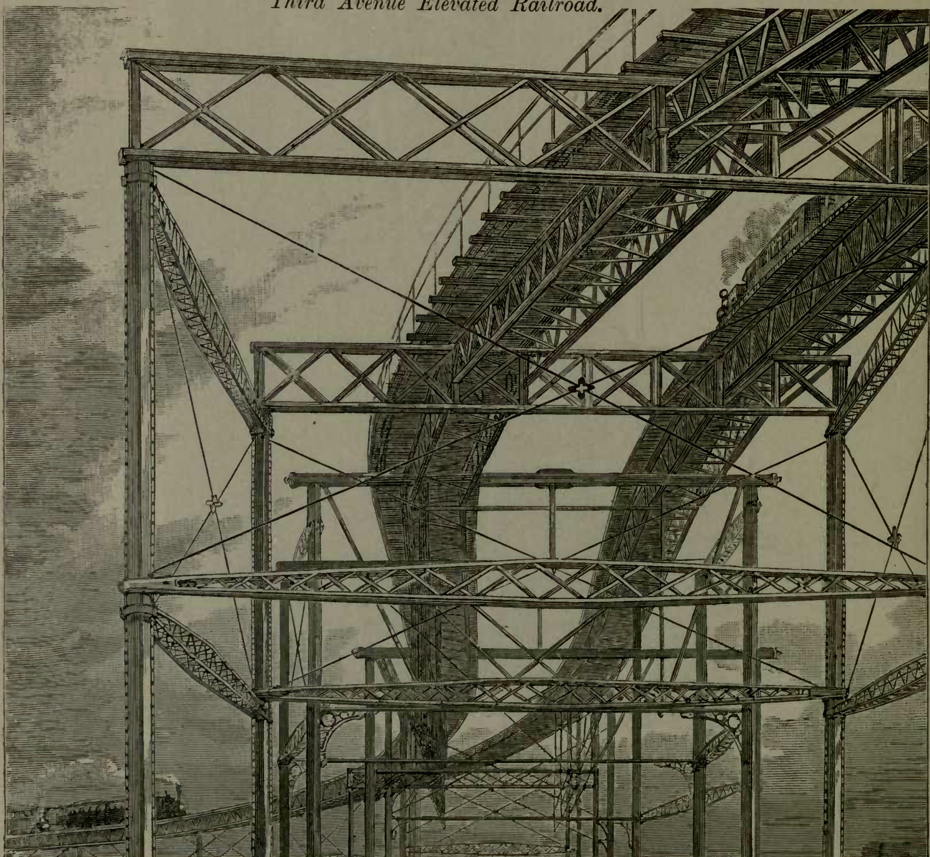
Twelve-Wheeler, Central Pacific Railroad.



Standard Passenger-Engine, Pennsylvania Railroad.

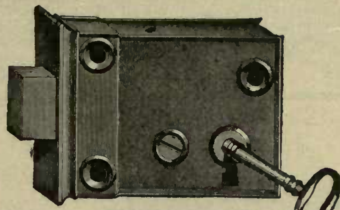


Third Avenue Elevated Railroad.

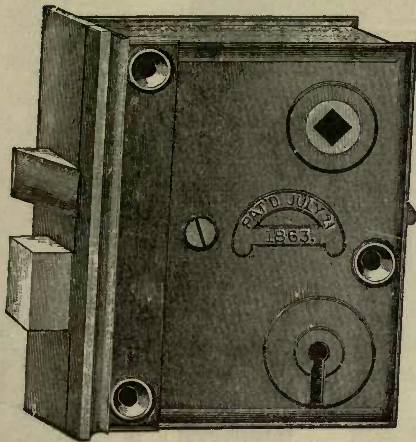


Curve at Eighth Avenue.

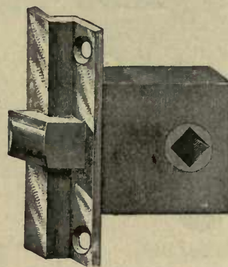
BUILDERS' HARDWARE.



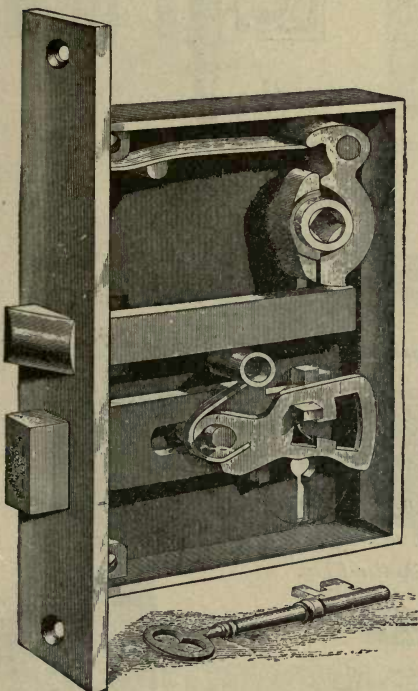
Rim Dead-Lock.



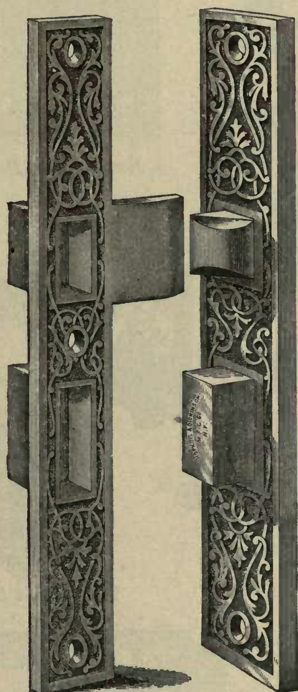
Rim Knob-Lock.



Rabbeted Knob-Catch.



Mortise-Lock, cover off.



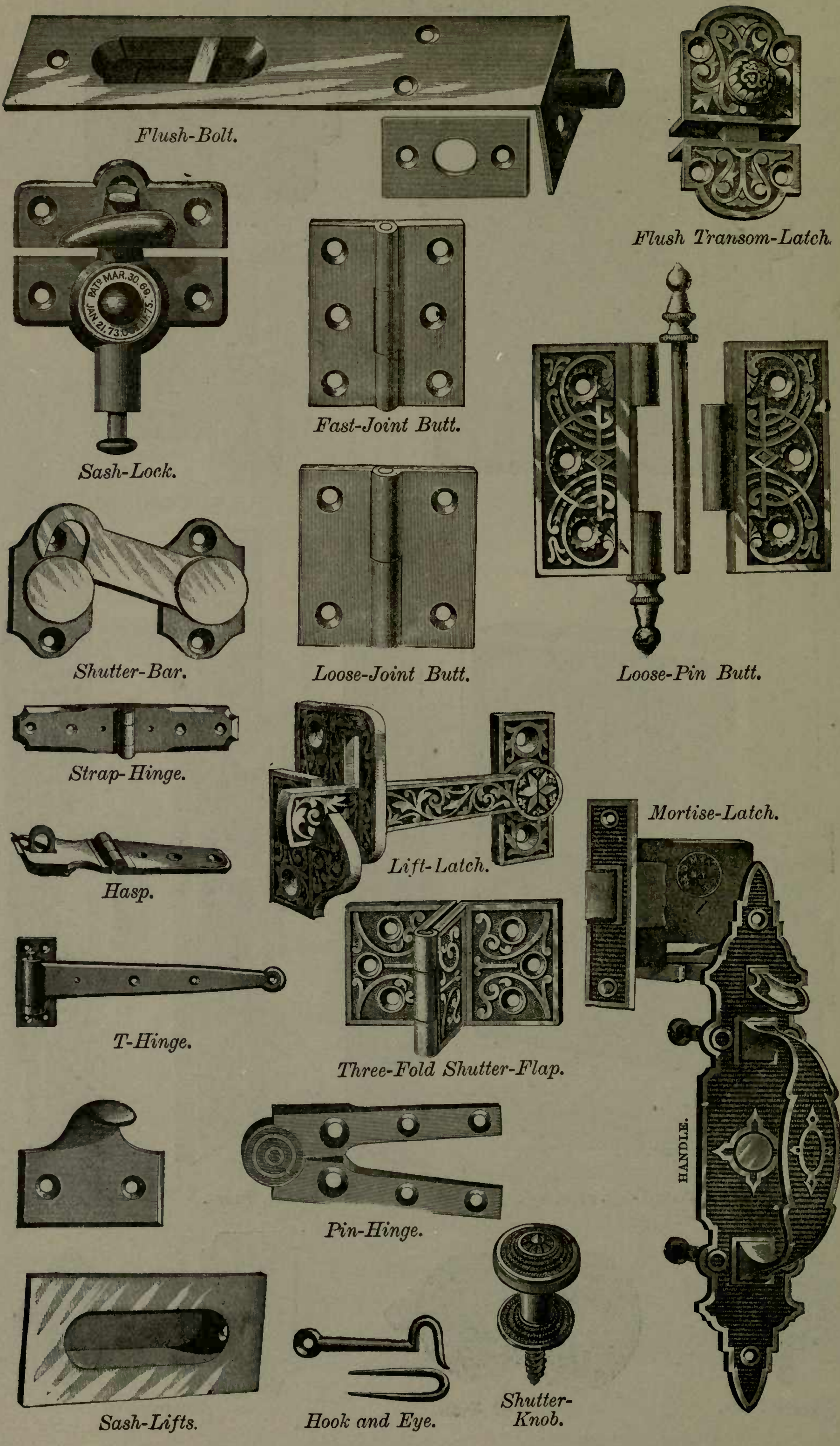
Boxed Strike, Front.

Front Sliding-door Lock.

Thumb-Piece.

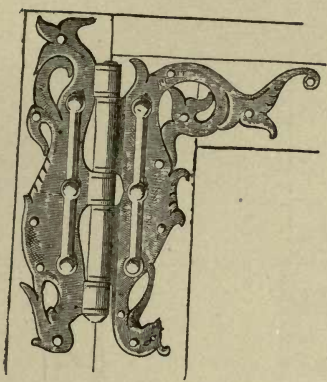
Knob and Rose.

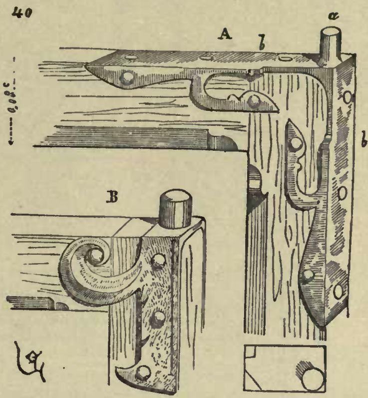
Escutcheons.

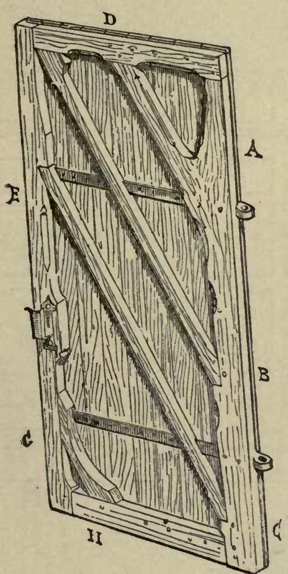

Flush-Bolt.
Flush Transom-Latch.
Sash-Lock.
Fast-Joint Butt.
Shutter-Bar.
Loose-Joint Butt.
Loose-Pin Butt.
Strap-Hinge.
Lift-Latch.
Mortise-Latch.
Hasp.
T-Hinge.
Three-Fold Shutter-Flap.
HANDLE.
Pin-Hinge.
Sash-Lifts.
Hook and Eye.
Shutter-Knob.

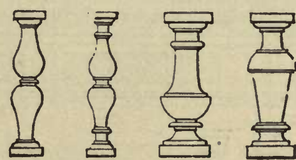
SCRAPS.

Examples of Ancient Hinges and Doors.

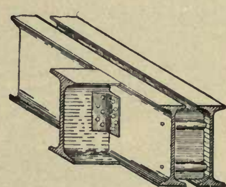


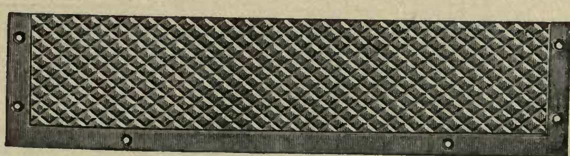




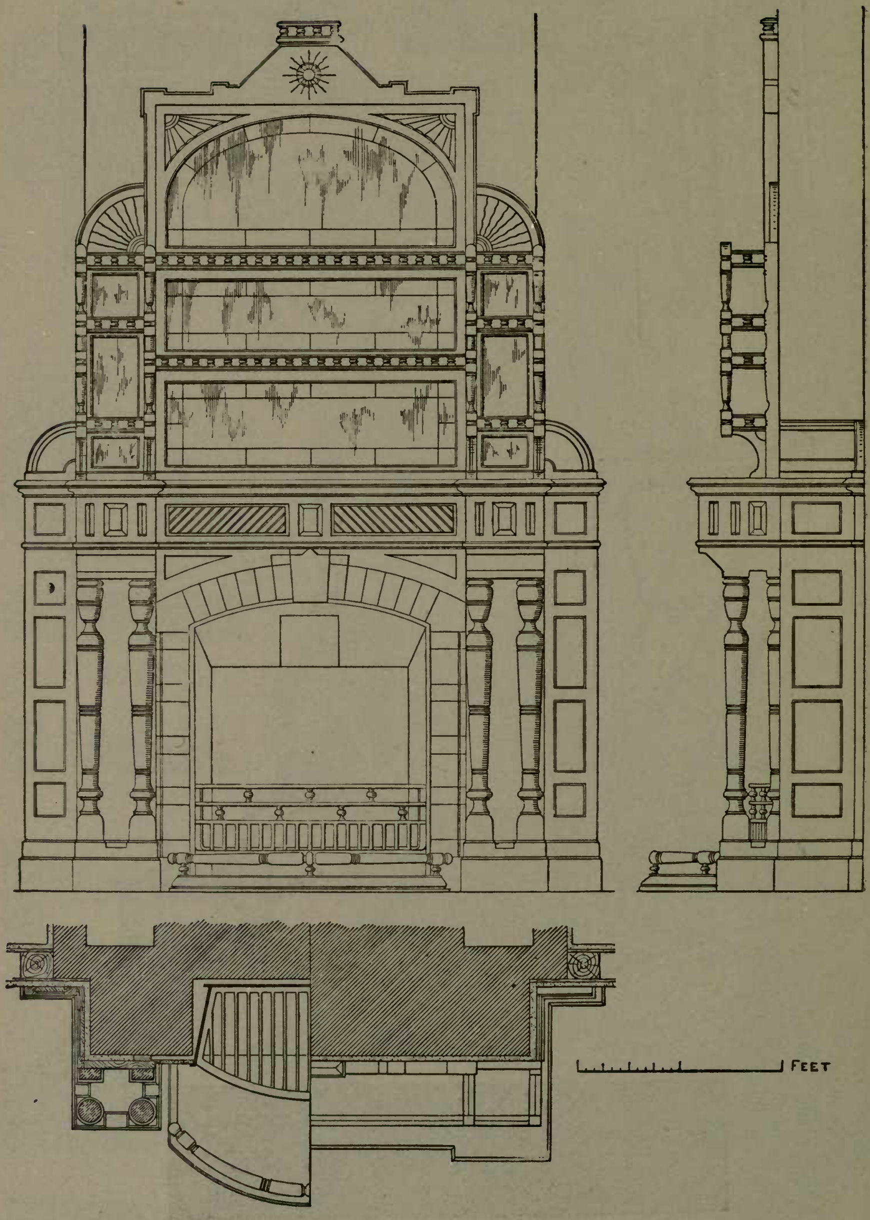


Balusters.

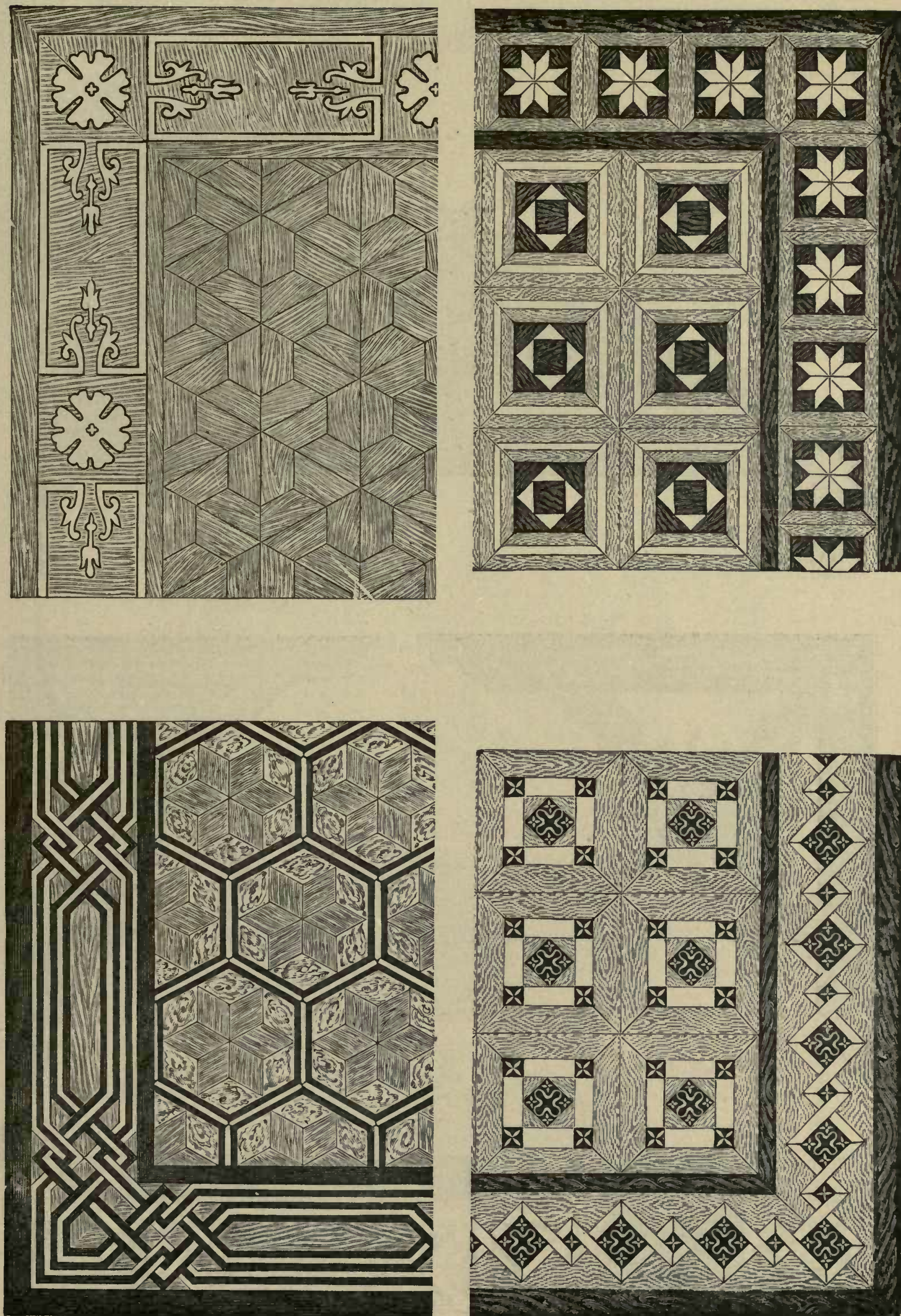




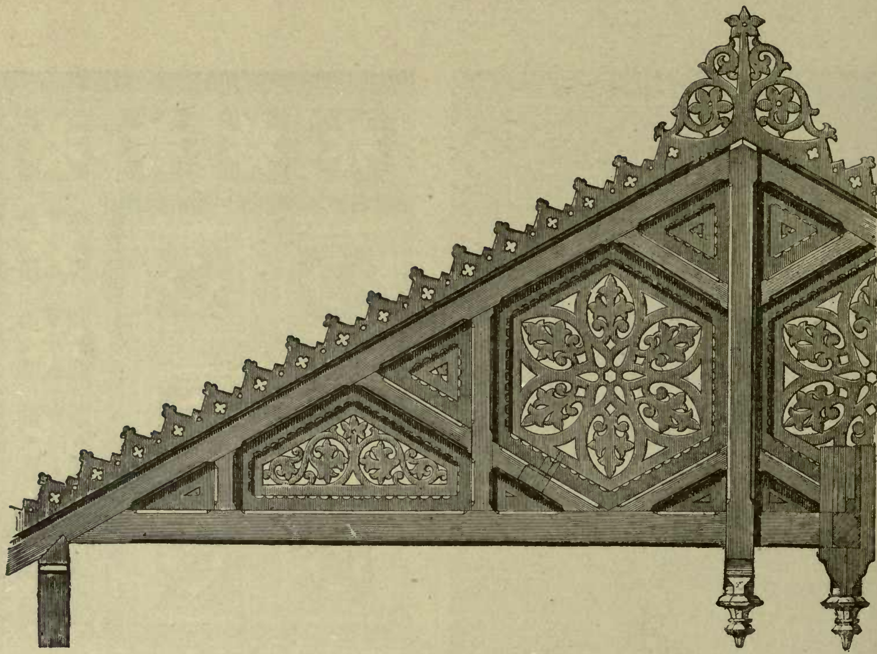
Cast-Iron Tread.

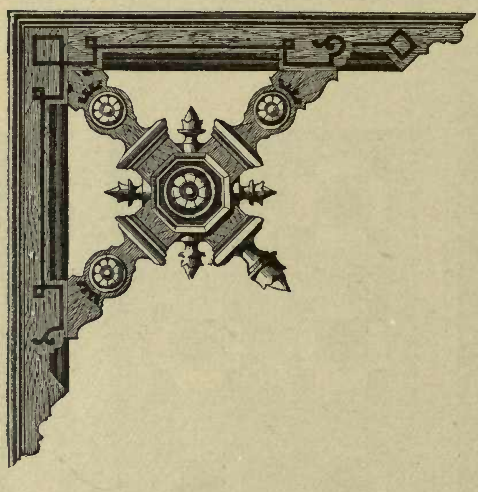


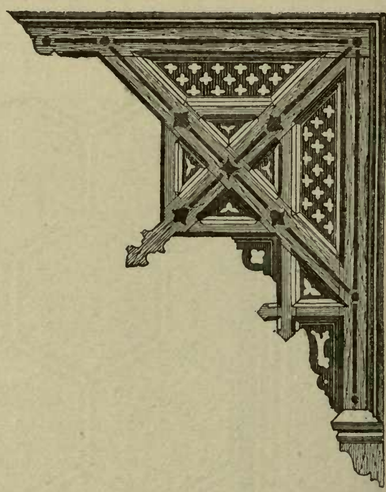
Plan, Section, and Elevation of a Wooden Mantel and Fire-Place.

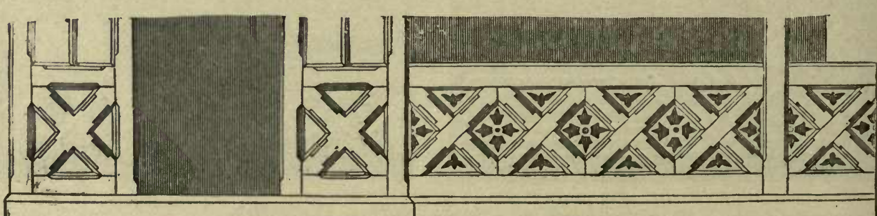


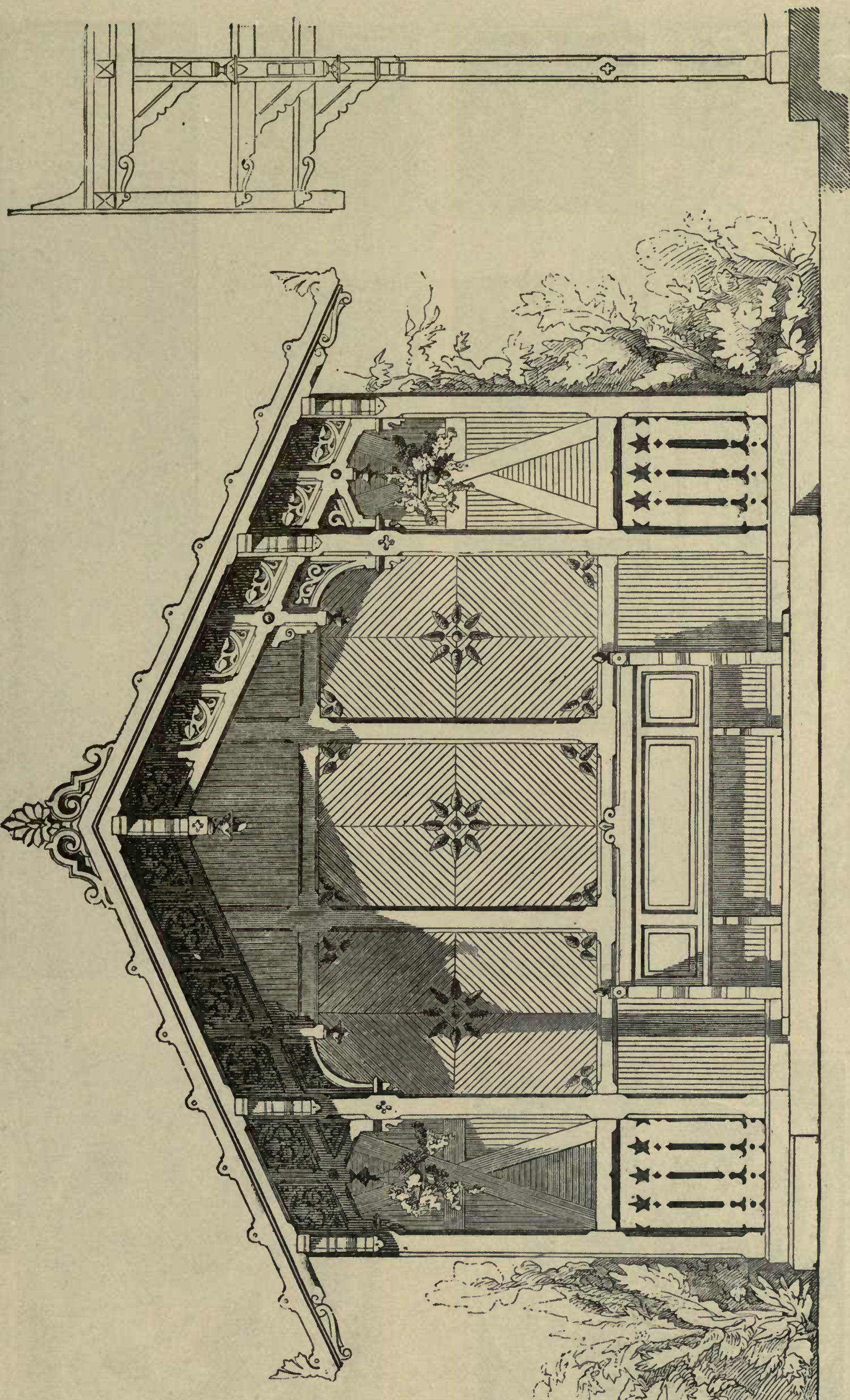
Examples of Inlaid Floors or Marquetry.

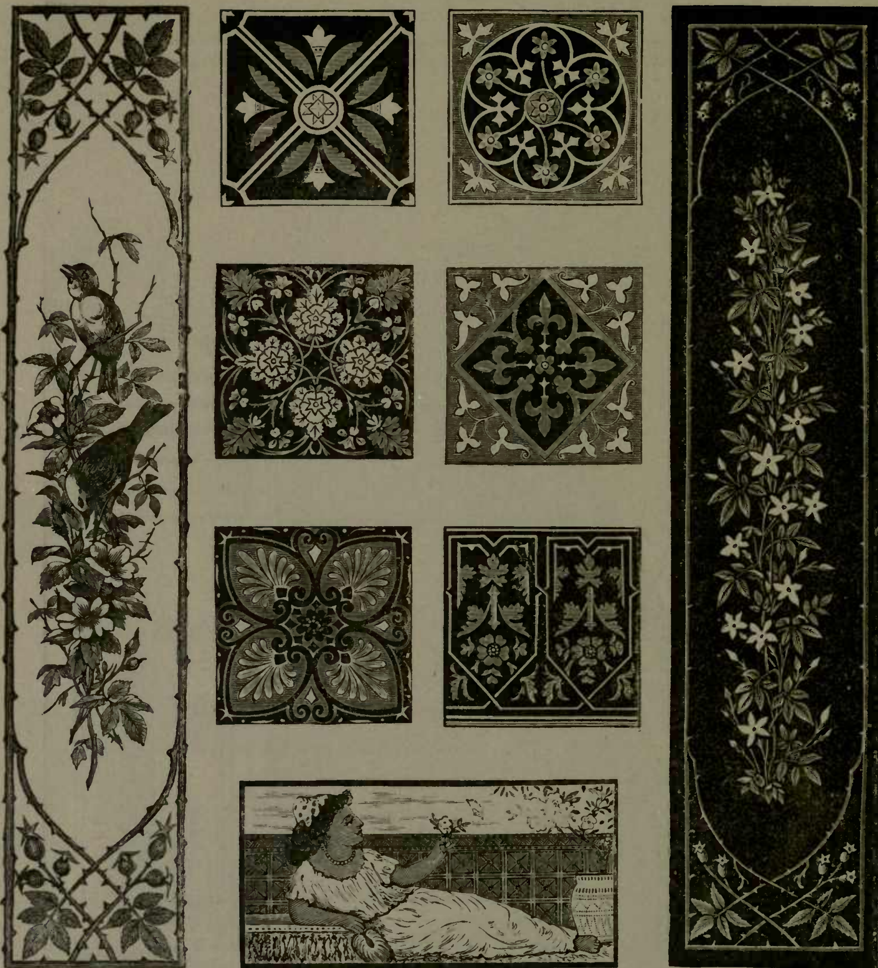




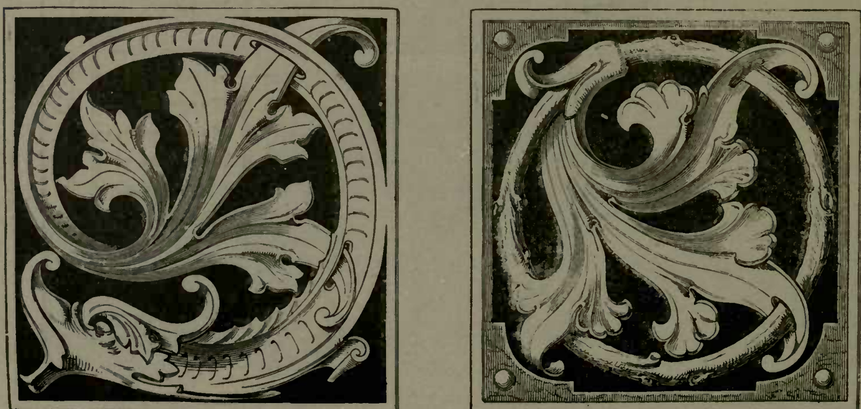






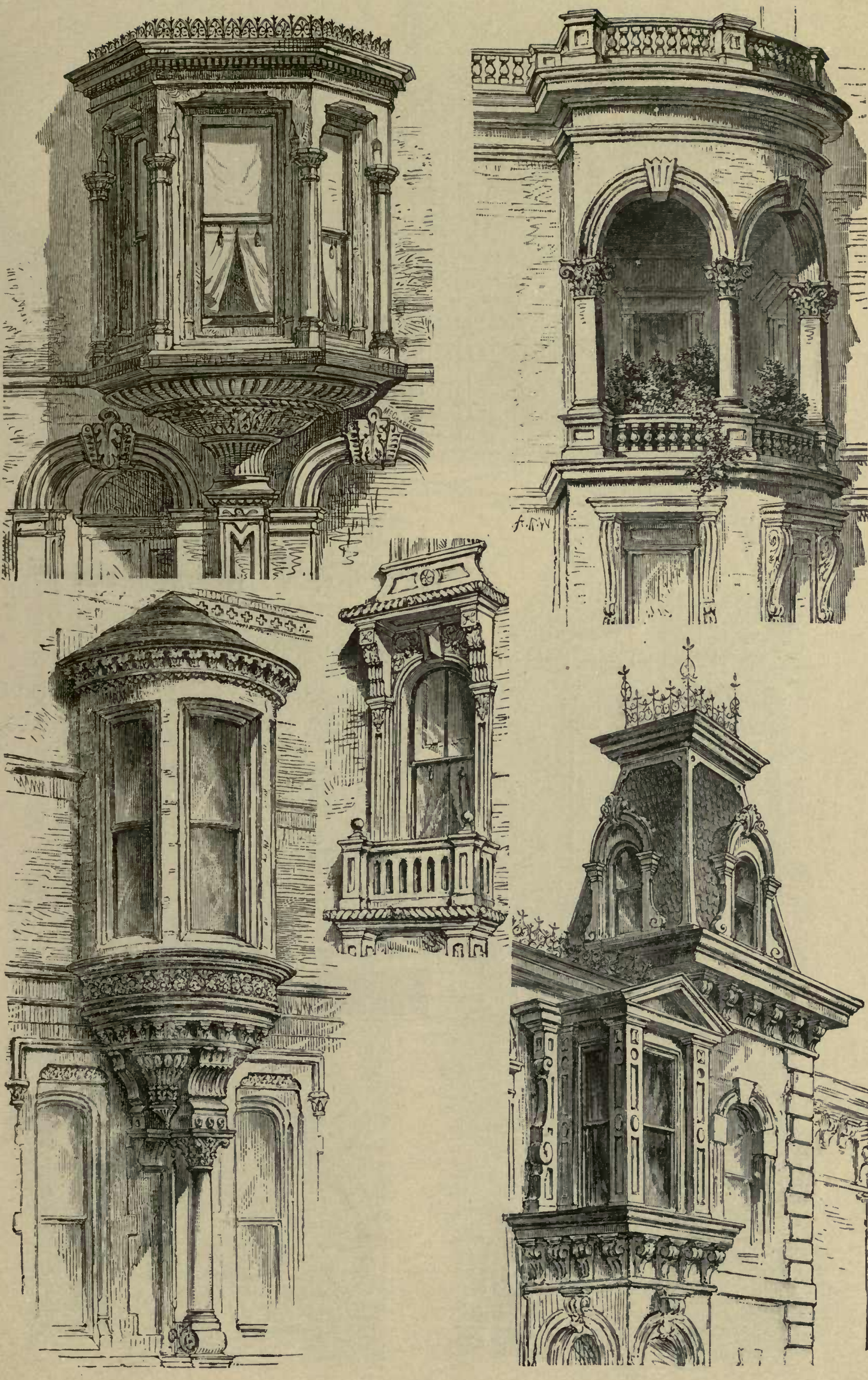


Enameled Tile.

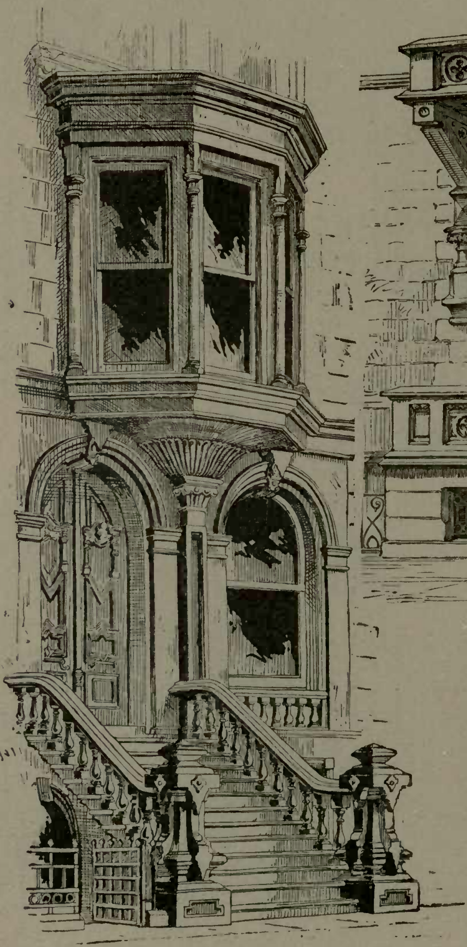


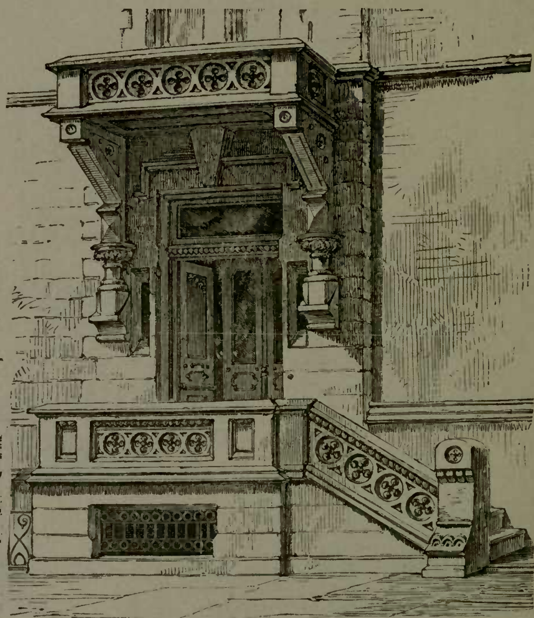
Terra Cotta.

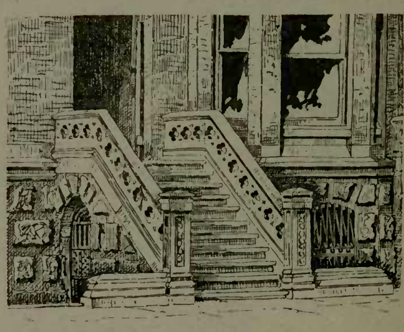
SCRAPS.

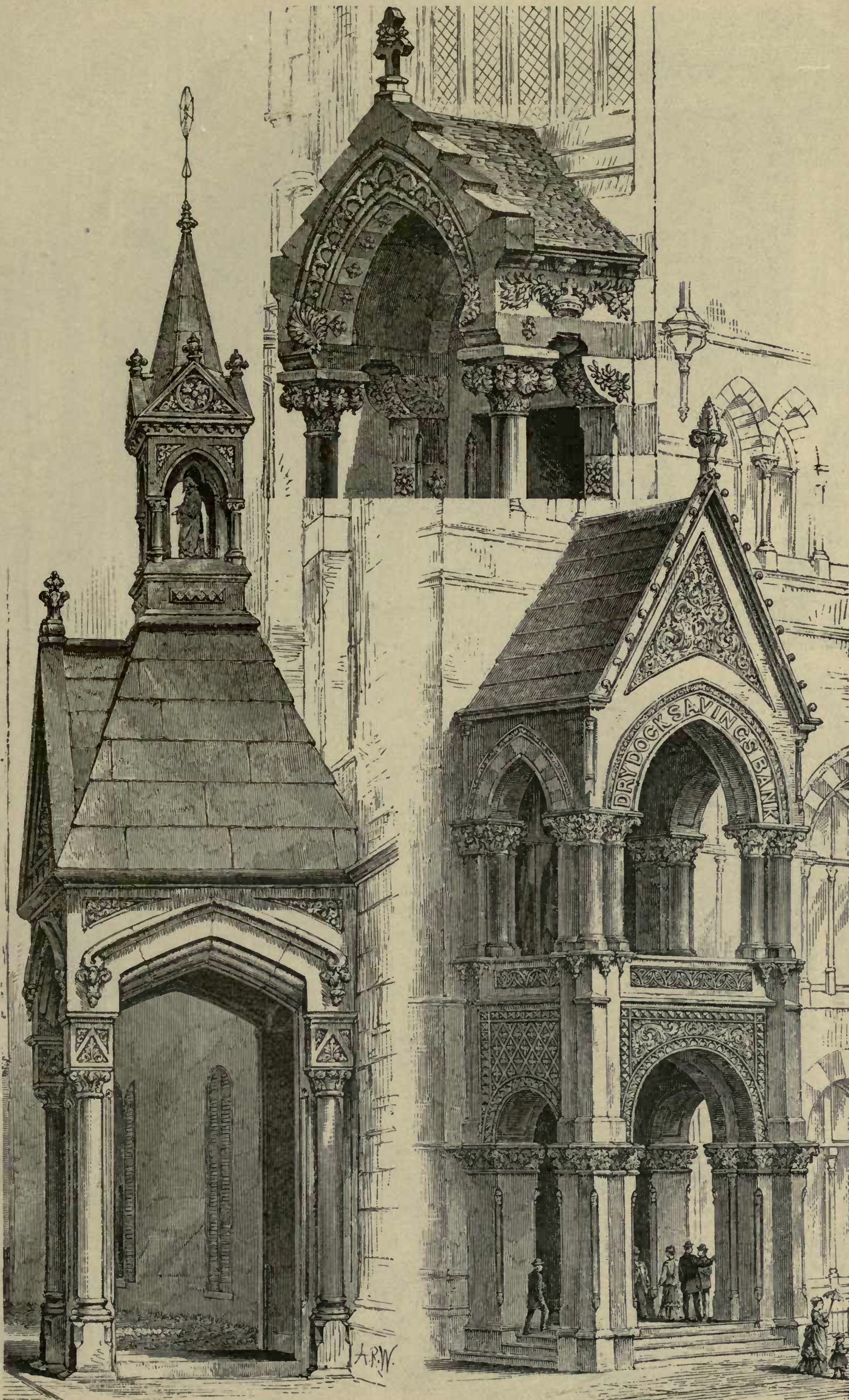


SCRAPS.

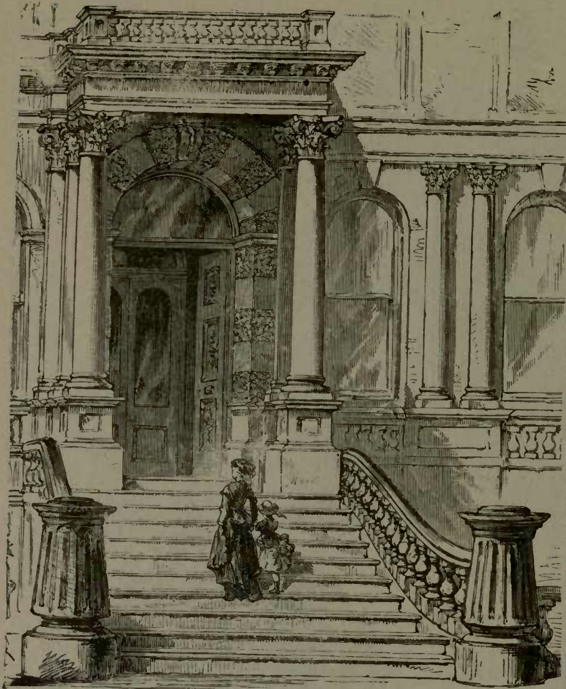


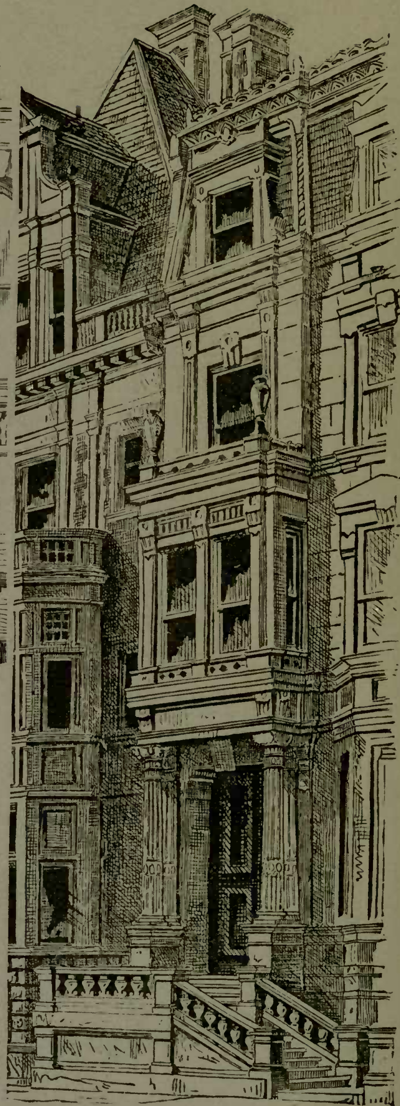


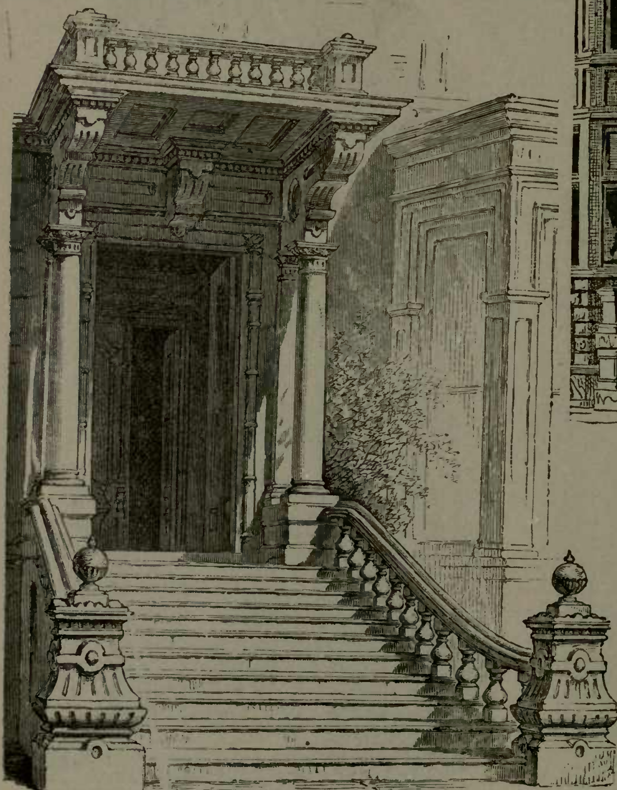


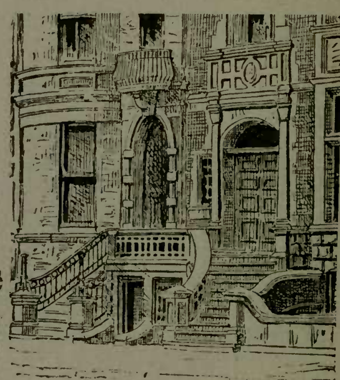

DRY DOCK SAVINGS BANK
A.R.W.

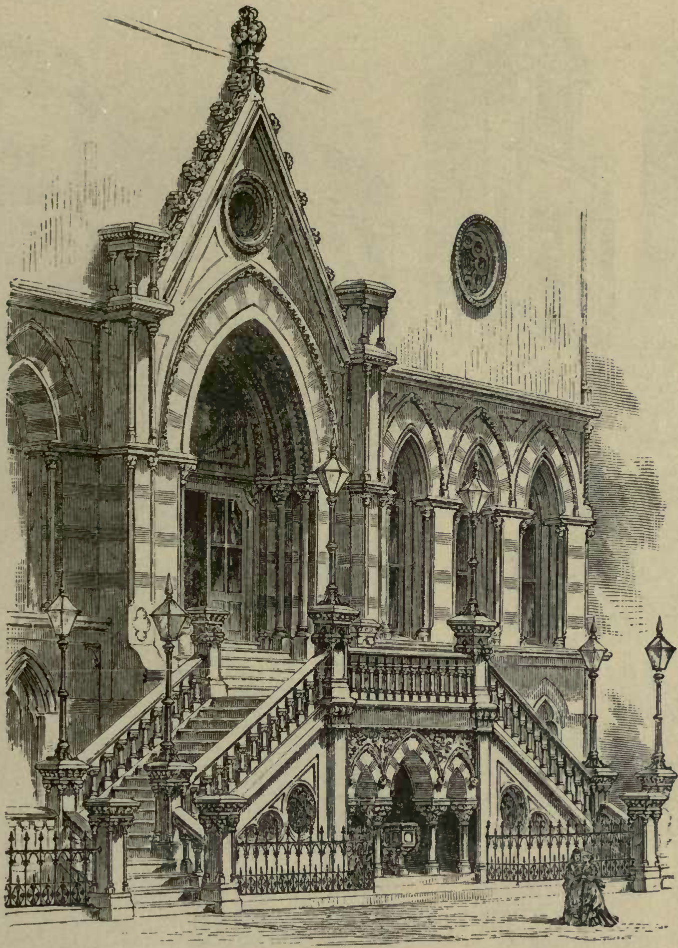
SCRAPS.

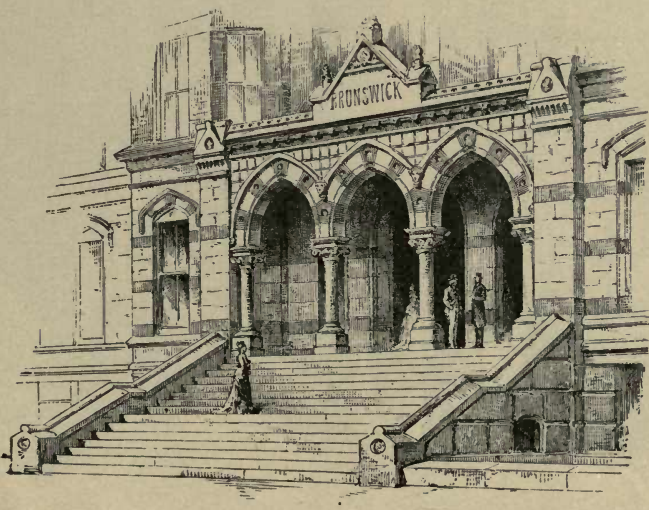


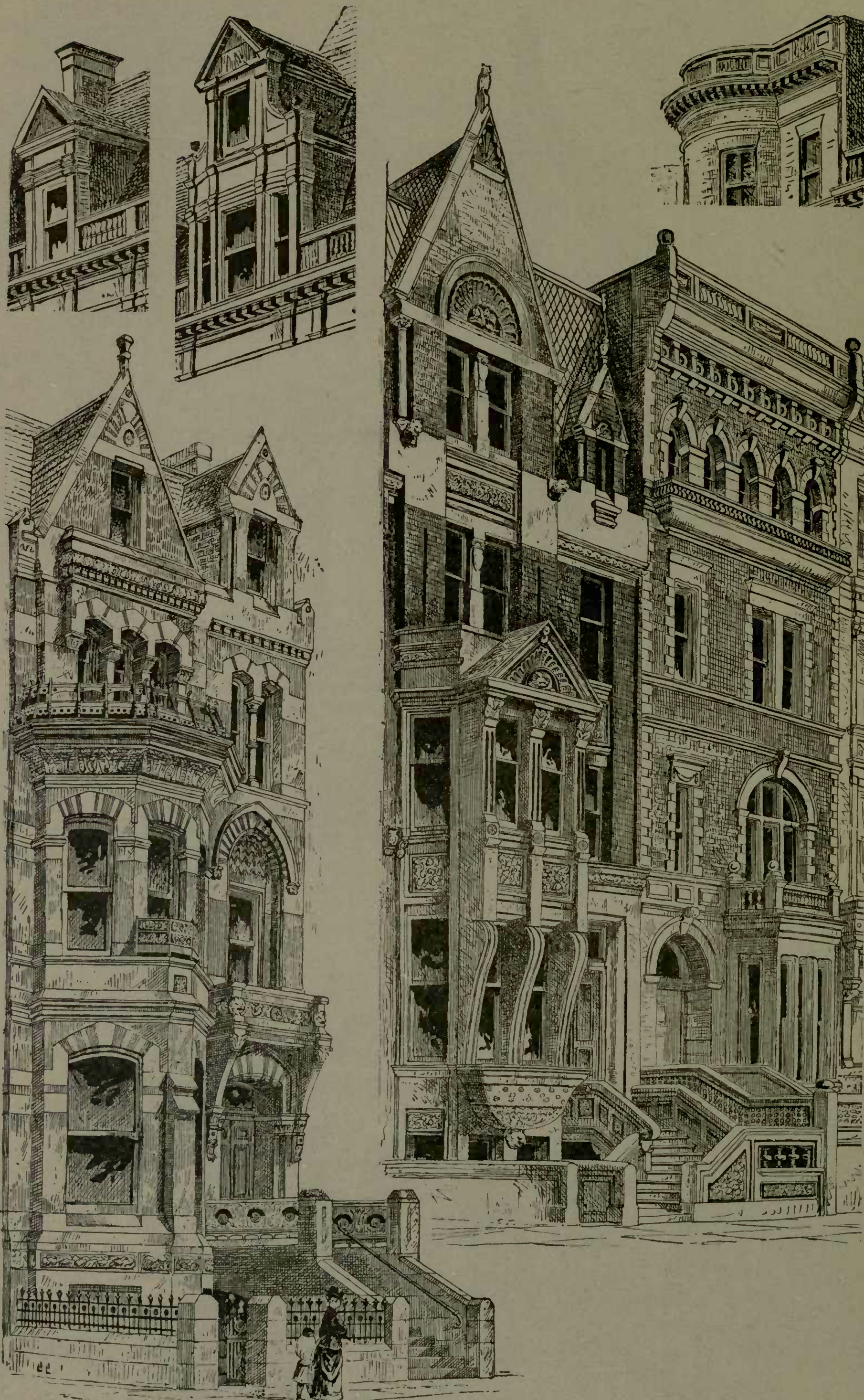




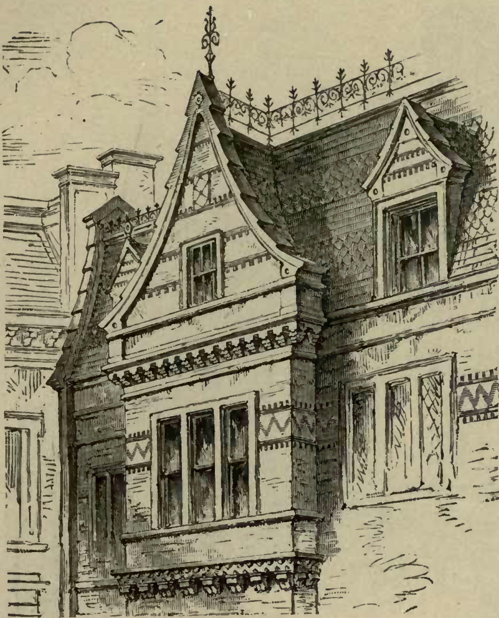


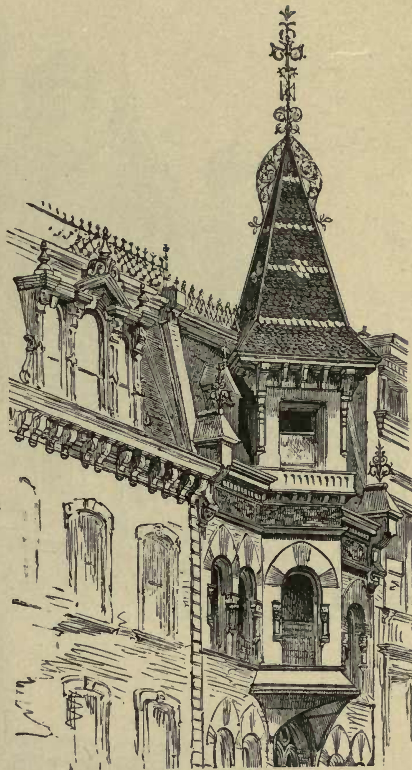


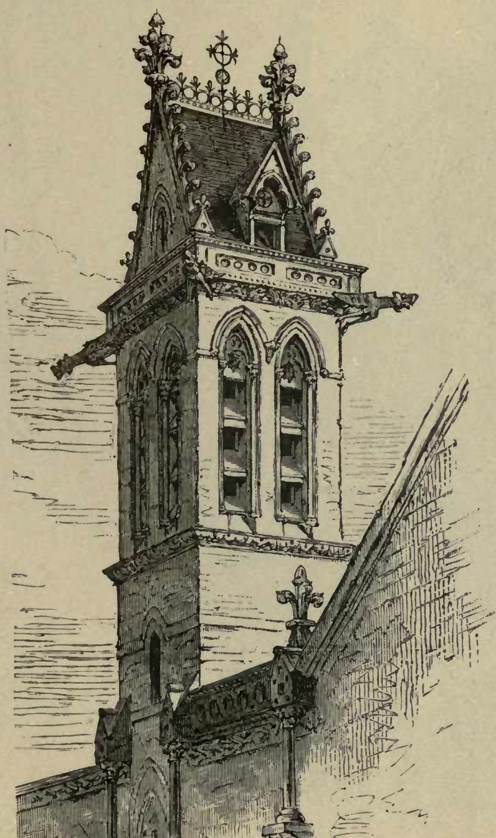

BRUNSWICK

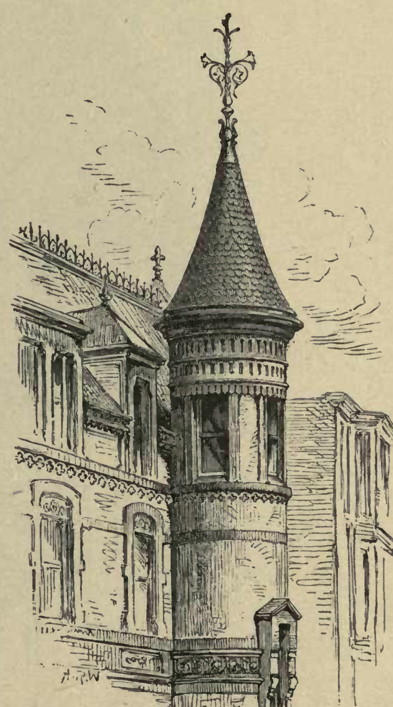


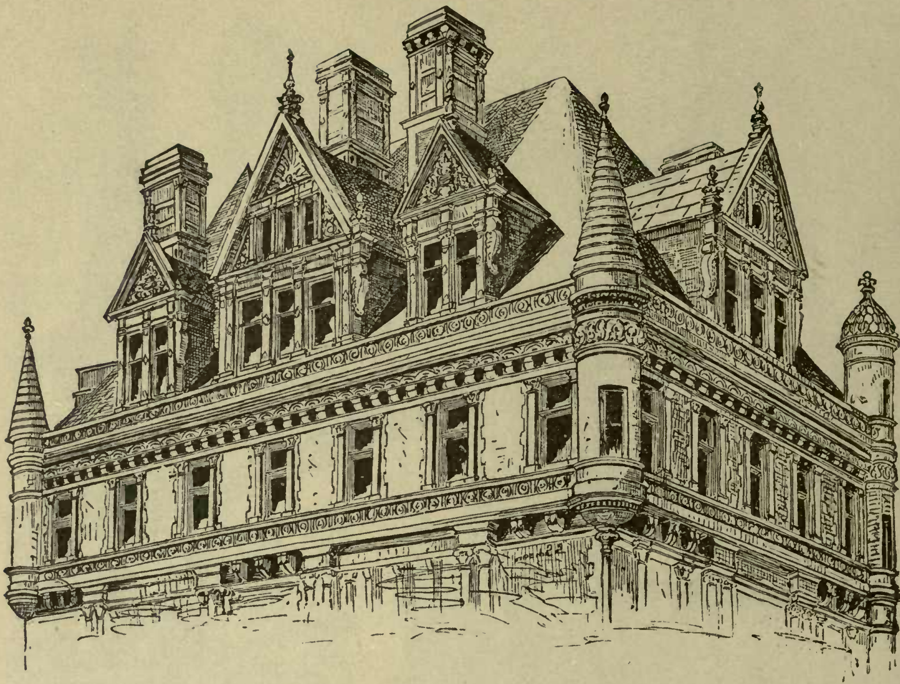
SCRAPS.

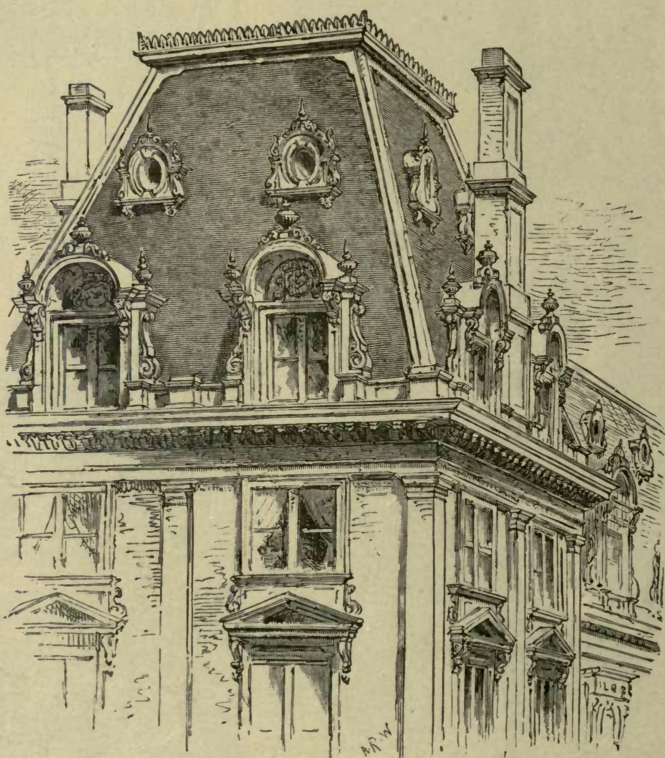


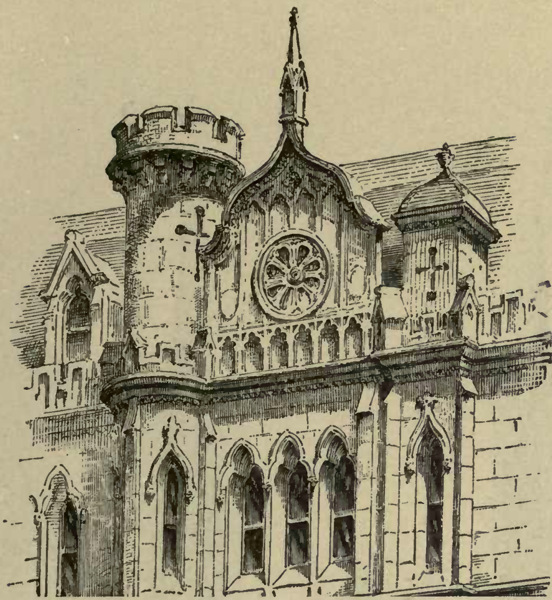


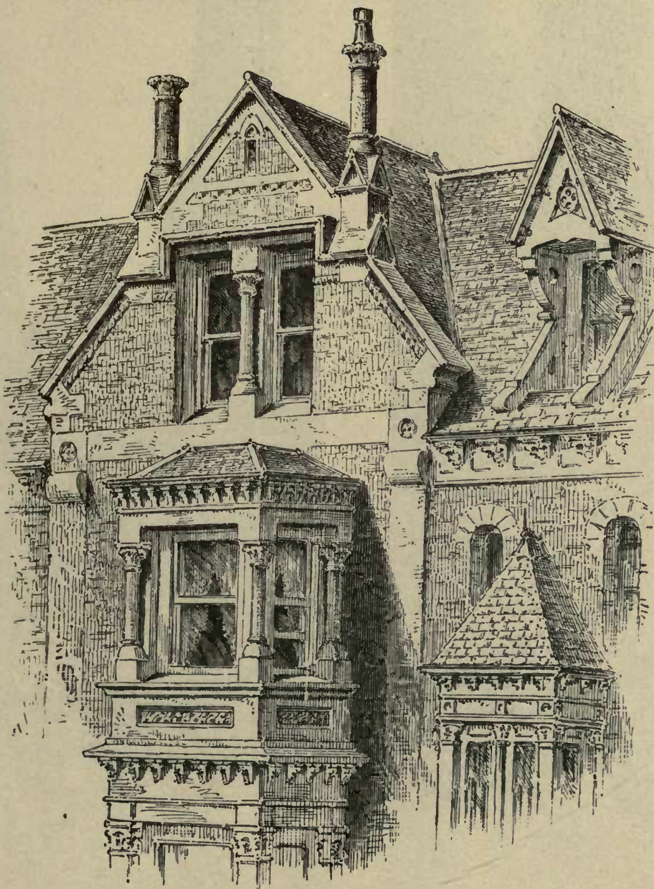


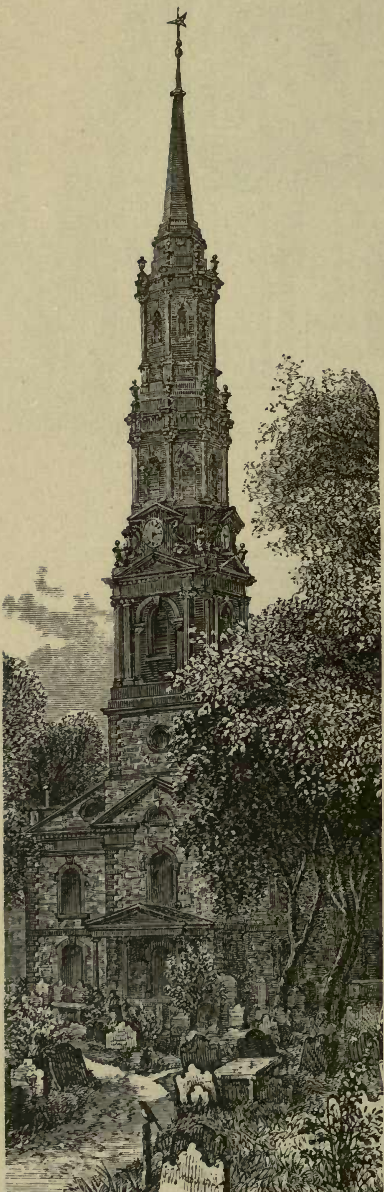


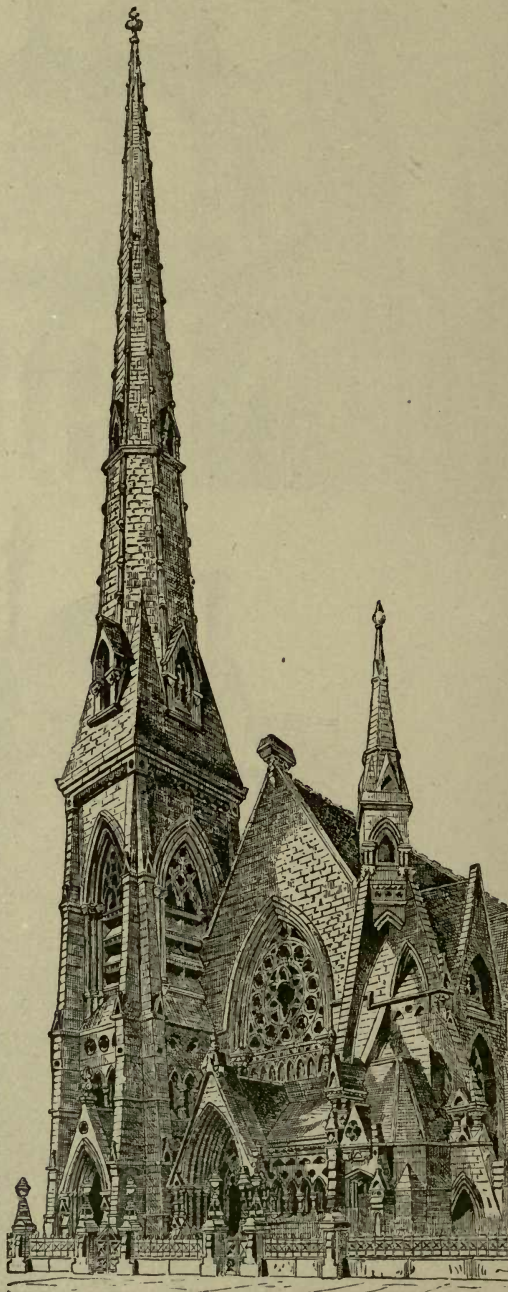


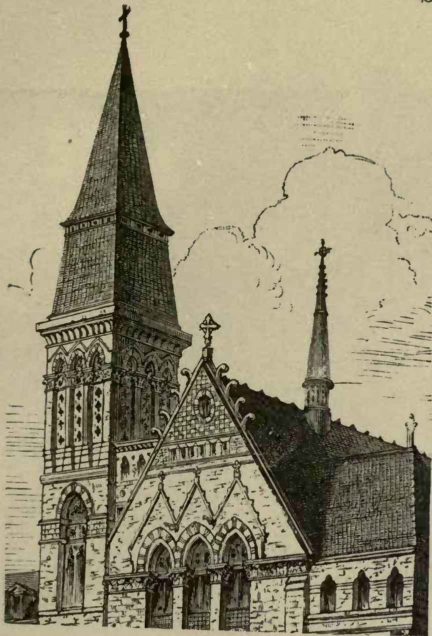


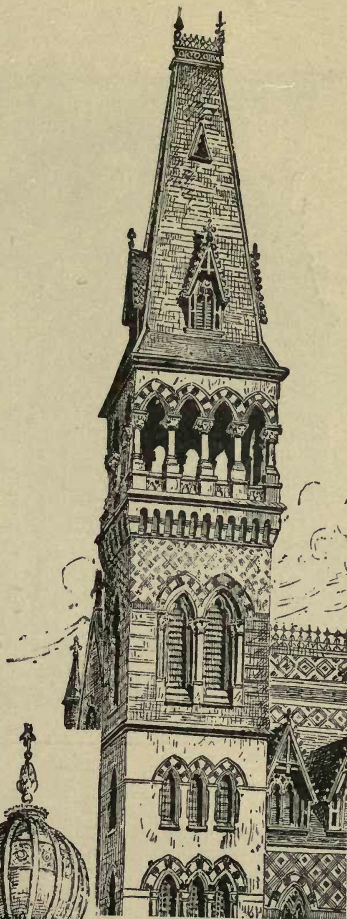


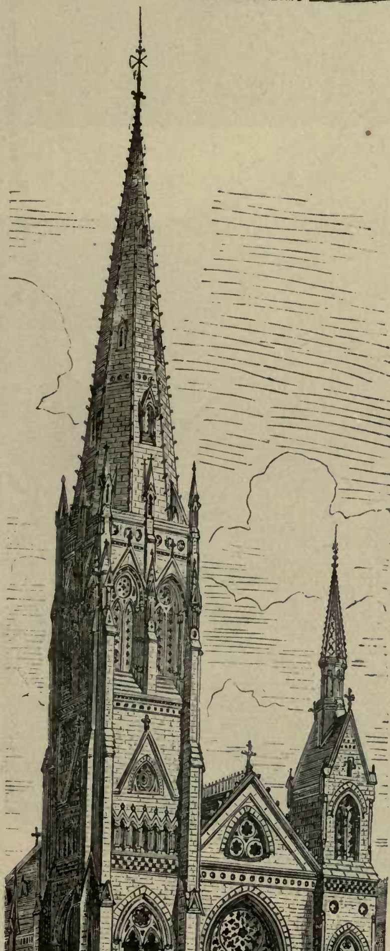


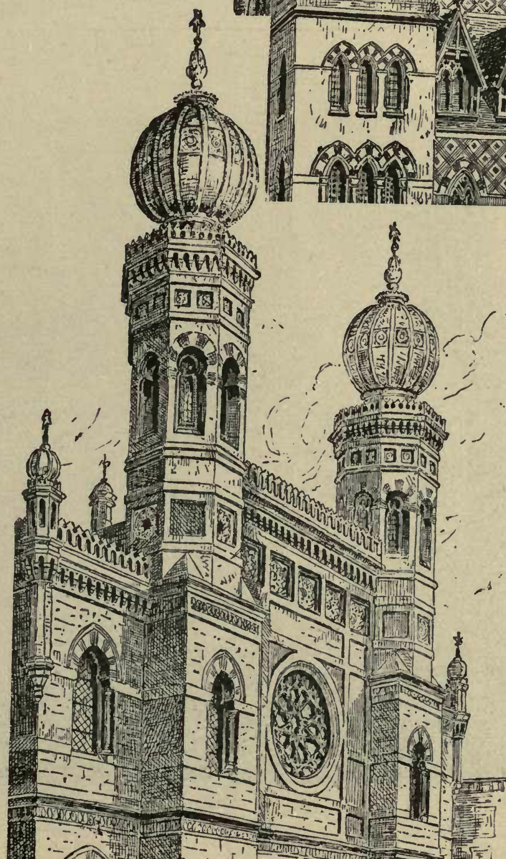


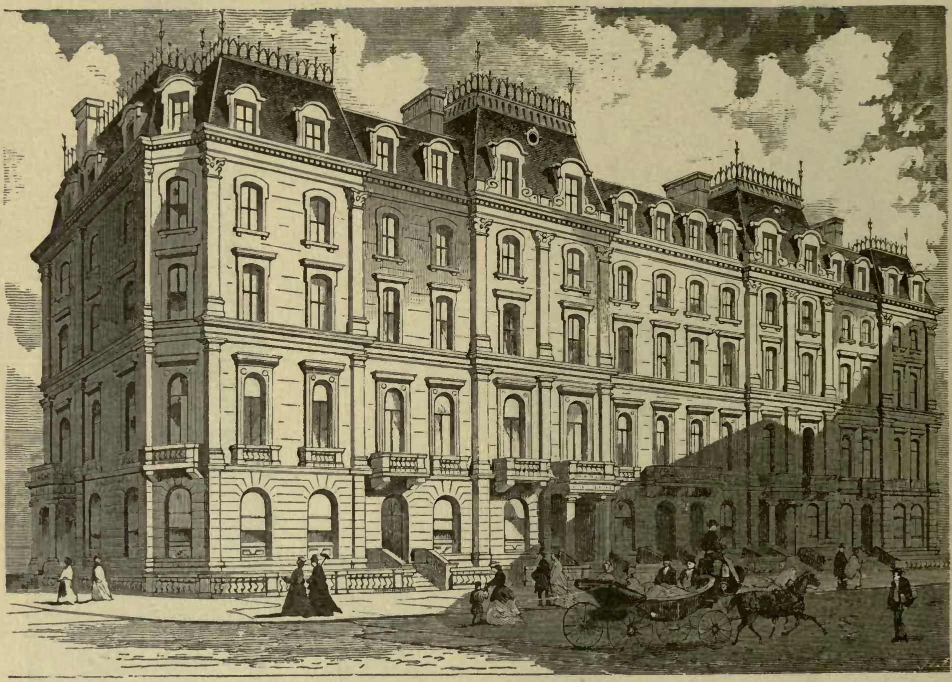


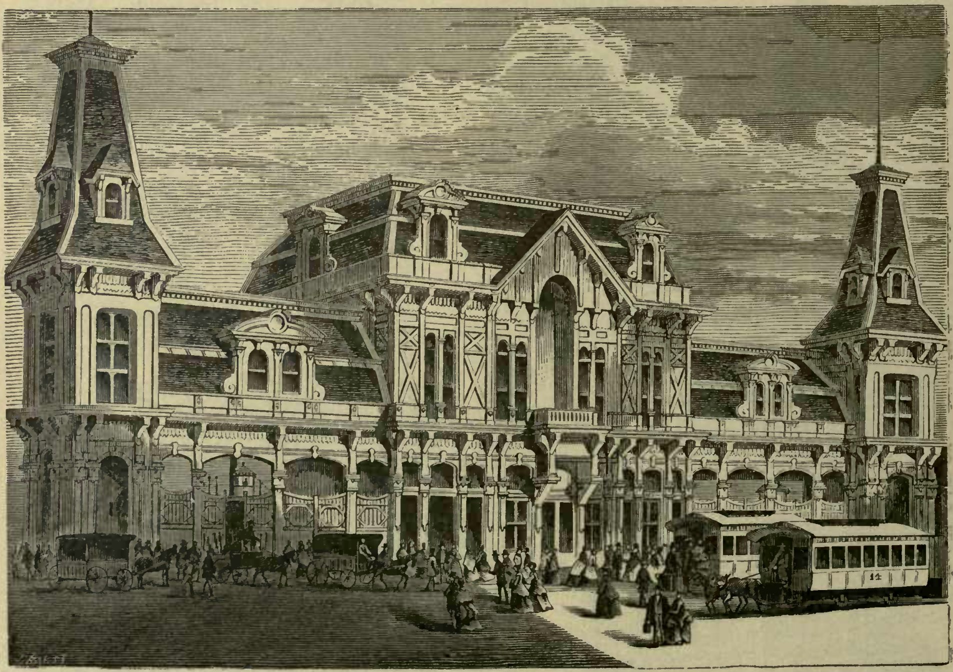


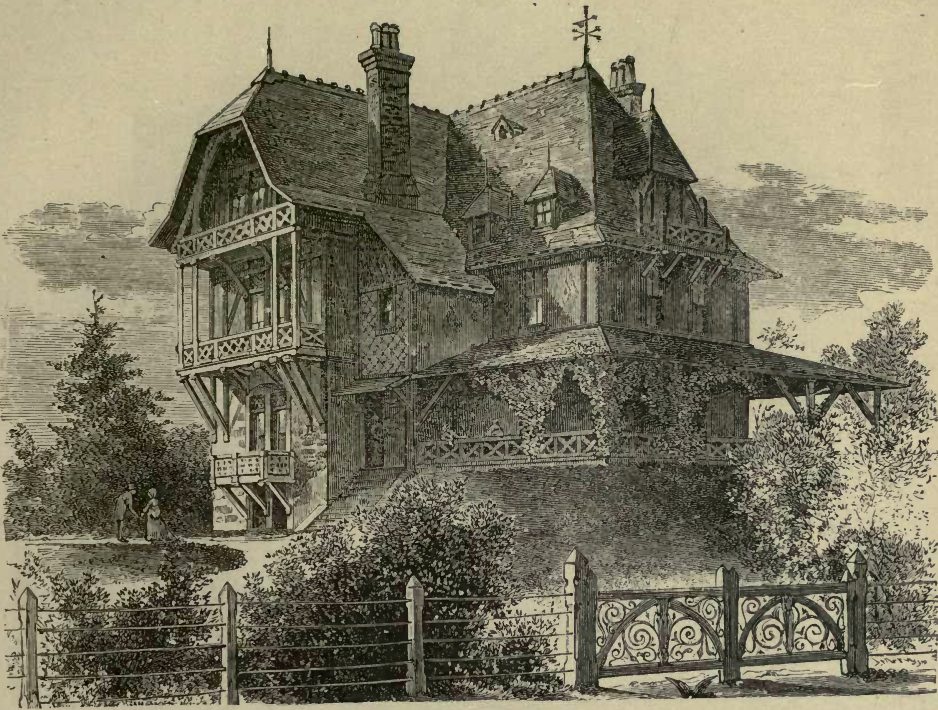


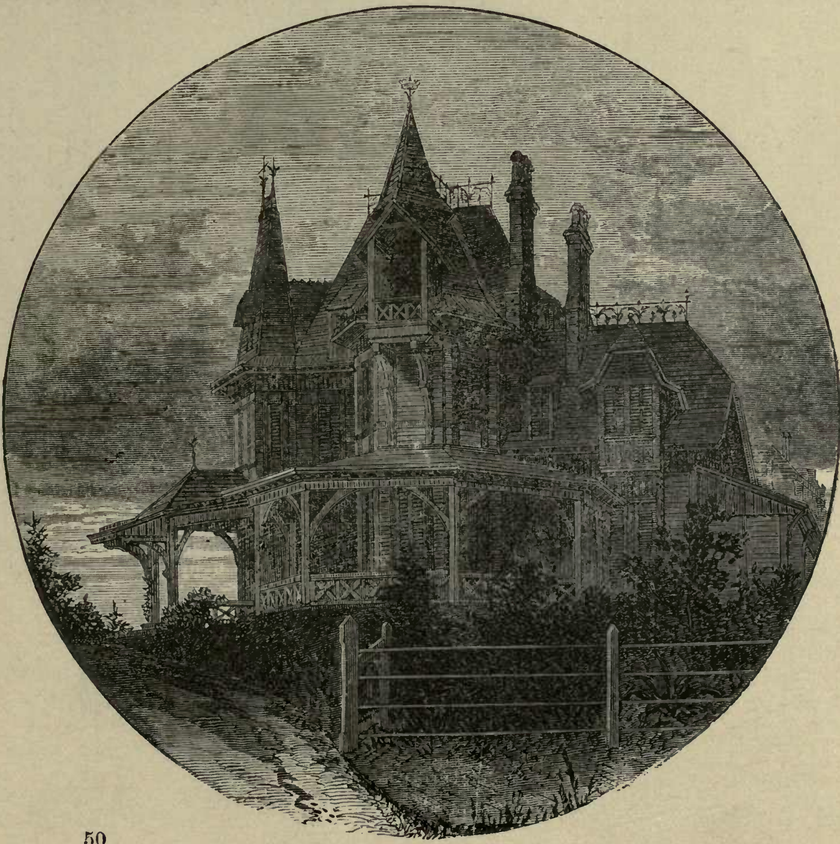


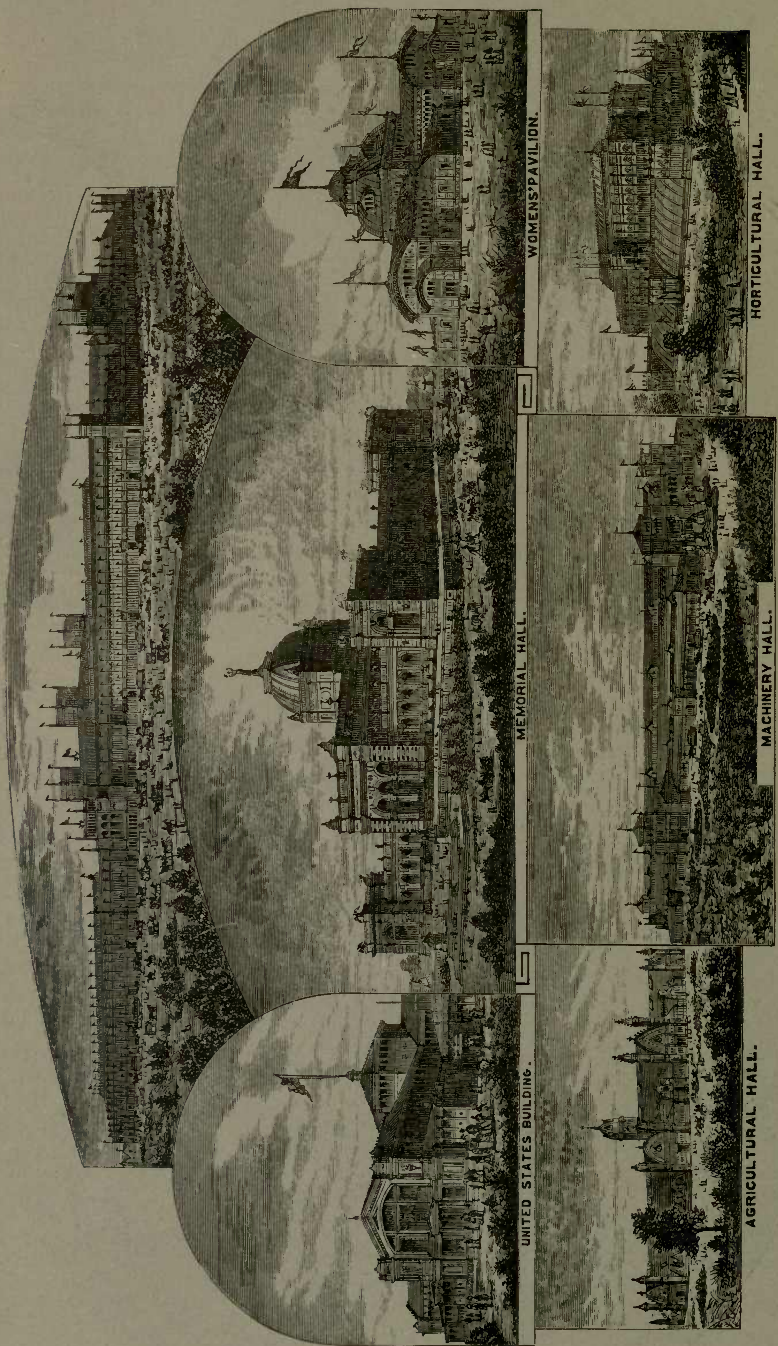




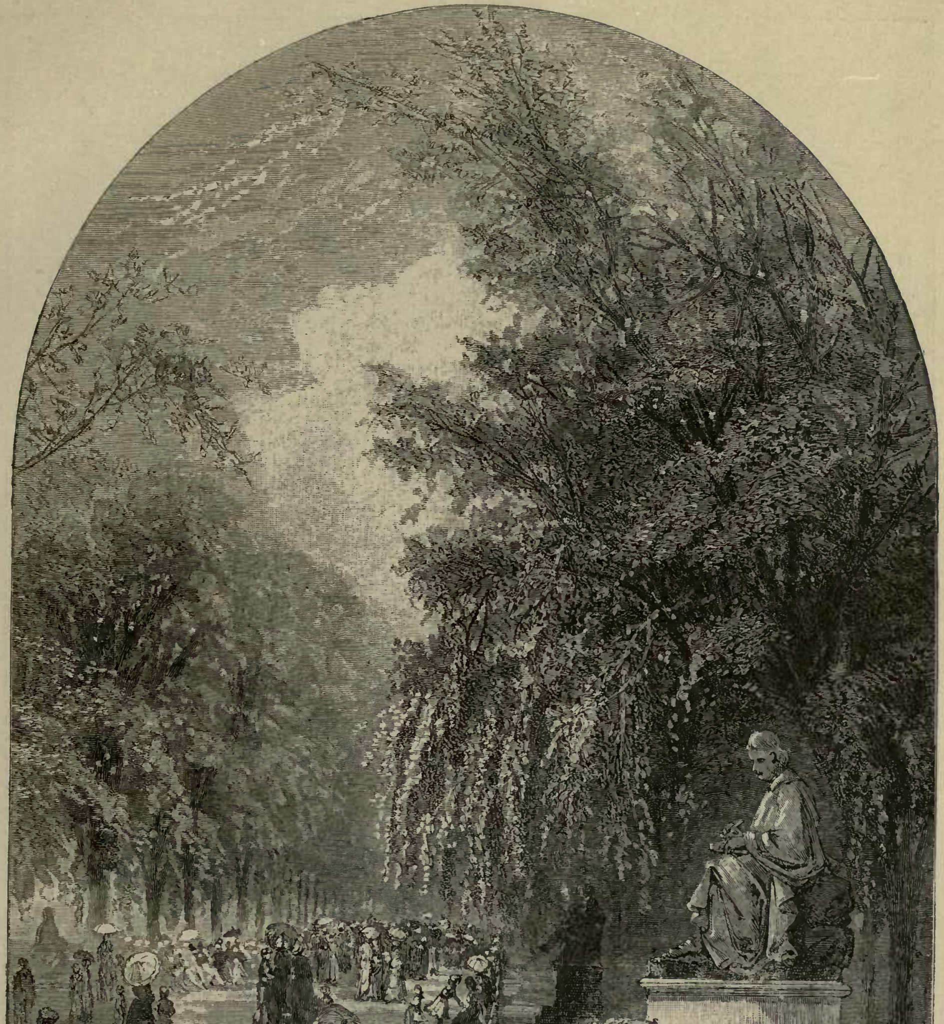




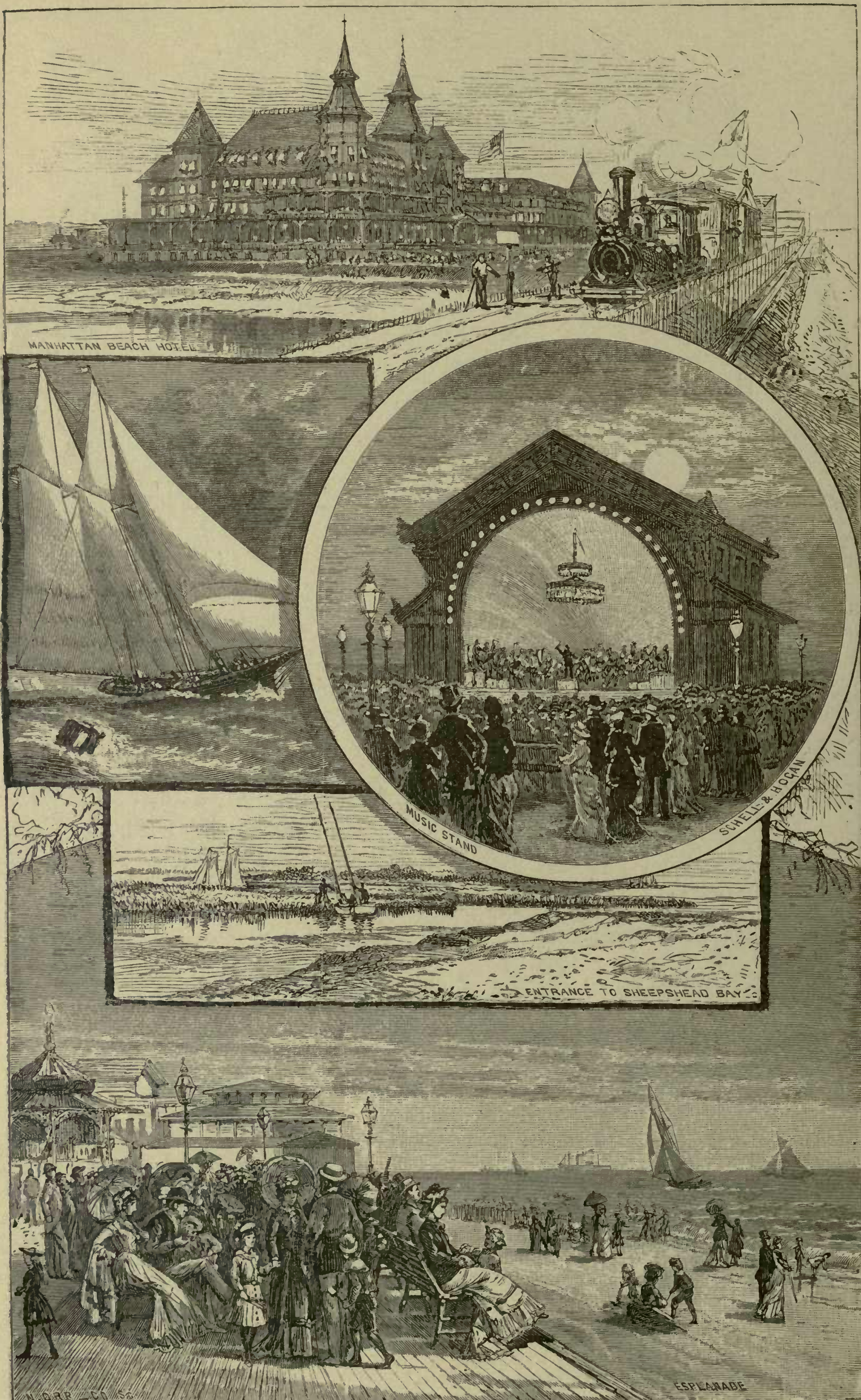




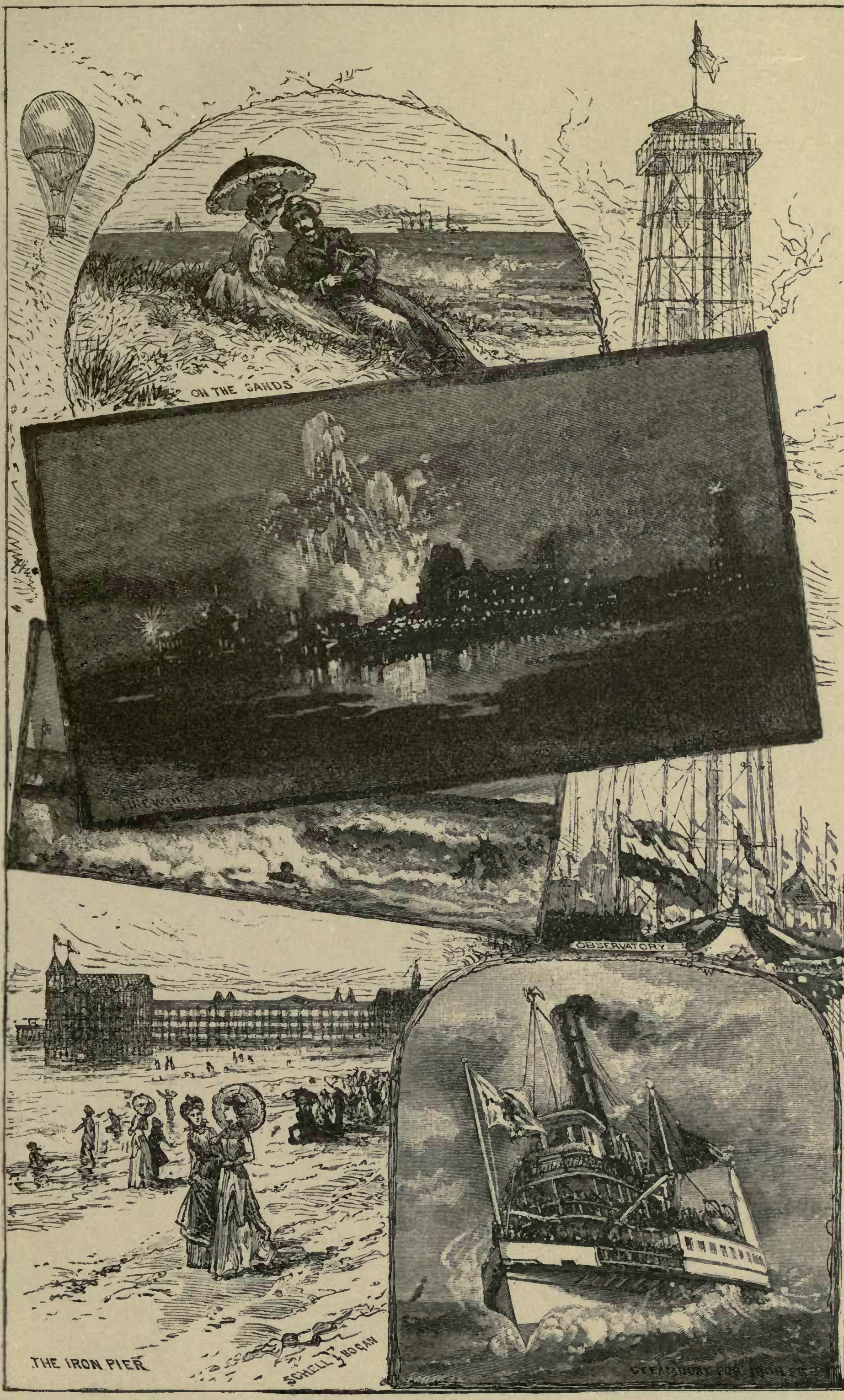
Buildings Centennial Exhibition, 1876.



Central Park, New York City.



Coney Island.



Coney Island.