Solution of 6 degree Polynomial

Write most general 6 degree

Equation as

$$ax^6+bx^5+cx^4+dx^3+ex^2+fx+g=0$$

call this equation 1

replace x by $t - (b/6a)$ so that the

term t^5 will be deleted and

depressed equation of form

$$ht^6+kt^4+mt^3+pt^2+qt+r=0$$

rewrite as

$$t^6 = -(kt^4+mt^3+pt^2+qt+r)/h$$

$$(t^3 + s)^2 =$$

$$-(kt^4+(m-2hs)t^3+pt^2+qt+r+hs^2)/h$$

Call this Equation 2

Now use condition for Quartic equation on RHS so that it becomes

A perfect square when

$$-64 (k/h)^3 (hs^2) - 16(k/h)^2 (p/h)^2$$

$$+16(k/h)(m-2sh)^2(p/h)$$

$$-16(k/h)^2(m-2sh)(q/h) -3(m-2sh)^4$$

$$=0$$

$t^6 = -(kt^4+mt^3+pt^2+qt+r)/h$

$(t^3 + s)^2 =$

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$-16(k/h)^2(m-2sh)(q/h) - 3(m-2sh)^4$

$=0$

Solve last equation for `s´ which will be quartic in s(call it resolvant quartic) so it may have no real

Solution on which case we may check

Original equation 1 in x for its real solution . if s has real value as solution then put this value in Equation 2 and hence we need to solve a cubic equation which can be solved by cardano method or direct

Viete substitution Good Luck