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# Airfield aggregation and route selection methods for strategic airlift

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# NAVAL POSTGRADUATE SCHOOL
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# THESIS

**AIRFIELD AGGREGATION AND ROUTE SELECTION METHODS FOR STRATEGIC AIRLIFT**

by

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December 1995

Thesis Advisor: David P. Morton

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Due the remarkable growth in the size and complexity of airlift operations, there is an increased need for planning tools to assist decision makers with issues ranging from selecting the number and types of aircraft for an airlift fleet to making informed decisions with respect to investing or divesting in overseas air bases. In Fiscal Year (FY) 94 research was initiated in the Operations Research Department of the Naval Postgraduate School in response to a request from the United States Air Force Studies and Analyses Agency and resulted in the development of a high fidelity strategic airlift optimization model called Throughput II. The model is formulated as a multi-period, multi-commodity linear programming model for determining the maximum on-time throughput of cargo and passengers that can be transported with a given fleet or given network, subject to appropriate physical and policy constraints. Troop and equipment movement requirements are specified by the Time Phase Force Deployment Data (TPFDD). An optimization model that utilizes the full level of detail available in a TPFDD would be of intractable size. Moreover, it is not necessary to build a model with such a fine level of detail in order to obtain the important insights required to assist decision makers. Therefore Throughput II replaces the potentially large set of airfields with a smaller set of centroids and schedules aircraft through these aggregated airfields. Currently route selection is performed manually, by an expert, who incorporates a variety of factors based on his/her experience. In this thesis we develop techniques for selecting a set of candidate routes for any deployment scenario without requiring historical data or extensive interaction with an expert. An analyst should be concerned about two potentially detrimental effects of these preprocessing procedures. First, infeasibility may be introduced by aggregation and second, Throughput II may provide suboptimal solutions since we consider a limited number of routes. To address these issues, a postprocessing step can be used to screen for constraint violations and to perform sensitivity analysis with respect to alternative routing options.

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# AIRFIELD AGGREGATION AND ROUTE SELECTION METHODS FOR STRATEGIC AIRLIFT


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Submitted in partial fulfillment
of the requirements for the degree of

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# ABSTRACT

Due the remarkable growth in the size and complexity of airlift operations, there is an increased need for planning tools to assist decision makers with issues ranging from selecting the number and types of aircraft for an airlift fleet to making informed decisions with respect to investing or divesting in overseas air bases. In Fiscal Year (FY) 94 research was initiated in the Operations Research Department of the Naval Postgraduate School in response to a request from the United States Air Force Studies and Analyses Agency and resulted in the development of a high fidelity strategic airlift optimization model called Throughput II. The model is formulated as a multi-period, multi-commodity linear programming model for determining the maximum on-time throughput of cargo and passengers that can be transported with a given fleet or given network, subject to appropriate physical and policy constraints. Troop and equipment movement requirements are specified by the Time Phase Force Deployment Data (TPFDD). An optimization model that utilizes the full level of detail available in a TPFDD would be of intractable size. Moreover, it is not necessary to build a model with such a fine level of detail in order to obtain the important insights required to assist decision makers. Therefore Throughput II replaces the potentially large set of airfields with a smaller set of centroids and schedules aircraft through these aggregated airfields. Currently route selection is performed manually, by an expert, who incorporates a variety of factors based on his/her experience. In this thesis we develop techniques for selecting a set of candidate routes for any deployment scenario without requiring historical data or extensive interaction with an expert. An analyst should be concerned about two potentially detrimental effects of these preprocessing procedures. First, infeasibility may be introduced by aggregation and second, Throughput II may provide suboptimal solutions since we consider a limited number of routes. To address these issues, a postprocessing step can be used to screen for constraint violations and to perform sensitivity analysis with respect to alternative routing options.

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# EXECUTIVE SUMMARY

The National Defense Authorization for Fiscal Year (FY) 1991 required the Department of Defense to conduct a comprehensive study to determine the mobility requirements for the United States Armed Forces. The Mobility Requirements Study (MRS) examined all aspects of the mobility question, from domestic transportation to intertheater to intratheater requirements. The MRS analyzes sea, air and amphibious lift, surface transportation and prepositioning requirements to provide Congress with an integrated plan for procuring the necessary lift assets and infrastructure for power projection in the 21st century. In January 1991, the Force Structure, Resource and Assessment Directorate (J8) of the Joint Staff assumed responsibility for the MRS. The Mobility Optimization Model (MOM) [Wing *et al.*, 1991] was developed by J8 with assistance from the Naval Postgraduate School (NPS) within a short period of six weeks. MOM was developed to determine airlift, sealift and prepositioning requirements. It modeled single onload and offload location with a time window for delivery. Even though MOM produces reasonable high level results, it is not suitable for answering the mobility questions currently sought by USAF Studies and Analyses Agency (USAF/SAA) because it does not model airlift assets, infrastructure and the associated constraints in sufficient detail.

A second linear programming (LP) model for strategic airlift was USAF/SAA's Throughput Model (Yost, 1994). This model was developed to determine an optimal airlift fleet mix based on given movement requirements and to compare different airlift fleets in terms of cargo throughput capabilities. Throughput proved to be a useful model but had some inherent limitations due to its static nature with respect to time. Important system details such as delivery time windows for specific units and unit closure dates are not incorporated. In FY 94 research was performed at NPS in response to a request from the USAF/SAA, and resulted in the development of a higher fidelity strategic airlift model called Throughput II (Lim, 1994) which has since been enhanced (Morton, *et al.*, 1995). In this research, the strategic airlift assets optimization problem is formulated as a multi-period, multi-commodity network-based linear programming model, with a large number of side constraints. The objective of the model is to minimize late deliveries subject to physical and policy constraints, such as aircraft utilization limits and airfield handling capacities.

A typical Time Phased Force Deployment Data (TPFDD) document specifies over 14000 specific movement requirements over 180 days from 120 Aerial Ports of Embarkation (APOE) to 90 Aerial Ports of Debarkation (APOD) through a network infrastructure with a large number of potential enroute stops. To attempt to build an optimization model that incorporates the full level of detail available in a TPFDD would result in a mathematical model of intractable size. Moreover, it is not necessary to build a model with such a fine level of detail in order to obtain the important insights required to assist decision makers. Therefore, Throughput and Throughput II both use data aggregated from the TPFDD for input. Both models, for example, replace the potentially large set of airfields with a smaller set of centroid airfields and schedule aircraft through these aggregate airfields.

We develop a mixed integer location-allocation centroid model that may be used as a preprocessing step. Aggregation of airfields may lead to infeasibilities in the resulting model with respect to ability to fly certain flight legs due to altered inter-airfield distances. This is the principle reason that we propose a centroid aggregation model that has the primary objective of minimizing the maximum centroid-satellite distance. A pure minimax model, will typically have many multiple optimal solutions. Some optima, however, are better than others with respect to criteria ignored by the minimax objective (e.g., the satellite-centroid for second furthest airfield). Because of this drawback, a secondary minisum objective term is included in the centroid selection model.

Currently, route selection is performed manually, by an expert, who incorporates a variety of factors including critical legs for specific aircraft types, aircrew flight time restrictions and commercial track avoidance of certain countries. Such a procedure is time consuming and may be adequate for flying to destinations with which USAF has extensive experience. However, more difficulties may arise when flying to an unfamiliar destination. We develop heuristic techniques for selecting the optimal routes for any deployment scenario without requiring historical data and extensive interaction with an expert. The route selection module is implemented in the Pascal programming language. Feasible routes are sorted with respect to their lengths for each origin-destination pair. Inefficient and infeasible routes, due to constraints such as crew flying hours or crew stages, are then eliminated. This automated process has the advantage that routes may be generated without requiring the extensive experience and knowledge of an expert. Nevertheless, there are certain system requirements (e.g., country permissions) that are difficult to embed in such a program, and expert knowledge may still be required to rule out the feasibility of certain routes.

The elimination and preprocessing step may still generate a large number of routes, and it may not be computationally practical to include all the candidate routes in the Throughput II model. For this reason, a method is applied to convert the large problem into a smaller problem of manageable size. This reduction is achieved by solving the linear program over a subset of its columns; the result of which may be suboptimal. Through standard LP sensitivity analysis we can test whether a certain route, not in the original subset, is attractive. If it is attractive, it can be put in the basis as a new column and is used to improve the route generation model. This procedure can be automated and is known as column generation; in our case, the route selection model will be the column generator. Including new attractive routes and iterating the Throughput II model can be continued until a satisfactory solution is achieved.

# I. INTRODUCTION

In spite of the sophisticated inventories and impressive capabilities of modern air forces, military aviation is still the youngest of all major military operations. Except for a few early attempts, such as using balloons and airships as instruments of war, air power has been an extremely effective means of achieving military objectives, mainly because of its great versatility, for the last 80 years. Among their many missions, airforces provide two key tasks: delivering firepower directly by air-to-surface and air-to-air weapons, and rapidly deploying combat units over substantial distances. While the use of military air power is relatively new, air mobility operations are of even more recent vintage. The substantial exploitation of military airlift dates only from 1938. Prior to that time, except for some primitive transportation efforts on a limited scale via air transport systems, the deployment and logistic support of military forces was undertaken mostly via surface transport systems.

There are three fundemental disadvantages of surface deployment, which cannot be eliminated by speed and efficiency provided by the newly developing ideas and technology of this century. Firstly, surface deployment over substantial distances is slower than air transport systems. Secondly, surface deployment is heavily restricted by geographical constraints. The third, and most important reason, concerns its susceptibility to enemy attacks. On the other hand, the airlift has less comparative capacity than the sealift does, but because of its speed and range, all the modern armed forces have incorporated airlift capability into their transportation systems.

During World War II, in addition to the effectiveness of fighting, attacking and bombing units of air forces, air transport operations proved to be an efficient means of projecting forces to strategic locations. This added another aspect to the obvious potential of air forces as a direct force, namely the use of air forces as an indirect force. The importance of this latter feature of air forces is easily recognized by the success of airlift operations which were totally impossible by other means in the same range and time frames

Despite prepositioned fuel, ammunition, and equipment, the magnitude of the United Nations/ United States airlift effort during Desert Shield and Desert Storm was unprecedented. By 10 March 1991, strategic airlift had moved over 500,000 people and 540,000 tons of cargo to the theater. At the height of the Desert Shield airlift, Military Airlift Command's (MAC's) cargo movement averaged 17 million ton-miles per day[1]. By comparison, during the 1973 Arab-Israeli War, US airlift moved 4.4 million ton-miles per day. Other historical comparisons include the World War II "Hump" at 0.9 million ton-miles per day, the Berlin Airlift at 1.7 million ton-miles per day, and Operation Just Cause at 2.0 million ton-miles per day (Gulf War Air Power Survey, 1993).

[1] US strategic airlift requirements and capabilities may be measured in terms of million-ton-miles per day, i.e., in multiples of the capacity to move one ton of cargo by air a distance of one mile in one day. This standard of measurement, though a useful planning tool and gauge of present and future strategic airlift capabilities, does not, of course, take into account such real world constraints as aircraft and crew exhaustion, weather, availability of airfields, overflight rights, and possible enemy action.

Due to the remarkable growth in the size and complexity of airlift operations, there is increased need for planning tools to assist decision makers with issues ranging from selecting the number and types of aircraft for an airlift fleet to making informed decisions with respect to investing or divesting in overseas air bases. Although it must be interpreted carefully, this "search for optimality" is a very important theme in Operations Research. The field of Operations Research provides tools for attempting to find the best, or optimal, solution to mathematical models of the problem under consideration.

## A. BACKGROUND

The National Defense Authorization for Fiscal Year (FY) 1991 required the Department of Defense to conduct a comprehensive study to determine the mobility requirements for the United States Armed Forces. The Mobility Requirements Study (MRS) examined all aspects of the mobility question, from domestic transportation to intertheater to intratheater requirements. The MRS analyzes sea, air and amphibious lift, surface transportation and prepositioning requirements to provide Congress with an integrated plan for procuring the necessary lift assets and infrastructure for force projection in the 21st century. In January 1991, the Force Structure, Resource and Assessment Directorate (J8) of the Joint Staff assumed responsibility for the MRS. The Mobility Optimization Model (MOM) (Wing *et al.*, 1991) was developed by J8 with assistance from the Naval Postgraduate School (NPS) within a short period of six weeks. MOM was developed to determine airlift, sealift and prepositioning requirements. For airlift, it modeled single onload and offload locations with a time window for delivery.

Even though MOM produces reasonable high level results it is not suitable for answering the mobility questions currently sought by USAF Studies and Analyses Agency (USAF/SAA) because it does not model airlift assets, infrastructure and the associated constraints in sufficient detail.

A second linear programming (LP) model for strategic airlift was USAF/SAS's Throughput Model (Yost, 1994). This model was developed to determine an optimal airlift fleet mix based on given movement requirements and to compare different airlift fleets in terms of cargo throughput capabilities. Throughput proved to be a useful model but had some inherent limitations due to its static nature with respect to time. Important system details such as delivery time windows for specific units and unit closure dates are not incorporated. In FY 94, research was performed at NPS in response to a request from the USAF/SAA, and culminated with development of a higher fidelity strategic airlift model called Throughput II (Lim, 1994). In this research, the strategic airlift assets optimization problem is formulated as a multi-period, multi-commodity network-based linear programming model, with a large number of side constraints. The objective of the model is to minimize late deliveries subject to physical and policy constraints, such as aircraft utilization limits and airfield handling capacities.

## B. PROBLEM STATEMENT

Executing a deployment - delivering units and equipment to theater efficiently and effectively - with limited aircraft and infrastructure assets is a formidable task. The National Command Authority (NCA), the Joint Chief of Staff, and major field commanders have access to the Worldwide Military Command and Control System (WWMCCS), which includes a command and control system called Joint Operational Planning and Execution System (JOPES). The Operational Plan (OPLAN) is entered as a requirement and JOPES generates a Time Phased Force Deployment List (TPFDL). This list called the Time Phased Force Deployment Data (TPFDD) document, and contains information pertaining to the force and movement requirements of the OPLAN. With this information, units able to supply force requirement can be identified and transportation arrangements can be made. A typical TPFDD specifies over 14000 specific movement requirements over 180 days from 120 Aerial Ports of Embarkation (APOE) to 90 Aerial Ports of Debarkation (APOD) through a network infrastructure with a large number of potential enroute stops. To attempt to build an optimization model that utilizes the full level of detail available in a TPFDD would result in a mathematical model of intractable size. Moreover, it is not necessary to build a model with such a fine level of detail in order to obtain the important insights required to assist decision makers. Therefore, Throughput and Throughput II both use data aggregated from the TPFDD for input. Both models, for example, replace the potentially large set of airfields with a smaller set of centroid airfields and schedule aircraft through these aggregate airfields.

A mixed integer programming model for centroid selection has been developed by AFSAA that may be used as a preprocessing step for either model (Yost, 1994). The basic idea in this model is to first define candidate centroid airfields based on their passenger and cargo flows and then to solve the model that minimizes a weighted sum of distances between the centroids and their satellite airfields. A primary shortcoming of such an aggregation procedure is that it may lead to infeasible solutions in the subsequent airlift optimization model Airfields aggregated into centroids may inappropriately "borrow" capacity from one another. As a result, it may be discovered on disaggregation that the solution to Throughput II has assigned too many sorties to an airfield. Such an outcome may occur, for example, when a relatively small capacity airfield with a large flow requirement is aggregated with another airfield that has a large capacity value. A second potential drawback in the current method of aggregating airfields is that the centroids are selected to minimize the sum of weighted distances to all corresponding satellite airfields, and the distances between airfields in Throughput II are determined from the centroid not the satellite. As a result, there are cases in which a particular aircraft type may be capable of flying a route which originates from the centroid but not from the satellite airfield. Conversely, certain routes may be eliminated unnecessarily due to airfield aggregation. As a result of these aggregation effects, the fidelity of the associated Throughput II model may be reduced.

Currently, route selection is performed manually, by an expert, who incorporates a variety of factors including minimum payload ferry ranges[2] for specific aircraft types, aircrew flight time restrictions and commercial track avoidance of certain countries. Such a procedure is time consuming but may be adequate for flying to destinations with which

[2] This is the longest distance which an aircraft can fly with minimum reasonable payload (20 % of its maximum capacity) without refueling.

USAF has extensive experience; however, difficulties may arise when flying to an unfamiliar destination.

In summary, the size of the infrastructure network is currently reduced by a preprocessing step that selects airfields via the centroid model and eliminates, by hand, a large number of infeasible or inefficient routes. The results of Throughput II have indicated that for some important deployment scenarios the airlift system may be infrastructure constrained, in part, due to the aircraft handling capability of certain enroute airfields. This makes the process of preselecting centroid airfields as well as delivery and recovery routes very important. It is desirable to minimize the effect of capacity borrowing. Moreover, it may be possible to improve the system's throughput by proper aircraft-route scheduling. An automated system that can generate routes for given origin-destination pairs could dramatically aid the analyst in the selection of deployment and recovery routes.

## C. METHODOLOGY

This thesis will consist of two main parts as illustrated in Figure 1.1. The first part is a preprocessing procedure that selects a set of centroids and candidate routes that will be used as input to Throughput II or any other appropriate optimization model. The second part is a postprocessing procedure that seeks to determine whether the Throughput II solution may be disaggregated into a valid solution for the original model and/or improved by providing Throughput II with additional routing options.

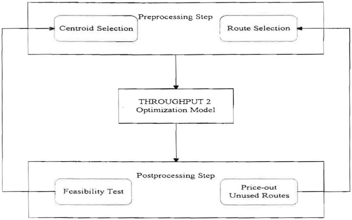



Figure 1.1. Route Selection Model

### 1. Preprocessing Modules

The preprocessing step consists of two distinct modules; first a model is solved to determine the centroids and then a heuristic[3] generates routes from the origin centroid airfields through enroute airfields, to the destination centroid airfields for all relevant aircraft types.

#### *a. Centroid Model*

We develop a location-allocation model with a primary objective to minimize the maximum centroid-satellite distance and a secondary objective to minimize the sum of the corresponding distances. Aggregation of airfields may lead to infeasibilities

[3] "A heuristic is a rule of thumb, strategy, or trick used to improve the efficiency of a system which tries to discover the solutions of complex problems."(Slagle, 1971).

in the resulting model, with respect to an aircraft's ability to fly certain flight legs, due to altered inter-airfield distances This is the primary reason that we propose a centroid aggregation model that has a primary objective of minimizing the maximum centroid-satellite distance. A pure minimax model, typically has many multiple optimal solutions. Some optima, however, are better than others with respect to criteria ignored by the minimax objective (e.g., the satellite-centroid for second furthest airfield). Because of this drawback, the secondary minisum objective term is included in the centroid selection model.

#### *b. Route Selection Model*

The route selection module is implemented in the Pascal programming language. Feasible routes are sorted with respect to their lengths for each origin-destination pair. Inefficient and infeasible routes, due to constraints such as crew flying hours or crew stages, are then eliminated. This automated process has the advantage that routes may be generated without requiring the extensive experience and knowledge of an expert. Nevertheless, there are certain system requirements (e.g., country permissions) that are difficult to embed in such a program, and expert knowledge may still be required to rule out the feasibility of certain routes.

### 2. Postprocessing Module

The final output of the preprocessing step is a set of candidate routes that can be used as input data for Throughput II or any other appropriate model such as the Stochastic Airlift Optimization Model (Goggins, 1995). However the preprocessing step may still generate a large number of routes, and it may not be computationally practical to include all the candidate routes in the Throughput II model. For this reason, a method is applied to convert this large problem into a smaller problem of manageable size. This reduction is achieved by solving the linear program over a subset of its columns, the result may be suboptimal. Through standard LP sensitivity analysis we can test whether a certain route, not in the original subset, is attractive. If it is attractive, it can be put in the basis as a new column and is used to improve the route generation model. This procedure can be automated and is known as column generation; in our case, the route selection model will be the column generator. This process of including new attractive routes in the Throughput II model, solving the corresponding LP, and testing for optimality can be continued until a solution to the original problem is achieved.

## D. SUMMARY

In this thesis we develop techniques for selecting the optimal routes for any deployment scenario without requiring historical data and extensive interaction with an expert. The results of Throughput II have indicated that for some important deployment scenarios the airlift system may be infrastructure constrained. This makes the process of preselecting centroid airfields very important. In the first part of this thesis we develop an efficient procedure for solving the combination minimax-minisum facility location model to determine centroid airfields that will help curb detrimental aggregation effects.

An analyst should be concerned about two potentially detrimental effects of the preprocessing procedure. Either infeasibility may be introduced by aggregation or Throughput II may yield a suboptimal solution since only a limited number of routes are considered. To address these issues, a postprocessing step includes

i. disaggregation and feasibility tests,

ii. sensitivity analysis with respect to alternate routing options.

Together, these measures assist Throughput II in providing tractible, as well as valid results.

# II. REVIEW OF THROUGHPUT II MODEL

The Throughput II strategic airlift optimization model was developed by Captain Lim Teo Weng (Lim, 1994) and has since been enhanced in ongoing research at the Naval Postgraduate School (Morton *et al.*, 1995). The mathematical formulation of Throughput II is contained in Appendix A. The Throughput II model is a multi-period model for determining the maximum on-time throughput of cargo and passengers that can be transported with a given fleet over a given network, subject to appropriate physical and policy constraints. The objective function minimizes the total weighted penalties for late deliveries and nondeliveries.

The preprocessing and postprocessing techniques developed in this thesis are utilized by the Throughput II optimization model as indicated in Chapter I. For this reason, we provide a review of Throughput II in this chapter.

## A. MODEL FEATURES

The Throughput II model has been designed to handle many of the airlift system's particular features and modes of operation. The model is a strategic airlift model, meaning that it considers inter-theater but not intra-theater deliveries. The major features of the airlift system currently captured by the model include (Morton *et al.*, 1995):

- Multiple origins and destinations: The model routes aircraft through multiple origin, enroute and destination airfields.

- Flexible routing structure: The air route structure supported by the model includes delivery and recovery routes with a variable number of enroute stops (usually between zero and three). This gives the model the option of short-range flights with heavier loads or long-range flights with lighter loads. For further routing flexibility, the model also allows the same aircraft to fly different delivery and recovery routes on the same mission.

- Aircraft-to-route restrictions: The user may impose aircraft-to-route restrictions; e.g., military aircraft may only use military airfields for enroute stops. This particular provision arises because the USAF Air Mobility Command (AMC) may call upon civilian commercial airliners to augment USAF aircraft in a deployment, under the Civil Reserve Airlift Fleet (CRAF) program. The model distinguishes between USAF and CRAF aircraft.

- Aircraft assets can be added over time. This adds realism to the model because CRAF and other aircraft may take time to mobilize and are typically unavailable at the start of a deployment.

- Delivery time windows: In a deployment, a unit is ready to move on its *available-to-load date* (ALD) and has to arrive at the theater by its *required-delivery-date* (RDD). This aspect of the problem has been incorporated in the model through user-specified time windows for each unit. The model treats the time window as "elastic" in that cargo may be delivered late, subject to a penalty.

## B. ASSUMPTIONS

The assumptions used in the model are as follows:

- Airfield capacity is represented by a single aggregate figure, called *Maximum-on-Ground* (MOG). The literal translation of MOG as the maximum number of planes that can be simultaneously on the ground at an airfield is misleading, because MOG is used to convey more than just the number of parking spaces. In actuality, airfield capacity depends on many additional factors such as availability of material handling equipment and various ground services capacities. Unfortunately, data are not currently available to support a multidimensional MOG modeling enhancement.

- Inventoried aircraft at origin and destination airfields are considered not to affect the aircraft handling capacity of the airfield. This assumption is not strictly valid since an inventoried aircraft takes up parking space even if it is not consuming services.

- Deterministic ground time: Aircraft turnaround times for onloading and offloading cargo and enroute refueling are assumed to be known constants, although they are naturally stochastic. This ignores the fact that deviations from the given service time can cause congestion on the ground. To offset the optimism of this assumption, an efficiency factor is used in the formulation of airfield capacity constraints.

## C. CONCEPTUAL MODEL FORMULATION

The primary decision variables are the number of sorties initiated, and the amount of cargo and passengers carried, for each unit, by each aircraft type, via each available route, in each time period. Additional variables are defined for the recovery flights, for aircraft inventoried at airfields, and for the possibility (at high penalty cost) of not delivering required cargo or passengers.

### 1. Objective Function

The objective function minimizes the total weighted penalties for late deliveries and nondeliveries subject to appropriate physical and policy constraints. The penalties are weighted according to two factors: the priority of the unit and the degree of lateness. The penalty increases with the amount of time late, and non-delivery has the most austere penalty.

The anticipated use of the model is for situations when the given airlift resources are insufficient for making all the required deliveries on time. On the other hand, if there are enough resources for complete on-time delivery, then the model's secondary objective function is to choose a feasible solution that maximizes unused aircraft. The motivation for the secondary objective is that if the available aircraft are used as frugally as possible, while still meeting the known demands and observing the known constraints, then the mobility system will be as well prepared as possible for unplanned breakdowns and unforeseen requirements, such as an additional nearly simultaneous regional contingency.

### 2. Constraints

The model's constraints can be grouped into five categories: demand satisfaction, aircraft balance, aircraft capacity, aircraft utilization, and airfield handling capacity.

- Demand Satisfaction Constraints: The cargo demand constraints attempt to ensure for each unit that the correct amount of cargo moves to the required destination within the specified time window. The passenger demand constraints do the same for each unit's personnel. The demand constraints have elastic variables for late delivery and non-delivery. The optimization will seek to avoid these options if it is possible with the available assets, or to minimize them if not.

- Aircraft Balance Constraints: These constraints keep physical count of aircraft by type (e.g., C17, C5, C141, etc.) in each time period. They ensure that the aircraft assets are used only when they are available.

- Aircraft Capacity Constraints: There are three different kinds of constraints on the physical limitations of aircraft -- troop carriage capacity, maximum payload, and cabin floor space -- which must be observed at all times.

- Aircraft Utilization Constraints: These constraints ensure that the average flying hours consumed per aircraft per day are within AMC's established utilization rates for each aircraft type.

- Aircraft Handling Capacity at Airfields: These constraints ensure that the number of aircraft routed through each airfield each day is within the airfield's handling capacity.

## D. LIMITATIONS

Two limitations of the optimization model are the inability to handle local traffic congestion and low fidelity with respect to certain aspects of the airlift system.

### 1. Time Resolution

With one-day time periods, the model can route aircraft in a manner that causes local congestion. Another limitation is that the model rounds the time (to the nearest day) at which an aircraft arrives at enroute and destination airfields.

### 2. Aircraft Reliability

Aircraft in need of repair can have an immediate impact on throughput capability, especially when airfield capacity resources are limited.

### 3. Deterministic Ground Time

The aircraft ground times used in the calculation of MOG consumption represent the expected times for onload/refuel/offload, resulting in optimistic throughput capability.

### 4. Airfield Aggregation

Throughput II uses data aggregated from the TPFDD for input. It replaces the large set of airfields with a smaller set of centroid airfields and schedule aircraft through these aggregate airfields.

### 5. Limited Route Structure

Route selection is performed manually, by an expert, who incorprates a variety of factors including minimum payload ferry ranges for specific aircraft types, aircrew flight time restrictions and commercial track avoidance of certain countries. Such a procedure is time consuming but may be adequate for flying to destinations with which USAF has extensive experience; however difficulties may arise when flying to an unfamiliar destination.

Airfield aggregation and limited route structure are examined in this thesis. Limitations involving aircraft reliability and stochastic ground times are examined by Goggins (1995).

# III. PREPROCESSING MODULES

The preprocessing step consists of two distinct modules where airfield aggregation and route selection are performed. The first module aggregates origin and destination airfields into selected centroid airfields. The second module generates the route set which will be used in the optimization model.

## A. CENTROID MODEL

As outlined in the introduction, aggregation of airfields may lead to infeasibilities in the solution produced by the resulting strategic airlift model with respect to borrowing airfield capacity and with respect to an aircraft's ability to fly certain flight legs due to altered inter-airfield distances. It is primarily for the latter reason that we propose a centroid aggregation model that has a primary objective of minimizing the maximum centroid-satellite distance. A pure minimax model will typically have many multiple optimal solutions, but some optima are usually better than others with respect to criteria ignored by the minimax objective (e.g., the satellite-centroid for second furthest airfield). As a result we incorporate a secondary minisum objective term in the centroid selection model to resolve multiple optimal solutions. Aggregation is not performed for enroute airfields, but is performed for origins and destinations separately.

The preprocessing step is based on a set of raw data provided by various sources such as the TPFDD and airfield infrastructure data. Due to some policy constraints or data inaccuricies, there are occasionally some triangle inequality violations for inter-airfield distances. As a data integrity check we ensure that given any two points $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ the great circle distance between $X$ and $Y$, which we denote by $|X - Y|$ has the following properties for all $X$ and $Y$:

1. Nonnegativity $|X - Y| \geq 0$
2. Symmetry $|X - Y| = |Y - X|$
3. Triangle Inequality $|X - Y| \leq |X - Z| + |Z - Y|$ for any $Z$

For the given data set, if the triangle inequality violation occurs,

$$|X - Y| > |X - Z| + |Z - Y| \quad \text{for any } X, Y, Z$$

an algorithm is used to replace this inequality with the following equality.

$$|X - Y| = |X - Z| + |Z - Y|$$

## 1. Mathematical Formulation

### a. *Index Sets*

$i \in I$ : Original airfields, e.g., Travis AFB

$j \in J$ : Candidate airfields that may be centroids

### b. *Data*

$d_{ij}$ : Great circle distances between airfield $i$ and centroid j

$k$ : Number of origin (or destination) airfields that will be used in the Throughput II model

c. ***Decision Variables***

$$x_{ij} = \begin{cases} 1 & \textit{if airfield is assigned to centroid } j \\ 0 & \textit{otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & \textit{if candidate airfield } j \textit{ is selected as a centroid} \\ 0 & \textit{otherwise} \end{cases}$$

d. ***Minimax Model***

$$\begin{aligned} \min \quad & \max_{(i,j) \subset I \times J} \{d_{ij} x_{ij}\} \\ s.t. \quad & x_{ij} \le y_j && \forall (i,j) \in I \times J \\ & \sum_{j \in J} y_j = k \\ & \sum_{j \in J} x_{ij} = 1 && \forall\, i \in I \\ & x_{ij} \in \{0,1\} && \forall (i,j) \in I \times J \\ & y_j \in \{0,1\} && \forall j \in J \end{aligned} \qquad (2.1)$$

The integer program in (2.1) is a variation of Schrage (1991) that may be converted to a linear integer programming problem by introducing a new decision variable $z$ via

$$\begin{aligned} \min \quad & z \\ s.t. \quad & z \ge d_{ij} \cdot x_{ij} && \forall (i,j) \in I \times J \\ & x_{ij} \le y_j && \forall (i,j) \in I \times J \\ & \sum_{j \in J} y_j = k \\ & \sum_{j \in J} x_{ij} = 1 && \forall\, i \in I \\ & x_{ij} \in \{0,1\} && \forall (i,j) \in I \times J \\ & y_j \in \{0,1\} && \forall j \in J \end{aligned} \qquad (2.2)$$

This model minimizes the maximum distance between centroids and their assigned airfields while ensuring that all airfields are assigned to only one centroid and the total number of centroids equals a prespecified value. For reasons detailed above, we use a secondary minisum objective to select among multiple optimal solutions. This can be stated mathematically,

$$
\begin{aligned}
z^* = \min \quad & z + \varepsilon \sum_{i \in I} \sum_{j \in J} d_{ij} \cdot x_{ij} \\
s.t. \quad & z \geq d_{ij} \cdot x_{ij} && \forall (i,j) \in I \times J \\
& x_{ij} \leq y_j && \forall (i,j) \in I \times J \\
& \sum_{j \in J} y_j = k && && (2.3) \\
& \sum_{j \in J} x_{ij} = 1 && \forall\, i \in I \\
& x_{ij} \in \{0,1\} && \forall (i,j) \in I \times J \\
& y_j \in \{0,1\} && \forall j \in J
\end{aligned}
$$

where $\varepsilon > 0$ is a sufficiently small prespecified constant. Model (2.3) contains a large number of binary variables that can quickly result in unreasonably long solution times, even for modest values of $|I|$ and $|J|$. As a result, we utilize a solution technique that solves a sequence of a set covering location models to find the optimal solution to (2.3).

#### *e. Set Covering Model*

The covering problem involves selecting the minimum number of centroids so that each of the original airfields is within a specified distance of some centroid (i.e., covered by that centroid) (Francis *et. al.*, 1992). The set covering model can be stated by introducing the following addition notation,

(1) Data

r Maximum allowed distance between a centroid and its assigned airfields

(2) Index Set

$\Omega_i = \{ j : d_{ij} \leq r \}$ set of centroids that may used to cover airfield I

$$z^* = \min \sum_{j \in J} y_j \qquad (2.4)$$

$$s.t. \sum_{j \in \Omega_i} y_j \geq 1 \qquad \forall i \in I$$

$$y_j \in \{0,1\} \qquad \forall j \in J$$

The key connection between the set covering model (2.4) and minimax / $\varepsilon$ -minisum model (2.3) is the following. If $z^*_{(2.4)}$, the optimal objective value to (2.4) for a specific value of $r$, exceeds $k$ ( the centroid budget in (2.3)) then we know $z^*_{(2.3)} > r$. Similarly if $z^*_{(2.4)} \leq k$ then we know $z^*_{(2.3)} \leq r$. Our goal, then, is to find the smallest value of $r$ for which (2.4) yields an objective of at most $k$. Such an approach proves to be computationally efficient because:

(i) the number of binary decision variables in (2.4) is $|J|$ while the number of binary variables in (2.3) is $|I| \cdot |J| + |J|$ and

(ii) empirical evidence has shown that a linear relaxation of (2.4) often provides a "near-integer" solution.

The algorithm for solving (2.3) via a sequence of set covering models as defined in (2.4) is shown in Figure 2.1. The main part of the algorithm solves (2.3) with $\varepsilon = 0$, and a postprocessing step solves an appropriately defined minisum model to resolve multiple optimal minimax solutions according to the secondary minisum objective.

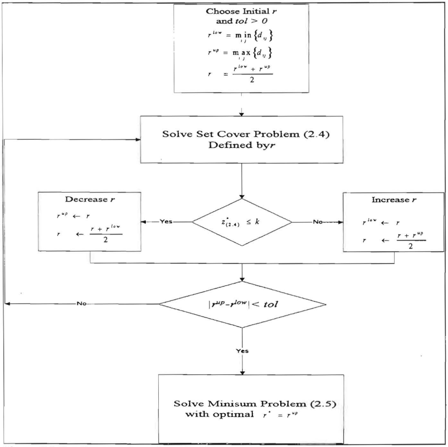



Figure 2.1 Set Covering Location Algorithm

The post-processing minisum model may be stated :

Index Set

$$\Omega = \left\{ (i, j) : d_{ij} \le r^* \right\}$$

$$
\begin{aligned}
\min \sum_{i \in I} \sum_{j \in J} & d_{ij} \cdot x_{ij} \\
s.t. \quad & x_{ij} \le y_j && \forall (i,j) \in \Omega \\
& \sum_{j \in J} y_j = k \\
& \sum_{j \in J} x_{ij} = 1 && \forall i \in I \qquad (2.5) \\
& y_j \in \{0,1\} && \forall j \in J \\
& 0 \le x_{ij} \le 1 && \forall (i,j) \in \Omega
\end{aligned}
$$

Note that due to the linearity of the objective the binary constraints on $x_{ij}$ can be relaxed in (2.5) (Schrage, 1991). This mixed integer program selects a solution from the set of optimal solutions to (2.1), because centroid assignments are approximately restricted by the set $\Omega$. General Algebraic Modeling System (GAMS) (Brooke *et. al.*, 1992) code for solving model (2.3) via the algorithm of figure 2.1 is in Appendix B. Computational results for this and alternative approaches are discussed in Chapter V.

## B. ROUTE SELECTION MODEL

The primary goal of strategic airlift planning is to satisfy Commander-in-Chief Central Command (CINCCENT)'s requirements by employing airlift resources effectively. To meet the goal, planners have to consider the entire airlift system and its interrelated parts. Each enroute stop and destination airfield must have adequate runways, taxiways, ramps and support facilities. Where crew changes are required, a sufficient number of qualified and properly rested aircrews must be available. Normal crew duty time limitations (unless otherwise specified in the governing OPORD/OPLAN) are 16 hours for a basic crew and 24 hours for an augmented crew.

The route selection model may be viewed as a search process or more specifically as a traversal of a tree structure in which each node represents an airfield and each arc represents a flight leg between the airfields represented by the nodes. The search process must find paths through the tree that connect the origin with the destination and obey given physical constraints for each aircraft type such as the appropriate airfield types it may utilize, the aircraft minimum payload ferry range, crew stages etc. Because of the extremely large number of possible routes that could be taken, we need to bound the number of enroute stops. Governing our search is a set of rules that defines the legal routes we can follow. These rules include the crew stages, enroute airfields, runway limitations in airfields and the type of airfield (e.g., military, civilian, joint). Suppose that we want to find some routes from node A to node G through a network of airfields, such as shown in Figure 2.2.

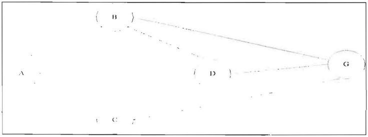



Figure 2.2 A Basic Route Selection Problem.

Finding a route involves six kinds of efforts:

- Each leg of the flight should be less than the minimum payload ferry range of the corresponding aircraft,
- The cumulative time including ground time spent at each stop, without a crew change should be less than 24 hours,
- Every airfield on the route should be compatible with the aircraft,
- The length of the route should be less than the upper bound[4] which is defined by the user,
- All airfields can be used by all types of aircraft as an APOD or an APOE but only specified aircraft as an enroute,
- The distance between two adjacent enroute airfields should be greater than a prespecified distance defined by the user.

The most obvious way to find a solution is to devise a bookkeeping scheme that allows an orderly representation of all possible paths. It is useful to note that the bookkeeping scheme must not allow a node to be revisited (i.e., a cycle) in the network. With cyclic paths eliminated, networks are equivalent to trees. The tree shown in Figure 2.3 is made from the network in Figure 2.2 by using ***breadth first*** search and eliminates certain routes (e.g., ADBG) by using physical and strategic constraints. Pseudo code for the route generation algoritm is described below and additional details are contained in Appendix D.

[4] Upper bound is defined by a parameter $\lambda$ times the great circle distance between origin and destination.

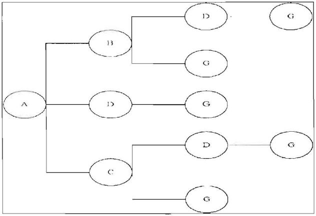



Figure 2.3 A Tree Derived from the Network of Figure 2.2

### 1. Pseudo Code of Route Generation Model

```
read input data files
compute the great circle distances between airfields and add a random number between 0-1[5]
for delivery/recovery routes
        for each aircraft
                determine the adjacency relation between airfields
                        if (distance < ferry range ) and (airfield1 <> airfield2 ) and
                          (aircraft is compatible with airfield ) then
                                adjacent = 1.0
                        otherwise
                                adjacent = 0.0
                        end if.
                route generation algorithm
                put the origin as first stage in the tree
                LOOP : stage 2 to stage 5
                        LOOP : if eof (previous stage) then exit (of LOOP)
                                node:= first(previous stage)
                                if node ≠ destination then begin
                                        select nodes which are accessible from node due to
                                        given constraints
                                        exclude any node which is found in the past route list
                                        add rest of the nodes to next stage
```

[5] If the geographic location values are not available based on security reason, program can just read the distance matrix provided by user. By adding a small random number, the program perturbs the data in order to facilitate breaking ties and having routes uniquely identified by their length.

```
                        end
                    goto LOOP
                    increment stage level
                end LOOP.

                LOOP start from the last stage to first stage
                    find the destination node
                    follow past route list through origin
                    record the route
                end LOOP.
                feasibility checks
                    if route length > upper bound then delete the route
                    if route is infeasible due to crew stages then delete the route
                manual screening of the routes by user
                enumerate routes based on their length and store for each aircraft
            end for
            manual route adding by user
        end for
        output the route set.
```

# IV. POSTPROCESSING MODULES

The final output of the combined centroid and route selection preprocessing step is a set of candidate routes that can be used as input data for Throughput II. However the airfield aggregation and subsequent route selection process may still generate a large number of routes, and it may not be computationally practical to include all the candidate routes in the Throughput II model. Thus, we utilize a method to convert the large problem into smaller problems of manageable size. The approach to this reduction is to solve a linear program optimized over a subset of its columns, the result of which may be suboptimal. Based on ideas from LP sensitivity analysis we can test whether the set of decision variables associated with a certain route, not in the original subset, is attractive. If it is attractive, it can be put into the basis as a new column. Moreover, we can examine why this route might be attractive and, if necessary, modify the rules that select the initial set of routes in the heuristic route selection model. The Throughput II model may be regarded as an example of the following LP model :

$$\begin{aligned} \min \quad & cx \\ s.t. \quad & Ax = b \\ & x \geq 0 \end{aligned} \qquad (3.1)$$

and its dual

$$\begin{aligned} \max \quad & \pi b \\ s.t. \quad & \pi A \leq c \\ & \pi \ unrestricted. \end{aligned} \qquad (3.2)$$

The optimality conditions for a linear program state that necessary and sufficient conditions for $x^*$ and $\pi^*$ to be an optimal primal-dual pair are,

| | | | |
|---|---|---|---|
| 1. | Primal Feasibility | : | $Ax = b$ |
| 2 | Dual Feasibility | : | $\pi A \leq c$ |
| 3. | Strong Duality | : | $cx = \pi b$ |

In standard column generation techniques, new columns are defined with respect to the original constraint set. Throughput II, however, does not fall into this framework. Instead, associated with each new route-aircraft combination at any point in time is a set of decision variables that appear in the original constraints plus an additional set of constraints that contain only the new decision variables. For example, one of the new constraints specifies that the amount of cargo that can flow along a new route is restricted by the capacity of aircraft flying this route. We denote the set of variables associated with the new routes $y$ and express their inclusion in model (3.1) via,

$$\begin{aligned} \min \quad & cx + fy \\ s.t. \quad & Ax + Fy = b \\ & Dy = 0 \\ & x, y \geq 0 \end{aligned} \qquad (3.3)$$

The fact that the right-hand-side of the new set of constraints is zero reflects our assumption that the new LP (3.3) is feasible if $y=0$ ; i.e., we are not required to fly the new routes. The dual of (3.3) is,

$$
\begin{aligned}
\max \quad & \pi b && (3.4)\\
s.t \quad & \pi A \le c \\
& \pi F + \gamma D \le f
\end{aligned}
$$

As a result of our non-standard form the usual optimality test regarding the sign of the reduced cost of a new column no longer directly applies. However, the following theorem provides an analogous optimality test for the framework outlined above.

**Theorem 3.1**

Let $x^*$ and $\pi^*$ be an optimal primal-dual pair for (3.1) and (3.2). If $\pi F_j - f_j \le 0 \quad \forall j$, then $(x^*, 0)$ and $(\pi^*, 0)$ form an optimal primal-dual pair for (3.3) and (3.4).

**Proof of Theorem 3.1:**

We will verify the Karush - Kuhn - Tucker (KKT) Optimality Conditions of

1. Primal Feasibility
2. Strong Duality
3. Dual Feasibility

of the primal-dual solution pair $(x'', y'') = (x^*, 0)$ and $(\pi'', \gamma'') = (\pi^*, 0)$ in (3.3) and (3.4).

**1. Primal Feasibility :**

$$Ax'' + Fy'' = b$$
$$Dy'' = 0$$
$$x'', y'' \geq 0$$

Now,

$Ax^* + F0 = b$ since $Ax^* = b$ and

$D0 = 0$

$(x^*, 0) \geq 0$ also follows from feasibility of $x^*$ in (3.1).

**2. Strong Duality :**

$$cx'' + fy'' = \pi'' b + \gamma'' 0$$
$$cx^* + f0 = \pi^* b + \gamma 0$$
$$cx^* = \pi^* b$$

follows from strong duality of $x^*$ and $\pi^*$ in (3.1) and (3.2).

**3. Dual Feasibility :**

$\pi'' A_j - c_j = \pi^* A_j - c_j \leq 0$ follows from dual feasibilty in (3.2).

$$\pi'' F_j - \gamma'' D_j - f_j = \pi^* F_j - f_j \leq 0$$

follows from the optimality test hypothesis. QED

From Theorem 3.1 we know that if we solve Throughput II and subsequently determine that the reduced costs of variables associated with routes not explicitly included in the original model pass the optimality test $\pi^* F_j - f_j \leq 0$ then the current solution is optimal with respect to all routes. If the optimality test is not satisfied then we can append the variables associated with attractive routes and re-solve the corresponding Throughput II model. To implement the column generation technique in Throughput II, note that the variables associated with the new routes are *X, Y, TONSUE,* and *TPAX.* Thus, summing the dual variables associated with constraints that these variables appear in computes the "$\pi l \cdot$" terms. Additionaly, only the *TONSUE* and *TPAX* variables have objective function coefficients (for late penalty computation). Consequently, only these columns have "*f*" terms associated. The complete GAMS code is included in Appendix C.

It is desirable to prevent, or at least recognize, when Throughput II generates solutions that are infeasible with respect to the original, disaggregated network. One example of such a violation may occur when a unit's origin airfield (and hence the unit) is aggregated with a centroid closer to the destination airfield; in this case, the origin-destination distance may be greater than the selected aircraft's minimum payload range, even though the centroid-destination distance was not longer than the minimum payload ferry range. In reality, therefore, this aircraft wold have to use a different route or carry a lighter load. Another potential infeasibility can arise when Throughput II inappropriately "borrow" airfield capacity between airfields that have been aggregated into a single airfield. A promising and important area for future research is to implement a disaggregation procedure that maps aircraft missions to the original network. With such a procedure in place, it will be possible to recognize and take steps to avoid the infeasibilities described above.

# V. RESULTS AND SUMMARY

This chapter reports the results for model instances based on two different data sets provided by AFSAA in the summer and fall of 1995. TPFDD data must be aggregated for use in Throughput II. This aggregation procedure is implemented in the Pascal programming language. It takes the input TPFDD, and then performs aggregation based on a strategy determined by several user-specified parameters. The flow diagram of the aggregation algorithm is in Appendix D and may be summarized as follows:

Step 1: Aggregate units that are to be shipped between the same origin-destination pair and have identical delivery time windows based on the *available-load-date* (ALD) and *required-delivery-date* (RDD).

Step 2: Eliminate units that have very small movement requirements (e.g., one passenger).

Step 3: Aggregate units that are to be shipped between the same origin-destination pair and have similar RDD values. The definition of "similar" is allowed to depend on when over the deployment period the units should be delivered. Greater latitude is permitted near the end of the scenario compared to the early days of the deployment. This is implemented by the user specifying parameters $R_1$, $R_2$, $R_3$, partitioning the planning period into segments with corresponding tolerance values $r_1$, $r_2$, $r_3$, where,

$$0 \leq r_1 < r_2 < r_3$$

If two units with the same origin and destination have RDD's in the same segment and if the difference between the RDD's is less than the corresponding $r$ value, the two units are aggregated. This is depicted in Figure 5.1.

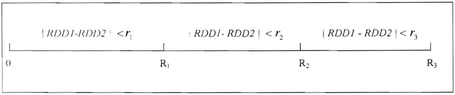



Figure 5.1. Unit Aggregation Over the Time Horizon

After the aggregation the RDD and ALD values are defined as:

$RDD_{new}$ = Max (RDD1,RDD2) ,

$ALD_{new}$ = Weighted Based on Movement Requirements (ALD1,ALD2)

Step 4: The results of the aggregation procedure are then used in the centroid model, followed by the route selection procedure which in turn provides input data for the Throughput II optimization model.

## A. RESULTS

### 1. Centroid Model

The data set that has been used for the centroid models consists of 117 airfields in the Continental United States (CONUS). All the models presented in Chapter III can be used to select the centroid airfields. In order to investigate the relative merits of these models three criteria have been defined. The first one is the computational time required to solve the model on an IBM RS6000/590 system using GAMS with the OSL solver. The second criterion is the average distance between the centroid airfields and their satellite airfields. This is an important issue in order to have an accurate delivery cycle time in the Throughput II optimization model. The last criterion is the primary objective value of the proposed model which is the maximum distance between the centroid airfields and their satellite airfields. The model that we develop allows the user to predefine certain centroid-satellite groups and effectively remove them from consideration in the centroid model. This is desirable feature because it allows the user to prevent the possibility of capacity borrowing at known system bottlenecks. The results are illustrated in Table 5.1.

| MODEL | TIME (RS6000/590 GAMS/OSL) | AVERAGE DISTANCE (nm) | MAXIMUM DISTANCE (nm) |
|---|---|---|---|
| YOST'S MODEL | 14 SECONDS | 326 | 794 |
| MINISUM MODEL | 10 MINUTES | 277 | 717 |
| MINIMAX MODEL[6] | 32 SECONDS | 370 | 600 |
| SUGGESTED MODEL | 87 SECONDS | 317 | 600 |

Table 5.1. Computational Results for Various Centroid Models.

Note that Yost's model is a minisum model with a limited number of airfields that may be used as centroids. As a result, in runs quickly but performs worse than the other models relative to the other criterion. The suggested model provides an attractive balance between the minisum model and the minimax model with respect to the average and maximum distance criterion and requires modest computational effort.

[6] This is the model that solved via set covering algorithm.

Recall that the centroid models require the user to define the (maximum) number of centroids that may be selected. Figure 5.2 illustrates how the maximum centroid-satellite distance varies with the number of centroids. As shown in Figure 5.2, there is a large distance when the centroid budget is small. Large range values are more likely to lead to infeasible (unflyable) routes in the route selection model.

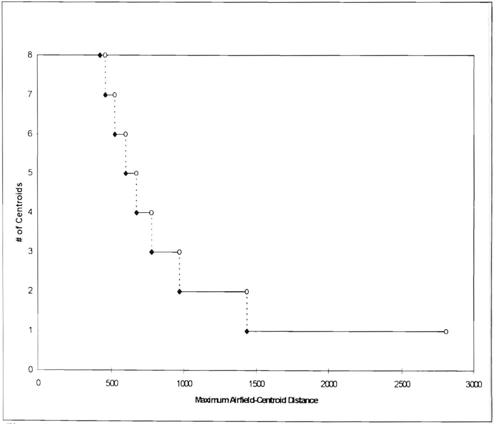


Figure 5.2. Computational Results for Centroid Model.

## 2. Route Selection Model

After we select the centroid airfields for origin and destination airfields via the centroid model, the route generation model constructs routes based on user-defined input parameters. It takes less than an hour to set up the input files for the first run. After this preparation, each run takes approximately five minutes.

The first input parameter is denoted $\lambda$ and is used to define the maximum route length between origin-destination pairs. This lambda value actually consists of the product of three different parameters. We assign a lambda value for each airfield and aircraft. To illustrate this process, if we call the resultant lambda value as the composite lambda value, then,

$$\lambda_{composite} = \lambda_{origin} * \lambda_{destination} * \lambda_{aircraft}$$

If we define each of these parameters as 1.0, then only the direct route (i.e., no enroute stops) will be generated. Using these three different parameters provides flexibility for selecting routes based on the type of aircraft or accessibility of different regions of the world with origin/destination dependent lambda values.

The second parameter of the route selection model is the restriction on the flight time of an aircrew. In this research we used 24 hours for the maximum flight time without a crew-change. The final input required from the user is the set of crew-stage airfields and enroute airfields that can be used during the airlift operation. Table 5.2 shows the number of infeasible routes for various $\lambda$ values for the C747 which has a minimum payload ferry range of 6500 nm for delivery routes and 9200 nm for recovery routes,

| Total Routes | Composite Lambda Value | Total Feasible Routes | Total Infeasible Routes Due To Flight Time Restrictions |
|---|---|---|---|
| | 1.041 | 116 | 0 |
| **7266** | 1.117 | 163 | 6 |
| **Delivery Routes** | 1.161 | 217 | 22 |
| | 1.184 | 254 | 27 |
| | 1.240 | 302 | 43 |
| | 1.000 | 32 | 0 |
| **4152** | 1.139 | 149 | 26 |
| **Recovery Routes** | 1.162 | 233 | 53 |
| | 1.185 | 274 | 73 |

Table 5.2. Route Selection Model Results.

We have performed experiments in order to show the "quality" of the routes that have been generated by the route selection procedure versus the routes with the same data set in Lim (1994). The computational results based on comparing manually generated routes and the model are shown in Table 5.3.

| Model | Number of Routes | Objective Function Value |
|---|---|---|
| **Route Selection Model** | 71 routes | 13.208 |
| **Manually Generated Routes** | 62 routes | 15.021 |

Table 5.3. Route Selection Model's Results.

The route selection model improves the routing options in Throughput II so that there is a 12% reduction in the weighted sum of late and undelivered cargo.

### 3. Postprocessing Module

One important issue in the postprocessing module is whether including additional routes in Throughput II will result in an improved solution. There are many decision variables associated with a single physical route through the network:

The route may be flown by several types of planes and at different points in time in the planning period. There are typically a large number of decision variables associated with routes not explicitly included in Throughput II that have attractive reduced costs (see Chapter II). However, many of these provide small if any improvement in the solution, if included. For this reason, we only augment the set of routing options with decision variables whose reduced costs are significantly attractive, relative to some tolerance. In the proposed method, Throughput II is solved with respect to the original routes and then the set of routes is augmented and the model is re-optimized.

When this is done, it is desirable to initialize the re-optimization with the basis from optimal solution of the original model. Table 5.4 illustrates the savings in computational effort that this technique provides relative to solving the larger model from scratch. In addition, Table 5.4 shows that the postprocessing scheme found routing options that significantly decreased the weighted sum of late and undelivered cargo.

| **Number of Routes** | **Model** | **Objective Function Value** | **Running Time (RS6000)** |
|---|---|---|---|
| **71 routes** | Throughput II | 13.208 | 10 minutes |
| **117 routes** | Throughput II | 8.0292 | 60 minutes |
| **117 routes** | Proposed Model | 8.0292 | 38.75 minutes |

Table 5.4. Computational Results of Sensitivity Analysis

## B. RECOMMENDATIONS AND FUTURE WORK

The techniques proposed in this thesis have been recently applied to a large TPFDD provided by AFSAA in August 1995 for a C17/C747 fleet mix study. Due to the enormous size of the data, which has approximately 14500 movement requirements, the preprocessing modules for centroid selection and route reduction were an invaluable tool for quickly generating an optimization model of tractable size. This modeling and data reduction exercise also revealed that certain additional features could further streamline the process for future analyses:

- User interface: Current aggregation and route selection modules are implemented as a Disk Operating System (DOS) application. Because of the structure of this operating system, the ease of use for an analyst would be greatly enhanced by implementing this software as a Windows application with user-friendly interfaces.

- Data Interface: The current model uses text files that must be manually entered by the analyst. Because this is a time consuming procedure, a data interface that permits easy modifications while ensuring data integrity would significantly decrease the model generation time.
- Route Generation Module: As indicated above, there is a set of decision variables associated with each route. It would be useful to develop a theory that utilizes reduced cost information from individual decision variables to indicate when the inclusion the set of decision variables will improve (or be likely to improve) the current solution.
- Feasibility Test Postprocessing Module: As discussed at the end of Chapter IV, deriving a procedure that maps Throughput II solutions to the original disaggregated network would prove valuable with respect to testing the feasibility of the proposed solution.

As shown above, much work remains to realize the full potential of pre and post-processing of the Throughput II model. The contributions of this thesis form a solid foundation for these improvments.

# APPENDIX A. THROUGHPUT II MODEL

The following is a brief summary of Throughput II (Morton *et. al.*, 1995).

## A. INDICES

| | |
|---|---|
| $u$ | indexes units, e.g., 82nd Airborne |
| $a$ | indexes aircraft types, e.g., C5, C141 |
| $t, t'$ | index time periods |
| $b$ | indexes all airfields (origins, enroutes and destinations) |
| $i$ | indexes origin airfields |
| $k$ | indexes destination airfields |
| $r$ | indexes routes |

## B. INDEX SETS

### 1. Airfield Index Sets

| | |
|---|---|
| $B$ | set of available airfields |
| $I \subseteq B$ | origin airfields |
| $K \subseteq B$ | destination airfields |

### 2. Aircraft Index Sets

| | |
|---|---|
| $A$ | set of available aircraft types |
| $A_{bulk} \subseteq A$ | aircraft capable of hauling bulk size cargo |
| $A_{over} \subseteq A_{bulk}$ | aircraft capable of hauling over-sized cargo |
| $A_{out} \subseteq A_{over}$ | aircraft capable of hauling out-sized cargo |

Bulk cargo is palletized on 88 x 108 inch platforms, which can fit on any plane. Over-sized cargo is non-palletized rolling stock, it is larger than bulk cargo and can fit on a C141, C5 or C17. Out-sized cargo is very large non-palletized cargo that can fit into a C5 or C17 but not a C141.

### 3. Route Index Sets

| | |
|---|---|
| $R$ | set of available routes |
| $R_a \subseteq R$ | permissible routes for aircraft $a$ |
| $R_{ab} \subseteq R_a$ | permissible routes for aircraft $a$ that use airfield $b$ |
| $R_{aik} \subseteq R_a$ | permissible routes for aircraft $a$ that have origin $i$ and destination $k$ |
| $DR_i \subseteq R$ | delivery routes that originate from origin $i$ |
| $RR_k \subseteq R$ | recovery routes that originate from destination $k$ |

### 4. Time Index Set

| | |
|---|---|
| $T$ | set of time periods |
| $T_{uar} \subseteq T$ | possible launch times of sorties for unit $u$ using aircraft $a$ and route $r$ |

The set $T_{uar}$ covers the allowed time window for unit $u$, which starts on the unit's available-to-load date and ends on the unit's required delivery date, plus some extra time up to the maximum allowed lateness for the unit.

## C. GIVEN DATA

### 1. Movement Requirements Data

| | |
|---|---|
| $MovePAX_{uik}$ | Troop movement requirement for unit $u$ from origin $i$ to destination $k$ |
| $MoveUE_{uik}$ | Equipment movement requirement in short tons (stons) for unit $u$ from origin $i$ to destination $k$ |
| $ProBulk_u$ | Proportion of unit $u$ cargo that is bulk-sized |
| $ProOver_u$ | Proportion of unit $u$ cargo that is over-sized |
| $ProOut_u$ | Proportion of unit $u$ cargo that is out-sized |

### 2. Penalty Data

| | |
|---|---|
| $LatePenUE_u$ | Lateness penalty (per ston per day) for unit $u$ equipment |

| | |
|---|---|
| $LatePenPAX_u$ | Lateness penalty (per soldier per day) for unit *u* troops |
| $NoGoPenUE_u$ | Non-delivery penalty (per ston) for unit *u* equipment |
| $NoGoPenPAX_u$ | Non-delivery penalty (per soldier) for unit *u* troops |
| $MaxLate$ | Maximum allowed lateness (in days) for delivery |
| $Preserve_{at}$ | Penalty (small artificial cost) for keeping aircraft *a* in mobility system at time *t* |

### 3. Cargo Data

| | |
|---|---|
| $UESqFt_u$ | Average cargo floor space (in sq. ft.) per ston of unit *u* equipment |
| $PAXWt_u$ | Average weight of a unit *u* soldier inclusive of personal equipment |

### 4. Aircraft Data

| | |
|---|---|
| $Supply_{at}$ | Number of aircraft of type *a* that become available at time *t* |
| $MaxPAX_a$ | Maximum troop carriage capacity of aircraft *a* |
| $PAXSqFt_{ua}$ | Average cargo space (in sq. ft.) consumed by a unit *u* soldier fro aircraft *a* |
| $ACSqFt_a$ | Cargo floor space (in sq. ft.) of aircraft *a* |
| $LoadEff_a$ | Cargo space loading efficiency (<1) for aircraft *a*. This accounts for the fact that it is not possible in practice to fully utilize the cargo space. |
| $URate_a$ | Established utilization rate (flying hours per aircraft per day) for aircraft *a* |

### 5. Airfield Data

| | |
|---|---|
| $MOGCap_b$ | Aircraft capacity (in narrow-body equivalents) at airfield *b* |

| | |
|---|---|
| $MOGReq_{ab}$ | Conversion factor to narrow-body equivalents for one aircraft of type $a$ at airfield $b$ |
| $MOGEff$ | MOG efficiency factor (<1), to account for the fact that it is impossible to fully utilize available MOG capacity due to randomness of ground times |

### 6. Aircraft Route Performance Data

| | |
|---|---|
| $MaxLoad_{ar}$ | Maximum payload (in stons) for aircraft $a$ flying route $r$ |
| $GTime_{abr}$ | Aircraft ground time (due to onload or offload of cargo, refueling, maintenance, etc.) needed for aircraft $a$ at airfield $b$ on route $r$ |
| $DTime_{abr}$ | Cumulative time (flight time plus ground time) taken by aircraft $a$ to reach airfield $b$ along route $r$ |
| $FltTime_{ar}$ | Total flying hours consumed by aircraft $a$ on route $r$ |
| $CTime_{ar}$ | Cumulative time (flight time plus ground time) taken by aircraft $a$ on route $r$ |
| $DaysLate_{uart}$ | Number of days late unit $u$'s requirement would be if delivered by aircraft $a$ via route $r$ with mission start time $t$ |

## D. DECISION VARIABLES

### 1. Sortie Variables

| | |
|---|---|
| $X_{uart}$ | Number of aircraft $a$ that airlift unit $u$ via route $r$ with mission start time during period $t$ |
| $Y_{art}$ | Number of aircraft $a$ that recover from a destination airfield via route $r$ with start time during period $t$ |

## 2. Aircraft Allocation and De-allocation Variables

| | |
|---|---|
| $Allot_{ait}$ | Number of aircraft $a$ that become available at time $t$ that are allocated to origin $i$ |
| $Release_{ait}$ | Number of aircraft $a$ available at origin $i$ in time $t$ that are not scheduled for any flights from then on |

## 3. Aircraft Inventory Variables

| | |
|---|---|
| $H_{ait}$ | Number of aircraft $a$ inventoried at origin $i$ at time $t$ |
| $HP_{akt}$ | Number of aircraft $a$ inventoried at destination $k$ at time $t$ |
| $NPlanes_{at}$ | Number of aircraft $a$ in the air mobility system at time $t$ |

## 4. Airlift Quantity Variables

| | |
|---|---|
| $TonsUE_{uart}$ | Total stons of unit $u$ equipment airlifted by aircraft $a$ via route $r$ with mission start time during period $t$ |
| $TPAX_{uart}$ | Total number of unit $u$ troops airlifted by aircraft $a$ via route $r$ with mission start time during period $t$ |

## 5. Elastic (Nondelivery) Variables

| | |
|---|---|
| $UENoGo_{uik}$ | Total stons of unit $u$ equipment with origin $i$ and destination $k$ that is not delivered in the prescribed time frame |
| $PAXNoGo_{uik}$ | Number of unit $u$ troops with origin $i$ and destination $k$ who are not delivered in the prescribed time frame |

## E. OBJECTIVE

minimize

$$\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenUE_u * DaysLate_{uart} * TonsUE_{uart} \tag{A.1}$$
$$+\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenPAX_u * DaysLate_{uart} * TPAX_{uart}$$
$$+\sum_{u}\sum_{i}\sum_{k}\left(NoGoPenUE_u * UENoGo_{uik} + NoGoPenPAX_u * PAXNoGo_{uik}\right)$$
$$+\sum_{a}\sum_{t} Preserve_{at} * NPlanes_{at}$$

The objective function minimizes the total weighted penalties incurred for late deliveries and non-deliveries. The model's secondary objective is to choose a feasible solution that maximizes unused aircraft.

## F. CONSTRAINTS

$$\sum_{a\in A_{bulk}}\sum_{r\in R_{uik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} = MoveUE_{uik}, \tag{A.2}$$
$$\forall\ u,i,k:\ MoveUE_{uik} > 0$$

$$\sum_{a\in A_{out}}\sum_{r\in R_{uik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq ProOut_u * MoveUE_{uik}, \tag{A.3}$$
$$\forall\ u,i,k:\ MoveUE_{uik} > 0$$

$$\sum_{a\in A_{ovr}}\sum_{r\in R_{uik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq \tag{A.4}$$
$$\left(ProOver_u + ProOut_u\right) * MoveUE_{uik},\quad \forall\ u,i,k:\ MoveUE_{uik} > 0$$

$$\sum_{u}\sum_{r\in R_{uik}}\sum_{t\in T_{uar}} TPAX_{uart} + PaxNoGo_{uik} = MovePAX_{uik}, \tag{A.5}$$
$$\forall\ u,i,k:\ MovePAX_{uik} > 0$$

$$\sum_{r} \sum_{r \in DR_i} X_{uart} + H_{ait} + Release_{ait} = \tag{A.6}$$

$$H_{ai,t-1} + Allot_{ait} + \sum_{r \in R_a} \sum_{t' \in [CTime_{ar}] - t} Y_{art'}, \qquad \forall\, a,i,t$$

$$\sum_{r \in RR_k} Y_{art} + HP_{akt} = HP_{ak,t-1} + \sum_{u} \sum_{r \in R_{ak}} \sum_{\substack{t' \in T_{uar} \\ t' \in [CTime_{ar}] - t}} X_{uart'}, \qquad \forall\, a,k,t \tag{A.7}$$

$$\sum_{t'=1}^{t} \sum_{i} Allot_{ait} \le \sum_{t'=1}^{t} Supply_{at}, \qquad \forall\, a,t \tag{A.8}$$

$$NPlanes_{at} = \sum_{t'=1}^{t} \sum_{i} Allot_{ait'} - \sum_{t'=1}^{t} \sum_{i} Release_{ait'}, \qquad \forall\, a,t \tag{A.9}$$

$$\sum_{r \in R_u} \sum_{t'=1}^{t} \sum_{u} K_{artt'} * X_{uart'} + \sum_{r \in R_u} \sum_{t'=1}^{t} K_{artt'} * Y_{art'} + \sum_{i} \sum_{t'=1}^{t} H_{ait'} \tag{A.10}$$

$$+ \sum_{k} \sum_{t'=1}^{t} HP_{akt'} \le \sum_{t'=1}^{t} NPlanes_{at}, \qquad \forall\, a,t$$

where

$$K_{artt'} = \begin{cases} t - t' + 1, & \text{if} \quad t' \le t < t' + CTime_{ar} - 1 \\ CTime_{ar}, & \text{if} \quad t \ge t' + CTime_{ar} - 1 \end{cases}$$

$$TPAX_{uart} \le MaxPAX_{a} * X_{uart}, \qquad \forall\, u,a,r,t: t \in T_{uar} \tag{A.11}$$

$$TonsUE_{uart} + PAXWt * TPAX_{uart} \le MaxLoad_{ar} * X_{uart}, \qquad \forall\, u,a,r,t: t \in T_{uar} \tag{A.12}$$

$$PAXSqFt_{a} * TPAX_{uart} + UESqFt_{u} * TonsUE_{uart} \le \tag{A.13}$$

$$ACSqFt_{a} * LoadEff_{a} * X_{uart}, \qquad \forall\, u,a,r,t: t \in T_{uar}$$

$$\sum_{u}\sum_{r \in R_a}\sum_{t \in T_{uar}} FltTime_{ar} * X_{uart} + \sum_{r \in R_a}\sum_{t} FltTime_{ar} * Y_{art} \leq \sum_{t} URate_a * NPlanes_{at}, \quad \forall a \tag{A.14}$$

$$\sum_{u}\sum_{a}\sum_{r \in R_a} \sum_{\substack{t' \in T_{uar} \\ t' + \lceil DTime_{abr} \rceil = t}} (MOGReq_{ab} * GTime_{abr} / 24) * X_{uart'} + \sum_{a}\sum_{r \in R_a} \sum_{t' + \lceil DTime_{abr} \rceil = t} (MOGReq_{ab} * GTime_{abr} / 24) * Y_{art'} \leq MOGEff * MOGCap_b, \quad \forall b,t \tag{A.15}$$

A.2 Demand satisfaction constraints for all classes of cargo

A.3 Demand satisfaction constraints for out-sized cargo

A.4 Demand satisfaction constraint for over-sized cargo

A.5 Demand satisfaction for troops

A.6 Aircraft balance constraints at origin airfields

A.7 Aircraft balance constraints at destination airfields

A.8 Aircraft balance constraints for allocations to origins

A.9 Aircraft balance constraints accounting for allocations and releases

A.10 Cumulative aircraft balance constraints

A.11 Troop carriage capacity constraints

A.12 Maximum payload constraints

A.13 Cargo floor space constraints

A.14 Aircraft utilization constraints

A.15 Airfield MOG constraints

# APPENDIX B. GAMS CODES FOR CENTROID MODEL

```
$TITLE SETCOVERING
$STITLE Yasin TURKER 29 JUL 1995
*-----------GAMS AND DOLLAR CONTROL OPTIONS---------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF INLINECOM { }
$ONMIXED
OPTIONS
   LIMCOL = 0 , LIMROW = 0 , SOLPRINT = OFF , DECIMALS = 2
   RESLIM = 18000, ITERLIM = 400000, OPTCR = 0.1 , SEED = 3141
   MIP = OSL
*--------------------------------------------------------------------
*include centroid model for origins
$INCLUDE origset.gms
SETS INDEX       /1*100/
    AF airfields
$INCLUDE airfield.set
I(AF) origins
$INCLUDE origin1.txt

ALIAS (I,IP,J);
ALIAS(AF,AFP,AFPP);

PARAMETER DIS (AF,AFP)
$INCLUDE dist1.txt

SCALAR K2  number of airfields which is allowed throughput model /4/
     RAD   radius of centroid
     RBEST best radius of centroid
     UPP upper value of centroid
     LOW lower value of centroid;
UPP=SMAX((I,J),DIS(I,J));
LOW=SMIN((I,J),DIS(I,J));
RAD=(UPP+LOW)/2;
RBEST=UPP;

VARIABLE
ZORIG    maximum number of centroids
Z1ORIG   objective function value of minisum model;

BINARY VARIABLE
YORIG(J) open centroid j;

EQUATIONS
OBJ      objective function of covering problem
OBJ1     objective function of minisum model
COVER(I) covering assignments for set covering model
```

```
ASSIGN1(I,IP) assignment for each airfield for an centroid
ASSIGN2  number of airfield which is allowed for throughput  model
ASSIGN3(I) make sure every airfield either be in centoid or alone;

*MINIMIZE
OBJ.. SUM(J,YORIG(J))=E=ZORIG;
COVER(I).. SUM(J,YORIG(J)$(DIS(I,J) LE RAD))=G=1;

MODEL ORIGCOV /OBJ,COVER/;

LOOP(INDEX$(NOT(((UPP - LOW) LE 5.0) AND (ZORIG.L EQ K2))),
  RAD=(UPP+LOW)/2;
  SOLVE ORIGCOV USING MIP MINIMIZING ZORIG;
  DISPLAY RAD,UPP,LOW;
  UPP=RAD$(ZORIG.L LE K2+0.1)+UPP$(ZORIG.L GE K2+0.9);
  LOW=RAD$(ZORIG.L GE K2+0.9)+LOW$(ZORIG.L LE K2+0.1);

);
DISPLAY YORIG.L,RAD,UPP,LOW,ZORIG.L;

POSITIVE VARIABLE
XORIG(I,IP) airfield i belongs to centroid j ;

XORIG.UP(I,J)$(DIS(I,J) LE RAD) =1.0;

*minisum model
OBJ1.. SUM((I,J))$(DIS(I,J) LE RAD),DIS(I,J)*XORIG(I,J))=E=Z1ORIG;
ASSIGN1(I,J)$(DIS(I,J) LE RAD).. XORIG(I,J)=L=YORIG(J);
ASSIGN2.. SUM(J,YORIG(J))=E=K2;
ASSIGN3(I).. SUM(J$(DIS(I,J) LE RAD),XORIG(I,J)) =E=1;
MODEL CENTROID/OBJ1,ASSIGN1,ASSIGN2,ASSIGN3/;
SOLVE CENTROID USING MIP MINIMIZING Z1ORIG;
DISPLAY YORIG.L,ZORIG.L,Z1ORIG.L,XORIG.L;

*include centroid model for destinatinations
$INCLUDE destset.gms
SETS  K(AF) airports
$INCLUDE destina1.txt

ALIAS (K,KP,L);
UPP=SMAX((K,L),DIS(K,L));
LOW=SMIN((K,L),DIS(K,L));
RAD=(UPP+LOW)/2;

SCALAR K1 number of centroid destination airfields is allowed in Throughput /8/;
VARIABLE
ZDEST     maximum number of centroids
Z1DEST    objective function value of minisum model;

BINARY VARIABLE
YDEST(L)  open centroid L;
```

```
EQUATIONS
OBJ4    objective function of covering problem
OBJ6    objective function of minisum model
COVER1(K)  covering assignments for set covering model
ASSIGN11(K,KP)  assignment for each airfield for an centroid
ASSIGN21   number of airfield which is allowed for throughput  model
ASSIGN31(K) make sure every airfield either be in centoid or alone;

*MINIMIZE
OBJ4..  SUM(L,YDEST(L))=E=ZDEST;
COVER1(K).. SUM(L,YDEST(L)$(DIS(K,L) LE RAD))=G=1;

MODEL DESTCOV /OBJ4,COVER1/;

LOOP(INDEX$(NOT(((UPP - LOW) LE 5.0) AND (ZDEST.L EQ K1))),
  RAD=(UPP+LOW)/2;
  SOLVE DESTCOV USING MIP MINIMIZING ZDEST;
  UPP=RAD$(ZDEST.L LE K1+0.1)+UPP$(ZDEST.L GT K1+0.9);
  LOW=RAD$(ZDEST.L GT K1+0.9)+LOW$(ZDEST.L LE K1+0.1);

);
DISPLAY YDEST.L,RAD,UPP,LOW,ZDEST.L;

POSITIVE VARIABLE
XDEST(K,KP) airfield i belongs to centroid L ;
XDEST.UP(K,L)$(DIS(K,L) LE RAD) =1.0;

*minisum model
OBJ6..  SUM((K,L) $(DIS(K,L) LE RAD),DIS(K,L)*XDEST(K,L))=E=Z1DEST;
ASSIGN11(K,L)$(DIS(K,L) LE RAD).. XDEST(K,L)=L=YDEST(L);
ASSIGN21..  SUM(L,YDEST(L))=E=K1;
ASSIGN31(K)..  SUM(L$(DIS(K,L) LE RAD),XDEST(K,L)) =E=1;
MODEL CENTROID1/OBJ6,ASSIGN11,ASSIGN21,ASSIGN31/;
SOLVE CENTROID1  USING MIP MINIMIZING Z1DEST;
DISPLAY YDEST.L,ZDEST.L,Z1DEST.L,XDEST.L;


*include set of units
SET UNIT  units in TPFDD
/
$INCLUDE unit1.txt
/;

SETS AB aircraft body type
*Aircraft are classified as WB (widebody), NB (narrow body),
*GM (ground manuverable), or TT (tactical).
 / WB, NB, GM, TT /

ENR(AF) enroute airfields
$INCLUDE enroute.txt
/;
```

```
*moving cargo requirements for units given origin and destination
*origins and destinations are not centroids
PARAMETER TONSUE(UNIT,I,K)
$INCLUDE moving1.txt

*moving passenger requirements for given origin and destination
*origins and destinations are not centroids
PARAMETER TPAX(UNIT,I,K)
$INCLUDE movepa11.txt

*mog capacity for all airfield  before aggregation
TABLE  MOG(AF,AB)
$INCLUDE mogcap.txt
;
scalar totton total cargo after aggregation
     tottpax total passenger after aggregation;
totton=sum((unit,i,k),tonsue(unit,i,k));
tottpax=sum((unit,i,k),tpax(unit,i,k));

*aggregation parameter
*aggregate cargo and passenger due to its origin and centroid
PARAMETER REPORT2(UNIT,J,L);
PARAMETER REPORT22(J,L);
REPORT2(UNIT,J,L)=SUM((I,K)$((XORIG.L(I,J) EQ 1.0) AND (XDEST.L(K,L) EQ 1.0)),
              TONSUE(UNIT,I,K));
REPORT22(J,L)=SUM(UNIT,REPORT2(UNIT,J,L));

PARAMETER REPORT3(UNIT,J,L);
REPORT3(UNIT,J,L)=SUM((I,K)$((XORIG.L(I,J) EQ 1.0) AND (XDEST.L(K,L) EQ 1.0)),
              TPAX(UNIT,I,K));
PARAMETER REPORT6(J,AB);
PARAMETER REPORT61(L,AB);
*aggregation of mog values into centroids first origin
REPORT6(J,AB)=SUM(I$(XORIG.L(I,J) EQ 1.0),MOG(I,AB));
*now destinations
REPORT61(L,AB)=SUM(K$(XDEST.L(K,L) EQ 1.0),MOG(K,AB));

PARAMETER REPORT4(J,L);
LOOP(UNIT,
   REPORT4(J,L)$((REPORT2(UNIT,J,L) GT 0.0) OR (REPORT3(UNIT,J,L) GT 0.0))=1;
);
PARAMETER REPORT5(*);
REPORT5(J)$((SUM(L,REPORT4(J,L))) GT 0.0 )=1.0;
REPORT5(L)$((SUM(J,REPORT4(J,L))) GT 0.0) =1.0;

FILE F1 /movement.txt/;
FILE F2 /distance.txt/;
FILE F3 /pairs.txt/;
FILE F4 /mogcap.dat/;
FILE F5 /origin11.txt/;
FILE F6 /destin11.txt/;
```

```
FILE F7 /distance.dat/;
FILE F8 /routelog.txt/;

PUT F1;
OPTION REPORT2:2:0:1;
OPTION REPORT3:2:0:1;
PUT "MOVEMENT REQUIREMENTS";
LOOP((UNIT,J,L)$((REPORT2(UNIT,J,L) GT 0.0) OR REPORT3(UNIT,J,L) GT 0.0),
   PUT /;
   PUT UNIT.TL:4;
   PUT J.TL:4;
   PUT L.TL:4;
   PUT "    ";
   PUT REPORT2(UNIT,J,L);
   PUT "    ";
   PUT REPORT3(UNIT,J,L);
);

PUT F2;
PUT "DISTANCES BETWEEN AIRFIELDS";
LOOP ((AF,AFP)$((ENR(AF) OR I(AF) OR K(AF)) AND
                  (ENR(AFP) OR I(AFP) OR K(AFP))) ,
    PUT /;
    PUT AF.TL:4;
    PUT AFP.TL:4;
    PUT "    ";
    PUT DIS(AF,AFP);
 );

PUT F3;
PUT "origin destination pairs for route selection model";
LOOP((J,L)$(REPORT4(J,L) EQ 1.0),
   PUT /;
   PUT J.TL:4;
   PUT L.TL:4;
);

PUT F4;
PUT "      WB  NB  GM  TT";
LOOP(ENR$(REPORT5(ENR) NE 1.0),
   PUT /;
   PUT ENR.TL:4;
   PUT "   ";
   PUT MOG(ENR,'WB'):2:0;
   PUT "   ";
   PUT MOG(ENR,'NB'):2:0;
   PUT "   ";
   PUT MOG(ENR,'GM'):2:0;
   PUT "   ";
   PUT MOG(ENR,'TT'):2:0;
);
```

```
LOOP(J$(REPORT5(J) EQ 1.0),
   PUT /;
   PUT J.TL:4;
   PUT "    ";
   PUT REPORT6(J,'WB'):2:0;
   PUT "   ";
   PUT REPORT6(J,'NB'):2:0;
   PUT "   ";
   PUT REPORT6(J,'GM'):2:0;
   PUT "   ";
   PUT REPORT6(J,'TT'):2:0;
);

LOOP(L$(REPORT5(L) EQ 1.0),
   PUT /;
   PUT L.TL:4;
   PUT "    ";
   PUT REPORT61(L,'WB'):2:0;
   PUT "   ";
   PUT REPORT61(L,'NB'):2:0;
   PUT "   ";
   PUT REPORT61(L,'GM'):2:0;
   PUT "   ";
   PUT REPORT61(L,'TT'):2:0;
   );

PUT F5;
PUT "ORIGIN CENTROID AIRFIELDS";
LOOP(J$(YORIG.L(J) EQ 1.0),
  PUT /;
  PUT J.TL:4;
);

PUT F6,
PUT "DESTINATION AIRFIELDS";
LOOP(L$(YDEST.L(L) EQ 1.0),
  PUT /;
  PUT L.TL:4;
);

PUT F7;
PUT "*DISTANCES BETWEEN AIRFIELDS";
PUT /;
PUT "/";
LOOP ((AF,AFP)$((ENR(AF) OR I(AF) OR K(AF)) AND
                    (ENR(AFP) OR  I(AFP) OR  K(AFP))) ,
  PUT /;
  PUT AF.TL:4;
  PUT "  .  ";
  PUT AFP.TL:4;
  PUT " =  ";
```

```
  PUT DIS(AF,AFP);
);

PUT /;
PUT "/.";

PUT F8;
F8.AP=1;
PUT "CENTROID MODEL RESULTS";
PUT /;
PUT "DATE    ";
PUT SYSTEM.DATE;
PUT /;
PUT "TIME      ";
PUT SYSTEM.TIME;
PUT /;
PUT "ORIGIN CENTROID AIRFIELDS";
PUT /;

LOOP(J$(YORIG.L(J) EQ 1.0),
  PUT J.TL:4;
  PUT /;
);

PUT "DESTINATION CENTROID AIRFIELDS";
PUT /;

LOOP(L$(YDEST.L(L) EQ 1.0),
  PUT L.TL:4;
  PUT /;
);

PUT "MOG VALUES FOR THE AIRFIELDS";
PUT /;
PUT "        WB   NB   GM   TT";
LOOP(ENR$(REPORT5(ENR) NE 1.0),
   PUT /;
   PUT ENR.TL:4;
   PUT "    ";
   PUT MOG(ENR,'WB'):2:0;
   PUT "   ";
   PUT MOG(ENR,'NB'):2:0;
   PUT "   ";
   PUT MOG(ENR,'GM'):2:0;
   PUT "   ";
   PUT MOG(ENR,'TT'):2:0;
 );

LOOP(J$(REPORT5(J) EQ 1.0),
   PUT /;
   PUT J.TL:4;
```

```
  PUT "   ";
  PUT REPORT6(J,'WB'):2:0;
  PUT "   ";
  PUT REPORT6(J,'NB'):2:0;
  PUT "   ";
  PUT REPORT6(J,'GM'):2:0;
  PUT "   ";
  PUT REPORT6(J,'TT'):2:0;
);

LOOP(L$(REPORT5(L) EQ 1.0),
  PUT /;
  PUT L.TL:4;
  PUT "   ";
  PUT REPORT61(L,'WB'):2:0;
  PUT "   ";
  PUT REPORT61(L,'NB'):2:0;
  PUT "   ";
  PUT REPORT61(L,'GM'):2:0;
  PUT "   ";
  PUT REPORT61(L,'TT'):2:0;
 );

DISPLAY ORIG.L,XDEST.L,TONSUE,TPAX,REPORT2,REPORT3,REPORT4,REPORT6,REPORT61;
```

# APPENDIX C. SENSITIVITY ANALYSIS FOR ROUTES

```
$TITLE COLUMN GENERATION
$STITLE Yasin TURKER  29 APR 1995
*-----------GAMS AND DOLLAR CONTROL OPTIONS---------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS
   LIMCOL = 0  , LIMROW  = 0   , SOLPRINT = OFF , DECIMALS = 2
   RESLIM =  18000, ITERLIM = 400000, OPTCR    = 0.1 , SEED     = 3141
*---------------------------------------------------------------------

*read candidate routes
SETS
$INCLUDE gamscol.dat

   DELR(GEN)  additional routes for column generation
   DDR(GEN)
   DRR(GEN)
   DRI(GEN,AF)
   DRK(GEN,AF)
   DRE(GEN,AF)
   DADJACENT(GEN,AF,AF)
   DRA(GEN,A);

DELR(GEN)$(CAN(GEN))=YES$(NOT R(GEN));
DDR(GEN)$(DRCAN(GEN))=YES$(NOT DR(GEN));
DRR(GEN)$(RRCAN(GEN))=YES$(NOT RR(GEN));
DRI(GEN,AF)$(RICAN(GEN,AF))=YES$(NOT RI(GEN,AF));
DRK(GEN,AF)$(RKCAN(GEN,AF))=YES$(NOT RK(GEN,AF));
DRE(GEN,AF)$(RECAN(GEN,AF))=YES$(NOT RE(GEN,AF));
DADJACENT(GEN,AF,AFP)$(ADJCAN(GEN,AF,AFP))=YES$(NOT ADJACENT(GEN,AF,AFP));
DRA(GEN,A)$(RACAN(GEN,A))=YES$(NOT RA(GEN,A));

MAXLEG(GEN)$DELR(GEN)=SMAX((AF,AFP)$DADJACENT(GEN,AF,AFP),DIST(AF,AFP));

LOOP((GEN,A)$(DELR(GEN) * DRA(GEN,A)),
  CURRENT(AF)=NO;

* current is set to the first airfield on the route
  CURRENT(AF)=YES$((DDR(GEN) * DRI(GEN,AF)) + (DRR(GEN) * DRK(GEN,AF)) );
  NEXT(AF)=NO;
  NEXT(AF)=SUM(CURRENT,DADJACENT(GEN,CURRENT,AF));
  FLTTIME(A,GEN)=SUM((CURRENT,NEXT),DIST(CURRENT,NEXT))/(100*SPD(A));
  WI(AF)=CURRENT(AF)*I(AF);
  WK(AF)=CURRENT(AF)*K(AF);
  CTIME(A,GEN,NEXT)=GTIME(A,'ONLD')$(CARD(WI) GT 0) +
            GTIME(A,'OFFLD')$(CARD(WK) GT 0) +
```

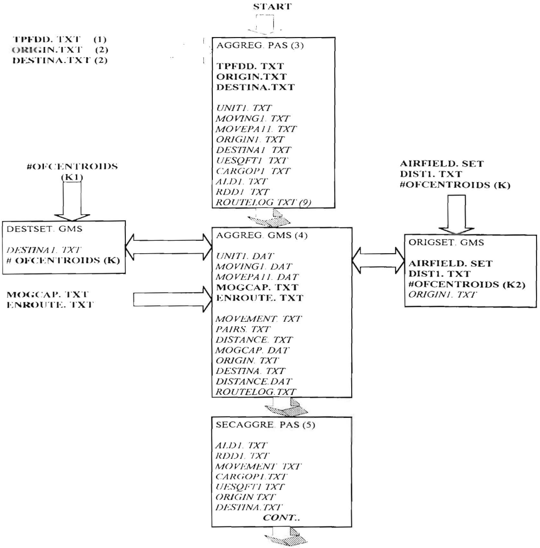
```
          SUM(CURRENT,DIST(CURRENT,NEXT))/SPD(A);
 KEND=1;
 LOOP(AFP$KEND,
  CURRENT(AF)=NO;
  CURRENT(AF)=YES$NEXT(AF);
  NEXT(AF)=NO;
  NEXT(AF)=SUM(CURRENT,DADJACENT(GEN,CURRENT,AF));
  FLTTIME(A,GEN)=FLTTIME(A,GEN)+
          SUM((CURRENT,NEXT),DIST(CURRENT,NEXT))/(100*SPD(A));
  CTIME(A,GEN,NEXT)=SUM(CURRENT,CTIME(A,GEN,CURRENT))+
            GTIME(A,'ENR')+
            SUM(CURRENT,DIST(CURRENT,NEXT))/SPD(A);
*  quit inner loop if NEXT is empty
  KEND= CARD(NEXT);
 );
);


MAXLOAD(A,GEN) $ (DELR(GEN) * DDR(GEN) * DRA(GEN,A)) =
     ACLOAD(A,'1')$(MAXLEG(GEN) LE ACRANGE(A,'1')) +
(ACLOAD(A,'1')+SLOPE(A,'2')*(ACRANGE(A,'1')-MAXLEG(GEN)))
        $(MAXLEG(GEN) GE ACRANGE(A,'1') AND
         MAXLEG(GEN) LE ACRANGE(A,'2'))    +
(ACLOAD(A,'2')+SLOPE(A,'3')*(ACRANGE(A,'2')-MAXLEG(GEN)))
        $(MAXLEG(GEN) GE ACRANGE(A,'2') AND
         MAXLEG(GEN) LE ACRANGE(A,'3'))    +
(ACLOAD(A,'3')+SLOPE(A,'4')*(ACRANGE(A,'3')-MAXLEG(GEN)))
        $(MAXLEG(GEN) GE ACRANGE(A,'3') AND
         MAXLEG(GEN) LE ACRANGE(A,'4'))    +
(ACLOAD(A,'4')+SLOPE(A,'5')*(ACRANGE(A,'4')-MAXLEG(GEN)))
        $(MAXLEG(GEN) GE ACRANGE(A,'4') AND
         MAXLEG(GEN) LE ACRANGE(A,'5'))    +
(ACLOAD(A,'5')+SLOPE(A,'6')*(ACRANGE(A,'5')-MAXLEG(GEN)))
        $(MAXLEG(GEN) GE ACRANGE(A,'5') AND
         MAXLEG(GEN) LE ACRANGE(A,'6'))    +
(ACLOAD(A,'6')+SLOPE(A,'7')*(ACRANGE(A,'6')-MAXLEG(GEN)))
        $(MAXLEG(GEN) GE ACRANGE(A,'6') AND
         MAXLEG(GEN) LE ACRANGE(A,'7'))    ;
$ONMIXED

* first delivery routes
KCONSUME(A,GEN,TP,T)$( (ORD(TP) LE ORD(T)) AND DELR(GEN) AND
(DDR(GEN)*DRA(GEN,A)) )=SUM(K$DRK(GEN,K), CTIME(A,GEN,K)/24);

KCONSUME(A,GEN,TP,T)$( (ORD(TP) LE ORD(T)) AND DELR(GEN) AND
(DDR(GEN)*DRA(GEN,A)) )=MIN( ORD(T)-ORD(TP)+1 , KCONSUME(A,GEN,TP,T));

* now recovery routes
KCONSUME(A,GEN,TP,T)$( (ORD(TP) LE ORD(T)) AND DELR(GEN) AND
(DRR(GEN)*DRA(GEN,A)) )=SUM(I$DRI(GEN,I), CTIME(A,GEN,I)/24);
```

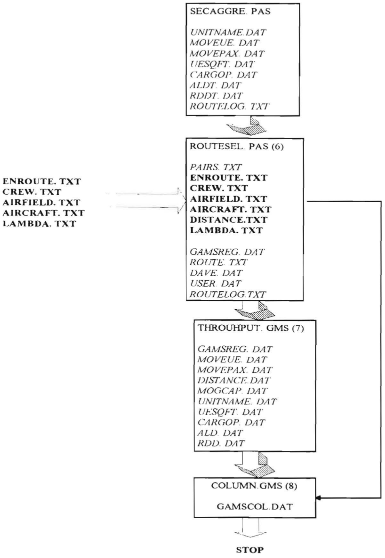
```
KCONSUME(A,GEN,TP,T)$( (ORD(TP) LE ORD(T)) AND DELR(GEN) AND
(DRR(GEN)*DRA(GEN,A)) ) =MIN( ORD(T)-ORD(TP)+1 , KCONSUME(A,GEN,TP,T));

*++++++++++++++++++ DYNAMIC SET DECLARATION +++++++++++++++++

* DSETX(U,A,GEN,T) is used to control the allowable combination
* of unit, aircraft, route, and start time for the following
* decision variables: X(U,A,GEN,T), TONSUE(U,A,GEN,T) and
* TPAX(U,A,GEN,T). A valid combination implies that the following
* must be satisfied:
* (1) it's a delivery route (2) aircraft A can fly route R
* (3) start time >= unit's ALD (4) delivery date <= RDD + MAXLATE
* (5) positive movement requirement (6) Maximum aircraft payload for
* the route must be greater than 25% its full load capability; this is
* for cost efficiency. (7) Suppply of aircraft type >= 0
DSETX(U,A,GEN,T)$DELR(GEN) = YES $( 1$DDR(GEN) AND 1$DRA(GEN,A) AND
   (ORD(T) GE ALD(U)) AND
   (ORD(T) LE (RDD(U) - ROUND(SUM(K$DRK(GEN,K),CTIME(A,GEN,K))/24)
      + MAXLATE)) AND
   (1$SUM((I,K)$(DRI(GEN,I)*DRK(GEN,K)),MOVEUE(U,I,K)) OR
      1$SUM((I,K)$(DRI(GEN,I)*DRK(GEN,K)),MOVEPAX(U,I,K)) )
   AND (1$(MAXLOAD(A,GEN) GT (0.25 * ACLOAD(A,"1")) ))
   AND SUM (TP$(ORD(TP) LE ORD(T)), SUPPLY(A,TP)) ) ;


* DSETDY(A,GEN,T) is used to control the allowable combination
* of aircraft, route, and start time for the decision variable
* Y(A,GEN,T). An allowable combination is one in which the
* aircraft-route is compatible and that the supply of aircraft
* type to date is greater than 0.
DSETY(A,GEN,T)$DELR(GEN) = YES $( 1$DRR(GEN) AND 1$DRA(GEN,A) AND
   SUM (TP$(ORD(TP) LE ORD(T)), SUPPLY(A,TP)) );
*define priceout parameters

PARAMETER TOTX(U,A,GEN,T)  price out value for x;
TOTX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=0.0;
*computations for TOTX
TOTX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTX(U,A,GEN,T)+
        SUM(I$(DSETX(U,A,GEN,T) *DRI(GEN,I)),ACBALI.M(A,I,T));
LOOP(TP,
TOTX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTX(U,A,GEN,T)-
      SUM(K$(1$DSETX(U,A,GEN,T) AND 1$DRK(GEN,K) AND
      ((ORD(T) + ROUND(CTIME(A,GEN,K)/24)) EQ ORD(TP))),
      ACBALK.M(A,K,TP)) );

TOTX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTX(U,A,GEN,T)-
            (FLTTIME(A,GEN)*ACURATE.M(A));
LOOP(TP,
TOTX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN) AND (ORD(T) LE ORD(TP)))=
   TOTX(U,A,GEN,T)+(KCONSUME(A,GEN,T,TP)*ACCONSUME.M(A,TP)) );
```

```
LOOP(TP,
TOTX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTX(U,A,GEN,T)+
       SUM(AF$(DRI(GEN,AF)*DDR(GEN)),
           MOGREQ(AF,A)*(GTIME(A,'ONLD')/24)*MOGUTILITY.M(AF,T))+
       SUM(AF$(1$DRK(GEN,AF) AND 1$DDR(GEN) AND
         (ORD(T) + ROUND(CTIME(A,GEN,AF)/24) EQ ORD(TP))),
         MOGREQ(AF,A)*(GTIME(A,'OFFLD')/24)*MOGUTILITY.M(AF,TP))+
       SUM(AF$(1$DRE(GEN,AF) AND 1$DDR(GEN) AND
         (ORD(T) + ROUND(CTIME(A,GEN,AF)/24) EQ ORD(TP))),
         MOGREQ(AF,A)*(GTIME(A,'ENR')/24)*MOGUTILITY.M(AF,TP)) );
*computation for TONSUE

PARAMETER TOTTON(U,A,GEN,T);
TOTTON(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=0.0;
TOTTON(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTTON(U,A,GEN,T)+
     SUM((I,K)$(ACSQFT(A) AND 1$DSETX(U,A,GEN,T) AND 1$DRI(GEN,I) AND
     1$DRK(GEN,K) AND 1$MOVEUE(U,I,K)),REQMTUE.M(U,I,K));

TOTTON(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTTON(U,A,GEN,T)+
     SUM((I,K)$(1$MOVEUE(U,I,K) AND ACCARGO(A,'OUT') AND
     1$DSETX(U,A,GEN,T) AND 1$DRI(GEN,I) AND 1$DRK(GEN,K)),
     OUTREQMT.M(U,I,K));

TOTTON(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTTON(U,A,GEN,T)+
    SUM((I,K)$(1$MOVEUE(U,I,K) AND ACCARGO(A,'OVER') AND
    1$DSETX(U,A,GEN,T) AND 1$DRI(GEN,I) AND 1$DRK(GEN,K)),
    OVERREQMT.M(U,I,K));

*includes cost coefficient
LOOP(K,
 TOTTON(U,A,GEN,T)$(ACSQFT(A) AND 1$DSETX(U,A,GEN,T) AND 1$DELR(GEN) AND
       1$DRK(GEN,K))=TOTTON(U,A,GEN,T)-
                 LATEPEN(U,'UE')*MAX(0,(ORD(T)+ROUND(CTIME(A,GEN,K)/24)-RDD(U))) );

*computations for variable
PARAMETER TOTY(A,GEN,T);
TOTY(A,GEN,T)$(DSETY(A,GEN,T) AND DELR(GEN))=0.0;
LOOP(TP,
TOTY(A,GEN,T)$(DSETY(A,GEN,T) AND DELR(GEN))=TOTY(A,GEN,T)-
     SUM(I$(1$DRI(GEN,I)  AND
     ((ORD(T)+ROUND(CTIME(A,GEN,I)/24)) EQ ORD(TP))),ACBALI.M(A,I,TP)) );

TOTY(A,GEN,T)$(DSETY(A,GEN,T) AND DELR(GEN))=TOTY(A,GEN,T)+
    SUM(K$(DSETY(A,GEN,T)*DRK(GEN,K)),ACBALK.M(A,K,T));

TOTY(A,GEN,T)$(DSETY(A,GEN,T) AND DELR(GEN))=TOTY(A,GEN,T)+
    (FLTTIME(A,GEN)*ACURATE.M(A));
   LOOP(TP,
   TOTY(A,GEN,T)$(DSETY(A,GEN,T) AND DELR(GEN) AND (ORD(T) LE
   ORD(TP)))=TOTY(A,GEN,T)+
     (KCONSUME(A,GEN,T,TP)*ACCONSUME.M(A,TP)) );
```

```
LOOP(TP,
TOTY(A,GEN,T)$(DSETY(A,GEN,T) AND DELR(GEN))=TOTY(A,GEN,T)+
     SUM(AF$(1$DSETY(A,GEN,T) AND 1$DRE(GEN,AF) AND 1$DRR(GEN) AND
     1$(ORD(T) + ROUND(CTIME(A,GEN,AF)/24) EQ ORD(TP))),
     MOGREQ(AF,A)*(GTIME(A,'ENR')/24)*MOGUTILITY.M(AF,TP)) );

*computations for variable TPAX
PARAMETER TOTTPAX(U,A,GEN,T);
TOTTPAX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=0.0;
TOTTPAX(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=TOTTPAX(U,A,GEN,T)+
   SUM((I,K)$(1$DSETX(U,A,GEN,T) AND 1$DRI(GEN,I) AND 1$DRK(GEN,K) AND
   1$MOVEPAX(U,I,K)),REQMTPAX.M(U,I,K));
*include cost
LOOP(K,
   TOTTPAX(U,A,GEN,T)$(DSETX(U,A,GEN,T)*DELR(GEN) * DRK(GEN,K))=
       TOTTPAX(U,A,GEN,T)-
              (LATEPEN(U,'PAX')*MAX(0,ORD(T)+ROUND(CTIME(A,GEN,K)/24)-RDD(U))) );


*warmup starting
X.M(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=1.0;
Y.M(A,GEN,T)$(DSETY(A,GEN,T) AND DELR(GEN))=1.0;
TONSUE.M(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=1.0;
TPAX.M(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=1.0;
ACWEIGHT.M(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=1.0;
ACSPACE.M(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=1.0;
ACPAX.M(U,A,GEN,T)$(DSETX(U,A,GEN,T) AND DELR(GEN))=1.0;

*define new route set base to price out values
LOOP((U,A,T),
R(GEN)$(DSETX(U,A,GEN,T) * DELR(GEN))=YES$((TOTX(U,A,GEN,T) LT 0.0) OR
(TOTTPAX(U,A,GEN,T) LT 0.0) OR (TOTTON(U,A,GEN,T) LT 0.0) OR (TOTY(A,GEN,T) LT 0.0)) );
DR(GEN)$DDR(GEN)=YES$R(GEN);

RR(GEN)$DRR(GEN)=YES$R(GEN);
RI(GEN,AF)$DRI(GEN,AF)=YES$R(GEN);
RK(GEN,AF)$DRK(GEN,AF)=YES$R(GEN);
RE(GEN,AF)$DRE(GEN,AF)=YES$R(GEN);
ADJACENT(GEN,AF,AFP)$DADJACENT(GEN,AF,AFP)=YES$R(GEN);
RA(GEN,A)$DRA(GEN,A)=YES$R(GEN);
RESULT('NEWR')=CARD(R);
DISPLAY RESULT;


SOLVE AIRLIFT USING RMIP MINIMIZING Z;
```

# APPENDIX D. FLOW DIAGRAM OF ROUTE SELECTION MODEL




SECAGGRE. PAS
UNITNAME.DAT
MOVEUE. DAT
MOVEPAX. DAT
UESQFT. DAT
CARGOP. DAT
ALDT. DAT
RDDT. DAT
ROUTELOG. TXT
ENROUTE. TXT
CREW. TXT
AIRFIELD. TXT
AIRCRAFT. TXT
LAMBDA. TXT
ROUTESEL. PAS (6)
PAIRS. TXT
ENROUTE. TXT
CREW. TXT
AIRFIELD. TXT
AIRCRAFT. TXT
DISTANCE.TXT
LAMBDA. TXT
GAMSREG. DAT
ROUTE. TXT
DAVE. DAT
USER. DAT
ROUTELOG.TXT
THROUHPUT. GMS (7)
GAMSREG. DAT
MOVEUE. DAT
MOVEPAX. DAT
DISTANCE.DAT
MOGCAP. DAT
UNITNAME. DAT
UESQFT. DAT
CARGOP. DAT
ALD. DAT
RDD. DAT
COLUMN.GMS (8)
GAMSCOL.DAT
STOP

1. Text file separated with tab and generated in EXCEL.
2. Preselected origin-destination airfields which will be considered during the aggregation If the user wants to include all the airfields that mentioned in TPFDD, then enter "No" for the including file question.
3. Unit aggregation based on preselected origin destination airfields , minimum movement requirements and the difference between the RDD values.
4. Centroid model for origin and destination airfields written in GAMS.
5. Unit aggregation based on centroid airfields.
6. Route selection model.
7. Optimization model.
8. Sensitivity Analysis.
9. Text file contains all the input and aggregation information based on this run.

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