PHYSICAL AND ELECTRICAL ENGINEERING
LABORATORY MANUALS

EXAMPLES IN
Electrical Engineering

EXAMPLES IN
ELECTRICAL ENGINEERING

EXAMPLES IN
ELECTRICAL ENGINEERING

BY

SAMUEL JOYCE, A.I.E.E.
LECTURER TO THE SENIOR CLASSES IN ELECTRICAL ENGINEERING
MUNICIPAL TECHNICAL SCHOOL, MANCHESTER

LONGMANS, GREEN, & CO.
LONDON, NEW YORK, AND BOMBAY
1896

PREFACE

MANY of the examples in the following pages have been collected during the past few years to illustrate the author's lectures to Advanced and Honours Students in Electrical Engineering, though the majority are here published for the first time.

Originally intended as a collection of exercises, the explanatory matter forming the bulk of the text was, however, found necessary to make the book more complete in itself, though it is not intended to act as a full treatise on the subject. These explanations, together with the tables at the end of the book, will, it is hoped, be found very useful by draughtsmen and others engaged in electrical machine design.

The author's best thanks are due to such writers as have been made use of, too numerous to mention by name; and also to two of his third-year students, Messrs. A. B. Mallinson and W. K. Meldrum, for many carefully executed diagrams.

Lastly, and not the least, the author's thanks are due to his friend Mr. E. S. Shoults, for considerable assistance in checking examples.

S. JOYCE.

LATCHFORD HOUSE, GREENHEYS,
MANCHESTER,
July, 1896.

NOTE.—The author will be much obliged if readers will kindly notify any errors that may have escaped observation.

CONTENTS

EXAMPLES IN ELECTRICAL ENGINEERING.

CHAPTER I.

SIMPLE CIRCUITS.

The Simple Electric Circuit consists of a conducting path, insulated so as to confine the electric flux as much as possible to that path. In it the relationship known as Ohm's Law holds good, viz.—

$$C = \frac{e}{R}$$

If e be stated in volts, R in ohms, then C will be in ampères. The value of e in the above expression must be considered as being the algebraic sum of all the electro-motive forces (E.M.F.) acting in the circuit, and will be called the effective or active E.M.F.

The resistance of a simple circuit may be composed of a number of separate resistances in series; in which case R will be the sum of all these other resistances, thus—

$$R = r_1 + r_2 + r_3 + r_4$$

and therefore $C = \dfrac{e}{r_1 + r_2 + r_3 + r_4}$

or $e = Cr_1 + Cr_2 + Cr_3 + Cr_4$

and each of these quantities may be called a potential difference, or P.D., for short. The E.M.F. e is thus the sum of all the P.D.'s, or is equal to—

$$\Sigma Cr$$

writing the P.D. $Cr_1 = v_1$, we have that—

$$e = v_1 + v_2 + v_3 + v_4$$

The Rate of doing Work.—The amount of work done in a circuit is equal to the quantity, Q, of electricity passed multiplied by the E.M.F. And since Q is the product of the current and the time, then the work done, W, in the time t will be—

$$W = eCt$$

from which the rate of doing work, or power, or activity, is—

$$P = \frac{W}{t} = \frac{eCt}{t} = eC$$

and as $e = Cr_1 + Cr_2 + Cr_3 + Cr_4$, etc., we have—

$$P = eC = C^2r_1 + C^2r_2 + C^2r_3 + C^2r_4, \text{ etc.}$$

which gives the distribution of energy expenditure round the circuit, and states that the whole rate of doing work in a circuit is equal to the sum of all the activities in the separate parts of the circuit.

The activity or power P is measured in watts, one watt being the activity when the product eC is unity. Owing to the values chosen for the volt and the ampère, the watt is equivalent to the $\frac{1}{746}$ part of the mechanical horse-power, or—

$$\text{H.P.} = \frac{\text{watts}}{746}$$

Calculation of Resistance.—The value of the resistance of a circuit is determined by the nature of the material of which that circuit is composed, by its temperature, and by the dimensions of the circuit; the relationship between these quantities being that—

$$R = \frac{L\rho}{a}$$

where L and a are respectively the length and sectional area (average if not uniform) measured in terms of the same unit, and ρ represents the resistivity at a certain temperature of the material; that is to say, ρ is the actual resistance in ohms of the unit cube of the material.

The term resistance has come more into general use than conductance, and represents the opposite idea, the relationship between them being—

$$\rho = \frac{1}{m}$$

where m stands for the conductivity, that is the actual conductance, in mhos of the unit cube.

Thus, the value of the resistance of a circuit can be expressed in two ways, viz.—

$$R = \frac{L\rho}{a} = \frac{L}{am} = \text{ohms}$$

and similarly the conductance may also be written—

$$K = \frac{am}{L} = \frac{a}{L\rho} = \text{mhos}$$

Thus, if the dimensions of a circuit, both electrical and mechanical, be known, its resistance or conductance can at once be found. And in cases where the whole resistance is made up of a number of items, we can find the value of each item r by inserting in the equation—

$$r = \frac{l\rho}{a}$$

the proper values for l, a, and ρ, the latter being possibly different for each item of the circuit, either on account of temperature or on account of difference of material.

The tables on page 209 will give all necessary data for these values.

Different Kinds of Circuits.—There are two main ways in which lamps are connected to form a circuit: the series and the parallel.

The *series* circuit is arranged thus—

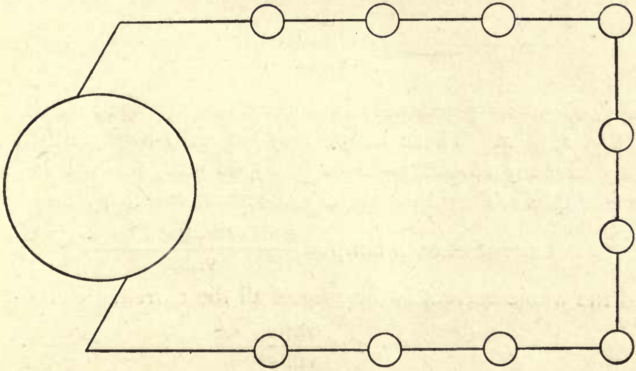

FIG. 1.

and the lamps are generally arcs, each taking such and such a current at a P.D., usually from 45 to 50 volts in the case of continuous currents.

The current in the circuit is that taken by any lamp, and is constant; that is, fixed in value for that particular circuit. The E.M.F. is that required by each lamp multiplied by the number of lamps and added to the P.D. required for the dynamo and leads.

The *parallel* circuit is arranged thus—

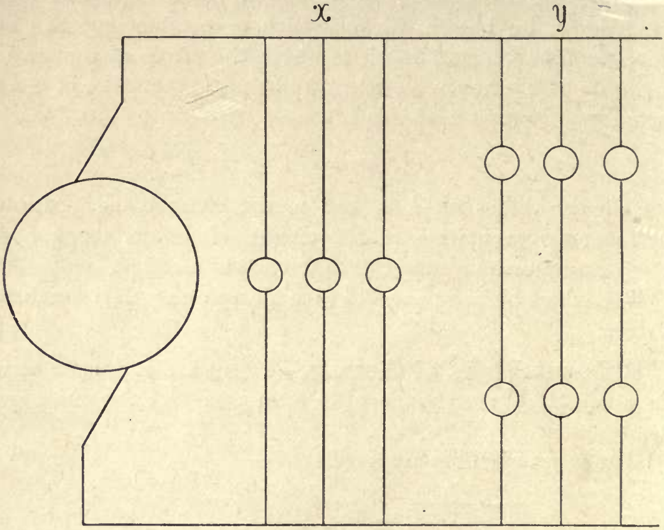

FIG. 2.

and the lamps are generally incandescents, arranged as shown in Fig. 2, at *x*. These lamps may generally be described as being so many 16 candle-power (c.p.), 60 watt, 100 volt lamps, for example. They thus all take the same current, viz.—

$$\text{current in each lamp} = \frac{\text{watts required by it}}{\text{volts}}$$

and the whole current is the sum of all the currents, or is—

$$n \times \frac{\text{watts}}{\text{volts}}$$

In some cases there are arc lamps arranged two in series as shown at *y*, Fig. 2. These lamps will be 10 ampère lamps, for example, and the current in that case will be 10 × half the number of lamps.

Calculation of Resistance of Parallel Circuit.—In considering the value as to resistance or conductance of a number of items in parallel, it is generally simplest to take the sum of all the values of current in ampères through each item of the parallel circuit due to a common P.D. For example, What is the resistance of four lamps in parallel having resistances respectively 100 ohms, 50 ohms, 25 ohms, and 10 ohms? Consider the common P.D. to be, say, 100 volts, whence the currents respectively will be 1 ampère, 2 ampères, 4 ampères, and 10 ampères, making a total of 17 ampères, which current is due to a P.D. of 100 volts acting upon the united resistances in parallel, which must consequently have the value $R = \frac{e}{C} = \frac{100}{17} = 5{\cdot}88$ ohms.

The direct calculation of conductance is simpler, since the total conductance is obviously the sum of all the conductances. Thus, the respective conductances are 0·01, 0·02, 0·04, and 0·1 mho, or the total conductance is 0·17 mho. Now, conductance is the reciprocal of resistance, and resistance is the reciprocal of conductance, or—

$$K = \frac{1}{R}. \quad \text{And } R = \frac{1}{K} = \frac{1}{\frac{1}{R}}$$

But the total conductance is the sum of all the separate conductances, or is $K = k_1 + k_2 + k_3 + k_4$, etc., which can be replaced by $K = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}$, etc. And as the resistance is the reciprocal of conductance, we have—

$$R = \frac{1}{K} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}}$$

which may be expressed in words: "The resistance of a number of items in parallel is equal to the reciprocal of the sum of the reciprocals of their separate resistances."

In the parallel or multiple-parallel system of distribution the circuit is always composed of a number of items in series, such as the resistances respectively of the dynamo, the conducting wires, and the lamps; and as these latter generally have to work at a fairly constant P.D., notwithstanding variations in the current, it becomes a question of great importance to know what size the leads must be to prevent more than a certain small loss of pressure in them. As a good compound dynamo gives a fairly constant P.D. at its terminals for all values of the current, its own resistance may be disregarded, and the terminal P.D. taken as the whole E.M.F. of the outside circuit. A few typical cases of this kind will render this clear.

Example I.—What must be the sectional area in square inches of the copper leads to convey the current required by 200 lamps in parallel, all situated at the far end of the leads, each lamp taking 60 watts at 100 volts? The distance from the dynamo to the lamps is 100 yards, and the loss along the leads is not to exceed 2 volts.

Solution.—Since the loss in the leads may be 2 volts, and is equal to the current multiplied by the resistance of the leads, *i.e.* the P.D. required to send the current through the leads, or Cr_l, it is necessary first to find the total current. Each lamp is a 60 watt, 100 volt, and therefore takes $\frac{60}{100} = 0{\cdot}6$ ampère; and the whole current is consequently $0{\cdot}6 \times 200$, or $= 120$ ampères. Therefore the P.D. required by the leads is $120r_l$, and must not exceed 2 volts; or—

$$r_l = \frac{2}{120} = 0{\cdot}01\dot{6}6 \text{ ohm}$$

which is the resistance of a certain length of wire, viz. 2 × 100 yards. We can write this resistance also in terms of its length, sectional area, and specific resistance, or—

$$r_l = \frac{2 \times 100 \times 36 \times 0{\cdot}00000067}{a}$$

where *a* is the sectional area required in square inches, and—

$$a = \frac{2 \times 100 \times 36 \times 0\cdot00000067}{0\cdot016\dot{6}} = 0\cdot2895 \text{ sq. inch}$$

which would be slightly larger than a $\frac{19}{10}$ stranded cable.

Example II.—If in the above question the resistance of the dynamo be 0·013 ohm, what will be the total E.M.F. in the whole circuit, the P.D. at the dynamo terminals, and the P.D. at the lamps at full load and at half load, when the lamps are supposed to vary one volt up or down, from 100 volts as an average?

Solution.—Neglecting the slight difference made in the total current when the P.D. at the lamp terminals varies slightly, and the shunt current of the dynamo, we have that at full load $C = 120\alpha$, loss in leads = 2 volts, P.D. at lamps is 99 volts, and E.M.F. total must be equal to the sum of these plus the P.D. required to send the current through the dynamo. And this last is 120 × 0·013, or 1·56 volt. Whence $E = 99 + 2 + 1\cdot56 = 102\cdot56$ volts.

Again, at half load only 100 lamps are in circuit, and C = 60 ampères; whence the loss in leads is only $60 \times 0\cdot01666 = 1$ volt, and the P.D. required for the dynamo resistance is $60 \times 0\cdot013 = 0\cdot78$ volt. The whole E.M.F. is then the sum of these, viz. 1·78, plus the P.D. at the lamp terminals, which is now 100 volts, and total E.M.F. = 101·78 volts. Also the P.D. at the dynamo terminals is in each case the sum of the P.D.'s required by the leads and the lamps. At full load this is $2 + 99 = 101$ volts, and at half load $1 + 100$, or, again, 101 volts. Thus the P.D. at the dynamo terminals is to be kept constant, at 101 volts, for all loads.

Example III.—What will be the diameter of the copper rods to convey 5000 ampères to an aluminium furnace at a distance of 7 yards from the dynamo terminals, if the loss along the leads is to be only $\frac{1}{8}$ volt?

Solution.—The P.D. required by the leads is 0·125 volt, and is equal to $5000r_l$, whence—

$$r_l = \frac{0·125}{5000} = 0·000025 \text{ ohm}$$

and the leads are $2 \times 7 \times 36 = 504$ inches long, or have a resistance of—

$$\frac{0·000025}{504} = 0·0000000496 \text{ ohm}$$

per one inch. Therefore their sectional area will be—

$$\frac{0·00000067}{0·0000000496} = 13·5 \text{ sq. inches}$$

and if circular bars, their diameter will be—

$$d = 2 \times \sqrt{\frac{a}{\pi}} = 2 \times \sqrt{4·3}$$

$$= 2 \times 2·073 = 4·146 \text{ inches}$$

Example IV.—What will be the loss of pressure at half load and full load in the leads in a parallel circuit running 40 arc lamps arranged two in series, and taking 12 ampères each, when the dynamo is kept at a terminal pressure of 102 volts, and is 50 yards from the lamps, the sectional area of the leads being 0·3 sq. inch?

Solution.—At full load the current will be—

$$C = 12 \times \frac{40}{2} = 240 \text{ ampères}$$

and at half load C = 120 ampères. The leads are $50 \times 2 \times 36 = 3600$ inches long, and therefore have a resistance—

$$r_l = \frac{L\rho}{a} = \frac{3600 \times 0·00000067}{0·3}$$

$$= 0·00804 \text{ ohm}$$

Therefore the loss of pressure at full load will be $0·008 \times 240 = 1·92$ volt, and at half load $0·008 \times 120 = 0·96$ volt nearly.

In simple series circuits there is no variation in the loss of pressure in the leads, etc., with variation of load since the current is constant. It is, then, only necessary to keep the loss of energy in the leads within a reasonable percentage of the whole activity. Thus in the following examples:—

Example V.—A series circuit consists of 60 ten-ampère arc lamps, spaced 100 yards apart, and taking an average of 49 volts each. What will be the diameter of the single copper leads required in order that the energy waste in them will be within $2\frac{1}{2}$ per cent. of the energy required by the lamps?

Solution.—The expenditure of energy in the lamps is at the rate of $60 \times 49 \times 10 = 29{,}400$ watts, and $2\frac{1}{2}$ per cent. of this is $\frac{2{\cdot}5}{100} \times 29{,}400 = 735$ watts, which is a loss of $73{\cdot}5$ volts, as the current is 10 ampères, and consequently their resistance is $7{\cdot}35$ ohms. The length of the leads is $60 \times 100 \times 36 = 216{,}000$ inches long, and thus the resistance per inch is—

$$\frac{7{\cdot}35}{216{,}000} = 0{\cdot}000034 \text{ ohm}$$

Thus the sectional area is $\frac{0{\cdot}00000067}{0{\cdot}000034} = 0{\cdot}0197$ sq. inch, whence the diameter will be—

$$\text{diameter} = 2 \times \sqrt{\frac{a}{\pi}} = 2 \times \sqrt{0{\cdot}00626}$$
$$= 0{\cdot}158 \text{ inch}$$

or about No. 8 B.W.G.

Example VI.—What will be the brake H.P. of engine required to run the above lamps if the loss in the dynamo itself is 150 volts, and only $\frac{9}{10}$ of the power received is converted into electricity?

Solution.—The activity in the lamps is $60 \times 49 \times 10 = 29400$ watts, and the loss in the leads is 735 watts. The loss in the dynamo is $10 \times 150 = 1500$ watts; whence the whole activity is 31,635, or the electrical H.P. is—

$$= \frac{31635}{746} = 42{\cdot}41$$

But this is only $\frac{9}{10}$ of the H.P. at the engine pulley, whence the brake H.P. of engine must be—

$$42{\cdot}41 \times \frac{10}{9} = 47{\cdot}13 \text{ H.P.}$$

Example VII.—What will be the E.M.F. required to send a current of 10 ampères through a circuit consisting of a dynamo whose resistance is 7 ohms, of $1\frac{1}{2}$ mile of leads of No. 6 B.W.G. = 0·203 inch diameter, and 80 lamps each of 2·5 ohms resistance? What will be the H.P. of engine required if the dynamo turns $\frac{9}{10}$ of the power received into electricity? And what will be the cost of 1 Board of Trade unit in the lamps if 1 mechanical H.P. cost twopence per hour?

Solution.—Total resistance of circuit is—

$$7 + 80 \times 2{\cdot}5 + \text{leads}$$

and the resistance of the leads is—

$$r_l = \frac{1{\cdot}5 \times 1760 \times 36 \times 0{\cdot}00000067 = 4 \times 7}{0{\cdot}203 \times 0{\cdot}203 \times 22}$$

$$= 1{\cdot}96 \text{ ohm}$$

or whole resistance is 208·96 ohms;

$$\text{whence the E.M.F. is } 10 \times 208{\cdot}96 = 2089{\cdot}6 \text{ volts}$$

$$\text{and the total activity is } 2089{\cdot}6 \times 10 = 20{,}896 \text{ watts}$$

and therefore the engine power must be—

$$20{,}896 \times \frac{10}{9} \times \frac{1}{746} = 31{\cdot}12 \text{ H.P.}$$

Thus the cost per hour's run will be 31·12 H.P. at *2d.*, or 62·24*d.*, which has to be paid for by the charge for the lamps, or is the cost of 200 × 10 × 10 watt-hours, or of 20 B.T.U.; whence the cost—

$$= \frac{62{\cdot}24}{20} = 3{\cdot}112d. \text{ per B.T.U.}$$

Insulation Resistance.—The insulation resistance of a circuit, *i.e.* the resistance between any conductor and earth, is measured for convenience in megohms, one megohm being 1,000,000 ohms. As far as insulation goes, all parts of a circuit are in parallel, and if R be the resistance of any one part to earth, then the resistance of a circuit consisting of *n* such parts, whether joined electrically in series or parallel, will only be $\frac{1}{n}$ of R. Fuse blocks, joints, fittings, switches, and connections generally are all sources of leakage, and usually bring down the insulation resistance very considerably, in spite of the fact that the resistance to earth of the cable employed may be many thousands of megohms.

Example VIII.—A circuit 500 yards long is made of cable having an insulation resistance of 3000 megohms per mile, and has 20 joints in it, each of 10,000 megohms resistance, and 250 fittings of 800 megohms each. What is the total insulation resistance?

Solution.—All the above items are in parallel, but as the cable is only 500 yards long at 3000 meghoms per mile, the resistance of this item will be—

$$3000 \times \frac{1760}{500} = 10560 \text{ meghoms}$$

The 20 joints in parallel will be $\frac{1}{20} \times 10000 = 500$ meghoms collectively, and the 250 fittings will together be—

$$\frac{1}{250} \times 800 = 3{\cdot}2 \text{ meghoms}$$

The whole insulation resistance is consequently all these in parallel, or will be—

$$\frac{1}{\frac{1}{10{,}560} + \frac{1}{500} + \frac{1}{3{\cdot}2}} = 3{\cdot}1 \text{ megohms}$$

being practically that due to the fittings alone, and quite independent of the quality of cable, the high insulation of which must, however, be regarded as an indication of its durability.

Example IX.—A central station makes a rule that the insulation resistance of any consumer's installation must not be less than 0·2 megohm. If there are 600 consumers, and the P.D. between the mains and earth is 100 volts, what will be the total leakage current?

Solution.—The total insulation will be $\frac{1}{600} \times 0{\cdot}2 = 333$ ohms, whence the leakage current is $\frac{100}{333} = 0{\cdot}3$ ampère.

In the measurement of activity by means of an ammeter and a voltmeter, connections may be made in two ways—

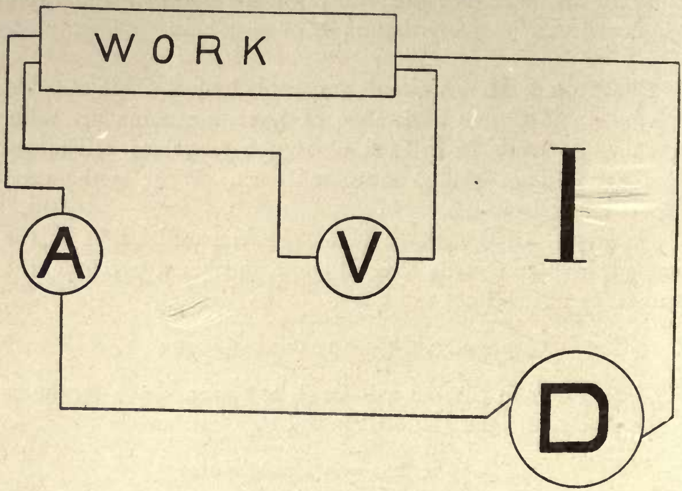

FIG. 3.

or—

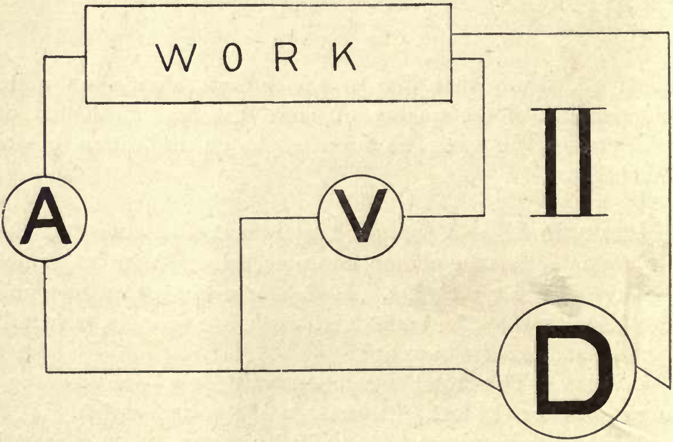

FIG. 3*a*.

In Case I., if the voltmeter be not of the electro-static type, the ammeter reads the current in the work added to the current in the voltmeter when simultaneous readings are taken; whilst the voltmeter reads the true P.D. at the terminals of the work.

Let R = resistance of voltmeter.
r = „ ammeter.
e = true P.D. at terminals of work.
C = „ current in work.

Then the current in the voltmeter will be $\frac{e}{R}$, and the ammeter reads $C + \frac{e}{R}$, whilst the voltmeter reads e.

The product of these two is $e\left(C + \frac{e}{R}\right) = ec + \frac{e^2}{R}$, and is the apparent activity in the work. But the true activity is obviously ec, and the difference is easily recognized as the power lost in the voltmeter. Thus the measured power is the true power plus the power required to actuate the voltmeter.

In Case II. similarly the measured power is the sum of the true power plus the power required to work the ammeter. It is thus necessary, in readings of this sort, to make corrections, especially when the activity to be measured is comparable in magnitude to the power required to work the instruments used. For example—

Example X.—It is required to know how many watts a certain incandescent lamp requires, connections being made as in Case I. (Fig. 3). The ammeter indicates a current of 0·66 ampère, whilst the voltmeter reads 101 volts, and has a resistance of 2000 ohms.

Solution.—Since 101 volts is the P.D. at the terminals, the current in the voltmeter is—

$$\text{Current in voltmeter} = \tfrac{101}{2000} = 0{\cdot}0505 \text{ ampère}$$

and the reading of the ammeter is 0·66 ampère; therefore the current in the lamp is 0·66 − 0·0505 = 0·06095 ampère. Thus the total activity is—

$$\text{Activity} = 0{\cdot}6095 \times 101 = 61{\cdot}5595 \text{ watts}$$

Connections as in Case II. (Fig. 3) do not give such reliable information as those in Case I., for the reason that in ascertaining the loss of volts in the ammeter its resistance must be accurately known; and as this is low and partly made up of variable contact resistances at the terminals, there is generally considerable uncertainty as to its exact value. An example is, however, given.

Example XI.—What is the resistance of a lamp when the ammeter reads 0·67 ampère and the voltmeter indicates 101 volts? Connections are made as in Case II. (Fig. 3), and the resistance of the ammeter is 0·75 ohm.

Solution.—The actual current in the lamp is given by the ammeter as 0·67 ampère, and consequently the loss of volts in the ammeter is $0·67 \times 0·75 = 0·5025$ ampère, and thus the P.D. at the terminals of the lamp will be—

$$101 - 0·5025 = 100·4975 \text{ volts}$$

from which its resistance is $\frac{100·4975}{0·67} = 150$ ohms

Note.—Power-meters and ohm-meters, which are virtually combinations of an ammeter with a voltmeter arranged so as to give, in the one case, the product of volts and ampères, and in the other case, the ratio of volts to ampères, can, in calibration, be corrected for errors due to the current in the voltmeter part, or P.D., required by the ammeter coil, but must be then connected up in such a way as to agree with such correction.

CHAPTER II.

DISTRIBUTION.

MULTIPLE WIRE CIRCUITS.

So far the examples given have only dealt with cases where the lamps are all connected in a bank at the end of the leads, remote from the generator. In a great many cases, however, the load is more or less uniformly distributed along the length of the circuit. If the arrangement be quite uniform, the case is simple. Consider two parallel mains lead and return, having lamps uniformly spaced between them, as in Fig. 4.

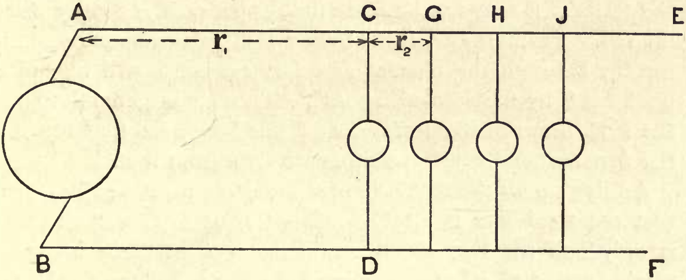

FIG. 4.

Let AB represent the terminals of a dynamo supplying current to a number of lamps, the nearest being situated at a distance AC from the dynamo, and any lamp being at a distance CG from its neighbour. Let the resistance of the leads from A to the first lamp be r_1 ohms lead and return, and the resistance of the lead and return between any two lamps be r_2 ohms. Now, all the lamps obviously cannot be at the same P.D., since there will be a loss of pressure in between each lamp and the next nearer the dynamo, of an amount equal to r_2 times the current flowing to all the lamps beyond the point under consideration. Thus, if the current taken by each lamp be C ampères, neglecting the slight variation of current due to the fact that all lamps are not at the same P.D., the loss of volts between $CG = r_2 \times 3C$, and similarly the loss between GH and HJ, will be $r_2 \times 2C$, and $r_2 \times C$. There will also be a loss of $r_1 \times 4C$ volts between the terminals of the dynamo and the nearest lamp. If e represents the P.D. at the dynamo terminals, the P.D. at the terminals of the four lamps, C, G, H, and J, will be respectively—

$$e - r_1 \times 4C$$
$$e - r_1 \times 4C - r_2 \times 3C$$
$$e - r_1 \times 4C - r_2 \times 3C - r_2 \times 2C, \text{ and}$$
$$e - r_1 \times 4C - r_2 \times 3C - r_2 \times 2C - r_2 \times C$$

In order that the lamps may not differ from one another by more than quite a small amount in brightness, it is necessary that the difference of P.D. between the nearest lamp C and the farthest E may not be more than 2 per cent., or say 2 volts on 100. Furthermore, it is obvious that this comparative uniformity between the nearest and farthest lamps will be quite unaffected by the loss along AC. Thus, it is general to fix the P.D. between the terminals of one lamp, viz. that nearest the dynamo, and cause the others to date from that.

Adding up all the losses between the lamps, it will be seen that the total loss is $r_2(3C + 2C + C)$, or $6r_2C$ volts, which is only half the loss which would have taken place had the whole current of 4C ampères been flowing all along the leads from C to E. From this follows a rule for ascertaining the loss along leads having a uniformly distributed load.

The loss of pressure in volts along leads carrying current to lamps uniformly distributed along their length is equivalent to the product of the whole resistance of the leads and the average current.

The above is the loss along that part of the leads bounded by the nearest and farthest lamp, and is independent of the loss along the part AC (Fig. 4), which is equal to r_1 times the whole current. Thus, calling R_1 and R_2 the resistances respectively of the parts of the leads in between the dynamo terminals and the nearest lamp and in between the nearest and furthest lamps, and C the total current, then the loss from the dynamo to the nearest lamp is—

$$R_1C$$

and the loss along the leads between the lamps is—

$$R_2\frac{C}{2}$$

Uniformity of brightness in the lamps dictates that the latter should not be more than, say, 2 volts on 100; but this condition will not be affected by the loss R_1C, provided the middle or some other convenient lamp be maintained at a certain agreed upon P.D., irrespective of the number of lamps in circuit. Economy of cost is the only consideration which will determine the loss along R_1, and this will be considered later on.

It is usual to distinguish between these two parts of a circuit by calling that part of a system of leads which carries the same current throughout its whole length and is not tapped anywhere, a *feeder;* whilst that part of a system which is tapped at intervals along its length, and consequently carries a current which is not the same in all parts, is called a *distributor.*

A circuit may consist of a number of distributors connected together, and is then called a distributing network. Each portion of such network will have to be supplied by its own feeder to maintain the required P.D.

Fig. 5 will render this clear, where the double line of conductors bounding the figure represents the distributing network, which is connected to the station, or electricity works, S, by the feeders S*a*, S*b*, etc. Each point—*a*, *b*, *c*, etc.—is called a feeding centre, and is kept at a fixed P.D. by manipulation of the feeders in the station. The current for the lamps situated in between *a* and *b* will be partly supplied by the feeder S*a* and partly by S*b*. The total current taken off the distributors between *a* and *b* being called C ampères, and being the same as that taken off the distributors between any two other feeding centres, it follows that each feeder will have to carry C ampères; and if R_f be the resistance of a feeder, lead and return, the loss of pressure in the feeder will be CR_f, an amount which is not necessarily the same for all feeders, as their lengths may be different, or they may not be at all times equally loaded.

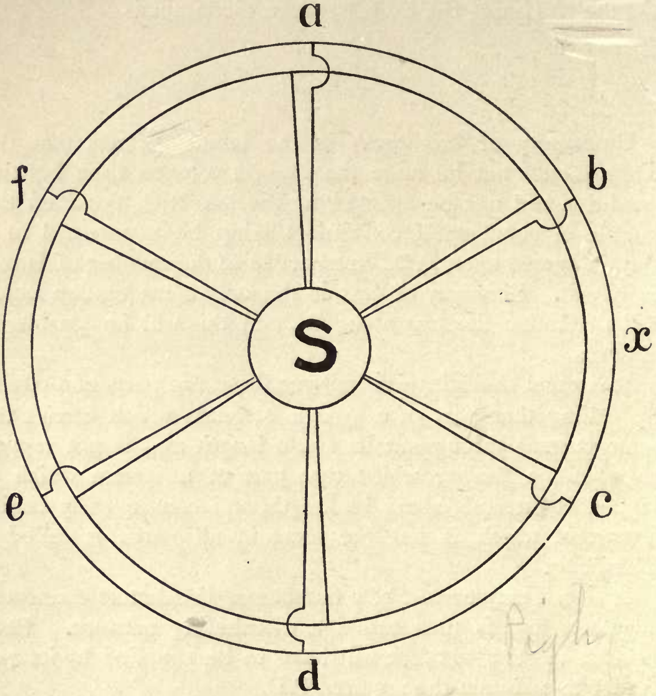

FIG. 5.

Figh

As the load between *b* and *c* (Fig. 5) is equally divided, the current in the distributors will be $\frac{C}{2}$ ampères at the feeding centres, but gradually falling till some point is reached about the middle of the distance *bc*, say at *x*, where the current will be zero. In order, then, to state the maximum difference of P.D. between any two lamps, it is necessary to allow that the distributor in between *a* and *b* has a mean current of $\frac{C}{4}$ ampères, and a length equal to half the distance *bc*, say *bx*. Calling R_d the resistance of distributor *ab*, lead and return, the maximum difference of P.D. between a lamp at *b* and a lamp at *x* will be—

$$\text{Loss of volts along distributors} = \frac{C}{4} \times \frac{R_d}{2} = \frac{CR_d}{8} \text{ volts}$$

The determination of the size of the distributors depends upon the difference of P.D. allowable between any two lamps, and has already been considered. But in determining the size of the feeders other considerations are involved, such as economy of cost of transmission. If the feeders be of very small sectional area, they will cost little for material, but will waste much in heating, and the whole yearly cost made up of interest on cost of cables and laying, and value of energy wasted may be very large. On the other hand, if the feeder be of large sectional area it will cost much, whilst the energy wasted may be very small, but the total yearly cost may again be great. It is therefore necessary to take some intermediate size of leads which shall have the yearly cost as small as possible. This was first pointed out by Sir William Thomson (now Lord Kelvin), whose results may be expressed in the following simple rule—

The size of the feeders must be such that the interest for one year on money lying idle in their cost and laying together with depreciation, must be equal to the value of the energy wasted in them during one year.

Professor Baily has pointed out a simple way of ascertaining that size of cable which gives the desired equality. Construct two curves of cost (per mile, say) of cable for one year, thus—

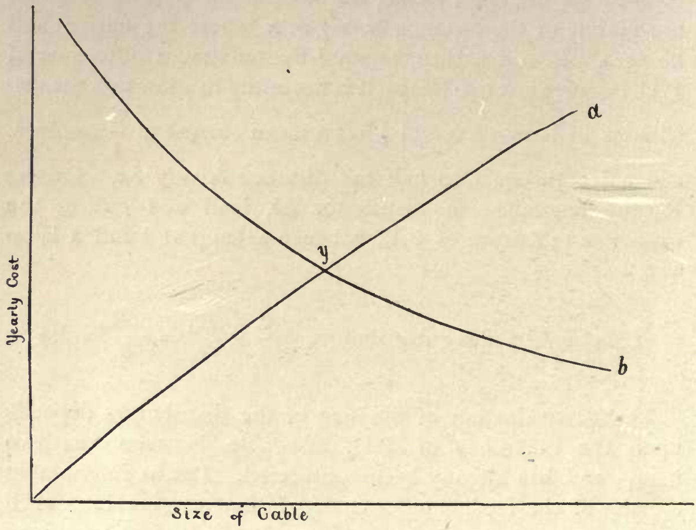

FIG. 6.

The curve *b* represents the cost of waste energy in the cable, whilst *a* represents the interest, etc., on the cost of the cable. The point *y*, where these two curves cut, shows where the two items are equal and also the size of cable.

In calculating out the loss in the feeders it is necessary to allow that no central station is on full load for all the twenty-four hours. It thus comes about that the current taken in the above calculation of waste in the cables must not be the full current; it should, in fact, be more nearly the—

$$\sqrt{\text{mean square}}$$

value when this can be ascertained.

A little consideration will at once show that strict adherence to Kelvin's law is not always desirable, seeing that where power is very cheap and materials and cost of laying very great, the most economical size of conductor might be such that the energy waste was excessive, and though of no consequence as energy, yet producing a dangerous rise of temperature on the conductors.

A good many safety regulations limit the rise in temperature of the conductors to 150° Fahr. as a maximum, thus permitting a possible difference between the air and the conductor of about 75° Fahr., and that for a current twice as large as the normal maximum. Since the heating is, other things being equal, proportional to the square of the current, it follows that the rise in temperature due to the maximum working current must not be more than one-fourth part of the 75° Fahr., or about 18° Fahr. From a large number of experiments on conductors of all kinds, Mr. Kennelley has discovered a rule connecting the diameter of the conductor and the current to be carried consistent with the fact that the rise in temperature shall be within this limit, viz.—

$$C = 560\sqrt{d^3}, \text{ or } d = \frac{\sqrt[3]{C^2}}{68}$$

thus indicating that the current density should decrease as the diameter increases. As, however, most Fire Office rules fix the current density at 1000 ampères per square inch, irrespective of the size of the conductor, it follows that though for small sizes there is a very high degree of safety, nevertheless for conductors exceeding $\frac{1}{5}$ sq. inch sectional area the allowable current would produce a greater rise in temperature than that allowed by Kennelley's rule. Thus, when the sectional area is $\frac{1}{10}$ sq. inch, Kennelley's rule would allow about 17 per cent. more current for safety than the Phœnix Fire Office; when the area is $\frac{1}{5}$ sq. inch, the Fire Office rule would be equivalent to the safe temperature rise; whilst at 1 sq. inch area, the Fire Office rules allow 50 per cent. more current than is consistent with the maximum allowable rise in temperature.

Example XII.—The sectional area of the copper conductors forming a system of distributors is $\frac{1}{4}$ sq. inch, and they are loaded with one 50-watt 100-volt lamp per yard of street. What must be the distance between the feeding centres, which are maintained at a P.D. of 100 volts, in order that the maximum difference between any two lamps shall not exceed 2 volts?

Solution.—Neglecting the small difference in the total current made by the fact that all lamps are not run at exactly the same P.D., viz. 100 volts, each lamp takes 0·5 ampère, and consequently the total current supplied by any feeder will be $\frac{L}{2}$ ampères, where L is the distance in yards between the two feeding centres. The resistance of distributor, lead and return, in between the feeding centres will be $2L \times$ resistance per yard, or—

$$2L \times 36 \times 4 \times 0{\cdot}00000067 = 0{\cdot}00019296L \text{ ohm}$$

and the loss of volts along half of L will be—

$$\frac{\frac{L}{2} \times 0{\cdot}00019296L}{8} = 2$$

$$\text{whence } L^2 = \frac{16}{0{\cdot}00009648} = 165{,}803$$

$$\text{and } L = 407 \text{ yards}$$

Example XIII.—What will be the maximum difference in volts between two lamps if the distance between two feeding centres is a quarter of a mile, the size of distributors is $\frac{1}{4}$ sq. inch sectional area, and current is taken off at the rate of $\frac{1}{6}$ ampère per foot of street?

Solution.—The maximum current in distributors is—

$$\tfrac{1}{6} \times \tfrac{1}{2} \times 13{,}200 = 110 \text{ ampères}$$

and therefore the average value is 55 ampères. The resistance of $\frac{1}{4}$ mile (lead and return) of distributor of $\frac{1}{4}$ sq. inch sectional area is—

$$4 \times 1320 \times 12 \times 0{\cdot}00000067 = 0{\cdot}04244 \text{ ohm}$$

whence the loss of volts is $0{\cdot}04244 \times 55 = 2{\cdot}334$ volts, which is consequently the maximum difference between two lamps.

Multiple Wire Systems.—In some cases the distributors are more than two in number, three or even five conductors being used. This method was first proposed by Dr. John Hopkinson. The connections are made as shown in Fig. 7, taking the five-wire system as an example—

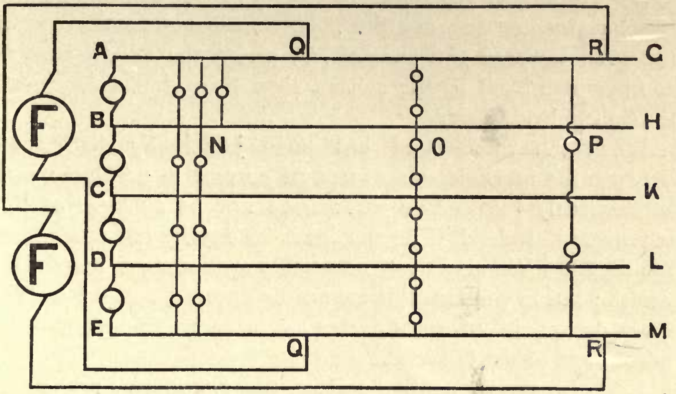

FIG. 7.

The five wires constituting the distributors are shown at AG, BH, CK, DL, EM, the outermost conductors AC and EM being considerably larger in sectional area than the three others. The object of the system is to gain the advantages of transmission of energy from the central station at a high pressure, say 400 volts, and nevertheless provide for the use of 100-volt lamps.

A three-wire system can do the same, using 200-volt lamps, which are now coming in. To transmit the same amount of energy at an E.M.F. of 400 volts, will only require one-sixteenth the weight of copper for the same percentage loss as on a 100-volt circuit. Lamps may be connected in various ways, as at N, 100-volt lamps in parallel between the conductors; as at O, two arc lamps being connected in series between each pair of distributors; and as at P, where 200-volt lamps are connected between the middle and outside wires.

The objects of the middle wires BH, CK, and DL, are—

(1) To enable the equality of P.D. between the wires to be maintained. This is done by introducing between each of them in the station a dynamo or other device, maintained at P.D., of 100 volts each.

(2) To prevent any want of equality in the numbers of the lamps connected between the parallels causing fluctuations in the P.D. between the parallels, by giving the excess current another path back to the station than through the lamps on the lightly loaded side.

When the whole load has been judiciously distributed between the parallels this excess of current is not large, and consequently the middle wires need not be so large as the outermost. Indeed, if all the loads be equally balanced, then the middle wires may be of so small a section as to practically vanish; but in practice this cannot be ensured, and the middle wires are made of such a size as to carry an agreed-upon percentage of the full load.

Like the simple parallel system, the distributors are fed by feeders connected to large dynamos running at a P.D. of 400 volts, plus the economical loss of volts in the feeders, conveying the current to the distributors at feeding centres as shown at QQ and RR.

Example XIV.—Compare two systems. No. 1. A two-wire system to supply current to a circle of distributors 4700 yards round, at the rate of one 50-watt 100-volt lamp per foot of street, and having ten feeding centres at equal distances round the circle, the station being at an average distance of 800 yards from the distributors. There is not to be more than 2 volts difference between any two lamps, and the pressure at the station end of the feeders may be 10 per cent. more than at the distributors.

No. 2. The same town to be supplied on the five-wire system with the same conditions; provision, of course, to be made for the connection of the distributors to the potential equalizers in the station.

Compare the weights of copper used in the two cases.

Solution.—No. 1. The distance between the feeding centres is 470 yards, whence the average current in the distributors is—

$$\tfrac{470}{2} \times 1·5 \times \tfrac{1}{2} = 176 \text{ ampères, say}$$

and thus the resistance of the distributor must be—

$$\frac{2 \times 2}{176} = 0·02273 \text{ ohm}$$

for a 2-volt drop. And as the length of distributor from a feeding centre to feeding centre is (lead and return)—

$$2 \times 470 \times 36 = 33,840 \text{ inches}$$

the resistance per one inch is—

$$\frac{0·02270}{33840} = 0·0000006712 \text{ ohm}$$

whence the sectional area is 1 sq. inch.

The feeders have to carry a current of 4 × 176 = 704, say 705 ampères, and are 800 yards long, with 10 volts loss. Thus the resistance is—

$$\frac{10}{705} = 0·01418 \text{ ohm}$$

or—

$$\frac{0·01418}{800 \times 2 \times 36} = 0·000000246 \text{ ohm per 1 inch}$$

and the sectional area is—

$$\frac{671}{246} = 2·728 \text{ sq. inches}$$

Taking 1 cubic inch of copper as 0·32 lb., the total weight of distributors will be—

$$\frac{4700 \times 36 \times 0·32 \times 2}{2240} = 48·34 \text{ tons}$$

and the feeders—

$$\frac{10 \times 800 \times 36 \times 2 \times 2·728 \times 0·32}{2240} = 224·4 \text{ tons}$$

Or the whole weight of copper in No. 1 = 272·74 tons.

In No. 2, as the outside wires will be at 400 volts P.D., the current will only have an average value of $\frac{1}{4}$ as much as in No. 1, or 44 ampères. Also the fall of P.D. may now be 8 volts, being only 2 per cent. as before. If the feeding centres are at the same distance apart, the sectional area of the distributors will thus be only $\frac{1}{16}$ that of No. 1, or $\frac{1}{16}$ sq. inch. But their length will be increased by 2 × 800 yards to connect them to the potential equalizers in the station; or the whole weight of outside wires will be—

$$\frac{(4700 + 800) \times 2 \times 36 \times 0{\cdot}32 \times \frac{1}{16}}{2240} = 3{\cdot}53 \text{ tons}$$

and allowing $\frac{1}{3}$ the section for the three middle wires each, their combined weights will be half as much as the outsides, or 1·76 ton.

The feeders also carry $\frac{1}{4}$ the current, and with four times the loss of volts they will likewise be $\frac{1}{16}$ the area of those in No. 1, or weight will be $\frac{272{\cdot}74}{16} = 14{\cdot}04$ tons, or the total weight will be 19·33 tons, as against 272·74 tons for No. 1.

CHAPTER III.

CIRCUITS CONTAINING MORE THAN ONE E.M.F.

In some cases there is more than one E.M.F. in the circuit, and the consideration of the magnitude of the phenomena set up will depend upon whether all the E.M.F.'s are in one direction, or whether some are opposed to the others. The cases where all the E.M.F.'s are in one direction have already been dealt with, and we will now deal with some cases where the effective or active E.M.F. is the difference between an applied E.M.F. and another E.M.F., due to the nature of the circuit, acting against the applied E.M.F., and consequently called Back E.M.F.

Let E = an E.M.F. applied to a circuit to (1) overcome the B.E.M.F., and (2) leave sufficient over to send the necessary current through the resistance of the circuit.

ϵ = the B.E.M.F., such as that possessed by chemical cells, running motors, arc lamps, etc.

e = the active E.M.F., which is equivalent to the difference between E and ϵ, or $= E - \epsilon$, and which is also equal to the product of the resistance of the circuit and the current passing.

Thus $E = e + \epsilon$, or $e = E - \epsilon$, and consequently the current, which will be produced by E in a circuit of resistance R and containing the B.E.M.F. ϵ, will be—

$$C = \frac{E - \epsilon}{R} = \frac{e}{R}$$

The resistance R may be composed of various items as before.

Secondary Batteries.—In the case of charging a Secondary Battery, the circuit must consist of a dynamo, the Secondary Battery, and the necessary connecting cables. Each of these items will have their own resistances, that of the Secondary Battery being composed of a number of items in series, each being the resistance of a single cell. In addition to this, each cell will have an E.M.F., which must be overcome in charging, and this E.M.F. is a quantity which changes with the state of charge, being small when the charge is low, and rising to a higher value as the charge increases. This rise of E.M.F. is partly due to actual chemical change, and partly due to a variable temporary increase of local resistance at the surface of the plates. It is, however, customary to refer to it on the whole as an increase of the B.E.M.F. The curve in Fig. 8 shows the values of this E.M.F., corresponding with different charges.

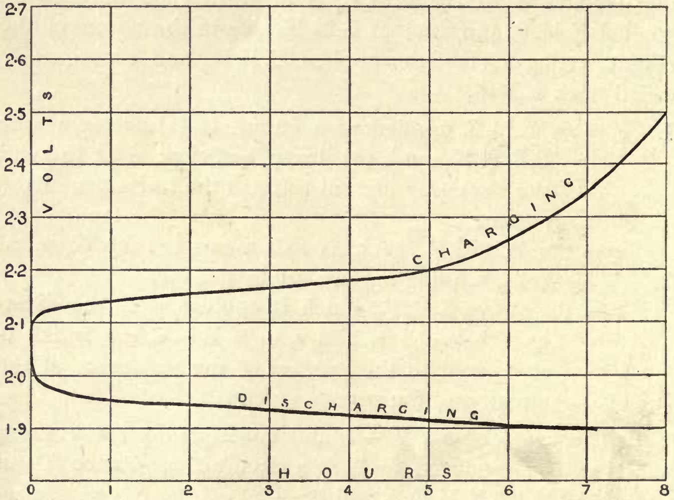

FIG. 8.

From this it will be observed that, though a cell may require 2·3 volts or so to overcome its B.E.M.F. when charging, it cannot give more than a fraction more than 2 volts for even a short time when discharging.

We may thus be required to state what E.M.F. a dynamo must produce in order to charge so many secondary cells with such and such a current; or we may have to find what will be the value of the current through the cells when charged throughout with a constant E.M.F.

Example XV.—What must be the E.M.F. to charge with 100 ampères a battery of 54 cells at an average B.E.M.F. of 2·3 volts each, and each having an internal resistance of 0·0004 ohm, when the resistance of the dynamo is 0·02 ohm and that of the leads is 0·03 ohm?

Solution.—The total B.E.M.F. is—

$$\epsilon = 54 \times 2{\cdot}3 = 124{\cdot}2 \text{ volts}$$

and the active E.M.F. required for the resistance of the circuit will be the product of current into the sum of all resistances, or—

$$\begin{aligned} e &= 100(0{\cdot}02 + 0{\cdot}03 + 54 \times 0{\cdot}0004) \\ &= 100\,(0{\cdot}0716) = 7{\cdot}16 \text{ volts} \end{aligned}$$

whence the applied E.M.F. must be—

$$\text{E} = 124{\cdot}2 + 7{\cdot}16 = 131{\cdot}36 \text{ volts}$$

Example XVI.—What will be the current through the circuit at the beginning and end of charging in the case of a battery of 53 cells, each of 0·0002 ohm internal resistance, charged by a dynamo at an E.M.F. of 135 volts constant, when the dynamo resistance is 0·015 ohm, and that of the leads is 0·025 ohm, if the charging is continued for 7 hours, the value of ϵ per cell rising from 2·1 volts to 2·35 volts?

Solution.—The B.E.M.F. at starting to charge will be $2{\cdot}1 \times 53 = 111{\cdot}3$ volts, and at the end of 7 hours will be $2{\cdot}35 \times 53 = 124{\cdot}55$ volts. The total resistance is—

$$53 \times 0{\cdot}0002 + 0{\cdot}015 + 0{\cdot}025 = 0{\cdot}0506 \text{ ohm}$$

the active E.M.F. at the start of charging will thus be—

$$e = \text{E} - \epsilon = 135 - 111{\cdot}3 = 23{\cdot}7 \text{ volts}$$

and at the end of charging—

$$e = 135 - 124{\cdot}55 = 10{\cdot}45 \text{ volts}$$

whence the current at starting will be—

$$c = \frac{23{\cdot}7}{0{\cdot}0506} = 468 \text{ ampères}$$

and at end of run is—

$$c = \frac{10{\cdot}45}{0{\cdot}0506} = 206 \text{ ampères}$$

Example XVII.—In the last example (XVI.), if the dynamo has to be run so as to produce 135 volts always, and it is desired that the current shall not exceed 200 ampères, what will be the value of the necessary resistance to be inserted (1) at starting, (2) at end of charging?

Solution.—The active E.M.F. at starting—

$$e = 23{\cdot}7 \text{ volts}$$

and at end of run is—

$$e = 10{\cdot}45 \text{ volts}$$

whence the total resistance at starting must be—

$$\frac{23{\cdot}7}{200} = 0{\cdot}1185 \text{ ohm}$$

and at end of run—

$$\frac{10{\cdot}45}{200} = 0{\cdot}05225 \text{ ohm}$$

But the resistance of the circuit is already 0·0506 ohm, whence the added resistance must be—

$$0{\cdot}1185 - 0{\cdot}0506 = 0{\cdot}0679 \text{ ohm at start}$$

$$\text{and } 0{\cdot}05225 - 0{\cdot}0506 = 0{\cdot}00165 \text{ ohm at end of charging.}$$

Example XVIII.—A battery of 55 cells has a discharging E.M.F. of 2·05 volts per cell at start, and gradually falling to 1·9 volts each cell at end of 7 hours' run. The internal resistance of each cell is 0·0004 ohm. The resistance of the leads from the cells to the lamps is 0·005 ohm. What number of cells must be used to supply 400 50-watt 100-volt lamps if the P.D. at the lamp terminals is to be kept at as nearly 100 volts as possible all the time?

Solution.—The lamps are to take 100 volts, and will also require $\frac{50}{100} \times 400 = 200$ ampères, neglecting the slight difference made by the P.D. at their terminals not always being exactly 100 volts.

The loss of volts along the leads is—

$$0{\cdot}005 \times 200 = 1 \text{ volt}$$

and the loss in the cells is—

$$0{\cdot}0004 \times 200 = 0{\cdot}08 \text{ volt per cell}$$

or the available E.M.F. per cell is $2{\cdot}05 - 0{\cdot}08 = 1{\cdot}97$ volt at start, or—

$$1{\cdot}90 - 0{\cdot}08 = 1{\cdot}82 \text{ volt at end of run}$$

The E.M.F. required for leads and lamps is to be—

$$100 + 1 = 101 \text{ volts}$$

whence the number of cells required will be—

$$\frac{101}{1{\cdot}87} = 51{\cdot}2, \text{ say } 51 \text{ at start}$$

and—

$$\frac{101}{1{\cdot}82} = 55{\cdot}4, \text{ or } 55 \text{ cells at end}$$

Motors.—In running motors we are also dealing with circuits containing a B.E.M.F. A motor is a dynamo caused to run by sending through its armature a current at sufficient E.M.F. to overcome the B.E.M.F., and leave sufficient to maintain that current through the resistance of the circuit. We have, then, as before, that the current in the armature will be—

$$C = \frac{E - \epsilon}{R}$$

R being the resistance of the whole circuit, and ϵ the B.E.M.F.

produced by the rotating armature. The part of the energy which is supplied to the motor that is converted into mechanical work is $C\epsilon$, and of this a part is expended in friction of various kinds in the motor itself; this part will be dealt with later on, and at present will simply be expressed as a waste in watts or horse-power. The difference between $C\epsilon$ and this waste is the available power of the motor, and is usually expressed as so many brake horse-power (B.H.P.).

Example XIX.—A motor having a resistance of 0·025 ohm, and producing a B.E.M.F. of 90 volts at a given speed, is connected by leads of 0·005 ohm resistance to a dynamo of resistance = 0·02 ohm. It is required to produce a B.H.P. of 7 H.P. What must be the E.M.F. of the dynamo if the waste in internal friction, etc., in the motor is 300 watts?

Solution.—The total mechanical power is—

$$C\epsilon = C \times 90 \text{ watts}$$

of which 300 watts are wasted in the motor, leaving 7 × 746 watts available at the brake. Therefore—

$$\begin{aligned} C \times 90 - 300 &= 7 \times 746 \\ C &= \frac{5522}{90} \\ &= 61{\cdot}35 \text{ ampères} \end{aligned}$$

And this current is produced through the resistance of the whole circuit by an active E.M.F—

$$e = C \text{ (resistance of circuit)}$$

or—

$$\begin{aligned} e &= 61{\cdot}35(0{\cdot}025 + 0{\cdot}005 + 0{\cdot}02) \\ &= 3{\cdot}0675 \text{ volts} \end{aligned}$$

whence the total E.M.F. must be—

$$90 + 3{\cdot}0675 = 93{\cdot}0675 \text{ volts}$$

Example XX.—In the above question, what will be the brake horse-power of the motor if the total E.M.F. of the dynamo is 95 volts, assuming the same speed and loss due to friction, etc.?

Solution.—Here the active E.M.F. will be—

$$e = 95 - 90 = 5 \text{ volts}$$

and consequently the current will be—

$$C = \frac{5}{0{\cdot}05} = 100 \text{ ampères}$$

therefore the total mechanical power is—

$$100 \times 90$$

and of this 300 watts is still the waste, leaving for brake power—

$$9000 - 300 = 8700 \text{ watts}$$

or—

$$\frac{8700}{746} = 11{\cdot}6 \text{ H.P. at brake}$$

Arc Lamps.—In the case of arc lamps, the energy required for the vaporization of the carbons is generally considered as made up of the product of the current and an E.M.F. at the crater, called a back E.M.F., the magnitude of which is about 39 volts for a continuous current arc; but which appears to be as low as 30 volts (virtual) for some alternating current arcs. Thus, in an arc lamp, part of the E.M.F. supplied is concerned in sending the current through the resistance of the circuit, and the remainder is to overcome the back E.M.F.

Example XXI.—An arc lamp has a B.E.M.F. of 39 volts, and is connected to leads at P.D. of 50 volts. The resistance of the lamp leads is 0·11 ohm, the resistance of the lamp coil is 0·09 ohm, and the carbons have a resistance of 0·08 and 0·12 ohm respectively, whilst the arc itself has a resistance of 0·1 ohm. What must be the resistance included in the circuit so that the lamp may take a current of 10 ampères?

Solution.—The available or active E.M.F. is—

$$e = 50 - 39 = 11 \text{ volts}$$

and since the current is to be 10 ampères, we have the total resistance must be—

$$\frac{e}{C} = \frac{11}{10} = 1{\cdot}1 \text{ ohm}$$

But the resistance of the lamp itself is—

$$0{\cdot}11 + 0{\cdot}09 + 0{\cdot}08 + 0{\cdot}12 + 0{\cdot}1 = 0{\cdot}5 \text{ ohm}$$

whence the additional resistance required must be—

$$1{\cdot}1 - 0{\cdot}5 = 0{\cdot}6 \text{ ohm}$$

$E = 4.44 \Phi f N 10^{-8}$

CHAPTER IV.

DYNAMOS AND MOTORS.

In questions relating to the proportions of parts of dynamos and motors we have to consider the relationship between the armature and magnet windings, etc. The electro-motive force generated in a revolving armature can be arrived at from the data of its parts. Thus, the whole generated E.M.F. is—

$$E = \frac{Nnw}{10^8}$$

for a Bipolar machine, whether drum or gramme wound, where—

N = whole number of c.g.s. magnetic lines which traverse the armature core,
n = the revolutions per second of the armature,
and w = the whole number of wires counting all round the circumference of the armature,
10^8 = the number of c.g.s. units of E.M.F., equivalent to the practical unit, the volt.

The factor Nn stands for the rate of cutting lines in c.g.s. lines per second.

The number of lines N is made up of the product of the sectional area of the iron armature core and the number of c.g.s. lines per unit of sectional area. It is very usual to call the number of c.g.s. lines per square centimetre the value of β, or—

$$\beta = \text{c.g.s. lines per sq. cm. of armature iron}$$

For different types of armature there are different values of β most suitable. Thus, for gramme or cylinder armatures, β

may range from 14,000 to 18,000 sq. cms., or even higher; whilst, owing to the necessity of passing the lines through the same size of air gap in the drum as in the cylinder type, we must keep the value of β for drums at about 9000 to 12,000 per sq. cm., though in some types of drum higher values can be used with advantage.

The speed n is limited by the strength of the structure, on the one hand, to not more than 3000 to 4000 feet per minute (roughly, 1500 to 2000 cms. per second) circumferential speed; or, on the other hand, to far smaller values where slow speeds are desirable for direct coupling, to slow-speed engines, for example.

The number of wires all round, w, is determined by the E.M.F. desired, and their size by the current to be carried. As the commutator connects the armature to the outside circuit in two parallels in Bipolar machines, the size of wire on the armature is that to carry half the current from the machine. No fixed rule can be stated for the relationship between the current and the size of wire, as this will depend upon the size of machine, the efficiency desired, and the rise in temperature allowable. This part of the subject is considered in a later chapter.

The whole E.M.F. E is the total volts generated by the armature, and, of course, a part of this is required to circulate the current in armature itself. Thus, if—

r_a = resistance of armature winding measured from brush to brush,

and C = total current in ampères in the armature, then Cr_a volts will be lost in the armature, and the difference $E - Cr_a$ is the terminal P.D. of the dynamo, in the case of a magneto machine or a shunt dynamo.

The resistance r_a is obviously the resistance of $\frac{1}{4}$ the whole length of wire upon it (since the two halves, and in parallel, and each half is half the resistance of the whole), plus any contact resistance at the brushes and in the leads to the brushes.

Leaving for the present the exact statement of the relationship between the armature and magnet windings, we will take a few simple examples.

Example XXII.—What will be the sectional area of iron required in the armatures respectively of two dynamos, one ring wound and the other drum wound, each having 240 wires all round, running at 1200 revolutions per minute, and giving a total E.M.F. each of 120 volts?

Solution.—In each case we have that the number of lines N required will be—

$$N = \frac{E 10^8}{nw}$$

and since $n = \frac{1200}{60} = 20$ revolutions per second

$$N = \frac{120 \times 10^8}{240 \times 20} = 2{,}500{,}000 \text{ lines total}$$

Now, the ring-wound armature may have β = say 15,000, whilst the drum may have β only 10,000, say.

$$\text{But } N = \beta a$$

whence in the ring type—

$$a = \frac{2{,}500{,}000}{15{,}000} = 166\tfrac{2}{3} \text{ sq. cms.}$$

and in the drum—

$$a = \frac{2{,}500{,}000}{10{,}000} = 250 \text{ sq. cms.}$$

area of iron core.

Example XXIII.—What is the total E.M.F. produced by a dynamo, having an armature ring or gramme wound with 204 wires all round, when $\beta = 16{,}000$, area of cross-section of iron core is 125 sq. cms., and speed is 25 revolutions per second?

Solution.—

$$E = \frac{Nnw}{10^8} = \frac{16{,}000 \times 125 \times 25 \times 204}{10^8} = 102 \text{ volts}$$

Example XXIV.—At what speed must the above armature be run so as to give 102 volts terminal pressure when the resistance of the armature is 0·02 ohm, the current is 100 ampères, and the value of β remains at 16,000?

Solution.—The loss of volts in armature resistance will be—

$$Cr_a = 100 \times 0{\cdot}02 = 2 \text{ volts}$$

whence the total generated E.M.F. must be—

$$E = 102 + 2 = 104 \text{ volts}$$

thus the speed—

$$n = \frac{E10^8}{Nw} = \frac{104 \times 10^8}{16{,}000 \times 125 \times 204} = 25{\cdot}49 \text{ revolutions per sec.}$$

or 1529·4 per minute.

Example XXV.—What is the value of β in an armature core, drum wound, 25 cms. diameter, with 7 cm. hole in the centre, and composed of 500 discs half a millimetre thick, if wound with 180 wires all round, and run at 1200 revolutions per minute, producing an E.M.F. of 175 volts at the terminals, when the resistance of the winding is 0·032 ohm, and the current is 100 ampères?

Solution.—The total generated E.M.F. is—

$$\begin{aligned} E &= 175 + Cr_a \\ &= 175 + 100 \times 0{\cdot}032 \\ &= 178{\cdot}2 \end{aligned}$$

and therefore—

$$N = \frac{178{\cdot}2 \times 10^8}{20 \times 180} = 4{,}950{,}000$$

But N = β × area, and area is—

$$a = (25 - 7) \times 500 \times 0{\cdot}05 = 450 \text{ sq. cms.}$$

and therefore—

$$\beta = \frac{4{,}950{,}000}{450} = 11{,}000 \text{ per sq. cm.}$$

Example XXVI.—An armature, gramme wound, has a core 36 cms. diameter, with 23 cm. hole, and is built up of 1000 discs, each 0·28 millimetre thick. There are 192 wires all round, each large enough to carry a current of 60 ampères. The speed is 781·2 revolutions per minute, and the resistance of the armature is 0·039 ohm. What is the value of β required to produce a terminal P.D. of 150 volts at a current of 120 ampères?

Solution.—At 120 ampères the loss of volts in the armature is—

$$\mathrm{C}r_a = 120 \times 0{\cdot}039 = 4{\cdot}68 \text{ volts}$$

whence the generated E.M.F.—

$$\mathrm{E} = 150 + 4{\cdot}68 = 154{\cdot}68 \text{ volts}$$

thus to produce this E.M.F.—

$$\mathrm{N} = \frac{154{\cdot}68 \times 10^8}{\frac{781{\cdot}2}{60} \times 192} = 6{,}188{,}000 \text{ lines}$$

and the sectional area of the iron core is—

$$a = (36 - 23) \times 1000 \times 0{\cdot}028 = 364 \text{ sq. cms.}$$

whence—

$$\beta = \frac{\mathrm{N}}{a} = \frac{6{,}188{,}000}{364} = 17{,}000 \text{ per sq. cm.}$$

Magnetization of Field-magnets.—So far we have been considering cases where the exact value of the current in the armature is given. In the majority of cases, however, the armature has to carry a current which is greater than that in the external circuit by the amount taken by the coils to magnetize the field-magnets. This magnetization is brought about in several ways, viz.—

1. By current from some independent source—called separate excitation, thus—

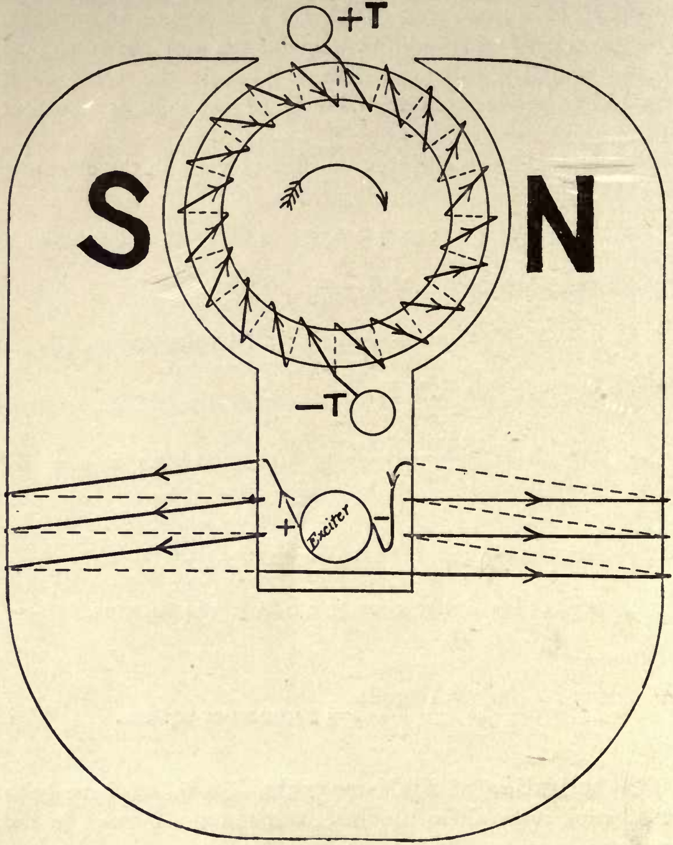

FIG. 9.

Here the data of the magnet-winding are of no particular importance in the present connection.

The current in the armature is the same as the current in the external circuit.

2. By a current from the armature of the machine itself

caused to circulate round the magnet-winding before going to the outside circuit, thus—

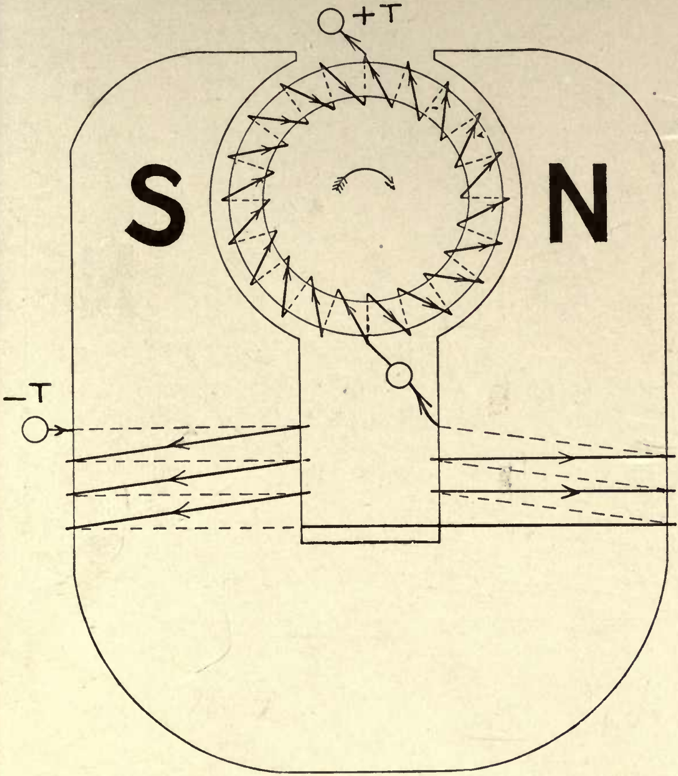

Fig. 10.

where + T and − T represent the terminals of the machine which are to be connected to the external circuit.

This is called *series* winding; and here there is a loss of volts in the resistance of the series coil, which has to be added to the loss in the armature to get the terminal P.D.

In this case also the current in the armature is the same as the current in the outside circuit.

We will call r_m the resistance of the series coil.

3. Sometimes the magnet legs are excited by a current derived from the terminals of the machine, called a *shunt* winding, thus—

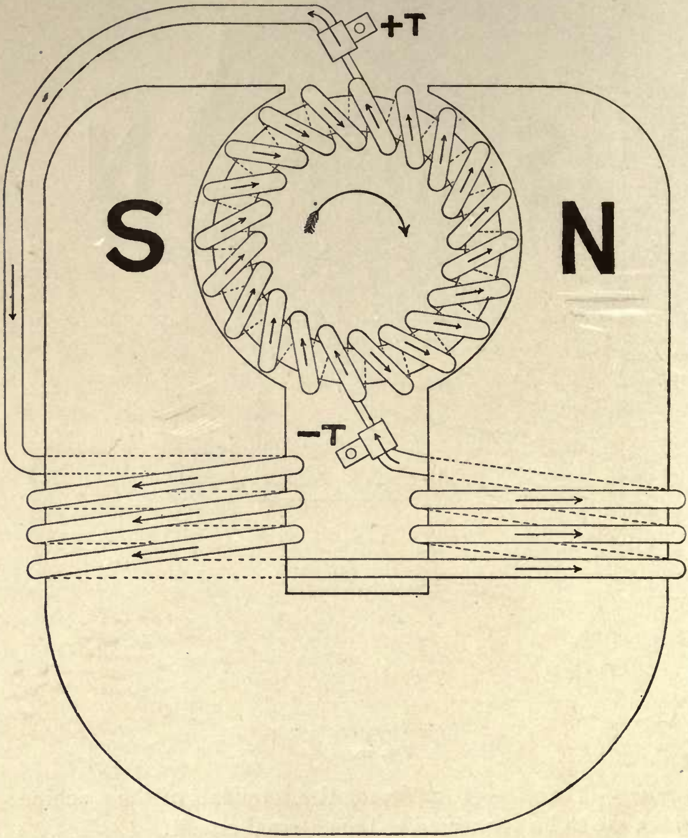

FIG. 11.

In this case it is clear that the armature has in it a current which is greater than that in the external circuit, by the amount taken by the shunt coil. The total resistance of the circuit is composed of the armature in series with the external circuit which is in parallel with the shunt coil. Calling r_s = the resistance of shunt coil, and r_a that of the armature, and the current in the external circuit = C ampères due to a terminal P.D. of e watts, we have that the current in the armature—

$$C_a = C + \frac{e}{r_s}$$

and the loss of volts in the armature will be—

$$C_a r_a = r_a\left(C + \frac{e}{r_s}\right)$$

whence the total generated E.M.F.—

$$E = e + r_a\left(C + \frac{e}{r_s}\right)$$

(4) There may, for certain reasons, to be considered later on, be a combination of shunt and series excitation, and it is possible to connect up these two coils in two different ways; firstly, thus—

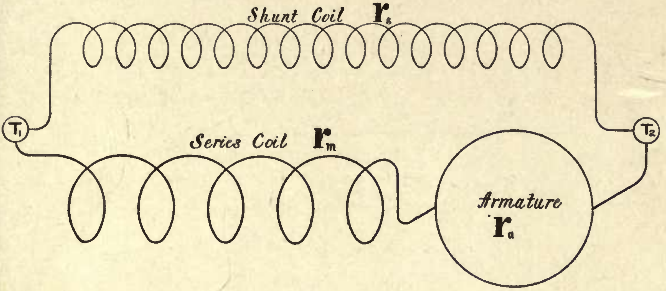

FIG. 12.

called "long shunt." Compound wound.

In this case, calling e = terminal P.D.; C = current in the external circuit, etc., as before, we have that the current in the shunt—

$$C_s = \frac{e}{r_s}$$

and the current in the series coil and armature will be both the same, viz.—

$$C_a = C + \frac{e}{r_s}$$

whence there will be a loss of volts in the series coil of—

$$r_m\left(C + \frac{e}{r_s}\right) \text{ volts}$$

and in the armature of—

$$r_a\left(C + \frac{e}{r_s}\right) \text{ volts}$$

The total loss in the machine will be—

$$(r_a + r_m)\left(C + \frac{e}{r}\right) \text{ volts}$$

and thus the total generated E.M.F.—

$$E = e + (r_a + r_m)\left(C + \frac{e}{r_s}\right)$$

Or, secondly, connections may be made thus—

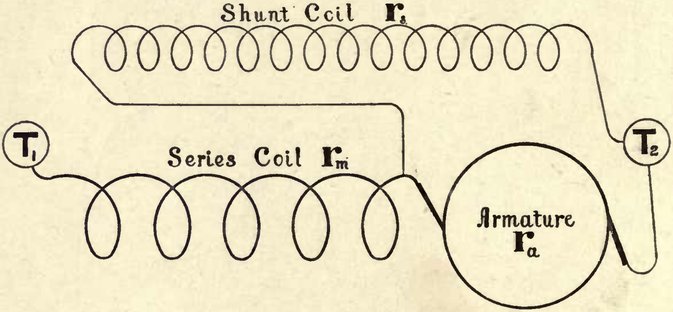

FIG. 13.

which is called "short shunt." Compound wound.

Here there will be a loss of volts in the series coil, due to the main current C, or this loss will be—

$$Cr_m \text{ volts}$$

whilst the current in the armature is greater than C, the external current, by the amount taken by the shunt coil.

Calling e the terminal P.D. as before, we have that the P.D. at the terminals of the shunt coil will be—

$$e + \mathrm{C}r_m$$

and thus the current in the armature will be—

$$\mathrm{C}_a = \mathrm{C} + \frac{e + \mathrm{C}r_m}{r_s}$$

and the loss of volts in the armature is thus—

$$r_a\left(\mathrm{C} + \frac{e + \mathrm{C}r_m}{r_s}\right)$$

whence the whole E.M.F.—

$$\mathrm{E} = e + \mathrm{C}r_m + r_a\left(\mathrm{C} + \frac{e + \mathrm{C}r_m}{r_s}\right)$$

There are other important differences between "long" and "short" shunt winding, which will have to be considered later on.

Example XXVII.—A shunt dynamo is to give a current of 50 ampères at a terminal P.D. of 104 volts. The resistance of the armature is 0·04 ohm, that of the shunt coil 52 ohms. There are 210 wires all round, and the sectional area of the core is 160 sq. cms. The speed is 1080 revolutions per minute. If the winding is gramme type, what will be the value of β?

Solution.—The current in the armature—

$$\mathrm{C}_a = \mathrm{C} + \frac{e}{s} = 50 + \frac{104}{52} = 52 \text{ ampères}$$

whence the loss of volts in the armature is—

$$52 \times 0{\cdot}04 = 2{\cdot}08 \text{ volts}$$

or the total generated E.M.F.—

$$\mathrm{E} = 104 + 2{\cdot}08 = 106{\cdot}08 \text{ volts}$$

to produce which E.M.F. the total number of lines—

$$N = \frac{106{\cdot}08 \times 10^8}{\frac{1080}{60} \times 210} = \frac{10{,}608{,}000{,}000}{0{\cdot}18 \times 210}$$

$$= 2{,}806{,}000 \text{ lines}$$

whence the value of the magnetic density is—

$$\beta = \frac{2{,}806{,}000}{160} = 17{,}530 \text{ per sq. cm.}$$

Example XXVIII.—At what speed must a series machine run to give a current of 10 ampères at a terminal P.D. of 1000 volts, if the resistance of the armature is 2 ohms, and that of the magnet coils 1·8 ohm, when the area of cross-section of the armature core is 150 sq. cms., and there are 17,500 c.g.s. lines per sq. cm.? The winding is ring type, and there are 2100 wires all round.

Solution.—The loss of volts in the armature is—

$10 \times 2 = 20$ volts, and in the magnet coil is—

$10 \times 1{\cdot}8 = 18$ volts

or a total loss of $\overline{38}$ volts; whence the generated E.M.F.—

$$E = 1000 + 38 = 1038 \text{ volts}$$

The total number of lines through the armature is—

$$N = 150 \times 17{,}500 = 2{,}625{,}000 \text{ lines}$$

whence the speed—

$$n = \frac{1038 \times 108}{2100 \times 2{,}625{,}000} = 18{\cdot}83 \text{ revolutions per second}$$

or 1129·8 per minute.

Example XXIX.—A long shunt compound dynamo is run so as to light 200 50-watt 100-volt lamps at the end of leads of resistance of 0·008 ohm. The resistance of the armature is 0·022 ohm, that of the series coil 0·015 ohm, and that of the shunt is 60 ohms. If the speed be 1200 per minute, what will be the sectional area of iron core required when $\beta = 17{,}085$, and there are 102 wires all round the armature?

Solution.—The current in the lamps is—

$$C = 100 \text{ ampères}$$

And there is a loss of 100 × 0·008, or 0·8 volts in the leads; whence the terminal P.D. is 100·8 volts; and thus the shunt current is—

$$C_s = \frac{100{\cdot}8}{60} = 1{\cdot}68 \text{ ampère}$$

and consequently the current in the series coil and armature will be—

$$C_a = 101{\cdot}68 \text{ ampères}$$

the loss of volts in the armature and series coil will be—

$$101{\cdot}68 \times (r_a + r_m)$$

or—

$$101{\cdot}68 \times (0{\cdot}022 + 0{\cdot}015) = 3{\cdot}76 \text{ volts}$$

then for the whole generated E.M.F.—

$$E = 100{\cdot}8 + 3{\cdot}76 = 104{\cdot}56$$

to produce which the number of lines total will be—

$$N = \frac{10{,}456{,}000{,}000}{20 \times 102} = 5{,}125{,}490$$

and as $\beta = 17{,}085$, the sectional area—

$$a = \frac{5{,}125{,}490}{17{,}085} = 300 \text{ sq. cms.}$$

Example XXX.—A short shunt compound dynamo is required to give a current of 200 ampères through 53 secondary cells of 0·00025 ohm each internal resistance, and having a B.E.M.F. when charging of 2·41 volts each cell. The leads to the cells are 0·005 ohm resistance; the series coil is 0·008 ohm; the armature is 0·012 ohm; and the shunt resistance is 50 ohms. At what speed must she run when there are 144 wires all round the armature, the sectional area of the armature core is 1100 sq. cms., and there are 17,100 c.g.s. lines per sq. cm.?

Solution.—The total B.E.M.F. of the cells is—

$$\epsilon = 2{\cdot}41 \times 53 = 127{\cdot}73 \text{ volts}$$

there is a loss in their internal resistance of—

$$200 \times 53 \times 0{\cdot}00025 = 2{\cdot}65 \text{ volts}$$

the loss in the leads is—

$$200 \times 0{\cdot}005 = 1 \text{ volt}$$

and that in the series coil is—

$$200 \times 0{\cdot}008 = 1{\cdot}6 \text{ volt}$$

Thus the P.D. at the terminals of the shunt coil is—

$$127{\cdot}73 + 2{\cdot}65 + 1 + 1{\cdot}6 = 132{\cdot}98, \text{ say } 133 \text{ volts}$$

and due to which the current in the shunt coil is—

$$C_s = \frac{133}{0{\cdot}50} = 2{\cdot}66 \text{ ampères}$$

Whence the current in the armature will be—

$$C_a = 202{\cdot}66 \text{ ampères}$$

and the loss in the armature—

$$202{\cdot}66 \times 0{\cdot}012 = 2{\cdot}43 \text{ volts}$$

and thus the total generated E.M.F. is—

$$E = 133 + 2{\cdot}43 = 135{\cdot}43 \text{ volts}$$

to produce which the speed must be—

$$n = \frac{13{,}543{,}000{,}000}{1100 \times 17{,}100 \times 144} = 5 \text{ revolutions per second}$$

or 300 per minute.

Pull on Conductors.—The relationship between the power required to drive an armature as a dynamo and the current and E.M.F. produced, is that the E.M.F. depends upon the speed only, as is shown from the formula we have already used, viz.—

$$E = \frac{Nnw}{10^8}$$

since for any given machine N and w are fixtures. The current which the machine will give depends upon the resistance of the circuit connected to it, and upon the power which the prime mover can put into it. There is a magnetic effect produced by the current in the armature windings, which acts so as to resist the motion. This will be understood by reference to Fig. 11.

The prime mover has to overcome the resistance to motion, called a drag on the winding, by exerting a pull on the one side of the belt attached to the dynamo. The magnitude of this pull can be simply expressed as—

$$P = CL\beta_g \times 0{\cdot}0000002244 \text{ pounds per wire}$$

where C = the current in ampères in any wire,
L = the length of a wire passing through the field in cms.
β_g = the strength of the magnetic field in the air gap in c.g.s. lines per sq. cm.

Owing, however, to the want of uniformity in the strength of the magnetic field round the armature, the actual pull on any wire will depend upon its position in the air gap. When, for instance, the wire under consideration is midway in between the polar horns, the pull will practically be zero, since there the field is very small; but when under a pole piece the pull will be considerable, though gradually varying from a smaller to a larger value as the wire moves round the polar arc. The average value of the pull is that given above, and the total pull on the belt, as far as production of electricity is concerned, will be the product of this average pull and the number of wires all round which can be considered to be under the magnetic influence, or cutting lines. This number is approximately equal to—

$$w_1 = w\frac{2\theta}{360} \times 1{\cdot}1 = 1{\cdot}1\frac{w\theta}{180}$$

for bipolar machines. Where w is the whole number of wires counted all round, θ is the angle of polar embrace, and 1·1 is a numeric to take in the stray field in between the poles.

The angle θ varies from 120° to 140° generally, though it is sometimes outside these limits.

Thus the total pull will be—

$$P = \frac{C}{2} L \beta_g w \frac{\theta}{180} \times 1{\cdot}1 \times 0{\cdot}0000002244$$

where C = the total current from the machine in ampères,
L = the length of any wire that is practically the length of the bore of the magnets,
β_g = density of lines c.g.s. per sq. cm. in the air gap,
w = total number of wires all round,
θ = the polar angle,

and the formula is only for a bipolar machine. Simplifying, we have—

$$P = CL\beta_a w\theta \times 0{\cdot}0000000006856$$

From the above, the rate of doing work can be found by considering that this pull is a weight of so many pounds lifted through a distance equivalent to the mean circumference of the armature once in the time taken by one revolution; thus let—

d = mean diameter in feet of the armature, *i.e.* diameter of core plus the depth of winding.
n = number of revolutions per second.

Then the above weight is lifted—

$$\pi d \text{ feet in } \frac{1}{60n} \text{ minutes}$$

or the work done in one minute is—

$$60Pn\pi d \text{ foot-pounds}$$

whence the power is—

$$\frac{60Pn\pi d}{33{,}000} \text{ horse-power}$$

$$= \frac{63 \times 22ndCL\beta_g w\theta \times 0{\cdot}0000000006856}{7 \times 33{,}000}$$

$$= \frac{3917{\cdot}7ndCL\beta_g w\theta}{10^{15}} \text{ horse-power}$$

For a dynamo this is, of course, only that part of the power required which is concerned in the production of electricity;

the power to overcome frictional losses of various kinds must be added. But in the case of a motor the above gives the mechanical power developed, part, however, of which is also lost in friction, etc.

In the case of motors the E.M.F. produced by the rotation of the armature is opposed to the supplied E.M.F., and is called a back E.M.F. ϵ. But as far as the motor is concerned this B.E.M.F. is one of the factors of the power, the other being the current; this B.E.M.F. has to be calculated exactly as though we were dealing with a dynamo.

Example XXXI.—At what speed will the armature of a series machine run, and what will be the brake H.P. if a current of 20 ampères be supplied to it at an E.M.F. of 400 volts? Its armature has a resistance of 0·25 ohm; its magnet winding 0·15 ohm. The loss in friction is 380 watts. There are 900 wires all round, and the total magnetic flux through the armature is two million lines.

Solution.—Since the total loss of volts in the machine is—

$$20 \times (0·25 + 0·15) = 8 \text{ volts}$$

there will be—

$$400 - 8 = 392 \text{ volts}$$

available for overcoming the B.E.M.F. And the speed necessary to produce this will be—

$$n = \frac{\epsilon 10^8}{\mathrm{N}w} = \frac{392 \times 10^8}{900 \times 2 \times 10^6}$$

$$= 21·77 \text{ revolutions per second}$$

Thus the total rate of doing work mechanically is—

$$\begin{array}{lrl} & 392 \times 20 = 7840 \text{ watts,} & \text{of which the} \\ \text{loss in friction is—} & \underline{380 \text{ watts,}} & \text{leaving—} \\ & 7460 \text{ watts,} & \text{or—} \end{array}$$

10 H.P. available at the brake.

Example XXXII.—A machine run as a shunt dynamo, with a current in the armature of 100 ampères, gives 100 volts at its terminals at a speed of 1200 revolutions per minute. The resistance of the armature is 0·035 ohm. What will be the speed if this machine be run as a motor off mains at 100 volts, and taking a current of 102 ampères total, when the resistance of the shunt coil is 50 ohms, and the loss due to friction, etc., is at the rate of 325 watts?

Solution.—As a motor the shunt coil takes 2 ampères, leaving 100 ampères for current through the armature. As a dynamo there was a loss in the armature of 100 × 0·035 = 3·5 volts, whence the total generated E.M.F. must have been 103·5 volts, at a speed of 1200 revolutions per minute. As a motor there will also be the same loss of 3·5 volts in the armature, leaving only 96·5 volts for overcoming B.E.M.F., which B.E.M.F. will be produced at a speed of—

$$\frac{96·5}{103·5} \times 1200 = 1118 \text{ per minute}$$

Since the strength of the magnetic field will be the same as before, also the total mechanical rate of doing work will be—

$C\epsilon = 100 \times 96·5 = 9650$ watts, of which the
loss in friction is— 325 watts, leaving—
9325 watts for external work, or—

$$\frac{9325}{746} = 12·5 \text{ H.P. at the pulley}$$

Example XXXIII.—A shunt machine running at 720 revolutions per minute as a dynamo, gives 50 volts P.D. at its terminals at a current of 200 ampères in the external circuit. Its armature resistance is 0·0147 ohm, and that of its shunt coil is 12·5 ohms. At what speed will it run as a motor, and what current will it take at a P.D. of 50 volts to develop 12 brake H.P., when ½ H.P. is lost in friction in the gearing, and 0·352 H.P. in the machine?

Solution.—The total rate of doing work by the motor must be $12 + 0·5 + 0·352 = 12·852$ H.P., or $12·852 \times 746 = 9588$ watts, which must be the product of the current in the armature and the B.E.M.F. produced. The shunt coil takes—

$$\frac{50}{12·5} = 4 \text{ ampères}$$

and thus the total current will be $C = C_a + 4$.

As the resistance of the armature is 0·0147 ohm, the loss of volts therein will be $C_a \times 0·0147$ volts, and thus—

$$\epsilon = 50 - 0·0147C_a$$

and $C_a\epsilon$ must be = 9588, or—

$$(50 - C_a \times 0·0147)C_a = 9588$$

or—

$$-0·0147C_a^2 + 50C_a - 9588 = 0$$

whence $C_a = 204$ ampères, at which current the loss in the armature is—

$$204 \times 0·0147 = 3 \text{ volts}$$
$$\text{leaving } 50 - 3 = 47 \text{ volts for } \epsilon$$

The total rate of doing work mechanically is—

$$204 \times 47 = 9588 \text{ watts}$$

and the speed is—

$$\frac{47}{53} \times 720 = 628·4 \text{ per minute}$$

Example XXXIV.—A 10-ampère series machine which gives 1000 volts total E.M.F. at a speed of 1000 revolutions per minute, is supplied with a current of 10 ampères to run as a motor, and has to lift a weight of 3 tons at the rate of 50 feet per minute. If there is a loss due to friction in the gearing of ½ H.P., and 0·32 H.P. lost in the friction, etc., in the motor, what will be the speed and P.D. of supply mains required if the resistance of the armature is 3·5 ohms, and that of the magnet coils 2·5 ohms?

Solution.—To lift 3 tons 50 feet per minute is doing work at the rate of—

$$\frac{3 \times 2240 \times 50}{33,000} = 10·18 \text{ H.P.}$$

and, adding the losses, the total H.P. will be—

$$10·18 + 0·5 + 0·32 = 11 \text{ H.P.}$$

or in watts = 8206, which is done by a current of 10 ampères against a B.E.M.F. of—

$$\tfrac{8206}{10} = 820·6 \text{ volts}$$

to produce which the speed will be—

$$\frac{820\cdot6}{1000} \times 1000 = 820\cdot6 \text{ revolutions per minute}$$

To send 10 ampères through the resistance of the machine will require a P.D. of—

$$10(r_a + r_m) = 10 \times 6 = 60 \text{ volts}$$

whence the P.D. at the motor terminals will have to be—

$$820\cdot6 + 60 = 881 \text{ volts, say}$$

Example XXXV.—The length of one wire on the surface of an armature is 20 cms., the mean diameter is 21 cms., the speed is 1200 revolutions per minute, and the density of lines in the air gap is 2291 per sq. cm. The angle of polar embrace is 135°, and there are 360 wires all round. If this machine be used to give a total current of 50 ampères, what will be the H.P. required to drive it if ½ H.P. is lost in various ways?

Solution.—Applying the formula on p. 50, we have—

$$\text{H.P.} = \frac{3917\cdot7 \times \frac{1200}{60} \times \frac{21}{30\cdot47} \times 50 \times 20 \times 360 \times 135 \times 2291}{10^{15}}$$

= 6·01 H.P., to which must be added ½ H.P. as above for friction, etc., or total H.P. required = 6·51 H.P.

Example XXXVI.—What will be the drag on one wire of a dynamo armature which gives a current of 800 ampères, and has an average magnetic field of 3500 c.g.s. lines per sq. cm. of air gap? The length of the polar bore is 80 cms., and the machine is bipolar.

Solution.—Using the formula given on p. 49, we have pull is—

$$P = CL\beta_g \times 0\cdot000002244 \text{ pounds}$$

and as the current here is the current in each wire, it is consequently 400 ampères, and—

$$\begin{aligned} P &= 400 \times 80 \times 3500 \times 0\cdot000002244 \\ &= 25\cdot1328 \text{ pounds} \end{aligned}$$

CHAPTER V.

ARMATURE WINDING.

In considering the size of wire on a given armature to carry a given current, we have to allow a certain temperature rise, and consequently have to take into account all the sources of heating. These are mainly threefold, viz.—

(*a*) The $C_a^2R_a$ loss due to the effect of the current in the armature on the resistance of the armature;

(*b*) The H loss due to molecular friction in the iron core, brought about by the changing of the magnetic flux;

(*c*) The F loss due to Foucault or eddy currents in the core, and other moving parts of the armature.

The energy developed in the armature due to these effects has to be got rid of at some reasonable temperature, and it is customary to fix a limiting difference of temperature between the surface of the armature and the outside air.

In calculating this temperature difference it is of course necessary to bear in mind that some part of the heating is got rid of in other ways than by convection and radiation from the outside surface of the winding. Thus, part is lost by conduction through the spider to the spindle, the winding to the commutator. Experience shows that this part so conducted away may be about $\frac{1}{4}$ of the total loss.

Mr. Esson has shown that the surface of an armature revolving at a circumferential speed of 3000 feet per minute, can dissipate energy at the rate of 0·00444 watts per sq. cm. of its surface covered by winding for every 1° C. temperature difference between that surface and the air. The corresponding figure for a stationary surface, such as a field-magnet coil, is 0·00281 watts per sq. cm. per 1° C. temperature difference.

A combination of these figures will give the rate of loss of heat for any armature in terms of the circumferential speed. Thus the increase in rate of cooling due to the speed of movement through the air of 3000 feet per minute (1500 cms. per second) is—

$$0{\cdot}00444 - 0{\cdot}00281 = 0{\cdot}00163 \text{ watts}$$

per sq. cm. per 1° C., or—

$$\frac{0{\cdot}00163}{3000} = 0{\cdot}000000543 \text{ watts}$$

per sq. cm. extra per 1° C. difference of temperature per 1 foot per minute of speed through the air; or the loss for any speed may be put down as—

$$0{\cdot}00281 + 0{\cdot}000000543 f \text{ watts}$$

per sq. cm. per 1° C., where f is the circumferential speed in feet per minute.

For a speed of $f = 3000$ feet per minute, we have that the total difference in temperature between the air and the armature surface will be—

$$T = \frac{W}{A \times 0{\cdot}0044} \text{ degrees Centigrade}$$

where W = that part (say, $\frac{3}{4}$) of the whole expenditure of energy in the armature, which is considered to be radiated from its surface, expressed in watts;
and A = the surface area in sq. cms. of the armature.

Similarly, for a magnet coil we have—

$$T = \frac{W}{A \times 0{\cdot}00281}$$

where A is the outside surface of the magnet winding, not including the cheeks of the bobbin, expressed in sq. cms.

The loss W is approximately $\frac{3}{4}(C_a{}^2R_a + H + F)$, and of these items the first is, of course, quite easily found, allowance being made that, the temperature being high, we have to employ a high value for ρ.

The hysteresis loss H is due to the fact that molecular friction in the iron causes heating when the iron core as a mass is rotated by the rotation of the armature, whilst the molecules of iron are themselves stationary. The effect produced is the same as if the molecules were rotated whilst the armature core remained stationary. In a bipolar field the armature core has all the magnetized molecules thus rotated once per revolution, and the value of the energy so expended has been found by Professor Ewing for all conditions of magnetization. The table on p. 214 gives these values expressed in ergs per cc. per cycle of change, one cycle corresponding to one revolution of an armature in a bipolar field. Now, as one watt-second is 10^7 ergs, we have that the total ergs expended in the armature per revolution multiplied by the speed in revolutions per second and divided by 10^7 will give the rate of loss in hysteresis in watts. Calling the value of the ergs per cc. per cycle h, we have that the total ergs per revolution will be $h\mathrm{V}$, when V is the bulk of the armature iron in cc. If, then, n be the revolutions per second, we have the hysteresis loss is—

$$\mathrm{H} = \frac{h\mathrm{V}n}{10^7} \text{ watts}$$

The other loss, viz. that due to eddy currents, F, is almost impossible of calculation in an armature. Only a small part of it is in the laminated iron core, the major part being in other conducting masses, such as the winding, end plates, spider and spindle, etc. Due precautions being taken to minimize it as much as possible, we shall assume in these pages that it is equivalent to that part of the total loss which is got rid of in other ways than by radiation directly from the surface of the winding.

Therefore we may now put the value of W as—

$$\mathrm{W} = \mathrm{C}_a^2\mathrm{R}_a + \mathrm{H}$$

and write the temperature rise as being—

$$\mathrm{T} = \frac{\mathrm{C}_a^2\mathrm{R}_a + \mathrm{H}}{\mathrm{A} \times 0{\cdot}00444} \text{ degrees Centigrade}$$

for a bipolar armature revolving at 3000 feet per minute circumferential speed.

In finding the area of the surface of the armature, we must only take that part of the surface which is exposed to the air. In the case of a drum armature this is easy, but in a ring or gramme armature, some care is required in calculating the true size of that part of the surface which is inside the cylinder. In all cases the surface should be considered as smooth, no allowance being made for corrugations due to the curved surface of the wires; for the coefficients taken are stated higher to allow for such irregularity.

Example XXXVII.—What will be the hysteresis loss in an armature ring wound, 25 cms. long, 25 cms. diameter, with a 15-cm. hole (all dimensions being those of solid iron)? The speed is 1226 revolutions per minute, in a field of 4 million c.g.s. lines the value of the ergs per cc. per cycle is 10,750 per cc. for this particular iron.

Solution.—The sectional area of the armature is—

$$(25 - 15) \times 25 = 250 \text{ sq. cms.}$$

whence the density of lines is—

$$\beta = \frac{4 \times 10^6}{250} = 16{,}000 \text{ per sq. cm.}$$

corresponding to which the ergs per cc. per cycle are given as 10,750. The number of cycles per second will be the revolutions per second, or—

$$n = \frac{1226}{60} = 20{\cdot}43$$

And the number of cc. of iron in the core will be—

$$V = 5 \times 25 \times 20 \times \pi = 7855 \text{ cc.}$$

whence we have that—

$$H = \frac{7855 \times 10750 \times 20{\cdot}43}{10^7} = 172{\cdot}51$$

say, 173 watts.

Example XXXVIII.—If the resistance of the armature in the last example is 0·0212 ohm when hot, and it is used to light 200 50-watt 100-volt lamps at a terminal pressure of 102 volts, the resistance of the shunt coil being 51 ohms, what will be the rise in temperature due to hysteresis and $C_a^2 r_a$ losses, if the eddy current loss is got rid of by conduction, etc.? The surface of the armature is 3675 sq. cms., and the circumferential speed is to be taken as 3000 feet per minute.

Solution.—The current taken by the lamps is—

$$\frac{200 \times 50}{100} = 100 \text{ ampères}$$

and that taken by the shunt coil is 2 ampères, whence the total current in the armature is—

$$C_a = 102 \text{ ampères}$$

and thus—

$$C_a^2 r_a = (102)^2 - 0\cdot0212 = 220\cdot5, \text{ say } 221 \text{ watts}$$

and the loss due to H has been found to be 173 watts, whence the total watts—

$$W = 221 + 173 = 394 \text{ watts}$$

and thus the temperature rise will be—

$$T = \frac{394}{0\cdot00444 \times 3675} = 24\cdot1^\circ \text{ C.}$$

Example XXXIX.—Suppose the armature mentioned in Example XXXVII. were replaced by a drum having as before 25 cms. length of iron in it, but having only a 7-cm. hole, and run at a density of only 11,000 c.g.s. lines per sq. cm., corresponding to which the value of ergs per cc. per cycle is 5900—

(1) What will be the temperature rise with the same winding as to size of wire and the same current at the same E.M.F., and what the speed? and—

(2) What would be the temperature rise and output if the current and the speed are to be the same as when a gramme armature?

(3) Also, what would be the value of the current to make the temperature rise the same and the speed the same as in the case of the gramme?

The surface area of the drum may be taken as 4000 sq. cms., and the variations in the speed are not supposed to affect the rate of cooling.

Solution.—(1) As a drum there will be a sectional area of the armature of—

$$(25 - 7)25 = 450 \text{ sq. cms.}$$

whence at $\beta = 11{,}000$ the total number of lines will be—

$$N = \beta a = 11{,}000 \times 450 = 4{,}950{,}000$$

The total generated E.M.F. was before equal to 102 volts for the terminal P.D. plus the loss in the armature of—

$$102 \times 0{\cdot}0212 = 2{\cdot}16 \text{ volts, or } 104{\cdot}16 \text{ total}$$

to produce which E.M.F. the drum will have to run at a speed inversely as the number of lines, or speed will be—

$$n = 20{\cdot}43 \times \frac{400}{495} = 16{\cdot}5 \text{ revolutions per second}$$

the volume of the drum core will be—

$$\frac{d_1^2\pi}{4}l - \frac{d_2^2\pi}{4}l$$

or—

$$V = \frac{25 \times 25 \times 22 \times 25}{4 \times 7} - \frac{7 \times 7 \times 22 \times 25}{4 \times 7} = 11{,}314 \text{ cc.}$$

Thus the hysteresis loss is—

$$H = \frac{5900 \times 11{,}314 \times 16{\cdot}5}{10^7} = 110 \text{ watts}$$

And since in question (1) the current is to be the same as before, we have that the $C_a^2r_a$ loss is 221 watts as before, whence the rise in temperature will be—

$$T = \frac{331}{4000 \times 0{\cdot}00444} = 18{\cdot}6^\circ \text{ C.}$$

(2) If the speed is to be the same as before, there will be a greater E.M.F. total in the ratio of—

$$\frac{495}{400}, \text{ or } \frac{495}{400} \times 104{\cdot}16 = 128{\cdot}9 \text{ volts}$$

whence with the same current the output will be—

$$100 \times 126{\cdot}74 = 12{,}674 \text{ watts}$$

as against only 10,200 for the ring winding.

But at this speed of 20·43 revolutions per second the hysteresis loss will be—

$$H = \frac{5900 \times 11{,}314 \times 20{\cdot}43}{10^7} = 136 \text{ watts}$$

or the total watts expended will be—

$$136 + 221 = 357 \text{ watts}$$

and the consequent temperature rise will be—

$$T = \frac{357}{4000 \times 0{\cdot}00444} = 20^\circ \text{ C.}$$

(3) If, however, the temperature rise is to be the same as for the ring armature at the same speed, the current may be larger, such that the new $C_a^2 r_a$ loss added to the H loss will make the heating such as to cause a temperature rise of 24·1° C. Thus the total watts will be—

$$\begin{aligned} W &= T \times A \times 0{\cdot}00444 \\ &= 24{\cdot}1 \times 4000 \times 0{\cdot}00444 \\ &= 428 \text{ watts} \end{aligned}$$

of which 136 watts are for hysteresis, leaving 292 watts for $C_a^2 r_a$. If the armature resistance be taken as 0·0212 ohm as before, then—

$$C_a^2 = \frac{292}{0{\cdot}0212}$$

$$\text{and } C_a = \sqrt{\frac{292}{0{\cdot}0212}} = 117 \text{ ampères}$$

Example XL.—An armature working at a density of 8000 lines per sq. cm. and giving a certain E.M.F. is run at half the speed with a density of 16,000 per sq. cm.: what will be the ratio of the watts lost in hysteresis in the two cases?

Solution.—At the higher speed and $\beta = 8000$, h is 2860 (p. 215) ergs per cc. per cycle; and at the lower speed and β 16,000, h is 10,400 ergs. But as the cycles per second are only half as many in the second case as in the first, the two losses will be 2860 in the first case to $\frac{10400}{2} = 5200$ in the second case, or as—

$$\frac{2860}{5200} = 0·5499, \text{ say } 0·55$$

or the loss in H in the high speed low density machine is only 0·55 as great as in the slow speed high density.

Multipolar Machines.—When the dynamo is not bipolar, it may be so wound that either the current or E.M.F. is proportionally increased. Thus, with what is called the parallel winding, the coils are connected up as before to a commutator, which then has as many pairs of brushes as there are pairs of poles, unless the commutator is cross-connected, when all the positive brushes can be replaced by one brush, and similarly all the negative brushes by another brush. Using the same symbols as before, and N still standing for the number of lines entering the armature from or leaving the armature towards any one pole piece, the E.M.F. will be—

$$E = \frac{Nnw}{10^8} \text{ as before}$$

but the armature is now in $2p$ parallels instead of 2 parallels, where p is the number of pairs of poles, and can consequently carry p times as much current if the wire be the same size, or—

$$C_1 = pC$$

where C_1 is the current from the machine with p pairs of poles, and C the current from a bipolar machine with the same value for N, etc.

On the other hand, with the series winding, which is to be preferred to the parallel for various reasons, the E.M.F. is increased in proportion to the number of pairs of poles, whilst the current-carrying capacity of the armature is the same as before, being only in 2 parallels whatever the number of poles. Thus the E.M.F. will be—

$$E = \frac{Nnwp}{10^8} \text{ volts}$$

the current being simply the same as in a bipolar machine having the same size of wire.

The effect on the hysteresis loss in the core is the same in both cases and may be found by taking the product of the volume of the armature into h, n, and p and dividing by 10^7—

$$\text{or } H = \frac{hVnp}{10^7} \text{ watts}$$

for both series or parallel windings.

Virtually the effect of the multiplicity of poles is the same as having p equal dynamos connected on the one hand in parallel and on the other in series.

Example XLI.—A six-pole dynamo has an armature core composed of 600 discs of iron 0·25 mm. thick, and having a radial depth of 7 cms. It is wound with wire which will carry 50 ampères, and has 540 wires all round. If the density of magnetic flux through the iron is 15,000 per sq. cm., and the speed is 6 revolutions per second, what will be the E.M.F. and current if connected up in the parallel method?

Solution.—The total value of lines is—

$$\begin{aligned} N &= 2 \times 600 \times 0{\cdot}25 \times 0{\cdot}1 \times 7 \times 15{,}000 \\ &= 3{,}150{,}000 \text{ lines} \end{aligned}$$

and the whole E.M.F. will be—

$$E = \frac{3{,}150{,}000 \times 6 \times 540}{10^8} = 102 \text{ volts}$$

As each wire will carry 50 ampères, the current, as an ordinary bipolar machine, would be 100 ampères; but there are now $2p = 6$ parallels in the armature, and thus the current is—

$$C = 6 \times 50 = 300 \text{ ampères}$$

Example XLII.—What will be the hysteresis loss in the above armature if the outside diameter of the core is 70 cms.?

Solution.—

$$H = \frac{hVnp}{10^7}$$

and *h* from the table on p. 216, is 9300. The volume of the core is—

$$V = 600 \times 0{\cdot}025 \times 7 \times \frac{22}{7} \times 63 = 20{,}790 \text{ cc.}$$

and thus—

$$H = \frac{9300 \times 20{,}790 \times 6 \times 3}{10^7} = 348 \text{ watts}$$

CHAPTER VI.

THE MAGNETIC CIRCUIT.

THIS can best be illustrated and explained by a comparison with the electric circuit. The quantities corresponding to conductivity and resistivity, E.M.F., and current, are called, in the magnetic circuit, permeability and reluctivity, magnetomotive force (M.M.F.), and magnetic flux respectively. Permeability has this difference from conductivity, that it has no fixed value in the so-called magnetic materials (like the corresponding value of electric conductivity, which has a fixed value for a given temperature quite irrespective of the magnitude of the current or electric flux), but varies with the variation of the magnetic flux even when the temperature is kept constant.

In the non-magnetic materials, however, the value of the permeability is independent of the value of the magnetic flux, even when this is confined to the same path, and so varies in density. The use of the idea of current-density is not so usual as to have caused the adoption of a symbol for its indication; but for purposes of comparison we shall make use of D, as standing for the density in ampères per sq. cm. The corresponding symbol very frequently used in magnetic measure is β, and stands for the number of c.g.s. lines of magnetic flux per sq. cm. of cross-section.

It is very usual to confine β to the magnetic density in iron or the other magnetic metals, but for sake of uniformity we shall not so restrict its use, but let it stand for the density of magnetic lines in any material. It has been shown before that we may write the relationship between the three quantities e, C, and R in the electric circuit in a simple way, viz.—

$$C = \frac{e}{R}, \text{ and } R = \frac{e}{C}$$

also—

$$R = \frac{l\rho}{a} = \frac{l}{am}$$

Adding now to our symbols the above-mentioned D, the ampères per sq. cm., we shall have that—

$$C = Da$$

and—

$$e = CR = DaR = Dl\rho = \frac{Dl}{m}$$

or, putting again C for the total electric flux—

$$e = \frac{Cl\rho}{a} = \frac{Cl}{am}$$

The corresponding quantities in the magnetic circuit we will call—

$\mathfrak{M}$ = magneto-motive force (M.M.F.) measured in terms of a unit, to be afterwards defined.

$N = \beta a$ = the whole magnetic flux, generally represented in the form βa, on account of the importance of knowing the value of the density.

The quantity corresponding to resistance is the magnetic reluctance, and may be represented by $\mathfrak{R}$, whose magnitude will have to be stated in terms of the permeability, since there is no unit in use called unit reluctivity. Thus—

$$\mathfrak{R} = \frac{l}{a\mu}$$

where μ stands for permeability in the same way as we have used m to stand for conductivity, that is the actual conductance of the unit cube. We thus call μ the actual permeance of the unit cube, measured in terms of its value for some particular material, viz. air, in which μ is always regarded as unity.

Consider a solenoid of infinite length having a current of C ampères passing through it. Let the turns or convolutions of wire on it be w per cm. length. It is found that there will be produced inside the solenoid a magnetic flux having a density of $\frac{4\pi}{10}Cw$ lines per sq. cm. of cross-section of core, provided the core consist of air, or some other so-called non-magnetic material. This $\frac{4\pi}{10}Cw$ is commonly called H, and, owing to the permeability of air, etc., being unity, it follows that—

$$H = \frac{4\pi}{10}Cw = \text{the magnetizing force per unit length}$$

and it is in this sense that H is generally used. The magnetic materials are chiefly iron, steel, nickel, and cobalt, all others being practically non-magnetic, or having a permeability of unity; some few, such as bismuth, have a permeability slightly less than unity, but only to a quite insignificant extent. The so-called magnetic materials have a permeability much greater than unity, not constant, however, but depending upon the density of magnetization; that is, upon the magnitude of the magnetic flux through a given cross-section. We must also class a vacuum as non-magnetic, and having a value of $\mu = 1$. Thus, if the core of our infinite solenoid be replaced by a bar of iron, we have in the first place expelled all the air, but the ether may be regarded as being still present, and we now have the effect of two cores in parallel, the original core with $\mu = 1$, and a new core of iron with μ much greater than unity. The magnetizing force is still—

$$\frac{4\pi}{10}Cw \text{ per cm. length}$$

and is undisturbed, and consequently we get a magnetic flux of density H through the original ether core, and a magnetic flux through the new core of magnitude—

$$I = \frac{4\pi}{10}Cw \times \mu_1$$

when μ_1 is the value of the permeability of the iron core.

Adding these two magnetic fluxes together, we have—

$$\beta = \mathrm{I} + \mathrm{H}$$

the density of magnetic flux in the composite core.

From a purely physical point of view it is important to distinguish between I and H; but as we can never actually separate them in practice, we invariably speak of β, their sum, and it is much more convenient to so consider the matter.

Though the idea of current density is sometimes used, we are not so familiar with it as we are with actual current total; but a little consideration will show that the magnitude of the E.M.F. required to produce a given current density D in a conductor depends simply upon the material and length of the conductor, or, more simply, for a given material of given length at a given temperature the E.M.F. required is proportional to the value of D, irrespective of the sectional area of conductor. Fortunately for the ease of calculation in the electric circuit, there is a very simple connection between the E.M.F. required and the value of D, viz. one of direct proportionality subject to the slight effect of temperature in altering the resistance of the conductor.

Thus the E.M.F. required for a given value of D was stated above as—

$$e = \frac{\mathrm{D}l}{m} \text{ or } = \frac{\mathrm{C}l}{am}$$

Similarly, calling the value of total magnetic flux $\mathrm{N} = \beta a$, we have that the M.M.F. required is—

$$\mathrm{H} = \frac{\mathrm{N}l}{a\mu} \text{ or } = \frac{\beta l}{\mu} \text{ in physical measure}$$

In the electric circuit the proportionality between e and D renders calculation easy without the use of tables; but in the magnetic circuit the want of direct proportionality between H and β renders the use of curves or tables absolutely necessary for exact work. The exact value of this relationship has been investigated by various observers, but we shall confine ourselves to the measurements made by Professor Ewing and the Drs. Hopkinson. A combination of their results for good qualities of iron is shown in the tables given, which have been read off from carefully drawn curves, having a general appearance as shown in Fig. 14, which gives the relationship between--

$$\frac{4\pi}{10}cw = \text{H and } \beta$$

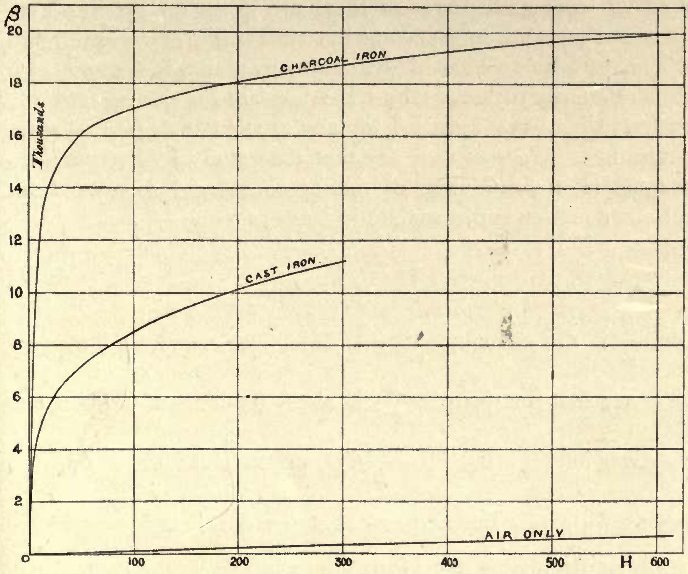

FIG. 14.

Ordinates stand for β, the c.g.s. lines per sq. cm. in the core of iron, and abscissæ for the magnetizing force $\frac{4\pi}{10}Cw = H$, required for every centimetre length of the iron. It will be noticed that when only feebly magnetized the iron has a much greater ratio of $\beta : H$ than when magnetized to a considerable extent, though at very low values of β this is not so noticeable. The three curves stand for this relationship for wrought iron, cast iron, and for air only. It has already been pointed out that the iron adds to the value in air an amount called I, which is the difference between the ordinate of the air curve and that of the iron curve. This addition is varying in value, gradually increasing with the value of β, till, as the curve indicates, it is likely that at a very high value of H, this addition will have reached a maximum, and will then be simply a constant addition to the value for air alone. Tables carefully read off from such curves as these are given on pp. 214–218. It is not possible to represent the relationship between β and H for the whole extent of the curves by a simple formula, and thus reference to these tables is necessary for finding out the magnetizing force required for any particular degree of magnetization. We can thus say that the value of H required to magnetize a given piece of matter considered separate from all other matter and complete in itself as being—

$$H = \frac{lN}{a\mu} = \frac{l\beta}{\mu}$$

But $\frac{\beta}{\mu}$ is the same quantity as H in our curve, and thus we may say that the quantity $\frac{N}{a\mu}$ is some function of β expressed in terms of H. But $H = \frac{4\pi}{10}Cw$, which is the same thing as saying $\frac{4\pi}{10}$ times the ampère-turns per cm. length, and as we really apply magnetizing forces by winding with wire to carry a current, it is most convenient to do away with the numeric $\frac{4\pi}{10}$ by making allowance for it in the tables, thus—

$$H = \frac{4\pi}{10}Cw, \text{ or } Cw = H\frac{10}{4\pi}$$

$$Cw = H \times 0{\cdot}795$$

or—

$$\text{ampère-turns per cm. length} = H \times 0{\cdot}8 = f(\beta)$$

nearly enough, since our knowledge of the exact values of β and H is not so exact as to require a distinction between 0·795 and 0·8. The convenience of this was first pointed out by Mr. Esson, and has since been extensively made use of by engineers. Thus, in the tables, we have put in values of β with the corresponding values of H and $f(\beta)$, this last being the mode of writing *function of* β. The permeability μ is also shown, being readily found from—

$$\mu = \frac{\beta}{H}$$

These tables are for the magnetic circuit what tables, showing the relationship between D (ampères per sq. cm.) and e the P.D. required in volts per cm. length of conductor, would be for the electric circuit.

Using the symbols—

H = the whole magnetizing force required by a magnetic circuit, in physical measure called M.M.F.

we have—

The ampère-turns total = $0{\cdot}8H = \mathfrak{M}$

and the ampère-turns required by the part $\Big\}$ = $0{\cdot}8H = \mathfrak{m}$ = magnetic P.D.

Thus, remembering that it is just as necessary to have a complete circuit for the circulation of the magnetic flux as it is in the case of the electric flux, we have—

$$H = \frac{Nl}{a\mu} = \frac{l\beta}{\mu} \text{ and } \mathfrak{M} = 0{\cdot}8\,\frac{l\beta}{\mu}$$

We must either have the whole circuit of one material of uniform quality and sectional area and of length l, in which case $\mathfrak{M}$ is the whole magnetizing force required; or if l be the length of a part only of the circuit having a definite property, we must then regard—

$$0{\cdot}8\,\frac{Nl}{a\mu} = \mathfrak{m}$$

the magnetic P.D. ampère-turns required by that part alone; whilst then the whole M.M.F. will be the sum of all such items as $\mathfrak{m}$.

Thus in a composite magnetic circuit—

$$\mathfrak{M} = \sum 0{\cdot}8 \frac{\mathrm{N}l}{a\mu}$$

$$0{\cdot}8 \left\{ \frac{l_1\mathrm{N}}{a_1\mu_1} + \frac{l_2\mathrm{N}}{a_2\mu_2} + \frac{l_3\mathrm{N}}{a_3\mu_3} \right\} \text{ etc.}$$

when the same total flux N circulates round the whole circuit.

The following comparison of the Electric and Magnetic Circuits will serve to sum up the previous considerations :—

ELECTRIC CIRCUIT.	MAGNETIC CIRCUIT.
C = Current or whole electric flux in ampères.	N = Magnetic flux in c.g.s. lines total.
$\mathrm{D} = \frac{\mathrm{C}}{a}$ = ampères per sq. cm. of section, or current density.	$\beta = \frac{\mathrm{N}}{a}$ = magnetic density or c.g.s. lines per sq. cm.
m = conductivity in mhos of the material of the circuit ; it is the actual conductance in mhos of the unit cube. It is constant for a constant temperature irrespective of D.	μ = permeability of the material of the circuit, and is the actual permeance of the unit cube. It is constant only for air and the non-magnetic materials having a value of unity (1). It is variable for the magnetic materials, and dependent upon the value of β.
$\rho = \frac{1}{m}$ = the resistivity of the material or actual resistance in ohms of the unit cube. It is constant subject to the constancy of m.	$r = \frac{1}{\mu}$ = the reluctivity of the material or actual reluctance of the unit cube being constant, and of value unity (1) for air and the non-magnetic materials; but varies with μ in the magnetic materials, according to the value of β.
$\mathrm{R} = \frac{l\rho}{a} = \frac{l}{am}$ = resistance in ohms of a circuit or part of a circuit of length l and area a made of material having resistivity ρ.	$\mathfrak{R} = \frac{lr}{a} = \frac{l}{a\mu}$ = reluctance of a circuit or part of a circuit of length l and area a made of material having a permeability μ.

ELECTRIC CIRCUIT.	MAGNETIC CIRCUIT.
$e = \frac{Cl}{am} = \frac{Dl}{m} = Dl\rho =$ the D.P. in volts required to send a current of C ampères through a circuit or part of a circuit.	$\omega =$ The magnetic P.D. required to send a magnetic flux through a circuit or part of a circuit. It is equal to $0{\cdot}8N\mathfrak{H} = \frac{Nl}{a\mu}0{\cdot}8$.
e can readily be calculated from any of the above relations.	ω can only be calculated for the non-magnetic materials where $\mu = 1$. For the magnetic materials reference has to be made to the tables for the value of μ, etc. By reference to the $f(\beta)$ column of the tables, the value of ω in ampère-turns per cm. length can readily be found, and calculation thereby much simplified.
E = the whole E.M.F. in the circuit, being the algebraic sum of all the P.D.s round the circuit, or the sum of all such quantities as— $D_1l_1\rho_1 + D_2l_2\rho_2 + D_3l_3\rho_3$ Though these P.D.s may be required distributed round the circuit E may be generated at one part, viz. the armature of a dynamo, for example.	$\mathfrak{M} =$ the whole M.M.F., being the algebraic sum of all the magnetic P.D.'s round the circuit, or being equal to— $0{\cdot}8 \times \left\{\frac{\beta_1 l_1}{\mu_1} + \frac{\beta_2 l_2}{\mu_2} + \frac{\beta_3 l_3}{\mu_3}\right\}$ Though these items may be required distributed round the whole circuit, the whole of $\mathfrak{M}$ may be produced at one part, viz. the magnet coils of a dynamo, for example.

In the electric circuit it is quite easy to confine the electric flux to a very definite path, because some materials (air, for example) are practically of infinite resistivity. In the magnetic circuit, however, air and the other so-called non-magnetic materials have a permeability which is, at the very best, only $\frac{1}{3000}$ as great as that of iron, the very best magnetic conductor we have. There is thus no possibility of magnetically insulating any circuit to anything like the perfect extent attainable in the electric circuit; and we have consequently to allow for some leakage of lines. The permeability of iron varies from about 3000 to 30 or so at such values of β as are convenient for general work, and thus the leakage of lines may often be as great even as if we had no better insulator than manganin, or such like, for the protection of the copper conducting path in the electric circuit. This leakage of lines is particularly the case in dynamos and such apparatus where the magnetic circuit is composite in character, and when the magnetic P.D. between point and point may be very great. Thus, if the permeance of the armature and air gaps in series be represented by u, and that of the surrounding air space be called w, these two permeances are in parallel, and are subjected to nearly the same magnetic P.D. Consequently, if N be the total magnetic lines through the armature core and air gaps, the magnetic P.D. between the polar faces will be $\frac{\mathrm{N}}{u}$, and this magnetic P.D. will produce a flux of $\frac{\mathrm{N}w}{u}$ lines through the surrounding air space. Now, these two fluxes have to flow in the magnet limbs, which will consequently carry, not N lines, but—

$$\mathrm{N} + \mathrm{N}\frac{w}{u} = \mathrm{N}_m$$

In their paper on dynamos in 1886 the Drs. Hopkinson chose to represent this fact in the very simple way of stating the lines N_m in the magnet limbs as being ν times the lines in the armature core; the value of ν depending upon the relationship of w to u. Thus, when $w = u$, $\nu = 2$, a value met with in such multipolar machines as the old pattern of Ferranti alternator. The value of w is, however, generally much less than u in a well-proportioned dynamo, some measured values of ν being given in the following table :—

Type of machine.	Value of ν.
Edison, Hopkinson, single magnet, drum	1·32
Siemens ,, ,, ,, ...	1·30
Phœnix over type ,, ,, gramme	1·32
Phœnix double magnet ,, ...	1·40
Manchester ,, ,, ,, ...	1·49
Victoria Mordey, 4 pole, ring or gramme	1·40
Ferranti, 16 pole, disc	2·00
Manchester with polar supports	1·5 to 1·60

Professor Forbes, many years ago, showed how a sufficiently exact approximation to the value of ν for practical purposes could be calculated from the drawing of the machine.

Thus, the same magnetic P.D. which is required between the two polar faces to produce N lines in the armature and air gaps, is also producing a magnetic flux through the very imperfectly magnetically insulating air which surrounds the dynamo, and which additional magnetic flux has to be accommodated in the magnet limbs, thereby causing the value of β in them to be greater, and making the value of the magnetizing force for that part of the circuit greater than would otherwise be the case.

We have, then, to consider the magnetic circuit of the dynamo as composed of a number of items, each of which may be different, either on account of material or the fact that a greater flux of lines has to be accommodated. This composite magnetic circuit consists of—

(1) The armature, which is almost invariably made of a good quality of wrought iron. The sectional area is easily found, as has been indicated previously. The average length of the path of magnetic lines can be found with sufficient exactness by taking the mean semi-circumference in the case of a bipolar machine, whether drum or gramme; or for a multipolar machine it is—

$$\frac{\text{mean circumference}}{2p}$$

where p is the number of pairs of poles.

(2) The iron to iron space, called the air gaps, though mostly occupied by the winding, etc., on the armature. The sectional area of this part is equivalent to the area of the curved polar surface, increased by a fringe of $0{\cdot}8 \times$ the iron to iron space. This increase is to take into consideration the spreading of the lines round the edges, and was first pointed out by the Drs. Hopkinson.

The length of this space is twice the iron to iron space.

(3) The magnet limbs, which may be wrought iron, cast iron, or cast steel. Their sectional area is easily found from the

drawing, and may vary in different parts, as, for instance, when the pole-pieces are of cast iron. The length is the same as that of a line drawn slightly nearer the centre of the machine than the centre of the limb, and bounded by the centre of the polar face on the one hand, and by the yoke on the other, taken twice over.

(4) The yoke, which is generally counted separately, either on account of difference of material or difference in sectional area. The area of section is easily found, whilst the length is that of a curved line joining the extremities of the lines representing the path through the magnet limbs.

Considerable experience and discretion is required to accurately fix these lengths.

Using the following symbols to represent these various measurements, viz.—

l_a, A_a, and μ_a for the armature, through which a total flux of N c.g.s. lines is passing, and in which, consequently, there is a magnetic density of $\beta_a = \frac{N}{A_a}$

l_g, A_g for the iron to iron space or air gaps. In these the value of μ is 1, and consequently need not appear; and the magnetic flux is also N, as through the armature. The density of lines in the gap is thus $\beta_g = \frac{N}{A_g}$

l_m, A_m, μ_m for the magnet limbs, through which a greater number of lines than N, viz. νN, will flow, and consequently the magnetic density is $\beta_m = \frac{N\nu}{A_m}$

l_y, A_y, and μ_y for the yoke, in which again the total flux is $N\nu$, and the density $\beta_y = \frac{N\nu}{A_y}$

Thus the total M.M.F. in physical measure will be—

$$\mathfrak{M} = \frac{Nl_a}{A_a\mu_a} + \frac{Nl_g}{A_g} + \frac{N\nu l_m}{A_m\mu_m} + \frac{N\nu l_y}{A_y\mu_y}$$

And we have shown above that the ampère-turns can be found by multiplying the physical measure by 0·8, whence—

$$\left.\begin{array}{c}\text{Ampère-turns}\\ \text{total}\end{array}\right\} = 0{\cdot}8\frac{Nl_a}{A_a\mu_a} + 0{\cdot}8\frac{Nl_g}{A_g} + 0{\cdot}8\frac{N\nu l_m}{A_m\mu_m} + 0{\cdot}8\frac{N\nu l_y}{A_y\mu_y}$$

Or again, to save trouble in calculation, we can call the $0{\cdot}8\frac{N}{A\mu}$ the value of $f(\beta)$ in ampère-turns, and put—

$$\text{Ampère-turns total} = l_af(\beta_a) + 0{\cdot}8l_g\beta_g + l_mf(\beta_m) + l_yf(\beta_y)$$

the 0·8 as a multiplier being retained in the case of the air gaps, since for air we have not made out a table or curve for reference, and consequently the 0·8 is not included there as it is in the $f(\beta)$ for iron.

Example XLIII.—How many ampère-turns will be required to produce a magnetic flux of 300,000 lines in a wrought-iron ring of 20 sq. cms. cross-sectional area, made of round bar iron, and being 50 cms. outside diameter?

Solution.—The value of—

$$\beta = \frac{300{,}000}{20} = 15{,}000$$

whence $f(\beta) = 22{\cdot}88$ ampère-turns per cm. length.

The length of the magnetic circuit is, nearly enough, the mean circumference of the ring. From its sectional area of 20 sq. cms. the diameter of the bar is 5·046 cms., whence, nearly enough, the mean diameter is $50 - 5{\cdot}046 = 45$ cms., say, and thus the circumference is—

$$45 \times \pi = 141{\cdot}35 \text{ cms.}$$

whence—

$$lf(\beta) = 141{\cdot}35 \times 22{\cdot}88 = 3238 \text{ ampère-turns}$$

Example XLIV.—Neglecting the air path or leakage lines, what will be the magnetizing force required to produce 3 million lines through a mass of cast iron 1 metre long and 517 sq. cms. sectional area?

Solution.—The value of $\beta = \frac{3 \times 10^6}{517} = 5802$, and the magnetizing force in physical measure—

$$H = \frac{Nl}{a\mu} = \frac{\beta l}{\mu}$$

corresponding to which value of β, $\mu = 290$, whence—

$$H = \frac{5800 \times 100}{230} = 2000$$

which is in physical measure, and will be converted into ampère-turns by multiplying by $\frac{10}{4\pi} = 0{\cdot}8$, or—

$$\text{Ampère-turns} = 0{\cdot}8 \times 2000 = 1600$$

Example XLV.—What will be the addition to the ampère-turns required for the iron ring in Example XLIII. if a saw slot, 1 mm. thick, be made through the iron bar in a plane containing the axis of the ring? Neglect leakage lines.

Solution.—There will be slightly less ampère-turns required for the iron, in as far as the iron is less in length by 1 mm. or 0·1 cm. As the magnetizing force is 22·88 per cm., we shall now require 2·288 less magnetizing force as far as the iron is concerned, viz. 2·288 ampère-turns less. But for the air gap, where the whole 300,000 may be taken as uniformly distributed over an area of 20 sq. cms., plus a fringe of 0·08 cm. all round, that is over an area equal to that of a circle 5·046 + 0·16 = 5·206, say 5·2 cms. diameter, which has an area of 21·23 sq. cms.

Thus the density is—

$$\beta_g = \frac{300{,}000}{21{\cdot}23} = 14{,}127 \text{ per sq. cm.}$$

and the ampère-turns required per cm. length will be $0{\cdot}8 \times 14{,}127 = 11{,}301$. But as the gap is only 0·1 cm. long the additional ampère-turns will be only 1130·1 ampère-turns for air space.

Thus the difference will be an addition of—

$$1130{\cdot}1 - 2{\cdot}28 = 1127{\cdot}82 \text{ ampère-turns}^{*}$$

---

[*] Strictly speaking, owing to leakage lines caused by the increased magnetic P.D. between the ends of the gap and points near to the gap, there will be leakage of lines outside the area of gap taken, and these lines will have to be accommodated in the iron part of the ring, thus making the ampère-turns for the iron greater than before.

Example XLVI.—What will be the ampère-turns required to magnetize a dynamo which has a ring armature 25 cms. diameter, with a 15-cm. hole, and built up of 1000 discs of iron 0·25 mm. thick, if the total magnetic flux through it is 4 million lines? The diameter of the bore is 27 cms., its length is 27 cms., and the angle of embrace is 135 degrees. The length of the two magnet legs together is 124 cms., with a sectional area of 415 sq. cms., that of the yoke being 54 cms. and 729 sq. cms. respectively. Both magnet legs and yoke are wrought iron, and the leakage coefficient $\nu = 1{\cdot}4$.

Solution.—Using symbols previously mentioned, we have—

$$A_a = (25 - 15)(1000 \times 0{\cdot}25 \times 0{\cdot}1) = 250 \text{ sq. cms.}$$

and also—

$$l_a = \frac{\pi}{2} \times \frac{25 + 15}{2} = 10\pi = 31{\cdot}5 \text{ cms. say}$$

thus—

$$\beta_a \frac{4 \times 10^6}{250} = 16{,}000$$

whence the ampère-turns required for the armature will be—

$$\begin{aligned} \varpi_a &= l_a f(\beta) = l_a f(16{,}000) \\ &= 31{\cdot}5 \times 39{\cdot}84 = 1254{\cdot}9 \end{aligned}$$

say 1255 ampère-turns for armature.

For the air gap we have—

$$l_g = 27 - 25 = 2 \text{ cms.}$$

and as the angle of embrace is 135°, the curved length of the pole-piece is—

$$\frac{135}{360} \times \pi \times 27 = 31{\cdot}8 \text{ cms.}$$

The length parallel with the shaft is 27 cms., and thus the area of the polar surface is—

$$31{\cdot}8 \times 27 = 858{\cdot}6 \text{ sq. cms.}$$

to which must be added the fringe all round of 0·8 times the iron to iron space, or in this case of 0·8 cm. all round, or—

$$0{\cdot}8(2 \times 27 + 2 \times 31{\cdot}8) = 94{\cdot}08 \text{ sq. cms.}$$

making—

$$A_g = 858{\cdot}6 + 94 = 952{\cdot}7, \text{ say, } 953 \text{ sq. cms.}$$

and thus—

$$\beta_g = \frac{4 \times 10^6}{953} = 4200 \text{ say}$$

whence the ampère-turns for gap $= 0{\cdot}8 l_g \beta_g$—

$$\varpi_g = 0{\cdot}8 \times 2 \times 4200 = 6720$$

Again, for the magnet legs we have the total number of lines—

$$N\nu = 1{\cdot}4 \times 4 \times 10^6 = 5{\cdot}6 \times 10^6$$

and as $A_m = 415$, we have—

$$\beta_m = \frac{5{\cdot}6 \times 10^6}{415} \doteq 13{,}494$$

corresponding to which $f(\beta) = 10{\cdot}9$ nearly enough, and the ampère-turns required will be—

$$\varpi_m = l_m f(\beta) = 124 \times 10{\cdot}9 = 1352 \text{ ampère-turns say}$$

Lastly, for the yoke there will be the same total flux, $5{\cdot}6 \times 10^6$, whence—

$$\beta_y = \frac{5{\cdot}6 \times 10^6}{729} = 7681$$

for which—

$$f(\beta) = 2{\cdot}12$$

and therefore $\varpi_y = 54 \times 2{\cdot}12 = 115$ ampère-turns say. Thus the whole magnetizing force will be in ampère-turns—

Armature	1255
Air gaps	6720
Magnet legs	1352
Yoke	115
	9442 ampère-turns total

Example XLVII.—The length of magnetic circuit of a transformer iron core is 50 cms. Its sectional area is 300 sq. cms., and the maximum value of the current allowed for magnetization is 0·15 ampère. How many turns of wire must the primary coil have if the total magnetic flux is 1,200,000 lines?

Solution.—

$$\text{Density, } \beta = \frac{12 \times 10^5}{300} = 4000$$

for which $f(\beta)$ is 1·24, and therefore the ampère-turns required—

$$lf(\beta) = 50 \times 1{\cdot}24 = 62$$

whence, the current being 0·15, the total turns must be—

$$w = \frac{62}{0{\cdot}15} = 413 \text{ say}$$

CHAPTER VII.

MAGNET COIL WINDINGS.

WE have seen in the previous chapter how to ascertain the magnetizing force required in ampère-turns, and will now proceed to show how the size of wire and number of turns or convolutions may be found. There are two main cases—

(1) That in which the current to be used is a fixture.

(2) That in which the P.D. available is fixed.

The first is simple, and can be easily disposed of. Let $\mathfrak{M}$ = the ampère-turns to be provided, and C the fixed current, then the number of turns is, of course—

$$w = \frac{\mathfrak{M}}{C}$$

The size of wire is then fixed, partly by the magnitude of the current to be carried, and partly by the fall of potential allowable. Generally this last is so limited that the expenditure of energy is so small as to make the heating negligible; but in any case the dimensions of the bobbin, or former, can be made such as to satisfy Mr. Esson's rule, given on p. 56.

The coils which mostly come under this, the "series," class, are those of ammeters, cut-outs, series-wound dynamos, series coils on compound dynamos, and independent coils generally.

In case (2), which may be called the "shunt" class, we have different conditions to take. A certain E.M.F., say E volts, is to set up such a current that this current, multiplied by the turns of wire, shall be equal to $\mathfrak{M}$, the ampère-turns required; or—

$$Cw = \mathfrak{M}$$

Since the whole resistance R will be—

$$R = wr$$

where r is the resistance of the mean turn, we have—

$$C = \frac{E}{wr}$$

and—

$$\mathfrak{M} = \frac{Ew}{wr} = \frac{E}{r}$$

or—

$$r = \frac{E}{\mathfrak{M}}$$

whence a simple rule :—

Shunt Coil.—*The resistance of the mean turn is equal to the P.D. at the terminals divided by the ampère-turns required; or is the ratio of the E.M.F. to M.M.F.*

It is easy in most cases to fix or find the length of the mean turn, say l, whence the sectional area of the wire will be—

$$a = \frac{l\rho}{r}$$

when ρ is the resistivity at the particular temperature to which the coil is to be raised. The diameter of the wire, if circular in section, is then—

$$d = \sqrt{\left(\frac{4a}{\pi}\right)} = \sqrt{\left(\frac{4l\rho}{r\pi}\right)}$$

Or, putting in the previously found value for the resistance of the mean turn, we have—

$$d = \sqrt{\left(\frac{4l\rho}{\pi\frac{E}{\mathfrak{M}}}\right)} = \sqrt{\left(\frac{4l\rho\mathfrak{M}}{\pi E}\right)}$$

when, of course, l and ρ must be in cms. or inches, according as d is wanted in cms. or inches. Also ρ should have the value corresponding to the temperature to which the coil is allowed to rise.

To find the number of turns, w, we must again have recourse to the formula of Mr. Esson, and make the total expenditure of energy—

$$\frac{E}{R} \text{ or } \frac{E}{wr}$$

such as to make no greater rise of temperature than may have been previously agreed upon.

A little investigation will at once show that the number of turns can be varied considerably without altering the value of $\mathfrak{M}$ when once the right size of wire has been found, provided the length of the mean turn be not thereby altered.

Thus with only one turn the energy waste will be—

$$\frac{E^2}{r}$$

and may be hopelessly large; but with a suitable number of turns, w, this waste will be reduced to—

$$\frac{E^2}{wr}$$

or to $\frac{1}{w}$ times its first value. Since, however, the ampère-turns are—

$$Cw \text{ and } C = \frac{E}{wr}$$

we have in both cases that—

$$\mathfrak{M} = \frac{E}{rw} \times w = \frac{E}{r}$$

whatever value w may have.

The number of turns it is possible to get on a coil is found when the size of the coil is known, and the depth to which it is possible to wind it. Thus let d_1 stand for the diameter of the covered wire (easily found for double cotton covering by adding 0·02″ to the diameter of the bare wire in inches). Let L stand for the length of the bobbin or former which may be occupied by wire, and D the depth to which the winding may be laid. Then there will be—

$$\frac{D}{d_1} \text{ layers}$$

each of—

$$\frac{L}{d} \text{ turns}$$

and—

$$w = \frac{LD}{d_1^2}$$

all quantities being, of course, expressed in the same units.

The value of $\mathfrak{M}$ for dynamos, etc., is calculated as shown in a previous chapter; whilst $\mathfrak{M}$ for ammeters, voltmeters, and other small instruments, is usually found in the first instance by experiment. Many ammeters and voltmeters of the magnetic type are alike in size of parts whether ammeter or voltmeter, being only different in the winding. Now, it is evident that, with all the different ranges of reading required, it is not possible to provide exactly any particular number of ampère-turns with any particular current or P.D. Thus all ammeters and voltmeters of this class have some power of adjustment that enables the nearest value of $\mathfrak{M}$ to be made use of. In ammeters it is not always possible to make the product $cw =$ the value of $\mathfrak{M}$ desired, and owing to the trade range in sizes of wires for voltmeters, it is not always possible to find that size in stock that will make the right value of resistance of mean turn. Thus both ammeters and voltmeters are calculated for on the assumption that the value of $\mathfrak{M}$ is variable within certain limits. In some instruments of the permanent magnet class $\mathfrak{M}$ ranges round 300 ampère-turns, whilst in some of the electro-magnet type it may range round 600 ampère-turns for ammeters, and about 400 ampère-turns for voltmeters.

CHAPTER VIII.

ARMATURE REACTIONS.

So far we have only considered the simplest case of dynamo-winding, and will now proceed to discuss the calculations for different types of machines. In order to appreciate what it is we have to allow for in a dynamo at full load, we shall have to consider the effect of the current in the armature and the magnetic circuit. In Fig. 15 is shown a section through a dynamo, with the wire on the field magnets and armature indicated by lines.

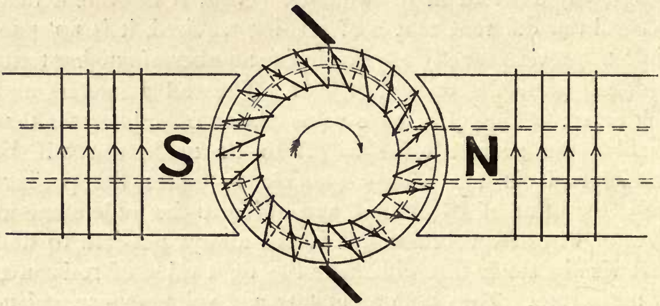

FIG. 15.

The direction in which the current flows in the field-magnet winding is indicated by arrows. By applying Dr. Fleming's rule for the direction of the E.M.F. in any wire on the armature, we are able to mark these wires to show the direction in which the current would flow if the brushes were connected. Thus, on connecting the brushes, a number of things will happen—

(1) Due to the resistance of the armature winding, we shall get a P.D. at the brushes which will be less than the whole generated E.M.F. by an amount which may be considerable at full load, though hardly noticeable at light load.

Thus, calling the whole E.M.F. = E, the current in the armature C_a, and its resistance r_a, we have that the value of the P.D. at the brushes—

$$e = E - C_a r_a$$

(2) In addition to this loss of volts in the armature resistance, we have to allow for the effect of the current in the armature winding as a M.M.F. acting in the magnetic circuit. Thus, with the brushes on the diameter of commutation, corresponding with no load, as shown in Fig. 15, we see that the armature winding and current produce a M.M.F., causing a magnetic flux at right angles to the flux set up by the magnets, and acting in a direction up the page, whilst the flux due to the magnet winding acts from right to left, the resultant flux through the armature will, of course, be a flux having a direction somewhere in between these two directions, or inclined to the horizontal line in the figure by an amount which will depend upon the relative magnitudes of the two M.M.F.'s.

Thus, if the armature current and number of windings be very large, we shall have the resultant magnetic flux taking a direction chiefly down the length of the page; the dynamo would then be a very bad one, and spark considerably. On the other hand, if the M.M.F. due to the magnet-field winding greatly predominates, we should have the resultant flux nearly horizontal as before taking any current from the armature. This state of affairs is that to be desired in all good dynamos and motors where sparkless collection among other advantages is wanted. In the best of machines, however, there is some distortion, and it is this we have next to consider. When a current is flowing in the armature the direction of the resultant flux will be as shown in Fig. 16, where a dotted line carried round the magnetic circuit indicates the path of the lines of force. The brushes have been adjusted so as to bear upon a diameter at right angles to the direction of flux through the core; that is upon the new diameter of commutation. Two things will now be noticed—

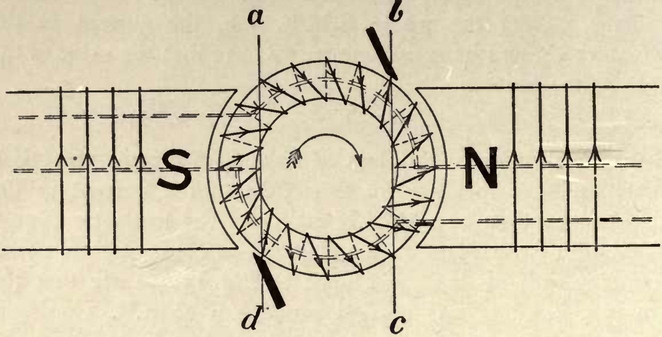

FIG. 16.

(1) The cross magnetic effect of the armature will now be somewhat decreased, being now only that due to the wires on the armature contained by the line *ad* or *bc*; and—

(2) The remainder of the winding on the armature, viz. that contained by the line *ab* or *dc*, is now a M.M.F. causing a magnetic flux in a direction parallel to the flux caused by the magnet winding, but in an opposite sense, or tending to directly decrease its magnitude. Since the value of the whole generated E.M.F. depends upon the value of the magnetic flux through the armature, this E.M.F. will no longer be as large as before a current was taken from the machine.

We must thus allow that the generated E.M.F. is now less than E by an amount, say E_1, and can write the terminal P.D.—

$$e = E - C_a r_a - E_1$$

In a great many cases we are required to produce a machine which will give a constant value of terminal P.D. e_1 at all loads, and in order to do this it will be necessary to make allowance for the reductions indicated in the above equation. In some cases, however, it is desirable to make a correction also for the loss along leads, at the distant end of which lamps are to be kept at a constant P.D., and we may then add to the above losses the loss along the leads, or Cr_l, making now—

$$e = E - C_a r_a - C r_l - E_1$$

Correction for these losses is commonly made by adding to the M.M.F. of the magnet-winding by adding a series coil to the shunt, thus compounding the machine. Now, such series winding has the effect, if connected as "long shunt," of increasing the effective resistance of the armature, and our equation will now stand—

$$e = E - C_a r_a - C_a r_m - C r_l - E_1$$

where r_m is the series-coil resistance. When, however, the connections are made "short shunt," the series coil acts as an increase in the resistance of the leads, and the resultant reduction of terminal P.D. is slightly different, viz.—

$$e = E - C_a r_a - C r_m - C r_l - E_1$$

There is, however, a more important difference between the two cases: whereas the P.D. at the terminals of the shunt coil is practically constant in the case of "long shunt," it is gradually increased from no load to full load with the "short shunt" connections, by an amount equal to the loss in the series coil, and thus the amount of series winding necessary in the case of "short shunt" is not quite so great as in "long shunt," when other things are equal.

We have, then, that the generated E.M.F.—

$$E = e + C_a r_a + C_m r_m + C r_l + E_1$$

putting C_m for the current in the series coil, which may be either C or C_a. Of these items we can have generally the most accurate knowledge, save in the case of E_1. The exact effect of the band of winding contained by the lines *ab* or *dc* in Fig. 16, called the "demagnetizing band," is somewhat difficult to predetermine. Its magnitude depends upon the position of the brushes, and this depends upon the exact distortion produced by the cross magnetizing band. This distortion can be found by plotting out on the drawing of the machine the path of the lines for every value of the current in the armature, and thus finding the diameter of commutation for every value of the current. Considerable experience and care are required for this, and it will be out of place to further consider it here.

An example of the ascertainment of the value of the necessary series winding, when all the above losses have been found, will serve to sum up the matter.

A dynamo is required to give a terminal P.D. of e volts at all values of current, from zero to C. ampères. In the first place, at very small output the whole generated E.M.F. need not be appreciably larger than e, and consequently a calculation must be made of the M.M.F. necessary to produce such a value of N that the generated E.M.F. = e, or—

$$N = \frac{e10^8}{nw}$$

which will require a M.M.F. of $\mathfrak{M}$ ampère-turns. The winding to produce this must be the shunt coil. In the second place, we have to allow that the losses at full load make the necessary generated E.M.F. to be E volts, and thus necessitate a greater value of magnetic flux, or—

$$N_1 = \frac{E10^8}{nw}$$

to produce which the M.M.F. will be larger, and may be called $\mathfrak{M}$.

Thus we have the additional M.M.F. required at full load is—

$$\mathfrak{M}_1 - \mathfrak{M} \text{ ampère-turns}$$

which must be supplied by the series coil in addition to that required to balance the demagnetization. The series M.M.F. is a product of the series turns into a current, which may be that in the main circuit (short shunt), or may be that in the armature (long shunt). In the latter event it will be noticed that when compounding has been accurately carried out for all the usual items, the P.D. at the terminals of the shunt coil is a constant at all loads (except when compounding for loss in the leads). But in the short shunt combination the P.D. at the shunt terminals will only be e volts at very small loads, and will increase to $e + Cr_m$ volts at full load, thus increasing the M.M.F. and making consequently less series winding necessary. This alteration of M.M.F., due to the shunt coil, is of very great importance in all cases where there is imperfect compounding as any defect. Both over- or under-compounding is much exaggerated.

A little consideration will show that it is hardly possible to produce a perfectly constant P.D. at the terminals by compound winding. For if we calculate the additional ampère-turns to be supplied by the series coil in, say, two portions, viz. one for half-load, and the other as a further addition for the other half-load to full load, we shall find that the latter exceeds the former. Thus, let the extra M.M.F. required for half-load be $\mathfrak{M}$; this is due to an increase in the total flux necessary to increase the generated E.M.F. by an amount equal to the sum of a number of items, viz.—

$$C_a r_a + C_m r_m + C r_l$$

which will be all nearly enough half the amounts for full load, whilst the other item, E_1, will be less than half, since it will be due to a demagnetizing band of less turns than at full load, and carrying only half the current. The items for the further increase up to full load will all be higher than for the first half-load, since the resistances will all be slightly higher, and also the demagnetizing band will have increased in number of turns as well as having been doubled in current; or $\mathfrak{M}_2$, the increase of magneto-motive force for the second half of full load, will be considerably greater than $\mathfrak{M}_1$. But both are produced by the increasing current flowing in coils of fixed number of turns, which will cause the additional ampère-turns to be simply proportional to the current. And also the relationship between the magnetic flux produced and the ampère-turns producing it is such as to make the additional effect of $\mathfrak{M}_2$ actually less than $\mathfrak{M}_1$ instead of greater.

It thus follows that if a dynamo is to give, say, e volts P.D. at its terminals at a certain speed at no load and also at full load, it must give a P.D. which is greater than *e* at the intermediate loads.

Consequently it is customary to fix upon two points, say quarter-load and three-quarter-load, and so wind as to produce the same P.D. at the terminals at these points, letting the P.D. be slightly less than this at no load and full load, but slightly higher at half-load.

In the above considerations it is sometimes necessary to ask ourselves what effect in a composite magnetic circuit a given increase or decrease of M.M.F. will have. It is very easy to find what increase or decrease of M.M.F. a given increase or decrease of magnetic flux will require; but the converse is a much more difficult problem, since the relationship between all the items of the circuit is at once altered as soon as the flux is altered.

ARMATURE REACTIONS IN A MOTOR.

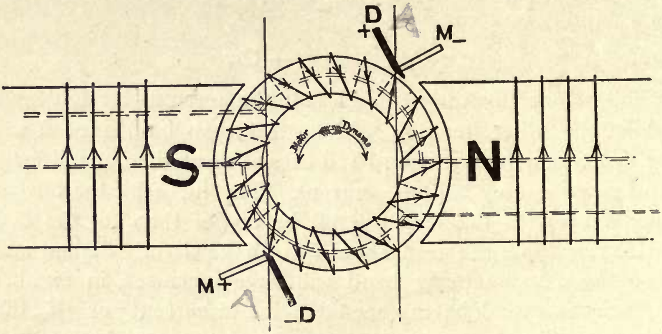

FIG. 17.

If in the above diagram DD represent brushes bearing on the commutator of a machine used as a dynamo, we shall have the current flowing as shown by the arrow when the direction of rotation is clock-wise, as indicated by the slope of the brushes. In order to cause rotation in the clock-wise direction, a pull will have to be supplied to the belt to overcome the drag between the current in the wires under the pole-pieces and the lines of force passing into and out of the armature. When, however, the diagram is taken to represent a motor supplied with current to flow through the circuit in the direction as shown in the figure, this drag will now be the moving force, and the armature will revolve in a counter clock-wise direction, to admit of which the brushes MM will have to be used sloping in the opposite sense to the dynamo brushes, but bearing upon the same point on the commutator, since the nature of the magnetic forces acting in the magnetic circuit is still unchanged. But as the armature is now revolving in the opposite sense to what it was before the case, the E.M.F. generated will also be opposite to that previously generated, and will, consequently, be against the E.M.F. of the source supplying the motor with power, which will thus have to exceed that generated by the amount required for the resistance of the armature coils.

It thus follows that, as in a dynamo so in a motor, the current in the armature produces a demagnetizing effect upon the magnetic circuit. But whereas this demagnetizing effect is disadvantageous in a dynamo, it actually tends to aid constancy of speed in a motor by reducing the generated E.M.F. for a given speed somewhat in the proportion required by a constant E.M.F. of supply.

The above diagram is due to Mr. Swinburne.

CHAPTER IX.

EFFICIENCY.

THE ratio of the useful expenditure of energy in a system to the whole energy is called the "efficiency;" but we have, in the many and various cases to be considered, to sometimes separate the parts of a system and consider them independently, as will be seen in the examples taken.

As a simple case, we may take a single cell used to light a single lamp. Let the total E.M.F. of the cell be E volts, its internal resistance be r ohms, whilst that of the lamp is R ohms. Then the current flowing through the circuit will be—

$$C = \frac{E}{R + r}$$

and the rate of doing work in the lamp is—

$$C^2R = \frac{E^2}{(R + r)^2}R$$

whilst the whole activity is—

$$C^2(R + r) = \frac{E^2}{(R + r)^2} \times (R + r)$$

the efficiency is then the ratio of that part of the energy concerned in lighting the lamp to the whole activity, or is—

$$\frac{\dfrac{E^2}{(R + r)^2}R}{\dfrac{E^2}{(R + r)^2}(R + r)} = \frac{R}{R + r}$$

Or we may split our circuit up into items, and consider the ratio of the useful energy in the lamps to that in the lamps plus that in the leads, thus getting the efficiency of distribution. Or we may consider the ratio of the energy given out by the cell to the whole energy produced by it, in which case we have the electrical efficiency of the cell or other machine.

Or, again, we may include the energy required to run the generator—a dynamo, for example—against the various sources of friction, and thus state the efficiency as the ratio of the energy delivered from the terminals to the energy put in at the pulley, thus getting the mechanical efficiency, or commercial efficiency.

In the case of motors, we can take the ratio of the power at the pulley to the power supplied at the terminals, which is again the commercial efficiency. On the other hand, we may regard the motor as a machine for the conversion of electrical energy into mechanical energy, whether useful or not, and so obtain the electrical efficiency.

With secondary cells we may, on the one hand, regard the cell as a store of quantity of electricity, and state the quantity efficiency as the ratio of the coulombs got out to the coulombs put in; or we may consider the cell as a store of energy, and state the energy efficiency as the ratio of the watt-hours got out to the watt-hours put in.

Also, as a dynamo and engine will be required for charging, we may have a combined efficiency of the whole apparatus, called the plant efficiency, and being the ratio of the watts got out to the power required to run the dynamo stated in watts.

I. Efficiency of Distribution and Transmission.

(*a*) In a simple parallel system, with all the lamps at the ends of the leads, if r_l = resistance of the leads, C the current through the lamps, and e the P.D. at the lamps, we have—

energy used in the lamps $= e\mathrm{C}$
,, wasted in leads $= \mathrm{C}^2 r_l$

and the efficiency will be—

$$\frac{e\mathrm{C}}{e\mathrm{C} + \mathrm{C}^2 r_l} = \frac{e}{e + \mathrm{C}r_l}$$

which is the same thing as the ratio of the useful P.D. to the total P.D.

(*b*) Simple parallel system, with the lamps uniformly distributed along the length of the leads. Here the loss of volts in the leads will be—

$$\frac{C}{2}r_l$$

making the energy waste in the leads $= \frac{C^2}{2}r_l$.

But the energy expended in the lamps will be—

$$C \times \left(e + \frac{C}{2}r_l\right) \text{ as an average}$$

where e is the P.D. at the far end; or may be put more conveniently in terms of the initial P.D., thus—

$$C \times \left(E - \frac{C}{2}r_l\right) \text{average}$$

where the efficiency is—

$$\frac{C\left(E - \frac{C}{2}r_l\right)}{CE}$$

(*c*) A parallel system with feeders. Let r_f be the resistance of the feeders, and r_d be the resistance of the distributors between two feeding centres. Then the total current will be, say, C ampères in the feeders, or an average of $\frac{C}{4}$ ampères in the distributors.

The loss of energy in the feeders will then be C^2r_f, whilst that in the distributors will be—

$$\frac{C^2}{16}r_d$$

If also E be the P.D. at the starting ends of the feeders, the mean P.D. at the lamps will be—

$$E - Cr_f - \frac{C}{4}r_d$$

whence the efficiency will be—

$$\frac{C\left(E - Cr_f - \frac{C}{4}r_d\right)}{CE}$$

Or, again, the ratio of average P.D. at lamps to P.D. at the dynamo.

(*d*) Series circuit containing n items, each of r ohms resistance, and having a constant current of C ampères. Calling r_l the resistance of the leads, the efficiency is—

$$\frac{C^2nr}{C^2(nr + r_l)}$$

again the ratio of the P.D. at the lamps to the total P.D.

II. Efficiency of dynamos, motors, etc.

(*a*) Magneto-dynamo. Here the only loss is in the armature and brush contacts, etc., the circuit being a simple series system. Let e be the total P.D., and r_a the resistance of the armature, etc.; then the output is eC, and the waste in the armature C^2r_a, whence the—

$$\text{electrical efficiency} = \frac{eC}{eC + C^2r_a}$$

which is also the ratio of the useful P.D. to the whole P.D.

From a theoretical point of view the efficiency of a magneto may be higher than that of a dynamo since there is no waste of energy to magnetize the field-magnets.

The other losses in the machine will consist of M the mechanical friction, H the hysteresis loss, and F the eddy currents. Calling all these W_1, we have the—

$$\text{mechanical efficiency} = \frac{eC}{eC + C^2r_a + W_1}$$

(*b*) Series dynamo. Calling as before e, C, and r_a respectively the terminal P.D. current and armature resistance, and r_m the resistance of the magnet coils, we have the—

$$\text{electrical efficiency} = \frac{eC}{eC + C^2(r_a + r_m)}$$

And again, if W_1 stands for all the other losses expressed in watts, then the—

$$\text{mechanical efficiency} = \frac{eC}{eC + C^2(r_a + r_m) + W_1}$$

(*c*) Shunt dynamo. Using the same symbols as before, and calling the resistance of the shunt coil r_s, we have the output is eC, but the current in the armature is greater than C by the amount taken by the shunt coil, viz. $\frac{e}{r_s}$, the waste in the armature being consequently—

$$\left(C + \frac{e}{r_s}\right)^2 r_a$$

Whilst the shunt-coil waste is $\frac{e^2}{r_s}$, and the—

$$\text{electrical efficiency} = \frac{eC}{eC + \frac{e^2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 r_a}$$

And W_1 having the same meaning as previously, the—

$$\text{mechanical efficiency} = \frac{eC}{eC + \frac{e^2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 r_a + W_1}$$

(*d*) Compound dynamo, short shunt. Here the output is eC, as before. The current in r_m is C ampères, but that in r_a is C + shunt current, or—

$$C + \frac{e + Cr_m}{r_s}$$

whence the various losses are $C_2 r_m$ in the series coil,

$$\frac{(e + Cr_m)^2}{r_s}$$

in the shunt coil, and—

$$\left(C + \frac{e + Cr_m}{r_s}\right)^2 r_a$$

in the armature, making the electrical efficiency—

$$= \frac{eC}{eC + C^2 r_m + \frac{(e + Cr_m)^2}{r_s} + \left(C + \frac{e + Cr_m}{r_s}\right)^2 r_a}$$

the mechanical efficiency being—

$$= \frac{eC}{eC + C^2 r_m + \frac{(e + Cr_m)^2}{r_s} + \left(C + \frac{e + Cr_m}{r_s}\right)^2 r_a + W_1}$$

(*e*) Compound dynamo, long shunt. The current in the shunt is $\frac{e}{r_s}$, and flows through both r_m and r_a, in addition to the main-circuit current C;

whence the—

$$\text{electrical efficiency} = \frac{eC}{eC + \frac{e_2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 (r_m + r_a)}$$

and the—

$$\text{mechanical efficiency} = \frac{eC}{eC + \frac{e^2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 (r_m + r_a) + W_1}$$

(*f*) A series motor. The electrical efficiency is the ratio of that part of the energy supplied to the machine, which is actually converted into mechanical work, whether useful or waste, to the whole energy supplied. Calling E the P.D. of supply, with C the current, and r_a and r_m having the values above, we have that the back E.M.F. of rotation is—

$$\epsilon = E - C(r_a + r_m)$$

and the whole rate of doing work mechanically is $C\epsilon$; whence the—

$$\text{electrical efficiency} = \frac{C\epsilon}{EC} = \frac{\epsilon}{E}$$

The mechanical efficiency is—

$$\frac{EC - C^2(r_a + r_m) - W_1}{EC}$$

or—

$$\frac{C\epsilon - W_1}{EC}$$

Since of the total rate of doing work mechanically the part W_1 being internal friction, etc., is wasted; M is direct mechanical friction, H is molecular friction, but F is a load on the armature of exactly the same nature as a closed secondary on a dynamotor armature.

(*g*) Shunt motor. Calling E the P.D. at which a current of C ampères is supplied, we have that of C ampères, the shunt coil takes $\frac{E}{r_s}$, leaving the—

$$\text{current in the armature} = C - \frac{E}{r_s}$$

$$\text{due to which the loss in } r_a \text{ is } \left(C - \frac{E}{r_s}\right) r_a \text{ volts}$$

$$\text{leaving the B.E.M.F} = \epsilon = E - \left(C - \frac{E}{r_s}\right) r_a$$

whence the—

$$\text{electrical efficiency} = \frac{\left\{E - \left(C - \frac{E}{r_s}\right) r_a\right\} \left(C - \frac{E}{r_s}\right)}{EC}$$

and the mechanical efficiency

$$= \frac{\left\{E - \left(C - \frac{E}{r_s}\right) r_a\right\} \left(C - \frac{E}{r_s}\right) - W_1}{EC}$$

(*h*) Compound motor, short shunt. Calling EC the supply as before, we have the shunt current is—

$$\frac{E - Cr_m}{r_s}$$

Whence the armature current is—

$$C - \frac{E - Cr_m}{r_s}$$

and the B.E.M.F.—

$$\epsilon = E - Cr_m - \left(C - \frac{E - Cr_m}{r_s}\right) r_a$$

and therefore the electrical efficiency is—

$$\frac{\left\{E - Cr_m - r_a\left(C - \frac{E - Cr_m}{r_s}\right)\right\} \left(C - \frac{E - Cr_m}{r_s}\right)}{EC}$$

whilst the mechanical efficiency—

$$= \frac{\left\{E - Cr_m - r_a\left(C - \frac{E - Cr_m}{r_s}\right)\right\} \left(C - \frac{E - Cr_m}{r_s}\right) - W_1}{EC}$$

(*k*) Compound motor, long shunt.

Here the shunt current is $\frac{E}{r_s}$
and the B.E.M.F.—

$$\epsilon = E - \left(C - \frac{E}{r_s}\right)r_m - \left(C - \frac{E}{r_s}\right)r_a$$

and the electrical efficiency

$$= \frac{\left[E - \left\{\left(C - \frac{E}{r_s}\right)\left(r_m + r_a\right)\right\}\right]\left(C - \frac{E}{r_s}\right)}{EC}$$

whilst the mechanical efficiency

$$= \frac{\left[E\left\{\left(C - \frac{E}{r_s}\right)\left(r_m + r_a\right)\right\}\right]\left(C - \frac{E}{r_s}\right) - W_1}{EC}$$

(*l*) Secondary battery. Calling the charging current C ampères and the time of charging t hours, the corresponding quantities for discharging being c_1 and t_1, we have the ampère hours put in $= ct$ and output $= c_1t_1$, whence the—

$$\text{quantity efficiency} = \frac{c_1t_1}{ct}$$

and may be as much as 100 per cent.

Considering, however, the fact that the B.E.M.F. of the cell is higher on charging than the available E.M.F for discharging, we have the energy put in is—

$$Cte$$

and that got out is $C_1t_1e_1$ when e and e_1 are respectively the E.M.F. to charge against a certain B.E.M.F. and resistance, and the P.D. available for use at the terminals on discharging. The

$$\text{energy efficiency is} = \frac{C_1e_1t_1}{Cet}$$

(*m*) Plant efficiency. This must include all sources of loss and give the ratio of—

$$\frac{\text{The useful work done in a given time}}{\text{The energy supplied to the prime mover for the same time}}$$

CHAPTER X.

VOLTMETERS AND THEIR RESISTANCE COILS.

In the following examples notice will be taken of the effect of temperature on the readings of voltmeters of the magnetic class. As was mentioned in the chapter dealing with the winding of coils, there is no difference between an ammeter and a voltmeter in point of shape, etc. There is, however, this important difference between them—that, whilst the former has its dial graduated in terms of the current passing through its coil, the dial of the latter is marked in terms of the product of the current through the coil into the resistance of that coil, such resistance being taken at the temperature at which the instrument was calibrated. Thus the readings of an ammeter are independent of the value of its resistance, but those of a voltmeter depend upon the resistance of its circuit, and will only be correct when that resistance has the value it had when the dial was marked. It is, therefore, the object of the instrument-maker to so contrive matters that the resistance of the voltmeter circuit shall vary as little as possible. Alteration in temperature may be produced in two ways, viz. by external means such as atmospheric variation, or variation due to locality; or by internal means such as the expenditure of energy in the coil due to the current through it. We will call the first of these the "temperature" error, and the second the "heating" error. The employment of a material such as manganin or constantan, will make both these errors quite small, or even negligible as far as mere temperature variation of resistance (T.V.R.) is concerned, though the actual heating may be very large, and even dangerous. For the energy waste in a coil of a given size producing a given value of ampère-turns will be directly proportional to the resistivity of the material of the wire. When the resistivity is very high, it is not easy to make an instrument coil of reasonable dimensions without excessive heating and waste of energy, and owing to the want of a material of low specific resistance and small or negative T.V.R., we are obliged to make a compromise between the two extremes, and wind that part of the instrument which is to produce the magnetic effect with wire of small resistivity in order to get large ampère-turns with small heating; and the rest with wire of small T.V.R., and having with advantage a high resistivity. The two things to be aimed at are, firstly, to get the requisite M.M.F. with small energy waste, and then to get the necessary freedom from variation of resistance by addition to the instrument resistance of a coil having a constant, or fairly constant, value of resistance. This is effected by making the working coil of the instrument of copper wire to get the maximum M.M.F. with the least energy waste, and by adding to the instrument so wound an extra coil or resistance of, say platinoid or manganin, materials having very small coefficients of T.V.R., and thus causing the effective total resistance to remain nearer a constant then if copper alone had been employed. The effect of so adding to the resistance of the working coil is twofold; firstly, the current through the instrument due to a certain P.D. will be decreased, and consequently the deflection for a given number of volts will be less than before, or the total reading of the instrument will be increased; and, secondly, there is the tendency to greater constancy in total resistance. The increase of total reading power is obviously proportional to the increase of resistance; thus if the extra coil have a resistance equal to that of the instrument's working coil, the instrument will read twice as much; or, more generally, if the extra coil be n times the resistance of the working coil, the total reading power will be $(n + 1)$ times the original. The T.V.R. of the whole combination is, on the other hand, inversely as the number of times the total resistance is increased if the material of the extra coil has no T.V.R. If the T.V.R. of the extra coil be important, having a value c say, whilst the T.V.R. of the working coil is b, and the extra coil has a resistance n times that of the working coil then the combined T.V.R. will be—

$$a = \frac{b \pm nc}{n + 1} \text{ per cent. per } 1^\circ \text{ C.}$$

when b and c are stated in values per cent. per 1° C. The plus sign is mostly the case, but when carbon or manganin is employed for the extra resistance, then c is negative, and by adjusting n so that—

$$nc = b$$

we shall obtain a system of perfectly constant resistance.

It thus follows that the value of an indication on a voltmeter in volts is found by multiplying the reading as marked on the dial by the ratio of the present resistance total to the total resistance when calibrated, or—

$$\text{volts} = \text{deflection} \times \frac{\text{present resistance}}{\text{resistance when calibrated}}$$

The deflection being of course marked by a number which is equal to the product of the resistance of the instrument when calibrated into the current which a given P.D. produces through the instrument.

The value of the present resistance depends upon the temperature and the T.V.R., values for which are given on A, p. 209.

Thus let R_τ be the resistance at which the instrument was calibrated (temperature, say τ° C.), and let R_t be the resistance at some other temperature t° C., which may be either higher or lower than τ° C. Let a be the T.V.R. per cent. per 1° C. Then, nearly enough for practical purposes—

$$R_t = R_\tau \left\{ 1 + \frac{a}{100}(t - \tau) \right\}$$

when the difference $t - \tau$ may be zero, positive when t is greater than τ, or negative when t is less than τ.

Similarly, the value of any indication on the voltmeter will be always proportional to its resistance at the time of taking a reading, and if we call the value per one division k volts, when the instrument is at the correct temperature we shall then have that the value of one division at any other temperature will be—

$$K = k\left\{1 + \frac{a}{100}(t - \tau)\right\}$$

CHAPTER XI.

ELECTRIC TRACTION, RAILWAYS, ETC.

THE application of the electric motor to tramways and other tractive work renders it necessary to ascertain the horse-power required for any particular case, and the data necessary for this purpose may now be discussed. In order to draw a car along a line, the mechanical pull required will depend upon the following things—

(1) The weight of the car.
(2) The condition of the line.
(3) The inclination of the line.

By the condition of the line is meant its smoothness and freedom from dirt or other obstruction. In the best possible cases of railway work the pull required to keep a ton weight of car in motion on a level line may be less than 10 lbs., but in some cases, even with grooved rails, the pull may be 15 to 20 lbs. in good cases, and even 40 lbs. or more in cases where dust or road grit has collected in the rails. Curves also cause a local increase of resistance to traction.

In order to allow for the inclination of the line we have to take into consideration the effect of gravity, which effect may be against us in going up an incline, but which is for us in going down an incline. It is seldom that the inclination is so much as to render it necessary to distinguish between the length of the actual distance gone and the true horizontal distance, and consequently we may write the pull due to gravity as being—

$$\frac{2240\text{W}}{n}$$

where n is the number of feet along the line for a rise or fall of 1 foot, this pull being + or − according as the inclination is up or down. We shall thus have the pull required to move a car of W tons weight along a line of 1 in n inclination with a resistance to traction of x lbs. per ton as being in pounds—

$$P = Wx \pm \frac{W2240}{n}$$

taking the + sign when going up hill and the − when coming down hill.

Again, the rate of doing work is proportional to the speed of running, and we have the horse-power required will be the pull multiplied by the distance gone through in feet per minute and divided by 33,000. Thus, if S = the speed in miles per hour, we have the horse-power is—

$$\text{H.P.} = \frac{88PS}{33000}$$

and this is the useful mechanical rate of doing work, and to it must be added that required to run and excite the motor and that required to transmit the power of the motor to the axle of the car.

In general the conditions are so disadvantageous to the high-speed motor, that in practice it is found that for every horse-power put into the motor-terminals we may get out from 60 to 80 per cent. as useful work to run the car; the lower figure is frequently exceeded, but the higher can only be attained under the best conditions, and with large motors. Calling this factor $\frac{100}{y}$, we have that the electrical power put into the motors must be—

$$\frac{100}{y}\left(\frac{88SP}{33000}\right) \text{ or, in full—}$$

$$\text{H.P. required} = \frac{100}{y}\left\{\frac{88S\left(Wx \pm \frac{W \times 2240}{n}\right)}{33000}\right\}$$

which is converted into watts by multiplying by 746. Now, this power is a product of volts and ampères, and it is simply a matter of convenience as to what the E.M.F. may be. The motor is in any case arranged with its gearing so that a given current through its armature means a given pull for traction, and the value of this pull per ampère can usually be stated or fixed. From this and the actual pull required we can readily find the current required to move the car, and can add that to overcome internal friction, etc. From the current and the speed of running we can next find the useful rate of doing work, and thence the B.E.M.F. of the motor. In the case of shunt motors this B.E.M.F. will be practically constant, being in any case proportional to the speed, and the cars will run at a nearly constant maximum speed both up and down hill, and on the level if supplied with a constant E.M.F. A knowledge of the electrical data will then enable us to ascertain the necessary increase of volts on account of internal losses, and to state the exact value of the P.D. of the mains, In some cases the tractive pull per ampère is not a constant, since with series motors the drag on the wires is increased by an increase of current both on account of that increase itself and also on account of the consequent increase of magnetic flux through the armature. When series motors are run off mains at a constant P.D., we shall have their speeds varying by an amount which is greater than in the case of shunt motors, having a practically constant excitation. Thus, at places on the line taking a heavy load, the extra current in the magnet coils will cause the speed to go down, since otherwise the B.E.M.F. of the motor would soon exceed that of the supply mains, were that possible. The tractive force being thus increased in a proportion more nearly as the square of the current, makes the series motor particularly suitable for overcoming steep gradients and other heavy loads with less heating and waste of energy than would be the case with a shunt motor, for the series motor will automatically reduce its speed up hill, and by increasing its field strength take less increase of current to produce the necessary tractive pull.

When getting up speed the car has to store an amount of energy, equivalent in foot pounds to half its mass, multiplied by the square of its velocity in feet per second. The extra pull required to do this will depend upon the time taken to attain the speed, and we can write the work to be stored in foot pounds, as—

$$\frac{W \times 2240}{2g}\left(\frac{S \times 5280}{3600}\right)^2 = K$$

When g is taken as 32, we can simplify this to—

$$K = WS^2 \times 75{\cdot}3 \text{ foot pounds}$$

where W = tons weight of car and S the speed to be got up in miles per hour.

Now, if this store of energy is to be acquired in m, minutes, we must remember that the car in gradually getting up the speed of S miles per hour will go through a distance of—

$$m \times \frac{S \times 5280}{60 \times 2} = l \text{ feet say}$$

since its average speed during the time will be $\frac{S}{2}$ if the acceleration be uniform. And through this distance a pull over and above that required to overcome the tractive resistance will be required of—

$$\frac{K}{l} \text{ pounds}$$

or the extra pull during the time of m, minutes, to get up speed of S miles per hour will be—

$$\frac{WS^2 \times 75{\cdot}3}{44Sm} = \frac{WS}{m} \times 1{\cdot}711 \text{ pounds}$$

Or if speed is to be attained in t seconds, the extra pull is—

$$\frac{102{\cdot}7\ WS}{t} \text{ pounds}$$

If also p be the pull per ampère, the current required will then be—

$$C = \frac{Wx \pm \frac{W2240}{n}}{p} \text{ ampères}$$

to which must be added the extra current to get up speed, or—

$$C_1 = \frac{102{\cdot}7\,WS}{tp} \text{ ampères}$$

The value of p the pull per ampère may be nearly a constant for all values of current in the case of a shunt motor; but in the case of a series motor it has more nearly the value—

$$p = \frac{\frac{n}{a}C^2}{b + C}$$

where n = speed per minute and a and b are constants, depending upon the size of the machine.

In questions of electric traction involving a knowledge of the value of the pull produced by a given current in the armature, or of finding the value of the current required to give a certain pull, we must be careful to distinguish between the shunt, compound, and series types of motors.

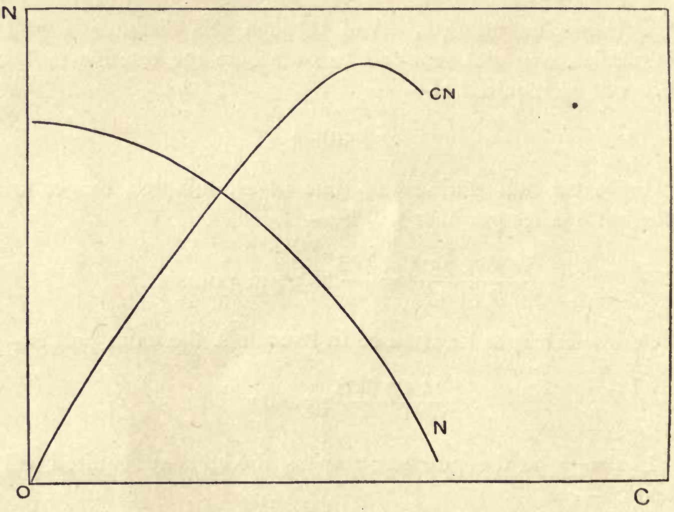

FIG. 18.

Shunt Motor.—When the E.M.F of supply is constant, we may regard the magnetizing force, acting upon the magnetic circuit as practically constant, subject to the fact that distance from the generating station will mean a lowering of the P.D. at the terminals of the shunt coil by the ohmic loss in the leads. The pull or torque is always proportional to the product of current and lines total; and these last should be constant in a shunt motor with constant P.D. at the terminals were it not for the fact that armature reactions produce a demagnetizing effect, and the curve of relationship between N and C, the current through the machine will be of the nature of the curve shown in Fig. 18, and consequently the pull per ampère will be decreasing slightly as the current increases. Thus at times of very large current, as at starting, the actual pull may not be very large, and the maximum pull or torque may never be great.

Compound Motor.—Taking next the case of a motor compounded for ordinary work, that is for constant speed with varying load, we find that the effect of the compounding series coil is to still further increase the demagnetizing effect of the current, with the result that the curve of relationship between N and C will be still more drooping at high values of C than that shown in Fig. 18, with the effect of a still further reduced pull under similar conditions.

But when the series-magnet winding is connected so as to magnetize the circuit, we may have this effect made, either to give a constant value of N for all values of C (in which case there will always be a constant pull per ampère, whatever value the current may have), or to actually produce an increase of lines with an increase of current, making the curve of N – C rise for an increase of C, and thus giving a pull per ampère increasing with the current. It will not be necessary to investigate this action in combination with the shunt coil, but to go on to consider the effect of a series coil alone.

Series Motor.—Here the value of N is constantly increasing with the current, and would have a curve of relationship similar to the βH curve for a composite magnetic circuit, were it not for the fact of demagnetization causing a tendency to drop at high values of C. Taking the shape of NC, as shown in Fig. 19, we may also plot a curve, showing the relationship between C and the product CN, as shown in the upper curve. The pull exerted is directly proportional to the ordinate of the second curve, and can be found from the nature of the winding and gearing employed. Calling the total pull for tractive effect, $p = a\text{CN} \times 10^{-6}$, where a is a constant depending upon the type of machine, we have from a knowledge of the value of p in pounds that the product—

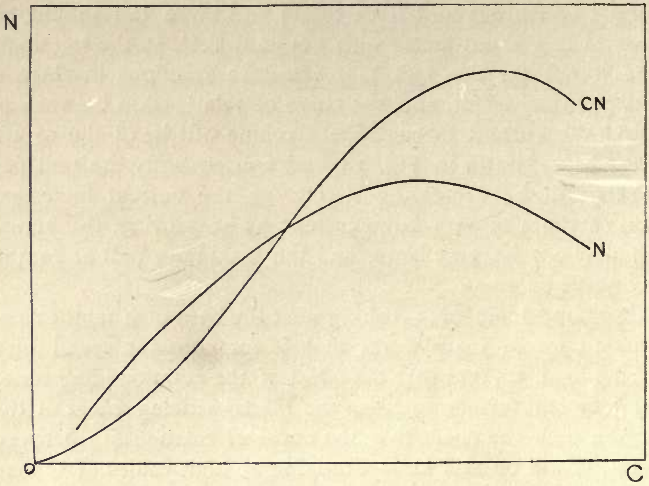

FIG. 19.

$$\text{CN} = \frac{p \times 10^6}{a}$$

and a reference to the curve will give the value of C and N corresponding with this product.

It is also clear that the pull will reach a maximum value, and the performance of the motor is thereby limited.

CHAPTER XII.

ALTERNATING CURRENT CIRCUIT.

WHEN the current in a circuit changes periodically in direction, passing through a set of cyclic changes of value, it is termed an alternating current, and, owing to the disturbing influence of the medium which surrounds the circuit, the usual relationship known as Ohm's Law requires modification before it can completely represent the true state of affairs. The manner of change of value of the current from time to time is different in different types of machinery, but is very frequently of a nature simply expressed as a sine function of the time. Such a current can be represented in all possible values by the curve in Fig. 20, where ordinates above and below the line O*t* represent the instantaneous values of the current in ampères corresponding with any instant in time marked along O*t*. Such instantaneous values can be represented by the expression—

$$C_i = C_m \sin \frac{2\pi}{\tau} t$$

where C_i is the value at the instant in time t, C_m the maximum value to which the current ever attains, and τ is the time in seconds taken to go through one complete cycle, or change from zero rising to a positive maximum, declining to zero, reversing and rising to a negative maximum, again declining to zero. The curve can be drawn from a table of natural sines by dividing the horizontal line into 360 parts corresponding to the 360 degrees in a complete revolution, and marking points at a distance above or below the horizontal equal to C_m times the natural sine of the angle. The circular measure of 360 degrees is 2π, and it thus follows that when t has such values as to make $\frac{t}{\tau} \times 2\pi$ equivalent to convenient angles, the curve may be easily drawn in. When $t = \tau$ the angle $= 2\pi = 360^\circ$.

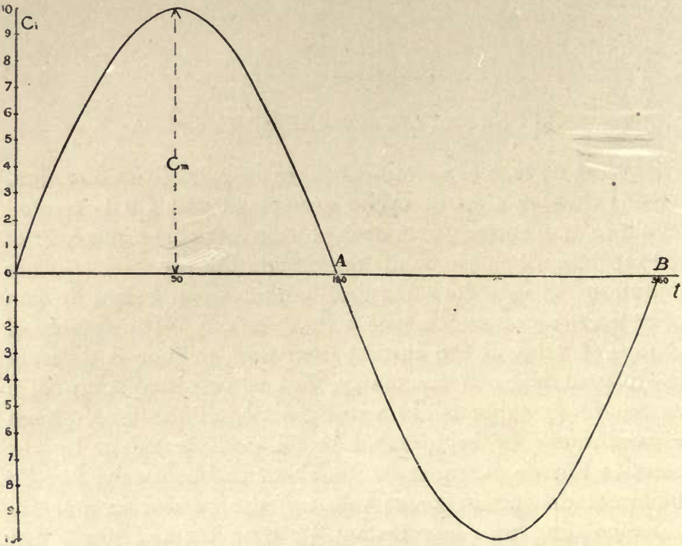

FIG. 20.

The time τ is called the time of one complete cycle or period, and ranges with different makers from $\frac{1}{20}$ second to $\frac{1}{130}$ second, though smaller values were used in past times. It has a very general value in English practice of either $\frac{1}{80}$ or $\frac{1}{100}$ second, the value $\frac{1}{130}$ being American. It is thus customary to describe the circuit as one having, for example, 100 periods per second; or to say it has a frequency or periodicity of so and so per second. Calling the frequency p periods per second, we may put instead of the above expression—

$$C_i = C_m \sin 2\pi pt$$

where p stands for $\frac{1}{\tau}$.

Now, it will be quite evident that in some cases the rapidity with which the current changes in value will materially affect the conditions of working; and it will be necessary to investigate as fully as possible the effect of any such disturbance for the different conditions possible. There are four ways in which an electric current can become manifest to us—

(1) Chemical.
(2) Thermal.
(3) Magnetic.
(4) Electrostatic.

And we will examine the effect produced for each of these considered separately.

I. **The Chemical Effect.**—With continuous or uni-directed currents the chemical change produced is always proportional to the strength of the current and the time during which it flows. It is also dependent upon the direction of flow, being of the nature of a solution where the current enters, and a deposition where the current leaves the circuit. Thus, if a current of one ampère be passed for the space of time of t seconds through a solution of copper sulphate, it will dissolve from the plate of copper at which it enters the solution, and deposit upon the plate at which it leaves the solution, an amount of copper which may be expressed as—

$$t \times 0{\cdot}00032959 \text{ gramme}$$

Thus, during the time from o to A in Fig. 20, we have the current flowing all in one direction; and if such current be passed through a solution of copper sulphate, it will cause a deposit of copper on one plate and a solution of copper from the other plate, which will be in amount equal to the product of the average strength of the current into the time, viz. $\frac{T}{2}$ seconds, and into the numeric used above (the electro-chemical equivalent of copper).

But if the circuit be not broken at the instant when the current becomes zero, the contrary effect will be produced, and there will now be a re-solution of copper from the plate which before received the deposit, and a deposition on the plate from which the copper was dissolved; and this reverse effect will be of exactly the same magnitude as the previous effect if the current be maintained for the next or second half-period.

It thus becomes apparent that an alternating current cannot produce any chemical effect; for what is done during any half-period is exactly undone during the next.

When, however, the current is "rectified" by a commutating device, timed so as to exactly reverse the connections at the instant the current changes in direction, we shall get a total chemical effect which will be proportional to the product of the time and the average value of the current, or is—

$$C_a t 0{\cdot}00032959 \text{ grammes of copper}$$

Now, it can be shown, by carefully measuring the area of the half-wave from 0 to A, that its average height is 0·6366 times C_m the maximum height, and we thus have that the amount of copper deposited by an alternating current which has been rectified is—

$$0{\cdot}6366 \times C_m \times 0{\cdot}00032959 \times t \text{ grammes}$$

and knowledge of the electro-chemical equivalent of any other material will similarly give the sum-total of the effect produced, as, for instance, in charging a secondary battery.

II. **Thermal Effect.**—This is, at any instant, proportional to the rate of expenditure of energy, and is independent of the direction of the current; thus, it is proportional to—

$$C_i^2 R$$

In order to find an expression for the average effect so produced, it is necessary to draw a curve which represents the quantity C_i^2R, and to find the average height of such a curve. Thus, if $C_m = 10$ ampères, and R be 1 ohm, this curve of energy expenditure will have zero values coinciding with the zero values of current, and will rise to a maximum of 100 at the time when the current is maximum. If now the length corresponding to half a wave be divided up into 18 parts, and the area of each such slice be taken, all such areas being added, it will be found to amount to 899·6, when the width of each slice is called unity, and its height taken as that of its middle value as marked on the scale of the diagram. The average height is consequently—

$$\frac{899{\cdot}6}{18} = 49{\cdot}95$$

which would have come out = 50 exactly, if we had been able to estimate values more exactly. Now, this 50 is the average rate of expenditure of energy in the circuit, and consequently is of the form—

$$C^2R$$

but we know R to be unity, whence it follows that the C^2 must be = 50, or the effect produced is the same as would be the case had a current of magnitude—

$$\sqrt{50} = 7{\cdot}071 \text{ ampères}$$

been maintained constant for the corresponding time. But this current is—

$$0{\cdot}7071 \times \text{the maximum}$$

and is also the—

$$\sqrt{(\text{mean square})}$$

It is called the "virtual" value, and is really the value about which we should know.

As far, then, as working in the circuit in heating it, the alternating current which follows a sine law has an effect which is the same as would be produced by a continuous current having a magnitude of—

$$0{\cdot}7071 \times C_m$$

and lasting for the same length of time.

From this value the maximum can be found by multiplying the virtual by $\frac{1}{0{\cdot}7071}$, or 1·414, or—

$$\text{max. } C = C \text{ virtual} \times 1{\cdot}414$$

Before going on to consider the magnetic effect in detail, it will be as well to point out that an electro-dynamometer will be affected by the value of the square of the current at any instant as with continuous currents, and will thus read a quantity which is this $\sqrt{\text{mean square}}$, or virtual value.

It follows from this that the rate of expenditure of energy in heating a circuit will be proportional to the product of the resistance in ohms into the square of the electro-dynamometer reading.

And also, when the current has been rectified, the electro-dynamometer will still read the virtual value, or—

$$0{\cdot}7071 \times \text{maximum}$$

But it has been shown that the chemical effect produced by a rectified current is only proportional to—

$$0{\cdot}6366 \times \text{maximum}$$

whence it follows that the dynamometer will indicate a value which is—

$$\frac{0{\cdot}7071}{0{\cdot}6366}, \text{ or } 1{\cdot}11 \text{ times too much}$$

III. **The Magnetic Effect.**—This may take place under circumstances which may be summed up in the following order, beginning with the condition where no magnetic effect can be produced:—

(*a*) No magnetic effect possible; circuit non-inductive.

(*b*) Magnetic effect set up in a medium which has a uniformly constant permeability—say air where $\mu = 1$—and which is also non-conducting.

(*c*) Medium having constant $\mu = 1$, but being conducting.

(*d*) Medium having variable value of μ, as in a closed iron circuit. Non-conducting, *i.e.* laminated.

(*e*) Medium with μ variable but conducting; closed circuit.

(*f*) Medium with μ variable, but of composite character, as in the case of an open-iron circuit. Non-conducting.

(*g*) Medium with variable μ, composite and conducting.

For the sake of simplicity, capacity is supposed to be absent in all the above cases.

(*a*) Non-inductive circuit. Here the only effect which is possible has already been considered in the section on the thermal effect. It will be as well, however, to complete that case by considering the value of the E.M.F. in the circuit.

In order to cause a current of C_i ampères to flow in a circuit having a resistance of R ohms, we know that an E.M.F. of—

$$E_i = C_iR \text{ volts}$$

will be necessary, and this being the only E.M.F. in the circuit, will necessarily be always proportional to the current, and is then said to be in phase with the current, having its zero values and the positive and negative signs at the times when the current has these values and signs.

When this is the case, we can represent the rate of expenditure of energy in the circuit by the product of the readings of an electro-dynamometer and an electro-static voltmeter, both of which instruments give the virtual or $\sqrt{\text{mean square}}$ values. Thus—

$$\text{watts} = C_v{}^2R = C_vE_v$$

where C_v and E_v stand for the virtual values, that is are 0·7071 times the maximum.

(*b*) Magnetic effect when the medium surrounding the circuit is, say air, having $\mu = 1$ and constant, and also being non-conducting.

Consider three rings superposed, the middle one carrying an alternating current. These rings are represented in section in Fig. 21.

The middle ring has a current flowing in it of a kind represented in Fig. 20, and consequently rising in a certain direction which we will call positive, when it is as indicated by the arrow in Fig. 21, where the middle ring has then a polarity of N face at the top and S face at the under side. Imagine the current growing from 0 upwards; as it grows lines of force will increase in number, threading through the upper ring so as to enter its section inwards from all points round it. An effect will consequently be produced which will be of the same kind as if the upper ring were being dropped upon the middle ring when that is producing a constant magnetic flux. By Fleming's rule it is easy to see that there will be an E.M.F. set up in the upper ring, having a direction contrary to that in the middle ring, or as shown by the arrows; and this E.M.F. will be greatest where the rate of change of lines is greatest, and will be least, or zero, when the rate of change of lines is zero. Thus, it will be greatest at the moment the current in the middle ring starts, and will be zero at the moment when that current has attained the maximum value.

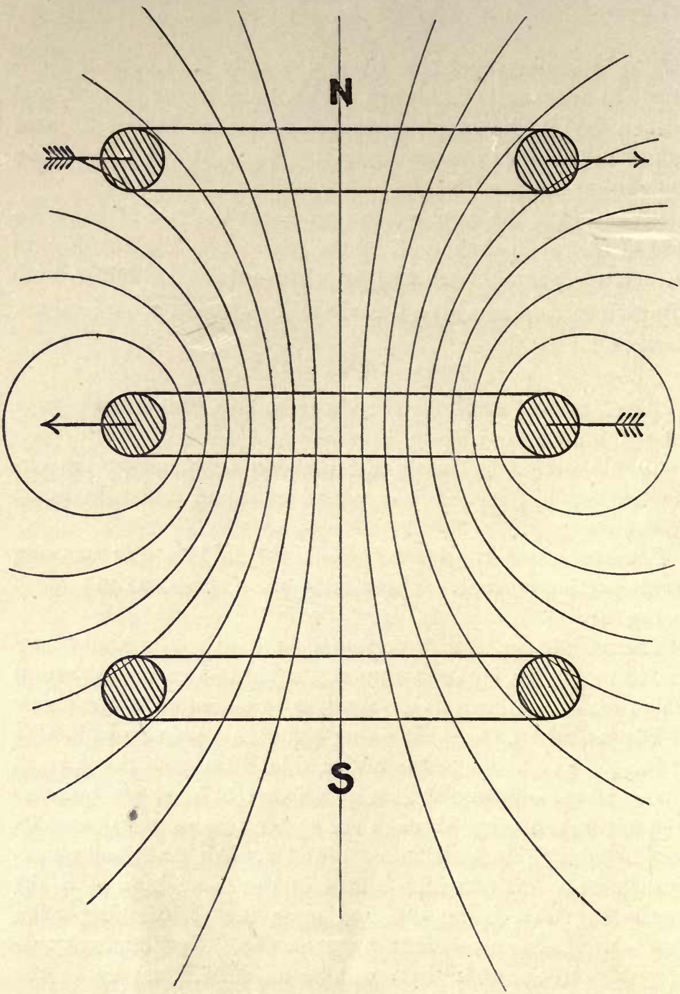

FIG. 21.

Again, when the maximum has been passed and the current in the middle ring is now decreasing, there will be produced in the upper ring an effect of the same kind as if that ring were being withdrawn upwards from the field produced by a constant current in the middle ring, and having a direction still shown by the arrows in the middle ring. Fleming's rule shows that such effect will be to generate in the upper ring an E.M.F. contrary in direction to that indicated in Fig. 20, and consequently similar in direction to that in the middle ring. Also the magnitude of such E.M.F. will be least when the rate of change of lines is least, and greatest when such change is greatest; that is, it will be least at the moment when the current in the middle ring is a maximum, and greatest when the current there is on the point of change from positive to negative.

A similar investigation will show that when the current in the middle ring is reversed in direction, we shall get an exactly similar state of affairs only in an opposite sense. And to some scale or other we can represent the relationship between the current in the middle ring and the E.M.F. set up in the top ring, as also that in the lower ring (which will obviously be of the same order), as is shown in Fig. 22, where C represents the value and direction of the current in the middle ring, and the dotted line marked ϵ represents the value (to some arbitrary scale) and direction of the E.M.F. induced in the other rings. In consequence of its opposing character, this E.M.F. is called a back E.M.F.; and just as it is induced in the upper and lower rings by the changing current in the middle ring, so and much more, therefore, will a similar E.M.F. be induced by the middle ring in itself—*self-induced*, and therefore called the—

B.E.M.F. of self-induction

Leaving for the present the determination of the exact value of such E.M.F., we have next to consider the conditions under which such a current as C in Fig. 22 can flow in a circuit having this magnetic property. It is quite clear that an

E.M.F. will be required at any instant freely acting in the circuit in such a direction as to do two things—

(1) Nullify any B.E.M.F. which may be present; and

(2) Leave sufficient over to send the current C against the ohmic resistance of R ohms.

This last quantity, RC_i, has been drawn in above C in Fig. 22, and all that remains to be done is to add, for all instants in time, RC_i to an amount equal and opposite to ϵ to get the value of E, the E.M.F. which must be supplied to or impressed upon the circuit. This has been done after the manner indicated by Dr. Barfield in 1886,[1] and the curve E so produced is marked in full line. It will now be noticed that this E curve does not coincide in phase with the C curve which it produces; in fact, the C curve *lags* after E by an amount AB, which we will call the angle λ. A

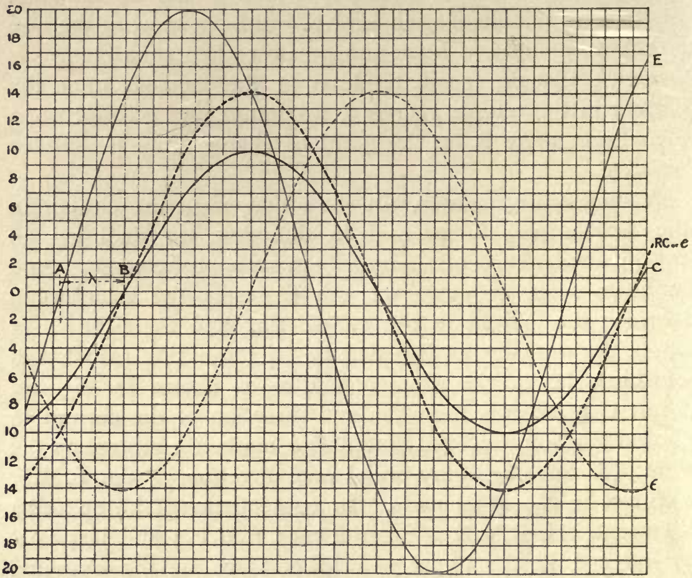

FIG. 22.

---

[1] "Industries," vol. i. p. 17.

little consideration will show that this lag λ will depend upon the relative values of RC and ϵ, since E partakes most in character and phase with that one which is of greatest value. Thus, when ϵ is zero, as in a non-inductive circuit, $E = RC$, and there is no lag. But when RC is very small and ϵ very large, then E is equal and opposite to ϵ, and there is a lag of practically $\lambda = 90°$.

Now, if we regard RC as a sine curve, it is clear that the opposite of ϵ must be a cosine curve; and as E is the sum of these two, we have that—

$$E = RC_m \sin \frac{2\pi}{\tau} t + \epsilon_{1m} \cos \frac{2\pi}{\tau} t$$

when RC_m is the maximum value of RC and ϵ_{1m} the maximum value of ϵ_1. Now, this can be shown to be equivalent to—

$$E = \sqrt{(R^2C^2_m + \epsilon^2_{1m})} \sin \left(\frac{2\pi}{\tau} t + \lambda \right)$$

where λ is an angle whose tangent is—

$$\frac{\epsilon_m}{RC_m}$$

Calling $RC_m = e$ the active E.M.F., we may write shortly that the impressed E.M.F.—

$$E = \sqrt{e^2 + \epsilon^2} \text{ in magnitude}$$

and has a phase-position which is λ degrees earlier than the current, λ being such an angle that—

$$\tan \lambda = \frac{\epsilon}{e} = \frac{\text{B.E.M.F.}}{\text{active E.M.F.}}$$

Thus, when the circuit is of such a nature that a very powerful magnetic field can be set up by a small current, and the resistance is also small, there is a considerable lag of the current after the E.M.F., and such E.M.F. is very much greater than that necessary to merely cause the current to flow in the ohmic resistance of the circuit. But again, should the resistance of the circuit be very large, so that the active E.M.F. RC required is large compared to the B.E.M.F. set up by the changing magnetism, then the current does not lag much, and E is only slightly larger than that required for RC.

This can also be graphically illustrated by remembering that the E.M.F. to be supplied, E, is the resultant of the other two E.M.F.'s which are present, viz. e and ϵ, which two are also at right angles. Thus we can construct a figure which shall show this resultant E.M.F., as is shown in Fig. 23, which shows the

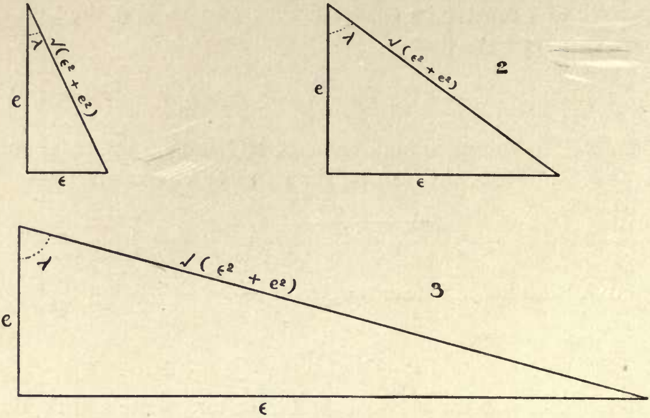

FIG. 23.

magnitude of the sum of e and ϵ, and also the nature of its position. Considering the phase-position of e to be represented by the vertical line, and remembering that the current C is in phase with e, it is clear from the three triangles in Fig. 23 that the difference in phase, as indicated by the departure from the vertical of line $E = \sqrt{e^2 + \epsilon^2}$, is least of all when ϵ is small compared with e, is very marked when ϵ is comparable to e in value, whilst, however large ϵ may become, the sum $E = \sqrt{(e^2 + \epsilon^2)}$ can never be more inclined to e than the horizontal position, or can never differ in phase from e by more than 90°.

Further investigation will also show that the rate of expenditure of energy in the circuit will depend upon the lag; being greatest when there is no lag, and least when the lag is large.

If the instantaneous values of the product of E and C be taken for the three possible cases, viz. no lag, small lag, and large lag, we shall get the waves of watts, as represented in Figs. 24, 25, 26, where the areas enclosed by the dotted lines W will represent the work done. It will be noticed, in the case of no lag, that all the work is positive that is done in the circuit, and it can be shown that the average height of the watts area is then—

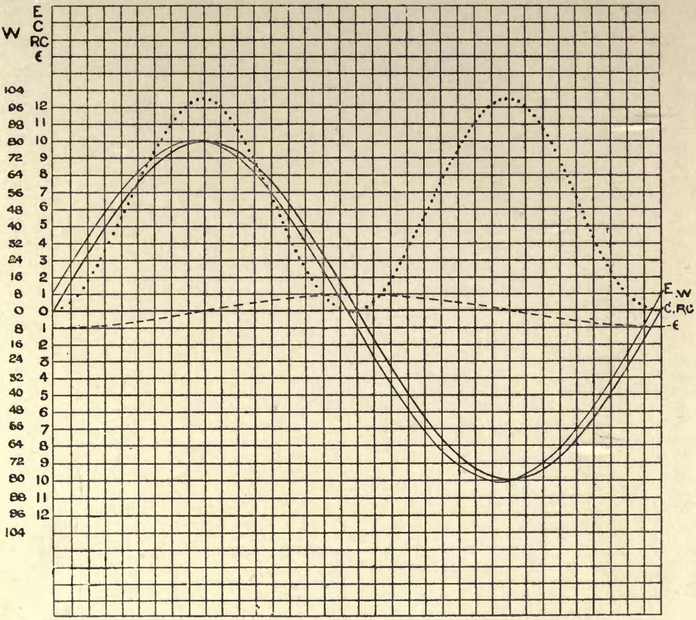

FIG. 24.

$$E_v C_v = \text{watts}$$

—the voltmeter reading multiplied by the ammeter reading.

But in the case of small lag there is part of the area of work marked on the lower part of the zero line and called negative; this must be understood as representing the case when the circuit is not receiving energy, but is giving it back to the engine, the resultant work being the difference between the +

and − areas, an amount which can be shown to be equal as an average value to—

$$E_vC_v \times \cos \lambda = \text{watts}$$

Lastly, in the case of maximum lag = 90° the + and −

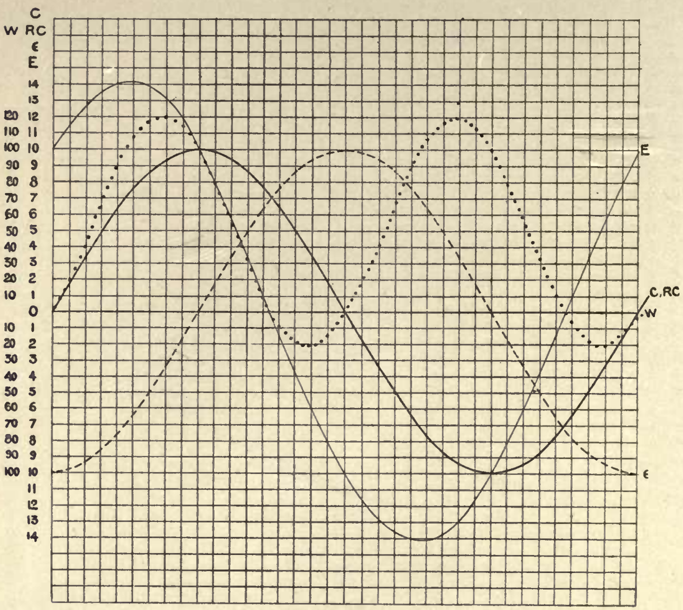

FIG. 25.

areas are equal, and consequently the work done nil. This can also be represented as—

$$\text{watts} = E_vC_v \times \cos \lambda$$

where $\lambda = 90°$ and $\cos \lambda = 0$. Such a condition is, of course, not absolutely attainable, though in some cases the lag is very nearly 90°.

The coefficient $\cos \lambda$ has been called by Dr. Fleming the *power-factor*, and though it cannot be directly measured, it is of great importance. As shown above, its value can be found from a knowledge of the ratio of the back E.M.F. to the active E.M.F., or of—

$$\tan \lambda = \frac{\epsilon}{e}, \text{ whence } \cos \lambda = \frac{e}{\sqrt{(e^2 + \epsilon^2)}} = \frac{e}{\mathrm{E}}$$

The active E.M.F. is easily found for any circuit whose resistance and current are known; and we will now proceed to ascertain ϵ.

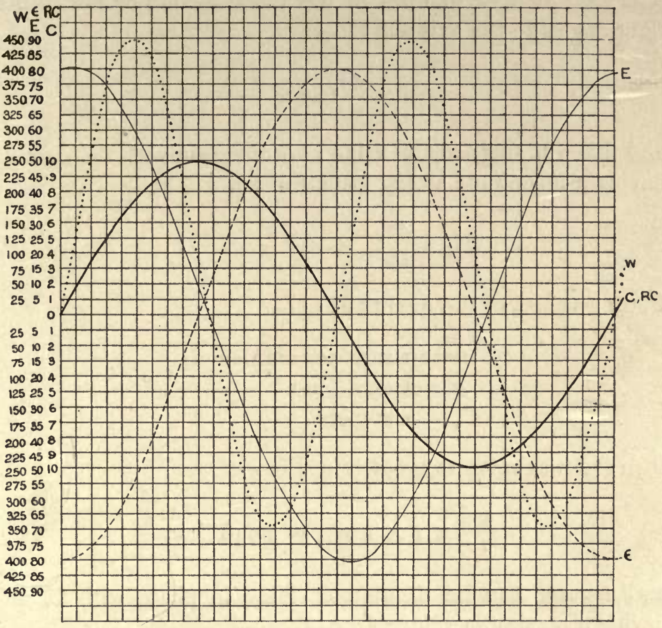

FIG. 26.

The magnitude of the B.E.M.F. *set up in a circuit of constant* $\mu = 1$.

Consider an infinite solenoid having no magnetic core, and being—

d cms. = mean diameter, *i.e.* *d* is the diameter to the middle of the depth of winding.

w = number of convolutions per cm. of length of coil.

The value of the magnetic density set up inside this solenoid will be—

$$\frac{4\pi}{10}Cw$$

when C is the instantaneous value of the current in ampères. The total number of lines in the plane section of the core at right angles to the axis will be—

$$N = \frac{4\pi}{10}Cw \times \frac{d^2\pi}{4}$$

and this will alternate in value and direction with the current, having a magnitude at any instant which will be—

$$N_i = \frac{4\pi}{10}C_m w \times \frac{d^2\pi}{4} \sin \frac{2\pi}{\tau}t$$

Putting values to the quantities of—

$w = 1$ turn per cm. length
$d = 2$ cms., and
$C_m = 1$ ampère

then the maximum value of N will be—

$$\frac{4\pi^2}{10} \times 1 \times 1 \times 1 = 3{\cdot}948 \text{ lines}$$

and a curve can be drawn coinciding in phase with C, and having a maximum value of 3·948, or—

$$N_i = 3{\cdot}948 \sin \frac{2\pi}{\tau}t$$

Now, the E.M.F. which a changing magnetic field can set up is, in volts, equal to the number of lines gained or lost per second multiplied by the number of turns or convolutions so gaining or losing lines, divided by 10^8. As before shown, the lines are changing at the greatest rate at the instant when they are zero—that is to say, the slope of the lines' curve is then a maximum.

Let the periodic time $\tau = \frac{1}{100}$ second, then the number of lines corresponding to the instant in time $0{\cdot}00002\dot{7}$ second after the zero value, will be—

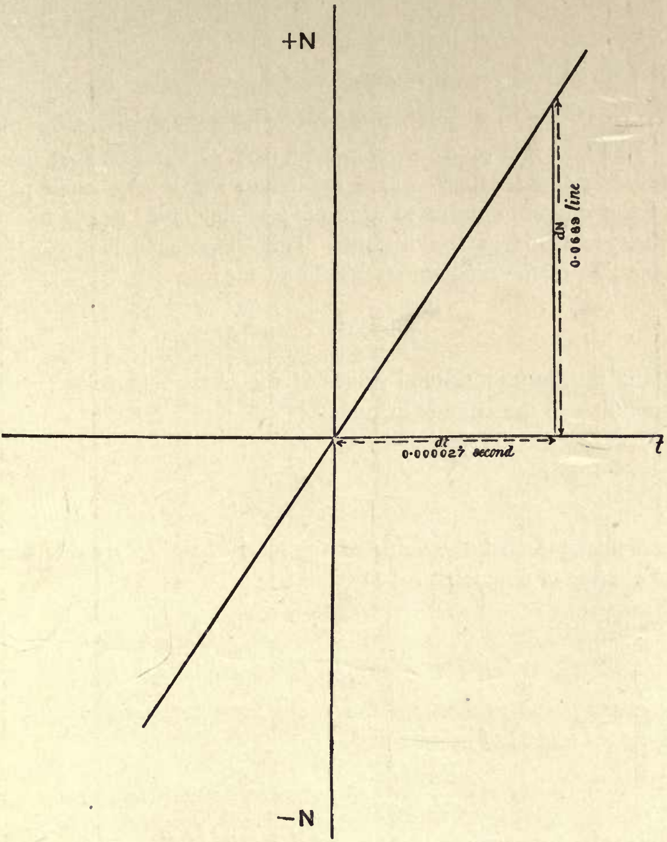

FIG. 27.

$$N_i = 3{\cdot}948 \times \sin\left(\frac{2 \times 3{\cdot}1416}{\frac{1}{100}} \times 0{\cdot}00002\dot{7}\right)$$

$$= 3{\cdot}948 \sin 0{\cdot}017452$$

which angle is in circular measure, and must be multiplied by $\frac{180}{\pi}$ to get degrees, or—

$$N_i = 3{\cdot}948 \sin\left(0{\cdot}017452 \times \frac{180}{\pi}\right)$$
$$= 3{\cdot}948 \sin 1^\circ$$

But sin 1° is 0·01745, whence the value of N_i is—

$$N_i = 3{\cdot}948 \times 0{\cdot}01745 = 0{\cdot}0689 \text{ line}$$

In Fig. 27 is given a magnified part of the curve of lines showing these values, which are taken for a very small part of the curve, in order to approximate the more nearly to the absolute slope at the instant. This slope is called, in the language of the differential calculus—

$$\frac{dN}{dt} \text{ when } dN \text{ and } dt$$

stand for the infinitesimal values of the change in lines corresponding to the change in time.

In our case the slope—

$$\frac{dN}{dt} = \frac{0{\cdot}0689}{0{\cdot}000027}$$

and we have that the value of the E.M.F. set up by this rate of change of magnetism is—

$$\epsilon = \frac{\frac{wdN}{dt}}{10^8} \text{ volts}$$

whence, putting values as above, we have for the E.M.F. per 1 cm. of length of infinite coil—

$$\epsilon = 1 \times \frac{0{\cdot}0689}{0{\cdot}000027} \div 10^8 = 0{\cdot}0000248 \text{ volt}$$

which is the maximum value of the B.E.M.F. curve, and, as has been previously shown, has a negative value in accordance with Lenz's Law.

Now the value—

$$\frac{dN}{dt} = \frac{0{\cdot}0689}{0\ 000027} = 2480$$

is the maximum rate of change of lines, and can be found from the maximum value of N = 3·948 by multiplying by $\frac{2\pi}{\tau}$ or 628·32. Thus—

$$\frac{dN}{dt}\text{ maximum} = 3{\cdot}948 \times 628\ 32 = 2480$$

and we can write the maximum value of the B.E.M.F. set up in a coil, subject to a magnetic field changing after the manner of a sine curve as—

$$\epsilon_{max} = \frac{2\pi N_{max} w \frac{1}{\tau}}{10^8}$$

$$= \frac{2\pi N_m w p}{10^8}$$

putting $p = \frac{1}{\tau}$ = periods per second.

And the virtual value of this, as it is a sine curve, can be found by multiplying 0·7071, or—

$$\epsilon_v = \frac{0{\cdot}7071 \times 2\pi N_m w p}{10^8}$$

$$= \frac{4{\cdot}443 N_m w p}{10^8} \text{ virtual volts per 1 cm. of length}$$

In ascertaining the value of ϵ for any circuit, it is necessary to know N, and this is generally the real difficulty. So far we have considered an infinite solenoid; but in practice we have quite finite circuits to handle, and it is sometimes very difficult to accurately state their dimensions. If we next consider a solenoid of finite length, it will be at once perceived that the conditions are now no longer so simple, though we shall have the same value for the E.M.F., set up in the middle cm. length of the coil, provided the ends be fairly remote. Thus in a—

Finite Solenoid without magnetic core, we have that the B.E.M.F. per cm. length, at and near the middle is—

$$\epsilon = \frac{4{\cdot}443 N_m w p}{10^8} \text{ volts virtual}$$

but near the ends, owing to the straying of lines, the value will not be so great. The coil will, in fact, act as though it had a length shorter than its actual length by an amount which is found to be very nearly equal to half the mean diameter. Thus in Fig. 28 is represented in section a coil having a total length l cms., a mean diameter of d cms., and let us say of w turns per cm. length. If this coil were part of an infinite solenoid of similar construction, it would have a B.E.M.F. per cm. of its length of—

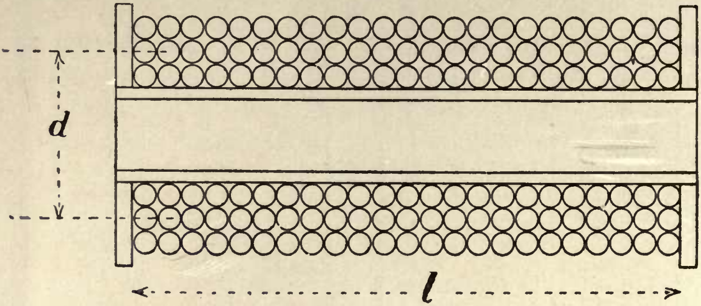

FIG. 28.

$$\epsilon = \frac{4\cdot443 N_m w p}{10^8} \text{ volts}$$

or—

$$\epsilon = \frac{4\cdot443 C_m w \frac{4\pi}{10} d^2 \frac{\pi}{4} w p}{10^8} \text{ volts virtual}$$

per cm. length, or a value of—

$$\epsilon = \frac{4\cdot443 C_m w^2 \frac{4\pi^2}{40} d^2 p}{10^8} \, l \text{ volts}$$

total. But as it is finite, its B.E.M.F. will approximately be—

$$\epsilon = \frac{4\cdot443 C_m w^2 \frac{\pi^2}{10} d^2 p}{10^8} \left(l - \frac{d}{2} \right) \text{ volts virtual}$$

which comes to saying that the effective number of lines set up in this coil is not uniformly $C_m\frac{4\pi}{10}w$ per sq. cm. of cross-sectional area, but is only—

$$C_m\frac{4\pi}{10}w \times \frac{l - \frac{d}{2}}{l}$$

per sq. cm. average. Nevertheless the whole M.M.F. in ampère-turns is at the maximum crest—

$$\mathfrak{M} = C_m\frac{4\pi}{10}wl$$

whence it follows that part only of this is concerned in causing the flux through the cylindrical air core, the rest obviously being required to send the lines through the surrounding air space.

Now, the reluctance of the air core can be expressed as—

$$\mathfrak{R}_c = \frac{l}{a\mu} = \frac{l}{\mu\frac{d^2\pi}{4}} = \frac{4l}{d^2\pi\mu}$$

and to send—

$$\frac{d^2\pi}{4}C_m\frac{4\pi}{10}w\frac{l - \frac{d}{2}}{l} \text{ lines}$$

through this reluctance will require a M.M.F. in ampère-turns of—

$$\frac{10}{4\pi} \times \text{N} \times \mathfrak{R}$$

or—

$$\frac{10}{4\pi} \times \frac{d^2\pi}{4}C_m\frac{4\pi}{10}w\frac{l - \frac{d}{2}}{l} \times \frac{4l}{d^2\pi} \text{ ampère-turns}$$

μ being unity; which may be simplified to—

$$C_m w\left(l - \frac{d}{2}\right) \text{ ampère-turns}$$

thus leaving $C_m w\frac{d}{2}$ ampère-turns to overcome the reluctance of the air space surrounding the coil: and as the same number of lines, viz.—

$$\frac{d^2\pi}{4} C_m \frac{4\pi}{10} w \frac{l - \frac{d}{2}}{l} \text{ lines}$$

traverse this air space, it follows that the reluctance of it will be—

$$\mathfrak{R}_s = \frac{\text{ampère-turns}}{0{\cdot}8 \times \text{lines}} = \frac{C_m w \frac{d}{2}}{\frac{10}{4\pi} \times \frac{d^2\pi}{4} C_m \frac{4\pi}{10} w \frac{l - \frac{d}{2}}{l}}$$

$$= \frac{1{\cdot}273 l}{d(2l - d)} = \text{reluctance of the air space}$$

This expression is not often required in considering the behaviour of an air-core coil, but will be required when dealing with a composite core.

We have next to consider the effect produced when the medium surrounding the coil, *including the coil itself*, is more or less conducting.

(*c*) Air core, μ constant and unity, but medium conducting. These conditions will be fulfilled by the coils in Fig. 28, if some or all of them be individually joined so as to make complete electric circuits.

We have seen that a given impressed E.M.F.—

$$E = \sqrt{(e^2 + \epsilon_1^2)} = \sqrt{(R^2C^2 + \epsilon_1^2)}$$

where ϵ_1 stands for the equal and opposite of the B.E.M.F., which is produced by the current C. It follows from this that the current which a given E.M.F. impressed on a circuit will produce must be sufficiently large to provide the necessary maximum number of lines to produce such a value of ϵ as will leave so much E.M.F. over as is required to send C ampères against the ohmic resistance R. In circuits as generally met with in transformers, impedance coils, and such like, the value of e is usually small compared to ϵ, and it is now necessary to consider the effect which will be produced by variations in the magnitude of e for a given value of E. Thus, let—

E = 100 virtual volts

e = CR, and R be, say 1, whilst C for some cause or other is made to range from 1 to 20

We have, then, that—

$$\epsilon_1 = \sqrt{E^2 - e^2}$$

and will be as shown in the following table:—

E	C	ϵ_1
100	1	99·995
100	2	99·979
100	3	99·954
100	4	99·919
100	5	99·874
100	6	99·819
100	7	99·754
100	8	99·679
100	10	99·498
100	15	98·868
100	20	97·979

which shows that e may range from 1 per cent. to 10 per cent. of E, without affecting the value of ϵ_1 produced by more than $\frac{1}{2}$ per cent., and may even be 20 per cent. of E, with only a trifle more than a corresponding reduction in ϵ, of 2 per cent.

It consequently follows that the number of lines produced by an alternating P.D. applied to a coil will depend almost entirely upon the P.D., and will be almost independent of the nature of the core or medium surrounding the coil, even when that medium is of such a nature as to make the current required to magnetize so large as to represent a P.D. required for the ohmic resistance of as much as 20 per cent. of the impressed P.D. In other words, this comes to saying that the constancy of the magnetic field produced is, as it were, the sole concern of the impressed E.M.F. Should anything arise tending to undo the magnetic effect first set up, the result will be such an adjustment of the current with respect to the E.M.F. as to make the magnetic flux produced, and consequently the B.E.M.F. still the same as at first, subject only to the restrictions indicated in the table on p. 135.

This important fact was first pointed out by the author in a course of lectures in the summer term at the Municipal Technical School, Manchester, in 1892, and was afterwards confirmed by some experiments mentioned by Prof. S. P. Thomson in a paper before the Physical Society in 1894.

Now, when the rings in Fig. 21 are considered to be closed, the E.M.F. set up in them can cause a current to flow, which current will obviously disturb the magnetic field set up by the middle coil unless, as indicated above, the current in that ring is altered in such a manner as to exactly balance such disturbance, and still leave sufficient to produce the original magnetic flux. Foucault currents or eddies set up in the mass of the copper coils, in the iron core, and in any other conducting body within the influence of the magnetic field are of this nature, as is also such a state as current flowing in one of the closed rings, which may thus represent a loaded secondary coil. To exactly balance the effect produced, there will have to flow in the middle ring a current which will be magnetically equal and opposite to these "secondary" currents. Such a current may be called the balancing current C_b, and when the current to be balanced does not lag after its own E.M.F. will come in opposite phase to the induced B.E.M.F., as is indicated in Fig. 29, where C_m stands for the current required to produce the magnetic flux before any "secondary" load came on, whilst C_b is such addition to C_m as will nullify the "secondary" load. The curve marked C_p is the sum of C_m and C_b, and may be taken as the current produced by the given E.M.F., acting in a circuit containing a given B.E.M.F. and secondary load.

The full consideration of such an effect will be deferred till the question of regulation in a transformer presents itself. Briefly we may sum up by saying that the effect of a conducting property in the system will only be to cause an alteration in magnitude and phase of the current, and affecting only in a very small degree the value of the magnetic disturbance set up.

And also that the number of lines produced in a given coil by a fixed alternating P.D. will be practically the same,

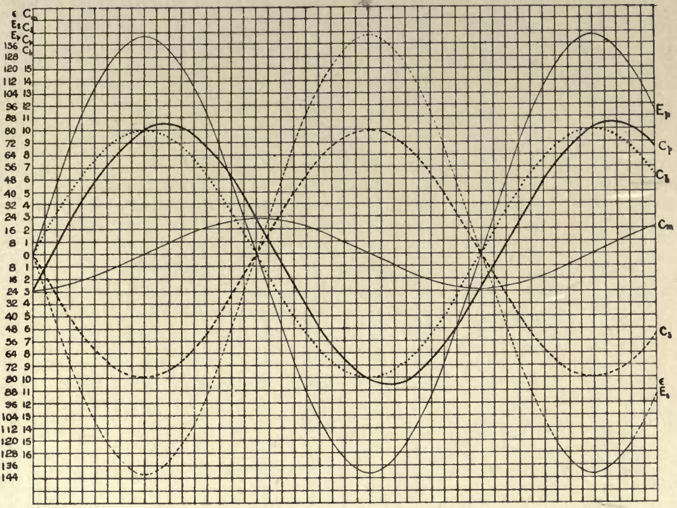

FIG. 29.

whether that coil has a core of air or of the best possible laminated iron.

(*d*) We have now to consider the case for a coil surrounded by a medium, such as iron, having a high but variable value of μ, and for simplicity so well laminated as to be non-conducting.

In iron of even the very best and softest quality it is found that the relationship between the magnetizing force and the flux produced is not of a simple nature; and further, that when the magnetizing force changes periodically, the flux changes periodically, but after a different law. In Fig. 30 is shown this relationship, the enclosed area being a measure of the work required to overcome the molecular friction in turning the particles of iron through a complete circle. Professor Ewing has published tests of a large number of samples of iron, and from his curves tables have been read off, showing the values of H required for numbers of points round each cycle for different maximum values of β.

Corresponding with these a table is given, showing the waste

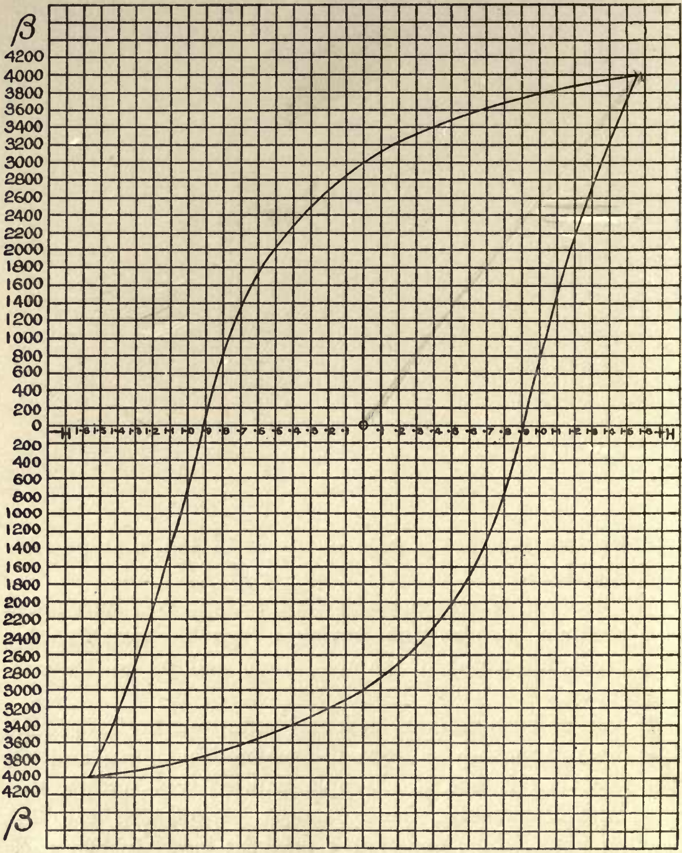

FIG. 30.

in friction in ergs per cubic centimetre per cycle, called h as before, for dynamo calculations.

Now, the nature of the motion of the iron particles will depend upon the change from moment to moment of the

E.M.F. acting in the circuit, and when this is of the simple sine curve nature, it is found that the value of β also changes after the manner of a sine curve. This is shown by the fact that the shape of the secondary curve of E.M.F. is practically a sine curve. Working backwards it is then easy, as was first pointed out by Mr. Evershed, to plot the corresponding values of current required when the curve of lines has been drawn. The cycle of magnetism in Fig. 30 is supposed to be that found when the iron has settled down to a steady state, and it will thus be seen that the manner of change is such as would be met with in going round the figure in a counter-clockwise direction. It thus becomes apparent that, owing to hysteresis, when β is zero, but becoming of positive magnitude, then C has to be of quite a considerable value, and does not increase nearly in accordance with the rise of β from 0 to maximum. Also that when the maximum β has been reached (which *always* coincides with the maximum value of the current), the current declines in value much more quickly than β, having even to become of opposite sign before β has again become zero. The reverse process takes place for the negative half wave of β, and a complete wave is shown in Fig. 31, where the current curve C is drawn to a larger scale than β for convenience. It will be noticed—

(1) That the value of C is very small, owing to the high value of μ, which is thousands of times what it would be in air.

(2) That the C curve does not coincide with β, though, of course, ϵ will have its customary position or lag 90° after β. Consequently, when the B.E.M.F. is the major item in determining E, E will still come in nearly opposite phase to ϵ, and consequently C will not lag so much after E as would have been the case with a similar air core.

This decreased lag is of great importance, as, together with the altered shape of the current curve, it determines the supply of energy which is wasted in hysteresis, in addition to that necessary for C^2R.

Now, we can easily draw in such a curve as that shown, and know its maximum value. But as indicated on an electrodynamometer we shall of course get its virtual value, and this will no longer be 0·7071 × maximum value. Thus, in such cases as impedance coils made with a closed iron core, and where the current is made fairly large by making the value of

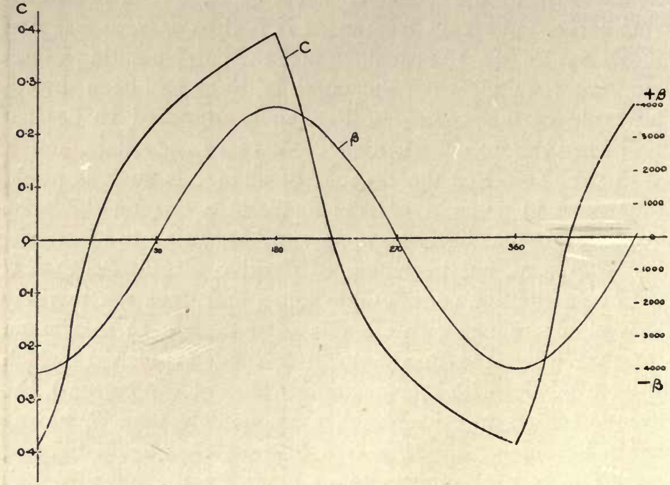

Fig. 31.

w small, this must be remembered. Owing mainly to the irregular shape of the C curve in such a circuit it is general to make impedance coils with an open iron circuit, the advantage of which will be indicated in the section devoted to the consideration of the open iron circuit.

(*e*) Consideration of the closed iron circuit when conducting, or having eddy current in the iron core, or a secondary load. The nature of the disturbance set up by Foucault currents, or a closed secondary will be of precisely the same character as in the other case considered; it is only necessary to point out that, as in general the magnitude of C_m, as we will call the current required to produce the flux in the iron, is very small, and lags little after E, whilst the current to balance the Foucault losses has a magnitude which is also small, but which is practically in phase with the impressed E.M.F., their sum is consequently larger than either, and lags less than C_m, whilst when there is a secondary load to balance there is practically no lag of the current after E, for even a small value of the necessary balancing current.

(*f*) Medium composite in character, as, for instance, in the case of an open iron circuit impedance coil. Here, as in the composite circuit of a dynamo, calculation has to be made for the several items. Thus for the straight iron core of length l and sectional area a, to be magnetized to a total of N lines, we have with a given winding of coil surrounding it, that the M.M.F. required will be—

$$Cwl = lf\left(\frac{N}{a}\right) = lf(\beta)$$

a reference to the tables being necessary to find the values of C_w corresponding to various values of β.

In addition to this there will be required a current to send the same lines through the air space. This may be found by considering the reluctance of the air space, as given in the formula—

$$\mathfrak{R}_s = \frac{1{\cdot}273l}{d(2l - d)}$$

where l and d now must be considered to have the values for the iron core. To force the lines through this space additional ampère-turns will be required of magnitude—

$$0{\cdot}8N\mathfrak{R}_s$$

or—

$$C_1wl = 0{\cdot}8N\frac{1{\cdot}273l}{d(2l - d)}$$

Now, that part of the current C, which is required for the iron core, will be, as indicated, a saw-tooth curve like that in Fig. 31, will lag somewhat after E, but not nearly to 90°, and will also be small in magnitude compared to C_1, which will be a sine curve, will be large, and will lag practically 90° after E. The total magnetizing current will consequently lag much, will partake mostly of the sine curve shape of C_1 and will, on the whole, be very large compared to the case of a closed iron circuit. An example will render this clear. Let the iron core (laminated perfectly) be 30 cms. long and 5 cms. in diameter. Let the value of β be 5000 total, as an average of maximum value throughout the whole length. Let there be 4 turns per cm. length of core. Then—

$$Cwl = lf(\beta) = C \times 4 \times 30 = 30 \times 1{\cdot}41 = 0{\cdot}35 \text{ ampère}$$

maximum value of the saw-tooth shaped curve for the iron core. The total number of lines produced will be a maximum of—

$$N = 5000 \times \frac{5 \times 5 \times 22}{4 \times 7} = 98{,}200$$

And the reluctance of the air space will be—

$$\mathfrak{R} = \frac{1{\cdot}273 \times 30}{5(60 - 5)} = \frac{1{\cdot}273 \times 30}{5 \times 55} = 0{\cdot}1388$$

Whence the ampère-turns required will be—

$$C_1wl = 0{\cdot}8 \times 0{\cdot}1388 \times 98{,}200$$

$$\text{and } C_1 = \frac{0{\cdot}8 \times 0{\cdot}1388 \times 98{,}200}{4 \times 30} = 90{\cdot}86 \text{ ampères}$$

maximum value.

Now, the 0·35 ampères required for the iron core will lag after E considerably less than 90°; but the 90·86 ampères, which is a sine curve, will lag 90°, and as its magnitude is so large compared to that for the iron, we shall get the real magnetizing current required, being practically a sine curve and lagging all but 90° after E, whilst its maximum value will only be slightly less than 91 ampères. Indeed, we may say that the iron core has nullified the reluctance of the core, and all we have to consider is the outer air space. Had there been no iron core the reluctance of the air core would have been—

$$\frac{l}{a\mu} = \frac{30}{19{\cdot}65} = 1{\cdot}526$$

and the current required would have been nearly 1000 ampères.

It is for this reason, amongst others, that open iron circuit impedance coils are generally used instead of those with a closed iron core. The advantages are—

(1) Current curve is practically a sine curve.
(2) Easy shape to wind and adjust.
(3) Small quantity of iron required.

The Hedgehog transformer of Mr. Swinburne is an example designed to obtain a large all-day efficiency. This will be more fully dealt with in the section on transformers.

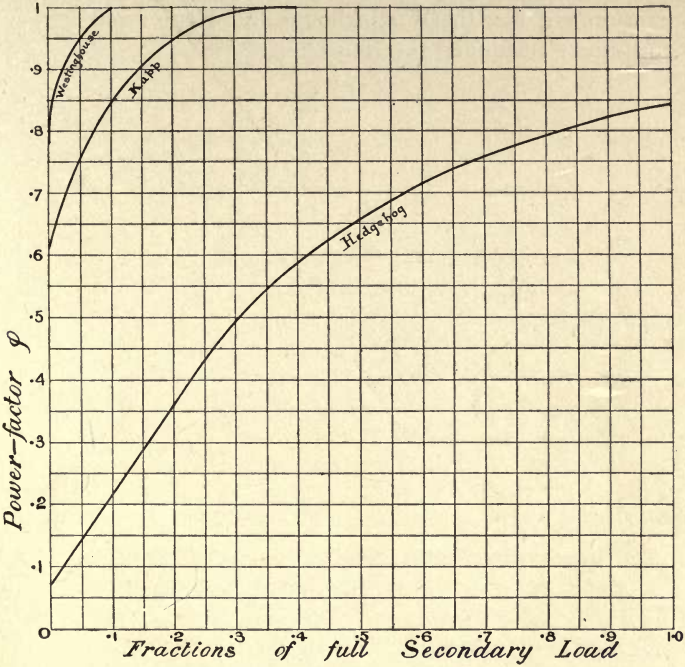

FIG. 32.

(*g*) When the medium of an open-iron circuit coil is conducting, as when imperfectly laminated or when containing a closed secondary circuit, we have the same balancing currents required. Only here such currents are no longer the only ones of any magnitude, and their effects in determining the total current are considerably reduced by the already large magnetizing current. It thus follows that even at very large values of the secondary load there is still a very considerable lag of C after impressed E.M.F. In some curves, showing the relationship between the power factor and various loads for closed-iron and open-iron circuit transformers, published by Dr. Fleming in 1892, this great difference is very apparent. For convenience of reference they are here reproduced in Fig. 32, where the relationship is shown for two closed-iron transformers, viz. the Westinghouse and Kapp; and for one open-iron transformer, the Hedgehog.

CHAPTER XIII.

IMPEDANCE COILS AND TRANSFORMERS.

Impedance Coils, or choking coils, as they are sometimes called, are pieces of apparatus designed to make use of the property of self-induced E.M.F., by introducing such an E.M.F. into a circuit acting so as to decrease the active E.M.F. without at the same time absorbing any power except that incidental to the properties of the materials employed. Thus, we may have a pair of mains at an alternating P.D. of 100 (virtual) volts, and it is desired to connect to these a lamp taking a current of 10 ampères at a P.D. of 40 volts. With a continuous current circuit one way is to introduce a resistance in series with the lamp of 6 ohms, but then the energy wasted in this resistance will be at the rate of 600 watts. Or a number of secondary cells may be put in circuit, having together an E.M.F. acting against that of the mains of about 60 volts. In this case there will be a storage of the energy not required by the lamp, only a small part being wasted in heating, and other losses peculiar to secondary batteries. With the alternating current it is equally possible to introduce a dead resistance of 6 ohms, and waste the same 600 watts; but we can introduce a similar B.E.M.F., as in the case of the secondary battery, the problem being to know how much? Remembering, then, that this B.E.M.F. is to be produced by the passage of the current through an inductive circuit, we have that—

$$E = \sqrt{(e^2 + \epsilon_1^2)}$$

or—

$$e = \sqrt{(E^2 - \epsilon_1^2)}, \text{ or } \epsilon_1 = \sqrt{(E^2 - e^2)}$$

and inserting values in this last, we have—

$$\epsilon_1 = \sqrt{(100 \times 100 - 40 \times 40)}$$
$$= \sqrt{(8400)} = 91{\cdot}65$$

This value is, however, slightly too large, since the very coil which is to produce it will have a resistance, and so absorb a small E.M.F. in becoming heated. In fact the impedance coil is also a resistance coil, and the effect of its own resistance should be allowed for as a resistance in series with the external circuit. This resistance of the coil is usually very small, compared to the resistance of the rest of the circuit, in order that the power wasted may be small.

Calling e_1 the active E.M.F. required by the coil itself, we then have—

$$\epsilon_1 = \sqrt{\{E^2 - (e + e_1)^2\}}$$

and all these values may be in either virtual or maximum values, as we please, provided they are all either the one or the other.

Transformers.—An unloaded transformer, that is with open secondary circuit, is simply a case of an impedance coil designed, as a rule, to take the minimum current and produce a B.E.M.F. differing from E the impressed volts by as small an amount as possible. In as far as, then, as it is unloaded, it can be dealt with in precisely the same manner as the impedance coils discussed above.

In allowing for the effects of eddy currents it has been pointed out that the disturbing effect produced by them had to be balanced (automatically) by the increase of the main current by an amount such as would produce ampère-turns magnetizing force sufficient for the purpose of overcoming the demagnetizing effect due to the eddy currents. A little consideration will show that the current taken from the secondary coil of a transformer will be of an exactly similar nature and will similarly have to be balanced by a corresponding increase of current in the primary. The consequence of this is that the current in the primary coil is not only variable in magnitude but is variable in phase.

As with impedance coils, there are two cases: the closed iron circuit and the open iron circuit.

Consider a simple type of closed iron circuit transformer wound so as to have the same number of turns on the primary, as on the secondary.

The current required on open secondary circuit will be of a kind represented in Fig. 31, including the two items—

(1) True magnetizing current; and,

(2) Balance for eddy currents.

The sum of these two will be a curve partaking most largely of the nature of the true magnetizing current, since the current to balance eddies is small and also lags due to self-induction in the core.

The phase position of this no-load current, which we will call C_o, will be after the primary E.M.F. by an amount less than 90° on account of the hysteresis, but will be very small in magnitude.

Of the two items of C_o the magnetic effect of the balance for eddy currents is nullified by the eddy currents, thus leaving only C_m, the true magnetizing current to be looked upon as causing so many ampère-turns to act upon the magnetic circuit, setting up an alternating magnetic field in the core which produces the E.M.F. in the secondary as an E.M.F. coinciding with the B.E.M.F. in the primary, and as we have already seen this will have a phase position approximately 180° after the primary E.M.F.

So far as the secondary is on open circuit, we get no effect due to its presence except in so far as that presence has probably increased the eddy current loss somewhat. But when the secondary is connected, as is most usual, to a non-inductive circuit, we get a current flowing in it which is in phase with its own E.M.F., but in contrary phase to the primary E.M.F. This current will, of course, mean so many ampère-turns acting upon the magnetic circuit, and these would produce a very serious disturbance were it not for their effect upon the momentary value of the B.E.M.F., causing the primary current to increase in magnitude and lag less. This increase is of exactly such a nature as would be produced by an additional current in the primary acting in contrary phase to the secondary current, and of such a magnitude as to produce ampère-turns exactly balancing the magnetic disturbance set up by the secondary current.

The real new primary current will, of course, be the sum of this balancing current C_b, and the original no-load current C_o, which, it must be remembered, is itself the sum of two items previously stated. In the case of a transformer without iron core and with no eddy current waste, there will then be only two currents to consider, viz., the current C_m to set up the necessary magnetic flux, a current lagging almost 90° after the primary E.M.F.; and the balance for secondary load, a current which is in phase with the primary E.M.F., and thus 90° apart from C_m. Calling the latter C_b, we shall have that the current at any time in the primary coil will be—

$$C_p \times \sqrt{(C^2_b + C^2_m)}$$

and its phase position will be after the primary E.M.F. by an angle whose tangent is approximately $\frac{C_m}{C_b}$, which shows that when C_b is large compared to C_m the lag of C_p after the primary E.M.F. may be almost nothing. In the case of the closed iron circuit type this is most marked; for even at as small a value of C_b as corresponds to only one-tenth of the full secondary load, we find the ratio of C_m to C_b so small as to make no appreciable lag of C_p after primary E.M.F.

But in the case of the no iron core or open iron circuit transformer, cm. is so large as to prevent the ratio of C_m to C_b ever becoming very small, even with a value of C_b corresponding to full secondary load. In the case of the open iron core we have C_p being composed of the sum of several items, viz.:—

(1) C_{mi} to magnetize the iron core: this is small in magnitude, is saw-toothed in shape, and lags less than 90° after the primary E.M.F.

(2) C_{ma} to magnetize the air space: this is large, is practically sinusoidal, and lags almost 90° after the primary E.M.F.

(3) C_b to balance the effects of secondary load and any

eddy current waste there may be in the whole apparatus: this is practically sinusoidal, and lags hardly at all after the primary E.M.F.

The sum of all these is mainly a sine curve, even when C_b is quite small; and its phase position is mainly determined by the relationship of C_{ma} to C_b. But as C_b never attains such a size compared to C_{ma} as to make—

$$\frac{C_{ma}}{C_b} = < 0{\cdot}3, \text{ or so}$$

it follows that there is always considerable lag of C_p after the primary E.M.F.

This is strikingly shown in the curves reproduced from Dr. Fleming's paper on Transformers, on p. 143, Fig. 32.

In transformer design care must be taken that the loss of volts in resistance is very small; that the loss of power in hysteresis and eddy currents is small; and that, above all, these last are small, inasmuch as they are a constant load and so very materially affect the efficiency when taken over a period of time during which the load varies considerably.

The various losses may be thus stated—

(1) Drop of volts in the primary coil, causing the B.E.M.F. ϵ, which is a measure of the magnetic flux set up, to differ from E the impressed E.M.F. This loss is such as to cause the E.M.F.—producing power of the transformer to be smaller than E, by the amount C_pR_p taken in its proper phase, and making the ratio of $\frac{\epsilon}{E}$ slightly less than unity. It is, in the closed iron circuit type, very small at no load, since, then, C_p is very small; and it only reaches a value of importance when the secondary is on full load. But in the open iron circuit type there is not nearly so large a variation in the value of C_p for various secondary loads, and so not as great an alteration in the primary loss of volts.

(2) The drop of volts due to ohmic resistance of the secondary, viz. C_sR_s, is always proportional to the secondary load in all types, and can be prearranged to have any value desired.

(3) A drop in the secondary due to the slightly inductive character of the winding. This will increase with the load being such as to make the effective E.M.F. in the secondary less than—

$$\epsilon_p \times \frac{w_s}{w_p}$$

where w_s and w_p stand for the number of secondary and primary turns, in as far as the secondary coil acts in the same manner as an impedance coil in its own circuit. For this reason the number of windings w_s should be kept small.

(4) There is also a drop in E.M.F. on full load, due to leakage of magnetic lines. It has already been shown that a fully loaded transformer has two oppositely acting magnetizing forces in the circuit, and if these be not evenly distributed round the magnetic circuit the consequence is evident polarity, followed by inequality of the magnetic flux through the coils. In a good closed iron type with the primary and secondary coils well intermingled along the magnetic circuit this drop can be kept to as low as $\frac{1}{10}$ per cent.

(5) The loss due to eddy currents in the core; and, in the case of open iron circuit types, due to eddy currents in the winding.

Mr. Alexander Siemens has pointed out how this loss may be arrived at in the case of a core composed of fine iron wires. The following formula is adapted from his paper on the subject, and is the value of the watts lost in eddy currents per cubic centimetre of iron—

$$F = \frac{6 \cdot 6 p^2 \beta^2 d^2}{10^{12}} \text{ watts per cc. of iron}$$

where d stands for diameter of wire in cms.
p ,, periods $\sim$ per second.
β ,, c.g.s. lines per sq. cm. of iron section.

Mr. Evershed has also shown how to express the loss when the iron core is composed of thin iron plates, viz. :—

$$F = \frac{p^2 \beta^2 t^2}{10^{11}} \text{ watts per cc. of iron}$$

where t is the thickness of plate in cms.

(6) The hysteresis loss due to molecular friction in the iron. This may be very accurately found in the way shown for the armatures of dynamos by means of the table on p. 214. Thus—

$$\mathrm{H} = \frac{h\mathrm{V}p}{10^7} = \text{hysteresis loss in watts total}$$

Calling the total induction in the iron of a transformer N c.g.s. lines, and using the symbols already mentioned, we have, neglecting the slight difference at no load between ϵ_p and E_p—

$$\mathrm{N}wp = \frac{\mathrm{E}_p \times 10^8}{4{\cdot}443 \times p}$$

and $\mathrm{N} = \beta a$, β being quite small, ranging from as low as 3000 to 6000, and seldom exceeding the latter, whence—

$$w_p a = \frac{\mathrm{E}_p \times 10^8}{4{\cdot}443 \times p \times \beta}$$

which gives an important product of the two unknown quantities in terms of known, or easily fixed values. Only experience can fix the relationship between the primary turns, and the sectional area of the core. On the one hand, if w_p be large we shall have, with a given weight of copper, a high resistance and large copper loss; but the iron losses H and F will be small. If, however, a be made large, the copper losses may be very small, but the iron losses will be large. The relationship between these two losses, copper and iron, will depend greatly upon the average output or load-factor. When the transformer is to be on full load always, then the highest efficiency will be attained when the copper losses are equal to the iron losses. But in general the conditions of supply render it impossible that the transformer can be on full load for more than a very small part of the 24 hours, whilst for a very large part it may be practically unloaded. The all-day efficiency can then be made higher by making the constant iron-losses smaller at the expense of large copper losses, which will only be of importance in themselves for the short times of high and full load. In this, of course, due regard must be had for a reasonable constancy in available secondary P.D.

Example.—A transformer for 40 ampères and 100 volts secondary output. In order that the efficiency at full load may be, say 95 per cent., it follows that the input must exceed the output by about 200 watts, which is to cover both the copper losses and the iron losses. If these be about equal, we must allow for something like 79 watts in hysteresis, from which, if the value of β be fixed, we can find the volume of iron. Thus from—

$$H = \frac{hVp}{10^7}$$

we have—

$$V = \frac{H \times 10^7}{hp}$$

and if β be 6000, $h = 1820$. Putting these values and 79 for H, we have, if $p = 80$ ∽ per second.

$$V = \frac{79 \times 10^7}{1820 \times 80} = 5425 \text{ cc.}$$

If the primary E.M.F. $E_p = 2000$ volts virtual, we have from—

$$\epsilon_p = \frac{4{\cdot}443 N w_p p}{10^8}$$

where N is the maximum induction, that—

$$N w_p = \frac{2000 \times 10^8}{4{\cdot}443 \times 80} = 562{,}700{,}000$$

and as $\beta = 6000$, we can put—

$$w_p a = 93{,}800$$

where a is the sectional area of the iron core in sq. cms. Taking this to be 155 sq. cms., we have from the previously found volume that the average length of the iron core must be—

$$\frac{V}{a} = \frac{5425}{155} = 35 \text{ cms.}$$

If the core is composed of stampings of the shape and size shown below, we shall require a pile of them 22·2 cms. of iron, not counting lamination.

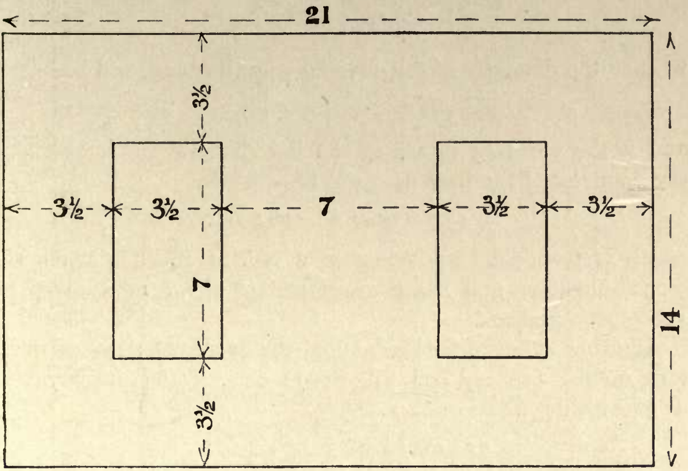

FIG. 33.

The total volume of iron in the above plates is—

$$(21 \times 14 - 7 \times 7)\ 22{\cdot}2 = 5439 \text{ cc.}$$

but as the magnetic induction cannot be taken as filling the iron into the corners, the amount under magnetic influence will be more nearly that previously found, viz. 5425 cc.

The sectional area is—

$$7 \times 22{\cdot}2 = 155{\cdot}4, \text{ say } 155 \text{ sq. cms.}$$

as was desired.

Of the spaces for winding we will allow that owing to superior insulation the space occupied by the primary coils will be 14·5 sq. cms. out of the total of 24·5 sq. cms., and that the secondary takes 8 sq. cms., leaving 2 sq. cms. for an insulating partition between the two coils. From—

$$w_p a = 93{,}800$$

putting $a = 155$, we have—

$$w_p = 606 \text{ say}$$

which number of wires has to go into the space of 14·5 sq. cms., or each wire will take up a room of—

$$\frac{14·5}{606} = 0·02392 \text{ sq. cm. area}$$

whence the diameter of the covered wire, if round, will be—

$$d = \sqrt{0·02392} = 0·154 \text{ cm. covered wire}$$

and if the covering increases the bar diameter by 0·07 cm., the diameter of the bare wire will be—

$$0·154 - 0·07 = 0·084 \text{ cm.}$$

which is practically equivalent to a No. 21 B.W.G., which is 0·032″ diameter, and has a resistance of about 11 ohms per 1000 feet warm.

Allowing 2·8 cms. for lamination, the average length of one turn on the primary coil will be 72 cms. if the primary be wound inside the secondary.

$$72 \text{ cms.} = 28·5 \text{ inches say}$$

whence the total length of the primary wire will be—

$$606 \times 28·5 \text{ inches} = 17271$$

or 1440 feet. Whence the resistance—

$$R_p = 1·44 \times 11 = 15·4 \text{ ohms say}$$

Again, the mean length of the secondary turn will be 76 cms., or 30 inches, and—

$$w_s = \frac{1}{20} w_p = \frac{606}{20} = 31 \text{ say}$$

The space to be filled by these 31 turns is 8 sq. cms., whence each wire will occupy a room of—

$$\frac{8}{31} = 0·258 \text{ sq. cms.}$$

and if the wire be square the size will be $\sqrt{0·258} = 0·507$ cm. square covered, which will admit of a bare wire of 0·437 cm. side of square, about equivalent to B.W.G. $6\frac{1}{2}$, and having a sectional area of 0·1909 sq. cm., and a resistance of about $0·28^{w}$ per 1000 feet warm.

The total length of secondary wire will be—

$$\frac{30 \times 31}{12} = 78 \text{ feet say}$$

whence—

$$R_s = 0{\cdot}28 \times 0{\cdot}078 = 0{\cdot}022 \text{ ohm.}$$

If, then, the maximum secondary current be 40 ampères, there will be a full load ohmic drop of—

$$40 \times 0{\cdot}022 = 0{\cdot}88 \text{ volts}$$

and a full load secondary copper loss of—

$$(40)^2 \times 0{\cdot}022 = 35{\cdot}2 \text{ watts, say } 36$$

Also, at full load the current in the primary coil will be—

$$C_p = \frac{C_s}{20} + C_m$$

where C_m is to include the magnetizing current giving the loss in hysteresis and also the current required to balance the eddy current loss in the core. In as far as these are losses they can easily be found, as shown below; but as they are very small compared to the secondary balancing current, they hardly at all affect the value of the primary copper loss. Thus, for example, corresponding to $\beta = 6000$ every centimetre length of the iron core will require 1·624 ampère turns, or the maximum magnetizing ampère turns on the primary will be—

$$35 \times 1{\cdot}624 = 56{\cdot}84$$

which must be produced by a current of—

$$\frac{54{\cdot}84}{606} = 0{\cdot}0937 \text{ ampères (maximum value)}$$

But this current is not a sine curve, and its virtual value will be greater than 0·707 times the maximum, or the virtual value will be, say 0·075 ampères.

Furthermore it is not in phase with the secondary balance, which has a virtual value at full load of—

$$\frac{40}{20} = 2 \text{ ampères}$$

Thus, including the hysteresis and eddy current losses, the full load primary current cannot be greater than 2·1 ampères virtual, if so much, and therefore the primary full load copper loss is—

$$(2{\cdot}1)^2 \times 14 = 62 \text{ watts say}$$

Using the formula on p. 150 for the eddy current loss, and taking the thickness of the stampings as 0·03 cms., we have the loss per cc. is—

$$\frac{80 \times 80 \times 6000 \times 6000 \times 0{\cdot}03 \times 0{\cdot}03}{10^{11}} = 0{\cdot}0020$$

whence the total eddy current waste will be—

$$5425 \times 0{\cdot}002 = 10{\cdot}85 \text{ watts}$$

say 11 watts.

Thus, the total output will be 40 × 100 watts, and the input will be $40 \times 100 + 79 + 62 + 36 + 11 = 4188$, which is within the prescribed limit.

If kept on full load the efficiency will be—

$$100 \times \frac{4000}{4188} = 95{\cdot}5 \text{ per cent.}$$

But when it is remembered that during a day of 24 hours a transformer on a lighting circuit is very variably loaded and may be practically on open secondary circuit for several hours, during which time the iron losses are constantly to be supplied, it will be apparent that the all-day efficiency or ratio of—

$$\frac{\text{Units paid for during 24 hours}}{\text{Units put in during 24 hours}}$$

will be a very different value to the full-load efficiency.

Consider, then, the case of the above transformer as on a circuit where it is—

On	No load.	$\frac{1}{10}$ load.	$\frac{1}{4}$ load.	$\frac{1}{3}$ load.	$\frac{1}{2}$ load.	Full load.
For	8 hrs.	4 hrs.	4 hrs.	4 hrs.	3 hrs.	1 hr.

We shall have the following table of outputs and inputs for

24 hours, the total full load copper loss being 98 watts, a loss which will vary practically inversely as the square of the load; and the iron loss equal to 90 watts, whatever the load.

Load.	Hours.	Output watt-hours.	Input = watt-hours. Output + iron loss + copper loss.	Total.
0	8	0	0 + 8 × 90 + 0	= 720
$\frac{1}{10}$	4	4 × 400 = 1600	1600 + 4 × 90 + 4 × 0·98	= 1964
$\frac{1}{4}$	4	4 × 1000 = 4000	4000 + 4 × 90 + 4 × 6·1	= 4385
$\frac{1}{3}$	4	4 × 1334 = 5336	5336 + 4 × 90 + 4 × 10·8	= 5739
$\frac{1}{2}$	3	3 × 2000 = 6000	6000 + 3 × 90 + 3 × 25	= 6345
1	1	1 × 4000 = 4000	4000 + 1 × 90 + 1 × 98	= 4188
		Output 20936		Input 23341

And all-day efficiency $= \dfrac{100 \times 20936}{23341} = 89{\cdot}6$ per cent., which is a very different thing to the 95·5 per cent. found above for the full-load efficiency, being even less than the efficiency if the transformer were kept on quarter load always, which would be over 91 per cent.

Example XLVIII.—A circuit has a B.E.M.F. of 1600 volts and a resistance of 20 ohms. What E.M.F. alternating will be required to send through it a current of 10 ampères?

Solution.—The relationship between the E.M.F. required E the B.E.M.F. ϵ and active E.M.F. $e = RC$, is that—

$$E = \sqrt{(e^2 + \epsilon^2)}$$

whence since—

$$e = RC = 20 \times 10 = 200$$

$$E = \sqrt{(1600)^2 + (200)^2} = 1612{\cdot}45 \text{ volts}$$

Example XLIX.—In the above circuit what will be the current if the alternating E.M.F. supplied is 2000 volts, and what will be the rate of doing work?

Solution—

$$e = \sqrt{E^2 - \epsilon^2}$$

or—

$$e = \sqrt{(2000)^2 - (1600)^2} = 1200 \text{ volts}$$

but—

$$e = \text{RC} \therefore \text{C} = \frac{1200}{20} = 60^a$$

and the rate of doing work is $C^2R = 72{,}000$ watts or 96·48 H.P.

Example L.—The P.D. at the terminals of a house connected to an alternating system of distribution is 100 volts. It is required to run 2 arc lamps, taking 34 volts each, in series off these terminals. If the lamps can be regarded as simple resistances of 3·4 ohms each, what will have to be the value of the B.E.M.F. of the choking coil to be used in series with them?

Solution.—Since each lamp takes 34 volts, and is equivalent to a resistance of 3·4 ohms, they are taking a current of 10 ampères, and when in series will require 68 volts, which is the value of R.C., the active E.M.F. required. Thus, since E = 100 we have—

$$\begin{aligned} \epsilon &= \sqrt{E^2 - e^2} = \sqrt{((100)^2 - (68)^2)} \\ &= \sqrt{(10000 - 4624)} = \sqrt{(5376)} \\ &= 73{\cdot}32 \text{ volts B.E.M.F.} \end{aligned}$$

Example LI.—In the last question, what is the saving effected, by using the choking coil instead of a non-inductive resistance, if the choking coil consumes energy at the rate of 30 watts?

Solution.—The total expenditure with a non-inductive resistance would be CE = 1000 watts, and with the choking coil it is only—

$$C^2R + 30$$

or—

$$100 \times 6{\cdot}8 + 30 = 710 \text{ watts}$$

whence the saving is—

$$1000 - 710 = 290 \text{ watts}$$

Example LII.—What will be the angle of lag of the current in Example L.?

Solution, the lag is such that $\tan \lambda = \frac{\epsilon}{e}$

or—

$$\tan \lambda = \frac{73 \cdot 32}{68} = 1 \cdot 078$$

whence $\lambda = 47^\circ$ about.

CHAPTER XIV.

EFFECTS OF CAPACITY.

THE properties of a condenser may be easily studied by reference to the water analogy suggested by Dr. Fleming, as a mechanical representation of the electrical conditions.

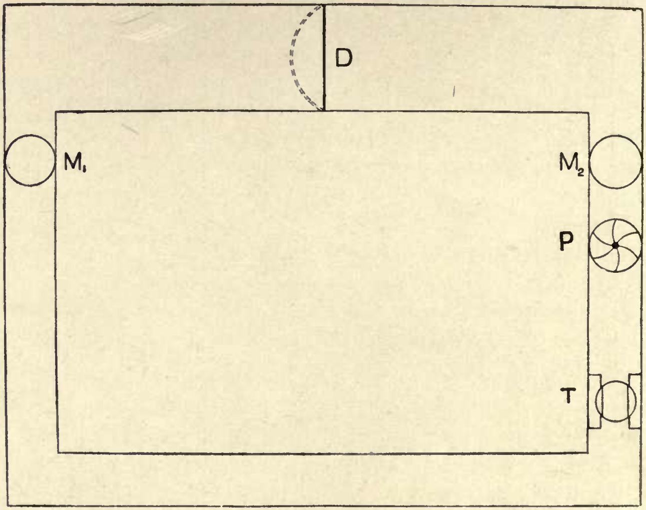

FIG. 34.

In Fig. 34 is represented a system of tubes containing water, divided at D by a flexible diaphragm of indiarubber, and having in each leg a water-meter, M_1 and M_2. A tap and pump are also placed in one leg; the tap to represent a switch in an electric circuit, and the pump corresponding to the electro-motive force.

First of all suppose the diaphragm removed. On working the pump a current of water will flow round the circuit in one direction or another, according to the rotation of the pump. This current will be indicated on the water-meters, and will be at the rate of so many gallons per minute—corresponding to an electrical effect of so many coulombs per second. This flow of water is constant in direction and in quantity, so long as the pump be rotated in one direction at the same rate.

If, however, the pump be replaced by a simple piston reciprocating in one of the tubes, we shall have the current of water constantly varying in direction, according as the piston is moving up or down; this reciprocating motion of the piston corresponds to an alternating E.M.F., and the alternating flow produced by it will not make any permanent record on the meters, since as much flows through in one sense as flows back in the other sense. The force urging the piston on its course will vary from point to point of the stroke, according as the mass of water is moving with or against it, and according to the resistance to the flow due to friction against the sides of the tube. This friction corresponds to resistance and the momentum of the water to the B.E.M.F. of self-induction.

But though the meters record nothing, they may be replaced by suitable mechanism indicating the amplitude of the reciprocating motion, which is a measure of the work-doing property of the moving mass of water.

When, however, the diaphragm is replaced we have a very different state of affairs. Formerly there was no obstruction to the motion of the water right round the circuit; but now, on rotating the pump, water will, let us say, be forced up the right side, causing the elastic diaphragm to swell out, as shown by the dotted line in the figure. Meter No. 2 will record the quantity of water required to do this as a quantity of water flowing *into* the diaphragm; but meter No. 1 will also record it as a quantity of water flowing *out* of the diaphragm. For however long the pump be worked the diaphragm will only become distended to an extent equal to the force exerted by the pump, supposing, of course, that its strength is such as not to admit of rupture by any force the pump can exert. Thus, there will be a flux of water on the one hand into the diaphragm, and on the other hand out of the diaphragm for a short time only; and the rate of flow will have been larger at the start when the elastic force exerted by the diaphragm was small, than at the close when the elastic force due to stretching is equal, and opposite to the force exerted by the pump, thus leaving no available force for urging water along the circuit.

Whilst still keeping the pump in rotation if the tap be closed, the stretched condition of the diaphragm may be preserved indefinitely, although the pump be afterwards stopped.

A careful examination will disclose the fact that the only difference now existing in the apparatus is this stretched condition of the diaphragm: not a store of water, for as much obviously flowed in as flowed out, but a power to do work by relaxing—a store of energy.

The above conditions correspond to that of the continuous-current circuit, and the flux of water which lasts for a short time only after the starting of the pump is called the "displacement current."

When the pump is now replaced by a reciprocating piston, we have a representation of the alternating-current circuit. Now, the diaphragm will be stretched first in one direction and then in the other, and the water-meters will, as before, be affected by a quantity of water surging to and fro in the tubes. The rate at which this quantity of water will be flowing in any one direction will depend upon the force of the piston and upon the elastic force of the diaphragm. When unstretched, the water will flow into the stretching diaphragm at a great rate, even though the force due to the piston may be small; but as the diaphragm becomes distended, the rate of flow into it will be small, and ultimately reach a zero value as the elastic force of the diaphragm equals that of the piston at the end of its stroke. As the pressure on the piston which caused it to move forwards is relaxed, allowing it to return to its starting position, the elastic force of the diaphragm causes water to flow back, slowly at first, but reaching a maximum rate as the piston reaches its starting-point (the middle of its total stroke), and then again declining to zero as the piston moves on its opposite stroke, the diaphragm now becoming distended in the opposite way. This can better be seen by reference to Fig. 35, where P is the reciprocating piston, D the diaphragm, and ABC represent the course of the piston. As it starts from B the diaphragm is unstretched as at B_1, and the rate at which water will flow into it is greatest; as the piston reaches C the diaphragm is distended to C_1, and the rate of flow is a minimum, having declined from a maximum, but being always in the same direction as the force applied to the piston. As the piston, displaced thus far by a force, moves back to B by a removal of that force, the water now flows out or back towards the piston, not away from it as heretofore, till the piston reaches B, when the diaphragm is now as at B_1 the rate of flow, however, being now a maximum, but declining to zero again, though still towards the piston, as the piston moves towards A, due to a reverse force to that which urged it from B to C. The diaphragm is now as at A_1 exerting a force away from the piston, and as the latter returns, due to the gradual reduction of the outside force acting on it, the water now flows away from the piston at a slow rate at first, but reaching the maximum again as the piston returns to B, when the diaphragm is once more unstretched.

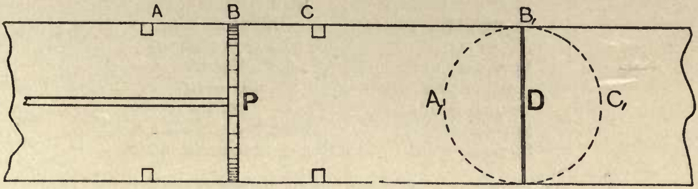

FIG. 35.

Calling the outside force which urges the piston along as equivalent to the alternating E.M.F. E, and the current of water which flows into the diaphragm as the current C, there being no resistance to the flow and no magnetic effect set up, we thus have the following order of effects—

E.M.F. – E.	Current – C.
Zero but rising to a + maximum then declining though still + to zero reversing and rising to a – maximum and declining to zero again.	A + maximum but declining to zero then reversing and rising again to a – maximum declining whilst still – to zero reversing and rising to a + maximum again.

and so on, which can best be indicated by plotting the curves of E.M.F. and current on the assumption that the variations of strength of E.M.F. are sinusoidal, as is done in Fig. 36, where E is the E.M.F. supplied and C the current, the circuit

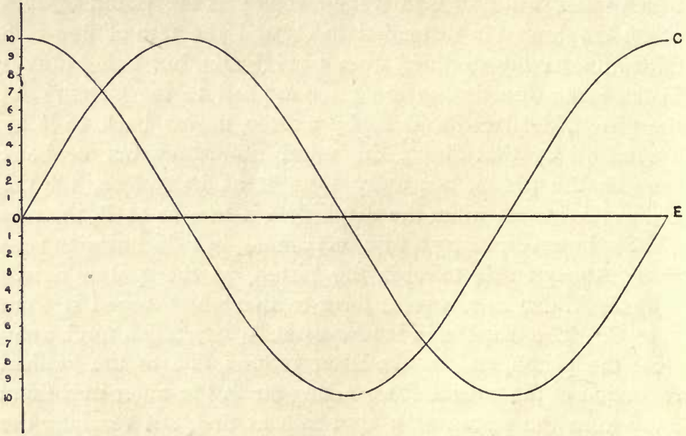

Fig. 36.

being supposed to be unable to set up any magnetic effect and to offer no resistance to the passage of the current, a condition of affairs which would correspond, on the water analogy, to a massless fluid moving in frictionless tubes.

Now, the degree to which the diaphragm can be distended by any force will depend upon its size and upon the material of which it is made; or its capacity depends upon its construction. Similarly the electric condenser has a capacity which is proportional to its area, inversely proportional to its dielectric thickness and also proportional to the property of the material of which it is made.

Thus the quantity of water required to distend the diaphragm so that its elastic force is equal and opposite to the unit force acting on the piston will be a measure of the capacity of the system. Also, in the electric circuit we may say that the quantity of electricity in coulombs required to strain the dielectric to an extent equal and opposite to the E.M.F. of 1 volt is a measure of the capacity of the condenser; and when that quantity is unity (1 coulomb) the condenser is considered to have unit capacity, or a capacity of 1 farad. Thus—

1 farad is that capacity which requires 1 coulomb to produce a P.D. of 1 volt between its two coatings.

But this quantity Q coulombs must, in an alternating-current circuit, be passed into, or out of, the condenser each quarter period, and therefore the average current flowing into, or out of, the condenser will be—

$$C_a = \frac{Q}{\frac{\tau}{4}} = \frac{KE_m}{\frac{\tau}{4}}$$

where K is the capacity in farads and E_m is the maximum value of the E.M.F. supplied to the condenser.

Or, putting $p = \frac{1}{\tau}$, we have the average current—

$$C_a = 4pKE_m$$

and thus the virtual current and maximum current will be—

$$C_v = 4{\cdot}44pKE_m$$

$$\text{and } C_m = 2\pi pKE_m$$

But if E be stated as the virtual E.M.F., then—

$$C_v = 2\pi pKE_v$$

And we have seen that the E.M.F. supplied to the condenser reaches its maximum value, 90 degrees, later than the maximum value of the current due to it, the condenser itself exerting an E.M.F. in the circuit, always equal and opposite to the supplied E.M.F. It is necessary to make a very careful distinction between these two E.M.F.s; that supplied to charge the condenser is of course that of the dynamo, or other generator, and is called, as before, the impressed E.M.F.; that exerted by the condenser is of the nature of a back E.M.F., and may be called the B.E.M.F. of the condenser, κ.

Thus, if a condenser be in a circuit having an E.M.F. of E_v volts, it will take a current—

$$C_v = 2\pi p K E_v \text{ ampères}$$

and will exert a B.E.M.F. of—

$$\kappa_v = \frac{C_v}{2\pi p K} \text{ volts}$$

With a perfect condenser having plates of some material offering no resistance to the flow of the charge into them, and with a dielectric of perfect insulating properties, this B.E.M.F. κ_v will be exactly equal and opposite to E_v, the impressed E.M.F. But since some E.M.F. is required to maintain the momentary charging current, and there is a small actual leakage from one set of plates to the other (in both cases causing heating), it follows that the impressed E.M.F. E_v must slightly exceed the B.E.M.F. κ_v, the difference of phase being also slightly less than 90 degrees.

Now, since the supplied E.M.F. lags 90° after the current in the condenser, and the condenser B.E.M.F. is equal and opposite, it follows that the current in a condenser lags 90° after its own B.E.M.F. Therefore the effect of capacity in a circuit is to oppose the effect of self-induction, as may be easily seen by reference to the following diagram (Fig. 37), where the current through a circuit is marked as C ampères, the circuit being able to set up a magnetic field of such magnitude as to cause a B.E.M.F. of self-induction of ϵ volts, and also possessing a capacity such as will give κ volts, which E.M.F. may now be called a forward E.M.F., in contradistinction to the B.E.M.F. of self-induction ϵ. To maintain the current C ampères through the circuit, the impressed E.M.F. E will thus have to be sufficient at any moment to (*a*) send C ampères through R ohms, (*b*) to nullify ϵ, and (*c*) to nullify κ; and when $\epsilon = \kappa$, it follows that $E = e = RC$ simply, or the capacity effect has counteracted the magnetic effect.

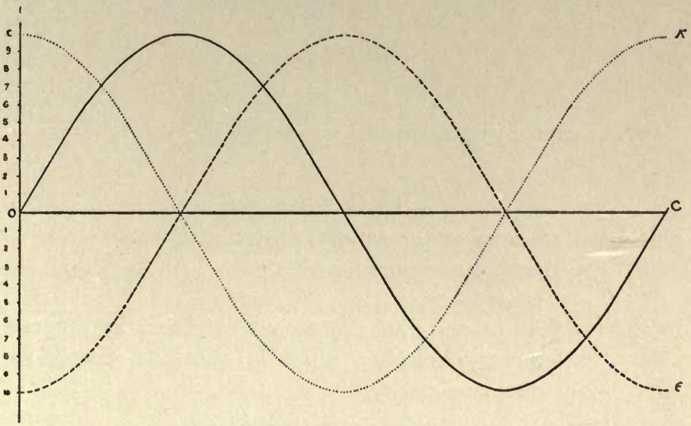

FIG. 37.

Now, we have seen that—

$$\epsilon_v = \frac{4{\cdot}44 N_m w p}{10^8} = \frac{2\pi N_v w p}{10^8} \qquad \text{(see p. 132)}$$

and also that—

$$\kappa_v = \frac{C_v}{2\pi p K} = \frac{C_m}{4{\cdot}44 p K}$$

whence it follows, that when the capacity is such as to make the circuit non-inductive in spite of a magnetic effect—

$$\frac{2\pi N_v w p}{10^8} = \frac{C_v}{2\pi p K}$$

whence—

$$K = \frac{C_v 10^8}{4\pi^2 p^2 N_v w}$$

or if N and C be maximum values, then—

$$K = \frac{C_m 10^8}{N_m w} \times \frac{1}{4\pi^2 p^2}$$

or—

$$K = \frac{C_r}{\epsilon_v} \times \frac{1}{2\pi p}$$

Now, the capacity of a condenser in farads is—

$$K = i\frac{A}{1{\cdot}131 \times 10^{13} \times d}$$

where A = the area of one set of coatings in sq. cms.
d = the distance between one set of coatings and the other in cms.
and i = that property of the dielectric placed in between the two coatings, which is called its specific inductive capacity.

The values of i for different materials are:—

Air	1
Mica	5
Flint Glass	6·5 to 13
Glass	3
Ebonite	2·5 to 3·1
Sulphur	2·8 to 3·8
Paraffin wax	about 2
Paper baked in paraffin oil	about 2·7 to 3

In constructing a condenser, the conductors are composed of thin sheets of metal, usually tinfoil, piled upon each other with some dielectric in between. In order to obtain the large surface required for even a small capacity, the sheets are made of a reasonable size, and interleaved with each other, as shown in section in Fig. 38, where the lines show the metal sheets, and the spaces the dielectric. It will be noticed that five alternate plates are connected together by the terminal block T_1, and are interleaved with the other six, these last being also all connected together by the terminal block T_2. In such an arrangement use is made of each side of each plate, and thus the surface of such a condenser is found by the products of half the number plates into the surface of one plate counting both sides. If, consequently, each plate measures 24 cms. by 48 cms., the area per plate will be 2304

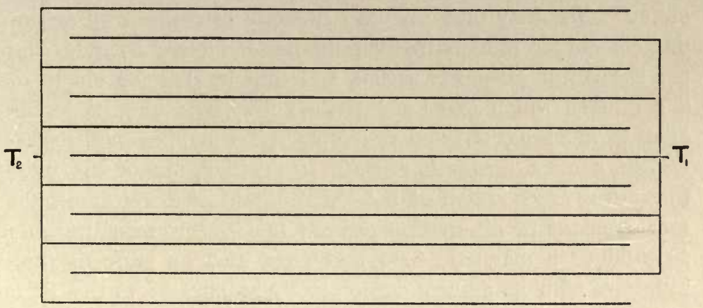

Fig. 38.

sq. cms. If now the dielectric be of oiled paper, 0·015 cm. thick, each piece of paper and plate of tinfoil will be, say 0·017 thick, or there will be $\frac{100}{0{\cdot}017} = 5882$ plates, or 2940 say connected to one terminal per metre thick, and taking the specific inductive capacity as 3, the capacity per metre thick will be—

$$K = 3 \times \frac{2940 \times 2304}{1{\cdot}131 \times 10^{13} \times 0{\cdot}015} = 0{\cdot}000119 \text{ farad}$$

or to make a condenser of 1 farad capacity would take a thickness of about 8400 metres, which gives some idea of the great size of a condenser to have so large a capacity.

Also, if used on a circuit working at 2000 virtual volts at 100 ∽ per second, the current taken will be—

$$\begin{aligned} C_v &= 2\pi p K E_v \\ &= 628 \times 1 \times 2000 = 1{,}256{,}000 \text{ ampères} \end{aligned}$$

to which should be added such current as may leak from one set of plates to the other, a current which will probably be almost in phase with the E.M.F. supplied, and consequently lagging 90° after the condenser current.

Use of Condensers for modifying the effect of self-induction. We have already seen that the nature of the B.E.M.F., set up by a condenser, enables it to oppose the E.M.F. of self-induction when the condenser is placed in series with the circuit. We may thus say that capacity in series with an inductive circuit renders that circuit non-inductive by nullifying the E.M.F. of magnetic action, and thus leaving the circuit in a condition which can be described fully by saying its resistance is R ohms, it requires simply RC volts, but can, nevertheless, exert a magnetic effect. In practice, one of the most interesting cases is that of the shunt circuit of a watt-meter for measurement of alternating power; this circuit must be able to produce a magnetic field, and must also be perfectly non-inductive with its current consequently in phase with the P.D. at its terminals.

Condensers are, however, sometimes used in parallel with a circuit the properties of which they are desired to modify. The circuit to which a condenser is a shunt is, however, unaltered in any way, the main effect is in the generator and leads connecting it to the circuit and its shunting condenser. Thus, if we have a circuit such as the primary of an unloaded open-iron circuit transformer, we have seen that the current taken by it will be large, but will lag almost 90° after the E.M.F. supplying it. This current will not necessarily mean any great waste of energy in the transformer, but the loss in the connecting leads and dynamo armature will be a considerable item. If now a condenser be connected across the primary terminals, the dynamo will have to supply through the leads a current such as will be determined by the size of the condenser, but coming 90° earlier than the E.M.F., and consequently opposed to the transformer current. The current, therefore, which flows in the leads and armature will be the sum of these two, and may be almost zero when the condenser current is equal to the transformer current, being only so large as will, in combination with the E.M.F., represent the waste in the transformer and condenser.

We have seen that the value of the current flowing into a condenser is—

$$C_v = 2\pi p K E_v$$

whence in this case we must make the capacity such as will take this current C_v equal to that required by the unloaded transformer at a P.D. of E_v volts, or—

$$K = \frac{C_v}{E_v} \times \frac{1}{2\pi p}$$

where C_v stands for the so-called "magnetizing" current of the open-iron circuit transformer.

The use of condensers for the above purpose was proposed by Mr. Swinburne, in conjunction with his "hedgehog" transformer, but very few are in use.

It is clear from the above that a condenser in parallel with a circuit tends to make the whole combination non-inductive, rather by rendering the apparent resistance larger, than by opposing any B.E.M.F. in the circuit.

In systems using concentric cables, the capacity of the cable considered as two conducting masses intimately associated but separated by the insulating covering on the inner conductor, is a shunt to the circuit supplied by those leads, and so may modify the effect of self-induction in the circuit. Messrs. Cowan and Still have published the values of the capacity of a few cables, viz.—

For $\frac{19}{18}$ concentric cables the capacity per mile is—

British Insulated Wire Co. (paper insulation) ...	0·31	microfarad
Glover and Co. (vulcanized rubber)	0·615	,,
,, ,, (diatrine)	0·315	,,

This capacity will take a current coming 90° earlier than the impressed E.M.F., which current must be added to the current supplied through the cable to the load at the far end. When the load current is lagging much, due to self-induction, the total current will be, as shown above, the difference between the two, and therefore may be zero.

Example LIII.—A circuit supplied from an alternator takes a current of 10 ampères at a pressure of 100 volts, its resistance being, however, only 5 ohms. Find the lag of the current and the capacity in microfarads of the condenser which must be used in series with the circuit in order that the current may be

20 ampères, and not lag at all. There are 100 periods per second.

Solution.—The active E.M.F. will be 50 volts, whence the B.E.M.F. ϵ is $\sqrt{10000 - 2500} = 86{\cdot}6$ volts, whence there will be a lag of current after the E.M.F. such that $\tan \lambda = \frac{86{\cdot}6}{50}$ or $\lambda = 60°$. In order to reduce this lag to zero, and thus make the current 20 ampères, the capacity will have to be—

$$K = \frac{20}{86{\cdot}6} \times \frac{1}{2\pi 100} = 0{\cdot}000367 \text{ farad}$$

or 367 microfarads, a capacity which could only be made in a size of about 25 cubic feet.

Example LIV.—An unloaded open-iron circuit transformer is shunted by a condenser of 30 microfarads capacity, and appears to take a negligibly small current. It is supplied at 2000 volts off a circuit, having 80 periods per second. What is the true magnetizing current?

Solution.—It is clear that the condenser current is equal to the magnetizing current as, being in opposite phase, it has just nullified it. Now, the current taken by such a condenser will be—

$$2\pi p KE = \frac{44}{7} \times 80 \times 0{\cdot}00003 \times 2000 = 30{\cdot}17 \text{ ampères}$$

which is, therefore, the true magnetizing current.

Example LV.—A concentric main six miles long, having a capacity of 0·7 microfarad per mile, is supplying energy to an inductive load, taking 10 ampères and lagging almost 90°. If the P.D. is 1000 volts, what is the indicated current where there are 100 ∽ per second?

Solution.—The current taken by the main on account of capacity will be—

$$C = 2\pi p KE = \frac{44}{7} \times 100 \times 4{\cdot}2 \times 10^{-6} \times 1000 = 2{\cdot}64 \text{ ampères}$$

which current will be almost in opposite phase to that in the load, whence the apparent current will be the difference or—

$$10 - 2{\cdot}64 = 7{\cdot}36 \text{ ampères}$$

Example LVI.—A concentric cable is 12 miles long, and is connected to an alternating P.D. of 10,000 volts. If the capacity is 1·5 microfarad per mile, what will be the current supplied, the circuit working at 80 ∽ per second?

Solution.—Total capacity is $12 \times 1{\cdot}5 = 18$ microfarads, whence the current taken is—

$$C = 2\pi K p E = \frac{44}{7} \times 0{\cdot}000018 \times 80 \times 10{,}000 = 90{\cdot}5 \text{ ampères}$$

Example LVII.—Two condensers of $\frac{1}{3}$ microfarad capacity each are connected to an alternating P.D. of 50 volts at 225 ∽ per second. What will be the current taken—

(*a*) when the two condensers are in parallel?

(*b*) when in series?

Solution.—In parallel the total capacity will be $\frac{2}{3}$ microfarad, since virtually the surface of the condenser is now double that of either. Thus the current will be—

$$\frac{44}{7} \times 0{\cdot}000000667 \times 50 \times 225 = 0{\cdot}047 \text{ ampère}$$

But when in series the effect is the same as making the thickness of the dielectric twice as much while the surface remains the same. The capacity is then only $\frac{1}{6}$ microfarad, and thus the current will be a quarter of that in parallel, or 0·0117 ampère.

CHAPTER XV.

REGULATING RESISTANCES.

In order to produce desired variations in the manner of working of various types of electrical machinery, it is sometimes necessary to employ resistance coils connected in circuit with the machine to be regulated. For this and other purposes it may be necessary to provide resistances calculated to effect the following—

(1) To replace some device cut out of circuit;

(2) To regulate and vary the speed of a motor.

(1) The value of a resistance to replace a bank of lamps, an arc lamp, a motor, or other device which requires when working a P.D. of E volts, and takes a current of C ampères, is simply—

$$R = \frac{E}{C} \text{ ohms}$$

This resistance must, of course, be made of such shape, size, and material, that it will be able to carry the current for the length of time it is to be in circuit without overheating. Practice varies considerably in this respect, the rise in temperature allowed ranging from quite a few degrees above the surrounding air to a temperature quite sufficient to discolour paper. No rule can be given for the size of wire or current density, as a great deal depends upon the spacing of the wires, or the degree to which the convolutions of the spirals are separated from one another. With a fair amount of spacing, however, a current ranging from $\frac{1}{8}$ to $\frac{1}{16}$ of that required to melt the wire employed according to the table of fusing currents determined by Mr. Preece, may be taken as reasonable. The materials usually employed are German silver, platinoid, iron, and manganin, the latter having the advantage of constancy of resistance in spite of variation of temperature.

(2) The resistance employed to vary and regulate the speed of a motor is usually combined with a set of resistances and switch contacts to enable the motor to be gradually introduced into the circuit, so that excessive current through the motor on the one hand, and sudden demand on the generator on the other hand may be avoided.

In all motors the tendency to rotate is due to a pull on the armature conductors by the poles of the magnets, the pull depending directly upon the strength of the current and the strength of the magnetic field. Thus, whatever be the magnitude of the current in the armature, there will be little or no pull on its conductors if the strength of the magnetic field be small. It thus follows that in starting a motor the first thing to see to is production of a strong magnetic field. In the case of a shunt motor, the switch for introducing it into circuit is, therefore, arranged so as to, first of all, make circuit with the shunt coil; then the armature is introduced into the circuit in series with a considerable resistance, in order that the current may be only a small fraction of that at full load. In this way the two essentials to rotation are produced without any sudden strain either on the motor or generator. In the event of there being no load on the pulley the motor now starts, and will soon reach a speed near the maximum possible; but if there be a load on the pulley, the torque due to the small armature current may not be sufficient to produce rotation. In this case the switch has to be moved over several contacts, cutting out of circuit a resistance at each step till the current through the armature is sufficient to produce rotation. Regulation of speed is effected in either of two ways, or sometimes by both combined, viz. :—

(*a*) Reduction of the E.M.F. available for the armature by resistance.

(*b*) Reduction of the strength of the magnetic field by introduction of resistance into the shunt circuit.

(*c*) A combination of the two previous methods.

Fig. 39 will illustrate this. The dynamo D has one terminal connected directly to the one brush of the motor M, to which is also connected one end of its shunt coil as usual. The other terminal of the dynamo is connected to a contact arm A, which may be brought into contact with either of the contact blocks 1, 2, 3, 4, 5, 6, or 7, or may be off any of them, when circuit will then be broken. The other brush of the motor is disconnected from the other end of the shunt coil, and is connected to one end of the series of resistance coils placed between the contact blocks; the shunt coil is connected to some part of the resistance coils, as shown at B. If now the contact arm be upon block marked *a*, there will be no circuit,

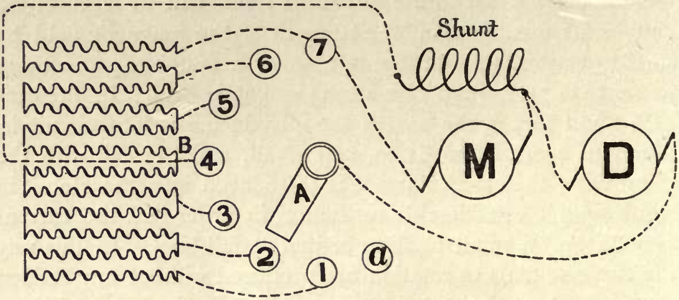

Fig. 39.

but when contact is made with block 1, the armature and shunt coil will then be in circuit, the armature current, however, being quite small as the coils between B and block 1 have a resistance which is large relatively to the armature, but not so large as to make much reduction in the shunt current. Thus there will be produced a considerable field strength, and also some armature current. The resistance now in circuit is such as to prevent the motor running at anything like full speed since the loss of pressure in the resistance coils will very considerably reduce the E.M.F. available for overcoming the full B.E.M.F. of rotation; but on moving the arm A over to block 3 or 4, the resistance in series with the armature is considerably reduced, and the shunt is now directly connected to the dynamo, this position of the switch being that intended for the slowest speed of running, since the shunt is now strongest, and the armature has still considerable external resistance. It is from this point onwards that the double regulation comes in; for as the arm A is moved onwards towards block 7, the shunt is being made gradually weaker, necessitating an increase of speed in the armature to give the required B.E.M.F., whilst at the same time there is less and less drop of pressure in external resistance till the arm A is on block 7, when the armature then has the full P.D. of the dynamo, and will, consequently, run fastest.

In calculating the magnitude of the resistance coils to be inserted in a switch of the above type it is as well to bear in mind that the current taken by the motor depends mainly upon the demand made upon it at the pulley, and may reach the maximum value even at a slow speed when probably the demand for work may be large. Thus all the coils between blocks 4 and 7 may have to carry the full current, and must consequently be of such a size as to prevent overheating. The coils between contacts 1 and 4 must also be made of such size as will make them quite safe at any current they may have, the largest value of course being such as the full P.D. can produce through the whole external resistance with the armature of the motor stationary; these coils, though not intended generally to be left in circuit for any length of time, are frequently so left, and should therefore not be so slender as to risk fusion. As to the value in ohms, a good deal depends upon the range of regulation required, bearing in mind that in any case such regulation is very wasteful of power since any not used by the motor is dissipated in heating the coils.

Let the value of the resistance between blocks 1 and 4 be called R ohms and that between 4 and 7 = r ohms, the resistance of the armature being R_a and that of the shunt coil R_s. For the present we will leave out of consideration the effect of the resistance in the shunt coil, and deal only with that in the armature.

Calling the dynamo P.D. E volts and the maximum value of the motor armature current C_a ampères, it is first of all necessary that $R + r$ shall be so large that—

$$C = \frac{E}{R + r} = \text{the starting current}$$

may be less than C_a, or certainly not greater than it. Next we will suppose that the switch has reached the contact block 4, and that the motor is so loaded that it is taking a current of C_a ampères. Under these circumstances there is a loss of pressure in the external resistance and in the armature, neglecting the shunt current, of—

$$C_a(R_a + r) \text{ volts}$$

leaving—

$$E - C_a(R_a + r) = \epsilon$$

the speed being nearly proportional to the B.E.M.F. ϵ. Thus, by making $C_a(R_a + r)$ any desired proportion of E, the speed can be thereby reduced in the same proportion.

Next consider that the arm A has reached the block 7, then the value of ϵ will be simply—

$$E - C_a R_a$$

and the speed will be normal. But with the switch connections as shown in Fig. 39, p. 176, the shunt coil has now a resistance inserted in it, the effect of which is to reduce the magnetizing force, and therefore the magnetic field, but not in the same proportion, since the magnetic circuit is composite, and consequently the various items of it are altering in relative importance. Thus, whereas the air gap was, at full excitation, a certain large proportion of the whole circuit, it is now relatively much more important, since, due to reduction of magnetic density, the permeability of all the iron items has increased. The exact effect of such reduction of magnetizing force is not capable of direct calculation, since the law connecting β and μ is not easily stated in symbols; and, moreover, the greater effect of armature reactions at reduced excitation renders the problem a very difficult one. We can, however, form an approximate estimate of such effect by remembering that, initially, the reduction of lines will be always less than the reduction of magnetizing force. But, counting armature reactions as more important at low inductions, we shall be very near the truth in assuming that the field falls off directly as the magnetizing force.

Thus in the above case the magnetizing force is proportional to the shunt current, which is as a maximum—

$$C_s = \frac{E}{R_s}$$

and as a minimum—

$$C_{s1} = \frac{E}{R_s + r}$$

If the speed be *n* at the point when the switch arm is on block 4, when the shunt gets the full P.D., and the armature has the resistance *r* in series, then this will be the slowest speed if the motor be so loaded as to take the full armature current C_a. But when all the resistance is out of the armature circuit and inserted in the shunt, the speed will have increased from its former value to—

$$n_1 = n \times \frac{E - C_a R_a}{E - C_a(R_a + r)} \times \frac{R_s + r}{R_s}$$

and whence—

$$r = \frac{\left(\frac{n_1}{n} - 1\right)(ER_s - C_a R_a R_s)}{E - C_a R_a + \frac{n_1}{n} C_a R_s}$$

from which it will be seen that in any case the shunt effect is small. But in the simpler case, where the shunt coil is always at the full P.D. of the circuit, then—

$$\frac{n_1}{n} = \frac{E - C_a R_a}{E - C_a(R_a + r)}$$

and—

$$r = \frac{C_a R_a - E + \frac{n_1}{n} E + \frac{n_1}{n} C_a R_a}{\frac{n_1}{n} C_a}$$

In all cases n_1 stands for the maximum speed, and n for the slowest speed. The difference is not considerable between the two cases, the resistance r required to vary the speed from 1 to 2 being about 8 per cent. less when the shunt is connected as shown in Fig. 39, than when the shunt always has the full P.D. at its terminals.

Example LVIII.—A shunt motor having $R_s = 50$ ohms, $E = 100$ volts, $C_a = 100$ ampères, and $R_a = 0{\cdot}02$ ohm, is required to have a speed ranging from 600 to 1200. Find the value of the regulating resistances required.

(*a*) When the shunt always has the full P.D. 100 volts;

(*b*) When the shunt has the P.D. at its terminals varied by the same resistance as in the armature circuit.

Solution.—$\frac{n_1}{n} = 2$, whence in case (*a*) the resistance to be inserted will be—

$$r = \frac{2 - 100 + 200 + 4}{200} = \frac{106}{200} = 0{\cdot}53 \text{ ohm}$$

and in case (*b*)—

$$r = \frac{5000 - 100}{100 - 2 + 10000} = \frac{4900}{10098} = 0{\cdot}485 \text{ ohm}$$

CHAPTER XVI.

PRIME MOVER COSTS.

In most forms of heat engine the consumption of fuel may be divided up into two items, one to run the engine idle, and the other to supply the output. The latter is generally represented by a regular constant amount for every actual brake horse-power hour, whilst the former is also generally nearly a constant, bearing some relationship to the difference between full load consumption and idle consumption. The actual consumption for any output is of course the sum of these two. Calling—

B = the maximum brake H.P. of the engine;
b = consumption per actual B.H.P. hour;
a = a factor such that $a(bB)$ gives the idle or no load consumption,

we have the actual consumption as being—

$$b(aB + x)$$

where x = the load on the brake in horse-power.

The values of these constants will depend greatly upon the size of the engine, and may be taken to have the following values for steam and gas engines of the usual sizes employed in electric lighting.

For Steam Engines.	For Gas Engines.
$a = 0{\cdot}3$ to $0{\cdot}4$.	$a = 0{\cdot}16$ to $0{\cdot}28$.
$b = 1{\cdot}7$ to 3 lbs. of coal per hour.	$b = 12$ to 18 cubic feet of gas per hour.

The smaller the engine in each case the higher the value to be taken.

If, now, we call z pence the price per unit, either pound of coal or cubic foot of gas, of fuel, we can write the cost of one hour as being—

$$zb(a\text{B} + x)$$

And the cost of 1 B.T.U. will be found by putting for x the actual B.H.P. required, with the particular gearing and dynamo to give an output of 1 B.T.U.; the cost of 1 B.T.U. thus varying from a somewhat large value at small load to lower values at full load.

Example LIX.—What will be the cost of fuel for 10 B.T.U. from a dynamo of 90 per cent. commercial efficiency, when run at full load of 100^a at 100^v, by a gas engine in which $a = 0{\cdot}2$ and $b = 15$, the price of gas being *2s. 3d.* per 1000 cubic feet? The gas engine has a maximum power of 15 B.H.P.

Solution.—The H.B.P. required will be—

$$\frac{1000}{746} \times 10 \times \frac{100}{90} = 14{\cdot}74 = x$$

whence the cost of fuel per hour will be—

$$zb(a\text{B} + x)$$

where z is—

$$\frac{27}{1000}\text{ penny}$$

or—

$$= \frac{27}{1000} \times 15\,(0{\cdot}2 \times 15 + 14{\cdot}74)$$

$$= \frac{27}{1000} \times 266{\cdot}1 = 7{\cdot}185\text{ pence}$$

which is the cost of 10 B.T.U., whence the fuel cost per B.T.U. = $0{\cdot}7185$ pence.

Example LX.—If with the above engine the average load is only 3 B.T.U. when the commercial efficiency of the dynamo is only 80 per cent., find the fuel cost of 1 B.T.U.

Solution.—Here the B.H.P. required is—

$$\frac{1000}{746} \times 3 \times \frac{100}{80} = 5{\cdot}23 = x$$

and the cost of fuel per hour is—

$$\frac{27}{1000} \times 15\,(0{\cdot}2 \times 15 + 5{\cdot}23) = 3{\cdot}33 \text{ pence}$$

which is the cost of 3 units or 1·11 pence per B.T.U.

Example LXI.—Find the cost of running the above dynamo by a steam engine having the following particulars. $a = 0{\cdot}4$ and $b = 2{\cdot}5$, $B = 15$ when the cost of coal is 0·043 pence per lb.

Solution.—(1) on full load of 10 B.T.U. the B.H.P as before will be 14·74 whence cost of fuel will be—

$$0{\cdot}043 \times 2{\cdot}5\,(0{\cdot}4 \times 15 + 14{\cdot}74) = 2{\cdot}23 \text{ pence}$$

or only 0·223 pence per B.T.U.

(2) But on load of 3 B.T.U, the B.H.P. required = 5·23 as before and the cost of 1 hour will be—

$$0{\cdot}043 \times 2{\cdot}5\,(0{\cdot}4 \times 15 + 5{\cdot}23) = 1{\cdot}207 \text{ pence}$$

or only 0·402 pence per B.T.U.

Example LXII.—A large gas engine having $a = 0{\cdot}16$, $b = 12$, and of 200 B.H.P., is used to run a dynamo which has a commercial efficiency of 50 per cent. at an output of 7 B.T.U. per hour, and 95 per cent. at 140 B.T.U. per hour. The price of gas is 2*s.* 6*d.* per 1000; find the cost of gas per B.T.U. at an output of 7 and 140 B.T.U. per hour.

Solution.—(1) At 7 B.T.U. per hour the horse-power required is—

$$\frac{7 \times 1000}{746} \times \frac{100}{50} = 18{\cdot}7$$

whence the cost of gas per hour will be—

$$\frac{30}{1000} \times 12(0{\cdot}16 \times 200 + 18{\cdot}7) = 18{\cdot}28 \text{ pence}$$

or—

$$\frac{18 \cdot 28}{7} = 2 \cdot 61 \text{ pence per 1 B.T.U.}$$

(2) At 140 B.T.U. horse-power required is—

$$\frac{140 \times 1000}{746} \times \frac{100}{95} = 198$$

and cost per hour is then—

$$\frac{30}{1000} \times 12(0 \cdot 16 \times 200 + 198) = 82 \cdot 8 \text{ pence}$$

or—

$$\frac{82 \cdot 8}{140} = 0 \cdot 59 \text{ pence per 1 B.T.U.}$$

It will be seen from the foregoing examples that the cost of fuel will depend very much upon the extent to which the machinery can be loaded. Large engines and dynamos run to supply only a small fraction of their full power will be very wasteful, and so run up the cost of generation. To the cost of the fuel must also be added the cost of attendance, lubrication, etc., all of which items become still more costly as the load goes down. Thus, though at times of full load the cost of fuel may be but a fraction of a penny per B.T.U., the average cost is much increased by the, to a large extent, unavoidable waste at times of light load. The following example will illustrate this.

Example LXIII.—The average pressure generated in a central station is 430 volts, and the system is a five-wire network maintained at 400 volts between the outers. At full load there are 10,000 30-watt lamps in use. The dynamos have an average efficiency of conversion of 92 per cent. The station is on full load for 2 hours, on half load for 8 hours, on $\frac{1}{10}$ load for 6 hours, and runs idle for the rest of the 24 hours. One mechanical brake H.P. costs 1·75 pence per hour for wages, coal, oil, etc., at full load; costs 2·5 pence at half load, and 6 pence at $\frac{1}{10}$ load, whilst at no load the cost of running is

15*s.* per hour. What should be the price charged per B.T.U., in order that there may be a profit of 10 per cent.?

Time hrs.	Output watts.	Watts generated.	Brake H.P. required.	Total cost for time in pence.	B.T.U. paid for.
2	300,000	$\frac{430}{400} \times 300{,}000$ = 322,500	$\frac{100}{92} \times \frac{322500}{746}$ = 469·95	@ 1·75 pence per hour = 1644·8	600
8	150,000	$\frac{430}{400} \times 150{,}000$ = 161,250	$\frac{100}{92} \times \frac{161250}{746}$ = 234·97	@ 2·5*d.* per hr. = 4699·4	1200
6	30,000	32,250	46·995	@ 6*d.* per hr. = 1691·82	180
8	0			@ 15*s.* hour = 1440	0
				Total cost 9475 pence	Total units sold 1980

whence the cost per unit is $\frac{9475}{1980}$ = 4·78 pence, and to pay 10 per cent. the price must be 5·258 pence.

EXAMPLES.

CHAPTER I.

1. The suspension of a moving-coil galvanometer is made of platinoid. It is $3\frac{1}{2}$ inches long, 0·0005 inch thick, and 0·004 inch wide. If platinoid will be 20 times the resistance of copper, what is the resistance of this suspension?

2. What relationship will the resistance of the above suspension have to the resistance of the coil, which is 1 inch wide and $2\frac{1}{4}$ inches long, being wound with 400 turns of copper wire 0·002 inch diameter?

3. What will be the E.M.F. required to send a current of 10 ampères through a circuit consisting of a dynamo of resistance of 7 ohms, $1\frac{1}{2}$ mile of leads of No. 6 B.W.G., and 80 lamps each of $2\frac{1}{2}$ ohms resistance? State the loss in the dynamo and in the leads, and give the horse-power of engine required, if the dynamo converts $\frac{9}{10}$ of the power received into electricity. Also if the lamps are paid for at so much per B.T.U. in the lamps, what will be the cost of 1 B.T.U. if a mechanical horse-power costs twopence per hour?

4. What size of leads will be required to keep 400 60-watt 100-volt lamps at a P.D. of 100 volts, if the dynamo is 170 feet from the lamps, has a resistance of 0·019 ohm, and produces a total E.M.F. of 106 volts?

5. How many secondary cells would be required to run the lamps in the above question when each cell has a resistance of 0·00007 ohm, and an E.M.F. of 2·05 volts at the start, but falling to 1·92 volt at the end of discharge? The lamps are connected to the cells by the same leads as above.

6. In the above question the numbers of cells do not come out exactly, but as whole numbers have to be used, find what compensating resistances would be necessary to make the current exactly right.

7. A series dynamo has a resistance of 10 ohms, and sends a current of 12 ampères through 40 arc lamps taking 48 volts each, and equal to 1000 candle-power, all connected in series on a circuit of a total length of $\frac{1}{4}$ mile

of No. 9 B.W.G. What will be the horse-power of engine required if the dynamo has a frictional loss of $\frac{3}{4}$ horse-power?

8. What will be the horse-power of a dynamo to run 40 incandescent lamps in series each of 1000 candle-power, and taking 65 ampères each at 48 volts, if they are on a circuit $\frac{1}{4}$ mile long of No. 3 B.W.G., and the resistance of the dynamo is 1·25 ohm, with a friction loss of $3\frac{1}{2}$ horse-power?

9. A dynamo has an armature resistance of 0·042 ohm, and is required to run 300 16-candle-power 50-watt lamps at the end of leads which are 0·01333 ohm resistance. The total current is 150 ampères. What will be the reading of a voltmeter connected to the terminals of the dynamo, and what percentage is this P.D. of the whole generated E.M.F.?

10. What will be the respective resistances of two bars, one of copper and one of aluminium, but both the same weight and the same length?

11. A pair of conductors, 1 square inch sectional area, conveys 400 ampères to a distance of $\frac{3}{4}$ of a mile from the dynamo. If the load at the far end requires a P.D. 410 volts, find the P.D. required at the dynamo.

12. In a secondary battery the lead lugs connecting cell to cell are $\frac{1}{2} \times 1$ inch in section, and 10 inches long each. If there are 54 cells joined in series, find the resistance of these connections.

13. If the cells in the above question have 15 plates each cell, each plate being 10 inches square and a quarter of an inch distant from the next, find the resistance of the electrolyte in each cell if the resistance of one cubic inch of the solution is 3 ohms.

14. Neglecting the E.M.F. of the cells in the above questions, find the E.M.F. required to send a current of 100 ampères through the mere resistances of the connections and electrolyte if all the cells are in series.

15. The connections on a switch-board are made of rod brass, and have an aggregate length of 27 feet. They are to carry 150 ampères, and the drop of pressure over all is not to exceed half a volt. Find the sectional area of the brass.

16. A standard manganin resistance of 0·001 ohm is 20 inches long. It is made of sheet metal $\frac{1}{16}$ inch thick. How wide will it be?

17. An arc-light circuit consists of 60 lamps in series, each taking 10 ampères. Each lamp has the positive carbon 18 inches long and $\frac{1}{2}$ inch diameter, and the negative carbon 12 inches long and $\frac{7}{16}$ inch diameter. Neglecting the effect of ^ heating, find the loss of energy in the carbons alone.

18. If the arc lamps in the above question be evenly spaced along a circuit at a distance of 30 yards apart, and the cable employed is $\frac{7}{16}$ S.W.G., what will be the total E.M.F. required if each lamp takes 45 volts in addition to the drop in the carbons?

increased temperature due to

19. What will be the nearest cable of strands of S.W.G. copper to make leads to convey 600 ampères a distance of 30 yards so that there shall not be more than 1·26 volt lost in the lead and return?

20. A motor taking current of 100 ampères at a P.D. of 2000 volts is run from a dynamo 8 miles away. If the loss in the leads is not to be more than 10 per cent., what will be the nearest cable of S.W.G. wire, and what will have to be the P.D. at the dynamo?

21. If the power in the last question be transmitted at 20,000 volts, what will then be the size of the bare solid conductor employed with only 5 per cent. loss of pressure?

22. Find the horse-power of engine required to run 1000 16-candle-power lamps taking 100 volts and 0·6 ampère each. There is a loss in the leads of 1 volt, and the dynamo gives out $\frac{9}{10}$ of the power received by it.

23. If all the above lamps are replaced by arcs in series, taking 10 ampères and 50 volts each of 900 candle-power, so as to give twice the total light, and if the loss in the leads be 4 volts, and the dynamo gives out 85 per cent. of the power received by it, find the horse-power of engine required, and the number of lamps.

24. In the calibration of an ammeter to read 2000 ampères, what will be the smallest power required, if a number of secondary cells be used in parallel, each cell giving 2 volts?

25. A street 4 miles long is to be supplied with 50 arc lamps per mile, each taking 10 ampères and 45 volts. The current is supplied by 4 dynamos, one to each mile of street, the leads being respectively 1, 2, 3, and 4 miles, lead and return, of $\frac{1}{16}$ square inch sectional area. If the mechanical efficiency of each dynamo be 90 per cent., and its resistance 10 ohms, find the E.M.F. of each, and the power required to drive each.

26. What size of leads will be required to send current to a number of 98-volt lamps from a dynamo giving 200 ampères at 100 volts?

27. A current is required to be sent a distance of 50 yards to 400 lamps, taking 0·6 ampère at 100 volts each, from a dynamo at P.D. of 101 volts. What will be the size of leads required?

28. How many incandescent, and how many arc lamps, will be required to give together 20,000 candle-power for 20,000 watts? Each arc gives 2000 candle-power for 500 watts, and each incandescent 16 candle-power for 60 watts.

CHAPTER II.

29. The distance between the feeding centres of a two-wire system of distributors is 400 yards, and current is required at the rate of 1 ampère per yard. What will be the size of the distributors if no two lamps are to

differ by more than 2 volts? And what will be the size of the feeders if the loss along them is not to exceed 10 volts, and the station is an average of 500 yards distant from any feeding centre?

30. Two parallel mains running round a district are of $\frac{1}{4}$ square inch sectional area, and are connected to the station by feeders $\frac{1}{2}$ square inch sectional area at points 400 yards apart. Each feeder has to convey 400 ampères to the mains, and there are 5 pairs, of lengths respectively of 100, 200, 300, 400, and 500 yards. If the P.D. at each feeding centre is maintained at 204 volts, what will be the P.D. in the station at the ends of each pair of feeders?

31. In the above circuit, what is the maximum difference in volts between any two lamps at full load?

32. The feeding centres in a two-wire system are 500 yards apart. What must be the sectional area of the distributors, so that there may not be more than 2 volts difference between any two lamps if the full load is at the rate of 1 ampère per yard of street?

33. The distributors in a street 1200 feet long are connected to feeders midway. The load is equal to one 48-watt 100-volt lamp per yard of street, and the feeding centre is maintained at 101 volts. Find the size of the distributors when the furthest lamps are at a P.D. of 99·5 volts at full load.

34. A system of distributors is $\frac{1}{2}$ square inch sectional area, and the load is one 50-watt 100-volt lamp per yard of street. If the feeding centres are maintained at 101 volts, find the distance between them so that the drop in the distributors shall not be more than $1\frac{1}{2}$ volt.

35. A street 500 yards long is supplied with current at the rate of 1 ampère per yard from a 200-volt circuit. What will be the size of leads if the dynamo is connected at the middle of the street, and the difference between any two lamps is not to be more than 3·5 volts? Also, if the loss in the feeders is 7 volts, what will be the horse-power required at the dynamo terminals?

36. In a five-wire system the outers are 1 square inch sectional area. What will be the maximum distance allowable between the feeding centres if no two lamps are to differ by more than $1\frac{1}{2}$ volt? There are an average of four 0·6-ampère lamps in series per yard of street.

37. If the distance between the feeding centres be 400 yards on a three-wire circuit running two 200-volt lamps in series per foot of street, each lamp taking 0·3 ampère, what must be the size of the outers if no two lamps are to differ by more than 2 volts?

38. In a three-wire system running 200-volt lamps, two in series between the outers, the feeding centres are 500 yards apart and supply a maximum current of 500 ampères each. If the outers are $\frac{1}{3}$ square inch sectional area,

what is the P.D. between the terminals of the poorest lamp when the feeding centres are kept at 410 volts?

39. A district is to be supplied with current at the rate of 1 ampère per yard of street, the system is two-wire with 100-volt lamps, and the feeding centres 600 yards apart. What will be the current supplied by each feeder, and what size must the distributors be in order that no two lamps may differ by more than 1 volt?

40. If the feeders in the last question are 700 yards long on an average, and the loss along them is 10 volts, find the weight of copper per mile of street.

41. A district takes ½ ampère per yard of street, using 200-volt lamps on a two-wire system. If the feeding centres are 600 yards apart, what current will they supply, and what will be the size of the distributors in order that no two lamps may differ by more than 2 volts?

42. If the feeders in the above case be 700 yards long, find the weight of copper per mile of street with 20 volts lost in feeders.

43. A district is supplied with two 100-volt lamps in series on a three-wire system at the rate of ½ ampère per yard of street. The middle wire is ¼ the section of the outers, and the feeders are 600 yards apart. If no two lamps are to differ more than 1 volt, find the size of the outers. Also find the weight of copper per mile of street, if there be a loss of 20 volts in the feeders, which have an average length of 700 yards.

44. If ¼ ampère per yard of street be supplied on a three-wire system with two 200-volt lamps in series, with feeders 600 yards apart, find the size of the distributors so that no two lamps shall differ more than 2 volts. And if 40 volts be lost in the feeders which are an average of 700 yards long, and the middle wire be ¼ the area of the outers, find the weight of copper per mile of street.

45. If the system be three-wire, with the outers 1 square inch sectional area, and the middle wire ½ square inch area, the feeders being 600 yards apart, what may be the maximum load in order that no two lamps may differ by more than 2 volts? Each lamp is for 200 volts, and takes 0·3 ampère.

46. A current is required along a street 2000 feet long at the rate of 1 ampère per 10 feet. Find the sectional area of conductors, so that there shall be only 2 volts difference between any two lamps. 1000 feet of copper 1 square inch section is 0·008 ohm—

(*a*) When current is supplied at one end of the street.

(*b*) When current is supplied midway. (*City Guilds Examination.*)

CHAPTER III.

47. A secondary battery of 54 cells has a resistance per cell of 0·0002 ohm and an E.M.F. of from 2 volts at start to 1·85 volt at the end of a run. What will be the current sent through a bank of lamps of 2 ohms resistance 70 yards away, connected by leads of 0·02 ohm resistance?

48. Find the speed at start and end of charging for a dynamo to send 150 ampères through the above cells. The dynamo produces $\frac{1}{10}$ volt per revolution per minute, is separately excited, and has armature resistance 0·021 ohm. In charging each cell has B.E.M.F. ranging from 2·2 to 2·4 volts, and the battery is 20 yards away, and connected by leads of 0·0004 ohm per yard.

49. What resistance should be introduced into the circuit on starting to charge in order to keep the current 150 ampères, though the dynamo runs at the higher speed?

50. Find the E.M.F. required to charge 108 cells in two parallel with 200 ampères each cell, if the B.E.M.F. is 2·3 volts each. Each cell has resistance 0·001 ohm, that of the dynamo being 0·005 ohm, and leads 0·01 ohm.

51. Find the E.M.F. required to charge 57 cells in series, each having B.E.M.F. of 2·3 volts and 0·0005 ohm resistance. The dynamo = 0·03 ohm, and leads 0·04 ohm, and current 100 ampères.

52. What will be the electrical efficiency of the above cells when discharging at 100 ampères through lamps on the above leads, each cell giving an average E.M.F. of 1·9 volt? Also state the P.D. at the lamp terminals.

53. If a dynamo gives 100 volts at 1000 revolutions, at what speed must it run to charge 54 cells at 50 ampères, each cell having B.E.M.F. 2·2 volts and 0·005 ohm internal resistance when the leads = 0·01 ohm?

54. If the above cells are discharged at 50 ampères, the E.M.F. of each being 2 volts at start, falling to 1·8 volts at the end of run, what percentage of the power put in is utilized?

CHAPTERS IV. TO IX.

55. What will be the brake horse-power of a motor series wound having armature resistance, 0·025 ohm, series coil 0·025 ohm, when running at 1200 revolutions per minute with a current of 50 ampères supplied at an E.M.F. of 100 volts? The electrical loss in the machine is $\frac{1}{3}$ of the friction losses.

56. In the above motor what will be the value of the total flux through

the armature if there be 360 wires all round? Also find the density in the air-gap, if the polar angle is 135 degrees, the bore is 22 cm. diameter, and length 20 cm.

57. In the above motor, what will be the pull in pounds on any one wire?

58. A motor is supplied with 150 ampères at 100 volts. The magnets are shunt wound to 40 ohms, and the armature resistance 0·01 ohm. If 1 horse-power is lost in various frictions, what will be the brake horse-power, and what the mechanical efficiency?

59. If the above motor armature has 120 wires all round, find the total lines, and the density in the air-gap, if the polar angle is 140 degrees, the diameter and length of bore respectively 30 cm. and 25 cm., and the speed 1200 per minute.

60. A machine separately excited gives 100 volts P.D. at 15 revolutions per second when run as a dynamo. The armature resistance is 0·015 ohm, the watts lost in eddy currents = 120, in hysteresis = 180, and in mechanical friction = 150. At what E.M.F. and current will it run, at the same speed as an unloaded motor?

61. A magneto machine gives 50 volts as a dynamo at no load. The armature resistance is 1 ohm, and 125 watts are lost in various frictions. Find the E.M.F. and current required to run it as an unloaded motor at the same speed.

62. A shunt dynamo having armature resistance 0·02 ohm, shunt resistance 40 ohms, when run on open circuit at 20 revolutions per second, gives P.D. of 100 volts. If it be separately excited at 100 volts, and run as a motor at the same speed, to work a crane lifting 3 tons, at the rate of 30 feet per minute, what will be the current taken, and E.M.F. required, if the crane has a friction loss at the rate of 200 pounds per ton lifted, and the mechanical friction of the motor is equal to 150 watts, the hysteresis loss 130 watts, and eddy current loss 170 watts? Assume that there is no armature reaction.

63. What is the commercial efficiency of the above combined crane and motor?

64. The E.M.F. produced by a separately excited machine when run as a dynamo is proportional to the speed, and is 100 volts at 1000 revolutions per minute. The armature resistance is 0·04 ohm. The loss due to hysteresis is proportional to the speed, and is 120 watts at 1000 per minute. The eddy current loss, however, is 80 watts at 1000 revolutions per minute, and is proportional to the square of the speed. The mechanical-friction loss varies directly as the speed, and is 110 watts at 1000 revolutions. If the above machine be supplied with current at 100 volts, at what speed will it run, and what current will it take (*a*) unloaded, (*b*) loaded to 2 brake horse-power?

65. A machine has armature resistance of 0·005 ohm, and shunt resistance 25 ohms, and is separately excited at 100 volts. If run at 600 per minute as a dynamo, it gives 100 volts on open circuit, the eddy current loss being 250 watts, the mechanical friction 300 watts, and hysteresis loss 400 watts. When supplied at 100 volts, what will be the speed and current required to run as an unloaded motor?

66. A series dynamo gives a current of 12 ampères, and is required to run a series motor of 1 brake horse-power. The dynamo has 360 wires all round, and a resistance of 1·3 ohm, with one million lines through the armature. The resistance of the motor is also 1·3 ohm, and ⅕ horse-power is lost in various internal frictions. At what speed must the dynamo run, and what will be the P.D. at the motor terminals, the electrical efficiency of the dynamo, and the electrical and mechanical efficiency of the motor?

67. A series motor produces a B.E.M.F. of 94 volts, and is supplied with 100 ampères at a P.D. of 100 volts. If ½ horse-power is lost in various frictions, what will be the brake horse-power, and the electrical and mechanical efficiency?

68. A motor has an armature resistance of 3 ohms, with 3·6 million lines through it. The shunt coil is 750 ohms, and ½ a horse-power is lost in various frictions. If the armature has 720 wires all round, and the machine is supplied with 15 ampères at 550 volts, find the speed, the brake horse-power, and the electrical and mechanical efficiency?

69. A series motor has 240 wires all round the armature, with a resistance of 0·08 ohm, and 22 million lines through it. The magnet winding is 0·12 ohm, and the machine is to give 100 brake horse-power when taking a current of 150 ampères. If n be the revolutions per second, and the mechanical friction loss is $100n$, the hysteresis loss $80n$, and the eddy current loss is $3n^2$, find the E.M.F., the speed, and electrical and mechanical efficiency.

70. A series machine has 140 wires all round the armature, which is 222·2 square centimetres sectional area, with a density of 18,000, and a speed of 1200 per minute. If the P.D. at the terminals for a current of 100 ampères is 106 volts, what would be the electrical efficiency as a dynamo, and also as a motor at the same speed and current? Also, if the friction loss is ⅔ horse-power, in both cases, state the brake horse-power as a motor and mechanical efficiency, both as dynamo and motor.

71. A series dynamo, when giving a current of 10 ampères, has a P.D. at its terminals of 1000 volts at a speed of 1040 per minute. The armature resistance is 1·8 ohm and magnet resistance 2·2 ohms. At what speed will it run as a motor if supplied with 10 ampères at a P.D. of 1000 volts? The internal friction losses are ½ horse-power. Also give electrical efficiency as a dynamo, the brake horse-power and electrical and mechanical efficiency as a motor.

72. What would be the E.M.F. required to run the above motor at 1040 revolutions per minute? and what would then be its brake horse-power and electrical and mechanical efficiency, the internal friction being 400 watts?

73. An armature running at 20 revolutions per second has 96 wires all round, and 4 million lines through it; its resistance is 1·05 ohm, and the shunt magnet coil 20 ohms. If 200 ampères be passed through the armature, what E.M.F. will be required to run it as a motor? Also if the hysteresis and eddy current losses are together equal to 300 watts, and ½ a horse-power is lost in mechanical friction, what will be the brake horse-power, and the electrical and mechanical efficiency?

74. A crane is to lift 1 ton at the rate of 500 feet per minute, and has a friction loss of 5 horse-power. It is run by a motor having 6 million lines through the armature, which has 400 wires all round, and a resistance of 0·05 ohm. There is a loss of ¾ of a horse-power in mechanical friction, and 500 watts in iron losses. The magnets are series wound to 0·08 ohm. If it is supplied at 600 volts, what will be the current taken, the speed, the brake horse-power, the electrical and mechanical efficiency of the motor, and the plant efficiency?

75. A motor with ring armature 16½ inches mean diameter and 120 wires all round runs in a field of 3000 lines per square centimetre at 800 revolutions per minute. The length of the bore is 16⅛ inches, its diameter being 17 inches, and the curved length of the polar face 22 inches. The resistance of the armature is 0·024 ohm, the shunt coil being 48 ohms. Half a horse-power is lost in mechanical friction, and 400 watts in iron losses. If the armature be supplied with a current of 200 ampères, what will be the pull on one side of the belt on a pulley 1 foot mean diameter? Also give the brake horse-power and the electrical and mechanical efficiency.

76. A mechanical contrivance requires 100 horse-power, and wastes 15 horse-power in friction. If run by a series motor having armature resistance 0·06 ohm, magnet coil 0·08 ohm, 360 wires all round the armature and 18 million lines through it, 1¼ horse-power lost in friction, 600 watts in iron, and supplied with a P.D. of 700 volts, find the speed, the current taken, the electrical and mechanical efficiency of the motor, and the plant efficiency.

77. A compound dynamo to give P.D. of 105 volts and a current of 90 ampères is to have 92 per cent. electrical efficiency. If N be about 3 million lines, the speed 1100 per minute, and half the loss is to be in the armature, the rest in the magnets divided as ⅔ in the shunt and ⅓ in the series coil, find the wires all round, and the resistances of the armature, shunt, and series coils.

78. A shunt motor has the shunt resistance 750 ohms, armature resistance 2 ohms, and N 4·2 million. If supplied with 15 ampères at 500

volts, find the brake horse-power, the speed, and the electrical and mechanical efficiency, if there be 720 wires all round the armature, and the friction losses are equivalent to 300 watts.

79. The magnetic circuit of a dynamo has reluctance = 0·0025, and an average of 5 million C.G.S. lines through it. Find the size of wire for a shunt winding, if the average length of one turn is 30 inches, and the P.D. at the terminals is 400 volts.

80. In the above question, what number of turns will be required on the shunt coil so that the energy waste shall not exceed 600 watts?

81. A dynamo takes 8000 ampère turns, and produces a terminal P.D. of 100 volts. If the average length of one turn on the shunt coil is 2 feet, find the size of wire required.

82. An armature running at 18 revolutions per second, has 288 wires all round, and a resistance of 0·025 ohm. Find N so as to give 70 ampères at 100 volts.

83. The electrical efficiency of a dynamo is 98 per cent., and mechanical efficiency 93 per cent. If the output be 200 units, what power is wasted in resistance, and what power in mechanical and other frictions?

84. A drum armature is 3 feet in diameter, with a 10-inch hole and 3·8 feet length of iron parallel to the shaft. It runs at 400 revolutions per minute, and is worked at such a density that the hysteresis loss is 5000 ergs per cubic centimetre per cycle. Find the hysteresis loss in watts.

85. If the resistance of the above armature be 0·005 ohm, the current 600 ampères at P.D. of 440 volts, shunt resistance 110 ohms, eddy current loss 2 horse-power, and mechanical friction 2½ horse-power, find the mechanical efficiency.

86. A shunt dynamo has electrical efficiency 95 per cent., and gives an output of 20 units. If there is an equal loss in the shunt and in the armature, and the P.D. is 100 volts, find the resistances of the armature and shunt coils.

87. If the mechanical efficiency of the above dynamo is $\frac{9}{10}$, what power must be wasted in other ways than in resistance?

88. A shunt dynamo has magnet coil 33 ohms resistance, and armature 0·005 ohm, and gives current to 500 60-watt 100-volt lamps, with a terminal P.D. of 101 volts. Find the electrical efficiency; and if the mechanical efficiency is 81 per cent., what horse-power of engine will be required, and what power is lost otherwise than electrically?

89. Two series machines, each having armature resistance 0·025 ohm, and magnet resistance 0·025, also N is 5 million lines, and the current 100 ampères. They are connected together by leads of 0·1 ohm resistance. Each machine has a frictional loss of 600 watts, which may be considered to be independent of the speed. If one machine be run as a

dynamo by an engine of 15 brake horse-power, at what speed will they run, and what will be the brake horse-power of the motor when the current is 100 ampères? Also state the ratio of the power given out by the motor to the power received by the dynamo.

90. In the last question, what brake horse-power engine would be required to run the dynamo if the motor be used so as to give out 15 brake horse-power, the current and losses being the same as before? Also state the speeds.

91. A series motor is to give 50 brake horse-power at a speed of 600 revolutions per minute. Its armature has 720 wires all round, is $\frac{2}{3}$ ohm resistance, and has 12 million lines through it. If the field coils have a resistance of 0·833 ohm, find the E.M.F. required to supply the above power through leads $2\frac{1}{2}$ ohms resistance, the friction losses being 2 horse-power.

92. How many ampère-turns will be required on each limb on a Manchester-type machine having $N = 4{\cdot}2 \times 10^6$, $A_a = 280$, l_a 35, A_g 1620, l_g 2·1, A_p 587, l_p 35, A_m 294, l_m 26, $\nu = 1{\cdot}5$? The pole-pieces are of cast iron, and magnet cores wrought iron.

93. A shunt dynamo to give 310 volts has the length of the average turn on the magnet coils 1 metre, and is of the single-magnet type. It also has the following dimensions: l_a 70, A_a 1400, N 21·6 millions, l_g 2·4, A_g 4000, l_m 240, A_m 2700. If $\nu = 1{\cdot}4$, and the magnets are wrought iron wound with 2000 turns of wire, what will be the shunt resistance at a temperature of 35° C.?

94. A single-magnet bipolar dynamo has the pole-pieces of cast iron and yoke of wrought iron, on which is placed the magnet winding. The armature resistance is 0·024 ohm hot, with 144 wires all round, and N = 7 million lines at no load. It is to give 200 ampères at a P.D. of 120 volts, and has the following dimensions: l_a 40, A_a 450, l_g 2·5, A_g 2300, l_p 80, A_p 1300, l_m 76, A_m 700, with $\nu = 1{\cdot}43$. Its shunt coil has 4000 turns, each of 45 inches average length, and the series coil is to be 0·01 ohm resistance hot. If the brushes are lead forward 6 wires on the armature at full load, find the windings of the series coil, the size of the shunt wire, and the electrical efficiency at full load. The shunt coil is connected "long shunt."

95. An armature, gramme wound, for a bipolar-field is required to carry 100 ampères at a P.D. of 100 volts. It has the following dimensions: N = 4 millions, n 20·43, w 126 each, being 80 cms. average length and 0·24 sq. cms. sectional area. The iron stampings are 25 cms. diameter with 15-cm. hole, and make altogether 25 cms. length of iron parallel to the spindle. If the over-all surface is 3675 sq. cms., what will be the rise in temperature above the air on account of copper and hysteresis losses, neglecting the heating due to eddy currents? Take 1 cc. of copper to be 0·000002 ohm resistance.

96. A drum armature for bipolar-field is 25 cms. diameter with 7-cm. hole, and has 25 cms. length of iron parallel to the shaft. It runs at 20·43 revolutions per second in a field of $4·95 \times 10^6$ lines, and is wound with 102 wires all round, each of 0·3 sq. cm. sectional area. Each convolution has an average length of 135 cms. If $\rho = 0·000002$, and there be the same rise in temperature as in the gramme armature above, find the maximum current allowable, neglecting the eddy current loss.

97. The magnetic system of the gramme armature in question 95 above has the following dimensions: l_a 31, A_a 250, l_g 2, A_g 950, l_m 170, A_m 415, all the iron parts being charcoal iron. The leakage factor is 1·4. Find the size of shunt wire required if the average turn is 3 feet long, and the P.D. at the terminals is 100 volts, the maximum temperature of the shunt coil being 35° C.

98. Find the size of shunt wire for a dynamo having the following dimensions: Armature ring wound, $w = 252$, l_a 37, A_a 137, diameter of core 10⅝ ins., bore 11⅜ ins., and length of 9 ins. Magnet limbs are of cast iron, 9 ins. × 7 ins. area and 6½ ins. apart, with no polar projections, l_m 100. Speed 1000 per minute to produce 97 volts on open circuit. One turn on the shunt coil is 40 ins. long, and the coils rise to a temperature of 40° C. Leakage factor 1⅓.

99. If the armature resistance in the above machine is 0·07 ohm, and the shunt coil 42 ohms, find the number of series turns to keep the P.D. of 97 volts at 70 ampères. Allow the series coil to be 0·04 ohm resistance, and the lead of the brushes 20 degrees. The connections are to be "long shunt."

100. A dynamo is to generate 118 volts, and has 17,080 lines per square centimetre in the armature, which has an area of 480 sq. cms., and runs at 720 revolutions. The average length of 1 coil on the (ring wound) armature is 46 ins., and the size of conductor 0·072 sq. in. If the machine be bipolar, and gives a current of 200 ampères, find the P.D. at the terminals at 35° C.

101. A dynamo has 200 wires all round the armature, which has a resistance of 0·03 ohm, an area of 250 sq. cms. at a density of 17,000, and is to run 200 half-ampère 16-candle-power lamps at the end of leads of ⅕ sq. in. area and 20 yards long. At what speed must she run to keep the P.D. at the lamps at 100 volts?

102. A shunt dynamo gives a terminal P.D. of 216 volts at a current of 50 ampères. Its armature has 7 million lines through it, and 160 wires all round, each coil being 4 feet long, and bipolar ring wound. It runs at 1200 revolutions, and the shunt resistance is 108 ohms. Find the size of wire on the armature at a temperature of 35° C.

CHAPTER X.

103. A voltmeter coil has a core ½ inch diameter, and can be wound to a depth of ¼ inch. It is 1·2 inch long, and requires about 300 ampère-turns to produce a deflection to the full extent of the scale. What size of wire will be required to make the instrument read 120 volts total, and what will be the energy wasted in the coil if wound with copper wire covered with silk to a radial depth of 0·001 inch?

104. A certain type of instrument wound as an ammeter to read 20 ampères total has 40 turns of wire on its coil. The length of the mean turn is 6 inches, and the diameter of the wire 0·2 inch. What is the waste of energy at full reading?

105. If the space for winding in the above instrument is 2 inches by 1¼ inch, find the size of wire and total waste of energy to make it read as a voltmeter to 20 volts with the coil wound with copper, and having an extra coil to increase the reading up to 120 volts wound with manganin.

106. How much will it cost a central station to keep an indication of 105 volts on each of two voltmeters for a year of 8000 hours, if the cost of a B.T.U. is 2⅝ pence, and interest on the first cost of instrument is to be at the rate of 5 per cent. per annum?—

(1) A magnetic instrument costing £10, and having a resistance of 2100 ohms.

(2) A hot-wire instrument costing £7, and having a resistance when in use of 470 ohms.

107. An instrument is wound with copper wire, and has a resistance of 1855·7 ohms at 0° C. Its dial is divided into 120 divisions, and 37·5 milli-ampères gives a deflection of 75 divisions. What will be the constant of this instrument as a voltmeter at 20° C., and what resistance should the extra coil of manganin have to increase the range to 2400 volts? Also state the T.V.R. with the extra coil in circuit if manganin is of constant resistance.

108. A voltmeter is wound with copper, and has a resistance of 2000 ohms at the temperature at which it was calibrated to read 120 volts total. It is first connected to the terminals of a dynamo in a hot engine-room, where its resistance rises to 2200 ohms. It is then taken outside, where the temperature is so low as to make its resistance only 1800 volts, and connected to the end of the circuit, but on the dynamo side of the load. The leads have a resistance of 0·02 ohm, and the lamps take 100 ampères. If the true P.D. at the dynamo is 102 volts, what will the instrument read in the two cases?

109. A voltmeter of 1500 ohms resistance is connected to the terminals

of a lamp. An ammeter connected in the main circuit reads 0·6 ampère. If the voltmeter indicates 150 volts, what is the resistance of the lamp?

110. An instrument reads 10 milli-ampères for 100 divisions deflection at 15° C., being wound with copper to a resistance of 100 ohms. What will it read as a voltmeter at 25° C., when in circuit with a manganin extra coil of 2900 ohms resistance?

111. A voltmeter to read 100 volts requires 1000 ampère-turns. If the average turn of wire be 3 inches long, find the size of wire required.

112. Find the diameter of copper wire for a voltmeter requiring from 300 to 400 ampère-turns, reading 60 volts total, if the mean diameter of the winding is ¾ inch.

113. A voltmeter requires 750 ampère-turns, and reads 120 volts total. If the mean turn is 6 inches long, find the diameter of copper wire required.

114. If only one-fourth part of the resistance of the above instrument is to be upon the working coil, the rest being of manganin on an extra coil, find the diameter of the copper wire required for the instrument and the total resistance, if the waste is 8 watts.

CHAPTER XI.

115. A train weighs 120 tons, and is to go at the rate of 25 miles per hour. The resistance to motion on the level is 25 lbs. per ton; the gearing is such that 1 ampère gives a pull of 10 lbs. The line is divided into four stages, viz. level, up an incline of 1 in 480, up incline of 1 in 348, and down incline of 1 in 960. Find the current required for each stage, going and returning, and also state the horse-power.

116. The resistance to motion on the level is 30 lbs. per ton, and the train weighs 100 tons. 1 ampère gives a tractive force of 20 lbs. Find the current, the E.M.F., and horse-power required to run both ways on the line in the previous question.

117. If the E.M.F. of supply is 500 volts, find the current required to move a train of 200 tons weight at a speed of 50 miles per hour both on the level and up and down an incline of 1 in 200. The resistance to motion on the level is 15 lbs. per ton.

118. In the last question find the total current to get up full speed in five minutes.

119. Find the current required to run a car of 12 tons weight at 10 miles per hour on the level and up and down inclines of 1 in 200 and 1 in 80, if the resistance to motion on the level is 25 lbs. per ton, and the E.M.F. of supply is 100 volts.

120. For the above car find the extra current required to get up full speed in half a minute.

121. A car weighing 10 tons runs up an incline of 1 in 100. The motor is geared to give a pull of 10 lbs. per ampère. If the resistance to motion on the level be 30 lbs. per ton, find the current required. Also give the horse-power for a speed of 4 miles per hour.

122. A tram-car, weighing 6 tons empty, starts empty, on a level line, picking up 20 persons halfway along. On reaching an incline up 1 in 50, 10 more persons get on. Fifteen persons get off when the car is going down 1 in 100. What will be the current required on the level both empty and loaded; on incline up 1 in 50 loaded; and down 1 in 100 loaded as above? The tractive force required is 25 lbs. per ton on the level, and 1 ampère gives a pull of 8 lbs. Also state the horse-power required for the various conditions for a uniform speed of 6 miles per hour. Take 20 persons to one ton.

123. A series dynamo has the following relationship between N in the armature and the current C :—

C	2	4	6	8	10	12	13	14	16	18
N	0·90	1·5	1·85	2·1	2·3	2·44	2·45	2·41	2·26	1·8

where N is in millions of lines. The armature has 500 wires all round, and a resistance of 0·77 ohm. The pull exerted as a motor is $4 \times C \times N \times 10^{-6}$ lbs. If supplied with an E.M.F. of 300 volts, find the current required as a motor to haul a car 3 tons in weight on level, and up and down incline of 1 in 150. Also give the speed of the car and the horse-power required. The resistance to traction on the level is 25 lbs. per ton.

124. A series dynamo of 0·15 ohm resistance is supplied with current at 440 volts, and has the following relationship between C and N :—

C	10	26	50	75	100	120	140	150	160	180	200	220	240	250
N	4	8	12	14·8	16·7	17·7	18·1	18·2	18·1	17·5	16·6	15·3	13·4	12

the induction being in millions of lines. If the tractive force exerted by it is $0{\cdot}5CN \times 10^{-6}$ lbs., find the C required to run a train of 50 tons weight on level line, taking 20 lbs. per ton; the maximum speed in miles per hour, and the revolutions per second of the motor.

125. With the above motor and train, give the C required, the maximum speed of train and of motor to go up an incline of 1 in 200.

126. For the same motor and train, find the maximum speed of train, the C required, and the speed of the motor to go down an incline of 1 in 200.

127. Again, with the same train, find the shortest time in which the maximum speed can be attained on the level line.

128. A shunt motor, working at 100 volts, has 200 wires all round the armature and the following relationship between the current in the armature C_a and the magnetic flux N :—

C_a	6	10	20	30	40	50	60	70	80	86
N	2·485	2·4825	2·475	2·46	2·4375	2·4023	2·3525	2·2823	2·17	2·0075

N being given in millions of lines. The armature resistance is 0·12 ohm, and that of the shunt 50 ohms. The pull exerted is $0·8CN \times 10^{-6}$ lbs. Find the motor speed, the current required, and the speed of the car to run on the level if the car weighs 4 tons and resistance to motion is 25 lbs. per ton.

129. With the same motor and car, find the current required, speed of car and motor to go up incline of 1 in 200.

130. For what current will the speed of the above motor be slowest?

CHAPTERS XII. TO XIV.

131. The maximum value of the alternating E.M.F. in a circuit is 2820 volts, and the resistance is 100 ohms, being non-inductive. What will be the average value of the current; and if the terminals of the circuit be connected to an electrostatic voltmeter, what will it indicate?

132. A transformer has a primary resistance 12 ohms, a secondary resistance 0·02 ohm. It is supplied with a virtual P.D. of 2010 volts at the primary. What will be the P.D. at the secondary terminals when running 50 lamps taking 0·6 ampère each, the ratio of transformation being 20 to 1, and the magnetizing current negligible?

133. A circuit has 10 ohms resistance and a B.E.M.F. of 20 volts: what will be the current through it if connected to an alternating P.D. of 29 volts as indicated on a hot-wire voltmeter?

134. What will be the horse-power supplied to a circuit having a B.E.M.F. of 100 volts virtual, and a resistance of 1 ohm, when connected to a dynamo producing a maximum E.M.F. of 200 volts alternating?

135. A circuit has a resistance of 0·015 ohm, and takes a current of 50 ampères when connected to an alternating P.D. of 60 volts. Find the B.E.M.F.

136. An alternating P.D. of 100 volts virtual is supplying a current of 12 ampères to two 40-volt lamps connected by a switch with two choking coils, in such a manner that when the switch is on contact No. 1, the two lamps in series are in series with coil x; but when the switch is on contact

No. 2, only one lamp is in circuit, and is in series, with the two coils x and y in series. Find the B.E.M.F. which the coils x and y must have.

137. What must be the maximum magnetic flux in the cores of the coils x and y in the previous question, if they have each 200 convolutions of wire, and negligible resistance, there being 80 complete cycles per second?

138. A transformer is to supply current to one hundred 60-watt 100-volt lamps, and the secondary resistance is 0·107 ohm. The primary has 20 times as many convolutions as the secondary, and its resistance is 6 ohms. The magnetizing current is 0·8 ampère, and lags practically 90° after the primary E.M.F. What must be the P.D. at the primary terminals to give the lamps 100 volts?

139. A transformer is working off mains kept at a P.D. of 1015 volts. The resistance of its primary coil is 13 ohms, and that of the secondary 0·11 ohm. If the ratio of transformation is 10 to 1, what will be the P.D. at the secondary terminals, when the secondary current is 20 ampères, and the primary magnetizing current is 0·5 ampère lagging 90°?

140. With the above transformer, what should the primary terminal P.D. be to make the secondary P.D. exactly 100 volts?

141. A transformer with 9·6 to 1 ratio of transformation is to give 100 ampères at a secondary P.D. of 205 volts. The secondary resistance is 0·035 ohm, and that of the primary 3 ohms. It is connected to the station by leads having a total resistance of 4 ohms. Find the P.D. at the station end, at zero load, half-load, and full load, when the magnetizing current may be neglected.

142. A transformer has an output of 10 units at full load, with a loss of 0·1 unit in the iron, and 0·1 unit in the copper. What will be the all-day efficiency if run on a load factor of?—

Hours	10	4	6	2	1	1
Load	0	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	1

143. A transformer has to convert from 1000 volts to 200 volts, giving a secondary current of 60 ampères. The primary resistance is 0·75 ohm, the secondary being 0·021 ohm. What will be the efficiency at $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and full load, if there be an iron loss of 120 watts?

144. If the above transformer be used on a load factor thus—

Hours	16	$\frac{1}{2}$	$\frac{1}{2}$	2	3	2
Load	0	1	$\frac{3}{4}$	$\frac{6}{10}$	$\frac{1}{2}$	$\frac{1}{4}$

what will be its all-day efficiency?

145. The primary coil of a closed iron circuit transformer is to work at P.D. of 2050 volts. The secondary is to give a maximum of 50 ampères, at a P.D. of 101 volts. The secondary resistance is 0·018 ohm, and the primary 8 ohms. If the iron loss is 140 watts, what will be the efficiency of conversion at full load, $\frac{1}{2}$, and $\frac{1}{10}$ load? And if employed on a load diagram as under, what will be the all-day efficiency?—

Hours	$\frac{1}{2}$	$\frac{1}{2}$	1	2	4	2	14
Load	1	0·9	0·75	0·5	0·4	0·2	0

146. A transformer has a secondary current of 400 ampères, the resistance of the secondary 0·0015 ohm, and the primary 0·8 ohm, with 20 times as many turns as the secondary. If the iron loss is two-thirds of a horse-power, and the magnetizing current is 5 ampères lagging nearly 90° after the primary E.M.F., what will be the efficiency at full, $\frac{1}{4}$, $\frac{1}{10}$, and $\frac{1}{20}$ load?

147. A number of open iron circuit transformers are supplied by a concentric cable 5 miles long, and having a capacity of 2 microfarads per mile. The total magnetizing current is 20 ampères lagging practically 90°. The cable has a resistance of 4 ohms, and the P.D. at the transformers is to be 2020 volts. Find the P.D. in the station and the capacity of the shunting condenser required to reduce the current in the cable to practically zero. The dynamo is 140 ∽ per second.

CHAPTER XV.

148. A series motor which requires 50 ampères at 400 volts has a resistance of 0·8 ohm. It is to have a starting and regulating resistance such that the starting current cannot exceed 30 ampères, and the speed may range from 200 to 800. Find the values of the resistances required, and state the maximum power for each speed with a current of 50 ampères.

149. A shunt motor having R_s 35, R_a 0·05, and running at 800 when supplied at 120 volts, is required to range in speed from 100 to 800. The full-load current is 100 ampères. What will be the value of the regulating resistance required, arranged so that the shunt is always at a P.D. of 120 volts? Also, if there be a friction loss of 800 watts proportional to the speed, what will be the brake horse-power at slow and high speeds with full current in each case?

150. A shunt motor is to have a regulating resistance arranged so as to insert resistance in the shunt circuit as it is removed from the armature circuit. $R_s = 25$, $R_a = 0·02$, and the full-load current is 250 ampères at 100 volts. The speed is to range from 200 to 600. Find the value of the resistance.

CHAPTER XVI.

151. A gas-engine consumes when on full load three times as much gas as when running idle. At full load it gives 5 horse-power at the pulley, and runs a dynamo having R_s 50 ohms and R_a 0·3 ohm, giving 100 volts. Find the average cost of 1 B.T.U. for a run of 12 hours, if on no load 2 hours, $\frac{1}{4}$ load 3 hours, $\frac{1}{2}$ load 6 hours, and full load of 40 sixty-watt lamps for 1 hour. The dynamo has a friction loss of 300 watts, and each brake horse-power on the engine takes 18 cubic feet of gas costing 3 shillings per 1000.

152. A steam-engine has a no-load loss equivalent to $\frac{3}{10}$ its maximum brake power, which is 200 horse, for each of which the consumption of coal is 1·8 lb. per hour, costing 15 shillings per ton. The friction load attached to it is 20 horse-power, and useful load 25 ampères at 400 volts for 5 hours. Find the average cost of 1 B.T.U. during the run.

ANSWERS.

1. 23·45 ohms.
2. $\frac{1}{23}$R of coil.
3. 2089·6 volts, 7 volts in dynamo, 19·6 volts in leads, 31·12 H.P., 3·112 pence.
4. 0·4557 sq. in.
5. 53 or 54.
6. 0·000916 ohm at start, and 0·00555 ohm at finish.
7. 33·69 H.P.
8. 179 H.P.
9. 102 volts; 94 per cent.
10. 2 to 1.
11. 435·472 volts.
12. 0·018 ohm.
13. 0·000535 ohm.
14. 4·7 volts.
15. 0·272 sq. in.
16. 5·76 ins.
17. 1746 to 3492 watts.
18. 2894 or 3070 volts.
19. $\frac{61}{11}$ S.W.G.
20. $\frac{91}{13}$ S.W.G.; 2185·6 volts.
21. About No. 13 S.W.G.
22. 90·26 H.P.
23. 28·5 H.P., 36 lamps.
24. 5·3 H.P.
25. E.M.F. 2364, 2378, 2392, and 2406 volts; power 35·2, 35·4, 35·6, 35·8 H.P.
26. 0·804 sq. in.
27. 0·578 sq. in.
28. 8 arcs, 272 incandescents.
29. 0·4824 sq. in., 0·9648 sq. in.
30. 207·86, 211·72, 215·58, 219·44, 223·29 volts.
31. 3·86 volts.
32. 0·7545 sq. in.

33. 0·3087 sq. in.
34. 498 yds.
35. 0·432 sq. in., 138·7 H.P.
36. 1290 yds.
37. 0·22 sq. in.
38. 202·73 volts.
39. 600 amps., 2·17 sq. ins.
40. 83·3 tons.
41. 300 amps., 0·5427 sq. in.
42. 20·7 tons.
43. 0·5427 sq. in., 21·93 tons.
44. 0·144 sq. in., 5·69 tons.
45. 1·82 amps. per yard of street.
46. 1·6 sq. in., 0·4 sq. in.
47. 53·1 amps. to 49·1 amps. at end of run.
48. 1259·7 and 1367·7 per min. at end.
49. 0·072 ohm.
50. 141 volts.
51. 140·95 volts.
52. 93·6 per cent., 101·45 volts.
53. 1328 per min.
54. 77·2 per cent.
55. 6·03 H.P.
56. 2291 per sq. cm.
57. 0·2566 lbs.
58. 18·47 H.P., 91·8 per cent.
59. 4,105,208; 4032.
60. 100·0675 volts; 4·5 amps.
61. 56·25 volts and 2·5 amps.
62. 101·08 volts, 56·62 amps.
63. 79 per cent.
64. 998·76 per min. and 3·1 amps.; 992·752 per min. and 18·119 amps.
65. 599·715 and 9·5 amps.
66. 30·05 per sec.; 90·18 volts, 83·3 per cent., 82·7 per cent., and 68·9 per cent.
67. 12·1 H.P., 94 per cent., and 90·27 per cent.
68. 19·568 per sec., 9·18 H.P., 87·7 per cent., and 83·06 per cent.
69. 540·576 volts, 9·67 per sec., 94·4 per cent., and 92·0 per cent.
70. 94·64 per cent. dynamo, 94·91 per cent. motor; 14·3 H.P., 90·6 per cent. as dynamo and 90·6 per cent. as motor.
71. 960 per min., 96·15 per cent. as dynamo, 12·35 H.P., 96 per cent., and 92·2 per cent.
72. 1080 volts, 13·4 H.P., 96·29 per cent., and 92·59 per cent.
73. 79·8 volts, 19·68 H.P., 94·3 per cent., and 90·2 per cent.
74. 53·84 amps., 24·7 per sec., 38·93 H.P., 98·83 per cent., 89·9 per cent., and 78·3 per cent.

75. 367·35 lbs., 28·5 H.P., 94·6 per cent., and 91·3 per cent.

76. 10·56 per sec., 111·07 amps., 97·7 per cent., 95·94 per cent., and 81·549 per cent.

77. 200 wires; R_s 39·8 ohms; R_a 0·0467, R_m 0·0156 ohm.

78. 8·65 H.P., 15·6 per sec., 90·0 per cent., and 86·0 per cent.

79. 0·00054 sq. in. area.

80. 6650.

81. 0·0419 in. diameter.

82. 1,962,770.

83. 5·49 H.P. in res., and 14·7 H.P. in frictions.

84. 2337.

85. 96·6 per cent.

86. 0·0125 ohm, and 18·9 ohms.

87. 1·56 H.P.

88. 96·3 per cent., 50·1 H.P., 7·88 H.P.

89. Dynamo 21·18 per sec., motor 17·18 per sec., 10·71 H.P., 71·4 per cent.

90. 19·29 H.P., dynamo 27·58 per sec., motor 23·58 per sec.

91. 1043·5 volts.

92. 7500.

93. 46 ohms.

94. Shunt wire 0·066 in. diameter, series 30 convolutions, 92·8 per cent.

95. 23·08° C.

96. 148·9 amps.

97. 0·057 in. diameter.

98. No. 15 S.W.G.

99. 96 convolutions.

100. 115·24 volts.

101. 730·2 per min.

102. 0·0081 sq. in.

103. 0·00224 in. diameter; 2·14 watts.

104. 2·2 watts.

105. 0·0143 in. diameter, 11·5 watts.

106. (*a*) £1 nearly, (*b*) £2 10*s*. 0*d*.

107. 38,000 ohms; 0·0199 per cent.

108. 92·72 in station; 111·11 volts outside.

109. 300 ohms.

110. 30·0388 volts total.

111. 0·005 in. diameter.

112. 0·0031 in. diameter.

113. 0·0056 in. diameter.

114. 0·0112 in. diameter; 1800 ohms.

115. Level, 300 amps., 200 H.P.; up 1 in 480, 356 amps., 237·3 H.P.; up 1 in 384, 370 amps., 246·6 H.P.; down 1 in 960, 272 amps., 181·3 H.P.; up 1 in 960, 328 amps., 218·6 H.P.; down 1 in 384, 230 amps., 153·3 H.P., down 1 in 480, 244 amps., 162·6 H.P.

116. Level 150 amps., 80 H.P.: up 1 in 480, 173·3 amps., 92·4 H.P.; up 1 in 384, 179·1 amps., 95·52 H.P.; down 1 in 960, 138·3 amps., 73·76 H.P.; up 1 in 960, 161·6 amps., 86·16 H.P.; down 1 in 384, 120·8 amps., 64·4 H.P.; down 1 in 480, 126·6 amps.; 67·52 H.P. The E.M.F. in all cases being 397·8 volts.

117. 596·8 amps., 1042 amps., 151·1 amps.

118. 1732 amps.

119. Level 59·68 amps., up 1 in 200, 86·53 amps., down 1 in 200, 32·8 amps., up 1 in 80, 126·5 amps., down 1 in 80, put on brake.

120. 82 amps.

121. 52·4 amps., 5·589 H.P.

122. Level, empty 18·75 amps., loaded 21·875 amps., 2·4 H.P. and 2·8 H.P., up 1 in 50, 65·43 amps., and 8·376 H.P.; down 1 in 100, 2·193 amps., and 0·28 H.P.

123. Level 8·7 amps., 1504 ft. per min., 3·41 H.P.; up 1 in 150, 12·3 amps., 1317 ft. per min., 4·78 H.P.; down 1 in 150, 2·7 amps., 1426 ft. per min., and 1·077 H.P.

124. 115 amps.; 24·4 miles per hour, 6·71 revolutions per sec.

125. 177 amps., 23·56 miles per hour, 6·5 revolutions per sec.

126. 64 amps., 31·45 miles per hour, 8·69 revolutions per sec.

127. 3·05 mins.

128. 54·2 amps., 2165 ft. per min., 19·6 revolutions per sec.

129. 77 amps., 2273 ft. per min., 20·53 revolutions per sec.

130. About 42 amps.

131. 17·76 amps., 1993 volts.

132. 99 volts.

133. 2·1 amps.

134. 13·4 H.P.

135. 60 volts.

136. x 60 volts, y 31·65 volts.

137. x 84,375, y 27,630.

138. 2020·5 volts.

139. 99·24 volts.

140. 1022·2 volts.

141. 1968, 2021·55 and 2075·1 volts.

142. 95·51 per cent.

143. 92·4, 95·7, 97·3, and 97·5 per cent.

144. 93·6 per cent.

145. 95·5, 93·9, 78·0, and all-day 86·7 per cent.

146. 97·38, 94·7, 88·41, and 79·36 per cent.

147. 2020 volts, 1·25 microfarad.

148. 13⅓ ohms starting, 5·4 ohms regulating, 6 H.P. slow, and 24·1 H.P. full speed.

149. 1·9 H.P. and 14·3 H.P.

150. 0·252 ohm.

151. 2·21 pence.

152. 1·35 pence.

SPECIFIC RESISTANCE OF VARIOUS MATERIALS AT 0° C., WITH TEMPERATURE VARIATIONS.

Material.	Ohms per cubic inch.	Ohms per cubic centimetre.	Temperature variation of resistance per cent. per degree Centigrade.	Weight in pounds per cubic inch.
Copper	0·00000063	0·0000016	0·388 increase	0.320
Iron	0·00000405	0·0000104	0·453 ,,	0·280
Aluminium ...	0·00000114	0·0000029	0·390 ,,	0·092
Mercury	0·00003666	0·0000943	0·072 ,,	0·489
Lead	0·00000780	0·0000200	0·387 ,,	0·410
Platinum	0·00000353	0·0000089	0·247 ,,	0·828
Platinoid	0·000014	0·000034	0·021 ,,	
German silver ...	0·0000082	0·000021	0·044 ,,	0·30
Manganin ...	0·000018	0·000044	0·002 decrease	
Brass	0·0000028	0·0000072		0·290
Platinum-silver ...	0·0000095	0·000024	0·031 increase	
Carbon (arc lamp)	0·0017 to 0·0034	0·0044 to 0·0086	0·052 decrease	
Sulphuric acid— solution 1·1 sp. g.	3·33	8·45	2·2 ,,	
,, 1·2 sp. g.	2·62	6·66	2·2 ,,	

VALUES OF THE SPECIFIC RESISTANCE OF COMMERCIAL COPPER AT VARIOUS TEMPERATURES.

Temperature degrees.		Resistance of one cubic centimetre in ohms.	Resistance of one cubic inch in ohms.
Cent.	Fahr.		
0	32	0·00000160	0·00000063
5	41	0·00000163	0·00000064
10	50	0·00000167	0·00000065
15	59	0·00000169	0·00000067
20	68	0·00000172	0·00000068
25	77	0·00000176	0·00000069
30	86	0·00000179	0·00000071
35	95	0·00000182	0·00000072
40	104	0·00000186	0·00000073
45	113	0·00000189	0·00000074
50	122	0·00000193	0·00000076
55	131	0·00000196	0·00000077
60	140	0·00000200	0·00000078
65	149	0·00000203	0·00000079
70	158	0·00000207	0·00000081
75	167	0·00000210	0·00000082
80	176	0·00000214	0·00000084
85	185	0·00000217	0·00000085
90	194	0·00000221	0·00000087
95	203	0·00000224	0·00000088
100	212	0·00000228	0·00000090

Values of H corresponding to Various Values of β for Cyclic Changes in β as a Sine-Function of the Time, and having Maxima of 1000, 2000, etc., to 7000.

The values are given for each hundred β for a complete half-period. Values for the other half-period can be easily put in by reversing the signs.

The hysteresis loss *h* in ergs per cc. per cycle corresponding with these values of the cyclic βH curve are given in the table of values of β and H on pp. 214–218.

β	H	β	H	β	H	β	H
0	0·49	600	0·72	900	0·61	400	− 0·20
100	0·55	700	0·75	800	0·42	300	− 0·28
200	0·59	800	0·78	700	0·23	200	− 0·36
300	0·62	900	0·80	600	0·07	100	− 0·42
400	0·66	1000	0·83	500	− 0·07	0	− 0·49
500	0·69						
0	0·62	1100	0·88	1900	0·90	900	− 0·28
100	0·65	1200	0·90	1800	0·72	800	− 0·34
200	0·67	1300	0·92	1700	0·55	700	− 0·38
300	0·69	1400	0·95	1600	0·40	600	− 0·42
400	0·72	1500	0·97	1500	0·26	500	− 0·46
500	0·74	1600	1·00	1400	0·16	400	− 0·50
600	0·76	1700	1·03	1300	0·05	300	− 0·53
700	0·78	1800	1·05	1200	− 0·04	200	− 0·56
800	0·80	1900	1·08	1100	− 0·14	100	− 0·59
900	0·83	2000	1·12	1000	− 0·21	0	− 0·62
1000	0·85						
0	0·83	1600	1·09	2900	1·06	1400	− 0·45
100	0·85	1700	1·11	2800	0·80	1300	− 0·49
200	0·87	1800	1·13	2700	0·61	1200	− 0·53
300	0·88	1900	1·14	2600	0·46	1100	− 0·56
400	0·90	2000	1·16	2500	0·33	1000	− 0·60
500	0·91	2100	1·18	2400	0·21	900	− 0·63
600	0·92	2200	1·20	2300	0·11	800	− 0·66
700	0·94	2300	1·22	2200	0·0	700	− 0·68
800	0·96	2400	1·24	2100	− 0·08	600	− 0·71
900	0·97	2500	1·26	2000	− 0·15	500	− 0·74
1000	0·99	2600	1·27	1900	− 0·22	400	− 0·76
1100	1·01	2700	1·29	1800	− 0·27	300	− 0·79
1200	1·02	2800	1·31	1700	− 0·32	200	− 0·81
1300	1·04	2900	1·33	1600	− 0·37	100	− 0·82
1400	1·06	3000	1·35	1500	− 0·41	0	− 0·83
1500	1·07						

β	H	β	H	β	H	β	H
0	0·91	2100	1·20	3900	1·25	1900	− 0·55
100	0·92	2200	1·22	3800	1·00	1800	− 0·58
200	0·94	2300	1·23	3700	0·80	1700	− 0·60
300	0·95	2400	1·25	3600	0·65	1600	− 0·62
400	0·96	2500	1·26	3500	0·52	1500	− 0·65
500	0·98	2600	1·28	3400	0·39	1400	− 0·67
600	0·99	2700	1·30	3300	0·27	1300	− 0·70
700	1·00	2800	1·31	3200	0·17	1200	− 0·72
800	1·01	2900	1·33	3100	0·08	1100	− 0·74
900	1·02	3000	1·35	3000	0·00	1000	− 0·76
1000	1·03	3100	1·37	2900	− 0·07	900	− 0·78
1100	1·05	3200	1·38	2800	− 0·14	800	− 0·79
1200	1·07	3300	1·40	2700	− 0·20	700	− 0·81
1300	1·09	3400	1·42	2600	− 0·25	600	− 0·83
1400	1·10	3500	1·44	2500	− 0·31	500	− 0·84
1500	1·11	3600	1·46	2400	− 0·36	400	− 0·85
1600	1·12	3700	1·48	2300	− 0·40	300	− 0·86
1700	1·14	3800	1·50	2200	− 0·44	200	− 0·88
1800	1·15	3900	1·53	2100	− 0·47	100	− 0·90
1900	1·17	4000	1·56	2000	− 0·51	0	− 0·91
2000	1·18						

0	0·98	2600	1·27	4900	1·34	2400	− 0·68
100	0·99	2700	1·29	4800	1·08	2300	− 0·70
200	1·00	2800	1·31	4700	0·90	2200	− 0·72
300	1·01	2900	1·33	4600	0·74	2100	− 0·74
400	1·02	3000	1·35	4500	0·60	2000	− 0·75
500	1·02	3100	1·37	4400	0·46	1900	− 0·77
600	1·03	3200	1·38	4300	0·34	1800	− 0·78
700	1·04	3300	1·40	4200	0·22	1700	− 0·80
800	1·05	3400	1·42	4100	0·12	1600	− 0·81
900	1·06	3500	1·44	4000	0·00	1500	− 0·82
1000	1·07	3600	1·46	3900	− 0·08	1400	− 0·83
1100	1·08	3700	1·48	3800	− 0·17	1300	− 0·84
1200	1·10	3800	1·50	3700	− 0·24	1200	− 0·85
1300	1·11	3900	1·53	3600	− 0·30	1100	− 0·86
1400	1·12	4000	1·55	3500	− 0·36	1000	− 0·88
1500	1·13	4100	1·57	3400	− 0·40	900	− 0·89
1600	1·14	4200	1·59	3300	− 0·44	800	− 0·90
1700	1·15	4300	1·61	3200	− 0·47	700	− 0·91
1800	1·17	4400	1·63	3100	− 0·50	600	− 0·92
1900	1·18	4500	1·65	3000	− 0·54	500	− 0·93
2000	1·20	4600	1·67	2900	− 0·57	400	− 0·94
2100	1·21	4700	1·70	2800	− 0·59	300	− 0·95
2200	1·23	4800	1·72	2700	− 0·62	200	− 0·96
2300	1·24	4900	1·74	2600	− 0·64	100	− 0·97
2400	1·25	5000	1·76	2500	− 0·66	0	− 0·98
2500	1·26						

CYCLIC βH VALUES.

β	H	β	H	β	H	β	H
0	1·06	3100	1·42	5900	1·72	2900	– 0.75
100	1·07	3200	1·43	5800	1·40	2800	– 0·76
200	1·08	3300	1·45	5700	1·16	2700	– 0·77
300	1·09	3400	1·46	5600	0·94	2600	– 0·79
400	1·10	3500	1·48	5500	0·78	2500	– 0·80
500	1·11	3600	1·50	5400	0·62	2400	– 0·81
600	1·12	3700	1·52	5300	0·47	2300	– 0·82
700	1·13	3800	1·54	5200	0·37	2200	– 0·84
800	1·14	3900	1·56	5100	0·27	2100	– 0·85
900	1·15	4000	1·58	5000	0·17	2000	– 0·86
1000	1·16	4100	1·60	4900	0·08	1900	– 0·87
1100	1·17	4200	1·62	4800	0·00	1800	– 0·88
1200	1·18	4300	1·64	4700	– 0·10	1700	– 0·89
1300	1·20	4400	1·66	4600	– 0·17	1600	– 0·90
1400	1·21	4500	1·68	4500	– 0·24	1500	– 0·91
1500	1·22	4600	1·70	4400	– 0·28	1400	– 0·92
1600	1·23	4700	1·72	4300	– 0·33	1300	– 0·93
1700	1·24	4800	1·74	4200	– 0·37	1200	– 0·94
1800	1·25	4900	1·76	4100	– 0·42	1100	– 0·95
1900	1·26	5000	1·78	4000	– 0·46	1000	– 0·96
2000	1·27	5100	1·80	3900	– 0·49	900	– 0·97
2100	1·28	5200	1·83	3800	– 0·52	800	– 0·98
2200	1·29	5300	1·85	3700	– 0·56	700	– 0·99
2300	1·30	5400	1·88	3600	– 0·59	600	– 0·100
2400	1·31	5500	1·91	3500	– 0·62	500	– 0·101
2500	1·32	5600	2·00	3400	– 0·64	400	– 0·102
2600	1·34	5700	2·10	3300	– 0·67	300	– 0·103
2700	1·35	5800	2·20	3200	– 0·69	200	– 0·104
2800	1·36	5900	2·30	3100	– 0·72	100	– 0·105
2900	1·38	6000	2·40	3000	– 0·74	0	– 0·106
3000	1·40						

0	1·120	1800	1·296	3600	1·536	5400	1·890
100	1·128	1900	1·308	3700	1·552	5500	1·910
200	1·136	2000	1·320	3800	1·568	5600	1·940
300	1·144	2100	1·332	3900	1·584	5700	1·960
400	1·152	2200	1·344	4000	1·600	5800	1·980
500	1·160	2300	1·356	4100	1·620	5900	2·010
600	1·170	2400	1·368	4200	1·640	6000	2·050
700	1·180	2500	1·380	4300	1·660	6100	2·080
800	1·190	2600	1·394	4400	1·680	6200	2·110
900	1·200	2700	1·408	4500	1·700	6300	2·140
1000	1·210	2800	1·422	4600	1·718	6400	2·180
1100	1·220	2900	1·436	4700	1·736	6500	2·210
1200	1·230	3000	1·450	4800	1·754	6600	2·250
1300	1·240	3100	1·464	4900	1·772	6700	2·290
1400	1·250	3200	1·478	5000	1·790	6800	2·340
1500	1·260	3300	1·492	5100	1·810	6900	2·380
1600	1·272	3400	1·506	5200	1·830	7000	2·440
1700	1·284	3500	1·520	5300	1·860	6900	1·960

β	H	β	H	β	H	β	H
6800	1·600	5000	− 0·460	3300	− 0·830	1600	− 0·972
6700	1·380	4900	− 0·492	3200	− 0·840	1500	− 0·980
6600	1·190	4800	− 0·524	3100	− 0·850	1400	− 0·988
6500	1·000	4700	− 0·556	3000	− 0·860	1300	− 0·996
6400	0·840	4600	− 0·588	2900	− 0·870	1200	− 1·004
6300	0·720	4500	− 0·620	2800	− 0·880	1100	− 1·012
6200	0·590	4400	− 0·644	2700	− 0·890	1000	− 1·020
6100	0·440	4300	− 0·668	2600	− 0·900	900	− 1·030
6000	0·320	4200	− 0·692	2500	− 0·910	800	− 1·040
5900	0·200	4100	− 0·716	2400	− 0·918	700	− 1·050
5800	0·110	3000	− 0·740	2300	− 0·926	600	− 1·060
5700	0·000	3900	− 0·752	2200	− 0·934	500	− 1·070
5600	− 0·100	3800	− 0·764	2100	− 0·942	400	− 1·080
5500	− 0·200	3700	− 0·776	2000	− 0·950	300	− 1·090
5400	− 0·252	3600	− 0·780	1900	− 0·956	200	− 1·100
5300	− 0·304	3500	− 0·810	1800	− 0·962	100	− 1·110
5200	− 0·356	3400	− 0·820	1700	− 0·968	0	− 1·120
5100	− 0·403						

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

β Density in c.g.s. lines per sq. cm. area.	H in c.g.s. per cm. length.	$f\beta$ Ampère-turns per cm. length.	μ Permeability.	h Hysteresis loss in ergs per cc. per cycle.	$\beta/f\beta$ Product of density and ampère-turns per cm. length.
100	0·17	0·140	588	5	14
200	0·32	0·260	625	10	52
300	0·44	0·356	685	17	107
400	0·54	0·432	740	25	172
500	0·60	0·476	833	37	238
600	0·65	0·520	908	50	312
700	0·69	0·552	1014	65	386
800	0·76	0·584	1052	80	467
900	0·78	0·624	1154	97	561
1000	0·83	0·664	1204	115	664
1100	0·87	0·696	1264	134	765
1200	0·91	0·728	1318	155	873
1300	0·94	0·752	1382	177	978
1400	0·97	0·776	1443	200	1086
1500	0·99	0·796	1515	222	1194
1600	1·02	0·816	1568	245	1305
1700	1·04	0·836	1634	270	1421
1800	1·07	0·856	1682	295	1540
1900	1·09	0·872	1743	320	1656
2000	1·11	0·888	1802	345	1776
2100	1·13	0·904	1858	372	1898
2200	1·15	0·922	1913	400	2024
2300	1·17	0·940	1966	430	2162
2400	1·19	0·958	2117	460	2304
2500	1·22	0·976	2049	490	2440
2600	1·24	0·992	2096	520	2579
2700	1·26	1·01	2142	550	2727
2800	1·29	1·03	2170	580	2889
2900	1·31	1·05	2214	610	3045
3000	1·34	1·07	2238	640	3216
3100	1·36	1·09	2279	670	3379
3200	1·38	1·10	2318	700	3520
3300	1·40	1·12	2357	735	3696
3400	1·42	1·14	2394	770	3862
3500	1·44	1·16	2430	805	4065
3600	1·47	1·18	2448	840	4233
3700	1·49	1·20	2486	877	4440
3800	1·51	1·21	2516	915	4590
3900	1·53	1·22	2548	952	4758

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

β Density in c.g.s. lines per sq. cm. area.	H in c.g.s. per cm. length.	$f\beta$ Ampère-turns per cm. length.	μ Permeability.	h Hysteresis loss in ergs per cc. per cycle.	$\beta f\beta$ Product of density and ampère-turns per cm. length.
4000	1·55	1·24	2580	990	4960
4100	1·57	1·26	2611	1022	5166
4200	1·59	1·27	2641	1055	5346
4300	1·61	1·29	2670	1092	5567
4400	1·63	1·30	2699	1130	5781
4500	1·65	1·32	2726	1170	5940
4600	1·67	1·34	2754	1210	6145
4700	1·69	1·36	2780	1252	6392
4800	1·72	1·38	2790	1295	6604
4900	1·74	1·39	2816	1337	6811
5000	1·76	1·41	2841	1380	7040
5100	1·78	1·42	2865	1420	7242
5200	1·80	1·44	2889	1460	7488
5300	1·82	1·46	2912	1505	7738
5400	1·85	1·48	2918	1550	7992
5500	1·88	1·50	2925	1590	8250
5600	1·91	1·53	2931	1630	8556
5700	1·94	1·55	2938	1677	8835
5800	1·97	1·58	2944	1725	9140
5900	2·00	1·60	2950	1772	9445
6000	2·03	1·62	2955	1820	9744
6100	2·06	1·65	2961	1867	10065
6200	2·10	1·68	2955	1915	10416
6300	2·13	1·71	2952	1962	10773
6400	2·17	1·74	2949	2010	11110
6500	2·21	1·77	2941	2060	11505
6600	2·25	1·80	2938	2110	11880
6700	2·28	1·83	2933	2160	12261
6800	2·32	1·86	2930	2210	12662
6900	2·36	1·89	2923	2260	13041
7000	2·41	1·93	2904	2310	13496
7100	2·46	1·97	2886	2362	13987
7200	2·51	2·01	2868	2415	14457
7300	2·56	2·05	2851	2470	14965
7400	2·62	2·10	2824	2525	15510
7500	2·67	2·14	2808	2587	16050
7600	2·73	2·18	2783	2650	16598
7700	2·78	2·23	2769	2705	17171
7800	2·84	2·27	2746	2760	17721
7900	2·90	2·32	2723	2810	18328
8000	2·96	2·37	2703	2860	18944
8100	3·01	2·41	2691	2939	19521
8200	3·06	2·45	2679	3018	20090
8300	3·12	2·50	2660	3097	20750
8400	3·18	2·55	2641	3176	21420

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

β Density in c.g.s. lines per sq. cm. area.	H in c.g s. per cm. length.	$f\beta$ Ampère-turns per cm. length.	μ Permeability.	h Hysteresis loss in ergs per cc. per cycle.	$\beta f\beta$ Product of density and ampère-turns per cm. length.
8500	3·25	2·60	2615	3255	22100
8600	3·31	2·65	2598	3334	22790
8700	3·37	2·70	2581	3413	23490
8800	3·43	2·75	2565	3492	24200
8900	3·50	2·80	2542	3571	24720
9000	3·56	2·85	2528	3650	25650
9100	3·62	2·90	2513	3730	26390
9200	3·68	2·95	2500	3810	27140
9300	3·76	3·01	2473	3890	27993
9400	3·85	3·08	2441	3970	28952
9500	3·92	3·14	2423	4050	29830
9600	4·00	3·20	2400	4130	30730
9700	4·08	3·27	2377	4210	31719
9800	4·16	3·35	2355	4290	32830
9900	4·27	3·42	2318	4370	33858
10000	4·37	3·50	2288	4450	35000
10100	4·48	3·59	2254	4535	36259
10200	4·60	3·68	2217	4620	37536
10300	4·71	3·77	2187	4705	38831
10400	4·82	3·86	2157	4790	40147
10500	4·96	3·96	2117	4875	41580
10600	5·07	4·06	2090	4960	43036
10700	5·20	4·16	2057	5045	44512
10800	5·33	4·27	2026	5130	46116
10900	5·46	4·37	1995	5215	47633
11000	5·60	4·48	1963	5300	49280
11100	5·73	4·59	1937	5390	50949
11200	5·87	4·70	1908	5480	52640
11300	6·03	4·83	1873	5570	54579
11400	6·20	4·96	1838	5650	56544
11500	6·40	5·12	1797	5750	58880
11600	6·60	5·28	1757	5840	61248
11700	6·85	5·48	1707	5930	64116
11800	7·10	5·68	1662	6020	67024
11900	7·36	5·88	1617	6110	69972
12000	7·60	6·08	1579	6200	72960
12100	7·90	6·32	1531	6300	76472
12200	8·20	6·56	1487	6400	80032
12300	8·50	6·80	1437	6500	83640
12400	8·90	7·12	1393	6600	88288
12500	9·20	7·36	1351	6700	92000
12600	9·60	7·68	1312	6800	96768
12700	10·0	8·00	1270	6900	101600
12800	10·4	8·32	1231	7000	106496
12900	10·8	8·64	1192	7100	111456

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

β Density in c.g.s. lines per sq. cm. area.	H in c.g.s. per cm. length.	$f\beta$ Ampère-turns per c.m. length.	μ Permeability.	h Hysteresis loss in ergs per cc. per cycle.	$\beta f\beta$ Product of density and ampère-turns per cm. length.
13000	11·3	9·04	1151	7200	117520
13100	11·7	9·36	1115	7300	122616
13200	12·2	9·76	1082	7400	128820
13300	12·7	10·1	1047	7500	134330
13400	13·2	10·5	1015	7600	140700
13500	13·7	10·9	986	7700	147150
13600	14·2	11·3	958	7800	153680
13700	14·8	11·8	926	7900	161600
13800	15·4	12·3	896	8000	169740
13900	16·1	12·8	863	8100	177920
14000	16·9	13·5	839	8200	189000
14100	17·8	14·2	793	8310	200220
14200	18·7	14·9	759	8420	211580
14300	19·7	15·7	725	8530	224510
14400	20·7	16·5	695	8640	237600
14500	21·8	17·4	665	8750	252300
14600	23·0	18·4	636	8860	278640
14700	24·2	19·3	608	8970	283710
14800	25·5	20·4	580	9080	301920
14900	26·8	21·4	555	9190	318860
15000	28·6	22·8	524	9300	340000
15100	30·0	24·0	503	9410	362400
15200	31·5	25·2	482	9520	383040
15300	33·1	26·4	461	9630	403920
15400	35·0	28·0	440	9740	431200
15500	37·1	29·6	419	9850	458800
15600	39·2	31·3	398	9960	488280
15700	41·5	33·2	379	10070	521240
15800	43·8	35·0	361	10180	553000
15900	46·8	37·4	341	10290	594660
16000	48·8	39·0	321	10400	624000
16100	53·2	42·5	303	10510	684250
16200	56·7	45·3	285	10620	733860
16300	60·7	48·5	270	10730	790550
16400	65·0	52·0	256	10840	852800
16500	69·9	55·9	238	10950	922340
16600	75·2	60·1	221	11060	997660
16700	80·7	64·5	207	11170	1077150
16800	87·0	69·6	193	11280	1169280
16900	93·0	74·4	181	11390	1257360
17000	100·0	80·0	170	11500	1360000
17100	107·0	85·5	159	11620	1462050
17200	115·0	92·0	149	11740	1582400
17300	123·0	98·4	141	11860	1702320
17400	131·0	104·0	133	11980	1809600

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

β Density in c.g.s. lines per sq. cm. area.	H in c.g.s. per cm. length.	$f\beta$ Ampère-turns per cm. length.	μ Permeability.	h Hysteresis loss in ergo per cc. per cycle.	$\beta f\beta$ Product of density and ampère-turns per cm. length.
17500	140	112	125	12100	1960000
17600	149	119	117	12220	2094400
17700	158	126	111	12340	2230200
17800	167	133	106	12460	2367400
17900	176	140	101	12580	2506000
18000	186	148	96	12700	2664000
18100	196	156	90	12820	2823600
18200	211	168	86	12940	3057600
18300	226	180	80	13060	3294000
18400	244	195	71	13180	3588000
18500	260	208	69	13300	3848000
18600	276	220	67	13420	4092000
18700	293	234	65	13540	4375800
18800	311	248	64	13660	4662400
18900	330	264	60	13780	4989600
19000	350	280	54	13900	5320000
19100	372	297	51	14020	5672700
19200	396	316	48	14140	6067200
19300	422	337	45	14260	6504100
19400	448	358	43	14380	6945200
19500	476	380	41	14500	7410000
19600	505	404	39	14620	7918400
19700	537	429	36	14740	8451300
19800	575	460	34	14860	9108000
19900	609	487	32	14980	9691300
20000	650	520	31	15100	10400000

MAGNETIC PROPERTIES OF SOFT GREY CAST IRON.

β Density in c.g.s. lines per sq. cm.	H c.g.s. per cm. length.	$f\beta$ Ampère-turns per cm. length.	μ Permeability.
3000	1·5	1·2	2000
3100	1·9	1·5	1695
3200	2·3	1·8	1391
3300	2·7	2·2	1241
3400	3·1	2·5	1096
3500	3·5	2·8	1009
3600	3·9	3·1	923
3700	4·3	3·4	860
3800	4·7	3·8	808
3900	5·1	4·1	764
4000	5·5	4·4	727
4100	5·9	4·7	695
4200	6·3	5·0	666
4300	6·7	5·4	641
4400	7·1	5·7	619
4500	7·6	6·1	592
4600	8·1	6·5	568
4700	8·7	7·0	540
4800	9·3	7·4	516
4900	10·0	8·0	490
5000	10·7	8·6	467
5100	11·6	9·3	439
5200	12·5	10·0	416
5300	13·5	10·8	392
5400	14·5	11·6	372
5500	15·8	12·6	348
5600	17·1	13·7	327
5700	18·5	14·8	307
5800	20·0	16·0	290
5900	21·6	17·3	274
6000	23·2	18·5	258
6100	24·8	19·8	246
6200	26·5	21·2	234
6300	28·2	22·5	223
6400	30·0	24·0	213
6500	32·0	25·6	203
6600	34·0	27·2	194
6700	36·4	29·1	186
6800	38·4	30·7	178
6900	40·8	32·6	174
7000	43·2	34·6	162

MAGNETIC PROPERTIES OF SOFT GREY CAST IRON.

β Density in c.g.s. lines per sq. cm.	H c.g.s. per cm. length.	$f\beta$ Ampère-turns per cm. length.	μ Permeability.
7100	45·9	36·7	155
7200	48·6	38·9	148
7300	51·7	41·4	141
7400	54·9	43·9	135
7500	58·6	46·8	129
7600	62·3	49·8	123
7700	67·2	53·7	117
7800	70·2	56·1	111
7900	74·3	59·4	106
8000	78·5	62·8	102
8100	82·7	66·1	98
8200	86·9	69·5	94
8300	91·1	72·8	91
8400	95·4	76·3	89
8500	99·8	79·8	86
8600	104·2	83·3	83
8700	108·9	87·1	80
8800	113·6	90·8	77
8900	118·6	94·8	74
9000	123·7	98·9	72
9100	129·5	103·6	70
9200	135·6	108·4	68
9300	141·9	113·5	66
9400	148·3	118·6	64
9500	154·8	123·8	61
9600	161·4	129·1	59
9700	168·0	134·4	57
9800	174·7	139·7	55
9900	181·5	145·2	54
10000	188·5	150·8	53
10100	195·8	156·6	51
10200	203·2	162·5	49
10300	210·8	168·6	48
10400	219·0	175·2	47
10500	228·0	182·4	45
10600	238·0	190·4	44
10700	248·0	198·4	43
10800	259·0	207·2	42
10900	272·0	217·6	40
11000	288·0	230·4	39

DETAILS OF CONDUCTORS.

S.W.G.	Diameter of each wire.	Diameter of the strand.	Area.	Resistance at 60° F.	Weight.
	Inch.	Inch.	Square inches.	Per 1000 yards.	Per 1000 yards.
				Ohms.	lbs.
7/0	0·500	...	0·196349	0·1246	2270
6/0	0·464	...	0·169093	0·1446	1955
5/0	0·432	...	0·146574	0·1669	1694
4/0	0·400	...	0·125663	0·1946	1454
3/0	0·372	...	0·108686	0·2251	1256
2/0	0·348	...	0·095114	0·2572	1099
1/0	0·324	...	0·082447	0·2967	953
1	0·300	...	0·070685	0·3461	817
2	0·276	...	0·05982	0·4089	691
3	0·252	...	0·04987	0·4905	576
4	0·232	...	0·04227	0·5787	488
5	0·212	...	0·03529	0·6931	405
6	0·192	...	0·0289	0·8450	334
7	0·176	...	0·0243	1·0056	281
8	0·160	...	0·0201	1·2168	232
9	0·144	...	0·0163	1·5022	188
10	0·128	...	0·0128	1·9012	148
11	0·116	...	0·0105	2·3150	122
12	0·104	...	0·0085	2·8800	98
13	0·092	...	0·0066	3·6803	76
14	0·080	...	0·0050	4·8673	58
15	0·072	...	0·0040	6·0089	47
16	0·064	...	0·0032	7·6049	37
17	0·056	...	0·0024	9·9332	28
18	0·048	...	0·0018	13·5198	21
19	0·040	...	0·0012	19·4697	14·5
20	0·036	...	0·0010	24·0354	11·7
21	0·032	...	0·0008	30·422	9·3
22	0·028	...	0·0006	39·729	7·1
3/25	0·020	0·042	0·00096	25·955	11
3/24	0·022	0·022	0·00116	21·454	13·5
3/23	0·024	0·024	0·00138	18·026	16
3/22	0·028	0·028	0·00188	13·243	22
3/21	0·032	0·032	0·00246	10·144	28
3/20	0·036	0·036	0·00311	8·0118	36
3/19	0·040	0·040	0·00384	6·4899	44
3/18	0·048	0·048	0·0055	4·5066	64

DETAILS OF CONDUCTORS.

S.W.G.	Diameter of each wire.	Diameter of the strand.	Area.	Resistance at 60° F.	Weight.
	Inch.	Inch.	Square inches.	Per 1000 yards.	Per 1000 yards.
				Ohms.	lbs.
7/25	0·020	0·060	0·0022	11·124	25·5
7/24	0·022	0·066	0·0027	9·1952	31·5
7/23	0·024	0·072	0·0032	7·7256	37
7/22	0·028	0·084	0·0043	5·6757	51
7/21½	0·030	0·090	0·0050	4·9445	58
7/21	0·032	0·096	0·0057	4·3460	66
7/20½	0·033	0·099	0·0064	4·0864	75
7/20	0·036	0·108	0·0072	3·4336	84
7/19	0·040	0·120	0·0089	2·7813	104
7/18	0·048	0·144	0·0129	1·9314	149
7/17	0·056	0·168	0·0175	1·4190	203
7/16	0·064	0·192	0·0229	1·0864	266
7/15	0·072	0·216	0·0290	0·8584	336
7/14	0·080	0·240	0·0358	0·6953	415
7/13	0·092	0·276	0·0474	0·5257	549
7/12	0·104	0·312	0·0606	0·4114	701
7/11	0·116	0·348	0·0754	0·3307	872
7/10	0·128	0·384	0·0918	0·2716	1062
7/9	0·144	0·432	0·1162	0·2146	1343
7/8	0·160	0·480	0·1435	0·1752	1660
7/6	0·192	0·576	0·2067	0·1207	2390
19/24	0·022	0·110	0·0073	3·3877	85
19/23	0·024	0·120	0·0087	2·8463	101·5
19/22	0·028	0·140	0·0119	2·0910	138
19/21	0·032	0·160	0·0156	1·6011	180
19/20	0·036	0·180	0·0197	1·2650	228
19/19	0·040	0·200	0·0244	1·0247	282
19/18	6·048	0·240	0·0351	0·7115	406
19/17	0·056	0·280	0·0478	0·5228	553
19/16	0·064	0·320	0·0624	0·4002	722
19/15	0·072	0·360	0·0790	0·3162	914
19/14	0·080	0·400	0·0976	0·2561	1128
19/13	0·092	0·460	0·1290	0·1937	1491
19/12	0·104	0·520	0·1649	0·1515	1906
19/11	0·116	0·580	0·2052	0·1218	2372
19/10	0·128	0·640	0·2498	0·1000	2888
19/9	0·144	0·720	0·3162	0·07906	3655
19/8	0·160	0·800	0·3904	0·06406	4513
19/7	0·176	0·880	0·4724	0·05292	5461

DETAILS OF CONDUCTORS.

S.W.G.	Diameter of each wire.	Diameter of the strand.	Area.	Resistance at 60° F.	Weight.
	Inch.	Inch.	Square inches.	Per 1000 yards.	Per 1000 yards.
				Ohms.	lbs.
37/24	0·022	0·154	0·0143	1·7396	165
37/23	0·024	0·168	0·0171	1·4647	198
37/22	0·028	0·196	0·0233	1·0737	270
37/21	0·032	0·224	0·0304	0·8222	352
37/20	0·036	0·252	0·0386	0·6496	446
37/19	0·040	0·280	0·0476	0·5262	550
37/18	0·048	0·336	0·0686	0·3654	793
37/17	0·056	0·392	0·0934	0·2684	1080
37/16	0·064	0·448	0·1220	0·2055	1410
37/15	0·072	0·504	0·1544	0·1624	1785
37/14	0·080	0·560	0·1906	0·1315	2203
37/13	0·092	0·644	0·2521	0·09947	2914
37/12	0·104	0·728	0·3221	0·07783	3723
37/11	0·116	0·812	0·4008	0·06265	4633
37/10	0·128	0·896	0·4880	0·05138	5641
37/9	0·144	1·008	0·6176	0·04060	7140
37/8	0·160	1·120	0·7625	0·03288	8815
61/24	0·022	0·198	0·02378	1·0551	275
61/23	0·024	0·216	0·02831	0·8865	327
61/22	0·028	0·252	0·03854	0·6583	446
61/21	0·032	0·288	0·0503	0·4987	572
61/20	0·036	0·324	0·0637	0·3940	736
61/19	0·040	0·360	0·0786	0·3191	909
61/18	0·048	0·432	0·1132	0·2216	1309
61/17	0·056	0·504	0·1541	0·1628	1781
61/16	0·064	0·576	0·2013	0·1246	2327
61/15	0·072	0·648	0·2548	0·09850	2945
61/14	0·080	0·720	0·3145	0·07979	3636
61/13	0·092	0·828	0·4160	0·06033	4809
61/12	0·104	0·936	0·5316	0·04721	6145
61/11	0·116	1·044	0·6614	0·03795	7646
61/10	0·128	1·152	0·8053	0·03116	9309
91/18	0·048	0·528	0·1692	0·14857	1956
91/17	0·056	0·616	0·2302	9·10915	2661
91/16	0·064	0·704	0·3007	0·08357	3476
91/15	0·072	0·792	0·3806	0·06603	4400
91/14	0·080	0·880	0·4699	0·05348	5432
91/13	0·092	1·012	0·6215	0·04044	7185
91/12	0·104	1·144	0·7942	0·03164	9181
91/11	0·116	1·276	0·9881	0·02543	11422

SQUARES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1·0	1·000	1·020	1·040	1·061	1·082	1·103	1·124	1·145	1·166	1·188	2	4	6	8	10	13	15	17	19
1·1	1·210	1·232	1·254	1·277	1·300	1·323	1·346	1·369	1·392	1·416	2	5	7	9	11	14	16	18	21
1·2	1·440	1·464	1·488	1·513	1·538	1·563	1·588	1·613	1·638	1·664	2	5	7	10	12	15	17	20	22
1·3	1·690	1·716	1·742	1·769	1·796	1·823	1·850	1·877	1·904	1·932	3	5	8	11	13	16	18	22	24
1·4	1·960	1·988	2·016	2·045	2·074	2·103	2·132	2·161	2·190	2·220	3	6	9	12	14	17	20	23	26
1·5	2·250	2·280	2·310	2·341	2·372	2·403	2·434	2·465	2·496	2·528	3	6	9	12	15	19	22	25	28
1·6	2·560	2·592	2·624	2·657	2·690	2·723	2·756	2·789	2·822	2·856	3	7	10	13	16	20	23	26	30
1·7	2·890	2·924	2·958	2·993	3·028	3·063	3·098	3·133	3·168	3·204	3	7	10	14	17	21	24	28	31
1·8	3·240	3·276	3·312	3·349	3·386	3·423	3·460	3·497	3·534	3·572	4	7	11	15	18	22	26	30	33
1·9	3·610	3·648	3·686	3·725	3·764	3·803	3·842	3·881	3·920	3·960	4	8	12	16	19	23	27	31	35
2·0	4·000	4·040	4·080	4·121	4·162	4·203	4·244	4·285	4·326	4·368	4	8	12	16	20	25	29	33	37
2·1	4·410	4·452	4·494	4·537	4·580	4·623	4·666	4·709	4·752	3·796	4	9	13	17	21	26	30	34	39
2·2	4·840	4·884	4·928	4·973	5·018	5·063	5·108	5·153	5·198	5·244	4	9	13	18	22	27	31	36	40
2·3	5·290	5·336	5·382	5·429	5·476	5·523	5·570	5·617	5·664	5·712	5	9	14	19	23	28	33	38	42
2·4	5·760	5·808	5·856	5·905	5·954	6·003	6·052	6·101	6·150	6·200	5	10	15	20	24	29	34	39	44
2·5	6·250	6·300	6·350	6·401	6·452	6·503	6·554	6·605	6·656	6·708	5	10	15	20	25	31	36	41	46
2·6	6·760	6·812	6·864	6·917	6·970	7·023	7·076	7·129	7·182	7·236	5	11	16	21	26	32	37	42	48
2·7	7·290	7·344	7·398	7·453	7·508	7·563	7·618	7·673	7·728	7·784	5	11	16	22	27	33	38	44	49
2·8	7·840	7·896	7·952	8·009	8·066	8·123	8·180	8·237	8·294	8·352	6	11	17	23	28	34	40	46	51
2·9	8·410	8·468	8·526	8·585	8·644	8·703	8·762	8·821	8·880	8·940	6	12	18	24	29	35	41	47	53
3·0	9·000	9·060	9·120	9·181	9·242	9·303	9·364	9·425	9·486	9·548	6	12	18	24	30	37	43	49	55
3·1	9·610	9·672	9·734	9·797	9·860	9·923	9·986				6	13	19	25	31	38	44	50	57
3·1								10·05	10·11	10·18	1	1	2	3	3	4	5	5	6
3·2	10·24	10·30	10·37	10·43	10·50	10·56	10·63	10·69	10·76	10·82	1	1	2	3	3	4	5	5	6
3·3	10·89	10·96	11·02	11·09	11·16	11·22	11·29	11·36	11·42	11·49	1	1	2	3	3	4	5	5	6
3·4	11·56	11·63	11·70	11·76	11·83	11·90	11·97	12·04	12·11	12·18	1	1	2	3	3	4	5	6	6
3·5	12·25	12·32	12·39	12·46	12·53	12·60	12·67	12·74	12·82	12·89	1	1	2	3	4	4	5	6	6
3·6	12·96	13·03	13·10	13·18	13·25	13·32	13·40	13·47	13·54	13·62	1	1	2	3	4	4	5	6	7
3·7	13·69	13·76	13·84	13·91	13·99	14·06	14·14	14·21	14·29	14·36	1	2	2	3	4	5	5	6	7
3·8	14·44	14·52	14·59	14·67	14·75	14·82	14·90	14·98	15·05	15·13	1	2	2	3	4	5	5	6	7
3·9	15·21	15·29	15·37	15·44	15·52	15·60	15·68	15·76	15·84	15·92	1	2	2	3	4	5	6	6	7
4·0	16·00	16·08	16·16	16·24	16·32	16·40	16·48	16·56	16·65	16·73	1	2	2	3	4	5	6	6	7
4·1	16·81	16·89	16·97	17·06	17·14	17·22	17·31	17·39	17·47	17·56	1	2	2	3	4	5	6	7	7
4·2	17·64	17·72	17·81	17·89	17·98	18·06	18·15	18·23	18·32	18·40	1	2	3	3	4	5	6	7	8
4·3	18·49	18·58	18·66	18·75	18·84	18·92	19·01	19·10	19·18	19·27	1	2	3	3	4	5	6	7	8
4·4	19·36	19·45	19·54	19·62	19·71	19·80	19·89	19·98	20·07	20·16	1	2	3	4	5	5	6	7	8
4·5	20·25	20·34	20·43	20·52	20·61	20·70	20·79	20·88	20·98	21·07	1	2	3	4	5	5	6	7	8
4·6	21·16	21·25	21·34	21·44	21·53	21·62	21·72	21·81	21·90	22·00	1	2	3	4	5	6	7	7	8
4·7	22·09	22·18	22·28	22·37	22·47	22·56	22·66	22·75	22·85	22·94	1	2	3	4	5	6	7	8	9
4·8	23·04	23·14	23·23	23·33	23·43	23·52	23·62	23·72	23·81	23·91	1	2	3	4	5	6	7	8	9
4·9	24·01	24·11	24·21	24·30	24·40	24·50	24·60	24·70	24·80	24·90	1	2	3	4	5	6	7	8	9
5·0	25·00	25·10	25·20	25·30	25·40	25·50	25·60	25·70	25·81	25·91	1	2	3	4	5	6	7	8	9
5·1	26·01	26·11	26.21	26·32	26·42	26·52	26·63	26·73	26·83	26·94	1	2	3	4	5	6	7	8	9
5·2	27·04	27·14	27·25	27·35	27·46	27·56	27·67	27·77	27·88	27·98	1	2	3	4	5	6	7	8	9
5·3	28·09	28·20	28·30	28·41	28·52	28·62	28·73	28·84	28·94	29·05	1	2	3	4	5	6	7	9	10
5·4	29·16	29·27	29·38	29·48	29·59	29·70	29·81	29·92	30·03	30·14	1	2	3	4	6	7	8	9	10

SQUARES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5·5	30·25	30·36	30·47	30·58	30·69	30·80	30·91	31·02	31·14	31·25	1	2	3	4	6	7	8	9	10
5·6	31·36	31·47	31·58	31·70	31·81	31·92	32·04	32·15	32·26	32·38	1	2	3	5	6	7	8	9	10
5·7	32·49	32·60	32·72	32·83	32·95	33·06	33·18	33·29	33·41	33·52	1	2	3	5	6	7	8	9	10
5·8	33·64	33·76	33·87	33·99	34·11	34·22	34·34	34·46	34·57	34·69	1	2	4	5	6	7	8	9	11
5·9	34·81	34·93	35·05	35·16	35·28	35·40	35·52	35·64	35·76	35·88	1	2	4	5	6	7	8	10	11
6·0	36·00	36·12	36·24	36·36	36·48	36·60	36·72	36·84	36·97	37·09	1	2	4	5	6	7	9	10	11
6·1	37·21	37·33	37·45	37·58	37·70	37·82	37·95	38·07	38·19	38·32	1	2	4	5	6	7	9	10	11
6·2	38·44	38·56	38·69	38·81	38·94	39·06	39·19	39·31	39·44	39·56	1	3	4	5	6	8	9	10	11
6·3	39·69	39·82	39·94	40·07	40·20	40·32	40·45	40·58	40·70	40·83	1	3	4	5	6	8	9	10	11
6·4	40·96	41·09	41·22	41·34	41·47	41·60	41·73	41·86	41·99	42·12	1	3	4	5	6	8	9	10	12
6·5	42·25	42·38	42·51	42·64	42·77	42·90	43·03	43·16	43·30	43·43	1	3	4	5	7	8	9	10	12
6·6	43·56	43·69	43·82	43·96	44·09	44·22	44·36	44·49	44·62	44·76	1	3	4	5	7	8	9	11	12
6·7	44·89	45·02	45·16	45·29	45·43	45·56	45·70	45·83	45·97	46·10	1	3	4	5	7	8	9	11	12
6·8	46·24	46·38	46·51	46·65	46·79	46·92	47·06	47·20	47·33	47·47	1	3	4	5	7	8	10	11	12
6·9	47·61	47·75	47·89	48·02	48·16	48·30	48·44	48·58	48·72	48·86	1	3	4	6	7	8	10	11	13
7·0	49·00	49·14	49·28	49·42	49·56	49·70	49·84	49·98	50·13	50·27	1	3	4	6	7	8	10	11	13
7·1	50·41	50·55	50·69	50·84	50·98	51·12	51·27	51·41	51·55	51·70	1	3	4	6	7	9	10	11	13
7·2	51·84	51·98	52·13	52·27	52·42	52·56	52·71	52·85	53·00	53·14	1	3	4	6	7	9	10	12	13
7·3	53·29	53·44	53·58	53·73	53·88	54·02	54·17	54·32	54·46	54·61	1	3	4	6	7	9	10	12	13
7·4	54·76	54·91	55·06	55·20	55·35	55·50	55·65	55·80	55·95	56·10	1	3	4	6	7	9	10	12	13
7·5	56·25	56·40	56·55	56·70	56·85	57·00	57·15	57·30	57·46	57·61	2	3	5	6	8	9	11	12	14
7·6	57·76	57·91	58·06	58·22	58·37	58·52	58·68	58·83	58·98	59·14	2	3	5	6	8	9	11	12	14
7·7	59·29	59·44	59·60	59·75	59·91	60·06	60·22	60·37	60·53	60·68	2	3	5	6	8	9	11	12	14
7·8	60·84	61·00	61·15	61·31	61·47	61·62	61·78	61·94	62·09	62·25	2	3	5	6	8	9	11	13	14
7·9	62·41	62·57	62·73	62·88	63·04	63·20	63·36	63·52	63·68	63·84	2	3	5	6	8	10	11	13	14
8·0	64·00	64·16	64·32	64·48	64·64	64·80	64·96	65·12	65·29	65·45	2	3	5	6	8	10	11	13	14
8·1	65·61	65·77	65·93	66·10	66·26	66·42	66·59	66·75	66·91	67·08	2	3	5	7	8	10	11	13	15
8·2	67·24	67·40	67·57	67·73	67·90	68·06	68·23	68·39	68·56	68·72	2	3	5	7	8	10	12	13	15
8·3	68·89	69·06	69·22	69·39	69·56	69·72	69·89	70·06	70·22	70·39	2	3	5	7	8	10	12	13	15
8·4	70·56	70·73	70·90	71·06	71·23	71·40	71·57	71·74	71·91	72·08	2	3	5	7	8	10	12	14	15
8·5	72·25	72·42	72·59	72·76	72·93	73·10	73·27	73·44	73·62	73·79	2	3	5	7	9	10	12	14	15
8·6	73·96	74·13	74·30	74·48	74·65	74·82	75·00	75·17	75·34	75·52	2	3	5	7	9	10	12	14	16
8·7	75·69	75·86	76·04	76·21	76·39	76·56	76·74	76·91	77·09	77·26	2	4	5	7	9	11	12	14	16
8·8	77·44	77·62	77·79	77·97	78·15	78·32	78·50	78·68	78·85	79·03	2	4	5	7	9	11	12	14	16
8·9	79·21	79·39	79·57	79·74	79·92	80·10	80·28	80·46	80·64	80·82	2	4	5	7	9	11	13	14	16
9·0	81·00	81·18	81·36	81·54	81·72	81·90	82·08	82·26	82·45	82·63	2	4	5	7	9	11	13	14	16
9·1	82·81	82·99	83·17	83·36	83·54	83·72	83·91	84·09	84·27	84·46	2	4	5	7	9	11	13	15	16
9·2	84·64	84·82	85·01	85·19	85·38	85·56	85·75	85·93	86·12	86·30	2	4	6	7	9	11	13	15	17
9·3	86·49	86·68	86·86	87·05	87·24	87·42	87·61	87·80	87·98	88·17	2	4	6	7	9	11	13	15	17
9·4	88·36	88·55	88·74	88·92	89·11	89·30	89·49	89·68	89·87	90·06	2	4	6	8	9	11	13	15	17
9·5	90·25	90·44	90·63	90·82	91·01	91·20	91·39	91·58	91·78	91·97	2	4	6	8	10	11	13	15	17
9·6	92·16	92·35	92·54	92·74	92·93	93·12	93·32	93·51	93·70	93·90	2	4	6	8	10	12	14	15	17
9·7	94·09	94·28	94·48	94·67	94·87	95·06	95·26	95·45	95·65	95·84	2	4	6	8	10	12	14	16	18
9·8	96·04	96·24	96·43	96·63	96·83	97·02	97·22	97·42	97·61	97·81	2	4	6	8	10	12	14	16	18
9·9	98·01	98·21	98·41	98·60	98·80	99·00	99·20	99·40	99·60	99·80	2	4	6	8	10	12	14	16	18

RECIPROCALS OF NUMBERS FROM 1000 TO 9999.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0·0010000	$\overline{9901}$	$\overline{9804}$	$\overline{9709}$	$\overline{9615}$	$\overline{9524}$	$\overline{9434}$	$\overline{9346}$	$\overline{9259}$	$\overline{9174}$	9	18	27	36	45	55	64	73	82
11	0·0009091	9009	8929	8850	8772	8696	8621	8547	8475	8403	8	15	23	30	38	45	53	61	68
12	0·0008333	8264	8197	8130	8065	8000	7937	7874	7813	7752	6	13	19	26	32	38	45	51	58
13	0·0007692	7634	7576	7519	7463	7407	7353	7299	7246	7194	5	11	16	22	27	33	38	44	49
14	0·0007143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	10	14	19	24	29	33	38	43
15	0·0006667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4	8	13	17	21	25	29	33	38
16	0·0006250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	7	11	15	18	22	26	29	33
17	0·0005882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	20	23	26	29
18	0·0005556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	9	12	15	17	20	23	26
19	0·0005263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	11	13	16	18	21	24
20	0·0005000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	0·0004762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	7	9	11	13	15	17	20
22	0·0004545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
23	0·0004348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	4	5	7	9	11	13	14	16
24	0·0004167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
25	0·0004000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	14
26	0·0003846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	3	4	6	7	8	10	11	13
27	0·0003704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
28	0·0003571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
29	0·0003448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
30	0·0003333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	7	9	10
31	0·0003226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
32	0·0003125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	0·0003030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	0·0002941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
35	0·0002857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
36	0·0002778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	2	3	4	5	5	6	7
37	0·0002703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	4	4	5	6	6
38	0·0002632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
39	0·0002564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	4	5	6
40	0·0002500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	0·0002439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	1	2	2	3	3	4	5	5
42	0·0002381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
43	0·0002326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	0·0002273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
45	0·0002222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0	1	1	2	2	3	3	4	4
46	0·0002174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
47	0·0002128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
48	0·0002083	207	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	3	3	3	4
49	0·0002041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	0·0002000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
51	0·0001961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0	1	1	2	2	2	3	3	3
52	0·0001923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3	3
53	0·0001887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	0·0001852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3	3

N.B.—Three zeros follow the decimal point in the reciprocal of any four figure whole number except the number 1000.

NOTE.—Numbers in difference columns to be subtracted, not added.

RECIPROCALS OF NUMBERS FROM 1000 TO 9999.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	0·0001818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
56	0·0001786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
57	0·0001754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	2	2	2	2	3
58	0·0001724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	1	1	1	1	2	2	2	3
59	0·0001695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
60	0·0001667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	3
61	0·0001639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	0·0001613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	0·0001587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
64	0·0001563	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
65	0·0001538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
66	0·0001515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0	0	1	1	1	1	2	2	2
67	0·0001493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
68	0·0001471	1468	1466	1474	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	2	2	2
69	0·0001449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	2	2	2
70	0·0001429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
71	0·0001408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
72	0·0001389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
73	0·0001370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	2	2
74	0·0001351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
75	0·0001333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
76	0·0001316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
77	0·0001299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	1	1	1	1	1	1
78	0·0001282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
79	0·0001266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
80	0·0001250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
81	0·0001235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
82	0·0001220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	0·0001205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
84	0·0001190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
85	0·0001176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
86	0·0001163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	0·0001149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
88	0·0001136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
89	0·0001124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	1	1	1	1	1	1
90	0·0001111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
91	0·0001099	1098	1096	1095	1094	1093	1092	1091	1089	1088	0	0	0	0	1	1	1	1	1
92	0·0001087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	0·0001075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
94	0·0001064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
95	0·0001053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	0·0001042	1041	1040	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
97	0·0001031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
98	0·0001020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
99	0·0001010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	0	1	1	1	1

N.B.—Three zeros follow the decimal point in the reciprocal of any four-figure whole number except the number 1000.

NOTE.—Numbers in difference columns to be subtracted, not added.

NATURAL TANGENTS.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965

NATURAL TANGENTS.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728
62	1·8807	8887	8907	9047	9128	9210	9292	9375	9458	9542
63	1·9626	9711	9797	9883	9970	0057	0145	0233	0323	0413
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878
71	2·9042	9208	9375	9544	9714	9887	0061	0237	0415	0595
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646
78	4·7046	7453	7867	8288	8716	9152	9594	0045	0504	0970
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140
80	5·6713	7297	7894	8502	9124	9758	0405	1066	1742	2432
81	6·3138	3859	4596	5350	6122	6912	7920	8548	9395	0264
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	0285
83	8·1443	2636	3863	5126	6427	7769	9152	0579	2052	3572
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	0349	0366	0384	0401	0419	0436	0454	9471	0488	0506
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45	7071	7083	7096	7 108	7120	7133	7145	7157	7169	7181
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128
66	9135	9143	9150	9157	9164	8171	9178	9184	9191	9198
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998
89	9998	9999	9999	9999	9999	1·000 nearly.	1·000 nearly.	1·000 nearly.	1·000 nearly.	1·000 nearly.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	6465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	3441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

INDEX

THE END.

PRINTED BY WILLIAM CLOWES AND SONS, LIMITED, LONDON AND BECCLES.