

## CASE 2

In this case the SSV is put to the test. The results of this test will be compared to the expected results, and changes will be made. Also the tension on the axis will be calculated to make sure it doesn't exceed the maximum tension allowed for this material. A 2D drawing of the SSV will show all the dimensions.

### 1. FIRST TEST

To test the SSV we let it roll downhill for 2 meters and see how far it goes. We compare this to the expected results.

To create a new Sankey diagram out of our test, we are going to compare our test with the Simulink simulation. This simulation predicts that our SSV is going to drive 30 meters after it went down of a slope for 2 meters. In reality, this is only 7.16 meters. The difference between the simulation and the real test can be explained by the fact that we didn't levy the gear losses in the simulation and that the other losses are bigger than predicted. We adapted our values in the Simulink file so that this simulation gives also  $\pm 7.16$  meters. We simulated the race (in Simulink) again but now with the adapted forces. With this new values for the forces, we can calculate the new power losses and create a new Sankey diagram.

#### Old values:

$$E_{\text{sun}} = 800 \text{ W/m}^2$$

$$A_{\text{solar panel}} = (0.03 * 0.04) * 30 = 0.036 \text{ m}^2$$

$$\Rightarrow E_{\text{solar panel}} = 0.036 * 800 = 28.8 \text{ W}$$

$$I_{\text{short circuit current}} = 0.88 \text{ A}$$

$$U_{\text{at workingpoint}} = 7.02 \text{ V}$$

$$C_{rr} = 0.012$$

$$C_w = 0.5$$

$$\text{Mass} = 0.75 \text{ kg}$$

#### New values:

$$E_{\text{sun}} = 800 \text{ W/m}^2$$

$$A_{\text{solar panel}} = (0.03 * 0.04) * 30 = 0.036 \text{ m}^2$$

$$\Rightarrow E_{\text{solar panel}} = 0.036 * 800 = 28.8 \text{ W}$$

$$I_{\text{short circuit current}} = 0.88 \text{ A}$$

$$U_{\text{at workingpoint}} = 7.02 \text{ V}$$

$$C_{rr} = 0.018$$

$$C_w = 0.5$$

$$\text{Mass} = 0.816 \text{ kg}$$

DIAGRAM 1: SSV ON FULL SPEED

Calculations

$$P_{\text{solar panel with sun}} = 0.88 * 7.02 = 6.18 \text{ W}$$

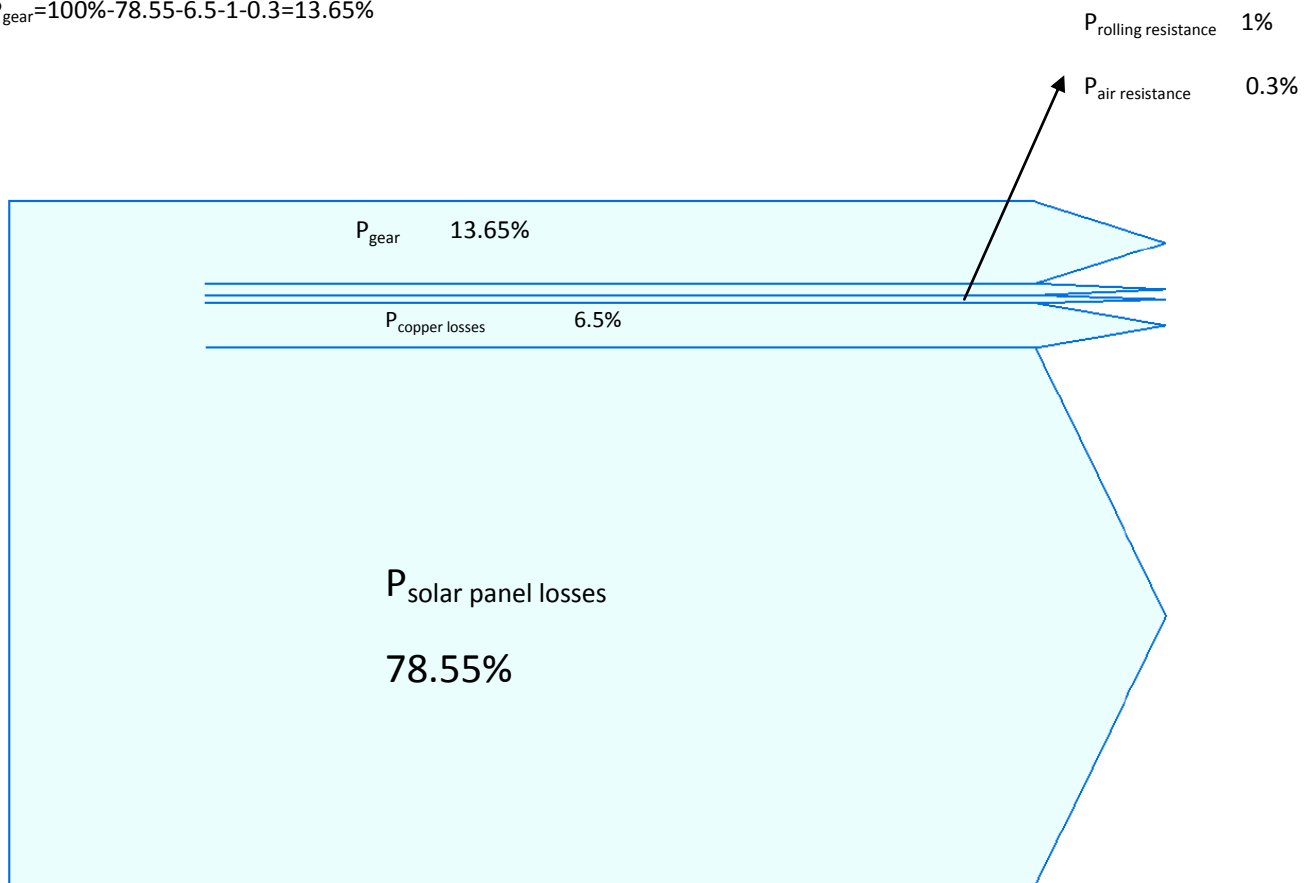
$$P_{\text{losses}} = 28.8 - 6.18 = 22.62 \text{ W} \rightarrow 78.55 \%$$

$$P_{\text{coper losses}} = I^2 * R = (0.75)^2 * 3.32 = 1.87 \text{ W} \rightarrow 6.50 \%$$

$$P_{\text{rolling resistance}} = m * g * C_{rr} * v_{\text{max}} = 0.816 * 9.81 * 0.018 * 2.066 = 0.298 \text{ W} \rightarrow 1 \%$$

$$P_{\text{air resistance}} = \frac{1}{2} * A * \rho * C_w * v * v^2 = 0.5 * 1.2041 * 0.033 * 0.5 * (2.066)^3 = 0.087 \text{ W} \rightarrow 0.3 \%$$

$$P_{\text{gear}} = 100\% - 78.55 - 6.5 - 1 - 0.3 = 13.65\%$$



## DIAGRAM 2: SSV ON HALF SPEED ON THE SLOPE

$$P_{\text{copper losses}} = I^2 \cdot R = (0.85)^2 \cdot 3.32 = 2.3987 \quad \text{W} \rightarrow 8.33 \%$$

$$P_{\text{rolling resistance}} = m \cdot g \cdot C_{rr} \cdot v_{\text{max}} = 0.816 \cdot 9.81 \cdot 0.018 \cdot 1.033 = 0.149 \text{ W} \rightarrow 0.517 \%$$

$$P_{\text{air resistance}} = \frac{1}{2} \cdot A \cdot \rho \cdot C_w \cdot v \cdot v^2 = 0.5 \cdot 0.033 \cdot 1.2041 \cdot 0.5 \cdot (1.033)^3 = 0.011 \text{ W} \rightarrow 0.038 \%$$

$$P_{\text{slope resistance}} = m \cdot g \cdot (\% \text{ of slope}) \cdot v = 0.816 \cdot 9.81 \cdot \tan(7.2) \cdot 1.033 = 1.044 \text{ W} \rightarrow 3.6 \%$$

$$P_{\text{gear losses}} : P_{\text{max speed}} = T \cdot \omega \Leftrightarrow T = P / \omega_{\text{max speed}} \quad \text{with}$$

$$- \quad P = 0.1365 \cdot 28.8 \text{ W} = 3.9312 \text{ W}$$

(the power losses because of the gear are 13.65% of 28.8 W at maximum speed (see diagram 1). This is 3.9312 W)

$$\begin{aligned} - \quad \omega_{\text{max speed/motor}} [\text{rpm}] &= 2\pi \cdot f \cdot (60/2\pi) \cdot \text{gear ratio} \\ &= 2\pi \cdot (\text{speed/circumference}) \cdot (60/2\pi) \cdot \text{gear ratio} \\ &= 2\pi \cdot (2.066/0.08\pi) \cdot (60/2\pi) \cdot 5 \\ &= 2466.105 \text{ rpm} \end{aligned}$$

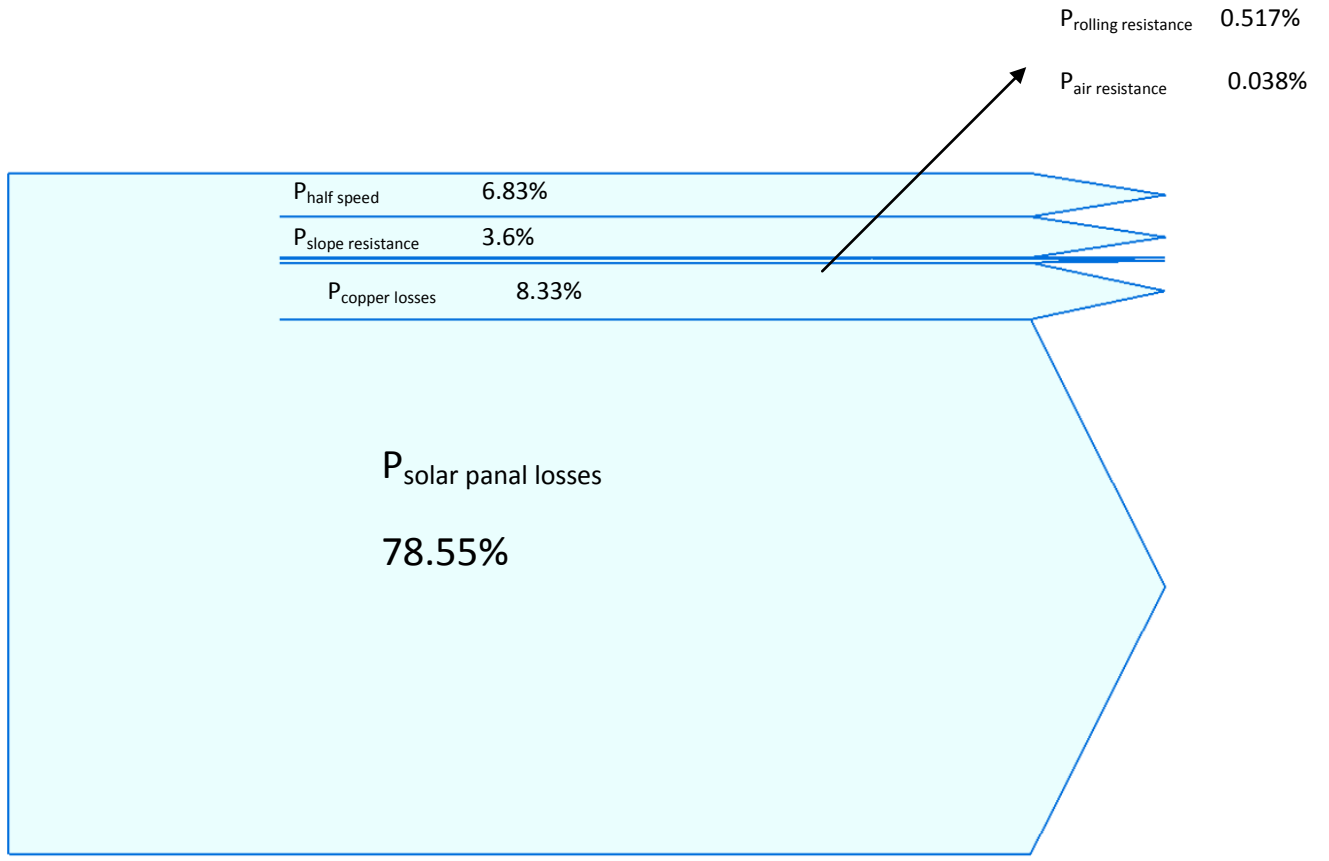
$$\Leftrightarrow T = 3.9312 \text{ W} / 2466.105 = 1.594 \cdot 10^{-3}$$

$$T \text{ remains constant} \rightarrow P_{\text{half speed}} = T \cdot \omega_{\text{half speed}} \quad \text{with}$$

$$\begin{aligned} - \quad \omega_{\text{half speed/motor}} [\text{rpm}] &= 2\pi \cdot f \cdot (60/2\pi) \cdot \text{gear ratio} \\ &= 2\pi \cdot (\text{speed/circumference}) \cdot (60/2\pi) \cdot \text{gear ratio} \\ &= 2\pi \cdot (1.033/0.08\pi) \cdot (60/2\pi) \cdot 5 \\ &= 1233.053 \text{ rpm} \end{aligned}$$

$$\Leftrightarrow P_{\text{half speed}} = (1.4656 \cdot 10^{-3}) \cdot 1233.053$$

$$= 1.966 \text{ W} \rightarrow 6.83 \%$$



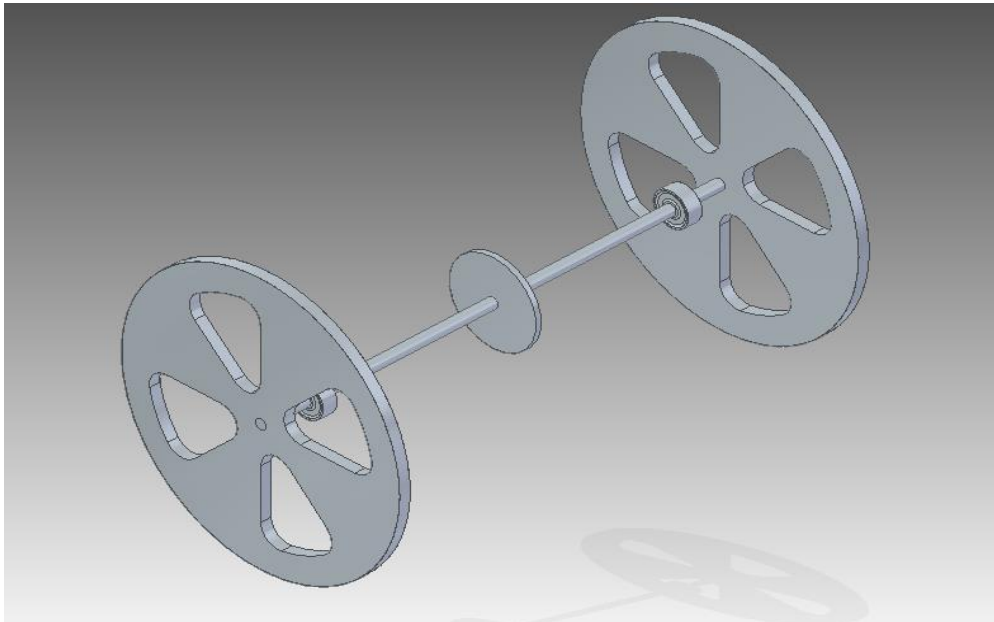
## 2. CRITICAL FORCE

You have to dimension the critically loaded components very carefully. Therefore, a thorough analysis is required. We examine the drive shaft of the SSV in different situations.

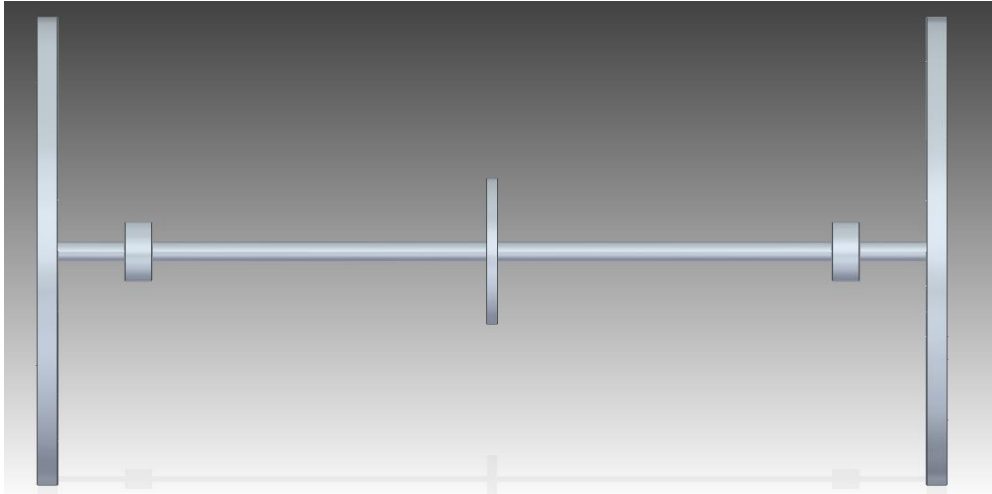
In figure 1 and 2 you can find sketches of the axle.

### A. FIRST SITUATION:

The SSV accelerates from standstill and the motor delivers maximum torque. The wheels do not slip.

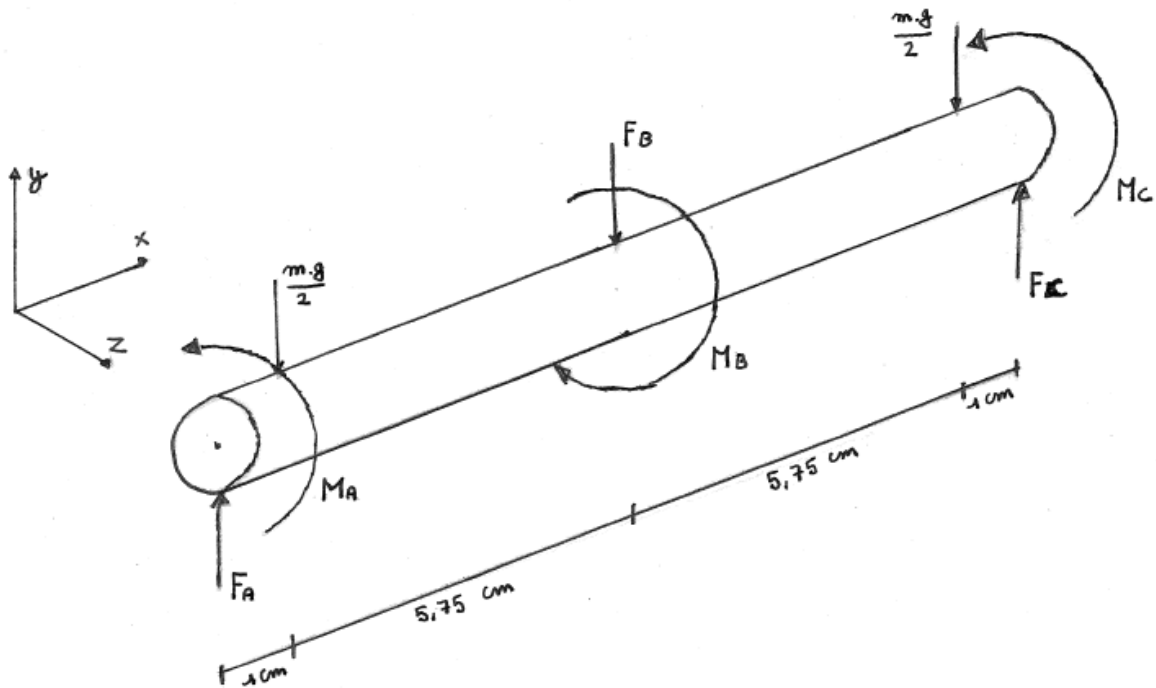


FIGUUR 1 3D VIEW AXIS



FIGUUR 2 2D VIEW AXIS

In figure 3 you can find a sketch of the axle with the forces working on it.



FIGUUR 3 SKETCH OF AXLE WITH FORCES

There are no losses to role and air resistance so there is power available to accelerate.

Data:

$$m_{\text{car}} = 1 \text{ kg}$$

Gear ratio = 3.5 (calculated model)

$$T_{\text{mot}} = 23.2 \text{ mNm}$$

$$T_B = T_{\text{mot}} * \text{Gear ratio} = 81.2 \text{ mNm}$$

$$m_{\text{axle+gear}} = 0.01 \text{ kg}$$

Solution:

$$\underline{\Sigma F_x = 0}$$

$$F_A + F_C - m_{\text{car}} * g/2 - m_{\text{car}} * g/2 - F_B = 0$$

$$F_A = F_C; F_B = \text{Mass}_{\text{axle+gear}} * g = 0.1 \text{ N}$$

$$2F_A = m_{\text{car}} * g + F_B$$

$$F_A = (1 \text{ kg} * 9.81 \text{ N/kg} + 0.1 \text{ N})/2$$

$$F_A = F_B = 4.955 \text{ N}$$

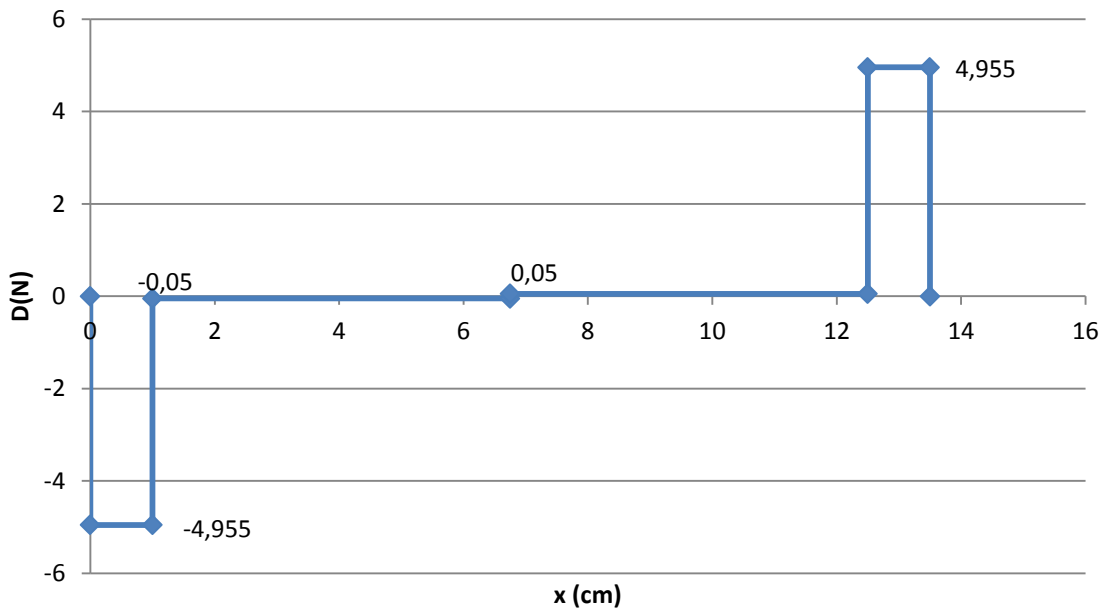
$$\underline{\Sigma T = 0}$$

$$T_A + T_C = T_B$$

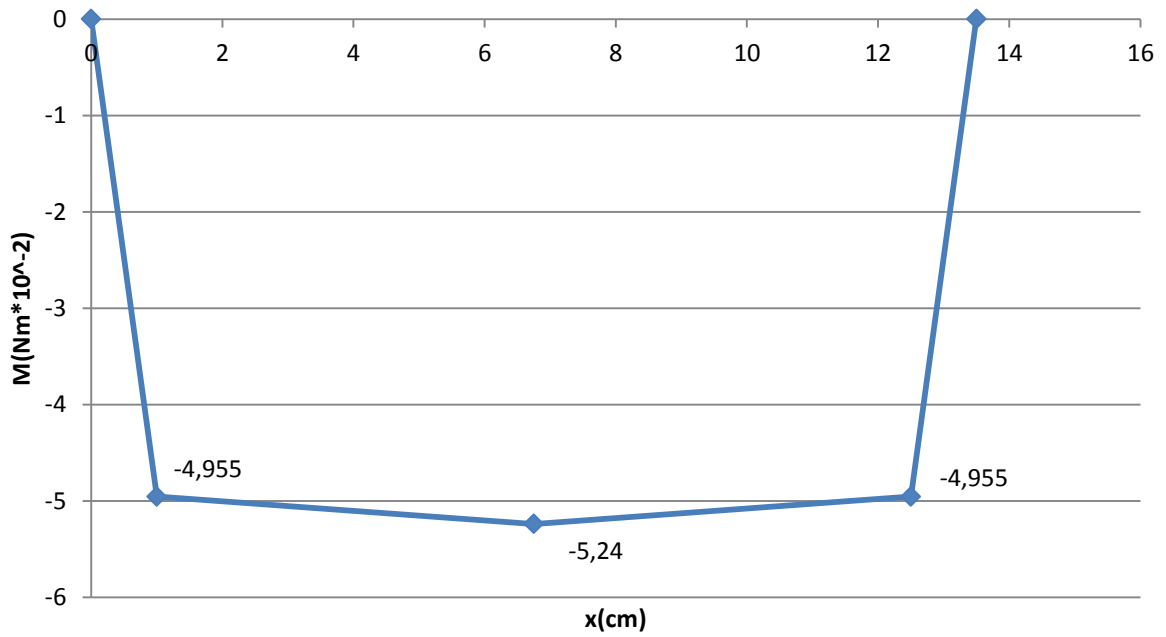
$$T_A = T_C = T_B / 2 = 40.6 \text{ mNm}$$

Below you can find several diagrams to show the forces working on the axle. On the vertical axle you can find the force in Newton, on the horizontal axle you find the length of the axle in cm.

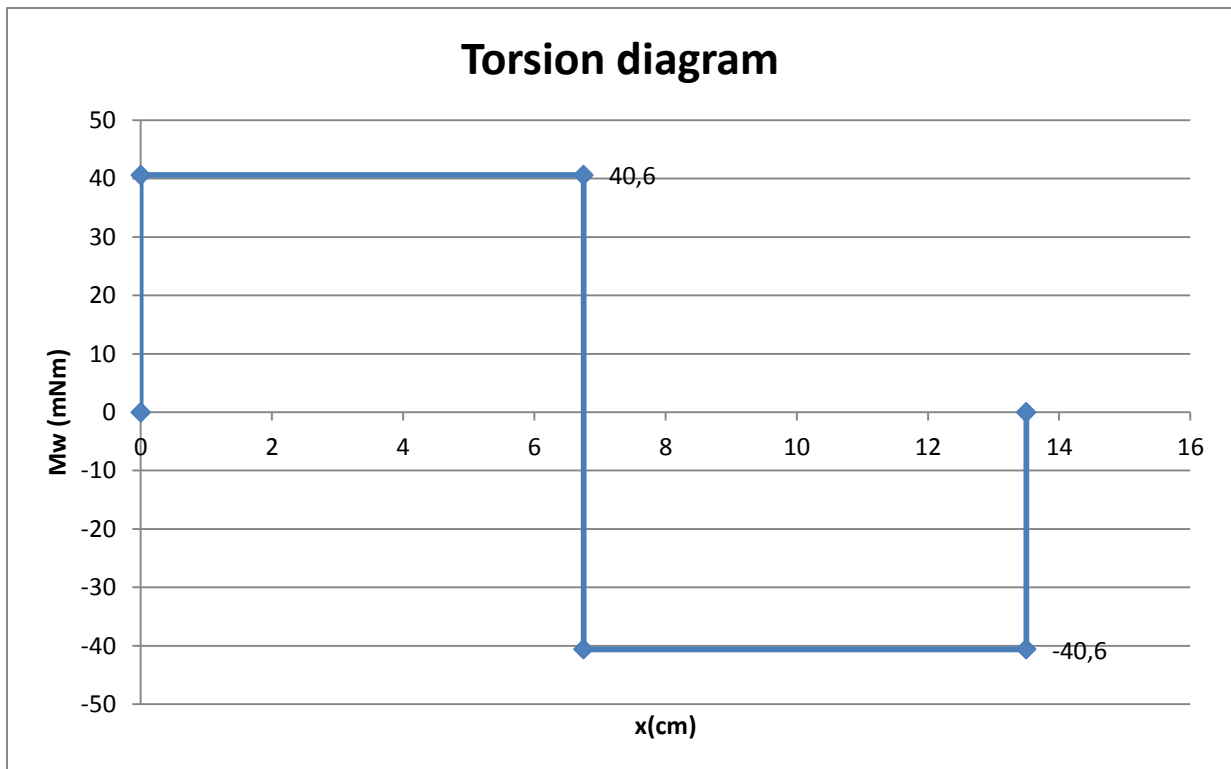
### Shear force diagram



### Bending moment diagram

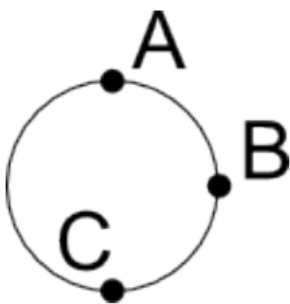






WE CALCULATE THE MISES STRESS ON 3 POINT A, B AND C AS SHOWN ON THE FOLLOWING FIGURE.

We calculate Von Mises on the point which seems most critical to us, which is on 5 centimeter from the edge of the axle, this is because the bending and torsion have the greatest values on this spot and we will ignore the shear stress in the calculation of the Von Mises stress.



Shear stress:

$$\tau = \frac{D}{A} = \frac{0.05}{\pi * 0.0015^2} = 7073.55 Pa$$

We will ignore the shear stress to calculate the Von Mises stress.

Bending stress:

$$\sigma = \frac{M * y}{I} = \frac{-5.24 * 0.0015}{\frac{\pi * 0.0015^4}{4}} = -9.88 * 10^8 Pa$$

As we can see the bending stress is a lot bigger than the value found for the shear stress, so it was a good choice to ignore the shear stress.

The value found is the value which is valid for A and B although they have an opposite value, the value in B is 0 because B is part of the neutral line.

Torsion stress:

$$\tau = \frac{T * r}{J} = \frac{40.6 * 0.0015}{\frac{\pi * 0.003^4}{32}} = 7,66 * 10^9 Pa$$

It's again obvious that this value is a lot bigger than the value found for the shear stress.

The value found is the Torsion stress in A, B and C.

Von Mises Stress:

To calculate the Von Mises stress we use a simplified formula instead of the full Von Mises formula.

$$\sigma_{VM} = \sqrt{\sigma^2 + 3 * \tau^2}$$

In point A this gives:

$$\sigma_{VM} = \sqrt{(-9.88 * 10^8)^2 + 3 * 7,66 * 10^9^2} = 1.330 * 10^{10} Pa$$

In point B this gives:

$$\sigma_{VM} = \sqrt{0^2 + 3 * 7,66 * 10^9^2} = 1.326 * 10^{10} Pa$$

In point C this gives:

$$\sigma_{VM} = \sqrt{9.88 * 10^8^2 + 3 * 7,66 * 10^9^2} = 1.33 * 10^{10} Pa$$

*Examine the material of your shaft and check whether the calculated stress remains below the maximum allowed stress.*

Our shaft is a RVS (304) threaded rod. The distance of the shaft is 135 millimeters and it has a diameter of three millimeters. The characteristic are listed below.

RVS 304

**Density** = 7.86 (kg/m<sup>3</sup>) \* 10<sup>3</sup>

**Modulus of elasticity E** = 193 (GPa)

**Shear modulus  $G = 75$  (GPa)**

**Yield  $\sigma_F$**

Stretch = 207 (MPa)

Pressure = 207 (MPa)

**Tensile strength  $\sigma_M$**

Stretch= 517 (MPa)

Pressure= 517 (MPa)

**Poisson-factor = 0.27**

**Thermal expansion =  $17 (10^{-6}) / ^\circ\text{C}$**

**Bron:** [https://cygnus.cc.kuleuven.be/bbcswbdav/pid-9948116-dt-content-rid-6425555\\_3/orgs/j-C130023-B-1213/Course%20Documents/Instructies%20Sterkteleer%20Nederlandstalig/Instructies%20sterkteleer/Scanned\\_document\\_14-02-2011\\_18-05-09.pdf](https://cygnus.cc.kuleuven.be/bbcswbdav/pid-9948116-dt-content-rid-6425555_3/orgs/j-C130023-B-1213/Course%20Documents/Instructies%20Sterkteleer%20Nederlandstalig/Instructies%20sterkteleer/Scanned_document_14-02-2011_18-05-09.pdf)

## B. SECOND SITUATION:

The speed is maximal; the torque is smaller.

Execute the same steps as in the first situation.

There now are severe losses to friction on the wheels, so there is no more power to accelerate, this means we are at maximum speed.

### Data:

$v_{\max} = 3.26 \text{ m/s}$  (calculated with matlab @  $m_{\text{car}} = 1 \text{ kg}$  and gear ratio = 3.5)

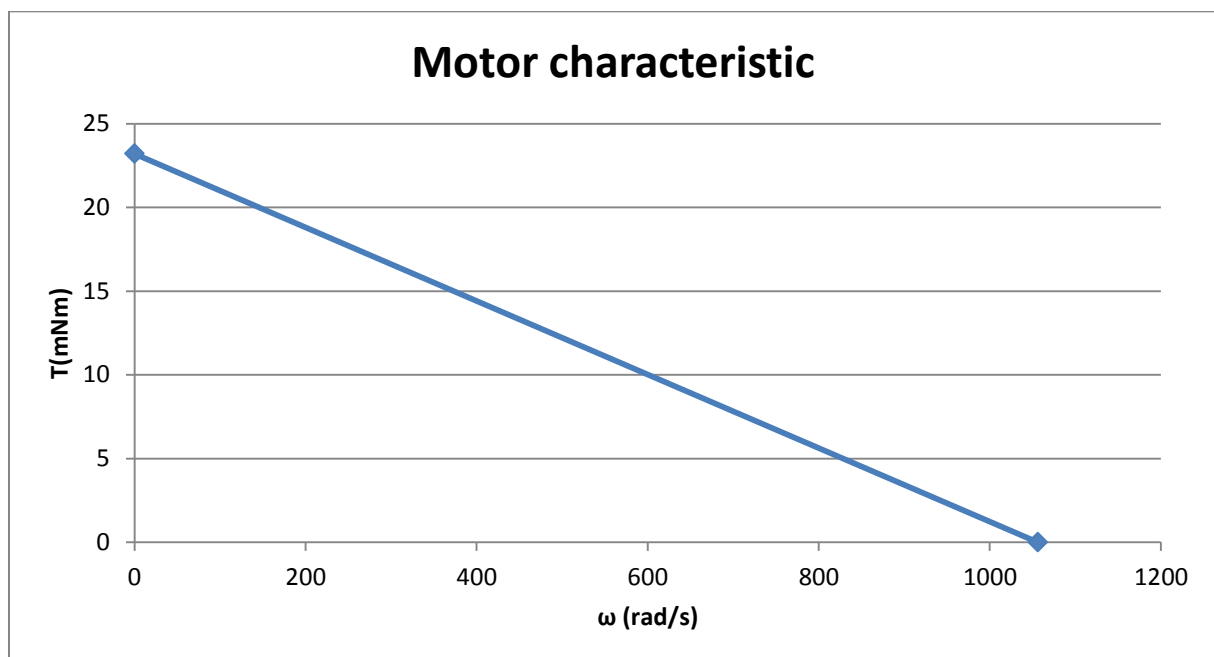
$O_{\text{wheel}} = 2\pi r = 0.2513 \text{ m}$  ( $O$ = Circumference)

### Solution:

$v_{\max} / O_{\text{wheel}} = 3.26 / 0.2513 \text{ rotations/s} = 12.97 \text{ rotations/s}$

$\omega_{\text{wheel}} = \omega_{\text{axle}} = 12.97 \text{ rotations/s} * 2\pi = 81.5 \text{ rad/s}$

$\omega_{\text{motor}} = \omega_{\text{axle}} * \text{gear ratio} = 81.5 \text{ rad/s} * 3.5 = 285.25 \text{ rad/s}$



$$T_{\text{mot}} = 23.2 - (23.2 * \omega_{\text{mot}}) / 1056$$

$$T_{\text{mot}} = 16.9 \text{ mNm}$$

$$T_{\text{as}} = 16.9 \text{ mNm} * 3.5 = 59.2 \text{ mNm}$$

$$F = F_r + F_w$$

$$F_w = \frac{1}{2} * C_w * A * \rho * v_{\max}^2$$

$$F_w = \frac{1}{2} * \frac{1}{2} * 1.2 * 0.036 * 3.26^2 = 0.115 \text{ N}$$

$$F_r = C_{rr} * N = 0.012 * 1 * 9.81 = 0.118 \text{ N}$$

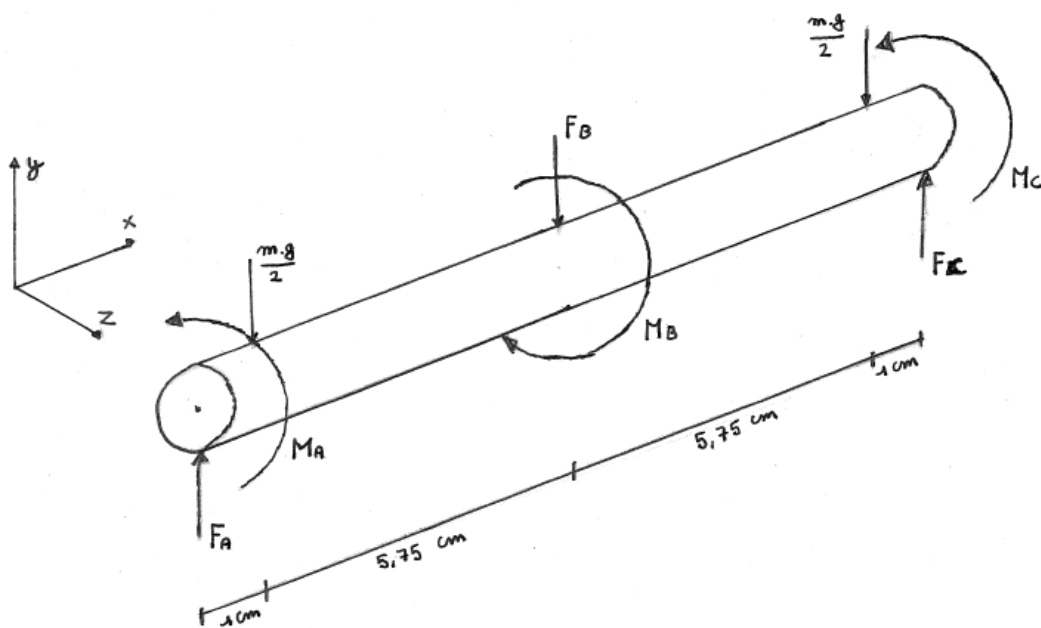
$$F = 0.115 + 0.118 = 0.233 \text{ N}$$

$$P = F * v_{\max}$$

$$P = 0.233 \text{ N} * 3.26 \text{ m/s} = 0.76 \text{ W}$$

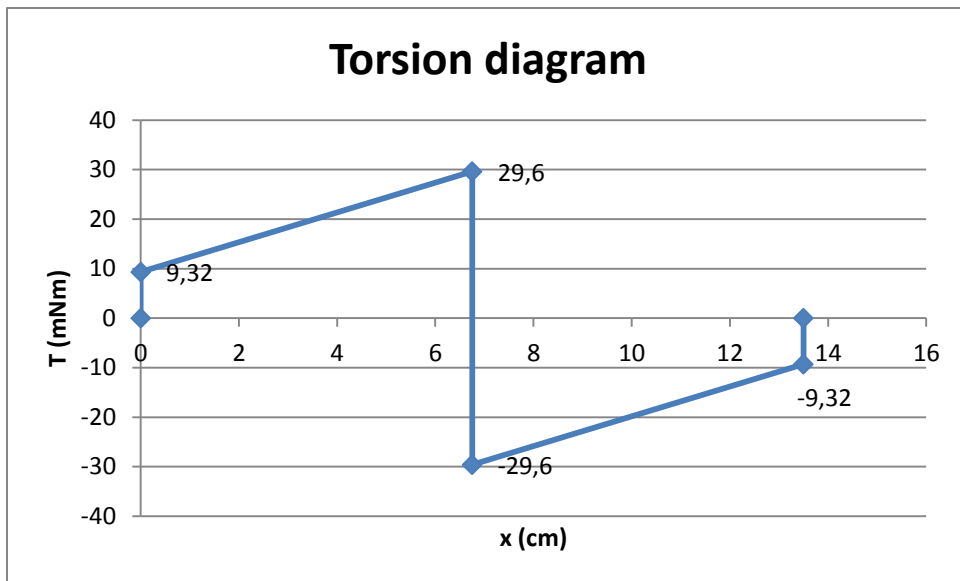
$$P = T * \omega_{\text{wheel}}$$

$$T = P / \omega_{\text{wheel}} = 0.76 \text{ W} / 81.5 \text{ rad/s} = 9.32 \text{ mNm}$$



FIGUUR 4 SKETCH AXLE WITH FORCES AT MAXIMAL SPEED

The shear force diagram and bending moment diagram are the same as in situation a, only the torsion diagram is different.



#### Von Mises

In the second situation the Shear stress and the Bending stress will be the same.

Shear stress:

$$\tau = \frac{D}{A} = \frac{0.05}{\pi * 0.0015^2} = 7073.55 Pa$$

We will be able to ignore the shear stress once again.

Bending stress:

The value found is the value which is valid for A and B although they have an opposite value, the value in B is 0 because B is part of the neutral line.

Torsion stress:

$$\tau = \frac{T * r}{J} = \frac{29.6 * 0.0015}{\frac{\pi * 0.003^4}{32}} = 5.58 * 10^9 Pa$$

This is the only value which is different from the first situation.

The value found is the Torsion stress in A, B and C.

Von Mises Stress:

In point A this gives:

$$\sigma_{VM} = \sqrt{-9.88 * 10^{8^2} + 3 * 5.58 * 10^{9^2}} = 9.715 * 10^9 Pa$$

In point B this gives:

$$\sigma_{VM} = \sqrt{0^2 + 3 * 5.58 * 10^{9^2}} = 9.66 * 10^9 Pa$$

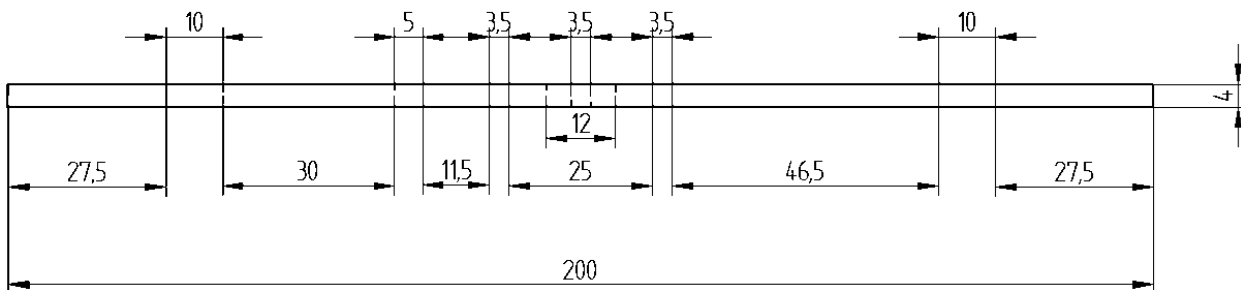
In point C this gives:

$$\sigma_{VM} = \sqrt{9.88 * 10^{8^2} + 3 * 5.58 * 10^{9^2}} = 9.715 * 10^9 Pa$$

### 3. 2D TECHNICAL DRAWING

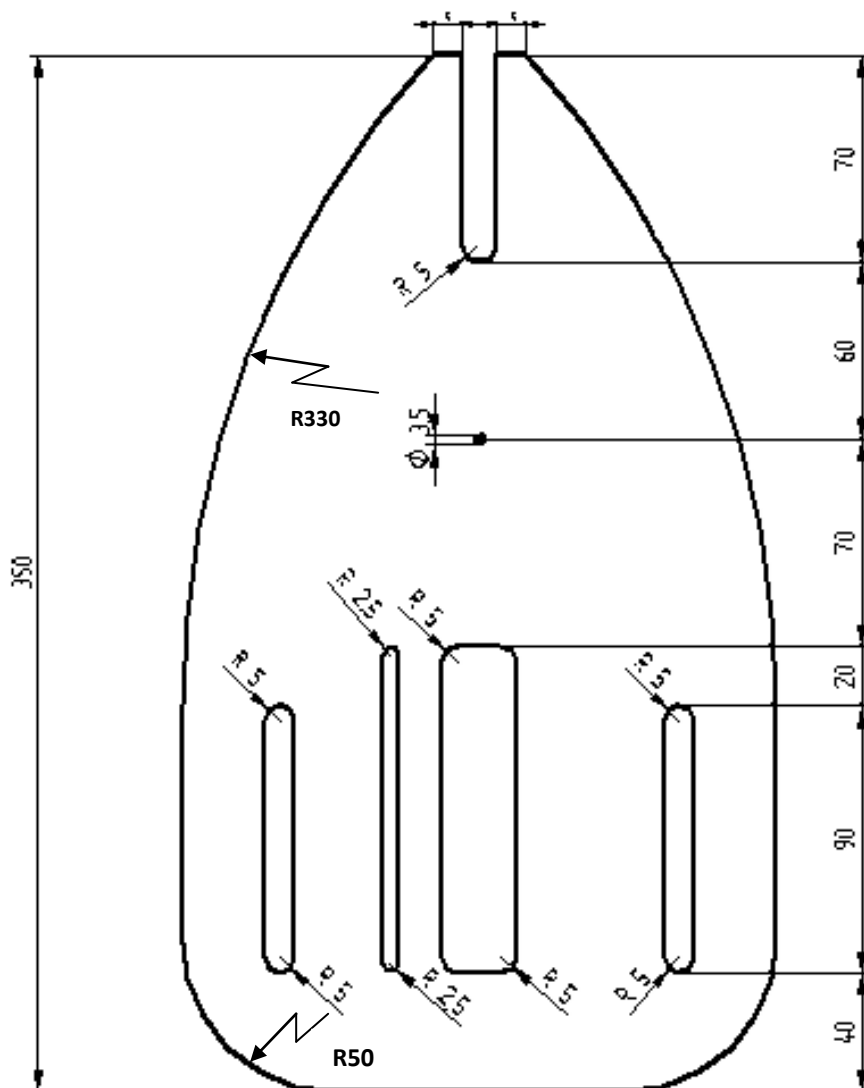
Figure 5 shows a 2D technical drawing of the SSV. The dimensions are indicated.

Front view:



FIGUUR 5 FRONT VIEW

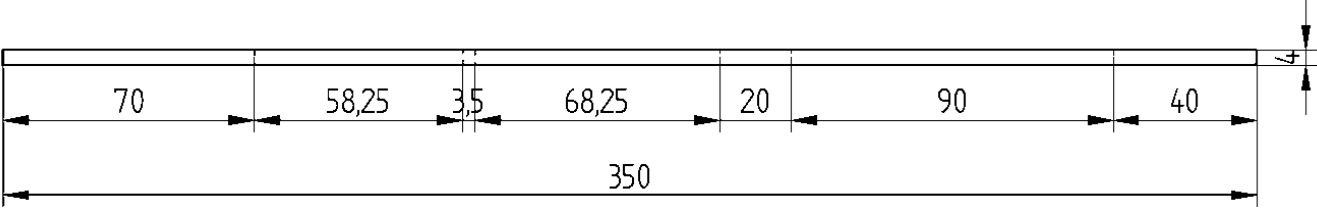
Top view:



FIGUUR 6 TOP VIEW



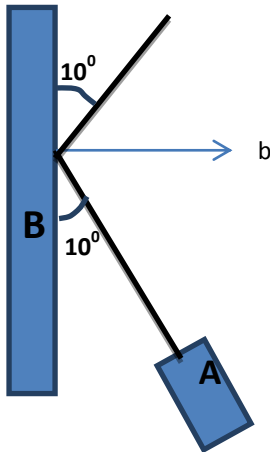
Left-side view



FIGUUR 7 LEFT SIDE VIEW

#### 4. COLLISION

To understand the impact of a collision we make a small exercise. If the SSV collides with the side of the track with an angle of  $10^\circ$  and maximum speed, what will the impulse be? Considering that the collision is elastic. How long does the collision need to last for the force to remain below 10N?



Data: mass = 0.81657kg

$V_A = 2,5 \text{ m/s}$

The impulse assuming an elastic collision

$$L'_b - L_b = \int_0^T \langle F_{ext} \rangle dt$$

With :

$$L_b = m * v_{A,b} = 0,81657\text{kg} * 2,5 \text{ m/s} * (-\sin(10)) = -0,81657 * 2,5 * \sin(10)$$

$$L'_b = m * v'_{A,b}$$

$$E = 1 = - \frac{(v'_{B,b} - v'_{A,b})}{v_{B,b} - v_{A,b}} = - \frac{0 - v'_{A,b}}{0 - v_{A,b}} \rightarrow -v_{A,b} = v'_{A,b} \rightarrow v'_{A,b} = 0,81657 * 2,5 * \sin(10)$$

$$\rightarrow L'_b - L_b = 0,81657 * 2,5 * \sin(10) - (-0,81657 * 2,5 * \sin(10))$$

$$= 2 * 0,81657 * 2,5 * \sin(10)$$

$$= 0,709 = \int_0^T \langle F_{ext} \rangle dt$$

Time of the collision when force below 10 N

$$0,709 = \int_0^T \langle F_{ext} \rangle dt$$

$$\Leftrightarrow 0,709 = F_{\text{ext}} * T$$

$$\text{with } F_{\text{ext}} < 10N$$

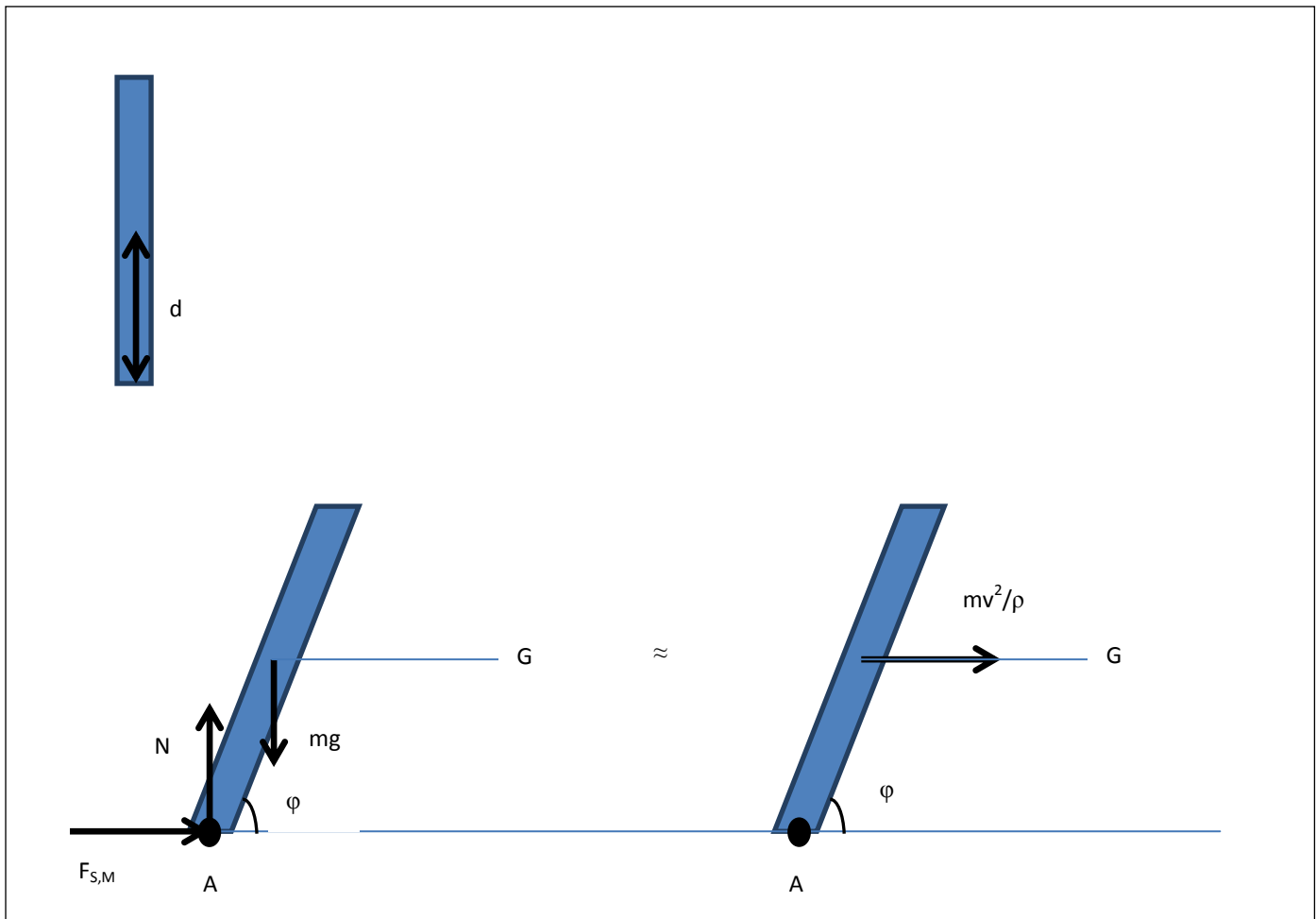
$$\Leftrightarrow T > \frac{0,709}{10} = 0,0709 \text{ s}$$

$$T > 0,0709 \text{ s}$$

## 5. CYCLIST EXERCISE

Another exercise is this one about a cyclist. A cyclist is riding with a speed of 50km/h. On a crossroad he needs to turn left, the radius of the turn is 10m. What is the necessary inclination angle? Does he have to reduce his speed for a safe turn? And what is the maximum possible speed to make this turn? The following numbers are given: the mass of the cyclist is 60kg, the mass of the bike 12kg, the centre of gravity is 1,5m above the ground and the static coefficient of friction between wheels and ground is 0,3.

**Assumption: not static**



**Data**

$$v = 50 \text{ km/h} = 13.89 \text{ m/s}$$

$$\rho = 10 \text{ m}$$

$$\left. \begin{array}{l} m_{\text{cyclist}} = 60 \text{ kg} \\ m_{\text{bicycle}} = 12 \text{ kg} \end{array} \right\} m = 60 + 12 = 72 \text{ kg}$$

$$d = 1.5 \text{ m}$$

$$u_s = 0.3$$

### Questions

- What is the necessary inclination angle?
- Does he have to reduce his speed to make a safe turn?
- What is the maximum possible speed?

### Solution

$$\underline{\Sigma F_y = \dots}$$

$$-mg + N = 0$$

$$\rightarrow N = mg = 72 * 9.81 = 706.32 \text{ (N)}$$

$$F_{s,M} = u_s * N$$

$$\rightarrow F_{s,M} = u_s * mg = 0.3 * 72 * 9.81 = 211.90 \text{ (N)}$$

$$\underline{\Sigma F_x = \dots}$$

$$F_{s,M} = m * v^2 / \rho$$

$$F_{s,M} = u_s * mg$$

$$v = (u_s * g * \rho)^{(1/2)} = (0.3 * 9.81 * 10)^{(1/2)} = 5.42 \text{ (m/s)} \quad \rightarrow \text{(c)}$$

$$\underline{M_A = \dots}$$

$$m * g * d * \cos \varphi = (mv^2 / \rho) * d * \sin \varphi$$

$$m * g * d * \cos \varphi = ((m * (u_s * g * \rho)) / \rho) * d * \sin \varphi$$

$$\tan \varphi = 1 / u_s = 1 / 0.3 \rightarrow \varphi = 73.30 \text{ (}^\circ\text{)} \quad \rightarrow \text{(a)}$$

From question c, we can conclude that the cyclist have to reduce his speed to make a safe turn. The maximum possible speed without slipping away is 5.42 m/s and the cyclist drive at a speed of 13.89 m/s, so it's impossible.