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Linear Momentum and Collisions

Module Overview

Acknowledgments

This presentation is based on and includes content derived from the following OER resource:

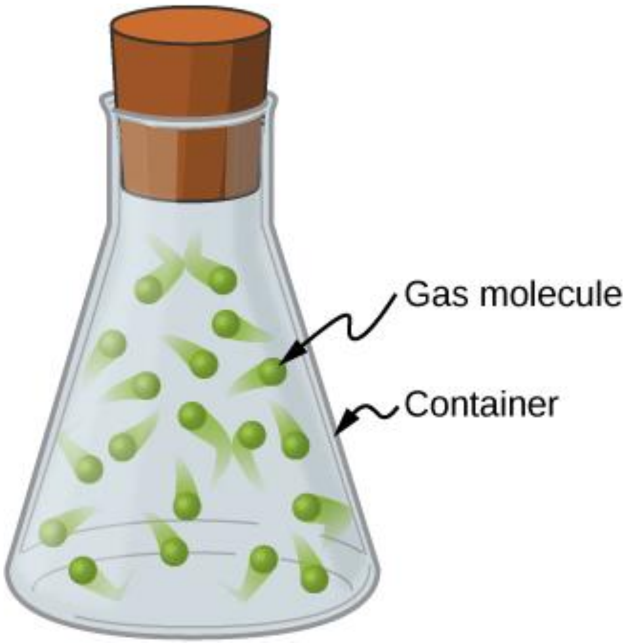
University Physics Volume 1

An OpenStax book used for this course may be downloaded for free at:
https://openstax.org/details/books/university-physics-volume-1

Linear Momentum

Momentum, $\vec{\mathbf{p}}$, is a physical vector quantity that describes both the mass and velocity of an object or system. It is defined as $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$.

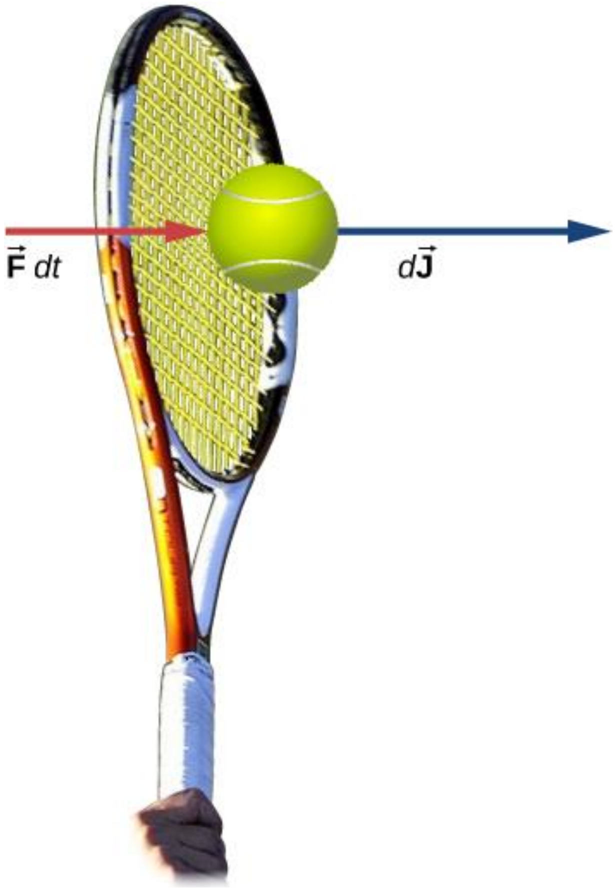
Momentum is very useful for determining whether an object's motion is difficult to change. A very light object has little momentum even at a very high velocity, making it easy to change its directions.



(University Physics Volume 1. OpenStax. Fig. 9.4)

Impulse and Collisions

Impulse, $\vec{\mathbf{J}}$, is the product of force and the time over which it is applied. If a force $\vec{\mathbf{F}}(t)$ is applied to an object, then the impulse over an interval dt is given by $d\vec{\mathbf{J}} \equiv \vec{\mathbf{F}}(t)dt$. The total impulse is obtained by integration, $\vec{\mathbf{J}} = \int_{t_i}^{t_f} \vec{\mathbf{F}}(t)dt$, or the impulse can be obtained from an average force, $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{ave}\Delta t$.



(University Physics Volume 1. OpenStax. Fig. 9.6)

Effect of Impulse

The impulse applied to an object results in a change in its momentum. This can be shown by integrating Newton's second law over time. The result is known as the **impulse-momentum theorem**.

$$\vec{\mathbf{F}}(t) = m\vec{\mathbf{a}}(t)$$

$$\vec{\mathbf{J}} = \int_{t_\mathrm{i}}^{t_\mathrm{f}} \vec{\mathbf{F}}(t)dt = m\int_{t_\mathrm{i}}^{t_\mathrm{f}} \vec{\mathbf{a}}(t)dt$$

$$\vec{\mathbf{J}} = m[\vec{\mathbf{v}}(t_\mathrm{f}) - \vec{\mathbf{v}}(t_\mathrm{i})]$$

$$\vec{\mathbf{J}} = m\Delta\vec{\mathbf{v}} = \Delta\vec{\mathbf{p}}$$

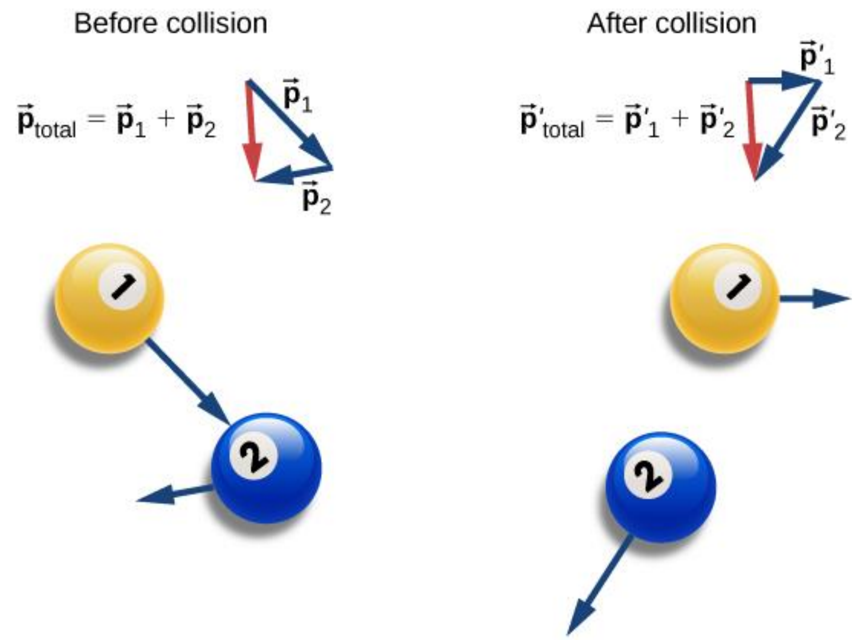
Momentum and Force

The force applied to an object is equal to the rate of change of its momentum, $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$. This restatement of Newton's second law is how Newton originally expressed the relationship between the forces on an object and its motion. When the mass of the system remains constant, the equation reduces to its more familiar form, $\vec{\mathbf{F}} = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}}$.

Conservation of Linear Momentum

When two objects interact, they are acted on by equal and opposite forces according to Newton's third law. This results in equal and opposite changes in momentum, $\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0$.

Since the time derivative of the sum of the momenta is zero, the total momentum is conserved, $\sum \vec{\mathbf{p}}_i = \text{constant}$.



(University Physics Volume 1. OpenStax. Fig. 9.14)

Requirements for Momentum Conservation

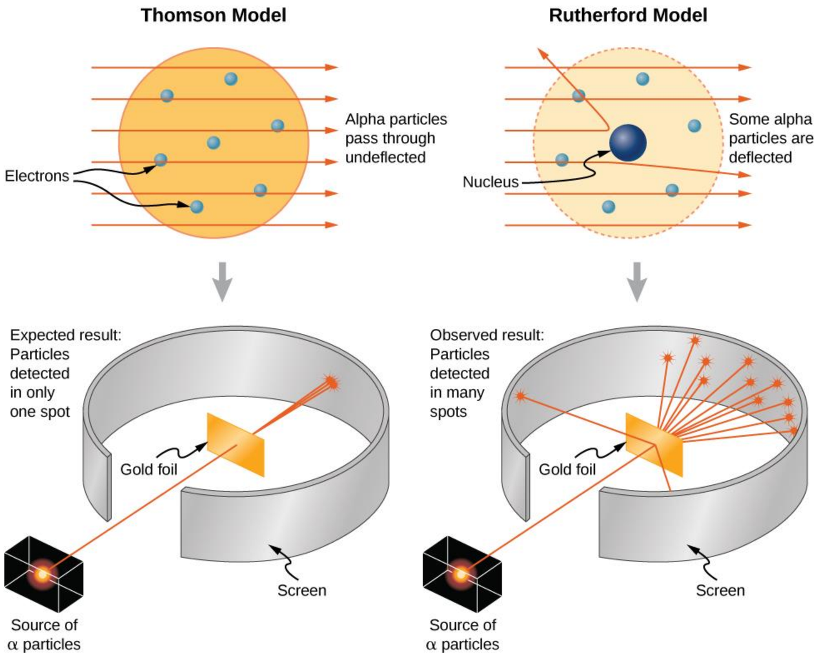
A **system** is the collection of objects in whose motion we are interested. Two conditions must be met for momentum to be conserved in a system:

- **The total mass of the system must remain the same.** If mass is lost from the system, the momentum that mass carried leaves the system as well.
- **The net external force must be zero.** All internal forces are balanced by their equal and opposite force according to Newton's third law. External forces add momentum because they have no third-law pair within the system.

A system that meets these requirements is called a **closed system**. **The law of conservation of momentum states: In a closed system, the total momentum never changes.** This is one of the most important concepts in physics.

Types of Collisions

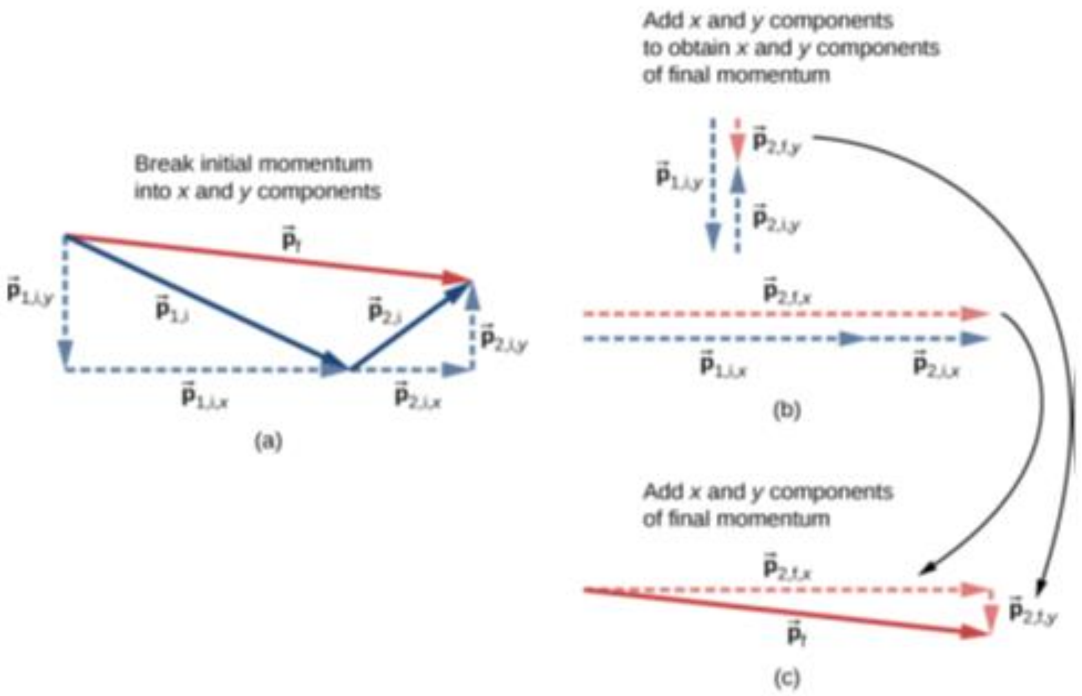
An **explosion** is a type of interaction where one particle becomes many and the system's kinetic energy increases. When objects interact without loss of kinetic energy, the collision is called **elastic**. When objects collide and stick together, they lose kinetic energy, and the collision is called **inelastic**. In the extreme case, a **perfectly inelastic** collision occurs when objects stick together and are motionless.



(University Physics Volume 1. OpenStax. Fig. 9.21)

Collisions in Multiple Dimensions

Collisions most often occur in two or three dimensions. In this case, each component of the total momentum is conserved, $p_{\mathrm{f},x} = p_{1,\mathrm{i},x} + p_{2,\mathrm{i},x}$ in the *x*-direction, and $p_{\mathrm{f},y} = p_{1,\mathrm{i},y} + p_{2,\mathrm{i},y}$ in the *y*-direction.



(University Physics Volume 1. OpenStax. Fig. 9.2)

Internal and External Forces on an Extended Body

An extended object or system is made up many particles, N. The object experiences an **external force**, $\vec{\mathbf{F}}_{\text{ext}}$. The jth particle in the system experiences an **internal force**, $\vec{\mathbf{f}}_j^{\text{int}}$, and some portion of the total external force, $\vec{\mathbf{f}}_j^{\text{ext}}$. These forces change the momentum of the particle according to Newton's second law, $\vec{\mathbf{f}}_j^{\text{int}} + \vec{\mathbf{f}}_j^{\text{ext}} = \frac{d\vec{\mathbf{p}}}{dt}$.

The net force on the system is $\vec{\mathbf{F}}_{\text{net}} = \sum_{j=1}^{N} \vec{\mathbf{f}}_j^{\text{int}} + \sum_{j=1}^{N} \vec{\mathbf{f}}_j^{\text{ext}} = \sum_{j=1}^{N} \frac{d\vec{\mathbf{p}}_j}{dt}$. The sum over internal forces vanishes by Newton's third law, and the sum over external forces is simply the total external force. As a result, $\vec{\mathbf{F}}_{\text{ext}} = \sum_{j=1}^{N} \frac{d\vec{\mathbf{p}}_j}{dt}$.

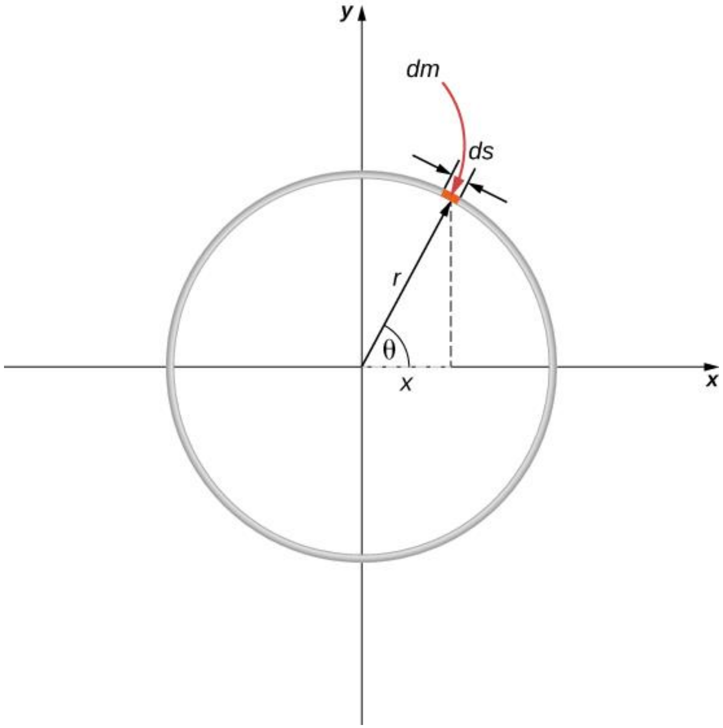
Center of Mass

The **center of mass** of an extended object or system is the point that behaves like a point particle, with mass M equal to the mass of the extended body. It is defined as a mass-weighted position, $\vec{\mathbf{r}}_{\mathrm{CM}} \equiv \frac{1}{M}\sum_{j=1}^{N} m_j\, \vec{\mathbf{r}}_j$. Its acceleration and velocity are given by $\vec{\boldsymbol{a}}_{\mathrm{CM}} = \frac{d^2}{dt^2}\left(\frac{1}{M}\sum_{j=1}^{N} m_j\, \vec{\mathbf{r}}_j\right)$ and $\vec{\mathbf{v}}_{\mathrm{CM}} = \frac{d}{dt}\left(\frac{1}{M}\sum_{j=1}^{N} m_j\, \vec{\mathbf{r}}_j\right)$.

The center of mass obeys Newton's second law, with the force equal to the total force acting on the object and the momentum equal to the total momentum of the object, $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}_{\mathrm{CM}}}{dt}$.

Center of Mass of Continuous Objects

For a continuous object, the mass of the individual particles can be treated as infinitesimals, dm. Instead of summing over discrete particle masses, the center of mass can be found by integrating over all infinitesimal mass elements in the object, $\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\, dm$.



(University Physics Volume 1. OpenStax. Fig. 9.30)

Center of Mass and Conservation of Momentum

An extended object or system is subject to conservation of momentum, with the center of mass acting as a single particle with mass M. This is written as $M\vec{\mathbf{v}}_{\mathrm{CM,f}} = M\vec{\mathbf{v}}_{\mathrm{CM,i}}$. Furthermore, since conservation of momentum only applies to closed systems, the mass of the system must not change, $\vec{\mathbf{v}}_{\mathrm{CM,f}} = \vec{\mathbf{v}}_{\mathrm{CM,i}}$. As long as no external forces are acting on the system, the velocity of the center of mass never changes.

(University Physics Volume 1. OpenStax. Fig. 9.31)

Rocket Propulsion

A rocket of mass m travels at speed v. To propel itself, it ejects a small amount of fuel , dm_g, at speed u.

The momentum of the rocket increases by an amount equal to that of the ejected fuel, but the rocket's mass also decreases by the mass of the ejected fuel. The net effect is to create a change in the velocity of the rocket described by the **rocket equation**.

$$p_{\mathrm{i}} = p_{\mathrm{f}} = p_{\mathrm{rocket}} + p_{\mathrm{gas}}$$

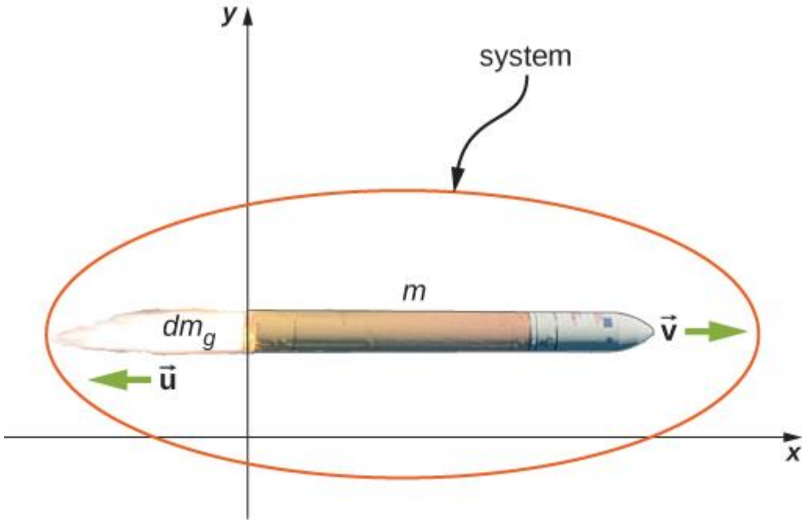
$$mdv = -dmu$$

$$\int_{v_{\mathrm{i}}}^{v_{\mathrm{f}}} dv = -u \int_{m_i}^{m} \frac{1}{m} dm$$

$$\Delta v = u\ln(\frac{m_i}{m})$$

A Rocket in a Gravitational Field

During launch, a rocket must also contend with the downward force of gravity. The change in momentum due to gravity is $dJ = -mgdt$. Adding this term to the previous analysis results in a modified expression for the infinitesimal change in velocity, $dv = -u\frac{dm}{m} - gdt$. Integrating this expression gives the equation for the rocket's change in velocity, $\Delta v = u \ln\left(\frac{m_i}{m}\right) - g\Delta t$.



(University Physics Volume 1. OpenStax. Fig. 9.33)

How to Study this Module

- Read the syllabus or schedule of assignments regularly.
- Understand key terms; look up and define all unfamiliar words and terms.
- Take notes on your readings, assigned media, and lectures.
- As appropriate, work all questions and/or problems assigned and as many additional questions and/or problems as possible.
- Discuss topics with classmates.
- Frequently review your notes. Make flow charts and outlines from your notes to help you study for assessments.
- Complete all course assessments.

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