SCHOOL PHYSICS

AVERY

SHELDON AND COMPANY
NEW YORK — CHICAGO.

SCHOOL PHYSICS

A NEW TEXT-BOOK
FOR HIGH SCHOOLS AND ACADEMIES

Slate

BY

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AUTHOR OF A SERIES OF PHYSICAL SCIENCE TEXT-BOOKS

SHELDON AND COMPANY
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DR. AVERY'S PHYSICAL SCIENCE SERIES.

FIRST PRINCIPLES OF NATURAL PHILOSOPHY.

ELEMENTS OF NATURAL PHILOSOPHY.

SCHOOL PHYSICS.

ELEMENTS OF CHEMISTRY.

COMPLETE CHEMISTRY.

This contains "The Elements of Chemistry," with an additional chapter on Hydrocarbons in Series, or Organic Chemistry. It can be used in the same class with "The Elements of Chemistry."

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PREFACE.

In this book will be found an unusual number of problems. It is not intended that each member of each class shall work all of the problems. It is hoped that they are sufficiently numerous and varied to enable you to select what you need for your particular class. No author can make a comfortable Procrustean bedstead.

For several years there has been a growing tendency in the high schools of the country to indulge in laboratory methods. An effort has been made to adapt this book to such needs. The author has no sympathy with the idea that the pupil should have set before him the impossible task of rediscovering all the physical truths known to modern science. Even were there no other obstacle, individual life is too short, and but a small part of that short period can be given to a high-school course in natural philosophy. Nor has he any more sympathy with the notion that the high-school laboratory should attempt the full work of the technological school. High-school laboratory work has its limitations in the capacities of the pupils, in the time at their disposal, in laboratory equipment, etc. Still it affords a needed variation from the old method in which the author stated facts *ex cathedra* to be accepted and memorized by the pupils, and from the less objectionable plan in which the teacher performed all the experiments and the pupil simply observed and admired. Pedagogic practice, having swung from one end of the arc to the other, is now settling down to the golden mean. May it prove that in its excursions it held fast to all the good it found and left the rest behind.

If, at the beginning of an experiment, John Tyndall could ask, "For what shall I look?" we may be permitted to suggest that the pupil, ignorant of scientific truths and experimental methods, and without manipulatory skill, needs a text-book something like this to save even his laboratory practice from degenerating into chaotic waste. An effort has been made by the author of this book to introduce the pupil into what is a new world to him, to give him a few elementary lessons in the ways of that world, trusting that, in later years, other hands will guide him over more rugged paths and into higher realms. But, no matter how well an author's work may have been done, it can never take the place of the living and live teacher; it may be a help, but it certainly cannot be a substitute.

Each pupil is expected to perform as many of the laboratory exercises as possible. The classroom work must be kept ahead of the laboratory work; i.e., the pupil must come to the laboratory with some knowledge of the principles involved in the work that he is required to perform. Even then, there will be a grievous waste of time and effort unless the teacher is judicious, vigilant and firm. For instance, it will not be easy, at the beginning, to lead pupils to appreciate the importance of making all measurements as accurately as possible with the given instruments, and to realize that there is value in repeated measurements of the same quantity. Much of the benefit to be derived from laboratory practice hangs upon the cultivation of habits of accuracy of observation, the formation of habitually systematic methods, and the development of an ability to reason from observed particulars to general laws. The ability to generalize from observed phenomena should not be expected of many, and must not be demanded.

The divisions of the class for laboratory practice should be so small that the teacher may get to each pupil at short intervals to check gross errors at the beginning, and thus prevent much waste of time. Ten or twelve is perhaps a fair limit for the size of such divisions. Pupils should be taught neatness and dexterity of manipulation, held to a rigid accountability for the care and condition of all apparatus used by them whether it belongs to them or to the school, required to make accurate notes of their work as it proceeds, and encouraged to write them up neatly and fully in note-books of prescribed form. They should be taught to put their records into tabular form when the nature of the work will permit such a form of record. Additional to all of this is the work of enforcement; questions, discussions, supplementary experiments, and problems. All of this demands so much of time and enthusiasm from the teacher, that the school authorities ought not to forget that "to give good instruction in the sciences requires of the teacher more work than to give good instruction in mathematics and the languages," and that "the teacher should be absolutely at liberty, not only during the physics hours, but also during several other hours of the week, to arrange for and to direct the experiments, unvexed by any care of schoolrooms or of pupils save those actually engaged in laboratory work." If the school has not a regularly equipped physical laboratory, as is desirable, a room should be set aside for the exclusive use of classes in experimental physics, and fitted for such use as well as the circumstances of the case will allow.

The author has taken special pains to select experiments and exercises that may be performed with simple and inexpensive apparatus, and many of them have been devised expressly for this work. The author has not disdained any aid that he could draw from any source, but he cheerfully acknowledges his especial obligation to Professor Barker's "Physics," the Harvard College pamphlet, and its amplification in the admirable work of Messrs. Hall and Bergen. Special acknowledgments are also due to Professor Dayton C. Miller of the Case School of Applied Science, who has read the work in manuscript and in proof-sheets ; to Mr. George T. Hanchett of Pawtucket, R. I., for valuable assistance in the chapter on Electricity and Magnetism; and to Mr. Henry C. Muckley, assistant superintendent of the public schools of Cleveland, for his many valuable suggestions, and for his aid in correcting proof. To the many others who have given assistance in many ways, the author tenders assurances of grateful appreciation.

The author would be glad to receive any suggestions from any who may use this book, or to answer any inquiries concerning the study or apparatus. He may be addressed at Cleveland, O.

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CHAPTER I.

MATTER.

I. DIVISIONS OF MATTER.—THE DOMAIN OF PHYSICS.

"Read Nature in the language of experiment."

1. **Science** *is classified knowledge.* General information is valuable, but it is only when facts are classified that the knowledge becomes scientific knowledge.

2. **Matter** *is anything that occupies space or "takes up room."* Its existence is made known to us through the senses. Substances are the different kinds of matter, as water, wood, silver, etc. A body is any separate portion of matter, as a book, a table, or a star.

Structure of Matter.

Experiment 1.—Heat the mercury in the bulb of a common thermometer. The bulb remains full, but the liquid rises in the tube. There seems to be more mercury than there was before. How can this be? There must be a greater number of molecules, the molecules must be larger, or *they must be further apart.*

Experiment 2.—Make a common goose-quill pop-gun. Notice that when you use it the air confined between the two wads is compressed, or made to occupy about half its original space. The air particles were reduced in size or in number, or were crowded together more closely. *Perhaps the matter of which a body is made does not actually fill all the space which the body seems to occupy.*

Experiment 3. — Rub the smooth handle of a fine awl over a piece of fine wire gauze, and the gauze seems to present a continuous surface. Perhaps it is the fault of the instrument in your hand, and not the fault of the gauze. Rub the point of the awl over the gauze, and you soon find openings between the metal threads. *But the openings are there, whether you can feel them or not.*

Experiment 4. — Rub the point of a fine sewing needle over the surface of a window pane. The glass seems to be continuous in its structure, and the needle cannot get through. Perhaps it is the fault of the instrument you are using. Try one more delicate. Let a ray of light fall upon the glass, and *it easily finds a passageway between the solid molecules.* Rays of light are often used by scientific men as instruments for their work.

3. Structure of Matter. — Many facts, some of which will be considered later, indicate that *matter is not continuous;* that any sensible portion of it is a group of very small particles; that no two of these are in actual contact; and that the minute particles of each group are held together by certain attractive forces, to which we must give careful consideration and earnest study.

(*a*) When you look at a brick wall from a considerable distance, it has an apparent uniformity of structure. You cannot see that it is made of many bricks, separated by mortar-filled spaces. This is the fault of your sense of vision. As you come nearer, you see what you did not see before, — the individual and separated bricks. *But such is the structure of the wall, whether you can see it or not.*

4. Divisions of Matter. — *Matter exists in atoms, molecules, and masses.* It is very important that we clearly understand what these words mean, or we shall have trouble in trying to understand much that is to follow.

5. An Atom *is the smallest quantity of matter that can enter into combination and thus form molecules and masses.* It is the chemical unit of matter, and is considered indivisible. In nearly every case, an atom is a part of a molecule.

(*a*) We may say that atoms are the smallest particles of matter that can exist. They seldom exist alone, but quickly unite with others like themselves to form *elementary molecules,* or with others unlike themselves to form *compound molecules.* For example, one atom of oxygen combines with another like itself to form an elementary molecule of oxygen, while one atom of oxygen combines with two of hydrogen to form a compound molecule of water. There are more than seventy kinds of atoms now known.

6. A Molecule *is a quantity of matter so small that it cannot be divided without changing its nature.* It is the physical unit of matter, and can be divided only by a chemical process. Atoms make molecules ; molecules make masses.

(*a*) A molecule is so very small that the smallest particle of matter visible in the best of modern microscopes contains millions of molecules. If a drop of water could be magnified until it appeared to be as large as the earth on which we live, each molecule in the drop thus magnified would still look smaller than a base-ball. Even in dense solids, molecules are separated by spaces that are large as compared to their own size. Tait assumes it as probable that the molecule itself does not occupy so much as five per cent. of its share of the whole space. This signifies that the distances between molecules is about three times the diameter of a molecule.

(*b*) Some compound molecules are very complex. The common sugar molecule contains forty-five atoms of three kinds. Elementary molecules make elementary masses or substances. Compound molecules make compound masses or substances. The nature of the molecule determines the nature of the substance.

(*c*) Molecules are believed to be in ceaseless motion, but ever subject to the constraining action of certain molecular forces. Many of the phenomena observed in matter are due to these molecular motions, as will more clearly appear further on.

7. A Mass *is any quantity of matter that is composed of molecules.* Masses are elementary or compound. An elementary substance is called *an element.* There are as many elements as there are kinds of atoms. Compound substances are innumerable.

(*a*) We may take a lump of salt, which is a mass, and break it into many pieces; each piece will be a mass. We may take one of these pieces and crush it to finest powder; each grain will still be a mass. We may imagine one of these grains of powdered salt to be divided into so many parts that any further division will change them from salt to something else; these particles of salt, so small that further division would change their nature, are molecules. If one of these molecules is divided, it ceases to be salt; we have instead an atom of sodium and an atom of chlorine.

(*b*) The quantity of matter constituting a mass is not necessarily great. A drop of water may contain a million animalcules. Each animalcule is a mass as truly as the greatest monster of the land or sea. The dewdrop and the ocean, clusters of grapes and clusters of stars, are equally masses of matter.

(*c*) The term "mass" also has reference to the quantity of matter in a body. This double use of the word is unfortunate.

8. Forms of Motion. — It is probable that each of these three divisions of matter has its own form or mode of motion. *The motion of a mass is often called molar or mechanical motion.* The motion of a bullet is an example. *The motion of the molecules in a mass constitutes heat.* If a bullet strikes a target, the shock that destroys the molar motion of the bullet increases the vibration of the molecules of which the bullet is composed. These molecular vibrations constitute heat. When a bullet is thus stopped, it is heated, and the production of heat is explained only in this way. These molar and molecular motions give to matter the power of doing work, the scientific name of which power is *energy.*

The motion of atoms within the molecule has not been proved.

(*a*) Fancy a million flies surrounded by an imaginary shell. If each fly represents a molecule, the contents of the shell represent a mass. Imagine this shell to be thrown through the air. The motion of the shell represents molar motion. As the shell is moving through the air, the flies are moving slowly among themselves within the shell. This motion of the flies represents molecular motion, and is a very different thing from the motion of the shell. When the shell strikes the ground, the molar motion is destroyed, but the molecular motions are increased, for the flies are set in much more rapid motion by the shock. This is just about what happens when the bullet is fired against a target.

Changes in Matter.

Experiment 5. — Hold a piece of platinum wire in the flame of an alcohol or of a Bunsen lamp. It becomes hot, glows, emits light. There has been a change in the platinum. Remove the wire and allow it to cool. Can you see that the wire was permanently changed in any way by the experiment?

Experiment 6. — With forceps, hold a piece of magnesium wire in the flame of an alcohol or of a Bunsen lamp. It becomes hot, glows, emits light. At the end of the experiment do you notice any permanent change in the magnesium wire?

9. Physical and Chemical Changes. — Any change that alters the constitution of the molecule, and thus affects the identity of the substance, is a *chemical change.* Other changes in matter are *physical changes.*

10. Phenomena, etc. — Any directly observed change in matter is a *phenomenon.* A supposition (or scientific guess) advanced in explanation of phenomena is an *hypothesis.* The value of an hypothesis increases with the variety of the phenomena for which it can offer an exclusive explanation. As this variety increases, the hypothesis rises to the rank of a *theory.* When the theory has acquired so high a degree of probability that it is accepted by the judicious as an established truth, i.e., when it is easier for men to believe it than to doubt it, it becomes a *law*, e.g., the law of gravitation. In the words of Mr. Huxley, "Law means a rule which we have always found to hold good, and which we expect always will hold good."

Force.

Experiment 7. — Mount each of two 4 × 8 inch boards on the trucks of a pair of roller skates (preferably with ball or roller bearings), adjusting the parts so that the two carriages will run in straight lines when set in motion on a level surface. See that the bearings are well oiled. Provide two smooth, straight boards, 6 ft. × 6 in. Cut two wooden blocks with thickness equal to the elevation of the body of the carriage; i.e., so that they may just slip under the mounted boards when the carriages rest on smooth surfaces. Nail one of these blocks across the face of each of the smooth boards at its end. Raise the other end of one of the boards a little, until by trial you find that one of the carriages will roll down the incline with a velocity as nearly uniform as is attainable, thus eliminating largely the resistance due to friction. Fasten the board in this position. Similarly adjust the other board to the other carriage, and place it side by side with the first board. If the second carriage runs less freely than the first, the second board will require a greater incline than the first board. Provide two pieces of elastic tape or of black-rubber tubing, each $\frac{3}{4}$ of a yard long. Stitch or clamp together one end of each. Stretch the joined pieces, and fasten their free ends at points 3 yards apart. If the pieces stretch unequally, trim the edges of the stiffer piece until the seam that joins the two pieces shall be midway between the fixed ends. Loosen the fastenings at the ends of the tape, and rip the seam at the middle; the tensions of the two pieces, when equally

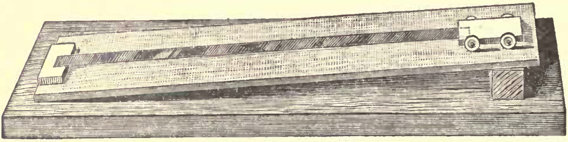


FIG. 1.

stretched, will pull with equal forces. Tack one end of each elastic to the top of one of the blocks at the lower end of the smooth board, and the other end of the elastic to the under side of the board that constitutes the body of the carriage that was adjusted for that board; the elastic, when stretched, will be parallel to the long board. Similarly fasten the other elastic to the other block and carriage. Load one carriage with a chalk box filled with iron nails, scraps of lead, or other heavy material; load the other with a box of chalk. Draw the carriages toward the upper ends of their respective boards so as to stretch the elastics considerably and equally; release them at the same time, and notice the speeds at which they are drawn by the equal forces. Try to find some relation between the two velocities and the weights of the two loaded carriages. Transfer part of one load to the other carriage until they will be moved with equal velocities, and determine the approximate relation between the weights of the two loaded carriages.

11. Force. — Every phenomenon necessarily implies a change; every change necessarily implies a cause. The causes that produce phenomena, or changes in matter, are called forces. The word "force" is difficult of satisfactory definition. As generally used in physical discussions, *force signifies the immediate cause that produces, or tends to produce, a change in the velocity or direction of motion of a body;* i.e., a push or a pull. Pushes are often called pressures; and pulls, tensions. Forces act on matter. Equal forces produce equal velocities when applied for the same length of time to equal masses. Matter may now be defined as that which can exert force or be acted on by a force.

(*a*) If the intensity of a force varies in successive intervals of time, it is said to be *variable;* if its intensity does not change, it is said to be *constant.* Forces acting between masses of matter separated by sensible distances are called *molar forces;* e.g., gravitation. Forces acting between molecules separated by insensible distances are called *molecular forces;* e.g., cohesion. Forces acting between the atoms of molecules are called *atomic forces;* e.g., chemism.

12. Experiments. — A physical experiment is the production of physical phenomena under conditions that are controlled by a scientific student. It is a question addressed to Nature in the only language that she understands. The value attached to her replies involves a firm belief in the "constancy of nature;" i.e., that under the same physical conditions the same physical results will always be produced. Its purpose is to discover or to illustrate some physical truth.

(*a*) If the experiment simply shows how something acts, it is *qualitative;* if it shows how much something acts, it is *quantitative.*

13. Physics *is the branch of science that treats of the laws and physical properties of matter and of those phenomena that depend upon physical changes.* It is essentially an experimental science. With the explanation that energy signifies the power of doing work, physics, in its most general sense, may be defined as the science that treats of matter and energy.

II. THE PROPERTIES OF MATTER.

14. Properties of Matter. — *Any quality that belongs to matter, or is characteristic of it, is called a property of matter.* Any property that can be shown without a chemical change is a physical property.

15. Extension *is that property of matter by virtue of which it occupies space.* It has reference to the qualities of length, breadth, and thickness. It is an essential property of matter, involved in its very definition.

(*a*) All matter must have these three dimensions. We say that a line has length, a surface has length and breadth; but lines and surfaces are mere conceptions of the mind, and have no material existence. The third dimension, which affords the idea of solidity or volume, is necessary to every form of every kind of matter.

16. Measurement of Extension. — There are two linear units in use in this country, — the English yard and the international meter. From these are derived units of area and of volume.

17. The Yard is the distance between two certain marks on gold plugs set in a certain bronze bar, when the bar is at a temperature of 62° F. This bar is kept in the Tower of London.

(*a*) The divisions of the yard, as feet and inches, together with its multiples, as rods and miles, are in familiar use. The units of surface are squares whose sides are some one of the units of length, as the square yard or the square mile. The units of volume are cubes whose edges are some one of the units of length, as the cubic yard or the cubic inch.

(*b*) The standard gallon contains 231 cubic inches.

18. The Meter. — The international meter is the distance between two certain lines on a certain platiniridium bar, when the bar is at the temperature of 0° C. It is equal, as nearly as can be ascertained, to 39.37 inches, or "three feet, three inches and three eighths." This bar is kept at Sèvres, near Paris. Like the Arabic system of notation and the table of United States money, the divisions and multiples of the meter vary in a tenfold ratio, hence some of the great advantages of the system based upon it. This system is in familiar use by the people of most of the civilized countries of the world and by scientists of all nations. The scientific unit of length is the centimeter, — the one-hundredth part of the meter.

(*a*) The United States government very carefully preserves, at the office of standard weights and measures in Washington, three accurate copies of the international meter. These are authorized by congress as the standards of length for this country. The length of the yard is determined by the relation above stated.

19. Metric Measures of Length. Ratio = 1 : 10.

DIVISIONS.	*Millimeter*	(*mm.*) =	.001 *m.* =	0.03937	inch.
	Centimeter	(*cm.*) =	.01 *m.* =	0.3937	“
	Decimeter	(*dm.*) =	.1 *m.* =	3.937	inches.
UNIT.	*Meter*	(*m.*) =	1. *m.* =	39.37	“
MULTIPLES.	Dekameter	(*Dm.*) =	10. *m.* =	393.7	“
	Hektometer	(*Hm.*) =	100. *m.* =	328 feet 1 inch.	
	Kilometer	(*Km.*) =	1000. *m.* =	0.62137	mile.
	Myriameter	(*Mm.*) =	10000. *m.* =	6.2137	miles.

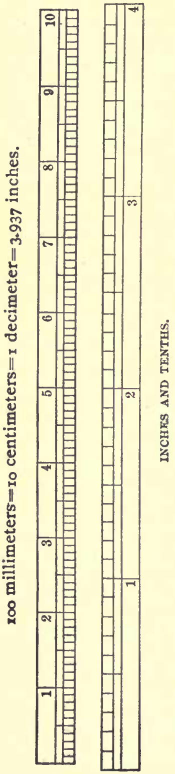


FIG. 2.

The table may be read, “10 millimeters make 1 centimeter, 10 centimeters make 1 decimeter,” etc. The denominations most used in practice are printed in Italics. The system of nomenclature is very simple. The Latin prefixes *milli-*, *centi-*, and *deci-*, signifying respectively .001, .01, and .1, and already familiar in the mill, cent, and dime of United States money, are used for the divisions; while the Greek prefixes *deka-*, *hekto-*, *kilo-*, and *myria-*, signifying respectively 10, 100, 1,000, and 10,000, are used for the multiples of the unit. Each name is accented on the first syllable.

20. Metric Measures of Surface and of Volume. — As with English measures, the metric units of surface and volume are surfaces or cubes whose sides or edges respectively are some one of the units of length, as the square meter or the cubic centimeter. For square measures, the ratio is $1 : 10^2 = 1 : 100$; thus, one hundred square millimeters make one square centimeter, etc. For cubic measures, the ratio is $1 : 10^3 = 1 : 1000$; thus, one thousand cubic centimeters make one cubic decimeter, etc.

21. Metric Measures of Capacity. Ratio = 1 : 10. — For many purposes, such as the measurement of articles usually sold by dry or liquid measure, a smaller unit than the cubic meter is desirable. *For such purposes, the cubic decimeter has been selected as the standard, and when thus used is called a liter* (pronounced *leeter*).

In value it is intermediate between the liquid and the dry quarts.

DIVISIONS.	Milliliter	(*ml.*) =	1 cu. cm.	= 0.061022 cu. in.
	Centiliter	(*cl.*) =	10 "	= 0.338 fluid oz.
	Deciliter	(*dl.*) =	100 "	= 0.845 gill.
UNIT.	*Liter*	(*l.*) =	1000 "	= 1.0567 liquid qts.
MULTIPLES.	Dekaliter	(*Dl.*) =	10 cu. dm.	= 9.08 dry qts.
	Hektoliter	(*Hl.*) =	100 "	= 2 bu. 3.35 pks.
	Kiloliter	(*Kl.*) =	1 cu. m.	= 264.17 gals.

LABORATORY EXERCISES.

Apparatus, etc., Needed. — A notebook made of good paper, and having some of its pages ruled in little squares; paper; pencil; a school rule; a yardstick graduated to eighths of an inch; a meter stick graduated to millimeters; a quart measure; a liter measure; a glass vessel graduated to cubic centimeters, i.e., a graduate (see Fig. 3).

1. With a yardstick, measure the length of your laboratory.

2. From the table given in § 19, compute the equivalent of that length in meters and decimals thereof.

3. With a meter stick, measure the length of your laboratory, and compare the result with that obtained by computation.

4. With a meter stick, measure the door of your laboratory, and make an outline sketch thereof, using the scale of 1 : 20.

5. With a yardstick, measure the width of your laboratory. Draw a ground plan of the room, using the scale of one inch to the yard.

6. Make the necessary measurements and compute the capacity of the room (*a*) in cubic feet, (*b*) in cubic meters, (*c*) in gallons, (*d*) in liters.

7. With the meter stick, measure the length of this leaf of your book. Place the stick on its edge so as to bring the graduation as close as possible to the object to be measured. Bring, not the end of

the rod, but one of the centimeter marks, even with one end of the leaf, and from the stick read the length of the page accurately to 0.1 mm. You can divide the smallest division on the scale into tenths by the eye.

8. As an exercise in subdividing distances by the eye, let the teacher draw a fine line, curved or straight, on cross-section paper, designate certain lines as axes of coördinates, and require each pupil in succession to record in tabular form the locus of each point where the given line crosses one of the lines ruled on the paper.

9. Make two fine marks with a sharp knife on a table-top or other board, as far apart as is convenient, the distance being more than a meter. Measure as accurately as possible the distance between the marks, estimating fractions of millimeters to tenths, and expressing the results in meters. Do this ten times. Measure the same distance in inches, estimating fractions of the smallest division on the scale to tenths. Express these results in inches and decimals of an inch. Do this ten times. Divide the average number of inches by the average number of meters; the quotient will be the number of inches in a meter. Express in millimeters the measures that you took in meters, and divide the average number of millimeters by the average number of inches; the quotient will be the number of millimeters in an inch. Compare your results with the table given in § 19.

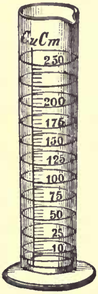


FIG. 3.

10. With the graduate, measure 250 cu. cm. of water, and pour it into the liter measure. See how often you can repeat the work without overflowing the measure. It will require careful attention to tell just when the water level reaches the required mark. The liquid climbs up the sides of the glass, so that it is difficult to tell where the water-level really is. The eye of the observer should be placed on the level of the required mark on the graduate.

11. Compute the number of cubic centimeters in a quart. Test your result by the actual measurement of water or of dry sand.

Impenetrability.

Experiment 8. — Pass a funnel (or a funnel-tube) and a bent tube, as shown in Fig. 4, through the cork of a bottle. *Be sure that all joints are air-tight.* The delivery-tube is best made of glass, which may be bent when heated to redness in an alcohol or gas flame. Place the end of the delivery-tube in a tumbler of water. Pour water through the funnel. As it runs into the bottle, *air will be forced out*, and may be seen bubbling through the water in the tumbler. Directions for glass working may be found in Avery's Chemistry, Appendix 4.

Experiment 9. — Thrust a lamp chimney into water. The water will rise inside the chimney, entering at the lower end, and pushing the air out at the top. Repeat the experiment, closing the upper end of the chimney with the hand (or use an inverted tumbler). *The water cannot rise as before*, because the vessel is filled with air that cannot escape.

FIG. 4.

22. Impenetrability *is that property of matter by virtue of which two bodies cannot occupy the same space at the same time.*

(*a*) Illustrations of this property are very simple and abundant. Thrust a finger into a tumbler of water; it is evident that the water and the finger are not in the same place at the same time. Drive a nail into a piece of wood; the particles of wood are either crowded more closely together to give room for the nail, or some of them are driven out before it. Clearly, the iron and the wood are not in the same place at the same time. The familiar method of measuring the volume of an irregular solid by immersing it in a liquid and then measuring the volume of the liquid displaced by it, implies the impenetrability of matter.

23. Mass and Weight. — The mass of a body is its quantity of matter. *The weight of a body is, in general terms, the measure of the earth's attraction for it.* The weight of a body varies as its mass, and with the position of the body relative to the earth's surface. The mass of a given body is constant; its weight is not. The word "mass" signifies matter; the word "weight" signifies force.

(*a*) If the given body could be carried to the moon, its weight there would be the measure of the attraction existing between the body and the moon; but as the mass of the moon is less than that of the earth, the attraction between the body and the moon would be less than that between that body and the earth. The mass of the given body would be the same as it was on the earth, but its weight would be less.

24. Measurement of Mass and Weight. — Unfortunately we still have two systems of measurement, — one practically limited in use to the United States and the British Empire; the other, international. The English unit of mass is the quantity of matter contained in the avoirdupois pound. The international unit of mass is the kilogram, a certain piece of platiniridium deposited at Sèvres, near Paris. For many scientific uses, this unit is too large; and the gram, which is the one-thousandth part of the kilogram, is generally used.

(*a*) The mass of a gram was intended to be, and is very nearly, equal to the quantity of matter in one cubic centimeter of distilled water at the temperature of 4° C. As with the meter, the United States government carefully preserves a standard kilogram.

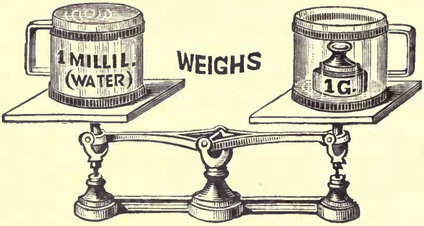


FIG. 5.

(*b*) The units of weight measure the attractions of the earth for these units of mass, and receive the same names. *Under like conditions*, a comparison of weights may be substituted for a comparison of masses, since at any one place the weight varies as the mass. Unfortunately we have in common use pounds Troy, avoirdupois, and apothecaries', the use varying with the nature of the transaction. On the other hand, the kilogram is definite, having but a single value.

25. Metric Measures of Weight. Ratio = 1 : 10.

DIVISIONS.	Milligram	(*mg.*)	= 0.0154 grain.
	Centigram	(*cg.*)	= 0.1543 “
	Decigram	(*dg.*)	= 1.5432 grains.
UNITS.	*Gram*	(*g.*)	= 15.432 “
MULTIPLES.	Dekagram	(*Dg.*)	= 0.3527 oz. avoirdupois.
	Hektogram	(*Hg.*)	= 3.5274 “ “
	Kilogram	(*Kg.*)	= 2.2046 lbs. “
	Myriagram	(*Mg.*)	= 22.046 “ “

(*a*) A five-cent nickel coin weighs five grams. A cubic centimeter of water weighs one gram.

CLASSROOM EXERCISES.

1. How much water by weight will a liter flask contain?

2. If sulphuric acid is 1.8 times as heavy as water, what weight of the acid will a liter flask contain?

3. If alcohol is 0.8 times as heavy as water, how much will 1,250 cu. cm. of alcohol weigh?

4. What part of a liter of water is 250 g. of water?

5. What is the weight of a cubic decimeter of water?

6. What is the weight of a deciliter of water?

7. How many gallons of water may be held by a vessel 18 × 19 × 20 inches in dimensions?

8. How many liters of water may be held by a vessel measuring 25 × 35 × 75 cm.?

LABORATORY EXERCISES.

Additional Apparatus, etc. — A fairly delicate balance (see Fig. 6); English and metric weights; two rectangular wooden blocks, 2 × 3 × 4 inches; an iron ball an inch or two in diameter; a baseball; a croquet ball; a pair of compasses with pencil point; a penknife; a teacupful of lead bullets; a bottle; wire.

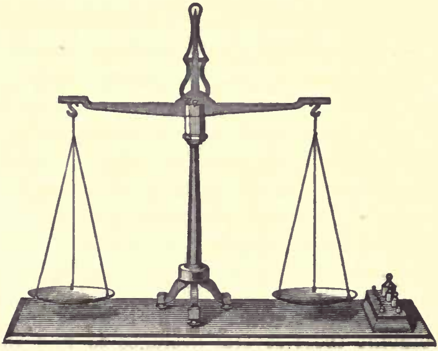


FIG. 6.

1. Measure the mass of each of the three balls in English weight units.

2. Compute the metric equivalents of these three weights.

3. Weigh the three balls, using metric standards, and compare results with those found by computation.

4. Place a meter stick on the table, and by its edge place two rectangular blocks (chalk boxes will answer for rough work). Place a croquet ball between the blocks. Move the blocks as near each other as possible with the ball between them, keeping one face of each block in contact with the straight edge of the meter stick. (*a*) What is the diameter of the ball? (*b*) What is the area of its surface?

5. (*a*) In similar manner, measure the diameter of a base-ball. (*b*) On paper, draw a circle of that diameter. (*c*) Compute the area of that circle. (*d*) With a sharp penknife, cut out the circle, and pass the base-ball through the hole.

6. (*a*) In similar manner, measure the diameter of the iron ball. (*b*) Compute its volume. (*c*) Compute the weight of the same volume of water. (*d*) Measure out and weigh that volume of water, and compare its weight with the computed result. (*e*) Iron is how many times as heavy as water? (*f*) Place the iron ball in a tumbler or beaker filled with water; catch and measure the water that runs over. (*g*) How does this measure compare with the computed volume of the ball?

Caution. — The pupil must clearly understand that all measurements made by him in this course are extremely rude, and scarcely comparable with the measurements of precision made by scientists. Measurements to the hundred-thousandth of an inch and the twenty-thousandth of a second are frequently made, and no painstaking is deemed too great if it will increase the degree of accuracy attained.

7. From a coil of moderately fine wire (say, No. 30) of unknown length, cut off a piece exactly a meter long, and weigh it carefully. Weigh the rest of the coil, and from the two weights compute the length of the wire. Verify your result by actual measurement. If the error of your result exceeds 1 per cent., repeat the work.

8. Weigh a dry, clean bottle. Fill the bottle with cold water, wipe its outside surface dry, and weigh the filled bottle in metric units. From the weight of the water, determine the capacity of the bottle. Test the result by measurement with the graduate.

9. Weigh each of five bullets at least three times. For each bullet take the average of the several weighings as the true weight. Combine these several averages to find the weight of the average bullet. Count the bullets on hand. Multiply the weight of the aver-

age bullet by the number of bullets, and compare the result with the mass of all the bullets as determined by weighing them together.

Indestructibility.

Experiment 10. — Into a glass tube 2 cm. in diameter, and 15 or 20 cm. in length, having one end closed and rounded like a test-tube, place 20 mg. of freshly burnt charcoal. Draw the upper part of the tube out to a narrow neck. Fill the tube with dry oxygen, and seal the tube by fusing the neck. Weigh the tube and its contents very carefully. By gradually heating the rounded end of the tube, the charcoal may be ignited, and, with sufficient care, entirely burned without breaking the tube. When the charcoal has disappeared, weigh the tube and its contents again. *The chemical changes that led to the disappearance of the charcoal have caused no change in the weight of the materials used.*

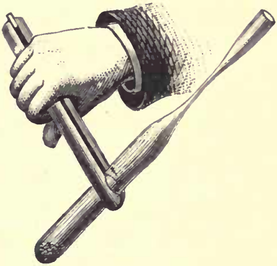


FIG. 7.

26. Indestructibility *is that property of matter by virtue of which it cannot be destroyed.* The science of chemistry is based on this fact, the "conservation of matter."

Inertia.

Experiment 11. — Upon the tip of the forefinger of the left hand place a common calling card. Upon this card, and directly over the finger, place a cent. With the nail of the middle finger of the right hand let a sudden blow or "snap" be given to the card. A few trials will enable you to perform the experiment so as to *drive the card away, and leave the coin* resting upon the finger. Repeat the experiment with the variation of a bullet for the cent and the open top of a bottle for the finger tip.

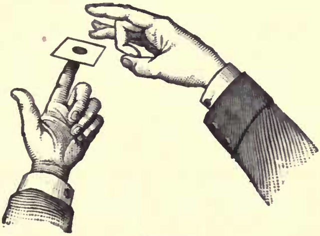


FIG. 8.

Experiment 12. — Suspend a heavy weight by a string not much stronger than is necessary to carry the load. Attach a similar string to the under side of the weight. Pull steadily downward on the lower string; in a majority of cases the *upper* string will break, for it has to support the weight and resist the pull, while the lower string has only to resist the pull.

Support the weight as before. Pull suddenly downward on the lower string: in a majority of cases the *lower* string will break, as if *there was not time enough for the pull to pass through the weight and reach the upper string.*

Experiment 13. — Suspend an iron ball weighing at least 10 pounds by a long, stout string from a firm support. Safety-valve weights may be bought for a few cents a pound, and answer admirably for many such purposes. Tie a string strong enough to carry a weight of several pounds to the ball, and with sudden motion pull the ball horizontally. If the pull is sudden enough, *the string will break* without giving much motion to the ball. This "hanging back" of the ball is very important, and must at least be given a name. Replace the stout string by a thread, and by a series of gentle, well-timed pulls, set the ball swinging. When it is in rapid motion, try to stop that motion by a single pull on the thread. It will be seen that *the ball can go ahead as well as hang back.*

27. Inertia *signifies the tendency of matter at rest to remain at rest, and of matter in motion to move with uniform velocity in a straight line.*

(*a*) Illustrations of the inertia of matter are so numerous, that there should be no difficulty in getting a clear idea of this property. The "running jump" and "dodging" of the playground, the frequent falls which result from jumping from cars in motion, the backward motion of the passengers when a car is suddenly started and their forward motion when the car is suddenly stopped, the difficulty in starting a wagon and the comparative ease of keeping it in motion, illustrate inertia. By virtue of inertia, a cannon ball pierces the hardened steel armor of a battle ship; and because of the same property, it is well not to kick the cannon ball, even when it is resting on a smooth and level surface.

Porosity.

Experiment 14. — Pour 30 cu. cm. of water into a long test-tube. Carefully add 20 cu. cm. of strong alcohol, holding the tube so that the latter may run down its side and rest upon the water without mixing with it. Gently bring the tube into a vertical position, mark the height of the liquid in the tube, close the mouth of the tube with the thumb, and thoroughly shake the two liquids together. *Notice again the height of the liquid contents of the tube.* It looks as if some of the water and some of the alcohol had been forced into the same space, in spite of the impenetrability of matter.

Experiment 15. — Fill a glass tumbler with large shot or peas, and then see how much well-dried sand or salt you can add. *Perhaps what happens here is analogous to what happened in Experiment 14.*

28. Porosity *is that property of matter by virtue of which spaces exist between the molecules.* A body does not completely fill the space it seems to occupy. As a result of this, we have the possibility of an interpenetration of two bodies, the molecular volumes of one occupying the intermolecular spaces of the other, so that the resultant volume is less than the sum of the constituent volumes.

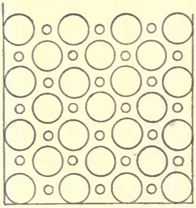


FIG. 9.

(*a*) When iron is heated, the molecules are pushed further apart, the pores are enlarged, and we say that the iron has expanded. When a piece of iron or lead is hammered, it is made smaller, because the molecules are forced nearer together, thus reducing the size of the pores. Cavities or cells, like those of bread or sponge, are not properly called pores.

29. Strain and Stress. — *Any change in the shape or size or volume of a solid is called a strain.* Thus, if a mass of metal becomes compressed, or bent, or twisted, or distorted in any way, it is said to experience a strain. The use of the word "strain" to designate a force that produces a deformation is common in literature, but incorrect in mechanics. *The force that produces a strain is called a stress.*

Elasticity.

Experiment 16. — Squeeze a rubber ball. Stretch a rubber band. Stretch a spiral spring. Bend a thin strip of steel, wood, or whalebone. In each case *the volume or form is restored to its initial condition when the distorting force ceases to act.*

Experiment 17. — Firmly fasten one end of a piece of spring-brass wire, about No. 27 and about 1 m. long (e.g., grip it in a hand vise), so that the wire hangs vertical. To the lower end of the wire fasten a weight of 75 or 100 g. To this weight attach a pointer so that it extends horizontally from the direction of the wire. Turn the weight through a considerable angle, thus twisting the wire. Release the weight, and *notice the rapid movements of the pointer of the torsional pendulum.*

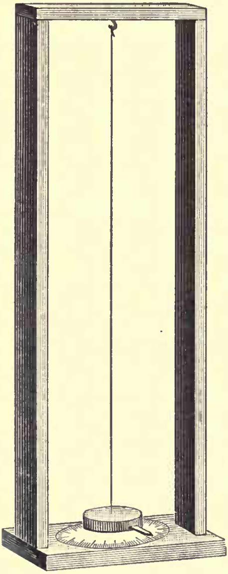


FIG. 10.

30. Elasticity *is that property of matter by virtue of which bodies resume their original form or size when that form or size has been changed by any external force.* There is an elasticity of volume and an elasticity of form or figure. The former is peculiarly a property of gases and liquids; the latter, of solids.

(*a*) The elasticity of a body may be developed by pressure, by pulling, by bending, or by twisting.

(*b*) All bodies possess this property in some degree, because all bodies, solid, liquid, or aëriform, when subjected to pressure (within limits varying with the substance), will resume their original size upon the removal of the pressure. Fluids have no elasticity of form; on the other hand, all fluids have perfect elasticity of size. The ratio of the numerical value of a stress to the numerical value of the strain produced by it is called *the coefficient of elasticity.*

(*c*) Solids that have little or no ability to resume their original shape after the action of a stress are said to be inelastic or plastic. In the case of even highly elastic solids, when the strain exceeds a certain value, called the limit of elasticity, the substance acts like a plastic solid. A body under stress has reached its limit of elasticity when any further stress will cause a permanent alteration of form or size.

Molecular Attraction.

Experiment 18. — Dip a finger into water. Upon removing it, notice that it is wet, that water adheres to it. Hold the finger pointing downward, and notice that a drop of water gathers at the finger tip. That drop is composed of many particles that cling together, or cohere. *Something makes the water particles cling to each other and to the finger*, in spite of the force of gravity.

Experiment 19. — Hold the end of a stick of sealing wax in a candle flame. Notice that *the fused drop increases in size* until it becomes so heavy that the molecular forces can no longer counteract its weight.

Experiment 20. — Cut a lead bullet so as to present two flat, clean surfaces. Press the two parts together with a slight twisting motion. *They will cling together.* Lead disks are made for this purpose. When used, their opposing surfaces should be flat and bright.

FIG. 11.

Experiment 21. — Take a sheet of gold leaf in your fingers, and try to pick the metal off with the fingers of the other hand. *Some of the gold will stick to your fingers.*

31. Cohesion and Adhesion. — *Cohesion is the force that holds together like molecules; adhesion is the force that holds together unlike molecules.* The distinction is traditional rather than necessary.

(*a*) Exhibitions of this force are more noticeable in solids than in liquids; in aëriform bodies it seems to be wanting.

(*b*) This is the force that holds bodies together, and gives them form. Were its action suddenly to cease, brick and stone and iron would crumble to finest powder, and all our homes and cities and selves fall to hopeless ruin. This force acts only at insensible (molecular) distances. Let the parts of a body be separated by a sensible distance, and we say that the body is broken. If the molecules of the parts can again be brought within molecular distance of each other, cohesion will again act, and hold them there. This may be done by simple pressure, as in the cases of wax, freshly cut lead, broken ice, and many powders; it may be done by welding or melting, as in the case of iron.

32. Hardness *is that property of matter by virtue of which some bodies resist any attempt to force a passage between their particles.* The relative hardness of two substances is determined by finding out which of them will scratch the other; e.g., we know that glass is harder than copper because it will scratch copper.

(*a*) The hardness of many solid substances is increased by raising the body to a high temperature and suddenly cooling it. The process of giving a body a suitable degree of hardness is called *tempering.* Some substances, like copper, are made softer by raising to a high temperature and slowly cooling. This process is called *annealing.*

Tenacity.

Experiment 22. — Cut several strips of manilla paper about 5 by 25 cm. Turn each end of each strip over, and fasten the edges with glue so as to make a good hem at each end. In the loop at one end of the paper strip insert a stout rod the length of which exceeds the width of the paper strip. Fasten this rod by a stout string or wire bail to a nail in a board or table-top. Similarly fasten the other end of the strip to the hook of a good spring-balance, held as shown in Fig. 12. Pull steadily with the balance and in a line with its length, so as to avoid, as far as possible, all friction of the sliding bar to which the hook is attached. Watch the index of the balance all the time, looking directly down upon it so as to avoid the error of parallax. Continue to pull until the paper breaks. Be careful that the recoil of the hook does not injure your hands. Repeat the experiment with several similar strips, recording after each test the maximum reading of the index. If the index does not rest over the zero mark when the balance is in a horizontal position, the proper correction should be made for each reading taken. *The average of these several readings will be a fair expression of the strength of the paper.* Make a similar series of tests with similar strips of paper twice as wide. Compare the two average results. From your data, compute the strength of a strip 2.2 cm. wide and experimentally verify the result.

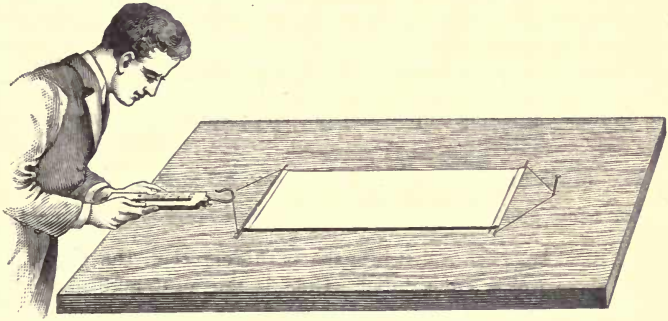


FIG. 12.

33. Tenacity *is that property of matter by virtue of which some bodies resist a force tending to pull their particles asunder.* Its measure is the ratio between the breaking weight and the area of the cross-section of the body broken. It varies with different substances, with the form of the body, with the temperature, and with the duration of the pull.

(*a*) Like hardness and other characteristic properties of matter, tenacity is a variety of cohesion. For any given material, it has been found that *tenacity is proportional to area of cross-section;* e.g., a rod with a sectional area of a square inch will carry twice as great a load as a rod of the same material with a sectional area of a half square inch; a rod 10 cm. in diameter will carry four times as great a load as a similar rod 5 cm. in diameter. The explanation of this is simple. Imagine these rods to be cut across, and it will be evident that on each side of the cut the first rod will expose twice as many molecules as will the second, and that the third will expose four times as many as the fourth. But, for the same material, each molecule has the same attractive force. Doubling the number of these attractive molecules, which is done by doubling the sectional area, doubles the total attractive force, which, in this case, is called *tenacity;* quadrupling the sectional area quadruples the tenacity; etc. Hence the law.

34. Malleability *is that property of matter by virtue of which some bodies may be rolled or hammered into sheets.*

(*a*) Steel has been rolled into sheets thinner than the paper upon which these words are printed. Gold is the most malleable metal, and, in the form of gold leaf, has been beaten so thin that 300,000 sheets, placed one upon the other, measured but an inch in height.

Ductility.

Experiment 23. — Heat the middle of a piece of glass tubing about 6 inches long in an alcohol flame until red-hot. Roll the ends of the glass slowly between the fingers, and, when the heated part is soft, *quickly draw the ends asunder.* That the fine glass wire thus produced is still a tube, may be shown by blowing through it into a glass of water, and noticing the bubbles that will rise to the surface.

35. Ductility *is that property of matter by virtue of which some bodies may be drawn into wire.*

(*a*) Platinum wire has been made $\frac{1}{30000}$ of an inch in diameter. Glass, when heated to redness, is very ductile.

(*b*) Malleability and ductility are closely related, so that most substances that have great malleability also have high ductility. But this rule is not universal; lead and tin are very malleable, but only slightly ductile.

(*c*) Ductility involves tenacity; i.e., although a substance may be tenacious without being ductile, it cannot be ductile without being tenacious. The tenacity of most metals is increased by the process of wiredrawing. Cables made of twisted iron or steel wires are stronger than iron chains or rods of equal weight and length. Steel wire with a tenacity of more than 92 tons per square inch of cross-section has been made, and wire with a tensile strength of 70 or 80 tons per square inch is readily procurable.

LABORATORY EXERCISES.

Additional Apparatus, etc.—A decimeter rule divided to 0.2 mm.; a diagonal scale; dividers (see Fig. 13); wire; wire gauge (see Fig. 14); calipers (see Figs. 15 and 16); a solid cylinder; a metal tube; a tumbler; tin can; a glass vessel graduated to 0.1 cu. cm.; wooden blocks, rods, and bars; clevis; a tin-can cover for scale-pan; two pieces of $\frac{3}{4}$-inch gas-pipe each 4 inches long; a straightedge; a wheat straw; two small vises; sand-bag weights; soldering tools and materials; cutting pliers.

1. What is the length of a full line as printed in this book? Place the graduated edge (not the side) of the decimeter rule on the paper, with some plainly visible mark, as 0.5 or 1 cm. (not the end of the rule), at one end of the printed line. Always use a rule in this way for accurate measurements.

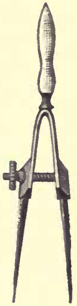


FIG. 13.

2. Draw on paper three straight lines with lengths of 2.57 inches, 3.34 inches, and 6.4 centimeters respectively.

3. Tightly pinch the leaves of this book (inside the covers) between two small blocks that come flush with them at the top. Remove some of the leaves so that those that remain make a layer just 1 cm. thick. Count the leaves, and compute in decimals of a millimeter the thickness of an average leaf of this book.

4. Determine the gauge numbers of four pieces of wire. Use the notches around the edge of the steel plate, and not the larger circular openings at the inner end of the notches. If you have a micrometer caliper, determine the diameter of each piece. The best wire gauge to use is that known as the Brown and Sharpe, "B & S." Introduce the wire into the slit which admits it with a very slight pressure, and note the number corresponding to that slit. Be sure that the wire is not rusty, dirty, or bruised at the point where it is gauged. It is convenient to buy such wire wound on spools.

5. Wind 25 turns of No. 30 annealed wire around a cylinder an inch or more in diameter, being careful that the successive turns are as close as possible to each other. Measure the total width of the wire band on the cylinder, and compute the diameter of No. 30 wire. Compare your result with the table given in the appendix.

6. With the outside calipers, measure the diameter of the iron ball used in Exercise 6 on p. 24. Compare the result then obtained and recorded in your notebook with that now found.

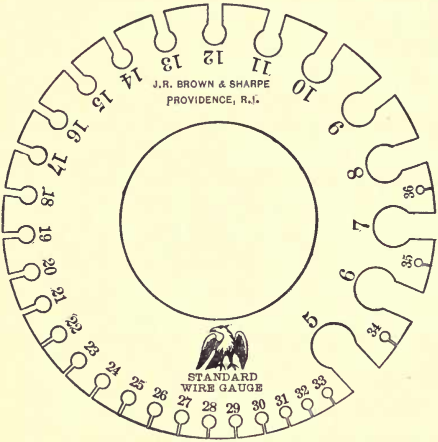


FIG. 14.

7. Measure the diameter and length of a small cylinder. If, by measuring the rod at short intervals with the calipers, you find that its diameter is not uniform, use the average of your several measurements. (*a*) Compute the surface area of the cylinder. (*b*) Compute the volume of the cylinder.

8. Measure the length and the inside and outside diameters of a metal tube. If you have no inside calipers, bend a piece of annealed wire into a V-shape, and use that. (*a*) Compute the total surface area of the tube. (*b*) Compute the volume of metal in the tube. (*c*) Test your result by the displacement of water.

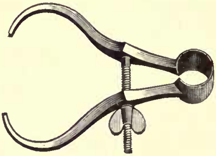


FIG. 15.

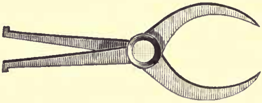


FIG. 16.

9. (*a*) With the inside calipers, measure the diameter of the tumbler on the inside, at the bottom and at the top. From these measurements determine the average diameter of the tumbler. Place a straightedge across the top of the tumbler in the line of a diameter. Measure the perpendicular

distance from the bottom of the tumbler to the under side of the straightedge. Compute the capacity of the tumbler in cubic centimeters. (*b*) Fill the tumbler with water and pour the water into the graduate, and thus test the accuracy of your previous measurements and computation.

10. Using the wire gauge, select coils of No. 30 wire, brass, copper, iron, and steel. From each coil cut off two pieces each about 1 m. long. Wrap one end of a wire twice about the middle of a piece of gas-pipe, and fasten by twisting the end around the body of the wire, being careful not to make any kink in the body of the wire. Similarly fasten the other end of the wire to the other piece of gas-pipe. Through each piece of gas-pipe pass a 60 cm. piece of wire much stouter than the No. 30 iron or steel, bend these wires sharply at the ends of the gas-pipe, and twist the ends together so as to make a bail in the shape of an isosceles triangle of which the pipe shall be the shorter side. From one of these bails suspend a tin can or basin (the weight of which, with its bail and gas-pipe, has been ascertained); support the other bail so that the can shall be only a few inches above the floor or table. Place a weight of 200 or 300 g. in the pan, and add weights of 10 or 5 g. each until the load breaks the wire. In like manner determine the breaking strength of the other piece of the same kind of wire. Make like tests with the other kinds of wire. Tabulate your results in some such way as the following:—

BREAKING STRENGTH OF NO. 30 WIRE.

	Copper.	*Brass.*	*Iron.*	*Steel.*
First test . . .	———	———	———	———
Second test . .	———	———	———	———
Averages . . .	———	———	———	———

11. Provide two pieces of hard wood, *A* and *B*, and place them on the table as shown in Fig. 17, with their ridges parallel, and a meter apart. Upon these blocks place a wooden bar, *W*, 1.2 cm. (½ in.) square and about 104 cm. long. Midway between the supporting ridges place a small clevis, *C*, made of sheet metal and carrying a scale-pan. In some cases it may be convenient to pass the string through a hole in the table, so that the scale-pan will hang below the table-top. Provide a straightedge a meter long, and faced on the edge at each end with metal of the same thickness as the clevis. Place the straight-

edge on *W*, with its ends over the ridges of *A* and *B*. If there is no deflection or "sag" in *W*, the straightedge will just touch the upper surface of *C*; otherwise the distance between *C* and the straightedge will measure the deflection. Place a known weight (say, 100 g.) in the scale-pan, and to this weight add the weight of the clevis and the scale-pan, and call the sum the load. Measure the deflection of *W* caused by the load. Double the load, and measure the deflection thus caused. Record these data.

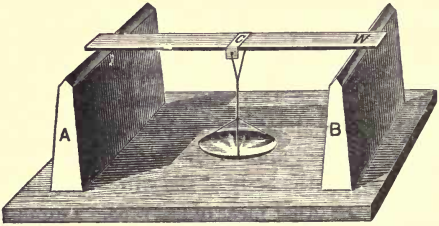


FIG. 17.

For quick work, each end of a cord or annealed wire may be fastened to the knob of a known weight, and the cord, thus loaded, placed across the middle of the bar.

Instead of measuring the deflection thus directly, using a straightedge, the deflection may be magnified and more easily read in this way: Solder a pin to the further end of the clevis so that it shall project horizontally and at right angles to the length of the wooden bar. Select a straight straw 30 cm. or more in length. Trim the smaller end of the straw so that it may act as a pointer or index. Thrust a needle through the straw a little less than five-sixths of its length from the pointed end. Measure, from the needle toward the nearer end of the straw, a distance just one fifth the length of the part on the other side of the needle, and mark the point thus located by a pen mark on the straw. Thrust the end of the needle carrying the straw into the cork of a bottle so that the needle shall be horizontal when the bottle is upright. Place the bottle so that the height of the needle shall be a little less than that of the pin carried by the clevis, while the straw is parallel with the wooden bar. Adjust the bottle so that the ink mark shall come under the pin on the clevis. Tie or tack any convenient graduated scale so that it shall stand on end against the vertical face of a block, and set it so that the index end of the straw shall stand opposite one of the marks near the bottom of the scale. This mark is to be considered the zero mark. A deflection of 1 cm. at the middle of the bar will move the index over 5 cm. of the scale. Read deflections from the edge of the scale. After each weighing, see whether the index returns to the zero of the scale or not.

Replace *W* by another bar made of the same kind of wood, with the same length and thickness, but 2.4 cm. wide. Load it to produce the same deflections as before, and record the data. In similar manner try the narrow bar with the supports 0.5 m. apart, and the broad bar on edge with the supports 1 m. apart. From all the data thus obtained and recorded, concerning the stiffness of beams carrying loads midway between their supports, answer the following questions: What is the relation between the load and the deflection? Between the length of the beam and the deflection? Between the breadth of the beam and the deflection? Between the thickness of the beam and the deflection?

12. Experimenting with wooden rods of different lengths, widths, and thicknesses, supported at their ends and loaded in the middle, show that the breaking strength varies directly as the width and as the square of the thickness, and inversely as the length, or that it does not; i.e., verify or disprove the formula: —

$$\text{Breaking strength} \propto \frac{wt^2}{l}.$$

(The sign $\propto$ is read "varies as" or "is proportional to.")

13. Grip one end of a small straight wire, a yard or so in length, in a small vise. Support the vise firmly, so that the wire may hang vertically. Grip the free end of the wire in another small vise of known weight. Near the lower end of the wire, fix a horizontal pointer (a stiff bristle stuck on with shoemaker's wax will answer), and mark its level on a card fixed upright. Call this level the zero of the card scale. Add a sand bag of the same weight as the vise, and mark the level of the pointer on the card. Remove the sand bag and see if the pointer comes back to zero. Add sand bags or equal weights in succession, marking the successive levels of the pointer on the scale, removing the weights after each trial, and giving the pointer a chance to return to the zero mark. When you find that the wire has been permanently stretched, stop the test. Record the elongation after each weight. Repeat with a similar wire, twice as long. Can you discover any law for the stretching of wire? If so, record it. Verify that law by similar experiments with other wires.

14. Using a rod of clear ash, 36 inches long and half an inch square, determine the relation between the force used to twist the rod and the amount of torsion produced; i.e., if a certain force produces a torsion of 5°, how much torsion will twice that force produce?

15. Using the same rod or a similar one, determine the relation between the length of the rod and the amount of the torsion.

16. Using the same rod and one of the same kind of wood and the same length, but $\frac{3}{4}$ of an inch square, verify the statement that torsion varies as $\frac{1}{d^4}$, d representing the diameter of the rod. How does the formula as given agree with the statement that the torsion varies inversely as the square of the cross-section.

III. THE THREE CONDITIONS OF MATTER, ETC.

36. Conditions of Matter. — *Matter exists in three conditions or forms, — the solid, the liquid, and the aëriform.* Liquid and aëriform bodies are fluids.

37. A Solid *is a body whose molecules change their relative positions with difficulty.* Such bodies have a strong tendency to retain any form that may be given to them, and can sustain pressure without being supported laterally. A movement of one part of such a body produces motion in all of its parts.

38. A Fluid *is a body whose molecules easily change their relative positions.* Fluids cannot sustain pressure without being supported laterally. The term comprehends liquids, gases, and vapors.

Cohesion of Liquids.

Experiment 24. — Suspend a clean glass plate of about four inches area from one end of a scale-beam, and accurately balance the same with weights in the opposite scale-pan. The supporting cords may be fastened to the plate with wax. Beneath the plate place a saucer so that when the saucer is filled with water the plate may rest upon the liquid surface, the scale-beam remaining horizontal. Be sure that there are no air-bubbles under the plate. Carefully add small weights to those in the scale-pan. Notice that the water beneath the plate is raised above its level. Add more weights until the plate is lifted from the water. Notice that the under surface of the plate is wet. These molecules on the plate have been torn from their companions in the saucer, the adhesion between the water and the plate being greater than the cohesion of the water. The weights added to the original counterpoise were needed to overcome the cohesion of the water molecules.

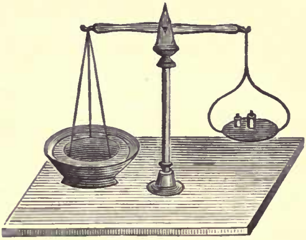


FIG. 18.

39. A Liquid *is a body whose molecules easily change their relative positions, yet tend to cling together.* Such bodies adapt themselves to the form of the vessel containing them, but do not retain that form when the restraining force is removed. Their free surfaces are always horizontal. Water is the best type of liquids.

40. An Aëriform Body *is one whose molecules easily change their relative positions, and tend to separate from each other almost indefinitely.* Such bodies are of two kinds, — gases and vapors. Gases remain aëriform (i.e., retain the form of air) under ordinary conditions, while vapors resume the solid or liquid form at ordinary temperatures. Atmospheric air is the most familiar type of aëriform bodies.

41. Changes of Condition. — Many substances, like iron and gold and water, may be made to exist in all of these three forms by suitable adjustments of temperature and pressure. The identity of ice, water, and steam, is familiar to all.

(*a*) Experiments with electric discharges in high vacuums have given results which, in the minds of many, prove the existence of a fourth condition of matter. For matter in this extremely thin or attenuated form, the name "radiant matter" has been proposed.

Solution.

Experiment 25. — Into a beaker half full of water, drop a few lumps of sugar. Stir the contents of the glass until the solid disappears.

Experiment 26. — Mix 50 g. of pulverized ammonium nitrate and 25 g. of pulverized ammonium chloride (sal-ammoniac). Put the mixture into 75 cu. cm. of cold water in a beaker, and stir the substances together with a small test-tube containing a little cold water. Notice that the solids disappear. Carefully observe the condition of the water in the test-tube.

42. Solution *is the transformation of matter from the solid or gaseous form to the liquid form by means of a liquid called the solvent or menstruum.* The process is essentially a change of molecular condition. When the change is from the solid to the liquid form, there is an absorption of heat with a consequent fall of temperature, as is strikingly seen in freezing mixtures. The solution of a gas in a liquid is accompanied by a release of heat and a consequent rise of temperature. When the solvent has dissolved as much of a given substance as it can, *the solution is said to be saturated.*

(*a*) The solubility of any solid in any liquid is constant at a given temperature, and may be determined by experiment. The solubility of any gas is also constant under the same conditions; it varies with temperature, pressure, and the nature of the solvent. With a mixture of gases, each is dissolved in the same quantity as if it were present alone, and under the same pressure as in the mixture.

Crystallization.

Experiment 27. — Dissolve about a quarter of a pound of alum in a pint of hot water. Hang several strings in the solution, and set the vessel containing it aside for the night. In the morning notice the regularity and similarity of the crystals that have formed on the strings.

43. Crystallization. — Many solids forming slowly from the liquid or aëriform condition, and thus having great freedom of molecular motion, assume regular forms, with a certain number of plane surfaces symmetrically arranged, and with a definite internal structure. *Such bodies are called crystals.* Thus quartz crystallizes in hexagonal prisms terminated by hexagonal pyramids.

FIG. 19.

(*a*) The internal structure is exhibited in the cleavage, and in the appearance of sections cut from the crystal and viewed by polarized light. The planes of cleavage are certain definite planes along which separation is most easily effected.

(*b*) The form and structure of crystals are of great importance to the chemist and the mineralogist, as the nature of many substances may be ascertained thereby. (See definition of "crystallography" in "The Century Dictionary.")

LABORATORY EXERCISES.

Additional Apparatus, etc. — Bottles containing concentrated solutions of potassium nitrate (saltpeter) and of ammonium chloride (sal ammoniac); several pieces of window glass 4 or 5 inches square; a magnifying lens, preferably mounted (the glasses used in botanical study will answer admirably); a Bunsen or an alcohol lamp; test-tube; iodine; Hessian crucible; brimstone; saucer; bottle; round or "rat-tail" file; Florence flask; corks and cork-borers; glass tubing; retort-stand; two plain tumblers of thin glass and of the same size; waste-jar.

1. (*a*) Slowly warm a piece of thoroughly cleaned glass over the lamp; hold the glass horizontal, and pour a little of the solution of saltpeter upon it. Move the glass quickly so as to spread the liquid over its surface, and then hold it over the waste-jar so as to drain off the surplus solution. When a cloudy patch appears on the glass, examine it carefully through the lens, make a drawing of what you see, and label it "KNO_3 Crystals."

(*b*) Take a similar course with the other solution, and label the drawing "NH_4Cl Crystals."

2. Drop a single crystal of iodine into the bottom of a test-tube (Fig. 20), and heat it gently. After the tube has been well filled with the beautiful iodine vapor, allow it to cool. With a magnifying lens, examine the iodine crystals that form on the walls of the test-tube.

FIG. 20.

3. Melt about 200 g. of sulphur (brimstone) in a Hessian crucible (Fig. 21), and allow it to cool until a crust forms over it. Through a hole pierced in this crust, pour out the still liquid sulphur. When the crucible is cool, break it, and with a magnifying glass examine the needle-shaped sulphur crystals with which it is lined. The crucible may be saved by pouring all of the melted sulphur into a pasteboard box, and allowing the crystals to form there.

4. Fill a clear glass tumbler with fresh hydrant or well water. Fill a similar vessel with water that has recently been well boiled. Set both in a moderately warm, quiet place, and let them stand over night. Examine the walls of the two tumblers, and account for the difference in their appearance.

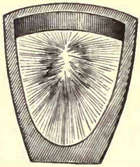


FIG. 21.

5. With a round file, work a notch in the edge of a saucer, and a hole about a centimeter in diameter in the middle of the bottom. Invert the saucer in an earthenware or tin pan, and cover it with water. Fill a bottle with water, and stand it upside down with its mouth around the hole in the saucer. Fill a Florence flask with water, and, holding it under water, close its mouth with a cork carrying a bent glass delivery-tube. Keeping the flask and tube full of water, thrust the free end of the tube through the notch in the edge of the saucer, and place the flask on the retort-stand. Be sure that flask, tube, and bottle are full of water. Heat the flask carefully until the water has boiled for several minutes. The collection bottle will be found to

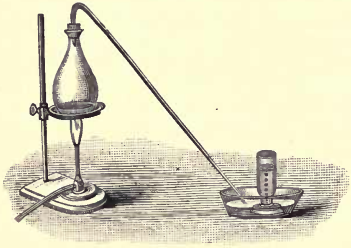


FIG. 22.

contain something besides water; it is air. Can you imagine whence it came? Repeat the experiment with water that has been recently boiled in an open vessel. Do you collect any air now? Perhaps the boiling of water expels air that it holds in solution. Think about it. Has your work given you any information as to the solubility of air in water? As to the porosity of water?

Superficial Molecules.

Experiment 28. — Fill a tumbler brimming full of water. With a pipette (Fig. 23), add more water, drop by drop and patiently, until the water in the tumbler is actually heaped up higher than the edges of the glass. Try to imagine an invisible skin stretched over the liquid surface to keep it from overflowing the edge of the tumbler.

Fig. 23.

Experiment 29. — Carefully place a fine sewing needle upon the surface of water. With care, and perhaps repetition, the needle may be made to float. If you have serious trouble in making it float, draw it between the fingers or wipe it with an oily cloth. A hair-pin bent slightly near the tips may be used to lower the needle so that neither end shall touch the water before the other. Closely examine the surface of the water. Notice that the needle rests in a little depression or bed, just as it would if the surface of the water was a thin skin or membrane.

Fig. 24.

Experiment 30.—Blow a soap-bubble without detaching it from the pipe or tube. Leave the tube open, and notice that the film contracts, diminishing the size of the bubble, and expelling some of the air from it. The current of air from the interior of the bubble may be made to deflect the flame of a candle.

Experiment 31. — Float two sewing needles on the surface of water about a quarter of an inch apart, and let a drop of alcohol fall upon the water between them. Notice that the needles separate as if they had been supported on a stretched membrane, and the membrane had been cut so that its parts might separate, each carrying its needle with it.

Experiment 32. — Drop a few small pieces of camphor upon the surface of clean, warm water. Notice their peculiar gyratory motions.

Experiment 33. — Moisten a small bit of paper, and stick it to the concave side of a watch crystal near the edge, as an indicator. Dip the part of the convex surface on the side indicated by the paper into alcohol, so that not more than a sixth of the rim shall be wet. Holding the crystal so that the adhering drop of alcohol shall be under the paper bit, float it on the surface of a shallow dish of water a foot or more in diameter. The glass will skim across the surface of the water with the segment that was wet with alcohol astern.

44. Superficial Films. — The molecular forces of a liquid are strikingly manifested at its surface, so that every liquid may be regarded as bounded by a superficial film. This film is physically different from the interior of the liquid mass, and is a seat of energy. Two of the properties of these films are called surface viscosity and surface tension.

45. Surface Viscosity. — *The superficial film of a liquid is, as a rule, exceedingly viscous as compared with the interior mass.* It is comparatively hard to move or break. To this toughness of the superficial film, the floating of a needle or the walking of an insect on water must in part be ascribed, for the depth of the dimple is not sufficient to account for the support afforded to so heavy a body. A solution of soap in water has greater surface viscosity than has pure water, hence its adaptability to the formation of bubbles.

(*a*) The surface viscosity of a solution of gum arabic is sufficient to enable frothing when the solution is shaken, but not enough for the formation of bubbles; that of water is so little that pure water will not froth; and that of alcohol is so eminently feeble that alcohol is often used in pharmacy to mix with superficially viscous liquids for the purpose of checking or preventing frothing. To the same property is attributed the smoothing of a rough sea by pouring oil upon it. The new surface is comparatively rigid, and is not so easily broken into surf.

46. Surface Tension. — Experiments show that *a liquid surface (as the surface that separates water from air, or oil from water) is in a state of tension* similar to that of a membrane stretched equally in all directions. This tension is practically independent of the form of the surface. It depends on the nature and temperature of the liquid, diminishing as the temperature rises. Pure water has a surface tension higher than that of any other substance that is liquid at ordinary temperatures, except mercury; hence the mixture of any other liquid with water lessens the surface tension of the water, as was shown in Experiments 31 and 33.

(*a*) In a liquid film, such as a soap-bubble, "it is possible that no part of the liquid may be so far from the surface as to have the potential and density corresponding to the interior of a liquid mass;" i.e., the film may be *mostly surface*. The exterior and the interior surfaces of the bubble act like two sheets of india-rubber stretched equally in length and breadth. Their tendency to contract forces air from the interior of the bubble, and repays the work performed, or energy expended, in increasing the surfaces when the bubble was blown.

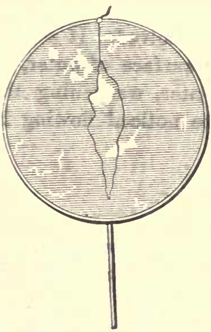


FIG. 25.

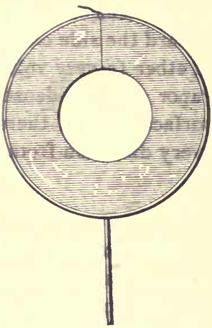


FIG. 26.

Surface tension may be studied under very favorable conditions by using soap or collodion films. If a roughened ring is dipped into a strong solution of Castile soap, to which glycerin has been added, a plane film will be found stretched across it. To such a ring, tie a loop of thread and secure another film, as shown in Fig. 25. With a hot wire, puncture the film inside the thread loop, and the tension of the film will pull the thread outward in all directions, as shown in Fig. 26. Such films may be made to stretch themselves in singularly beautiful forms on wire skeletons of cubes, pyramids, cylinders, etc.

(*b*) The tension of the superficial film tends to reduce the contained liquid to the form that gives the greatest volume with the least area of surface; hence the spherical form of soap-bubbles, air-bubbles in water, raindrops, shot, etc. The various forms assumed by liquid masses under the influence of surface tension are conveniently studied by relieving them of the influence of gravity by floating them in liquids of their own density, and with which they will not mix. Thus, one may make a mixture of alcohol and water of the same density as olive oil. Masses of olive oil placed in such a mixture will neither rise nor sink. If left free, they will assume the globular form. When limiting conditions are imposed upon them, they assume geometrical forms of great interest, all having the smallest superficial area possible under the conditions imposed.

(*c*) When camphor floats on water, solution is likely to take place more rapidly on one side of each piece than on the other. The surface tension becomes weaker where the camphor solution is the stronger; and the lump, being pulled in different directions by unequal surface tensions, moves in the direction of the strongest tension, i.e., toward the side on which the least camphor is dissolved. If a drop of ether (a very volatile liquid) is held near the surface of water, its vapor will condense on the surface of the water, weakening the surface tension there. Surface currents may be noticed flowing in every direction from under the drop of ether.

Capillarity.

Experiment 34. — Partly fill a thin, clean beaker with water, and a similar beaker with clean mercury. Notice that the upper surfaces of the two liquids are level *except at the edges near the glass.* Notice, further, that the water is lifted at the edge by the glass, and that the mercury is depressed.

Experiment 35. — Support a clean glass rod vertically in the water, and notice that the liquid is lifted by the rod, as shown at *a* in

Fig. 27. Remove the rod. *Notice that it is wet.* Wipe the rod dry, and place it similarly in the mercury. Notice that this liquid is depressed by the rod. Remove the rod, and *notice that it was not wetted by the mercury.*

Smear the glass rod with oil, and place it in the water, as before. Notice that the water is depressed thereby. Remove the rod, and notice that it is not wetted by the water. Place a clean strip of tin, lead, or zinc, in the mercury. Notice that the mercury is lifted. Remove the strip, and notice that the strip was wetted by the mercury.

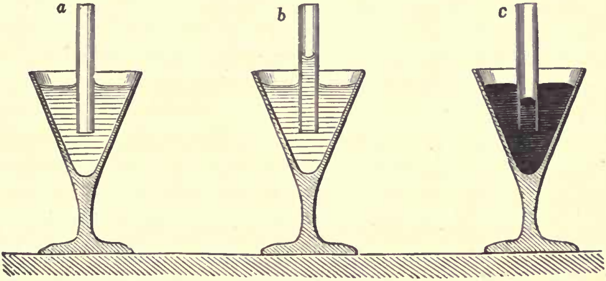


FIG. 27.

47. Capillary Attraction. — The excess of the attraction of one of two fluids, one of which is generally air, for the wall of a vessel with which they have a common line of contact, is called *capillary attraction;* it is proximately accounted for by surface tension. The common surface of the wall and of the more attracted fluid makes the acuter angle with the common surface of the fluids. The truth suggested by our experiments is general: *all liquids that wet the sides of solids placed in them will be lifted, while those that do not will be pushed down.*

Experiment 36. — So place two small, clean glass plates in a shallow dish of clean water, that the angle included between the plates shall be very acute, and that the edges in contact shall be vertical. The vertical edges not in contact may be held apart by a thin strip of wood placed between them, the whole being held together by a rubber band placed horizontally around the plates. Notice the rise of the liquid between the plates, and the outline of the hyperbola traced upon them by the surface of the lifted liquid.

Experiment 37. — Wet the inner surfaces of several clean glass tubes of small and different diameters (1 mm. and less) to remove the adhering air-film. Support the tubes vertically in pure water. Notice that the water rises in the tubes, as shown at *b* in Fig. 27; that, the less the diameter of the tube, the greater the elevation of the water; and that the free surface of the water in the tube is concave.

Remove the tubes, and similarly support them in clean mercury. Notice that the mercury is depressed in the tubes, as shown at *c* in Fig. 27; that, the less the diameter of the tube, the greater the depression; and that the free surface of the mercury in the tubes is convex.

Experiment 38. — Make a tapering capillary tube by drawing out a glass tube that has been heated to redness. Clean the tube thoroughly, and into its larger end introduce a drop of water. Notice the concave form of the two free liquid surfaces, and the motion of the water toward the smaller end of the tube.

Empty the tube, and introduce a drop of mercury into the smaller end. Notice the convex form of the two free liquid surfaces, and the motion of the mercury toward the larger end of the tube.

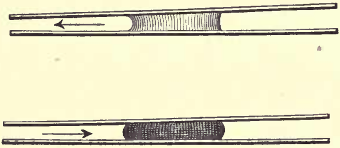


FIG. 28.

48. Capillary Tubes. — The rise of liquids in capillary tubes is explained by the action of cohesion as a force acting at insensible distances, and producing a tension of the superficial film of the liquid. This tension produces an upward pull where the liquid surface is concave, and a downward pressure where the liquid surface is convex. The effect of this tendency is, in the case of water, partly to neutralize the downward pull of gravity. The following facts have been experimentally established: —

(1) *Liquids ascend in tubes when they wet them, i.e., when the liquid surface is concave; and they are depressed when they do not wet them, i.e., when the liquid surface is convex.*

(2) *The elevation or the depression varies inversely as the diameter of the tube.*

(3) *The elevation or the depression decreases as the temperature rises.*

(*a*) The extreme range of the forces that produce capillary action seems to lie between a thousandth and a twenty-thousandth part of a millimeter. Familiar illustrations of capillary action are numerous, such as the action of blotting-paper, sponges, lamp-wicks, etc.

Absorption.

Experiment 39. — Fill a large test-tube with dried ammonia (see Chemistry, § 67) by displacement over mercury. Heat a piece of charcoal to redness, and plunge it into the mercury. When it is cool, slip it under the mouth of the test-tube, and let it rise into the ammonia

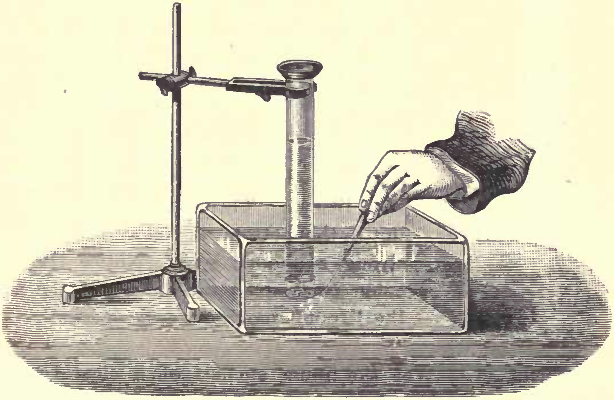


FIG. 29.

atmosphere. Notice that the mercury rises in the tube as if the gas was absorbed by the charcoal.

49. Absorption. — Some solids have the power of taking up or absorbing gases. Thus, a porous body like charcoal has the ability to condense on its surface a large quantity of some gases through the molecular attraction exerted between its surface and the molecules of the gas. Boxwood charcoal is able thus to absorb ninety times its volume of ammonia gas. This absorption is increased by pressure, and decreased by a rise of temperature.

Diffusion.

Experiment 40. — Half fill a jar with water. Through a long-stemmed funnel (Fig. 30) reaching to the bottom of the jar, pour a strong aqueous solution of copper sulphate (blue vitriol). The plane of separation between the colored and the colorless liquids is clearly visible. Allow the jar to stand undisturbed for several weeks, observing it from day to day. The plane of demarcation between the strata becomes blurred, the liquids mix, the solution becomes uniform.

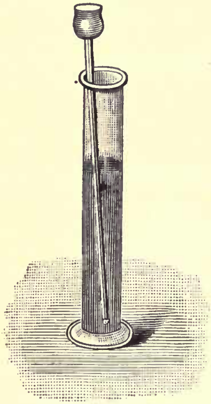


FIG. 30.

Experiment 41. — Partly fill a test-tube or other tall glass vessel with water tinted with blue litmus. Through a funnel-tube reaching to the bottom of the vessel, drop a little strong sulphuric acid. Notice the reddish color (caused by the action of the acid on the litmus) moving *slowly upward.*

Experiment 42. — Wet the inner surface of a clear tumbler or beaker with strong ammonia water, leaving a few drops of the liquid in the bottom. Cover it with a sheet of writing paper. Moisten the inner surface of a like vessel with strong hydrochloric (muriatic) acid. Invert the second vessel over the first, mouth to mouth, so that the contents of the two vessels shall be separated only by the paper. Each vessel is filled with an invisible gas. Remove the paper, and notice that the invisible gases quickly diffuse into each other and form a dense cloud.

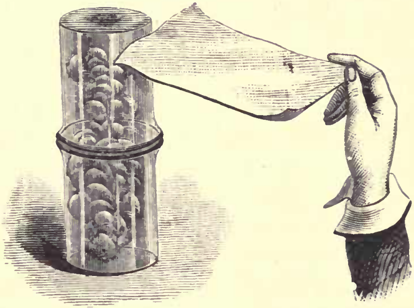


Fig. 31.

If two bottles are filled with different gases, as oxygen and hydrogen, and the bottles connected by a glass tube two or three feet long, with the bottle containing the lighter gas (hydrogen) above the other, the gases still mix by diffusion through the tube, but the process, of course, requires more time.

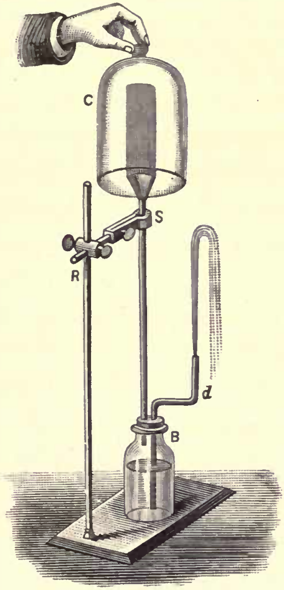


Fig. 32.

Experiment 43. — Cement a small porous battery-cup to a large funnel-tube, mouth to mouth. Pass the end of the funnel-tube snugly through the cork of a bottle, *B*, partly filled with water, and provided with a delivery-tube, *d*, drawn out to a jet, as shown in Fig. 32. When a bell-glass, *C*, containing hydrogen is placed over the porous cup, that gas diffuses inward so much more rapidly than the air can diffuse outward, that an increased pressure is exerted on the surface of the water. If all the joints are tight, water will be thrown from the jet. The experiment may be simplified by allowing the tube to dip into water in an open vessel. Bubbles will then rise through the water.

50. Diffusion. — *The gradual and spontaneous mixing of two fluids that are placed in contact is called diffusion.* It takes place without application of external force, and even in opposition to the force of gravity. It is explained only by the motions and attractions of the molecules of the two fluids.

(*a*) Some liquids, such as mercury and water, do not mix at all when placed in contact. Other liquids, such as chloroform and water, mix only in certain proportions. The chloroform takes up a little water, and the water takes up a little chloroform, but even the two mixed liquids will not mix. Still other liquids, and all gases, mix in all proportions. When two such fluids are placed in contact, diffusion begins of itself, and goes on continuously until the fluids are in a state of uniform mixture.

(*b*) Even with our most powerful microscopes, we cannot follow these motions or detect any currents. The motions are molecular, not molar.

51. Kinetic Theory of Gases. — A perfect gas consists of free, elastic molecules in constant and rapid motion. Each molecule moves in a straight line and with a uniform velocity, until it strikes another molecule or the vessel in which the gas is contained. When these molecules encounter each other, they behave much as billiard balls would do if no energy were lost in their collisions. Each molecule travels a very small distance between one encounter and another, so that it is every now and then changing its velocity both in magnitude and direction. The magnitude of the velocity may be computed, and one direction is just as likely as any other.

(*a*) One result of this motion of free molecules is, that, if in any part of the containing vessel the molecules are more numerous than in a neighboring region, more molecules will pass from the first region into the second than will pass in the opposite direction; i.e., the gas will diffuse itself equally through the vessel. Even when two gases are placed in the same vessel, each gas diffuses itself in the same way that it would if the other gas was not present; but the molecules of the two gases will encounter each other, and every collision will check the process. Thus the interdiffusion of two gases is slower than the equalization of the density of a single gas. It is said that in a given vessel a given stage in the diffusion of liquids requires as many days as a like stage in the diffusion of gases requires seconds.

(*b*) A second result is, that the blows that the molecules thus strike upon the walls of the containing vessel are so numerous, that their total effect is a continuous, constant force or pressure.

Osmose.

Experiment 44. — Tie a piece of wet parchment-paper over the mouth of a large funnel-tube, and pour a saturated solution of copper sulphate into the stem of the tube until the liquid a little more than fills the bulb. Support the funnel-tube in a clear glass vessel of water, adjusting the height of the tube so that the two liquids shall stand at the same level. Watch the apparatus, and soon you may notice that the bluish tint appears in the water in the outer vessel, and that the liquid is rising in the stem of the funnel-tube. Evidently both liquids are passing through the parchment membrane, the greater flow being inward.

52. Osmose. — *The tendency of fluids to pass through porous partitions and to mix is called osmose.*

(*a*) When two solutions differing in strength and composition are separated by a porous diaphragm, they pass with unequal rapidities. The action of the fluid that passes with the greater rapidity is called *endosmosis;* that of the other fluid is called *exosmosis.*

(*b*) Soluble substances have a wide range of diffusibility. Bodies of rapid diffusibility through porous membranes, like common salt and sugar, generally have a crystalline form, and are called *crystalloids.* Bodies of slow diffusibility through porous membranes generally have the amorphous, glue-like character that gives them the name of *colloids.* Colloids are often separated from crystalloids by placing the mixture in a vessel having a parchment-paper bottom, and suspending it in another vessel containing water. This process is called *dialysis.*

CLASSROOM EXERCISES.

1. What is science?

2. What is matter?

3. Define the several divisions of matter.

4. What is the difference between a hypothesis and a theory?

5. (*a*) What is a gram? (*b*) A liter?

6. On what property of matter does compressibility depend?

7. If you thrust a knitting-needle into a mass of dough, is the hole thus made a pore? What is a pore?

8. What is the difference between a fluid and a liquid?

9. Are molecules of water larger or smaller than those of steam? Give a reason for your answer.

10. Are intermolecular spaces greater in water or in steam? Give a reason for your answer.

11. Considered with reference to the three conditions of matter, are cohesion and heat coöperative, or antagonistic?

12. Why can you not blow a soap-bubble with pure water?

13. Tell how, with two pieces of glass and a plate of water, you can produce a hyperbolic curve.

14. Upon what property do most of the characteristic properties of matter depend? Name five universal and three characteristic properties of matter. Define inertia.

15. State definitely how you could separate a solution of loaf-sugar from a solution of gum-arabic with which it was mixed. What is the name of the process employed? After such separation, how could you separate the sugar from its water of solution?

16. Find out how many pounds you weigh. Express that weight in kilograms.

17. State the kinetic theory of gases.

18. Does the number of steps that a man takes in traveling a mile vary directly or inversely with the length of the steps; i.e., does $n \propto l$ or $n \propto \frac{1}{l}$?

19. If No. 27 spring-brass wire breaks under a load of 15 pounds, calculate the breaking strength of No. 25 brass wire. (See table of wire-gauge numbers in the appendix.)

20. If No. 27 spring-brass wire has a breaking strength of 15 pounds, and No. 30 annealed iron wire one of 5 pounds, compute the ratio between the tenacities of spring-brass and of annealed iron.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Draughtsman's triangle; proportional dividers; cardboard; Castile soap; glycerin; glass funnel; rubber tubing; clay pipe; a good balance that weighs to centigrams, and has a centigram "rider;" a silver coin; nitric acid; salt; ammonia water.

1. Press one side of the triangle firmly against the edge of a ruler resting on a sheet of paper. Trace a pencil line along one of the other sides of the triangle. Without allowing the ruler to move, slide the triangle along its edge, and trace another line along the other edge, as before. In like manner draw several more lines, all of which will be parallel.

2. Draw *AB*, a line 7.3 cm. long. Divide it into 5 equal parts. From *A* draw an indefinite straight line, *AX*, making an angle of 30 or 40 degrees with *AB*. Set the dividers to any convenient length, say 2 cm., and, measuring from *A*, lay off on *AX* as many equal distances as the number of parts into which *AB* is to be divided; i.e., 5 such equal distances. Mark these equidistant points on *AX*, in succession, *a*, *b*, *c*, *d*, and *e*. Draw the straight line *Be*. Using the triangle and rule as in Exercise 1, draw lines through *d*, *c*, *b*, and *a*, parallel to *Be*. These parallel lines will divide *AB* into 5 equal parts, as required. With the dividers, test the equality of the several parts of *AB*.

3. Divide a line 7 cm. long into 3 equal parts. Set the index of the proportional dividers at the division on the scale for thirds. Open the dividers until the points of the longer legs rest upon the ends of the given line. The distance between the points of the shorter legs may be laid off in succession from one end of the given line, which will thus be divided into thirds, as required.

4. Carefully heat a tumbler, and half fill it with boiling water. Cover it with cardboard. Invert a second tumbler over the first. Watch the apparatus for a few minutes. If you notice any change in the appearance of the upper tumbler, find out whether it is due to a change in the inner or the outer surface of the glass. What property of the cardboard is thus illustrated?

5. Make of No. 24 iron wire a skeleton of a square pyramid with edges 5 cm. long, and attach a handle, as shown in Fig. 33. Also make two wire rings

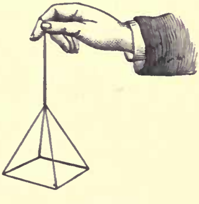


FIG. 33.

6 cm. in diameter and with wire handles. Make a soap-bubble solution as follows: Dissolve 10 g. of Castile soap, in fine shavings, in 400 cu. cm. of warm water, recently boiled, shaking the mixture from time to time. When the soap is dissolved, allow the solution to stand for several hours. Pour off the clear liquid, and to it add 250 cu. cm. of good glycerin, shaking the two thoroughly together.

(*a*) Slip a piece of rubber tubing over the shank of a glass funnel about 10 cm. across the top. Dip the edges of the funnel into the solution, catch a film, and blow as large a bubble as you can.

(*b*) Blow a bubble with a common clay pipe. Detach it from the pipe, and catch it on one of the iron rings. Bring the other ring into contact with the bubble on the other side, and draw the bubble into cylindrical form.

(*c*) Immerse the pyramidal frame into the solution, and try to secure a film on each side, thus forming a hollow, regular pentahedron.

6. Weigh accurately a clean United States silver coin, which is an alloy containing ten per cent. of copper. Place two or three drops of strong nitric acid on the coin, and allow it to stand until the action of the acid on the coin seems to cease. Wash the coin thoroughly in a tumbler of pure water. Measure the water in the graduate (cubic centimeters), and determine the number of drops in a cubic centimeter. Divide this water into two equal parts. To one part, add a few drops of a strong solution of common salt (brine); the milky appearance indicates the presence of a silver compound. To the other part of the water, add a few cubic centimeters of ammonia water; the blue tint indicates the presence of a copper compound. Weigh the coin again, and ascertain how much of it was eaten off. Compute the weight of silver and of copper in each drop of the measured liquid.

CHAPTER II.

MECHANICS: MASS PHYSICS.

I. MOTION AND FORCE.

53. Mechanics *is the branch of physics that treats of forces and their effects.*

(*a*) Mechanics is commonly divided into kinematics and dynamics, and the latter into statics and kinetics. Some of these distinctions seem "artificial, unscientific, and confused," and they will not be rigidly observed in the present book.

54. Motion, Velocity, and Acceleration. — *Motion is change of position.* A body has a motion of translation when any point in it moves along a straight line, and a motion of rotation when any point in it describes a circular arc about some other point in it as a center. *Velocity is rate of motion*, and its magnitude is expressed by saying that it is such a distance in such a time, as ten miles an hour, or one meter a second. Velocity may be uniform or variable. The velocity of a body at any instant is the distance it would pass over in the next unit of time if left wholly free from any outside influence. Thus, the velocity of a falling body at the end of the third second of its fall is the distance it would pass over in the fourth second if it could be freed from the attraction of the earth and the resistance of the air. A variable velocity is accelerated or retarded. *The change of velocity per unit of time (i.e., the rate of change of velocity) is called acceleration.* Acceleration is positive or negative (+ or −) respectively as the velocity is accelerated or retarded. If the acceleration remains constant, the velocity is uniformly accelerated or retarded according to the algebraic sign of the acceleration.

(*a*) A body passing over unit of space in unit of time has unit velocity. The velocity per second multiplied by the number of seconds measures the distance traversed in any given time by a body moving with a uniform velocity. Representing these functions by l for distance, v for velocity per second, and t for time counted in seconds, we have

$$l = vt. \qquad (1)$$

From this fundamental formula we derive algebraically the following: —

$$v = \frac{l}{t}, \text{ and } \quad t = \frac{l}{v}.$$

If two of these values are known, they may be substituted in one of these formulas, and the third value obtained thence. If a body moves at the rate of 50 feet per second for 12 seconds, and the distance traversed is desired, formula (1) is applicable: —

$$l = vt;\ l = 50 \times 12;\ l = 600, \text{ the number of feet.}$$

(*b*) Represent constant acceleration by a. In t seconds, a body starting from rest will have acquired a velocity represented by at.

$$v = at. \qquad (2)$$

This is the formula for a body starting from a state of rest, and having a uniformly accelerated velocity. Half the sum of the initial and the final velocities is the average velocity. In the case now under consideration, the initial velocity was zero, and the final velocity was at; therefore, the average velocity of a body starting from rest, and gaining a velocity uniformly accelerated for t seconds, is $\frac{0 + at}{2}$ or $\frac{1}{2}\, at$.

The average velocity multiplied by the number of time-units equals the distance traversed; therefore, $l = \frac{1}{2}\,at \times t$, or

$$l = \tfrac{1}{2}\,at^2. \qquad (3)$$

$l = \frac{1}{2}\,\frac{v^2}{a}$

From this formula we derive algebraically the following:—

$$a = \frac{2\,l}{t^2}, \text{ and } t = \sqrt{\frac{2\,l}{a}}.$$

Equating the values of t in equations (2) and (3), we may deduce the following:— ?

$$l = \frac{v^2}{2\,a}. \qquad (4)$$

(*c*) To find the distance passed over in any particular unit of time, it may be necessary to subtract the distance traversed in $t - 1$ units, from the distance traversed in t units, the whole time. Representing this distance traversed in a single time-unit by l', we have

$$l' = \tfrac{1}{2}\,at^2 - \tfrac{1}{2}\,a(t-1)^2;$$

therefore, $$l' = \tfrac{1}{2}\,a(2\,t - 1). \qquad (5)$$

— single unit

(*d*) Suppose that a body moving with a uniformly accelerated velocity starts from rest and passes over 7 meters in the first second. How far does it move in the next 3 seconds? If the body moves 7 meters in the first second under the conditions stated, its average velocity for that second is 7 meters, and its velocity at the end of that time is 14 meters. All of this velocity is gained in this single second; hence, $a = 14$. Starting from rest, it moves 4 seconds; hence, $t = 4$. Substituting these values in formula (3),

$$l = \tfrac{1}{2}\,at^2;\ l = \tfrac{1}{2} \times 14 \times 16 = 112,$$

the distance passed over in 4 seconds. From this, subtract the distance passed over in the first second, and we have 105, the number of meters passed over in the second, third, and fourth units of time, as called for. This solution illustrates the method of applying physical formulas to the solution of physical problems.

(*e*) For want of a fixed point for reference, it is impossible to determine absolute motion. All the members of our solar system have very complicated motions, and the most distant stars seem to have a general drift through space. We are therefore obliged to deal exclusively with relative motion. Unless otherwise specified, the motions spoken of in this book are relative to some point on the earth. The point from which motion or its measurement starts is called the *origin.*

55. Laws for Accelerated Motion. — From the foregoing we derive the following laws for the motion of bodies starting from rest, and having a uniformly accelerated velocity: —

(1) *The velocity at the end of any unit of time equals acceleration multiplied by the number of time-units.* (Formula 2.)

(2) *Acceleration equals twice the distance traversed in the first unit of time;* when $t = 1$, formula (3) becomes $l = \frac{1}{2} a$.

(3) *The distance traversed in any single unit of time equals half the acceleration multiplied by one less than twice the number of time-units.* (Formula 5.)

(4) *The total distance traversed in any given time equals half the acceleration multiplied by the square of the number of time-units.* (Formula 3.)

56. Graphic Representation of Motions. — *A straight line may definitely represent uniform motion in a straight line,* the direction of the line indicating the direction of the motion, and the length of the line representing the magnitude of the motion.

(*a*) Any convenient unit of length may be chosen to represent any unit of velocity, but, when the scale has been determined, it should not be changed in any given discussion. For example, two motions, one having an easterly direction and a magnitude of 10 yards per second, and the other having a southerly direction and a magnitude of 15 yards per second, may be fully represented by a horizontal line 2 inches long and a vertical line 3 inches long, the chosen scale of magnitudes being 5:1; i.e., each inch of the length of either line representing a velocity of 5 yards per second.

(*b*) In indicating a line by the letters at its extremities, the order of the letters is that in which the line is to be drawn.

57. The Composition of Motions. — A motion may be the resultant of two or more component motions, as the motion of a person who is walking on the deck of a moving ship. Under such conditions, several distinct cases may arise.

(*a*) When two motions have the same direction, the magnitude of the resultant motion is the sum of the magnitudes of the components, and the direction will be unchanged; e.g., when the brakeman on a railway freight train that is running from Cleveland to Buffalo, at the rate of 20 miles per hour, runs at the rate of 4 miles per hour along the car-tops toward the locomotive, he is really approaching Buffalo at the rate of 24 miles per hour.

(*b*) When two motions have opposite directions, the magnitude of the resultant motion will be the arithmetical difference of the magnitudes of the components, and the direction will be that of the greater component; e.g., when the brakeman above mentioned runs toward the rear of the train, he is approaching Buffalo at the rate of 16 miles per hour.

(*c*) When two component motions have different directions, the finding of the resultant involves the application of a principle known as the *parallelogram of motions.* The lines that properly represent the components are made adjacent sides of a parallelogram. The diagonal drawn from the angle included between these sides represents the resultant in both magnitude and direction. Thus, let AB and AC represent the two component motions. Draw BD and CD to complete the parallelogram. From A, the included angle, draw the diagonal AD. This diagonal will be a complete graphic representation of the resultant. The resultant will be greater than the difference between

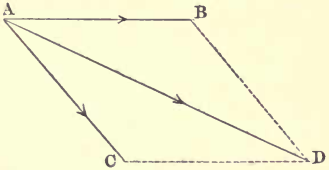


FIG. 34.

the components, and less than their sum. If each component had a velocity of 25 meters per second, which was represented by lines 25 millimeters in length, and *AD* has a length of 45 millimeters, then the result of compounding the two as indicated will be motion with a velocity of 45 meters per second, and in the direction of *AD*.

(*d*) When the included angle, as *BAC*, is a right angle, the resultant line is the hypotenuse of a right-angled triangle, and its magnitude will be the square root of the sum of the squares of the components. When the components are equal, and include an angle of 120 degrees, the resultant divides the parallelogram into two equilateral triangles, and is equal to either of the components. In other cases the magnitude of the resultant may be determined by a careful construction of the parallelogram and a careful measurement of the diagonal, or, more accurately, by the processes of plane trigonometry.

(*e*) When there are three or more components, the resultant of any two may be compounded with a third; the resultant thus obtained may be compounded with a fourth component; etc. The diagonal of the last parallelogram thus constructed will represent the resultant of all the components.

58. The Resolution of Motions. — The converse of the process described in the last paragraph, i.e., the finding of two or more motions that may be substituted as an equivalent for a given motion, is called the resolution of motions. It most frequently consists in finding the sides of a parallelogram the diagonal of which represents the given motion.

(*a*) It is evident that, for a given diagonal, an infinite number of parallelograms may be constructed.

(*b*) When the direction of the two components or the magnitude of the two components is prescribed, or the direction and the magnitude of one of the components are prescribed, the problem becomes determinate.

59. Momentum. — So far, motion has been considered with reference to its speed and direction. But the result of the action of a force upon a body depends upon the mass of the body as well as upon its velocity. If m represents the mass of the body, and v its velocity, the product, mv, will represent its quantity of motion. *This product is called momentum.*

(*a*) The momentum of a body having a mass of 20 pounds and a velocity of 15 feet is twice as great as that of a body having a mass of 5 pounds and a velocity of 30 feet.

Note.—The expression "mass into velocity," or "mass multiplied by velocity," need not disturb the pupil's ideas. The multiplier must be abstract, and the product must be of the same kind as the multiplicand. Still, by a sort of ellipsis, the abbreviated phrase means the same as the longer one, and is more commonly used.

CLASSROOM EXERCISES.

1. Find the momentum of a 500-pound ball moving 500 feet a second.

2. By falling a certain time, a 200-pound ball has acquired a velocity of 321.6 feet. What is its momentum?

3. A boat that is moving at the rate of 5 miles an hour weighs 4 tons; another that is moving at the rate of 10 miles an hour weighs 2 tons. How do their momenta compare?

4. What kind of motion is caused by a single, constant force? Illustrate your answer.

5. A stone weighing 12 ounces is thrown with a velocity of 1,320 feet per minute. An ounce ball is shot with a velocity of 15 miles per minute. Find the ratio between their momenta.

6. An iceberg of 50,000 tons moves with a velocity of 2 miles an hour. An avalanche of 10,000 tons of snow descends with a velocity of 10 miles an hour. Which has the greater momentum?

7. Two bodies weighing respectively 25 and 40 pounds have equal momenta. The first has a velocity of 60 feet a second. What is the velocity of the other?

8. Two balls have equal momenta. The first weighs 100 Kg., and moves with a velocity of 20 m. a second. The other moves with a velocity of 500 m. a second. What is its weight?

9. Three men start from Cleveland: the first goes 10 miles eastward; the second goes 15 miles southward; the third goes 18 miles

southwesterly. Represent these journeys by lines, using a scale of 1 inch to 3 miles, and indicating directions by arrowheads.

10. A railway train moves at the rate of 40 miles an hour. Express its velocity per second in feet.

11. If the mean distance of the earth from the sun is 92,390,000 miles, and it requires 16 minutes 36 seconds for a ray of light to pass over the diameter of the earth's orbit, what is the velocity of light, expressed in miles per second?

12. A body at rest receives a constant acceleration of 20 feet per second. How far will it move in 6 seconds, and what will be its velocity at the end of that time?

13. Draw a circle and ascertain its area. (See appendix.)

14. Find the volume of a sphere that will just pass through the circle that you draw.

15. If the breaking weight of No. 27 spring-brass wire is 15 lbs., determine the diameter of a wire of the same material and quality that can just carry a load of 50 Kg.

LABORATORY EXERCISES.

Additional Apparatus, etc. — A protractor; a metric rule; three balls; an elastic cord.

1. Draw a circle 3 inches in diameter, and divide its circumference into arcs of 10 degrees each.

2. Draw a triangle with a base line 7.27 inches long, and with angles at the extremities of this line measuring 23 and 32 degrees respectively. Measure the altitude and compute the area of the triangle.

3. Two forces, capable of giving a certain body velocities of 35 m. and of 56 m. respectively, act on that body at an angle of 25 degrees with each other. Determine the magnitude of the resultant velocity, and its direction relative to that of the smaller component. (In your drawings represent the direction of each velocity by an arrowhead.)

4. Resolve a velocity of 50 m. into two components that make with its direction angles of 20 and 45 degrees respectively. Use a scale of 1 : 500. Determine the magnitude of each component.

5. Resolve a velocity of 20 m. into two components with magnitudes of 13 and 17 m. respectively. Use a scale of 1 : 200. Determine the angle that each component makes with the given velocity.

6. Resolve a velocity of 35 m. into two components, one of which

shall have a magnitude of 24.5 m., and make an angle of 63 degrees with the given velocity. Use a scale of 1:350. Determine the magnitude of the other component and the angle included between the two components.

7. Draw two lines bisecting each other at right angles, and mark the ends of the lines to represent the cardinal points of the compass, as in a map. From the intersection of the two lines draw another line to represent the velocity of a United States cruiser steaming south of southeast at the rate of 19 miles an hour. Determine the rate of the southerly and the easterly motions of the ship. Record on your diagram the scale used.

8. Carefully weigh a meter of No. 30 copper wire, such as was used in Exercise 10, p. 35, and from the data ascertained in that exercise calculate the length of a piece of such wire that would just break under its own weight when suspended by one end.

9. From the table given in the appendix, ascertain the diameter of No. 30 wire. Calculate the breaking strength of a copper rod 1 sq. cm. in cross-section, the quality of the copper being the same as that of the wire used in Exercise 8.

10. On a level table, connect two balls of equal mass by an elastic cord or band. Separate the balls until the cord has been stretched to about double its ordinary length, and mark the positions of the balls. Release the balls simultaneously, and mark the place where they meet. If they meet midway between their positions as first marked, show that the given force has produced equal momenta in the two balls.

11. Repeat Exercise 10, using balls one of which weighs twice as much as the other. If one ball moves twice as far as the other, show that $l = 2l'$; $v = 2v'$; $mv = 2mv' = m'v'$.

60. Laws of Motion. — The following propositions, known as Newton's Laws of Motion, are so important, and so famous in the history of physical science, that they ought to be remembered by every student: —

(1) *Every body continues in its state of rest or of uniform motion in a straight line unless compelled to change that state by an external force.*

(2) *Every change of motion (momentum) is in the direction of the force impressed, and is proportionate to it.*

(3) *Action and reaction are equal and opposite in direction.*

61. First Law of Motion. — The first law of motion results directly from inertia, and suggests the following definition: *Force is that which changes or tends to change a body's state of rest or motion.*

(*a*) It is impossible to furnish perfect examples of this law, because all things within our reach or observation are acted upon by some external force.

62. Second Law of Motion. — The second law of motion is sometimes given as follows: *A given force will produce the same effect, whether the body on which it acts is in motion or at rest; whether it is acted on by that force alone or by others at the same time.* In the law as given by Newton (§ 60), the word "motion" is doubtless used in the sense of "momentum."

(*a*) The law as given by Newton points out that forces may be compared by comparing the momenta that they produce in equal times. Representing force by f, mass by m, and acceleration by a, we have $f = ma$.

If the forces act on equal masses, the changes of momenta will vary with the changes of velocity, i.e., as the acceleration; hence, the acceleration that a force generates may be used to measure that force (see § 106).

63. Elements of a Force. — In treating of forces, we have to consider three things: —

(1) *The point of application*, or the point at which the force acts.

(2) *The direction*, or the right line along which it tends to move the point of application.

(3) *The magnitude*, or value when compared with a given standard, or the relative rate at which it is able to produce motion in a body free to move.

64. Measurement of Forces. — It frequently is desirable to compare the magnitudes of two or more forces. That they may be compared, they must be measured; that they may be measured, a standard of measure or unit of force is necessary. Units of force are of two kinds.

65. The Gravity Unit. — A force may be measured by comparing it with the weight of some known quantity or mass of matter. Although the force of gravity varies at different places, this is a very simple and convenient way, and often answers every purpose. *The gravity unit of force is the weight of any standard unit of mass, as the kilogram or pound.*

(*a*) As the force of gravity exerted upon a given mass is variable, it will not suffice, when scientific accuracy is required, to speak of a force of 10 pounds, but we may speak of a force of 10 pounds at the sea-level at New York City.

66. The Absolute Unit. — *The absolute unit of force is the force that, acting for unit of time upon unit of mass, will produce unit of acceleration (i.e., change of velocity).*

The foot-pound-second (F.P.S.) unit of force is the force that, applied to one pound of matter for one second, will produce an acceleration of one foot per second. It is called a *poundal.*

The centimeter-gram-second (C.G.S.) unit of force is the force that, acting for one second upon a mass of one gram, produces an acceleration of one centimeter per second. It is called a *dyne.*

(*a*) Absolute units are invariable in value. Gravity units may easily be changed to absolute units. At New York the force of gravity acting upon one pound of matter left free to fall will produce an acceleration of 32.16 feet per second for every second that it acts; consequently, at New York a force of one pound equals 32.16 poundals. Since the same force produces an acceleration of 980 centimeters per second, it appears that the weight of a gram at New York corresponds to a force of 980 dynes.

(*b*) A force is measured in poundals or dynes by multiplying the number of units of mass moved by the number representing the acceleration produced, only such units being used as are indicated by the initials F.P.S. or C.G.S. respectively. The acceleration may be determined by dividing the total velocity that the force has produced by the number of seconds that the force has acted.

(*c*) The simplest way of measuring a force is to use a dynamometer, of which the spring-balance (Fig. 35) is a familiar example. The dynamometer may be graduated in pounds, grams, poundals, or dynes.

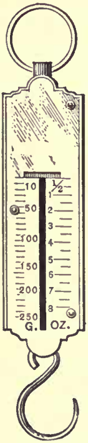


Fig. 35.

CLASSROOM EXERCISES.

1. A railway train 120 yards long moves at the rate of 30 miles an hour. How long will it take to pass completely over a bridge 120 feet long?

2. At the sea-level at New York a force of 25 pounds equals how many poundals?

3. Under the same conditions, a force of 5 Kg. equals how many dynes?

4. A poundal equals how many dynes?

5. Compare the momentum of a 64-pound cannon ball moving with a velocity of 1,300 feet per second, with that of an ounce bullet moving with a velocity of 400 yards per second.

6. If a beam 3 m. long, 10 cm. wide, and 5 cm. thick, is bent 0.5 cm. by a certain load, how much would a similar beam 4 m. long be depressed by the same load?

7. What property of matter is illustrated in the removal of dust from a carpet by beating?

67. Graphic Representation of Forces.— *Forces may be represented by lines*, the point of application determining one end of the line, the direction of the force determining the direction of the line, and the magnitude of the force determining the length of the line.

(*a*) It will be noticed that these three elements of a force (§ 63) are the ones that define a line. By drawing the line as above indicated, the units of force being numerically equal to the units of length, we have a complete graphic representation of the given force. The unit of length adopted in any such representation may be determined by convenience; but, the scale once determined, it must be adhered to throughout the problem. Thus, the diagram represents two forces applied at the point *B*. These forces act at right angles to each other. The arrowheads indicate that the forces represented act from *B* toward *A* and *C* respectively. The force that acts in the direction, *BA*, being 20 lbs., and the force acting in the direction, *BC*, being 40 lbs., the line, *BA*, must be one-half as long as *BC*. The scale adopted being 1 mm. to the pound, the smaller force will be represented by a line 2 cm. long, and the greater force by a line 4 cm. long.

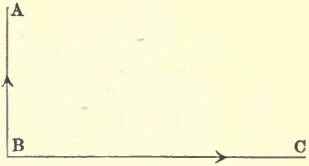


FIG. 36.

68. Resultant Motion.—*Motion produced by the joint action of two or more forces is called resultant motion.*

The single force that will produce an effect like that of the component forces acting together is called the *resultant.* The single force that, acting with the component forces, will keep the body at rest is called the *equilibrant.* The resultant and the equilibrant of any set of component forces are equal in magnitude, and opposite in direction.

The point of application, direction, and magnitude of each of the component forces being given, the direction and magnitude of the resultant force are found by a method known as the composition of forces.

Experiment 45. — Suspend two similar spring-balances, *A* and *B*, from any convenient support, as shown in Fig. 37. From the wooden rod carried by their hooks, suspend a known weight. Be sure that the dynamometers hang vertical, and therefore parallel. Record the readings of the dynamometers. Carefully measure the distances, *CD* and *DE*, and record them. If the dynamometers are accurate, the work has been carefully done, and the weight of the rod is inconsiderable, the results should show that

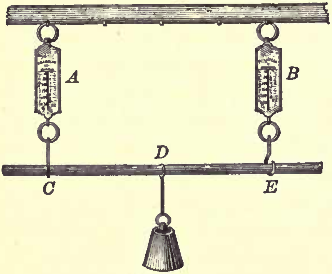


FIG. 37.

$$W = A + B, \text{ and that } \frac{A}{B} = \frac{DE}{CD}.$$

If the weight of the rod is considerable, place the rod in the hooks, and notice the readings of the dynamometers. Then hang the weight from the rod, and represent the increase in the readings by *A* and *B*. The result should be as given above.

69. Composition of Forces. — Under composition of forces there are several cases, of which the more important are the following: —

(1) *When the component forces act in the same direction and along the same line. The magnitude of the resultant is then the sum of the given forces.* Example: Rowing a boat down-stream.

(2) *When the component forces act in opposite directions and along the same line. The magnitude of the resultant is then the difference between the given forces.* Motion will be produced in the direction of the greater force. Example: Rowing a boat up-stream.

(3) *The resultant of two forces that act in the same direction along parallel lines has a magnitude equal to the sum of the magnitudes of the components, and its point of application divides the line joining the points of application of the components inversely as the magnitudes of said components.* This principle is illustrated by Experiment 45.

(4) *When two equal parallel forces act at different points on a body and in opposite directions, the arrangement constitutes what is called a couple. It produces rotary motion, and the components can have no resultant.*

(*a*) If a magnetic needle is placed in an east and west position, the attraction of the north magnetic pole of the earth attracts one end of the needle and repels the other with equal parallel forces, the effect of which is to turn the needle upon its pivot until it is in a north and south position. The attraction and the repulsion constitute a couple.

(5) *When the component forces have a given point of application (i.e., when they are "concurring forces") and act at an angle with each other, as when a boat is rowed across a stream, the resultant may be ascertained by the "parallelogram of forces."*

70. Parallelogram of Forces. — In the diagram, let *AB* and *AC* represent two forces acting upon the point, *A*. Draw the two dotted lines to complete the parallelogram. From *A*, the point of application, draw the diagonal, *AD*. *This diagonal will be a complete graphic representation of the resultant.* If two forces, such as those represented in the diagram, act simultane-

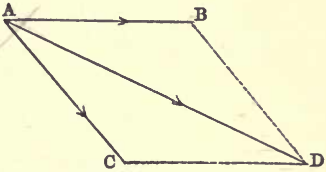


FIG. 38.

ously upon a body at *A*, that body will move over the path represented by *AD*, and come to rest at *D*. The process is very similar to the composition of motions mentioned in § 57.

(*a*) When the two component forces act at right angles to each other, the determination of the numerical value of the resultant is like that of finding the length of the hypotenuse of a right-angled triangle; it is the square root of the sum of the squares of the two components. (See § 57, *d*.)

Experiment 46. — The principle of the parallelogram of forces may be verified as follows: *H* and *K* represent two pulleys that work with very little friction. Fix them to the frame of the blackboard. Knot together three silk cords; pass two of them over the pulleys; suspend three weights, *P*, *Q*, and *R*, as shown in the figure. *R* must be less than the sum of *P* and *Q*. When the apparatus has come to rest, take the points, *A* and *B*, so that $AO : BO :: P : Q$. Complete the parallelogram, *AOBD*, by drawing lines upon the board. Draw the diagonal, *OD*. It will be found by measurement that $AO : OD :: P : R$; or that $BO : OD :: Q : R$. Either equality of ratios affords the verification sought.

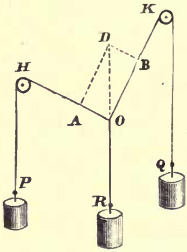


FIG. 39.

Experiment 47. — Modify the experiment by supporting two spring-balances, *A* and *B*, from *P* and *S*, two nails in the frame of the blackboard. Hook them with a third dynamometer, *C*, into a small ring, *Z*, as shown in Fig. 40. Pull steadily on *I* in some downward direction. Mark on the board the centers of the rings, *Z* and *I*, and record the readings of the three dynamometers. Remove the apparatus, and through the points indicated draw on the board the lines, *ZP*, *ZS*, and *ZI*. Using any convenient scale, lay off the lines, *ZE*, *ZA*, and *ZI*, proportional to the readings of the respective dynamometers. Complete the parallelogram, *ZETA*. Draw the diagonal, *ZT*, measure its length, and determine the magnitude that it represents according to the scale adopted. If the work has been accurately done, *ZI* and *ZT* will be equal in value, and form a straight line. *ZT* is the resultant, and ZI is the equilibrant, of the components, ZE and ZA. Place the apparatus horizontal and repeat the work.

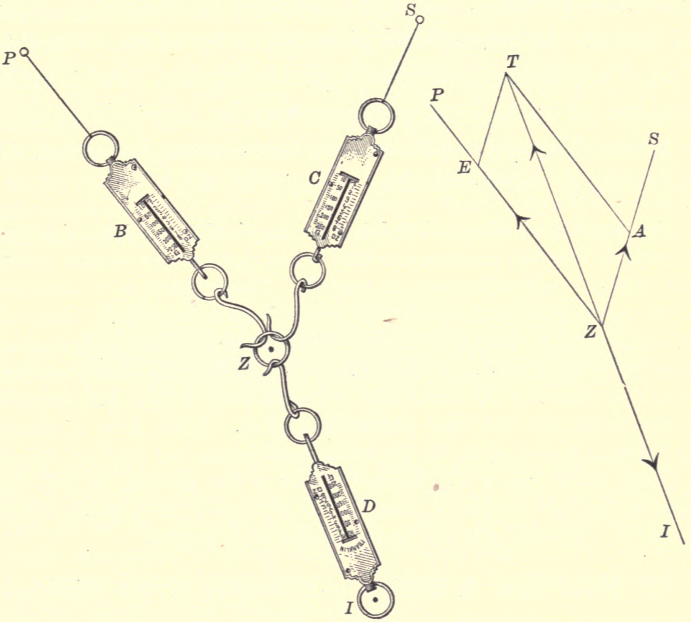


FIG. 40.

71. Composition of More than Two Forces.—If more than two forces concur, the resultant of any two may be combined with a third, their resultant with a fourth, and so on. The last diagonal will represent the resultant of the given forces. As is indicated by Fig. 41, it is not necessary that all of the forces act in the same plane.

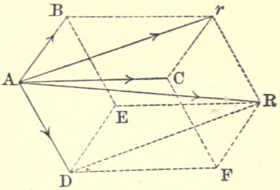


FIG. 41.

72. Resolution of Forces. — *The operation of finding the components to which a given force is equivalent is called the resolution of forces.* It is the converse of the composition of forces. Represent the given force by a line. On this line as a diagonal, construct a parallelogram. An infinite number of such parallelograms may be constructed with a given diagonal. Other conditions must be added to make the problem definite. (See § 58, *b*.)

(*a*) By way of illustration, let it be required to resolve a force of 20 pounds into two components that act at right angles to each other, one of them to be a force of 12 pounds. The problem is to construct a rectangle one side of which shall measure 12 units, and the diagonal of which shall measure 20 units. Draw a vertical and a horizontal line intersecting at *A*. From *A*, measure off 12 units on the vertical line, thus securing the point, *B*. From *B*, draw a line, *BX*, parallel to the horizontal line that passes through *A*. From *A* as a center, and with a radius equal to 20 units, describe an arc cutting *BX* at a point, which mark *C*. From *C*, draw a line parallel to *AB*, and intersecting the horizontal line drawn through *A* at a point, which mark *D*. *AB* and *AD* will be the components sought.

73. Third Law of Motion. —Examples of the third law of motion are very common. When we strike an egg upon the table, the reaction of the table breaks the egg. The action of the egg may make a dent in the table. The reaction of the air, when struck by the wings of a bird, supports the bird if the action is greater than the weight. The oarsman urges the water backward with the same force that he urges his boat forward. In springing from a boat to the shore, muscular action tends to drive the boat adrift; the reaction, to put the passenger ashore. These illustrations suggest the idea that every action of a force develops another force opposite in direction, so that two forces, instead of one, are apparently in action.

Reaction.

Experiment 48.—Make a railway of two wooden strips $1\frac{1}{2}$ inches by $\frac{1}{4}$ inch, and about 6 feet long, fastened together by three or five crosspieces, as shown in Fig. 42. The distance between the rails should be about an inch. Place the railway on a board, and fasten down the middle crosspiece with a screw. Spring up the ends, and support them by books or wooden blocks. At the toy shop, get several large glass marbles, or other elastic balls, and place them on the middle of the railway. Bring one ball to the highest point of the track, and let it roll down against the others. Ball No. 1 gives its motion to No. 2, and comes to rest; No. 2 gives it to No. 3, and in turn comes to rest. The energy is thus passed through the line to No. 7, which is driven some distance on the up grade, as to the position shown by the dotted line at 8. From 8, this ball rolls down grade, and passes its energy along the line, forcing No. 1 up the grade to a lesser distance than before. The balls will repeat their motions several times, until they are finally brought to rest by friction, etc.

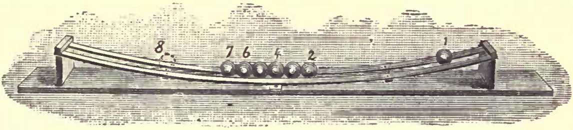


FIG. 42.

Experiment 49. — Repeat Experiment 48 after replacing the middle ball by a lead ball of the same size.

Experiment 50. — This action of ivory or glass balls is due to the fact that they are elastic, and are flattened by the blow. To show that this is so, smear a flat stone or iron plate with paint. Before the paint becomes dry, place one of the glass balls on the smeared surface, and notice the size of the round spot thus made. Then drop the ball from a height of several inches, and notice that the spot is larger than before.

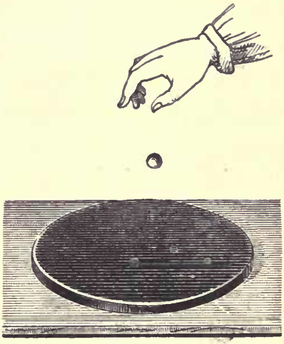


FIG. 43.

74. Elasticity and Reaction.—The effects of action and reaction are modified largely by elasticity, but never so as to destroy their equality.

(*a*) From any convenient support, as the door-frame, hang, by strings of equal length, two clay balls of equal mass, or two such bags of shot or of sand, so that they will just touch each other. If one is drawn aside and let fall against the other, both will move forward, but only half as far as the first would had it met no resistance. The gain of momentum by the second is due to the action of the first. It is equal to the loss of momentum by the first, which loss is due to the reaction of the second.

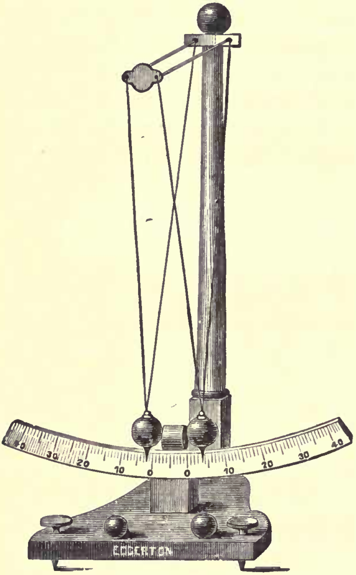


FIG. 44.

(*b*) If two glass or ivory balls, which are elastic, are similarly placed, and the experiment repeated, it will be found that the first ball will give the whole of its motion to the second, and remain still after striking, while the second will swing as far as the first would have done if it had met no resistance. In this case, as in the former, it will be seen that the first ball loses just as much momentum as the second gains. These balls may be suspended by gluing a narrow strip of leather to each, leaving a little loop at the middle of the strip for the fastening of the string.

75. Reflected Motion *is the motion produced in a body by the reaction between it and another body against which it strikes.* A ball rebounding from the wall of a house or from the cushion of a billiard table is an example of reflected motion.

76. Law of Reflected Motion. — The angle, *ABD*, included between the direction of the moving body before it strikes the reflecting surface, and a perpendicular to that surface drawn from the point of contact, is called the angle of incidence. The angle between the perpendicular and the direction of the moving body after striking is called the angle of reflection. *When the bodies are perfectly elastic, the angle of incidence is equal to the angle of reflection, and lies in the same plane. When the elasticity of the bodies is imperfect, the angle of reflection is greater than the angle of incidence.* If a glass or ivory ball is shot from *A* against an elastic surface at *B*, the center of the semicircle, it will be reflected back to *C*, making the angles, *ABD* and *CBD*, equal. If the ball or the body at *B* is not perfectly elastic (e.g., if a lead ball is used), the path of the ball after reflection will make with the normal line, *BD*, an angle greater than *ABD*.

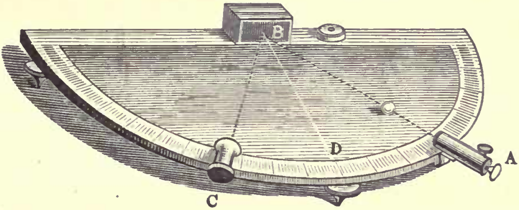


FIG. 45.

77. Curvilinear Motion. — When a ball at the end of a string is whirled around the hand, there is a consciousness of a pull on the string. A moment's observation and reflection impress one with the fact that the ball is deflected by the tension of the string from the rectilinear path which it tends to follow in accordance with the first law of motion, and is thus constrained to move in a curved line. There are evidently two forces involved in the production of such a motion, — a tangential component, which sets the ball in motion; and a centripetal component, as exhibited in the tension of the string.

The resistance offered by the body to its deflection from a rectilinear path is due to its inertia, and is commonly called by the ill-chosen name "centrifugal force." From this point of view, *centrifugal force may be defined as the reaction of a moving body against the force that makes it move in a curved path.*

(*a*) Examples and effects of this so-called centrifugal force may be suggested as follows: the sling, wagon turning a corner, railway curves, water flying from a revolving grindstone, broken fly-wheels, erosion of river-beds, a pail of water whirled in a vertical circle, the inward leaning of the circus horse and rider, the centrifugal drying apparatus of the laundry or the sugar refinery, difference between polar and equatorial weights of a given mass, elongation of the equatorial diameter of the earth, etc.

(*b*) "The student cannot be too early warned of the dangerous error into which so many have fallen who have supposed that a mass has a tendency to fly outwards from a center about which it is revolving, and therefore exerts a centrifugal force which requires to be balanced by a centripetal force." — *Tait.*

78. Measurement of Centrifugal Forces. — The laws of centrifugal force may be studied or illustrated by means of the whirling table and accompanying apparatus, some of which is represented in Fig. 46. It may be shown that

$$\textit{Centrifugal Force} = \frac{mv^2}{r},$$

in which m represents the mass of a body moving in a circular path; v, its velocity; and r, the radius. Of course, the numerical result will represent absolute units of force.

This formula justifies the following laws of centrifugal force: —

(1) *The force varies directly as the mass.*

(2) *The force varies directly as the square of the velocity (radius being constant).*

(3) *The force varies inversely as the radius (velocity being constant).*

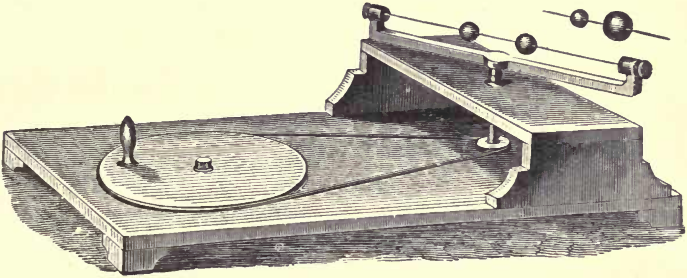


FIG. 46.

CLASSROOM EXERCISES.

1. Represent graphically the resultant of two forces, 100 and 150 pounds respectively, exerted by two men pulling a weight in the same direction. Determine its value.

2. In similar manner, represent the resultant of the same forces when the men pull in opposite directions. Determine its value.

3. Suppose an attempt is made to row a boat at the rate of 4 miles an hour directly across a stream flowing at the rate of 3 miles an hour. Determine the direction and velocity of the boat.

4. A flag is drawn steadily downward 64 feet from the masthead of a moving ship. During the same time, the ship moves forward 24 feet. Represent the direction and length of the actual path of the flag.

5. A sailor climbs a mast at the rate of 3 feet a second. The ship is sailing at the rate of 12 feet a second. Over what space does he actually move during 20 seconds?

6. A foot-ball simultaneously receives three horizontal blows, — one from the north, having a force of 10 pounds; one from the east, having

a force of 15 pounds; and one from the southeast, having a force of 25 pounds. Determine the direction of its motion.

7. Why does a cannon recoil or a shot-gun "kick" when fired? Why does not the velocity of the gun equal the velocity of the ball?

8. If the river mentioned in Exercise 3 is a mile wide, how far did the boat move, and how much longer did it take to cross than if the water had been still?

9. A plank 12 feet long has one end on the floor, and the other end raised 6 feet. A 50-pound cask is being rolled up the plank. Resolve the gravity of the cask into two components,—one perpendicular to the plank, to indicate the plank's upward pressure; and one parallel to the plank, to indicate the muscular force needed to hold the cask in place. Find the magnitude of this needed muscular force.

10. To how many poundals is a force of 60 pounds equal?

11. To how many dynes is a force of 60 kilograms equal?

12. What is meant by a force of 10 pounds? To how many poundals is it equal?

13. A force of 1,000 dynes acts on a certain mass for one second, and gives it a velocity of 20 cm. per second. What is the mass in grams? *Ans.* 50.

14. A constant force, acting on a mass of 12 g. for one second, gives it a velocity of 6 cm. per second. Find the force in dynes.

15. A force of 490 dynes acts on a mass of 70 g. for one second. What velocity will be produced? *Ans.* 7 cm. per second.

16. Show how the principle of resolution of forces and the principle of the couple may be applied to explain the action of a windmill.

17. Determine, in absolute units, the centrifugal force of a 10-pound body moving with a velocity of 20 feet per second, in a circle of 5 feet radius. *Ans.* 800 poundals.

18. Without drawing a diagram, find the numerical value of the resultant of two concurrent forces of 5 Kg. and 12 Kg. respectively, acting at right angles to each other.

19. Resolve a force of 30 pounds into two component forces acting at a right angle, one of them being a force of 18 pounds.

20. Determine the point of application of the resultant of two forces of 8 and 11 pounds respectively, acting in the same direction along parallel lines 57 inches apart.

21. A railway train is moving southeastward at the rate of 30 miles

an hour. (*a*) How fast is it moving eastward? (*b*) How fast is it moving southward?

22. A mass of 10 Kg. moves with a velocity of 20 m. a second in a circular path with a radius of 10 m. What is its centrifugal force?

23. (*a*) What will be the effect upon the motion of a boat if air is blown directly against the sail from a huge bellows placed astern? (*b*) What will be the effect if the air is drawn in by a valve under the bellows, and forced out directly backward?

LABORATORY EXERCISES.

Additional Apparatus, etc. — Two small pulleys that work with very little friction; two good spring-balances; a fish-line or other stout, flexible cord; a sheet of paper; four thumb-tacks; boards; hammer and nails; brace and bits; hand-saw.

1. Bore a small hole through a meter stick at the middle of its length and near one edge. Through this hole, pass a wire nail of such size that it may carry a good load and yet allow the stick to turn freely upon it as an axis. Make a stout wire clevis that shall come down on each side of the stick and support each end of the nail without rubbing the sides of the stick. Through the upper part of the clevis, pass the hook of a spring-balance, and support the dynamometer so that the stick may hang at an elevation convenient for observation. As the stick hangs in the clevis with the hole near its upper edge, load one end of it with putty until the stick hangs horizontal. Note the reading of the dynamometer. Over each end of the stick, slip a loose single loop of silk thread, the weight of which may be ignored; from them hang unequal but known weights. Shift the position of the weights until the loaded apparatus hangs in equilibrium. Note the distance of each loop from the middle of the stick, and the reading of the dynamometer. Determine the ratio between the sum of the two suspended weights and the difference between the two observed readings of the dynamometer. Using the scale of 1 to 10, draw a horizontal line to represent the meter stick, and mark its middle point *F*. On this line, represent the points of application of the two suspended weights, and mark them *P* and *W*. Using any convenient scale, draw lines from *P* and *W*, and place arrowheads to represent the forces acting at those points. In similar manner, draw and mark a line to represent the equilibrant of those forces. Add a reference to the paragraph of this book that treats of the subject chiefly illustrated by this exercise.

2. Two forces pulling toward the east act on two points of a rigid body 5 feet asunder. Represent this body by a vertical line, on which locate the two points of application, using the scale of an inch to the foot. One of these forces has a magnitude of 7 pounds, and the other has one of 11 pounds. Using the scale of a half-inch to the pound, draw graphic representations of these forces, their resultant and their equilibrant.

3. Arrange apparatus as shown in Fig. 39. Make *P* equal 5 pounds, and *Q* equal 7 pounds. Add unknown weights at *R* until the cords, *OH* and *OK*, include an angle that is acute or not very obtuse. Back of the cords around *O*, support a board of such thickness that the face of the board shall just touch the cords. With thumb-tacks, fasten a sheet of paper to this board; and on the paper, with rule and pencil, draw lines from *O* in the direction of *H*, *K*, and *R*. Using the scale of a half-inch to the pound, represent forces acting from *O* along the lines, *OH* and *OK*, with magnitudes of 5 and 7 pounds respectively. Complete the parallelogram, and draw the diagonal, *OD*. Lay off from *O*, and on the line, *OR*, a distance equal to *OD*. Which line on your paper represents the gravity of the unknown weight? From the length of this line compute the mass of *R*. Place *R* on the balance or hang it from the spring-balance, and thus determine its mass by weighing. Compare these two results. Does *OD* form a continuation of the straight line, *RO?* If it does not, your work has been ill done. If *ROD* is a straight line, what does *OD* represent?

4. Provide two strings, each about a foot long. Tie one end of one string to the ring of a spring-balance. Tie one end of the other string to the ring of another spring-balance. By these strings, support the dynamometers from nails driven at points about one corner of the blackboard, and corresponding more or less closely to *H* and *K* in Fig. 39. Cut off two pieces of cord, each about 2 feet long. Tie a small loop that will not slip at each end of each cord. Pass the first cord through the loop at one end of the second cord. Pass the loops at the ends of the first cord over the hooks of the two dynamometers. To the free end of the second cord, attach a weight of 10 pounds. The loop at the upper end of the cord that carries the weight will slip along the length of the cord that joins the dynamometers until the apparatus is in equilibrium. Place the board carrying a fresh sheet of paper as was done in the last exercise. On the paper, mark three intersecting lines as indicated by the strings between the dynamometers and the load, and mark their common point of

intersection *Z*, as in Fig. 40. Prolong the vertical line upward 5 inches to a point, which mark *T*. Construct the parallelogram of which *ZT* is a diagonal, and the two sides of which, meeting at *Z*, coincide with the lines already drawn upon the paper. Mark the left-hand corner of the parallelogram *E*, and the opposite corner *A*. What does *ZT* represent? What is the scale adopted? On that scale determine the magnitudes represented by the lengths of *ZE* and *ZA*. Compare these magnitudes with the readings of the dynamometers, and determine the magnitude of the error of your work. Record a reference to the paragraph in the text-book illustrated and approximately verified by this exercise.

5. Change the point of support of one of the dynamometers so as to increase the angle included between the component forces. Repeat the experiment.

6. Change the point of support of one of the dynamometers so as to make the angle between the components very acute. Repeat the experiment. Compare the diagram made with those made in Exercises 4 and 5.

7. Without using a protractor, lay off an angle of 60°.

8. Two forces, of 7 and 16 units respectively, have a resultant of 21 units. By construction and measurement, determine the angle between the two components.

II. WORK AND ENERGY.

79. Work. — In physical science, the word "work" *signifies the overcoming of resistance of any kind.* Work implies a change of position, and is independent of the time taken to do it. When a force causes motion through space, i.e., when it moves a body, it is said to do work on that body. When the expansive force of steam presses against the piston of an engine and overcomes the resistance, i.e., when it moves the piston, it does work.

(*a*) A man who is supporting (not lifting) a heavy weight may be putting forth great effort, but he is not doing work in the scientific sense of that expression.

80. Units of Work. — It is often necessary to represent work numerically; hence the necessity for a unit of measurement. Four work-units are in use; viz., the foot-pound and the kilogrammeter (which are gravitation units), and the foot-poundal and the erg (which are absolute units).

(1) *The foot-pound is the amount of work required to raise one pound one foot high against the force of gravity.* It is the unit in common use among English-speaking engineers. See Appendix 1.

(2) *The kilogrammeter is the amount of work required to raise one kilogram one meter high against the force of gravity.*

(3) *The foot-poundal is the amount of work done by a force of one poundal in producing a displacement of one foot.* It is numerically equal to the foot-pound multiplied by the acceleration of gravity expressed in feet per second; thus, at New York, a foot-pound is equivalent to 32.16 foot-poundals.

(4) *The erg is the amount of work done by a force of one dyne producing a displacement of one centimeter.* It is the unit in common use among scientists. Since there are 980,000 dynes in the weight of one kilogram of matter at New York, a kilogrammeter there equals 98,000,000 ergs. A foot-poundal is equivalent to 421,402 ergs; a foot-pound is equivalent to 32.16 times that many ergs.

(*a*) To get a numerical estimate of work done, we multiply the number of units of force by the number of units of displacement:

$$\text{Work done} = fl.$$

Since the resistance overcome is numerically equal to the force acting, the work done may be computed by multiplying, in a similar manner, the resistance by the space:

$$\text{Work done} = wl.$$

In this formula, w represents the resistance; and l, the length or distance. When the body is simply lifted against the force of gravity, w represents weight. A weight of 25 pounds raised 3 feet, or one of 3 pounds raised 25 feet, represents 75 foot-pounds. A weight of 15 kilograms raised 10 meters represents 150 kilogrammeters.

81. Activity and Horse-Power. — In measuring work done, no consideration is given to the time taken. In considering an engine or other agent that is to do the work, the time required is a very important thing. *The activity of an agent is the rate at which it can do work, and is measured by the work it can do in unit time. The unit in most common use for the measurement of activity is the horse-power. It represents the ability to do* 550 *foot-pounds per second.*

$$H.P. = \frac{pounds \times feet}{550 \times seconds}.$$

(*a*) The practical unit of electrical activity is the watt, which is equal to 10^7 ergs per second. One horse-power equals 746 watts, or 746×10^7 ergs per second.

82. Energy *is the power of doing work*, and is possessed by bodies by virtue of work having been done upon them. If a falling cannon ball can overcome a greater resistance than a flying base-ball, it has more energy; more work was done upon it.

The two fundamental ideas with which physics concerns itself are matter and energy.

(*a*) "We are acquainted with matter only as that which may have energy communicated to it from other matter. Energy, on the other hand, we know only as that which, in all natural phenomena, is continually passing from one portion of matter to another. It cannot exist, except in connection with matter." — *Maxwell.*

83. Types of Energy. — It is a general and familiar fact that bodies in motion can do work on other bodies. A cannon ball falling toward the earth has energy, because of its mass and its velocity. Even before it began to fall, it had a power of doing work, because work had been done in lifting it into its elevated position. Thus there are two types of energy, which may be designated as energy of motion and energy of position. *Energy of motion is called kinetic energy; energy of position is called potential energy.* Energy that is not kinetic is potential.

(*a*) A falling weight or running stream possesses energy of motion; it is able to overcome resistance by reason of its mass and velocity. On the other hand, before the weight began to fall, it had the power of doing work by reason of its elevated position with reference to the earth. When the water of the running stream was at rest in the lake among the hills, it had a power of doing work, an energy that was not possessed by the waters of the pond in the valley below. In either case, work had to be done to lift the body into its elevated position, and thus to endow it with potential energy. In bending a bow, or in elongating the spring of a dynamometer, or in winding up a watch, work is performed in distorting the bow or the spring, and, by virtue of this distortion, the instruments possess potential energy.

(*b*) Kinetic and potential energies are interconvertible. Imagine a ball thrown upward with a velocity that will keep it in motion for two seconds. At the end of one second it has lost some of its initial velocity, and hence some of its kinetic energy; but it has gained an elevated position, and has therefore acquired some potential energy. This potential energy just equals the loss of kinetic energy, and exists by virtue of that loss. At the end of another second it has no velocity, and, therefore, no kinetic energy. But the energy with which the ball began its upward flight has not been annihilated; it has been wholly converted into potential energy. If at this moment the ball is caught, all of the original kinetic energy may be kept in store as potential energy.

(*c*) If we ignore the disappearance (not loss) of the energy expended in overcoming the resistance of the air, the ball would, when permitted to fall, reach the level from which it started with its original velocity and kinetic energy. At the start, at the finish, or at any intermediate point of either its ascent or descent, the sum of the two types of energy is the same. It may be all kinetic, all potential, or partly both, but the sum of the two is constant.

(*d*) The pendulum affords a good and simple illustration of kinetic and potential energy, their equivalence and convertibility. When the pendulum hangs at rest in a vertical position, as *Pa*, it has no energy at all. Considered as a mass of matter separated from the earth, it certainly has potential energy; but considered as a pendulum, it has not. If we draw the pendulum aside to *b*, we raise it through the space, *ah*; that is, we do work upon it. The energy thus expended is now stored up as potential energy, ready to be reconverted into energy of the kinetic type, whenever we let it drop. As it falls the distance, *ha*, in passing from *b* to *a*, this reconversion is gradually going on. When the pendulum reaches *a*, its energy is all kinetic, and just equal to that spent in raising it from *a* to *b*. This kinetic energy now carries it on to *c*, lifting it again through the space, *ah*. Its energy is again all potential, just as it was at *b*. If we could free the pendulum from the resistances of the air and friction, the energy originally imparted to it would swing to and fro between the extremes of all potential and all kinetic; but, at every instant, and at every point of the arc traversed, the total energy would be an unvarying quantity, always equal to the energy originally exerted in swinging it from *a* to *b*.

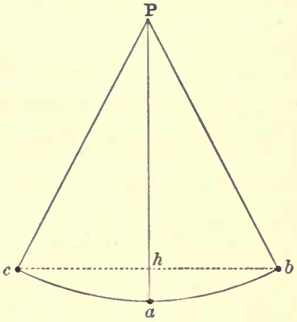


FIG. 47.

Velocity and Energy.

Experiment 51. — Into a pail full of moist clay or stiff mortar, drop a bullet from the height of one yard. Notice the depth to which the bullet penetrates. Drop the bullet from a height of four yards. It will strike the clay with twice the velocity (§ 107) and penetrate four times as far as it did before. This suggests that perhaps an increase in the velocity of a given body increases the energy of that body more rapidly than it increases the momentum; that the mass being the same, the energy varies as the square of the velocity; $E \propto v^2$.

84. Relation of Velocity to Energy. — Any moving body can overcome resistance, or perform work; it has energy. We must acquire the ability to measure this energy. In the first place, we may notice that its definition indicates that it may be measured by the units used in measuring work; i.e., in units of force and displacement. In the next place, we may notice that the direction of the motion is unimportant. A body of given weight and velocity can at any instant do as much work when going in one direction as when going in another. This energy may be expended in penetrating an earth-bank, knocking down a wall, or lifting itself against the force of gravity. Whatever the work actually done, the manner of expenditure does not change the amount of energy expended. *We may therefore find to what vertical height the given velocity would lift the body* (§ 110), *and thus determine its energy in foot-pounds or kilogrammeters.*

85. Kinetic Energy Measured in Gravitation Units. — Representing the weight of a body by w, and the vertical height to which its velocity can carry it by l, it is evident that the kinetic energy can do $w \times l$ units of work. When we come to study the laws of falling bodies, we shall find that

$$l = \frac{v^2}{2g},$$

in which g represents the acceleration due to gravity, i.e., 32.16 feet or 980 cm. (see § 107, d). Substituting this value of l in the equation given above, we have

$$\textit{Kinetic Energy} = \frac{wv^2}{2g}.$$

If w is measured in pounds and v in feet, this measures the energy in foot-pounds; if w is measured in kilograms and v in meters, it measures the energy in kilogram-meters.

(a) In measuring work, we consider resistance and the distance through which it is overcome; in measuring energy, we consider the force and the distance through which it acts. A foot-pound of energy is the amount of energy that must be expended in doing, or that is capable of doing, a foot-pound of work. Similarly, a kilogrammeter of energy is the amount of energy that must be expended in doing, or that is capable of doing, a kilogrammeter of work.

86. Kinetic Energy Measured in Absolute Units. — We have already seen (§ 62, a) that a force may be measured by the momentum it produces.

$$f = ma.$$

But the measurement of work (§ 80, a) introduces the additional factor, l, representing the number of units of displacement. Introducing this factor into the equation above, we have, for work or kinetic energy,

$$K.E. = fl = mal.$$

In § 54 (b) we have

$$l = \frac{v^2}{2a}.$$

Substituting this value of l in the second member of the equation above, we have

$$K.E. = fl = \frac{mav^2}{2a} = \tfrac{1}{2}mv^2.$$

(a) If m is measured in pounds and v in feet, this formula gives a numerical expression for foot-poundals; if m is measured in grams and v in centimeters, it gives that expression in ergs. Foot-poundals may be reduced to foot-pounds by dividing by 32.16, the value of g; ergs may be reduced to kilogrammeters by dividing by 98,000,000.

87. Potential Energy Measured. — In the case of a body raised above the surface of the earth, its potential energy may be measured,

(1) In gravitation units, by the product $w \times h$.

(2) In absolute units, by the product $m \times h \times g$.

88. Conservation of Energy. — In § 83 (*d*) it was stated that, were it not for friction and the resistance of the air, the pendulum would vibrate forever; that the energy would be indestructible. Energy is withdrawn from the pendulum to overcome these impediments, but the energy thus withdrawn is not destroyed. What becomes of it will be seen when we study heat. The truth is that *energy is as indestructible as matter; this is what is meant by the conservation of energy.*

(*a*) Energy cannot be created; it cannot be destroyed. Taking the universe as a whole, its quantity is unchangeable. For the present we must admit that a given amount of energy may disappear, and escape our search, but it is only for the present. We shall soon learn to recognize the fugitive even in disguise.

(*b*) Transformations of energy are constantly recurring, and it is the prime duty of every student of physical science to watch for them and to try to recognize them in every phenomenon.

CLASSROOM EXERCISES.

1. What is the horse-power of an engine that will raise 8,250 pounds 176 feet in 4 minutes?

2. A ball weighing 192.96 pounds is rolled with a velocity of 100 feet a second. How much energy has it? *Ans.* 30,000 foot-pounds.

3. A projectile weighing 50 Kg. is thrown obliquely upward with a velocity of 19.6 m. How much kinetic energy has it?

4. Two bodies weigh 50 pounds and 75 pounds respectively, and have equal momenta. The first has a velocity of 750 feet per second. What is the velocity of the second?

5. A body weighing 40 Kg. moves at the rate of 30 Km. per hour. Find its kinetic energy.

6. What is the horse-power of an engine that can raise 1,500 pounds 2,376 feet in 3 minutes? *Ans.* 36 H.P.

7. A cubic foot of water weighs about $62\frac{1}{2}$ pounds. What is the horse-power of an engine that can raise 300 cubic feet of water every minute from a mine 132 feet deep.

8. A body weighing 100 pounds moves with a velocity of 20 miles per hour. Find its kinetic energy.

9. A weight of 3 tons is lifted 50 feet. (*a*) How much work was done by the agent? (*b*) If the work was done in a half minute, what was the necessary horse-power of the agent?

10. How long will it take a 2-horse-power engine to raise 5 tons 100 feet?

11. How far can a 2-horse-power engine raise 5 tons in 30 seconds?

12. What is the horse-power of an engine that can do 1,650,000 foot-pounds of work in a minute?

13. What is the horse-power of an engine that can raise 2,376 pounds 1,000 feet in 2 minutes?

14. If a perfect sphere rests on a perfect horizontal plane in a vacuum, there will be no resistance, other than its own inertia, to a force tending to move it. How much work is necessary to give to such a sphere, under such circumstances, a velocity of 20 feet a second, if the sphere weighs 201 pounds? *Ans.* 1,250 foot-pounds.

15. A railway car weighs 10 tons. From a state of rest it is moved 50 feet, when it is moving at the rate of 3 miles an hour. If the resistances from friction, etc., are 8 pounds per ton, how many foot-pounds of work have been expended upon the car? (First find the work done in overcoming friction, etc., through 50 feet, which is 50 foot-pounds $\times 10 \times 8$. To this, add the work done in giving the car kinetic energy.)

16. Determine, by the composition of forces, whether three concurring forces with magnitudes of 5, 6, and 12 pounds, respectively, can be in equilibrium.

17. Explain why a soap-bubble blown at one end of a tube contracts, and forces a current of air out of the other end of the tube.

18. A railway train moves past a station at the rate of 20 miles an hour. A mail agent throws out a parcel, in a direction perpendicular to the track and with a horizontal velocity of 20 feet per second. Determine the velocity of the parcel at the beginning of its flight.

19. A constant force acting on a mass of 15 grams for 4 seconds gives it a velocity of 20 cm. per second. Find the magnitude of the force in dynes.

20. A 250-pound projectile is fired from a 12-ton gun with an initial velocity of 1,420 feet per second. Determine the velocity of the gun's recoil.

21. What is the centrifugal force of a 20-pound mass moving uniformly once in 5 seconds around a circle 6 feet in diameter?

22. A man pushes at the rear of a street car and in the line of its motion with a force of 50 pounds. How much work does he perform while the car moves 10 feet?

23. A man pushes at one corner of the rear platform of a street car with a force of 50 pounds, and in a direction that makes an angle of 45° with the car's line of motion. How much work does he perform while the car moves 10 feet?

24. A man pushes directly against the side of a street car with a force of 50 pounds. How much work does he perform while the car moves 10 feet along the track?

25. A man pushes against the corner of the front platform of a street car with a force of 50 pounds, and in a direction that makes an angle of 135° with the track. How much work does he perform while the car moves forward 10 feet?

26. Determine the magnitude of the kinetic energy of a body having a mass of 50 pounds, and a velocity of 30 feet per second, (*a*) in gravitation units; (*b*) in absolute units.

27. If a man does 1,056,000 foot-pounds in a working day of 8 hours, what horse-power represents his working power?

Ans. $\frac{1}{15}$ H.P.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Whirling table (Fig. 46) and attachments; wooden blocks; rubber cord; speed counter.

1. Determine the ratio between the circumferences of the driver wheel and the follower (i.e., the two wheels that carry the belt) of the whirling table by counting the revolutions made by the spindle, *c*, while the large wheel revolves once.

2. To the whirling table, attach a disk on which rests a ball. Rotate the apparatus and make a record of the consequent phenomenon. Connect the ball by a stiff elastic cord to the center of the

disk. Rotate the apparatus with increasing speed, and make a note of what takes place. (See Fig. 48.)

FIG. 48.

3. To the whirling table, attach a frame carrying two tubes containing mercury and water, and supported at an angle, as shown in Fig. 49. Rotate the apparatus rapidly, and make a record of anything peculiar that attracts your notice.

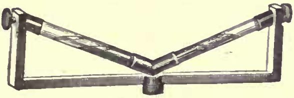


FIG. 49.

4. To the whirling table, attach the apparatus consisting of flexible hoops, as shown in Fig. 50. The hoops are firmly fixed at the bottom, but the central spindle passes freely through the holes where the hoops cross each other at the top. Rotate the apparatus rapidly, and record any result noticed that has any bearing on the shape of the earth.

5. Replace the flexible hoops by an inflexible iron ring about a foot in diameter. From the upper point of this ring, suspend successively a skein of thread, a looped chain, and a globular glass vessel containing some mercury and some ink-colored water. In each case, rotate the apparatus, and record your observations, as you are supposed to do in all such cases.

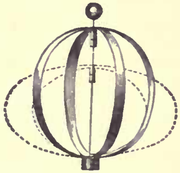


FIG. 50.

6. From the extremities of different axes, successively suspend wooden solids of various geometrical forms; e.g., sphere, oblate spheroid, prolate spheroid, cylinder, etc. Rotate and record as before.

7. To the whirling table, attach the frame carrying two balls of equal mass (see Fig. 46) free to slide on a wire, and connected by a thread. By trial, find a position for the balls such that the joined balls will be on opposite sides of the center of rotation, and yet not slide toward either end of the wire when the spindle is put in rapid rotation. Measure the distance of each ball from the center of rotation. What is the tendency of each ball? What case under composition of forces is thus illustrated?

8. Change the balls for two joined balls of unequal but known masses. Place them one on each side of the center of rotation, and so that they will retain their positions when the spindle is rapidly rotated. When these positions have been determined, measure the

distances from the center of rotation to the centers of the two balls, and see how the ratio between the distances compares with the ratio between the masses of the balls. What do you suppose to be the purpose of this exercise?

9. Fig. 51 represents a very valuable attachment for the whirling table. A ball of known mass slides on a horizontal wire. To this ball is attached a flexible cord that turns around a pulley at the bottom, divides into two, passes over the pulleys at the top, and carries adjustable and slotted disk weights. The middle cord passes through the slots of the weights. The cord and the center of the disks must lie in the center of rotation. Rotate the apparatus with gradually increasing speed until the outward pull of the ball just begins to lift the load. Keeping this speed constant, count the number of revolutions that the driving wheel makes in 10 seconds. Find the average of several such trials. Measure the horizontal distance between the center of the ball and the axis of rotation, and thence compute the velocity of the ball in the recent trials. Compare the result with the formula given in § 78. Repeat the experiment, using a "speed counter" to determine the number of revolutions made by the spindle in 10 seconds, and computing anew the velocity of the ball.

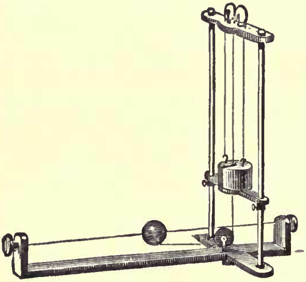


FIG. 51.

Replace the ball by one twice as heavy and at the same distance from the center of rotation. Double the load carried by the vertical cords. Counting as before, determine the rate of rotation necessary to lift the load. Compare the result with the first law given in § 78.

Determine the load that may be lifted by the ball when the driver makes twice as many revolutions as before; when it makes three times as many revolutions. Compare results with the second law given in § 78.

Taking any of these trials as a standard for comparison, move the ball to a point twice as far from the center of rotation, and turn the driving wheel half as fast. How will the two velocities of the ball compare? At this slower rate of rotation, determine the load that may be lifted. Compare the result with the third law given in § 78.

Tabulate all of your results as usual.

10. Float upon water two blocks of wood, one of which is twice as heavy as the other. Connect them by a stretched rubber cord. Release the blocks, and they will move toward each other, but with unequal velocities. Determine how much faster one moves than the other, and compare their momenta.

11. On a page of cross-section paper (i.e., paper ruled in squares 1 mm. or 0.1 inch on a side, which can be purchased at nearly any optician's) select arbitrarily some corner of a square; mark it O, and call it "the origin of coördinates." Mark the right-hand end of the horizontal line passing through the origin, X, and the left-hand end of the same line, X'. Call the line, $X'X$, "the axis of abscissas." Mark the upper end of the vertical line passing through the origin, Y, and the lower end of the line, Y'. Call the line, $Y'Y$, "the axis of ordinates." From O, count off, on the axis of abscissas, three spaces to a point, which mark M. On the vertical line, measure two spaces upward to a point, which mark P The distance OM is called "the abscissa" of the point, P. The distance MP is called "the ordinate" of the point, P. Negative abscissas would be measured from O toward X', and negative ordinates would be drawn downward from the axis of abscissas.

Using the same axes of coördinates, locate points having coordinates with the following values: —

Points.	*Abscissas.*	*Ordinates.*	*Points.*	*Abscissas.*	*Ordinates.*
a	– 3	12	*e*	4	– 2
b	– 2	4	*f*	6	– 2.4
c	0	0	*g*	8	– 2.67
d	2	– 1.3	*h*	10	– 2.86

Through the points thus located, draw a line with as nearly uniform a curve as possible.

III. GRAVITATION, ETC.

89. Gravitation. — *Every particle of matter in the universe has an attraction for every other particle. This attractive force is called gravitation.*

(*a*) Gravitation is unaffected by the interposition of any substance. During an eclipse of the sun, the moon is between the sun and the earth. But at such a time, the sun and earth attract each other with the same force that they do at other times.

(*b*) Gravitation is independent of the kind of matter, but depends upon the quantity or mass, and the distance. Mass does not mean size. The planet Jupiter is about 1,300 times as large as the earth, but it has only about 300 times as much matter, because it is only 0.23 times as dense.

90. Law of Gravitation. — *The mutual attraction between two bodies varies directly as the product of their masses, and inversely as the square of the distance between their centers of mass.* For example, doubling this product doubles the attraction; doubling the distance, quarters the attraction; doubling both the product and the distance halves the attraction.

(*a*) Represent the attraction between two units of mass at unit distance by a. Then, in the case of two bodies containing respectively m and n units of mass, the attraction of either for the other at unit distance will be mna. If the distance be increased to d units, the attraction will be $\frac{mna}{d^2}$.

(*b*) Notice that this attraction is mutual. The earth draws the falling apple with a force that gives it a certain momentum; the apple draws the earth with an equal force, that gives to it an equal momentum.

91. Gravity. — The most familiar illustration of gravitation is the attraction between the earth and bodies upon or near its surface. This particular form of gravitation is commonly called gravity. Its measure is weight. Its direction is that of the plumb line, i.e., vertical.

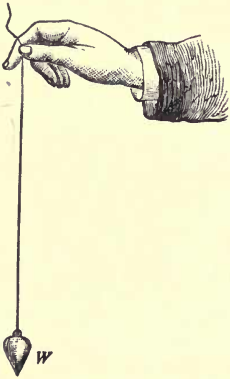


FIG. 52.

92. Weight. — As the mass of the earth remains constant, doubling the mass of the body weighed doubles the product of the masses, and consequently doubles the weight. When we ascend from the surface of the earth, there is nothing to interfere with the working of the law of universal gravitation ; but when we descend below the surface, we leave behind us particles of matter the attraction of which partly counterbalances that of the rest of the earth. The weight of a body at one place on the surface of the earth differs from its weight at another place, because the earth is not a perfect sphere and its density is not uniform.

93. Law of Weight. — *Bodies weigh most at the surface of the earth. For bodies in the earth's crust, the weight varies approximately as the distance from the center. For bodies above the earth's surface, the weight varies inversely as the square of the distance from the center.*

(*a*) Let the heavy black line of Fig. 53 represent a spherical shell of uniform thickness and density with bodies at *c* and *e* within the shell, and at *i* and *n*, without the shell. For such conditions, these propositions have been established : —

(1) The attraction of the matter composing the shell draws bodies within the shell, as at *c* and *e*, equally in all directions.

(2) The attraction of the matter composing the shell pulls bodies outside the shell, as at *i* or *n*, just as it would if the entire mass of the shell were concentrated at its center, *c*.

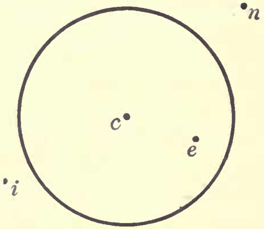


Fig. 53.

In assuming that such a shell with a radius of 4,000 miles represents the earth, we ignore the variation between polar and equatorial diameters, the variation in the density of the earth's crust, and the possible variation of its thickness. Still, make the assumption. Then, at a depth of 15 miles, a body weighing 100 pounds at the surface would weigh about 100 pounds $\times \frac{3985}{4000}$. At an elevation of 4,000 miles above the surface (8,000 miles from the center), it would weigh 100 pounds $\times \frac{4000^2}{8000^2} = 25$ pounds.

CLASSROOM EXERCISES.

1. Suppose the earth to be solid. How far below the surface would a 10-pound ball weigh only 4 pounds?

Solution. — As the weight is to be reduced six tenths, it must be carried 0.6 of the way to the center.

Ans. 4,000 miles × 0.6 = 2,400 miles.

2. On the same supposition, what would a body weighing 550 pounds on the surface of the earth weigh 3,000 miles below the surface? *Ans.* 137½ pounds.

3. Two bodies attract each other with a certain force when they are 75 m. apart. How many times will the attraction be increased when they are 50 m. apart? *Ans.* 2¼.

4. Given three balls. The first weighs 6 pounds, and is 25 feet distant from the third. The second weighs 9 pounds, and is 50 feet distant from the third. (*a*) Which exerts the greater force upon the third? (*b*) How many times as great? *Ans.* $\frac{8}{3}$.

5. A body at the earth's surface weighs 900 pounds. What would it weigh 8,000 miles above the surface?

6. How far above the surface of the earth will a pound avoirdupois weigh only an ounce? *Ans.* 12,000 miles.

7. At a height of 3,000 miles above the surface of the earth, what would be the difference in the weights of a man weighing 200 pounds and of a boy weighing 100 pounds? *Ans.* 32.65 pounds.

8. Find the weight of a 180-pound ball 2,000 miles above the earth's surface.

9. (*a*) If the earth was solid, would a 50-pound cannon ball weigh more 1,000 miles above the earth's surface, or 1,000 miles below it? (*b*) How much?

10. If the moon was moved to three times its present distance from the earth, what would be the effect (*a*) on its attraction for the earth? (*b*) On the earth's attraction for it?

11. A team pulling northeast with a force of 800 pounds, moves a

railway car 12 feet along a track running north. How much work is done by the team?

12. How far above the surface of the earth must 2,700 pounds be placed to weigh 1,200 pounds? *Ans.* 2,000 miles.

13. What effect would it have on the weight of a body to double the mass of the body and also to double the mass of the earth?

14. A 50-pound ball moving with a velocity of 75 feet per second strikes a 200-pound ball squarely, and rebounds with a velocity of 25 feet. What velocity was given to the 200-pound ball by the collision?

94. Center of Mass. — A body's center of mass is the point about which all the matter composing the body may be balanced. It is also called the center of inertia. In some cases it is also the center of gravity.

(*a*) The force of gravity tends to draw every particle of matter toward the center of the earth, or downward in a vertical line. We may, therefore, consider the effect of this force upon any body as the sum of an almost infinite number of parallel forces, each of which is acting upon one of the particles of which that body is composed. We may also consider this sum of forces, or total gravity, as a resultant force, *GP*, acting upon a single point, just as the force exerted by two horses harnessed to a whiffletree is equivalent to another force equal to the sum of the forces exerted by the horses, and applied at a single point at or near the middle of the whiffletree. This single point, *G*, which may be regarded as the point of application of the force of gravity acting upon a body, is called the *center of mass* of that body; in other words, the weight of a body and its mass may be considered as concentrated at a single point.

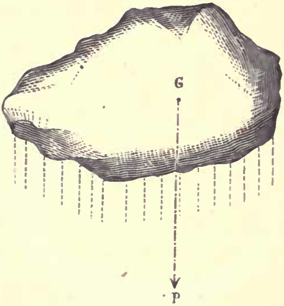


FIG. 54.

(*b*) When a body is acted on by any force, there is, owing to the inertia of each particle, a series of reactions in the opposite direction, the resultant of which has its application at a point called the *center of inertia.*

(*c*) Any force acting on a body at its center of mass tends to produce a motion of translation in the direction of that force; but, if the force acts on the body at any other point, it and the reaction at the center of mass form a couple that tends to produce rotary motion of the body.

(*d*) In a freely falling body, no matter how irregular its form or how indescribable the curves made by any of its projecting parts, the *line of direction* in which the center of mass moves is a vertical line.

95. To find the Center of Mass. — In a body suspended from a point, the center of mass will be brought as low as possible, and will, therefore, lie in a vertical line drawn through the point of support. This fact affords a ready means of determining this point experimentally.

(*a*) Let any irregularly shaped body, as a stone or chair, be suspended so as to move freely. Drop a plumb line from the point of suspension, and make it fast or mark its direction. The center of mass will lie in this line. From a second point, not in the line already determined, suspend the body; let fall a plumb line as before. The center of mass will lie in this line also. But to lie in both lines, it must lie at their intersection.

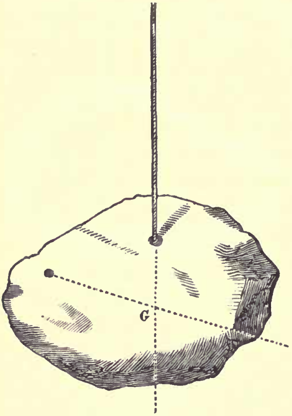


Fig. 55.

(*b*) If a flat piece of cardboard can be balanced on the point of a pin, its center of mass lies vertically above the point of support, midway between the two sides of the cardboard. When a body is of uniform density and regular shape, its center of mass and its center of figure will coincide; e.g., the center of mass of a sphere of uniform density is at its center of volume.

96. May be Outside the Body. — In some bodies, as a ring or box or hollow sphere or cask, the center of mass does not lie in the matter of which the body is composed.

(*a*) This fact may be illustrated by the "balancer," represented in Fig. 56. The center of mass lies a little above the line joining the two heavy balls, and thus under the foot of the waltzing figure. But the point, wherever found, will have the same properties as if it lay in the mass of the body.

97. The Base. — *The side on which a body rests is called its base.* If the body is supported on legs, as a chair, the base is the polygon formed by joining the points of support.

98. Equilibrium. — A body supported at a single point will rest in equilibrium when a vertical line passing through its center of mass, i.e., the line of direction, also passes through the point of support. A body supported on a surface will rest in equilibrium when the line of direction (§ 94, *d*) falls within its base. In general terms, *a body is in equilibrium when the resultant of all the forces acting on it is zero.* The center of mass will be supported when it coincides with the point of support, or is in the same vertical line with it. When the center of mass is supported, the whole body is supported and rests in a state of equilibrium.

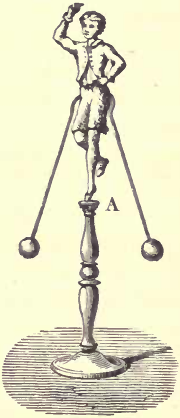


FIG. 56.

(*a*) When the line of direction falls without the base, weight and reaction of support become forces that form a couple and overturn the body.

Experiment 52. — With the point of a penknife blade, make a hole of 2 or 3 mm. diameter in the large end of an egg. In the small end, prick a pinhole. Blow the contents of the shell out through the larger hole. Rinse and dry the shell. Drop a little pulverized rosin or melted sealing wax through the larger hole into the smaller end of the egg. Support the egg in a small tin can (that may be obtained from any kitchen) or in any other convenient way, and pour a few grams of melted lead through the larger hole and into the smaller end. The lead will not run out through the pinhole even if the rosin or sealing wax is not used. The larger hole may be neatly concealed with a piece of thin paper put on with flour paste. Try to make your "magical egg" lie on its side.

99. Kinds of Equilibrium. — There are three kinds of equilibrium: —

(1) *A body supported in such a way that, when slightly displaced from its position of equilibrium, it tends to return to that position, is said to be in stable equilibrium.* Such a displacement raises the center of mass.

(2) *A body supported in such a way that, when slightly displaced from its position of equilibrium, it tends to fall further away from that position, is said to be in unstable equilibrium.* Such a displacement lowers the center of mass.

(3) *A body supported in such a way that, when displaced from its position of equilibrium, it tends neither to return to its former position nor to fall further from it, is said to be in neutral or indifferent equilibrium.* Such a displacement neither raises nor lowers the center of mass.

100. Stability. — *When the line of direction falls within the base, the body stands; when without the base, the body falls over.* The stability of a body is measured by the amount of work that must be done to overturn it. This amount may be increased by enlarging the base, or by lowering the center of mass, or both.

(*a*) Let Fig. 57 represent the vertical section of a brick placed upon its side, its position of greatest stability. In order to stand the brick upon its end, *g*, the center of mass, must pass over the edge, *c*; that is to say, the center of mass must be raised a distance equal to the difference between *ga* and *gc*, or the distance, *nc*. But to lift *g* this distance is the same as to lift the whole brick vertically a distance equal to *nc*. Draw similar figures for the brick when placed upon its edge and upon its end. In each case, make *gn* equal to *ga*, and see that the value of *nc* decreases. But *nc* represents the distance that the brick, or its center of mass, must be raised, before the line of direction can fall without the base and the body be overturned. To lift the brick, or its center of mass, a small distance involves less work than to lift it a greater distance. Therefore, the greater the value of *nc*, the more work required to overturn the body, or the greater its stability. But this greater value of *nc* evidently depends upon a larger base, a lower position for the center of mass, or both.

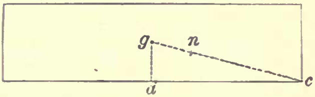


FIG. 57.

(*b*) When the body rests upon a point, as does the sphere, or upon a line, as does the cylinder, a very slight force is sufficient to move it, no elevation of the center of mass being necessary.

CLASSROOM EXERCISES.

1. Why does a person stand less firmly when his feet are parallel and close together than when they are more gracefully placed?

2. Why can a child walk more easily with a cane than without?

3. Why will a book placed on a desk-lid stay there, while a marble will roll off?

4. Why is a ton of stone on a wagon less likely to upset than a ton of hay similarly placed?

5. If a falling body near the surface of the earth gains an acceleration of 32.16 feet, what would be its acceleration 240,000 miles from the center of the earth?

6. A body is simultaneously acted upon by two forces, one of which would give it a velocity of 100 feet per second northward, while the

other would give it a northeasterly velocity of 75 feet per second. Determine the magnitude and direction of the resultant velocity.

7. A given force, acting for ten minutes upon a body weighing 100 pounds, produces a velocity of a mile a minute. Determine the magnitude of the force in poundals.

8. How many gallons of water, each weighing 8 pounds, can a 100-horse-power engine raise to a height of 200 feet in 10 hours?

9. Why have the Egyptian pyramids great stability?

10. Why is it easier for a baby elephant than for a baby boy to learn to walk?

11. Where is the axis of rotation of a carriage wheel? Where should the center of mass of a carriage wheel be?

12. A boy placed a step-ladder as shown in Fig. 58, and it stood. Why? He then climbed to its top, and it fell. Why?

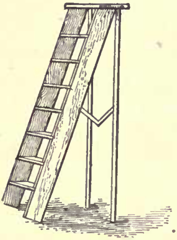


FIG. 58.

LABORATORY EXERCISES.

Additional Apparatus, etc.—Plumb bob; chalk; cardboard; box as described below; screw-eye; mortar.

1. Drive small tacks into the frame of a slate at adjacent corners. Tie the middle of a stout thread to one of the tacks. Fasten a small weight to one end of the thread, and support the apparatus from the other end. Mark on the slate the direction in which the thread crosses it. Similarly support the slate by the other tack, and mark the direction of the thread by another line. Place the intersection of the two lines over the end of the finger, and see if the slate is balanced. The point thus located approximately represents what?

2. Cut a rectangle from cardboard. Draw its two diagonals. Balance the cardboard to see how near the center of mass coincides with the center of area. Can the center of mass lie on the surface of such a body?

3. Drive a wire nail into a vertical support, and cut off the head of the nail. Bore several holes through an irregularly shaped board near its edges. Using one of these holes, hang the board on the nail. From the nail, hang a chalked plumb line. When the plumb line has come to rest, "snap" it so as to make a vertical line on the board. Change the position of the board, the nail passing through another

of the holes. Chalk the line, suspend and "snap" as before. Place the intersection of the two chalk lines over the end of the finger, and see if the board then balances. Using another hole, similarly chalk another line, and see if the three lines have a common point of intersection.

4. Fill a box, about 2 × 4 × 8 inches (a shallow cigar box will do), with stiff mortar, and nail down the cover. Stand it on end, so that the 8″ edges will be vertical. (It is common for architects and mechanics to indicate "feet" by one tick, and "inches" by two ticks, as in the preceding sentence.) Insert a small screw-eye or hook at the middle of one of the 4″ × 8″ surfaces; it will be approximately on a level with the center of mass. Tack a small wooden strip to the table, at the foot of this side, to keep the box from slipping when pulled. Tie one end of a cord to this screw-eye, and the other end to the hook of a spring-balance. Hold the dynamometer so that its axis and the string lie in a straight line that is perpendicular to the side of the box. Carefully observing the scale of the dynamometer, pull steadily until the box falls over. Record the maximum reading of the scale. On one of the 2″ × 8″ faces, draw and bisect a diagonal. What is the difference between half the length of the diagonal and half the height of the box as it was standing? Record that difference. What does it represent?

Instead of using the spring-balance, one may pass the cord over a pulley adjusted at the level of the screw-eye, attach a scale-pan of known weight to the free end of the cord, and add weights until the box begins to overturn.

If the screw-eye pulls out, pass a cord or wire around the box at the proper level, and attach the other cord to it at the proper place. A wire with a loop at one end may be passed through the box from one face to the opposite side, and there made fast before filling the box. The loop will then take the place of the screw-eye. It is especially desirable thus to connect the opposing 2″ × 4″ and the 2″ × 8″ faces.

Place the box so that its 4″ edges shall be vertical. Ascertain and record the force necessary to overturn the box by a horizontal pull, as before. On one of the 2″ × 4″ faces, draw and bisect a diagonal. Find how much the semi-diagonal exceeds 2″, the semi-altitude of the box. Record this difference.

Transfer the screw-eye to the middle of one of the 2″ × 8″ sides, and place the box with its 8″ edges vertical. Ascertain and record the force necessary to overturn the box, as before. On one of the 4″ × 8″ faces, draw and bisect a diagonal. Find how much the semi-diagonal exceeds 4″, the semi-altitude. Record this difference.

Place the box so that its 2″ edges shall be vertical. Ascertain and record the force necessary to overturn the box, as before. Find how much the semi-diagonal of the 2″ × 4″ face exceeds 1″, the semi-altitude. Record this difference.

Transfer the screw-eye to the middle of one of the 2″ × 4″ sides, and place the box with its 4″ edges vertical. Ascertain and record the force necessary to overturn the box, as before. Find how much the semi-diagonal of a 4″ × 8″ face exceeds 2″, the semi-altitude, and record that difference.

Place the box so that its 2″ edges shall be vertical. Ascertain and record the force necessary to overturn the box, as before. Find how much the semi-diagonal of the 2″ × 8″ surface exceeds 1″, the semi-altitude, and record that difference.

Weigh the box. Multiply its weight by the excess of the semi-diagonal over the semi-altitude in each case. What do these products represent? What do they measure? If weight is measured in pounds and decimals thereof, and diagonals and altitudes in feet and decimals thereof, these products will represent what kind of units? Compare these several products with the corresponding forces used in overturning the box. Do you discover any relation between them?

5. Cut a piece of board 20″ long, 3″ wide at one end, and 7″ wide at the other end. Find a point on the surface of the board as near as possible to the center of mass, and over it paste a patch of black paper an inch in diameter. On the same side of the board, and a foot or so from the other paper, paste a patch of red paper about 2″ in diameter. Toss the board up edgewise in the open air, so that it will turn end over end, carefully observing the motion of the two paper patches relative to each other. Record and explain what you see.

IV. FALLING BODIES.

101. Freely Falling Bodies. — When a body is left unsupported and free to move under the influence of the force of gravity and without any resistance, it is a freely falling body.

(*a*) Unless the body falls from a very great height, the change in the intensity of the attraction due to the change of distance from the center of the earth is so small that it may, without sensible error, be disregarded. It is, therefore, common to consider gravity a constant force; hence, if we ignore the resistance of the air, the laws for falling bodies will be the same as for uniformly accelerated motion.

Experiment 53. — From the upper window, drop simultaneously an iron and a wooden ball of the same size. Be careful that your fingers do not "stick" to one ball longer than to the other. Notice that the two balls of different weight strike the ground at practically the same time.

102. Velocities of Falling Bodies. — When a feather and a cent are dropped from the same height, the cent reaches the ground first. This is not because the cent is heavier, but because the feather meets with more resistance from the air in proportion to its mass. If this resistance can be removed or equalized, the two bodies will fall equal distances in equal times, or with the same velocity. The resistance may be avoided by dropping them in a glass tube from which the air has been removed. The resistances may be nearly equalized by making the two falling bodies of the same size and shape but of different weights, as in the preceding experiment.

FIG. 59.

Experiment 54. — Tack a strip of wood half an inch square to the straight edge of a plank 16 feet long.

Fasten metal strips an inch wide to the sides of the wooden strip so as to make a double-track way which should be straight and smooth. Divide the edge of the plank on one side of the track into 16 foot-spaces, plainly marked. Raise one end of the plank a foot higher than the other. Place a glass or an iron ball at the top of the inclined track. Notice how often the classroom clock ticks in a second. Place a finger on top of the ball, thus holding it ready for a start. Repeat the word "tick" in unison with the clock until you "feel" the rhythm of its swing, and, just at the moment of a "tick," lift the finger from the ball, which will begin to roll down the track. Notice and record the position of the ball at the end of successive seconds. To locate the ball at the end of the allotted period, place on the upper side of the half-inch strip a wooden block just wide enough to hold its position, and just thick enough to produce an easily audible click when struck by the ball. By trial, place this block so that the tick and the click shall coincide. Repeat your observations, and average the results of similar trials. The greater the number of carefully conducted trials, the more valuable will be your averages.

The ball will roll down the inclined plane, about 1 foot in the first second, 4 feet in 2 seconds, 9 feet in 3 seconds, 16 feet in 4 seconds, etc. The average results may be tabulated as follows: —

Number of Seconds.	*Spaces fallen each Second.*	*Velocities at the End of each Second.*	*Total Number of Spaces fallen.*
1	1	2	1
2	3	4	4
3	5	6	9
4	7	8	16
t	$2t-1$	$2t$	t^2

Representing the velocity gained each second (acceleration) by a, and, consequently, the value of each of our spaces by $\frac{1}{2}a$, we have, from the above, the already familiar formulas, $l' = \frac{1}{2}a(2t-1)$; $v = at$; and $l = \frac{1}{2}at^2$ (see § 54).

103. Unimpeded Fall. — By giving a greater inclination to the plane used in Experiment 54, the ball will roll more rapidly, and our unit of space will increase from one foot, as supposed thus far, to two, three, four, or five feet, and so on; but the number of such spaces will remain as indicated in the table above. By disregarding the resistance of the air, we may say that when the plane becomes vertical, the body becomes a freely falling body. Our unit of space has now become 16.08 feet, or 490 centimeters. It will fall this distance during the first second, three times this distance during the next second, five times this distance during the third second, and so on.

104. Galileo's Device.— The laws of accelerated motion, as given in § 55, were first experimentally verified by Galileo. To avoid the difficulty of accurate observation of the very rapid motion of a freely falling body, he used an inclined plane, down the grooved edge of which a heavy ball was made to roll.

(*a*) Let *AB* represent a plane so inclined that the velocity of a body rolling from *B* toward *A* will be readily observable. Let *C* be a heavy ball. The gravity of the ball may be represented by the vertical line, *CD*. But *CD* may be resolved into *CF*, which represents a force acting perpendicular to the plane and producing pressure upon it but no motion at all, and *CE*, which represents a force acting parallel to the plane, the only force of any effect in producing motion. It may be shown geometrically that

$$EC : CD :: BG : BA.$$

By reducing, therefore, the inclination of the plane, we may reduce the magnitude of the motion-producing component of the force of gravity, and thus reduce the velocity. This will not affect the laws of the motion, that motion being changed only in amount, not at all in character.

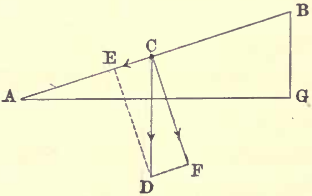


FIG. 60.

105. Atwood's Device.— The Atwood machine consists essentially of a wheel or pulley, *R*, over the grooved edge of which are balanced two equal weights suspended by a long silk thread which is both light and strong. The axle of this wheel is preferably supported upon the circumferences of four friction wheels, r, r, r', r', for greater delicacy of motion. As the thread is so light that its weight may be disregarded, it is evident that the weights will be in equilibrium whatever their position. This apparatus is supported upon a pillar seven or eight feet high. A weight or rider placed upon one of the weights produces motion with a moderate but uniformly accelerated velocity.

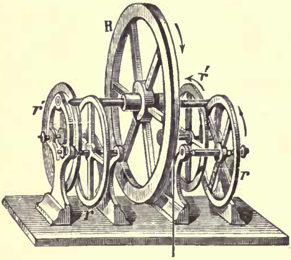


FIG. 61.

(*a*) Suppose that the balanced masses weigh 49.5 grams each, and that the rider weighs 1 gram. The total mass moved is 100 grams, and the force acting is the weight of 1 gram. When this force of 1 gram moves a mass of 1 gram (freely falling body), it produces a velocity too great for easy observation; when the same force acts on the mass 100 times as great, it produces a velocity only $\frac{1}{100}$ as great as it produced in the other case, when the gram mass fell alone. Thus we may produce variations in acceleration as we desire.

106. Acceleration Due to Gravity. — In the latitude of New York, a freely falling body gains a velocity of 32.16 feet, or 980 centimeters, during the first second of its fall. It makes a like gain of velocity during each subsequent second of its fall. *This distance is, therefore, called the acceleration due to gravity, and is generally represented by the letter g.*

107. Formulas for Falling Bodies. — Since the motion of a freely falling body is uniformly accelerated motion, the formulas for freely falling bodies may be derived from those for uniformly accelerated motion (§ 54) by substituting the definite quantity, g, for the indefinite quantity, a. Hence, we have for bodies starting from rest: —

$$(1)\quad v = gt.$$
$$(2)\quad l' = \tfrac{1}{2}g\,(2t-1).$$
$$(3)\quad l = \tfrac{1}{2}gt^2.$$

(a) For bodies rolling down an inclined plane, these formulas may be made applicable by multiplying the value of g by the ratio between the height and the length of the plane.

(b) If $t = 1$, formula (1) becomes $v = g$; i. e., the velocity of a body falling freely for one second from a state of rest equals the acceleration.

(c) If $t = 1$, formula (3) becomes $l = \frac{1}{2}g$, i.e., the space traversed in a second by a body freely falling from a state of rest equals half of the acceleration.

(d) From formula (1) we derive the value, $t = \frac{v}{g}$. Substituting this value in formula (3), we have $l = \frac{1}{2}g \times \frac{v^2}{g^2} = \frac{v^2}{2g}$. Hence, $v = \sqrt{2gl}$, showing that the velocity of a falling body varies as the square root of the distance it has fallen.

(e) None of these formulas involve any expression for mass, thus indicating that the velocity of a falling body is not affected by its mass.

108. Laws of Falling Bodies. — For bodies starting from rest, these formulas may be translated as follows: —

(1) *The velocity of a freely falling body at the end of any second of its descent is equal to* 32.16 *feet* (980 *cm.*) *multiplied by the number of the second.*

(2) *The distance traversed by a freely falling body during any second of its descent is equal to* 16.08 *feet* (490 *cm.*) *multiplied by one less than twice the number of the second.*

(3) *The distance traversed by a freely falling body dur-*

ing any number of seconds is equal to 16.08 *feet* (490 *cm.*) *multiplied by the square of the number of seconds.*

109. Initial Velocity of Falling Bodies. — A body may be thrown downward as well as dropped. In such a case the effect of the throw must be added to the effect of gravity.

$$v = gt + V;\ l' = \frac{1}{2}g(2t-1) + V;\ l = \frac{1}{2}gt^2 + Vt.$$

110. Bodies thrown Upward. — When a body is thrown vertically upward, gravity diminishes its velocity every second by g. The time of the ascent may be found by dividing the initial velocity by the acceleration of gravity:

$$t = \frac{v}{g}.$$

Projectiles.

Experiment 55. — From a strip of wood shaped like a meter stick or common lath, cut a piece about 10 cm. long. Cut equal notches at two corners, a and e, as shown in Fig. 62. Nail the middle of this piece across the end of the rest of the lath, thus making a **T**-shaped form. Clamp the other end of the long leg firmly in a vise so that the edge, ae, and the corresponding edge of the long piece, shall be horizontal and several feet above a level floor. Place lead bullets at a and e. Strike the long piece a sharp, horizontal blow near the cross-piece. One of the bullets will be shot horizontally, and the other will be dropped nearly vertically. Will the bullets strike the floor at the same time? Repeat the experiment several times, and do not expect more than approximate agreements.

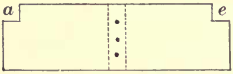


FIG. 62.

111. Projectiles. — Every projectile is acted upon by an impulsive force and the force of gravity. The path of a projectile is a parabolic curve, the resultant of these forces.

(*a*) Suppose a ball to be thrown horizontally. Its impulsive force will give a uniform velocity, and may be represented by a horizontal line divided into equal parts, the magnitude of each part being equal to that of the velocity. The force of gravity may be represented by a vertical line divided into unequal parts, representing the spaces 1, 3, 5, 7, etc., over which gravity would move it in successive seconds. Constructing parallelograms of forces, we find that at the end of the first second the ball will be at *A*, at the end of the next second at *B*, at the end of the third at *C*, at the end of the fourth at *D*, etc. The resistance of the air modifies the nature of the curve somewhat. The horizontal distance, *GE*, is called the range of the projectile.

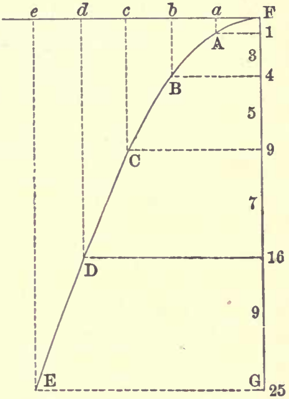


FIG. 63.

CLASSROOM EXERCISES.

1. What will be the velocity of a body after it has fallen 4 seconds?

Solution: — $v = gt = 32.16 \times 4 = 128.64$. *Ans.* 128.64 feet.

2. A body falls for several seconds. During one of these seconds it passes over 530.64 feet. Which one is it? $l - \frac{1}{2}g(2t-1)$

Solution: — $l' = \frac{1}{2}g(2t-1)$

$$530.64 = 16.08 \times (2t-1);\ \therefore t = 17.$$

Ans. 17th second.

3. A body was projected vertically upward with a velocity of 96.48 feet. How high did it rise? 3 32.16

Solution: —
$$t = \frac{v}{g} = \frac{96.48}{32.16} = 3.$$
$$l = \frac{1}{2}gt^2 = 16.08 \times 9 = 144.72.$$
Ans. 144.72 feet.

4. How far will a body fall during the third second of its fall?

5. How far will a body fall in 10 seconds? *Ans.* 1,608 feet.

6. How far in $\frac{1}{2}$ second? *Ans.* 4.02 feet.

7. How far will a body fall during the first second and a half of its fall?

8. How far in 12½ seconds?

9. A body passed over 787.92 feet during its fall. What was the time required? *Ans.* 7 seconds.

10. What velocity did it finally obtain?

11. A body fell during 15½ seconds. Give its final velocity.

12. In an Atwood machine, the weights carried by the thread are 7½ ounces each. When the "rider," which weighs one ounce, is in position, what is the acceleration?

13. A stone is thrown horizontally from the top of a tower 257.28 feet high, with a velocity of 60 feet a second. Where will it strike the ground? *Ans.* 240 feet from the tower.

14. When a fishing line with a heavy sinker is thrown into a stream with a rapid current, it often is carried down stream to the full length of the line, and held suspended in the water. Draw a diagram showing the forces acting on the sinker, and how equilibrium is secured. Neglect the buoyancy of the water.

15. A body is thrown directly upward with a velocity of 80.4 feet. (*a*) What will be its velocity at the end of 3 seconds, and (*b*) in what direction will it be moving?

16. In Fig. 63, what is represented by the following lines: *F*1? *Fa? Aa? Fc? Dd?*

17. A body falls 357.28 feet in 4 seconds. What was its initial velocity? *Ans.* 25 feet.

18. A ball thrown downward with a velocity of 35 feet per second reaches the earth in 12½ seconds. (*a*) How far has it moved, and (*b*) what is its final velocity?

19. (*a*) How long will a ball projected upward with a velocity of 3,216 feet continue to rise? (*b*) What will be its velocity at the end of the fourth second? (*c*) At the end of the seventh?

20. A ball is shot from a gun with a horizontal velocity of 1,000 feet, at such an angle that the highest point in its flight is 257.28 feet. What is its range? *Ans.* 8,000 feet.

✓ 21. A body was projected vertically downward with a velocity of 10 feet. It was 5 seconds falling. Required the entire space passed over. *Ans.* 452 feet.

22. Required the final velocity of the same body. *Ans.* 170.8 feet.

23. A body was 5 seconds rolling down an inclined plane, and passed over 7 feet during the first second. Give (*a*) the entire space passed over, and (*b*) the final velocity.

24. A body rolling down an inclined plane has, at the end of the

first second, a velocity of 20 feet. (*a*) What space would it pass over in 10 seconds? (*b*) If the height of the plane was 800 feet, what was its length? *Ans.* (*b*) 1,286.4 feet.

25. A body was projected vertically upward, and rose 1,302.48 feet. Give (*a*) the time required for its ascent, and (*b*) the initial velocity.

26. A body projected vertically downward has, at the end of the seventh second, a velocity of 235.12 feet. How many feet did it traverse in the first 4 seconds? *Ans.* 297.28 feet.

27. A body falls from a certain height; 3 seconds after it has started, another body falls from the height of 787.92 feet. From what height must the first fall if both are to reach the ground at the same instant? *Ans.* 1,608 feet.

28. A body falls freely for 6 seconds. What is the space traversed during the last two seconds of its fall?

29. When a body is thrown upward, does its velocity vary directly or inversely with the number of time-units it has been rising?

30. See definition of "parabola" in the dictionary. When would the curved line, *FCE* in Fig. 63, become parallel with the vertical line, *FG?*

LABORATORY EXERCISES.

Additional Apparatus, etc. — Inclined track; iron balls; pendulum; electromagnets; a voltaic battery; turnbuckle; blade of hacksaw; screws; screw-driver; needles; thread.

1. Graduate to centimeters the edge of the plank used in Experiment 54, using the part outside the track that was not graduated to feet. Elevate one end of the plank as in that experiment.

On the vertical face of a movable wooden support, and more than a meter from its foot, tack a horizontal strip of soft wood, the thickness of which is not less than the radius of the iron ball mentioned below. Into the vertical face of this strip press a stout needle nearly to its eye. The hole through the needle should be vertical. An inch or so above the needle set a common screw. Fasten one end of a thread that will pass through the eye of the needle, and having a length of about 110 cm., to a small iron ball. Pass the other end of the thread through the eye of the needle, fasten it to the screw, and wrap it around the screw until the center of the ball hangs about a meter below the needle.

Instead of using the needle as above described, a small, firm cork may be fastened to the vertical face of the strip, and a threaded needle drawn vertically through the middle of the cork. The thread may then be fastened to the screw, although the cork will probably pinch it firmly enough to support the ball. Such cork supports are easily provided, and convenient for quick adjustment.

Swing the ball as a pendulum, and count the number of times it passes through its arc in 60 seconds. If the number of swings exceeds 60, turn the screw so as to unwind some of the thread, and increase the distance between the ball and the needle, or the under side of the cork. Swing the ball and count as before, continuing the adjustment until the ball makes 60 swings in 60 seconds. Place the pendulum so that as it swings, and as the ball rolls down the inclined track, both may be observed at the same time.

With the assistance of a friend who understands such things, fix two electromagnets that are on the same circuit so that their attraction will hold an iron ball at the upper end of the inclined track, and the iron pendulum-bob at one end of the arc through which it is to swing. Close the circuit of the battery, and bring the two iron balls into their respective positions, separated from the magnets only by bits of thin paper. The attraction of the magnets will hold the balls, one at the top of the track, and the other at the end of its arc.

Break the circuit, and the two iron balls will be released simultaneously. By trial, adjust the inclination of the track so that the ball will roll the whole length in 4 seconds, as measured by the swings of the pendulum. By repeated trials, verify the accuracy of the figures in the second and fourth columns of the table given in Experiment 54.

2. From the frame of a small pulley running with little friction suspend a weight of about 2 pounds. Place the wheel of the pulley so that it will run on a No. 10 wire tightly stretched between opposite sides of the laboratory, one end of the wire being a little higher than the other. The wire may be tightened with a turnbuckle. Just above the wire, and parallel with it, stretch a cord. From the upper end of the wire start the pulley with its load, and note the point where it is at the end of 3 seconds. If the distance traversed in the 3 seconds is not at least 9 feet, increase the inclination of the wire. Mark the point where the pulley is at the end of the third second by a strip of paper hung from the cord so that its lower end will be struck by the top of the pulley as it passes. Mark the point on the cord above the starting point of the pulley by tying a thread

there. Divide the intervening distance into 9 equal parts. Hang similar paper strips from the cord, at distances of 5 such equal parts and of 8 such equal parts above the strip already hung, and of 7 such equal parts and 16 such equal parts below it.

Swing the pendulum that vibrates seconds. As its thread passes a vertical line on the wooden support drawn downward from the needle, start the pulley, and see if it taps the successive strips as the pendulum successively passes the vertical line. If the weight carried by the pulley is of iron, the weight and the pendulum-bob may be simultaneously released as in Exercise 1.

NOTE. — A good Atwood machine is an expensive piece of apparatus, and unfortunately many schools and laboratories have none. If any particular school is thus equipped, the teacher should provide for its use in the verification of the laws of falling bodies, the approximate determination of the acceleration due to gravity, etc.

3. Modify Experiment 53 by using two iron balls of different mass supported by the attraction of two electromagnets that are in the same circuit. Tie the magnets to a stick, and hold them and the magnetically supported balls from an upper window. When you are sure that the balls are at the same level, break the circuit. A co-worker standing on the ground will report whether the different masses make the journey in the same time.

4. A ball is thrown horizontally with a velocity of 20 feet a second. Using any convenient scale and the cross-section paper, map the position of the ball at the end of half seconds for 5 seconds. Bend the thin blade of a hack-saw or other flexible bar so that its edge will pass through as many of these points as possible. Along the side of the blade, trace a pencil mark to represent the path of the ball. See Exercise 11, p. 95, and compare your curve with Fig. 63. If any point thus located lies much out of the curve, reëxamine your work for that point. On your diagram, lay off and measure coördinates for the point that represents the position of the ball at the end of 3.25 seconds and 5.5 seconds from the beginning of its fall. Compare the results of your work with corresponding results obtained by computation. In tracing the curve, run the pencil to the lower end of the saw blade.

5. From a rectangular wooden block about 30 × 23 × 4 cm., cut a semi-cycloid, thus shaping the piece marked *B* in Fig. 64. Cut a groove in the curved edge, and fasten the block against the blackboard so that its long edge shall be vertical. A small ball that has rolled down the cycloidal path will be projected with a con-

stant horizontal velocity and an accelerated vertical velocity. Let one of two pupils working together adjust, by repeated trials, a ruler so that the projected ball will just touch it, and thence determine and mark the point passed over by the center of the ball. In this way determine the loci of points sufficiently numerous to plot on the blackboard the path described by the center of the projected ball. From the center of the ball at the lowest point of its cycloidal path draw a horizontal line, and mark off a number of equal spaces upon it. These will represent the horizontal motions of the ball in equal intervals of time. From each division on this line, draw a vertical line, l, to the plotted path, and measure the lengths of these lines. They represent the spaces fallen in the several intervals of time. Show that, for each interval of time, $l = kt^2$, k being some constant. If the horizontal intervals are made equal to the horizontal speed of the ball per second, k will equal $\frac{1}{2}g$. From the measurements made on the blackboard, plot the curve on cross-section paper.

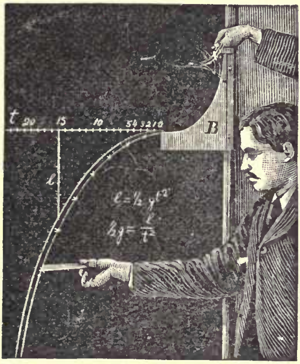


FIG. 64.

V. THE PENDULUM.

112. A Simple Pendulum *is a single material particle supported by a line without weight, and capable of oscillating about a fixed point.* Such a pendulum has a theoretical but not an actual existence. Its properties may be approximately determined by experimenting with a small lead ball suspended by a fine thread.

113. Motion of the Pendulum. — When the pendulum is drawn from its vertical position, the force of gravity, MG, is resolved into two components, one of which, *MC*, produces pressure at the point of support, while the other, *MH*, acts at right angles to it, producing motion toward *N*. As the pendulum approaches *N*, its kinetic energy increases. This energy carries the weight beyond *N* toward *O*, against the action of the continually increasing component, *OP*. By the time the pendulum has arrived at *O*, its kinetic energy has been wholly transformed into potential energy. Then *OP* pulls the weight toward *N* again, transforming the potential energy into kinetic, which, in turn, carries the weight once more toward *M*. Thus the pendulum oscillates for an indefinite time by the alternate action of gravity and its acquired energy of motion.

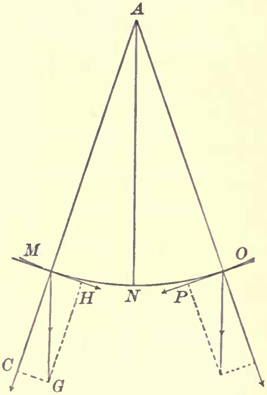


FIG. 65.

114. Definitions. — The motion from one extremity of the arc through which a pendulum swings to the other is called an *oscillation*. The time occupied in moving over this arc is called the *time* or *period of oscillation*. The angle measured by half this arc is called the *amplitude of oscillation*. The trip from *M* to *O* is an oscillation. The angle, *MAN*, is the amplitude of oscillation.

(*a*) The motion from *M* to *O* and back again, one "swing-swang," is sometimes called a "complete vibration." The time occupied by the round trip, or in passing from any point to its next passage in the same direction through the same point, is sometimes called a "complete period."

Experiment 56. — Suspend three lead bullets and a small iron ball as shown in the accompanying figure. The lengths of the threads, measured between the points of support and the centers of the balls, should be as $1:\frac{1}{4}:\frac{1}{9}$; e.g., 1 yard, 9 inches, and 4 inches respectively. Set one of the pendulums swinging through a small arc, and count the oscillations made in 10 seconds. Set the same pendulum swinging through a somewhat larger arc, and count the oscillations as before. Record and compare results. Repeat the experiments with each of the pendulums, recording and comparing results in each case. Note the effect of amplitude or of mass on the period of oscillation.

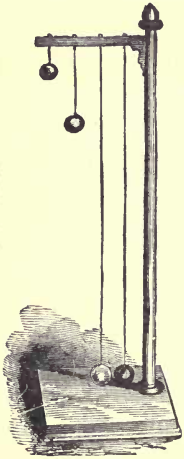


Fig. 66.

From your notes, or by fresh experiment, determine the period of each pendulum, and observe the relation between the period of oscillation and the length of the pendulum.

Place a magnet under the iron ball, so that when the latter swings it will just clear the end of the magnet. Swing the iron pendulum, and count the number of oscillations made in 10 seconds. The attraction of the magnet being added to that of the earth, the acceleration is increased and the period is lessened.

115. Laws of the Pendulum. — When the amplitude of oscillation does not exceed three degrees, the period of oscillation depends mainly upon the length of the pendulum and the acceleration due to gravity. Representing period by t, and length by l, the relation is expressed by the formula

$$t=\pi\sqrt{\frac{l}{g}}.$$

The following laws are consistent with this formula and with the results of numberless experiments: —

(1) *At any given place, the vibrations of a given pendulum are isochronous*, i.e., are made in equal periods.

(2) *The period of oscillation is independent of the material or the mass of the pendulum.*

(3) *The period of oscillation varies directly as the square root of the length.*

(4) *The period of oscillation varies inversely as the square root of the acceleration.*

116. The Compound Pendulum. — *Any pendulum other than the simple or ideal pendulum is a compound pendulum.* In its most common form, it consists of a slender rod, flexible at the top, and carrying at the bottom a heavy mass of metal known as the *bob*.

117. The Seconds Pendulum. — *At any given place, a seconds pendulum is one that makes a single oscillation in a second.* At the sea-level, its length is about 39 inches at the equator and about 39.2 inches near the poles. Its value at the sea-level at New York may be found by making $t=1$, and $g=980.19$ cm., in the formula

$$t=\pi\sqrt{\frac{l}{g}},$$

and solving the equation for the value of l.

(*a*) The length of the seconds pendulum being known, the length of any other pendulum may be found when the period of oscillation is given, or the period of oscillation may be found when the length is given. As the seconds pendulum is inconveniently long, use is often made of one one-fourth as long, which oscillates in half seconds.

Center of Oscillation.

Experiment 57. — Drive a small wire nail through a flat board of any form, at some point near its edge, as shown in Fig. 67. Hold the ends of the wire by the finger and thumb, and allow the board to hang in a vertical plane. Fasten a small bullet to the end of a thread, and pass the thread over the wire so that the bullet hangs close to the board. Move the hand that supports the wire horizontally and in the plane of the board. Board and bullet will swing as pendulums. If one swings more rapidly than the other, lengthen or shorten the string until they swing together. With the thread at this length, and board and bullet hanging in equilibrium, mark the point on the board opposite the center of the ball. Holding the board by the wire as before, move it with varied, sudden, and irregular motions in the plane of the board. The bullet will not quit the marked place on the board.

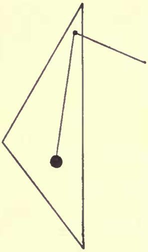


FIG. 67.

118. Centers of Suspension and Oscillation.

118. Centers of Suspension and Oscillation. — In every pendulum not simple, the parts near the center of suspension tend to move faster than those further away, and force the latter to move more rapidly than they otherwise would. Between these, there is a particle that moves, of its own accord, at the rate forced upon the others. This particle fulfills all the conditions of a simple pendulum that has the period of the compound pendulum. Its position is called the *center of oscillation or percussion.*

(*a*) Fig. 68 represents a wooden bar, suspended so as to have freedom of motion about the point *S*, which thus becomes the center of suspension. *G* indicates the center of mass, and *O* the center of oscillation. *S* and *O* are interchangeable; i.e., if the pendulum is suspended from its center of oscillation, the period remains the same.

FIG. 68.

119. The Real Length of a Pendulum. — If we consider the length of the compound pendulum to be the distance between the centers of suspension and oscillation, all the laws of the simple pendulum become applicable to the compound pendulum.

120. Uses of the Pendulum. — The continued motion of a clock is due to the force of gravity acting upon the weights, or to the elasticity of the spring. But the weights have a tendency toward positively accelerated motion, and the spring toward negatively accelerated motion. Either defect would be fatal in a timepiece. The properties of the pendulum set forth in the first law enable us to regulate this motion, and to make it available for the desired end.

The pendulum is also used to determine the relative and absolute acceleration of gravity at different places, and in this way the figure of the earth. Having at any given place a pendulum of known length, its period may be determined and the value of *g* computed from the formula given in § 115.

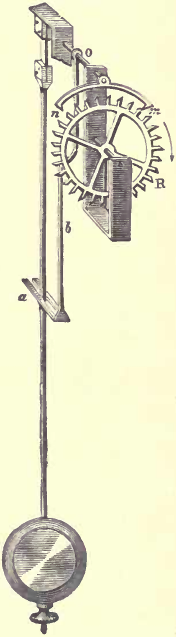


FIG. 69.

(*a*) A pendulum has a strong tendency to maintain its plane of oscillation, a fact that has been used in the experimental demonstration of the rotation of the earth upon its axis. The chief function of the wheel-work of a clock is to register the number of the vibrations of the pendulum. If the clock gains time, the pendulum is lengthened by lowering the bob; if it loses time, the pendulum is shortened by raising the bob.

CLASSROOM EXERCISES.

No.	Inches.	Oscillations.	Period.	No.	Cm.	Oscillations.	Period.
1	?	20 per min.	?	11	99.33	?	?
2	?	30 “	?	12	?	?	2 sec.
3	30	?	?	13	?	?	2 min.
4	16	?	?	14	24.83	?	?
5	?	?	$\frac{1}{4}$ sec.	15	?	8 per sec.	?
6	?	?	$\frac{1}{4}$ min.	16	397.32	?	?
7	39.37	? per min.	?	17	11.03	?	?
8	?	10 “	?	18	?	?	10 sec.
9	10	? per sec.	?	19	2,483.25	?	?
10	?	1 per min.	?	20	?	?	4 sec.

21. How will the periods of oscillation of two pendulums compare, their lengths being 4 feet and 49 feet respectively? *Ans.* As 2 : 7.

22. Of two pendulums, one makes 70 oscillations a minute, the other, 80 oscillations a minute. How do their lengths compare? *Ans.* As 64 : 49.

23. If one pendulum is 4 times as long as another, what are their relative periods of oscillation?

24. The length of a seconds pendulum being 39.1 inches, what must be the length of a pendulum to oscillate in $\frac{1}{5}$ second?

25. How long must a pendulum be to oscillate (*a*) once in 8 seconds? (*b*) In $\frac{1}{8}$ second?

26. How long must a pendulum be to oscillate once in $3\frac{1}{2}$ seconds?

27. Find the length of a pendulum that will oscillate 5 times in 4 seconds. *Ans.* 25.02 + inches.

28. A pendulum 5 feet long makes 400 oscillations during a certain time. How many oscillations will it make in the same time after the pendulum rod has expanded 0.1 of an inch?

29. At Paris, $g = 981$ cm. Determine the length of the seconds pendulum at that place.

30. A lead ball is suspended as a pendulum. From the center of the ball it is 83 inches to its center of suspension. The pendulum oscillates 206 times in 5 minutes. From the formula given in § 115, determine the acceleration due to gravity at the time and place of the experiment. *Ans.* 32.188 feet.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Wooden bars, etc., for pendulums; a pendulum-clock; a piece of sheet iron; mercury; a telegraph sounder or an electric bell.

1. Set up a pendulum of length as great as you can conveniently. Set up another that oscillates just twice as often in a given time. Determine the ratio between the lengths of the two pendulums. Shorten the shorter pendulum until it oscillates three times as fast as the other. Determine the relative lengths as before. Shorten the shorter pendulum again until it oscillates four times as fast, and find the ratio as before. In your notebook, record the data obtained, using the following form, and placing the ascertained ratios in the places of x, y, and z: —

Relative Numbers of Oscillations.	*Relative Lengths.*
1	1
2	x
3	y
4	z
5	?
6	?

Can you see any law or rule governing in such cases? Try, without experiment, to put the proper figures in the places of the two interrogation points.

2. Set up the pendulum used in Exercise 1, p. 115, or a similar one, and adjust its length so that it will oscillate 60 times in 60 seconds. The eye of the needle should be not much larger than is necessary for the thread used. Measure the distance from the center of the bob to the needle. From the data thus secured, compute the value of g.

3. Set up a similar pendulum, but with a shorter thread. Adjust the length of the pendulum by turning the screw until the pendulum oscillates 60 times in 30 seconds. Measure the length of the pendulum. Compare its period with that of the one used in Exercise 2. Compare its length with that of the one used in Exercise 2. How do these comparisons tally with the statement made in § 115 (3)?

4. Shorten the thread of the pendulum used in Exercise 3 until the pendulum oscillates 60 times in 20 seconds. Measure the length of the pendulum. Compare its period with that of those used in Exercises 2 and 3. Compare its length with the lengths of those. How do these results conform to the law referred to above?

5. Using these pendulums successively, test the accuracy of the law given in § 115 (1).

6. Using either of these pendulums and another prepared by you for that purpose, test the accuracy of the second law given in § 115.

7. On a stout thread, fasten 5 or 6 lead bullets at successive intervals of 10 cm., and suspend the combination as a pendulum. Swing it as a pendulum. Does the string retain its rectilinear form while the compound pendulum is oscillating? Account for any observed difference in this respect between this pendulum and those previously used.

8. Through the laboratory meter stick or a similar strip of wood, drill or burn a small hole 3 cm. from one end. Using this as a center of suspension, locate the center of oscillation. Determine the real length of the meter-stick pendulum. Suspend a bullet by a single thread, and adjust its length so that it will swing with the same period as the meter-stick pendulum. Compare the length of this pendulum with the distance between the centers of suspension and oscillation of the other pendulum.

9. Remove the dial of a clock, and study the movements of the escapement (*mn* in Fig. 69), and of the escapement wheel, *R*. What does it enable the lifted weights or the coiled spring of the clock to do to the pendulum? What does it enable the pendulum to do to the weights or the spring? What would happen to the weights or to the spring if the escapement should be suddenly removed? What would happen to the pendulum if the escapement should be removed? How many times must the pendulum oscillate that the escapement wheel may turn around once?

10. Suspend a heavy metallic ball by two fine wires that are gripped at the lower horizontal edge of a metallic clamp. To the bottom of the ball, solder a short pointed iron or platinum wire. (See Fig. 44.) Make a slight depression in a small plate of sheet iron, and solder a small copper wire to the plate. Fasten the plate beneath the pendulum bob so that the wire pointer will just touch a drop of mercury placed in the depression in the plate. The mercury globule

should be so placed that when the pendulum swings, its pointer shall, at each oscillation, touch the mercury, but not the plate. Connect the wire that is soldered to the plate with one pole of a voltaic cell or battery. Connect the other pole of the battery through a telegraph sounder or single-stroke electric bell with the end of the pendulum wire where it protrudes above the supporting clamp. Swing the pendulum. As the pointer passes through the mercury, it will "close the circuit" of the battery, and the sounder will click or the bell will strike. Adjust the length of the pendulum until it gives 60 signals in 60 seconds. Make this pendulum a permanent feature of the laboratory.

11. From a board, cut two isosceles-triangular pieces with sides of 4 and 24 inches. Insert a small screw-eye at the pointed end of one, and another screw-eye at the middle of the 4-inch side of the other. Suspend the wooden pieces by the screw-eyes, and swing them as pendulums. Determine the period of each, and thence compute the real lengths of the two pendulums.

VI. SIMPLE MACHINES.

121. Machines. — *In mechanics, the word "machine" signifies an instrument for the conversion of motion or the transference of energy.* Thus, a machine may be designed to convert rapid motion into slow motion; e.g., a crowbar. There are six simple machines, — the lever, the wheel and axle, the pulley, the inclined plane, the wedge, and the screw.

FIG. 70.

122. Weight and Power. — The action of a machine involves two forces, the weight and the power. *The* *power signifies the magnitude of the force that acts upon one part of the machine; the weight signifies the magnitude of the force exerted by another part of the machine upon some external resistance.* The general problem relating to machines is to find the ratio between power and weight; i.e., to determine the "mechanical advantage" of the machine.

(a) It is common in elementary discussions to neglect friction, and to assume that the parts of the machine are perfectly rigid and without weight.

123. A Machine cannot Create Energy. — *No machine can create or increase energy.* In fact, the use of a machine is accompanied by a waste of the energy that is needed to overcome the resistances of friction, the air, etc. A part of the energy exerted must, therefore, be used upon the machine itself, thus diminishing the amount that can be transmitted or utilized for doing the work in hand.

124. General Laws of Machines. — The operations of a machine are subject to the principles of "the conservation of energy;" the work done by the power equals the work done on the weight.

(1) *The power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves:* $Pl = Wl'$.

(2) *The power multiplied by its velocity equals the weight multiplied by its velocity:* $Pv = Wv'$.

125. Efficiency of Machines. — As was hinted in § 123, part of the work done upon a machine is expended in overcoming resistances that correspond to waste, — resistances other than that which the machine was designed to overcome, such as friction, etc. *The ratio that the useful work done by the machine bears to the total work done on the machine is called the efficiency of the machine.* If this ratio could be brought up to unity, we should have a perfect machine, — the impossible thing that would supply "perpetual motion."

(*a*) Whenever we find that a machine does less work than was done upon it, we should bear in mind that the missing energy has not been destroyed. Mechanical energy has been transformed into a familiar form of molecular energy, and exists somewhere in the form of heat.

126. Impediments to Motion. — The impediments to motion most frequently met in the use of machines result from rigidity or from friction. The first of these is familiarly illustrated in the stiffness of a pulley rope. The second will receive further consideration in the following paragraph. Due allowance must be made for these hindrances in all close calculations of the useful work of any machine. If the machine is designed merely to support a load, the greater the impediments, the less the power required; if the machine is designed to move a load, the greater the impediments, the greater the power required.

127. Friction *is the resistance that a moving body meets from the surface on which it moves*, and may be rolling or sliding. It is due partly to the adhesion of bodies, but more largely to their roughness. Friction proper is independent of the velocity of the motion and of the area of contact. It depends upon the nature of the two surfaces and upon the pressure upon them, and varies directly as such pressure. The quotient arising from dividing the force necessary to keep the body in motion by the normal pressure that the body exerts on the surface over which it moves, i.e., *the ratio between the friction and the pressure, is called the coefficient of friction.*

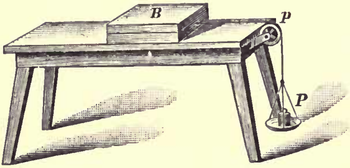


FIG. 71.

(*a*) Friction is generally lessened by polishing and lubricating the surfaces that move upon each other, and often by making the two bodies of different material. The axles of railway cars are made of steel, the boxes in which they turn are made of brass, the surfaces are made smooth and kept oiled. In spite of all these precautions, the axle often becomes heated by friction to such an extent as to render it necessary to stop the train.

128. A Lever *is an inflexible bar freely movable about a fixed axis called the fulcrum.* Every lever is said to have two arms. The power arm is the perpendicular distance from the fulcrum to the line in which the power acts; the weight arm is the perpendicular distance from the fulcrum to the line in which the weight acts. If the arms are not in the same straight line, the lever is called a bent lever.

(*a*) There are three classes of levers, depending upon the relative positions of power, weight, and fulcrum.

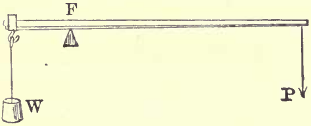


FIG. 72.

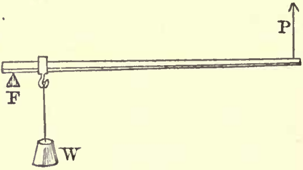


FIG. 73.

(1) If the fulcrum is between the power and weight (*PFW*), the

lever is of the first class (Fig. 72); e.g., crowbar, balance, steelyard, scissors, pincers.

(2) If the weight is between the power and the fulcrum (*PWF*), the lever is of the second class (Fig. 73); e.g., cork-squeezer, nutcracker, wheelbarrow.

(3) If the power is between the weight and the fulcrum (*WPF*), the lever is of the third class (Fig. 74); e.g., fire-tongs, sheep-shears.

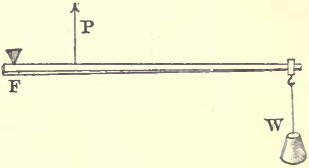


FIG. 74.

(*b*) In the bent lever, represented in Fig. 75, and acted upon by two forces not parallel, the arms are not *FP′* and *FW′*, but *FP* and *FW*.

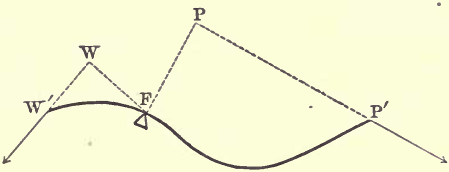


FIG. 75.

129. Mechanical Advantage of the Lever. — The general laws of machines may be adapted to the lever as follows: *A given power will support a weight as many times as great as itself as the power arm is times as long as the weight arm.*

(*a*) The ratio between the arms of the lever will be the same as the ratio between the velocities of the power and the weight, and the same as the ratio between the distances moved by the power and the weight. If the power arm is twice as long as the weight arm, the power will move twice as fast and twice as far as the weight does. The power and weight are inversely proportional to the corresponding arms of the lever:

$$P : W :: \overline{WF} : \overline{PF}.$$

The power multiplied by the power arm equals the weight multiplied by the weight arm:

$$P \times \overline{PF} = W \times \overline{WF}.$$

NOTE. — In all experimental work, the lever should be loaded so as to be in equilibrium before the power and weight are applied. It is to be noticed that, when we speak of the power multiplied by the power arm, we refer to the abstract numbers representing the power and power arm. We cannot multiply pounds by feet, but we can multiply the number of pounds by the number of feet.

130. The Moment of a Force *with respect to a given point is its tendency to produce rotation about that point, and is measured by the product of the numbers representing respectively the magnitude of the force and the perpendicular distance between the given point and the line of the force.*

(*a*) In the case of the lever represented in Fig. 72, the weight arm is 8 mm., and the power arm is 30 mm. Suppose that the power is 4 grams and represent the weight by x. Then the moment of the force acting on the power arm will be represented by $(4 \times 30 =)$ 120, and the moment of the force acting on the weight arm by $8x$.

131. Moments Applied to the Lever. — Sometimes several forces act upon one or both arms of a lever, in the same or in opposite directions. Under such circumstances, *the lever will be in equilibrium when the sum of the moments of the forces tending to turn the lever in one direction is equal to the sum of the moments of the forces tending to turn the lever in the other direction.* Representing the moments of the several forces acting upon the lever represented in the figure by their respective letters and numerical values,

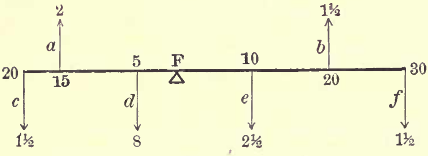


FIG. 76.

$$\begin{array}{rl|l} & b+c+d=a+e+f & 30+30+40=30+25+45. \\ \text{or} & c+d-a=e+f-b. & 30+40-30=25+45-30. \end{array}$$

132. The Balance *is essentially a lever of the first class, having equal arms.* The beam carries a pan at each end, — one for the weights used, the other for the article to be weighed.

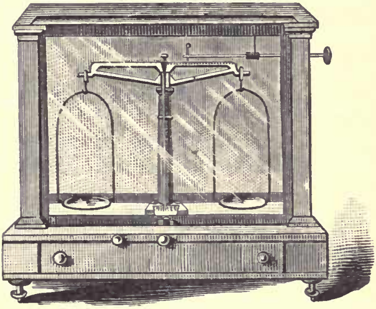


FIG. 77.

(*a*) Dishonest dealers sometimes use balances with arms of unequal lengths. When buying, they place the goods on the shorter arm; when selling, on the longer. The cheat may be exposed by changing the goods and weights to the opposite sides of the balance. The true weight may be found by weighing the article first on one side and then on the other, and taking the geometrical mean of the two false weights; that is, by finding the square root of the product of the two false weights.

(*b*) The true weight of a body may be found with a false balance in another way. Place the article to be weighed in one pan, and counterpoise it, as with shot or sand placed in the other pan. Remove the article, and place known weights in the pan until they balance the shot or sand in the other pan. These known weights will represent the true weight of the article in question.

133. Compound Lever. — Sometimes it is not convenient to use a lever sufficiently long to make a given power support a given weight. A combination of levers, called a compound lever, may then be used. Hay scales may be mentioned as a familiar illustration of the compound lever. In this case we have the following statical law: —

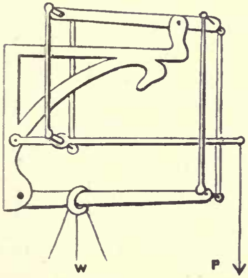


FIG. 78.

The continued product of the power and the lengths of the alternate arms, beginning with the power arm, equals the continued product of the weight and the lengths of the alternate arms, beginning with the weight arm.

CLASSROOM EXERCISES.

1. If a power of 50 pounds acting upon any kind of machine moves 15 feet, (*a*) how far can it move a weight of 250 pounds? (*b*) How great a load can it move 75 feet?

2. If a power of 100 pounds acting upon a machine moves with a velocity of 10 feet per second, (*a*) to how great a load can it give a velocity of 125 feet per second? (*b*) With what velocity can it move a load of 200 pounds?

3. A lever is 10 feet long with its fulcrum in the middle. A power of 50 pounds is applied at one end. (*a*) How great a load at the other end can it support? (*b*) How great a load can it lift?

Ans. (*b*) Anything less than 50 pounds.

4. The power arm of a lever is 10 feet. The weight arm is 5 feet. (*a*) How long will the lever be if it is of the first class? (*b*) If it is of the second class? (*c*) If it is of the third class?

5. A bar 12 feet long is to be used as a lever, keeping the weight 3 feet from the fulcrum. (*a*) What class or classes of levers may it represent? (*b*) What weight can a power of 10 pounds support in each case?

6. The length of a lever is 10 feet. Four feet from the fulcrum and at the end of that arm is a weight of 40 pounds; two feet from the fulcrum, on the same side, is a weight of 1,000 pounds. What force at the other end will counterbalance both weights?

Ans. 360 pounds.

7. At the opposite ends of a lever 20 feet long, two forces are acting whose sum is 1,200 pounds. The lengths of the lever arms are as 2 to 3. What are the two forces when the lever is in equilibrium?

8. The length of a lever is 8 feet, and its fulcrum is in the center. A force of 10 pounds acts at one end; 1 foot from it is another of 100 pounds; 3 feet from the other end is a force of 100 pounds. The direction of all the forces is downward. Where must a downward force of 80 pounds be applied to balance the lever?

Ans. 3 feet from the fulcrum.

9. The length of a lever, *ab*, is $6\frac{1}{4}$ feet. The fulcrum is at *c*. A downward force of 60 pounds acts at *a*; one of 75 pounds, at a point, *d*, between *a* and *c*, $2\frac{3}{4}$ feet from the fulcrum. Required the amount of equilibrating force acting at *b*, the distance between *b* and *c* being $\frac{3}{4}$ of a foot.

10. On a lever, *ab*, a downward force of 40 pounds acts at *a*, 10 feet from fulcrum, *c*; on the same side, and $6\frac{1}{2}$ feet from *c*, a 56-pound force, *d*, acts upward. The distance, *bc*, is 3 feet. A downward force of 96 pounds acts at *b*. (*a*) Where must a fourth force of 28 pounds be applied to balance the lever, and (*b*) what direction must it have?

11. A beam 18 feet long is supported at both ends. A weight of 1 ton is suspended 3 feet from one end, and a weight of 14 hundred-weight 8 feet from the other end. Give the pressure on each point of support. *Ans.* $2{,}288\frac{8}{9}$ pounds at one end.

12. The length of a lever is 3 feet. Where must the fulcrum be placed so that a weight of 200 pounds at one end shall be balanced by 40 pounds at the other end?

13. In one pan of a false balance, a roll of butter weighs 1 pound 9 ounces; in the other, 2 pounds 4 ounces. Find the true weight.

14. A and B, at opposite ends of a bar 6 feet long, carry a weight of 300 pounds suspended between them. A's strength being twice as great as B's, where should the weight be hung?

15. A and B carry a quarter of beef weighing 450 pounds on a rod between them. A's strength is $1\frac{1}{4}$ times that of B's. The rod is 8 feet long. Where should the beef be suspended?

16. The length of a lever is 16 feet. At one end is a weight of 100 pounds. What power applied at the other end, $3\frac{1}{2}$ feet from the fulcrum, is required to move the weight?

17. A power of 50 pounds acts upon the long arm of a lever of the first class. The arms of this lever are 5 and 40 inches respectively. The other end acts upon the long arm of a lever of the second class. The arms of this lever are 6 and 33 inches respectively. (*a*) Figure the machine. (*b*) Find the weight that may be thus supported. (*c*) What power will support a weight of 4,400 kilograms?

18. A uniform bar of metal 10 inches long weighs 4 pounds. A weight of 6 pounds is hung from one end of the bar. Determine the position of the fulcrum upon which the loaded bar will balance.

LABORATORY EXERCISES.

Additional Apparatus, etc. — A handful of wheat; weights made of bags containing sand; a tin can of known weight to serve as a scale-pan; wooden bars, blocks, and board as described below; plumbago powder; a stout coverless dry-goods box to replace the frame shown in Fig. 81, and a wooden cylinder about 8 inches in diameter and long enough to reach across the box from side to side.

1. Weigh five samples of wheat, each containing 20 grains. Determine the weight of the average grain of wheat and the number of such grains of wheat in a bushel of 60 pounds.

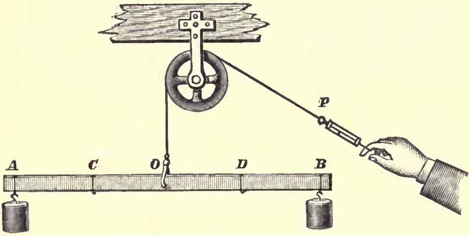


FIG. 79.

2. Support a wooden bar, preferably graduated (the yardstick or the meter rod will answer admirably), by a pin and clevis at the middle of its length, as shown in Fig. 79. Put the bar in equilibrium (as in all such experimental cases), and provide stops 2 or 3 inches below each end of the bar to limit its oscillations. Support equal and known weights by thread loops at equal distances from the middle of the lever, and compare the reading of the dynamometer with the sum of the suspended weights. Do they agree? If not, why not? Make the necessary correction.

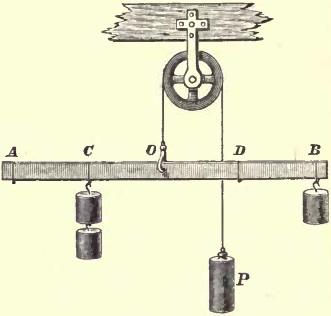


FIG. 80.

3. Modify the apparatus used in Exercise 2 by removing the dynamometer and adding a counterpoise, as shown in Fig. 80. Replace the weight at *A* with one twice as heavy, and shift its position until the bar is in equilibrium. Note the distances of *C* and *B* from *O*. Using either form of apparatus, load the two arms of the lever with weights of varying ratios,

and note the agreement or disagreement of your results with the several statements made in §§ 129 and 131.

4. Provide two additional fixed pulleys and use the apparatus in an experimental verification of the equations given in § 131.

5. Take two points at *slightly* different distances from O, the fulcrum of the balance-beam. Suspend an unknown weight from one of these points, and counterpoise it with known weights at the other point so taken. Verify the statements made in § 132 (*a*).

6. From one of the points taken as directed in Exercise 5, suspend a tin can, and put the lever in equilibrium. From the other of those two points, suspend a body of unknown weight, and find its true weight by the process of double weighing, as described in § 132 (*b*).

7. Get a board 4 or 5 feet long and about 1 foot wide; also a wooden block about 2 × 4 × 8 inches. Plane the board on one side, and the block on one of its 2 × 8 inch and on one of its 4 × 8 inch faces. Insert a small screw-eye or screw-hook at the middle of one of its 2 × 4 inch faces. Weigh the block. Attach a cord to the screw-eye so that the block may be drawn lengthwise on the board, the other end of the cord being attached to a spring-balance or to a scale-pan, as shown in Fig. 71. Place the board horizontal, with its rough surface up. Place the 2 × 8 inch rough surface of the block on the board, and draw the block, using the spring-balance or sufficient weights, and keeping the cord horizontal. Ascertain what force is necessary to start the load. Determine the force that will just maintain the sliding motion while you keep tapping on the table. Determine the coefficient of friction for these two surfaces. Find the averages of several tests.

Place the block upon its 4 × 8 inch rough surface, and repeat the work. How does the coefficient of friction now compare with that obtained in the first set of tests?

Turn the board over, and place the 2 × 8 inch smooth face of the block upon it. Make a similar set of tests.

Place the block on its 4 × 8 inch smooth face, and make another set of tests. How does the coefficient obtained in the last set of tests compare with that of the third set? How do the coefficients of the third and fourth sets compare with those of the first and second sets?

Repeat the third and fourth sets of tests with weights on the blocks so that the load moved shall be successively 2, 3, and 4 times the weight of the block.

Smear the smooth surfaces of the board and the block with powdered graphite or plumbago, such as is sold for chains of bicycles, and repeat the tests with the heavier loads previously used.

Record all of the conclusions that you draw from this series of experiments.

8. Place the block with its broad and smooth face upon two round lead pencils that lie parallel upon the smooth face of the board. Determine the coefficient of rolling friction, and compare it with the coefficient of sliding friction for the same surfaces.

9. Wind the draw-cord several times around a cylinder, and arrange apparatus as shown in Fig. 81. Load the cylinder with four equal weights carried on cords, so that the weight of the cylinder and its load shall equal the weight of the block and some one of its loads as used in Exercise 7. Determine the coefficient of rolling friction, and compare it with that obtained for sliding friction under an equal pressure.

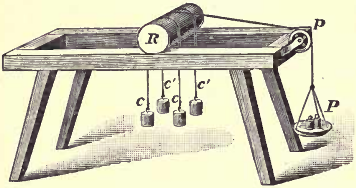


Fig. 81.

10. Prepare a slightly tapering pine rod about 15 inches long and about 1 inch square at the larger end. Balance it upon a knife-edge or other sharp support to determine the distance of the center of mass from the ends of the rod. The block marked *A* in Fig. 17 will answer as a fulcrum for this and the other purposes of this exercise. Indicate the line of support by a pencil mark across the rod. Weigh the rod accurately. Half an inch from the heavy end of the rod, suspend by a thread loop a weight of 20 grams, and so adjust the fulcrum that the lever thus loaded will balance. In all such cases, see that the fulcrum-edge is exactly crosswise the length of the lever. Measure the distances between the fulcrum and the weight, and the fulcrum and the center of mass. Determine the moment of the 20 grams and of the weight of the lever, and see how the two compare. Shift the position of the 20 grams weight, and repeat the work. Increase the suspended weight to 25 or 30 grams, and repeat the previous tests. Record your conclusions.

11. Suspend a weight of 100 grams 5 centimeters from one end of a meter bar, and a weight of 500 grams 5 centimeters from the other end. Find the point from which the bar thus loaded must be suspended in order that the "system" may just balance. From the

principles of §§ 94 and 131, calculate the weight of the meter bar. Verify the result by actual weighing.

134. The Wheel and Axle *consists of a wheel united to a cylinder in such a way that they may turn together on a common axis.* It is a modified lever of the first or second class.

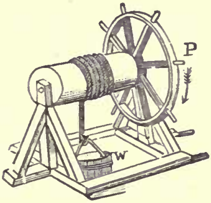


FIG. 82.

(*a*) Considered as a lever, the fulcrum is at the common axis, while the arms of the lever are the radii of the wheel and of the axle. The usual arrangement is to take *ac*, the radius of the wheel, as the power arm, and *bc*, the radius of the axle, as the weight arm.

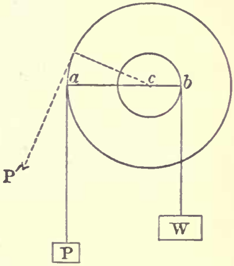


FIG. 83.

135. Mechanical Advantage of the Wheel and Axle. — Evidently, what was said concerning the advantage of the lever is equally applicable here: —

$P : W :: \overline{WF} : \overline{PF}$, or

$P : W :: r : R$,

the radii of the wheel and of the axle respectively being represented by R and r. But

$$r : R :: d : D, \text{ and } r : R :: c : C.$$

In other words, *the mechanical advantage of this machine equals the ratio between the radii, diameters, or circumferences of the wheel and of the axle.*

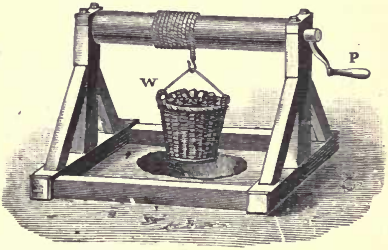


FIG. 84.

136. Modifications of the Wheel and Axle. — It is not necessary that an entire wheel be present, a single spoke or radius being sufficient for the application of the power, as in the case of the windlass (Fig. 84) or the capstan (Fig. 85).

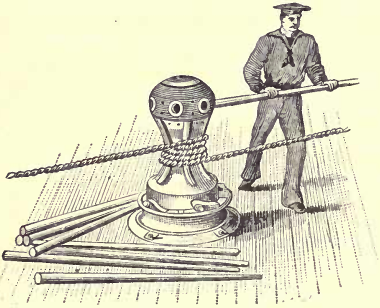


FIG. 85.

(*a*) In the differential or Chinese windlass, different parts of the cylinder have different diameters, the rope winding upon the larger and unwinding from the smaller parts. By one revolution, the load is lifted a distance equal to the difference between the circumferences of the two parts of the axle.

(*b*) The advantage of the wheel and axle may be increased by combining several, so that the axle of the first may act on the wheel of the second, and so on. The arrangement is closely analogous to the compound lever. The transmission of motion may be effected in three or more ways: —

(1) By the friction of their circumferences, as in some sewing machines.

(2) By bands or belts, as in a turning lathe, bicycle, or sewing machine.

(3) By teeth or cogs, as in Fig. 86. In any case, the advantage may be computed by applying the general laws of machines (§ 124).

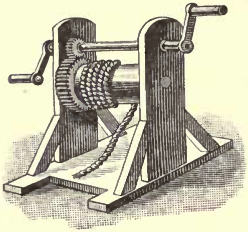


FIG. 86.

CLASSROOM EXERCISES.

1. The pilot wheel of a boat is 3 feet in diameter; the axle, 6 inches. The resistance of the rudder is 180 pounds. What power applied to the wheel will move the rudder?

2. Four men are hoisting an anchor of 1 ton weight. The barrel of the capstan is 8 inches in diameter. The circle described by the handspikes is 6 feet 8 inches in diameter. How great a pressure must each of the men exert?

3. With a capstan, four men are raising a 1,000-pound anchor. The barrel of the capstan is a foot in diameter. The handspikes used are 5 feet long. Friction equals 10 per cent of the weight. How much force must each man exert to raise the anchor?

4. The circumference of a wheel is 8 feet; that of its axle, 16 inches. The weight, including friction, is 85 pounds. How great a power will be required to raise it?

5. A power of 70 pounds, on a wheel whose diameter is 10 feet, balances 300 pounds on the axle. Give the diameter of the axle.

6. An axle 10 inches in diameter, fitted with a winch 18 inches long, is used to draw water from a well. (*a*) How great a power will it require to raise a cubic foot of water which weighs 62½ pounds? (*b*) How much to raise 20 liters of water?

7. A capstan whose barrel has a diameter of 14 inches is worked by two handspikes, each 7 feet long. At the end of each handspike a man pushes with a force of 30 pounds; 2 feet from the end of each handspike a man pushes with a force of 40 pounds. Required the effect produced by the four men.

8. How long will it take a horse, working at the end of a bar 7 feet long, the other end being in a capstan which has a barrel of 14 inches' diameter, to pull a house through 5 miles of streets, if the horse walks at the rate of 2½ miles an hour?

9. Give a good definition and illustration of "inductive reasoning." (Get your information from any available source, but get it.)

137. A Pulley *is a wheel having a grooved rim for carrying a rope or other line, and turning on an axis carried in a frame, called a pulley block.* The pulley is fixed if the block is stationary (Fig. 87); the pulley is movable if the block moves during the action of the power (Fig. 88).

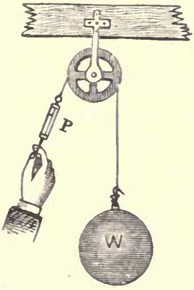


FIG. 87.

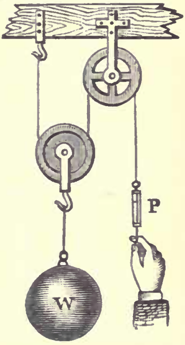


FIG. 88.

(*a*) The pulley is a lever with equal arms of the first or second class, but, when it moves, the attachments of the forces are moved. The underlying fact that enables the pulley to afford any mechanical advantage is the uniformity of the tension of the cord in all of its parts, the pulley itself serving only to diminish the friction.

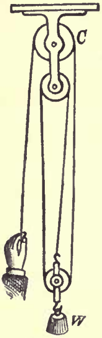


FIG. 89.

138. Systems of Pulleys. — Combinations of pulleys are made in great variety. In the forms most commonly used, one continuous cord passes around all the pulleys. Frequently two or more sheaves are mounted in the same block and turn on the same axis, as in the common block and tackle, shown in Fig. 90.

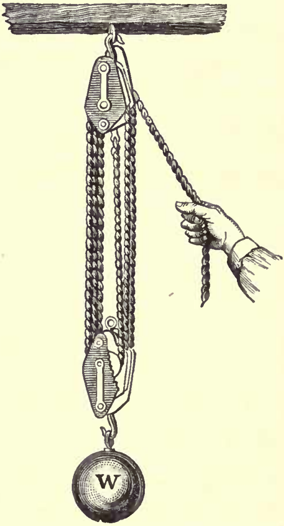


FIG. 90.

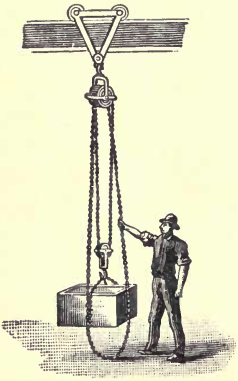


FIG. 91.

(*a*) Another arrangement, sometimes seen on board merchant ships, requires a separate cord for each pulley. (See Fig. 102.)

(*b*) In the differential pulley, an endless chain is reeved upon a solid wheel that has two grooved rims and is carried in a fixed block above, and upon a pulley below, as is shown in Fig. 91. The two rims of the single wheel in the upper block have different diameters, and carry projections to keep the chain from slipping on them. When the chain is pulled down until the upper wheel turns once upon its axis, the chain between the two pulleys is shortened by the difference between the circumferences of the two rims of the upper wheel, and the load is lifted half that distance. This device avoids the use of inconveniently long ropes or chains. In Fig. 91, the hoisting apparatus is hooked into the triangular frame of a traveler which is supported by rollers on the railway overhead.

139. Mechanical Advantage of the Pulley. — With the ordinary arrangement of pulleys, like the block and tackle, the part of the cord to which the power is applied carries but a part of the load, the magnitude of that part varying inversely as the number of sections into which the movable pulley divides the load. *With pulleys thus arranged, a given power will support a weight as many times as great as itself as there are parts of the cord supporting the movable block.*

$$W = P \times n.$$

(*a*) In the case of the differential pulley, the mechanical advantage may be determined by the laws given in § 124.

(*b*) In all experiments to determine the mechanical advantage of a system of pulleys, as in all similar experiments, see that the apparatus is in equilibrium before applying P and W.

140. An Inclined Plane *is a smooth, hard, inflexible surface, inclined so as to make an oblique angle with the horizon.*

FIG. 92.

(*a*) When a body is placed on an inclined plane, the gravity pull is resolved into two component forces. One of these acts perpendicularly to the plane, producing pressure on it, the other component tending to produce motion down the plane. To resist this last-mentioned tendency, and thus to hold the body in its position, a force may be applied in three ways: —

(1) In a direction parallel to the length of the plane.

(2) In a direction parallel to the base of the plane; i.e., horizontal.

(3) In a direction parallel to neither the length nor the base.

141. Mechanical Advantage of the Inclined Plane. — The mechanical advantage to be derived from the use of an inclined plane varies with the three conditions above given.

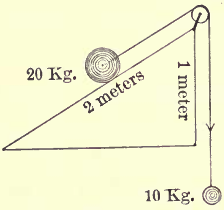


FIG. 93.

(1) *When a given power acts parallel to an inclined plane, it will support a weight as many times as great as itself as the length of the plane is times as great as its vertical height.*

(2) *When a given power acts horizontally, it will support a weight as many times as great as itself as the horizontal base of the plane is times as great as its vertical height.*

(3) When the power acts in a direction parallel to neither the length nor the base, no law can be given. The ratio of the power to the weight may be determined trigonometrically, or, with approximate accuracy, by resolving the force of gravity, the construction and measurement being carefully done.

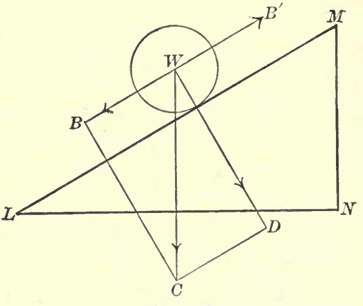


FIG. 94.

(*a*) In Fig. 94, *LM* represents an inclined plane on which a ball is to be supported by a force acting parallel to the plane. Represent the gravity of *W* by the vertical line, *WC*, and resolve it into two components. WD produces pressure on the plane, and WB draws the body down the plane. A force represented by WB', the equilibrant of WB, will just balance the downward pull of WB, and hold the ball in position. From the similarity of the triangles, CWB and LMN, it may be proved that

$$WB : WC :: MN : ML.$$

Careful construction and measurement will give the same result. But WB, or its equal, WB', represents the power, and WC represents the weight of the body. MN represents the height of the plane, and ML its length. Therefore

$$P : W :: h : l.$$

By similarly resolving the force of gravity into two components, one perpendicular to the plane and the other horizontal, the second law as given above may be established.

CLASSROOM EXERCISES.

1. With a fixed pulley, what power will support a weight of 50 pounds?

2. With a movable pulley, what power will support a weight of 50 pounds?

3. With block and tackle, the fixed block having four sheaves and the movable block having three, what weight may be supported by a power of 75 pounds? If an allowance of $\frac{1}{5}$ is made for friction and rigidity of ropes, what is the maximum weight that may be thus supported?

4. With a system of five movable pulleys, one end of the rope being attached to the fixed block, what power will raise a ton?

5. If, in the system mentioned in Exercise 4, the rope is attached to the movable block, what power will raise a ton? If an allowance of 25 per cent is made for friction and rigidity of ropes, what power will be required?

6. With a pulley of six sheaves in each block, what is the least power that will support a weight of 1,800 pounds, allowing $\frac{1}{4}$ for friction? What will be the relative velocities of P and W?

7. Figure a set of pulleys by which a power of 50 pounds will support a weight of 250 pounds.

8. A boy who can lift only 100 pounds wishes to put a barrel of flour (196 pounds) into a wagon-box 5 feet above the ground. He backs the wagon to one end of a plank 20 feet long and weighing 125 pounds. Show that he can, without help, use the plank as an inclined plane for his purpose, and state how much force he exerts (*a*) in getting the plank into position, and (*b*) how much in lifting the flour? (*c*) How much work does he perform in lifting the flour?

Ans. (*a*) $62\frac{1}{2}$ pounds; (*b*) 49 + pounds.

9. How much energy must be expended to pull a 100-pound weight up an inclined plane 10 feet, the vertical ascent accomplished being 6 feet, and the coefficient of friction being 0.2?

10. The base of an inclined plane is 10 feet; the height is 3 feet. What force, acting parallel to the base, will balance a weight of 2 tons?

11. An incline has its base 10 feet; its height, 4 feet. How heavy a ball will 50 pounds power roll up?

12. How great a power will be required to support a ball weighing 40 pounds on an inclined plane whose length is 8 times its height?

13. A weight of 800 pounds rests upon an inclined plane 8 feet high, being held in equilibrium by a force of 25 pounds acting parallel to the base. Find the length of the plane.

14. A load of 2 tons is to be lifted along an incline. The power is 75 pounds. Give the ratio of the incline that may be used.

15. A 1,500-pound safe is to be raised 5 feet. The greatest power that can be applied is 250 pounds. Give the dimensions of the shortest inclined plane that can be used for that purpose.

16. A weight of 400 pounds is being raised by a block and tackle. One end of the rope is fastened to the upper block. Each block has two sheaves and weighs 10 pounds. What is the pressure on the support of the upper block? Disregard the weight of the rope.

Ans. 522 pounds.

142. A Wedge is a triangular prism of hard material, fitted to be driven between objects that are to be separated, or into anything that is to be split. *It is simply a movable inclined plane, or two such planes united at their* *bases.* The power is generally applied in repeated blows on the thick end or "head." For a wedge thus used, no definite law of any practical value can be given, further than that, with a given thickness, the longer the wedge, the greater the mechanical advantage.

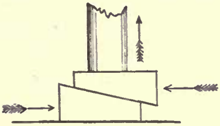


FIG. 95.

FIG. 96.

143. A Screw *is a cylinder, generally of wood or metal, with a spiral ridge (the thread) winding about its circumference. The thread works in a nut, within which there is a corresponding spiral groove to receive the thread.* That the screw is a modified inclined plane, may be shown by winding a triangular piece of paper around a cylinder, as shown in Fig. 98.

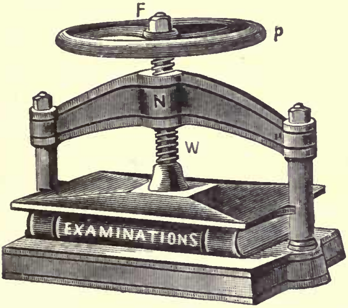


FIG. 97.

(*a*) The power is generally applied by a wheel or a lever, and moves through the circumference of a circle. The distance between two consecutive turns of any one continuous thread, measured in the direction of the axis of the screw, is called the *pitch of the screw.*

FIG. 98.

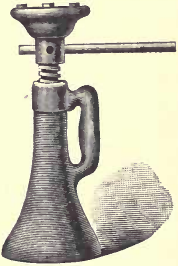


FIG. 99.

(*b*) The screw is largely used where great resistances are to be overcome, as in raising buildings, compressing hay or cotton, propelling ships, etc. It is also used in accurate measurements of small distances, of which application the spherometer, and the micrometer calipers, afford good illustrations.

144. Mechanical Advantage of the Screw. — *With the screw, a given power will support a weight as many times as great as itself as the circumference described by the power is times as great as the pitch of the screw.*

145. Compound Machines. — We have now considered each of the six traditional simple machines. When any two or more of these machines are combined, the mechanical advantage may be found by computing the effect of each separately, and then compounding them; or by finding the weight that the given power will support, using the first machine alone, considering the result as a new power acting upon the second machine, and so on.

CLASSROOM EXERCISES.

1. A bookbinder has a press, the screw of which has a pitch of $\frac{1}{3}$ of an inch. The nut is worked by a lever that describes a circumference of 8 feet. How great a pressure will a power of 15 pounds applied at the end of the lever produce, the loss by friction being equivalent to 240 pounds?

2. A screw has 11 threads for every inch in length. If the lever is 8 inches long, the power 50 pounds, and friction absorbs $\frac{1}{8}$ of the energy used, what resistance may be overcome by it?

3. A screw with threads $1\frac{1}{4}$ inches apart is driven by a lever $4\frac{1}{2}$ feet long. What is the mechanical advantage of the apparatus?

4. How great a pressure will be exerted by a power of 15 pounds applied to a screw whose head is 1 inch in circumference, and whose threads are $\frac{1}{8}$ of an inch apart?

5. At the top of an inclined plane that rises 1 foot in 20 is a wheel and axle. The radius of the wheel is $2\frac{1}{2}$ feet; radius of axle, $4\frac{1}{2}$ inches. What load may be lifted by a boy who turns the wheel with a force of 25 pounds?

6. In moving a building, the horse is harnessed to the end of a lever 7 feet long, acting on a capstan barrel 11 inches in diameter. On the barrel winds a rope belonging to a system of 2 fixed and 3 movable pulleys. What force will be exerted by 500 pounds power, allowing $\frac{1}{4}$ for loss by friction?

7. In raising a building, why do the men who work the jackscrews pull upon the levers by a series of jerks instead of steady pulls?

LABORATORY EXERCISES.

Additional Apparatus. — Pulleys and cords that are strong enough to support at least 100 pounds.

1. Experimentally determine the ratio of power to weight with pulleys arranged as shown in Fig. 100.

2. Determine the loss due to friction and to the rigidity of the ropes used in Exercise 1.

3. Experimentally determine the ratio of power to weight with pulleys arranged as shown in Fig. 101.

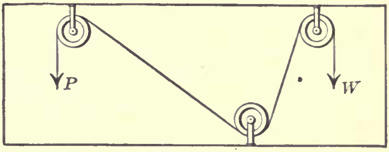


FIG. 100.

4. Show how the work done at *P*, in Exercise 3, compares with the work done at *W*, and account for any difference if you find any to exist.

5. Experimentally determine the ratio between *P* and *W* with pulleys arranged as shown in Fig. 102. Determine the static law of such a combination.

6. The height of an inclined plane is $\frac{1}{5}$ its horizontal base. A globe weighing 250 Kg. is supported in place by a force acting at an angle of 45° with the base. The pressure of the globe upon the plane is less than 250 Kg. By construction and measurement, determine the magnitude of the supporting force.

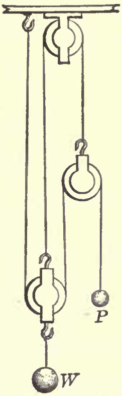


FIG. 101.

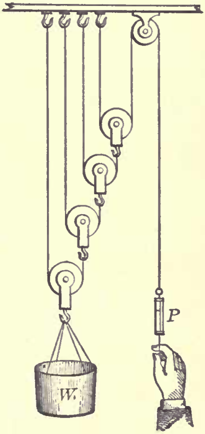


FIG. 102.

7. With the conditions as given in Exercise 6, except that the pressure of the globe upon the plane is more than 250 Kg., determine the magnitude of the supporting force.

8. Place the board used in Experiment 7 so that it may be used again as an inclined plane. Tie one end of a cord to the carriage used in the same experiment, and the other end to a spring-balance. The dynamometer and the cord are to be used in pulling the carriage up the incline. Varying the load and the inclination of the plane in each case, verify the statements made in § 141. Be sure that the board does not bend or sag under the load. Watch for error in the zero point of the balance when held in different positions, and, if any is detected, make correction for it. To determine the correction to be made for friction, find first the pull necessary to move the carriage up the incline at a uniform speed, and then the pull which will allow it to move down the incline at a uniform speed. The difference between these two pulls will be twice the force required to overcome the friction, and the average of the two pulls will be the force that would be required if friction could be eliminated.

9. Arrange an inclined plane so that its base shall be $1\frac{1}{3}$ times its height. Draw a diagram for the resolution of the force of gravity and determine the tendency of a ball that weighs $7\frac{1}{2}$ pounds to roll down the plane, and the pressure of the ball on the plane. Suspend the ball by two spring-balances, so that one of them is drawn parallel to the plane and the other perpendicular to it when the ball is just lifted off the plane. Note the reading of the dynamometers and compare them with the computed results.

VII. THE MECHANICS OF LIQUIDS.

146. Compressibility and Elasticity of Liquids. — Liquids are nearly incompressible. When the pressure is removed, the liquids regain their former volume, showing thus their perfect elasticity. The practical incompressibility of liquids is of great mechanical importance.

Liquid Pressure.

Experiment 58. — Fill a small bottle with water, hold a Prince Rupert drop in its mouth, and break off the tapering end of the "drop." The whole "drop" will be instantly shattered, and the force of the concussion transmitted in every direction to the bottle, which will be thus broken. These "drops" are not expensive.

Experiment 59. — Tie a piece of thin sheet rubber (such as you can get from the druggist or dentist, or from a broken toy balloon) over the large end of a lamp-chimney. Reinforce the other end by winding upon it a dozen turns of wrapping twine, and fit it with a fine-grained cork or rubber stopper through which passes snugly a bit of glass tubing. (See Chemistry, Appendices 4 [*b*] and 9.) Connect the glass tubing and a supported funnel by two or three feet of rubber tubing. Fill the apparatus with water, loosening the cork for a moment to allow the escape of air. See that the funnel is still half full of water and elevated above the chimney. Notice the effect of the water pressure on the sheet rubber. Hold the chimney in various positions, keeping the center of the sheet rubber at a uniform distance below the level of the funnel, and notice whether the elastic sheet is stretched more or less when the liquid pressure upon it is horizontal, upward, or downward. Then try it at varying distances below the level of the water in the funnel, and determine whether such vertical distance or "head" has any relation to the pressure.

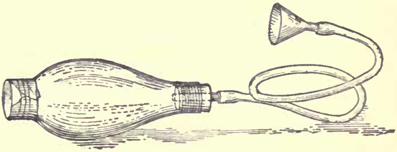


FIG. 103.

Experiment 60. — To the cork of Experiment 59, fit a bit of glass tubing that has been drawn to a jet at the outer end. Hold the chimney in different positions and at different depths, adding water as may be necessary to keep a constant level in the funnel.

147. Transmission of Pressure. — *Fluids transmit pressures in every direction.*

(*a*) Fig. 104 represents a number of balls placed in a vessel. Imagine these balls to have perfect freedom of motion and perfect elasticity. It is evident that if a downward pressure, say of 10 grams, is applied to 2, it will force 5 and 4 toward the left, and 6, 7, and 8 toward the right, thus forming lateral pressure. This motion of 5 will force 1 upward, and 9 downward, etc. Owing to the perfect elasticity and freedom of motion, there will be no loss, and the several balls will be moved just as if the original pressure had been applied directly to each one. The pressure will be thus transmitted to all of the balls without loss, and the total pressure exerted on the sides of the vessel will equal 10 grams multiplied by the number of balls that touch the sides. It makes no difference with the result whether the pressure exerted by 2 was the result of its own weight only, this weight together with the weight of overlying balls, or both of these with still additional pressure.

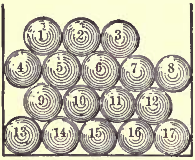


FIG. 104.

(*b*) Disregarding viscosity, we may consider a fluid to be made up of molecules having the perfect elasticity and freedom of motion assumed for the balls just discussed. Hence, when pressure is applied to one or more of the molecules of a fluid, the pressure will be transmitted as now explained.

148. Pascal's Law. — *Pressure exerted anywhere upon a liquid inclosed in a vessel is transmitted undiminished in all directions, and acts with the same force upon all equal surfaces, and in a direction at right angles to those surfaces.*

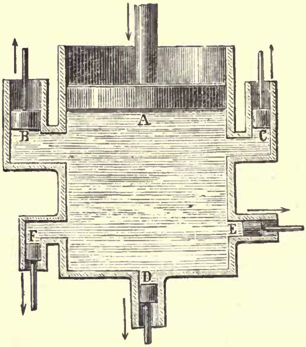


FIG. 105.

(*a*) Provide two communicating tubes of unequal sectional area. When water is poured into these, it will stand at the same height in both tubes, — a fact which of itself partly confirms the law above given. If, by means of a piston, the water in the smaller tube is subjected to pressure, the pressure will force the water back into the larger tube, and raise its level there. To prevent this result, a piston must be fitted to the larger tube, and held there with a greater force. If, for example, the smaller piston has an area of 1 sq. cm., and the larger piston an area of 16 sq. cm., a weight of 1 Kg. may be made to support a weight of 16 Kg.

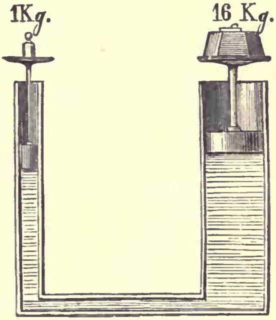


FIG. 106.

149. The Hydraulic Press. — Pascal's law finds an important application in the hydraulic press, in the more common forms of which the pressure of a piston operated by a lever is transmitted through a pipe to a piston of larger area. The press is represented in section by Fig. 107, and in perspective by Fig. 108.

(*a*) If the power arm of the lever is ten times as long as the weight arm, a power of 50 Kg. will exert a pressure of 500 Kg. upon the water beneath the piston, *a*. If this piston has a sectional area of 1 sq. cm., and the piston in *B* has an area of 500 sq. cm., then the pres-

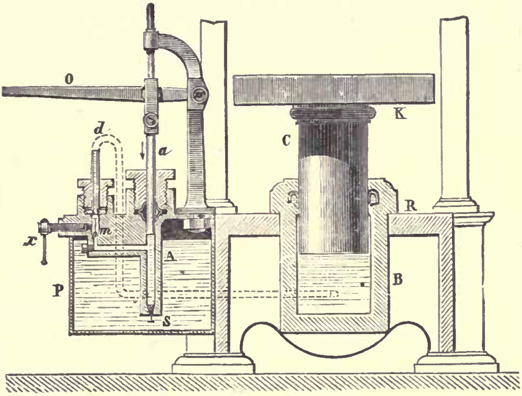


FIG. 107.

sure of 500 Kg. exerted by the small piston will produce a pressure of 500 Kg. × 500 or 250,000 Kg. upon the lower surface of the large piston.

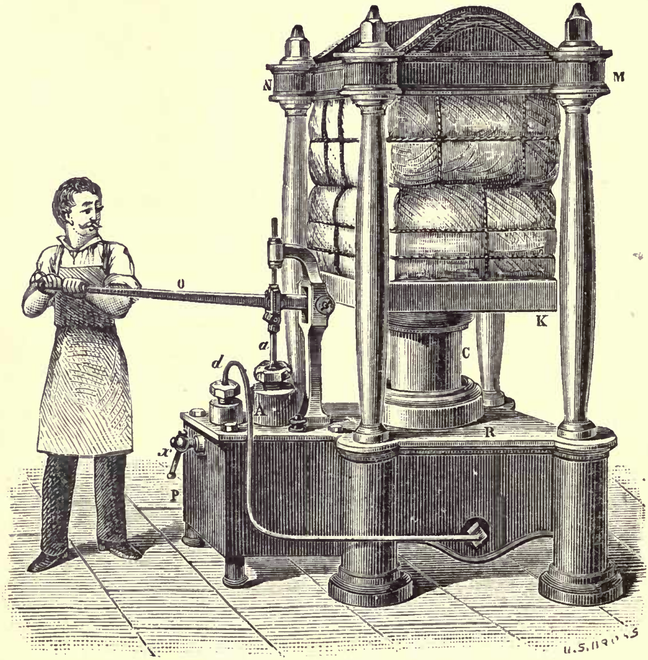


Fig. 108.

Pressure due to Gravity.

Experiment 61. — Make a small hole in the bottom of a tin fruit-can or similar vessel. Push the can downward into water until the open mouth of the can is "near the water's edge." The liquid will spurt upward through the hole in a little jet. Why?

Experiment 62. — Get a lamp-chimney, preferably cylindrical. With a diamond or a steel glass-cutter, cut a disk of window glass a little larger than the cross-section of the lamp-chimney. Pour some fine emery powder on the disk, and rub one end of the chimney upon it, thus grinding them until they fit accurately. With wax, fasten a thread to the center of the ground surface of the disk, and draw that surface against the ground end of the chimney. Holding the chimney in the hand, or supporting it in any convenient way, place it in water as shown in Fig. 109. The upward pressure of the water will hold the disk in place. Pour water carefully into the tube; the disk will fall as soon as the weight of the water in the chimney, plus the weight of the disk, exceeds the upward pressure of the water.

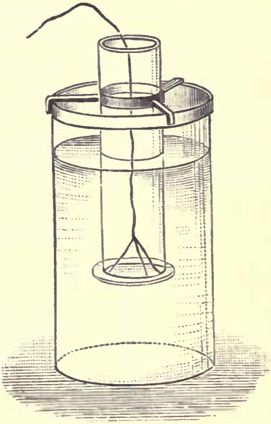


Fig. 109.

Experiment 63. — Into a U-tube, pour enough mercury to fill each arm to the depth of 3 or 4 cm. Place the U-tube upon a table, and hold it upright by any convenient means. Back of it, and resting against it, stand a card having a horizontal line, *a*, drawn on it to mark the level of the mercury in the two arms of the tube. To one arm, attach the neck of a funnel by means of a bit of rubber tubing. The funnel may be held by the ring of a retort stand. Pour water slowly into the funnel until it is nearly full, and mark the level of the water by a suspended weight or other means. In one arm, the mercury will be depressed below the line marked on the card; in the other arm, it will be raised above it an equal distance. Mark these two mercury levels by dotted horizontal lines on the card. Remove the funnel and replace it by a funnel- or thistle-tube, making the connection by means of a perforated cork. Pour water into the funnel-tube until it stands at the level indicated by the suspended weight, *being careful that no air is*

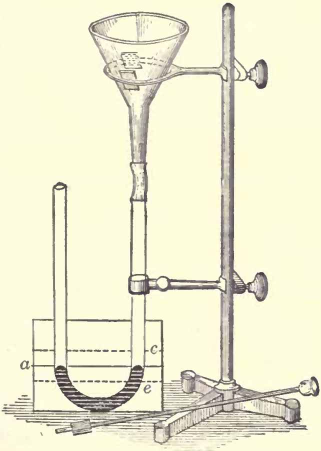


Fig. 110.

confined in the tubes. Although much less water is in the funnel-tube than was in the funnel, it forces the mercury into the position indicated by the dotted lines on the card. The downward pressure of the water in each case is measured by a mercury column with a height, *ce*, equal to the vertical distance between the two dotted lines.

Experiment 64. — Provide several glass vessels, open at each end, and having equal bases, but varying shapes and capacities. In any convenient way, support one of them, as *M* in Fig. 111. Close the lower end of the vessel with a glass or metal disk, ground to fit it water-tight, the disk being supported by a thread carried from one end of a balance-beam. Place known weights in the scale-pan at the other end of the beam, so that the disk shall be held firmly in place. Pour water carefully into the upper or open end of the vessel, until the pressure loosens the disk and allows a little to escape. By an index rod, suspended weight, or other convenient means, mark the upper level of the water at the moment when some of the liquid begins to escape below. Repeat the experiment with the other vessels in succession, using the same counterpoise in each case. The disk will be loosened when the water has reached the marked level,

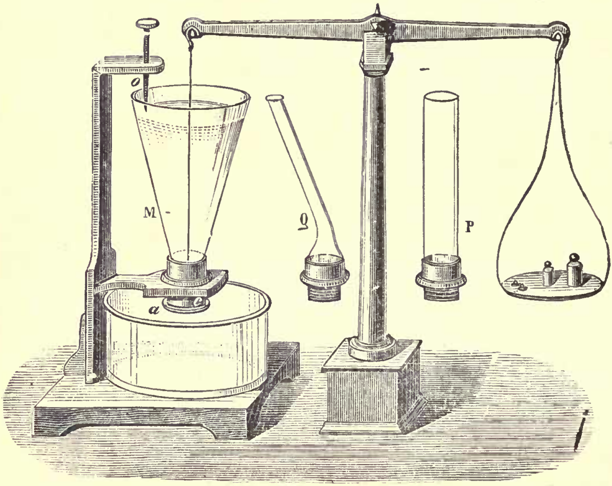


Fig. 111.

although the quantity of water used varies. The glass vessels for this experiment may be easily secured by using glass tubing, a glass funnel, corks, lamp chimneys, etc. (See Avery's Chemistry, Appendix 4.)

150. Liquid Pressure due to Gravity. — *The downward pressure of a liquid is independent of the shape of the containing vessel and of the quantity of the liquid. It is proportional to the depth of the liquid and the area of the base.*

151. Rules for Liquid Pressure.

(1) *To find the downward or the upward pressure on any submerged horizontal surface, find the weight of an imaginary column of the given liquid, the base of which is the same as the given surface, and the altitude of which is the same as the depth of the given surface below the surface of the liquid.*

(2) *To find the pressure upon any vertical surface, find the weight of an imaginary column of the liquid, the base of which is the same as the given surface, and the altitude of which is the same as the depth of the center of the given surface below the surface of the liquid.*

(*a*) A cubic foot of water weighs 62.42 lbs. or about 1,000 oz.

Liquid Level.

Experiment 65. — Remove the jet from the cork used in Experiment 60, and insert in its place a glass tube about two feet long. Holding the chimney on the table-top with this glass tube upright, fill the apparatus with water. Does the water stand at a higher level in the funnel, or in the tube? Raise and lower the funnel, and for each position notice the relation between the liquid levels in the funnel and the tube.

152. Communicating Vessels. — When any liquid is placed in one or more of several vessels communicating with each other, *it will not come to rest until it stands at the same height in all of the vessels.* This principle is embodied in the familiar expression, "Water seeks its level." The principle is illustrated, on a large scale, in the system of pipes by which water is distributed in cities.

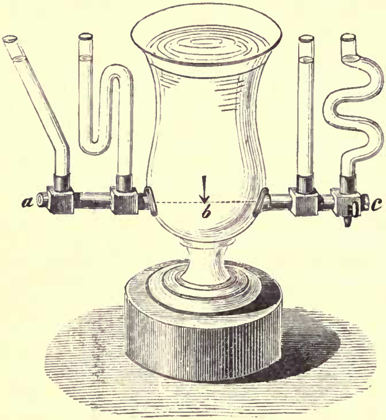


Fig. 112.

CLASSROOM EXERCISES.

1. What will be the pressure on a dam in 20 feet of water, the dam being 30 feet long?

2. What will be the pressure on a dam in 6 m. of water, the dam being 10 m. long?

3. Find the pressure on one side of a cistern 5 feet square and 12 feet high, filled with water.

4. Find the pressure on one side of a cistern 2 m. square and 4 m. high, filled with water.

5. A cylindrical vessel having a base of a square yard is filled with water to the depth of two yards. What pressure is exerted upon the base?

6. A cylindrical vessel having a base of a square meter is filled with water to the depth of 2 meters. What pressure is exerted upon the base?

7. What will be the upward pressure upon a horizontal plate a foot square at a depth of 25 feet of water?

8. What will be the upward pressure upon a horizontal plate 30 cm. square at a depth of 8 m. of water?

9. A square board with a surface of 9 square feet is pressed against the bottom of the vertical wall of a cistern in which the water is $8\frac{1}{2}$ feet deep. What pressure does the water exert upon the board?

10. A cubical vessel with a capacity of 1,728 cubic inches is two-thirds full of sulphuric acid, which is 1.8 times as heavy as water. Find the liquid pressure on one side of the vessel.

11. A conical vessel has a base with an area of 237 sq. cm. Its altitude is 38 cm. It is filled with water to the height of 35 cm. Find the pressure on the bottom. *Ans.* 8,295 g.

12. In Exercise 11, substitute inches for centimeters, and then find the pressure on the bottom.

13. What is the total liquid pressure on a prismatic vessel containing a cubic yard of water, the bottom of the vessel being 2 by 3 feet?

14. The lever of a hydraulic press is 6 feet long, the piston rod being 1 foot from the fulcrum. The area of the tube is half a square inch; that of the cylinder is 100 square inches. Find the weight that may be raised by a force of 75 pounds.

15. What is the pressure on the bottom of a pyramidal vessel filled with water, the base being 2 by 3 feet, and the height 5 feet?

16. What is the pressure on the bottom of a conical vessel 4 feet high, filled with water, the base being 20 inches in diameter?

17. At what depth in water will the liquid pressure be 1 Kg. per square centimeter?

18. A closed cylindrical vessel 30 cm. high is filled with water. At the middle of its height, a bent tube communicates with the interior of the vessel. Water stands in this tube at a height of 50 cm. above the middle of the opening into the cylinder. What is the liquid pressure per square centimeter on the upper end of the cylinder? On the lower end?

19. An upright cylindrical jar having a base of 100 sq. cm. and a height of 20 cm. is filled with water. An open tube 1 sq. cm. in cross-section passes through the cover, rises 30 cm. above it, and is filled with water. (*a*) What is the weight of the water in the jar and tube? (*b*) What is the liquid pressure on the square centimeter of the base that lies exactly beneath the tube? (*c*) What square centimeter of the base has a greater pressure? (*d*) A less pressure?

20. (*a*) In the case of the jar and tube described in Exercise 19, what is the liquid pressure upon that square centimeter of water at

the level of the under side of the cover and beneath the tube? (*b*) Is the liquid pressure against each square centimeter of the cover greater, or less, than this? (*c*) What is the total liquid pressure against the cover?

21. In the case of the jar and tube considered in Exercise 20, subtract the total liquid pressure against the cover from the total liquid pressure against the base, and compare the result with the weight of the water in the jar and tube.

LABORATORY EXERCISES.

Additional Apparatus, etc.—Stout glass tubing; an acid bottle; a wine bottle; linseed oil; rubber stoppers; mercury; tall hydrometer jar.

1. Bend a piece of glass tubing into the shape shown in Fig. 113, and support it upright in any convenient way. If you remove the top and bottom from a box, and cut a slot in one of the remaining sides, you will have a cheap and convenient support. Remove the funnel from the apparatus used in Experiment 59, and connect the rubber tubing to the glass tube at *B*. Half fill the tube with water colored with red ink, and it becomes a pressure gauge. The two liquid levels will lie in the same horizontal plane. Mark this level on the open arm of the gauge, and make it the zero of a scale extending upward. Remember that an elevation of the liquid level above the zero mark measures half the difference between the two liquid levels of the gauge. Place the chimney in water at such a depth that the liquid pressure exerted upon the rubber diaphragm, and transmitted through all the coilings of the rubber tubing, shall depress the surface at *m* and raise that at *n*, until the difference of their levels, *on*, is, say, 1 cm. Note the depth of the diaphragm below the level of the water. Hold the chimney in as many different positions as is convenient, but with the center of the diaphragm at the same depth, and note the reading of the gauge. Sink the chimney to a greater depth until *on* becomes successively 2 cm., 3 cm., etc., up to the limit of the gauge. Compare your results with §§ 147 and 150, and indicate any and all of the statements therein made that your work confirms.

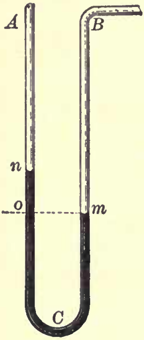


FIG. 113.

2. Cut the bottoms from a large bottle, and from another bottle of about equal height but much less diameter. Close their mouths by corks perforated by bits of glass tubing. Support the bottomless bottles by thrusting their necks downward through two holes bored in the top of a box. With rubber tubing, connect the glass tubes that pass through the corks, making thus two communicating vessels. Half fill the bottles with water, and mark the liquid level on each bottle. Pour a measured quantity of oil into the smaller bottle until it forms a layer several centimeters thick. The water-levels have been changed. Pour measured quantities of the oil into the other bottle until the water is restored to its marked levels. How do the thicknesses of the two oil layers compare? How do the volumes of the two oil layers compare? How would the ratio between the weights of these oleaginous additions differ from the ratio between their volumes? With the calipers, measure the internal diameters of the two bottles; compute the cross-section areas of the two oil cylinders. How does the ratio between these areas differ from the previously determined ratios for volume and weight? How does the downward pressure per square centimeter in one branch correspond to the pressure per square centimeter in the other branch?

3. Provide a stout glass tube of the shape shown in Fig. 114. Pour mercury into the upper end until it stands at a depth of 1 or 2 cm. in the bend at *a*. Provide a hydrometer jar with a depth as great as the length, *ce*, and nearly fill it with water. Lower the long leg of the tube into the water about $\frac{1}{3}$ of its length. Measure the vertical distance between the levels of the water within and without the tube, and call it *w*. Measure the vertical distance between the two mercury-levels, and call it *m*. Lower the tube until $\frac{2}{3}$ of the long arm is in the water. Determine the distance between the two water-levels, and call it *w′*; determine the difference in the two mercury-levels, and call it *m′*. Lower the tube until the bend at *c* rests on the edge of the hydrometer jar, and, as before, determine the differences of the water-levels (w'') and of the mercury-levels (m''). What does the difference in the mercury-levels in each case represent? From the data secured, test the equality of these ratios: $\frac{w}{m}=\frac{w'}{m'}=\frac{w''}{m''}$. If you find that the quantities are proportional, finish the following incomplete expression

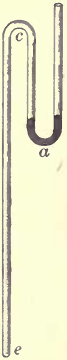


FIG. 114.

of the relation: In the body of a liquid, the upward pressure varies as . . .

4. Considering m, m', and m'' as abscissas, and w, w', and w'' as ordinates, give a graphic representation of the data obtained in Exercise 3. Is your line straight, or curved? If it is straight, what does that fact show? If it is curved, what does that fact show?

Principle of Archimedes.

Experiment 66. — Suspend a stone or brick by a slender cord or fine wire from the hook of a spring-balance, and note the reading of the scale. Transfer the suspended load from air to water, and note the reading. Transfer the load to a strong brine, and note the reading. Transfer the load to kerosene, and note again the reading. It seems as if the liquids help to support the stone, with a buoyant force of varying magnitude.

Experiment 67.— From one end of a scale-beam, suspend a cylindrical metal bucket, *b*, with a solid cylinder, *a*, that fits accurately into it hanging below. Counterpoise with weights (shot or sand) in the opposite scale-pan. Immerse *a* in water, and the counterpoise

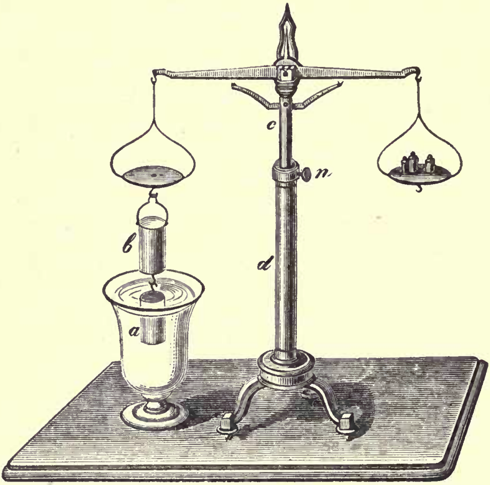


Fig. 115.

will descend, as if *a* had lost some of its weight. Carefully fill *b* with water. It will hold exactly the quantity displaced by *a*. Equilibrium will be restored.

Experiment 68. — For rough work, a spring-balance may take the place of the beam-balance; a tin pail may take the place of *b*; a piece of stone suspended beneath the pail by strings tied to the ears of the pail may take the place of *a*; a larger tin pail filled with water and set in a tin pan may take the place of the vessel of water shown in Fig. 115. Note the weight of the smaller pail, with and without the suspended stone. Lower the apparatus so that the stone shall be immersed in the water, and note the reading of the scale. Determine the loss of weight resulting from the immersion of the stone. The volume of water forced from the pail and caught in the pan is equal to what other volume? Remove the pan, immerse the stone as before, pour the water from the pan into the upper pail, and note the reading of the scale. To what other reading is it equal? To what is the weight of the water displaced by the stone equal?

Experiment 69. — Modify the experiment again as follows: Instead of the suspended bucket, *b*, place a tumbler upon the scale-pan. Instead of the cylinder, *a*, suspend any convenient solid heavier than water, as a potato. Counterpoise the tumbler and the potato with weights in the other scale-pan. Provide an overflow-can by inserting a spout about 6 cm. long and 7 or 8 mm. in diameter in the side of a vessel (as a tin fruit-can) about an inch below the top of the can. This spout should slope slightly downward. Fill the can with water and catch the overflow from the spout in a cup. Throw away the water thus caught. Wait a minute for the spout to stop dripping and then carefully immerse the potato in the water of the can, catching in the cup every drop of water that overflows. Wait a minute for the spout to stop dripping. The equilibrium of the balance is destroyed, but it may be restored by pouring into the tumbler the water that was displaced by the potato and caught in the cup.

Experiment 70. — Provide a wooden cube just 5 cm. on an edge. Coat it with shellac varnish, or dip it into hot paraffine. Weigh it; also weigh a saucer. Place a beaker or tumbler in the saucer, and fill it with water. Stick two pins or needles into one face of the cube, and, using them as handles, immerse the cube in the water of the beaker. Remove the beaker, and weigh accurately the saucer and its liquid contents. Pour the water from the saucer into a graduate, and measure it in cubic centimeters. How does its volume compare with the volume of the wooden cube? What should that quantity of water weigh according to § 24 (*a*)? Subtract the weight of the saucer empty from the weight of the saucer with the liquid overflow. How do these two weights of the water compare? Has your work been well done? The weight of the wood is what fractional part (expressed decimally, of course) of the weight of the water?

153. Archimedes' Principle. — (It is evident that, when a solid is immersed in a fluid, it will displace exactly its own volume of the fluid.) Immerse a solid cube one centimeter on each edge in water, so that its upper face shall be level and one centimeter below the surface of the liquid, as shown in Fig. 116. The lateral pressures upon any two opposite vertical surfaces of the cube, as *a* and *b*, are clearly equal and opposite. Their resultant is zero. They have no tendency to move the solid. The vertical pressures on the other two faces, *c* and *d*, are not equal. The upper face sustains a pressure equal to the weight of a column of water having a base one centimeter square (i.e., the face, *d*) and an altitude equal to the distance, *dn*. This imaginary column of water has a volume of one cubic centimeter and a weight of one gram. The downward pressure on *d* is one gram. As the face, *c*, has the same area and is at twice the depth, the upward pressure upon it is two grams. The resultant of the two vertical and opposite forces acting on the cube is an upward pres-

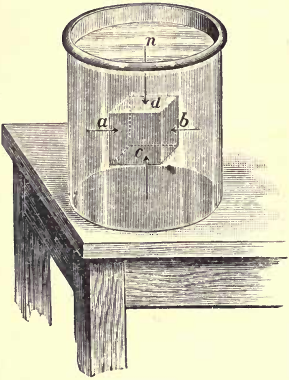


FIG. 116.

sure of one gram; i.e., the cube is partly supported by a buoyant force of one gram, which is the weight of the cubic centimeter of water that it displaces. No matter what the depth to which the block is immersed, this net upward pressure, or buoyant effect, is always the same. This truth, discovered by Archimedes, may be stated thus: (*A body is buoyed up by a force equal to the weight of the fluid that it displaces.*) Hence the apparent weight of a body in a fluid (e.g., water or air) is less than its true weight. This buoyant effect is often spoken of as a "loss of weight."

(*a*) In the above discussion, the effect of atmospheric pressure is left out of the account. It affects equally the top and bottom of the block.

Flotation.

Experiment 71. — Place the tin can mentioned in Experiment 69, upon one scale-pan, and fill it with water, some of which will overflow through the spout. Do not let any of the water fall upon the scale-pan. When the spout has ceased dripping, counterpoise the vessel of water with weights in the other scale-pan. Place a floating body on the water. This will destroy the equilibrium, but water will overflow through the spout until the equilibrium is restored. This shows that the floating body has displaced its own weight of water.

Experiment 72. — Place a fresh egg in a vessel of fresh water; it is a little heavier than the water, and will sink. Place it in salt water; it is a little lighter than the brine, and will float. Carefully pour the fresh water on the salt water in a tall, narrow vessel. Place the egg in the water; it will descend until it reaches a layer of the liquid with a density like its own, and *there it will float.*

154. Floating Bodies. — When a solid is immersed in a liquid it falls under one of three cases, according as the weight of the solid is less than, equal to, or greater than that of the displaced liquid. In the first case, the buoyant effect of the liquid (§ 153) exceeds the weight of the body, and the body rises to the surface and floats. In the second case, buoyancy and weight are equal and opposite, and their resultant is zero; the body is in equilibrium in any part of the liquid. In the third case, the weight exceeds the buoyancy, and the body sinks. But in any case, Archimedes' principle is strictly true. A floating body is only partly immersed, and the volume of liquid displaced by it is only a fraction of its own volume. In order that it may float at rest, the forces acting upon it must be in equilibrium; i.e., the upward and the downward pressures must be equal. Consequently, the law of flotation is: *A floating body will sink in a liquid until it displaces a weight of the liquid equal to its own weight.*

(*a*) Sometimes a heavy substance is given such a shape that it displaces enough of a lighter fluid to float thereon. Thus, an iron kettle or an iron ship floats on water, although iron is much heavier than water.

(*b*) Just as the gravity of a body may be considered as acting upon a single point called the center of mass, so the buoyant effort of a fluid may be considered as acting upon a single point called the center of buoyancy. *The center of buoyancy is situated at the center of mass of the displaced fluid.*

CLASSROOM EXERCISES.

1. How much weight will a cubic decimeter of iron lose when placed in water?

2. How much weight will it lose in a liquid 13.6 times as heavy as water?

3. If the cubic decimeter of iron weighs only 7,780 g., what does your answer to Exercise 2 signify?

4. How much weight will a cubic foot of stone lose in water?

5. If 100 cu. cm. of lead weighs 1,135 g., what will it weigh in water?

6. If a brass ball weighs 83.8 g. in air, and 73.8 g. in water, what is its volume?

7. A cubical vessel 20 cm. on an edge has fitted into its top a tube 2 cm. square and 10 cm. high. Box and tube being filled with water, (*a*) what is the weight of the water? (*b*) What is the liquid pressure on the bottom of the vessel? (*c*) If the weight and pressure differ, explain the difference.

8. In Fig. 117, the line, *ABC*, represents the surface of water that has been distorted from its level condition. Show what force acts on any water particle in the distorted surface, as the one at *B*, and moves it so that the surface becomes level. At what moment does that force vanish?

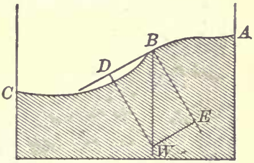


FIG. 117.

155. Density and Specific Gravity. — The density of a substance is its mass per unit of volume. *The specific gravity of a substance is the ratio between the weight of any volume of the substance and the weight of a like volume of some other substance taken as a standard;* i.e., it is the ratio of its density to that of some standard substance. For solids and liquids, the standard is distilled water at its temperature of maximum density (4° C. or 39.2° F.); for gases and vapors, the standard is hydrogen or air under a barometric pressure of 76 centimeters, and at the temperature of 0° C.

(*a*) Since the weights of bodies are proportional to their masses, specific gravity is equivalent to relative density. The term "density" has nearly displaced "specific gravity" in scientific works.

(*b*) To illustrate, in the simplest way, what is meant by density (i.e., specific gravity), suppose that 1 cu. cm. of marble weighs 2.7 g. Since 1 cu. cm. of water weighs 1 g., the marble is 2.7 times as heavy as water, volume for volume. In shorter phrase, the density of marble is 2.7. To avoid the difficulty of obtaining just a unit volume of the substance, the principle of Archimedes is utilized, as will be illustrated.

156. To Find the Density of a Solid Heavier than Water. — The most common way of determining the density of such a body, if it is insoluble in water, is to find its weight in air (w); find its weight when immersed in water (w'); divide the weight in air by the loss of weight in water.

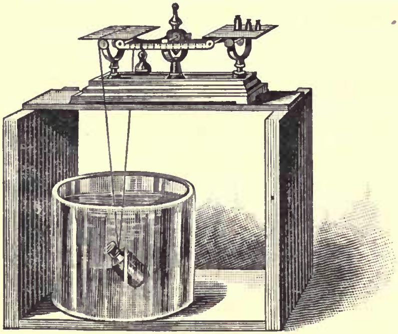


FIG. 118.

$$D = \frac{w}{w - w'}.$$

(*a*) This method is illustrated by the following example : —

(1)	Weight of the solid in air	(w)	113.4 g.
(2)	“ “ “ “ “ water	(w')......	79.14 g.
(3)	“ “ equal bulk of water	($w-w'$)...	34.26 g.
(4)	Density “ the solid (1)÷(3)............		3.31

157. The Hydrometer. — Instruments called hydrometers are made for the more convenient determination of densities. There are hydrometers of constant volume, and hydrometers of constant weight. The Nicholson hydrometer of constant volume is a hollow cylinder carrying at its lower end a basket, *d*, heavy enough to keep the apparatus upright in water. At the top of the cylinder is a vertical rod carrying a pan, *a*, for holding weights, etc. The whole apparatus must be lighter than water, so that a certain weight (W) must be put into the pan to sink the apparatus to a fixed point marked on the rod (as *c*). The given body, which must weigh less than W, is placed in the pan, and enough weights (w) added to sink the point, *c*, to the water line. It is evident that the weight of the given body is $W-w$. The given body

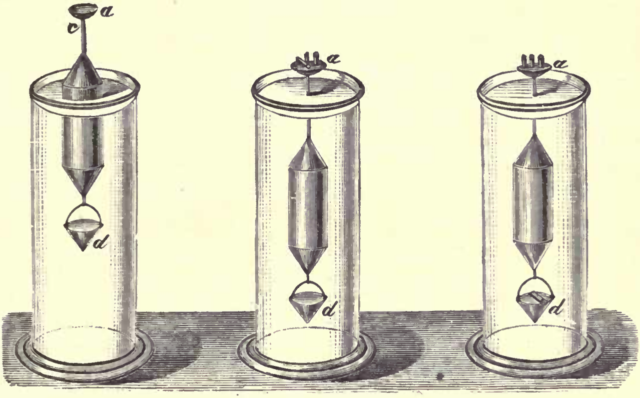


FIG. 119.

is now taken from the pan and placed in the basket, when additional weights, x, must be added to sink the point, *c*, to the water line.

$$D=\frac{W-w}{x}.$$

158. To Find the Density of a Solid Lighter than Water. — Fasten to it another body heavy enough to sink it in water. Find the loss of weight for the combined mass when weighed in the water. Do the same for the heavy body. Subtract the loss of the heavy body from the loss of the combined mass. Divide the weight of the given body by this difference.

159. To Find the Density of a Solid Soluble in Water. — Determine the density of the given solid with reference to some liquid, the density (d) of which is known, and in which the solid is not soluble. Multiply the result obtained by any of the processes previously described by the density of the liquid used.

$$D = \frac{wd}{w - w'}.$$

160. To Find the Density of a Liquid. — There are several methods of finding the density of a liquid, but the principle in each is that already given.

(*a*) Four of these methods are given here; others will be found in the Laboratory Exercises.

(1) Weigh a flask first, empty; next, full of water; then, full of the given liquid. Subtract the weight of the empty flask from the other two weights; the results represent the weights of equal volumes of the given substance and of the standard. Divide as before. A flask of known weight, graduated to measure 100 or 1,000 grams or grains of water, is called a *specific-gravity flask.* Its use avoids the first and second weighings above mentioned, and simplifies the work of division.

(2) Find the loss of weight of any insoluble solid in water and in the given liquid. Divide the latter loss by the former. A solid thus used is called a *specific-gravity bulb.*

(3) The Fahrenheit hydrometer of constant volume is made of glass, the bulb at the bottom being loaded with mercury or shot. Its weight (W) being accurately determined, the instrument is placed in water, and a weight (w) sufficient to sink a marked point on the rod to the water-line is placed in the pan. The weight of water displaced by the instrument $= W + w$. The hydrometer is then removed, wiped dry, and placed in the given liquid. A weight (x) sufficient to sink the hydrometer to the marked point is placed in the pan.

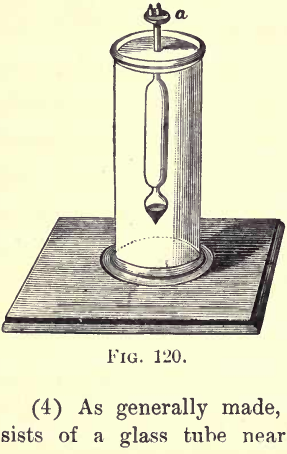


FIG. 120.

$$D = \frac{W + x}{W + w}.$$

(4) As generally made, a hydrometer of constant weight consists of a glass tube near the bottom of which are two bulbs.

The lower and smaller bulb is loaded with mercury or shot. The tube and upper bulb contain air. The point to which it sinks when placed in water is marked zero. The tube is graduated, the scale being arbitrary, and varying with the purpose for which the instrument is intended. Such hydrometers are used to determine the degree of concentration of certain liquids, as acids, alcohols, milk, solutions of sugar, etc. According to their uses, they are known as *acidometers*, *alcoholometers*, *lactometers*, *saccharometers*, etc.

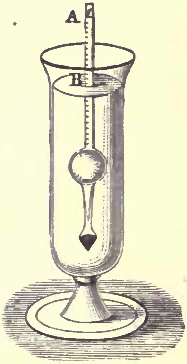


FIG. 121.

161. To Find the Density of a Gas. — The density of an aëriform body is found by comparing the weights of equal volumes of the standard (air or hydrogen) and of the given substance. The method is strictly analogous to that first given for liquids. The determination of the density of gases presents many practical difficulties which cannot be considered in this place.

NOTE. — The weight of any solid or liquid, in grams per cubic centimeter, represents its density.

The weight of a cubic foot of any solid or liquid is equal to 62.421 pounds avoirdupois multiplied by its density.

The weight of a cubic centimeter of any solid or liquid is equal to 1 gram multiplied by its density.

The weight of a liter of any liquid or a cubic decimeter of any solid is equal to 1 kilogram multiplied by its density.

162. Water Power. — An elevated body of water is a storehouse of potential energy. As the water runs to a lower level, it may be made to turn a wheel, and thus to move machinery, etc., a good illustration of the conversion of potential into kinetic energy.

(*a*) Water-wheels are of different kinds, their relative advantages depending upon the nature of the water-supply and of the work to be done. In the overshot wheel (Fig. 122), the water falls into buckets at the top, and by its weight, aided by the force of the current, turns the wheel. Such wheels have been made nearly 100 feet in diameter. The little water that they need must have a considerable fall. In the breast wheel, the water is received at or near the level of the axis; thus the weight of the water and the force of the current are turned to account. In the undershot wheel, the water acts upon a few float-boards at the bottom, the force of the current turning the wheel. The most efficient form of water-wheel is the turbine, one form of which is shown in Fig. 123. The wheel, *B*, and the inclosing case, *D*, are placed

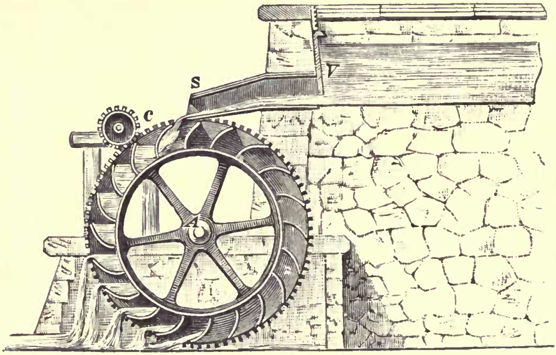


FIG. 122.

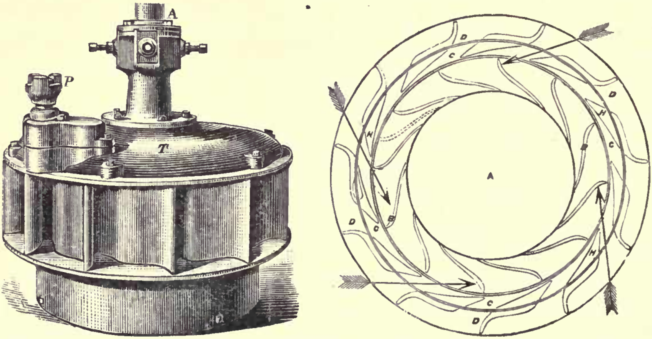


FIG. 123.

on the floor of a penstock wholly submerged in water under the pressure of a considerable head. The water enters, as shown by the arrows, through openings in *D*, which are so constructed that it strikes the buckets of *B* in the direction of greatest efficiency. After leaving the buckets, the "dead water" escapes from the central part of the wheel, sometimes by a vertical draft-tube. The weight of the water in this tube increases the velocity with which the water strikes the buckets. A central shaft, *A*, is carried by the wheel, and communicates its motion to the machinery above. The wheel itself rests upon a central pivot carried by cross-arms from the bottom of the outer case. The case, *D*, is covered with a top, *T*, which protects the wheel from the vertical pressure of the water.

CLASSROOM EXERCISES.

NOTE. — Be on the alert to recognize Archimedes' Principle in disguise. Consider the weight of water $62\frac{1}{2}$ pounds per cubic foot.

1. What is the density of a body that floats with half of its volume under water?

2. Assuming the density of aluminium to be 2.6, determine the weight of an aluminium sphere 25 cm. in diameter.

3. A piece of metal weighing 52.35 g. in air is placed in a cup filled with water. The overflowing water weighs 5 g. What is the density of the metal?

4. A solid weighing 695 g. in air loses 83 g. when weighed in water. (*a*) What is its density? (*b*) How much would it weigh in alcohol that has a density of 0.792?

5. A 1,000-grain bottle holds 708 grains of benzoline. Find the density of the benzoline.

6. A solid that weighs 2.4554 ounces in air, weighs only 2.0778 ounces in water. Find its density.

7. A specimen of gold that weighs 4.6764 g. in air, loses 0.2447 g. weight when weighed in water. Find its density.

8. A ball weighing 970 grains, weighs in water 895 grains, in alcohol 910 grains. Find the density of the alcohol.

9. A body loses 25 grains in water, 23 grains in oil, and 19 grains in alcohol. Required the density of the oil and of the alcohol.

10. A body weighing 1,536 g., weighs in water 1,283 g. What is its density?

11. Calculate the density of sea water from the following data: —

Weight of bottle empty	3.5305 g.
" " filled with distilled water . .	7.6722 g.
" " " sea " . .	7.7849 g.

12. Determine the density of a piece of wood from the following data: weight of wood in air, 4 g.; weight of sinker, 10 g.; weight of wood and sinker under water, 8.5 g.; density of sinker, 10.5.

13. A piece of a certain metal weighs 3.7395 g. in air; 1.5780 g. in water; 2.2896 g. in another liquid. Calculate the densities of the metal and of the unknown liquid.

14. Find the density of a piece of glass, a fragment of which weighs 2,160 grains in air, and 1,511½ grains in water.

15. A lump of ice weighing 8 pounds is fastened to 16 pounds of lead. In water, the lead alone weighs 14.6 pounds, while the lead and ice weigh 13.712 pounds. Find the density of the ice.

16. A piece of lead weighing 600 g. in air weighs 545 g. in water, and 557 g. in alcohol. Find (*a*) the density of the lead; (*b*) the density of the alcohol; (*c*) the volume of the lead.

17. A person can just lift a 300-pound stone in the water. What is his lifting capacity in the air (density of the stone, 2.5)?

18. A liter flask holds 870 g. of turpentine. Required the density of the turpentine.

[In the next two exercises, the weight of the empty flask is not taken into account.]

19. A liter flask containing 675 g. of water had its remaining space filled with fragments of a mineral, and was found to weigh 1,487.5 g. Required the density of the mineral.

20. A liter flask was four-fifths filled with water; the remaining space being filled with sand, the weight was found to be 1,350 g. Required the density of the sand.

21. A weight of 1,000 grains will sink a certain Nicholson hydrometer to a mark on the rod carrying the pan. A piece of brass plus 40 grains will sink it to the same mark. When the brass is taken from the pan and placed in the basket, it requires 160 grains in the pan to sink the hydrometer to the same mark on the rod. Find the density of the brass.

22. A Fahrenheit hydrometer, which weighs 2,000 grains, requires 1,000 grains in the pan to sink it to a certain depth in water. It re-

quires 3,400 grains in the pan to sink it to the same depth in sulphuric acid. Find the density of the acid.

23. A certain body weighs just 10 g. It is placed in one of the scale-pans of a balance, together with a flask full of pure water. The given body and the filled flask are counterpoised with shot in the other scale-pan. The flask is removed, and the given body placed therein, thus displacing some of the water. The flask, being still quite full, is carefully wiped and returned to the scale-pan, when it is found that there is not equilibrium. To restore the equilibrium, it is necessary to place 2.5 g. with the flask. Find the density of the given body.

24. What would a cubic foot of coal (density, 2.4) weigh in a solution of potash (density, 1.2)?

25. 500 cu. cm. of iron (density, 7.8) floats on mercury. With what force is it buoyed up?

26. A piece of cork weighing 2.3 g. was attached to a piece of iron weighing 38.9 g. Both were found to weigh in water 26.2 g., the iron alone weighing 33.9 g. in water. Required the density of the cork.

27. A piece of wood weighing 300 grains has tied to it a piece of lead weighing 600 grains; together they weigh in water 472.5 grains. The density of lead being 11.35, (*a*) what does the lead weigh in water? (*b*) What is the density of the wood?

28. A Fahrenheit hydrometer weighs 618 grains. It requires 93 grains in the pan to sink it to a certain mark on the stem. When wiped dry and placed in olive oil, it requires only 31 grains to sink it to the same mark. Find the density of the oil.

29. A platinum ball weighs 330 g. in air, 315 g. in water, and 303 g. in sulphuric acid. Find (*a*) the density of the ball; (*b*) the density of the acid; (*c*) the volume of the ball.

30. A hollow ball of iron weighs 1 Kg. What must be its least volume to float on water?

31. A body whose density is 2.8 weighs 37 g. Required its weight in water.

LABORATORY EXERCISES.

Additional Apparatus, etc. — A piece of rock-salt; naphtha; pine rod, loaded and graduated as described below; rectangular prism of hard wood; piece of brimstone; bottle with ground-glass stopper; kerosene; wooden cylinder and support; Y-tube; pinchcock.

1. Determine the density of two solids heavier than water, the solids being supplied by the teacher.

2. Determine the density of a solid lighter than water.

3. Determine the density of an unknown liquid without using a specific-gravity flask or a hydrometer.

4. Rock-salt is soluble in water, and insoluble in naphtha. Determine the density of a specimen of rock-salt.

5. Determine the density of a liquid, using a specific-gravity bulb.

6. Redetermine the density of one of the solids used in Exercise 1, using a Nicholson hydrometer.

7. Get a rectangular block of hard wood about 5 × 6 × 7 centimeters. The exact dimensions are not essential. Weigh the block on a spring-balance suspended from some firm support (not held in the hand). Measure the dimensions of the block as accurately as possible, and compute its volume in cubic centimeters and cubic inches. Determine the weight of the block in grams per cubic centimeter, and in ounces per cubic inch.

8. Provide a water-proofed wooden cylinder about 1 cm. in diameter and about 20 cm. long, and a support for holding the cylinder upright in the water in such a way that it may easily move up and down without tipping much from a vertical position. Accurately measure the length of the cylinder. When the cylinder is floating upright in water, joggle it a few times, and see that it comes to rest each time at the same depth. Accurately measure the length of the submerged part of the cylinder, and from the two measurements, compute the density of the cylinder.

9. Make a rod of white pine or other light wood, just 1 cm. square and about 30 cm. long. Graduate one side of the rod to millimeters, with the zero of the scale at the loaded end. In one end, bore a hole, and pound in enough sheet lead to make the rod stand on end when floated in water and with about half of it immersed. Fill the rest of the cavity with putty, and dip the rod into hot paraffine. Place the rod in water, and read from the scale the depth to which it sinks.

Using it as a hydrometer of constant weight, determine the density of alcohol and of a 20-per-cent solution of common salt.

10. Paste a strip of writing paper around the upper end of the rod used in Exercise 9, one edge of the paper overlapping the end of the stick so as to make a small cup. Float the rod as before, and place enough shot or sand in the cup to bring one of the graduations exactly to the water-level. Add successively weights of 1 g., 2 g., 3 g., etc., and at each addition, note how much the rod sinks. Record the teachings of the experiment.

11. Fill the tin can used in Experiment 69, until water overflows through the spout. Weigh a small beaker, and place it under the spout. Weigh a piece of roll sulphur (brimstone) about 5 cm. long, suspend it by a fine thread, and lower it into the water in the tin can, catching in the beaker every drop of water that overflows. Weigh the beaker with its liquid contents, and by subtraction find the weight of water it contains. Suspend the sulphur by the thread from the scale-pan, and weigh it in water. Record your data as follows: —

(*a*) Weight of sulphur in air = ?
(*b*) Weight of beaker = ?
(*c*) Weight of beaker and water = ?
(*d*) Weight of displaced water $(c-b)$ = ?
(*e*) Weight of sulphur in water = ?
(*f*) Loss of weight in water $(a-e)$ = ?

Compare (*d*) with (*f*), and record the fact thus indicated. From the recorded data, determine the density of the sulphur.

12. Provide a bottle that will hold two or three ounces of water, and that has a ground-glass stopper; a thread with which to suspend the bottle; a cloth with which to wipe the bottle; a delicate spring-balance; water; kerosene. Without any other apparatus or supplies, determine the density of the kerosene.

13. Fill a bottle like that used in Exercise 12 with water, and put the stopper firmly into place. Without removing the stopper or adding to your material, determine the density of the kerosene.

14. Get a glass U-tube with an internal diameter of 8 or 10 mm. and having arms that are close together and about 50 cm. long (see Fig. 110); a meter stick graduated to millimeters; a small funnel for pouring liquids into the U-tube; some support that will hold the U-tube upright; water; kerosene. Without additional material, determine the density of the kerosene.

15. Get a lead or glass Y-tube (i.e., a three-way tube), each arm of which has a length of about 5 cm., and an internal diameter of about 5 mm.; two pieces of glass tubing each having an external diameter about equal to that of the Y-tube, and a length of 50 cm.; two pieces of rubber tubing about 5 cm. long, for connecting the two straight tubes to the Y-tube; a piece of rubber tubing about 10 cm. long to attach to the other arm of the Y-tube, and a pinchcock (see Chemistry, Appendix 20) or other device for closing the free end of the tube; the meter stick used in Exercise 14; two tumblers; water; kerosene. Without additional material, determine the density of kerosene. Make any necessary corrections for capillary action. Remember that you can easily suck air from or through the apparatus.

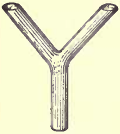


FIG. 124.

16. Using the apparatus shown in Fig. 110, and a meter stick, determine the density of mercury.

17. Wind closely 10 m. of No. 22 spring-brass wire upon a rod about 3 cm. in diameter, and suspend the spiral thus formed in front of a vertical meter stick or other scale, as shown in Fig. 125. To the lower end of the spring, the extremity of the wire having been bent into a horizontal index, attach two small scale-pans arranged so that one shall be about 10 cm. below the other. Place a glass of water upon an adjustable stand or easily movable blocks so that the lower pan may be kept immersed while the upper pan is always above the water. In the upper pan, place a small solid that will sink in water, and note the elongation of the spring as indicated by the movement of the index over the scale, lowering the

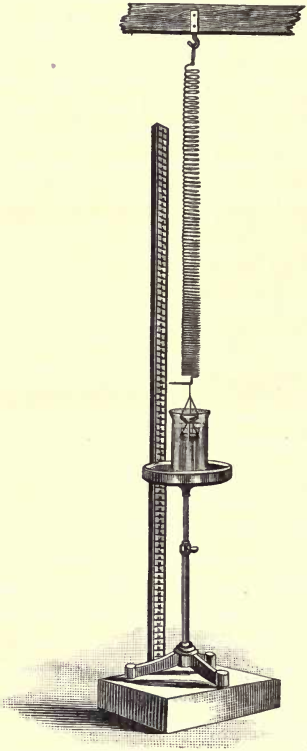


FIG. 125.

dish of water as may be necessary to keep the lower pan submerged and freely suspended. Then place the solid in the lower pan, and similarly weigh it in water. Determine the density of the solid.

18. Place a 1-gram weight in the upper pan of the *Jolly balance* described in Exercise 17, and note the elongation of the spring. Assuming that the spring will stretch proportionally for other loads, obtain in grams the two weights of another small solid that will sink in water, and determine its density.

19. Place a cork in the upper pan of the Jolly balance, and a sinker in the lower pan, and weigh them. Fasten the sinker to the cork, place both in the lower pan, and weigh them. Determine the density of the cork.

20. Remove both pans from the Jolly balance, and suspend the glass stopper of a bottle from the lower end of the spring. With this apparatus, determine the density of kerosene.

VIII. THE MECHANICS OF GASES.

163. Pneumatics *is the branch of physics that treats of the mechanical properties of gases*, and describes the machines that depend for their action chiefly on the pressure and elasticity of air.

(*a*) As water was taken as the type of liquids, so atmospheric air will be taken as the type of gases. All statements made in Section VII. concerning fluids, apply to gases as well as to liquids.

Note. — It is taken for granted that the school has an air-pump, an instrument that will soon be described, and the simpler pieces of apparatus that generally accompany it.

Weight of Air.

Experiment 73. — On a delicate balance, carefully weigh a thin glass or metal vessel that will hold several liters, and that may be closed by a stopcock. Pump the air from the vessel, close the stopcock, remove the vessel from the pump and carefully weigh it again.

Its loss of weight measures the weight of the air removed. If more convenient, the following may be substituted for the foregoing:—

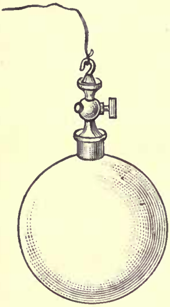


FIG. 126.

Into a half-liter Florence flask, put about 100 cu. cm. of water. Place the flask on a sand-bath and boil the water. The steam will expel the air from the flask. After the boiling has continued for some time, remove the lamp, and, when the boiling has ceased, loosely cork the flask. Cool the upper part of the flask by putting a wet cloth around it; the water will boil again. Then tightly close the flask with a rubber stopper. When the flask has cooled to the temperature of the room, shake it. The peculiar "water-hammer" sound of the water indicates a good vacuum in the flask. Weigh the flask and its contents. Loosen the cork and weigh again. The increase of weight is the weight of the air admitted to the flask. Subtract the volume of the water from the capacity of the flask to find the volume of the air thus weighed and compute the weight of the air per cubic centimeter. Record the readings of the thermometer and barometer at the time and place of the experiment.

Experiment 74.—Draw out a piece of glass tubing to a jet, and push it through a perforation in a cork that snugly fits a bottle. Slip a short piece of snugly fitting rubber tubing over the outer end of the glass tubing, and insert the cork so that the jet shall project into the bottle. Remove by suction as much air as possible from the bottle, pinch the rubber tubing tightly, place it under water, and remove the pressure. Something will force water into the bottle, forming the "fountain in vacuo," as shown in Fig. 127.

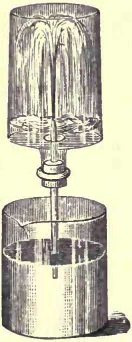


FIG. 127.

FIG. 128.

Experiment 75.—Fill a tumbler with water, place a slip of thick paper over its mouth and hold it there while the tumbler is inverted; the water will be supported when the hand is removed from the card.

Experiment 76. — Over the upper end of a cylindrical receiver, tie tightly a wet bladder or sheet of writing paper and allow it to dry. Then exhaust the air. The bladder will be forced inward, bursting with a loud noise. Replace the bladder with a thin sheet of india-rubber. Exhaust the air. The rubber sheet will be pressed inward, and nearly cover the inner surface of the receiver.

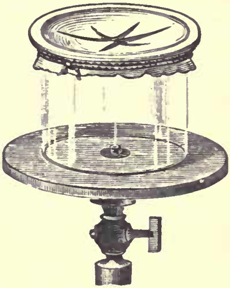


FIG. 129.

Experiment 77. — The Magdeburg hemispheres are accurately fitting, metallic vessels, generally three or four inches in diameter. Their edges are provided with projecting lips, and fit one another air-tight; the lips prevent sidewise slipping. Grease the edges to make more sure of a tight joint, fit the hemispheres to each other, and exhaust the air with a pump. Close the stopcock, remove the hemispheres from the pump, attach the second handle, and, holding the hemispheres in different positions, try to pull them apart. When you are sure that the pressure that holds them together is exerted in all directions, place them under the receiver (i.e., the bell-glass) of the air-pump, and exhaust the air from around them. The pressure seems to be removed, for the hemispheres fall apart of their own weight.

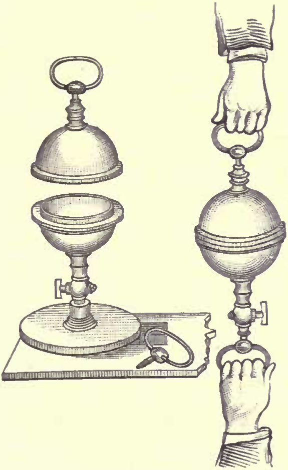


FIG. 130.

Experiment 78. — Connect the lamp-chimney apparatus used in Experiment 59 by a thick-walled rubber tube, and partly exhaust the air with the air-pump or by suction. Hold the chimney in different positions, and notice that the pressure that pushes in the rubber diaphragm is exerted equally in all directions. Any change of pressure will be shown by a change in the form of the rubber cup.

164. The Air. — *These experiments show that air has weight, that it exerts great pressure at the surface of the earth, and that this pressure is transmitted equally in all directions*, in accord with Pascal's law. Under ordinary atmospheric conditions, a liter of air weighs about 1.3 grams; a cubic foot weighs about an ounce and a quarter. As the atmospheric pressure is due to the weight of the overlying air, it follows that atmospheric pressure must decrease as we ascend from the sea-level.

Atmospheric Pressure.

Experiment 79. — Into one end of a piece of stout glass tubing about 1 m. long, and with a bore of about 1 cm., closely press a good cork or rubber stopper. Fill the tube with water; close the open end with the forefinger; invert the tube over the water-bath, and, when the end is under water, remove the finger. Note whether the water falls away from the corked end of the tube. Loosen or remove the cork, and note the result.

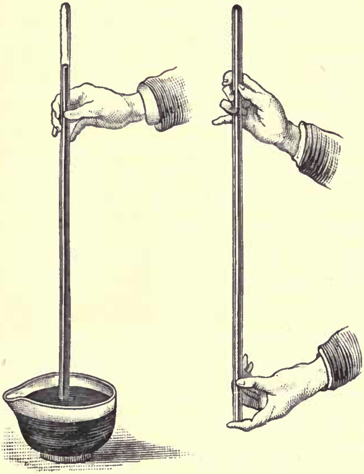


FIG. 131.

Experiment 80. — Fill with mercury a stout glass tube closed at one end and about 50 cm. long; a long "ignition tube" will answer. Invert it at the mercury-bath as shown in Fig. 131. Note whether the mercury falls away from the closed end of the tube.

Experiment 81. — Select a stout glass tube about 80 cm. long, several millimeters in internal diameter, and closed at one end. Twist a piece of paper into the shape of a hollow cone, and, using it as a funnel, fill the tube with mercury. With an iron wire, remove any air-bubbles that you see in the tube. Close the open end with the finger, and invert the tube at the mercury-bath, as shown in Fig. 131. When the finger is removed, the mercury falls away from the upper end of the tube, and finally comes to rest at a height of about 30 inches (or 76 cm.) above the level of the mercury in the bath, leaving a vacuum at the upper end of the tube. This historical experiment was first performed in 1643, by Torricelli, Galileo's pupil. If the tube is supported upright, the height of the sustained mercury column may be found to vary from day to day. If it is placed under a tall bell-glass and the air exhausted, the column will fall as the atmospheric pressure on the surface of the mercury decreases.

Experiment 82. — Modify the last experiment by selecting a tube open at both ends. Thoroughly soak in water such a membrane as comes tied over the stoppers of perfumery bottles, and tie it tightly over one end of the tube. When the membrane is thoroughly dry, fill the tube with mercury, and invert it at the mercury-bath as before. After measuring the height of the supported liquid column, prick a pinhole through the membrane, and notice what takes place.

165. Atmospheric Pressure. — In spite of the tendency of liquids to seek their level, we see that something supports a liquid column of great weight in the Torricellian tube. The removal of the air from the surface of the mercury in the bath shows that the pressure of the atmosphere is this supporting force. Since the size of the tube will not affect the height of the column, we may assume that the tube has a cross-section of one square centimeter. Then the supported mercury will measure 76 cu. cm. As the density of mercury is 13.596, this quantity of mercury will weigh 13.596 times 76 grams. The weight thus sustained shows that *the atmospheric pressure at the sea-level is approximately* 1,033.3 *grams per square centimeter, or* 14.7 *pounds per square inch.* For rough work or "in round numbers," it is often said that this pressure, which is called an atmosphere, is a kilogram per square centimeter, or fifteen pounds per square inch.

(*a*) Pascal carried a Torricellian tube to the top of a mountain, and there found that the mercury column was three inches shorter, showing that, as the weight of the atmospheric column diminishes, the counterbalanced column of mercury also diminishes. He then took a tube 40 feet long, and closed at one end. Having filled it with water, he inverted it over a water-bath. The water in the tube came to rest at a height of 34 feet. The weights of the two columns were equal. Experiments with still other liquids gave corresponding results, all of which strengthened the theory that the supporting force is atmospheric pressure, and left no doubt as to its correctness.

(*b*) Since mercury is more than ten thousand times as heavy, bulk for bulk, as air under the ordinary conditions of temperature and atmospheric pressure, a mercury column about 76 cm. (or 30 inches) tall is able to counterbalance an air column of equal cross-section reaching from the bottom to the top of our atmosphere.

166. The Barometer. — *A Torricellian tube, firmly fixed to an upright support and properly graduated, constitutes a mercurial barometer.* The zero of the scale is at the surface of the mercury in the cistern.

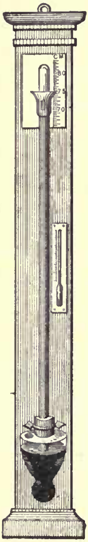


FIG. 132.

(*a*) When scientific accuracy is required, the cistern of the barometer is made with a flexible leather bottom that can be raised or lowered by a screw until the surface of the mercury just touches the point of an index, with which the zero of the scale coincides. For the more accurate reading of the scale, some instruments are provided with verniers. In every case, the extreme height of the convex surface of the mercury should be taken, and, if the scale has no vernier, the divisions of the scale should be subdivided by the eye as accurately as possible. The height of the barometer is, in such cases, corrected for temperature, for variations of gravity, for capillarity, for expansion of the scale, for elevation above sea-level, etc.

(*b*) The aneroid barometer is an easily portable instrument, and avoids the use of any liquid. It consists of a circular metallic box, exhausted of air, the corrugated diaphragm of which is held in a state of tension by springs. Varying atmospheric pressures cause movements of the diaphragm. These movements, being multiplied by delicate levers and a fine chain wound around a pinion, move the index pointer over a graduated scale. Such barometers are made so delicate that they show a difference in atmospheric pressure when transferred from an ordinary table to the floor. Their very delicacy involves the necessity for careful usage or frequent repairs.

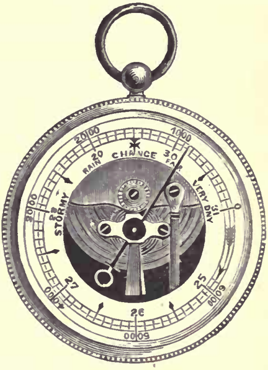


FIG. 133.

167. Barometric Variations. — Observation shows frequent variations in the barometric readings. Some slight changes are found to be periodic, but the greater changes follow no known law. *The utility of a barometer depends largely upon the fact that these irregular variations correspond to changes in the pressure of the air column* that rests on the surface of the mercury in the cistern, and, therefore, signal coming meteorological changes.

(*a*) The falling of the mercury generally indicates the approach of foul weather; a sudden fall denotes the coming of a storm. The rising of the mercury indicates the approach of fair weather or the "clearing up" of a storm.

(*b*) Sometimes the "barometer falls" and the looked-for storm does not appear. In such a case, it should be remembered that the barometer announced only a diminution of atmospheric pressure, and that we, influenced by experience, inferred the coming of a storm. Barometric declarations are reliable; inferences from those declarations are subject to error.

(*c*) The barometer is also used for the approximate determination of altitudes above sea-level.

CLASSROOM EXERCISES.

1. Give the pressure of the air upon a man the surface of whose body is 20 square feet.

2. A soap-bubble has a diameter of 4 inches. Give the pressure of the air upon it.

3. What is the weight of the air in a room 30 by 20 by 10 feet?

4. What will be the total pressure of the atmosphere on a decimeter cube of wood when the barometer stands at 760 mm.?

5. How much weight does a cubic foot of wood lose when weighed in air?

6. (*a*) What is the pressure on the upper surface of a Saratoga trunk $2\frac{1}{2}$ by $3\frac{1}{2}$ feet? (*b*) How happens it that the owner can open the trunk?

7. (*a*) What effect would it have upon the height of the barometer column if the barometer tube was enlarged until it had a sectional area of 1 sq. cm.? (*b*) Assuming that the density of mercury is 13.6, and that the barometer stands at 760 mm., what is the atmospheric pressure per square centimeter of surface? *Ans.* 1,033.6 g.

8. A certain room is 10 m. long, 8 m. wide, and 4 m. high. (*a*) What weight of air does it contain? (*b*) What is the pressure upon its floor? (*c*) Upon its ceiling? (*d*) Upon each end? (*e*) Upon each side? (*f*) What is the total pressure upon the six surfaces? (*g*) Why is not the room torn to pieces?

9. An empty toy balloon weighs 5 g. When filled with 10 l. of hydrogen, what load can it lift? A liter of hydrogen weighs 0.0896 g.

Elastic Force.

Experiment 83. — Tightly close the opening of a toy balloon, football, or other rubber bag, only partly filled with air. Place it under the receiver of an air-pump, as shown in the accompanying figure, and exhaust the air from the receiver. The flexible wall of the bag will be pushed back by the innumerable impacts of the moving molecules against the confining surface. The observed phenomenon is in strict accord with the kinetic theory of gases, § 51.

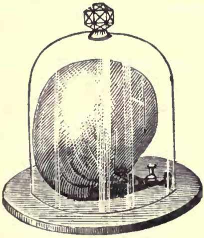


FIG. 134.

Experiment 84. — For the rubber bag of Experiment 83, substitute successively a dish containing soap-bubbles, and a bottle with its mouth opening under water in a tumbler. Exhaust the air as before, and notice the effect of the molecular impacts on the liquid walls of the confined air.

Experiment 85. — Half fill a small bottle with water, and close the neck with a cork through which a small tube passes. The lower end of this tube should dip into the liquid; the upper end should be drawn out to a jet. Apply the lips to the upper end of the tube, and force air into the bottle.

Experiment 86. — Place the bottle, arranged as above described, under the receiver of an air-pump, and exhaust the air from the receiver. Water will be driven in a jet from the tube.

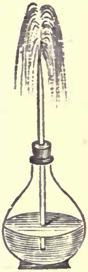


FIG. 135.

Experiment 87. — Apparatus dealers supply "bursting squares" made of thin glass, and sealed under the ordinary atmospheric pressure. Place one of these "squares" upon the plate of the air-pump, cover it with wire netting as a protection against accident, and over all place a bell-glass. Exhaust the air from the bell-glass, and the elastic force of the air confined in the square will burst the thin glass walls outward.

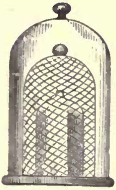


FIG. 136.

168. Elastic Force of Gases. — When a glass vessel (see Fig. 126) is open, the atmospheric pressure on the outer surface is exactly balanced by the pressure on the inner surface. Closing the stopcock will not destroy the equality of pressures; the elastic force of the confined gas will just equal the pressure of the surrounding atmosphere. If the stopcock is closed when the gas is under a pressure of two atmospheres, the equality will still continue, each being about thirty pounds per square inch. In neither case will the vessel be subjected to any stress by the gas within or without. *The elastic force of a gas supports and equals the pressure exerted upon it.*

Relation of Volume to Pressure.

Experiment 88. — Provide a stout glass tube more than a meter long, bent as shown in Fig. 137, the short arm being closed. Pour a small quantity of mercury into the tube so that its surfaces in the two tubes are in the same horizontal line. By holding the tube nearly level, bubbles of air may be passed into the short arm or from it until the desired result is secured. The air in the short arm will then be under a pressure of one atmosphere. Fasten the tube to an upright support, and place a scale graduated to millimeters by each arm, the zero of each scale being just at the mercury levels. Pour mercury into the long arm of the tube, thus increasing the pressure on the air confined in the short arm. When the vertical distance between the levels of the mercury in the two arms is one fourth the height of the barometric column at the time and place of the experiment, the pressure upon the confined air will be $\frac{5}{4}$ atmospheres, i.e., a pressure of about 95 cm. of mercury; the elastic force of the confined air just supports this pressure, and must, therefore, be $\frac{5}{4}$ atmospheres; the volume of the confined air is $\frac{4}{5}$ what it was under a pressure of one atmosphere. If more mercury is added

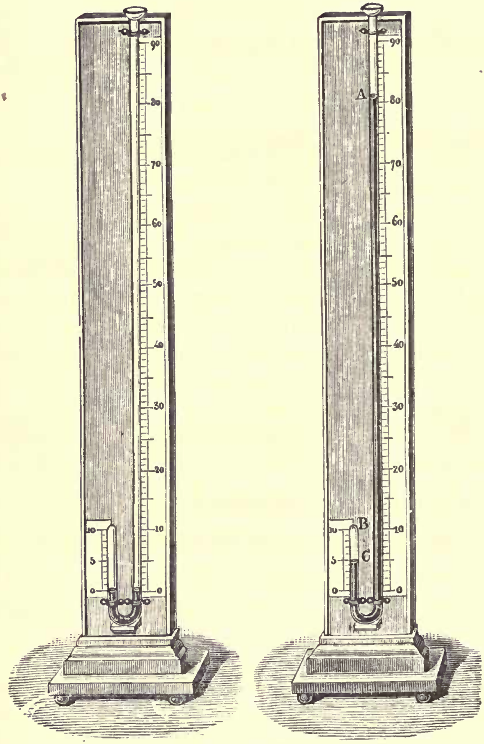


Fig. 137.

until the vertical distance between the two mercurial surfaces is half the height of the barometric column, the pressure and the elastic force will be $\frac{3}{2}$ atmospheres, or about 114 cm. of mercury; the volume of the confined air will be $\frac{2}{3}$ what it was under a pressure of one atmosphere. When mercury has been poured into the long arm until the vertical distance, *CA*, is equal to the height of the barometric column, the pressure will be two atmospheres, or about 152 cm. of mercury, and the volume of the confined air will be half what it was under a pressure of one atmosphere.

Experiment 89. — Nearly fill the barometer tube used in Experiment 81, or a similar tube more than half as long and graduated to cubic centimeters, with mercury, and invert it over a mercury-bath as shown in Fig. 138. Lower the tube into the tank until the mercury within the tube and without it is at the same level. The confined air is under the same pressure as the external air; e.g., 76 cm. of mercury. Note the volume of the confined air. Raise the tube until this volume of the confined air is doubled, and measure the height of the mercury column in the tube. It will be found that the confined air is under a pressure half that of the external air; e.g., 38 cm. of mercury.

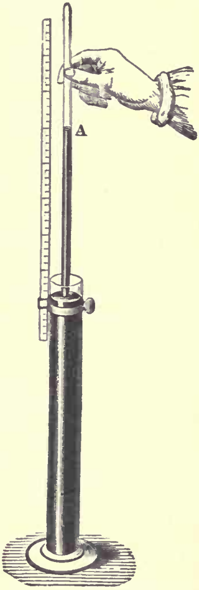


FIG. 138.

Addenda. — Suppose the volume of gas confined in each of the last two experiments measured 5 cm. under a pressure of one atmosphere. Arrange the data just obtained in the following form, and complete the table: —

Pressures.	*Volumes.*	*Products.*
76	5	380
95	4	?
114	$3\frac{1}{3}$	?
152	$2\frac{1}{2}$	?
38	10	?

169. Boyle's Law. — When the temperature remains constant, *the volume of a gas varies inversely as the pres-*

sure upon it; i.e., the product of a given volume of gas by its pressure is constant.

$$V \propto \frac{1}{P}, \text{ or } V \times P = \text{a constant quantity.}$$

(*a*) Later experiments have shown that Boyle's law is only approximately true, and that all gases deviate from it as they near the point of liquefaction. This law is often called Mariotte's.

CLASSROOM EXERCISES.

1. Under ordinary conditions, a certain quantity of air measures one liter. Under what conditions can it be made to occupy (*a*) 500 cu. cm.? (*b*) 2,000 cu. cm.?

2. Under what circumstances would 1 cubic foot of air, at the freezing temperature, weigh about 2½ ounces?

3. Into what space must we compress (*a*) a liter of air to double its elastic force? (*b*) Two liters of hydrogen?

4. A barometer standing at 30 inches is placed in a closed vessel. How much of the air in the vessel must be removed that the mercury may fall to 15 inches?

5. A vertical tube, closed at the lower end, has at its upper end a frictionless piston that has an area of 1 square inch. The weight of this piston is 5 pounds, and it confines 24 cubic inches of dry steam. (*a*) What is the elastic force of the confined steam? (*b*) If the piston is loaded with a weight of 10 pounds, what will be the volume of the confined steam?

6. Mercury stands at the same level in both arms of a tube like that shown in Fig. 137. The barometer rises, and thereupon is noticed a difference in the heights of the two mercury columns in the **J**-tube. In which arm does the mercury stand the higher? Why?

Siphon.

Experiment 90. — Place a pail of clean water on the table, and an empty water pail on the floor. Place one end of a piece of thick-walled rubber tubing, about a yard long, in the water. Hold the other end of the tubing below the level of the table-top, and fill the tube with water by suction. Notice the transfer of water from one pail to the other. Be careful that the flexible walls of the tubing do not close upon each other at the edge of the upper pail, and thus cut off the flow.

Experiment 91. — Change the positions of the pails, placing the one containing water on the table. Gradually lower the rubber tubing into the water, allowing air to escape from the upper end as water enters at the lower end. When the tube is filled with water, pinch one end of it tightly, and carry it below the level of the table-top. Raise and lower this end of the tubing to see if the distance of the opening below the edge of the upper pail has anything to do with the rate of flow.

Experiment 92. — Using a bent glass tube of small internal diameter and two tumblers or bottles, arrange apparatus to transfer water from a higher to a lower level, essentially as in Experiment 91. Put the apparatus in operation on the plate of the air-pump, cover it with a bell-glass, and quickly exhaust the air. The flow of liquid ceases, but begins anew when air is again admitted.

170. The Siphon *is essentially a tube with unequal arms, used to carry liquids from one level, over an elevation, to a lower level by means of atmospheric pressure.* It is generally set in action by filling it with the liquid, closing its ends, placing the end of the shorter arm in the liquid to be moved, bringing the end of the longer arm to a lower level, and then opening the ends. The flow will continue until the liquids stand at the same level, or until air enters the tube at the end of the shorter arm.

171. Explanation of the Siphon. — The vertical distance from the level of the upper liquid to the highest point of the tube (*ab*) is the length of one arm (see Fig. 139); the vertical distance from the highest point of the tube to the lower end of the tube, or to the level of the liquid into which it dips (*cd*), is the length of the other arm. The second of these must exceed the first.

Consider the horizontal layer of molecules in the tube at the levels, a and d. The atmospheric pressures, whether direct or transmitted by the liquids in accordance with Pascal's law, will be upward and equal; represent them by p. The weight of the water in the short arm produces a downward pressure at a; represent this by w. The resultant of these forces acting at a is $p - w$. Similarly, the weight of the water in the long arm produces a downward pressure at d; represent this by w'. The resultant of these forces acting at d is $p - w'$. These two resultants act against each other, $p - w$ being the greater. The resultant of these resultants is their difference; $(p - w) - (p - w') = w' - w$. Thus we see that the liquid is pushed through the tube by a net force equal to the weight of a liquid column whose height is the difference between the two arms of the siphon.

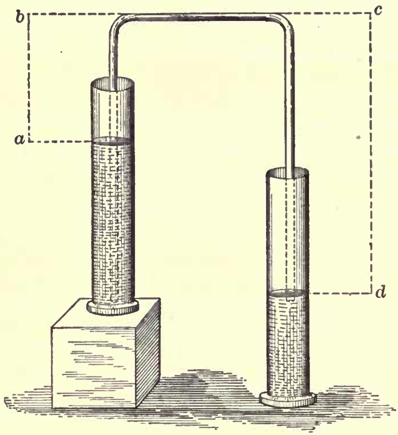


FIG. 139.

(a) Suppose the siphon to have a cross-section of 1 sq. cm.; that $ab = 10$ cm.; that $cd = 22$ cm. Then the upward pressure at a and at d will be 1,033 g.; the downward pressure at a will be 10 g.; the downward pressure at d will be 22 g. Then $p - w = 1{,}023$ g.; $p - w' = 1{,}011$ g. The resultant of these two upward and antagonistic pressures $(w' - w)$ is a force of 12 g. tending to push the water from a to d.

(b) If the downward pressure at a is as great as the atmospheric pressure, the liquid will not flow. Hence, the elevation over which water is to be siphoned must be less than 34 feet.

Pumps.

Experiment 93. — Every one knows that a liquid may be sucked up through a straw or other tube. Modify the familiar experiment by passing a glass tube snugly through the cork of a bottle. Fill the bottle with water, and close it with the perforated cork. Be sure that no air is left in the bottle. The tube should dip an inch or so into the water. Try to suck water from the bottle.

172. The Lift Pump or suction-pump consists of a cylinder or barrel, piston, two valves, and a suction-pipe, the lower end of which dips below the surface of the liquid to be raised. The piston works practically air-tight in the cylinder, and has an outlet-valve that opens upward. The inlet-valve is at the upper end of the suction-pipe, and also opens upward. When the piston is drawn upward, its valve is closed by the pressure of the air above, and a partial vacuum is formed in the cylinder below. The elastic force of the air in the cylinder being thus reduced, *the atmospheric pressure forces water up the suction-pipe*, driving the air above it through the lower valve. When the piston is pushed down, the inlet-valve is closed, and the confined air escapes through the outlet-valve. As the piston continues its work, the air is gradually removed from the cylinder and suction-pipe, and the transmitted pressure of the atmosphere pushes the water up to take its place and to restore the disturbed equilibrium.

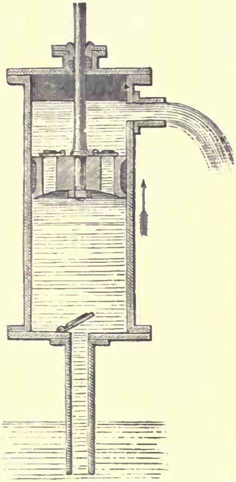


FIG. 140.

(*a*) Theoretically the piston may be 34 feet above the level of the water in the well, but, owing to mechanical imperfections, the practical limit for a pump lifting water by suction is about 28 vertical feet. The height to which water may be lifted above the piston depends only upon the strength of the pump and the power applied.

173. The Force Pump. — The operation of the force-pump is similar to that of the suction-pump. The outlet-valve generally opens from the cylinder, the piston being made solid. When the piston is raised, water is forced into the barrel by atmospheric pressure. When the piston is forced down, the inlet-valve is closed, the water being forced through the other valve into the discharge-pipe. When next the piston is raised, the outlet-valve is closed, preventing the return of the water above it, while atmospheric pressure forces more water from below into the barrel.

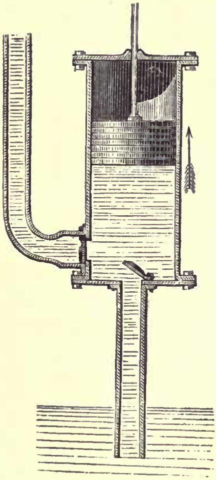


Fig. 141.

(*a*) For the purpose of securing steadiness for the stream as it issues from the delivery-pipe, the water usually passes into an air-chamber. The elasticity of the confined and compressed air largely takes up the pulsating effect due to the successive pushes of the piston, and forces the water from the nozzle of the delivery-pipe in a continuous stream. Fire-engines and nearly all steam-pumps have such attachments.

174. The Air Pump is an instrument for removing a gas from a closed vessel. Fig. 142 shows the essential parts of one of the many forms.

(*a*) The glass receiver, *R*, fits accurately upon the ground plate. The edge of the receiver is often greased to insure an air-tight joint.

From *R*, a tube, *tt*, leads to the cylinder, *C*, in which moves a piston, *P*. Two valves open from the receiver. The outlet-valve, *v′*, is in the piston; the inlet-valve, *v*, may be carried by a rod that passes through the piston. Of course the piston, valve, and all sliding parts of the pump, must work air-tight. A down-stroke of the piston carries down the valve-rod, and closes *v*; the elastic force of the air confined beneath *P* opens *v′*, and some of the air escapes to the upper side of the piston. The next up-stroke of the piston closes *v′* and lifts the valve-rod, and thus opens *v*. The upward motion of the valve-rod is closely limited by a shoulder near its upper end, the piston sliding upon the rod during the greater part of its up-and-down movements. The air that passes up through *v′* is forced out through an opening (preferably closed by a valve) at the top of the cylinder, and the elastic force of the air in *R* and *t* fills again the lower part of *C*. By the continued working of the piston, the mass and the elasticity of the air in *R* are reduced, a vacuum being approached more and more closely. As only a fractional part of the residual air is removed at each stroke, a perfect vacuum is out of the question; moreover, there is a limit arising from the unavoidable imperfections of the apparatus.

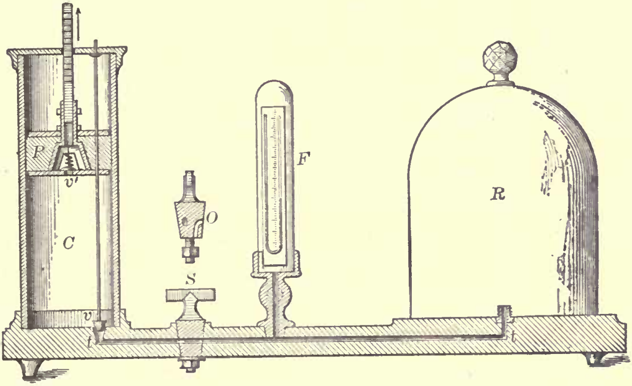


FIG. 142.

(*b*) In Fig. 142, *F* is a glass vessel communicating with the receiver. It contains a siphon-barometer or mercurial gauge to indicate the degree of rarefaction obtained. A stopcock at *S*, when turned one way, cuts off communication between *C* and *R*, thus reducing the risk that air will reënter the receiver; when turned the other way, it readmits air to *R*.

175. The Condensing Pump is an instrument for compressing a gas into a closed vessel, as in pumping air into a pneumatic tire of a bicycle, or oxygen or hydrogen into the cylinders commonly used for stereopticon purposes, or charging water with carbon dioxide for sale as "soda water."

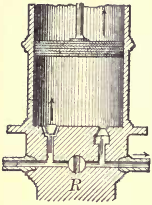


Fig. 143.

(*a*) It differs from the air-pump chiefly in the facts that the valves are made strong enough to endure high pressures, and that they open toward the receiver. For some purposes, the piston is made solid, and both valves are placed at the bottom of the cylinder, as shown in Fig. 143. The stopcock at *R* is closed while the pump is in operation. By connecting each of the lateral tubes with a closed tank, gas may be transferred from one to the other.

CLASSROOM EXERCISES.

1. How high can water be raised by a perfect lift-pump, when the barometer stands at 30 inches? The density of mercury is 13.6.

2. If a lift-pump can just raise water 28 feet, how high can it raise alcohol having a density of 0.8?

3. Water is to be taken over a ridge 12.5 m. higher than the surface of the water. (*a*) Can it be done with a siphon? Why? (*b*) With a lift-pump? Why? (*c*) With a force-pump? Why?

4. Will a given siphon carry water over a given elevation more rapidly at the top, or at the bottom, of a mountain? Why?

5. If water rises 34 feet in an exhausted tube, how high will sulphuric acid (density, 1.8) rise under the same circumstances?

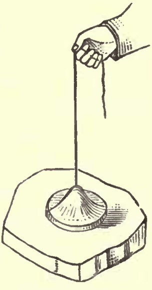


Fig. 144.

6. The "sucker" consists of a circular piece of thick leather with a string attached to its middle. Being soaked thoroughly in water, it is firmly pressed upon a flat stone to drive out all air from between the leather and

the stone. Unless the stone is too heavy, it may be lifted by the string. Is the stone really pulled up, or pushed up? Explain your answer.

7. If the capacity of the cylinder of an air-pump is $\frac{1}{4}$ that of the receiver, (*a*) what part of the air will remain in the receiver at the end of the fourth stroke of the piston? (*b*) How will its elastic force compare with that of the external air?

8. How high can a liquid with a density of 1.35 be raised by a perfect lift-pump when the barometer stands at 29.5 inches?

9. Over how high a ridge can water be continuously carried in a siphon, the minimum standing of the barometer being 69 cm.?

10. What is the greatest pull that can be resisted by Magdeburg hemispheres (*a*) 4 inches in diameter? (*b*) 8 cm. in diameter?

11. How can you arrange a single lift-pump to raise water from the bottom of a well 50 feet deep?

12. Construct the apparatus shown in Fig. 145, filling each of the three bottles half full of water. Be sure that all joints made by the corks of the three bottles are air-tight. Blow into the tube, *f*, until a jet is formed at *n*. Explain the continued action of the apparatus.

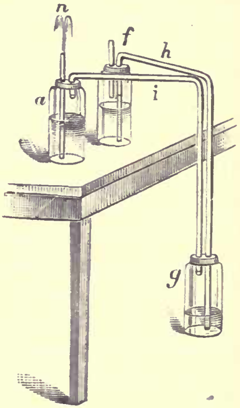


FIG. 145.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Small test-tube; pen-filler; sulphuric ether; pinchcock; lead counterpoise.

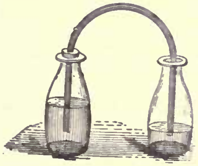


FIG. 146.

1. Partly fill two bottles with water. Connect them by a bent tube that fits closely into the mouth of one, and loosely into the mouth of the other. Place the bottle under the receiver and exhaust the air. Note and record what takes place. Admit air to the receiver. Note and record what takes place. Write an explanation of the phenomena.

2. Fill a test-tube with water and invert it in a tumbler of water. With a pen-filler, introduce a few drops of sulphuric ether, a very volatile and extremely inflammable liquid, into the test-tube. The ether will rise to the top of the tube. Place the tumbler and the test-tube under the receiver and exhaust the air. The water in the test-tube falls. Readmit air to the receiver, and note the contents of the test-tube. Record your conclusions concerning the effect of pressure upon the molecular condition of sulphuric ether.

3. With a short piece of rubber tubing, connect the short arms of two L-shaped glass tubes, and set up the apparatus as a siphon. While the water is flowing, perforate the rubber wall between the glass tubes. Note and explain the effect.

4. With apparatus like that used in Experiments 88 and 89, and noting the reading of the mercurial barometer at the time, verify Boyle's law between the limits of half an atmosphere and two atmospheres, by showing the constancy of the product of volume into pressure.

5. Fill the U-tube used in Experiment 63 with mercury to a depth of about 30 cm., and attach a rubber tube about 40 or 50 cm. long firmly to one end of the tube. Force air steadily from the lungs through the rubber tube; pinch the tube tightly; rest; increase the pressure on the mercury if you can without injury or pain; close the rubber tube with a pinchcock; and compute your maximum "lung power" in pounds per square inch.

6. With the same apparatus, suck air from the tube, and compute your maximum "force of suction" in pounds per square inch. Explain the use of quotation marks in the last sentence.

7. With the same apparatus, measure the pressure under which illuminating gas is delivered at the laboratory. Will you get more accurate results by using mercury, or water?

8. What objection is there to using the same apparatus for measuring the pressure under which water is supplied at the laboratory? Devise some means for that problem, and solve it.

9. Repeat Experiment 73. Determine the capacity of the vessel used. Compare the reading of the pressure-gauge of the air-pump with that of the barometer at the time of the experiment, and determine what part of the air in the vessel was removed. Determine the density of air under the thermometric and barometric conditions of the experiment. If you have no vessel like that mentioned in Experiment 73, use a 2-liter bottle, connecting its rubber stopper

with the pump by a thick-walled rubber tube. After exhaustion, close the tube with a pinchcock. Make all connections air-tight, using glycerin for that purpose. Be sure not to omit any of the apparatus from one weighing, that is included in the other.

10. Fill with mercury the tube used in Experiment 81, and by direct weighing and measurement, determine the weight of the mercury per centimeter of the length of the tube. Measure the length of the barometric column in the same tube and compute the weight of the mercury supported by atmospheric pressure.

11. Make a siphon with parallel arms, each about 50 cm. long, by bending a piece of glass tubing twice at right angles. Place one branch of the tube in a large vessel of water at least 40 cm. deep, and support the siphon so that its short arm shall be 10 cm. long. (See § 170.) Start the siphon by suction, and measure with a graduate the water that flows in two minutes. Determine the difference between the lengths of the two arms. Raise the siphon until its short arm is 15 cm. long. Set the siphon in action. Measure the water that flows in two minutes, and determine the difference between the lengths of the two arms, as before. As this difference decreases, does the amount of water delivered also decrease? Continue the work, successively testing with short arm lengths of 20, 25, 30, 40, and 45 cm. Tabulate your results. What is the relation between rate of flow and the difference in the lengths of the two arms of the siphon?

12. Using the differences in lengths of the arms as ordinates, and the rate of flow as abscissas, map the line that represents this relation. From this line, measure the abscissa of the point that has an ordinate corresponding to a length of 35 cm. for the short arm of the siphon, and determine the rate of flow that it represents. Set the siphon used in Exercise 11 so that its short arm shall measure 35 cm., and by direct experiment test the accuracy of the result computed from the curve.

13. With the apparatus used in Experiment 88, measure the compression of the air confined under different pressures until the excess of mercury in the long arm is about equal to the barometric column. Avoid touching the short arm, or in any way changing the temperature of the confined air. From the data thus obtained, map upon cross-section paper a line representing the relation between volume and pressure. Let each vertical division of the paper represent a pressure of 2 cm. of mercury, and each horizontal division a com-

pression of 0.5 cm. Is the line thus mapped a straight line? If so, what does that show? If the line is curved, what does the varying downward slope indicate? From the line as drawn, do you conclude that equal increments of pressure produce equal compressions, or otherwise?

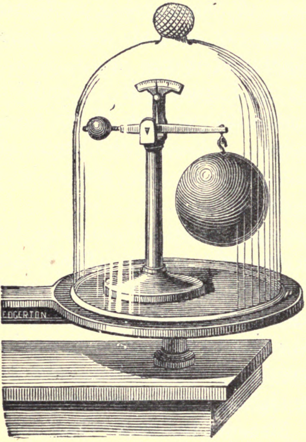


Fig. 147.

14. Using a lead ball as shown in Fig. 147, counterpoise the globe of Experiment 73, filled with air and with stopcock closed. Cover the apparatus with a bell-glass and exhaust the air. The globe seems to be heavier than the ball which previously balanced it. Record a description and explanation of what you do and of what followed.

CHAPTER III.

ACOUSTICS: MASS PHYSICS.

I. THE NATURE OF SOUND, ETC.

176. Sound *is the mode of motion that is capable of affecting the auditory nerve.*

(*a*) We have to deal with sound only as a physical phenomenon; not as a physiological or psychological process.

Cause of Sound.

Experiment 94. — Suspend a small pith-ball so that it shall rest lightly against the edge of a bell or bell-jar. Strike the bell so that it will give forth a sound. Such a bell or a tuning-fork may be sounded by tapping it with a hammer made by slipping a piece of rubber tubing over a stout iron wire, or by thrusting the handle of a cheap tapering pen-holder into the hole at the center of a circular eraser. A better way, however, is to make it vibrate by drawing a resined violin-bow across the edge or end. Notice that the ball acts as if the edge of the bell was in vibration. Touch the edge of the sounding bell lightly with the finger-nail.

Experiment 95. — Sound a tuning-fork and just touch a water surface with one of its prongs. Notice the spray.

FIG. 148.

Experiment 96. — The chimneys of student-lamps often break at the contracted part near the bottom. Close such a broken chimney at the broken end with cork or wax. At the toy store, buy a wooden whistle like that shown in the upper part of Fig. 148, and cut it off at the point indicated by the dotted line between the first and second finger-holes. Fit the end of the whistle into the opening of a cork that will tightly close the open end of the broken chimney. Inside the tube, place a small quantity of precipitated silica, a very light powder that you can buy from a dealer in chemical supplies, or a like quantity of the dry dust obtained by filing a cork. Close the tube with the cork and whistle, hold the tube horizontal and shake it endwise so as to distribute the powder evenly, and then blow the whistle. The powder is agitated in a peculiar manner, rising in thin vertical plates, and coming to rest in little piles transverse to the axis of the tube. Was the motion of the air in the tube one of vibration, or one of translation?

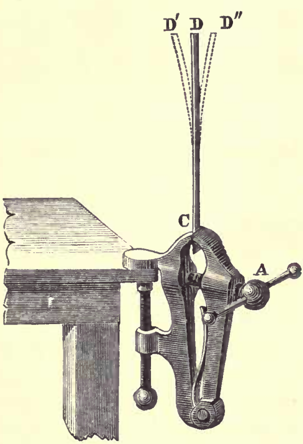


Fig. 149.

Experiment 97. — Grasp one end of a straight spring made of hickory or steel in one end of a vise, as shown in Fig. 149. Pluck the free end of the spring so as to produce a vibratory motion. If the spring is long enough, the vibrations may be seen. Lower the spring in the vise to shorten the vibrating part of the rod, and pluck it again. The vibrations are reduced in amplitude, and increased in rapidity. Continued shortening of the spring will render the vibrations invisible and audible; they are lost to the eye, but revealed to the ear.

177. Cause of Sound. — From these experiments, it is reasonable to conclude that *sound is caused by the rapid vibrations of a material body.* In fact, all sounds may be traced to such vibrations. Bodies that emit sounds are called sonorous.

(*a*) A glass plate that has been blackened by holding it over a petroleum or a camphor flame may be arranged so as to slide easily in the grooved frame, *F*. A triangular piece of tinsel or a short piece of the hairspring of a watch attached by wax to the end of one of the prongs of the fork will make a good style for our purpose. When the fork is made to vibrate, the style placed against the smoked plate, and the plate drawn along rapidly in the grooves, the undulating line traced on the glass represents the vibratory movement of the prong.

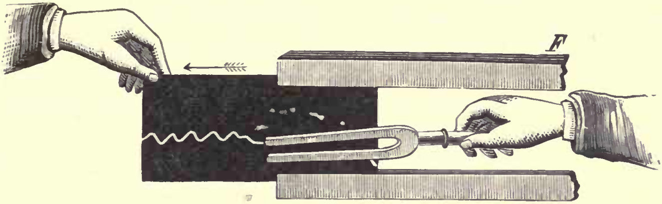


FIG. 150.

Wind or Wave?

Experiment 98. — Provide a tube four or five yards long, and about four inches in diameter. A few lengths of common spout from the tinner's will answer. Furnish it with a funnel-shaped piece, having an opening about an inch in diameter. Place the tube on a table with a candle flame opposite the opening at *B*. With a book, strike a sharp blow upon the table opposite the opening at *A*. The flame will be agitated and perhaps blown out. Something went from *A* to *B*. Did it go through the tube?

FIG. 151.

Experiment 99. — Close the opening at *A* and repeat the experiment; the flame is not put out. Remove the tube and repeat the blow; the flame is not put out.

Experiment 100. — Replace the tin tube by a rubber tube of the same length and with an internal diameter of about 10 or 15 mm. Insert the neck of a funnel at the end of the tube at *A*. Get a few inches of glass tubing that will fit snugly into the rubber tubing. Heat the middle of the glass in a flame until it softens. Slowly draw the ends asunder until the softened part is reduced to a diameter of about 2 mm. Break the tube at this narrow neck and push the large end of one piece into the rubber tube at B. Place a *small* flame opposite the small opening of the glass tube. Strike a blow in front of the funnel at A and notice that a puff or pulse of air blows the flame. Make a loose loop in the rubber tube and repeat the experiment. Clap the hands at A and notice the series of puffs at B. While an assistant is clapping his hands at A, pinch the rubber tube. Notice that the puffs at B cease while the tube is thus pinched, and reappear as soon as the tube is released. The tube is necessary. Whatever agitated the candle flame did go through the tube. Was this something a wind, or a wave?

Experiment 101. — Replace the tin tube. Dissolve as much potassium nitrate (saltpeter) as you can in half a cupful of hot water. Soak a piece of blotting-paper in this liquid and dry it. This "touch-paper" burns with much smoke but no flame. Burn the paper in the tube near A, filling that end of the tube with smoke. Repeat Experiment 98. No smoke issues at B; *it was not a wind that passed through the tube.*

178. Propagation of Sound. — Sound is ordinarily propagated through the air. Tracing the sound from its source to the ear of the hearer, we may say that the first layer of air is struck by the vibrating body. The particles of this layer give their motion to the particles of the next layer, and so on until the particles of the last layer strike upon the drum of the ear.

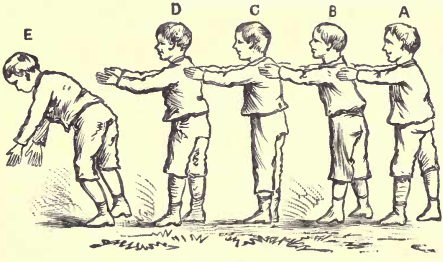


Fig. 152.

(*a*) This idea is beautifully illustrated by Professor Tyndall. He imagines five boys placed in a row, as shown in Fig. 152. "I suddenly push *A*; *A* pushes *B* and regains his upright position; *B* pushes *C*; *C* pushes *D*; *D* pushes *E*; each boy, after the transmission of the push, becoming himself erect. *E*, having nobody in front, is thrown forward. Had he been standing on the edge of a precipice, he would have fallen over; had he stood in contact with a window, he would have broken the glass; had he been close to a drumhead, he would have shaken the drum. We could thus transmit a push through a row of a hundred boys, each particular boy, however, only swaying to and fro. Thus also we send sound through the air, and shake the drum of a distant ear, while each particular particle of the air concerned in the transmission of the pulse makes only a small oscillation."

The Medium of Sound.

Experiment 102. — Place a small music box or alarm clock on a thick layer of felt, cotton wool, or other inelastic material on the plate of the air-pump, and cover it with a bell-glass. Exhaust the air and notice that while the motion of the mechanism is plainly visible, the sound is scarcely audible.

Experiment 103. — In a large glass globe provided with a stopcock, suspend a small bell by a thread. Pump the air from the globe; shake the globe, and notice that the sound of the bell is very faint. Readmit the air; shake the globe, and notice that the sound of the bell is heard distinctly. A large, wide-mouthed bottle, with a perforated rubber stopper, rubber tubing, and pinchcock, may be substituted for the globe and stopcock. See Experiment 73.

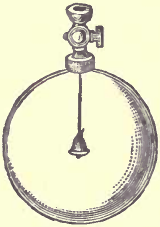


FIG. 153.

Experiment 104. — Fill a tumbler with water and place it on an empty crayon box. Stick the stem of a tuning-fork into a small wooden disk, sound the fork, and hold it with the disk resting upon the surface of the water. The vibrations will be transmitted by the water, and the sound of the fork will be heard as if coming from the box. Other liquids may be similarly tested.

Experiment 105. — Provide a wooden rod about half an inch square and five or six feet long. Place one end of this rod (preferably made of light, dry pine) against the panel of a door; hold the rod horizontal, and place the handle of a vibrating tuning-fork against the other end. Notice the sound given out by the panel. The common "string telephone" is a more familiar illustration of the transmission of sound by a solid.

179. Sound Media. — *Any elastic substance may be the medium for the transmission of sound.* Liquids and solids are better conductors of sound than gases are. The scratching of a pin may be heard through a long wooden beam; and the gentle tap of a hammer, through a water-pipe a mile or more in length. The nature of the vibrations in sound-media demands careful consideration.

Vibratory Motion.

Experiment 106. — Grip one end of the meter stick in a vise, as shown in Fig. 149. Pluck the free end, and notice that the vibrating end returns periodically to the starting point. Suspend a lead bullet by a long thread, swing it as a pendulum, and notice that the ball returns periodically to the starting point. Swing the ball as a conical pendulum, and notice that the ball, moving in a circular path, returns periodically to the starting point. Twist the torsional pendulum (Experiment 17), and notice that the index returns periodically to the starting point.

Experiment 107. — Fasten an elastic cord to a ball, or buy a "return ball" at a toy shop. Hold the end of the cord in one hand, and, with the other hand, pull the ball down and let it go. The ball swings up and down in the direction of the length of the cord. Notice that the speed of the ball varies much as does that of a common pendulum, and that the ball returns periodically to the starting point.

180. Vibrations. — When the parts of a body move so that each returns periodically to its initial position, the body is said to be in vibration. *The motion made in the interval between two successive passages in the same direction through any position is called a vibration.* The vibration may be transverse, torsional, or longitudinal, the classification having reference to the direction of the vibration relative to the length of the vibrating body.

(*a*) A vibration is analogous to a double or "complete" oscillation, as defined in § 114 (*a*). When the reciprocating movement is comparatively slow, as that of a pendulum, the term "oscillation" is commonly used; the term "vibration" is generally confined to rapid reciprocations or revolutions, as that of a sonorous body. The three kinds of vibration have the same manner of moving, the changes in velocity being the same as take place in the swings of a pendulum.

Pendular Motion.

Experiment 108. — Let a pupil take a ball-and-thread pendulum to the further side of the room. With a slight circular motion of the hand that supports the end of the thread, let him cause the ball to move in a circular path, thus forming a conical pendulum. When the speed of the ball has become uniform, count the swings that the ball makes around the circle in 30 seconds. Then place your eye on a level with the ball and observe it; i.e., look at the ball along a line of sight that is in the plane of the circle. *The ball will appear to move from side to side in a straight line that coincides with a diameter of the circle, and to vary its velocity as a common pendulum does.* Next, swing the same ball as a common pendulum, and count the vibrations that it makes in 30 seconds. A conical pendulum and a common pendulum of the same length have the same period. When the common pendulum is viewed from beneath, i.e., when the line of sight is in the plane of vibration as before, the ball again appears to move in a straight line and with a like varying velocity. This apparent motion and its relation to the real motion are very interesting and instructive. Let the circle shown in Fig. 154 represent the path described by the conical pendulum; then will the diameter, *A G*, represent the apparent rectilinear path. Suppose that the ball goes around the circle in two seconds. Divide the circumference into any number of equal parts, as 12. The ball will move over each of these equal arcs in $\frac{1}{6}$ of a

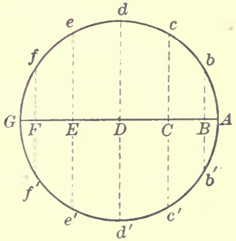


FIG. 154.

second. To one who is looking at this motion in the plane of the paper, the ball appears to go from *A* to *B* while it really goes from *A* to *b;* it appears to go from *B* to *C* while it really goes from *b* to *c;* etc. When the ball is at *d*, it is moving across the line of sight, and, therefore, appears to have its greatest velocity, just as a common pendulum does, at the middle of its arc. When it is at *A* or *G*, it is moving in the line of sight, and, therefore, appears to be at rest, although it is really moving with its uniform velocity. From a study of the figure, it will be seen that the ball appears to go from *A* to *G* and back in the two seconds in which it really goes around the circle. The unequal lengths, *AB*, *BC*, . . . *FG*, give a fair idea of the varying speed of a common pendulum.

181. Simple Harmonic Motion. — If, while a particle moves in the circumference of a circle with uniform velocity, a point moves along a fixed diameter of the circle so as always to be at the foot of a perpendicular drawn from the particle to the diameter, as described in Experiment 108, the motion of the point along the diameter is called a *simple harmonic motion.* The radius of the circle, or the distance from the middle to the extremity of the swing, is called the *amplitude* of vibration; the time intervening between two passages of the particle in the same direction through any point is called the *period* of vibration.

Wave Forms.

Experiment 109. — Drop a pebble into a tub of water. Waves will be seen moving on the surface of the water from the center of disturbance, and in concentric circles, toward the sides of the tub. A small cork floating on the surface rises and falls with the water, but is not carried along by the advancing waves of troughs and crests.

Experiment 110. — Tie one end of a soft cotton rope about 20 feet long to a fixed support, and hold the other end in the hand. Move the hand up and down with a quick, sudden motion, so as to set up a series of waves in the rope, as shown in Fig. 155, in which each curved line may be considered an instantaneous photograph of a rope thus shaken.

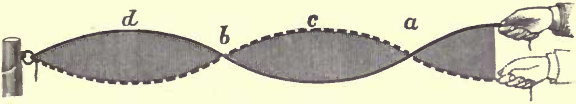


FIG. 155.

182. Waves of Crests and Troughs. — When a person sees waves approaching the shore of a lake or ocean, there arises the idea of an onward movement of great masses of water. But if the observer watches a piece of wood floating upon the water, he may notice that it merely rises and falls without approaching the shore. Again, he may stand beside a field of ripening grain, and, as the breezes blow, he will see a series of curved forms or waves pass before him. There is no movement of matter from one side of the field to the other; the grain-ladened stalks merely bow and raise their heads. Most persons are familiar with similar wave movements in ropes, chains, and carpets. *Each material particle has a simple harmonic motion, vibratory, not progressive. The only thing that has an onward movement is the pulse or wave, which is only a form or change in the relative positions of the particles of the undulating substance.*

(*a*) By fixing a pencil at the end of a lath firmly held at the other end, and vibrating in a horizontal plane, the pencil may be made to mark a nearly straight line, *ab*, on a sheet of paper or cardboard. By moving the paper while the rod is vibrating, the pencil may be made to trace a sinusoidal curve or wavy line like that shown in Fig. 156. The construction of such a curve, and its relation to the simple harmonic motion of the pendulum, will be further illustrated in the exercises. The distance from crest to crest (1 to 5), or from trough to trough (3 to 7), or from any point to the next point at which the vibrating particle was in the same stage of vibration or in the same phase (A to 4, or 2 to 6, or 4 to B), is called a *wave-length.* Evidently, the disturbance, i.e., the wave, advances just one wave-length in the time required for one vibration; this time is called the *vibration-period.*

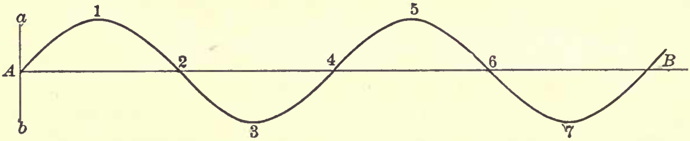


FIG. 156.

Experiment 111. — Make a spiral spring about 12 feet long by closely winding No. 18 spring-brass wire on a rod about half an inch in diameter. Fasten one end of the spiral to a hook on the wall, or clamp it in a vise, and tie short pieces of bright-colored strings into several of the coils. Holding the other end of the spiral in the hand, insert a finger-nail or knife-blade between two turns of the wire near the hand, and pull one of them further from the other. Suddenly release the coil, and a pulse will run along the spiral. Each coil swings to and fro, the coils being crowded closely together at one place, and more widely separated at another, as shown in Fig. 157.

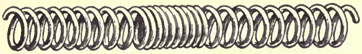


FIG. 157.

Experiment 112. — Tightly tie a sheet of writing paper over the large end of the tube used in Experiment 98, and hold a candle flame in front of the small end. Tap the paper diaphragm, and notice the consequent flickering of the flame.

183. Waves of Condensation and Rarefaction. — The advancing paper diaphragm or other vibrating body crowds the layers of air immediately in its front, thus setting up a condensation or push along the length of the tube, as explained in § 178. When the paper swings with its pendulum-like motion in the opposite direction, the nearest layers of air follow it, thus setting up a rarefaction. As the paper diaphragm continues to vibrate, a series of condensations and rarefactions is sent along the tube, as shown in Fig. 158, which compare with Fig. 154. The air particles are crowded unusually at *A* and *G*, where their velocity is the least, and are separated more widely at *D*, where their velocity is the greatest. Just as a water wave consists of two parts, the crest and the trough, so a sound wave consists of two parts, a condensation and a rarefaction. The particles in a sound wave move with simple harmonic motion forward and backward in the line of propagation, and not across it. The vibrations are longitudinal, not transverse.

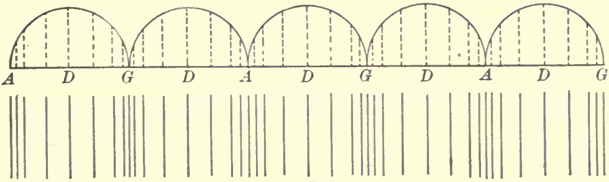


FIG. 158.

(*a*) A series of complete sound waves, such as would be set up in the open air, consists of alternate condensations and rarefactions advancing in the form of concentric spherical shells, at the common center of which is the sounding body. Any radius of the sphere is a line of propagation of the sound.

(*b*) The distance from any point to the next point that is in the same phase, as from condensation to condensation or from rarefaction to rarefaction, is a wave-length. The wave advances one wave-length in the time required for one vibration, or in a wave-period.

(*c*) A sinusoidal curve like that shown in Fig. 156 is commonly used to represent a sound wave. The parts above the horizontal line represent condensations, while the parts below that line represent rarefactions. Perpendicular distances, from the curve to the line AB, i.e., ordinates, show the relative amount of condensation or rarefaction at any point of the curve; but it must not be inferred that amplitudes are as great relative to wave-lengths as would be indicated by the curves. The curve is merely a symbol for the sound wave, not a picture of one.

(*d*) Fig. 159 represents apparatus devised by Mach for the illustration of the pendular motions of the particles of a medium transmitting waves of any kind, longitudinal or transverse, stationary or progressive. A wooden framework about 2.2 m. long and 1.2 m. high is arranged to support 17 or more pendulums with bifilar suspensions 0.9 m. long. At the top of the frame are two parallel bars joined by pivoted crossbars, after the manner of the familiar parallel ruler, and so that they may be separated about 10 cm. One string of the bifilar suspension of each pendulum is fastened to the inner face of the fixed bar, and the other string to the inner face of the movable bar. When the two bars are as far apart as possible (as shown in the figure), the balls can swing lengthwise of the frame; i.e., longitudinally. When the two bars are brought close together the balls can swing only transversely.

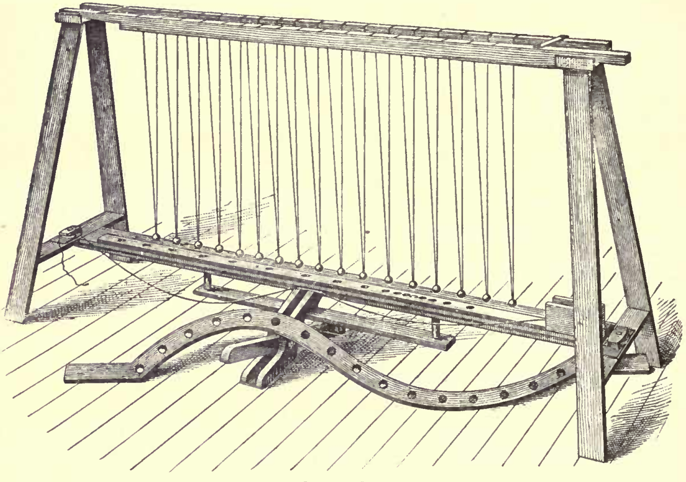


FIG. 159.

A light frame worked by the foot-lever carries a wave "pattern." The holes in this pattern are at varying distances from each other, one part representing a rarefaction and another part representing a condensation, as is clearly shown in the figure. When the pattern is raised by the lever, the pendulums may be placed in the holes. When the pattern is lowered, the balls swing longitudinally, their relative motions representing a stationary sound wave, such as the wave in an organ-pipe, and giving a vivid idea of the alternate condensations and rarefactions.

A block, shown at the lower right-hand part of the figure, may be drawn along the framework by the string attached to it. This block has upon its upper surface a groove of such depth that, as it is drawn under the balls from one end of the row to the other, each ball is pulled from its position of rest and released as the block passes under it. Thus is produced the form of a progressive wave of condensation and rarefaction.

The two bars that support the pendulums may be brought together so that the balls can swing only transversely, and the pattern previously used may be replaced by the sinusoidal pattern that represents one complete wave. Such a sinusoidal pattern is shown in the figure. When this pattern is raised by the foot-lever and the balls placed in the holes and the pattern then dropped, the pendular motions give a clear representation of a stationary transverse wave.

A long board, just wide enough to reach from the pattern supports to the top of the balls may be placed on edge so that all of the balls are pushed about 10 cm. to one side of their positions of rest. When this board is drawn endwise, the balls are successively released, and the pendular motions represent a progressive transverse wave.

CLASSROOM EXERCISES.

1. State clearly the difference between a transverse and a longitudinal wave. Illustrate.

2. The velocity of sound being given as 1,145 feet per second, what is the wave-length of a tone due to 458 vibrations per second?

3. It is a common experiment for one of two boys in swimming to hold his head under water while another at a distance strikes two stones together under water. The loudness of the sound heard by the first boy is painful and sometimes injurious, even when the dis-

tance is so great that the sound would be scarcely heard in the air. Explain.

4. If a blow is struck with a hammer upon one end of a long iron pipe, a listener at the other end may hear two sounds instead of one. Explain.

5. What is the difference between the physical and the physiological definitions of the word "sound"?

6. What is the difference between an oscillation and a vibration?

7. What is the difference between a motion of translation and one of vibration? Illustrate.

8. Why is the motion of a particle of the medium through which a sound is passing properly described as "pendular"?

9. How does a pendular motion differ from a simple harmonic motion?

10. What word accurately describes the curve that is used as a symbol of a sound wave?

LABORATORY EXERCISES.

Additional Apparatus, etc. — A wooden rod about half an inch square and 12 feet long; cardboard; India ink; drawing instruments; heavy plank and blocks; hinge; clock spring; pocket tuning-fork.

1. Draw a graphic representation of a series of waves of troughs and crests as follows: With a radius equal to the amplitude of vibration, draw a circle. Draw its vertical diameter, *AI*. Beginning at *A* or *I*, divide the circumference of the circle into any number of equal parts, say 16. Through these divisions draw horizontal lines intersecting *AI* at the points *B*, *C*, *D*, *E*, *F*, *G*, and *H*, and prolong them indefinitely. See Fig. 154. Similarly extend tangents to the circle at *A* and *I*. On the extension of the line passing through *E*, the center of the circle, lay off successively 16 equal parts. Consider the beginning of the first of these equal parts as the origin of co-ordinates, and number thence the successive divisions on the axis of abscissas, from 1 to 16 inclusive. Through these 16 points, draw vertical lines, terminating the successive verticals at the points where they intersect the successive horizontal lines. Mark these successive intersections 1′, 2′, 3′, etc., to 15′. Then the ordinates of 1′ and 7′ will be equal to *EF;* those of 2′ and 6′ will be equal to *EG;* those of 3′ and 5′ will be equal to *EH*, and that of 4′ will be equal to *EI*. The ordinates of 8′ and 16′ will be zero. The ordinates of 9′ and 15′ will be nega-

tive and equal to *ED;* those of 10′ and 14′ will be negative and equal to *EC;* those of 11′ and 13′ will be negative and equal to *EB;* that of 12′ will be negative and equal to *EA*. Join these several loci by drawing a curve through them, and we have the wavy line known as a sinusoidal curve, the outline of the wave as required. What is the distance from *O* to 16 called? What is the distance from 4 to 4′ called? What is the point 4′ called? What is the point 12′ called? Can you see any connection between the motion of a drop of water in an oscillatory wave and the motion of a pendulum?

2. Slightly stretch a solid rubber cord about 0.5 cm. in diameter between a hook in the ceiling and another in the floor. With a ruler, tap the cord near the lower end, timing the blows so that the cord shall vibrate as a whole. Count the number of vibrations made in 60 seconds. Repeat the test twice and determine the average of the three trials.

Tap the cord more rapidly until it vibrates in two segments. Repeat the test and determine as before the average number of vibrations made in 60 seconds.

Similarly, make the cord vibrate in three and then in four segments, recording the numbers of vibrations as before. Measure the length of the cord between the hooks, tabulate the segment-lengths and the numbers of vibrations per second for each of the four tests. Determine the relation between the segment-lengths and the vibration-numbers.

3. Hold one end of a slender wooden rod between the teeth, while another pupil holds the stem of a vibrating tuning-fork against the other end of the rod. See if the fork is audible without the intervention of the rod.

4. Cut a slit 1 mm. × 4 cm. in a postal card. Place a ruler below Fig. 156 and parallel with the printed lines. Place the edge of the card against the edge of the ruler, so that the slit shall be at right angles to the line, *AB*, at its end. *AB* should show through the slit at its middle point. Slide the card with steady motion along the edge of the ruler, observing the apparent motion of the black line seen through the slit. How does that apparent motion up and down the slit compare with the simple harmonic motion of the pendulum?

5. From a piece of stiff cardboard, cut a disk 31 cm. in diameter, and from its center draw a circle 0.5 cm. in diameter. Divide the circumference of this little circle into twelve equal parts, and number the points of division consecutively from 1 to 12. With dot 1 as a

center, draw a circle with a radius of 7.5 cm., using the pen-compasses and India ink. With dot 2 as a center, draw a circle with a radius of 7.8 cm. With dot 3 as a center, draw a circle with a radius of 8.1 cm. Continue to draw such eccentric circles, using the numbered dots in succession as centers and increasing the radius by 0.3 cm. each time. Go thus around the little circle twice, when you will have 24 circles drawn upon the cardboard, as indicated in Fig. 160. Cut a hole exactly at the center and mount the disk upon the spindle of a whirling table. Cut a narrow slit about 10 cm. long in a card, and hold it so that the slit lies parallel to a radius of the disk and close to it. The short arcs of the circles seen through the slit look like a series of dots, each of which may be taken to represent an air particle. Still viewing the dots through the slit, rotate the disk and you will get a very vivid idea of the way in which air particles actually move when set in motion by a sound wave.

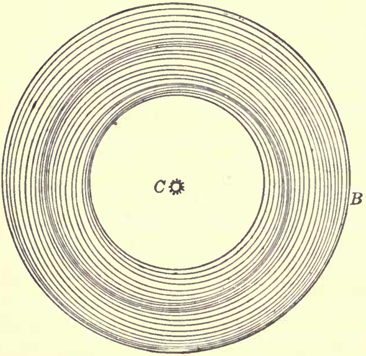


FIG. 160.

6. From a two-inch plank, cut a baseboard about 75 × 20 cm. To one edge of this base and about 20 cm. from one end, screw an upright, and at the upper end of the upright support a short horizontal shelf, one edge of which shall be over and parallel with the middle line of the baseboard. Paste one end of a small strip of stout paper to one end of a piece of glass about 15 cm. long and 10 cm. wide, so that the projecting part of the paper may serve as a handle for moving the glass, which is to be lightly smoked. Provide a wooden block shaped like that shown in Fig. 161, the greatest length of which is about 30 cm. long and the thickness of which at the part marked *a* is about 2.5 cm. The several faces of the thicker part of this block have holes, into any one of which the handle of a tuning-

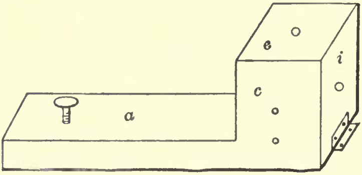


FIG. 161.

fork may be driven. The face, *i*, carries a hinge for fastening the block to the baseboard, and near the end of *a* is a large screw-eye extending into the baseboard. Fig. 162 shows a pendulum supported from the edge of the shelf already mentioned. The pendulum is made of a straightened piece of clock-spring with a lead bullet-bob at the lower end. The under side of the bob carries a style. This pendulum will vibrate across a line drawn along the middle of the baseboard. Place the smoked glass on the baseboard so that its length shall be parallel with the length of the board, and so that the pendulum shall hang near the end that carries the paper strip. Drive the handle of the tuning-fork into the hole in the block at the face marked *i*, and fasten the block at the other end of the smoked glass so that the style at the end of the fork shall come as near as possible to the style of the pendulum, and shall swing parallel with it when prong and pendulum are both in motion. Adjust the height of the shelf so that the pendulum will be long enough to make between 100 and 150 complete oscillations per minute. Then adjust the pendulum at the clamp on the edge of the shelf so that the style carried by the bob may just touch the glass and cut a line on its smoked surface. Accurately count the number of vibrations that the adjusted pendulum makes in a minute. Loosen the screw at the further end of the block that carries the fork, and adjust the height so that the style at the end of the prong just grazes the surface of the glass. Tack a thin strip of wood at the long edge of the glass to serve as a guide for the latter. Set pendulum and fork in vibration and quickly draw the glass lengthwise. From the two traces on the glass, count the number of vibrations of the fork that correspond to one vibration of the pendulum, and thence compute the vibration-number of the fork.

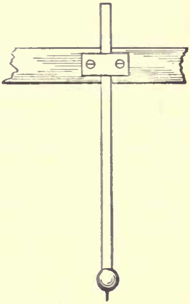


FIG. 162.

II. VELOCITY, REFLECTION, AND REFRACTION OF SOUND.

Experiment 113. — Provide three similar rubber tubes, each about 3 m. long; fill one of them with sand, and with equal tension stretch the three side by side, suspended between supports at their ends. Strike the two empty tubes simultaneously with a ruler and near one end; notice that a wave runs along each tube to the other end where it is reflected; the waves return to the starting point at practically the same time; they travel with equal velocities. Increase the tension of one of the tubes and repeat the experiment. The increase of the elastic force increases the velocity of the wave. Similarly, send a wave along the sand-filled tube, and notice that the waves travel perceptibly more slowly in the heavier tube.

184. The Velocity of Sound depends upon two considerations, — the elasticity and the density of the medium. *It varies directly as the square root of the elasticity, and inversely as the square root of the density.*

$$v = \sqrt{\frac{E}{D}}.$$

(*a*) In solids, elasticity is measured by the modulus of elasticity (E), which is the reciprocal of the coefficient of elasticity (e). In liquids and gases, elasticity is measured by the resistances they offer to compression.

(*b*) The velocity of the wave motion may be found by multiplying the wave-length by the number of vibrations per second, or the wave-length may be found by dividing the velocity by the number of vibrations.

(*c*) Careful experiment has established the fact that *the velocity of sound in air at the freezing temperature* (0° C. or 32° F.) *is about* 332 *m., or* 1,090 *feet per second.* Oxygen is sixteen times as dense as hydrogen. Under the same pressure, the elasticity is the same; hence, sound travels four times as fast in hydrogen as it does in oxygen. A change of pressure on a gas will change elasticity and density equally, and, therefore, will not affect the velocity of sound transmitted by the gas. If a confined portion of any gas is heated, its elasticity is increased without any change of density. Hence, a rise of temperature without barometric change increases the velocity of sound in the air. The added velocity is about 0.6 m., or 2 feet for each degree that the centigrade thermometer rises; or 0.33 m. or 1.12 feet for each degree that the Fahrenheit thermometer rises.

(*d*) Owing to the high elasticity of liquids and solids as compared with their densities, they transmit sound with great velocities. In water at 8° C., sound travels at the rate of 4,708 feet per second, and the velocity is considerably affected by changes of temperature; in glass, the velocity is 14,850 feet, and in iron it is 16,820 feet; in lead, a metal of high density and low elasticity, the velocity of sound is 4,030 feet per second.

Reflection.

Experiment 114. — Repeat Experiment 110, and notice that the waves successively started by the hand are turned back at the other end of the rope and meet the advancing waves. When any part of the rope is equally urged in opposite directions by a direct and a reflected wave, the resultant of the two forces is zero, and the rope at that point remains at rest.

Experiment 115. — Slip the loops at the ends of the wire spiral used in Experiment 111 over hooks screwed into the sides of two boxes. Separate the boxes so as to support and slightly stretch the spiral, fastening the boxes by nailing them down or by loading them with sand. Start a pulse in the spiral, and notice that the wave runs to the other end, is turned back or reproduced in the same medium, moves along the spiral to its starting point, and so continues its journeys to and fro until its energy is dissipated. It looks as though a wave motion might be reflected (§ 76) as well as a motion of translation.

Experiment 116. — Hold a lamp reflector or other large concave mirror directly facing the sun, so as to bring the rays of light to a focus. Move a piece of paper until you find the place where a spot on the paper is most brilliantly illuminated by the reflected rays, and measure the distance of this focus, *F*, from *A*, the center of the reflector (see Fig. 163). At some point, *W*, between *F* and *C*, the center of curvature of the reflector, hang a loud-ticking watch, and hunt for the point, *X*, at which the ear can most distinctly hear the ticking.

Use a glass funnel as an ear-trumpet. Keep watch and ear in these positions, and have the reflector removed. The ticking will be faint or inaudible.

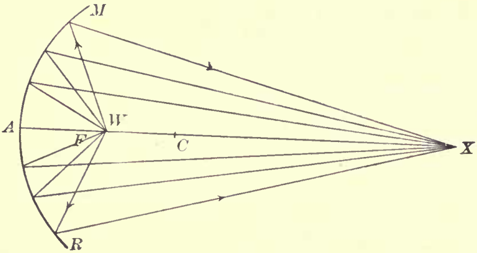


FIG. 163.

185. Reflection of Sound. — When a sound wave strikes an obstacle, it is reflected in obedience to the principle given in § 76. Fig. 164 represents two parabolic reflectors, *mn* and *op*. It is a peculiarity of such reflectors that rays starting from the focus, as *F*, will be reflected as parallel rays, and that parallel rays falling upon such a reflector will converge at the focus, as *F′*. Hence, two such reflectors may be placed in such a position that sound waves starting from one focus shall, after two reflections, be converged at the other focus. By such means, the ticking of a watch may be made audible at a distance of two or three hundred feet. *Two reflectors so placed are said to be conjugate to each other.* This principle underlies the phenomena of whispering galleries.

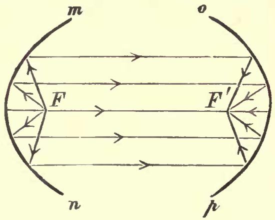


FIG. 164.

186. An Echo is a sound repeated by reflection so as to be heard again at its source. If the direct and reflected sounds succeed each other with great rapidity, as will be the case when the reflecting surface is near, the echo obscures the original sound and is not heard distinctly. Such indistinct echoes often interfere with distinct hearing in large halls and churches. Multiple or tautological echoes are due either to independent reflections by bodies at different distances, or to successive reflections, as between parallel walls.

(*a*) The time interval between a sound and its echo is the time required for a sound to travel twice the space interval between the source of the sound and the reflecting body. Suppose that a person can distinctly pronounce five syllables in a second. While one syllable is being spoken, the sound waves that constitute the first part of the syllable will have traveled one-fifth of 1,120 feet or 224 feet. If these waves are to be brought back to the ear of the speaker immediately after the syllable is completed, the reflecting surface should be about 112 feet distant. If it is nearer than this, the reflected sound will return before the articulation is complete and confusedly blend with it. If the reflector is 224 feet distant, there will be time to pronounce two syllables before the reflected wave returns. The echo of both syllables may then be heard; and so on.

Refraction.

Experiment 117. — Fill with carbon dioxide a large rubber toy balloon or other double-convex lens having easily flexible walls. Suspend a watch, and place yourself so that you can just hear its ticking. Have the gas-filled lens moved back and forth in the line between watch and ear until the ticking is much more plainly heard. Use a glass funnel as an ear-trumpet.

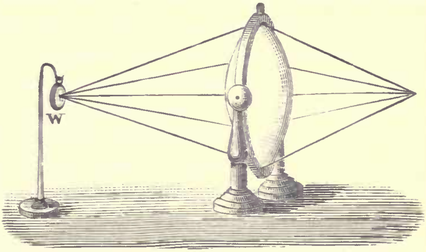


FIG. 165.

187. Refraction of Sound. — As explained in § 183 (*a*), the lines of propagation of sound are ordinarily radial or divergent. When such waves pass obliquely from one medium to another of different density, the line of propagation is bent, as will be more fully explained in the chapter on Light. *This bending of the lines of propagation is called refraction.* Such lines may be made less divergent or even converging, as in Experiment 117, and the energy of the waves concentrated at a focus.

CLASSROOM EXERCISES.

1. If 18 seconds intervene between the flash and report of a gun, what is its distance, the temperature being 0° C.?

2. Steam was seen to escape from the whistle of a locomotive, and the sound was heard 7 seconds later. The temperature being 15° C., how far was the locomotive from the observer?

3. What is the length of sound waves propagated through air at a temperature of 15° C. by a tuning-fork that vibrates 224 times per second?

4. Determine the temperature of the air when the velocity of sound is 1,150 feet per second.

5. Why will an open hand or a palm-leaf fan held back of the ear often aid a partly deaf person in hearing a speaker?

6. A shot is fired before a cliff and the echo heard 6 seconds later. The temperature being 15° C., determine the distance of the cliff.

7. A musical instrument makes 1,100 vibrations per second. Under what conditions will the sound waves be each a foot long?

8. How many vibrations per second are necessary for the formation of sound waves 4 feet long, the velocity of sound being 1,120 feet? Determine the temperature at the time of the experiment.

9. Taking the velocity of sound as 332 m., determine the length of the waves produced by a body vibrating 830 times per second.

10. When the velocity of sound is 1,128 feet, determine the rate of vibration of the vocal cords of a man whose voice sets up waves 12 feet long.

11. A person stands before a cliff and claps his hands and hears an echo in $\frac{3}{8}$ of a second. Determine the distance of the cliff from the man.

12. A sportsman fires his gun and $2\frac{1}{2}$ seconds later hears its report

the second time. The temperature being 0° C., how far away is the reflecting surface?

13. A stone is dropped down the shaft of a mine and 5 seconds later is heard to strike the bottom. The temperature being 15° C., what is the depth of the mine?

14. Why does sound travel more rapidly through the iron of a pipe than it does through the air contained in the pipe?

15. From the cyclopedia, cull the story of the prison built by Dionysius, the Syracusan tyrant, and explain its remarkable acoustic properties.

16. Two single-stroke electric bells on the same circuit are made to strike 5 times a second. When the bells are at the same distance from the hearer, 5 strokes per second are heard; when one of them is about 112 feet further away than the other, 10 strokes per second are heard; and when one of them is about 224 feet further away than the other, only 5 strokes per second are heard. Explain.

LABORATORY EXERCISES.

Additional Apparatus, etc. — A seconds pendulum; spy glass or opera glass; heavy hammer; long measuring tape; thermometer; two pistols; two or more good watches; cardboard; toy trumpet; a few lengths of tin water-spout.

1. Let two pupils, *A* and *B*, take positions about 900 feet apart, so that each can see the other. Let *A* set up a seconds pendulum with a heavy bob painted white, so that it shall swing across the line extending from him to *B*. Drive a stake beneath the pendulum bob, or indicate its lowest position in some other way that may be seen by *B*. Swing the pendulum, and, just as the pendulum passes the vertical, strike a board, stone, or anvil with a hammer that carries a white cloth, so that its motion may be easily visible. *B* observes these motions through a spy glass and shifts his position from time to time, signaling for other hammer strokes, until his distance from *A* is such that the sound produced when the pendulum passes the vertical in one direction is heard when the pendulum passes the vertical in the other direction. Mark the position of *B*. Measure the distance between *A* and *B*, and note the reading of the thermometer. If the wind is not blowing, this distance roughly indicates the velocity of sound in air at the observed temperature. It will be well to check the result obtained by reversing the experiment. Let *A* swing the pendulum. Let *B* watch it until *he feels* the motions, and then strike

a blow just as the pendulum passes the vertical. If A does not hear the sound just as the pendulum next passes the vertical, let him signal B to come nearer or go further away, continuing the work until the sound moving from B to A comes in on time. Let B measure the distance between his two stations, and thence determine the distance of his second position from A. The average of the two distances between A and B may be taken as the velocity of sound in still air at the temperature observed. If your result differs much from the velocity as computed from the data recorded in § 184 (*c*), you may know that your work has not been well done.

NOTE. — Exercises 1 and 2 may not be practicable in the immediate vicinity of a city school, but it is well worth the effort to make a Saturday scientific class-excursion into the country, for the purpose of executing them. If the experiments are performed in the cool air of a frosty morning, and repeated in the warmer air of early afternoon, a change in the velocity of the sound will be observed.

2. Place two pupils, C and D, each of whom has a pistol and a watch, and knows how to take care of them, a long distance apart, but in sight of each other. Let half of the pupils who have watches accompany C; the others who have watches should go with D. Let C fire his pistol, D and his party noting the interval between the appearance of the flash and the hearing of the report. Take the average of the observations made by the different members of the party, excluding any observation that differs very widely from most of the others. Then let D fire his pistol while C and his party observe the interval and determine their average. Measure the distance between the stations occupied by C and D, and note the reading of the thermometer. Record the average of the two averages as the time required for sound to travel that distance at that temperature. From such data compute the velocity of sound per second under the conditions of the experiment.

3. On opposite sides of the center of a disk of cardboard about 15 inches in diameter, cut out two sectors, as shown in Fig. 166. Mount the disk on a whirling table. Sit beside the apparatus, so as to turn the driving wheel with one hand, and with the other hold a toy trumpet so that its axis shall be inclined to the surface of the disk, about midway between center and circumference. Rotate the disk steadily and sound the trumpet at the same time. Let other pupils take positions in a distant part of the room, as indicated by the law of reflected motion, so that the sound waves from the trumpet reflected

by the disk will reach their ears. When the sectors pass before the mouth of the trumpet, the sound will become softer, and when the cardboard reflector passes, the sound will become stronger. Record a description and explanation of the experiment.

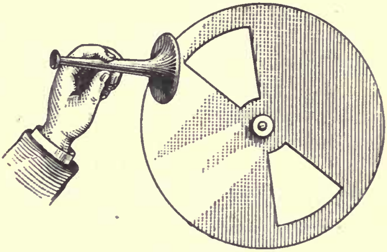


FIG. 166.

4. On a table like that shown in Fig. 45, lay two tin tubes, each about 150 cm. long and 10 cm. in diameter. A few lengths of water-spout will answer. The axes of the tubes should lie on radial lines (like *BA* and *BC*) that make equal angles with the radius, *BD*, drawn perpendicular to the reflecting surface at *B*. If you have no such table, draw the radial lines on any table-top, and properly place a piece of glass, as at *B*, to serve as a reflector. Suspend a watch at the outer end of one of the tubes and hold the ear at the outer end of the other tube. Notice the intensity of the sound caused by the ticking of the watch. Shift the inner end of one of the tubes from its position. Listen again and notice the relative intensity of the sound.

III. CHARACTERISTICS OF TONES.

188. Differences in Tones. — Sound waves differ in respect to amplitude, length, and form. These differences in the waves give rise to corresponding differences in the sensations that they produce. *Variations in amplitude correspond to differences in intensity or loudness; differences in wave-length correspond to differences in pitch; differences in wave-form correspond to differences in timbre or musical quality.*

Intensity.

Experiment 118. — Set a tuning-fork in feeble vibration by striking it gently; the sound that you hear will be faint. Strike the fork a harder blow; its prongs will vibrate with more energy and the sound that you hear will be louder. Gently pluck a guitar string; it vibrates to and fro across its place of rest, striking feeble blows upon the air and sending sound waves to the ear. Pluck the same string more vigorously; it vibrates with greater amplitude, striking the air with greater energy and sending to the ear sound waves of greater intensity than before.

189. Intensity and Amplitude. — *The intensity of a sound depends primarily upon the energy of vibration of the sonorous body*, and thence on the amplitude of the vibrating particles of the sound medium. The greater the amplitude, the greater the energy and the louder the sound.

(*a*) If the amplitude of the vibration of a sonorous body is doubled, the velocity with which it swings will be doubled, for the vibrations are as strictly isochronous as the oscillations of a pendulum. Since energy varies as the square of the velocity, it follows that *the intensity of sound varies as the square of the amplitude.* Since energy varies as the mass, it follows that the intensity of sounds generated in gases of little density (see Experiments 102 and 103) will have less intensity than sounds generated in heavier gases like air and carbon dioxide.

(*b*) If a smoked glass is drawn very slowly under a style carried by a prong of a vibrating tuning-fork, as shown in Fig. 150, the soot will be scraped from the glass and the area thus cleaned will be triangular. As the sound of the fork grows feebler, the swings of the prong become shorter and the trace tapers off.

Experiment 119. — Whisper into one end of a length (50 feet) of garden hose. A person listening with his ear at the other end of the hose can distinctly hear what is said although the sound is inaudible to a person holding the middle of the hose.

190. Intensity and Distance. — In the open air, a sound wave expands as a spherical shell, and its energy is distributed among the increasing number of air particles that constitute these successive shells or spherical surfaces. This number of air particles varies as the square of the radius of the sphere. The energy of any given number of these air particles must, therefore, vary inversely as the square of such radius; in other words, *the intensity of sound varies inversely as the square of the distance from the sonorous body.*

(*a*) If the sound wave is not allowed to expand as a spherical shell, its energy cannot be thus diffused and its intensity will be conserved. Hence, the efficiency of speaking-tubes and speaking-trumpets.

(*b*) The law above given is true only when the distance is so great in comparison with the dimensions of the sounding body that the latter may be considered a center from which sound waves proceed along radial lines. A person 10 feet from a passing railway train does not hear a sound four times as loud as that heard by a person 20 feet from the train.

Experiment 120.—Strike a tuning-fork held in the hand. Notice the feeble sound. Strike the fork again and place the end of the handle upon a table. The loudness of the sound heard is remarkably increased.

Experiment 121.—Strike the fork and hold it near the ear, counting the number of seconds that you can hear it. Strike the fork again with equal force; place the end of the handle on the table and count the number of seconds that you can hear it.

191. Intensity and Area.—When a vibrating body is small or thin, the particles of the air readily flow around it instead of being set into vibration by it. Hence, the sound of a small tuning-fork is feebler than that of a large one. *When the sonorous body has a large surface, its vibrations set up well-marked condensations and rarefactions, and the consequent sound is correspondingly intense.*

(*a*) In the sonometer, piano, violin, guitar, etc., the sound is due more to the vibrations of the resonant bodies that carry the strings than to the vibrations of the strings themselves. The strings are too thin to impart enough motion to the air to be sensible at any considerable distance; but as they vibrate, their tremors are carried by the bridges to the material of the sounding apparatus with which they are connected. These larger surfaces throw larger masses of air into vibration and thus greatly intensify the sound. It necessarily follows that the energy of the vibrating body is sooner exhausted.

(*b*) The intensity of tones is also affected by resonance and interference, as will be subsequently explained.

Pitch.

Experiment 122. — Draw a finger-nail across the tips of the teeth of a comb, slowly the first time and rapidly the second time. Notice the difference in the sounds produced. If one is louder than the other, is that the only difference?

Experiment 123. — The Savart wheel, shown in Fig. 167, consists of a heavy metal toothed wheel that may be put in rapid revolution by pulling a cord wound upon its axis. Set such a wheel in rapid motion and hold the edge of a card against its teeth. As the speed of the wheel diminishes, the shrill tone produced by the rapid vibrations of the card correspondingly falls in pitch.

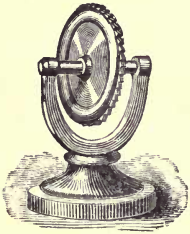


FIG. 167.

Experiment 124. — From a piece of stiff cardboard, cut a disk $8\frac{1}{2}$ inches in diameter. From the same center, draw four circles with radii of $2\frac{1}{4}$ inches, $2\frac{3}{4}$ inches, $3\frac{1}{4}$ inches, and $3\frac{3}{4}$ inches respectively. Divide the inner of these circumferences into 24 equal parts, the second into 30, the third into 36, and the fourth into 48. At each division, punch a $\frac{3}{16}$-inch (5 mm.) hole. Cut a hole at the center and mount the perforated disk on the spindle of a whirling table, and *you have a simple form of the siren.* See Fig. 168.

Rotate the disk slowly, blowing meanwhile through a tube of about $\frac{3}{16}$-inch bore, the nozzle of the tube being held opposite the interior ring of holes. As each successive hole comes before the end of the tube, a puff of air goes through the disk. As the speed of the disk increases, the puffs become more frequent, and finally blend into a whizzing sound in which the ear can detect a smooth tone. As the disk is given an increasing velocity, this tone rises in pitch. With a given rate of rotation of the apparatus, the pitch will rise as the tube is moved outward in succession from the inner to the outer circle of perforations. Does it not appear that the pitch of a sound rises with the frequency of the vibrations that produce it?

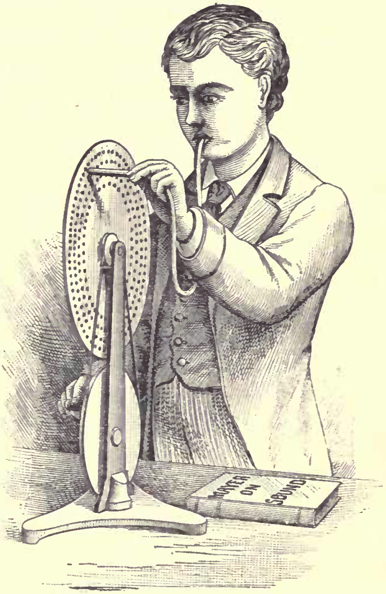


FIG. 168.

192. Pitch *is the characteristic of a sound or tone by which it is recognized as acute or grave, high or low. It depends upon the rapidity of the vibrations by which the sound is produced;* the more rapid the vibrations, the higher the pitch.

(*a*) Since, in a given medium, all sounds travel with the same velocity, the rate of vibration determines the wave-length. If the sounding body vibrates 224 times per second, 224 waves will be started each second. If the velocity of the sound is 1,120 feet, the total length of these 224 waves must be 1,120 feet, or the length of each wave must be 5 feet. If another body vibrates twice as fast, it will crowd twice as many waves into the 1,120 feet; each wave will be only $2\frac{1}{2}$ feet long. Thus, wave-length may be used to measure pitch; the greater the wave-length, the lower the pitch.

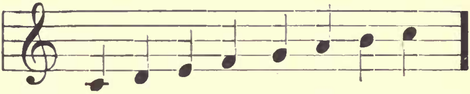
(*b*) If the sounding body and the listening ear approach each other, the sound waves will beat upon the ear with greater rapidity. This is equivalent to increasing the rapidity of vibration of the sounding body. The opposite holds true when the sounding body and the ear recede from each other. This explains why the pitch of the whistle of a railway locomotive is perceptibly higher when the train is rapidly approaching the observer than when it is rapidly moving away from him.

193. The Range of Hearing of different persons varies. The lower limit for most persons is probably represented by about 30 vibrations per second, although some experimenters place it at 16 vibrations. Similarly, the upper limit varies from 38,000 to 41,000 vibrations per second. Tones of musical value lie between the limits of 27 and 4,000 vibrations per second.

(*a*) Everybody understands the differences in the range of the human voice, that one can sing bass and another one soprano, the difference depending upon the rate of vibration of the vocal cords. It is equally true, but not equally well known, that some persons are unable to hear low sounds that are distinctly audible to most persons, while the hearing apparatus of others is unable to respond to sounds of high pitch or short wave-length, which are easily heard by the greater number. Some persons whose hearing is considered fairly sensitive have never heard the shrill chirping of the cricket.

194. An Interval *is the difference or distance in pitch between two tones, and is described by the ratio between the vibration-numbers of the two tones.* Thus, the interval of an *octave* is represented by the ratio 2 : 1; a *fifth*, 3 : 2; a *fourth*, 4 : 3; a *major third*, 5 : 4; and a *minor third*, 6 : 5.

195. A Musical Scale is a definite, standard series of tones for artistic purposes, and lying within a limiting interval. In constructing such a series, the first step is the adoption of such a limiting interval for the division of the possible range of tones into convenient sections of equal length. In modern music, this limiting interval is the octave.

196. The Gamut. — Starting from any tone arbitrarily chosen, and called the keynote, the interval of an octave may be traversed by seven definite steps, thus giving a series of eight tones that are very pleasing to the ear. The eighth tone of this group becomes the first tone (i.e., the keynote) of the group or octave above. The intervals between these tones are not equal, as will soon appear more clearly. *This familiar series of eight tones is called the gamut or major diatonic scale.* The series may be repeated in either direction to the limits of audible pitch. The names and relative vibration-numbers of these tones, and the intervals between them, are as follows: —

Relative names	1	2	3	4	5	6	7	8
Absolute names	C_3	D_3	E_3	F_3	G_3	A_3	B_3	C_4
Syllables	*do*	*re*	*mi*	*fa*	*sol*	*la*	*si*	*do*
Relative vibration-numbers .	24	27	30	32	36	40	45	48
Vibration-ratios	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Intervals		$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

(*a*) The initial tone or keynote of such a series may have any number of vibrations, and whatever pitch is assigned to *C*, the number of vibrations of any tone may be found by multiplying the vibration-number for *C* by the vibration-ratios given above. Physicists assign to C_3, sometimes called "middle C," 256 vibrations per second ($256 = 2^8$). Musicians and makers of musical instruments in this country and Europe have adopted the "international pitch," which gives for standard A_3, 435 vibrations per second. This corresponds to 258.6 vibrations for C_3 (§ 198).

(*b*) When two tones with vibration-numbers as 1 : 2 are sounded together, the character of the combination is the same as that of either tone alone. It is the interval most readily produced by the human voice, and seems to have a foundation in nature; such an interval is called an *octave*. When three tones with vibration-numbers as 4 : 5 : 6 are sounded together (e.g., *C*, *E*, *G*), a new quality seems to be added, and the combination produces a very pleasing sensation. The tones are in harmony, or in accord with each other. *Such simultaneous sounding of three or more concordant tones constitutes a chord*, of which there are several kinds. The three tones above mentioned (i.e., C, E, G) constitute a *major chord.* A combination of three tones with vibration-numbers as 10 : 12 : 15 (e.g., E, G, B) constitute a *minor chord.*

197. Diatonic Scales. — The major diatonic scale is built upon three major chords, and the minor diatonic scale upon three minor chords, with the octave of the various tones.

(a) The method of building up the major diatonic scale is as follows: Assign to C_3, as the first of three tones with vibration-numbers as 4 : 5 : 6, any number of vibrations, as 256 per second. Its octave will have 512 vibrations; and the tones of the major chord will have respectively $256 \times \frac{4}{4}$, $256 \times \frac{5}{4}$, $256 \times \frac{6}{4}$, vibrations per second. Designating these four tones by their absolute names, we have: —

C_3	D_3	E_3	F_3	G_3	A_3	B_3	C_4
256	?	320	?	384	?	?	512.

Taking C_4 as the third tone of another major chord, we have: —

$$4:5:6 = ?:?:512 = 341\tfrac{1}{3}:426\tfrac{2}{3}:512.$$

Assigning the two vibration-numbers thus found to F_3 and A_3, we have: —

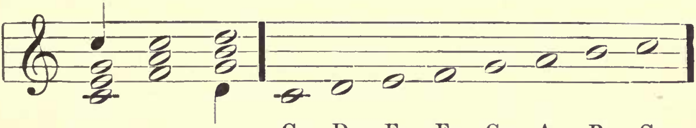
C_3	D_3	E_3	F_3	G_3	A_3	B_3	C_4
256	?	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	?	512.

Again, starting with $G_3 = 384$, as the first tone of another major chord, we have the series: —

$$4:5:6 = 384:?:? = 384:480:576.$$

The tone with 480 vibrations will be called B_3, as it lies between those already called A_3 and C_4. The last tone, having 576 vibrations, must be placed beyond C_4, but the tone an octave lower, with a vibration-number half as great, 288, falls between C_3 and E_3. This lower tone we will call D_3.

The three chords and the complete series formed from them, in musical notation, with their respective vibration-numbers, which will be found to be in the ratios already given, are:—



C_3	D_3	E_3	F_3	G_3	A_3	B_3	C_4
256	288	320	341.3	384	426.6	480	512

(*b*) A minor diatonic scale may be constructed from three minor chords, founded upon any assigned pitch, in the same manner as described above for the major scale.

198. Chromatic Scale.—In music, other tones of the simple scale already described are needed as the beginnings of similar diatonic scales. For instance, D_3 may be used as the keynote. With this, three chords may be formed, using the intervals 4 : 5 : 6, as follows:—

$$4:5:6 = 288:380:432$$
$$4:5:6 = 384:480:576$$
$$4:5:6 = 432:540:2 \times 324.$$

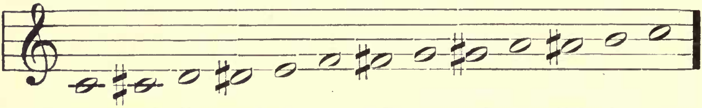
The new complete series is:—

D_3	?	?	G_3	?	B_3	?	D_4
288	324	380	384	432	480	540	576.

Thus four new tones are introduced by using D_3 as the keynote. Any other tone of the first scale, or any of these new tones, may be used as new keynotes. If we form the twenty-four scales ordinarily used in music, twelve major and twelve minor, no fewer than seventy-two tones, within the limits of the octave, will represent them. To use so many tones in each octave of keyed instruments, such as the piano and organ, is a practical impossibility. As many of these tones differ from each other but little, musicians have agreed to make a compromise by giving up the simple perfection of the intervals of the chords described, and to divide the octave into twelve equal intervals, called semi-tones. *The series of thirteen semi-tones, separated by the twelve equal intervals, constitutes the modern chromatic scale.*

(*a*) The eight tones nearest those already described are named as we have already designated them, while the five interpolated tones, corresponding to the black keys on the piano keyboard, are called *sharps* of the tones immediately below them or *flats* of the tones next above them. The compromising process between theory and practice, or the principle by which the octave is divided into twelve equal intervals, is called *equal temperament*. In this system, the only perfect interval is the octave, and all chords are slightly "out of tune."

(*b*) The interval in this scale is $\sqrt[12]{2} = 1.05946$. Any tone being given, the next above is found by multiplying by 1.05946, or the next below by dividing by the same number. The equal tempered chromatic scale founded upon the international pitch, $A_3 = 435$ vibrations, as universally used in music, is as follows: —



C_3	C_3♯	D_3	D_3♯	E_3	F_3	F_3♯	G_3	G_3♯	A_3	A_3♯	B_3	C_4
258.6	274.0	290.3	307.5	325.8	345.2	365.8	387.5	410.6	435	460.9	488.3	517.3

Tones and Overtones.

Experiment 125. — Repeat Experiment 114, using the soft cotton rope or the long wire spiral used in Experiment 115. By properly timing the motion of the hand, the rope may be made to vibrate as a whole. Doubling the rapidity of motion of the hand, the rope or spiral divides itself into two vibrating segments, separated from each other by a point of apparent rest called a "node." Trebling or quadrupling the rapidity of the motion of the hand causes the rope or spiral to divide into three or four segments separated by a corresponding number of nodes. In each case, the period of the hand must synchronize with that of the rope or spiral and of the several segments, but by a little practice one may so time the motions of the hand as to bring out the segmental vibrations just described.

Experiment 126. — Bow or pluck the string of a sonometer (see Fig. 171) near its end, thus setting it in vibration as a whole. The string will have the appearance of a single spindle as shown in Fig. 169, and will sound the lowest tone that it is capable of producing. Lightly touch the wire at its middle point with the tip of the finger or the beard of a quill; the wire will vibrate in halves (Fig. 170) and sound a tone an octave above that previously heard. Sound the sonometer again, touch the string as before, and try to distinguish

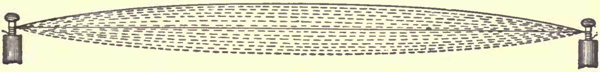


FIG. 169.

FIG. 170.

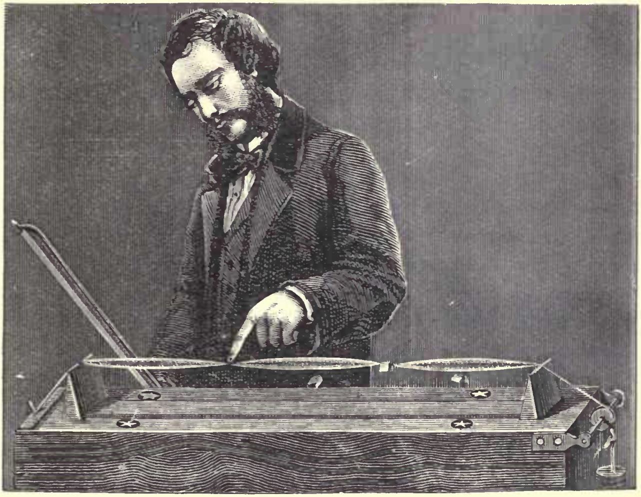


FIG. 171.

both tones as coming simultaneously from the apparatus. Again set the string in vibration and touch it at one-third its length. The vibrating string divides into thirds as shown in Fig. 171, and emits a tone that the trained ear recognizes as the fifth of the octave above that first sounded. Probably both sounds will be heard at the same time. In similar manner, a string sufficiently long may be made to vibrate in any aliquot part of its whole, as fourths, fifths, ninths, tenths, etc. The string should be touched at *n* and bowed at *v*, as shown in Fig. 172.

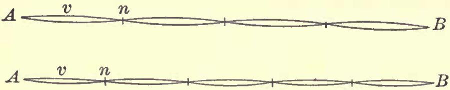


FIG. 172.

199. Fundamental Tones and Overtones. — *The tone that is sounded by a body vibrating as a whole, i.e., the lowest tone that such a body can produce, is called its fundamental or primary tone. The tones produced by the vibrating segments of sonorous bodies are called overtones, partial tones, or harmonics.* The partial tones are named first, second, third, etc., in the order of their vibration-numbers, beginning with the fundamental.

(*a*) It is customary to regard both ends of the string as nodes. The points of greatest vibration, midway between the nodes, are called *anti-nodes.* If little Λ-shaped riders, made of slips of paper bent in the middle, are placed on a string and the string is then made to vibrate in segments, the riders at the nodes will remain in position while those at the anti-nodes will be thrown off as shown in Fig. 171.

(*b*) The interval from the fundamental to the first overtone is an octave; to the second, an octave and a fifth; to the third, two octaves; to the fourth, two octaves and a major third; to the fifth, two octaves and a fifth, etc.

200. Quality or Timbre is the characteristic by which we distinguish one tone from another of the same intensity and pitch. The middle *C* of a piano is essentially different from the same tone of an organ, and any tone of a flute is distinguishable from any tone of a violin. *The physical basis of quality is wave-form*, and is due to the number, relative intensities, and relative phases of the overtones that accompany the fundamental.

(*a*) The well-trained ear can detect several tones when a piano-key is struck. In other words, the sound of a vibrating piano-wire is a compound tone (see Experiment 138). The sound of a tuning-fork is a fairly good example of a simple sound. By sounding simultaneously the necessary number of forks, each of proper pitch and with appropriate relative intensity, Helmholtz showed that the compound sounds of musical instruments, including even the most wonderful one of all, the human voice, may be produced synthetically. Simple tones lack the richness that is so highly prized in musical instruments.

(*b*) The way in which a single string can simultaneously give rise to several tones, i.e., how the segmental vibrations are imposed upon the fundamental, may be explained as follows: In Fig. 173, *AB* represents a string which, when vibrating as a whole, sounds its fundamental, and assumes the form *ACB*.

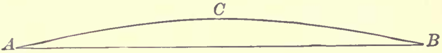


FIG. 173.

Fig. 174 represents the same string sounding its fundamental and its first overtone. In this case the fundamental is represented by the dotted line, while the resultant compound tone is represented by the continuous line, *ACB*. While *AB* vibrates as a whole, its halves, *AC* and *CB*, vibrate in opposite directions, and with doubled rapidity.

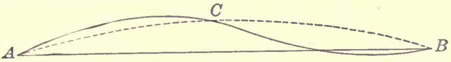


FIG. 174.

Fig. 175 represents the compounding of the same fundamental with its second overtone. The fundamental is represented by the dotted line as before,

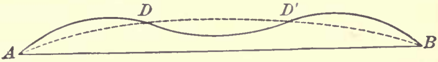


FIG. 175.

while the resultant compound tone is represented by the continuous line, *ADD′B*. While *AB* vibrates as a whole, its thirds, *AD*, *DD′*, and *D′B*, vibrate in alternately opposite directions, and with trebled rapidity. The difference in the three wave-forms is manifest in the figures. Such combinations may be made in almost endless variety, each combination representing a compound tone that varies from all of the others.

201. The Graphic Method of studying sounds, which fairly meets even the exacting demands of physicists, and is largely used by them, may be briefly explained thus: Suppose the smoked plate of Fig. 150 to be a sheet of smoked paper fastened upon the surface of a cylinder that is so mounted that, when it is turned by a crank, the screw cut upon the axis moves the cylinder endwise, as shown in Fig. 176. *Such an instrument is called a vibroscope.*

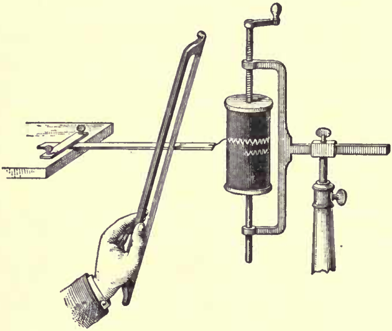


FIG. 176.

(*a*) When the style of a vibrating tuning-fork just touches the paper, and the crank is turned, the vibrations will be traced in the form of a sinusoidal spiral upon the smoked surface, the amplitude, length and form of each wave being truthfully recorded. The cylinder may be turned by clockwork. By counting the number of waves traced in one second, we obtain directly the vibration-number of the fork. By various ingenious and delicate devices, the wave-forms that correspond even to a very complex tone may thus be secured for study or illustration. Such a record may be written parallel with that of a tuning-fork of known frequency (i.e., vibration-number), and comparative study thus facilitated. For instance, if the record of a phonautograph (see dictionary) shows that while the fork recorded

70 vibrations, a singing voice recorded 180, and the vibration-number of the fork is known to be 100, a simple proportion ($70:180::100:x$) shows that the vibration-number of the voice was $257\frac{1}{7}$, indicating a tone almost identical with that of middle *C* of the pianoforte. Fig. 177 shows traces of several compound tones, each written below the sinusoidal tracing of the tuning-fork.

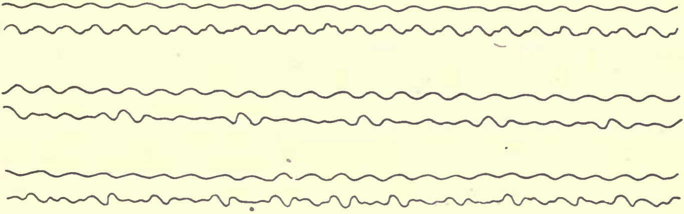


FIG. 177.

Manometric Flames.

Experiment 127. — From an inch board, cut a strip, *A*, 2 inches wide and 10 inches long. Cut another block, *B*, 2 inches square. Placing the point of an inch center-bit an inch from the end of *A*, bore a shallow hole, about $\frac{1}{8}$ of an inch deep, in one side of the strip. Bore a similar hole at the middle of one side of *B*. Place the point of a $\frac{1}{2}$-inch center-bit at the center of the shallow hole in *A*, and bore a hole through the wood. Bore two $\frac{3}{16}$-inch holes from the bottom of the shallow hole in *B* and through the wood, one directly through the block at the center, and the other obliquely downward from the lower edge of the hole. Stretch a piece of gold-beater's skin, or of the thinnest sheet rubber you can find (toy balloon) over the mouth of the shallow hole in *B*, gluing it there. Spread glue over the face of *A* around the shallow hole and screw *A* and *B* together, so that the two shallow holes shall come face to face with the elastic membrane between them. The "manometric capsule" is complete. Nail the other end of *A* to a base board, as shown in Fig. 178. Set a glass tube, *e*, into the $\frac{1}{2}$-inch hole of *A*, making the joint tight with a strip of paper smeared with glue and wrapped about the end of *e* before it is forced into the hole. Attach one end of a piece of large-sized rubber tubing to the glass tube, *e*, and the other end to a trumpet made by rolling up a piece of cardboard into a cone about 8 inches long and 2 inches across the mouth. Into the $\frac{3}{16}$-inch hole at the middle of *B*, tightly fit a glass tube, bent and drawn to a jet at the outer end. Into the other $\frac{3}{16}$-inch hole of *B*, tightly fit a straight glass tube, *c*, that may be connected with the house supply of illuminating gas. Turn on the gas and light it at the jet. If the air of the room is still, the flame will be comparatively steady. Hold a vibrating tuning-fork at the mouth of the trumpet, and notice the flickering of the flame. This flickering of the flame is the thing that we are to study, and its cause ought to be clearly apparent to the pupil.

From a board $\frac{1}{4}$ of an inch thick, 4 inches wide, and a foot long, cut a square piece marked *M*, and the two attached spindles, *H* and *K*. Taper the spindles so that the whole piece may be easily twirled, as shown in the figure. The blunt point of the shorter spindle should rest in a shallow pit, on a firm support. To the opposite sides of *M*, fasten, with tacks at the lower edge and with thread wound along the top and bottom borders, two pieces of thin silvered glass, thus completing the "revolving mirror." The support of the mirror should be at such a height that the flame may be seen reflected from the middle of the mirror.

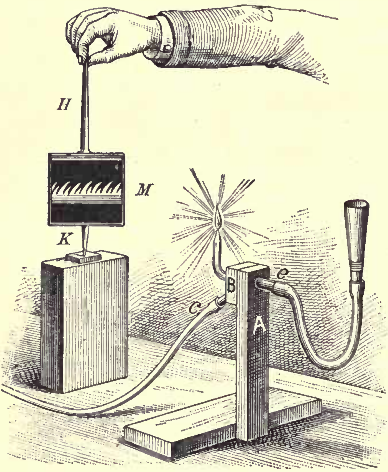


Fig. 178.

Rotate the mirror and notice that the steady flame appears as a luminous ribbon of uniform width. If the flame is agitated by the wind from the mirror, shield the flame with a lamp chimney. While twirling the mirror, sing into the mouth of the cone, and notice that the image becomes indentated, each tongue indicating an increase of pressure on the diaphragm of the capsule. Each projection of the image corresponds to the condensation of a sound wave, and each depression to the rarefaction.

The vibration of the flame may be seen without using the mirror, by quickly turning the head from side to side while looking at the flame, — an interesting experiment.

Experiment 128. — While the mirror is rotating, sound a tuning-fork at the mouth of the trumpet, and notice that the image resembles Fig. 179. Then sound a tuning-fork that is an octave higher and notice that the image resembles Fig. 180, in which twice as many tongues as before are crowded into the same space.

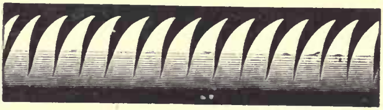


Fig. 179.

Experiment 129. — Remove the large rubber tubing and connect *e* with two trumpets, using a T-pipe or a Y-tube. Sound the two forks just used, and hold each at the mouth of a trumpet, so that their respective waves may be blended before they reach the diaphragm of the capsule. The image will resemble that shown in Fig. 181. Evidently this figure could not have been made by a simple vibration. The alternate condensations sent out by the fork of higher pitch unite with the condensations sent out by the fork of lower pitch, thus making the flame jump higher by their combined action on the diaphragm.

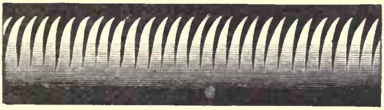


Fig. 180.

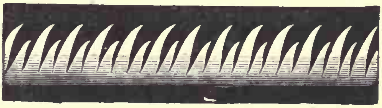


Fig. 181.

Note. — By singing different vowels to different tones, many different images may be produced in the rotating mirror.

202. The Optical Method of studying sounds is well illustrated by Mayer's adaptation of Kœnig's manometric flames, as employed in Experiment 127. This method, like the graphic, has the advantage of being independent of the sense of hearing. When the "manometric capsule" is connected by the tube, *e*, with a Helmholtz resonator (§ 205, *d*,), the flame will respond to the tone that affects the resonator. By using a series of such resonators in connection with the flame and mirror, the analysis of compound tones is made possible even for one who is deaf.

CLASSROOM EXERCISES.

1. If a musical sound is due to 144 vibrations, to how many vibrations will its third, fifth, and octave, respectively, be due?

2. If a tone is produced by 264 vibrations per second, what number will represent the vibrations of the tone a fifth above its octave?

Ans. 792.

3. A given tone is found to be in unison with the tone emitted by the inner row of holes of the siren described in Experiment 124 when the disk is turned at the uniform rate of 640 times in 30 seconds. Assigning 256 vibrations for middle *C*, name the given tone.

4. The vibrations of two tuning-forks are simultaneously recorded by a vibroscope. Comparison shows that 9 waves of one occupy the same space as 15 waves of the other. If the fork of lower tone is marked *D*, what should the other fork be marked?

5. Determine the vibration-number for each tone of a gamut the keynote of which has 261 vibrations.

6. Is there any difference in the pitch of a locomotive whistle when the locomotive is standing still, when it is rapidly approaching the observer, and when it is rapidly moving from him? If so, describe and explain it.

7. What is the vibration-number of the tone *G* next preceding that of a "violin-*A*" fork of 440 vibrations?

8. Why does the sound of a circular saw cutting through a board fall in pitch as the saw enters the board?

9. If an observer should approach a sounding organ-pipe with the velocity of sound, what would be the effect upon the pitch of the tone?

10. If an observer should recede from the source of a musical tone with a velocity a little less than that of sound, what would be the effect upon the pitch of the tone?

11. Suppose that when an orchestra has nearly finished a per-

formance, an observer should move away from the orchestra with a velocity twice that of sound. Describe his relation to the sounds previously executed by the orchestra.

12. A tube about 6 feet long is mounted at its middle on an axis that is perpendicular to the length of the tube. A reed is fixed at one end of the tube and may be sounded by air forced into the tube through an aperture at its axis of rotation. The tube is sounded while in rotation. An observer standing in a prolongation of the axis of rotation hears a tone of constant pitch. An observer standing in the plane of rotation hears a tone of varying pitch. Explain the difference.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Cardboard; punch.

1. Using the graphic method, show that the two prongs of a tuning-fork are moving in opposite directions at any given instant.

2. Make another disk for the siren used in Experiment 124, making eight circles of holes, each circle having in order the number of holes indicated by the relative vibration-numbers given in § 196. Put this disk upon the whirling table and rotate it at such a uniform speed that the puffs made by the inner circle of twenty-four holes shall give a smooth musical tone. Move the nozzle of the tube through which you blow over the several circles in succession and name the familiar series of tones that you hear.

3. Figure 182 represents two sets of sound waves with like periods and phases but different amplitudes. Draw a single curve to represent the resultant of the two series, remembering to make the ordinates of the resultant equal to the algebraic sum of the corresponding ordinates of the constituents.

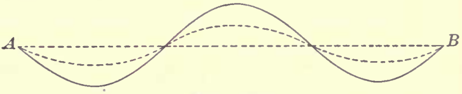


FIG. 182.

4. Figure 183 represents two such wave systems meeting in opposite phases. Draw the resultant curve and tell how the sound it represents corresponds to the sound represented by the resultant drawn in Exercise 3, and how it differs.

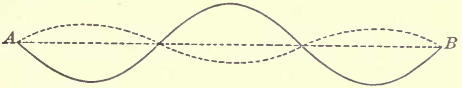


FIG. 183.

5. Figure 184 represents two wave systems of equal periods and amplitudes but of opposite phases. Draw the resultant and describe in a single word the sonorous effect that it represents.

FIG. 184.

6. Bow a sonometer-string vigorously, and while it is sounding lessen the tension. Explain the discordant groan-like sound that is produced.

7. Arrange apparatus as required by Exercise 4, page 225. To the outer end of one of the tubes connect, by a funnel, a piece of large-sized rubber tubing about a yard long and thrust the shank of a glass funnel into the outer end of the rubber tubing. At the outer end of the other tube, place the tube, *e*, of the manometric capsule and arrange apparatus as described in Experiment 127. Light the flame and rotate the mirror. Have a vibrating tuning-fork at the mouth of the glass funnel, and notice, directly and by reflection, the agitation of the flame. Shift the position of one of the tubes so that the angles of incidence and reflection shall be unequal, and repeat the experiment.

IV. CO-VIBRATION.

Experiment 130. — Support a soft cotton rope several yards long between two fixed supports, as the opposite sides of the room, or the floor and the ceiling. With a ruler, strike the rope a blow near one end so as to form a crest, as shown in Fig. 185. Vary the tension of the rope if necessary, until the crest is easily seen. Notice that the crest, *c*, travels from *A* to *B*, where it is reflected back to *A* as a trough, *t*. Strike the rope from above and thus start a trough which will be reflected as a crest.

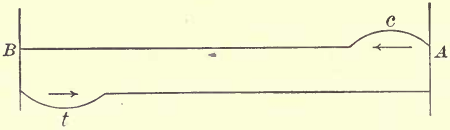


FIG. 185.

Experiment 131. — Start a trough from *A*. At the moment of its reflection as a crest at *B*, start a crest at *A* as shown in Fig. 186. The two crests will meet near the middle of the rope. The crest at the point and moment of meeting results from two forces acting in the same direction; *their resultant is greater than either of the components.*

Fig. 186.

Experiment 132. — Using the rope as previously described, start a crest at *A*. At the moment of its reflection at *B* as a trough, start a second crest at *A*. The trough and crest will meet near the middle of the rope. Vary the experiment by using the rope as shown in Fig. 155, timing the movements of the hand so that an advancing crest shall meet a returning trough near the middle of the rope. The rope particles at this point, being thus simultaneously acted upon by opposite forces, will remain at rest or nearly so. The resultant will be the difference of the components. *Thus, one wave may be made to destroy another wave.*

Fig. 187.

203. Coincident Waves. — Just as, when one crest coincides with another, the wave has an increased height, and when a crest coincides with a trough, the wave disappears, so, when the condensation of a sonorous wave coincides with another condensation, the actual motions of the particles of the sound medium are increased, and, when a condensation coincides with a rarefaction, said motions are reduced or destroyed. Such increased resultant motions of the material particles imply an increased loudness of the sound. Such diminished resultant motions imply an enfeebled sound or perhaps silence.

Sympathetic Vibrations.

Experiment 133. — Repeat Experiment 13, and vary it by setting the heavy pendulum in motion by the cumulative action of well-timed puffs of air from the mouth or from a hand-bellows.

Experiment 134. — Suspend several pendulums from a frame as shown in Fig. 66. Make two of equal length, so that they will vibrate at the same rate. Be sure that they will thus vibrate. The other pendulums are to be of different lengths. Set *a* in vibration. The swinging of *a* will produce slight vibrations in the frame, which will, in turn, transmit them to the other pendulums. As the successive impulses thus imparted by *a* keep time with the vibrations of *b*, this energy accumulates in *b*, which is soon set in perceptible vibration. As these impulses do not keep time with the vibrations of the other pendulums, there can be no such marked accumulation of energy in them, for many of the impulses will act in opposition to the motions produced by previous impulses, and thus weaken if not destroy them.

Experiment 135. — Tune the two strings of a sonometer to perfect unison. Place two or three paper "riders" upon one of the strings, and gently bow the other. The "riders" will be dismounted from the first string, even if the vibrations of the second string are not audible. The vibrant energy was carried from one string through the bridges of the sonometer to the other string and there accumulated. Change the tension of one of the strings, thus destroying the unison, and try to repeat the experiment. Notice that the sympathetic vibrations are not produced.

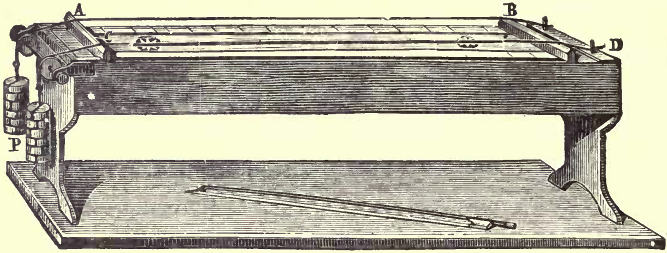


FIG. 188.

Experiment 136. — Place two mounted tuning-forks that are in perfect unison several feet apart, and with the openings of their resonant boxes facing each other. Sound one of the forks, and notice its pitch. After a second or two, touch the prongs to stop their motion. It will be found that the second fork has been set in motion and is giving forth a sound of the same pitch as that originally produced by the first fork. With wax, fasten a small weight to one of the prongs of the second fork. An attempt to repeat the experiment will fail. When the two forks are in unison, their periods are the same. The second and subsequent pulses sent out by the first fork strike the second fork, already vibrating from the effect of the first pulse, in the same phase of vibration, and thus each adds its effect to that of all its predecessors. If the forks are not in unison, their periods will be different, and but few of the successive pulses can strike the second fork in the same phase of vibration; the greater number will strike it at the wrong instant.

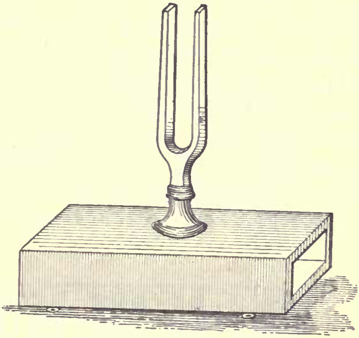


Fig. 189.

Experiment 137. — Fig. 190 represents Mayer's "sound-mill," which consists of four small resonators attached to the ends of a small cross. The cross is carefully balanced on a vertical pivot. The resonators are made of aluminium (a very light metal), and accurately tuned to unison with a mounted tuning-fork. When the "sound-mill" is placed in front of the opening of the resonant box of the fork with which it is in unison, and that fork sounded, the four resonators turn upon their pivot, — a veritable acoustic reaction wheel.

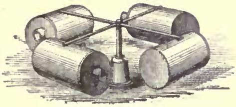


Fig. 190.

204. Sympathetic Vibrations. — It has been shown repeatedly that the motion of a body may produce sound. The last few experiments show that sound may produce motion. The most important feature now to be noticed is that *the sonorous body accumulates only the particular kind of vibration that it is capable of producing.*

(*a*) By the aid of sympathetic vibrations we are able to analyze compound tones, as is shown by the following experiment: —

Experiment 138. — Take your seat before the keyboard of a piano. Press and hold down the key of "middle C," marked 1 in Fig. 191, which represents part of the keyboard. This will lift the damper from the corresponding piano wire, and leave it free to vibrate. Strongly strike the key of C_2, an octave below. Hold this key down for a few seconds, and then remove the finger. The damper will fall upon the vibrating wire and bring it to rest. When the sound of C_2 has died away, a sound of higher pitch is heard. The tone corresponds to the wire of 1, which wire is now vibrating. These vibrations are sympathetic with those that produced the first overtones of the wire that was struck. These vibrations in the wire of 1 prove the presence of the first overtone in the vibrating wire of C_2. In similar manner, successively raise the dampers from the wires of 2, 3, 4, and 5, striking C_2 each time. These wires will accumulate the energy of the waves that correspond to the respective overtones of the wire of C_2, and give forth each its proper tone. Thus we analyze the sound of C_2, and prove that overtones are blended with the fundamental.

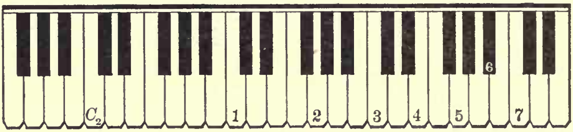


FIG. 191.

Some of these tones of higher pitch, thus produced by vibrations sympathetic with the vibrations of the segments of the wire of C_2, are feebler than others. This shows that the quality of a tone depends upon the relative intensities as well as the number of the overtones that blend with the fundamental.

Resonance.

Experiment 139. — Hold a vibrating tuning-fork over the mouth of a cylindrical jar about 15 or 18 inches deep, and notice the feebleness of the sound. Pour in water, as shown in Fig. 192, and notice that, when the liquid reaches a certain level, the sound suddenly becomes much louder. The water has shortened the air-column until it is able to vibrate in unison with the fork. If more water is added, the sound will become weaker. If a fork of different pitch is used, the length of the resonant air-column will be changed, said length being about one-fourth the length of the wave produced by the fork.

205. Resonance.—*The increase of sound by the sympathetic vibrations of a body other than that by which it was originally produced is called resonance.* The apparatus used to produce such an effect is called a resonator.

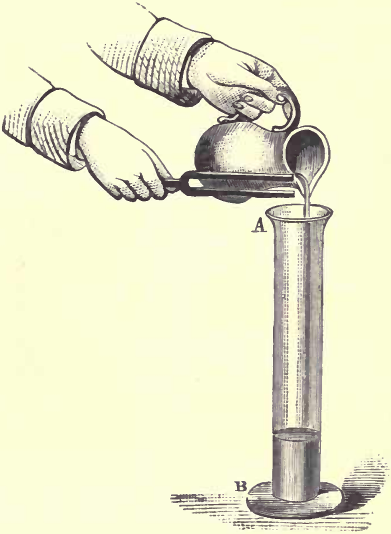


FIG. 192.

(*a*) Resonance occurs more or less prominently in connection with all sound, and is carefully utilized in musical instruments. Sounding-boards like that of the piano, and diaphragms like those of the phonograph and telephone, have a general resonance by virtue of which they are sensitive to any vibratory motion within the limits of ordinary audition. The audiphone is another illustration of the same fact. When the lower prong of the tuning-fork used in Experiment 139 vibrated outward, it started a condensation down the tube. This condensation and the succeeding rarefaction were reflected upward from the bottom of the tube, and returned to unite with other waves sent out by the prong. When the air-column was made of the right length, the reflected waves coincided with the other waves in like phases; i.e., condensation with condensation, and rarefaction with rarefaction. Under different circumstances, the direct and reflected waves may combine in different phases, and thus cause an enfeeblement of the sound.

(*b*) It is found by experiment that the diameter of the tube affects the length of the resonant air-column, so that an arbitrary correction has to be made. For a cylindrical vessel, the experimental column shortens as the diameter increases. The theoretical column equals the experimental column increased by about one-fourth of the diameter.

(*c*) Fig. 193 represents a Savart bell and resonator. The length of the resonant air-column is changed by means of the movable bottom of the resonator, which is to be adjusted by trial for resonant effect. When the bell is sounded, and its tone is just audible, the approach of the tube produces a marked reinforcement of sound.

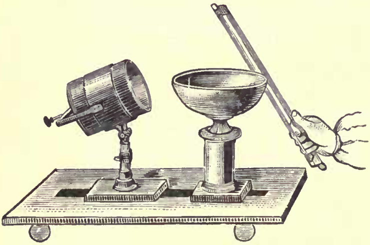


Fig. 193.

(*d*) Helmholtz constructed a series of resonators, each one of which responds powerfully to a single tone of certain pitch or wave-length. They are metallic vessels, nearly spherical, having an opening, as at *A* in Fig. 194, for the admission of the sound-waves. The funnel-shaped projection at *B* has a small opening, and is inserted in the outer ear of the observer. Such resonators are largely used in the analysis of complex tones.

Musical tones may thus be picked from sounds that are commonly reckoned as noises; e.g., the roar of the tempest or the hum of a busy street, or even from an atmosphere apparently in silence.

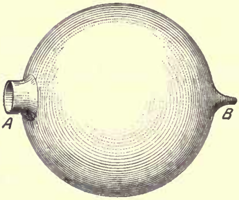


Fig. 194.

Interference.

Experiment 140. — Hold a vibrating tuning-fork near the ear, and slowly turn it between the fingers. During a single complete rotation, four positions of full sound and four positions of silence will be found. When a side of the fork is parallel to the ear, the sound is plainly audible; when a corner of a prong is turned toward the ear the waves from one prong destroy the waves started by the other.

Experiment 141. — Hold a vibrating tuning-fork at the mouth of a resonator, and slowly turn it upon its axis. Notice that, in certain positions of the fork, its tone is nearly inaudible. While the tube is in one of these positions, slip a paper tube over one of the prongs, as shown in Fig. 195, being careful not to touch it. The sound will be restored, because the interfering sound has been removed. When, by removing the paper tube, we restore the sound of the second prong, we demonstrate the almost paradoxical fact that *sound added to sound may produce silence.*

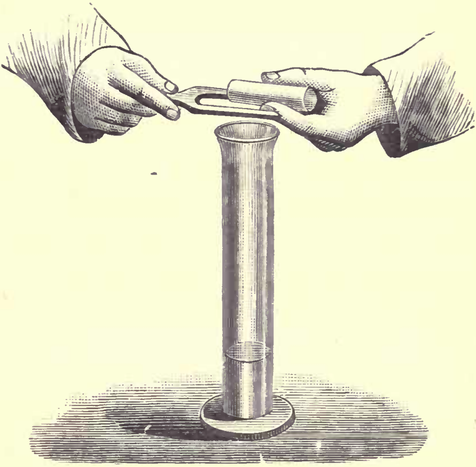


FIG. 195.

206. Interference. — The mutual action of waves upon one another so that the effects of their vibratory motions are increased, diminished, or neutralized is called interference. Thus, resonance is, properly speaking, a variety of interference. But, as a general thing, *interference signifies the coming together of different systems of waves in different phases, so that the vibratory motion of the resultant wave is less than that of the components.* In this sense, interference of sound signifies the union of two or more systems of sound waves in such a way as to weaken or destroy the sound.

(*a*) If, while a tuning-fork is vibrating, a second fork is set in vibration, the waves from the second must traverse the air already vibrant from the effects of the first. When two forks that have the same pitch are any number of whole wave-lengths apart, their waves will unite in like phases, and a reinforcement of sound will ensue, as indicated by Fig. 196. When the forks are an odd number of half wave-lengths apart, their waves will unite in opposite phases, and a silence, perfect or partial, will ensue, as indicated by Fig. 197. *Interference is the leading characteristic property of wave motion.*

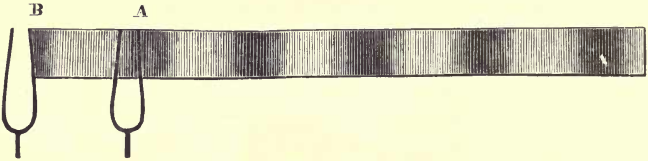


FIG. 196.

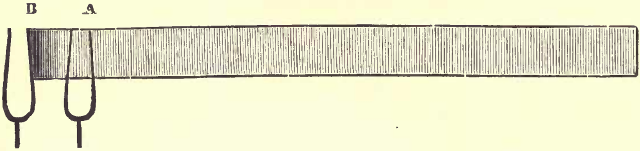


FIG. 197.

Beats.

Experiment 142. — Simultaneously sound two large tuning-forks that are in unison, and notice that the sound is as smooth as if only one fork was sounding. Load one of the prongs of one of the forks with wax, sound both forks, and notice that the sound is not smooth, but that a series of palpitations or beats is easily perceptible.

Experiment 143. — In a quiet room, strike simultaneously one of the lower white keys of a piano, and the adjoining black key. A similar series of beats will be heard.

207. Beats. — If two tuning-forks, *A* and *B*, vibrating respectively 255 and 256 times a second, are set in vibration at the same time, their first waves will meet in like phases, and the result will be an intensity of sound greater than that of either. After half a second, B having gained half a vibration upon A, the waves will meet in opposite phases, and the sound will be weakened or destroyed. At the end of the second, we shall have another reinforcement; at the middle of the next second, another interference. *This peculiar pulsation arising from the successive reinforcement and interference of two tones differing slightly in pitch is called a beat.* The number of beats per second equals the difference of the two vibration-numbers.

(*a*) In Fig. 198, the high crests and deep troughs represent, in the conventional manner, the phases where the two tones reinforce each other, while the intermediate portions of the curve similarly symbolize the interference phases.

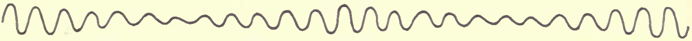


FIG. 198.

208. Noise and Music. — A noise is a sound so complex that the ordinary powers of the ear fail to resolve it into its constituent tones. A simple tone is incapable of resolution, by reason of its simplicity. A combination of sounds that may be easily resolved into simple tones is a musical sound. The distinction is often difficult. The combination of sounds heard in a boiler-shop is surely a noise, while the sound of a tuning-fork mounted on a resonant box is very nearly a simple tone.

CLASSROOM EXERCISES.

1. How can a deaf person determine whether a given tone is simple or compound?

2. If two tuning-forks, vibrating respectively 256 and 259 times per second, are simultaneously sounded near each other, what phenomena will follow?

3. A musical string, known to vibrate 400 times a second, gives a certain tone. A second string, sounded a moment later, seems to give the same tone. When sounded together, two beats per second are noticeable. Are the strings in unison? If not, what is the rate of vibration of the second string?

4. How many beats per second will be produced by the simultaneous sounding of two tones having vibration-numbers of 297 and 308 respectively?

5. Show that the length of a resonant air-column must be about $\frac{1}{4}$ the wave-length of the tone that it reinforces, or some odd multiple of that length.

6. The notion is very common, but without foundation in fact, that the sound heard when a conch shell is held to the ear is a lingering remnant of the roar of the ocean from which the shell came. Explain the sound that is thus heard.

7. A tuning-fork produces a strong resonance when held over a jar 15 inches long. (*a*) Find the wave-length of the fork. (*b*) Find the wave-period. Ignore the influence of the diameter of the jar.

8. A tuning-fork held over a tall glass jar, into which water is slowly poured, receives its maximum reinforcement of sound when the resonant air-column is 64.8 cm. long. Assuming that the fork is accurately tuned to give an exact number of vibrations per second, noting the fact that the thermometer records a temperature of 16° C., and keeping in mind the probability of slight experimental error, determine the vibration-number of the fork. *Ans.* 132.

9. Refer to § 205 (*b*), and show that, representing the velocity of sound in air by v, the length of the resonant air-column by l, the diameter of the resonant column by d, and the number of vibrations of the fork by n,

$$v = 4\left(l + \frac{d}{4}\right)n.$$

10. If a tube 4 cm. in diameter and 50 cm. long responds most

loudly to a certain fork, what is the wave-length of the tone of that fork?

11. One of two tuning-forks, each tuned to 512 vibrations per second, is loaded with wax. The forks are simultaneously sounded, and 20 distinct beats are heard in 10 seconds. What is the vibration-number of the loaded fork?

LABORATORY EXERCISES.

Additional Apparatus, etc. — Resonant-tube and two tuning-forks, as described; wooden rod; piano; music-box and wraps for the same; guitar; glass funnel; T-tubes.

1. In a manner similar to that employed in Exercise 1, page 214, represent two series of waves. Draw the second circle immediately under the first and with equal diameter. Divide the circumference of the first circle into eight, and that of the other into twelve, equal parts. Use the same scale in laying off the equal parts on the axes of abscissas for the two curves. When the two curves of sines are drawn, prolong the vertical lines downward and draw a third axis of abscissas. Construct a curve that will represent the form of the wave formed by compounding the two waves represented by the curves previously drawn; i.e., make each ordinate equal to the algebraic sum of the two corresponding ordinates.

2. Using a resonant jar and a tuning-fork the vibration-number of which is known, determine the velocity of sound in air. (Apply the formula developed in the solution of Exercise 9, on page 254.)

3. Stretch a string horizontally between two fixed supports. From this string suspend two bullet pendulums by threads about a meter long. Swing one of these pendulums across the direction of the horizontal string. Describe and explain the result that you think the exercise was intended to bring to your notice.

4. Remove the cover from a piano, depress the pedal so as to lift the dampers from all the wires, hold the lips near the wires, and sing the vowel, *a*, with the sound it has in "fate," and prolong the tone. Listen for the sympathetic response of the piano. Repeat the experiment, singing the same vowel with the sound it has in "father," and then the vowel, *o*, with the sound it has in "tone."

5. Fig. 199 represents two series of sound waves traveling together. The full line represents one series and the dotted line another. What phenomenon would result from such a combination of tones as is here

represented? Describe the condition of affairs as represented at *A*, *B* and *C* respectively. Draw a single curve to represent the resultant of combining these two tones.

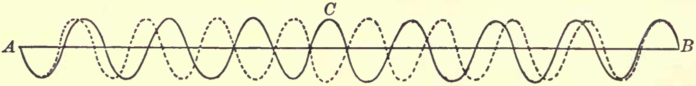


FIG. 199

6. Fig. 200 represents two systems of sound waves having equal amplitudes and periods that are as 1:2. In the first diagram, the waves start together; in the second, the shorter wave is a quarter of a wave-length behind; in the third, it is a half wave-length behind; in the fourth, it is three-fourths of a wave-length behind. Draw curves representing the compound waves resulting from these combinations, and repeat each form twice, thus getting four curves, each representing a series of three compound waves. Carefully note the differences in the wave-forms that result from combining like waves in different phases.

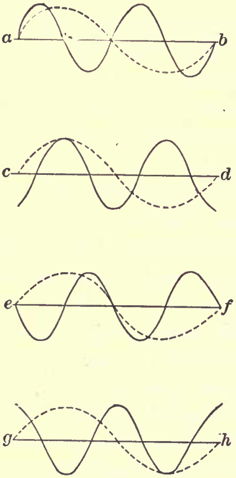


FIG. 200.

7. Construct a single curve that shall represent the form of sound waves composed of the combined motions of the harmonics represented in Fig. 201.

8. Get a glass tube about $\frac{3}{4}$ of an inch in diameter and 12 inches long. Into this

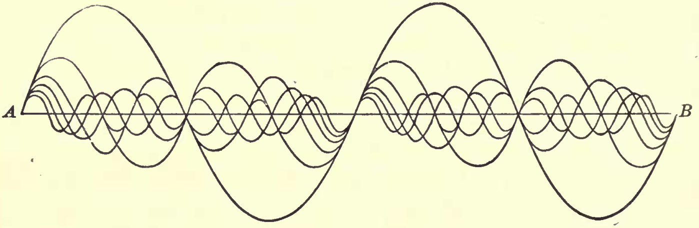


FIG. 201.

tube thrust a neatly fitting cork. Move the cork with a ramrod until, by trial, you have adjusted the tube for maximum resonance with a tuning-fork, e.g., one marked "Philharmonic A." Support the tube with its mouth close to the disk of the siren shown in Fig. 168, and facing one of the circles of holes. Hold the nozzle of the tube on the other side of the disk and just opposite the mouth of the resonant tube. Turn the tube with gradually increasing speed, and blow air through the tube. When the sound is at its maximum, the siren-tone will be in unison with the tone of the fork by which the resonant tube was tuned. Determine the vibration-number of the fork.

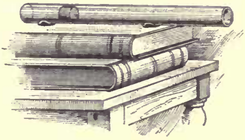


FIG. 202.

9. Support a wooden rod about an inch square and three feet long with its lower end resting upon the cover of a music-box that is sounding. Wrap the music-box in cotton-wool and manifold layers of woolen cloth, until no sound from the box can be heard. Carefully balance a guitar or violin upon the top end of the rod. Describe and explain the consequent phenomenon.

10. Place two tuning-forks having frequencies of 512 and 516 respectively upon the table, and sound them together. They will give four beats per second. Make sure of the fact by trial. Place your ear in a line with the forks, and have one of the sounding forks steadily moved toward the other at the rate of two feet per second. How many beats per second do you hear? Then have one of the sounding forks moved directly away from the other at the uniform rate of two feet per second. How many beats per second do you hear? Explain.

11. With glass funnel and T-tubes and rubber tubing, arrange apparatus as shown in Fig. 203. A sound wave entering at *o* will divide its energy, part passing by way of *a* and part by *c*, and uniting at *e*. If the two paths between *b* and *e* are of equal lengths, the waves will unite at *e* in like phases, and the ear at *s* will hear the sound without serious diminution. If, however, one path is longer than

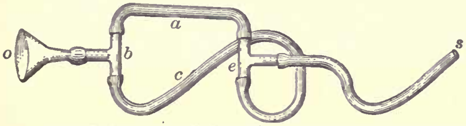


FIG. 203.

the other by a half wave-length of the sound entering at *o*, or any odd multiple thereof, the waves will unite at *e* in opposite phases, and an interference more or less nearly complete will result. Provide a tuning-fork with a frequency of about 512; determine, by experiment or computation, the length of its waves at the temperature of the room, and make one branch of the apparatus a half wave-length longer than the other. This difference will be about a foot. Sound the fork at *o* and hold the ear at *s*, and slip one end of *c* back and forth over the glass tubing, until the adjustment is such that the sound heard at *s* is very feeble or *nil*. When a good interference is secured, sound the fork again, and pinch the rubber tubing at *a* or *c*. Two **Y**-tubes may be used instead of the **T**-tubes, and the tube between *b* and *e* may consist in part of a bent glass tube that will slide inside the rubber tube.

V. THE LAWS OF VIBRATION.

209. Vibrations of Strings. — The laws of the vibrations that give rise to musical tones are most conveniently studied by means of stringed instruments, especially the sonometer (Fig. 204). The vibrations may be transverse, torsional, or longitudinal, as stated in § 180. Of these, the transverse vibrations are the most important. When used for the production of musical tones, strings are

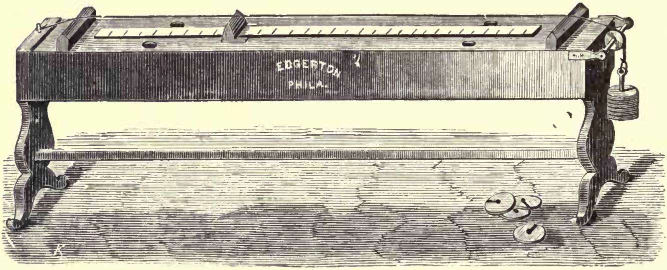


FIG. 204.

fastened at their ends, stretched to proper tension, and then made to vibrate by bowing, as in the violin; by plucking, as in the guitar or banjo; or by striking them with a light hammer, as in the piano or dulcimer. The manner of producing the vibrations has little effect upon the tone, which is chiefly determined by the length, diameter, density, and tension of the string itself.

Experiment 144. — Remove the sliding bridge of the sonometer, stretch one of the strings and pluck or bow it near its end. Notice the pitch of the tone. Place the sliding bridge at the middle of the scale on the sonometer box so as to halve the length of the string; then bow as before. Notice that the pitch of the tone is an octave higher. If desirable, the experiment may be modified by tuning the string until its tone is in unison with a certain tuning-fork or with the tone of the siren when the air-stream is directed against the inner row of holes. Then the tone of the half string will be in unison with another fork an octave higher in pitch or with the siren-tone when the air stream is directed against the outer row of holes, the speed of rotation being the same.

Experiment 145. — Stretch two wires of the same diameter and material with unequal but known weights, and, with the movable bridge, shorten the wire that carries the smaller weight until it sounds in unison with the other. Notice that the lengths of the strings vary as the square roots of the weights or tensions.

Experiment 146. — Stretch two iron wires of different diameters, upon the sonometer, with equal tension. With the sliding bridge, shorten the heavier wire until it is in unison with the other wire. From the scale on the sonometer box, read the length of the vibrating part of the heavier wire. Measure the diameters of the two wires and notice that the diameters of the wires are inversely proportional to their lengths.

Experiment 147. — Stretch, with equal tension, a brass and a steel wire of the same diameter, and, with the sliding bridge, shorten one of the wires until the two are in unison. Ascertain, in the easiest way, the densities of brass and steel, and notice that the lengths of the strings vary inversely as the square roots of the densities of the materials.

210. Laws of Vibrations of Strings. — These experiments indicate the following facts relative to musical strings : —

(1) *Other conditions being the same, the vibration-numbers vary inversely as the lengths.*

(2) *Other conditions being the same, the vibration-numbers vary directly as the square roots of the tensions.*

(3) *Other conditions being the same, the vibration-numbers vary inversely as the diameters.*

(4) *Other conditions being the same, the vibration-numbers vary inversely as the square roots of the densities.*

(*a*) The third and fourth laws may be consolidated as follows : — *Other conditions being the same, the vibration-numbers vary inversely as the square roots of the weights per linear unit.*

(*b*) In instruments like the piano, the length, diameter, and density of the strings are determined once for all by the manufacturer, the tension being adjusted by the piano tuner.

Air Columns.

Experiment 148. — Make a reed-pipe by cutting a piece of wheat straw eight inches (20 cm.) long so as to have a knot at one end. At *r*, about an inch from the knot, cut inward about a quarter of the straw's diameter; turn the knife blade flat and draw it toward the knot. The strip, *rr′*, thus raised is a reed; the straw itself is a reed-pipe. When the reed is placed in the mouth, the lips firmly closed around the straw between *r* and *s* and the breath driven through the apparatus, the reed vibrates and produces vibrations in the air-column of the wheaten pipe. Notice the pitch of the tone thus produced. Cut off two inches from the end of the pipe at *s*. Blow through the

FIG. 205.

pipe as before and notice that the pitch is raised. Cut off two inches more, sound the pipe, and notice that the pitch is still higher.

211. Vibrations of Air Columns. — Experiments 96, 139, and 148 show that when gases are confined in tubes they may be made to vibrate as sonorous bodies. The air-column may be set in vibration by a vibrating tongue, as in the reed-pipe of Experiment 148, or in reed instruments like the melodeon, accordion, clarinet, etc., or by the fluttering of air particles driven against the edge of an opening in a tube, as in the whistle, fife, flute, or organ-pipe. Whatever the way of producing the vibrations, the dimensions of the air-column itself determine the tone. In Experiment 148, we saw that the air-column, and not the straw tongue, determined the pitch.

Experiment 149. — Fit a cork loosely as a piston into the end of a glass tube about 2 cm. in diameter and 30 cm. long. Blow across the open end of the tube so as to produce a steady tone. It may be more easy to do this if you use a mouthpiece made by flattening the end of a piece of brass tubing. Notice the pitch of the tone produced, and measure the length of the air-column in the tube. By trial, determine the lengths of the air-columns that will give the tones of the gamut, and compare the relative lengths with the relative vibration-numbers given in § 196.

Experiment 150. — Provide two glass tubes of the same diameter (about 2.5 cm.), one being half as long as the other (e.g., 10 cm. and 20 cm.). Blow across the end of the longer tube so as to produce its lowest tone while the other end of the tube is open. Notice the pitch. Stop one end of the shorter tube with the hand, and blow across the open end so as to produce the lowest tone. Notice that the pitch of the short stopped-pipe is the same as that of the long open-pipe. Of course, if the school is provided with an assortment of organ-pipes (as is desirable), it is better to use them.

Experiment 151. — Procure an open organ-pipe, at least one side of which is made of glass. While the pipe is emitting its fundamental tone, lower a small ring with a paper bottom, on which a little fine sand has been strewn, as shown in Fig. 206. Just inside the upper part of the pipe, the sand dances right merrily, its motion becoming less energetic as the ring approaches the middle of the pipe. When the ring is lowered below this point, the agitation of the sand steadily increases. Vary the experiment by closing the open end of the pipe with a cover or plug perforated for the thread and moving the sand-strewn membrane up and down as before. The only place where the sand is not agitated is at the closed end of the pipe.

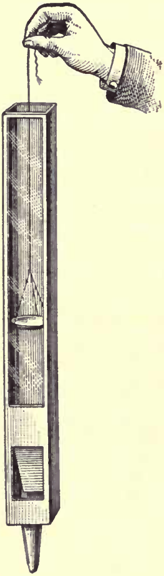


FIG. 206.

212. Laws of Vibrations of Air-Columns. — Careful and elaborate tests have verified what is suggested by our rude experiments, namely, that —

(1) *The vibration-numbers of air-columns vary inversely as their lengths.*

(2) *The pitch of a closed-pipe is an octave below that of an open-pipe of the same length.*

(*a*) The two ends of an open-pipe sounding its fundamental are places of maximum motion, i.e., the middle points of ventral segments; while at the middle of the pipe, where the direct and reflected pulses cross each other, there is no motion. This middle point is a node. The length of the air-column is half the wave-length. In the stopped-pipe sounding its fundamental, the node is at the end, and the length of the air-column is a fourth of the wave-length.

(*b*) By increasing the pressure of the blast that blows the pipe, overtones may be produced. The change of nodes and ventral segments may be shown by the sand-strewn membrane as before.

(*c*) If a hole is made in the side of a pipe at a point occupied by a node, the point is at once changed to the middle of a ventral segment, and there is a corresponding change of pitch. This action is familiarly shown in the fife and flute.

Vibrating Rods.

Experiment 152. — Hold a steel rod, a meter in length, at its middle point, and rub one of its halves with a piece of resined leather. The rod will emit its fundamental tone. Repeat the experiment successively with two other steel rods, one having the same length and a different diameter, the second rod being just half as long. The second rod will give a tone of the same pitch as the first, while the shorter rod will sound a tone an octave above.

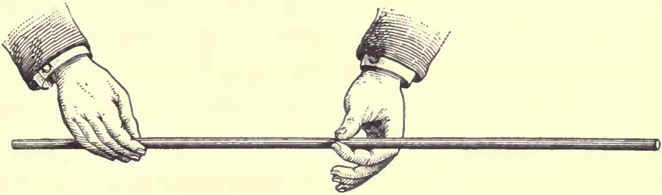


Fig. 207.

Experiment 153. — That the rubbing of a rod, as in Experiment 152, produces longitudinal vibrations in the rod is prettily shown by the apparatus devised by Kœnig, and represented in Fig. 208. A brass rod, *AB*, is supported by a clamp at its middle point, *c*. An ivory ball is suspended so as just to touch the end of the rod. When the rod is rubbed with resined leather at *d*, the vibrations thus set up in the rod repel the elastic ball in a very energetic manner. The rod may be clamped in a vise, and the ball suspended in any convenient way.

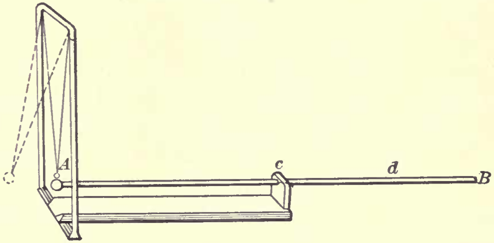


Fig. 208.

213. The Vibrations of Rods may be transverse, longitudinal, or torsional. Transverse vibrations are familiarly illustrated in the music-box, jews'-harp, xylophone, tuning-fork, etc. Longitudinal vibrations were illustrated in Experiment 153. By clasping a vertical glass tube with one hand, and rubbing the upper half with a wetted cloth held in the other, it is possible to produce longitudinal vibrations that will shatter the lower part of the tube. For longitudinal vibrations of rods of any given material, the vibration-numbers are inversely proportional to the lengths of the rods. Such rods may also be made to vibrate in segments, and then the vibration-numbers are inversely proportional to the lengths of the segments. If a violin-bow is drawn around a rod that is clamped at one end, the rod will twist and untwist with vibrations that are as isochronous as those of a tuning-fork, emitting a tone a little lower than that produced by longitudinal vibrations of the same rod having the same number of segmental divisions.

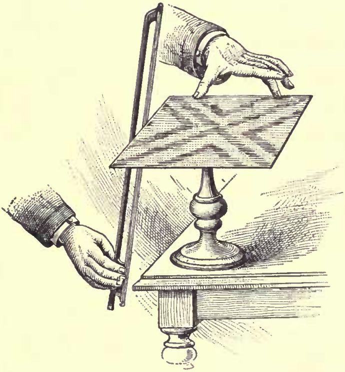


FIG. 209.

Vibrating Plates.

Experiment 154. — Support, as shown in Fig. 209, a glass or brass plate, square or round, and strew it evenly with fine sand. Place the finger at any point on the edge of the plate (e.g., at the middle of one side) so as to form a node there, and draw a violin-bow at a point properly chosen (e.g., near the adjacent corner). The sand immediately begins to dance on the plate and arrange itself along nodal lines. By changing the nodal points and bowing properly, other sand-figures may be produced, one of which is shown in Fig. 209.

214. Vibrations of Plates. — The method of studying the vibrations of plates as just illustrated is due to Chladni, after whom the plates and figures are named. The arrangement of nodal lines is determined by the relative positions of the point that is bowed and the point that is touched with the finger. The figures may be produced in multitudinous variety. As the complexity of the figures produced on a given plate increases, the pitch of the corresponding tone rises, *the same figure always answering to the same tone.*

Experiment 155. — Draw a violin-bow across the edge of a large goblet nearly full of water on the surface of which cork dust or powdered sulphur has been evenly sifted. The glass bell will emit a musical tone and the surface of the water will disclose the mode of the vibration. When the bell sounds its fundamental tone, it vibrates in four segments, and the surface of the water tells the story by a record like that shown in Fig. 210. A few vigorous strokes of the bow would set up vibrations of amplitude sufficient to break the bell.

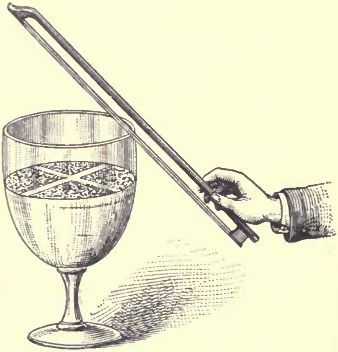


Fig. 210.

215. Vibrations of Bells. — Just as a tuning-fork may be looked upon as a rod bent into a U-shape, so a bell may be considered as a disk bent into a cup shape. Like a disk, it sounds its fundamental tone when vibrating in four segments, and the number of segments is always even.

CLASSROOM EXERCISES.

1. A musical string vibrates 200 times a second. State what takes place when the string is lengthened or shortened with no change of tension, and what change takes place when the tension is made more or less, the length remaining the same.

2. A certain string vibrates 100 times a second. (*a*) Find the vibration-number of a similar string, twice as long, stretched by the same weight. (*b*) Of one that is half as long.

3. A string sounding C_3 is 18 inches long. Must it be lengthened or shortened and how much to give the tone D_3?

4. A certain string vibrates 100 times per second. Find the vibration-number of another string that is twice as long, and weighs four times as much per foot, and is stretched by the same weight.

5. A certain string sounds the tone C_2 when it is stretched by a weight of four pounds. What weight must it carry to give the tone F_2.

6. A musical string five feet long emits a tone in unison with that of a fork that is known to vibrate 256 times a second. What will its vibration-number be when it has been shortened two feet?

7. A sonometer string is stretched by a load of 16 pounds. What load must be given to it so that it may sound a tone an octave lower?

8. A tube open at both ends is to produce a tone corresponding to 32 vibrations per second. Taking the velocity of sound as 1,120 feet, find the length of the tube. If the number of vibrations is 4,480, find the length of the tube.

9. Find the length of an organ-pipe the waves of which are four feet long, the pipe being open at both ends. Find the length, the pipe being closed at one end.

10. What will be the relative vibration-numbers of two strings of the same length, diameter, and tension, one being made of catgut and the other of brass, the density of brass being nine times that of catgut?

11. If three organ-pipes of the same dimensions are filled respectively with gases of different densities, e.g., air, hydrogen, and carbon dioxide, would the several tones emitted agree or differ in pitch? Why?

12. How would you experimentally determine whether a vibrating tuning-fork has nodes or not?

LABORATORY EXERCISES.

Additional Apparatus, etc. — Two spring-balances, each of which will carry a load of 30 pounds; four hard wood triangular prisms, each about 5 cm. long and with a triangular altitude of 2.5 cm.; spring-brass wire, No. 24 and No. 27; a tuning-fork with pitch of middle *C*, and another an octave lower; a glass tube and a brass rod as described below; mercury cup; two electromagnets; voltaic cell.

1. Set two screws horizontally into the end of a table-top, and about 15 cm. apart. Anneal the two ends of a piece of No. 24 spring-brass wire, about 150 cm. long, and fasten one of the ends to one of the screws, and the other end to the hook of one of the spring-balances. To the ring of the balance, attach a stout, soft iron wire, about 50 cm. long. Within easy reach of the free end of this second wire, set a stout screw, so that its head shall project about 2.5 cm. above the top of the table, and fasten the free end of the wire to it. In similar manner, stretch a No. 27 spring-brass wire parallel with the No. 24 wire. Prop the dynamometers so that they will lie flat on their backs, and slip one of the hard wood prisms under each end of each of the two brass wires. Put the No. 24 wire under a tension of 20 pounds, and move the prism away from the dynamometer hook until the wire, when plucked midway between the prisms, gives a tone that is in unison with the fork of lower tone. In doing this, press the wire lightly against the ridge of the moved prism, hold the ear near the wire and let the rough overtones die away, leaving the fundamental which is to be tested. Measure the distance between the two ridges; i.e., the length of the vibrating wire. Shift the prism again and determine the length of the wire when its tone is in unison with the fork of tone an octave higher. Reduce the tension of the wire to 5 pounds, and determine the length of the wire when it is in unison with the fork of lower tone. Put the No. 27 wire under a tension of 10 pounds, and determine the lengths of the wire when it is in unison with each of the forks. Consult the table of wire gauges in the appendix, and notice that the area of cross-section for No. 24 wire is almost exactly twice that of No. 27 wire, and remember that the weight per linear unit varies as the area of cross-section. How do your results correspond to the laws given in § 210?

2. Replace the No. 27 wire of Exercise 1 with a second No. 24 wire like the first, and put it under a tension of 6 pounds. Increase the tension of the first wire to 24 pounds. Move the bridge until the

second wire gives a musical tone. Move the other bridge until the other wire has the same length. With the siren, determine the vibration-numbers of the two shortened wires, and see how the ratios between them compare with the ratios between the two tensions.

3. Bring the siren into unison with a tuning-fork. Turning the wheel regularly for 10 seconds at the rate that gives unison, determine the number of puffs per second, and thus determine the vibration-number of the fork.

4. Close one end of a glass tube about 1.5 m. in length and 4 cm. in internal diameter, with a stopper. Scatter along the length of the tube a small quantity of precipitated silica or cork dust. Fasten a thin cork that neatly fits into the glass tube to one end of a brass rod or tube about 2 m. in length and 1 cm. in diameter. Lay the glass tube on the table, and push the cork at the end of the brass rod about 50 cm. into it. Clamp the rod between two grooved blocks at its middle so that it shall lie along the axis of the glass tube. Rub the outer end of the rod with resined leather so as to produce a shrill sound and to agitate violently the powder in the tube. Change the length of the air-column by moving the glass tube endwise until the powder divides into segments separated by clearly marked nodes. From the distance between these nodes, determine the wave-length of the sound in the air. From the length of the rod, determine the wave-length of the sound in brass. From these results, determine the relative velocities of sound in brass and in air.

5. Solder a wire pointer to the under side of a steel sonometer-wire at its middle. Place a small cup of mercury below the wire so that the pointer shall just touch its surface. Cover the surface of the mercury with a thin layer of mixed alcohol and glycerin to keep the metal surface clean. Place the end of an electro-magnet, *E*, a

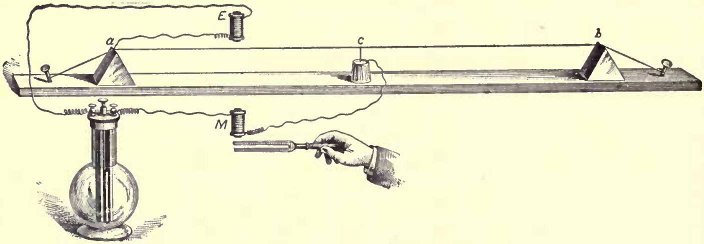


FIG. 211.

little above the wire and midway between the pointer and the end, and another electro-magnet, *M*, at a distance of several feet. Put the apparatus into circuit with a voltaic cell, as shown in Fig. 211, and vibrate the wire. Support a tuning-fork that is nearly in unison with the wire so that the face of one of its prongs shall be near the end of *M*. Adjust the tension or the length of the wire until the response of the fork shows that the wire and the fork are in unison.

6. Select two tuning-forks of the same tone, and, by means similar to those used in Exercise 5, cause one to respond to the other. Notice the persistence of the vibrations.

CHAPTER IV.

HEAT: MOLECULAR PHYSICS.

I. NATURE OF HEAT, TEMPERATURE, ETC.

216. Heat *is a form of energy into which all other forms of energy are convertible. It consists in the agitation of the molecules of matter, and is generally recognized by the sensation of warmth to which it gives rise.*

(*a*) When the molecular agitation of a body is increased, the body is heated; when it is lessened, the body is cooled. See §§ 6 (*c*), 8, and 51.

217. The Temperature *of a body is its state considered with reference to its ability to communicate heat to other bodies.* When two bodies are brought together, there is a tendency toward an equalization of temperature. If there is an actual transfer between them, the one that gives the greater amount of heat has the higher temperature, and the one that receives it has the lower temperature.

(*a*) Water flows from a point of high to one of low level. Electrification flows from a point of high to one of low potential. Heat flows from a point of high to one of low temperature.

(*b*) An addition of heat may increase the velocity of the molecular motion or it may do another kind of work. When a body receives heat, its temperature generally rises, but sometimes a change of condition results instead. When a body gives up heat, its temperature falls or its physical condition changes.

Experiment 156. — Into one basin put hot water; into a second basin put ice-cold water; into a third basin put water at the temperature of the room. Put the right hand into the hot water and the left hand into the cold water, and hold them there for some time. Transfer the right hand from the hot water to the water in the third basin, and that water will seem cold. Transfer the left hand from the cold water to the water in the third basin; the water that felt cold to the right hand will feel warm to the left hand.

218. An Unsafe Standard. — Experiment 156 shows that our sensations may not be trusted as a measure of temperature. They are of even less value in the measurement of heat. A body feels hot when it is imparting heat to us; it feels cold when it is drawing heat from us.

Experiment 157. — Provide a ring (or a sheet of tin with a hole cut in it) and a metal ball that, at the ordinary temperature, will just pass through the opening. Heat the ball and it will no longer pass through.

Experiment 158. — Connect, by a perforated cork, a piece of glass tubing about 50 cm. long to a Florence flask. Put water that has been colored with red ink into the apparatus so that it partly fills the upright tube. Mark the level of the water in the tube by a rubber band or in some other convenient way. Immerse the flask in hot water and carefully observe the level of the water in the tube.

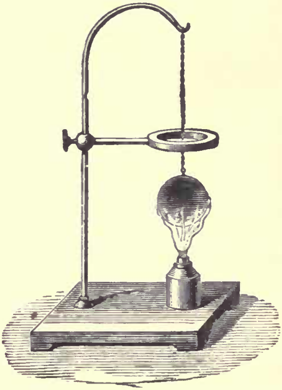


FIG. 212.

219. Expansion. — Experiments 157 and 158 show that one effect of heating a body is to increase its volume. This increase is the immediate result of the increase of the molecular motions; the amount of the expansion is definitely related to the increase of temperature.

220. A Thermometer *is an instrument for measuring temperatures.* In its most common form, it consists of a liquid-filled bulb and a tube of uniform bore, as illustrated in Experiment 158. The liquid generally used is mercury or alcohol. The upper part of the tube is freed from air and hermetically sealed. A scale of equal parts is added for the measurement of the rise or fall of the liquid in the tube.

(*a*) An air thermometer consists essentially of a large glass bulb at the upper end of a tube of small but uniform bore, the lower end of which dips into colored water. When the bulb is heated, some of the expanded air escapes in bubbles through the liquid; when the bulb cools, some of the liquid rises in the tube. Of course, the tube has its scale of equal parts. Any slight change of temperature affects the elastic force of the air in the bulb and changes the height of the liquid column. For a small change of temperature, the movement of the index is comparatively large; i.e., the instrument has great sensitiveness.

FIG. 213.

(*b*) The differential thermometer shows the difference in temperature of two neighboring places by the expansion of air in one of two bulbs that are connected by a bent glass tube containing some liquid not easily volatile. It is an instrument of simple construction and great sensitiveness.

(*c*) Mercury freezes at about – 39° C. For temperatures lower than – 38° C., an alcohol thermometer is generally used. Mercury boils at about 350° C. Temperatures higher than 300° C. are generally measured by the expansion of a metal rod or by using an air thermometer with a porcelain or platinum bulb.

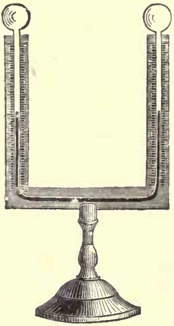


FIG. 214.

221. Graduation of the Thermometer. — In every thermometer there are two fixed points called the freezing-point and the boiling-point. The first of these indicates the temperature of melting ice; the other, the temperature of steam as it escapes from water boiling under an atmospheric pressure of 76 centimeters of mercury. The distance between the fixed points is divided into equal parts according to different arbitrary scales.

(*a*) The two scales chiefly used in this country are the centigrade (or Celsius) and Fahrenheit's. For these scales, the fixed points, determined as just explained, are marked as follows: —

	Centigrade.	*Fahrenheit.*
Freezing-point,	0°	32°
Boiling-point,	100°	212°

The tube between these two points is divided into 100 equal parts for the centigrade scale, and into 180 for Fahrenheit's. Either scale may be extended beyond either fixed point as far as is desired. The divisions below zero are considered negative; e.g., – 10° signifies 10 degrees below zero. The scales are designated by their respective initial letters, as 5° C., or 41° F. In the Reaumur scale, which is little used in this country, the freezing-point is marked 0° and the boiling-point, 80°. Unless otherwise stated, the thermometer readings given in this book are in centigrade degrees.

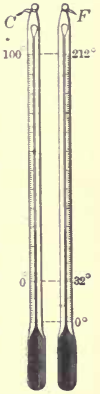


FIG. 215.

(*b*) Since 0° C. corresponds to 32° F., and an interval of 1 centigrade degree equals an interval of 1.8 Fahrenheit degrees, we may reduce centigrade readings to Fahrenheit readings by multiplying the number of centigrade degrees by 1.8 and adding 32. Similarly, we may reduce Fahrenheit degrees to centigrade degrees by subtracting 32 from the number of Fahrenheit degrees and dividing the remainder by 1.8.

222. Absolute Zero of Temperature. — *The temperature at which the molecular motions constituting heat wholly cease is called the absolute zero.* It has never been reached, but theoretical considerations indicate that it is 273° below the centigrade zero. (§ 232, *a*.) It is often convenient as an ideal starting-point or standard of reference.

(*a*) Temperature, when reckoned from the absolute zero, is called absolute temperature. Absolute temperatures are obtained by adding 273 to the readings of a centigrade thermometer, or 460 to the readings of a Fahrenheit thermometer.

CLASSROOM EXERCISES.

1. The difference between the temperatures of two bodies is 36 Fahrenheit degrees. Express the difference in centigrade degrees.

2. The difference between the temperatures of two bodies is 35 centigrade degrees. Express the difference in Fahrenheit degrees.

3. (*a*) Express the temperature 68° F. in the centigrade scale. (*b*) Express the temperature 20° C. in the Fahrenheit scale.

4. What is the corresponding centigrade reading for 50° F.?

5. How would it affect the readings of the school thermometer (mercurial) if the bulb should permanently contract?

6. How would the readings of a mercury thermometer be affected if the tube was gradually enlarged from the bulb upward; i.e., if the bore of the tube was made slightly conical?

7. When a centigrade thermometer indicates a temperature of 15°, what should be the reading of a Reaumur thermometer hanging by its side?

8. Suppose that one of the flat faces of a tin can was painted with a mixture of lampblack and oil, the opposite face of the can being left bright; that the can thus prepared was filled with hot water and hung between the bulbs of the differential thermometer with the painted side facing one of the bulbs; and that the liquid moved toward the bulb that was opposite the bright face of the can. What inference would you draw concerning the effect of the paint on the facility with which tin at a given temperature emits heat?

9. Suppose that when the Florence flask used in Experiment 158 was immersed in hot water, the level of the liquid in the tube fell a little before it began to rise. How would you explain such an effect?

10. What effect would a diminution of the bore of a thermometer tube have upon the sensitiveness of the instrument?

11. What is in the upper part of a thermometer tube?

12. What relation has a flow of heat between two bodies to the relative temperatures of the two bodies?

13. Describe very briefly the molecular agitations of a body at a temperature of $-273°$.

14. What is the absolute temperature of this room at this time?

LABORATORY EXERCISES.

Additional Apparatus, etc.— One or two chemical thermometers (Fig. 215) graduated from 0° to 100°; Bunsen burner or alcohol lamp; the boiler apparatus described below; a tin pail; ice or snow; barometer.

1. Hold a chemical thermometer in the right hand as a pen is generally held, but with the bulb uppermost. Strike the right hand against the palm of the left hand so as to separate some of the mercury in the stem from the main column. Measure, in divisions of the scale and at different parts of the scale, the length of the mercury thus separated from the main column to determine the accuracy of the calibration of the thermometer. Do not imagine that this experiment gives any adequate idea of the tedious painstaking necessary for such calibration as is required for work of precision.

2. Nearly fill any convenient metal vessel with clean ice in small pieces, the smaller the better. For this and other experiments with heat, snow may be used instead of ice. Fill the spaces between the lumps of ice with water, and insert the lower end of a thermometer until the zero mark is within 1 or 2 mm. of the water surface. Place the eye so that the line of vision cuts the thermometer at right angles at the top of the mercury column. When the mercury has fallen to its lowest point, record the reading of the thermometer, estimating fractions of the divisions of the scale with the eye as closely as possible.

3. Make, of sheet-copper, a boiler with a diameter of 10 cm. and a height of 15 cm. The top of the cylinder should not be wired, but should be left flexible. About 2 cm. from the top, insert a sheet-copper tube, *a*, about 5 cm. long and about 6 mm. in diameter, so that it shall slope slightly upward from the boiler. Provide three reliable legs, or other means of supporting the boiler about 20 cm. above the table. Make a sheet-copper conical cover that fits the top of the boiler snugly and internally (as a tin pail cover fits), and that tapers upward for 30 cm. to an opening about 2.5 cm. in diameter. This opening at the top of the cover should be so constructed that it may be closed with a cork, *c*. About 2 cm. below the top of the cover, insert a tube, *b*, like that inserted in the boiler proper, but only 2 cm. long.

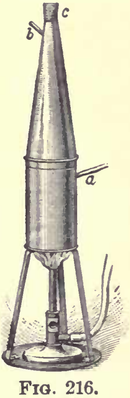


FIG. 216.

Fill the boiler with water to the depth of 3 or 4 cm. Cork the tube, *a*. Pass a centigrade thermometer through a perforated cork that closes *c*, and push it down until the point marked 100° is not more than 2 or 3 mm. above the cork, unless that would bring the bulb close to the water. In that case, adjust the thermometer so that its bulb shall be about 3 cm. above the water. Boil the water (but not violently), guarding against the streaming of the flame up the sides of the boiler. When the mercury has risen as far as it will, record the reading of the thermometer. Note the reading of the barometer, and correct the reading of the thermometer by allowing 1° for each 27 mm. that the barometer column exceeds 760 mm., or falls short of it. Hold a wadded handkerchief over the mouth of the tube, *b*, and note the effect of added pressure upon the boiling-point of the water.

Allow the thermometer to cool in air, and then redetermine its freezing-point, as in Exercise 2. If that point varies from the one previously found, consider the newly found point as the true one. Heat the ice or snow, stirring it with the thermometer during liquefaction, and observing the thermometric reading.

If the tests of the thermometer develop an error of 1° or more in the fixed points, get another thermometer, or correct subsequent readings for the observed errors. In the estimation of such errors, the error at the 50° mark may be taken as the mean between the errors at 0° and 100°; at 25°, three times as much influence should be allowed for the error at 0° as for that at 100°.

II. THE PRODUCTION AND TRANSFERENCE OF HEAT, ETC.

223. Sources of Heat. — The sun is the great source of thermal energy, but man is able to transform other forms of energy into heat.

Experiment 159. — Rub a metal button on the floor or carpet. It soon becomes uncomfortably warm.

Experiment 160. — Place a nail or coin on an anvil or stone and hammer it vigorously. It soon becomes too hot to handle. In this way, blacksmiths sometimes heat iron rods to redness.

Experiment 161. — Hold a piece of iron or steel against a dry grindstone or an emery-wheel in rapid revolution. The shower of sparks noticeable is due to the fact that the small particles of metal torn off by the grindstone are heated to incandescence.

Experiment 162. — Half fill a stout four-ounce bottle with mercury at the temperature of the room. Cork the bottle, wrap it in several thicknesses of paper to protect it from the heat of the hand, and shake it vigorously. Remove the cork, insert the bulb of the thermometer, and see if there has been any change in the temperature of the mercury. Cork the bottle and shake it as before, but longer and more vigorously. Take the temperature of the mercury again.

Experiment 163. — Place a bit of tinder in the cavity at the end of the piston of the "fire-syringe" represented in Fig. 217. Put the piston into the open end of the cylinder and force it in, compressing the confined air as abruptly as possible. Promptly remove the piston. The tinder will probably be on fire.

Experiment 164. — Cut a thin slice from a stick of phosphorus under water. Carry the slice on the knife-blade, and press it between the folds of a handkerchief to dry it. Moving it again upon the knife-blade, place it upon a brick. Carefully place a single crystal of iodine upon the phosphorus. Some of the potential energy of chemical separation will be transformed into the kinetic energy of heat.

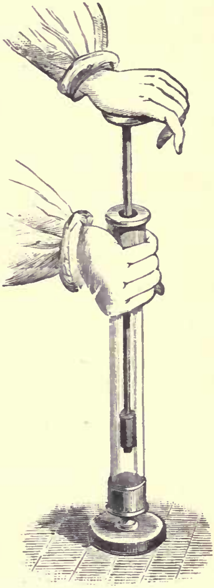


FIG. 217.

224. Production of Heat. — The experiments just given illustrate some of the methods by which other forms of energy are transformed into heat. The ignition of a common friction-match illustrates the transformation of mechanical energy and of chemical action into heat. Such transformations are continually taking place, and the attention has only to be called to the subject that they may be recognized.

(*a*) The fire in the lamp or stove or grate affords familiar illustration of the transformation of chemical action into heat. The heating of saws and other tools by use, and of carriage or car axles when not properly lubricated; the warming of hands by rubbing them together; the conversion of the energy of the electric current into heat by the electric lamp; the old process of striking fire with flint and steel, are a few of the many illustrations that may be drawn from common life.

Diffusion of Heat.

Experiment 165. — Thrust an iron poker into the fire. The end of the poker that is held in the hand soon grows warm. If it is not a familiar fact that the rod has been heated through its whole length, and that there is a gradual rise of the temperature of the successive parts from the end held in the hand to the end that is in the fire, test the rod for the acquisition of such information. Hold the hand over the stove and it is warmed by the ascending current of heated air. Hold the hand in front of the stove and, in some way that we shall understand better by and by, it receives heat from the fire.

225. Diffusion of Heat. — Heat is transferred from one point to another in two ways, *conduction* and *convection*.

(*a*) It is often said that heat is transferred in a third way, viz., by *radiation*. A heated body, like the sun, may communicate periodic disturbances to a medium called the ether, and another body, like the earth, may absorb these disturbances and be heated thereby. The energy thus transmitted is often called *radiant heat*, although it is not heat at all, simply radiant energy that may be transformed into heat by absorption by ordinary matter. This so-called radiant heat will be considered in the next chapter.

Conductivity of Solids.

Experiment 166. — Turn up the four sides of a sheet of writing paper and, without cutting the corners, make a basin about 2 cm. deep. Half fill the paper pan with water and place it on the top of a hot stove. You can boil the water without burning the paper.

Experiment 167. — Instead of the iron poker of Experiment 165, use a glass rod or a wooden stick. The end in the fire may be melted or burned without rendering the other end uncomfortably warm.

Experiment 168. — Put a silver and a German-silver spoon into the same vessel of hot water. The handle of the former will become hot much sooner than that of the latter.

Experiment 169. — Place a bar of iron and a similar one of copper end to end so as to be heated equally by the flame of a lamp. Fasten small balls (or nails) by wax to the under surfaces of the bars at equal distances apart. More balls will be melted from the copper than from the iron. Give plenty of time and consider the number of balls that are melted off, and not the promptness with which a ball or two may fall at an early stage of the experiment.

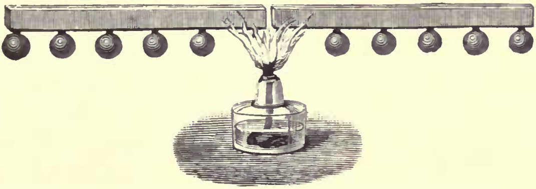


FIG. 218.

Experiment 170. — Modify Experiment 169, by providing two stout wires of iron and of copper, each 40 or 50 cm. long. Twist them together for about 10 cm., and spread the untwisted parts so as to form a fork with a twisted handle. Balance this fork on a coverless crayon box, and place a lamp flame under the overhanging handle. After several minutes, test the two metals with similar friction-matches and ascertain the respective distances at which the matches will be ignited by simple contact without rubbing.

226. Conduction *is the mode by which heat is transmitted from points of high temperature to points of low temperature by passing from one particle to the next particle*, or by which it is transmitted to a distance by raising the temperature of the intermediate particles without any sensible motion of them. The conduction of the heat is very gradual and as rapid through a crooked as through a straight bar.

(*a*) The power of conducting heat is called *thermal conductivity.* Some of the preceding experiments show that different substances have different conductivities, although part of the differences observed may have been due to the facts that equal quantities of some substances require unequal additions of heat for equal increments of temperature, and that some substances part with their heat by radiation more rapidly than others.

(*b*) Among solids, metals are the best conductors both of heat and of electricity. Owing to lack of continuity, powdered substances have low conductivities, as have wood, leather, flannel, and organic substances generally. The relative thermal conductivities of some metals are as follows: —

Silver	100	Iron	12
Copper	74	Lead	9
Gold	53	Platinum	8
Brass	24	German silver	6
Tin	15	Bismuth	2

Conductivity of Liquids.

Experiment 171. — Fasten a piece of ice at the bottom of an ignition tube. A loosely wound coil of soft wire that snugly fits the tube will hold it in place. Cover the ice to the depth of several inches with water. Hold the tube obliquely and apply the flame of a lamp below the upper part of the water. The water there may be made to boil without melting the ice below. Instead of using ice and water, pack the tube full of moist snow if you can get it.

Experiment 172. — Pass the tube of an air thermometer or of an inverted mercury thermometer through a cork in the neck of a funnel. Cover the thermometer bulb to the depth of about half an inch with water. Upon the water, pour a little sulphuric ether and ignite it. The thermometer below will scarcely be affected, although the water above may be boiling. Stir the water and note the prompt movement of the thermometer index. To prevent the downward conduction of the heat by the funnel, the ether may be placed in a small porcelain cup supported by a wire frame so that its bottom dips into the water directly over the bulb.

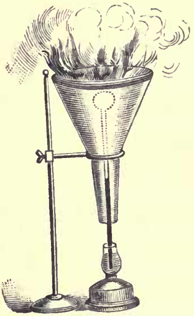


FIG. 219.

227. Conductivity of Fluids. — Experiments 171 and 172 indicate that water has a very low conductivity, a fact that applies to liquids generally. The one marked exception to this rule is the liquid metal, mercury. The conductivity of copper is about five hundred times that of water. Gases have still less, if any, thermal conductivity.

Fluid Currents caused by Heat.

Experiment 173. — Put a small quantity of oak filings or fine sawdust into a glass vessel of water. Heat the water by a lamp placed below, and notice the liquid currents as indicated by the motion of the oak particles.

Experiment 174. — With a lamp chimney or other large glass tube, a perforated cork, two pieces of glass tubing 4 and 15 inches long respectively, a bit of rubber tubing, a small lamp or a candle, and two coverless crayon boxes, arrange apparatus as shown in Fig. 220. Partly fill the apparatus with water, and add a small quantity of fine paper raspings or of a paste made by moistening some aniline dye with a drop of water. Carefully heat the tube as shown in the figure, and explain any observed movement of the water.

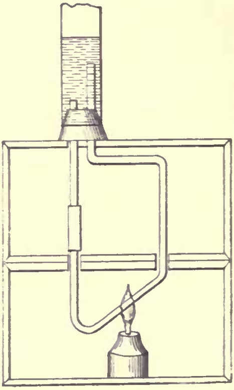


Fig. 220.

Experiment 175. — Cut a square of stiff writing paper, 15 cm. on each edge, and draw its diagonals. From the four corners, cut the paper along the diagonals to within 1.5 cm. of the middle of the square. From the corners, bring four alternate paper tips together and thrust a pin through them and the middle of the square and into the end of a penholder or lead pencil. Hold the paper wind-wheel thus made over a stove or metal plate that is very hot, the wooden handle being vertical and above the paper. One of the components into which the force of the ascending current of heated air is resolved sets the wheel in rotation.

228. Convection. — When a portion of a fluid is heated above the temperature of the surrounding portions, it expands and thus becomes specifically lighter. It then rises, while the cooler and heavier portions of the fluid rush in from the sides and descend from higher points. In this way, all of the fluid becomes heated. *This mode of transferring heat by the mechanical motion of heated fluids is called convection*, and the currents thus established are called convection currents.

(*a*) Convection currents are applied to the heating of houses, etc., by hot-water pipes or hot-air furnaces, and constitute the basis of the most common forms of house and mine ventilation, the draft of chimneys, etc. The Gulf Stream and the trade-winds are grand convection currents.

CLASSROOM EXERCISES.

1. Why are hot metal utensils commonly handled with cloth "holders"?

2. Why does oil-cloth on a cold floor feel colder to bare feet than carpet does, both being at the same temperature?

3. Is a good conductor of heat or a non-conductor preferable for keeping a body warm? For keeping it cool?

4. Explain the non-conducting character of hollow walls, double doors, and double windows.

5. Why is water heated more quickly when placed over a fire than when placed under one?

6. Fur and clothing of loose texture enclose much air in their structure. What bearing, if any, has this on their warmth?

7. Why are tin tea-kettles and boilers often given copper bottoms?

8. Draw a diagram illustrating the heating of a house by hot-water pipes. Explain your diagram.

9. Draw a diagram showing how a house is supplied with hot water from a boiler connected with the kitchen range or stove. Explain your diagram.

10. Draw a diagram showing how a house is heated by a hot-air furnace, representing the furnace, cold-air duct, and hot-air pipes. Explain your diagram.

III. EFFECTS OF HEAT.

229. Expansion is the first visible effect of heat upon bodies.

Expansion of Solids.

Experiment 176. — With a hack-saw, cut a piece from one side of a large link of an iron chain, and force the ends of the opened link slightly together so that the small piece may be pressed hard enough to hold it in place. Heat the opposite side of the link. The metal will expand and the piece will fall out of its place.

Experiment 177. — Grip one end of an iron rod about 1 cm. in diameter and 30 cm. long so that the rod shall be horizontal. Pass a pivot through the hole near the end of the meter stick used in Exercise 8, p. 126, and into a board. The iron rod should lightly touch the vertical edge of the stick a little below the level of the pivot. Heat the iron rod. The motion of the lower end of the meter stick indicates that the rod is increasing in length.

Experiment 178. — Get a straight, compound bar consisting of a strip of brass and a strip of iron, each 2 or 3 mm. thick, 2 cm. wide, and about 20 cm. long, bound face to face with wire, or riveted together at intervals of about 2.5 cm. Move the bar back and forth in a hot flame so that the two metals may be heated in their whole length and equally. As the bar becomes curved, notice which of the metals has expanded the more. Cool the bar to the temperature of the room and examine its form. Bury it for a few minutes in a freezing mixture of ice and salt, and again examine its form.

FIG. 221.

230. Expansion of Solids. — Almost without exception, solids expand when heated and contract when cooled, the amount of expansion varying with the increase of the temperature and the nature of the substance.

(*a*) The energy of the expansion and contraction of solids is very great and enables many industrial applications.

Expansion of Fluids.

Experiment 179. — Nearly fill a Florence flask with water and place it upon the ring of a retort stand. Support a long straw by a thread tied near one end, so that a light weight hung from the short arm may float upon the liquid surface in the neck of the flask and support the long arm of the straw in a horizontal position. Heat the water in the flask. Its expansion will be shown by the motion of the long arm of the straw.

Experiment 180. — Close by fusion one end of each of three similar glass tubes, 15 or 20 cm. long. Put water into one, alcohol into another, and glycerin into the third, using equal quantities of the liquids. Place the three tubes in a vessel of hot water and notice that the liquids expand unequally.

Experiment 181. — Partly fill a toy balloon with air and tightly tie the opening. Hold the balloon over a hot stove. The expanded air fills the balloon. (See § 168.)

Experiment 182. — Close a bottle with a cork through which passes a glass tube of small bore and about 30 cm. long. Warm the bottle between the hands, and place a drop of ink at the end of the tube. As the air in the bottle contracts, the ink will move down the tube, forming a liquid index. By heating or cooling the bottle, the index may be made to move up or down.

Experiment 183. — To a Florence flask, fit air-tight a delivery-tube that terminates under water as shown in Fig. 22. Heat the air in the flask, and, in a graduated test-tube, "collect over water" the air driven from the flask. Do not let the flask cool until the delivery-tube has been removed from the water.

Experiment 184. — Fill two Florence flasks of the same size and shape, one with air, and the other with coal-gas. Arrange them as described in Experiment 183, and immerse them to the same depth in hot water, collecting separately the gases driven out by expansion. Compare the volume of the gases thus collected.

231. Expansion of Fluids. — As illustrated by the experiments just given, liquids and gases expand when heated, and contract when cooled, the amount of expansion varying with the increase of temperature. In the case of liquids, the amount of expansion also varies with the nature of the substance. The rate of expansion is practically the same for all gases, and greater than it is for solids or liquids.

(*a*) Substances that crystallize on cooling, expand as they approach the temperature of solidification; i.e., a given quantity of matter occupies more space when it has a crystalline structure than it does when it has a liquid form. Ice is a good example of such a substance.

Experiment 185. — Pour recently boiled water into the apparatus of Experiment 182, until the tube is half full. Pack the bottle in a mixture of salt and finely broken ice. Observe the liquid level in the tube. The water contracts, then expands, then freezes and expands.

232. Coefficient of Expansion. — *The elongation per unit of length for each degree that the temperature is raised above* 0°, *is called the coefficient of linear expansion.* Similarly, *the increase in volume per unit of volume for a change of one degree of temperature, is called the coefficient of cubical expansion.* It is generally determined by dividing the increment of volume by the original volume as above indicated. It may be taken as three times the coefficient of linear expansion.

(*a*) For solids, the coefficient is nearly constant for different temperatures. For liquids, the coefficient is more variable. That of mercury, for example, is regular between – 36° and 100°, but for higher temperatures it gradually increases in value. Water exhibits the most remarkable variation from regularity. As water is heated from 0° to 4°, it gradually contracts, so that 4° is the temperature of maximum density for water. As the temperature is raised above 4°, the water expands, slightly at first, but more and more rapidly as it approaches the boiling point. For gases under constant pressure, the coefficient is nearly constant, with a value of $\frac{1}{273}$ or 0.00366. (See

§ 222.) For solids, the linear expansion is the more conveniently measured; for fluids, the cubical.

CLASSROOM EXERCISES.

1. Why do wheelwrights heat the tires of wheels before setting them?

2. The bulging walls of buildings are often straightened by passing iron rods from one wall to the opposite wall, placing nuts at the ends of the rods, heating the rods, and tightening up the nuts. Explain.

3. Why is at least one end of a long iron bridge generally supported upon rollers?

4. Why is a gallon of alcohol worth more in cold than in warm weather, the market price being the same?

5. Explain the draft of a lamp chimney.

6. What is the temperature of the surface water of a pond that is just about to freeze? Of the water at the bottom of the pond?

7. A certain quantity of gas is measured at 0°. To what temperature must it be heated, the pressure being constant, so that its volume may be doubled?

8. A mass of air at 0°, and under an atmospheric pressure of 30 inches, measures 100 cubic inches; what will be its volume at 40°, under a pressure of 28 inches?

Solution: — First suppose the pressure to change from 30 inches to 28 inches. The air will expand, the two volumes being in the ratio of 28 to 30 (§ 171). 100 cu. in. $\times \frac{30}{28} = 107\frac{1}{7}$ cu. in. Next, suppose the temperature to change from 0° to 40°. The expansion will be $\frac{40}{273}$ of the volume at 0°; the volume of the air at 40° will be $1\frac{40}{273}$ times its volume at 0°.

$107\frac{1}{7} \times 1\frac{40}{273} = 122\frac{536}{637}$. *Ans.* $122\frac{536}{637}$ cu. in.

Alternate Solution: —

$$\left.\begin{array}{r} 28 : 30 \\ 273 : 273 + 40 \end{array}\right\} :: 100 : x.$$

9. At 150°, what will be the volume of a gas that measures 10 cu. cm. at 15°?

$273 + 15 : 273 + 150 :: 10 : x$. *Ans.* 14.69 cu. cm.

10. If 100 cu. cm. of hydrogen is measured at 100°, what will be the volume of the gas at − 100°?

$273 + 100 : 273 - 100 :: 100 : x$. *Ans.* 46.37 cu. cm.

11. A liter of air is measured at 0° and 760 mm. What volume will it occupy at 740 mm. and 15.5°?

$$\left.\begin{array}{l}273 : 273 + 15.5\\ 740 : 760\end{array}\right\} :: 1{,}000 : x. \qquad \textit{Ans. } 1{,}085.34 \text{ cu. cm.}$$

12. A rubber balloon that has an easy capacity of a liter contains 900 cu. cm. of oxygen at 0°. What will be the volume of the oxygen when it is heated to 30°? *Ans.* 998.9 cu. cm.

13. A certain weight of air measures a liter at 0°. How much will the air expand on being heated to 100°? *Ans.* 366.3 cu. cm.

14. A gas has its temperature raised from 15° to 50°. At the latter temperature, it measures 15 liters. What was its original volume? *Ans.* 13,374.6 cu. cm.

15. A gas measures 98 cu. cm. at 185° F. What will it measure at 10° C. under the same pressure? *Ans.* 77.47 cu. cm.

16. A certain quantity of gas measures 155 cu. cm. at 10°, and under a barometric pressure of 530 mm. What will be the volume at 18.7°, and under a barometric pressure of 590 mm.?

17. A gallon of air (231 cubic inches) is heated, under constant pressure, from 0° to 60°. What is the volume of the air at the latter temperature? *Ans.* 281.77 cu. in.

18. The bulb and tube of an air thermometer were filled with boiling water. The bulb being placed in water that contained ice, the level of the water in the tube fell for a time and then rose. Explain. At what temperature did the contraction cease and the expansion begin?

Liquefaction.

Experiment 186. — Place snow or finely broken ice and a thermometer in a vessel of water. The thermometer will fall to the freezing-point, but no further. Apply heat, so as to melt the ice very slowly, and stir the mixture constantly. The temperature does not rise until all of the ice is melted, or it rises so little that we may feel sure that there would be no rise if each particle of water could be kept in contact with a particle of ice.

Experiment 187. — Put a little water into a beaker, and determine its temperature. Add a small quantity of sodium sulphate, and stir with a thermometer. Notice the fall of temperature during the process of solution. Repeat Experiment 26.

233. The Liquefaction of a solid is effected by fusion or by solution. In either case heat is required to overcome the force of cohesion, and disappears in the process. Sometimes the absorption of heat involved in the liquefaction is disguised by the evolution of heat due to chemical action between the substances used.

(*a*) The action of freezing-mixtures, e.g., one weight of salt and two or three of snow or pounded ice, depends upon the fact that heat is absorbed or disappears in the solution of solids.

Solidification.

Experiment 188. — Place a thermometer in a small glass vessel containing water at 30°, and a second thermometer in a large bath of mercury at – 10°. Immerse the glass vessel in the mercury. The temperature of the water gradually falls to 0°, when the water begins to freeze, and its temperature becomes constant. The temperature of the mercury rises while the water is freezing.

234. Solidification. — When a liquid changes to a solid, the energy that was employed in maintaining the characteristic freedom of molecular motion against the force of cohesion is released and appears as heat. The amount of heat that reappears during solidification is the same as that which disappears during liquefaction.

235. Laws of Fusion. — It has been found by experiment that the following statements are true: —

(1) *A solid begins to melt at a certain temperature that is invariable for a given substance under constant pressure. This temperature is called the melting-point of that substance. In cooling, such liquids solidify at the melting-point.*

(2) *The temperature of a melting solid or of a solidifying liquid remains at the melting-point until the change of condition is completed.*

(3) *Substances that contract on melting have their melting-points lowered by pressure, and vice versa.*

(*a*) It is possible to reduce the temperature of a liquid below the melting-point without solidification, but when solidification does begin, the temperature quickly rises to the melting-point.

Vaporization.

Experiment 189. — Pour a few drops of ether upon the bulb of a thermometer, or into the palm of the hand, and notice the rapid fall of temperature. See that there is no flame near enough to ignite the inflammable vapor.

Experiment 190. — Wet a block of wood and place a watch-crystal upon it. A film of water may be seen under the central part of the glass. Half fill the crystal with sulphuric ether, and evaporate it rapidly by blowing over its surface a stream of air from a small bellows. So much heat disappears that the watch-crystal is frozen to the wooden block.

Experiment 191. — In a vessel of sulphuric ether, place a test-tube containing water. Force a current of air through the ether. (Fig. 222.) Rapid evaporation is thus produced and, in a few minutes, the water is frozen. See Exercise 2, page 198.

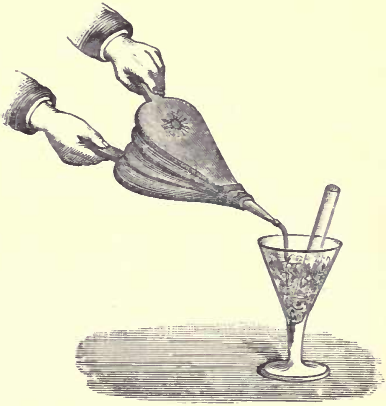


FIG. 222.

236. Vaporization *is the process of converting a substance, especially a liquid, into a vapor.* This change of condition may be effected by an addition of heat, or by a diminution of pressure, or both. When it takes place slowly and quietly, the process is called *evaporation*. When it takes place so rapidly that the liquid mass is visibly agitated by the formation of vapor bubbles within it, the process is called *ebullition*. The heat that produces the change of condition disappears in the process.

237. Condensation. — The liquefaction of gases and vapors is effected by a withdrawal of heat or by an increase of pressure, or both. In either case, the energy that was employed in maintaining the aëriform condition is released and appears as heat. The amount of heat that reappears during liquefaction is the same as that which disappears during vaporization.

238. Laws of Evaporation. — Experiments show that the rapidity of evaporation —

(1) *Increases with a rise of temperature.*

(2) *Increases with an increase of the free surface of the liquid.*

(3) *Increases as the atmospheric or other pressure upon the liquid decreases, it being very rapid in a vacuum.*

(4) *Increases with the rapidity of change of the atmosphere in contact with the liquid.*

(5) *Decreases with an increase of the vapor of the same substance in the atmosphere in contact with the liquid.*

(*a*) Water may be frozen by its own rapid evaporation under a low pressure. When liquefied carbon dioxide is relieved from pressure, it evaporates very rapidly; the correspondingly rapid absorption of heat reduces the temperature to about −90°, and freezes much of the gas to a snow-like solid. By evaporating liquefied hydrogen, a temperature of −243° has been obtained.

239. Dew-Point. — A space is said to be in a state of saturation with respect to a vapor when it contains as much of that vapor as it can hold at that temperature. The vapor then has the maximum elastic pressure for that temperature. The quantity of vapor required for saturation increases rapidly with the temperature. When a body of moist air is cooled, the point of saturation is gradually approached; when it has been reached, any further cooling causes a condensation of the vapor to dew, fog, or cloud, according to circumstances. *The temperature at which this condensation occurs is called the dew-point.* An instrument for determining the ratio between the actual amount of water vapor present in the air, and that required for saturation is called a *hygrometer.* The branch of physics that relates to the determination of the humidity of the atmosphere is called *hygrometry.*

(*a*) The ratio between the amount of watery vapor present in the air and the quantity that is required for saturation at the temperature of observation is called the *relative humidity.* This ratio is generally expressed in percentages, as 75 per cent, or 0.75.

Boiling-Point.

Experiment 192. — In a beaker half full of water, place a thermometer and a test-tube half filled with ether. Heat the water. When the thermometer shows a temperature of about 60°, the ether will begin to boil. The water will not boil until the temperature rises to 100°. The temperature will not rise beyond this point.

FIG. 223.

Experiment 193. — Place a thermometer in a metal dish half filled with water, and place a lamp beneath the dish. Be careful that the bulb of the thermometer is covered with water, and that it is not less than 4 or 5 cm. above the bottom of the vessel. Notice the rise of the thermometer. Soon the formation and condensation of minute steam-bubbles in the liquid will produce the peculiar sound known as singing or simmering, the well-known herald of ebullition.

Finally, the water becomes heated throughout, the bubbles increase in number, grow larger as they ascend, burst at the surface, and disappear in the atmosphere. Notice that the temperature remains stationary after ebullition begins.

Experiment 194. — When the water used in Experiment 193 has partly cooled, dissolve in it as much common salt as possible, heat it again, and notice that it does not boil until the temperature is noticeably higher than before.

240. Laws of Ebullition. — It has been found by experiment that the following statements are true : —

(1) *A liquid begins to boil at a certain temperature that is invariable for a given substance under constant conditions. This temperature is called the boiling-point of that substance. In cooling, such vapors liquefy at the boiling-point.*

(2) *The temperature of the boiling liquid or of the liquefying vapor remains at the boiling-point until the change of condition is completed.*

(3) *An increase of pressure raises the boiling-point, and vice versa.*

(4) *The boiling-point is affected by the character of the surface of the vessel containing the liquid, — an effect of cohesion.*

(5) *The solution of a salt in a liquid raises its boiling-point, additional energy being required to overcome the cohesion involved in the solution.*

(*a*) It is possible to heat water above its true boiling-point without ebullition, by confining the steam and thus increasing the pressure, but when the pressure is relieved, the superheated vapor immediately expands and its temperature is reduced. Hence, in determinations of the boiling-point, the thermometer is never immersed in the liquid but in the vapor just above it. Strictly speaking, *the boiling-point is the temperature at which the elastic force of the vapor is equal to the pressure of the atmosphere.*

(*b*) The temperature of the water in a steam-boiler is higher than 100° whenever the pressure (recorded by the gauge) is greater than one atmosphere. At ten atmospheres, the temperature is about 180°. Owing to the effect of atmospheric pressure upon the boiling-point of water, the latter may be used in the determination of altitudes above the sea-level. A thermometrical barometer for this purpose consists of a portable apparatus for boiling water, and a very sensitive thermometer, and is called a *hypsometer*.

(*c*) A drop of water on a smooth metal surface at a high temperature may rest upon a cushion of its own vapor, without coming into contact with the metal. A liquid in this *spheroidal state* is at a temperature below its boiling-point. When the metal cools so that the vapor pressure will not support the globule, the liquid comes into contact with the metal surface, and is converted into steam with great rapidity. Many boiler explosions are due to such causes.

(*d*) Whenever the boiling-point of a substance is lower than its melting-point, the substance vaporizes directly without previous liquefaction. Such a change is called *sublimation*. The pressure at which the melting-point and the boiling-point of any substance coincide is called the *fusing-point pressure*. If the fusing-point pressure of a solid substance is greater than the atmospheric pressure, it will sublime when heated unless the pressure upon it is increased. Carbon dioxide sublimes under any pressure less than three atmospheres. Conversely, if the fusing-point pressure is less than the atmospheric pressure, sublimation may be secured by reducing the pressure. Iodine sublimes at pressures less than 90 mm. of mercury, and ice can not be melted at a pressure of less than 4.6 mm. Such substances evaporate at temperatures below their melting-points.

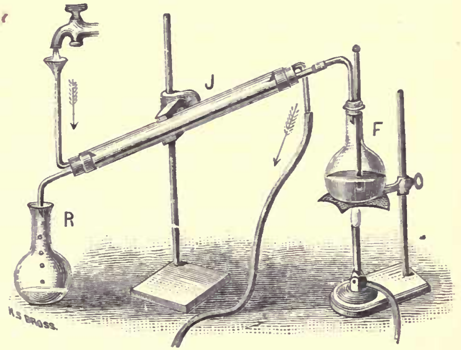


FIG. 224.

Distillation.

Experiment 195. — Partly fill with strong brine a Florence flask the cork of which carries a delivery-tube and a thermometer. Pass the delivery-tube through a "water jacket," *J*, kept cool substantially as shown in Fig. 224. Heat the liquid in the flask until it just boils, and taste the distilled water that collects in *R*.

Experiment 196. — Place a teaspoonful of alcohol in a saucer and apply a flame; the alcohol burns. Mix 50 cu. cm. of alcohol and 50 cu. cm. of water. Test a teaspoonful of the mixture as before; it does not burn. Place the rest of the mixture in the flask of the apparatus described in Experiment 195, and heat the mixture to about 90° C. See if the liquid that collects in *R* will burn. If it will not, empty the contents of *R* into *F*, and interpose between *F* and *J* a bottle that is partly immersed in a bath of boiling water and repeat the experiment.

241. Distillation is an application of volatilization and subsequent condensation for various purposes, such as the extraction of the essential principle of a substance from the liquid in which it has been macerated.

(*a*) The most common distillation process consists in placing the distillable liquid in a metal retort, generally made of copper. When heat is applied, vapors rise into the movable head of the retort, the neck of which is connected with a spiral tube called the "worm." The worm being kept cool by flowing water, the vapors of the more easily volatile constituents of the liquid pass into it, are condensed, and make their exit as a liquid, while the solid and non-volatile liquid constituents remain behind in the retort. The whole apparatus is called a "still."

(*b*) Fractional distillation is the process of separating liquids that have different boiling-points. The mixture is heated in a retort that allows constant observation of the temperature, and the distillates obtained between certain temperatures are collected separately. The most volatile constituent of the mixture will be found chiefly in the "fractions" first collected. By redistillation of the first fraction, this more volatile liquid may be obtained in comparative or absolute purity.

CLASSROOM EXERCISES.

1. At high elevations, water boils at temperatures too low for ordinary culinary purposes. How may persons living there heat water sufficiently for boiling meats and vegetables?

2. For the extraction of gelatine from bones by the action of hot water, a higher temperature than 100° is required. How may the water be heated sufficiently for such purposes?

3. In sugar refining, it is desirable to evaporate the saccharine liquid at a temperature considerably lower than 100°. Indicate a way in which this may be done.

4. Under ordinary pressure can ice be made warmer than 0°?

5. Solid type-metal floats on melted type-metal. Does melted type-metal expand or contract on solidifying? What effect has this quality upon the use of the metal in making type?

6. At the summit of Mount Washington, water boils at a temperature of about 94°; at the summit of Mont Blanc, at 86°; at the level of the Dead Sea, at 101°. Explain these differences in the boiling-points of water.

7. If the smooth, dry surfaces of two pieces of ice are pressed together for a few seconds, the pieces will be frozen together when the pressure is removed. Explain this result, which is called *regelation*.

8. When the air of a room is artificially heated, the temperature may become considerably higher than the dew-point of the air in the room. Under such circumstances, the rapid evaporation of moisture from the person causes a disagreeable sensation in the lips, tongue, skin, etc. How may such results be avoided?

9. Sulphur begins to melt at 115°. At what temperature does melted sulphur begin to solidify?

10. How may sea-water be made fit for drinking?

11. A drop of water may be placed on a very hot platinum plate, and the plate so held that a candle-flame may be seen between the water and the plate. Explain.

12. How may a thermometer, a fire, and a dish of water, be used to determine the elevation of a place above the sea-level?

13. Water standing in a slightly porous vessel acquires a temperature lower than that of the surrounding atmosphere. Explain.

14. The temperature of islands and of the borders of the ocean and great lakes is more equable than that of inland regions of the same latitude. Point out the dependence of this fact upon the physical properties of water.

15. The inner surface of the upper part of a bottle that contains iodine or gum-camphor is generally covered with minute crystals. What conclusion concerning the physical properties of iodine and camphor do you draw from this fact?

16. What effect has the humidity of the atmosphere upon the dew-point?

LABORATORY EXERCISES.

Additional Apparatus, etc. — Two air thermometers; kerosene; hydrometer jar with perforations and jacket; candle; expansion apparatus as described below; one of the thin, nickel-plated, brass vessels, larger at the top than at the bottom, such as are sold at hardware stores as lemonade "shakers."

1. Connect a small glass funnel by rubber tubing to the stem of an air thermometer. (Fig. 224.) The diameter of the bulb should be 4 or 5 cm. and the bore of the stem, 3 or 4 mm. Pour water into the funnel and work it down with a wire until the bulb is full and the liquid stands at the height of about 2 cm. in the stem. Similarly, fill a like bulb with kerosene. Immerse both bulbs in water almost boiling-hot. Notice the liquid levels in the stems at the instant of immersion and a few minutes later. Record all of your conclusions from the observed phenomena, not omitting to state what was measured by the rise of the liquids in the stems.

2. Drill a hole through the wall of a tall hydrometer jar near its top and another near the bottom. Close the holes with perforated corks carrying thermometers, so that the bulbs of the thermometers shall be inside the jar. Fill the jar with ice-cold water and notice that the thermometers give like readings. As the water is warmed to the temperature of the room, observe the thermometers, record your observations, and explain any change in the thermometric readings, and any difference between the readings of the two thermometers.

3. Around the middle of the hydrometer jar mentioned in Exercise 2, place a jacket and fill the jacket with a mixture of finely broken ice and salt. Record and explain changes in thermometric readings as before.

4. On a day when the doors and windows are closed, ascertain the temperature of the laboratory near the ceiling and near the floor. Record your observations and explain any difference that you find.

5. On a day when the air in the laboratory is warmer than that outside, stand an outer door slightly ajar, and with a candle flame,

seek for inward and outward air-currents. If you find them, explain their production and show that they have an important relation to artificial ventilation.

6. Half fill a Florence flask with water. Boil the water until the steam drives the air from the upper part of the flask. Cork the flask so that no air can enter, and quickly remove the lamp. Support the inverted flask upon the ring of a retort stand and place a pan below it. By this time, the water will have stopped boiling. Pour cold water upon the flask. Record and explain the consequent phenomena. Without again heating the water, repeat the drenching several times and finally immerse the flask in cool water.

7. Get a cylindrical tube, *cd* (Fig. 225), made of "galvanized" iron or other sheet metal. It should be about 2.5 cm. in diameter and about 60 cm. long. Near each end of the tube, insert a tube about 6 mm. in diameter and about 3 cm. long, as shown at *a* and *i*. At the middle of the main tube, insert a tube about 1.5 cm. in diameter and about 1 cm. long, as shown at *e*. Get a brass tube about 6 mm. in diameter and about 6 mm. longer than the tube, *cd*. Solder a fine steel wire in a diametral position across one end of the brass tube. Accurately measure the length of the brass tube and place that tube inside the larger tube so that it shall be supported by short perforated corks that close the ends of the latter. The end that carries the steel wire should be at the end of the jacket marked *c*. Upon a baseboard about 1 m. long and 15 cm. wide, erect five posts, *s*, *x*, *m*, *n* and *w*. As *m* and *n* are to carry the brass tube and its jacket, they have **V**-shaped notches at their upper ends; *m* is made a few millimeters longer than *n*, so that the water of condensation will run out at *i*. A common flat-headed screw is set in the vertical face of *w* so that, when the jacket is in position, the end of the brass tube will rest against the head of the screw. The distance between *w* and *x* is such that the latter may carry a right-angled lever, *l*, with its short arm resting in a vertical

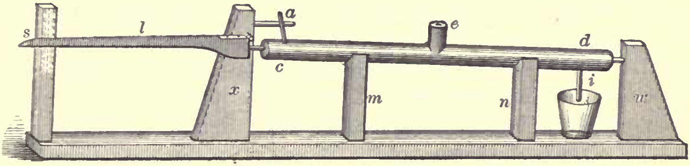


FIG. 225.

position against the wire soldered to the end of the brass tube. This lever may be made of a tapering piece of wood about 1.5 cm. square near the fulcrum end. A "machine" screw set into the face of x makes a good fulcrum. The short arm of the lever should be faced with a metal strip, the free surface of which lies in a vertical line through the center of the fulcrum when the long arm is horizontal. A metal casting of the shape indicated by Fig. 226, is desirable for carrying the index-arm of the lever. The lever should turn upon the fulcrum screw by its own weight but without looseness. The post, s, carries a millimeter scale over which the long arm of the lever moves. The large tube may be kept from turning upon its axis by a peg inserted in x, to which the tube at a may be tied.

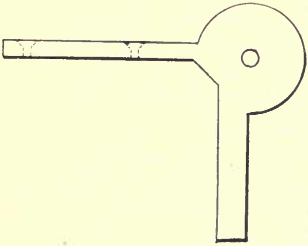


FIG. 226.

Place the large tube in position. Adjust the brass tube so that it projects about 3 mm. at each end and so that the steel wire is horizontal. When the short arm of the bent lever rests against the steel wire, adjust the screw in w until the long arm of the lever is horizontal. Pass the bulb of a thermometer through a perforated cork that closes the short tube at e; do not let it touch the brass tube. Place a tumbler below the exit tube at i. Connect the inlet tube at a, by a piece of rubber tubing 75 or 80 cm. long, to the boiler described in Exercise 3, page 275, and shield the adjusted apparatus from the heat of the lamp and boiler. Then measure the horizontal distance from the center of the fulcrum screw to the edge of the millimeter scale from which readings are to be taken, and the vertical distance from the center of the same screw to the steel wire at the end of the brass tube, and find the ratio between the two distances. This ratio should be not less than 20. Note the reading of the scale and the temperature inside the jacket. Generate steam in the boiler and let it flow through the jacket for a few minutes after the mercury has ceased to rise in the thermometer. When the movement of the long arm of the lever ceases, take the readings of the millimeter scale, the thermometer, and the barometer. Test the accuracy of the thermometric reading by the temperature as computed from the boiling-point of water at the observed atmospheric pressure. Detach the rubber tube at a and allow the apparatus to cool. Press the brass tube against the head of the screw in *w* and see if the index returns to its original position, as it should. From the data obtained, calculate the coefficients of linear and cubical expansion for brass.

Represent the actual elongation of the bar by *e;* the temperature observed at the beginning of the experiment by *t;* the highest temperature by *t′;* the length of the brass tube before heating by *l;* and the coefficient of linear expansion by *k*. Then we have, by definition:—

$$k = \frac{e}{(t' - t)\,l}; \text{ whence } e = k\,(t' - t)\,l.$$

This last algebraic expression shows why *k* is called a coefficient.

8. Determine the boiling-point of a saturated solution of saltpeter.

9. Put a little water at the temperature of the laboratory into a nickel-plated cup, the outer surface of which should be brightly polished. Breathe upon the polished surface, and notice that the moisture-film is evanescent. Place the bulb of a thermometer in the water and add ice. Stir the mixture continually. Note the temperature of the cooling water at the moment when the moisture-film clearly appears on the outer surface of the cup at a point that cannot be affected by your breath. If the ice is not all melted, remove the residue from the cup. As the water slowly warms, note the temperature at which the moisture-film begins to disappear. Take the mean of the two observed temperatures and call it the "dew-point." To your other records, add your observation of the weather and the out-door temperature.

IV. THE MEASUREMENT OF HEAT.

242. Calorimetry is the process of measuring the amount of heat that a body absorbs or gives out in passing through a change of temperature or of physical condition.

243. A Thermal Unit, *or a heat-unit, is the quantity of heat required to raise the temperature of unit mass of water one degree.* The unit most commonly used is *the* *quantity of heat required to raise the temperature of one gram of water from 0° to 1°. This water-gram-degree unit is called a therm, or a small calory.*

(*a*) A large calory is the quantity of heat required to raise the temperature of a kilogram of water from 0° to 1°. Unless otherwise specified, the calory mentioned in this book is the small calory.

244. Latent Heat. — In considering changes of condition of matter, we have spoken of the disappearance and reappearance of heat. When heat thus disappears, molecular kinetic energy is transformed into the potential form; when it reappears, the reverse transformation takes place. Because this molecular kinetic energy affects temperature, it is called *sensible heat.* Because this molecular potential energy does not affect temperature, it is called *latent heat.*

(*a*) So much of the added heat as is used to increase the rapidity of molecular motions is kinetic and appears as sensible heat. So much of it as is used to oppose cohesion (disgregation) and to overcome pressure becomes potential, and disappears as latent heat. When ether evaporates, the potential energy needed to establish the aëriform condition is obtained by the transformation of kinetic energy and at the expense thereof; hence, the disappearance of sensible heat, or the fall of temperature. When steam is condensed, the potential energy that is no longer required to maintain the aëriform condition is transformed into kinetic energy; hence, the increase of sensible heat. These terms, "sensible" and "latent," are reminiscences of the old theory that heat is a kind of matter.

Experiment 197. — Add a kilogram of finely broken ice (0°) to a kilogram of water at 80°. The ice will melt, and the temperature of the two kilograms of water will be about 0°. The 80,000 calories given out by the hot water were used in simply melting the ice.

245. The Latent Heat of Fusion *of a substance is the quantity of heat that is required to melt one gram of the substance without raising its temperature;* i.e., the quantity of heat that is expended in the molecular work involved in the change from the solid to the liquid condition. The latent heat of fusion of ice is about eighty calories.

Ice at 0° + *latent heat of fusion* = *water at* 0°.

(*a*) From the above statement it necessarily follows that the heat required to melt any weight of ice would warm 80 times that weight of water one degree, or the same weight of water 80 degrees, provided there was no change of physical condition.

Experiment 198. — To the end of the delivery-tube of a Florence flask containing water, attach a "trap" like that shown in Fig. 227, so that the water that condenses in the delivery-tube may be retained in the trap. (Instead of using the trap, the delivery-tube may be kept hot by a steam-jacket, for which purpose the apparatus shown in Fig. 224 or Fig. 225 may be easily adapted.) Boil the water, and when steam passes rapidly from *a*, the lower tube of the trap, dip *a* into a beaker of known weight and containing water of known weight and temperature. The temperature of the water in the beaker should be considerably lower than that of the room, and the end of the tube that leads steam from the trap to the beaker should not dip into the water so much that the condensation of the steam may not be plainly heard. The beaker should be covered with a piece of cardboard, perforated for the admission of the tube, *a*, and of the thermometer, and should be shielded from the heat of the lamp and flask. After the flow of steam has been continued for some time, remove the beaker, stir its contents with the thermometer thoroughly, and take the temperature quickly but carefully. Ascertain the exact increase in the weight of the water in the beaker, and compute the amount of heat derived from the condensation of each gram of steam. Suppose that at the beginning of the experiment the water in the beaker weighed 400 g., and had a temperature of 0°, and that at the end of the experiment the weight was 420 g., and the temperature 30°. The 400 g. of water received 12,000 calories that came from the 20 g. of steam. In cooling from 100° to 30°, the condensed steam parted with 1,400 calories. The remaining 10,600 calories came from the latent heat of the steam; i.e., each gram of steam at 100° gave out 530 calories in condensing to water at the same temperature. This result is subject to correction for radiation, absorption, etc.

FIG. 227.

246. The Latent Heat of Vaporization *of a substance is the quantity of heat that is required to vaporize one gram of that substance without raising its temperature.* The latent heat of the vaporization of water is about 537 calories.

Water at 100° + *latent heat of vaporization* = *steam at* 100°.

(*a*) From the above statement it necessarily follows that the heat required to vaporize any weight of water would warm 537 times that weight of water one degree, or n times that weight of water $\frac{537}{n}$ degrees, provided there was no change of physical condition.

Experiment 199. — Cut a piece of sheet lead about 5 × 30 cm., wind it into a loose roll, and suspend it by a thread in a vessel of boiling water. In a few minutes the lead will have the temperature of 100°. Transfer the lead to a thin metal vessel, containing a weighed quantity of water sufficient to cover the lead, and of known temperature. Stir the water with a thermometer, and note the temperature of the water when it reaches its maximum. Multiply the weight of the warmed water by its increase of temperature, to ascertain the number of calories transferred by the lead. Divide the number of calories by the fall of the temperature of the lead, to find the heat capacity of the lead roll. Divide this capacity by the weight of the lead, to find the specific heat of lead. Remember that, for work of precision, such results would have to be corrected for radiation, absorption by the vessel, etc.

247. The Specific Heat *of a substance is the ratio between the amount of heat required to raise the temperature of any weight of that substance one degree, and the amount of heat required to raise the temperature of the same weight of water one degree.* It indicates the number of calories absorbed or emitted by one gram of that substance while undergoing a change of one degree of temperature.

(*a*) The force of cohesion differs considerably for different substances. Consequently, when heat is added, the part thereof that is employed against cohesion in giving new positions to the molecules, and that is thus transformed from kinetic to potential energy (i.e., from sensible to latent heat), is different for different substances. The quantities of sensible heat remaining after such transformations being thus different, the several substances have different specific heats.

(*b*) The specific heat of hydrogen is 3.409; of ice, 0.505; of steam, 0.48; of oxygen, 0.2175; of iron, 0.1138; of lead, 0.0314. Water in its liquid form has a higher specific heat than any other substance except hydrogen.

Experiment 200. — Pour 400 cu. cm. of water at the temperature of 20° into 400 cu. cm. of water at the temperature of 60°, and contained in a thin liter flask. The temperature of the mixture will not vary much from 40°. Remember that allowance must be made for loss of heat by radiation and for absorption of heat by the vessel. The cool water gains and the warm water loses equal amounts of heat; i.e., 8,000 calories; the thermal capacity of water is practically the same at different temperatures.

248. The Thermal Capacity *of a body is the number of calories required to raise its temperature one degree.* It is the product of the mass into the specific heat, and has direct reference to the amount of heat the body absorbs or gives out in passing through a given range of temperature.

CLASSROOM EXERCISES.

1. One kilogram of water at 40°, 2 Kg. at 30°, 3 Kg. at 20°, and 4 Kg. at 10°, are thoroughly mixed. Find the temperature of the mixture. *Ans.* 20°.

2. One pound of mercury at 20° was mixed with one pound of water at 0°, and the temperature of the mixture was 0.634°. Calculate the specific heat of mercury.

3. What weight of water at 85° will just melt 15 pounds of ice at 0°? *Ans.* 14.117 pounds.

4. What weight of water at 95° will just melt 10 pounds of ice at – 10°? *Ans.* 8.947 pounds.

5. What weight of steam at 125° will melt 5 pounds of ice at – 8°, and warm the water to 25°?

6. How many grams of ice at 0° can be melted by 1 g. of steam at 100°?

7. Equal masses of ice at 0° and hot water are mixed. The ice is melted, and the temperature of the mixture is 0°. What was the temperature of the water?

8. Ice at 0° is mixed with ten times its weight of water at 20°. Find the temperature of the mixture. *Ans.* 11° nearly.

9. One pound of ice at 0° is placed in 5 pounds of water at 12°. What is the result?

10. What temperature will be obtained by condensing 10 g. of steam at 100° in 1 Kg. of water at 0°?

11. A gram of steam at 100° is condensed in 10 grams of water at 0°. Find the resulting temperature. *Ans.* 58° nearly.

12. If 200 g. of iron at 300° is plunged into 1 Kg. of water at 0°, what will be the resulting temperature? *Ans.* 6.67°.

13. A body with a weight of 80 g., and a temperature of 100°, is immersed in 200 g. of water at 10°, and raises the temperature of the water to 20°. What is the specific heat of the body?

14. How many pounds of steam at 100° will just melt 100 pounds of ice at 0°? *Ans.* 12.55 + pounds.

15. What will be the result of mixing 5 ounces of snow at 0° with 23 ounces of water at 20°?

16. What weight of steam at 100° would be required to raise the temperature of 500 pounds of water from 0° to 10°? *Ans.* 7.97 pounds.

17. What weight of mercury at 0° will be warmed one degree by placing in it 150 g. of lead at 300°?

18. If 4 pounds of steam at 100° is mixed with 200 pounds of water at 10°, what will be the resultant temperature?

19. A pound of sulphur can melt only one-fifth as much ice as a pound of water at the same temperature. What does this show concerning the specific heats of water and sulphur?

20. Explain the difference between thermal capacity and specific heat.

21. If there was no water on the earth, would the differences in temperature between day and night, and between summer and winter, be greater or less than they now are? Why?

22. From a good dictionary or any other available source of information, get an idea of the operation of an ice-machine or of a refrigerating machine, and then show that when work is done upon a gas there is an increase of sensible heat, and that when work is done by a gas there is a decrease of sensible heat.

23. Tubs of water are sometimes placed in cellars to "keep the frost away" from vegetables, the freezing-point of which is a little below 0°. Explain the effect of the water in this respect.

24. The cylinder of a pump that forces air into the pneumatic tire of a bicycle is heated in the process. Explain.

LABORATORY EXERCISES.

Additional Apparatus, etc. — 1.5 Kg. of mercury; 5 balls of different metals, each about an inch in diameter; a cake of beeswax; a copper dipper as described below; $1\frac{1}{4}$ pounds of No. 13 shot; ice or snow.

1. Pour quickly, and through the shortest possible air space, 1.5 Kg. of mercury at 100°, into 500 g. of water at 0°. Stir the liquids thoroughly together with a thermometer, and, from the resultant temperature, determine the specific heat of mercury.

2. Place small and similar balls made severally of iron, copper, tin, lead, and bismuth, in a bath of linseed oil, and heat them to a temperature of 180°, or 200°. When they have all had time to acquire the temperature of the bath, wipe them dry, place them upon a cake of beeswax about half an inch thick, and, from what you see, arrange the five metals in the order of their several specific heats.

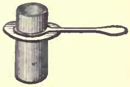


FIG. 228.

3. Provide a sheet-copper dipper, 4 cm. in diameter, and 10 cm. deep, and encircled about 2 cm. from the top by a flat flange of the same material, and about 4 cm. wide. A handle should be fastened to this flange. Accurately determine the weight of the "shaker" used in Exercise 9, page 299, which we shall hereafter call a calorimeter. Into the dipper, put about 500 g. of very fine shot that has been accurately weighed. Fill the cylindrical part of the boiler described in Exercise 3, page 275, to the depth of about 6 cm. with water, and cork the side tube, *a*. Place the dipper in the boiler, the flange of the dipper resting upon the top of the boiler. Cover the dipper with a piece of cardboard that has a hole, through which push the bulb of a thermometer down into the shot. Boil the

water, stir the shot frequently and thoroughly with any convenient instrument, and observe the rise of temperature of the shot.

Put about 100 cu. cm. of water into the calorimeter. Cool the water with ice or snow until its temperature is 7 or 8 degrees below that of the laboratory. When the mercury column of the thermometer becomes stationary, note the temperature of the shot. Remove the thermometer, and allow it to cool in air to about 40°, and then ascertain the temperature of the water in the calorimeter, stirring the water with a thermometer until the thermometer and all parts of the water have a uniform temperature. In the meantime, stir the shot at frequent intervals. Bring the mouth of the dipper to the mouth of the calorimeter, and quickly pour the shot into the water, being careful not to spill the shot or to splash the water. Stir the shot and water quickly and thoroughly, and take the temperature of the contents of the calorimeter. Weigh the calorimeter and its contents, and, deducting the weights of the vessel and the shot, ascertain the weight of the water heated by the shot. From the data now secured, calculate the specific heat of lead. Remember that for work of precision, corrections would have to be made for the loss of heat by radiation, and by absorption by the calorimeter, etc.

4. Repeat Exercise 3, using brass-filings instead of shot.

V. THE RELATION BETWEEN HEAT AND WORK.

249. Correlation of Heat and Mechanical Energy. — When heat is produced, some other kind of energy disappears, and *vice versa*. The most important of these transformations are those between heat and mechanical energy. We are able to effect a total conversion of mechanical energy into heat, but we are not able to bring about a total conversion of heat into mechanical energy.

Experiment 201. — Pass a bent glass tube through the air-tight cork of a flask half full of water, and let it dip beneath the surface of the water. Heat the flask. The heat will raise some of the water to the end of the tube, where it may be caught as shown in Fig. 229.

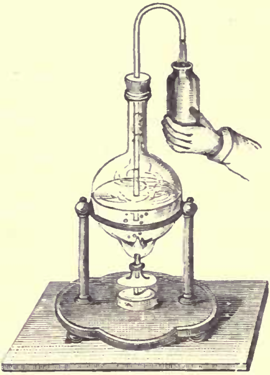


FIG. 229.

Experiment 202. — To the spindle of a whirling-table, attach a brass tube about 10 cm. long and closed at the lower end. Partly fill the tube with alcohol and cork the open end. Press the tube between two pieces of board hinged together as shown in Fig. 230. The grooves on the inner faces of the boards should be faced with leather. Rotate the apparatus, pressing with the clamp upon the tube. Friction transforms the mechanical energy into heat, and the vapor from the alcohol thus boiled may drive out the cork with explosive violence.

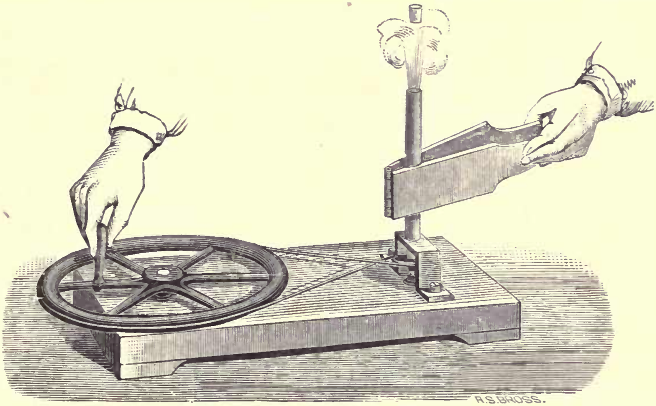


FIG. 230.

250. Joule's Principle. — *The disappearance of a definite amount of mechanical energy is accompanied by the production of an equivalent amount of heat.*

251. — The Mechanical Equivalent of Heat *signifies the numerical relation between work-units and equivalent heat-*

units. The quantity of heat that will raise the temperature of one pound of water one Fahrenheit degree is equivalent to about 778 foot-pounds. For centigrade degrees the equivalent is 1.8 times as great, or about 1,400 foot-pounds. The mechanical equivalent of a calory is about 427 gram-meters, or 4.2×10^7 ergs.

252. The Heat Equivalent of Chemical Union has a determinative relation to the comparative fuel-values of substances.

(*a*) The numerical values given below indicate that the combustion of a given weight of the substance in oxygen yields heat enough to warm so many times its own weight of water one centigrade degree, or 1.8 times that many Fahrenheit degrees. For example, the combustion of a gram of pure carbon develops 8,080 calories: —

Hydrogen	34,462	Carbon	8,080
Petroleum	12,300	Alcohol	6,850

253. The Steam Engine is a powerful device for utilizing the energy involved in the elasticity and expansive force of steam as a motive power. It is a real heat-engine, transforming heat into mechanical energy. In its modern forms, it has many complicated accessories for increasing its efficiency and adapting it to the special uses to which it is put, but the fundamentally important parts are the cylinder, piston, and slide-valve, diagrammatically

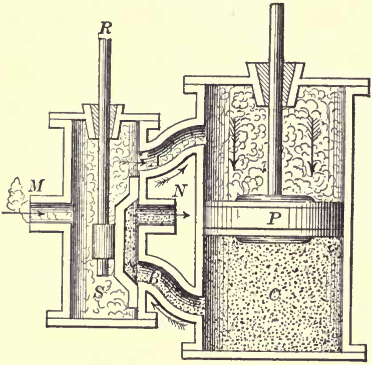


FIG. 231.

represented in Figs. 231 and 232, in which the steam-chest is represented as being at a distance from the cylinder, simply for the purpose of making clear the communicating steam passages. The piston, *P*, is moved to and fro in the cylinder by the pressure of the steam which is applied to its two faces alternately. This alternate application of the steam pressure is effected by the slide-valve, inclosed in a steam-chest, and moved by the valve-rod, *R*. The slide-valve covers the exhaust-port, *N*, and one of the other two ports, *A* and *B*.

(*a*) Steam from the boiler enters the steam-chest at *M*. When the valve is in position, as shown in Fig. 231, "live" steam passes through the induction-port, *A*, into the cylinder, and pushes the piston, as indicated by the arrows, forcing out the "dead" or exhaust steam by the eduction-port, *B*, and the exhaust-port, *N*. As the piston nears the end of its journey in this direction, the valve-rod, *R*, is moved by an "eccentric," or other device, and shifts the valve into position, as shown in Fig. 232. This movement of the slide-valve changes *B* to an induction-port, by which "live" steam is admitted to the other face of the piston, pressing it in the direction indicated by the arrow, and forcing the "dead" steam out through *A* and *N*. Then the slide-valve is pushed back to its former position by the rod, *R*, and the alternating movement of the piston thus continued. The piston-rod and the valve-rod work through steam-tight packing boxes.

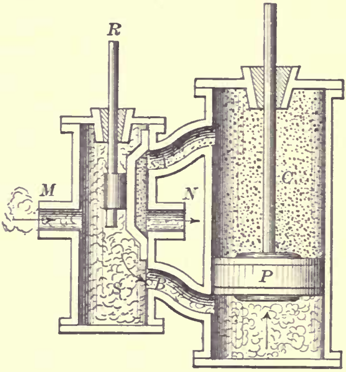


FIG. 232.

(*b*) The outer end of the piston-rod carries a transverse bar or cross-head that slides between two guide-bars, so that the motion of the piston-rod is always along the axis of the cylinder. The same end of the piston-rod is pivoted to a pitman or connecting-rod, the other end of which is attached to a crank on the shaft. The pitman receives the reciprocating motion of the piston-rod, and imparts a rotary motion to the crank-shaft. This shaft carries a heavy fly-wheel, the accumulated energy of which carries the shaft across the two "dead-points" when the piston is at one end or the other of the cylinder. The fly-wheel otherwise tends to steadiness of motion, and often serves as a belt-pulley. In large engines, the length of the cylinder is generally horizontal.

(*c*) When the exhaust steam escapes through N into the air, the engine is said to be a "high-pressure" or a "non-condensing" engine; when it is led to a chamber and there condensed by a spray of cold water for the purpose of removing the back pressure of the atmosphere, the engine is said to be a "low-pressure" or a "condensing" engine. As the water is pumped from the condenser, a partial vacuum is maintained. Sometimes the engine expands its steam in two, three, or four successive stages, and in two, three, or four distinct cylinders, the first taking steam directly from the boiler, and the others taking it from the exhaust-port of the cylinder working at the next higher pressure. Such engines are called respectively, "double-expansion," "triple-expansion," and "quadruple-expansion" engines. The live steam may be cut off from the cylinder during the latter part of the travel of the piston, leaving the steam in the cylinder to expand with decreasing pressure to the end of the stroke. The point of "cut-off" may be fixed, as at three-fourths of the stroke, or it may be variable with the nature of the work. In the latter case, the cut-off device may be adjusted automatically.

(*d*) More heat is carried to the cylinder of a steam-engine than is carried from it. The piston does work at every stroke, and every stroke annihilates heat. With a given supply of steam, the engine will give out less heat when it is made to labor than when it runs light.

(*e*) With all of its merits and all of its improvements, the modern steam-engine utilizes less than 15 per cent of the heat energy developed by the combustion of the fuel.

(*f*) Good steam-engines are now easily accessible from nearly every school, and should be studied in detail, and by direct inspection. The action of the pitman, the crank, the crank-shaft, the fly-wheel, and the dead-points may be illustrated by almost any sewing-machine.

CLASSROOM EXERCISES.

1. Show that the high latent heat of water has an important relation to the fact that when the temperature of the atmosphere rises above 0°, all the ice and snow of winter do not melt in a single day.

2. If a cannon ball weighing 192.96 pounds, and moving with a velocity of 2,000 feet per second, could be suddenly stopped and all its kinetic energy converted into heat, to what temperature would that heat warm 100 pounds of ice-cold water?

Solution: — $K.E. = \frac{wv^2}{2g} = \frac{192.96 \times 2000^2}{64.32} = 12{,}000{,}000$, the number of foot-pounds. Division of the number of foot-pounds by 778 gives the number of heat-units (pound-Fahrenheit) developed. This number divided by 100 gives the number of heat-units for each pound of the water, and consequently the number of Fahrenheit degrees that it will raise the temperature. This, added to 32°, the initial temperature, will give the temperature called for.

3. A steam-engine raises 8,540 Kg. to a height of 50 m. How many calories are thus expended?

4. One gram of hydrogen is burned in oxygen. To what temperature would a kilogram of water at 0° be raised by the combustion?

5. From what height must a block of ice at 0° fall that the heat generated by its collision with the earth would just melt it if all of the heat was utilized for that purpose?

6. Show that to raise the temperature of a pound of iron from 0° to 100° requires more energy than to raise 7 tons of iron a foot high.

7. To what height could a ton weight be raised by utilizing all the heat produced by burning 5 pounds of pure carbon?

Ans. 28,280 feet.

8. Find the height to which it could be raised if the coal had the following percentage composition: —

carbon, 88.42; hydrogen, 5.61; oxygen, 5.97.

9. With what velocity must a leaden bullet strike a target that its temperature may be raised 100° by the collision, supposing all its energy of motion to be spent in heating the bullet? (Specific heat of lead, 0.0314; $g = 980$ cm.)

10. The specific heat of tin is .056 and its latent heat of fusion is 25.6 Fahrenheit degrees. Find the mechanical equivalent of the amount of heat needed to heat 6 pounds of tin from 374° F. to its melting point, 442° F., and to melt it.

CHAPTER V.

RADIANT ENERGY: ETHER PHYSICS.

I. NATURE OF RADIATION.

254. The Ether. — Physicists are generally of the opinion that all space is filled with an incompressible medium of extreme tenuity and elasticity. *This hypothetical medium is called the ether.* The variety of the phenomena for which the ether hypothesis offers the only explanation that modern science can accept (see § 10) is so great that the unproved existence of the ether is confidently accepted.

(*a*) It has been estimated that the density of the ether is 9.36×10^{-19}, which is enormously great as compared with that which air would assume in interstellar space. It has been estimated that its rigidity is about 0.000,000,001 that of steel, so that masses of ordinary matter readily pass through it. Its structure is assumed to be continuous instead of granular like that of ordinary matter (see § 3). It is regarded as an incompressible substance pervading all space and penetrating between the molecules of all ordinary matter which are embedded in it and connected with one another by its means. It has been compared to an impalpable and all-pervading jelly through which the particles of ordinary matter move freely; through which heat and light waves are constantly throbbing; which is constantly being set in local strains and released from them, and being whirled in local vortices, thus producing the various phenomena of electricity and magnetism.

255. Radiant Energy. — Since the ether fills all intermolecular spaces, it follows that the vibrating molecules of a body must communicate their motion to it. The periodic disturbances thus communicated to the ether are propagated through it in the form of waves that are assumed to be transverse, and with a velocity of about 186,000 miles per second. Conversely, when these ether-disturbances reach a body, they may communicate their energy to the molecules of that body, and thus increase the total energy of that body. *The transference of energy by means of periodic disturbances in the ether (without regard to the precise nature of those disturbances) is called radiation. The energy thus transferred is called radiant energy.*

(*a*) The mechanism of radiation involves two correlative processes, emission and absorption, the former term referring to the communication of disturbances to the ether, and the latter to the reception of disturbances from the ether. Any increase in the vibratory molecular energy of a body increases its total radiation. Any increase in the rapidity of those molecular vibrations correspondingly increases the number of the ether disturbances in a unit of time, i.e., increases the wave-frequency. There is, therefore, an evident analogy between the phenomena of radiation and those of sound.

(*b*) The energy of radiation is measured by totally absorbing it and determining the heating effect produced by it. Lampblack is the most efficient substance known for such absorption.

256. **A Ray** *is a line along which radiant energy is propagated ;* i.e., the straight line perpendicular to the wave-front. A collection of parallel rays is called *a beam.* A collection of converging or diverging rays is called *a pencil.*

(*a*) The expressions, rays, beams, and pencils, are traces of an exploded theory. So far as they pertain to the wave theory, they are merely convenient geometrical conceptions, having no material existence.

257. Incident Radiation may be transmitted, reflected or absorbed by the body upon which it falls. When a body absorbs radiant energy, it is heated thereby.

Experiment 203. — Take a white-hot poker into a dark room. You are conscious of the sensations of heat and light, and readily attribute both sensations to the energy radiated by the poker; i.e., you say that the poker emits heat and white light. The light gradually becomes reddish, and finally fades from view. There is a continuous change from the emission of white light and much heat to that of no light and less heat.

258. Radiant Energy is Recognized by its phenomena, which may be classified as luminous, thermal, and chemical.

(*a*) Not even in theory can we assign limits to the length of the ether undulations. Some of these waves are competent to excite the optic nerve and to produce vision; some are not. This ability and inability are matters of wave-length. The variety of the effects must not be permitted to obscure the identity of the cause. All of these ether vibrations are of the same nature. Most of the properties and phenomena of radiant energy are most conveniently studied by luminous effects, which constitute the chief subject-matter of this chapter.

II. LIGHT: VELOCITY AND INTENSITY.

259. Light. — *The portion of radiant energy that is capable of producing the effect of vision constitutes light.*

(*a*) The longest recognized ether wave is $3{,}000 \times 10^{-6}$ cm.; the shortest is 18.5×10^{-6} cm. These wave-lengths correspond respectively to vibration frequencies of 10×10^{12} and $1{,}622 \times 10^{12}$. The radiant energy that constitutes light lies within the comparatively narrow limits of 7.6×10^{-5} cm., and 3.9×10^{-5} cm., wave-lengths that respectively correspond to vibration-frequencies of 392×10^{12} and 757×10^{12}.

While the total range already observed is more than seven octaves, the range within the limits that correspond to light is little more than one octave.

260. Visible Bodies are visible because of the light that they send to the eye of the observer. This is true whether the body shines by its own or another's light, i.e., whether it is self-luminous like a "live" coal, or illuminated like a "dead" coal.

(*a*) Light makes luminous bodies visible but is itself invisible. The illumination of the path of a sunbeam entering a darkened room is due to the reflection of light by the dust motes floating in the air. When a sunbeam is sent through moteless air, its path is imperceptible.

NOTE. — For many experiments in light, a darkened room is desirable. The windows should be provided with opaque curtains so arranged that the sunlight may be quickly and completely excluded from the classroom and laboratory.

Rectilinear Propagation.

Experiment 204. — Provide six blocks $1\frac{1}{2} \times 2\frac{1}{2} \times 3\frac{1}{2}$ inches, and three other pieces of wood each $\frac{1}{8} \times 3\frac{1}{2} \times 4$ inches. Place three postal cards one over the other on a board and perforate them with a stout needle about half an inch below the middle of one end. Pare off the rough edges of the holes with a sharp knife, and again pass the needle through each hole to make its edge smooth and even. From these materials, make three screens like that shown at *A* or *B* in Fig. 242. Place the three screens parallel to each other and with their blocks separated by two of the other blocks. Pass a thread through the holes in the screens and carefully put it under tension to be sure that the perforations are in a straight line. If necessary, adjust the screens for that purpose. Remove the thread without disturbing the adjustment. On the remaining block, place a lighted candle of such length that its flame is at the height of the perforations in the cards. Place eye and candle so that the flame may be seen through the screen perforations. Move one of the screens a little so that the three holes are not in a straight line; the candle flame cannot be seen as it was before.

261. Radiant Energy is Propagated *along straight lines when the medium is homogeneous, i.e., when it has a uniform composition and density.*

(*a*) The familiar experiment of "taking sight" depends upon this fact, for we see objects by the light that they send to the eye. A small beam of light that enters a darkened room illuminates a straight path. The use of fire-screens and sunshades illustrates the same fact.

262. Transparency, etc. — *According to the freedom with which they transmit light, bodies are classified as transparent, translucent, and opaque.* Transparent bodies, as glass, transmit light so freely that objects may be seen through them distinctly. Translucent bodies, as oiled paper, transmit light so imperfectly that objects seen through them appear indistinct. Opaque bodies cut off the light entirely, and prevent objects from being seen through them at all. No sharp line of separation can be drawn between these classes.

(*a*) When light falls upon a mass of small particles or thin films of substances that are usually transparent, as finely-pounded glass or ice, or froth, or foam, or cloud, most of the light is reflected. What passes through one particle or film is reflected by another. Such masses are brilliantly white in sunlight. As the light is reflected and not transmitted, such masses are opaque.

Shadows.

Experiment 205. — Hold a lead pencil between the flame of an ordinary lamp and a sheet of paper about two feet distant from the lamp, first with the edge of the flame toward the pencil, and then with the side of the flame toward the pencil. Notice the difference in the appearance of the screen.

Experiment 206. — Coat with asphaltum varnish the lower half of the outer surface of the chimney of a lamp that has a large flat flame.

At the height of the flame, scrape the varnish from a spot 3 or 4 mm. in diameter. Place the chimney on the lighted lamp with the clear spot opposite the middle of a screen of light-colored paper and about 2 m. from it. Instead of being varnished, the chimney may be smoked, or a sheet of cardboard may be rolled into a hollow cylinder large enough to surround the lamp; a hole may be cut in the cardboard at the proper height. Hang a croquet ball midway between the lamp and the screen. If the room is not darkened, place the ball and the screen between the lamp and the window. Prick a pin-hole through the darkened section of the screen and look through it toward the lamp. From the further side of the screen, prick a series of such holes about an inch apart and in a straight line, looking through each hole before another is pricked. When you have pricked a hole through which you can see the luminous spot on the lamp-chimney, examine the other side of the screen and notice that the pin-hole is outside the darkened section.

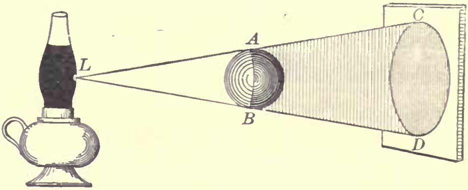


FIG. 233.

Experiment 207.—Replace the chimney used in Experiment 206 by one that is clear, and see that the side of the flame is turned toward the ball. Examine the darkened section on the screen and notice that its central disk is equally dark in all its parts, and surrounded by a ring of varying darkness. Beginning at the middle of the disk, prick pin-holes as before, examining each in succession, and avoiding those pricked in Experiment 206. Notice that

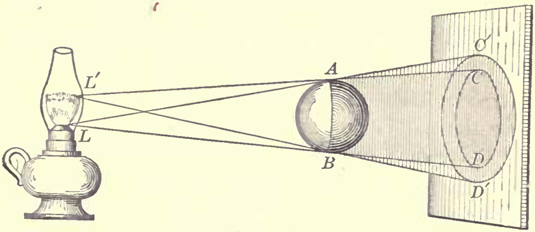


FIG. 234.

you cannot see the flame through any hole in the central disk, that you can see part of the flame through any hole in the annular space, and that you can see the whole of the flame through any hole outside the annular space.

263. A Shadow *is the darkened space from which an opaque body cuts off light.* If the source of light has considerable magnitude there will be a region of complete shadow, called the *umbra*, surrounded by a partial shadow, called the *penumbra.* No light enters the umbra; the penumbra receives light from a part of the luminous surface.

264. An Image *is an optical counterpart of an object* and may be formed by passing light through a small aperture, by reflecting it with a mirror, or by refracting it with a lens. When the light actually comes from the image to the eye, the image is *real.* Such an image may be received on a screen. When the light seems to come from the image to the eye but does not, the image is *virtual.* All virtual images are optical illusions.

Inverted Images.

Experiment 208. — Place the opened end of an empty tin fruit-can upon a hot stove and leave it there just long enough to melt off the mutilated cover. Make a good sized nail-hole at the center of the other end. Cover the nail-hole with tin-foil, and the other end of the can with thin tracing cloth or paper. Prick a pin-hole in the tin-foil, and turn it toward a candle flame. Upon the paper may be seen an inverted image the size of which will depend upon the distance of the flame from the pin-hole. The image will be seen more plainly if the room is darkened, or a dark cloth used (after the manner of a photographer) to shut the outside light from the eyes and the screen.

Experiment 209. — Bore a hole about 3 cm. in diameter in the side of a wooden box, paste or tack tin-foil over the hole, and prick the tin-foil with a pin. Invert the box over a lighted candle of such height that its flame will be at the level of the pin-hole. The box should be so large that the candle cannot set it on fire. Darken the room, and hold a paper screen before the hole in the tin-foil. Move the screen backward and forward and notice that, in any position, the length of the flame is to the length of the image of the flame as the distance from the pin-hole to the flame is to the distance from the pin-hole to the image. Replace the tin-foil by another piece from which has been cut a triangle 1 or 2 mm. on a side. Notice that the change in the shape of the aperture does not make much difference with the form of the image. Replace this tin-foil by another piece from which has been cut a triangle about 2 cm. on a side. Notice that the image is brighter and that its outline is less distinct.

265. Images by Apertures. — If light from a highly luminous body is admitted to a darkened room through a small hole in the shutter and there received upon a white screen, it will form an inverted image of the object.

Fig. 235.

As the rays are straight lines, they cross at the aperture; hence, the inversion of the image. The darkened room constitutes a camera obscura of simple form. The image of the school playground at recess is very interesting, and is easily produced.

(*a*) When the aperture is circular, each point of the luminous surface sends a cone of rays to the screen, and illuminates a circular spot upon it. When the aperture is triangular, or square, each point sends a pyramid of rays, and illuminates a triangular or a square spot. In any case, there will be an image of the aperture thrown upon the screen for every point in the luminous surface. As these superposed images of the aperture are symmetrically placed with reference to the corresponding points of the luminous surface, and as they overlap each other, they build up an image of the luminous body regardless of the shape of the aperture. The idea may be more easily comprehended by imagining the building up of a star-shaped figure by using small, round wafers. The smaller the wafers, the sharper the outline of the star. Remembering that the size and shape of our analogic wafer depend upon the size and shape of the aperture, it ought not to be difficult to understand the phenomena presented by images like those now under consideration. The aperture or the wafer may be so large as to destroy all resemblance between the image and the luminous object, and to substitute therefor a resemblance to the aperture or wafer itself.

266. The Velocity of Light *is about* 186,000 *miles* (3×10^{10} cm.) *per second.* For terrestrial distances, the passage of light is, therefore, practically instantaneous.

(*a*) At equal intervals of 42 h. 28 min. 36 s., the nearest of Jupiter's satellites passes within his shadow and is thus eclipsed. This phenomenon would be seen from the earth at equal intervals if light traveled instantaneously from planet to planet. In 1675, a Danish astronomer noticed that the interval between successive eclipses as observed was longer during the half year when the earth was passing from conjunction to opposition, as from E to E', than it was when the earth was passing over the other half of its orbit, as from E' to E. If, when the earth was at E, the time of successive eclipses was computed a year in advance, the eclipse observed six months later when the earth was at E', seemed to be 16 min. 36 sec. behind time, the time apparently lost being regained in the next six months. This led irre-

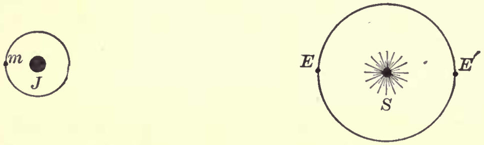


FIG. 236.

sistibly to the conclusion that it requires 16 min. 36 sec. for light to pass over the diameter of the earth's orbit from *E* to *E′*. This distance being approximately known, the velocity of light is easily computed. The velocity of light has been measured by other means, giving results that agree substantially with that above recorded.

Intensity of Illumination.

Experiment 210. — Make three cardboard screens, *A*, *B*, and *C*, respectively 5 cm., 12 cm. and 17 cm. on a side. Draw a line parallel to each edge of *B* and *C*, and at a distance of 1 cm. therefrom, thus inscribing squares 10 cm. and 15 cm. on a side. Divide the smaller inscribed square into four squares, each the size of *A*, and the larger inscribed square into nine such squares. Mount the three screens so that they stand upright with their middle points at the height of the cleared spot on the lamp-chimney used in Experiment 206. Instead of the asphaltum coat on the lamp-chimney, a perforated cardboard screen may be used as shown in Fig. 237. The screens may be conveniently supported by soft-wood rods, each having a fine slit sawed in one end and a sewing-needle thrust half-way into the other end. Place *A* about 30 cm. from the perforation. Set *C*, parallel to *A* and at such a distance that the shadow of *A* just covers its nine squares. Then place *B* so that the shadow of *A* just covers its four squares. Determine the relative distances of *A*, *B*, and *C* from the source of light. Remove *A* and notice that the light that previously fell upon it now falls upon *B*. Remove the second screen and notice that the light that previously fell upon *A* and *B* now falls upon *C*.

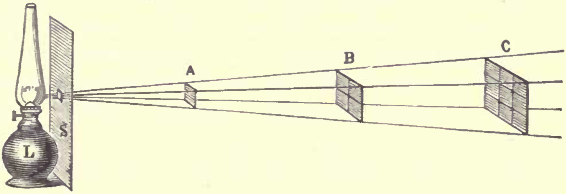


FIG. 237.

267. The Intensity of Radiation that falls upon a surface —

(1) *Varies inversely as the square of the distance between this surface and the source of radiation.*

(2) *Varies with the angle that the incident radiation makes with this surface*, being at a maximum when the surface is perpendicular to the direction of propagation.

(*a*) In Experiment 210, the light that fell upon *A* was diffused over four times the area at *B*, at twice the distance; and nine times the area at *C*, at three times the distance. With the same quantity of light diffused over nine times the area, the intensity of the illumination, i.e., the quantity of light per unit of surface, is only $\frac{1}{9}$ as great.

Photometry.

Experiment 211. — Arrange apparatus in a darkened room as shown in Fig. 238, where *S* represents a screen of white paper or cardboard, and *R*, a small rod placed upright a few inches from *S* (a cheap pen and pen-holder, or a lead pencil held by a bit of wax on the table will answer). The two flames should be on opposite sides of a plane perpendicular to the screen and passing through *R*, and at equal angular distances from it; they should be at the same level, and the flat lamp-wick should stand diagonally to the screen. Place *C* about 20 inches from *S*, and move *L* until the two shadows upon *S* nearly touch and are of equal darkness. The candle and the lamp are now throwing equal amounts of light upon the screen. If the distance from *S* to *L* is twice that from *S* to *C*, then *L* is four times as powerful a light as *C*; if the distance is three times as far, *L* is nine times as powerful. Apparatus thus used constitutes a *Rumford photometer*.

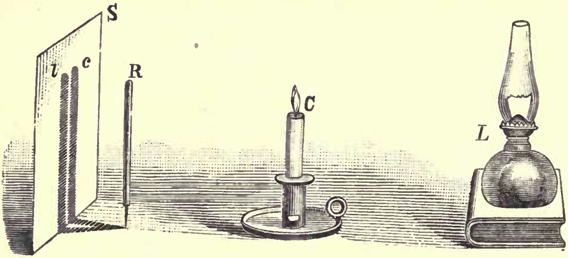


Fig. 238.

Experiment 212. — Drop some melted paraffine upon a piece of heavy, unglazed white paper, making a spot about an inch in diameter. Remove the excess of paraffine with a knife, and heat the spot with a flat-iron or can of water. Support the paper as a vertical screen. When the paraffined disk is viewed by transmitted light, it appears brighter than the surrounding paper; when viewed by reflected light, it appears darker. Place a lighted standard candle (see § 268) at one end of a table, and a lamp or gas-flame at the other end. Place the screen between them, and arrange the pieces so that the middle points of the candle flame, the translucent disk, and the lamp-flame are in a straight line that is perpendicular to the screen. If the lamp-flame is flat, set it diagonally to the screen. Move the screen along the line between the candle and the lamp until its two sides are equally illuminated; i.e., until the paraffined spot is invisible, or until the contrast between it and the rest of the screen is the same on both sides when viewed at the same angle. Find the ratio between the distances of candle and lamp from the screen, and square the ratio to find the candle-power of the lamp. Apparatus thus used constitutes a *Bunsen photometer*.

268. Photometry is the measurement of the relative amounts of light emitted by different sources. The usual process is to determine the relative distances at which two sources of light produce equal intensities of illumination. The standard in general use is the light given by a sperm candle (of the size known as "sixes") when burning 120 grains per hour. The result is expressed by saying that the light tested has so many candle-power.

CLASSROOM EXERCISES.

1. Describe the shadow cast by a wooden ball (*a*) when the source of light is a luminous point; (*b*) when the source of light is a white-hot iron ball smaller than the wooden ball; (*c*) when it is of the same size; (*d*) when it is larger.

2. Do sound waves or water waves the more closely resemble waves of light? Why?

3. State clearly your idea of the carrier of radiant energy.

4. Explain the formation of inverted images by small apertures.

5. Draw figures to illustrate the effect that doubling the distance of an opaque body from a source of light has upon the shadow of the former.

6. A coin is held 5 feet from a wall and parallel to it. A luminous point, 15 inches from the coin, throws a shadow of it upon the wall. How does the size of the shadow compare with that of the coin?

7. An opaque screen, 3 inches square, is held 12 inches in front of one eye; the other eye is shut; the screen is parallel with a wall 100 feet distant. What area on the wall may be concealed by the screen?

8. A standard candle is 2 feet and a lamp is 6 feet from a wall. The shadows on the wall are of equal intensity. What is the candle-power of the lamp?

9. An electric arc lamp 100 feet north of me and one 200 feet south of me illuminate opposite sides of a sheet of paper in my hand and render invisible a grease spot on the paper. How do the illuminating powers of the lamps compare?

10. If you hold a sheet of paper with a greased spot on it between you and the light, the spot will look lighter than the rest of the sheet. Why is this?

11. If you hold the sheet in front of you when you are turned away from the light, the spot will look darker than the rest of the sheet. Why is this?

12. Study the shadows cast by an electric arc lamp, and write a very brief description of the penumbra of the shadows.

13. Describe the shape in space of the umbra and the penumbra of the moon's shadow. Draw an illustrative figure.

14. When has an umbra an infinite length, and when a finite length?

15. The length of the umbra of the moon's shadow is a little longer than the radius of the moon's orbit. On the figure drawn for Exercise 13, indicate the position in space occupied by your city (*a*) when a total eclipse of the sun is visible there; (*b*) when a partial eclipse of the sun is visible there.

16. What does the great velocity of light indicate as to the density and the elasticity of the ether?

LABORATORY EXERCISES.

Additional Apparatus, etc.—A chalk-line; a standard candle; five "Christmas" candles; cardboard, 20 × 30 cm.; a slender wooden rod; two small kerosene lamps; two pieces of looking-glass, 10 cm. square, tied to the vertical faces of two rectangular blocks.

1. Place a yardstick vertically against the wall of the room. Hold one end of a foot rule at the eye, sight along the upper side of the rule, and bring it into line with the lower end of the yardstick. Keeping the rule in this position, hold a lead pencil vertically across its further end so that the upper end of the pencil is in line with the upper end of the yardstick. Carefully measure the length of the part of the pencil that projects above the rule, and compute the distance of your eye from the yardstick.

2. From a point on a blackboard or floor, draw two lines that diverge 1 inch in 10 feet, and make them as long as possible. It will be convenient to use a chalked line for this purpose. The included angle will represent (fairly well) the angle subtended by the diameters of the moon and the sun as observed from the center of the earth. With the apex of the angle as a center, draw a circle 4 inches in diameter to represent the earth. At the distance of 10 feet, draw a circle 1 inch in diameter to represent the moon. Imagine the long lines extended until they have diverged sufficiently to include between them a circle 400 inches in diameter to represent the sun. Compute the distance of the center of this circle from the apex of the angle. Assume the diameter of the earth to be 8,000 miles. From the figure as thus completed in imagination, compute the diameters of the moon and of the sun, and their distances from the earth. Shade the triangular space between the moon and the center of the earth. Is it possible for the moon's umbra to envelop the whole earth? Under the assumed conditions, what phenomenon would be seen by an observer looking toward the moon from the portion of the earth's surface included in the shaded triangle? By an observer on the earth's surface just outside that shaded part?

3. Arrange a Rumford photometer by setting up a rod 20 cm. long and 1 cm. in diameter, 5 cm. in front of a white cardboard screen 20 cm. tall and 10 cm. wide. Mount a short candle upon a wooden block, and four other candles of the same kind and length in a straight row upon another block. Place the block with the single candle on one side of the median plane, and the block with the four candles

on the other side. The outside candles of the set of four should be equally distant from the rod. After the candles have burned for some minutes, carefully trim their wicks so that the flames shall be of the same size. Stand in the median plane and so adjust the distance of the single candle as to get shadows of equal intensity as described in Experiment 211. Measure and record the distances of the two sources of light from the screen and compare the result with the statements of § 267.

4. Instead of the candles of Exercise 3, use two small kerosene lamps. Place them at equal distances from the rod, and turn the edges of their flames toward the rod. Turn one of the flames down until the shadows are of equal darkness. Turn one lamp so that the side of its flame is toward the rod. Fix the attention on the middle of the blurred shadow. If the two shadows are of equal darkness, record the fact. If they are not, move one of the lamps until they are. Then measure and record the distance of each lamp from its shadow. Record your conclusions as to the perfect transparency of a lamp-flame.

5. Using a Bunsen photometer, compare the illumination of the four candles used in Exercise 3 with that of a standard candle. Interchange the positions of the lights, and record the average distances.

6. Place two plane mirrors so that an observer standing in the plane of the screen can simultaneously see the images of both faces of the paraffined spot, and compare them for equality of illumination. Thus, determine the candle-power of a kerosene lamp.

III. REFLECTION OF RADIANT ENERGY.

Experiment 213. — Paint the outside of a pint tin-pail with lampblack, and fill the vessel with hot water. Support the pail a few inches above the table and, on the table near by, lay a sheet of tin-plate. Between the pail and the sheet, place a glass or wooden screen that has an aperture about 2 cm. in diameter so that radiant energy from the pail may pass through the aperture and fall upon the tin reflector on the table. Place one bulb of a differential thermometer so that the energy radiated directly from the pail will be cut off by the screen, while that reflected by the sheet of tin, in accordance with the law stated in § 76, will fall upon it. Notice the effect of the reflection. Move the bulb out of the line of reflection, and notice the effect.

Experiment 214. — About two feet from an air thermometer, place an inverted flower-pot. Midway between the two, place a board or glass screen that reaches from the table to a height of several inches above the bulb of the air thermometer. Upon the flower-pot, place a very hot brick. Notice that the heat of the brick has little effect upon the thermometer. Then hold a sheet of tin-plate over the screen so that energy radiated obliquely upward from the brick may be reflected obliquely downward toward the thermometer. By properly adjusting the position of the reflector, the thermometer may be quickly affected.

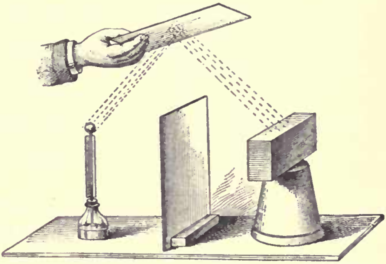


FIG. 239.

269. Reflection of Radiant Energy *is the sending back of incident ether waves by the surface on which they fall into the medium from which they come.* The reflection may be irregular or regular.

(*a*) The proportion of the incident energy that is reflected increases with the angle of incidence and with the degree of polish of the reflecting surface, and varies according to the nature of the reflecting substance.

NOTE. — The laboratory should be provided with a porte-lumiere, which consists of a plane mirror so mounted and fitted with adjusting appliances that the direction of light reflected from the mirror may be easily controlled. The mirror is placed on the outside of the shutter of a darkened window and operated from within, sunlight being reflected through the aperture in the shutter.

Experiment 215. — Let a beam of light pass through an opening in the shutter of a darkened room, and fall upon a sheet of drawing paper lying on the table-top. The light will be scattered, and will illuminate the room. With a hand mirror, reflect the beam downward into a tumbler of water into which a teaspoonful of milk has been stirred. The milky water will scatter the light, and illuminate the room as if it was self-luminous.

270. Irregular Reflection or Diffusion results from the incidence of radiant energy upon an irregular surface, as is illustrated by Fig. 240. Bodies are made visible to the eye mainly by the light that they thus diffuse.

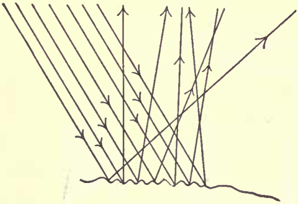


FIG. 240.

Experiment 216. — Repeat Experiment 215, allowing the beam of light to fall upon a mirror instead of drawing paper. Most of the light will be reflected in a definite direction, and will brilliantly illuminate a small part of the enclosing wall. Reflect the beam downward into a tumbler of clear water; the tumbler will be visible but the room will not be illuminated as it was by the milky water.

271. Regular Reflection results from the incidence of radiant energy upon a polished surface. When a beam of light falls upon a mirror, the greater part of it is reflected in a definite direction as is illustrated by Fig. 241, and forms an image of the object from which it came. A perfect mirror would be invisible.

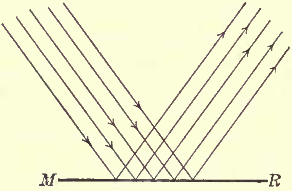


FIG. 241.

Law of Reflection.

Experiment 217. — Provide a semi-circular table like that shown in Fig. 45. With a sharp knife, cut a line on the silvered surface of a piece of looking-glass about 5 × 10 cm., perpendicular to one of the long edges, and at its middle point. With a thread or fine rubber band, fasten the reflector to the vertical face of the block at *B* so that the lower end of the line on the looking-glass shall rest upon the normal line, *DB*. Place a lighted candle at one end of a radius, as at *A*, and set one of the postal card screens used in Experiment 204, near the edge of the table, and so that light from the candle will pass through the hole and fall upon the line marked on the mirror. Similarly, place a like screen on the other side of *BD*, and move it about until, when looking through the hole in it, an image of the hole in the screen near *A* is seen in the mirror directly in line with the knife-mark on the mirror. Mark the points on the table directly under the perforations in the screens, and through them draw radial lines. From the graduated edge of the table or with a protractor, ascertain which of these radii makes the greater angle with the radius, *BD*, perpendicular to the face of the mirror.

Experiment 218. — Using the screens and blocks provided for Experiment 204, arrange apparatus as shown in Fig. 242. At the middle of the middle block, place a bit of window glass, *m*, painted on the under side with black varnish. On the blocks that carry the screens, place bits of glass, *n* and *o*, of the same thickness as the black mirror. Light from the candle will pass through *A*, be reflected at *m*, and pass through *B*. Place the eye in such a position that the spot of light in the mirror may be seen through *B*. Mark this spot with a needle held in place by a bit of wax. Place a piece of stiff writing paper upright upon *m* and *n*, mark the positions of *B* and of *m*, and draw on the paper a straight line joining these two points. The angle between this line and the lower edge of the paper coincides with the angle *Bmn*. Reverse the paper, placing it upon *m* and *o*. It will be found that the same angle coincides with *Amo*. The complementary angles, *Amo* and *Bmn*, being thus equal, the angle of incidence equals the angle of reflection.

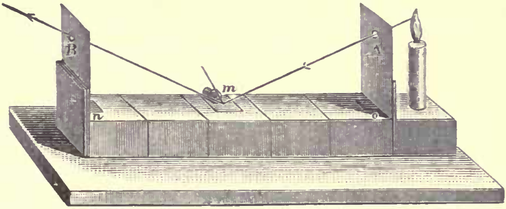


Fig. 242.

272. Law of the Reflection of Radiant Energy. — *The angle of incidence and the angle of reflection are equal, and lie in the same plane.*

273. Explanation of Reflection. — Consider a beam of light as made up of a number of ether waves moving forward in air and side by side, as represented by the rays *A*, *B*, and *C*. Imagine a plane, *MN*, normal to these rays, attached to the waves and moving forward with them. *Such a plane is called a wave-front.* It continues parallel to itself and moves forward in a straight line. As the wave-front advances beyond *MN*, the ray, *A*, strikes the reflecting surface, *RS*, and is turned back into the air in accordance with the law just given. In the interval of time that passes before the ray, *C*, arrives at *P*, the ray, *A*, traveling with unchanged speed as before, passes over the distance, *MO*, equal to the distance, *NP*. This changes the direction of the plane that is attached to the waves, and sets it in the new position indicated by *OP*. Lines drawn from *M*, *Q*, and *P*, perpendicular to *OP*, will represent the new direction of propagation, i.e., the paths of the reflected rays. From Fig. 243, it may easily be proved that the angles of incidence and of reflection are equal.

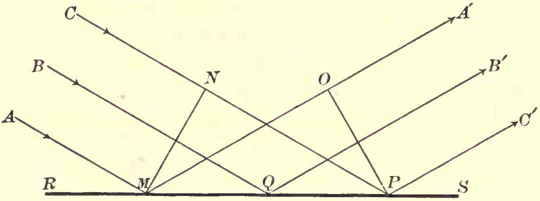


FIG. 243.

274. Apparent Direction of Bodies. — Every point of a visible object sends a cone of light to the eye. The pupil of the eye is the base of the cone. *The point always* *appears at the real or apparent apex of the cone.* If the path of the light from the point in question to the eye is straight, the apparent position of the point is its real position. If the path is bent by reflection, or in any other manner, the point appears to be in the direction of the light as it enters the eye.

Experiment 219. — Place a jar of water 10 or 15 cm. back of a pane of glass placed upright on a table in a dark room. Hold a lighted candle at the same distance in front of the glass. The jar will be seen by light transmitted through the glass. An image of the candle will be formed by light reflected by the glass. The image will be seen in the jar, giving the appearance of a candle burning in water. The same effect may be produced in the evening by partly raising a window, and holding the jar on the outside and the candle on the inside. This experiment suggests an explanation of many optical illusions, such as "Pepper's ghost," etc.

275. Plane Mirrors. — If an object is placed before a plane mirror, a virtual image appears behind the mirror. Each point of this image seems to be as far behind the mirror as the corresponding point of the object is in front of the mirror. Hence, images seen in still, clear water are inverted.

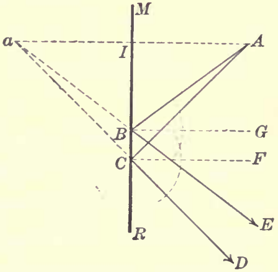


FIG. 244.

(*a*) In Fig. 244, *AB* and *AC* represent any two luminous rays proceeding from *A* and incident upon the plane mirror, *MR*. From the points of incidence, draw the perpendiculars, *BG* and *CF*. Draw *BE* so that the angle, *GBE*, is equal to the angle, *GBA*. Then will *BE* represent the path of the light reflected at *B* (§ 272). Similarly, draw *CD* to represent the path of the light reflected at *C*. Prolong *DC* and *EB* until they intersect at a. Draw Aa. From this figure, it may be proved geometrically that AIB is a right angle and that $AI = aI$.

276. The Construction for the Image produced by a plane mirror depends upon the fact that the image of an object may be located by locating the images of a number of well chosen points in the surface of the object.

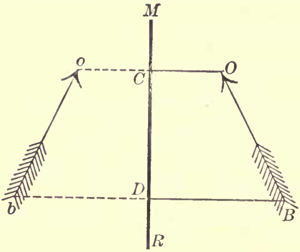


FIG. 245.

(*a*) In Fig. 245, OB represents an arrow in front of the mirror, MR. From the ends of the arrow, draw OC and BD perpendicular to the face of the mirror, and prolong them indefinitely. Take oC equal to OC and bD equal to BD. Join o and b. The image is virtual, erect (i.e., not inverted), and of the same size as the object.

Experiment 220. — Hinge together two rectangular pieces of looking-glass, each about 7 × 10 cm., by pasting cloth along two short edges, and set them on the table with an angle of 906 between them. Set a "Christmas" candle or a bright-headed pin between the mirrors and about 3 cm. from the apex of the angle, and count the visible images. Reduce the angle to 60°, and count the images. Reduce the angle to 45°, and count the images.

277. Multiple Images. — By placing two plane mirrors facing each other, we may produce an indefinite series of images of an object between them. Each image acts as a material object with respect to the other mirror, in which we see an image of the first image, etc. When the mirrors are placed so as to form with each other an angle that is an aliquot part of 360 degrees, the number of images is one less than the quotient obtained by dividing four right angles by the included angle, provided that quotient is an even number.

(*a*) The mirrors will give three images when placed at an angle of 90°; five at 60°; seven at 45°. When the mirrors are placed at right angles, the object and the three images will be at the corners of a rectangle as shown at *A*, *a*, *a′* and *a″*.

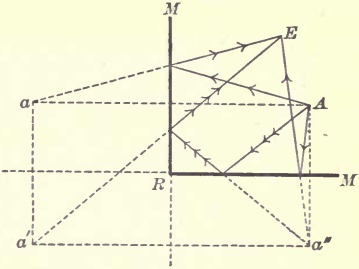


FIG. 246.

Experiment 221. — Let a small beam of light fall perpendicularly upon a concave mirror. Strike two blackboard erasers together in front of the mirror, and notice that the light converges at a point not far from the mirror.

278. A Focus is a point at which light converges, in which case it is called a *real focus;* or it is a point from which light appears to proceed, in which case it is called a *virtual focus.*

279. Concave Mirrors are generally spherical; i.e., the reflecting surface is a small part of the inner surface of a spherical shell. The center of the sphere, *C*, is the *center of curvature* of the mirror. *A*, the middle point of the mirror, is called the *center or vertex of the mirror.* Any straight line passing through *C* to or from the mirror is called an *axis* of the mirror. *ACX*, the axis that passes through *A*, is called the *principal axis;* all other axes are called *secondary axes.* The angle, *MCR*, is called the *aperture* of the mirror. A concave mirror

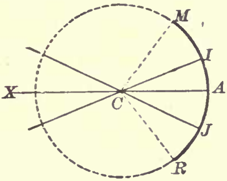


FIG. 247.

increases the convergence or decreases the divergence of light that falls upon it, as is shown in Fig. 248.

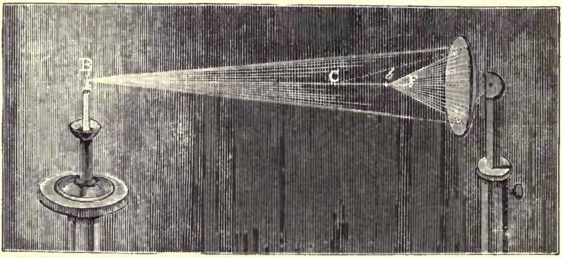


FIG. 248.

Experiment 222. — Arrange conjugate parabolic reflectors of polished brass as shown in Fig. 164. Place a hot iron ball at one focus, and a bit of gun-cotton that has been blackened with lampblack at the other focus. Repeat the experiment, holding a differential thermometer so that one bulb will be at the focus, the second bulb being in a direct line with the ball and, therefore, nearer to it. Notice which bulb receives the more heat.

280. The Foci of Concave Mirrors may be in front of the mirror, in which case they are *real;* or they may be behind the mirror, in which case they are *virtual.*

(*a*) The location of these foci gives rise to several cases: —

(1) The incident rays may be parallel to the principal axis, as they will be when the radiating point is at an infinite distance. Solar rays are practically parallel. Suppose a spherical mirror of small aperture to be held facing the sun. The ray that follows the principal axis will fall upon the mirror perpendicularly at A, and be reflected back upon itself. Other rays will be reflected as shown in Fig. 249, intersecting at F, a point midway between C and A. This focus of rays

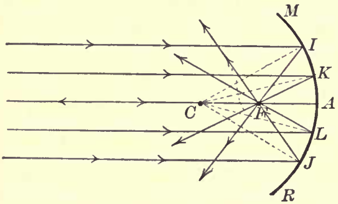


FIG. 249.

parallel to the principal axis is called the *principal focus* of the mirror. The distance, *FA*, is called the *principal focal length* or distance of the mirror.

(2) When the rays diverge from the center of curvature they strike the mirror perpendicularly, and are reflected back upon themselves. The radiant point and the focus coincide.

(3) When the rays diverge from a point beyond the center of curvature, as *B*, the focus falls on the same axis, at a distance from the mirror greater than that of the principal focus, and less than that of the center of curvature. If the radiant point is at *B*, the focus falls at *b*, as shown in Fig. 250.

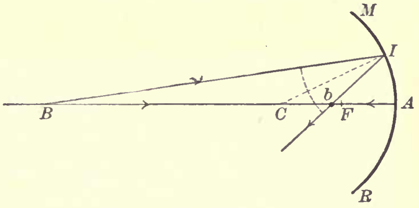


FIG. 250.

(4) When the rays diverge from a point at a distance from the mirror greater than that of the principal focus and less than that of the center of curvature, we have the converse of the third case. The focus falls on the same axis beyond the center of curvature. If the radiant point is at *b* (Fig. 250), the focus falls at *B*. Foci that are thus interchangeable are called *conjugate foci.* (See § 185.) This illustrates "the principle of reversibility."

(5) When the rays diverge from a point at a distance from the mirror less than that of the principal focus, the reflected rays diverge as if from a point back of the mirror. This point, *b*, is a virtual focus.

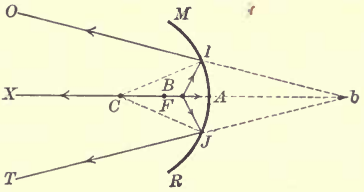


FIG. 251.

(6) When the rays diverge from the principal focus, the reflected rays are parallel and there is no focus, real or virtual. This is the converse of the first case.

(*b*) The convergence of parallel rays at the principal focus is only approximately true with a spherical mirror; it is strictly true with a parabolic mirror. In order that the difference between the spherical and the parabolic mirror may be reduced to a minimum, the aperture of the former should be small. The light from a luminous point at the focus of a parabolic mirror is reflected in truly parallel lines. The head lights of railway locomotives are thus constructed. Parabolic mirrors would be more common if they were less expensive.

(*c*) The sum of the reciprocals of the conjugate focal distances is equal to the reciprocal of the principal focal distance. Representing the radius of curvature by r, the distance of the luminous point from the mirror by f, and the distance of the focus from the mirror by f',

$$\frac{1}{f}+\frac{1}{f'}=\frac{2}{r}.$$

Concave Mirror Images.

Experiment 223. — In a dark room, hold a candle between the eye and the concave side of a bright silver spoon held a little ways in front of the face. Notice that the inverted image of the flame is in front of the spoon. Place the spoon between the flame and your face but so as to allow the face to be illuminated by the candle. Notice the image of the observer.

Experiment 224. — Place a concave mirror facing the sun, and hold a bit of paper so that its illumination by the reflected light is of the greatest intensity obtainable, thus locating the principal focus of the mirror. Measure this focal distance. Then stand directly in front of the mirror and at a considerable distance from it. Notice that the image of yourself is inverted, diminished and real. If you are not sure that the image is real, have some one hold his outspread fingers between the image and the mirror. Approach the mirror, and notice that your image increases in size until your eye is at the center of curvature. Continue your approach, and notice that when your eye is between the center of curvature and the principal focus, no image is to be seen. The image is behind you and, therefore, invisible. When your eye is between the principal focus and the center of the mirror, your image is erect, magnified and virtual.

Experiment 225. — In front of a concave mirror, and at a distance equal to the radius of curvature, place a box that is open on the side toward the mirror. Within this box, hang an inverted bouquet of bright-colored flowers. The observer should stand in front of the mirror and some ways back of the box. By giving the mirror a certain inclination, easily determined by trial, an image of the invisible bouquet will be seen just above the box. A glass vase may be placed upon the box to hold the imaged flowers.

Experiment 226.—Place a lighted candle in front of a concave mirror so that the flame is in a secondary axis of the mirror, and at a distance greater than the focal length and less than the radius of curvature. Place a tracing-cloth or oiled-paper screen as shown in Fig. 252, and, with a blackened card, shield it from the direct light of the candle. Adjust the positions of the candle and the screen until a good image of the former is projected on the latter. If the outer edge of the image is indistinct, place before the mirror a paper curtain with a circular opening of such size that the aperture of the exposed part of the mirror does not exceed 10°. Notice that the image is less intensely illuminated, but that its outline is more sharply defined.

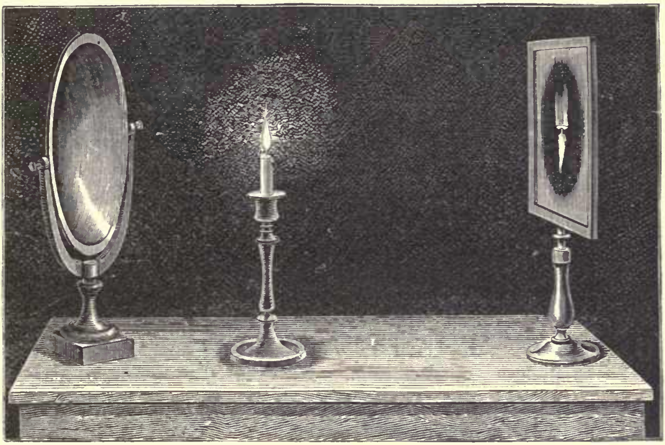


FIG. 252.

281. Images formed by Concave Mirrors consist of the conjugate foci of the several points in the surface of the object presented to the mirror and may, therefore, be real or virtual. The construction of figures to illustrate the formation of images under different conditions may be easily performed by selecting a few determinative points, as the ends of an arrow, and determining the foci of those points under the given conditions.

(*a*) The focus of each point chosen may be determined by tracing two rays from the point, and locating their real or apparent intersection after reflection by the mirror. The two rays most convenient for this purpose are the one that lies along the axis of the point, and the one that lies parallel to the principal axis of the mirror. The first of these is reflected back upon itself, and the focus must, therefore, lie in that line. The other is reflected through the principal focus, and the construction of equal angles of incidence and reflection is, therefore, unnecessary. The process is illustrated in Fig. 253. Following the order of the cases discussed in § 280, it will be found that: —

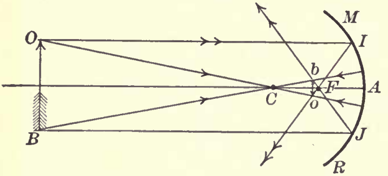


FIG. 253.

(1) When the object is at a distance so great that the incident rays may be considered parallel (e.g., solar rays), the image is formed at the principal focus.

(2) When the object is at the center of curvature, the image is real, inverted, of the same size as the object, and at the center of curvature.

(3) When the object is at a distance from the mirror somewhat greater than the center of curvature, as beyond *C*, the image is real, inverted, smaller than the object, and at a distance from the mirror greater than that of the principal focus and less than that of the center of curvature, as between *F* and *C*.

(4) When the object is at a distance from the mirror greater than that of the principal focus and less than that of the center of curvature, as between *F* and *C*, the image is real, inverted, larger than the object, and

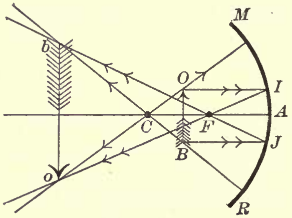


FIG. 254.

at a distance from the mirror greater than that of the center of curvature, as beyond *C*. This is the converse of the third case.

(5) When the object is at a distance from the mirror less than that of the principal focus, as between *F* and *A*, the image is virtual, erect, and larger than the object.

(6) When the object is at a distance from the mirror equal to that of the principal focus, the reflected rays are parallel and no image is formed. This is the converse of the first case.

282. The Spherical Aberration of a concave mirror is the deviation of some of the reflected light from the focus, as is shown in Fig. 255. It arises from the curvature of the mirror, and causes an indistinctness or blurring of the image. Parabolic mirrors are free from this defect; spherical mirrors are partly freed from it by reducing their apertures so that the curvature conforms closely to the curvature of a paraboloid.

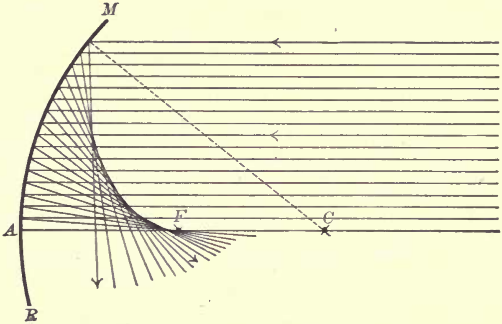


Fig. 255.

Experiment 227.—Hold the convex side of a bright silver spoon toward you, and bring the spoon and a candle into the positions described in Experiment 223. Notice that the erect image of the flame is back of the spoon. Place the spoon between the flame and your face but so as to allow the face to be illuminated by the candle. Notice the image of the observer.

283. A Convex Mirror is generally a part of the outer surface of a spherical shell. It increases the divergence, or decreases the convergence of light that falls upon it. The foci are virtual; the principal focus is midway between the center of the mirror and the center of curvature. The foci may be located and the images determined by processes closely similar to those used for concave mirrors, as is sufficiently illustrated by Fig. 256. Such an image is erect, diminished, and virtual.

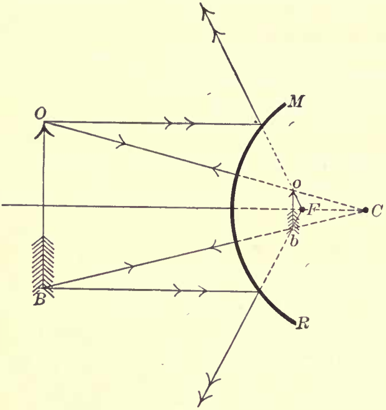


Fig. 256.

CLASSROOM EXERCISES.

1. What must be the angle of incidence that the angle between the incident and the reflected rays shall be a right angle?

2. Copy Fig. 245, and add lines to show that the rays that form the image for the right eye of the observer are different from the rays that form the image for the left eye.

3. With a radius of 4 cm., describe ten arcs of small aperture to represent the sections of spherical concave mirrors. Mark the centers of curvature, and the principal foci, and draw the principal axes. Find the conjugate foci for points in the principal axis designated as follows: (*a*) At a distance of 1 cm. from the mirror; (*b*) 2 cm. from the mirror; (*c*) 3 cm. from the mirror; (*d*) 4 cm. from the mirror; (*e*) 6 cm. from the mirror. Make five similar constructions for points not in the principal axis. Notice that each effect is in consequence of the equality between the angle of incidence and the angle of reflection.

4. Illustrate by a diagram the image (*a*) of an object placed at the

principal focus of a concave mirror; (*b*) of one placed between that focus and the mirror; (*c*) of one placed between the focus and the center of the mirror.

5. Write a brief discussion of the formation of images by concave mirrors under six different conditions, following the order of the cases discussed in § 280. Draw carefully constructed diagrams for the cases not illustrated in answering Exercise 4.

6. Rays parallel to the principal axis fall upon a convex mirror. Draw a diagram to show the course of the reflected rays.

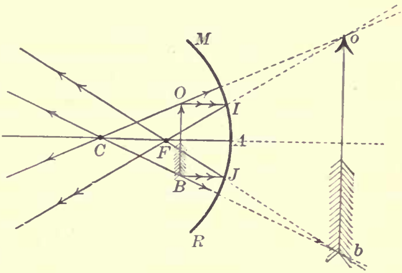


FIG. 257.

7. Why do images formed by a body of water appear inverted?

8. (*a*) What kind of a mirror always makes the image smaller than the object? (*b*) What kind of a mirror may make it larger or smaller, and according to what circumstances?

9. A man stands before an upright plane mirror and notices that he cannot see a complete image of himself. (*a*) Could he see a complete image by going nearer the mirror? Why? (*b*) By going further from it? Why?

10. When the sun is 30° above the horizon, its image is seen in a tranquil pool. What is the angle of reflection?

11. A person stands before a common looking-glass with the left eye shut. He covers the image of the closed eye with a wafer on the glass. Show that when, without changing his position, he opens the left and closes the right eye, the wafer will still cover the image of the closed eye.

12. The distance of an object from a convex mirror is equal to the radius of curvature. Show that the length of the image will be one-third that of the object.

13. From the formula given in § 280 (*c*), deduce an algebraic expression for the value of one of the conjugate focal distances in terms of the principal focal distance and the other conjugate focal distance, and write its significance in words.

14. Similarly show that as one of the conjugate focal distances increases, the other decreases, and that when one of them becomes

infinite, the other coincides in value with that of the principal focal distance.

15. Considering the same formula, notice that if f is less than half the radius, $\frac{1}{f}$ is greater than the principal focal distance, and that f', therefore, has a negative value. What would such a negative value indicate as to the positional character of the focus?

16. Given three points, *A*, *B* and *C*, not in a straight line. Show, by a diagram, how to place a plane mirror at *C* so that light proceeding from *A* shall be reflected to *B*.

17. Show by a figure how, with the aid of plane mirrors, one can see around, or apparently see through an opaque object.

18. How should a plane mirror be placed so that the images seen in it shall have their natural positions relative to the horizon?

19. Study a description of a kaleidoscope in a dictionary or cyclopædia, and determine the number of mirrors used in such an instrument that gives pentagrammatic designs.

20. Refer to Fig. 255, and show that the aberration of a spherical mirror may be lessened by reducing its aperture.

21. Determine the position of the image of a vertical object formed by a plane mirror placed at an angle of 45° with the horizon.

22. The sun's rays are straight and practically parallel. How then does daylight reach every nook and corner of a room from which the sun is never visible, e.g., a room that has only a northern exposure?

23. Remember that watery particles in the air may reflect light as well as does suspended crayon-dust, and explain the production of the halos often seen around the street lamps upon foggy nights.

LABORATORY EXERCISES.

Additional Apparatus, etc.—Bright tin-plate, 8 × 30 cm.; plane and curved mirrors; shawl-pins; cardboard; several large sheets of Manila paper, and one of thin white paper; a wooden cube, 4 or 5 cm. on an edge.

1. Lay a sheet of paper on the table top, and draw on it a straight line, *AB*, about 15 cm. long, and mark its middle point *C*. From *C*, draw *CD* at right angles to *AB*, and 10 or 15 cm. long. Through *D*, draw *MN*, a line parallel to *AB*. Place the rectangular block and mirror used in Experiment 217 so that the back of the mirror (the silvered side) is exactly over *AB*, while the lower end of the vertical

line that is marked on the mirror rests upon *CD*. Much of the success of the exercise depends upon the correct placing of the mirror. If the block is 1 or 2 cm. shorter than the mirror, it will be easier to place the mirror properly. Set a shawl-pin upright at *E*, about 3 cm. from *D* toward *N*. Bring the eye nearly to the level of the paper on the other side of *D*, and into such a position that the image of the pin shall be in line with the line scratched on the mirror. If the scratched line and the image are not parallel, straighten up the pin. Set another shawl-pin upright in this line of vision and in *DM*, and mark its position *F*. Draw the lines, *CE* and *CF*. Measure *DE* and *DF*, and see how they compare. If your work has been done accurately, they will be equal, but accurate work must not be expected. The mirror may not be a perfect plane; certain lines may not be perfectly straight, and others may not be perfectly parallel; the right angles called for may vary a little from 90°; and, with all the rest, a personal error is nearly inevitable. Therefore, determine the ratio that *DF* bears to *DE*, shift *E* to a new position a little further from *D*, and repeat the work. Then shift the position of *E* again, and repeat the work. The average of the three ratios for *DF* and *DE* will be pretty close to unity if your work has been well done. With *DF* and *DE* equal, the work establishes the equality of the angles of incidence and of reflection. Measure the angles with a protractor.

2. Continue working with apparatus and diagram as in Exercise 1. Move the pin from *F* to *G*, 2 or 3 cm. toward *M*. In line with *G* and the image of the pin at *E* and near *AB*, set a pin, and mark its position *G'*. Move the pin from *G* to *H*, 2 or 3 cm. toward *M*. In line with *H* and the image of the pin at *E*, and near *AB*, set a pin (that may be moved from *G'*), and mark its position *H'*. Remove the pins from *H* and *H'*. Bring the eye to the other side of *D*, and place it so that the pin at *E* covers its image. Set a pin near *AB*, and in line with the pin at *E* and its image. Mark the position of this pin *O*. Remove the block, the mirror, and the pins at *E* and *O*. Draw a line through the pinholes at *E* and *O*, and produce it indefinitely back of *AB*. The part that lies back of *AB* should be broken or dotted, and so in all similar cases (see Figs. 244 and 245). Similarly, produce *FC* and the lines drawn through the pinholes at *G* and *G'*, and at *H* and *H'*. The four dotted lines should intersect at a common point which mark ***e***. The fact that the image remained at *e* whether the eye was in line with *E*, or *F*, or *G*, or *H*, shows clearly that the image has a definite location as long as the object and the mirror remain stationary. If

AB is perpendicular to *Ee*, as it should be, tell what that fact signifies. If *AB* bisects *Ee*, as it should, tell what that fact signifies.

3. Draw a straight line, *AB*, across a 30 × 50 cm. sheet of thin white paper midway between its shorter edges. Set the block and mirror used in Exercise 1 with the foot of the line on the mirror at *C*, the middle point of *AB*. Draw on the paper in front of the mirror a triangle, *EFG*, no side of which is less than 10 cm. long, and no corner of which is less than 7 or 8 cm. from the mirror. Let one corner of the triangle come 1 or 2 cm. nearer the edge of the paper than the end of the mirror does. Paste white paper over one face of a cubical wooden block about 4 cm. on an edge, and draw a vertical ink-mark across the middle of the papered face. This ink-mark is to replace the shawl-pins used in Exercises 1 and 2. Place this block with its papered face toward the mirror, and with the foot of the inked line over *E*. Place a school rule or other straight-edge on the paper and, sighting along its horizontal edge, bring the edge into line with the eye and the image of the vertical line. Be sure that the mirror or the wooden cube has not been moved from its proper position, and try the alignment of the edge of the rule again. If it is correct, hold the rule in place and, with a well sharpened pencil, trace a line on the paper along that edge of the rule that is just under the edge used in taking sight. Mark this line E_1E_1. Move the rule into a new position as far as possible from the line just drawn, bring its edge into alignment with the image of the vertical line as before, and trace along its edge a line which mark E_2E_2. Then place the rule about midway between the two lines drawn on the paper, bring its edge into alignment with the image as before, and trace, as before, a third line which mark E_3E_3. Be sure that the mirror and the wooden cube are still in their original positions; if they have been moved, test the accuracy of the work already done.

Place the wooden cube with the foot of its vertical line over *F* and, in like manner, draw three lines which mark F_1F_1, F_2F_2 and F_3F_3. Similarly, place the vertical mark over *G*, trace three lines toward its image, and mark them G_1G_1, G_2G_2, and G_3G_3.

Remove the wooden cube and the mirror with its block, and prolong the lines E_1, E_2 and E_3 back of the mirror as in Exercise 2. They should intersect at a common point which mark *e*, the location of the image of the point, *E*. Similarly, determine the points, *f* and *g*, the location of the images of *F* and *G*. Draw the triangle, *efg*. Fold the sheet of paper along the line *AB*, and hold the folded sheet against the window; compare the size and shape of the two triangles, and their positions relative to the line *AB*. Compare your results with § 276.

4. Balance the block and mirror upon a rule so that the face of the mirror crosses the rule at its middle. Place the eye so that the further end of the rule may be seen by looking obliquely downward and over the upper edge of the mirror. If the block back of the mirror is visible, move the mirror up until it conceals the top of the block, or use a thinner block. Adjust the mirror so that the further end of the image of the rule as seen in the mirror coincides with the end of the rule as seen over the mirror. The length of the rule is now perpendicular to the face of the mirror. Look at the images of the several divisions of the scale in front of the mirror and notice the distance of each image back of the mirror.

5. On a sheet of paper, draw two lines, *AB* and *CD*, that bisect each other at right angles at *E*. Make each line about 30 cm. long. Place the block and mirror with its reflecting surface over *AB*, and with its middle over *E*. See that the image of *DE* visible in the mirror is in line with the part of *EC* that is visible back of the mirror, as in Exercise 4. This is a delicate adjustment for the normal position of the mirror. Set the face of the mirror so that it crosses *AB* at *E* making acute angles with *AB*. Notice that the image of *DE* is turned more than the mirror is. Set a shawl-pin upright back of the mirror, and in line with the image of *DE* as seen in the mirror. Mark its position *O*. Draw a line along the face of the mirror through *E*, and mark it *MR*. Remove the block and mirror. Draw the line, *EO*. Measure the angles, *AEM* and *CEO*, and see how they compare. Place the mirror again across *AB* at *E*, increasing the angle, *AEM*. Construct the two angles that measure the rotation of the mirror and of the image as before. Measure and compare them. Once more, place the mirror across *AB* at *E*, and turn it until the image of *DE* coincides with *EB*. Mark on the paper the position of the face of the mirror, and compare the magnitude of the angle that it makes with *AE* with the magnitude of *CEB*. State in general but exact terms the effect that the deflection of a plane mirror has upon the deflection of an image seen in it.

6. With a radius of 12.5 cm., draw *MR*, the arc of a circle, and through its middle point, *C*, draw a tangent, *AB*, extending about 5 cm. each way from *C*. Draw the radius *Cc*. Cut a rectangular piece of bright tin-plate about 8 × 30 cm., and mark a line across its

face parallel to the two short edges and midway between them. Bend the tin until its long edge coincides with the circular arc on the paper, and hold it in that shape by tying a string around it. Take pains to have the marked line on the concave side, to avoid "kinking" the tin, and to secure a uniform curve. Place this curved reflector over the circular arc with the foot of its marked line at *C*, and with the concave side toward the light. To test the accuracy of the mirror, see that the image of *cC* is in a straight line with *cC* itself. Notice the caustic curves reflected on the paper, compare them with Fig. 255, and remember them when you come to the consideration of Exercise 13. About 4 cm. from *C*, and 2 cm. from *cC*, set a pin upright, as in Exercise 1. Mark its position *E*. Bring the eye to the level of the paper and near the reflector, and set a second pin at about the same distance from *C*, and in line with *C* and the image of the pin at *E*. Mark this position *F*. Remove the curved mirror, and place the rectangular block and plane mirror used in Exercise 1 as therein directed. The pin at *F*, the line over *C*, and the image of the pin at *E* should be in a straight line. Remove the block and mirror. From *C* and through *E* and *F*, draw lines of equal length, *CG* and *CH*. Draw *GH* intersecting *Cc* at *D*. With a protractor, measure the angles, *DCG* and *DCH*; they should be about equal. Measure the lengths of *DG* and *DH*, and find the ratio between them; it should be nearly unity. Shift *E* successively to two new positions, and repeat the work for each. Find the average of the three ratios between *DG* and *DH*; it should be about unity. If it is, the work shows that the relation between the angles of incidence and reflection is the same for a concave mirror that it is for a plane mirror. See the remarks made under Exercise 1.

7. Repeat Exercise 6, using the convex side of the mirror.

8. Place a cardboard screen close behind a candle-flame. Hold a concave mirror so that a sharp image of the flame is projected on the screen, making image and flame coincide as nearly as possible. Image and flame will be nearly at the center of curvature of the mirror. Describe the image. Determine the focal length of the mirror.

9. Place the flame between the center of curvature of the mirror used in Exercise 8 and its principal focus, and adjust the screen so that the image is sharply outlined on it. Determine the distances of the flame and image from the mirror. Change positions of the flame and screen, holding them so that an image may still be formed on the screen. Does the principle of "reversibility" hold good in this case?

10. Place the flame as near as possible to the mirror used in Exercise 8 and still get an image on the screen. What is the limiting distance, expressed in general terms?

11. Support the mirror used in Exercise 8 in a vertical position at one end of a long table. Directly in front and at the other end of the table (the distance being greater than the radius of curvature), place a lighted candle. Place a small paper screen so that a clearly defined image of the flame may be projected upon it. Measure the distances of the object and the image from the mirror, and describe the image. Move the candle successively into new positions nearer the mirror, projecting and describing the image, and making measurements as above directed for each position. Compare your results with the statements of your answer to Exercise 5, page 341.

12. In similar manner, experimentally determine the effect that the position of the object has upon the character of the images formed by a convex mirror, and write a brief discussion of the results attained.

13. Put a pint of milk into a shallow circular dish of bright tin that will hold a quart or more, and hold a lamp so that a catacaustic curve is formed on the surface of the milk. Account for the existence of the curve, and determine the relation that the reflected rays bear to the caustic.

IV. REFRACTION OF RADIANT ENERGY.

Experiment 228. — Hold a double-convex lens in the sun's rays, so that the bright focus on the side opposite the sun shall fall upon some easily combustible material like the tip of a friction-match. A spectacle-glass will answer, but a larger lens, like that of a reading-glass, is desirable; the larger the lens, the better. Compare the effects when dark-colored paper and white paper are held at the focus. Then compare the effects with white paper that is clean, and the same paper after it has been blackened with a lead pencil, or smeared with lampblack. A lens thus used is called a "burning-glass."

Experiment 229. — Fill a large glass globe with water. An aquarium globe will answer. Hang the filled globe in the bright sunshine and, at the focus of the liquid lens, hold a bit of gun-cotton that has been blackened with lampblack. The gun-cotton may be thus ignited. The experiment will work equally well if a large double-convex lens of ice is used instead of the globe of water.

Experiment 230. — Place a coin on the bottom of a tin pan 15 cm. or more in diameter and 4 or 5 cm. deep, and lay a slender straight rod across the top of the pan. Hold the eye vertically above the coin, bring the rod nearly into the line of vision, and note the apparent distance of the coin below the rod. Have water poured into the pan, and estimate the apparent displacement of the coin if any is noticed. Empty the pan and replace the coin. Rest the head against the edge of a shelf or other convenient fixed support, close one eye, and have the pan adjusted so that its side just hides the coin from view. Have water poured carefully into the pan until the coin is visible. The light coming from the coin to the eye is bent downward somewhere and somehow. Note the depth of water in the pan. Empty and wipe the pan, and repeat the experiment using kerosene instead of water, and compare the depths of the two liquids.

Experiment 231. — Procure a clear glass bottle with flat sides not less than 10 cm. broad. The larger the face of the bottle the better; a rectangular glass battery-jar is still better. Cover one face of the bottle with paper, and then cut out as large a circle as possible. On this clear circular space, mark horizontal and vertical diameters intersecting at *i*, as shown in Fig. 258. With a protractor, graduate the four quadrants from 0° at the ends of the vertical diameter to 90° at the ends of the horizontal diameter. Fill the bottle with clear water to the level of *i*, and hold it so that a

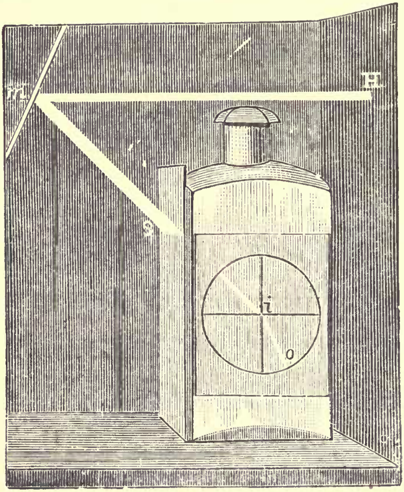


Fig. 258.

horizontal sunbeam passes through the clear sides of the bottle above the water. Notice that the path of the beam through the bottle is straight. The path may be made plainly visible by clapping together two blackboard erasers so as to scatter dust through the air around the bottle. Turn the bottle with its clear face to the front, and raise it so that the beam passes through the water. Put a sheet of black paper back of the bottle, and notice that the beam is still straight. In a card, cut a slit about 5 cm. long and 1 mm. wide. Place the card against the bottle as shown in the figure. Reflect the beam through this slit so that it falls upon the surface of the water at *i*. Notice that the reflected beam is straight until it reaches the water, but that it is bent as it obliquely enters the water. From the scale on the paper, read the angles that the beam makes with the vertical diameter, above and below *i*.

Experiment 232. — Repeat Experiment 231, using a rectangular paper-weight of glass instead of the bottle of water, and notice whether the deviation of the light from its straight path is more or less in the glass than it was in the water.

284. Refraction of Radiant Energy *signifies a retardation of the ether waves, and may be manifested by a change of direction.* When light is incident on the surface of a transparent medium, part of it is reflected as already explained. Another part of the incident light enters the medium, and generally pursues therein a changed direction. *This part is said to be refracted.*

(*a*) There is a change of direction, i.e., the radiant energy is deviated, when it falls obliquely upon the interface that separates two media, as air and water, and passes from one to the other; or when it passes through a medium the density of which is not uniform, as the atmosphere.

(*b*) In Fig. 259, *LA* represents a ray of light propagated in air, falling obliquely upon the surface of water at *A*, and deviated by the water from *AE* to *AK*. Draw *CD* perpendicular to the refracting surface at the point of incidence. *LAC* is the angle of incidence; *KAD*, the angle of refraction; and *KAE*, the angle of deviation. From A as a center and with unity as a radius, describe a circle, and draw mn and pq perpendicular to CD. Then mn is the sine of the angle of incidence; pq is the sine of the angle of refraction.

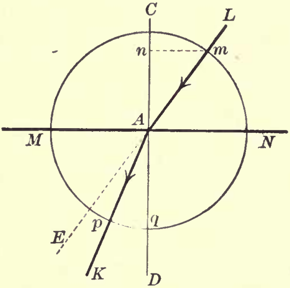


FIG. 259.

(*c*) For any two media, the quotient arising from dividing the sine of the angle of incidence by the sine of the angle of refraction is constant, and is called the *index of refraction.* The following table gives the indices of the substances named: —

Air (0° and 760 mm.) . .	1.00029	Crown-glass	1.515 to 1.563
Water	1.3324	Flint-glass	1.541 to 1.710
Alcohol	1.3638	Diamond	2.44 to 2.755
Carbon disulphide . . .	1.6442	Lead chromate . .	2.974

The relative index of refraction for any two of these media may be found by dividing the index of one as given above, by the index of the other. For ordinary purposes, the index of refraction of gases may be neglected; the index of refraction for light passing from air may be considered as $1\frac{1}{3}$ for water; $1\frac{1}{2}$ for crown-glass; $1\frac{3}{5}$ for flint-glass; $2\frac{1}{2}$ for diamond; and 3 for lead chromate. In this sub-paragraph, refraction has been discussed as if light was homogeneous, i.e., had but one wave-frequency. Subsequently, proper consideration will be given to the fact that light is complex in this respect.

(*d*) The following propositions are often given as the "laws of refraction":

(1) *When radiant energy passes obliquely from one medium to another of greater refractive power, it is bent, at the point of incidence, toward a line that is perpendicular to the surface that separates the two media.*

(2) *When radiant energy passes obliquely from one medium to another of less refractive power, it is bent from the perpendicular.*

(3) *The incident and refracted rays are in a plane that is perpendicular to the refracting surface.*

(4) *For any two media, the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction.*

(*e*) The determination of the direction of the refracted ray may be illustrated as follows: — Let LA represent a ray passing from air into water at *A*. Through *A*, draw *CD* perpendicular to the refracting surface. The index of refraction for the two media is $\frac{4}{3}$. From *A* as a center and with radii that are to each other as 4:3, draw concentric circles. Prolong *LA* to *E*. From *v*, the intersection of *AE* with the circumference of the inner circle, draw *vp* parallel to *CD*. Through *p*, the intersection of this line with the circumference of the outer circle, draw *AK*, the line sought. Drawing *pq*, *vw*, and *mn* perpendicular to *CD*, it may be shown geometrically that

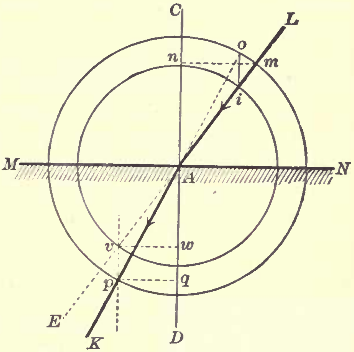


FIG. 260.

$$\frac{\sin LAC}{\sin KAD} = \frac{mn}{pq} = \frac{4}{3},$$ the index of refraction for air and water.

(*f*) If the ray passes in the opposite direction, i.e., from water into air, the process is the reverse of that just indicated. Let *KA* represent the incident ray. Through *p*, draw *pv*. Through *v*, draw *EA* and prolong it to *L*. *AL* is the direction sought. In some cases it will be more convenient to use the equivalent process of continuing *KA* to *o*, drawing *oi* parallel to *CD*, and drawing the refracted ray from *A* through *i*.

(*g*) According to the forms and relative positions of their refracting surfaces, there are three kinds of refractors; plates, prisms and lenses.

Total Reflection.

Experiment 233. — Place the bottle used in Experiment 231, upon a block on the table. Invert the card so that its horizontal slit is near the bottom of the bottle. Place a mirror on the table and close to the card. With a hand-mirror, reflect a sun-beam downward upon the mirror on the table so that it will be reflected obliquely upward, passing through the slit in the card and through the water toward *i*. Diffuse crayon-dust through the air near the bottle, and notice the refraction of the beam as it leaves the water. Bring the slit gradually higher, changing the position of the hand mirror so that, as the rays pass through the slit and upward through the water to *i*, the angle of incidence is gradually increased. Notice that the angle of refraction increases more rapidly than the angle of incidence. As the angle of incidence changes from 47° or 48° to 49° or 50°, closely observe the refracted light which approaches the refracting surface more and more closely. When the angle of incidence has a certain magnitude, the refracted ray coincides with the surface of the water; i.e., the angle of refraction has reached its maximum value, 90°. If the angle of incidence is still further increased, the light cannot emerge at *i*, but will be reflected downward as if the plane between the water and the air was a perfect mirror.

Experiment 234. — Place a bright spoon in a tumbler of water with the handle leaning from you. Hold the tumbler considerably above the level of the eye. Notice that you see not only the lower part of the spoon in the water but also an image of the shank of the spoon above the upper surface of the water. The free liquid surface glistens and reflects as does a mirror.

FIG. 261.

285. Total Reflection. — When a ray of light passes obliquely from a medium of higher to one of lower refractive power, the angle of refraction is always greater than the angle of incidence. The angle of incidence may be increased until the angle of refraction is 90°, as represented by the ray, *DFM*. A further increase in the angle of incidence cannot result in an increase of the angle of refraction. Consequently, the ray cannot obey the laws of refraction,

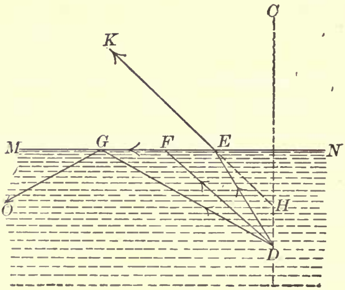


FIG. 262.

but does obey the laws of reflection as represented by the ray, *DGO*. *It is totally reflected at the point of incidence back into the former medium.* The angle of incidence at which the effect changes from refraction to internal reflection is called the *critical angle.*

(*a*) The magnitude of the critical angle varies with the media employed. For (air and) water, it is about 48½°; for crown-glass, about 41°; for diamond, about 24°. The reflection is called "total" because all of the incident light is reflected, which is never the case in ordinary reflection.

(*b*) To construct the critical angle, draw concentric circles as in Fig. 260, the ratio of their radii being the index of refraction for the media used. Remember that the emergent ray must graze the surface of the water, and reverse the process described in § 284 (*f*). At the point where *AN* intersects the inner circle, erect a perpendicular. The point where this normal intersects the outer circle will lie in the prolongation of the ray incident at *A*.

286. Cause of Refraction. — It is easy to conceive of the motions of the ether as being hindered by the particles of the matter that is permeated by the ether. Thus, when ether waves that constitute light are transmitted through glass, they are hindered by the molecules of the glass, and impart some of their motion to those molecules; i.e., a part of the light is absorbed. When a beam of light, as

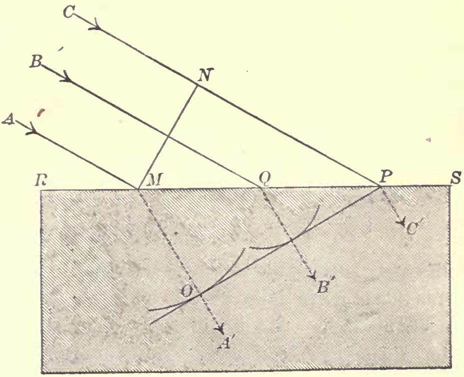


FIG. 263.

represented by the rays *A*, *B*, and *C*, moves forward in the air, the wave-front, *MN* (see § 273), continues parallel to itself and moves forward in a straight line. As the wave-front advances beyond *MN*, the ray, *A*, enters the glass, while *B* and *C* are still in the air. The advance of *A* in the glass is retarded by the glass so that, while *C* is passing in air from *N* to *P*, *A* traverses the shorter path, *MO*. This retardation of *A* and the corresponding retardation of *B* change the direction of the plane that is attached to the waves, and set it in the new position indicated by *OP*. All of the rays having entered the glass, the wave-front again moves forward in a straight course, normal to *OP*, representing the new direction of propagation. *In passing into the glass the direction of the beam was changed, a direct result of a change of speed at the surface of the glass. This phenomenon is called refraction.* The beam was bent toward a perpendicular to the bounding surface, *RS*. When the beam emerges from the glass, similar changes will take place in inverse order, and the beam will be bent from the perpendicular to the refracting surface.

(*a*) The index of refraction is numerically equal to the ratio between the velocity of the incident light and the velocity of the refracted light.

Refraction by Plates.

Experiment 235. — Draw a straight line of such length that it extends both ways beyond the ends of a piece of thick plate-glass placed upon it. Look obliquely through the glass and from the side of the line, and notice the apparent displacement of the part of the line seen through the plate.

287. Refraction by Plates. — When radiant energy passes through a medium bounded by parallel planes, the refractions at the two surfaces are equal and contrary in direction. The direction after passing through the plate is parallel to the direction before entering the plate; *the rays merely suffer lateral aberration.* Objects seen obliquely through such plates appear slightly displaced from their true position.

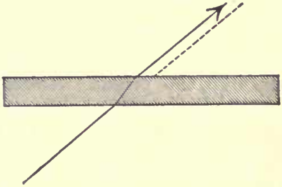


FIG. 264.

288. A Prism is a transparent body with two refracting surfaces that lie in intersecting planes. The angle formed by these planes is called the refracting angle.

(*a*) Let *mno* represent the principal section of a prism. A ray of light from *L* is refracted at *a* and *b*, and enters the eye in the direction *bE*. The object, being seen in the direction of the ray as it enters the eye, appears to be at *l*. An object seen through a prism seems to be moved in the direction of the refracting angle; the rays are bent away from the refracting angle.

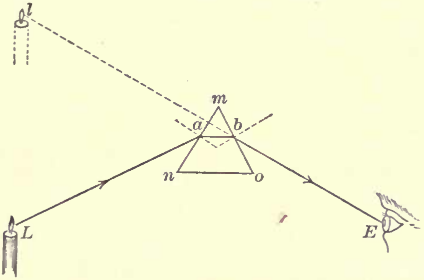


FIG. 265.

(*b*) Cathetal prisms readily yield the phenomena of total reflection as shown in Fig. 266, and are often used when light is to be turned through a right angle.

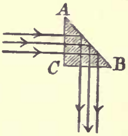


FIG. 266.

Experiment 236. — Make a monochromatic light by sprinkling a little table-salt on the wick of an alcohol lamp. Place the flame on the level of the perforation in one of the postal card screens used in Experiment 204. Back of this screen, place another so that the light from the lamp passing through the perforation of the first makes a spot on the second. Mark the position of the spot. Behind the first screen and close to it, hold a glass prism with its refracting edge uppermost, horizontal, and parallel to the screen. Adjust its height so that the rays passing through the perforation also pass through the prism. Notice that the spot of light on the second screen is moved downward. Mark the new position of the spot.

289. The Angle of Deviation of rays thus refracted is the difference in direction between the incident and emergent rays. In Experiment 236, this angle may be roughly described as the angle between lines drawn from the perforation in the first screen to the two marked positions of the luminous spot on the second screen. In Fig. 265, imagine *La* extended until it intersects *El* at *x*. The angle *Lxl* is the angle of deviation. The angle varies with the magnitude of the refracting angle of the prism, its index of refraction, the wave-length of the light used, and the angle of incidence. Other conditions being similar, a prism gives the least deviation when the angles of incidence and of emergence are equal.

(*a*) The position of a prism for minimum deviation is easily determined by looking through it at an object, as in Fig. 265, and turning the prism until the changing apparent position, *l*, comes to a standstill, and begins to move backward.

290. A Lens is a transparent body the two refracting surfaces of which are curved, or one of which is curved and the other plane. Lenses are generally made of crown-glass which is free from lead, or of flint-glass which contains lead and has greater refractive power. The curved surfaces are generally spherical.

(*a*) With respect to their surfaces, lenses are of two classes, with three varieties of each: —

(1.) Double-convex,
(2.) Plano-convex,
(3.) Meniscus,
Thicker at the middle than at the edges; converging.

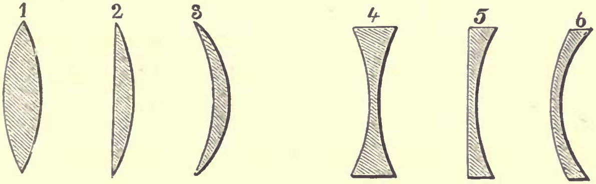


FIG. 267.

The double-convex (biconvex or magnifying) lens may be taken as the type of these; its effects may be considered as produced by two prisms with their bases in contact.

(4.) Double-concave,
(5.) Plano-concave,
(6.) Concavo-convex,
Thinner at the middle than at the edges; diverging.

The double-concave (biconcave) lens may be taken as the type of these; its effects may be considered as produced by two prisms with their refracting edges in contact.

(*b*) A double-convex lens may be described as the part common to two spheres that intersect each other. The centers of the limiting spherical surfaces, as *c* and *C*, are the *centers of curvature*. The straight line, *XY*, passing through the centers of curvature is the *principal axis* of the lens. In the plano-lenses, the principal axis is a line drawn from the center of curvature normal to the plane surface. A point on the principal axis so taken that rays passing through it pierce parallel elements of the refracting surfaces is called the *optical center*. A ray passing through the optical center suffers no change of direction other than a slight lateral aberration that may be disregarded. When the two spherical surfaces are of equal curvature, the

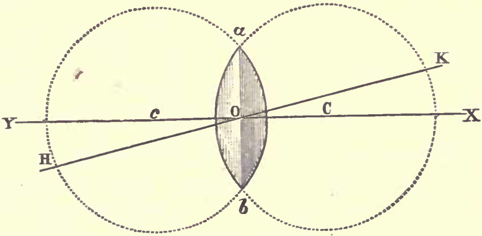


FIG. 268.

optical center is at equal distances from the two faces of the lens, i.e., at its center of volume. For the plano-lenses, the optical center lies on the curved surface; for the meniscus, it lies outside the lens and on the convex side; for the concavo-convex lens, it lies outside the lens and on the concave side. Any straight line, other than the principal axis, passing through the optical center is a *secondary axis.*

(*c*) To trace a ray through a lens, we have only to apply the principles already explained. For example, let *LN* represent a glass biconvex lens (index of refraction, $\frac{3}{2}$) with centers of curvature at *C* and *C′*, and *AB*, the incident ray. From *B* as a center, draw the arcs, *mn* and *op*, making the ratio of their radii equal to the index of refraction, i.e., 2 : 3. Draw the normal, *C′B*. Draw *st* parallel to *C′B*. Draw the straight line *tBDy; BD* is the path of the ray through the lens. From *D* as a center, draw the arcs, *uv* and *wx*, using the same radii as for *mn* and *op*. Draw the normal, *CD*. Draw *yz* parallel to *CD*. Draw *DzA′*, the path of the ray after emergence.

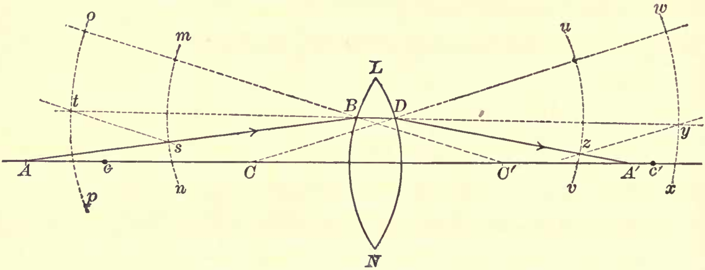


FIG. 269.

Experiment 237. — Hold one of the large lenses of an opera glass or optical lantern in the sun's rays. Notice the converging pencil formed by the light (after passing through the lens) as it passes through air made dusty by striking together two blackboard erasers. The focus and its distance from the lens may be seen. Measure this distance. Hold a similar lens by the other, face to face. Notice that the light after passing through both lenses converges more quickly, lessening the distance of the focus from the lens.

291. The Foci of Convex Lenses may be determined experimentally, but some of their properties are more conveniently studied by the diagrammatical tracing of rays in accordance with the principles and processes already studied. To locate the focus for light diverging from any point, it is necessary to determine the point of intersection of two emergent rays. The problem is much simplified by considering the axis that passes through the point of divergence as the path of one of these rays.

(*a*) For converging lenses, the reciprocal of the principal focal distance equals the sum of the reciprocals of any pair of conjugate focal distances.

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{p'}.$$

(*b*) Experimental work with convex lenses, and a careful study of this formula, develop frequent analogies to the phenomena of concave mirrors, and give rise to several cases as follows:—

(1) When the incident rays are parallel to the principal axis, their focus is called the *principal focus*. With a biconvex lens of crown-glass (index of refraction, $\frac{3}{2}$) the principal focus is at the center of curvature, i.e., the focal length of the lens is equal to the radius of curvature. With a plano-convex lens, the focal length is twice the radius of curvature. In either case, the focus is real.

(2) When the incident rays diverge from a point more than twice the focal distance from the lens, a real focus is formed on the other side of the lens, and at a distance greater than the focal length and less than twice the focal length. (See A and A', Fig. 269.)

(3) When the incident rays diverge from a point at twice the focal distance from the lens, a real focus is formed on the other side of the lens and at the same distance from it. These two points, as c and c' in Fig. 269, are called *secondary foci*.

(4) When the incident rays diverge from a point distant from the lens more than the focal length and less than twice the focal length, a real focus is formed on the other side of the lens and at a distance greater than twice the focal length. This is the converse of the second case. Two foci that are thus interchangeable, like A and A' in Fig. 269, are called *conjugate foci*. The secondary foci are conjugate.

(5) When the incident rays diverge from the principal focus, the

emergent rays will be parallel, and no focus, real or virtual, will be formed. This is the converse of the first case.

(6) When the incident rays diverge from a point nearer the lens than the principal focus, the emergent rays are still diverging, and a virtual focus is formed back of the radiant point.

(7) When the incident rays are converging, a real focus is formed on the other side of the lens at a distance less than the focal length. This is the converse of the sixth case.

(*b*) Each pupil should draw a figure to illustrate each of the foregoing cases.

292. The Foci of Concave Lenses may be located by processes already studied. Such lenses have their centers of curvature, their primary and secondary axes, and their optical centers the same as convex lenses.

(*a*) For diverging lenses, the reciprocal of the principal focal distance equals the difference of the reciprocals of any pair of conjugate focal distances.

$$\frac{1}{f}=\frac{1}{p}-\frac{1}{p'}.$$

(*b*) Experimental work with concave lenses, and a careful study of this formula, develop frequent analogies to the phenomena of convex mirrors, and give rise to several cases as follows: —

(1) When the incident rays are parallel to the principal axis, the emergent rays diverge as if they came from a virtual focus, which is called the principal focus. With a biconcave lens of glass (index of refraction, $\frac{3}{2}$), the principal focus is at the center of curvature. With a plano-concave lens, the focal length is twice the radius of curvature.

(2) When the incident rays are diverging, the focus is virtual and at a distance from the lens less than the focal length. As the radiant point approaches the lens, the focus also approaches the lens.

(3) When the incident rays are converging, the effects are varied according to the degree of convergence. If the point of convergence is nearer the lens than the principal focus, a real focus will be formed at a distance greater than the focal length of the lens. If the point of convergence is at the principal focus, the emergent rays will be parallel, and no focus will be formed. If the point of convergence is further from the lens than the principal focus, a virtual focus will be formed.

(*b*) Each pupil should draw a figure to illustrate each of the foregoing cases.

Images.

Experiment 238. — Repeat Experiment 228, and measure the focal length of the lens.

Experiment 239. — Place a candle, a convex lens, and a screen in line as shown in Fig. 270, the distance of the candle from the lens being a little greater than the focal length of the lens. Adjust the position of the screen until a sharply defined image of the candle is projected upon it. Place the eye back of the screen and have the screen removed; the inverted image may be seen suspended in mid-air. Burn touch-paper under the image, and notice its projection on the screen of smoke. Replace the screen first used.

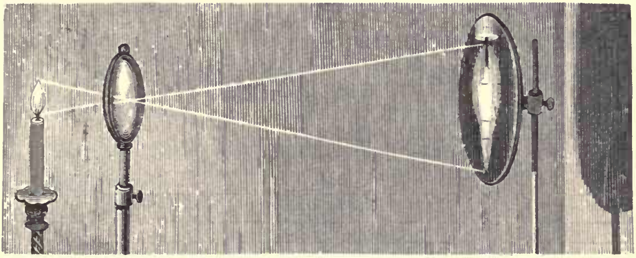


FIG. 270.

Experiment 240. — With candle and screen in positions as described in Experiment 239, adjust the position of the lens so that the flame and the image of the flame are of the same size. Measure the distance of the screen from the candle, and compare a quarter of that distance with the focal length of the lens.

293. Images Formed by Lenses consist of the conjugate foci of the several points in the surface of the object presented to the lens and may, therefore, be real or virtual. The construction for such images is closely analogous to the process employed with mirrors.

(*a*) The focus of each point chosen may be determined by tracing two rays from the point, and locating their real or apparent intersection after emerging from the lens. The two rays most convenient for this purpose are the one that lies along the secondary axis of the point, and the one that lies parallel to the principal axis of the lens. For example, from *A* and *E*, extremities of an arrow, draw the secondary axes, *AOa* and *EOe*. From *A*, draw *AB* parallel to the principal axis, *XY*. Determine the direction of *BD* by construction. From *D*, draw the path of the emergent ray through the principal focus, *F*. It intersects the secondary axis at *a*, the conjugate focus of the radiant point, *A*. In similar manner, the conjugate focus of the point, *E*, may be located at *e*. The points, *a* and *e*, mark the extremities of the image of the object, *AE*.

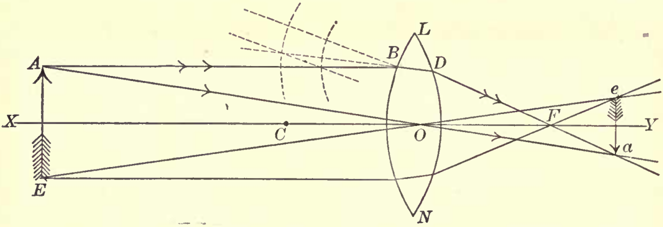


FIG. 271.

(*b*) An examination of Fig. 271 shows that the linear dimensions of object and image are directly as their respective distances from the center of the lens; they will be virtual or real, erect or inverted, according as they are on the same side of the lens, or on opposite sides.

Experiment 241. — Select a large biconvex lens, and cut a cardboard disk of the same diameter. Punch a ring of small holes near the circumference of the disk, and cut a hole about 2 cm. in diameter at its center. Cut a notch in the border of one of the small holes as a distinguishing mark. Place the lens in the path of a beam from the porte lumière, and cover one of its faces with the perforated disk. Hold a screen near the lens and so that the refracted rays fall upon it. Slowly move the screen away from the lens until you find the focus of the light that passes through the small holes. Moving the screen still further, you will find another focus for the light that passes through the central opening in the disk. Notice that the rays that pass through the marginal holes cross before reaching the screen, and form a ring of luminous spots around the central image.

294. Spherical Aberration.—The rays that pass through a spherical lens near its edge are deviated more than those that pass nearer the center. They, therefore, converge nearer the lens. *This unequal deviation is called spherical aberration.* The indefiniteness of focus causes a blurring of the image. In practice, the marginal rays are often cut off by an annular screen called a diaphragm, or the curvature of the lens is lessened toward its edge. A lens thus corrected for spherical aberration is called *aplanatic.*

CLASSROOM EXERCISES.

1. Remembering the varying density of the earth's atmosphere, draw a diagram showing that the sun may be seen before it has astronomically risen, and after the true sunset, i.e., after it has dipped below the western horizon.

2. (*a*) Name, and illustrate by diagram, the different classes of lenses. (*b*) Explain, with diagram, the action of the burning-glass.

3. Draw circles so that parts of their circumferences may represent the curved surface of a meniscus, a biconcave, and a concavo-convex lens.

4. (*a*) Describe the phenomena of total reflection. (*b*) Show, with diagram, how the secondary axes of a lens mark the limits of the image.

5. Construct the critical angle for air and water.

6. Show how a beam of light may be bent at a right angle by a glass prism.

7. Trace a ray through a biconcave lens, using the process employed in § 290 (*c*) for the biconvex lens.

8. Trace a ray through a biconvex lens for the location of its principal focus.

9. Trace a ray through a biconcave lens for the location of its principal focus.

10. Through what point does the line joining the conjugate foci of a convex lens always pass?

11. Construct the images formed by a convex lens under the six following cases and describe each image: (*a*) when the incident rays are practically parallel; (*b*) when the object is a little beyond a secondary focus; (*c*) when the object is at a secondary focus; (*d*) when the object is between secondary and principal foci; (*e*) when the object is at the principal focus; (*f*) when the object is between the principal focus and the lens.

12. (*a*) The focal distance of a convex lens being 6 inches, determine the position of the conjugate focus of a point 12 inches from the lens. (*b*) 18 inches from the lens.

13. The focal distance of a convex lens is 30 cm. Find the conjugate focus for a point 15 cm. from the lens.

14. If an object is placed at twice the focal distance of a convex lens, how will the length of the image compare with the length of the object?

15. A small object is 12 inches from the lens; the image is 24 inches from the lens and on the opposite side. Determine (by construction) the focal distance of the lens.

16. A candle-flame is 6 feet from a wall; a lens is between the flame and the wall, 5 feet from the latter. A distinct image of the flame is formed upon the wall. (*a*) In what other position may the lens be placed, that a distinct image may be formed upon the wall? (*b*) How will the lengths of the images compare?

17. Why does a sphere under water look like a spheroid?

18. When clear glass is pounded into small particles, it becomes opaque. Explain.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Lenses; spy-glass; wooden cubes grooved and fitted as described in Exercise 12.

1. Trace a ray passing obliquely from air into glass (index of refraction, 1.5).

2. Trace a ray passing obliquely from glass into air.

3. Trace a ray passing obliquely from air into water (index of refraction, 1.3).

4. Trace a ray passing obliquely from water into air.

5. Trace a ray passing obliquely from air into diamond (index of refraction, 2.5).

6. Trace a ray passing obliquely from water into glass.

7. Trace a ray passing through water toward air so that the angle of incidence is 50°. (See § 285, *a*.)

8. Hold a test-tube partly immersed in water so that its length is slightly inclined from a vertical. Look down through the water upon the immersed part of the air-filled tube, and notice that it looks like highly polished silver. Fill the test-tube with water, and write an explanation of the change in its appearance.

9. Make a concave air-lens and place it in water. Make a convex water-lens and place it in air. Compare the effects of the two lenses upon beams of light that pass through them.

10. Adjust a candle-flame and a screen on opposite sides of a biconvex lens so that a sharp image of the flame is projected on the screen when flame and screen are at equal distances from the lens. Record the focal length of the lens, the names of the positions occupied by flame and screen, and a description of the image. Exchange the positions of the flame and the screen, and test the principle of "reversibility." Slowly diminish the distance between lens and candle until it is impossible to place the screen so as to obtain an image. Measure this distance, compare it with the focal distance of the lens, and indicate the significance of the comparison. With the lens at four different distances from the screen, form images. In each case, measure the linear dimensions of flame and image, and the distances from the lens to flame and image. Find the ratio between the distances and the corresponding dimensions, and compare the ratios.

11. Focus a spy-glass or small telescope on an object a mile or more distant. The rays coming from the object to the eye will be practically parallel. Place a lens, the focal length of which you are to measure, in front of the telescope. Paste a small-type newspaper-clipping on a piece of cardboard, and look at it through the telescope and lens. Adjust the position of the cardboard so that the printing appears distinct. Measure the distance of the cardboard from the lens. Obtain the average of several such trials. Record a discussion of the proposition that this average distance is the focal length of the lens.

12. Provide two wooden cubes with edges about 4 cm. long and with the grain of the wood, cut in each block a groove about 2 cm. deep and wide enough to admit the edge of the meter rod as shown in Fig. 272. Provide common screws so that the blocks may be fixed in any position on the rod. Provide a circular biconvex spec-

tacle-lens having a known focal length of not less than 12 cm. and not more than 16 cm. Mount this lens on one of the grooved blocks, *e*. It may be held in place between slotted brass strips each fastened to the block by a screw. Upon the other block, mount a cardboard screen about 8 cm. square. This screen, *s*, may be carried in a groove sawed across the block, *a*. As an object the image of which is to be projected upon the screen, cut a small cross in the varnish on the lamp-chimney or in the paper cylinder mentioned in Experiment 206. Give the top of the cross some distinguishing shape or mark so that you can tell whether its image is erect or inverted. Support the meter rod horizontally so that the center of the lens shall be at the height of the center of the cross, and so that *m*, the end of the meter rod, shall be just under the cross. Set the screen at a distance from the cross equal to about three times the focal length of the lens. Slide the lens along the rod, seeking for it a position that gives upon the screen a clear image of the cross. If no such position for the lens can be found, move the screen 1 or 2 cm. further from the object, and renew the search for the desired position of the lens. If necessary, move the screen further and further from the object until you are able to place the lens so that a distinct image is obtained. When it is found, record a description of the image, as erect or inverted, and magnified, diminished, or the same size. Read from the rod the distance from the cross to the lens, and enter it as the first number in a second column headed "Object-distances." Make a corresponding entry of the distance from the screen to the lens in a third column headed "Image-distances." Without changing the image-distance, try to secure a distinct image with the lens in any other position. If you can, make the three entries as before. When you have exhausted the possibilities of object-distance with this first image-distance, move the screen 10 cm. further from the cross, secure a good image, and record the description and distances as before. Move the

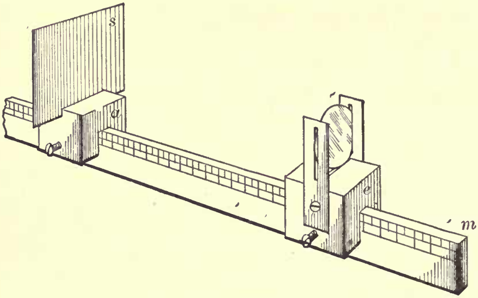


FIG. 272.

screen 10 cm. further from the cross, adjust the lens, and make the record as before. Continue the work until you have recorded at least five pairs of distances. Compare your results with those of Exercise 11, p. 364. To your tabular-record, add a fourth column, each entry in which is the sum of the two recorded distances. Such sums will represent the distances of image from object. Head the column "Total distances." In a fifth column, enter the quotients obtained by dividing the several total distances by the focal length of the lens. Try to get a quotient as small as 3, and if you fail, give a good reason for your failure.

Using the records made in this exercise, test the accuracy of the statement that the reciprocal of the focal length equals the reciprocal of the object-distance plus the reciprocal of the image-distance, or of the equivalent statement that the product of the object and image distances equals the sum of those distances multiplied by the focal length.

If you were told that in such an experiment the object-distance was 20 cm. and the image-distance 60 cm., could you theoretically determine any other object-distance and image-distance for the same positions of object and screen?

13. In similar manner, experimentally determine the effect that the position of the object has upon the character of the images formed by a concave lens, and write a brief discussion of the results attained.

14. Focus a magnifying glass on a finely divided decimeter scale. Hold a similar scale at the distance of distinct vision (about 25 cm.). With one eye, look through the lens at the first scale and with the other eye look directly at the other scale. By continued trial, the eyes will become accustomed to the unusual conditions, and the two images will appear as if one was superposed on the other. Then count how many divisions of the magnified scale correspond to a certain number of divisions of the other scale. Divide the latter number by the former to determine the magnifying power of the lens.

15. Determine the focal distance of the lens used in Exercise 14 and call it f. Call the distance taken in that exercise as the distance of distinct vision, v. Determine the value of $\frac{v}{f} + 1$ and compare it with the magnifying power of the lens as determined in Exercise 14.

V. SPECTRA, CHROMATICS, ETC.

Analysis.

Experiment 242. — Admit a sunbeam through a small opening in the shutter of a darkened room. In the path of the beam, place a prism, as shown in Fig. 273. Instead of the colorless image of the sun at *E*, there appears upon the white screen a many-colored band changing gradually from red at the lower end, through all the colors of the rainbow, to violet at the upper end.

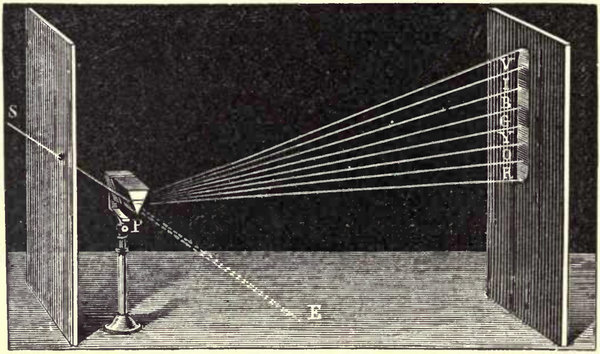


Fig. 273.

Experiment 243. — Paste a strip of white paper 3 × 0.2 cm. upon a black card. Upon a similar card, paste, end to end, strips of red, of white, and of blue paper, each 1 × 0.2 cm. In a well-lighted room, place the first card with the white strip vertical. Hold a prism with its refracting edges vertical, and look through it at the white strip. On its way to the eye, the beam of white light from the strip will be separated into differently colored parts. You will see a colored band instead of the white strip. Similarly, view the tri-color strip on the other card, and carefully compare the three colored bands that correspond to the three parts of the strip.

295. Dispersion. — *The separation of differently colored rays by refraction is called dispersion.* Experiment 242 shows that white or colorless light, like that of the sun, is a mixture of radiations of varying color and refrangibility. The differences in deviation arise from differences of wave-length, the angle of deviation increasing as the wave-length diminishes.

296. Spectra. — *The many-colored image of the sun projected on the screen in Experiment* 242 *is called a spectrum.* It is called a solar spectrum when the source of light is under consideration, and a prismatic spectrum when the method of producing it is under consideration. Most incandescent bodies emit light of varying wave-length and refrangibility.

(*a*) These prismatic colors are generally described as violet, indigo, blue, green, yellow, orange, and red. The initial letters of these terms form the meaningless, mnemonic word "vibgyor."

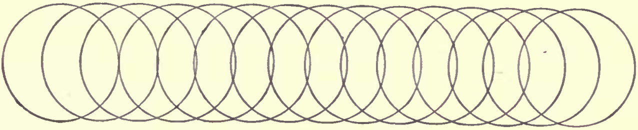


Fig. 274.

In fact, the gradations of color are imperceptible. The differently colored images of the sun overlap, as shown in Fig. 274. Consequently, such an opening in the shutter gives an impure spectrum.

Synthesis.

Experiment 244. — Repeat Experiment 242, and hold a second prism in a reversed position close behind the first. The light dispersed by the first prism will be reunited by the second, and emerge as colorless light. The two prisms have the effect of a plate with its refracting faces parallel.

Experiment 245. — Let light that has been dispersed by a prism fall upon an achromatic convex lens as shown in Fig. 275. It will be refracted to a focus and recombined to form white light. Hold a card between the prism and the lens so as to cut off the red light and notice the focus of what remains. Similarly cut off the violet light,

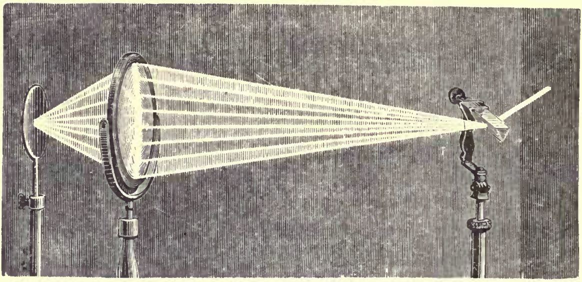


FIG. 275.

and again notice the focus of what remains. A concave mirror may be used to reflect the light to a focus instead of using the lens as above described.

Experiment 246.—Make a "Newton disk," painting the prismatic colors in proper proportion as indicated by Fig. 276, or pasting sectors of properly colored paper upon cardboard. It is better to divide the surface given to each color into smaller sectors arranged alternately as shown in Fig. 277. Fasten this disk to a large top, or to a whirling table, and cause it to revolve rapidly. The "persistence of vision" blends the colors, and the disk appears grayish white.

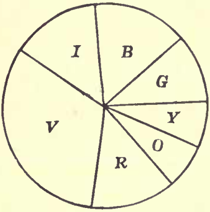


FIG. 276.

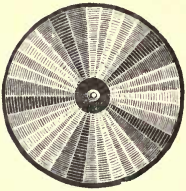


FIG. 277.

Disks about 15 or 20 cm. in diameter may be cut from colored paper, and a hole cut at the center of each of such size that the disks may be slipped over the spindle of the whirling table. By cutting each along a radial line, the several disks may be worked into each other as shown in Fig. 278, and in such a way as to expose the several colors to view in any desired proportion.

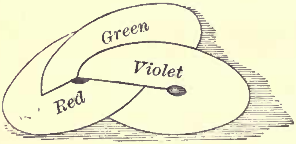


FIG. 278.

Experiment 247. — Hold a hand mirror near the dispersing prism so as to reflect the refracted light to a distant wall or ceiling. Give a rapid, angular motion to the mirror so that the spectrum moves to and fro very quickly in the direction of its length. The spectrum changes to a band of white light with a colored spot at each end.

297. The Composition of White Light. — We have now shown, by both analysis and synthesis, that *white light is composed of the prismatic colors.* We have decomposed white light into its constituents, and recombined these constituents into white light.

Experiment 248. — With a double-convex lens in a darkened room project on the screen an image of the aperture in the shutter. The white image will be fringed with color.

298. Chromatic Aberration. — Because of their greater refrangibility, the focus of the violet rays is nearer the lens than the focus of the red rays, as illustrated in Fig. 279. If the screen is as near the lens as the focus marked v, the outer fringe is red; if the screen is as far from the lens as the focus marked r, the outer fringe is violet. *This difference in the deviation of differently colored rays is called chromatic aberration.*

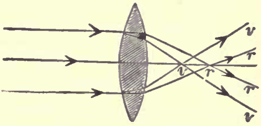


FIG. 279.

(*a*) A double-convex lens of crown-glass may be combined with a plano-concave lens of flint-glass so as to overcome the dispersive effect for some of the colors without overcoming the converging effect. As such a compound lens forms an image that is nearly free from the fringe of spectral colors, it is called an *achromatic lens.*

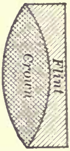


FIG. 280.

Experiment 249. — Gradually raise the temperature of a platinum wire by an electric current. The first radiations emitted are those of "obscure heat"; i.e., they affect the nerves of general sensation only. The vibrations increase in frequency and amplitude with the temperature, and, at about 525°, the eye perceives the wire as a dark red line. As the temperature continues to rise, waves of shorter and shorter wave-length are added, while those previously emitted are increased in amplitude. The wire successively appears orange and yellow and then becomes white hot, the light emitted being exceedingly complex.

299. Color *is a property of light, and depends upon wave-length.* Thus, the relation between color and light is the same as that between pitch and sound.

(*a*) The wave-lengths that correspond to the several prismatic colors as they appear in the solar spectrum are as follows: —

Violet,	4,059	Green,	5,271	Orange,	5,972
Indigo (violet-blue),	4,383	Yellow,	5,808	Red,	7,000
Blue (cyan-),	4,960				

These magnitudes are for the middle points of the several colors, and represent ten-millionths of a millimeter. Light of only one wave-length is said to be *monochromatic or homogeneous.*

(*b*) An incandescent body emits light with wave-lengths that grade imperceptibly from values less to values greater than any of those given above. When the wave-lengths are much less or greater than those above given, the radiation is incapable of exciting vision. Within the limits of visibility are an indefinitely great number of wave-lengths, and a correspondingly great number of colors. When light of all grades of refrangibility within these limits is blended, as it is in sunlight, the resultant effect is white or colorless light. When the light that corresponds to some of the prismatic colors is wanting, the resultant effect of blending what is present is colored light. Many artificial lights are deficient in some of these wave-lengths.

(*c*) Since the wave-length for the extreme red is approximately twice that of the extreme violet, it may be said that the range of the visible spectrum is only about one octave. The full spectrum, from the extreme ultra-violet to the longest waves yet recognized, embraces more than seven octaves. These invisible spectra have been explored with delicate thermoscopes and by photography. Wave-lengths twenty times that of the visible red, and corresponding to the temperature of melting ice, have thus been detected in the radiation of the surface of the moon. The method of phosphorescence is also employed, while fluorescence is made use of in studying the ultra-violet region.

(*d*) Every sensation of light that the human eye experiences is the effect of impressing about five hundred trillion (5×10^{14}) waves upon the ether each second. If the frequency of the ether waves is much lower, the result is chiefly heat.

Color of Bodies.

Experiment 250. — Repeat Experiment 242, and hold a piece of deep red glass between the slit in the shutter and the prism. Notice that the intensity of illumination is reduced less in the red than in any other part of the spectrum.

Experiment 251. — Paste three strips of paper, one white, one vermilion-red, and one aniline-violet, each about 3 × 0.2 cm., upon sheets of black cardboard. Successively place these strips in a strong light, and look at them through a prism held with its refracting edge parallel to the length of the strips. Carefully compare the coloring of the images of the three strips thus viewed.

Experiment 252. — Paint three narrow strips of cardboard, one vermilion-red, one emerald-green, and the other aniline-violet. Be sure that the coats are thick enough thoroughly to hide the cardboard. When dry, hold the red strip in the red of the solar spectrum; it appears red. Move it slowly through the orange and yellow; it grows gradually darker. In the green and colors beyond, it appears black. Repeat the experiment with the other two strips, and carefully notice the effects.

Experiment 253. — Make a loosely wound ball of candle-wick; soak it in a strong solution of common salt in water; squeeze most of the brine out of the ball; place the ball in a plate, and pour alcohol over it. Take it into a dark room and ignite it. Examine objects of different colors, as strips of ribbon or cloth, by this yellow light. Only yellow objects will have their usual appearance.

Experiment 254. — In a clear tumbler or large beaker of water, dissolve a little white castile soap, or stir a few drops of an alcoholic solution of mastic. Hold the vessel in the hand, and examine the liquid by transmitted sunlight. Notice that it appears yellowish-red. In a small test-tube, either liquid will appear colorless. Place a black screen behind the vessel and examine the liquid by reflected sunlight. Notice that it appears blue.

300. The Color of a Body *depends upon the light that the body reflects or transmits to the eye.* The color of the light thus sent to the eye depends partly upon the nature of the incident light, and partly upon the nature of the body. Some bodies have a power that may be described as selective absorption, reflecting or transmitting light of certain wave-lengths, and absorbing the others. When a house is painted yellow, the painters lay on not a yellow color but a substance that absorbs from sunlight all the colors except yellow. If the light incident upon a body has only the wave-lengths that the body absorbs, the body can send no light to the eye and, therefore, appears black.

(*a*) A red ribbon is red because it reflects light of the particular wave-length that corresponds to the sensation of redness, and absorbs the rest. A white ribbon is white because it reflects the same proportion of all the light that constitutes sunlight. A piece of blue glass is blue because it transmits or reflects light of the particular wave-length that corresponds to the sensation of blueness, and absorbs the rest. Glass that absorbs none of the incident light is colorless.

(*b*) The earth's atmosphere freely transmits yellow and red light, and freely reflects blue light after the manner of the solutions used in Experiment 254. The blue of the sky is due to light thus reflected. When the sun is near the horizon, its light traverses a thicker layer of air than it does at noon. Hence the predominance of yellow and red in the light of the morning and evening hours.

Complementary Colors.

Experiment 255. — Repeat Experiment 246, and receive the red and orange light upon a prism of small refracting angle placed behind the lens. The prism will deflect the red and orange, and form a reddish colored image at *n*. The violet, indigo, blue, green, and yellow light, not caught by the prism, will unite at *f* to form a greenish image. When the prism is removed, the reddish light that fell at *n*, and the greenish light that fell at *f*, unite to form white light. By shifting the position of the prism, other parts of the beam may be deflected to the side, giving rise to various pairs of colors, as orange and blue, etc. Each of these pairs of colors has this property in common, that when united they form white light.

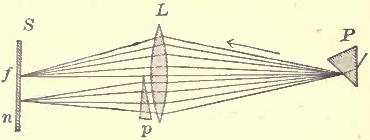


FIG. 281.

Experiment 256. — Again repeat Experiment 246, holding a paper screen in front of the lens. Mark the positions of the yellow and blue parts of the spectrum on the paper, and cut narrow slits across the spectrum so as to allow the yellow and the blue light to pass through the lens. These two simple colors will be blended by the lens, forming a light that is nearly white. The effect of mingling any two colors may be determined in this way.

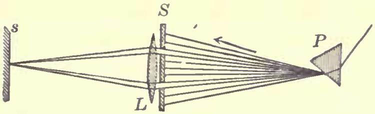


FIG. 282.

Experiment 257. — Lay a piece of blue paper and a piece of yellow paper, each about 5 cm. square, upon a black horizontal surface and about 5 cm. apart. Hold a piece of plate glass 10 or 15 cm. above the colored papers and in a vertical plane that passes between them. Looking obliquely downward, you may see one of the papers by light that the glass transmits, and an image of the other paper by light that the glass reflects. By trial, you can find positions for the glass and eye such that the object seen by the transmitted light and the image produced by the reflected light overlap each other with a blending of their colors.

301. Complementary Colors *are any two colors the blending of which produces white light.* If all the colors of the solar spectrum are divided into two parts and the colors in each part are blended, each resultant color evidently has what the other needs to make white light. Either of such colors is said to be complementary to the other. When complementary colors are placed in proximity, each heightens the effect of the other, by contrast.

(*a*) Any two colors standing opposite each other in Fig. 283 are complementary to each other. If such colors are blended, the resultant is white light; if any two alternate colors are blended, the resultant will be the color that appears between them in the figure.

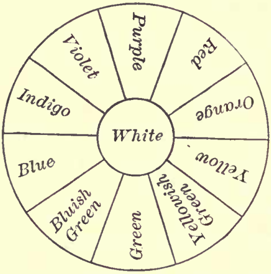


FIG. 283.

Experiment 258. — Cut holes 8 cm. in diameter at the middle of two boards each 18 × 10 cm. and, in each case, remove part of the strip remaining at the middle of one of the long edges. Thinly coat the circular edges with melted beeswax or paraffine. Provide four pieces of clear window glass, 10 × 18 cm., and fasten one of them with marine glue to each side of each board, thus covering the openings. The glue and the surfaces to be joined should be heated. The glue may be thinned, if necessary, with naphtha. Fill one of these "chemical tanks" with a solution of copper sulphate, and the other with a solution of potassium dichromate. Repeat Experiment 242, and hold the yellow solution between the shutter and the prism. Notice that the solution absorbs the radiations of shorter wave-length and thus cuts the violet, indigo and blue from the spectrum. Change the tanks, and notice that the blue solution absorbs the radiations of greater wave-length and thus cuts the yellow, orange and red from the spectrum. If both solutions are interposed, the green alone will be freely transmitted.

Experiment 259. — With a yellow-colored crayon, draw a broad mark on the blackboard. Along the same line, draw a similar mark with a blue crayon. Also mix a small quantity of chrome-yellow with a like quantity of some ultramarine-blue pigment. The blending of the blue and the yellow colors in Experiment 256, gave a white; the blending of the yellow and the blue pigments gives a green.

302. Mixing Pigments is a very different thing from mixing colors, as has just been illustrated. In the majority of cases, the scattering of incident light takes place not only at the surface of bodies but also at distances below the surface. This distance is generally small but in some cases it is considerable. When sunlight falls obliquely upon a piece of blue glass, part of the incident light is reflected at the anterior surface of the glass; the color of this reflected light is white. Another part of the incident light is reflected from the posterior surface; the color of this light that has twice traversed the thickness of the glass is blue, the radiations of other wave-lengths having been absorbed. The difference in the colors may be seen by receiving the reflected light upon a screen. In the case of pigments, most of the scattered light comes from below the surface. In Experiment 259, the yellow pigment removed most of the violet, indigo and blue by such absorption. The blue pigment similarly removed most of the yellow, orange and red. (Compare Experiment 258.) The radiations that escaped both were of the particular wave-length that constitutes green: —

~~vib~~g~~yor~~.

Experiment 260. — Fill the bulb of an air thermometer with clear water. Cut a circular opening (somewhat smaller than the bulb) in a large sheet of cardboard. Reflect a sunbeam into a darkened room so that it shall pass through the opening in the cardboard and fall upon the water-filled bulb. Adjust the position of the bulb until circular spectra are thrown by the bulb back upon the cardboard screen.

303. A Rainbow *is a solar spectrum formed by water-drops.* The necessary conditions are: —

(1) A shower during sunshine.

(2) That the observer shall stand with his back to the sun, and facing the falling drops.

(*a*) The center of the circle of which the rainbow forms a part is in the prolongation of a line drawn from the sun through the eye of the observer. This line is called the *axis of the bow*.

(*b*) Often, a second bow is visible. The inner or *primary bow* is much brighter than the other; the outer or *secondary bow* has the order of colors reversed, as indicated in Fig. 284.

(*c*) The rays of sunlight incident upon the rain-drops are refracted as they enter the drop, internally reflected, and chromatically dispersed, as illustrated in Experiment 260. The drop at *V* has an angular distance of 40° from *EO*, the axis of the bow, and sends violet rays to the eye at *E*, and red rays below the eye. Other drops, at the same angular distance from *EO*, send violet light to the eye and, therefore, form a violet-colored circular arc of which *OV* is the radius of curvature. Similarly, the angle of deviation for red rays is such that the drop, *R*, at an angular distance of 42° from *EO*, sends red rays to the eye of the observer and violet rays above the eye. Other drops at the same angular distance send red light to the eye and, therefore, form a red-colored circular arc, of which *OR* is the radius of curvature. The primary bow, therefore, has an angular width of 2°, the other prismatic colors ranging in regular order between the violet and the red.

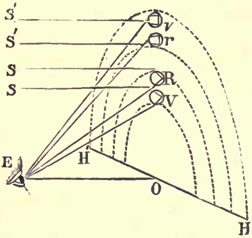


Fig. 284.

(*d*) The secondary bow involves two reflections within the rain-drops, as shown at *r* and *v*. The drops that send these rays to the eye are at the angular distances of 51° and 54° respectively from *EO*. As some light is lost at each reflection, the secondary bow is fainter than the primary.

Pure Spectra.

Experiment 261. — Cut a very narrow slit, 2 or 3 cm. long, in a piece of tin or of tin-foil, and fasten the sheet over the opening in the shutter of a darkened room so that the slit shall be horizontal. Hold a prism about 1.5 m. from the slit and with its edges horizontal. Looking through the prism at the slit, turn the prism about its axis until the colored image of the slit is at the least angular distance from the slit itself. The colors of the image will show with a greater distinctness than before observed.

Experiment 262. — Change the position of the tin-foil used in Experiment 261, so that the slit shall be vertical. Using a convex lens with a focal distance of about 30 cm., project an image of the slit upon a white screen at a considerable distance. Place a glass or a carbon disulphide prism near the lens, and between it and the screen. See that its edges are vertical, and that it is properly placed for minimum deviation. Shift the position of the screen so that the rays from the prism fall normally upon it, but keeping it at the same distance from the lens. The spectrum visible upon the screen will be more distinct than any before observed.

304. A Pure Spectrum is made up of a succession of colored images with little or no overlapping. The first requisite in preventing the overlapping, like that of the impure spectrum described in § 296 (*a*), is that the slit be very narrow.

(*a*) A *spectroscope* is an instrument used to produce a spectrum of the light from any source, and for its study. It affords a delicate means of chemical analysis and is one of the most powerful aids to modern science. In one of its simple forms it consists of, —

(1) A collimator, *C*, a tube with an adjustable slit with parallel edges at the outer end through which the light enters, and at the other end a collimating lens that brings the rays into a parallel beam.

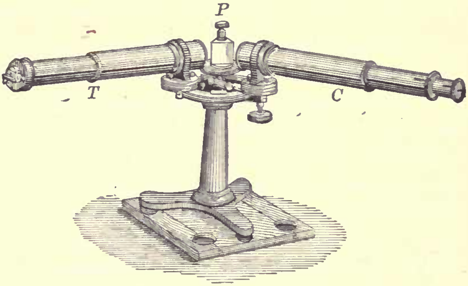


FIG. 285.

(2) A prism, *P*, or a series of prisms, that receives the radiation from *C*, and disperses it, thus forming a spectrum.

(3) A telescope, *T*, through which the magnified image of the spectrum is viewed. The spectrum is received directly upon the retina of the eye and may be distinctly seen even when the radiation is feeble.

It is often necessary to determine the position of certain lines that appear in the spectrum (§ 307). In such cases, the spectroscope is provided with a third tube that carries a collimating lens, and a transparent plate on which a fine scale has been engraved. Light, as from a candle, enters the outer end of this tube, passes from the collimating lens at the inner end, and is reflected from the face of the prism so that it enters the telescope with the light that is being examined. Thus the spectrum and the image of the scale are viewed simultaneously and in close juxtaposition.

A pocket form of the spectroscope, often called a *direct-vision spectroscope*, has two telescoping tubes. The inner tube carries a series of three or more prisms made of different kinds of glass, and so placed as to overcome the deviation of the light from a straight path and yet to preserve the dispersion. The outer tube carries an adjustable slit for the admission of light which, after dispersion, is received by the eye at the other end of the instrument. Such an instrument is not very expensive, and may be made to answer for the purposes of this book.

Spectrum Analysis.

Experiment 263. — Examine a candle-flame with a spectroscope, and notice that the colored spectrum is continuous through all the prismatic colors. Evidently, the radiation is extremely complex.

Experiment 264. — Dip a platinum wire or a strip of asbestos into a solution of sodium chloride (common salt), and hold it in the almost colorless flame of a Bunsen burner. The sodium vapor colors the flame yellow. Examine this sodium flame with a spectroscope, and notice that the spectrum consists of a bright yellow line instead of the continuous multi-colored band. If your spectroscope was of high dispersive power, it would show that the sodium line is really double. These two fine lines represent wave-frequencies of 508.3×10^{12} and 509.3×10^{12} respectively. If the flame is similarly colored with a solution of chloride of lithium, the bright line spectrum will have a carmine color. If the flame is colored with strontium nitrate, the crimson flame will yield a spectrum composed of bright lines the colors and positions of which are different from those of either of the spectra previously examined. If the flame is colored with a mixture of these three substances, the spectrum will show all of the bright lines previously observed.

305. Spectrum Analysis.—It has long been known that when certain substances are heated they give colored flames, the yellow of sodium, the lilac of potassium, etc., being familiar. Each of the chemical compounds used in Experiment 264 has a metallic base, sodium, lithium, or strontium. The vapors of these metals yielded the spectra observed. Like tuning-forks, free molecules have definite vibration-periods; e.g., the ether waves set up by incandescent sodium have the same frequency whether the sodium is solar, stellar or terrestrial. The characteristic frequency of the radiation thus established determines the relative position of the corresponding spectrum. As the spectra of such substances are characteristic, i.e., no two of them are alike, they may be used for the identification of the several substances that produce them. This method of *analyzing composite radiations, or of identifying substances by the spectra of their incandescent vapors, is called spectrum analysis.*

(*a*) As a condition necessary for the production of the spectrum, the temperature must be so high that the substance to be examined may be vaporized, disassociated, and made incandescent. If a compound gas or vapor is not disassociated at the temperature employed, it gives its own spectrum instead of the spectra of its constituent elements. Having mapped the spectra of all known substances, the presence of new lines in any spectrum would indicate the presence of a substance previously unknown. The quantity of material required for such examination is exceedingly small, a hundred-millionth of a milligram of strontium giving the spectrum characteristic of that element. The chemist is thus provided with a method of qualitative analysis of far greater power than any previously known. By it, the chemistry of the stars has been studied, and the extreme generality of the diffusion of the elements in nature has been shown, and several new elements have been discovered. The method has been successfully applied in the industrial arts.

Experiment 265. — Remove the objective from an optical lantern (§ 323). From the lantern, send a beam of electric or calcium light through a narrow vertical slit in a tin screen. Beyond the screen, place a double convex lens to receive the light that passes through the slit. Beyond the lens, place a prism so as to throw a spectrum on a screen still beyond. Place a Bunsen burner or an alcohol lamp between the lantern and the slit and, in its almost colorless flame, hold a bit of sodium. The metal will burn giving an intense yellow to the flame. Notice that the yellow of the spectrum, instead of being more intensely illuminated, is marked by a dark band. Then hold a piece of tin between the lantern and the flame and so as to cut off the light of the lantern from the upper part of the slit. The upper part of the slit is now traversed by light from the sodium-colored flame, and the lower part of the slit by light from both the lantern and the flame. The image of the slit is inverted, and two parallel spectra are thrown on the screen. One of these is the bright-line spectrum of sodium; the other shows a dark line on a continuous spectrum. Notice that the bright line of one spectrum falls in the same relative position as the dark line of the other spectrum.

Experiment 266. — Place some common salt on the wick of an alcohol lamp. The sodium of the salt will give a yellow tinge to the flame. Let a beam of electric or calcium light pass through this yellow flame and fall upon the collimator slit of the spectroscope; study its spectrum. A dark line crosses the spectrum, which thus becomes discontinuous. Shut off the lantern light, and the sodium flame again gives its bright-line spectrum as before. Turn on the lantern light, and remove the sodium-colored flame. Notice that the spectrum is continuous. Replace the sodium flame, and notice that the dark line of the discontinuous spectrum falls in the same relative position as the bright line of the sodium spectrum, just as if the sodium vapor absorbed light of the same refrangibility as that it emits.

306. Kinds of Spectra. — From the foregoing, it appears that a spectrum may be continuous or discontinuous, and that a discontinuous spectrum may be a bright-line spectrum or a dark-line spectrum. For obvious reasons, dark-line spectra are sometimes called reversed spectra, or absorption spectra.

307. The Fraunhofer Lines. — A spectrum of sunlight is crossed by dark lines, many hundreds of which have been counted and accurately mapped. The more conspicuous of these dark lines are distinguished by letters of the alphabet, as shown in Fig. 286. A few of these dark lines in the solar spectrum are due to absorption in the earth's atmosphere, but by far the greater number originate in the selective absorption of the solar atmosphere itself.

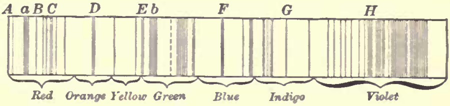


FIG. 286.

(*a*) In accordance with the principles illustrated by the experiments immediately preceding, and as more fully explained in the paragraph immediately following, it is supposed that the nucleus of the sun would give a continuous spectrum if it was not surrounded by gases and metallic vapors that absorb some of the rays to which their own spectra correspond. Just as the D-line corresponds to sodium, so the greater number of the Fraunhofer lines have been identified in the spectra of known terrestrial substances. The presence of at least thirty-six elements in the sun's atmosphere has been thus established, the identity of the absent wave-frequencies indicating the identity of the absorbing media.

(*b*) The indices of refraction given in § 284 (*c*) are for light that has the particular wave-frequency that corresponds to the Fraunhofer D-line.

308. Laws of Spectra. — The following laws have been established : —

(1) *Incandescent solids and liquids give continuous spectra.* This is true of vapors and gases also when they

are under great pressures. The spectrum from the flame of a candle, of kerosene, or of illuminating gas is continuous, being due to the incandescent carbon particles suspended in the flame.

(2) *Incandescent rarefied vapors and gases give discontinuous spectra consisting of colored bright lines or bands.* These lines or bands have a definite position for each substance and are, therefore, characteristic of it. Thus the sodium spectrum consists of bright yellow lines, corresponding in position to the D-line as shown in Fig. 286.

(3) *If light from an incandescent solid or liquid passes through a gas at a temperature lower than that of the incandescent body, the gas absorbs rays of the same degree of refrangibility as that of the rays that constitute its own spectrum.* This absorption is somewhat analogous to that mentioned in § 204. The result is a spectrum continuous except as interrupted by dark lines that occupy the position that the bright lines in the spectrum of the gas itself would occupy. Thus, the emission spectrum of sodium corresponds in position to the D-line; the sodium flame absorbs light of the same refrangibility, and its absorption spectrum falls in the same position.

(*a*) If the source of radiant energy under spectroscopic examination is approaching the observer, the effect of the motion will be the same as if the wave-length was shortened; the characteristic lines will be moved toward the violet end of the spectrum. If the source of radiation is moving from the observer, the opposite effects will follow. Compare § 192 (*b*). Such displacement of spectra lines has enabled investigations of the motions of even the "fixed" stars.

Experiment 267. — Hold a pane of glass between the face and a hot stove; the glass shields the face from the heat of the stove. Hold the glass between the face and the sun; the glass does not shield the face from the heat of the sun.

309. Thermal Effects may be detected throughout the length of the visible spectrum and beyond in each direction, i.e., in the infra-red spectrum and in the ultra-violet spectrum. The infra-red radiation is of longer, and the ultra-violet radiation is of shorter wave-length than that of any part of the visible spectrum. The former is present in the spectrum from any hot body; the latter, in that from a body at a high temperature, as the incandescent carbons of an arc electric light. When radiant energy is considered with reference to its heating effects, it is sometimes called "radiant heat," a term that is evidently misleading, but that has acquired a good foot-hold in the literature of science. Similarly, the radiation of the infra-red region is spoken of as "obscure heat."

(*a*) Lenses and prisms of rock-salt are generally used in the study of the heating effects of radiant energy, as glass absorbs much of the energy of the longer ether waves, as was shown in Experiment 267. When a solar spectrum is produced with a rock-salt prism, the maximum heating effect is found in the infra-red region, but with a normal spectrum (diffraction-spectrum, § 315), the maximum heating effect coincides somewhat closely with the maximum luminous effect.

(*b*) When the heating effects of radiant energy rather than the luminous effects are under consideration, the ability freely to transmit the ether waves constitutes *diathermancy;* the corresponding inability constitutes *athermancy.* In other words, diathermanous corresponds to transparent, and athermanous to opaque. Glass, water and alum transmit light, but absorb nearly all of the energy radiated from a vessel filled with boiling water, i.e., they are transparent and athermanous. A solution of iodine in carbon disulphide is opaque and diathermanous. Dry air is very diathermanous; watery vapor is decidedly athermanous.

310. Theory of Exchanges. — All bodies at temperatures above absolute zero must radiate energy that may be converted into heat. When two bodies at different temperatures are placed near each other, one gains and the other loses heat by radiation until both have the same temperature. Each radiates to the other, but while the inequality of temperature continues, the hotter body gives more than it receives, and vice versa. This is a brief statement of *Prevost's theory of exchanges.*

Absorption, etc.

Experiment 268. — When there is snow on the ground, and the sun is shining, spread a piece of white cloth and a similar piece of black cloth on the snow, and notice whether the snow melts more rapidly under one than under the other.

Experiment 269. — Focus a sunbeam on the clear glass bulb of an air thermometer, and notice the feeble effect produced. Coat the bulb with candle soot, and repeat the experiment. Notice the greatly increased effect.

Experiment 270. — Secure two similar pieces of tin-plate at least 10 cm. square. Coat one face of one of the pieces with lampblack (candle-soot). Support the two plates, with the painted surface vertical, facing the other plate, and about 10 cm. from it. With small bits of shoemaker's wax, fasten a small ball to the middle of the outer face of each plate. Hold a hot body, as a "soldering iron," midway between the plates. Notice which ball first falls. Repeat the experiment several times to make certain whether the effect was accidental or due to the lampblack.

Experiment 271. — Provide two bright tin cans of the same size and shape. In the cover of each, make a hole and insert the bulb of a chemical thermometer. Blacken the outside of one can with lampblack. Fill both cans with hot water from the same vessel and, consequently, of the same temperature. At the end of half an hour, pass the bulb of the thermometer through the holes in the covers, and ascertain the temperature of the water in each can. It will be found that the blackened can has radiated its heat more rapidly than the other. Fill both cans with cold water, and set them in front of a hot fire or in the sunshine. The temperature of the water in the blackened can rises more rapidly than that of the water in the other can.

Experiment 272. — Secure a piece of stoneware (Fig. 287) of any black and white pattern. Notice carefully what parts absorb the most radiant energy. Heat the plate intensely, view it in a darkened room, and notice carefully what parts radiate the most energy (Fig. 288).

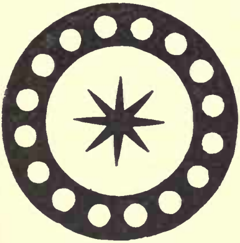


FIG. 287.

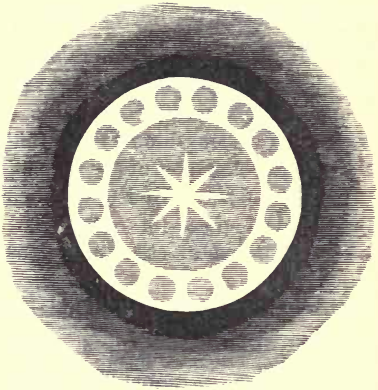


FIG. 288.

311. Radiation, Reflection and Absorption. — Bodies differ greatly in absorbing power. From the nature of the case, a good absorber is a poor reflector. Lampblack is a substance of maximum absorbing and of minimum reflecting power. Further than this, *the emission and the absorption of radiant energy go hand in hand*, good absorbers being good radiators, and good reflectors being poor radiators, and vice versa.

312. Chemical Effects may be detected throughout the length of the visible spectrum and beyond in each direction. The chemical changes upon which ordinary photography depends are most stimulated by the violet and ultra-violet rays; this, however, is not true of all chemical changes, and even infra-red photography has been accomplished. By exciting molecular agitation of the molecules of sulphide of zinc with an electric current of about 10,000 alternations per second, Nikola Tesla demonstrated, in 1894, the actinic value of "cold rays" by taking photographs by phosphorescent light.

(*a*) It, therefore, appears that the long-time division of the spectrum into three parts, heat, light and actinism, was ill founded; that from one end of the spectrum to the other, the radiation differs intrinsically in wave-length only; and that the observed diversity of effect is due to the character of the surface upon which the radiation falls.

(*b*) Lenses and prisms of quartz are generally used in the study of the chemical effects of radiant energy as they absorb less of the energy of the short ether waves than do those of glass.

313. Change of Vibration-Frequency. — When solutions of certain substances, such as esculin and sulphate of quinine, are exposed to ultra-violet radiation, the solutions lower the rate of vibration to that of an opalescent blue light. This property of lowering the vibration-frequency of ultra-violet radiation to the range of vision is called *fluorescence.* Another class of substances, such as the sulphides of barium, calcium, and strontium, are luminous when carried from sunlight into a dark room and, for a long time after, present the general appearance of a hot body cooling. This property of shining in the dark after exposure to light is called *phosphorescence.* What is correctly termed phosphorescence has nothing to do with phosphorus, the luminosity of which in the dark is due to slow oxidation. The radiations that excite this luminosity are those of high wave-frequency, so that phosphorescence is a species of fluorescence that lasts longer after the excitation has ceased than the species just described. The property has been taken advantage of for the production of what are called "luminous paints." The luminous rays of an electric arc may be absorbed by a solution of iodine in carbon disulphide, and the residual infra-red rays reflected or refracted to a focus. A piece of platinum or of charcoal at such a focus of non-luminous radiation may be heated to incandescence. This raising of the vibration-frequency of infra-red radiation to the range of vision is called *calorescence.*

CLASSROOM EXERCISES.

1. Why is the rainbow a circular arc instead of a straight band?

2. What does a wave-length of red light measure in centimeters?

3. Taking the velocity of light to be 186,000 miles per second and the wave-length of green light to be 0.00002 of an inch, how many waves per second beat upon the retina of an eye exposed to green light?

4. How may spherical and chromatic aberration caused by a lens be corrected?

5. What name is given to the differential deviation by refraction of rays of different wave-frequencies?

6. Why is a rainbow never seen at noon?

7. Describe Fraunhofer's lines, and tell how they may be produced.

8. Under what circumstances will a spectrum be (*a*) continuous, (*b*) bright-line; (*c*) dark-line?

9. Why do not the sun's rays heat the upper atmosphere of the earth as they pass through it?

10. Show that the glass walls and roof of a greenhouse are a trap for solar heat.

11. Why is it oppressively warm when the sun shines after a summer shower?

12. Why is there greater probability of frost on a clear than on a cloudy night?

13. Explain the fact that the glass of a window may remain cold while the sun's radiations are pouring through it and heating objects in the room.

14. What is meant by the statement that water is transparent and athermanous?

15. What is meant by "radiant heat," and what by "obscure heat"? Why are these terms objectionable?

16. How can you cut out the short-wave radiations of an arc electric lamp? How can you cut out the long-wave radiations?

17. Why does a polished tea-urn remain warm longer than a similar one with a roughened surface?

18. Explain the apparent unsteadiness of an object seen across the top of a hot stove.

19. Show that the watery vapor in the atmosphere acts as a blanket for terrestrial objects.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Porte-lumière or optical lantern; crown-glass, flint-glass, and carbon disulphide prisms; a wooden block, grooved and painted black; cyan-blue and orange colored glass; picric acid; copper sulphate; ammonia water; fine platinum wire; induction tube; voltaic cell; Plücker-tube; two Leslie cubes; pane of window glass about 9 × 12 inches; lampblack; India-ink; tin-foil; white-lead; chemical tank; alum; iodine; carbon disulphide.

1. Cut a very narrow slit with straight, smooth edges in a piece of cardboard. Fasten the cardboard across the opening of a porte-lumière or an optical lantern, with the slit vertical. With a convex lens that has a focal distance of about 30 cm., project an image of the slit upon a white screen at a considerable distance. Place a prism (60°) of crown-glass between the lens and screen, close to the lens and with its edges vertical. Turn the prism about its axis until it is in the position of least deviation. Without changing its distance from the lens, set the screen so that the light refracted by the prism falls perpendicularly upon it. Describe the image and mark on the screen its limits, and the position of any characteristic points. Replace the crown-glass prism successively with flint-glass and carbon disulphide prisms, and note any changes in the spectrum. Set a second prism near the first and with

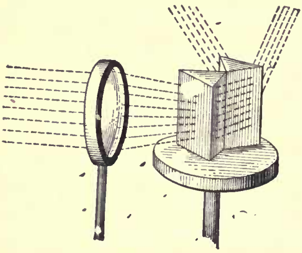


FIG. 289.

its base turned the same way, as shown in Fig. 289, so that the dispersion effect of the second may be added to that of the first, and note any changes in the spectrum.

2. Shorten the slit used in Exercise 1 and hold the second prism with its edges perpendicular to the edges of the first, so that the differently colored light emergent from the first shall be received upon a face of the second, each color by itself. See if these rays of differing wave-lengths are refracted equally by the second prism.

3. Arrange apparatus as described in Exercise 1, and cut a narrow vertical slit from the screen so that light of some one color may pass through the slit. Receive this light upon a prism behind the screen, and see if there is any further dispersion, or any production of new colors. Explain the result of your experiment.

4. Cut a slit 2.5 cm. long and 2 mm. wide in each of two pieces of black cardboard, and support the two cards in a groove cut in a blackened piece of wood. The width of the groove should be just twice the thickness of the cards, so that the distance between the slits may be adjusted by moving the cards in the groove. Repeat Experiment 245, and hold the perforated cardboard screen between the lens and the prism, and adjust the slits so that light of only two colors falls upon the lens. Note the color of the spot formed by the synthesis of these colors. Try other pairs of colors, and note the resultant color in each case. Especially, try to find as many combinations as possible that yield white light.

5. Place the three disks represented in Fig. 278 upon the whirling table, and fasten them in position. Turn the spindle rapidly, and note the color of the blending. Change the proportions of the exposed colors, and blend them again. Continue the work with a view of determining the accuracy of the statement that any color of the spectrum may be produced by the composition of these three colors.

6. With a variety of similar disks, demonstrate the effect of blending complementary colors.

7. Admit two sunbeams to a darkened room, and cause them to overlap on a white screen. Hold a piece of blue glass so that one of the beams passes through it. Pass the other beam through a piece of glass so chosen that the blending of the two colored beams produces a white spot on the screen. Shut off one of the beams, and determine the effect of sending the other through both of the pieces of glass.

8. While observing the solar spectrum with a spectroscope, hold a

piece of cyan-blue glass over the slit of the instrument, and note the effect. Then try a piece of orange-colored glass. Then study the effect when the two pieces are superposed in front of the slit. Successively try test-tube portions of a solution of picric acid, and of an ammoniacal solution of copper sulphate.

9. Make a loop about 2 mm. in diameter at the end of a fine platinum wire. Fuse a small bit of common salt into this loop. Place an alcohol flame just beyond the slit of a spectroscope. Hold the bead of salt in the edge of the flame nearest the spectroscope and a little below the level of the slit. Examine the spectrum, and map the position of any bright lines that you observe. Devise some way of producing dark lines that occupy the same positions in the spectrum.

10. With an induction coil, illuminate a Plücker-tube containing some known gas, and examine its spectrum with a spectroscope. Record a description of the spectrum. (See § 506.)

11. Make a cubical metal vessel with edges of about 7 or 8 cm., and vertical faces made respectively of polished brass, sheet lead, bright tin-plate, and tin-plate that has been coated with lampblack. Leave a small opening in the upper face. Such a vessel is called a *Leslie cube.* Fill it with water, and bring the temperature to 10°. Place the cube 3 or 4 cm. from an air thermometer or from one bulb of a differential thermometer, and note the effect upon the thermometer. Raise the temperature successively to 20°, 30°, 40°, etc.; bring it within the same distance of the thermometer, and note the effect in each case. Record a comparison of the readings of the mercury thermometer in the cube with the indications of the air thermometer, and a clear statement of the relation between them.

12. Repeat one of the tests of Exercise 11, and then interpose a pane of window glass between the cube and the thermometer. Explain the effect produced by the screen.

13. With the same apparatus, test the absorptive powers of tin-foil, lampblack, India-ink, and white-lead by successively coating the bulb of the air thermometer with such substances.

14. Test the radiating powers of tin, lampblack, India-ink, and white-lead by successively turning faces of the Leslie cube thus coated toward the bulb of the air thermometer, being careful that the temperature and the distance of the cube are the same in each instance. Try to find some relation between the absorbing powers and the radiating powers of these several substances. Also similarly test the

radiating powers of faces of the cube that have been coated with unglazed white paper and with white cotton cloth.

15. Repeat Experiment 228, holding a "chemical tank" (see Experiment 258) so that the sun's rays shall pass through the two circular windows before they fall upon the lens. Fill the tank with a solution of alum in water, and repeat the experiment. Determine the effect, if any, that the presence of the tank, empty or filled, has upon the result at the focus. Empty out the alum-water and fill the tank with a solution of iodine in carbon disulphide, and repeat the experiment.

VI. INTERFERENCE, DIFFRACTION, POLARIZATION, ETC.

Experiment 273. — In any convenient clamp, firmly press together the centers of two pieces of clean, thick, plate-glass. Look obliquely at the glass, and a beautiful play of colors will be seen surrounding the point of greatest pressure. If the glass is illuminated by the monochromatic light of a sodium flame (see Experiment 253), yellow bands separated by dark bands will be seen.

314. Interference of Light. — We have seen that two wave-motions may combine in such a way as to neutralize each other (§ 206), and that such an interference is a peculiarity of wave-motion. The fact that light may thus neutralize light is strong confirmation of the wave-theory.

(*a*) In the historical experiment of which Experiment 273 is a modification, a plano-convex lens of little curvature was pressed upon a flat piece of glass. When looked at from above, the center of the lens thus used is surrounded by rainbow-like bands of color, known as Newton rings. Of the light that falls vertically upon the lens, some is reflected at the curved surface, and some from the upper surface of the plate under the lens. These latter rays have to traverse twice the wedge-shaped film of air between the lens and the plate. Whenever the thickness of the air-film is such that the two sets of reflected waves unite in opposite

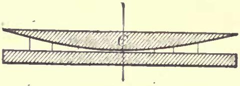


FIG. 290.

phases, interference is the result. If the apparatus is observed by white light, and the red rays are destroyed at a certain distance from the center of the lens, the color perceived at that distance will be complementary to the destroyed red, and will form a circular green band. If the apparatus is observed by red light, a dark ring will appear at the same distance. At another distance from the center of the lens, the violet rays will be destroyed, and the circular band seen at that distance will be due to the combination of the other constituent rays of the light used.

(*b*) Interference colors similarly produced by reflection are often seen in soap bubbles, in small quantities of oil that have been spread over large sheets of water, in mica, selenite, ice, and other crystals. Certain striated surfaces, like those of mother-of-pearl, some kinds of shells and feathers, etc., owe their beautiful colors to the interference of reflected light (see § 315, *b*).

Diffraction.

Experiment 274. — With a fine needle, rule a number of fine parallel lines upon a piece of glass that has been coated with India-ink. Take pains to cut through the ink to the glass. Cut a slit 2 mm. wide in a black card, and hold it at arm's length in front of a flame. Hold the glass close to the eye and, through the scratched lines, look at the slit. Notice the series of spectra on each side of the slit.

Experiment 275. — Throw a sunbeam through a very small opening in the shutter of a darkened room. Receive the beam upon a convex lens of short focal length, placing a piece of red glass between the aperture and the lens. Place an opaque screen with a sharp edge beyond the focal distance of the lens, as at *a*, so as to cut off the lower part of the cone of homogeneous light, and project the upper part thereof upon a screen at *b*. *A faint light is seen on the screen below the level of* a *and, therefore, within the geometrical shadow.* The part of the screen immediately above the level of *a* contains a series of dark and light bands, as shown at *B*, which is a front view of the screen at *b*.

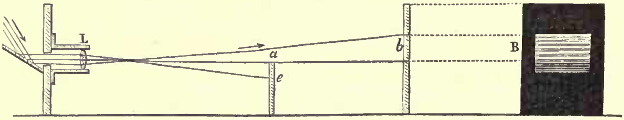


Fig. 291.

315. Diffraction.—When water waves strike an obstacle, part of the energy of the wave is expended in producing a second set of waves that seem to circle outward from the side of the obstacle as a center. The original wave (primary) passes directly onward, while the secondary waves wind around behind the obstacle. In similar manner, sound bends around a corner, but sound-shadows may be produced if the wave-length is sufficiently small, or if the obstacle is of great size compared with the length of the sound waves. So ether waves are modified when they traverse a minute opening or narrow slit, or impinge upon an obstacle, e.g., a hair, so small as to be comparable with the wave-length. The phenomena are identical when the scale of the experiment is the same. If a beam of monochromatic light is passed through a narrow slit and received upon a screen in a dark room, a series of alternately light and dark bands or "fringes" is seen; if white light is employed, a series of colored spectra is obtained. Thus it appears that, under proper conditions, rays may be bent and caused to penetrate into the shadow. *The interference-phenomena resulting from this action are called diffraction.*


FIG. 292.

(*a*) As the primary and secondary waves cut each other, they unite at some points, crest with crest and, at other points, crest with trough. At the latter points, we have interference of light and the effects of colors produced thereby as explained above. The halos sometimes seen around the sun and moon are due to the diffraction of light by watery globules in the atmosphere. The colors often seen on looking through a feather or one's half-closed eyelashes at a distant source of brilliant light are also due to diffraction.

(*b*) When lines are ruled on the surface of glass, the ruled lines become opaque, the spaces between them remaining transparent. A system of close, equidistant, parallel lines ruled on glass, or on polished (speculum) metal, constitutes a *diffraction grating.* Lines are ruled for this purpose at the rate of 10,000 to 20,000 or even 40,000 to the inch. Such gratings yield *interference or diffraction spectra*, which are much used in spectroscopic work, and afford a simple means for measuring the wave-length of ether vibrations. In these spectra, the colors are distributed in their true order and extent according to their differences in wave-length; while, in prismatic spectra, the less refrangible (red) rays are crowded together, and the more refrangible (violet and blue) are correspondingly dispersed. For this reason, the diffraction spectrum is called a *normal spectrum.* The upper part of Fig. 293 represents a normal spectrum, and the lower part, a prismatic spectrum. Comparing the two, the "irrationality of dispersion" of the prismatic spectrum is seen.

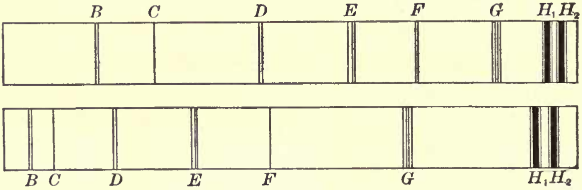


FIG. 293.

316. Irradiation *is the apparent enlargement of a strongly illuminated object when seen against a dark ground.* Thus, when the two equal circles shown in Fig. 294 are carefully observed in a good light, one seems to be larger than the other.

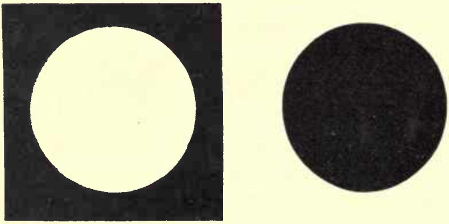


FIG. 294.

(*a*) Irradiation increases with the brightness of the object, diminishes as the illumination of the object and that of the field of view approach equality, and vanishes when they become equal. This effect is very perceptible in the apparent magnitude of stars, which look much larger than they otherwise would; also in the appearance of the new moon, the illuminated crescent seeming to extend beyond the darker portion, as if the new moon was holding the old moon in its arms.

Polarization.

Experiment 276. — While looking through the plates of a pair of tourmaline tongs, turn one of the plates in its wire support. The intensity of the light transmitted will vary as the plate is turned. When little or no light is transmitted, the plates are said to be "crossed."

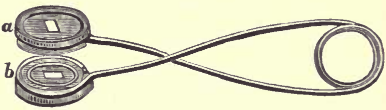


FIG. 295.

Experiment 277. — Write your name on a sheet of paper, and cover it with a crystal of Iceland spar. The lines will appear double, as shown in Fig. 296. Place the crystal over a dot on the paper, hold the eye directly over the dot, and slowly turn the crystal around; one of the two images of the dot will revolve about the other image. Prick a pin-hole through a card, and hold the card against one side of the crystal, look through the crystal at the pin-hole, and rotate the crystal as before.

FIG. 296.

Experiment 278. — Look through one of the plates of the tourmaline tongs (Fig. 295) at the two images of the dot formed by the double refraction of the Iceland spar as described in Experiment 277. One of the images will be much fainter than the other. Turn the tourmaline plate slowly around, and notice that one image grows fainter and the other brighter, the maximum brightness of one being simultaneous with the extinction of the other.

317. Polarization of Light. — Common white light is a highly complex form of radiant energy, comprising not only an indefinite number of wave-lengths, but also an indefinite number of modes of ether vibration. When a rope is shaken as described in Experiment 110, the vibrations of the wave thus produced will lie in a vertical plane; when the hand is moved horizontally, the vibrations will lie in a horizontal plane. It thus appears that a transverse wave is capable of assuming a particular side or direction; a longitudinal wave is not. In like manner, a single row of ether particles engaged in propagating a linear transverse wave may describe any one of a variety of paths, each perpendicular to the line of propagation of the radiation. For example, each particle may vibrate in a straight line, parallel to the wave-front and indifferently in any plane about the line of propagation, as represented in Fig. 297. If all the ether particles in the row under consideration successively vibrate along lines lying in the same plane, the radiation is said to be plane-polarized, and the wave thus constituted is called a plane-polarized wave. *A change by which the transverse vibrations of luminous waves are limited to a single direction is called polarization of light.* This change may be produced in several ways.

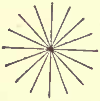


FIG. 297.

(*a*) Light may be polarized: —

(1) By reflection from the surface of glass, water, and other non-metallic substances. The degree of polarization reaches its maximum when the angle of incidence has a certain value depending upon the substance, and called the *angle of polarization*. For glass, this angle is $54\frac{1}{2}°$.

(2) By transmission through a series of transparent plates of

glass placed in parallel position at the proper angle to the incident ray.

(3) By double refraction, as in the case of Iceland spar or of a plate cut in a certain way from a tourmaline crystal. A beam of light, falling upon such a crystal, is generally split into two parts polarized at right angles. One of these parts obeys the regular law of refraction, and is called the *ordinary ray;* the other does not, and is called the *extraordinary ray.* A prism of Iceland spar prepared in such a way that one beam of polarized light is totally reflected and extinguished, while the other beam passes through as polarized light, is called a *Nicol prism.* Nicol prisms and tourmaline plates are largely used in experiments with polarized light.

(*b*) Light that has passed through a tourmaline plate differs so much from ordinary light that it may be stopped by a similar plate, as was seen in Experiment 276. For the sake of simplicity, imagine the indifferently placed planes of vibration, as represented in Fig. 297, to be resolved into two that lie at right angles to each other, as shown in Fig. 298. Then the action of the first tourmaline plate may be compared to that of a vertical-bar grating that allows the vibrations in a vertical plane to pass, but absorbs the vibrations that lie in a horizontal plane. Evidently, the vibrations that pass one such grating, as T, will pass others similarly placed, but will be stopped by one that is crossed, as at T'. The part of the beam that lies between T and T' represents plane polarized light.

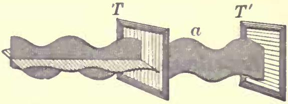


FIG. 298.

(*c*) Polarized light presents to the unaided eye the same appearance as common light. An instrument for producing and testing polarized light is called a *polariscope.* It consists of two characteristic parts; one, used to produce polarization and called the *polarizer;* the other, used to test or to study the polarized light and called the *analyzer.* Apparatus that serves for either of these purposes will serve for the other. The Nicol prism is generally preferred for both purposes. Some of the color-effects due to the interference of polarized light are very beautiful.

(*d*) The plane of polarization may be rotated by passing plane-polarized light through certain substances, some substances producing a right-hand rotation and others a left-hand rotation. This property of polarized light has been applied to the estimation of the commercial value of sugar by the amount of rotation produced by a solution of it of known strength. The devices for the precise measurement of the amount of rotation involve advanced scientific principles. When plane polarized light is passed through a plate of quartz cut perpendicularly to the axis, the plane of polarization is turned through an angle that varies with the thickness of the plate and the wave-length of the light. Thus, if a pebble spectacle-lens is placed between crossed tourmaline plates, the dark field is brightened, and generally colored, by the transmitted light.

(*e*) Light may be circularly and elliptically polarized as well as plane polarized.

Note. — For a fuller discussion of the polarization of radiant energy, the pupil is referred to some special work on Light. The subject is interesting and the phenomena are beautiful.

VII. A FEW OPTICAL INSTRUMENTS.

Experiment 279. — Stick two needles into a board about 6 inches apart. Close one eye, and hold the board so that the needles shall be nearly in range with the open eye and about 6 and 12 inches respectively from it. One needle will be seen distinctly while the image of the other will be blurred. Fix the view definitely on the needle that appears blurred and it will become distinct, but you cannot see both clearly at the same time.

Experiment 280. — Cover half of a white sheet of paper with a sheet of black paper. Fix the eye intently on the middle of the white surface for fifty or sixty seconds. Keep the eye fixed on the same point, and suddenly remove the black paper. The newly exposed part of the sheet appears more brilliantly illuminated than the other.

Experiment 281. — Stick a bright red wafer upon a piece of white paper. Hold the paper in a bright light and look steadily at the wafer, for some time, with one eye. Turn the eye quickly to another part of the paper or to a white wall, and a greenish spot, the size and shape of the wafer, will appear. The greenish color of the image is complementary to the red of the wafer. If the wafer is green, the image afterwards seen will be of a reddish (complementary) color.

Experiment 282. — Close one eye and try to thread a needle. Bend a stout wire at a right angle, and try to pass one end of it through a ring held at arm's length, one eye being closed.

Experiment 283. — Prick a pin-hole in a card, hold it near the eye, and look through the pin-hole at a pin held at arm's length. As the pin is slowly moved toward the eye, the visual angle (§ 318, *e*) increases and the pin seems to grow larger.

318. The Human Eye, optically considered, is an arrangement for projecting inverted, real images upon a screen made of nerve filaments. This image is the origin of the sensation of vision. The luminous waves transfer their energy to the nerve filaments, they transmit it to the brain and, in some mysterious way, the sensation follows.

(*a*) The most essential parts of this instrument are contained in the eyeball, a nearly spherical body, about an inch in diameter, and capable of being turned considerably in its socket by the action of various muscles. It is represented in section from front to back by Fig. 299. The greater part of the outer coat is tough and opaque, and is called the "white of the eye" or the *sclerotic coat*, *S*; the front part of the coat is a hard, transparent structure called the *cornea*, *C*. The cornea is more convex than the rest of the eyeball, and fits into the sclerotic as a watch-crystal does into its case. The chief part of the second tunic of the eye is the *choroid coat*, *N*, which is opaque and intensely black, and absorbs all internally reflected light. The third or inner tunic is the *retina*, *R*, an expansion of the optic nerve which enters the eyeball from behind. These several tunics or coats form a kind of camera filled with solid and liquid refractive media. The *crystalline lens*, *L*, a solid biconvex body, is suspended in the middle of this camera and directly in the axis of vision. Its shape is shown

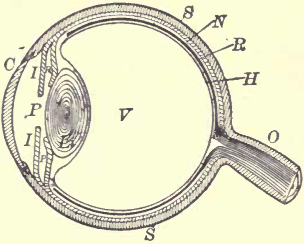


FIG. 299.

in the figure; it tends to flatten with age. With its capsule, it divides the eye into two compartments, and is chiefly instrumental in bringing the rays of light to a focus on the retina. The larger chamber of the eyeball is filled with a transparent, jelly-like substance, *V*, that resembles the white of an egg, is called the *vitreous humor*, and is enclosed in the delicate *hyaloid membrane*, *H*. The chamber between the cornea and the lens is filled with a more watery liquid, the *aqueous humor*. This anterior chamber is partly divided into two compartments by an annular curtain, *I*, called the *iris*. This curtain is opaque, and its color constitutes the color of the eye. The circular opening in the iris is called the *pupil*. The iris acts as a self-regulating diaphragm, dilating the pupil and thus admitting more light when the illumination is weak; contracting the pupil and cutting off more light when the illumination is strong.

(*b*) That vision may be distinct, the image formed on the retina must be clearly defined, well illuminated, and of sufficient size. Without our consciousness, the muscular action of the eye changes the curvature of the crystalline lens so that rays from near or distant objects may be focused on the retina. Instead of moving the screen, the refractive power of the lens is changed. This power of "accommodation," or automatic adjustment for distance, is limited. When a book is held close to the eyes, the rays from the letters are so divergent that the eye cannot focus them upon the retina. The near point of vision is generally about $3\frac{1}{2}$ inches from the eye. As parallel rays are generally brought to a focus on the retina when the eye is at rest, the far point for good eyes is infinitely distant. Owing to the small size of the pupil, rays from a point 20 inches or more distant are practically parallel. The near point of some eyes is less than $3\frac{1}{2}$ inches, while the far point is only 8 or 10 inches. Such eyes are myopic and their owners are *near-sighted;* the retina is too far back, the eyeball being elongated in the direction of its axis. The remedy is in concave glasses. The near point of some eyes is about 12 inches and the far point is infinitely distant. Such eyes are hypermetropic and their owners are *far-sighted*. In such eyes the retina is too far forward, the eyeball being flattened in the direction of its axis. The remedy is in convex glasses. When the diminished power of accommodation for near objects is an incident of advancing years, and due to the progressive loss of elasticity in the crystalline lens, the eyes are presbyopic and their owners are *old-sighted*. The cause of the difficulty is different from that of far-sightedness, but the remedy is the same.

(*c*) The impression upon the retina does not disappear instantly when the action of the light ceases, but continues for about an eighth of a second. The result is what is called the *persistence of vision.* If the impressions are repeated within the interval of the persistence of vision, they appear continuous. This phenomenon is well illustrated by the luminous ring produced by swinging a firebrand around a circle, and in the action of the common toy known as the thaumatrope or the zoetrope.

(*d*) The retina is thickly studded with microscopic projections called *rods and cones*, the terminal elements of the optic nerve. According to a theory that is as yet purely provisional, these end-organs are tuned to sympathetic vibrations with the ether vibrations that severally correspond to violet, green, and red (§ 299, *a*), and by combining these effects in suitable proportions, the several color-sensations are produced. When any of these end-organs are inoperative or when they are not equally sensitive, the person is affected with *color-blindness,* i.e., he is unable to recognize certain colors, generally red. Sometimes these terminal elements seem to become tired of vibrating at a given rate and thus to become insensible to a certain color. (See Experiments 280 and 281.) Hence, what is known as subjective color is due to a retinal fatigue. In the middle of the retina and in the axis of the eye is a little rounded elevation called the *yellow spot* or *macula lutea.* It is the most sensitive part of the retina. On the nasal side of the yellow spot is the entrance of the optic nerve and its central artery. As this part of the retina lacks the visual function that characterizes the rest of its surface, it is called the *blind spot.*

(*e*) The estimation of distances by the eye is a matter of judgment and is chiefly based upon experience. This experience relates to the amount of muscular effort exerted in adjusting the eye for distinct vision, and in turning the two eyes inward so that their axes meet at the object, thus forming the *optical angle* (see Experiment 282); to the comparison of the angle formed by lines drawn from the extremities of the object to either eye and called the *visual angle* with the visual angle subtended by objects of known size and distance; and to the observation of changes of color and brightness produced by the varying thickness of the air through which the object is viewed.

(*f*) The estimation of the size of distant objects is also a matter of judgment, based upon the known or supposed distance of the object. The ratio between the size of object and image equals the ratio between the distance of each from the lens, and the mind unconsciously bases its conclusions on this fact.

Stereoscopic Effect.

Experiment 284. — Close the left eye, and hold the right hand so that the forefinger hides the other three fingers. Without changing the position of the hand, open the left and close the right eye. The hidden fingers become visible in part. It is evident that the images upon the retinas of the two eyes are different.

Experiment 285. — Place a die on the table directly in front of you. Looking at it with only the left eye, three faces are visible, as shown at *A*, Fig. 300. Looking at it with only the right eye, it appears as shown at *B*. If, in any way, we combine two such drawings, so as to produce images upon the retinas of the two eyes like those produced by the solid object, we obtain the idea of solidity.

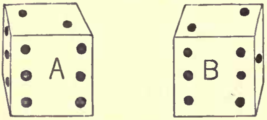


Fig. 300.

319. The Stereoscope *is an instrument for illustrating the phenomena of binocular vision*, and for producing from two nearly similar pictures of an object the effect of a single picture with the appearance of relief and solidity that pertains to ordinary vision.

(*a*) The stereoscopic view or slide shows, side by side, two pictures taken under a slightly different angular view. It is the office of the stereoscope to blend these two pictures. As in ordinary vision, each eye sees only one of the pictures, but the two images conveyed to the brain unite into one. The diaphragm, *D*, prevents either eye from seeing both pictures at the same time. Rays of light from *B* are refracted by the half-lens, *E′*, so that they seem to come from *C*, beyond the plane of the

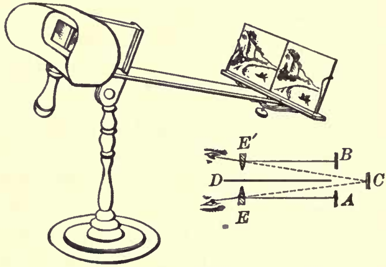


Fig. 301.

pictures. In the same way, rays from *A* are refracted by *E* so that they also seem to come from *C*. The two slightly different pictures, thus seeming to be in the same place at the same time, are successfully blended; the picture "stands out," or has the appearance of solidity.

320. The Photographer's Camera corresponds to the camera-obscura described in § 265. A darkened box, *DE*, adjustable in length, takes the place of the darkened room, and an achromatic convex lens is substituted for the aperture in the shutter.

(*a*) A ground-glass plate is placed in the frame at *E*, which is adjusted so that a well-defined inverted image of the object in front of *A* is projected upon the glass plate as shown in Fig. 302. This adjustment is completed by moving the lens and its tube by the toothed wheel at *D*. When the focusing is satisfactory, *A* is covered, the ground-glass plate is replaced by a chemically prepared sensitive plate, *A* is uncovered, and the image projected on the chemical film. The chemical changes that the light produces in the film are made visible by a process called "developing," and made permanent by a process called "fixing."

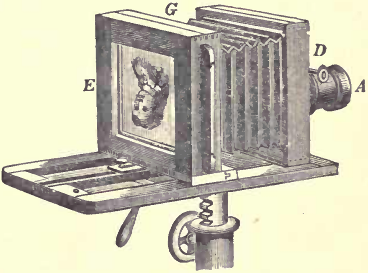


FIG. 302.

Microscope.

Experiment 286. — Provide two small biconvex lenses about 4 cm. in diameter, one with a focal length of about 3 cm. and the other with a focal length of about 5 cm. Mount each by inserting its edge in a slit in a large cork. Place a small bright object in front of the lens of shorter focal length and close to it, and adjust a screen on the other side of the lens so that a sharp image of the object will be projected on it. Place the other lens back of the screen, and at a distance from it less than the focal length of the lens. Remove the screen, and look through the second lens toward the first. Adjust the second lens until you can see a virtual image of the real image of the object.

321. A Microscope *consists of a lens or a combination of lenses used to observe small objects*, often so minute as to be invisible to the unaided eye.

(*a*) *The simple microscope* is generally a single convex lens, and is often called a *magnifying glass*. The object is placed between the lens and its principal focus. The lens increases the visual angle. The image is virtual, erect, and magnified.

(*b*) The *magnifying power* of a lens is the ratio between the length of the object and the length of its image.

(*c*) *The compound microscope* consists essentially of two lenses or systems of lenses. One of these, *O*, called the *objective*, is of short focus. The object, *AB*, being placed slightly beyond the principal focus, a real image, *cd*, magnified and inverted is formed. The other lens, *E*, called the *eyepiece* or ocular, is so placed that the image, *cd*, lies between it and its focus. A magnified, virtual image of the real image, *cd*, is formed by the eyepiece and seen by the observer at *ab*. Eyepiece and objective are placed at opposite ends of a tube and are generally compound, the objective consisting of two or three achromatic lenses, and the eyepiece of two or more simple lenses. The instrument varies widely in construction, and is often provided with many accessories or special devices applicable to particular uses.

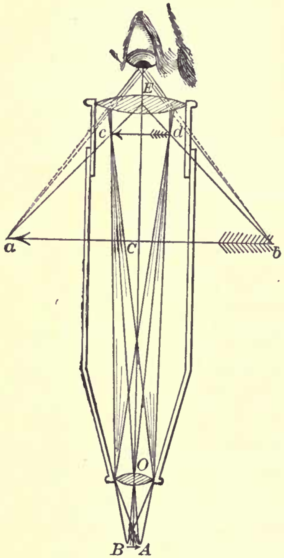


Fig. 303.

Experiment 287. — Using a biconvex lens about 10 cm. in diameter and with a focal length of about 40 cm., project a sharp image of a distant object on a screen. Back of the screen, place the lens of

3 cm. focal length that was used in Experiment 286. The distance of the lens from the screen should be less than the focal length of the lens. Remove the screen, and look through the second lens toward the first. Adjust the second lens until you can see a virtual image of the real image of the object.

322. A Telescope *is an instrument designed for the observation of distant objects*, and consists essentially of an objective for the formation of an image of the object and of an eyepiece for magnifying this image. The optical parts are generally set in a tube so arranged that the distance between the objective and the eyepiece may be adjusted for distinct vision. A telescope is refracting or reflecting according as its objective is a convex lens or a concave mirror, and astronomical or terrestrial according as it is designed for the observation of celestial or terrestrial objects.

(*a*) *The astronomical refractor* consists essentially of a large convex lens objective of long focus, and a convex lens eyepiece of short focus,

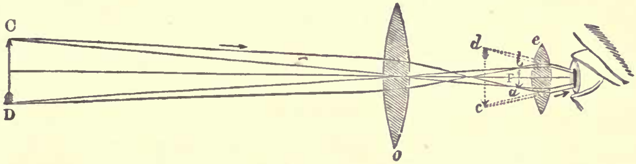


FIG. 304.

as is shown in Fig. 304. The objective is made large that it may collect many rays, to the end that its real and diminished image, *ab*, may be so bright that it may be considerably magnified without too great loss of distinctness. This real image formed by the objective is magnified by the eyepiece, as in the case of the compound microscope. The visible image is a virtual image of the real image.

(*b*) *The spy-glass* or terrestrial telescope avoids the inversion of the image by the interposition of two double-convex lenses, *m* and *n*, between the objective and the eyepiece. The rays diverging from the inverted image at *I* cross between *m* and *n*, and form an erect, magnified, virtual image at *ab*.

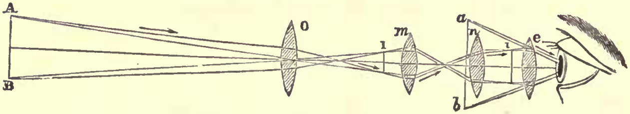


FIG. 305.

(*c*) *The Galilean telescope* has a double-concave eye-lens that intercepts the rays before they reach the focus of the objective. The rays from *A*, converging after refraction by *O*, are rendered diverging by

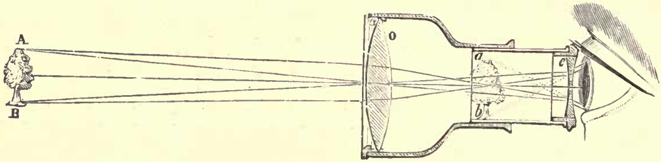


FIG. 306.

C; they seem to diverge from *a*. In like manner, the image of *B* is formed at *b*. The image, *ab*, is erect and very near. Two Galilean telescopes placed side by side constitute an *operá-glass*.

(*d*) *The reflecting telescope* has as an objective a concave mirror, technically called a speculum. The images formed by the speculum are brought to the eyepiece in several different ways. Sometimes the

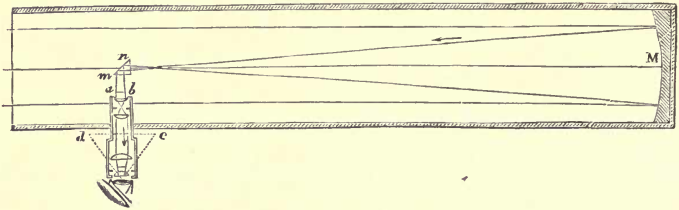


FIG. 307.

eyepiece consists of a series of convex lenses placed in a horizontal tube, as shown in Fig. 307. The rays from the mirror may be reflected by a cathetal prism, *mn*, and a real image formed at *ab*. This image is magnified by the eyepiece, and a virtual image formed at *cd*. The Earl of Rosse built a telescope with a mirror that was six feet in diameter, and had a focal distance of fifty-four feet.

(*e*) The magnifying power of a telescope depends upon the ratio between the focal length of the objective and that of the eyepiece, and may be changed by changing one eyepiece for another.

Optical Projection.

Experiment 288. — Reflect a horizontal beam of sunlight into a darkened room. In its path, place a piece of smoked glass on which you have traced the representation of an arrow, *AB* (Fig. 308), or written your autograph. Be sure that every stroke of the pencil has cut through the lampblack and exposed the glass beneath. Place a convex lens beyond the pane of glass, as at *L*, so that rays that pass through the transparent tracings may be refracted by it, as shown in the figure. It is evident that an image will be formed at the foci of the lens. If a screen, *SS*, is held at the positions of these foci, *a* and *b*, the image will appear clearly cut and bright. If the screen is held nearer the lens or further from it, as at *S′* or *S″*, the picture will be blurred.

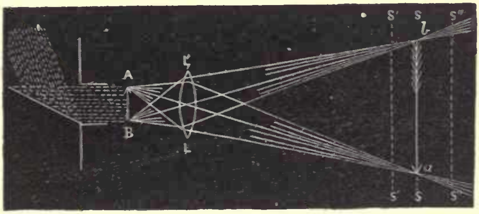


FIG. 308.

323. The Optical Lantern is an instrument for projecting on a screen magnified images of transparent photographs, paintings, drawings, etc.

(*a*) The light is placed at the common focus of a concave mirror, and of a convex lens called the *condenser*. A powerful beam of light is thus thrown upon *ab*, the transparent object, technically termed a "slide." A compound objective, *m*, is placed at a little more than its focal distance beyond the slide. A real, inverted, magnified image of the picture is thus projected upon the screen, *S*. The tube carrying *m* is adjustable, so that the foci may be made to fall upon the screen, and thus render the image distinct. By inverting the slide, the image is seen right side up. Solar and electric microscopes act in nearly the same way, the chief difference being in the source of light. An optical lantern is often called a *magic lantern.*

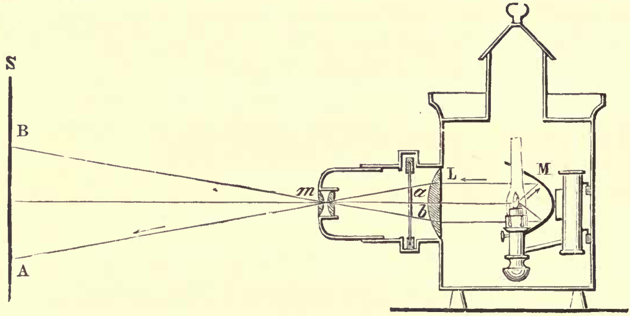


FIG. 309.

(*b*) Two matched lanterns placed so that their images coincide constitute a *stereopticon.* The use of such an instrument avoids the delay and unpleasant effect of moving the pictures across the screen in view of the audience when the slides are changed, and enables the production of many interesting "dissolving" effects that are impossible with a single lantern.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Two pieces of heavy plate-glass, about 8 cm. square; a small iron clamp; three spring clothes-pins; a spectacle-lens made of quartz, and a similar one made of glass.

1. In a good light, press together two pieces of clean plate glass with a clamp at their centers, and explain the appearance of colors in the glass.

2. Spring a clothes-pin upon each of three corners of the glass plates used in Exercise 1, and support the plates by an iron clamp at the fourth corner. Let a beam of sunlight from the porte-lumière fall upon the face of the plate so as to make the angle of incidence 45°. Receive the beam reflected from the plate upon a convex lens so that an image of the opening in the shutter will be projected on the screen. Vary the pressure at the clamp, and explain the change of colors on the screen.

3. Look through the two plates of the tourmaline tongs (Fig. 295) at the bright sky. Turn one of the plates in its supporting ring and observe the changes of brightness. When the plates are so adjusted that the view through them is the darkest, slip successively between them a quartz spectacle-lens and a similar lens made of glass, noting the effect of each, and explaining the effect of one.

4. Project a solar spectrum upon a white screen, and look at it intently for 50 or 60 seconds. Then have some one suddenly cut off the light that yields the spectrum, and turn up the lamp or gas-light. During these changes, keep your eyes fixed on the screen watching for any change that may take place in the appearance of the spectrum. Describe and explain any such change that takes place.

5. Fasten a thread to a disk of paper of some bright color. Place this disk upon a sheet of white paper and in a strong light. Look intently at the colored disk for 20 or 30 seconds. Suddenly pull away the colored disk without moving the eye. Describe and explain the after-image.

6. While a friend is looking intently at a distant object, look obliquely into his eye, holding a candle-flame on the other side of it. If the flame is properly held, three images of it will be seen; one erect and bright, reflected from the cornea; another erect and less bright, reflected from the anterior surface of the crystalline lens; and a third, inverted, reflected from the posterior-surface of the lens. When the eye that is being studied changes its adjustment for the observation of an object held near it, the first image of the candle-flame is unchanged, while the second and third become smaller, the change being greater in the second than in the third.

7. Close the left eye and look steadily at the cross below, holding the book about a foot from the face. The dot is plainly visible as well as the cross. Keep the eye fixed on the cross and move the book slowly toward the face. When the image of the dot falls on the "blind spot" of the eye, the dot disappears. Hold the book in this position for a moment and see if the changing convexity of the crystalline lens throws the image of the dot off the blind spot, making the dot again visible.

CHAPTER VI.

ELECTRICITY AND MAGNETISM.

(*Ether Physics continued.*)

I. GENERAL VIEW.

A. STATIC ELECTRICITY.

324. Electricity is the common cause of a large variety of phenomena, including apparent attractions and repulsions of matter, heating, luminous and magnetic effects, chemical decomposition, etc.

(*a*) The true nature of electricity is not yet well understood. Little more can be said at this point than that it is the agent upon which certain phenomena depend, and that "it behaves like an incompressible fluid filling all space and yet entangled in an ether that has the rigidity necessary to propagate the enormously rapid and minute oscillatory disturbances that constitute radiation, while, at the same time, it allows the free motion of ordinary matter through it."

(*b*) The phenomena of electricity are generally classified as static or dynamic, and considered under the heads, frictional electricity or current electricity. Owing to their common cause and for reasons of convenience, little effort will be made to maintain the distinction in this work.

(*c*) "Experiments with electricity produced by friction are very beautiful and of great theoretical interest, but many of them are troublesome to perform, and their practical importance is very small."

Experiment 289. — Draw a silk ribbon about an inch wide and a foot long between two layers of warm flannel and with considerable friction. Hold the ribbon near the wall, and notice the unusual attraction. Place a sheet of paper on a warm board, and briskly rub it with india-rubber. Hold it near the wall, as you did the ribbon, and notice the effect.

Experiment 290. — Cut a number of pith-balls about 1 cm. in diameter. Whittle them nearly round, and finish by rolling them between the palms of the hands. Cover one of these balls with gold-leaf, suspend it by a silk fiber, and call it an *electric pendulum.* Briskly rub a stout stick of sealing-wax with warm flannel, and bring it near the electric pendulum. Notice the attraction.

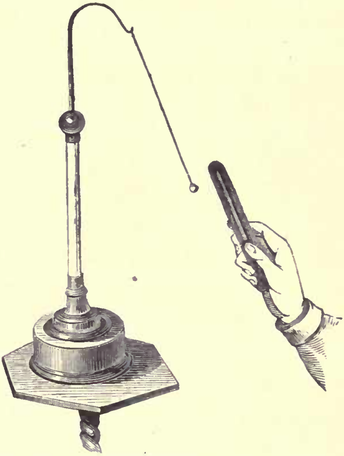


Fig. 310.

Experiment 291. — To the middle of a straw about a foot long, fasten with wax a short piece of straw as shown in Fig. 311. Fasten two disks of bright-colored paper at the ends of the straw, and balance the apparatus upon the point of a sewing-needle, the other end of which is thrust into the cork of a glass vial. Rub the sealing-wax as before, and hold it near one of the paper disks. The straw may be made to follow the wax round and round. A paper hoop or an empty egg-shell may be made to roll after the rubbed rod.

Fig. 311.

Experiment 292. — Repeat the last two experiments, using a glass rod or tube that has been rubbed with a silk pad. The ends of the glass should be rounded or smooth; a long ignition-tube will answer. The effect may be increased by smearing lard on one side of the pad, and applying a coat of the amalgam that may be scraped from bits of a broken looking-glass. Small scraps of paper, shreds of cotton and silk, feathers and

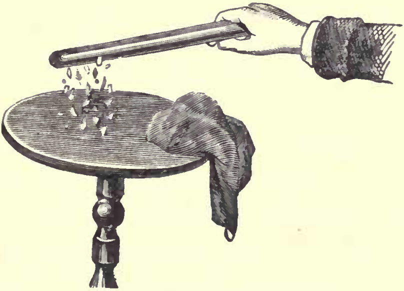


Fig. 312.

gold-leaf, bran, sawdust, and other light bodies may be similarly attracted.

Experiment 293. — Place an egg in an egg-cup, and balance a yardstick upon it. The end of the stick may be made to follow the rubbed rod round and round. Place the blackboard pointer or other stick in a wire stirrup (Fig. 313) or stiff paper loop suspended by a stout silk thread or a narrow silk ribbon. It may be made to imitate the actions of the balanced yardstick.

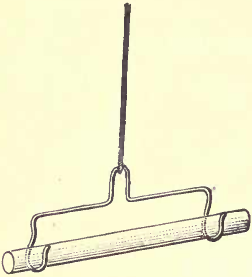


FIG. 313.

Experiment 294. — Suspend the rubbed sealing-wax or glass rod in the stirrup, and hold the pointer or your hand near it. Evidently *the action is mutual,* i.e., each body attracts the other.

325. Electrification. — Bodies that are endowed with the power of attracting other bodies as just illustrated are said to be *electrified.* Any substance may be electrified by suitable means. The state or condition thus established is called *electrification*, and may be brought about in a variety of ways.

Experiment 295. — Support a meter stick upon a glass tumbler. Bring an electrified glass rod to one end of the stick, and hold some small pieces of gold-leaf or paper a few centimeters under the other end of the stick. The gold-leaf or paper will be attracted and repelled by the stick as it previously was by the glass itself. The electrification passed along the stick from end to end.

326. Conductors and Insulators. — *Substances that easily permit the transference of electrification along them are said to be good conductors.* No substance is so good a conductor as not to offer some resistance to the transfer. No substance is so poor a conductor that the electrification cannot be forced through it, but there are some that offer resistances so great that they are called *insulators*, or *non-conductors*. A conductor supported by an insulator is said to be *insulated*. An insulated body that is electrified is said to have a *charge*, or to be charged.

(*a*) In the following table, the substances named are arranged in the order of conductivity:—

Conductors.
Metals.
Charcoal.
Graphite.
Acids.
Salt water.
Vegetables.
Animals.
Linen.
Cotton.
Dry wood.
Paper.
Silk.
India-rubber.
Porcelain.
Glass.
Sealing-wax.
Vulcanite.
Insulators.

(*b*) The fact that a conductor in the air may be insulated shows that air is a non-conductor. Dry air is a very good insulator (at least 10^{26} times as good as copper), but moist air is a fairly good conductor. All experiments in static electricity are, therefore, more successfully performed in clear, cold weather when the atmosphere is dry. Resistance and conductivity will be more specifically considered in subsequent pages.

(*c*) A medium intervening between two electrified bodies, i.e., a substance, solid, liquid or gaseous, through or across which electric force is acting, is called a *dielectric*. The dielectric plays an important part in the phenomena of electrification.

Kinds of Electrification.

Experiment 296. — Suspend several pith-balls by fine linen threads from an insulating support, and touch them with an electrified rod. The rod repels the balls, and the balls repel each other.

Experiment 297. — Electrify a suspended pith-ball by contact with a rubbed rod. Notice that the ball is repelled by the rod, and attracted by the cloth with which the rod was rubbed.

Experiment 298. — Bring an electrified glass rod near a pith-ball electroscope as before, and notice that, after contact, the

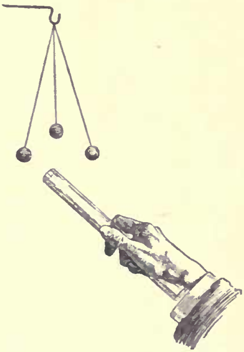


FIG. 314.

ball is actively repelled. Similarly charge a second ball with an electrified rod of sealing-wax. Bring the two balls near each other, and notice their mutual attraction. Charge the two balls as before. Bring the glass rod near the ball that is repelled by the sealing-wax, and notice the attraction. Bring the sealing-wax near the ball that is repelled by the glass rod, and notice the attraction.

327. Opposite Electrifications. — As just illustrated, electrification may be manifested by repulsion as well as by attraction, and is of two kinds, opposite in character. The electrification developed by rubbing glass with silk is called positive; that developed by rubbing sealing-wax with flannel is called negative. *Bodies similarly electrified repel each other; bodies oppositely electrified attract each other.*

(*a*) The statement that there are two kinds of electrification does not necessarily imply that there are two kinds of electricity. It is, however, very convenient to speak of one kind of electrification as caused by a charge of one kind of electricity, and the other kind of electrification as caused by a charge of an opposite kind of electricity.

(*b*) Any substance mentioned in the following electric series is positively electrified when rubbed with any substance that follows it, and negatively electrified when rubbed with any substance that precedes it in the list: —

1. Cat's fur,
2. Flannel,
3. Ivory,
4. Quartz,
5. Glass,
6. Cotton,
7. Silk,
8. The hand,
9. Wood,
10. Metals,
11. Caoutchouc,
12. Sealing-wax,
13. Resin,
14. Sulphur,
15. Gutta-percha,
16. Gun-cotton.

Thus, cat's fur is always positively electrified, and gun-cotton negatively, when rubbed with any other substance mentioned in the list. Glass is positively electrified when rubbed with silk, and negatively when rubbed with flannel.

(*c*) The electrification of the rubbed body is equal in amount to that of the body with which it is rubbed, but opposite to it in character.

328. Electrification by Conduction *is the process of charging a body by putting it in contact with an electrified* *body.* The charge thus produced is of the same kind as that of the communicating body.

329. The Electroscope *is an instrument for detecting and testing electrification.* The electric pendulum, or the balanced straw of Experiment 291, constitutes a simple and efficient electroscope. The gold-leaf electroscope represented in Fig. 315 is a common form of a more sensitive instrument. A metallic rod passes through the cork of a glass vessel, and terminates on the outside in a ball or a disk. The lower end of the rod carries two strips of gold-leaf or of aluminium-foil that hang parallel and close together. When an electrified object is brought near the knob or into contact with it, the metal strips below become similarly charged and are, therefore, mutually repelled.

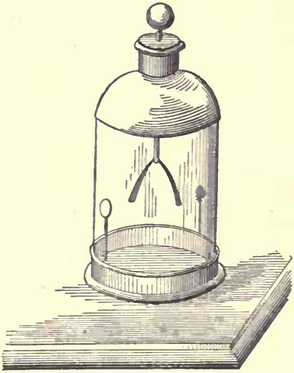


Fig. 315.

(*a*) A *proof-plane* may be made by cementing a bronze cent or a disk of gilt paper to a thin insulating handle, as a glass tube or a vulcanite rod. Slide the disk of the proof-plane along the surface of the electrified body to be tested, and quickly bring it into contact with the knob of the gold-leaf electroscope, the leaves of which will diverge. Positively charge the proof-plane by contact with a glass rod that has been electrified by rubbing it with silk, and transfer the second charge to the electroscope. If the leaves diverge more widely,

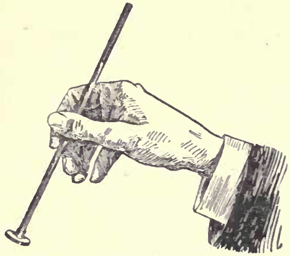


Fig. 316.

the first charge was positive. If the leaves collapse, repeat the test, using the negative charge from a rod of sealing-wax rubbed with flannel instead of the positive charge from the glass rod. If the leaves are thus made to diverge more widely, the first charge was negative.

330. Electrical Units. — There are two systems of electrical units derived from the fundamental "C.G.S." units, one set being based upon the attraction or repulsion exerted between two quantities of electrification, and the other upon the force exerted between two magnet poles. The former are termed *electrostatic* units; the latter, *electromagnetic* units. Distinctive names have not yet been adopted for the electrostatic units.

331. The Electrostatic Unit of Quantity *is the quantity of electrification that exerts through the air a force of one dyne on a similar quantity at a distance of one centimeter.* The force may be attractive or repulsive.

332. Law of Electric Action. — The force that is mutually exerted between two charges varies directly as the product of the charges, and inversely as the square of the distance between them. The two charges are supposed to be collected at two points.

$$f = \frac{Q \times q}{d^2}.$$

Distribution of the Charge.

Experiment 299. — Make a conical bag of linen, supported, as shown in Fig. 317, by an insulated metal hoop five or six inches in diameter. Electrify the bag. A long silk thread extending each way from the apex of the cone will enable you to turn the bag inside out without discharging it. Test the inside and outside of the bag, using the proof-plane. Turn the bag and repeat the test. Whichever surface of the linen is external, no electrification can be found upon the inside of the bag. Vary the experiment by the use of a hat suspended by silk threads. Notice that the greatest charge is obtained from the edges; less from a curved or flat surface; none from the inside.

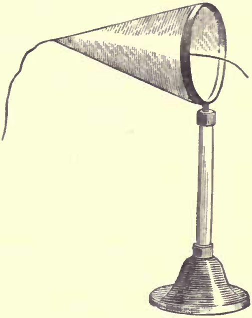


FIG. 317.

Experiment 300. — Fasten one edge of a large sheet of tin-foil to a horizontal glass rod or tube. Connect a lower corner of the tin-foil by a fine wire to the knob of an electroscope. Charge the tin-foil lightly, and notice the divergence of the leaves of the electroscope. Slowly turn the rod so as to roll the tin-foil upon it. As the area of the electrified surface is reduced, notice the increase in the divergence of the leaves.

Experiment 301. — Prick a pin-hole at each end of an egg, and blow out the contents of the shell. Paste tin-foil or Dutch-leaf smoothly over the entire surface of the empty shell. Fasten the two ends of a white silk thread with wax near the ends of the shell, so that the shell may be suspended with its greater diameter horizontal. Charge this insulated egg-shell conductor. With a proof-plane, carry a charge from the side of the conductor to the knob of the gold-leaf electroscope, and notice the degree of divergence of the leaves. In like manner, carry a charge from the smaller end of the conductor, and notice the greater divergence of the leaves.

Experiment 302. — Cement the end of a small glass tube to the middle of a pin, and hold the head of the pin against the knob of a gold-leaf electroscope. Observe the collapse of the leaves.

333. Distribution of the Charge. — As the electrification is self-repulsive, *the charge lies wholly upon the outer surface.* The amount of electrification per unit of surface is called the *surface density.* Whenever a charge is communicated to a conductor, the electrification distributes itself over the surface of the conductor until it reaches a condition of equilibrium. A change in the area of the surface works a corresponding change in the surface density, as was shown in Experiment 300. The distribution is a function of the surface, independent of the substance of the conductor, and greatest where the curvature is the greatest. On a sphere, the density is uniform; on an egg-shaped conductor, it is greatest at the smaller end.

(*a*) Since any charge is self-repulsive, there must be, at every point of the surface of a charged conductor, an outward pressure against the surrounding dielectric. The surface density increases with the curvature, but the repulsion increases still more rapidly, varying as the square of the density. When the density becomes about a hundred electrostatic units per square centimeter, the electrification cannot be retained upon the conductor, and sparks fly into the surrounding air. The discharge takes place most readily where the density is the greatest; i.e., where the curvature is the greatest, as at *a point.* Since the air in contact with such a point is similarly electrified, and, therefore, repelled, an air-current passes from the point, and the charge is dissipated by *convection.* As a general thing, points and sharp edges are avoided in apparatus for use with static electricity, but they are sometimes purposely provided.

334. Process of Electrification. — *When two dissimilar substances are brought into contact and then separated, they are equally and oppositely electrified.* If the substances are poor conductors, they must be rubbed together; i.e., contact must be made at every point in order to secure electrification over the entire surface. If the substances are good conductors, the opposite and equal electrifications flow to the point last in contact, and pass by conduction from one to the other. Evidently the resultant, in this case, is zero.

335. Electrification and Energy. — When two dissimilar substances are brought into contact, they become oppositely electrified. When they are subsequently separated, work is done against their mutual electric attraction. This work represents the increased potential energy of the system. That energy is at zero when the bodies are in contact, and at its maximum when they are at an infinite distance from each other. If the charged bodies are similarly electrified, work is done against their mutual electric repulsion. Then the potential energy of the system varies from zero at an infinite distance between the bodies to a maximum when the two are in contact.

336. Electrical Field and Lines of Force. — *The space surrounding an electrified body and through which the electrical force acts is called an electrical field of force.* We may imagine lines drawn in this field, each indicating the direction in which a unit of electrification would move if placed in the field. Evidently, we may draw an indefinite number of such lines, but in order to "map" an electrical field and to show the relative intensity of different parts of it, it has been agreed that one line shall be drawn through each square centimeter of surface for each dyne of force exerted in the field. If one such line representing a force of one dyne cuts each square centimeter of surface, the field is said to be of unit intensity; i.e., *a unit of electrification in a field of unit intensity would be acted upon by a force of one dyne tending to move it along a line of force.*

(*a*) We may further imagine two electrified bodies as immersed in an electrical field of force, and connected by elastic lines of force that tend to shorten and that are self-repellent. In such a field, there will be a stress parallel to the lines of force, and of the nature of a tension; also a stress perpendicular to the lines of force, and of the nature of a pressure.

337. Potential. — In a general way, it may be said that *potential represents degree of electrification*, or that it is the relative condition of a conductor that determines the direction of a transfer of electrification to it or from it. The direction of the transfer depends, not upon quantity or upon surface density, but upon relative potential.

(*a*) In dealing with masses of matter and the force of gravitation, it is easy to understand that the potential energy of a unit mass at a given point is an attribute of that point, and that the condition at the point is due only to the existence of attracting bodies, and is the same whether the unit mass is actually there or not. This attribute of the point is called *the gravitation potential at the point.* The potential at a point is zero when a unit particle if placed there would have no potential energy, as when the point is at an infinite distance from all attracting masses. Similarly, if a charge is placed anywhere in an electrical field, it has a potential energy due to the work done upon it in carrying it thither. This attribute of the point is called *the electrical potential at the point.* The charge thus placed is subject to the action of the electrical force that tends to move it to another point where its potential energy will be less; i.e., to move it from a point of higher to a point of lower potential. *An electrostatic unit difference of potential exists between two points when an erg of work is involved in moving unit charge from one point to the other.*

(*b*) Relative potential is analogous to level. As the sea-level is taken as the zero from which altitudes are measured, so the surface of the earth is taken as the zero of electric potential. As water tends to flow from higher to lower levels, and as heat tends to flow from places of higher to places of lower temperature, so electrification tends to flow from places of higher to places of lower potential until an equalization is reached. In the latter case, the flow is called a *current of electricity.* If the quantity of electrification is limited, the current is temporary, as in the discharge of a Leyden jar. If the difference of potential is maintained, the current is continuous, as in the case of a voltaic cell.

338. Equipotential Surfaces. — Surrounding a unit positive charge as a center, there is a surface such that it will require the expenditure of an erg of work to carry a unit negative charge from the center to any point of that surface, as from A to P. Further from the center, there is another surface such that it will require the expenditure of an erg of work to carry a unit negative charge from the first surface to the second, as from P to Q; i.e., such that there is unit difference of potential between the two surfaces. *Such surfaces as these, throughout which the potential is everywhere the same, are called equipotential surfaces.* Such surfaces are everywhere perpendicular to the lines of force that cut the electrical field.

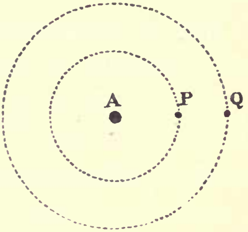


FIG. 318.

(*a*) When a charge is moved from any point to another point in the same equipotential surface, no work is done upon it. When a charge is moved from one such surface to another, the work done is independent of the path of transfer. If such a surface was to be rendered impenetrable, a particle could lie upon it without tendency to move along it in any direction. If any two points in such a surface were to be joined by a conductor, no flow of electrification would take place. The closed lines in Fig. 319 are equipotential lines drawn, of course, upon equipotential surfaces, about two similarly electrified spheres, the quantity of electrification at A being twice that at B. The lines radiating from A and B represent lines of force.

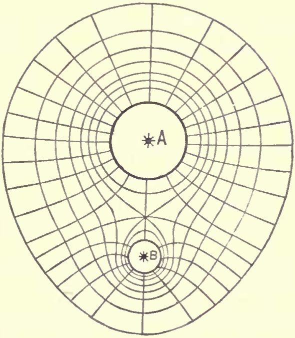


FIG. 319.

Experiment 303. — Charge a gold-leaf electroscope positively until its leaves diverge slightly. Similarly charge a like electroscope until its leaves diverge widely. The potential of the second charge is higher than that of the first. Connect the knobs of the two electroscopes by an insulated conductor. The change in the divergence of the leaves shows that electrification has passed from a place of higher to a place of lower potential.

339. Electromotive Force. — Whenever a positive charge is placed upon a conductor, it raises the potential at the point of application, and there is a flow of electrification until the surface of the conductor is an equipotential surface. If two conductors at different potentials are connected by a wire, a transfer of electrification will take place until the difference of potential disappears. *Whatever its nature, the agency that tends to produce such a transfer is called electromotive force.*

Electrostatic Induction.

Experiment 304. — Electrify a glass rod by rubbing it with silk, and bring it near the electroscope but without making contact. The leaves diverge. When the rod is removed, the leaves fall together. Repeat the experiment, holding a glass plate between the rod and the electroscope.

Experiment 305. — Bring a metallic sphere positively charged near an insulated cylindrical conductor with hemispherical ends and provided with pith-ball and linen thread electroscopes as shown in Fig. 320. The divergence of the pith-balls shows electrification at the ends but not at the middle of the conductor. With the proof-plane and gold-leaf electroscope, examine the condition of the conductor at the points *A*, *B*, and *m*, and compare your results with the representations in the figure. Remove the sphere from the vicinity of the conductor, or discharge it by touching it with the hand. All signs of electrification on the conductor disappear, showing that the charges at *A* and *B* were opposite and equal.

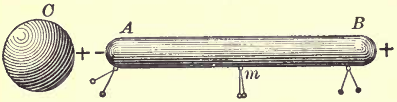


FIG. 320.

Experiment 306. — Electrify the insulated conductor, *AB*, as in Experiment 305. Touch it with the finger, thus connecting it with the earth and making it of indefinite length; its positive electrification is so diffused as to be insensible. Remove first the finger and then the electrified sphere. The negative electrification being no longer held at *A* by the attraction of the positive electrification at *C*, diffuses itself over the cylinder, and the balls at each end of the cylinder diverge, all being charged negatively.

Experiment 307. — Suspend two egg-shell conductors (see Experiment 301), as shown in Fig. 321. Be sure that the shells are in contact. Bring an electrified glass rod near one of them, and slide one of the loops along the supporting rod until the shells are about 10 cm. apart. Hold the electrified rod between the shells. It will attract one and repel the other, showing that they are oppositely electrified. Bring the shells into contact again, and charge them similarly, as indicated in Experiment 306.

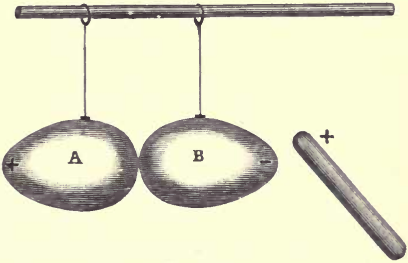


FIG. 321.

Experiment 308. — Charge one of the egg-shells of Experiment 307, and suspend it above the knob of a gold-leaf electroscope and at such a distance that the leaves of the latter diverge but slightly. Provide a plate of beeswax or of sulphur, the thickness of which is a little less than the distance between the shell and the knob, and pass a gas flame over its surface to remove all electrification from it. Hold the plate between the shell and the electroscope without touching either. The leaves of the electroscope diverge more widely, as if the electric force passed more readily through the plate than through the air.

340. Electrification by Induction. — The collapse of the leaves of the electroscope in Experiment 304 showed that there was no transfer of electrification from the rod to the electroscope. Whenever an electrified body is brought into the vicinity of an unelectrified conductor, thus placing the latter in an electrical field, and subjecting the intervening dielectric to a condition of strain, the unelectrified conductor becomes electrified. A dissimilar electrification appears on the side nearer the electrifying conductor, and similar electrification upon the further side. *Electrification produced in this way, by the influence of an electrified body and without contact with it, is called electrification by induction.*

(*a*) A charged body surrounded by a dielectric (e.g., the air) induces an equal and opposite charge on the inner surface of the enclosure containing the charged body and the dielectric (e.g., the walls of the room). An induced charge is opposite in kind to the charge of the inducing body.

(*b*) The amount of inductive effect that takes place across an intervening medium depends upon the nature of that medium; it is a function of the dielectric. The relative powers of different substances to transmit electrical inductive effects is called *specific inductive capacity*, or the *dielectric constant*. The introduction of a dielectric plate increases the inductive effect when the dielectric constant of the plate is greater than that of air.

Experiment 309. — Charge a gold-leaf electroscope to a high potential, i.e., until its leaves diverge widely. Bring the electric pendulum of Experiment 290, or a similar metallic ball similarly suspended, into contact with the knob of the electroscope, and notice the diminished divergence of the leaves. The charge being distributed over a larger surface, the potential is lowered.

341. The Capacity *of a conductor is the amount of electrification required to raise its potential from zero to unity, i.e., the ratio of its charge to its potential.* The unit of capacity is the capacity of a conductor that requires unit quantity to produce unit difference of potential: it is called a *farad;* one-millionth of a farad is called a *microfarad.* The capacity of a simple conductor is dependent upon its size and shape, and upon the form and position of neighboring conductors that may act upon it inductively.

(*a*) Under like conditions, the capacities of spheres are proportional to their radii.

Experiment 310. — Spread a sheet of tin-foil upon a pane of glass supported on a tumbler. Charge the tin-foil by repeated sparks from the electrophorus (§ 404) until it will receive no more. Count the number of sparks that the tin-foil will receive.

Experiment 311. — Lay a sheet of tin-foil upon the table so that it will be in electrical connection with the earth. Over it place the glass and foil used in Experiment 310. Charge the upper sheet as before, and notice that it will receive a much greater number of sparks. Touch the lower sheet of tin-foil with a finger of one hand, and the upper sheet with a finger of the other hand, thus discharging the apparatus. A pricking sensation will be caused by the discharge.

342. A Condenser *consists of a pair of conductors slightly separated by a dielectric.* If one of these conductors is connected to earth, it requires a much larger quantity of electrification to raise the potential of the other from zero to unity, i.e., the capacity of the other is greatly increased. A condenser is, therefore, a device for increasing the electrical density without increasing the potential, i.e., for accumulating a large charge with a small electromotive force. The smaller the distance between the conducting surfaces, the greater the capacity of the condenser.

(*a*) When a charge is given to a conductor on one side of the dielectric, it induces an opposite charge in the conductor on the other side, as in Experiment 305. By their mutual attraction, these opposite charges are "bound" at the surface of the dielectric, thus leaving the first conductor free to receive another charge, which acts inductively upon the second conductor as the original charge did; and so on, successively. This process necessarily results in an increasing strain of the dielectric; an over-charge may break it.

(*b*) The nature of the dielectric has a great effect on the capacity of the condenser. The specific inductive capacity of a dielectric may now be defined as the ratio of the capacity of a condenser with air insulation to the capacity of a similar condenser using the dielectric in question. For instance, changing the dielectric from air to ebonite more than doubles the capacity of the condenser.

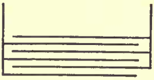


FIG. 322.

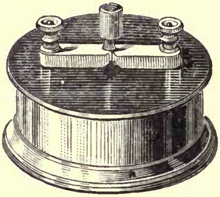


FIG. 323.

(*c*) Condensers of the flat type (Fig. 322), consisting of tin-foil conductors separated by thin, flat dielectric sheets (usually of mica), are much used. To obtain large area, and hence great capacity, they are arranged alternately in two series. A condenser of this type (Fig. 323), having a capacity of one micro-farad, weighs 6 or 7 pounds. The plug serves to connect the coatings when the instrument is not in use.

343. The Leyden Jar.

343. The Leyden Jar. — The most common and, for many purposes, the most convenient form of condenser is the Leyden jar. This consists of a glass jar, coated within and without for about two-thirds its height with tin-foil, and a metallic rod that communicates by means of a small chain with the inner coat, and terminates above in a knob or a disk. The upper part of the jar, and the wooden or ebonite stopper that closes the mouth of the jar and supports the rod, are generally coated with sealing-wax or shellac-varnish to lessen the deposition of moisture from the air. Evidently, it may be considered as a flat condenser rolled into cylindrical form.

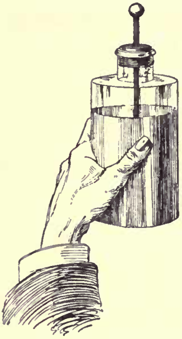


FIG. 324.

(*a*) The jar may be charged by holding it in the hand as shown in

Fig. 324, or otherwise placing the outer coat in electrical connection with the earth, and bringing the knob into contact with a charged body. If the outer coat is insulated so that the repelled electrification cannot pass to the earth, the jar cannot be very highly charged. To discharge the jar, pass a stout wire through a piece of rubber tubing and bend it into a **V** shape, or, in some other way, provide the wire with an insulating handle. Bring one end of the wire into contact with the outer coat, and then bring the other end into contact with the knob. It is well to provide the wire "discharger" with metal balls at its ends.

(*b*) That the phenomenon of electrification pertains to the dielectric and not to the conducting plates may be shown with a Leyden jar with movable coats. The parts being put together in proper order and the jar charged, the inner coat, *C*, is removed with a glass rod, and the glass vessel, *B*, lifted from the outer coat, *A*. Tests show that *A* and *C* are not electrified, and that *B* is electrified. By placing thumb and forefinger on the inner and the outer surfaces of *B*, a slight shock may be felt. When the parts are put together, the condenser is highly electrified, and may be discharged in the usual way. After a Leyden jar is discharged, a "residual charge" gradually accumulates, as if the glass was strained and slowly returned to its normal condition. The time-interval required for the residual charge is lessened by tapping the jar and thus facilitating the molecular readjustments. The metallic coats simply provide the means for the prompt discharge of the superficial layers of the molecules of the dielectric.

(*c*) A number of Leyden jars having their coats connected constitutes an *electric battery*.

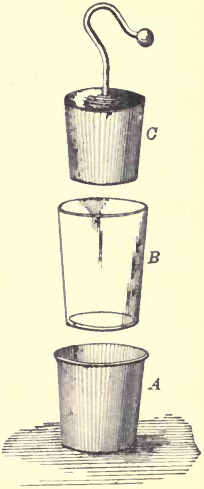


FIG. 325.

344. Nature of Electricity. — The phenomena of electrification indicate that electricity is a perfectly incompressible substance of which all space is completely full, and the question arises, *is it not identical with the ether?* It has recently been suggested that the ether is made up of two equal opposite constituents, each endowed with inertia, and connected to the other by elastic ties which the presence of gross matter generally weakens and sometimes dissolves, and that these two constituents of the ether are positive and negative electricity. According to this provisional hypothesis, and the general belief of physicists, *electricity is a form of matter rather than a form of energy.* A full discussion of the ultimate nature of electricity is beyond the province of this book, but it is safe for us to say that *electricity is that which is transferred from one body to another in the process of electrifying them.*

345. Theory of Electrification. — When electricity is transferred from one body to another and the bodies are separated (see §§ 334 and 335) against their mutual attraction, the intervening medium is thrown into a state of strain indicated by the lines of force. *This state of strain in the dielectric constitutes electrification.* Whatever the real nature of electricity, and whether the phenomena of attraction and repulsion are explained on the hypothesis that the elastic ether is strained by the separation of the electrified bodies and tends to recover its normal condition, or on any other hypothesis, *electrification results from work done, and is a form of potential energy.*

CLASSROOM EXERCISES

1. How can you show that there are two opposite kinds of electrification?

2. How would you test the kind of electrification of an electrified body?

3. (*a*) What is a proof-plane? (*b*) An electroscope? (*c*) Describe one kind of electroscope. (*d*) Another kind.

4. Why do we regard the electrifications produced by rubbing two bodies together as opposite and equal?

5. Why is it desirable that a glass rod used for electrification be warmer than the atmosphere of the room where it is used?

6. Two small balls are charged respectively with + 24 and − 8 units of electrification. With what force will they attract one another when placed at a distance of 4 centimeters from one another?

Ans. 12 dynes.

7. If these two balls are then made to touch for an instant and then put back in their former positions, with what force will they act on each other? *Ans.* Repulsion of 4 dynes.

8. At what distance from a small sphere charged with 28 units of electrification must you place a second sphere charged with 56 units that one may repel the other with a force of 32 dynes? *Ans.* 7 cm.

9. (*a*) Having a metal globe positively electrified, how could you with it negatively electrify a dozen globes of equal size without affecting the charge of the first? (*b*) How could you charge positively one of the dozen without affecting the charge of the first?

10. Suppose two similar conductors to be electrified, one with a positive charge of 5 units and the other with a negative charge of 3 units. They are made to touch each other. When they are separated, what will be the charge of each?

Ans. One unit of positive electrification.

11. In what way may an electric charge be divided into three equal parts?

12. When a pin or needle is held with its point near the knob of a charged gold-leaf electroscope, the leaves quickly collapse. Explain.

13. A Leyden jar standing on a plate of glass cannot be highly charged. Why?

14. Will you receive a greater shock by touching the knob of a charged Leyden jar when it is held in the hand or when it is standing on a sheet of glass? Explain.

15. Imagine that the knob of a gold-leaf electroscope is connected by wire to the knob of a Leyden jar, and that a given amount of electrification is communicated to the knob of the jar. Will the divergence of the leaves of the electroscope be greater when the jar is held in the hand or when it is standing on a sheet of glass? Explain.

16. Show by a diagram that electrostatic induction always precedes electric attraction, and explain why the repulsion between the opposite electrifications does not neutralize the attraction.

LABORATORY EXERCISES.

Additional Apparatus, etc. — A rubber comb; metal pipe; tin pail.

1. Quickly pass a rubber comb through the hair and determine whether the electrification of the comb is positive or negative.

2. Provide an insulated egg-shell conductor as described in Experiment 301, which the teacher will electrify by induction, using a glass rod that has been rubbed with silk or with flannel. Determine the kind of electrification of the conductor experimentally, and thence determine theoretically whether the glass rod was rubbed with silk or with flannel.

3. Show that an electric charge is self-repulsive by blowing a soap-bubble on a metal pipe and then electrifying it. Compare the change in the size of the bubble with that noticed in Experiment 30.

4. Bring an electrified body near the knob of a gold-leaf electroscope; touch the knob with the finger; remove the finger; remove the electrified body. Bring a rod that is positively charged near the knob and, from the increased or diminished divergence of the leaves, determine whether the electrification of the first body was positive or negative.

5. Twist some tissue paper into a loose roll about six inches long. Stick a pin through the middle of the roll into a vertical support. Present an electrified rod to one end of the roll, and thus cause the roll to turn about the pin as a horizontal axis. Give this piece of apparatus a scientific name.

6. From a horizontal glass rod or a tightly stretched silk cord, suspend a fine copper wire, a linen thread, and two silk threads, each at least a meter long. To the lower end of each, attach a metal weight of any kind. Place the weight supported by the wire upon the disk of a gold-leaf electroscope. Bring an electrified rod near the upper end of the wire; the gold leaves diverge instantly. Repeat the experiment with the linen thread; the leaves diverge soon. Repeat the experiment with the dry silk thread; the leaves do not diverge at all. Rub the rod upon the upper end of the silk thread; no divergence follows. Wet the second silk cord thoroughly, and repeat the experiment; the leaves diverge instantly. Record the teaching of the experiment.

7. Prepare two wire stirrups, *A* and *B*, like those shown in Fig. 313, and suspend them by threads. Electrify two glass rods by rubbing them with silk, and place them in the stirrups. Bring *A* near *B*.

Notice the repulsion. Repeat the experiment with two sticks of sealing-wax that have been electrified by rubbing with flannel. Notice the repulsion. Place an electrified glass rod in *A*, and an electrified stick of sealing-wax in *B*. Notice the attraction. Give the law illustrated by these experiments.

8. Place a gold-leaf electroscope inside an insulated tin pail and electrify the pail. Describe and explain the indications given by the electroscope.

9. Insulate a tin pail, and run a fine wire from its edge to the knob of an electroscope. Suspend a metal ball by a silk thread, electrify it, and lower it into the pail without contact. Notice and account for the divergence of the leaves of the electroscope. Touch the pail with a finger. Notice and account for the collapse of the leaves. Remove the finger and withdraw the ball. Notice and account for the divergence of the leaves. If the ball is negatively charged, what is the final charge of the electroscope?

B. CURRENT ELECTRICITY.

Experiment 312. — Partly fill a tumbler with a solution made by slowly pouring one part of sulphuric acid into ten parts of water. Place a strip of zinc, 2 × 10 cm., in the tumbler of dilute acid, and *notice the bubbles that rise*. Apply a flame to them as they reach the surface of the liquid, and notice that they burn with slight puffs. Hydrogen is evolved by the chemical action between the zinc and the acid.

Experiment 313. — Take the zinc from the tumbler of acid and, while it is yet wet, rub thereon a few drops of mercury, thus *amalgamating the zinc*. The amalgamated surface will have the appearance of polished silver. Replace the zinc in the acid, and notice that no bubbles are given off. Place a copper strip, 2 × 10 cm., in the solution, being careful that it does not touch the zinc. No bubbles appear on either the copper or the zinc. Bring the strips together at their upper ends as shown in Fig. 326. Bubbles now arise from the copper. Connect the metals above the liquid by a piece of copper wire about No. 18. The same results are observed.

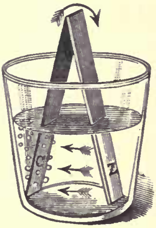


FIG. 326.

NOTE. — Always make such connections secure, metal to metal, and with large area of contact. Each metal strip may be bent at the top so as to clasp the edge of the tumbler, leaving the part on the inside long enough to reach very nearly to the bottom.

346. Suspicion. — It seems as though a metallic contact is necessary to bring about this phenomenon of bubbles on the copper. We have a complete circuit of materials, copper strip, wire, zinc strip, and acid. Perhaps we do not see all that is taking place in the system.

Experiment 314. — Solder a wire 50 centimeters long to each strip. This gives a better electrical contact than simply twisting the wire about the strip. Place the strips in the acid, and bring the free ends of the wires into contact with the tongue, one above and one below it, being sure that there is no acid on the wires. A bitter, biting taste is felt. Make sure that this taste disappears when either strip is removed from the solution; when either wire is disconnected from the tongue; or when the circuit is broken at any point.

Experiment 315. — Hold the two wires over a compass-needle as shown in Fig. 327. No change appears. Bring the two ends of the wire into contact, and thus close the circuit. The needle instantly flies around as though it was trying to place itself at right angles to the wire. Break the circuit, and the needle swings back to its north and south position. Twist the wires together, and bend the conductor into a loop so that the current passes above the needle in one direction and beneath the needle in the other direction, as shown in Fig. 328. The deflection of the needle will be greater than before. If the wire is formed into a loop that makes several turns about the needle, the deflection will be

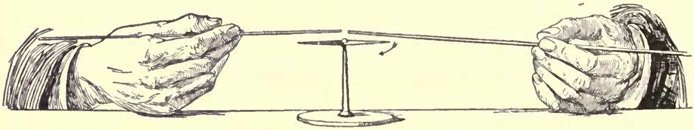


FIG. 327.

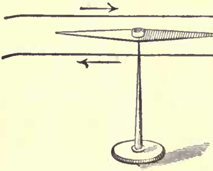


FIG. 328.

greater still. Continued investigations with this simple apparatus will show that the hydrogen bubbles cling tenaciously to the copper, and that, by this "polarization of the cell," its electrical power is much diminished.

Experiment 316. — Put the cover of a tin spice-box into a fire and thoroughly melt the tin coating from the iron plate. The cover thus prepared is to be used as a mold for casting a zinc plate 6 mm. thick. Place the mold on a fire-shovel and hold it over a hot fire, preferably a gas or gasoline burner. Fill the mold with zinc clippings, and when they have melted, place in the liquid metal a copper wire about 28 cm. long, bent as shown in Fig. 329. Turn out the flame and allow the zinc to cool. Remove the zinc plate from the mold. If the work has been properly done, the hook of the wire will be embedded in the zinc, and the straightened wire will support the plate from its edge as shown in Fig. 330. Smooth the rough edges of the plate with a file, and amalgamate the zinc with mercury.

FIG. 329.

FIG. 330.

Invert a common tumbler on a square board of soft pine, about 1.5 cm. thick, and large enough to serve as a cover for it. Run a pencil around the edge of the tumbler and draw the diagonals of the inscribed and circumscribed squares, as shown in Fig. 331. Bore holes as shown at *a*, *b*, *c*, and *d* just large enough to admit an electric (arc) light carbon. Cut four such carbons to lengths that are equal and less than the depth of the tumbler. If the carbons are copper-coated,

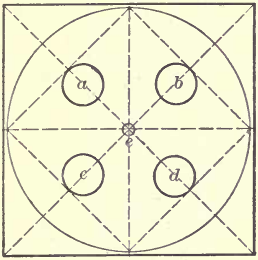


FIG. 331.

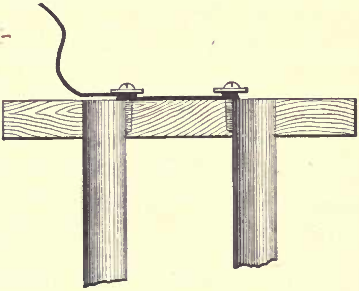


FIG. 332.

dissolve the copper with nitric acid from all of the rod except 1.5 cm. at the upper end. Insert one end of each carbon into one of the holes, and connect the four carbons by a copper wire as shown in Figs. 332 and 333. Pass the wire of the zinc plate through a small hole at the middle of the board, so that the plate may be suspended in the tumbler as shown in Fig. 334. Wedge the wire in place. Be careful that the wire from the zinc does not touch the wire from the carbons on the top of the cover. It will be well to insulate the former wire, by slipping over it a piece of soft rubber tubing of very small bore and two or three inches long. The tubing may be held in place by a kink in the wire. Wire that has been insulated with cotton and paraffine may be used for supporting the zinc, the end that is to be embedded in the zinc being scraped bare before the casting.

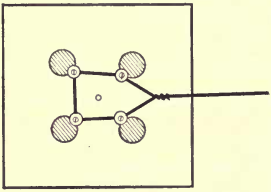


FIG. 333.

Prepare a solution as follows: slowly pour 167 cu. cm. of sulphuric acid into 500 cu. cm. of water, and let the mixture cool. Dissolve 115 g. of potassium dichromate (bichromate of potash) in 335 cu. cm. of boiling water, and pour the hot solution into the dilute acid. When this liquid is cool, fill the tumbler about two-thirds full with it, and place the carbons and zinc therein. Adjust the height of the plate as shown in Fig. 334, and be sure that the zinc does not touch any of the carbons. The zinc and carbon should be kept in the fluid no longer than is necessary. It is well to provide a second tumbler in which to drain them. Each pupil should make at least one of these cells; he will find three or four of them very useful. The cost of the cell need not exceed twenty-five cents. Using this cell, repeat Experiment 315. Notice the direction of the deflection of the needle. Reverse the cell connections, and notice that the needle deflects in the opposite direction.

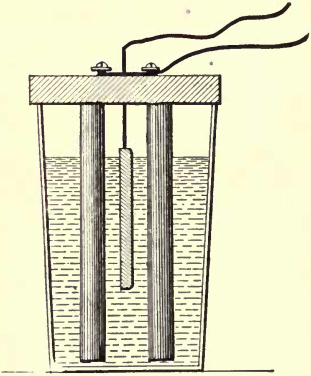


FIG. 334.

347. Certainty. — We are now sure that something unusual is going on in the wire. *This something is called a current of electricity.* Its exact nature is not yet known, but much has been learned about its properties and the laws by which it is governed. There is a difference of potential between the plates, and the chemical action between the liquid and one or both of the plates, or some other cause, tends to maintain that difference. *The containing vessel, the plates, and the exciting liquid constitute a voltaic cell.*

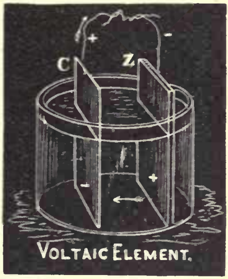


FIG. 335.

348. Direction of Current. — We cannot conceive a current without direction. The actual direction of current-flow is not known, but, for the sake of convenience and uniformity, electricians assume that the current flows from the carbon through the wire to the zinc, and from the zinc through the liquid to the carbon.

349. Plates, Poles, etc. — The entire path traversed by the current, including liquids as well as solids, is called *the circuit.* The plate that is the more vigorously acted upon by the liquid is called the *positive plate;* the other is called the *negative plate.* The free end of the wire attached to the negative plate is called the *positive pole* or electrode; that of the wire attached to the positive plate is called the *negative pole* or electrode. When the two electrodes are joined, *the circuit is closed;* when they are separated, *the circuit is broken.* When several cells are connected so that the positive plate of one is joined to the negative plate of the next, as zinc to carbon, and so on, as shown in Fig. 336, they are said to be grouped or joined *in series.* When all of the positive plates are connected on one side, and all of the negative plates are connected on the other side, as shown in Fig. 337, the cells are said to be joined *in parallel*, *or in multiple arc.* A number of cells joined in either way is called a *voltaic battery.*

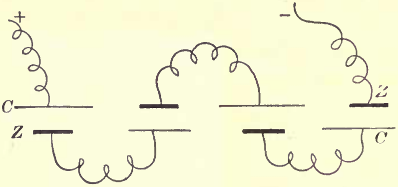


FIG. 336.

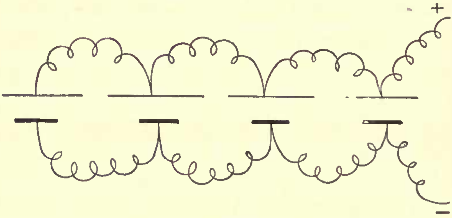


FIG. 337.

(*a*) The nomenclature of plates and poles is a little perplexing, but the possible confusion may be avoided by remembering that in any part of an electric circuit, a point from which the current flows is called positive (+) and a point toward which the current flows is called negative (–).

NOTE. — The representation of the zinc and carbon plates, as at *Z* and *C* in Fig. 336, is the conventional way of representing a voltaic cell.

Experiment 317. — Provide a flat piece of soft pine wood about 10 cm. square and 3 cm. thick, and wind on evenly one layer of No. 16 cotton-covered or insulated copper wire, covering the whole block. Secure the two ends of the wire by double-pointed tacks. Place a small pocket compass upon the block thus wound, and turn the block until the coils of wire are parallel to the needle when the

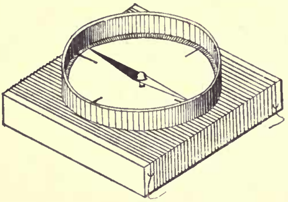


FIG. 338.

circuit is open. Then pass a current through the coil. The deflection of the needle is much stronger than before, although, owing to the weakening of the cell, the deflection falls off after a time. The instrument we have made is called a *galvanoscope*. If a pocket compass can be spared for this exclusive use, it is well to mount it in a grooved block, and to attach the terminals of the wire to the bases of two binding posts, as shown in Fig. 339.

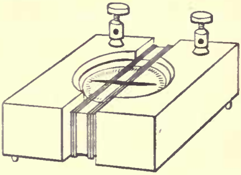


FIG. 339.

Experiment 318. — Interpose 20 feet of No. 30 (or finer) iron wire in the circuit of a voltaic cell. Connect it so that the current will flow from the carbon through the galvanoscope, through the iron wire, and back to the battery. In other words, connect the wire and galvanoscope in series. The deflection will be less than before. Keep the current on just long enough to read the galvanoscope; otherwise, the diminished deflection may be due more to the weakening of the cell than to the interposition of the wire.

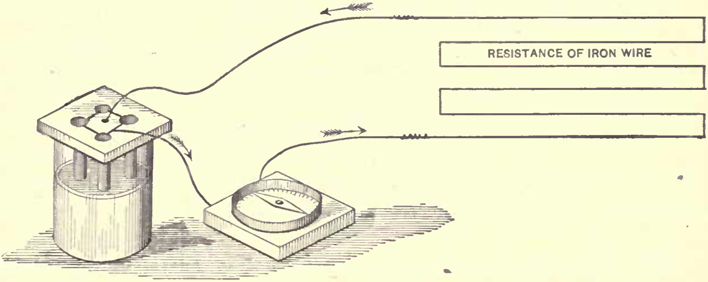


FIG. 340.

350. Resistance. — The interposition of the iron wire appears to diminish the electrical effect, or to resist the current flow. This property exists in all substances, and its manifestation is accompanied by a transformation of electrical energy into heat. *The property of a conductor* *by virtue of which the passage of an electric current through it is diminished, and part of the electric energy dissipated is called resistance.*

(*a*) We know nothing of the nature of electrical resistance, and perhaps can best define it, as we soon shall (§ 361, *c*), in terms of difference of potential and current strength.

(*b*) The word "resistance" is also applied to a material device, such as a coil of wire, introduced into an electric circuit on account of the resistance that it offers to the passage of the current.

Experiment 319. — Provide 20 feet of No. 30 iron wire, 20 feet of No. 30 copper wire, 60 feet of No. 30 iron wire, and 20 feet of No. 20 iron wire. Repeat Experiment 318 with each of these wires, in each case noting the deflection of the galvanoscope, *G*.

Each wire may be coiled on a board, care being taken that adjacent coils do not touch. Coiled or uncoiled, the wires may be connected as in Fig. 341, and the free end of *F* touched at 1, 2, 3, and 4 successively. Give the cell a moment's rest between successive contacts.

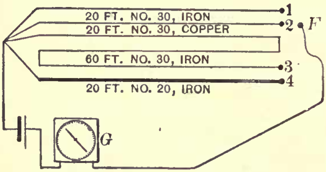


FIG. 341.

351. The Ohm *is the practical unit of resistance.* It is the resistance of a column of pure mercury one square millimeter in cross-section and 106.3 centimeters long, and at a temperature of 0°. A thousand feet of No. 10 copper wire, or 9.3 feet of No. 30 copper wire, has a resistance of very nearly an ohm, — an important "rough and ready" standard.

(*a*) The ohm has been repeatedly determined by societies and congresses of electricians. The British Association determined its magnitude with great care, but there was an error in the method that long passed unnoticed. The international ohm, defined above, is equal to 1.013+ B.A. ohms. Resistance boxes and other apparatus measuring in B.A. ohms are common; their results should be corrected as above indicated.

(*b*) A million ohms is called a *megohm;* one-millionth of an ohm is called a *microhm.*

352. Laws of Resistance. — Three important laws have been experimentally established: —

(1) *Other things being equal, the resistance of a conductor is directly proportional to its length.*

(2) *Other things being equal, the resistance of a conductor is inversely proportional to its area of cross-section,* or to the square of its radius or diameter.

(3) *Other things being equal, the resistance of a wire depends upon the material of which it is made.* At a given temperature, resistance is directly proportional to a constant that is different for different substances.

(*a*) This constant, K, is called *the specific resistance* or the *resistivity* of the material. The specific resistance of a substance is the resistance at 0° C. of a cubic centimeter of the substance, i.e., of a conductor made of the substance, 1 cm. long and 1 sq. cm. in cross-section. It varies widely with temperature and other considerations, and is practically measured in microhms. The reciprocal of resistivity is called *conductivity*, and is measured in a unit called the *mho.* Tables of resistances, etc., are given in the appendix.

(*b*) The laws of resistance may be expressed algebraically as follows: —

$$R = \frac{Kl}{\pi r^2},$$

in which R represents the resistance of the wire in ohms; K, the resistivity of the material; and l and r, the length and radius in centimeters.

Experiment 320. — Heat a long, fine iron wire to dull redness by an electric current, and dip a loop of the hot wire into ice-cold water. The resistance of the cooled part of the wire is lessened, the current is increased thereby, and the uncooled part of the wire becomes highly incandescent.

353. Effect of Temperature on Resistance. — The resistance of metals and of most other substances increases as the temperature rises. But the resistance of some substances, notably carbon and electrolytes, is lowered by heating. The "cold" resistance of the carbon filament of an incandescent electric lamp is much greater than the resistance of the same filament when the lamp is lighted.

(*a*) Suppose a wire at any point to become reduced to *half* its diameter. The cross-section will have an area $\frac{1}{4}$ as great as in the thicker part. The resistance here will be 4 times as great, and the number of heat units developed will be 4 times as great as in an equal length of the thicker wire. But 4 times the amount of heat spent on $\frac{1}{4}$ the amount of metal will warm it to a degree 16 times as great. In other words, the heat developed by a given current in different parts of a wire of uniform material and varying size is inversely proportional to the fourth power of the diameters.

CLASSROOM EXERCISES.

1. What is the resistance of a No. 10 copper wire 1,000 feet long? (Consult the table in the appendix.)

2. What is the resistance of 800 feet of German silver wire, No. 4?

3. What is the resistance of 750 feet of iron wire, No. 8?

4. What is the resistance of 350 feet of silver wire, No. 14?

5. What is the resistance of 6,050 feet of copper wire, No. 25?

6. There is a "fault" in a telegraph line 3,590 feet long and made of No. 14 iron wire. By means of electrical instruments, it is found that the resistance of the wire from one end to the fault is 1.75 ohms. How far is the fault from the end of the line?

7. What is the resistance of the whole line mentioned in Exercise 6?

8. How far away would the fault have been, had the line been of No. 14 copper wire?

9. Determine the diameter of a copper wire that has a resistance of 2 ohms per thousand feet.

10. What is the resistivity of a wire 50 mils in diameter, 900 feet long and having a resistance of 46.2 ohms? Of what material mentioned in the table on page 593 might the wire be made?

354. Analogy. — In many respects, it is convenient to compare the flow of electrification through a wire to the flow of water through a horizontal pipe. Such a comparison yields the following analogues: —

Functions.	*Hydraulic Units.*	*Electromagnetic Units.*
Pressure.	Head in feet.	Volt.
Quantity.	Pound.	Coulomb.
Rate of flow.	Pounds per second.	Coulombs per second, or ampere.
Resistance.	No definite unit.	Ohm.
Work.	Foot-pound.	Joule.
Rate of work.	Foot-pounds per second, or horse-power.	Volt-ampere, or watt.

355. The Volt. — Just as a head of water supplies a hydraulic pressure that causes the liquid to flow through a pipe in spite of friction, so there is an electrical pressure that forces a current through a conductor in spite of its resistance. As hydraulic pressure might be called water-moving force, so electrical pressure is called electromotive force (E.M.F.). *The unit of electrical pressure is called the volt*, and is almost the same as the electromotive force of a cell consisting of a copper and a zinc plate immersed in a solution of zinc sulphate.

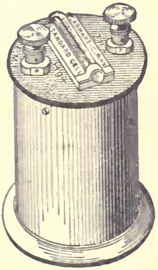


FIG. 342.

(*a*) The E.M.F. of a Daniell cell is about 1.1 volts; of a fresh chromic acid cell, 2 volts; and of a Leclanché cell, 1.5 volts. The E.M.F. of a Carhart-Clark standard cell (see Fig. 342) is 1.44 volts at 15°; conversely, a volt is about 0.7 of the E.M.F. of a Carhart-Clark standard cell at that temperature. A standard cell should never be used on a closed circuit.

Experiment 321. — Prepare a block for a galvanoscope, winding it closely with ten layers of No. 34 insulated copper wire, thus making an instrument of high resistance. Arrange a circuit as shown in Fig. 343, employing about ten feet of No. 30 iron wire. As the copper wire, *f*, is slid along the iron wire from *a* to *x*, the deflection of the galvanoscope will decrease.

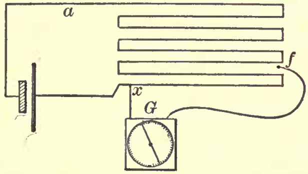


FIG. 343.

356. Difference of Potential. — When the stop-cock of a vessel like that shown in Fig. 344 is closed, water will stand at the same level in the vertical tubes, *a*, *b*, and *c*. There is no difference of pressure at different points along the tube *B*, and, therefore, no flow of water. When the stop-cock is opened, the pressure at *C* is relieved, and the greater pressure at the bottom of *A* results in a flow along the horizontal pipe. The variations in liquid pressure at different points along *B* is now shown by the differences of level in *a*, *b*, and *c* (Fig. 345). The pressure becomes less as we pass from *A* toward *C*. The analogous phenomenon is shown in Experiment 321, where the galvanoscope reveals the differences of electric pressure, or potential, at different points of the circuit.

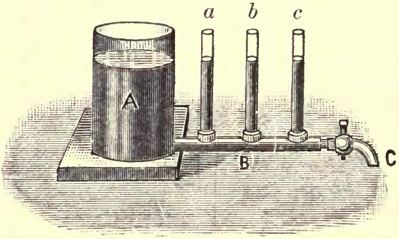


FIG. 344.

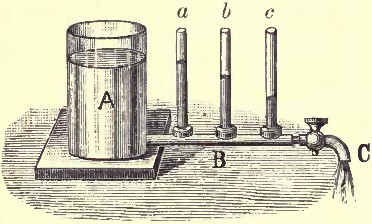


FIG. 345.

(*a*) Difference of potential is a different thing from electromotive force. The electromotive force of a circuit is the total electrical pressure existing therein, while the difference of potential is merely the difference of electrical pressure between two points on the circuit. A generator of electricity for arc lights may have an electromotive force of 3,000 volts, while the difference of potential between the terminals of an arc lamp in circuit with it is only 45 volts.

Experiment 322. — Connect several similar cells in series, as shown in Fig. 346. Put a No. 40 iron wire, *mn*, and a larger copper wire, *ef*, in circuit as shown. Slide the end of the copper wire along the iron wire from *n* toward *m* until the latter becomes red hot.

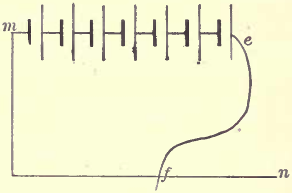


FIG. 346.

357. The Ampere. — In Experiment 322, we gradually reduced the length of the circuit, and thus reduced its resistance. As the resistance was reduced, the electromotive force of the battery sent a correspondingly increased current through the wire. This increase of current strength was manifested by the increased heating effect. *The unit of rate of flow, or current strength, is the ampere*, which may be defined as the current flowing in unit of time (second) through a wire having unit resistance (ohm), and between the two ends of which unit difference of potential (volt) is maintained.

(*a*) A 1-ampere current passes a coulomb of electricity each second, and will electrolytically deposit 0.001118 of a gram of silver, or 0.0003287 of a gram of copper in a second. A thousandth of an ampere is a *milliampere*.

358. The Coulomb. — Just as quantity of water may be measured in pounds, so *quantity of electrification is measured in coulombs.* The coulomb may be defined as the quantity of electrification carried past any point by a

1-ampere current in one second. The unit is rather large for practical purposes, and is but little used.

359. The Joule *is the electrical unit of work*, and represents the energy of one coulomb delivered under a pressure of one volt, or the work done in one second in maintaining a current of one ampere against a resistance of one ohm.

$$Joules = volts \times coulombs.$$

It is equivalent to 10^7 ergs.

360. The Watt *is the unit of electrical activity or power*, and represents the rate of working in a circuit when the electromotive force is one volt and the current is one ampere. One horse-power equals 746 watts.

$$Watts = volts \times amperes.$$

It is equivalent to 10^7 ergs per second.

361. Ohm's Law. — Representing current strength by C, voltage by E, and resistance by R, the numerical relations of these functions of an electrical current are expressed by the formula,

$$C = \frac{E}{R}, \quad \text{or} \quad E = C \times R, \quad \text{or} \quad R = \frac{E}{C}.$$

Any two of these being known, the third may be found.

(*a*) Applied to an electric generator (as a dynamo or voltaic cell), we may represent the resistance of the external circuit by R and the internal resistance of the generator itself by r. Then

$$C = \frac{E}{r + R}.$$

Thus, if the E.M.F. of a chromic acid cell is 2 volts, the internal resistance of the cell is 1.5 ohms, and the wire resistance is 0.5 ohms,

$$C = \frac{2}{1.5 + 0.5} = 1.$$

The current strength will be 1 ampere.

(*b*) Representing algebraically the definition of the watt, we have

$$W = E \times C. \qquad (1)$$

Substituting, in this equation, the above given value of E, we have

$$W = R \times C^2. \qquad (2)$$

Substituting, in the same equation, the above given value of C, we have

$$W = \frac{E^2}{R}. \qquad (3)$$

(*c*) In the light of Ohm's law, resistance might be defined as the ratio between E. M. F. and current strength, or as electric pressure divided by electric flow.

362. Joule's Law. — The work done by an electric current is equal to the product of the strength of the current, C; the fall of potential, E; and the time, t.

$$W = CEt.$$

Since, by Ohm's law, $E = CR$, we have the following equivalent expression : —

$$W = C^2Rt.$$

If E is measured in volts, C in amperes, and R in ohms, W will be expressed in joules per second, or watts. Since a small calory equals 4.2 joules,

$$H = \frac{C^2Rt}{4.2} = C^2Rt \times 0.24,$$

in which formula, H represents the number of small calories.

Experiment 323. — Join equal lengths of iron wires of different sizes end to end, and pass a gradually increasing current through them. The smallest wire will be most heated.

Experiment 324. — Join, end to end, equal lengths of iron and copper wires of the same size, and increase the current that passes through them until the iron wire is red-hot. Ascertain the thermal condition of the copper wire.

Experiment 325. — Send the current of a few cells in series through a chain made of alternate links of silver and platinum wires of the same size. The platinum links grow red-hot, while the silver links remain comparatively cool. The specific resistance of platinum is about six times that of silver, and its specific heat is about half as great; hence, the rise of temperature in wires of equal thickness traversed by the same current is about twelve times as great for platinum as for silver.

Experiment 326. — Pass a suitable current through a long, fine iron wire, and thus heat it to dull redness. By means of a sliding contact, progressively shorten the iron wire part of the circuit, as in Experiment 322. As the resistance decreases, the current increases, until the iron wire that remains in circuit is melted.

363. Distribution of Heat in the Circuit. — Since, at any instant, the current strength is uniform at every part of the circuit, it follows from the last formula given in § 362 that the heat developed in any part of the circuit will be proportional to the resistance of that part of the circuit. As the fall of potential is proportional to the resistance, *the heat energy developed in any part of the circuit is proportional to the fall of potential through that part of the circuit.*

364. Shunts. — When part of a circuit consists of two branches, each branch is said to be a shunt to the other. The current flowing through such a circuit will divide,

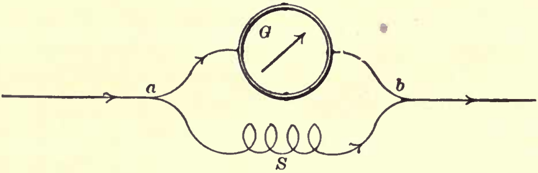


FIG. 347.

part of it going one way, and the other part the other way.

(*a*) Under such circumstances, the current that flows through the branches will be inversely proportional to the respective resistances of the branches. To illustrate, suppose that the branch that carries the galvanometer, *G*, has a resistance of 900 ohms, and that the branch that carries the coil, *S*, has a resistance of 100 ohms. Then 0.9 of the current will flow through *S*, and 0.1 through *G*.

(*b*) The introduction of a shunt lessens the resistance of the circuit. The conductivity of the circuit between *a* and *b* is the sum of the conductivities of the two branches, and conductivity is the reciprocal of resistance. Representing the resistance of the branched circuit by *R*, that of one branch by *r*, and that of the other branch by *r′*, we have

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r'}; \text{ whence } R = \frac{rr'}{r + r'}.$$

For instance, in the case of the galvanometer and the coil above mentioned,

$$R = \frac{900 \times 100}{900 + 100} = 90, \text{ the number of ohms.}$$

CLASSROOM EXERCISES.

1. A copper wire is carrying a 5-ampere current. The resistance of this wire is 2 ohms.

(*a*) How many volts are necessary to force the current through the wire?

Solution: — $E = C \times R = 5 \times 2 = 10$, the number of volts.

(*b*) How much energy is consumed in the wire?

Solution: — $W = E \times C = 10 \times 5 = 50$, the number of watts; or
$W = R \times C^2 = 2 \times 25 = 50$, the number of watts.

2. An incandescence lamp is connected with an electric generator (dynamo) 300 feet away by a No. 18 copper wire that is carrying a 1-ampere current. A fine coil galvanoscope, used as described in Experiment 321, would show differences in potential between the ends of the two wires running to the lamp, and between the two terminals of the lamp itself. What is the loss of voltage due to the line?

Solution: — The table of resistances given in the appendix shows that the resistance of the 600 feet of wire is 3.83466 ohms.

$$E = C \times R = 1 \times 3.83466 = 3.83466, \text{ the number of volts.}$$

If the lamp took 1 ampere at 100 volts, the line loss would be nearly 3.8 per cent.

3. What would be the proper size of copper wire to supply a group of lamps 400 feet away, and taking 15 amperes, so that the line loss shall be 2 volts.

Solution: — The resistance of the line would be,

$$R = \frac{E}{C} = \frac{2}{15} = 0.1333, \text{ the number of ohms.}$$

Its resistance in ohms per foot must be $(0.1333 \div 800 =)$ 0.0001666, and the resistance per 1,000 feet, 0.1666 ohms. From the table, we find that No. 2 is the nearest size of wire.

4. The wire loss of an electric motor is 156 watts. If the resistance of the motor is 2 ohms, what current flows?

Solution: —

$$W = R \times C^2; \; C = \sqrt{\frac{156}{2}} = 8.83, \text{ the number of amperes.}$$

5. How many foot-pounds per minute equal a watt? *Ans.* 44.236.

6. How many horse-power will be absorbed by a circuit of arc lamps, taking 9.6 amperes at 2,900 volts pressure?

Ans. 37.32 H.P., nearly.

7. If the electric generator that develops the current described in Exercise 6 wastes 10 per cent. of the power delivered to it, how much work was done upon it? *Ans.* 41.46 H.P.

8. A group of incandescence lamps absorbs 21 amperes. The line loss is limited to 1.5 volts.

(*a*) What is the resistance of the line? *Ans.* 0.07143 ohms.

(*b*) How many watts are lost? *Ans.* 31.5 watts.

(*c*) If the line is 800 feet from source of supply to lamps, what is the nearest size of copper wire to use? *Ans.* No. 0000.

9. An incandescence lamp absorbs 0.5 amperes at 110 volts, and gives out 16 candle-power. An arc light absorbs 10 amperes at 45 volts, and produces 2,000 candle-power. Which light is the more economical? Determine the electrical energy per candle-power absorbed by each? *Ans.* Incandescence, 3.4375 watts; arc, 0.225 watts.

10. Which has the more energy, an arc light generator capable of

delivering 10 amperes at 900 volts pressure, or an electro-plating machine that produces 1,800 amperes at a pressure of 5 volts?

11. What mechanical horse-power is necessary for 50 incandescence lamps, each taking 0.5 amperes at 110 volts, allowing 10 per cent loss for transformation from mechanical into electrical energy?

Ans. 4.09 H.P.

12. What energy is absorbed by a coil of wire of 23 ohms resistance, through which 3.5 amperes is flowing? *Ans.* 281.75 watts.

13. A coil of wire of resistance 37 ohms is subjected to a pressure of 110 volts. What energy is expended? *Ans.* 327.02 watts.

14. A dynamo receives 525 H.P. of mechanical energy, and delivers 350,000 watts at a pressure of 10,000 volts. The line that completes the circuit has a resistance of 14 ohms. (*a*) Determine the current strength. (*b*) What is the line loss in volts? (*c*) in watts? (*d*) What is the efficiency of the dynamo?

LABORATORY EXERCISES.

Additional Apparatus, etc.—Mercury; sulphuric acid; nitric acid; copper sulphate; glass tumblers; porous cup; a zinc and a copper plate; sheet lead; sheet iron; tin-plate; galvanoscope.

1. Put the cell described in Experiment 313, with clean and unamalgamated plates into circuit with a galvanoscope. Notice the movement of the needle, tap the galvanoscope lightly, and record the position in which the needle comes to rest. Observe for a minute what takes place at the surface of each plate. Amalgamate the zinc plate, carefully remove any adhering mercury drops, replace the zinc in the acid, being careful that it is as far from the copper as it was before. When the circuit is again closed through the galvanoscope, record the deflection of the needle, and observe for a minute what takes place at the surface of the plates. At intervals of two minutes, record the successive deflections of the needle. If any bubbles are visible on either or both of the plates at the end of ten minutes, rub them off without removing the plates from the acid. Take care that no mercury comes in contact with the copper, and record the deflection of the needle. Remove the copper plate from the acid, rub it thoroughly, replace it in the acid, and record the deflection of the needle. Remove the copper plate again, dip it into nitric acid, and amalgamate it. Put it back into the acid, and record the deflection of the needle. Record the teachings of your experiment.

2. Solder one end of 50 cm. of insulated copper wire, No. 20, to one end of a zinc plate 10 cm. long, 2.5 cm. wide, and 0.5 cm. thick. Similarly, solder a like wire to a piece of sheet copper 10 cm. square. Weigh the plates and their wires carefully to 0.1 of a gram. Put the zinc plate into a porous cup about 10 cm. deep and 4 cm. wide, and nearly fill the cup with dilute sulphuric acid. Put the cup into a glass tumbler about 10 cm. deep and 8 cm. wide. Pour a saturated solution of copper sulphate into the tumbler until it stands at the same level as the acid in the cup. Amalgamate the zinc, and put it back into the acid. Clean the copper plate, bend it so that it will partly encircle the porous cup, and put it into the copper sulphate solution. Put the cell into circuit with a low resistance galvanoscope. Watch carefully for bubbles on this copper plate. Record the deflection of the needle at intervals of five minutes for half an hour. Take the cell to pieces and clean its several parts. Weigh the plates carefully as before. Name the cell, and compare the constancy of its current with that of the cell used in Exercise 1. Account for any change in the weight of either plate.

3. Cut 2 × 10 cm. strips of zinc, lead, iron, copper, and tin-plate, and provide a carbon plate or half of an electric light carbon rod, and attach to each a copper wire 40 or 50 cm. long. Successively use different pairs of these as plates of similar voltaic cells, connect each cell with the galvanoscope, determine and record the direction of the current and the magnitude of the deflection, and make as many different combinations as possible. Arrange the given materials in an electromotive series, i.e., so that if any two are used as plates of a voltaic cell, the current will flow through the wire from the former to the latter. When the series is completed, using dilute sulpuric acid as the exciting liquid, go over the work again using a dichromate solution, and ascertain whether any change in the series is required.

4. Wind four or five layers of No. 20 insulated copper wire upon the edge of a board 25 cm. square. Slip the wire from the board, and tie together the several turns of the wire at the corners of the rectangle. Bend one end of the wire into a hook and solder it to the middle of the pointed half of a sewing-needle as shown at *m* in Fig. 348. Straighten the other end at a right angle, as shown at *n*. Bend a narrow strip of brass at a right angle, and in one arm make an indentation that will hold a globule of mercury. Support the brass L with the indented arm horizontal, and from it hang the wire rectangle. A globule of mercury insures a good connection at *m*, and the straightened part of

the wire dips into a cup of mercury at *n*. Adjust the form of the supporting hook so that the sides of the rectangle are vertical or horizontal, and place the face of the rectangle in a north and south plane. Pass the current of a battery of 3 cells through the apparatus, and notice that the rectangle turns into an east and west plane. Reverse the current and notice the effect. Make a record of this motion of the wire rectangle, and reserve it for future study.

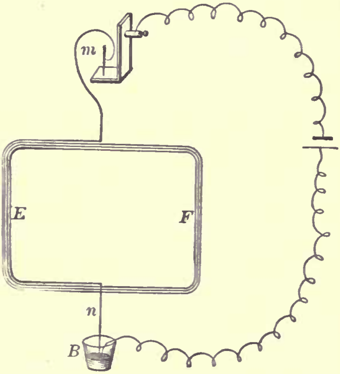


FIG. 348.

5. Wind four or five layers of No. 20 insulated copper wire upon the edge of a board 10 × 20 cm. Slip the wire from the board, and tie as directed in Exercise 4. Place this coil in the circuit between the battery and the mercury cup at *n*, Fig. 348. Call the larger wire rectangle *A*, and the smaller one *B*. Hold *B* with one of its 20 cm. sides vertical and near one side of *A*. Record the effect as manifested by the motion of *A*, when the current flows upward through the adjacent sides of the two rectangles; when the current flows downward through both; and when it flows upward in one and downward in the other. Formulate a general expression of the action of parallel currents upon each other. (*a*) When they flow in the same direction. (*b*) When they flow in opposite directions. The consideration of the interaction between currents as herein illustrated constitutes the subject-matter of *electrodynamics.*

6. Hold the rectangle *B* of Exercise 5 within *A* so that a long side of the former makes an angle with the lower side of the latter. Record the effect. Change the angle several times, recording the effect in each case. Formulate a general expression for the mutual action of currents that are not parallel (*a*) when they flow toward the point of intersection or from it; (*b*) when one flows toward the intersection and the other from it.

7. Wind some No. 16 insulated copper wire into a close spiral about 4 cm. in diameter and 15 cm. long. Bend its ends as indicated in Fig. 349. Put it into the circuit of the battery as directed for the

rectangle of Exercise 4 and hold a bar magnet near one of its ends. Trace the current through the solenoid.

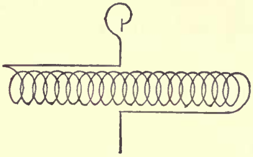


FIG. 349.

8. Pass two stout copper wires separately through a cork about 2 cm. in diameter. About 2 cm. from the smaller end of the cork, connect the copper wires with a short piece of very fine iron wire. Wrap the edge of a strip of paper about 5 cm. wide around the cork so as to make a paper cup with the iron wire inside. Fill the cup with fine gunpowder, and close the other end with a cork or a paper cap. Place this torpedo at a safe distance, connect it by stout copper wires to a voltaic battery, and send through the wires a current that will heat the iron wire and explode the torpedo. State some industrial application of electricity that is illustrated by this exercise. Cut the leading wires at three or four points and join them with short pieces of fine iron wire. Tie the fuse of a fire-cracker around each piece of iron wire, and send a current that shall ignite all of the fuses.

9. Make a torpedo similar to the one described in Exercise 8. Instead of interposing the high-resistance iron wire, bend the copper wires until their ends nearly but not quite touch. Place the torpedo at a safe distance, lay leading wires, and explode the torpedo by a spark from an induction coil or an electric machine (§§ 403, 405).

C. MAGNETISM.

Experiment 327.—Wrap a piece of writing paper around a large iron nail, leaving the ends of the nail bare. Wind fifteen or twenty turns of stout copper wire around this paper wrapper, taking care that the coils of the wire do not touch each other or the iron. It is well to use insulated wire. Put this spiral into the circuit of a voltaic cell, and dip the nail into iron filings. Some of the filings will cling to the ends of the nail in a remarkable manner. Upon breaking the circuit, the nail instantly loses its newly acquired power, and drops the iron filings.

Experiment 328.—Draw a sewing-needle four or five times from eye to point across one end of the nail of Experiment 327, while the current is flowing through the wire wound upon it. Dip the needle into iron filings. Some of the filings will cling to each end of the needle.

Experiment 329. — Cut a thin slice from the end of a vial cork and, with its aid, float the needle of Experiment 328 upon the surface of water. The needle comes to rest in a north and south position. Turn it from its chosen position and notice that, after each displacement, it resumes the same position, and that the same end of the needle always points to the north.

Experiment 330. — Break the tangs from a few flat, worn-out files. Smooth the ends and sides of the files on a grind-stone. Get some good-natured dynamo tender to magnetize these hard-steel bars and three or four stout knitting-needles. You can magnetize the needles yourself by winding upon them successively, evenly, and from end to end, a layer of insulated No. 20 wire, and sending a current from a voltaic battery through the wire. Freely suspend these permanent magnets at a considerable distance from each other and so that each can turn in a horizontal plane. The knitting-needles may be thrust through two corners of triangular pieces of paper to the third corner of which the end of a horse-hair is fastened by wax. The heavier magnets may be placed in stout paper stirrups similarly supported, or they may be floated upon water, as shown in Fig. 350. The suspended magnets will come to rest in a north and south line. Mark the north-seeking end of each magnet so that it may be distinguished from the other.

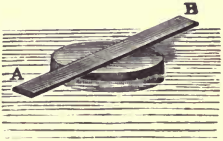


FIG. 350.

Experiment 331. — Suspend a bar of iron as you did the magnets in Experiment 330. Bring one end of a magnet near one end of the iron bar, and notice the attraction. Try the other end of the iron bar. Bring the other end of the magnet successively near the two ends of the iron bar, noticing the effect in each case.

365. A Magnet *is a body that has the property of attracting iron or steel, and that, when freely suspended, tends to take a definite position, pointing approximately north and south.*

(*a*) One of the most valuable iron ores is called magnetite ($Fe_3 O_4$). Occasional specimens of magnetite attract iron. Such a specimen is called a *lodestone.* It is a natural magnet.

(*b*) Artificial magnets have all the properties of natural magnets, and are more powerful and convenient. They may be temporary or permanent. Temporary magnets are made by passing electric currents around soft iron, as in Experiment 309, and are called electromagnets. Permanent magnets are made of hardened steel, as in Experiment 328. The most common forms of artificial magnets are the *bar magnet* and the *horseshoe magnet.* The first of these is a straight bar of iron or steel; the second is U-shaped, as shown in Fig. 351. Several similar thin steel bars, separately magnetized and fastened together side by side and with like poles in contact, constitute a *compound magnet.* A piece of iron placed across the two ends of a horseshoe magnet is called an *armature.* The process of making a magnet is called *magnetization.*

FIG. 351.

366. Magnetic Substances. — It appears to be clearly established that all matter is subject to the magnetic force as universally as it is to the force of gravitation. Substances that are attracted, as iron is, are called *paramagnetic;* substances that are repelled, as bismuth is, are called *diamagnetic.* Paramagnetic substances are sometimes called magnetic. Diamagnetic substances are more numerous than paramagnetic substances; diamagnetic effects are more feeble than paramagnetic effects.

367. Magnetic Poles. — When a bar magnet is dipped into iron filings, the magnetic effect is seen to be at maximum at the ends of the bar, and to diminish rapidly toward the middle, at which point no filings are sustained (see Experiment 328). The ends of the freely suspended magnet also point toward the poles of the earth. *These ends of the magnets are called poles*, and the magnet is said to exhibit polarity. A distinguishing mark is put on the end that turns toward the north, and that end is called the marked, north-seeking, or + pole. The other end is called the unmarked, south-seeking, or − pole. *A unit magnetic pole is a pole that exerts a force of one dyne upon a like pole at a distance of one centimeter.*

(*a*) For purposes of discussion, a theoretical magnet is assumed, long and indefinitely thin and uniformly magnetized. Such a magnet may be looked upon as a pair of poles united by a bar exerting no action, the whole magnetic effect being concentrated at the poles. When it is freely suspended, the line that joins the poles is called the *magnetic axis.*

Magnetic Needles.

Experiment 332. — Repeat Experiment 29 using the sewing-needle of Experiment 329. The needle will assume a north and south position.

Experiment 333. — Straighten a piece of watch-spring about 15 cm. long by drawing it between thumb and finger. Heat the middle of this steel bar to redness in a flame and bend it double. Bend the ends back into a line with each other, as shown in Fig. 352. Magnetize each end separately and oppositely. Wind a waxed thread around the short bend at the middle to form a socket, and balance the needle upon the point of a sewing-needle thrust into a cork. A little filing, clipping, or loading with wax may be necessary to make it balance. The needle will point north and south.

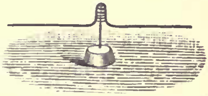


FIG. 352.

Experiment 334. — Pass a knitting-needle through a small cork from end to end and so that the cork shall be at the middle of the needle. Thrust a sewing-needle or half of a knitting-needle through the cork at right angles to the knitting-needle to serve as an axis of support. Place the ends of the axis upon the edges of two glass goblets or other convenient objects. Push the knitting-needle through the cork until it balances upon the axis like a scalebeam. Magnetize the knitting-needle, and notice that the marked end seems to have become heavier.

368. Magnetic Needles. — *A small bar magnet suspended in such a manner as to allow it to assume its chosen position relative to the earth is a magnetic needle.*

(*a*) The needle may turn in a horizontal or in a vertical plane. If it turns freely in a horizontal plane, it is a horizontal needle; e.g., the mariner's or the surveyor's compass. If it turns freely in a vertical plane, it constitutes a dipping-needle (Fig. 353). A magnetized sewing-needle, suspended at its center of mass by a fine thread or hair or an untwisted fiber will serve as a *dipping-needle*. Two magnets fastened to a common axis and with their poles reversed constitute an *astatic needle* (Fig. 354). An astatic needle assumes no particular direction with respect to the earth if the two needles are equally magnetized.

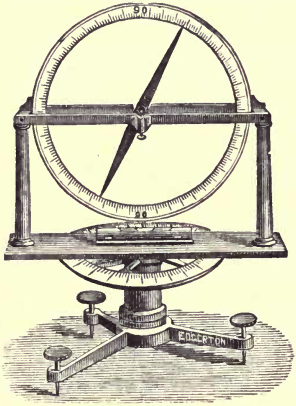


FIG. 353.

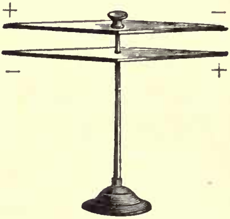


FIG. 354.

Magnetic Field.

Experiment 335. — Lay a bar magnet on the table between two wooden strips of the same thickness as the magnet. Cover the magnet with a sheet of paper or cardboard, or a plate of glass. With a dredge-box or muslin bag, sprinkle uniformly over the plate the finest filings of wrought iron that you can obtain. Gently tap the plate to facilitate the movement of the filings. They will arrange themselves in lines that seem to proceed from the poles, to curve outward through the air, and to complete their circuit through the magnet, as shown in Fig. 355. Place a short magnet (e.g., a piece of a magnetized sewing-needle suspended by a silk fiber) just above the filings, and move it into different positions. At every point, the magnet will place itself parallel to a tangent to the curves, with its marked end always pointing in the same direction relative to the curves.

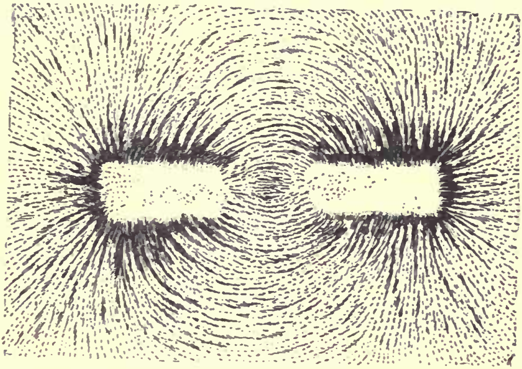


FIG. 355.

Experiment 336. — Similarly map out the "magnetic phantom" curves when the opposite poles of two bar magnets are brought near each other. The result will be like that represented in Fig. 356. The

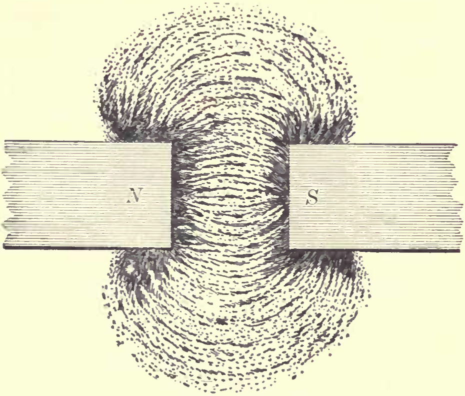


FIG. 356.

lines from one magnet seem to interlock with those from the other as if by mutual attraction.

Experiment 337. — Similarly produce the phantom when the like poles of two bar magnets are brought near each other. The result will be like that represented in Fig. 357. The lines now seem to repel each other.

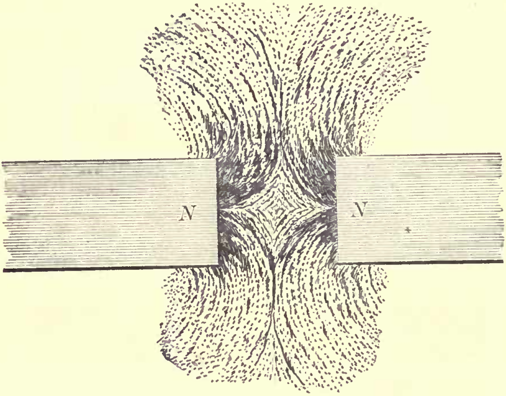


FIG. 357.

369. Magnetic Field and Lines of Force. — *The space surrounding a magnetized body and through which the magnetic force acts is called a magnetic field.* The iron filings could not have arranged themselves in their definite phantom curves except under the action of some force or forces. We may imagine lines drawn in the magnetic field, each indicating the direction in which a marked pole would move. *Such lines are called magnetic lines of force.* They are assumed to flow from the marked to the unmarked pole outside the magnet, and in the opposite direction inside the magnet, so as to form closed loops, or complete circuits. By agreement among physicists, as many lines are drawn through each square centimeter of surface as there are dynes in the force of that part of the field. Each line, therefore, represents a force of one dyne, and the closeness of the lines indicates the intensity of the field.

(*a*) A number of lines of force traversing a magnetic field is called a flow or *flux of force. The unit of flux is called a weber*, and represents one line of force. A flux of 10,000 lines of force would be a flux of 10,000 webers. *The unit of strength of field, or intensity of flux, is called a gauss*, and represents the number of lines of force per square centimeter. With a flux of 24,000 webers in 12 square centimeters, the intensity of flux would be 2,000 gausses. A field is of unit strength when a unit magnetic pole placed in it is acted upon with a force of one dyne. A pole which, when placed in a field of unit strength, is acted upon by a force of one dyne is sometimes said to be of unit *magnetic mass.*

(*b*) By agreement, the direction in which a marked pole would move in a field is called positive. When a magnet is placed in a magnetic field, the marked pole tends to move in a positive direction, and the unmarked pole in a negative direction. The total effect is that of a couple, and tends to produce rotation. The universal tendency of a magnetic needle thus to turn upon its pivot so as to place its axis in a north and south line indicates that the earth is surrounded by a magnetic field.

(*c*) The magnetic action that takes place in a magnetic field has been happily illustrated by supposing the lines of force to be stretched elastic threads that tend to shorten along their lengths, and that are self-repellent. Compare § 336 (*a*). This conception of magnetic lines of force suggests that unlike poles ought to attract each other (see Fig. 356), and that like poles ought to repel each other (see Fig. 357).

(*d*) The magnetic lines of force form closed circuits, passing through the air from one pole to the other. The shorter the air-path, the more numerous the lines of force. Hence, the advantage of the U-shaped or horseshoe magnet. When a bar of soft iron is placed across the poles, the magnetic circuit is all iron, and the lines of force that traverse it are at a maximum.

(*e*) A magnetic pole tends to repel its own magnetization, to develop opposite polarities there, and thus to weaken itself. To maintain the strength of the magnet, an armature is used to connect opposite poles, and thus to provide a closed circuit of magnetic material. Two bar magnets placed near each other in reversed position may be given a closed magnetic circuit, and thus preserved, by placing two armatures at their ends.

Experiment 338. — Place the end of a bar magnet upon a thin plate of glass, and bring a few tacks to the under side of the plate. The magnetic lines of force seem to pass readily through the glass; the attraction is not perceptibly weakened. In like manner, try paper, mica, sheet zinc, and sheet iron. The paramagnetic iron seems to act as a magnetic screen; the other substances do not.

370. Magnetic Transparency. — Substances other than those that are paramagnetic allow the free action of magnetic forces through them. This quality is sometimes called magnetic transparency.

Experiment 339. — Suspend one of the bar magnets at a consider able distance from the others. Bring one end of another magnet held in the hand near one end of the suspended magnet, and notice the attraction or repulsion. Also notice the designations of the poles that are brought into proximity. Satisfy yourself that —

N repels N,
S repels S,
N attracts S,
S attracts N.

371. Law of Magnetic Poles. — (1) *Like magnetic poles repel each other; unlike magnetic poles attract each other.*

(2) *The force exerted at different distances between two poles of the same magnetic mass is inversely proportional to the squares of the distances.*

(3) *The force exerted at a given distance between two poles is directly proportional to the product of the magnetic masses of the poles.*

(*a*) Representing the force acting by F, the magnetic masses by m and m', and the distance between the poles by d, and measuring all magnitudes in absolute units, we may summarize the last two laws thus: —

$$F = \frac{mm'}{d^2}.$$

If, in this equation, we make m, m', and d each equal to unity, F will also be equal to unity; i.e., a unit pole exerts unit force on a unit pole at unit distance. The absolute C.G.S. unit magnetic pole is, therefore, a pole of such magnetic mass that, when placed a centimeter distant from another unit pole, the force between them shall be one dyne, as was stated in § 367.

372. Magnetic Potential is essentially analogous to electrostatic potential. At any point, it is measured by the work done against the magnetic forces in moving a unit magnetic pole from an infinite distance to the given point. The difference of magnetic potential between two points is measured by the amount of work required to move a unit magnetic pole from one to the other. If this work is one erg, there is unit difference of potential between the two points. See § 338.

Magnetization.

Experiment 340. — Place a short rod of soft iron in a paper stirrup, and suspend it by a thread over and near the poles of a strong horseshoe magnet that is supported in a vertical plane. Bring first one end of a bar magnet, and then the other end, near one end of the soft iron rod, and determine whether the suspended iron is a magnet or not. If it is, ascertain which of its poles is over the marked pole of the horseshoe magnet.

Experiment 341. — Support a bar magnet in a horizontal position 6 or 7 cm. above the table. Bring a quantity of iron tacks to one pole so that as many as possible will be supported. Place a similar magnet on the table, parallel to the first but with its poles in reversed position. Test the upper magnet for increase or decrease of lifting power.

373. Magnetization. — Any magnetic substance is magnetized by bringing it into contact with a magnet, or simply by placing it in a magnetic field. In the latter case, it is said to be magnetized by *induction*. If the substance is paramagnetic, its magnetic axis will coincide with the lines of force of the field. The amount of magnetization developed depends upon the nature of the substance and the strength of the field.

(*a*) With a given field, iron receives the greatest amount of magnetization, steel coming next. As the magnetizing force increases, the magnetization produced also increases, rapidly at first but more and more slowly. When the magnetization ceases to increase, the substance is said to be *saturated;* the saturation point is, therefore, a function of the magnetizing force.

Theory of Magnetization.

Experiment 342. — Heat a magnetized needle to redness and, when it has cooled, test it for magnetism. Put a magnetized knitting-needle into active vibration (see Fig. 149), and subsequently test it for magnetism. In each case, the magnetization will be weakened or destroyed.

Experiment 343. — Magnetize a piece of watch-spring about 10 cm. long, and ascertain how large a nail it will support. Bring the two ends of the magnets into contact. The ring thus formed manifests no polarity, thus showing the equality of the opposite poles. Break the magnet at its middle, and test the strength of magnetization of the two new poles developed at the point of fracture.

Experiment 344. — Nearly fill a slender glass tube with steel filings, and close the ends of the tube with corks. Draw the marked pole of a strong magnet from the middle of the tube to one end, and the unmarked pole from the middle to the other end, and repeat the stroking several times. One end of the tube will attract and the other will repel the marked pole of a suspended magnetic needle; i.e., the filled tube has become a magnet. Thoroughly shake up the filings; the tube loses its magnetic properties, as if the actions of the many little magnets in the tube were neutralized through their indiscriminate arrangement.

374. Theory of Magnetic Polarization. — When a magnet is broken, each piece becomes a magnet, the newly developed poles being of strength nearly equal to that of the original poles. The subdivision of the magnet may be carried on indefinitely, and with the same results. This suggests that the magnetic property must reside in the smallest particle capable of existing by itself, i.e., the molecule. It is easy to imagine that if the steel particles in the shaken tube of Experiment 344 could be restored to their former positions, the tube would again act as a magnet. It may be assumed that *the molecules of a magnetic substance are always magnets; that the substance does not exhibit magnetic properties when the magnetic axes of the molecules are turned indifferently in every direction; and that the process of magnetization consists in turning the molecules so that their axes point in the same direction.* When the axes of all the molecules have thus been set parallel, the maximum of magnetization has been secured.

(*a*) Magnetization changes the length but not the volume of the bar magnetized. When soft iron is rapidly magnetized and demagnetized, it becomes heated, and sometimes a clicking sound is produced at each change.

Magnetic Properties of Electric Currents.

Experiment 345. — Repeat Experiment 315, and test the accuracy of the following rules: —

(1) To determine the direction of the deflection of the needle, hold the open right hand over or under the conducting wire, but so that the wire is between the hand and the needle, so that the palm of the hand is toward the needle, and so that the fingers point in the direction of the current; the marked end of the needle will turn in the direction of the extended thumb.

(2) To determine the direction of the current, hold the open right hand over or under the conducting wire, but so that the wire is

between the hand and the needle, so that the palm of the hand is toward the needle, and so that the thumb is extended in the direction in which the marked end of the needle is deflected; the fingers will point in the direction of the current.

Experiment 346. — Dip a short part of a stout copper wire that is carrying a large current into fine iron filings. A cluster of the filings will cling to the wire.

Note. — If you cannot obtain an electric-light or a trolley-wire current for the next experiment, connect a number of similar cells in parallel. Make the external circuit of very heavy wire, and have the paper in place around the wire, and the dredge-box ready. Close the circuit and perform the experiment quickly.

Experiment 347. — Around a vertical conductor carrying a heavy current, place a piece of paper cut as shown in Fig. 358, and sprinkle fine iron filings on the paper. Notice that the iron particles arrange themselves in distinct circular whirls around the wire, as shown in Fig. 359. Experimentally establish the tangential relation between a short magnet and these curves, as in Experiment 335. Hold the closed right hand so that the extended thumb points in the direction of the current in the wire; then the fingers will indicate the direction of the lines of force in the surrounding field. Bend the upper part of the conducting wire, and pass it vertically downward through the paper. Sprinkle iron filings as before. Notice that the magnetic lines of force around the two parallel parts of the wire circle in opposite directions, clockwise in one case, and counter-clockwise in the other. If the conducting wire is bent into the form shown in Fig. 360, the lines of force will pass around the wire from one face of the loop to the other, and in the direction indicated by the "rule of thumb" just given.

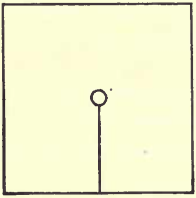


FIG. 358.

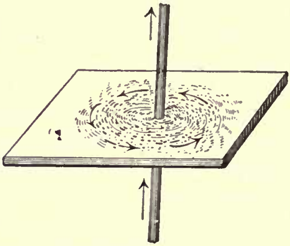


FIG. 359.

Experiment 348. — Bend a piece of stout copper wire into the form shown in Fig. 360. Suspend it at its highest point by a long thread and so that its free ends just dip into mercury contained in separate, shallow dishes. Put the apparatus into the circuit of a strong electric current, making connections with the mercury cups. Bring a bar magnet near either face of the circular conductor. The loop will turn upon its vertical axis, seeking a position at right angles to the magnet, and acting as if it was a disk magnet with poles at its faces.

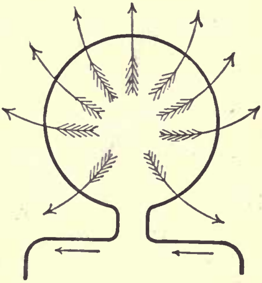


FIG. 360.

Experiment 349. — Coil some No. 12 copper wire through holes in a board, as shown in Fig. 361, and pass a strong current through it. Sprinkle iron filings as before and note the effect. Such a coil of conducting wire, wound so as to afford a number of equal and parallel circular electric circuits arranged upon a common axis, is called a *solenoid.*

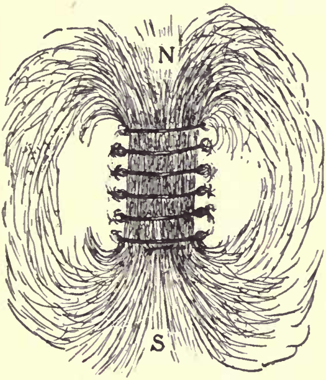


FIG. 361.

Experiment 350. — Wind the middle part of about 3 meters of No. 20 insulated copper wire around a rod about 1.5 cm. in diameter, forming thus a solenoid about 10 cm. long. Bring the ends of the wire along the axis of the solenoid, and bend them at right angles near the middle. Solder small plates of sheet copper and amalgamated sheet zinc to the ends of the wire. Support the solenoid and plates by a large flat cork on the surface of dilute sulphuric acid, as shown in Fig. 362. The floating cell will take position so that the axis of the solenoid extends north and south. Test the ends of the solenoid for polarity, using a bar magnet for that purpose.

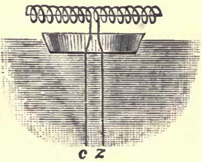


FIG. 362.

Experiment 351. — Repeat Experiment 315, using the floating cell of Experiment 350 instead of the compass, needle.

Experiment 352. — Prepare a second solenoid similar to that described in Experiment 350, omitting the plates. Put it into an electric circuit, and use it as you did the bar magnet in Experiment 350.

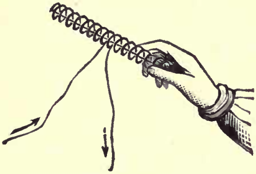


Fig. 363.

375. The Magnetic Character of an Electric Current has been shown by the deflection of the magnetic needle, by the production of phantom curves, and by other experiments. The passage of an electric current through a solenoid gives it many of the properties of a cylindrical bar magnet.

(*a*) The polarity of the solenoidal magnet may be determined by holding it in the right hand so that the fingers point in the direction of the current; then the extended thumb will point toward the marked or north-seeking pole of the magnet. The lines of force flow from the marked to the unmarked pole outside the solenoid, and in the opposite direction inside the solenoid. Looking at one end of a solenoid so that the current circles around clockwise, you may imagine that the enclosed lines of force go forward, just as the right-hand turning of a corkscrew is related to its forward motion. Two solenoids freely suspended act toward each other as did the suspended magnets of Experiment 339.

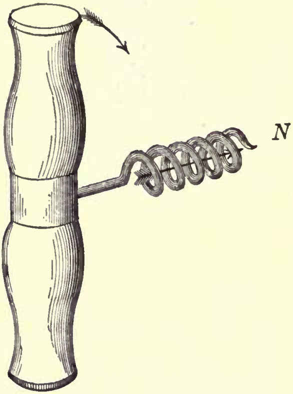


Fig. 364.

376. Magnetomotive Force. — The influence to which these magnetic lines of force are due is called *magneto-*

motive force (M.M.F.). The magnetomotive force of a magnetic circuit is directly proportional to the number of amperes in the electric circuit surrounding it, and to the number of turns that the electric circuit makes around the magnetic circuit; i.e., *the magnetomotive force is proportional to the ampere-turns.* The unit of magnetomotive force is called a *gilbert*, and corresponds to 0.7958 ampere-turns.

Experiment 353. — Place a strip of sheet iron in the solenoid of Experiment 350, as shown in Fig. 365, and repeat that experiment. Notice that most of the lines of force are gathered into the iron and issue from its ends. Notice that the lines curve outward and tend to return, forming closed loops or complete magnetic circuits. Change the iron from the inside of the solenoid to the outside, and repeat the experiment. Notice that the iron again gathers in the lines of force as if it offered an easier path for them.

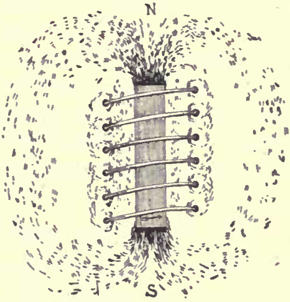


FIG. 365.

377. Permeability. — Some substances are capable of receiving more lines of force than others with the same magnetomotive force. This relative capacity is called *permeability*, which may be defined as the ratio between the number of lines of force that pass through a given area of the substance and the number of lines of force that pass through a like area of the inducing field alone. It is a sort of magnetic conductivity.

(*a*) The permeability of paramagnetic substances is greater than unity; that of diamagnetic substances is less than unity. Bismuth has the lowest permeability of any known substance. When a paramagnetic substance is placed in a uniform magnetic field, its permeability being greater than that of the field, an increased number of lines of force are crowded into it, as shown in Fig. 366. When a diamagnetic substance is placed in such a field, its permeability being less than that of the field, the number of lines that pass through it is diminished, as shown in Fig. 367.

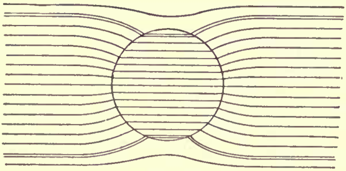


FIG. 366.

(*b*) If a small compass is put into a closed bottle, an outside magnet will affect it, but if it is put into a hollow iron ball, an outside magnet will not affect it. Soft iron acts as a magnetic screen (§ 370) because of its high permeability. Watches are sometimes protected from magnetic influence by soft iron shields in the shape of inside cases.

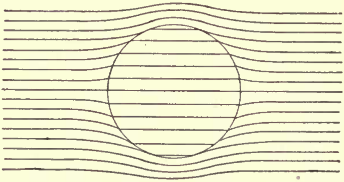


FIG. 367.

378. Reluctance and Reluctivity. — Like electric currents, magnetic lines of force flow in the greatest quantity through paths of least resistance. Magnetic resistance is called *reluctance*, and its unit is the *oersted*. Specific magnetic resistance (specific reluctance) is called *reluctivity*. Reluctivity is the reciprocal of permeability.

379. The Analogue of Ohm's Law. — In § 361, we have a formula that shows the mathematical relations between amperes, volts, and ohms. The corresponding relations for magnetic units is expressed by the equation, —

$$webers = \frac{gilberts}{oersteds}. \tag{1}$$

From this we may derive the other two,—

$$\textit{gilberts} = \textit{webers} \times \textit{oersteds}, \text{ and} \qquad (2)$$

$$\textit{oersteds} = \frac{\textit{gilberts}}{\textit{webers}}. \qquad (3)$$

These formulas are much used in magnetic calculation.

Coercive Force.

Experiment 354.—Prepare three small bars of the same size, one of soft iron, one of soft steel, and one of hardened steel. Bring one end of a good bar magnet into contact with one end of the iron bar, and dip the other end of the iron bar into iron filings. Lift the magnet and the iron bar without breaking the contact between them. Notice the quantity of filings lifted by the iron. Break the contact, and notice the quantity of filings dropped by the iron. Repeat the test with the other two bars in succession. It will be found that the iron will lift the most and the hardened steel the least, but that, when the contact is broken, the hardened steel holds the most and the iron the least.

Experiment 355.—With the end of a good bar magnet, write your name upon the blade of a hand saw. The invisible characters may be made visible by sifting fine iron filings upon the blade.

Experiment 356.—Lay a thin piece of well-hardened steel (it may be cut from a saw-blade) in the solenoid, and repeat Experiment 353. After the current has been interrupted, jar the solenoid, and notice that some of the filings are still held in their positions. The hard steel has become a permanent magnet.

380. Coercive Force and Remanance.—The persisting magnetomotive force shown by the steel is due to what is called the *coercive force* of the steel, which acts as a sort of magnetic inertia, resisting magnetization at the outset and tending to retain it afterward. The number of lines of force per square centimeter in the body of the steel is called its *remanance.*

(*a*) Coercive force is measured in gilberts; remanance, in gausses.

Experiment 357. — Make a helix about 15 cm. long by neatly winding three layers of No. 18 insulated copper wire upon a rod about 2 cm. in diameter. Remove the rod, pass a few threads through the opening of the helix, and tie them on the outside so as to hold the turns of wire in place. Put the helix into the circuit of a voltaic cell, and bring it near a magnetic needle. The deflection of the needle shows the magnetic power of the helix. Nearly fill the opening in the helix with straight pieces of soft iron wire, and again test its magnetic power. The deflection of the needle will be much greater than before.

381. An Electromagnet *is a bar of iron magnetized by an electric current*, substantially as just shown. When the current was passed through the helix used in Experiment 357, some of the lines of force leaked out at the sides, as indicated by Fig. 368, and few of them extended from end to end. The soft iron core, by reason of its high permeability, diminished this leakage of lines of force, and greatly increased their number, as shown in Fig. 369.

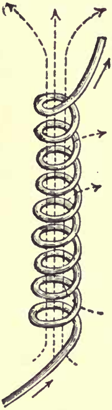


FIG. 368.

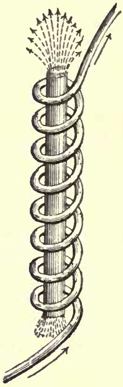


FIG. 369.

(*a*) Since the lines of force are perpendicular to the direction of the current, the iron core is to be placed at right angles to the wire; the effect is increased by increasing the number of turns of the wire. The direction of polarity in the core depends only upon the direction of the current in the helix, and may be determined by the thumb or the corkscrew rule already given.

(*b*) When an electromagnet is U-shaped, the coils around the two ends of the bent iron core are so wound that if the coil should be straightened either coil would appear as a continuation of the other, i.e., the current would circle around the core in the same direction in the two coils. Such magnets are often made by connecting an end of the core of one spool-shaped magnet, by a straight soft iron bar called a yoke, to one end of the core of a similar magnet, a screw passing through the bar into an end of each straight core. The two spools are then connected as above indicated.

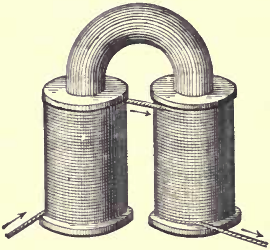


FIG. 370.

(*c*) To produce the best effect, the resistance of the electromagnet should be equal to that of the rest of the circuit. If several electromagnets are used on the same circuit, the sum of their resistances should equal the resistance of the rest of the circuit.

(*d*) If the iron of the magnet core is of commercial quality, it is not wholly demagnetized when the current is interrupted. The magnetization thus retained after withdrawal from a magnetic field is called *residual magnetism.*

382. Ampere's Theory of Magnetism. — As an electric current is surrounded by a whirl of lines of magnetic force, so we may conceive a magnetic line of force as surrounded by an electrical current-whirl. This would imply, as Ampere long ago suggested, that magnetism is simply a vortical electric current, and that a magnetic field is something like a whirlpool of electricity. Fig. 371 represents a vertical conductor carrying an electric current, and surrounded by a magnetic line of force, which is in turn surrounded by electric whirls; the magnetic line of force is an electric vortex-ring. It is not difficult to conceive the vortex-ring as made up of ether whirls. As the phenomena of magnetism belong to the molecules, these electrical whirls must be rotations perpendicular to the magnetic axes of

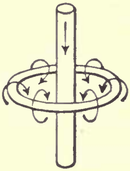


FIG. 371.

the molecules. Ampere's theory supposes that electric currents circle round the molecules of a magnetic substance, thereby polarizing them, and that when all these magnetic axes face in the same direction the substance is magnetically saturated.

(*a*) The great importance of the relation between magnetic lines of force and electric currents will appear more plainly in the following section.

Terrestrial Magnetism.

Experiment 358. — Place a small dipping-needle over the marked end of a long, horizontal bar magnet, and move it slowly toward the other end of the bar, observing the changes in the position of the dipping-needle. Similar changes would be observed if you could carry the dipping-needle from far southern to far northern latitudes.

Experiment 359. — Take a bar of very soft iron about 75 cm. long, and make sure by trial that its ends will not attract bits of soft iron. Then hold the bar in a meridian plane, and with its north end depressed below the horizon a number of degrees approximately corresponding to the latitude of the place of the experiment, i.e., give it the position of a dipping-needle. Tap the rod on its end with a mallet or wooden block, and test it for magnetic polarity.

383. Terrestrial Magnetism. — A magnetic field is recognized by the fact that it gives a definite direction to a magnet freely suspended in it. The directive tendency of the compass, and other phenomena, show that the earth is surrounded by such a field. In fact, these phenomena are such as might be expected if we knew that a bar magnet four or five thousand miles long extended nearly north and south through the earth's center. This terrestrial magnetism is explained as being due to equatorial electric currents produced by the action of the sun, and modified by the motion of the earth.

(*a*) The angle that the axis of a dipping-needle makes with a horizontal plane is called the *inclination* or *dip* of the needle. The dip is 90° at the magnetic poles of the earth, and 0° at the magnetic equator, and, at any given place, does not differ greatly from the latitude. Lines passing through points on the earth's surface where the inclination has the same value are called *isoclinic lines.* The inclination of the needle is subject at most places to secular, annual, and diurnal changes.

(*b*) The magnetic poles of the earth do not coincide with its geographical poles and, consequently, in some places, the magnetic needle does not point to the geographical north. The angle that the axis of a compass-needle makes with the geographical meridian at any place is called the *declination* or *variation* of the needle at that place. When the marked end of the needle lies east of the meridian, the variation is easterly, and *vice versa.* Lines drawn through places on the earth where the declination is the same are called *isogonic lines*, as is shown in Fig. 372. The particular isogonic line for which the declination is zero is called an agone or an *agonic line.* These lines are very irregular, being apparently affected by local conditions. The American agone, in 1890, entered the United States near Charleston, passed through the mountains of North Carolina and West Virginia, and near Columbus, Toledo, and Ann Arbor, and is slowly moving westward. The declination of the needle is subject to periodic changes, secular, annual, and diurnal, and to irregular variations or perturbations. The mariner or the surveyor must recognize not only the declination of his needle but also the changes in its declination.

FIG. 372.

(*c*) The magnetic intensity of the earth is also an element that varies from point to point at the same time, and from time to time at the same place. Lines drawn through places on the earth where the force of terrestrial magnetism is the same are called *isodynamic lines.*

(*d*) The determination of these three magnetic elements is the object of governmental magnetic surveys.

(*e*) The observed coincidences between magnetic storms, i.e., sudden disturbances of the earth's magnetism, and solar storms indicate a connection, the nature of which is not yet well understood.

CLASSROOM EXERCISES.

1. What part of a magnet might properly be designated by the term equator?

2. Explain the increase of lifting power manifested in Experiment 357.

3. How can the intensity of different parts of a magnetic field be roughly estimated from the behavior of a magnetic needle?

4. Show that the influence of the earth's magnetism upon a magnetic needle is merely directive.

5. If a wire coil of 220 turns carries a 3-ampere current, what is its magnetomotive force? *Ans.* 829 + gilberts.

6. A rectangular bar of steel 1 × 3 cm. and 30 cm. long, is bent into a circle, and upon it is wound 40 turns of wire. A 5-ampere current is passed through the wire. (*a*) Determine the M.M.F. (*b*) Assume the reluctance to be 0.00593 oersteds, and divide the M.M.F. by the reluctance to determine the flux of force in webers. (*c*) Determine the intensity of flux in gausses. (*d*) Assume the permeability to be 1,684 and determine the reluctivity. (*e*) Divide the M.M.F. by the length of the bar to determine the magnetizing force in gausses.

LABORATORY EXERCISES.

Additional Apparatus, etc. — A collection of bar magnets; a hack-saw blade; several sheets of paper about 50 cm. square; a compass with a needle 2 or 3 cm. long.

1. By observations of the North Star or in any other convenient way, mark a true north and south line on the laboratory table. Place a magnetic needle in this line, and determine the direction and magnitude of the declination.

2. Float a magnet on water as shown in Fig. 350. The float should be the lightest that will carry the load with safety, and the body of water should be so large that surface tension will not urge the float toward the side of the vessel. When the magnet is at rest near the

middle of the liquid surface, determine the tendency of the magnet to drift toward the north or south. Repeat the experiment with a variety of magnets, and try to find one that always floats in one direction, i.e., one in which the marked pole is stronger or weaker than the other. If you cannot find such a magnet, strongly magnetize the blade of an old hack-saw, and test it on the float. If you have not yet found that for which you seek, break the blade in the middle, and test each half. If necessary to the success of your search, break one of the halves in two, and repeat the tests. Make very careful notes of any magnet that you find to have more magnetism of one kind than of the other.

3. Fasten a large sheet of paper upon the table-top. Place a bar magnet about 20 cm. long and 1 sq. cm. in cross-section upon the middle of the paper with its marked end pointing toward the north. Place a small compass on the paper at the northeast corner of the magnet, move it away from the magnet in the direction in which the marked end of the needle points, tracing upon the paper the path of the middle point of the compass, and indicating by arrow-heads upon the line thus traced the direction in which the compass is moved. Continue the movement of the compass, continually changing the direction of the motion of the compass as indicated by the direction in which the marked end of the needle points, until the traced path reaches the edge of the paper or returns to the magnet. Repeat the work, starting near the same corner but 1 cm. nearer the middle of the magnet. Taking the successive starting points nearer and nearer the middle of the magnet, continue to draw such lines until you come within about 3 cm. of the middle of the bar. Similarly, trace an equal number of lines on the other side of the magnet. The curves may be traced in an "off-hand" way, their general character being of more importance than exact details. Trace the outline of the magnet on the paper, and indicate its polarity.

4. Place the magnet used in Exercise 3 upon a clean sheet of paper, and with its marked end pointing toward the south. Trace lines as indicated in that exercise.

5. Place two similar bar magnets on a clean sheet of paper, parallel to each other, about 15 cm. apart, and with their marked ends pointing toward the north. In manner similar to that prescribed for Exercise 3, trace the lines of force, including several lines between the two magnets. Compare this diagram with those drawn in Exercises 3 and 4, specify their characteristic features, and explain their significance.

6. Map a magnetic field as in Experiment 335. Carefully remove the magnet and wooden strips. Over the filings, carefully place a sheet of printing-paper that has been wet with a solution of tannin. Over this, place a sheet of heavy blotting-paper. Place a board on the blotting-paper and a weight on the board. When the printing-paper is removed, some of the iron filings will adhere to it. When the paper is dry, brush off these filings. The ink-like markings on the paper make a permanent copy of the map.

II. ELECTRIC GENERATORS, ELECTROMAGNETIC INDUCTION, ETC.

384. Introductory. — We have seen that when two bodies at different electric potentials are connected by a conductor, an electric current transfers along the conductor the state of strain that constitutes electrification. If that strain is reproduced as fast as it is relieved, the difference of potential will be constant, and the current will be continuous and uniform. The devices considered in the preceding section are incapable of producing a current adequate to the demands of the age in which we live. It is the purpose of this section to indicate how such currents are produced.

385. Voltaic Cells are among the most common and important of "electric generators," and have been devised in great variety. Some of them are dry, some have one liquid, and others have two. Some are constant and strong while they last, but require frequent renewals; others are effective for short periods only, and require time for their own recovery. Each has its advantages and its disadvantages, so that one is the better for one purpose, and another for another.

(*a*) When commercial zinc is used as one of the plates of a cell, the chemical action shown in Experiment 312, and known as *local action*, contributes nothing to the current. It is probably due to the presence of particles of carbon, iron, etc., in the zinc. The zinc and these foreign particles in the zinc act as the plates of minute voltaic cells, the currents flowing in short circuits from the zinc through the liquid to the foreign particles, and thence back to the zinc. This local action is prevented by using pure zinc, or by amalgamating commercial zinc as in Experiment 313.

(*b*) The *polarization* of the cell, i.e., the accumulation of the hydrogen film on the negative plate, interferes with the action of the cell and diminishes the available current by increasing the resistance of the circuit, and by setting up a counter electromotive force that may reduce, stop or even reverse the flow of the current. The various devices for removing the hydrogen, or for preventing its accumulation, constitute the most essential differences between the different forms of cells. These devices may be classified as physical (e.g., the mechanical agitation of the liquid, or the roughening of the plate to lessen the adhesion of the gas), and chemical (e.g., the use of some agent, like nitric or chromic acid or manganese dioxide, for the oxidation of the hydrogen before it reaches the negative plate).

Note. — The oxidation of hydrogen yields water.

(*c*) A few forms of cells are mentioned, although it is impossible to give descriptions of all or many. For such descriptions, the pupil is referred to some technical work on the subject.

(1) The *Smee* cell consists of a silver or a lead plate suspended between two zinc plates immersed in dilute sulphuric acid. Polarization is partly prevented by giving the negative plate a rough coat of finely divided platinum.

(2) The *potassium dichromate* cell (see Experiment 316) consists of zinc and carbon plates immersed in a solution of potassium dichromate in dilute sulphuric acid. The action of the sulphuric acid on the dichromate liberates chromic acid which oxidizes the hydrogen, and thus prevents polarization. This cell is very convenient for quick use, and valuable for "all-around" work. It is sometimes called the *Grenet* cell. A similar cell that employs *sodium dichromate*

instead of potassium dichromate is more enduring in its action. A solution of chromic acid is much used and is more economical than either.

(3) In the *Grove* cell, a cylindrical plate of zinc is immersed in dilute sulphuric acid, and carries a porous cup that contains strong nitric acid in which a platinum strip is immersed. The hydrogen evolved at the zinc plate is oxidized by the nitric acid.

(4) The *Bunsen* cell differs from the Grove in a substitution of carbon for platinum, and in the larger size of the plates. Like the Grove cell, it is little used, the fumes that come from the nitric acid being choking and corrosive.

(5) In the *Leclanché* cell, a zinc rod is immersed in a saturated solution of ammonium chloride (sal-ammoniac). In this solution is also a porous cup that contains a bar of carbon tightly packed in a mixture of granular carbon and manganese dioxide. The hydrogen evolved is oxidized by the dioxide, but so slowly that the cell must be given frequent intervals of rest to recover from polarization. This cell is much used for working telephones, electric bells, etc., i.e., on circuits that are open most of the time.

(6) The *Daniell* cell consists of a zinc plate immersed in dilute sulphuric acid contained in a porous vessel outside of which is a perforated copper plate surrounded by a solution of copper sulphate. The hydrogen is taken up by the sulphate before it reaches the copper plate. Polarization being wholly prevented, this cell is one of the most constant known.

(7) The *gravity* cell is a modification of the Daniell. The liquids are kept separate by their different densities, thus dispensing with the porous cup. It is commonly used on closed circuits. This is the form of cell most used for telegraphic purposes in the United States.

(*d*) Every cell has an internal resistance that consists chiefly of the resistance of the liquid or liquids used. The voltage of the cell is largely taken up in overcoming this internal resistance, thus greatly lessening the energy available at the electrodes. If R is the resistance of the circuit outside the cell, and r is the resistance of the cell itself, then Ohm's law becomes

$$C = \frac{E}{r + R}.$$

Refer to Fig. 335, and notice that the liquid prism between the plates is part of the circuit; that when the plates are separated, the length of the liquid conductor, and the internal resistance of the cell, are increased (see § 352); that when one of the plates is lifted partly from the liquid, the area of cross-section is reduced, and the resistance increased.

Experiment 360. — Upon each end of a 4-inch piece of soft, round iron-rod 1 inch in diameter, drive a vulcanite or hard-wood collar about $1\frac{1}{2}$ inches in diameter. Upon the spool thus formed, wind about 6 feet of No. 8 insulated copper wire, being careful first to insulate the iron core with paper. Fasten a rectangular piece of soft iron, *a*, to a piece of whalebone and support it, as shown in Fig. 373, over *M*, the electromagnet just described. Place *M* in the circuit of a battery of six or more similar cells joined in series. The whalebone magnetoscope will enable you to make a rough estimate of the pull of the electromagnet.

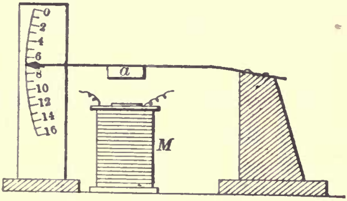


FIG. 373.

Experiment 361. — Connect the cells of the battery in parallel, and repeat Experiment 360.

Experiment 362. — Make an electromagnet similar to that of Experiment 360, but using about 250 feet of No. 24 insulated copper wire, and with it, repeat Experiments 360 and 361.

Experiment 363. — Connect the terminals of the high resistance galvanoscope described in Experiment 321 to the poles of a single cell, and record the deflection of the needle. Next, put the galvanoscope in circuit with a battery of six similar cells joined in parallel, and record the deflection of the needle. Then put the galvanoscope in circuit with a battery of the same cells joined in series, and record the deflection of the needle. From the records, determine which method of joining cells is most effective with a high external resistance.

386. Advantages of Grouping in Parallel. — Some of the foregoing experiments indicate what is a general truth, that, when the external resistance is small, the grouping of electric generators in parallel will give a greater current than will a series grouping of the same generators.

(*a*) With such a grouping, the available difference of potential between the terminals of the system is not increased, but the internal resistance is diminished (see § 448). For instance, with n cells thus grouped, we have

$$C = \frac{E}{\frac{r}{n} + R}.$$

It is evident that the less the value of R, the greater will be the effect of n in increasing the value of C.

387. Advantages of Grouping in Series. — Our experiments also indicate that when the external resistance is great, the grouping of electric generators in series will give a greater current than will a parallel grouping of the same generators.

(*a*) With such a grouping, the voltages of the several generators are added together for the total available difference of potential, and the internal resistances are added together for the total internal resistance of the system. With n cells thus grouped, we have

$$C = \frac{nE}{nr + R}.$$

It is evident that when R is small, the effect of n upon the value of C must also be small, but that when R is large, the effect of multiplying r is more than counterbalanced by the corresponding multiplication of E.

(*b*) Having a given number of similar cells and a certain known external resistance, the maximum current may be obtained by joining the cells in such a way as to make the resistance of the battery as nearly equal as possible to the resistance of the external part of the circuit.

CLASSROOM EXERCISES.

1. Determine the current strength of a battery of five cells joined in parallel, each having an E.M.F. of 2 volts and an internal resistance of 0.5 ohms, (*a*) when the external resistance is 0.1 ohm; (*b*) when the external resistance is 500 ohms. *Ans.* (*a*) 10 amperes.
(*b*) Nearly 0.004 amperes.

2. Determine the current strength of a battery made up by coupling

the same 5 cells in series, (*a*) when the external resistance is 0.1 ohm; (*b*) when the external resistance is 500 ohms.

Ans. (*a*) 3.846 + amperes.
(*b*) 0.0199 + amperes.

3. Connect in parallel 8 voltaic cells, each having an E.M.F. of 2 volts, and an internal resistance of 8 ohms, the total external resistance being 16 ohms. Determine the current strength. *Ans.* 0.1176 amperes.

4. Compute the current strength of the same 8 cells connected in series. *Ans.* 0.2 amperes.

5. Compute the current strength of the same 8 cells when joined in two rows, each row being a series of four cells, and the rows being joined in multiple arc. *Ans.* 0.25 amperes.

6. Each of ten given cells has an electromotive force of 1 volt and an internal resistance of 5 ohms. What is the current strength of a single cell, the external resistance being 0.001 of an ohm?

Ans. 0.19996 + amperes.

7. The ten cells above mentioned are joined in parallel. The external resistance is 0.001 of an ohm. What is the current strength of the battery? *Ans.* 1.996 + amperes.

8. The ten cells above mentioned are joined in series, the external resistance remaining the same. What is the current strength of the battery? *Ans.* 0.19999 + amperes.

9. What is the current strength given by one of the above mentioned cells when the external circuit has a resistance of 1,000 ohms?

Ans. 0.00099502 amperes.

10. When the ten cells are joined in parallel with an external resistance of 1,000 ohms, what is the ampere yield of the battery?

Ans. 0.0009995 amperes.

11. When the ten cells are joined in series with an external resistance of 1,000 ohms, what is the current strength of the battery?

12. Six cells, each having an E.M.F. of 2 volts and an internal resistance of 0.8 of an ohm, are joined in series. When the circuit is closed, the wire connections aggregate 6 feet of No. 8 copper wire. (*a*) What is the total resistance of the circuit? (*b*) What is the current strength of the battery? *Ans.* (*a*) 4.80386 ohms.
(*b*) 2.498 amperes.

13. Suppose the same six cells to be joined in parallel, the wire resistance being the same as before. (*a*) What is the total resistance of the circuit? (*b*) What is the current strength of the battery?

Ans. (*a*) 0.1371 ohms.
(*b*) 14.6 amperes.

14. The terms "tandem" and "abreast" are sometimes used to describe the methods of grouping cells that we have studied. Which term refers to grouping in series?

15. Two voltaic cells give equal currents on "short circuit," i.e., when the external resistance is very small. How can you experimentally ascertain whether their electromotive forces are equal?

16. Review Laboratory Exercises 4 and 7, page 453, and indicate the direction of the magnetic lines of force of the rectangle, of the solenoid and of the magnet, and show how they may be made to account for the observed phenomena.

Electromagnetic Induction.

Experiment 364. — For a galvanoscope more delicate than any we have yet used, procure two soft pine blocks, 4 cm. square and 2 cm. thick. On the square faces of each, nail or glue a thin piece of wood, 6 cm. square. (These pieces may be cut from a cigar box.) The channel around the edges of the blocks will be 2 cm. wide and 1 cm. deep. Through the middle of each block, from face to face, bore a hole at least 1.5 cm. in diameter. Wind the grooves full of No. 36 insulated copper wire, and mount the blocks, *A* and *B*, on a base-board with their opposing faces about 1 cm. apart, as shown in Fig. 374. Connect the wires of the two coils so that a current flowing through the wire will circle around the coils in the same direction; i.e., connect them in series.

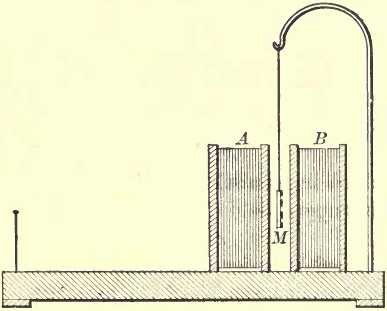


FIG. 374.

Straighten and magnetize four or five pieces of watch spring each 1.5 cm. long, and fasten them with thin shellac varnish to the back of a piece of looking glass, 1.5 cm. square and as thin as you can get. See Fig. 375. From a support made of brass wire, suspend the mirror, *M*, by a strand of silk, the lightest that will carry the load. A single silk fiber may be strong enough. The mirror when suspended should hang midway between the two coils, and directly in line with the holes through the two coils. So adjust the base of the galvanoscope that the coils are parallel to the mirror when the latter is freely suspended between them, and protect the apparatus from air currents by a glass cover. A feeble current passing through the coils will deflect the delicately suspended needles, as was roughly illustrated in Experiment 315. By placing a bar magnet on the table so as partly to neutralize the directive tendency of the terrestrial magnetism, the sensitiveness of the galvanoscope may be increased.

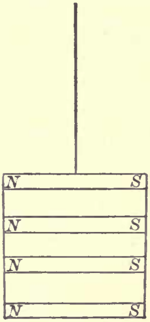


FIG. 375.

Stick a pin into the end of the base-board and in line with the centers of the openings in the coils, as appears more plainly in Fig. 376. The eye may be so placed that the pin will cover its image in the mirror. The slightest deflection of the mirror will be manifested by the destruction of this coincidence. Indicate the polarity of the suspended magnets by marking the letters *N* and *S* near the edges of the base-board between the coils *A* and *B*. Put the galvanoscope into circuit with a single cell, and note the deflection of the mirror. Record on the base-board of the instrument the fact that "This instrument shows a deflection of the *N* end of the needle toward the east when the zinc plate of a cell is connected with the free terminal of coil *B* (or of *A*, as the case may be).

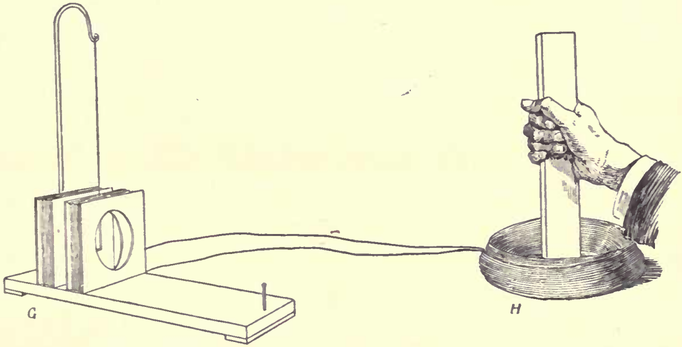


FIG. 376.

Experiment 365. — Make a coil of wire with many turns of No. 36 insulated copper wire, as shown at *H* in Fig. 376. The coil should have an internal diameter of about 3 cm., and a cross-section area of at least 1 sq. cm. Connect the terminals of the coil with the terminals of the galvanoscope. Level the galvanoscope, and see that its needle-mirror is freely suspended as directed in the preceding experiment. Thrust the end of a bar magnet at least 1.5 cm. in diameter into the coil, *H*, thus filling the coil with lines of force. An electric pulse deflects the mirror of the galvanoscope. That the deflecting current was of momentary duration is shown by the fact that the mirror returns to its first position. When it has come to rest, remove the magnet from the coil. The mirror is turned the other way and comes to rest as before, thus showing that the direction of the second current was opposite to that of the first, and that its duration was but momentary. Repeat the experiment, making the motions of the magnet more rapid. Notice that the pulses are more marked than before. Repeat the experiment again, using a low resistance solenoid that carries a current of electricity, as shown in Fig. 377, instead of the bar magnet. Then place the solenoid inside the coil, *H*, and break, and make the battery circuit. Place a soft iron rod inside the solenoid and again break and make the circuit, noticing any increase in the deflections of the needle.

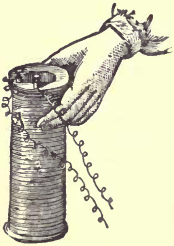


FIG. 377.

That the galvanoscope may be free from disturbing magnetic influence, see that all knives, keys, watches and other articles of iron or steel are kept at a considerable distance from it, and that the coil, *H*, is so far removed that the magnet or the solenoid may not have any perceptible direct influence upon it. It will be well to wind the wire of the coil, *H*, upon a spool as shown in Fig. 378.

388. Induced Currents. — When the number of magnetic lines of force that pass through a closed coil of wire is changed, as in Experiment 365, pulses of electricity are generated in the coil. The rapidity with which the coil is filled or emptied has a marked effect upon the intensity of the pulses generated. *These momentary currents are said to be induced in the coil; i.e., they are induced currents.*

Experiment 366. — Connect the coil, *H* (Fig. 376), to the galvanoscope, *G*. Make another coil of No. 20 insulated copper wire, the same size as *H*, and call it *A*. Connect *A* with the battery, and determine, by the corkscrew rule, which side of it is north, and so mark it.

Consider the deflections of *G* to the right as +, and deflections to the left as −. Consider lines of force going through *H* from its upper side as +, and lines that flow in the opposite direction as −.

Bring the north end of a magnet to the coil, *H*.
“ “ south “ “ “ “ “ “ “ “
“ “ north side of the coil *A* to the coil, *H*.
“ “ south “ “ “ “ “ “ “ “ “

Lay *A*, north side down upon *H*, and make the circuit.
“ *A*, “ “ “ “ “ “ break “ “
“ *A*, south “ “ “ “ “ make “ “
“ *A*, “ “ “ “ “ “ break “ “

In each of these cases, record the deflection of *G*, and the direction of the lines of force passing through *H*. When a magnet is used, the direction of the flux may be determined by the marked polarity of the magnet. When the coil, *A*, is used, the direction of the flux may be determined by the corkscrew rule. Remember that there can be no deflection of *G* without a change in the number of lines of force in *H*. In each case, record your answer to these two questions: —

(1) Was there an increase or decrease in the flux of force in *H*?

(2) What deflection of *G* results from an increase of that flux in *H*, and what from a decrease?

Experiment 367. — Place a soft iron core in the coil *H* as shown in Fig. 378, and repeat Experiment 366. Notice that the deflections are now much more marked.

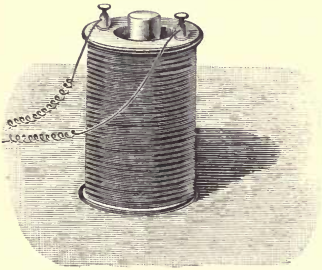


FIG. 378.

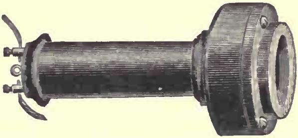


FIG. 379.

Experiment 368. — Place the coil, *H*, in circuit with a telephone receiver instead of the galvanoscope. When the circuit of A is made or broken, a distinct click may be heard in the receiver which is a delicate detector of pulses of electricity. One may be bought at a low price, or borrowed.

389. Laws of Induced Currents. — The following laws have been established : —

(1) *An increase in the number of the lines of force passing through a closed coil induces a current in one direction through the wire of the coil; a decrease in the number of the lines of force induces a current in the other direction.*

(2) *The electromotive force of the induced currents depends upon the rapidity of change in the number of lines of force that pass through the coil.*

390. The Magneto. — A number of permanent magnets might be fastened to a wheel so that the revolution of the wheel would carry the ends of the magnets in front of a closed coil of wire, corresponding to the coil, H, of our recent experiments. As each magnet approaches the coil, a current would be generated within the coil; as it recedes from the coil, a current of opposite direction would be generated in the coil. We should thus obtain an alternating current of electricity from mechanical power. *Such a device for inducing electric currents in wire coils or bobbins, by variations in the relative positions of the coils and of permanent magnets, is called a magneto-electric machine, or simply a magneto.*

(*a*) The fundamental process in the generation of electric currents from mechanical power consists in revolving closed conductors in a magnetic field in such a way as to vary the number of lines of force passing through them, i.e., by successively filling and emptying closed coils. The mechanical motion may move the coils, or the source of the magnetic flux, or it may simply move a mass of iron that forms a ready path for the lines of force. The magneto made it practicable to obtain electric currents from mechanical power, an advance step because mechanical power is cheaper than the chemicals used in a voltaic battery. The magneto is of great historical interest, but it has been largely displaced by the more efficient dynamo.

391. The Dynamo, or dynamo-electric machine, differs characteristically from the magneto in that the former employs a field of force due to the influence of electromagnets, while the latter utilizes permanent magnets.

392. The Simple Dynamo. — Of course, it makes no difference whether the coil or the flux of force moves, provided that the number of lines of force that pass through the coil is continually changing. Suppose a single loop of wire to turn upon a horizontal axis, and between the opposite poles of two magnets, *N* and *S*, as shown in Fig. 380. When the loop stands in a vertical plane, as indicated by the heavy black line, the magnetic lines of force between the pole-pieces thread through the loop in the greatest possible number. When the loop has been turned upon its axis through ninety degrees, until it lies in a horizontal plane, as indicated by the dotted lines in the figure, the lines of force run parallel to the plane of the loop, and none thread through it. During this quarter revolution of the loop, the number of lines of force that pass through the loop was decreasing, and an electric current was thereby in-

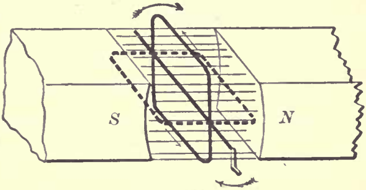


FIG. 380.

duced in the loop, as indicated by the arrows. During the next quarter revolution of the loop, the number of lines of force threading the loop was increasing, but as they passed through the loop from the other side, the current induced in the loop had the same direction as before. During the next half revolution, the induced current will flow through the loop in the opposite direction. The current, therefore, reverses twice for each revolution of the loop.

393. The Direct Current Dynamo, one of the most important of the modern, practical devices for the transformation of mechanical into electrical energy, consists essentially of three parts: an *armature* made of coils of wire, which may be revolved in a magnetic field, and thus successively filled with lines of force and emptied of them; a *commutator* for giving a uniform direction to the alternating currents induced in the coils by their rotation in the field of force; and a large *electromagnet* as a source of flux of force.

394. The Armature. — If the revolving coil is composed of many turns of wire instead of a single loop, the electromotive force generated by the revolution will be multiplied by the number of turns. If the loop is filled with soft iron, which has a greater magnetic permeability than air, the number of lines of force gathered into the space traversed by the coil will be increased, and the electric effect thereby augmented. *A soft iron cylinder or ring upon which coils of insulated copper wire have been wound and arranged for rapid rotation in a magnetic field is called an armature.*

(*a*) The Siemens or shuttle armature, represented in Fig. 381, consists of a coil of wire wound in two broad grooves plowed on opposite sides of an iron cylinder. Such armatures are largely used in the magnetos used for "calling up" on telephone circuits, but they are not well adapted for large currents, because the "local currents" (often called Foucault currents) generated in the iron core absorb energy, and transform it into heat. This heat increases the internal resistance of the coil, and is objectionable in other ways. Moreover, with such an armature, the current falls to zero twice every revolution and, for many purposes, such a current is useless.

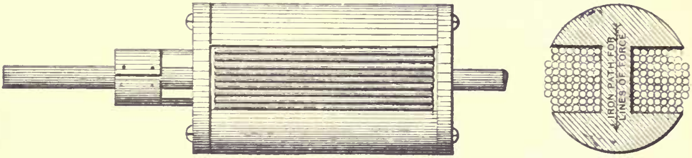


FIG. 381.

(*b*) The drum armature differs from the shuttle armature chiefly in that it employs many coils instead of one. The cylindrical iron core is made of thin disks of soft iron insulated from each other, thus minimizing the "local currents" and the heating effects thereof. The insulation for this purpose sometimes consists of tissue paper, sometimes of varnish, and sometimes only of the oxidation on the surfaces of the metal. On the cylinder thus built up, many separate coils are wound lengthwise, as is shown in Fig. 382. These separate coils are joined in series, and the several junctions connected to insulated bars, the extremities of which are grouped around the shaft of the armature as shown at the left of the figure. Brass bands around the outside of the cylinder hold the coils in place.

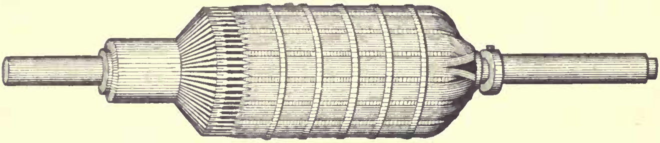


FIG. 382.

(*c*) The Brush or ring armature consists of eight or more coils wound in grooves upon an annular core, as shown in Fig. 383. The core is laminated, i.e., built up by winding a thin band or ribbon of soft iron in successive layers, each layer being insulated from the next. The wedge-shaped projections that separate the coils are made by thin pieces of iron placed crosswise between successive layers of the long band as the ring is built up. Coils radially opposite are joined in series, and the terminals of each such pair are carried to the commutator on the shaft of the armature.

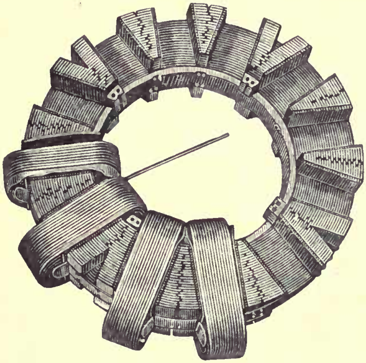


Fig. 383.

(*d*) Armature coils are sometimes wound upon arms or spokes that project radially from a central hub, or are set in succession on the face of a disk and near its circumference.

395. The Commutator. — If the connections of the armature coils are reversed at the moment when the current in the coils is reversed, the induced currents will all flow in the same direction in the external circuit. *The special device for thus changing the connections of armature coils is called a commutator.*

(*a*) The commutator of the Siemens armature consists of the two halves of a metal collar around the armature shaft, and two metal strips or "brushes." The two halves of the collar, i.e., the "commutator segments," *m* and *n*, are separated from the shaft, *s*, that carries them by a bushing of insulating material, and are separated from each other as shown in Fig. 384. One end of the armature coil is connected with one segment, and the other end with the other segment. The brushes, *bb′*, are held by fixed supports so that their free ends rest lightly on the segments. The points of contact are diametrically opposite.

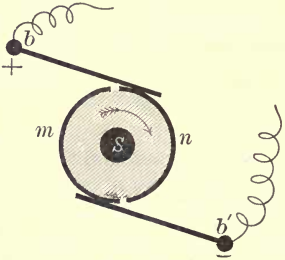


Fig. 384.

Consider *b* and *b′* the terminals of the dynamo, and that they are connected by a wire that constitutes the external circuit. Remember that *m* and *n* are connected through the armature coil. Assume that the connections of the terminals of the armature coil with the commutator segments are such that current flows through the coil and passes out by way of *n* and *b*. As the armature is turned a little further, the current in the coil is reversed, and flows out through *m* instead of *n*. But the same rotation of the shaft that carries both the armature and the commutator has now brought *m* into contact with *b* so that the current continues to flow through *b* which thus remains the + terminal as long as the shaft is turned in the direction indicated by the arrow.

(*b*) There are many different ways of connecting armature coils with their commutators, each one of which may call for careful study. The following may be taken as typical of most of them: in Fig. 385, the numbered loops represent the armature coils joined in series as in the ring armature. The heavy broken circle represents commutator segments on which rest the ends of the brushes, *b* and *b′*. The brushes divide the coils into two sections. In the figure, coils numbered 2, 3, 4, 5, 6, and 7 constitute the right-hand section, and coils 8, 9, 10, 11, 12, and 1 constitute the left-hand section. The six coils of each section are connected in series, and the two sections are connected in parallel. As the armature turns between the pole-pieces, *N* and *S*, and in the direction indicated by the arrow, the currents induced in the coils of the right-hand section will flow in the direction indicated by the arrow-heads. At the same time, the currents induced in the coils of the left-hand section will flow in the opposite direction, as indicated by the arrow-heads. Imagine a current returning from the external circuit and entering the generator at *b′*, the negative terminal. It will find two paths of equal resistance, one through the right-hand section, and the other through the left-hand section. In either case, the direction of the current freshly induced in those sections coincides

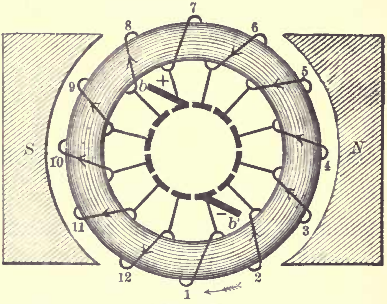


FIG. 385.

with its own. It, therefore, divides itself equally between the two paths and flows from the generator at *b*, the positive terminal.

396. The Field Magnet. — The electromagnet that supplies the flux of force must have a current to excite it. This current is sometimes supplied from an outside source, as is diagrammatically shown in Fig. 386. Such a dynamo is said to be *separately excited*. Often all of the current from the armature is carried around the coils of the field magnet, thus forming a *series* dynamo, as is shown in Fig. 387. Sometimes a part of the current from the armature is carried through a shunt circuit consisting of many turns of wire that is smaller than the wire of the main circuit, as is shown in Fig. 388. Such a dynamo is said to be *shunt wound*. Sometimes, for purposes of regulation, the field magnet is encircled by both series and shunt coils, as is shown in Fig. 389, or by either of those with a separately excited coil. Such a dynamo is said to be *compound wound*.

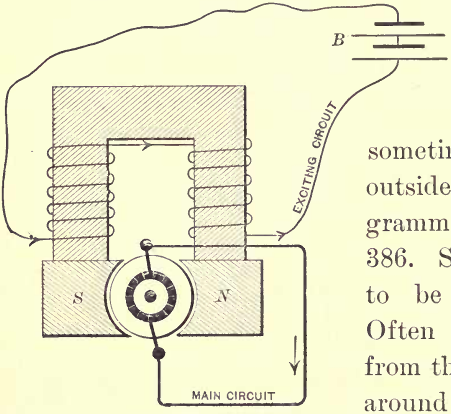


Fig. 386.

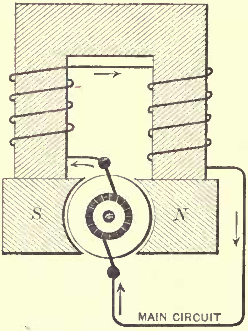


Fig. 387.

For arc lighting, a current that is constant under varying load is needed; it is generally secured by a "regulator" connected with the dynamo, as shown at *R* in Fig. 442. The regulator may be automatic in its action. For incandescence lighting, a potential that is constant under varying load is needed; it is generally secured by compound winding.

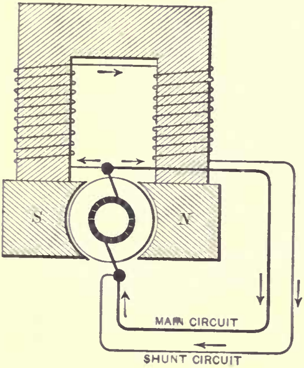


FIG. 388.

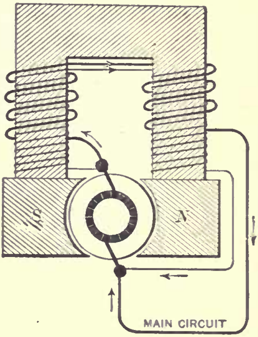


FIG. 389.

(*a*) When the armature of a "self exciting" dynamo, i.e., one that has not an exciting current from an external source, is put in motion, the feeble *residual magnetism* of the cores of the field magnets induces feeble currents in the armature coils. These currents flow around the magnets, intensifying their power, and thus increasing the E.M.F. of the machine. The current thus strengthened further energizes the field magnet. Thus, the machine "builds up" its current until the magnets have reached the limit of excitation. Many dynamos, especially those used for the generation of alternating currents, have more than two pole-pieces.

(*b*) Lines of force generated by the field magnet, and that do not pass from pole to pole, are termed the stray-field or the leakage lines. The stray-field between the pole-pieces and the bed-plate of a dynamo may become the source of serious loss and annoyance.

(*c*) Fig. 390 represents the Brush dynamo complete. A shaft runs through the machine from end to end, carrying a pulley, *P*, at one end, a commutator, *c*, at the other end, and a wheel armature, *R*, at the middle. The armature carries eight or more helices of insulated wire, *H H*, connected in pairs as described in § 394 (*c*). As the shaft is turned by the action of the belt upon the pulley, the armature and the commutator are turned with it. The armature coils are thus carried rapidly across the four poles of the field magnets, *M M*, traversing the intenser parts of the magnetic field, and cutting the lines of force.

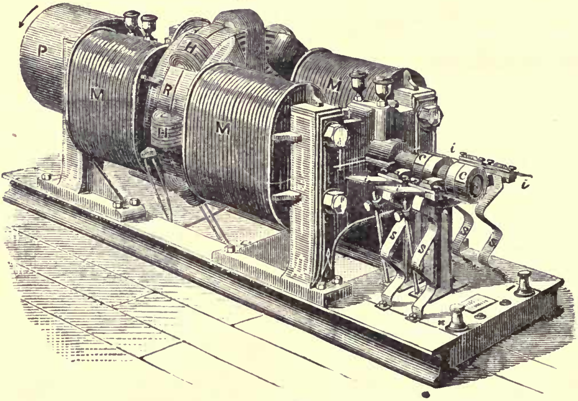


Fig. 390.

CLASSROOM EXERCISES.

1. What is an induced electric current? How is it produced?

2. How are induced currents made continuous?

3. Give some proof that the condition of a wire when it closes an electric circuit is different from the condition of the same wire when the circuit is open.

4. Why are the field magnets of dynamos generally provided with iron cores?

5. What advantage is there in making the field-magnet cores of cast iron instead of pure soft iron?

6. Upon what does the E.M.F. of a dynamo depend?

7. What is the difference between a magneto and a dynamo?

8. When a dynamo is in operation, its field magnets are likely to become heated. Does this increase or diminish the efficiency of the machine, and why?

9. How are the field magnets of a shunt-wound dynamo energized?

LABORATORY EXERCISES.

Additional Apparatus, etc.— A 1-ohm resistance coil of No. 30 iron wire; a resistance coil of No. 30 German-silver wire; two magnetoscopes; voltaic cells; plates of antimony, and of zinc, and of copper; a copper cartridge-shell; hydrochloric acid; pieces of electric light carbon; a telephone receiver; German-silver wire; two semicircles of soft iron; an iron rod 45 cm. long; a brass tube, and one of rubber.

1. Given the two electrodes of a concealed voltaic battery, determine which of the wires is connected to the zinc plate.

2. Connect a Leclanché cell and a freshly prepared dichromate cell in parallel, and place a galvanoscope in the circuit, between the carbons. (*a*) Determine and record the direction of the current. (*b*) Place the galvanoscope between the zincs, and determine the direction of the current. (*c*) Why should this current flow? (*d*) Is it advisable to connect cells that are dissimilar in parallel, and why? (*e*) Suppose the E.M.F. of the dichromate cell to be 2 volts, and that of the Leclanché cell, 1.42 volts. Suppose the resistance of the dichromate cell to be 0.6 of an ohm, that of the Leclanché cell to be 0.8 of an ohm, and that of the wire and galvanometer to be 1 ohm. What is the resultant current in the system? Remember that when two electromotive forces are in opposition, their difference is all that is available.

3. Prepare a voltaic cell by immersing a strip of amalgamated zinc in a copper cartridge-shell filled with dilute sulphuric acid. Connect this cell in parallel with a large copper and zinc pair and, by means of the galvanoscope, determine whether either cell forces current over into the other. Make several trials, using acid of the same strength in both cells. If carefully done, it will be found that the E.M.F.'s are nearly equal.

4. Form a cell with antimony and copper plates, and dilute sulphuric acid. Insert a galvanoscope in the circuit. Note and record the deflection. Prepare a jar of dilute hydrochloric acid. Lift the plates from one jar to the other without otherwise changing the apparatus. Note and record the deflection. Which is attacked more vigorously by sulphuric acid, antimony or copper? Which by hydrochloric acid?

5. Provide a glass tube about 1 cm. internal diameter. Insert a wire in each end, and fill the tube with pieces of pounded electric light carbon. Pass a current from a cell through the apparatus, interpos-

ing a low resistance galvanoscope. By means of a wooden rod, compress the powdered carbon. Why is the deflection largely increased? Why is a low resistance galvanoscope used?

6. Calculate the length of No. 30 iron wire that has a resistance of 1 ohm. Procure the wire, and wind it on a board, being careful that adjacent turns do not touch. With this addition to your apparatus, determine whether the resistance of a telephone receiver is greater or less than an ohm, and whether it differs much or little from an ohm.

7. Wind 4 ounces or more of No. 30 insulated German-silver wire upon a spool. Connect this bobbin in series with a single cell and a low resistance galvanoscope. Record the deflection of the latter. Change the low resistance block of the galvanoscope for one of high resistance. Again note and record the deflection. Explain the increased deflection of the needle that accompanies the increased resistance of the circuit.

8. Prepare two semicircles of $\frac{1}{2}$-inch soft iron rod, two inches in diameter, and file the faces smooth, so that they will fit together nicely.

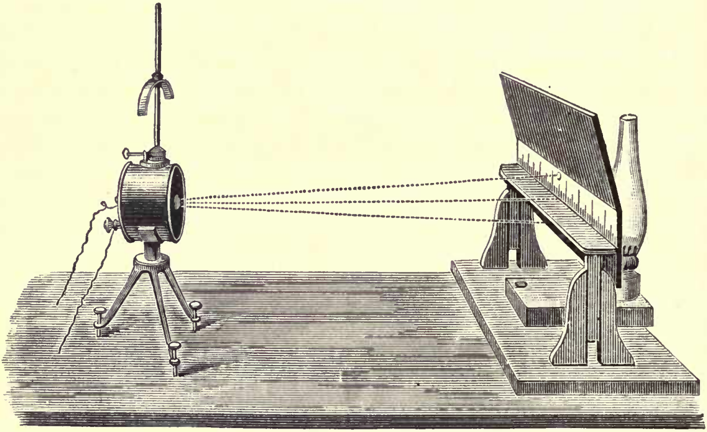


Fig. 391.

Prepare two bobbins of approximately equal weight, one made of about 20 turns of No. 16 wire, and the other of No. 30 wire, both insulated. Slip the bobbins on one of the iron semicircles. Connect the terminals of the fine wire coil to those of the mirror galvanometer (see Experiment 364). Arrange the galvanometer so that the mirror reflects its beam upon a ground glass scale about a meter away, as shown in Fig. 391. A thin line of black paint drawn vertically across the center of the mirror will aid in exactly locating the spot of light upon the scale. Connect the terminals of the coarser coil to those of a single cell. Open and close the primary circuit, and note the magnitude of the kick of the galvanometer. Repeat with the other half of the ring in place, and notice that the deflection is increased. Explain this increase, remembering the effect of permeability upon the number of lines of force in a magnetic circuit.

9. Take a rod of iron ¼ of an inch in diameter and 18 inches long, and wind upon one end a primary, such as was used in the ring of Exercise 8; upon the other end of the rod, wind a secondary coil. By means of opening and closing the primary circuit, and noticing the deflection of a galvanoscope properly connected to the secondary, determine at what points along the rod the lines of force are the greatest in number. Why is the maximum deflection found when the primary and secondary are close together? On paper, map the lines of force as you think they must exist in space. Cover the primary coil with a piece of glass, and pass a current of ten or more amperes through it. Sprinkle the glass with filings, and prove or disprove your theory.

10. Clamp to a vertical board two magnet bobbins (see Exercise 12) joined in series as shown in Fig. 392. Support one end of the armature, *bm*, by an elastic band, *ab*. Pass a current through the bobbins, and notice the pull upon *ab*. Looking at the upper ends of the bobbins, notice whether the current circles around the two bobbins in the same direction or not, as clockwise or counter-clockwise. Turn one of the bobbins upside down, changing the connections in this respect. Ascertain which connection gives the greater pull upon the armature, *bm*, and, with the bobbins thus joined, bring the movable soft iron yoke, *cd*, into position as shown in the figure. Explain why this improves the magnetic circuit, so that the upper armature is pulled harder than before, and probably drawn down

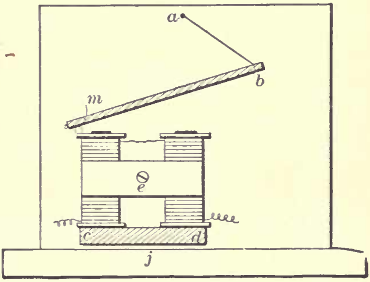


FIG. 392.

with a sharp click. Remember that all magnetic circuits tend to contract themselves, and to make their reluctance as small as possible.

11. Prepare an exploring coil with several hundred turns of No. 30 insulated copper wire and an internal diameter of 5 cm. Place this coil in the field of a strong electromagnet, connect its terminals with those of a telephone receiver, and make and break the magnet circuit. Determine the points where the flux is the heaviest by moving the coil about and repeating the experiment. Be sure to have the face of the coil at right angles to the supposed direction of the magnetic lines of force. Mount the coil on a stiff wire that passes through the center of the coil, and lies in the plane of the coil. The plane of the coil may be easily changed in the magnetic field by rolling the supporting wire between the thumb and forefinger. Place the face of the coil in the position that you think is at right angles to the lines of force in the location you have chosen. Make and break the magnet circuit, and notice the strength of the click. Now turn the coil a quarter turn and repeat. If you have estimated rightly, there should now be no click. Place the magnet on a piece of paper, and draw a line under the coil parallel to its face whenever you find a position where no click occurs, and thus map out the field. Sprinkle the paper with filings, and verify your map.

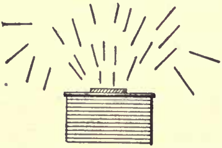


FIG. 393.

12. Make two magnetoscopes like that shown in Fig. 373. An ordinary carriage-bolt about 7 cm. long may be used as the core, and a soft iron nut may answer as the armature. With the two magnetoscopes, a voltaic battery, and a supply of insulated No. 20 copper wire, arrange apparatus so that you can exchange telegraphic signals with another pupil at another table, or in another room.

397. The Alternator is a dynamo designed for the generation of alternating currents. It has collecting rings instead of a commutator so that the current is delivered just as it is generated (§ 395), and a small direct current dynamo for energizing its field magnets, the pole-pieces of which are generally very numerous. Fig. 394 represents one form of the machine.

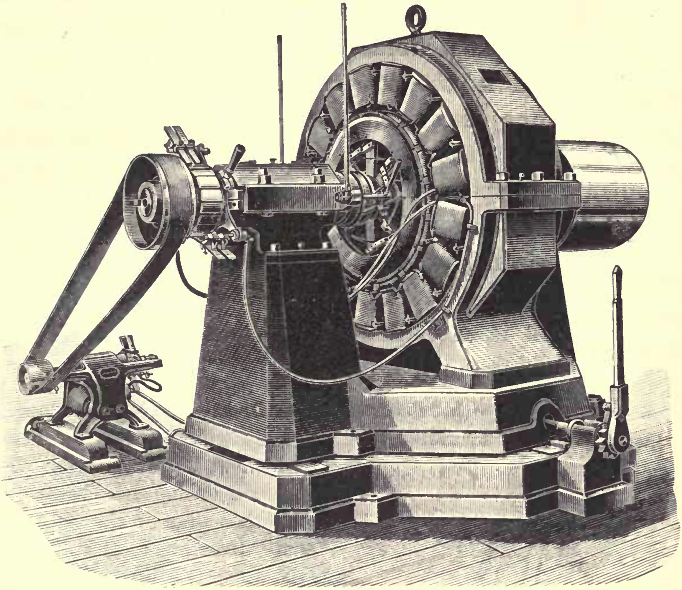


FIG. 394.

398. Tesla's Oscillator is a combined prime motor and electric generator, and produces alternating currents without rotary motion of the generating coils. The motive force may be that of steam or of compressed air. The machine is represented in section by Fig. 395. The powerful field magnets, *MM*, are excited by a current from an outside

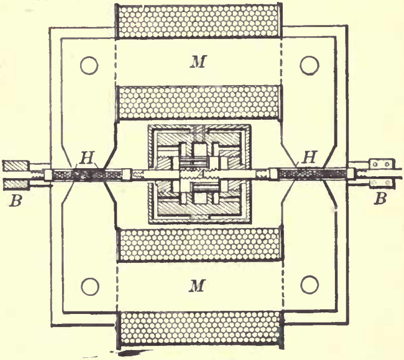


FIG. 395.

source. The generating coils are mounted on the piston-rod, *A*, and rapidly vibrate back and forth in the direction *BB*, and in the powerful magnetic fields, at *HH*. The oscillating piston-rod slides endwise in its supports at *BB*. The action of the motive power is somewhat peculiar, and depends largely on the inertia of the oscillating parts. The stroke of the piston-rod is from $\frac{1}{64}$ to $\frac{3}{8}$ of an inch, according to the pressure used and the nature of the current desired. The output of the machine relative to its weight is exceedingly large, and the machine gives promise of commercial value.

Experiment 369. — Mount a metal clock-wheel upon wooden bearings, and solder to its axle a wire crank by which it may be turned. Provide two metal springs. The upper end of one should rest upon the toothed edge of the wheel, and "snap" from one tooth to the next as the wheel is turned. The upper end of the other should rest on the axle of the wheel. Consider the fixed ends of these springs as the terminals of this "interrupter." Put this apparatus into the circuit with a voltaic battery and the galvanoscope that has a coil of No. 16 wire. Turn the wheel, and notice the deflection of the needle.

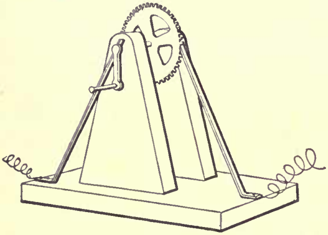


FIG. 396.

399. Alternating Currents have some peculiar properties largely due to the constantly fluctuating field of force that surrounds their conductors. The pulsating current produced by the interrupter has many of the properties of the alternating current, and will facilitate our investigations.

(*a*) The current does not wholly cease when the spring of the interrupter snaps from tooth to tooth. As the circuit is broken, the encircling magnetic lines of force (§ 382) are decreased in number, and that very decrease tends to continue the current as explained in § 400. In brief, the current does not have time wholly to die away before the spring is on the next tooth of the wheel.

Self-Induction.

Experiment 370. — Double a piece of No. 24 insulated copper wire about 100 feet long, and wind it upon a wooden rod as shown in Fig. 397. Join the ends of this wire in the series circuit of the apparatus arranged for Experiment 369. Turn the wheel of the interrupter rapidly, and note the deflection of the galvanoscope. Remove the No. 24 wire from the circuit, straighten it, and wind it upon an iron rod so as to form an electromagnet. Put this electromagnet into the circuit, and repeat the experiment. Notice that the deflection of the galvanoscope is less, and that the sparks at the wheel of the interrupter are greater than before.

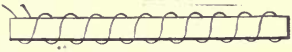


FIG. 397.

Experiment 371. — Place the coil and core of Experiment 367 in the circuit of a voltaic battery, and insert a galvanoscope as shown at *G* in Fig. 398. When the circuit is closed by depressing the key, *K*, part of the battery current is shunted from *m* to *n* through the galvanoscope, and deflects its needle. Force the needle back to zero, and place some obstacle to prevent its moving again in that direction, but leaving it free to move in the opposite direction. Break the circuit at *K*, and notice that the needle does swing in the opposite direction, showing that a current passed through it from *n* to *m*. This current was not the battery current, for the battery circuit was open.

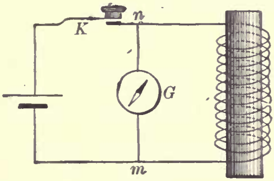


FIG. 398.

400. Self-Induction. — *When the number of lines of force in a coil is increasing, an electromotive force opposite to that of the inducing current is established, thus weakening the direct current; when the number is decreasing, an electromotive* *force that coincides in direction with that of the inducing current is established, thus strengthening the direct current.* In consequence of this, we may have an opposition to the current-flow other than the resistance of the circuit, namely, the opposing, self-induced electromotive forces. The coil manifests a conservative tendency, opposing sudden changes. When the doubled wire of Experiment 370 was wound upon the wooden rod, every part of it lay adjacent to another part that was carrying current in the opposite direction. The magnetic lines of force generated by one part neutralized the lines of force that circled in the opposite direction around the adjacent part; i.e. the circuit was non-inductive. In the other case, the lines of force circled in the same direction around adjacent parts of the wire, and assisted each other in setting up an opposing, self-induced electromotive force that greatly weakened the current that produced them. In Experiment 371, the self-induced electromotive force generated by the action of the several turns of the coil upon one another at the moment of opening the circuit acted through the coil in the same direction that the battery current did, and, consequently, sent an induced current through the galvanoscope in the opposite direction.

(*a*) Such a coiled circuit is said to have a *reactance.* This reactance has much the effect of resistance, but it depends upon other considerations, chiefly the frequency of the pulsations, and upon a certain constant called the *coefficient of self-induction.* This coefficient depends upon the shape, coiling, and coring of the circuit and, in practice, is determined only by experiment. Self-induction coefficients are measured by a unit called the *henry.*

401. Reactance and Impedance. — As alternating currents are fluctuating in value, their measure must be that of averages. The chosen average is the square root of the arithmetical mean of the squares of all its values. This "square root of a mean square" applies to current and to voltage, C and E. For a true alternating current (i.e. one that increases and diminishes by what is called the law of sines), the numerical relations may be represented thus:

$$C = \frac{E}{\sqrt{R^2 + (2\,\pi n L)^2}},$$

in which R represents the ohmic resistance, n the frequency of alternation, and L the coefficient of self-induction. *The expression* $2\,\pi n L$ *represents the reactance. The apparent resistance*, i.e., $\sqrt{R^2+(2\,\pi n L)^2}$, *is called the impedance*, *and is measured in ohms.*

(a) The mathematical relations of resistance, reactance and impedance may be easily remembered by considering the first and second of these functions as the two sides of a right angled triangle of which the impedance is the hypothenuse, i.e. the "square root of the sum of the squares of the other two sides."

Experiment 372. — Wind about twenty turns of No. 18 insulated copper wire around a $\frac{1}{2}$-inch iron rod, or (preferably) around a bundle of iron wires, and put the coil into circuit with a pulsating current. The lines of force inside the coil and in the core fluctuate in value with the current. On the outside of this coil, and carefully insulated from it, wind 300 or 400 feet of No. 28 insulated copper wire. Place one of the terminals of this outer or secondary coil above the tongue, and the other terminal below it. When the pulsating current flows through the inner or primary coil, currents are induced in the secondary coil, and produce distinct shocks in the tongue.

402. The Transformer. — By suitably winding and coring primary and secondary coils, an alternate current at one voltage may be received by the primary, and a current at a voltage higher or lower as desired delivered from the secondary. When the primary is made of a few turns of large wire, and the secondary is made of many turns of small wire, the voltage is increased, and *vice versa*. *Coils so wound and properly cored are called transformers.*

(*a*) Transformers are largely used when currents of high voltage are to be carried great distances, and delivered at a pressure suitable for use. With a given resistance in the line, the loss in watts is less with a small current and high voltage than it is with large current and low voltage. With a given line loss, high voltage currents enable the use of small conductors, and copper wire is expensive.

(*b*) When the electric energy is transformed from a current of low voltage and many amperes to one of high voltage and few amperes, the apparatus is called a "step up" transformer. Similarly, when the voltage is decreased, the apparatus is called a "step down" transformer.

403. The Induction Coil is a modification of the apparatus used in Experiment 372, and is often called the Rhumkorff coil. Receiving a large current of small electromotive force, it delivers a small current at a high pressure, sometimes hundreds of thousands of volts, i.e., it is a "step up" transformer.

(*a*) In the diagram shown in Fig. 399, *M* represents a core of iron

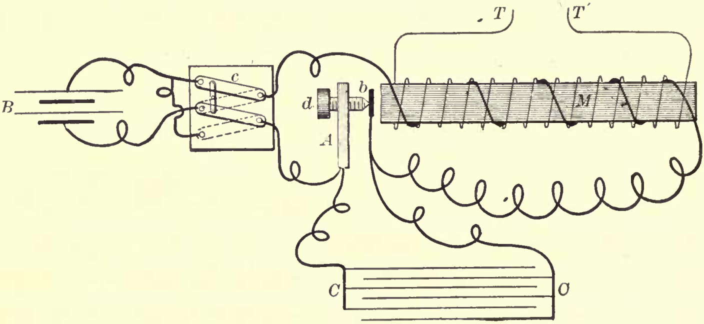


FIG. 399.

wires upon which is wound a primary coil of coarse wire that is in circuit with the voltaic battery. In this primary circuit, are a commutator, *c*, for changing the direction of the current, and an automatic interrupter, *b*. Wound upon the primary coil, and very carefully insulated from it, is a secondary coil made of very many turns of fine wire, the terminals of which are marked *T T'*. If the coil is designed to give sparks between *T* and *T'*, the condenser, *CC*, is added. This consists of sheets of tin-foil separated by sheets of paraffined or shellac-varnished paper. The alternate sheets of tin-foil are joined in parallel; the two groups are connected to the primary circuit on opposite sides of the interrupter. The condenser is generally placed in the base that carries the coil. A simple form of the instrument is shown in Fig. 400.

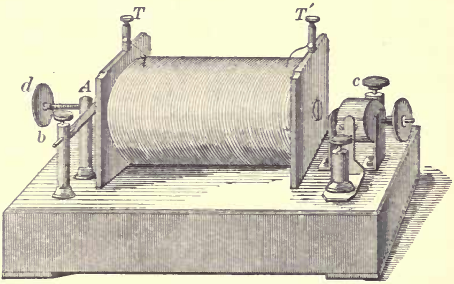


FIG. 400.

(*b*) The current passes through the commutator, *c*, up the post, *A*, through the adjusting screw, *d*, and across to the spring interrupter, *b*, which rests against the end of *d*, and is carried by another post, as shown in Fig. 400. Thence it passes to the primary coil, magnetizing the iron-core, and making its way back to the generator. The iron core thus magnetized attracts the soft iron hammer at the end of the spring, thus breaking the circuit at *b*. When the current is broken, the core is demagnetized, and the elasticity of the spring throws *b* against the end of *d*, again making the circuit. Thus the spring vibrates between the end of the core and the end of the screw, making and breaking the circuit with great rapidity, and inducing currents in the secondary coil. Owing to the permeability of the iron core which intensifies the flux of force through the coils, and to the great number of turns in the wire of the secondary coil, the electromotive force of the induced currents is very high.

(*c*) The self-induction of the primary coil when the circuit is made at *b* develops a counter E.M.F. that opposes the battery current, and thereby lessens the E.M.F. of the induced current. When the circuit is broken, the counter E.M.F. reinforces that of the battery current, so that, for an instant, the latter may be increased by its own interruption. One effect of this is to strengthen the sparks noticeable at *b*.

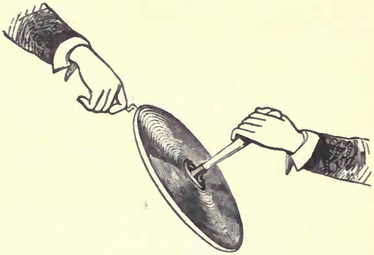
(*d*) One effect of the condenser is to make the interruption of the battery current at *b* more abrupt and, therefore, to increase the E.M.F. of the induced current. Immediately after the interruption of the battery current, the condenser sends a reverse current through the primary coil and battery, thus demagnetizing the core more rapidly, increasing the rate of change in the intensity of the flux through the coils and, in this second way, increasing the E.M.F. of the induced current. Consequently, the E.M.F. of the direct current induced in the secondary coil when the primary circuit is broken is higher than the E.M.F. of the reverse current induced in the secondary coil when the primary circuit is made.

(*e*) The length of the spark depends upon the E.M.F. of the induced secondary current. The difference of potential necessary to produce a spark 1 cm. or more in length between parallel plates in air under ordinary barometric conditions is about 30,000 volts per centimeter. For shorter sparks, the difference of potential has a greater value.

High Potential Phenomena.

Experiment 373. — Connect a voltaic battery with the primary of an induction coil. Bring the terminals of the secondary within a few millimeters of each other, and notice the rapid succession of sparks that strike across the gap filled with air, one of the best of insulators. With a good coil, plates of glass and other non-conductors may be thus perforated. We have not noticed this property of electricity before because we have not had a current of sufficiently high E.M.F.

Experiment 374. — In a shallow tin pan (e.g., a common pie-tin), melt equal quantities of rosin and shellac. Stir the substances together, avoid ignition and the formation of bubbles, and, when the tin is filled, set it aside to cool. Cut a disk of sheet tin a little less in diameter than the resin plate, and fasten a piece of sealing-wax at its center for a handle. Whip the plate briskly with a catskin, or rub it with warm flannel. Place the tin disk upon the resin plate, and touch the former with a finger. Place a number of small bits of paper upon the disk. Lift the disk by its handle; the charged paper bits are repelled. Bring a knuckle to the edge of the disk; an electric spark may be seen. Such a discharge in air requires a force of about 130 electrostatic units per centimeter of length, although the E.M.F. per unit length is greater for small than for great distances. The disk may be charged many times without repeating the excitation of the resinous plate. The apparatus may be improved by making the disk of wood, rounding its edge, covering it with tin-foil, and smoothing down the latter with a paper-folder or a finger-nail.



404. An Electrophorus consists of a plate of resinous material or of vulcanite resting on a metallic bed-piece, and a movable metallic cover provided with an insulating handle. It is used as illustrated in the preceding experiment. As the surface of the resin plate is uneven, the metallic cover touches it at but a few points; as the material is non-conducting, scarcely any electrification passes from the former to the latter. The two disks and the thin layer of air between them constitute a condenser (§ 342). The negatively electrified resin plate acts by induction on the disk, holding positive electrification "bound" at its lower surface, and repelling the negative which escapes through the finger. When the plate thus charged is removed from the resin plate, the bound electrification is set free.

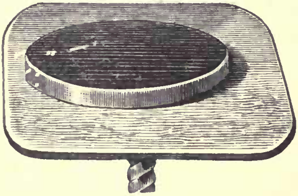


Fig. 401.

405. Electric Machines for developing statical electrification in large quantities depend upon either friction or induction for their operation, and are made in great variety.

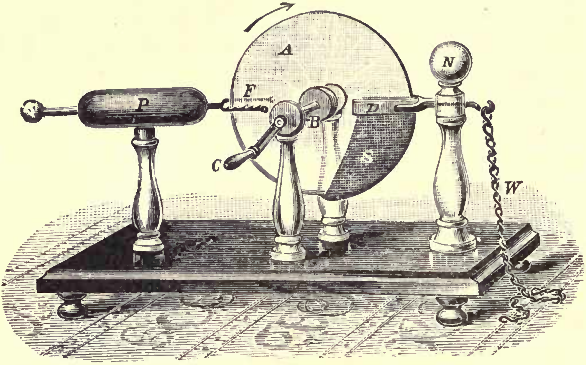


Fig. 402.

(*a*) The frictional electric machine usually consists of a plate of glass, *A*, which is revolved between stationary cushions, *D*, the surfaces of which are covered with amalgam. The parts of the plate thus positively electrified are successively brought between two metallic combs, *F*, the pointed teeth of which nearly touch the plate. The prime conductor, *P*, is electrified by induction, the negative electrification escaping by air-convection from the pointed teeth to the oppositely electrified plate, thus neutralizing its electrification and leaving the prime conductor positively charged. The negative conductor, *N*, that carries the cushions is

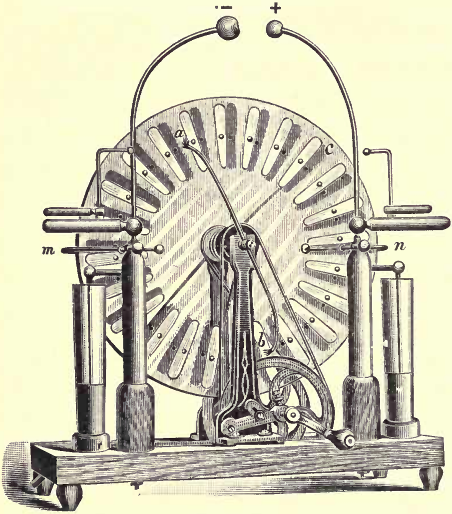


Fig. 403.

generally connected to earth, as shown in Fig. 402. The potential energy of the electrification thus obtained is the equivalent of the kinetic energy expended in turning the crank, minus that transformed into useless heat.

(*b*) The induction machines may almost be described as continuous electrophori. The Wimshurst machine (Fig. 403), which may be taken as a representative of the class, consists for the most part of two equal glass disks that revolve in opposite directions. Sector-shaped strips of tin-foil are fastened to the outer surfaces of the plates, and act as carriers of electrification and, when opposite each other, as field plates or inductors. Two conductors are placed at right angles to each other, obliquely across the plates, one at the front and the other at the back. The ends of these conductors carry tinsel brushes that lightly touch the sectors as they pass. The discharging circuit is provided with combs that face each plate, and that are connected with small Leyden jars. The distance between the balls of the discharging circuit may be regulated by insulated handles. This machine is almost wholly free from "weather troubles."

The tin-foil strips or carriers on the rear plate of a Wimshurst machine are represented in Fig. 404 by the outer row of strips; those on the front plate, by the inner row. The diagonal conductor that faces the rear plate is represented by *cd*; the one that faces the front plate, by *ab*. The strips from which the arrows proceed are charged positively; the others, negatively. The strips at the top of the rear plate are represented in the diagram as being positively charged; those at the bottom as being negatively charged. These conditions are reversed for the front plate. The maximum charge upon one of the tin-foil strips or carriers is represented as six units. The opposite motions of the two plates are represented by the two large, curved arrows. As the carrier at *a* moves into the position shown in the diagram, it comes under the inductive influence of the positively

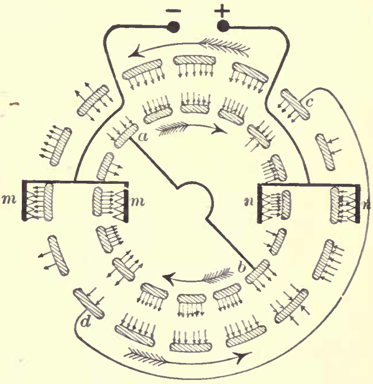


FIG. 404.

charged carrier opposite it on the rear plate. At this instant, it touches the brush of the diagonal conductor, and a transfer of positive electrification from *a* to *b* leaves the carrier at *a* negatively charged. At the same instant and in the same way, the carrier at *b* is positively charged. Similar effects are also produced in the carriers at *c* and *d*. Thus, the carriers of both plates come to *m* and *n*, the combs of the discharging circuit, similarly charged, positively at *n*, and negatively at *m*. The inductive action of these carriers upon the discharging circuit electrifies its two sides oppositely.

(*c*) The phenomena of static electricity that may be exhibited with a machine like that just described are very beautiful, and their study is very enticing. Some of them were known to man before the dawn of authentic history. Static electricity opens a field for deep study that has been of great value in theoretical research, and is now full of promise, but the greater and rapidly growing industrial importance of current electricity, and the necessary limitations of a work like this, compel the author to refer the pupil to other texts for a fuller discussion of the subject than he can give here. He does so with a feeling of sadness that utility should thus dominate beauty.

High Voltage Currents.

Experiment 375. — Wind several turns of wire upon a piece of glass tubing inside of which is an unmagnetized sewing-needle. Discharge a Leyden jar through the wire, and test the needle to see if it has been magnetized.

Experiment 376. — Wind ten or more turns of insulated wire, No. 22, on the outside of a thin glass tumbler, being careful that the turns do not touch each other. A coating of shellac varnish will help to hold the wire in place. Wind a smaller coil of ten or twelve turns of similar wire, bringing one end of the wire up through the coil, and being careful that it does not touch any

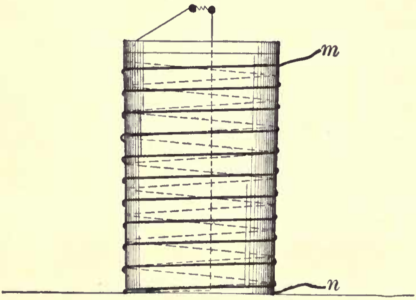


FIG. 405.

of the convolutions. Tip the two ends of this wire with bullets, and adjust them so that they will be within about 1 cm. of each other. Place the second coil in the tumbler, as shown in Fig 405, and fill the tumbler with high grade kerosene. Connect *n*, the lower end of the outer wire, with the tin pan of the electrophorus. Charge the disk of the electrophorus, and discharge it through *m*, the upper end of the outer coil. Notice the spark between the terminals of the inner coil. Support an iron rod inside the inner coil, being careful that it does not touch the wire. Repeat the experiment, and notice that the "striking distance" between the terminals of the inner coil may be increased.

Experiment 377. — Connect one terminal of the outer coil of the apparatus used in Experiment 376 to a terminal of the secondary of an induction coil. Set the latter in operation, and discharge the other terminal of its secondary into the other terminal of the outer coil of the tumbler. Notice the series of sparks between the terminals of the inner coil of the tumbler, and that the sparks there keep step with those of the induction coil.

406. Identity. — The experiments just given indicate the remarkable similarity between current electricity at high voltage, and static electricity. Each can overcome the enormous resistance of an insulator like the air, and each lends itself to electromagnetic induction in the same way. Many facts tend inevitably to the conclusion that *the two kinds of electricity are identical.*

Nature of Electric Discharge.

Experiment 378. — Let another pupil push a pin through a visiting card. Examine the card, and try to tell from which side of the card the perforation was made. Perforate the card by the spark of an induction coil, examine it carefully, and try to tell from which side the perforation was made. Similarly examine the perforations made in a card by the discharges of an electric machine, and of a Leyden jar. What do you infer from your comparison of the perforations?

Experiment 379. — Wind two or three layers of paper upon *MN* (Fig. 406), a bar of soft iron, and about fifty turns of No. 22 insulated copper wire upon the paper. Twist loops in the wire at *A* and *B*. Tip the ends of the wire with bullets, and bring them very near each other, as at *C*. Ground the wire at *B*, i.e., put it into electrical connection with the earth, and discharge the electrophorus or a Leyden jar into the loop at *A*. Notice the sparks at *C*.

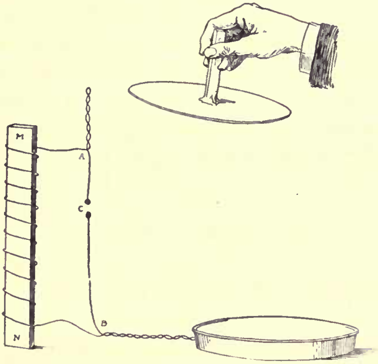


FIG. 406.

Experiment 380. — Straighten the wire of Experiment 379, and bend it into a long loop returning on itself as in Fig. 407. Adjust the knob terminals at *c* for the same distance as in Experiment 379. Ground *b*, and discharge the electrophorus or Leyden jar into *a*, as before. You will find great difficulty in getting a spark at *c*, and may not be able to do so at all.

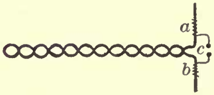


FIG. 407.

407. Oscillatory Discharge. — The sparks between the knobs, as observed in Experiment 379, show that, for some reason, the electricity preferred the path through the air at *C*, with a resistance of millions of ohms, to the path through the wire coiled upon *MN*, with a resistance of only a small part of an ohm. If the flow of electricity from a point of high to one of low potential was of the nature of a direct current, it would have followed Ohm's law, and passed in the greatest quantity through the path of least resistance. The formula applicable in such a case,

$$C = \frac{E}{R},$$

offers no possible explanation of the phenomenon observed.

On the other hand, if the flow was like that of an alternating current, it would be governed by the law that is expressed thus : —

$$C = \frac{E}{\sqrt{R^2 + (2\,\pi n L)^2}}.$$

The fact that in Experiment 380, the electricity flowed through the wire of low resistance instead of forcing its way through the enormous resistance of the air as it did in Experiment 379, suggests that the impedance of the circuit coiled around the iron was the cause of the apparently paradoxical choice of path, for, when we got rid of that impedance, the choice was what we should have expected. *If this hypothesis is correct, the impedance must have been very great.* Studying the mathematical expression for impedance, as given above,

$$R^2 + (2\,\pi n L)^2,$$

we notice that the value of R (the resistance of the wire) is too small to account for much of the great magnitude. The rest of the expression is the square of the reactance. Studying the values therein involved, we find that n is the only factor that can be great enough to account for the magnitude assumed for the impedance. This factor represents the frequency of alternation, and its magnitude must be measured by hundreds of thousands in order that the impedance may be sufficiently great to force the flow through the enormous resistance of the insulating air as was done in Experiment 379.

Recent investigations have done much to sustain conclusions like those now indicated, and to justify the statement that *in an electric discharge the flow surges back and forth thousands of times in the brief interval measured by the duration of the spark.* This oscillatory movement is a basis of important investigations now in progress, and maps out the line by which static electricity may become a thing of practical utility as well as of phenomenal beauty.

408. Atmospheric Electricity. — The surface of the earth is electrified. The electrification is generally negative but, in time of rain, it may become locally positive. Moreover, the electrical density varies greatly at different times and places. The origin of the earth's electrification is not known with certainty, but it "is influenced very largely, as it would seem, by external matter somewhere; probably at a distance of not many radii from its surface."

409. The Electrical Function of Clouds seems to be to collect and to concentrate the diffused electrification of the atmosphere. Suppose a thousand spherical watery particles, each having a unit charge, to coalesce to form a waterdrop. The diameter of this drop will be ten times that of a single particle, its capacity will be ten times as great, but its charge will be a thousand times as great; in other words, its potential will be increased a hundred-fold. *The condensation and aggregation of charged vapor particles must result in the production of a very high potential.*

410. A Lightning Flash is simply a disruptive discharge between two surfaces oppositely and highly electrified. The discharge may be from cloud to cloud, or from cloud to earth. The charged surfaces and the intervening air are analogous to a huge Leyden jar. Like the discharge of the jar, the lightning flash is oscillatory. A lightning flash a kilometer long corresponds to a difference of potential of about thirteen million electrostatic units.

(*a*) The sound that follows a lightning flash constitutes *thunder.* The sudden expansion and compression of the heated air along the line of discharge is followed by a violent rush of air into the partial vacuum produced. When the observer is about equally distant from the two surfaces between which the discharge takes place, a short and sharp thunderclap is heard; when one end of the path of discharge is considerably further from the observer than the other, so that there is a perceptible difference in the time that the sound requires to reach the ear from the different parts of the path, there is a prolonged roll or rattle. Sometimes near-by hills and clouds reflect the sound so as to produce a continuous or rumbling roar. One-fifth the number of seconds that intervene between seeing the flash and hearing the roar approximately indicates the number of miles that the observer is from the discharge.

(*b*) The induced charge on the earth tends to accumulate on buildings, trees, and other elevated objects, thus reducing the thickness of the dielectric, intensifying the attraction between the opposite electrifications, and increasing the liability of such elevated bodies. Statistical studies seem to show a minimum liability to accident from lightning stroke in thickly settled communities, and that the danger in the country is five times as great as that in the city. Such studies have also shown that there is no foundation for the popular notion that "lightning never strikes twice in the same place" except the fact that lightning often leaves nothing to be struck a second time. At the same time, it is well to remember that "Heaven has more thunders to alarm than thunderbolts to punish," and that *one who lives to see the lightning flash* need not concern himself much about any personal injury from that flash.

Experiment 381.—Twist together two wires, one of iron and one of German-silver, and attach their free ends to the terminals of a galvanoscope. Heat the junction of the two wires. The deflection of the needle indicates that an electric current was generated. Cool the junction of the dissimilar metals with ice. The opposite deflection of the needle shows that the current now generated flows in a direction that is the reverse of the first.

411. Thermo-electric Pile. —Two dissimilar metals joined and used like those of Experiment 381, constitute a *thermo-electric pair*. Antimony and bismuth are the metals generally used for the purpose. Many such pairs connected in series and having their ends exposed constitute a *thermo-electric pile*. Such a pile with conical reflectors is represented in Fig. 408. When its terminals are connected to the terminals of a delicate galvanoscope, the combination constitutes a *thermoscope* of great sensitiveness.

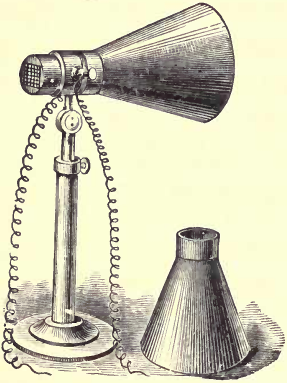


FIG. 408.

CLASSROOM EXERCISES.

1. (*a*) How is impedance measured? (*b*) How is the coefficient of self-induction measured? (*c*) Upon what does the latter depend?

2. Show, by the formula for impedance, that the static discharge is of a rapidly alternating nature.

3. Explain self-induction. How does it interfere with the flow of an alternate current?

4. (*a*) Define reluctance, and write the mathematical expression therefor. (*b*) Show geometrically the relation between reactance, impedance and resistance.

5. Sketch the connections of the induction coil. Explain the action of the automatic current interrupter.

6. Describe the electrophorus, and explain its action.

LABORATORY EXERCISES.

Additional Apparatus, etc.—A gold-leaf electroscope in which the knob shown in Fig. 315 is replaced by a metal disk about 15 or 20 cm. in diameter; 50 small glass tumblers; sheet zinc; sheet copper; tin plate; Geissler tube; an incandescence electric lamp; rosin or pitch.

1. Place 50 small glass tumblers in a circle. Into each tumbler, put a small strip of clean zinc, and a similar strip of copper. Nearly fill each tumbler with water, and connect the battery in series. Solder a thin copper wire about 50 cm. long to the zinc plate at the end of the series, and a similar wire to the copper plate at the other end of the series. Lay a sheet of thin paper that has been well soaked in melted paraffine upon the disk of a gold-leaf electroscope. Place an electrophorus cover upon the paraffined paper, thus making a condensing electroscope. Bring one of the battery terminals into contact with the disk of the electroscope, and the other terminal into contact with the disk of the electrophorus. Remove the wires, and lift the electrophorus cover and the paper from the disk, thus reducing the capacity of the lower disk and raising its potential. If you notice any evidence of an electric charge on the leaves of the electroscope, determine whether that charge is positive or negative.

2. Substitute dilute sulphuric acid for the water that was used as the exciting fluid in the "crown of cups" of Exercise 1, and complete the circuit through a resistance of several hundred ohms. By a wire, connect a point in this circuit near the terminal zinc with the disk of the electroscope, arranged as in Exercise 1. Similarly, connect a point in this circuit near the terminal copper with the electrophorus cover. Try to charge the electroscope as in Exercise 1. If you succeed, test the character of the charge, and record your conclusions as to a permanent difference of potential at different points in the circuit of a voltaic battery.

3. Attach a Geissler tube to the secondary terminals of an induction coil. Put the coil in operation, and notice the discharge through the tube, and the difference from its discharge through air. Measure the maximum length of the spark obtainable with the coil, and compare it with the length of the longest discharge that you can get through a Geissler tube.

Present a magnet pole to the Geissler tube, and notice the deflection of the discharge. Reverse the polarity of the primary of the induction coil. Notice that the discharge is now deflected in an opposite direction. Does this throw any light on the question whether the alternating pulses of the induction coil are of equal

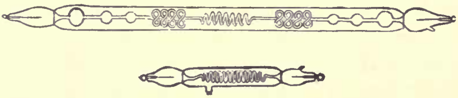


FIG. 409.

strength or not? Study the discharge through the tube with reference to the different appearances of the two ends of the tube. Reverse the coil, and notice the corresponding reversal in the positions of the violet tint and the scintillations. On reversing the coil, the lights in the tube reverse. If the tube was excited by a true alternate current, would such differences be noted?

4. Suspend a tin plate about 10 cm. square from each binding post of the secondary of a strong induction coil, as shown in Fig. 410. Let the plates hang parallel to each other and about 8 cm. apart. Start the coil. Darken the room, and hold a small Geissler tube in the electrostatic field of force between the plates, with the ends of the tube near but not touching them. The tube glows brightly. Touch the plates with the ends of the tube. Notice the increased brightness. Quickly lay the tube in a dark corner, and notice the after-glow.

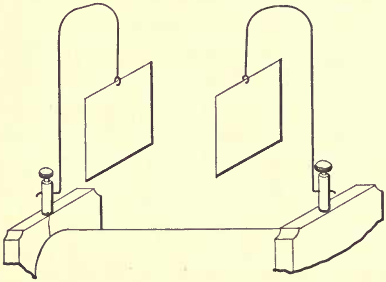


FIG. 410.

5. Grasp a 110-volt incandescence lamp firmly in the hand, keeping the fingers away from the brass cap. Let some one else charge the lamp with an electrophorous. Discharge the lamp by touching the brass cap, keeping an eye on the filament. When the discharge takes place, the filament swings around the bulb as if it were sweeping off the charge from the surface of the glass and delivering it to the cap. As there is danger of breaking the filament, it is well to use an old lamp. Repeat the experiment in the dark, and notice the brilliant glow of the lamp when discharging.

6. Paste strips of tin-foil on a microscope slide as shown in Fig. 411. Discharge the induction coil through these strips, and view the spark through the microscope with a $\frac{3}{4}$-inch or a 1-inch objective. Notice the general resemblance of the discharge to the discharge in a vacuum. Note the purple vaporous negative pole, and the scintillating positive pole, as shown in Fig. 412. Bring the pointed ends of the tin-foil strips on the slide as near together as you

FIG. 411.

can without contact. Focus the microscope well, using a $\frac{1}{4}$-inch or a $\frac{1}{5}$-inch objective. Be careful that the discharge does not enter the end of the objective instead of leaping the gap between the strips, a possible circumstance that would not injure you as much as it would your experiment. Watch the points through the microscope, turn on the current, and notice that the negative pole is rapidly eaten away. Quickly reverse the connections of the primary switch, and notice that what was the positive pole is now eaten away.

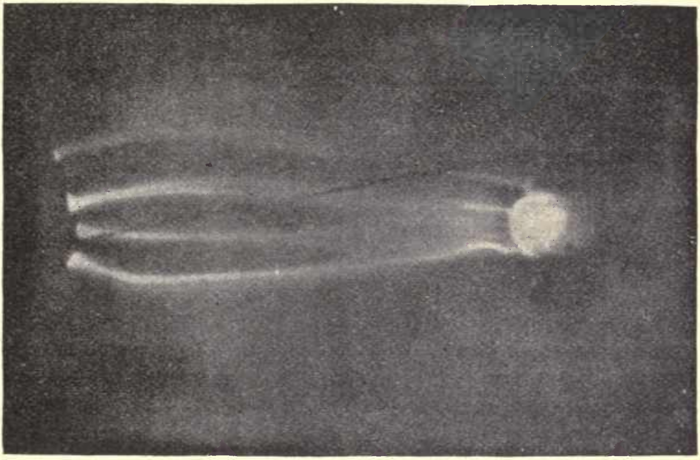


FIG. 412.

7. Connect the outside coating of a Leyden jar by a wire to one of the terminals of an induction coil. Bring the knob of the jar near the other terminal of the coil and allow sparks to pass between them for a minute. Remove the jar, and connect its two coatings with the fingers. A smart shock shows that the jar is charged. Bring the knob of the jar into contact with the free terminal of the coil instead of allowing the discharge to spark across. It will be found impossible thus to charge the jar. Why?

8. Support two metal balls, *a* and *b* (Fig. 413), between the terminals of an induction coil, put the coil in operation, and determine the limiting length of the discharge between the balls. Then connect a Leyden

jar to the terminals, as shown in Fig. 413. Start the coil again, and notice that the spark will not strike across so long a gap, but that it is a much hotter, "fatter" spark. Open the circuit at x, and insert the oil transformer of Experiment 376. It will be found to work in a satisfactory manner.

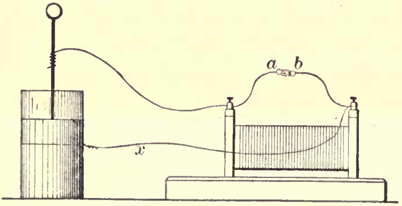


FIG. 413.

9. Devise a suitable experiment to determine whether the flame of a carbonaceous substance, e.g., rosin or pitch, or dry air is the better conductor for the high potential discharge between the terminals of the secondary of an induction coil. Describe the experiment in detail. To what conclusion does your experiment lead you?

10. On a thin hard rubber or glass tube 10 cm. long, wind evenly a layer of No. 16 insulated copper wire. Insulate it thoroughly, and wind on a secondary coil of ten layers of No. 30 insulated copper wire. Place the ends of the secondary wire above and below the tongue, or connect them to the terminals of a telephone receiver. Pass a battery current through the primary coil. On making and breaking the primary circuit, electric pulses are detected in the secondary circuit. Place an iron core in the tube. The pulses are much increased in intensity. Why? Slip a brass tube over the iron core, place it in the rubber tube, and repeat the experiment. The pulses are now of the same intensity that they were before the core was inserted. Slit the brass tube along its length, and repeat the experiment. The pulses are now as strong as they were before the tube was used. Remember that the induced current acts in direct opposition to the lines of force inducing it, and tends to neutralize them. Such an induced current flows through the brass tube which corresponds to a closed secondary coil, absorbing energy, and tending to neutralize the magnetic effect of the primary current.

III. ELECTRICAL MEASUREMENTS.

412. Electrostatic Units necessarily relate to quantity, potential difference, and capacity; they have already been defined. Since

Quantity = capacity × difference of potential,

any one may be calculated when the other two are known.

(*a*) For practical convenience, certain multiples and submultiples of these absolute C.G.S. units are in common use among electricians. Their names, and their values in absolute electrostatic units are as follows:—

Practical Units.

Denomination.	*Name.*	*Value.*
Quantity	Coulomb	3×10^9
Potential difference	Volt	$\frac{1}{3} \times 10^{-2}$
Capacity	Farad	9×10^{11}
Current	Ampere	3×10^9
Resistance	Ohm	$\frac{1}{9} \times 10^{-11}$
Work (volt-coulomb)	Joule	10^7 ergs
Activity (volt-ampere)	Watt	10^7 ergs per second

(*b*) Various methods have been devised for measuring electrostatic quantity, one of the simplest of which is with the Kinnersley electrical air-thermometer, shown in Fig. 414. When a spark passes between the balls within the larger tube, the confined air is expanded, and the liquid column in the smaller communicating tube rises, and thus approximately indicates the quantity of the charge.

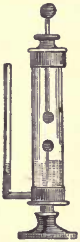


FIG. 414.

(*c*) Instruments for measuring differences of potential by electrostatic action are called *electrometers*. The gold-leaf electroscope is an electrometer when it is used to indicate equality of potential by equality of divergence of the leaves; or in that of two bodies dissimilarly electrified by bringing them into contact, and observing zero divergence. One of the best-known instruments of this class is Coulomb's torsion-balance, which consists essentially of a gilt ball, *i*, carried at the end of a horizontal shellac needle that is suspended by a fine silver wire from the top of a tube that rises from the cover of the enclosing glass cylinder. A vertical insulating rod passing through the cover carries a handle, *a*, and a gilt ball, *e*, at its ends. The tube is turned until *i* just touches *e*. When the ball, *e*, is electrified, it repels *i* through a certain angle. This angle of deflection is approximately proportional to the force of repulsion, and the force is proportional to the amount of electrification. As the capacity of the ball is constant, the charge that it receives must vary as the potential of the body by which it was charged.

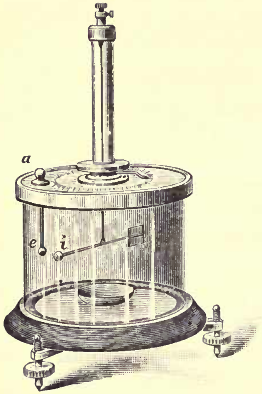


FIG. 415.

413. Electromagnetic Units constitute a system based on the electromagnetic actions of the current. The C.G.S. electromagnetic unit of quantity is the quantity which, flowing per second through a circular arc a centimeter in length and a centimeter in radius, exerts a force of a dyne on a unit magnetic pole at the center. The C.G.S. electromagnetic unit of quantity has a value 3×10^{10} times as great as the corresponding electrostatic unit, a ratio that represents the velocity of light.

(*a*) This ratio or its square applies to the other units, so that, for the practical units, we have the following values in absolute electromagnetic units: —

Coulomb	10^{-1}	Ampere	10^{-1}
Volt	10^{8}	Ohm	10^{9}
Farad	10^{-9}		

(*b*) The wonderful advance made in the last few years by electrical science is largely due to the adoption of definite electrical units, and the general practice of making exact electrical measurements.

414. The Galvanometer *is an instrument for determining the strength of an electric current* by means of the deflection of a magnetic needle around which the current flows. When a galvanoscope is provided with a scale so that the deflections of its needle may be measured, it becomes a galvanometer.

(*a*) The *astatic galvanometer* consists of an astatic needle supported by an untwisted fiber so that one of its needles is nearly in the center of the coil through which the current passes while the other needle is just above the coil. *When the deflections of the needle are less than* 10° *or* 15°, *they are very nearly proportional to the strengths of the currents that produce them.* A current that deflects the needle 6° is about three times as strong as one that deflects it 2°.

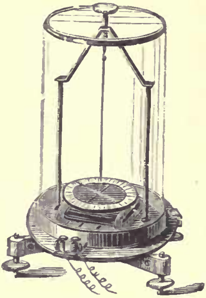


Fig. 416.

(*b*) The *tangent galvanometer* consists of a very short magnetic needle suspended so as to turn in a horizontal plane, and with its point of support at the center of a vertical hoop or coil of copper wire through which the current is passed. The diameter of the hoop or coil is not less than ten times the length of the needle. Owing to the shortness of the needle, pointers of aluminium or of glass fiber are generally cemented across it at right angles, as shown in Fig. 417. In use, the hoop is placed in the plane of the magnetic meridian, the current that is to be measured is sent through the hoop, and the deflection of the needle is read from the scale. *The strength of the current is proportional to the tangent*

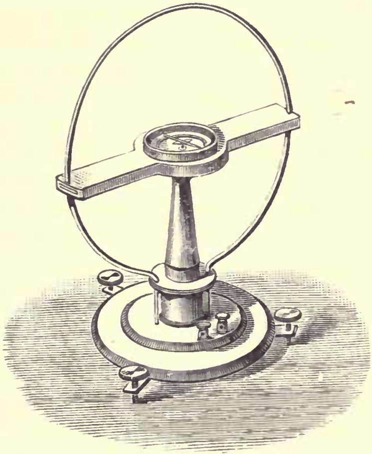


Fig. 417.

of the angle of deflection. For example, suppose that a certain current gives a deflection of 15°, and that another current gives a deflection of 30°. The amperes are not in the ratio of 15 : 30 but in the ratio of tan 15° : tan 30°. The values of such tangents may be obtained from a table of natural tangents. If a current of known strength, C, gives a deflection of m degrees, and another of unknown strength, x, gives a deflection of n degrees, the value of x may be found from the proportion, —

$$C : x :: \tan m : \tan n.$$

A table of natural tangents is given in the appendix.

(*c*) Any sensitive galvanometer, the needle of which is directed by the earth's magnetism, and in which the frame on which the coils are wound is capable of being turned round a vertical axis, may be used as a *sine galvanometer.* The coils are set parallel to the needle (i.e., in the magnetic meridian). The current is then sent through the coils, deflecting the needle. The coil is then turned until it overtakes the needle, which once more lies parallel to the coil. Two forces are now acting on the needle and balancing each other, viz., the directive force of the earth's magnetism, and the deflecting force of the current flowing through the coil. At this moment, *the strength of the current is proportional to the sine of the angle through which the coil has been turned.* The values of the sines may be obtained from a table of natural sines. Such a table is given in the appendix.

(*d*) The *mirror galvanometer* (Fig. 391) has a very short needle rigidly attached to a small concave mirror that is suspended by a delicate fiber in the center of a vertical coil of small diameter. A curved magnet, carried on a vertical support above the coil, serves to counteract the earth's magnetism, and to bring the needle into the plane of the coil when the latter does not coincide with the magnetic meridian, or to direct it within the coil. A beam of light from a lamp passes through a small opening under a millimeter scale about a meter from the mirror, falls upon the mirror, and is reflected back upon the scale. The curved magnet enables the operator to bring the spot of reflected light to the zero mark at the middle of the scale. A current passing through the coil turns the needle and its mirror, thus shifting the spot of light to the right or left of the zero point. The current through the galvanometer may be reduced by shunt to any desirable extent, thus extending the serviceable range of the instrument. The apparatus was devised by Sir William Thomson, now

Lord Kelvin, for use in connection with the Atlantic cable, and is exceedingly sensitive. The current produced by dipping the point of a brass pin and the point of a steel needle into a drop of salt water, and closing the external circuit through this instrument sends the spot of light swinging way across the scale.

(*e*) In the Deprez-d'Arsonval *dead-beat reflecting galvanometer*, a movable coil is suspended between the poles of a strong, permanent U-magnet that is fixed. The coil consists of many turns of fine wire the terminals of which above and below serve as the supporting axis. Within the coil is an iron tube that is supported from the back, and that serves to concentrate the magnetic field. The passage of a current turns the coil, and sets it so that its plane encloses a larger number of lines of force. This movement of the coil turns the mirror by means of which the angles of deflection are read with a telescope and scale. When the galvanometer is short circuited, the oscillations of the coil induce currents that quickly bring it to rest. It is simple in construction, and almost wholly independent of the magnetic field surrounding it.

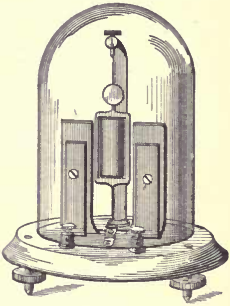


FIG. 418.

(*f*) In the *differential galvanometer*, the coil is made of two separate wires wound side by side. If two equal currents are sent through these wires in opposite directions, the needle will not be deflected. If the currents are unequal, the needle will be deflected by the stronger one with a force corresponding to the difference of the strengths of the two currents. It is much used in null methods of measurement.

(*g*) The *ballistic galvanometer* is provided with a heavy needle that has a slow rate of oscillation, and that is strongly magnetized and placed in a strong directing field. It is used for measuring currents that are variable in the time of measurement, and in measuring condenser capacity. The discharge causes the needle to give a sudden kick. *The quantity of electrification discharged and the capacity of the condenser are respectively proportional to the sine of half the angle of deflection of the needle.*

(*h*) A galvanometer of low resistance, empirically calibrated, i.e., graduated for the direct measurement of electric currents and giving its readings in amperes, is called an *ammeter*, or ampere-meter. Any galvanometer that is wound with wire of sufficient size safely to carry the current to be measured, and properly calibrated, may be used as an ammeter. Fig. 419 shows one of the common forms in practical use.

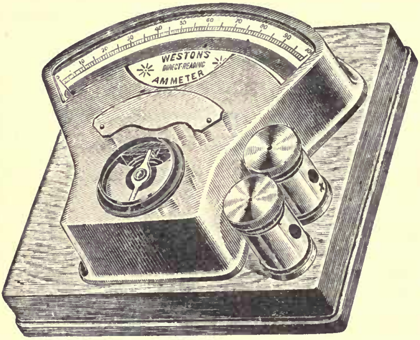


Fig. 419.

(*i*) An *electrodynamometer* is a galvanometer with two coils, at least one of which is movable, and through at least one of which the current to be measured passes. It is very useful in the measurement of alternating currents and voltages. Fig. 420 shows a form of the instrument that is much used in practice. The torque of the movable coil is resisted by a spiral spring. The deflection caused by the current is indicated on a properly graduated dial, and from such indications the current or voltage is computed. Such an instrument calibrated for direct currents measures the square root of mean square values of alternating currents.

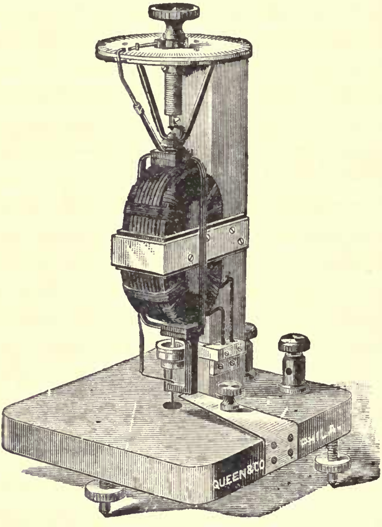


Fig. 420.

(*j*) If a galvanometer is put in shunt circuit between two points of different potentials, current will pass through it, and the current thus passing may be used to measure the difference of potential. A galvanometer of high resistance, calibrated so as to indicate in volts the difference of potential between its terminals, is called a *volt-meter*. The resistance must be high so as to reduce the shunted current, for, if the shunted current is large, the difference of potential between the terminals will be lowered, a change of the very function that is to be measured. In general, a volt-meter is a galvanometer properly calibrated, and wound so as to have a resistance of from 5,000 to 50,000 ohms.

(*k*) If the movable coil of an electrodynamometer is made of very fine wire suitable for voltage measurement, and the stationary coil of coarse wire suitable for current measurement, and the high-resistance coil is put in parallel with the main circuit, and the low-resistance coil is put in series, the apparatus may be used to measure, in watts, the rate of working, or the electrical activity of the current. Such a device is called a *wattmeter*.

(*l*) Electric current being a merchantable commodity, it is often desirable to measure both the rate at which the electrical energy is delivered and the time that it is delivered, i.e., the number of watt-hours. This is accomplished by a modification of the wattmeter. The current swings an armature coil with complete revolutions in the field of a stationary coil. These revolutions are counted by a registering apparatus that gives direct readings in watt-hours. The tendency of the armature to turn is directly proportional to the current. The rotary motion of the armature is retarded by a copper disk that revolves between magnet poles as shown in Fig. 421. The motion of the disk-conductor in the magnetic field develops in the disk eddy or local currents that produce the required drag or brake on the revolving armature.

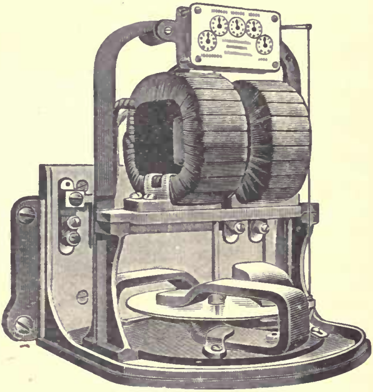


FIG. 421.

(*m*) The resistance of a galvanometer should correspond to that of the rest of the circuit; i.e., a high resistance galvanometer should be used on a high resistance circuit, and vice versa.

415. Resistance Coils are made of wires of known resistance for use with galvanometers in measuring resistances.

Insulated and doubled wires are wound upon spools, and the terminals of each spool connected to heavy brass blocks, *A*, *B*, *C*, etc., on the top of the box that carries the spools. This style of winding destroys the magnetic effects, and reduces the self-induction of the coils. When the brass plugs are inserted, as shown in Fig. 422, the coils are short-circuited, i.e., practically, the whole of a current passing from block to block goes through the plug, but when a plug is withdrawn the current passes through the corresponding coil. Such coils with resistances of 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500 ohms, etc., severally are connected to form a resistance-box as shown in Fig. 423. By withdrawing the proper plugs, one may throw into the circuit any resistance desired, from a single ohm up to the full capacity of the box.

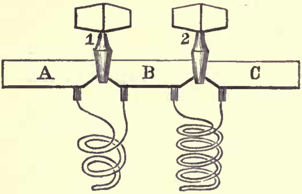


FIG. 422.

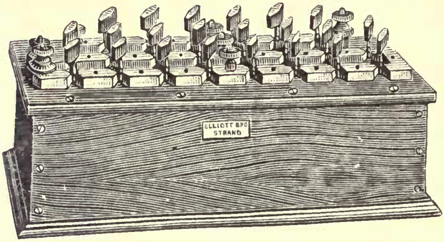


FIG. 423.

(*a*) Formerly, German-silver wire was generally used for resistance coils because its resistance is high and little influenced by temperature. "Platinoid" wire, an alloy of German-silver and tungsten, is now generally used for this purpose, as it is much harder, has a much higher specific resistance, and a lower temperature coefficient, and is not expensive.

416. The Measurement of Resistance is done in several ways according to the nature and magnitude of the resistance. Much use is made of the following important principle: *The fall of potential between two points on a conductor is proportional to the resistance of the conductor between those points.*

(*a*) In Experiment 305, we observed a certain deflection of the galvanoscope with a wire of unknown resistance in the circuit. By removing such an unknown resistance, and inserting known resistances until the same deflection of the same galvanoscope with the same cell is obtained, we may determine the resistance of the wire first used. This method is called resistance measurement by *substitution.* Its chief defect arises from the variation in the power of the cell.

(*b*) The method explained above may be used with any galvanometer of sufficient sensitiveness, but with a tangent galvanometer, the process may be shortened. Suppose the tangent galvanometer and an unknown resistance, *R*, to be included in the circuit, as in Fig. 424, and that the deflection is *a* degrees. Substitute for *R* any known resistance, *r*, which gives a deflection of *b* degrees. Since the strengths of the currents are proportional to *tan a* and *tan b* respectively, the resistance, *R*, may be calculated by the inverse proportion:

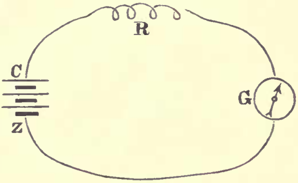


FIG. 424.

$$\tan a : \tan b :: r : R.$$

(*c*) Another method is to divide the circuit into two branches so that a part of the current flows through the given resistance and round one set of coils of a differential galvanometer, the other part of the current being made to flow through known resistances and then round the other set of coils in the opposing direction. When we have matched the unknown resistance by an equal known resistance, the currents in the two branches will be equal, and the needle of the differential galvanometer will show no deflection. With an accurate instrument, this method is very reliable. When the instrument is not known to be accurate, a better way is to balance the given resistance with other resistances, known or unknown. Then substitute known resistances for the given resistance until the deflection of the galvanometer again is *nil*. Compare § 132 (*b*).

(*d*) The method that has the most general application is that known as the *Wheatstone bridge*. In Fig. 425, we have a quadrangle of resistances. The four conductors, *m*, *n*, *p*, and *x*, that form the sides are called the "arms;" the conductor that joins *C* and *D* and carries the galvanometer, *G*, is called the "bridge." The current divides at *A*, and reunites at *B*. The fall of potential through *n* and *x* is evidently the same as the fall through *m* and *p*. The resistances of the arms may be so adjusted that, when the bridge-circuit is closed at *K*, there will be no deflection of the needle of *G*. Under such circumstances, *C* and *D* are at the same potential, and it may be shown that the resistances of the four arms "balance" by being in proportion, thus: —

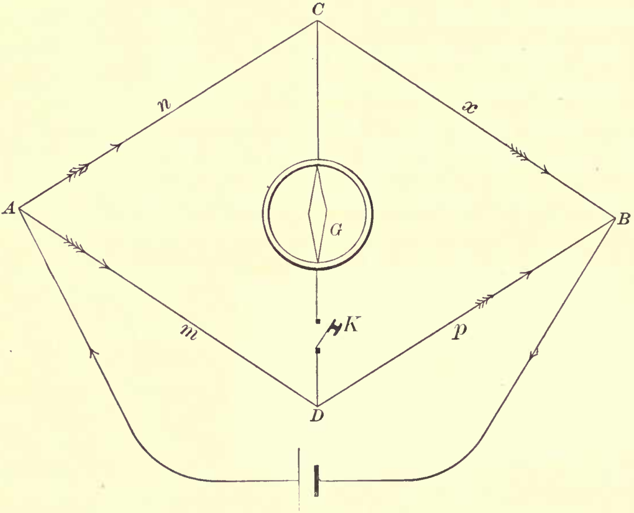


FIG. 425.

$$m : n :: p : x.$$

When three of these resistances are known, the other one may be calculated, or if the ratio of $m : n$, and the value of p are known, the value of x may also be determined. Under the conditions described, it is also true that $m : p :: n : x$.

The practical working of this method may be illustrated as follows: Suppose that x is the resistance to be determined. The other three arms are built up from the coils of a standard resistance-box. The arms, m and n, are called the "balance-arms," and are so cut into the circuit by the removal of plugs that the ratio between their resistances is decimal, as 10 or 100, thus simplifying the solution of the proportion. To illustrate, suppose that the resistance of m is 10 ohms; that of n, 100 ohms; and that of p, 15 ohms, as represented by Fig. 426. Then the resistance of x is 150 ohms. If the resistance of n is 10 ohms; that of m, 100 ohms, and that of p, 464 ohms, the resistance of x is 46.4 ohms. In using the bridge, the battery circuit should always be made by depressing the key, k, before K, the key of the galvanometer branch, is depressed. This avoids the sudden "throw" of the galvanometer needle, in consequence of self-induction, when the circuit is closed.

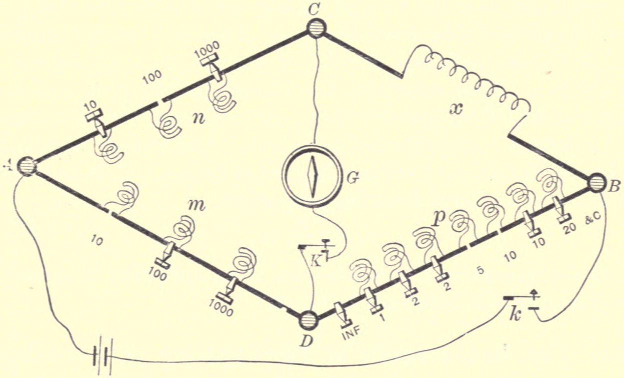


FIG. 426.

417. The Measurement of Internal Resistance.

— The best way of determining the internal resistance of a voltaic cell is to join two similar cells in opposition to one another, so that they send no current of their own. Then measure their united resistance (as if it were the resistance of a wire) as just described. The resistance of one cell will be half that of the two.

418. The Measurement of E.M.F., or of difference of potential, is generally made with a volt-meter, or by comparison with the E.M.F. of a standard cell.

(*a*) Represent the known E.M.F. of a Daniell cell by E, and that of the given cell or battery by x. Connect the given cell to the galvanometer, and note the number of degrees of deflection that it produces. Represent this deflection by a. Then add enough resistance, R, to bring the deflection down to b degrees (e.g., 10 degrees less than before). Then substitute the Daniell for the given cell in the circuit, and adjust the resistances of the circuit until the galvanometer shows a deflection of a degrees, as at first. Add enough resistance, r, to bring the deflection down to b degrees as before. E, R and r being known, x may be found from the proportion,

$$r : R :: E : x,$$

because the resistances that produce an equal reduction of current are proportional to the electromotive forces.

419. The Measurement of the Capacity of a condenser is accomplished by placing the given condenser, K, a standard condenser, K', and known resistances, r and r', in the arms of a Wheatstone bridge, as shown in Fig. 427.

(*a*) By pressing the key on the stop, a, the current flows through the point, B, and charges the condensers, the greater quantity going to the condenser of greater capacity. The resistances, r and r', are adjusted so that there is no deflection of the needle at G. When the key is pressed on the other stop, c, there is a rush of current from the condensers that is retarded by the resistances, r and r'. Any change or inequality of the voltage of the condensers is instantly shown by the galvanoscope. Assuming that the capacity of K is less than that of K', its voltage will fall more rapidly unless the resistance, r, is greater than r'. Since capacity equals quantity divided by potential,

$$K = \frac{Q}{V},$$

it follows that if the resistances are adjusted so that the voltages are the same for both (i.e., no deflection at G) K will be directly proportional to Q. As the condensers discharge in the same time, their capacities are proportional to the currents sent through r and r'. As these currents have the same E.M.F., each is inversely proportional to the resistances through which it flows. Hence

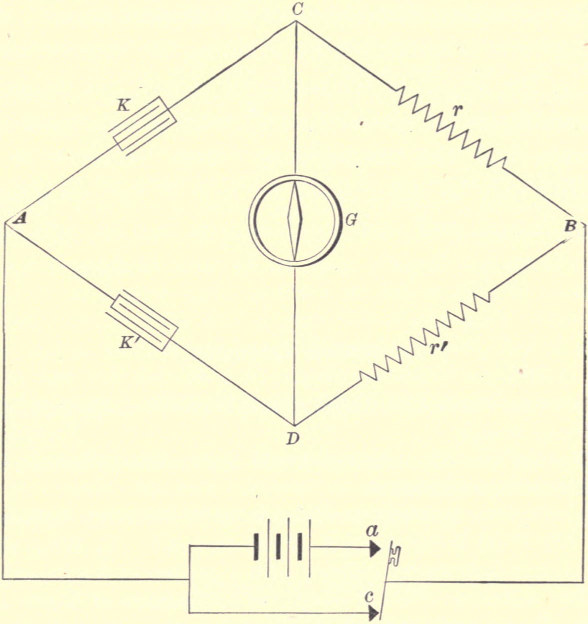


FIG. 427.

$$K : K' : : r' : r.$$

Three of these values being known, K is easily determined.

420. The Measurement of Magnetic Functions.—Magnetic flux is measured directly in webers by the use of a little exploring coil of many turns of fine wire connected to a ballistic galvanometer. The quantity of the current generated by suddenly jerking the exploring coil from the magnetic field is thus measured, and from this the number of lines of force is calculated. Magnetomotive force is usually calculated directly from the ampere-turns. Magnetizing force is calculated by dividing the magnetomotive force by the length of the magnetic circuit in centimeters. Permeability is calculated directly from the quotient of the induction and the magnetizing force. Curves of permeability may be determined for iron at various stages of magnetization, and each kind of iron has its own curve. Such curves are largely used in dynamo design.

CLASSROOM EXERCISES.

1. Draw an illustrative diagram, and deduce the formula for the Wheatstone bridge.

2. Explain why an ammeter should have a low resistance, and a volt-meter a high resistance.

3. A volt-meter that has a resistance of 26,000 ohms indicates 37 volts. (*a*) What is the strength of the current? (*b*) What voltage would such an instrument indicate with a current of 3 milliamperes?

4. Two volt-meters, one of which has a resistance of 25,000 ohms, and the other a resistance of 15,000 ohms, are connected in series across 110 volts. (*a*) What current flows through the system? (*b*) What voltage does the first instrument indicate? (*c*) the second instrument?

Ans. (*a*) 0.00275 amperes; (*b*) 68.75 volts; (*c*) 41.25 volts.

5. I have a volt-meter that measures up to 15 volts and that has a resistance of 3,500 ohms. I want to use it on a 115-volt circuit, and, therefore, put it in series with a resistance of 24,000 ohms. With the two thus connected, and with a voltage of 110, (*a*) What is the current strength? (*b*) What is the indication of the volt-meter? (*c*) By what must the reading of the volt-meter be multiplied to get the actual voltage? Suppose the voltage to be increased to 117. (*d*) What is the reading of the volt-meter? (*e*) Is there any change in the multiplier used to give the correct voltage?

6. What resistance must be put in series with the volt-meter of Exercise 5, so that the multiplier shall be 10? *Ans.* 31,500 ohms.

7. How many watts is taken by a station volt-meter that indicates 110 volts and uses a 0.002-ampere current?

8. I have an ammeter that indicates milliamperes up to 100. It has a resistance of 6 ohms. I desire to put it on a circuit that I know to have a current of 6 or 7 amperes. As the instrument will not safely carry more than 0.1 of an ampere, I put it in a shunt, as shown in Fig. 432. What must be the resistance of *R*, the other branch of the circuit, so that the instrument shall have a multiplier of 100; i.e., so that a current of 6.5 amperes will produce a reading of 65 milliamperes? Evidently, with such a current and with the shunts properly adjusted, 0.065 of an ampere will pass through the milliammeter, and 6.435 amperes through *R*.

9. At an electric light station, I am called upon to measure the resistance of one of the field-magnet coils of a dynamo. I have a volt-meter and an ammeter, and a small dynamo (the exciter of an alternator) that will furnish a 15-ampere current at any desired voltage from 150 to 225. With this outfit, how shall I measure the resistance of the coil?

10. Draw a sketch of the connections of a series dynamo, and show how you would arrange a volt-meter to measure the voltage necessary to force the current through the field magnets.

LABORATORY EXERCISES.

Additional Apparatus, etc. — Three Daniell cells; galvanoscopes and galvanometers; volt-meter; ammeter; resistance-box; rheochord as described below; current reverser; 2m. of fine platinum wire; double connector; wires of unknown resistance; Wheatstone slide-bridge as described below.

1. Solder one end of a piece of No. 20 insulated copper wire, 50 cm. long, to one end of a piece of zinc 10 × 2.5 × 0.5 cm., and amalgamate the zinc. Solder a similar wire to a piece of sheet copper 10 × 10 cm. Put the zinc into a porous cup 4 or 5 cm. in diameter and 10 cm. deep, and fill the cup to the depth of 8 cm. with dilute sulphuric acid. Put the copper plate into a glass vessel 7 or 8 cm. in diameter and 10 cm. deep, bending it slightly to fit the inner surface of the tumbler. Put the porous cup and its contents into the glass vessel, and fill the latter to the depth of 8 cm. with a saturated solution of copper sulphate.

Connect the terminals of this Daniell cell with the terminals of a low resistance galvanoscope, and record, at intervals of 5 minutes for half an hour, the deflections of the needle. Ascertain whether the current strength is practically constant after the porous cup is wet through.

2. Make a resistance frame (rheochord) as follows: Nail two uprights, each 25 × 3 × 1 cm. to the edge of a plank 100 × 10 × 4 cm., and screw the ends of a meter stick to the upper ends of the uprights, as shown in Fig. 428. Set two small metal binding-posts at *a*, and another pair at the same level at *b*. Set similar binding-posts, in pairs, at *c*, *d*, and *e*. Connect the base of the inner post at *a* with the base of the inner post at *b* by No. 30 German-silver wire. It is well to solder the wire to the posts. Lead a similar wire around the outer edges of the uprights, and connect its ends to the outer posts at *a* and *b*.

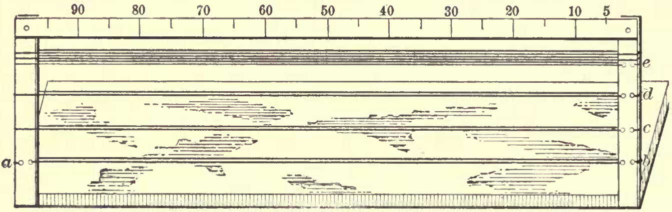


FIG. 428.

From one of the posts at *c*, lead a similar wire around both uprights to the other post at *c*. In like manner, connect the posts at *d* by No. 28 German-silver wire. In like manner, connect the posts at *e* by 10 turns of insulated No. 30 copper wire, laying the wire on carefully so as to prevent the current from leaking across from one turn to the next (short-circuiting). All of the posts should be firmly fixed, and the wire should be drawn tight.

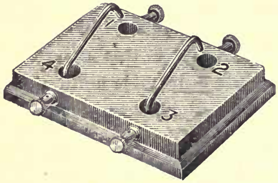


FIG. 429.

Near each corner of a wooden block about 10 cm. square, bore a centimeter hole, about a centimeter deep, and number the holes in succession, 1, 2, 3, and 4. These holes are for mercury cups. Set sharp little spikes at the corners of the opposite face of the block, to fix it to the table wherever it is placed. Set metal binding-posts at the corners of the block so that the screw of each penetrates to one of the cups. Provide two stout copper wires, amalgamated at their ends, and bent so that they may connect any of the holes with either of the two adjoining holes. When mercury is placed in the cups, the block constitutes a "current reverser" or mercury commutator.

Connect diagonally opposite mercury cups (as 1 and 3) of the commutator to the terminals of a Daniell cell. From one of the other cups (as 2), lead a wire to one of the terminals of a low resistance galvanoscope, and connect the other terminal of the galvanoscope to one of the binding-posts (at *b*) of the rheochord. From the other binding-post at *b*, lead a wire to the fourth mercury cup. The galvanoscope should be at least a meter from the other parts of the apparatus so that its needle may be unaffected by them, and the wires leading to and from it should lie close together. Connect the two binding-posts at *a* by a short, stout copper wire. Complete the circuit by placing the two bent copper wires so that one of them shall connect cups 2 and 3, while the other connects 1 and 4. Trace the direction of the current through the galvanoscope. Change the bent wires of the commutator so that one of them connects 1 and 2, while the other connects 3 and 4. Trace the direction of the current through the commutator. By this time, the porous cup of the cell will probably be wet through, and the current nearly constant.

While the current is flowing through 200 cm. of No. 30 German-silver wire, record the deflection of the needle of the galvanoscope; reverse the current and record the deflection; record the average of the two deflections. With a copper wire or a double connector like that shown in Fig. 430, short circuit the two wires near *a*, so that the current shall flow through 180 cm. of the German-silver wire. Record the three deflections as before. Make similar successive records for 160, 140, 120, 100, 80, and 60 cm. of the German-silver wire, ending with the record for 200 cm., taken again to be sure that the current has not fallen off while the measurements were in progress. Tabulate all of your records, and notice whether they indicate, in a general way, any dependence of electrical resistance upon the length of the conductor.

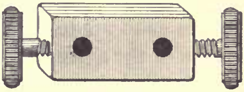


FIG. 430.

3. Using the apparatus arranged for Exercise 2, change the connections at the rheochord from *b* to *d*, so as to put the 200 cm. of No. 28 German-silver wire into the circuit. Record the deflection; reverse

the current, and record the deflection; and record the average of the two deflections. Compare this average with the averages obtained in Exercise 2 for the several lengths of No. 30 wire, and estimate the length of the latter that has a resistance equal to that of the 200 cm. of No. 28 wire. Carefully measure the diameters of the two wires, and compute the ratio between the areas of their cross-sections. Determine the relation between cross-section and resistance.

4. Using the apparatus arranged for Exercise 3, change the connections from *d* to *c*, and connect the two binding-posts at *b* by copper wires to the two binding-posts at *d*, so as to provide for the current, two equal parallel branches of No. 30 German-silver wire. Record the deflections before and after reversal, and their average, as before. Estimate the length of No. 30 wire, as used in Exercise 2, that has a resistance equal to that of the two 200-cm. pieces in multiple arc as used in this exercise.

5. Determine the length of No. 30 German-silver wire that has a resistance equal to that of 20 m. of No. 30 copper wire.

6. Wind into spiral coils two equal lengths (e.g., 100 cm.) of fine platinum wire, and put them into the arms of a Wheatstone bridge. Balance the bridge. Heat one of the spirals in a Bunsen or alcohol flame, and notice that the deflection of the needle indicates that the balance has been destroyed. While the spiral is still heated, balance (roughly) the bridge again, and determine whether the resistance of the wire was increased or decreased by the rise of temperature.

7. Place in the circuit of the Daniell cell of Exercise 1, a galvanometer, a set of resistance coils, and a conductor, *X*, of unknown resistance. Adjust the known resistances so that the needle shows a deflection of about 45°, and record the exact reading. Remove *X* from the circuit. Add known resistances to make the deflection the same as before. Repeat the work twice, adjusting the known resistances so as to produce deflections of about 43° and 47°, and take the average of the three totals of added resistance as the resistance of *X*. This is called the method by substitution.

8. To a table-top or other board, tack two stout metal strips, *AC* and *BD*, with a meter-stick between them, as shown in Fig. 431. Tack a similar metal strip, *EF*, 90 cm. long, in position as shown. Solder metal binding-posts at the ends of these strips, and at the middle of *EF*. The resistance of the strips is negligible. Tightly stretch a German-silver wire, No. 26, over the face of the meter stick, and solder it to the faces of the metal strips at *r* and *s*. One of the

terminals of a sensitive galvanoscope is to be connected to *EF;* the other galvanoscope wire is to make a sliding contact with the German-silver wire, dividing it into two variable parts, *m* and *p*. Put the apparatus into the circuit of a voltaic cell, as shown in the figure. Interpose a conductor of unknown resistance at *x* and a known resistance of approximately the same value at *n* (the better this guess at the equality of resistances, the less the liability of error in the results attained). You have a Wheatstone bridge, easily comparable to that shown in Fig. 425. Make the sliding contact at a point on *rs* that causes a deflection to the right, and note its position on the meter-scale; find a position that causes a deflection to the left. As the point of contact at which the bridge will balance is between these points, it is easy to locate it definitely. When the contact is made at such a point on *rs* that there is no deflection of the needle, read the values of *m* and *p*, directly from the meter-scale, and determine the resistance of *x*. Repeat the work with two slightly different values for *n*, and take the average of the three computed values of *x*.

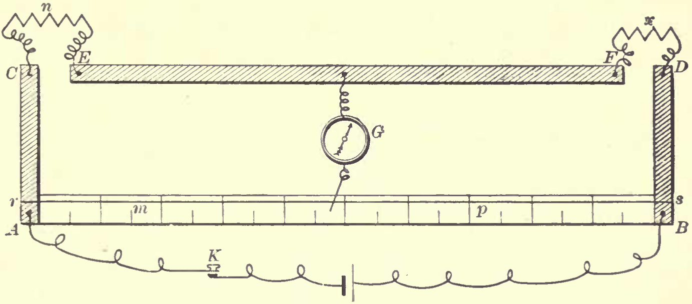


FIG. 431.

Note. — In practice, the galvanoscope should be placed at a distance from the rest of the apparatus, the connecting wires being kept near together.

9. Make a Daniell cell similar to that of Exercise 1, with cups of the same size but with plates of sheet metal and 10 × 0.5 cm. in size. Take care that the liquids are of the same depth in the two cells. Put the large-plate cell in circuit with a galvanoscope, inserting the commutator as in some of the preceding exercises. Set the plates as

far apart as possible, and record the deflections before and after reversal, and their average. Repeat the work with the plates as near together as possible. Repeat both tests with the small-plate cell. Put 200 cm. of No. 30 German-silver wire into the circuit, and repeat the work with the two cells in succession. Repeat these latter tests, using a galvanoscope of higher resistance. From your record, determine the effect of the size of the plates, and of the distance between the plates upon the current strength, and whether the addition of an external resistance has any effect upon the sensitiveness of the current to changes in the size and relative positions of the plates.

10. Replace the small plates of the cell described in Exercise 9 by plates like those of the cell of Exercise 1. Join the two like cells in parallel, and put into the circuit a galvanoscope, and 200 cm. of No. 30 German-silver wire. Record the deflections as previously directed. Make the tests with galvanoscopes of high and of low resistances. Repeat the tests with the cells joined in series. Remove the German-silver wire from the circuit, and repeat the tests. From your record, determine under what conditions it is better to join cells in parallel, and when it is better to join them in series.

11. Join the two Daniell cells of Exercise 10 in parallel, and substitute the battery for the unknown resistance, *x*, of Exercise 8. Remove the battery used in that exercise, or leave it open circuited at *k* (Fig. 426). Compare the resistance of the battery with that of the same cells in series, and with that of one of the cells.

Note.— The accurate measurement of the resistance of a cell on closed circuit is a difficult problem.

12. Place a volt-meter that indicates tenths of a volt in a shunt circuit around a resistance, *R*, as shown in Fig. 432. Assume that the resistance of the instrument is so high that the current shunted through it is negligible. Determine by computation the resistance of *R* when a 2-ampere current flowing through it gives a reading of 0.2 at the volt-meter. Try to verify your result experimentally. Notice that the readings of the volt-meter multiplied by 10 give the current

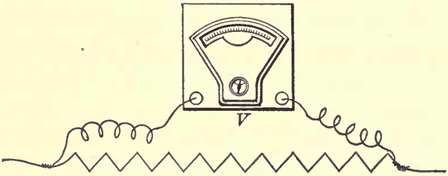


Fig. 432.

strength in amperes. Determine the value that should be given to *R* so that the volt-meter may be used as an ammeter giving direct readings in amperes.

Note. — Many ammeters are made on this principle of shunting a high resistance galvanometer.

IV. SOME APPLICATIONS OF ELECTRICITY.

Incandescence Lighting.

Experiment 382. — Place a few centimeters of No. 36 platinum wire across the terminals of a battery of several bichromate cells in series. The wire will be heated to incandescence, and may be melted. Lift one of the plates partly from the liquid, and notice the diminished brilliancy of the light emitted by the incandescent wire. By gradually lowering the plate into the liquid as the cells weaken, the brilliancy of the platinum wire may be kept nearly uniform. Notice the progressive oxidation of the wire. Try to continue the experiment until the wire breaks down by oxidation, noting the length of time taken. Repeat the work with No. 36 iron wire, and compare the lasting qualities of the two wires.

421. Incandescence Lamps operate essentially on the principle illustrated in Experiment 382, the current being sent through some substance that, because of its high resistance, becomes intensely heated and brilliantly incandescent. The only suitable substance known for such a resistance filament is carbon, which, carefully prepared and bent into a loop, is enclosed in a glass bulb from which the air is exhausted to prevent oxidation, i.e., combustion. At the best, the filament gradually deteriorates and finally breaks, thus ruining the lamp. The ends of the carbon filament are cemented to short plati-

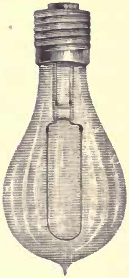


FIG. 433.

num leading-in wires that are imbedded in the glass by the fusion of the latter. These platinum wires are connected to the metallic fittings of the lamp in such a way that, when the bulb is screwed into its supporting socket, the connections are properly made. Some such lamps are provided with turn-offs, for open-circuiting the lamp.

(*a*) As incandescence lamps are generally connected in parallel, as shown in Fig. 434, they require a heavy current at a comparatively low voltage. Such currents require large conductors that are generally made of copper. The "hot" resistance of the carbon filaments varies from about 25 to 250 ohms, according to the voltage of the current and the candle-power of the lamp. In what is known as "the Edison 3-wire system," two dynamos are connected in series, as shown in Fig. 435. The lamps on each side of the middle or neutral wire,

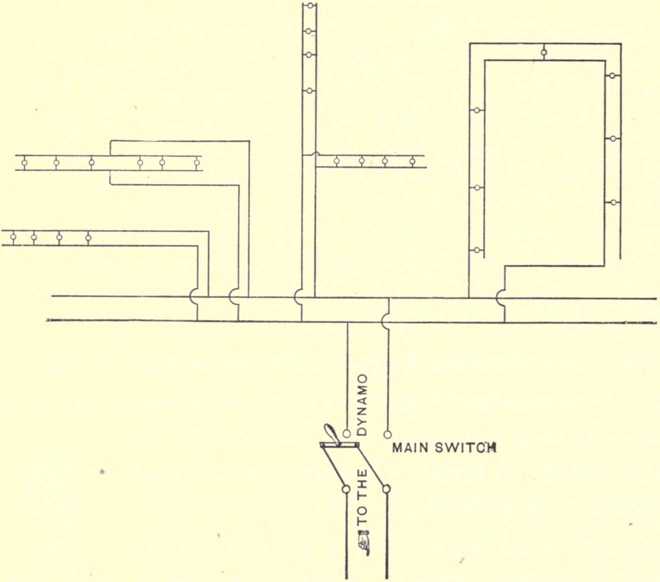


FIG. 434.

N, are made as nearly equal as possible in number and resistance. When they thus balance, *N* carries no current, and the voltage of the second dynamo is added to that of the first. Under such conditions, the effect is the same as if the lamps of each pair were in series, the doubled resistance being met by the doubled potential difference of the two generators. Doubling thus the voltage doubles the current, and quadruples the energy delivered. This enables a division of the area of cross-section of the line-wires by 4, and results in a saving of $\frac{5}{8}$ of the weight and cost of the mains. If lamps are turned out on one side of the middle main, current flows along *N* to supply the required excess.

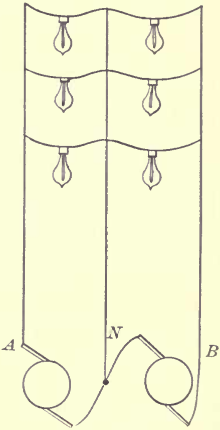


FIG. 435.

Note. — The arches at points where conductors cross each other in Fig. 434 indicate, in the conventional way, that the wires cross without contact. Dynamos are often represented by commutator circles and brushes as shown in Fig. 435.

(*b*) With lamps placed in parallel, the greater the number of the lamps in use, the less the resistance of the circuit. The current is usually operated at 110 volts, and each 16-c.p. lamp takes about 0.5 of an ampere. The expenditure is, therefore, nearly 3.5 watts per candle-power. Evidently, an increase of current will increase the number of watts expended in the lamp, and the quantity of light produced. The greater the number of watts expended, the higher the temperature of the filament, and the greater the efficiency of the lamp. But excessive temperatures weaken the filament and shorten its time of service, so that in practice efficiency is sacrificed to some extent for the sake of a greater durability.

(*c*) The dynamos designed for direct use with such lamps are generally shunt- or compound-wound. As they must deliver currents that are of constant potential but of strength that varies with the number of lamps in use, some regulating device is necessary for the shunt-wound dynamo; the compound-wound dynamo is self-regulating.

(*d*) Incandescence lamps are often placed on the secondary circuit of a "step down" transformer, the primary circuit of which carries the high-voltage current of an alternator. The primary coils of several transformers may be put in series, as shown in Fig. 436, or in multiple arc, as shown in Fig. 437.

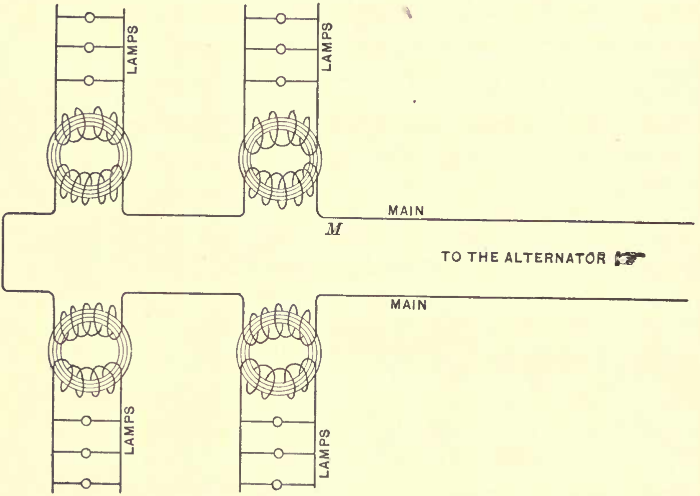


FIG. 436.

(*e*) When electric lamps are supplied by an electric lighting company, the customer sometimes pays a fixed rental per lamp per day, and sometimes a certain price per watt-hour for the current actually delivered. For this purpose, a wattmeter, like that described in § 414 (*l*), is often used; the Edison meter depends upon the electrolytic action of a part of the current that is shunted for that purpose. See § 429 (*a*).

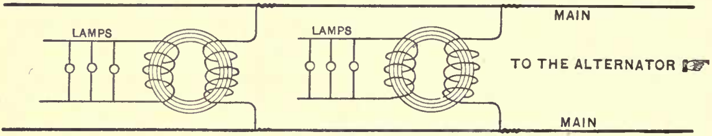


FIG. 437.

Caution. — In experimenting with an incandescence electric lighting current, remember that a low resistance placed across the mains will receive an enormous current. Many a galvanoscope and other piece of apparatus has been ruined in this way. Never "ground" an electric lighting wire.

The Electric Arc.

Experiment 383. — Connect one of the terminals of the battery to a small file, and draw the other terminal along the rough surface. A series of minute sparks is produced as the circuit is rapidly made and broken. When such a luminous effect is larger and more lasting, the band of light between the terminals is called an *electric arc*.

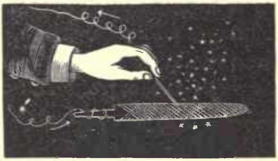


FIG. 438.

Experiment 384. — Connect one end of a 3-pound coil of insulated copper wire, No. 20, to one of the mains of a direct current, incandescence lighting circuit, and the other end to a short piece of No. 6 copper wire. Connect another piece of No. 6 copper wire to the other main. Bring the free ends of the No. 6 wires into contact, and slowly separate them. A flashing arc will follow the wires for a short distance and then break. Bring the wires into contact again, separate them, and try to maintain the arc. Notice that the arc is tinted green by the vapor of the copper. The wires will become hot and their ends will be melted.

Experiment 385. — Tip one of the terminals with a screw or other piece of steel, and connect the other terminal to a block of commercial zinc. Set up an arc between the steel and the zinc. The pyrotechnic effect is very striking. Replace the steel and zinc with two pieces of electric light carbon. Notice that the terminals are not melted, and that the light is a brilliant white. View the arc through a piece of smoked glass, and try to discover, from their appearance, which of the tips is the hotter.

422. The Voltaic Arc is the most brilliant luminous effect of an electric current. When carbon rods that form part of the circuit of a strong electric current are separated, as in Experiment 385, *their tips glow with a brilliancy greater than that of any other light under human control,* *and the temperature of the intervening arc is unequalled by that of any other source of artificial heat.*

(*a*) It is necessary to bring the carbons into contact to start the light. The tips of the carbons become intensely heated, and the carbon begins to volatilize. When the carbons are separated, the current passes through this intervening layer of vapor and the accompanying disintegrated matter which acts as a conductor of very high resistance. The intense heat of the voltaic arc is due to the conversion of the energy of the current and not to combustion; the arc may be produced in a vacuum where there could be no combustion.

(*b*) The constitution of the voltaic arc may be studied by projecting its image on a screen with a lens. Three parts will be noticed:

1. The dazzling white, concave extremity of the positive carbon.
2. The less brilliant and more pointed tip of the negative carbon.
3. The globe-shaped and beautifully colored aureole surrounding the whole.

(*c*) There is a transfer of matter across the arc in the direction of the current, the positive carbon wasting away more than twice as rapidly as the negative. Most of the light is radiated from the crater at the end of the positive carbon.

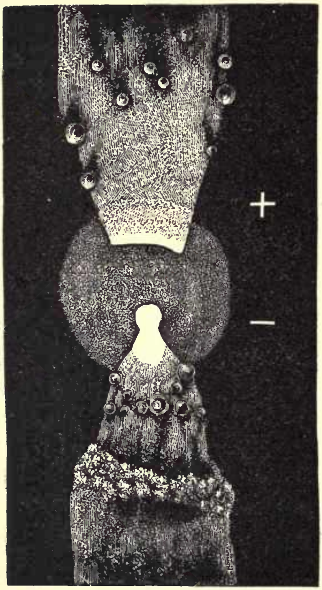


FIG. 439.

423. The Arc Lamp is essentially a device for automatically separating the carbons when the current is turned on, for "feeding" the carbons together as they are burned away at their tips, and, in some cases, for short-circuiting the lamp in case of irregularity or accident.

Such lamps of from one to two thousand candle-power require an expenditure, at the dynamo, of about three-fifths of a horse-power per lamp. A common form of the arc lamp is shown in Fig. 440.

(*a*) In the arc lamp as ordinarily supplied for commercial uses, the distance between the carbon tips is about $\frac{1}{8}$ of an inch. Such lamps require a current of from 9 to 10 amperes, and have a potential difference between the carbons of 45 to 50 volts. They are generally operated in series, so that the current passes in succession through all the lamps on the circuit. The resistance of the circuit is thus increased by the successive addition of lamps. As many as 125 arc lamps have been worked in series. The E.M.F. of the current required is often 3,000, and occasionally 6,000 volts. Such currents must be handled with care, as dangerous results might follow ignorance or neglect.

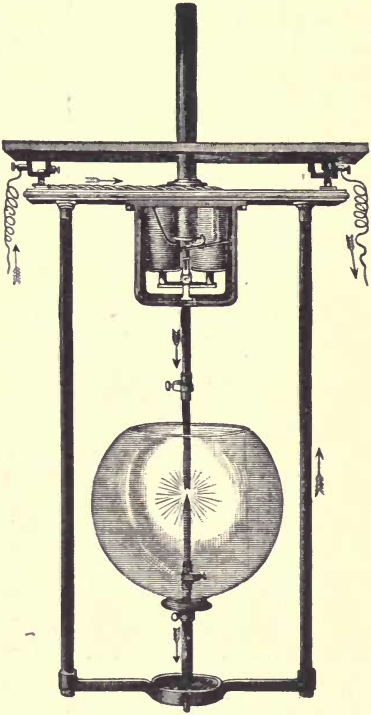


FIG. 440.

(*b*) The mechanism for separating and feeding the carbons consists chiefly of a clutch-washer, *w*, a clutch, *c*, and a solenoid or "sucking magnet," *S*, doubly and oppositely wound. One of these windings is of coarse wire in series with the carbons; the other constitutes a high resistance shunt across the arc. The two cores of the solenoid and their connecting yoke move freely up and down, under the alternating influence of magnetic attraction and gravity. At the start, the carbons are in contact. When the current is turned on, the series magnet lifts *c*; *c* lifts one edge of *w*, thus causing it to clutch and to lift the rod that carries the upper carbon. The arc being

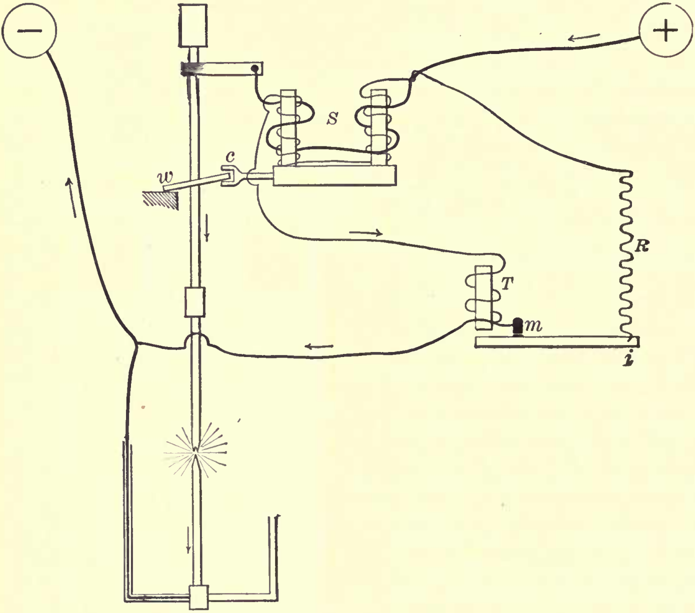


FIG. 441.

thus established, the greatly increased difference of potential between the arc terminals sends more current through the oppositely wound shunt circuit, thus weakening the lifting power of *S*. As the carbons wear away and the arc grows longer, the gradually increasing potential difference between the terminals of the arc gradually forces more current through the shunt winding of the solenoid, antagonizing the lifting effect of the series magnet until gravity pulls down the cores and the clutch. When *w* falls into a horizontal position, it releases its grip on the carbon rod and allows it to slip down, thus reducing the length of the arc, strengthening the current through the series coils, and reducing the current through the shunt coils. The clutch is immediately lifted and the fall of the carbon thus arrested. So delicately have these devices been adjusted that the feeding of the carbons is as imperceptible as the movement of the hour hand of a watch. The current of the shunt circuit may be made to traverse an electromagnet at *T*, so that when the arc becomes abnormally long, as it will if the carbon does not feed properly, an iron bar pivoted at *i* is attracted until it closes a short circuit at *m*. *R* represents a resistance properly adjusted. In some lamps, the carbons are separated at the start, the shunt magnet brings them into contact, and the series magnet separates them, thus establishing the arc. The shunt then feeds the carbons as before. Sometimes the "sucking magnet" has but a single core. Many search lights are made without any automatic mechanism, the carbons being fed by hand.

(*c*) Since arc lamps are operated in series, any particular lamp that is to be extinguished must be short-circuited. The dynamo is provided with appliances for maintaining a uniform current strength regardless of the number of lamps in use on its circuit. When the number of lamps is increased, the voltage of the dynamo is correspondingly increased. The connections of a system of arc lights are diagrammatically shown in Fig. 442, in which *m* and *n* represent short-circuiting or cut-out switches; *C*, the commutator of the dynamo; *F*, the field-magnet coils; and *R*, a regulating resistance. Any lamp may be cut out by closing a switch at *m*. By closing the switch at *n*, the current is diverted from the field magnets. This destroys the magnetic field and, of course, destroys the current. When *n* is open, the

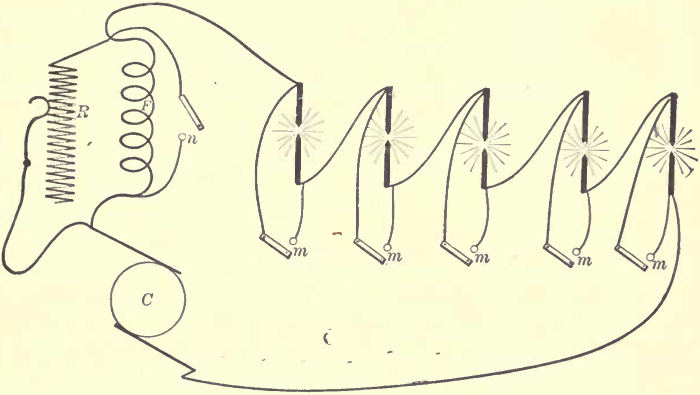


FIG. 442.

proportion of the current that is sent through F (and, consequently, the strength of the magnetic field) may be regulated by the resistance that is thrown into the shunt circuit at R.

(*d*) Some arc lamps are made to operate on incandescence circuits at constant potential. They are extinguished by open-circuiting them. Incandescence and arc lamps are often operated from central lighting stations.

Experiment 386. — Separate the terminals of a bichromate cell in illuminating gas escaping from an ordinary burner. It will be difficult to make the spark light the gas. Interpose a large, low-resistance electromagnet in the circuit, and renew the attempt. Explain the increased magnitude of the spark. Try to light the gas with a spark from the electrophorus; from an induction coil; and from a static electric machine.

424. Electric Gas Lighting is often effected by sparks from the interrupted circuit of a voltaic battery, in which circuit is a "kicking coil," as illustrated in Experiment 386, or by sparks from the secondary of an induction coil, or from a machine for the generation of static electricity. Burners like that shown in Fig. 443 are connected in series in such a circuit.

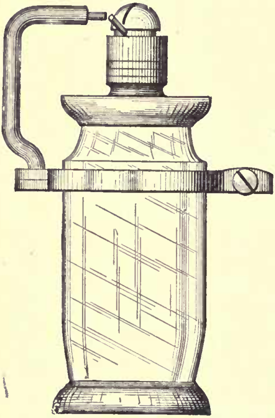


FIG. 443.

425. Electric Welding has become a common application of the electric current. A suitable transformer changes an alternating current of high voltage into a current of many amperes, the small electromotive force of which is adequate for the low resistance of the metals to be welded. It is found that when metals thus heated are welded, the union is unusually firm and perfect. When a weld so made is finished off with machine-tools, the line of union cannot be detected by the eye. Railway tracks are sometimes made continuous by this process.

Electric Motors.

Experiment 387. — Fasten 4 iron strips to the face of a wooden cylinder 4 cm. long and 6 cm. in diameter, parallel to the axis of the cylinder, and at equal distances from each other. Support the axle so that, as the cylinder turns, the iron strips will pass near the end of an electromagnet, as shown in Fig. 444. When the cylinder is rotated, a square nut on its axle acts as a cam, forcing the vertical spring to the right, and closing an electric circuit at the tip of the set-screw, *s*, four times for each revolution. The metal support that carries *s* is connected to the binding-post, *b*. The metal support that carries the vertical spring is connected to one terminal of the electromagnet, the other terminal of which is connected to the binding-post, *a*. The nut is set so that the circuit is broken at *s* just as one of the iron strips on the face of the cylinder comes to the end of the core of the magnet. The momentum of the rotating cylinder carries it over the dead point until the next corner of the nut forces the vertical spring into contact at *s*. The apparatus will be improved by placing a small fly-wheel at the other end of the axle. All of the metal parts, except the core of the magnet, and the 4 strips on the cylinder, would better be made of brass. Place this apparatus in the circuit of several cells joined in series, and set the cylinder in rotation; adjust the position of the nut if necessary to secure a continuous motion.

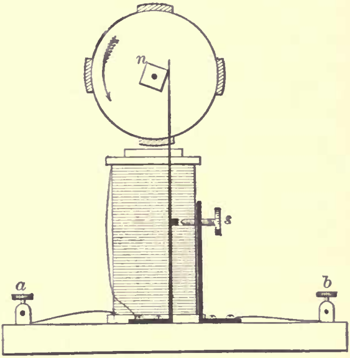


FIG. 444.

Experiment 388. — Connect a small battery-motor (one may be bought for a dollar or less) to a number of cells joined in series, and interpose a low-resistance galvanoscope as indicated in Fig. 445. Hold the shaft of the motor to prevent its rotation, and note the reading of the galvanoscope. Then permit the motor shaft to revolve, and again note the reading of the galvanoscope. The resistance of the circuit seems to be greater when the armature is in motion than when it is at rest.

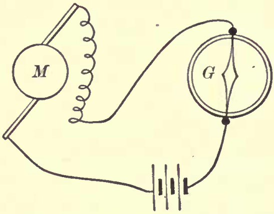


FIG. 445.

426. An Electric Motor *is a device for doing mechanical work at the expense of electric energy.* As made for industrial use, it is generally similar to the dynamo in form and construction, and is often identical with it. The current from a dynamo, perhaps miles distant, is sent through the armature of the motor (the binding-posts of one machine being connected to the binding-posts of the other), and causes the motor armature to revolve in a direction opposite to that in which it would revolve if the motor was acting as a dynamo. This assumes that the motor is series wound. It thus appears that the motor is based upon the principle of the reversibility of the dynamo. The pulley on the armature shaft is belted or geared to other machinery.

(*a*) When the armature of a dynamo revolves in the magnetic field, the motion develops a magnetic field for the armature coils that acts in opposition to that of the field magnets, thus opposing the motion of the armature and acting as an addition to friction, etc., as a drag or counter torque. Hence, the transformation of mechanical foot-pounds into electrical watts. Conversely, when a current is sent through the armature of a dynamo or motor at rest, the opposition between the magnetic field of the field magnets and the magnetic field of the armature coils produces a repulsion that causes the rotation of the armature. Hence the transformation of watts into foot-pounds. Any direct-current dynamo will act as an efficient motor when it is supplied with a current of the same strength and potential as that which it yields when acting as a dynamo.

(*b*) The E.M.F. of the inverse current generated in the armature acts in direct opposition to the E.M.F. of the direct current. Representing the E.M.F. of the inverse current by *e*, Ohm's law, as applicable to this case, is as follows: —

$$C = \frac{E - e}{R}.$$

Evidently, it is not well to turn the whole voltage of the actuating current suddenly upon a motor; the full current might do injury to the motor before its armature could acquire sufficient speed to produce the inverse E.M.F. (*e*) that is necessary to reduce the current to a safe magnitude.

(*c*) Electric motors are made in great variety of form, and for almost countless purposes. In our cities and large villages, they are placed on the circuits of powerful dynamos at central "power houses," that correspond to electric lighting stations, and that are often identical with them. The convenience, cleanliness and economy of the electric motor have led to its common use for the operation of light machinery, such as fly and ventilating fans, sewing machines, lathes, printing presses, etc. On the larger scale, the motor is used for the propulsion of street cars, and is even displacing the locomotive engine on some railways. As a generator and as a motor, the dynamo is revolutionizing more than one department of the industrial world.

427. An Electric Bell consists mainly of an electromagnet, *E* (Fig. 446), and a vibrating armature that carries a hammer, *H*, that strikes a bell. One terminal of the magnet coils is connected to the binding-post, and the other terminal to the flexible support of the armature. The armature carries a spring that rests lightly against the tip of an adjustable screw at *C*. This screw is connected to the other binding-post. The bell is connected to a battery of 2 or 3 cells in series, a key, a push-button, *P*, or some other device for closing the circuit being placed in the line.

(*a*) When the circuit is closed by pushing the button at *P*, the magnet attracts the armature and causes the hammer to strike the bell. This movement of the armature breaks the circuit at *C*. *E*, being thus demagnetized, no longer attracts its armature, which is thrown back against the end of the screw by the elasticity of the spring that supports it. It is then again attracted and released, thus vibrating rapidly and striking a blow upon the bell at *H* at every vibration (see § 403, *a*). The rapidity of the strokes depends largely upon the length of the pendulum-like hammer (see § 115).

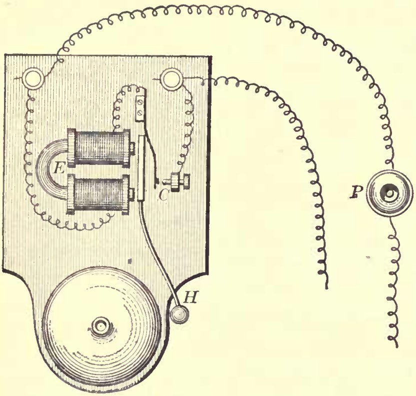


FIG. 446.

428. A Fire Alarm Box contains a crank which the person who "turns in" the alarm is to pull down once. This motion of the crank winds up a spring that drives a train of wheel-work that puts in revolution a make-and-break wheel. The circumference of this wheel has a series of notches arranged to correspond to the number of the box; e.g., if the number of the box is 371, there will be three notches, separated by a longer space from a succession of seven notches, which are followed at a similar distance by one notch. This notched wheel and an arm that presses upon the circumference of the wheel are in the circuit. When the wheel revolves, the circuit is broken as each notch passes under the arm. When the circuit is broken, an electromagnet at the fire station is demagnetized. The armature of the magnet is then drawn back by a spring, and strikes one blow upon a bell. A single revolution of the make-and-break wheel in the distant alarm box gives a succession of 3 strokes, 7 strokes and 1 stroke, thus indicating the number, 371. The wheel-work may turn the make-and-break wheel two or three times, thus repeating the alarm signal. The location of Box 371 being known, valuable time is saved in determining the vicinity of the fire.

Electrolysis.

Experiment 389. — Put a solution of sodium sulphate, or any other neutral salt, that has been colored with an infusion of purple cabbage into a V-tube about 1.5 cm. in diameter, and supported in any convenient way. Close the ends of the tube with corks that carry platinum wires terminating in narrow strips of platinum foil that reach nearly to the bend of the tube. Put this apparatus in the circuit of 2 or 3 cells joined in series. In a few minutes, the liquid at the positive electrode will be colored red, and that at the negative electrode, green. If, instead of coloring the solution, a strip of blue litmus paper is hung near the positive electrode it will be reddened, while a strip of reddened litmus paper hung near the negative electrode will be colored blue. These changes of color are chemical tests; the appearance of the green or blue denotes the presence of an alkali (caustic soda in this case), while the appearance of the red denotes the presence of an acid.

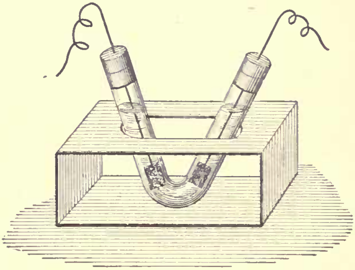


FIG. 447.

Experiment 390. — Arrange apparatus as shown in Fig. 448. The glass vessel may be made from a glass funnel, or by cutting the bottom from a wide mouthed bottle, and may be supported in any convenient way. The platinum electrodes should be about 2 cm. apart and covered with water (H_2O) to which a little sulphuric acid has been added to increase its conductivity. Fill two test-tubes with acidulated water, and invert them over the electrodes. When the circuit is closed, bubbles of oxygen escape from the positive electrode, and bubbles of hydrogen from the negative. The volume of hydrogen thus collected will be about twice as great as that of the oxygen. When a sufficient quantity of the gases has been collected, they may be tested; the hydrogen, by bringing a lighted match to the mouth of the test-tube, whereupon the hydrogen will burn; the oxygen, by thrusting a splinter with a glowing spark into the test tube, whereupon the spark will kindle into a flame. If the gases thus separated are mixed, and an electric spark produced in the mixture, the ions will recombine with explosive violence.

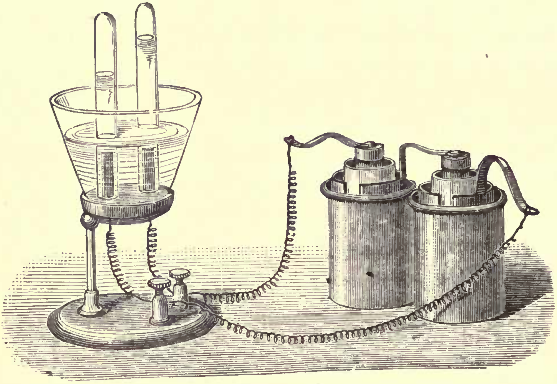


FIG. 448.

Experiment 391. — Repeat Experiment 389, placing a solution of copper sulphate instead of sodium sulphate in the V-tube. When the circuit is closed, copper will be deposited upon one of the electrodes, while oxygen will be evolved at the other. Reverse the current, and notice the disappearance of the copper from the electrode where it was first deposited. Copper may be dissolved from the platinum foil with nitric acid if desirable.

Experiment 392. — Melt some tin, and pour the melted metal slowly into water. Dissolve some of this granulated tin in hot hydrochloric acid, and add a little water. Into this hot solution of tin chloride, introduce electrodes made of tinned iron (tin-plate). Pass the current of the battery joined in series through the liquid, and notice the remarkable tree-like growth of tin crystals. Modify the experiment by successively using solutions of lead acetate, and of silver nitrate.

429. Electrolysis, etc. — The decomposition of a chemical compound, called the *electrolyte*, into its constituent parts, called *ions*, by an electric current is called *electrolysis.* When, for example, water is electrolyzed, the hydrogen collects at the negative electrode, called the *cathode;* such an ion is called a *cation*, and is said to be *electropositive.* The oxygen similarly collects at the positive electrode, called the *anode;* such an ion is called a *anion*, and is said to be *electronegative.*

(*a*) In battery or in electrolytic bath, the metallic or electropositive ion is carried with the current through the electrolyte. Similarly, when a chemical salt is electrolyzed, the metallic base is carried to the cathode, while the acid constituent appears at the anode. The amount of chemical decomposition effected in a given electrolytic bath in a given time is proportional to the current strength. This principle has been utilized in devices for the commercial measurement of electric energy.

Experiment 393. — Fasten a copper wire to a silver coin, and a similar wire to a piece of sheet copper of about the same size. Suspend the two pieces of metal in a tumbler containing a solution of copper sulphate. Connect the wire that carries the silver to the negative terminal of a strong battery of cells joined in parallel, and the other wire to the other terminal. Close the circuit, and notice that a firm, hard copper coating is deposited upon the silver. Reverse the current until the copper is removed from the silver. Then connect the cells of the battery in series, and notice that copper is deposited upon the silver as a spongy mass instead of a firm coating.

430. Electrometallurgy is the art or process of depositing certain metals, such as gold, silver and copper, from solutions of their compounds by the action of an electric current. Its most important applications are electroplating and electrotyping. In electroplating, an adherent film of metal is thus deposited on some other material (strictly speaking, on metallic substances only). In electrotyping, the metallic film deposited in the bath is not adherent.

(*a*) For plating with gold, a solution of the cyanide of gold is generally used; for plating with silver, a solution of the cyanide of silver is generally used.

(*b*) The most common forms of electrotypes are copies of medals, jewelry, silverware, woodcuts, and pages of composed type. The metal most used is copper. The form to be copied is molded in wax; the face of the mold is dusted with powdered plumbago, in order to make it a conductor; the mold thus prepared is immersed in a solution of copper sulphate, and subjected to the action of a current as illustrated in Experiment 393. When the copper film is thick enough (say as thick as an ordinary visiting card), it is removed from the mold, and strengthened by filling up its back with melted type-metal. The copper film and the type-metal are made to adhere by means of an alloy of equal parts of tin and lead. The copper-faced plate thus produced is an exact reproduction of the form from which the mold was made. When used for printing, it is more durable than the type from which it was copied.

(*c*) Current for electrometallurgical processes is generally provided by specially constructed dynamos of low voltage. Such dynamos are called electroplating machines, or simply *platers*.

Secondary Cells.

Experiment 394. — Arrange apparatus as in Experiment 390. After the passage of the current for a few minutes, disconnect the battery and put a galvanoscope in its place. The deflection of the needle shows that the "water voltameter" is developing an electric current, and illustrating the reversibility of electrolytic action.

Experiment 395. — Fit neatly to a large tumbler two pieces of sheet lead as large as can be used without contact between them. To each lead plate, solder a copper wire about 50 cm. long. Fill the tumbler with dilute sulphuric acid. Connect the wires to the terminals of a high-resistance galvanoscope, and see if there is any deflection of the needle. Free one of the lead-cell wires from the galvanoscope. Put the lead-cell and the galvanoscope in series in the circuit of a battery of several cells joined in series. Note the deflections of the galvanoscope needle for several minutes. Quickly throw the battery out of the circuit, and connect the lead-plate cell with the galvanoscope. Note the deflection of the needle, and the direction and permanency of the current now flowing from the lead-plate cell. Open and close this circuit to make sure that the deflection of the needle is caused by a current from the lead plates, and not by any sticking of the instrument.

431. A Secondary or Storage Battery is a combination of cells each of which consists essentially of two plates of metallic lead coated with red oxide of lead, and immersed in dilute sulphuric acid. Sometimes the oxide is plastered on the roughened surface of the lead, and sometimes it is packed in little pockets made in the lead plates for that purpose. When such a cell is "charged" by passing an electric current through it, the electrolysis of the liquid liberates oxygen and hydrogen. One of these ions peroxidizes the coating of one of the plates; the other ion reduces, i.e., deoxidizes, that of the other plate, thus storing up chemical energy to be given back as an electric current when the poles of the charged cell are connected, and the chemical action is reversed. Such a cell or battery is often called an *accumulator*.

(*a*) In a charged secondary battery, the two plates are unlike, and the potential energy of chemical separation is converted into the kinetic energy of an electric current, just as with an ordinary or "primary" battery. When a secondary battery has run down, the passage of a current through it will restore the plates to their former effective condition; when a primary battery has run down, a current will not thus restore the plates. Thus, the great advantage of the secondary cell lies in the fact that no costly materials are consumed, the lead and acid being as useful at the end of the operation as at the beginning, and the coal consumed for the operation of the dynamo that delivers the charging current being much less expensive than the zinc that is consumed in the primary battery. Owing largely to the mechanical weakness of the lead plates, the storage battery has not yet proved the commercial success that was expected a few years ago.

(*b*) The secondary cell has a low internal resistance, and an E.M.F. of about 2 volts.

(*c*) The condition of the plates of a charged secondary cell is closely analogous to that of the polarized plates of a primary cell. The ions have a tendency to reunite by virtue of their chemical affinity, and thus to set up an opposing E.M.F., as was illustrated in Experiment 371. In the electrolysis of water, this E.M.F. is about 1.45 volts. Consequently, an E.M.F. of more than 1.45 volts is necessary for the decomposition of water.

Telegraph.

Experiment 396. — Connect two telephone receivers, two batteries and two keys as shown in Fig. 449. Both batteries are on open circuit. When the key is depressed at 2 or 3, and thus raised at 1 or 4, clicks will be heard at *T* and *R*. Trace the path of the current in each case. It would be easy to devise a code of signals for communication with such apparatus between two distant stations.

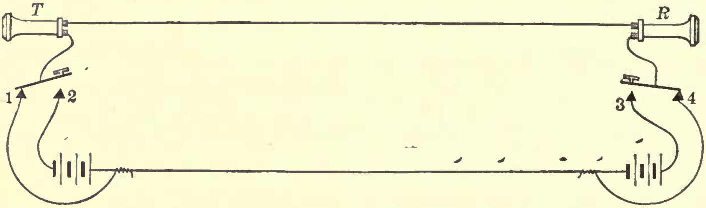


FIG. 449.

Experiment 397. — Support a metal cylinder, *C*, upon an axle. Pivot a metal bar at *a* (Fig. 450) so that the style, *s*, at its other end may rest upon the cylinder. Connect battery wires to the axle of the cylinder, and at *a*, interposing a key, *K*. Make a paste by boiling starch in water. Dissolve about 3 g. of potassium iodide in 3 or 4 cu. cm. of hot water, and add a little of the paste. Prepare a long ribbon of white paper, and soak it in the starch and iodide solution. While the paper is moist, fasten one end of it to a spool, *S*, and turn the handle so as to draw the paper between the style and cylinder. While the paper is moving over the surface of *C*, make and break the circuit at *K* so as to inscribe a series of blue dots and dashes on the paper at *s*. With *K* at one station and *s* at another, it would be easy for a person at *K* to send a dot and dash message to a person at *s*. Consult the code of signals given on page 564, and, with your apparatus, write the word *Morse*.

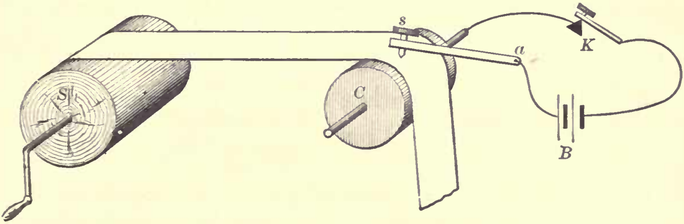


FIG. 450.

Experiment 398. — Arrange a line between two stations as shown in Fig. 451, using galvanoscopes, *G* and *G′*, instead of the telephone receivers used in Experiment 396. Each of the keys consists of two metal springs (e.g., 1 and 2), which are fastened to a board at one end, passing under the metal strip, *n*, with which they are in contact, and over the metal strip, *m*. When any of these keys is depressed, it makes contact with *m* or *m′*, and breaks contact with *n* or *n′*. When 1 is depressed, the needles at *G* and *G′* will be turned in one direction; when 2 is depressed, the needles will be turned in the other direction. Simi-

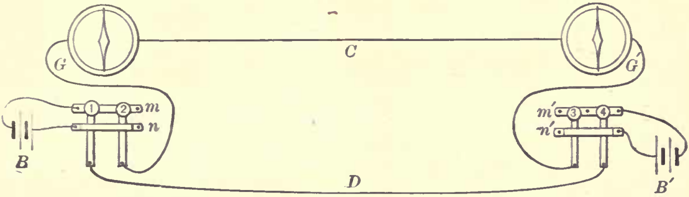


FIG. 451.

larly, depressing 3 or 4 will cause opposite deflections of the needles. Trace the course of the current for a depression of each of the four keys. Call a deflection of the needle in one direction a dot, and a deflection in the opposite direction a dash, and signal the word *Kelvin.*

432. The Electromagnetic Telegraph is a device for transmitting intelligible messages at a distance by means of interrupted electric currents. It consists essentially of a line-wire or main conductor ; a battery or dynamo for the generation of the current ; a transmitter or key ; and an electromagnetic receiving instrument. The use of the telegraph on a commercial scale is chiefly due to S. F. B. Morse of New York. The system devised by him about 1844 is still in general use.

(*a*) The Morse code of signals is as follows : —

LETTERS, ETC.			FIGURES.
a - —	*k* — - —	*u* - - —	1 - — — -
b — - - -	*l* ——	*v* - - - —	2 - - — - -
c - - -	*m* — —	*w* - — —	3 - - - — -
d — - -	*n* — -	*x* - — - -	4 - - - - —
e -	*o* - -	*y* - - - -	5 — — —
f - — -	*p* - - - - -	*z* - - - -	6 - - - - - -
g — — -	*q* - - — -	*&* - - - -	7 — — - -
h - - - -	*r* - - -	, - — - —	8 — - - - -
i - -	*s* - - -	? — - - — -	9 — - - —
j — - — -	*t* —	. - - — — - -	0 ——

To prevent confusion, a small space is left between successive letters, a longer one between words, and a still longer one between sentences, thus : —

H	e	w	i	l	l	c	o	m	e	a	t	t	e	n	.
- - - -	-	- — —	- -	—	—	- - -	- -	— —	-	- —	—	—	-	— -	- - — — - -

(*b*) The line-wire is most commonly made of iron, coated with zinc or copper. It connects the apparatus at the several stations and is carefully insulated. When the stations are far distant from each other, the ends of the line-wire are connected to large metallic plates buried in the earth (see Fig. 457), or otherwise "grounded." Whether the earth is considered as a conductor, so that the part between the grounded plates constitutes the return part of the circuit, or as a great "reservoir of electricity" from which current is drawn at one end of the line and into which current is discharged at the other, its use greatly reduces the cost and the resistance of the circuit.

(*c*) The battery generally consists of many gravity cells joined in series. A dynamo is often used instead.

(*d*) The transmitter or key is a current interrupter manipulated by the operator. It consists essentially of a metal lever, *L*, pivoted at *aa*, and connected to the line by the screw at *m* which is insulated from the base, and the screw at *n* which is connected to the base and lever. When the handle of the lever is depressed, a platinum point on the under side of the lever makes contact with another platinum point at *e*, and closes the circuit. The motion of the lever is limited by a set-screw at its further end. When the key is not in use, the switch, *s*, is turned for the purpose of closing the circuit at that point.

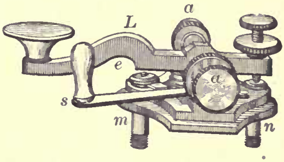


FIG. 452.

(*e*) The Morse register is represented in Fig. 453. The armature, *A*, is supported at the end of a lever, and over the cores of the magnet bobbins, *M*. A spring, *S*, lifts the armature when the cores are demagnetized on the breaking of the circuit by the operator at the key. When *A* is pulled down by *M*, a style or pencil at *P* is pressed against *R*, a paper ribbon that is drawn along by clock work. This style may be made to record upon the paper a dot-and-dash communication sent by the operator at a key, perhaps hundreds of miles away. Such registers are little used nowadays, most operators reading by sound, i.e., determining the message from the clicks of the magnet armature.

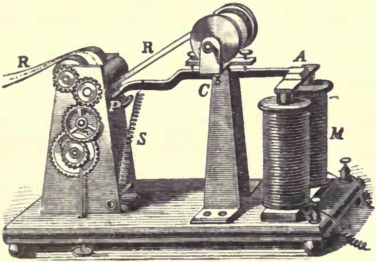


FIG. 453.

(*f*) In the Morse system, just described, a given wire can transmit only one message at a time. By what is known as the *duplex system*, a wire may be made to convey two messages, one each way, at the same time, without conflict. By what is known as the *quadruplex system*, a wire may be made to carry four messages, two each way, at the same time. The *multiplex system* enables the sending of six or more messages in the same direction at one time.

(*g*) Experiment 398 illustrates the action of a receiver often used with long *submarine cables*. The minute motions of the needle are rendered visible by the use of a mirror galvanoscope. (See Fig. 391.) There are several telegraphic-printing systems, the object of which is to print the message directly upon paper as it is received. *Fac-simile-telegraphy* has also been accomplished. In the so-called *rapid system*, the message is first prepared by punching a series of holes in a strip of paper, each perforation or group of perforations representing a letter. This strip of paper is rapidly passed under metal points connected with the line-wire. At each perforation, a point passes through the paper and closes the circuit. At the other end of the line, a band of chemically prepared paper is drawn rapidly under a style connected with the line-wire. The current that is interrupted at the sending station makes a series of stains on the prepared paper at the receiving station, as is illustrated in Experiment 397. As the transmission and recording are automatic, the messages may be sent in rapid succession.

Experiment 399. — Put a key and the apparatus shown in Fig. 392 in series in the circuit of a voltaic cell. Keeping the Morse alphabet in mind, try to signal one of the words similarly used in Experiments 397 and 398. Consider a short interval between two clicks to be a dot, and a longer interval to be a dash.

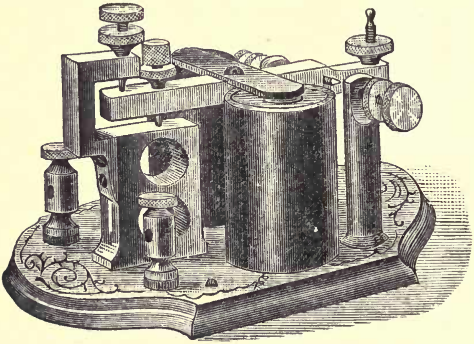


FIG. 454.

433. The Sounder is a telegraphic receiver consisting of an electromagnet, and a pivoted armature that plays up and down between its stops as the circuit is alternately made and broken. The message is "read by sound," i.e., from the clicks made by the armature, substantially as indicated in Experiment 399. A sounder generally has a resistance of from 3 to 5 ohms.

Experiment 400. — Fasten a wire to the apparatus used in Experiment 399, so that when the armature descends, the free end of the wire will be dipped into mercury in the cup at *c*. Arrange the apparatus as shown in Fig. 455, placing a sounder or a telephone receiver at *T*. As the key is worked at *K*, the secondary or "local" circuit is made and broken at *c*, and clicks are produced by the instrument at *T*.

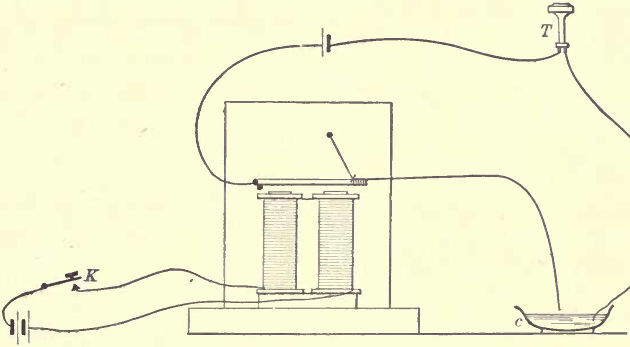


Fig. 455.

434. The Relay. — With a long main-line and many instruments in circuit, the resistance may be so great as to render the main-battery current so feeble that it cannot operate the sounder with sufficient energy to render

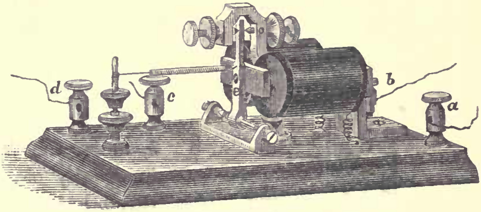


Fig. 456.

the signals distinctly audible. This difficulty is met by introducing a "local battery," and a "relay" at each station on the line. The relay (Fig. 456) is an electromagnet made of many turns of fine wire of which the terminals, *a* and *b*, are connected with the main line. This magnet operates an armature lever, *e*, the end of which strikes against a metal contact-piece and thus closes the local circuit through the terminals, *c* and *d*. The resistance of relays varies from 50 to 500 ohms. The "Western Union" standard relay has a resistance of 150 ohms.

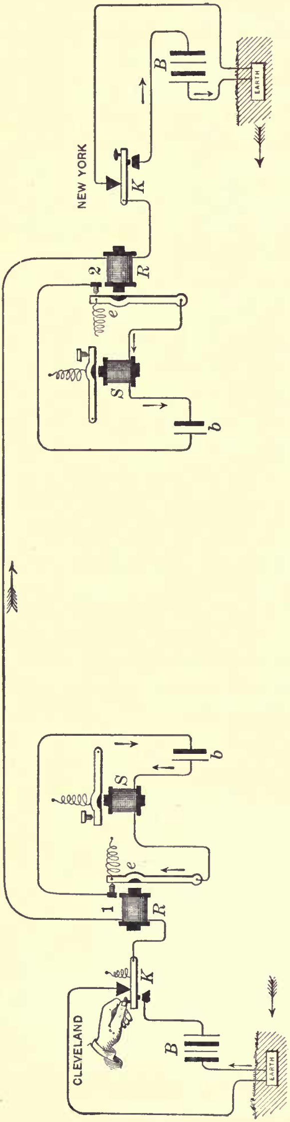


FIG. 457.

(*a*) The arrangement of instruments is best studied at a telegraph station, one or more of which may be found at almost any town or railway station. The general features of the "plant" are represented by the diagram shown in Fig. 457. The pupil will probably find the key, sounder and relay on a table, and the local battery, *b*, under the table. The keys being habitually closed, the current passes through all relays on the line, the current being continuous except when a message is being sent from some office. When an operator, in sending a message, opens his key, the breaking of the circuit demagnetizes the relays, and allows their springs to draw back the armature levers, *e*. This breaks each local circuit, and demagnetizes each sounder, the spring of which raises its armature. Things are now as shown in the diagram, which also represents the condition of affairs at every other station on the line. When a message is sent from any station, each relay lever, *e*, acts as a key to its local circuit, it and the sounder armature working in correspondence with the motions of the key at the sending station. Of course, the message may be read from any sounder on the line.

(*b*) If the local circuit at New York (see Fig. 457) is lengthened so as to reach thence to Boston, and the local battery, *b*, is increased to the size of a main battery, *B* (ground connections being made, of course), the relay at New York will transmit to Boston the message received from Cleveland. In such cases, the relay at New York becomes *a repeater*.

Experiment 401. — Put a telephone receiver in circuit with a battery and two electric-light carbon pencils as shown in Fig. 458. Vary the resistance of the circuit by pressing the points of the pencils together, and notice the harsh, grating sound heard in the telephone.

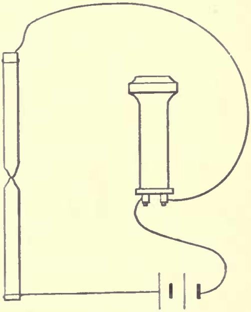


FIG. 458.

435. The Microphone *is an instrument for augmenting small sounds.* Its action is based on the fact that when substances of low conductivity are placed in an electric circuit, the resistance of the circuit is diminished by even a very small pressure.

(*a*) In one of the simplest forms of the microphone, a piece of charcoal, *b* (Fig. 459), is held between two other pieces of carbon, *a* and *c*, in such a way as to be affected by the slightest vibrations carried to it by the air or any other medium. The external pieces being put into circuit with a battery, and a telephone receiver held at the ear, "the sounds caused by a fly walking on the wooden support of the microphone appear as loud as the tramp of a horse."

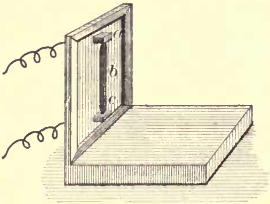


FIG. 459.

Experiment 402. — Wind 4 oz. of No. 22 insulated copper wire into a coil, *A*, that has an internal diameter of 1 cm., and connect its terminals to those of a telephone receiver (Fig. 379). Put a similar coil, *B*, in the circuit of a series battery of three or four voltaic cells. Lay *B* on a block of iron (e.g., a stove cover or a flat-iron), place *A* upon it, and insert a soft iron core vertically in the openings of the two coils. Place a similar iron bar on end, close to the coils and parallel with the iron core. Holding the telephone receiver to the ear, quickly lay a soft iron strip upon the upper ends of the two iron rods. A click will be distinctly heard in the telephone receiver. Modify the experiment by replacing the second iron rod by a piece of gas pipe of the same length, and with an internal diameter a little greater than the external diameter of the coils. Put the iron core and the two coils that it carries into the gas pipe, and stand the combination on end, close the magnetic circuit with a soft iron strip, and listen at the telephone as before.

436. The Telephone *is an instrument for the transmission of articulate speech to a distant point by the agency of electric currents.* The process consists essentially in the transmission of electric pulses that agree in period and phase with sound waves in air. These pulses, by means of an electromagnet, cause a plate to vibrate in such a way as to agitate the air in a manner similar to the original disturbance, and thus to reproduce the sound.

(*a*) The Bell telephone receiver (see Fig. 379) is a magneto-electric device, and is represented in section by Fig. 460. *A* is a permanent bar magnet around one end of which is wound a coil, *B*, of carefully insulated fine copper wire. The terminals of *B* are connected to the binding-posts at *D*. A soft, flexible sheet-iron disk or diaphragm, *E*, is held by a conical mouth-piece or ear trumpet across the face of *B*, near to but not quite touching the end of *A*. The outer case is made of wood, or of hard rubber.

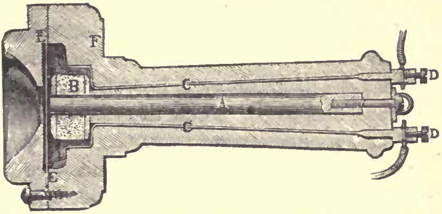


Fig. 460.

(*b*) When a person speaks into the mouth-piece, the sound waves beat upon the diaphragm and cause it to vibrate. The nature of these vibrations depends upon the loudness, pitch and timbre of the sounds uttered. Each vibration of the diaphragm modifies the magnetic circuit of the receiver, varying the lines of force that pass through *B*, and thus generating electric pulses in the wire when the circuit is closed. When *E* approaches *B*, a current flows in one direction; when *E* moves the other way, the current flows in the opposite direction. "It is as if the close approach and quick oscillation of the piece of soft iron fretted or tantalized the magnet, and sent a series of electrical shudders through the iron nerve."

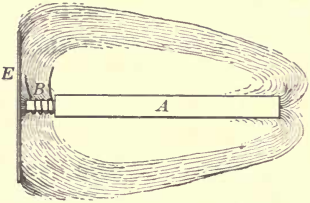


Fig. 461.

(*c*) The currents generated in a Bell telephone as just described may be sent through a similar instrument, at a distance of several

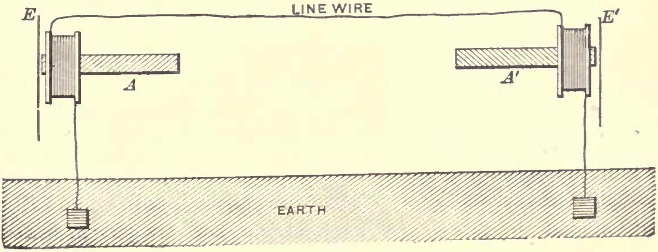


Fig. 462.

miles, perhaps. The earth may form part of the circuit, as shown in Fig. 462, but a return wire or complete metallic circuit is preferable. When the current generated at E flows in such a direction as to reinforce the magnet at A', the latter attracts E' more strongly than it did before. When the current flows in the opposite direction, it weakens the magnetism of A', which then attracts E' less. The disk, therefore, flies back. Thus, the vibrations of E' keep step with those of E, and produce sound waves that are very faithful counterparts of those that fell upon E. The sound thus produced at E' is feeble, but, when the receiving instrument is held close to the ear of the listener, the sound is clear, and the articulation remarkably distinct. Conversation may be carried on between moderately distant stations with this apparatus, no battery being necessary.

437. The Transmitter *is a microphone adapted for the transmission of telephonic messages* and, in general practice, is so used, better results being thus secured than is possible when the transmitter and receiver are identical in form.

(*a*) In the Blake transmitter, a diaphragm is supported back of a mouthpiece, as in the Bell telephone. Back of the center of the diaphragm is the point of a spring, *m*, that carries a small platinum ball that makes gentle contact with the diaphragm. Back of this is a spring, *n*, that is insulated from *m*, and that carries a carbon button, *B*, that rests lightly against the platinum ball. The ball, the button, and the primary of an induction coil are put in series in the circuit of a voltaic battery, *D*, as shown in Fig. 463. The varia-

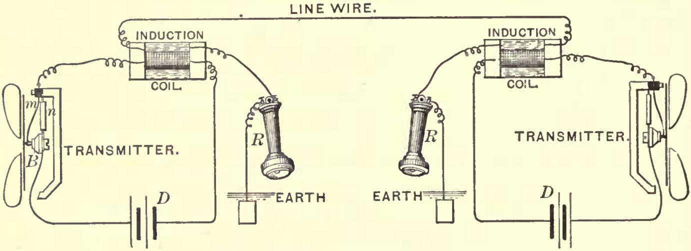


FIG. 463.

tions in the resistance of this circuit, caused by the varying pressure and surface contact between the platinum and the carbon, cause variations in the current that flows through the primary of the induction coil, and thus induce currents in the secondary of the coil. One of the terminals of the secondary is connected through a receiving instrument, *R*, to the earth, and the other terminal to the line-wire leading to another station, as diagrammatically indicated in Fig. 463. As previously suggested, a complete metallic circuit is preferable to the earth connections. The induced currents correspond closely to the pulsations produced in the primary current by the vibrations of the diaphragm of the transmitter, and in turn set up vibrations in the diaphragm of the receiver at the other station which, accordingly, sends off sound waves similar to those that disturbed the diaphragm of the transmitter. An electric bell, shown at *E* in Fig. 464, is placed at each station. It is rung by a small magneto at the sending station for the purpose of attracting the attention of the person at the receiving station. When the speaker or the listener lifts the receiver, *B*, from the hook that carries it, as shown in Fig. 464, the upward motion of the hook cuts the magneto and the bells from the circuit, and completes the connections substantially as shown in Fig. 463.

(*b*) The long distance transmitter, represented at *C* in Fig. 464, differs from the Blake transmitter chiefly in the use of a carbon that is granular instead of hard, and in the use of two or three cells instead of one.

(*c*) In most cities and villages, the telephones are connected by wires with a central station, called a telephone exchange. The exchange may thus be connected with hundreds of houses in all parts of the city, or even in different cities. Upon request by

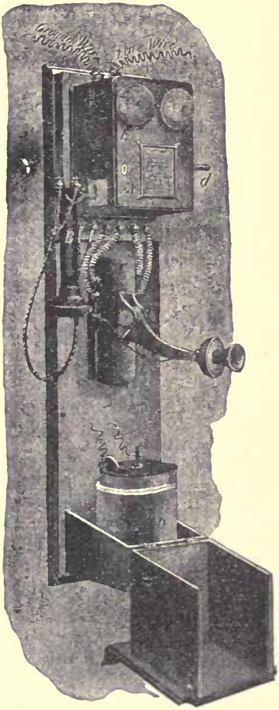


FIG. 464.

telephone, the attendant at the central station connects the line from any instrument with that running to any other instrument. Long distance telephony has been so nearly perfected that it is common to carry on conversation between places as far distant as Boston or New York and Chicago.

438. The Bolometer is an instrument designed for measuring very small variations in the intensity of radiant energy by the alteration of electrical resistance produced in a metallic conductor by changes of temperature.

(*a*) Two narrow and very thin strips of platinum or of iron are made two adjacent arms of a Wheatstone bridge, the other two arms being any convenient adjustable resistance. A delicate galvanometer is placed in the "bridge." The radiation to be measured is allowed to fall upon one of these strips, while the other strip is shielded from it by a screen. The absorption of the radiant energy by the strip increases the resistance of the strip, and thus destroys the balance and deflects the galvanometer needle. With such an instrument, changes of temperature as minute as a hundred-thousandth of a degree centigrade have been measured, and still smaller variations may be detected.

439. A Lightning Rod is a metallic conductor placed on a building as a protection from lightning. When an electrified cloud floats over a building, the latter is oppositely electrified by induction. The electrification of the building escapes from the pointed conductor, and tends to neutralize the electrification of the cloud. Its action may proceed too slowly to keep down the rapidly rising potential of the cloud and to prevent the disruptive discharge, but even then the rod tends to protect the building by offering a path of less resistance. The discharge does not always follow the path of least resistance, but the protection is probable. The discovery of the oscillatory character of the discharge has largely modified the character of the protection recommended.

(*a*) In lightning protection, the following facts should be kept in mind: the surface of the rod is of more importance than sectional area; iron is, at least, as good as copper; the upper end should have several branches terminating in sharp points that are plated, or otherwise protected from rust or corrosion; very little reliance should be placed on the so-called "area of protection"; a rod towering high above the roof is not as good as many smaller ones along the ridge of the roof; the conductor should be continuous and run to earth by the most direct path, avoiding sharp bends, and going deep enough to be sure of a good connection with a stratum that is always moist; owing to the high potential involved and the brief duration of the discharge, all possible paths should be provided, and metallic surfaces should be independently connected to earth; bury terminal earth-plates in damp earth or running water, and do not imagine that you can overdo the matter of making a good ground connection; insulators should not be used to hold the rod in position. Next to a poor "ground," the chief defects likely to occur are blunted points, and breaks in the continuity of the rod. For ordinary dwellings in city blocks (not unusually exposed) lightning rods are hardly necessary, but scattered houses, especially those built on hillsides, should be thus protected.

(*b*) According to Professor Barker, the cheapest way to protect an ordinary house "is to run common galvanized-iron telegraph-wire up all the corners, along all the ridges and eaves, and over all the chimneys, taking these wires down to the earth in several places and, at each place, burying a load of coke around the wire in order to establish at each point an efficient connection with the ground."

CLASSROOM EXERCISES.

1. A dynamo is feeding 16 arc lamps that have an average resistance of 4.56 ohms. The internal resistance of the dynamo is 10.55 ohms. What current does the dynamo yield with an E.M.F. of 838.44 volts?
Ans. 10.04 amperes.

2. If a wire about 18 inches long is attached to one electrode of a cell, and the other electrode is momentarily touched with the other end of the wire, a minute spark may be noticed at the instant of breaking the circuit. If the wire is bent into a scalariform or ladder-like shape

and the experiment repeated, the spark will be greater than before. If the form of the external circuit is again changed by winding the wire into a spiral (as shown in Fig. 465), the spark will be still greater. Explain the increase in the spark.

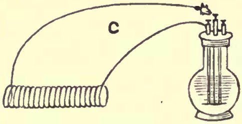


FIG. 465.

3. A dynamo is run at 450 revolutions, developing a current of 9.925 amperes. This current deflects the needle of a tangent galvanometer, 60°. When the speed of the dynamo is sufficiently increased, the galvanometer shows a deflection of 74°. What is the current developed at the higher speed? *Ans.* 20 amperes.

4. The current running through the carbon filament of an incandescence lamp was found to be 1 ampere. The difference of potential between the two terminals of the lamp was found to be 30 volts. What was the resistance of the lamp?

5. The hot resistance of a 16-c.p. incandescence lamp is 240 ohms. The lamp is placed in a 110-volt circuit. (*a*) What current flows through the lamp? (*b*) How many watts are expended in the lamp? (*c*) What is the consumption of energy per candle power?

Ans. (*a*) 0.46 ampere; (*b*) 50.6 watts; (*c*) 3.16 watts.

6. I want to place, in series, 10 incandescence lamps, each of 25 ohms resistance; the line-wire is to be 200 feet long and its resistance must be not more than 2 per cent. of the resistance of the lamps. Determine the size of wire that should be used. *Ans.* No. 23.

7. I want to place the same lamps abreast. The line-wire is to be 200 feet long and have a resistance of not more than 2 per cent. of that of the lamps. Determine the size of wire that should be used.

Ans. No. 4.

8. The resistance of the normal arc of an electric lamp is 3.8 ohms. The current strength is 10 amperes. What is the difference of potential between the carbon tips? *Ans.* 38 volts.

9. The resistance of the arc lamp above mentioned, when the carbons are held together, is 0.62 ohm. When it is burning with normal arc and a 10-ampere current, what is the difference of potential between the terminals of the lamp? *Ans.* 44.2 volts.

10. A dynamo has an E.M.F. of 206 volts, and an internal resistance of 1.6 ohms. Find the current strength when the external resistance is 25.4 ohms. *Ans.* 7.6 amperes.

11. A dynamo has an internal resistance of 2.8 ohms. The line-

wire has a resistance of 1.1 ohms and joins the dynamo to 3 arc lamps in series, each lamp having a resistance of 3.12 ohms. Under such conditions, the dynamo develops a current of 14.8 amperes. What is the E.M.F.? *Ans.* 196.25 volts.

12. A dynamo, run at a certain speed, gives an E.M.F. of 200 volts. It has an internal resistance of 0.5 of an ohm. In the external circuit are 3 arc lamps in series, each having a resistance of 2.5 ohms. The line wire has a resistance of 0.5 of an ohm. I want a current of just 25 amperes. Must I increase or lessen the speed of the dynamo?

13. With an external resistance of 1.14 ohms, a dynamo develops a current of 81.58 volts and 29.67 amperes. What is the internal resistance of the dynamo? *Ans.* 1.61 ohms.

14. Upon trial, it was found that a dynamo that was known to have an internal resistance of 4.58 ohms developed a current of 157.5 volts and 17.5 amperes. What was the resistance of the external circuit? *Ans.* 4.42 ohms.

15. Three incandescence lamps having a resistance of 39.3 ohms each (when hot) were placed in series. The total resistance of the circuit outside of the lamps was 11.2 ohms. The current measured 1.2 amperes. What was the E.M.F.? *Ans.* 154.92 volts.

16. A dynamo supplies current for two incandescence lamps in series, each having a hot resistance of 97 ohms. The other resistances of the circuit amounted to 12 ohms. The current in the first lamp was 1 ampere. What was the current carried by the carbon filament of the second lamp? What was the E.M.F.?

17. A dynamo is required to furnish a 9.6-ampere current for 60 arc lamps. Assuming that a copper wire 1 square inch in cross-section will safely carry 2,000 amperes, determine the proper size of wire for the Gramme ring armature of the dynamo. In such an armature, each wire carries half the current. *Ans.* No. 15.

18. What E.M.F. must be generated by the dynamo mentioned in Exercise 17 if the resistance of the line-wire is 5 ohms, the internal resistance of the machine is 16 ohms, and the difference of potential between the terminals of each lamp is 45 ohms?

19. Assume that the E.M.F. of a bipolar dynamo may be determined by the formula,

$$E = \frac{nks}{100,000,000},$$

in which E represents the E.M.F.; n the number of lines of force passing effectively through the armature; k the number of conductors in series on the armature; and *s* the number of revolutions that the armature makes per second. Determine the E.M.F. of a dynamo the armature of which makes 780 revolutions per minute, has 120 conductors in series, and is threaded by 7,200,000 lines of force.

Ans. 112.32 volts.

20. What effect would a slowing of the speed of a dynamo have upon its E.M.F.?

21. Suppose that the armatures of two dynamos rotate at the same speed in fields of like intensity. The armatures differ only in that one has twice as many bobbins in series as the other. How will the dynamos compare in E.M.F.?

22. Suppose that the armature of a dynamo is wound with No. 8 copper wire, that it revolves at a uniform speed, and that the potential difference at the terminals of the dynamo is constant. What is to prevent our drawing from the machine a current that is almost unlimited as to strength?

23. Suppose an adjustable resistance, *R*, to be cut into the magnetizing circuit of a shunt-wound dynamo as shown in Fig. 466. How would an increased resistance affect the number of lines of force that pass through the armature, *A*? Could the E.M.F. of the dynamo be thus controlled?

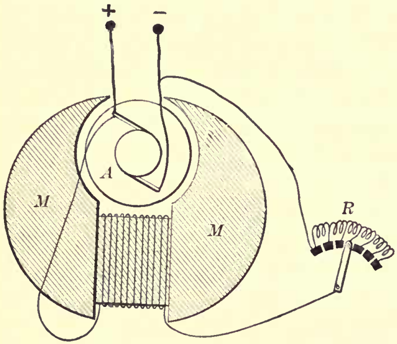


FIG. 466.

24. A certain dynamo yields a current of 9.6 amperes at almost any voltage desired. I want to send a 2-ampere current through a coil that has a resistance of 2 ohms. What resistance must be put in parallel with the coil between the terminals of the dynamo?

Ans. 0.526 + ohms.

25. A system of 127 incandescence lamps, each fed at a potential difference of 110 volts, and with a current of 0.5 of an ampere, is 620 feet from the dynamo. Allowing for an absorption of four per cent. of the energy of the current by the line-wire, (*a*) what must be the voltage of the dynamo? The difference between the voltage of the dynamo and that of the lamps is the fall of potential due to the resistance of the wire. Applying Ohm's law, divide this fall of poten-

tial by the current required, and thus (*b*) determine the total resistance of the line-wire. (*c*) Determine the resistance of the line-wire per foot. (*d*) Consult the table in the appendix, and ascertain the proper size of the line-wire. *Ans.* (*a*) 114.58 + volts.

26. The field magnet of a shunt-wound dynamo is to be made of No. 14 copper wire. Such a wire cannot carry a current of more than 15 amperes without danger of becoming so hot as to injure its insulation. Assume that the dynamo delivers current at 110 volts, and determine the length of wire that must be used in the field magnets to keep them from over-heating.

27. Fire insurance rules allow No. 0000 ("four-naught") copper wire to carry a current of 175 amperes, which is sufficient for 350 lamps at 110 volts. Suppose that one hundred such lamps are to be placed a mile from a central station, (*a*) What will be the resistance of the line if it is made of the wire above mentioned? (*b*) What must the voltage at the station be?

Ans. (*a*) 0.53856 ohms; (*b*) 136.928 volts.

28. I have two telegraph sounders. One of them is made with a few turns of coarse wire; the other of many turns of fine wire. Trying them on a long line of great resistance, I find that one works satisfactorily while the other will not work at all. Which sounder works? Explain the difference in the results secured with the two instruments.

29. A telegraph line is to be operated between Boston and Chicago. The high resistance of so long a line requires a current of such high potential that there is great difficulty in maintaining the insulation. How may this difficulty be removed?

30. I find that I can hear readily through a telephone receiver that is in circuit with several miles of uncoiled wire, but that when the wire is wound upon an iron core, thus making an electromagnet, I cannot hear through the telephone at all. Explain.

V. ELECTROMAGNETIC CHARACTER OF RADIATION.

440. Electromagnetic Rotation, etc. — When a beam of polarized light is passed through a magnetic field, the plane of polarization is rotated in the direction in which an electric current would have to circulate around the beam to produce the existing magnetism. If the beam is reflected back so as to traverse the magnetic field twice in opposite directions, the magnetic rotation is doubled. When a beam of polarized light is reflected from the polished pole of an electromagnet, the plane of polarization is rotated in a direction contrary to that of the magnetizing current. A dielectric under electrostatic stress becomes doubly refracting. These and other facts suggest that there is some definite relation between electricity and light.

441. Electrical Oscillations. — If an elastic cord is stretched between two supports and its middle pulled to one side, it may be allowed to return to its normal position so gently as to cause no vibrations in the cord, or it may be released so as to set up vibrations, the frequency of which will depend largely upon the tension of the cord. Similarly, when a Leyden jar is discharged through a dry linen thread, there is a gentle flow of electrification from one coat to the other, and equilibrium is quietly restored. On the other hand, when the charged jar is discharged through a short metal conductor, the charge rushes suddenly from one coat to the other, overdoes its work, and surges back and forth with gradually weakening energy until the two coats are at the same potential.

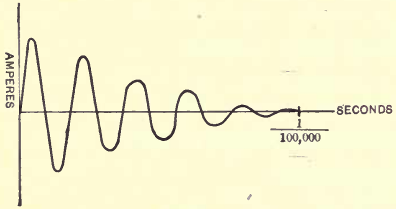


Fig. 467.

(*a*) Fig. 467 is a graphic representation of such electrical oscillation, the heights of the waves being proportional to the magnitudes of the flow, and the lengths being proportional to the times required for the successive oscillations. Fig. 468 shows a similar curve for an ordinary alternating current. Comparing the time axes (abscissæ) of the two curves, it is seen that the curve of the Leyden jar may be completed in 0.00001 of a second, while the alternate current requires 0.004 of a second for a single pulse. Further comparison of the curves shows a marked difference in the rate of change, the time interval required, in the first case, for a drop from a maximum value in one direction to a maximum in the other direction being almost infinitesimal. It also appears that the alternating current curve is strictly periodic and repeats itself, while the Leyden jar oscillations gradually die away. It has been shown that when the resistance of the circuit is negligible, the number and periods of these oscillations per second may be represented thus: —

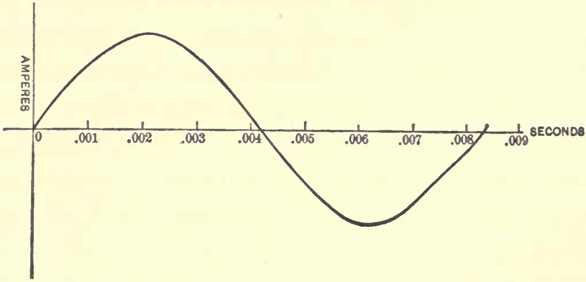


FIG. 468.

$$n = \frac{1}{2\,\pi\,\sqrt{KL}}, \text{ or } t = 2\,\pi\,\sqrt{KL}.$$

In this formula, *n* represents the frequency, and *t* the period of oscillation; *K*, the capacity; and *L*, the coefficient of self-induction. An increase in the values of *K* and *L* diminishes the value of *n*. By discharging a large condenser through a high inductive resistance, the frequency of the electrical oscillations has been reduced to 400 per second, under which conditions the electric spark emitted a musical tone.

442. Electromagnetic Waves. — With currents of slowly varying strength, magnetic lines of force surround the conductors that carry the currents. When the direction of the current is changed, the direction of the lines of force is changed. These lines of force spread out from a wire carrying an increasing current much like the ripples on a pond moving outward from the center of disturbance. As the current dies away, these magnetic lines of force are called in again. As the current reverses its direction, other lines of force opposite in direction from the former ones are sent out. This alternating action sets up a series of waves that travels outward. So, when the rapidly oscillating motions of the Leyden jar discharge are applied to a conductor, the magnetic ripples that are propagated never return to the conductor that generates them, but continue on through space like the waves of radiant energy. Just as a vibrating tuning-fork expends its energy in producing air waves, so a discharging condenser sets up vibrations in the medium of electrical action, whatever that medium may be.

(*a*) In recent years, Hertz and Nikola Tesla have experimented with these electromagnetic waves, and have shown that they may be reflected, refracted, and polarized, and that they possess all the transmittive properties of radiant energy. They have also shown that their velocity is identical with that of light, and that indices of refraction are the same for electromagnetic waves as they are for the shorter waves that are familiarly recognized as radiant energy.

Experiment 403. — Provide two Leyden jars that are alike. Paste a strip of tin-foil so that it touches the inner coat of one of the jars, passes over the edge, and nearly touches the outer coat as shown at *a* in Fig. 469. From the knob and the outer coat of this jar, *J*, lead two wires. Tie these wires with silk thread to the edges of a plate of glass that is 6 or 7 cm. wide and supported in a vertical plane. Connect a wire to the outer coat of the other jar, *L*, bend the wire into a loop, and provide a terminal ball, *B*, as shown in the figure. Bend the wire so that *B* is brought very near to the knob, *A*, without quite touching it. Place *L* near *J*, with its wire loop in a plane that is parallel to that of the suspended glass plate. Charge *L* by an induction coil or an electrical machine so that a stream of sparks snaps across *AB*. Connect the two wires carried by *J*, by a spring clip, *C*.

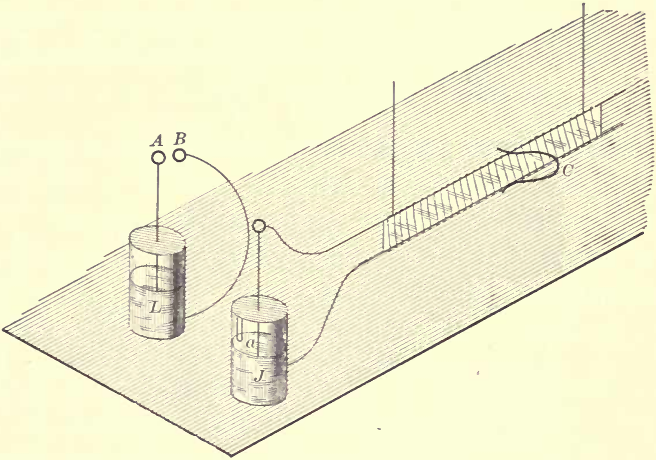


FIG. 469.

Move the clip back and forth along the wires until you find for it a position such that as sparks pass between *A* and *B*, other sparks will snap across the gap at *a*. The two jars will then be in "electrical resonance."

443. Electrical Resonance. — When certain relations of capacity and reactance exist between two electric circuits, electrical vibrations of high frequency in one of the circuits will set up electrical vibrations in the other circuit. The best results are obtained when the receiving circuit is so adjusted that its oscillations are synchronous with those of the generating circuit. *A receiving circuit so adjusted is called an electrical resonator.* Electrical resonance is analogous to acoustic resonance, and has been of great value in recent researches on the electromagnetic theory of light. The adjustment of capacity and self-induction so as to harmonize the vibration-frequency of the two circuits, i.e., to establish resonance, is often a matter of great delicacy and difficulty. In very exact adjustments, minute changes completely destroy the balance, just as they do in acoustic resonance.

444. The Hertz Experiments. — In his study of electric waves, Hertz, the first successful investigator in this field, used an "oscillator" that consisted of an induction coil, the terminals of which carried two metal rods, *t* and *t′*, that ended in small metal balls, and that were provided with large movable metal plates or spheres, *A* and *B*. The capacity, *K*, of the system was determined from the size of the plates or the spheres, *A* and *B*, and the coefficient of self-induction, *L*, from the dimensions of the rods, *t* and *t′*. Within certain limits, the value of *L* could be adjusted by varying the position of *A* and *B* upon the rods. The values of *K* and *L* being known, the period of a complete oscillation could be determined (see § 441, *a*). To reduce the wave-length to convenient values (say a meter or two), the oscillations were made rapid; i.e., *K* and *L* were made small.

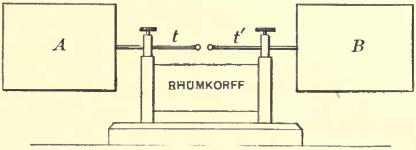


FIG. 470.

(*a*) The action of this apparatus generates two sets of waves of the same period; one set being electrostatic, and the other, electromagnetic. The effects produced by these two sets of waves may be separately studied. The electrostatic lines of force extend between *A* and *B*. The electromagnetic lines of force are concentric about the rods, *t* and *t′*. Hertz's resonator was an open wire ring terminally provided with small balls, micrometrically adjustable so that the distance between them could be accurately measured. When this simple resonator is made to enclose the electromagnetic lines of force set up by the oscillator, induction sparks pass between the terminals *m* and *n*. Adjustment for synchronous effect, and placing the resonator so that its plane passes through the axis of the oscillator, raise the sparks to their maximum value.

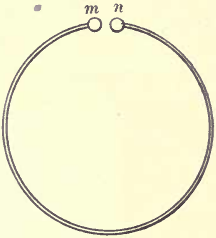


FIG. 471.

(*b*) Anywhere in the field, sparks may be obtained between any two metallic objects. Hertz placed two rods in the same line and with their ends near together, and used them as a receiver. He attached sheets of tin-foil to the rods, and thus increased their capacity, so that sparks were obtained at a distance of 20 or 30 m. from the oscillator. He found that non-conductors are transparent to the electric radiations, and that conductors act as reflectors. With waves reflected from a zinc-covered wall, and by moving the resonator about, he found points of maximum and minimum disturbance corresponding to the loops and nodes of a vibrating string. The wave-lengths thus determined, multiplied by the frequency of vibration as calculated from *K* and *L*, the constants of the oscillator, gave a velocity for electric wave propagation that is practically the same as the velocity of light. He also demonstrated the rectilinear propagation of the waves, and focused them with a metallic parabolic mirror. He refracted them with a prism of pitch, and established the index of refraction for radiations of certain wave-lengths.

(*c*) Using more delicate apparatus, Lodge has detected electromagnetic waves that had passed through floors and walls, and at a distance of hundreds of feet.

445. The Tesla Experiments. — By means of his oscillator (§ 398), in which the armature coils are shot, very rapidly and shuttle-fashion, in and out of the magnetic field, Nikola Tesla has generated alternating currents of higher frequency, potential, and regularity than any previously employed, and has greatly augmented the Hertzan effects and added to them.

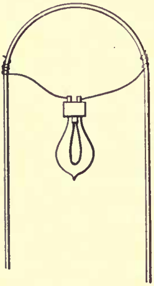


Fig. 472.

(*a*) He has shown that such currents flow mostly on the outer surface of the conductor, as though ether vortices were rolled along the wire as a rubber band may be rolled along a pencil, and that the impedance of a stout copper rod may be hundreds of times as great as that of an incandescent lamp filament, although the ratio of their resistances is the reverse. For instance, he short-circuited a lamp by a copper rod as shown in Fig. 472. Any ordinary current, direct or alternating, would melt the rod before the lamp began to glow, but Tesla's current brilliantly illuminated the lamp and left the rod cool. In fact, with Teslaic currents, the resistance of the lamp filaments counts for little or nothing, and the filament may as well be short and thick as long and thin. More than even this, the filament and conducting wires may be wholly omitted. Vacuum tubes of ordinary glass held in the hand near the terminal of a high frequency coil became luminous, as shown in Fig. 473, clearly showing that, as the current-frequency rises, electrodynamic conditions are left for those that are electrostatic, and that space itself "is all circuit if we properly direct the right kind of impulses through it." Tesla led a small cable around the walls of a room 40 × 80 feet in size, and connected its ends to the terminals of an oscillator. In the middle of the room, he placed a coil-wound resonator three or four feet high and provided with two adjustable condenser plates. These plates stood on edge above the end of the coil and facing each other, much as if they were cymbals resting upon the head of a bass drum. By adjusting the condenser plates, the resonator was so attuned that the periodicity of the induced current kept step with that of the cable current. When current from the oscillator was sent along the cable around the room, powerful sparks poured in dense streams across the space between the cymbal-like plates of the attuned condenser in the middle of the room. A potential difference of 200,000 or 300,000 volts is easily developed in this way, and the high-tension currents pass through the human body without injury. With a similar resonator similarly placed and properly attuned, an ordinary 16-c.p. lamp was well illuminated. When an 110-volt lamp was attached by one terminal to a wire circle, and the circle held above the resonator coil, the lamp immediately lighted up. Such an illumination of such a lamp calls for the expenditure of more than a tenth of a horse-power, so that at least that amount of energy was transmitted through free space, i. e., without any wire. By connecting to earth one terminal of a coil in which rapidly vibrating currents are thus produced, and leaving the other terminal free in space, Tesla produces striking effects, purple fiery streams and lightning discharges pouring into the air from the tip of the wire "with the roar of a gas well."

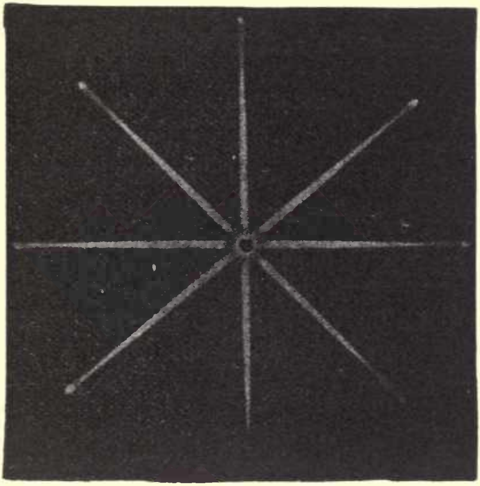


FIG. 473.

Mr. Tesla has secured his results chiefly because he has "learned the knack of loading electrically on the good-natured ether a little of the protean energy of which no amount has yet sufficed to break it down or put it out of temper." His experiments point toward the possibility of telephony without any wire, i.e., by induction through the air and, in fact, some such transmission of intelligence has been seriously propounded, not as a mere theoretical possibility but as a problem in practical electrical engineering.

446. The Electromagnetic Theory of Light. — Optical and electrical phenomena seem to call for media that have identical properties; i.e., they indicate that the medium of the one is identical with the medium of the other. The experimental proof that 3×10^{10} centimeters represent alike the velocity of light and the velocity of electric waves, indicates that the ether is the common medium. This, taken in connection with the close relation established between specific inductive capacity and index of refraction, and that between electric conductivity and optical opacity, and the experiments of Hertz and of Tesla, "appear to prove beyond a question that light is itself an electrical phenomenon and that optics is a department of electrics. To produce radiation, it is only necessary to produce electric oscillations of sufficiently short period." This theory of light as an electromagnetic disturbance was propounded in 1865 by Maxwell; if recent investigations do not wholly establish it, they certainly give it very strong support.

(*a*) It has been calculated that a condenser of one microfarad capacity might generate electric waves 1,900 Km. long; a pint Leyden jar, waves 15 or 20 m. long; and a tiny thimble-like jar, waves 1 m. long. Continuing this process to ascertain the size of a circuit that will give wave-lengths comparable to those of light, physicists come to the suggestion that the ether waves that affect the retina of the human eye may be produced by the electric oscillations of circuits of atomic dimensions. An atom of sodium vibrates five hundred million times in one millionth of a second. If we could produce electric oscillations and maintain them at this rate, we could produce light.

(*b*) Light is ordinarily produced by combustion, less than one per cent of the emitted energy being visible. "The problem of the age is to convert some other form of energy entirely into the energy of light. That this is possible in theory, Rayleigh long ago showed.

That it is actually accomplished in Nature, Langley's remarkable measurements upon the glowworm abundantly confirm. Now that the mechanism of the process is before us, it would seem not impossible eventually to create and to maintain electric oscillations of the frequency required for light alone. When this is done, the problem of the economical production of artificial light will have been solved."

447. Yesterday, To-day, To-morrow. — In the light of what has lately been accomplished by the blending of theory and practice, and of the promise that comes from the state of unrest in which electrical science now exists, it seems a fitting final word to suggest that constant study is the price of a clear understanding of conditions that prevail in the domain of electricity. "Its theoretical problems assume novel phases daily. Its old appliances ceaselessly give way to successors. Its methods of production, distribution, and utilization vary from year to year. Its influence on the times is ever deeper, yet one can never be quite sure into what part of the social or industrial system it is next to thrust a revolutionary force. Its fanciful dreams of yesterday are the magnificent triumphs of to-morrow, and its advance toward domination in the twentieth century is as irresistible as that of steam in the nineteenth."

APPENDIX.

1. Mensurative, etc.

$\pi = 3.14159$.

Circumference of a circle $= \pi D$.

Area of a circle $= \pi R^2$.

Surface of a sphere $= 4\pi R^2 = \pi D^2$.

Volume of a sphere $= \frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3$.

Meters × 3.2809 = feet.

Feet × 0.3048 = meters.

Inches × 2.54 = centimeters.

Cubic inches × 16.386 = cubic centimeters.

Cubic centimeters × 0.06103 = cubic inches.

Kilogrammeters × 7.2331 = foot-pounds.

2. Table of Materials in Electromotive Order.

(*Electrochemical Series.*)

Direction of current through wire.

DILUTE SULPHURIC ACID.	DILUTE HYDROCHLORIC ACID.	SOLUTION OF SULPHIDE OF POTASSIUM.	SOLUTION OF CAUSTIC POTASH.
1. Zinc.	1. Zinc.	1. Zinc.	1. Zinc.
2. Cadmium.	2. Cadmium.	2. Copper.	2. Tin.
3. Tin.	3. Tin.	3. Cadmium.	3. Cadmium.
4. Lead.	4. Lead.	4. Tin.	4. Antimony.
5. Iron.	5. Iron.	5. Silver.	5. Lead.
6. Nickel.	6. Copper.	6. Antimony.	6. Bismuth.
7. Bismuth.	7. Bismuth.	7. Lead.	7. Iron.
8. Antimony.	8. Nickel.	8. Bismuth.	8. Copper.
9. Copper.	9. Silver.	9. Nickel.	9. Nickel.
10. Silver.	10. Antimony.	10. Iron.	10. Silver.
11. Gold.	11. . . .	11. . .	11. . . .
12. Platinum.	12. . . .	12. . .	12. . . .
13. Carbon.	13. . . .	13. . . .	13. . . .

Direction of current through liquid.

For any given solution, the farther apart any two materials are in the electromotive table, the stronger will be the electrical effect of a cell with plates made of such materials.

3. Table of Resistivities. — Represent the length of a conducting wire measured in feet by l, its diameter measured in thousandths of an inch (mils) by d, and its resistance measured in ohms by r. In the formula

$$r = \frac{Kl}{d^2},$$

K represents a constant that depends upon the material of the wire and, for the substances considered, is as given in the following table of resistivities: —

Silver	9.84	Mercury	58.24
Copper	10.45	Platinum	59.02
Zinc	36.69	Iron	63.35
German-silver	128.29		

These values of K are computed for the temperature of 20°. Thus the resistance of 1,000 feet of No. 0000 copper wire at 20°, is $10.45 \times 1{,}000 \div 460^2 = 0.049 +$ ohms.

4. Dimensions and Functions of Copper Wires. — In the table given on the next two pages, the second column gives the diameters in mils, i.e., thousandths of an inch; the third column in millimeters. The fourth column gives the equivalent number of wires each one mil in diameter. By multiplying the numbers in the sixth column by 5.28, the resistances per mile may be found. The resistance for any other metal than copper may be found by multiplying the resistance given in the table by the ratio between the resistivity of copper and that of the given metal. The resistances given in the table are for pure copper wire. Ordinary commercial copper wire has a lower conductivity than that of pure copper. Consequently, the resistances of such wires will be greater than those given in the table.

Dimensions and Functions of Copper Wires.

Gauge Number.	Diameter.		Circular Mils.	Sectional Area in Square Inches.	Weight and Length Density = 8.9.		Resistance at 24°.			Capacity in Amperes.
	Mils.	Millim.			Lbs. per 1000 Ft.	Feet per Lb.	Ohms per 1000 Ft.	Feet per Ohm.	Ohms per Lb.	
0000	460.000	11.684	211600.00	0.166190	639.33	1.56	0.051	19929.700	0.0000785	312.
000	409.640	10.405	167805.00	0.131790	507.01	1.97	0.063	15804.900	0.000125	262.
00	364.800	9.266	133079.40	0.104520	402.09	2.49	0.080	12534.200	0.000198	220.
0	324.950	8.254	105592.50	0.082932	319.04	3.13	0.101	9945.300	0.000315	185.
1	289.300	7.348	83694.20	0.065733	252.88	3.95	0.127	7882.800	0.000501	156.
2	257.630	6.544	66373.00	0.052130	200.54	4.99	0.160	6251.400	0.000799	131.
3	229.420	5.827	52634.00	0.041339	159.03	6.29	0.202	4957.300	0.001268	110.
4	204.310	5.189	41742.00	0.032784	126.12	7.93	0.254	3931.600	0.002016	92.3
5	181.940	4.621	33102.00	0.025998	100.01	10.00	0.321	3117.800	0.003206	77.6
6	162.020	4.115	26250.50	0.020617	79.32	12.61	0.404	2472.400	0.005098	65.2
7	144.280	3.665	20816.00	0.016349	62.90	15.90	0.509	1960.600	0.008106	54.8
8	128.490	3.264	16509.00	0.012966	49.88	20.05	0.643	1555.000	0.01289	46.1
9	114.430	2.907	13094.00	0.010284	39.56	25.28	0.811	1233.300	0.02048	38.7
10	101.890	2.588	10381.00	0.0081532	31.37	31.38	1.023	977.800	0.03259	32.5
11	90.742	2.305	8234.00	0.0064670	24.88	40.20	1.289	775.500	0.05181	27.3
12	80.808	2.053	6529.90	0.0051286	19.73	50.69	1.626	615.020	0.08237	23.0
13	71.961	1.828	5178.40	0.0040671	15.65	63.91	2.048	488.250	0.13087	19.3
14	64.084	1.628	4106.80	0.0031469	12.41	80.59	2.585	386.800	0.20830	16.2
15	57.068	1.450	3256.70	0.0025578	9.84	101.63	3.177	306.740	0.33133	13.6
16	50.820	1.291	2582.90	0.0020286	7.81	128.14	4.582	243.250	0.52638	11.5
17	45.257	1.150	2048.20	0.0016086	6.19	161.59	5.183	192.910	0.83744	9.6
18	40.303	1.024	1624.30	0.0012757	4.91	203.76	6.536	152.990	1.3312	8.1

Dimensions and Functions of Copper Wires. — *Continued.*

GAUGE NUMBER.	DIAMETER.		CIRCULAR MILS.	SECTIONAL AREA IN SQUARE INCHES.	WEIGHT AND LENGTH DENSITY = 8.9.		RESISTANCE AT 24°.			CAPACITY IN AMPERES.
	Mils.	Millim.			Lbs. per 1000 Ft.	Feet per Lb.	Ohms per 1000 Ft.	Feet per Ohm.	Ohms per Lb.	
19	35.390	0.899	1252.40	0.0009836	3.78	264.26	8.477	117.960	2.2392	6.7
20	31.961	0.812	1021.50	0.0008023	3.09	324.00	10.394	96.210	3.3438	5.7
21	28.462	0.723	810.10	0.0006363	2.45	408.56	13.106	76.300	5.3539	4.8
22	25.347	0.644	642.70	0.0005048	1.94	515.15	16.525	60.510	8.5099	4.0
23	22.571	0.573	509.45	0.0004001	1.54	649.66	20.842	47.980	13.334	3.2
24	20.100	0.511	504.01	0.0003173	1.22	819.21	26.284	38.050	21.524	2.8
25	17.900	0.455	320.40	0.0002516	0.97	1032.96	33.135	30.180	34.298	2.4
26	15.940	0.405	254.01	0.0001995	0.77	1302.61	41.789	23.930	54.410	2.0
27	14.195	0.361	201.50	0.0001583	0.61	1642.55	52.687	18.980	86.657	1.7
28	12.641	0.321	159.79	0.0001255	0.48	2071.22	66.445	15.050	137.283	1.4
29	11.257	0.286	126.72	0.0000995	0.38	2611.82	83.752	11.940	218.104	1.2
30	10.025	0.255	100.50	0.0000789	0.30	3293.97	105.641	9.466	349.805	1.0
31	8.928	0.227	79.71	0.0000626	0.24	4152.22	133.191	7.508	557.286	0.84
32	7.950	0.202	63.20	0.0000496	0.19	5236.66	168.011	5.952	884.267	0.70
33	7.080	0.180	50.13	0.0000394	0.15	6602.71	211.820	4.721	1402.78	0.60
34	6.304	0.160	39.74	0.0000312	0.12	8328.30	267.165	3.743	2207.98	0.50
35	5.614	0.143	31.52	0.0000248	0.10	10501.35	336.810	2.969	3583.12	0.42
36	5.000	0.127	25.00	0.0000196	0.08	13238.83	424.650	2.355	5661.71	0.35
37	4.453	0.113	19.83	0.0000156	0.06	16691.06	535.330	1.868	8922.20	0.27
38	3.965	0.101	15.72	0.0000123	0.05	20854.65	675.220	1.481	15000.50	0.25
39	3.531	0.090	12.47	0.0000098	0.04	26302.23	821.789	1.174	22415.50	0.21
40	3.144	0.080	9.89	0.0000078	0.03	33175.94	1074.110	0.931	35803.80	0.17

5. Table of Natural Tangents.

ARC.	TANGENT.	ARC.	TANGENT.	ARC.	TANGENT.	ARC.	TANGENT.
0°	.000	23°	.424	46°	1.036	69°	2.61
1	.017	24	.445	47	1.07	70	2.75
2	.035	25	.466	48	1.11	71	2.90
3	.052	26	.488	49	1.15	72	3.08
4	.070	27	.510	50	1.19	73	3.27
5	.087	28	.532	51	1.23	74	3.49
6	.105	29	.554	52	1.28	75	3.73
7	.123	30	.577	53	1.33	76	4.01
8	.141	31	.601	54	1.38	77	4.33
9	.158	32	.625	55	1.43	78	4.70
10	.176	33	.649	56	1.48	79	5.14
11	.194	34	.675	57	1.54	80	5.67
12	.213	35	.700	58	1.60	81	6.31
13	.231	36	.727	59	1.66	82	7.12
14	.249	37	.754	60	1.73	83	8.14
15	.268	38	.781	61	1.80	84	9.51
16	.287	39	.810	62	1.88	85	11.43
17	.306	40	.839	63	1.96	86	14.30
18	.325	41	.869	64	2.05	87	19.08
19	.344	42	.900	65	2.14	88	28.64
20	.364	43	.933	66	2.25	89	57.29
21	.384	44	.966	67	2.36	90	Infinite.
22	.404	45	1.000	68	2.48		

6. Table of Natural Sines.

Arc.	Sine.	Arc.	Sine.	Arc.	Sine.	Arc.	Sine.
0°	0.000	23	0.391	46°	0.719	69°	0.934
1	0.017	24	0.407	47	0.731	70	0.940
2	0.035	25	0.423	48	0.743	71	0.946
3	0.052	26	0.438	49	0.755	72	0.951
4	0.070	27	0.454	50	0.766	73	0.956
5	0.087	28	0.469	51	0.777	74	0.961
6	0.105	29	0.485	52	0.788	75	0.966
7	0.122	30	0.500	53	0.799	76	0.970
8	0.139	31	0.515	54	0.809	77	0.974
9	0.156	32	0.530	55	0.819	78	0.978
10	0.174	33	0.545	56	0.829	79	0.982
11	0.191	34	0.559	57	0.839	80	0.985
12	0.208	35	0.574	58	0.848	81	0.988
13	0.225	36	0.588	59	0.857	82	0.990
14	0.242	37	0.602	60	0.866	83	0.993
15	0.259	38	0.616	61	0.875	84	0.995
16	0.276	39	0.629	62	0.883	85	0.996
17	0.292	40	0.643	63	0.891	86	0.998
18	0.309	41	0.656	64	0.899	87	0.999
19	0.326	42	0.669	65	0.906	88	0.999
20	0.342	43	0.682	66	0.914	89	0.999
21	0.358	44	0.695	67	0.921	90	1.000
22	0.375	45	0.707	68	0.927		

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