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SMITHSONIAN

CONTRIBUTIONS TO KNOWLEDGE.

VOL. XIX.

EVERY MAN IS A VALUABLE MEMBER OF SOCIETY, WHO, BY HIS OBSERVATIONS, RESEARCHES, AND EXPERIMENTS, PROCURES KNOWLEDGE FOR MEN.—SMITHSON.

CITY OF WASHINGTON:
PUBLISHED BY THE SMITHSONIAN INSTITUTION.
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THIS volume forms the nineteenth of a series, composed of original memoirs on different branches of knowledge, published at the expense, and under the direction, of the Smithsonian Institution. The publication of this series forms part of a general plan adopted for carrying into effect the benevolent intentions of JAMES SMITHSON, Esq., of England. This gentleman left his property in trust to the United States of America, to found, at Washington, an institution which should bear his own name, and have for its objects the "*increase* and *diffusion* of knowledge among men." This trust was accepted by the Government of the United States, and an Act of Congress was passed August 10, 1846, constituting the President and the other principal executive officers of the general government, the Chief Justice of the Supreme Court, the Mayor of Washington, and such other persons as they might elect honorary members, an establishment under the name of the "SMITHSONIAN INSTITUTION FOR THE INCREASE AND DIFFUSION OF KNOWLEDGE AMONG MEN." The members and honorary members of this establishment are to hold stated and special meetings for the supervision of the affairs of the Institution, and for the advice and instruction of a Board of Regents, to whom the financial and other affairs are intrusted.

The Board of Regents consists of three members *ex officio* of the establishment, namely, the Vice-President of the United States, the Chief Justice of the Supreme Court, and the Mayor of Washington, together with twelve other members, three of whom are appointed by the Senate from its own body, three by the House of Representatives from its members, and six persons appointed by a joint resolution of both houses. To this Board is given the power of electing a Secretary and other officers, for conducting the active operations of the Institution.

To carry into effect the purposes of the testator, the plan of organization should evidently embrace two objects: one, the increase of knowledge by the addition of new truths to the existing stock; the other, the diffusion of knowledge, thus increased, among men. No restriction is made in favor of any kind of knowledge; and, hence, each branch is entitled to, and should receive, a share of attention.

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The Act of Congress, establishing the Institution, directs, as a part of the plan of organization, the formation of a Library, a Museum, and a Gallery of Art, together with provisions for physical research and popular lectures, while it leaves to the Regents the power of adopting such other parts of an organization as they may deem best suited to promote the objects of the bequest.

After much deliberation, the Regents resolved to divide the annual income into two parts—one part to be devoted to the increase and diffusion of knowledge by means of original research and publications—the other part of the income to be applied in accordance with the requirements of the Act of Congress, to the gradual formation of a Library, a Museum, and a Gallery of Art.

The following are the details of the parts of the general plan of organization provisionally adopted at the meeting of the Regents, Dec. 8, 1847.

DETAILS OF THE FIRST PART OF THE PLAN.

I. To INCREASE KNOWLEDGE.—*It is proposed to stimulate research, by offering rewards for original memoirs on all subjects of investigation.*

1. The memoirs thus obtained, to be published in a series of volumes, in a quarto form, and entitled "Smithsonian Contributions to Knowledge."

2. No memoir, on subjects of physical science, to be accepted for publication, which does not furnish a positive addition to human knowledge, resting on original research; and all unverified speculations to be rejected.

3. Each memoir presented to the Institution, to be submitted for examination to a commission of persons of reputation for learning in the branch to which the memoir pertains; and to be accepted for publication only in case the report of this commission is favorable.

4. The commission to be chosen by the officers of the Institution, and the name of the author, as far as practicable, concealed, unless a favorable decision be made.

5. The volumes of the memoirs to be exchanged for the Transactions of literary and scientific societies, and copies to be given to all the colleges, and principal libraries, in this country. One part of the remaining copies may be offered for sale; and the other carefully preserved, to form complete sets of the work, to supply the demand from new institutions.

6. An abstract, or popular account, of the contents of these memoirs to be given to the public, through the annual report of the Regents to Congress.

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II. To increase Knowledge.—*It is also proposed to appropriate a portion of the income, annually, to special objects of research, under the direction of suitable persons.*

1. The objects, and the amount appropriated, to be recommended by counsellors of the Institution.

2. Appropriations in different years to different objects; so that, in course of time, each branch of knowledge may receive a share.

3. The results obtained from these appropriations to be published, with the memoirs before mentioned, in the volumes of the Smithsonian Contributions to Knowledge.

4. Examples of objects for which appropriations may be made:—

(1.) System of extended meteorological observations for solving the problem of American storms.

(2.) Explorations in descriptive natural history, and geological, mathematical, and topographical surveys, to collect material for the formation of a Physical Atlas of the United States.

(3.) Solution of experimental problems, such as a new determination of the weight of the earth, of the velocity of electricity, and of light; chemical analyses of soils and plants; collection and publication of articles of science, accumulated in the offices of Government.

(4.) Institution of statistical inquiries with reference to physical, moral, and political subjects.

(5.) Historical researches, and accurate surveys of places celebrated in American history.

(6.) Ethnological researches, particularly with reference to the different races of men in North America; also explorations, and accurate surveys, of the mounds and other remains of the ancient people of our country.

I. To diffuse Knowledge.—*It is proposed to publish a series of reports, giving an account of the new discoveries in science, and of the changes made from year to year in all branches of knowledge not strictly professional.*

1. Some of these reports may be published annually, others at longer intervals, as the income of the Institution or the changes in the branches of knowledge may indicate.

2. The reports are to be prepared by collaborators, eminent in the different branches of knowledge.

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3. Each collaborator to be furnished with the journals and publications, domestic and foreign, necessary to the compilation of his report; to be paid a certain sum for his labors, and to be named on the title-page of the report.

4. The reports to be published in separate parts, so that persons interested in a particular branch, can procure the parts relating to it, without purchasing the whole.

5. These reports may be presented to Congress, for partial distribution, the remaining copies to be given to literary and scientific institutions, and sold to individuals for a moderate price.

The following are some of the subjects which may be embraced in the reports:—

I. PHYSICAL CLASS.

1. Physics, including astronomy, natural philosophy, chemistry, and meteorology.
2. Natural history, including botany, zoology, geology, &c
3. Agriculture.
4. Application of science to arts.

II. MORAL AND POLITICAL CLASS.

5. Ethnology, including particular history, comparative philology, antiquities, &c.
6. Statistics and political economy.
7. Mental and moral philosophy.
8. A survey of the political events of the world; penal reform, &c.

III. LITERATURE AND THE FINE ARTS.

9. Modern literature.
10. The fine arts, and their application to the useful arts.
11. Bibliography.
12. Obituary notices of distinguished individuals.

II. To diffuse Knowledge.—*It is proposed to publish occasionally separate treatises on subjects of general interest.*

1. These treatises may occasionally consist of valuable memoirs translated from foreign languages, or of articles prepared under the direction of the Institution, or procured by offering premiums for the best exposition of a given subject.

2. The treatises to be submitted to a commission of competent judges, previous to their publication.

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DETAILS OF THE SECOND PART OF THE PLAN OF ORGANIZATION.

This part contemplates the formation of a Library, a Museum, and a Gallery of Art.

1. To carry out the plan before described, a library will be required, consisting, 1st, of a complete collection of the transactions and proceedings of all the learned societies of the world; 2d, of the more important current periodical publications, and other works necessary in preparing the periodical reports.

2. The Institution should make special collections, particularly of objects to verify its own publications. Also a collection of instruments of research in all branches of experimental science.

3. With reference to the collection of books, other than those mentioned above, catalogues of all the different libraries in the United States should be procured, in order that the valuable books first purchased may be such as are not to be found elsewhere in the United States.

4. Also catalogues of memoirs, and of books in foreign libraries, and other materials, should be collected, for rendering the Institution a centre of bibliographical knowledge, whence the student may be directed to any work which he may require.

5. It is believed that the collections in natural history will increase by donation, as rapidly as the income of the Institution can make provision for their reception; and, therefore, it will seldom be necessary to purchase any article of this kind.

6. Attempts should be made to procure for the gallery of art, casts of the most celebrated articles of ancient and modern sculpture.

7. The arts may be encouraged by providing a room, free of expense, for the exhibition of the objects of the Art-Union, and other similar societies.

8. A small appropriation should annually be made for models of antiquity, such as those of the remains of ancient temples, &c.

9. The Secretary and his assistants, during the session of Congress, will be required to illustrate new discoveries in science, and to exhibit new objects of art; distinguished individuals should also be invited to give lectures on subjects of general interest.

In accordance with the rules adopted in the programme of organization, each memoir in this volume has been favorably reported on by a Commission appointed

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for its examination. It is however impossible, in most cases, to verify the statements of an author; and, therefore, neither the Commission nor the Institution can be responsible for more than the general character of a memoir.

The following rules have been adopted for the distribution of the quarto volumes of the Smithsonian Contributions:—

1. They are to be presented to all learned societies which publish Transactions, and give copies of these, in exchange, to the Institution.

2. Also, to all foreign libraries of the first class, provided they give in exchange their catalogues or other publications, or an equivalent from their duplicate volumes.

3. To all the colleges in actual operation in this country, provided they furnish, in return, meteorological observations, catalogues of their libraries and of their students, and all other publications issued by them relative to their organization and history.

4. To all States and Territories, provided there be given, in return, copies of all documents published under their authority.

5. To all incorporated public libraries in this country, not included in any of the foregoing classes, now containing more than 10,000 volumes; and to smaller libraries, where a whole State or large district would be otherwise unsupplied.

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SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
240

PROBLEMS OF ROTARY MOTION

PRESENTED BY

THE GYROSCOPE,

THE PRECESSION OF THE EQUINOXES,

AND

THE PENDULUM.

BY

BREVET MAJ.-GEN. J. G. BARNARD,
COLONEL OF ENGINEERS U. S. A., A.M., LL.D., MEMBER OF NATIONAL ACADEMY OF SCIENCES.

[ACCEPTED FOR PUBLICATION, OCTOBER, 1871.]

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The three following papers were read at intervals before the National Academy of Sciences, and subsequently presented to the Smithsonian Institution for publication.

JOSEPH HENRY,
Secretary Smithsonian Institution.

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THE PRECESSION OF THE EQUINOXES AND NUTATION AS RESULTING FROM THE THEORY OF THE GYROSCOPE.

In a paper published in the American Journal of Science, in 1857, and in Barnard's American Journal of Education[1] [No. 9] of the same year, I remarked:—

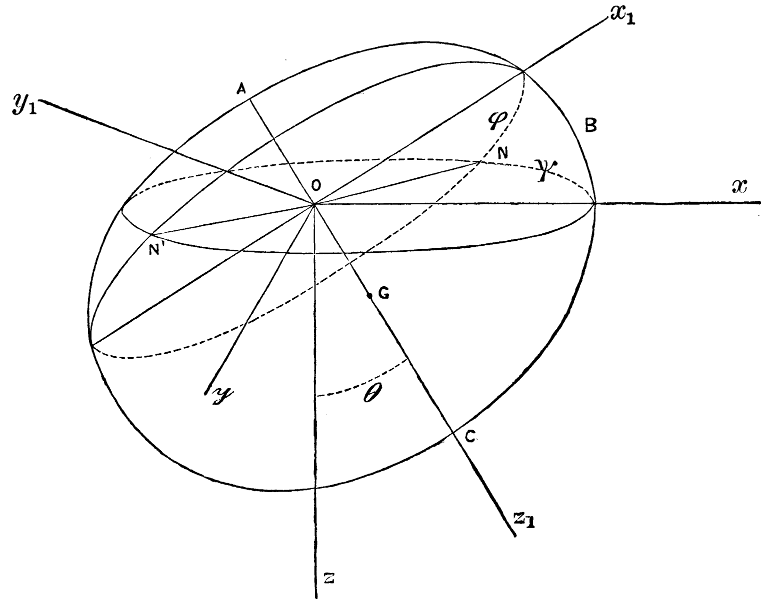
"The analogy between the minute motions of the gyroscope and that grand phenomenon exhibited in the heavens, the 'precession of the equinoxes,' is often remarked. In an ultimate analysis, the phenomena, doubtless, are identical," &c.

It is the object of the present paper to deduce the analytical expressions of this phenomenon directly from the theory of the gyroscope.

A brief summary of the processes used and results arrived at in the paper referred to is necessary as a preliminary.

Let A, B, C, D (Fig. 1) be a solid body of any shape, retained by the fixed point O (within or without its mass). Ox, Oy, and Oz are the three co-ordinate axes, *fixed in space*, to which the motion of the body is referred. Ox_1, Oy_1, Oz_1 are the three *principal axes* belonging to the point O, and which, of course, partake of the

Fig. 1.



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[1] "The Phenomena of the Gyroscope Analytically Examined."

body's motion. The position of the body at any instant of time is determined by those of the moving axes.

For the purpose of determining the positions of the axes Ox_1, Oy_1, and Oz_1, with reference to the (fixed in space) axes Ox, Oy, Oz, three auxiliary angles are used.

If we suppose the moving plane of $x_1\,y_1$, at the instant considered, to intersect the fixed plane of xy in the line NN' and call the angle $xON=\psi$, and the angle between the planes xy and $x_1 y_1$ (or the angle $zOz_1)=\theta$, and the angle $NOx_1=\phi$ (in the figure these angles are supposed *acute* at the instant taken), these three angles will determine the positions of the axes Ox_1, Oy_1, Oz_1 (and hence of the body) at any instant, and will themselves be functions of the time; and the rotary velocities about the axes of x_1, y_1, and z_1, may be expressed in terms of them and of their differential coefficients.

When a body is a solid of revolution, revolving with an angular velocity n, about its axis of figure, and acted upon by the accelerating force of gravity (the fixed point O being in the axis of figure), the general equations of rotary motion (by processes fully developed in the paper referred to)[1] take the form

1. $$\left\{\begin{aligned} &\sin^2\theta\frac{d\psi}{dt}=\frac{Cn}{A}(\cos\theta-\cos\omega)\\ &\sin^2\theta\frac{d\psi^2}{dt^2}+\frac{d\theta^2}{dt^2}=\frac{2Mg\gamma}{A}(\cos\theta-\cos\omega)\end{aligned}\right.$$
$$d\phi=ndt+\cos\theta d\psi$$

In which

M is the mass of the body.

A its moment of inertia about an equatorial axis through O.

C " " " " its axis of figure.

g the force of gravity.

γ the distance OG from centre of gravity to the point of support.

ω the initial value of θ, or its value at the instant when the body has no other motion than the rotation n about its axis of figure.

Eliminating $\frac{d\psi}{dt}$ between the first two equations (1), and putting

2. $$\frac{A}{M\gamma}=\lambda,\text{ and }\frac{C^2n^2}{2A^2g}=\frac{2\beta^2}{\lambda},\text{ we get}$$

3. $$\sin^2\theta\frac{d\theta^2}{dt^2}=\frac{2g}{\lambda}[\sin^2\theta-2\beta^2(\cos\theta-\cos\omega)](\cos\theta-\cos\omega),$$

and the first equation (1) becomes

4. $$\sin^2\theta\frac{d\psi}{dt}=2\beta\sqrt{\frac{g}{\lambda}}(\cos\theta-\cos\omega)$$

The quantity $\beta=\frac{Cn}{2A}\sqrt{\frac{\lambda}{g}}=\frac{Cn}{2\sqrt{AM\gamma g}}$, may be very great in consequence of the rotary velocity, n, being great, or (n being small) in consequence of the ratio

---

[1] The analysis therein used was mostly taken from Poisson as far as equations (of that paper) (9), (10), (11), corresponding to (7), (8), (9) of this; but the subsequent developments were original.

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$\frac{C}{2\sqrt{AM\gamma g}}$, being very great. In the phenomena of the gyroscope, the first condition obtains; in the case of the earth, attracted by the sun or moon (n being small), it is easy to show that the alternative condition is fulfilled.[1]

Putting $\frac{d\theta}{dt}$ equal to zero in equation (3) we get $\theta=\omega$ for the *maximum* of θ, and for the minimum, the equation,

$$\cos\theta=-\beta^2+\sqrt{1+2\beta^2\cos\omega+\beta^4}$$

in which, if β is very great, the value of $\cos\theta$ differs but slightly from that of $\cos\omega$. Hence by introducing a new variable u, equal to $\omega-\theta$, and deducing the values of $d\theta$ and (by development) of $\sin^2\theta$ and $\cos\theta$ (neglecting the higher powers of u) and substituting in (3) and (4), they become (omitting, as relatively small, $\cos\omega$ in the factor $\cos\omega+4\beta^2$).

5. $$\sqrt{\frac{g}{\lambda}}\,dt=\frac{du}{\sqrt{2u\sin\omega-4\beta^2u^2}}$$

6. $$\frac{d\psi}{dt}=2\beta\left(\frac{g}{\lambda}\right)^{\frac{1}{2}}\frac{u}{\sin\omega}$$

Equation (5) gives by integration and putting $\beta\left(\frac{g}{\lambda}\right)^{\frac{1}{2}}=k$.

7. $$u=\frac{1}{2\beta^2}\sin\omega\sin^2 kt$$

which substituted in (6) gives

8. $$\frac{d\psi}{dt}=\frac{1}{\beta^2}k\sin^2 kt$$

9. $$\psi=\frac{1}{2\beta^2}kt-\frac{1}{4\beta^2}\sin 2kt$$

If we make $\omega=90°$, $\sin\omega=1$, in equation (6), deduce the value of dt, and substitute in (5) we get,

10. $$d\psi=\frac{u\,du}{\sqrt{\frac{1}{2\beta^2}u-u^2}}$$

the differential equation of the cycloid, generated by a circle of which the diameter is $\frac{1}{2\beta^2}$, and having a chord $\frac{\pi}{2\beta^2}$.

---

[1] For the earth the moments of inertia, A and C, with reference to principal axes through the centre, differ very little. The value of β may therefore be approximately written $\frac{n}{2}\sqrt{\frac{C}{Mg\gamma}}$, and the denominator is to be replaced (17) by $\frac{L}{\sin\theta}$. Substitute the value of L (19) and put, for the sun, $\frac{S}{r^3}=n_1^2$ (25), and the value of β becomes $\frac{n}{2n_1}\sqrt{\frac{C}{3(C-A)\cos\theta}}$, which is very large.

The value as depending on the moon's attraction is (28a), $\frac{n}{2n_2}\sqrt{\frac{C(1+\eta)}{3(C-A)\cos\theta}}$, of the same order of magnitude as before.

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If the value of ω is not 90°, the diameter of this circle will be $\frac{1}{2\beta^2}\sin\omega$; but the quantity $\frac{\pi}{2\beta^2}$ then measures an angle of an arc of a small circle having a radius $=\sin\omega$; and the chord of the curve is reduced in the same proportion as its sagitta, and the curve is still a cycloid.

The axis of figure *gyrates* therefore about the vertical through O as if it was attached to the circumference of a small circle of the minute diameter specified, the centre of which circles moves with a uniform horizontal velocity, which velocity is the *mean* rate of gyration; or the motion may be compared to that of a cone having its vertex at O and diameter of base $=\frac{1}{2\beta^2}\sin\omega$ (the axis of figure constituting an element of this cone) rolling upon a fixed conical surface, all the elements of which make, with the vertical, the angle ω; but this imaginary cone *is not fixed in the body* (save in the exceptional case of the moments of inertia A and C being equal). For the rotary velocity of the body is n, while that of the cone is $2\beta\sqrt{\frac{g}{\lambda}}=\frac{Cn}{A}$.

Since the rotary velocities of the body and cone are different, the *instantaneous axis* cannot move along the chord of the cycloid, *nor with uniform velocity.*

The common methods of investigating the Precession of the Equinoxes, founded upon the incipient rate of motion of the instantaneous axis, involve this error, which does not become apparent, simply because the moments of inertia A and C are, for the earth, so nearly equal.

The instantaneous axis will describe a prolate cycloid having the same chord as the common one $\left(\frac{\pi}{2\beta^2}\right)$, and a sagitta $=\frac{1}{2\beta^2}\sin\omega\left(\frac{A}{C}-1\right)$.

The mean rate of gyration is given by the coefficient of t in equation (9); it is $\frac{1}{2\beta}\left(\frac{g}{\lambda}\right)^{\frac{1}{2}}$, or substituting values (2) for β and λ;

$$\frac{Mg\gamma}{Cn} \tag{11.}$$

Thus far I have supposed that at the origin of time, or at the moment when the accelerating force commenced to act, the body had no other motion than a rotation, n, about its axis of figure. It remains to prove, that if, at this instant, there are *small* (compared to n) velocities about either or both the other principal axes, the rate of gyration will be the same.

The solution will be perfectly general if we suppose at this instant a velocity, m, about the axis of x only, and assume at same moment $\theta=\omega$, $\phi=90°$, we should get, instead of equations (3) and (4), the two following:—

$$\sin^2\theta\,\frac{d\theta^2}{dt^2}=\left[\frac{2Mg\gamma}{A}\sin^2\theta-\frac{2Cmn}{A}\sin\omega-\frac{C^2n^2}{A^2}(\cos\theta-\cos\omega)\right.$$
$$\left.-m^2(\cos\theta+\cos\omega)\right](\cos\theta-\cos\omega) \tag{12.}$$

$$\sin^2\theta\,\frac{d\psi}{dt}=\frac{Cn}{A}(\cos\theta-\cos\omega)+m\sin\omega$$

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Substituting in these $\omega-u$ for θ, rejecting all small quantities of the second order (among which is m^2), and introducing β and λ (see equations 2)

13. $$\frac{\lambda}{g}\frac{du^2}{dt^2}=2u\left(\sin\omega-2\beta\sqrt{\frac{\lambda}{g}}m\right)-4\beta^2u^2$$

14. $$\frac{d\psi}{dt}=2\beta\sqrt{\frac{g}{\lambda}}\frac{u}{\sin\omega}+\frac{m}{\sin\omega}$$

The integral of (13) (using k with its already given value) in (7) is

15. $$u=\frac{1}{2\beta^2}\left(\sin\omega-\frac{2\beta^2}{k}m\right)\sin^2 kt$$

Substitute in (14) and integrate

16. $$\psi=\frac{1}{2\beta^2}kt-\left(\frac{1}{4\beta^2}-\frac{m}{2k\sin\omega}\right)\sin 2kt$$

The coefficient t in (16) is identical with that of equation (9), $=\frac{Mg\gamma}{Cn}$, showing that although the character of the gyratory motion is altered, and the axis of figure, instead of moving on a common cycloid (which forms cusps) and coming periodically to rest, moves along a *prolate* cycloid or even *without undulation*, yet the rate of gyration is unchanged.

If $\omega=90°$ and $m=\frac{Cn}{Mg\gamma}$, u and $\frac{du}{dt}$ become zero for all values of t, and the body gyrates horizontally without nutation.[1]

In all that precedes, the revolving body has been supposed retained by a fixed point in its axis of figure, but *not* at its centre of gravity, while the accelerating force, being gravity itself, acts through that centre.

If, instead, the fixed point by which the body is retained is the centre of gravity, and the accelerating or disturbing forces any other whatever (provided their direction is invariable and their resultant acts through a fixed point of the axis), the

---

[1] This is the case referred to in the preceding paragraph in which the moment of the accelerating force (or couple) is equal to that of (what I have styled in the work before referred to) the "deflecting force," which has for its value the expression $\frac{Cnm}{M\gamma}$.

That this case should arise, a determinate relation between m, n, and g, expressed by the equation $\frac{mn}{g}=\frac{M\gamma}{C}$, is necessary.

When this relation exists, the movement may be represented by the rolling of a conical surface (the locus, in the body, of the instantaneous axis), described about the axis of figure with the angle (approximately) equal to $\frac{Mg\gamma}{Cn^2}$, upon another, all of the elements of which make, with the vertical, the angle $\omega-\frac{Mg\gamma}{Cn^2}$, (when ω is not 90°, the centrifugal as well as the deflecting force affects the relation between m, n, and g).

But no such relation is *essential* to the gyration expressed by (11); and, in the case of the precession of the equinoxes, the supposition of rolling cones is not realized. There are, probably, no two instants of time at which the precessional movements of the axis are identically the same.

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equations will be precisely the same, the moments of inertia A and C, referring to principal axes through the centre of gravity, and Mg expressing the intensity of the resultant of the forces, and γ the distance from the centre of gravity of the point through which the resultant acts.

In expression (11) $Mg\gamma$ is the moment of the force with respect to the point O, divided by the sine of the angle (θ) which its direction makes with the axis of figure. Denote that moment by L. Then the expression for the velocity of gyration (11) becomes,

17.
$$\frac{L}{Cn \sin \theta}$$

If the body in question, like the earth, is acted upon by forces, the resultant of which does not pass through its centre of gravity, its movements about that centre are precisely the same as if that centre were fixed; in other words, it will gyrate about the line connecting its centre and the origin of the force with a velocity denoted by expression (11). In the case of the earth, however, the direction of the disturbing force and its moment are constantly changing, and I have to assume something not proved in what foregoes, viz., that the *elementary gyration at each moment of time* will be likewise expressed by (11); an assumption *not* (probably) strictly true, since, when the forces are constant in direction and intensity, equation (14) shows (the value of u, equation (15) being substituted) that the gyratory velocity, though its mean is always expressed by (11), varies at each instant unless the value of m has a certain relation to that of k.

Since the integral of these varying elementary displacements shows, under all circumstances of constantly directed force (though these elementary motions of the axis exhibit all possible directions with regard to that of the force), a mean rate of gyration expressed by (11), we may assume that the fact will hold good though the direction and moment of the force change.[1]

In the case of the earth there is probably no instant of time at which it is revolving exactly about its axis of figure; the quantity m has, for it, in all cases, a finite (though exceedingly small) value; neither observation nor (scarcely) analysis can detect the minute diurnal (nearly) nutations which belong to the diurnal cycloidal movement; and hence the presumption that the gyration is *at all instants* perpendicular, or nearly so, to the direction of the force, and hence that even its elementary values vary little from expression (11).[2]

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[1] Such an assumption is made in all the investigations not, like Laplace's, purely analytical, without always giving the true grounds on which it should be based.

[2] In reality, if the moment L *remains the same* for different values of θ, the elementary *displacement* produced by the gyration is independent of θ, for, though the expression $\frac{L}{Cn \sin \theta}$ varies inversely as sin θ, yet the radius of the small circle on which the displacement takes place increases in like proportion. Again, that a revolving body should gyrate around a given axis it is not *necessary* that the accelerating force should be always parallel in direction to that axis, but that it should remain in the moving plane through the axis of figure and the given axis. The general equations of rotation would be the same.

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Let a, be the equatorial diameter of the earth.
b, be the polar diameter.
ρ, its variable density.
C, its moment of inertia about the polar axis.
A, its moment of inertia about an equatorial one.
$e = \frac{a-b}{a}$, earth's ellipticity.
S, the absolute attractive force of the sun, or its attraction upon a unit of mass at a unit's distance.
r, the mean distance of centres of sun and earth. x, y, z being rectangular co-ordinates of any element of the earth's mass, dm; the origin being the earth's centre, the axis of z the polar one, of x an equatorial one in a plane passing through the sun's centre, of y an equatorial one perpendicular to this plane.

The moment of the sun's attractive force upon the earth is shown in various works on precession (*vide* Mr. Airy's "Figure of the Earth," Encyc. Metropolitana) to be (θ being the angle of earth's axis with line drawn to sun).

$$18. \qquad \frac{3S}{r^3}\iiint \rho\,(x^2 - z^2)\,dx\,dy\,dz \sin\theta \cos\theta$$

the integral being taken through the spheroid.

The quantity under the signs of integration may be written

$\rho\,(x^2+y^2)\,dm - \rho\,(y^2+z^2)\,dm$, the integral of the first term of which is C, and of the second A.

Hence the moment of the sun's force (18)

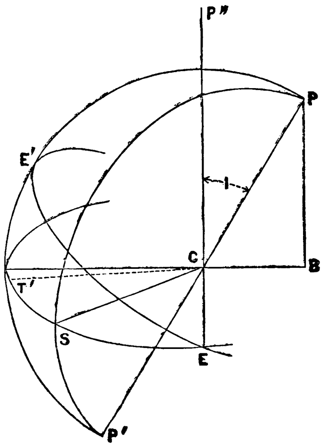
$$19. \qquad \frac{3S}{r^3}(C-A)\sin\theta\cos\theta = L$$

Hence the gyration produced upon the earth by the sun's force about the line of its direction is (17)

$$20. \qquad \frac{3S}{r^3 n}\,\frac{C-A}{C}\cos\theta;\ \text{and in time } dt,$$

$$21. \qquad \frac{3S}{r^3 n}\,\frac{C-A}{C}\cos\theta\,dt$$

Fig. 2.



Let EST be a great circle in the plane of the ecliptic, EE' an equatorial one, $PE'TP'$ a great circle through the tropics, PSP' one through the sun in any position, C the centre of the earth, and PCP' its axis. SCP' is the angle θ. If the sun moves in the ecliptic from E (the equinox) towards T with an angular velocity n_1, $n_1 t$ will be the value of the arc ES. In the spherical triangle $P'ST$, right-angled at T, we have cos $P'S$ (or cos θ)=cos $P'T$ cos TS=sin TE' sin SE.

TE' is the inclination of the equator to the ecliptic; call this I. Then

$$\cos\theta = \sin I \sin n_1 t$$

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The elementary gyration about the line *SC* will be therefore, (21)

22. $$\frac{3S}{nr^3}\frac{C-A}{C}\sin I \sin n_1 t\, dt.$$

If this rotation about *SC* is decomposed into components about the lines *TC* and *EC*, they will be

23. $$\frac{3S}{nr^3}\frac{C-A}{C}\sin I \sin^2 n_1 t\, dt.$$

24. $$\frac{3S}{nr^3}\frac{C-A}{C}\sin I \sin n_1 t \cos n_1 t\, dt.$$

The component (23) represents a rotation of the pole about *TC*, the radius of its motion being *PB*, or cos *I*. To obtain the actual value as an arc *of a great circle*, of this minute displacement, it must be multiplied by cos *I*; and to *refer this to the pole of the ecliptic as angular motion*, it must be divided by sin *I*.[1] Performing these operations, integrating, and remembering that by Kepler's laws $\frac{S}{r^3}=\frac{4\pi^2}{T^2}=n_1^2$ (T=number of units of time in one year), we get, for precession,

25. $$\frac{3}{2}\frac{n_1^2}{n}\frac{C-A}{C}\cos I.t-\frac{3}{4}\frac{n_1}{n}\frac{C-A}{C}\cos I \sin 2n_1 t.$$

And for nutation

26. $$\frac{3}{4}\frac{n_1}{n}\frac{C-A}{C}\sin I \cos 2n_1 t.$$

The first term of (25) is the mean solar precession; making $t=\frac{2\pi}{n_1}$, it gives for the annual solar precession

27. $$3\frac{n_1}{n}\pi\frac{C-A}{C}\cos I.$$

Expression (26) is the solar nutation, and the second term of (25) gives the *equation of the equinoxes in longitude*, or the fluctuating term of the precession corresponding to the nutation.

These expressions correspond to those obtained by the ordinary solutions. They differ from most of them, however, in having *C* in the denominator instead of *A*, an error of those solutions I have alluded to before, which, however real, analytically, exerts no important influence on the result.

By the above method the precession is the integral of the components of gyration about a solstitial diameter of the ecliptic, which line itself, by the process of precession, has an angular motion equal to that precession, the real effect

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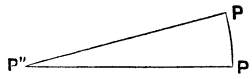
[1] 

In the spherical triangle $PP''P$ in which P'' is the pole of the ecliptic and PP an arc of a great circle through which the pole P has moved (equal to (23) × cos I), and the sides $P''P$ are $=I$, the angle $PP''P$ (or the elementary precession)$=\frac{PP}{\sin I}$.

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being a revolution about an axis (the pole of the ecliptic) perpendicular to the plane of that angular motion. In other words, if we integrate directly equation (23) and make $t=\frac{2\pi}{n_1}$, and $\frac{S}{r^3}=n_1^2$, we shall get

$$28. \qquad 3\frac{n_1}{n}\pi\frac{C-A}{C}\sin I;$$

and this will be the total angular motion of the pole P about the solstitial line TC *in one revolution of the sun;* but by this very motion of the pole the equinoxes have moved an angle measured by *this displacement referred to the pole of the ecliptic*—that is, by the angle expressed by (27)—and the solstitial line TC has of course, undergone the same movement, and the next annual gyration will be about the consecutive line $T'' C$, and so on; producing a continuous motion of the pole P about the pole of the ecliptic P''.

To obtain the precession due to the moon, it is necessary to substitute in (19) for $\frac{S}{r^3}$, $\frac{M^1}{(r)^3}$, in which M^1 is the attractive force of the moon and (r) its mean distance. But T^1 (time of moon's revolution) is, by Kepler's laws, $\frac{2\pi(r)^{\frac{3}{2}}}{(M^1+E)^{\frac{1}{2}}}=\frac{2\pi(r)^{\frac{3}{2}}}{[(1+\eta)M^1]^{\frac{1}{2}}}=\frac{2\pi}{n_2}$ (calling the mean angular velocity of the moon n_2 and the ratio of earth's mass to that of moon's mass, η) (28*a*); hence $\frac{M^1}{(r)^3}=\frac{n_2^2}{1+\eta}$.

If i is the inclination of the moon's orbit to the equator during any one revolution (regarded as constant for that time), we should obtain for the precession and nutation, *referred to the pole of the moon's orbit*, expressions analogous to (25) and (26).

Although the moon's disturbing effect, as above expressed, is almost exactly double that of the sun, yet the larger divisor n_2, introduced by integration, renders the value of (26) and of the fluctuating term of (25) very small for the moon—say about $\frac{1}{6}$th the corresponding values for the sun. Hence these terms are usually disregarded in the lunar expressions.

The elementary precession due to the moon *about the pole of its own orbit* would be by (25)

$$29. \qquad \frac{3}{2}\frac{n_2^2}{n(1+\eta)}\cdot\frac{C-A}{C}\cos i\,dt.$$

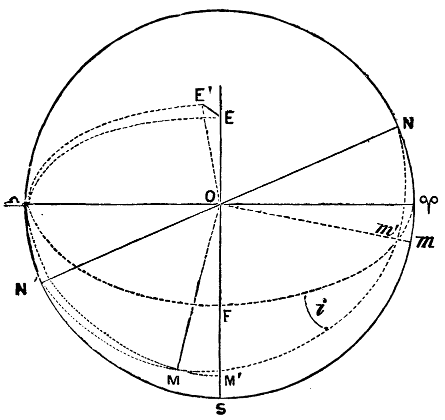
From this, by the usual methods, can be deduced the real precession and nutation. But it will be more in harmony with the object of this paper, and indeed more elegant, to reduce the *gyration* produced by the moon directly to precession and nutation.

If we substitute for n_1, $\frac{n_2}{1+\eta}$, and $\sin i$ for $\sin I$, in (28) we shall get, for the total gyration *about the line of greatest declination*, produced by one revolution of the moon in its orbit, the expression:

$$30. \qquad 3\frac{n_2}{n(1+\eta)}\pi\frac{C-A}{C}\sin i.$$

In which the line of greatest declination is regarded as stationary during the single revolution, and taking a consecutive position for the next; but it will be in harmony with the fact, and allowable, to regard the line as in continuous motion and the above amount of gyration to be uniformly spread over the time $\frac{2\pi}{n_2}$, of the revolution, producing thus an elementary gyration, in the time dt, of

31. $$\frac{3}{2}\,\frac{n_2^2}{n(1+\eta)}\,\frac{C-A}{C}\sin i\,dt.$$



Let ♈S♎ be a great circle in the plane of the ecliptic; ♈♎ the line of equinoxes, NON' the line of moon's nodes, ♈F♎ the equator, and $Nm'MN'$ the moon's orbit crossing the equator at m'. The line of the moon's maximum declination, OM, will be 90° from the line Om'.

The pole E of the earth is supposed to undergo a displacement by gyration about OM represented by EE'; the precession produced will be the angle EOE'; the nutation, the angle E♎E'.

In the spherical triangle Nm'♈ the angle at N is $= I'$, the inclination of moon's orbit to ecliptic; the angle at ♈ is the supplement of I (inclination of the equator) and the angle at m' is i (or the variable inclination of the moon's orbit to the equator), and the side ♈N is $= n_3 t$ (calling the angular velocity of the moon's node n_3); therefore,

32. $$\cos i = \cos I' \cos I + \sin I' \sin I \cos n_3 t$$

and

33. $$\text{tang } m'♈ = \frac{\sin n_3 t}{\sin I \cot I' - \cos I \cos n_3 t}.$$

In the spherical triangle mm'♈

34. $$\text{tang } m♈ = \cos I \text{ tang } m'♈$$

OS is the line of maximum declination of the sun, or the solstitial diameter of the ecliptic about which the annual gyration produced by the sun is made. As the inclination I' of the moon's orbit is small, the arc MM', drawn through M♎, is approximately equal to m♈, and the angle MO♎ differs immaterially from the complement of m♈; hence by (33) and (34)

35. $$\text{tang } MM' = \frac{\cos I \sin n_3 t}{\sin I \cot I' - \cos I \cos n_3 t} = \frac{\cos I \sin I' \sin n_3 t}{\sin I \cos I' - \cos I \sin I' \cos n_3 t}.$$

If the gyration about OM (31) is decomposed into components about OM' and O♎, we shall have for the first (calling the coefficient of $\sin i.dt$, K)

36. $$K \sin i \cos MM' dt,$$

and for the second

37. $$K \sin i \sin MM' dt.$$

To refer the *displacement* expressed by (36) to the pole of the ecliptic O, as angular motion, we must (see note page 8) multiply by cos $M'F$ or (from which it differs but slightly) cos i and divide by sin I; we thus obtain for the elementary *precession*

38. $$\frac{K}{\sin I} \cos MM' \sin i \cos i \, dt,$$

and for nutation

39. $$K \sin MM' \sin i \, dt.$$

The maximum value of the arc MM' is about 11° 56′; the line MO describing during an entire revolution of the moon's nodes an elliptical cone about OS of which the minor semi-diameter (SM') is 5° 8½′ (about), and the semi-major 11° 56′. By conceiving the elementary motion of the pole of the earth (or its gyration) as at each instant about the line MO, as it makes its conical revolution, the undulating nature of that motion, or the "nutation," is easily conceived.

Approximate values of sine and cosine of MM' may be determined from (35); which, substituted with those of i (32) in (38) and (39), will enable us to integrate and obtain very accurate expressions for the lunar precession and nutation.⑴ But these expressions, nearly free from errors of approximation, may be more elegantly determined as follows: When the angle of the moon's orbit with the equator is minimum, the angle $i=I-I'$; when maximum $i=I+I'$ (epochs corresponding to $n_3t=0$ and $n_3t=\pi$); the angle MM' is zero, and the corresponding *rates* of precession are by (38)

(*a*) $$K\frac{\sin 2(I-I')}{2 \sin I} \text{ for } n_3t=0.$$

(*b*) $$K\frac{\sin 2(I+I')}{2 \sin I} \text{ for } n_3t=\pi.$$

When $n_3t=\frac{1}{2}\pi$, we have

$$\cos MM'=\frac{\sin I}{\sqrt{1-\cos^2 I \cos^2 I'}}, \quad \sin MM=\frac{\cos I \sin I'}{\sqrt{1-\cos^2 I \cos^2 I'}}$$

$\cos M'F=\cos I \cos^2 I'$, $\cos i=\cos I \cos I'$, $\sin i=\sqrt{1-\cos^2 I \cos^2 I'}$ (the three first being the residuals of exact analytical expressions after omission of quantities of inappreciable magnitude).

Hence, by (38), the rate of precession for $n_3t=\frac{1}{2}\pi$ is

(*c*) $$K \cos I \cos^2 I' \; *$$

Assume the formula for precession to be

$$K(Pt+P' \sin n_3t+P'' \sin 2n_3t);$$

---

* When the moon's orbit intersects the ecliptic in a solstitial line, the elementary precession $K \cos i$ (29) *about its own pole* is reduced, with but slight error, to the same about the ecliptic pole, by simple multiplication by $\cos I'$: a result coinciding with the above.

⑴ See Additional Notes, p. 51.

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the *rates* of precession will be

(a') $\qquad K(P+n_3P'+2n_3P'')$ $\qquad$ for $n_3t=0$

(b') $\qquad K(P-n_3P'+2n_3P'')$ $\qquad$ " $n_3t=\pi$

(c') $\qquad K(P-2n_3P'')$ $\qquad$ " $n_3t=\frac{1}{2}\pi$

Equating (a) to (a'), &c., we deduce

$$P=\tfrac{1}{2}\cos I\,(\cos^2 I'+\cos 2I')=\cos I\,(1-\frac{3}{2}\sin^2 I')$$

$$P'=-\frac{1}{2n_3}\,\frac{\sin 2I'\cos 2I}{\sin I}=-\frac{1}{n_3}\cos I\sin 2I'\cot 2I$$

$$P''=-\frac{1}{4n_3}\cos I\,(\cos I'-\cos 2I')=-\frac{1}{4}n_3\cos I\sin^2 I'.$$

Hence the formula for precession may be written

44. $$\frac{3}{2}\,\frac{n_2^2}{n\,(1+\eta)}\,\frac{C-A}{C}\cos I\Big[(1-\frac{3}{2}\sin^2 I')\,t-\frac{1}{n_3}\sin 2I'\cot 2I\sin n_3t - \frac{1}{4n_3}\sin^2 I'\sin 2n_3t\Big].$$

Similarly we would get for the nutation

45. $$\frac{3}{2}\,\frac{n_2^2}{n\,(1+\eta)}\,\frac{C-A}{C}\cos I\,\frac{\sin I'}{n_3}\cos n_3t.$$

The ratio of actual lunar precession to what it would be were the moon's orbit in the ecliptic, is therefore expressed by

$$1-\frac{3}{2}\sin^2 I'=0.99 \text{ (very nearly).}$$[1]

The third term of (44) indicates a slight periodical variation from the true elliptic motion referred to in the next paragraph. There should be a corresponding term in (45) which may be obtained by the same process, but they are both too minute to enter into computations.

---

[1] It is worthy of remark that the formulæ of Laplace [3100] and [3101] (Bowditch) contain no such coefficient qualifying the mean lunar precession, though one is found in all the more popular solutions; neither do they contain the term (quite minute) in $2n_3t$ of (44), but, on the other hand, contain terms in $2n_2v$ (corresponding to the terms in $2n_1t$ of 25 and 26), which, referred to in the fourth par. (page 9), are generally omitted as inappreciable.

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If we multiply the coefficient of sin n_3t in (44) by sin I, we shall have the value as an arc of a great circle of this fluctuating displacement called *the equation of the equinoxes in longitude.* The coefficient thus modified will represent the minor semi-diameter, and that of the nutation proper (45), the major semi-diameter of the ellipse of nutation; they have the ratio cos $2I$: cos I nearly. This ellipse has its major axis (equal to about 18″ of arc) directed towards the pole of the ecliptic. The period of its description $\frac{2\pi}{n_3}$ is that of the revolution of the moon's nodes. At the same time a minute ellipse of semi-annual nutation due to the sun is superimposed upon this. It has its longer axis (about one second of arc) likewise directed to the pole of the ecliptic. The smaller axis is to the major as cos I: 1.

It is easy to show that the precession caused by the sun and moon is equal (with slight difference due to the ratio we have just been considering) to what it would be if those bodies were uniformly distributed in solid rings over circles (in the plane of the ecliptic) about the centre of the earth, having radii equal to their mean distances. In this case, unless there was a particular relation between the couple producing the initial rotation of the earth and that arising from the attraction of the two rings, there would be an extremely minute nutation, of which the period would be (see equation 9) $\frac{2\pi}{2k}=\frac{2\pi}{n}\frac{A}{C}$; which is almost identical with the siderial day $\left(\frac{2\pi}{n}\right)$, the ratio being $\frac{A}{C}$: 1. If we suppose the primitive rotation of the earth to be that alone, about its axis of figure n, then the nutation will exhibit the common cycloidal motion of equation (10); but its total amount would be but about $\frac{1}{55}$ second of arc.

An explanation of the deviation of rifled projectiles will be found in what is said (pages 8 and 9) in reference to the conversion of gyration about a shifting axis into precession about an axis perpendicular to the plane of motion of the first. Elongated rifled projectiles, while they maintain almost unaltered their "angle of elevation," are found to deviate with great uniformity from the vertical plane of projection, in a direction corresponding to the twist of the gun, while spherical projectiles (fired from rifled guns) having precisely the same rotary motion, do not so deviate; showing that the cause of the phenomenon is something else than the direct action of friction or pressure of the air.

In the remarks made in the paragraphs just referred to, it is explained how the consecutive small annual gyrations about the line from the centre of the earth to the sun (in the tropics) become, in their integral, the movement which we call precession, about an axis perpendicular to the plane of motion of that line, inasmuch as each small primary gyration causes a corresponding shifting of the line about which it takes place. Something very similar occurs to produce the deviation of elongated projectiles. In issuing from the gun, the resultant of the atmospheric resistance (denoted by the arrow R) coincides with the axis of the projectile, and it has no other effect than to retard the motion of translation; but the action of gravity causes the trajectory to curve downwards, and the direction of the atmospheric resistance becomes oblique to the axis of the projectile (at a'), and (in

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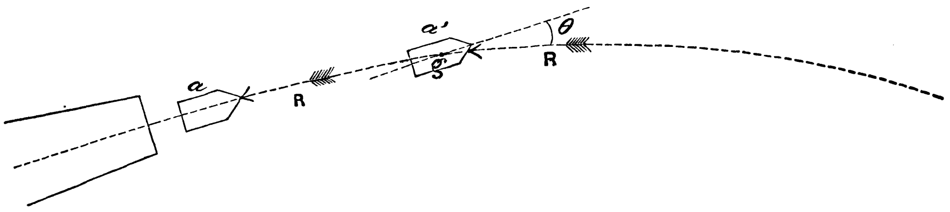
almost all forms of projectiles) passes *above*, not through, the centre of gravity g. We have then the essential conditions of gyration, viz., a solid of revolution revolving rapidly about its axis, and a dynamic "couple" (*i. e.*, the inertia of the projectile's motion of translation acting through its centre of gravity, and the resistance of the air acting through a point of the axis more or less distant from g) tending to turn the projectile upwards about a horizontal (or "equatorial") axis through g, and there is in fact, at each instant, an elementary gyration about a line through g, parallel to R (the atmospheric resistance). If this line retained an invariable direction, the integral effect of these elementary gyrations would be to revolve *down* the axis of the projectile, and we should ultimately find it assuming horizontal and even sub-horizontal directions. But such cannot be the case; the direction of the axis is no sooner deviated, laterally, from its original direction, than a (nearly) corresponding change takes place in the direction of the resistance R (since from the elongated form of the projectile, the direction of its motion follows pretty nearly that of its axis) and in that of the line (parallel to R) about which gyration takes place. The integral of such a series of elementary gyrations, accompanied by a *corresponding* horizontal angular motion of the line about which they take place, is angular motion about a line perpendicular to the plane in which that line shifts direction, that is, about a vertical. Hence the vertical direction (or "elevation") of the axis of the projectile remains constant, or nearly so, while its horizontal direction undergoes a progressive angular precession (if I may so term it), and the deviation of rifled projectiles is thus seen to have analogy with the precession of the equinoxes.[1]



In what precedes I do not profess to throw new light on a subject so thoroughly studied as the Precession of the Equinoxes; my object has been rather to make evident the analogy that exists between "the minute motions of the gyroscope and that grand phenomenon exhibited in the heavens," and to show how a common analysis applies to both.

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[1] It is quite probable that there are other causes of deviation, the friction of the air being (in case of long ranges) one. Experimental facts are needed for a full discussion of this subject.

ON THE

MOTIONS OF FREELY SUSPENDED AND GYROSCOPIC PENDULUMS, AND ON THE PENDULUM AND GYROSCOPE AS EXHIBITING THE ROTATION OF THE EARTH.

LET the point of suspension of the pendulum be taken as the origin of rectangular co-ordinates, the axis of z vertical (downwards), and those of x and y in the plane of the horizon, the former directed to the east, the latter to the north.

The forces which act on the pendulum (considered as concentrated in its centre of oscillation) are

1st. Gravity, the resultant of the earth's attraction, and of the centrifugal force of its rotation.

2d. The tension of the pendulum cord.

3d. The force of inertia, the components of which are represented by the differential coefficient $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$, $\frac{d^2z}{dt^2}$; and

4th. The disturbing forces arising from the earth's rotation.

5th. The resistance of the air.

If we represent the length of the string by l, its tension by N, and the force of gravity by g, and neglect the forces named in the 4th and 5th categories, we shall have the three equations:

$$(1)\qquad \begin{aligned} \frac{d^2x}{dt^2} &= -\frac{Nx}{l} \\ \frac{d^2y}{dt^2} &= -\frac{Ny}{l} \\ \frac{d^2z}{dt^2} &= -\frac{Nz}{l}+g. \end{aligned}$$

The forces due to the earth's rotation which disturb the *relative* motions of a projectile, or any material particle moving near the earth's surface, have been expressed by Poisson (Journal de l'Ecole Polytechnique Cahier, 26) as follows:—[1]

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[1] These expressions are perfectly general and applicable to *all* problems which involve relative motions near the earth's surface, whether of solids or fluids. They (or their equivalent) appear in the work of Laplace (Mec. Cel., Vol. IV.), as well as of Poisson (from whom I have quoted them), in the investigations of the motions of projectiles. It is somewhat extraordinary that both of these great analysts should have failed to perceive their remarkable application in the motions of the gyroscope and pendulum, to the exhibition of the earth's rotation, although the latter has seemed to desire such an exhibition in the sentence: "Quoique la rotation de la terre soit maintenant établie

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$$(2) \qquad \begin{aligned} X &= 2n\left(\frac{dy}{dt}\sin\lambda + \frac{dz}{dt}\cos\lambda\right) \\ Y &= -2n\,\frac{dx}{dt}\sin\lambda \\ Z &= -2n\,\frac{dx}{dt}\cos\lambda. \end{aligned}$$

In which λ is the latitude and n the angular velocity of rotation of the earth.

These analytical expressions make their appearance in the transformations of the equations of motion, near the earth's surface, expressed in co-ordinates referring to *fixed* axes, into others referring to the moving axes which are used in this analysis. But these forces can be obtained and their origin better understood by the following considerations.

The centrifugal force of a material point at rest on the earth's surface at the given latitude will be $\frac{n^2 r^2 \cos^2\lambda}{r\cos\lambda}$ (r being the earth's radius). If it has a small relative velocity $\frac{dx}{dt}$ to the east, the centrifugal force will become $\frac{\left(n\,r\cos\lambda + \frac{dx}{dt}\right)^2}{r\cos\lambda}$ Subtracting the former expression from the latter (omitting $\frac{dx^2}{dt^2}$) we get for the centrifugal force arising from the relative velocity $\frac{dx}{dt}$ the expression $2n\frac{dx}{dt}$. The component of this in the direction of the axis of y will be $-2n\sin\lambda\frac{dx}{dt}$, which corresponds to the value of Y (equations 2) of Poisson, and is to be added to the second member of the second of equations (1).

The component of the force just calculated in the direction of the axis of Z is

$$-2n\cos\lambda\frac{dx}{dt}.$$

This force corresponding to Poisson's value of Z is to be added to the second member of the third of equations (1).

A body moving on a meridian of the earth's surface from south to north will have the moment of its quantity of motion, with reference to the earth's axis, diminished; in virtue of which it will press with a certain force towards the east

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avec toute la certitude que les sciences physiques comportent, cependant une *preuve directe* de ce phénomène doit interesser les géomètres et les astronomes." The former, in making a partial application to the pendulum, of his investigations, absorbed, apparently, in the single object of proving that the accuracy of the instrument as a measure of time was not affected, has inadvertently assumed that the disturbing force normal to the plane of oscillation is "trop petite pour écarter sensiblement le pendule de son plan et avoir aucune influence appreciable sur son mouvement." (Journal de l'Ecole Poly. Cahier, 26, p. 24.) It is true, indeed, that the force he mentions, even if permitted free action, will have but an inappreciable influence upon the time, and none whatever when, as in the chronometer, the plane is constrained to fixedness; but the effect is *cumulative* in changing the azimuth of the freely suspended pendulum.

These same disturbing forces, introduced along with the attractions of the sun, moon, and earth, into the general equations of equilibrium of fluids, produce in a very simple manner the differential equations for the tidal motions. (*Vide* American Journal of Science, 1860.)

and the moment of the force thereby developed is equal to $\frac{d}{dt}$ (the moment of its quantity of motion). This moment for the pendulum in any latitude λ, is $nr^2 \cos^2 \lambda$, of which the differential coefficient, taken with reference to λ as a function of t, is $-2nr^2 \sin \lambda \cos \lambda \frac{d\lambda}{dt}$, which is the moment of the force required. Dividing by the radius of rotation, $r \cos \lambda$, we have $+2nr \sin \lambda \frac{d\lambda}{dt}$ for the expression of the force which (acting positively in the direction of the axis of x) is to be taken with the plus sign.

In the system of rectangular co-ordinates which I am using $\frac{dy}{dt}$ corresponds to the velocity expressed by $r \frac{d\lambda}{dt}$; and substituting it therefor, we have for a disturbing force in the direction of the axis of x the expression

$$2n \sin \lambda \frac{dy}{dt}$$

In almost precisely the same way it may be shown that a body falling towards the centre of the earth with a velocity $\frac{dz}{dt}$ will have the moment of its quantity of motion diminished by $2nr \cos^2 \lambda \frac{dz}{dt}$, giving rise to the force

$$2n \cos \lambda \frac{dz}{dt}$$

The sum of these two expressions constitutes the disturbing force X of Poisson, and is to be added to the second member of the first of equations (1), and these equations become[1]

$$(3) \qquad \begin{aligned} \frac{d^2x}{dt^2}+\frac{Nx}{l}&=2n \sin \lambda \frac{dy}{dt}+2n \cos \lambda \frac{dz}{dt} \\ \frac{d^2y}{dt^2}+\frac{Ny}{l}&=-2n \sin \lambda \frac{dx}{dt} \\ \frac{d^2z}{dt^2}+\frac{Nz}{l}&=g-2n \cos \lambda \frac{d'x}{dt} \end{aligned}$$

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[1] There are really other disturbing forces (comparatively slight indeed) than the X Y and Z of Poisson (equation 2), as appears from the following considerations :—

Draw a line through the origin of co-ordinates parallel to the axis of the earth, and project the moving body on the plane of $y\,z$. The distance of the projection from the line will be $y \sin \lambda + z \cos \lambda$, the distance of the body from the plane of $y\,z$ being x: hence there will be a centrifugal force relatively to this line, due to the earth's rotation, tending to increase the ordinates $x\,y\,z$ by its components

$$n^2 x$$
$$n^2 \sin \lambda\, (y \sin \lambda + z \cos \lambda)$$
$$n^2 \cos \lambda\, (y \sin \lambda + z \cos \lambda)$$

With these expressions added, respectively, to the second members of equations (3), they correspond to those found in Carmichael (Calcul. of Operations), who quotes from Galbraith and Houghton (Proc. R. Irish Acad., 1851). They express forces of the second order in minuteness, compared with those expressed by equations (2), and, insensible in their effects, are neglected in all discussions. They are noticed here only to recognize their existence and to show their origin.

Since $x^2+y^2+z^2=l^2$, we have $x\,dx+y\,dy+z\,dz=O$. Hence multiplying equations (3), respectively, by dx, dy, and dz, and adding, we have

$$\frac{dx\,d^2x+dy\,d^2y+dz\,d^2z}{dt^2}=gdz$$

$$(3)_2 \qquad \frac{dx^2+dy^2+dz^2}{dt^2}=2gz+c$$

This expression is independent of n, and the velocity at any point of the path depends, in the same way as does that of a pendulum vibrating over a motionless earth, upon the height of fall. The plane of vibration of the chronometer pendulum is maintained in a fixed relative position, thereby differing from a "freely suspended" pendulum. It will be seen hereafter, in treating of the gyroscope pendulum, that the forces which maintain this relative fixedness are equivalent to a force varying directly as the angular velocity, applied at the centre of gravity, normally to the path. Such a force will have no influence upon the velocity. Hence the time of vibration of the chronometer pendulum is not affected by the earth's rotation, nor by the azimuth angle of the plane of vibration.[1]

Multiplying the first of equations (3) by y, and the second by x, and adding, we get:—

$$y\frac{d^2x}{dt^2}-x\frac{d^2y}{dt^2}=2n\sin\lambda\left(y\frac{dy}{dt}+x\frac{dx}{dt}\right)+2n\cos\lambda y\frac{dz}{dt}$$

Integrating:—

$$(4) \qquad y\frac{dx}{dt}-x\frac{dy}{dt}=n\sin\lambda\,(x^2+y^2)+C+2n\cos\lambda\int yaz$$

The above (4) expresses that the moment of the quantity of motion about the axis of z is equal to a constant C (depending upon any arbitrarily given initial value) increased by what is due to the constant angular motion $n\sin\lambda$, and by the area $2\int ydz$ (in the case of ordinary plane vibration this is the projection on

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[1] This conclusion is not invalidated by the introduction of the disturbing forces of the order n^2 referred to in note to p. 17, for, since the arc of vibration of the chronometer pendulum is exceedingly small, z may be considered as equal to l, the pendulum's length, and y as very minute. Those forces will thence be

$$n^2x$$
$$\tfrac{1}{2}\,n^2l\sin 2\lambda$$
$$n^2\,l\cos^2\lambda$$

The third of these is an *increment to gravity*, and the first tends to prolong vibrations in the prime vertical. The second is null in its effects, since, being always positive, it retards the vibration in one direction as much as it accelerates it in the other. But they are all *inappreciably* minute, the last being, for the seconds pendulum, an increment to the force of gravity at the equator of about $\frac{1}{1.800.000.000}$, decreasing the time of vibration by about $\frac{1}{3.600.000.000}$. The first has the contrary tendency to increase the time of vibrations if made in a prime vertical (or any other plane than a meridian), but its effect is equally inappreciable even when (as in the prime vertical) it is a maximum.

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the plane of yz of the circular segment included between the arc and chord of vibration), multiplied by $n \cos \lambda$. This multiplier, which is the component of the earth's rotation about a diameter of the earth normal to that passing through the locality, indicates that the term $2n \cos \lambda \int y\, dz$ expresses a disturbance produced by this complementary component. As this term for vibratory motion is small and periodic, passing through nearly equal positive and negative values in the course of a double vibration, it follows that $n \sin \lambda$ expresses the mean increment of angumortion, or, in other words, that, to the plane or spherical vibrations exhibited by the pendulum over a motionless earth, there is, superadded, in consequence of this rotation, a uniform azimuthal motion measured by the earth's rotating velocity multiplied by the sine of the latitude. This is the material fact or peculiar feature of the freely suspended pendulum, and we see that it is exhibited by equation (4) *generally* for all *ordinary* vibrations, whether plane or spherical. We shall see hereafter, however, that the disturbing term of equation (4) $2n \cos \lambda \int y\, dz$ expresses a tendency to a like motion about the complementary axis, and that, on the supposition of an infinite velocity, this tendency may be realized, and the plane of motion, by the joint effect of the two components, turn around a parallel to the earth's axis, with an angular velocity equal and contrary in direction to n.

In the case of *very small* deviations from the vertical, the equations (3) may be solved as follows: The variations of z then become of the second order of minuteness compared with those of x and y, and omitting them we have between x and y and their differentials the relations

$$(4)_2 \qquad \begin{aligned} \frac{d^2x}{dt^2}+\frac{Nx}{l}-2n \sin \lambda \frac{dy}{dt}&=0 \\ \frac{d^2y}{dt^2}+\frac{Ny}{l}+2n \sin \lambda \frac{dx}{dt}&=0 \end{aligned}$$

the integrals of which are (Gregory Examp. p. 390),

$$(5) \qquad \begin{aligned} x&=+\Sigma\,(D \cos \beta t+E \sin \beta t) \\ y&=-\Sigma\,(D \sin \beta t-E \cos \beta t) \end{aligned}$$

in which to β is given both the values obtained from the quadratic equation

$$a_0-a_2\,\beta^2=a_1\,\beta \; *$$

in which $a_0=\frac{g}{l}$, $a_1=-2n \sin \lambda$, $a_2=1$; hence solving the quadratic

$$\beta=-\frac{1}{2}\,\frac{a_1}{a_2}\pm\sqrt{a_0+\frac{1}{4}\left(\frac{a_1}{a_2}\right)^2}$$

Substituting the values of a_0, &c., and omitting the second term under the radical as inappreciably small, we have

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* This equation, $\frac{g}{l}-\beta^2=2n \sin\lambda\, \beta$, can be got by substituting the integrals $x=C \cos(\beta t-\varepsilon)$, $y=C \sin(\beta t-\varepsilon)$ in the given equations.

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$$\beta = n \sin \lambda \pm \sqrt{\frac{g}{l}}$$

Representing these two values by β_1 and β_2, equations (5) may be put in the form (by writing for D_1 and E_1, $C_1 \cos \varepsilon_1$ and $C_1 \sin \varepsilon_1$, &c., and reducing)

$$\begin{aligned} x &= \quad C_1 \cos (\beta_1 t - \varepsilon_1) + C_2 \cos (\beta_2 t - \varepsilon_2) \\ y &= -C_1 \sin (\beta_1 t - \varepsilon_1) - C_2 \sin (\beta_2 t - \varepsilon_2) \end{aligned}$$

Assuming for $t=0$, $x=0$ and $\frac{dy}{dt}=0$, it will give $\varepsilon_1 = \varepsilon_2 = \frac{1}{2}\pi$, and the above become

$$\begin{aligned} x &= C_1 \sin \beta_1 t + C_2 \sin \beta_2 t \\ y &= C_1 \cos \beta_1 t + C_2 \cos \beta_2 t \end{aligned}$$

Instead of the arbitrary constants C_1 and C_2 we may write $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$, at the same time substituting the values of β_1 and β_2 and developing; by which the preceding equations become (putting $n \sin \lambda = n'$)

$$(6) \qquad \begin{aligned} x &= A \cos \sqrt{\frac{g}{l}}\, t \sin n' t + B \sin \sqrt{\frac{g}{l}}\, t \cos n' t \\ y &= A \cos \sqrt{\frac{g}{l}}\, t \cos n' t - B \sin \sqrt{\frac{g}{l}}\, t \sin n' t \end{aligned}$$

If we transfer the co-ordinates now referring to (relatively) *fixed* axes, to others *moving* with the relative angular velocity n', that is, if we transfer to axes making at any instant the angle $n \sin \lambda\, t$ with the fixed ones, the new co-ordinates will have the values

$$\begin{aligned} x' &= x \cos n't - y \sin n't \\ y' &= x \sin n't + y \cos n't \end{aligned}$$

or, substituting values of x and y,

$$x' = B \sin \sqrt{\frac{g}{l}}\, t$$

$$y' = A \cos \sqrt{\frac{g}{l}}\, t$$

From which we may obtain

$$A^2 x'^2 + B^2 y'^2 = A^2 B^2$$

which is the equation of an ellipse, having A and B for semi-transverse and semi-conjugate axes. If $B=0$ the ellipse becomes a right line, hence the earth's rotation causes an azimuthal motion of this line, or of the axes of the ellipse if the motion is elliptical, equal to the component of that rotation about the local axis and in the reverse direction.

The motions of the "gyroscope pendulum," which is but the ordinary gyroscope with an exceedingly long arm (or distance γ, of my analysis, from the point of support in the axis to the centre of gravity), are indicated by equations precisely similar to the above, deduced from an identical analysis; always assuming, as in the solutions just given, that the arcs of vibration are small, so that *vertical* motions may be disregarded. To prove this, I refer to my expression (h) and the context, in my analysis of the "Gyroscope."[1]

$$(\alpha) \qquad -\frac{Cn}{\gamma M}v_s$$

for the deflecting force, as I call it (a force due to the rotation of the disk with angular velocity n, and acting, at the centre of gravity, normally to the plane of angular motion of the disk-axis, or of the arm of the gyroscope pendulum), in which M is the mass, C its moment of inertia about the disk-axis, γ the distance from its centre of gravity to point of suspension, and v_s the angular velocity.

Disregarding the vertical motions represented by $\frac{dz}{dt}$ on account of the smallness of the arcs, $\frac{1}{l}\frac{dx}{dt}$ and $\frac{1}{l}\frac{dy}{dt}$ (I substitute l for the γ mentioned above) would represent very nearly the components of angular velocity of the centre of gravity. Substituting these for v_s we shall get the components of the "deflecting force," and the equations of motion will be,

$$\frac{d^2x}{dt}+\frac{Nx}{l}=-\frac{Cn}{l^2M}\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2}+\frac{Ny}{l}=+\frac{Cn}{l^2M}\frac{dx}{dt}$$

$$\frac{d^2z}{dt}+\frac{Nz}{l}=+g$$

These equations are identical in all but the value of the coefficients with (3), when transformed to $(3)_2$, under the same license. Of course the motions of the gyroscope pendulum would have the same solutions, the mean azimuthal motion of the nodes of its orbit being expressed by half the coefficient of $\frac{dy}{dt}$, $-\frac{Cn}{2l^2M}$, or, since the moment of inertia A, of the gyroscope, with reference to a principal axis through the point of support, is (l being supposed to be very large compared to the dimensions of the disk) very nearly l^2M, the mean azimuthal motion is more simply expressed by $\frac{Cn}{2A}$.

This may be more generally proved as follows: The first of the general differential equations (equations 4 of my analysis) of gyroscopic motion is

$$\sin^2\theta\frac{d\psi}{dt}=\frac{Cn}{A}(\cos\theta-c)$$

in which θ, counted from the inferior vertical, is the variable inclination, and ψ the azimuth angle of the disk-axis or pendulum arm, and c a constant depending on initial values of θ and $\frac{d\psi}{dt}$.

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[1] See American Journal of Science, 1857, and Barnard's American Journal of Education, 1857.

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Develop $\cos\theta = -(1-\sin^2\theta)^{\frac{1}{2}}$ and we get

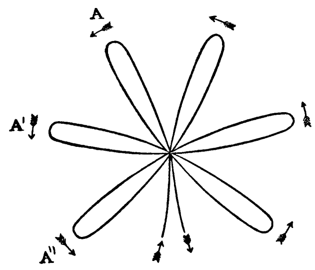
$$\frac{d\psi}{dt} = -\frac{Cn}{2A}\left(1+2\frac{1-c}{\sin^2\theta}-\tfrac{1}{4}\sin^2\theta-\tfrac{1}{8}\sin^4\theta-\&c.\right)$$

For ordinary ranges of pendulum vibration the terms involving positive powers of $\sin\theta$ (which express an excess of nodal motion for large excursions) may be omitted, and we have

$$\frac{d\psi}{dt} = -\frac{Cn}{2A}+\frac{Cn}{A}\,\frac{1-c}{\sin^2 0}$$

Thus we see that for small vibrations, whether spherical or plane, the azimuthal motion is made up of a uniform progression of the nodes $\frac{Cn}{2A}$, and a fluctuating term which represents the angular velocity *in the orbit.* Indeed we have, in the second term, the motions of the spherical pendulum.

If we suppose the pendulum to have been propelled from a state of rest in the vertical, $\frac{d\psi}{dt}$ must have a finite value when θ is indefinitely small, and c must hence be unity. Hence we see that at the very outset the initial value $\frac{Cn}{2A}$ must be attributed to $\frac{d\psi}{dt}$, and that the pendulum reacquires it at every return excursion, that is, whenever θ diminishes indefinitely. Hence the pendulum continues to pass through the vertical at every return. The horizontal projection of the curve would be a series of loops radiating from a common centre. For each complete vibration the integral of $\frac{Cn}{2A}dt$ would represent the entire angular motion of the nodal axis (much exaggerated in the diagram) from A to A', &c., and the integral of the remaining terms should be 2π. These loops are in fact but the path a pencil attached to a common pendulum would trace upon a paper beneath, turning with uniform angular velocity about the projection of the point of suspension.



Though the numerator of the fraction $\frac{Cn(1-c)}{A\sin^2 0}$ becomes zero for the case just considered, it is evident that, at the moment of passing through the vertical, the limiting value must be considered infinity, and that the integral through the infinitely short time of passage must be π; for the azimuthal position undergoes, at that instant, an increment (or decrement) of a semi-circumference. There is an identical case in the spherical pendulum. Regarding plane as the final limit of narrowing spherical vibrations, it is evident that the azimuthal velocity of passage by the vertical becomes very great and has its limit infinity when they pass *through* the vertical.

This expression $\frac{Cn}{2A}$(equal to $\beta\sqrt{\frac{g}{\lambda}}$ nearly) is a very different thing from the "mean precession" of the gyroscope, $\frac{1}{2\beta}\sqrt{\frac{g}{\lambda}}$, given in my analysis. The latter is

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the mean azimuthal motion of the body itself, the former that of the nodes of its orbit. The latter is strictly true only for very great values of β; the former, rightly interpreted, is always true, though it has no special applicability except for (as in the gyroscope pendulum) small values of β. The harmony of the two expressions is easily shown.

Practically the gyroscope is made up of not only a rotating disk, but a non-rotating frame. In estimating the "deflecting force," therefore, in the expression (α) C should apply to the disk alone, and M and γ to the entire mass of disk frame and stem.

The solutions that have been given of equations $(4)_2$ for the freely suspended pendulum are restricted to very small motions; the following is general.

Transfer equations (4) to polar co-ordinates by substituting for x, y, z the values[1]

$$x=l \sin \phi \sin \theta$$
$$y=l \cos \phi \sin \theta$$
$$z=l \cos \theta$$

in which ϕ denotes the azimuth of the pendulum measured from the north, and θ its deviation from the vertical, and we get

$$\frac{d\phi}{dt}=n \sin \lambda+\frac{C}{l^2 \sin^2 \theta}-\frac{2n \cos \lambda}{\sin^2 \theta}\int \cos \phi \sin^2 \theta \, d \theta \tag{7}$$

in which C is a constant depending on arbitrary initial values of $\frac{d\phi}{dt}$, the final term corresponding to the last term of (4).

At the equator we have $\lambda=0$, and the azimuthal velocity expressed by the third term of (7) becomes $-\frac{2n}{\sin^2 \theta}\int \cos \phi \sin^2 \theta \, d\theta$, which being periodic, produces but very minute change in the plane of vibration. If the pendulum is propelled, from a state of rest in the vertical, in the direction measured by the angle ϕ from the meridian, this angle will be but very slightly affected by the minute values of the above expression during the *outward* excursion, and the increment which $\frac{d\phi}{dt}$ receives will be *almost* exactly neutralized (*quite* so if $\cos \phi$ were absolutely invariable) during the return, and the angular velocity due to the term will again become zero; which cannot happen unless the pendulum *again* pass through the vertical on its return, in which case ϕ will be as little varied during the return; (*otherwise* ϕ will, during the return, pass through all possible values from 0 to $\frac{1}{2}\pi$, and integration is impracticable). Hence we may assume ϕ as constant, and, as in any other latitude, the term in question is, multiplied by $\cos \lambda$, the same as at the equator, we may generally integrate that term *for plane vibrations*, considering ϕ constant, and putting $C=O$. Equation 7 thus becomes,

$$\frac{d\phi}{dt}=n \sin \lambda-n \cos \lambda\frac{\theta-\sin \theta \cos \theta}{\sin^2 \theta} \tag{8}$$

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[1] The following analysis, as far as equation (9), is modified from Galbraith and Houghton. Proc. R. I. Acad.

If the amplitudes of the vibration are very minute, so that θ may be substituted for sin θ in (7), the integral becomes

$$\text{(9)} \qquad \frac{d\phi}{dt}=n \sin \lambda-\frac{2}{3} n \cos \lambda \cos \phi\ \theta$$

expressing the precession in azimuth, $n \sin \lambda$, precisely as it results from the former analysis (equation 6). The slight periodic disturbance expressed by the second term of (9) escaped that analysis, however, owing to the omission of the terms involving dz.

In the above integrals the angle ϕ must be taken at 180° greater or less on one side of the vertical than on the other, and the parts of $\frac{d\phi}{dt}$ expressed by it will have contrary signs. Hence the curve described in each complete excursion, disregarding the superadded uniform azimuthal motion expressed by the first term of the second member of (9), will have the form of an excessively attenuated leminiscate, or figure of 8.(1)

For greater amplitudes equations (8) will apply until θ becomes nearly equal to 180°; if θ equals or exceeds 180°, it *cannot be assumed* that the pendulum will pass through the zenith (the condition for ϕ to remain nearly constant), and the integral becomes inapplicable and erroneous.

The foregoing integrals involve the condition that the pendulum shall pass through the vertical, and imply that vibration is induced by propulsion from a state of rest in the vertical. But, in the usual form of the experiment for exhibiting the rotation of the earth, the pendulum starts from a state of *relative* rest at the extremity of the initial vibratory arc.

If we disregard the symbolic integral of (7), as may be done, since the minute periodic disturbance it measures has no influence upon the permanent azimuthal motion, that equation will become

$$(a) \qquad \frac{d\phi}{dt}=n \sin \lambda+\frac{C}{l^2 \sin^2 \theta}$$

The second term of the second member is identically the equation of the "spherical pendulum." The latter, we know, exhibits an azimuthal motion of the apsides of its orbit, very minute when C is small, but incomparably greater than the horary azimuthal motion when, C being large, the conjugate dimensions of the orbit approaches equality to the transverse, and of which the limit corresponding to perfect equality of these dimensions is for one vibration,

$$(a)_2 \qquad \pi\left[\frac{2}{\sqrt{1+3 \cos^2 \theta_2}}-1\right]$$

as may be deduced from the expression for U_π, par. 731, Peirce, Analyt. Mech., or from expression [79] and [83] of Méc. Cél. (Bowditch), by making $a=b$ and determining the corresponding values of c and dt.

In the case under consideration the value of C will be determined by making $\frac{d\phi}{dt}=0$ for the commencement of motion, $\theta=\theta_2$, hence

$$C=-n\, l^2 \sin \lambda \sin^2 \theta_2$$

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(1) See Additional Notes, p. 51.

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The pendulum will not move in a plane passing through the vertical, but on a conical surface differing slightly from such a plane, and there will ensue a slight apsidal motion reverse to, and diminishing, the apparent horary motion.

(*b*) The equation $\frac{d\phi}{dt}=\frac{C}{l^2\sin^2\theta}=\frac{C}{l^2-z^2}$ is usually solved by the aid of elliptic integrals (*vide* Prof. Peirce's Analyt. Mech., p. 418); but for present objects the ordinary processes of integration are preferable.

From equations $(3)_2$ and (4) may easily be deduced, neglecting terms containing n, and bearing in mind that

$$x^2+y^2=l^2-z^2,\ x\,dx+y\,dy=-z\,dz,$$

$$(c)\qquad dt=\frac{-ldz}{\mp\sqrt{(l^2-z^2)(c+2gz)-C^2}}$$

in which the upper or lower sign of the radical is to be taken according as dz is positive or negative, that is, as the pendulum is descending or ascending. The quantity under the radical may be put in the form (*vide* Mec. Celeste, Bowditch, Vol. I. p. 52).

$$2g\,(a-z)\,(z-b)\left(z+\frac{f}{2g}\right)$$

in which

$$\frac{f}{2g}=\frac{l^2+a\,b}{a+b}$$

$$(d)\qquad C^2=\frac{2g\,(l^2-a^2)\,(l^2-b^2)}{a+b}$$

$$c=2g\frac{(l^2-a^2-ab-b^2)}{a+b}$$

a and b being the greatest and least values of z.

If now we transfer the origin of co-ordinates to the lowest point of the spherical surface by substituting for z, a, and b, $l-u$, $l-\alpha$, $l-\beta$, and replace C and dt in (*b*) by the values above found, we shall have (putting $l+\frac{f}{2g}=\rho$)

$$d\phi=\tfrac{1}{2}\frac{\sqrt{(l^2-a^2)\,(l^2-b^2)}}{a+b}\ \frac{ldu}{u(2l-u)\,[(\rho-u)\,(u-\alpha)\,(\beta-u)]^{\frac{1}{2}}}$$

the varying sign of the radical being understood. If we develop the two factors $(2l-u)^{-1}$ and $(\rho-u)^{-\frac{1}{2}}$, and multiply the results, we shall have

$$d\phi=\sqrt{\frac{(l^2-a^2)(l^2-b^2)}{a+b}}\int\frac{ldu}{u[-\alpha\beta+(\alpha+\beta)\,u-u^2]^{\frac{1}{2}}}\Big[\frac{1}{2l\rho^{\frac{1}{2}}}-\frac{1}{4l\rho^{\frac{1}{2}}}\Big(\frac{1}{l}+\frac{1}{\rho}\Big)u$$
$$+\frac{1}{8}\frac{1}{l\rho^{\frac{1}{2}}}\Big(\frac{1}{l^2}+\frac{1}{l\rho}+\frac{3}{2\rho^2}\Big)\,u^2+\frac{1}{16l\rho^{\frac{1}{2}}}\Big(\frac{1}{l^3}+\frac{1}{l^2\rho}+\frac{3}{2l\rho^2}+\frac{15}{6\rho^3}\Big)u^3+\&c.\Big]$$

Strike out the common factor l, and remove the factor $2\rho^{\frac{1}{2}}$ into the denominator of the first radical factor (which factor then becomes $\sqrt{(l-a)(l-b)}=\sqrt{\alpha\beta}$), and the above integral becomes (*vide* Hirsch, Integral Tables, pp. 160–164), writing U for $-\alpha\,\beta+(\alpha+\beta)\,u-u^2$,

$$(e)\quad \phi=\tfrac{1}{2}\cos^{-1}\left(\sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2-4\alpha\beta}}\cdot\frac{\sqrt{U}}{u}\right)+\frac{1}{4}\sqrt{\alpha\beta}\left(\frac{1}{l}+\frac{1}{\rho}\right)\cos^{-1}\frac{2\sqrt{U}}{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}$$
$$+\frac{1}{8}\sqrt{\alpha\beta}\left(\frac{1}{l^2}+\frac{1}{l\rho}+\frac{3}{2\rho^2}\right)\left[-\sqrt{U}+\frac{1}{2}(\alpha+\beta)\cos^{-1}\frac{2\sqrt{U}}{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}\right]$$
$$+\frac{1}{16}\sqrt{\alpha\beta}\left(\frac{1}{l^3}+\frac{1}{l^2\rho}+\frac{3}{2l\rho^2}+\frac{15}{16\rho^3}\right)\left[-\left(-\frac{1}{2}u+\frac{3}{4}(\alpha+\beta)\right)\sqrt{U}+\right.$$
$$\left.\left(\frac{3}{8}(\alpha+\beta)^2-\frac{1}{2}\alpha\beta\right)\cos^{-1}\frac{2\sqrt{U}}{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}\right]+\&c.,+\text{cons't.}$$

In order that the arcs in the above expression should continually increase with the time, the positive or negative sign must be applied to the radical $\sqrt{U}$ according as u is increasing or diminishing: taken from $u=\alpha$ to $u=\beta$ (or the converse), the arcs all become $=\pi$, and the non-circular functions vanish.

Hence the azimuthal angle passed over by the pendulum in its motion from a lowest to a highest point of its orbit (or the converse) is expressed by

$$\phi_{\frac{1}{2}\pi}=\frac{1}{2}\pi\left[1+\frac{1}{2}\sqrt{\alpha\beta}\left(\frac{1}{l}+\frac{1}{\rho}\right)+\frac{1}{4}\sqrt{\alpha\beta}\left(\frac{1}{l^2}+\frac{1}{l\rho}+\frac{3}{2\rho^2}\right)\frac{\alpha+\beta}{2}\right.$$
$$(f)\quad+\frac{1}{8}\sqrt{\alpha\beta}\left(\frac{1}{l^3}+\frac{1}{l^2\rho}+\frac{3}{2l\rho^2}+\frac{15}{6\rho^3}\right)\left(\frac{3(\alpha+\beta)^2}{8}-\frac{1}{2}\alpha\beta\right)$$
$$\left.+\frac{1}{16}\sqrt{\alpha\beta}\left(\frac{1}{l^4}+\frac{1}{l^3\rho}+\frac{3}{2l^2\rho^2}+\frac{15}{6l\rho^3}+\frac{105}{24\rho^4}\right)\left(\frac{5(\alpha+\beta)^3}{16}-\frac{3\alpha\beta(\alpha+\beta)}{4}\right)+\&c.\right]$$

The sum of the terms after unity included in the brackets is the ratio by which the azimuth angle exceeds a quadrant; or, if the integral is taken through an entire revolution (relatively *to the apsides*), it, multiplied by 2π, is angle of advance of the apsides per revolution.

For motion nearly oscillatory, of whatever amplitude (*i. e.*, α being small and β arbitrarily large), or for spherical motions of considerable amplitude (α and β taken within limits not exceeding say one-third of l, corresponding to a swing of over 90°), ρ, always greater than $2l$, differs but slightly from that magnitude. Giving ρ that value, and taking the angle ϕ for a complete vibration, or a semi-apsidal revolution, we have the formula,

$$(g)\qquad \phi_{\pi}=\pi\left[1+\frac{3}{2}\cdot\frac{\sqrt{\alpha\beta}}{2l}+\frac{1.3.5}{1.2.4}\,\frac{\sqrt{\alpha\beta}.\frac{1}{2}(\alpha+\beta)}{(2l)^2}+\&c.\right]$$

If α and β are both small and nearly equal, and θ the angle of which they are the versed sines, then $\frac{\sqrt{\alpha\beta}}{l}=\frac{1}{2}\sin^2\theta$, and the apsidal motion corresponding to the second term of the above (the following terms neglected) becomes $\frac{3}{8}\pi\sin^2\theta$; agreeing with the expression $(a)_2$ on p. 24, when developed for the same case.

If the pendulum moves *nearly* horizontally in a great circle, that is, if $\alpha+\beta=2l$ and $\alpha\beta=l^2$ (nearly), then ρ, C, and [illegible] each infinitely great, and (f) becomes

$$\phi_{\frac{1}{2}\pi}=\tfrac{1}{2}\pi\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{16}+\frac{1}{32}+\&c.\right]=\pi$$

which denotes that the line of apsides moves through a quadrant while the pendulum is passing, through 180° azimuth, from a highest to a lowest point; in other words, that the highest and lowest points are diametrically opposite, and the apsides are *apparently* stationary. This theorem is true (as shown by Prof. Peirce), whatever be the inclination of the great circle, though it cannot be *generally* made evident by the above formulæ.

If in (c) we make transformations and substitutions already described, develop the factor $(\rho - u)^{-\frac{1}{2}}$, integrate between the limits $u=\alpha$, $u=\beta$, and double the result, we shall have for the time of one vibration, or one semi-orbital revolution,

$$(h) \quad T=2\pi\frac{l}{\sqrt{2g\rho}}\Big[1+(\tfrac{1}{2})^2\frac{\alpha+\beta}{\rho}+\left(\frac{1.3}{2.4}\right)^2\left(\frac{\alpha+\beta}{\rho^2}\right)^2-\frac{1.3}{2.4}\,\frac{\alpha\beta}{2\rho^2}+\left(\frac{1.3.5}{2.4.6}\right)^2\frac{(\alpha+\beta)^3}{\rho^3}$$
$$-\frac{1.3.5}{2.4.6}\cdot\frac{3\alpha\beta(\alpha+\beta)}{4\rho^3}+\&c.\Big]$$

and if $\alpha=0$, that is, if the motion is purely oscillatory, then $\rho=2l$, and this becomes

$$(i) \qquad T=\pi\sqrt{\frac{l}{g}}\Big[1+(\tfrac{1}{2})^2\frac{\beta}{2l}+\left(\frac{1.3}{2.4}\right)^2\left(\frac{\beta}{2l}\right)^2+\&c.\Big]$$

and more generally for any value of α and β not exceeding the versed sine of thirty or forty degrees:

$$(j) \qquad T=\pi\sqrt{\frac{l}{g}}\Big[1+\left(\frac{1}{2}\right)^2\frac{\alpha+\beta}{2l}-\frac{1.3}{2.4}\,\frac{\alpha\beta}{(2l)^2}+\left(\frac{1.3}{2.4}\right)^2\left(\frac{\alpha+\beta}{2l}\right)^2\pm\&c.\Big]$$

When α and β are both small, as in most pendulum experiments, the terms after the first in the brackets may be omitted, and we have the ordinary expression for the time $\pi\sqrt{\frac{l}{g}}$.

In the expression (g), omitting all the terms in brackets after the second, it is evident that that term will measure the apsidal motion for the time $\pi\sqrt{\frac{l}{g}}$; and hence that the total integral of equation (a) taken through that time will be,

$$\pi\Big[n\sin\lambda\sqrt{\frac{l}{g}}\pm\frac{3\sqrt{\alpha\beta}}{4l}\Big]$$

and if we denote by ϕ' the angle of azimuth of the apsidal line measured from its initial direction, we shall have, at any time t, substituting for α and β, $l\,(1-\cos\theta_1)$ and $l\,(1-\cos\theta_2)$,

$$(k) \qquad \phi'=\Big[n\sin\lambda\pm\frac{3}{4}\sqrt{\frac{g}{l}(1-\cos\theta_1)\,(1-\cos\theta_2)}\Big]t$$

or, since θ_1 is always small,

$$k') \qquad \phi'=\Big[n\sin\lambda\pm\frac{3}{4}\sqrt{\frac{g\,(1-\cos\theta_2}{2l}}\sin\theta_1\Big]t$$

The angle θ_2 being given, θ_1 is determined for the *ordinary* pendulum experiment by the consideration that the constant C, of which the value is found p. 24, is the moment of the quantity of motion. As c is very minute, the actual velocity at the lowest point will be, sensibly, $\sqrt{2gl(1-\cos\theta_2)}$, and hence the moment at that point will be $l\sin\theta_1\sqrt{2gl(1-\cos\theta_2)}=C=-nl^2\sin\lambda\sin^2\theta_2$, from which deducing the value of $\sin\theta_1$ and substituting in (k'), we have

$$(l) \qquad \phi'=n\sin\lambda\left(1-\frac{3}{8}\sin^2\theta_2\right)t$$

The second term in brackets is the retardation from the true horary motion; it being implied, of course, that the amplitude of oscillation is preserved unimpaired. Though small, this retardation would be sensible, especially if θ_2 had a considerable magnitude, say eight or ten degrees, though inappreciable for very small oscillations.

Practically, the resistance of the air constantly diminishes the value of θ_2, and failure to procure a perfect state of rest to the pendulum before it is set free, or currents of air, may give quite different values to C and c, and determine the character of the orbital motion to be progressive instead of retrograde; and it is generally observed that the conjugate dimension of the orbit increases (probably owing to the resistance of the air) as θ_2 diminishes. In this way the apsidal motion due to the orbit may acquire a value quite considerable compared to the proper horary motion, which will apparently be sensibly retarded or accelerated. By observing, at any period of the experiment, the value of θ_1 and θ_2 and the direction of the orbital motion, the coefficient of t in the formula (k') will give the theoretical rate of azimuthal motion at that instant.

If the orbital motion is retrograde and

$$(m) \qquad \sin\theta_1=\frac{4}{3}n\sin\lambda\sqrt{\frac{2l}{g(1-\cos\theta_2)}}$$

the line of the apsides would be stationary. For Columbia College, where $n\sin\lambda=00004747$, with a pendulum of 26 feet in length, and a value of θ_2 of 6°, this would give $\theta_1=3',3$, the actual semi-axes of the projection of the orbit being about two feet nine inches and one-third of an inch.

It would generally be sufficiently accurate to substitute for $1-\cos\theta_2$, $\frac{1}{2}\sin^2\theta_2$, by which formula ($k'$) would become more simply,

$$(n) \qquad \phi'=\left[n\sin\lambda\pm\frac{3}{8}\sqrt{\frac{g}{l}}\sin\theta_1\sin\theta_2\right]t$$

in which it is seen that the deviation from the proper horary motion is proportional to the area of the projection of the orbit.

The above, or (k'), expresses the azimuthal motion as it would be were there no other forces acting than those included in the investigation, in which case θ_1 and θ_2 would be invariable. In point of fact there are practically numerous disturbing forces, of which, however, the resistance of the air is the most considerable, and through which θ_1 and θ_2 are incessantly changing, and it is not improbable that the *change of shape* of the orbit may, in itself, cause some variation of direction of the line of apsides:[1] a matter which cannot be decided until the problem of the spherical pendulum is solved with the resistance taken into account.

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[1] An investigation of the simpler case of the plane elliptical motion of a body attracted by a central force proportional to the distance, in a medium which resists either directly or as the square

To get a clearer idea of what is expressed by the periodic term of (7), $-2\,n\cos\lambda \int\cos\phi\,\sin^2\theta\,d\theta$ (which corresponds to the integral $\int y\,dz$ of (4)), we must revert to the latter equation. Conceive the pendulum propelled from a state of rest in the vertical, with a very great angular velocity, denoted by v, in the plane of the meridian. Were the earth motionless, it would continue to whirl in this plane, passing through the zenith at every revolution. Introduce the element of the earth's rotation, and the two terms of equation (4) containing n take effect, by the first of which the plane of revolution moves in azimuth with the angular velocity $n\sin\lambda$. The second expressing that there will be an increase of the moment of the quantity of motion about the vertical after a time T proportionate to the area generated in that time $\int y\,dz$. Under these conditions this area is cumulative, and at the end of one revolution expresses the area of the circle of radius l. Let us suppose that the plane of motion turns about a line parallel to the complementary terrestrial axis with an angular velocity $n\cos\lambda$. At the end of the time T (supposed very small) the plane will make with the meridian the angle $n\cos\lambda T$, and as the quantity of motion in its own plane is vl, its moment referred to a vertical axis will, from zero, have become $vl^2\sin(n\cos\lambda T)$, or, substituting the small arc for its sine,

$$vl^2\,n\cos\lambda T$$

But T, for one revolution, is expressed by $\frac{2\pi}{v}$ hence the above becomes

$$2n\cos\lambda\,\pi\,l^2$$

The area of the circle which is generated in the same time is πl^2 and is expressed by the integral $\int y\,dz$, and it is easy to show for each successive revolution that the area $\int y\,dz$ multiplied by $2n\cos\lambda$ corresponds to an increment of the moment of quantity of motion about the vertical which it would receive from a turning of the plane about the complementary axis through the angle $n\cos\lambda\,t$.

Hence, for the particular case under consideration, the second term of second member of equation 4 expresses an angular motion about the complementary axis of which $n\cos\lambda$ is the velocity. The resultant of this, and the azimuthal component, is rotation about an axis parallel to that of the earth, and opposite in direction to the earth's rotation.

The above theorem can be analytically demonstrated. The quantity N, expressive of the tension of the cord, is made up of the centrifugal force due to the pendulum's relative angular motion and of the variable component of the force of gravity (neglecting, as we have done, quantities of the order n^2). If this centrifugal force is so great that the component of gravity may be neglected, $\frac{N}{l}$ will

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of the velocity, indicates no apsidal motion accompanying the decrease of parameters of the orbit. Neither, however, does it indicate the enlargement of the minor axis (initially very small) so universally observed in the pendulum experiments.

reduce to the constant v^2 (v being the impressed angular velocity of pendulum movement), and the second and third of equations (3) will yield the equation

$$\frac{d^2}{dt^2}(y\cos\lambda - z\sin\lambda) = -v^2\,(y\cos\lambda - z\sin\lambda) - g\sin\lambda$$

the integral of which is

$$y\cos\lambda - z\sin\lambda = -\frac{g\sin\lambda}{v^2} + A\cos vt + B\sin vt$$

The above is independent of n; $y\cos\lambda - z\sin\lambda$ is the value of the new co-ordinate of y' when the axis of y is changed to parallelism to that of the earth. Hence, as thus transformed, this co-ordinate is unaffected by the earth's rotation, the plane of pendulum motion must turn (if it turns at all) about such an axis.

The assumption for $t=0$, of $y=0$, $z=l$, $\frac{dy}{dt}=lv$, $\frac{dz}{dt}=0$, gives

$$\begin{cases} A = -\left(l - \frac{g}{v^2}\right)\sin\lambda \\ B = l\cos\lambda \end{cases}$$

(10) hence $y\cos\lambda - z\sin\lambda = -\frac{g}{v^2}\sin\lambda\,(1-\cos vt) + l\sin(vt-\lambda)$

The second term of the second member of the above gives precisely the value which the first member would have, were the plane of pendulum motion stationary or were it turning with *any* angular velocity about an axis parallel to the earth's. The first term is, owing to the assumed high value of v, very minute, and is *periodic*, the period being equal to $\frac{2\pi}{v}$, the time of the pendulum's revolution in its circular orbit. Owing to its minuteness and periodicity, this term may be neglected, and equation (10) becomes

$$y\cos\lambda - z\sin\lambda = l\sin(vt-\lambda)$$

to satisfy which, and at the same time the three differential equations (3) (omitting g, as we have found reason to do in (10), and also omitting terms containing n_2, in the developments), requires the following values for the co-ordinates:—

$$x = l\sin nt\cos(vt-\lambda)$$
$$y = l\,[\cos\lambda\sin(vt-\lambda) + \sin\lambda\cos nt\cos(vt-\lambda)]$$
$$z = l\,[-\sin\lambda\sin(vt-\lambda) + \cos\lambda\cos nt\cos(vt-\lambda)]$$

Changing the axes of y and z by turning them through the angle λ, we should have for new co-ordinates,

$$x' = l\sin nt\cos(vt-\lambda)$$
$$y' = l\sin(vt-\lambda)$$
$$z' = l\cos nt\cos(vt-\lambda)$$

If we now change the plane of yz by moving it through the variable angle nt about the axis of y', we get,

$$x'' = 0$$
$$y'' = l\sin(vt-\lambda)$$
$$z'' = l\cos(vt-\lambda)$$
$$y''^2 + z''^2 = l^2$$

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These values show that the plane of pendulum revolution turns about an axis parallel to the earth's with the relative angular velocity n; or, in other words, that the plane preserves its parallelism to itself in space.

If we had a *succession of pendulums* rigidly connected with each other, the disturbing effect of gravity would be eliminated. Such a succession would be simply a *gyroscope*, and the gyroscope, mounted in gimbals and set running in a meridian plane, would exhibit the apparent rotation of its disk around an axis parallel to the earth's axis equal and contrary to that of the earth; which is simply saying that as the earth revolves the plane of the disk maintains its parallelism to itself, and if we suppose its axis directed at a star in the plane of the equator, it would follow that star so long as the rotation of the disk is sustained.[1]

The pendulum experiment, in its ordinary form, exhibits not the whole rotation of the earth, but only one component of it; the component which belongs to an axis passing through the locality. It is perhaps quite as interesting and important, as being the only experimental demonstration we can have of a principle difficult of comprehension, but as fundamental to mechanics, since its enunciation by Euler, as the corresponding one of the decomposition of linear velocities, viz., that of the decomposition into distinct components, of rotary velocities. The plane of the pendulum appears to turn relatively to the surface of the earth simply because the earth turns just so much underneath it, the earth really *revolving* about the local axis with a certain calculable component of velocity. The earth turns at the same time with another component of velocity about another axis (the complementary one), and the joint effect, or the resultant of the two components, is the rotation about the polar axis. The second component, very great as we approach the equator, where the first vanishes entirely, is not exhibited by the pendulum, and is only detected by analysis as a slight disturbance. Convert the pendulum into the gyroscope, however, and this second component appears equally with the first.

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[1] Owing to the friction of the gimbals, there would be, *practically*, besides the motion above described, a motion of the axis in the plane of the meridian, north or south, according to the direction of the disk rotation; this angular motion might be greater or less than the equatorial motion, but would be, with a well-constructed apparatus, independent of it, at least for the brief time during which a gyroscope experiment would last.

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ON THE

INTERNAL STRUCTURE OF THE EARTH CONSIDERED AS AFFECTING THE PHENOMENA OF PRECESSION AND NUTATION.

THE equations of precession and nutation are, as is well known, entirely independent of any particular law of density, and are functions only of the absolute values of the moments of inertia about the equatorial and polar axes A and C,[1] and are independent indeed of the figure of the earth, except so far as it affects the values of these moments.

Moreover, if the earth, instead of being solid throughout, is (as supposed by most geologists) a solid shell inclosing a fluid nucleus, it is only necessary (leaving out of consideration the pressure that may be exerted on the interior surface by the fluid) that the shell should have these moments of inertia. Mr. Poinsot[2] has obtained as the results of calculation for a homogeneous spheroid, values of precession and nutation identical with those of observation, by taking the ellipticity at $\frac{1}{308.65}$ and η (the ratio of mass of moon to that of the earth), at $\frac{1}{88}$.

We have, assuming a uniform density, indicating by a and b the equatorial and axial radii, and by e the ellipticity:—

$$C=\frac{8}{15}\pi\, a^4 b=(\text{approx.})\ \frac{8}{15}\pi\, b^5\,(1+4e)$$

$$A=\frac{4}{15}\pi\,(a^4 b+a^2 b^3)=\frac{8}{15}\pi\, b^5\,(1+3e)$$

$$(46)\qquad \frac{C-A}{C}=\frac{e}{1+4e}=e-4e^2$$

If e is taken at $\frac{1}{308.65}$ then $\frac{C-A}{C}=\frac{1}{312.7}$. But all meridian measurements of

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[1] I assume, of course, the equality of all moments of inertia, A, about the equatorial axes, and overlook all questions as to the non-symmetry of the earth with respect to its axis of figure or to the equator; for, in fact, neither the rotation of the earth nor any observable celestial phenomena reveal it.

[2] Connaissance des temps, 1858.

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the earth indicate an ellipticity greater rather than less than $\frac{1}{300}$;[*] and the latest determination makes it $\frac{1}{295}$,[†] by giving which value to e we obtain $\frac{C-A}{C}=\frac{1}{299}$.

Therefore the observed precession is to that which would result in a homogeneous spheroid, from the formulas, with the latest determined value of e introduced, as 299 : 311.7, provided the relative mass of the moon be but $\frac{1}{88}$.[‡]

The value of $\frac{C-A}{C}$ would be the same for a homogeneous *shell* of which the interior surface had the *same ellipticity*, e, as the exterior; or it would be the same for a shell of which all the elementary strata had the same ellipticity, in which the density, constant through each stratum, should vary according to any law, from stratum to stratum. The ratio of 299 : 312.7, so nearly unity ($=0.96=\frac{24}{25}$ nearly), while the ratio of mean to surface density of the earth is so high, indicates nearly uniform ellipticity of stratification, and hence fluidity of *origin;* while, on the other hand, the considerable inequalities in the equatorial axes indicated in the note below are incompatible with the hypothesis of *actual* fluidity beneath a *thin* crust, and are, to the measure of their probability, a disproof of it.

The effect upon the axial movements of such a shell which would result from the pressures of an internal fluid has been made the subject of an elegant mathematical investigation by W. Hopkins, F.R.S., in the Philosophical Transactions of 1839–40–42. On the supposition of a uniform density of shell and fluid, and the same ellipticity for inner and outer surfaces of the shell, the precession will be the same

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[*] Airy, "Figure of the Earth," Encyc. Metrop.; Guillemin: Mädler, Am. Journ. of Science, Vol. 30, 1860, makes the polar compression of greatest meridian $\frac{1}{292.109}$

of smallest meridian $\frac{1}{302.004}$

(Article translated by C. A. Schott, U.S. Coast Survey, from Prof. Heis' "Astronomie, Météorologie et Géographie," Nos. 51, 52. 1859.)

[†] Appendix "Figure of the Earth" to the "Comparisons of Standards of Length," published 1866 by the British Ordnance Survey, gives for a "spheroid of revolution," $\frac{a-c}{c}=\frac{1}{295}$; for a spheroid of three axes, $\frac{a-c}{c}=\frac{1}{285.97}$, $\frac{b-c}{c}=\frac{1}{313.38}$, $\frac{a-b}{c}=\frac{1}{3269.5}$. The probabilities of the latter supposition to the former being 154 : 138.

[‡] There is yet great uncertainty as to the relative mass of the moon, and as long as that point is unsettled, so is also the ratio of observed to calculated precession. Laplace, from observations of the tides at Brest, fixed it at $\frac{1}{75}$, which number is adopted by Pontécoulant. Former determinations from the observed nutations make it $\frac{1}{80.75}$; but $\frac{1}{87}$ was the determination from the coefficient of nutation of Lindenau. Guillemin gives $\frac{1}{88}$, and these two last numbers coincide nearly with that used by Poinsot. A discussion by Mr. Wm. Ferrell, member of National Academy of Sciences, of tidal observations made for a series of years at the port of Boston, as well as those at Brest, gives results confirmatory of the larger ratio of Laplace. Serret (Annales de l'Observatoire Imp. 1859) assumes $\frac{1}{83}$ and deduces $\frac{C-A}{C}=\frac{1}{306}=0.00327$. These ratios are adopted by Thomson and Tait, §§ 803, 828. Archdeacon Pratt ("Figure of the Earth," 4th ed. 1871) adheres to Laplace's determination.

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and the nutation essentially the same as for a homogeneous spheroid; but for the actual case of a heterogeneous fluid contained in a heterogeneous shell he finds that the ellipticity of the inner surface of the shell must be less than that of the exterior in the proportion of the *observed* precession to the precession of a homogeneous spheroid of same external ellipticity, a proportion which he assumes to be $\frac{7}{8}$, in accordance to the then received numbers for ellipticity, and for the mass of the moon.

The fulfilment of the condition, in the actual constitution of the earth, is improbable; for the isothermal surfaces are in all probability (and indeed by his own mathematical conclusions) of progressively *greater* ellipticity from the surface inwards. He finds, however, the requisite decrease of ellipticity in the fluid surfaces of *equal density*, assuming the well-known hypothetical law of density of Laplace $\rho = A \frac{\sin qb}{b}$; but we know not the influence which pressure has upon solidification, and it seems probable that the interior surface of the shell would conform nearly to the surfaces of equal temperature. The demand which he makes for 800 or 1000 miles thickness of shell is therefore a minimum (for the data used), while the more *probable* result is a much greater thickness or even entire solidity. But however elegant may be Mr. Hopkins' analysis, the basis of the structure is but slender, and those results have not been generally accepted as fully decisive of the question.

Sir Wm. Thomson, F.R.S., brings forward (Philosophical Transactions, 1863; also Thomson's and Tait's Treatise on Natural Philosophy, 1867) arguments against the popular theory of a thin crust, which are more forcible. A thin crust would itself undergo tidal distortion, and the height of the *apparent* tides of the ocean be thereby much reduced, while the *actual* precession would be diminished in the same ratio; that is, the differential forces of the sun and moon would expend themselves in producing these solid tides instead of producing precession.

From a theoretical investigation (given in a separate paper in the same volume)[1] of the deformation experienced by a homogeneous elastic spheroid under the influence of any external attracting force, he arrives at the result that, if the earth had no greater rigidity than steel or iron, it would yield about $\frac{2}{5}$ as much to tide-producing influences as if it had no rigidity—more than $\frac{3}{4}$ as much if its rigidity did not exceed that of glass. Moreover, the apparent ocean tides (or difference of high and low water level) would be (if H is the measure for a perfectly rigid earth) 0.59 H, if the earth had the rigidity of iron or steel only; $\frac{1}{4\frac{1}{2}}$H if it had that of glass.

As to *precession*, the centrifugal force of the crowns of the tidal elongation would balance $\frac{7}{9}$ of the dynamic couple resulting from the sun's or moon's attraction if the earth had only the rigidity of glass, and $\frac{2}{5}$ if it had only that of steel.

"That the effective tidal rigidity, and what we call the precessional effective

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[1] See also "Treatise on Natural Philos.," § 832 *et seq.*

rigidity of the earth, may be several times as much as that of iron (which would make the phenomena, both of the tides and precession, sensibly the same as if the earth were perfectly rigid), it is enough that the actual rigidity should be several times as great as the actual rigidity of iron throughout 2000 or more miles thickness of crust."

A theorem fundamental to the establishment of the above propositions is, that a revolving spheroid destitute of rigidity, a homogeneous fluid one, for instance, would have no precession. Sir W. Thomson does not mathematically demonstrate this theorem, but by use of an hypothesis gives an elegant illustration of its truth, for which, though to me it is convincing, I prefer to substitute the following demonstration.

Such a spheroid, all the particles of which revolve about an axis with a common angular velocity n, and attract each other by the law of universal gravitation, would have the form of an ellipsoid of revolution, the ellipticity of its meridional section being $\frac{5}{4}\frac{n^2}{g}$.[*] (See "Figure of the Earth," Encyc. Metrop., par. 33, by Prof. Airy.) Attracted by the sun, its tides would be expressed by the terms of [2316] Méc. Cél., Book IV (Bowditch). Of these three terms, the first (a function of the declination *only* of the attracting body) and the third (the semi-diurnal oscillation) express tidal elevations symmetrically distributed on each side of the equator, which would, hence, exert no influence through the centrifugal forces of their masses, upon precession. The second therefore, or *the diurnal tide*, is alone to be considered.

Conceive a meridian plane passed through the sun at any declination, the "couple" exerted by its attraction would be exerted wholly to turn the spheroid about an equatorial axis normal to this plane. We have therefore to investigate what dynamic couple, with reference to this same axis, will be exerted by the centrifugal force of the diurnal tidal protuberance. As the calculation involves the state of things at but a single instant of time, the angle, $nt+\varpi-\psi$, may be written ϖ and counted from the meridian of the sun: ρ, the uniform density of the fluid, taken as unity. The height, y, of the diurnal tide will be expressed for all parts of the spheroid by

$$(47) \qquad y=\frac{15S}{2r^3g}\sin\theta\cos\theta\sin\lambda\cos\lambda\cos\varpi\,[†]$$

in which λ is the polar distance or complement of the latitude of the locality. and θ the declination of the sun. If, with Laplace, we put $\cos\lambda=\mu$, and $\sin\lambda=\sqrt{1-\mu^2}$, the mass of the elementary column of height y will be $yd\mu\, d\varpi$, and its centrifugal

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[*] g being the force of gravity *at the equator of the hypothetical spheroid.*

[†] The expression, in the original, for the diurnal oscillation, is

$$\frac{3L}{r^3g\left(1-\frac{3}{5\rho}\right)}\sin V\cos V\sin\theta\cos\theta\cos(nt+\varpi-\psi)$$

The notation of my paper on the precession of the equinoxes is substituted, and the assumed value of ρ introduced.

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force (the radius of the spheroid being taken at unity, and the variation, assumed slight, due to ellipticity, disregarded) $n^2 y\, d\mu\, d\varpi \sqrt{1-\mu^2}$. The component of this tending to tilt the spheroid about the axis in question is $n^2 y\, d\mu\, d\varpi \sqrt{1-\mu^2} \cos\varpi$, and its moment $n^2 y\, d\mu\, d\varpi\, \mu \sqrt{1-\mu^2} \cos\varpi$.

Substituting the value of y (47), the above becomes

$$n^2 \frac{15S}{2r^3g} \sin\theta \cos\theta\, d\mu\, d\varpi\, \mu^2 (1-\mu^2) \cos^2\varpi$$

Integrating, first with reference to μ from $\mu=-1$ to $\mu=+1$, then with reference to ϖ from 0 to 2π, we get, as the expression for the couple due to "the centrifugal force of the crowns of the tidal elongation," resisting the sun's action,

$$(48) \qquad 2\pi \frac{n^2 S}{gr^3} \sin\theta \cos\theta$$

We have found (19) for the moment of the sun's force, producing precession, the expression

$$\frac{3S}{r^3} (C-A) \sin\theta \cos\theta$$

and (46), $(C-A)=\frac{8}{15}\pi e$ (b being taken at unity) and e, as already stated, is for a homogeneous fluid spheroid $=\frac{5}{4}\frac{n^2}{g}$. Making these substitutions, the above expression becomes identical with (48). The precessional force of the sun is, therefore, exactly neutralized by the centrifugal force of the tidal swelling.

The theorem could, doubtless, be demonstrated for a revolving fluid spheroid in equilibrium, of which the density of the strata varies. Without extending any further the mathematical analysis, it will be sufficient to remark that the calculation of the tidal elevations is, identically, that of equilibrium of form of the revolving body subjected to a foreign attraction, and in the calculation the motion of rotation is disregarded, and the centrifugal force, which expresses its entire effect upon the form, alone considered. Under this point of view, equilibrium of form is, necessarily, equilibrium (or stability) of *position*. For if any effective turning force exists, it must, in order not to interfere with equilibrium of form, either be so distributed as to give each individual particle of the spheroid its proper relative quantity of turning motion, or it must be a distorting force. The first alternative cannot be admitted; the second is excluded by the hypothesis of equilibrium. Hence, there can be no turning (or precessional) force.

The accuracy of the foregoing analysis is complete,[1] except that the consideration of *relative motion* of the particles is excluded. But Laplace shows (p. 604, Vol. II, Bowditch) that as the depth of the ocean increases, the expressions for the tidal

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[1] There are slight errors of approximation: 1st, in the tidal expression (47) itself; 2d, in the above integration which disregards the variation of the radius; and, 3d, in the value of $C-A$. They neutralize each other in the final result.

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oscillations given by the dynamic theory approximate rapidly to those of the "equilibrium theory," with which, when the depth is very great, or the spheroid wholly fluid, they are essentially identical. Moreover, he shows (p. 219, Vol. I) that the *vertical* motions of the particles, when the depth is small, may be disregarded. When the spheroid is wholly fluid, *all* the relative motions of the particles are of the same order as the vertical ones and exceedingly minute; and the forces of inertia thereby developed are insensible compared with those we have been considering.[1]

By parity of reasoning the truth of Sir W. Thomson's propositions concerning a solid but yielding spheroid is made evident; for exactly in the same ratio to the tides of a fluid spheroid that the solid tidal elevations are produced (the actual ellipticity of the earth being nearly that of equilibrium with the centrifugal forces), will the precessional couple due to the tide-producing attraction be neutralized by their centrifugal action.[2] That a thin solid crust, such as geologists generally assume, would yield and exhibit tidal elongations, seems without calculation very probable; but if Sir W. Thomson is correct as to the rigidity required in even a wholly solid earth, the hypothesis of a thin crust must be abandoned, and it would seem indeed that rigidity several times as great as the actual rigidity of iron throughout 2000 or more miles thickness of crust would be incompatible with a very high internal temperature.

Without having recourse to Sir W. Thomson's profound analysis, the necessity, in order that there shall be no sensible solid tidal wave, of a very high rigidity

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[1] The foregoing demonstration does not conflict with Laplace's theorem that ocean tides do not affect the precession; for his theorem applies only to a shallow ocean over a rigid nucleus, of which ocean the precessional couple, by altered attractions, pressures, and centrifugal forces due to generation of living forces in the fluid, is transferred to the nucleus. I have already alluded to the minuteness of the motions of the particles of a fluid spheroid. The remarks apply, *à fortiori*, to those of an elastic *solid*. *Vibratory* motions, properly speaking, cannot exist, for the elastic forces extremely minute are always held (sensibly) in equilibrium by the distorting forces. The solid surface would oscillate in the same sense that the ocean tides oscillate, *i. e.*, by a "forced" tide-wave.

[2] "It is interesting to remark," say Thomson and Tait (§ 848, "Treatise, &c."), "that the popular geological hypothesis of a thin shell of solid material, having a hollow space within it filled with liquid, involves two effects of deviation from perfect rigidity which would influence in opposite ways the amount of precession. The comparatively easy yielding of the shell must render the effective moving couple due to sun and moon much smaller than it would be if the whole interior were solid, and, on this account, must tend to diminish the amount of precession and nutation. But the effective moment of inertia of a thin solid shell, containing fluid in its interior, would be much less than that of the whole mass if solid throughout; and the tendency would be to much greater amounts of precession and nutation on this account."

The co-efficient of precession of the "thin solid shell" would be (p. 34) the same, nearly, as that of the spheroid of which the homogeneous strata have the same ellipticity. Its precession-resisting couple (48) due to tidal distortion would be just what is necessary to develop its proportional influence upon the precession *of that shell*, upon which the fluid contents can exert influence only through their pressure. This is identically Prof. Hopkins' problem. The *thin* shell of popular geological hypothesis would, however, be subject to tidal distortions scarcely inferior in magnitude to those of a wholly fluid spheroid; by which, as we have seen, the sun and moon's "moving-couple" is wholly neutralized throughout the whole spheroid.

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for the earth, may be made evident from the following considerations: A rod of steel extending towards the sun from the centre to the surface of the earth, would be elongated by the differential force of the sun's attraction $0^{ft}.975$, or one foot, nearly. The height of the solar tide of a homogeneous fluid spheroid is $1^{ft}.355$; but the *mutual attraction* of the elevated particles produces $0^{ft}.793$ of this, and the remaining $0^{ft}.542$ is the proper measure of the *direct* action of the solar force. In the case of the rod the elastic forces of the steel *alone* are considered; in the spheroid gravitation is the sole binding force. The maximum extension of the rod per unit of length would be expressed by the decimal .000000055 corresponding to a tensile force of 1.87 lbs. (taking the coefficient of elasticity at 34 millions lbs.) per square inch.(1) The necessity of the extreme rigidity demanded by Sir W. Thomson is recognized when it is seen how excessively minute would be the elastic forces developed in the production of distortion, in a rigid earth spheroid, commensurable with fluid tide-waves.[1]

In a paper "On the Secular Cooling of the Earth" (Trans. R. S. E., 1862, and Appendix to "Treatise, &c."), Sir W. Thomson applies a solution of Fourier to the determination of the interior temperature and its rate of increase downwards,

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(1) See Additional Notes, p. 51.

[1] M. Delaunay, President of the French Academy, after quoting (Comptes rendus 1868) from the paper of Sir W. Thomson to which I have already referred, the results of Hopkins and some corroborating remarks from Sir W. Thomson's paper (referred to above), says: "Ainsi, on le voit, l'objection mise en avant par M. Hopkins, contre les idées genéralement admises par les geologues sur la fluidité intérieure du globe terrestre, est regardée par plusieurs savants anglais comme parfaitement fondée. Je suis d'un avis diametralement opposé: je crois que l'objection de M. Hopkins ne repose sur aucun fondement réel." M. Delaunay then refers to an experiment made under his direction with a glass vase 0^m 24 in diameter, as furnishing decisive proof that the "viscosity" of a liquid as perfectly fluid as water even, is sufficient to cause it to take up the rotary motions of its enveloping shell, provided that those motions are relatively slow, as are those which constitute the precession and nutation of the earth; and he goes on to say: "Hence it does not appear to me possible to admit that the effect of the perturbing forces to which precession and nutation are due extend only to a portion of the mass of the terrestrial globe; the entire mass ought to be carried along (entrainée) by the perturbing actions, whatever may be the magnitude attributed to the interior fluid portion, and consequently the consideration of the phenomena of precession and nutation can furnish no datum for estimating the greater or less thickness of the solid crust of the globe."

M. Delaunay seems to be unaware that Sir W. Thomson coincides with Prof. Hopkins only in *this* (as the sequel of the very paper quoted shows), that he demands a great thickness of crust, and, moreover, that the interior, to the depth of this crust, shall be not merely "solid," but possessing a rigidity "several times as great as that of iron." I have endeavored to show that Sir W. Thomson's argument is irrefragable; but, based upon wholly different considerations, it is certain that no degree of "viscosity" assigned to an internal liquid will refute it.

I have remarked, at the outset of this discussion, that Prof. Hopkins' results "have not been generally accepted as decisive;" but I cannot admit that, as a test of their tenability, the experiment of M. Delaunay possesses the crucial character which he attributes to it. Viscosity, considered as an accelerating force tending to impart to a fluid the rotary motions of an enveloping shell, is directly proportional to the surface of contact, and inversely to the *mass* of contained liquid; in other words, it varies inversely as the diameter of the enveloping shell. The effect of viscosity of the fluid contents of the earth compared to those contained in a similar spherical envelope of only ten inches diameter, would be expressed (nearly enough) by the fraction $\frac{1}{40,000,000}$.

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assuming a uniform primitive melting temperature of 7000° Fah., and a lapse of 100 millions of years since the cooling process commenced.

"The rate of increase of temperature from the surface downwards would be sensibly $\frac{1}{51}$ of a degree per foot for the first 100,000 feet or so. Below that depth the rate of increase per foot would begin to diminish sensibly. At 400,000 feet it would have diminished to about $\frac{1}{141}$ of a degree per foot. At 800,000 feet it would have diminished to less than $\frac{1}{50}$ of its initial value, that is to say, to less than $\frac{1}{2550}$ of a degree per foot; and so on, rapidly diminishing. Such is, on the whole, the most probable representation of the earth's present temperature, at depths of from 100 feet, where the annual variations cease to be sensible, to 100 miles, below which the whole mass, or all except a nucleus cool from the beginning, is (whether liquid or solid) probably at, or very nearly at, the proper melting temperature for the pressure at each depth."

The high rigidity demanded is difficult to conceive of in connection with a temperature in the solidified mass "at or near" that of melting, extending downwards indefinitely towards the centre of the earth. Hence Poisson's reasoning, which results in showing that the earth to have become thoroughly cooled and to have been subsequently reheated, *superficially*, harmonizes better with the demand for rigidity than that of Leibnitz, which supposes it to be now cooling from a (throughout) incandescent liquid state. In the latter case the law of actual temperature as deduced from Fourier's formulæ (as expressed above by Sir W. Thomson) would extend to the centre; in the former case only to the unmelted portion or to the "nucleus cool from the beginning."

Referring to the increase of temperature, with depth observed in mines, &c., Poisson remarks: "Fourier et ensuite Laplace ont attribué ce phénomène à la chaleur d'origine que la terre conserverait à l'époque actuelle et qui croîtrait en allant de la surface au centre, de tel sort qu'elle fût excessivement élévée vers le centre * * * en vertu de cette chaleur initiale la temperature serait aujourd'hui de plus de 2000 degrés à une distance de la surface égale seulement au centième du rayon; au centre elle surpasserait 200,000 degrés. * * * Mais quoique cette explication ait été généralement adoptée, j'ai exposé, dans mon ouvrage, les difficultés qu'elle presente, et qui m'ont paru la rendre inadmissible, * * * je crois avoir demontré que la chaleur developpée par la solidification de la terre a du se dissiper pendant la durée de ce phénomène, et que *depuis longtemps il n' en subsiste plus aucune trace.*" (Théorie de la Chaleur.)

Poisson, as is well known, attributes the increase of temperature, with depth, observed in the earth's crust, to the passage, at a remote period, of the solar system through hotter stellar regions, the temperature of which, he argues, should differ from place to place. Even a hypothetical case of the illustrious author, conforming to his theory, of an increased superficial temperature, 5000 centuries ago, of 200° C. diminishing by cooling (by transition to cooler stellar regions) to 5°, 500 centuries ago, and, subsequently, to the actual mean temperature, would scarcely meet the present demands for time, of palæontology; while the determinations of the conductivity of the earth's crust made near Edinburgh show, according to Sir W. Thomson, a necessity for increments of temperature of 25°, 50°, and 100°

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(Fah.), for past periods of only 1250, 5000, and 20,000 years, and authorize him to pronounce Poisson's hypothesis impossible, without destruction of life, or relegation of the date to so remote an era as to demand an intensely heated stellar region.

A reheating *after* solidification which should again fuse the surface to great depths would be, it seems to me, as "inadmissible" for the origin of observed subterranean temperatures, as the heat originally "developed by the solidification of the earth." Hence, the hypothesis of a reheating to fusion of the surface by impact of meteoric bodies would be likewise excluded by Poisson's theory of internal temperatures.

In the paper referred to (p. 39), Sir Wm. Thomson discusses the probable circumstances of solidification of the earth, assuming the known crust-materials (granite, &c.), in a molten state, as the constituent, and reasons that in consequence of the great condensation of granite in freezing, solidification must commence at the centre, and that "there could be no complete permanent incrustation all round the surface "till the globe is solid, with, possibly, the exception of irregular, comparatively "small spaces of liquid;" such separation of constituents in the process of crystallization taking place in all liquids composed of heterogeneous materials, and, indeed, is observable in the lava of modern volcanoes. He infers from the probable phenomena developed, into the discussion of which he goes at some length, "results sufficiently great and various to account for all that we see at present, and "all that we learn from geological investigation, of earthquakes, of upheavals, and "subsidences of solid, and of eruptions of melted rock."

Still we would, if possible, find reason to attribute a lower than "the proper melting temperature" to the solidified interior. Ice, indeed, preserves its rigidity unimpaired up to the point of fusion, and there may be a few other substances that have the like property; but it seems to be an exceptional one. The known constituents of the earth's crust certainly do not possess it, at least under ordinary pressures. If, as suggested by Prof. Joseph Le Conte (Am. Journal of Science, Nov. Dec. 1872), the "conductivity" be increased by pressure and *condensation*, such diminished temperatures may obtain.

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NEW ADDENDUM.[1]

A FEW words are in place here concerning the results of the late Prof. Hopkins' investigation (against which M. Delaunay's objections, (note, page 38,) are especially directed), briefly stated, pages 34, 35. They are as follows: First FOR HOMOGENEOUSNESS. "Supposing the earth to consist of a homogeneous spheroidal shell (the ellipticities of the outer and inner surfaces being the same) filled with a fluid mass of the same uniform density as the shell;" then, "the precession will be the same, whatever be the thickness of the shell, as if the whole earth were homogeneous and solid."

SECOND, FOR HETEROGENEOUSNESS, his result may be thus expressed:

$$(a) \qquad P_1 - P' = P_1 \left\{ 1 - \frac{\varepsilon}{\varepsilon_1} \left(1 + \frac{s}{1 + \frac{h}{q^5 - 1}} \right) \right\};$$

"where P_1 denotes the precession of a solid homogeneous spheroid of which the ellipticity $=\varepsilon_1$, that of the earth's exterior surface, and P' the precession of the earth, supposing it to consist of an interior heterogeneous fluid contained in a heterogeneous spheroidal shell, of which the interior and exterior ellipticities are respectively ε and ε_1, the transition being immediate from the entire solidity of the shell to the perfect fluidity of the interior mass."

In the multiplier of $\frac{\varepsilon}{\varepsilon_1}$, second member of ($a$), q is the ratio of external to internal polar radius of the shell; s depends on the varying ellipticity and density of the strata of equal density of the shell; h depends on the density of the fluid interior. For a thin crust the coefficient in question is unity nearly; for a thick one it will be somewhat greater if ε be less than ε_1.

It cannot fail to be observed that, under the conditions just before expressed for *homogeneousness—i. e.*, equality of external and internal ellipticities—we get from the formula (s becoming zero) *the same result*, *i. e* , $P' = P_1$, as for that case.

In accordance with rational hypothesis as to the internal condition of the earth, equalities of ellipticities for the surfaces of a *thin* crust (and corresponding equality of densities), or closely approximate equalities would be expected. The necessity for a *thick* crust arises, therefore, from the alleged discrepancy between the observed and calculated annual precessions (50 seconds and 57 seconds), which, according to Prof. Hopkins, makes $\frac{P_1 - P'}{P_1} = \frac{1}{8}$, nearly, assuming the moon's mass $\frac{1}{70}$, and the earth's ellipticity $\frac{1}{300}$. (The real discrepancy is probably very much less. See page 34, *et sequentia.*)

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[1] The original ADDENDUM, hurriedly written while the work was in the printer's hands, has been, in what follows, somewhat modified and amplified.

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If, in applying the expression (*a*), the symbolic fraction, for a first approximation, be omitted, we have, according to above assumption of discrepancy, $\varepsilon = \frac{7}{8}\varepsilon_1$. This value of ε will be *in excess;* hence, the thickness of crust deduced from it will err the other way, and a determination on this basis will give a thickness which must, *in fact*, be exceeded.

The limit of solidity, proceeding inwards, may and probably does depend upon both temperature and pressure. Isothermal surfaces Prof. Hopkins finds to have *increasing* ellipticities. Surfaces of equal pressure, deduced from the hypothetical law of density, $A\,\frac{\sin qb}{b}$, have diminishing ellipticities, and if $qb_1 = 150°$ the above law agrees sufficiently well with the actual ellipticity and ratio of surface to mean density of the earth. This law for $\varepsilon = \frac{7}{8}\varepsilon_1$ demands a thickness of crust of $\frac{1}{4}$ the radius, or 1000 miles. This is a *minimum*, since the actual surface of solidification (lying between this and the corresponding isothermal surface) would have greater (and hence too great) ellipticity.

Before commenting upon this application, and upon the real meaning of the formula, I return to the case of homogeneousness. Some of the results arrived at by the analysis of Prof. Hopkins may be illustrated by the following considerations: The fluid spheroid, treated of p. 36, is subjected, by the attraction of the sun, to the distortion expressed by (47). This distortion, as shown by the form of the expression, is equivalent to an exceedingly slight rotational displacement[1] *of figure* about an equatorial axis, such as would be caused by displacing through a still more minute angle the planes of diurnal rotation. It is one of the beautiful results of the analysis to show that the change in the direction of the centrifugal force due to this slight obliquity of the planes of rotation is equivalent to turning forces at all points of the fluid exactly proportional to their distances from the equatorial axis.

Let now a rigid shell, exactly conforming internally to the external surface of the fluid, be applied, and the whole turned back until the planes of rotation are restored to perpendicularity to their axis; the precessional effect of the attracting body now operâtes upon the whole mass; for there are no longer counteracting tidal protuberances. If we take that part of (47) which is due to the *direct* action of the sun, viz., $\frac{3S}{r^3 g}\sin\theta\cos\theta\sin\lambda\cos\lambda\cos\varpi$ (for, the protuberances being repressed by the shell, the pressures on its interior which replace them will arise only from the direct action), and estimate it as a pressure and calculate the elementary couples for an internal ellipticity, e, we shall find the integral couple (and this corresponds with Prof. Hopkins' result) to be identical with (48) viz., exactly that due to the

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[1] The required angle of the displacement is the height of the tidal wave (47), for $\varpi = 0$, divided by $\frac{dr}{d\lambda}$ for an ellipse of ellipticity, e, ($2e\sin\lambda\cos\lambda$). Prof. Hopkins shows that the corresponding divergence of the planes from perpendicularity develops a couple $= \frac{4}{15}\pi n^2 e$ multiplied by the sine of twice this arc (or twice the arc itself). Performing the operations we get, $\frac{24}{15}\pi e\,\frac{S}{r^3}\sin\theta\cos\theta$, in which we have the solar couple (19) and (48), which causes the displacement, since $e = \frac{15}{8\pi}(C - A.)$

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couple which the sun would exert on the fluid mass considered as a solid.[1] It would increase the precessional force of the shell in the ratio $\frac{q^5}{q^5-1}$ of the analysis. By virtue of this pressure the fluid tends to transform its own precession into an augmented precession of the shell.

It requires, however, but an extremely minute angular separation of the axes of the shell and fluid to generate counter-pressures equivalent to those which caused the separation.[2] The divergence cannot, therefore, be progressive, but is simply a minute oscillation of the two axes, or a rotation around each other. In the latter form it appears in the analysis which, otherwise, gives to the internal fluid mass a precession *identical with that of the enveloping shell.*

Prof. Hopkins confines his analysis for the case of homogeneousness to equal ellipticities for the bounding surfaces of the shell. Excepting the case of sphericity for the inner surface, the result would be the same—viz., an unchanged precession, however the ellipticities might differ.

I now return to the formula (a) and remark, that it is an inaccurate expression for a slight difference (P_1—P') due to the fact that the spheroid is *heterogeneous*—that it is *not capable of being made a test of internal fluidity, or a measure of thickness of crust.*

I have already shown that for homogeneousness the couple due to pressure on the inner surface of the shell is identical with the sun-couple upon the fluid mass solidified, a result approximately true (as will be shown hereafter) if the density of the fluid strata vary. Hence, if we take the sum of the sun-couple exerted on a shell of interior and exterior ellipticities, ε and ε_1, and of the pressure-couple developed in the fluid,[3] and divide by the moment of inertia of the entire mass and by ω, we shall have the rate of gyration of the entire mass considered as a solid.

Referring to Prof. Hopkins' analysis and symbolism, the quotient will be[4]

$$(v) \qquad \frac{3}{2}\frac{\mu}{r^3\omega}\sin 2\Delta \frac{\int_a^{a_1} \rho' \frac{d\,(a^5\,\varepsilon')}{da'}\,da' + 2a^3\,\varepsilon \int_0^a \rho' a' da'}{\sigma\,(a_1) = \int_0^{a_1} \rho' \frac{da^5}{da'}\,da'}$$

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[1] The lever arm is also $2e \sin \lambda \cos \lambda$. Multiply the above by this arm, by g, by the elementary surface $d\,\mu\,d\,\varpi$, and, again, by $\cos \varpi$, and we get the elementary component tending to tilt the shell. The integral, with proper substitutions, is equivalent again to (19) or (48).

[2] There is another process which may take effect in neutralizing internal pressure. I have remarked (last par. p. 6), that, considered as a perfectly rigid body, the precessional motions of the earth cannot be *precisely* those assumed. In fact, our imperfect integrals of the conditional differential equations present the anomaly of a varied motion in which the generating force *does no work;* no yielding to the tilting couple having place. There are necessarily some, too minute to be detected, nutational movements. In case the precessional force were augmented by so large a ratio as $\frac{q^5}{q^5-1}$ would be for a *thin* shell, *these* nutational movements would surpass in magnitude those necessary to generate the required counteracting pressures.

[3] I use provisionally Prof. Hopkins' computations for this, involving $\int_0^a \rho' a' da'$; its erroneousness will appear hereafter.

[4] The symbols μ, Δ, ω, correspond to S, θ, n, of p. 7; ρ' is the density of stratum, solid or fluid, for which ε' is the ellipticity, and a' the polar radius; a_1 is the external, and a the internal polar radius of the shell.

Denote the moment of inertia of the entire spheroid by $I_1 = \frac{8}{15}\pi\,\sigma\,(a_1)$

" " " " " " shell " $I' = \frac{8}{15}\pi[\sigma(a_1)-\sigma(a)]$

" " " " " " nucleus " $I'' = \frac{8}{15}\pi\,\sigma\,(a)$

Then $$\sigma\,(a_1) = \frac{I_1}{I'}\Big(\sigma\,(a_1) - \sigma\,(a)\Big)$$

and the above expression, reduced to precession, will become

(x) $$\frac{I'}{I_1}\Big(1 + s + \frac{h}{q^5 - 1}\Big)\,P$$ (1)

Prof. Hopkins gets for the precession of the same spheroid considered as fluid within the shell, (his symbolic abbreviations used in both cases)

(y) $$\frac{(\gamma_2)}{(\gamma_1) + (\gamma_2)}\Big(1 + s + \frac{h}{q^5 - 1}\Big)\,P$$

In this last expression (γ_1) and (γ_2) denote coëfficients of gyration which one and the same couple (*i. e.* the centrifugal force, by pressure on the shell and by reaction on the fluid mass—*the assumption being made* that the latter, having its proportionate force on each particle, gyrates as a solid) produce upon the shell and fluid mass respectively. They should be therefore inversely proportional to the respective moments of inertia of the shell and nucleus, rendering the expressions (x) and (y) identical.

But this apparent identity is brought about by *assuming* that Prof. Hopkins' expression for the pressure-couple on the shell arising from the sun's attraction on the fluid to be identical (or at least approximately so) with that which would be exerted on the same heterogeneous fluid solidified; by which assumption I introduce in (x), $\frac{h}{q^5-1}$, (which is Prof. Hopkins' symbolic abbreviation of

$$\left.\frac{2a^3\int_0^a \rho' a' da'}{\sigma\,(a_1)-\sigma\,(a)}\right):\ \text{instead of}\ \frac{\int_0^a \rho' \frac{da^5\,\varepsilon'}{da}\,da'}{\varepsilon\,[\sigma\,(a_1)\,\sigma\,(a)]}$$

which latter expression belongs to the case just specified, of the *solidified* fluid.

Now in the case of nature—*i. e.* the earth with the received hypothetical laws of density and ellipticity, the two expressions differ in a ratio (about 4 : 3) so greatly exceeding unity, as to forbid the assumption of approximate equality. The error of the first expression will be better appreciated by referring to the quan-

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[1] The interpretation of (x) and (y) is obvious. P is the coefficient of precession for a *homogeneous* shell of uniform ellipticity ε; instead thereof let the shell be *heterogeneous* with same internal surface, but of an *external* ellipticity ε_1. P for such a shell will have a fractional increment denoted by the ratio s. By the pressure of the internal fluid the precessional coefficient of this shell will be still further increased by a ratio *denoted* (a result of the analysis) by $\frac{h}{q^5 - 1}$. But the shell is constrained to carry along and to take up a common precession with the nucleus, and the coefficient will be thereby diminished in a ratio $\frac{I'}{I_1}$ of (x) or (according to Prof. Hopkins) the corresponding expression of (y). Since $P = P_1\frac{\varepsilon}{\varepsilon_1}$, expression (a) is readily deducible from (y).

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tities (γ_1) and (γ_2) in (y). The brief account given, p. 43, will show how, as resulting from the analysis, a rotating homogeneous fluid enveloped and confined by a shell reacts, with a *practical* rigidity conferred by rotation, against the shell, (when the respective axes of rotation are slightly separated) and thereby receives an angular motion "precisely as if it were solid," (Phil. Trans., 1839, p. 394). It is clear that, through this interaction, the shell, likewise, must receive an angular motion, and that these several angular motions must be in inverse ratio to the respective movements of inertia of shell and nucleus; and so, for homogeneousness, the analysis makes them. When we come to heterogeneousness, the same *modus operandi* is (and rightly) attributed to the fluid; and again, most clearly, the relative angular motions of shell and fluid should be in above-mentioned inverse ratio; whereas their ratio is quite differently computed to be $\frac{h}{q^5-1}$ of which the value has just been given. The error (for there is clearly one) is in the computation of the pressure-couple developed in the fluid and exerted upon the shell by centrifugal force, when their axes of rotation are slightly separated.

The same error of computation (exhibiting itself by the identical symbol, $\frac{h}{q^5-1}$) enters into the expression for the pressure-couple developed by solar attraction, and introduces *again* the above symbol as the third term of the second factor of (y).

Without going into lengthy discussion, it is sufficient to remark that both the centrifugal force and the foreign attraction produce in the strata of equal density of a heterogeneous fluid, *special configurations*, and expend themselves in so doing; and, moreover, that the prior establishment of these forms of equilibrium is assumed, and necessarily assumed, in the analysis. It is, therefore, unwarrantable to integrate (as is done) through the fluid mass these forces, as free forces, to get its pressure upon the shell.[1]

I have shown, I think, that in the expression (y) the first factor should be corrected to be, as it is in (x), $\frac{I'}{I_1}$.

The correction consists in substituting for $\frac{(\gamma_1)}{(\gamma_2)}$ in the *first* factor of (y), $\frac{\int_0^a \rho' \frac{da^5}{da}\, da'}{\sigma(a_1)-\sigma(a)}$ instead of $\frac{h}{q^5-1}$ [or its value, p. 45]. The *same* correction for $\frac{h}{q^5-1}$, introduced into the second factor, would require $\varepsilon \int_0^a \rho' \frac{da^5}{da} da'$ to be substituted for the second term of numerator of (v). But that (v) should belong to entire solidity we require $\int_0^a \rho \frac{d\, a^5 \varepsilon'}{da'} da'$.(2) Now these quantities differ inappreciably, for taking the entire

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[1] It is obvious that the taking account of the internal motions by which the configurations produced by foreign attraction adapt themselves to the diurnal rotation, does not meet the point made.

[2] Although the errors in the two cases have the same expression, it does not follow that the corrections should be *identical.* The configurations of strata of like density due to foreign attraction are superinduced on the *previously established* configurations due to the centrifugal force, which have the varying ellipticity ε'. The correction should involve this ellipticity, and it is probable that the above symbol is, if we disregard the slight *internal motions*, the true one. Indeed, I think I may venture to affirm, that, given a heterogeneous fluid wholly enveloped by a rigid bounding surface,

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integrals from zero to a_1 and multiplying by $\frac{8}{15}\pi$, the last becomes (Thomson and Tait, § 825),

$$\frac{2}{3} M a_1^2 (\varepsilon_1 - \tfrac{1}{2} m) = \mathfrak{C} - \mathfrak{A}, \text{ for the earth as actually constituted.}$$

And the first (deduced from Hopkins, Phil. Trans., 1840, pp. 203 and 204), is (nearly)

$$\frac{1}{3} M a_1^2 \varepsilon_1 = C - A, \text{ for same spheroid with } \textit{uniform} \text{ internal ellipticities.}$$

($M =$ mass of the earth).

[The value of the first, using the constants of density of Archdeacon Pratt, "Figure of the Earth," 4th ed., p. 113, is somewhat *less* than this last expression.] Now $m = \frac{1}{289}$ (ratio of centrifugal force to gravity) and hence $2(\varepsilon_1 - \frac{1}{2}m)$ is very little less than ε_1. For a fluid *nucleus*, the inequality would be still less.

Hence it appears that both (x) and (y) (when corrected) express very nearly the precession of the solidified earth; and, moreover, that the effect upon precession due to the variation of internal ellipticity is very small, the precession of the earth considered as rigid being, essentially, that corresponding to uniform ellipticity; or what is the same thing, that of a *homogeneous spheroid of its external form.*

This also appears in the comparison of the value of $\frac{C-A}{A}$, as established from observation, and the resulting *calculated* ellipticity. The first is .00327 and the second $\frac{1}{297}$ (Thomson and Tait, § 828). Now a homogeneous spheroid of the latter ellipticity would have for $\frac{C-A}{C}$ a value $(e - 4e^2)$ of .00332; a difference of about $\frac{1}{66}$. Variations in the constants which enter into the expressions for internal density give rise to variations in the *calculated* ellipticity—and, of course, in the resulting precession; but if the *external* ellipticity is defined by a rigid shell, the effect of *internal* variation is, in the case in hand, almost *nil.* Hence, had the hypothetical consolidation of the earth, of p. 43, been carried to the very centre, no material approximation to the desired correction of $\frac{1}{8}$ in the calculated precession would have been found.[1] In fact, the problem for heterogeneousness

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subjected to a foreign attraction, and a condition of static equilibrium assumed, the pressure-couple exerted by the fluid on the shell cannot differ from that which the attraction would exert on the solidified fluid.

In the case of homogeneousness, I have arrived (p. 43: the results, though based on an ellipticity corresponding to fluid equilibrium, hold good for any *small* ellipticity) at the exact expression for the pressure-couple, from the function expressing the tidal protuberance due to the foreign attraction. The tidal configuration of the heterogeneous earth, wholly liquefied, would result from the transcendental analysis of Hopkins, pp. 203, 204, or of Thomson and Tait, § 822–824, and the maximum height would be one foot, very nearly; but the pressure function cannot be readily deduced. It would depend on gravity and [§ 825] "the value of $C - A$ may be determined solely from a knowledge of surface or external gravity, or from the figure of the sea level without any data regarding the internal distribution of density."

[1] It is curious, to say the least, that there should be ground for the remark that the expressions (a) and (y), which latter, with its author's valuation of $\frac{(\gamma_1)}{(\gamma_2)}$, may be writen $\left\{1 + \frac{s}{1 + \frac{h}{q^5 - 1}}\right\} P$, give,

was practically solved under Prof. Hopkins' treatment of it when it was shown that, for homogeneousness, the precession for internal fluidity was the same as for solidity, and when it appeared that the analysis applied would exhibit the action of the heterogeneous fluid as if disposed in strata of like ellipticity with that of the inner shell surface. The erroneous computation of pressures has merely the effect of exaggerating in a like ratio the densities of all the fluid strata. Hence, and hence only, a resulting precession differing materially from that which would result from solidity.

In conclusion, I remark, 1st. The analysis of Prof. Hopkins, in its application to a homogeneous fluid and shell, seems to establish (and the result is confirmed by its harmony with tidal phenomena as developed in p. 43) that the rotation imparts to the fluid a *practical* rigidity[1] by which it reacts upon the shell as if it were

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with decreasing internal ellipticities, for fluidity of nucleus *less precession than would belong to solidity.* This is obvious since his $\frac{h}{q^5-1}$ is greater than the ratio $\frac{I''}{I'}$ (nearly) which should take its place on the latter hypothesis.

[1] I do not concur with Sir William Thomson in the opinions quoted in note, p. 38, from Thomson and Tait, and expressed in his letter to Mr. G. Poulett Scrope ("Nature," February 1st, 1872), so far as regards fluidity, or imperfect rigidity, within an infinitely rigid envelope. I do not think the rate of precession would be affected.

That no increase arises from fluidity I have endeavored to show; and it is unquestionably a corollary of Prof. Hopkins' investigations. As regards imperfect rigidity, Sir William Thomson bases his argument upon the assumption that "the whole would not rotate as a rigid body round one 'instantaneous axis' at each instant, but the rotation would take place internally, round axes deviating from the axes of external figure, by angles to be measured in the plane through it and the line perpendicular to the ecliptic in the direction towards the latter line. These angular deviations would be greater and greater the more near we come to the earth's centre. * * * * * Hence the moment of momentum round the solsticial line would be sensibly less than if the whole mass rotated round the axis of figure."

If I do not misunderstand his language, Sir William Thomson assumes that the same *bending* distortion which would ensue from the application of a couple to the external portions of a non-rotating spheroid, would, equally and *identically,* take place in a rotating one: thus causing the angle made by the planes of the external rings of matter and the solsticial line to be increased; with a corresponding diminution of the component of Cn about this line.

In the case specified by him (an extreme one) while sensible and important nutational movements would ensue, the mean *precession* would be insensibly affected; but I do not think precisely *such* elastic yielding would take place.

As an extreme case of an infinitely rigid and infinitely *thin* shell containing matter completely destitute of rigidity, take the fluid spheroid of p. 36, and conceive it enveloped by such a shell. It is *still,* as shown, p. 43, susceptible (and susceptible *only*) of the extremely minute deflections of its planes of rotation by which precession is completely annihilated. Confer now upon the contents of the shell rigidity, uniform, or varying from surface to centre, continuously or discontinuously, in any arbitrary manner, and you have every possible case of imperfectly rigid matter contained within a perfectly rigid crust. I can attribute no other effect to the conferred rigidity than a restoration of the *lost* precession—in whole or in part; nor can I suppose the shell enveloping imperfectly rigid matter to change its obliquity *more* than that which contains the fluid; regard being had to conditions of *equilibrium* without reference to living forces generated.

I must remark that this hypothetical case, though as admissible for argument as any other form of "preternaturally rigid" crust, is exceptional. With a shell of *finite* moment of inertia, having some comparable relation to that of the fluid contents, the precession, instead of being annihilated, would be that due to the entire mass a solid mass, while its pressure imparts to the shell the requisite couple to preserve the precession unchanged.

2d. The same practical rigidity is, with entire reason, attributed to the heterogeneous fluid by which (leaving out of view minute *relative* oscillations which do not affect the mean resultant in other natural phenomena and should not in this) the shell and fluid take a common precession.

3d. The two masses retaining their configurations, mutual relations, and rotary velocities, essentially unaltered by the hypothesis of internal fluidity, it would be a violation of fundamental mechanical principles were the resulting precession not identical with that due to the entire mass considered as solid.

4th. The common and identical precession of fluid and shell resulting from the analysis, is indispensable to any conception of *precession* for the earth as composed of thin shell and fluid; for otherwise internal equilibrium would be destroyed and the "figure of the earth" cease to have any assignable expression. The entire mass, fluid and solid must (without invoking the aid of "viscosity"), be "carried along in the precessional motion of the earth." The analysis I have examined demonstrates the possibility and exhibits the *rationale* of such a community of precession, but fails in the attempt to exhibit a test of the existence or absence of internal fluidity.

5th. The powerful pressures that would be exerted upon a *thin* and rigid shell would probably produce in it noticeable nutational movements;[1] while if the shell be *not* of a rigidity far surpassing that of the constituents of the cognizable crust, the "precessional motion of the earth" would, owing to the neutralizing effect of tidal protuberances, scarcely be observable.

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[1] *Vide* p. 44, and note 2: without reference to conventional "Nutation" which is but a form of *precession*. In connection with these *relative* motions of shell and fluid, it is in place to allude to the "Vindication of Mr. Hopkins' method against the strictures of M. Delaunay," by the late Archdeacon Pratt ("Figure of the Earth," 4th ed., p. 132). He reasons, that, if at any moment, the crust and fluid be arranged as to density, "exactly as if they had been hitherto one solid mass and be moving alike, this state cannot possibly continue." For the shell will be acted upon not only by the foreign attraction but by the fluid pressure, and will "begin to move quicker," with a precession due to *both* the thence arising couples. That this should not occur requires, he estimates, the counteracting centrifugal force of a tidal protuberance, in a crust supposed 100 miles thick, of *seventy-four feet.*

The writer does not seem to be aware that the author whom he vindicates finds no such relative acceleration of the shell (*vide* p. 44, § 2, of this Addendum) as resulting from the pressure; and, strangely, for an authority on the "Figure of the Earth," fails to recognize that "an elevation of the outer surface of the crust"—that is a tidal distortion—of *a single foot,* would relieve the shell from all pressure. This is, perhaps, a natural result of the use of an expression (Prof. Hopkins') for the pressure which disregards the influence of "Figure."

ADDENDUM TO NOTE 1, PAGE 38.

THE apparent antagonism between the theorem of the text and that of Laplace suggests a few additional words. The theorem of Laplace is that "in whatever manner the waters of the ocean act upon the earth, either by their attraction, their pressure, their friction, or by the various resistances which they suffer, they communicate to the axis of the earth a motion which is *very nearly equal* to that it would acquire from the action of the sun and moon upon the sea, if it form a solid mass with the earth." (Méc. Cél., Bowditch [3345].)

The theorem is demonstrated in two distinct, quite different, manners. The last demonstration is founded upon the principle of the "conservation of areas;" and as the result of *this* demonstration the proposition is stated in the above quoted words.

The first demonstration is purely analytical, and, after stating that "this fluid" (*i. e.* of the ocean) "acts upon the terrestrial spheroid by its pressure and by its attraction," Laplace proceeds to find the analytical expressions for the precession and nutation-producing couples due to this pressure and to this attraction as they are modified by the attraction of the sun and moon upon the fluid. He then proceeds to calculate these couples for the material substance of the ocean, considered as rigidly connected (or forming a solid mass) with the earth. He finds the couples, so calculated, respectively, identical in the two cases, and epitomizes the result as follows: "the phenomena of the precession of the equinoxes and the nutation of the earth's axis *are exactly the same* as if the sea form a solid mass with the spheroid which it covers." [3287.]

But this demonstration is limited by the assumption that "the sea wholly covers the terrestrial spheroid or nucleus, that is of a regular depth, and suffers no resistance from the nucleus;" and *both* demonstrations imply an ocean of (relatively) small depth.

Under the last *mentioned* treatment of the subject the proposition of Laplace and that which I demonstrate are but the extreme phases exhibited by the solution of a problem, according as the datum be that the depth of the sea is minute (in which case its entire precession-producing couple, not effectively exerted upon its own mass, is *almost* wholly *transferred* to the solid nucleus); or that the nucleus is very small, in which case the lost precession-producing couple of the fluid is but in small part transferred to the nucleus—or wholly disappears with the vanishing of the latter.

I think, however, that the last-mentioned (first in point of order) demonstration of Laplace is not as general as the language quoted [3287] would indicate. The omission of variations of the radius-vector, R, in all the integrations gives rise to errors which do not seem to me to be identical in the two processes by which the couples are calculated, when the variation of depth is very small.

An apt illustration of the above remarks is derived from the supposition—as admissible as any other—that, the depth, γ, is constant. In this case, *whatever be the ellipticity* of the solid nucleus, the value of y (the height of the diurnal or precession-affecting tide, see [2253] [3333] and also p. 36) is zero, and, of course, the couples [3272] and [3273] become zero—as will be found by performing the integrations in those equations. So, of course, do expressions [3284] and [3285] become zero, with γ made constant. But these last *should not* be zero except for the case of sphericity of the nucleus.[1] The expressions do not seem to me capable of sustaining the inference which I have quoted [3287], when the depth of the sea is uniform; a case which most naturally presents itself to the mind.

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[1] A shell of slight internal ellipticity and small *uniform* thickness has a precessional coëfficient of four-fifths the value of that of a shell bounded by surfaces of equal ellipticity. Hence *in general* the variations of R (or the ellipticity) produce effects insensible compared to those of the depth.

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Mr. Airy (Tides and Waves, Art. 127) bases his demonstration of the theorem exclusively upon the principle of the conservation of areas, remarking at the outset, "if the earth and sea were so entirely disconnected that one of them could revolve for any length of time with any velocity, increasing or diminishing in any manner, while the other could revolve with any other velocity changing in any other manner, we could pronounce nothing as to the effect of the fluctuation" (tidal) "upon precession."

A spheroidal nucleus wholly covered by an ocean of uniform depth, suffering no resistance, does not seem to me to lack much for fulfilling the above conditions.

If *velocities* are generated in the waters of the ocean by solar (or lunar) attraction, the centrifugal forces due to them might be looked to (though not alluded to by Laplace) as agents for transferring, from the fluid to the nucleus, the precession-producing couples due to the fluid mass, especially in the above hypothetical case. It will be found, however, by reference to the expressions [2260], that they give rise to no couple, and are, moreover, very minute.

The motion which the displacements [2260] [2261] indicate is a slight oscillation of the axis of the fluid envelope, moving as a solid, about the axis of the nucleus, the angular distance between these axes being slightly less than 2 seconds: it is, I presume, that which a non-rotating shell would have were the attracting body, with constant distance and declination, to move, with angular velocity n, in right ascension. In the case in hand it is the fluid shell which revolves and, suffering no change of form, would be itself affected by its proper precessional couple to the exclusion of the oscillation above described.

ADDITIONAL NOTES.

NOTE TO PAGE 11.

(1) The process indicated is a more legitimate carrying out of the methods peculiar to this paper than what follows in the text. The tangent of MM' (35) may be (approx.) taken for the sine, and the cosine taken constant at unity, as may also be the cos I'.

From (32) we may calculate by developing and neglecting terms in which $\sin^2 I'$ enters

$$\sin i = (1 - \cos^2 i)^{\frac{1}{2}} = \sin I - \sin I' \cos I \cos n_3 t$$

$$\sin i \cos i = \sin I \cos I - \sin I' \cos 2I \cos n_3 t - \tfrac{1}{2} \sin^2 I' \sin 2I \cos^2 n_3 t$$

Introducing these values in (38) and (39), and integrating we get expressions identical with 44 and 45, except a (practically) immaterial difference in the coefficient of t in the first which becomes $1 - \frac{1}{2} \sin^2 I'$ instead of $1 - \frac{3}{2} \sin^2 I'$.

NOTE TO PAGE 24.

(1) The foregoing interpretation of the symbolic integral in (7), adopted with hesitation from authors cited, is based on assumed constancy of the angle ϕ; but this angle necessarily varies, slowly indeed, but progressively, by the azimuthal motion measured by $n \sin \lambda$. The conditions for the formation of a leminiscate are not, therefore, rigidly fulfilled. It will be found, however, taking into account a complete excursion, that the slight increment which will enure to the moment of the quantity of motion, $\sin^2\theta \frac{d\phi}{dt}$, from this cause on one side of the vertical, will be neutralized on the other, in consequence of the opposing signs of $\cos \phi$, in opposite azimuths; or, at least, the *resultant* increment or decrement will be a quantity of the second order in minuteness, and hence, affecting only in the same degree the azimuthal motion $\frac{d\phi}{dt}$.

NOTE TO PAGE 39.

(1) The differential attraction of the sun on any length $d\chi$ of the rod, at distance χ from the earth's centre is $\left(\frac{S}{(r-\chi)^2} - \frac{S}{r^2}\right) d\chi$, r being sun's distance. Integrate from $\chi = \chi$ to $\chi = R$ (the earth's radius).

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$$(1) \qquad S\left(\frac{1}{r-R}-\frac{R}{r^2}-\frac{1}{r-x}+\frac{x}{r^2}\right)$$

The above divided by the coefficient of elasticity E will give the elongation per unit of length at any point. Multiply by dx and integrate from $x=0$ to $x=R$, and the total elongation is

$$\frac{S}{E}\left\{\frac{R}{r-R}-\frac{R^2}{2r^2}+\log\left(1-\frac{R}{r}\right)\right\}.$$

Since $\log\left(1-\frac{R}{r}\right)=-\frac{R}{r}-\frac{1}{2}\frac{R^2}{r^2}-\frac{1}{3}\frac{R^3}{r^3}-$ &c., the foregoing will reduce, approximately to $\frac{2}{3}\frac{S}{E}\frac{R^3}{r^3}$.

If $M=$ earth's mass, $g=$ gravity at its surface, and $\frac{1}{n}$ the ratio of M to S, and m the ratio of length of rod of weight E (per square inch section) to R, we shall have: $S=ngR^2$, $E=mgR$, and the above expression for total elongation becomes:

$$(2) \qquad \frac{2}{3}\frac{n}{m}R\frac{R^3}{r^3}$$

Take $\frac{R}{r}=\frac{4000}{92000000}=\frac{1}{23000}$; $R=4000\times5280$ feet; $n=316000$; $m=.472$ (the latter value based on $E=34$ mill'ns lbs. per square inch, and a steel rod of that section to weigh 3.4 lbs. per foot length) and the total elongation (2) becomes $0^{\text{ft}}.975$. The *maximum* extension per unit of length is at the centre, and is found by putting $x=0$ in (1) and dividing by E. It is $\frac{S}{E}\frac{R^2}{r^3}=\frac{n}{m}\frac{R^3}{r^3}=$ .000000055, indicating a strain of $1^{\text{lb}}.87$ per square inch. The ratio of the *total* elongation (2) to the total length of the rod R is two-thirds of the above, indicating *about* that ratio for the ellipticities of superficial and central strata of a steel globe distorted by the sun's attraction; a result thus rudely calculated which differs little from that given in Thomson and Tait, § 837.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
241

A CONTRIBUTION

TO THE

HISTORY OF THE FRESH-WATER ALGÆ

OF

NORTH AMERICA.

BY

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[ACCEPTED FOR PUBLICATION, FEBRUARY, 1872.]

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THE following memoir was referred for examination to Dr. John Torrey and Dr. F. A. P. Barnard, of Columbia College, New York. They recommended its publication provided certain changes were made in the manuscript. These having been made by the author, the work is published as a part of the series of "Smithsonian Contributions to Knowledge."

JOSEPH HENRY,
Secretary, S. I.

WASHINGTON, October, 1872.

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PREFACE.

Of all the various branches of Natural History, none has been more enthusiastically and more successfully prosecuted in the United States than Botany. The whole field has been most thoroughly occupied, save only as regards certain of the lower cryptogams, and amongst the latter, it is the *fresh-water Algæ* which alone can be said to have been almost totally neglected. In this fact lies my apology for offering to the scientific public the following memoir.

In doing this, so far from thinking that the work contains no error, I hasten to disarm criticism, and to ask with solicitude for a favorable reception, in view of the difficulties of the investigation, which I have conducted alone, and almost unaided.

The investigation was first undertaken in connection with my elementary studies of Materia Medica and Therapeutics, and has since been prosecuted at intervals amidst the distractions of medical teachings and practice, and in some cases without immediate access to authorities. The field covered is so wide that it is almost impossible to exhaust it, and, if it were not for rapidly increasing professional engagements, I would gladly devote more time to the subject; but, as it is, I must leave to others to carry on the work thus begun.

While saying this, it is but just to state that nothing here published has been done hastily, but that all is the result of arduous and conscientious investigation.

A very large part of my material has been of my own gathering, and was studied whilst fresh; but I am indebted to several persons for aid by collections.

First of all, I desire to offer my thanks to Dr. J. S. Billings, U. S. A., and to Professor Ravenel, of South Carolina; to the former for assistance in various ways, and for collections made near Washington City; to the latter for very large collections made in Texas, South Carolina, and Georgia. I am also indebted to Mr. C. F. Austin for a large collection gathered in Northern New Jersey, to Mr. William Canby for some beautiful specimens obtained in Florida, to Professor Sereno Watson for Rocky Mountain plants, and to Dr. Frank Lewis for a number of White Mountain desmids.

These various collections were partly dried and partly preserved in a watery solution of carbolic acid or of acetate of alumina, both of which I have found more or less satisfactory preservatives.

The present investigations embrace all families of the fresh-water algæ except the *Diatomaceæ*, which, as every one knows, are so numerous as to constitute in

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themselves a special study. As I have paid no attention to these plants, they are of course not included in this memoir.

In the synonymy I have generally followed Prof. Rabenhorst. The original descriptions of the forms, especially those of the older authorities, are very frequently so meagre and obscure, that the species cannot be recognized by them with any certainty. Prof. Rabenhorst has gone over the ground most carefully, with access to the whole literature of the subject and probably to all extant type specimens, and his decisions are, no doubt, as accurate as the circumstances will allow. To attempt to differ from them, to go behind his work to the original sources and make fresh interpretations, would cause endless confusion. I have, therefore, nearly always contented myself with his *dictum*, and have referred to him as the authority for the names used.

The following references were omitted through a misunderstanding from the first portion of the text.

Page 14. *Cælosphærium dubium*, Grunnow. Rabenhorst, Flora Europ. Algarum, Sect. I. p. 55.
" 15. *Merismopedia convoluta*, Brébisson. Rabenhorst, Flora Europ. Algarum, Sect. I. p. 58.
" 18. *Oscillaria chlorina*, Kützing. Rabenhorst, Flora Europ. Algarum, Sect. I. p. 97.
" 18. *O. Fröhlichii*, Kützing. Rabenhorst, Flora Europ. Algarum, Sect. I. p. 109.
" 19. *O. nigra*, Vaucher. Rabenhorst, Flora Europ. Algarum, Sect. I. p. 107.
" 19. *O. limosa*, Agardh. Rabenhorst, Flora Europ. Algarum, Sect. I. p. 104.
" 21. *Chthonoblastus repens*, Kützing. Rabenhorst, Flora Europ. Algarum, Sect. I. p. 132.
" 22. *Lyngbya muralis*, Agardh. Harvey, Nereis Boreali-Americana, pt. III. p. 104.

In the text after the "*Habitat*," a name is quoted as the authority therefor; if such a name be in brackets, it signifies that the specimens were simply collected by such individual, but that the identification was made by some one else; when there is not a name *uninclosed* in brackets, it is meant that the identification was made by the author of this memoir.

Since the present memoir has gone to press, I have received from the author a copy of "Algæ Rhodiaceæ. A list of Rhode Island Algæ, collected and prepared by Stephen T. Olney, in the years 1846–1848, now distributed from his own herbarium."

In the introduction to this list, Mr. Olney says: "Of the fresh-water species, I have few for distribution. These were obtained mainly in the environs of this city, and were placed in twenty-seven small vials in Goadsby's solution, and sent to Prof. Harvey, who submitted them to the judgment of the most learned English botanist in this particular department, G. H. K. Thwaites, Esq., then of Bristol, England. The large number of species found in this collection, in so limited a range, and collected within a very short period, is surprising, and shows what more persistent collections will develop. I have not time to collate the numerous publications of the lamented Prof. Bailey, or I might have made the list of this portion of Rhode Island plants more complete."

The chlorosperms of this list are as follows:—

Porphyra vulgaris, Ag.-Harv. Ner Bor. Am. 3. 53. Newport.
Bangia fuscopurpurea, Lyngb.-Harv. Ner. Bor. Am. 3. 54. Southern Rhode Island.

Enteromorpha intestinalis, LYNGB.-HARV. Ner. Bor. Am. 3. 56. Providence to Newport.
Enteromorpha compressa, GREV.-HARV. Ner. Bor. Am. 3. 56. Southern Rhode Island.
Enteromorpha clathrata, GREV.-HARV. Ner. Bor. Am. 3. 56. Newport.
Ulva latissima, L.-HARV. Ner. Bor. Am. 3. 59. Providence.
Ulva lactuca, L.-HARV. Ner. Bor. Am. 3. 60. Providence.
Tetraspora lacunosa, CHAUV.-HARV. Ner. Bor. Am. 3. 61. *T. perforata*, BAILEY Mss. Providence.
Tetraspora lubrica, AG. Providence.
Batrachospermum pulcherrimum, HASS. Providence.
Batrachospermum moniliforme, ROTH.-HARV. Ner. Bor. Am. 3. 63. Providence.
Chætophora endivæfolia, AG.-HARV. Ner. Bor. Am. 3. 69. Providence.
Draparnldia glomerata, AG.-HARV. Ner. Bor. Am. 3. 72. Providence.
Stigeoclonium minutum, KÜTZ. Providence.
Cladophora rupestris, L.-HARV. Ner. Bor. Am. 3. 74. Newport.
Cladophora glaucescens, GRIFF.-HARV. Ner. Bor. Am. 3. 77. Rhode Island.
Cladophora refracta, ROTH.-HARV. Ner. Bor. Am. 3. 79. Southern Rhode Island.
Cladophora Rudolphiana, AG.-HARV. Ner. Bor. Am. 3. 80. Providence.
Cladophora gracilis, GRIFF.-HARV. Ner. Bor. Am. 3. 81. Little Compton.
Cladophora fracta, HARV. Ner. Bor. Am. 3. 82. Rhode Island, Bailey.
Chætomorpha ærea, DILLW.-HAR. Ner. Bor. Am. 3. 86. Newport, etc.
Chætomorpha Olneyi, HARV. Ner. Bor. Am. 3. 86. Little Compton.
Chætomorpha longiarticulata, HARV. Ner. Bor. Am. 3. 86. Little Compton.
var. crassior, HARV. Ner. Bor. Am. 3. 86. Little Compton.
Chætomorpha sutoria, BERK.-HARV. Ner. Bor. Am. 3. 87. Newport.
Zygnema malformatum, HASS. 1. 147. Providence.
Zygnema catenæforme, HASS. 1. 147. Providence.
Zygnema Thwaitesii, OLNEY, n. s. Near *Z. subventricosum*, Providence.
Zygnema longatum, HASS. 1. 151. Providence.
Zygnema striata, OLNEY, n. s. "Cells evidently striated," Thwaites. Providence.
Tyndaridea bicornis? HASS. 1. 162. Providence.
Tyndaridea insignis? HASS. 1. 163. Providence.
Mesocarpus parvulus, HASS. 1. 169. Providence.
Mougeotia genuflexa, AG.-HASS. 1. 173. Providence.
Vesiculifera concatenata, HASS. 1. 201. Providence.
Vesiculifera æqualis, HASS. 1. 205. Providence.
Vesiculifera bombycina, HASS. 1. 208. Providence.
Vesiculifera Candollii, HASS. 1. 208. Providence.
Bulbochæte Thwaitesii, OLNEY, n. s. Providence.
Lyngbya majuscula, HARV. Bor. Am. 3. 101. Providence.
Sphæroplea virescens, BERK. Providence.
Sphæroplea punctalis, BERK. Providence.
Tolypothrix distorta, KÜTZ.-HASS. 1. 240.
Calothrix confervicola, AG.-HARV. Ner. Bor. Am. 3. 105. Providence.
Calothrix scopulorum, AG.-HARV. Ner. Bor. Am. 3. 105. Providence.
Hyalotheca dissiliens, BREV.-RALFS. Des. 51. (*Gloeoprium.*) Providence.
Hyalotheca mucosa, EHRH.-RALFS. Des. 53. Providence.
Didymoprium Grevillii, KÜTZ.-RALFS. Des. 61. Rhode Island, Bailey.
Didymoprium Borreri, RALFS. Des. 58. Rhode Island, Bailey.
Desmidium Swartzii, AG.-RALFS. Des. 61. Throughout United States, Bailey.
Aptogonum Baileyi, RALFS Des 209. Worden's Pond, Rhode Island, Bailey.
Micrasterias rotata, RALFS. Des. 71. Providence.
Micrasterias radiosa, AG.-RALFS. Des. 72. Maine to Virginia, Bailey.
Micrasterias furcata, RALFS. Des. 73. Worden's Pond, Rhode Island, Bailey.
Micrasterias Crux-Melitensis, RALFS. Des. 73. Maine to Virginia, Bailey.
Micrasterias truncata, BREB.-RALFS. Des. 75. United States, Bailey.

Micrasterias foliacea, Bailey-Ralfs. Desm. 210. Worden's Pond, Rhode Island, Bailey.
Micrasterias Bayleyi, Ralfs. Desm. 211. Rhode Island, Bailey.
Euastrum oblongum, Ralfs. Des. 80. Rhode Island, Bailey.
Euastrum crassum, Kütz.-Ralfs. Des. 81. Rhode Island, Bailey.
Euastrum ansatum, Ehrh.-Ralfs. Des. 85. *E. binale* Kntz. Providence.
Euastrum elegans, Kütz.-Ralfs. Des. 89. Providence.
Euastrum binale, Ralfs. Desm. 91. Providence.
Cosmarium cucumis, Corda.-Ralfs. Desm. 93. United States, Bailey.
Cosmarium bioculatum, Ralfs. Des. 95. Providence.
Cosmarium Meneghinii, Breb.-Ralfs. Des. 96. United States, Bailey.
Cosmarium crenatum, Ralfs. Des. 96. Providence.
Cosmarium amœnum, Breb.-Ralfs. Des. 102. Providence.
Cosmarium ornatum, Ralfs. Des. 104. Providence.
Cosmarium connatum, Breb.-Ralfs. Des. 108. Providence.
Cosmarium Cucurbita, Ralfs. Des. 109. Providence.
Cosmarium granditaberculatum, Olney, n. s.; "near *C. cucumis*, but with large tubercles on the frond." Providence.
Staurastrum orbiculare, Ralfs. Des. 125. Providence.
Staurastrum hirsutum, Ralfs. Des. 127. Providence.
Staurastrum Hystrix, Ralfs. Des. 128. Providence.
Staurastrum gracile, Ralfs. Des. 136. Providence.
Staurastrum tetracerum, Ralfs. Des. 137. United States, Bailey.
Staurastrum cyrtocerum, Breb.-Ralfs. Des. 139. Providence.
Tetmemoras Brêbissoni, Ralfs. Des. 145. Providence.
Tetmemoras granulatus, Ralfs. Des. 146. Providence.
Penium margaritaceum, Breb.-Ralfs. Des. (*Closterium* Ehr.) Providence.
Penium Digitus, Breb.-Ralfs. Des. 151. (*Closterium lamellosum.*)
Docidium nodulosum, Breb.-Ralfs. Des. 155. Maine to Virginia, Bailey.
Docidium Baculum, Breb.-Ralfs. Des. 158. United States. Bailey.
Docidium nodosum, Bailey-Ralfs. Des. 218. United States, Bailey.
Docidium constrictum, Bailey-Ralfs. Des. 218. Worden's Pond, Bailey.
Docidium verrucosum, Bailey-Ralfs. Des. 218. Rhode Island, Bailey.
Docidium verticillatum, Bailey-Ralfs. Des. 218. Worden's Pond, Bailey.
Closterium Lunula, Ehrh.-Ralfs. Des. 163. New England, Bailey.
Closterium moniliferum, Ehrh.-Ralfs. Des. 163. New England, Bailey.
Closterium striolatum, Ehrh.-Ralfs. Des. 173. New England, Bailey.
Closterium cuspidatum, Bailey-Ralfs. Des. 219. Worden's Pond, Bailey.
Pediastrum tetras, Ralfs. Des. 182. New England, Bailey.
Pediastrum heptactis, Ralfs. Des. 183. Providence.
Pediastrum Boryanum, Menegh.-Ralfs. Des. 187. Maine to Mexico, Bailey.
Pediastrum ellipticum, Hass.-Ralfs. Des. 188. Maine to Virginia, Bailey
Scenedesmus quadricauda, Breb.-Ralfs. Des. 190. Maine to Virginia, Bailey.
Scenedesmus obtusus, Meyen.-Ralfs. Des. 193. Maine to Virginia, Bailey.

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INTRODUCTION.

ALTHOUGH beset with difficulties in the outset, no branch of natural science offers more attractions, when once the study is fairly entered upon, than the fresh-water algæ. The enthusiasm of the student will soon be kindled by the variety and beauty of their forms and wonderful life processes, and be kept alive by their abundance and accessibility at all seasons of the year; for unlike other plants, the winter with them is not a period of counterfeited death, but all seasons, spring, summer, autumn, and winter alike, have their own peculiar species. They have been found in healthy life in the middle of an icicle, and in the heated waters of the boiling spring; they are the last of life alike in the eternal snow of the mountain summit and the superheated basin of the lowland geyser.

In their investigation, too, the physiologist can come nearer than in almost any other study to life in its simplest forms, watching its processes, measuring its forces, and approximating to its mysteries. Sometimes, when my microscope has revealed a new world of restless activity and beauty, and some scene of especial interest, as the impregnation of an œdogonium, has presented itself to me, I confess the enthusiastic pleasure produced has been tempered with a feeling of awe.

To any on whom through the want of a definite pursuit the hours hang heavy, to the physiologist who desires to know cell-life, to any student of nature, I can commend most heartily this study as one well worthy of any pains that may be spent on it.

An aquarium will often, in the winter time, give origin to numerous interesting forms, but it is not a necessity to the fresh-water algologist; besides his microscope and its appliances, all that he absolutely needs is a few glass jars or bottles and the fields and meadows of his neighborhood.

The great drawback to the investigation of these plants has been the want of accessible books upon them. In the English language there is no general work of value, and the various original memoirs are separated so far and wide in the Continental and English journals, as to be of but little use to most American readers. The *Flora Europœum Algarum Aquœ Dulcis et Submarinœ*, of Prof. Rabenhorst, has done much to facilitate the study, and its cheapness brings it within the reach of all. It merely gives, however, brief diagnoses of the various species, but with the present memoir will, I trust, suffice for the American student, at least until he is very far advanced in his researches.

A certain amount of experience and knowledge of the subject greatly facilitates the collection of these plants, but scarcely so much as in other departments of cryptogamic botany, since most of the species are so small that the most experienced algologist does not know how great the reward of the day's toil may be until he places its results under the object glass of his compound microscope. In order to aid those desirous of collecting and studying these plants, I do not think I can do better than give the following hints as to when and where to find, and how to preserve them.

There are three or four distinct classes of localities, in each of which a different set of forms may be looked for. These are: stagnant ditches and pools; springs, rivulets, large rivers, and other bodies of pure water; dripping rocks in ravines, &c.; trunks of old trees, boards, branches and twigs of living trees, and other localities.

In regard to the first—*stagnant waters*—in these the most conspicuous forms are oscillatoriæ and zygnemaceæ. The oscillatoriæ may almost always be recognized at once, by their forming dense, slimy strata, floating or attached, generally with very fine rays extending from the mass like a long, delicate fringe. The stratum is rarely of a bright green color, but is mostly dark; dull greenish, blackish, purplish, blue, &c. The oscillatoriæ are equally valuable as specimens at all times and seasons, as their fruit is not known, and the characters defining the species do not depend upon the sexual organs. The zygnemas are the bright green, evidently filamentous, slimy masses, which float on ditches, or lie in them, entangled amongst the water plants, sticks, twigs, &c. They are only of scientific value when in fruit, as it is only at such times that they can be determined. Excepting in the case of one or two very large forms, it is impossible to tell with the naked eye with certainty whether a zygnema is in fruit or not; but there are one or two practical points, the remembrance of which will very greatly enhance the probable yield of an afternoon's search. In the first place, the fruiting season is in the spring and early summer, the latter part of March, May, and June being the months when the collector will be best repaid for looking for this family. Again, when these plants are fruiting they lose their bright green color and become dingy, often yellowish and very dirty looking—just such specimens as the tyro would pass by. The fine, bright, green, handsome masses of these algæ are rarely worth carrying home. After all, however, much must be left to chance; the best way is to gather small quantities from numerous localities, keeping them separate until they can be examined.

Adhering to the various larger plants, to floating matters, twigs, stones, &c., in ditches, will often be found filamentous algæ, which make fine filmy fringes around the stems, or on the edges of the leaves; or perchance one may meet with rivulariæ or nostocs, &c., forming little green or brownish balls, or indefinite protuberances attached to small stems and leaves. These latter forms are to be looked for especially late in the season, and whenever seen should be secured.

In the latter part of summer, there is often a brownish, gelatinous scum to be seen floating on ditches. Portions of this should be preserved, as it frequently contains interesting nostocs and other plants.

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In regard to large *rivers*, the time of year in which I have been most successful in such localities is the latter summer months. *Springs* and small bodies of clear water may be searched with a hope of reward at any time of the year when they are not actually frozen up. I have found some exceedingly beautiful and rare algæ in such places as early as March, and in open seasons they may be collected even earlier than this. The desmids are most abundant in the spring, and possibly most beautiful then. They, however, rarely conjugate at that time, and the most valuable specimens are therefore to be obtained later—during the summer and autumn months; at least, so it is said; and the experience I have had with this family seems to confirm it. *Rivulets* should be watched especially in early spring, and during the summer months.

From the time when the weather first grows cool in the autumn, on until the cold weather has fairly set in, and the reign of ice and snow commences, is the period during which the algæ hunter should search carefully all wet, *dripping rocks*, for specimens. Amongst the stems of wet mosses—in dark, damp crevices, and little grottos beneath shelving rocks—is the algæ harvest to be reaped at this season. Nostocs, palmellas, conjugating desmids, sirosiphons, various unicellular algæ, then flourish in such localities. My experience has been, that late in the autumn, ravines, railroad cuttings, rocky river-banks, &c., reward time and labor better than any other localities.

The vaucherias, which grow frequently on wet ground, as well as submerged, fruit in the early spring and summer in this latitude, and are therefore to be collected at such times, since they are only worth preserving when in fruit.

In regard to algæ which grow on trees, I have found but a single species, and do not think they are at all abundant in this latitude. Farther south, if one may judge by Professor Ravenel's collections, they are the most abundant forms.

Although perhaps of but little interest to the distant collector, yet for the sake of those living nearer, I will occupy a few lines with an account of the places around Philadelphia which will best repay a search for fresh-water algæ. As is well known, below the city, there is what is known as the "Neck," a perfectly level extent of ground lying in the fork between the rapidly approaching rivers, Schuylkill and Delaware. This is traversed by numerous large ditches, and, especially just beyond the city confines, has yielded to me an abundant harvest. My favorite route is by the Fifth Street cars to their terminus, then across the country a little to the east of south until the large stone barn, known as "Girard's Barn," is reached. A large ditch lies here on each side of the road, which is to be followed until it crosses the Pennsylvania Railroad, then along this to the west, until the continuation of Tenth Street crosses it. Here the ditches cease, and the steps are to be turned homeward. From Girard's barn to the crossing just alluded to, ditches great and small lie all along and about the route, ditches which have often most abundantly rewarded my search, and enabled me to return home richly laden. The best season for collecting here is from March to July, and again in October, when some of the nostocs may be looked for.

Crossing the river Delaware to the low country below and above the city of Camden, the collector will find himself in a region similar to that just described,

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and like it cut up by numerous ditches, in which are pretty much the same forms as in the "Neck." But by taking the Camden and Atlantic cars for twenty to forty miles into New Jersey to what is known as the "Pines," he will get into a very different country; low, marshy, sandy grounds, with innumerable pools, and streams whose dark waters, amber-colored from the hemlock roots over which they pass, flow sluggishly along. I have been somewhat disappointed in my collections in such localities. Fresh-water algæ do not appear to flourish in infusion of hemlock, and consequently the streams are very bare of low vegetable life. On the other hand, in pools in the more open places, my search has been repaid by finding some very curious and interesting forms, which apparently are peculiar.

North of Philadelphia are several places, which at certain seasons will richly reward the microscopist. Along the Delaware River, there is a similar country and flora to that of the "Neck." But back from the river things are quite different. The North Pennsylvania Railroad passes near Chelten Hills, some eight miles or so from the city, through some deep rock cuttings, which are kept constantly dripping by numerous minute springs bursting from between the strata. At the proper season, these will yield an abundant harvest. Besides these, there is also a stream of water with ponds running along by the road, which should be looked into. I have seldom had more fruitful trips than some made very early in the spring to this locality; but then it was in little pools in the woods, and especially in a wooded marsh or meadow to the left of the road, some distance beyond the station, that I found the most interesting forms.

The Schuylkill River and its banks have afforded materials for many hours of pleasant work. In the river itself a few very interesting forms have been found; but it is especially along its high banks that the harvest has been gathered.

The dripping rocks and little wood pools in the City Park are well worth visiting; but the best locality is the western bank, along the Reading Railroad, above Manayunk, between it and the upper end of Flat Rock tunnel. Down near the river, at the lower end of the latter, will be found a number of beautiful, shaded rocky pools, which, in the late summer, are full of *Chaetophora* and other algæ. Along the west rocks of the river side of the bluff, through which the tunnel passes, are to be found, late in the fall, numerous algæ. It is here that the *Palmella Jessenii* grows in such abundance.

West of the city, in Delaware and Chester Counties, is a well wooded and watered, hilly country, in which, here and there, numerous fresh-water algæ may be picked up.

As to the preservation of the algæ—most of the submerged species are spoiled by drying. Studies of them should always, when practicable, be made whilst fresh. Circumstances, however, will often prevent this, and I have found that they may be preserved for a certain period, say three or four months, without very much change, in a strong solution of acetate of alumina.

An even better preservative, however, and one much more easily obtained, is carbolic acid, for I have studied desmids with great satisfaction, which had been preserved for five or six years in a watery solution of this substance. In regard to the strength of the solution I have no fixed rule. Always simply shaking up

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a few drops of the acid with the water, until the latter is very decidedly impregnated with it, as indicated by the senses of smell and taste.

Almost all species of algæ which are firm and semi-cartilaginous, or almost woody in consistency, are best preserved by simply drying them, and keeping them in the ordinary manner for small plants. The fresh-water algæ which bear this treatment well belong to the *Phycochromophyceæ*, such as the *Nostocs*, *Scytonema*, &c., the true confervas not enduring such treatment at all. When dried plants are to be studied, fragments of them should be soaked for a few minutes in warm, or for a longer time in cold water.

The only satisfactory way that algæ can be finally prepared for the cabinet is by mounting them whole or in portions, according to size, for the microscope. Of the best methods of doing this, the present is hardly the time to speak; but a word as to the way of cleaning them will not be out of place. Many of them, especially the larger filamentous ones, may be washed by holding them fast upon an ordinary microscope slide, with a bent needle or a pair of forceps, and allowing water to flow or slop over them freely, whilst they are rubbed with a stiffish camel's-hair pencil or brush. In other cases, the best plan is to put a mass of the specimens in a bottle half full of water, and shake the whole violently; drawing off the water from the plants in some way, and repeating the process with fresh additions of water, until the plants are well scoured. At first sight, this process would seem exceedingly rough, and liable to spoil the specimens, but I have never seen bad results from it, at least when practised with judgment. The water seems so to envelop and protect the little plants that they are not injured.

After all, in many instances it appears impossible to clean these algæ without utterly ruining and destroying them—the dirt often seeming to be almost an integrant portion of them; so that he who despises and rejects mounted specimens, simply because they are dirty and unsightly, will often reject that which, scientifically speaking, is most valuable and attractive.

In finally mounting these plants, the only proper way is to place them in some preservative solution within a cell on a slide. After trial of solution of acetate of alumina and various other preservative fluids, I have settled upon a very weak solution of carbolic acid, as the best possible liquid to mount these plants in. Acetate of alumina would be very satisfactory were it not for the very great tendency of the solution to deposit minute granules, and thus spoil the specimens. As every one knows, the great difficulty in preserving microscopic objects in the moist way is the perverse tendency of the cells to leak, and consequently slowly to allow entrance to the air and spoil the specimen.

As I have frequently found to my great chagrin, the fact that a slide has remained unchanged for six months, or even a year, is no guarantee that it will remain so indefinitely. It becomes, therefore, exceedingly important to find some way of putting up microscopic objects that can be relied on for their preservation. Where carbolated glycerine jelly or Canada balsam can be used, the solid coating which they form around the specimens constitutes the best known protection. Except in the case of the diatoms, however, these substances so shrivel and distort the fresh-water algæ immersed in them as to utterly ruin them. I lost so many specimens

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by the old ways of mounting, that, becoming disheartened, I gave up all idea of making a permanent cabinet, until a new cement, invented by Dr. J. G. Hunt, of this city, was brought to my notice. This is prepared as follows:—

"Take damar gum, any quantity, and dissolve it in benzole; the solution may be hastened by heat. After obtaining a solution just thick enough to drop readily from the brush, add enough of the finest dry oxide of zinc—previously triturated in a mortar with a small quantity of benzole—until the solution becomes white when thoroughly stirred. If not too much zinc has been added, the solution will drop quickly from the brush, flow readily, and dry quickly enough for convenient work. It will adhere, if worked properly, when the cell-cover is pressed down, even when glycerine is used for the preservative medium. Keep in an alcohol-lamp bottle with a tight lid, and secure the brush for applying the cement in the lid of the bottle."

Its advantages lie in the circumstance, that the glass cover can be placed upon the ring of it whilst still fresh and soft, and that in drying, it adheres to both cover and slide, so as to form a joint between them of the width of the ring of cement, and not, as with asphaltum, gold size, &c., simply at the edge and upon the outside of the cover. It is readily to be seen how much less liability to leakage must result from this. The method of mounting with it is as follows: A ring of any desired size is made, by means of an ordinary Shadbolt's turn-table, upon a slide, which is then placed to one side to dry. When required for use, the specimen, cover, &c., being all prepared and ready, the slide is again placed upon the turn-table and a new ring of cement put directly upon the old one. The specimen is immediately placed within the cell thus formed, and the requisite quantity of the carbolated water placed upon it. The cover, which must be large enough to entirely or nearly cover the cement ring, is now picked up with the forceps, the under side being moistened by the breath to prevent adhesion of air-bubbles, and placed carefully in position. It is now to be carefully and equably pressed down with some force. By this, any superfluous water is squeezed out and the cover is forced down into the cement which rises as a little ring around its edge. The pressure is best made with a stiff needle, at first on the centre and then upon the edges of the cover, which may finally be made slowly to revolve underneath the needle point. The slide may then be put aside to dry; or, better, an outside ring of the cement thrown over its edge in the usual manner. Where a deep cell is required, several coats of the cement should be placed one over the other, each being allowed to dry in turn. If time be an object, and only a shallow cell be necessary, the first ring of cement may be dispensed with, and the whole mounting of the specimen be done in a few minutes. Even with this cement and the utmost care in mounting, the cabinet should be occasionally inspected, for there will always be some slides into which air will penetrate. When such are found, efforts may be made to stop the leak by new rings of cement overlaid upon the old, but very often entire remounting of the specimen is the only satisfactory cure.

The classification which I have adopted in this memoir is that of Professor Rabenhorst. I have finally selected it, not as being absolutely natural, but as convenient, and as rarely doing much violence to the natural relations of the various species.

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Our knowledge of the life-history of the algæ must make very many advances before the true system can be developed, and abstinence from adding to the present numerous classifications is an exhibition of self-control not very common.

There are, however, certain great groups, which are already plainly foreshadowed, and which no doubt will be prominent points in the perfected classification. Amongst these are the *Conjugatœ*, or those plants in which sexual reproduction occurs by the union of two similar cells. In the present paper all the plants of this family described are together, since the diatoms are not noticed; but in Rabenhorst's work the latter plants are very widely separated from their fellows, and this seems to me the weak point of the Professor's system.

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FRESH-WATER ALGÆ OF THE UNITED STATES.

CLASS **PHYCOCHROMOPHYCEÆ.**

Plantæ uni- vel multicellulares, in aqua vigentes vel extra aquam in muco matricali nidulantes, plerumque familias per cellularum generationes successivas ortas formantes.

Cytioderma non siliceum, combustibile.

Cytioplasma phycochromate coloratum, nucleo destitutum, granulis amylaceis plerumque nullis.

Propagatio divisione vegetativa, gonidiis immobilibus vel sporis tranquillis.

Unicellular or *multicellular* plants living in water, or incased in a maternal jelly out of it, mostly in families formed from successive generations of cells.

Cytioderm not siliceous, combustible.

Cytioplasm an endochrome, brown, olivaceous, fuscous, &c., destitute of nucleus, mostly without starch granules.

Propagation by vegetative division, by immovable gonidia or tranquil spores.

The phycochroms are plants at the very bottom of the scale, distinguished by the simplicity of their structure and the color of their protoplasm, which, instead of being of the beautiful green that marks chlorophyll, is fuscous, or yellowish, bluish, brownish, or sometimes particolored, and rarely greenish, but of a shade very distinct from the chlorophyll green, more lurid, bluish or yellowish, or olivaceous in its hue. The nucleus appears to be always wanting. The cell wall is oftentimes distinct and sharply defined, but in many instances it is not so, the walls of different cells being fused together into a common jelly in which they are imbedded. In a large suborder the wall is replaced by a sheath, which in some genera surrounds cells with distinct walls, in others, cells without distinct walls, and in still others, a long cylindrical mass of endochrome, which may be looked upon as a single cell.

Many of the phycochroms are unicellular plants in the strictest sense of the word, but more often the cells are conjoined, so as to form little families, each cell of which is in a sense a distinct individual capable of separate life, yet the whole bound together into a composite individual. Rarely the phycochrom is a multicellular plant in the stricter use of the term. Increase takes place by the multiplication of cells by division, and also by the formation of enlarged thick-walled cells, to which the name of spores has been given, although it is entirely uncertain whether they are or are not the result of sexual action. There are numerous peculiar forms of cell multiplication by division occurring in these plants, the discussion of which will be found scattered through the remarks on the various families and genera.

The method of reproduction, and in fact the life history in general, of the phycochroms, is still involved in such mystery, that I am not aware that absolute sexual generation has been demonstrated in any of them. This being the case, it is not to be wondered at that many have conjectured as possible, and some have roundly asserted as true, that the phycochroms are merely stages in the life history of higher plants; that they are not species, and, consequently, that any attempt at describing such is little more than a busy idleness. In regard to some of them it has certainly been rendered very probable that they are merely fixed stages of higher plants. On the other hand, in the great bulk of the forms, no proof whatever has been given that they are such. They all certainly have fixed, definite characters, capable of being expressed and compared, so that the different forms can be defined, recognized, and distinguished. If, therefore, future discoveries should degrade them as subordinate forms, names will still be required, and definitions still be necessary to distinguish them one from the other, so long as they are common objects to the microscopist.

If *Nostoc commune*, for example, were proven to be a peculiar state or development of *Polytricum commune*, I conceive it would be still known as *Nostoc commune*. But, as previously stated, no proof whatever has as yet been furnished for the vast majority of the plants of this family, to show that they bear any such relation to higher plants; and until some such proof is forthcoming, certainly the only scientific way to act, is to treat them as distinct species.

ORDER **Cystiphoræ.**

Plantæ unicellulares. Cellulæ singulæ vel plures in familias consociatæ.

Unicellular plants. Cells single or consociated in families.

In this order the cells are oblong, cylindrical, spherical, or angular. They are sometimes single, or more commonly are united by a common jelly into families, which sometimes are surrounded by distinct coats. The mucus or jelly, in which the cells are imbedded, is mostly, but not always, colorless, and varies in firmness from semifluid to cartilaginous. The division of the cells may take place either in one, two, or three directions or planes.

FAMILY CHROOCOCCACEÆ.

Character idem ac ordine.

Characters those of the order.

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Genus CHROOCOCCUS, NÆGELI.

Cellulæ globosæ ovales vel a pressione mutua plus minus angulosæ, solitariæ vel in familias consociatæ, liberæ (a vesica matricali non involutæ); cytiodermate achromatico, homogeneo, sæpe in muco plus minus firmo confluente; cytioplasmate æruginoso vel pallide cæruleo-viridi, non rare luteolo vel aurantiaco, interdum purpurascente. Generationum successivarum divisio alternatim ad directiones tres.

Syn.—Protococcus, AG. et KTZ., &c., ex parte. *Pleurococcus*, MENGH.
Globulinæ et *Protosphæriæ*, TURPIN, ex part.

Cells globose, oval, or from mutual pressure more or less angular, solitary, or consociated in free families (not involved in a maternal vesicle); Cytioderm achromatic, homogeneous, often confluent into a more or less firm mucus; cytioplasm æruginous or pale bluish-green, not rarely yellowish or orange, sometimes purplish. Successive generations arising by alternate division in three directions.

C. refractus, WOOD.

C. cellulis in familias solidas arcte consociatis, plerumque subquadratis, sæpius triangularibus, rare angulosis; familiis sæpius lobatis; cytiodermate tenui, vix visibile, achroo; cytioplasmate subtiliter granulato, subfusco vel subluteo vel olivaceo, valde refrangente.

Diam.—Cell $\frac{1}{6000}''$—$\frac{1}{3000}''$, rare in cellulis singulis $\frac{1}{2000}''$; famil. $\frac{1}{1500}''$—$\frac{1}{170}''$.

Syn.—*C. refractus*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 122.

Hab.—In rupibus irroratis prope Philadelphia.

Cells closely associated together into solid families, mostly subquadrate, very often triangular, rarely multiangular; families often lobed; cytioderm thin, scarcely perceptible, transparent; cytioplasm finely granular, brownish, olivaceous, or yellowish, highly refractive.

Remarks.—The color of this species varies from a marked almost fuscous brown to a light yellowish-brown, the lighter tints being the most common. The cells are remarkable for their powerful refraction of the light, resembling often oil as seen under the microscope, especially if they be the least out of the focus. They are very closely joined together to form the families, many of which are composed only of four cells. Often, however, a large number of the cells are fused together into a large, irregular, more or less lobate family, and these sometimes are closely joined together into great irregular masses. I have occasionally seen large single cells with very thick coats, whose protoplasm was evidently undergoing division. Are such a sort of resting spore? The color of the protoplasm varies. Perhaps the more common hue is a sort of clay tint. Bluish-olive and a very faint yellowish-brown are not rarely seen. The species grows abundantly on the wet rocks along the Reading Railroad between Manayunk and the Flat Rock tunnel.

Fig. 5, pl. 5, represents different forms of this species; those marked *a*, magnified 750 diameters; *b*, 470 diameters; *c*, 950 diameters.

C. multicoloratus, WOOD.

C. in strato mucoso inter algas varias sparsus; cellulis singulis et sphæricis, vel 2–4 (rare 8) aut angulis aut semisphæricis aut abnormibus in familias oblongas consociatis; cytiodermate crasso, hyalino, haud lamelloso; tegumentis plerumque nullis, interdum subnullis; cytioplasmate plerumque homogeneo, interdum subtiliter granulato, vel luteo-viride vel cæruleo-viride vel luteo vel subnigro, vel brunneo, vel saturate aurantiaco, sæpe ostro tincto.

Diam.—Cell., sing. sine tegm., $\frac{7}{9000}''$ cum teg. $\frac{2}{1500}''$; cell. in famil. sing. $\frac{2}{4500}''$—$\frac{4}{4500}$.″ Fam. long. $\frac{7}{4500}''$—$\frac{10}{4500}''$; lat. $\frac{6}{4500}''$—$\frac{7}{4500}''$.

Syn.—*C. multicoloratus*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 122.

Hab.—in rupibus humidis prope Philadelphia.

C. occurring scattered in a mucous stratum with other algæ; cells spherical and single, or else angular semi-spherical or irregular and associated together in oblong families of from 2–4 (rarely 8); inner coat thick, hyaline, not lamellate; outer coat generally wanting, sometimes indistinctly present; endochrome mostly homogeneous, sometimes minutely granular, either a yellowish-green or bluish-green, or yellowish or brown, or blackish, sometimes tinged with bright lake.

The cells of this species do not appear to have any tendency to unite to form large masses or fronds. On the contrary they are generally very distinct. Their color varies very much, in a larger number of instances they were a decided yellowish-green, tinged at some point or other with a beautiful lake. When several cells are formed by division out of one cell, a similar division of the very thick surrounding gelatinous coat follows separating them finally entirely one from the other. I have seen a single cell which appears to be an encysted form of this, of which I give a drawing.

Fig. 6, pl. 5, represents different forms of this species magnified 260 diameters.

C. thermophilus, WOOD.

C. cellulis singulis aut geminis vel quadrigeminis et in familias consociatis, oblongis vel subglobosis, interdum angulosis, haud stratum mucosum formantibus; tegumento crassissimo, achroo, haud lamelloso, homogeneo; cytioplasmate viride, interdum subtiliter granulato, interdum homogeneo.

Diam.—Cellulæ singulæ sine tegumento longitudo maxima $\frac{1}{1500}$″, latitudo maxima $\frac{1}{2300}$″.

Syn.—*C. thermophilus*, WOOD, American Journal Science and Arts, 1869.

Hab.—Benton Springs, Owen Co., California (Mrs. Parz.).

Cells single, geminate, or quadrigeminate and consociated into families, oblong or subglobose, sometimes angular, not forming a mucous stratum; tegument very thick, transparent, not lamellate, homogeneous; cytioplasm green, sometimes minutely granulate, sometimes homogeneous.

Remarks.—Remarks upon this species will be found under the head of *Nostoc calidarium*, Wood.

Genus GLOEOCAPSA, KTZ.

"Cellulæ sphericæ aut singulæ aut numerosæ in familias consociatæ; singulæ tegumento vesiculiforme (cytiodermate tumido) inclusæ, post divisionem spontaneam in cellulas duas filiales factam utraque tegumento se induit, dam ambæ tegumento matricali involutæ remanent; cellularum harum filialium iterum in duas cellulas divisione continuo repetita, tegumentum ataviæ restat et sese extendens familiam totam circumvelat. Cytioderma crassum, sæpe crassissimum, cellulæ lumen crassitie æquans vel superans, achromaticum vel coloratum, plerumque lamellosum; lamellæ vel strata non raro discedentia. Cytioplasma æruginosum, cæruleo-viride, chalybeum, rufescens, luteo-fuscum, &c. Cellularum divisio directione ad tres dimensiones alternante. Cellulæ generationum ultimarum minores quam priorum sunt." (Rab.)

Syn.—*Globulina et Bichatia*, TURPIN, ex part.
Gloeocapsa, KTZ., ex part.
Microcystis, MENEGH., ex part.

Cells spherical, either single or associated in numbers into families; the single cell included a vesiculiform tegument (the tumid cytioderm); this cell then undergoing division into two daughter-

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cells, each of which has a distinct tegument, the whole being surrounded with that of the old mother-cell. This process of division is then repeated again and again, the original cell-wall remaining and surrounding the family thus formed. Cytioderm thick, often very thick, equalling, or exceeding in diameter the cavity of the cell, achromatic or colored, mostly lamellated, lamellæ or strata not rarely separating. Cytioplasm of various colors, æruginous, bluish-green, chalybeate, reddish, yellowish-fuscous, &c. Division of the cells occurring in three directions. The last generation of cells smaller than the earlier ones.

G. sparsa, WOOD.

G. in strato mucoso sociis algis variis sparsa; cellulis sphæricis, vel oblongis vel ovatis, 2–8 in familias consociatis; familiis subglobosis vel subovatis, interdum numeroso-aggregatis; tegumentis internis aureofuscis, firmis, rarissime coloris expertibus, homogeneis, vel lamellosis; tegumentis externis achromaticis, rare subachromaticis, plerumque vix visibilibus; cytioplasmate homogeneo.

Diam.—Max. cell. oblong. sine tegum. long., $\frac{1}{3000}''$; lat., $\frac{1}{7500}''$; cell. glob., sine tegum., $\frac{1}{5000}''$; cum tegum., $\frac{1}{1875}''$; fam., $\frac{1}{750}''$.

Syn.—*G. sparsa,* WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 123.

Hab.—In rupibus irroratis prope Philadelphia.

G. scattered in a mucous stratum composed of various algæ; cells spherical, or oblong, or ovate, associated together in families of from 2–8; families subglobose or subovate, sometimes aggregated together in large numbers; inner tegument yellowish-brown, firm, rarely colorless, homogeneous or lamellate; external tegument achromatic, rarely subachromatic, generally scarcely visible.

Remarks.—This species was found in a rather firm, grumous or gelatinous coating of a light brown color, growing on the rocks at Fairmount Water Works, chiefly composed of a very minute nostochaceous plant, but contained numerous other algæ. The color of the tegument is yellowish-brown, sometimes with some red in it, sometimes with something of a greenish tint. This inner colored coat is not generally more than once or twice lamellate, often it is not at all so. This species seems somewhat allied to *G. styophila*, but differs slightly in the form of the cell, and more especially in not having a distinct thallus, and in the families being small and containing but few cells.

Fig. 7, pl. 8, represents this species, magnified 750 diameters.

Genus CÆLOSPHÆRIUM, NÆGELI.

Thallus parvus, e cellulis minimis in familias periphericas consociatis vel in stratum periphericum simplex et in muco tegumentis celerrime confluentibus formato nidulantibus compositus. Cellularum divisio, initio generationum serierum, in omnem fit directionem, tum denique alternatim ad superficiei sphæricæ utramque directionem.

Thallus small, composed of very small cells consociated into peripheral families, or in a simple peripheral layer, inclosed in their quickly confluent teguments. Division of the cells at first in all directions, afterwards only in each direction on the surface of the sphere.

C. dubium, GRUN. ?

C. thallo microscopico, subgloboso vel enorme, natante, congregato; cellulis globosis aut subglobosis; cytioplasmate pallide ærugineo, subtiliter granulato.

Diam.—Cell. plerumque $\frac{1}{6000}'' = .00016''$; rare $\frac{1}{4000}'' = .00025''$; fam. $\frac{10}{12000}''$—$\frac{40}{12000}'' =$ .00083″—.0033″.

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Hab.—In aquis stagnis, prope Philadelphia.

Thallus microscopic, subglobose or irregular, floating, aggregated in great numbers; cells globose or subglobose; cytioplasm finely granulate, pale æruginous green.

Remarks.—I found this beautiful little plant forming a dense scum on a stagnant brick-pond, below the city, in the month of July. The scum was of the "color of pea-soup," and so thick was it, that I think a quart of the plants might have been readily gathered. The fronds were of various sizes, and many of them were apparently undergoing division—some of them seemed to have little fronds in their interior. They were composed of an exceedingly transparent firm jelly, in which the cells were placed, often so as to leave the central parts of the frond empty, merely forming a sort of filament-like layer around the edge. Rarely they were in such numbers as to be crowded together over the whole surface of the frond. In some of the younger fronds the cells formed a little ball within the jelly, instead of being scattered through its outer portion. I have seen some large single cells three or four times the size of the ordinary frond cell, swimming amongst the plants, of which they are apparently the reproductive gonidia. Their cell-coats are very firm and thick. The fronds themselves are often closely aggregated together into little masses, and I think it probable that there is a state of the plant, in which the jelly becomes softened and the fronds more or less fused together in protococcus-like masses. This plant appears to be the same as the European *C. dubium*, but differs from the description in the fronds not attaining to anything like the size. It is very probable, however, that this depends upon age or circumstances of growth, and that American plants may be found as large as the European.

Genus MERISMOPEDIA, MEYEN.

Cellulæ globosæ, aut oblongæ, aut ovales, tegumentis confluentibus, 4, 8, 16, 32, 64, 128 in familias tabulatas, unistratas consociatæ. Thallus planus, tenuis, plus minus quadratus, in aqua libere natans. Cellularum divisio in planitiei utramque directionem.

Cells globose, oblong, or oval, joined together by their confluent coats into tabular families of 4, 8, 16, 32, 64, 128. Thallus, a more or less quadrate plane, swimming free in the water. Division of the cells occurring in all directions in the one plane.

M. nova, WOOD.

M. thallo membranaceo, distincte limitato, cellulis numerosissimis composito; cellulis ovalibus, arcte approximatis, 16 in familias consociatis, dilute cæruleo-viridibus, interdum medio constrictis; thalli marginibus rectis, integris.

Syn.—*M. nova*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 123.

Diam.—Cell. ad. $\frac{1}{4000}''$ = 0.0025''.

Hab.—In flumine Schuylkill, prope Philadelphia.

Thallus membranaceous, distinctly limited, composed of very numerous cells; cells oval, closely approximated, consociate in families of 16, light bluish-green, sometimes constricted in the middle; margin of the thallus straight and entire.

Remarks—The only specimens I have ever seen of this species were found growing in the Schuylkill River adherent to, or entangled in, a lot of filamentous algæ.

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The frond is very sharply defined, and, under a low power, is of a uniform bluish-green tint. The cells are associated in primary families of 16, of a number of which the thallus is composed. The species appears to be most closely allied to *M. mediterranea*, Næg., from which it differs very essentially in the size of the fronds, and perhaps even more closely to *M. glauca*, the only character separating it from which is the straight margin. I have myself some doubts whether it ought not to be considered as merely a form of *M. glauca*.

Fig. 8, pl. 8, represents this species, magnified 400 diameters.

M. convoluta, Breb.

M. thallo membranaceo, oculis nudis visibili, plus minus convoluto; familiis e cellulis geminis et in subfamilias dispositis, 256 compositis, interdum familiis duabus in familia gemina conjunctis; cellulis sphæricis aut oblongis; cytioplasmate homogeneo, viridi.

Diam.—Cell. $\frac{1}{6000}''$ = 0.00017''; fam. long. $\frac{1}{180}''$ = .06''; lat. $\frac{1}{250}''$ = 0.04''.

Hab.—In aquis quietis prope Philadelphia.

Thallus membranous, visible to the naked eye, more or less folded; families composed of 256 geminate cells, arranged in subfamilies, sometimes two of these families conjoined with a composite family; cells spherical or oblong; cytioplasm homogeneous, green.

Remarks.—When my *Prodromus* was published, the only specimens of this plant which I had seen were contained in a mounted slide given me by my friend Dr. J. Gibbons Hunt, of this city. Since then I have found it growing in a very shallow, quiet, but fresh, sweet pool at Spring Mills, making a distinct green layer upon the mud many feet in extent. Of course, there were millions of specimens in this layer. The fronds are irregular in shape, often somewhat ovate, sometimes subquadrate, variously torn, and not rarely somewhat lobate. Their edges are frequently very sharply defined and rendered firm and prominent by several rows of cells being crowded closely together along them. The cells in the body of the frond are arranged in large parallelogrammatic families, composed of 256 cells. There are 16 cells on each side, the families being parallelogrammatic rather than square, owing to the oblong shape of the cells. This cell family is composed of four subfamilies, each containing 64 cells. These are again subdivisible into four more or less distinct groups of 16 cells each. The cells are, finally, generally closely geminate, each pair being very distinctly separated from its neighbors. In certain stages of growth, as immediately after a general division of the cells, two of the large cell-families spoken of are often temporarily joined together to form a huge family of 512 cells, but soon separate one from the other.

Order **Nematogeneæ.**

Plantæ multicellulares vel pseudo-multicellulares. Cellulæ filum (trichoma) formantes et plerumque vagina tubulosa homogenea vel lamellosa inclusæ. Trichomata aut simplicia aut ramificata.

Plants multicellular or pseudo-multicellular. Cells forming a filament, and generally included in a tubular lamellate or homogeneous sheath. Filaments either simple or branched.

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FAMILY OSCILLARIACEÆ.

Trichomata simplicia haud vero multicellularia, sed distincte articulata, plerumque vaginata, motionibus variis prædita.

Filaments simple, not strictly multicellular, but distinctly articulate, mostly vaginate, moving in various ways.

Genus OSCILLARIA, BOSC.

Trichomata simplicia, plerumque distincte articulata, rigida, recte vel parum curvata, rarius circinata vel spiraliter convoluta, plerumque læte colorata, motu triplici pædita, in muco matricali nidulantia vel vaginula tubulosa angustissima utroque fine aperta inclusa; articuli fronte disciformes. (R.)

Filaments simple, mostly distinctly articulate, rigid, straight, or somewhat curved, very rarely circinate or spirally convolute, capable of three motions, floating in a maternal jelly, and shut up in a fine tubular sheath, open at both ends; joints from the front disciform.

The oscillaria are very peculiar plants, which flourish almost in every situation in which fresh water is to be found. The purest springs are not always free from their presence, although they occur most abundantly in stagnant pools and ditches, where animal or vegetable matters are undergoing decay. When viewed in mass, floating upon some foul pool, few objects in the vegetable world are better calculated to excite disgust. A dark, slimy scum reeking with its putrescent surroundings, they seem to offer nothing of pleasure or interest. But, when brought home to the table of the microscopist and placed beneath his object-glass, they startle the observer by the wonders of their life-history. Living rods, writhing, twisting, bending, curling, creeping, gliding hither and thither; incessant, apparently causeless, motion, occurring too in what to most minds is the very type of fixity and passivity—a plant. No marvel, then, that they are so famous.

The structure of an oscillatoria is about as simple as it can be. An outside colorless cellulose sheath, which is probably in the uninjured filament closed at the end, although, as seen by the microscope, violence and age have often torn it open. Within is a long rod of variously colored endochrome, distinctly articulated by, at great or less intervals, breaks in the color, which appear as dark lines under a low power, but, under a higher objective, are revealed as narrow linear portions of protoplasm lighter and more transparent than the rest. Frequently at the joints there is a marked tendency to separation between the successive articles, and a very decided contraction of the endochrome on each side, so as to leave a little gutter, or dividing trench. The endochrome is sometimes homogeneous, sometimes contains numerous granules, which are, however, never amyloid in their nature.

The color of the endochrome varies very greatly in the different species. Slate color, blue, greenish, olivaceous, are among the most common hues. According to Dr. Ferdinand Cohn (Botan. Zeitung, 1867, p. 38; Sitzung, 13th Dec. 1866, der Schlesischen Gesellschaft für vaterländische Cultur), the coloring matter of the oscillatoria consists of true chlorophyll, and a substance which he calls *Phycocyan*, but which he states to be different from *Phykokyan* of Kützing, the *Phychochrom* of Nægeli, and also from *Phycocyan* of the latter authority. The chlorophyll is, of course, soluble in ether and alcohol but not in water; but the *Phycocyan* (Cohn) is insoluble in alcohol and ether, but soluble in water after the death of the oscillatoria. It is precipitated out of its solution by acids, alcohol, and metallic salts, as a blue jelly, but potash and ammonia throw it down as in a colorless, gelatinous mass. I have myself frequently noticed that oscillatoria after death will yield a bluish coloring matter to water, but thought that such coloring matter was the result of a partial decomposition, and I think that Professor Cohn has by no means established as a fact that his *Phycocyan* exists in the oscillatoria during life.

As to the method of reproduction of these plants, we are as yet almost entirely in the dark. Individuals do multiply by the breaking up of the internal endochrome into masses or sections through a separation at the joints. These little masses frequently grow immediately into new individuals. Sometimes, however, they roll themselves into a ball, but whether they then have the power of coating themselves with a protective wall and passing into a sort of resting spore or not, I cannot say.

The specific characters of the oscillatoria are derived from the color, form, mode, and place of growth, &c., of the large common mass, its thickness, consistency, the absence or presence of radii, &c. Descending to the individual filament, the characters are drawn from the size, the color, the length of the articulations, and the shape of the uninjured ends. Thus, it is to be noted, whether the latter are gradually narrowed (attenuated), or preserve their size to the very point, whether they are acutish or obtuse, rounded or truncate, whether they are straight or constantly curled. The activity and modes of motion are also to be remarked. Some species merely glide across the field of the microscope, some are constantly curling and uncurling at their ends, some bending to and fro almost like a pendulum, some are very sluggish, others very active and restless.

After all, however, it must be confessed that the specific characters are very unsatisfactory, much more so than in any other phycochroms which I have studied.

A very large number of European forms have been described, some few of which I have been able to recognize. I have also ventured to name a few forms apparently distinct, but have refrained from going farther into their specific study, because I have found it so unenticing, and my time has been so limited.

Professor Bailey, in Silliman's Journal, N. S., vol. iii., states that he has identified a few species of this family, although with great hesitation and doubt. At the time he wrote there were really no known grounds upon which specific unity could be predicated in these plants, and I therefore think that his identifications are of but little value, although holding the most profound respect for his abilities as a naturalist. The list he gives is as follows:—

O. tenuissima, Ag. Warm Springs of Washita.

O. tenuis, Ag. Providence, Rhode Island. West Point, New York. Culpepper County, Virginia.

O. decorticans, Gener. Common everywhere on pumps, &c.

O. muscorum, Ag. West Point, New York.

O. nigra, Vauch. West Point, New York.

O. corium, Ag.

O. chlorina, Kützing.

O. interdum in strato sordide viridi natante, interdum in aqua diffusa; trichomatibus rectis, vivide moventibus, vel articulatis et cum cytioplasmate granulato, vel inarticulatis et cum cytioplasmate haud granulato; cytioplasmate hyalino, interdum coloris fere expertibus, interdum dilutissime viride; apiculo haud attenuato, obtuse rotundato, recto; articulis diametro subæqualibus.

Diam.—$\frac{1}{7000}''$—$\frac{1}{9000}''$ = .00014''—.0001''.

Hab.—In stagnis prope Philadelphia.

Sometimes swimming on the water as a dirty-greenish stratum, sometimes diffused in the water; filaments straight, actively moving, either articulated and having the cytioplasm filled with blackish granules, or else neither articulate nor granulate, cytioplasm hyaline, almost colorless, or with a faint greenish tint; ends of the filaments not attenuate, straight, obtusely rounded; joints about equal to the diameter.

Remarks.—I found this species in the month of August, 1869, in one of the stagnant brick-ponds below the city. It occurred as a sort of floating scum, or else diffused through the water, which was then opaque and greenish. It resembled so a protococcus in gross appearance that I did not think of its being an oscillatoria until I placed it under the microscope. The filaments are almost colorless, and, in most instances, are very distinctly granulate and articulate. The dissepiments are in such cases clear and transparent, perfectly free from granules. This form is very close to the descriptions of the European *O. chlorina*, Ktz., but differs somewhat from descriptions, chiefly in habit of growth. The filaments, when in mass, are often seen to be curved under the restraining force of the glass cover, but when free I think always straighten themselves.

Fig. 1, pl. 1, represents a single filament, magnified 750 diameters.

O. Fröhlichii, Ktz.?

O. strato indefinito, tenue, viride; trichomatibus læte viridibus, subrectis, vivide oscillantibus, ad genicula nonnihil pellucidis et leviter contractis et rarissime granulatis; articulis diametro 2, 3, 4 plo brevioribus; cytioplasmate obscure aut distincte minutissime granulato; apiculo haud attenuato, late rotundato.

Diam.—$\frac{1}{2500}''$—$\frac{1}{1500}''$ = 0.00066''—0.0004.

Hab.—In flumine Schuylkill.

Stratum indefinite, thin, green; filaments bright green, straightish, vividly oscillating, somewhat pellucid at the joints, where they are slightly contracted and very rarely granulate; articles 2, 3, 4 times shorter than the diameter, cytioplasm obscurely or distinctly very minutely granulate; apex not attenuate, broadly rounded.

Remarks.—I found this species growing upon the bottoms of the shallows in the Schuylkill River and its larger tributaries, forming a somewhat badly defined stratum, rather, indeed, a coating on the mud than a definite stratum. The motion is exceedingly active, the filaments bending and gliding, and their apices constantly curling and extending in all directions. The apices are very blunt. The filaments are not often seen woven and twisted together into a mass composed simply of themselves, but are stuck together loosely, each filament remaining straightish, with numerous little masses of mud between them. I have not been

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able to identify the species positively, but have referred it with doubt to *O. Fröhlichii.*

Fig. 2, pl. 1, represents the end of a filament.

O. nigra, VAUCH.

> O. strato plus minus compacto, amplo, plerumque natante, atro-viride, cum radiis longis; trichomatibus plerumque flexuosis; apice obtuse rotundato; articulis diametro $\frac{3}{4}$ plo brevioribus; dissepimentis distincte granulatis; cytioplasmate pallide cæsio.

Diam.—$\frac{1}{3500}''$—$\frac{1}{4000}''$.

Hab.—In fossis stagnis prope Philadelphia.

> Stratum more or less compact, ample, broad, mostly floating, blackish-green, with long radii; filaments mostly flexuous; apices obtusely rounded; joints $\frac{3}{4}$ shorter than broad; dissepiments distinctly granulate; cytioplasm pale-grayish.

Remarks—This species is found in thick, rather loose strata, floating, especially when old, on stagnant waters, or adhering to plants, &c., or the muddy shores and bottom of ditches, foul aquaria, &c. The color of the stratum is a very dark blackish-green, with a peculiar, glossy, repulsive appearance. The single filaments are of a pale-bluish neutral tint, sometimes a little greenish, very much curved and entangled, or more rarely straightish. Their motion is active. The measurements do not quite equal those given by European authorities, but otherwise the plant agrees well with their descriptions.

Fig. 3*a*, pl. 1, represents the mass of the plant as seen with the naked eye; fig. 3*b*, shows a number of filaments slightly magnified; fig. 1*c*, a broken portion of a filament magnified 260 diameters, with the sheath projecting beyond the endochrome; fig. 1*d*, the end of a filament still more highly magnified.

O. limosa, AGARDH.

> O. trichomatibus subrigidis et subrectis, vivide oscillantibus, cæruleo-viridibus, in stratum mucosum læte saturate viride et modice longe radians et natans collectis et intertextis, distincte articulatis; articulis diametro subæqualibus, interdum duplo brevioribus (post divisionem), ad genicula distincte constrictis; dissepimentis haud granulatis; apiculo obtuso, haud attenuato, interdum recto, interdum curvato; cytioplasmate granulato.

Diam.—$\frac{1}{6000}''$

Hab.—In stagnis prope Camden, New Jersey.

> Filaments straightish and somewhat rigid, vividly oscillating, bluish-green, interwoven into a bluish-green, floating stratum, with moderately long radii, distinctly articulate; articles about equal to the diameter, or after division one-half shorter, at the joints distinctly constricted; dissepiments not granulate; apices obtuse, not attenuate, sometimes straight, sometimes curved; cytioplasm granulate.

Remarks.—I have found this species floating on foul ditches near Kaighn's Point, New Jersey, in the month of May. The color of the stratum is a very pure deep-green; the single filaments vary from a rather bright deep-green to a pale blue-green, according to the power under which they are seen. The apices are not at all attenuate. The constriction at the articles is scarcely visible with a lower power than $\frac{1}{5}$th. The stratum is rather thin, with a good deal of dirt adhering to its bottom.

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When grown in a bottle, the plant appears as a very thin stratum growing up the sides. The agreement of this plant with the descriptions of the European *O. limosa* is very close, so that I do not think it can be separated from it, although in *O. limosa* the dissepiments are said to be distinctly granular.

Fig. 4, pl. 1, represents a filament of the American plant magnified 1250 diameters. The color and form are closely counterfeited, but the characteristic separation of the endochrome into parts at the joints is decidedly exaggerated.

O. neglecta, WOOD.

O. trichomatibus modice brevibus, aut dilute purpuraceo-plumbeis aut plumbeo-cinereis, plerumque rectis, aut stratum mucosum atro-purpureum haud distincte radiante formantibus, aut in strato gelatinoso haud radiante subplumbeo dispersis et cum algis aliis intermixtis, rare oscillantibus sed lente sese moventibus; articulis diametro fere 4 plo brevioribus; dissepimentis plerumque haud granulosis, rare indistincte granulosis; apiculo obtuse rotundato, interdum breviter nonnihil attenuato.

Syn.—*O. neglecta*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 124.

Diam.—$\frac{1}{1500}''$ = .0066.

Hab.—In stagnis prope Philadelphia.

Filaments rather short, of a dilute purplish-lead color, or leaden-gray, generally straight, either forming a mucous, blackish-purple stratum without marked rays, or diffused with other algæ in a gelatinous mass, rarely oscillating but gliding; articles about four times shorter than broad; joints for the most part not granulate, rarely indistinctly granulate; ends obtusely rounded, occasionally short, somewhat attenuate.

Remarks.—I have found this plant in the shallow ditches along the track of the Norristown Railroad above Manayunk, growing in two different ways. In the one it forms a distinct, soft, gelatinous, floating stratum of a very dark purplish color, consisting of nothing but interwoven filaments, and provided with long rays. In the other, the plant is largely mixed with diatoms and other algæ into a thick, gelatinous stratum without rays, whose color is a dirty slaty tint, which, however, is not all distinctive, and often varies as the proportion of the different constituents varies. The color of the single filaments is a slaty, almost neutral tint. The cytioplasm is remarkable for the numerous very minute spots more transparent and with less color than the surrounding parts. The ends of the filaments are often abruptly obtuse, frequently however there is a very short taper. Motion does not appear to be very active, and seems especially to be gliding, rather than a bending to and fro of filaments.

Fig. 5*a*, pl. 2, is an outline drawing of a filament magnified 450 diameters; 5*b* is a portion of a filament.

O. imperator, WOOD.

O. in strato mucoso, plerumque natante, olivaceo-atro, longe radiante; trichomatibus rectis aut subrectis, tranquillis, dilute viridibus vel saturate olivaceis, haud oscillantibus, sed ambulantibus; apiculis nonnihil attenuatis, late rotundatis vel subtruncatis, curvatis; articulis diametro 5–12 plo brevioribus, ad genicula indistincte contractis; cytioplasmate homogeneo, olivaceo-viride; vaginis firmis, ad genicula distincte transverse striatis.

Syn.—*O. imperator*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 124.

Diam.—.002''.

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Hab.—In stagnis prope Philadelphia.

O. occurring in an olive-black; mucous stratum, mostly swimming and with long rays; filaments straight or straightish, light-green or deep-olive, tranquil, not oscillating, but moving with a gliding motion; ends somewhat attenuate, broadly rounded or subtruncate, curved; articles 5-12 times shorter than broad, slightly contracted at the joints; cytioplasm homogeneous, olive-green; sheaths firm, distinctly transversely grooved at the joints.

Remarks.—The strata of this species are often of great extent, and resemble more masses of spirogyra than of the ordinary oscillatoria. They are very loose in texture and are very slimy, whilst their edges are fringed by the long tranquil rays. In certain conditions of growth, the endochrome of the filaments is so dense as to render them very opaque and the articulations very obscure. The sheaths when emptied show the marks of the joints very distinctly; but, at times, when gorged with cytioplasm, scarcely can the sheath itself be seen. The color of the filament is also affected by the state of the protoplasm, so that it varies from a lightish-green with an olive tint to a very decided dark olive. This species seems to be closely allied to the European *O. princeps*, from which, however, it differs in its motion, which is always very slow and merely gliding, its color, the distance of the dissepiments, and the much longer curvature of the ends. It grows everywhere in the ditches around the city; when mature, generally floating upon the surface with an adherent under-stratum of dirt, but, in its earlier history, often adhering to the bottom.

Fig. 6*a*, pl. 1, is a drawing of the end of a filament; fig. 6*b*, represents a small fragment of a filament, showing the tendency to take a roundish or barrel shape; much of the endochrome has been squeezed out by the injury which has broken the filaments.

Genus CHTHONOBLASTUS, Ktz.

Phormidii trichomata fasciatim congesta et vagina communi mucosa apice clausa vel aperta inclusa. Tales fasciculi numerosi in stratum (quasi thallum) gelatinosum, passim amoso-divisum aggregati. Vaginæ communes achromaticæ, sæpe lamellosæ, plus minus ampliatæ, rarius indistinctæ et subnullæ, evacuatæ, plerumque valde intumescentes. Trichomata Phormidii modo oscillantia, articulata et vaginata, rigida, recta vel parum curvula, in fasciculos funiformes plus minus dense contorta, apice soluta et divaricata. Cellulas propagatorias observare mihi contigit. (R.)

Filaments fasciately placed together and included in a common mucous sheath with open or shut apex. A number of these fasciculi aggregated in a gelatinous stratum (pseudothallus), which is gelatinous, and here and there ramosely divaricate. Common sheath colorless, often lamellate, more or less enlarged, rarely indistinct and nearly wanting, when empty mostly markedly intumescent. Filaments oscillating like to those of Phormidium, articulate and vaginate, rigid, straight, or a little curved, more or less densely entangled into cord-like fasciculi, with the apex dissolved and dissevered.

Ch. repens, Ktz.

Ch. terrestris, strato plus minus expanso, saturate ærugineo-chalybeo aut olivaceo-fuscescente, mucoso-membranaceo; trichomatibus æqualibus in fasciculos filiformes, sæpe valde elongatos, e vaginæ communis apertura penicillatim exsertos congestis; articulis diametro æqualibus dissepimentis granulatis, apiculo obtuse recto. (R.)

Species, mihi ignota.

Hab.—Common on damp earth. West Point, New York; Bingham, Massachusetts; Providence, Rhode Island; Baily, Silliman's Journ., N. S., vol. iii.

Terrestrial, stratum more or less expanded, deep æruginous chalybeate, or olivaceous fuscous, mucous membranaceous; filaments equal, in filiform fasciculi, which are often much elongate and penicillately exserted from the open common sheath; joints as long as broad, the dissepiments granulate; the apex obtuse, straight.

Genus LYNGBYA, AGARDH.

Trichomata inarticulata vel breve articulata, cellulis perdurantibus instructa. Vaginæ sæpe coloratæ, crassæ, sæpe lamellonæ.

Filaments not articulate, or shortly so, furnished with heterocysts. Sheaths often colored, thick, often lamellate.

"**L. muralis,** AG.

Filaments somewhat rigid, thickish, tortuous, very long, interwoven in a bright, grass-green stratum; annuli strongly defined. *Ag.* Syst., p. 74; *Harv.* Man. Ed., p. 160; *Conf. muralis. Dillw.*, tab. 7, E. Bot. t. 1554. β. aquatica.

Hab.—Var. β. in pools of fresh water, Whalefish Island, Davis Straits. Dr. Lyall.

The specimens are mixed with turfy soil. Except in the submerged habitat, this agrees with the ordinary form. Intermixed with threads of the usual size and structure are others cohering in pairs, as in *L. copulata,* Harv., which is obviously only a state of this widely dispersed species. I have not received specimens of the ordinary *L. muralis* from America; but no doubt it is common on damp walls, &c., as in Europe generally."

I have never identified this species, and have simply copied Harvey's account of it from the Nereis Boreali Americana, pt. III. p. 104.

L. bicolor, WOOD.

L. trichomatibus simplicibus, in cæspites nigro-virides vel cæruleo-virides dense intricatis, varie curvatis, plerumque inarticulatis, interdum breviter articulatis et ad genicula contractis; cytioplasmate dilute cæruleo-viride, plerumque copiose granulato, sæpe interrupto; cellulis perdurantibus cylindricis, sæpe elongatis, saturate brunneis, sparsissimis; vaginis firmis, achrois, in trichomata matura modice crassis.

Syn.—*L. bicolor*, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, 124.

Diam.—$\frac{1}{1700}''$.

Hab.—In flumine Schuylkill prope Philadelphia.

L. with the filaments closely interwoven into a blackish or bluish-green mat; filaments variously curved, simple, mostly inarticulate, sometimes shortly articulate with the joints contracted; endochrome light bluish-green, mostly very granulate, often interrupted; heterocysts cylindrical, often elongate, deep brown, very few; sheaths firm, transparent, in old filaments moderately thick.

Remarks.—This species is abundant in the shallow water of the Schuylkill River, near Spring Mills, where it forms dark waving tufts a half inch or more in height, which are adherent either to the bottom of the stream or to some firm support, such as large growing plants, sticks fixed in the mud, &c. When examined with the microscope, these tufts are seen to be composed of innumerable, very long, motionless, greatly curved filaments. They do not seem to be attached to their support, but in the denser parts are woven into a very thick mat, which apparently adheres *en masse* to the fixed body. These filaments are very rarely articulate, but, when they are, the joints are shorter than broad. The endochrome is mostly very granulate; sometimes, however, it is much more homogeneous. The sheaths in the old filaments are rather thick, and frequently partially empty; the exterior of such sheaths has often a rough, ragged look. The larger cells are very few in number. They are elongated cylinders with concave ends. I have found this plant in the Schuylkill River, just above Fairmount dam, in a younger state, and apparently without heterocysts. The threads near their ends had their endochrome distinctly articulate, like an oscillatoria, but elsewhere the protoplasm was continuous. It often contains numerous large granules resembling minute starch grains, which however fail to exhibit the reaction with iodine.

Fig. 7, *a*, pl. 1, represents a portion of the filament slightly magnified; fig. 7, *b*, a heterocyst from the same specimen more magnified; fig. 7, *c* and *d*, are drawings from another specimen from the same locality, each magnified 800 diameters; fig. 8, pl. 1, represents the form alluded to in the text as having been found in the Schuylkill River just above the dam.

FAMILY NOSTOCHACEÆ.

Trichomata simplicia, e cellulis distinctis composita, interdum vaginata, articulata, in gelatina immersa, cellulis perdurantibus, et interdum sporis porro instructa.

Filaments simple, composed of distinct cells, sometimes vaginate, imbedded in jelly; furnished with heterocysts and sometimes with spores also.

Remarks.—The nostochaceæ are plants of simple construction, consisting of a more or less firm jelly in which are imbedded serpentine filaments, composed of numerous cells. These cells are mostly more or less globose, especially in the true nostocs, so that the filament has a moniliform aspect. They have not distinct walls, or at least any that can be distinctly seen by ordinary powers of the microscope, and are sometimes closely connected, sometimes rather widely separated. No nuclei are usually discernible; I have, however, seen in some instances central spots, which were possibly of that nature. The filaments themselves are of various length, almost always tortuous, sometimes widely separated, sometimes closely interwoven. The gelatinous portions of the fronds are of various consistence—sometimes semifluid, sometimes very firm, almost cartilaginous.

The order is divisible into two families—the *Nostocs* proper and the *Spermosireæ*. In the former, the outer portion of the frond is condensed and firm, forming a sort of outer coat or epidermis, which is sometimes quite distinct, but in other instances can scarcely be said to exist.

In the filaments of a true nostoc are placed at irregular intervals cells, which are mostly larger than the others, and have thick, distinct walls. These cells contain very little or no chlorophyllous protoplasm. They are often, but by no means always, provided with numerous exceedingly attenuated, hair-like processes, or quiescent cilia. These bodies were supposed by Kützing to have some sexual value, and received from him the name of *Spermatia*. But, as their functions are entirely unknown, the name of heterocysts, first applied by M. Allman, is preferable. They are the "connecting cells" of Thwaites. No one has as yet demon-

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strated the existence of anything indicating sexuality in the nostocs proper, or shown any body at all worthy to be looked upon as a spore.

Their ordinary method of reproduction is simply a slight modification of that of growth. If a fragment of an actively growing nostoc is placed under the microscope, the filaments of it are seen to be irregular and distorted, thicker in one place than another, the cells misshapen, and sometimes apparently lumped and fused together. The formation of new filaments is taking place in such cases by the simultaneous growth and longitudinal segmentation of the cells of the old, and this may occur through the whole or in only a portion of the length of the latter. (Pl. 2, fig. 10.)

The filament of a nostoc is, in other words, capable of a double growth or development, the result in one instance being increase in its length, in the other the production of a new form like itself. The first of these is brought about by a transverse division of the cells, so that out of each single cell two are formed, placed end to end, each daughter-cell at first only half the size of their parent, but soon attaining to its full stature. In the other case great increase in the size of the cell occurs almost consentaneously with a longitudinal or lateral segmentation, the cell dividing in the direction of its length, instead of transversely, so as to form two cells lying side by side instead of end to end. The misshapen filaments alluded to simply represent different stages of this change, which goes on until two perfect filaments lie side by side, to be finally more or less widely separated by the jelly which they secrete around themselves.

This process of growth continues until the plant has arrived at its mature size, when it ceases. During this time the inner portion of the frond has been becoming more and more liquid, and finally the outer epidermis bursts and the thoroughly softened inner portion is discharged. In this way, innumerable filaments are set free, which are endowed with a power of motion similar to, but much less active than, the gliding of the oscillatoria, by means of which they are diffused in the water. Scattered in this way, carried hither and thither by currents, each minute thread, fixing itself to some object, at last becomes the centre from which a new plant is formed in a manner similar to that already described.

In the second division of the *Nostochaceæ*, the jelly is always much less firm than in the true nostocs, and is not condensed in the outer portions. The fronds are therefore soft, almost diffluent, and entirely shapeless. The filaments themselves also differ from those of the true nostocs. There are no fixed differences in the vegetable cells or heterocysts, however, although the former are apt to become more cylindrical and the filament consequently less moniliform. It is especially in the possession of distinct reproductive sporangial cells that the differences are to be found. These are much larger than the ordinary cells, from which, in their first appearance, they are not distinguishable; but, when the frond has attained a certain age, the spore-cells begin to enlarge both in diameter and length, and finally assume a form and size apparently fixed within narrow bounds for each species, and surround themselves with distinct, often quite thick coats. It is very possible that the production of new individuals may take place by a detachment of portions of the frond and subsequent growth, as described in the *Nostocs* proper, but increase of the species does certainly occur by means of these so-called spores. The growth of the plant takes place in the same way as in the true nostocs. The filaments increase in length by transverse division and consequent multiplication of the cells, whilst new filaments are formed by the consentaneous longitudinal division of all the cells of a filament.

The spores of a *Cylindrospermum* have the power of germinating after prolonged desiccation, they having been successfully cultivated even from specimens long preserved in the herbarium. Their development has been carefully and successfully studied by M. Thuret. According to this authority the first change consists in an elongation of the spore, which ruptures the wall of the sporangium, pushing a portion of it before it. Directly after this the spore undergoes division, so that out of it is formed a little torulose filament, composed of four or five cells. Growth takes place at both ends, but more rapidly at the free one. The new cells formed are smaller than those which arise directly from the spore, but, finally, all the articles assimilate. The wall of the sporangium remains attached for a long time to the end of the filament forming a little cap to it. The heterocysts, according to Thuret, at first are indistinguishable from the ordinary cells, but after awhile the granules in them begin to disappear, the color to pale, the outer wall to become apparent and grow thicker, until at last a perfect "connecting cell" is educed. I have, myself, carefully watched the early development of the spores of a cylindrospermum, and can confirm, in all essential particulars, the description Thuret has given of the process. Fig. 10, pl. 2, represents a partially formed filament, to which the empty sporangium is still attached.

As no sexual reproduction has as yet been shown to exist among the *Nostochaceæ*, it is very evident that their whole life-history is not comprised within the changes which have been detailed. It has long been known that the gonidia of many lichens have the power of independent existence, *i. e.* that when they are discharged from their thallus they can continue to live and multiply, if circumstances favor them, without giving origin to a new thallus. This, and the great similarity of structure between the nostocs and the lichen genus *Collema*, has suggested a possibly close relation between the two. The first observer, I believe, who asserted that they were different stages of the same plant was Dr. Hermann Itzigsohn.

His observations are, however, rendered of so little value by his own statements that it is not necessary to review them here. Thus, he says, that after seven years' observation he had yet to see a true one called algæ, that the *Desmidiæ* are, at least, two-celled, &c. &c. The most weighty observations upon this subject are those of Professor Julius Sachs and of J. Baranetzky—the former published in the *Botanische Zeitung* for 1855, the other in the Bulletin of the St. Petersburg Academy for 1867.

Professor Sachs states that he watched a whole bed of *Nostoc commune* developing into *Collema bulbosum.* He says that the peculiar Collemoid threads first appeared as little lateral offshoots or prolongations from the cells of the nostoc filament, and rapidly developed into well-formed collemoid filaments. Every possible stage from the typical nostoc to the typical collema was seen repeatedly.

The development of the distinguishing threads of the collema out of the ordinary nostoc-cell has never been confirmed by any other observer; but it seems to me that it must be at least provisionally accepted, although De Bary expresses some doubt of it. (*Morphol. und Physiol. der Pilze*, *Flechten*, &c., p. 290.)

The researches of M. Baranetzky were directed to the developing of a nostoc out of a collema. Hicks and other observers had previously stated that they had seen this, but none of them had given sufficient details as to the method of their observations, to be fully convincing.

M. Baranetzky placed sections of actively growing fronds of *Collema pulposum*, Ach, upon smooth, damp earth, using all proper precautions to prevent external influence. After some days the sections became less transparent and intensely green from the crowding of the gonidia, which were now arranged in curved rows closely rolled together into balls. Upon the upper surface of the section appeared little gelatinous balls or warts, which contained gonidia in rows, and gradually developed typical nostoc forms, whilst on the edges of the sections appeared little colorless wart-like masses of jelly, in which, after some time, appeared gonidia, some of which developed into the typical nostoc form, others into true collemoid plants.

Mr. Baranetzky further states that he watched the body of the section gradually change by the continual growth and increase of the rows of gonidia, before alluded to, and by the disappearance of the collemoid threads, until at last the whole mass of the tissue of the lichen had been converted into a true nostoc, which was finally identified as *Nostoc vesicarium*, D. C.[1]

I have no observations of my own to offer upon this subject; but think enough has been done to show not only that the nostocs proper have very close relations with the collemoid lichens, but that they are probably a peculiar phase in their life-history. This being the case, it may seem a perfectly superfluous work to indicate species amongst the nostocs. To any one who has given much study to the fresh-water algæ, the reply to this will immediately suggest itself; namely, that in the present state of the science it seems impossible to avoid it; they are so commonly thrust at one by collectors, amateurs, &c., are so distinct, are so often the subject of tongue and pen, that they must have a name. The idea that attaches to the term species is at present not a very definite one; that there are, however, amongst the nostocs fixed forms, which do not change into one another, and can readily be distinguished, I have no doubt. Such forms are herein described. If they be only life stages of lichens, I have no doubt that it will finally be found that each so-called species of nostoc has its own peculiar so-called species of lichen, from which it alone springs, and into which alone it can develop. It seems to me, then, that as yet no cause for abandoning the specific names of the

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[1] In order to aid any one desirous of going over this subject more thoroughly, a list of papers is appended:—

Ventenab und Cassini Opuscula Phytolog., 1817, vol. ii. p. 361.
Dr. Hermann Itzigsohn. Botanische Zeitung, 1854, p. 521.
Prof. Julius Sachs. Botanische Zeitung, 1855, p. 1.
Bayrhoffer. Botanische Zeitung, 1857.
Hicks. Journal of Microscopical Science, 1861, p. 90.
Baranetzky. Bulletin de la Société des Sciences Nat., St. Petersburg, vol. xii. p. 418.

nostocs has been shown, but only reason to study also their relations with the various collema.

In regard to the *Spermosireæ*, there is as yet no direct proof whatever connecting them with lichens. It is very possible that they are not so closely related to the true nostocs as is generally believed, so that the probabilities of their being lichens are at present so remote, that for the systematist to refuse to take note of their distinct forms, seems to me most unwarrantable.

SUBFAMILY NOSTOCEÆ.

Thallus peridermate plus minus distincto instructus, sporis destitutus.

Thallus provided with a more or less distinct integument, and destitute of spores.

Genus NOSTOC, VAUCHER, (1803.)

Thallus gelatinosus, varie coloratus, aut globosus vel subglobosus aut foliaceo-membranaceus et irregulariter expansus, sæpe bullatus. Trichomata plus minus moniliformia. Cellulæ perdurantes exacte sphæricæ vel rare oblongæ.

Thallus gelatinous, variously colored, either globose or subglobose, or foliaceously membranous and indefinitely expanded, often a bulla. Filaments more or less moniliforme. Heterocysts exactly spherical or rarely oblong.

a. Thallus globosus vel subglobosus, vel disciformis.

Thallus globose, subglobose or discoid.

N. Austinii, WOOD, (sp. nov.)

N. subglobosum, parvum, plerumque magnitudine ovorum piscium, rare ad 2″, fuscescente, vel nigrescente, interdum durum interdum submolle, superficie sæpe corrugata; trichomatibus varie curvatis, dense intricatis vel distantibus et laxissime intricatis, viridibus, fuscescentibus, subplumbeis vel luteo-brunneis, in thallis minoribus sæpe distincte vaginatis, in thallis majoribus haud vel indistincte vaginatis; articulis maturis globosis, sæpe didymis, crasse granulatis; cellulis perdurantibus articulorum diametro æqualibus vel paulo majoribus, globosis, interjectis vel terminalibus, plerumque sparsis.

Diam.—Cell. Veg., $\frac{2}{7500}''$—$\frac{4}{12000}''$ = .0026″—00033″; cell. perdurant, .00033″.

Hab.—in rupibus irroratis, New Jersey. (Austin.)

Subglobose, small, mostly the size of fish-eggs, but reaching the diameter of nearly two lines, fuscous or blackish, sometimes very hard, sometimes much softer; surface often corrugated; filaments variously curved, densely intricate or distantly and loosely interwoven, greenish, fuscous, subplumbeis or yellowish-brown, in the smaller fronds often distinctly vaginate, in the larger indistinctly or not all vaginate; mature joints globose, often didymous, coarsely granulate; heterocysts equal to the diameter of the other joints or a little larger, globose interspersed or terminal.

Remarks.—The fronds of this distinct species vary greatly in appearance; the larger of them are often almost colorless, and, when viewed with the microscope, are seen to be composed of a transparent colorless jelly, with remarkably large filaments scattered through it. These filaments are generally without sheaths, though occasionally a sheath can be faintly traced. The smaller fronds are much firmer than the larger and are more decidedly colored. Some of them are entirely opaque, looking simply black when viewed by transmitted light under the microscope. In these the filaments are densely crowded together, often misshapen and provided with distinct broad brownish sheaths: every gradation exists between these forms and the first described fronds. The heterocysts are quite uniform in size, agreeing in diameter with the largest vegetative cells, they are always single. This species is most nearly allied to *N. ichthyoon*, Rabenh.; from which it is separated by the differences in the sheaths, the greater size of the filaments, and the single heterocysts. It gives me great pleasure to dedicate the species to Mr. Austin, by whom it was collected near Gloucester, New Jersey, growing amidst mosses on rocks.

N. pruniforme, (Roth,) Agh.

N. magnum, gregarium, noncohærens, globosum, magnitudine pisi, pruni majoris et ultra, olivaceum vel saturate ærugineum, ætate provecta fusco-nigrescens, haud raro cavum, lævissimum, intus aquosum, peridermate coriaceo subachroo; trichomatibus subæqualibus, hic illic tumidis, laxe intricatis; articulis globosis, plerumque compressis, sæpe didymis, arcte connexis; cellulis perdurantibus articulis duplo majoribus, plerumque terminalibus, rarius interjectis. R. Species mihi ignota.

Diam.—Artic. 0.00024″—0.0003″; cell. perdur. 0.0003—0.00045″. (R.)

Syn.—*N. pruniforme*, (Roth,) Ag. Rabenhorst, Flora Europ. Algarum, Sect. II. p. 168.

Hab.—Maine. Leidy.

Large, gregarious, not cohering, globose, varying from the size of a pea to a large plum, or even beyond this, olivaceous or deep ærugineous, in old age blackish fuscous, often hollow, very smooth, within watery, periderm coriaceous, somewhat transparent; filaments subequal, here and there swollen, laxly intricate; articles globose, mostly compressed, often twofold, closely connected; heterocysts twice the size of the vegetative cells, mostly terminal, rarely interspersed.

Remarks.—I have never found this species; but some years since some specimens, sent to the Academy of Natural Sciences of Philadelphia from Maine, were identified by Professor Joseph Leidy as belonging to it.

N. verrucosum, (Linn.) Vauch.

N. magnum, gregarium, bipollicare et ultra, subglobosum, sæpe lobatum, verruculosum, irroratum, initio solidum, postremo cavum, vesiciforme, saturate brunneo-viride; peridermate membranaceo-coriaceo, olivaceo-fuscescente; trichomatibus varie curvatis, centralibus parcioribus et laxissime intricatis, periphericis densius intricatis; articulis oblongis, rare globosis, arcte connexis, crasse granulatis; cellulis perdurantibus interstitialibus vel terminalibus, sphæricis, articulorum diametro duplo majoribus.

Diam.—Cell. vegetativ. .000166″; cell. perdurant. .000233″.

Syn.—*N. verrucosum*, (Linn.) Vauch. Rabenhorst, Flora Europ. Algarum, Sect. II. p. 176.

Hab.—In fonte, Centre County, Pennsylvania.

Large, subglobose, often lobed, warty; gregarious, two inches in diameter, growing under water, fixed, in the beginning solid, afterwards hollow, bladder-shaped; periderm membranaceous, coriaceous, olivaceous-fuscous; filaments variously curved, centrally fewer, and laxly intricate, towards the outside much more close; articles oblong, rarely globose, closely connected, coarsely granulate; heterocysts interstitial or terminal, spherical, twice the size of the other joints.

Remarks.—In the summer of 1869, I found a nostoc growing in great abundance in a very cold, large, limestone spring in Centre County, Pennsylvania, which I

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have referred to *N. verrucosum* with some little hesitation. Some of the fronds were smoothish, others very decidedly warty. My specimens are old plants, which have become hollow by the discharge of their internal contents. It is possibly on this account I have not been able to verify the minute description given by Professor Rabenhorst. As the latter may not be accessible to some of those who consult these pages, I append the latter part of it, which differs from that given by myself from the American plants.

"Trichomatibus flexuoso-curvatis, quasi triplici ordine; centralibus parcioribus, laxissime implicatis, apices versus plus minus attenuatis, articulis oblongis, subdistantibus, periphericis densius sæpe densissime intricatis, basi haud raro cellulis biseriatis, articulis globosis, arcte connexis, extremis (nonnisi in thallo vetusto occurrunt) subflagelliformibus, articulis oblongis, cylindraceis sphæricisque simul immixtis, distantibus; cellulis perdurantibus sphæricis interjectis terminalibusque, nonnunquam pluribus simul seriatis articulorum diametro duplo triplove majoribus."

According to Professor Harvey (Nereis Bor. Amer., part iii. p. 114), this species has been collected by Dr. Lyall in pools of fresh water, Isle of Disco, and at Beechey Island, Arctic Regions; also by Mr. Fendler at Sante Fé, New Mexico.

N. alpinum, KTZ.

N. rupestre, immersum; thallo suborbiculare, erecto, membranaceo, ad $\frac{1}{4}=\frac{3}{4}$ unciam lato, ad lineas duas vel tres crasso, tenaci, saturate olivaceo-fusco, lævi, sæpe rugoso-plicato, cum margine integro et plerumque incrassato; trichomatibus varie curvatis, laxe vel nonnihil dense implicatis; articulis fuscis vel dilute ærugineis plerumque globosis, sæpe subtiliter granulatis, arcte connexis; cellulis perdurantibus sphæricis plerumque articulorum diametro paulo majoribus, interdum subæqualibus, interjectis vel terminalibus.

Diam.—Artic. vegetativ. .00016″—00023″; cell. perd. .00026.

Syn.—"*N. alpinum,* KTZ. Phycol. General., p. 206, No. 10." RABENHORST, Flora Europ. Algarum, vol. ii. p. 174.
"*N. Sutherlandi,* DICKIE." HARVEY, Nereis Boreali Americana, part iii. p. 114.
"*N. cristatum,* BAILEY." HARVEY, Nereis Boreali Americana, part iii., 1857, p. 114.

Growing attached by its margin to the rocks in running water; thallus suborbicular, erect, membranaceous $\frac{1}{4}$—$\frac{3}{4}$ an inch high and 1—3 lines thick, very tenacious, deep olive-green, smooth, often rugosely plicate especially at the base, with the margin entire, rounded, and mostly thickened; filaments variously curved, laxly or somewhat densely interwoven; articles fuscous or greenish, mostly globose, often finely granulate, closely connected; heterocysts spherical, generally a little larger than the ordinary cells, sometimes about equal to them, interspersed and terminal.

Remarks.—This interesting little plant was found in the mountain rivulets near West Point, New York, by the late Prof. Bailey, and received from him the specific name *cristatum*, first published in Harvey's work on the North American Algæ. I have myself seen it growing in very great abundance in rapid mountain streams in the central portions of this State. It is doubtless, therefore, an inhabitant of the whole Alleghany range. In the low country, east or west of these mountains and their outlying hills, I do not know of its having been found. I have very recently received specimens of a nostoc from Sereno Watson, Esq., undoubtedly belonging to this species, which were collected by himself, in cold streams in the Clover Mountains, Nevada, at an altitude of 11,000 feet. Under the name of *M.*

Sutherlandii a nostoc has been described by Mr. Dickie, which was collected in the neighborhood of Baffin's Bay, and must be referred to this species, although the description given of it is very imperfect. Again, *N. alpinum*, KTZ., appears to be in all respects similar to the North American forms. So that this cosmopolitan little plant seems only to ask for a cold shelter, and it flourishes. The Alps, the Alleghanies, the Rocky Mountains, and the cold North are its homes. To those who believe in a single centre for a species, the suggestion that it has spread across the globe, through the arctic regions, and followed our mountain chains southward, will of course present itself.

As I have seen it, the plant is very abundant where it grows, five, six, twelve, or more of the little fronds adhering to a single pebble. The frond is generally longer than broad, the margin sometimes sinuous but never, as I have seen it, lobate or incised. It appears finally to burst and discharge its inner portion, whilst the outer cortical portion, now a little vesicle containing a globule of air, is set free and floats down the stream.

N. depressum, WOOD, (sp. nov.)

N. enormiter suborbiculare, minutum, gregarium et interdum aggregatum muscos immersos adhærens, mangitudine seminis sinapeos vel parvius, durum, elasticum, subnigris; peridermate firme, achroo; trichomatibus plerumque laxe intricatis, haud vaginatis; articulis globosis, plerumque modice arcte connexis, rare distantibus; cellulis perdurantibus globosis, ceteris paulo majoribus.

Diam.—Artic. veget. max: .0002″; cell. perdurant. max .00029.

Hab.—In rivulis, New Jersey (Prof. Austin).

Irregularly suborbicular, gregarious and sometimes aggregated, elastic, blackish, about the size of a mustard-seed, or smaller, adhering to immersed mosses; periderm firm, translucent; filaments not vaginate, mostly loosely interwoven; joints globose, generally rather closely connected, rarely distant; heterocysts rather larger than the other.

Remarks.—This plant was found by Prof. Austin attached to a brook-moss (*Dichelyma*), growing in a rapid rivulet in Northern New Jersey.

The minute fronds sometimes are so thin and spread out as to be almost foliaceous. The species I take to be most nearly allied to *N. lichenoides* of Europe, from which it is, however, apparently distinct. In the American plant the filaments and heterocysts are a little larger, and the frequent elliptical cells of the European plant are wanting. The frond also apparently does not grow so large as the European, and is further distinguished by its flat, discoid form. In many of the specimens examined the filaments are very thick, irregular, and contorted, the cells being fused together. In other instances, a filament will be plainly double, and every grade between these conditions is present. This is plainly owing to a process of growth, to the cells enlarging and dividing laterally so as to form new filaments.

N. sphæricum, (POIRET,) VAUCH.

N. globosum, interdum oblongum vel ovale, gregarium, sæpius aggregatum, raro tamen confluente, durum, elasticum (in ætate provecta intus molle et subaquosum?), olivaceum, magnitudine seminis sinapeos, ad cerasi parvi; peridermate firmo, pellucido; trichomatibus intricatis, luteolis, aut prasinatis aut dilute cæruleis; articulis plerumque subquadratis, interdum transverse

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subovalibus, arcte connexis; cytioplasmate granulato; cellulis perdurantibus interjectis terminalibusque, sphæricis.

Diam.—Artic. diam. long. $\frac{1}{8000}''$ = .000125"; transv. $\frac{1}{6000}''$ = .00017"; cell. perdurant. $\frac{1}{3400}''$ = .00029".

Syn.—*N. sphæricum* (POIRET.) VAUCH. RABENHORST, Flora Europ. Algarum, Sect. II. p. 167.

Hab.—In fontibus, prope Philadelphia.

Globose, sometimes oblong or oval, gregarious, but rarely confluent, hard, elastic (in advanced age within soft and watery?), olivaceous, varying from the size of a mustard-seed to that of a small cherry; periderm firm, pellucid; filaments intricate, yellowish, greenish or bluish; articles mostly subquadrate, sometimes transversely suboval, closely connected; cytioplasm granular; heterocysts interspersed or terminal, spherical.

Remarks.—The specimens from which the above diagnosis was prepared were found at Spring Mills, adhering to mosses and twigs in the water. The fronds were remarkable for their firmness and elasticity. The color was a dull, rather greenish, olive; that of the filaments varied from a decided greenish to a marked yellowish, or sometimes an almost silvery bluish tint. The heterocysts were rather few in number, and were either terminal or interstitial, sometimes they were without, sometimes with evident endochrome. The length of the general articulations varied a good deal, it was, however, mostly less than their breadth, which seems quite constant. When kept in water in the house, this species softens, and the periderm as it were peels off, allowing the interior to disperse itself as it gradually becomes more and more diffluent. Most of the fronds afforded ample evidence of their method of growth by the presence of filaments in every stage of division.

Fig. 10, pl. 2, represents filaments of this species.

N. cæruleum, LYNGB.

N. minimum, sæpe microscopicum, enormiter globosum vel subglobosum, affixum, gregarium, sejunctum vel aggregatum; trichomatibus valde inæqualibus; articulis elongato-cylindraceis, vel acute ellipticis, vel perfecte ellipticis, vel globosis, vel subglobosis, vel subquadrangulis, sejunctis et nonnihil distantibus vel arcte connectis aut confluentibus; cellulis perdurantibus globosis, passim interjectis terminalibusque, ceteris duplo vel subduplo majoribus.

Diam.—Cell. perdurant, .000303; cell. vegetat. plerumque .00012—000166"; rarius .0001—.00021.

Syn.—*N. cæruleum,* LYNGB. RABENHORST, Flora Europ. Algarum, Sect. II. p. 167.

Hab.—Inter muscos, New Jersey (Prof. Austin).

Very small, often microscopic, irregularly globose or subglobose, affixed, gregarious, separate or aggregated; filaments very unequal; articles elongate-cylindrical, or acutely elliptical or perfectly so, or subglobose, or globose, or subquadrangular, separate and somewhat distant or closely connected or confluent; heterocysts globose, interspersed or terminal, double or about double the size of the other cells.

Remarks.—I am indebted to Mr. Austin for specimens of this species collected by him in Northern New Jersey. The fronds grow attached to moss and are very minute, the largest I have seen being not more than half a line in diameter. The filaments are remarkable for their inequality, which is often very perceptible in different parts of the same filament. I have referred my specimens to *N. cæruleum*—

the only differences between them and the European plant are that they are not so large, and do not agree in color, many of them being browner; but these are certainly insufficient grounds for separating them. Prof. Rabenhorst speaks of observing the contents of heterocysts dividing up so as to form a little colony of cells, which finally break through the maternal wall. I have only studied mounted specimens, but have seen very clearly heterocysts in which this process was taking place.

N. punctatum, Wood, (sp. nov.)

N. terrestre; thallo expanso orbiculare vel nonnihil irregulare, tenuissimo, ærugineo, parvo membranaceo, pellucidulo; trichomatibus laxe intricatis, varie curvatis, articulis globosis vel sæpius ellipticis, plerumque medio pellucidulis, laxe connexis; cellulis perdurantibus terminalibus vel interjectis.

Diam.—Cell. vegetat. $\frac{2}{12000}$″ = .000166; cell. perdur. $\frac{4}{12000}$″ = .00033.

Hab.—In terrestre, New Jersey, (Prof. Austin.)

Terrestrial; thallus expanded, irregular or orbicular, very thin, æruginous, small, membranous, pellucid; filaments loosely interwoven, variously curved, joints globose or often elliptical, mostly pellucid in the centre, loosely connected; heterocysts terminal or interspersed.

Remarks.—Mr. Austin has kindly sent me the only specimens I have seen of this species; they are labelled "Damp Ground, Sept." The fronds, which are often aggregated, are very small and exceedingly thin, especially in their central portions, where they are quite translucent; in form they are often circular, sometimes quadrangular, sometimes quite irregular. As to size, most of them are not more than two lines in diameter, some three, or possibly five lines. The margins are often reflexed and thickened, especially in the smallest fronds. Two kinds of filaments are visible; 1st, those which I take to be in a perfected quiescent state; 2d, those which are in active growth. The former are composed of globose, or more commonly elliptical joints, which are remarkable for the possession of a central translucent, almost colorless spot, the endochrome apparently being arranged in a ring around the outer part of the cell. This is, however, occasionally wanting. The filaments, which are in active growth, are very irregular in form, often much broader than the others; their cells very irregular and sometimes fused together into one mass. The measurements given in the diagnosis were taken from the filaments of the first kind.

b. Thallus indefinite expansus.

Thallus indefinitely expanded.

N. Cesatii, Bals.

N. terrestre; thallo longe lateque expanso, gelatinoso-membranaceo, viridi-flavescente; trichomatibus flexuoso-curvatis, sublaxe implicatis, pallide ærugineis; articulis sphæricis, laxe vel arctius connexis; cellulis perdurantibus sphæricis, et interjectis et terminalibus.

Diam.—Artic. .00016—.0002; cell. perdur.—.00033″.

Syn.—*N. Cesatii*, Bals. Rabenhorst, Flora Europ. Sect. II. p. 175.

Hab.—In terrestre, Kansas (Prof. Parry); Texas (Prof. Ravenel).

Terrestrial; thallus broadly and indefinitely expanded, gelatinous-membranaceous, yellowish-green; filaments flexuously curved, rather laxly implicate, pale-greenish; articles spherical, laxly or more closely connected; heterocysts spherical, both interstitial and terminalibus.

Remarks.—This plant was sent to me by Dr. C. C. Parry, from whose letter the following is extracted: "I send enclosed specimens of a singular land *Alga?* which I met with in this vicinity; lightly attached to bare patches of soil interspersed with buffalo grass. In the adjoining bluffs are cretaceous shales full of seams and layers of selenite, from the decomposition of which the bottom soil becomes strongly impregnated with various saline matters. The present season has been characterized by unusual quantities of rain, causing extensive floods over what is usually a dry, arid district."

The agreement between the mature forms is essentially perfect. There can be scarcely any doubt as to the identification, although I have not seen the American plant in its young state. The fronds appeared to be 1—2 lines in thickness, with its surface smooth, or sometimes with close subparallel ridges or wrinkles.

According to Rabenhorst, the young European *N. cesatii* is in the beginning globose, and pale golden-yellow; soon, however, bursting and spreading out into an indefinitely expanded thallus.

Among the algæ collected by Prof. Ravenel in Texas is a *Nostoc*, labelled "On Mud Flats, Cedar Bayou, Harris Co.," which comes so close to *N. cesatii*, that I think it must be referred to it. It differs only in being more olivaceous, somewhat firmer and in the size of the heterocysts—the largest of the latter which I have examined, attaining the size only of .00027″. The largest vegetative cells are .00017 in diameter.

N. calcicola? Ag.

N. thallo irregulariter expanso, enormiter sublobato, tenue, membranaceo, cartilagineo, elastico, pellucido, aut laete viride, vel brunneo, vel dilute viride, irregulariter undulato plicato vel bullatoo; peridermate plerumque subnullo; trichomatibus cum filis leptothrichoideis ramosis intermixtis, flexuosis, plerumque distantibus, rarissime e cellulis biseriatis compositis; cellulis subglobosis, oblongis, ovalibus, cum ceteris ellipticis intermixtis, plerumque laxe connexis; cellulis perdurantibus spnæricis, interjectis et terminalibus.

Diam.—Art. $\frac{1}{7000}$″—$\frac{1}{10000}$″ = .0001″4—.0001″; cell. perdur. $\frac{3}{10000}$″—$\frac{1}{5000}$″ = .0003″—.0002″.

Syn.—*N. calcicola*, Ag. Rabenhorst, Flora Europ., Sect. II. p. 174.

Hab.—In rupibus, Georgia. (Prof. Ravenel.)

Thallus irregularly expanded, membranaceous, thin, cartilaginous, elastic, pellucid, bright green, pale green or brownish, thin, irregularly undulately plicate or bullate; periderm mostly scarcely distinguishable; filaments intermixed with branched leptothrix filaments, flexuous, mostly distant, very rarely composed of biseriate cells; cells subglobose, oblong, oval, intermingled with elliptical ones, mostly loosely connected; heterocysts spherical, interspersed or terminal.

Remarks.—This species is one of those sent me by Dr. Billings. It was collected near Catoosa Springs, Georgia, by Prof. H. W. Ravenel. In the dried state it is of a dirty olive-green, and very much wrinkled and irregular on its surface. The largest specimens are about an inch long. There is no very distinct periderm, although in some places the filaments are placed more closely together on the outer portions of the frond. This plant seems to agree with the descriptions of the

European *S. calcicola*, from which it differs somewhat, however, in having its heterocysts both terminal and among the cells, and also somewhat in their size.

N. calidarium, Wood.

N. thallo maximo, indefinite expanso, aut membranaceo-coriaceo vel membranaceo-gelatinoso vel membranaceo, aut læte virdi vel sordide olivaceo-viridi vel olivaceo-brunneo, irregulariter profunde laciniato-sinuato, ultimo eleganter laciniato; trichomatibus inæqualibus, interdum flexuoso-curvatis, plerumque subrectis et arcte conjunctis, in formis duabus occurrentibus: forma altera parva, viridi, articulis cylindricis, cum cellulis perdurantibus hic illic interjectis, vaginis interdum obsoletis, sæpius diffluentibus; forma altera maxima, articulis globosis vel oblongis, aurantiaco-brunnea, cellulis perdurantibus ab articulis ceteris haud diversis.

Diam.—Formæ primæ articuli maximi $\frac{1}{10000}$ unc.; cellulæ perdurantis $\frac{1}{6000}$ unc. Formæ secundæ articuli long. $\frac{1}{2000}$ to $\frac{1}{3000}$ unc., lat. $\frac{1}{5000}$ to $\frac{1}{6500}$, articuli globosi $\frac{1}{3500}$ to $\frac{1}{4000}$ unc.

Syn.—*N. calidarium*, Wood, American Journal of Science and Arts, 1869.

Hab.—"Benton Springs, Owen's Valley, California" (Mrs. Partz).

Thallus very large, indefinitely expanded, either membrano-coriaceous or membrano-gelatinous or membranaceous, either bright green or dirty olive-green or olive-brown, irregularly profoundly laciniately sinuate, finally elegantly laciniate; filaments unequal, sometimes flexuously curved, but mostly straightish and closely conjoined, occurring in two forms; the one small, green, with cylindrical joints, the heterocysts scattered here and there, the sheaths sometimes absent, often diffluent; the other form very large, with globose or oblong articles, orange-brown, the heterocysts not different from the other cells.

Remarks.—Numerous specimens of this species were received from Mrs. Partz, who collected them in Benton's Spring, a thermal water situated in the extreme northern point of Owen's Valley, California, sixty miles southwest from the town of Aurora. The following extract from a letter of Mrs. Partz describes the place and mode of their growth more minutely.

"I send you a few samples of the singular vegetation developed in the hot springs of our valley. These springs rise from the earth in an area of about eighty square feet, which forms a basin or pond that pours its hot waters into a narrow creek. In the basin are produced the first forms, partly at a temperature of 124°—135° Fahr. Gradually in the creek and to a distance of 100 yards from the springs are developed, at a temperature of 110°—120° Fahr., the Algæ, some growing to a length of over two feet, and looking like bunches of waving hair of the most beautiful green. Below 100° Fahr., these plants cease to grow, and give way to a slimy fungus growth, though likewise of a beautiful green, which, finally, as the temperature of the water decreases, also disappears. They are very difficult to preserve, being of so soft and pulpy a nature as not to bear the least handling, and must be carried in their native hot water to the house, very few at a time, and floated upon paper. After being taken from the water and allowed to cool they become a black pulpy mass. But more strange than the vegetable are the animal organizations, whose germs, probably through modifications of successive generations, have finally become indigenous to these strange precincts. Mr. Partz and myself saw in the clear water of the basin a very sprightly spider-like creature running nimbly over the ground, where the water was 124° Fahr., and on another occasion dipped out two tiny red worms."

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In regard to the temperatures given, and the observation as to the presence of animal life in the thermal waters, Mr. William Gabb, of the State Geological Survey, states that he has visited the locality, knows Mrs. Partz very well, and that whatever she says may be relied on as accurate.

The color of the dried specimen varies from a very elegant bluish-green to dirty-greenish and fuscous-brown. After somewhat prolonged soaking in hot water, the specimens regained apparently their original form and dimensions, and were found to be in very good condition for microscopical study.

The plant in its earliest stages appears to consist simply of cylindrical filaments, which are so small that they are resolved with some difficulty into the component cells by a first-class one-fifth objective. Fronds composed entirely of filaments of this description were received. Some of these were marked as "first forms," and as having grown in water at a temperature of 160° Fahr. Probably these were collected immediately over the spot where the heated water bubbled up. At this temperature, if the collection made is to be relied on as the means of judging, the plant does not perfect itself. To the naked eye these "first forms" were simply membranous expansions, of a vivid green color and indefinite size and shape, scarcely as thick as writing-paper, with their edges very deeply cut and running out into a long, waving, hair-like fringe. Other specimens, which grew at a much lower temperature, exactly simulated those just described, both in general appearance and microscopical characters. These, I believe, were the immature plant.

The matured fronds, as obtained by the method of soaking above described, were "gelatinous membranous," of a dirty-greenish or fuscous-brown at their bases, and bright green at their marginal portions, where they were deeply incised and finally split up into innumerable hair-like processes. Proximally they were one, or even two, lines in thickness, distally they were scarcely as thick as tissue paper. Their bases were especially gelatinous, sometimes somewhat translucent, and under the microscope were found to have in them only a few distant filaments.

Two sets of filaments were very readily distinguished in the adult plant. The most abundant of these, and that especially found in the distal portions of the fronds, were composed of uniform cylindrical cells, often enclosed in a gelatinous sheath. The diameter of such filaments varies greatly; in the larger the sheaths are generally apparent, in the smaller they are frequently indistinguishable.

In certain places these filaments are more or less parallel side by side, and are glued together in a sort of membrane. It is only in these cylindrical filaments that I have been able to detect heterocysts, which are not very different from the other cells; they are about one-third or one-half broader, and are not vesicular, but have contents similar to those of the other cells. In one instance only was I able to detect hairs upon these heterocysts.

The larger filaments are found especially near the base and in the other older portions of the frond. Their cells are generally irregularly elliptical or globose, rarely are they cylindrical. They are mostly of an orange-brown color; and there exists a particular gelatinous coating to each cell rather than a common gelatinous sheath to the filament. These larger threads are apparently produced from the smaller filaments by a process of growth.

Near the base and in the under portions of the fronds, these filaments are scattered in the homogeneous jelly in which they run infinitely diverse courses. In the upper portions of the frond, and at some little distance from the base, the adjoining cells are very close to one another, and pursue more or less parallel courses, with enough firm jelly between to unite them into a sort of membrane.

This plant certainly belongs to the *Nostochaceæ*, and seems a sort of connecting link between the genera *Hormosiphon* of Kützing and *Nostoc*.

The best algologists now refuse to recognize the former group as generically distinct; and the characters presented by this plant seem to corroborate that view.

Adherent to, and often more or less imbedded in, the fronds of the Nostoc, were scattered frustules of several species of diatoms, none of which was I able to identify. In some of the fronds there were numerous unicellular Algæ, all of them representatives of a single species belonging to the genus *Chroococcus*, Nägeli. This genus contains the very lowest known organisms—simple cells without nuclei, multiplying, as far as known, only by cell-division. These cells are found single or associated in small families; and in certain species these families are united to form a sort of indeterminate gelatinous stratum. In these species the families are composed of but very few cells, surrounded by a very large, more or less globular or elliptical mass of transparent, firm jelly. The species is very closely allied to *Chroococcus turgidus*, var. *thermalis*, Rabenh., from which it differs in the outer jelly not being lamellated.

The technical description of this plant will be found in the proper place.

Fig. 2 *a*, pl. 2, represents the most mature and largest filament; Fig. 2 *b*, a small filament from the same frond, each magnified 800 diameters. Fig. 2 *c*, represents portions of the upper surfaces of fronds.

N. comminutum, KTZ.

N. thallo indefinite expanso, gelatinoso, natante, modo viride, plerumque sordide ferrugineo; trichomatibus flexuosis, plerumque subdense intricatis; articulis globosis (ante divisionem factam subcylindricis), subtiliter granulatis, interdum læte viridibus, plerumque ferrugineis aut luteo-fuscescentibus aut fuscis; cellulis perdurantibus globosis, articulorum diametro duplo majoribus, interjectis aut terminalibus.

Diam.—Artic. $\frac{1}{6000}''$; cell. perdur. $\frac{1}{3000}''$.

Syn —*N. comminutum*, KTZ. RABENHORST, Flora Europ. Algarum, Sect. II. p. 179.

Hab.—In fossis natante, prope Philadelphia.

Thallus indefinitely expanded, gelatinous, floating, mostly sordidly ferruginous, sometimes greenish; trichomata flexuous, mostly subdensely intricate; joints globose (before division subcylindrical), minutely granulate, sometimes bright green, sometimes ferruginous, yellowish-fuscous, or fuscous; heterocysts globose, about twice as long as ordinary joints, both interspersed and terminal.

Remarks.—This species is to be found floating on the surface of the ditches below the city in the latter part of August and September, forming a repulsive, ferruginous, slimy scum. The periderm is not very apparent, and indeed the separate fronds are not distinct. The filaments are very long, mostly closely intricate, very much curved; in some places they are more sparse. Their color is mostly a sort of yellowish ferruginous-green, sometimes they are, decidedly, almost purely ferruginous, more rarely a bright green. This plant agrees pretty well with the descriptions of the European *Nostoc comminutum*, and I believe is the same species; if, however, *N. lacustre* of Kützing is distinct from *N. comminutum*, this is also; but I incline to the opinion that they are all different forms of one plant.

Fig. 3, pl. 2, represents a single filament magnified 800 diameters.

N. commune, VAUCH.

N. terrestre, thallo irregulariter expanso, difformi, undulato-plicato, tremulo, intus aquose gelatinoso, ætate provecta plerumque excavato, peridermate subcoriaceo firmo, olivaceo, luteo-fuscescente vel luteo-fusco cincto; trichomatibus flexuoso-curvatis, pallide æruginеis, laxe implicatis, æqualibus vel subæqualibus, haud raro a basi ad medium usque cellulis biseriatis compositis; articulis sphæricis vel e mutua pressione subquadrangularibus, laxe connexis, passim distantibus, puncto centrali turbato præditis; cellulis perdurantibus globosis, articulorum diametro duplo majoribus, interstialibus terminalibusque.

Diam.—Cell. vegetat. .00012″—.00016″; cell. perdurant. .00025″—.00033″.

Syn.—*N. commune*, VAUCH. RABENHORST, Flora Europ. Algarum, Sect. II. p. 175.

Hab.—In terrestre, New Jersey. (Austin.) "Rio Bravo. Schott." Harvey.

Terrestrial; thallus irregularly expanded, shapeless, undulate-plicate, tremulous, within of the consistence of thin jelly, in advanced age mostly hollow; periderm subcoriaceous, firm, olivaceous, yellowish-fuscous; filaments flexuously curved, pale green, laxly implicate, equal or subequal, not rarely composed of a double series of cells from their base to their middle; articles spherical or subquadrangular from mutual pressure, loosely connected, here and there distant, furnished with a central spot; heterocysts globose, twice as large as the vegetative articles, interstitial and terminal.

Remarks.—The only specimens I have seen of this species are very old ones, which have burst and discharged their central portions. I have consequently preferred to copy the diagnosis of Prof. Rabenhorst. My specimens agree pretty closely with it. The filaments, and also the single cells, are closer together than his words would seem to indicate. My measurements of the heterocysts, as given above, are larger than those of Prof. Rabenhorst. They agree, however, with his text, which his own measurements do not. I am indebted to Prof. Austin for specimens of this species, which he collected in Northern New Jersey. According to Professor Harvey this plant was collected by Dr. Schott along the Rio Bravo, where it is common on dry flats after rains.

SUBFAMILY SPERMOSIREÆ.

Thallus sine peridermate, interdum nullus. Trichomata sporis instructa.

Thallus without any periderm, sometimes absent. Filaments furnished with spores.

Genus ANABÆNA, BORY.

Trichomata moniliformia, evaginata; sporis sphæricis, aureis vel aureo-fuscis, plerumque singulis, cum cellulis vegetativis vel perdurantibus conjunctis.

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Filaments moniliform, without sheaths; spores spherical, yellow or yellowish-fuscous, mostly single, variously placed as to the heterocysts and ordinary cells.

Remarks.—The characters which I have given are somewhat different and less exacting than those of Prof. Rabenhorst, otherwise our American species would hardly be covered by the diagnosis. Professor Harvey in his Phycologia Britannica states that A. Jussieu had preoccupied the name, *Anabœna*, by applying it to a genus of *Euphorbiaceæ*. The date of Bory's name is, however, 1823, whilst that of Jussieu is 1824. Hence, it is the latter which must be changed.

A. gelatinosa, Wood.

A. thallo mucoso gelatinoso, indefinite expanso, dilutissime brunneo, nonnihil pellucido; trichomatibus haud vaginatis, leviter flexuoso-curvatis, nonnihil distantibus, haud intricatis, aut dilute aureis aut dilute cæruleo-viridibus; articulis globosis, homogeneis; cellulis perdurantibus articulorum diametro fere æqualibus, globosis, vel rare oblongis; sporis terminalibus, singulis, globosis (fusco-brunneis?).

Syn.—*A. gelatinosa,* Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, 126.

Hab.—Prope Philadelphia.

Thallus gelatinous, mucous, indefinitely expanded, somewhat pellucid, with a brownish tinge; filaments not vaginate, somewhat curved, rather distant, not intricate, either a light golden-yellow or light bluish-green; joints globose, homogeneous; heterocysts about equal to the filaments in diameter, globose or rarely oblong; spores terminal, globose.

Remarks.—The color of the shapeless mass of jelly of which the frond is composed is a light brown with, in places, a decided reddish or flesh-colored tint. The heterocysts are either interstitial or terminal, no hairs were detected on them; they are mostly globose and only occasionally are they oblong.

Fig. 4, pl. 2, represents a filament of this species magnified 750 diameters; the color of the endochrome of the large spore was possibly due to its being dead.

A. flos aquæ, (Lyngb.) Ktz.

A libere natans, submembranacea, æruginea; trichomatibus plus minus curvatis, sæpius circinatis; articulis sphæricis vel e mutua pressione modo ellipticis modo oblongo-quadratis; cellulis perdurantibus ellipticis singulis vel geminis; cytioplasmate pallide æruginėo granulato turbato; sporis exacte globosis aureo-fulvis lucidis, singulis interjectis, articulorum diametro subduplo majoribus. R. Species mihi ignota.

Diam.—Artic. 0.00017″—0,00025″; diam. long cell. perd. 0.00048″—0.00053″; spor. 0.00032″—0.0004″.

Syn.—*A. flos aquæ,* (Lyngb.) Ktz. Rabenhorst, Flora Europ. Algarum, Sect. II. p. 182.

Hab.—"Round Pond, West Point, New York." Prof. Bailey. Silliman's Journal, N. S., vol. iii. 18

Swimming free, submembranaceous, æruginous; filaments more or less curved, very often circinnate; articles spherical, or, from mutual pressure, elliptical or oblong quadrate; heterocysts elliptical, single or geminate; cytioplasm pale æruginous, granulate; spores exactly globular, golden-fulvous, bright, singly interspersed, nearly twice the diameter of the joints.

A. gigantea, Wood.

A. thallo nullo, trichomatibus singulis et numeroso-consociatis, natantibus, rectis, in ætate juveni spiraliter convolutis; articulis plerumque subglobosis, arcte connexis, granulosis; cel-

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lulis perdurantibus interjectis, articulis vegetativis subæqualibus, utroque polo punctiforme incrassatis, subsphæricis; sporis subsphæricis.

Syn.—*A. gigantea*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, 145.

Hab.—In stagnis natante, prope Philadelphia.

Diam.—Artic. vegetat. max. $\frac{11}{24000}$. Heterocysts $\frac{1}{2000}$ = .0005. Spor. lat. $\frac{11}{12000}$ = Long. $\frac{1}{1000}$ = .001.

Thallus wanting; filaments occurring floating singly on water or in great numbers, straight, but in the young state often spirally convolute; articles mostly subglobose, closely connected, granular, heterocysts subspherical, interstitial, a very little larger than the vegetative cells, thickened at each end in a punctiform manner; spore subspherical.

Remarks.—This plant was found by myself, late in the summer, floating upon a brick-pond below the city, forming a part of a thick, dirty-green, "pea-soup colored," almost pulverulent scum. The filaments, though occasionally in great numbers, were never, that I saw, joined together by any jelly so as to form a frond.

Fig. 5, pl. 3, represents a short filament of this species magnified 750 diameters.

Genus CYLINDROSPERMUM, Ktz.

Sporæ ante cellulam terminalem ortæ.

Spore developing from the next to the terminal cell.

C. minutum, Wood.

C. trichomatibus dilute ærugineis, plerumque flexuoso-curvatis et intricatis, interdum subrectis; articulis cylindricis, ad genicula plus minus constrictis, homogeneis vel granulatis; cellulis perdurantibus terminalibus, hirsutis, globosis; sporis ellipticis, diametro 2—3 plo longioribus, subtilissime granulatis.

Syn.—*C. minutum*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, 126.

Diam.—Artic. $\frac{1}{9000}''$; spor. long. $\frac{1}{1630}''$; transv. $\frac{1}{4000}''$.

Hab.—In stagnis prope Philadelphia.

Filaments light æruginous-green, generally curved and intricate, sometimes straightish; articles cylindrical, more or less constricted at the joints, homogeneous or granulate; heterocysts terminal, hirsute, globose; spores elliptical, 2—3 times longer than broad, very minutely granulate.

Remarks.—This species was found by myself at Spring Garden, New Jersey. With a number of other algæ it formed a ferruginous-brown gelatinous mass, growing in a deep, shaded, very stagnant pool. In most instances the filaments were closely interwoven, and sometimes formed minute greenish balls, just large enough to be visible to the unassisted eye. In other instances they were mixed up with various algæ in little indefinite masses. There is apparently a stage in the life of the plant, when it consists of a single filament enclosed in a little capsule, for mixed in with the rest of the gelatinous scum were little microscopic, subglobose masses, with a firm outer periderm and a single filament coiled up in the centre. The color of the filaments was generally a faint bluish-green, sometimes, however, with a yellowish tint. The spores were decidedly yellowish.

Fig. 6, pl. 2, represents a fragment of a filament with the spore magnified 800 diameters.

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C. flexuosum, (Ag.) Rabenh.

C. strato gelatinoso, saturate viride, indefinite expanso; trichomatibus æqualibus, pallide vel saturate cæruleo-viridibus, plerumque valde flexuosis et intricatis, sæpius circinatim vel fasciatim convolutis, interdum subrectis, et fasciatim contextis; articulis oblongis, ad genicula plus minus contractis, homogeneis vel granulatis, distinctis; cellulis perdurantibus terminalibus, subglobosis, rare hirsutis, nonnunquam in trichomatis utroque fine; sporis oblongo-cylindricis, diametro 2—3 plo longioribus, distincte granulatis.

Diam.—Spor. $\frac{5}{12000}''$ = .000416"; cell. veget. $\frac{1}{6000}''$ = .000166".

Syn.—*C. flexuosum,* (Ag.) Rabenhorst, Flora Europ. Algarum, Sect. II. p. 188.

Hab.—In locis irroratis, prope Philadelphia.

Stratum gelatinous, deep green, indefinitely expanded; filaments equal, pale or deep bluish-green, mostly very flexuous and interwoven, often circinnately or fasciately convolute; sometimes straightish and in bundles; articles oblong, more or less contracted at the joints, homogeneous or granulate, distinct; heterocysts terminal, subglobose, rarely hirsute, sometimes at both ends of the filament; spores oblong-cylindrical, 2 or 3 times longer than broad, distinctly granulate.

Remarks.—The color of the filaments in young specimens is deeper than in the older, which, however, grew in a much darker locality. The young spores are a yellowish-green, afterwards they are of a sort of yellowish reddish-brown. In one instance two spores were seen closely conjoined together at the end of a filament. In some filaments one or more heterocysts occur interstitially. Often one or more filaments will be seen coiled together like a rope. On the banks of the Schuylkill River I have found this species in two localities in the latter part of September. In the one instance it grew along the Reading Railroad, just above the Flat Rock tunnel, in a dark little grotto, formed by shelving rocks. In the other case, it was on wet ground by a horse-trough very near the west end of the upper bridge at Manayunk.

Fig. 1*a*, pl. 3, represents a filament, magnified 450 diameters.

Fig. 1*b*, a portion of a filament, magnified 800 diameters.

C. macrospermum, Ktz.

C. trichomatibus curvatis vel subrectis, pallide æruginеis; articulis cylindricis vel subcylindricis (in forma Europæa "globosis vel ellipticis"), ad genicula plus minus constrictis, passim confluentibus; cellulis terminalibus plerumque ellipticis vel ovatis, diametro paulo vel subduplo longioribus; sporis elliptico-oblongis vel oblongo-cylindraceis, viridibus (in formam Europæam maturam "saturate fuscis"), subtiliter granulosis, diametro duplo longioribus.

Diam.—Trich. cell. transv. $\frac{1}{8000}''$ = .00003"; spor. .00046"—.00054".

Syn.—*C. macrospermum,* Ktz. Rabenhorst, Flora Europ. Algarum, Sect. II. p. 186.

Hab.—In rivulis, South Carolina. (Prof. Ravenel.)

Filaments curved or straightish, pale æruginous; articles cylindrical or subcylindrical (in European species "globose or elliptical"), more or less constricted at the joints, here and there confluent; terminal cells mostly elliptical or ovate, a little longer or about twice as long as broad; spores elliptical-oblong or oblong cylindrical, greenish (in mature European specimens deep fuscous), finely granular, about twice as long as broad.

Remarks.—I have received this species from Professor Ravenel, who collected it near Aiken, South Carolina, in the month of September; with it was the following note: "In bottom of shallow, slowly running streams, adhering to ground or fallen leaves, &c., gelatinous green." The specimens agree well with the description of the European form, except that I have never seen the joints globose or elliptical, but *always* cylindrical, as they are said to be *sometimes* in the typical specimens. The color of the spores also is not "fuscous," but that probably depends upon their not being fully mature.

Fig. 7, pl. 2, represents the spore of this species with the neighboring heterocyst, magnified 750 diameters.

C. comatum, WOOD (sp. nov.)

C. terrestre, stratum gelatinosum ærugineum interdum brunneo tinctum, formans; trichomatibus flexuosis, intricatis, haud spiralibus, æqualibus; articulis breve cylindraceis, diametro æqualibus ad plus duplo longioribus, plerumque sejunctis, pallide ærugineis, obscure granulatis; cellulis terminalibus subglobosis; sporis oblongo-cylindricis, diametro fere duplo longioribus, granulatis, luteo-brunneis; membrana crassa, distincte granulata.

Diam.—Spor. transv. $\frac{5}{12000}''$ = .00042″. Long. $\frac{11}{12000}''$ = .00092″. Artic. .0001″.

Hab.—In terra uda; Niagara, Canada.

Growing on the ground, forming a gelatinous stratum of an æruginous color, sometimes tinged on edges with brown; filaments flexuous, equal, intricate, not spiral; joints shortly cylindrical, equal to or more than twice as long as the diameter, mostly separated, pale æruginous, obscurely granulate, terminal cells subglobose; spores oblong-cylindrical, about twice as long as broad, granulate, yellowish-brown; membrane thick, distinctly granulate.

Remarks.—I found this *Cylindrospermum* growing upon the ground in the marshes which border the Niagara River just above the Canadian Falls. It formed a bright, æruginous, gelatinous, but firmish, almost membranous, stratum.

The filaments are often quite long, and are composed of short, cylindrical cells, mostly placed rather far apart. The terminal cells are remarkable for being abundantly provided with long, flexible, hair-like processes, upon the ends of which are minute lobular bodies (cells?). These appendages are so minute as to make it difficult to determine their structure, and although I have studied them with a $\frac{1}{25}$th immersion lens, giving a power of nearly 2500 diameters, there are some points about them still undetermined. I do not know whether they or the little globules are hollow or not. I do feel pretty certain, however, that the little globules are distinct bodies, and that they finally drop off, leaving the naked hair behind. Is it possible that they have any sexual significance? The spore-wall is thick, and under a high power is seen to be distinctly granulate. The granules are of course small, but in the perfected spore can plainly be seen with an eighth objective projecting out from the margin.

Fig. 8, pl. 2, represents the spore-end of a filament, magnified 1375 diameters.

Genus DOLICHOSPERMUM, THWAITES.

Sporæ ellipticæ, oblongæ vel cylindraceæ, inter cellulas vegetativas ortæ, sæpe in seriebus connexæ, a cellulis perdurantibus disjunctæ.

Spores elliptical, oblong, or cylindrical, occurring amidst the vegetative cells, often connected in series, separated from the heterocysts.

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Syn.—Sphærozyga, (AUCTORES, *partim.*)
Dolichospermum, THWAITE'S MSS. Mr. J. RALFS on the *Nostochineæ,* Ann. Mag. Nat. Hist. 1850, p. 335.

Remarks.—This genus differs from *Sphærozyga* in that the spores have no relation, in regard to position, with the heterocysts. Professor Rabenhorst, in his Flora, does not acknowledge it; but it is very evident that he has neither seen the original paper of Mr. Ralfs, nor the species upon which the genus was founded, for he mentions none of the latter, either as good species or synonyms, and the memoir itself is not included in his bibliographical list. The generic characters given by myself are essentially those of the original description, with the exception that the filaments in the latter are said to be aggregated into a stratum, which is not true of the American forms herein described.

D. subrigidum, WOOD.

S. natans; trichomatibus singulis, rectis aut subrectis, minimis, dilute viridibus; articulis cylindraceis aut subglobosis, distinctis; sporis cylindraceis, in medio gradatim nonnihil constrictis, singulis aut duplicis, sine cellulis perdurantibus inter se; cellulis perdurantibus breve cylindraceis, singulis, distinctis.

Syn.—Sphæroziga subrigidum, WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 123.

Diam.—Cell. veg. trans. $\frac{1}{6000}''=.00016''$; spor. transv. $\frac{1}{4300}''—\frac{1}{4500}''=.00023''—.00022''$; long. $\frac{1}{1500}''=.00066''$; cell. perd. transv. $\frac{1}{4500}''=.00022''$.

Hab.—In stagnis prope Philadelphia.

S. Floating; filaments single, straight or straightish, very small, light green; articles cylindrical or subglobose, distinct; spores single or double, in the middle gradually a little constricted, not having a heterocyst between them; heterocysts shortly cylindrical, single, distinct.

Remarks.—I have found this species growing in the scum floating upon the ditches below the city. The filaments are always, as I have seen them, scattered. They seem always to be nearly straight, or entirely so, and indeed preserve their straightness so constantly as to suggest the name given the species. The spores are very distinct, and all that I have seen were greenish, cylindrical, and constricted in the middle, so that their sides are concave. Their position does not seem to be uniform, any further than that they are amongst the ordinary cells. The heterocysts are large, almost equalling the spores in diameter; I have never detected hairs on them. This species appears to be most nearly allied to *D. Thwaitesii* of Ralfs, from which it differs in not forming a stratum, and in the great proportionate diameter of the heterocysts. I have never seen any measurements of *D. Thwaitesii.*

Fig. 2, pl. 3, is a filament, magnified 975 diameters.

D. polysperma, (KTZ.)

S. trichomatibus plerumque subsolitariis, sed interdum consociatis et intricatis, dilute cæruleo-viridibus, subrectis aut varie curvatis et flexuosis; articulis aut subsphæricis aut breve cylindricis; cellulis perdurantibus globosis aut latissime ellipticis, articulorum diametro paulo vel duplo majoribus; sporis plus minus elongatis, cylindraceis—in ætate immatura, sparse granulatis, dilute cæruleo-viridibus, et cum membrana haud distincta,—in ætate matura dense granulatis et cum membrana subcrassa.

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Diam.—Artic. $\frac{1}{6000}''$ = .00016″; spor. $\frac{4}{15000}''$—$\frac{5}{15000}''$ = .00026″—.00033″.

Syn.—*S. Carmichælii*, HARVEY, Phycol. Brittanica, T. cxiii.

S. polysperma, (KTZ.) RABENHORST, Flora Europ. Algarum, Sect. II. p. 192.

Hab.—In stagnis, New Jersey.

S. filaments mostly subsolitary, but sometimes associated and interwoven together, light bluish-green, straightish, or variously curved and flexuous; articles either subspherical or shortly cylindrical; heterocysts globose or very broadly elliptic, a little larger to twice as large in diameter as the ordinary joints; spores more or less elongate, cylindrical, in the uncertain condition sparsely granulate, light bluish-green, with the membrane not distinct, in the mature state densely granulate, and with a thickish membrane.

Remarks.—I found this species growing in a brownish jelly, with various other algæ, in a pool east of Camden, New Jersey. The filaments were mostly scattered, but in some places numbers of them were collected in little masses. In some filaments almost all the cells were developed into spores, so that a single thread contained ten or even more spores. In by far the larger number of such cases there was between each pair of spores a heterocyst; sometimes, however, the latter was wanting, and the spores would be attached to one another.

My specimens differ somewhat from the European form, but are too close to separate from them. They equally resemble, however, *S. Carmichœlii.* Indeed, I cannot see any sufficient reason for separating the species. *S. Carmichœlii* is, to be sure, a salt-water plant. I have, however, received specimens collected by Dr. Lewis, near Stonington, which I believe grew in salt water, and which agree in every respect with my fresh-water specimens.

Fig. 3, pl. 3, represents a portion of a filament, magnified 750 diameters.

FAMILY RIVULARIACEÆ.

Thallus gelatinosus, mollis vel induratus, vel crustaceus, interdum calce impletus, subglobosus vel amorphus. Trichomata ad oscillarium morem articulata, vaginata, sed interdum ætate provecta cum vaginis in gelatinam matricalem confluentibus et haud visibilibus, simplicia vel pseudoramosa, superne attenuata, sæpius in apicem piliformem longe producta, parallela vel radiatim disposita, cellula basale hyalina globosa et interdum cellulis interstialibus instructa. Sporæ (*manubria*, KTZ.), singulæ plerumque inter cellulam perdurantem basilarem et cellulas vegetativas positæ, sæpe permagnæ, cylindricæ, plerumque pachydermaticæ.

Vegetatio terminalis. Propogatio sporis tranquillis.

Thallus gelatinous, soft, or indurated, or crustaceous, sometimes filled with lime, subglobose or amorphous. Trichomata articulated like an oscillatoria, vaginate, but sometimes, when old, with the sheaths confluent in the maternal jelly and not visible, simple or pseudoramose, attenuated above, often with the apex prolonged into a long hair, parallel or radiately disposed, furnished with globose hyaline, thick-walled basal cells, and sometimes with interstitial cells. Spores cylindrical, generally placed between the basal and vegetative cells, often very large, mostly with thick coats.

Vegetation tranquil. Propagation by means of tranquil spores.

Remarks.—In the *Rivulariaceæ* the thallus is always small; but is most generally in the various species somewhat definite in form and size. Its consistency in our North American forms varies from that of an exceedingly soft, formless jelly to that of a gristly mass. The maternal jelly is usually colorless, sometimes brownish or yellowish. There is never any condensation of the outer portion of the frond into a periderm. The filaments commonly radiate from the centre to the circumference; sometimes, especially in the softer fronds, they are simply parallel with one another. The sheaths vary in their breadth, firmness, and distinctness. These little plants grow chiefly in the water; some species are said to live in the air in exceedingly damp places, but I have not as yet met with any such. They appear to prefer cold climates, although I have received specimens from South Carolina. With us, I have only found them in the late autumn and winter months. As to their life-history very little appears to be known; I have not been able to make any observations myself upon this point, nor to obtain access to the papers[1] by De Bary, almost the only sources of such information, and therefore pass by the subject.

Genus NOSTOCHOPSIS, Wood.

Trichomata ramosa cum cellulis perdurantibus aut in lateribus sessilibus aut in ramulorum brevissimorum apicibus dispositis. Vaginæ nullæ. Thallus definitus.

Syn.—Nostochopsis, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869.

Thallus definite; filament branched; heterocysts sessile upon the sides of the filaments, or raised upon the apices of short branches; sheaths none.

Remarks.—The curious plant upon which this genus is founded has the habit of a nostoc. The outer portion of the frond is condensed, so as to give the appearance of a periderm; but there is, in reality, no true periderm. The consistence of the thallus is that of a firm, gelatinous mass. The trichomata or filaments radiate from the inner part of the frond towards the outer surface, but many of them take their origin in the outer portions of the thallus. In most places they are distinctly articulated, and, indeed, the joints being swollen and moniliform, in some parts they almost seem to be composed of globular cells, resembling somewhat the filaments of a nostoc; on the other hand, in certain portions they are not at all articulated, and this for long distances. No sheaths are anywhere visible. The heterocysts are, strangely enough, never placed in the continuity of the filaments. Sometimes they are sessile immediately upon the latter, sometimes they are raised upon very short branches. They are globose, with rather thick walls. Possibly, however, I am mistaken in believing these bodies to be heterocysts, for they may be rather of the nature of spores, as is somewhat indicated by their thick walls, and often apparently dense contents. Their round shape, and the absence of anything else representing heterocysts, has induced me, however, so to consider them. In my Prodromus I placed this plant provisionally amongst the nostocs; but the radiation of the filaments from within outwards, and especially their being branched, on second thought seem to me to indicate a closer relation with the *Rivulariaceæ*. The genus appears to be a sort of connecting link between the two families.

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[1] "Flora," 1863.

N. lobatus, WOOD.

N. thallo vivide viride aut luteo-viride, cavo, enormiter lobato, natante, modice magno, firmo, gelatinoso; trichomatibus plerumque longis, flexuosis, dilute viridibus, plerumque articulatis, partim inarticulatis, cylindricis aut sub-moniliformibus, sparse granulatis.

Diam.—Trichom. $\frac{1}{15000}'' = .00006''$—$\frac{1}{7500}'' = .00013''$; cell perdum. $\frac{1}{3750}'' = .00026''$.

Syn.—*N. lobatus,* WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869.

Hab.—In Schuylkill Flumine, prope Philadelphia.

Thallus bright green or yellowish-green, hollow, irregularly lobed, floating, moderately large, firm, gelatinous; filaments mostly long, flexuous, dilute green, mostly articulate, partly inarticulate, cylindrical or somewhat moniliform, sparsely granulate.

Remarks.—I found this plant floating upon the Schuylkill River just above Manayunk. The hollow frond was buoyed up by a bubble of gas contained within it. It was an irregular, flattened, somewhat globose mass, of a bright green color and about half an inch in diameter. It seems very probable that in its earlier condition, it was a solid attached frond. The long slender filaments are often very tortuous, but run a pretty direct general course towards the outer surface.

Fig. 6 *a*, pl. 3, represents a section of the frond slightly magnified; *a*, *b*, *c*, portions of filaments magnified 800 diameters.

Genus GLOIOTRICHIA, J. AG. (1842.)

Trichomata e planitie orta pseudoramosa, distincte vaginata; vaginæ amplæ, basi plerumque saccatæ, transverse undulato-plicatæ, plus minus constrictæ, apice apertæ, non laciniatæ. Sporæ magnæ cylindricæ.

Filaments springing from a plane, pseudoramose, distinctly vaginate; sheath ample, mostly saccate at the base, transversely undulately plicate, more or less constricted, open at the apex, not laciniate. Spores large, cylindrical.

Remarks.—This genus was, I believe, first indicated by Professor Agardh in his *Algæ Maris Mediterranei et Adriatici,* a work to which I have not access. On account of this, and also because I have not seen any of the typical species of the genus, I have preferred simply copying the generic characters given by Professor Rabenhorst. If my understanding of "e planitie orta" is correct, I do not think it true. Professor Rabenhorst's own figure of *Rivularia* shows that the filaments do not all arise on one plane; although he asserts the character equally for that genus. In our American species the filaments do not all arise on one plane, nor can they be spoken of as "*pseudoramosa.*"

G. incrustata, WOOD.

G. globosa vel subovalis, firma, solida, ad pisi minimi magnitudinem, dilute viridis, crystallophora; trichomatibus rectis aut leviter curvatis, in pilum productis, viridibus aut flavescentibus, sæpe infra læte viridibus sed supra flavescentibus, haud ordinatim articulatis; articulis inferioribus in trichomatibus maturis brevibus, plerumque compressis; pilo apicale recto aut leviter curvato, plerumque indistincte articulato, sæpe interrupto; vaginis amplis, achrois, saccatis, interdum valde constrictis; sporis cylindricis, sæpe curvatis, diametro ad 9 plo longioribus; cellulis perdurantibus sphæricis.

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Diam.—Trichom. cum vag. $\frac{7}{7500}''$—$\frac{9}{7500}''$; sporis max. $\frac{3}{7500}''$—$\frac{4}{7500}''$; cell. perd. $\frac{7}{15000}''$.

Syn.—*G. incrustata,* WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 128.

Hab.—Schuylkill River, plantas aquaticas adhærens.

Frond globose or suboval, firm, solid, about the size of a very small pea, light green, crystal bearing; filaments straight or slightly curved, produced into long hairs, green or yellowish, sometimes bright green in their proximal portions but yellowish above, not regularly articulate; lower articles in the mature filament short, and generally compressed; apical seta straight or slightly curved, mostly indistinctly articulate, frequently interrupted; sheath ample, transparent, saccate, sometimes strongly constricted; spores cylindrical, frequently curved, about 9 times as long as broad.

Remarks.—I found this species growing attached to some little plants, either in the Schuylkill near Spring Mills, or else in the spring itself, I do not know which. The roundish fronds varied in size from a mustard-seed to that of a half-grown pea. They were of a decided green color, but appeared grayish from the amount of carbonate of lime in and upon them. The larger balls, when cut in two, were distinctly separable into a central and cortical part. The former was more gelatinous and contained fewer of the filaments than the latter. The filaments mostly arose in sets together, *i. e.* there were one or more zones or planes in which the bases of the filaments were placed together. This, however, was not strictly the case, as there were almost always some scattered trichomata. The matured filaments are very distinct. Their sheaths are very large, and often saccate, with wavy, loose-looking margins; sometimes they are suddenly transversely constricted, once or more in their length; sometimes they look as if a tight spiral band were wound around; sometimes they are entirely free from any constrictions. These sheaths are open above, appearing as though they had been melted away. The spore is long and cylindrical, and is highly granular. The endochrome is generally articulated below, the joints are often so nearly globular in the lower portions as to give a moniliform appearance; sometimes the articles are compressed. The upper portion of the trichoma is frequently interrupted, and if at all articulated is very irregularly and indistinctly so. The younger filaments have their endochrome variously and irregularly interrupted. The basal cells are globular. I believe the formation of new filaments and the consequent growth of the frond take place by distal portions of the projecting endochrome separating from the parent filament, then forming a basal cell, and lastly a sheath. (See Plate 00.) The carbonate of lime does not exist as a definite incrustation, but in the form of semi-crystalline masses scattered through the frond. This species seems to come closer to *G. boryana* than any described species, from the description of which it differs, in the color of thallus, in the latter being always solid (at least so I have found it late in the fall, when the spores were fully perfected), in its habit of inclosing crystals of carbonate of lime, in the curved spores; and, doubtless, a comparison of the specimens would show still more important differences.

Fig. 4 *a*, pl. 3, represents a section of a frond moderately magnified; fig. 4 *b*, the basal end of a filament magnified 460 diameters; fig. 8 *c*, filaments magnified 260 diameters.

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G. angulosa, (Roth.) J. Agh.

G. globoso-angulosa, cava, viridi-fuscescens, ad cerasi magnitudinem; trichomatibus strictis, torulosis, superne leviter flexuosis, passim interruptis; articulis inferioribus plus minus compressis, diametro duplo triplove longioribus; vaginis amplis, achrois hic illic leviter constrictis; sporis plus minus elongatis, oblongo-ovatis vel ellipsoideo-cylindricis, diametro 3–6–10 plo longioribus, ærugineo-fuscescentibus, nonnunquam leviter curvatis, cytioplasmate subtiliter granuloso, turbato. (R.) *Species mihi ignota.*

Diam.—Cell. perd. 0.00036″—0.0005″. Spor. max. 0.00059″. (R.)

Syn.—*G. angulosa,* (Roth.), J. Agardh., Rabenhorst, Flora Europ. Algarum, Sect. II. p. 201.

Hab.—Hudson River prope West Point. (Bailey.)

Globose angular, hollow, greenish-fuscous, attaining the size of a cherry; filaments strict, torulose, above somewhat flexuose, here and there interrupted; inferior joints more or less compressed, 2–3 times longer than their diameter; sheath ample, colorless, here and there slightly constricted; spores more or less elongate, oblong-ovate or ellipsoidal-cylindrical, 3–6–10 times longer than the diameter, æruginous-fuscous, sometimes slightly curved, cytioplasm very minutely granulate.

Genus RIVULARIA, (Roth.) Agh.

Thallus et trichomata eadem quæ Gloiotricha, sed vaginæ arctissimæ, sæpe in gelatinam matricalem confluentes, quasi nullæ.

Thallus and filaments similar to those of Gloiotricha, but the sheaths very close, often confluent in the gelatinous matrix and apparently wanting.

Remarks.—The characters given above are those of Professor Rabenhorst. Flora Europ. Algarum, Sect. II. p. 206

R. cartilaginea, Wood.

R. subglobosa, parva, cartilaginea, saturate brunnea vel subatra, solitaria in plantis aquaticis:—trichomatibus maturis-sterilibus, rectis aut subrectis, cylindricis, elongatis, haud articulatis; cytioplasmate sæpe interrupto; vaginis arctis et distinctis; cellulis perdurantibus globosis, diametro subæqualibus:—trichomatibus fertilibus—rectis aut subrectis, supra spora cellulis 8–9 instructis; sporis elongatis, rectis, cylindricis; vaginis nonnihil crassis, arctis:—trichomatibus immaturis breve articulatis; vaginis subamplis.

Diam.—Trich. cum vag. $\frac{1}{2000}''$; spor $\frac{1}{3000}''$.

Syn.—*R cartilaginea,* Wood, Proc. Am. Philos. Soc., 1869, p. 128.

Hab.—In palude, Northern Michigan.

Frond subglobose, small, cartilaginous, deep brown or blackish, solitary upon aquatic plants; mature sterile filaments, cylindrical, elongated, not articulated, their cytioplasm frequently interrupted, their sheaths close and distinct, their heterocysts globose and about equal to them in diameter; fertile filaments straight or nearly so, above the spores furnished with 8 or 9 cells; spores elongate, straight, cylindrical; sheaths rather thick, close; immature filaments shortly articulate, their sheaths rather large.

Remarks.—The frond of this species grows attached to the leaves of water-plants, and has its under side markedly flattened so that it is somewhat semi-globose. The filaments which compose the mass of the very firm frond are elongated, cylindrical, and of nearly or entirely uniform diameter throughout. The sheaths are close, distinct, rather thin, open above, and, in many instances, almost or even entirely empty. Scattered amongst such filaments are the fertile ones. These have at their base an elongated cell, in which is the long cylindrical spore, which varies

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very greatly in length in the various filaments, but is almost always shorter than the cell containing it. Just beyond the spore is a series of distinct, variously shaped cells, about seven in number, which are, as I have seen them, empty. In the outer portions of the frond occur what I believe to be young filaments. These are distinguished by their rapidly decreasing in diameter towards their distal end, by their being distinctly articulated, by their basal cell not being distinctly separated as in the older filaments, and by their sheaths being more ample.

These various filaments composing the fronds do not arise from any one place, but commence at very different distances from the centre, and pursue a more or less straight course to the circumference of the frond, from which they often project.

Fig. 9, pl. 2, represents a section of the frond moderately magnified; fig. 9 *b*, is a drawing of the basal part of a filament magnified 800 diameters.

Genus ZONOTRICHA.

Thalli pulvinato-hemisphærici, sæpe confluentes, calce prægnantes, plus minus indurati, basi plani affixa, ætate provecta plerumque excavati, intus zonati; zonis concentricis, variegatis; trichomata pseudoramosa, gracilia, inæqualia, apice hyalina et plus minus longe cuspidata vel in pilum producta. Vaginæ firmæ, homogeneæ vel longitudinaliter plicato-fibrillosæ, apice integræ vel dilatatæ et in fibrillas solutæ. Sporæ ignotæ.

Thalli pulvinately hemispherical, often confluent, impregnated with lime, and more or less indurated, fixed by the flattened base, in advanced age mostly excavated, zoned within; zones concentric variegated; filaments pseudoramose, slender, unequal, their apices hyaline and more or less cuspidate or prolonged into a hair; sheaths firm, homogeneous, or longitudinally plicately fibrillose, their apices entire or dilated and dissolved in fibrillæ. Spores unknown.

Z. mollis, WOOD (sp. nov.)

Z. interdum subhæmispherica sed gregaria et in stratum nonnihil mammillosum confluens, submollis, cinerea vel griseo-carnea, parcezonata; trichomatibus longissimis, angustis, flexuosis; vaginis arctis, decoloratis, non fibrosis, firmis; trichomatibus internis articulatis, sæpe interruptis; articulis disjunctis, diametro æqualibus ad 4 plo longioribus; cellulis perdurantibus singulis globosis.

Diam.—Trich. c. v. $\frac{2}{12000}''$ = .00017″. Sine vag $\frac{1}{12000}''$ = .000084″.

Hab.—In saxis irroratis, "Cave of the Winds," Niagara, Wood.

Z. sometimes subsemispherical but gregarious and confluent into a somewhat mammillate, rather soft stratum, ashy or grayish flesh-colored, sparsely distinctly zoned; filaments very long, narrow, flexuous; sheaths close, colorless, not fibrillose, firm; internal filament articulated, often interrupted; joints separated, equal to 4 times longer than the diameter; heterocysts single globose.

Remarks.—Every American tourist is familiar with that most wonderful spot, the so-called "Cave of the Winds," at Niagara. It is simply a place where it is possible to go underneath a portion of the great cataract, and then round upon the rocky debris outside of it. Growing upon these rocks, eternally wet and glistening with foam and spray, I found this and the following species. The present form was much the most abundant, making a slippery, grayish, or grayish flesh-colored coating to many of the rocks, dotted here and there with the rigid, blackish fronds of

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its fellow. This coating was not at all uniform, but was covered with mammillated masses, and consequently varied from two to six lines in thickness. Internally, it was striated or radiated, but not so evidently as the following species, and presented several distinct variegated zones. It was quite soft to the touch, as well as readily broken or crushed, and under the microscope was seen to contain very little lime salt. When dried it has a pronounced sebaceous appearance. The filaments composing it are remarkable for their great length, often apparently running from the bottom to the top of the frond. They are rarely if ever branched, and appear never to be furnished with any heterocysts save at their enlarged base. I have never seen any distinct hairs terminating them, their ends always appearing broken and open. They are often quite flexuous or even tortuous. The internal filament is remarkable for having its articles so distinctly separated. It is often very much interrupted, and in specimens preserved in carbolic-acid water is of an orange-brown color.

Fig. 3, pl. 4, represents a single filament magnified 260 diameters.

Z. parcezonata, WOOD, (sp. nov.)

Z. nigro-viridis, enormiter semiovalis, ad 6″ longa, dura, lubrica, non fragilis, calce prægnans, intus a basi distincte radiata, parce et sæpe obsolete zonata; trichomatibus modice longis, subrectis; trichomatibus internis cylindricis inarticulatis vel articulatis, et interdum moniliformibus; articulis longis et cylindricis vel brevibus et globosis; vaginis amplis, fibrillosis; cellulis perdurantibus basalibus et interjectis, his oblongis vel cylindricis, illis globosis et sæpe geminis.

Diam.—Cell. perd. basal. $\frac{1}{6000}''=.00017''$; trichom. cum vag. $\frac{1}{4000}''-\frac{3}{8000}''=.00025''-.00037''$. Sine vag. .00006″—.00008″.

Hab.—In saxis irroratis. "Cave of the Winds," Niagara.

Var.—Z. cinerea.

Blackish green, irregularly semioval, to 6 lines long, hard, slippery, not fragile, impregnated with lime, internally distinctly radiate, sparsely and often obsoletely zoned; filaments moderately long, straightish; internal filament cylindrical, not articulated or articulated, sometimes moniliform; joints long and cylindrical, or short and subglobose; sheath ample, fibrillose; heterocysts basal and interposed in the body of the filament; the former globose, often geminate; the latter oblong or cylindrical.

Var.—Cineritious in color.

Remarks.—I found this plant growing on rocks as glossy, blackish, very hard and slippery fronds or masses, which varied in size from that of very small shot to nearly half an inch in length. The larger ones were not nearly so high as long, and presented irregular, almost bossellated upper surfaces. The filaments are often very evidently and frequently pseudoramose. The external surface of the broad sheath is covered with numerous fibrillæ, which envelop and seem sometimes to wrap it round and round. The color of the frond internally, when broken, is mostly a dark chocolate, and the surface presents a radiated appearance, with but two or three zones at most, and, in the very dark specimens, even these are not evident. No signs of spores have been found. Certain specimens which I obtained growing with the others, instead of being blackish in color, are grayish, but agree in all other respects with their fellows. This gray color depends, I believe, upon the deposit of an immense quantity of lime salts, which in such specimens constitute by far the larger portion of the frond.

Fig. 4, pl. 4, represents a section of frond, slightly magnified.

It is either this, or the preceding species, which is referred to by Professor Bailey in Silliman's Journal, vol. iii, under the name of *Rivularia calcarea*, Sm. The present form may possibly be that plant, but not having been able to find any description sufficiently well made out to make identification possible, I have described both species as new.

Z minutula, Wood, (sp. nov.)

Z. minutissima, nigro-viridis, subglobosa, haud distincte zonata, nonnihil mollis, muscicola, calce non prægnans; trichomatibus internis, breve articulatis, distinctissime fasciculatim pseudoramosis; vaginis crassis, amplis, sæpe dilute aurantiaco-brunneis, apice plerumque coloris expertibus fissis et apertis; cellulis perdurantibus ovato-globosis.

Diam.—Trich. intern. .00012″—.00021″; cell. perd. .00025.″

Hab.—In lacu, " Clear Pond," muscis affixa, Adirondack Mountains.

Very small, blackish-green, subglobose, not distinctly zoned, rather soft, growing on mosses, not impregnated with lime; internal filaments shortly articulate, very distinctly fasciculately pseudoramose; sheaths thick, ample, often pale orange-brown, with their apices mostly colorless, torn and open; heterocysts ovately globose.

Remarks.—The locality in which I found this plant is in the heart of the Adirondack wilderness. The little frond in none of my specimens is larger than a mustard-seed, and is not distinctly zoned. The plants were collected in the beginning of July, and very possibly are not fully grown, as the season of general growth opens very late in its parent lake. Very possibly, later in the year, it may be found larger and distinctly zoned. The general appearance of the plant, the character of its sheath, and the marked branching habit of the filaments have caused me to place it in this genus.

Genus DASYACTIS, Ktz.

Thallus gelatinosus, mollis, non zonatus. Trichomata matura sæpe haud vaginata. Sporæ nullæ.

Thallus gelatinous, soft, homogeneous, not zoned. Mature filaments often not vaginate. Spores absent.

D. mollis, Wood.

D. parva, ad magnitudinem pisi minimi, enormiter subglobosa, mollis, gelatinosa, dilute viridis; trichomatibus plerumque subrectis, partim distincte, partim indistincte articulatis; vaginis, in trichomatibus maturis haud visibilibus, in trichomatibus juvenibus supra subamplis; cellulis perdurantibus sub-globosis, globosis, vel ellipticis, diametro duplo majoribus, plerumque singulis sed interdum bi vel triseriatis.

Diam.—Trich. $\frac{1}{6000}$″—$\frac{1}{4500}$″; cell. perd. $\frac{1}{1800}$″.

Syn.—*D. mollis*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 128.

Hab.—In palude plantas aquaticas adhærens, Northern Michigan.

Frond small, about the size of a small pea, irregularly subglobose, soft, gelatinous, light green; filaments generally straightish, partly distinctly, partly indistinctly articulate; sheaths in the mature filament not perceptible; in the young filaments rather large in the upper portion; heterocysts subglobose or globose or elliptic, twice as large as the filament, generally single but sometimes bi or tri-seriate.

Remarks.—I found this species growing attached to the little leaves of various minute cryptogamic and phaneerogamic water-plants, in a small bog, near the mouth of Carp River, in Northern Michigan. The frond is somewhat translucent, with a slightly greenish tint, and has a soft, gelatinous consistency. The matured trichoma or filaments are more or less radiating, very long, generally nearly straight and parallel. Their joints or articles are long, mostly not very distinctly separated, and often are entirely wanting. The sheaths are entirely lost, no traces of them being perceptible. They seem to be altogether melted down into the homogeneous jelly, in which the filaments are imbedded. The basal cell is large, mostly globular, and very prominent. On the edges of the frond may frequently be seen small, evidently immature filaments, which have no distinct basal cell. Around the basal portion of these young trichoma there is a well-marked close sheath, which near the apex is wanting. In their immature filaments the joints are mostly very short, rather distinctly separated, almost globular.

Fig. 5, pl. 4.

Genus MASTIGONEMA, Schwabe.

Trichomata articulata, sursum flagelliformia vel subulata, simplicia vel pseudoramosa (nonnunquam fasciculatim pseudoramosa), procumbentia vel erecta, in thallo indistincto cæspitoso-aggregata; vaginæ arctæ et homogeneæ vel amplæ et plus minus distincte lamellosæ, apice plerumque apertæ, interdum laciniatæ.

Filaments articulate, superiorly flagelliform or subulate, simple, or falsely branched, sometimes fasciculately so, procumbent or erect, cæspitosely aggregated into a sort of thallus; sheaths close and homogeneous or ample, and more or less distinctly lamellate, the apex for the most part open, sometimes laciniate.

M. fertile, Wood, (sp. nov.)

M. cæspitosum, cum algis alteris intermixtum; trichomatibus simplicibus, elongatis, flexuoso-curvatis, apice truncatis; trichomatibus internis viridibus, sæpe interruptis, interdum distincte articulatis interdum inarticulatis; articulis diametro 3–5 plo longioribus; vaginis modice arctis, firmis, achrois, crassis, coloris expertibus, apice truncatis et apertis; sporis cylindricis, sparsis, in filamento unico sæpe pluribus, in cellulis inclusis; cellulis perdurantibus globosis, interdum compressis trichomatis diametro fere æqualibus.

Diam.—Filam. $\frac{1}{3000}''$ = .00033″; spor. $\frac{1}{6000}''$ = .000166″.

Hab.—In stagnis. Alleghany Mountains, Centre County, Pennsylvania.

Cæspitose, intermixed with other algæ; filaments simple, elongate, flexuously curved, truncate at the apex; internal filament green, often interrupted, sometimes articulated, sometimes not articulate; joints 2–3 times longer than their diameter; sheath moderately close, thick, firm, transparent, and colorless, truncate and open at the apex; spores cylindrical, scattered, each contained in a cell, frequently several in a filament; heterocysts globose, sometimes compressed, about equal in diameter to the filament.

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Remarks.—I found this plant in a stagnant pool in "Bear Meadows," forming a filamentous, felty mass with *Œdogonium echinatum* and other algæ. The variously curved and interlaced flexible filaments are always simple and of uniform, or nearly uniform, diameter through their whole length; excepting that in some instances there are small, local, bulbous enlargements of the sheath. Though the ends of the filaments in all the specimens I have seen are abruptly truncate, it is very possible that in the young trichoma the apex is prolonged into a long hair as in most of the *Mastigonema.* The inner filament is sometimes very distinctly articulated, often, however, it is not at all so. The sheaths are firm, not at all lamellate, and generally project beyond the inner trichoma. The spores are cylindrical, yellowish, with a pretty distinct, although very close coat. They are always inclosed in distinct cells, and are mostly several in a filament, placed at intervals in its length.

This is the first instance, at least that I know of, in which a species of this genus has been found in fruit, and it is interesting to note the resemblance of the spores to those of the more commonly fruiting rivularias. At the same time the peculiar arrangement of the spores is remarkable, and if the other species of *Mastigonema* should be found to have the more common exclusively basal arrangement of spores, I think it would afford good ground for considering *M. fertile* as the type of a new genus. Moreover, the filaments are not united into a distinct thallus, and also want the apical hair of *Mastigothrix*, so that it is very probable that they represent an undescribed genus. Until, however, the fructification of the European species is elucidated, it seems best to forbear multiplying names.

Fig. 1, pl. 4, represents a single filament of this species.

M. halos, Wood, (sp. nov.)

M. cæspitulis; trichomatibus simplicibus, in ætate matura valde elongatis et cum vaginis truncatis et apertis,—in ætate immatura modice brevibus et in setam modice longam achroam productis; trichomatibus internis breve articulatis, subtiliter granulatis continuis vel varie interruptis; vaginis firmis, modice crassis, sæpe distincte lamellosis, coloris expertibus; cellulis perdurantibus subglobosis.

Diam.—Sine vag. = .0003″; cum vag. = .0005″.

Hab.—In æstuario, Stonington, Conn. (Dr. F. Lewis.)

In little tufts; filaments simple, in mature state greatly elongate, and with the sheath truncate and open,—in the young condition shorter and often ending in a rather short seta; internal filament shortly articulate, minutely granular, continuous or variously interrupted; sheath firm, rather thick, often distinctly lamellated, colorless; heterocysts subglobose.

Remarks.—This species is an inhabitant of salt, or at least brackish water, having been collected in Stonington Inlet by Dr. Frank Lewis. The filaments are very long and always simple; forming apparent exceptions to this, I have seen once or twice a number of young filaments so united as to give the appearance of having been produced from one old one, and in other cases young filaments growing from the side of an old one; but I believe those are always set free so soon as they attain a certain size. In one instance there were large, globular cells, with very thick walls, produced, and lying free, in the sheath. Are these spores? They are well shown in figure 2 *b*, pl. 5. Associated with them were a number of similar cells which had not obtained as yet the outer thick wall. The color of the filaments is in my specimens of a rich golden brown; but, as they have been preserved in carbolic acid water, I cannot speak positively as to the original tint. The heterocysts are subglobose, sometimes compressed, sometimes somewhat triangular. They about equal in diameter the internal filament.

Fig. 2, pl. 5, represents a small cluster of youngish filaments of this species.

M. sejunctum, Wood, (sp. nov.)

M. thallo cæspitulo, molle, parasitico; trichomatibus simplicibus, plerumque inarticulatis, sed, interdum breve, interdum longe, articulatis, continuis, rarius interruptis, apice attenuatis, flavo-olivaceis aut viridibus, sparse granulatis; vaginis plerumque amplis et distinctis, hyalinis, sæpius valde undulatis, apice plerumque valde amplificatis et in fibrillas solutis; cellulis perdurantibus diametro subæqualibus; sporis nullis.

Diam.—Trichom $\frac{1}{6000}'' = .00016''$; cum vag. $\frac{1}{2000}'' = .0005''$.

Hab.—In plantarum aquaticarum foliis, Carp River, Michigan.

Thallus somewhat cæspitose, soft, parasitic; filaments simple, mostly inarticulate, but sometimes shortly sometimes long articulate, continuous or more rarely interrupted, attenuate at the apex, yellowish-olive or greenish, sparsely granulate; sheaths mostly ample and distinct hyaline, often strongly undulate, the apex mostly much amplified and dissolved into fibrillæ; heterocysts about equal to the filament in diameter; spores wanting.

Remarks.—This species was found in the Carp River bog, growing on the edges of minute leaves, so as to form little prominences or thickenings of the margin. The trichomata are quite distinct from one another, and can scarcely be said to be united into a frond, although they all appear to radiate from the base, where they are consolidated into a dense mass. The sheaths are generally quite distinct, much broader than the cytioplasm, and are not sensibly dilated below. In most specimens they are very distinctly alternately dilated and contracted, or in other words, undulated. This is especially the case when the sheaths are quite wide. Above, they are rapidly and widely dilated, are distinctly fibrillose, and appear to gradually melt away. The cytioplasm is rarely articulated, and, when it is so, the joints are scarcely longer than broad, and are most generally confined to the distal end of the filament. The species appears to be most nearly allied to *M. Bauerianum*, Grun., from which, however, it is quite distinct.

Fig. 2 *a*, pl. 4, represents this species magnified 250 diameters; fig. 2 *b*, a single filament magnified 800 diameters.

M. elongatum, Wood.

M. initio subglobosum, postea sæpe nonnihil fusinum, nigro-viride, lubricum, firme; trichomatibus ærugineis, valde elongatis, flagelliformibus, interdum inarticulatis sed sæpius breve articulatis, interdum ad genicula valde constrictis, apice interdum truncatis sed plerumque in pilum, longum, achroum, flexuosum, productis; vaginis achrois, arctis, sæpe apice truncatis; cellulis perdurantibus globosis vel subglobosis.

Diam —$\frac{2}{7500}'' = .00026.''$

Syn.—*M. elongatum*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 128.

Hab.—In aquario.

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Thallus at first subglobose, afterwards frequently fusiform, blackish-green, slippery, firm; filaments æruginous, very elongate, sometimes not articulated, but more generally shortly articulated, sometimes strongly contracted at the joints; apices sometimes truncate but generally produced into a long, flexuous, translucent hair; sheath transparent, close, frequently truncate at the apex; heterocysts globose or subglobose.

Remarks.—This species grew in my aquarium on some brook-moss, which I obtained from a spring above Manayunk. It forms little nodules of the size of a pin's head upon the wire-like stems, or sometimes longer fusiform masses, which are apparently produced by the coalescence of a number of the little globes. The color of these fronds, which are very firm, is a blackish-green. The filaments radiate from the base in all directions, and at the apex are tipped with a very long hair-like flexuous point, or they are truncate, apparently from the breaking off of this terminal seta. The endochrome is not unfrequently interrupted within the sheath. When it is articulated, the joints are usually about as long as broad, and frequently are distinctly separated from one another. The sheath is sometimes quite apparent and distinctly truncate and open above, but in other instances is with difficulty perceived anywhere, and above is lost in the long hyaline point. At the points of attachment of the frond the filaments are so densely crowded as almost to appear to be coalescent, though I believe they are never really so; yet it is often almost impossible to separate them one from another by pressure on the glass cover, without entirely mashing and distorting the filaments.

Fig. 1 *a*, pl. 5, represents a section of a frond of this species slightly magnified; fig. 1 *b*, a single filament magnified 460 diameters.

Genus MASTIGOTHRIX, KTZ.

Trichomata singula, plerumque sparsa, parasitica intra thallum Chætophorarum aliarumque algarum, flagelliformia, in apicem piliformem achroum hyalinum cuspidata, distincte articulata, arcte vaginata, basi cellula perdurante instructa. (R.)

Filament single, mostly scattered, parasitic within the thallus of Chætophora or other algæ, flagelliform, with the apex produced in a hyaline hair, distinctly vaginate, furnished with a basal heterocyst.

Remarks.—I have simply copied the generic description of Prof. Rabenhorst, although it seems to me more than doubtful whether the place of growth is any generic character whatever. I have relied more on the long hyaline apical hair, although our American form does grow in a gelatinous palmella like jelly.

M. fibrosa, WOOD.

M. dilute vel cæruleo-viridis, vel olivaceo-viridis, vel sub-æruginea, infra haud articulata, sursum sæpe breve articulata, apice in trichomata matura in setam hyalinam, distincte articulatam, longam, producta; vaginis achroois—in filamento immaturo, supra distinctis, latis, hyalinis, infra modice crassis, arctis,—in trichomata matura infra arctis, indistinctis, supra in fibrillis dissolutis, apice absentibus; cellulis perdurantibus globosis, interdum geminis.

Diam.—Trichone $\frac{1}{2250}''$; cell. perdur. $\frac{7}{18000}''$—$\frac{4}{18000}''$.

Syn.—*M. fibrosa,* WOOD, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 129.

Hab.—Prope Philadelphia.

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Light bluish-green, or olivaceous-green, apex in the mature filament prolonged into a long, distinctly articulated hyaline seta; sheath transparent—in the immature filament distally, broad, and distinct although hyaline, below rather thick and close; in the mature filament below close, indistinct, above dissolved in fibrillæ and wanting at the apex; heterocysts globose, sometimes geminate.

Remarks.—This plant was found growing with other low algæ in a thick jelly, which clothed some wet, dripping rocks near Manayunk. In the young filaments the sheath is produced above into a broad, thick, gelatinous-looking portion, the cavity of which is often scarcely apparent. The cytioplasm in such filaments is mostly of a light bluish-green color, is granular and not very apparent. In older filaments, the trichoma above is prolonged into a long, curved hyaline point, and the sheath just below the base of this is split into a number of fibrillæ. No spores were perceived. The increase of the species appears to take place in the following manner: Near the middle of the filament a tumid swelling forms, in the centre of which appears after awhile a constriction, and this increases until at last there are shaped out the bases of two filaments. Then the heterocysts appear, and finally the two halves of the original trichoma separate—each a perfect filament. (Fig. 3, pl. 5.) Sometimes, instead of a pair of filaments being thus formed, but a single base is shaped out at the place of swelling, and the original filaments split, as it were, thus giving origin to a second trichoma, which for awhile appears as a branch of the former, but is soon detached from it. In some specimens there are two heterocysts, unless the proximal of these, which is a light orange-clay color, represents a spore.

Fig. 3, pl. 5, represents different forms of this species.

Family SCYTONEMACEÆ.

Trichomata articulata, sæpe moniliformia vel submoniliformia, vaginata, pseudoramosa, cellulis limataneis, ad pseudoramulorum basin, vel interstitialibus, plerumque pachydermaticis instructa. Vaginæ e stratis pluribus (etsi non semper distinctis) formitæ, superficie læves, corrugatæ vel exasperatæ, crustatæ, nonumquam stratis exterioribus in fibrillas discedentibus, haud raro passim intumescentes vel ocreatæ.

Vegetatio non terminalis; cellularum vegetativarum divisio ad unam directionem, initio in trichomatibus medio, postea in utroque fine sæpe alternans. Cellulæ limitaneæ ad utrumque polum locello lucido instructæ.

Propagatio gonidiis plerumque exultima generatione ortis. Gonidia plerumque numerosa seriata e vagina se exserunt tumque in singula secedunt. (R.)

Filaments equal, articulate, often moniliform or submoniliform, vaginate, pseudoramose, furnished with heterocysts which are either interstitial or at the base of the branches, and are mostly thick-walled. Sheaths formed of numerous strata (not always distinct), their surface smooth, corrugate, or roughened, the exterior stratum sometimes breaking up into fibrillæ, not rarely intumescent or ochreate.

Vegetation not terminal; division of the cells occurring in one direction, in the beginning in the middle of the trichoma, afterwards often alternately at each end. Heterocysts furnished with a translucent spot at each end.

Propagation mostly by gonidia arising from the last generation Gonidia mostly numerously seriate, passing out of the sheaths and then separating one from the other.

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Remarks.—The *Scytonemaceæ* are simple or branched filamentous plants, which grow in water, or in the air, upon tree-trunks, rocks, fences, &c., in moist localities. A number of individuals of one or more species are almost always associated to form on the ground little mats, or in the water attached or floating masses of varying color and characters according to the species. The individual filaments are composed of two distinct parts, the inner the protoplasmic matter, the outer the cellulose sheath. The former of these is a long cylindrical mass, which is occasionally interrupted by a distinct thick-walled cell, spoken of in this memoir as the *heterocyst*, or "*cellulis perdurantibus.*" The inner filament is composed of colored protoplasm, which is sometimes homogeneous, but in other cases is distinctly granular. It is most generally articulated after the manner of an *oscillatoria*, but occasionally it is continuous for a great portion of its length, and in one species, which is here described, although very possibly not belonging in the family, there are, at regular intervals, partitions running across from one side to the other of the sheath, so that the inner filament may be said to be made up of a number of cells.

The heterocysts are of various shapes, globular, compressed, cylindrical, oblong, &c. &c. They are mostly provided with a bright colorless spot at each end. Their number varies according to the species. Sometimes they are single, in other cases there are several of them arranged in series. They are placed either at the origin of the branches, or are scattered apparently without definite arrangement in the length of the filament. In the one case, they are known as "*basal*," in the other as "*interstitial.*" In any species, either of these methods, or both of them, may prevail; but a certain amount of specific value attaches to the situation of the heterocysts. Their function is totally unknown, although some have imagined them to have a sexual significance and even to be spermatozoids, but there is no proof whatever of the truth of such suppositions, and it is, I think, very certain that these heterocysts are not of the nature of spores.

The sheath of the *Scytonema* is composed of one or more strata, which are often very distinct from one another, but are more often, perhaps, not so. It is opaque or translucent, aud has its outer surface smooth, or tubercular, fibrillate or roughened in some way.

The specific characters in this family can best be commented upon under two heads—namely, those which are discoverable with the unaided eye, and those which the microscope alone can reveal. The points to be observed under the first of these are as follows: The place of growth of the plant, whether in the air or in the water, and, if it live in the air, to what it is attached—whether to stones, dead wood, or living trees, and it is possible that in some cases it may be found that certain species of *Scytonema* inhabit only certain species of trees. If the plant be in water, it must be noted whether it be attached or floating. Then the habit of growth must be looked at, including in this the size and thickness of the masses of filaments, whether they be flocculent, turfy, crustaceous, membranous-gelatinous, &c., their softness or rigidity, their color, as well as the arrangement in them of the filaments. To discover the latter, it will generally be necessary to use a low power of the microscope, and at the same time the mode and profusion of branching of the individual plant should be studied.

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The second class of characters are those discoverable only with the higher powers. They are divisible into two sets; those afforded by the inner filament and those derived from the sheaths. In the first of these the points to be noted are, the diameter of the filament; its color, whether it be or be not articulated, and if it be the length of the joints; whether it is uniform or moniliform; whether it be homogeneous or granulate; then the heterocysts should be examined as to their size, position, arrangement, shape, number, and color.

The diameter of the sheath, its homogeneousness, its color, firmness, and the condition of its outer surface are to be included in the specific study.

Genus SCYTONEMA.

Trichomata cæspitoso-congregata vel fasciculata, plus minus pseudoramosa, cellulis interstitialibus instructa; vaginæ gelatinoso-membranaceæ, e stratis (interdum obsoletis) pluribus cylindraceis compositæ; cellulis perdurantibus singulis.

Filaments cæspitosely-congregate or fasciculate, more or less pseudoramose; furnished with interstitial cells; sheaths gelatinous-membranaceous, composed of many cylindrical, sometimes obsolete, strata; heterocysts single.

a. Terrestres vel aquaticæ.

a. Terrestrial or aquatic.

S. simplice, WOOD, (sp. nov.)

S. in strato modice crasso, subtomentoso, nigro-viride; trichomatibus valde elongatis, flexuoso-curvatis, parcissime pseudoramosis vel sæpe sine pseudoramulis; pseudoramulis geminis vel singulis, plerumque elongatis; trichomatibus internis modo distincte articulatis, modo inarticulatis, apice interdum brevissime articulatis, granulosis, pallide viridibus, ad genicula sæpe nodosis vel disjunctis, articulis plerumque diametro æqualibus ad 7 plo longioribus; vaginis plerumque supra truncatis et apertis, pellucidis, sæpe coloris expertibus, interdum dilute aureo-brunneis; cellulis perdurantibus cylindricis, interjectis, diametro 2–5 plo longioribus.

Diam.—Trich. cum vag. $\frac{1}{2500}''$—$\frac{1}{1500}''$ = .0004″—.00066″; sine vag. $\frac{1}{7500}''$—$\frac{2}{7500}''$ = .00013″ —.00026″.

Hab.—In lignis irroratis, South Carolina. (Ravenel.)

S. in a moderately thick, somewhat tomentose, blackish-green stratum; trichomata very elongate, flexuously curved, very sparsely branched or frequently without branches; branches geminate or single, mostly elongate; internal filament partly distinctly articulate, partly inarticulate, granular, pale-greenish, in its apex sometimes very shortly articulate, sparsely granular, often nodose or disjoined at the joints; articles mostly from equal to to 7 times longer than the diameter; sheaths thick, transparent, often colorless, sometimes pale yellowish-brown, mostly open and truncate at apex; heterocysts cylindrical, interspersed, 2–5 times longer than their diameter.

Remarks.—I am indebted to Professor Ravenel for specimens of this species. They are preserved in solution of acetate of alumina and accompanied by the following label: "Adhering to the wet sides of a wooden gutter, leading water from a spring, September 29, 1869: Aiken, South Carolina." The filaments are remarkable for the fewness of their branches. Generally, indeed, there are no

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branches whatever, and I have never seen more than a single pair, or, at most, three branches to a filament. The mass of filaments is blackish-green, somewhat tomentose and quite shiny in appearance. The articles are often very long, and the internal filament is frequently in such cases enlarged into a sort of globular node at the joint. Not at all rarely there is a very decided break in the endochrome at the joints.

This species is very close to *S. Austinii*, from which, however, I think it sufficiently distinct. The points of difference are in the much firmer, much more colored and opaque, and rougher sheath of that species; in the swollen ends of the internal filament of *S. Austinii*, and its shorter articles, with the absence of nodes or distinct interruption of the endochrome at the joints. The heterocysts are also quite different in the two forms, whilst the filaments of *S. simplice* are much the longer.

S. Austinii, Wood, (sp. nov.)

S. rupicola, strato tomentoso, cæspitoso, crasso, fusco-nigro; trichomatibus adscendentibus, curvatis, plerumque simplicibus; trichomatibus internis ærugineis vel fuscescentibus, articulatis vel inarticulatis, fine sæpe valde incrassatis; articulis diametro plerumque multo brevioribus, interdum longioribus; vaginis rubido- vel aureo-fuscescentibus, sæpe sub-opacis, firmis, indistincte lamellosis, in apice plerumque achrois et coloris fere expertibus, superficie subrugosa et hirta; cellulis pedrurantibus breviter cylindricis, vel subquadratis vel subglobosis, interdum valde compressis et diametro multo brevioribus.

Diam.—Fil. cum. vag. .0006″—.0008″; sine vag. .00016″—.0004″.

Hab.—In rupibus, "Little Falls, New Jersey." (Austin.)

S. growing on rocks, stratum tomentose, and somewhat turfy, brownish-black; trichomata ascending, mostly simple, curved; internal filament æruginous or fuscous, articulate or inarticulate, often very much thickened at the ends; articles much shorter to longer than their diameter; sheaths reddish or yellowish-fuscous, at the apex colorless and transparent, firm, indistinctly lamellate; surface rough; heterocysts shortly cylindrical, subquadrate or subglobose, sometimes strongly compressed and much shorter than broad.

Remarks.—This plant occurs as a blackish stratum of one or two lines in thickness, forming a sort of miniature turfy cushion upon the rock. When examined with the hand-glass, this layer is seen to be composed of a great number of ascending curved filaments whose color, in some specimens, is a reddish-brown; in others, apparently younger, yellowish-brown. Under the compound microscope the sheaths in the older filaments are seen to be much roughened externally and irregular in outline. The young sheaths are smooth. The filaments are mostly simple, since I have not seen more than a half dozen having even a single branch.

The heterocysts are scattered at irregular intervals, and are remarkably irregular in form—sometimes much shorter than broad, sometimes several times as long. As the ends of the filaments are approached the internal filament suddenly swells out and increases sometimes to twice the diameter it has in the central part of the filament. In the filament proper it rarely attains a diameter of more than .0003″, and is commonly about .00025″, whereas at the ends it very generally approaches the maximum .00042″.

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S. immersum, WOOD, (sp. nov.)

S. immersum cum algis alteris intermixtum et plantas aquaticas adhærens; trichomatibus elongatis; pseudoramulis plus minus distantibus, plerumque geminis, et e basi divergenter adscendentibus, brevibus aut elongatis; trichomatibus internis læte æruginеis, interdum distincte articulatis, interdum inarticulatis, apice obtuse rotundato, æruginео; articulis diametro subæqualibus vel brevioribus; vaginis amplis, hyalinis, coloris expertibus; cellulis perdurantibus distinctis, singulis, interjectis, subcylindricis, diametro interdum fere duplo brevioribus, interdum duplo longioribus.

Diam.—Sin. vag. $\frac{5}{12000}'' = .000415''$. Cum vag. $\frac{9}{12000}'' = .00075''$.

Hab.—In aquis quietis, Cumberland County, New Jersey.

S. immersed, intermixed with other algæ and adhering to aquatic plants; filaments elongate; branches mostly geminate, more or less distant, short or elongate; internal filaments bright æruginous, sometimes distinctly articulate, at others not so, apex obtusely rounded æruginous; joints about equal to the diameter or shorter; sheath ample hyaline, colorless; heterocysts distinct, single, interjected, subcylindrical, sometimes about half as long as broad, sometimes nearly twice as long.

Remarks.—I found this plant in September, 1869, in Shepherd's Mill Pond, near Greenwich, Cumberland County, New Jersey, forming, with other algæ, a flocculent, greenish-black, slimy coating to the stems and finely dissected leaves of *Ranunculus aquatilis.* The branches are very few in number in most specimens, and when they are more plentiful are apt to be short and abortive. Their apices do not differ materially from their other portions.

Fig. 9, pl. 2 *a*, represents a portion of a filament of this specimen magnified 750 diameters; fig. 2 *b* a whole filament magnified 260 diameters.

S. Nægelii, KTZ. (?)

S. cæsptoso-floccosum, bryophilum, nigro-viride; trichomatibus, plerumque sparse pseudoramosis, pseudoramulisque elongatis et intricatis; trichomatibus internis breviter articulatis, sæpe interruptis, sæpe nonnihil moniliformibus, viridibus aut in ætate provecta brunneis; articulis sæpe sejunctis, diametro plerumque brevioribus, subtiliter granulatis; pseudoramulis plerumque singulis; vaginis modice arctis, interdum subamplis, haud distincte lamellosis, modice crassis, hyalinis, coloris expertibus aut in ætate provecta dilute fusco-brunneis; cellulis perdurantibus nonnihil reniformibus, plerumque nullis, basilaribus.

Diam.—Fil. cum vag. plerumque $\frac{5}{7500}''$—max. $\frac{5}{6000}''$; sine vag. $\frac{5}{12000}''$, cell. perdurant. lab. $\frac{5}{12000}''$—long. $\frac{3}{12000}''$.

Syn.—*S. Nægelii* (KTZ.), RABENHORST, Flora Europ. Algarum, Sect. II. p. 252.

Hab.—In fonte, prope Belvidere, Centre County, Pennsylvania.

Growing in small, blackish-green woolly mats attached to mosses; filaments mostly sparsely branched, with the branches elongate and intricate; internal filament shortly articulate, often somewhat moniliform, often interrupted, green, or, in mature state, brownish; joints often disjoined, mostly shorter than the diameter, finely granulate; branches mostly single; sheaths moderately close, sometimes ample, not distinctly lamellate, rather thick, hyaline, colorless, or, in old age, light fuscous brown; heterocysts mostly wanting.

Remarks.—I found this plant in the large spring that supplies Bellefonte with water, growing attached to mosses, so as to form little dark-green mats around

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their stems and branches. These mats never exceeded an inch in length in any specimens that came under my notice. The filaments themselves are apparently not much branched and are densely interwoven. The sheaths are close, rather thick, not lamellate, of uniform diameter, except in that they are occasionally locally swollen, and are truncate and open at the end. The internal filaments are frequently much interrupted, and in the younger plants are of a deep green. The joints are in many instances much separated, and in most cases very distinct. The filaments indeed show a remarkable tendency to break up at the joints, so as to form a series of dish-like gonidia, so that the articles, or endochrome masses, may be generally described as strongly compressed spheres. In all the specimens that I have examined, I have seen but a single heterocyst. This was at the base of a branch, was somewhat reniform, and about three-fifths as long as broad. I have referred this species, doubtfully, to *S. Naegelii*, Ktz., the only account of which that I have met with, or know of, is a brief diagnosis in Rabenhorst's Flora, in which many of the essential characters are omitted.

Fig. 6, pl. 8, represents a portion of a filament of this species.

S. thermale, Ktz.

S. strato tenue, nigrescente; trichomatibus flexuoso-curvatis, intricatis, parce pseudoramosis, internis pallide ærugineis, sæpe coloris fere expertibus, passim interruptis, plerumque inarticulatis sed sæpe indistincte et interdum distincte articulatis, granulosis; articulis diametro brevioribus vel subæqualibus; pseudoramulis plerumque brevibus, geminis, in diametro trichomatibus æqualibus vel subæqualibus et interdum usque ad medium conjunctis, basi coalitis, sæpe e basi divergentibus; vaginis crassis, indistincte lamellosis, vel luteo-fuscis vel fuscis, sed passim fere coloris expertibus, plerumque vix pellucidulis, in ramulorum apice sæpe hyalinis et coloris fere expertibus; cellulis perdurantibus, subquadratis vel cylindricis, singulis, interjectis.

Diam.—Tr. cum vag. $\frac{5}{12000}''$—$\frac{7}{12000}''$ = .00042″—.00058; sine vag. $\frac{1}{6000}''$ = .000166″—$\frac{1}{4000}''$ = .00025.

Syn.—*S thermale*, Ktz., Rabenhorst, Flora Europ. Algarum, Sect II. p. 250.

Hab.—In terra argillacea, South Carolina. (Ravenel.)

Stratum thin, blackish; filaments flexuously curved, intricate, sparingly branched; internal filament pale-greenish, often almost colorless, here and there interrupted, mostly inarticulate, but often indistinctly and sometimes distinctly articulate, granular; joints shorter or about as long as broad; branches geminate, mostly short, equal or subequal to the filament in diameter, coalescent at the bases, rarely so even to their middle, mostly divergent from the base; sheath thick, indistinctly lamellate, yellowish-fuscous, and scarcely semitransparent, but here and there nearly colorless and pellucid, generally so in the apices of the branches; heterocysts subquadrate or cylindrical, single, interspersed.

Remarks.—I am indebted to Professor Ravenel for specimens of this species preserved in solution of acetate of alumina. The label reads, "Damp surface of hard clay, Sept. 25, 1869." The sheaths are quite thick and scarcely translucent, so that the color of the inner filament seen through them is that of themselves. Curiously enough, one of these dark sheaths will for a space lose its color and be very transparent, in such places and in the apices of the branches, the inner filament is often a decided pale-green; at other times it is almost colorless. The end of the sheaths are mostly closed, but I have seen them open, with the inner filament project-

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ing. The branches are nearly always short, and divergent from their united bases. The heterocysts have frequently one of their ends rounded; and are quite numerous. This species corresponds too closely to Rabenhorst's description of *Scytonema thermale* to be separated, but it is possible a comparison of specimens might show decided differences—the description of the European form is not very full. The American plants seem to approximate most closely the *Var. intextum.* I have seen a single branch given off only in one instance.

Fig. 1, pl. 6, represents a filament of this species magnified 260 diameters; fig. 1 *b*, the outline of a heterocyst magnified 750 diameters.

S. Myochrous, Ag.

S. strato tenui, pannoso-tomentoso, obscure fusco (nonnunquam subsericeo); trichomatibus validissimis, fuscis, lucidis, leniter curvatis, adscendentibus, internis ærugineis, apice (articul. term. 5–6) rubellis, distincte articulatis; pseudoramulis plerumque geminis, sæpe longissimis flaccido-erectis, trichomate dimidio circiter tenuoribus; trichomatis vaginis crassis, distincte lamellosis, firmis, pulchre luteo-fuscis, superficie lævissimis, ramulorum semper pallidioribus (luteis, rarius achrois), apice sæpe achrois, clausis et obtuso-rotundatis; cellulis perdurantibus oblongis vel subcylindricis, achrois, trichomatis interni diametro subæqualibus. (R.) Strato obscure olivaceo, trichomatibus parce pseudoramosis, ad $\frac{1}{85}'''$ crassis; pseudorannulis singulis, vaginis achrois vel luteolis; vag. trich. luteo-fuscis. (R.) Species mihi ignota.

Diam.—Trichom 0.0011″—0.0014″; ramulorum ad 0.00068″. (R.)

Syn.—*S. Myochrous*, Agardh; Var. *Contextum*, Carmichael. Rabenhorst, Flora Europ. Algarum, Sect. II. p. 254.

Hab.—" Foot of Crow's-nest, West Point." Bailey. Silliman's Journal, N. S. vol. iii.

Strato thin, pannosely tomentose, obscurely fuscous (sometimes somewhat silky); filaments very strong, fuscous, bright, slightly curved, ascending; the internal æruginous, distinctly articulate with the apex (terminal 5–6 joints) reddish; branches mostly geminate, often very long, flaccidly erect, about one-half thinner than the filament; sheath of the filament thick, distinctly lamellate, firm, beautifully yellowish-fuscous, surface very smooth; sheath of the branches always paler (luteous or rarely colorless) with the apex colorless, short and obtusely rounded; heterocysts about equal in diameter to the internal filament. Stratum obscurely olivaceous, filaments sparsely branched, about $\frac{1}{85}'''$ thick; branches single, with the sheaths transparent or yellowish; sheath of the trichoma luteo-fuscous.

S. calotrichoides. Kützing(?).

S. cæspitosum, mucosum, plerumque cum algis variis intermixtum; trichomatibus plus minus curvatis; pseudoramulis plerumque geminis, varie curvatis, simplicibus, elongatis; trichomatibus internis modo distincte articulatis, modo inarticulatis, interdum moniliformibus, luteo-viridibus vel æruginei, granulosis; articulis plerumque diametro brevioribus sed interdum permulto longioribus, haud rare vel subglobosis vel valde compressis; cellulis perdurantibus singulis, subcylindricis; vaginis plerumque pellucidulis, distincte lamellosis, in trichomatibus plerumque rubido-vel luteo brunneis sed interdum coloris expertibus, in pseudoramulis hyalinis, coloris expertibus vel dilutissime luteis vel dilute luteo-brunneis.

Diam.—Cum vag. max. $\frac{3}{4000}''$=.00075″; plerumque $\frac{2}{4000}''$ =.00045″; sine vag. $\frac{1}{7500}''$—$\frac{1}{4000}''$; pseudoram. $\frac{1}{2000}''$ = .0005″.

Syn.—*S. calotrichoides*, Ktz. Rabenhorst, Flora Europ. Algarum, Sect. II. p. 252.

Hab.—South Carolina. (Ravenel.)

Cæspitose, mucous, mostly intermixed with various algæ; filaments more or less curved; branches mostly in pairs, elongate, simple, variously curved; internal filament partly distinctly articulate, partly not articulate, sometimes moniliform, yellowish-green or æruginous, granular; joints mostly shorter than the diameter, sometimes much longer, sometimes subglobose or strongly compressed; heterocysts single, subcylindrical; sheaths distinctly lamellate, mostly reddish or yellowish-brown, but sometimes colorless, in branches hyaline, colorless, or with a very faint yellowish tint, or sometimes brownish.

Remarks.—The specimens, from which the above description was drawn up, were sent me by Professor Ravenel from South Carolina. The extremities of the sheaths are either closed, or open. The branches are almost always in pairs, and sometimes three or four are given off together, but this is not common. They are often nearly or quite colorless; the main filament is generally a sort of brown—sometimes quite bright from the predominance of the yellow hue. Although my specimens do not precisely agree with the descriptions of the European *S. calotrichoides*, yet the disagreement does not seem sufficient or sufficiently constant to separate specifically the two forms; the most important of the differences is in the coloration of the sheaths and heterocysts, which in the American plant are commonly, but not universally, respectively brownish and greenish.

The label, which Professor Ravenel has attached to some of the specimens, reads, "In wet, boggy places, on rotten pine boards, Sept. 25, 1869."

Fig. 2, pl. 6, represents a filament of this plant magnified 250 diameters.

S. cataracta, Wood.

S. rupicola, cæspitosum, fusco-atrum, longe et late expansum; trichomatibus flexuosis, flexilibus, fere 0.25″ longibus, vage pseudoramosissimis, superficie lævibus; pseudoramis elongatis, singulis, rarissime geminis, liberis, interdum fuscis, sæpius hyalinis, apice plerumque truncatis et rare nonnihil attenuatis et sæpe barbais sed haud rubellis; trichomatibus internis ærugineis, tenuissimis, plerumque distincte articulatis; articulis diametro plerumque brevioribus, sed interdum longioribus, sæpe sejunctis, sæpe subglobosis; vaginis crassis et firmis; cellulis perdurantibus et basilaribus et interjectis, singulis, rarissime geminis.

Diam.—Trich. cum vag. plerumque .00045″; max. .0011″; sine vag. max. .00013″.

Syn.—*S. cataracta,* Wood, Prodromus, Proc. Am. Phil. Soc., p. 129, 1869.

Hab.—In flumine Niagara prope cataractam.

S. forming on rocks an extended turf-like stratum of a brownish-black color; filaments flexuous, flexible, almost 0.25″ long, irregularly branched, their surface smooth; branches elongate, single, rarely in pairs, free, sometimes fuscous, frequently hyaline, their apices generally truncate, rarely somewhat attenuate, frequently provided with enlargements, never reddish; cytioplasm æruginous, very thin, generally distinctly articulate; articles mostly shorter than broad, but sometimes longer, frequently disjoined, often subglobose; sheaths thick and firm; heterocysts both basal and interjected, single, extremely rarely geminate.

Remarks.—This species grows abundantly in the Niagara River, on the rocks below the great cataract. It is really in little tufts, but these are in many cases placed so closely as to form a broad turf-like coating to the stones. Often, however, the tufts are in smaller patches, and are of sufficient length to wave with the eddies and currents in the water. The branches are almost always given off singly since I have examined some hundreds of specimens, and have only in one instance detected them in pairs. The apices of the branches, and indeed of the main filaments, are beautifully colorless and hyaline, and not unfrequently a branch will have this hyaline sheath for a long distance. The extreme ends are mostly truncate and open, and, often near them, the sheaths will have marked swellings; a condition which, for want of a better term, I have spoken of as being *barbate.* Sometimes near the end of the filament the diameter of the sheath will be suddenly lessened. The large cells are both interstitial and placed at the bases of the branches; they are more or less oblong or quadrangular, sometimes being scarcely longer than broad, but in other cases several times longer. At their position there is very generally a sort of globular enlargement of the filament. The sheath is sometimes very obscurely lamellate. The color of the older filaments is a dark, almost chocolate-brown. This is apparently the species referred to by Professor Bailey as being *Scytonema ocellatum* of Harvey, in Silliman's Journal, vol. iii. N. S., although that plant, according to Professor Rabenhorst, belongs to the genus *Sirosiphon.*

Fig. 1 *a*, pl. 7, represents a portion of a filament, magnified 280 diameters; fig. 1 *b*, a whole filament slightly magnified.

S. dubium, WOOD (sp. nov.)

S. immersum, in floccis mucoso-tomentosis olivaceo-nigris plantas aquaticas adhærens, vel in strato mucoso et nonnihil tomentoso dispositum; trichomatibus valde elongatis et arcte intricatis, varie curvatis, plerumque sparse pseudoramosis; pseudoramulis plerumque singulis, et plus minus distantibus et modice brevibus, vel interdum brevissimis et abortivis et nonnihil confertis; trichomatibus internis sæpe in pseudocellulis distinctis contentis, interdum continuis et indistincte articulatis vel inarticulatis, plerumque dilute cæruleo-viridibus sed interdum læte ærugineis, subtiliter granulatis; vaginis arctis plerumque modice crassis et firmis, hyalinis, coloris expertibus; cellulis perdurantibus cylindricis, diametro 2–6 plo longioribus.

Diam.—Cum vag. $\frac{3}{12000}''$—$\frac{5}{12000}''$ = .00025″—.0004″.

Hab.—In aquis quietis, Cumberland County, New Jersey.

Immersed, adhering to water plants in olive-black tomentose flocculent masses, or arranged in a mucous and somewhat tomentose stratum; trichomata very long and closely interwoven, variously curved, mostly sparsely branched; branches generally single, more or less distant, and moderately short, sometimes very short, abortive, and somewhat crowded; internal filament often contained in distinct cell-like apartments, sometimes continuous and indistinctly articulate, or not at all articulate, finely granulate, mostly a pale bluish-green, sometimes a bright æruginous color; sheath close, mostly rather thick and firm, hyaline colorless; heterocysts cylindrical, 2–6 times longer than broad.

Remarks.—I found this plant, September, 1869, in Shepherd's Mill Pond, near Greenwich, Cumberland County, New Jersey. It formed dark, ugly, somewhat slimy, tomentose flocculi adhering to, and binding together, the finely-dissected leaves of *Ranunculus aquatilis.* The filaments are very long, slender, and sparsely branched. The branches are given off at right angles, or nearly so, but are frequently sharply bent just above their origin. They are often, but not always, rather short. The most remarkable character that the plant possesses is that in many filaments there are very distinct regular partitions stretching across from

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side to side, so that the interior is divided, as it were, into successive cell-like chambers, in which the colored protoplasm is contained. This character seems almost to separate the plant from the genus *Scytonema*, but I have deemed it insufficient grounds for indicating a new genus. Since writing the preceding remarks, I have received specimens of this species from Professor Ravenel, who collected them in South Carolina, near the town of Aiken. They agree in all respects, except that they form a dark, mucous, somewhat tomentose coating to pieces of wood.

Fig. 3 *a* represents the outline of a series of the cells alluded to, magnified 750 diameters, and figs. 3 *b* and 3 *c*, portions of filaments magnified 460 diameters.

b. Arboricolæ.

b. Growing on trees.

S. cortex, Wood.

S. minutissimum, stratum tenue submembranaceum formante; trichomatibus sparse pseudoramulosis, pseudoramulisque repentibus et plus minus concretis, viridibus aut dilute fuscis, varie curvatis, haud rigidis; cytioplasmate viride, articulato, rare distincte granuloso; articulis diametro longioribus aut brevioribus; vaginis arctis, nonnihil tenuibus, plerumque coloris expertibus, sed interdum dilute fuscis; cellulis perdurantibus et singulis et geminis, et basalibus et interjectis, globosis vel subglobosis.

Diam.—Trich. cum vag. $\frac{2}{7500}''$—$\frac{3}{7500}''$.

Syn.—*Scytonema cortex,* Wood, Prodromus, Proc. Am. Philos. Soc., 1869, p. 130.

Hab.—South Carolina.

S. very minute, forming a thin, submembranaceous stratum; filaments sparsely branched, together with the branches, creeping and more or less concreted together by their sides, green or light brown, variously curved, not rigid; cytioplasm (internal filament) articulate, rarely distinctly granulate; joints longer or shorter than broad; sheaths close, rather thin, transparent, generally colorless but sometimes light brown; heterocysts globular or subglobular, single or in pairs, basal or otherwise.

Remarks.—I have specimens of this species collected in South Carolina by Professor Ravenel, who found it growing on the bark of *Platanus occidentalis.* The thin, almost membranous stratum which it forms, is of a dark olive-black, and has to the eye a sort of minutely warty appearance. The filaments are so involved and so adherent, one to the other, that I have not been able to separate any length of them, nor are the branches distinguishable from the main filaments. The sheaths are rather thin, and often not very apparent.

Fig. 4, pl. 6, represents this species.

S. Ravenelii, Wood.

S. lignicola, breve cæspitosum, viride-nigrum; trichomatibus plerumque repentibus, vel fusco-olivaceis vel aureo-fuscis, modice pseudoramosis; ramis ascendentibus, rigidis, flexuosis rare pseudoramulosis, vel fusco-olivaceis vel aureo-fuscis, rarissime cum apicibus subachrois; trichomatibus internis coloris expertibus, granulosis, sæpe vagina erumpentibus, plerumque articulatis; articulis diametro longioribus aut brevioribus; vaginis arctis, crassibus, fusco-olivaceis vel aureo-fuscis, plerumque supra truncatis et apertis, superficie nonnunquam irregularibus; cellulis perdurantibus subquadratis vel subglobosis singulis aut rare geminis, interjectis; in stato juvene trichomatibus internis ærugineis, vaginis tenuibus.

Diam.—Trich. cum vag. $\frac{9}{7500}''$—$\frac{6}{7500}''$; ram cum vag. $\frac{4}{7500}''$—$\frac{6}{7500}''$; trich. sine vag. $\frac{1}{2000}''$—2.0005.″

Syn.—*S. Ravenelii*, WOOD, Prodromus, Proc. Am. Philos. Soc., 1869, p. 130.

Hab.—In cortice, South Carolina.

S. Forming little turfy spots of a greenish color, on bark; filaments mostly creeping, either brownish-olive or yellowish-brown, moderately branched; branches ascending, rigid, flexuous, very rarely provided with secondary branchlets, either brownish-olive or yellowish-brown, rarely subtransparent at the apex; cytioplasm colorless, granular, often extending out beyond the sheaths, generally articulate; joints longer or shorter than broad; sheaths close, thick, brownish-olive or yellowish-brown, for the most part truncate at their ends and open, their surface sometimes irregular; heterocysts subquadrate, single, interstitial.

Remarks.—I am indebted to Prof. H. W. Ravenel for specimens of this very distinct species. Some of these are labelled as having grown on the twigs of a celtis in South Carolina, other specimens are on the bark of a willow. The branches, which are mostly shortish, simple, and variously curved, are sent up in great numbers by the creeping stems, and, like the stems themselves, are mostly free, but not unfrequently are closely adherent by their edges.

The internal trichoma or cytioplasm, owing to the great thickness of the sheaths, is not very apparent within these latter, but not unfrequently projects for a distance beyond them, when it is seen to be colorless, very granular, and mostly, but not always, distinctly articulated. In the young plant the filaments are bright-green, often not more than $\frac{1}{3500}$ of an inch in thickness, and have the sheath very thin, or may be almost imperceptible. It affords me great pleasure to dedicate this species to Professor Ravenel, not as an acknowledgment merely of his aid in my studies of this hitherto neglected branch of the North American Flora, but rather of the great services he has rendered science in some of its kindred branches.

Fig. 4, pl. 5, represents the end of a filament of this species magnified some 450 diameters.

Genus TOLYPOTHRIX, KTZ.

Trichoma scytonemacea cum cellulis perdurantibus seriatis.

Filament similar to that of scytonema, but with the heterocysts seriate.

T. distorta, (MÜLLER) KÜTZ.

T. cæspitoso-floccosa, læte et pulchre viridis; trichomatibus intertextis, læte viridibus, modo distincte articulatis modo inarticulatis; articulis diametro brevioribus sæpe aut sub-nullis aut nullis; pseudoramulis singulis; vaginis arctis, homogeneis, vitreis; cellulis perdurantibus basilaribus et interdum interjectis, pachydermaticis, plerumque in parallelogrammæ enormis forma, plerumque 4-seriatis, subachrois, interdum sparsissime granulatis.

Diam.—$\frac{1}{1800}''$—$\frac{1}{3000}''$.

Syn.—*T. distorta*, (MÜLLER) KTZ. RABENHORST, Flora Europ., Algarum, Sect. II. p. 275.

Hab.—In aquario, Philadelphia, Wood. Rhode Island (Olney) Thwaites. Warden's Pond, Rhode Island; Reservoir Pond, West Point; Fourth Lake, Madison, Wisconsin, Bailey.

Flocculent cæspitose, bright, beautiful green; filaments interwoven, bright green, partly distinctly articulate, partly continuous; articles shorter than long, often very indistinct, sometimes absent; branches single; sheaths close, homogeneous, glassy; heterocysts basilar, sometimes interspersed, thick-walled, mostly irregularly parallelogrammatic, mostly 4-seriate, semitransparent, sometimes very sparsely granulate.

Remarks.—This species grew spontaneously in the aquarium of my friend Dr. Frické, to whom I am indebted for specimens of it, forming little, bright-green balls adherent to the various aquatic plants. It approaches so very closely the European *T. distorta*, that I have considered it as a mere variety of it, although it differs in having the heterocysts mostly arranged in fours, and also apparently in their shape—they being in our plant mostly parallelogrammatic.

Fig. 1 *a*, pl. 8, represents a section of heterocysts magnified 800 diameters; fig. 1 *b*, a portion of filament magnified 800 diameters.

Genus PETALONEMA, BERK. (1833.)

Scytonematis trichomata vaginis crassissimis e stratis numerossissimis brevioribus, infundibuliformi dilatatis, imbricatis et plerumque dilutissime coloratis compositis. (R.)

Syn.—Arthrosiphon, KTZ. (1845.)

"Filaments stratified, decumbent, free, simple, or branched. Tube or sheath very wide, flattened, longitudinally and transversely striate and crenulate at the edge; endochrome olivaceous annulated, here and there interrupted by a heterocyst. Branches issuing in pairs, formed by the division and protrusion of the endochrome of the original filament.

"When placed under the microscope the filaments present the appearance of a cylindrical central column, containing annulated, olive-colored endochrome, and a wide wing-like border at each side of the column. This border or sheath is obliquely striate, the striæ running in an arch from the margin toward the centre, where they become parallel, and are then continued longitudinally downward along the medullary column, till lost in the density. The margin of the wing is closely crenulate and in age transversely striate at the crenatures as though jointed. Such is the apparent structure; the real structure seems to be, that an annulated central filament is inclosed within a number of compressed, trumpet-mouthed gelatino-membranaceous tubular sheaths, one arising within the other, and successively developed as the growth proceeds. These sheaths, thus concentrically arranged, are indicated by arching longitudinal striæ; and the mouths of the younger sheaths, projecting slightly beyond those of the older, form the crenatures of the margin." HARVEY.

P. alatum, BERK.

A. pulvinato-crustaceus, rupicola, varie coloratus; trichomatibus internis ærugineis, curvatis, parce pseudoramosis, modo continuis, modo torulosis, submoniliformibus, apice plerumque paulum incrassatis, sæpe roseolis, rotundatis; articulis distinctis, granulosis, diametro subæqualibus vel paulo brevioribus; vaginis stratis internis, aureis vel aureo-fuscentibus, externis achrois, vitreis; cellulis perdurantibus interjectis et ad pseudoramulorum basin, plerumque solitariis, subglobosis vel oblongis, dilute fuscis. (R.) *Species mihi ignota.*

Diam.—Trich. intern. 0.00016″—0.00032″; vag. 0.00377″. (R.)

Syn.—Arthrosiphon alatus, (GREV.) RABENH. Flora Europ. Algarum, Sect. II. p. 265.
Petalonema alatum, BERKELEY. HARVEY, Nereis Boreis Americana, part iii. p. 99, Smithsonian Contributions, 1846.

Hab.—"On dripping rocks under Biddle Stairs, Niagara Falls." (Harvey.)

"This forms strata of a dark chestnut-brown color and of indefinite extent on the surface of rocks or soil exposed to the constant drip of water. The filaments are decumbent, lying without order in the gelatinous matrix in which they are developed, and which forms the groundwork of the stratum. They appear to be unattached to the soil, and each filament may be about half an inch in length; but they are commonly found broken off at the inferior end, or the lower part decays whilst the upper continues to grow. They are slightly curved, in serpent-like fashion, never quite straight; at first they are simple, but now and then emit lateral branches, which issue at considerable angles and generally in pairs. When a filament is about to branch, a rupture takes place in the side of the sheath, and the endochrome issues in two portions, one connected with the upper, the other with the lower half of the filament; these form the nuclei or medullary portion of two new branches and become duly invested with a membranous sheath, and gradually put on the aspect of the adult filament. The endochrome is granular, dark-brown, and annulated at short intervals, the transverse rings being placed very close together in the youngest portions, and less closely in the older, where they are distant from each other about twice the diameter of the column. This annulated endochrome is interrupted at certain fixed places, where an ellipsoidal cell is formed, separating the endochrome of the lower from that of the upper portions." HARVEY.

Remarks.—I have never seen either the genus or species, and therefore am forced to copy the descriptions of both from Rabenhorst and Harvey.

FAMILY SIROSIPHONACEÆ.

Thallus ramosus, e cellulis pachydermaticis aut uni vel pluri seratis et in vagina ampla inclusis formatus, interdum cellulis perdurantibus instructus. Ramificatio vera fit cellularum vegetativarum quarundam divisione in axis longitudinalis directionem, qua ex re cellulæ duæ sororiæ gignuntur; cellula inferior in trichomatis continuitate permanet, superior divisione continua repetita in eandem directionem se ad ramum explicat.

Propagatio adhuc ignota.

Frond branched, formed of thick-walled cells in an ample sheath, sometimes furnished with heterocysts. Cells uni- or multi-seriate. Branches formed by a longitudinal division of certain cells, so as to form two sister cells; the inferior of which remains as part of the trichoma, whilst the other, by repeated divisions, grows into a branch.

Propagation not known.

Remarks.—The *Sirosiphonaceæ* are the most complex in their organization of all the *Phycochromophyceæ*, in so far as the protoplasm within the sheaths is everywhere broken up into a number of distinct cells, each of which is provided with a thick coat or wall as well as in the circumstance of the frond having more perfect branching. The so-called pseudo-branches in the other families are more truly comparable to distinct fronds or thalli remaining attached to the parent thallus than to distinct branches, whilst among the sirosiphons the branches really belong to the original thallus. The heterocysts are much more frequently absent than present, only one of the known American species being furnished with them. The sheaths are generally not so distinctly sheaths as among the oscillatoria, &c., for, instead of being distinct tubes, they appear rather in most cases as masses of firm jelly, the outer portion of which is hardened almost into a periderm, and in the inner part of which the cells are imbedded. Their color varies from the transparent colorlessness of glass to a dark opaque-brown. Their surface is perhaps most frequently smooth, but at times is tuberculate or otherwise roughened. I have never seen anything like spores about them.

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These plants grow in the majority of cases in the air, in such situation as on the face of dripping rocks, on the trunks and branches of trees, on moist ground, &c.; but some of the species are found in the water, either attached or floating. They generally form little mats of indefinite extent, but occasionally the filaments are united more closely into an almost membranaceous stratum.

The species are, I think, in most instances readily distinguished, the characters being partly discoverable with the unaided eye and partly microscopic. The points to be attended to in the first category are the size, color, form, and consistency of the mats of fronds, and the place of growth. In the second are included the general shape of the frond and its size and method of branching; the general shape, color, and size of the cells, the thickness of their walls and the method of their arrangement, both in the main thallus and the branches, also the form, &c., of the end cells of the branches; the heterocysts, their absence, or, if present, their frequency, size, shape, color, and position; the sheaths, their color and firmness, and the character of their surface.

Genus SIROSIPHON, Ktz.

Trichomata torulosa, vaginata, plerumque ramossissima et aureo- vel olivaceo-fusca, e cellulis pachydermaticis 1–2–3 vel pluri-seriatis formata et cellulis interstitialibus (sæpe nullis) subglobosis vel oblongis coloratis instructa. Vagina plerumque crassissima, firma, pulchre aureo-fusca, lutea vel olivacea, in apicem obtusum plus minus attenuata.

Filament torulose, sheathed, mostly very much branched, yellowish, or olivaceous-fuscous, formed of thick-walled 1–2–3 or many seriate cells and furnished with interstitial cells (often wanting) which are globose or oblong and colored. Sheaths mostly very thick, firm, beautiful golden fuscous, clay-colored or olivaceous, more or less attenuate at the obtuse apex.

a. Cellula in trichomatibus plerumque in serie simplice vel duplici ordinata.

a. Cells mostly arranged in a simple or double series in the filament.

S. scytenematoides, Wood.

S. strato submembranaceo, nigro-viride, sæpe interrupto, cum superficie inæquale; trichomatibus sæpe arcte intricatis, flexuosis aut varie curvatis, haud rigidis, plerumque vix ramosis; cellulis uniseriatis, interdum interruptis, arctis, irregulare quadrangulis, diametro subæqualibus aut 1–3 plo brevioribus, haud distincte granulatis, cæruleo-viridibus; vaginis amplis, haud distincte lamellosis, superficie enormiter corrugatis et hirtis, plerumque coloris expertibus sed interdum dilute brunneis.

Diam.—Sine vag. max. $\frac{5}{7500}''$ = .00066″; cum vag. max. $\frac{10}{7500}''$ = .0013″.

Syn.—*S. scytenematoides,* Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 134.

Hab.—South Carolina. (Ravenel.)

S. In a submembranaceous, blackish-green, frequently interrupted stratum, with an uneven surface; filaments often closely intricate, flexuous or variously curved, not rigid, mostly sparsely branched; cells uni-seriate, sometimes interrupted, close, irregularly quadrangular, about equal in length to their diameter, or about 1–3 times shorter, not distinctly granulate, bluish-green; sheaths ample, not distinctly lamellate, their surface rough and corrugate, transparent, mostly colorless, sometimes light-brown.

Remarks.—This species was collected in South Carolina by Prof. Ravenel, who found it in the month of February growing on the limbs of *Myrica cerifera.* The

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blackish-green layer, which it makes upon the bark is very peculiar, being almost membranaceous, and especially in the dried state, presenting a rough, somewhat warty surface. The trichomata have the sheath more distinctly in the form of a hollow cylinder, or, in other words, more plainly a sheath, than any other species I have seen of the genus; the cells are also without any apparent walls, and are placed very closely together, so that the whole filament looks very like a *scytonema.*

Fig. 1, pl. 9, represents a portion of a frond magnified 260 diameters.

S. pellucidulus, Wood.

S. immersus; trichomatibus ramossissimis, solitariis vel subsolitariis; ramis plerumque unilateralibus, ramulosis; ramulorum apicibus late rotundatis, haud attenuatis; cellulis in seriebus simplicibus dispositis, in trichomatibus nonnihil rotundatis, in ramulis sæpe angularibus, plerumque compressis, diametro æqualibus—4 plo brevioribus; terminalibus cylindricis et obscure articulatis; cellulis interstitialibus nullis; vaginis arctis, hyalinis, haud lamellosis; cytioplasmate ærugineo vel brunneo, minute granulato.

Diam.—Trich. cum vag. $\frac{8}{7500}''$ = .00106″; sine vag. = .0008″.

Syn.—*S. pellucidulus,* Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 133.

Hab.—In stagnis, prope Hibernia, Florida. (M. W. Canby.)

S. immersed; filaments very much branched, solitary or subsolitary; branches mostly unilateral, branched; apices of the branches not attenuate, broadly rounded; cells disposed in a simple series, in the trichoma somewhat rounded, in the branches frequently angular, mostly compressed, from equal to 4 times shorter than the diameter; terminal cell cylindrical, obscurely articulate; interstitial cells none; sheath close, hyaline, not lamellate; cytioplasm æruginous or brown, minutely granulate.

Remarks.—This species was collected by Mr. William Canby in a little marsh pool near Hibernia, Florida. The branches are given off in abundance, mostly in a unilateral manner, are often very long, and about equal in diameter to the main filament, and give origin to numerous branchlets. The sheaths are very transparent and very close. I have never seen them in any way lamellate or fibrous, or of any color. The cells do not have very apparent walls. In the main filament and branches they are globose, or, more commonly, very much compressed, but in the newer branches, and sometimes in the older, they are very angular. The few cells near the end of the branches are so shaped as to remind one of the phalanges of the fingers. The last cell is cylindrical and has a number of cells indicated in it. The color of the young cells varies from a deep bluish-green to a ferruginous-brown—that of the older from a light bluish-green to ferruginous-brown.

Fig. 2 *a*, and 2 *b*, pl. 8, represent portions of filaments of this species.

S. compactus, (Ag.) Ktz.

S. strato expanso, tomentoso, fusco-nigro; trichomatibus elongatis ramulisque adscendentibus, apice interdum paullum attenuatis sed sæpe clavatis, obtusis; trichomatibus internis e cellularum serie simplici formatis, et plerumque moniliformibus; cellulis diametro subæqualibus vel brevioribus, subglobosis vel subquadratis, sæpe compressis; cytioplasmate dilute cæruleoviride, subtiliter granulatis; cellulis apicalibus cylindricis et oscillarium modo, sæpe indistincte,

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articulatis; vaginis firmis, aureo- vel rubido-fuscis, in ramulis sæpe subluteis, haud distincte lamellosis; cellulis perdurantibus plerumque modice numerosis, singulis, subglobosis, sæpe valde compressis, dilute fuscentibus.

Diam.—Plerumque $\frac{10}{12000}$″—$\frac{13}{12000}$″ = .0008″—.001″; max. $\frac{4}{3000}$″ = .0013″; cell. perdurant. $\frac{7}{12000}$″ = .00058″.

Syn.—*Scytonema compactum,* AGARDH. Syst. p. 38, N. 3. HARVEY'S Manual, p. 154.
Hassalia compacta, HASSAL, Fresh-water Algæ, p. 232, t. lxviii. f. 3.
Sirosiphon compactus, (AG.) KTZ. RABENHORST, Flora Algarum, Sect. II. p. 287.

Hab.—In rupibus calcareis, New Jersey. (Austin.) Prope Salem, Mass. (Russel.)

Stratum expanded, tomentose, fuscous-black; filaments and branches ascending, with their obtuse ends sometimes slightly attenuate but often clavate; internal filaments composed of a single series of cells, mostly moniliform; cells shorter than or nearly as long as broad, subglobose or subquadrate, often compressed; apical cell cylindrical and articulate somewhat like an oscillatoria; cytioplasm light bluish-green, finely granulate; sheath firm, reddish or yellowish-brown, yellowish in the branches and near the ends; heterocysts mostly rather numerous, single, subglobose, brownish.

Remarks.—The specimens from which the above description was drawn up were received from Messrs. Austin and Russell, and have been considered as identical with the European *S. compactus*, although not in absolute agreement with the descriptions thereof. The most important of the differences are in the matter of size, the measurements given by Prof. Rabenhorst not equalling those attained to by the American plant.

The differences, however, do not seem sufficient to separate the forms, and, in the absence of European specimens, the two have been considered one species. The sheaths in the older portions of the filaments are nearly opaque, but in the branches and younger portions they are quite translucent. The heterocysts sometimes are truncate at one end. The internal cells are rarely arranged in a double series, such arrangement is, however, much more common in the specimens received from near Salem, than in those found in Northern New Jersey. Mr. Russell's specimens are labelled as growing on shaded and moist rocks in patches two or three inches wide.

Fig. 3 *a*, pl. 8, represents the end of a filament of this magnified 150 diameters; 3 *b*, a fragment magnified 250 diameters; 3 *c*, a heterocyst magnified 860 diameters.

S. Crameri, BRÜGG.

S. cæspitibus, tomentosis, spatiose expansis, fusco-nigris; trichomatibus vage ramosis; ramis plerumque singulis, sæpe elongatis, sæpe clavatis; cellulis internis uniseriatis, diametro subæqualibus vel brevioribus, interdum subglobosis, sæpe subquadratis, in ætate provecta sæpe e pressione mutua valde compressis et transverse oblongis, aureo-fulvis vel in ætate juvene interdum ærugineis; cellulis terminalibus in massam subcylindricam coalescentibus; cellulis perdurantibus nullis; vaginis aureo-fuscis in ætate provecta plus minus subopacis et distincte lamellosis, in ætate juvene plus minus pellucidis et sæpe coloris expertibus.

Diam.—Trich. cum vag. max plerumque .002″; interdum .00225″; ram. .0015″—.0025″; trich. sine vag. .00083″.

Syn.—*S. Crameri,* BRÜGG. RABENHORST, Flora Europ. Algar., Sect. II. p. 288.

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Hab.—In rupibus irroratis inter muscis minutis. Mount Tahawus (*vulgo* Mount Marcy),[1] alt. 5000 feet.

Forming a blackish, widely expanded, tomentose turfy covering to rocks; filament with scattered branches; branches mostly single, often elongate and clavate; cells uniseriate, about equal, or shorter than long, sometimes subglobose, often subquadrate; in advanced age often strongly compressed and transversely oblong from mutual pressure, yellowish, or sometimes, when young, greenish; the apical cells coalescent into an irregularly cylindrical mass; heterocysts wanting; sheaths yellowish-brown; at maturity more or less subopaque, and distinctly lamellate; in youth more or less transparent, and sometimes colorless.

Remarks.—Near the top of Mount Tahawus, in the Adirondack Mountains, there is, at an altitude of about five thousand feet, a steep slope of bare rock, the bed of an old landslide, over portions of which water is continually dripping. In such places the plant under consideration flourishes, forming with some very minute mosses a blackish, turfy coating to the rock of many feet, or even yards, in extent. The specimens agree well with the descriptions of the European plant, which also grows at about the same altitude as the American. They have, however, one peculiarity not noted in description of the European form, namely, that oftentimes the sheath of a branch widens out until it is actually much larger than the main filament. The color of the cells in the European form is said to be æruginous; but I conceive this depends somewhat upon the age of the specimens and is scarcely of primary value. The only other difference worth noticing is that my measurements exceed somewhat those given of the European plant. I do not think, however, there is any good ground for separating the forms as distinct species.

The finding of an Alpine plant growing on a mountain half way across the world from its first discovered home, at practically the same altitude, is a matter worth noting as a fact in Botanical Geography.

S. neglectus, Wood.

S. immersus; trichomatibus subsolitariis, longis usque ad lineas quatuor, cylindricis, ramossissimis; ramulis singulis; cytioplasmate interdum ærugineo, plerumque aureo-brunneo; cellulis uniseriatis rarissime biseriatis, subglobosis, interdum sejunctis sed plerumque arcte connectis et moniliformibus, modo confluentibus, haud distincte pachydermaticis; cellulis terminalibus elongato-cylindricis, sæpe nonnihil oscilatorium modo articulatis; cellulis interstitialibus nullis; vaginis interdum brunneis, plerumque coloris expertibus.

Diam.—Trichom. cum vag. $\frac{1}{570}''=.0017''$; sine vag. $\frac{1}{1000}''=.001''$.

Syn.—*S. neglectus*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 133.

Hab.—In stagnis, New Jersey.

S. immersed, subsolitary, attaining a length of 4 lines, cylindrical, very much branched; branches single; cytioplasm æruginous, mostly yellowish-brown; cells uniseriate, very rarely biseriate, subglobose, sometimes separate but more frequently closely united and moniliform; terminal cell an elongated cylinder, often articulate somewhat like an oscillatoria; interstitial cells wanting; sheaths transparent, sometimes brown, mostly colorless.

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[1] "*Tahawus*," cloud splitter. The Indian names of the American mountains ought to be retained, in spite of the fact that some vulgar land surveyor has defiled the Adirondacks with the names of politicians, through whose influence he hoped for patronage.

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Remarks.—This plant was found in a very stagnant pool, forming, with various other species of algæ, a gelatinous, æruginous-brown stratum, through which the single plants were thickly scattered, without anywhere forming the major portion of the mass. The plants themselves are large enough to be distinguished by the unaided eye. Under the microscope the sheaths are seen to be exceedingly transparent and colorless, except in the older part of the filament, where they are often dark brown and opaque; but even in such case, the edges are translucent and lighter colored.

The internal cells or globose masses rarely have distinct coats, and even when such were apparent, as in the older portions of the plants, there appeared to be a communication between the cells. The original main stem is rather short, shorter often than numerous branches into which it breaks up. Very often the apices of the branches are colorless and entirely empty, consisting simply of sheath; often, however, they are occupied by a cylinder of protoplasm, which is sometimes articulated more or less distinctly like an oscillatoria.

Fig. 4, pl. 8, represents a fragment of a filament with a small branch.

S. lignicola, Wood.

S. strato expanso, tomentoso, atro; trichomatibus ramossissimis, arcte intertextis; ramulis abbreviatis vel elongatis, subrectis aut varie curvatis, apicibus obtuse rotundatis vel subacuminatis; trichomatum et ramulorum cellulis uni-vel biseriatis, rare in trichomatibus maturis multiseriatis, plerumque pachydermaticis, dilute vel saturate ærugineis, enormibus, plerumque homogeneis; cellulis terminalibus in trichomatibus immaturis elongatis, cylindricis, sæpius nonnihil oscillatorium modo articulatis, granulosis; vaginis sat amplis, haud achrois, vel luteo-brunneis vel fuscentibus vel ferrugineis.

Diam.—Trich. cum vag. max. $\frac{1}{1500}'' = .00066''$.

Syn.—*S. lignicola,* Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 133.

Hab.—South Carolina. (Ravenel.)

Occurring in an expanded, tomentose, black stratum; filaments very much branched, closely interwoven, branches abbreviate or elongate, nearly straight or variously curved, their apices obtusely rounded or subacuminate; cells 1–2 seriate, mostly thick-walled, light or deep æruginous, irregular, mostly homogeneous; terminal cells elongate, cylindrical, frequently articulate somewhat like an oscillatoria, granulate; sheaths somewhat ample, not transparent, light bright, fuscous or ferruginous.

Remarks.—I have seen dried specimens only of this plant, which were collected by Prof. H. W. Ravenel, in South Carolina. It is said to grow on old boards, and appears to be a very distinct species. There are frequently two or three very short, stubby branches arising together. The apices of the filaments and branches are in some cases filled with endochrome to the end, and are broadly rounded at the apex. In other cases the sheath of the filament extends a distance beyond the endochrome, and is finally rapidly diminished to a point. The cells within the filaments are of various shapes, sometimes globular, sometimes quadrangular, more often irregular. The original specimens from which this description was written were collected in April. I do not know whether they grew immersed, or merely on boards exposed to the weather. I have since received

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specimens collected in the month of August, which grew on boards over which spring water was constantly running. These specimens agree perfectly with the others, except that the filaments are larger and the elongated apical cell is wanting; differences which I believe to be due to the specimens collected in August being older than those first received.

Fig. 2 *a* and 2 *b*, pl. 9, were taken from the types, whilst fig. 2 *c*, pl. 9, from the August specimens.

a. Cellulæ plerumque in serie duplici vel multiplici.

a. Cells generally in double series, or multiple series.

S. argillaceus, Wood, (sp. nov.)

S. strato tenui, expanso, subnigro, submembranaceo; trichomatibus brevibus, dense intricatis et sæpe nonnihil concretis, ramosis, irregularibus; pseudoramulis brevibus, varie curvatis, nonnihil rigidis, plerumque ascendentibus, apice nonnihil attenuatis; cellulis subglobosis, sæpe compressis, plerumque in serie simplici sed interdum in serie duplici, vel rare multiplici; cellulis apicalibus valde elongatis, cylindricis, scytonemæ trichomatibus internis similibus; vaginis crassis, firmis, in trichomatibus maturis saturate rubido-brunneis, in ramulis sæpe luteo-brunneis et in apice hyalinis et fere coloris expertibus; cellulis perdurantibus nullis.

Diam.—$\frac{5}{6000}''=.000833''$.

Hab.—In palude argillacea, South Carolina. (Ravenel.)

Stratum thin, expanded, blackish, submembranaceous; filaments short, densely intricate, and frequently somewhat concreted, giving origin to numerous branches, irregular; branches short, variously curved, somewhat rigid, mostly ascending, apex somewhat attenuate; cells subglobose, often compressed, mostly in simple series, sometimes in double, rarely even in multiple; apical cells elongate, cylindrical, resembling the inner filament of a scytonema; sheath thick, firm, in the mature filament deep reddish-brown, in the branches yellowish-brown, at the apices of the branches nearly colorless and transparent; heterocysts absent.

Remarks.—I am indebted to Prof. Ravenel for this plant, which was found by him on a moist clay bank near Aiken, South Carolina, August, 1869. It forms a thin, somewhat membranous, dark stratum, the filaments of which are so closely united that it is almost impossible to tease them apart with needles. Neighboring filaments are often united at the edges so as to form distinct bundles, and even the branches are sometimes concreted, although, generally, as seen under the microscope, they project from the mass in all directions. The surface of the filaments is mostly rough and ragged with fibrillæ and membranous projections. In the older filaments the cells are often entirely absent. They are mostly single, but sometimes multiple in the filaments; in the branches they are often partially double. The ends of the older branches are often broken and empty, whilst those of the younger are rounded. The color of the cells, as I have seen it, does not strikingly differ from that of the sheaths.

Fig. 3 *a*, pl. 9, represents a portion of an old frond magnified 460 diameters, and fig. 3 *b*, the end of a younger branch. No. 79. Collection of Ravenel, Aug. 1869.

S. guttula, Wood.

S. in maculis subnigris, parvis, tenuibus, plerumque rotundatis, interdum enormibus, dispositus; trichomatibus arcte intertextis, ramossissimis, rigidis, inæqualibus, subcylindricis, nonnihil contortis; ramulis abbreviatis vel nonnihil elongatis, apice obtuse rotundatis; ramulorum et trichomatum cellulis tri-multiseriatis, plerumque pachydermaticis, ferrugineo-fuscis, enormiter globosis, homogeneis; cellulis apicalibus interdum breve cylindricis, haud articulatis; vaginis sat amplis, luteo-brunneis vel dilute ferrugineo-brunneis.

Diam.—Max. trich. cum vag. $\frac{1}{750}''=.0013''$.

Syn.—*S. guttula*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 132.

Hab.—South Carolina, in Taxodium distichum. (Prof. Ravenel.)

Arranged in small, thin, black spots, which are generally round, but sometimes irregular: filaments closely interwoven, very much branched, rigid, unequal, subcylindrical, somewhat contorted; branches abbreviate or somewhat elongate, apex obtusely rounded; cells of the trichoma and branches 3 to many seriate, mostly with thick coats, ferruginous-fuscous, irregularly globose, homogeneous; apical cells sometimes shortly cylindrical, not articulate, sheaths ample, yellowish-brown.

Remarks.—This species was found growing on the bark of *Taxodium distichum*, by Prof. H. W. Ravenel, in South Carolina, and by him given to Dr. Billings, U. S. A., to whom I am indebted for specimens. It forms on the bark minute roundish, blackish, dot-like spots of about a line in diameter, or sometimes, apparently, by the coalescence of two or more of these spots, larger irregular patches. The habit of the plant is a rigid one. The main stem is often irregular in size, variously bent and rebent, and mostly gives off a number of branches, which frequently nearly equal the main filament in size, and like it are bent in various directions. They also frequently give origin to numerous short branches. In some instances, there is a distinct apical cell, which is cylindrical, but only two or three times longer than broad; in many cases, however, this cylinder being wanting, the ordinary cells extend to the extreme apex.

Fig. 4 *a*, pl. 8, represents a filament, and fig. 4 *b*, the end of a branch magnified 460 diameters.

S. acervatus, Wood.

S. in guttulis minutissimis, subcrustaceis, nigris, in strato subcontinuo sæpe aggregatis; trichomatibus parvis et brevibus, rigidis, admodum inæqualibus, prostratis, tuberculis, arcte et dense ramossissimis, viridibus aut aureis aut brunneis; ramulis brevibus, plerumque haud ramulosis, erectis aut ascendentibus, sæpe abbreviatis et papilliformibus, obtusis, sæpe lateraliter connatis; cellularum serie in trichomatibus multiplici in ramulis plerumque simplici; cellulis subglobosis vel subangularibus, viridibus, haud distincte granulosis, in ramulorum apice sæpe breve cylindricis et interdum obsolete articulatis; vaginis aureis, nonnihil hyalinis.

Diam.—Trich. max. $\frac{15}{7500}''$; ram. $\frac{3}{7500}''$—$\frac{4}{7500}''$.

Syn.—*S. acervatus*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 132.

Hab.—South Carolina, in cortice (Ilex opaca). (Prof. H. W. Ravenel.)

Arranged in drops, which are very minute, subcrustaceous, black, and frequently aggregate into a subcontinuous stratum; filaments small and short, prostrate, rigid, somewhat unequal, tuberculate, densely and closely branched, green or golden or brown; branches short, for the most part not branched, erect or ascending, frequently abbreviate, and papilliform, obtuse; series of cell multiple in trichoma, mostly simple in the branches; cells subglobose or subangular, green, not distinctly granulate, in the apices of the branches frequently shortly cylindrical and sometimes obsoletely articulate; sheaths golden, somewhat hyaline.

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Remarks.—This species was found in winter by Prof. H. W. Ravenel in South Carolina, growing upon the bark of *Ilex opaca*, forming minute, firm, crustaceous, roundish dots or masses, much smaller than a mustard-seed, but in some cases so closely aggregated as almost to make a continuous stratum. When one of these dots is placed under the microscope, the branches are seen presenting their ends upon all sides, reminding one of some varieties of coral, and between these are blackish matters, which prevent the whole dot from being seen. These branches are frequently placed very close to one another, and cohere by their edges so as to make a sort of membrane or a solid mass. The filaments themselves are mostly obscured in the dense mass of branches which clothe them. This species seems to be closely allied to *S. coralloides*, and I am not certain whether it is distinct or not. It is certainly very much smaller.

S. pulvinatus, BRÉB.

S. pulvinatus, humectatus, saturate olivaceo-niger, ad tres lineas crassus; trichomatibus crassissimis, ramossissimis, fuscescentibus, enormiter curvatis; ramulis polymorphis pro ætate crassitie magnitudineque variis, apice plerumque obtuse rotundatis; trichomatum cellularum serie multiplici, ramulorum 2–4 plici; vaginis crassis, luteo-fuscis ad saturate-fuscis, vel pellucidis vel non pellucidis, interdum rugoso-tuberculis.

Diam.—Trich. cum. vag. max. .0042″.

Syn.—*S. pulvinatus*, (BRÉB.) RABENHORST, Flora Europ. Algar., Sect. II. p. 290.

Hab.—In rupibus prope Philadelphia. Wood.

In moist, deep olive-black cushion-like masses of two or three lines thick; filaments very thick, much branched, brownish, irregularly curved; branches polymorphous, varying in thickness and size, mostly with their apices obtuse; cells of the filament many seriate, of the branches two to four seriate; sheaths thick, yellowish-fuscous to deep fuscous, pellucid or opaque, sometimes rugose-tuberculate.

Remarks.—I have received specimens of this species found by Mr. Austin in Northern New Jersey, growing on the exposed face of rocks.

The size attained to exceeds that given by Mr. Rabenhorst for the European form. The color of the cytioplasm varies from an almost verdigris-green to fuscous.

Besides these specimens, Dr. I. Gibbons Hunt has given me fresh ones of a *Sirosiphon* which he found growing on the face of dripping rocks along the Wissahickon Creek, near this city. These are much smaller in every way than their more northern brethren, and differ in other respects, I think, sufficiently for a distinct variety. The filaments and branches are much flatter than in Mr. Austin's specimens. I append a description.

(*Var.* **parvus.**)

S. trichomatibus in cæspite saturate olivaceo-nigro arcte intertextis; trichomatibus crassissimis, enormiter ramosissimis, luteo-fuscescentibus, varie curvatis; ramulis polymorphis, apice plerumque obtuse rotundatis; trichomatum cellularum serie multiplici, ramulorum 1–4 plici; cytioplasmate granulato, plerumque saturate fuscescente, interdum læte viride; vaginis crassis, dilute luteo fuscescentibus, interdum achrois.

Diam.—Trichom. cum. vag. max. $\frac{1}{325}''$ = .03″

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Filaments closely interwoven into a deep olive-black turfy mass, very thick, irregularly and frequently branched, yellowish-fuscous, variously curved; branches polymorphous, their apices mostly obtusely rounded; series of cell in filament multifold, in branches 1–4 fold; cytioplasm granulate, mostly deep brown, sometimes bright green; sheaths thick, light yellowish-brown, sometimes transparent.

Remarks.—The fronds are very irregular in form and size, much branched, and so closely interwoven that they mostly cannot be separated without breaking. The branches are sometimes short and stumpy, sometimes they are very long. The color of the cells approaches somewhat to a chocolate, at times with a little red in it so as to give something of a mahogany tint. The walls of the cells are mostly very thick, but they are often lost in the general mass of the frond. In the branches, the cells are often so closely crowded as to almost obliterate their walls. In a few specimens I have found the cells to be of a bright green color, instead of that just mentioned. The exact meaning of this I do not know; it would scarcely seem to indicate immaturity, for I have found it in the oldest portion of large fronds, whose other parts were of the normal color.

Fig. 1, pl. 10 represents a filament of this variety magnified 160 diameters.

I have received from Prof. Ravenel certain dried algæ, labelled *Stigonema Ravenelli*, BERKELEY, which appear to me to belong to this genus. In what place Berkeley described them, if ever, I do not know, nor why he placed them in the genus *Stigonema.* The following is a description of the species:—

S. strato sub-nigro; trichomatibus arcte intertextis, ramossissimis, enormibus, varie curvatis; ramulis brevibus et sublongis, varie curvatis, latis, apice nonnihil attenuatis et obtusis; trichomatum et ramulorum cellulis arctis, enormibus, in serie duo-multiplici enormiter dispositis; cytio-plasmate homogeneo, læte viride; vaginis aureis, lucidis.

Diam.—Max. trich. cum vag. $\frac{14}{7500}''$.

When dried blackish; filaments closely interwoven, very much branched, irregular, and variously curved; branches short or largish, variously curved, broad, their apices somewhat attenuated and obtuse; cells of the filament and its branches very close, irregular, irregularly arranged in a twofold or multiple series; endochrome homogeneous, bright green; sheath yellow, semitranslucent.

Remarks.—This plant was collected by Prof. Ravenel on the now famous Lookout Mountain. It is of a thick, bushy habit, and appears to form turf-like mats of a line or two in thickness and of a blackish color. The filaments throw off in all directions very numerous branches, some of which are short and stumpy, others quite long, and are themselves the parents of numerous secondary branches. The longer branches often rival the main filament in size, and like it vary continually, in being irregularly expanded and contracted. There is never a long, articulated cell, not even in the apices of the branches. The apices are often somewhat attenuated, and are always more or less obtuse. The cells are of a bright green color, are very irregular in form, and are often very irregularly arranged in rows of from two to five, both on the main filament and branches. The base of the filament often gives origin to several small, cylindrical, root-like processes.

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Fig. 4 *a*, pl. 9, represents a frond of this plant magnified 125 diameters; fig. 4 *b*, a fragment magnified 460 diameters.

Professor Bailey, in American Journal of Sciences, vol. iii., new series, states that he has found two species of the genus *Stigonema*, namely, *St. atrovirens*, AG. and *St. mammillosum*, AG.; the former growing on wet rocks at Indian Falls, Putnam County, New York; the latter at Round Pond, near West Point. I have no personal knowledge of the genus, but, according to authorities, it belongs to the lichens rather than the algæ, apothecia having been detected in various species.

CLASS **CHLOROPHYLLACEÆ.**[1]

Plantulæ aquaticæ vel aëreæ, uni-, bi-, vel multicellulares, aut singulæ aut consociatæ, familias formantes.

Vegetatio terminalis vel non terminalis.

Ramificatio aut nulla aut vera, sed cellularum non divisione, potius prolificatione.

Cytioderma non siliceum, combustibile, sæpius e stratis successivis compositum, substantiam gelatinosam plerumque liquidam exsudans.

Cytioplasma chlorophyllosum, chlorophylli loco nonnunquam erythrino vel substantia oleosa coccinea, carnea aut rufescente coloratum, nucleo (centrali vel laterali) plerumque præditum, granulis amylaceis rarissime carens.

Multiplicatio fit cellularum divisione vegetativa. Fœcundatio plerumque sexualis.

Propagatio fit aut oosporis vel zygosporis aut gonidiis tranquillis vel agilibus.

Aquatic or *aërial* uni-, bi-, or multicellular plants occurring singly, or consociated in families.

Vegetation terminal or not so.

Branches either wanting, or if present, true branches, although formed rather by a process of proliferation than division of the cells.

Cytioderm not siliceous, combustible, often composed of successive strata.

Cytioplasm chlorophyllous, sometimes colored by an oily crimson, flesh-colored or yellowish-red substance, in the place of the chlorophyl, generally furnished with a nucleus (either lateral or central), very rarely without starch granules. Growth occurring by the division of the cells. Fecundation generally sexual.

Propagation taking place by oospores or zygospores, or by tranquil or motile gonidias.

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[1] The description of this Class and Order is that of Prof. Rabenhorst.

ORDER **Coccophyceæ.**

Algæ unicellulares. Cellulæ aut singulæ (plerumque perfecte segregatæ) aut plures in familias consociatæ, tegumentis involutæ vel nudæ, aut ramificatione aut vegetatione terminali destitutæ. Propagatio fit aut cellularum divisione aut zoogonidiis.

Unicellular algæ. Cells either single (mostly entirely segregate), or mostly consociated in families, walled or clothed with teguments, destitute of branches or terminal vegetation. Propagation by means of zoospores, or by the division of the cells.

FAMILY PALMELLACEÆ.

Algæ unicellulares sensu latiori. Cellulæ aut singulæ aut numerosæ, familias constituentes, in muco matricali plus minus firmo, stratum gelatinosum amorphum, sæpius figuratum, tubulosum (Hormospora) varie divisum et perforatum (Tetraspora), quasi ramificatum (Hydrurus) formante nidulantes, vel nullo (Rhaphidium, Dactylococcus). Cytioderma plerumque tenue, sæpius tegumento gelatinoso aut homogeneo aut lamelloso præditum. Cytioplasma homogeneum, ætate provecta plerumque distincte granulosum, viride, aut rubescens aut fuscescens, vesicula chlorophyllosa semper instructum (excepto Rhaphidio).

Multiplicatio fit cellularum divisione vegetativa, propagatio gonidiis ex ultima cellularum generatione transitoria cytioplasmatis divisione varia ortis. Gonidia tegumentis liberata, polo autico ciliis vulgo binis plerumque instructa et alacriter circumvagantia. (R.)

Algæ unicellular in a broad sense. Cells either single or numerous, constituting families, imbedded in a jelly to form a gelatinous stratum which is amorphous or shaped, as tubular (Hormospora), variously divided and perforate (Tetraspora), falsely branched (Hydrurus), or sometimes is wanting (Rhaphidium, Dactylococcus). Cytioderm mostly thin, often furnished with a gelatinous or homogeneous or lamellate tegument. Cytioplasm homogeneous, mostly at maturity distinctly granular, green-reddish or fuscous, always furnished with a chlorophyllous vesicle (except Rhaphidium).

Multiplication taking place by a vegetative division of the cells, propagation by transitory gonidia arising by various divisions of the protoplasm from the last vegetative generation. Gonidia without integument, mostly furnished with two cilia at the anterior end, and moving about actively.

Genus PLEUROCOCCUS, MENGH. (RABENH.)

Cellulæ globosæ vel e mutua pressione angulosæ, plerumque nucleo instructæ, tum singulæ tum in familias consociatæ. Cytioderma firmum, sæpe crassum, læve, hyalinum; cytioplasma homogeneum viride vel oleosum rubrum. Multiplicatio cellularum vegetativarum divisione in directionem ad omnes dimensiones alternantem. Propagatio fit gonidiis intra sporangia ortis.

Cells globose or angular from mutual pressure, mostly furnished with a nucleus, sometimes single, sometimes aggregated into families. Cytioderm firm, often thick, smooth, hyaline; cytioplasm homogeneous-green or oleaginous-red. Multiplication occurring by a vegetative division of the cells alternately in three directions. Propagation by means of gonidia, formed within sporangia.

P. seriatus, WOOD, (sp. nov.)

P. corticolus, strata pulverula, rubido-brunnea, nonnihil crustacea formans; cellulis enormiter subglobosis, vel ovalibus, læte aurantiacis, interdum viride tinctis, haud distincte nucleatis, in seriebus singulis rectis vel curvatis conjunctis; tegumentis crassis, haud lamellosis, coloris expertibus.

Diam.—$\frac{4}{7500}''$—$\frac{9}{7500}''$ = .00053″—.0012″.

Hab.—In palude. New Jersey. (Austin.)

Growing on bark, forming a reddish-brown, somewhat crustaceous powdery mass; cells irregularly subglobose, or oval, bright orange, sometimes tinged with green, not evidently nucleated, conjoined in single straight or curved series; tegument thick, lamellate, or not so, colorless.

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Remarks.—I am indebted to Mr. Austin for specimens of this little plant, which he found growing in a swamp near Closter, Northern New Jersey, on a young pin oak. It forms a sort of crustaceous powder, with little aggregations here and there, of a dull reddish-brown color. When these little masses are broken up, they are found to be composed of little series of very closely joined cells, generally a half dozen to a dozen in the row. I believe that at certain states of their growth these cells are green, as many of them have a very decided green tint on their edges, and I have seen one or two of them quite green.

Fig. 2, pl. 10, represents this species magnified 460 diameters.

P. pulvereus, Wood, (sp. nov.)

P. cellulis minimis, cæruleo-viridibus, enormiter subglobosis, vel angulosis, in familias numerosas consociatis; familiis e cellulis numerossissimis et dense confertis compositis, irregularibus, interdum confluentibus, plerumque pseudotegumentis hyalinis involutis, in strato pulvereo læte viridi aggregatis.

Diam.—$\frac{1}{24000}$″—$\frac{3}{24000}$″ = .00004″—.00013″.

Hab.—In fonte. "Boiling Springs," prope Bellefonte, Pennsylvania.

Cells very small, bluish-green, irregularly subglobose, oval, or angular, associated in numerous families; families composed of very numerous and densely crowded cells, irregular, sometimes confluent, mostly surrounded by a false hyaline tegument, aggregated into a bright green pulverulent stratum.

Remarks.—In Centre County, Pennsylvania, two miles from Bellefonte, there is a very large and beautiful limestone spring, which is a favorite roadside watering place, and is laid down on the maps as "Boiling Springs." Forming a stratum over most of the bottom of this spring is the little plant here described. The stratum is in places nearly an inch in thickness, and when lifted by the hand is found to be dry and crumbly, instead of mucous and tenacious. Under the microscope it is seen to be composed of vast numbers of irregular masses or families of cells imbedded in a firm jelly, which projects so as to form a sort of transparent coat to the whole mass; this cast I have spoken of in the description as a false tegument. The cells themselves are exceedingly small and furnished with an excentric point, which is probably a nucleus.

Genus PALMELLA.

Cellulæ globosæ vel ovales vel oblongæ, tegumentis plus minus crassis in mucum gelatinosum, sæpius mox confluentibus involutæ, thallum difforme efficientes. Cellularum divisio directione in omnes dimensiones alternante.

Cells globose, oval, or oblong, surrounded with a more or less thick integument generally very soon confluent into a firm or soft jelly. Thallus shapeless. Division of the cells alternately in all directions.

P. Jesenii, Wood.

P. thallo indefinite expanso, initio dilute aut læte viride, molle, pellucidulo; ætate provecta firmo, tuberculoso, saturate olivaceo-viride; cellulis globosis vel ellipticis,—in thalli ætate immaturo, plerumque singulis aut geminis, sæpe distantibus,—in ætate provecta sæpe in familias connexis, plerumque confertis; tegumentis in thalli ætate immaturo plerumque diffluentibus, ætate provecta plerumque distinctis.

Diam.—Cell. glob. max. $\frac{1}{3500}'' = .00028''$; cell. oblong. long. max. $\frac{1}{2500}'' = .0004''$.

Syn.—*P. Jesenii*, WOOD, Prodromus, Proc. Am. Philos. Soc., 1869, p. 134.

Hab.—In rupibus irroratis, prope Philadelphia.

Thallus indefinitely expanded, in the beginning soft and pellucid, afterwards firm, tubercular, deep olive-green; cells globose or elliptical; in the immature thallus, single or geminate, frequently scattered; in the mature thallus often closely conjoined into families, mostly crowded; in the young thallus the teguments of the cells are mostly diffluent, afterwards distinct.

Remarks.—This little plant was found along the banks of the Schuylkill River, just above Flat Rock tunnel, near Manayunk, forming in the early winter a gelatinous mass of two to three lines in thickness, irregularly and interruptedly spread over the face of wet, dripping rocks. In what appeared to be the younger portions, the jelly was often quite soft and almost colorless, and had the cells scattered rather sparsely and distantly through it. The cells were but partially filled with chlorophyl, the vacuole left containing often numerous granules, and had distinct walls, being, as it were, merely immersed in the general maternal jelly. In the older fronds the texture is more firm, the color a deep green, and the bright green cells are mostly surrounded by a thick, very distinct tegument. They are also largely arranged in little families of two, four, or even eight cells, surrounded by a common integument. The oldest fronds are of a deep olive, almost blackish color, markedly tuberculate upon their upper surface and very firm in texture. They are surrounded by very distinct, firm, dark brown coats (a simple coat often involving two or more cells), and arranged in groups or families. As shown by the microscope in the superficial portion of such fronds, the jelly is of a yellowish-fuscous color, and the cells are themselves of a dark brown tint. The number of cells in the individual families varies from two to a dozen or more. Even in these old, firm fronds, the interior portions are frequently composed of greenish cells, without any distinct teguments or coat. In such cases the cells are mostly oblong or elliptical, and very much crowded together. This species appears to come closest to *P. Brébissonii*, KTZ., from which it differs, however, in its habit of growth and the size of its cells.

Fig. 3 *a*, pl. 10, represents a fragment of the upper surface of an old frond magnified 750 diameters; fig. 3 *b*, when taken from the inner jelly of similar fronds.

P. dura, WOOD, (sp. nov.)

P. thallo enormiter subgloboso, enormiter minute lobato vel verrucoso, cæruleo-nigro, nonnihil crustaceo, minuto; cellulis arctissime confertis, plerumque enormiter oblongis, sæpe in seriebus irregulare dispositis, cæruleo-viridibus vel luteo-brunneis; tegumentis haud distinctis; sporis globosis vel ovalibus.

Diam.—Cell. $\frac{1}{12000}'' = .00008''$—$\frac{1}{6000}'' = .00016''$; spor. $\frac{7}{12000}'' = .00058''$—$\frac{6}{7500}'' = .0008''$

Hab.—In fonte prope Philadelphia.

Thallus irregularly subglobose, irregularly minutely lobate or warty, bluish-black, somewhat crustaceous, minute; cells densely crowded, mostly irregularly oblong, often arranged irregularly in series, bluish-green or yellowish-brown; coats not apparent; spores globose or oval.

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Remarks.—I found this plant growing in the large spring at Spring Mills in March or April. The fronds were in the form of little blackish balls attached to the stems of mosses in the water. They varied in size from the minutest speck, scarcely visible to the naked eye, up to ten lines in diameter; they are globose, very firm and hard, and the larger look almost as if they were aggregations of smaller ones. They are gregarious. The spores are mostly borne on the edges of the frond, sometimes they appear to be imbedded in its substance. At first they are of an intense bluish-green, but afterwards they appear to be yellowish-brown. None of the cells, as I have seen them, have their contents granulate.

Fig. 5 *a*, pl. 10, represents a section of a frond magnified 460 diameters; fig. 5 *b*, a section of the edge of an old frond, developing spores.

P. hyalina, LYNGB.

> "Fronds from a quarter of an inch to an inch in diameter, somewhat globose, but at length frequently more or less elongated into an ovate or even cylindrical form. Substance gelatinous and very tender, of a pellucid, watery appearance. Granules numerous, globose, green. The fronds are produced at first on rocks and stones at the bottom of streams, and afterwards become disengaged and float on the surface."

Remarks.—Professor Bailey states that he has found this species from Rhode Island to Wisconsin. Whether it is identical with the *P. hyalina* of Brébisson, or not, I cannot say.

Genus PAGEROGALA,[1] WOOD.

Thallus solidus, gelatinosus, indefinitus, exalbidus, nonnihil pellucidulus, nodulis dense aggregatis et sæpe confluentibus formatus. Cellulæ globosæ, confertæ, in familias consociatæ. Familiæ tegumentis tenuibus et membranaceis involutæ, in nodulorum centro positæ.

Thallus solid, indefinite, gelatinous, whitish, somewhat pellucid, composed of closely aggregated nodules which are often indistinct. Cells globose, crowded in families. Families surrounded by a thin membranaceous coat and placed in the centre of the gelatinous nodule.

Remarks.—This curious plant was found by myself floating as indefinite masses of milk-white jelly on a mountain spring near Bear Meadow, Centre County, Pennsylvania. The largest of these gelatinous masses was six inches long. On taking them out of the water they were seen to be composed of somewhat irregular nodules, which in some portions of the mass were very distinct one from the other, but in other parts were confluent into an almost uniform jelly. When the nodules were separated it was discovered that each contained a membranous very delicate sack of a pale green color, which the microscope showed to be really a cell family. Their interior was hollow, or at least only partially filled with a transparent fluid, and they contained all round their exterior portion a layer of round, closely placed cells. In some instances the outer membrane was ruptured, and the sac only contained a few cells, which could often be seen to be moving freely in the inner liquid. The sac membrane is thin and delicate, colorless, and marked with curious, regular wrinkles or folds. In those portions of the common gelatinous mass, where the nodules were lost, I could not find any of these sacs.

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[1] Πάγερος, frozen; γαλα, milk.

No opportunity was afforded to study the development of this plant; but there can be but little doubt that the globular, thickish-walled cells are finally discharged by a rupture of the membrane and escape from the softening jelly into the water, each to be a possible starting point for a new frond.

I have given this curious plant the name of *Pagerogala*, from its milky whiteness. Floating in the water it offered so close a resemblance to the spawn of frogs, though more opaque, that my companion, a most excellent naturalist, insisted, until its true nature was absolutely demonstrated, that I was simply wasting my time collecting the spawn of an amphibian.

P. stellio, (sp. nov.)

Diam.—Frond $\frac{1}{3}$ inch; cells $\frac{4}{3000}''$—$\frac{1}{2000}''$.

Genus TETRASPORA, Link.

Thallus gelatinosus, membranaceus vel submembranaceus, initio saccato-clausus, ætate provectiori vel postea explanatus. Cellulæ globosæ (vel anguloso-rotundatæ) plus minus distantes sed in familias magnas unistratas consociatæ; tegumentis crassis in mucum homogeneum cito diffluentibus. Cellularum divisio in planitiei duas directiones alternans.

Propagatio fit gonidiis mobilibus.

Thallus gelatinous, membranous or submembranous, in the beginning a short sack, afterward expanded. Cells globose, or angularly so, more or less distant but consociated in a single stratum into large families. Tegument thick, very rapidly diffluent into a homogeneous mucus. Division occurring in two directions in the one plane.

Propagation by means of zoospores.

T. lubrica ? (Roth) Ag.

T. thallo gelatinoso-membranaceo, lubrico, dilutissime viride, tubuloso sed sæpe postea explanato, simplice vel ramoso, undulato-sinuoso, sæpe lacunis munerosis perforato; cellulis globosis vel ellipticis, læte viridibus, interdum singulis sed plerumque quaternis vel geminis, locello achroo hyalino parietali sæpe præditis; cytiodermate tenuissimo, haud distincte visibile.

Diam.—Cell. $\frac{1}{2000}''$—$\frac{1}{4000}''$ = 0.00025″—0.0005″.

Syn.—*T perforata*, Harvey. Bailey, Silliman's Journal, N. S. vol. iii.
T. lubrica, (Roth) Ag. Rabenhorst, Flora Europ. Algarum, Sect III. p. 41.

Hab.—Northern Atlantic States.

Thallus gelatinoso-membranaceous, slippery, very dilute green, tubular, but often finally expanded, simple or branched, undulately-sinuate, often perforated with numerous holes; cells globose or elliptical, bright green, sometimes single but mostly in pairs or fours, furnished with a parietal transparent hyaline space; cytioderm very thin, not distinctly visible.

Remarks.—This little plant is very common around this city, growing usually in limpid, quiet water, such as springs, little rushy pools, and clean ditches. The frond is a translucent, light green or scarcely greenish, very slippery jelly, with the edges often very markedly undulate. It is very rarely simple, but on the contrary is often very much and very irregularly branched, frequently indeed consisting of several broad portions united by narrow necks. It is an irregular sack, generally profusely perforate, and often with large imperfect portions. I think it finally in many instances becomes expanded and open. It is sometimes found lying on the bottom, but more frequently floats on the surface of the water. The breadth of the frond varies from two or three lines to an inch. The length often reaches several inches. The cells are mostly globular; but, immediately after

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division, they are elliptical. They are of a bright green color and almost always have a conspicuous rounded granule within them; sometimes, but not commonly, at one end there is a hyaline space or vesicle, similar to that seen in zoospores. I have watched the production of zoospores in a plant gathered late in November. The outer wall of the cell is always so thin as to be scarcely perceptible, and when the zoospore is beginning to move, it looks as though the whole cell were rocking, the thin outer coating being lost to sight. After a considerable period of vain effort the zoospore escapes from the thick gelatinous mass which surrounds it. It is biciliated, roundish, and furnished with a hyaline space at the end.

I have observed a *Tetraspora* growing in rapidly running water, which some would no doubt consider distinct, but which seems to me rather a variety. The saccate frond was of a very vivid green, erect, buoyed up by an air-bubble contained in its upper end. Its shape was that of a long sack widened very much above, and below constricted into a fine point, by which it was firmly attached. In some instances it attained a length of seven or eight inches. In all other respects these plants agreed with the others found in quiet water.

The species of this genus are to me not at all well-defined in any work which I have had access to. The plant now under consideration abounds everywhere in this neighborhood, and is without doubt the one identified by Prof. Bailey as *T. gelatinosa* (Vauch), of which, however, he afterwards states that Prof. Harvey, to whom he had sent specimens, writes that it is a distinct species, and proposes to call it *perforata*. In my Prodromus I referred the plant to *T. lubrica* (Roth). My reasons for doing this were that the size of the cells corresponds very closely with the measurements of that species as given by Prof. Rabenhorst, and the absence of anything that seemed to me definite in the descriptions of the two species. Moreover, if the possession of a parietal hyaline spot be not simply an accident of growth, it would indicate that the plant belongs to *P. lubrica.* I do not think, however, that any importance is to be attached to this, as the vacuole is often absent, and, although Prof. Rabenhorst makes no mention of it, is, in all probability, present in certain states or stages of *T. gelatinosa.* My own conviction is, at present, that *T. gelatinosa* and *T. lubrica* are very probably synonyms. If they be distinct, the plant from which the above description was taken is referrible to *T. perforata* (Harvey), which, if not new, is a form of *T. lubrica* rather than *T. gelatinosa.* If *T. lubrica* and *T. gelatinosa* be united, no grounds are left for sustaining the separateness of *T. perforata.*

Whilst botanizing in a primeval glade and forest, known as Bear Meadows, in this State, I came across a spring, covered with a *Tetraspora*, which appears to represent the *T. gelatinosa* type. It formed great masses half an inch in thickness, at first attached, afterwards floating and covering the surface of the pool for several feet each way. When young these masses were elongated and were formed of numerous lobes attached often by very slender pedicles, and having their margins thickened and undulated so as to give a beautiful waved appearance to the light green mass. Under the microscope the structure was similar to that of the other form, except that the cells varied more and attained a greater size. Their diameters ranged from $\frac{1}{3750}'' = 0.00027''$ to $\frac{1}{1500}'' = 000066''$.

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I have also received from Prof. Ravenel specimens of a *Tetraspora*, which may be the young of a variety of this species, but which is very possibly distinct. If the specimens are adult, it certainly is. They consist of numerous little fronds not more than a third of an inch in length, often composed of several subcylindrical arms, as it were, radiating from a central portion, and attaining a length of a third of an inch or so. These fronds are irregularly perforate, and are composed of cells agreeing perfectly in form, size, and arrangement with the more ordinary forms of *T. lubrica*.

T. bullosa, (ROTH) AG.

T. thallo membranaceo-saccato, obovato, sinuoso-bulloso, unciam usque palmam longo, postea explanato, dilacerato, saturate viridi, plus minus verrucoso; cellulis subsphericis (post divisionem factam hemisphæricis vel angulosis) geminis vel quaternis, confertis, granulosis. (R.) *Species mihi ignota.*

Diam.—Cell. ante divis. 0.00032″—0.00049″; post divis. 0.00022″—0.00029″. (R.)

Syn.—*T. bullosa,* (ROTH) AG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 39.

Hab.—"Salem, North Carolina. Schweinitz, Newburgh, New York." Bailey, Silliman's Journal, New Series, vol. iii.

Thallus membranaceous saccate, obovate, sinuosely-bullose, from one to six inches in length, afterwards expanded, torn, deep green, more or less verrucose; cells subspherical (after division hemispherical or angular) in twos or fours, crowded, granular.

Genus DICTYOSPHÆRIUM, NÆG.

Thallus gelatinosus plus minus liquidus, libere natans, sæpe quasi nullus. Cellulæ vesicula chlorophyllosa unica et locello achroo parietali prædita, tegumentis crassis in gelatinam homogeneam confluentibus involutæ, filis propriis subtilibus dichotome divisis, e familiarum centro ad peripheriam radiantibus connexæ. Cellularum divisio ad omnes directiones.

Propagatio fit gonidiis mobilibus

Thallus gelatinous, more or less liquid, swimming free, often almost wanting. Cells furnished with a single chlorophyllous vesicle and a lateral transparent spot, surrounded with thick coats, which are confluent into a homogeneous jelly and united by very fine filaments, which are dichotomously divided and radiate from the centre to the peripheral families. Division of the cells occurring in all directions.

Propagation by motile gonidia.

D. pulchellum, WOOD, (sp. nov.)

D. thallo subgloboso vel subovale, interdum subnullo, interdum indistincte lobato; cellulis globosis plerumque sparsis sed interdum nonnihil confertis.

Diam.—Cell. $\frac{1}{4000}$″ = 0.00025″; thalle plerumque $\frac{1}{300}$″ = 0.0033″; interdum $\frac{1}{185}$″ = 0.0054.″

Hab.—In stagnis prope Philadelphia.

Thallus subglobose or suboval, sometimes indistinctly lobate, sometimes almost wanting; cells globose, mostly scattered, but sometimes rather crowded.

Remarks.—I found this little plant, one August day, floating, in company with *Closterium acerosum*, in a brick-pond below the city. The little fronds are mostly roundish, or longer than broad, with a distinct outline, sometimes, however, the constituent jelly seems to fade into the surrounding water. There is never a distinct outer coat. The lateral transparent spot in the cells is mostly very evident, sometimes it is wanting, however. Occasionally there is a very distinct blackish "eye spot." The threads which join the cells are very delicate, and I have never been able to absolutely demonstrate their meeting in the centre of the frond, although I believe they do so. In mounted specimens, even when preserved in carbolic acid water, they disappear after a time. I have never seen zoospores or any other reproductive bodies.

Genus RHAPHIDIUM, Ktz.

Cellulæ fusiformes vel cylindraceæ, utrinque (plerumque) sensim sensimque cuspidatæ vel acuminatæ, rarius obtusatæ, rectæ vel varie curvatæ, singulæ, geminæ vel fasciculatim aggregatæ, medio decussatim vel radiatim conjunctæ, rarius binæ sub polis lateraliter connexæ, ceterum liberæ. Cytioderma tenue, læve. Cytioplasma viride, subtiliter granulosum, locello pallidiori vel achroo, centrali, rarius laterali, præditum. Cellularum divisio ad unam directionem. (R.)

Cells fusiform or cylindrical, generally very gradually cuspidate or acuminate at the ends, rarely obtuse, straight or variously curved, single, geminate, or fasciculately aggregate, decussate in the centre or radiately conjoined, rarely two laterally united at the end, other cells free. Cytioderm thin, smooth. Cytioplasm green, very finely granular, furnished with a central or rarely lateral transparent vacuole. Division of the cells occurring only in one direction.

R. polymorphum, Fresen.

R. cellulis rectis vel varie curvatis, singulis, vel 2–4–8–16 fasciculatim collocatis, gracilibus, sæpe gracillimis, nonnunquam medio paullum turgidis, subventricosis, nonnunquam paullum constrictis, apices versis, sensim attenuatis, acutissimis.

Diam.—$\frac{1}{7500}''=.00013''$.

Syn.—*R. polymorphum*, Fresen., Rabenhorst, Flora Europ. Algarum, Sect. III. p. 44.

Hab.—Prope Philadelphia, Wood.

Cells straight or variously curved, single or 2–4–8–16 fasciculately joined together, slender, often exceedingly so; sometimes slightly turgid in the centre, subventricose, sometimes slightly constricted; the apices gradually attenuate, very acute.

Var. **falcatum.**

Cellulis fusiformibus, gracillibus, utroque fine acutissime cuspidatis, curvatis vel semilunaribus, 4–16 fasciculatim congregatis.

Syn.—*Ankistrodesmus falcatus.* (Corda.) Rabenhorst, Flora Europ. Algarum, Sect. III. p. 45.

Hab.—South Carolina, Georgia, Florida, Rhode Island. (Bailey.)

Cells fusiform, slender, at each end very acutely cuspidate, curved or semilunar, 4–16 fasciculately congregate

Remark.—Fig. 3, pl. 7, represents different forms of *R. polymorphum.*

Family PROTOCOCCACEÆ.

Algæ unicellulares sensu strictissimo, chlorophyllosæ, et vegetatione terminali et ramificatione vera carentes, sine cellularum generatione vegetativa. Vivunt aut singulæ, segregatæ aut in familias consociatæ. Harum familiarum cellulæ numero aut indefinitæ semper se augentes (tum sensu vero familiæ nomen ferunt), aut definitæ, se non augentes (quæ cœnobium dicuntur).

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Propagatio fit gonidiis, quæ intra cellulam matricalem cytiogenesi libera oriuntur et duplicis indolis sunt; altera majora, quæ macrogonidia, altera minora quæ microgonidia dicuntur; illa oblonga, polo antico plerumque rostelliformi-producta, pallidiora, ciliis vibratoriis prædita, polo postico truncato-rotundata, obscure viridia, individuum propagant; hæc forma similii, itidem mobilia, brevi postea in statum quiescentem transeunt, druique in sporas perdurantes (*Hypnosporas*, BRAUN) transmutantur. (R.)

Unicellular algæ, in the strictest sense of the word, chlorophyllous, without terminal growth or true branching, without a vegetative generation of cells. They live either single, segregate, or consociated into families. The cells of these families, either indefinitely increasing in number (then families in the true sense of the term), or of definite number (then forming a cœnobium).

Propagation by means of gonidia arising within the mother-cell by free cell-formation; gonidia of two kinds; the one larger, macrogonidia—the other smaller, microgonidia; the former oblong, mostly produced into a pale bicilate beak anteriorly, rounded and greenish at their hinder end, developing into the individual plant; the microgonidia similar to these and also motile, but passing after a short time into a quiescent state, and at last into resting spores or hypnospores.

Genus PROTOCOCCUS, AG. 1824.

Cellulæ sphæroideæ, segregatæ, cytiodermate tenui, hyalino, absque tegumentis, libere natantes vel extra aquam in stratum tenue pulvereum cumulatæ. Cytioplasma initio homogeneum, denique granulosum, viride vel rubellum.

Spheroidal cells, segregate, cytioderm thin, hyaline, without integument, swimming free or collected out of water into a thin pulverulent stratum. Cytioplasm in the beginning homogeneous, finally granular, green, or reddish.

Remarks.—I have introduced this genus as given by Professor Rabenhorst in his Flora Europæa Algarum for the purpose of describing a little plant, upon which I have made some observations. As the notes were originally drawn up as a description of a species, I leave them in that form. I believe it has never before been described.

Protococcus, (sp. nov. ?)

P. aquaticus; cellulis globosis vel angulis, viridibus in stratum pulvereum cumulatis vel in familias arcte conjunctis; cytiodermate plerumque distincto; sporis rotundatis, tegumentis duobus vel tribus protectis; tegumentis externis, crassibus; zoogonidiis ovalibus, vel subrotundatis, vel subellipticis, ciliis duobus instructis.

Diam.—Max. spor. perdurant. $\frac{7}{7500}''$ = .00093″; microg. $\frac{4}{7500}''$ = .00053″.

Aquatic; cells green, globose or angular, accumulated in a green pulverulent stratum, often closely united into families; cytioderm mostly not distinct; resting spores round with two or three thick coats; zoospores oval or roundish, or somewhat elliptical, furnished with two cilia.

Remarks.—I found this species growing in a spring near Hestonville, West Philadelphia, in the month of March. The large winter spores are round, with thick coats. Except in one instance, in which the color was a decided reddish-brown, all that I have seen have been green. How they are produced I do not know. The history of their development into the plant appears to be as follows: The first change is the rupture of their outer thick coat (fig. 4 *b*, pl. 7) from which the spore finally escapes still clothed with a coat of moderate thickness. The green contents next divide into a number of oval bodies (fig. 6 *b*, pl. 7) which

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grow, and, at the same time, separate from one another. Whilst these changes have been taking place the spore coat has been becoming gelatinous and enlarging, so that it continues to enclose its progeny. In this way a family of oval cells is formed (fig. 4 *b*, pl. 7). So far, I think, is positive. The next step I have never actually seen, but believe to be the escape of these oval bodies as zoospores (fig. 4 *c*, pl. 7) which are of very various sizes and are elliptical, globose, or oval. They have a tolerably well-marked bright vacuole at their beak, and after swimming about actively for a time finally settle down, lose their cilia, and undergo division. They seem often to cluster together before thus becoming quiescent, so as to make little colonies (fig. 5, pl. 7).

Genus CHLOROCOCCUM, Fries.

Cellulæ sphæroideæ, aut singulæ, liberæ, vesicula chlorophyllosa et locello laterali pallidiori cavo? instructæ, limbo hyalino et tegumentis sæpe amplissimis cinctæ, aut plures in stratum vel acervulos cumulatæ.

Propagatio fit zoogonidiis cytioplasmatis divisione ortis, e cytiodermatis abaviæ (intellige tegumentum extremum) rupturis excedentibus.

Cells spheroidal, either single, free, furnished with a chlorophyllous vesicle and a paler lateral (hollow?) spot, with a hyaline nimbus and surrounded by a wide coat; mostly accumulated together into strata or little heaps. Propagation by means of zoospores, which are formed by a division of cytioplasm and escape from their general tegument (the cytioderm of the original cell).

Remarks.—But a few weeks after the commencement of my study of fresh-water algæ, a friend, a young microscopist, asked me to look at his aquarium, as the water of it had become stagnant, opaque, and green. On examining a little of the water with the microscope it was found to be full of what I now know to have been either one of the forms already described under this genus, or else one undescribed, but still embraced within its limits. There were two sets of bodies, the one motile the other at rest. The motile forms (Fig. 5, pl. 3) were globular or pyriform, and generally contained a large, roundish, green, distinct mass. They were of course provided with cilia, although at that time I was not able to demonstrate their presence. These bodies, even when moving, appeared to have a distinct wall. After a time they settled down and assumed the quiescent state. The outer coat now rapidly enlarged so as to leave a considerable space between it and the green endochrome, which rapidly underwent division, forming two or more new cells which were still surrounded by the enlarged maternal coat. The number of daughter-cells enclosed in the parent cell varied. A considerable quantity of the water was allowed to stand in a glass jar, exposed to the light. In a very few days all the motile forms had disappeared. The contents of the vessel were allowed slowly to evaporate. The jar being tall and narrow it was some weeks now before this process was completed, before which consummation *hœmatococcus* forms were abundantly developed.

Instead of being green, and surrounded by a distant, almost sac-like wall, the cells had acquired a dark brownish-red color, were very opaque, and were protected by a thick wall, whose surface was quite rough. Unfortunately, I did not measure either the active gonidia or their progeny, the quiet cells, but I found the general

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diameter of these hæmatococcus cells to be one twelve-hundredth of an inch (.00083″).

MM. Famnitzin and Boranetzky, in a recent paper ("Zur Entwickelungsgeschichte der Gonidien und Zoosporenbildung der Flechten," Mem. de L'Académie Impériale des Sciences de St. Petersbourg, 1868, Annals and Mag. Nat. History, Feb. 1869), state as the result of direct observation that this genus of algæ, so called, is really a stage in the life history of the gonidia of lichens. These gentlemen took thin slices of lichen thalli containing gonidia, and placed them upon pieces of fir and linden bark, which had been previously boiled to kill any plants that might be growing on them. These were then put in a glass jar inserted over a vessel containing water, in such way that they would be constantly exposed to a very damp atmosphere, and at the same time communication with the external air would be impossible. In another set of experiments, pieces of the lichens were allowed to lie for a long time in water, until the component filaments were decomposed into a gelatinous mass, in which the still green vigorous gonidia were imbedded. These pap-like (*breiige*) masses were then washed with pure water and smeared upon pieces of linden bark. The results obtained were identical in the two cases. The gonidia were at first provided each with a distinct nucleus and a well-marked lateral vacuole, and resembled closely the first form of cystococcus. The next change was a division of their contents into a large number of roundish masses, with the disappearance both of the vacuole and of the central nucleus. The cell-membranes were next ruptured, and the endochrome, protruding through the opening, formed a little ball sitting upon the parent cell. In doing this it doubled in size, so that the part without was as large as the part within, although the latter still filled the cell. The contents finally escaped, but were yet surrounded by a very thin membrane, which soon, however, ruptured, and freed the biciliated zoospores into which the endochrome had in the mean time resolved itself. These zoospores remained a long time in the motile state, but finally settled down, dropping their cilia, and became little round cells, which grew to three or four times their original size. Further development was not made out.

Certain of the gonidia, belonging to a lichen of the genus *Physcia*, failed to produce zoospores, but their endochrome, divided so as to form a number of quiescent cells, which either ruptured very early the original cell-membrane and became free in the water, or else remained bound together by it into a family for a longer period. In these researches MM. Famnitzin and Boranetzky employed lichens of three genera, namely *Physcia*, *Cladonia*, and *Evennia*, and claim, as above stated, that their investigations prove that they developed the algæ genus *Cystococcus* of Nægeli (*Chlorococcum*, Fries), from the gonidia.

Genus POLYEDRIUM, NÆGELI, (1849.)

Cellulæ singulæ, segregatæ, libere natantes, compressæ, 3–4–8 angulares, angulis plus minus productæ, nonnunquam radiatim elongatæ, aut integræ aut bifidæ, plerumque armatæ, a latere oblongo-ellipticæ, utroque polo rotundatæ vel subtruncatæ. Cytioderma tenue, læve. Massa chlorophyllacea plerumque granulosa, per cellulæ lumen æqualiter distributa, nonnunquam guttulis oleosis rubris 1–4 mixta.

Propagatio adhuc ignota. (R.) *Genus mihi ignotum.*

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Cells single, segregate, swimming free, compressed, 3–4–8-angled, more or less produced as to their angles, sometimes radiately elongate, either entire or bifid; mostly armed, oblong-elliptical when viewed laterally, at each end rounded or subtruncate. Cytioderm thin, smooth; chlorophyl mostly granular, equally distributed through the cell, sometimes mixed with reddish oil-drops.

Propagation unknown.

Remarks.—This genus was described by Nægeli in his "Gattungen Einzelliger Algen," and, although I have never seen any specimen of it, it claims a place here, because one species has been found in this country by Prof. Bailey.

P. enorme, (RALFS) DE BARY.

P. tetraëdiicum, angulis productis achrois profunde bilobis, nonnunquam repetito-bilobis, lobis mucronatis. (R.)

Diam.—0.0011″—0.0016″. (R.)

Syn.—*P. enorme,* (RALFS) DE BARY. RABENHORST, Flora Europ. Algarum, Sect. III. p. 62. *Staurastrum enorme,* RALFS, British Desmidieæ.

Hab.—Florida. Bailey.

"Frond irregular or quadrate, spinous; end view three or four-lobed; lobes broad, more or less emarginate or bifid, and terminated by spines, which are either simple or branched. Sometimes the front view differs but little from the end one, usually, however, there is a slight constriction or sinus at the junction of the segments, but I have never observed any difference in the endochrome at that part. The spines, which are almost confined to the angles, are irregular, some simple and some branched. The end view has three or four broad and very irregular lobes; these are spinous and more or less emarginate, and frequently one lobe is much broader and more spinous than the others. The spines on such lobe form two groups, separated by the notch; they vary much in size and are either simple and subulate, or else forked; sometimes the forked spines are again divided at the apex."—*Ralfs' British Desmidieæ,* p. 141.

Genus SCENEDESMUS, MEYEN.

Cellulæ polymorphæ, utroque polo æquales vel inæquales, sæpe in cornu spiniforme productæ, in ætate perfecto 2–16 aut in seriem simplicem aut parenchymatice arcte conjunctæ et cœnobium constituentes; cytioplasmate initio homogeneo, postea granuloso, vesicula chlorophyllosa centrali vel sublaterali et sæpe locello achroo laterali instructo.

Propagatio fit cytioplasmatis divisione succedanea, unde gonidia oriuntur, quæ intra cellulam matricalem jam in cœnobium planum sese conjungunt et membranæ matricalis ruptura vel dissolutione prodeunt.

Cells polymorphous, equal or unequal at the ends, often produced into a spine-like horn, in the perfect state 2–16 closely conjoined, either as a simple series or in a parenchyma-like manner so as to form a cœnobium. Cytioplasm in the beginning homogeneous, afterwards granular, furnished with a central or sublateral chlorophyllous vesicle, and often with a lateral transparent spot.

Propagation occurring as a succedaneum to the division in the cells, whence arise gonidia, which, already within the mother-cell, join themselves into a cœnobium, and are finally set free by the rupture and dissolution of the maternal cell-wall.

Remarks.—According to Unger, in the genus *Scenedesmus* the cells never exist singly, but always in families.

Two of the species here described as representatives of the genus certainly do not conform to this, for I have frequently seen them both separate and in cœnobia or families. The latter were exactly like those of the European forms, at least in one of the two species, and I do not therefore think it justifiable to indicate a new genus. Moreover, I have certainly seen single cells, belonging to a species which agrees precisely in its characters with a European form, save only in the occasional existence of these single cells.

I have never studied the method of propagation, but it is said to occur by the division of the cytioplasm of a large cell into a minute cœnobium composed of two or more cells, which remains for some time within the walls of the mother-cell, but is finally set free by the solution of the latter.

The cells are mostly much longer than broad, cylindrical, elliptical, or oval, but in one species herein described they are habitually globular.

a. Cellulæ inermes.

a. Cells unarmed.

S. obtusus, MEYEN.

S. cellulis oblongis vel ovatis, utroque polo obtusis, 4–6–8 modo arcte modo laxe in seriem simplicem aut rectam aut duplicem obliquam conjunctis, diametro 3–5 plo longioribus. (R.)

Diam.—Transv. max. 0.00023″—0.00028″. (R.)

Syn.—*S. obtusus,* MEYEN. RABENHORST, Flora Europ., Algarum, Sect. III. p. 63.

Hab.—Georgia: Rhode Island, Bailey.

Cells oblong or ovate, obtuse at each end, 4–6–8, partly closely partly laxly conjoined into a simple series either straight or oblique and double, 3–5 times longer than broad.

Remark.—I have never met with this species.

S. acutus, MEYEN.

S. cellulis fusiformibus, vel ovato-fusiformibus vel ovatis, utrinque acutis sed inermibus, interdum singulis sed plerumque in seriem aut simplicem rectam aut duplicem inordinate alternantem dispositis, arcte concretis, diametro 2–4 (6?) -plo longioribus.

Diam.—Trans. vag. max. ? .00016″.

Syn.—*S. acutus,* MEYEN. RABENHORST, Flora Europ. Algarum, Sect. III. p. 64.

Hab.—Prope Philadelphia, Wood. Rhode Island, Bailey.

Cells fusiform, or ovate-fusiform or ovate, acute at each end but unarmed; sometimes single but mostly conjoined into a single straight series or into an irregularly alternate double series, 2–4 times longer than broad.

Remarks.—This species is common around Philadelphia. Our specimens agree very well with the descriptions and figures of the European, excepting that occasionally a cell is single, and that none which I have measured have attained the size given by Prof. Rabenhorst as the maximum, namely, 0.00023″. According to Rabenhorst, *S. obliquus,* Ktz., is only a variety of *S. acutus,* Meyen. It has been found by Prof. Bailey in South Carolina, Georgia, and Rhode Island.

b. Cellulæ armatæ.

b. Cells armed.

S. polymorphus, Wood.

S. cellulis fusiformibus, aut ovalibus aut ellipticis aut globosis, singulis aut 2–8 conjunctis, plerumque utroque polo aculeo unico, interdum aculeis duobus, instructis: apicibus obtusis, acutis, vel acutissimis; aculeis gracillimis, rectis, modice elongatis, inclinatis.

Diam.—$\frac{1}{2500}''$—$\frac{1}{7500}''$; plerumque $\frac{1}{4000}''$.

Syn.—*S. polymorphus*, Wood, Prodromus, Proc. Am. Philos. Soc., 1869, p. 135.

Hab.—In aquis quietis prope Camden, New Jersey.

S. cells fusiform, or oval, or elliptic, or globose, single or 2–7 conjoined, furnished in most cases with a single spine, sometimes 2, at each end; ends obtuse, acute, or very acute; spines exceedingly slender and acute, straight, moderately long, inclined.

Remarks.—This plant was found in a quiet pool, filling the water in such numbers as to make it opaque and very green. The color of the cells, as first obtained, under the microscope, was a vivid green, but, the water containing them having been placed in a dish, during the slow desiccation which followed the color of the cells changed to a golden yellow.

Fig. 1, pl. 11, represents different forms of this species magnified 450 diameters.

S. quadricauda, (Turpin) Bréb.

S. cellulis oblongo-cylindricis, utroque polo obtuse rotundatis, 2–4–8 arctissime conjunctis, ordine aut simplici recto aut duplice alternante, omnibus rectis, medianis inermibus vel his illisve apice uno alterove aculeo curvato instructis, extimis utroque apice sæpius item dorso armatis.

Diam.—0.00035″—0.00039″; long. 0.00091″.

Syn.—*S. quadricauda,* (Turpin) Bréb. Rabenhorst, Flora Europ. Algar., Sect. III. p. 65.

Hab.—Rhode Island, Bailey. Pennsylvania, Wood.

Cells oblong-cylindrical, obtusely rounded at each end, 2–4–8 very closely conjoined either in a single straight series or a double alternating one, all straight, the median unarmed or some of them with the apex furnished with a curved spine, the external with both apices and sometimes the dorsum thus armed.

Remark.—Fig. 2, pl. 11, represents this species magnified 750 diameters.

S. rotundatus, Wood, (sp. nov.)

S. cellulis globosis vel subglobosis, spinulis longissimis, rectis, gracillimis, acutissimis, 3–6 armatis, aut singulis aut geminis aut 3–4 arcte duplice conjunctis.

Diam.—$\frac{1}{6000}''$ to $\frac{1}{3000}''$.

Hab.—In aquis quietis prope Philadelphia. (Dr. Chapman.)

Cells globose or subglobose, armed with three to five very long, slender, acute, straight spines, single or in pairs, or three to four closely conjoined in a twofold rank.

Remarks.—The cells of this species are globular, and, when more than two, they are arranged in two rows placed at right angles one to the other. The contents of the cells are markedly granular, and the endochrome a bluish-green, and from the surface of the walls project outwards, very long and fine, rigid hair-like spines.

It seems scarcely correct to place this plant in the genus *Scenesdesmus*, but I do

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not know any other genus to which it is more closely allied, and do not feel disposed to indicate a new one for it.

Fig. 3, pl. 11, represents a cell-family magnified 250 diameters.

Genus HYDRODICTYON, Roth. (1800.)

Cellulæ oblongo-cylindricæ, in cœnobium reticulato-saccatum connexæ, omnes fertiles; aliæ procreant macrogonidia, quæ jam intra cellulam matricalem in cœnobium filiale se connectunt; aliæ microgonidia, quæ multo minora, cellulæ matricalis membranam perrumpunt, polo antico ciliis vibratoriis binis et puncto rubro laterali prædita sunt, brevi postea in globulos protococcoideos tranquillos transformata sporas perdurantes efficiunt.

Cells oblong-cylindrical, joined into a reticulated saccate cœnobium, all fertile; some producing macrogonidia, which join themselves into a cœnobium within the parent cell; the others producing microgonidia, which are furnished with two vibratile cilia and a lateral red spot, and which, escaping from the parent cell, are, after a brief period of motile life, transformed into protococcoid thick-walled spores.

Remarks.—The genus *Hydrodictyon* comprises, as far as known, but a single species, which is common to North America and Europe. It grows in great abundance in the neighborhood of Philadelphia, especially in the ditches and stagnant brick-ponds in the low grounds below the city known as the "Neck." There it very frequently forms floating masses several inches in thickness and many feet in extent, so that with the aid of a rake it could be gathered by the bushel. When thus in mass the color is very generally dingy and yellowish, although the fronds, when in active vegetative life, are mostly of a bright, beautiful green. The plant is in greatest profusion in June and July, after which time it gradually disappears, until in the autumn it is scarcely to be found, but early in the spring it reappears. The very young fronds are minute, oval, cylindrical, filmy-looking, closed nets, with the meshes not appreciable to the eye; when growth takes place, the fronds enlarge until finally they form beautiful cylindrical nets two to six inches in length, with their meshes very distinct and their ends closed. In the bright sunlight they, of course, by virtue of the life-functions of their chlorophyl, liberate oxygen, which being set free in the interior of the net, and its exit barred by the fine meshes, collects as a bubble in one end of the cylinder and buoys it up, so that, the heavier end sinking, the net is suspended, as it were, vertically in the water. I know of few things of the kind more beautiful than a jar of limpid water with masses of these little nets hanging from the surface like curtains of sheen in the bright sunlight. A few cells collected in the fall or early spring, if put into a preserving-jar and the water occasionally changed, will multiply, and in a little while become a source of frequent pleasure to the watcher.

As the fronds increase in size they are always in some way or other broken up, so that, instead of being closed cylinders, they appear as simple open networks of less or greater extent. The extreme length to which the frond attains is, I think, very rarely over twelve inches, with meshes of about a third of an inch in length. The construction of the frond is always the same. It is composed of cylindrical cells united end to end in such a way as to form polygonal, and mostly pentagonal meshes, the size of which varies with the age of the plant. These cells, which are closely conjoined but have no passage-ways between them, are capable of independent life, so that the hydrodictyon may be looked upon as an elaborate type of a cell-family, one in which cells are conjoined in accordance with a definite plan, so as to make a body of definite shape and size, yet in which each cell is an independent being, drawing nothing from its neighbors. The cells themselves are cylindrical, with a thickish cellulose wall, and have no nuclei. Their chlorophyllous protoplasm is granular, and is placed in the exterior portion of the cell, forming thus, within the outer wall, a hollow cylinder, in which are imbedded starch granules, and whose interior is occupied with watery contents. The hydrodictyon cell, when once formed, is capable of growth, but not of going through the usual process of cell multiplication by division, so that the adult frond is composed of just as many and indeed the same cells, as it had in its earliest infancy.

No true sexual reproduction has as yet been discovered in the water-nets. There have been described, however, two forms or methods in which the species multiplies, both of them occurring by means of motile zoosporoid bodies. In the one case these develop immediately into the new plant, whilst in the other before doing so they pass through a resting stage. Of the life-history of the latter, the *microgonidia*, I have no personal knowledge.

The investigation of the production and development of the *macrogonidia*, however, has occupied considerable of the time devoted by myself to the microscope, and I have seen large numbers of specimens in almost all the stages of development. I have never been able to detect, however, any decided motion in the *macrogonidia*.

They are formed in the protoplasmic stratum, already alluded to as occupying the outer portion of the interior of the hydrodictyon cell. The first alteration in this, presaging their formation, is a disappearance of the starch granules, and a loss of the beautiful, transparent green color. Shortly after this, even before all traces of the starch-grain are gone, there appear in the protoplasm numerous bright spots placed at regular intervals; these are the centres of development around which the new bodies are to form. As the process goes on, the chlorophyl granules draw more and more closely around these points, and at the same time the mass becomes more and more opaque, dull, and yellowish-brown in color. This condensation continues until at last the little masses are resolved into dark hexagonal or polygonal plates, distinctly separated by light, sharply defined lines. In some, the original bright central spot is still perceptible, but in others it is entirely obscured by the dark crowded chlorophyl. The separation of these plates now becomes more and more positive, and they begin to become convex, then lenticular, and are at last converted into free, oval, or globular bodies. When these are fully formed, they are said to exhibit a peculiar trembling motion, mutually crowding and pushing one another, compared by M. Braun to the restless, uneasy movement seen in a dense crowd of people in which no one is able to leave his place. Whilst the process just described has been going on, the outer cellulose wall of the hydrodictyon cell has been undergoing changes, becoming thicker and softer and more and more capable of solution, and by the time the gonidia are formed it is enlarged and cracked, so that room is afforded them to separate a little distance from one another within the parent cell. Now the movements are said to become more active—a trembling jerking which has been compared to the ebullition of boiling water. There is, however, with this a very slight change of space, and in a very short time the gonidia arrange themselves so as to form a little net within the parent cell, a miniature in all important particulars of the adult hydrodictyon. The primary cell-wall now becomes more and more gelatinous, and soon undergoes complete solution, so that the new frond is set free in its native element. As previously stated, in my investigations I have never seen the peculiar motion above described, the newly formed gonidia simply separating and arranging themselves without my being able to perceive any motion, or exactly how they fell into position.

It is evident that when the species is multiplied in the way just described, the birth of the new frond is consentaneous with the death of the old cell. But when the hydrodictyon disappear in the fall, it is months before they reappear in the spring. It is, therefore, evident there must be some other method of reproduction. This slow development of new fronds takes place, according to Pringsheim, by means of little motile bodies which he calls "*Dauerschwärmer*," which has been translated into English *chronispores* (*statospores*, Hicks). M. Braun stated already some years since that sometimes, instead of the hydrodictyon producing the ordinary reproductive bodies (*macrogonidia*), there are formed in the cells much smaller and more active bodies, the *microgonidia*. The changes which occur in the production of these are very similar to those already described as happening when the macrogonidia are formed. When the *chronispores* are once formed, however, they, instead of uniting together escape in a free distinct condition into the water. They are now small ovate bodies, with a large anterior transparent space, to which are attached a pair of cilia, and their life and history, according to Pringsheim, is as follows: For a few hours they move about very actively in the water, and then, dropping their cilia, and acquiring an outer cellulose wall, pass into a quiescent stage, in which they closely resemble protococcus granules. They are capable of living in this state for a long time, if kept in water. They can also endure desiccation if the light be excluded during the process, but, if it be present, they wither and die, and cannot be revivified.

After a longer or shorter period, but never shorter than three months, according to Pringsheim, they recommence their life, provided they be in water. For four or five months after this the chief change consists simply in an increase in size. The dark-green protoplasm is arranged around the exterior of the cell, within are the more fluid colorless contents, the whole body still looking like a protococcus cell. After a size of about $\frac{1}{40}$ mm. is attained, the endochrome divides successively into several portions. The external layers of the surrounding wall now give way in some spot and allow the inner layers to protrude and form a sort of hernial sac, into which the several endochrome masses soon pass, at the same time assuming the well-known characters of true zoospores. From two to five of these

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bodies are thus produced out of each original microgonidium. They are large, ovate, biciliate, and, generally, soon escaping from the hernial sac, move about actively in the water for a few minutes. Sometimes, however, they settle down within the generative utricle. In either case, after a little time, they become motionless, lose their cilia, and develop into polyhedral cells, which are structurally remarkable for having their angles prolonged into long horn-like appendages. Under favorable circumstances, at the end of a few days, the bright green endochrome of these undergoes similar changes to those described as presaging the production of the microgonidia, and is finally formed into zoospores, which, in from twenty to forty minutes, unite, within the polyhedron or large cell, into a *Hydrodictyon*, which is finally set free by a solution of the cellulose coat of the polyhedron. The network thus formed differs in no essential way from that which arises in the better known way, except that it is composed of much fewer cells. It is generally a closed sac; but when the polyhedron, out of which it is developed, is small, it is sometimes merely an open network. Its after-history appears to be identical with that of the ordinary hydrodictyon frond.

H. utriculatum, Roth.

Species unica.

Syn.—*H. utriculatum*, Roth. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 66.

Hab.—In aquis quietis. West Point, Bailey. Weehawken, (Mr. Walters.) "Waterholes between Van Horn's Mills and Mueote on the Mexican boundary, Dr. Bigelow. Pennsylvania, Wood.

Genus PEDIASTRUM, Meyen, (1829.)

Cœnobium planum, disciforme, libere natans, e celluarum strato unico, rarius centro entro duplicato, continuo vel perforato formatum. Cellulæ polygoniæ, periphericæ sæpe bilobæ, lobis cuneatis et simplicibus et bidentatis, nonnunquam in cornua productis.

Cœnobium plain, discoid, swimming free, formed of cells in a single, rarely in the centre double stratum, which is continuous or perforate; cells polygonal, the peripheral often bilobed, the lobes cuneate, either simple or bidentate, sometimes produced into a horn.

Remarks.—The cœnobium or cell-family, or colony, in the genus *Pediastrum* is always discoid, and has generally a more or less truly circular outline. The cells are mostly in a single stratum, but in some species there are two, more or less, complete strata superimposed one upon the other. The arrangement of the cells in this stratum, or these strata, varies greatly, as does also their number. They are mostly more or less polyhedral, and often have their margins scooped out or their angles prolonged. This may occur in such a way that the projecting point of one cell fits into the hollow in its neighbor, and the cœnobium be rendered entire, or, no such relation existing between the parts of adjacent cells, the cœnobium may be perforated with regular or irregular openings. The outer or marginal cells are often deeply notched externally, and frequently are prolonged into acute or obtuse lobe-like processes. The walls of the cells are, in adult specimens, quite thick. The contents consist of chlorophyl, protoplasm, starch granules, &c.

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There are generally one or more hyaline spaces, besides a distinct chlorophyl vesicle, but no distinct nuclei.

At certain periods of their existence the Pediastrums produce both macrogonidia and *microgonidia*. The life-history of the former is very similar to that of the same bodies in the water-nets. The ultimate fate of the microgonidia has not as yet been determined, but in all probability they go through cycles of change similar to those seen in the lives of the corresponding bodies in the *Hydrodictyon utriculatum*. I have not had an opportunity of watching the development of either of these reproductive forms, but, according to MM. Braun, Pringsheim, &c., their life-history, as far as known, is as follows: In most cases, all the cells of a pediastrum produce their macrogonidia simultaneously, or within a very short period of time, so that the cœnobium will be left emptied of its contents as a mere shell, the outer skeleton of its former self. When a cell is about to give birth to these reproductive bodies, the endochrome divides into two parts; each of which then undergoes a similar binary division. This is repeated once, twice, thrice, or oftener, until the endochrome is divided into 8–16–32–64 gonidial masses, the number of which, generally, but not always, corresponds to the number of cells in the colony, to which the parent-cell belongs. After the division of the endochrome is completed, a slit occurs in the outer strata of the wall of the mother-cell through which a hernial protrusion of the inmost stratum occurs. The protruded part now rapidly enlarges until at last there is formed a sort of hourglass-shaped sac, one portion of which is within, the other part without, the old parent-cell. Whilst this has been going on a portion of the gonidia have escaped from the parent-cell into the outer free portion of the sac, and each end of the hourglass, therefore, contains some of them. The sac with its contents now gradually withdraws itself more and more from the parent-cell until at last it lies a free globose vesicle in the water. The gonidia occupy the centre, and M. Braun states, that, although he has never been able to demonstrate any cilia upon them, yet they have an active swarming motion. At first, they are irregularly heaped together in the nearly filled sac; but the latter rapidly enlarges and elongates, and the gonidia in a little while arrange themselves in a flat, tabular group within it, and cease to move. Then the several individuals of this group begin to develop, becoming emarginate and assuming the form of the parent-cell, until, finally, they have all grown into the shape which is peculiar to the adult cells of the species, and after a few hours have closely cohered to form a young cœnobium.

The microgonidia are formed in a very similar way by the dividing of the endochrome, the cracking of the outer membrane, and the protrusion and final escape of the inner. They are, however, much smaller and more numerous than the macrogonidia. When the parent vesicle first escapes into the water, they are crowded in its centre, and are nearly globose. As it enlarges, however, they elongate more and more, and finally become distinctly bi- or, more rarely, uni-ciliate. The cilia are much longer than the body, and are attached to the smaller end, which is prolonged into a pointed, transparent beak, about equal to the green portion in length. The microgonidia now become more and more restless, they, moving about very actively, and after awhile bursting the parent sac, escape into the water.

What becomes of them after this, as has been stated, is a mere matter of conjecture. M. Braun[1] and others have described unicellular forms of several of the multicellular species of *Pediastrum*, and Pringsheim suggests that these are really polyhedrons developed out of these microgonidia, as is seen in the water-nets. This, of course, may or may not be the case.

P. Boryanum, (Turpin) Mengh.

P. cœnobio orbiculari, oblongo vel elliptico, magnitudine vario, continuo, læte viridi, e cellulis 4–8–16–32–64 (rarissime 128) composito (cellularum strato simplici, nonnunquam medio duplicato); cellulis periphericis plus minus profunde emarginatis vel bilobis, lobis cornutis, cornibus achrois hyalinis, abbreviatis vel elongatis, teretibus, obtusis vel subobtusis, interdum capitellato-incrassatis, centralibus arctissime concretis, polygonis (4–6 angularibus), in antica parte modo angulo prominulo modo plane truncatis, modo leviter repandis, omnium membrana decussatim punctata. (R.)

Diam.—Transv. cell 0.000795″; rarius 0.00088″—0.00094″. (R.)

Syn.—P. Boryanum, (Turpin,) Menghini. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 74.

Hab.—Georgia, Florida, Rhode Island, Bailey; Pennsylvania, Wood.

Cells arranged in one or more circles round one or two central cells; the inner variable, generally concave at one side, the outer tapering into two long subulate points, the notch narrow. L. 1–2083″ to 1–1633″; B. 1–2733″ to 1–2222″. (Archer.)

P. Selenæa, Ktz.

P. cœnobio orbiculari, integro, e cellulis 8–16 (rarius 31 = 1 + 5 + 10 + 15, Ktz.) formato; cellulis periphericis angustis, lunatis, acute lobatis, disci cellulis leviter excisis, centrali unica 5-angulari, omnium membrana firma, subcrassa, ætate provecta rubescente. (R.)

Diam.—Cœnobii 0.00124″—0.0035″; cell. (distantiæ interlobos) 0.00026″—0.00069″. (R.)

Syn.—P. Selenæa, Ktz. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 73.

Hab.—Rhode Island, Bailey.

Cells crescent-shaped, arranged in one or more circles round one or two central ones, connecting medium colored. (A.)[2]

P. pertusum, Ktz.

P. cœnobio orbiculari, lacunis pertuso, magnitudine vario, e cellulis plerumque 1 + 5 + 10 + 15 (in formis quibusdam ad 64) composito; cellulis periphericis basi tantum laxe connexis, ad medium usque bilobis, lobis rectis, in cornua hyalina modo subacuta modo obtusa vel truncata plus minus productis, centralibus plus minus exacte quadrangularibus, et in antica parte et utrinque emaginatis, omnibus lævibus, locellis pallioribus finis instructis. (R.)

Diam.—Transv. cell. perfecte evolut. circiter 0.00065″—0.00089″. (R.)

Syn.—P. pertusum, Ktz. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 75.

Cells arranged in circles round one or two central ones; inner cells quadrangular, sides concave and leaving angular vacant intervals; the outer cells with square bases, externally triangularly notched, the subdivisions tapering to an acute point. L. 1–2266″; B. 1–3268″. (A.)

P. constrictum, Hassal.

P. cœnobio orbiculari vel suborbiculari, læte viridi, continuo, lævi ?, e cellulis 16 (ad 1 + 5 + 10) vel 32 (ad 1 + 6 + 10 + 15) formato; cellulis periphericis irregulariter bilobis, sinu

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[1] The best exposition of this genus is to be found in Braun's Unicellular Algæ.

[2] The letter A used here signifies that the description is copied from Mr. Archer in Prichard's Infusoria.

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angusto, lobis inæqualibus, basi plerumque constrictis, in cornua subcrassa obtusa productis, centralibus polygonis, in antica parte repandis. (R.)

Syn.—*P. ellipticum*, Hassal. Rabenhorst, Flora Europ. Algarum, Sect. III, p. 77.

Hab.—South Carolina, Georgia, Rhode Island, Bailey.

Cells varying in number and arrangement; outer cells suddenly contracted into two short, cylindrical, obtuse processes. L. 1–1754″ to 1–906″; B. 1–1515″ to 1–1020″.

β, Processes of the lobes truncately emarginate. (A.)

P. Ehrenbergii, (Corda) Braun.

P. cœnobio et orbiculari et oblongo, perfecte clauso, e cellulis 8 vel 16 composito et quadrato, e cellulis 4, late cuneatis, profunde lobatis, exacte cruciatim dispositis formato; cellulis periphericis cuneatis a basi truncata ad apicem usque concretis, profunde bilobis sinu angusto, lobis sæpe oblique truncatis, plus minus sinuato-excisis, angulis interioribus ad duplum longioribus, omnibus acutis vel breviter appendiculatis; cellulis centralibus aut singulis aut pluribus (2–5–6 v. 8), omnibus flavo-viridibus, polygonis, uno latere repandis vel profunde incisis. (R.)

Syn.—*P. Ehrenbergii*, (Corda,) Braun. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 77.

Hab.—South Carolina, Georgia, Florida, Rhode Island, Bailey.

Frond minute; cells eight (seven disposed in a single series round a central one), bilobed, angular. L. 1–2900″; B. 1–2500″. (A.)

P. simplex, Meyen.

P. cellulis periphericis ovato-cuspidatis, 8–10–16 basi tantum concretis, circulum simplicem constituentibus, centralibus sæpe nullis. (R.)

Syn.—*P. simplex*, Meyen. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 71.

Monactinus octonarius, Bailey, Smithsonian Contributions.

Hab.—South Carolina, Rhode Island, Bailey.

Var.—**duodenarius.**

Cœnobio clathrato, cellulis periphericis 12, centralibus 4, regulariter cruciatum dispositis. (R.)

Syn.—*Monactinus duodenarius*, Bailey.

Inner cells four, somewhat triangular, enclosing a central, quadrate vacant interval, and four broadly lanceolate vacant intervals between them and the outer series, to which they are united by their terminal angles, outer cells twelve, subovate, truncate below, much attenuated, acuminate. (A.)

Family VOLVOCINEÆ.

Cœnobia mobilia, globosa, subglobosa vel quadrangulo-tabulata, e cellulis viridibus cilia bina agilia gerentibus, intus vesica duplici contractibili præditis composita, membrana (tegumento, chlamyde) communi achroa hyalina plus minus ampliata involuta.

Propagatio aut sexualis, monoica vel dioica (adhuc in paucis tantum generibus probata); cellulis cœnobii aut omnibus aut quibusdam genus masculinum vel feminum exhibentibus, illis in fasciculos spermatozoideorum (antheridia), has in oosporas episporio inclusas, non mobiles commutatis, aut non sexualis, gonidiis agilibus, (et macrogonidiis et microgonidiis—etiam zoogonidia vocantur). Macro- et microgonidia (cellulæ primordiales) cytioplasmatis divisione simultanea et multiplici orta; priora numero definita (2–4–8–16, &c.), majora oblonga vel rotundata, polo antico plus minus rostriformi producta, ciliis binis per vesicæ membranam exsertis, puncto (ocello Ehrberg. stigma) sanguineo centrali vel parietali et locellis (vacuolis) sæpe binis contractibilibus instructa; ultima numero indefinita, multo minora, pallide vel sordide viridia vel luteola, apice ciliis instructa, plerumque jam intra cellulam matricalem vivide vacillantia, postea membranæ ruptura libere erumpentia, examinantia. (R.)

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Cœnobium mobile, globose, subglobose or in square tables, composed of green cells which have two motile cilia and a double contractile vesicle. The common tegument surrounding the cœnobium hyaline, and more or less amplified.

Propagation either sexual or non-sexual. The sexual monæcious or diæcious; either all or some of the cells of the cœnobium exhibiting male and female characters. The male cells containing spermatozoids, the female finally converted into a quiet oospore. Non-sexual propagation taking place by means of motile gonidia (both macrogonidia and microgonidia, by some called zoogonidia). Macro- and micro-gonidia arising by the simultaneous and repeated division of the cytioplasm; the first definite in number (2–4–8–16, &c), the larger, oblong or rounded, with the anterior end more or less rostellate, with two cilia exserted through the membrane of the vesicle, furnished with a central or parietal red spot, and often with two contractile vacuoles; the microgonidia indefinite in number, much the smaller, pale or dirty green or luteolous, furnished at the apex with cilia, mostly even within the mother-cell, moving rapidly, and finally escaping on the rupture of the membrane.

Genus CHLAMYDOCOCCUS, A. Braun.

Cellulæ globosæ, vel subglobosæ (4–8 in cœnobium fugacissimum conjunctæ), cytiodermate subcrasso firmo, cytioplasmate granuloso, fusco-rubro vel puniceo (in evolutionis gradibus quibusdam in colorem viridem mutato). Macrogonidia 2–4–8, rotundata, polo antico rostriformi producta, duo cilia longissima gerentia, nucleo centrali rubro, globulis amylaceis 4–6, non semper visibilibus instructa, tegumento amplissimo hyalino plerumque ovoideo vestita. Microgonidia multo minora, numerosa, luteola vel sordide viridia, apice rubella, ciliis binis instructa, intra tegumentum matricali alacriter vacillantia, denique membranæ ruptura elabentia. (R.)

Cells globose, or subglobose (4–8 conjoined in a very fugitive cœnobium), cytioderm thickish, firm, cytioplasm granular, brownish-red or puniceus, in certain stages of evolution changed into green. Macrogonidia 2–4–8, rounded, the frond end bearing very long cilia, furnished with a central reddish nuclei and with four to six, not always perceptible, starch granules, clothed with a very ample, hyaline, mostly ovoidal tegument. Microgonidia much the smaller, numerous, luteolous or sordid green, the apex reddish, furnished with two cilia, moving actively within the maternal tegument, and at last escaping by the rupture of the membrane.

Ch. nivalis (Bauer, Ag.). A. Braun.

Ch. globulis, 0.004″—0.00135″. (R.)

Hab.—In nive æterna, Greenland. Rocky Mountains.

Syn.—*Ch. nivalis* (Baur, Ag.). A. Braun. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 97.

Globules, 0.004″—0.00135″ in diameter.

Remarks.—I have never seen any good specimens of this plant, merely some cells mounted in Canada balsam, and therefore ruined for scientific study, which had been collected by Dr. Kane in one of his Arctic voyages. I have also had some indications of plants in a little parcel sent me by Mr. Sereno Watson, who informs me he has seen the red snow very abundant in the higher peaks of the Rocky Mountains. It is a matter of presumption rather than determination, therefore, that the species is identical with the European.

Genus VOLVOX, Ehrb.

Cœnobium exacte sphæricum, continuo rotatum et agitatum, globum cavum quasi fingens, e cellulis numerossissimis æquali distantia peripherice dispositis, gelatina matricali connexis, puncto rubro laterali, locellis (vacuolis) binis contractibilibus necnon ciliis binis longe exsertis instructis, vesica communi hyalina circumcinctis compositum.

Propagatio duplex ist, aut non sexualis aut sexualis; illa fit cellulis quibusdam certa distantia intumescentibs, multipartitis, in cœnobia filialia intra cœnobium matricale evolutis, postea libere erumpentibus; hæc cellulis masculis multipartitis in fasciculos spermatozoideorum mobilium, contractilium, pyriformium, ciliis binis instructorum, postea liberorum evolutis; cellulis femineis intumescentibus, non divisis, sed post fœcundationem in oosporas immobiles episporio duplici circumdatas postremo rubras evolutis. (R.)

Cœnobium exactly spherical, continually rotating and agitated, looking like a hollow globe, composed of very numerous cells, which are arranged on the periphery at equal distances, and are connected by the maternal jelly, and surrounded by a common hyaline bladder; they are also furnished with a lateral red point, with two contractile vacuoles, as well as two long exserted cilia.

The propagation is both sexual and non-sexual. In the latter, certain distant cells enlarge greatly, divide into numerous parts, and evolve within the parent cœnobium daughter-cœnobia, which are finally set free. In the sexual propagation certain molecular cells undergo a multipartite division into fasciculi of spermatozoids, which are motile, contractile, pyriform, and furnished with two cilia; the feminine cells are enlarged, and do not undergo division, but after fecundation develop into immovable oospores, which are finally red, and are surrounded by a double episporium or coat.

V. globator, (LINN.) EHRB.

V. cœnobiis majoribus ad $\frac{1}{3}'''$, cellulis numerossissimis (ad 12,000); cœnobiis filialibus semper octo intra matricale fructificatione non sexuali evolutis; fructificatione dioica; cœnobiis masculis fasciculos spermatozoideorum numerosos rubescentes foventibus (= *Sphærosira volvox*, Ehrb.); cœnobiis femineis cellulas sexuales (oogonia) 20–40 post fœcundationem in totidem oosporas globosas rubras episporio hyalino stellato circumdatas foventibus (= Volvox stellatus, Ehrb.). (R.)

Syn.—*V. globator*, (LINNÉ,) EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 97.

Hab.—In stagnis. United States.

Larger cœnobium, about $\frac{1}{3}''$ in diameter, composed of very numerous (about 12,000) daughter-cœnobia, always 8 within the maternal one, evolved without sexuality; fructification diæcious; male cœnobium giving origin to numerous reddish spermatozoids (= Sphærosphæra Volvox, Ehrb.); female cœnobium, giving origin to from 20–40 sexual cells, which, after fecundation, develop into the same number of globose red oospores surrounded by a stellate hyaline episporium.

Remarks.—Some of my friends tell me they have found this species abundantly around Philadelphia. I have not been so fortunate, and have seen but a few scattered specimens, which have afforded no opportunity of studying their development and life-history.

ORDER **Zygophyceæ.**

Algæ aut uni- aut pseudomulti-cellulares, sine vegetatione terminali et ramificatione vera. Cellulæ singulæ aut geminatæ aut seriatim conjunctæ. Multiplicatio fit cellularum divisione in unam directionem.

Propagatio fit zygosporis conjugatione cellularum similium binarum ortis.

Algæ either uni- or pseudomulti-cellular, without terminal growth or true branches. Cells segregate or geminate, or arranged in a single row. Multiplication taking place by a division of the cells in one direction.

Propagation by zygospores, formed by the conjugation of two similar cells.

FAMILY DESMIDIACEÆ.

Algæ unicellulares, sine ramificatione vel vegetatione terminali. Cellulæ forma admodum varia, plerumque in medio plus minus profunde constrictæ et in duas semicellulas symmetricas divisæ.

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liberæ vel in fascias filiformes aut tæniiformes arcte conjunctæ aut in muco matricali nidulantes et in familias indefinitas consociatæ. Cytioderma non siliceum, plus minus firmum, læve aut varie asperatum (striatum, costatum, aculeatum, &c.). Massa chlorophyllacea in laminales axiles vel parietales, sæpe e centro radiantes, distributa.

Propagatio non sexualis per divisione transversa in eandem directionem repetita; sexualis per zygosporas, quæ per cellularum binarum conjugationem oriuntur.

Unicellular algæ, without branches or terminal growth. Cells of very various forms, mostly more or less profoundly constricted in the middle and divided into two symmetrical semicells, free or conjoined in filiform or tæniform fascia, or involved in the maternal jelly so as to form indefinite families. Cytioderm not siliceous, more or less firm, smooth, or variously roughened (striate, costate, aculeate, &c.) Chlorophyl masses in axillary or parietal lamina, which often radiate from the centre.

Non-sexual propagation by repeated transverse division in one direction; sexual by zygospores which are formed by the conjugation of two cells.

Remarks.—Of all the fresh-water algæ, with the exception of the diatoms, this family has attracted most attention, owing, not only to the beauty and variety of its forms, but also to their universal presence and abundance, and the ease with which their most wonderful life-histories are observed. They are exclusively, as far as known, denizens of fresh-water, and preferably that which is pure and limpid. Although Mr. Ralfs states that they never grow in stagnant water, I have often found them in great abundance in such, yet never in that which was actually putrid. The same authority is also too sweeping, at least as far as this country is concerned, in stating they are never found in woods, although they are really most abundant in the open country. My experience has taught me to look for them in brick-ponds, small mountain lakes, springy fens, ditches, and, in the fall, growing among mosses and in the thick jelly composed of unicellular algæ on the face of dripping rocks, or, to sum up in a word, they dwell in quiet, shallow waters, for I have never found them in rapidly moving or very deep water.

The single cell, of which a desmid is composed, is mostly divided into two very marked similar portions, the exact counterparts one of the other, which by some have been asserted to be distinct cells. Their close union and connection, and their inherent oneness are, however, so apparent that it is needless here to spend time in demonstrating that they really are halves of one individual cell. They contain together all the parts found in the typical vegetable cell; an outer cellulose wall, chlorophyllous protoplasm, a nucleus, starch granules and semiliquid contents. The cell-wall, or cytioderm, as it is called in this memoir, varies in thickness and firmness. During life it is mostly, if not always, colorless; but in certain species in the dead empty frond is of a reddish-yellow. The markings upon it are various, and are not infrequently altogether absent; they are such as fine or coarse punctations, granulations of various size, striæ, furrows or elevated ribs, tubercles, obtuse or sharp simple or forked spines, hair-like processes, umbonations, &c. &c. These markings are within narrow limits constant in each species, and more or less peculiar, so that they afford valuable characters to the systematist. The cytioderm itself is mostly composed of cellulose free from appreciable inorganic matters, but in certain species contains a large amount of silex. Thus,

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according to De Barry, if *Closterinm lunula* be carefully burnt upon a slide, a perfect hyaline silex cast of the cells is left.

The chlorophyl is variously placed in the cell, sometimes it is arranged in lamina, sometimes in spirals, sometimes in the form of radii from a central mass. These different methods afford good generic characters, and will be dwelt upon more in detail under the various genera. The color of the chlorophyl during active life is a vivid green, which, as the vital forces lessen, changes to a faded yellowish tint.

Nægeli and others affirm that there is always a central nucleus in the desmid, and probably do so with truth, although in many instances I have found it impossible to demonstrate its presence from the size and opaqueness of the frond, crowded with endochrome, &c. In a large number of cases, however, it is very apparent.

As ordinarily viewed under the microscope the two most striking peculiarities presented by these little plants are the motion of the whole desmid in the water and the various movements exhibited within the fronds. The general movement is most apparent in the larger species, which exist free and distinct in the water, especially in the boat-shape closteria. It mostly consists of a steady, stately, slow onward movement, with sometimes backward oscillations. By virtue of it, desmids in a bottle will often congregate in such positions as are most exposed to light. There have been various theories advanced as to the cause of this motion. Ehrenberg believed that he had found foot-like processes protruding from the end of the frond and giving the motile power. Others, such as Rev. Mr. Osborne and Mr. Jabez Hogg, have attributed the movements to the presence of cilia, but I think have failed so entirely to establish this that their views are more than problematical. That the motion is due to vital actions, taking place especially under the action of light, is as much as can be at present affirmed with any certainty, though it is probable that the immediate agents are endosmotic currents of gas or water.

The movements of the contents within the cells are chiefly of two kinds. Taking *Closterium lunula* as an illustrative example, there will be found on examination with an $\frac{1}{8}$th objective, a narrow, very transparent, and therefore often not very apparent layer or zone lying immediately within the cell-wall, between it and the endochrome, and dipping inward in the middle of the frond so as to communicate with the nucleus. In this zone are protoplasm, watery fluid, and scattered granules. In the ends of the fronds the different portions of this zone, meeting and widening, fill up the whole of the cavity, and within the space thus occupied by them, there is a globular, sharply defined, still more transparent vacuole. This, some have thought to be a closed sac, with a distinct wall, but it seems really to be a vacuole lying in the midst of the inner protoplasm, which with a few granules occupies more or less completely the transparent zone already described. Sometimes the chlorophyl encroaches upon this zone at the ends so as to more or less completely surround the vacuole, within which are always found watery fluid and granules. In the protoplasmic zone and its vacuole active movements are probably always present during active life. Streams of protoplasm appear to be constantly passing to and fro between the nucleus and the ends of

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the cell along the outer zone, and granules can be always seen passing backwards and forwards with an unsteady motion.

When the streams of protoplasm are setting very actively from the centre towards one end, there will often be an accumulation of the protoplasm there, and a consequent decided lessening in the size of the vacuole, which will again expand as the return currents arouse themselves. Within the vacuoles are seen more or less numerous smaller or larger granules in active busy motion, swarming over and about one another with an unsteady hurrying to and fro.

A form of motion, similar in appearance to this, but probably of different significance, is seen in most desmids when in an unhealthy feeble condition. I have seen it most marked in *Cosmarium margaritaceum.* In such fronds the endochrome has lost its deep green color, and become shrunken, and lying within it is a great space containing myriads of minute blackish particles swarming about actively. This peculiar state and appearance is by no means confined to the desmids, for I have seen it very highly developed both in species of *Spirogyra* and *Œdogonium.* It appears to be connected with decay. Is it possible that these minute particles are foreign to the plant, vibrionic in nature?

In regard to the nature of the movements seen within a healthy desmid, some have viewed them as exceedingly mysterious, the result of the presence of cilia, &c.; but these views have been so thoroughly exploded that it is scarcely necessary even to mention them here. The movements are, in truth, precisely parallel to the so-called cyclosis of the higher plants. Protoplasmic germinal matter, wherever it exists, be it in animal or vegetable, has as one of its distinguishing characters the power of active, spontaneous, apparently causeless movements, and it is simply the carrying out of this power or attribute which has attracted so much attention in the desmids, because it is in them so readily seen.

There are, in this family, two distinct methods in which the species are multiplied one with, the other without, the intervention of anything like sexuality. The non-sexual method of increase is really a modification of an ordinary vegetative process, a peculiar cell multiplication by division. In such fronds as those of the genus *Cosmarium*, which are composed of two evident halves connected by a longer or shorter isthmus, the first step in the process is an elongation of this neck. In a very short time there appears around the centre of this a constriction, and I believe an actual rupture of the outer coat. By this time a new wall has formed inside each half of the isthmus, and stretches also across its cavity, forming with its fellow a double partition wall, separating the two halves of the old frond. Rapid growth of the newly formed parts now takes place, the central ends become more and more bulging as they enlarge, and in a little time two miniature lobules have shaped themselves at the position of the old isthmus. These are at first small, colorless, and destitute of all markings, looking, as Mr. Ralfs says, like condensed gelatine. They, however, rapidly increase in size and firmness, their contents assuming a green color and their walls taking on the peculiar markings of the species. At last, the parts thus formed having assumed the shape and appearance of the original lobules, the two fronds, which have been developed out of one, separate, mostly before the new semicells have acquired their full size.

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What part the nucleus has in the process just described I have never actually demonstrated, but have little doubt but that it undergoes a division in the very commencement, so that the new nucleus of each secondary frond is formed out of one-half of the old one.

In proportion as the form of the desmid becomes simpler, so do the peculiarities of its cell multiplication become less. In those species which are simple cylindrical cells, there appears to be nothing peculiar in the method of dividing, which, however, always takes place through the centre of the cell, and subsequent growth occurs, generally, only in the newly formed part.

True sexual reproduction apparently does not take place as freely in this family as the former process, for whilst I have seen hundreds of cells undergoing the latter, it has not been my good fortune to meet with conjugating specimens on more than two or three occasions.

The process has, however, been studied very closely by De Bary, Braun, Hofmeister, and others, and appears to consist generally in a rupture of the outer wall of two cells and the protrusion of delicate processes from an inner, often newly formed coat, with subsequent union of these, and consequently of the two cells, and afterwards a condensation of the contents in the enlarged connecting passage. The connecting passage between the fronds is really a sporangium in which the spore is perfected, the contents of the cells finally condensing it into a firm globe and secreting around themselves a thick coat.

The after-history of this spore has been very successfully studied by M. Hofmeister, whose observations were made upon *Cosmarium tetraophthalmum*, which he watched conjugating and forming a sort of resting spore which was perfected early in the month of July. This was composed of a thick outer coat and green endochrome lying within as a distinct ball, nowhere in contact with the investing membranes. In three weeks' time this chlorophyllous protoplasm had divided into ellipsoidal masses, or primordial cells, which soon surrounded themselves with cellulose walls and became distinct free cells in the granular fluid which filled the cavity of the original spore. In August, each of these masses was divided into two and in the month of September the process was repeated, so that out of the original endochrome eight strongly flattened primordial cells were produced. Division in some specimens ceased here, and in others took place once more, so that by the following spring all of the living *Sporangia* contained eight or sixteen green daughter-cells, each of them discoid in outline with a strongly marked central notch. These daughter-cells were finally set free by the solution of the spore wall, as *Cosmaria* of minute size, but agreeing in all other characters with the specific form to which they belonged.

According to Braun, in the larger, more or less lunate *Closteria*, conjugation occurs in the following method: Two fronds approach one another in such a way that they lie back to back. In the middle of each of them, there then appears an annular line or trench reaching through the cell wall, and accompanied by a distinct separation of the endochrome into two halves. Whilst these changes have been progressing there has also formed a new double wall at the position of the trench, so that out of the two *Closteria* two pairs of separate equal cells have been formed. Near to the larger or central end of each of these now appears a pouting transparent nipple-like process. The corresponding opposing processes enlarging and meeting coalesce, so that the upper half of one closterium, in the form of a daughter-cell, is finally united with the upper half of the other closterium, and the two lower halves are also joined together. Thus from a single pair of fronds arise two conjugating pairs of cells, and finally two sporangia, in each of which a spore is perfected.

This process does not seem, however, to be universal amidst the *Closteria*, for in many, if not all, of the smaller species, a pair of fronds produces a single sporangium.

In the genus *Palmogloea*, in which I have had an opportunity to study the development of the spores, the process closely simulates that seen in certain of the *Spirogyra*. The contents of the cells first became broken up and confused, and almost simultaneously the nucleus disappeared (fig. 4, pl. 11) the cells became swollen at one side and slightly bent backward so as to form jutting processes, which meeting grew together, became confluent and developed into a sporangium much larger than either of the parent cells. Into this sporangium the contents of the latter passed and soon became converted into a thick-walled spore (fig. 00, pl. 00) often completely filling the cavity, and apparently with its wall adherent to that of the latter.

Genus PALMOGLŒA, Ktz. (1843).

Cellulæ oblongæ, ellipticæ vel cylindricæ, utroque polo rotundatæ, medio non constrictæ, plerumque in muco gelatinoso nidulantes, liberæ, singulæ vel in familias consociatæ, lamina chlorophyllacea axili vel excentrica, ætate provecta medio constricta, denique divisa præditæ. (R.)

Syn.—*Mesotænium*, Nægeli.

Cell oblong, elliptical or cylindrical, rounded at each end, not constricted in the middle, mostly swimming in a gelatinous mucus, free, single or associated in families, chlorophyl lamina axillary or excentric, in the early state constricted, and at length divided in the middle

Remarks.—The above diagnosis of the genus is that given by Prof. Rabenhorst, and agrees essentially with that of De Bary, Nægeli, &c. In the species herein described however, the axillary lamina of chlorophyl were not so pronounced, for the green coloring matter seemed often to surround the cavity of the cell, and in other specimens was broken up and diffused through it.

P. clepsydra, Wood.

P. saxicola et bryophila, in gelatina achroa interdum dilute viride nidulans; cellulis cylindricis, cum polis obtuse truncato-rotundatis, diametro 2–3 plo longioribus; lamina chlorophyllacea axili, plerumque indistincte, sæpe nulla; plasmate dilute viride; nucleo plerumque distincto; zygosporis subfuscis aut subglobosis aut enormiter in clepsydræ forma; membrana externa enormiter excavata et sulcata.

Diam.—$\frac{13}{7500}''$.

Syn.—*P. clepsydra*, Wood, Prodromus, Proc. Amer. Philosophical Soc. 1869.

Hab.—In rupibus et in muscis irroratis ad Chelten Hills, prope Philadelphia.

P. living on rocks and mosses, swimming in a transparent, sometimes light-green jelly; cells obtusely truncated, rounded at the ends, 2–3 times longer than broad; chlorophyl lamina axillary, mostly indistinct, often wanting; endochrome light-green; nucleus generally distinct; zygospore subfuscous, either globose or of an irregular form, somewhat resembling that of an hour-glass; external coat irregularly excavated and sulcate.

Remarks.—This species was found along the North Pennsylvania Railroad, near Chelten Hills, growing amid mosses on the rocky juttings over which the water was dripping. It occurs as a rather firm, transparent jelly, mostly of a light greenish tint, in which the cells are often placed quite thickly. They are cylindrical, mostly straight, but sometimes slightly curved, and often completely filled with a light greenish endochrome. The central lamina is irregular, and mostly not at all pronounced. In some cells the endochrome is much broken up, so that the interior is filled with little green masses with light spaces between them. In these cells the nucleus is generally not perceptible, whilst in the others it is very well marked. The zygospore is often globular, sometimes it is irregularly elliptical, with a constriction in the centre, so as to give it somewhat of an hour-glass shape. The outer coat mostly fits pretty closely on the inner contents, and is very often distinctly marked with little pits, some round, some irregular in shape; in other cases, instead of being thus pitted, the spores seem to be marked with deep curved furrows.

Fig. 4, pl. 11, represents this plant in different stages of growth. (See Explanation of Plates.)

Genus PENIUM, Bréb. (1848.)

Cellulæ cylindricæ vel fusiformes, rectæ, utroque polo rotundatæ vel truncato-rotundatæ (nec emarginatæ nec excisæ), medio sæpius constrictæ. Lamina chlorophyllacea axilis, ex transverso conspecta radiatim-divergens, radii sæpe furcati, granula amylacea plerumque longitudinaliter seriata includens. Individua in aqua libere natantia, singula, sparsa vel in massa gelatinosa consociata. Cellulæ membrana lævis vel granulata, achroa vel fuscescens vel rubicunda, sæpius longitudinaliter striata. (R.)

Syn.—*Netrium*, Nægeli.
Cylindrocystis, Mengh.
Closterium, *partim*, Ehrenberg.

Cells cylindrical or fusiform, straight, rounded at each end, or truncately rounded (not emarginate or excised), medianly often constricted. Chlorophyl lamina axillary, when seen transversely radiately divergent, arms often forked, and containing starch granules, mostly longitudinally striate. Individuals swimming free in the water, scattered and single, or associated in gelatinous masses. Cell membranes smooth or granulate, transparent or fuscous or reddish, often longitudinally striate.

a. *Lamina chlorophyllacea peripherice lobata vel radiatim expansa.*

a. *Chlorophyl lamina, lobate on the periphery or radiately expanded.*

P. Digitus, (Ehrb.) Bréb.

P. cellulis ovato-cylindricis, diametro 3–5 plo longioribus, utroque polo parum attenuatis, subtruncato-rotundatis; laminis chlorophyllaceis peripherice lobatis, medio interruptis.

Diam.—$\frac{13}{7500}''=.00173''$—$\frac{22}{7500}''=.0029''$.

Syn.—*P. Digitus*, (Ehrb.) Bréb. Rabenhorst, Flora Europ. Algar., Sect. III. p. 118.

Cells ovately cylindrical, 3–5 times as long as broad, at each end slightly attenuate, subtruncately rounded; chlorophyl lamina lobate on the periphery, interrupted in the middle.

Remarks.—This species is probably widely diffused through the temperate portions of North America. I have found it abundantly near Philadelphia, as well as among the Alleghanies, and have received specimens from Dr. Lewis, collected in Saco Lake, Northern New York; Prof. Bailey also notes it as occurring in Georgia. There is one form of it which resembles somewhat in outline the modern coffin, one end being much broader and much more rapidly narrowed than the other. There is no distinct vacuole at the end, at least in any specimen I remember to have seen, although frequently large numbers of moving granules can be detected in that portion of the frond.

Fig. 6, pl. 20, represents the outline of a frond of this species.

P. lamellosum, Bréb.

P. cellulis oblongo- vel fusiformi-cylindricis, diametro 5–6 plo longioribus, medio sæpe leviter constrictis, utroque polo magis attenuatis, obtuso rotundatis. (R.)

Diam.—0.0023″—0.0029″. (R.)

Syn.—*P. lamellosum,* Bréb. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 118.

Hab.—Rhode Island. (Olney) Thwaites.

Cells oblong or fusiform cylindrical, 5–6 times longer than broad, often slightly constricted in the centre, more attenuate at the ends, obtusely rounded.

Remarks.—I have never recognized this species, but it is one of those sent over by Mr. Olney, and identified by Prof. Thwaites.

b. Lamina chlorophyllacea integerrima.

b. Chlorophyl lamina entire.

* *Cellulæ in medio plus minus constrictæ.*

* *Cells more or less constricted in the middle.*

P. margaritaceum, Ehrb.

P. elongato cylindricum, diametro 8–9 plo longius, medio plerumque leviter constrictum, utroque polo rotundato-truncatum; cellulæ membrana nodulis seriatis quasi margaritacea; locellus in medio (circiter) utriusque cruris corpusculis mobilibus in more Closteriorum repletus. (R.)

Diam.—0.00098″—0.0011″. (R.)

Hab.—Rhode Island. (Olney) Thwaites; Bailey. Florida. Bailey.

Elongate cylindrical, 8–9 times longer than broad, in the centre generally slightly constricted, at each end roundly truncate; membrane of the cells somewhat pearly with seriate granules; vacuole about in the centre of each crus, filled with moving granules, as in closterium.

Remarks.—I have not seen this desmid, but it is in Prof. Bailey's list; it was also among those sent by Mr. Olney to Prof. Thwaites.

P. minutum, Cleve.

P. cylindricum, gracile, diametro 5–7 plo longius, læve, ad polos obtusissimos (latissime rotundatos) parum attenuatum, medio leviter constrictum. (R.) *Species mihi ignota.*

Diam.—0.00044″—0.00063″. (R.)

Syn.—*Docidium minutum,* Ralf's British Desmid.
P. minutum, Cleve. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 122.

Hab.—South Carolina; Florida. Bailey. Rhode Island. (S. T. Olney) Thwaites.

Frond slender, suture not prominent; segments four to six times longer than broad, somewhat tapering, inflation obsolete, sides straight, ends entire; e. f. without puncture. L. $\frac{1}{212}''$; B. $\frac{1}{1582}''$. (Archer.)

b. Cellulæ in medio non constrictæ.

b. Cells not constricted in the middle.

P. interruptum, BRÉB.

"P. cellulis late lineari-cylindricis, diametro 5-6 plo longioribus, utroque polo subito cuneato-acutatis, apicibus obtuso-rotundatis; laminis chlorophyllaceis longitudinalibus saturate viridibus, ætate provecta fasciis transversis tribus pallidis interruptis." (R.) *Species mihi ignota.*

Diam.-0.00147″—0.00177″. (R.)

Syn.—P. interruptum, BRÉB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 119.

Hab.—In fossis, South Carolina, prope Grahamsville. Prof. Bailey.

Cells broadly linearly cylindrical, 5–6 times longer than broad, at each end suddenly cuneately sharpened, the apex obtusely rounded; longitudinal chlorophyl lamina deep green, in advanced age interrupted by three transverse pale fascia.

P. Jenneri, RALFS.

P. ab P. Brebissonii vix discernendum, cellulis cylindricis, utroque polo rotundatis, lævibus, diametro 2½–5 plo longioribus; zygosporis plerumque globosis, membrana fuscescente subgranulata. (R.) *Species mihi ignota.*

Diam.—0.00057″—0.0006″. (R.)

Syn.—P. Jenneri, RALFS. RABENHORST, Flora Europ. Algarum, Sect. III. p. 120.

Hab.—In fossis, Florida. Prof. Bailey.

Scarcely distinguishable from P. Brébissonii, cells cylindrical, rounded at each end, smooth, 2½–5 times longer than broad; zygospores mostly globose, membrane somewhat fuscous, subgranulate.

P. Brébissonii, (MENGH.) RALFS.

P. in massa mucosa indefinite expansa sæpe cum algis alteris intermixtis; cellulis perfecte cylindricis, interdum nonnihil curvatis sed plerumque rectis, diametro 2½–4 plo longioribus, utroque polo late rotundatis, in medio non constrictis; "zygosporis angularibus vel rotundatis, membrana fuscente, subtiliter granulata."

Diam.—$\frac{5}{7500}'' = .00066''$.

Syn.—P. Brêbissonii, (MENGH.) RALFS. RABENHORST, Flora Europ. Algarum, Sect. III. p. 120.

Hab.—In fossis, South Carolina. (Prof. Ravenel.)

In an indefinitely expanded mucous mass, intermixed with other algæ; cells perfectly cylindrical, sometimes slightly curved, but generally straight, at each end broadly rounded, not constricted in the middle; "zygospores angular or rounded, membrane fuscous, finely granulate."

Remarks.—Among the numerous desmids which I have received from Prof. Ravenel are some which, I think, must be referred to *P. Brebissonii*, although they do not nearly equal the size of the European form, nor even the diameter given above, which is almost the lowest limit of the mature foreign plant. I believe, however, Prof. Ravenel's specimens are immature.

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Mr. Ralfs' description of the conjugation is as follows: The process of the conjugation in this species differs from that in the rest of this genus; for, as in *Hyalotheca dissiliens*, the conjugation cells enter into the formation of the containing cell and are permanently attached to the sporangium, instead of being detached, as commonly happens, in the Desmids. The sporangium is at first cruciform, then quadrate, and finally orbicular.

P. closterioides, Ralfs.

P. cellulis anguste lanceolatis, diametro maximo 5–6 plo longioribus, a medio in apices subtruncato-rotundatos sensim attenuatis; laminis chlorophyll. saturate viridibus, medio fascia transversa pallida interruptis. (R.) *Species mihi ignota.*

Diam.—0.00159″—0.00175″. (R.)

Syn.—*P. closterioides*, Ralfs. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 121.

Hab.—Prope Grahamsville, South Carolina. Prof. Bailey.

Cells narrowly lanceolate, 5–6 times longer than the greatest diameter, sensibly attenuate from the middle into the subtruncate apices; chlorophyl lamina deep green, interrupted by a median pale band.

Genus CLOSTERIUM, Nitsch.

Cellulæ interdum cylindricæ sed sæpius fusiformes et utroque polo attenuatæ, plus minus lunulatim curvatæ, in medio haud constrictæ sed stria transversa unica vel 2–5 impressæ. Cytioderma tenue, sat firmum, læve vel plus minus distincte striatum et interdum longitudinaliter costatum. Cytioplasma chlorophyllosa plerumque in laminis longitudinalibus disposita, et sub cellulæ polis locello achroo, plerumque globoso et corpusculis plus minus numerosis se vivide moventibus impleto instructa.

Cells sometimes cylindrical, but more often fusiform and attenuate at each end, more or less lunately curved, in the centre not constricted but marked with from 1–5 transverse striæ. Cytioderm thin, moderately firm, smooth or more or less distinctly striate, and sometimes longitudinally costate. Chlorophyllous cytioplasm mostly arranged in longitudinal lamina, and furnished at each end with a clear space, which is mostly globose, and contains more or less numerous actively moving corpuscles.

a. *Zygosporæ globosæ, rarissime angulares; cellulæ crura aut non aut minus producta.*

a. *Zygospores globose, very rarely angular; crura of the cells not at all, or only slightly, produced.*

1. *Cellulæ cylindricæ, ad utrumque polum vix vel paullum attenuatæ, rectæ vel leviter curvatæ, apicibus rotundatis vel truncatis.*

1. *Cells cylindrical, not at all or but slightly attenuated at the ends, straight or slightly curved, the apex rounded or truncate.*

C. striolatum, Ehrb.

C. anguste lanceolato-fusiforme, leviter arcuatum, 8–12 plo fere longius quam latum, utroque polo paulum sensimque attenuatum, apicibus truncatis sæpe fuscescentibus; membrana distinctissime striata, vacuata fuscescente; vesiculis chlorophyllaceis 5–7 (in quoque crure); locello apices versus sito, submagno, corpuscula 12–20 includente. (R.)

Diam.—$\frac{1}{58}$″—$\frac{1}{48}$″ = 0.00152″—0.00187″. (R.)

Syn.—*C. striolatum*, Ehrb. Rabenhorst, Flora Europ. Algarum, Sect III. p. 125.

Hab.—In aquis quietis, Centre County, Pennsylvania. Wood. Saco Pond, New Hampshire. (Lewis)

Narrowly lanceolately-fusiform, slightly bent, 8–12 times longer than broad, sensibly attenuated at the ends, which are truncate and often somewhat fuscous; membrane very distinctly striate, when empty somewhat fuscous; chlorophyl globules 5–7 (in each limb); vacuole placed in the bent apex, moderately large, including 12–20 corpuscles.

Remarks.—The measurements given are those of Prof. Rabenhorst. Our American forms agree well with them.

C. angustatum, Ktz.

C. gracile, sublineare, diametro 16–18 plo longius, ad polos levissime attenuatum, apicibus late truncatis; costis longitudinalibus paullulum prominulis 4–5, interstitiis circiter $\frac{1}{450}'''$ latis; vesiculis chlorophyllaceis in quoque cruro 6–7; locello ab apice subremoto mediocri, corpusculis 12–20 impleto. (R.)

Diam.—$\frac{1}{101}''$—$\frac{1}{84}''$ = 0.00081″—0.0010″. (R.)

Syn.—*C. angustatum,* Ktz. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 126.

Hab.—In aquis quietis, prope Philadelphia, Pennsylvania. Wood. Rhode Island. Bailey. New Hampshire. (Lewis)

C. slender, sublinear, 16–18 times longer than broad, very slightly attenuate at the ends, which are broadly truncate; with from 4–5 somewhat prominent longitudinal ribs, the interstices about $\frac{1}{450}'''$ broad; chlorophyl globules in each limb 6–7; vesicle subremote from the apex, moderate, containing from 12–20 corpuscles.

C. juncidum, Ralfs.

C. elongatum, anguste lineare, diametro 20–35 plo longius, leviter arcuatum, utroque polo vix attenuatum; apicibus truncatis; cytiodermate luteolo, interdum longitudinaliter striato.

Diam.—$\frac{1}{2500}''$ = .0004″.

Syn.—*C. juncidum,* Ralfs. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 127.

Hab.—In fossis, South Carolina. (Ravenel) In lacu Saco, New Hampshire. (Lewis)

Elongate, narrowly linear, 20–35 longer than broad, slightly bent, scarcely narrowed at the ends; apices truncate; cytioderm yellowish-brown, sometimes longitudinally striate.

Remarks.—I am indebted to Prof. Ravenel for specimens of this species, by whom they were found on the slimy surface of a half dried-up ditch, associated with numerous other desmids. The specimens are all smaller than the measurements of Rabenhorst, but much larger than those given by Mr. Ralfs. None of the plants have any chlorophyl granules—a circumstance probably simply dependent upon the stage of their development. The longitudinal striæ are in none of the specimens very distinct, and in many cannot be demonstrated.

Since writing the above I have seen specimens collected by Dr. Lewis in "Saco Pond," near the Crawford House, New Hampshire.

Mr. Archer (Pritchard's *Infus.*, p. 749) lays stress upon the fronds being straight in the middle, with the ends curved downwards; but I have seen numerous specimens in which the curve was through the whole length.

Fig. 2 *o*, pl. 12, represents one of the specimens collected by Prof. Ravenel in South Carolina.

2. *Cellulæ cylindricæ, dorso plus minus convexæ, ventre subplanæ, nunquam ventricoso—inflatæ.*

2. *Cells cylindrical, with the dorsum more or less convex, the belly straightish, never ventricosely inflated.*

C. Lunula, (MÜLLER) EHRB.

C. permagnum, sublæve (striæ subtilissimæ vel indistinctæ), semilunare, dorso alte convexum, ventre subplanum, apicibus attenuatis rotundatis; vesiculis chlorophyllaceis numerosis sparsis; locello distincto subapicali corpuscula numerosa includente. (R.)

Diam.—$\frac{1}{27}''$—$\frac{1}{20}''$ = 0.00032″—0.0045″. (R.)

Syn.—*C. Lunula,* (MÜLLER,) EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 127.

Hab.—South Carolina, Georgia, Florida. Prof. Bailey. Pennsylvania. Wood.

Very large, smoothish (striæ very fine or indistinct), semilunar, dorsum strongly convex, belly straightish, the ends attenuate and rounded; chlorophyl globules numerous, scattered; vesicle distinct, subapical, including numerous corpuscles.

C. acerosum, (SCHRANK) EHRB.

(*Var. nov.* **maximum.**)

C. lineare-fusiforme, sub-rectum aut leve curvatum, utroque fine sensim et paullulum attenuatum, diametro 15–24 plo longiore; apicibus angustissime truncatis, achrois; membrana haud striata; vesiculis chlorophyllaceis 11–14 in quoque crure, in serie axilli simplici collocatis; locello apicali parvo, corpuscula numerosa includente; zygosporis globosis.

Diam.—Transv. max. $\frac{13}{7500}''$ = .0017″; zygosp. $\frac{20}{7500}''$ = .0027″.

Syn.—*C. acerosum,* (SCHRANK) EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 128.

Hab.—Pennsylvania; Wood. South Carolina, Georgia, Florida; Bailey.

Linear, fusiform, straightish, or slightly curved, at each end sensibly little by little attenuate, 15–24 times longer than broad; apices narrowly truncate, transparent; membrane not striate; chlorophyl globules 11–14 in each limb placed in a simple axillary series; apical vesicle small, containing numerous corpuscles; zygospores globose.

Remarks.—The desmid, described above, was found in New Jersey, near Camden. It differs from the typical form of *C. acerosum* in its size, proportionate length to breadth, and in not being striate. The European "*formâ major*" (RABENH.) appears, however, to exceed it in transverse diameter, and, according to some authors, certain fronds of the species are not striate, and all authorities agree that at times the striæ are exceedingly delicate. For these reasons, I think, our American form must be regarded simply as a variety. As far as can be judged from the rude figure, it is this species which Prof. Bailey identifies as *C. tenue*, KTZ., in Silliman's Journal for 1841.

Fig. 5, and 5 *a*, pl. 11, represent this species magnified 250 diameters; 5 *b* represents the sporangium with portions of the dead fertile fronds still attached.

C. areolatum, WOOD, (sp. nov.)

C. fusiforme, subrectum vel nonnihil curvatum, lateris ventralis medio sæpe paullulum concavum, diametro 9–10 plo longius, utrinque modice attenuatum; apicibus truncato-rotundatis; membrana crassa, et firma, rubido-brunnea, profunde distante striata, et minutissime sed distincte granulata vel areolata; suturis medianis distinctissimis·4–10.

Diam.— 0.0024″.

Hab.—In aquis puris quietis; Northumberland Co., Pennsylvania.

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Fusiform, straightish, or very slightly curved, the ventral side often a little concave in the middle, 9–10 times longer than broad, moderately attenuated at each end; the apices truncately rounded; cell-membrane reddish-brown, thick and firm, distantly profoundly striate, and very minutely but distinctly granulate or areolate; median sutures very distinct, 4–10 in number.

Remarks.—I found this species growing in a quiet pool of pure water, in a wild, deeply wooded ravine, near Danville, Central Pennsylvania. It was in great abundance, forming a translucent greenish jelly, one or two gills of which might have been readily gathered. Unfortunately, I had no microscope with me and cannot, therefore, determine at all as to the arrangement of the endochrome, the carbolic acid, used as preservative, having entirely disarranged this by the time I got the fronds upon a slide. The empty frond is of a reddish-brown color. The membrane is quite thick and firm, and is marked with very prominent broad striæ or grooves. In a number of cases I have counted these and always found nine present upon one face of the frond. There are also upon the surface numerous minute markings not fairly visible with a lower power than a $\frac{4}{10}$th objective. Under this glass they appear as minute punctations. An eighth resolves them into granules mostly of an oblong shape, arranged more or less regularly in longitudinal rows. Very generally, each side of the stria or groove has a close row of larger and more distinct granules forming a sort of border to it. In truth, the surface of the frond is covered with broad longitudinal bands of these granules, and the narrow smooth spaces between them constitute the stria spoken of. This species is very closely allied to *C. turgidum*, Ehrb., agreeing pretty well with it in general outline and size. I think, however, the peculiar markings upon the membrane are sufficient to separate it, and do not doubt that if fresh specimens were at hand, differences would be found to exist also in the arrangement of the cell-contents. The turning up of the ends, generally so marked in *C. turgidum*, is mostly entirely absent in this species, rarely there is some tendency to it.

Fig. 6, pl. 11, represents in outline a frond magnified 160 diameters; Fig. 6 *a*, the end of an empty cell, magnified 1375 diameters; the color of this is, perhaps, a little too dark.

C. lineatum, Ehrb.

C. valde elongatum, gracile, quater vicies-tricies longius quam latum, distincte striatum, e medio recto cylindrico utrinque valde attenuatum, apices versus leviter incurvum, obtuso-truncatum; vesiculis chlorophyllaceis in quoque crure 20–21, in seriem unicam axilem distributis; locello parvo, ab apice remoto, corpusculis 10–12 impleto. (R.)

Diam.—$\frac{3}{2000}'' = .0015''$.

Syn.—*C. lineatum*, Ehrb. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 130.

Hab.—Pennsylvania, Wood.

Very much elongate, slender, distinctly striate, from the centre straight and cylindrical, at each end very greatly attenuate, apex bent, slightly incurved, obtusely truncate; chlorophyl globules 20–21 in each limb, placed in a simple axillary series; vacuole small, remote from the apex, containing from 10–12 corpuscles.

Remarks.—The American forms agree well with the above description; some of them, however, are a little more curved in the central portion than it would imply.

Fig. 1, pl. 12, is a drawing of an American plant, magnified 160 diameters.

C. Cucumis, EHRB.

C. oblongum, turgidum, leviter curvum, læve, diametro 4–7 plo longius, apicibus obtusis. (R.)

Syn.—*C. Cucumis*, EHRENBERG, Verbreit. b. 28, IV. F. 28. RABENHORST, Flora Europ. Algarum, Sect. III. p. 138.

Hab.—New York; Ehrenberg.

Oblong, turgid, slightly curved, smooth, 4–7 times longer than broad, the apex obtuse.

Remarks.—I have no knowledge of this species other than that in the above short description, which has been copied from Rabenhorst's works.

3. *Cellulæ semilunares, plerumque magis curvatæ quam in Sect.* 1 *et* 2, *ventre semper tumidæ, ventricoso-inflatæ.*

3. *Cells semilunar, mostly more curved than in Sect.* 1 *and* 2, *with the belly always tumid, ventricosely inflated.*

C. Ehrenbergii, MENGH.

C. fusoideo-semilunare, ventre inflato, ceterum ut in C. Lunula. (R.)

Diam.—Lat. $\frac{13}{4500}$″ = .0029″. Long. $\frac{21}{4500}$″ = .0042.″

Syn.—*C. Ehrenbergii*, MENGH. RABENHORST, Flora Europ. Sect. III. p. 131.

Hab.—Prope Philadelphia.

"Frond large, stout, about five or six times as long as broad, lunately curved, extremities tapering; upper margin very convex, lower concave with a *conspicuous central inflation;* ends broadly rounded; large granules, numerous, scattered; fillets several; e. f. colorless, without striæ, central suture not evident. Sporangia orbicular, smooth, placed between the but-slightly-connected empty conjugated fronds, the endochrome during the process of conjugation emerging from the opened apex of a short conical extension from each under side of each younger segment (or shorter cone) of each pair of recently divided fronds, the conjugating fronds being produced immediately previously by the self-division of a pair of old fronds—two sporangia being thus the ultimate produce of the two original fronds. L. $\frac{1}{68}$″; B. $\frac{1}{400}$″. Archer." Pritchard's *Infusoria*, p. 748.

Remark.—Fig. 2, pl. 12, represents a plant of this species magnified 160 diameters.

C. moniliferum, (BORY) EHRB.

C. semilunare, plus minus curvatum, diametro maximo 6–9 plo longius, ventre inflato, utroque polo sensim attenuatum, apicibus achrois obtusis, vesiculis chlorophyllaceis in serie unica longitudinali axili dispositis, in quoque crure 7–10; locello apicali submagno, corpuscula numerosa includente (corpusculum in quoque locello unicum mobile ellipsoideum, magnitudine lineæ partem millesimam æquans, cetera mobilia per totum corporis distributa observavit cl. Perty.) (R.) *Species mihi ignota.*

Diam.—0.0019″—0.0022″. (R.)

Syn.—*C. moniliferum*, RABENHORST, Flora Europ. Algarum, Sect. III. p. 131.

Hab.—Georgia; Rhode Island; Bailey.

"Frond smaller than C. Ehrenbergii, stout, five or six times as long as broad, lunately curved, extremities tapering, upper margin convex, lower concave, with a *central inflation*, ends rounded; large granules, conspicuous, in a single longitudinal series; e. f. colorless, without striæ, suture not evident. L. $\frac{1}{75}''$—$\frac{1}{60}''$. B. $\frac{1}{510}''$—$\frac{1}{466}''$. Archer." Pritchard's *Infusoria*, p. 748.

C. Leibleinii, KTZ.

C. priore minus, semilunare, magis incurvum, ventre inflato, ad utrumque polum largius attenuatum, apicibus achrois acutis; vesiculis chlorophyll. in quoque crure 5–6, in serie simplici axillari dispositis; locello magno, apices versus sito, corpuscula numerosa includente. (R.)

Diam.—$\frac{13}{7500}''$.

Syn.—*C. Leibleinii*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 132.

Hab.—Georgia; Rhode Island; Bailey. Pennsylvania; Wood.

"Frond somewhat stout, distance between the extremities six or eight times the breadth, crescent-shaped, much curved, rapidly attenuated, upper margin very convex, lower very concave, often with a slight central inflation; ends subacute; large granules, in a single series; fillets few or indistinct; e. f. somewhat straw-colored, without striæ; suture evident. Sporangium orbicular." Archer.

Remark.—Fig. 6, pl. 12, represents this plant, magnified 260 diameters.

4. *Cellulæ maxime curvatæ, ventre non tumidæ.*

4. *Cells most curved, the belly not tumid.*

C. Dianæ, EHRB.

C. anguste fusiforme, semilunare, utroque polo valde attenuatum, apicibus subacutis; cytiodermate achroo (vel dilutissime umbrino), striis subtilissimis medio interruptis prædito, in media parte striis transversalibus 3–5; vesiculis in quoque crure 6–7, in serie unica axili dispositis; laminis chlorophyllaceis pluribus, sæpe flexuosis; locello indistincto, corpusculis pluribus vivide mobilibus. (R.)

Diam.—Lat. $\frac{4}{7500}''=.00053''$. Long. $\frac{62}{7500}''=.00082''$.

Syn.—*C. Dianæ*, EHRENBERG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 133.

Hab.—Georgia; Florida; Rhode Island; Bailey. Pennsylvania; Wood.

Frond crescent-shaped, six or eight times as long as broad, much curved, rapidly attenuated; upper margin very convex, lower very concave without a central inflation; ends subacute with a very slight emargination at the upper outer extremity; large granules in a single series; empty frond, somewhat straw-colored, or faintly reddish, without striæ, suture evident. (A.)

Remarks.—Mr. Archer marks *C. Venus*, KTZ., as a doubtful synonym of this species; not having Prof. Kützing's work at hand, I do not know whether *C. Venus*, KTZ. is really the following species or not. The two forms *here known* as *C. Dianœ*, EHRB. and *C. Venus*, KTZ. are, however, I think sufficiently distinguished. Fig. 4, pl. 12, represents this species of desmid.

C. Venus, KTZ.

C. parvum, plus minus gracile, semicirculare, octies-duodecies longius quam latum, in apices subacutos æqualiter sensimque attenuatum; cytiodermate tenui, læve; laminis chlorophyllaceis obliteratis; vesiculis in quoque crure 3–4; locello distincto corpusculis 4–6 repleto. (R.)

Diam.—.0004".

Syn.—*C. Venus*, Kützing. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 134.

Hab.—South Carolina. (Ravenel.)

Small, more or less slender, semicircular, eight to twelve times longer than broad, equally and very perceptibly attenuate at both apices; cytioderm thin, smooth; chlorophyllous lamina obliterated; vesicles in each crus 3–4; vacuole distinct, containing 4–6 corpuscles.

Remark.—Fig. 7, pl. 11, represents in outline a frond magnified 450 diameters.

C. parvulum, Næg.

C. parvum, semicirculare, medio non tumidum, gracile, anguste lanceolatum, sexies-octies longius quam latum, apicibus acutis; cytiodermate tenui, lævissimo, vacuato nonnunquam luteolo-fuscescente et subtiliter striato; vesiculis uniseriatis, in quoque crure 2–4, varius 1–7; laminis chlorophyllaceis 4–5. (R.)

Diam.—Max. 0.00026″—.00062″ (R.) (.0008″ W.)

Syn.—*C. parvulum*, Nægeli. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 134.

Hab.—Prope Philadelphia, Wood.

Small, semicircular, not swollen in the middle, slender, narrowly lanceolate, six to eight times longer than broad, with the apices acute; cytioderm thin, very smooth, when empty somewhat yellowish-fuscous and finely striate; vesicles uniseriate, in each crus 2–4, rarely 1–7; chlorophyllous lamina 4–5.

Remarks.—I have referred to this species a desmid which I have found about Philadelphia, and which agrees in all respects with the description of Prof. Rabenhorst except in attaining a larger size.

Fig. 5, pl. 12, represents this plant magnified 450 diameters.

C. Jennerii, Ralfs.

C. cylindraceo-fusiforme, semilunare, læve, utrinque modice attenuatum, sexies-octies longius quam latum, apicibus obtuse rotundatis; vesiculis in quoque crure 5–7, in serie unica axili dispositis; laminis chlorophyllaceis 2–3; locello subæquali magno, corpusculis numerosis impleto. (R.)

Diam.—0.00057″. (R.)

Syn.—*C. Jennerii*, Ralfs. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 134.

Hab.—Rhode Island, Bailey.

Frond small, distance between the extremities six or seven times the breadth, crescent-shaped, much curved, gradually tapering (sometimes with an obscure central constriction); upper margin very convex, lower very concave without a central inflation; ends obtuse, rounded; large granules in a single series; e. f. colorless without striæ. L. $\frac{1}{281}$″. B. $\frac{1}{1730}$″. Archer. Pritchard's *Infusoria*.

b. Zygosporæ plerumque quadrangulares, cellularum crura longe vel longissime producta, sæpe setiformia.

b. Zygospores mostly quadrangular, crura of the cells greatly produced, often setiform.

C. rostratum. Ehrb.

C. corpore lanceolato-fusiformi, utrinque valde et longe attenuato, leviter curvato, striato; cornibus setaceis singulis corpus vix æquantibus, sæpius longe brevioribus; cytiodermate dilute umbrino vel luteolo, dense striato; vesiculis uniseriatis, in quoque crure 5–6; locello oblongo, sæpius indistincto, corpusculis 12–15 vivide se moventibus. (R.)

Diam.—0.0008″. (0.0009″–0.0016.″ R.)

Syn.—*C. rostratum*, EHRENBERG. RABENHORST, Flora Europ. Algarum, Sect. III. p. 135.

Hab.—In fossis, prope Philadelphia; Wood.

Body lanceolate-fusiform, ăt each end greatly and for a long distance attenuated, slightly curved, striate; crura setaceous and scarcely as long as the body and sometimes much shorter; cytioderm light or luteolous, densely striate, vesicles uniseriate, 5–6 in each crus; vacuoles oblong, often indistinct, containing from 12–15 actively moving granules.

Remark.—Fig. 3, pl. 12, is a drawing of this species, magnified 260 diameters.

C. setaceum, EHRB.

C. corpore anguste lanceolato, recto vel subrecto, distincte striato, utrinque in rostrum setaceum, levissime incurvum, obtusum, longissime porrecto; singulo rostro corpore 3–4 plo longiore; et vesiculis et locello indistinctis. (R.)

Diam.—Max. (plerumque) 0.0004″—0.00044.″ (R.)

Syn.—*C. setaceum*, EHRENBERG, Flora Europ. Algarum, Sect. III. p. 136.

Hab.—Stonington. (Lewis) Pennsylvania; Wood. Georgia; Florida; Providence, Rhode Island; Bailey.

Frond very slender, -from twenty to twenty-five times as long as broad, *narrow-lanceolate; upper and lower margins nearly equally and but slightly convex;* each extremity tapering into a very long and slender setaceous colorless beak, longer than the body, ultimately curved downwards, ends obtuse; e. f. colorless, *striæ close, faint,* central suture *solitary.* Sporangium cruciform. L. $\frac{1}{116}$″. B. $\frac{1}{2381}$″. Archer. Pritchard's *Infusoria.*

C. Amblyonema, EHRB.

C. filiforme, cylindricum, læve, utroque fine parum attenuatum, apice rotundum. (R.)

Syn.—*C. lineatum*, EHRB. BAILEY, American Journal of Science and Arts, 1841, p. 303.

C. Amblyonema, EHRB. Verbreit. p. 123. RABENHORST, Flora Europ. Algarum, Sect. III. p. 138.

Hab.—West Point, New York; Providence, Rhode Island, Bailey.

Filiform, cylindrical, smooth, gradually attenuated at each end; the apex rounded.

Remarks.—I have never recognized a specimen of this species, nor have I had access to the original description of Ehrenberg.

Genus TETMEMORUS.

Cellulæ cylindricæ vel fusiformes, rectæ, medio distincte constrictæ, utroque polo anguste incisæ cytioderma sat firmum, plerumque granulatum vel punctatum.

Cells cylindrical or fusiform, straight, distinctly constricted in the middle, narrowly incised at each end. Cytioderm firm, mostly punctate or granulate.

T. Brébissonii, (MENGH.) RALFS.

T. diametro 4–6 plo longior; a fronte cylindricus, utroque polo non attenuatus sed rotundato-truncatus; a latere fusiformis et a medio in apices rotundatos sensim attenuatus; cytiodermate striato-punctato.

Diam.—$\frac{12}{7500}$″ = .0016″.

Syn.—*T. Brébissonii*, MENGHEINI, RABENHORST, Flora Europ. Algarum, Sect. III. p. 139.

Hab.—In fossis, Atlantic States.

Four to six times longer than broad; from the front cylindrical, not attenuate at the truncately rounded ends; viewed laterally fusiform, attenuated from the middle to the rounded ends; cytioderm striately punctate.

Remarks.—The central constriction is more apparent in the lateral than front view. When the frond is full of endochrome the punctæ on the outer wall are not apparent, but when it is empty they are seen to be small, and closely arranged in stria-like rows. This species extends through all the Atlantic sea-board States. Prof. Bailey has found it in South Carolina and Florida, as well as in Rhode Island. I have collected it in Centre County, of this State.

Fig. 3, pl. 21, represents an empty half frond of this species; 3 *a* the outline of the frond.

T. granulatus, (BRÉB.) RALFS.

T. habitu Tetm. Brébissonii, sed major et cytiodermate irregulariter granulato-punctato. (R.)

Diam.—$\frac{1}{750}$″ = .0013″. (.00155″. R.)

Syn.—*T. granulatus*, (BRÉBISSON.) RALFS. RABENHORST, Flora Europ. Algarum, Sect. III. p. 140.

Hab.—Prope Philadelphia; Wood. Rhode Island; (S. T. Olney.) Thwaites. South Carolina; Bailey.

Frond somewhat longer than T. Brebissonii, about five or six times longer than broad; in both f. v. and s. v. fusiform, the constriction a very shallow groove, ends with a hyaline lip-like projection extending beyond the notch; endochrome with a longitudinal series of large granules; e. f. punctate, the puncta scattered, except near the constriction; where they are disposed in two transverse rows. Sporangium orbicular, smooth, margin finely striated, placed between the deciduous empty fronds. L. $\frac{1}{130}$″. B. $\frac{1}{649}$″. Archer. Pritchard's *Infusoria.*

Remark.—Fig. 8, pl. 12, represents this species magnified 450 diameters.

T. giganteus, WOOD.

T. maximus, oblongus, diametro 3 plo longior; apicibus haud attenuatis, late rotundatis; suturis profundis, linearibus; cytiodermate irregulariter granulato-punctato.

Diam.—$\frac{24}{7500}$″ = .0031″.

Syn.—*T. giganteus*, WOOD. Proc. Acad. Nat. Sci., 1869.

Hab.—In stagnis, Centre County, Pennsylvania.

Very large, oblong, 3 times longer than broad; with the ends not attenuate but broadly rounded; suture profound, linear; cytioderm irregularly granulately punctate.

Remarks.—I found this beautiful desmid in a stagnant pool in Bear Meadows, Centre County, in the month of August. It is very different in its outline from its nearest ally, *T. granulatus.* The diameter is preserved uniform until near the end, where there is an alteration in the line of the margin, so as to cause some contraction, which is, however, wanting in some specimens. The ends are therefore broad and obtuse. The size is also double that of *T. granulatus.*

Fig. 7, pl. 12, represents a frond of this species magnified 260 diameters.

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T. levis, (Kutz.) Ralfs.

T. Brébissonii formis similis sed parvior, 3–4 plo longior quam latus; cytiodermate plerumque levissimo, interdum indistinctissime punctato.

Diam.—$\frac{1}{1500}''=.00066''$.

Syn.—*T. levis,* Kützing. Ralfs. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 140.

Hab.—In aquis quietis, prope Philadelphia.

Similar in form to T. Brébissonii, but smaller, 3–4 times longer than broad; cytioderm mostly very smooth, sometimes indistinctly punctate.

Remarks.—Prof. Rabenhorst states that the cytioderm of this species is very smooth, and Mr. Ralfs says that he has failed to detect any punctations, but also states that "Mr. Jenner and Mr. Ross assure me that they (punctæ) are scattered as in *T. granulatus.*" I have no doubt of their existence in certain individuals, whilst in other cases they appear to be absent.

Genus PLEUROTÆNIUM, Nægeli (1849).

Cellulæ singulæ in aqua natantes, rectæ vel subrectæ, cylindricæ vel fusiformes, valde elongatæ, utroque polo rotundatæ vel truncatæ, medio leviter constrictæ, ex transverso circulares. Cytioplasma chlorophyllaceum in laminis longitudinalibus pluribus dispositum, et sub utroque polo locello rotundato corpusculis se vivide moventibus impleto instructum.

Cells single, swimming in water, straight or nearly so, cylindrical or fusiform, very much elongate, rounded or truncate at each end, in the end view with a circular outline. Chlorophyllous protoplasm arranged in longitudinal laminæ and furnished at each end with a round vacuole containing actively moving corpuscles.

Remarks.—This genus appears to include the main portion of the species, which have been described under the name of *Docidium;* the remainder being representatives of a number of genera. I have not had access to the original description of *Docidium,* and do not know in what year it was published; but, according to De Bary, *Docidium* is much the older name ("Ueber de Conjugat.," p. 75). M. De Bary states, however, that he prefers the name of Nægeli, because that authority first defined the genus and his name expresses very clearly the character of it, as well as from the circumstance that the name *Docidium* having been made to cover a heterogeneous mass of species, its retention might cause confusion. I confess to thinking that this action of De Bary is not in accordance with the recognized laws of priority, but, in the absence of the original description, have thought best to follow it.

P. trabecula, (Ehrb.) Nægeli.

P. sæpe valde elongatum, octies vicies-longius quam latum, cylindraceum, utroque fine lævissime attenuatum aut incrassatum, juxta medium constrictum sæpius bigibbum (quasi biundatum), apicibus late truncatum; cytiodermate tenui lævi, achroo. (R.)

Diam.—$\frac{1}{750}''=.0013''$.

Syn.—"*Docidium Ehrenbergii.* Ralfs." Bailey, Microscopical Observations. Smithsonian Contributions.

Pleurotænium trabecula, (Ehr.) Nægeli. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 141.

Hab.—South Carolina, Georgia, Florida; Bailey. Pennsylvania; Wood.

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(*Docidium Ehrenbergii.* RALFS.) Frond slender, *linear;* suture forming a very sharply defined rim; segments 8–12 times longer than broad, basal inflation having another smaller one above it, sides otherwise straight, parallel; ends crenate, owing to a number of emarginations on the edge of the truncate extremities, from three to five of the crenations being usually visible; e. f. punctate or rough with minute granules. Sporangium suborbicular or elliptic, or slightly angular, smooth, placed between the deciduous empty fronds. Ciliated zoospores formed by segmentation of the cell contents, and their emission effected through the opened apex of each one, two or three, especially, formed lateral tubes arising from beneath the base of one of the segments. Archer.

Remarks.—This species is quite common around Philadelphia; but I do not remember ever to have seen one with the cell-wall granulate. The smaller of the two umbonations near the centre is often wanting or exceedingly small, and the crenulations in the ends are very often obsolete.

Fig. 9, pl. 12, represents a cell of this species magnified 160 diameters.

P. Baculum, (BRÉB.) DE BARY.

P. priori simile, sed gracilius, angustius et plerumque longius, medio tantum semel constrictum; cytiodermate lævi. (R.) *Species mihi ignota.*

Diam.—0.00054″—0.0009″. (R.)

Syn.—*P. Baculum,* (BRÉB.) DE BARY. RABENHORST, Flora Europ. Algarum, Sect. III. p. 141.

Hab.—Georgia; Bailey.

Frond slender, suture not prominent; segments very many times longer than broad, basal inflation very conspicuous, solitary, sides otherwise straight, very nearly parallel, large granules of the endochrome in a single series; ends entire; e. f. without puncta. L. $\frac{1}{111}$″. B. $\frac{1}{1937}$″.

P. breve, WOOD.

P. robustum, diametro 4–8 plo longius, in medio distincte constrictum sed haud undulatum, utroque polo nonnihil attenuatum; apicibus truncatis et nonnihil rotundatis; cytiodermate crassissimo, dense granulato-punctato; marginibus vel rectis, vel breve undulatis.

Diam.—.0038″—.00095″.

Syn.—*P. breve,* WOOD. Proc. Acad. Nat. Sciences, 1869.

Hab.—District of Columbia. (Billings.)

Robust, 4–8 times longer than broad, distinctly constricted but not undulated in the middle, slightly attenuated towards the ends; apex truncate and somewhat rounded; cytioderm very thick, densely minutely granulate; margins either straight or shortly undulate.

Remarks.—This species was sent to me by Dr. Billings, who obtained it near Washington, D. C. The margins are sometimes straightish, but in other fronds there are three or more distinct short undulations, or rounded projections in each half margin. The cell-wall is excessively thick, especially at the end—in many cases much thicker than the drawing.

Fig. 2, pl. 21, represents an empty frond of this plant magnified 750 diameters.

P. crenulatum, (EHRB.) RABENHORST.

P. robustum, cylindraceo-subclavatum, octies-duodecies longius quam latum, medio undulato-nodulosum, stricturæ mediæ margine tumido, apicibus late truncatis, altero sæpe crenulato; cytiodermate granulato-punctato. (R.) *Species mihi ignota.*

Diam.—0.0023″. (R.)

Syn.—*P. crenulatum,* (EHRB.) RABENHORST, Flora Europ. Algarum, Sect. III. p. 142.

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Docidium nodulosum, Bréb. Ralfs. British Desmidiæ, p. 155.
Closterium trabecula, Bailey. American Journal of Science, 1841.

Hab.—In aquis quietis, South Carolina; Georgia; Florida; Rhode Island; Bailey. Pennsylvania; New Jersey; Wood.

(*Docidium nodulosum.*) Frond very stout, the thickened sutures forming a projecting rim; segments four to six times as long as broad, scarcely attenuated, regularly inflated at intervals so as to form an undulated margin, the basal inflation the most prominent, the others, as they approach the ends, less so, where they are indistinct or wanting; ends entire; e. f. coarsely punctate. L. $\frac{1}{50}$″. B. $\frac{1}{428}$″. Archer. Pritchard's *Infusoria.*

Remarks.—I have found this species in "Shepherd's Mill Pond," near Greenwich, Cumberland County, New Jersey, and also in a Spring in the Philadelphia Park, near Columbia bridge.

Fig. 1, pl. 21, represents the outline of a frond of this species magnified 160 diameters.

P. clavatum, (Ktz.) De Bary.

P. subcylindraceum, multoties (16–24) longius quam latum, ad utrumque polum sensim incrassatum, subclavatum, apicibus late truncatis; cytiodermate firmo achroo, dense et irregulariter granulato-punctato. (R.) *Species mihi ignota.*

Diam.—Max. 0.00165″—0.00147″; min. = 0.0010″—0.00092″. (R.)

Syn.—*P. clavatum*, (Ktz.) De Bary. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 141.
Docidium clavatum, Kützing. Ralfs, British Desmidiæ. Archer. Pritchard's *Infusoria.*

Hab.—South Carolina; Georgia; Bailey.

Frond slender, suture scarcely prominent, segments eight to ten times as long as broad, slightly clavate near the ends, and ultimately somewhat attenuated, basal inflation sometimes solitary, sometimes having another slight one above it; ends entire; e. f. punctate. L. $\frac{1}{65}$″. B. $\frac{1}{813}$″.

P. undulatum, (Bailey.)

D. læve, gracile cylindricum, undulatum, latitudine 18–20 plo longius, medio modice constrictum; cruribus et basi et apice truncatis et crenatis. (R.) *Species mihi ignota.*

Syn.—*Docidium undulatum*, Bailey. Micros. Observ. p. 36.

Hab.—Florida, Bailey.

"Segments eight to ten times longer than broad, constricted six to eight times at regular intervals throughout their entire length, with the base and ends crenate, smaller than *D. nodulosum*, Bréb., with more frequent and deeper constrictions. The same characters distinguish it from *D. nodosum* and *D. constrictum.*"

P. nodosum, (Bailey.)

D. validissimum, undulatum, spinulis sparsis hirsutum, medio valde constrictum, diametro 8–10 plo longius; cruribus e basi dilatata leviter attenuatis 4-undatis, apicibus quasi productis, latissime truncatis; locello apicali ratione parvo, rotundo, corpusculis paucis (ut videtur) repleto. (R.) (*Species mihi ignota.*)

Syn.—*Docidium nodosum*, Bailey. Micr. Observ., pl. 1, fig. 4. Ralfs, British Desmids, p. 218.

Hab.—United States; Bailey.

"Frond stout; segments with four prominent nodes separated by constrictions; end view crenate. An end view shows that each node is not a simple swelling, but really formed by whorls of tubercles. 'This species is easily recognized by the deep indentations in its outline, corresponding to the constrictions which separate the transverse rows of knob-like projections. It is one of the largest species in the genus,' Bailey." Ralfs.

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P. constrictum, (Bailey)

D. subvalidum, læve, latitudine 10–12 plo longius, medio valde constrictum, stricturæ margine non prominente; cruribus a basi tumida in apicem late truncatum non attenuatis, 4 undulatis. (R.) (*Species mihi ignota.*)

Syn.—*Docidium constrictum,* Bailey. Ralfs, British Desmids, p. 218.

Hab.—Rhode Island, Bailey.

"Frond stout, segments with moderately deep constrictions, which separate four equal, gently curving prominences; end view entire. 'This species is at once distinguished from *D. nodosum* by the cross section of the nodes being a simple circle instead of an indented one,' Bailey." Ralfs.

P. verrucosum, (Bailey)

D. validum, granuloso-verrucosum, latitudine 10–12 plo longius, undulatum, apicibus integris truncatis. (R.) (*Species mihi ignota.*)

Syn.—*Cosmarium verrucosum,* Bailey, Amer. Journ. Sci. and Arts, 1846.
Docidium verrucosum, Ralfs, Brit. Desm. p. 218. Bailey, Micr. Observ. p. 28.

Hab.—Rhode Island; Bailey.

"Segments, with numerous whorls of small prominences, which give the margins an undulated appearance, all the undulations are equal. 'This is a very pretty species with a waved outline, caused by the slight projections, which are arranged in numerous transverse rings,' Bailey." Ralfs.

P. hirsutum, (Bailey)

D. spinuloso-hirsutum, medio valde constrictum, diametro 10–12 plo longius; cruribus et basi et apice subdilatatis, truncatis. (R.) (*Species mihi ignota.*)

Syn.—*Docidium hirsutum,* Bailey, Micr. Observ. p. 36.

Hab.—Florida; Bailey.

"Segments many times longer than broad, slightly inflated at the base, surface hirsute. A small species resembling *D. Ehrenbergii* in its form, but strongly hirsute on its outer surface." Bailey.

Genus TRIPLOCERAS, Bailey.

Cellulæ singulæ, rectæ, valde elongatæ, processus magnorum seriebus transversis armatæ, utroque polo trilobatæ, lobis acute bidentatis.

Syn.—*Triploceras,* Bailey, Microscopical Observations, p. 37, Smithsonian Contributions, 1850.

Cells single, straight, very much elongate, armed with transverse series of large processes, trilobate at each end, lobes acutely bidentate.

T. verticillatum, Bailey.

T. cellulis subcylindricis, sed utroque fine leviter angustatis et nonnihil fusiformibus, modice robustis, diametro 12–20 plo longioribus; processibus lateralibus robustis, magnis, apice emarginatis.

Diam.—Cum process. $\frac{11}{7500}'' = .00146''$; sine process. $\frac{17}{15000}'' = .00113''$.

Syn.—*T. verticillatum,* Bailey. Microscopic Observations. Smithsonian Contributions, 1850.
Docidium verticillatum, Ralfs, British Desmids, p. 218.
Pleurotænium verticillatum, Rabenhorst, Flora Europ. Algar., Sect. III. p. 148.

Hab.—Rhode Island, New Jersey, Georgia, Florida; Bailey. Saco Lake, (Dr. Lewis) Wood.

Subcylindrical, but slightly narrowed at each end, and therefore somewhat fusiform, moderately robust, 12–20 times longer than broad; lateral processes large, robust, with their apices emarginate.

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T. gracille, Bailey.

T. cellulis subcylindricis, utroque fine vix angustatis, gracillimis, diametro 25–30 plo longioribus; processibus lateralibus brevibus, conicis.

Diam.—Cum process. $\frac{6}{7500}''$ = .008″; sine proc. $\frac{9}{15000}''$ = .0006″.

Syn —*T. gracille*, Bailey, Smithsonian Contributions.
Docidium pristidæ, Hobson, Magazine Natural History, v. p. 168.
Pleurotænium gracile, Rabenhorst, Flora Europ. Algar., Sect. III. p. 144.

Hab.—In iisdem cum antecedente locis.

Subcylindrical, scarcely narrowed at the ends, 25–30 times longer than broad; lateral processes, short, conical.

Genus SPIROTÆNIA, Bréb.

Cellulæ rectæ, cylindricæ vel subfusiformes, sæpe in muco gelatinoso aggregatæ, medio haud constrictæ, utroque polo rotundatæ vel acuminatæ. Cytioplasma chlorophyllaceum in laminis spiralibus dispositum.

Cells straight, cylindrical or subfusiform, often aggregated in a gelatinous mucus, not constricted in the middle, rounded or acuminate at each end. Chlorophyllous cytioplasm arranged in spiral lamina.

Sp. bryophila, (Bréb.) Rabenhorst.

Sp. mimina, bryophila; cellulis in gelatina matricali consociatis, oblongo-cylindricis, rectis vel subcurvatis, bis vel ter longioribus quam latis, utroque polo rotundatis; lamina chlorophyllacea singula anfractu 1–2½.

Diam.—$\frac{1}{3000}''$ = .00033″ (0.00024″—.00029″. R.)

Syn.—*Spirotænia bryophila*, (Bréb.) Rabenhorst, Flora Europ. Algarum, Sect. III. p. 146.

Hab.—Prope Philadelphia; Wood.

"(S. muscicola (De Bary)) Frond cylindrical two to four times as long as broad, ends rounded; endochrome a single, broad, smoothly defined, widely wound spiral band, its revolutions very few (one or two)." (A.)

Remarks.—I found this beautiful little desmid on the North Pennsylvania Railroad, near Chelten Hills, growing amongst some mosses which were kept constantly wet by overhanging dripping rocks. It formed little transparent masses of almost colorless jelly looking much like drops of dew. It agrees well with the descriptions of the European form, except that there were generally from 2–2½ turns of the spiral, and the cells exceed somewhat the measurements of Prof. Rabenhorst. The cells are closely placed in the jelly.

Fig. 10, pl. 12, represents some plants of this species.

Sp. condensata, (Bréb.) Rabenhorst.

Sp. cellulis cylindraceis, rectis (vel leviter curvatis) octies vel decies longioribus quam latis, utroque polo rotundatis; laminis chlorophyll. singulis, anfractibus subarctis (plerumque 8–12).

Diam.—0.00075″.

Syn.—*Sp. condensata*, (Bréb.) Rabenhorst, Flora Europ. Algarum, Sect. III. p. 146.

Hab.—Florida; Rhode Island; Bailey. Pennsylvania; Wood.

Frond cylindrical, two to four times as long as broad, ends rounded; endochrome a single, broad, closely wound spiral band, its revolutions numerous. L. $\frac{1}{208}''$. Br. $\frac{1}{1048}''$. Archer. Pritchard's *Infusoria.*

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Remarks.—The only specimens that I have seen of this species were found in a spring in the Philadelphia City Park, near Columbia bridge.

Fig. 11, pl. 12, was drawn from one of these specimens.

Genus SPHÆROZOSMA, Corda.

Cellulæ compressæ, medio transversim profunde incisæ, itaque bilobatæ, in quoque lobo massa chlorophyllosa quadriradiata nucleum amylaceum involvente præditæ, in filum planum tæniiformem literaliter isthmis conjunctæ. Zygosporæ globosæ vel ovales, glabræ. (R.)

Syn.—*Isthmosira*, Ktz.
Odontellæ, spec., Ehrb.
Isthmiæ, spec., Meneg.
Spondylosium, Bréb.

Cells compressed, transversely very deeply incised in the centre and therefore bilobate, furnished in each lobe with a quadriradiate mass of chlorophyl surrounding a starch grain, conjoined laterally by isthmuses in a tæniform fascia.

Remarks.—I have never found any species of this genus in America. Professor Bailey has, however, detected the following:—

Sph. excavatum, Ralfs.

Sph. plerumque nudum (sine tubo mucoso) sph. vertebratum multo minus; cellulis diametro duplo-longioribus, medio excavato-constrictis, a latere ellipticis utroque polo rotundatis; lobis brevibus truncato-rotundatis, lævibus vel granulato-denticulatis; isthmis binis parvis verruciformibus; zygosporis plerumque ovatis. (R)

Latit. filor. 0.00047″—0 00032″. (R.)

Syn.—*Sph. excavatum*, Ralfs, British Desmids, p. 67.

Hab.—Florida; Georgia; South Carolina; Rhode Island; Bailey.

"Joints longer than broad, having a deep sinus on both sides and two sessile glands at each margin at their junction, very minute, seldom more than twenty-five joints in the filament, which is fragile, and finally separates into single joints; at their junction, in the front view are two minute glands or processes, situated one near each angle, and nearly invisible before the escape of the endochrome. The joints are nearly twice as long as broad and much constricted in the middle; the constriction is like an excavation or broad sinus on each side, so that the margins of the filaments appear sinuated. The endochrome is pale bluish-green with minute scattered granules. The transverse view is oblong with four sessile glands, two on each side and situated near the ends."—Ralfs' *Brit. Desm.*, p 67.

Sph. pulchrum, Bailey.

Sph. cellulis oblongo-quadrangularibus, diametro duplo-brevioribus, acute incisis, arcte connexis; lobis oblongis rectis, apice rotundatis; isthmis nullis, vagina mucosa ampla disncta. (R.)

Syn.—*S. pulchrum*, Bailey. Ralfs, British Desmid., p. 209 (Cum icone).

Hab.—West Point, New York; Princeton, New Jersey; Bailey.

"Joints twice as broad as long, deeply incised on each side; junction margins straight, connected by short bands."

Remark.—"Prof. Bailey informs me that this species is twice as large as *Sph. vertebratum*," Ralfs.

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Sph. serratum, BAILEY.

Sph. cellulis diametro duplo brevioribus, profunde et acute excisis, arcte conjunctis; lobis utrinque cuspidatis, paulum conniventibus; isthmis nullis; vagina crassa. (R.)

Syn.—*Sph. serratum,* BAILEY, Micros. Observation. Smithsonian Contributions, 1850. Cum icone.

Hab.—South Carolina; Georgia; Florida; Bailey.

"Joints broader than long, deeply notched or divided into two transverse portions with acute projecting ends, which give a serrated outline to the chain." Bailey.

Genus HYALOTHECA.

Cellulæ brevæ, cylindricæ, medio non profunde constrictæ, a latere disciformes, in fila confervacea sine isthmis arcte conjunctæ et vagina mucosa ampla achroa inclusæ. Massa chlorophyllosa in quaque semicellula 4–8, 5–10 radiata.

Cells short, cylindrical, not profoundly constricted in the middle, disciform in the end view, closely united without intervening isthmuses into a confervoid filament, which is inclosed in an ample mucous sheath. Chlorophyl masses in each cell 4–8, 5–10 radiates.

H. disilliens, (SMITH) BRÉB.

H. fasciis prælongis; cellulis oblongo-quadrangularibus, diametro sub-duplo brevioribus, interdum ante divisionem subæqualibus, angulis nonnihil rotundatis, plerumque medio obsolete constrictis, sæpe haud constrictis.

Diam.—0.00089″—0.00098″. (R.)

Syn.—*H. disilliens,* (SMITH) BRÉB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 152.

Hab.—South Carolina; Florida; Rhode Island; BAILEY. Rhode Island (S. T. Olney), Thwaites. Pennsylvania; Wood.

Filament very long, cells oblong, quadrangular, about one-half as long as broad, sometimes before division as long as broad, angles somewhat rounded, mostly obsoletely constricted in the middle, often not constricted.

Remarks.—The specimens which I have identified as *H. disilliens,* agree with the various figures and descriptions of the European form, in every thing except that in many cases there is no constriction whatever in the centre of the cell, and when the constriction does exist, it is never so pronounced, as some of the descriptions indicated. The plant is very common about Philadelphia, growing in springs and ditches.

Fig. 12, pl. 12, represents this part of a filament of this species.

H. mucosa, (MERT.) EHRB.

H. fasciis confervaceis, minus fragilibus; cellulis quadrangularibus, diametro æqualibus vel subæqualibus, medio non constrictis, ad utrumque finem (annuliformi-bicarinatis) bidentatis. (R.) *Species mihi ignota.*

Diam.—0.00073″—0.0008″. (R.)

Syn.—*Gloeoprium mucosum,* HASSAL, Fresh Water Algæ, p. 346.
H. mucosa, (MERT.) EHRB. RABENHORST, Flora Europ. Algarum, Sect. III. p. 132.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Filament scarcely fragile, mucous sheath very broad; joints about as broad as long, not constricted, but having at one of the ends a minute bidentate projection on each margin, the

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adjoining ends of the next joint being similar, these projections being produced by an annular grooved rim. L. $\frac{1}{1250}$″—600″. B. $\frac{1}{1250}$″—$\frac{1}{1111}$″. (Archer)

Genus BAMBUSINA.

Cellulæ oblongo-orculiformes, in filamenta articulata nodosa dense conjunctæ, medio vitta transversa carinis duabus annuliformibus limitata instructæ, itaque superne et inferne bidentatæ, fronte circulares, supra et infra dente unico prominente. (R.)

Cells oblong-orculiform, densely united into an articulate nodose filament, surrounded by two median bands.

B. Brébissonii, Ktz.

B. filamentis nodoso-articulatis; cellulis diametro duplo longioribus. (R.)

Diam.—0.00077″—0.00092″. (R.)

Syn.—*B. Brébissonii,* Kützing. Rabenhorst, Flora Europ., Algarum, Sect. III. p. 152.

Hab.—South Carolina. (Ravenel) Wood. South Carolina; Georgia; Florida; Rhode Island. Bailey.

(Didymoprium Borreri, (Ralfs)) Joints inflated, barrel-shaped, longer than broad, without a thickened border at their junction; angles bicrenate, crenatures rounded; transverse view circular; sporangium elliptic, formed within the (for some time) persistent extensions from the conjugating joints, which do not previously break up into single joints, but couple, still united in the filament, in a confused or zigzag manner, some of the joints remaining unchanged. L. $\frac{1}{939}$″. B. $\frac{1}{1030}$″.

Remarks.—The specimens which I have seen agree well with the descriptions, except in regard to size; some of the cells which I measured were more than $\frac{1}{1200}$ of an inch in diameter.

Genus DIDYMOPRIUM.

Cellulæ oblongo-ellipticæ, modice compressæ, ancipites, angulis porrectis inciso-bidentatis, in filamenta articulata biconvexa et torta sine isthmo arcte conjunctæ, et in vagina mucosa inclusæ. Cytioplasma chlorophyllosa cellulæ a fronte cruciatim disposita, cujus crura e laminis duabus parietalibus divergentibus granum amylaceum unicum involventibus formantur.

Cells oblong-elliptical, moderately compressed, two-edged, with the produced angles incisely-bidentate, closely united into a biconvex and twisted filament, which is inclosed in a mucoid sheath, cytioplasm so placed as to be cruciate when viewed from the front (end), each crus composed of two parietal divergent lamina, each of which contains a single starch granule.

D. Grevillii, Ktz.

D. cellulis oblongis diametro duplo brevioribus, saturate viridibus. (R.)

*Diam.*0.0024″—0.0031.″ (R.)

Syn.—*D. Grevillii,* Kützing. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 153.

Hab.—Georgia, South Carolina; Bailey. Pennsylvania; Wood.

Sheath distinct; joints broader than long, with a thickened border at their junction; angles bidentate; teeth angular; transverse view broadly elliptic. Sporangium orbicular, formed within one of the two conjugating joints, the endochrome passing over from one by a narrow connecting tube produced between the otherwise, but little altered, broken-up single joints. (A.)

Remark.—Fig. 13, pl. 12, represents the end view of a broken filament of this species.

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Genus DESMIDIUM.

Cellulæ oblongo-tabulares, medio inciso-bilobæ, lobis integris vel irregulariter dentatis, a fronte tri- vel quadrangulares, angulis obtuse rotundatis, in fila angulosa, prælonga, torta, fragiles arcte connexæ. Massa chlorophyllosa (a cellulæ fronte visa) 3–4 radiata; quisque radius e laminis duabus lateralibus divergentibus compositus. Zygosporæ globosæ vel oblongæ, glabræ.

Cells oblong-tabular, medianly incisely bilobate, with the lobes entire or irregularly dentate, as seen from the front tri- or quadrangular, and having the angles obtusely rounded, closely conjoined into an angular, fragile, twisted filament. Chlorophyl (as seen from the front) 3–4 radiate; each radius composed of two lateral divergent lamina; zygospores globose or oblong, smooth.

D. Swartzii, Ag.

D. cellulis a fronte triangularibus, diametro 2–3 plo brevioribus. (R.)

Diam.—0.00096″—0.00189″. (R.)

Syn.—*D. Swartzii,* Agardh. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 154.

Hab.—In aquis quietis, Atlantic States. Florida; Georgia; South Carolina; Rhode Island; Bailey. New York; Edwards. Pennsylvania; Wood.

Filament triangular, equal, with a single longitudinal waved, dark line, formed by the third angle; joints in front view somewhat quadrangular, broader than long, with two slightly angular crenatures on each lateral margin, united at the whole of their end margins by a thickened border, end view triangular; endochrome three-rayed. Archer. Pritchard's *Infusoria.*

D. quadrangulatum, Ktz.

D. quadrangulare, cellulis oblongo-quadrangularibus, diametro 2–3 plo brevioribus, lobis dentiformibus obtusis, a fronte sinuato-quadrangularibus, angulis late rotundatis, lateralibus excavatis. (R.) *Species mihi ignota.*

Diam.—0.0021″—0.0029″.

Syn.—*D. quadrangulatum,* Kutzing. Rabenhorst, Flora Europ. Algarum, Sect. III, p. 155.

Filament quadrangular, varying in breadth from its twisting, having two longitudinal waved lines; joints in f. v. broader than long, with two somewhat rounded crenatures on each lateral margin, united by the whole of their end margins; e. v. quadrangular; endochrome four rayed. L. $\frac{1}{1244}$″. B. $\frac{1}{603}$″—$\frac{1}{455}$″. (Archer)

D. aptogonium, Bréb.

D. fasciis plerumque subbrevibus, nudis, perforatis; cellulis quadrangularibus, inciso-bilobis, lateralibus concavis, lobis crenatis, a fronte triangularibus (nonnunquam biangularibus), centro concavo, angulis rotundatis protensis isthmum brevissimum triplicem efficientibus. (R.) *Species mihi ignota.*

Diam.—0.00089″—0.00147″. (R.)

Syn.—*Aptogonium desmidium,* Ralfs, British Desmids.
D. aptogonium, Brébisson. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 155.

Hab.—Georgia; South Carolina; Bailey.

Joints in f. v. quadrangular, broader than long, with two rounded crenatures on each lateral margin, united at the outer portion only of each end margin by mutual projections, thus producing intervening central oval foramina. Archer.

Genus APTOGONIUM, Ralfs.

Cellulæ 3–4 angulares vel compressæ, non constrictæ; margine laterali planæ vel crenatæ, in fascias perforato-articulatas, angulares conjunctæ. (R.)

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Cells 3–4 angular or compressed, not constricted, their lateral margins plain or crenate, conjoined into angular perforately articulate fascia.

A. Baileyi, Ralfs.

"Filament not crenated; joints about equal in length and breadth.

Syn.—Odontella? tridentata, Bailey. *In lit. cum icone* (1846).

Hab.—Worden's Pond, Rhode Island; near Princeton, New Jersey, with sporangia," Bailey.

"Filament triangular; joints excavated at their junction like those of Aptogonum desmidium. The joints are not bicrenate, hence the margins of the filament are entire, a character which distinguishes it from that species. The end view is triangular, with rounded angles." Ralfs, *British Desmidieæ*, p. 208.

Genus COSMARIUM, (Corda)

Cellulæ oblongæ, oblongo-cylindricæ, ellipticæ, vel orbiculares, medio transverse plus minus constricæ, utroque polo obtusæ vel rotundatæ et integræ, a vertice ellipticæ. Zygosporæ muricatæ vel verrucosæ.

Cells oblong cylindrical, elliptical or orbicular, more or less transversely constricted in the middle, obtuse or rounded, and entire at each end, viewed from the end elliptical. Zygospore warty or muricate.

1. *Cellulæ sejunctæ.*

1. *Cells separate.*

a. *Cellulæ ellipticæ, vel subellipticæ; semicellulæ medio nonventricosæ.*

a. *Cells elliptical or subelliptical; semicells medianly not ventricose.*

* *Cytiodermate granuloso vel verruculoso.*

* *Cytioderm granular or warty.*

C. margaritiferum, (Turp.) Mengh.

C. paulo longius quam latum, profunde constrictum; sinu amplo, vel modice angusto, interdum intra excavato; semicellulis semiorbicularibus, vel reniformibus vel nonnihil quadrangulis dorso plerumque late rotundatis; cytiodermate verruculoso.

Diam.—Max. $\frac{4}{2500}''$ = .0006″ (0.00073″—0.0012″. R.)

Syn.—Euastrum margaritiferum, Ehrb. Bailey, Silliman's Journal, 1841.
Cosmarium margaritiferum (Turpin), Meneghini. Rabenhorst, Flora Europ. Algar., Sect. III. p. 157.

Hab.—In aquis quietis, South Carolina; Florida; Mexico; Bailey. Pennsylvania, Wood.

A little longer than broad, profoundly constricted; sinus ample or moderately narrow, sometimes widened on the inside; semicells semiorbicular, reniform or somewhat quadrangular; dorsum mostly broadly rounded; cytioderm warty.

Remarks.—I have found a form of this species growing in the vicinity of this city, which I at first was disposed to look upon as distinct, but which, in truth, grades into the typical form. In it the cells are almost quadrangular, often with their basal angles acute. The margin of the frond in *C. margaritiferum*, as it occurs with us, is sometimes distinctly serrate or, more correctly, crenulate from the presence of the granulations. The granules are larger than in *C. botrytis*, but smaller than in *C. tetrophthalmum.* When viewed laterally the semi-cells are roundish, or nearly so (according to Ralfs' elliptical), and closely connected by a very broad neck. I have never seen the sporangia, but, according to Mr. Ralfs, they are orbicular and inclosed in a granulated cell.

Fig. 8, pl. 21, represents half of an empty frond of this species magnified 750 diameters; and fig. 21, pl. xii., a frond densely filled with living endochrome

C. Botrytis, (BORY) MENGH.

C. late ovale, profunde constrictum, diametro plerumque $1\frac{1}{4}$—2 plo longius; sinu angusto, lineare; semicellulis nonnihil triangularibus, apice interdum truncatis, interdum late rotundatis; cytiodermate minute granulato.

Diam.—$\frac{1}{536}''$ = 0.0019″ (0.0014″—0.0023″). (R.)

Syn.—*C. Botrytis,* (BORY) MENEGHENI. RABENHORST, Flora Europ. Algarum, Sect. III. p. 158.

Hab.—Pennsylvania, Wood.

C. broadly oval, profoundly constricted, $1\frac{1}{4}$–2 times longer than broad; sinus narrow, linear; semicells somewhat triangular, with the apex sometimes truncate, sometimes broadly rounded; cytioderm minutely granulate.

Remarks.—In this species the semicells, as viewed transversely, are broadly elliptic in outline. The end view presents a longer narrower ellipse. Their outline, when seen from the front, varies remarkably from that of a very broad semioval to distinctly triangular with a truncate apex. The granules are small and arranged regularly, sometimes they are very obscure. I have often seen the endochrome so arranged as to leave a large pyriform central vacuole in each cell, communicating with the narrow margin between it and the cell-wall. This vacuole was apparently filled with a transparent fluid, in which were minute granules in immense numbers, in constant active motion circling among one another and passing out, into and along the marginal connecting space. According to Ralfs, the sporangia of this species are large ($\frac{1}{435}''$), with branched spines.

Fig. 5, pl. 21, represents an empty frond of this species; 5 *a*, outlines of semicells to show the variations, and fig. 14, pl. 12, represents a frond crowded with endochrome, magnified 460 diameters.

C. ovale, RALFS.

C. magnum, ovale, compressum, profunde constrictum, diametro subduplo longius, ambitu integerrimum vel crenatum, a vertice late ellipticum; semicellulis basi paulo latioribus quam longis, triangulo-rotundatis, disco punctatis, margine verrucis margaritaceis achrois hyalinis in series 4 ordinatis. (R.)

Diam.—Long. 0.0053″—0.0067″. Lat. plerumque 0.0041″. (R.)

Syn.—*C. ovale,* RALFS, British Desmidieæ, p. 98.

Hab.—South Carolina; Rhode Island; Bailey. Cobble Mountain, Pa. (Lewis) Wood.

Frond very large, elliptic, nearly twice as long as broad, constriction very deep, linear; segments somewhat broader than long, somewhat triangular, rounded at ends, rough near the margin, with a band of large pearly granules, producing a dentate appearance, the disc punctate; e. v. elliptic. (A.)

C. Brébissonii, MENEGH.

C. paulo longius quam latum; semicellulis semicircularibus, diametro paulo longioribus, angulis inferioribus obtusis, approximatis, ventre modice concavis subplanis, dorso latissime rotundatis; cytiodermate muricato, muricibus conicis in ordinibus regularibus collocatis. (R.)

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Diam.—Semicell, 0.0019"—0.0022". (R.)

Syn.—*C. Brébissonii*, MENEGHENI. RABENHORST, Flora Europ. Algarum, Sect. III. p. 158.

Hab.—White Mountains, New Hampshire, (Dr. F. W. Lewis) Wood.

Frond somewhat longer than broad, constriction deep, linear; segments semiorbicular, rough all over, with somewhat elongate conical scattered pearly granules; e. v. elliptic. (A)

Remark.—Fig. 6, pl. 21, represents an empty frond of this species, magnified 750 diameters.

C. suborbiculare, WOOD.

C. parvum, suborbiculare, paulo longius quam latum, cum margine enormiter crenato vel crenato-undulato; semicellulis a latere orbicularibus, a vertice ellipticis; sinu extrorsum angustissimo sed introrsum nonnihil excavato; cytiodermate crasso, sparse verruculoso; granulis in semicellulis singulis subdistantibus et in seriebus elongatis, duabus (interdum unica) externis curvatis, et in seriebus duabus internis brevibus et rectis.

Diam.—Lat. $\frac{14}{12000}''$ = .0012"; lat. $\frac{16}{12000}''$ = .0013".

Syn.—*C. orbiculare*, WOOD, Proceed. Acad. Nat. Sc. 1870.

Hab.—In lacu "Saco," New Hampshire, (Lewis.)

Small, suborbicular, a very little longer than broad, with the margin irregularly crenate, or crenate undulate; semicells from the side orbicular, from the vertex elliptical; sinus very narrow, but within somewhat excavated; cytioderm thick, sparsely coarsely granulated; granules subdistant, in each cell arranged in one or two curved marginal series and in a central group of two or three short rows.

Remarks.—The arrangement of the granules in this desmid is peculiar, one, or sometimes two rows of large obtuse pearly granules are placed at rather wide intervals along the whole outer margin, and then in the centre of each semicell is a group of two or three, or even more short straight rows of three or four similar but rather smaller granules. The isthmus is rather broad and short; sometimes it has on it one or two granules.

Fig. 9, pl. 21, represents an empty frond of this species, magnified 750 diameters; 9 *a*, the outline of the end view of the same.

C. tetrophthalmum, (KTZ.) BRÉB.

C. tertiam partem circa longius quam latum, profunde constrictum; sinu angusto, plerumque sublineare; ambitu obtuse crenato; semicellulis nonnihil semicircularibus, ventre subplanis, dorso rotundatis; cytiodermate verruculoso; verruculis magnis, obtusis, subordinatim dispositis.

Diam.—$\frac{30}{12000}''$ = .0025".

Syn.—*C. tetrophthalmum*, (KÜTZING), BRÉBISSON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 159.

Hab.—New Jersey; Wood.

About one-third longer than broad, deeply constricted; sinus narrow, mostly sublinear; margin obtusely crenated; semicells somewhat semicircular, belly nearly even, dorsum rounded; cytioderm warty; prominences large, obtuse, arranged somewhat regularly.

Remarks.—The only specimens I have seen, and I believe the only ones hitherto found on the continent, were collected by myself in "Shepherd's Mill Pond," near Bridgeton, Cumberland County, New Jersey.

Fig. 7 *a*, pl. 21, represents the outline of a frond magnified 460 diameters.

C. amœnum, BRÉB.

C. mediocre, oblongum cylindricum, leviter compressum, diametro duplo fere triplove longius, utroque polo rotundatum, medio profunde constrictum, sinu angusto, lineari, ambitu granulis margaritaceis achrois obsessum, a vertice ellipticum; semicellulis oblongo-rotundatis, dorso alte convexis, lateribus vero rectis parallelis, angulis inferioribus rectis et subacutis; cytiodermate granuloso-verrucoso, verrucis hyalinis in series regulares dispositis. (R.) *Species mihi ignota.*

Long. 0.0017"—0.1016"; lat. 0.00087". (R.)

Syn—*C. amœnum*, BRÉBISSON. RABENHORST, Flora Europ. Algar., Sect. III. p. 159.

Hab.—Florida; Bailey. Rhode Island (S. T. Olney); Thwaites.

Frond twice as long as broad, sides parallel, ends rounded, constriction deep, linear; segments rough with crowded obtuse papilla-like pearly granules; s. v. much compressed, about thrice as long as broad; e. v. elliptic. (A.)

** *Cytiodermate glabro.*

** *Cytioderm smooth.*

C. Cucumis, CORDA.

C. ovale ellipticum, utroque polo late rotundatum, tertiam partem vel duplo longius quam latum, profunde constrictum; sinu lineari; semicellulis angulis inferioribus rotundatis, cytiodermate glabro, haud punctato.

Diam.—Max. long. $\frac{1}{375}''$ = 0.0026"; lat. $\frac{13}{7500}''$ = .0019".

Syn.—*C. Cucumis*, CORDA. RABENHORST, Flora Europ. Algarum, Sect. III. p. 161.

Hab.—South Carolina; Georgia; Florida; Bailey. Pennsylvania; Wood. Saco Lake, (Lewis.)

Oval or elliptic, at each end broadly rounded, one-third to twice as long as broad, profoundly constricted; sinus linear; semicells with their inferior angles rounded; cytioderm smooth, not punctate.

Remarks.—This species is very abundant around Philadelphia. The semicells generally each contain two large globular masses placed near the median line, which are sometimes hidden by the crowded endochrome.

Figs. 15, 15 *a*, pl. 12, represent this species with their endochrome in different conditions; 15 *b*, represents a monstrous frond, which had attempted to divide. but had not succeeded in so doing.

C. depressum, BAILEY.

"Elliptical, binate, division in the plane of the longest axis. Segments entire, nearly twice as long as broad, rounded above, very much flattened at base.

Hab.—Lakes in Florida.

This species resembles *C. bioculatum*, BRÉB.; but the segments are much closer together, and are angular, not rounded at the basal extremities." BAILEY. Microscopical Observations. Smithsonian Contributions.

C. pyramidatum, BRÉB.

C. mediocre, ovale vel subovale, utroque polo truncatum, medio profunde constrictum, duplo

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fere longius quam latum; semicellulis breviter pyramidatis, angulis inferioribus rotundatis, apice (dorso) modo truncatis modo rotundatis, a vertice late ellipticis; cytiodermate punctato vel subtilissime granulato. (R.)

Long. 0.0021″—0.0037″. Lat. max. 0.0026″.

Syn.—*C. pyramidatum*, Brébisson. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 162.

Hab.—Georgia; Florida; Bailey. Pennsylvania; Wood.

Frond scarcely twice as long as broad, suboval; constriction deep, linear; segments pyramidal, rounded at basal angles, somewhat truncate at the ends, punctate; e. v. broadly elliptic. (A.)

Remark.—Fig. 14, pl. 13, is a drawing of this species.

C. bioculatum, Bréb.

C. parviter, circiter tam longum quam latum vel paulo longius, profunde constrictum, sinu extrorsum ampliato; semicellulis diametro duplo latioribus, elliptico-prope hexagonis angulis obtuse rotundatis, integerrimis aut levissime crenulatis; cytiodermate lævi vel subtilissime punctato. (R.) *Species mihi ignota.*

Long. 0.00069″. Lat. 0.00066″. (R.)

Syn.—*C. bioculatum*, Brébisson. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 163.

Hab.—Rhode Island, (S. T. Olney) Thwaites.

Frond minute, about as long as broad, constriction deep, producing a gaping notch at each side; segments about twice as broad as long, elliptic, smooth; s. v. compressed s. v. elliptic. Sporangium orbicular with conical spines. L. $\frac{1}{1416}''$; B. $\frac{1}{1773}''$. (A.)

C. Meneghenii, Bréb.

C. parvum, tam longum quam latum, modo paulo-longius, modo paulo-brevius, profunde constrictum, sinu lineari, extrorsum non ampliato; semicellulis subquadratis, leviter sinuato-hexagonis; angulis rotundatis, cytiodermate lævi vel subtillissime punctato. (R.)

Long. $\frac{1}{87}'''$—$\frac{1}{65}'''$ = 0.00103″—0.0013″; lat. $\frac{1}{109}'''$—$\frac{1}{100}'''$ = 0.00081″—0.00089″. (R.)

Syn.—*C. Meneghenii*, Brébisson. Rabenhorst, Flora Europ. Algar., Sect. III. p. 163.

Hab.—Pennsylvania; Wood.

Frond very minute, rather longer than broad, constriction linear; segments subquadrate, bicrenate at the sides and ends, smooth; e. v. elliptic. (A.)

Remark.—Fig. 18, pl. 12, represents a frond of this species, magnified 750 diameters.

C. crenatum, Ralfs.

C. oblongum, tertiam partem circa longius quam latum, profunde constrictum, sinu lineari angusto; semicellulis e basi lata subsemicircularibus, dorso plus minus depressis vel truncatis, ambitu crenatis vel regulariter undulato crenatis, crenis 10–14; cytiodermate punctato. (R.) *Species mihi ignota.*

Long. 0.0021″—0.0023″; lat. 0.0015″. (R.)

Syn.—*C. crenatum*, Ralfs, British Desmidieæ, p. 96.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Frond slightly longer than broad, constriction linear; segments semiorbicular, ends and sides broadly rounded, crenate or minutely undulate at margin; e. v. elliptic. Sporangium orbicular, spinous; spines elongate, slender, swollen at the base and divided at the apex. L. $\frac{1}{416}''$; B. $\frac{1}{571}''$.

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C. undulatum, CORDA.

C. submediocre, oblongum, diametro subduplo longius, utroque polo late rotundatum, ambitu leviter sinuato-undulatum, profunde constrictum, sinu lineari extrorsum paullum ampliato; semicellulis semiorbicularibus, et dorso et lateribus late rotundatis, margine undulato-crenatis, crenis 9, sublatis; cytiodermate lævi; zygosporis sphæricis spinis elongatis, apice bi-tri-fidis obsitis. (R.) *Species mihi ignota.*

Long. 0.0024″. Lat. 0.0017″. (R.)

Syn.—*C. undulatum,* CORDA. RABENHORST, Flora Europ. Algarum, Sect. III. p. 165.

Hab.—South Carolina; Rhode Island; Bailey.

Frond rather larger than that of *C. crenatum,* slightly longer than broad, constrictions linear; segments semiorbicular, ends and sides broadly rounded, crenate or minutely undulate at the margin; e. v. elliptic. Sporangium orbicular, spinous; spines elongate, slender, swollen at the base and divided at the apex. (A.)

b. Semicellulæ medio-ventricoso inflatæ.

b. Semicells medianly ventricose.

* *Cytiodermate lævi.*

* *Cytioderm smooth.*

C. sublobatum, (BRÉB.) ARCHER.

C. parvum, oblongo subquadratum, diametro subduplo longius, sinu angusto lineari; semicellulis subquadratis, e basi dilatata ad verticem sensim angustatis, angulis et inferioribus et superioribus rotundatis, dorso late truncatis lateribusque leviter sinuatis; cytiodermate lævissimo. (R.)

Long. 0.00179″—0.00196″. Lat. max. 0.0015″—0.00157″. (R.)

Syn.—*C. sublobatum,* (BRÉBISSON) ARCHER. Pritchard's Infusoria, p. 731.

Hab.—Georgia; Florida; Rhode Island; Bailey.

Frond scarcely twice as long as broad, oblong; constriction linear, segments subquadrate, somewhat wider at the base, lateral and end margins slightly concave, smooth and transverse vein cruciform. (A.)

* * *Cytiodermate granulato.*

* * *Cytioderm granulate.*

C. ornatum, RALFS.

C. parvum, plerumque tam longum quam latum; semicellulis reniformibus, diametro duplo longioribus, angulis inferioribus una cum lateribus rotundatis; dorso sub-producto late truncatis; cytiodermate granulato-verruculoso; zygosporis longe spinosis, spinis elongatis apice furcatis. (R.) *Species mihi ignota.*

Long. 0.0016″—0.0015″. Lat. 0.0016″. (R.)

Syn.—*C. ornatum,* RALFS, British Desmidieæ, p. 104.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Frond in f. v. about as long as broad, constriction deep, linear; segments semiorbicular or subreniform, with a central truncate projection at the ends produced by the continuation of a central inflation, rough towards the margin and on the inflation with pearly granules; e. v. with a rounded lobe on each side. Sporangium orbicular, spinous; spines elongated, dilated at the base and slightly divided at the extremity, $\frac{1}{613}$″. (Archer.)

C. commissurale, (BRÉB.)

C. minutum, fere dimidio latius quam longum, profundissime constrictum, sinu amplo basi excavato; semicellulis anguste reniformibus, diametro paene triplo longioribus, leviter incurvis,

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angulis rotundatis, dorso truncato-rotundatis, margine crenulato-dentatis, a dorso oblongis, medio ventricosis, utroque polo plus minus tumidis; cytiodermate granulato margaritifero. (R.)

Long. 0.0010″—0.0012.″ Lat. 0.0013″—0.0015″. (R.)

Syn.—*C. commissurale*, Brébisson. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 170.

Hab.—In lacu. White Mountains, New Hampshire; (Dr. F. W. Lewis)

Frond small, in f. v. one-third broader than long; constriction very deep, rounded; segments narrow-reniform, with a central, somewhat truncate projection, produced by the continuation of the central inflation, rough on the inflation and on the extremities, with somewhat large pearly granules, e. v. three times longer than broad, constricted between the central inflation and the rounded extremities. Sporangium as in *C. ornatum.* (A.)

Remarks.—I have seen but a single specimen of this species which differed from the typical form, in having the sinus very narrow in its outer portion, and in being shorter.

Fig. 16, pl. 13, represents the frond of this specimen, magnified 750 diameters.

C. cælatum, Ralfs.

C. suborbiculare, profunde constrictum; sinu angustissimo lineari; semicellulis inciso-crenatis, angulis rotundatis, a vertice medio nonnihil inflatis; cytiodermate granulato, granulis in series regulariter circulares positis.

Diam.—Long. $\frac{20}{12000}$″ = .0017″. Lat. $\frac{17}{12000}$″ = .0014″.

Syn.—*C. cælatum*, Ralfs, British Desmidieæ, p. 103.

Hab.—In stagnis prope Aiken, South Carolina. (Ravenel.)

Suborbicular, profoundly constricted; sinus very narrow, linear; semicells inciso-crenate, angles rounded, when seen from the end somewhat inflated in the middle; cytioderm granulate, granules placed in circular series.

Remarks.—This species was collected by Prof. Ravenel in a quiet ditch near Aiken, South Carolina, sparsely scattered amidst innumerable diatoms and desmids. The number of the crenations appears to vary. In the few individuals I have seen there were six end ones besides the two very broad basal ones, if the latter can be called crenatures. Ralfs gives six as the total number, and yet every one of his figures has many more. So I think the number a character of but little importance. The circular arrangement of the granules is not so positive and regular in the specimens I have seen, as is represented in the figure of Ralfs, otherwise the agreement is perfect.

C. Broomei, Thwaites.

C. subparvum, plerumque tam longum quam latum, nonnunquam paulo longius, obtuse quadrangulare, profunde constrictum, sinu angustissimo lineari; semicellulis oblongo-quadrangulis, diametro duplo longioribus, angulis et inferioribus et superioribus obtuse rotundatis, ventre subplanis, dorso latissime truncatis et sæpius leniter retusis vel plane convexis; cytiodermate granulato-margaritaceo, granulis in seriebus subrectis collocatis. (R.)

Long. 0.00194″—0.0022″. Lat. max. .002″, thick .0015″.

Syn.—*C. Broomei*, Thwaites. Ralfs, British Desmidieæ, p. 103.

Hab.—Georgia; Bailey. Prope Philadelphia; Wood.

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Frond in f. v. about as long as broad, constriction deep, linear; segments quadrilateral, ends straight, angles rounded, rough all over with minute granules; e. v. twice as long as broad, slightly inflated at the middle and rounded at the ends. Sporangium orbicular, smooth. (A.)

Remarks.—The only specimens which I have seen were found in a brick-pond below the city in the month of June. They agree well with the descriptions, excepting in that I should describe their central inflation as pronounced. The sinuses also are ampliate or hollowed out within. The granulations are quite large, and are arranged somewhat irregularly in rows.

Fig. 15, pl. 13, is a view of the front of the frond magnified 460 diameters; fig. 10, pl. 21, the outline of the lateral view.

c. *Cellulæ fusiformes, cylindricæ vel ovales, in medio leviter constrictæ.*

c. *Cells fusiform, cylindrical, or oval, lightly constricted in the middle.*

C. Thwaitesii, Ralfs.

C. mediocre, diametro bi-triplo longius, fusiformi-cylindraceum, medio leviter constrictum, ambitu integerrimum, utroque polo rotundatum; semicellulis e cylindraceo subconicis, e medio in apicem sensim sensimque (sed modice) attenuatum; cytiodermate lævi vel indistincte punctato. (R.) *Species mihi ignota.*

Long. 0.00267″—0.00287″. Lat. max. 0.0012″. (R.)

Syn.—*C. Thwaitesii,* Ralfs, British Desmidieæ, p. 109.

Hab.—Florida; Bailey.

Frond in f. v. two or three times longer than broad; constriction a very shallow groove; segments subcylindrical, with rounded ends; endochrome scattered; e. v. circular, or very slightly compressed; e. f. not punctate, or puncta very indistinct. (A.)

C. connatum, Bréb.

C. validum, submagnum, leviter compressum, diametro duplo circa longius, subpanduriforme, plus minus constrictum, utroque polo late rotundatum, a vertice lato ellipticum; semicellulis subhemisphæricis, ambitu æquabiliter rotundatis, integerrimis; cytiodermate punctato. (R.) *Species mihi ignota.*

Long. 0.0035″. Lat. max. 0.00165″—0.0019″. (R.)

Syn.—*C. connatum,* Brébisson. Ralfs, British Desmidieæ, p. 108.

Hab.—Florida; Bailey.

Frond large, in f. v. about one-half longer than broad; constriction shallow; segments about two-thirds of a circle, coarsely punctate, and with a distinct, sometimes striated, border; e. v. circular. (A.)

d. *Cellulæ in familias connexæ.*

d. *Cells united into families.*

C. Quimbyii, Wood. (sp. nov.)

C. cellulis parvis, sub-ellipticis, medio profunde constrictis, in familias copulis hyalinis connexis; semicellulis a fronte ellipticis et diametro subduplo longioribus, a vertice ellipticis, a latere rotundatis; sinu lato; marsis chloro-phyllaceis in quaque semicellula singulis; cytiodermate tenue, glabro.

Diam.—Long. $\frac{1}{1000}''$ = 0.001″. Lat. a fronte $\frac{3}{4000}''$ = 0.00075″; a latere $\frac{5}{12000}''$ = 0.00042″.

Hab.—In aquis puris, New Jersey.

Cells small, subelliptical, profoundly constricted in the middle, joined by translucent bands into families; semicélls seen from the front elliptical, and nearly twice as long as broad, from the vertex elliptical, from the side roundish; sinus broad; chlorophyl masses single in each cell; cytioderm thin, smooth.

Remarks.—This plant was found by my friend Mr. Quimby growing in a beautiful spring above Camden, upon whose bottom it formed a gelatinous, translucent, greenish mass. The cells resemble in shape those of *C. cucumis*, although much smaller. They are joined by bands into little families, in which the original parent-cell is generally very distinct, it, or rather the two cells into which it first divides, remaining in the centre of the group. The bands are so hyaline that their edges can alone be distinctly seen, and hence the latter often look as though they were threads—there appearing to be two parallel threads, or two threads crossing one another, or a single thread, according as the band is flat, twisted, or on edge.

It gives me great pleasure to dedicate the species to my friend Mr. Quimby, by whom it was collected.

Fig. 9, pl. 1, represents one of the family groups of this plant.

Genus EUASTRUM, EHRB.

Cellulæ vel oblongæ vel ellipticæ, medio profunde incisæ, symmetrice sinuatæ, vel lobatæ, tumoribus inflatis circularibus (rare obsoletis) instructæ, utroque polo sinuato-emarginatæ vel inciso-bilobatæ, a vertice ellipticæ.

Cells either oblong or elliptic, profoundly incised in the middle, symmetrically sinuate or lobed, provided with circular inflated protuberances (which are rarely absent), at each end sinuately emarginate or incisely-bilobate, from the vertex elliptic.

A. Lobo polares in apice late sinuato-excisi.

A. Polar lobe with its apex broadly sinuately excised.

E. multilobatum, WOOD.

E. magnum, fere duplo longius quam latum, medio profunde constrictum, et cum sinu modice amplo; a latere medio ventricosum et duplo biumbonatum, ad verticem dilatatum et emarginatum; semicellulis a fronte trilobatis, lobis sinus amplissimis inter se sejunctis; lobi basale distincte late emarginato, lobo centrale obtuso, lobo polare late leviter sinuato-emarginato; semicellulis a vertice quinque lobulatis; cytiodermate lævi.

Diam.—Long. $\frac{57}{12000}''=.00475''$. Lat. $\frac{30}{12000}''=.0025''$.

Syn.—*E. multilobatum*, WOOD, Proc. A. N. S., 1869.

Hab.—In lacu "Saco;" New Hampshire; (Lewis.)

E. large, about twice as long as broad, in the centre profoundly constricted, with the sinus moderately large; from the lateral view somewhat enlarged and doubly biumbonate in the middle; semicells from the front trilobate, the lobes separated by very wide sinuses, the basal lobe broadly emarginate, the central lobe obtuse, the end lobe broadly and shallowly sinuately emarginate; semicells from the vertex five-lobed; cytioderm smooth.

Remarks.—The basal lobes of this beautiful desmid are distinctly five lobulate, the lateral lobules being longer and broader than the others, which, instead of being emarginate, are obtuse. The sinuses, separating lobes and lobules, are very broad, with very obtuse angles. When the desmid is viewed from two-thirds round, so as to show the anterior and posterior lobules especially, it presents an

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outline in which all the sinuses are of similar form, and the central and basal lobes are about equal size; whereas, when viewed from the front, the basal lobe is much the broader. When the desmid is viewed from the side it is seen to be enlarged in the centre, and provided with two distinct umbonations each side of the comparatively narrow central sinus.

Fig. 10, pl. 12, represents the front view of a frond of this plant; fig. 5, pl. 20, the outline of a two-thirds view, and fig. 5 *a*, the outline of a lateral view, all magnified 450 diameters.

E. verrucosum, Ehrb.

E. magnum, late ovatum, vix longius quam latum, medio profunde constrictum, sinu extrorsum dilatato; semicellulis trilobatis, lobis triangularibus, divergentibus, apice late et profunde sinuatis; a latere ovato-oblongum, sinuato-lobatum, lobis octo in apice rotundatis, polaribus singulis porrectis, lateralibus ternis; cytiodermate granulato-verrucoso. (R.) *Species mihi ignota.*

Long. 0.0036″—0039″. (R.)

Syn.—E. verrucosum, Ehrenberg. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 179.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey.

Frond somewhat longer than broad, rough all over with conic granules; segments 3-lobed, somewhat divergent, all the lobes broad, cuneate, with a very broad, shallow, or external sinus. Empty frond; f. v. segments with one large circular basal inflation on surface, one smaller on each side, and two others on the end lobe; s. v. segments inflated at the base, narrowed into a short neck, end dilated with a central sinus; e. v. oblong, with three inflations at each side, one at each end, end lobe having 4 divergent lobelets. (A.)

E. gemmatum, Bréb.

E. mediocre, diametro duplo longius, profunde constrictum, sinu angusto lineari, a vertice ovato-oblongum, ambitu sinuato-lobatum, lobis 8 conformibus, rotundatis; semicellulis trilobatis, basi tumoribus 3 in seriem dispositis, lobis in apice profunde emarginatis, lobulis rotundatis, lobo polari dilatato et paulum producto; cytiodermate in tumoribus et lobulis granulato-punctato, cæterum lævi. (R.) *Species mihi ignota.*

Long. 0.00224″—0.0029″. Lat. 0.00157″—0.0017″. (R.)

Syn.—E. gemmatum, Brébisson. Rabenhorst, Flora Europ. Algar., Sect. III. p. 180.

Hab.—Rhode Island; Bailey.

Frond scarcely twice as long as broad; segments 3-lobed, lateral lobes horizontal, deeply emarginate, the protuberances minutely granulate; end lobe dilated, its dilatations inclined upwards, and minutely granulate; ends with a deep rounded emargination. Empty frond slightly punctate; f. v. segments with three granulate inflations near the base; tr. v. broadly elliptic, with three granulate inflations at each side and one at each end; e. v. end lobe cruciform, lobelets rounded, granulate. (A.)

E. oblongum, (Grev.) Ralfs.

E. magnum, diametro duplo triplove longius, oblongum, profunde constrictum, sinu angusto, a latere oblongo-lanceolatum, utroque polo truncato leniter retusum, ambitu undulato-sinuatum; semicellulis (fronte) sinuato-quinquelobis, basi et in quoque lobo tumore instructis, lobis lateralibus in apice dilatato sinuato-retusis, inferioribus latioribus quam superior., lobo polari late cuneato in apice profunde inciso, angulis omnibus rotundatis, cytiodermate lævi; zygosporis globosis verrucosis, verrucis obtuse conicis achrois hyalinis. (R.) *Species mihi ignota.*

Long. 0.0057″—0.0065″. Lat. max. 0.00346″.

Syn.—*E. oblongum,* (GRÉVILLE) RALFS' British Desmidieæ, p. 80.

Hab.—Rhode Island; Bailey.

Frond rather more than twice as long as broad, smooth, oblong; segments 5-lobed; lobes nearly equal, cuneate; lateral lobes, or the basal only, with a broad, shallow, marginal concavity, all their angles rounded, terminal notch linear.

Empty frond; f. v. seg. punctate, with three large inflations, on surface near the base, two others above and two on terminal lobe; tr. v. three times as long as broad, with three subdistant marginal inflations at each side, and one at each end, in β, broader in proportion, more elliptic, and inflations close; e. v. end lobe notched at opposite external margins. Sporangium orbicular, beset with numerous conical tubercles. (A.)

B. Lobi polares evidenter discreti et in apice anguste incisi.

B. End lobes evidently separated and narrowly incised in the centre.

E. crassum, (BRÉB.) KTZ.

E. oblongum, diametro subtriplo longius, profunde constrictum, sinu angusto lineari, e vertice subquadrangulare, utroque polo profunde excisum, angulis rotundatis; semicellulis (fronte) trilobis, basi et in quoque angulo tumore instructis, lobis lateralibus latissimis unisinuatis, lobo polari paullum prominente, in apice bifido, segmentis late rotundatis; cytiodermate distincte punctato, punctis in series transversas ordinatis. (R.)

Long. 0.0051″—0.0073″. Lat. max. 0.0041″. (R.)

Syn.—*E. crassum,* (BRÉBISSON) KÜTZING RABENHORST, Flora Europ. Algarum, Sect. III. p. 181.

Hab.—United States.

Frond about twice as long as broad, subquadrilateral, smooth; segments 3-lobed; basal lobes very broad, with a very broad, shallow marginal sinus, in which there is sometimes a slight intermediate rounded projection; end lobe creneate, rounded, terminal notch linear.

Empty frond; f. v. punctate, segments with three inflations below and two above; tr. v. two or three times longer than broad, with three lobes or inflations at each side and one at each end; e. v. end lobe sinuate at opposite external margins. (A.)

E. ornatum, WOOD.

E. oblongum, diametro duplo longius, profunde constrictum, sinu angusto lineari; semicellulis a fronte trilobatis; lobis basalibus latissimis, nonnihil sinuato-emarginatis, angulis plus minus productis et rotundatis; lobo polari medio profunde lineare inciso, segmentis late rotundatis; semicellulis a latere bilobatis, lobis basalibus profunde emarginatis et cum angulis plus minus acutis; cytiodermate distincte ordinatim punctato.

Diam.—$\frac{35}{12000}$″ = .00029″.

Syn.—*E. ornatum,* WOOD, Proc. A. N. S., 1869.

Hab.—Saco Lake; New Hampshire. Lewis.

E. oblong, twice as long as broad, profoundly constricted; semicells from the front trilobate; basal lobe very broad, slightly sinuately-emarginate, angles more or less produced and rounded; polar lobe medianly profoundly linearly incised, segments broadly rounded; semicells bilobate at the sides, basal lobes profoundly emarginate and with the angles more or less acute; cytioderm distinctly regularly punctate.

Remarks.—This species is close to *E. crassum,* from which it differs in the proportionate length, being only twice instead of three times as long as broad; in the size being only three-fourths as large; and especially in the peculiar lateral splitting, as it were, of the basal lobes.

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Fig. 12, pl. 21, represents the front view of an empty half frond of this species, magnified 450 diameters; fig. 12 *a*, the side view of an empty frond.

E. affine, RALFS.

E. E. humerosum affine, paulo minus; semicellulæ quinquelobæ; lobi basales quales in E. humerosum sed tumores quatuor in seriem transversam simplicem dispositi, lobi intermedii valde abbreviati eorumque basi tumoribus duobus instructi, lobus polaris magis porrectus et in apice minus dilatatus; cytioderma subtilissime punctatum sublæve. (R.)

Long. 0.0038″—0.0041″. (R.)

Syn.—E. affine, RALFS, British Desmideæ, p. 82.

Hab.—South Carolina; Georgia; Bailey.

Frond about twice as long as broad; segments 3-lobed; basal lobes slightly emarginate, having intermediate between them and the end lobe on each side a tubercle representing middle lobes, the upper margin of which is horizontal; end lobe exserted, dilated, its notch linear.

Empty frond; f. v. minutely punctate; the segments with four basal inflations, two above and two on end lobe; tr. v. elliptic, with four inflations on each side and one at each end; e. v. end lobe emarginate at opposite; e. v. end lobe emarginate at opposite external margins, producing four shallow lobulets. (A.)

E. Didelta, (TURPIN) RALFS.

E. robustum, diametro duplo longius etiam supra, in sectione transversa ellipticum, ambitu undulato-crenatum, in utroque latere crenis quaternis; semicellulis pyramidalibus, quinquelobis, tumoribus 9 in series tres alternantibus ordinatis, lobis inferioribus oblique truncato-rotundatis nonnunquam leniter retusis, intermediis subadscendentibus, rotundatis, lodo polari minus dilatato, bifido, segmentis rotundato-truncatis, conniventibus, in apice tumidis; cytiodermate distincte punctato, punctis modo irregulariter sparsis modo in seriebus rectis collocatis. (R.)

Long. 0.0055″. Lat. 0.00279″.

Syn.—E. Didelta, (TURPIN) RALFS, British Desmideæ, p. 84.

Hab.—South Carolina; Georgia; Rhode Island; Bailey. Pennsylvania; Wood.

Frond rather more than twice as long as broad; segments pyramidal, inflated at the base and again at the middle, end scarcely dilated, rounded, its notch linear.

Empty frond punctate; f. v. segments with several inflations in lines and two at the end; tr. v. elliptic with four inflations at each side and one at each end; e. v. end lobe entire at margin. Sporangium orbicular, with subulate spines. (A.)

Remark.—Fig. 13, pl. 21, represents this species.

E. ampullaceum, RALFS.

E. diametro duplo longius; semicellulis trilobis, ad basin tumidis, e basi latissima subito in lobi polaris collum attenuatis, lobis basalibus maximis integris, loco loborum intermediorum processu deutiformi, lobo polari cuneato, in apice bifido, segmentis late truncato-rotundatis; cytiodermate subtiliter punctate. (R.)

Long. 0.0035″—0.0038″. Lat. max. .0026″; lat. in colli (lobi polar.) 0.00085″. (R.)

Syn.—E. ampullaceum, RALFS, British Desmideæ, p. 83.

Hab.—South Carolina; Florida; Bailey.

Frond rather more than one-half longer than broad; segments obscurely 3-lobed, short, with broad inflated base; basal lobes not emarginate, having on each upper side a small intermediate tubercle between each and the end lobe; end lobe exserted and dilated, its notch linear. Empty frond minutely punctate; f. v. narrow elliptic, with several inflated protuberances, ends scarcely dilated, rounded; tr. v. with four inflations at sides and one at each end. (A.)

circulare, Hassal.

E. mediocre, diametro duplo longius; semicellulis trilobis (at non semper distincte), ad basin versus tumoribus quinis aut pluribus in series duas v. tres alternantes aut singulo centrali, quaternis semicirculariter ordinatis instructis, lobis basalibus sinuato-emarginatis, subito in lobum polarem apice paullum dilatatum attenuatis; cytiodermate subtiliter punctato. (R.)

Syn.—*E. circulare,* Hassal, Fresh-Water Algæ, p. 383.

Hab.—Providence, Rhode Island; Bailey.

"Frond about twice as long as broad, tapering upwards into a neck, end not dilated, its notch an acute incision. Empty frond, segments with five basal inflations, four in a half circle around the fifth and two others at the extremity." Archer.

(*Var.* **Ralfsii**)

Semicellula tumoribus minimis 11 in series tres alternantes ordinatis.

Hab.—Saco Lake, New Hampshire; (F. W. Lewis) Wood.

E. Jenneri, nobis. Frond scarcely twice as long as broad; segments 3-lobed, basal portions subquadrate, emarginate at the sides; end lobe, its notch linear. Empty frond punctate, segments with several inflations arranged in alternate lines. (Archer.)

E. insigne, Hassall.

E. subgracile, diametro duplo-triplove longius, a vertice fere quadratum, lateribus concavis, angulis rotundatis; semicellulis basi inflatis, integris, e basi subreniformi in collum elongatum citius attenuatis, lobo polari dilatato bifido truncato; cytiodermate subtiliter punctato. (R.)

Long. 0.0039″—0.0043″. Lat. max. 0.00236″. (R.)

Syn.—*E. insigne,* Hassall, Fresh-Water Algæ, p. 21.

Hab.—Florida; Rhode Island; Bailey.

Frond rather more than twice as long as broad; segments inflated at base, sides entire, without lateral tubercles, and tapering into a long slender neck; end lobe dilated, its notch linear. Empty frond minutely punctate; f. v. segments with two inflations at the base; f. v. narrower, gradually tapering to the end, which is considerably dilated; projections rounded, with a sinus between; tr. v. subquadrate, slightly concave at sides, with a rounded lobe at the centre of each end; e. v. end lobe with a sinus at opposite external margins, angles thus protruded into four divergent rounded lobelets. (A.)

E. Ralfsii, Rabenh.

E. mediocre, leviter compressum, medio inflatum, diametro duplo circiter longius; semicellulis pyramidalibus, e basi ventricosa in lobum polarem rectum truncatum sinuato-attenuatis; cytiodermate subtiliter punctato, punctis in lineas rectas ordinatis. (R.)

Syn.—*E. ansatum,* Ehr. et auctores. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 184.
E. Ralfsii, Rabenhorst, Flora Europ. Algarum, Sect. III. p. 184.

Hab.—South Carolina; Rhode Island; Bailey. White Mountains, New Hampshire (F. W. Lewis).

"*E. ansatum, Ehrb.* Frond about twice as long as broad; segments inflated at the base, tapering upwards without sinuations into a neck, end not dilated, rounded, its notch linear. Empty frond punctate; f. v. segments turgid on the surface, at the middle without circular inflations; tr. v. elliptic, with a single large inflation at each side; e. v. end lobe entire at the margin, its divisions circular. (A.)

Remarks.—I have seen only a very few specimens in a gathering made in Saco Lake, New Hampshire, by Dr. Lewis, which differ considerably from the typical form in the proportion of the breadth and length. There are also certainly four, if not more, umbonations on the face of each half-cell. These are nowhere distinctly spoken of as existing, and Mr. Archer states there are none visible in the front view of *E. ansatum.* They are, however, represented in the side view of the original figure, and are said to be very noticeable by Mr. Archer himself, when the desmid is so looked at. In the Saco Lake specimens they are always seen in the front view with great difficulty, and in some cases I failed entirely to demonstrate them, so that they do not afford a good character for the indication of a new species.

Fig. 1, pl. 13, represents a front view of a Saco specimen, magnified 450 diameters.

C. Lobi polares non-evidenter discreti.

C. End lobes not evidently distinct.

E. elegans, (Bréb.) Ktz.

E. minus, oblongum, diametro duplo longius, utroque polo bifidum, segmentis introrsum rotundatis; semicellulis sursum modice attenuatis, utroque margine laterali bi- vel tri- sinuatis, sinu superiori vel intermedio profundiori, sub polo utrinque dente acuto prominente; cytiodermate subtiliter punctato, punctis irregulariter sparsis; zygosporis globosis aculeatis, aculeis elongato-subulatis. (R.)

Long. 0.0012″.—0.002″. Lat. max. circiter 0.0011″. (R.)

Syn.—*E. elegans,* (Brébisson,) Kützing. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 185.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey. White Mountains, (F. W. Lewis).

Frond minute, scarcely twice as long as broad, oblong; segments with their basal portion emarginate at the sides, connected by a broad neck with the terminal portion; ends protuberant, rounded, acutely emarginate at the centre, pouting; s. v. with an inflation at the base of the segments, sides concave, ends rounded. Sporangium orbicular, spinous. (A.)

Remarks.—According to Prof. Rabenhorst *E. rostratum,* Ralfs, which is noted as an American species by Bailey, is a variety of *E. elegans.* Its peculiarities, according to Rabenhorst, are as follows: "Forma magis evoluta, profundior sinuata, segmentis polaribus latioribus, angulis acutis, dente paulo longiore."

Fig. 14, pl. 21, represents the outline of the frond as viewed laterally; fig. 2, pl. 13, a front view of the frond, magnified 750 diameters.

E. binale, (Turpin) Ralfs.

E. minimum, diametro paulo vel subduplo longius, in sect. transversa oblongo-cylindricum, medio tumidum, utroque polo rotundatum; semicellulis indistincte trilobis, lobis basalibus

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latis, rotundatis vel sinuato-bi-tricrenatis; lobo polari abbreviato late truncato, leviter emarginato, angulis acutis lateraliter plus minus porrectis; cytiodermate subtilissime punctato.

Syn.—*E. binale,* (TURPIN) RALFS, British Desmideæ, p. 90.

Hab.—Florida; Bailey. Rhode Island, (S. T. Olney,) Thwaites. Pennsylvania; Wood.

Frond very minute, scarcely twice as long as broad, oblong; segments with their basal portion either entire or bicrenate at the sides, slightly contracted beneath the ends; ends dilated, not protuberant beyond the angles, its central notch acute, broad; tr. v. with two lateral inflations, ends truncate, angles rounded. (A.)

Remark.—Fig. 3, pl. 13, represents the front view of a frond, magnified 750 diameters.

Genus MICRASTERIAS, AG. (1827).

Cellulæ compressæ, profunde constrictæ, a fronte orbiculares vel late ellipticæ, a vertice fusiformes cum utroque polo acuto, semicellulæ tri- vel quinque-lobæ; lobi basales aut integri aut pluripliciter inciso-lobulati; lobus polaris aut integer aut sinuatus aut emarginatus, et interdum angulis productus et bifidus. Cytioplasma chlorophyllacea in cellulæ lumen subæqualiter distributa, granula amylacea sparsa involvens. Cytioderma plerumque læve, nonnunquam punctatum, granulatum vel mucronatum.

Zygosporæ globosæ, ætate provecta aculeis simplicibus, apice bi-multi-fidis, nonnunquam repetito-multifiidis armatæ.

Cells compressed, profoundly constricted, viewed from the front orbicular or broadly elliptic, from the vertex fusiform with acute ends. Semicells 3- or 5-lobed; basal lobes either entire or many times incisely-lobulate; end lobe either entire or sinuate or emarginate, and sometimes with its angles produced and bifid. Chlorophyllous cytioplasm distributed nearly uniformly in the cavity of the cell, surrounding scattered starch granules. Cytioderm mostly smooth; sometimes punctate; granulate or mucronate.

Zygospores globose, at maturity armed with simple spines, whose ends bifid or multifid, and sometimes repeatedly multifid.

A. Semicellulæ trilobæ. Lobi basales horizontales; lobus polaris valde dilatatus, dorso plane convexus, truncatus vel leviter retusus, a lobis basalibus sinu amplissimo discretus.

A. Semicells trilobate. Basal lobes horizontal; end lobe strongly dilated, with the back convex, truncate, or slightly retuse.

M. arcuata, BAILEY.

M. mediocris, quadrangularis, paulo latior quam longa, profunde pinnatifida; lobis basalibus angustis elongatis, arcuatis, in apicem acutum attenuatis, divergentibus; lobis polaribus angustusimis, utrinque graciliter productis, in apicem acutum attenuatis, in medio dorso modice retusis. (R.)

Syn.—*M. arcuata,* BAILEY, Microscopical Observations: Smithsonian Contributions, vol. ii.

Hab.—In stagnis. Florida; Bailey.

"Quadrangular, segments three-lobed, the basal lobes long and arcuate, subtended by the transverse projections from the ends of the slightly notched terminal lobes." (Bailey.)

M. expansa, BAILEY.

M. mediocris, tam longa quam lata, lobis stellatim expansis; lobis basalibus angustis in apicem acutum attenuatis, divergentibus, rectis; lobis polaribus e basi angusta sensim dilatatis, in medio dorso late sinuatis, angulis acutis (sed muticis). (R.)

Syn.—*M. expansa,* BAILEY, Microscopical Observations: Smithsonian Contributions, vol. ii.

Hab.—In stagnis, Florida; Bailey.

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Segments three-lobed, basal lobes long, subconical, acute; terminal lobes slender, forked at the end, with the divisions much shorter than the basal lobes. (Bailey.)

M. quadrata, Bailey.

M. arcuatæ similis, sed duplo major, semicellularum lobi basales minus arcuati, basi inflati, apice bidentati et cytioderma irregulariter granulatum. (R.)

Diam.—0.0043″—0.0049″.

Syn.—*M. quadrata,* Bailey, Microscopical Observations: Smithsonian Contributions, vol. ii.

Large quadrangular, three-lobed, basal lobes elongated, slightly curved, bidentate; terminal lobes with two slender transverse bidentate projections. Bailey.

M. disputata, Wood.

M. magna, fere tam longa quam lata, subpinnatisecta, sinu acuto, lobis æqualibus; semicellulis profunde trilobis, lobis basalibus in apicem acute bidentatum valde attenuatis; lobo polari valde dilatato, dorso rotundato, angulis lateralibus acutissimis.

Long. $\frac{37}{7500}$″ = .005″. Lat. $\frac{30}{7500}$″ = .004″.

Syn.—*Micrasterias incisa,* Ktz. Bailey, Microscopical Observations: Smithsonian Contributions, 1850.

Haud *Micrasteria incisa,* Kützing, Spec. Algarum, p. 171.

Tetrachastrum Americanum, Archer, Pritchard's Infusoria, 1860, p. 725.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey. Pennsylvania; Wood.

M. large, about as long as broad, subpinnatisected; sinuses acute; semicells profoundly trilobate; basal lobes strongly attenuate into the acutely bidentate apex; distal lobes strongly dilated, rounded, with their lateral angles bidentate; end lobe broadly dilated, lateral angles very acute.

Remarks.—This desmid was first figured by the late Prof. Bailey in his Microscopical Observations (Smithsonian Contributions), as *M. incisa* of Ktz., and Rabenhorst, in his Flora Europæa Algarum, confirms this identification. He has probably, however, never seen the plant itself, but merely accepts the opinion of Professor Bailey. Mr. Archer (Pritchard's *Infusoria*), thinks the American plant is certainly distinct from the European, and this seems to me correct. The points of difference are—the American form is nearly twice the size of the European, the sinuses are much more widened outwardly, and the lobes are reduced rapidly in breadth to a mere point at the end, the dorsum of the distal lobes is also, I believe, more rounded. In his description of *T. Americanum*, as he calls it, Mr. Archer states the end lobe has its angles bidentate. In the only specimen I have seen, the angles end in a very sharp, almost spine-like point. Dr. Leidy found the species abundantly at Newport, Rhode Island, and his figure agrees with mine in this respect. In regard to the name, as there is already an *M. Americanum*, the specific name of Archer cannot be adopted, and for a similar reason it would not do to call it *M. Baileyi.* I have then been forced to give it a new title.

Fig. 4, pl. 13, was drawn by myself from the single specimen I have seen; fig. 4 *a* was drawn by Dr. Leidy from a Newport specimen.

M. oscitans, Ralfs.

M. magna, pæne tam longa quam lata, subpinnatisecta, a vertice elliptico fusiformis, utroque polo bifida; lobis basalibus horizontalibus conico-productis, apice bifidis; labo polari a lobis basalibus sinu amplo ac rotundato discreto, plus minus convexo, haud raro truncato, rarius leviter retuso, utrinque producto acuminato, plerumque bidentato. (R.)

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Diam.—0.0047″. Long. 0.0039″. (R.)

Syn.—*M. oscitans*, RALFS, British Desmidieæ, p. 76. RABENHORST, Flora Europ. Algarum, Sect. III. p. 119.
M. pinnatifida KTZ. RABENHORST, Flora Europ. Algarum, Sect. III. p. 119.

Hab.—Florida; Rhode Island; Bailey.

Frond about as broad as long, pinnatifid; lateral lobes separated from the terminal by a rounded sinus, horizontal, conical, their extremities bidentate; end lobe short, broad, its lateral projections short, conical, usually bidentate, narrower and shorter than the lateral lobes; ends convex at the centre; tr. v. fusiform, e. f. punctate. (A.)

Remarks.—According to Prof. Rabenhorst *M. pinnatifida*, Ktz., is a variety of *M. oscitans*, different from the typical form only in being smaller, and in having the lobes narrower.

B. Semicellulæ 3-vel 5-lobæ, plerumque radiatim inciso-lobulatæ. Lobi basales assurgentes aut non aut minus a lobo polari remoti.

B. Semicells 3, or 5-lobate, mostly radiately incisely lobulate. Basal lobes assurgent, either close to, or but slightly remote from the end lobes.

* *Semicellulæ trilobæ.*

* *Semicells trilobate.*

M. Americana, (EHRB.) KTZ.

M. magna, oblonga, subpinnatiseeta, lobis polaribus paulum remotis, pæne duplo longior quam lata; cytiodermate spinuloso unde laborum margines dentato-serrati conspiciuntur; cellula e latere conspecta oblonga, in medio leviter constricta, utroque polo bicornuta; semicellulæ basi tumore plus minus distincto instructæ, fere quinquelobæ, lobis basales latissimi iisdemque profunde bilobati, lobulis late excisis, segmentis dentato-serratis; lobis polaribus plus minus productis, in medio late excisis, segmentis profunde bifidis. (R.)

Diam.—0.0041″. Long. circa 0.0051″. (R.)

Syn.—*M. Americana*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 189.

Hab.—In stagnis, South Carolina; Florida; Bailey.

Frond angular elliptic, more or less punctate; segments 3-lobed; lateral lobes broad, cuneate, their margins concave, inciso-serrate; and lobe broad, cuneate, end exserted, bipartite at the angles, the subdivisions narrow, and minutely dentate at the extremities; end concave. (A.)

Remark.—Fig. 17, pl. 12, represents a plate of this species.

M. Baileyi, RALFS.

M. parva, oblonga, granulata; semicellulis trilobis, lobis basalibus a lobo polari sinu amplo discretis, excisura acute triangulari in duas lacinias partitis, laciniis e basi latiori in apicem truncatum bidentatum attenuatis; lobo polari e basi angusta longe porrecto, sursum valde dilatato, in vertice leviter et late sinuato, angulis truncato, bidentato. (R.)

Syn.—*M. Baileyi*, RALFS, British Desmidieæ, p. 211.

Hab.—New York; Rhode Island; South Carolina; Florida; Bailey.

Frond granulated; segments three-lobed; lobes bipartite, end one much exserted. (Ralfs.)

M. ringens, BAILEY.

M. mediocris, oblonga, margine granulata; semicellulis trilobis; lobis lateralibus bipartitis, laciniis divaricatis, apice obtusis, truncatis vel bidentatis; lobo polari e basi angusta sursum valde dilatato, exserto, in vertice leniter sinuato, angulis truncato. (R.)

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Syn.—*M. ringens*, Bailey, Microscopical Observations, pl. 1, fig. 11: Smithsonian Contributions, vol. ii.

Hab.—Florida; Bailey.

Oblong, segments three-lobed, coarsely granulated near the edge; basal lobes subdivided by a deep notch into two rather broad and obtuse or slightly bidentate projections; terminal lobes exserted, emarginate; extremities bidentate or obtuse.

* * *Semicellulæ quinque-lobatæ.*

* * *Semicells 5-lobed.*

M. truncata, (Corda) Breb.

M. magna, orbicularis, aut lævis aut subtiliter punctata; semicellulis quinquelobis, lobis inter se sinu obtusangulo subangusto discretis, basalibus et intermediis inciso-lobulatis, segmentis acute bidentatis; lobo polari late cuneato, in dorso truncato, modo leviter convexo, modo leviter retuso, angulis aut bidentatis aut integris. (R.)

Diam.—0.003″ Long. .0036″.

Syn.—*M. truncata*, (Corda,) Brebisson. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 191.

Hab.—Georgia; Florida; Rhode Island; Bailey. Pennsylvania; Wood. Rhode Island (S. T. Olney); Thwaites.

Frond orbicular, smooth; segments 5-lobed; basal and middle lobes obscurely bipartite, extremities bidentate; end lobe very broadly cuneate, bidentate at the angles, and with a slightly central concavity. (A.)

Remarks.—The dimensions given above were taken from the largest specimens I have seen, but do not at all equal those given by Prof. Rabenhorst, his breadth is .0041″. According to the same authority, *M. crenata*, Bréb., is merely a variety of this species.

Fig. 15, pl. 21, represents the outline of a frond of this plant.

M. furcata, Ag.

M. permagna paulo longior quam lata, levis; semicellulis quinque lobis (pæne 7-lobis); lobis omnibus rectis; lobis basalibus angustioribus, bilobulatis, lobulis bifidis, sinu obtusangulo vel acutangulo, segmentis linearibus bidentatis (denticulis sæpe inæquilongis); lobis intermediis duplo latioribus, inciso-bilobis, lobulis iisdem ac loborum basalium; lobo polari nonnihil anguste cuneato, prominulo, in apice plus minus profunde sinuato-vel undulato inciso, angulis bidentatis.

Diam.—$\frac{60}{7500}$″ = .008″.

Syn.—*M. rotata*, Ralfs, British Desmidieæ, p. 71.
M. furcata, Agardh. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 191.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey. New Jersey; Wood.

M. very large, a little longer than broad, smooth; semicells 5-lobed (scarcely 7-lobed); lobes all straight; basal lobe narrower than the intermediate, bilobulate, lobules bifid, their sinuses acute or obtuse, segments linear, bidentate; teeth often long and unequal; intermediate lobes twice as wide as the basal, bilobate, their lobules of the same form as the basal lobe; end lobes narrowly cuneate, prominent, more or less profoundly sinuately or undulately cut at the apex, angles bidentate.

Remarks.—According to Rabenhorst and others, there is a European form of this species in which the marginal teeth are wanting. This may exist in this

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country, but I have never met with it. All the specimens which have come under my notice were obtained in "Shepherd's Dam," near Greenwich, Cumberland County, New Jersey. None of them were as large as the maximum of the European measurements of which Rabenhorst gives 0.0109″ as the diameter.

Fig. 5, pl. 13, represents a frond of this species, magnified 260 diameters.

M. denticulata, Bréb. ?

M. permagna, paulo longior quam lata, lævis; semicellulis quinquelobis (pæne 9 lobis); lobis intermediis et basalibus simillimis, bilobatis, lobulis item in lobulis bifidis duobus divisis; lobo polare angusto, cuneato, in apice plus minus inciso; margine minute denticulato.

Diam.—Lat. .0092″. Long. .011.″

Syn.—*M. denticulata*, Brébisson. Ralfs, British Desmidieæ, p. 70, et Archer, Pritchard's Infusoria.

M. denticulata, Brébisson. ? Rabenhorst, Flora Europ. Algarum, Sect. III. p. 192.

Hab.—Pennsylvania; Wood. Florida; Bailey.

Very large, a little longer than broad, smooth; semicells with five lobes (scarcely 9); basal and intermediate lobes alike bilobate, lobules also divided into two bifid lobules; end lobe narrow, wedge-shaped, more or less incised at its apex; margin minutely denticulate.

Remarks.—Prof. Rabenhorst gives *M. denticulata*, Bréb. as merely a variety of *M. furcata*, Ag., stating that it only differs from the latter in the marginal incisions and teeth. Not having access to the original description of Brébisson I cannot express an opinion as to whether Prof. R. is correct or not, but the specimen from which the above description was drawn up (and which is figured on plate 13) certainly differs from *M. furcata* very essentially in the arrangement of its lobes, and is, I feel confident, *M. denticulata*, Bréb. of Ralfs and Archer.

Fig. 6, pl. 13, is a drawing of this plant, as seen by myself, magnified 260 diameters.

M. radiosa, Ag.

M. maxima, orbicularis, lævis, antecedenti simillima, differt inprimis segmentis ultimis tumidis in apicem bi- tri- fidum attenuatis, lobo polari vix prominulo, apice sinuato, ad utrumque angulum bi- tri- dentato. (R.) *Species mihi ignota.*

Diam.—0.0076″. (R.)

Syn.—*M. radiosa*, Agardh. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 192.

Hab.—Florida; Bailey.

Frond orbicular, smooth; segments 5-lobed; basal lobes twice, middle lobes generally thrice dichotomous, ultimate subdivisions inflated, attenuate towards the end, bidentate; end lobes emarginate, its angles dentate. (A.)

M. fimbriata, Ralfs.

M. magna, orbicularis, lævis (nonnunquam superficie aculeis singulis sparsis); semicellulis quinquelobis, lobis omnibus confertis, basalibus angustioribus, repetito bilobulatis, lobis intermediis duplo latioribus, repetito-bilobulatis, lacinulis extremis leviter emarginatis, in angulis spinis elongatis armatis; lobo polari prominulo, in apice obtuse sinuato-vel-undulato-emarginato, angulis lateralibus rotundatis, ad marginem superiorem spinis singulis vel geminis obsito (rarius nudo). (R.) *Species mihi ignota.*

Diam.—.0051″—.0078″. (R.)

Syn.—*M. fimbriata*, RALFS, British Desmidieæ, p. 71, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 193.

Hab.—South Carolina; Florida; Bailey.

Frond orbicular, smooth; segments 5-lobed, basal lobes twice, middle lobes thrice dichotomous; ultimate subdivisions acutely bidentate; end lobe very slightly exserted, its angles very slightly produced, bidentate, ends emarginate. In transverse view is seen an inflated protuberance just over the central isthmus, which may possibly exist in other species of *Micrasterias*. (A.)

M. papillifera, BRÉB.

M. orbicularis, superficie lævis, margine extremo dentato papillifera; semicellulis quinquelobis; lobis basalibus et intermediis æquilatis, bilobatis; lobulis bifidis, laciniis linearibus bidentatis, dentibus papilliferis; lobo polari vix prominulo, in apice sinuato, angulis et margine dentato-mucronatis. (R.) *Species mihi ignota.*

Diam.—0.0045″. (R.)

Syn.—*M. papillifera*, BRÉBISSON. RALFS, British Desmidieæ, p. 72, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 194.

Hab.—Florida; Rhode Island; Bailey.

Frond orbicular, having the principal sinuses bordered by a row of minute granules, otherwise smooth; segments 5-lobed; basal and middle lobes twice dichotomous, their ultimate shallow subdivisions terminated by two, sometimes three, gland-like teeth; end lobe emarginate, its angles dentate. Sporangium as in *M. denticulata*, but considerably smaller. (A.)

M. granulata, WOOD (sp. nov.)

M. magna, suborbicularis, arcte granulata; semicellulis quinquelobis, lobis inter se sinu angusto discretis, basalibus et intermediis plerumque integris, lobo polari supra valde dilatato, in dorso medio leviter retuso; marginibus valde crenatis.

Diam.—Long. $\frac{1}{240}''$ = .0043″. Lat. $\frac{43}{12000}''$ = .0036″.

Hab.—South Carolina, (Ravenel)

Large, suborbicular, closely granulate; semicells 5-lobed, lobes separated by narrow sinuses; basal and intermediate lobes mostly entire; end lobe distally broadly dilated, broadly and very shallowly emarginate; margin of frond strongly crenate.

Remarks.—The only specimens of this species that I have seen were collected by Prof. Ravenel in a shallow ditch near Aiken, South Carolina, where they formed a greenish, gelatinous mass, with numerous desmids and diatoms. It is most closely allied to *M. truncata*, from which it is separated by its entire lateral lobes, by its granulated surface, and its crenated margins. It also does not apparently attain as large a size as that species. The granules are very small in the central portion of the frond, but become larger as they approach the margin.

Fig. 16, pl. 21, represents an empty frond of this species, magnified 460 diameters.

M. Jenneri, RALFS.

M. magna, oblonga, plerumque subtiliter granulata; semicellulis quinquelobis; lobis basalibus et intermediis æquilatis, confertis, cuneatis, bilobulatis; lobo polari late truncato vel late rotundato, in medio interdum leviter et obtuse emarginato, interdum nonnihil profunde emarginato.

Diam.—Lat. $\frac{45}{7500}''$—$\frac{75}{12000}''$ = .006″—.0062″. Long. $\frac{65}{7500}''$—$\frac{75}{12000}''$ = .0062″—.0087″.

Syn.—*M. Jenneri*, RALFS, British Desmidieæ, p. 76.

Hab.—Prope Philadelphia; Wood. South Carolina; (Ravenel)

Large, oblong, for the most part finely granulate; semicells 5-lobed; lobes wedge-shaped; basal and intermediate, about equally broad; end lobe broadly truncate or broadly rounded, in the middle sometimes slightly and obtusely emarginate, sometimes rather deeply emarginate.

Remarks.—I have found this species near Philadelphia, and also received it from Prof. Ravenel, by whom it was collected in South Carolina. The American plant differs from the typical form in not having the ultimate lobules emarginate, they being merely a little hollowed out in the centre, and sometimes scarcely this. The angles in some specimens are also more acute. Mr. Archer, however, speaks of a variety occurring in England, in which these lobules are not emarginate, and I do not think characters can be found separating the American from the European forms. The median suture is in all the specimens very narrow and deep, a mere line, as it were, extending nearly to the centre.

Fig. 7, pl. 13, represents a frond of this species.

M. Torreyi, BAILEY.

M. permagna, oblongo-orbicularis, lævis, profundissime lobata; semicellulis quinquelobis, lobis basalibus profunde bifidis, laciniis inferioribus apice bidentatis, superioribus integris, lobis intermediis profunde trifidis, laciniis superioribus bidentatis, inferioribus integris, lac. omnibus lanceolatis acuminatis, inferioribus paulum incurvis, superioribus recurvis; lobo polari non prominente, e basi angusta seusim dilatato, in vertice acute sinuato, angulis integris acuminatis. (R.) *Species mihi ignota.*

Syn.—*M. Torreyi*, BAILEY. RALFS, Brit. Desmidieæ, p. 210.

Hab.—Prope Princetown, New Jersey; Bailey.

Frond smooth; segments 5-lobed; basal lobes bifid, middle lobes trifid, the subdivisions nearest the opposite segments and those nearest the terminal lobe bidentate at the apex; the intermediate three terminating in acute points; all somewhat inflated and tapering; terminal lobe narrow, not exserted, spreading at the angles into divergent tapering points, ends slightly emarginate. (A.)

M. foliacea, BAILEY.

M. parva, subquadrata, lævis; semicellulis trilobis, lobis lateralibus profunde bifidis (unde rectior semicell. quinquelobæ), lobulis inæqualiter inciso-dentatis, lobulis inferioribus rectis, superioribus recurvis; lobo polari plus minus prominente, anguste cuneato, in vertice plus minusve emarginato, angulis aut acutis integris aut productis, bidentatis. R. *Species mihi ignota.*

Syn.—*M. foliacea*, BAILEY. RALFS, British Desmidieæ, p. 210.

Hab.—"Worden's Pond, Rhode Island; Bailey."

Frond subquadrate, smooth; segments 3-lobed; lateral lobes deeply bipartite, inciso-dentate, their margins concave, inciso-serrate; end lobe broad, cuneate, and exserted, bipartite at the angles, the subdivisions narrow, and minutely dentate at the extremities; end concave. (A.)

Genus STAURASTRUM, MEYEN.

Cellulæ libere natantes, in medio plus minus profunde constrictæ; semicellulæ a vertice 3-6 angulares vel radiatæ. Cytioderma aut læve aut punctatum aut verrucosum aut aculeatum, nonnunquam ciliis vel pilis obsessum.

Cells swimming free, more or less profoundly constricted in the middle; semicells when seen from the vertex 3 to 6 angular or radiate. Cytioderm either smooth or punctate, or verrucose or aculeate, sometimes covered with hairs or cilia.

A. CYTIODERMA LÆVE VEL RARISSIME SUBTILITER PUNCTATUM.

CYTIODERM SMOOTH OR VERY RARELY VERY FINELY PUNCTATE.

1. *Semicellularum anguli rotundati.*

Angles of the semicells rounded.

St. muticum, BRÉB.

St. a fronte orbiculare, læve, profunde constrictum, nudum, vel muco plus minusve firmo involutum; semicellulis ellipticis, a vertice conspectis 3–4 angularibus (rarius quinquangularibus) angulis rotundatis, lateribus leviter sinuato-retusis; zygosporis aculeatis, aculeis elongatis, subulatis, furcatim fissis. (R.) *Species mihi ignota.*

Diam.—0.0013″—0.000147″. (R.)

Syn.—*S. muticum,* BRÉBISSON. RABENHORST, Flora Europ. Algar., Sect. III. p. 200.

Hab.—South Carolina; Rhode Island; Bailey.

Segments in f. v. elliptic, smooth, without spines; e. v. with three or four broadly rounded angles, sides concave. Sporangium beset with numerous elongate somewhat stout spines, forked at the apex. (A.)

St. orbiculare, (EHRB.) RALFS.

St. suborbiculare, læve, sæpius muco matricali involutum; semicellulis divergentibus, semiorbicularibus, dorso nonnunquam elevatis, angulis plus minus late rotundatis, lateribus plus minus sinuato-retusis; zygosporarum aculeis elongatis, subulatis, integris. (R.)

Diam.—.002″.

Syn.—*St. orbiculare,* (EHRB.) RALFS, British Desmidieæ, p. 125. RABENHORST, Flora Europ. Algarum, Sect. III. p. 200.

Hab.—Rhode Island; Bailey. Pennsylvania; Wood. Rhode Island; (S. T. Olney) Thwaites.

Segments in f. v. semiorbicular, smooth, without spines; e. v. with three broadly rounded angles, sides slightly concave. (A.)

Remarks.—Fig. 17, pl. 21, represents the outline of the end view of a frond of this species. Fig. 8, pl. 13, is a drawing of the front view of a living frond.

2. *Semicellularum anguli mucronati vel aristati.*

Angles of the semicells mucronate or bristly.

St. longispinum, (BAILEY) ARCHER.

St. magnum triangulare, læve, angulis in aculeos geminos validos subulatos longe productum, lateribus subplanum. (R.) *Species mihi ignota.*

Syn.—*Didymocladon longispinum,* BAILEY, Microscopical Observations.

Hab.—Florida; Bailey.

"Large, smooth, triangular, with two long spines at each angle." Bailey.

St. dejectum, BRÉBISSON.

St. læve, parvum, sinu amplo, obtusangulo (vel acutangulo); semicellulis ellipticis (vel subtriangularibus), dorso nonnihil convexo, utroque fine in aculeum achroum rectum vel varie curvatis productis; a vertice triangularibus (vel quadrangularibus), angulis sæpe rotundatis aculeo interdum obsoleto imposito.

Diam.—Lat. $\frac{10}{1200}$″—$\frac{12}{12000}$″ = .0008″—.001″. Long. $\frac{10}{12000}$″—$\frac{12}{12000}$″ = .0008″—.0001″.

Syn.—*Staurastrum dejectum,* BRÉBISSON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 203.

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Hab.—South Carolina; (Ravenel) Wood.

Smooth, small; sinus ample, obtuse angled (sometimes acute angled?); semicells elliptic (or subtriangular?), with the dorsum slightly convex, at the angles with a straight or curved transparent spine; from the vertex triangular (or quadrangular?), angles often rounded, with a sometimes obsolete spine superimposed.

Remarks.—This species was collected near Aiken, South Carolina, by Prof. Ravenel, who found it forming with various diatoms and desmids a slimy mass in a feebly running ditch. It agrees very well with the European form, except that it is not so large (at least the largest I ever measured did not come up to the size of their transatlantic brethren), neither does it appear to vary quite so much. In the description, I have placed in brackets those characters in which the European form varies, and the specimens I have seen do not.

Fig. 18, pl. 21, represents outline of end of a semicell, magnified 750 diameters. Fig. 9, pl. 13, a front view, and 9 *a* the end view, of the living frond, magnified diameters.

St. aristiferum, Ralfs.

St. læve, St. cuspidatum quodammodo simile, et eadem magnitudine sed isthmo destitutum; semicellulis tumidis, in media parte subrotundatis, lateraliter in lobum, basi constrictum, apice aristatum productis, lobis divergentibus, a vertice tri-quadrilobo-radiatis, radiis strictis æquidistantibus cruciatim dispositis, interstitiis profunde excisis. (R.) *Species mihi ignota.*

Diam.—Incl. arist. 0.0014″. (R.)

Syn.—*St. aristiferum*, Ralfs, British Desmidieæ, p. 123. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 204.

Hab.—Georgia; Rhode Island; Bailey.

Segments smooth, in f. v. prolonged at each lateral extremity into a mamillate projection, which is terminated by a subulate, acute straight awn, the awns divergent, e. v. with three or four angles; angles inflated mamillate, terminated by an awn, sides deeply concave in the centre. (A.)

St. Lewisii, Wood.

St. læve; sinu amplissimo, spinulo parvo armato et cum angulo obtuso; isthmo nullo; semicellulis a fronte late triangularibus, a vertice triangularibus et cum angulis nonnihil tumidis, et rotundatis; angulis spino maximo, robusto, acuto armatis.

Diam.—Long. cum. spin. $\frac{1}{400}''=.0025''$; lat. cum. spin. $\frac{27}{12000}''=.00225''$. Sine spin.: long. $\frac{1}{600}''=.001666''$; lat. $\frac{13}{12000}''$ .001666″. Spin.: long. $\frac{1}{1500}''=.000666''$

Syn.—*St. Lewisii,* Wood, Proc. Acad. N. S. 1870.

Hab.—In lacu Saco; (Lewis) Wood.

Smooth, with a very ample sinus, which is armed with a small spine and has a very obtuse angle; isthmus absent; semicells from the front broadly triangular, from the vertex triangular, with the angles somewhat tumid and rounded; angles armed with a very large acute robust spine.

Remarks.—This desmid is most closely allied to *St. aristiferum*, Ralfs, but differs from it in outline as seen from the front, there being no mamellation of the ends. The spines in the sinuses are always wanting in the European species.

Fig. 19, pl. 21, represents the outline of the end of a semicell, magnified 750 diameters. Fig. 11, pl. 13, represents the perfectly formed frond, magnified 750 diameters.

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B. CYTIODERMA GRANULATUM VEL VERRUCOSUM.

CYTIODERM GRANULATE OR WARTY.

1. *Semicellulæ a vertice 3–7 angulares; anguli plus minus radiatim elongati.*
Semicells seen from the vertex 3-7 angled; angles more or less radiately produced.

St. margaritaceum, EHRB.

St. mediocre, granulatum; semicellulis convergentibus, subfusiformibus, in medio tumidis, utrinque productis, truncatis, a vertice orbicularibus, 5–7 radiatis, radiis obtuse truncatis achrois, hyalinis, granulato-margaritaceis. (R.) *Species mihi ignota.*

Diam.—0.00135″—0.0017″. (R.)

Syn.—*St. margaritaceum,* (EHRB.) MENEGHENI. RABENHORST, Flora Europ. Algarum, Sect. III. p. 206.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey.

Segments in f. v. gradually widening upwards, rough with pearly granules, outer margin convex, produced at each side into a colorless, more or less attenuate, short process, having the granules in transverse lines, blunt and entire at the apex, e. v. circular, bordered by from five to seven short, narrow, obtuse, colorless, granulate marginal rays. (Archer.)

St. dilatatum, EHRB.

St. parvum, granulatum; semicellulis rectis, cylindrico-fusiformibus, non tumidis, utroque fine obtusis vel subtruncatis, a vertice 3–4–5 radiatis, radiis latioribus, truncatis vel rotundatis, achrois, hyalinis, granulato-margaritaceis. (R.) *Species mihi ignota.*

Diam.—0.0008″—0.0011″. (R)

Var. **alternans.**

Semicellulis ellipticis rectis, utroque fine rotundatis, a vertice triradiatis, radiis obtusis, alternantibus cum semicellulæ inferioris. (R.)

Var. **tricorne.**

Semicellulis fusiformibus, nonnunquam in medio subtumidis, haud raro isthmo distincto conjunctis, a vertice 3–4 angularibus, angulis truncatis vel obtusis, plus minus radiatim productis. (R.)

Hab.—Georgia; Florida; Rhode Island; Bailey.

Syn.—*S. alternans,* BRÉBISSON. *Var. alternans et tricorne.* RABENHORST, Flora Europ. Algarum, Sect. III. p. 207.

Remarks.—Prof. Rabenhorst considers *St. alternans* and *tricorne*, as simple varieties of *St. dilatatum*, whilst both Archer and Ralfs describe them as distinct. I have not seen either of them, and am not therefore entitled to offer an opinion. Mr. Archer describes the two species as follows:—

St. alternans, BRÉB.

Segments in front view elliptic or oblong, two or three times as broad as long, separated by a wide sinus, twisted, unequal; rough with very minute pearly granules; e. v. with three obtuse and rounded angles, forming short, not colorless rays, alternating with those of the other segments, sides concave. L. $\frac{1}{1037}$″. Br. $\frac{1}{1106}$″.

St. tricorne, BRÉB.

Segments in f. v. somewhat fusiform, often twisted, rough with minute puncta-like granules, tapering at each side into a short, usually colorless process, blunt or divided at the apex;

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e. v. tri- or quadriradiate, processes short, usually colorless, sides somewhat concave. Sporangium orbicular, beset with spines ultimately branched at the apex. L. $\frac{1}{1275}$"—$\frac{1}{972}$". B. $\frac{1}{948}$".

2. *Semicellulæ triangulares; anguli non producti, obtusi vel rotundati.*

Semicells triangular; the angles not produced, obtuse or rounded.

St. punctulatum, Bréb.

St. parvum, punctulato-granulosum; semicellulis enormiter ellipticis, dorso late rotundatis, a vertice triangularibus; angulis non productis, obtuse rotundatis; lateribus modice retusis.

Diam.—Lat. $\frac{1}{7500}$" = .0012".

Syn.—*S. punctulatum*, Brébisson. Ralfs, British Desmidieæ. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 208.

Hab.—Pennsylvania; Wood.

Small, punctulate-granulate; semicells irregularly elliptic, with the dorsum broadly rounded from the vertex triangular; angles not produced, obtusely rounded; sides somewhat retuse.

Remarks.—This desmid is exceedingly common around Philadelphia, growing in the greatest abundance upon the face of wet dripping rocks. It is represented, fig. 10, pl.13.

St. crenatum, Bailey.

St. duplo circiter longius quam latum, in medio utrinque exsectione profunda rotundata; semicellulis e basi cuneata flabelliformibus, margine superiore crenatis, a vertice triangularibus, angulis rotundato-truncatis, crenatis, lateribus sinuatis glabris. (R.) *Species mihi ignota.*

Syn.—*St. crenatum*, Bailey. Ralfs, British Desmidieæ, p. 214. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 220.

" Segments cuneate; outer margins crenate; end view with three truncate and crenate angles."

3. *Semicellulæ vertice* 3–7 *radiatæ; radii in apice plerumque bi- tri- fidi vel bi- tri- spini.*

Semicells 3–7 *radiate at the vertex; radii bi- or tri-fid, or bi- or tri-spinous at the apex.*

St. polymorphum, Bréb.

St. semicellulis ellipticis, subtiliter granulatis vel tenuissime spinulosis, in medio magis minusve inflatis, haud raro ventricosis, rectis, nonnunquam incurvis, utrinque processu plus minus elongato, lineari, in apice 3–4 fido vel spinulis 3–4 tenuissimis instructis, a vertice 3–4–5–6–7 radiatis, radiis achrois, aut trifidis aut rotundatis, trispinis. (R.) *Species mihi ignota.*

Syn.—*St. polymorphum*, Brébisson. Ralfs, British Desmidieæ, p. 135. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 209.

Diam.—Long. 0.001". Lat. 0.00087". (R.)

Hab.—Florida; Bailey.

Segments in f. v. broadly elliptic or almost circular, rough with minute granules (sometimes with a few minute scattered spines), processes short, stout, tipped by three or four divergent spines; e. v. with three, four, five, or six angles each produced into a short, stout process. Sporangium orbicular, beset with elongate spines, forked or branched at the apex. Archer.

Var. **cyrtocerum.** (*St. cyrtocerum*, Bréb.)

Majus, ad $\frac{1}{59}$''', longum, semicellulis introrsum ventricosis, dorso late rotundatis, utrinque processu elongato, plerumque incurvo apice bi- vel tri-cuspidato instructis, a vertice triradiatis, radiis rectis vel leniter curvatis, in apice aut bi- aut tri-cuspidatis. (R.)

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Syn.—*Var. St. cystocerum*, Brébisson. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 210.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Segments in f. v. subcuneate, gradually widening upwards, truncate at the end margin, rough with minute granules, the lateral processes incurved, divided at the apex; e. v. triradiate, processes short, curved, sides slightly concave. L. $\frac{1}{800}''$. B. $\frac{1}{500}''$. (Archer.)

St. parodoxum, Meyen.

St. semicellulis inflatis, dorso rotundatis vel rectilinearibus, angulis superioribus in radium elongatum achroum hispidum, apice trifurcatim productis, sæpius radio æquali interposito a vertice tri- vel quadriradiatis, radiis strictis, trifurcatis, longitudine corporis diam. æquantibus vel superantibus. (R.)

Diam.—Cum rad. .0015″.

Syn.—*St. parodoxum*, Meyen. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 210.

Hab.—In lacu Saco, New Hampshire; (Lewis) Wood.

Semicells inflated, dorsum rounded or rectilinear, with superior angles produced into elongate, transparent, hispid radii with trifurcate apices, often furnished also with intermediate equal radii; from the vertex three or four radiate, radii straight, trifurcate, equalling or longer than the diameter of the body.

Remarks.—I am indebted to Dr. Lewis for specimens of this species, which he collected at Saco Lake.

Fig. 20, pl. 21, represents the end view of an empty frond.

St. arachne, Ralfs.

St. parvum, gracile, granulato-asperum; semicellulis introrsum ventricoso-globosis, angulis superioribus in cornu gracile, incurvum, apice obtusum, elongatis, a vertice pentagonis, quinque-radiatis, radiis elongatis linearibus achrois, obtusis, rectis vel leniter curvatis asperis. (R.)

Diam.—Sine rad. .0005″, cum rad. .00167″.

Syn.—*St. arachne*, Ralfs, British Desmidieæ, p. 136. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 210.

Hab.—In lacu Saco, New Hampshire, (Lewis) Wood.

Segments in f. v. suborbicular, rough with minute granules, lower margin turgid, outer convex, tapering at each side into an elongate, slender, incurved process having the granules thereon in transverse lines, entire at the apex; e. v. circular, bordered by five slender, linear, colorless marginal rays.

Remark.—Fig. 21, pl. 21, represents an outline of the end view of the semicell.

St. gracile, Ralfs.

St. mediocre, granulato asperum, granulis in series transversas ordinatis; semicellulis ventre valde inflatis, dorso truncatis, angulis in cornu rectum achroum gracile apice trifidum productis, a vertice triradiatis, lateribus sinuatis. (R.) *Species mihi ignota.*

Diam.—0.0022″. (R.)

Syn.—*St. gracile*, Ralfs, British Desmidieæ, p. 136. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 211.

Hab.—South Carolina; Florida; Georgia; Rhode Island; Bailey. Rhode Island; (Olney) Thwaites.

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Segments in f. v. triangular, ends truncate, rough with minute granules, tapering at each side into elongate, straight, slender, horizontal processes, terminated by three or four minute spines; e. v. triradiate, processes straight, sides concave. (A.)

C. CYTIODERMA PILOSUM, SPINULOSUM VEL ACULEATUM.

CYTIODERM PILOSE, SPINULOSE OR THORNY.

St. polytrichum, PERTY.

St. mediocre, tam longum quam latum, profunde constrictum, sinu acutangulo ampliato, superficie undique setosum; semicellulis ellipticis vel subellipticis, divergentibus, dorso subplanis, ventre tumidis, margine setoso-ciliatis, a vertice triangularibus, angulis obtusis, lateribus subrectis. (R.)

Diam.—$\frac{13}{7500}''$ = .0017″.

Syn.—*St. polytrichum*, PERTY. RABENHORST, Flora Europ. Algarum, Sect. III. p. 214.

Hab.—Prope Philadelphia; Wood.

Moderately large, about as long as broad, profoundly constricted, with the acute angled sinus widened, surface everywhere furnished with setæ; semicells elliptical or subelliptical, divergent, the dorsum nearly plane, their belly swollen, the margin setose-ciliate, from the vertex triangular, the angles obtuse.

Remarks.—This desmid appears to be rare in this country, as it probably is also in Europe. I have seen but a single specimen, which I found amongst other algæ near Chelten Hills, north of the city. It agreed in all respects with the description of Rabenhorst, as given above.

Fig. 12, pl. 13, is a drawing of this plant, also fig. 23, pl. 21.

St. Ravenelii, WOOD. (sp. nov.)

St. mediocre, paulo longius quam latum; semicellulis a fronte ellipticis, a vertice triangularibus cum lateribus convexis vel leniter retusis et angulis rotundatis; isthmo connexivo subnullo, lato; sinu acutangulo; cytiodermate spinis acutis, robustis numerosis armato.

Diam.—Long. $\frac{17}{12000}''$ = 0.0014″. Lat. $\frac{1}{1000}''$ = 0.001″.

Hab.—South Carolina; (Ravenel) Wood.

Mediocre, a little longer than broad; semicells from the front elliptical, from the vertex triangular, with the sides convex or slightly retuse, and the angles rounded; connecting isthmus obsolete, broad sinus acute-angled; cytioderm armed with numerous acute robust spines.

Remark.—Fig. 22, pl. 21, represents the front view of an empty frond of this plant; fig. 22 *a*, the side view, and fig. 22 *b*, the end, all magnified 750 diameters.

St. hirsutum, (EHRB.) BRÉB.

St. magnum, tertiam partem circiter quam longius quam latum, plus minus dense spinulosum, sinu plus minus lineari, acutangulo; semicellulis late ellipticis vel subsemiorbicularibus, spinis tenuibus strictis hirsutis, a vertice triangularibus, angulis obtuse rotundatis, lateribus rectis vel leniter convexis. (R.) *Species mihi ignota.*

Diam.—Sine spinis 0.0015″. Zygospor. 0.0022″. (R.)

Syn.—*St. hirsutum*, (EHRENBERG) BRÉBISSON. RABENHORST, Flora Europ. Algarum, Sect. III. p. 211.

Hab.—Florida; Rhode Island; Bailey. Rhode Island; (S. T. Olney) Thwaites.

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Segments in f. v. semiorbicular, separated by a linear constriction, covered with very minute, very numerous close set hair-like spines; e. v. with three broadly rounded angles, the spines evenly and numerously scattered; sides slightly convex. Sporangium orbicular, beset with short spines, branched at the apex. (A.)

St. Hystrix, Ralfs.

St. parvum, tertiam partem longius quam latum, angulis aculeatum (cæterum læve), sinu acutangulo; semicellulis subquadratis, angulis late rotundatis, dorso planis, a vertice 3–4 angularibus, angulis late rotundatis, plus minus dense aculeatis. (R.) *Species mihi ignota.*

Diam.—0.001″—0.00089″. (R.)

Syn.—*St. Hystrix,* Ralfs, British Desmidieæ, p. 128. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 213.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Segments in f. v. subquadrate, extremities somewhat rounded, end margin nearly straight, furnished with a few scattered, subulate, acute spines, chiefly confined to the lateral extremities; e. v. with three or four broadly rounded angles, the spines scattered, chiefly confined to the extremities, sides concave. L. $\frac{1}{1075}$″—$\frac{1}{1020}$″. Br. $\frac{1}{1165}$″—$\frac{1}{954}$″.

St. Cerberus, (Bailey) Archer.

St. parvum, tam longum quam latum, sinu rotundato, superficie lævi; semicellulis oblongis utroque fine sinuato-truncatis, angulis in aculeum cuspidatum productis, in medio sursum et deorsum prominentiis geminis in aculeum elongatis instructis, a vertice triangularibus, angulis in apice truncato- vel sinuato-bi-cuspidatis, sub apice aculeis geminis brevibus præditis. (R.) *Species mihi ignota.*

Diam.—Cum. acul. 0.00114″—0.0013″. (R.)

Syn.—*Didymocladon Cerberus,* Bailey, Microscopical Observations.

St. Cereberus, (Bailey) Archer. Rabenhorst, Flora Europ. Algar., Sect. III. p. 215.

Hab.—Florida; Bailey.

Small, deeply constricted, segments three-lobed, lobes with four teeth, two of which project upwards and two downwards at each truncated angle. (A.)

D. CYTIODERMA PROCESSIBUS NUMEROSIS, APICE PLERUMQUE TRUNCATIS ET DENTATO-FISSIS MUNITUM.

CYTIODERM WITH NUMEROUS PROCESSES, WHOSE APICES ARE MOSTLY TRUNCATE AND DENTATELY TORN.

St. furcigerum, Bréb.

St. validum, submagnum, circiter tam longum quam latum, læve vel subtiliter granulatum, plerumque profundissime constrictum, sinu angusto lineari; semicellulis oblongo-ellipticis, plus minus tumidis, angulis in processus bifurcum aut rectum aut divergentem longe productis, dorso processibus similibus 2, 3, 4, instructis, omnibus processibus achrois granulato-dentatis, granulis in series transversas ordinatis, a vertice 3-, 4-, 6-, 7-, 8-, 9-angularibus vel radiatis, angulis plus minus tumidis, in processus crassum achroum asperum in apice fissum productis. (R.) *Species mihi ignota.*

Long. Sine process. 0.0018″—0.0019″; c. pr. 0.003‴—0.0032″. Lat. sine proc. 0.00185″; c. pr. 0.0027″. (R.)

Syn.—*Staurastrum furcigerum,* Brébisson. Rabenhorst, Flora Europ. Algar., Sect. III. p. 219.

Didymocladon furcigerus, Ralfs, British Desmidieæ.

Hab.—South Carolina; Florida; Rhode Island; Bailey.

St. munitum, Wood.

St. submagnum, fere ½ plo longius quam latum, medio leviter constrictum, semicellulis a fronte

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enormiter hexagonis, angulis in processus rectos et divergentes productis, dorso processibus similibus 4–5 instructo; semicellulis a vertice polygonis vel suborbicularibus margine processibus numerosis, plerumque 9 instructo; dorso processibus 5–8 instructis; processibus omnibus similibus, granulato-dentatis, apice achroo simplicibus, bifurcatis vel fissis.

Diam.—A vertice cum processibus, $\frac{51}{12000}''$ = .00475″. Sine process. $\frac{25}{12000}''$ = .002″.

Syn.—*St. munitum*, Wood, Proceed. Ac. Nat. Sc., 1869.

Hab.—In lacu Saco, New Hampshire; (Lewis) Wood.

S. rather large, about one-half longer than broad, slightly constricted in the middle; semicells from the front irregularly hexagonal, the angles prolonged in straight divergent processes, and the surface furnished with four or five similar ones; semicells from the vertex polygonal or suborbicular, the margin furnished with numerous processes, mostly about nine, and also with 5–8 on the dorsum; processes all similar, granulate-dentate, their transparent apices simple, bifurcate or torn.

Remarks.—This species is most closely allied to *St. furcigerum*, Bréb., from which it is at once distinguished by the orbicular vertex. The constriction between the semicells is also very different. In *St. munitum* it is a gradual, not very deep, hour-glass contraction; in *St. furcigerum* it is very narrow and linear.

Fig. 13 *a*, pl. 13, is a front view of this plant magnified 260 diameters; fig. 13 *b*, the end view of the same.

St. eustephanum, (EHRB.) RALFS.

St. laterum integrorum angulis productis apice spinulosis, spinularum furcatarum corona media dorsali. (R.) *Species mihi ignota.*

Syn.—*Desmidium eustephanum.* EHRENBERG, Verbreitung und Einfluss der Mikrosk. Lebens in Süd- und Nord-Amerika, t. 4, f. 23.

Staurastrum eustephanum, (EHRB.) RALFS, British Desmidieæ, p. 215.

Stephanoxanthium eustephanum, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 221.

Staurastrum eustephanum, RALFS. RABENHORST (*loc. cit.*)

Hab.—West Point, New York; Bailey.

End view triangular with six emarginate spines on the upper surface; each angle terminated by a short ray tipped with spines. (Ralfs)

St. senarium, (EHRB.) RALFS.

Antecedenti simile sed laterum parietibus spinulis furcatis binis (sex), corona dorsali senaria. (R.) *Species mihi ignota.*

Syn.—*Desmidium senarium*, EHRENBERG, Verbreitung. T. IV.

Stephanoxanthium senarium, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 220.

Staurastrum senarium, (EHRB.) RALFS, British Desmidieæ. RABENHORST, (*loc. cit.*)

Segments smooth in end view with three angles, each terminating in a short process tipped by minute spines, without lateral processes, but with six others confluent at their bases on the upper surface, divergent and forked. (Archer.)

Genus XANTHIDIUM, EHRB.

Cellulæ singulæ vel geminæ concatenatæ, inflato-rotundatæ, profunde constrictæ; semicellulæ compressæ, oblongæ, hemisphæricæ vel subquadrangulares, centro in tuberculum rotundatum vel truncatum et denticulatum protuberantes, ex transverso oblongo-rotundatæ. Cytioderma firmum setis, aculeis vel spinis simplicibus aut bi- tri-furcato-divisis armatum. Massa chlorophyllacea radiatim expansa. Zygosporæ armatæ. (R.)

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Cells single or geminately concatenate, inflated, profoundly constricted; semicells compressed, oblong, hemispherical or subquadrangular, protruding in the centre as a rounded truncate or denticulate tubercle. Cytioderm firm, armed with setæ, or simple, or bi- tri-furcately divided spines. Chlorophyl radiately expanded. Zygospores armed.

Remark.—It has so happened that I have identified but a single species of this genus.

X. aculeatum, Ehrb.

X. parvum, singulum, sparsum, diametro ipse subæquale, ex obliquo ellipsoideum, diametro duplo longius, constrictione obtusa lineari, semicellulis oblongis subreniformibus, basi subplanis, dorso late rotundatis, tuberculo centrali minus elevato, truncato, margine autem crenato-dentato; cytiodermate undique aculeis subulatis obsito. (R.) *Species mihi ignota.*

Diam.—(Sine aculeis) 0.0025"—0.0029". (R.)

Syn.—*Xanthidium aculeatum*, Ehrenberg. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 222.

Hab.—Prope Savannah, Georgia; Bailey.

Frond in f. v. broader than long; constriction deep, linear; segments somewhat reniform; spines subulate, short, scattered, chiefly marginal; central protuberance cylindrical, truncate, border minutely dentate. (A.)

X. Arctiscon, Ehrb.

X. semicellulis globosis, binis, aculeatis, aculeis numerosis undique sparsis crassis asperis apice trilobis. (R.) *Species mihi ignota.*

Syn.—*X. Arctiscon*, Ehrenberg. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 224.

Hab.—America borealis; Ehrenberg.

Frond in f. v. about as long as broad; constriction forming a wide notch; segments narrowed at the base, with broadly rounded ends; spines numerous, restricted to the outer margin, scattered, elongate, stout, terminated by three or four diverging points. (Archer.)

X. armatum, (Bréb.) Ralfs.

X. maximum, validum, solitarium vel binatim conjunctum, diametro plerumque duplo longius; semicellulis subcordatis vel angulari-rotundatis tuberculo centrali subelevato, truncato, margine granulato-dentato præditis; cytiodermate verruculoso et processibus sæpius geminatis truncatis apice inciso-furcatis instructo. (R.)

Syn.—*Xanthidium armatum*, (Brébisson) Ralfs, British Desmidieæ et Rabenhorst, Flora Europ. Algarum, Sect. III. p. 222.

Hab.—South Carolina; Florida; Bailey. Saco Lake; (Lewis) Wood.

Frond large, in f. v. twice as long as broad; constriction deep, linear; segments broadest at the base; ends rounded or somewhat truncate; spines in pairs, principally marginal, short, stout, terminated by three or four divergent points; central projections cylindrical truncate, the border dentate; e. f. punctate. Sporangium large, orbicular, with depressed tubercles, perhaps immature. L. $\frac{1}{180}''$. B. $\frac{1}{270}''$. (A.)

Remark.—Fig. 17, pl. 13, is a front view of a frond, magnified 260 diameters.

X. bisenarium, Ehrb.

X. semicellulis globosis subangulosis, binis, aculeatis; aculeis fasciculatis, fasciculis in quovis globulo senis. *Species mihi ignota.*

Syn.—*X. bisenarium*, Ehrenberg. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 224.
X. Brébissonii, Ralfs. Archer, Pritchard's Infusoria, p. 736.

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Hab.—America; Ehrenberg.

Frond in front view broader than long; constriction deep, acute not linear; segments subelliptic, sometimes irregular spines subulate, geminate, marginal, central protuberance cylindrical, truncate border minutely dentate. L. (not including spines) $\frac{1}{416}''$. B. $\frac{1}{408}''$ to $\frac{1}{365}''$.

X. cristatum, Bréb.

X. parvum, læve; semicellulis subhæmispherico-reniformibus, utroque polo aculeo unico incurvo, ambitu aculeis octo geminatis, a dorso ovato-ellipticis, utroque polo aculeis ternis, in medio plerumque aculeo abbreviato. (R.) *Species mihi ignota.*

Diam.—0.00196″. R.

Syn.—*X. cristatum*, Brébisson. Ralfs, British Desmidieæ, et Rabenhorst, Flora Europ. Algarum, Sect. III. p. 224.

Hab.—South Carolina; Georgia; Florida; Bailey.

Frond rather longer than broad; constriction deep, linear; segments subreniform or truncate at ends; spines straight or curved, subulate, marginal, one at each side, at the base of the segment, solitary, the others geminate, in four pairs; central protuberance short, conical. (A.)

X. coronatum, Ehrb.

X. semicellulis subglobosis binis, aculeatis, ubique asperis, aculeis crassis apice truncatis tridentato-coronatis quatuor utrinque dorsalibus, uno utrinque latere medio. (R.) *Species mihi ignota.*

Syn.—*X. coronatum*, Ehrenberg, Verbreitung, p. 138, et Rabenhorst, Flora Europ. Algarum, Sect. III. p. 224.
Asteroxanthium coronatum, Kützing. Rabenhorst, (*loc. cit.*)

Hab.—America; Ehrenberg.

Remark.—Mr. Archer appears to think that this species is simply a form of *Staurastrum furcigerum.* (Bréb.); see Pritchard's Infusoria, p. 743.

X. fasciculatum, Ehrb.

X. parvum, singulum, constrictione profunde lineari; cytiodermate lævi vel sublævi; semicellulis oblongo-reniformibus vel hexagonis, diametro duplo longioribus, ambitu aculeis gracilibus geminatis 4–6, a dorso ellipticis, utroque polo aculeis quatuor instructis. (R.) *Species mihi ignota.*

Diam.—0.00228″—0.00256″. (R.)

Syn.—*X. fasciculatum*, Ehrenberg. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 223.

Hab.—South Carolina; Georgia; Florida; Rhode Island; Bailey.

Frond about as long as broad; constriction deep, linear; segments somewhat reniform or subhexagonal, twice as broad as long, spines slender, subulate geminate, marginal, in four or six pairs; central protuberance short, conical, somewhat truncate. (A.)

Genus ARTHRODESMUS, Ehrb.

Cellulæ profunde constrictæ; semicellulæ compressæ aut oblongæ, utroque polo aculeo subulato firmo instructæ, aut quadrangulares, angulis in aculeum rectum vel curvum productis, a dorso vel ellipticæ vel fusiformes. Massa chlorophyllacea in fascias quatuor radiantes disposita. (R.)

Cells profoundly constricted; semicells compressed or oblong, furnished at each end with a subulate spine, or else quadrangular with the angles produced into straight or curved spines, the dorsal aspect, elliptic or fusiform. Chlorophyl masses disposed in four radiating fascia.

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Remarks.—I have found only a single undescribed species of this genus, but the following European forms have been detected in this country by Prof. Bailey. The genus appears to be, as Prof. Rabenhorst says, scarcely distinguishable from *Xanthidium* or *Staurastrum.*

A. octocornis, Ehrb.

A. parvus, lævis, constrictione lata excavata; semicellulis trapezoideis, inciso-quadriradiatis, radiis in aculeum acutissimum strictum porrectis, a latere elongato-ellipticis, diametro fere triplo longioribus, utroque polo aculeum singulum gerentibus. (R.)

Diam.—0.00065″. (R.)

Syn.—*Xanthidium octocorne*, Ralfs. Bailey, Microscopical Observations, p. 29.
Arthrodesmus octocornis, Ehrenberg. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 223.

Hab.—Florida; Rhode Island; Bailey.

Frond smooth, minute, about as long as broad; constriction a wide notch; segments much compressed, trapezoid, each angle terminated by one or two straight, subulate, acute spines, the intervals between the angles concave. (A.)

a. Spine solitary at each angle. L. $\frac{1}{1351}$″. B. $\frac{1}{1538}$″. (A.)
b. Larger spines geminate at each angle. L. $\frac{1}{1020}$″. B. $\frac{1}{906}$″. (A)

A. quadridens, Wood.

A. late ovalis, vel suborbicularis, paulum longior quam latus, cum margine crenato-undulato; semicellulis nonnihil reniformibus, utroque fine aculeo subulato, modice robusto, acuto, recurvo, armatis; cytiodermate cum verruculis paucibus modice minutis in seriebus paucibus dispositis instructo; semicellulis a vertice acute ellipticis, et cum margine crenato et superficie sparse verruculosa.

Diam.—Lat. $\frac{3}{4000}$″ = .00075″; long. $\frac{5}{4000}$″ = .00125″.

Syn.—*A. quadridens*, Wood, Proc. A. N. S. 1869.

Hab.—In lacu Saco, (Lewis) Wood.

Broadly oval or suborbicular, a little longer than broad, with the margin crenately undulate; semicells somewhat reniform, at each end armed with a subulate, moderately robust, acute, recurved large spine; cytioderm with a few smallish tubercles arranged in three or four rows; semicells from the vertex acutely elliptical, with the margin crenate and the surface sparsely warty.

Remarks.—This species approximates *A. divergens*, from which it differs in the arrangement of its granules, its attaining not one-half the size, and, I believe, in the larger and more robust spines.

Fig. 2, pl. 20, represents an empty frond of this species.

A. Incus, (Bréb.) Hassal.

A. parvus tam longus quam latus, constrictione lineari obtusa vel late excisa; semicellulis oblongo-quadrangularibus, angulis externis aculeatis, internis rotundatis inermibus, aculeis longis singulis divergentibus. (R.)

Diam.—Max. 0.00098″. Long. 0.00091″. Spor. (sine acul.) 0.00085″.

Syn.—*A. Incus*, (Brébisson) Hassal, Fresh-Water Algæ, p. 357, et Rabenhorst, Flora Europ. Algarum, Sect. III. p. 226.

Hab.—Georgia; Florida; South Carolina; Rhode Island; Bailey.

Frond minute, smooth, as long, or longer than broad, constrictions a deep notch or sinus; segments with inner margin turgid, outer truncate; spines subulate, acute; sporangium orbicular, spinous; spines subulate. (Archer)

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A. convergens, Ehrb.

A. lævis mediocris, profunde et anguste constrictus, aculeis convergentibus armatus; semicellulis ellipticis vel ovato-oblongis, nonnunquam reniformibus, utroque fine aculeo longo firmo incurvo instructis. (R.)

Diam.—0.00185″—0.0016″. (R.)

Syn.—*A. convergens*, Ehrenberg. Rabenhorst, Flora Europ., Algarum, Sect. III. p. 227.

Hab.— South Carolina; Georgia; Florida; Rhode Island; Bailey.

Frond smooth, broader than long; constriction deep, acute; segments elliptic, each having its lateral spines curved towards those of the other; ends convex. L. $\frac{1}{1539}$″—$\frac{1}{598}$″ B. $\frac{1}{1477}$″—$\frac{1}{584}$″. (Archer)

Family ZYGNEMACEÆ.

Cellulæ cylindricæ, æquipolares, similes, in familias filamentosas arcte conjunctæ, et cytioblasto centrali plasmate plerumque radiante involuto, et plasmate chlorophylloso aut effuso, aut effigurato, aut (plerumque) in fascias spirales ordinato, et granulis amylaceis instructæ. Filum simplex. Propogatio fit zygosporis conjugatione cellularum binarum ortis. Conjugatio triplici modo, aut lateralis aut scalariformis vel genuflexa. Vegetatio fit divisione transversali repetita.

Cells cylindrical, the same at both ends, closely conjoined into filamentous families, furnished with a central cytioblast wrapped up in generally radiating protoplasm, and with chlorophyllous protoplasm effused in shapeless masses or arranged in spiral filaments, and also with scattered starch-granules. Filament simple. Propagation takes place by means of zygospores, arising from the conjugation of two cells. Conjugation occurring in three ways, lateral, scalariform, and genuflexuous. Growth taking place by means of transverse division of the cells.

Remarks.—The family under consideration is among the commonest and most widely diffused of all the fresh-water algæ. In almost every ditch or spring, or dripping moss-covered rock representatives of it are to be found, so that wherever quiet water is they may be confidently looked for. The single filaments are so minute that frequently the unaided eye cannot distinguish them, but multiplication with them is such a rapid process, that wherever found they are in great masses. These masses, when growth is active, are of a beautiful intense green, glistening and shining with the gelatinous matter which coats the threads and makes the mass so slippery. They may be found in greater or less abundance at all seasons, but as the specific characters are largely of sexual origin, non-conjugating specimens are of little value. For this reason, *Zygnemas* are only worth gathering when in fruit. The spores appear to be formed only in the spring and early summer, at least these are the only times in which I have found fertile filaments. In this neighborhood I have collected them in excellent condition as early as the beginning of April and as late as the latter part of June. Further south, conjugation of course commences earlier, and fine fruiting specimens received by myself from Mr. Canby were collected in Florida by him in February.

When conjugating freely the mass of *Zygnema* or *Spirogyra* loses its beautiful bright green color and become dingy and even brownish, often very dirty looking. The collector soon learns to pass by the beautiful vivid mass, as comparatively worthless, and fasten upon the pale, wan, sickly, apparently dying specimens as prizes worthy of a place in his cabinet.

In the *Zygnemaceæ* the individual plant, as ordinarily considered, is a filament composed of a varying number of cells placed, end to end, all alike, and each of them apparently independent of its associates. Each cell in one sense is, therefore, a perfect, complete individual, capable of living dissociated from its companions. How far the life of one of these cells is influenced by that of its neighbors is uncertain, probably to a slight extent, possibly not at all. At any rate, they are so far independent that the filament is rather a composite body than a unit of life. These cells are cylindrical, with the ordinary cellulose wall, which can commonly be stained blue by iodine and sulphuric acid, and is often distinctly composed of layers, but never has any "secondary markings," each layer being precisely like that superimposed upon it. Outside of the wall is a jelly-like sheath, which is mostly not discernible from its thinness and transparency, although it no doubt exists, as is proven by the slipperiness of the general mass. The primordial utricle is always present. The chlorophyl is variously arranged, most generally in bands, either straight or spiral, sometimes in definite irregular masses, sometimes diffused through the cell. Imbedded in it are, at certain seasons, numerous minute, generally shining, granules, which are either minute specks of starch, or little drops of oil. Besides these there are contained in it, especially in the bands of chlorophyl, more or less numerous comparatively large, oval or roundish bodies, with a distinct outline and a deeper color than the surrounding portions. These masses are protoplasm, dyed with chlorophyl-green, and are believed to be especially active in the formation of starch. At times, iodine turns them simply brown; at others it colors their inner portions blue and their outer brown, showing them to contain starch. The general cavity of the cell is occupied by fluid, in which is placed the nucleus. This is mostly single, but rarely, according to Nægeli and other authorities, double, and De Bary states that he has seen three nuclei in a single cell. I have never seen more than one, and think that even this is not rarely absent, having certainly repeatedly failed to demonstrate its presence. It is colorless, often with a nucleolus, transparently bright, irregular in form, placed in the centre of the cell with numerous arms radiating out from it, some of them ending within the cell, others connecting it with the primordial utricle. De Bary states that this nucleus occasionally is tinged green with chlorophyl, I do not remember ever to have seen it so.

I have not infrequently seen numerous minute dark granules, similar to those seen in *Closterium,* scattered through the inside of the cell, in active motion. Sometimes they are to be found collected in vast numbers near the ends of the cells, dancing and swarming about one another, and passing off in small streams from one end to the other, coasting along close to the primordial utricle, in a word, exhibiting precisely the same motions as are so common among the desmids.

The *Zygnema* filament grows in length by a process of cell multiplication by division of the simplest kind. It seems to be somewhat uncertain whether the nucleus always divides into two as a part of the process or not. These plants multiply both by the separation of cells and their subsequent growth, and, by means of resting spores, the so-called *Zygospores.*

The first appearance of separation of the cells is an evident disposition to the rounding off of the ends of the cells. The corners are first rounded and separated and this continues until only the centres of the ends are in apposition, and in a little while even these separate. This certainly, at least, is the process in certain species; but I have thought, that in other cases cells were separated by a simple splitting of the end wall, each cell retaining its half of the partition.

The *zygospores* are produced by a process of union of two cells, to which the name of *conjugation* has been given. Very rarely, if ever, is there any difference between the cells before conjugation, and it has not existed in any species which has come under my notice; but, after conjugation, the receiving cell is frequently enlarged, the other remaining cylindrical. De Bary, however, states that he has found a small but constant difference between the fertile and sterile cells of *Spirogyra Heeriana.*

The first perceptible change in a cell about to produce a resting spore, appears to be a loosening of the primordial utricle from the outer wall, and a contraction of it upon the cell contents, which thus are crowded together and more or less deformed. Simultaneously with this, or a little after or before it, the side wall of the cell is ruptured and a little pullulation or process is pushed out, which directly coats itself with cellulose and rapidly enlarges to a considerable diameter, at the same time growing in length until it meets a similar process pushing out from an opposing cell, or has attained as great a length as its laws of development will allow. When two processes meet they become fused together, the end walls are ruptured, and the contents of one cell passing over are received within those of the other, or else the contents of both cells meet within the connecting tube and there fuse together. This is the more common mode of conjugation, in which two cells of distinct filaments become joined together by a connecting tube. It is evident, that, if the filaments are fertile to their fullest extent, there will be as many of these connecting tubes as there are pairs of cells in the filaments, and a ladder-like body will be formed, the original filaments corresponding to the side pieces, the connecting tubes to the rounds. Hence this method of conjugation has received the name of *scalariform.*

In the so-called "*lateral conjugation*," instead of cells of different filaments joining, adjacent cells of one filament unite together to complete the process. The union of the two cells appears to take place in several ways. In accordance with one plan (fig. 1 *a*, pl. 14), connecting tubes, pushed out from near the ends of the cells, grow for a short distance nearly at right angles to the long axis of the filaments, and then bend at a right angle to themselves so as to run parallel to the filament-cells. The ends of these processes are, of course, opposed to one another, and coming in contact fuse together so as to form a continuous tube for the passage of the endochrome. Another method by which neighboring cells are sometimes connected is by the formation of coadjacent pouch-like enlargements of the opposing ends, and a subsequent fusion of these newly formed enlargements by the absorption of the end wall between them. (See fig. 2, pl. 14.)

Sometimes I think the union of two neighboring cells is facilitated by a curved neck forming to one or both of them, so that they are bent at an angle to one another, and can readily be united by means of a straight tube.

There is still another method of conjugation, the so-called *genuflexuous*, in which, instead of a connecting tube being formed as the medium of union, two cells of opposing filaments become sharply bent backwards, so that their central portions are strongly thrust forward as obtuse points, which, coming in contact, adhere and allow of a passage-way between the cells being made by the absorption of their cohering walls.

A curious modification of, or departure from, the ordinary method of conjugation is sometimes seen, in the union of three instead of two cells. This is, I think, very rare, but has been seen by Meyen in the genus *Zygnema*, as well as by Schleiden and De Bary in *Spirogyra*. I myself have observed it once or twice in the latter genus. One of the cells plays the part of the female, receiving the contents of the other two, so that in it the primordial utricles of the three, with their contracted protoplasm, are fused into a zygospore.

The zygospore, however formed, varies in shape, but is mostly oval or globular, sometimes cylindrical, and when ripe is in most if not all species of a dark brownish color. It is described both by Pringsheim and De Bary as having three coats, but I have frequently found it impossible to demonstrate the presence of all of these, and I believe that not rarely one of them is absent. The outer coat is developed first and is the thickest and firmest. Occasionally it is double, *i. e.* composed of two distinct layers or parts, as in *Sp. protecta*, in which species the outer of these layers is the thickest, firmest, and most evident, whilst the inner layer is translucent and much less apparent. The second coat contains the coloring matter, which is sometimes brown, sometimes decidedly yellowish. The inner coat is not readily seen. It is elastic, thin, and is the last of the three to be formed.

The principal contents of the ripe spore are protein compounds (protoplasm), oil-drops, starch granules, and pigment. The oil is generally much more abundant than the starch, and not rarely the minute, bright drops entirely replace the little granules. According to Prof. De Bary, the pigment frequently, but not always, reacts with sulphuric acid, as does that of the fungal family, *Uredineæ*, striking with it a deep blue.

The germination of the spore, both in the genus *Spirogyra* and *Zygnema*, is very simple. The first step is an elongation and growth of the protoplasmic central mass, together with the inner transparent cellulose coat, and a consequent rupturing of the outer two coats, through which the newly forming plant protrudes and finally escapes. In this way in the genus *Spirogyra* an elongated club-shaped cell arises, one end of which is much larger than the other and contains all the chlorophyl. Sometimes a nucleus is perceptible in this cell, sometimes it is not. The larger end now becomes cut off by a partition wall from the smaller; if no nucleus has been previously apparent it now becomes so, and the first stage of development is completed. The filament after this grows by a simple repetition of the process of division in the larger end and the cells formed out of it. The smaller end undergoes little or no change. In the genus *Zygnema*, the cell that first emerges from the germinating spore is a perfect one, similar in all respects to those seen in the fully formed filament, which is developed out of it, by a simple process of cell division.

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Besides the true *Zygospores*, Hassall many years since described bodies (Fresh-water Algæ, vol. i. pp. 132, 156, 170), which he found in filaments of this family, and which resemble in all respects ordinary *Zygospores*, but are produced each in a single cell without any aid from a second cell. He affirmed that he had observed this phenomenon especially in two species, *Spirogyra mirabilis* and *Zygnema notabilis*. These observations were doubted by some, whilst others, as Alexander Braun, supposed that there was a division of the cell protoplasm into two distinct portions, and then a conjugation of these within the original cell, and that Mr. Hassall had overlooked these changes. Prof. De Bary, however, states that he has seen a great many instances of this production of spores without conjugation (all in one species), and that there can be no doubt that Hassall's observations are substantially correct, and that no division of the primordial utricle such as was imagined by Prof. Braun takes place. Spores formed in this manner, as yet have not been seen to develop. There is, therefore, no certainty that they are capable of doing so. It is possible that they are merely the results of abortive attempts at reproduction, wanting the power of development because not fertilized.

Pringsheim and others have drawn from these bodies strong argument against the idea, that conjugation is to be looked upon at all as a sexual process.

The arguments both for and against regarding conjugation as the simplest expression of sexual life are ably elaborated by De Bary, *Untersuchungen über die Familie der Conjugatem*, p. 57, to which I must refer those desirous of following the subject further, contenting myself with expressing an agreement with the conclusions there arrived at, namely, that in conjugation the first dawnings of sexuality are to be found. Looking at it in this light Prof. De Bary states his conviction that the spores formed in the manner last described, bear the same relation to the true *Zygospore* that the bud of a *Phanerogam* does to its seed, or the *Zoospore* of an *Œdogonium* does to its resting spore.

Quite a number of bodies have been described by the older authorities as being found within the cells of plants of this family, which more recent observers have proven to be parasitic. Such are the "Spermatic spheres," transparent spheres motile by virtue of vibratile cilia, various monads, &c. &c., bodies for which it has been claimed, from time to time, that they were sexual elements, spermatozoids.

Genus SPIROGYRA, Link.

Cellulæ vegetativæ cylindricæ, fasciis chlorophyllosis spiralibus instructæ. Conjugatio aut lateralis aut scalariformis aut et lateralis et scalariformis.

Syn.—*Spirogyra et Rhynchonema*, Kützing, Rabenhorst, et auctores.
Salmacis, Bory.
Zygnema (partim), Hassall.

Vegetative cells cylindrical, furnished with spiral chlorophyl bands. Conjugation either lateral or scalariform or both lateral and scalariform.

Remarks.—The genus *Spirogyra*, as defined above, has been divided by Kützing, Rabenhorst, and others into two genera, the characters being drawn from the method of union of the conjugating cells; in the one case the neighboring cells of a single filament (*Rhynchonema*), in the other cells of distinct filaments (*Spirogyra*),

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uniting to form the spore. This at first sight appears to be a good ground for separation, but there are certain species in which, undoubtedly, both the former and the latter method of conjugation take place indifferently. Such species make a third group so precisely between the two others as, to my mind, to fuse them together and necessitate either the acknowledgment of three genera or the denial of more than one. The latter seems to me the more philosophical course.

A. CONJUGATIO LATERALIS (RHYNCHONEMA).

A. CONJUGATION LATERAL.

Sp. elongata, WOOD.

Sp. articulis vegetativis diametro 7–20 plo longioribus; articulis sporiferis multo brevioribus, valde tumidis; cytiodermate utroque fine protenso et replicato; fascia unica, laxissime spirali; anfractibus plerumque 7; sporis ellipticis, diametro 1–2½ plo longioribus.

Diam.—Spor. $\frac{8}{7500}''$ = .00106″. Artic. vegetat. $\frac{4}{7500}''$ =.0005″.

Syn.—*Rhynchonemaelongatum,* WOOD, Prodromus, Proc. Amer. Philos. Soc. 1869, p. 137.

Hab.—In aquis limpidis, prope Philadelphia.

Sterile joints 7–20 times longer than broad; fertile joints much shorter, greatly swollen; cell wall at each end produced or folded in; chlorophyl filament 1, spiral lax; turns mostly 7; spores elliptical, 2–2½ times longer than broad.

Remarks.—I found this species about the middle of March, fruiting in a little pool near Chelten Hills, six or eight miles north of this city. It did not form a distinct stratum by itself, but was floating, intermingled with great numbers of other filamentous algæ, such as *fragillariæ*, *zygnemæ*, &c. It seems to be most closely allied to the European *R. minimum;* it however not only attains a somewhat larger size but also differs from that plant in the proportionate length of the sterile cells, in the number of the turns of the chlorophyl spiral in the cell, and in the proportionate length and breadth of the spore.

Fig. 1, pl. 14, represents portions of sterile filaments magnified 450 diameters; 1 *a*, a part of a fertile filament, magnified 450 diameters.

Sp. pulchella, WOOD.

Sp. articulis sterilibus diametro 2–3 plo longioribus; sporiferis nonnihil tumidis; fascia unica, anfractibus 3–4; sporis ellipticis, diametro fere duplo longioribus; cytiodermate utroque fine protenso et replicato.

Diam.—Artic. Steril. $\frac{4}{7500}''$—$\frac{9}{7500}''$=.00033″—.0013″. Spor. $\frac{9}{7500}''$—$\frac{10}{7500}''$=.0012″—.00133″.

Syn.—*Rhynconemapulchellum,* WOOD, Prodromus, Proc. Amer. Philos. Soc. 1869, p. 138.

Hab.—In stagnis, prope Philadelphia.

Sterile' joints 2–3 times longer than broad; fertile joints somewhat swollen; chlorophyl band one; turns of spiral 3–4; spores elliptical, almost twice as long as broad; cell wall at each end produced or folded in.

Remarks.—This species was found by myself fruiting in April, 1869, in stagnant ditches below the city, and in similar localities near Camden, New Jersey. It did not occur in masses but singly, intermixed with great numbers of other fruiting spirogyras. Most of the filaments seen were about .0010″ in diameter; in but a single instance did they come much short of this. This species differs from *R. elongatum*, among other points, in the shortness of the tubes connecting the fertile

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cells. I have never been able to identify an entirely sterile filament of this species; the measurements and description of the sterile cells were taken from infertile cells in filaments, which in other places had produced spores.

Fig. 2, pl. 14, represents a fertile filament, magnified 260 diameters.

B. CONJUGATIO SCALARIFORMIS (SPIROGYRA VERA).

a. Cytiodermate utroque fine protensum et replicatum.

a. Cytioderm folded in at the ends.

* *Fascia spiralis unica.*

* *Spiral filament single.*

Sp. Weberi, KTZ. ?

Sp. saturate viridis, lubrica; articulis vegetativis diametro 3–20 plo longioribus; fructiferis nonnihil inflatis; fascia dentata, plerumque unica sed fasciis duabus in quavis cellula; spiræ anfractibus 3-8; cytiodermate plerumque utroque fine protenso et replicato; zygosporis ellipticis.

Diam.—Artic. steril. $\frac{6}{7500}''$—$\frac{9}{7500}''$ = .0008″—.0012″.

Syn.—*S. Weberi,* KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 233.

Hab.—In stagnis, prope Philadelphia.

Deep green, slippery; sterile joints 3–20 times longer than broad; fertile joints not swollen; chlorophyl filaments mostly single, but sometimes two in certain cells, dentate; turns of the spiral 3–8; cytioderm protruded or infolded at the ends; zygospores elliptical.

Remarks.—This species, which is abundant around Philadelphia in stagnant ditches, I have found fruiting in the month of April. The number of spirals frequently varies even in the same filament. The infolding of the walls at the end of the cells is very often wanting in the fertile cells and occasionally is absent from one end of an ordinary vegetative cell. The American form agrees pretty well with the European, but is, however, larger, and also attains in its cells a greater proportionate length and has more turns of its chlorophyl spirals. The lower limits of the American form are, however, so overlapped by the upper limits of the European, that it seems to me they must be considered identical.

Fig. 19, pl. 12, represents a pair of fertile filaments of this species, magnified 260 diameters; 19 *a*, part of a sterile filament, magnified 260 diameters; 19 *b*, outline of a couple of fertile cells, magnified 260 diameters.

Sp. protecta, WOOD.

Sp. saturate viridis, lubrica; articulis sterilibus diametro 6 plo longioribus; sporiferis vix tumidis; cytiodermate utroque fine protenso et replicato; fascia unica; anfractibus 6; sporis oblongis vel ellipticis: membrano crassissimo.

Diam.—Art. steril. $\frac{11}{7500}''$ = .00146″; spor. lat. $\frac{10}{7500}''$—$\frac{12}{7500}''$ = .00133″—.0016″ long. $\frac{25}{7500}''$ = .0033″.

Syn.—*Sp. protecta,* WOOD, Prodromus, Proc. Am. Philos. Soc. 1869, p. 131.

Sp. deep green, slippery; sterile joints 6 times longer than broad; fertile cells scarcely swollen; cell wall folded in at the ends; chlorophyl band single; turns 6; spores oblong er elliptical, spore wall very thick.

Remarks.—I found this species in the latter part of April fruiting in a ditch in a meadow a little south of the mouth of Wissahicon Creek, near this city, and as late as the 25th of May in the "neck" below the city. It is remarkable

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for the very great thickness of the walls of the spore. There are two very apparent coats separated by a thin not very evident one. The outer is the thickest; it is very thick, firm, and nearly colorless. The inner coat is of a decided orange-brown. The parent-cells which give origin to these spores are slightly enlarged in diameter. Sometimes the spores, instead of being elliptical, are irregular in shape.

Fig. 3 *a*, pl. 14, represents a sterile filament, magnified 250 diameters; fig. 3, a mature spore, magnified 450 diameters.

Sp. insignis, (HASSALL) KTZ.

Sp. articulis sterilibus diametro 5–14 plo longioribus; fasciis 2 (rarius 1–3), laxe spiralibus, angustis, crenatis; articulis fructiferis nonnihil tumidis; cytiodermate utroque fine replicato vel protenso; zygosporis rubido-brunneis, ovato-ellipticis.

Diam.—0.0015″.

Syn.—*Zygnema insigne*, HASSALL, Fresh-Water Algæ, p. 440.
Spirogyra insignis, (HASSALL) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 235.

Hab.—In stagnis, prope Philadelphia.

Sterile joints 5–14 times longer than broad; chlorophyl filaments mostly 2 (rarely 1–3), laxly spiral, narrow, crenate; fertile joints somewhat enlarged; cytioderm at each end folded in or produced; zygospores reddish-brown, ovate elliptical.

Remark.—Fig. 6, pl. 16, represents this species.

b. Cytioderma cellulæ fine nec protensum nec replicatum.
Cytioderm not infolded in the end of the cell.

* *Fasciæ spirali unicæ (raro duæ).*
Chlorophyl band single (rarely two).

Sp. longata, (VAUCH.) KTZ.

Sp. dense cæspitosa, læte luteolo-viridis, valde lubrica; articulis sterilibus diametro 2–6 plo longioribus, fertilibus sæpe tumidis abbreviatis; fascia spirali lata, dentata; anfractibus sublaxis 2–5; zygosporis ellipticis.

Diam.—0.001″.

Syn.—*Conjugata longata*, VAUCHER, Histoire des Conferves d'Eau douce, p. 71.
Sp. longata, (VAUCH.) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 238.

Hab.—In stagnis, prope Philadelphia; Wood. Rhode Island; (S. T. Olney) Thwaites.

Densely cæspitose, bright yellowish-green, very slippery; sterile joints 2–6 times longer than broad; fertile articles swollen, often abbreviate; chlorophyl filaments broad, dentate; turns of the spiral somewhat loose, 2–5; zygospores elliptical.

Remarks.—According to Prof. Rabenhorst, this species attains in Europe a diameter of .0011″ and the cells a length of 8 times their breadth. The same authority also describes the fertile cell as being either not swollen, or moderately so ("aut non aut modice tumidis"). In all the specimens of our American forms which I have seen, the sporangial cells are very decidedly swollen.

Fig. 4, pl. 14, represents portions of sterile filaments, magnified 250 diameters, and fig. 4 *a*, a part of a fertile pair of filaments containing immature spores enlarged 260 diameters.

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Sp. quinina, (AG.) KÜTZ.

Sp. saturate viridis, valde lubricata; articulis sterilibus diametro 1–6 plo longioribus; articulis fertilibus vel haud tumidis vel nonnihil tumidis; fascia unica; spiræ anfractibus modo densioribus, modo laxioribus, nonnunquam laxissimis, plerumque 3, interdum 1½–4; cytiodermate cellulæ utroque fine nec protenso nec replicato; zygosporis aut globosis aut ovalibus aut cylindricis.

Diam.—Artic. steril. $\frac{10}{7500}$″—$\frac{13}{7500}$″ = .0013″—.0017″; sporis $\frac{11}{7500}$″ = .0014″.

Syn.—*Sp. quinina,* (AGARDH) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 240.

Hab.—In stagnis, prope Philadelphia.

Deep green, very slippery, sterile articles 1–6 times longer than broad; fertile joints scarcely or not at all tumid; chlorophyl filament single; turns of the spiral sometimes denser, sometimes laxer, sometimes very lax, mostly 3 in number, sometimes 1½–4; cytioderm neither infolded nor protruded at the end; zygospores polymorphous, globose, elliptical or cylindrical.

Remarks.—This species is very abundant in the ditches around Philadelphia, especially in the "neck" below the city. I have found it fruiting profusely in the month of April. The spores vary very much in form, some of them being globose, others elliptic, and still others cylindrical, with obtusely rounded ends. All these forms may occur in a single filament. The spore cell also varies in the amount of its enlargement. In many cases it preserves its cylindrical shape completely; in other instances it is markedly swollen.

Figs. 4 *e*, 4 *c*, pl. 19, represent portions of sterile filaments of this species; figs. *a*, 4 *b*, and 4 *d*, portions of fertile filaments.

†† *Fasciæ spiralæ duæ vel plures.*

†† *Chlorophyl filaments two or many.*

Sp. decimina, (MULLER) KTZ.

Sp. sordide viridis, lubrica; articulis sterilibus diametro (0.00135″—0.00159″) plerumque duplo-, quadruplo fere longioribus, nonnunquam subæqualibus, fertilibus aut non aut modice tumidis; fasciis spiralibus plerumque 2, latis, decussatis, rarius 1 vel 3, anfractibus laxis 1–1½; zygosporis aut ovalibus aut late ellipticis vel subglobosis. (R.)

Syn.—*Sp. decimina,* (MÜLLER) KUTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 242.

Hab.—Prope Philadelphia.

Dirty green, slippery; sterile joints mostly from 2–4 times as long as broad (0.00135″—0.00159″), sometimes about as long as broad; fertile joints either moderately or not at all swollen; spiral filaments mostly 2, broad, decussating, rarely 1–3, turns loose 1–1½; zygospores either oval, broadly elliptic, or subglobose.

Remarks.—I find this species marked in one of my note-books as having been found by myself near this city. I have no distinct recollection of seeing it, and, having preserved neither figure, specimen, nor description, am forced to content myself with copying the description of Prof. Rabenhorst.

Sp. dubia, KTZ.

Sp. viridis in fructe dilute viridis; articulis sterilibus cylindricis diametro 1½–2½ plo longioribus; fasciis spiralibus 2–3, angustissimis, nodosis, anfractibus laxis 1–2 (= 3–6); cytiodermate utroque fine nec protenso nec replicato, nonnihil crasso; zygosporis polymorphis, aut subglobosis aut ovalibus, aut subcylindricis, diametro æqualibus aut $\frac{3}{4}$ plo longioribus; articulis fertilibus cylindricis, haud tumidis.

Diam.—Art. steril. $\frac{15}{7500}'' = .002$; spor. $\frac{15}{7500}'' — .002''$.

Syn.—*Sp. dubia*, Kutzing. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 243.

Hab.—In stagnis, prope Philadelphia.

Green, in fruit light green; sterile joints cylindrical, 1½–2½ times longer than broad; spiral filaments 2–3, very narrow, nodose, lax, turns 1–2; cytioderm neither infolded nor protruded at the end, rather thick; zygospores polymorphous, either subglobose, oval, or subcylindrical, as broad as long to $\frac{3}{4}$ times longer; fertile articles cylindrical, not enlarged.

Remarks.—I have found this species growing in the ditches below the city, fruiting abundantly in May. When in this condition it forms masses of a dirty, lightish, yellowish-green. The spores mostly fill pretty well the fertile cells. My specimens do not agree completely with the descriptions given of the European. The two forms, however, completely overlap one another, except in one character, namely, the shape of the sporangial cell. I have never seen it swollen or at all tumid in American specimens, whilst in the European it is said to be "modice tumidis." This difference alone does not, however, seem to me sufficient to characterize a new species. I have seen specimens of this plant collected by Dr. Lewis at Cobble Mountain. They agree well with the Philadelphia specimens, except in attaining a little larger size, .0021″, and in the sterile filaments having their walls very thick. The character of non-inflation of sporangial cells is perfectly preserved.

Fig. 4, pl. 17, represents this species.

Sp. rivularis, (Hassall) Rabenh. (*non* Ktz.)

Sp. saturate viridis, lubrica; articulis sterilibus diametro 7–11 plo longioribus; fertilibus cylindricis aut vix tumidis; cytiodermato tenuissimo, utroque fine nec protenso nec replicato, fasciis 4, laxe spiralibus, modice angustis, nodulosis et serratis, anfractibus 2½; zygosporis ellipticis, diametro 2–2½ longioribus.

Diam.—Art. ster. $\frac{9}{7500}''—\frac{11}{7500}'' = .0012''—.00146''$; spor. $\frac{10}{7500}''—\frac{12}{7500}''$.

Syn.—*Zygnema rivularis*, Hassall, Fresh-Water Algæ, vol. i. p. 144.
Spirogyra rivularis, (Hassall) (*non* Kützing) Rabenhorst, Flora Europ. Algarum, Sect. III. p. 243.

Hab.—In rivulis, Florida; (Canby) Wood.

Deep green, slippery; sterile articles 7–11 times longer than broad, fertile cylindrical or slightly tumid; cytioderm very thin, neither infolded nor protruded at the end; chlorophyl filaments 4, laxly spiral, moderately narrow, nodose and serrulate, turns 2½; zygospores elliptical, 2–2½ times longer than broad.

Remarks.—This species was collected by Mr. Wm. Canby in Pine Barren Run, near Hibernia, Florida. It is rather smaller than the European forms, but does not appear to be distinct from them. Rabenhorst, indeed, states that there are only two or three chlorophyl spiral bands in a cell, but Hassall in the description of the type states distinctly that in some instances there are four bands, and also figures the plant so.

Fig. 5 *a* and *b*, pl. 17, represents sterile cells of this species, magnified 260 diameters. Fig. 5 *c* is an outline of a pair of fertile cells enlarged to the same extent.

Sp. parvispora, WOOD.

Sp. articulis sterilibus diametro 2–4 plo longioribus; fructiferis haud tumidis, diametro 1–$2\frac{1}{2}$ plo longioribus; fasciis spiralibus 4, angustis, nodosis, anfractibus pluribus; zygosporis parvissimis, ellipticis, diametro $1\frac{1}{4}$–2 plo longioribus; cytiodermate utroque fine nec protenso nec replicato.

Diam.—Art. steril $\frac{23}{7500}''$=.003″; spor. diam. transv. $\frac{15}{7500}''$—$\frac{17}{7500}''$ = .002″—.0023″, long. $\frac{21}{7500}''$—$\frac{30}{7500}''$.

Syn.—*S. parvispora,* WOOD, Prodromus, Proc. Am. Philos. Soc. 1869, p. 139.

Hab.—In stagnis, Hibernia, Florida. (WM. CANBY)

Sterile joints 2–4 times longer than broad; fertile not tumid, 1–$2\frac{1}{2}$ times longer than broad; chlorophyl bands 4, narrow, nodose; turns many; zygospores very small, elliptical, $1\frac{1}{4}$–2 times longer than broad; cell wall not infolded at the end.

Remarks.—I am indebted to Mr. Wm. Canby for specimens of this species, which he collected in a pond in the Pine Barrens near Hibernia, St. John's River, Florida. It is remarkable for the comparatively small size of the spores, which do not nearly fill the perfectly cylindrical mother-cells; indeed they are only about as long as the latter are wide. This species closely resembles *S. majuscula*, but is larger, does not, that I have ever seen, vary like it in the number of spores, and is especially separated from it by the very small size of the latter.

Fig. 7, pl. 15, represents a fertile pair of filaments of this species magnified 125 diameters.

Sp. majuscula, KTZ.

Sp. pallide et sordide viridis, fructus tempore fuscescens; articulis sterilibus diametro (0.0022″—0.0025″) $2\frac{1}{2}$–4–10 plo longioribus; cytiodermate tenui homogeneo; fasciis 3–4–5 (rarius 7), modo subrectis longitudinalibus, modo laxissime spiralibus, nodosis; zygosporis globosis vel ovalibus. (R.)

Syn.—*S. majuscula,* KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 244.

Hab.—Prope Philadelphia. ?

Pale and sordid green, fuscescent at the time of fruiting; sterile joints $2\frac{1}{2}$–4–10 times longer than broad (.0022″—0 0025″); cytioderm thin, homogeneous; spiral filaments 3–4–5 (rarely 7) partly straightish and longitudinal, partly laxly spiral, nodose; zygospores globose or oval.

Remarks.—Shortly after I commenced to study the fresh-water algæ, I found below the city a fruiting *Spirogyra*, of which I preserved only a drawing, which I have since identified as apparently specifically one with the European *S. majuscula*, it differing only in not being quite so large; my measurement was $\frac{1}{500}'' = 0.002''$. Not having any specimens at hand, I have copied the description from the work of Prof. Rabenhorst.

Fig. 1, pl. 15, was copied from the drawing alluded to.

Sp. nitida, (DILLW.) LINK.

Sp. cæspitibus, lubricis, saturate viridibus; articulis sterilibus post divisionem diametro subæqualibus, ante divisionem 2–3 plo longioribus; articulis fertilis aliis simillibus, haud tumidis; fasciis spiralibus 4 (3–4 R.), modice latis, anfractibus 1–2; zygosporis ellipticis.

Diam.—0.0025″.

Syn.—*S. nitida,* (Dillwyn) Link. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 245.

Hab.—Prope Philadelphia.

Occurring in lubricous turfy masses, of a deep green color; sterile joints after division about as long as broad, before division 2–3 times longer; fertile joints similar to the others, not tumid; spiral filaments 4, moderately broad, turns 1–2; zygospores elliptic.

Remarks.—This species appears to be somewhat rare, at least I have found it but once, and then only in small quantity. Rabenhorst states that there are occasionally only three spirals, and his maximum diameter is 0.0031″; he also speaks of the fertile joints as "vix tumidis."

Sp. diluta, Wood.

Sp. articulis sterilibus diametro subæqualibus ad duplo longioribus, fructiferis haud tumidis; fasciis spiralibus 5, angustissimis, laxis, valde nodosis; anfractibus plerumque ½, interdum 1; zygosporis sparsis, late ellipticis vel ovatis aut globosis; cytiodermate modice tenue, in utroque fine nec protenso nec replicato.

Diam.—Artic. steril. $\frac{23}{7500}$″ = .003″.

Syn.—*S. diluta,* Wood, Prodromus, Proc. Am. Philos. Soc. 1869, p. 139.

Hab.—In stagnis, prope Philadelphia.

Sterile joints about as long as broad to twice longer, fertile cells not swollen; chlorophyl bands 5, exceedingly narrow, lax, strongly nodose; turns mostly ½, sometimes 1; zygospores few, broadly elliptical, ovate or globose; cell-wall moderately thin, not infolded at the ends.

Remarks.—I have found this species several successive seasons growing in the ditches in the Neck, below the city, especially in the neighborhood of the large stone barn, built by the great millionaire, and still known as "Girard's Barn." The spirals are very narrow and slender, and are moderately close to one another. They are chiefly made up of a number of chlorophyl nodules, the connecting thread between which is often very faint. In all the fruiting specimens, as I have seen them, the spores have been very few in number, most of the cells of the fertile filaments appearing to have aborted, so that they are simply empty. In most cases only about every third or fourth cell contained a spore.

Fig. 2, pl. 15, represents this species.

Sp. setiformis, (Roth) Ktz.

Sp. saturate viridis, lubrica; articulis sterilibus diametro paullum brevioribus ad 1½ plo longioribus; articulis fructiferis haud inflatis; fasciis 3–8, latis, dentatis, interdum nonnihil remotis, sed sæpe arcte et dense conjunctis, nodosis; zygosporis globosis vel late ovalibus.

Diam.—.0035″.

Syn.—*S. setiformis,* (Roth) Kutzing. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 246.

Hab.—In stagnis, prope Philadelphia.

Deep green, slippery; sterile joints a little shorter to one and a-half times longer than broad; fertile joints not inflated; spiral filaments 3–8, broad, dentate, sometimes somewhat remote, sometimes closely and densely conjoined, nodose; zygospores globose or broadly oval.

Remarks.—None of the descriptions which I have seen of this species state the number of the spiral filaments, but the other characters of the American form so agree with those of the European plant that it is probable that this one does also. The plant is not uncommon in the Neck, fruiting in the spring.

Fig. 3 *a*, pl. 15, represents part of a sterile filament of this species; 3 *b*, portion of a pair of fertile filaments, both magnified 125 diameters.

Sp. crassa, Ktz.

Sp. læte viridis, denique sordide viridis; articulis sterilibus diametro subæqualibus, post divisionem interdum fere ½ plo brevioribus, ante divisionem sæpe fere 2 plo longioribus; cytiodermate tenui, homogeneo, utroque fine nec protenso nec replicato; fasciis spiralibus 4, dentatis vel tuberculatis, sæpe arctis, subtransversis, tenuibus; anfractibus 1½–4; cellulis fructiferis aliis simillimis, haud inflatis; zygosporis globosis vel ellipticis.

Diam.—Max. .0065."

Syn.—*Sp. crassa,* Ktz. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 246.

Hab.—In stagnis, prope Philadelphia.

Bright green, but finally a dirty green; sterile articles about as long as broad, sometimes after division only half as long, sometimes before division twice as long; cytioderm thin, homogeneous, not infolded or produced at the ends; spiral filaments 4, dentate or tuberculate, often close, subtransverse, thin; turns from 1½ to 4; fertile cells very like the others, not inflated; zygospores globose or elliptical.

Remarks.—This species is very common in the neighborhood of this city, occurring in springs, &c., but especially in the ditches in the Neck. It forms long, lubricous masses, of a bright green color, readily distinguishable by the size of the filaments, which are separated with ease by the unaided eye. I have gathered it repeatedly, in fruit, from the middle of April to the middle of June. In this state the mass has lost its bright green color, and when the filaments are closely examined, even without a glass, minute dark points mark the positions of the spores.

Fig. 4 *a*, pl. 15, represents part of a filament commencing reproduction; 4 *b*, filaments which have matured the spores; 4 *c*, a pair of conjugating filaments.

Genus ZYGNEMA.

Cellulæ vegetativæ cylindricæ. Massa chlorophyllacea initio effusa et subhomogenea, postea distincte granulosa aut per cellulæ lumen distributa, granula amylacea duo centralia involvens, aut in corporibus duobus (in quaque cellula) plus minusve distincte stellatim radiantibus juxta nucleum centralum granum amylaceum unicum involventibus collocata. Conjugatio scalariformis vel lateralis.

Vegetative cells cylindrical. Chlorophyl masses in the beginning effused and subhomogeneous, afterwards distinctly granular, either distributed throughout the cavity of the cell, involving two central starch granules, or gathered together into two masses (in each cell), with more or less distinctly stellate radii and a central starch granule placed near the nucleus, one on each side of it.

Z. insigne, (Hassall) Ktz.

Z. cæspitibus et plerumque natantibus vel in aqua diffusis, saturate viridibus vel sæpe sordide flavo-viridibus; articulis sterilibus diametro circiter æqualibus vel duplo longioribus; conjugatione scalariforme (et sæpe simul laterali, R.); zygosporis globosis; sporodermate lævi.

Diam.—Cell. $\frac{19}{15000}''$ = .00126"; spor. $\frac{14}{15000}''$—$\frac{16}{15000}''$ = 0.00093"—0.00016".

Syn.—*Tyndaridea insignis*, Hassall, Fresh-Water Algæ, vol. i. p. 163.

Zygnema insigne, (Hassall) Kützing. Rabenhorst, Flora Europ. Algæ, Sect. III. p. 249.

Hab.—In stagnis, prope Philadelphia; Wood. Rhode Island; (S. T. Olney) Thwaites.

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Cæspitose and mostly floating or diffused in the water, deep green, or a dirty yellowish-green; sterile joints about as long as broad, or twice as long; conjugation scalariform (according to Rabenhorst sometimes at the same time lateral); zygospores globose; spore coat smooth.

Remarks.—This species is very common around Philadelphia, forming great masses in the ditches of the "Neck," growing in the semistagnant water along the railroads, and forming with other algæ slimy coatings on the dripping rocks of the Wissahicon and various railroad cuttings. At certain times the cells are found crowded with endochrome, at other times they are almost empty. At certain seasons this plant multiplies with great rapidity after a somewhat peculiar fashion. Constrictions first appear in the filament at the junctions of the cells, which thus look as though their ends were rounding off. This goes on until the ends of the cells are greatly rounded, and are attached simply by their central parts, which soon separate. In this way (fig. 8b, pl. xv.) the filament is resolved into its component cells, or more generally into as many pairs of cells as compose it, which when once set free in the water rapidly grow into filaments by the ordinary process of cell multiplication by division. In most cases the zygospores are placed in one of the parent-cells, but I have seen instances in which some of them were formed in the connecting tubes.

Fig. 8, pl. 15, represents this species.

Z. cruciatum,(VAUCH.) AG.

Z. pallide viride, siccatum fuscescens vel fusco-nigrescens; articulis sterilibus brevicylindricis diametro (0.0016″—0.00195″) æqualibus vel dimidio longioribus, rarius duplo longioribus, post divisionem factam haud raro dimidio brevioribus, fructiferis non tumidis; zygosporis plerumque globosis, maturis obscure fuscis, sporodermate subtiliter punctatis. (R.) *Species mihi ignota.*

Syn.—*Zygnema cruciatum,* (VAUCHER) AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 251.

Tyndaridea cruciata, HASSALL, Fresh-Water Algæ, vol. i. p. 160.—HARVEY. BAILEY, Microscopical Observations, p. 21.

Hab.—Northern States; Virginia; Florida; Bailey.

Pale green, when dried subfuscous or blackish fuscous. Sterile joints shortly cylindrical, equal or a little longer, or more rarely twice as long as broad (diam. 0.0016″—0.00195″), after division sometimes shorter than broad; fruiting cells not tumid; zygospores mostly globose; when mature, obscure fuscous, their coat minutely punctate.

Genus SIROGONIUM, KTZ.

"Cellulæ vegetativæ cylindricæ, sporiferæ subinflatæ orculiformæ. Fasciæ chlorophyllosæ longitudinales, parietales, leviter flexuosæ, nodosæ (plerumque 2–3, rarius 4 in quaque cellula), granula amylacea 7–8 involutæ. Copulatio genuflexa, sine tubo connexivo." R. In specie Americana fasciæ chlorophyllosæ spirales et Spirogyræ illis similes.

Vegetative cells cylindrical, spore bearing cells somewhat inflated, or orculiform. Chlorophyl filament longitudinal, parietal, somewhat flexuous, nodose (mostly 2–3 rarely 4 in each cell), containing 7–8 starch granules; conjugation genuflexuous, without any connecting tubes. (Rabenhorst). In American species the chlorophyl filament spiral and like to that of Spirogyra.

Remarks.—This genus was originally made by Kützing to contain a single species, which possesses the characters given in the diagnosis of Prof. Rabenhorst

I have met with an American plant, which has some of these characters, and at the same time others which have been supposed to belong to the genus *Spirogyra.* It unites the method of reproduction of *Sirogonium* and the arrangement of the chlorophyl band of *Spirogyra*, standing as it were midway between them. It is not midway, however, but much nearer *Sirogonium*, for the passage from a very loose spiral to a longitudinal flexuous filament is a brief one, and although in some cells of *S. retroversum* the spiral makes a number of turns; in other long cells it scarcely gets around once, in other words the chlorophyl band is nearly straight. On the other hand, the reproduction is strictly that of *S. strictum*, at least in all cases which have come under my notice. There is, therefore, but one of two things to be done, either to unite *Sirogonium* with *Spirogyra*, or else to give up the arrangement of the chlorophyl as an essential character of the former genus. The great variance, in the latter respect, in our American species, greatly weakens the value of any such character, and I have, therefore, preferred the latter of the two courses.

S. retroversum, WOOD.

S. articulis sterilibus diametro 7–15 plo longioribus; fasciis spiralibus 1, rare 2, latis, granulatis; anfractibus 1–9; articulis fertilibus valde tumidis, retroversis; conjugatione genuflexa et sine tubo connexivo; cytiodermate nonnihil crasso, utroque fine protenso vel replicato; sporis ellipticis.

Diam.—Art. steril. $\frac{11}{7500}''$ =.00146″; spor. lat. $\frac{10}{7500}''$—$\frac{12}{7500}''$=.00133″—.0016″; long. $\frac{25}{7500}''$ = 0033″.

Syn.—*S. retroversum,* WOOD, Prodromus, Proc. Am. Phil. Soc. 1869, p. 139

Hab.—In stagnis, prope Philadelphia.

Sterile joints 7–15 times longer than broad; chlorophyl band 1, rarely 2, broad, granulate; turns 1–9; fertile article very tumid, retroverted; fertile cells scarcely swollen; cell wall folded in at the ends; chlorophyl band single; turns 6; spores oblong or elliptical, spore wall very thick.

Remarks.—I have found this species growing in stagnant ditches in the Neck below the city. In fruit the cells are almost always very markedly bent backwards, and have a broad pouch-like dilatation in front. The spores are elliptical, and, as I have seen them, greenish and with a thin coat, but may not have been completely matured.

Fig. 1, pl. 16, represents this species.

Genus MESOCARPUS, HASSALL.

Cellulæ massa chlorophyllosa initio diffusa, postea in fasciam longitudinalem, haud raro flexuosam contracta; nucleum centralem et granum amylaceum unicum vel duo involvens. Zygospora globosa vel ovata, in tubo connexivo inter cellulas binas plus minus genuflexas formata.

Chlorophyl mass in the beginning diffused in the cell, afterwards contracted into an often flexuous fascia, and involving a central nucleus and one or more starch granules. Zygospore globose or ovate, formed in the connecting tube between two more or less bent cells.

M. scalaris, HASSALL.

M. cellulis sterilibus diametro 3–6 plo longioribus, fertilibus valde curvatis; zygosporis ovalibus.

Diam.—Max. $\frac{85}{7500}$ = .0011″.

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Syn.—*M. scalaris*, HASSALL, Fresh-Water Algæ, vol. i. p. 166, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 257.

Hab.—In fossis, prope Philadelphia.

Sterile cells 3–6 times longer than their diameter, fertile strongly curved; zygospores oval.

Remarks.—This species is abundant in the stagnant ditches near Camden. It agrees well with the descriptions of the European form. I have, however, never seen it in the state in which it has "fuscous spores." They have always been greenish, but very possibly were not fully matured.

Fig. 5, pl. 15, represents a pair of cells of this species just commencing to conjugate.

M. parvulus, HASSALL.

M. cellulis diametro (0.00031″—0.00041″) 5–12 plo longioribus; zygosporis globosis, plerumque 0.00062″ latis, sporodermate fusco lævi. (R.) *Species mihi ignota.*

Syn.—*M. parvulus*, HASSAL, Fresh-Water Algæ, vol. i. p. 169, et RABENHORST, Flora Europ. Algarum, Sect. III. p. 257.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Cells 5–12 times longer than their diameter (0.00031″—0.00041″); zygospores globose, mostly 0.00062″ broad, spore coat fuscous smooth.

Genus PLEUROCARPUS, A. BRAUN (1855).

Cellulæ eædem quæ in *Mesocarpo;* copulatio lateralis et sporifera, nonnunquam genuflexa et plerumque sterilis. (R.)

Cells like those in *Mesocarpus;* conjugation lateral and sporiferous, somewhat genuflexuous and mostly sterile.

P. mirabilis, BRAUN.

P. cellulis diametro (0 0011″—0.0013″) 2–5 plo longioribus; zygosporis subglobosis, fuscis, lævibus. (R.) *Species mihi ignota.*

Syn.—*Mougeotia genuflexa*, AGARDH. BAILEY, Silliman's Journal. New Series, vol. iii.
Pleurocarpus mirabilis, A. BRAUN. RABENHORST, Flora Europ. Algarum, Sect. III. p. 258.

Hab.—West Point, New York; Providence, Rhode Island; Detroit, Michigan; Fort Winnebago, Wisconsin; Bailey.

Cells 2–5 times longer than their diameter (0.0011″—0.0013″); zygospores subglobose, fuscous, smooth.

ORDER **Siphophyceæ.**

Algæ unicellulares. Cellula utriculiformis, plerumque ramulosa; ramuli vegetatione terminali prædíti, sæpe demum septo discreti, et alteri in oosporangia, alteri in antheridia transmutantur. Cytioplasma viride, granulosum, mucilaginosum, vesiculis chlorophyllosis et granulis amylaceis repletum. Propogatio fit aut cytiogenesi libera, aut zoogonidiis aut oosporis.

Unicellular algæ. Cells utriculiform, mostly branched; branches with a terminal vegetation, often finally cut off by a partition wall and transformed into antheridia or oosporangia. Cytioplasm green, granular, mucilaginous, filled with chlorophyl vesicles and starch granules. Propogation either by forming minute spores by free cell formation, or by zoospores, or by oospores.

FAMILY HYDROGASTREÆ.

Plantulæ minimæ, terrestres, gregariæ. Cellula initio globosa, postea clavato- vel pyriformi-intumescens, basi attenuata elongata et in ramulos subtillissimos hyalinos partita. Cytioplasma mucilaginosum, ætate provecta gonidia divisione simultanea transformatum. Cytioderma lamellosum ætate provecta dilabens et contabescens et gonidia liberans.

Plants very small, terrestrial, gregarious. Cells in the beginning globose, afterwards clavate or pyriform, with an elongated, attenuated base, divided into very fine, hyaline branches. Cytioplasm mucilaginous, at maturity transformed by a simultaneous division into gonidia. Cytioderm lamellate, at maturity wasting, withering away and setting free the gonidia.

Remarks.—The *Hydrogastreæ* are curious little unicellular plants, which grow upon wet earth. The matured frond is swollen up at one end to form a subglobular or pyriform head, whilst at the other end it is produced into a long, much-branched, very fine root-like portion which enters the earth and maintains the little plant in its upright position. The green endochrome is contained almost entirely in the head, and forms generally a coat or layer in the outer portion of its cavity, the inner part of which appears to be occupied by a watery fluid.

The only specimens which I have seen of this family were found growing in the mud left by the receding water of a recently drawn mill pond, by Dr. Billings, U. S. A. When I got them they were thoroughly dried up, and consequently no opportunity of studying their development was afforded. According to Kützing and Braun, the species is propagated ordinarily by the breaking up of the chlorophylous layer of protoplasm lining the wall of the cell into a larger number of very small globular spores. These, although not endued with the power of motion, seem from their method of formation and history to be homologous with zoospores. In most cases they are set free by the membrane of the parent-cell becoming gelatinously softened, swelling up, collapsing, and finally dissolving away. The little protococcoid cells then enlarging, develop at one end a hyaline prolongation which penetrates into the ground. Growth and development continuing the upper end of the cell swells up into the ovate or globular head, whilst the lower becomes the hyaline, branching, root-like portion of the new frond. No indication of this method of reproduction was discoverable in the plants which Dr. Billings sent me. The evident affinities of the family with the *Vaucheriaceæ* render it exceedingly probable that there is in it some method of sexual reproduction, as yet undiscovered, allied to that which occurs in the latter. In some of the specimens sent me, there were what appeared to be resting-spores (pl. XVI., fig. 2 *a*), occupying the whole of the cavity of the cell, from which they appeared to be finally discharged by a decay and rupture of the outer coat or wall. How these bodies were formed, and whether they really have power to reproduce the species I cannot tell.

Genus HYDROGASTRUM, DESV.

Character idem ac familiæ.
Characters that of the family.

H. granulatum, (LINN.) DESV.

H. plerumque gregarium, sæpe aggregatum, haud raro confluens; cellula e globoso-pyriformi, magnitudine seminis papaveris vel sinapios et ultra, prasino-viridi superficie pulverulenta. (R.)

Syn.—*Botrydium argillaceum*, WALLROTH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 265.

Hydrogastrum granulatum, (LINNÆUS) DESV. RABENHORST, *loc. cit.*

Hab.—Delaware; (Dr. Billings) Wood; West Point, New York; Providence and Newport, Rhode Island; Bailey.

Mostly gregarious, often aggregate, not rarely confluent; cells pyriform, of the size of a poppy or mustard seed and larger; pea-green; surface pulverulent.

Remarks.— The above description is taken from Rabenhorst's work, and applies to the specimens collected by Dr. Billings in the State of Delaware, excepting that I did not discover any of them to be confluent, nor was their surface distinctly pulverulent. Prof. Kützing gives as a comparative character between this and *H. Wallrothii*, the smaller size of the spores; but Prof. R. says nothing about this. There were no spores in any of the American specimens, and I think it somewhat uncertain whether or not the plant is or is not either of the European species. It is very probable that it will be discovered that the only true specific characters are sexual, and consequently have not as yet been made out in any of the forms. Certainly the descriptions of the species as at present given seem to me not to contain any reliable characters.

Fig. 2 *a*, pl. 16, represents a very young state of our American plant; fig. 2 is the perfected frond, both magnified ninety diameters; fig. 2 *a* shows what is supposably a perfected resting spore magnified 160 diameters.

FAMILY VAUCHERIACEÆ.

Algæ monoicæ, cæspitosæ, unicellulares. Cellula vegetiva (thallus) vegetatione terminali, utriculi formi-elongata et ampliata, prominentiis plus minus elongatis ramosa.

Propagatio aut sexualis, fit oosporis ope spermatozoidiorum fecundatis, aut non sexualis zoogonidiis. Fructificatio triplex (melius organa fructificationis tria):—

1. *Sporangium* terminale, ex thalli apice plerumque globoso-clavato-tumido formatum, septo discretum, cytioplasmate obscure viridi, demum in zoogonidium (zoosporam, Thur.) unicum permagnum, ciliis vibratoriis dense obsitum abeunte farctum.

2. *Oogonium* (oosporangium) laterale, sessile vel prominentia, plus minus elongata vel simplici vel partita pedicellatum, cytioplasmate ætate provecta in oosporam singulam transmutato fetum.

3. *Antheridium* laterale, sessile vel e ramuli lateralis parte suprema septo discreta formatum, in quo *spermatozoidea* (antherozoidea, Thur.) numeros issima nascuntur, denique erumpunt. Spermatozoidea oblonga, ciliis duobus inæquilongis, subpolo antico ortis instructa. (R.)

Monæcious algæ, cæspitose, unicellular. Vegetative cells (thallus) growing at the ends, elongate, utriculiform, and ampliate, more or less profusely branched.

Propogation either sexual, with oospores which are fecundated by spermatozoids, or non-sexual, by means of zoospores. Organs of fructification of three kinds:—

1. *Sporangia*, which are terminal and mostly formed from the separation of clavately swollen, globose apex of the thallus (often of a branch) by means of a partition; in the sporangium arises a single, very large zoospore, which is densely clothed with cilia.

2. *Oogonia* (oosporangia), lateral, sessile or pedicelate simple bodies, whose cytioplasm is finally converted into an oospore.

3. *Antheridium* lateral, sessile, or formed out of the end of a branch; the spermatozoids formed in them oblong, furnished with two unequal cilia, arising near the front end.

Remarks.—The *Vaucheriaceæ* are amongst our most common fresh-water algæ. They occur generally in the form of vast numbers of individuals interwoven into broad mats, which have often both a felty look and feel. When growth is going on rapidly, these mats are of a beautiful vivid green; but when the process of sexual reproduction has checked the life of the individual they become dingy and dirty looking. The thallus is composed of a single cell and is almost always branched. The branches never have, at least in any of our species, a definite arrangement, save only in that they always arise from the side and not from the point of the thallus. In the European species, *V. tuberosa*, however, the branches are said to arise both from the point and sides of the frond.

The frond cell is generally nearly uniform in diameter and has a thick outer wall, which is composed of cellulose, as is proven by the action upon it of iodine and sulphuric acid and of the iodo-chloride of zinc solution. Within the cell are chlorophyllous protoplasm, starch granules, watery fluid, and a few scattered raphides or inorganic crystals. There is never any nucleus. The protoplasm is often very granular, and is mostly collected in a thick green layer upon the inner surface of the cell wall, leaving the centre of the cell free for the more watery contents.

Growth, except in the very young fronds, consists exclusively in an increase in length, and takes place only at the ends of the thallus or in the portions near it. The branches are almost always simple, but are said in some species to give origin to secondary branchlets, and even, at times, to tertiary ones. They grow in the same manner as the main thallus, *i. e.* by additions to their ends.

When the thallus of a *Vaucheria* is ruptured by external injury, or, at times, when it is dying from some hidden cause, a number of bright green globes of various sizes are formed out of the endochrome. These appear to have the power of independent existence for some time, but whether or not they ever actually grow into new thalli I am unable to state.

M. Walz asserts that he has observed in certain species the formation of a quiet spore without the intervention of sexual organs, and that the process is as follows. The end of a long or short twig swells up, and the chlorophyl and protoplasm from the neighboring parts accumulate in the enlarged portion. A partition wall then forms at the base of the latter, which is thus changed into a closed chamber, a sporangium. The green contents then slowly gather themselves together into a denser and denser ball, becoming more and more separated, in so doing, from the wall of the sporangium, and finally secreting around themselves a distinct membrane. After the formation of a spore in this way, the sporangium opens at the apex and allows it to escape. The spore, after remaining quiet for some time in the water, at last germinates into a new frond, in a similar manner to an ordinary zoospore. In my earlier studies of fresh-water algæ, I noticed something very similar to this in one of our species, but convinced myself that the little body was nothing but a zoospore, whose normal development had been perverted by untoward influences, and therefore paid no more attention to the matter. It is probable that the life-history of the bodies observed by M. Walz is capable of the same explanation.

Although I have very frequently cultivated Vaucherias, I have never been so fortunate as to see them form their zoospores, nor indeed to see a zoospore in its motile state. The life-history of these bodies has, however, been fully and repeatedly worked out by other observers. It is described by such as occurring in the following manner. One end of a branch first enlarges into a bulbous, often conical, point, into which the neighboring endochrome crowds itself. This point is next divided off by a partition wall from the remainder of the thallus and constitutes the *zoosporangium*, the contents of which rapidly condense into one or two masses, generally oval in shape, each of which eventually forms a zoospore. When the latter are matured, the apex of the *zoosporangium* opens, and the little bodies within slowly and gradually emerge, without any apparent cause for their motion. Sometimes, according to Cohn, instead of this steady outward passage, there are repeated forward and backward movements of the zoospores within the case. The zoospore after its perfection is generally oval, and very large. Within it there are one or more vacuoles, and surrounding it is a layer of colorless protoplasm. It is remarkable for having its whole surface densely covered with short cilia. Its period of motile life appears to be very brief; according to Walz, that of the zoospore of *V. sericea*, Lyngb., lasts only from one-half to one and a half minute, after which time the cilia are lost and a cellulose wall secreted around the mass. Germination takes place by the growth of the cylindrical thread out from each end of the zoospore.

True sexual reproduction takes place in this family by means of *antheridia* and *oogonia*, male and female organs. All known species are mostly if not absolutely monæcious, both organs being contained in the one individual and always placed in proximity. All of the species in which the development and structure of the sexual organs have been studied, agree in the essential points.

The first appearance of the *antheridium* is as a little pouch projecting out from the side of the thallus. This increases in size and soon assumes the peculiar shape of the species. At the same time there is a diminution, according to M. Walz, of the chlorophyl in the antheridium, so that, when the partition wall forms and shuts off the cavity of the latter from that of the thallus, there are only a very few scattered green granules remaining. The antheridium at the time of separation contains, therefore, only transparent protoplasm, which soon becomes granular, and shortly afterwards exhibits the moving spermatozoids, which appear to be formed out of the thick layer of protoplasm that lines the inner surface of the cell wall. The point of the antheridium opens so soon as the spermatozoids are perfected, and allows them to escape.

The formation of the *oogonia* takes place very similarly to that of the antheridia. There is the same little protrusion from the side of the thallus in the commencement of the process, the same after-growth and increase of this pouch, and the same formation of a separating wall between it and the main body of the frond. A very marked difference, however, is to be found in the contents of the two, the oogonium from the very commencement being crowded with chlorophyl and oil globules. When the oosporangium is completed, the end of it opens, and, at the same time, the contents gather themselves into a dense protoplasmic ball, which lies in the

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centre. The spermatozoids, which are at this time already free in the water, are very minute, longish, ellipsoidal or ovate masses, provided with two unequal cilia. These commonly both arise together from one end of the body, and are directed in opposite directions—one backwards, the other forwards. According to M. Walz, however, in *V. sericea* the cilia arise from the opposite ends. According to De Bary, the spermatozoids of *V. aversa*, Hassall, contain reddish pigment-granules. M. Walz states that he has twice seen the process of impregnation in *V. sericea*, Lyngb., and describes it essentially as follows: After the bursting of the antheridium and the formation of the opening in the oogonium, the spermatozoid clustered around the little orifice in the latter, but were apparently debarred entrance by the presence of a glutinous jelly. After a time, however, one, and then another, forced a passage through this obstacle until finally a number gained access to the protoplasmic ball within. Over this they swarmed, pushing it and retiring and butting against it until some of them actually forced their way into it and were absorbed by it. Impregnation being now completed, the oospore acquired a very sharp definite outline, and secreted in a very short time a membrane around itself. The changes which followed during its maturing consisted of the acquiring of a thick coat and the replacing of the chlorophyl within by a reddish-brown coloring matter. The ripened resting spore of almost all the *Vaucheria* is provided with three coats, of which the middle is the thickest. The contents consist of protoplasm, reddish-brown pigment, and numerous oil globules.

Genus VAUCHERIA.

Genus unicum, character idem ac familiæ.

The only genus of the family, having the same characters.

V. sessilis, (VAUCH.) DE CANDOLLE.

V. laxe intricata, pallide et subsordide viridis; thallo capillari, parce ramoso; oogoniis 2–3 approximatis, rarius singulis, ovatis vel ovali-oblongis, plus minusve obliquis, rostratis; antheridio intermedio, ramuli modo brevi hamato, modo recto subulato, subclavato, modo elongato et incurvato, haud raro circinato sustentato; oosporis maturis fusco-punctatis, membrana triplici involutis. (R.)

Syn.—*V. sessilis,* (VAUCH.) DE CANDOLLE. RABENHORST, Flora Europ. Algar., Sect. III. p. 267.

V. cæspitosa, (VAUCH.) AGARDH. RABENHORST, *loc. cit.*

Hab.—Salem, North Carolina; Schweinitz. Common at West Point, New York; Waterville, Maine; Culpepper Co., Va.; Bailey.

Laxly intricate, pale and subsordid green; thallus capillary, sparsely branched; oogonia 2–3, approximate, rarely single, ovate or oval-oblong, more or less oblique, rostrate; antheridia intermediate, sustained upon branches partly shortly hamate, partly straight subulate, subclavate, partly elongate and incurved, and not rarely circinnate; oospores at maturity, fuscous-punctate, surrounded by a three-fold membrane.

Remark.—I think I found this species near Philadelphia in my earliest researches, but cannot speak certainly, having preserved neither notes nor specimens.

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V. velutina, Ag.

V. thallo repente, ramulis erectis, numerosis, fastigiatis, in cæspitem velutinum læte viridem intricatis; oogoniis lateralibus singulis, globosis, sessilibus, antheridio paulo longiore unico subulato leviter incurvato consociatis (R.) *Species mihi ignota.*

Diam.—Oogonii 0.0023″—0.0027″. (R.)

Syn.—*V. velutina,* Agardh. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 274.

Hab.—Salem, North Carolina; Schweinitz. Common at West Point, New York; Waterville, Maine; Culpepper Co., Va.; Bailey.

"Filaments exceedingly tough, interwoven into a dense, velvety, green stratum, pellucid below and creeping over the mud; branches near the extremity erect, fastigiate, and more or less crooked; vesicles solitary, globular, on short lateral peduncles." Carmichael.

V. geminata, (Vauch.) De Candolle.

V. obscure vel sordide viridis, in cæspites dense intricata; thallo capillari, tenaci, dichotomo; oogoniis duobus (rarius 1 vel 3), ovatis vel obovatis, oppositis, distincte pedunculatis, antheridio intermedio subulato, plus minus recurvo; oosporis maturis fusco-maculatis, sporodermate achroo e stratis tribus composito involutis; sporangiis in eodem vel proprio thallo, cyathiformi-ampliatis truncatis et angulato-cornutis. (R.)

Syn.—*V. geminata,* (Vauch.) De Candolle. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 269.

Hab.—In stagnis, prope Philadelphia; Wood.

Obscure or sordid green, densely interwoven into a turfy mass; thallus capillary, tenacious, dichotomous; oogonia two (rarely 1–3), ovate or obovate, opposite distinctly pedunculate, antheridia intermediate, subulate, more or less recurved; oospores at maturity spotted with fuscous, their coat transparent and composed of three strata; sporangia in the same or a separate thallus swollen cup-shaped, truncate and horned at the angles.

Remarks.—I have found this species in fruit but once, then it grew in a ditch below the city. Not having mounted any of it, nor having written a description of it at the time, I have been forced to simply copy that of Prof. Rabenhorst.

V. polymorpha, Wood.

V. in cæspites dense intricata; thallo capillari, tenui; antheridiis corniculatis ex ramuli lateralis apice formatis; ramulis fertilibus interdum et oogoniis et antheridiis instructis, interdum antheridiis solum; oogoniis plerumque geminis, interdum singulis, globosis vel ovatis, sæpe breve rostratis, plerumque distincte pedunculatis sed rarius sessilibus; oosporis enormiter subglobosis vel ovatis; sporodermate achroo, e stratis duobus composito.

Syn.—*V. polymorpha.* Wood, Prodromus, Proceedings Amer. Philos. Society, 1869, p. 140.

Hab.—In aquis, prope "Buffalo Bayou," Texas; (Ravenel.)

Cæspitose; thallus hair-like, thin; antheridia corniculate, formed of the apex of lateral branches; fertile branches sometimes furnished both with oogonia and antheridia, sometimes with antheridia alone; oogonia sometimes single but mostly in pairs, occasionally shortly rostrate, generally distinctly pedunculate but sometimes sessile; oospores irregularly subglobose or ovate, surrounded by a transparent double spore coat.

Remarks.—This species was collected by Prof. Ravenel near the city of Houston, Texas. As I received the mass, it was labelled as being obtained from "a shallow slimy pool formed by drippings from the side of a ravine near Buffalo Bayou." The species probably grows in the water, evidently forming turfy mats. It is

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remarkable from the fact that, whilst in many cases the little branches which produce the antheridia give origin to the spores also, in others they do not; so that there are numerous antheridia, which are unconnected with any female organs. When a branch does produce both of the reproductive organs it usually forks into three short branchlets, thus giving origin to a pair of sporangia and a single curved, hooked antheridia. Sometimes, however, there is but a single female branchlet, and I have even seen a sporangium, immediately sessile upon a branch, which at its apex gave origin to a male organ. In the coat of the perfected spore, I have not been able to find more than two distinct strata.

Figs. 3 and 3 *a*, pl. 20, represent sporangia and antheridia of this species; 3 *b*, a simple, young and only partly formed antheridia, magnified 160 diameters; 3 *c*, a perfected spore magnified 260 diameters.

V. sericea, LYNGBYE.

V. aquatica vel terrestris, cæspitosa, vel sordide vel læte vel luteolo-viridis; thallis tenuibus, dense intricatis, laxe et vage ramosis, ramisque sæpe adscendentibus vel erectis; oogoniis sessilibus vel brevissime pedicellatis, 1–6 seriatis, unilateralibus, oblique et enormiter ovalibus, ore laterali producto rostellatis; antheridiis in thallo ipso juxta oogoniis sessilibus, cylindraceo-subclavatis, deflexis; spermatozoideis oblongis, puncto rubro notatis (teste de Bary), in utroque polo cilio unico præditis.

Syn.—*V. aversa*, HASSALL, Fresh-Water Algæ, p. 54.
V. sericea, LYNGBYE. RABENHORST, Flora Europ. Algarum, Sect. III. p. 271.

Hab.—Prope Philadelphia; Wood.

Aquatic or terrestrial, occurring in turfy mats of a yellowish, dirty, or bright green color; fronds thin, densely intricate, laxly and vaguely branched, often together with the branches ascending or erect; oogonia sessile or very shortly pedicellate, 1–6 seriate, unilateral, obliquely irregularly oval, their lateral mouths produced into a rostellum or beak; antheridia sessile upon the thallus itself near the oogonium, somewhat cylindrical, subclavate, deflexed especially in age; spermatozoids (according to De Bary) oblong, marked with a red point and furnished with a single cilia at each end.

Remarks.—I can perceive no constant differences between *V. sericea*, Lyng. and *V. aversa*, Hass. The extreme forms differ somewhat, but both are very common about Philadelphia, and everywhere grade into one another. Prof. Rabenhorst thinks that the two forms are scarcely distinct, and states that the most characteristic differences are, that in *V. aversa*, the thallus is much thicker, and the oogonia larger and more erect, whilst the oospores are smaller and consequently do not fill the cavity of their case. These differences are, except the last, simply differences in size, and seem to me to depend simply upon circumstances of growth. The relatively smaller size of the spore is a very frail hook indeed to hang a species upon.

The plant grows in springs and actively running water abundantly in this neighborhood; also on very wet ground, especially on that which is habitually overflowed, such as the face of dams, neighborhood of springs, &c. In the water, it is frequently on the ground, but also often clothes such objects as stones, largish sticks, &c.

ORDER **Nematophyceæ.**

Algæ multicelullares, chlorophyllosæ, membranaceæ vel filamentosæ, ramificatione aut instructæ aut destitutæ. Propogatio fit aut oosporis aut zoogonidiis, sed nunquam conjugatione.

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Multicellular, chlorophyllous algæ, membranaceous or filamentous, furnished with or destitute of branches. Propagated by oospores or zoospores, never by conjugation.

Family ULVACEÆ.

Thallus membranaceus vel foliaceus, vel filiformis (Schizomeris?) rarius crustaceus, e cellularum strato unico formatus, aut expansus aut tubuloso- vel vesiculoso-concretus.

Propogatio fit zoogonidiis, cytioplasmatis divisione repetita ortis. Zoogonidia oblonga, polo antico ciliis vel binis vel ternis vel quarternis instructa.

Thallus membranous or foliaceous, rarely crustaceous, composed of a single stratum of cells, either expanded or tubularly or vesicularly concreted.

Propagation by means of zoogonidia, formed by the repeated division of the cytioplasm. Zoogonidia oblong, furnished with two, three, or four cilia at the anterior end.

Genus PROTODERMA, Ktz.

Thallus crustaceus, indeterminatus, substrato arcte adhærens, e cellulis anguloso-rotundatis, irregulariter ordinatis, arcte connexis compositus.

Propagatio ignota.

Thallus crustaceous, indeterminate, closely adherent to the substratum, composed of closely conjoined irregularly arranged angularly rounded cells.

Propagation unknown.

P. viride, Ktz.

P. viride, lubricum.

Syn.—*P. viride,* Kutzing. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 307.

Hab.—In aquario; Wood.

Green; slippery.

Remark.—I have seen a plant, which I take to be this species, growing on the glass and on pebbles in the aquarium of my friend, Dr. Frické.

Genus ULVA, Linn.

Thallus membranaceus, plane expansus, angustus vel latus, nonnunquam latissimus, magis minusve undulato- crispatus, sæpe laciniatus, haud raro perforatus, e cellularum strato unico formatus, callo disciformi parvo affixus, ætate provecta sæpe libere natans. Cellulæ anguloso-rotundatæ, cœloplas maticæ, parenchymatice connexæ.

Vegetatio cellularum divisione in duas directiones repetitia. Propogatio fit zoogonidiis, in cellulis quibusdam cytioplasmatis divisione 4, 8–16 ortis, ciliis vibratoriis quaternis longitudine corporis longitudinem vix superantibus instructis.

Thallus membranous, expanded, narrow or broad, sometimes very broad, more or less undulately curled or crisped, often laciniate, not rarely perforate, formed of a single stratum of cells, fastened by a small discoid thickened portion, in advanced age often swimming free. Cells angularly globose, joined into a sort of parenchyma.

Growth occurring by the repeated division of the cells in two directions. Propagation by zoospores, 4–8–16 of which are formed at once by a division of the endochrome of certain cells, and are furnished with four vibratile cilia scarcely longer than the body.

U. merismopedioides, Wood.

U. ampla, membranacea, late expansa, dilute viridis, tenuis, radiatim et enormiter plicata, ambitu sæpe subrotundata; margine undulato, interdum subcrenato; cellulis enormiter ovalibus vel angularibus, nucleo destitutis, quarternariis et in familias *Merismopediarum* modo obscure associatis.

Diam.—Cell. max. $\frac{5}{12000}''$ = .00041′, plerumque $\frac{2}{12000}''$—$\frac{3}{12000}''$ = .00016′ — .00025.

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Syn.—*U. merismopedioides*, WOOD, Botanical Report of the United States Geological Exploration of the Fortieth Parallel, p. 415.

Hab.—In torrentibus, Diamond Range (alt. 6000 ft.), Rocky Mountains; (Sereno Watson) Wood.

Thallus ample, broadly expanded, membranaceous, dilute green, thin, radiately and irregularly plicate with its outline often somewhat rounded; its margin undulate or at times almost crenate; the cells irregularly oval or angular, destitute of nucleus, quarternary and obscurely arranged in families after the manner of a merismopedia.

Remarks.—The largest fronds of this species that have come under my notice are about three inches long by two broad, thin, easily torn, and not all gelatinous. The portion by which they have been attached is very evident, near one of the margins, and from it broad undulations or folds radiate. Sometimes the frond is split up into palmate, lobe-like parts.

The cells are not closely approximate, but are placed in a homogeneous translucent membrane, in such a way as to remind one of a *Merismopedia.*

I do not feel certain that this plant is distinct from *U. orbiculata* of Rabenhorst, though for the present I have preferred so to consider it. His description is very brief and incomplete, as is also the original one of Thuret, which I have consulted. Prof. R., however, gives *U. latissima* of authors as a synonym of *U. orbiculata*, and certainly this plant is distinct from *U. latissima*, Harvey, of our coast. Again it seems impossible that a plant growing near the summit of the Rocky Mountains should be identical with one found on the coast of France. Prof. Sereno Watson found this plant growing on rocks in a mountain stream of the Diamond Range, at an altitude of 6000 feet.

Genus ENTEROMORPHA, LINK.

Thallus membranaceus, tubulosus vel utriculiformis, basi affixus (saltem initio, postea sæpe libere natans), e cellularum strato unico compositus, sæpe ramosus, haud raro ramosissimus. Propogatio fit zoogonidiis. Hæc zoogonidia proceantur in cellulis quibusdam 8–16 cytioplasmatis divisione repetita, in polo antico rostriformi ciliis duobus corpus duplo superantibus prædita. (R.)

Thallus membranaceous, tubular or bladder-shaped, affixed by the base (at least in the beginning, often afterwards floating freely), composed of a single stratum of cells, often branched, not rarely very much branched. Propagation by means of zoospores, 8–16 of which are formed by the repeated division of the protoplasm of a cell. Their anterior beak-like portion provided with two cilia whose length is not less than twice that of the body.

E. intestinalis, (LINN.) LINK.

E. teres, forma et magnitudine admodum varia, sæpe pedalis etiam supra, leptoderma, saturate vel pallide viridis, filiformis vel intestiniformis, plana vel bullosa; cellulis 3–5–6 angularibus. (R.) *Species mihi ignota.*

Diam.—0.00048″ — 0.0008″. (R.)

Syn.—*E. intestinalis*, (LINNÆUS) BAILEY, Silliman's Journal, N. S., Vol. III., et RABENHORST, Flora Europ. Algarum, Sect. III. p. 312.

Hab—Hudson River, from Newburgh to New York City; Narragansett Bay, Rhode Island; Bailey.

Terete, very various in size and shape, often a foot or more in length, smooth, deep or pale green, filiform or intestiniform, plain or bullose; cells 3–5–6 angular; their diameter 0.00048′ — 0.0008″.

Genus SCHIZOMERIS, Ktz.?

Thallus filliformis, cylindricus, hic illic valde contractus, basi attenuata affixus. Vegetatio fit cellularum divisione initio in duas postea in tres (?) directionem. Propogatio fit zoogonidiis. Zoogonidia in thalli juvenis cellulis orta, ovata, polo antico ciliis tribus instructa.

Thallus filiform, cylindrical, here or there strongly contracted, adnate by the strongly contracted base. Growth in the beginning by the division of the cells in two directions, afterwards in three directions. Zoogonidia formed in the cells of the young thallus, ovate, their anterior end furnished with three cilia.

Remarks.—The plant from which the above generic description has been drawn up grows abundantly in our ditches below the city. Whether it really belongs to the genus *Schizomeris* or is the representative of a new group is somewhat uncertain. I have never seen the European plant, but, if I understand the descriptions of it, the cells in it are all arranged in a single plane. This certainly is not the case in the old plants of our North American form, for in them the cells are so placed as to make a thick opaque filament, the outside of which everywhere presents the outer walls of cells. The life history of the European species has not been at all worked out, and I have refrained from actually indicating a new genus, in the absence of absolute knowledge upon the subject, because the specific characters of the two plants are so much alike.

I have had some opportunities for studying the life history of our American plant. The zoöspore (Fig. 1 *c*. pl. XVII.) is of the ordinary conical or ovate form, with a very decided transparent anterior end, from which arise three cilia. As the number three is a rare one for cilia to exhibit, I have examined several zoospores with care, and am very certain that they had no more or less. It is, therefore, probable that the number is fixed for the species, although just possible that my finding several individuals in agreement was accidental. The zoospore after a period of free life, during which its motion is very active, becomes quiescent, and, its cilia withering away, attaches itself by its smaller end to some twig, stone, or other support. At the same time it appears to change its shape somewhat, growing longer and narrower, and the smaller end spreading out to form a little foot. Simultaneously with these changes the young plant acquires a cellulose coat, and so becomes a perfect cell, in which I have never been able to detect any nucleus. After a while the cell thus formed divides transversely into two, which, of course, lay end to end. Each of these cells then grows until it attains a certain size, and then the transverse division is repeated. In this way the process goes on until finally a long filament is produced, which is composed of but a single series of cells. These cells are much broader than long, and are placed end to end, so that the cylindrical frond is made up as it were of disks laid one upon the other. When the filament has in this way reached a certain stage of development, one of two things occurs, either the cells begin to divide at right angles to the plane of their previous division, or else the production of zoospores takes place. In the first instance each cell divides into two, four, or more cells. This division, I believe, occurs in three if not all directions, so that each original cell is represented by a number of cells, and a sort of compound filament arises, out of which the matured large trichoma is formed by a continuation of growth, and, perhaps, by a repetition of the division. I have never been able to discover that any reproductive process whatever takes place in this compound filament, and am very confident it never produces zoospores. It is very possible, however, that it may in some way give origin to resting spores, although, as above stated, no indication of this has ever come under my notice. The zoospores are formed in the young fronds as follows: The endochrome in the cell concerned gradually separates in the ordinary manner into several distinct masses, which soon assume a more or less irregularly globular or pyriform shape. Whether the number of these masses is fixed for the single cell or not I am unable to state. These changes occur almost simultaneously in a number of consecutive cells, commencing with the most distal and rapidly spreading towards the base of the filament. When they are pretty well advanced, the walls of the cells undergo some alteration, probably a gummy degeneration, whereby they become soluble in the water. As the division of the endochrome occurs first in the most distal cells of the filament, so does also this change in the cellulose coat. When the endochrome masses are well shapen and distinct, they begin to exhibit motion, becoming uneasy, restless, changing their position, rolling on themselves, and pushing against one another. At the same time solution of the cell walls commences, the partitions between the cells disappearing, and the outer walls spreading. These changes go rapidly forward, and in a little while the zoospores stream out from the fading end of the frond, jostling and crowding as though eager to enter upon their new life.

Fig. 1 *a*, pl. 17, represents the basal portion of an old filament which has failed to form zoospores, magnified 125 diameters. Fig 1 *b* was drawn from a young filament during the process of forming zoospores; owing to their rapid motion, the cilia of the latter could not be seen. This figure is enlarged 250 diameters. Fig. 1 *c* represents a zoospore which has just become quiescent, and still retains its cilia, although they have lost their motile power. Fig. 1 *d*, *e*, *c*, represent the very young plant in different stages of growth. They are all magnified 450 diameters.

S. Leibleinii, KTZ. ?

S. læte viridis vel saturate nigro-viridis.

Diam.—Max. $\frac{1}{125}''$. = .08″.

Syn.—*S. Leibleinii*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 311.

Hab.—In fossis, prope Philadelphia.

Bright green to deep blackish-green; largest diameter of the frond $\frac{1}{125}''$.

Remarks.—Owing to the profusion of zoospores produced by a single filament at one time, it is very usual to find large numbers of the younger plants attached so closely to some central body as to form dense masses of a beautiful green color. The support of these small masses is often entirely concealed, and I have frequently seen them moving freely about the jar, without any apparent cause, until the mystery was solved by finding that some unfortunate snail carried the forest on his back.

The oldest filaments are perfectly opaque, showing, under the microscope, by transmitted light, no trace of their structure.

The species is exceedingly common in the later summer and early fall months in the ditches and sluggish streams around the city, especially in the Neck.

FAMILY CONFERVACEÆ.

Fila articulata aut simplicia aut ramosa, vegetatione terminali non limitata instructa. Articuli plerumque plus minusve elongati, sed nonnunquam diametro breviores, cylindrici, rarius tumidi. Cytioderma plerumque manifesto lamellosum. Massa chlorophyllosa granulata, vesiculas amylaceas involvens, parietalis vel in ætate provecta sæpe in cellulæ centro contracta.

Vegetatio fit utriculi primordialis divisione semper in unam eandemque (transversam) directionem repetitia. Propagatio fit zoogonidiis.

Filaments articulate, simple or branched, growth terminal, unlimited. Joints mostly more or less elongated, but sometimes shorter than long, cylindrical rarely tumid. Cytioderm mostly plainly lamellate, chlorophyl masses granular, surrounding fine starch granules, parietal or often in the centre of the cell.

Growth taking place by division of the primordial utricle always in one direction, namely transversely. Propagation by means of zoospores.

Genus CONFERVA, (LINN.) LINK.

"Fila articulata simplicia. Articuli cylindrici. Massa chlorophyllosa homogenea vel granulata, vesiculas amylaceas involvens. Propogatio ignota." (R.)

Threads articulate simple. Articles cylindrical. Chlorophyl mass homogeneous or granulate, including amylaceous vesicles.

Remarks.—A large number of forms of the genus *Conferva* have been described as distinct species by Kützing and other authors. The characters assigned to these species, however, do not seem to me in any way distinctive. I cannot believe it possible at present to recognize, define, and describe species in this genus, and believe that further studies must be made in their life-history, and other characters discovered before the different forms can be separated. Probably, as was the case with the *Œdogoniaceæ*, when their sexual life is made out, in it will be found the vital differences. No doubt there are many species common to Europe and America, but I have been entirely unable to determine them. Among the very earliest of my observations upon the fresh-water algæ, before experience had taught how and what to observe, was one made upon what I suppose was a species of this genus. I have never met with the plant since, but as the observation has direct bearing upon the method of propagation, I mention it here, imperfect as it unfortunately is. The plant was found growing on the mud along the Schuylkill River, near Gray's Ferry Bridge, below the city. The filaments were simple, of great length, and uniform in diameter; fig. 7 *a*, pl. 18, represents a portion of one magnified 500 diameters. The cells varied from about as long as broad to three times as long. The amount of endochrome in the cells also varied very much. In most of them, it was not nearly sufficient to fill the cavity, and was arranged as a central superficial band. Many of the cells were seen engaged in the production of zoospores. (Fig. 7 *b*, pl. 18.) Such were well filled with endochrome, which gradually condensed itself into a globular or pyriform mass in the centre of the cell. This, after a short time, began to exhibit activity, rolling upon itself and finally pushing about as much as its confined quarters would allow, until at last it

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escaped into the water, through the cell wall. Each cell in this way gave origin to a single zoospore. The walls did not melt away in the water, and, as a number of consecutive cells underwent these changes at the same time, the filament or a portion of it was left as an empty shell. The zoospores were of the usual shape, with a bright anterior spot or beak. The number of cilia was not noted. After a time they settled down generally in clusters, attaching themselves to some foreign particle, dropping their cilia and acquiring a cellulose wall. (Fig. 7 *e*, pl. 18.) They then elongated, underwent the ordinary cell division in a transverse direction, and, by the repetition of this, gradually grew into filaments similar to that from which they sprang.

Fig. 7 *d*, pl 18, represents a young filament just formed in this manner, magnified 500 diameters.

Genus CLADOPHORA, KTZ. (1843.)

Fila cellularum serie simplici formata, varie ramosa. Rami filo centrali similes. Cytioderma plerumque crassum, lamellosum. Cytioplasma parietale.

Filaments composed of a simple series of cells and variously branched. Cytioderm mostly thick and lamellate. Cytioplasm parietal.

Remarks.—The *Cladophora* are branched plants of rather rigid habits, which grow both in salt and fresh water. They are readily recognizable by their comparatively still appearance, the absence of gelatinous matter about them, and by the want of regularity in their branching. A large number of species have been described, most of which are marine. They are exceedingly difficult to define, and it is very possible that their hitherto undiscovered sexual reproduction may be finally found to afford the only true characters. I have identified two European forms as growing near this city, and a third has been recognized by Prof. Harvey, as found in our northern States.

I have never seen the production of zoospores in this family, but they are said to be formed by the simultaneous division of the layer of chlorophyllous protoplasm, which fills the outer part of the cell cavity. They exhibit the power of very active motion even before their exit from the cell, which occurs through a papilloid orifice, mostly at the end of the cell, sometimes in its side. Their cilia are sometimes two, sometimes four in number, and their life-history appears to be precisely similar to that of other zoospores.

Cl. glomerata, (LINN.)

Ramuli fili primarii in parte superiore atque ramorum ordinis secundi et tertii plerumque fasciculato- vel penicilliformi-aggregati. Cellulæ maximæ vegetæ cytioplasmate cellularum parieti retiformi- vel subspiraliter applicato. Cellulæ fructiferæ semper terminales, inferiores semper steriles videntur. (R.)

Syn.—*Cl. glomerata,* (KÜTZING) RABENHORST, Flora Europ. Algarum, Sect. III. p. 337.

Hab.—Lake Ontario; Pickering. Falls of Niagara; Lakes Erie, Huron, and Michigan; Fourth Lake, near Madison, Wisconsin; Bailey.

"Filaments tufted, bushy, somewhat rigid, much branched, bright grass-green; branches crowded, irregular, erecto-patent, repeatedly divided; ultimate ramuli secund, subfasciculate; articulations 4–8 times as long as broad."

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Remarks.—Prof. Harvey says (Smithsonian Contributions): "I have received North American specimens from Milton, Saratoga County, N. Y., and from Lake Erie; also from the Mexican Boundary Surveying Expedition."

Cl. fracta, Dillw.

Clad. prima juventute affixa sed postea libere natans et cæspites formans; ramis ramulisque sparsis, divaricatis, nonnunquam refractis; ramulorum cytioplasmate non spiraliter ordinato; cytiodermate sæpe crassissimo; cellulis fertilibus haud terminalibus, plerumque in ramulorum medio, aut eorum basi.

Syn.—*Cl. fracta,* (Dillw.) Rabenhorst, Flora Europ. Algarum, Sect. III. p. 334.

Hab.—In flumine Schuylkill, prope Philadelphia; Wood. West Point, New York; Providence, Rhode Island; Bailey.

In the young state fixed, but afterwards floating free and forming matted masses; branches and branchlets scattered, divaricate, somewhat refracted; cytioplasm of the branches not spirally arranged; cytioderm often very thick; fertile cells not terminal, mostly in the middle of the branches, sometimes in their base.

Cl. brachystelecha, Rabenhorst.

C. per totam vitam innata, obscure viridis, sicca pallida, pygmæa, 2–4, rarius 6 linea longa, ramosissima, intricata, plerumque culmigena; ramis primariis $\frac{1}{30}'''$—$\frac{1}{40}'''$ = 0.00295″—0.0022″ crassis, ramulis ultimis $\frac{1}{60}'''$—$\frac{1}{70}'''$ = 0.00147″—0.00128″ crassis; articulis diametro 4–12 plo longioribus; cytiodermate subcrasso, hyalino, subtiliter plicato-striato; cytioplasmate imprimis cellularum superiarum laxe spiraliter ordinato. (R.)

Syn.—*Cl. brachystelecha,* Rabenhorst, Flora Europ. Algarum, Sect. III. p. 343.

Hab.—Prope Philadelphia; Wood.

Fixed through the whole life, obscure green, pale when dried, dwarfish, 2–4, rarely 6 lines long, very much branched, intricate, mostly attached to culms; primary branches 0.00295″—0.0022″ thick, ultimate ramuli 0.00147″—0.00128″ thick; articles 4–12 times longer than thick; cytioderm thickish, hyaline, subtilely plicately striate; cytioplasm, especially of the upper cells, laxly spirally arranged.

Remarks.—I have notes of having identified this species at some time, but, having kept neither specimens nor detailed memoranda, have simply copied the description of Prof. Rabenhorst.

Family OEDOGONIACEÆ.

Algæ monoicæ vel dioicæ. Fila articulata aut simplicia aut ramosa, cellula basali obovato-clavata, basi plerumque lobato-partitia vel scutata innata. Propagatio fit tum zoogonidiis tum oosporis fecundatione sexuali ortis. Zoogonidia formantur singula in quavis cellula, forma late ovali vel globosa, polo antico achroo corona ciliorum vibratoriorum prædita.

Oogonia singula vel plura (2–5) continua, plus minusve tumida, in quoque oospora singula, matura rubro- aut flavo-fusco-colorata, ante germinationem in zoosporas plerumque quatuor dilabens se format.

Antheridia brevi-filiformia, 1–2–3–10–articulata, plerumque singula aut oogonio aut filo vegeto insidentia aut in individuis variis sæpe cellula obovato-clavata subtentata.

Monæcious or diæcious algæ. Filaments articulate, either simple or branched, fixed by the basal cell which is obovate-clavate, mostly with its base lobately parted or shield shaped.

Propagation sometimes by zoospores, sometimes by resting spores, the result of sexual impregnation. Zoospores formed simply in certain cells, broadly oval or globose, their anterior end transparent, and furnished with a crown of vibratile cilia. Resting spores single or in series of from two to five, more or less tumid, single in each sporangium, at maturity reddish or yellowish fuscous, before germination dividing themselves into (mostly four) zoospores.

Antheridia shortly filiform, 1–2–3–10 articulate, mostly single, either upon the sporangium or vegetation cell.

Remarks.—The *Œdogoniaceæ* have been by previous writers simply divided into two genera, *Œdogonium* and *Bulbochæte.* The plants represented by these two divisions have certainly many characters in common, as in the production of their zoospores and spermatozoids as well as in their peculiar method of cell division. Yet they are so very diverse in some particulars in regard to the latter, as well as in their habit of growth and in the formation of their sporangia, that it has seemed to me that the differences between them were more than sufficient to characterize merely genera, and that to each of these groups should be awarded the rank of a sub-family.

Again, in the old genus of *Œdogonium*, we have very distinct groups, separated by differences in the most important of all the characteristic portions of the plant—the sexual apparatus. These groups are the so-called *Monœcious*, *Gynandrous*, and *Diœcious Œdogonia;* the *monœcious* division comprising those plants in which one individual gives origin both to the female and male germs; the *gynandrous*, those species in which the plant that produces the female germ gives origin also to a peculiar zoospore, the so-called *androspore*, which, after a period of motile life, settles down and develops a dwarf plant, the *andrœcium*, in which the *spermatozoids* are developed; and the *diœcious* group containing species in which the male and female plants are distinct individuals. Dr. Pringsheim states (*Morphologie der Œdogon.*, p. 43) that these groups pass into one another, but in my opinion, by his own showing, they are sharply distinct. The nearest approach to such passage is between the first and second groups, and consists simply in the fact that in certain species the androspore when it settles down develops into a one-celled instead of a two or three-celled antheridium. This to me does not seem to indicate a union of the groups, for the essential difference is not in the form or complexity of the antheridium, but in the circumstance that in the one case the female filament develops a spermatozoid capable of fertilizing the germ, whilst in the other it gives rise to a body which does not possess that power at all, but does have the capability of giving origin to a second plant, in which the spermatozoid is developed. The groups, therefore, appear to be sharply and distinctly definable.

In the *Bulbochætiæ* but a single genus has as yet been discovered, and this is distinctly gynandrous, but it seems probable that hereafter other plants of this subfamily will be found which are monæcious or diæcious, so that we will have in the two subfamilies two parallel groups of genera.

For the reasons above indicated I have ventured to divide the family into two subfamilies, the one comprising three, the other a single genus. The peculiarities of growth, production of zoospores, and sexual development will be found described under the particular subfamilies.

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Subfamily ŒDOGONIEÆ.

Filamenta stricta, haud ramosa, sine setis veris, sed sæpe apice setiforma, elongata, hyalina.

Filaments simple, not ramose, without true seta, but often with their apex seta-like, elongate, hyaline.

Remarks.—The *Œdogoniaceæ* are small filamentous plants, whose size is sufficient to render them visible to the unaided eye, and yet not sufficient to make each individual distinctly apparent. They grow mostly in quiet water, attached to almost any and every thing that can afford a foothold, fringing with apparent indifference stones, twigs, sticks, dead leaves, bits of glass, boards, etc. I have seen such masses of them crowding the whole surface of a *physa* as to entirely conceal the animal and its shell, and present the curious spectacle of a perambulating, waving forest of bright green. The individual filament is composed of cylindrical cells, which are always without a nucleus, and have their chlorophyl diffused instead of being collected into bands or stripes. The walls are mostly quite thick and marked near the distal end with circular striæ, whose numbers bear relation to the edge of the cell, for these striæ are the results of the peculiar method of cell multiplication by division, each one marking one such division. When an œdogonium cell has attained sufficient maturity and is about to divide, the first perceptible change is the appearance of a little circular line or streak near its distal end. About the same time and in the same place a fine partition is formed by an outgrowth from the primordial utricle, a probably double delicate wall of condensed protoplasm separating the upper end of the parent cell from the lower or main portion. The upper end now begins to develop into a new cell. This development takes place by the formation of an entirely new layer of cellulose inside the little cell, *i. e.* between the new primordial utricle and the old cell wall, and afterwards by the lengthening of this layer by interstitial deposit in the usual way; the thick wall of the parent cell in no way directly participates in the growth (fig. 2 *b*, pl. 17). It is evident that as the new wall grows the old cell wall must be as it were raised up upon it, borne away as a little capping from the basal portion of the parent cell. Consequently when a young cell is watched during this process the little line-like incisure of the parent cell is seen to widen until it becomes an evident trench, and this trench grows wider and wider, until at last it is so broad as to be no longer a trench, and the little end of the parent cell simply caps its offspring. When the latter has fulfilled its allotted period oı growth, the process is repeated, the line of separation appearing this time just below the edge of the first cap. It is plain that the second new cell when formed must have a double cap crowning its extremity. At each repetition a new layer is added to the thickening cap, until at last it may be composed of six distinct layers, each projecting just beyond the next older one. Under the microscope the increased thickening of the distal end of a cell bearing such a crown-piece is not sufficiently evident to at first attract attention, whilst each edge of a layer appears as a stria. It is plain that the number of these striæ represents the number of

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times division has occurred; if there be four striæ, four times; six striæ, six times, &c.

Besides this method of development, in many species new cells are formed by a sort of pullulation, occuring in the end cell of the filament. The primordial utricle appears to rupture the wall of the distal extremity of the latter and grow out into a little pullulation, or teat, which very soon becomes separated from the parent cell, by the reformation, as it were, of the end wall of the latter. The new little cell thus formed coats itself with cellulose, and rapidly grows, especially in length, always, however, or at least for a length of time, remaining of a smaller diameter than the cell from which it sprang. By a repetition of this process a succession of cells is formed, each one of which, like the successive joints of the field telescope, is a little smaller than its proximal neighbor and contains less chlorophyl, until finally the cells are reduced to exceedingly fine, perfectly transparent, colorless cylinders, which together form a seta or hair.

Reproduction takes place among the *Œdogoniaceæ*, both by means of zoospores and sexual organs. The former of these are quite peculiar, and, therefore, require especial notice.

Only a single one is ever produced in a cell, and there is consequently no division of the chlorophyllous protoplasm preceding their formation. The first change noticeable is a sort of confusion of the cell contents, the protoplasmic portion of which loosens itself, as it were, from the walls, and collects in a mass at the distal end of the cell. This mass after a short time assumes a more or less irregularly globose shape, and simultaneously the parent cell begins to separate from its distal neighbor. This separation appears to take place commonly by a solution of an exceedingly fine ring of the wall of the parent-cell, just at the origin of the transverse partition separating the two cells, and it is therefore brought about not by a splitting of the end partition wall, but by a circumcision of the side walls of the cell, and consequently the cavity of the latter is thrown open, the end wall remaining with and closing the distal cell, whose contents have not undergone change. On the other hand, observation leads me to think that sometimes there is a splitting of the end wall. According to my observation, sometimes the filament is completely broken in two, but very commonly the two cells remain attached by one corner, opening from one another as it were on a hinge-joint (fig. 2 *f*, pl. 17).

The gathering of the protoplasm, already spoken of, into a ball, is a slow process, and the escape of this ball, through the opening formed in the manner described, takes place even more slowly. The motion is not at all perceptible, with a power of a thousand or twelve hundred diameters. During the passage the ball becomes more or less twisted and deformed, but as it emerges the uncompressed portion shortens and swells out, and when the mass of protoplasm is at last free in the water, it soon assumes a globular or regularly ovate shape. The mother-cell, thus bereft of its contents, is left dead and void. The primordial utricle indeed still remains within, but it has lost all its wonderful powers, and is nothing but a shrunken, twisted, or folded dead membrane. What is the cause of the motion of the zoospore within the cell it is very difficult to determine. It certainly is not vibrating cilia. When the zoospore first escapes, it is, as already stated, an irregular lump of strongly chlorophyllous protoplasm, homogeneous or with one or more roundish masses of darker green within it. As it assumes its shape, however, a very distinct transparent spot appears at its smaller end. Whether this is an absolute vacuole or not, I have never been able to satisfy myself, but I am rather inclined to believe that it contains highly refractive transparent protoplasm. As this spot is perfected the cilia make their appearance. Whether they are actually first formed there, or whether, as is more probable, they are formed inside the cell, and are so folded against the general mass as to be invisible, I have never determined. Dr. Pringsheim, however, figures them within the cell. I have seen them in their early development long before motion commenced in them, but they were always perfectly formed as soon as apparent. They are present in great numbers, making a crown or ring around the edge of the transparent beak-like end. When they commence to vibrate, their action is at first very slow, and the waves of motion run through them deliberately from one cilium to the other, but soon, however, the motile impulses succeed one another more and more rapidly, until the general mass of the zoospore begins to tremble, then to rock, and finally darting off the little body hastens hither and thither through the water. The zoospore of an *Œdogonium* is always readily distinguished from most other similar bodies by its large size and peculiar motion, which is a forward movement combined with a distinct rolling on its long axis. After a time the zoospore, coming in contact with some speck of matter to which it can attach itself, ceases its movements, the cilia rapidly wither away, and the end to which they have been attached swells out or elongates into a broad, or narrow, simple, bifid, or trifid process, placed at an angle to the main axis of the cell, so as to form the so-called foot, the holdfast that anchors and fixes the new plant. Whilst this is taking place, the general form of the zoospore alters into that of a cylinder, a cellulose wall is secreted all about it, and the first cell of the new plant is complete. As soon as this cell is sufficiently matured, it begins to undergo division in the manner already described, and to develop into the new filament.

In regard to the time when these zoospores are given off most abundantly, and the circumstances that influence the process, I can only state that it occurs when there is least tendency to the production of resting spores, probably in youngish plants, and I have thought was favored by a full supply of light, with a moderate temperature.

Sexual reproduction occurs among the *Œdogoniaceæ* in accordance with three distinct types, to which the name of *monœcious*, *diœcious*, and *gynandrous* has been severally applied. The characteristic differences are to be looked for in the production of the antheridiæ or male plant, the female germ being always prepared in essentially the same way. In most instances two cells are requisite for the production of the latter. At first there is nothing by which cells set apart for the formation of the female germ can be distinguished from ordinary cells. The proximal one of the pair finally, however, undergoes changes similar to those seen when a zoospore is to be formed, namely, a sort of confusion of the endochrome, and finally a gathering of it into a mass at the distal end of the cell. Instead of there being a solution of the side wall of the cell, however, the end wall

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undergoes absorption, so that the cavities of the two cells are more or less completely thrown into one. All or nearly all of the contents of the proximal cell now slowly pass into the distal one, which thus becomes crowded with chlorophyllous protoplasm. At or before this period, the distal receiving cell undergoes a change in form, widening out greatly, and sometimes appearing actually to shorten, so that it is in most instances resolved into a more or less regular globose or oval cell. As the sporangium or spore-case thus formed perfects itself the endochromes of the two cells become completely fused into one mass, which gradually condenses and assumes a regular shape, until, in the form of the perfected female or receptive germ, it is a dark, opaque ball more or less completely filling the sporangial cell. At the same time, in order to afford passage for the male germ, an opening is formed through the walls of the sporangium. This happens in two ways. The simplest of these is by the formation of one or more circular openings or pores in the wall. This pore is sometimes below, sometimes above the equatorial line. Its position, numbers, and form afford good specific characters. The second method is by the development of a little trap-door entrance at the distal end of the spore-case. This method is unknown in our American flora, and, never having seen it, I must refer to the papers of Pringsheim for details.

The above-described mode of origin of the sporangium is the common one. In *O. mirabile*, WOOD, however, but one cell is concerned. This cell grows to an enormous size, far beyond that of its fellows, and its endochrome collects into the upper half of it, to be at last shut off from the lower half of the cell by the formation of a new cellulose partition or end wall; or, in other words, the parent cell divides by a modified process of cell division, different from that common in the family. The distal daughter-cell contains all the endochrome. After the changes are completed, the appearance is the same as ordinarily presented, namely, an empty cell surmounted by the sporangium. Sometimes, even in plants in which the ordinary process occurs elsewhere, a single cell appears at times to have sufficient vitality to develop into a sporangium without aid from its neighbor, so that the latter will preserve its integrity, and the resting spore finally lie in proximity to a cell full of endochrome.

In the *monœcious Œdogoniaceæ*, a single filament produces both the male and female germs. Certain cells appear to be set apart to develop into sporangia, whilst others give origin to the spermatozoids. No such plants have as yet been detected in North America, and I, therefore, pass on without speaking more in detail.

The second method in which the spermatozoids are produced is the most common in our flora; it is the so-called *gynandrous* plan. In this the single filament produces the female germs directly and the male germs indirectly. The former arise in the way previously described, whilst the latter are the resultant of a complex series of life actions, as follows: One of the main cells of the originating filament, differing in no perceptible way from its fellows, instead of like them developing new cells, divides up by a simple process of cell division into two or more cells, each one of which contains very largely of chlorophyllous protoplasm. The protoplasm within each of these secondary or daughter-cells soon condenses into an irregularly ovate or conical mass, which often, even within the cell, may be seen to have the transparent beak of the zoospore (pl. 18, fig. 2 *d*). Inside of the cell the *androspore*, as it is called, shows no cilia, but when it is set free by a more or less complete solution of the cell wall, it assumes the form of the ordinary *Œdogonium* zoospore, with a crown of cilia, whose vibrations soon cause it to dart through the water. These androspores are of course much smaller than an ordinary zoospore, and after a period of active motion, they attach themselves to the parent filament, generally either on or near the sporangial cell. Their first life-actions, after settling, are precisely like those of the zoospore, namely, dropping of the cilia, enlargement of the smaller end into the so-called "foot," an elongation of the general mass, and the secretion of an outer coating of cellulose. In this way a peculiar-shaped, somewhat ovate cell is formed, which contains a great quantity of rich protoplasm with mostly a small amount of chlorophyl. From such cells are developed the mostly two- or three-celled, perfect antheridia, which in *gynandrous Œdogonia* are generally to be seen, during the period of fructification, in numbers attached to the filament, mostly in the neighborhood of the sporangium. Their protoplasmic contents are remarkable for the activity of their movements, and I have seldom seen more beautiful and rapid cyclosis than they display—currents setting in all directions—particles actually brushing against one another (pl. 17, fig. 2 *h*). The spermatozoids are formed in the distal cell, sometimes one, sometimes more. In the species *O. mirabile*, WOOD, (pl. 18, fig. 2 *g*, 2 *b*) in which I have most carefully studied their origin, two are produced in the single cell. This cell is in the commencement of the process, although comparatively poor in chlorophyl, crowded with a rich solid protoplasm, which divides into two distinct masses, somewhat in the manner seen in the commencement of ordinary cell division. As there is no distinct nucleus, of course there are no precedent nuclear changes. The masses thus formed gradually assume a more or less perfectly globular shape inside the cell, although I have never been able to see that they there develop cilia, and finally are set free by the lifting up of the end of the mother-cell, like a little trap-door. Their mode of escape through the exit thus offered is similar to that of the ordinary zoospore, which they resemble, except that they are much smaller, are much less rich in chlorophyl, and have the anterior clear space less defined. They are said to be furnished with a crown of cilia similar to that of the zoospore. I myself have never seen these, but do not doubt their existence.

In the *diœcious Œdogonia* there are distinct filaments, male and female, one of which produces the oosporangium with its contained germ, whilst the other gives rise directly to the spermatozoids.

The resting spore which develops after impregnation is variously shaped, but in most instances is round or oval. It is often, if not always, furnished with two coats, the outer of which is thick, firm, and frequently provided with surface appendages, such as tubercles, ridges, spines, etc. Besides these there is also, probably, a very delicate inner coat. The spore appears to be set free from its case by the decay of the latter, there being never, at least that I have seen, any regular dehiscence. Although I have made several attempts, it has never been my good fortune to observe anything like germination of these resting spores. Prof. Chr. Vaupell, however, has published an account of the manner as observed by himself. Some water containing fruitful

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Œdogonia was allowed to dry in a glass, towards the close of September, and the greenish residue was placed in water in the following January. By March the resting spores were everywhere in active germination. The first change was a rupture of the two outer coats and the escape, through the slit, of the contents, still surrounded by a very delicate hyaline membrane. By this time the protoplasm had divided into usually four (sometimes only two or three) greenish masses, each of which was oval in shape and had its own extremely thin, hyaline coat, and was therefore a perfect cell. The old outer shell of the spore laid discarded in the water and soon decayed, and in a little while the hyaline sac surrounding the four daughter-cells itself disappeared, leaving them exposed and naked. After awhile each of these cells opened at one end by means of an annular split, cutting off the apex of the wall and allowing it to lift off like a little lid. Through the circular opening thus made, the contents now emerged. The point of the inner mass was colorless and directed towards the orifice, and the whole moved vigorously backwards and forwards until it finally escaped, as a perfected zoospore. This little body simulated very closely the ordinary zoospore, both in appearance and life-history, growing, after a brief period of activity, into an ordinary filament, in precisely the same manner as the zoospores.

Genus ŒDOGONIUM.

Antheridia et oogonidia in individuo unico.

Antheridia and oogonidia in the same individual.

Remark.—No species of the genus *Œdogonium*, as here defined, has as yet been discovered in this country.

Genus PRINGSHEIMIA.

Dioica. Antheridia et oogonidia in individuis distinctis orta.

Diœcious Antheridia and oogonidia arising in distinct individuals.

P. inequalis, Wood.

P dioica; cellula basali biloba; plantis femineis quam plantis masculis permulto majoribus; oogoniis enormiter globosis vel subovoideis, poro laterale supra medium posito instructis; oosporis forma eadem, sed paulo minoribus.

Syn.—*Œdogonium inequale,* Wood, Proc. Amer. Philos. Soc., 1869, p. 141

Hab.—In stagnis, prope Philadelphia.

O. diœcious, basal cell bilobate; female plant very much larger than the male plant; oosporangium irregularly globose or subovoidal, opening by a lateral pore above the middle; resting spores of the same form as the sporangium, but a little smaller.

Remarks.—This plant seems to be more closely allied to *O. gemelliparum*, Pringsheim, than to any other species. It agrees with it in the inequality of the male and female plants, in the shape of the sporangium, and the position of the lateral pore. The diameter of the female plant is often nearly four times that of the male, and the difference in length is much more apparent. The mother-plant is composed of from 3-6 cells in the most distal of which the spermatozoids are formed. I am not able to state how many of these bodies are formed in a single cell, having only seen the latter when more or less completely emptied, but, judging from the relative sizes, there must be several. In a cell containing a single spermatozoid, that body moved about freely, and at last escaped, apparently through an orifice in the end wall of the cell. It made two attempts before getting out, and during its passage was distinctly constricted in the middle. It resembled in appearance an ordinary zoospore, but was of course much smaller, and was nearly devoid of color, having but a slight greenish tint. I found this species growing abundantly in the stagnant ditches of the Neck, below the city.

Fig. 1 *a*, pl. 18, represents a young female plant; 1 *b*, a fertile plant with immature spores. 1 *c* was taken from the supposed male plant alluded to in the text. The latter figure is magnified 450 diameters, the others 250.

Genus ANDROGYNIA.

Gynandra. Androsporæ in plantis femineis ortæ; postea hanc affixæ et in antheridiis se formant.

Gynandrous. Androspores arising in the female plant; after affixing themselves to this and developing into antheridia.

A. multispora, Wood.

A. oogoniis singulis, vel binis vel ternis continuis, globosis instructa; poro laterale distale; oosporis globosis, oogonii lumen replentibus; antheridiis plerumque pluribus, planta feminea insidentibus, cellula inferiore multo majoribus.

Syn.—Oedogonium multispora, Wood, Proc. Amer. Philos. Soc., 1869, p. 141.

Hab.—In stagnis, prope Philadelphia.

Oosporangia single or bi- or triseriate, globose with a distal lateral pore; oospore globose, about the same size as the sporangial cavity; antheridia bi- or tricellular, curved, with the lower cell much the largest, generally adhering in considerable numbers to all parts of the female plant.

Remarks.—This species differs from its nearest European congeners, *Œdogon. Rothii* and *Œ. depressum*, very markedly in the bicellular antheridia. I have never seen the spermatozoids actually emerging from their mother-cell, but have seen in the terminal antheridial cell a pair of oval bodies, which I took to be those bodies.

Fig. 3, pl. 17, was taken from a filament of this species magnified 500 diameters. It shows spores in different stages of maturity, with an empty basal cell in one case, and in the others without. Also male plants, one of them containing partially formed spermatozoids. The small arrows indicate the direction of cyclotic currents.

A. mirabilis, Wood.

A. rare setigera; articulis diametro 2–8 plo longioribus; oogoniis plerumque singulis, rare geminis, nonnihil ovatis, infra latis sed supra contractis et medio tumidis; poris lateralibus duobus supra medium positis; oosporis aut late ovalibus aut subglobosis; sporodermate haud signato; antheridiis plerumque bicellularibus, interdum tricellularibus, plerumque in filo vegetativo infra oogonium aut in oogonio insidentibus; spermatozoideis singulis et geminis.

Diam.—Artic. veget $\frac{3}{7500}''$—$\frac{13}{7500}''$=.0004″—0017″. Spor. $\frac{18}{7500}''$—$\frac{20}{7500}''$.0024″—0027″.

Syn.—Œdogonium mirabile, Wood, Proc. Amer. Philos. Soc., 1869, p. 142.

Hab.—In rivulis quietis, prope Philadelphia.

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A. rarely setigerous; articles 2–8 times longer than broad; oosporangia mostly single, rarely geminate, subovate, in the lower portion broad, in the middle swollen, in the upper part contracted; the 2 lateral pores situated above the middle; oospore subglobose or broadly ovate, its coats without markings; antheridia generally bicellular, sometimes tricellular, numerous, placed generally upon the female filament either upon or below the oosporangia.

Remarks.—This species was found growing in a rather stagnant brook in the meadow by "Robinson's Knoll," at the junction of the Schuylkill River and Wissahickon Creek, near Philadelphia. The filaments, which vary very greatly in size, are in their early history attached to dead leaves and sticks, but finally, I think, float free in the water. The larger, fruit-bearing filaments are remarkable for their crookedness. None of the threads that I have seen ended in a seta-like portion.

The fruit is produced in abundance, but very rarely is there more than a single spore in any one place. The method of the formation of the sporangia differs from that of all the other *Œdogonia* which have come under my notice. Instead of two cells being concerned but one cell is employed. The cell (fig. 2 *a*, pl. 18) that is to be used for such a purpose grows much beyond the ordinary size, until it is nearly or quite twice as large as its neighbors. All the time it is well filled with chlorophyllous protoplasm. This now contracts and finally is all packed into the upper half of the cell. At or even before this time the lateral openings become apparent. There are two of them, situated just in the angle where the cell at its upper end commences to contract to the size of its fellow. At this time I think fertilization takes place, although I have never actually seen the spermatozoids enter the orifices. The cell (fig. 2 *b*, pl. 17) now divides into two by forming a wall separating the lower empty half from the upper full one, which is to be the sporangium. The contents of the latter now condense into a ball, and it itself becomes more tumid in the middle. Finally a reddish-brown broadly globular spore (fig. 2 *c*, pl. 18) is formed. I have not been able to make out more than one distinct thick coat. The surface of the spore is smooth. The androspores are formed in a cell (fig. 2 *d*, pl. 18) which has grown beyond the normal size and then divided into four or five short cells, each of which gives origin, I believe, to a single androspore in its interior. The antheridia are numerous, from 2 to 6 being commonly attached to the lower portion of the sporangium, or to the cells just beneath it. They (fig. 2 *e*, pl. 18) have a rather large foot, and are generally curved at the base. The distal of the two cells composing them is crowned with a little cap, and produces one or sometimes two spermatozoids. These (figs. 2 *b* and 2 *g*, pl. 18) during their escape are always very much squeezed out of shape, but when free become globular or slightly pear-shaped. They are highly transparent and contain a few green granules. Their motion is at first slow, but soon becomes very active. The mode of egress from the cell is obtained by the cutting off of the upper end of it, the little cap opening like a trap-door. After this cell has been emptied, sometimes a second similar one is formed, which bears it aloft. I have never seen spermatozoids produced by this second cell.

A. Huntii, WOOD.

Filuma plerumque in setam longam, terminalem coloris expertam productum; oogoniis plerumque singulis, globosis, interdúm nonnihil hexagoniis, medio nonnihil tumidis, poro laterale infra medium posito; oosporis globosis, oogonii lumen haud replentibus, superficie lineis elevatis spiralibus quatuor instructa; antheridiis bicellularibus (interdum tricellularibus ?).

Diam.—Spor. $\frac{1}{500}''=.002''$.

Syn.—*Œdogonium Huntii,* WOOD, American Naturalist, 1868.

Hab.—In aquario meo.

Filaments mostly produced into a long apical seta; oogonia mostly single, globose, sometimes somewhat hexagonal, somewhat tumid in the middle, the lateral pore placed below the middle; oospore globose, not filling the cavity of the spore case, its surface with four spiral elevated lines or ridges; antheridia bicellular (sometimes tricellular ?).

Remarks.—This little plant appeared in my aquarium some years since, forming a delicate fringe upon the various aquatic plants growing therein. Its color is a bright yellowish green, deepening to a very dark green in cells which are crowded with granular protoplasm. The filaments vary very greatly in size, the largest I have seen were $\frac{1}{600}$ of an inch in diameter. They are provided with long, terminal seta, which are much more universally present than in any of the other species I have met with. The first step in the formation of a spore is the emptying of a cell into its distal neighbor, so that each spore case is placed at the end of an empty cell. These sporangia may be single or they may be in series of two or more, separated only from one another by the eruptive cells just spoken of. The color of the mature spore is a very dark reddish-brown. The antheridia is bicellular, slightly curved, somewhat stipate, with a distinct foot. Its most common position is on the vacated cell just below the spore case. The zoospores, as I have seen them, are always globose.

I have named this species after my friend, Dr. J. Gibbons Hunt, a well-known microscopist of this city, to whom I am greatly indebted for aid in my earlier microscopic studies.

Fig. 2, pl. 17, represents different forms and parts of this plant. 2 *a* shows the end of a filament and the long seta-like lip. 2 *b* was taken from two cells, one of which had just undergone division, and shows very plainly the method of procedure; lying as it were between the cells, and bearing the end of the lower one upon it, is the new little cell. Fig. 2 *c* represents a fertile filament with two mature spores and one not fully grown. Fig. 2 *d* was drawn from a filament just forming a spore, and shows the male plant *in situ.* Fig. 2 *e* represents a male plant (magnified some 1300 diameters) with the outer terminal cell scarcely more than a primordial utricle. The contents of the lower cell were in a state of intense motion; and the arrows are meant to indicate the directions of the currents. Fig. 2 *f* represents a portion of a filament with a zoospore just escaped and still quiescent.

A. echinata, WOOD (sp. nov.)

A. valde elongata; articulis diametro 6–14 plo longioribus; oogoniis globosis, plerumque depressis, ad .0014″ crassis; oosporis oogonii forma et ejus lumen replentibus, valde aculeatis; poro laterale supra medium posito; antheridiis bicellularibus ?

Diam.—Spor. $\frac{13}{12000}''=.001''$. Cell. $\frac{1}{3000}''-\frac{1}{2000}''=.00033''-.0005''$.

Hab.—In stagnis, Alleghany Mountains.

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O. gynandrous, very elongate; joints 6–14 times longer than broad; sporangia globose, mostly depressed, about .0014″ in diameter; oospores of the same form as sporangia, whose cavity they almost fill; covered with sharp spines; the lateral pore placed above the middle; antheridia bicellular?

Remarks.—I found this distinct species in a little stagnant pool in the wilderness, known as Bear Meadows, in Centre County, of this State. The filaments are very long, and were matted together into a sort of fibrous mass. The male plants were few in number, and were attached to the female plant in the neighborhood of the sporangia. I have not seen any composed of more than two cells. They are furnished with a well-marked foot, above which there is a short neck. As I have seen them they are nearly straight.

I have not been able to make out more than one coat to the spores. This coat is very thick, and is furnished with numerous thorn-like spines. These are very sharp at the points, but at their bases are mostly very robust.

Fig. 3, pl. 18, represents a spore of this plant magnified 750 diameters.

SUBFAMILY BULBOCHÆTEÆ.

Filuma ramosum, setis strictis hyalinis achrois e basi bulbosa et plus minus elongatis instructum.

Filaments branching, furnished with straight, hyaline, more or less elongated seta, arising from a bulbous base.

Remarks.—The *Bulbochæteæ* are at once separated from their allies the *Œdogonieæ* by their bushy, branched habit of growth. The shape of the individual cell is also entirely different, for instead of being regularly cylindrical they are almost always markedly dilated at their distal end, so as to be somewhat clavate, nor is the filament or its branches ever ended by a long seta-like series of narrow colorless cells. Many or all of the cells are, however, furnished with a single very long unicellular unbranched hair. These hairs are colorless, hyaline, and provided with a markedly and abruptly bulbous base. The *Bulbochæteæ* grow in similar positions to their allies, but are not nearly so common, nor when present do they grow in such abundance, very rarely, if ever, forming the dense forest-like fringes or the matted masses that some species of the *Œdogonieæ* do. They are reproduced both by zoospores and resting spores.

The manner of the development of and growth of the plant from the zoospore is very peculiar. I have never myself studied it, but Prof. Pringsheim gives the following account: When the zoospore first settles down it produces a cell closely resembling that of an *Œdogonium.* The first change which occurs in this cell is the formation of a small, conical, transparent, colorless space at the apex, which space in a little while becomes separated from the mother-cell by a distinct partition-wall, and at the same time the apex itself is ruptured, and the point of the little growing cone pushed through the opening. This rupture does not take place irregularly, but by a sort of circumscribed dertiscence, similar to that of the *Œdogonium*, the top of the mother-cell being lifted up like a little trap-door, and finally pushed aside as the new conical cell grows elongate and becomes converted into a hair. After the formation of this apical hair, the mother-cell undergoes division in a manner similar to that of an *Œdogonium.* Near its distal end a

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circular slit appears, and at the same time a partition forms, so that from the mother-cell are developed a small apical and a large basal daughter-cell. The history of the former of these is simply one of growth as regards the main axis. It increases in size but does not give origin to new cells. All such cells are formed out of the basal daughter-cell, which, as already described, divides into a new apical and basal cell—the apical only to grow in the main filament—the basal to divide anew. It is always the basal cell that undergoes division, throughout the whole life-history of the plant, one cell alone contributing to the growth of the main filament. The filament thus formed bears upon its distal end the hair which grew upon the original spore-cell, and this hair is, save only the basal cell, the oldest part of the filament. The cell upon which it rests is the next oldest, the next to it in position, the next in age, and so on (from older to younger) down to the basal cell, the oldest of all, lying next to the latest born.

Although the cells of the main filaments do not contribute to its development, yet it is from them that the lateral branches are formed. The production of a branch begins by the appearance of a clear space near the apex of the cell, but this clear space is placed, not exactly at the apex, but a little to one side. It soon becomes distinctly conical, enlarges, bursts through the old cell-wall, is cut off by a cellulose partition from its parent, and develops into a hair similar to that first formed, but placed at an angle to the long axis. It is remarkable that the opening for the exit of the growing hair occurs, not by a circular transverse slit, but by a longitudinal one, the two halves of the old cell-walls separating as the little cone pushes its way between them and persisting as a sort of sheath to its base. When the hair is perfected the cell from which it grew undergoes division in the usual way, save only that the cutting off of the old wall is done obliquely instead of transversely, so that the partition is oblique instead of horizontal, and the new cell grows at an angle to the old, instead of in the line of its axis. The new cell, consequently, is the starting point to a branch at an angle to the main filament. This branch, like the main filament, grows only by the repeated divisions of its primal basal cell, and bears aloft its seta. Secondary branches may arise from it precisely in the way that it arose from the parent stem, and thus at last is formed the bushy plant of the *Bulbochæteæ*.

The zoospores closely resemble those of the *Œdogonieæ*, and are oval or globose masses of chlorophyllous protoplasm, with a transparent space at the smaller end, surmounted by a crown of cilia. Their mode of formation and whole life-history are also similar to that of the *Œdogonieæ* zoospores, up to the time when in their germination they begin to produce new cells.

Sexual reproduction amongst all the known *Bulbochæteæ* is similar in its general aspect to that seen among the gynandrous *Œdogonieæ*, but differs considerably in detail. The oogonia are mostly formed in lateral branches. Their position in these branches varies in the various species.

Since any cell from the next to basal to the most distal of all crowned with the terminal seta may be converted into a oogonium, according to Pringsheim, the cell which is to form the oogonium arises in the usual way, by the division of a cell into two daughter-cells. The new daughter-cell, which is to develop into the sexual part, does not, however, rupture the old wall of the mother-cell, but grows out beyond it, and there dilates. The new cell is therefore divisible into two parts, a proximal cylindrical portion, contained within the walls of the mother-cell, and a distal more or less globular piece beyond the latter. The chlorophyllous protoplasm now collects in this dilated portion, leaving the basal cylindrical part bare and empty. The oogonium is not, however, formed directly from this upper portion (the primitive oogonium, as it may be called), but a new wall forms within the latter and then it undergoes division much as did the primary cell. In this way it is that the upper and lower portions of the old wall, *i. e.* that of the primitive oogonium, remain as a sort of basal sheath and cap to the fully-formed sporangium. The little hole by which the spermatozoids find entrance to the contents of the oogonium is always formed in the upper half of the wall of the latter.

As stated, all the species of *Bulbochœteæ* as yet known are gynandrous. The antheridia resemble those of similar *Œdogonieæ*, and their life-history is very similar. The development of the resting spores is said to take place as follows: The first change is in the color of the spore, the bright red becoming green, especially near the margins of the cavity. The outer wall is then ruptured and the spore grows into a long oval body, whose contents are chiefly green with a sprinkling of the original red. The protoplasm of this oval body gradually divides into four masses, which become more and more distinct, until they are at last well formed zoospores, similar to those produced in the more ordinary method, except, perhaps, that they are redder. They are finally set free in the water by a solution of the cell wall surrounding them, and enter upon a brief free existence, to settle down after a little and grow into a fully-formed plant.

Genus BULBOCHÆTE.

Androsporæ in planta femineâ ortæ, postea hanc affixæ et in antheridiis se formantes.

Androspore arising in the female plant, afterwards affixed to it and developing into the antheridia.

B. ignota, Wood.

B. sparse ramosa, elongata; articulis diametro max. ($\frac{1}{1300}''$ = .00077") $1\frac{1}{2}$–3 plo longioribus; oogoniis long. $\frac{1}{400}''$ = .0025", lat. $\frac{11}{6000}''$ = .0018", interdum lateralibus et sessilibus, interdum inter ramulorum cellulas vegetativas positis, dissepimento nullo; oosporis ovalibus, longitudinaliter nonnihil oblique et distante costatis, in ætate provecta aurantiaco-brunneis, sporodermate crasso; antheridiis 3–4 cellularibus, stipitatis.

Syn.—*B. ignota*, Wood, Prodromus, Proc. Amer. Philos. Soc., 1869.

Hab.—In aquis quietis, prope Philadelphia.

B. sparsely branched, elongate with the joints $1\frac{1}{2}$–3 times longer than broad ($\frac{1}{1300}''$ = .00077"); oosporangia .0025" long by .0018" broad, sometimes lateral and sessile, sometimes placed upon the apex of a branch, sometimes situated in the length of the branches between their cells; the empty cell which supports the sporangium without dissepiment; oospores, oval, filling rather closely the cavity of the spore-case, longitudinally somewhat obliquely and distantly costate, when mature orange brown; spore-coat rather thick; antheridia 3–4 celled, scarcely stipitate.

Remarks.– When I described and figured this species I had never seen the mature fruit, but very recently Mr. Quimby has communicated specimens to me.

The color of the spore is orange brown, and the thick coat is slightly tinged with yellowish. The mature oosporangium is somewhat flattened at the sides, not so elliptical as the young spore, which I have figured.

Fig. 5 *a*, pl. 18, represents a fragment of a filament showing young sporangial cells magnified 260 diameters; 5 *b*, represents a branch with a youngish spore in it, magnified 460 diameters; fig. 5 *c*, was taken from a male plant.

B. dumosa, Wood.

B. articulis diametro 1½–2 plo longioribus; oogoniis plerumque in ramorum brevissimorum apicibus positis sed interdum lateralibus, plerumque setam terminalem gerentibus; oosporis enormiter ovalibus aut ovatis, nonnihil indistincte longitudinaliter oblique subarcte striatis; antheridiis bicellularibus, stipite instructis, cellula basale medio tumida, supra sæpe contracta.

Syn.—*B. dumosa,* Wood, Prodromus, Proc. Amer. Philos. Soc., 1869, p. 142.

Hab.—In aquario meo.

Joints 1½–2 times longer than broad; oosporangia generally placed upon the ends of short branches but sometimes lateral, mostly carrying a terminal seta; resting spores irregularly oval or ovate, somewhat indistinctly obliquely longitudinally and rather closely striate; antheridia bicellular, furnished with a little stipe, their basal cell tumid in the middle, frequently contracted above.

Remarks.—This species appeared spontaneously during the latter part of the winter upon some large fresh-water algæ which I was cultivating. It branches irregularly and sometimes somewhat profusely, so as to have quite a bushy habit. The antheridia appear to produce a single spermatozoon in the terminal cell; at least as far as my observation has gone this is true. I think I have always found the distal cells of fertile plants emptied of their contents, as though they had furnished the androspores which had grown into the antheridia. This species is closely allied to *B. gracilis*, of Pringsheim, from which it differs in the position of the oogonia, in the relative breadth and length of the cell, and the number of cells composing the antheridia.

Fig. 6 *a*, pl. 18, represents a filament of this species magnified 260 diameters; 6 *b*, a male plant magnified 750 diameters.

B. Canbyii, Wood.

B. permagna ad .035″ longa, sparse ramosa; articulis sterilibus diametro 2–8 plo longioribus: oogoniis lateralibus vel in ramulorum apicem positis, transverse enormiter ovalibus; oosporis, transverse enormiter ovalibus, plerumque nonnihil triangularibus, oogonii lumen replentibus; sporodermate crasso, haud costato, enormiter punctato; antheridiis bicellularibus.

Diam.—Cell. steril. $\frac{5}{7500}$″—$\frac{8}{6500}$″ = .00066—001. Spor. transv. $\frac{17}{7500}$″ = .00226.

Syn.—*B. Canbyii,* Wood, Proc. Amer. Philos. Society, 1869, p. 142.

Hab.—In aquis quietis, prope Hibernia, Florida; (Wllliam Canby).

B. very large, attaining a length of more than one-third an inch, sparsely branched; sterile joints 2 to 8 times longer than broad; oosporangia lateral or placed upon the ends of branches, irregularly transversely oval; oospores of a similar shape, often a little triangular, filling the cavity of the sporangium; spore coat thick, not costate but irregularly punctate.

Remarks.—It affords me great pleasure to dedicate this very handsome species to Mr. William Canby, by whom it was collected in Florida, as an acknowledgment of favors received, and as a testimony of respect and high regard for him personally, and as being among the foremost students of American phanerogamic botany.

This species is more nearly allied to *S. minor* than to any other of the European forms, but differs from it very essentially in size and habit. It is always, as I have seen it, except in very young plants, sparsely and mostly dichotomously branched, and attains a very great length, at times probably exceeding the third of an inch. The spore is mostly sessile upon the distal ends of the cells of the filament; in all such cases I have noticed that the cell upon which it was borne was divided in its middle by a partition into two cells. Not unfrequently the spore is raised upon a short branch. The male plants are attached to the female filaments generally in the neighborhood of the sporangium, to which they sometimes fasten themselves immediately. They are shortly stipitate, and composed of two cells. The mature spore is transversely oval, now and then slightly triangular, and is nearly of the color of burnt sienna. Its coat is thick, often slightly yellowish, and has on its outer surface irregular punctations, looking like corrosions. These are not detachable, except when the ruptured spore is more or less completely emptied of its contents. The sporangium closely invests the spore, and when the latter is matured undergoes a circular division, so that the top falls off and allows the spore to escape.

Fig. 6 *c*, pl. 16, represents a portion of a filament, magnified 260 diameters, with a young sporangium and young male plants attached; 6 *b*, represents a very young plant, magnified 260 diameters. Fig. 6 *a*, was taken from a mature plant, and shows the mature spore. Fig. 6 *e*, shows in outline a sporangium and male plants attached; whilst 6 *d*, was drawn from a sporangium which had perfected its spore and undergone the natural dehiscence.

FAMILY CHROOLEPIDEÆ.

Algæ aereæ, aureo-, aurantiaco- vel rubro-fusco-coloratæ, siccatæ sæpe canæ. Fila varie ramosa, cytiodermate crasso vel subcrasso, firmo, subcartilagineo prædita, in pulvinulos minutos vel in stratum tenue aut incrassato-tomentosum densissime aggregata vel implicata. Cytioplasma oleosum vel granulosum, aut rubellum, aureum, aut flavo-fuscum, interdum viride tinctum, post mortem plerumque expallescens. Propagatio fit zoogonidiis.

Ærial algæ. Golden orange, or reddish fuscous, often grayish when dried. Filaments variously branched, furnished with a thick, or thickish, subcartilaginous cytioderm, densely aggregated into minute cushions, or a thin or tomentosely thickened stratum. Cytioplasm granular or containing oily particles, reddish-golden, or yellowish-fuscous, sometimes tinged with green; after death often colorless or nearly so. Propagation by zoospores.

Remarks.—The plants of this family are so different from the others of the order, that it is a matter of considerable doubt whether or not they should be classified with them. They rarely possess distinct, well-pronounced chlorophyl, and form mats or strata of some shade of reddish, grayish, or brownish, so that they are very different in appearance from the other *Confervaceæ*.

I do not think their position can be certainly fixed until their life-history has been more fully developed. In assigning them this place I have simply followed Prof. Rabenhorst.

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The only specimens that have come to my notice are in a dried condition, and consequently no possible opportunity has been afforded of studying the manner of reproduction. No one has as yet, at least to my knowledge, discovered any sexual reproduction in the family, but the method in which the zoospores are produced has been carefully studied, especially by Drs. Caspary (*Regensburg Flora*, 1858) and Hildebrand (*Botanische Zeitung*). The little motile bodies are not produced in the cells indiscriminately, but in certain ones set apart for the purpose, to which the name of zoosporangia is very applicable. These are large, globular, thick walled cells, which are generally provided with a protuberance at the top and marked by transverse wrinkles. They are most frequently situated upon the end of the filament or one of its branches, but are rarely placed in the middle of the thread, and still more rarely the cell next below the zoosporangium elongates itself sideways and upwards into a thread, so that the reproductive cell is left as a lateral one-celled branch or process. When the zoosporangium is sufficiently matured the endochrome breaks up into a number of minute masses, the future zoospores. Finally the crowning papilla of the mother-cell ruptures and allows the contents to escape as a well-formed vesicle, containing the perfected zoospores. It is said, however, that sometimes the vesicle is wanting, and the zoospores are discharged into the water. In the ordinary course, after a little while the vesicle lying in the water bursts and sets its motile contents free. The zoospores themselves are very small, according to Hildebrand, $\frac{5}{500}$–$\frac{6}{500}$mm. in length, by $\frac{1}{500}$–$\frac{2}{500}$mm. in breadth. In accordance with the same authority they are, when first discharged, cylindrical, but in a little while become flattened, and shaped like a flaxseed. They are biciliate and contain a large number of small, orange-colored particles. From thirty-two to sixty-four of them are formed in one zoosporangium, and neither light nor time of day appear to have any influence upon their birth. Hildebrand states that their motile life lasts from eighteen to thirty-six hours, but according to Caspary, after continuing in motion for about an hour, they grow sluggish, sink, become globular, then elongate themselves and shortly undergoing transverse division, actively commence to form the new filament.

Genus CHROOLEPUS, AG.

Fila distincte articulata, intricata, enormiter ramosa.

Filaments distinctly articulate, intricate, irregularly branched.

C. aureum, (LINNÉ.) KTZ.

C. filis ramossimis, in stratum aureo-brunneum, ad duas tres lineas crassum, cæspitosum et molle intricatis vel in cæspitulos aggregatis; articulis enormibus, diametro sesqui-, duplo triplove longioribus.

Diam.—Max = .001″.

Syn.—*C. aureum*, (LINNÉ.) KÜTZING. RABENHORST, Flora Europ. Algarum, Sect. III. p. 371.

Hab.—Little Falls, New York; Godwinsville, New Jersey; (Austin). Texas; (Ravenel).

Filaments very much branched, interwoven to form a yellowish-brown softish mat, two or three lines in thickness; joints irregular, 1½–3 times longer than broad.

Remarks.—I am indebted to Mr. Austin for specimens which are labelled "Forms dense yellow-brown cushions on rocks, at Little Falls, New York and

Godwinsville, New Jersey." As dried, the plant is in extended, gray, felt-like masses. The walls of the articles as seen with the microscope are thick and irregular, and the joints themselves are also very irregular, the end ones being often swollen and rounded so as to give the branches a sort of bulbous termination.

Among the Algæ collected in Texas by Prof. Ravenel, is a dried specimen (No. 100), labelled "On Bark, Houston, Texas," which I cannot separate from this species. It occurs in small tufts, which, as dried, are of a very decided orange, and, no doubt, were still brighter during life. The articles are not so irregular as in Mr. Austin's specimens, but excepting in this and color when dried they agree very well. Besides these I have several specimens from the same source, which are in extended mats and agree in all respects with their northern brethren.

Our American form appears to attain a greater diameter in its individual filaments than does the European variety, but I know of no other character separating it from the latter; and consequently must consider them identical. The measurement given is an extreme one, .009″ being commonly the limit.

Genus BULBOTRICHIA, Kutz.

Fila indistincte articulata, achroa, firma, ramosa; rami in apice intumescentes, sporangia constituentes.

Filaments indistinctly articulate, translucent, firm, branched; the ends of the branches swollen so as to form sporangia.

B. albida, Wood (sp. nov.).

B. strato albido, coriaceo vel crustaceo; filis arcte intertextis, enormiter ramossissimis, coloris expertibus; sporangiis viridibus.

Hab.—In muscis, Northern New Jersey; (Austin.)

Forming a white leathery or crustaceous stratum; thread closely interwoven, irregularly and plentifully branched, colorless; sporangia greenish.

Remarks.—This curious little plant, which was sent me by Prof. Austin, occurs in minute white patches growing on mosses at the base of stumps in woods. Sometimes these are encrusted abundantly with the carbonate of lime, when they are hard and crustaceous. The sporangia appear to vary greatly in size; sometimes they resemble very closely a single spore (probably their commencing stage). The bases of the branches are rarely, if ever, furnished with the bulbous swelling, given by Rabenhorst as a generic distinction, but such enlargements do occasionally occur in the course of the filaments and branches. The filaments are composed of a series of cells, which are in places long, and have their end walls thin and not readily seen.

Fig. 5, pl. 16, represents a part of a plant magnified 460 diameters.

Family CHÆTOPHORACEÆ.

Algæ aquaticæ vel palustres, rarius terrestres, plerumque monoicæ vel dioicæ. Fila varia, sæpe dichotome ramosa, haud raro fasciculatim ramulosa, plerumque in cæspites vel pulvinulos cumulata, in muco gelatinoso subliquido vel firmo nidulantia. Propagatio fit tum oosporis, tum zoogonidiis. Zoogonidia oriuntur aut singula aut geminis aut cytioplasmatis divisione 8–16 in quoque sporangio.

Aquatic, paludal, or rarely terrestrial algæ, mostly monæcious or diæcious. Filaments various, often dichotomously, but not rarely fasciculately branched, mostly aggregated into turfy masses or little cushions, and generally surrounded by a firm or subliquid gelatinous mucus. Propagation both by zoospores and resting spores. Zoospores arising either singly or by the division of the cytioplasm into 8–16 in each sporangium.

Genus STIGEOCLONIUM.

Fila articulata, simpliciter ramosa; rami ramulique sparsi, rarius fasciculatim approximati, in apicem acutum, sæpe piliferum achroum attenuati et plerumque longe protensi, sæpius ramellis brevibus subulatis instructi. (R.)

Filaments articulate, simply branched; branches and branchlets sparse, rarely fasciculately approximated, with their ends acute and frequently prolonged into an attenuate transparent seta or hair, and very often furnished with short subulate branches.

Remarks.—Plants which are certainly referable to this genus are abundant in every place in which I have ever looked for fresh-water algæ. I confess, however, that although very much time has been given to their study, I have not been able to make out any distinct specific characters, nor any identifications from the diagnoses of M. Rabenhorst. In a certain spring northeast of the city, there grows one of these forms, which I have closely watched for several seasons. In the earlier state it appears at times to possess the characters of a young *Chætophora* (pl. 19, fig. 1), forming a small gelatinous base out of which the threads soon escape as they lengthen. It constituted a sort of mucoid layer adhering to the boards lining the stones with waving masses of projecting filaments six or even eight inches in length. The filaments were mostly about $\frac{1}{2250}''$ in diameter and much interlaced.

The cells varied greatly in length, some being scarcely as long as broad, whilst others were eight or ten times longer. The short cells were generally densely filled with endochrome, whilst the long ones were nearly empty. The branches often ended abruptly, but were more frequently tipped with a long seta-like point. The method of branching is as varied as can be imagined, as is shown by fig. 4, pl. 16, and fig. 1, pl. 20, all taken from different plants of this species. I have frequently seen the production of zoospores, but no other method of reproduction. In all cases a single motile body (fig. 4, pl. 16) was formed in each cell. These minute bodies are globular or pyriform, and within the cell exhibit no motion whatever. Their escape takes place very slowly through a lateral slit in the wall. No cause of the motion is visible, and during the passage the zoospore is often very much squeezed out of shape. According to Braun (Verjungung), these zoospores possess a red eye-spot. I had not read his description at the time my observations were made, but did not notice any. The zoospores germinated in the usual way, elongating and growing into a cell with a transparent seta-like end, and finally undergoing repeated divisions to form the plant.

M. Braun states that he has observed another process, in which the contents of a single cell undergoes a perpendicular division, so as to form four small zoospores, which escape from the cell in the same way as the larger one, and further says that he has never known these microgonidia to germinate.

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Genus DRAPARNALDIA, Ag.

Fila articulata ramosa, e cellulis magnis, maxime hyalinis, fascia chlorophyllosa latiuscula ornatis, semper sterilibus formata, fasciculis penicillato-ramulosissimis, e cellulis minoribus fertilibus compositis, plus minus dense obsessa. Articuli terminales omnium ramulorum inanes achroi steriles, in pilum hyalinum plus minus elongati.

Filaments articulated, branched, formed of large cells which are chiefly hyaline, but furnished with a transverse chlorophyllous fascia, more or less densely clothed with penicillately ramulose fasciculi, formed of smaller fertile cells. Terminal articles of all the joints empty, transparent, sterile, and elongate, in a more or less hyaline hair.

D. glomerata, (Vauch.) Ag.

D. filis ramisque primariis achrois vel subachrois, ad 0.00147″ crassis, articulis inferioribus diametro æqualibus vel paulo brevioribus, geniculis manifesto constrictis, fasciis chlorophyllosis angustis dilute viridibus; ramis primariis subrectangulo-patentibus, sæpe oppositis; ramulorum fasciculis confertis, patentibus, alternantibus vel oppositis, dense ramellosis, subovalibus, obtusis. (R.)

Syn.—*D. glomerata,* (Vaucher) Agardh. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 38.

Hab.—Rhode Island; (S. T. Olney) Thwaites.

Filament and primary branches colorless or subcolorless, and reaching 0.00147″ in diameter, lower articles about as long or a little shorter than broad, manifestly constricted at the joints, chlorophyl fascia narrow, light green; primary branches subrectangularly patent, often opposite; fasciculi of branches crowded, patent, alternating, or opposite, densely ramellose, suboval, obtuse.

Remarks.—According to M. Thwaites the true *Dr. glomerata* grows in Rhode Island, as he so identified specimens sent to him by Mr. Olney. These specimens were, however, in all probability dried, and if this was so, I confess not to attaching much weight to the identification. The *Draparnaldia*, common near Philadelphia, is at once so like and yet so different from the description of *D. glomerata*, that I am unable to fully satisfy myself whether it be a variety of the European species or distinct from it. It differs very greatly in the thickness of the stem and primary branches. I have given above Prof. Rabenhorst's description of the European variety, and now append one of the plant growing in this neighborhood.

Var. **maxima.**

Dr. filis achrois, ad 0.004″ crassis, articulis plerumque diametro duplo longioribus, in medio sæpe valde tumidis; ramis primariis achrois vel subachrois, oppositis vel alternantibus vel ternatis, elongatis, dense ramellosis, cum ramulis lanceolatis; ramulorum extremorum fasciculis dense ramelosis, ovatis vel late lanceolatis, plerumque confertis; ramulorum articulis inferioribus plerumque diametro (ad $\frac{1}{1875}$″) subæqualibus, articulis superioribus diametro duplo aut triplo longioribus, plerumque piliferis.

Hab.—Prope Philadelphia; Wood.

Filament transparent, attaining a diameter of 0.004″, its articles mostly twice as long as broad, strongly swollen in the middle; primary branches colorless or subcolorless, opposite, alternate or ternate, elongate, densely ramellose with the ramuli lanceolate; fasciculi of extreme branches densely ramellose, ovate, or broadly lanceolate, mostly crowded, inferior articles of the branches mostly about as long as broad ($\frac{1}{1875}$″), superior articles two to three times as long, mostly piliferous.

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Remarks.—In this form there are almost always numerous little clusters of branchlets, growing immediately from the main stem or large branches; such clusters are more rigid, more open, more broadly ovate, and less markedly piliferous than the others.

D. plumosa, (Vaucher) Agardh.

D. filis ramisque primariis hyalinis, plerumque $\frac{1}{50}'''$ = 0.00179″ crassis; articulis diametro æqualibus vel dimidio brevioribus, rarius paulo longioribus, geniculis vix aut modice constrictis, fasciis chlorophyllosis angustis læte viridibus; articulis inferioribus ramulorum diametro ($\frac{1}{198}'''$—$\frac{1}{223}'''$) æqualibus vel subduplo longioribus, pæne torulosis, superioribus cylindricis ad $\frac{1}{243}'''$ attenuatis, diametro duplo triplo-quintuplo longioribus, plerumque non piliferis; ramulorum *fasciculis dense ramellosis, elongatis, acute lanceolatis, erecto-subappressis.* (R.)

Syn.—*Dr. plumosa,* (Vaucher) Agardh. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 382.

Hab.—In rivulis et aquis quietis.

Filament and primary branches hyaline, mostly $\frac{1}{50}'''$ = 0.00179″ in diameter; articles as long as broad or one-half shorter, rarely a little longer, scarcely or slightly constricted at the joints, chlorophyl fascia bright green, narrow; lower articles of the branches about as long as broad ($\frac{1}{198}'''$—$\frac{1}{223}'''$) or nearly twice as long, somewhat torulose, the upper ones cylindrical, as small as $\frac{1}{243}'''$, two to five times longer than broad, mostly not piliferous; *fascicles of branches densely branched, elongate, acutely lanceolate, actually subappressed.*

Remarks.—I have found a *Draparnaldia* frequently, which I believe to represent the European *D. plumosa.* As I have preserved, however, no specimens or descriptions, I have simply copied the description of Prof. Rabenhorst.

D. Billingsii, Wood.

D. valde gelatinosa; filis et ramis primariis achrois ad $\frac{30}{7500}''$ crassis, sparsissime ramosis, articulis diametro 2–6 plo longioribus, sæpe medio valde tumidis; fasciis chlorophyllis dilute viridibus, sæpe nullis aut subnullis; ramulorum fasciculis distantibus, late ovalibus vel late triangularibus, alternantibus vel oppositis vel triplice verticellatis, sparse ramosis, patentissimis; ramulis pilis longissimis robustis terminalibus instructis; oosporis globosis, moniliforme conjunctis; sporodermate crasso.

Syn.—*D. Billingsii,* Wood, Proc. Am. Philos. Soc., 1869, p. 143.

Hab.—In aquis quietis, prope Philadelphia.

Frond very gelatinous, filament and primary branches attaining a diameter of $\frac{1}{250}''$, very sparsely branched, their articles 2–6 times longer than broad, often very much swollen in the middle; chlorophyl band light green, frequently almost or entirely wanting; fascicles of branches distant, broadly oval or triangular, alternate, opposite, or in whorls of three, very open; ultimate branchlets terminating in a long, robust, hyaline hair; resting spores globose, with thick walls, arranged in long moniliform sometimes branched filaments.

Remarks.—I found this plant about the middle of March, 1869, floating on the surface of a little pool in the woods near Chelten Hills, a few miles north of Philadelphia. To the naked eye it appears as a gelatinous mass, resembling a *Tetraspora,* but when closely examined this translucent jelly is seen to be filled with rather distant greenish points, which are the little clusters of branches. The largest specimens I have seen had attained a length of nearly two inches. The filaments are very transparent and have the branches placed at long intervals.

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The ultimate branch groups are ovate or oval, and are remarkable for their openness, the branchlets being few in number and widely separated. Most of the ultimate branchlets are prolonged into a remarkably strong long hair.

The cells of the main filaments are beautifully transparent, and are sometimes cylindrical but more generally are barrel-shaped. Both secondary and primary branches are often arranged singly, sometimes in pairs, not unfrequently in threes. When placed between two plates of glass and examined closely by the unaided eye, this species is readily distinguishable from our other *Draparnaldia*, by its fasciculi of branches being so widely separated as to be not at all confused with one another.

I have a single specimen which I believe to be in fruit. The resting spores (fig. 6, pl. 14) are in long branched chains. They are more or less globose, with a very thick outer transparent wall, and an inner green endochrome, which very probably becomes brownish at maturity. Except when they are branched, these series of spores remind one very strongly of the filaments of some nostocs.

I dedicate this very beautiful species to Dr. J. S. Billings, U. S. A., to whom I am under the greatest obligations for aid in the prosecution of this research, and whom I have ever found to unite the greatest scientific liberality with a strong enthusiasm for and able prosecution of the study of these lower vegetable forms.

Since describing this species I have received the Microscopical Journal for 1869, containing Dr. Hicks's paper upon *D. cruciata*. The original description in the Linnæan Transactions had escaped my notice. *D. cruciata* and *D. Billingsii* are exceedingly closely related, yet if Dr. Hicks's description and figures be accurate they are probably distinct. Thus in the last species the ramuli are not placed at right angles to the main filament, nor are they ever in fours, both of which are given as characters of *D. cruciata*. They are, on the contrary, in *D. Billingsii* at various angles, and commonly arise singly, but not unfrequently in pairs, and very rarely in threes. It is worthy of remark, on the other hand, that the figures of Dr. H. do not entirely agree with his description, as in no case are there more than two and frequently but a single branch at one place. The cells of the main filament are also more barrel-shaped in our species than one would infer to be the case with *D. cruciata*.

After all, however, I think it very possible that both forms belong to the one species.

Fig. 6, pl. 14, represents a small portion of the frond with fertile branches magnified 460 diameters.

Genus CHÆTOPHORA, Schrank.

Fila articulata ramique primarii radiatim dispositi, e cellulis vegetativis elongatis, fascia chlorophyllosa in morem Draparnaldiæ et Stigeoclonii ornatis compositi, sursum in ramulos numerosissimos, brevius articulatos, articulis extremis attenuatis sæpe inanibus non aut vix piliferis instructos, fasciculatos plus minus dense congestos divisi, massa gelatinosa firma, coriacea vel dura involuti, thallum globosum vel subglobosum aut plane expansum varie lobatum et fissum constituentes. (R.)

Filaments articulated, with the primary branches radiately disposed, composed of elongated vegetative cells, ornamented with a chlorophyllous fascia like a Draparnaldia or Stigeoclonium, distally resolved into very numerous fasciculate, more or less densely congested branches, with shorter joints, their end joints alternate, often empty, either not or scarcely piliferous; surrounded by a firm coriaceous or hard jelly, so as to form a globose, subglobose, or expanded thallus.

Remarks.—I have never seen the production of the zoospores in this genus, but they are said to arise one in a cell, and to escape by a sort of lateral splitting of the wall.

C. elegans, (ROTH) AGARDH.

Ch. thallo globoso vel subgloboso, pisi vel cerasi magnitudine, dilute vel saturate viridi, nitido, superficie lævi vel quasi tuberculata, elastice molli, nonnunquam indurato; fasciculorum ramulis laxis vel confertis, articulis extremis brevi-cuspidatis, sæpe piliferis.

Syn.—C. elegans, (ROTH) AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 384.

Hab.—United States.

Thallus globose or subglobose, of the size of a pea or cherry, light green, with the surface smooth or quasituberculate, elastic but soft, sometimes indurated; branches of the fasciculi lax or crowded; end articles shortly cuspidate, often piliferous.

Remarks.—One of the commonest of our fresh-water algæ is a plant belonging to this genus, which I think is probably the *C. elegans* of Roth. I am, however, unable to discover any characters separating *C. pisiformis*, *C. elegans*, and perhaps *C. tuberculosa*, and hardly know by which of the three names our American form should be known. Our plant grows generally in shaded pools, springs, and ditches in great abundance, adhering as little translucent balls to grasses, leaves, twigs, or anything that may be in the water. The size of the frond varies from the young one, not so large as a pin's head, to the old matured one, which may be nearly an inch in diameter. The color also varies greatly. It is always some shade of a pure green. The surface is mostly smooth, but sometimes it is so puckered up as to be a mass of large flat tubercles. It is these forms that I suppose to represent *C. tuberculosa.* The thallus is generally elastic, but at the same time soft, so that although readily compressed and pushed out of shape, it is entirely mashed with some difficulty, especially as, owing to its slipperiness, it constantly escapes from the grasp.

In regard to the individual filaments, the method of their branching and the proportionate length and breadth of the cells vary very much in different individuals and probably at different ages of the same individual.

Fig. 5, pl. 6, represents rather indifferently well a young individual of this species.

C. endiviæfolia, (ROTH) Ag.

Ch. thallo lineari, subplano, semipollicari vel pollicari, nonnunquam valde elongato, læte vel obscure viridi, dichotomo-subreticulatum-laciniato (nonnunquam habitu *Ricciæ fluitantis*); filis ramisque primariis plerumque achrois, passim viridi-zonatis, parallelis; ramulorum fasciculis lateralibus, plus minus densis, divaricato-patentibus; articulis plus minus tumidis, diametro æqualibus vel subæqualibus; geniculis constrictis; cytioplasmate granuloso effuso. (R.) *Species mihi ignota.*

Syn.—C. endiviæfolia, (ROTH) AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 383.

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Hab.—South Carolina; (Ravenel) Wood. Rhode Island; (S. T. Olney) Thwaites.

Thallus linear, flattish, of half to a whole thumb's breadth, sometimes greatly elongate, bright or obscure green, dichotomously subreticulately laciniate (sometimes with the habit of *Riccia fluitans*); filament and primary branches mostly colorless, sometimes zoned with green, parallel; lateral fasciculi of branches more or less dense, divaricately patent; joints more or less tumid, diameter equal or subequal; joints constricted; cytioplasm effused granulate.

Remarks.—I have never seen a living or well-preserved specimen of this species, and have, therefore, here simply copied the description of Prof. Rabenhorst. Prof. Ravenel has sent to me dried algæ labelled, and I think correctly, as belonging to this species, but their condition did not allow any scientific study of them.

Genus PILINIA, Ktz.

Fila articulata, erecta, simplicia vel dichotome ramosa, basi affixa, in stratum crustaceum subspongiosum, fragile aggregata. Propagatio adhuc ignota.

Filaments articulate, erect, dichotomously branched, fixed by the base, aggregated into a somewhat spongy fragile crustaceous stratum. Method of propagation unknown.

P. diluta, Wood, (sp. nov.)

P. rupicola, in strato cano-viridi disposita; filis ramisque fasciculatis, apice obtusis; articulis diametro 1½ plo–3½ plo longioribus.

Diam.—Max. 0.0004″.

Hab.—In fontibus maximis, prope Bellefonte, Centre County, Pennsylvania; Wood.

Growing on stones and rocks, forming a grayish-green stratum; filaments and branches fasciculate, with the apices obtuse; joints 1½–3½ times longer than broad.

Remarks.—Near Bellefonte, Centre County, Pennsylvania, there issues from the limestone rocks the largest spring I have ever seen, giving rise to a creek-like torrent, which supplies the city with water, and passes on scarcely diminished in volume. In this spring grows the curious algæ under consideration, forming a somewhat lubricous crustaceous and stony stratum on the stones and rocks in the basin. This stratum is of a grayish-green color, and is quite friable, breaking in the direction of the filaments with the greatest possible readiness. When placed under the microscope it is seen to be composed of filaments whose course is a direct one from the under to the upper surface. They are apparently rigid, preserving their courses, and not being intermatted. They are composed of cylindrical, confervoid cells, and are dichotomously branched, and yet when viewed as a whole the filament and its branches form a sort of fasciculus. The basal cell or cells appear to be globular. When I collected this plant I was forced by circumstances to put the specimens in carbolic-acid water for future study, and, therefore, I have had no opportunity of studying their method of reproduction. I am not altogether satisfied in referring this plant to the *Pilinia*, and yet all the most important of the characters given by Rabenhorst are preserved by it. It certainly, however, differs very greatly from *P. rimosa*, Ktz.

Genus APHANOCHÆTE, Braun.

Fila distincte articulata, prostrata, repentia, interdum in stratum irregulare plus minusve concreta; ramulis repentibus vel adscendentibus; cellulis chlorophyllaceis, apice vel dorso setigeris. Propagatio zoogonidiis.

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Threads distinctly articulate, prostrate, creeping, sometimes more or less concreted into an irregular stratum; branches creeping or ascending; chlorophyllous cells with the dorsum or apex setigerous. Propagation by zoospores.

Remarks.—Sexual reproduction has not as yet been discovered in this genus. According to Dr. Braun (Verjüng., Translation of the Ray Society, p. 184, &c.) two zoospores are generally formed in a cell by a division of its contents parallel to the septa, but occasionally this division not taking place, the cell contents are resolved into a single zoospore. The zoospores themselves are nearly globular, biciliate, and unprovided with any reddish eye-spot.

A. repens, Braun.

A. filis procumbentibus plerumque simplicibus; articulis cylindricis aut tumidis, diametro subæqualibus ad 1–2 plo longioribus; setis e cellularum dorso egressis, plerumque singulis sed interdum geminis, interdum nullis.

Diam.—Artic. $\frac{3}{12000}''$—$\frac{5}{12000}'' = .00025$—$.0004''$.

Syn.—*A. repens*, Braun. Rabenhorst, Flora. Europ Algarum, Sect. III p. 391.

Hab.—In Œdogoniis, prope Philadelphia; Wood.

Filaments procumbent, mostly simple; articles cylindrical or tumid, from as long as broad to twice as long; seta arising from the back of the cells, generally single, sometimes geminate, sometimes wanting.

Remarks.—The specimens from which the above description was drawn up, were found growing on the filaments of *Œdogonium mirabile*, Wood. They were remarkable for the rarity with which they were branched, for in but two or three cases out of a great number, were any branches detected. The articles were frequently twice as long as broad. In both these particulars the plant differs from the typical European *A. repens*, but the descriptions of that form are so short and imperfect that I have preferred retaining the name for the American plant.

Fig. 5, pl. 14, represents an ordinarily formed specimen magnified 460 diameters. It had been kept for some time in weak carbolic-acid solution, and although the green of the chlorophyll was perfectly preserved, the stumps only of the setæ were visible. How long the perfect setæ are I cannot at present say, not having made any notes on the fresh specimens.

Genus COLEOCHÆTE, Bréb. (1844).

Fila articulata ramosa aut in pulvinulum conjuncta aut in thallum planum subdisciformem parenchymaticum concreta; articuli oblongi, antice plus minus dilatati, angulo superiori vel dorso sæpe in setam basi vaginatam producti. Propagatio fit tum oosporis fœcundatione sexuali ortis, tum zoogonidiis. Zoogonidia in quaque cellula fructifera unica, forma subglobosa vel late ovalia, polo antico ciliis vibratoriis binis instructa. (R.)

Filaments articulated, branched, either conjoined into a little cumulated mass or parenchematously concreted into a plain subdisciform thallus; articles oblong anteriorly, more or less dilated, often furnished with a long seta on their dorsum or superior angle. Propagation occurring by means of oospores, formed by sexual organs or by zoospores. Zoospores subglobose or broadly oval, formed singly in the fertile cell, furnished at their anterior pole with vibratile cilia.

Remarks.—I have seen a large number of specimens of, as I believe, two distinct species of this genus, but never having found any fruiting fronds, have not been

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able to identify them. One of the forms grows in this immediate locality, and is very probably *C. scutata*, Bréb. The other was collected in Northern Michigan. It is characterized by its frond never being disciform, although composed of a single plane of cells parenchematously united.

CLASS **RHODOPHYCEÆ.**

Algæ multicellulares, vegetatione terminalis non limitata prædítæ plerumque trioicæ.

Thallus e cellularum seriebus vel stratis singulis vel pluribus compositus, aut nudus aut e cellularum strato corticatus, forma quam maxime varius; membranaceus (Porphyridium), crustaceus (Hildenbrandtia), filamentosus et verticillatim ramosus (Batrachospermum, Thorea), fasciiformis (Bangia), foliaceus, etc.

Cytioplasma plerumque rhodophyllo (Cohn), rarius phycho-chromate coloratum, granula amyloidea vel amylacea et sæpe guttulas oleosas includens.

Propagationis organa triplicis indolis, saepissime in plantas distinctas disposita.

1. *Organa mascula* vel *antherida* e fasciculis cellurarum plerumque moniliformibus ramosis, denique in spermatozoidea vel spermatia fœcundantia (*Sporidia* I. Ag.) oblonga vel ovalia, achrod, immobilia dissolutis formata.

2. *Organa feminea* vel *cystocarpia* Ktz. e soris nonnunquam moniliformibus formata, qui e placenta saepissime corticali evolvuntur, nudi vel cuticula mucilaginosa vel involucro inclusi, denique sporas (*polysporas*) numerosas immobiles mox germinantes emittunt. Foecundatur cystocarpium statu primordiali ope organi piliformis (*trichogyne* Thuret et Bornet) quorum spermatia copulantur.

3. *Tetrasporangia* e cellula corticali unica valde intumescente formata, divisione utriculi primordialis cruciata quadrilocularia; in quoque loculo (*cellulis secundariis, sororiis*) spora unica (*tetraspora*) se format, quae sine fœcundatione germinat. (R.)

Multicellular algæ, mostly triæcious, furnished with unlimited not terminal vegetation.

Thallus composed of cells in rows or in a simple or multiple stratum, either bare or provided with cortical strata of cells, exceedingly various in form; membranaceous (Pophyridium), crustaceous (Hildenbrandtia), filamentous and verticillately branched (Batrachospermum, Thorea), fasciate (Bangia), foliaceous, &c.

Cytioplasm mostly rhodophyllous, rarely phycochromatously colored, including amyloid granules or starch and frequently oil drops.

Propagation by means of three immotile organs, generally placed upon distinct plants.

1. *Antheridia* composed of mostly moniliformly branched fascicles of

cells, which dissolve into oblong, oval, transparent immotile spermatozoids (*Sporidia* Ag.).

2. *Cystocarpia* Ktz., or *Pistillidia*, formed of somewhat moniliform sori, which are evolved from a generally cortical placenta, and are naked or surrounded by a mucilaginous cuticle or involucre, and finally emit numerous immotile spores (*polyspores*), which quickly germinate. The fecundation of the cystocarpia occurs in their primordial state by contact of the spermatia with a piliform organ known as *trichogonia*.

3. *Tetrasporangia* formed of single, greatly swollen cortical cells, becoming cruciately quadrilocular by division of the primordial utricle; in each loculus (*secundary or sister cells*) a single spore (*tetraspore*) forms, which germinates without fecundation.

FAMILY PORPHYRACEÆ.

Thallus mucoso-membranaceus, foliaceus vel filamentosus, e cellularum seriebus vel strato unico formatus, plerumque purpurascens, valde lubricus.

Vegetatio fit cellularum divisione in duas vel omnes directiones repetita.

Propagatio fit tetrasporis. Cystocarpia nondum observata.

Thallus mucous-membranous, foliaceous or filamentous, formed of cells in series or in a single stratum, mostly purplish, very slippery.

Growth taking place by repeated division of the cells in two or all directions.

Propagation by means of tetraspores. Cystocarps not yet observed.

Remarks.—The only species of this family as yet observed in North America can hardly be said to have a definite thallus. They are rather multitudes of cells heaped together and closely attached to one another into a shapeless expanded mass.

Genus PORPHRYDIUM, NAEG. (1849).

Thallus mucoso-membranaceus, subcrustaceus, longe lateque expansus, e cellulis globosis vel polyedricis compositus. Propagatio adhuc ignota.

Thallus mucous-membranous, subcrustaceous, long and widely expanded, composed of globose or polyhedral cells. Propagation unknown.

P. cruentum, (AG.) NAEG.

P. thallo saturate purpuro-sanguineo, lubrico; cellulis anguloso-rotundatis. (R.)

Diam.—0.00027″—0.00035″. (R.)

Hab.—New York.

Syn.—*P. cruentum,* (AGARD.) NAEGEL. RABENHORST, Flora Europ. Algarum, Sect. III. p. 397.

Thallus deep crimson purple, slippery; cells angled and rounded.

Remarks.—The only specimen I have seen of this species was a little speck, adherent to a bone picked up on Governor's Island, in New York Harbor. It is very probable that it was a recent arrival, brought over, perchance, by some emigrant. For it I am indebted to Dr. Billings, U. S. A. The description and measurements given above are copied from Prof. Rabenhorst's work. My specimen agrees well with it.

P. magnificum, Wood.

P. cellulis globosis vel subglobosis, sæpe nonnihil polygonis et in massam indefinite expansam confluentibus; cytioplasmate purpureo, granulato; cytiodermate crasso, haud lamelloso.

Diam.—Cell cum. tegum. $\frac{8}{24000}$—$\frac{14}{24000}$. Tegum. $\frac{1}{30000}$—$\frac{1}{16000}$.

Syn.—*P. magnificum,* Wood, Proc. Am. Philos. Soc., 1869, p. 144.

Hab.—In terra humida, Texas; Prof. Ravenel.

Cells globose or subglobose, often somewhat polygonal and conjoined into an indefinite mass; endochrome purple, granulate; cell wall thick, not laminate.

Remarks.—This species, which was collected in Texas by Prof. Ravenel, growing, I believe, on wet sand, is very distinct from the European plant, differing essentially in size and form. In some instances the cells have a greenish tint, but this is possibly owing to immaturity, as such cells seem smaller than others. The whole mass to the eye has a very rosy purple tint, and although under the microscope it appears much darker and more purple, yet it often retains some of the roseate hue. At the edges of the masses the dark-reddish color often gives way to a very decided greenish tint, presenting an appearance which is very well represented in the drawing of the preceding species, in M. Mengehini's *Monographia Nostochinearum Italicarum, &c., Memoire della Reale Academia delle Scienze di Torrino.* The cells are often closely united by their thick coats into a very coherent mass. With the ordinary cells I have occasionally seen other larger ones, of an orange color, with very thick walls. Are these resting spores?

Fig. , pl. 19, represents single cells of this plant magnified 750 diameters.

Family CHANTRANSIACEÆ.

Thallus filamentosus. Fila articulata, e cellularum serie unica formata, ramosa, stricta, nuda, raro passim corticata, rami superne fasciculatim ramellosi; articuli cylindrici. Cytioderma, homogeneum, maxime hyalinum. Cytioplasma homogeneum, plerumque purpurascens. Propagatio fit polysporis immobilibus, ovalibus, in ramellorum apice vel lateraliter formatis, corymboso aggregatis. Antheridia subglobosa, terminalia. Tetraspora raro observatæ.

Thallus filamentous. Threads articulate, formed of a single series of cells, branched, straight, bare, rarely here and there articulate; branches above fasciculately branched; joints cylindrical. Cytioderm homogeneous, mostly hyaline, cytioplasm homogeneous, mostly purplish. Propagation by immovable oval polyspores formed on the ends of the branches or laterally and corymbosely aggregate. Antheridia subglobose terminal. Tetraspores rarely observed.

Genus CHANTRANSIA, Fries.

Familiæ genus unicum.

The only genus of the family.

C. expansa, Wood.

C. cæspitosa, in lapide stratum saturate violaceo-purpureum lubricum, indefinite expansum, formans; filis purpureis, modice ramosis, fere 2 lineas longis et ramis plerumque strictis et rectis, sæpe elongatis; ramulis fertilibus brevibus, ascendentibus; articulis diametro 3–8 plo

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longioribus, extremis obtusis; polysporis in ramellis lateralibus racemosim et confertim cumulatis, ovalibus vel nonnihil obovatis.

Diam.—Fil. $\frac{1}{2500}''=.0004''$. Spor. transv. $\frac{1}{3650}''=.00027$ long. $\frac{1}{2500}=.0004''$.

Syn.—*C. expansa*, WOOD, Prodomus, Proc. Amer. Philos. Soc., 1869.

Hab.—In rivulis, prope Philadelphia.

Cæspitose, forming a dark purple, slippery, indefinite stratum on stones; filaments purple, moderately branched, almost 2 lines long, together with the branches strict and straight, often elongate; infertile branches sometimes very few, sometimes very numerous; fertile branches short, ascending; joints 3–8 times as long as their diameter, the final articles obtusely rounded: polyspores racemose, crowded on the fertile branches, oval or somewhat ovate.

Remarks.—This species was found growing in a running stream, forming a felty slimy coating upon large stones, looking so much like a stratum of *Oscillatoria*, that when I gathered it I thought it probably was a representative of that genus. The stratum, however, when carefully examined, is seen to be made up of an indefinite number of minute, very closely approximate tufts. The color was a dark dull purple. The plant may possibly be the *Chantransia violacea*, of KÜTZING, which it resembles in many particulars, but it is nearly twice as long and the filaments are considerably thicker. Its habit of growth also seems to be essentially different from that of the European plant, so that I have finally decided to consider it a distinct species. The exact locality of its growth is in a thickly-shaded portion of the stream that runs along the North Pennsylvania Railroad, just this side of Chelten Hills.

Fig. 2, pl. 19, represents a filament magnified 125 diameters; fig. 2 *a*, a part of a fertile branch magnified 460 diameters.

C. macrospora, WOOD (sp. nov.).

C. cæspitosa, subpollicaris, olivaceo-grisea vel saturate violaceo-purpurea; filis ramosis et ramis plerumque strictis et rectis, et elongatis; articulis diametro 3–8 plo longioribus; ramulis fertilibus brevissimis; polysporis singulis vel geminis, sparsis, sæpe distantibus, globosis, interdum nonnihil ovalibus.

Diam.—Fil. plerumque .0008—max. .001. Polysp. .0009.

Hab.—South Carolina; (Ravenel).

Cæspitose, about an inch long, olive-gray to deep-violet purple; filaments a good deal branched, with the branches mostly straight and elongated; fertile branches very short; articles 3–8 times longer than broad; spores single or geminate, few, often distant, globose, or sometimes slightly oval.

Remarks.—I am indebted to Prof. Ravenel for specimens of this species preserved in carbolic-acid water. They are labelled, "Dull olive green, growing against wooden boards in spring, Nov. 5, 1869. Aiken, South Carolina." The most of the mass is of the color noted, or at least approaches it, but a portion is almost blackish purple. The species is a very distinct one, characterized by the larger diameter of its articles and spores, by the paucity and shape of the latter, as well as by its variance in coloration. In some old specimens the cell wall is distinctly lamellate. I have only seen fruit on the purple filaments. The

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spores, apparently not mature, have a greenish-brownish tint. I have also received from Prof. Ravenel dried algæ, which, apparently, are the same species as those from which this description has been written, but which, not being in fruit, cannot be absolutely identified. They are, as dried, of a bright bluish-green, and attain the length of an inch and a half or more.

Fig. 3, pl. 19, represents a part of a branch of this plant magnified 460 diameters.

FAMILY BATRACHOSPERMACEÆ.

Algæ dioicæ. Thallus filamentosus, articulatus, ramosus, aut violaceus, violaceo-purpureus vel cæruleo-viridis, muco matricali involutus; filis primariis ramisque e cellularum serie unica centrali primaria et seriebus numerosis secundariis parallelis continuis vel interruptis externis compositis, aut ramulorum fasciculis verticillatis globoso vel subgloboso dense conglobatis æquali distantia obsitis, aut ramulis simplicibus vel dichotomis dense ubique vestitis. Vegetatio terminalis.

Diæcious algæ. Thallus filamentous, articulate, branched, violet or violet-purple or bluish-green, covered with mucous; primary filament and branches composed of a single central series of cells, and numerous external, parallel, continuous, or interrupted secondary series; either furnished with globosely or subglobosely densely conglobate, equally distant verticillate fasciculi of branches, or everywhere densely covered with simple or dichotomous branches. Vegetation terminal.

Genus BATRACHOSPERMUM, ROTH, 1800.

Thallus filamentosus, moniliformis, e cellularum serie unica medullari, accessoriis parallelis corticata compositis, ramulorum fasciculis subgloboso-conglobatis obsessus.

Thallus moniliform, composed of a simple series of medullary cells and cortical accessory parallel series, clothed with subglobosely conglobate fasciculi of branches.

Remarks.—The *Batrachosperms* are amongst the very largest of the fresh-water algæ, forming gelatinous branched masses from a few inches to even more than a foot in length. The fronds are very freely and very irregularly branched, and are evidently composed throughout, *i. e.*, both in regard to the main filaments and the branches, of two portions, a central axis and much more slender short transverse branchlets, which often end in a long hair, and are arranged more or less exclusively in groups, so as to form, to the naked eye, at regular intervals, little balls or knots, the whole plant thus presenting a sort of moniliform aspect. Sometimes, however, these glomeruli are placed so closely together, and grow so large that they become confluent, and the branch to which they are attached appears as a uniform thick and very gelatinous cylindrical cord.

The axis both of the stem and the branches of a *Batrachosperm* consist originally of but a single series of cells. The development of new cells takes place in two ways, the one of which results simply in an increase in the length of the axis, the other in the production of branches. The first of these is the ordinary process of cell multiplication by division, and occurs only in the end cells, so that no new cells are ever formed in the central portions of the axis, which increases in length solely by the addition of new cells at the end, and by longitudinal growth of the old ones. The first step towards the formation of a branch is the production of a little pouch-like protrusion near the upper end of a cell. This increases in size and soon being cut off from the parent-cell by a partition, forms a complete cell, the starting point of a new branch. If this cell has been formed alone, without companions, it is the beginning of a main branch, and divides after a very brief period transversely, the new cell thus arising in a little while itself divides, and so the process goes on until the axis of a large branch, similar to the parent axis is developed, and which, like the parent axis, increases only by a division of the end cell and longitudinal growth of the central ones.

When a glomerulus is to be formed instead of a single pouch, a number appear around the upper end of a cell, and become cut off as new cells. Each of these is the starting point of a new row of cells, which not only grows, at least up to a certain point, by the division of the end cells, but which also gives rise to a large number of branches in a way precisely similar to that in which it itself was developed, *i. e.*, by the formation of little lateral protrusions, &c. These secondary branches have a life-history similar to that of the branch whose offspring they are. They continually give origin to new branchlets in the way just described, which branchlets themselves produce fresh offshoots, and so it goes on until at last the forest of branchlets making up the dense glomerulus is evolved. It has been just stated that the original axis of the main filament or any branch is composed of a single simple series of large cells; when an old *Batrachosperm* is placed under the microscope, however, it is at once evident that the axis is in reality formed of such a series lying in the centre and covered over and often hidden by numerous longitudinal series of smaller cells. These latter do not belong to the original axis, but are secondary additions to it, and arise in this way. Whilst a glomerulus is being developed certain of the basal cells of its constituent branches give origin in the usual manner to branchlets, which, instead of growing outward to form a part of the glomerulus, grow upwards or downwards, closely hugging and finally enveloping the original axis, and at last forming a distinct cortical layer to it.

Very frequently in well-advanced *Batrachosperms* there will be seen scattered among the glomerulus large, round, firm, dense balls composed of a great number of small closely-attached cells. These are the reproductive bodies. According to H. Graf zu Solms-Laubach (*Botanische Zeitung*, 1867, p. 161), they are the result of sexual reproduction, and are developed from *antheridia* and *trichogonia* (female organs) in the following manner:—

The antheridia are small roundish cells full of a colorless protoplasm, which is remarkable for the very numerous bright granules which it contains. They occur either scattered or in groups, and are placed upon the upper ends of peculiar ovate cells, also filled with a colorless protoplasm. Most frequently there is a single antheridium to the basal cell, sometimes two; the latter number appears never to be exceeded. When matured, the antheridia open and allow their contents to escape in the form of roundish or flattened bodies, which never, as far as known, acquire cilia, and have, therefore, no power of spontaneous motion. These bodies, which are believed to be spermatozoids, are unprovided with anything like an external membrane, and are composed of protoplasm identical with that in the antheridium.

Whilst these changes are occurring, certain cells in other localities are being transformed into female organs, to which our author applies the name of *Trichogonia*.

These are borne upon cells similar to those supporting the antheridia. At first they are not markedly different from the other cells, but soon undergo a very rapid growth. This is not, however, regular, and is not partaken of by a band of tissue about one-third way from the basal end, so that at last a long somewhat flask-shaped cell is produced, with a very marked contraction at the point indicated, separating it into two portions. The wall of this cell is thin but very distinct, and the cavity is filled with a homogeneous or very sparsely granular protoplasm, which is continuous through the narrow neck-like portion. After a time there appear one or more large irregular vacuoles, with actively moving corpuscles in them, and at the same time the neck appears to be stopped with a slimy substance. Careful examination with reagents shows that this is cellulose, and that it does not completely block the passage-way through the isthmus. At this time there appear lying upon the free end of the trichogonia globular or flattened bodies, without external membrane, corresponding in all respects with those already described as being produced in the antheridia. The end of the trichogonium generally enlarges at this period into a sort of roundish knob, and by and by the end wall between this and one of these globules becomes absorbed, so that there is a free communication between the two. Whilst this is going on the globule acquires a thin, delicate coat, and there appears in it a vacuole similar to those preexisting in the trichogonium.

The first result of this impregnation of the trichogonium is the deposit of new cellulose, and the complete blocking up of the passage-way through the isthmus or narrowed portion. Already before the fecundation, the upper cells of the branches supporting the trichogonia have produced numerous branchlets, which growing upwards more or less completely cover that organ. After impregnation the cells near to the trichogonium become much larger and broader, their vacuoles disappear, and are replaced by a dense granular dark greenish-brown protoplasm.

These cells now show a great activity in the production of numerous branches in the usual way, but it is the upper two alone which, with the *trichogonium* that they support, are concerned in the formation of the fruit glomerulus. These put out all over their surface an immense number of protrusions, which soon in the ordinary way become the parents of as many twigs or branchlets, which growing and branching, precisely as do the vegetative branches, soon become excessively crowded. The base of the trichogonium participates also in this production of branches, and at last a dense ball is formed of pseudoparenchymatous tissue by the forced adhesion of the crowded twigs. The central cells of the glomerulus thus formed are very large and bladder-like. The outer part of the ball is composed of innumerable radiating rows of small cells, the end cell of each branch being roundish so as to present a convex external face. At maturity these cells open and allow their contents to escape as round masses, which appear to have no membrane, but begin at once to grow and secrete cellulose. Their after-history has not been made out with absolute certainty, but they are believed to directly develop the new plant.

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B. moniliforme, (Roth.)

B. pollicare, bi- tripollicare, raro pedale, muco gelatinoso plus minus firmo involutum, violaceum, fuscum, rufo-brunneum, purpureum vel cæruleo-viridiscens, vage ramossissimum; ramulorum articulis omnibus conformibus, oblongo-subclavatis, extremis nonnunquam setigeris; internodiis nudis vel ramulis accessoriis singulis sparsis instructis.

Diam.—Tetrasp. globulus $\frac{27}{4500} = .006$.

Syn.—*B. moniliforme*, Roth. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 405.

Hab.—In aquis puris, Michigan; Gray. New York; Bailey. Virginia; Jackson, Alabama; Tuomey. South Carolina; (Ravenel) Pennsylvania; New Jersey; Wood.

One inch to a foot in length, clothed with a more or less firm gelatinous mucus, violet, fuscous, reddish-brown, purple, or bluish-green, vaguely and profusely branched; joints of the branches similar, oblong-subclavate, the outer ones sometimes setigerous; internodes naked or furnished with a few scattered accessory branchlets.

Remarks.—This species is very abundant in fresh, cool rivulets, in springs, in limestone waters, in pine-barren streams, and even occasionally in ditches, wherever I have botanized in Pennsylvania and New Jersey. It varies greatly in size, in color, and other particulars.

The branchlets, as I have observed them, are most generally not setigerous, but at times they are provided with seta of moderate length.

I have found numerous fruiting fronds, but in none of them was the fruit in great abundance, not nearly so much so as in the Rocky Mountain species.

B. vagum, (Roth) Agardh.

B. vage ramossissimum, uni- vel tripollicare, fuscum vel ærugineum; internodiis inferioribus ramellis numerosis obessis, superioribus nudis vel subnudis; ramulorum articulis extremis setis longissimis instructis.

Diam.—Tetrasp. globulus $\frac{15}{4500} = .00333$.

Syn.—*B. vagum*, (Roth) Agardh. Rabenhorst, Flora Europ. Algarum, Sect. III. p. 406.

Hab.—In aquis quietis, Uintah Mountains, Nevada; (S. Watson).

Vaguely branched, one to three inches long, brownish or æruginous; internodes—the inferior covered with a dense mass of branchlets—the superior naked, or nearly so; last articles of the branchlets provided with an extremely long seta.

Remarks.—I have received from Mr. Sereno Watson some half a dozen dried algæ, which I have referred to *B. vagum*, with some doubt. They are labelled as having grown in shallow water, in a beaver pond, in Pack's Cañon, Unitas, Uintah Mountains, Nevada, at an altitude of 7000 feet. All the descriptions of *B. vagum* which I have seen are singularly imperfect; in none is it stated how large the spore masses grow, and how plentifully the branchlets are provided with seta. As far as the descriptions go, however, my specimens agree with them, and I have, therefore, refrained from indicating a new species. The plants are remarkable for the profusion and extreme length of the seta, and for the quantity of fruit which they produce. The fruit masses are small but very compact, scarcely more than half the size of those of the preceding species. The verticles of branchlets are often completely joined, and as it were almost swallowed up by the mass of intervening scattered branchlets which arise directly from the main axis. In the distal

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portions of the fronds, however, the glomeruli are more fasciculate and more distinct, for although sometimes so close as to be almost confluent at their spreading edges, at their bases they are distinct. This species very probably attains a much larger size than indicated by my specimens, and possibly varies as much in color as *B. moniliforme*.

Genus TUOMEYA, Harvey.

"Frond cartilaginous, continuous, solid, at first transversely banded, afterwards annularly constricted; composed of a longitudinal axis, and two strata of peripheric cells. Axis columnar, consisting of several longitudinal cohering filaments, beset with closely placed whorls of moniliform ramelli, whose branches anastomose horizontally and vertically into a cellular peripheric membrane, which is coated externally with moniliform filaments, gradually developed. Fructification probably in the superficial filaments.

T. fluviatilis, Harvey.

Hab.—On stones, in rivers and streams. River in Alabama; Prof. Tuomey. Near Fredericksburg, Virginia; Prof. Bailey.

Fronds tufted, an inch or two in height, scarcely as thick as a hog's bristle, much and irregularly branched, bushy; the branches alternate or secund, scattered or crowded, twice or thrice divided, and set with scattered patent ramuli which are slightly constricted at the interstices, and taper to an obtuse point. When young the branches and ramuli are perfectly cylindrical, and when examined under a low power of the microscope show a surface composed of minute, dotlike cells, placed close together, and marked at short intervals with dark-colored transverse bands. These bands disappear under a higher magnifying power. They are indications of the nodes of the axis of the frond seen through the peripheric stratum. In old, fully developed specimens the branches and ramuli are annularly constricted at short intervals, the nodes becoming swollen, whilst the internodes remain unchanged. When a young branch is bruised between two pieces of glass the axis may be readily extracted. It consists of several parallel longitudinal jointed threads combined together at closely-placed nodes, from which issue horizontal dichotomous filaments, composed of roundish or angular cells. These excurrent filaments spread both horizontally and vertically, and their branches anastomose into a cellular mass or fleshy membrane, which forms the inner peripheric stratum. In young plants a portion of the frond, between the axis and periphery, is hollow, but in older ones the cavity is quite filled up with cells. The external surface of the cellular periphery is clothed with a coat of moniliform filaments gradually developed, and forms what is above called the second peripheric stratum. These are found only in fully-grown specimens; they consist of much smaller cells than those of the inner stratum; they are more strongly colored, and I consider them to be connected with fructification. The color is a dark olive. The substance is brittle, rigid when dry, and the plant scarcely adheres to the paper. The generic name is in memory of the late Prof. Tuomey, of Tuscaloosa."

Remarks.—I have no knowledge of this plant, and have simply copied the description of Prof. Harvey; Smithsonian Contributions, 1846.

Family LEMANEACEÆ.

Algæ rivulares vel fluviatiles. Thallus e prœmbryone confervacea enascens, setaceus, subsimplice vel fasciculatim ramosus, cavus, nodosus, e cellularum stratis internis et corticatis formatus. Noduli plerumque papillarum corona instructi. Polysporæ numerosæ, in seriebus ramosis moniliformibus fasciculatim aggregatæ, sine fecundatione germinantes.

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Algæ growing in streams and rivers. Thallus developing from a confervoid prothalloid filament, setaceous, almost simple or fasciculately branched, hollow, nodose, composed of internal and cortical strata of cells. Nodules generally provided with a corona or papilla. Polyspores numerous, fasciculately aggregated in branched moniliform series, germinating without fecundation.

Genus LEMANEA, Bory.

Genus unicum.

The only genus.

Remarks.—The plants belonging to the genus *Lemanea* are quite peculiar in aspect and habit. They grow exclusively in fresh water, especially frequenting streams whose current is rapid, and whose waters are chilled by the mountain air. Their frail, tubular, scarcely-branched fronds offer but little resistance to the water, whilst their lower end is swollen into a sort of discoid root, which adheres firmly to the stones. The frond is mostly blackish or brownish, and is formed of two distinct portions or layers, of which the outer or cortical is composed of small closely cohering, colored cells; the inner of much larger cells, which have thick colorless walls, and are placed so as to leave more or less numerous interspaces. In the immature frond there is also a longitudinal central column, besides some slender many-jointed filaments, passing obliquely through the cavity, but as final development takes place these seem to disappear. The mature frond is alternately contracted and expanded throughout its length. In the narrow portions the inner tissue often blocks up the tube entirely, whilst the dilated parts are loosely filled with the spores, which are produced within the frond. The spores themselves are oval, thickish-walled cells, whose endochrome changes from greenish to a very decided yellow during the process of maturing. They are joined together to form rows or series, which are not simple, but are very much branched, so that from a central basal row arises a complex bush-like mass (pl. 20, fig. 4). These spore-clusters are always distinct, a number of them existing in each sporangial node of the frond.

Dr. B. Wartmann described, nearly twenty years ago, very fully the way in which the spores germinate and develop into the frond. The first step, according to this authority, consists in the elongation of the spore and the projection of one end, which is soon cut off by the formation of a transverse partition, and constitutes a new cell. This multiplying in no strikingly peculiar way soon develops into a branched confervoid filament. A large number of these filaments are generally produced in one place at one time and form a very apparent greenish layer. Finally certain cells in branches of these filaments swell up and become very much broader than their fellows, undergoing, at the same time, division so rapidly that they become very short. By and by they divide also in the direction of their breadth, so that instead of a simple series of cells there arises a compound mass. This is the beginning of the new frond. At first it is dependent upon the parent filament, but soon acquires a root-like process at the base and develops rapidly into the complex cartilaginous plant.

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L. torulosa, (ROTH) AG.

L. subsimplex, plerumque arcuata, cartilaginea et nonnihil rigida, 1–2 pollices longa; nodulis approximatis, papillis applanatis, plerumque 4–6 enormiter verticellatis, vel nonnihil sparsis, interdum nonnihil confluentibus; sporis ovalibus.

Diam.—Sporis. transv. max. $\frac{8}{12000}''$—$\frac{14}{12000}''$.

Syn.—*L. torulosa* (ROTH) AG. RABENHORST, Flora: Europ. Algarum, Sect. III. p. 411.

Hab.—In flumine, Kentucky; (Short) Harvey. Pennsylvania; Virginia; New York; New Jersey; Wood.

Subsimple, mostly arcuate, cartilaginous and somewhat rigid, 1–2 inches long; nodules approximate, with their papules applanate, mostly 4–6, irregularly verticillate or somewhat scattered, sometimes slightly confluent; spores oval.

Remarks.—This plant attains a length of about two inches, and grows in masses attached to rocks, often forming a sort of turfy covering to them, in rapidly running water. In mass it has a grayish or blackish appearance. The filament has a grayish groundwork, with a dark band at the position of the nodes, which are enlarged and inclose the spores. The transverse outline of the filament is a very irregular circle. I have found this species very abundant in the rapid water of the Schuylkill, just above Flat Rock Tunnel, on the Reading Railroad, eight or nine miles above Philadelphia. Prof. E. D. Cope has sent me specimens collected by himself in swift streams in Western Virginia, and Mr. Austin has obtained it in similar situations in Northern New Jersey. Mr. Austin has also sent me specimens collected in Canada West.

L. fluviatilis, AG.

L. simplex vel parce ramosa, quatuor uncias longa (interdum spithamea?), recta vel subrecta; nodulis subremotis, papillis verticillatis magnis obssesis; sporis globosis vel subellipticis.

Diam.—Spor. $\frac{10}{72000}''$—$\frac{13}{12000}''$.

Syn.—*L. fluviatilis*, AGARDH. RABENHORST, Flora Europ. Algarum, Sect. III. p. 411.

Hab.—In rivulis, Alabama; T. M. Peters.

Simple or sparsely branched, 4 inches long (sometimes growing of a span length?), straight or nearly so; nodules rather distant, papillæ verticillate, large, prominent.

Remarks.—The only specimens I have seen of this species were sent me by Prof. Ravenel. This plant is larger and heavier than *L. torulosa*, from which it is also readily distinguished by its very large prominent papillæ. These are in slightly irregular whorls of three or more. The spores vary in shape from that of a globe to that of a somewhat four-sided ellipse; in the latter case being sometimes nearly twice as long as broad. Prof. Rabenhorst speaks of the plant attaining the length of a span. I have never seen it over four inches.

L. catenata, KTZ.

L. ad uncias 5 longa, regulariter constricta, simplex, compressa, arcuata, in massa obscure violacea; papillis nullis; sporis enormiter ovalibus vel subglobosis.

Diam—Spor. transv. max $\frac{12}{12000}'' = .001''$.

Syn.—*L. catenata*, KÜTZING. RABENHORST, Flora Europ. Algarum, Sec. III. p. 412.

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Hab.—In rivulis frigidis montanis Diamond Range, Rocky Mountains; (Sereno Watson).

About 5 inches long, regularly constricted, simple, compressed, arcuate, in mass obscure violet; papules wanting; spores irregularly oval or subglobose.

Remarks.—I have received specimens of the plant from which the above diagnosis was drawn, from Mr. Sereno Watson, labelled "Mountain stream, Diamond Range, altitude 6500 feet." In the dried state they are closely interwoven into a dark purple, rigid thin mass. When soaked out they preserve the same color in mass, but each individual stem has a general light yellowish, neutral ground tint, with dark-purplish or greenish-black bands at regular intervals. At the position of these bands the filament is nearly round and contracted, whilst between them it is compressed and enlarged. The spores are placed, not at the swelling, but at the constrictions, corresponding to the dark rings in position. They are quite irregular in shape, and of a faint yellow tint. The filaments between the little knots of spores appear to be hollow. Their walls are everywhere very thin when compared with *L. torulosa*, hence they are more flaccid. The species agrees in every respect with Prof. Rabenhorst's diagnosis of *L. catenata*, KTZ., a native of cold mountain streams of Germany and Switzerland. I regret, however, very greatly that I have had no opportunity of comparison with European specimens, or a fuller description.

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SUPPLEMENT.

The following species, of which the author has not seen specimens, were inadvertently omitted from their proper places in the monograph. They are all contained in the *Nereis Boreali-Americana* of Prof. Harvey. The following descriptions and remarks are simply copied from the work mentioned.

Tetraspora lacunosa, Chauv.

Frond at first tubular, then flat, or irregularly lobed, membranaceo-gelatinous, pale-green, everywhere pierced with roundish holes of various sizes. *Chauv. Alg. Norm. Breb. Alg. Fal. p.* 11, *t.* 1. *Kütz. Sp. Alg. p.* 227. *T. Godeyi, De Breb. Kütz. Tab. Phyc. t.* 30, *f.* 3. *T. perforata, Bailey, M.S.*

Hab.—In fresh-water streams. Abundant near Westpoint, Prof. Bailey; Providence, Rhode Island, Mr. Olney. (v. s. in Herb. T.C.D.)

Frond at first funnel-shaped, afterwards splitting open, and then flat, expanding upwards and irregularly lobed, everywhere pierced with roundish holes of various sizes, large and small intermixed. These holes increase in size and numbers with age, and thus at last the frond becomes an open network. The substance is very gelatinous, but rather firmer than in some other species of the genus. The color is a pale green; and the hyaline gelatinous membrane is filled with roundish granules set in fours.

Kützing's figure of *T. Godeyi* answers well to our plant. I have not seen any authentic specimens of *T. lacunosa*, which is referred by Kützing to his *T. lubrica, var. β.*, but the description given of it applies to the American plant. When carefully dried, it forms a very pretty object for the herbarium. (*Chlorospermeæ*, p. 61.) (*Harvey*, p. 61.)

Nostoc (Hormosiphon) arcticum, Berk.

Fronds foliaceous, variously plaited, green or brownish; filaments at length (their gelatinous envelope being dissolved) free. *Berk. in Proc. Lin. Soc. fide An. Nat. Hist. 2d Ser. vol.* 10, *p.* 302.

Hab.—On the naked soil, in boggy ground. Assistance Bay, lat. 75° 40′ N. Dr. Sutherland. (v.s.)

"*Fronds* foliaceous, variously plicate, sometimes contracted into a little ball. Gelatinous envelope at length effused; connecting cells at first solitary, then three together; threads, which are nearly twice as thick as in *N. commune*, breaking up at the connecting cells, so as to form new threads, each terminated with a single large cell, the central cell becoming free." *Berk. l. c.*

"It grows," says Dr. Sutherland, "upon the soft and almost boggy slopes around Assistance Bay; and when these slopes become frozen at the close of the season, the plant lying upon the surface in irregularly plicated masses becomes loosened, and if it is not at once covered with snow, which is not always the case, the wind carries it about in all directions. Sometimes it is blown out to sea, where one can pick it up on the surface of the ice, over a depth of probably one hundred fathoms. It has been found at a distance of two miles from the land, where the wind had carried it. At this distance from the land it was infested with *Poduræ*, and I accounted for this fact by presuming that the insects of the previous year had deposited their ova in the plant upon the land, where also the same species could be seen in myriads upon the little purling rivulets, at the side of which the Nostoc was very abundant." At p. 205 of his Journal, Dr. Sutherland further mentions having tried it as an article of food, and found it preferable to the Tripe de Roche of the arctic hunters. Its nutritive qualities are probably equal to those of the jelly derived from other Algæ. (*Chlorospermeæ*, p. 113.)

Nostoc flagelliforme, BERK. and CURT.

Terrestrial; frond cartilaginous, linear, very narrow, compressed and often channelled, much branched, irregularly dichotomous; branches solid, densely filled with moniliform curved threads. *Berk. and Curt.* No. 3809.

Hab.—On naked aluminous soil, at San Pedro, Texas, Mr. Charles Wright. (v.s.)

Fronds several inches in length, half a line in diameter, lying prostrate on the surface of the soil, much branched in an irregularly dichotomous manner; branches exactly linear, compressed, often channelled on one or both sides, thinned in the middle and incrassated to the edge. *Substance* firm and elastic, cartilaginous, solid, densely filled with moniliform, curved or curled, interlaced threads, which are set longitudinally in the frond, and lie nearly parallel to each other. *Color* dark olive.

A very curious and most distinctly marked species, differing from others of this genus, much in the same manner that *Chætophora endiviæfolia* does from the ordinary globose forms of *Chætophora*. (*Chlorospermeæ*, p. 115.)

Nostoc microscopicum, CARM.

Fronds densely aggregated, very minute, globose or oblong, immersed in a blackish crust; filaments few. *Carm. in Hook. Brit. Fl.* 2, *p.* 399. *Harv. Man. Ed.* 1, *p.* 184. *N. muscorum, Hass. Br. Fr. Wat. Alg. p.* 292, *t.* 74, *fig.* 4.

Hab.—"Stones in a small stream, Baffin's Bay," Dr. Sutherland, *fide* Prof. Dickie.

I have not seen American specimens. In Britain this species grows among mosses on exposed calcareous rocks, but not in water. The above specific character is taken from the British plant. The fronds are rarely more than the tenth of an inch in diameter, and contain two or three beaded filaments lying in a copious transparent jelly. (*Chlorospermeæ*, p. 115.)

Genus HYDRURUS, AG.

Frond fixed at base, cylindrical or compressed, elongated, branched, gelatinous. *Structure:* seriated, but separate, cellules, filled with bright-green endochrome, inclosed in gelatinous parallel tubes, ranged longitudinally in the frond, and surrounded by a common gelatinous envelope.

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Of this genus several species have been described by authors, all having a close resemblance to each other, and all very variable in ramification. Indeed it is almost impossible to fix characters by which they can be permanently kept apart; and instead of adding another specific name to the already too numerous list, I prefer to consider the American specimens received as constituting a luxuriant variety of the best known of the established species. All previously recorded species or varieties of these plants are natives of rapid rivers and streams in various parts of Europe. (*Chlorospermeæ*, p. 118.)

Hydrurus penicillatus, var. **occidentalis,** Harv.

Frond very long (1–2 feet or more), much branched; branches very irregular, scattered or crowded, wormlike, tapering to a fine point, naked or clothed with feathery villous ramuli; cells ellipsoidal or pear-shaped, twice as long as their diameter.

Hab.—On the rocky bottom of rivers and streams, in a strong current. Santa Fe, New Mexico, Mr. Fendler, February to April, 1847. (v.s. in Herb. T.C.D.)

Fronds attached at base, one or two feet long, from one to four lines in diameter, very much and irregularly branched; branches scattered or crowded, simple or divided, a foot or more in length, attenuated to a fine point, sometimes smooth and naked, but generally densely clothed with slender, villous ramenta, spreading to all sides. The gelatinous tubes or sheaths in which the cells are seriated are very obvious, and lie close together in longitudinal, parallel strata. The cells are of large size, bright-green color, and variable shape; some are twice as long as others.

This I had at first supposed to be a new species, but now regard it as a very gigantic state of *H. penicillatus*, Ag., which under various forms and of various sizes is common in alpine streams in Europe. I fear characters derived from the shape and size of the cellules are not more to be depended upon than are those taken from the ramification. (*Chlorospermeæ*, p. 118.)

Draparnaldia opposita, Ag.

Frond vaguely much branched; joints of the main filament as long as broad, or shorter; pencils of ramuli mostly opposite, densely set, lanceolate-acuminate in outline, plumose, bi-tripinnate, the apices much attenuated. *Ag. Syst. p.* 59. *Kütz. Sp. Alg.* 357. *Lyngb. Hyd. Dan. tab.* 65, *fig. A. Batrachospermum Americanum, Schweinitz.*

Hab.—In clear streams. New York, Professor Bailey. New Jersey, Mr. Jackson. (v.s.)

Frond 2–3 inches long, gelatinous, capillary, irregularly much branched; the branches patent, lateral, more or less divided, and set with lesser ramuli. Main filaments with short articulations, as long as their breadth, or shorter, transversely banded. At every two or three nodes and sometimes at every node a pair of opposite penicillato-multifid ramuli are thrown off. These are bright green, ovato-lanceolate in outline, much acuminated and twice or thrice pinnate, their pinnules somewhat constricted at the nodes, and tapering at the apex into long, needle-like, hyaline points. Their cells are commonly nucleated and filled with endochrome.

Whether this be permanently distinguishable from *D. glomerata* is doubtful. It has externally the aspect of that species, but its microscopic characters are nearer those of *D. plumosa.*

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GEOGRAPHICAL LIST OF SPECIES.

CLASS **PHYCOCHROMOPHYCEÆ.**

ORDER CYSTIPHORÆ.

Family CHROOCOCCACEÆ.

Chroococcus

refractus, *Wood.* *Hab.* near Philadelphia.
multicoloratus, *Wood* *Hab.* near Philadelphia.
thermophilus, *Wood.* *Hab.* Benton Springs, Owen Co., California.

Gloeocapsa

sparsa, *Wood.*

Cælosphærium

dubium, *Grün.* *Hab.* near Philadelphia.

Merismopedia

nova, *Wood.* *Hab.* near Philadelphia.
convoluta, *Bréb.* *Hab.* Spring Mills, Montgomery Co., Pa.

ORDER NEMATOGENEÆ.

Family OSCILLARIACEÆ.

Oscillaria

chlorina, *Ktz.* *Hab.* near Philadelphia.
corium, *Ag.* *Hab.* New York.
decorticans, *Gener.* *Hab.* Northern U. States.
Fröhlichii, *Ktz.* *Hab.* Schuylkill River, near Philadelphia.
imperator, *Wood.* *Hab.* near Philadelphia.
limosa, *Ag.* *Hab.* near Camden, New Jersey.
muscorum, *Ag.* *Hab.* West Point, New York.
neglecta, *Wood.* *Hab.* near Philadelphia.
nigra, *Vauch.* *Hab.* New York; Philadelphia.
tenuis, *Ag.* *Hab.* Rhode Island; New York; Virginia.
tenuissima, *Ag.* *Hab.* Warm Springs of Washita.

Chthonoblastus

repens, *Ktz.* *Hab.* New York; Massachusetts; Rhode Island.

Lyngbya

bicolor, *Wood.* *Hab.* Schuylkill River, near Philadelphia.
muralis, *Ag.* *Hab.* Whale Fish Islands, Davis Straits, British America.

Family NOSTOCHACEÆ.

Sub-Family NOSTOCEÆ.

Nostoc

Austinii, *Wood.* *Hab.* New Jersey.
alpinum, *Ktz.* *Hab.* Alleghany Mountains; Clover Mts., Nevada; Baffin's Bay, British America.
calcicola, *Ag.* *Hab.* Catoosa Springs, Georgia.
calidarium, *Wood.* *Hab.* Benton Springs, Owen Co., California.
cæruleum, *Lyn.* *Hab.* New Jersey.
Cesatii, *Bals.* *Hab.* Kansas.
comminutum, *Ktz.* *Hab.* near Philadelphia.
commune, *Vauch.* *Hab.* New Jersey; Rio Bravo.
depressum, *Wood.* *Hab.* New Jersey.
punctatum, *Wood.* *Hab.* New Jersey.
pruniforme, *Agh.* *Hab.* New Jersey.
verrucosum, *Vauch.* *Hab.* Maine.
sphæricum, *Vauch.* *Hab.* Centre Co., Pennsylvania.

Sub-Family SPERMOSIREÆ.

Anabæna

gelatinosa, *Wood.* *Hab.* near Philadelphia.
gigantea, *Wood.* *Hab.* near Philadelphia.
flos-aquæ, *Ktz.* *Hab.* Round Pond, West Point, New York.

Cylindrospermum

comatum, *Wood.* *Hab.* Niagara, Canada.
flexuosum, *Rab.* *Hab.* near Philadelphia.
macrospermum, *Ktz.* *Hab.* South Carolina.
minutum, *Wood.* *Hab.* near Philadelphia.

Dolichospermum

polyspermum, *Ktz.* *Hab.* near Philadelphia.
subrigidum, *Wood.* *Hab.* New Jersey.

Family RIVULARIACEÆ.

Nostochopsis

lobatus, *Wood.* *Hab.* Schuylkill River, near Philadelphia.

Gloiotrichia

angulosa, *Roth.* *Hab.* Hudson River, near West Point.

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Gloiotrichia

incrustata, *Wood*. *Hab*. Schuylkill River near Philadelphia.

Rivularia

cartilaginea, *Wood*. *Hab*. Northern Michigan.

Zonotrichia

minutula, *Wood*. *Hab*. Clear Pond, Adirondack Mountains.
mollis, *Wood*. *Hab*. Cave of the Winds, Niagara.
parcezonata, *Wood*. *Hab*. Cave of the Winds, Niagara.

Dasyactis

mollis, *Wood*. *Hab*. Cass River, Northern Michigan.

Mastigonema

elongatum, *Wood*. *Hab*. Philadelphia.
fertile, *Wood*. *Hab*. Alleghany Mountains, Centre Co., Pennsylvania.
halos, *Wood*. *Hab*. Stonington, Connecticut.
sejunctum, *Wood*. *Hab*. Cass River, Northern Michigan.

Mastigothrix

fibrosa, *Wood*. *Hab*. near Philadelphia.

Family SCYTONEMACEÆ.

Scytonema

Austinii, *Wood*. *Hab*. Little Falls, New Jersey.
calotrichoides, *Ktz*. *Hab*. South Carolina.
cataracta, *Wood*. *Hab*. Niagara River, Niagara.
cortex, *Wood*. *Hab*. South Carolina.
dubium, *Wood*. *Hab*. Cumberland Co., New Jersey.
immersum, *Wood*. *Hab*. Cumberland Co., New Jersey.
Myochrous, *Ag*. *Hab*. West of Crow's Neck, West Point.
Nægelii, *Ktz*. *Hab*. near Bellefonte, Centre Co., Pennsylvania.
Ravenellii, *Wood*. *Hab*. South Carolina.
simplice, *Wood*. *Hab*. Aiken, South Carolina.
thermale. *Hab*. South Carolina.

Tolypothrix

distorta, *Mul*. *Hab*. near Philadelphia; West Point, N. Y.; Rhode Island; Madison, Wisconsin.

Family SIROSIPHONACEÆ.

Sirosiphon

acervatus, *Wood*. *Hab*. South Carolina.
argillaceus, *Wood*. *Hab*. South Carolina.
compactus, *Ag*. *Hab*. Salem, Massachusetts; New Jersey.
Crameri, *Br*. *Hab*. Mount Tahawus, Adirondack Mountains.

Sirosiphon

guttula, *Wood*. *Hab*. South Carolina.
lignicola, *Wood*. *Hab*. South Carolina.
neglectus, *Wood*. *Hab*. New Jersey.
pellucidulus, *Wood*. *Hab*. near Hibernia, Florida.
pulvinatus. *Hab*. Northern New Jersey.
scytenematoides, *Wood*. *Hab*. South Carolina.

Stigonema

Ravenellii, *Berkeley*. *Hab*. Lookout Mountains, Georgia.

CLASS **CHLOROPHYLLACEÆ**.

ORDER COCCOPHYCEÆ

Family PALMELLACEÆ.

Pleurococcus

pulvereus, *Wood*. *Hab*. Boiling Springs, near Bellefonte, Centre Co., Pennsylvania.
seriatus, *Wood*. *Hab*. New Jersey.

Palmella

dura, *Wood*. *Hab*. near Philadelphia.
hyalina, *Lyn*. *Hab*. From Rhode Island to Wisconsin.
Jesenii, *Wood*. *Hab*. near Philadelphia.

Pagerogalla

stellio, *Wood*. *Hab*. Bear Meadows, Alleghany Mountains, Centre Co., Pennsylvania.

Tetraspora

bullosa, *Roth*. *Hab*. Salem, North Carolina.
gelatinosa, *Roth*. *Hab*. Salem, North Carolina; Newburgh, New York.
lubrica, *Roth*. *Hab*. Northern Atlantic States.

Dictyosphærium

pulchellum, *Wood*. *Hab*. near Philadelphia.

Rhaphidium

falcatum. *Hab*. near Philadelphia.
polymorphum, *Fr*. *Hab*. near Philadelphia.

Family PROTOCOCCACEÆ.

Polyedrium

enorme, *Ralfs*. *Hab*. Florida.

Scenedesmus

acutus, *Meyen*. *Hab*. Rhode Island; near Philadelphia.
obtusus, *Meyen*. *Hab*. Georgia; Rhode Island.
polymorphus, *Wood*. *Hab*. near Philadelphia.
quadricauda, *Turp*. *Hab*. Rhode Island; Pennsylvania.
rotundatus, *Wood*. *Hab*. near Philadelphia.

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Hydrodictyon

utriculatum, *Roth*. *Hab.* West Point and Weehawken, New York; Mexican Boundary; Pennsylvania; New Jersey.

Pediastrum

Boryanum, *Tur*. *Hab.* Rhode Island; Pennsylvania; Georgia; Florida.
constrictum, *Hassall*. *Hab.* South Carolina; Georgia; Rhode Island.
duodenarius. *Hab.* South Carolina; Rhode Island.
Ehrenbergii, *Corda*. *Hab.* Rhode Island; South Carolina; Georgia; Florida.
pertusum, *Ktz*. *Hab.* Rhode Island.
Selenæa, *Ktz*. *Hab.* Rhode Island.

Family VOLVOCINEÆ.

Chlamydococcus

nivalis. *Hab.* Greenland; Rocky Mountains.

Volvox

globator, *Linn*. *Hab.* United States.

ORDER ZYGOPHYCEÆ.

Family DESMIDIACEÆ.

Palmoglœa

clepsydra, *Wood*. *Hab.* near Philadelphia.

Penium

Brébissonii, *Men*. *Hab.* South Carolina.
closterioides, *Ralfs*. *Hab.* South Carolina.
Digitus, *Ehrb*. *Hab.* Pennsylvania; New York; Georgia.
interruptum, *Bréb*. *Hab.* near Grahamsville, South Carolina.
Jenneri, *Ralfs*. *Hab.* Florida.
lamellosum, *Bréb*. *Hab.* Rhode Island.
margaritaceum, *Ehrb*. *Hab.* Rhode Island.
minutum, *Cleve*. *Hab.* Rhode Island; South Carolina; Georgia.

Closterium

acerosum, *Schr*. *Hab.* South Carolina; Georgia; Florida.
Amblyonema, *Ehrb*. *Hab.* West Point, New York; Providence, Rhode Island.
angustatum, *Ktz*. *Hab.* Rhode Island; New Hampshire; Pennsylvania.
areolatum, *Wood*. *Hab.* Northumberland Co., Pennsylvania.
Cucumis, *Ehrb*. *Hab.* New York.
Dianæ, *Ehrb*. *Hab.* Georgia; Florida; Pennsylvania; Rhode Island.
Ehrenbergii, *Men*. *Hab.* Philadelphia.
Jennerii, *Ralfs*. *Hab.* Rhode Island.
juncidum, *Ralfs*. *Hab.* Saco Lake, New Hampshire; South Carolina.

Closterium

Leibleinii, *Ktz*. *Hab.* Georgia; South Carolina; Pennsylvania.
lineatum, *Ehrb*. *Hab.* Pennsylvania.
Lunula, *Müller*. *Hab.* South Carolina; Florida; Georgia; Pennsylvania.
maximum, *var*. *Hab.* Pennsylvania.
moniliferum, *Bory*. *Hab.* Georgia; Rhode Island.
parvulum, *Næg*. *Hab.* near Philadelphia.
rostratum, *Ehrb*. *Hab.* near Philadelphia.
setaceum, *Ehrb*. *Hab.* Stonington, Connecticut; Providence, Rhode Island; Pennsylvania; Georgia; Florida.
striolatum, *Ehrb*. *Hab.* Centre Co., Pennsylvania.
Venus, *Ktz*. *Hab.* South Carolina.

Tetmemorus

Brébissonii, *Men*. *Hab.* Atlantic States.
giganteus, *Wood*. *Hab.* Centre Co., Pennsylvania.
granulatus, *Bréb*. *Hab.* Rhode Island; Pennsylvania; South Carolina.
levis, *Ktz*. *Hab.* near Philadelphia.

Pleurotænium

Baculum, *Bréb*. *Hab.* Georgia.
breve, *Wood*. *Hab.* District of Columbia.
clavatum, *Ktz*. *Hab.* South Carolina; Georgia.
constrictum, *Bailey*. *Hab.* Rhode Island.
crenulatum, *Ehrb*. *Hab.* Rhode Island; New Jersey; Pennsylvania; South Carolina; Georgia; Florida.
gracile, *Rab*. *Hab.* Florida.
hirsutum, *Bailey*. *Hab.* United States.
nodosum, *Bailey*. *Hab.* South Carolina; Georgia; Florida; Pennsylvania.
Trabecula, *Ehrb*. *Hab.* Pennsylvania; New Jersey; South Carolina; Georgia; Florida.
undulatum, *Bailey*. *Hab.* Florida.
verrucosum, *Bailey*. *Hab.* Rhode Island.

Triploceras

gracile, *Bailey*. *Hab.* Rhode Island; New Jersey; New Hampshire; Florida; Georgia.
verticillatum, *Bailey*. *Hab.* with the last.

Spirotænia

bryophila, *Bréb*. *Hab.* near Philadelphia.
condensata, *Bréb*. *Hab.* Pennsylvania; Rhode Island; Florida.

Bambusina

Brébissonii, *Ktz*. *Hab.* Florida; Georgia; South Carolina; Rhode Island.

Didymoprium

Grevillii, *Ktz*. *Hab.* Pennsylvania; South Carolina; Georgia.

Sphærozosma

excavatum, *Ralfs.*	*Hab.* Rhode Island; South Carolina; Georgia; Florida.
pulchrum, *Bailey.*	*Hab.* New York; New Jersey.
serratum, *Bailey.*	*Hab.* South Carolina; Georgia; Florida.

Hyalotheca

disilliens, *Smith.*	*Hab.* Rhode Island; Pennsylvania; South Carolina; Florida.
mucosa, *Mert.*	*Hab.* Rhode Island.

Desmidium

aptogonium, *Bréb.*	*Hab.* South Carolina; Georgia.
quadrangulatum, *Ktz.*	*Hab.* South Carolina.
Swartzii, *Ag.*	*Hab.* Atlantic States.

Aptogonium

Baileyi, *Ralfs.*	*Hab.* Rhode Island; New Jersey.

Cosmarium

amœnum, *Bréb.*	*Hab.* Florida; Rhode Island.
bioculatum, *Bréb.*	*Hab.* Rhode Island.
Botrytis, *Bory.*	*Hab.* Pennsylvania.
Brébissonii, *Men.*	*Hab.* White Mountains, New Hampshire.
Broomei, *Thw.*	*Hab.* Pennsylvania; Georgia.
cælatum, *Ralfs.*	*Hab.* near Albany, New York; South Carolina.
commissurale, *Bréb.*	*Hab.* White Mountains, New Hampshire.
connatum, *Bréb.*	*Hab.* Florida.
crenatum, *Ralfs.*	*Hab.* Rhode Island.
cucumis, *Corda.*	*Hab.* New Hampshire; Pennsylvania; South Carolina; Georgia; Florida.
depressum, *Bailey.*	*Hab.* Florida.
margaritiferum, *Turp.*	*Hab.* Pennsylvania; South Carolina; Florida; Mexico.
Meneghenii, *Bréb.*	*Hab.* Pennsylvania.
ornatum, *Ralfs.*	*Hab.* Rhode Island.
ovale, *Ralfs.*	*Hab.* Pennsylvania.
pyramidatum, *Bréb*	*Hab.* Pennsylvania; Georgia; Florida.
Quimbyii, *Wood.*	*Hab.* near Philadelphia.
sublobatum, *Bréb.*	*Hab.* Rhode Island; Georgia; Florida.
suborbiculare, *Wood.*	*Hab.* Lake Saco, New Hampshire.
tetropthalmum, *Ktz.*	*Hab.* New Jersey.
Thwatesii, *Ralfs.*	*Hab.* Florida.
undulatum, *Corda.*	*Hab.* Rhode Island; South Carolina.

Euastrum

affine, *Ralfs.*	*Hab.* South Carolina; Georgia.
ampullaceum, *Ralfs.*	*Hab.* South Carolina; Florida.

Euastrum.

binale, *Turp.*	*Hab.* Florida; Pennsylvania; Rhode Island.
circulare, *Hassal.*	*Hab.* Rhode Island.
crassum, *Bréb.*	*Hab.* United States.
Didelta, *Turp.*	*Hab.* South Carolina; Georgia; Pennsylvania; Rhode Island.
elegans, *Bréb.*	*Hab.* United States.
gemmatum, *Bréb.*	*Hab.* Rhode Island.
insigne, *Ralfs.*	*Hab.* Florida; Rhode Island.
multilobatum, *Wood.*	*Hab.* Saco Lake, New Hampshire.
oblongum, *Greville.*	*Hab.* Rhode Island.
ornatum, *Wood.*	*Hab.* Saco Lake, New Hampshire.
Ralfsii, *Rabenh.*	*Hab.* South Carolina; New Hampshire; Rhode Island.
verrucosum, *Ehr.*	*Hab.* Rhode Island; South Carolina; Georgia; Florida.

Micrasterias

Americana, *Ehrb.*	*Hab.* Florida; South Carolina.
arcuata, *Bailey.*	*Hab.* Florida.
Baileyi, *Ralfs.*	*Hab.* New York; Rhode Island; South Carolina; Florida.
denticulata, *Bréb.*	*Hab.* Pennsylvania; Florida.
disputata, *Wood.*	*Hab.* Atlantic States.
expansa, *Bailey.*	*Hab.* Florida.
fimbriata, *Ralfs.*	*Hab.* South Carolina; Florida.
foliacea, *Bailey.*	*Hab.* Worden's Pond, Rhode Island.
furcata, *Ag.*	*Hab.* Atlantic States.
granulata, *Wood.*	*Hab.* South Carolina.
Jenneri, *Ralfs.*	*Hab.* near Philadelphia.
oscitans, *Ralfs.*	*Hab.* Florida; Rhode Island.
papillifera, *Bréb.*	*Hab.* Florida; Rhode Island.
pinnatifida, *Ktz.*	*Hab.*
quadrata, *Bailey.*	*Hab.* Florida.
radiosa, *Ag.*	*Hab.* Florida.
ringens, *Bailey.*	*Hab.* Florida.
Torreyi, *Bailey.*	*Hab.* near Princeton, New Jersey.
truncata, *Corda.*	*Hab.* Atlantic States.

Staurastrum

alternans, *Bréb.*	*Hab.* Georgia; Florida; Rhode Island.
arachne, *Ralfs.*	*Hab.* Saco Lake, New Hampshire.
aristiferum, *Ralfs.*	*Hab.* Georgia; Rhode Island.
Cerberus, *Bailey.*	*Hab.* Florida.
crenatum, *Bailey.*	*Hab.* Rhode Island.
cyrtocerum, *var.*, *Bréb.*	*Hab.* Florida.
dejectum, *Bréb.*	*Hab.* New York; South Carolina.
dilatatum, *Ehrb.*	*Hab.* Southern Atlantic States.
eustephanum, *Ralfs.*	*Hab.* West Point, New York.
furcigerum, *Bréb.*	*Hab.* South Carolina; Florida; Rhode Island.

Staurastrum

gracile, *Ralfs.*	*Hab.* South Carolina; Georgia; Florida; New York; Rhode Island.
hirsutum, *Ehrb.*	*Hab.* Florida; Rhode Island.
Hystrix, *Ralfs.*	*Hab.* Rhode Island.
Lewisii, *Wood.*	*Hab.* Saco Lake, New Hampshire.
longispinum, *Arch.*	*Hab.* Florida.
margaritaceum, *Ehrb.*	*Hab.* Saco Lake, New Hampshire.
munitum, *Wood.*	*Hab.* Saco Lake, New Hampshire.
muticum, *Bréb.*	*Hab.* South Carolina; Rhode Island.
orbiculare, *Ehrb.*	*Hab.* Rhode Island; Pennsylvania.
paradoxum, *Mey.*	*Hab.* Saco Lake, New Hampshire.
polymorphum.	*Hab.* Florida.
polytrichum, *Per.*	*Hab.* near Philadelphia.
punctulatum, *Bréb.*	*Hab.* Pennsylvania.
Ravenellii, *Wood.*	*Hab.* South Carolina.
senarium, *Ehrb.*	*Hab.* America.
tricorne, *Men.*	*Hab.* Georgia; Florida; Rhode Island.

Xanthidium

aculeatum, *Ehrb.*	*Hab.* near Savannah, Georgia.
Arctiscon, *Ehrb.*	*Hab.* North America.
armatum, *Bréb.*	*Hab.* South Carolina; Florida; New Hampshire.
bisenarium, *Ehrb.*	*Hab.* America.
cristatum, *Bréb.*	*Hab.* Southern Atlantic States.
coronatum, *Ehrb.*	*Hab.* America.
fasciculatum, *Ehrb.*	*Hab.* South Carolina; Georgia; Florida; Rhode Island.

Arthrodesmus

convergens, *Ehrb.*	*Hab.* South Carolina; Georgia; Florida; Rhode Island.
Incus, *Bréb.*	*Hab.* Georgia; Florida; South Carolina; Rhode Island.
octocornis, *Ehrb.*	*Hab.* Florida; Rhode Island.
quadridens, *Wood.*	*Hab.* Lake Saco, New Hampshire.

Family ZYGNEMACEÆ.

Spirogyra

crassa, *Ktz.*	*Hab.* near Philadelphia.
decimina, *Mül.*	*Hab.* near Philadelphia.
diluta, *Wood.*	*Hab.* near Philadelphia.
dubia, *Ktz.*	*Hab.* near Philadelphia.
elongata, *Berk.*	*Hab.* near Philadelphia.
insignis, *Has.*	*Hab.* near Philadelphia.
longata, *Vauch.*	*Hab.* Rhode Island; near Philadelphia.
majuscula, *Ktz.*	*Hab.* near Philadelphia.
nitida, *Dill.*	*Hab.* near Philadelphia.
protecta, *Wood.*	*Hab.* near Philadelphia.
parvispora, *Wood.*	*Hab.* Hibernia, Florida.
pulchella, *Wood.*	*Hab.* near Philadelphia.
quinina, *Ag.*	*Hab.* near Philadelphia.
rivularis, *Hassall.*	*Hab.* Florida.
setiformis, *Roth.*	*Hab.* near Philadelphia.
Weberi, *Ktz.*	*Hab.* near Philadelphia.

Zygnema

insigne, *Hassal.*	*Hab.* Rhode Island; near Philadelphia.
cruciatum, *Vauch.*	*Hab.* Virginia; Florida; Northern States.

Sirogonium

retroversum, *Wood.*	*Hab.* near Philadelphia.

Mesocarpus

scalaris, *Hassall.*	*Hab.* near Philadelphia.
parvulus, *Hassall.*	*Hab.* Rhode Island.

Pleurocarpus

mirabilis, *Braun.*	*Hab.* New York; Rhode Island; Michigan; Wisconsin.

Order SIPHOPHYCEÆ.

Family HYDROGASTREÆ.

Hydrogastrum

granulatum, *Linn.*	*Hab.* Delaware.

Family VAUCHERIACEÆ.

Vaucheria

aversa, *Hassall.*	*Hab.* near Philadelphia.
geminata, *Vauch.*	*Hab.* near Philadelphia.
polymorpha, *Wood.*	*Hab.* Texas.
sessilis, *Vauch.*	*Hab.* New York; Maine; Virginia; North Carolina.
velutina, *Ag.*	*Hab.* New York; Maine; Virginia; North Carolina.

Order NEMATOPHYCEÆ.

Family ULVACEÆ.

Protoderma

viride, *Ktz.*	*Hab.* Philadelphia.

Ulva

merismopedioides, *Wood.*	*Hab.* Diamond Range, Rocky Mountains.

Enteromorpha

intestinalis, *Linn.*	*Hab.* Hudson River; Narragansett Bay.

Schizomeris

Leibleinii, *Ktz.*	*Hab.* near Philadelphia.

Family CONFERVACEÆ.

Conferva.

Hab. United States.

Cladophora

brachystelecha, *Rab.* *Hab.* near Philadelphia.
fracta, *Dill.* *Hab.* Pennsylvania; New York; Rhode Island.
glomerata, *Linn.* *Hab.* Lakes Ontario, Erie, Huron, and Michigan.

Family ŒDOGONIACEÆ.

Androgynia

echinata, *Wood.* *Hab.* Florida.
Huntii, *Wood.* *Hab.* near Philadelphia.
mirabilis, *Wood.* *Hab.* near Philadelphia.
multispora, *Wood.* *Hab.* near Philadelphia.

Pringsheimia

inæqualis, *Wood.* *Hab.* near Philadelphia.

Bulbochæte

Canbyii, *Wood.* *Hab.* Hibernia, Florida.
dumosa, *Wood.* *Hab.* near Philadelphia.
ignota, *Wood.* *Hab.* near Philadelphia.

Family CHROOLEPIDEÆ.

Chroolepus

aureum, *Ktz.* *Hab.* New York; New Jersey; Texas.

Bulbotrichia

albida, *Wood.* *Hab.* Northern New Jersey.

Family CHÆTOPHORACEÆ.

Stigeoclonium.

Hab. Eastern United States.

Draparnaldia

Billingsii, *Wood.* *Hab.* near Philadelphia.
glomerata, *Vauch.* *Hab.* Rhode Island.
maxima, *var.*, *Wood.* *Hab.* near Philadelphia.

Drapernaldia

plumosa, *Vauch.* *Hab.* near Philadelphia.

Chætophora

elegans, *Roth.* *Hab.* Eastern United States.
endiviæfolia, *Roth.* *Hab.* Rhode Island; South Carolina.

Pilinia

diluta, *Wood.* *Hab.* Centre County, Pennsylvania.

Coleochæte.

Hab. Eastern United States.

Aphanochæte

repens, *Braun.* *Hab.* near Philadelphia.

CLASS RHODOPHYCEÆ.

Family PORPHYRACEÆ.

Porphrydium

cruentum, *Ag.* *Hab.* New York.
magnificum, *Wood.* *Hab.* Texas.

Family CHANTRANSIACEÆ.

Chantransia

expansa, *Wood.* *Hab.* near Philadelphia.
macrospora, *Wood.* *Hab.* South Carolina.

Family BATRACHOSPERMACEÆ.

Batrachospermum

moniliforme, *Roth.* *Hab.* Eastern United States.
vagum, *Roth.* *Hab.* Uintah Mountains, Nevada.

Tuomeya

fluviatilis, *Harv.* *Hab.* Alabama; Virginia.

Family LEMANEACEÆ.

Lemanea

catenata, *Ktz.* *Hab.* Diamond Range, Rocky Mountains.
fluviatilis, *Ag.* *Hab.* Alabama.
torulosa, *Roth.* *Hab.* Virginia; Kentucky; Pennsylvania; New Jersey.

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BIBLIOGRAPHY.

Agardh (Karl Adolf).

Dispositio Algarum Sueciæ. Lundæ, 1810–12.

Anmärkningar om slägtet Lemania, samt beskrifning om tvenne nya arter deraf. Kongl. Svenska Vetenskaps Akademiens Handlingar, xxxv., 1814, p. 33.

Beskrifning af en ny art Conferva. Ibid., 1814, pp. 195–200.

Synopsis Algarum Scandinaviæ, adjecta dispositione universali Algarum. Lundæ, 1817.

De metamorphosi Algarum. Isis von Oken., 1820. Regensburg Flora, vi., 1823.

Species Algarum rite cognitæ cum synonymis, differentiis specificis et descriptionibus succinctis. Griphiæ, 1823–28.

Systema Algarum. Lundæ, 1824.

Aufzählung einiger in den Ostreichischen Ländern gefundenen neuen Gattungen und Arten von Algen, nebst ihrer Diagnostik und beigefügten Bemerkungen. Regensburg Flora, x., 1827.

Einige Bemerkungen über Hrn. Dr. Meyen's kritische Beiträge zum Studium der Süsswasser Algen. Regensburg Flora, 1829.

Icones Algarum Europæarum. Leipzig, 1828-35.

Agardh (Jakob Georg).

Observationer pa Sporidiernes rörelse hos de gröna Algerne. Stockh. Acad. Handl., 1836.

Beobachtungen über die Bewegung der Sporidien in den grünen Algen. (Translation.) Regensburg Flora, 1840.

Bidrag till en noggrannare kännedom af propagations—organerne hos Algerne. Stockh. Acad. Handl., 1836.

Observations sur la propagation des Algues. (Translation.) Annal. de Scien. Natur., vol. vi., 1836.

Anadema, ett nytt slägte bland Algerne (Anadema, genus novum confervearum familiæ). Stockh. Acad. Handl., 1846.

Algologiska Bidrag. Öfversigt af Konigl. Vetenskaps Akademiens Förhandlingar, Sjefte Argangen, 1849.

Nya Algformer. Ibid., Arg. xi., 1854.

Species, genera et ordines Algarum. Lundæ, 1848-63.

Allman (George James).

On a new Genus of Algæ belonging to the family Nostochineæ. Annals and Magazine of Nat. History, 1843, vol. xi. p. 161.

On an undescribed Alga allied to Coleochæte scutata. British Association Report, 1847.

On an apparently undescribed genus of Fresh-Water Algæ. Ibid., 1847.

On two undescribed Algæ. Proceedings Royal Irish Acad., 1847.

On microscopic Algæ as a cause of the phenomenon of the coloration of large masses of water. Phytologist, vol iv., 1852.

Alquen (F. d').

Notes on the Structure of Oscillatoriæ, with a description of a new species, possessing a most remarkable locomotive power not Cilia. Journal Microscopical Soc., iv., 1856.

Amici (Giovanni Battista).

Descrizione di un' Oscillaria. Firenze, 1833.

Andrejewsky (Erastes).

Ueber die Vegetation in den Bädern von Abano. Graefe und Walther's Journal für Chirurgie und Augen Krankheiten, 1831. Annalen der Chemie und Pharmacie, 1832.

Note sur les végétaux qui croissent autour et dans les eaux thermales d'Abano. Annals Scien. Nat., Bot.) 1835. (Translation.) Edinburgh New Philos. Journal, xix., 1835.

Archer (William).

Description of two new species of Staurastrum. Dublin Nat. History Soc. Proceedings, 1856–9. Nat. History Review (Dublin), vi., 1859. Journ. Micros. Society, 1860.

Notice on some cases of abnormal growth in the Desmidiaceæ. Ibid., 1856–9. Ibid., 1859. Ibid., 1860.

Observations on the genera Cylindrocystis, Mesotænia, and Spirotænia. Microscop. Journal, N. S., vol. viii.

On the conjugation of Spirotænia condensata. Ibid., N. S., vol. viii.

Catalogue of Desmidiaceæ (Dublin) Nat. Hist. Rev., vol. iv. Microscopical Journal, vol. vi.

Supplementary Catalogue of Desmidiaceæ (Dublin), with descriptions and figures of a new genus and four new species. Natural Histor. Rev., vol. v., 1858.

Notice of the occurrence near Dublin of a Unicellular Alga, believed to be allied to that alluded to by Hofmeister (Ann. Nat. Hist. 3 ser. vol. i., Jan. 1858). Nat. History Review, v., 1858. Dublin Zool. Bot. Assoc. Proceedings, i., 1859.

On a new species and genus of the Desmidiaceæ, with some remarks on the genera Micrasterias and Euastrum. Nat. Hist. Rev., vol. vi.

On the occurrence of Zoospores in the family Desmidiaceæ. Dub. Nat. Hist. Soc. Proc. iii. Journ. Micros. Society, 1860. Nat. Hist. Review, vol. vii.

On Asteridium occurring in Penium digitata. Microscopical Journ., vol. vii.

Record of the occurrence new to Ireland, with a note of a peculiar condition of the Volvocinaceous Algæ Stephanosphæra pluvialis, &c. Ibid., vol. vi.

On two new species of Saprolegnieæ. Ibid., vol. vii.

Description of a new species of Cosmarium and of a new species of Xanthidium. Dubl. Nat. Hist. Soc. Proc. iii., 1859–62. Nat. Hist. Review, vol. vii., 1860.

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Archer (William).—*Continued.*

Description of a new species of Micrasterias, with remarks on the distinction between M. rotata (Ralfs) and M. denticulata (Bréb.). Dublin Nat. Hist. Soc. Proc., vol. iii. Micros. Soc. Journ. ii., 1862.

Description of a new species of Cosmarium (Corda), of Staurastrum (Meyen); of two new species of Closterium (Nitzsch), and of Spirotænia (Bréb.) Dublin Nat. Hist. Soc. Proc. iii. Microscop. Soc. Journ., vol. ii., 1862.

On a new species of Ankistrodesmus (Corda), with remarks as regards Closterium Griffithii (Berk), and C. subtile (Bréb.).

On Closterium aciculare (West). Microscopical Soc. Journal, ii., 1862.

An endeavor to identify Palmoglœa macrococca (Ktz.), with description of the plant believed to be meant and of a new species, both referrible to the genus Arthrodesmus (Ehrb.). Dubl. Nat. History Soc. Proc. iv., 1862. Hedwigia, 1864.

Description of a new species of Cosmarium (Corda), and of Arthrodesmus (Ehrb.). Dub. Nat. Hist. Soc. Proc., iv., 1862–3.

Observation on Micrasterias Mahabuleshwarensis (Hobson), and on Docidium pristidæ (Hobson). Dubl. Nat. Hist. Soc. Proc., iv., 1862–63. Microscopical Journ., N. S., vol. vi.

Description of a new species of Docidium from Hong Kong. Dubl. Nat. Hist. Soc. Proc., iv. Microscop. Journ., N. S., vol. vi.

Description of two new species of Cosmarium, of Penium, and of Arthrodesmus. Microscop. Journ. N. S., vol. iv., Hedwigia, 1864.

Description of two new species of Staurastrum. Microscopic Journ., N. S., vol. vi., 1839.

On a new genus and species of Desmidiaceæ. Ibid., N. S., vol. vi.

On some cases of abnormal growth in Desmidiaceæ. Ibid., N. S., vol. vi.

Ardissone (Francesco).

Enumerazione delle Alghe della marca di Ancona. Fano, 1866.

Areschoug (Johann Erhart).

De Hydrodictyo utriculato. Lundæ, 1839.

Ueber die Vermehrungsart des Wassernetzes (Hydrodictyon utriculatum Roth). Linnæa, vol. xvi., 1842.

Om Achlya prolifera, växanda pa lefvande fisk. Stockholm, Ofversigt Kon. Vetensk. Akad. Förhand. 1844.

Algarum minus rite cognitarum pugillus primus, tab. 12. Linnæa, 1842.

Die Arten der Gattung Ulva. Nova Acta R. Soc. Scient. Upsal. Ser. III., vol. i.

Copulationem hos Zygnemaceæ. Stockholm, Öfversigt K. Ventensk. Akad. Forhand. 1853. Regensburg Flora, 1855.

Arrondeau (Theodore).

Essai sur les Conferves des Environs de Toulouse. Act. Soc. Linnæa de Bordeaux, 3 Series, tom. iv.

Observations sur l'Organization du Zygnema orbiculare, Hassal. Sessions des Congrès Scientifique de France, xix., 1852.

Bailey (John Whitman).

A sketch of the Infusoria of the family Baccilaria, with some account of the most interesting species which have been found in a recent tour in the U. States. American Journal of Sciences, First Series, vol. xli. p. 184. Vol. xlii. p. 88. Vol. xliii. p. 321.

Bailey (John Whitman).—*Continued.*

On some new species of American Desmidiaceæ from the Catskill Mountains. American Geologist and Nat. Assoc. Reports, 1843. Amer. Journ., N. S., vol. i., 1846.

Notes on the Algæ of the United States. Ibid., New Series, vol. iii. pp. 80, p. 329 Vol. vi. p. 37.

Microscopical Observations made in South Carolina, Georgia, and Florida. Smithsonian Contributions, vol. ii.

Notes on new species and localities of Microscopical Organisms. Ibid., 1853.

Balfour (John Hutton).

Observations on the Spores of Cryptogams, and on the reproductive processes in some Algæ and Fungi. Proceedings of the Royal Society of Edinburgh, Session of 1867–8.

Bary (Heinrich Anton de).

Bericht über die Fortschritte der Algenkunde in den Jahren 55, 56, 57. Botanishe Zeitung, 1858.

Sur la génération sexuelle des Algues. Annal. Scien. Natur. (Bot.), tome v., 1856.

Beitrag zur Kenntniss der Achlya prolifera. Botanische Zeitung, 1852.

Über die Copulation der Desmidiacien. Berlin, Botanische Zeitung, 1857. Regensb. Flora, 1857, p. 693.

Ueber das vortweltliche kleinste Süsswasser Leben in Ægypten. Monatsbericht der Kon. Akad., 1853.

Beitrag zur Kenntniss der niedersten Algenformen nebst Versuchen ihre Entstehung betreffend. Sitzungsbericht d. k. Akad. zu Wien, Bd. xi., 1853.

Beitrag zur Naturgeschichte der Schwärmsporen. Verhandlungen des naturhist. Vereins der preussisch. Rheinlande und Westphalens. Bonn, 1855.

Untersuchungen über die Familie der Conjugaten. Leipzig, 1858.

Über die Algengattungen Œdogonium und Bulbochæte. Verhandlungen der Senckenberg naturwissen. Gesselschaft zu Frankfurt. 1854.

Beitrag zur Kenntniss der Nostocaceen, insbesondere der Rivularieen. Regensburg Flora, 1863.

Einige neue Saprolegnieen. Pringsheim's Jahrbuch für wissens. Botanik, bd. 2, 1860, p. 169.

Bary (Heinrich Anton de) *et* **M. Woronin.**

Beitrag zur Kenntniss der Chytridieen. Freiburg, 1863, also Berichte der naturforschenden Gesellschaft in Freiburg, Band. iii., Heft. ii.

Bauer (Francis).

Microscopical Observations on Red Snow. The Journal of Science and Arts (Royal Inst. Gr. Br.), vol. vii., 1819.

Some experiments on the Fungi which constitute the coloring matter of the Red Snow discovered in Baffin's Bay. Philosophical Transactions, London, 1820.

Microscopical Observations on the Suspension of Muscular Movements in Vibrio tritica. Philosophical Transactions, 1823. Ann. Sci., vol. ii., 1824.

Beggiato (Francesco Secondo).

Delle terme Euganee memoria. Padova, 1838.

Berkeley (Miles Joseph).

Gleanings of the British Algæ, being an Appendix to the Supplement to English Botany. London, 1833.

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Berkeley (Miles Joseph).—*Continued.*

Description of Closterium Griffithii, n. sp. Annals and Magazine of Natural History, 1854.

On a new vegetable Parasite on fishes. Gardener's Chronicle, 1864. Annal de Scien. Natur., vol. ii., v. Ser.

Note on the recent discoveries in relation to the Microgonidia of Fresh-water Algæ. Journal of Linnean Society i., 1857.

Biasoletto (Bartolommeo).

Di alcune Alghe microscopiche Soggio. Trieste, 1832.

Ueber die Metamorphose der Algen. Regensburg Flora, 1838.

Ueber microscopische in chemischen Solutionen entstehende Algen. Isis von Oken, 1833.

Bory de St. Vincent (Jean Baptiste M. A. G.).

Memoire sur les genres Conferva et Byssus. Bordeaux, 1797.

Essai monographie sur les Oscillaries. Paris, 1827.

Braun (Alexander).

Ueber das Wassernetz, Hydrodictyon utriculatum, Roth. Verhandlungen der Schweizerischen Gesellschaft für die gesammte Naturwissenschaft, 1847.

Ueber das Vorkommen beweglicher Samen bei den Algen. Ibid., 1847.

Betrachtungen über die Erscheinung der Verjüngung in der Natur, inbesondere in der Lebens und Bildungsgeschichte der Pflanze, Leipsig, 1851. Translated and republished by the Ray Society, London.

Ueber Spirulina Jenneri. Botanische Zeitung, 1852.

Chlamydococcus pluvialis bei Berlin. Ibid.

Algarum unicellularum Genera nova et minus cognita, præmissis observationibus de Algis unicellularibus in Genere. Leipzig, 1855.

Ueber Chytridium, eine Gattung einzelliger Schmarotzergewächse auf Algen und Infusorien. Abhandlungen der K. Akademie der Wissenschaft zu Berlin, 1855.

Ueber neue Arten der Gattung Chytridium und über die damit verwandte Gattung Rhizidium Monatsbericht. 1856, p. 587.

Ueber einige mikroskopischen Algen. Botanische Zeitung, 1856.

Ueber Protoccus pluvialis. Botanische Zeitung, 1856.

Brebisson (L. Alphonse de).

Description de deux nouveaux genres d'Algues fluviatiles. Annales des Sciences Naturelles, 1844, vol. i.

Liste des Desmidiées observées en Basse-Normandie. Paris, 1856, avec 2 planch.; also Mem. Soc. Scienc. Nat. de Cherbourg, 1854.

Brebisson (L. Alphonse de) *et* Godey.

lgues des Environs de Falaise. Mémoires de la Société Académique de Falaise, (Bot.) 1835.

Brongniart (Adolphe Theodore) *et* Bory de St. Vincent.

Botanique du voyage autour du monde sur la Coquille. Cryptogames, vol. ii. de texte, avec atlas gr. in folio, de 100 planches color. et noir. Paris, 1829.

Brügger (Chr. G.).

Erster Bericht über das kleinste Leben der rhätischen Algen. Jahresbericht viii. der naturforschenden Gesellschaft Graubündens. Churwalden, 1863.

Busk (George).

On the structure of Volvox globator. Microscopical Transactions, vol. i.

On the occurrence of Sarcina ventriculi in the human stomach. Microscopic Journ., vol. i., 1842.

Candolle, See **De Candolle.**

Carradori (Gioracchino).

Della transformazione del Nostoc in Tremella verrucosa. Prato, 1797.

Carter (H. J.).

Note on a species of Nostoc from Sind. Journ. of the Bombay Branch of the Royal Asiatic Society, v., 1855.

On Fecundation in the two Volvoces and their specific differences; on Eudorina, Spongilla, Astasia, Euglena, and Cryptoglena. Annals and Mag. Nat. Hist., 1859, p. 1.

On Fertilization in Eudorina elegans and Cryptoglena. Ibid., 1858, p. 237.

On specific character, Fecundation, and abnormal development in Œdogonium. Ibid., 1857, p. 29.

Carus (Carl Gustav).

Beitrag zur Geschichte der unter Wasser an verschiedenen Thierkörpern sich erzeugenden Schimmel- oder Algengattung. Nova Acta Acad. Cæs. Leopold.-Car. Natur Cur. Bd. xi., Bonn, 1823.

Caspary (Robert).

Vermehrungsweise von Pediastrum ellipticum. Botanishe Zeitung, viii., 1850.

Description of a new British Alga belonging to the genus Schizosiphon. Annals of Natural History, vi., 1850.

Ueber die Zoosporen der Gattung Chroolepus. Regensburg Flora, 1857. Berlin Botanische Zeitung, 1857.

The Zoospores of Chroolepus. Quarterly Journal of Microscopical Science, July, 1859. Annal. Scien. Natur. (Bot.), vol. ix., 1858. (Translated from Regensburg Flora).

Cesati (Vincenzo barone).

Ueber die Vermehrung von Hydrodictyon utriculatum Roth. Hedwigia, 1852.

Chauvin (François Joseph).

Observations microscopique sur la Conferva zonata. Mémoires Société Linnéenne de Normandie, 1826–7.

Observations microscopique sur la mode de reproduction de la Conferva rivularis. Sessions des Congrès scientifique de France, 1833.

Examen comparatif des Hydrophytes non articulées de la France et de l'Angleterre. Ibid., 183.

Recherches sur l'organisation, la fructification et la classification de plusieurs genres d'algues, avec la description de quelque espèces inédites. Caen, 1842.

Cienkowski (L.).

Algologishe Studien. Botanische Zeitung, 1856.

Ueber einige chlorophyllhaltige Gloeocapsen. Botanische Zeitung, 1865, p. 21.

Die Pseudogonidien. Pringsheim's Jahrbuch, Bd. i., 1858, p. 370.

Rhizidium Confervæ glomeratæ. Botanische Zeitung, 1857.

Cleve (P. T.).

Bidrag till kännedomen om Sveriges sötvattensalger af familjen Desmidieæ. Öfvers af k. Vet. Akad. Forhandl. Arg. 20, N. 10. Hedwigia, 1864.

Om de Svenska arterna af Slägtet Vaucheria, De Cand. Stockholm, 1863. Hedwigia, 1864.

Försok till en monografi öfver di Svenska arterna af algenfamilien zygnemaceæ. Nova acta Reg. Societatis Scientarum Upsaliensis, 1868.

Iakttagelser öfver den hvilande Œdogonium (Œdogodium Sporens). Öfvers af k. Vetenskap-Akad., p. 247, 1864. Translated in Microscopic Journal.

Cohn (Ferdinand Julius).

Zur Lehre vom Wachsthum der Pflanzenzelle. Acad. Cæs. Leopold. Nova Acta, xxii., 1847.

Nachträge zur Naturgeschichte des Protococcus nivalis Ktz. Ibid., xxii., 1847.

Ueber die Entwickelungsgeschichte der Pflanzenzelle. Uebersicht der Schlesischen Gesellschaft für väterlandische Cultur, 1849.

Ueber blutähnliche Färbungen durch mikroskopische Organismen. Ibid., 1850.

On a new genus of the family Volvocineæ. Annals and Mag. Nat. Hist., 1852, translated from Siebold and Kolliker's Zeitschrift für wissenschäftliche Zoologie, vol. iii., 1852.

Ueber Keimung der Zygnemeen. Uebersicht der Schles. Gesell. für väterl. Cultur, 1852.

Ueber Protococcus crustaceus Ktz. Hedwigia, 1851.

Untersuchungen über die Entwickelungsgeschichte der mikroskopischen Algen und Pilze. Nova Acta Akad. Cæs. Leopold., xxiv., 1854. Micros. Soc. Journ., iii., 1855.

Ueber die Fortpflanzung von Sphæroplea annulina. Monatsbericht der k. Preus. Akad. Wissensch. zu Berlin, 1855. Ann. and Mag. Nat. Hist., 1856. Ann. Sci., Natur. (Bot.), v., 1856.

Ueber das Geschlecht der Algen. Uebersicht der Schles. Gesell. väterl. Cultur., 1855. Report of the British Association for the Advancement of Science, 1855.

Beobachtungen über den Bau und die Fortpflanzung von Volvox globator. Uebers. Schl. Gesell. väterl. Cultur., 1856. Annals and Magaz. Nat. Hist., 1857. Ann. Sci. Natur. (Bot.), 1856. Comptes Rendus, xliii. Micros. Soc. Journ., 1857.

Ueber einige neue Algen Schlesiens. Jahresbericht des naturwissen. Vereins zu Breslau, 1857.

Ueber lebendige Organismen in Trinkwasser. Ibid., 1853.

Ueber contractile und irritable Gewebe der Pflanzen. Ibid., 1860.

Contractile Gewebe im Pflanzenreiche. Ibid., 1861. (Translated) Annals and Mag. Nat. Hist., xi., 1863.

Ueber das Verhältniss der Zellen in den niederen Pflanzen und Thieren. Ibid., 1861.

Ueber rothen Schnee. Ibid., 1861.

Ueber die Algen des Carlsbader Sprudels und deren Antheil an der Bildung des Sprudelsinters. Ibid., 1862.

Ueber die Verbreitung der Algen, insbesondere in den Meerens Europa's. Ibid., 1862.

Verhalten der grünen mikroskopischen Pflanzen und Thiere zum Lichte. Ibid., 1863. Microscop. Journ., N. S., vol. vii.

Beiträge zur Physiologie der Phycochromaceen und Florideen. Schultzes Archive für Mikrosc. Anatomie, 1867.

Ueber Chlamydomonas marina, Cohn. Hedwigia, 1865.

Comelli (Francesco).

Intornuo alle alghe microscopiche del Dr. B. Biasoletto Relazione. Udine, 1833.

Corda (August Karl Joseph).

Observations microscopiques sur les Animalcules des eaux thermales de Carlsbad. Almanach de Carlsbad, 10e année, 1840.

Die Algen. Sturm's Deutschland Flora, 2e Abtheilung 1829–32.

Corti (Bonaventura).

Osservazioni microscopische sulla Tremella. Lucca, 1774.

Crouan (P. L., *and* **H. M.,** *brothers***).**

Observations microscopiques sur le genre Mesogloia Agardh. Annal. Scien. Natur. (Bot.), vol. iii., 1835.

Observations microscopiques sur la dissémination et la germination des Ectocarpes et sur le Conferva scutulata. Ibid. vol. xii.

Note sur le genre Spirulina. Memoires de la Société Imper. des Scien. Nat. de Cherbourg, ii., 1854.

Currey (Frederick).

On Stephanosphæra pluvialis. Microscopic Journ., vol. vi., 1858, p. 131.

On some British Fresh-Water Algæ. Ibid., p. 207.

Dareste (Camille).

Mémoire sur la Coloration de la mer de Chine. Annal. Scien. Natur., vol. i., 1854.

Mémoire sur les Animalcules et autres corps organisés qui donnent a la mer une couleur rouge. Annal. Scienc. Nat. (Zool.), 1855.

Davaine (C.).

Conferve parasite sur le Cyprinus Carpio. Mémoires de la Soc. de Biologie, 1851.

Recherches physiologiques et pathologiques sur les Bactéries. Comptes Rendus, Paris, 1868, tome lxvi.

Recherches sur les infusoires du sang dans la maladie connue sous le nom de sang de rate. Ibid., lvii., 1863. Journ. de Pharmacie, xliv., 1863.

De Candolle (Augustin Pyramus P.).

Flore Française. Tom. vi.

Rapport sur les Conferves. Journal de Physique, de Chimie et de l'Histoire Naturelle, liv., 1802.

Notice sur la Matière qui a coloré le lac de Morat en Rouge au Printemps de 1825. Mémoires de la Société Nat. de Genève, tome iii., 1826. Edinburgh Journ. Scien., 1827.

Derbes (Alphonse) *et* **Antoine Joseph Jean Solier.**

Sur les organs reproducteurs des Algues. Annal. Scien. Natur. (Bot.), xiv., 1850–1851.

Mémoire sur quelques points de la physiologie des algues. Paris, 1856.

Dillenius (John Jacob).

Historia Muscorum. A General History of Water Mosses and Corals, containing all the known species, their names, places of growth, and seasons. 85 tab. London, 1763.

Dillwyn (Lewis Weston).

Synopsis of the British Confervæ. London, 1800–1810.

Dippel (Leopold).

Beiträge zur Lösung der Frage "Kommt der Zellmembran blos ein Wachsthum von Aussen nach Innen zu, oder besitzt dieselbe zugleich ein solches von Innen nach Ausen. Botanische Zeitung, ix., 1851.

Zur Primordialschlauchfrage. Regensburg Flora, xxxix., 1856.

Ueber die Fortpflanzung der Vaucheria sessilis. Regensb. Flora, xxxix., 1856.

Zelltheilung der Ulothrix zonata. Abhandlungen der naturforschenden Gesellschaft zu Halle, 1867, p. 43.

Druce (T. C.).

On the reproductive processes in the Confervoideæ. Micros. Journal, 1860, p. 71.

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Duby (Jean Etienne).

Botanicon Gallicum. Tome ii. Paris, 1828.

Singulier mode de multiplication du Pediastrum ellipticum. Supplément à la Bibliothèque Universelle de Genève. Archives des Scienc. Nat. et Phys., tome xvii., 1851.

Reproduction des Algues. Bibliothèque Universelle des Sciences. Genéve, vi., 1836.

Dufour (Louis).

Elenco delle Alghe della Liguria. Genova, 1864.

Duffin (A. B.).

Some account of Protoplasm. Microscop. Journal, 1860, vol. iii., N. S., p. 251.

Ducluzeau (J. A. P.).

Essai sur l'histoire naturelle des Conferves des Environs de Montpellier. Montpellier, 1805.

Dujardin (Félix).

Histoire Naturelle des Zoophytes, avec pl. 22. Paris, 1841.

Sur quelques Végétaux inférieurs et particulier, sur le Nostoc. Annales Françaises et Étrangers d'Anatomie et de Physiologie (Laurent et Bazin). Paris, 1839.

Ehrenberg (Christian Gottfried).

Die Infusionsthierchen als vorkommene Organismen. Mit 64 Tafeln. Leipzig, 1838.

Mikroskopische Analyse des curländischen Meteorpapiers von 1686 und Erläuterung desselben als ein Product jetzt lebender Conferven und Infusorien. Mit 2 Tafeln. Berlin, 1839.

Verbreitung und Einfluss des mikroskopischen Lebens in Süd- und Nord-Amerika. Mit 4 Tafeln. Berlin, 1843. Monatsbericht der k. Preus. Akad., 1841.

Passat-Staub und Blut-Regen, ein grosses organisches unsichtbares Wirken und Leben in der Atmosphäre. Mit 7 Tafeln. Berlin, 1849.

Mikrogeologie. Das Erden und Felsen schaffende Wirken des unsichtbar kleinen selbsständigen Lebens auf der Erde. Leipzig, 1854.

Nebst Fortsetzung (Bogen 1–22). Daselbst, 1856, gr. fol. 88 Seiten, nur mit Umschagtitel nicht beendigt.

Ueber die seit 27 Jahenr noch wohl erhaltenen Organisations-Präparate des mikroskopischen Lebens. Mit 3 Tafeln. Berlin, 1862.

Zur Mikrogeologie, von Christian Gottfried Ehrenberg. Seiner Majestät Friedrich Wilhelm IV., König von Preussen zugeeignet. Ein und vierzig Tafeln mit über viertausend grossentheils colorirten Figuren, gezeichnet vom Verfasser. Daselbst, 1854, gr. fol. 31 Seiten mit 41 Tafeln und 49 Blättern dazu gehöriger Erklärung.

Mikroscopische Algen und Bryozoa als Begleiter der im Feuerstein Fossilen Infusorien. Monatsberichte k. Preuss. Akad. Wissens. zu Berlin, 1836, p. 114.

Uebersicht des Mikroscopischen Lebens in Californien. Ibid., 1852, p. 423.

Beitrag zu Bestimmung des Stationären Mikros. Lebens in bis 20,000 Fuss Alpenhöhe. Monatsber. Akad. Wissen. 1858, p. 426.

[For a further list of microscopic papers by Ehrenberg, 10 double columns, see Register für die Monatsberichte der König. Akademie der Wissenschaften zu Berlin vom Jahre 1836 bis 1858. Berlin, 1860.]

Ettingshausen (Konstantin von).

Die Proteaceen der Vorwelt. Sitzungberichte der Mathemat. Naturwissen. Classe der Kaiser. Akad. der Wissenschaften. Wien Bd. vii., 1851 und band viii., 1852.

Famintzin (A.) *und* **J. Boranetzky.**

Zur Entwickelungsgeschichte der Gonidien und Zoosporenbildung der Flechten. Mémoires de l'Academie Impériale des Sciences de St. Petersbourg, 1867.

Die Wirkung des Lichtes auf Algen und einige andere ihnen nahe verwandte Organismen. Pringsheim's Jahrbuch für Wissens. Botan. Bd. vi.

Influence de la lumière artificielle sur le Spirogyra Orthospira. Annales des Sciences Naturelle, vol. vii., 1867.

Influence de la lumière sur le movement des Chlamedomonas, Oscillatoria, &c. Ibid.

Fischer (Leopold Heinrich).

Beiträge zur Kenntniss der Nostochaceen. Berlin, 1853.

Der Nostochaceen. Botan. Zeitung, 1853.

Fleischer (Johann Gottlieb).

Ueber Protococcus roseo-persicinus Kütz. Wurtemb. Jahrb. xvii., p. 55.

Flotow (Julius von).

Chroolepis Körberi. Fw. Botanische Zeitung, viii., 1850.

Flotow (Julius von) *und* **Ferdinand Julius Cohn.**

Ueber Hematoccocus (Protococcus) pluvialis. Nova Acta, Bonn, 1843–1850.

Focke (Gustav Waldemar).

Physiologischen Studien. Heft i., 1847. Heft ii., 1854. Bremen.

Uber die Copulation der Bacillarien und Desmidiaceen-Regensb. Flora et Berlin Botan. Zeit. 1857.

Frauenfeld (George *Ritter* **von).**

Die Algen der dalmatischen Küste. Wien, 1854.

Fresenius (Johann Baptist Georg Wolfgang).

Ueber Sphæroplea anulina. Botanische Zeitung, 1851.

Ueber den Bau und das Leben der Oscillarien. Museum Senckenb.. 3 bande.

Ueber die Algengattungen Pandorina, Gonium, und Raphidium. Abhandlungen Senckenb. naturh. Gesselsch., 1857.

Fries (Elias Magnus).

Systema Orbis Vegetabilis. Pars I., Plantæ Homonemeæ. Lundæ, 1825.

Corpus Florarum provincialium Sueciæ.—I. Flora Scanica. Upsaliæ, 1835.

Gaebel (E.).

Beitrag zur Kenntniss der Spermatozoidien. Hedwigia, 1868.

Gaillon (Benjamin).

Essai sur les causes de la couleur verte des ventres. Rouen, 1821.

Aperçu d'histoire naturelle et observations sur les limites qui séparent le règne végétal du règne animal. Boulogne, 1838.

Observations microscopiques et physiologiques sur l'Ulva intestinalis. Précis Analytique des Travaux de l'Academie de Rouen, 1821.

Observations microscopiques sur le Conferva comoïdes. Dillw. Am. Sci. Nat., 1824.

Girod-Chantrans (Julius).

Recherches chimiques et microscopiques sur les Conferves, Bisses, Tremelles, etc. Paris, 1802.

Glurup.

Species, genera et ordines Algarum. Botanische Zeitung, 1852, p. 791.

Goethe (Johann Wolfgang von).

Mittheilungen aus der Pflanzenwelt. Nova Acta, Bonn, Bd. xv., 1831.

Göppert (Heinrich Robert).

Bemerkungen über das Vorkommen von Pflanzen in heissen Quellen und in gewöhnlich warmen Boden. Wiegman's Archives, 1837.

Göppert (Heinrich Robert) *und* **Ferdinand Julius Cohn.**

Ueber die Algen Schlesiens. Uebersicht der Arbeiten und Veränderungen der Schlesischen Gesellschaft für väterlandische Cultur. Breslau, 1850.

Goodsir (John).

On the Conferva which vegetates on the skin of the Goldfish. Annals and Mag. Nat. Hist., vol. ix., 1842, p. 333.

Gravenhorst (Johann Ludwig Christian).

Einiges aus des Infusorienwelt. Nova Acta, Bonn, bd. xvi., 1832, p. 841.

Gray (John Edward).

On the arrangement of the Families and the Genera of Chlorospermous Algæ. Annals and Mag. Nat. Hist., 1861, p. 404.

Greville (Robert Kaye).

Scottish Cryptogamic Flora. Edinburgh, 1823–29.

Griffith (John William) *and* **Arthur Henfrey.**

Micrographic Dictionary, Ed. 2. London, 1860.

Grunow (A.).

Specimen Floræ Cryptogamæ Septeminsularem Algæ. Abhandlungen des zoolog.-botanischen Vereins, 1861.

Die Desmidaceen und Pediastreen einiger Österreichischen Moore, nebst einigen Bemerkungen über beide Familien im Allgemeinen. Verhandlungen der k. k. zoolog.-botanischen Gesellschaft in Wien, 1858.

Ueber neue oder ungenügend gekannte Algen. Ibid., 1860–61–62.

Ueber die in Rabenhorsts Decaden ausgegebenden Süswasser-Diatomaceen und Desmidiaceen von der Insel Banka, &c. Rabenhorst's Beiträgen, Heft ii., Leipzig, 1865.

Hannover (Adolph).

Ueber eine contagiöse Confervenbildung auf dem Wassersalamander. Archiv für Anatomie, Physiologie, und wissensch. Medecin. (J. Müller), 1839.

Fernere Erläuterung der contagiösen Confervenbildung auf Fröschen und Wassersalamandern. Ibid., 1840.

Hartig (Theodore).

Ueber die Funktionen des Zellkerns. Botanische Zeitung, xii., 1854.

Ueber das Verhalten des Zellkerns bei der Zellentheilung. Ibid., 1854.

Beiträge zur Entwickelungsgeschichte der Pflanzenzelle. Botanische Zeitung, 1855.

Ueber die Conjugation der Spirogyren. Regensburg, Flora.

Harvey (William Henry).

Manual of the British Algæ. London, 1844.

Harvey (William Henry).—*Continued.*

Index Generum Algarum. A Systematic Catalogue of all the Genera, Marine and Fresh-water. London, 1860.

Phycologica Britannica, or a History of the British Seaweeds. London, 1846–1851.

Nereis Borealis Americanæ. Part III. Chlorospermeæ. Smithsonian Contributions.

Hassal (Arthur Hill).

Observations on the genera Zygnema, Tyndarideæ, and Mougeotia, with descriptions of new species. Annals and Mag. Natur. History, &c., 1842.

Observations on a new group (Vesiculaspermæ), genus (Vesiculifera), and subgenus (Vesiculifera composita) of Fresh-water Confervæ, with descriptions of species mostly New. Ibid., 1842.

Observations on the growth, reproduction, and species of the branched Fresh-water Confervæ, mostly new, with observations on some of the genera. Ibid., 1843.

Observations on some points in the anatomy and physiology of the Fresh-water Algæ. Ibid., 1848.

Observations on the genus Mougeotia, or two new genera of Fresh-water Algæ, and on Tyndaridea, with descriptions of species. Ibid., 1843.

Observations on the growth and reproduction of Enteromorpha intestinalis. Proceedings of the Linnæan Society of London, 1849.

Essay on the distribution, vitality, structure, modes of growth and reproduction, and uses of the Freshwater Confervæ. Ibid., 1849.

A History of the British Fresh-water Algæ. 2 vols., 100 plates. London, 1845.

Hawlett (Frederick).

On the structure and circulation of Nitella translucens. Intellectual Observer, London, 1866.

Hazslinkszky (Friedrich).

Die Algen Flora Ungarns und seiner Bundesländer. Mathematische Naturwissen. Mittheilungen. der Ungarischen Akademie d. Wissenchaften, Pest. bd. v. 1868.

Hedwig (Romanus Adolf).

Tremella Nostoch. Commentatio. Lipsiæ, 1798. Archiv für die Botanik (Rœmer), ii. (1799–1801).

Bemerkungen über das Studium und die Untersuchung der Wassergewächse, nebst Beschreibung zweier neuen Conferven (Conferva globulifera und C. fenestralis) und der Spongia Rothii. Ibid., iii., 1803–5.

Hermann (Johann).

Botrydium argillaceum Wallr., ob Alge oder Flechte. Regensburg Flora, 1868, p. 129.

Hicks (John Bruxton).

On Fresh-Water Algæ. Microscop. Journ., N. S., vol. vii.

On the Diamorphosis of Lyngbya, Schizogonium, and Prasiola. Ibid., N. S., vol. i.

On the Motionless Species (Statospores) of Volvox globator. Ibid., N. S., vol. i.

Remarks on Mr. Archer's Paper on Algæ. Ibid., vol. iv. p. 253.

On the Amæboid Form of Volvox globator. Ibid., 1860, p. 99.

Observations on the Gonidia and confervoid filaments of Mosses, and on the relation of their Gonidia to those of Lichens and of certain Fresh-water Algæ. Trans. Linnæan Soc., 1862, p. 567.

Hicks (J. B.).—*Continued.*

On Draparnaldia cruciata. Quarterly Journal of Microscopical Science, vol. ix., N. S., p. 383.

Observations on vegetable amæboid bodies. Ibid., 1862.

Hillebrandt (Franz).

Ueber ein Chroolepus mit Zoosporenbildung. Sitzungsberichte naturhist. Vereins des preussischen Rheinlandes und Westphalens. 1861, p. 33. Botanische Zeitung, 1861.

Mykologische Beiträge (Achlya, &c.). Pringsheim's Jahrbuch für wissen. Botan., Bd. vi., p. 249.

Hilse (Dr.).

Beiträge zur Algen- und Diatomeen-Kunde Schlesiens, insbesondere Strehlens. Breslau, 1860.

Hobson (J.).

Notes on Indian Desmideæ. Microscopical Journal, N. S., vol. iii., 1863, p. 168.

Hoffman (Jakob Friedrich).

Deutchland's Flora oder Botanisches Taschenbuch. Tome ii. Cryptogamen. Erlangen, 1795.

Neue Beobachtungen über Bacterien mit Rücksicht auf Generatio spontanea. Botanische Zeitung, 1863, p. 304.

Hofmeister (Wilhelm Friedrich Benedict).

Ueber die Mechanik der Protoplasma Bewegungen. Verhandlungen des naturhistor. medicinischen Vereins zu Heidelberg. 1865, Bd. iii. Regensburg Flora, 1867, p. 7.

Ueber die Fortpflanzung der Desmidieen und Diamoteen. Leipzig, Berichte ix. 1857. Ann. Nat. Hist., 1858.

Hooker (Joseph Dalton).

Crytogamia Antarctica. London, 1847.

Hooker (William Jackson).

British Flora, vol. ii. London, 1853.

Hornschuch (Christian Friederich).

Einige Beobachtungen und Bemerkungen über die Entstehung und Metamorphose der niederen vegetabilischen Organismen. Nova Acta, Bd. x. 1831.

Ueber die Entstehung und Metamorphosen der niederen vegetablischen Organismen. Regensb. Flora, xviii., 1835.

Husemann (Theodore).

De Animalibus et Vegetalibus in Corpore Humano parasitantibus, Dissertatio inauguralis. Berolini, 1854.

Itzigsohn (Hermann).

Kryptogamischer Ursprung der Miasmen. Botanische Zeitung, x., 1852.

Sur l'Existence des spermatozoïdes dans certaines Algues d'eau douce. Annal. des Scien. Natur. (Bot.) vol. xvi., 1852.

Ueber die Sporenbildung der Chætophoren. Botanische Zeitung, x., 1852.

Ueber Ulothrix cylindrocapsa, Itz. Hedwigia, 1852, und Regensburg Flora, 1852.

Phykologische Studien. Nova Acta, 1857, bd. xxvi.

Skizzen zu einer Lebensgeschiete des Hapalosiphon Braunii. Nova Acta. Bonn, Bd. xxv.

Algologische Mittheilungen. Sitzung der Gesellschaft naturforschender Freunde zu Berlin. Nov. 19, 1867.

Spermatosphærien und Spermatozoen der Spirogyra arcta, Ktz. Hedwgia, 1852.

Itzigsohn (Hermann).—*Continued.*

Bemerkung zu Rivularia gigantea, Grent. Ibid., 1853.

Ueber den männlichen Geschlechts-apparat bei Spirogyra und einigen andern Conferven. Botanische Zeitung, 1853.

Die Nostoc-Diamorphose. Ibid.

Die Fortpflanzung der Oscillarien. Ibid.

Ein Wort über Hyalotheca und Micrasterias. Ibid., 1853, p. 485.

Ueber die Sporenbildung der Cladophora. Ibid., 1852, pp. 549, 569, 723, 785.

Ueber das Studium der Süsswasseralgen. Regensb. Flora, 1850.

Zur Entwickelunggeschichte von Cladosporium herbarum. Hedwigia, 1855.

Ueber die Algengattung Psichohormium. Regensburg Flora, 1854.

Wie verhält sich Collema zu Nostoc und zu den Nostochineen. Botan. Zeitung, 1854.

Die Gloeocapsen und Chroococcaceen Diamorphose. Botans. Zeitung, 1854, xii.

Vegetabilische Zelle und Sexus, eine hypothetische Andeutung. Botanische Zeitung, xv., 1857.

Zur Naturgeschichte der Sarcina ventriculi, Goodsir. Archiv für Pathol. Anatomie (Virchow), xiv. 1858.

Gloeocapsa und Cladonia. Botan. Zeit. xiii., 1855.

Nachträgliche Bemerkungen über die Spermatozoïdeen der Vaucheria. Botan. Zeit. xii., 1854.

Jenner (Edward).

Flora of Tunbridge Wells. Tunbridge Wells, 1845.

Jessen (Carl Friederich Wilhelm.)

Prasiolæ generis Algarum Monographia. Dissertatio inauguralis botanica. Kiliæ, 1848.

Juratzka (J.).

Ueber Conferva aureo-fusca, Kütz. Verhandlungen der k. k. zoologisch-botan. Gesellschaft, Wien, 1864. Bd. 14.

Karsten (Hermann).

Beitrag zur Kenntniss des Zellenlebens. Botanische Zeitung, 1849.

Ueber die Bewegungsorgane und die Bewegungserscheinungen der Oscillatorieen. Regensb. Flora, 1856.

Die Fortpflanzung der Conferva fontinalis. Botanische Zeitung, 1852.

Chlamydococcus pluvialis. Botanische Zeitung, 1852.

Botanische Untersuchungen aus dem physiologischen Laboratorium der landwirthschaften Lehranstalt in Berlin. Berlin, 1865.

Berichtigung zu Pringsheim's Nachtrag zur Kritik und Geschichte der Untersuchungen über das Algengeschlecht. Botanische Zeitung, 1860.

Zur Geschichte der Befruchtung der Algen. Botanische Zeitung, 1857.

Kickx (Jean).

Flore cryptogamique des Flandres. Gand et Paris, 1867.

Kingsley (C. H.).

On cilia upon the surface of Oscillatoria. Microscop. Journal, 1855, p. 243.

Kniphof (John Hieronymus).

Physikalische Untersuchungen des Peltzes, [auf Wiesen]. Erfurt, 1753.

Kraus (G.) *et* A. Millardet.

Etudes sur la materie colorante des Phycochromacees et Diatomées. Mémoires de la Société Nat. de Strasbourg, 1868.

Kühn (Julius).

Ueber eine im Innern des Faulthierhaares lebende Alge, Pleurococcus Bradyhi. Abhandlung naturforsch. Gessellschaft. Halle, 1864. Hedwigia, 1864.

Kützing (Friedrich T.).

Algologische Mittheilungen. Regensburg Flora, 1833.

Beitrag zur Kenntniss über die Entstehung und Metamorphose der niedern vegetabilischen Organismen, nebst einer systematischen Zusammenstellung der hieher gehörigen niedern Algenformen. Linnæa, viii., 1833. Annals Sci. Natur., ii. (Bot.) 1834.

Ueber Sphærotilus natans, eine neue Süsswasseralge. Linnæa, viii., 1833

Ueber Gloionema, Agh. Ueber eine neue Gattung der Confervaceen (Ulothrix). Regensb. Flora, xvi., 1833.

Ueber die Bildung organischer Materie wässerigen Rhabarbertinctur. Journal für praktische Chemie (Erdmann), i., 1834.

Verwandlung der Microcystis umbrina, Ktz., in Alysphæria chlorina, Turpin. Regensburg Flora, xvii., 1834.

Die Umwandlung niederer Algenformen in höhere, so wie auch in Gattungen ganz verschiedener Familien und Klassen höherer cryptogamen mit zelligem Bau. Naturkundige Verhandl. van de Holland. Maatschappij der Wetensch. te Haarlem, i., 1841.

Ueber ein neues Botrydium (B. Wallrothii). Acad. Cæs. Leop. Nova Acta, xix., 1842.

Exposition systematique des genres et des organismes végétaux inférieurs. Annales des Scien. Nat., vol. ii., 1834.

Formation du genre Ulothrix. Ibid , vol. i., 1834.

Observations sur le genre Gloionema, Agd. Ibid.

Phycologia generalis oder Anatomie, Physiologie und Systemkunde der Tange. Leipzig, 1843.

Phycologia Germanica. Nordhausen, 1845.

Species Algarum. Leipzic, 1859.

Tabulæ Phycologicæ oder Abbildungen der Tange, Band 1 bis xv., mit 1500 Tafeln. Nordhausen, 1845–1866.

Lacoste (C. M., van der Sande), *en* W. F. R. Suringar.

Nieuw Beschrevene en voor onze flora nieuwe Zoetwater-Wieren, verzameld in Drenthe, 9–20, Juli, 1859.

Landsborough (David).

British Sea-Weeds, with Notes of the Fresh-water Algæ. London, 1857.

Laurent (J. L. M.).

Note sur la reproduction du Volvox globator. Paris, Soc. Philom. Proc. Verb, 1848.

Sur les deux sortes de corps reproducteurs du Volvox globator. Ibid. 1849.

Note sur le Volvox globator. Comptes Rendus et Les Memoires de la Société de Biologie, 1850.

Lawson (George).

Report on the Musci and Desmidieæ collected during a trip to West Lomond Hill, Fife. Proceedings Edinburgh Botanical Soc., 1855.

On Macadamia, a new Genus of Proteaceæ. Transactions of the Edinb. Bot. Soc., vi., 1860.

Note on Lemanea variegata Agd. Ibid., 1863. Edinb. New Philosoph. Journ. xviii., 1863.

Lawson (George).—*Continued.*

On the Structure and Development of Botrydium granulatum. Edinb. Botan. Soc. Trans., vol. vi., p. 424.

Lebret (E.).

Note sur les Conferves qui croissent dans les bassins de l'établissement thermal de Néris. Ibid. 1850.

Leidy (Joseph).

On the existence of Entophyta within healthy animals as a natural condition. Proc. Acad. Nat. Sc., Philad., 1848–9. Annals and Mag. Nat. Hist., 1850.

A Flora and Fauna within Living Animals. Smithson. Contributions, 1853.

Lespinasse (G.).

Les zoospores et les antherozoides des Algues, histoire de la decouverte, du mouvement et des fonctions physiologiques de ces organes. Act. de Linn. Soc. de Bordeaux, 1861.

Liantenant.

Beschreibung einiger neuen Algen. Regensburg Flora, 1831–1834.

Beiträge zur Algenkunde. Ibid., 1839–1840.

Lindsay (William Lauder).

Protophyta of Iceland. Microscop. Journ., N. S., vol. vii. p. 97.

Protophyta of New Zealand. Ibid., vol. vii. p. 197.

Linke (Heinrich Friedrich).

Handbuch zur Erkennung der nutzbarsten und am häufigstenvorkommenden Gewächse. Berlin, 1829–33.

Linné (Carl von).

Systema Naturæ. (Various editions.)

Lobb (E. G.).

On the Self-Division of Micrasterias denticulata. Micros. Soc. Trans. ix., 1861.

On Cosmarium radiatum. Microscop. Journ., N. S., vol. vi. p. 52.

Lüders (Johannes E.).

Ueber Abstammung und Entwickelung des Bacterium Termo Duj. Vibrio lineola Ehrb. Botanische Zeitung, 1866, pp. 33 and 49.

Lyngbye (Hans Christian).

Account of a New Arrangement of the Algæ. Edinb. Phil. Journ., ii., 1820.

Tentamen Hydrophytologiæ danicæ, adjectis simul speciebus norvegicis. Havniæ, 1819.

Efterretning om Hr. Gaillons mikroskopische og physiologiske Undersögelser med Conferva comoides Dillw. Örsted, Tidsskrift, iv., 1826.

Mackay (James Townsend).

Flora Hibernica. Dublin, 1836.

Martens (Georg von).

Reise nach Venedig, 1824. Translated in Bull. Sci. Nat., vol. vi., 1825.

Algologische Mittheilungen, zumal über Jürgens algologische Sammlung. Regensb. Botanische Zeitung, 1830.

Ammauria jungermannioides. Ibid. 1836.

Amphicoa pustulata. Ibid. 1836.

Martius (Carl Friedrich Philipp von.)

Flora Cryptogamica Erlangensis sistens vegetablia eclaisse ultima Linn. in agro Erlangensi hucusque detecta auctore. Norimbergæ, 1817.

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Meneghini (Giuseppe).

Conspectus Apologiae Euganeae. Patavii, 1837.

Cenni sulla organografia e fisiologia delle Alghe. Padova, 1838.

Synopsis Desmidearum hucusque cognitarum auctori. Linnæa, 1840.

Monographia Nostochinearum Italicarum, addito specimine de Rivulariis. Memoire della Reale Accademia delle Scienze di Torino, 1842.

Alghe Italiene e Dalmatiche illustrate. Padova, 1842–46.

Mettenheimer (C.).

Ueber Leptothrix ochracea, Ktz. Abhandlungen Senckenb. naturf. Gesellschaft, Abhandl. 1857.

Meyen (Franz Julius Ferdinand).

Ueber die eigenthümliche Säfte-Bewegung in den Zellen der Pflanzen. Acad. Cæs. Leop. Nova Acta, xiii., 1826.

Kritische Beiträge zum Studium der Süsswasseralgen Flora, x., 1827.

Ueber die Priestley'sche grüne Materie, wie über die Metamorphose des Protococcus viridis in Priestleya botryoides und in Ulva terrestris. Linnæa, ii. 1827, pp. 388–409.

Ueber das Genus Spirogyra Linn., über die Bewegung und Metamorphose der Sp. princeps insbesondere. Linnæa, ii., 1827.

Beiträge zur Physiologie und Systematik der Algen. Acad. Cæs. Leop. Nova Acta, xiv., 1828.

Beobachtungen über einige niedere Algenformen. Acad. Cæs. Leop. Nova Acta, xiv., 1828.

Nachträge zu meinen "Beobachtungen über einige niedere Algenformen." Oken, Isis, xxiii., 1830.

Micheli (Pierantonio).

Nova plantarum genera juxta Tournefortii methodum disposita. Florentiae, 1729.

Millardet.

Germination des zygospores des genera Staurastrum et Closterium et sur nouv. gen. Mémoires de la Société des Scien. Nat. de Strasbourg. 1868.

Montagne (Jean François Camille).

Sur l'existence des tétraspores dans une Algue de la tribus des Zygnémées. Comptes Rendus, xxi., 1845. Ann. Nat. Hist., 1845.

Quelques mots sur le Nostoc pruniforme, espèce nouvelle, tout à la fois pour la Flore Français. Revue Botanique, i., 1845-6.

Exploration scientifique de l'Algerie. Algues. Paris, 1846.

Ueber Thwaitesia, eine neue Gattung des Algen. Regensburg Flora, 1846.

Un dernier mot sur le Nostoc edule de la Chine. Revue Botanique, ii., 1846–7.

Résumé succinct. des observations faites jusq'ici sur le rubéfaction des eaux. Memoir de la Soc. de Biologie, 1849.

Note sur la station insolite de quelques Floridées dans les eaux douces et courantes des ruisseaux de la Guyane. Comptes Rendus, 1850.

Sur l'algue des œufs de limace. Mem. de la Soc. Biolog., iii., 1851.

Sur la coloration rouge des substances alimentaires par la présence du Monas prodigiosa. Mem. de la Soc. Biolog., iv., 1852.

Résumé des observations faites sur la rubéfaction des eaux; suivi d'une note de M. Alph. De Candolle sur la distinction des deux règnes organisés. Bibl. Univ. Archives, xix. 1852.

Montagne (Jean François Camille).—*Continued.*

Note sur deux Algues nées pendant les expérience de Boussingault, relatives a l'action du salpêtre sur la vegetation. Comptes Rendus, xlii., 1856.

Sylloge generum specierumque Cryptogamarum, quas in variis operibus descriptas iconibusque illustratas, nunc ad diagnosim reductas, nonnullasque novas interjectas, etc. Paris, 1856.

Montagne (Jean François Camille) *et* Rayer.

Nouvelle espéce d'Algue dans les urines. Mem. Soc. Biolog., 1849.

Mont-Sainct (Thomas).

Lettre missive à un sien amy sur le sujet du fait prodigieux [d'une] pluye rouge comme sang. Le Jardin Senonois. Sens. 1604.

Morren (Auguste *et* Charles F. A.)

Recherches physiologiques sur les hydrophytes de la Belgique. 3e Mémoire, contenant les recherches physiologiques, botaniques, zoologiques, et chemiques sur l'influence qu'exercent la lumière, les algues et les animalcules de couleur verte ou rouge, contenus dans les eaux stagnantes et courantes, sur la quantité et qualité des gaz que celles-ci peuvent contenir. Bruxelles Acad. Sci. Mem. xiv., 1841.

Morren (Charles).

Expériences sur l'absorption de l'azote par les animalcules et les algues. Paris, Comptes Rendus, 1854. Ann. Sci. Nat. (Zool.), 1854.

Morren (Charles F. A.).

Essais pour déterminer l'influence qu'exerce la lumière sur la manifestation et les développements des êtres végétaux et animaux, dont l'origin avait été attribuée à la génération directe, spontanée, ou équivoque. Ann. Sci. Nat. (Zool.) 1835, iii. iv.

Histoire d'un genre nouveau de la tribu des Confervées nommé Aphanizomène. Bruxelles, Acad. Sci. Bull. ii., 1835, iii. 1836.

Mémoire sur les Clostéries. Ann. Sci. Nat (Bot.), v., 1836. Bruxelles Acad. Scien. Bull. ii. 1835.

Recherches physiologiques sur les hydrophytes de la Belgique. 1er Mémoire: Histoire d'un genre nouveau de la tribu des Confervées, nommé par l'auteur Aphanizomène. Bruxelles, Acad. Sci. Mém., xi., 1838.

De l'existence des infusoires dans les plantes. Bruxelles Acad. Sci. Bull., vi., 1839. Annal. Nat. History, vi., 1841.

Recherches physiologiques sur les Hydrophytes de Belgique. 2e Mémoire: Histoire du genre Hydrodictyon, Roth. Brux. Acad. Sci. Mém., xiv., 1841.

Recherches physiologiques sur les Hydrophytes de Belgique. 4e Mémoire: Recherches sur la rubéfaction des eaux, suivies d'observations sur les apparences végétales que prennent les animalcules des familles des Monadines, Cryptomonadines, et Astasiées. Ibid., xiv., 1841.

Recherches physiologiques sur les Hydrophytes de Belgique. 5e Mémoire; Histoire du genre Hæmatococcus, Agardh. Ibid., xiv., 1841.

Recherches physiologique sur les Hydrophytes de Belgique. 6e Mémoire; Histoire du genre Tessararthra, *Ehr.* Ibid., xiv. 1841.

Müller (Karl).

Ueber die Conferva orthotrichi, Dillw. Regensburg Flora, 1841.

Einige Bemerkungen uber die Sarcina ventriculi, Goodsir. Botan. Zeitung, v., 1847.

Müller (Karl) *et* Ernst Hampe.

Plantæ Müllerianæ Australasicæ—Algæ. Linnæa, 1854.

Müller (Otho Friedrich).

Vermium terrestrium et fluviatium succincta historia. Hafniae et Lipsiae, 1773–7.

Animalcula Infusoria, fluviatilia et marina. Hafniae et Lipsiae, 1786.

Naegeli (Carl).

Die neueren Algensysteme und Versuch zur Begründung eines eignen Systems der Algen und Florideen. Mit 10 lith. Tafeln. Zurich, 1847.

Gattungen einzelliger Algen. Zurich, 1849.

Beiträge zur wissenschaftlichen Botanik. Leipzig, 1860.

Nave (Johann).

Ueber die Entwickelung und Fortpflanzung der Algen. Brünn. Jahresheft naturwiss. Sect., 1859.

Die Algen Mährens und Schlesiens. Verhandlungen des naturforsch. Vereines in Brünn. 1864.

Algologische Notizen. Botanische Zeitung, 1861.

Ueber die Grenzen und Berührungspuncte des Thierreichs und der Pflanzenwelt. Brünn. Verhandl., i., 1862.

Nees von Esenbeck (Christian Gottfried).

Bemerkungen über Conferva coactilis und Conferva ægagrophila. Regensburg Flora, 1824, p. 273.

Ueber einen blau-rothen Farbstoff der sich bei der Zersetzung von Oscillatorien bildet. Journ. de Pharmacie, xxii., 1836.

Nicolucci (Giustiniano).

De quibusdam Algis aquæ dulcis. Neapoli, 1843.

Nitzsch (Christian Ludwig).

Beitrag zur Infusorienkunde oder Naturbeschreibung der Zakarien und Bacillarien. Halle, 1817.

Nylander (William).

Notula circa Spermosiram et Nodulariam Algarum genera. Annal. Scien. Nat. (Bot.), vol. xv., 1861.

Œrsted (Andreas S.).

Beretning om en Excursion til Trindelen, en alluvialdannelse i Odensefjord. Kjöbenhaven, 1842.

Olivi (Giuseppe).

Dell' Ulva atropurpurea. Padova, 1793.

Perini.

Ueber die Befruchtung und den Generationswechsel der Algen. Botanische Zeitung, 1856.

Perty (Maximilian).

Bemerkungen über die Bacillarieen. Bern, Mittheil., 1848.

Ueber verticale Verbreitung miscroscop. Lebensformen. Ibid., 1849.

Ueber die Identität einiger Algen- und Infusoriensippen. Ibid., 1863.

Microscopische Organismen der Alpen und der Italienischen Schweiz. Ibid. 1849.

Die Bewegung durch schwingende mikroskopische Organe im Thier- und Pflanzenreiche. Nebst Erörterungen über Sporozoidien, Infusorien, und Bacillarien, und über die Elementarstructur der Halcyonella fluviatilis, var. Nymphaeæ. Bern, 1848.

Ueber den gefärbten Schnee des St. Gotthard, vom 16—17 Feb. 1850. Ibid., 1850.

Zur Kenntniss kleinster Lebensformen nach Bau, Functionen, Systematik, mit Specialverzeichniss der in der Schweiz beobachteten. Mit xvii. lith. Tafeln. Bern, 1852.

Peterstein (J.).

Ueber das Studium der Schwämme und Versuche dieselben aufzubewahrene, Œstereichisches Botanische Wochenblatt, 1852.

Petrovsky (André).

Études algologiques. Annal. Scien. Natur. (Bot.), vol. xvi., 1862. Bulletin Society Imper. Nat. de Moscow, 1861.

Antwort auf H. Weissec's Bemerkungen zu den Études algol. Ibid. 1862.

Pollini (Ciro).

Sulle Alghe viventi nelle terme Euganei, Lettera. Milano, 1817.

Pringsheim (Natan).

Ueber Keimung der ruhenden Sporen und über eine Form beweglicher Sporen bei Spirogyra. Regensburg Flora, 1852.

Ueber die Befruchtung und Vermehrung der Algen. Berlin, Monatsbericht, 1857.

Ueber Fortpflanzung von Cœlastrum Naegeli. Regensburg Bot. Zeitung, 1852. Ann. Sci. Nat. (Bot.), xi., 1853.

Ueber die Befruchtung und Keimung der Algen. Berlin, Monatsb. der k. Akad. der Wiss. 1855. Ann. Nat. Hist., 1855. Ann. Scien. Nat. (Bot.), 1855. Mic. Jour. vi. 1858.

Ueber die Befruchtung und den Generationswechsel der Algen. Berlin, Monatsb. der k. Akad. der Wiss. 1857. Microscopical Journal, 1858. Annal. Sci. Naturelle, vol. v., 1856.

Beiträge zur Morphologie und Systematik der Algen.

I. Morphologie der Œdogonien. Jahrbuch für wissens., Botanik., Bd. i., p. 11.

II. Saprolegnieen. Ibid., p. 284.

III. Die Coleochateen. Ibid., Bd. ii., p. 1.

IV. Nachträge zur Morphologie der Saprolegnieen. Ibid., p. 205.

Ueber die Dauerschwärmer des Wassernetzes und einige ihnen verwandte Bildungen. Berlin Monatsber. der königl. Akad., 1860. Ann. Sci. Natur. (Bot.), 1860. Journal Microscopical Sci., ii., 1862.

Nachtrag zur Critik und Geschichte der Untersuchungen über das Algengeschlecht. Pringsheim's Jahrbuch Botan., ii., 1860.

Prinsep, Macaire, et Marcet.

Analyse de la neige rouge du Pole. Mémoires de la Société de Genéve, Tome 4, 1828.

Pritchard (Andrew).

A History of Infusiora, including Desmidiaceæ and Diatomaceæ, British and foreign. 4th ed., nearly 1000 pages, 40 pl., mostly Diatoms and Infusoria. London, 1861.

Rabenhorst (Ludwig).

Deutschlands Kryptogamen-flora, oder Handbuch zur Bestimmung der kryptogamischen Gewächse Deutschlands, der Schweiz, des Lombardisch-Venetianischen Königreichs und Istriens. Band ii², Algen. Leipzig, 1847.

Beiträge zur nähern Kenntniss und Verbreitung der Algen. Heft i., mit 7 Tafeln. Leipzig, 1863, Heft ii., mit 5 Tafeln, Leipzig, 1865.

Systematische Uebersicht der auf meiner italienischen Reise beobachteten Kryptogamen. Regensburg Flora, 1850.

Die Algen Sachsens. Botanische Zeitung, 1851, 2, 3, 8–9–60.

Erklarung der Tab. ii. mit Spirogyra olivascens R., Sirosiphon panniformis, Ktz., S. torulosus nov. sp. und Scytonema salisburgense R. Hedwigia, 1853.

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Rabenhorst (Ludwig).—*Continued.*

Algologisches Curiosum (Einfluss der Dampfmaschinen auf die floren). Ibid.

Kryptogamen-flora von Sachsen, der Ober-Lausitz, Thüringen, und Nordböhmen, mit Berücksichtigung der benachtbaren Länder. Erste Abtheil. Leipzig, 1863.

Species novæ Algarum. Hedwigia, 1854.

Beitrag zur Cryptogamenflora Süd-Africas; Pilze und Algen. Allgem. Deutsch. Nat. Hist. Zeit., i., 1855.

Bemerkungen zu Observation des êtres microscopique de l'atmosphere terrestre, M. A. Baudrimont. Ibid., 1855.

Ein neues Prodigium (Palmella mirifica). Ibid., ii., 1856.

Ralfs (John).

The British Desmidieæ. London, 1848.

On the Genera Epiculina and Coleochæte. Annals and Mag. Nat. Hist., 1845 and 46, p. 308.

Note on Coleochæte scutata. Phytologist, ii., 1845.

On the British Desmideæ. Ibid., 1844, pp. 187, 256, 465; 1845, pp. 149, 401.

Remarks on the Species of Desmidium. Ibid., vol. xi., 1843, p. 372.

On the mode of growth in Oscillatoria and allied Genera. Ann. Nat. Hist. iii., 1849. Edinburgh Bot. Soc. Trans., iii., 1850.

On the mode of growth in Calothrix and allied Genera. Ann. Nat. Hist., iii., 1849. Edinb. Bot. Soc. Trans., iii., 1850.

On the Nostochineæ. Ann. Nat. Hist., v., 1850. Edinb. Bot. Soc. Trans., iv., 1853.

Remarks on Dickieia. Ann. Nat. Hist., viii., 1851. Edinb. Bot. Soc. Trans., iv., 1853.

On Chantransia, Fries. Ann. Nat. Hist., viii., 1851. Edinb. Bot. Soc. Trans., iv. 1853.

Reichardt (Henrich Wilhelm).

Ueber Conferva aureo-fulva Ktz. Abhandlungen des zoologisch-botanischen Vereins zu Wien, 1864.

Ueber ein massenhaftes Auftreten von Cladophora viadrina Kütz. in Galizien. Verhand. d. k. k. zoolog.-botan. Gessellschaft zu Wien, Bd. xiv. p. 778.

Reichert (Karl Bogislaus).

Saftströmung in den Pflanzenzellen mit Rücksicht auf die Contraktilitätsfrage. Müller Archives, 1866, p. 417, and Zeitschrift Gesam. Naturwissen. Halle, 1866, p. 502. Regensburg Flora, 1867, p. 409.

Reinicke (Fr.).

Die Bewegung der Oscillarien, besonders der Spirulina. Beiträge zur neueren Mikroscopie, Zweiter Heft., 800. Dresden, 1860.

Reinsch (Paul).

De Speciebus Generibusque nonnullis novis ex Algarum et Fungorum classe. Abhandlungen von der senkenbergischs. naturforsch. Gesellschaft, 1867, p. 111.

Die Algenflora des mittleren Theiles von Franken. Diagnosen von ein und fünfzig entdeckten neuen Arten und drei neuen Gattungen. Mit 13 Tafeln. Nürnberg, 1867.

Retschy.

Ueber Tremella nostoc. Regensburg Flora, 1847.

Richter (P.).

Description of Pleurotænium nobile n. sp. Hedwigia, 1865.

Ripart.

Recherches sur l'organization du genre Isomeria Ktz. Ann. Sci. Nat., 5 ser. (Bot.), tom. vii., 1867.

Notice sur les Algues recoltes, &c., dans les Pyrénés. Bulletin de la Soc. Botanique de France, vol. xv. Avril, 1868.

Observations sur le Mougeotia genuflexa, Ag. et sur la formation de ses spores. Annal. Scien. Natur. (Bot.), vol. ix., 1868.

Robin (Charles).

Note sur la presence de la Sarcine dans l'œil. Mémoires de la Soc. de Biologie, 1854.

Rose (August).

Ueber Bulbochæte setigera. Hedwigia, 1852. His most important papers omitted.

Rossman (Julius).

Ueber Urin-Sarcina. Regensburg Flora, 1857.

Bemerkung über Sarcina Welckeri. Virchow Archiv., xiv., 1858.

Einige Nachträge zu dem Verzeichniss der Wetterauer Algen von *G. Theobald.* Wettera. Gesell. Jahresb. 1860, 1861.

Roth (Albrecht Wilhelm).

Tentamen Floræ germanicæ. Leipzig, 1788.

Bemerkungen über den inneren Bau der Conferven und ihrer Vermehrungsart. Schrader Journ. Botanik, i., 1800.

De Conferva verticillatis. Schrader Journ. Botan., i. 1800.

Observations on the Rivularieæ and the Plants allied to them. Annals of Botany, i., 1805.

Schacht (Hermann).

Die Spermatozoiden in Pflanzenreich. Mit 6 Tafeln. Braunschweig, 1864. Nat. Histor. Rev., 1864.

Ueber der Zellstoffäden. Bulletin Soc. Bot. Franc., 1864.

Schaffner (Dr.).

Ueber Identität der Fenster und Schleimhaut Conferven. Regensburg Botanische Zeitung, 1844.

Ueber Schleimhaut Conferven. Ibid., 1845.

Ueber die fossilen Algen im grünen Jaspis. Regensb. Flor., xlv., 1859.

Schenk (August).

Ueber das Vorkommen von Infusorien im Darmcanal der Schmeissfliege. Virchow's Archiv, xiii., 1858.

Algologische Mittheilungen. Würzburg Verhandl. viii., 1858; ix., 1859.

Ueber das Vorkommen contractiler Zellen im Pflanzenreiche. Würzburg, 1858.

Zur Kenntniss der geschlechtlichen Fortpflanzung der Gattung Vaucheria. Würzburger naturwissen. Zeitschrift, vol. ii., p. 201.

Scherer (Johann Andreas).

Beobachtungen über das pflanzenähnliche Wesen in den Karlsbäden und Töplitzer Wässern. Dresden, 1787.

Schmarda (Ludwig Karl).

Kleine Beiträge zur Naturgeschichte der Infusorien. Wien, 1846.

Schneider (Anton).

Symbolæ ad Infusoriorum Historiam naturalem. Dissertatio Inauguralis. Berlin, 1854.

Beiträge zur Naturgeschichte der Infusorien. Müller's Archiv., 1854. Ann. Nat. Hist. xiv., 1854.

Schrank (Franz von Paula).

Ueber die Oscillatorieen. Nova Acta, 1823, Bd. xi., p. 522.

Schultze (Max Sigismund).

Innere Bewegungserscheinungen bei Diatomeen der Nordsee aus den Gattungen Coscinodiscus, Denticella, Rhizosolenia. Müller Archiv., 1858. Journ. Microsc. Soc. vii., 1859.

Ueber die Bewegung der Diatomeen. Schultze's Archiv. für Microscop. Anatomie, 1865, p. 376.

Das Protoplasma der Rhizopoden und der Pflanzenzellen. Leipzig, 1863.

Schwedener.

Gonidien und Fasern vieler Flechten stehen nicht in genetischem Zusammenhange sondern letzte sind Wucherungen von Pilzfaden auf Algenformen. Verhandlungen allgem, Schweiz. naturforsch. Gesellschaft Rheinfelden, 1867.

Secondat (Jean Baptiste, *Baron* de).

Sur une espece Ulva, qui croit dans la fontaine bouillante de Daix. Observations. Paris, 1750.

Shuttleworth (Robert James).

Nouvelles observations sur la matière colorante de la neige rouge. Bibl. Univ. xxv., 1840. Edinb. New Phil. Journ., xxix., 1840. Froriep, Notizen, xvi., 1840.

Simony (Friedrich).

Ueber Cladophora Sauteri. Verhandlung. des zoolog.-bot. Vereins in Wien, Bd. ii., 1853.

Smith (*Rev.* William).

Observations on the conjugation of Closterium Ehrenbergii. Ann. Nat. Hist., v., 1850.

On the germination of the spore in Conjugatæ. Ann. Nat. Hist., viii., 1851.

On the stellate bodies occurring in the cells of Freshwater Algæ. Microscop. Soc. Trans., i., 1853. Phytologist, iv., 1853.

Stein (Friedrich).

Auf die Entwickelungs-Geschichte der Infusionsthiere. Leipzig, 1854.

Stitzenberger (Ernst).

Notizen über Protococcus crustaceus. Hedwigia, 1855.

Ueber die Periodizität im Leben der Süsswasseralgen.

Spirulina und Artherospira, n. gen. Verhandlungen der allgem. schweizerischen Gesell. für die gesammten Naturwissenschaften, 1854. Hedwigia, 1854.

Stoitzner (Carl).

Verzeichniss der im Elisenthal in Mähren gesammelten Algen. Verhandlungen des zoologisch-botan. Verein, Wien, 1866, Band xvi., p. 319.

Strange (John).

Lettera sopra l'origine della Carta naturale di Corton. Pisa, 1764.

Suhr (Johann Nicholaus von).

Beschreibung neuer Algen. Beiträge zur Algenkunde. Regensb. Bot. Zeitung (Flora), 1831–34–36.

Suringar (W. F. R. R.).

De Sarcine (Sarcina ventriculi). Harlem, 1865.

Notice sur l'Histore des faisceaux chlorophylliques de la Spirogyra. Archives Nierland. des Sciences exact. et nat. La Haye, 1868.

Observationes Phycologicæ in floram batavam. Leovandiæ, 1857.

Thuret (Gustave).

Recherches sur les zoospores des Algues et des anthéridies des cryptogames. Paris, 1851.

Note sur le mode de reproduction du Nostoc verrucosum. Annal. Scien. Natur. (Bot.), vol. ii., 1844.

Recherches sur les zoospores des Algues. Ibid., p. 1.; vol. xiv., 1850; Part II., vol. xvi. Bruxelles Acad. Sci. Bull., xvii., 1846.

Note sur les spores de quelques Algues. Ibid., 1845.

Note sur la synonomie des Ulva lactuca et lactissima. Botanische Zeitung, 1856.

Ueber neue Algen bei Cherbourg. Ibid., 1856.

Observations sur la reproduction de quelques de Nostochinees. Cherbourg, Mém. Soc. Sci., v. 1857. Ann. Nat. Hist., ii., 1858, pp. 1–9.

Thwaites (George Henry Kendrick).

On an apparently undescribed state of the Palmelleæ, with a few remarks on germination in the lower plants. Annals and Mag. Nat. Hist., vol. ii., 1848, p. 312.

Description of Coccochloris Brébisonii. A new species of Palmellæ in Conjugation. Ibid., 1849, p. 243.

Mode of the formation of spores in a species of Vesiculifera. Ibid., 1846.

On the early stages of development of Lemanea fluvatilis, Ag. Transactions of the Linnæan Soc., 1851, p. 399.

Trevisan (Vittore, *Conte*).

Prospetto della Flora Euganea. Padova, 1842.

Saggio di una monografia delle Alghe coccotalle. Padova, 1848.

Unger (Franz).

Algologische Beobachtungen. Nova Actæ, Bonn, 1832, Bd. xvi.

Mikroskopische Beobachtungen. Ibid., 1837, Bd. xviii.

Sur l'Oscillatoria labyrinthiformis, Ag. Annal. Soc. Nat. (Bot.), vol. xi., 1839.

Die Pflanze in Momente der Thierwerdung. Wien, 1843.

Ueber Vaucheria clavata. Agdh. Sitzungsbericht. der kais. Akad. der Wissen. Wien, 1852, vol. viii.

Beiträge zur Kenntnissz der niedersten Algenformen, nebst Versuchen ihre Entstehung betreffend. Ibid., vol. xi., 1853.

Beiträge zur Kenntniss der niedersten Algenformen. Denkschriften der k. Akademie der Wissenschaften. Wien, 1854.

Vaucher (Jean Pierre Etienne).

Mémoires sur les graines des Conferves. Paris, 1860.

Histoire des Conferves d'eau douce, suivie de l'histoire des Trémelles et des Ulves d'eau douce. Genève, 1803.

Vogt.

Édward Desor's Excursions et séjour dans les glaciers et les hautes régions des Alpes. Neuchatel, 1844.

Wallich (Dr. G. C.).

Descriptions of Desmidiaceæ from Lower Bengal. Ann. and Mag. Nat. Hist., 1860, p. 184 and p. 273.

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Wallroth (C. Friedrich Wilhelm).

Annus botanicus, sive supplementum tertium ad Curtii Sprengelii Floram Halensem. Halæ, 1815.

Flora Cryptogamica Germaniæ. Pars posterior continens Algas et Fungos. Norimbergæ, 1833.

Walz (J.).

Beiträge zur Kenntniss der Zoosporenbildung bei den Algen. Botanische Zeitung, 1868, p. 497.

Wartman (B.).

Beiträge zur Anatomie und Entwicklungsgeschichte der Algengattung Lemanea. Mit 4 Tafeln. St. Gallen, 1854.

Watts (Henry).

On the Fresh-water Algæ of Victoria. Transactions and Proceedings of the Royal Society of Victoria. 1865.

Weber (Friedrich) *und* **Daniel Matthias Heinrich Mohr.**

Naturhistorische Reise durch einen Theil Schwedens. Göttingen, 1804.

Weise (Dr. J. F.).

Resultate v. vergleichenden mikrosk. Untersuchungen von mehr denn 30 verschiedenen Proben der sog. Schwarzen-Erde. Moscow, 1855.

Welwitsch (F.).

Systematische Aufzählung der Süsswasser Algen Unterösterreichs. Abhandlungen des zoolog. botan. Vereins, Wien, 1857.

Synopsis Nostochinearum Austriæ inferioris. Wien, 1836.

Wiegmann (A. F.).

Ueber Entstehung von Entomostraceen und Podurellen aus der Priestleyschen grünen Materie, Verwandlung derselben in kryptogamische Gewächse, und dieser wieder in die oben genannten Thiere. Nova Acta, Bonn, 1821, Bd. x., p. 717.

Fortsetzung. Ibid., Bd. xi., p. 541.

Willan (Robert).

Observations on the Sulphur-water at Croft near Darlington.

[A conferva described on p. 9–10.]

Williamson (W. C.).

On structure of Volvox globator. Microscopical Transactions, vol. i.

Wood (Horatio C.).

Notes on some Algæ from a Californian Hot-Spring. American Journ. Science, vol. xlv., N. Series.

Prodromus of a Study of North American Fresh-Water Algæ. Transactions of the American Philosophical Society, June, 1869.

Wyman (Jeffries).

Observations and experiments upon living organisms in heated Water. American Journ. Science, N. S., vol. xliv.

Zanardinii (Giovanni).

Synopsis Algarum in Mari Adriatico, cui accedunt Monographia Siphonearum. Memoire della Reale Accademia delle Scienze de orino. Series 2, tome iv., 1852.

Ueber Cladophora Heufleri. Verhandlungen des zoolog.-botan. Vereins in Wien, Bd. ii., 1853.

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CORRECTIONS OF THE PLATES.

Plate IV., for *Zonotricha* read *Zonotrichia*.
Plate VIII., for *um* in the terminations of the specific name of figures 2, 3, and 4, read *us*.
Plate IX., fig. 3, for *argillacea* read *argillaceus*.
Plate X., fig. 4, for *Botryococcus pulchellus* read *Dictyosphærium pulchellum*.
Plate XI., fig. 5, 6, and 7, for *Cosmarium* read *Closterium*.
Plate XII., fig. 1 and 20, for *Cosmarium* read *Closterium*.
Plate XV., fig. 8, for *insignis* read *insigne*.
Plate XVI., fig. 4, for *Bulbotricha* read *Bulbotrichia*.
Plate XVIII., fig. 1, for *Pringsheimii* read *Pringsheimia*.
Plate XXI., fig. 7, for *tetraopthalmum* read *tetropthalmum*.

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EXPLANATION OF PLATES.

PLATE I.

Fig. 1. A single filament of *Oscillatoria chlorina*, Kützing, magnified 750 diameters.

Fig. 2. The end of a filament of an *Oscillatoria* supposed to be identical with *O. Fröhlichii* Kützing.

Fig. 3. *O. nigra*, Vaucher.
Fig. 3 *a*. Represents a portion of a mat or mass of *Oscillatoria nigra*, Vaucher; there is too much green in the color.
Fig. 3 *b*. Represents several filaments separated from the edge of the mass and slightly magnified.
Fig. 3 *c*. A portion of a filament.
Fig. 3 *d*. A portion of another filament still more highly magnified. The color in 3 *c* is more natural than that of 3 *d*.

Fig. 4. A portion of a filament of *O. limosa*, Agardh, magnified 1250 diameters. The articles in this filament are more distinctly separated than natural.

Fig. 5. *O. neglecta*, Wood.
Fig. 5 *a*. An outline view of a filament, magnified 450 diameters.
Fig. 5 *b*. A full figure of the same, magnified 500 diameters.

Fig. 6 *O. imperator*, Wood.
Fig. 6 *a*. Represents the end of a filament, magnified 250 diameters. In the centre of the plate is a fragment (marked simply fig. *b*.), out of which the endochrome has been partially squeezed to show the markings of the sheath at the joints.

Fig. 7. *Lyngbya bicolor*, Wood. Fig. 7 represents a moderately magnified portion of a filament.
Fig. *a* (near to fig. 8) represents a portion of an ordinary filament very slightly magnified.
Fig. 7 *c*. A portion of a filament containing a heterocyst, magnified 800 diameters.
Fig. 7 *d*. A broken end of a filament showing the sheath extending beyond the endochrome, magnified 800 diameters.

Fig. 8. A variety(?) of *Lyngbya bicolor*, Wood, from the Schuylkill River, magnified 200 diameters.
Fig. 9. *Cosmarium Quimbyii*, Wood. The bands between the cells are too heavy and prominent.

PLATE II.

Figs. 1 *b* and 1 *c*. Different stages of germination of the spore of a *Cylindrospermum* of unknown species, magnified respectively 800 and 1200 diameters.
Fig. 1 *a*. A chain of spores, believed to belong to the same species; one of these spores has commenced to germinate.

Fig. 2. A portion of the upper surface of a frond of *Nostoc calidarium*, Wood.

Fig. 2 *b*. A "first form" filament of the same species.
Fig. 2 *a*. A filament from an old frond of the same plant.
Fig. 2 *c*. Fragments of tissue from the upper surface of a mature, actively growing plant of the same species.

Fig. 3. A filament of *Nostoc comminutum*, Ktz., magnified 800 diameters.

Fig. 4. A filament of *Anabæna gelatinosa*, Wood, magnified 750 diameters, showing the large globular body at the end, supposed to be a spore.

Fig. 5. A filament of *Anabæna gigantea*, Wood, magnified 750 diameters.

Fig. 6. A portion of a filament of *Cylindrospermum minutum*, Wood, magnified 800 diameters. The number has been omitted from this figure on the plate; the figure is immediately under *A. gigantea*, Wood; the hairs on the heterocyst are too coarse and rigid.

Fig. 7. A spore and outline of heterocyst of *Cylindrospermum macrospermum*, Ktz., magnified 750 diameters.

Fig. 8. The end of a filament of *Cylindrospermum comatum*, Wood, magnified 1375 diameters. The appendages to the heterocyst are coarser than natural.

Fig. 9 *a*. A section of an immature frond of *Rivularia cartilaginea*, Wood.
Fig. 9 *b*. The base of a fertile filament, showing the spore and basal cells, magnified 800 diameters.

Fig. 10. An ordinary filament of *Nostoc sphæricum*, Poiret.
Figs. 10 *a* and 10 *c*. Filaments enlarging preparatory to longitudinal division.
Fig. 10 *b*. A filament already partially divided into two.

PLATE III.

Fig. 1 *a*. *Cylindrospermum flexuosum* (Ag.), a fertile filament, magnified 450 diameters.
Fig. 1 *b*. The end of a fertile filament, magnified 750 diameters.

Fig. 2. *Dolichospermum* (*Sphærozyga*) *subrigidum*, Wood, magnified 975 diameters.

Fig. 3. Portion of a fertile filament of *Dolichospermum* (*Sphærozyga*) *polysperma*, Ktz., magnified 750 diameters.

Fig. 4 *a*. A section of a frond of *Gloiotrichia incrustata*, Wood, showing youngish filaments, masses of lime, and an organic body of unknown nature, all inclosed in a transparent jelly.
Fig. 4 *c*. Single filaments with immature spores, magnified 260 diameters.
Fig. 4 *b*. The base of a filament, showing the nearly matured spore, and empty cells situated beyond it.

Fig. 5. *Chlorococcus* of undetermined species.
Fig. 5 *a*. The motile state.
Fig. 5 *b*. The condition of the plant after having lost its cilia and commenced its quiescent life.
Figs. 5 and 5 *c*. Different stages in this life after division.
Fig. 5 *d*. The *Hæmatococcus* or resting condition, the form assumed by the plant during slow desiccation.

Fig. 6. *Nostochopsis lobatus*, Wood.
Fig. 6 *a*. Part of a section, from within outwards, of the frond, showing the tortuous branched filaments, without sheaths in the gelatinous matrix.
Fig. 6 *b*. A portion of a fertile filament with the lateral spores.
Fig. 6 *c*. A sterile filament.

Fig. 7. *Protococcus* of undetermined species.
Fig. 7 *a*. A cell supposed to belong to the resting or winter condition of the plant.

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Fig. 7 *b*. The first breaking-up of the contents of the large cell into a brood of cells.
Figs. 7 *d* and 7 *c*. Different stages in the life of the latter brood-cells and their progeny.

PLATE IV.

Fig. 1. *Mastigonema fertile*, Wood, a single plant (*not* magnified 750 diameters, as marked on the plate).
Fig. 1 *b*. A heterocyst magnified 750 diameters, also a spore cell and spore similarly amplified.

Fig. 2 *a*. A portion of the frond of *Mastigonema sejunctum*, Wood, amplified 250 diameters.
Fig. 2 *b*. A single filament magnified 800 diameters.

Fig. 3. A single filament of *Zonotrichia mollis*, Wood, enlarged 260 diameters.

Fig. 4. A section of the frond of *Zonotrichia parcezonata*, Wood, magnified a few diameters.

Fig. 5. The base of a filament of *Dasyactis mollis*, Wood.
Fig. 5 *a*. Section of the frond, magnified 450 diameters.
Figs. 5 *b* and 5 *c*. [illegible]oung filaments; each magnified 450 diameters.

Fig. 6. Fronds of *Cœlosphærium dubium*, Wood.

PLATE V.

Fig. 1 *a*. A portion of the frond of *Mastigonema elongatum*, Wood, slightly magnified to show the filaments radiating from the fragment of matter to which they are attached.
Fig. 1 *b*. A single filament magnified 460 diameters.

Fig. 2 *c*. A cluster of youngish filaments of *Mastigonema halos*, Wood.
Fig. 2 *b*. A portion of an older filament to show the spore-like divisions of the endochrome, magnified 460 diameters.

Fig. 3. A pair of young connate filaments of *Mastigothrix fibrosa*, Wood, magnified 450 diameters.
Fig. 3 *a*. An old filament magnified 800 diameters.
Fig. 3 *b*. A young filament with heterocyst, enlarged 450 diameters.
Fig. 3 *d*. A filament with two basal cells magnified 450 diameters.

Fig. 4 *a*. A portion of filament of *Scytonema Ravenellii*, Wood, magnified 160 diameters.
Fig. 4 *b*. The end of a branch magnified 450 diameters.

Figs. 5 *a*, *b*, *c*, &c. Different forms of *Chroococcus refractus*, Wood. Fig. 5 *h* is not a good one. I was not able to express well the peculiar translucent shining tint, and the artist who copied my drawing failed even more decidedly in simulating it; the pink shade is altogether wrong, I never saw any such color in the plant.

Fig. 6. Different forms of *Chroococcus multicoloratus*, Wood.
Fig. 6 *c*. Represents what was thought to be possibly a hybernating form of the species.

PLATE VI.

Fig. 1 *a*. A portion of a frond of *Scytonema thermale*, Ktz., magnified 260 diameters.
Fig. 1 *b*. Outline sketch, showing the form of the heterocyst, magnified 750 diameters.

Fig. 2. Trichoma or frond of *Scytonema callitrichoides*, Ktz., amplified 250 diameters.

Fig. 3 *a*. Outline sketch showing the cells or chambers of *Scytonema dubium*, Wood, magnified 750 diameters.
Figs. 3 *b* and 3 *c*. Portions of the filaments or trichoma, magnified 460 diameters.

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Figs. 4 and 4 *b*. *Scytonema cortex,* Wood. Portions of filaments, magnified 750 diameters.

Fig. 5. Piece of a small twig with a well-formed and also a very young frond of *Chætophora elegans,* Ag., growing upon it, magnified a few diameters.

PLATE VII

Fig. 1. A perfect trichoma of *Scytonema cataractum,* Wood.
Fig. 1 *b*. A terminal part of a filament, magnified 250 diameters.

Fig. 2 *a*. A portion of a filament of *Scytonema immersum,* Wood, magnified 750 diameters.
Fig. 2 *b*. Nearly a whole filament or trichoma, amplified 260 diameters.

Fig. 3. *Rhaphidium polymorphum,* Wood, different forms, magnified 750 diameters.

Fig. 4. *Protococcus* of undetermined species.
Fig. 4 *a*. The largest and most mature form, probably the hibernating or winter cell.
Fig. 4 *b*. The same, commencing its active life.
Fig. 4 *d*. Colony cells believed to have been developed out of the cell represented by fig. 4 *b*, magnified 750 diameters.
Figs. 4 and 4 *c*. The motile state of the species.

PLATE VIII.

Fig. 1 *b*. Portion of the frond of *Tolypothrix distorta* (Müller), magnified 500 diameters.
Fig. 1 *a*. Heterocysts magnified 800 diameters.

Fig. 2. Portion of a frond of *Sirosiphon pellucidulus,* Wood, magnified 260 diameters.
Fig. 2 *a*. End of the branch.

Fig. 3. Portion of a frond of *Sirosiphon compactus* (Ag.), magnified 260 diameters.
Fig. 3 *a*. End of a filament, magnified 160 diameters.
Fig. 3 *c*. Portion of a filament showing the heterocyst magnified 460 diameters.

Fig. 4. Portion of a frond of *Sirosiphon neglectus,* Wood.

Fig. 5. Frond of *Sirosiphon guttula,* Wood.
Fig. 5 *b*. End of a branch, magnified 460 diameters.

Fig. 6. Portion of a frond of *Scytonema Nægelii,* Ktz.

Fig. 7. Different forms of *Glœocapsa sparsa,* Wood, magnified 700 diameters.

Fig. 8. *Merismopedia nova,* Wood, magnified 400 diameters.

PLATE IX.

Fig. 1. Fragment of a frond of *Sirosiphon scytenematoïdes,* Wood.

Figs. 2 and 2 *c*. Portions of fronds of *Sirosiphon lignicola,* Wood, magnified 260 diameters.
Fig. 2. The end of a branch of the same, magnified 460 diameters.

Fig. 3 *a* Portion of a very old frond of *Sirosiphon argillaceus,* Wood, magnified 460 diameters.
Fig 3 *b*. A terminal branch of a growing frond of the same.

Fig. 4 *a*. A frond of *Stigonema Ravenelii,* Berkeley, magnified 125 diameters; also a fragment of the same plant, magnified 450 diameters.

PLATE X.

Fig. 1 *a*. A frond of *Sirosiphon pulvinatus*, Bréb., var. *parvus*, from a specimen collected by Dr. J. G. Hunt, near Philadelphia. The ground color of this figure is too yellow.

Fig. 1 *b*. A fragment of the same, magnified 460 diameters.

Fig. 2. A row of cells of *Pleurococcus seriatus*, Wood, magnified 460 diameters.

Fig. 3 *a*. A portion of the old external part of a mass of *Palmella Jessenii*, Wood, magnified 750 diameters.

Fig. 3 *b*. A fragment from the interior of such a mass of the same amplification.

Fig. 3 *c*. A portion of the soft jelly of a young actively growing mass, magnified 750 diameters.

Fig. 4. A frond of *Dictyosphærium pulchellum*, Wood, magnified 460 diameters. I at first referred this plant to the genus BOTRYOCOCCUS, and distributed some specimens under that generic title, and so marked my original drawing.

Fig. 5. A slice of a youngish frond of *Palmella dura*, Wood, magnified 460 diameters.

Fig. 5 *b*. A fragment from an old frond, showing the spores in various stages of growth. The color of the large spores is not nearly dark enough, it should be much more brownish.

PLATE XI.

Fig. 1. Different forms of *Scenesdesmus polymorphus*, Wood, magnified 450 diameters.

Fig. 2. *Scenedesmus quadricauda*, Bréb., magnified 750 diameters.

Fig. 3. *Scenedesmus rotundatus*, Wood, magnified 750 diameters.

Fig. 4. Ordinary vegetative cells of *Palmoglœa clepsydra*, Wood, in different stages or conditions of life-history, magnified 750 diameters. Those cells which have the endochrome much broken up are believed to be preparing for conjugation.

Fig. 4 *a*. A pair of cells uniting in conjugation.

Fig. 4 *b*. Cells which have united so that the young spore is very apparent with the empty semi-cells of the parents attached to it.

Fig. 4 *c*. A more advanced spore and empty semi-cells.

Figs. 4 *d* and 4 *e*. Matured or nearly matured spores, as seen with different focussing; in the first the upper surface of the spore is especially brought out. All these figures, except 4 *b*, are magnified 750 diameters.

Figs. 5 and 5 *a*. Different forms of *Closterium acerosum* (Schr.), magnified 250 diameters.

Fig. 5 *b*. Empty conjugating cells with nearly matured spore.

Fig. 6. Outline of *Closterium areolatum*, Wood, magnified 160 diameters.

Fig. 6 *a*. End of a dead, empty frond, enlarged 1375 diameters.

Fig. 7. Outline of *Closterium Venus*, Ktz., magnified 450 diameters.

(These last three species are incorrectly labelled on the plate, *Cosmarium*.)

PLATE XII.

Fig. 1. *Closterium lineatum*, Ehrb. (Incorrectly labelled on the plate *Cosmarium*.) Magnified 160 diameters.

Fig. 2. *Closterium Ehrenbergii*, Menegh., magnified 160 diameters.

Fig. 3. *Closterium rostratum*, Ehrb., magnified 260 diameters.

Fig. 4. *Closterium Dianæ*, Ehrb., magnified 260 diameters.

Fig. 5. *Closterium parvulum*, Næg., magnified 450 diameters.

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Fig. 6. *Closterium Leibleinii,* Ktz., magnified 260 diameters.

Fig. 7. *Tetmemorus giganteus,* Wood., magnified 260 diameters.

Fig. 8. *Tetmemorus granulatus* (Bréb.), magnified 450 diameters.

Fig. 9. *Pleurotænium Trabecula* (Ehrb.), magnified 160 diameters.

Fig. 10. *Spirotænia bryophila* (Bréb.).

Fig. 11. *Spirotænia condensata,* Bréb.

Fig. 12. *Hyalotheca dissiliens,* Bréb.

Fig. 13. *Didymoprium Grevillii,* Ktz.
Fig. 13 *a.* End view.

Fig. 14. *Cosmarium Botrytis* (Bory.), magnified 460 diameters.

Fig. 15. *Cosmarium Cucumis,* Corda.
Fig. 15 *d.* A frond in which the neck or isthmus has begun to elongate previous to division.
Fig. 15 *b.* An abnormal frond which has attempted division, but in which the inner semicells of the new frond have failed to form perfectly and to separate.

Fig. 16. *Euastrum multilobatum,* Wood; front view.

Fig. 17. *Micrasterias Americana* (Ehrb.).

Fig. 18. *Cosmarium Meneghenii,* Bréb., magnified 750 diameters. The sinus should be very narrow but distinct, instead of being absent as in the figure.

Fig. 19. *Spirogyra Weberi,* Ktz., portions of conjugating filaments, magnified 260 diameters.
Fig. 19 *a.* A portion of a sterile filament, magnified 160 diameters.
Fig. 19 *b.* Conjugating cells with nearly mature spores, magnified 260 diameters.

Fig. 20. *Closterium juncidum,* Ralfs., magnified 260 diameters.

Fig. 21. *Cosmarium margaritiferum* (Turp.), magnified 460 diameters.

PLATE XIII.

Fig. 1. Front view of *Euastrum Ralfsii,* Rabenh., magnified 450 diameters.

Fig. 2. Front of *Euastrum elegans,* Bréb., enlarged 750 diameters.

Fig. 3. Front view of *Euastrum binale* (Turp.), magnified 750 diameters.

Fig. 4. Front view of *Micrasteria disputata,* Wood, drawn from a Philadelphia specimen.

Fig. 4 *a.* The same after a figure drawn by Dr. Jos. Leidy, from a Newport specimen.

Fig. 5. *Micrasterias furcata,* Agardh., front view, magnified 260 diameters.

Fig. 6. Front view of *Micrasterias denticulata,* Bréb., magnified 260 diameters.

Fig. 7. *Micrasteria Jenneri,* Ralfs. Front view.

Fig. 8. *Staurastrum orbiculare* (Ehrb.). Front view.

Fig. 9. *Staurastrum dejectum,* Bréb. Front view, magnified 750 diameters.

Fig. 10. *Staurastrum punctulatum,* Bréb. Front view.
Fig. 10 *a.* View from the apex.

Fig. 11. Front view of *Staurastrum Lewisii,* Wood, magnified 750 diameters.

Fig. 12. Front view of *Staurastrum polytrichum,* Perty.

Fig. 13 *a.* Front view of *Staurastrum munitum,* Wood.
Fig. 13 *b.* End view of the same.

Fig. 14. *Cosmarium pyramidatum,* Brébisson. Front view.

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Fig. 15. *Cosmarium Broomei*, Thw. Front view, magnified 460 diameters.

Fig. 16. *Cosmarium commissurale*, Bréb. Front view, magnified 750 diameters.

Fig. 17. *Xanthidium armatum*, Bréb. Front view, magnified 260 diameters.

PLATE XIV.

Fig. 1. A sterile cell of *Rhynchonema elongatum*, Wood, magnified 450 diameters.

Fig. 1 *a*. Portion of a filament containing a fertile cell, with the spore nearly matured, amplified 450 diameters.

Fig. 2. A filament of *Rhynchonema pulchellum*, Wood, containing both fertile and sterile cells, magnified 260 diameters.

Fig. 3. The ripened spore of *Spirogyra protecta*, Wood, magnified 450 diameters.

Fig. 3 *a*. Outline of conjugating filaments, and figure of a sterile filament, enlarged 250 diameters.

Fig. 4. Sterile cells of *Spirogyra longata* (Vauch.), magnified 250 diameters.

Fig. 4 *a*. Fertile filaments, magnified 260 diameters.

Fig. 5. A filament of *Aphanochæte repens*, Wood, which has lost its cilia, magnified 460 diameters.

Fig. 6. A fertile branch of *Draparnaldia Billingsii*, Wood, showing the chains of spores, magnified 460 diameters.

PLATE XV.

Fig. 1. Portion of a filament of *Spirogyra majuscula*, Ktz., containing cells with mature spores and others just commencing the process of conjugation.

Fig. 2. A portion of a sterile filament of *Spirogyra diluta*, Wood, magnified 125 diameters, also the outline of a pair of conjugating filaments of the same amplification.

Fig. 2 *b*. Conjugating filaments of *Spirogyra diluta*, Wood, magnified 125 diameters.

Fig. 3 *a*. Portion of a sterile filament of *Spirogyra setiformis* (Roth) Ktz., magnified 125 diameters.

Fig. 3 *b*. Conjugating filaments of the same species, similarly amplified.

Fig. 4 *a*. Cells of *Spirogyra crassa*, Ktz., preparing for conjugation.

Fig. 4 *c*. Conjugating cells of the same plant in the first stage of union.

Fig. 4 *b*. Conjugating cells containing nearly matured zygospores, enlarged 125 diameters.

Fig. 5. Filaments of *Mesocarpus scalaris*, Hassall, commencing the process of conjugation, magnified 125 diameters.

Fig. 6. Sterile cells of *Spirogyra insignis* (Hassall) Ktz.

Fig. 6 *b*. Conjugating filaments of the same species.

Fig. 7. Conjugating filaments of *Spirogyra parvispora*, Wood, containing nearly matured spores, magnified 125 diameters.

Fig. 8. Portion of an ordinary sterile filament of *Zygnema insigne* (Hassall) Ktz.

Fig. 8 *a*. Fertile filaments of the same, magnified 250 diameters.

Fig. 8 *b*. Sterile filament in which multiplication of the species is taking place by the separation of the cells, magnified 260 diameters.

PLATE XVI.

Fig. 1 *a*. Cells of *Sirogonium retroversum*, Wood, just commencing the process of conjugation.

Fig. 1 *b*. Sterile cells.

Figs. 1 *d* and 1 *e*. Outlines of fertile cells; all of these figures are magnified 260 diameters.

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Fig. 2. A matured frond of *Hydrogastrum granulatum* (Linn.), enlarged 90 diameters.

Fig. 2 *a*. A resting spore of the same, enlarged 160 diameters; also a minute, very young frond, magnified 90 diameters.

Fig. 4. A branch of a *Stigeoclonium*, emitting zoospores, enlarged 460 diameters.

Fig. 5. *Bulbotrichia albida*, Wood, magnified 460 diameters.

Fig. 6 *a*. A fertile branch of *Bulbochæte Canbyii*, Wood, showing the matured spore, magnified 260 diameters.

Fig. 6 *b*. A young plant.

Fig. 6 *c*. A branch with young male plants, and a forming zoosporangium, magnified 260 diameters.

Fig. 6 *d*. The empty cup left after the discharge of the oospore.

Fig. 6 *e*. Outline of sporangium.

Fig. 9. A young plant of *Stigeoclonium*.

The globular figures in the lower part of the plate are separate cells of *Porphrydium magnificum*, Wood, magnified 760 diameters. The numbering of the figures at the bottom of the plate are wrong, fig. 3 should read fig. 4, 4, 5, &c.

PLATE XVII.

Fig. 1 *a*. The basal portion of an old frond of *Schizomeris Leibleinii*, Ktz., ? magnified 120 diameters.

Fig. 1 *b*. A filament emitting zoospores, magnified 250 diameters.

Fig. 1 *c*. A perfected zoospore.

Figs. 1 *d*, 1 *e*, 1 *c*. Young plants formed by the germination of the zoospore, magnified 450 diameters.

Fig. 2 *a*. The distal end of a filament of *Œdogonium Huntii*, Wood.

Fig. 2 *b*. Cells showing the formation and growth of a new cell.

Fig. 2 *c*. A portion of filament containing spores in different conditions of maturity.

Fig. 2 *d*. A young female plant with attached dwarf plant.

Fig. 2 *f*. Cells emitting a zoospore, magnified 250 diameters.

Fig. 2 *g*. The perfected zoospore.

Fig. 2 *h*. Outline sketch of a young male plant, magnified about 1200 diameters. The arrows are meant to represent cyclotic currents.

Fig. 3. A fertile filament of *Œdogonium multispora*, Wood, showing spores in different states of maturity, and dwarf male plants.

Fig. 4 *c*. Sterile cells of *Spirogyra dubia*, Ktz., enlarged 260 diameters.

Figs. 4 and 4 *d*. Outline sketches of cells containing spores, magnified 260 and 160 respectively.

Figs. 5 *a* and 5 *b*. Sterile cells of *Spirogyra rivularis* (Hassall), magnified 260 diameters.

Fig. 5 *c*. Outline sketch of conjugating cells with spore similarly amplified

PLATE XVIII.

Fig. 1 *a*. A young female plant of *Pringsheimia inæquale*, Wood, magnified 250 diameters.

Fig. 1 *b*. A portion of an adult female plant, containing immature spores, and showing in outline in the upper sporangium the orifice through which the spermatozoa enter, magnified 250 diameters.

Fig. 1 *c*. The supposed, young male plant, magnified 450 diameters.

Fig. 2. *Œdogonium mirabile*, Wood. A portion of a filament with a partially matured spore.

Fig. 2 *a*. A portion of a female plant, showing the beginning of the development of the female germ, *i. e.* the formation of a very large cell.

Fig. 2 *b*. A further stage of the process, showing the cell divided into an upper and lower portion, with the outline of the attached male plant.

Fig 2 *c*. A fertile filament containing a matured spore. All of these figures are magnified 160 diameters.

Fig. 2 *d*. A couple of cells, one of which has divided into four daughter-cells, each of which contains a nearly perfected androspore, magnified 460 diameters.

Figs. 2 *b* and 2 *g*. Different views of dwarf male plants discharging spermatozoids, the first figure offering a profile view of the cap, the second a view from behind, magnified 400 diameters.

Fig. 2 *e*. A three-celled dwarf male plant, magnified 460 diameters.

Fig. 3. Matured spore of *Œdogonium echinatum*, Wood, uncolored and magnified 750 diameters.

Fig. 4. Spores in sporangia of a Florida *Œdogonium* of undetermined species.

Fig. 5. *Bulbochæte ignota*, Wood.

Fig. 5 *a*. Branches of a frond, showing different stages in the early development of the female germs.

Fig. 5 *b*. Sporangium containing a nearly matured spore. All magnified 460 diameters.

Fig. 6 *a*. Part of a frond of *Bulbochæte dumosa*, Wood, with female germs and dwarf male plants in different stages of development, magnified 260 diameters. The fine markings on the spores have not been reproduced in the chromo-lithograph from my drawing.

Fig. 6 *b*. Male plant discharging spermatozoid, magnified 750 diameters.

Fig. 7 *a*. Part of a sterile filament of a *Conferva* of unknown species.

Fig. 7 *b*. The same discharging zoospores.

Fig. 7 *c*. A cluster of germinating zoospores.

Fig. 7. A young plant. All these figures are magnified 500 diameters.

PLATE XIX.

Fig. 1. *Stigeoclonium*, showing chætophoroidal stage.

Fig. 2. Portion of a fertile filament of *Chantransia expansa*, Wood, magnified 125 diameters.

Fig. 2 *b*. A fragment of a fertile branch, magnified 260 diameters.

Fig. 3. A portion of a fertile filament of *Chantransia macrospora*, Wood, magnified 460 diameters.

Fig. 4 *a*. Outline of some fertile cells of *Spirogyra quinina*, Ag.

Fig. 4 *b*. Filaments in an advanced stage of conjugation.

Figs. 4 *c* and 4 *e*. Fragments of sterile filaments.

PLATE XX.

Fig. 1. *Stigeoclonium*, found near Philadelphia.

Fig. 2. *Arthrodesmus quadridens*, Wood, as viewed from the end, and magnified 259 diameters, also a front view of similar amplification.

Figs. 3 and 3 *a*. Different forms of fructification of *Vaucheria polymorpha*, Wood, showing the emptied antheridia and fertile sporangium.

Fig. 3 *b*. An immature antheridium.

Fig. 3 *c*. Spore of same species.

Fig. 4. Section through fertile node of *Lemanea torulosa* (Roth).

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Fig. 5. Outlines of *Euastrum multilobatum*, Wood. The lower figure represents a lateral view; the upper a two-thirds view.

Fig. 6. Outline of *Penium digitus*, Bréb.

PLATE XXI.

Fig. 1. *Pleurotœnium crenulatum* (Ehrb.), an outline view, magnified 160 diameters.

Fig. 2. *Pleurotœnium breve*, Wood, the empty dead frond, magnified 750 diameters.

Fig. 3. *Tetmemorus Brêbissonii* (Mengh.), an empty semicell.
Fig. 3 *a*. Outline of a whole frond.

Fig. 4 *b*. *Closterium lineatum* (Ehrb.), an empty semicell, magnified 260 diameters.

Fig. 5. *Cosmarium Botrytis* (Bory), an empty frond, magnified 750 diameters.
Figs. 5 *a* and 5 *b*. Outlines of semicells to show the variety of form.

Fig. 6. *Cosmarium Brêbissonii*, Menegh., an empty frond, magnified 750 diameters and outline of apex view.

Fig. 7 *a*. *Cosmarium tetropthalmum* (Ktz.), outline of the empty frond, magnified 460 diameters.

Fig. 8. *Cosmarium margaritiferum* (Turp.), view of an empty semicell, magnified 750 diameters; the outline of this should be more regular.

Fig. 9. *Cosmarium suborbiculare*, Wood, an empty frond, magnified 750 diameters.
Fig. 9 *a*. An outline of end view of similar amplification.

Fig. 10. *Cosmarium Broomei*, Thw., lateral outline of the frond.

Fig. 11. *Micrasterias Jennerii*, Ralfs, an empty semicell.

Fig. 12. *Euastrum ornatum*, Wood, front view, magnified 450 diameters.
Fig. 12 *a*. Lateral view.

Fig. 13. *Euastrum Didelta*, Turpin, outline of the front view.

Fig. 14. *Euastrum elegans*, Bréb., outline of the lateral view.

Fig. 15. *Micrasterias truncata*, Corda, outline of front view, magnified 260 diameters.

Fig. 16. *Micrasterias granulata*, Wood, front view of an empty frond, magnified 460 diameters.

Fig. 17. *Staurastrum orbiculare*, Ehrb., outline of the end view.

Fig. 18. *Staurastrum dejectum*, Bréb., outline of the end view, magnified 750 diameters.

Fig. 19. *Staurastrum Lewisii*, Wood, outline of the end view, magnified 750 diameters.

Fig. 20. *Staurastrum paradoxicum*, Mey., outline of end view.

Fig. 21. The five radiate figure is an end view of *Staurastrum Arachne*, Ralfs, the triradiate of *Staurastrum paradoxum*, Meyen, magnified 750 diameters.

Fig. 22. *Staurastrum Ravenellii*, Wood, Front view of the empty frond, magnified 460 diameters
Fig. 22 *a*. The side view of an empty semicell, magnified 750 diameters.
Fig. 22 *b*. The end view with the same amplification.

Fig. 23. *Staurastrum Polytrichum*, Perty, outline of the frond as seen from the end.

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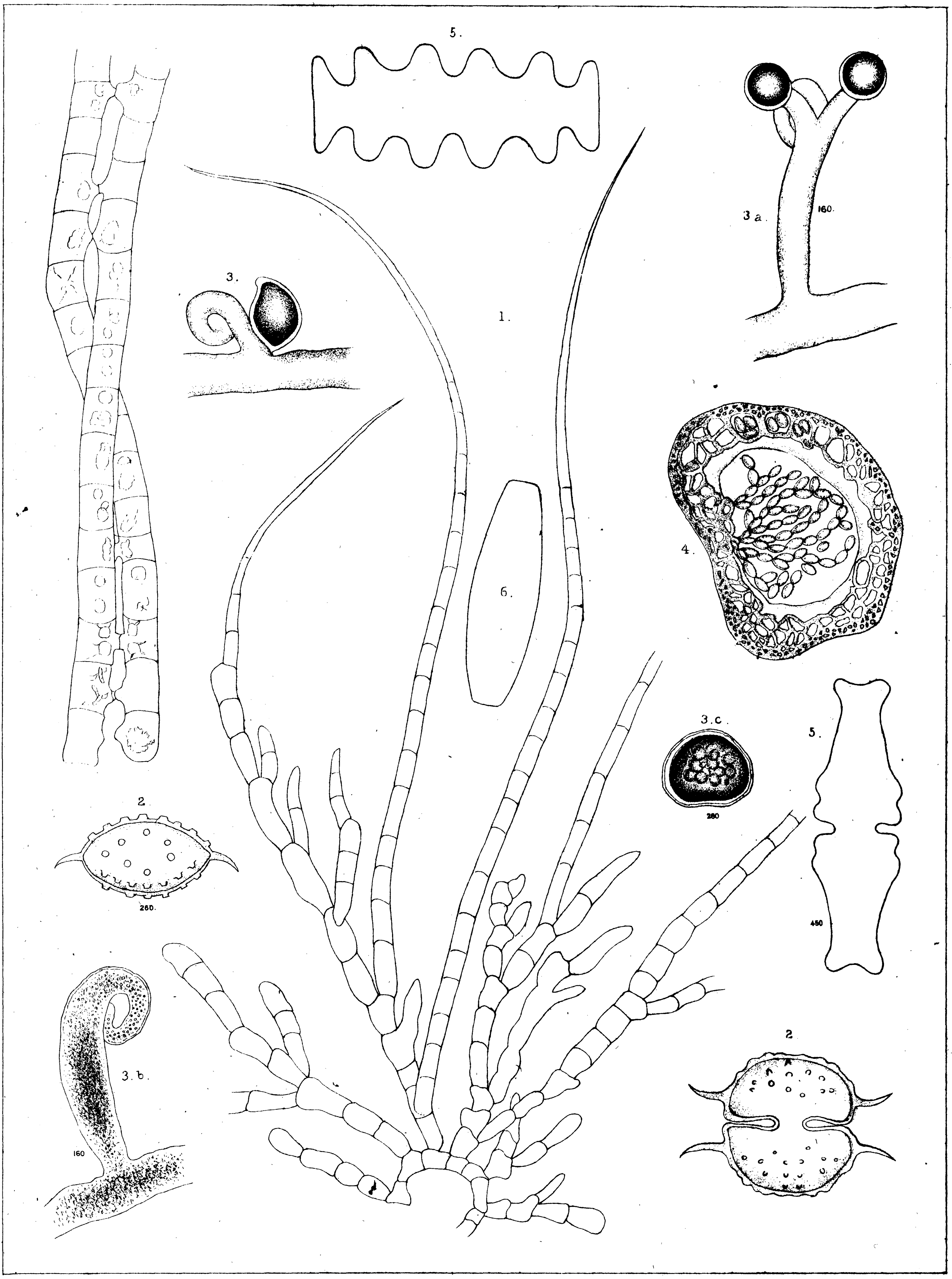

AFTER NATURE BY DR. H. C. WOOD.
T. SINCLAIR & SON, PHILADELPHIA

Fig. 1. STIGEOCLONIUM.
" 2. ARTHRODESMUS QUADRIDENS.

Fig 3. VAUCHERIA POLYMORPHA.
" 4. LEMANEA TORULOSA.

Fig. 5. EUASTRUM MULTILOBATUM.
" 6. PENIUM DIGITUS.

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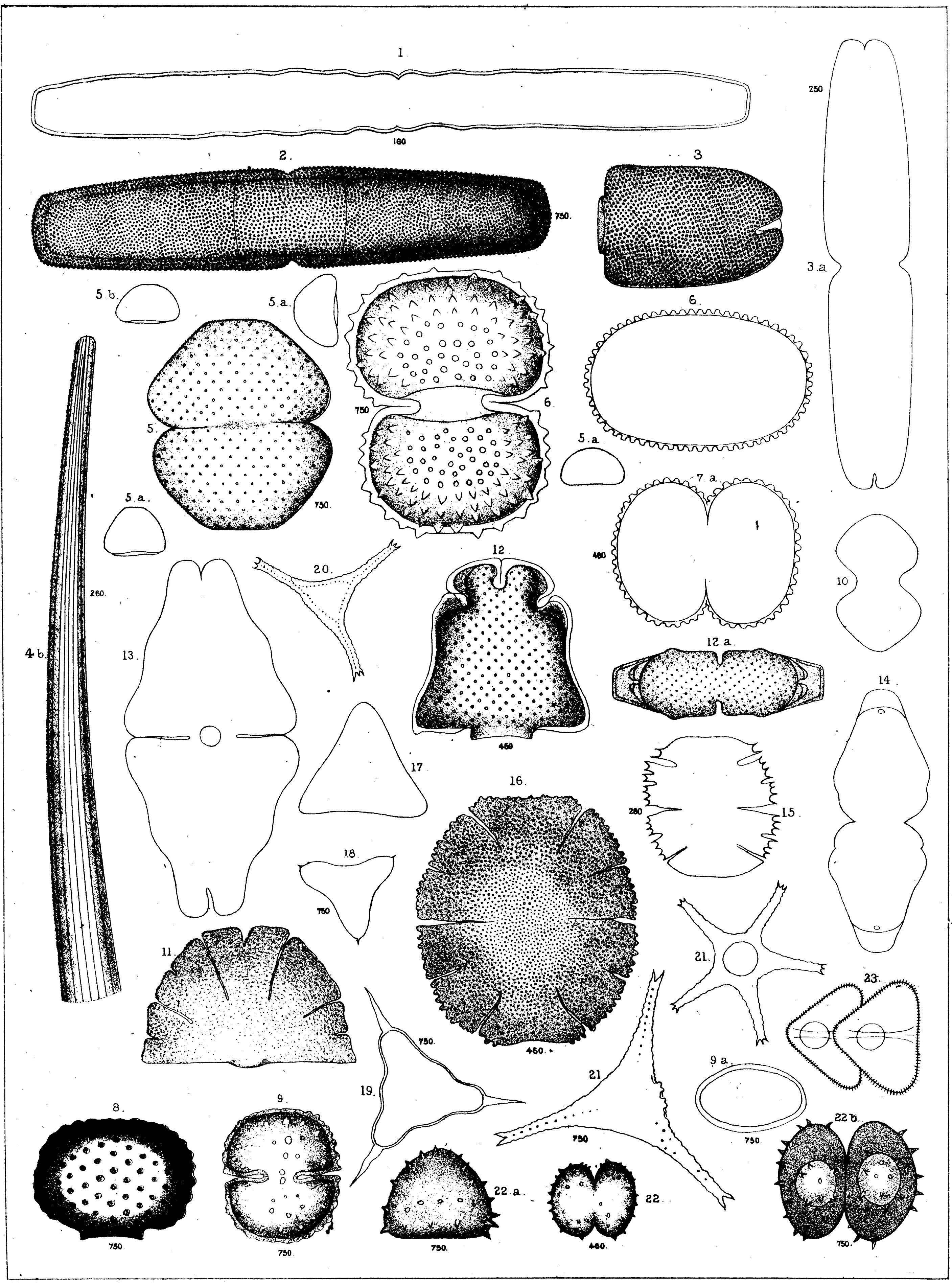

AFTER NATURE BY DR. H. C. WOOD.

T. SINCLAIR & SON, PHILADELPHIA.

Fig. 1. PLEUROTŒNIUM CRENULATUM.
" 2. PLEUROTŒNIUM BREVE.
" 3. TETMEMORUS BRÉBISSONII.
" 4. CLOSTERIUM LINEATUM.
" 5. COSMARIUM BOTRYTIS.

Fig. 6. COSMARIUM BRÉBISSONII.
" 7. COSMARIUM TETRAOPTHALMUM.
" 8. COSMARIUM MARGARITIFERUM.
" 9. COSMARIUM SUBORBICU-

Fig. 10. COSMARIUM BROOMEI.
" 11. MICRASTERIAS JENNERII.
" 12. EUASTRUM ORNATUM.
" 13. E. DIDELTA.
" 14. E. ELEGANS.
" 15. MICRASTERIAS TRUNCATA.

Fig. 17. STAURASTRUM ORBICULARE.
" 18. ST. DEJECTUM.
" 19. ST. LEWSII.
" 20. ST. PARADOXICUM.
" 21. ST. ARACHNE.

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SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
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AN
INVESTIGATION
OF THE
ORBIT OF URANUS,

WITH GENERAL TABLES OF ITS MOTION.

BY
SIMON NEWCOMB,
PROFESSOR OF MATHEMATICS, UNITED STATES NAVY.

[ACCEPTED FOR PUBLICATION, FEBRUARY, 1873.]

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ADVERTISEMENT.

In the investigation of the Orbit of Uranus which forms the subject of the accompanying memoir, as well as in that of the Orbit of Neptune previously published in the Smithsonian Contributions, a large amount of arithmetical computation has been required, especially in the reduction and comparison of observations. The cost of this, in accordance with the spirit of the Institution in advancing science, has been defrayed from the income of the Smithson fund.

As required by the rules of the Institution, the accompanying memoir was referred to competent authority for examination, and the persons selected for this purpose were Professor J. H. C. Coffin, of the Nautical Almanac Office, and Professor Asaph Hall, of the Naval Observatory.

JOSEPH HENRY,
Secretary S. I.

WASHINGTON, 1873.

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PREFACE.

THE present work was undertaken as far back as the year 1859. But the labor devoted to it at first amounted to little more than tentative efforts to obtain numerical data of sufficient accuracy, and to decide upon a satisfactory method of computing the general perturbations of the planet. The elements of Neptune employed in the earlier computations were found to deviate too widely from the truth to be used in computing the perturbations of Uranus with the first order of accuracy, and it became necessary to correct them. This was done during the years 1864 and 1865, and the investigation was printed by the Smithsonian Institution in the latter year. It was then found that the adopted elements of Uranus also differed too widely from the truth to serve as the basis of the work, and they were provisionally corrected by a series of heliocentric longitudes derived from observations extending from 1781 to 1861. Finally it was found that the adopted method of computing the perturbations, that of the "variation of elements," though not deserving of the disfavor into which it has fallen of late years, was practically inapplicable to the computation of the most difficult terms, namely, those of the second order with respect to the disturbing forces. Indeed, it appeared to the author that the only method of computing those terms which was at the same time general, practicable, and fully developed, was that of Hansen. But, were this method adopted, all that had previously been done would have been useless, even for the purpose of comparison and verification, owing to the expression of the coordinates in terms of a disturbed mean anomaly. It appeared to the author that, although this form of theory led to expressions having fewer terms than the other, it was not without its relative disadvantages. Other considerations being equal, he conceived that astronomers generally would greatly prefer to see the perturbations expressed directly in terms of the time, owing to the ease with which the results of different investigators could then be compared, and with which corrections to the theory may be introduced.

Under these circumstances the method described in the first chapter of the present paper was worked out. The question how much it contains that is essentially new is one that the author has never closely examined: it is, however, certain

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that the mode of considering the subject is well known, being that employed by La Place, Herschel, De Pontécoulant, Encke, and perhaps others. The method of forming the required derivations of the perturbative function from the analytical development of that quantity, he has not seen elsewhere.

With these improved elements and methods the work was recommenced in 1868. The earlier investigations being merely provisional, it has not been deemed necessary to present them in the present work. Some of the results, corrected for errors of the older elements, are, however, given for the purpose of comparison.

Although this investigation has absorbed the greater part of the author's leisure for more than five years, it is only through the aid of the Smithsonian Institution and Nautical Almanac that he has been enabled to bring it to a conclusion within that time. At an early stage of the work Professor Henry responded favorably to a request for aid by the employment of computers; it was, however, not found practicable to use such aid until the perturbations had been completed, and the provisional theory concluded. Then, the comparison of theory and observation, and the construction of the tables, involved a large amount of mechanical computation, and on this part of the work a number of persons have been employed by the Institution at various times, among whom may be mentioned Professor F. W. Bardwell, of the University of Kansas, and Dr. C. L. F. Kampf, late of the Observatory of Leiden. Every part of the work has, however, been done under the author's immediate direction, and, as nearly as possible, in the same way as if he had done it himself, a result which, in one or two cases, has been attained only by the expenditure of an amount of labor approximating that saved by the employment of the computer.

In presenting the steps of the investigation, the end has been kept constantly in view to render as easy as possible the detection and correction of any error, or the introduction of any alteration in the elements or other data. It is, of course, impossible to present the steps of the computation with any approach to fulness without far transcending the limits of the printed work: The results given are, therefore, those which it was supposed would be most useful to the future investigator of the same subject. There is reason to believe that the original computations will ultimately become the property of the National Academy of Sciences, so that they may always be referred to for the clearing up of any difficulty in the printed text.

The author's acknowledgments are due to Professor J. H. C. Coffin, Superintendent of the Nautical Almanac, and Mr. E. J. Loomis, of the Nautical Almanac Office, for reading the proof sheets of the last twelve tables during the absence of the former abroad.

WASHINGTON, July 31, 1873.

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ERRATA.

Pages 100 to 105. In computing the latitude from the provisional theory the values of the secular terms of $\delta\eta$ and δk on page 97 have been interchanged. The provisional latitude, therefore, requires the correction

$$-0''.53\ T \sin \mathrm{v} + 0''.53\ T \cos \mathrm{v}$$

where

$$\mathrm{v} = g + 12^\circ\, 45' + 2e \sin g.$$

This correction is not applied in the subsequent investigation. Its effect would have been to change the value of b deduced on p. 176 by probably $0''.2$ or $0''.3$. The effect on the other elements of latitude would have been much smaller, and therefore unimportant.

Page 122, line 15. Add: the corrections in the sixth column being omitted.

Page 151. Add foot-note: In forming these comparisons the corrections to the heliocentric longitude in the sixth column of the provisional ephemeris, pages 100 to 105, are not applied.

Page 159. Equation 7. In this equation the coefficients of $\delta\lambda$ and $\delta\rho$ have been multiplied by $\frac{1}{5}$, instead of $\frac{2}{5}$, the factor of δl. The effect of this error enters into all the subsequent results, but in the comparisons of theory and observation it is corrected.

Page 184. The element here represented by $\varkappa$ (kappa) is the same which, in the preceding chapters, has by mistake been represented by k, and which is defined on p. 24. The k of Chapter VIII is, therefore, not the same with that of preceding chapters.

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ON THE ORBIT OF URANUS.

INTRODUCTION.

THE connection of the planet Uranus with the most brilliant astronomical achievement of the century lends a peculiar interest to its theory. The researches of Adams and Le Verrier showed that the observed motions of that planet were represented, at least approximately, by the action of a theoretical planet having the longitude of Neptune. Peirce showed that the action of Neptune itself accounted for these motions within the limits of possible error of the observations used by Le Verrier. It remains to be seen whether the agreement between theory and observation still subsists when the comparatively few observations used by those investigators are reduced with the more refined data now at our disposal, and when the great mass of additional observations made both before and since the date of Le Verrier's researches are included.

The circumstances connected with the discovery of Neptune have been so exhaustively recounted by a number of authors that it would be difficult to add anything not already familiar to astronomers without transcending our present limits. I shall therefore confine myself to such an account of previous researches on the theory of Uranus as may give an idea of their nature and extent, and facilitate their comparison with the methods and results of the present investigation.

The perturbations used by Bouvard in his tables are those of the Mécanique Céleste. Although not affected with any striking error, the numerical methods adopted in their computation are necessarily too rough to allow of much interest attaching to their comparison with the results of the more recent researches.

It is essential to a clear understanding of subsequent researches that we classify the methods which have been or may be adopted in the computation of the general perturbations of the planets. This computation comprises two distinct operations: (1) the development of the disturbing forces, or some quantities of which these forces are functions; (2) the integration of the equations of motion under the influence of these forces. In each of these operations three methods have been employed.

In developing the perturbative function, we have first the purely analytic method used by the great geometers of the last century. In this method this function is developed in powers of the eccentricities and mutual inclination of the orbits of the two planets, and the numerical coefficients are found by substituting the values of the elements in these expressions. It is only applicable when the eccentricities

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and mutual inclination are small, and has for that reason fallen, of late, into a certain disrepute. The extended tables published by Le Verrier[1] have, however, added so much to its facility for use that it is not wholly unworthy of attention.

At the other extreme stands the purely mechanical method, in which special values of the disturbing force are computed for many combinations of the mean anomalies of the two planets, and the values of the coefficients in the general expression for the force thence deduced.

Between these two stands what I conceive we may designate as the Cauchy-Hansen method, in which the development is made mechanically with respect to the one planet, but the eccentric anomaly of the other is retained as an undetermined quantity. The germ of this method is found in several papers, by Cauchy, in the earlier volumes of the *Comptes Rendus* of the French Academy, which have since been combined into a homogeneous memoir by Puiseux.[2] The object had in view by these authors is only the computation of inequalities of long period. But Hansen has taken up the essential principle of the method, first, in his prize memoir on the perturbations of comets, crowned by the French Academy of Sciences, about 1848, and afterwards in his "*Auseinandersetzung einer zweckmässigen Methode zur Berechnung der Störungen der kleinen Planeten*,"[3] and applied it to the general development of perturbations.

Among the three methods of integration, the first in point of analytical elegance and generality, but the last in order of convenience in use, is that of the variation of elements, a method with which the name of La Grange is inseparably associated.

In the second the direct integration of the differential equations which express the perturbations of longitude, latitude, and radius vector is effected by special devices.

In the first of these methods the problem is presented in this form: The equations of motion being completely integrated for the action of the principal forces only, how must the arbitrary constants of integration vary in order that the same expressions may represent the motion of the planet under the influence of the disturbing forces? In the second method, the same thing being presupposed, the question is, what expressions must be added to the integrals of undisturbed motion in order that the sum may represent the integrals of the disturbed motion?

The third is Hansen's method, in which the co-ordinates are partly expressed in terms of a certain function of the time known as the disturbed mean anomaly, determined by the condition that the true longitude in the disturbed orbit shall be the same function of the disturbed time that the longitude in the elliptic orbit is of the simple time.

Although the last two methods have a great advantage over the first in the computation of the periodic perturbations, I conceive the first to be best adapted to the computation of the secular variations, and perhaps, of terms of very long period in the mean longitude and the elements of the orbit.

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[1] *Annales de l'Observatoire Impérial de Paris.* Tome I.

[2] *Annales de l'Observatoire Impérial de Paris.* Tome VII.

[3] *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften.* Band V. VI, VII.

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In his researches on the motion of Uranus, the first thing done by Le Verrier was to recompute the perturbations by Jupiter and Saturn. It will sufficiently describe his method of doing this to say that in the developments he used the purely mechanical method for the action of Saturn, and the algebraic development of the perturbative function for the action of Jupiter, while in the integration he used the method of the variation of elements. After completing the perturbations of the first order he made the earliest attempt at a complete determination of those of the second order. Beginning with the terms of this order which arise from the secular variations of the elements, he determines them by recomputing the terms of the first order for the epoch 2300, and assuming that the general term will then be given by interpolating between the two terms thus found, supposing them to increase uniformly with the time. This proceeding has the sanction of such high authority that it is worth while to call attention to its want of rigor. The differential coefficient of each element being given in the form

$$\frac{da}{dt} = k \cos bt,$$

k being a function of the elements, the perturbation of the first order will be

$$\delta a = \frac{k}{b} \sin bt.$$

When we take into account the variation of k, and suppose it of the form $k_0 + k't$, the process is equivalent to supposing that in this case

$$\delta a = \frac{k_0 + k't}{b} \sin bt,$$

whereas it really contains the additional term,

$$\frac{k'}{b^2} \cos bt,$$

which appears to be neglected in the process in question. It will be seen that the neglected coefficient is equal to the secular variation of the term during the time that its argument requires to increase by an amount equal to the unit radius. It is therefore the more important the longer the period of the inequality.

To obtain the periodic terms of the second order Le Verrier begins by determining the ten principal terms of the perturbations of the elements of Saturn produced by Jupiter. Next he takes up the terms in the mean longitude of Uranus which depend on the square of the mass of Saturn. The only sensible terms he finds are

$$\begin{aligned} &- 1''.17 \sin (\zeta' - 3\zeta) - 0''.35 \cos (\zeta' - 3\zeta) \\ &+ 0''.43 \sin (\zeta'' - 4\zeta' + 4\zeta) - 0''.21 \cos (\zeta'' - 4\zeta' + 4\zeta), \end{aligned}$$

ζ, ζ', and ζ'' being the mean anomalies of Uranus, Saturn, and Jupiter, respectively. The terms depending on the product of the masses of Jupiter and Saturn are then taken up. Fifteen arguments are found the coefficients of which vary from a small fraction of a second to one or two seconds, while a single one of long period amounts to 32''.

When the method of variation of elements is used, it is necessary not only to determine these variations to quantities of the second order, but, in the transforma-

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tion of the perturbations of the elements into perturbations of the co-ordinates, to carry this transformation to terms of the second order also. This Le Verrier avoids by showing that the terms of the lowest order with respect to the eccentricities thus introduced are destroyed by certain terms in the perturbations of the elements, so that it is only necessary to omit both classes of terms. These terms are of that fictitious class which disappear of themselves by a simple change of elements. When, instead of the eccentricity and longitude of the perihelion, we take h and k, which represent the products of the eccentricity into the sine and cosine of this longitude respectively, these terms disappear of themselves both from the perturbations of the elements and of the co-ordinates. It is not likely that any of the neglected terms of this class exceed 0″.1.

As soon as the elements of Neptune were known, the nature of its general action on Uranus became of interest. This subject was taken up by Prof. Peirce, whose results are found in the Proceedings of the American Academy of Arts and Sciences, Vol. I, pp. 334–337. This paper is accompanied with a comparison of his theory of Uranus with observations, to which similar comparisons of the theories of Adams and Le Verrier are added. This comparative exhibit is of sufficient interest to be given here. The numbers given are probably excesses of computed over observed longitudes.

RESIDUAL DIFFERENCES BETWEEN THE THEORETICAL AND OBSERVED LONGITUDES OF URANUS, FROM THE THEORIES OF PEIRCE, LE VERRIER, AND ADAMS.

Year.	From Le Verrier's best orbit of Uranus from the modern observations without any external planet.	From Le Verrier's original theory with his best orbit of hypothetical planet, of which the mass is $\frac{1}{9322}$.	From Adams's original theory with his second hypothetical planet of which mass is $\frac{1}{6666}$.	From Peirce's theory of Neptune adopting for its mass		
				That of Struve from his own observations of the satellite $\frac{1}{14496}$.	That deduced by Peirce from Bond's & Lassel's observations combined $\frac{1}{18780}$.	That deduced by Peirce from Bond's observations of Lassel's satellite $\frac{1}{19840}$.
	″	″	″	″	″	″
1690	+ 289.0	— 19.9	+ 50.0	— 124.7	+ 13.0	+ 0.8
1715	+ 279 6	+ 5.5	— 6.6	— 99.6	+ 10.0	+ 8.7
1756	+ 230 9	— 4.0	— 4.0	— 102.4	— 12.7	+ 4.0
1769	+ 123 3	+ 3.7	+ 1.8	— 67.0	— 16.0	— 6.0
1782	+ 20 5	+ 2 3	0.0	— 18.3	— 5.6	— 3.0
1787	+ 2.0	— 1.2	— 0.2	— 4.7	— 1 2	— 0.5
1792	— 7.8	+ 0 3	— 1.1	+ 1.6	+ 0 5	+ 0.3
1797	— 6.7	— 1.0	— 0.5	+ 3 3	+ 0.8	+ 0.3
1803	— 3 4	+ 0.8	+ 1 6	+ 3.2	+ 1.2	+ 0.8
1808	+ 3.8	+ 0.8	0.0	— 1.3	— 0.6	— 0.4
1813	+ 4.5	— 0 9	— 1.0	— 2.3	+ 1.1	— 0.3
1819	+ 3.8	+ 0.4	— 2 2	+ 0.9	+ 0.7	+ 1.0
1824	— 7.6	— 5 4	+ 1.7	— 1 6	— 1.9	— 2.0
1829	— 7.8	— 2.2	+ 2.0	+ 2 5	+ 1.3	+ 0.8
1835	— 4 5	— 0.8	— 1.2	+ 3.9	+ 2 4	+ 2.0
1840	+ 0.7	+ 2 2	+ 1.3	— 1.3	— 1.3	— 1.1
1845	+ 6.5	— 0.3	. . . . .	— 2.8	— 1 2	— 0.9

In this paper Professor Peirce presents the results of a complete computation of the general perturbations of Uranus by Neptune in longitude and radius vector, but without any details whatever of the investigation, or any statement of the methods employed. The minuteness of the residuals in the last column of the preceding table shows that employing these perturbations by Neptune, and those of Le Verrier by Jupiter and Saturn, we had a theory of Uranus from which quite accurate tables might have been constructed. But this never seems to have been done. The ephemeris of Uranus in the American Nautical Almanac was intended to be founded on this theory, but the proper definitive elements do not seem to have been adopted in the computations, as the ephemeris does not correspond with the theory.

Although twenty-five years have elapsed since the epoch of these researches, I am not aware of any published work of importance on the theory of Uranus during the interval. Mr. T. H. Safford has, however, made a very extended investigation of the subject, but has published nothing more than a brief general description of his work, which may be found in the Monthly Notices of the Royal Astronomical Society, Vol. 22. Like Professor Peirce, he took Le Verrier's perturbations by Jupiter and Saturn, but, instead of using general perturbations by Neptune, he computed the effect of the action of this planet by mechanical quadratures for the whole period of the observations of Uranus, and thus corrected the elements and the mass of Neptune from modern observations alone. The mass in question deduced was

$$\frac{1}{20039}$$

Mr. Safford does not give the representation of the modern observations, but presents the following comparison of the ancient ones, alongside which we place for comparison the corresponding numbers of Peirce's theory and those of the present investigation.

Excess of Observation over Theory.

Date.	No. of obs.	Safford.	Peirce.	Newcomb.
1690	1	+ 5″.0	— 0″.8	— 11″
1715	3	— 4.2	— 8.7	— 8
1750	2	— 1.2	.....	+ 2.9
1753	1	— 0.2	.....	
1756	1	— 0.9	— 4.0	
1764	1	+ 0.4		
1769	8	+ 4.5	+ 6.0	— 1.4

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CHAPTER I.

METHOD OF DETERMINING THE PERTURBATIONS OF LONGITUDE, RADIUS VECTOR, AND LATITUDE OF A PLANET BY DIRECT INTEGRATION.

LET us conceive a plane determined by the condition that it shall pass through the sun and contain the tangent to the orbit of a planet at any moment. If the planet were acted on by the sun alone, the position of this plane would be invariable, but, under the influence of the disturbing forces of the other planets, it is subject, at each instant, to a motion of rotation around the radius vector of the planet. We may regard this as the instantaneous plane of the planet's orbit. The disturbing and the disturbed planet will each have its own instantaneous plane.

Let us now put:—

v, the longitude of a planet counted from a determinate point in the instantaneous plane of its orbit.

v, its distance from the node of intersection of its own orbit with that of another planet.

γ, the mutual inclination of the two orbits.

σ, $\sin \frac{1}{2} \gamma$.

r, the radius vector of the planet.

ρ, its logarithm.

μ, the attractive force of the sun upon unit of matter at unit distance.

a, the mean distance corresponding to the observed mean motion of the planet, determined by the condition

$$a^3 = \frac{\mu(1+m)}{n^2},$$

m and n being as usual the mass and mean motion.

a_0, the value of a corrected for the constants introduced by the perturbations, so that, as in the elliptic motion, we have

$$\rho = \log a + f(l, e, \varpi),$$

we shall have in the disturbed motion

$$\rho = \log a_0 + f(l, e, \varpi) + \text{periodic terms only.}$$

a_1, the mean distance of an outer planet, whether it be a disturbing or disturbed planet.

$\mathfrak{w}$, the logarithm of a.

α, the ratio of two mean distances, taken less than unity.

R, the perturbative function.

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h, the coefficient of any term of $\frac{a_1}{m'}R$, so that we have

$$R = \Sigma \frac{m'h}{a_1} \cos N$$

m' being here the mass of the disturbing planet.

λ, the mean distance of the planet from the node, or the mean value of v.

ω, the distance of the perihelion from the node.

g, the mean anomaly.

l, the mean longitude, or the mean value of v.

ψ, the angle of eccentricity so that $e = \sin\psi$.

r_0, the radius of the planet in the undisturbed ellipse.

r_1, the quotient of r_0 divided by the mean distance, which is a function of the eccentricity and mean anomaly only.

T, the time after the epoch 1850, Jan. 0, Greenwich mean noon, counted in Julian centuries.

ν, the integrating factors of the periodic terms, or the ratio $\frac{n}{N}$, N being the change of the angle in unit of time.

u, the eccentric anomaly, and, in the tables, the argument of latitude.

We have for the value of R

$$R = \frac{m'}{\sqrt{r^2 - 2rr'(\cos\mathrm{v}\cos\mathrm{v}' + \sin\mathrm{v}\sin\mathrm{v}'\cos\gamma) + r'^2}} - \frac{m'r}{r'^2}(\cos\mathrm{v}\cos\mathrm{v}' + \sin\mathrm{v}\sin\mathrm{v}'\cos\gamma)$$

or, if we suppose r replaced by its value in ρ, namely

$$r = c^\rho$$

we shall have

$$R = m'f(\mathrm{v}, \mathrm{v}', \rho, \rho', \gamma).$$

With this value of R it is well known that the differential equations for the longitude and radius vector of a planet are

$$\begin{aligned} r\frac{d^2r}{dt^2} - r^2\frac{dv^2}{dt^2} + \frac{\mu(1+m)}{r} &= \mu\frac{\partial R}{\partial\rho}; \\ r^2\frac{d^2v}{dt^2} + 2r\frac{dr}{dt}\frac{dv}{dt} &= \mu\frac{\partial R}{\partial\mathrm{v}}. \end{aligned} \qquad (1)$$

If we multiply the first of these equations by $2\frac{d\rho}{dt}$ and the second by $2\frac{dv}{dt}$ and add them together, putting, for brevity,

$$\frac{\partial R}{\partial\rho}\frac{d\rho}{dt} + \frac{\partial R}{\partial\mathrm{v}}\frac{dv}{dt} = D'_tR, \qquad (2)$$

and then integrate, we shall have

$$\frac{dr^2}{dt^2} + r^2\frac{dv^2}{dt^2} - \frac{2\mu(1+m)}{r} = 2\mu\left(C + \int D'_t R\,dt\right)$$

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C being the arbitrary constant added to the integral. Adding this equation to the first of equations (1) we have

$$\tfrac{1}{2}\frac{d^2(r^2)}{dt^2}-\frac{\mu(1+m)}{r}=\mu\left(2C+2\int D'_{\mathrm{t}}R dt+\frac{\partial R}{\partial\rho}\right) \qquad (3)$$

Let us now represent by r_0 that elliptic value of r which satisfies the equation

$$\tfrac{1}{2}\frac{d^2(r_0^2)}{dt^2}-\frac{\mu(1+m)}{r_0}=2\mu C.$$

Subtracting this equation from the last we have

$$\tfrac{1}{2}\frac{d^2(r^2-r_0^2)}{dt^2}-\mu(1+m)\left(\frac{1}{r}-\frac{1}{r_0}\right)=\mu\left(2\int D'_{\mathrm{t}}R\,dt+\frac{\partial R}{\partial\rho}\right).$$

in which no constant is to be added to the integral, and both sides of the equation are of the order of the disturbing forces. As there is a decided advantage in taking the logarithm of the radius vector as the variable instead of r itself, we substitute for the latter its value

$$r=c^{\rho},\quad r_0=c^{\rho_0}$$

and put

$$\delta\rho=\rho-\rho_0.$$

Then

$$r^2=c^{2\rho_0+2\delta\rho}=r_0^2\,c^{2\delta\rho}=r_0^2\left(1+2\delta\rho+\frac{2^2}{1.2}\delta\rho^2+\text{etc.}\right)$$

$$\tfrac{1}{2}(r^2-r_0^2)=r_0^2\delta\rho+r_0^2\delta\rho^2+\text{etc.}$$

$$\frac{1}{r}-\frac{1}{r_0}=-\frac{\delta\rho}{r_0}+\frac{\delta\rho^2}{2r_0}+\text{etc.}$$

Substituting these values in the above equation, carrying the development only to terms of the second order, and transposing those terms to the right hand side of the equation, and putting $\mu'=\mu(1+m)$, we find

$$\frac{d^2(r_0^2\delta\rho)}{dt^2}+\frac{\mu'}{r_0^3}(r_0^2\delta\rho)=\mu\left(2\int D'_{\mathrm{t}}R dt+\frac{\partial R}{\partial\rho}\right)-\frac{d^2(r_0^2\delta\rho^2)}{dt^2}+\frac{\mu\delta\rho^2}{2r_0}, \qquad (4)$$

an equation which gives the perturbations of radius vector.

The general mode of solving this equation by successive approximation is familiar. The principles on which the successive approximations are made being the same, we shall begin by assuming that we have obtained first approximations to the values of $\delta\mathrm{v}$, $\delta\mathrm{v}'$, $\delta\rho$, $\delta\rho'$, $\delta\gamma$, and that from these we wish to pass to a second approximation. We must first carry this approximation into the functions of R in the second member of (4). To effect this we must show how, from the development of R in terms of the elements and the time, we may form its successive derivatives with respect to the quantities which enter into it. R, while originally a function of v, v', ρ, ρ', and γ, is, in its developed form, a function of λ, λ', ω, ω', e, e', ϖ, ϖ', and γ, the development being effected by substituting for the first set of quantities their values in terms of the second. The substitution is as follows:

$$\begin{aligned}\mathrm{v}&=\lambda+Fg,\\ \mathrm{v}'&=\lambda'+Fg', \qquad (5)\\ \rho&=\varpi+\phi g,\\ \rho'&=\varpi'+\phi g',\end{aligned}$$

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Fg being the equation of the centre, and ϕg the part of ρ depending on the eccentricity in the elliptic motion. It follows that if we express the developed expression for R as a function of λ, λ', g, g', ϖ, ϖ', which we may do by putting

$$\omega = \lambda - g, \quad \omega' = \lambda' - g';$$
$$a = c^{\varpi}, \qquad a' = c^{\varpi'};$$

we shall have by successive differentiation

$$\begin{aligned}
\frac{\partial R}{\partial \lambda} &= \frac{\partial R}{\partial \mathrm{v}}\,\frac{\partial \mathrm{v}}{\partial \lambda} = \frac{\partial R}{\partial \mathrm{v}} \\
\frac{\partial^2 R}{\partial \lambda^2} &= \frac{\partial^2 R}{\partial \mathrm{v}^2}\,\frac{\partial \mathrm{v}}{\partial \lambda} = \frac{\partial^2 R}{\partial \mathrm{v}^2} \\
\frac{\partial R}{\partial \varpi} &= \frac{\partial R}{\partial \rho}\,\frac{\partial \rho}{\partial \varpi} = \frac{\partial R}{\partial \rho} \\
\frac{\partial^2 R}{\partial \varpi^2} &= \frac{\partial^2 R}{\partial \rho^2}\,\frac{\partial \rho}{\partial \varpi} = \frac{\partial^2 R}{\partial \rho^2} \\
\text{etc.} & \qquad \text{etc.} \qquad \text{etc.}
\end{aligned} \tag{6}$$

and in general

$$\frac{\partial^{m+n+m'+n'} R}{\partial \lambda^m\, \partial \varpi^n\, \partial \lambda'^{m'}\, \partial \varpi'^{n'}} = \frac{\partial^{m+n+m'+n'} R}{\partial \mathrm{v}^m\, \partial \rho^n\, \partial \mathrm{v}'^{m'}\, \partial \rho'^{n'}}$$

Thus, by expressing the developed R in the above form, we may find the derivative of any order with respect to v, v', ρ and ρ', by taking the corresponding derivative with respect to λ, λ', ϖ and ϖ'.

The developed R is usually expressed in the form

$$R = \Sigma \frac{m'h}{a_1} \cos (i'\lambda' + i\lambda + j'\omega' + j\omega)$$

a_1 being the mean distance of the outer planet, whether disturbing or disturbed, and h a function of e, e', α, and γ. Substituting for ω its value in g, this equation will become

$$R = \Sigma \frac{m'h}{a_1} \cos \Big((i' + j')\, \lambda' + (i + j)\, \lambda - j'g' - jg \Big).$$

Putting for brevity

$$N = i'\lambda' + i\lambda + j'\omega' + j\omega,$$

the formulæ (6) give

$$\begin{aligned}
\frac{\partial R}{\partial \mathrm{v}} &= -\Sigma \frac{m'h}{a_1} (i + j) \sin N \\
\frac{\partial^2 R}{\partial \mathrm{v}^2} &= -\Sigma \frac{m'h}{a_1} (i + j)^2 \cos N \\
\frac{\partial R}{\partial \rho} &= \Sigma\, m' \frac{\partial \dfrac{h}{a_1}}{\partial \varpi} \cos N
\end{aligned} \tag{7}$$

and in general

$$\frac{\partial^{u+u'+n+n'} R}{\partial \mathrm{v}^u\, \partial \mathrm{v}'^{u'}\, \partial \rho^n\, \partial \rho'^{n'}} = \Sigma \pm m' (i + j)^u (i' + j')^{u'} \frac{\partial^{n+n'} \dfrac{h}{a_1}}{\partial \varpi^n\, \partial \varpi'^{n'}} \begin{matrix} \cos \\ \sin \end{matrix} N$$

The formation of the derivatives in the second member of this equation demands attention. In the analytic development of the perturbative function each value of h is composed of a series of terms each of the form

$$E \times A,$$

E being a function of the eccentricities and mutual inclination, and A a function of α of the form

$$(0)\, \alpha^{s-\frac{1}{2}} b_s^{(i)} + (1)\, \alpha^{s+\frac{1}{2}} \frac{\partial b_s^{(i)}}{\partial \alpha} + (2)\, \alpha^{s+\frac{3}{2}} \frac{\partial^2 b_s^{(i)}}{\partial \alpha^2} + \text{etc.} + \alpha^{\frac{2s+2n-1}{2}} \frac{\partial^n b_s^{(i)}}{\partial \alpha^n}, \qquad (8)$$

(0), (1), etc., being numerical coefficients connected with the coefficients $V^{(i)}$ tabulated by Le Verrier, in Tome I of his Annales de l'Observatoire, by the relation

$$(n) = \frac{V_n^{(i)}}{1.2.3 \ldots n},$$

and $b_s^{(i)}$ being, as usual, the coefficient of cos $i\phi$ in the development of

$$(1 - 2\alpha \cos \phi + \alpha^2)^{-s}$$

in multiples of cos ϕ, and $n - 1$ the sum of the exponents of the eccentricities in E.

It would have been much more convenient if in effecting this development the derivatives of $b_s^{(i)}$ had been taken with respect to ν instead of α. In fact the derivative $\frac{\partial^n b_s^{(i)}}{\partial \alpha^n}$ when expressed in terms of the derivatives with respect to ν is of the form

$$\alpha^n \frac{\partial^n b_s^{(i)}}{\partial \alpha^n} = n_1 \frac{\partial b_s^{(i)}}{\partial \nu} + n_2 \frac{\partial^2 b_s^{(i)}}{\partial \nu^2} + \text{etc.} + n_n \frac{\partial^n b_s^{(i)}}{\partial \nu^n}$$

Therefore, when expressed in terms of the derivatives with respect to ν, A will be of the form

$$\alpha^{s-\frac{1}{2}} \left((0)' \, b_s^{(i)} + (1)' \frac{\partial b_s^{(i)}}{\partial \nu} + (2)' \frac{\partial^2 b_s^{(i)}}{\partial \nu^2} + \text{etc.} \right),$$

from which the derivatives $\frac{\partial A}{\partial \nu}, \frac{\partial^2 A}{\partial \nu^2}$, etc., may be found with great facility.

As in the actual developments of R which we possess, the values of A are given in the form (8), we must find the expression for the first two derivatives of its several terms with respect to ν, which we easily do by the application of the symbolic formulæ

$$D_\nu = \alpha D_\alpha$$
$$D^2_\nu = \alpha (D_\alpha + \alpha D^2_\alpha).$$

Beginning with the case of $s = \frac{1}{2}$, we have

$$\frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \nu} = \alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha}$$

$$\frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \nu^2} = \alpha \frac{\partial b_{\frac{1}{2}}^{(i)}}{\partial \alpha} + \alpha^2 \frac{\partial^2 b_{\frac{1}{2}}^{(i)}}{\partial \alpha^2}$$

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$$\frac{\partial\left(\alpha^n \dfrac{\partial^n b^{(i)}_{\frac{1}{2}}}{\partial\alpha^n}\right)}{\partial x} = n\alpha^n \frac{\partial^n b^{(i)}_{\frac{1}{2}}}{\partial\alpha^n} + \alpha^{n+1} \frac{\partial^{n+1} b^{(i)}_{\frac{1}{2}}}{\partial\alpha^{n+1}}$$

$$\frac{\partial^2\left(\alpha^n \dfrac{\partial^n b^{(i)}_{\frac{1}{2}}}{\partial\alpha^n}\right)}{\partial x^2} = n^2\,\alpha^n \frac{\partial^n b^{(i)}_{\frac{1}{2}}}{\partial\alpha^n} + (2n+1)\,\alpha^{n+1} \frac{\partial^{n+1} b^{(i)}_{\frac{1}{2}}}{\partial\alpha^{n+1}} + \alpha^{n+2} \frac{\partial^{n+2} b^{(i)}_{\frac{1}{2}}}{\partial\alpha^{n+2}}$$

$$= n\,\frac{\partial\left(\alpha^n \dfrac{\partial^n b^{(i)}_{\frac{1}{2}}}{\partial\alpha^n}\right)}{\partial x} + \frac{\partial\left(\alpha^{n+1} \dfrac{\partial^{n+1} b^{(i)}_{\frac{1}{2}}}{\partial\alpha^{n+1}}\right)}{\partial x}$$

consequently we have for the derivatives of A from formulæ (8)

$$\frac{\partial A}{\partial x} = (0)\,\alpha\,\frac{\partial b^{(i)}_{\frac{1}{2}}}{\partial\alpha} + (1)\left(\alpha\,\frac{\partial b^{(i)}_{\frac{1}{2}}}{\partial\alpha} + \alpha^2\,\frac{\partial^2 b^{(i)}_{\frac{1}{2}}}{\partial\alpha^2}\right) + (2)\left(2\alpha^2\,\frac{\partial^2 b^{(i)}_{\frac{1}{2}}}{\partial\alpha^2} + \alpha^3\,\frac{\partial^3 b^{(i)}_{\frac{1}{2}}}{\partial\alpha^3}\right) + \text{etc.}$$

$$\frac{\partial^2 A}{\partial x^2} = (0)\left(\alpha\,\frac{\partial b^{(i)}_{\frac{1}{2}}}{\partial\alpha} + \alpha^2\,\frac{\partial^2 b^{(i)}_{\frac{1}{2}}}{\partial\alpha^2}\right) + (1)\left(\alpha\,\frac{\partial b^{(i)}_{\frac{1}{2}}}{\partial\alpha} + 3\alpha^2\,\frac{\partial^2 b^{(i)}_{\frac{1}{2}}}{\partial\alpha^2} + \alpha^3\,\frac{\partial^3 b^{(i)}_{\frac{1}{2}}}{\partial\alpha^3}\right) + \text{etc.} \tag{9}$$

The derivatives of A being formed in this way, those of h are immediately deduced from the equations

$$\frac{\partial h}{\partial x} = \Sigma E\,\frac{\partial A}{\partial x},$$

$$\frac{\partial^2 h}{\partial x^2} = \Sigma E\,\frac{\partial^2 A}{\partial x^2}.$$

When s is equal to $\frac{3}{2}$, A is of the form

$$\alpha\left\{(0')\,b^{(i)}_{\frac{3}{2}} + (1)'\,\alpha\,\frac{\partial b^{(i)}_{\frac{3}{2}}}{\partial\alpha} + (2)'\,\alpha^3\,\frac{\partial b^{(i)}_{\frac{3}{2}}}{\partial\alpha^2} + \text{etc.}\right\}$$

The quantity within parentheses is of the same form with A, in the case of $s = \frac{1}{2}$. If we represent it by A' we shall have

$$\frac{\partial A}{\partial x} = \alpha\left(\frac{\partial A'}{\partial x} + A'\right) = \alpha\,\frac{\partial A'}{\partial x} + A$$

$$\frac{\partial^2 A}{\partial x^3} = A + 2\alpha\,\frac{\partial A'}{\partial x} + \alpha\,\frac{\partial^2 A'}{\partial x^2}$$

A' being the same form with A, the derivatives $\frac{\partial A'}{\partial x}$ and $\frac{\partial^2 A'}{\partial x^2}$ will be of the form (9), substituting $\frac{3}{2}$ for the index $\frac{1}{2}$, and (0)', (1)', etc., for (0), (1), etc.

In the case of $s = \frac{5}{2}$ the derivatives are obtained in the same way, which is too simple to need elucidation.

We have now to pass from the derivatives of h to those of $\frac{mh}{a_1}$, the coefficients of the perturbative function. The form of these derivatives will depend not on whether the planet is disturbing or disturbed, but on whether it is an outer or inner one. Let us then suppose for the present, that a and v refer to the inner planet, and put v_1 for the logarithm of the mean distance of the outer one. We then have for the derivatives relatively to v

$$\frac{\partial^n \frac{h}{a_1}}{\partial v^n} = \frac{1}{a_1} \frac{\partial^n h}{\partial v^n},$$

and for the first derivative relatively to v, using the symbolic notation,

$$\frac{\partial \frac{h}{a_1}}{\partial v_1} = \frac{1}{a_1} (D_{a1} - 1)\, h.$$

The symbols in the second member being distributive, we have by successive differentiation

$$\frac{\partial^n \frac{h}{a_1}}{\partial v_1^{\,n}} = \frac{1}{a_1} (D_{v_1} - 1)^n h.$$

The quantity h is a function of α, the ratio of the mean distances or of $C^{v - v_1}$, C being the neperian base. Hence

$$D_{v_1} h = - D_v h,$$

which substituted in the last equation gives

$$\frac{\partial^n \frac{h}{a_1}}{\partial v_1^{\,n}} = \frac{(-1)^n}{a_1} (D_v + 1)^n h. \qquad (10)$$

This formula gives for the first two derivatives

$$\frac{\partial \frac{h}{a_1}}{\partial v_1} = -\frac{1}{a_1} \left(h + \frac{\partial h}{\partial v} \right)$$

$$\frac{\partial^2 \frac{h}{a_1}}{\partial v_1^{\,2}} = \frac{1}{a_1} \left(h + 2 \frac{\partial h}{\partial v} + \frac{\partial^2 h}{\partial v^2} \right).$$

Substituting in the general formulæ (7) these expressions for the derivatives relatively to v and v_1 we have expressions for the derivatives of R relatively to v, v', ρ, ρ', it being understood, however, that all the quantities are expressed in functions of the elements of elliptic motion.

In order to compute the perturbations of the second order we must carry R and such of its derivatives as enter into the differential equations (1) to quantities of the first order with respect to the perturbations. Let us then represent by v_0, v_0', ρ_0, ρ'_0, γ_0, the elliptic values of v, v', ρ, ρ', and γ, which we have assumed in the first approximation to the perturbations, and by $\delta\mathrm{v}$, $\delta\mathrm{v}'$, etc., the quantities to be added to v_0, v_0', etc., to make the true values of v, v′, etc., whether perturbations or corrections of the elements. We shall then have

$$\begin{aligned}
\delta R &= \frac{\partial R_0}{\partial \mathrm{v}_0}\,\delta\mathrm{v} + \frac{\partial R_0}{\partial \mathrm{v}'_0}\,\delta\mathrm{v}' + \frac{\partial R_0}{\partial \rho_0}\,\delta\rho + \frac{\partial R_0}{\partial \rho'_0}\,\delta\rho' + \frac{\partial R_0}{\partial \gamma_0}\,\delta\gamma \\
\delta\,\frac{\partial R}{\partial \mathrm{v}} &= \frac{\partial^2 R_0}{\partial \mathrm{v}_0^{\,2}}\,\delta\mathrm{v} + \frac{\partial^2 R_0}{\partial \mathrm{v}_0 \partial \mathrm{v}'_0}\,\delta\mathrm{v}' + \frac{\partial^2 R_0}{\partial \mathrm{v}_0 \partial \rho_0}\,\delta\rho + \frac{\partial^2 R_0}{\partial \mathrm{v}_0 \partial \rho'_0}\,\delta\rho' + \frac{\partial^2 R_0}{\partial \mathrm{v}_0 \partial \gamma_0}\,\delta\gamma \qquad (11) \\
\delta\,\frac{\partial R}{\partial \rho} &= \frac{\partial^2 R_0}{\partial \rho_0 \partial \mathrm{v}_0}\,\delta\mathrm{v} + \frac{\partial^2 R_0}{\partial \rho_0 \partial \mathrm{v}'_0}\,\delta\mathrm{v}' + \frac{\partial^2 R_0}{\partial \rho^2_0}\,\delta\rho + \frac{\partial^2 R_0}{\partial \rho_0 \partial \rho'_0}\,\delta\rho' + \frac{\partial^2 R_0}{\partial \rho_0 \delta \gamma_0}\,\delta\gamma
\end{aligned}$$

The value of $D'_{\mathrm{t}}R$ may be found either by equation (2), or by differentiating with respect to the time as introduced by the co-ordinates of the disturbed planet. When quantities of the first order only are considered the latter operation is very simple, but it is different when terms of the second order come in, because the true longitude of the planet is then expressed in terms not only of its own mean longitude, but also of the mean longitude of all the disturbing planets. The result can still be obtained in the same way by separating all the mean longitudes introduced by the co-ordinates of the disturbed planet from those introduced by the co-ordinates of the other until after the differentiation relatively to t'.

Let us now resume the equation (4), representing its second member by μQ, so that it becomes

$$\frac{d^2\,(r_0^{\,2}\,\delta\rho)}{dt^2} + \frac{\mu\,(1+m)}{r_0^{\,3}}\,r_0^{\,2}\,\delta\rho = \mu Q \qquad (12)$$

where

$$Q = 2\int D'_{\mathrm{t}}R\,dt + \frac{\partial R}{\partial \rho} - \frac{1}{\mu}\,\frac{d^2\,(r_0^{\,2}\,\delta\rho^2)}{dt^2} + \tfrac{1}{2}\,\frac{\delta\rho^2}{r_0}$$

By the operations already given Q has become a known function of the time.

It is well known that the integration of (12) may be effected by finding two values of $r_0^{\,2}\delta\rho$ which satisfy this equation when the second member is neglected, or, in other words, by finding two variables x and y which satisfy the equations

$$\frac{d^2x}{dt^2} + \frac{\mu(1+m)}{r_0^{\,3}}\,x = 0,$$

$$\frac{d^2y}{dt^2} + \frac{\mu(1+m)}{r_0^{\,3}}\,y = 0,$$

when the required integral is

$$r_0^{\,2}\,\delta\rho = \frac{\mu}{x\,\dfrac{dy}{dt} - y\,\dfrac{dx}{dt}}\left\{\,y\int xQdt - x\int yQdt\,\right\}.$$

The above differential equations are satisfied by the rectangular co-ordinates of the planet in its assumed elliptic orbit. The position of the axes of co-ordinates being arbitrary we shall take the line of apsides for the axis of X, the perihelion being on the positive side. If we put

$$e_0 = \sin\psi,$$

we have

$$x\,\frac{dy}{dt} - y\,\frac{dx}{dt} = \sqrt{a\mu\,(1+m)}\,\cos\psi = \frac{\mu\,(1+m)\cos\psi}{an}$$

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Let us, for convenience, replace x and y by two other variables ξ and η connected with them by the equations

$$x = a\xi,$$
$$y = a\eta \cos \psi.$$

ξ and η are then functions of the eccentricity and mean anomaly only, and may be developed according to the multiples of the latter. Substituting the last three expressions in the preceding value of $r_0{}^2 \delta\rho$ it becomes

$$r_0{}^2 \delta\rho = \frac{a^3 n}{1+m} \left\{ \eta \int \xi Q dt - \xi \int \eta Q dt \right\}.$$

If we put $r_1{}^2$ for the value of r_0 when the mean distance of the planet is put equal to unity, so that r_1, like ξ and η contains only the eccentricity and mean anomaly, we shall have

$$\delta\rho = \frac{n r_1{}^{-2}}{1+m} \left\{ \eta \int \xi a Q dt - \xi \int \eta a Q dt \right\} \tag{13}$$

We must now express ξ and η in terms of the time, or of the mean anomaly. Putting for the present u for the eccentric and v for the true anomaly, we have, by the theory of the elliptic motion,

$$x = r \cos v = a\,(\cos u - e),$$
$$y = r \sin v = a \cos \psi \sin u,$$

from which follow

$$\xi = \cos u - e,$$
$$\eta = \sin u.$$

As ξ and η are to be expressed in the form

$$\xi = \tfrac{1}{2}\, \Sigma p_i \cos ig,$$
$$\eta = \tfrac{1}{2}\, \Sigma q_i \sin ig,$$

the finite integrals extending to all values of i from $-\infty$ to $+\infty$, we shall deduce general expressions from p_i and q_i arranged according to the power of the eccentricity. Since

$$u = g + e \sin u,$$

we have by Lagrange's theorem

$$\cos u = \cos g - e \sin^2 g - \frac{e^2}{2!} \frac{\partial \sin^3 g}{\partial g} - \frac{e^3}{3!} \frac{\partial^2 \sin^4 g}{\partial g^2} - \text{etc.};$$

or

$$\cos\, u = -\sum_{n=0}^{n=\infty} \frac{e^n}{n!} \frac{\partial^{n-1} \sin^{n+1} g}{\partial g^{n-1}}$$

using the notation

$$n! = 1.2.3 \,.\,.\,.\,.\,.\, n = \Gamma(n+1).$$

We then have

$$0! = 1! = 1.$$

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Substituting in the general term of the above series for sin g its value in imaginary exponential functions

$$2 \sin g = \sqrt{-1}\,(c^{-g\sqrt{-1}} - c^{g\sqrt{-1}})$$

we find by the binomial theorem, using the notation of combinations,

$$\overset{s}{\underset{n}{C}} = \frac{n\,(n-1)\,\ldots\ldots\,(n-s+1)}{1.2.3\ldots s} = \frac{n!}{s!\,(n-s)!}$$

$$2^{n+1} \sin^{n+1} g = (\sqrt{-1})^{n+1} \Big\{ c^{-(n+1)\,g\,\sqrt{-1}} - \overset{1}{\underset{n+1}{C}} c^{-(n-1)\,g\sqrt{-1}} + \overset{2}{\underset{n+1}{C}} c^{-(n-3)\,g\sqrt{-1}} - \ldots\ldots$$

$$+ (-1)^n \overset{1}{\underset{n+1}{C}} c^{(n-1)\,g\sqrt{-1}} + (-1)^{n+1} c^{(n+1)\,g\,\sqrt{-1}} \Big\}$$

Differentiating $n - 1$ times with respect to g, and putting together the first and last terms, the one after the first, and that before the last, and so on, we find

$$-2^{n+1} \frac{\partial^{n-1} \sin^{n+1} g}{\partial g^{n-1}} = (n+1)^{n-1} (c^{(n+1)g\,\sqrt{-1}} + c^{-(n+1)g\,\sqrt{-1}})$$

$$- \overset{1}{\underset{n+1}{C}} (n-1)^{n-1}\,(c^{(n-1)g\,\sqrt{-1}} + c^{-(n-1)g\,\sqrt{-1}}) + \text{etc.}$$

Substituting for the exponentials their values in circular functions, and dividing by 2^{n+1} we have

$$\frac{\partial^{n-1} \sin^{n+1} g}{\partial g^{n-1}} = -\frac{1}{2^n} \Big\{ (n+1)^{n-1} \cos\,(n+1)\,g - \overset{1}{\underset{n+1}{C}} (n-1)^{n-1} \cos\,(n-1)\,g$$

$$+ \overset{2}{\underset{n+1}{C}} (n-3)^{n-1} \cos\,(n-3)\,g - \text{etc.} \Big\}$$

the series terminating at the last positive coefficient of g. Substituting this last value in the general term of the series which gives cos u, we have

$$\cos u = \sum_{n=0}^{n=\infty} \frac{e^n}{n!\,2^n} \Big\{ (n+1)^{n-1} \cos\,(n+1)\,g - \overset{1}{\underset{n+1}{C}} (n-1)^{n-1} \cos\,(n-1)\,g + \text{etc.} \Big\}$$

Let us now substitute for n another variable i, putting in the first term of the last factor $i = n + 1$, in the second $i = n - 1$, in the third $i = n - 3$, etc. The limits of finite integration with respect to i will then be

in the first term, $+1$ to $+\infty$,
in the second term, -1 to $+\infty$,
in the third term, -3 to $+\infty$,
etc. etc.

But all the coefficients of g will then be i, and the formula supposes the factor of cos $i\,g$ to vanish whenever i is zero or negative; whence, those elements of the finite integral in which i is negative must be omitted, and all the terms must be taken between the limits $+ 1$ and $+ \infty$. Making the proposed substitution we have

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$$\cos u = \sum_{i=1}^{i=\infty} \left\{ \frac{i^{i-2}}{(i-1)!\,2^{i-1}} e^{i-1} - \frac{i^{i}}{(i+1)!\,2^{i+1}} \underset{i+2}{\overset{1}{C}} e^{i+1} + \frac{i^{i+2}}{(i+3)!\,2^{i+3}} \underset{i+4}{\overset{2}{C}} e^{i+3} - \text{etc.} \right\} \cos ig$$

$$= \sum_{i=1}^{i=\infty} \frac{i^{i-2} e^{i-1}}{(i-1)!\,2^{i-1}} \left\{ 1 - \frac{i^2 e^2}{2^2 i(i+1)} \underset{i+2}{\overset{1}{C}} + \frac{i^4 e^4}{2^4 i(i+1)(i+2)(i+3)} \underset{i+4}{\overset{2}{C}} - \text{etc.} \right\} \cos ig$$

We have, therefore, for all values of i different from zero

$$p_i = p_{-i} = \frac{i^{i-2} e^{i-1}}{(i-1)!\,2^{i-1}} \left\{ 1 - \underset{i+2}{\overset{1}{C}} \frac{i\,e^2}{2^2(i+1)} + \underset{i+4}{\overset{2}{C}} \frac{i^3 e^4}{2^4(i+1)(i+2)(i+3)} - \text{etc.} \right\} \quad (14)$$

To obtain the value of p_0 we remark that the only constant term in $\cos u$ arises from the term $-e \sin^2 g$; its value is therefore $-\frac{1}{2}e$. The constant term in $\xi = \cos u - e$ is therefore $\frac{3}{2}e$, whence

$$p_0 = -3e. \quad (15)$$

The values of q_i may be obtained in a similar way by developing $\sin u$ by La Grange's theorem. But the development is rather more complex, and it is easier to derive them from p_i. Let us take up the equations

$$\xi = \cos u - e$$
$$\eta = \sin u$$
$$u - e \sin u = g$$

Considering u, like ξ and η, as a function of the independent variables e and g, we have by differentiation

$$\frac{\partial u}{\partial e} - \frac{\partial (e \sin u)}{\partial e} = 0$$

$$\therefore \frac{\partial u}{\partial e} = \frac{\partial (e\eta)}{\partial e} = \frac{\sin u}{1 - e \cos u} \quad (a)$$

$$\frac{\partial u}{\partial g} = \frac{1}{1 - e \cos u}.$$

$$\therefore \frac{\partial u}{\partial e} = \sin u \frac{\partial u}{\partial g} = -\frac{\partial \xi}{\partial u}\frac{\partial u}{\partial g} = -\frac{\partial \xi}{\partial g} \quad (b)$$

Comparing (a) and (b)

$$\frac{\partial \xi}{\partial g} = -\frac{\partial (e\eta)}{\partial e}$$

Putting in this equation for ξ and η their developed values this equation becomes

$$\Sigma\, i p_i \sin ig = \Sigma \frac{\partial (e q_i)}{\partial e} \sin ig$$

which gives by equating the coefficients of $\sin ig$

$$q_i = \frac{i}{e} \int p_i\, de. \quad (16)$$

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The following are special values of p_i and q_i, developed to the sixth power of the eccentricities, as derived from the preceding formulæ:

$$\begin{aligned}
p_0 &= -3e \\
p_1 &= 1 - \frac{3}{8}e^2 + \frac{5}{192}e^4 - \frac{7}{9216}e^6 \\
p_2 &= \frac{1}{2}e - \frac{1}{3}e^3 + \frac{1}{16}e^5 \\
p_3 &= \frac{3}{8}e^2 - \frac{45}{128}e^4 + \frac{567}{5120}e^6 \\
p_4 &= \frac{1}{3}e^3 - \frac{6}{15}e^5 \\
p_5 &= \frac{125}{384}e^4 - \frac{4375}{9216}e^6 \\
p_6 &= \frac{27}{80}e^5 \\
p_7 &= \frac{16807}{46080}e^6 \\
q_1 &= 1 - \frac{1}{8}e^2 + \frac{1}{192}e^4 - \frac{1}{9216}e^6 \\
q_2 &= \frac{1}{2}e - \frac{1}{6}e^3 + \frac{1}{48}e^5 \\
q_3 &= \frac{3}{8}e^2 - \frac{27}{128}e^4 + \frac{243}{5120}e^6 \\
q_4 &= \frac{1}{3}e^3 - \frac{4}{15}e^5 \\
q_5 &= \frac{125}{384}e^4 - \frac{3125}{9216}e^6 \\
q_6 &= \frac{27}{80}e^5 \\
q_7 &= \frac{16807}{46080}e^6
\end{aligned} \tag{16$'$}$$

Having the developed ξ and η in terms of time, let us resume the equation (13). As only purely linear operations are performed on Q in this equation, it follows that if we represent its several parts by Q_1, Q_2, etc., and by $\delta\rho_1$, $\delta\rho_2$, etc., the values $\delta\rho$ obtained by putting $Q = Q_1$, $Q = Q_2$, etc., we shall have

$$\delta\rho = \delta\rho_1 + \delta\rho_2 + \text{etc.}$$

We have, therefore, only to find the separate values of $r_0^2\,\delta\rho$ corresponding to the different terms of Q, and to take their sum. Let us then represent, as before, by

$$\frac{m'}{a_1} h \cos(i'\lambda' + i\lambda + j\omega' + j\omega)$$

any one term of R.

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We then have, considering only terms of the first order with respect to the disturbing forces,

$$D'_t R = -\frac{m'ihn}{a_1}\sin N, \qquad (17)$$

$$\int D'_t R = \frac{m'ih\nu}{a_1}\cos N;$$

$$\frac{\partial R}{\partial \rho} = m'\frac{\partial \dfrac{h}{a_1}}{\partial x}\cos N;$$

where we put for brevity,

$$\nu = \frac{n}{i'n' + in}$$

$$N = i'\lambda' + i\lambda + j'\omega' + j\omega.$$

Let us represent by Q_0 the terms in Q which are of the first order with respect to the disturbing forces, so that we have

$$Q_0 = 2\int D'_t R_0 + \frac{\partial R_0}{\partial \rho_0}.$$

The general term in R will then give rise in Q_0 to the term

$$m'\left\{\frac{2ih\nu}{a_1} + \frac{\partial \dfrac{h}{a_1}}{\partial x}\right\}\cos N.$$

In the case of the action of an outer on an inner planet this expression becomes

$$\frac{m'}{a_1}\left(2i\nu h + \frac{\partial h}{\partial x}\right)\cos N;$$

while in the contrary case it is

$$\frac{m'}{a_1}\left(2i\nu h - h - \frac{\partial h}{\partial x'}\right)\cos N,$$

both derivatives being taken with respect to the logarithm of the mean distance of the inner planet.

In the integration it will be more convenient to substitute for λ' and λ the mean longitudes counted from the perihelion of the disturbed planet. If we put

$$\lambda = g + \omega$$

$$\lambda' = l' + \omega$$

the angle N will become,

$$i'l' + ig + j'\omega' + (i + i' + j)\,\omega.$$

Since corresponding to each set of values of i' and i there are several values of j' and j, it will be convenient in the numerical computation to combine these different terms into a single one, because after forming the derivatives of R there is no need that ω, ω' and the other elements should appear in an analytical form. If we put

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k for the coefficient of $\frac{m'}{a_1}\cos N$ in the preceding general term of Q_0, this term will become

$$Q_0 = \frac{m'}{a_1} k \cos[j'\omega' + (i' + i + j)\,\omega] \cos[i'l' + ig]$$
$$- \frac{m'}{a_1} k \sin[j'\omega' + (i' + i + j)\,\omega] \sin[i'l' + ig]$$

If we put

$$k_c = \Sigma\, k \cos[j'\omega' + (i' + i + j)\,\omega],$$
$$k_s = \Sigma\, k \sin[j'\omega' + (i' + i + j)\,\omega],$$

the sign Σ being extended so as to include all values of j and j' which correspond to the given values of i and i', we shall have for the general terms of Q_0

$$\frac{m'}{a_1}\left\{ k_c \cos(i'l' + ig) + k_s \sin(i'l' + ig) \right\},$$

or, when we represent the angle $i'l' + ig$ by N_1

$$Q_0 = \frac{m'}{a_1}\left\{ k_c \cos N_1 + k_s \sin N_1 \right\}.$$

This we are to combine with the values of ξ and η

$$\xi = \tfrac{1}{2}\Sigma\, p_i \cos ig,\text{*}$$
$$\eta = \tfrac{1}{2}\Sigma\, q_j \sin jg,$$

in the general integral formula (13). If we substitute them in this formula, and represent by μ the coefficient of t in the value of N we shall have to integrate differentials of the form

$$\begin{matrix}\sin\\ \cos\end{matrix}\left(N_1 \pm ig\right).$$

in which the coefficient of the time t in the angle is $\mu + in$. Let us represent by ν_i the integrating factor

$$\frac{n}{\mu + in}.$$

The formula (13) will become by these substitutions, which, though a little complex, offer no difficulty,

$$\delta\rho = \frac{1}{16}\,\frac{m'ar_1^{-2}}{a_1(1+m}\,\sum_{i,j}^{2}{}_{-\infty}^{+\infty}\; p_i q_j \times$$
$$\left\{\begin{matrix} \{\nu_{+j}-\nu_{+i}\}\,\{k_c\cos[N_1+(i+j)g]+k_s\sin[N_1+(i+j)g]\} \\ +\{\nu_{+i}-\nu_{-j}\}\,\{k_c\cos[N_1+(i-j)g]+k_s\sin[N_1+(i-j)g]\} \\ +\{\nu_{+j}-\nu_{-i}\}\,\{k_c\cos[N_1-(i-j)g]+k_s\sin[N_1-(i-j)g]\} \\ +\{\nu_{-i}-\nu_{-j}\}\,\{k_c\cos[N_1-(i+j)g]+k_s\sin[N_1-(i+j)g]\} \end{matrix}\right\}$$

The sign Σ of finite integration here includes the separate combination of every value of i with every value of j, except those combinations which make the

---

* The indices i and j, in these equations, are not to be confounded with the coefficients of λ and ω in the general terms of R and Q. We need not use the latter at present.

coefficient of the time under the sign *sin* or *cos* vanish, and so render the corresponding value of ν infinite. These cases have to be treated separately.

To find, from the expression, the coefficient of the sine or cosine of cosine $N_1 + ug$ in $r_1^2 \delta\rho$, we put, in the four lines of this equation, as follows:

$$\begin{array}{lll} \text{In the first,} & i + j = u \therefore j = & u - i; \\ \text{“ second,} & i - j = u \therefore j = & i - u; \\ \text{“ third,} & -i + j = u \therefore j = & u + i; \\ \text{“ fourth,} & -i - j = u \therefore j = & -u - i. \end{array}$$

In the above expressions i and j being independent, and including all values from $-\infty$ to $+\infty$, i and u will also be independent, and include the same range of values. Substituting for j its value in u the coefficient of

$$\frac{1}{16} \frac{m' a r_1^{-2}}{a_1 (1 + m)} [k_c \cos (N_1 + ug) + k_s \sin (N_1 + ug)]$$

becomes

$$\Sigma \left\{ \begin{array}{l} p_i \, q_{(u-i)} \, (\nu_{(u-i)} - \nu_i \phantom{_{(u-i)}}) \\ + p_i \, q_{(i-u)} \, (\nu_i \phantom{_{(u-i)}} - \nu_{(u-i)}) \\ + p_i \, q_{(u+i)} \, (\nu_{(u+i)} - \nu_{-i} \phantom{_{(u-i)}}) \\ + p_i \, q_{-(u+i)} \, (\nu_{-i} - \nu_{(u+i)}). \end{array} \right.$$

Since $q_j = -q_{-j}$ this expression reduces immediately to

$$2\Sigma \left\{ \begin{array}{l} p_i \, q_{(i-u)} \, (\nu_i \phantom{_{(u-i)}} - \nu_{(u-i)}) \\ + p_i \, q_{(i+u)} \, (\nu_{(u+i)} - \nu_{-i} \phantom{_{(u-i)}}), \end{array} \right.$$

or, substituting $i - u$ for i in the second line

$$2\Sigma \, (p_i q_{(i-u)} + p_{(i-u)} q_i) \, (\nu_i - \nu_{u-i}).$$

Hence, writing N instead of N_1,

$$\delta\rho = \frac{1}{8} \frac{m' a r_1^{-2}}{a_1 (1 + m)} \Sigma^2_{i,u} (p_i q_{(i-u)} + p_{(i-u)} q_i) (\nu_i - \nu_{(u-i)}) [k_c \cos (N + ug) + k_s \sin (N + ug)] \quad (19)$$

This expression fails for the particular case $N = ug$, where the value of ν_{-u} will be infinite. If we take each term of Q of the form

$$\frac{m'}{a_1} (k_c^{(u)} \cos ug + k_s^{(u)} \sin ug),$$

and substitute in the general expression (13) it will be found that the terms in $r_1^2 \delta\rho$ which have the infinite values of ν as a factor are to be omitted, and replaced by

$$r_1^2 \delta\rho = \tfrac{1}{2} \frac{m' a n t}{a_1 (1 + m)} \{ \eta \, \Sigma p_u k_c^{(u)} - \xi \, \Sigma q_u k_s^{(u)} \} \quad (20)$$

The two parts of $r_1^2 \delta\rho$ thus found include all the terms of the first order with respect to the disturbing forces. But when terms of the second order are taken into account, we shall find terms in Q proceeding from secular variation in which the time appears as a factor, outside the signs sin and cos. Let us represent such of these terms as depend on any angle N by

$$Q = \frac{m' n t}{a_1} (k_c \cos N + k_s \sin N)$$

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and use the symbol ν_i, as before, to represent the ratio of the mean motion of the planet to the coefficient of t in the angle $N+ig$, so that if μ' represents the coefficients of t in N we have

$$\nu=\frac{n}{\mu'},$$

$$\nu_i=\frac{n}{\mu'+in}=\frac{1}{\frac{\mu'}{n}+i},$$

we find the expression

$$\int\xi\,Qdt=\frac{1}{4}\frac{m'\Sigma p_i}{a_1 n}\Big\{(\nu_i^2k_c-\nu_ik_snt)\cos(N+ig)+(\nu_{-i}^2k_c-\nu_{-i}k_snt)\cos(N-ig)$$

$$(\nu_i^2k_s+\nu_ik_cnt)\sin(N+ig)+(\nu_{-i}^2k_s+\nu_{-i}k_cnt)\sin(N-ig)\Big\}$$

$$\int\eta\,Qdt=\frac{1}{4}\frac{m'\Sigma q_j}{a_1 n}\Big\{(\nu_j^2k_c-\nu_jk_snt)\sin(N+jg)-(\nu_{-j}^2k_c-\nu_{-j}k_snt)\sin(N-jg)$$

$$-(\nu_j^2k_s+\nu_jk_cnt)\cos(N+jg)+(\nu_{-j}^2k_s+\nu_{-j}k_cnt)\cos(N-jg)\Big\}.$$

If we now put for brevity

$$\nu_i^2k_s+\nu_ik_cnt=c_i,$$

$$\nu_i^2k_c-\nu_ik_snt=s_i,$$

the general value of $r_0^2\delta\rho$ becomes

$$\delta\rho=\frac{1}{16}\frac{m'ar_1^{-2}}{a_1(1+m)}\times$$

$$\Sigma^2p_iq_j\left\{\begin{array}{l}(c_j-c_i)\cos(N+(i+j)g)+(s_i-s_j)\sin(N+(i+j)g)\\+(c_i-c_{-j})\cos(N+(i-j)g)+(s_{-j}-s_i)\sin(N+(i-j)g)\\+(c_j-c_{-i})\cos(N-(i-j)g)+(s_{-i}-s_j)\sin(N-(i-j)g)\\+(c_{-i}-c_{-j})\cos(N-(i+j)g)+(s_{-j}-s_{-i})\sin(N-(i+j)g)\end{array}\right\}$$

If, as before, we transform this expression by putting

in the first line $j=u-i$;
in the second " $j=i-u$;
in the third " $j=i+u$;
in the fourth " $j=i-u$·

the value of $r_1^2\delta\rho$ reduces to

$$\frac{1}{8}\frac{m'a}{a_1(1+m)}\times$$

$$\Sigma_{i,u}^2\left\{\begin{array}{l}p_iq_{(u-i)}(c_{(u-i)}-c_i)\cos(N+ug)+p_iq_{(u-i)}(s_i-s_{(u-i)})\sin(N+ug)\\p_iq_{(u+i)}(c_{(u+i)}-c_{-i})\cos(N+ug)+p_iq_{(u+i)}(s_i-s_{(u+i)})\sin(N+ug)\end{array}\right\}\qquad(21)$$

or, putting $i-u$ for i in the last line,

$$\delta\rho=\frac{1}{8}\frac{m'ar_1^{-2}}{a_1(1+m)}\times$$

$$\Sigma_{u,i}^2\{p_iq_{(u-i)}-p_{(u-i,}q_i\}\,\{(c_{(u-i)}-c_i)\cos(N+ug)+(s_i-s_{u-i})\sin(N+ug)\};$$

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to which expression is to be added, in lieu of the terms which will have infinite values of ν as a factor.

$$\frac{1}{4}\frac{m'ar_1^{-2}n^2t^2}{a_1(1+m)}\{\eta\Sigma p_u k_c^{(u)} - \xi\Sigma q_u k_s^{(u)}\} \qquad (22)$$

$k_c^{(u)}$ and $k_s^{(u)}$ being the factors of $\frac{m'}{a_1}nt \cos ug$ and $\frac{m'}{a_1}nt \sin ug$ in the expression for Q.

The formulæ 19, 20, 21, and 22 give the complete expressions for the perturbations of the logarithm of radius vector by successively substituting in it all the terms of Q.

Perturbations of Longitude.

We now pass to the perturbations of longitude. In the Mécanique Céleste (Première Partie, Liv. ii. Chap. vi.), Laplace gives an equation (Y) by which the perturbations of longitude, which are of the first order, may be derived from those of the radius vector without the formation of any other derivatives of R than those which enter into Q. But the formula does not seem easily adapted to the case in which the perturbations of the second order are taken into account, we shall therefore derive all the perturbations of longitude from the second of equations (1). By integration this equation gives

$$\frac{dv}{dt} = \frac{\mu}{r^2}\left\{\int\frac{\partial R}{\partial \text{v}}dt + C\right\}$$

C being the arbitrary constant of the integral. Representing, as before, by subscript zeros the values of the co-ordinates corresponding to the ellipse to which the orbit is supposed to reduce itself when the disturbing forces vanish, we have

$$\frac{dv_0}{dt} = \frac{a^2n\cos\psi}{r_0^{\,2}} = \frac{\mu C}{r_0^{\,2}},$$

because the constant to which the integral must reduce itself in the elliptic motion is $\frac{a^2n\cos\psi}{\mu}$. Subtracting the last equation from the preceding, and putting $v - v_0 = \delta v$, we find

$$\frac{d\delta v}{dt} = \frac{\mu}{r^2}\int\frac{\partial R}{\partial \text{v}}dt + \left(\frac{1}{r^2} - \frac{1}{r_0^{\,2}}\right)a^2n\cos\psi.$$

Developing $\frac{1}{r^2}$ to terms of the second order with respect to the disturbing force

$$\frac{1}{r^2} = \frac{1}{r_0^{\,2}}\left(1 - 2\delta\rho + 2\delta\rho^2 - \text{etc.}\right),$$

which, being substituted in the last equation by putting

$$\mu = \frac{a^3n^2}{1+m},$$

$$r_0^{\,2} = a^2r_1^{\,2},$$

gives

$$r_1^{\,2}\frac{d\delta v}{dt} = \frac{an^2}{1+m}(1-2\delta\rho)\int\frac{\partial R}{\partial \text{v}}dt - 2n\cos\psi\,(\delta\rho - \delta\rho^2), \qquad (23)$$

which is rigorous to quantities of the second order.

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The most convenient mode of making the numerical computation of the second order terms by means of this equation will depend upon circumstances. If the perturbations of longitude and radius vector of both planets are already known with a sufficient degree of approximation for the computation of formula (11), it will be more convenient to form at once the complete values of all the quantities which enter into the equations (12), (13), (19) to 22), and (23), so that no steps of the process shall have to be repeated. If such perturbations are not known, they must first be computed, and it will then be necessary to begin with the perturbations of the first order, and afterward add those of the second. There is, however, one class of terms of the second order which it will be most convenient to take account of from the beginning, namely, those arising from the constant term in $\delta\rho$ and $\delta\rho'$. This is effected by correcting the mean distances for an approximate value of these constants at the beginning of the computation, and then proceeding in the usual way. This is in fact what we have supposed to be done in the preceding investigation. The values of $\delta\mathrm{v}$, $\delta\mathrm{v}'$, $\delta\rho$, $\delta\rho'$ in formula (11) will then contain only periodic terms.

In computing the terms of the first order we determine the value of $\delta\rho$ from the equations (19) and (20), using the value of Q_0 in (18). Then those of δv are obtained by integrating the equation

$$\frac{d\delta v}{dt} = \frac{an^2 r_1^{-2}}{1+m} \int \frac{\partial R_0}{\partial \mathrm{v}}\, dt - 2n \cos\psi \frac{\delta\rho}{r_1^2}. \tag{24}$$

Having found the values of δv and $\delta\rho$ for both planets, they are to be substituted in (11), to obtain δR, $\delta \frac{\partial R}{\partial \mathrm{v}}$ and $\delta \frac{\partial R}{\partial \rho}$. But, rigorously, $\delta\mathrm{v}$ and $\delta\mathrm{v}'$ are not the same with δv and $\delta v'$, owing to the movement of the orbits of the planets, and the corrections for $\delta\gamma$ are also to be added. Considering, for the present, only the perturbations of the second order, which depend on δv, $\delta v'$, $\delta\rho$, and $\delta\rho'$, we may use the following equation for δR, and similar ones for its derivatives:

$$\delta R = \frac{\partial R}{\partial \mathrm{v}} \delta v + \frac{\partial R}{\partial \mathrm{v}'} \delta v' + \frac{\partial R}{\partial \rho} \delta\rho + \frac{\partial R}{\partial \rho'} \delta\rho'. \tag{25}$$

Having thus found δR, and hence $D'_t \delta R$ by differentiation, and then $\delta \frac{\partial R}{\partial \rho}$, we form the quantity

$$\delta Q = 2\int D'_t \delta R dt + \delta \frac{\partial R}{\partial \rho} - \frac{1}{\mu} \frac{d^2(r_0^2 \delta\rho^2)}{dt^2} + \tfrac{1}{2} \frac{\delta\rho^2}{r_0} \tag{26}$$

which is the difference between the value of Q_0 in (18) and that of Q in (12). The terms in $\delta\rho$ arising from δQ are then to be computed by the formulæ (19), (20), (21), and (22), when we shall have $\delta\rho$ accurate to quantities of the second order. Let us represent these additional terms by $\delta^2\rho$. Subtracting (24) multiplied by r_1^2 from (23), recollecting that the $\delta\rho$ which appears in the second term of the former is really $\delta\rho - \delta^2\rho$, we find, neglecting quantities of the third order,

$$r_1^2 \frac{d\delta^2 v}{dt} = an^2 \left\{ \int \delta \frac{\partial R}{\partial \mathrm{v}}\, dt - 2\delta\rho \int \frac{\partial R_0}{\delta \mathrm{v}}\, dt \right\} - 2n \cos\psi\, (\delta^2\rho - \delta\rho^2)$$

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from which the terms of δv of the second order are obtained by multiplying by r_1^{-2} and integrating.

Motion of the Orbital Planes.

The general theory of the motion of the planes of reference, especially of the motion of the instantaneous orbit, has been so often treated that I can scarcely hope to add anything essentially new to it. I shall, however, endeavor to present the differential equations of the motion in a simple and general form, and one in which the geometrical conceptions of the problem shall be made as clear as possible.

The orbital plane of each planet being at each moment osculatory to that part of the orbit which the planet is actually describing, its only motion is one of rotation around the radius vector of the planet as an instantaneous axis. This rotation may be resolved into two others around any pair of rectangular axes fixed in the moving plane. But the rotation produced by any one planet is most simply expressed when referred to axes, one of which coincides with the common node of the two orbits. The rotation produced by each separate planet must, therefore, be first referred to its node on the moving orbit, and then the combined rotations must be resolved into two around axes assumed at pleasure. To effect this, let us suppose positive rotation around an axis to be such that an observer looking from the origin along the positive direction of the axis sees the right hand side of the plane move downwards, and the left hand side upwards. Let us also denote the first axis in the order of longitude the principal axis, or that of X, and that 90° farther advanced the secondary axis, or that of Y. Let us now put

dq, the instantaneous rotation around the axis of X;

dp, the instantaneous rotation around the axis of Y. Let us also put, relatively to any disturbing planet,

$d\eta$, the instantaneous rotation around the ascending node of the disturbing planet on the orbit of the disturbed one.

dk, that around the corresponding secondary axis.

Then, from the known equations for the perturbations of the inclination and node of an orbit, we find, that, if any term of the perturbative function be represented, as before, by

$$\frac{m'h}{a_1}\cos\left(i'\lambda'+i\lambda+j'\omega'+j\omega\right),$$

the differential rotations η and k will be given by the equations

$$\frac{d\eta}{dt}=-\frac{m'h}{a_1}\frac{an}{\cos\psi}\left\{(i+j)\cot\gamma+(i'+j')\operatorname{cosec}\gamma\right\}\sin N$$

$$\frac{dk}{dt}=-\frac{m'an}{a_1\cos\psi}\frac{\partial h}{\partial\gamma}\cos N.$$

As R is actually developed, the mutual inclination γ does not explicitly appear, but is replaced by

$$\sigma=\sin\tfrac{1}{2}\gamma.$$

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Making this substitution, and putting also

$$i'' + i + j'' + j = -\iota.$$

these equations become

$$\begin{aligned} \frac{d\eta}{dt} &= \frac{m'an}{a_1 \cos\psi \cos\frac{1}{2}\gamma} \left\{ \frac{\iota h}{2\sigma} + (i + j)\,\sigma h \right\} \sin N \\ \frac{dk}{dt} &= -\frac{m'an \cos\frac{1}{2}\gamma}{2a_1 \cos\psi} \frac{\partial h}{\partial\sigma} \cos N. \end{aligned} \qquad (27)$$

To pass to the general rotations dp and dq, let us represent by θ_1, θ_2, etc., the longitudes of the ascending nodes of the several orbits of the disturbing planets on that of the disturbed planet. We shall then have

$$\begin{aligned} \frac{dq}{dt} &= \Sigma \cos\theta_i \frac{d\eta_i}{dt} - \Sigma \sin\theta_i \frac{dk_i}{dt} \\ \frac{dp}{dt} &= \Sigma \cos\theta_i \frac{dk_i}{dt} + \Sigma \sin\theta_i \frac{d\eta_i}{dt}. \end{aligned} \qquad (28)$$

These equations completely define the instantaneous motion of the orbital plane. They cannot, however, be rigorously integrated in their present form because p and q as integrals have no completely defined signification. To do this it is necessary to express the differential rotations dp, dq, etc., in terms of the differentials of any elements we may select to define the position of the orbital plane, and then to integrate the equations thus formed. But, for the purpose of constructing tables of the planets we may consider p, q, etc., to represent small rotations of the planes of which the powers and products may be neglected, and the integration is then quite simple.

Perturbations of the second order depending on the motion of the orbital planes.

R being a function of the five quantities of r, r', v, v', and γ, the motion of the orbital planes introduces terms of the second order by changing the values of v, v', and γ. These terms we have hitherto neglected. To investigate them let us refer the rotations of both planes as given by (28) to the node of the disturbing on the disturbed planet as the principal axis. If we represent by $d\eta$, dk, $d\eta'$, and dk' the rotations corresponding to this axis, and designate by the subscript 1, the quantities which refer to the disturbing planet whose action we are considering, and by 2, 3, etc., the other planets, the equations (28) will be replaced by these

$$\begin{aligned} \frac{d\eta}{dt} &= \frac{d\eta_1}{dt} + \Sigma \cos(\theta_i - \theta_1) \frac{d\eta_i}{dt} - \Sigma \sin(\theta_i - \theta_1) \frac{dk_i}{dt}, \\ \frac{dk}{dt} &= \frac{dk_1}{dt} + \Sigma \cos(\theta_i - \theta_1) \frac{dk_i}{dt} + \Sigma \sin(\theta_i - \theta_1) \frac{d\eta_i}{dt}, \end{aligned}$$

the summation commencing with $i = 2$.

By formulæ of the same kind we are to find the differential rotations $d\eta'$ and dk' of the orbit of the disturbing planet, produced by the action of all the planets.

These rotations will be around the same principal axis with the rotations $d\eta$ and dk, but around a secondary axis in the plane of the disturbing orbit, and therefore making an angle γ with the secondary axis of the disturbed orbit. A geometrical construction will now show quite simply that the infinitesimal rotations $\delta\eta$, δk, $\delta\eta'$, and $\delta k'$ will produce the following changes in v, v′, and γ.

$$\begin{aligned}\delta\mathrm{v} &= \cot\gamma\,\delta k - \operatorname{cosec}\gamma\,\delta k' \\ \delta\mathrm{v}' &= \operatorname{cosec}\gamma\,\delta k - \cot\gamma\,\delta k' \\ \delta\gamma &= \delta\eta' - \delta\eta\end{aligned} \qquad (29)$$

If we substitute these values in the general formulæ (11) the terms of the second order added to δR will be

$$\begin{aligned}\delta R = &\left(\frac{\partial R}{\partial\mathrm{v}}\cot\gamma + \frac{\partial R}{\partial\mathrm{v}'}\operatorname{cosec}\gamma\right)\delta k \\ &-\left(\frac{\partial R}{\partial\mathrm{v}}\operatorname{cosec}\gamma + \frac{\partial R}{\partial\mathrm{v}'}\cot\gamma\right)\delta k' \\ &+\frac{\partial R}{\partial\gamma}(\delta\eta' - \delta\eta).\end{aligned} \qquad (30)$$

The first two terms of this expression may be put into the form

$$\left\{\tfrac{1}{2}\left(\frac{\partial R}{\partial\mathrm{v}} + \frac{\partial R}{\partial\mathrm{v}'}\right)(\operatorname{cosec}\gamma + \cot\gamma) - \tfrac{1}{2}\left(\frac{\partial R}{\partial\mathrm{v}} - \frac{\partial R}{\partial\mathrm{v}'}\right)(\operatorname{cosec}\gamma - \cot\gamma)\right\}\delta k$$

$$-\left\{\tfrac{1}{2}\left(\frac{\partial R}{\partial\mathrm{v}} + \frac{\partial R}{\partial\mathrm{v}'}\right)(\operatorname{cosec}\gamma + \cot\gamma) + \tfrac{1}{2}\left(\frac{\partial R}{\partial\mathrm{v}} - \frac{\partial R}{\partial\mathrm{v}'}\right)(\operatorname{cosec}\gamma - \cot\gamma)\right\}\delta k.$$

But,

$$\operatorname{cosec}\gamma + \cot\gamma = \cot\tfrac{1}{2}\gamma = \frac{\cos\frac{1}{2}\gamma}{\sigma}.$$

$$\operatorname{cosec}\gamma - \cot\gamma = \tan\tfrac{1}{2}\gamma = \frac{\sigma}{\cos\frac{1}{2}\gamma}.$$

and in the general term of R, by (7)

$$\frac{\partial R}{\partial\mathrm{v}} = -\frac{m'h}{a_1}(i+j)\sin N$$

$$\frac{\partial R}{\partial\mathrm{v}'} = -\frac{m'h}{a_1}(i'+j')\sin N.$$

Making these substitutions, and putting, as before,

$$i+j+i'+j' = -\iota$$

the above value of δR reduces to

$$\begin{aligned}\delta R = &\frac{m'h}{2a_1}\{\iota\cot\tfrac{1}{2}\gamma\,(\delta k - \delta k') + (i+j-i'-j')\tan\tfrac{1}{2}\gamma\,(\delta k + \delta k')\}\sin N \\ &+\frac{m'\cos\frac{1}{2}\gamma}{2a_1}\frac{\partial h}{\partial\sigma}(\delta\eta' - \delta\eta)\cos N\end{aligned} \qquad (31)$$

The corresponding terms of $\delta\frac{\partial R}{\partial \mathrm{v}}$ and $\delta\frac{\partial R}{\partial \rho}$, and may be obtained in the same way by substituting $\frac{\partial R}{\partial \mathrm{v}}$ and $\frac{\partial R}{\partial \rho}$ for R in (30) and continuing the corresponding substitutions of the general terms of the derivatives of R as given on page 9.

The equation (31), besides being of the second order with respect to the disturbing forces, is also of the second order with respect to the mutual inclinations. For δk, $\delta k'$, $\delta\eta$, and $\delta\eta'$ are of the first order with respect to both quantities, and, whenever ι is not zero, h is a quantity of the second order, containing σ^2 as a factor. It is, therefore, only in exceptional cases that the terms of the second order depending on the motion of the orbital planes can become sensible.

Reduction of the longitude in the orbit to longitude on the ecliptic.

The integration of (23) gives a value of δv, which, added to the longitude in orbit corresponding to the pure elliptic motion gives the longitude in the disturbed orbit, counted from a fixed point in the moving plane of that orbit. The position of this fixed point is completely determined by the condition that the instantaneous rotation of the plane in question around the axis perpendicular to itself is always zero, so that the motion of the point of reference is always perpendicular to the direction of the plane. But, although this instantaneous rotation is zero, the integrated rotation is not rigorously zero when we consider the terms of the second order. It follows that the value of v, the longitude in orbit, and the position of the plane of the orbit do not rigorously determine the position of the planet: we must also know how the fixed point of reference has changed its position in consequence of the motions which the plane has undergone. Let us consider the relative positions of this plane at two epochs. If the fixed point were equally distant from the common node of the two planes, the integrated rotation of the plane around its own axis would be zero. But, these distances not being equal, their difference is a correction to be applied to the longitude of the planet in its orbit. Suppose, now, that at the end of any time the inclination of the actual orbit to the primitive orbit is ϕ, and the distance of its ascending node from the present position of the moving axis of x is θ. A rotation around the line of nodes will not change the quantity sought. But, if we represent the infinitesimal rotation around an axis perpendicular to it by dr we shall have

$$\cos\theta\, dp - \sin\theta\, dq = dr,$$

dq and dk being the instantaneous rotations around the respective axes of x and γ. By this rotation it is easy to see that the relative distance of any two fixed points, one on each plane, from the node, will be altered by the quantity,

$$dr\,(\operatorname{cosec}\phi - \cot\phi) = dr \tan\tfrac{1}{2}\phi,$$

the relative longitude of the fixed point on the moving plane being increased by this amount. The correction to the longitude in orbit from this cause is, therefore,

$$dl = dr \tan\tfrac{1}{2}\phi = \tan\tfrac{1}{2}\phi\,(\cos\theta\, dp - \sin\theta\, dq).$$

Counting the integrated values of p and q in a direction perpendicular to the moving plane we have

$$\sin\theta = \frac{\tan p}{\tan\phi}$$

$$\cos\theta = \frac{\tan q}{\tan\phi}$$

which, being substituted in the expression for dl, gives

$$dl = \frac{\cos\phi}{1+\cos\phi}(\tan q dp - \tan p dq).$$

The approximate value of the integrated correction is therefore

$$\delta l = \frac{1}{2}\int(q dp - p dq). \qquad (32)$$

For every pair of periodic terms in p and q, such as

$$q = s \sin \mu t, p = s \cos \mu t,$$

δl will contain the secular term $-\frac{1}{2} s^2 \mu t$, which will be confounded with the mean motion, and, if it were not so confounded, would in few or none of the larger planets amount to a second in a thousand years. If the secular terms in p and q be

$$q = st ; p = s't$$

δl will vanish. We hence conclude that these terms are entirely unimportant in the present state of astronomy, and that, if we consider the positions of the plane of the orbit at two epochs, we may consider the points of departure in them to be equally distant from their common node.

We have therefore only to consider the motion of the inclination and node due to the change of the position of the orbit and of the ecliptic. If we put

ϕ, the inclination of the orbit of the planet to the ecliptic,

θ, the longitude of its node counted on the ecliptic,

τ, the longitude of the same node counted from the same fixed point in the moving plane of the orbit from which v is counted,

Then, the longitude of the planet on the ecliptic, or L, will be given by the equation

$$\tan(L-\theta) = \cos\phi \tan(v-\tau),$$

or, when developed in powers of ϕ,

$$L = v + \theta - \tau + D, \qquad (33)$$

where D is the reduction to the ecliptic, the value of which is

$$D = -\tan^2 \tfrac{1}{2}\phi \sin 2(v-\tau) + \tfrac{1}{2}\tan^4 \tfrac{1}{2}\phi \sin 4(v-\tau) - \text{etc.}$$

Let us refer the instantaneous rotations of the orbit and of the ecliptic to the fixed points of reference in the two planes; q being the rotation around an axis passing through the sun and the fixed point, and p that around an axis in 90° greater longitude, and the accented quantities referring to the ecliptic. We then have

$$\begin{aligned}
\frac{d\phi}{dt} &= \cos\tau\frac{dq}{dt}+\sin\tau\frac{dp}{dt}\\
&\quad -\cos\theta\frac{dq'}{dt}-\sin\theta\frac{dp'}{dt}\\
\frac{d\theta}{dt} &= \operatorname{cosec}\phi\left(-\sin\tau\frac{dq}{dt}+\cos\tau\frac{dp}{dt}\right)\\
&\quad +\cot\ \phi\left(\ \ \sin\theta\frac{dq'}{dt}-\cos\theta\frac{dp'}{dt}\right)\\
\frac{d\tau}{dt} &= \cot\ \phi\left(-\sin\tau\frac{dq}{dt}+\cos\tau\frac{dp}{dt}\right)\\
&\quad +\operatorname{cosec}\phi\left(\ \ \sin\theta\frac{dq'}{dt}-\cos\theta\frac{dp'}{dt}\right)
\end{aligned}\tag{34}$$

If we differentiate (33) and substitute these values of $\frac{d\theta}{dt}$ and $\frac{d\tau}{dt}$, we shall have

$$\frac{dL}{dt}=\frac{dv}{dt}+\frac{dD}{dt}-\tan\tfrac{1}{2}\phi\left(\cos\tau\frac{dp}{dt}-\sin\tau\frac{dq}{dt}+\cos\theta\frac{dp'}{dt}-\sin\theta\frac{dq'}{dt}\right)\tag{35}$$

If we consider only quantities of the first order with respect to the disturbing forces, we may, in integrating, suppose τ and θ equal and constant, and ϕ constant. The integral will then be

$$L=v+D+\tan\tfrac{1}{2}\,\phi\,\{\cos\theta\,(\delta k+\delta k')-\sin\theta\,(\delta\eta+\delta\eta')\}\tag{36}$$

In the case of Uranus, $\tan\phi$ is so small that this equation will be sufficient for a long time before and after our epoch.

In the application of the method to other planets the mode of operation must depend on the circumstances of each particular case. The differential equations (34) between θ, τ, and ϕ are rigorous, and their integrals may be approximated to in various ways, out of which that best applicable to the particular case must be selected.

Expressions for the latitude.

If the position of the orbital plane and of the ecliptic were each determined by the preceding formulæ, there would be no perturbations of the latitude, the latitude itself being given rigorously by the equation

$$\begin{aligned}
\sin\beta &= \sin\phi\,\sin\,(v-\tau).\\
&= \sin\phi\,\cos\tau\,\sin v-\sin\phi\,\sin\tau\,\cos v.
\end{aligned}$$

But the instantaneous values of ϕ and τ, or of $\sin\phi\,\cos\tau$ and $\sin\phi\,\sin\tau$, are troublesome to tabulate; it will therefore, in practice, be found more convenient to use only their mean values, and to consider their changes from this mean as perturbations of the latitude. Representing by the sign δ the deviations from the mean values, which are of course arbitrary, we have

$$\cos\beta\delta\beta=\cos\phi\,\sin\,(v-\tau)\,\delta\phi-\sin\phi\,\cos\,(v-\tau)\,\delta\tau.$$

Let us substitute for $\delta\phi$ and $\delta\tau$ their values given by the integration of (34) to

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quantities of the first order, in which case θ and τ may be assumed equal. These values are

$$\begin{aligned} \delta\phi &= \sin\tau\,\delta p + \cos\tau\,\delta q \\ \sin\phi\,\delta\tau &= \cos\phi\,(\cos\tau\,\delta p - \sin\tau\,\delta q) \end{aligned}$$

the terms dependent on $\delta p'$ and $\delta q'$ being omitted because, being purely secular, they may be included in the mean values of ϕ and τ. Substituting in the expression for $\delta\beta$

$$\cos\beta\,\delta\beta = \cos\phi\,\{\sin v\,\delta q - \cos v\,\delta p\}. \qquad (37)$$

In the case of all the larger planets both $\cos\beta$ and $\cos\phi$ may here be put equal to unity, when the expression for $\delta\beta$ will become

$$\delta\beta = \sin v\,\delta q - \cos v\,\delta p. \qquad (38)$$

To develop this expression in purely periodic terms we must substitute for v its value in terms of the mean longitude or mean anomaly, namely,

$$v = l + 2e\sin g + \frac{5}{4}e^2\sin 2g + \text{etc.};$$

suppose the terms of δp and δq depending on any argument, N to be

$$\left.\begin{aligned} \delta p &= -\,a_s\sin N - a_c\cos N \\ \delta q &= a'_s\sin N + a'_c\cos N \end{aligned}\right\} \qquad (39)$$

and put π for the longitude of the perihelion, so that

$$l = \pi + g$$

then, to terms of the first order with respect to the eccentricities, we have

$$\begin{aligned} \delta\beta = &-e\,(a_s\cos\pi + a'_s\sin\pi)\sin N - e\,(a_c\cos\pi + a'_c\sin\pi)\cos N \\ &+\tfrac{1}{2}\,\{(a_s + a'_c)\cos\pi + (a'_s - a_c)\sin\pi\}\sin(N + g) \\ &+\tfrac{1}{2}\,\{(a_c - a'_s)\cos\pi + (a'_c + a_s)\sin\pi\}\cos(N + g) \\ &+\tfrac{1}{2}\,\{(a_s - a'_c)\cos\pi + (a'_s + a_c)\sin\pi\}\sin(N - g) \\ &+\tfrac{1}{2}\,\{(a_c + a'_s)\cos\pi + (a'_c - a_s)\sin\pi\}\cos(N - g) \\ &+\tfrac{1}{2}e\{(a_s + a'_c)\cos\pi + (a'_s - a_c)\sin\pi\}\sin(N + 2g) \\ &+\tfrac{1}{2}e\{(a_c - a'_s)\cos\pi + (a'_c + a_s)\sin\pi\}\cos(N + 2g) \\ &+\tfrac{1}{2}e\{(a_s - a'_c)\cos\pi + (a'_s + a_c)\sin\pi\}\sin(N - 2g) \\ &+\tfrac{1}{2}e\{(a_c + a'_s)\cos\pi + (a'_c - a_s)\sin\pi\}\cos(N - 2g) \end{aligned} \qquad (40)$$

The point of the orbit from which π and v are counted is entirely arbitrary, and, in considering the action of but a single planet, it will be most convenient to count them from the common node, in which case π must be replaced by ω, and δp and δq by δk and $\delta\eta$. Thus, deducing the perturbations of the latitude immediately from the formulæ (27), we shall have

$$\delta\beta = \sin \mathrm{v}\,\delta\eta - \cos \mathrm{v}\,\delta k.$$

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CHAPTER II.

APPLICATION OF THE PRECEDING METHOD TO THE COMPUTATION OF THE PERTURBATIONS OF URANUS BY SATURN.

Data of Computation.

THE elements of Uranus, adopted in this computation, were deduced from the comparison of nine normal heliocentric longitudes at intervals of 3697 days extending from 1781, December 26, to 1862, December 18, with corresponding provisional places derived from the elements given in the "Investigation of the Orbit of Neptune," with perturbations produced by Jupiter, Saturn, and Neptune. As the perturbations are to be entirely re-computed, and the elements to be re-corrected from more extended series of observations, all the details of this first approximation will be omitted. The resulting elements of Uranus are given in the following table, together with the adopted elements of Saturn, which are nearly the same as those employed in the theory of Neptune, except that the inclination and longitude of the node have been corrected to agree with observations:—

	Elements II. of Uranus.	Elements I. of Saturn.
π	168° 16′ 31″	90° 4′ 0″
ε	28 25 36.0	14 48 45.0
θ	73 11 58	112 20 0
ϕ	0 46 20	2 29 39.2
e	.0469276	.0560050
e in seconds,	9679.5	11551.9
n	15426.10	43996.13
m	$\frac{1}{21000}$	$\frac{1}{3501.6}$

In computing the perturbations of the radius vector, one of the largest terms will be a constant. To avoid the necessity of computing separately the perturbations of the second order, which depend on this constant, we shall include an approximate value of it in the mean distance. This approximate value is, in the action of an outer or an inner planet, $\delta \log a = -\frac{1}{6} m' M \alpha^2 D_\alpha\, b_{\frac{1}{2}}^{(0)}$. In the action of an inner or an outer planet, $\delta \log a' = +\frac{1}{6} mM (b_{\frac{1}{2}}^{(0)} + \alpha D_\alpha b_{\frac{1}{2}}^{(0)})$. M being the modulus of the system of logarithms.

Using the values of $b_{\frac{1}{2}}^{(0)}$ and $\alpha D_\alpha\, b_{\frac{1}{2}}^{(0)}$, which are found in different works relating to Celestial Mechanics, we find that the different planets produce the following changes in 6 log a, the units being those of the seventh place of decimals:—

		On Saturn.	On Uranus.
Action of	Venus,	+ 22	+ 22
“	Earth,	+ 24	+ 25
“	Mars,	+ 3	+ 3
“	Jupiter,	+10865	+8780
“	Saturn,		+3081
“	Uranus,	— 35	......
“	Neptune,	— 9	— 119
“	Sum,	+ .0010870	+ .0011792
$\delta \log a$		+ .0001812	+ .0001965

The uncorrected mean distance is deduced from the mean motion by the relation

$$a^3 = \frac{\mu(1+m)}{n^2}.$$

We thus have

	Saturn.	Uranus.
Uncorrected mean dist. (log)	0.979496	1.282901
Action of the planets	+ 181	+ 197
Corrected log a	0.979677	1.283098

The following functions of the elements are derived from the preceding elements by well known formulæ:—

γ (mutual inclination)	1° 57′ 24.4″
$\log \sin \frac{1}{2}\gamma = \sigma$	8.232373
$\log \cos \frac{1}{2}\gamma$	9.9999367
τ (long. of ascending node of Saturn on Uranus)	126° 44′ 51″
ω	41 31 40
ω'	323 18 21
$\omega' - \omega = (\omega)$	281 46 41
$\log \sin (\omega)$	—9.990759
$\log \cos (\omega)$	+9.309888
$\sin 2(\omega)$	—0.39966
$\cos 2(\omega)$	—0.91667
α	0.497249
$\frac{1}{\alpha^2}$	4.04438

The following functions of α, necessary in computing the coefficients h, are derived from Runkle's Tables, published by the Smithsonian Institution:—

Values of $\alpha^n D^n_\alpha b^{(i)}_{\frac{1}{2}}$.

i	$b^{(i)}_{\frac{1}{2}}$	$\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\alpha^3 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\alpha^4 D^4_\alpha b^{i}_{\frac{1}{2}}$	$\alpha^5 D^5_\alpha b^{(i)}_{\frac{1}{2}}$
0	2.14447	0.33969	0.5878	1.081	3.44	13.6
1	0.55207	.68314	.4990	1.177	3.40	13.8
2	0.20836	.47198	.7396	1.152	3.59	13.9
3	0.08687	.28491	.7123	1.463	3.68	14.5
4	0.03793	.16270	.5632	1.596	4.30	15.1
5	0.01702	.09010	.3998	1.485	4.87	16.9
6	0.00777	.04896	.2653	1.231	4.98	19.1
7	0.00359	.02624	.1682	0.940	4.60	20.5
8	0.00168	.01392	.1022	0.675	3.91	20.4
9	0.00079	0.00733	0.0615	0.463	3.11	18.8

Derivatives with respect to $(\log a = v)$ *of* $\alpha^n D^n_\alpha b^{(i)}_{\frac{1}{2}}$.

i	$n =$ 0 $D_v b^{(i)}_{\frac{1}{2}}$	1 $D_v(\alpha D_\alpha b^{(i)}_{\frac{1}{2}})$	2 $D_v(\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}})$	3 $D_v(\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}})$	4 $D_v(\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.33969	0.9275	2.257	6.68	27.4
1	.68314	1.1821	2.175	6.93	27.4
2	.47198	1.2116	2.631	7.05	28.3
3	.28491	0.9972	2.888	8.07	29.2
4	.16270	0.7259	2.722	9.09	32.3
5	.09010	0.4899	2.285	9.33	36.4
6	.04896	0.3143	1.762	8.67	39.0
7	.02624	0.1944	1.276	7.41	38.9
8	.01392	0.1161	0.879	5.93	36.0
9	0.00733	0.0688	0.586	4.50	31.2

Second derivatives.

i	$D^2_v b^{(i)}_{\frac{1}{2}}$	$D^2_v(\alpha D_\alpha b^{(i)}_{\frac{1}{2}})$	$D^2_v(\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}})$	$D^2_v(\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}})$
0	0.9275	3.184	11.19	47.4
1	1.1821	3.357	11.28	48.2
2	1.2116	3.843	12.31	49.4
3	0.9972	3.885	13.85	53.4
4	0.7259	3.448	14.53	59.6
5	0.4899	2.775	13.90	64.4
6	0.3143	2.076	12.19	65.0

Values of $\alpha^{n+1} D^n_\alpha b^{(i)}_{\frac{3}{2}}$

i	$\alpha b^{(i)}_{\frac{3}{2}}$	$\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\alpha^4 D^3_\alpha b^{(i)}_{\frac{3}{2}}$
0	1.865	2.674	8.104	30.8
1	1.267	2.844	7.77	30.8
2	0.761	2.412	7.63	29.9
3	0.433	1.790	6.92	28.7
4	0.240	1.224	5.73	26.8
5	0.130	0.792	4.41	23.5
6	0.070	0.493	3.20	19.6

Derivatives with respect to ($\log a = v$.)

i	$D_v(ab_{\frac{2}{2}}^{(i)})$ $= a_1 B_1^{(i)}$	$D_v(a^2 D a b_{\frac{3}{2}}^{(i)})$ $= 2a_1 B_2^{(i)}$	$D_v(a^3 D a b_{\frac{3}{2}}^{(i)})$ $= 6a_1 B_3^{(i)}$
0	4.539	13.452	55.1
1	4.111	13.46	54.1
2	3.173	12.45	52.8
3	2.223	10.50	49.5
4	1.464	8.18	44.0
5	0.922	5.99	36.7
6	0.563	4.19	29.2

Second derivatives.

i	$a_1 D_v B^{(i)}$	$a_1 D_v B_1^{(i)}$	$2a_1 D_v B_2^{(i)}$
0	4.539	17.99	82.0
1	4.111	17.57	81.0
2	3.173	15.62	77.7
3	2.223	12.72	70.5
4	1.464	9.64	60.4
5	0.922	6.91	48.7
6	0.563	4.75	37.6

	$a_1 E^{(i)}$	$a_1 E_1^{(i)}$	$2a_1 E_2^{(i)}$	$6a_1 E_3^{(i)}$	$a_1 D_v E^{(i)}$	$a_1 D_v E_1^{(i)}$	$2a_1 D_v E_2^{(2)}$
0	1.267	4.111	13.46	54.1	4.111	17.57	81.0
1	1.313	3.856	12.95	54.0	3.856	16.80	79.8
2	0.850	3.167	11.98	51.8	3.167	15.15	75.8
3	0.500	2.318	10.31	48.4	2.318	12.63	69.0
4	0.281	1.573	8.24	43.1	1.573	9.82	59.6
5	0.155	1.014	6.18	36.6	1.014	7.20	49.0

The notation $B_n^{(i)}$ and $E_n^{(i)}$ is that of Le Verrier in his development in the first volume of "*Annales de l'Observatoire Imperial de Paris.*"

Numerical expression of R and its derivatives.

We next proceed to the computation of the coefficients h and their derivatives. As an example of the most convenient form of computation we present in full that of the coefficient of $\frac{m'}{a_1}\cos\,(i\lambda'-(i-1)\lambda-\omega)$ in the expression of R for the action of Saturn on Uranus. In this computation I use the tables given by Le Verrier in his "*Annales de l'Observatoire,*" tome i, pages 358–383, comparing the development with that of Professor Peirce in the Astronomical Journal, vol. i, as a control.

$j = 0;\ j = 1.$

i	—3	—2	—1	0	+1	2	3	4	5	6
i'	+4	3	2	1	0	—1	—2	—3	—4	5
$(0) \times b_{\frac{1}{2}}^{(i)}$	+0.5212	+0.8334	+ 1.1041	0	—1.10414	—0.83344	—0.5212	—0.3034	—0.1702	—0.0929
$(1) \times aDab_{\frac{1}{2}}^{(i)}$	—0.2849	—0.4720	— 0.6831	—0.3397	—0.68314	—0.47198	—0.2849	—0.1627	—0.0901	—0.0490
$\Delta a_1\,(50)^{(i)}$	……	……	—16.1775	……	……	……	……	……	……	……
$a_1\,(50)^{(i)}$	+0.2363	+0.3614	—15.7565	—0.3397	—0.78728	—1.30542	—0.8061	—0.4661	—0.2603	—0.1419
$(0) b_{\frac{1}{2}}^{(i)}$	—13.55	—11.25	— 5.52	0.00	0.00	+ 2.92	+ 5.73	+ 6.83	+ 6.46	
$(1) \times aDa$	+ 8.54	+ 7.78	+ 4.78	+0.51	0.00	+ 1.18	+ 2.56	+ 3.25	+ 3.06	
$(2) \times a^2D^2a$	+ 1.43	+ 0.74	— 0.00	—0.59	— 1.00	— 2.22	— 2.85	— 2.82	— 2.40	
$(3) \times a^3D^3a$	— 0.73	— 0.58	— 0.59	—0.54	— 0.59	— 0.58	— 0.73	— 0.80	— 0.74	
$\Delta a_1(51)$	……	……	+ 48.53	……	……	……	……	……	……	
$a_1(51)$	— 4.31	— 3.31	+ 47.20	—0.62	— 1.59	+ 1.30	+ 4.71	+ 6.46	+ 6.38	
$(0) \times b_{\frac{1}{2}}^{(i)}$	—18.76	—13.33	— 4.42	0.00	+ 4.42	+13.33	+18.76	+19.41	+17.01	
$(1) \times aDa$	+13.10	+10.38	+ 4.10	—0.68	— 1.37	+ 2.83	+ 6.27	+ 7.48	+ 7.02	
$(2) \times a^2D^2a$	+ 1.43	0.00	— 1.00	—2.35	— 3.00	— 5.92	— 7.13	— 6.76	— 5.60	
$(3) \times a^3D^3a$	— 1.46	— 1.15	— 1.18	—1.08	— 1.18	— 1.15	— 1.46	— 1.60	— 1.48	
$\Delta a_1(52)$	……	……	+ 32.36	……	……	……	……	……	……	
$a_1(52)$	— 5.69	— 4.10	+ 29.86	—4.11	— 1.13	+ 9.09	+16.44	+18.53	+16.95	
$(0) \times aE_1{}^{(i)}$	—3.00	— 3.40	— 2.63	0.00	+ 2.63	+ 3 40	+ 3.00	+ 2.25	+ 1.55	
$(1) \times a_1E_1{}^{(i)}$	+2.32	+ 3.17	+ 3.86	+4.11	+ 3.86	+ 3.17	+ 2.32	+ 1.57	+ 1.01	
$\Delta a_1(60)$	……	……	+ 16.18	……	……	……	……	……	……	
$a_1(60)$	—0.68	— 0.23	+ 17.41	+4.11	+ 6.49	+ 6.57	+ 5.32	+ 3.82	+ 2.56	
$\frac{1}{2} e \times (50)$	+6.61[1]	+10.12	—441.22	—9.51	—50.049	—36.556	—22.573	—13.05	— 7.28	—3.97
$\frac{1}{8} e^3 \times (51)$	—0.09	— 0.07	+ 1.04	—0.01	— 0.035	+ 0.028	+ 0.103	+ 0.14	+ 0.14	
$\frac{1}{8} ee'^2 \times (52)$	—0.08	— 0.06	+ 0.45	—0.06	— 0.017	+ 0.138	+ 0.248	+ 0.28	+ 0.26	
$\frac{1}{2} ee^2 \times (60)$	—0.05	— 0.02	+ 0.14	+0.03	+ 0.051	+ 0.052	+ 0.043	+ 0.03	+ 0.02	
h	+6.39	+ 9.97	—439.59	—9 55	—50.050	—36.338	—22 179	—12.60	— 6.86	—3.60

The derivatives of h with respect to (log. $a = \nu$) are computed in precisely the same way by simply substituting for $b_{\frac{1}{2}}^{(i)}$, $\alpha D \alpha b_{\frac{1}{2}}^{(i)}$, etc., their derivatives with respect to ν as given in the above table of constants.

The quantities $\Delta a_1\,(50)^{(i)}$, etc., which appear in the third series of terms above express that part of the perturbations of Uranus caused by the action of Saturn

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[1] In units of the third place of decimals.

on the sun. They are each of the form $N \times \alpha^{-2}$, N being a numerical coefficient given by Le Verrier under the coefficient for each term. The derivative of this expression with respect to v is $-2N \times \alpha^{-2}$, so that for the corresponding terms in $D_v h$ and $D_v^2 h$ we have

$$\Delta D_v h = -2\Delta h$$
$$\Delta D_v^2 h = +4\Delta h$$

The values of h and its derivatives, corresponding to any one argument i' and i, are to be combined into two terms depending the one on the cosine, the other on the sine of the argument. Let us represent by g the mean anomaly of Uranus, and let us put l' for the mean longitude of Saturn counted from the perihelion of Uranus, or, more exactly, for the arc $\lambda' - \omega$. Put also

$$\begin{aligned} N &= ig + i'l', \\ i + i' + j &= j'', \\ P &= j\omega + j'\omega', \\ P' &= j''\omega + j'\omega'. \end{aligned}$$

Then, for each value of N there will be several values of P corresponding to different powers and products of the eccentricities and inclinations in h. Distinguishing these values and the corresponding values of h by subscript numerals, we shall have a series of terms of R of the following form—

$$R = \frac{m}{a_1} \left\{ \begin{array}{l} \quad h_1 \cos (N + P'_1) \\ + h_2 \cos (N + P'_2) \\ + h_3 \cos (N + P'_3) \\ + \text{ etc.} \qquad \text{etc.} \end{array} \right\}$$

and by putting

$$\left.\begin{aligned} h_c &= \quad h_1 \cos P'_1 + h_2 \cos P'_2 + h_3 \cos P'_3 + \text{etc.} \\ h_s &= -h_1 \sin P'_1 - h_2 \sin P'_2 - h_3 \sin P'_3 - \text{etc.} \end{aligned}\right\} \qquad (41)$$

The above terms may be condensed into

$$R = \frac{m}{a_1} h_c \cos N + \frac{m}{a_1} h_s \sin N,$$

which are of the form supposed in the preceding theory.

In order that the derivative of R, with respect to the true longitude of Uranus, may be expressed in the form

$$\frac{\partial R}{\partial \mathrm{v}} = \frac{m}{a_1} v_s \sin N + \frac{m}{a_1} v_c \cos N$$

we must, by (7), put

$$\left.\begin{aligned} v_s &= -(i + j_1)\, h_1 \cos P'_1 - (i + j_2)\, h_2 \cos P'_2 - \text{etc.} \\ v_c &= -(i + j_1)\, h_1 \sin P'_1 - (i + j_2)\, h_2 \sin P'_2 - \text{etc.} \end{aligned}\right\} \qquad (42)$$

j_1, j_2, representing the several values of j in the different terms which correspond to one and the same set of values of i and i'.

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To obtain the derivative with respect to γ we notice that all the appreciable terms in the different values of h, which depend upon the mutual inclination, are of the form

$$h = \sigma^2 A,$$

where $\sigma = \sin \frac{1}{2} \gamma$. These equations give

$$\frac{\partial h}{\partial \gamma} = \frac{\partial h}{\partial \sigma} \frac{\partial \sigma}{\partial \gamma} = A \sigma \cos \tfrac{1}{2} \gamma = \tfrac{1}{2} A \sin \gamma.$$

Consequently

$$\frac{\partial R}{\partial \gamma} = \tfrac{1}{2} A \sin \gamma \cos (N + P').$$

and the various terms depending on the same argument (i', i) may be condensed into two, exactly as in the case of R itself.

The different co-efficients h and $D_\iota h$, computed in the way already described, are given *in extenso* in the following table. At the top of each individual column is given the value of P, or of $j\omega + j'\omega'$, corresponding to the values of h below, and immediately under P is given its modified value, or P', to be used in condensing the terms, putting for brevity

$$(\omega) = \omega - \omega'.$$

P and P' are therefore regarded as constant angles the numerical values of the sines and cosines of which may be obtained from the values of ω and ω' already given.

The condensed h_c and h_s are given in the two right hand columns.

All the numbers are given in units of the third place of decimals.

VALUES OF h.

P P'	0 $= 0$	$\omega' - \omega$ (ω)		
i i'	h	h	h_c	h_s
—4, +4		+ 0.62		
—3, 3		+ 0.82		
—2, 2		+ 0.88		
—1, 1		+ 0.53		
0, 0	+1072.90	— 0.4975	+ 1072.80	
+1, —1	—3481.36	— 0.83	— 3481.42	— 1.33
2, —2	+ 205.92	— 9.63	+ 204.14	— 10.29
3, —3	+ 84.27	+ 1.62	+ 84.77	+ 0.79
4, —4	+ 35.80	+ 1.54	+ 36.24	+ 0.90
5, —5	+ 15.48	+ 1.2	+ 15.81	+ 0.81

P P'	$-\omega$ 0	$-\omega'$ $-(\omega)$	$\omega - 2\omega'$ $-2(\omega)$	$\omega' - 2\omega$ $+(\omega)$	ω' $\omega + \omega'$	ω $+2\omega$		
i i'	h	h	h	h	h	h	h_c	h_s
—3, +4	— 2.31	+ 6.39	0	0	0	0	— 1.01	— 6.26
—2, 3	— 3.42	+ 9.97	—0.95	0	0	+.02	— 0.51	— 9.40
—1, 2	— 3.54	—439.59	0	0	—.03	+.02	— 93.30	+430.35
0, 1	+287.30	— 9.55	+0.13	— .02	—.04	+.09	+285.20	+ 9.19
+1, 0	+ 58.38	— 50.05	—0.02	— .05	—.03	0	+ 48.15	+ 48.96
2, —1	— 39.760	— 36.338	—0.074	— .071	—.016	0	— 47.138	+ 35.531
3, —2	+ 35.13	— 22.179	—0.105	— .313	—.007	0	+ 30.63	+ 21.44
4, —3	+ 20.39	— 12.60	—0.11	+ .17	0	0	+ 17.96	+ 12.55
5, —4	+ 11.25	— 6.86	—0.10	+ .23	0	0	+ 9.98	+ 7.00
6, —5	+ 6.00	— 3.6	0	+ .22	0	0	+ 5.38	+ 3.78

VALUES OF h.

P P'	-2ω 0	$-\omega'-\omega$ $-(\omega)$	$-2\omega'$ $-2(\omega)$	0 $+2\omega$		
i i'	h	h	h	h	h_c	h_s
—1, 3	+0.04	— 0.27	—42.24	+0.11	+38.72	+17.03
0, 2	+0.06	+31.65	— 0.04	+0.18	+ 6.58	—31.15
+1, 1	—0.74	— 0.83	— 1.08	—0.91	— 0.03	+ 2.15
2, 0	+3.09	— 5.19	+ 1.89	+0.18	+ 0.32	+ 4.14
3,—1	+1.268	— 6.290	+ 2.082	+0.111	— 1.912	+ 5.216
4,—2	+4.16	— 5.458	+ 1.767	+0.061	+ 1.44	+ 4.576
5,—3	+3.13	— 4.03	+ 1.29	+0.04	+ 1.13	+ 3.39
6,—4	+2.11	— 2.74	+ 0.87	+0.02	+ 0.75	+ 2.31
7,—5	+1.35	— 1.76	+ 0.55	+0.01	+ 0.49	+ 1.49

P P'	-3ω 0	$-\omega'-2\omega$ $-(\omega)$	$-2\omega'-\omega$ $-2(\omega)$	$-3\omega'$ $-3(\omega)$	$-\omega'$ $3\omega-\omega'$	$-\omega$ 2ω		
i i'	h	h	h	h	h	h	h_c	h_s
+1,+2	0	—.12	0	—.11	—.14	+.02	+.17	+.06
2, 1	+.01	—.07	+.03	—.02	—.02	+.01	0	+.04
3, 0	+.18	—.46	+.34	—.08	—.02	+.04	—.15	+.213
4,—1	+.222	—.760	+.520	—.117	—.016	+.029	—.325	+.416
5,—2	+.424	—.847	+.558	—.122	—.012	+.021	—.176	+.490
6,—3	+.398	—.773	+.498	—.107	—.008	+.014	—.145	+.460
7,—4	+.327	—.627	+.395	—.086	—.005	+.009	—.107	+.379
8,—5	+.24	—.47	+.29	—.06	0	0	—.083	+.289

A P'	-4ω 0	$-\omega'-3\omega$ $-(\omega)$	$-2\omega'-2\omega$ $-2(\omega)$	$-3\omega'-\omega$ $-3(\omega)$	$-4\omega'$ $-4(\omega)$		
i i'	h	h	h	h	h	h_s	h_c
4, 0	0	—.04	+.04	—.02	0	—.04	0
5,—1	+.02	—.08	+.08	—.04	+.01	—.03	+.003
6,—2	+.039	—.105	+.106	—.047	+.008	—.045	+.028
7,—3	+.043	—.114	+.112	—.048	+.008	—.049	+.034
8,—4	+.042	—.106	+.103	—.044	+.007	—.044	+.032
9,—5	+.035	—.090	+.086	—.036	+.006	—.037	+.029

VALUES OF $D_{\mathfrak{v}}h$.

P P'	0 0	$\omega'-\omega$ (ω)		
i i'	$D_{\mathfrak{v}}h$	$D_{\mathfrak{v}}h$	$D_{\mathfrak{v}}h_c$	$D_{\mathfrak{v}}h_s$
—4,+4		+ 3.18		
—3, 3		+ 3.24		
—2, 2		+ 2.39		
—1, 1		+ 0.40		
0, 0	+ 171.93	— 2.075	+ 171.51	
+1,—1	+8749.59	— 2.69	+ 8749.12	— 3.02
2,—2	+ 467.72	+ 20.83	+ 472.46	+ 18.05
3,—3	+ 277.00	+ 2.87	+ 278.25	— 0.36
4,—4	+ 153.87	+ 4.62	+ 155.46	+ 1.41
5,—5	+ 82.12	+ 4.90	+ 83.54	+ 1.80

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VALUES OF $D_v h$.

P	$-\omega$	$-\omega'$	$\omega-2\omega'$	$\omega'-2\omega$	$+\omega'$	ω		
P'	0	$-(\omega)$	$-2(\omega)$	$+(\omega)$	$\omega+\omega'$	2ω		
$i \quad i'$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h_c$	$D_v h_s$
$-3, +4$	-9.28	$+19.4$	0	0	0	0	-5.32	-19.00
$-2, 3$	-9.74	$+18.5$	$+2.05$	$-.06$	$-.12$	$+.08$	-7.97	-17.43
$-1, 2$	-4.63	$+907.62$	$+.05$	$-.08$	$-.13$	$+.07$	$+180.44$	-888.65
$-0, 1$	-556.40	-26.17	$-.10$	$-.09$	$-.14$	$-.12$	-561.82	$+25.60$
$1, 0$	$+30.16$	-71.56	$+.04$	$-.19$	$-.11$	$+.02$	$+15.37$	$+69.86$
$2, -1$	$+265.17$	-86.47	$-.19$	$-.26$	$-.07$	0	$+247.58$	$+84.30$
$3, -2$	$+83.10$	-74.60	$-.42$	$+.63$	$-.04$	0	$+68.34$	$+73.49$
$4, -3$	$+68.34$	-54.9	$-.55$	$+.29$	$-.02$	0	$+57.70$	$+53.9$
$5, -4$	$+49.04$	-36.9	$-.58$	$+.68$	$-.01$	0	$+42.19$	$+36.7$
$6, -5$	$+32.1$	-23.2	$-.53$	$+.90$	0	0	$+28.1$	$+23.4$

P	-2ω	$-\omega-\omega'$	$-2\omega'$	0		
P'	0	$-(\omega)$	$-2(\omega)$	$+2\omega$		
$i \quad i'$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h_c$	$D_v h_s$
$-1, +3$	$+0.16$	-0.66	$+86.02$	$+0.47$	-78.76	-34.19
$0, 2$	$+0.32$	-64.55	$+0.15$	$+0.60$	-12.92	$+62.54$
$+1, 1$	$+3.27$	-2.70	$+4.69$	$+3.02$	-1.22	-2.26
$2, 0$	$+2.56$	-8.78	$+5.00$	$+0.60$	-3.74	$+6.00$
$3, 1$	$+13.745$	-15.870	$+7.232$	$+0.463$	$+3.933$	$+12.153$
$4, -2$	$+10.25$	-18.93	$+7.85$	$+0.32$	-0.77	$+15.07$
$5, -3$	$+10.76$	-18.07	$+6.99$	$+0.21$	$+0.69$	$+14.68$
$6, -4$	$+9.39$	-15.0	$+5.59$	$+0.13$	$+1.22$	$+12.4$
$7, -5$	$+7.3$	-11.5	$+4.21$	$+0.09$	$+1.2$	$+9.6$

P	-3ω	$-2\omega-\omega'$	$-\omega-2\omega'$	$-3\omega'$	$-\omega'$	$-\omega$		
P'	0	$-(\omega)$	$-2(\omega)$	$-3(\omega)$	$3\omega-\omega'$	$+2\omega$		
$i \quad i'$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h_c$	$D_v h_s$
$+1, +2$	$+0.03$	$+0.18$	$+0.05$	$+0.22$	$+.22$	$+.08$	-0.31	-0.17
$2, 1$	$+0.12$	-0.26	$+0.33$	-0.09	$-.08$	$+.17$	-0.08	-0.09
$3, 0$	$+0.21$	-0.89	$+0.98$	-0.31	$-.09$	$+.13$	-0.59	$+0.13$
$4, -1$	$+0.89$	-2.02	$+1.89$	-0.54	$-.09$	$+.13$	-0.84	$+0.69$
$5, -2$	$+1.08$	-3.00	$+2.53$	-0.67	$-.08$	$+.11$	-1.38	$+1.30$
$6, -3$	$+1.40$	-3.47	$+2.73$	-0.69	$-.07$	$+.09$	-1.33	$+1.68$
$7, -4$	$+1.46$	-3.44	$+2.60$	-0.62	$-.05$	$+.06$	-1.20	$+1.78$
$8, -5$	$+1.32$	-3.00	$+2.20$	-0.51	0	$+.04$	-0.99	$+1.62$

P	-4ω	$-3\omega-\omega'$	$-2\omega-2\omega'$	$-\omega-3\omega'$	$-4\omega'$		
P'	0	$-(\omega)$	$-2(\omega)$	$-3(\omega)$	$-4(\omega)$		
$i \quad i'$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h$	$D_v h_c$	$D_v h_s$
$4, 0$	$+.02$	$-.09$	$+.13$	$-.08$	$+.02$	$-.06$	$-.01$
$5, -1$	$+.05$	$-.22$	$+.30$	$-.18$	$+.04$	$-.14$	$-.02$
$6, -2$	$+.10$	$-.38$	$+.48$	$-.26$	$+.05$	$-.24$	$+.01$
$7, -3$	$+.15$	$-.52$	$+.62$	$-.32$	$+.06$	$-.30$	$+.04$
$8, -4$	$+.18$	$-.59$	$+.66$	$-.33$	$+.06$	$-.32$	$+.09$
$9, -5$	$+.19$	$-.59$	$+.65$	$-.31$	$+.05$	$-.31$	$+.11$

The values of $D_v^2 h$, needed in computing the perturbations of the second order with respect to the masses being obtained in the same way, by the simple substitution of the second derivatives of the functions b, $\alpha D\alpha b$, etc., for those functions themselves in the expressions for h, it is not necessary to present the details of the computation.

After obtaining h and its derivatives, it will be found convenient to change the arrangement of the terms. Hitherto we have kept in one series those in which the sum of the indices are a constant. Now, we shall put together all those in which the index of the disturbing planet has the same value, arranging the individual terms of each series according to the index of the disturbed planet. Thus, the index of the product of any term, as $h \cos N$, by any multiple of the mean anomaly of the disturbed planet, as jg, will be found in the same series with that of N itself, and j lines above and below.

The next process will be the formations of the required functions of the mean anomaly of Uranus, $\frac{dv}{dt}$, $\frac{d\rho}{dt}$, $\frac{a^2}{r^2}$, $\log r$. Their values are as follows:—

$r_1^{-2} =$	$\rho_0 = \nu$	$\frac{d\rho_0}{ndt} =$	$\frac{dv_0}{ndt} = 1$
1.001103	+.0005507		
+.093933 cos g	—.0468889 cos g	+.0468889 sin g	+.0938294 cos g
+.005507 cos $2g$	—.0016494 cos $2g$	+.0032988 sin $2g$	+.0055012 cos $2g$
+.000336 cos $3g$	—.0000732 cos $3g$	+.0002196 sin $3g$	+.0003357 cos $3g$
+.000020 cos $4g$	—.0000035 cos $4g$	+.0000142 sin $4g$	+.0000206 cos $4g$

Considering only those terms which are of the first order, the value of $D'_t R$ may be found in two ways, the agreement of which will afford a check upon the entire development of the perturbative function, and upon the computations of R and $\frac{\partial R}{\partial \mathrm{v}}$. These are (1) by direct differentiation, with respect to the time as contained in the mean anomaly of a single planet, whereby each term in R of the form

$$R = \frac{m}{a_1} h \cos N$$

will produce in $D'_t R$ the term

$$D'_t R = -\frac{m}{a_1} inh \sin N\cdot$$

and (2) by forming the expression

$$D'_t R = \frac{\partial R}{\partial \mathrm{v}} \frac{dv_0}{dt} + \frac{\partial R}{\partial \rho} \frac{d\rho_0}{dt}.$$

As several "mechanical multiplications," like those indicated in this last expression, are to be performed, the following example of the form of computation is presented. It exhibits the formation of the product of those terms of $\frac{\partial R}{\partial \mathrm{v}}$ in which $i' = -1$ by $\frac{dv_0}{dt}$.

i'	—2	—1	0	+1	2	3	4	5
$\frac{a_1}{m} \frac{\partial R}{\partial v}$ (sin)	+.02	+ 1.63	—287.42	+3481.14	+ 54.42	+ 6.988	+1.18	+0.10
× .046915	0	0	+ 08	— 13.48	+163.32	+ 2.553	+0.33	+0.06
	+.08	—13.48	+163.32	+ 2.55	+ 0.33	+ 0.056	0	0
× .002750	0	0	0	0.00	— 0.79	+ 9.576	+0.15	+ .02
	—.79	+ 9.58	+ 0.15	+ .02	0	0	0	0
× .000168	+.58	+ 0.01	0	0	0	— .048	+ .58	0
$\frac{a_1}{m'n} \frac{\partial R}{\partial \mathrm{v}} \frac{dv}{dt}$ (sin)	—.11	— 2.26	—123.87	+3470.23	+217.28	+19.125	+2.24	+0.18

The multipliers on the left are each one-half the coefficient cos jg in the expression for $\frac{dv_0}{dt}$, and each product is placed in the two columns corresponding respectively to $N+jg$ and $N-jg$.

All the derivations of R_0 necessary in the computation of the perturbations of the first order are given in the following tables. First we have the values of D'_tR obtained by direct differentiation, as indicated in the preceding formulæ. Next we have $\frac{\partial R}{\partial \mathrm{v}}$ and $\frac{\partial R}{\partial \rho}$, obtained by the formulæ (7) and (42). The products $\frac{\partial R}{\partial \mathrm{v}}$ by $\frac{dv_0}{dt}$ and of $\frac{\partial R}{\partial \rho}$ by $\frac{\partial \rho}{dt}$, being formed in the simple way just pointed out, and with the values of the component factors just given, their sum is next shown. This sum should agree accurately with D'_tR. The discrepancies are shown in the next two columns. The only apparently large discrepancy is found in the argument $5g'-5l$. It probably arises from the incompleteness of the computation of R and $\frac{\partial R}{\partial \mathrm{v}}$, so far as they depend on this argument. As the entire term does not amount to 0″.01, I have not sought to correct it.

The great value of this check arises from the fact that it gives a complete control of the correctness of the development of the perturbative function, *ab initio*, since the two valves of D'_tR are derived from different terms of that development. It also controls all the computations except that of $\frac{\partial R}{\partial \rho}$. This quantity being multiplied by quantities of the order of the eccentricities in the second value of D'_tR, an error in its value will produce a discrepancy of only $\frac{1}{40}$ its own amount in D'_tR, and may therefore be overlooked. The derivative in question must therefore be checked by a complete duplicate computation.

In the column next following are given the integrating factors ν, for which the expression is

$$\nu = \frac{n}{i'n'+in} = \frac{1}{i+i'\frac{n'}{n}}.$$

For each value of i' the values of ν are therefore the reciprocals of a series of numbers in arithmetical progression, the common difference being unity.

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	$D_t'R = \frac{m'}{a_1} n \times$		$\frac{\partial R}{\partial \mathrm{v}} = \frac{m'}{a_1} \times$		$\frac{\partial R}{\partial \rho} = \frac{m'}{a_1} \times$	
g *l'*	sin	cos	sin	cos	cos	sin
0, 0	0	0	....	+ 0.487	—1244.31	....
1,	— 48.15	+ 48.96	+ 10.20	+ 49.07	— 63.52	—118.82
2,	— 0.64	+ 8.28	+ 4.48	+ 3.20	+ 3.42	— 10.14
3,	+ 0.45	+ 0.64	+ 0.50	— 0.08	+ 0.74	+ 0.34
4,	+ 0.16	0	+ 0.06	— 0.06	+ 0.10	+ 0.01
— 2,—1	0	+ 0.08	+ 0.02	— 0.04	+ 0.08	— 0.05
— 1,	— 0.03	+ 2.15	+ 1.63	+ 1.34	+ 1.25	— 0.11
0,	0	0	— 287.42	— 0.10	+ 276.62	+ 34.79
1,	+3481.42	— 1.33	+3481.14	— 1.04	—5267.70	+ 4.85
2,	+ 94.28	+ 71.06	+ 54.42	+ 71.23	— 200.44	—119.83
3,	+ 5.74	+ 15.65	+ 6.988	+ 9.490	— 2.02	— 17.37
4,	+ 1.30	+ 1.66	+ 1.18	+ 0.42	+ 1.16	— 1.11
5,	+ 0.15	+ 0.15	+ 0.10	0	+ 0.17	— 0.01
—1,—2	+ 0.17	+ 0.06	+ 0.21	— 0.16	+ 0.14	— 0.11
0,	0	0	— 6.58	+ 30.99	+ 6.34	+ 31.39
1,	+ 93.30	—430.35	+ 96.81	—430.37	— 87.14	—458.30
2,	— 408.28	— 20.58	— 410.42	— 12.01	— 676.60	— 7.76
3,	— 91.89	+ 64.32	— 57.01	— 64.98	— 98.97	— 94.93
4,	— 5.76	+ 18.30	+ 1.45	+ 12.96	— 0.67	— 19.65
5,	+ 0.88	+ 2.45	+ 1.30	+ 1.04	+ 1.56	— 1.79
6,	+ 0.27	+ 0.17	+ 0.20	— 0.02	+ 0.28	— 0.04
1,—3	— 38.72	— 17.03	— 38.75	— 17.29	+ 40.04	— 17.16
2,	+ 1.02	+ 18.80	+ 5.31	+ 19.14	+ 8.48	— 26.83
3,	— 254.31	+ 2.37	— 254.15	— 0.02	— 363.02	— 0.43
4,	— 71.84	+ 50.20	— 51.49	+ 49.90	— 75.66	— 66.45
5,	— 5.65	+ 16.95	— 0.23	+ 13.0	— 1.82	— 18.07
6,	+ 0.87	+ 2.76	+ 1.29	+ 1.5	+ 1.48	— 2.14
7,	+ 0.34	+ 0.24	+ 0.27	0	+ 0.35	— 0.07
3,—4	+ 3.03	+ 18.78	+ 1.34	+ 18.78	+ 6.33	— 25.26
4,	— 144.96	+ 3.60	— 144.78	+ 1.5	— 191.70	— 2.31
5,	— 49.90	+ 35.00	— 38.64	+ 34.58	— 52.17	— 43.7
6,	— 4.50	+ 13.86	— 0.9	+ 11.2	— 1.97	— 14.7
7,	+ 0.75	+ 2.65	+ 1.1	+ 1.6	+ 1.31	— 2.2
8,	+ 0.35	+ 0.26	+ 0.3	+ 0.1	+ 0.36	— 0.1
5,—5	— 79.05	+ 4.05	— 78.9	+ 2.4	— 99.3	— 2.1
6,	— 32.3	+ 22.7	— 26.3	+ 22.3	— 33.5	— 27.2
7,	— 3.4	+ 10.4	— 1.1	+ 8.7	— 1.7	— 11.1
8,	+ 0.7	+ 2.3	+ 0.9	+ 1.5	+ 1.1	— 1.0
9,	+ 0.3	+ 0.3	+ 0.3	+ 0.1	+ 0.3	+ 0.1

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	$\frac{\partial R}{\partial v}\frac{dv_0}{dt}+\frac{\partial R}{\partial \rho}\frac{d\rho_0}{dt} = \frac{m'}{a_1} n \times$		Discrepancy.		ν	k_c	k_s
g l'	sin	cos	sin	cos			
0, 0	0	— 0.01	0	—.01	....	—1244.31	
1,	— 48.05	+ 48.92	+.10	—.04	+1.0	+ 32.78	— 20.90
2,	— 0.63	+ 8.27	+.01	—.01	+0.5	+ 4.06	— 1.86
3,	+ 0.45	+ 0.65	0	+.01	+⅓	+ 0.44	+ 0.08
4,	+ 0.11	— 0.03	—.05	—.03	+0.25		
—2,—1	— 0.03	+ 0.02	—.03	—.06	—0.206098	+ 0.08	— 0.08
—1,	— 0.05	+ 2.16	—.02	+.01	—0.259601	+ 1.23	— 1.23
0,	— 0.06	+ 0.09	—.06	+.09	—0.350623	+ 276.62	+ 34.79
+1	+3481.41	— 1.33	—.01	0	—0.539942	—1508.18	+ 5.78
2,	+ 94.20	+ 71.06	—.08	0	—1.173630	+ 20.85	—286.59
3,	+ 5.78	+ 15.63	+.04	—.02	+6.75940	— 79.56	+194.17
4,	+ 1.27	+ 1.66	—.03	0	+0.871126	— 1.10	+ 1.79
5,	+ 0.14	+ 0.11	—.01	—.04	+0.4656	+ 0.03	+ 0.13
—1,—2	+ 0.16	+ 0.10	—.01	+.04	—0.149162	+ 0.19	— 0.13
0,	+ 0.01	+ 0.07	+.01	+.07	—0.175312	+ 6.34	+ 31.39
+1,	+ 93.24	—430.37	—.06	—.02	—0.212580	— 47.47	—275.33
2,	— 408.28	— 20.60	0	—.02	—0.269971	— 897.05	+ 3.35
3,	— 91.90	+ 64.33	—.01	+.01	—0.369806	— 166.93	—142.50
4,	— 5.77	+ 18.17	—.01	—.13	—0.586813	— 7.44	— 41.13
5,	+ 0.91	+ 2.45	+.03	0	—1.42022	+ 4.05	— 8.75
6,	+ 0.29	+ 0.15	+.02	—.02	+3.3797	— 1.53	+ 1.09
1,—3	— 38.78	— 17.03	—.06	0	—0.132342	+ 29.78	— 12.65
2,	+ 1.02	+ 18.68	0	—12	—0.152528	+ 8.79	— 32.56
3,	— 254.29	+ 2.37	+.02	0	—0.179981	— 454.55	— 1.28
4,	— 71.88	+ 50.13	—.04	—.07	—0.219482	— 107.20	— 88.50
5,	— 5.69	+ 17.01	—.04	+.06	—0.281202	— 5.00	— 27.61
6,	+ 0.91	+ 2.73	+.04	—.03	—0.391210	+ 2.15	— 4.30
7,	+ 0.35	+ 0.19	+.01	—.05	—0.6426	+ 0.79	— 0.38
3,—4	+ 3.10	+ 18.78	+.07	0	+0.11893	+ 7.05	— 29.73
4,	— 145.0	+ 3.51	—.04	—.09	—0.13500	— 230.83	— 3.28
5,	— 49.9	+ 34.85	0	—.15	—0.15605	— 67.74	— 54.60
6,	— 4.7	+ 13.95	—.20	+.09	—0.18490	— 3.63	— 19.80
7,	+ 0.8	+ 2.62	+.05	—.03	—0.22685	+ 1.65	— 3.36
8,	+ 0.3	+ 0.2	—.05	—.06	—0.2934	+ 0.57	— 0.27
5,—5	— 79.3	+ 2.7	—.25	—1.35	—0.1080	— 116.4	— 3.5
6,	— 32.4	+ 22.7	—.10	0	—0.1211	— 41.3	— 32.7
7,	— 3.5	+ 10.5	—.10	+.10	—0.1377	— 2.6	— 13.9
8,	+ 0.8	+ 2.3	+.10	0	—0.1597	+ 1.3	— 2.6
9,	+ 0.3	+ 0.2	0	—.10	—0.1901	+ 0.5	— 0.2

The values of $\frac{a_1}{m}\int D'_tR_0dt$ are formed from $\frac{a_1}{m'n}D'_tR$ by simple multiplication by ν, and proper changes of sign. The values of k_c and k_s are then formed by adding the terms of $2\,\frac{a_1}{m'}\int D'_tR_0dt$ to the corresponding terms of $\frac{a_1}{m'}\,\frac{\partial R}{\partial\rho}$.

Perturbations of radius vector.

Let us now resume equation (19), and put for brevity

$$M=\frac{m'a}{a_1(1+m)}. \tag{44}$$

If we give to u the successive values 0, +1, —1, +2, —2, +3, —3, we have

$$r_1{}^2\delta\rho=M\times\left\{\begin{array}{l} \quad\frac{1}{4}\Sigma_i\,p_iq_i\,(\nu_i-\nu_{-i})\,\{k_c\cos N+k_s\sin N\} \\ +\frac{1}{8}\Sigma_i(p_iq_{(i-1)}+p_{(i-1)}q_i)(\nu_i-\nu_{(1-i)})\,\{k_c\cos(N+g)+k_s\sin(N+g)\} \\ +\frac{1}{8}\Sigma_i(p_iq_{(i+1)}+p_{(i+1)}q_i)(\nu_i-\nu_{-(i+1)})\,\{k_c\cos(N-g)+k_s\sin(N-g)\} \\ +\frac{1}{8}\Sigma_i(p_iq_{(i-2)}-p_{(i-2)}q_i)(\nu_i-\nu_{(2-i)})\,\{k_c\cos(N+2g)+k_s\sin(N+2g)\} \\ +\qquad\text{etc.}\qquad\text{etc.}\qquad\text{etc.}\qquad\text{etc.} \end{array}\right\}$$

the finite integral being taken with respect to all values of i from $-\infty$ to $+\infty$, and the terms in which the angles $N\pm ug$ vanish being omitted. Proceeding farther to expand with respect to i, if we collect similar terms we shall find the individual terms in $r^2\delta\rho$ to be as follows:

$$\begin{aligned} r_1{}^2\delta\rho=\tfrac{1}{2}M&\left\{\begin{array}{l}\quad p_1q_1\,(\nu_1-\nu_{-1}) \\ +\;p_2q_2\,(\nu_2-\nu_{-2})\end{array}\right\}\{k_c\cos N+k_s\sin N\} \\ +\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_1\;\;(\nu_1-\nu_0) \\ +(p_1q_2+p_2q_1)\;(\nu_2-\nu_{-1}) \\ +(p_2q_3+p_3q_2)\;(\nu_3-\nu_{-2}) \\ +\qquad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N+g)+k_s\sin(N+g)\} \\ +\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_1\;\;(\nu_0-\nu_{-1}) \\ +(p_1q_2+p_2q_1)\;(\nu_1-\nu_{-2}) \\ +(p_2q_3+p_3q_2)\;(\nu_2-\nu_{-3}) \\ +\qquad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N-g)+k_s\sin(N-g)\} \\ +\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_2\;\;(\nu_2-\nu_0) \\ +(p_1q_3+p_3q_1)\;(\nu_3-\nu_{-1}) \\ +\qquad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N+2g)+k_s\sin(N+2g)\} \\ +\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_2\;\;(\nu_0-\nu_{-2}) \\ +(p_1q_3+p_3q_1)\;(\nu_1-\nu_{-3}) \\ +\qquad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N-2g)+k_s\sin(N-2g)\} \\ +\tfrac{1}{4}M&\left\{\begin{array}{l}\qquad\quad p_0q_3\;\;(\nu_3-\nu_0) \\ +(p_1q_4+p_4q_1)\;(\nu_4-\nu_{-1}) \\ +(p_1q_2-p_2q_1)\;(\nu_2-\nu_1) \\ +\qquad\text{etc.}\qquad\text{etc.}\end{array}\right\}\{k_c\cos(N+3g)+k_s\sin(N+3g)\} \end{aligned}$$

$$+\tfrac{1}{4}M\left\{\begin{array}{l} \qquad\qquad p_0q_3\ (\nu_0 \ -\nu_{-3}) \\ +(p_1q_4+p_4q_1)\ (\nu_2 \ -\nu_{-4}) \\ +(p_1q_2-p_2q_1)\ (\nu_{-1}-\nu_{-2}) \\ + \qquad \text{etc.} \qquad\quad \text{etc.} \end{array}\right\}\{k_c\cos(N-3g)+k_s\sin(N-3g)\}$$

A law of the factors of $k_c\cos(N+ug)+k_s\sin(N+ug)$ which will be noticed in the above expression, is this: Representing this factor by K_u, we have

$$K^{\,i}_{\,u}=K^{\,(i+u)}_{\,-u},$$

the index i representing the coefficient of g in N, so that only half the values of K need be separately computed.

As the computation of $r_1{}^2\delta\rho$ from these formulæ can be arranged in such a way as to be very simple, the computation of the terms in which the index i' is -1 is here presented quite fully. The logarithms only are omitted, being used only in the cases in which they are more convenient than a table of products. In practice I find it convenient to write them in red ink immediately under the numbers which they represent.

First, to find M, it will be noticed that in the expression $\dfrac{m'a}{a_1(1+m)}$, the a in the numerator represents the mean distance of the *disturbed* planet, as deduced from the observed mean motion by the equation $a^3n^2=\mu\,(1+m)$ while a_1 represents the mean motion of the *outer* planet. When the outer planet is the disturbed one, the ratio $\dfrac{a}{a_1}$ would be unity, but that, to avoid a large class of second order terms, a_1 has been corrected for perturbations in the beginning (p. 32). In the case of Uranus disturbed by Saturn, we have in consequence

$$\log\frac{a}{a_1}=9.999803.$$

Whence

$$M=285.44$$

in units of the sixth place of decimals.

Computing the values of p_i and q_i from $(16)'$ we find, for Uranus,

$$\left.\begin{array}{lll} \frac{1}{2}M\ p_1q_1 & =+ & 142.56 \\ \frac{1}{2}M\ p_2q_2' & =+ & 0.0784 \\ \frac{1}{4}M\ p_0q_1 & =- & 10.044 \\ \frac{1}{4}M(p_1q_2+p_2q_1) & =+ & 3.3433 \\ \frac{1}{4}M(p_2q_3+p_3q_2) & =+ & 0.0028 \\ \frac{1}{4}M\ p_0q_2 & =- & 0.2358 \\ \frac{1}{4}M(p_1q_3+p_3q_1) & =+ & 0.118 \\ \frac{1}{4}M\ p_0q_2 & =- & 0.008 \\ \frac{1}{4}M(p_1q_4+p_4q_1) & =+ & 0.005 \\ \frac{1}{4}M(p_1q_2-p_2q_1) & -+ & 0.003 \end{array}\right\}\begin{array}{l}\text{In units of the sixth} \\ \text{place of decimals.}\end{array}$$

In the computation the first three lines are copied from previous pages.

$i' = -1$

i	—4	—3	—2	—1	0	+1	2	3	4	5
ν	**—0.1459**	**—0.1709**	**—0.20610**	**—0.25960**	**—0.35062**	**—0.53994**	**—1.17363**	**+6.75940**	**+0.87113**	**+0.4656**
		k_c	**+0.08**	**+1.23**	**+276.62**	**—1508.18**	**+ 20.85**	**— 79.56**	**—1.10**	**+0.03**
		k_s	**—0.08**	**—1.23**	**+ 34.79**	**+5.79**	**—286.59**	**+194.17**	**+1.79**	**+0.13**
	+142.56	$\nu_1-\nu_{-1}$	— .0887	— .14452	— .28034	— .82301	+7.29934	+2.04476	—6.2938	—0.5535
	+ 0.0784	$\nu_2-\nu_{-2}$	— .2047	— .3690	— .9675	+7.0190	+1.2218	+1.0055	+1.4913	—6.5186
	— 10.044	$\nu_1-\nu_0$	— .0535	— .0910	— .1893	— .63368	+7.9330	—5.8883	—0.4056	
	+ 3.3433	$\nu_2-\nu_{-1}$	— .1797	— .3338	— .9140	+7.1100	+1.4110	+1.6392	—6.4417	
	+ 0.0028	$\nu_3-\nu_{-2}$	— .39	—1.00	+6.96	+1.13	+0.82	+0.86	+1.41	
	— 0.2358	$\nu_2-\nu_0$	— .144	— .280	— .823	+7.299	+2.045	—6.294		
	+ 0.118	$\nu_3-\nu_{-1}$	— .369	— .968	+7.019	+1.222	+1.005	+1.490		
	— 0.008	$\nu_3-\nu_0$	— .33	— .91	………	+1.41	+1.64			
	+ 0.005	$\nu_4-\nu_{-1}$	—1.00	+6.97	………	+0.82	+0.86			
	+ 0.0003	$\nu_2-\nu_{-1}$	— .09	— .19	………	+7.93	—5.89			
	142.56	$\times(\nu_2-\nu_{-1})$	—12.35	—20.60	—39.965	—117.327	+1040.59	+291.49	—897.2	—78.8
	0.0784	$\times(\nu_3-\nu_{-2})$	— 0.02	— 0.03	— 0.075	+0.550	+0.10	+0.08	+0.1	— 0.5
		K_0	**—12.37**	**—20.63**	**—40.040**	**—116.777**	**+1040.69**	**+291.57**	**—897.1**	**—79.3**
	— 10.044	$\times(\nu_1-\nu_0)$	+0.532	+0.914	+1.901	+ 6.364	—79.68	+59.14	+ 4.1	
	+ 3.3433	$\times(\nu_2-\nu_{-1})$	—0.601	—1.126	—3.056	+23.770	+ 4.72	+ 5.48	—21.5	
	+ 0.0028	$\times(\nu_3-\nu_{-2})$	—0.001	—0.003	+0.020	+ 0.003	0	0	0	
		K_1	………	**—0.20**	**—1.135**	**+30.137**	**—74.96**	**+64.62**	**—17.4**	
		K_{-1}	………	**—0.07**	**—0.205**	**— 1.135**	**+30.14**	**—74.96**	**+64.6**	
	— .2358	$\times(\nu_2-\nu_0)$	+0.034	+0.066	+0.194	—1.721	—0.48	+ 1.48		
	+ .118	$\times(\nu_3-\nu_{-1})$	—0.043	—0.114	+0.828	+0.144	+0.12	+ 0.18		
		K_2	………	………	**+1.022**	**—1.577**	**—0.36**	**+ 1.66**		
		K_{-2}	………	………	**—0.009**	**—0.048**	**+1.02**	**+ 1.58**		
	— .008	$\times(\nu_1-\nu_0)$	+ .003	+ .007	………	—.011	—.013			
	+ .005	$\times(\nu_4-\nu_1)$	— .005	+ .035	………	+.004	+.004			
		K_3	………	………	………	**—.007**	**—.009**			
		K_{-3}	………	………	………	**—.002**	**+.042**			
		$K_0\ k_c$	— 1	— 26	—11076	+ 176117	+21698	—23197	+991	—2
		$K_1\ k_c$	0	0	0	—314	—45452	—1563	—5142	+19
		$K_{-1}\ k_c$	0	— 56	+1712	+628	+5964	—71	0	0
		$K_2\ k_c$	0	0	0	0	+ 282	+2378	—8	—132
		$K_{-2}\ k_c$	— 3	+ 72	+21	+126	0	0	0	0
		$K_3\ k_c$	0	0	0	0	0	0	+8	0
		$K_{-3}\ k_c$	+ 3	+ 1	0	0	0	0	0	0
		$r_1^2\,\delta\rho$ (cos)	**— 1**	**— 9**	**—9343**	**+176557**	**—17508**	**—22453**	**—4151**	**—115**
		$\times.046915$	0	0	0	—438	+8283	—821	—1053	—195
			0	—438	+8283	—821	—1053	—195	— 5	
		$\times.002751$	0	0	0	0	—26	+485	—48	—62
			—26	+485	—48	—62	—11	0	0	0
		$\times.000168$	+30	— 3	— 4	— 1	0	—2	+30	—3
		$\cos\psi\,\delta\rho$ (cos)	**+ 3**	**+ 34**	**—1112**	**+175235**	**—10315**	**—22986**	**—5227**	**—375**
		$K_0\ k_s$	+ 1	+ 26	—1393	—676	—298254	+56614	—1606	—10
		$K_1\ k_s$	0	0	0	— 39	+175	+21483	+12548	—30
		$K_{-1}\ k_s$	0	— 7	—6	—8637	—14556	+116	—2	0
		$K_2\ k_s$	0	0	0	0	+36	—9	+104	+322
		$K_{-2}\ k_s$	0	0	—293	—306	— 1	0	0	0
		$K+_3\ k_s$	0	— 11	0	0	0	0	0	—1
		$r_1^2\,\delta\rho$ (sin)	**+ 1**	**+ 8**	**—1692**	**—9658**	**—312600**	**+78204**	**+11044**	**+281**
		$\times.046915$	0	0	0	— 79	—453	—14666	+3669	+518
			0	— 79	—453	—14666	+3669	+518	+13	0
		$\times.002751$	0	0	0	0	— 5	—27	—860	+215
			— 5	— 27	—860	+215	+30	0	0	0
		$\times.000168$	— 5	— 53	+ 13	+2	0	0	—2	—3
		$\cos\phi\,\delta\rho$ (sin)	**— 9**	**— 151**	**—2992**	**—24186**	**—309359**	**+64029**	**+13864**	**+961**

In forming the next ten lines, it will be noticed that the value of ν_u corresponding to any vertical column is found u columns to the right. It is therefore necessary to extend the line ν two columns at each end. The extension on the right is, however, omitted for want of space. In performing the subtractions it will be convenient to copy the ν's again on the lower edge of a horizontal strip of paper, and, in forming the differences $\nu_u—\nu_u'$ to lay the strip above the line of ν's, and $u—u'$ columns to the right.

On the left of each line of differences is written the factor by which that line is to be multiplied.

The mode of formation of the K's is evident from the formula.

It will be seen that the same computation which gives K_u gives also K_{-u}, only the latter belongs u columns to the right.

Each k_c and k_s is multiplied in succession by all the K's which lie below it in the same column, but the product by K_u is to be written u columns to the right, and that by K_{-u} u columns to the left. The sum of the products in any one column gives the coefficient of $\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}(ig+i'l')$ in the development of $r_1^2\delta\rho$.

This quantity being multiplied by $\frac{dv_0}{ndt}=\frac{a^2\cos\psi}{r_0^2}$ we have $\cos\psi\delta\rho$, which only needs to be multiplied by $\sec\psi=1.001103$ to give $\delta\rho$. The units of $r_1^2\delta\rho$ and $\delta\rho$ correspond to the ninth place of decimals.

All the periodic terms are to be treated in this manner, all the series of values of k_c and k_s, including the constant term, being subjected to the same process. But, when i' and i are both zero, ν will be infinite. Here we simply omit the ν, treating it as if it were zero. We thus obtain the complete value of the terms with constant coefficients in $r_1^2\delta\rho$ and $\delta\rho$ which are given in the following table. The terms multiplied by the time are still to be computed. They are derived from (20), which may be put in the form

$$r_1^2\delta\rho=\tfrac{1}{2}M\{\eta\Sigma p_uk_c^{(u)}-\xi\Sigma q_uk_s^{(u)}\}nt.$$

This expression is computed thus:

u	p_u	$k_c^{(u)}$	q_u	$k_s^{(u)}$	$p_uk_c^{(u)}$	$-q_uk_s^{(u)}$
0	$-.140783$	-1244.31	0	0	$+175.18$	0
1	$+.99917$	$+\ 32.78$	$+.99972$	-20.90	$+\ 32.75$	$+20.89$
2	$+.0234$	$+\ 4.06$	$+.0234$	$-\ 1.86$	$+\ 0.09$	$+\ 0.04$

We have now

$$\Sigma p_uk_c^{(u)}=+208.02;\qquad \tfrac{1}{2}M\Sigma p_uk_c^{(u)}=+29689$$

$$-\Sigma q_uk_s^{(u)}=+\ 20.93;\qquad \tfrac{1}{2}M\Sigma q_uk_s^{(u)}=+\ 2988$$

$$\begin{aligned}r_1^2\delta\rho=&-210\,nt\\ &+2986\,nt\cos g\quad+29681\,nt\sin g\\ &+\ \ 70\,nt\cos 2g\quad+\ \ 697\,nt\sin 2g\\ &+\ \ \ \ 2\,nt\cos 3g\quad+\ \ \ \ 24\,nt\sin 3g,\end{aligned}$$

in units of the ninth place of decimals. The value of $\cos\psi\delta\rho$ is obtained from them by multiplying by r_1^{-2}, exactly as in the case of the constant terms.

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	$r_1^2\delta\rho$		cos $\psi\delta\rho$		log M_ν	v_c	v_s
g l'	cos	sin	cos	sin			
0, 0	—210 nt		—70 nt			+139$n't$	
1,	+2986 nt	+29681 nt	+2978 nt	+29633 nt			
2,	+70 nt	+697 nt	+209 nt	+2084 nt			
3,	+ 2 nt	+24 nt	+14 nt	+139 nt			
0, 0	—355045		—354347		$\propto$	0	
1,	+14875	—1503	—18411	—1496	2.45551	—2912	+14206
2,	—283	+49	—1536	—21	2.25448	—639	+457
3.	—20	— 1	—112	— 2	1.97839	—46	—8
—2,—1	— 1	+ 1	+ 3	— 9	1.7696	+ 1	+2
—1,	—10	+ 8	+34	—151	1.8698	+12	—99
0,	—9343	—1692	—1112	—2992	2.00035	—2876	+10
+1,	+176557	—9658	+175235	—24186	2.18786	+536512	+160
2,	—17508	—312600	—10315	—309359	2.52504	+1823	—23860
3,	—22453	+78204	—22986	+64029	3.28542	—13482	+18310
4,	—4151	+11044	—5227	+13864	2.39559	—293	+104
5,	—115	+281	—373	+961	2.1235	—13	0
—1,—2	— 1	+ 1	+ 1	— 2	1.6292	+ 9	+7
0,	—56	—276	+32	—100	1.6993	—329	—1551
+1,	+710	+3723	+1675	+3728	1.78303	+5874	+26114
2,	+20180	—13	+20583	+484	1.88683	—31660	+926
3,	+7797	+6512	+8824	+6796	2.02348	—6019	—6860
4,	+1534	+6202	+2054	+6230	2.22401	+243	—2172
5,	+2071	—5954	+2134	—5610	2.60786	+527	—421
6,	—653	+755	—550	+494	2.98438	—190	—20
1,—3	—151	+64	—137	+75	1.5772	—1464	+653
2,	—52	+221	+149	+229	1.63886	+231	—833
3,	+4348	+15	+4420	+88	1.71074	—13050	+1
4,	+1564	+1280	+1774	+1314	1.79691	—3225	—3126
5,	+133	+685	+214	+756	1.9045	—18	—1043
6,	—111	+232	—109	+272	2.0479	+144	—163
7,	—163	+87	—168	+100	2.2634	+50	—7
3,—4	—28	+121	+31	+122	1.5408	+181	—638
4,	+1223	+17	+1245	+41	1.5858	—5578	—58
5,	+485	+389	+545	+400	1.6488	—1721	—1540
6,	+38	+201	+63	+221	1.7225	—50	—590
7,	—25	+53	—23	+63	1.8012	+70	—100
8,	—15	+ 7	—16	+ 9	1.9229		
5,—5	+392	+12	+400	+18	1.4889	—2432	—74
6,	+175	+138	+194	+142	1.5385	—908	—760
7,	+14	+77	+22	+84	1.5946	—40	—340
8,	—10	+20	— 9	+24	1.6589	+40	—70
9,	— 5	+ 3	— 5	+ 4	1.7345		

Perturbations of Longitude.

The perturbations of the longitude are now to be computed by formulæ (24). To do this in the most simple way we remark that the numbers given on page 42, under the heading $\frac{\partial R}{\partial \mathrm{v}}$, are those represented in formula (42) by v_s and v_c. If we put

$$nt = t'$$

equation (24) may be put into the form

$$\frac{d\delta v}{dt'}, = r_1^{-2} \left\{ \frac{a}{1+m} \int \frac{\partial R}{\partial \mathrm{v}}\, dt' - 2 \cos \psi \delta\rho \right\};$$

but we have from (42)

$$\int \frac{\partial R}{\partial \mathrm{v}}\, dt = \Sigma \left(\frac{m'\nu}{a_1} v_c \sin N - \frac{m'\nu}{a_1} v_s \cos N \right).$$

If now we represent the numerical values of $\cos \psi\delta\rho$, already found, by

$$\Sigma\,(\rho_s \sin N + \rho_c \cos N),$$

and if we substitute these expressions in the above value of $\frac{d\delta v}{dt'}$, the latter will become

$$\frac{d\delta v}{dt'} = r_1^{-2} \Sigma\, \{(\mathrm{v}_s - 2\rho_s) \sin N + (\mathrm{v}_c - 2\rho_c) \cos N\},$$

where we put for brevity

$$\mathrm{v}_s = \quad M\nu v_c,$$
$$\mathrm{v}_c = - M\nu v_s.$$

The numerical expression for r_1^{-2} is given on page 40, and by multiplying the quantities within brackets by this expression, after the manner explained on pages 40 and 41, we form the terms of $\frac{d\delta v}{dt'}$. Multiplying each of these terms by its corresponding value of ν, changing *cos* to *sin* and *sin* to *cos*, we have the coefficients in the expressions for δv given on page 50.

As previously mentioned, before commencing the above computation, I had computed all the perturbations of Uranus by the method of "perturbations of the elements," using the formulæ developed in my Investigation of the Orbit of Neptune. The two results are here placed side by side, for the purpose of comparison. The discrepancies in the various coefficients, expressed in thousandths of a second, are shown in the sixth and seventh columns.

It will be seen that the largest discrepancies, and indeed the only ones (with a single exception) exceeding one-tenth of a second, occur in the coefficients of the terms $2g' - l$ aug $3g' - l$. Here the errors are almost certainly in the computation from perturbations of the elements. Owing to the long period of the term $3g' - l$ they would not become sensible in the course of any one century.

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PERTURBATIONS OF THE LONGITUDE OF URANUS PRODUCED BY THE ACTION OF SATURN, AND DEPENDING ON THE FIRST POWER OF THE DISTURBING FORCES.

	From comp. preceding δυ		From pert. of elements δυ		Discrepancy.		0.434294 δρ	
g, l′	sin	cos	sin	cos	sin	cos	cos	sin
	″	″	″	″	″	″		
0, 0	......	+ 10.9690t	......	+10.9645t	......	.0045t		
1,	— 1.230 nt	+ 12.231 nt	—1.228 nt	+12.271 nt	.002 nt	.040 nt	+13 nt	+128nt
2,	— 0.072 nt	+ 0.717 nt	—0.072 nt	+ 0.720 nt	0	.003 nt	+1 nt	+9nt
3,	— 0.004 nt	+ 0·043 nt	—0.004 nt	+ 0.043 nt	0	0	0	0
0, 0	......	......	......	......	....	....	—1541	
1,	+ 8.545	— 4.735	+ 2.844	+ 1.013	....	....	— 80	— 6
2,	+ 0.461	— 0.169	+ 0.133	+ 0.166	....	....	— 7	0
3,	+ 0.028	— 0.005	+ 0.013	+ 0.014	....	....	0	0
—1,—1	+ 0.036	+ 0.039	+ 0.032	+ 0.005	4	34	0	— 1
0,	+ 1.282	+ 0.718	+ 1.280	+ 0.719	2	1	— 5	— 13
1,	—20.817	+ 8.522	— 20.873	+ 8.595	56	73	+761	— 106
2,	—11.890	+143.463	— 11.093	+143.465	797	2	— 45	—1351
3,	+ 49.30	+115.86	+ 49.62	+116.08	320	220	—103	+ 280
4,	+ 2.133	+ 5.616	+ 2.195	+ 5.621	62	5	— 24	+ 61
5,	+ 0.126	+ 0.329	+ 0.109	+ 0.331	17	2	— 2	+ 4
0,—2	+ 0.017	— 0.017	+ 0.025	— 0.033	8	16	0	0
1,	+ 0.042	+ 0.814	+ 0.034	+ 0.818	8	4	+ 7	+ 16
2,	+ 4.110	— 0.009	+ 4.103	— 0.012	7	3	+ 89	+ 2
3,	+ 2.079	— 1.607	+ 2.106	— 1 676	27	69	+ 38	+ 30
4,	+ 0.648	— 1.830	+ 0.643	— 1.902	5	72	+ 9	+ 27
5,	+ 1.163	+ 2.956	+ 1.274	+ 2.991	111	35	+ 9	— 24
6,	+ 0.503	+ 0.378	+ 0.556	+ 0.445	53	67	— 2	+ 2
1,—3	+ 0.034	+ 0.012	+ 0.036	+ 0.015	2	3	0	0
2,	+ 0.037	— 0.041	+ 0.014	— 0.050	23	9	+ 1	+ 1
3,	+ 0.824	— 0.019	+ 0.812	— 0.017	12	2	+ 19	0
4,	+ 0.355	— 0.267	+ 0.351	— 0.263	4	4	+ 8	+ 6
5,	+ 0.047	— 0.165	+ 0.039	— 0.191	8	26	+ 1	+ 3
6,	— 0.026	— 0.063	— 0.028	— 0.066	2	3	0	+ 1
7,	— 0.053	— 0.032	— 0.053	— 0.018	0	14	— 1	0
3,—4	+ 0.006	— 0.022	— 0.005	— 0.023	11	1	0	+ 1
4,	+ 0.228	— 0.008	+ 0.221	+ 0.002	7	10	+ 5	0
5,	+ 0.103	— 0.077	+ 0.084	— 0.075	19	2	+ 2	+ 2
6,	+ 0.013	— 0.044	+ 0.013	— 0.057	0	13	0	+ 1
7,	— 0.005	— 0.013	— 0.001	— 0.015	4	2	0	0
8,	— 0.003	— 0.002	0	+ 0.002	3	4	0	0
5,—5	+ 0.074	— 0.003	+ 0.071	0	3	3	+ 2	0
6,	+ 0.038	— 0.027	+ 0.023	— 0.026	15	1	+ 1	+ 1
7,	+ 0.005	— 0.016	+ 0.005	— 0.025	0	9	0	0
8,	— 0.002	— 0.004	0	— 0.003	2	1	0	0

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Perturbations of Latitude.

These are computed from the formulæ (27) and (40), no reductions being made from δk and $\delta\eta$ to δp and δq, but the perturbations of the latitude being computed directly from the former by (40). We have only to represent the expressions for δk and $\delta\eta$ by

$$\begin{aligned} \delta k &= -\Sigma a_c \cos N - \Sigma a_s \sin N \\ \delta\eta &= \quad \Sigma a'_c \cos N + \Sigma a'_s \sin N \end{aligned}$$

and substitute ω for π in the equations (40) from which $\delta\beta$ is computed.

The principal steps of the computation are shown quite fully in the following table. The values of

$$\frac{\partial h}{\partial\gamma} = \frac{1}{2} \cos \tfrac{1}{2}\gamma \frac{\partial h}{\partial\sigma}$$

are first formed from those terms of h, on pages 37 and 38, which contain σ as a coefficient. Then, having for each original term of R

$$\frac{\partial R}{\partial\gamma} = \frac{m'}{a_1} \frac{\partial h}{\partial\gamma} \cos N$$

all the terms which have the same coefficients of λ and λ' in N are combined into two depending on g and l' as shown in the case of R on page 36. The coefficients of these terms, in units of the third place of decimals, are given in the columns headed $\frac{\partial R}{\partial\gamma}$.

The value of $\frac{\iota h}{2\sigma}$ sin N being formed for each term of R, all the terms depending on the same multiples of λ and λ' are combined into two, of which the coefficients are given under the proper heading. The terms of $(i+j)\,\sigma h$ sin N being formed in like manner, we have, by adding the last two expressions, all the quantities which enter into the formulæ (27). To integrate these equations thus forming the numerical values of δk and $\delta\eta$ we have only to multiply each term in the second, third, eighth, and ninth columns of the table by the corresponding values of $\frac{m'a\nu}{a_1 \cos\psi}$, for which we may use the value of $\frac{M\nu}{\sin 1''}$ already given.

The quantities given in the four columns under δk and $-\delta\eta$ show the values of $-a_s$, $-a_c$, $-a'_c$, $-a'_s$, corresponding to each argument. From these the terms of $\delta\beta$ are formed by equation (40) with the modification mentioned above.

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PERTURBATIONS OF THE LATITUDE PRODUCED BY SATURN.

g l′	$\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \times$		$\Sigma \frac{\iota h}{2\sigma} \sin N$		$\Sigma (i+j)\sigma h \sin N$		$\frac{dn}{dt} = Mn \times$		δk		$-\delta\eta$		$\delta\beta$	
i i′	cos	sin	sin	cos	sin	cos	sin	cos	sin ″	cos ″	cos ″	sin ″	sin ″	cos ″
0, 0	—10.82	0	0	0	0	+0.008	0.00	+ .008						+0.192
1,	— 3.45	— 2.92	—1.94	— 0.11	+ 0.17	+0.84	— 1.77	+ 0.73	+0.203	—0.172	—0.104	—0.043	+0.246	+0.209
2,	+ 1.42	—11.10	+1.30	+10.73	+ 0.07	+0.05	+ 1.37	+10.78	—0.042	—0.326	+0.042	—0.317	—0.049	+0.015
3,	+ 1.31	— 1.71	+1.31	+ 1.71	+ 0.01	0.00	+ 1.32	+ 1.71	—0.026	—0.033	+0.026	—0.033	—0.008	+0.003
—2,—1	+ 1.0	+ 0.1	—1.0	— 0.1	0.00	0.00	— 1.0	— 0.1	+0.012	—0.001	+0.012	—0.001	0.000	0.000
—1,	— 6.43	—52.6	+6.4	—52.8	+ 0.03	+0.02	+ 6.4	—52.8	—0.098	+0.804	—0.098	—0.804	—0.004	+0.018
0,	— 6.97	+ 7.06	+1.53	+ 5.14	— 4.91	0.00	— 3.38	+ 5.14	—0.144	—0.145	+0.070	+0.106	—0.661	+0.576
+1,	+46.67	0	0.00	0.00	+59.45	—0.02	+59.45	— 0.02	+1.482	0	—1.887	—0.001	+0.084	—0.026
2,	— 1.84	— 2.86	—0.99	— 0.20	+ 0.93	+1.22	— 0.06	+ 1.02	—0.127	+0.197	+0.004	+0.070	+2.218	+1.920
3,	+ 0.90	— 6.98	+0.78	+ 6.44	+ 0.12	+0.16	+ 0.90	+ 6.60	—0.358	—2 781	+0.358	—2.630	+0.091	—0.071
4,	+ 1.09	— 1.38	+1.09	+ 1.38	+ 0.02	+0.01	+ 1.11	+ 1.39	—0.056	—0.071	+0.057	—0.071	—0.048	+0.056
—1,—2	+ 7.6	— 1.5	—7.6	+ 1.5	0.00	0.00	— 7.6	+ 1.5	+0.067	+0.013	+0.067	+0.013	+0.003	+0.004
0,	+ 1.3	+10.9	—1.3	+10.7	— 0.11	+0.53	— 1.4	+11.2	+0.013	—0.112	+0.014	+0.116	+0.008	+0.034
+1,	— 0.27	+ 9.03	+1.70	+ 0.91	+ 1.65	—7.35	+ 3.35	— 6.44	—0.003	—0.112	—0.042	—0.080	+0.040	—0.042
2,	—14.51	0	0.00	0.00	— 7.01	—0.21	— 7.01	— 0.21	—0.230	0	+0.112	—0.003	0	—0.029
3,	— 2.85	— 2.29	—0.42	— 0.19	— 0.97	+1.11	— 1.39	+ 0.92	—0.062	+0.050	+0.030	+0.020	—0.063	—0.054
4,	+ 0.43	— 4.23	+0.44	+ 3.67	+ 0.02	+0.22	+ 0.46	+ 3.89	+0.015	+0.146	—0.016	+0.134	—0.080	—0.014
5,	+ 0.80	— 0.99	+0.80	+ 0.99	+ 0.02	+0.02	+ 0.82	+ 1.01	+0.067	+0.083	—0.068	+0.084	+0.008	—0.004
2,—3	— 1.17	+ 0.86	+1.22	+ 0.97	+ 0.09	+0.33	+ 1.31	+ 1.30	—0.010	—0.008	—0.011	+0.012	+0.050	—0.046
3,	— 8.54	0	0.00	0.00	— 4.34	0	— 4.34	0.00	—0.090	0	+0.046	0	+0.004	—0.010
4,	— 2.08	— 1.65	—0.13	— 0.15	— 0.88	+0.85	— 1.01	+ 0.70	—0.026	+0.022	+0.013	+0.009	—0.012	—0.012
5,	+ 0.14	— 2.50	+0.24	+ 2.00	0.00	+0.22	+ 0.24	+ 2.22	+0.002	+0.041	—0.004	+0.037	—0.010	—0.002
6,	+ 0.55	— 0.66	+0.55	+ 0.66	+ 0.02	+0.02	+ 0.57	+ 0.68	+0.013	+0.015	—0.013	+0.016	+0.004	—0.001
3,—4	— 0.72	+ 0.47	+0.82	+ 0.78	+ 0.09	+0.32	+ 0.91	+ 1.10	—0.005	—0.003	—0.006	+0.008	+0.022	—0.020
4,	— 4.80	0	0.00	0.00	— 2.48	—0.02	— 2.48	— 0.02	—0.038	0	+0.020	0	+0.002	—0.005
5,	— 1.41	— 1.09	—0.01	— 0.10	— 0.66	+0.59	— 0.67	+ 0.49	—0.013	+0.010	+0.006	+0.004	—0.004	—0.003
6,	+ 0.12	— 1.3	+0.12	+ 1.0	— 0.02	+0.19	+ 0.10	+ 1.2	+0.001	+0.014	—0.001	+0.013	+0.004	+0.002
5,—5	— 2.64	0	0	0	— 1.35	+0.04	— 1.35	+ 0.04	—0.017	0	+0.008	0		

$$\text{Secular terms} \begin{cases} \delta k = + 4''.77\ T \\ \delta\beta = - 4''.77\ T \cos v. \end{cases}$$

CHAPTER III.

PERTURBATIONS PRODUCED BY NEPTUNE AND JUPITER.

The perturbations of Uranus by Neptune were originally computed with elements of both planets quite different from those finally adopted. But the last computations, on which the concluded values of the perturbations depend, were made with the concluded elements of Neptune found in my investigation of the orbit of that planet.[1] They are as follows:

	°	′	″
π,	43	17	30
θ,	130	7	33
ϵ,	335	5	39
ϕ,	1	47	1.6
n,			7864.935
e,	0.0084962		
log a,	1.478141		
Mass,	$\frac{1}{17000}$		

Hence follow the following functions of the elements of Neptune and Uranus:

$$\alpha = 0.638195$$
$$\omega = 12^\circ \; 44' \; 58''$$
$$\omega' = 247 \; 45 \; 20$$
$$\gamma = 1 \; 30 \; 29.6$$
$$\sigma = \sin \tfrac{1}{2}\gamma = 0.013161$$
$$M = 37.522 \text{ (in units of 6th place of decimals).}$$

From these values of the elements are obtained the following values of the various terms in the development of the perturbative function, and of ν. As the developments have been formed on the same principle as in the case of Saturn, it is deemed unnecessary to give the details of the process. It is only necessary to remark that the indices i' and i are the coefficients of l' and g respectively, the mean longitude of Neptune, or l', being counted from the perihelion of Uranus.

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[1] Smithsonian Contributions to Knowledge, Vol. XV.

ACTION OF NEPTUNE.

	$R=\frac{m'}{a_1}\times$		$\frac{\partial R}{\partial v}=\frac{m'}{a_1}\times$		$\frac{\partial R}{\partial \rho}=\frac{m'}{a_1}\times$		$Q=\frac{m'}{a_1}\times$		ν
i' i	h_c	h_s	v_s	v_c	cos	sin	k_c	k_s	
0, 0	+1135.63	0	0	+ 0.202	+368.26	0	+368.26	0	
+ 1	— 18.07	— 1.25	+ 1.01	— 1.23	— 64.80	— 5.89	—100.94	— 8.39	+ 1 0
+ 2	+ 0.41	— 0.03	— 0.6	— 0.1	+ 2.98	+ 0.12	+ 3.82	+ 0.06	+ 0.5
+ 3	0	0	0	0	0	0	0	0	+ 0.33333
1,— 3	+ 0.21	— 0.01	+ 0.7	— 0.2	+ 1.6	+ 0.2	+ 2.1	+ 0.1	— 0.40158
— 2	— 5.21	— 0.62	— 4.90	+ 1.27	— 29 5	+ 0.6	— 43.5	— 1.1	— 0.67107
— 1	+ 134.19	+ 0.30	+133.98	— 0.08	+509.94	+ 0.77	+1057.49	+ 1.99	— 2.04023
0	— 25.45	—10 43	— 18.05	+ 0.02	— 84 57	—11.47	— 84.57	— 11.47	+ 1.96137
+ 1	+ 1.12	+ 1.19	+ 0.5	— 0.1	+ 5.83	+ 0.82	+ 7.31	+ 1.07	+ 0.66232
+ 2	— 0.05	+ 0.01	+ 0.02	+ 0.02	— 0.24	0	— 0.32	+ 0.02	+ 0.39843
2,— 4	+ 0.85	— 0.03	+ 2.1	0	+ 2.5	+ 0.1	+ 4.8	0	— 0.33554
— 3	+ 13.53	— 1.01	+ 27.7	+ 3.1	+ 18.7	— 2.4	+ 59.7	— 5.5	— 0.50497
— 2	+ 375.92	+ 0.36	+751.59	— 0.76	+9 7.82	+ 1.25	+2491.69	+ 2.72	— 1.02009
— 1	— 60.090	— 3.180	—117 903	+ 3.203	—171.79	—13.13	+5931.21	+309.82	+50.7820
0	+ 3.31	+ 0 46	+ 5 9	— 0.4	+ 12.24	+ 1.84	+ 12.24	+ 1.84	+ 0.98069
+ 1	— 0.09	— 0.01	— 0.09	+ 0.02	— 0.51	— 0.08	— 0.60	— 0.09	+ 0.49512
3,— 5	+ 0.83	— 0.04	+ 2 7	+ 0.2	+ 3.0	— 0.1	+ 5.5	— 0.2	— 0.28814
— 4	+ 12.09	— 0.95	+ 36.9	+ 3.8	+ 34.5	— 3.6	+ 73.6	— 6.7	— 0.40478
— 3	+ 200.37	— 0.04	+601.13	— 0.42	+717.94	+ 0.81	+1535.51	+ 0.65	— 0.68006
— 2	— 52 33	— 9.88	—150.02	+19.81	—190.33	—27.51	—635.23	—111.69	— 2.12559
— 1	+ 5.05	+ 1.56	+ 13.8	— 3.1	+ 20.12	+ 4.65	+ 1.0	— 1.2	+ 1.88844
0	— 0.24	— 0.07	— 0.58	+ 0.15	— 1 11	— 0.30	— 1.11	— 0.30	+ 0.65378
4,— 6	+ 0.75	— 0.07	+ 3.2	+ 0.3	+ 3.4	— 0.2	+ 5.7	— 0.4	— 0.25249
— 5	+ 9.47	— 0.79	+ 38.3	+ 4.0	+ 38.5	— 3.9	+ 70.4	— 6.6	— 0.33777
— 4	+ 111.16	— 0.12	+444.7	— 0.18	+512.22	+ 0.35	+965.7	— 0.1	— 0.51004
— 3	— 38.58	— 7.43	—149.10	+22.37	—178.15	—27 95	—419.12	— 74.36	— 1.04100
— 2	+ 5.205	+ 1.896	+ 19.54	— 5.50	+ 25.30	+ 7.19	—503.3	—185.4	+25.3910
— 1	— 0.36	— 0.17	— 1.28	+ 1.49	— 1.90	— 0.70	— 1.21	— 0.37	+ 0.96210
5,— 7	+ 0.65	— 0.07	+ 3.5	+ 0.4	+ 3.5	— 0.3	+ 5.5	— 0.5	— 0.22468
— 6	+ 6.97	— 0.59	+ 35.1	+ 3.4	+ 36.7	— 3.4	+ 60.9	— 5.5	— 0.28984
— 5	+ 63.03	— 0.16	+315.3	+ 0.1	+354.8	— 0.1	+612.0	— 0.7	— 0.40804
— 4	— 27.43	— 5.32	—133.4	+21.4	—153.6	—25.3	—304.8	— 54.6	— 0.68929
— 3	+ 4.84	+ 1.81	+ 23.0	— 7.0	+ 27.82	+ 8.57	+ 92.2	+ 32.7	— 2.21843
— 2	— 0.45	— 0.24	— 2.09	+ 0.90	— 2.77	— 1.14	+ 0.48	+ 0.57	+ 1.82073
— 1	+ 0.03	+ 0.02	+ 0.12	— 0.06	+ 0.18	+ 0.09	+ 0.15	+ 0.07	+ 0.6455
6,— 7	+ 4.93	— 0.41	+ 29 8	+ 2.7	+ 31.5	— 2.9	+ 49.0	— 4.4	— 0.2538
— 6	+ 36.16	— 0.17	+217.1	+ 0.4	+239.9	— 0.4	+387.4	— 1.1	— 0.3400
— 5	— 19.04	— 3 74	—111.6	+18.8	—125.7	—21.4	—223.8	— 40.6	— 0.5152
— 4	+ 4.14	+ 1.59	+ 23.9	— 7.8	+ 27.76	+ 9.06	+ 63.0	+ 22.6	— 1.0628
— 3	— 0.490	— 0.271	— 2.79	+ 1.30	— 3.42	— 1.55	+ 46.40	+ 26.01	+16.9273
— 2	+ 0.036	+ 0.026	+ 0.20	— 0.12	+ 0.28	+ 0.16	+ 0.14	+ 0.06	+ 0.9443
7,— 8	+ 3.41	— 0.28	+ 24.0	+ 2.1	+ 25.5	— 2.5	+ 37.8	— 3.5	— 0.2257
— 7	+ 20.90	— 0.16	+146.4	+ 0.4	+160 0	— 0.6	+245.3	— 1.3	— 0.2914
— 6	— 12.99	— 2.54	— 89 2	+15.4	— 98.9	—17.2	—163.1	— 29.8	— 0.4113
— 5	+ 3 40	+ 1.31	+ 23.0	— 7.7	+ 26.0	+ 8.8	+ 49.8	+ 18.0	— 0.6988
— 4	— 0.487	— 0.276	— 3.25	+ 1.61	— 3.79	— 1.87	— 12.83	— 6.99	— 2.3198
— 3	+ 0.044	+ 0.034	+ 0.28	— 0.19	+ 0.37	+ 0.24	— 0.09	— 0.12	+ 1.7577
8,— 9	+ 2.35	— 0.20	+ 18.8	+ 1.6	+ 19.5	— 2.1	+ 28.1	— 1.2	— 0.2032
— 8	+ 12.20	— 0.13	+ 96.6	+ 0 3	+105.0	— 0.7	+154.8	— 1.3	— 0.2550
— 7	— 8.8	— 1.71	— 68.9	+12.1	— 75.5	—13.1	—117.7	— 21.3	— 0.3423
— 6	+ 2.69	+ 1.05	+ 21.0	— 7.1	+ 23.2	+ 8.1	+ 40.0	+ 14.7	— 0.5205
— 5	— 0 45	— 0.26	— 3.5	+ 1.8	— 3.90	— 1.99	— 8.8	— 4.8	— 1.0855
— 4	+ 0.049	+ 0.039	+ 0.37	— 0.26	+ 0.45	+ 0.31	— 4.50	— 3.69	+12.6955
9,— 9	+ 7.0	— 0.1	+ 63.4	+ 0.1	+ 68.2	— 0.6	+ 96.8	— 1.0	— 0.2267
— 8	— 5 9	— 1.13	— 51 9	+ 9.1	— 56 3	— 9.9	— 84.0	— 15.2	— 0.2931
— 7	+ 2.07	+ 0.81	+ 18.1	— 6.4	+ 19 8	+ 7.0	+ 31.8	+ 11.7	— 0.4147
— 6	— 0.39	— 0.25	— 3.4	+ 1.9	— 3.78	— 2.02	— 7.1	— 4.1	— 0.7085
— 5	+ 0.050	+ 0.042	+ 0.43	— 0.32	+ 0.51	+ 0.36	+ 1.72	+ 1.38	— 2.4308
10,—10	+ 4.1	— 0.1	+ 43.	0	+ 44.3	— 0.5	+ 61.	— 0.8	— 0.2040
— 9	— 3.9	— 0.8	— 40.	+ 6.	— 42.0	— 7.0	— 60.	— 11.	— 0.2563
— 8	+ 1.6	+ 0.6	+ 14.7	— 5.6	+ 16.1	+ 6.0	+ 25.1	+ 9.3	— 0.3446
— 7	— 0.3	— 0.2	— 3.1	+ 1.9	— 3.5	— 1.9	— 5.8	— 3.4	— 0.5259
— 6	+ 0.05	+ 0.04	+ 0.46	— 0.37	+ 0.53	+ 0.40	+ 1.16	+ 0.96	— 1.1092

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The term of Long Period.

From the expressions for the perturbations of Uranus, subsequently given, it will be seen that several of the terms have very large coefficients, that of $\sin(2l'-g)$ being nearly an entire degree. The magnitude of most of the terms in which i' is even arises from the near approach to commensurability in the mean motions of the two planets. Twice the mean annual motion of Neptune exceeds that of Uranus by only 303″.8. The elements of the orbits of both planets will therefore, in consequence of their mutual action, be affected with a slow oscillation, having a period of about 4266 years. The employment of these large terms and the great inconveniences to which they will give rise, especially in the corrections of the elements of Uranus, may be avoided by the device employed in the theory of Neptune. The following are the essential features of this method:

First, all the perturbations arising from that portion of the perturbative function in which the coefficient of the time is $2n'-n$ or its multiples are considered and developed as perturbations of the elements.

Secondly, the arbitrary constants to be added to the integrals of these perturbations are so taken that the perturbations shall vanish at the epoch 1850.0.

In other words, the perturbations in question will be treated as producing secular variations of the elements of the orbit, only, instead of being developed in powers of the time, these variations will be retained in their rigorous form.

The formulæ for the computations of the perturbations in question, are as follows:

Let

$$\frac{m'}{a_1} h \cos(i'l' + il + j'\omega' + j\omega) = \frac{m'}{a_1} \cos N$$

be any term of the perturbative function, h being a function of α, e, and σ.

$$\begin{aligned} \sin\psi &= e \\ g &= \cos\psi \tan\tfrac{1}{2}\psi \\ \iota &= -(i' + i + j' + j) \\ \nu &= \frac{n}{i'n' + in}. \end{aligned}$$

For each such term, compute

$$\begin{aligned} A &= 2ih \\ L &= -3i\nu h - 2\frac{\partial h}{\partial \nu} + \frac{\partial h}{\partial e} \\ eW &= \cos\psi \frac{\partial h}{\partial e} \\ E &= -h(ig + j\cot\psi) \\ T &= \tfrac{1}{2}\frac{\partial h}{\partial \sigma} \\ I &= \tfrac{1}{2}\iota\frac{h}{\sigma} + (i+j)\sigma h. \end{aligned}$$

The corresponding perturbations of the elements may then be put into the form

$$\begin{aligned}
\delta \log a &= M\nu A \cos N + \delta n_0,\\
\delta l &= M\nu L \sin N + \delta l_0,\\
e\delta\pi &= M\nu e W \sin N + e\delta\pi_0,\\
\delta e &= M\nu E \cos N + \delta e_0,\\
\delta\gamma &= M\nu I \cos N + \delta\gamma_0,\\
\tan\gamma\, \delta\tau &= M\nu T \sin N + \tan\gamma\delta\tau_0.
\end{aligned}$$

Here, δn_0, δl_0, etc., are arbitrary constants so taken that $\delta \log a$, δl, etc., shall vanish at the fundamental epoch.

All the terms depending on the same values of i' and i are to be combined into a single one. And it will save labor to make this combination at as early a stage as possible in the computation; that is, to multiply the various values of h, $\frac{\partial h}{\partial n}$, $\frac{\partial h}{\partial e}$, $\frac{\partial h}{\partial \sigma}$, and E by the sines and cosines of $j'\omega' + j\omega$, and afterward proceed with the sums of the products according to the proper modification of the formulæ.

Thus are obtained the following long period perturbations of the elements of Uranus:

$$\begin{aligned}
\delta l = &- \overset{''}{3474.32} \sin(2l' - g) + \overset{''}{180.10} \cos(2l' - g)\\
&+ 146.72 \sin(4l' - 2g) - 54.10 \cos(4l' - 2g)\\
&- 8.97 \sin(6l' - 3g) + 5.03 \cos(6l' - 3g)\\
&+ 0.64 \sin(8l' - 4g) - 0.53 \cos(8l' - 4g)\\
&+ \text{constant} = 3320''.18.
\end{aligned}$$

$$\begin{aligned}
e\delta\pi = &- \overset{''}{484.96} \sin(2l' - g) + \overset{''}{0.73} \cos(2l' - g)\\
&+ 38.06 \sin(4l' - 2g) - 7.06 \cos(4l' - 2g)\\
&- 3.61 \sin(6l' - 3g) + 1.38 \cos(6l' - 3g)\\
&+ 0.33 \sin(8l' - 4g) - 0.15 \cos(8l' - 4g)\\
&+ \text{constant} = 465''.23.
\end{aligned}$$

$$\begin{aligned}
= &- \overset{''}{484.21} \cos(2l' - g) - \overset{''}{0.29} \sin(2l' - g)\\
&+ 38.21 \cos(4l' - 2g) + 7.16 \sin(4l' - 2g)\\
&- 3.61 \cos(6l' - 3g) - 1.40 \sin(6l' - 3g)\\
&+ 0.33 \cos(8l' - 4g) + 0.15 \sin(8l' - 4g)\\
&- \text{constant} = 158''.59.
\end{aligned}$$

$$\begin{aligned}
\delta n = &+ 2277 \cos(2l' - g) + 120 \sin(2l' - g)\\
&- 198 \cos(4l' - 2g) - 78 \sin(4l' - 2g)\\
&+ 18 \cos(6l' - 3g) + 7 \sin(6l' - 3g)\\
&+ \text{constant} = 630.
\end{aligned}$$

The variations of the elements which fix the position of the plane of the orbit are here omitted, because their nature is such that it is indifferent in which form they are developed.

These expressions are reduced to perturbations of the co-ordinates by the following formulæ. Express the usual developments of the longitude and logarithm of radius vector in the form

$$v = l + \Sigma\, V_i \sin ig;$$
$$\rho = \varpi + \Sigma\, R_i \cos ig.$$

Put also

$$V'_i = \frac{\partial V_i}{\partial e}$$

$$V''_i = \frac{i}{e} V_i$$

$$R'_i = \frac{\partial R_i}{\partial e}$$

$$R''_i = -\frac{i}{e} R_i.$$

Express any set of corresponding terms of the preceding perturbations in the form

$$\begin{aligned} \delta l &= L_s \sin N + L_c \cos N; \\ e\delta l - e\delta\pi &= F_s \sin N + F_c \cos N; \\ \delta e &= E_c \cos N + E_s \sin N; \\ \delta\varpi &= A_s \cos N + A_s \sin N. \end{aligned}$$

We shall then have

$$\begin{aligned} 2\delta v = {} & \Sigma\, (V''_i F_s + V'_i E_c) \sin (N + ig) + \Sigma\, (V''_i F_s - V'_i E_c) \sin (N - ig) \\ & + \Sigma\, (V''_i F_c - V'_i E_s) \cos (N + ig) + \Sigma\, (V''_i F_c + V'_i E_s) \cos (N - ig) \\ & + 2\,\delta l \end{aligned}$$

$$\begin{aligned} 2\delta\rho = {} & \Sigma\, (R'_i E_c - R''_i F_s) \cos (N + ig) + \Sigma\, (R'_i E_c + R''_i F_s) \cos (N - ig) \\ & + \Sigma\, (R'_i E_s + R''_i F_c) \sin (N + ig) + \Sigma\, (R'_i E_s - R''_i F_c) \sin (N - ig) \\ & + 2\delta\varpi \end{aligned}$$

The numerical values of V', V', R', and R'' are as follows:

$$\begin{array}{llll} V'_1 = & 1.99835 & V''_1 = & 1.99945 \\ V'_2 = & 0.11713 & V''_2 = & 0.11722 \\ V'_3 = & 0.00714 & V''_3 = & 0.00714 \\ V'_4 = & 0.00044 & V''_4 = & 0.00044 \\ R'_0 = & +0.02348 & & \\ R'_1 = & -0.99753 & R''_1 = & +0.99917 \\ R'_2 = & -0.07020 & R''_2 = & +0.07030 \\ R'_3 = & -0.00466 & R''_3 = & +0.00467 \end{array}$$

The final results of the entire computations are given in the following table: In the columns δv_1, we have the complete perturbations of the longitude computed by the direct method from the values of $\frac{\partial R}{\partial v}$, $\frac{\partial R}{\partial \rho}$, Q, etc., already given. Next we have, under the caption δv_2, the perturbations of the true longitude deduced from the long period perturbations of the elements, as set forth in the last paragraph, omitting the constants added to the perturbations. Under δv_3 we have the perturbations of the longitude deduced from all the remaining terms of the perturbations of the elements. The sum of the columns δv_2 and δv_3 shows the entire perturbations computed by the method of variation of elements. Thus, in δv_1 and $\delta v_2 + \delta v_3$ we have two complete sets of perturbations computed by methods entirely independent. The differences of the results, expressed in thousandths of a second, are given in the last two columns of the table.

This comparison gives rise to remarks similar to those suggested by the perturbations of Saturn computed by the same methods. The only terms in which the difference of results amounts to as much as one-tenth of a second are those of very long period, and those very nearly the period of Uranus, where a more accurate value is not at present of great importance, because the error will be compensated by the corrections of the element during several centuries.

Perturbations of the Longitude of Uranus produced by Neptune.

	δv_1		δv_2		δv_3		Discrepancy.	
l' g	sin	cos	sin	cos	sin	cos	sin	cos
	″	″						
0, 0		—0.4262 *t*						
—1	— .065 *nt*	—1.181 *nt*						
—2	— .004 *nt*	—0.069 *nt*						
0, 0								
—1	+ 0.697	— 0.088						
—2	+ 0.046	— 0.005						
—3	+ 0.003							
1,—3	+ 0.147	— 0.001	....	....	+ 0.146	—0.002	1	1
—2	+ 2.509	— 0.019	....	....	+ 2.509	—0.010	0	9
—1	+ 39.658	— 0.080	....	....	+39.673	—0.081	15	1
0	+ 4.257	— 0.511	....	....	+ 4 249	—0.478	8	33
—1	+ 0.280	— 0.032	....	....	+ 0.275	—0.032	5	0
—2	+ 0.017	— 0.002						
2,—4	+ 2.978	+ 0.027	+ 2.89	+ 0.03	+ 0.098	—0.002	10	1
—3	+ 49.015	+ 0.413	+ 47.22	+ 0.44	+ 1.797	—0.021	2	6
—2	+ 840.93	+ 7.388	+ 805.64	+ 7.43	+35.355	—0.028	65	14
—1	—3475.4	+ 180.36	—3474.32	+180.10	— 0.700	+0.095	380	165
0	— 162.07	+ 8.01	— 161.96	+ 8.01	— 0.067	+0.015	43	15
—1	— 9.447	+ 0.468	— 9.50	+ 0.47				
3,—5	— 0.076	0.000	....	....	— 0.077	—0.003	1	3
—4	— 1.162	0.000	....	....	— 1.153	—0.011	9	11
—3	— 17.286	+ 0.228	....	....	—17.285	+0.229	1	1
—2	— 22.085	+ 4.037	....	....	—22.077	+4.020	8	17
—1	— 0.673	+ 0.082	....	....	— 0.682	+0.079	9	3
0	— 0.037	+ 0.006						
4,—6	— 0.036	+ 0.002	— 0.015	+ 0.002	— 0.027	—0.003	6	3
—5	— 0.558	+ 0.037	— 0.25	+ 0.04	— 0.315	—0.007	7	4
—4	— 7.968	+ 0.750	— 4.08	+ 0.68	— 3.908	+0.059	20	11
—3	— 75.00	+ 12.832	— 69.55	+11.67	— 5.733	+1.067	283	95
—2	+ 146.78	— 54.218	+146.72	—54.10	+ 0.126	—0.079	66	39
—1	+ 6.960	— 2.579	+ 6.81	— 2.63				

Perturbations of the Longitude—*Continued.*

	δv_1		δv_2		δv_3		Discrepancy.	
l' g	sin	cos	sin	cos	sin	cos	sin	cos
	″	″						
5,— 7	—0.009	0.000	....	....	—0.015	—0.002	6	2
— 6	—0.103	+0.006	....	....	—0.113	—0.004	10	10
— 5	—0.986	—0.042	....	....	—0.994	—0.042	8	0
— 4	+3.366	—0.670	....	....	+3.370	—0.662	4	8
— 3	+3.210	—1.169	....	....	+3.227	—1.186	17	17
— 2	+0.077	—0.017	....	....	+0.075	—0.002	2	15
— 1	+0.005	—0.001						
6,— 7	—0.050	—0.004	0.00	0.00	—0.054	—0.002	4	2
— 6	—0.387	—0.020	+0.02	—0.01	—0.423	—0.013	16	3
— 5	+1.261	—0.320	+0.40	—0.14	+0.855	—0.169	6	11
— 4	+7.781	—2.825	+6.80	—2.54	+0.857	—0.318	124	33
— 3	—9.025	+5.073	—8.97	+5.03	+0.017	—0.022	72	65
— 2	—0.437	+0.246	—0.42	+0.26				
7,— 8	—0.027	—0.002						
— 7	—0.186	—0.003	....	....	—0.189	—0.009	3	6
— 6	+0.272	—0.046	....	....	+0.260	—0.047	12	1
— 5	—0.571	+0.217	....	....	—0.538	+0.202	33	15
— 4	—0.459	+0.250	....	....	—0.419	+0.255	40	5
— 3	—0.010	+0.005						
8,— 9	—0.013	0.000						
— 8	—0.084	—0.002	....	....	—0.093	—0.007	9	5
— 7	+0.125	—0.022	....	....	+0.115	—0.024	10	2
— 6	—0.194	+0.082	—0.038	+0.017	—0.145	+0.055	11	10
— 5	—0.839	+0.463	—0.66	+0.30	—0.110	+0.067	69	96
— 4	+0.647	—0.501	+0.64	—0.53				
9,— 9	—0.040	—0.001	....	....	—0.048	—0.005	8	4
— 8	+0.063	—0.011	....	....	+0.002	—0.011	61	0
— 7	—0.055	+0.020	....	....	—0.059	+0.025	4	5
— 6	+0.080	—0.049	....	....	+0.083	—0.054	3	5
— 5	+0.059	—0.050						
10—10	—0.020	0.000	....	....	—0.025	—0.005	5	5
— 9	+0.035	—0.005						
— 8	—0.027	—0.010						
— 7	+0.026	—0.017						
— 6	+0.092	—0.079						

The perturbations of the logarithms of the radius vector are given in a form similar to those of the longitude. Under $\delta\rho_1$ we have the complete perturbation. Under $\delta\rho_2$ the effect of the perturbations of long period. But under $\delta\rho_3$ we have only the difference between $\delta\rho_1$ and $\delta\rho_2$, it being deemed unnecessary to present in full the perturbations of the radius vector as computed by the other method. $\delta\rho_1$ being employed in computing δv_1 may, in fact, be regarded as completely checked by its affording a correct value of the latter.

In the last two columns $\delta\rho_3$ is reduced to common logarithms by multiplying the coefficients by the modulus 0.434294.

PERTURBATIONS OF THE LOGARITHM OF THE RADIUS VECTOR OF URANUS PRODUCED BY NEPTUNE.

l' g	$\delta\rho_1$		$\delta\rho_2$		$\delta\rho_3$		$M\delta\rho$	
	cos	sin	cos	sin	cos	sin	cos	sin
0, 0	+138	....	....	....	....	....	+ 60	0
—1								
—2								
1,—3	+ 4	0	....	....	....	....	+ 2	0
—2	+ 68	0	....	....	....	....	+ 30	0
—1	+523	+ 1	....	....	....	....	+227	0
0	— 68	— 6	....	....	....	....	— 30	— 3
+1	— 8	— 1	....	....	....	....	— 3	0
2,—4	+ 94	— 1	+92	— 1	+ 2	0	+ 1	0
—3	+1416	— 12	+ 1374	— 12	+ 42	0	+ 18	0
—2	+20025	—179	+19490	—180	+535	+ 1	+232	0
—1	+1663	+116	+ 1726	+120	— 63	— 4	— 27	— 2
0	+3912	+194	+ 3927	+194	— 15	0	— 7	0
3,—5	— 3	0	....	....	....	....	— 1	0
—4	— 38	0	....	....	....	....	— 17	0
—3	—527	— 6	....	....	....	....	—229	— 3
—2	—284	—54	....	....	....	....	—124	—23
—1	+ 16	+ 2	....	....	....	....	+ 7	+ 1
4,—6	— 1	0						
—5	— 19	— 1	— 8	....	— 11	— 1	— 5	0
—4	—254	— 22	— 118	— 20	—136	— 2	— 59	— 1
—3	—1759	—300	—1681	—282	— 78	—18	— 34	— 8
—2	—143	— 61	— 154	— 70	+ 11	+ 9	+ 5	+ 4
—1	—167	— 62	— 165	— 64	— 2	+ 2	— 1	+ 1
5,—6	— 4	0	....	....	....	....	— 2	0
—5	— 40	+ 1	....	....	....	....	— 17	0
—4	+103	+20	....	....	....	....	+ 45	+ 9
—3	+ 39	+15	....	....	....	....	+ 17	+ 6
6,—7	— 2	0	....	....	— 2	0	— 1	0
—6	— 17	+ 1	....	....	— 17	+ 1	— 7	0
—5	+ 42	+10	+ 10	+ 4	+ 32	+ 6	+ 14	+ 3
—4	+180	+65	+164	+ 61	+ 16	+ 4	+ 7	+ 2
—3	+ 13	+ 8	+ 14	+ 5	— 1	+ 3	0	+ 1
7,—7	— 8	0	....	....	....	....	— 3	0
—6	+11	+ 2	....	....	....	....	+ 5	+ 1
—5	—17	— 7	....	....	....	....	— 7	— 3
—4	— 5	— 3	....	....	....	....	— 2	— 1
8,—8	— 4	0	....	....	....	....	— 2	0
—7	+ 5	+ 1	....	....	....	....	+ 2	0
—6	— 7	— 3	....	....	....	....	— 3	— 1
—5	—19	—11	....	....	....	....	— 8	— 5
9,—9	— 2	0	....	....	....	....	— 1	0
—8	+ 3	0	....	....	....	....	+ 1	0
—7	— 2	— 1	....	....	....	....	— 1	0
—6	+ 2	+ 2	....	....	....	....	+ 1	+ 1

PERTURBATIONS OF THE LATITUDE PRODUCED BY NEPTUNE.

	$\frac{\partial R}{\partial \gamma} = \frac{m'}{a_1} \times$		$\Sigma \frac{ih}{2\sigma} \sin N$		$\Sigma (i+j)\sigma \times h \sin N$		$\frac{d\eta}{dt} = m'\alpha n \times$		δk		$\delta \eta$		$\delta \beta$	
i' i	cos	sin	sin	cos	sin	cos	sin	cos	sin	cos	cos	sin	sin	cos
									″	″	″	″	″	″
0 0	—21.65	0	0	0	0	0.00	0	0					0	—.012
1	+ 1.08	+2.73	— 4.00	—1.66	+ .01	—0.01	— 3.99	—1.67	— 0.008	+0.021	+0.030	—0.012	+.080	+.020
2	+19.32	—9.39	+19.32	+9.39	— .01	0	+19.31	+9.39	— 0.073	—0.036	—0.073	+0.036	+.016	—.002
1,—3	+14.58	+7.0	—14.7	+7.0	0	0	—14 7	+7.0	+0.045	—0.022	—0.045	—0.022	+.007	—.002
—2	— 1.16	—2.65	+ 4.04	—2.40	—0.07	+0.02	+ 3 97	—2.4	—0.006	+0.014	+0.020	+0.012	+.201	—.046
—1	—34.26	0	0	0	+1.76	0	+ 1.76	0 ...	—0.540	0	+0.028	0	—.008	—.037
0	+ 3.74	+2.32	— 2.80	—1.48	—0.24	0	— 3.04	—1.48	—0.057	+0.035	+0.046	—0.022	+.364	+.084
+1	+15.77	—7.87	+16.25	+7.75	0	0	+16.25	+7.75	—0.081	—0.040	—0.083	+0.040	+.060	+.007
2,—4	+10.52	+5.06	—10.6	+5.0	0	0	—10.6	+5.0	+0.027	—0.013	—0.027	—0.013	+.002	—.001
—3	— 2.60	—2.38	+ 3.65	—2.28	+0.36	+0.04	+ 4.01	—2.24	—0.010	+0.009	+0.016	+0.009	+.062	—.014
—2	—33.38	0	0	0	+9.88	—0.01	+ 9.88	—0.01	—0.264	0	+0.078	0	+.008	—.004
—1	+ 5.81	+2.04	— 2.12	—1 04	—1.53	+0.04	— 3.65	—1.00	[—2.283]	[+0.802]	[+1.431]	[—0.393]	+.326	+.075
0	+18.79	—9.52	+19.56	+9.32	+0.08	0	+19.64	+9.32	—0.143	—0.072	—0.049	+0.071	+.002	0
3,—5	+ 7.4	+3.6	— 7.4	+3.5	0	0	— 7.4	+3.5	+0.017	—0.008	—0.017	—0.008	+.002	0
—4	— 3.06	—2.00	+ 3.12	—2.0	+0.48	+0.05	+ 3.60	—2.0	—0.010	+0.006	+0.011	+0.006	+.023	—.004
—3	—24.4	0	0	0	+7.87	0	+ 7.87	0	—0.128	0	+0.041	0	—.004	+.041
—2	+ 6.58	+1.73	— 1.20	—0.54	—1.98	+0.26	— 3.18	—0.28	+0.108	—0 029	—0.053	+0.004	+.303	+.074
—1	+13.66	—7.28	+14.68	+7.00	+0.18	—0.04	—14.86	+6.96	—0.199	—0.106	—0.217	+0.101	—.090	—.026
0	— 1.51	+0.22	— 1.51	—0.22	0	0	— 1.51	—0.22	+0.008	+0.001	+0.008	—0.001	—.013	0
4,—5	— 2.96	—1.63	+ 2.53	—1.7	+0.5	0	+ 3.0	—1.7	—0.008	+0.004	+0.008	+0.004	+.022	—.004
—4	—17.4	0	0	0	+5.8	0	+ 5.8	0	—0.070	0	+0.023	0	—.008	+.006
—3	+ 6.34	+1.34	— 0.59	—0.22	—1.97	+0.30	— 2.56	+0.08	+0.051	—0.011	—0.020	—0.001	+.048	+.010
—2	+ 9.42	—5.37	+10 57	+5.04	+0.25	—0.07	+10.82	+4.97	[—1.852]	[—1 056]	[—2.127]	[+0.977]	—.047	0
—1	— 1.72	+0.32	— 1.72	—0.32	—0.02	+0.02	— 1.74	—0.30	+0.013	+0.002	+0.013	—0.002	—.001	0
5,—6	— 2.4	—1.40	+ 1.8	—1.4	+0.5	0	+ 2.3	—1.4	—0.005	+0.003	+0.005	+0.003	+.012	—.003
—5	—12.0	0	0	0	+4.0	0	+ 4.0	0	—0.038	0	+0.013	0	—.012	0
—4	+ 5.58	+1.01	— 0.22	—0.04	—1.75	+0.28	— 1.97	+0.24	+0.030	—0.005	—0.010	—0.001	—.109	—.028
—3	+ 6.25	—3.89	+ 7.43	+3.54	+0.30	—0.09	+ 7.73	+3.45	+0.107	+0.067	+0.131	—0.059	—.038	+.001
—2	— 1.66	+0.33	— 1.66	—0.33	—0.03	+0.01	— 1.69	—0.32	+0.024	+0.005	+0.025	—0.004	+.018	0
6,—6	— 8.4	0	0	0	+2.8	0	+ 2.80	0	—0.022	0	+0.007	0	—.010	+.002
—5	+ 4.63	+0.76	— 0.03	+0.04	—1.46	+0.24	— 1.49	+0.28	+0.019	—0.003	—0.006	—0.001	—.028	—.008
—4	+ 4.03	—2.79	+ 5.15	+2.45	+0.31	—0.10	+ 5.46	+2 35	+0.033	+0.022	+0.045	—0.019	—.010	0
—3	— 1.45	+0.31	— 1.45	—0.31	—0.04	+0.02	— 1.49	—0.29	[+0.190]	[+0.041]	[+0.195]	[—0.038]	+.015	0
7,—7	— 5.6	0	0	0	+1.9	0	+ 1.9	0	—0.012	0	+0.004	0	—.006	+.001
—6	+ 3.63	+0.61	0	0	—1.17	+0.20	— 1.17	+0.20	+0.012	—0.002	—0 004	—0.001	—.008	—,004
—5	+ 2.49	—2.00	+ 3.52	+1.68	+0.30	—0.10	+ 3.82	+1.58	+0.013	+0.011	+0.020	—0.008	+.014	0
—4	— 1.19	+0.27	— 1.19	—0.27	—0.04	+0.02	— 1.23	—0.25	—0.021	—0.005	—0.022	+0.004	+.002	0
8,—8	— 4.0	0	0	0	+1.3	0	+ 1.3	0	—0.008	0	+0.003	0	—.003	+.002
—7	+ 2.76	+0.50	0	0	—0.91	+0.16	— 0.91	+0.16	+0.007	—0.002	+0.002	0	—.003	—.002
—6	+ 1.49	—1.41	+ 2.38	+1.14	+0.27	—0.09	+ 2.65	+1.05	+0.006	+0.006	+0.011	—0.004	+.004	0
—5	— 0.97	+0.22	— 0.97	—0.22	0	+0.02	— 0.97	—0.20	—0.008	—0.002	—0.008	+0.002	+.002	0

Secular term, $\delta k = +1''.25\ T$
$\delta\beta = -1\,'.25\ T \cos v$

The terms δk and $\delta \eta$, which are inclosed in brackets, are of very long period, and are therefore omitted in forming the values of $\delta\beta$ in the last two columns

Perturbations produced by Jupiter.

The series in which these perturbations are expressed converge so rapidly that I deem it unnecessary to present the details of the computation. They have been computed by both methods, and the separate and independent results are given in the following table, where δv_1 represents the perturbations computed by the method developed in Chapter I, and δv_2 those computed by the method of variation of elements.

The apparently large discrepancy between the coefficients multiplied by the time arises from the circumstances that in the form of development the mean motion, and hence the mean anomaly, appears affected by the perturbation $31''.2t$. Accordingly when we enter the table which gives the true longitude in terms of the mean anomaly in the form

$$v = l + 2e \sin (l - \pi) + \text{etc.},$$

we may consider this quantity $31''.2t$ as a secular variation of $l - \pi$ producing in v the term

$$\delta v = 62''.4et \cos (l - \pi).$$

In δv_1 this term is left in its primitive form, while in δv_2 the value of l is supposed to include this term, and the secular terms are only those which arise from the secular variation of the eccentricity and perihelion.

It is also to be remarked that the terms which are independent of the mean longitude of Jupiter, or those in which $i' = 0$, are not comparable, as they correspond to slightly different elliptic elements in the two theories.

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PERTURBATIONS OF URANUS BY JUPITER.

	δv_1		δv_2		Diff.		$\cos \psi\delta\rho$		$M\delta\rho$	
g l′	sin	cos	sin	cos	sin	cos	cos	sin	cos	sin
	″	″	″	″						
0, 0	....	+31.2116t	....	+31.1982t						
1	— 0.160nt	+37.585nt	— 0.1622nt	— 1.5406nt	2					
2	— 0.010nt	+ 2.207nt	— 0.0095nt	— 0.0899nt	0					
3	....	+ 0.135nt	— 0.0006nt	— 0.0054nt	0					
0, 0	....	....	....	....	...	...	—10089	....	—4387.	0
1	+25.657	— 1.859	— 1.346	+ 1.361	...	...	—491.5	— 1.9	— 213.7	— 0.8
2	+ 1.397	— 0.087	— 0.030	+ 0.086	...	...	— 32.8	+ 1.1	— 14.2	+ 0.5
3	+ 0.072	— 0.005	....	....	...	...	— 1.8	0	— 0.8	0
—1,—1	+ 0.027	+ 0.009	+ 0.017	+ 0.011	10	2	0.0	— 0.5	0.0	— 0.2
0	+ 1.269	+ 0.002	+ 1.232	+ 0.001	37	1	— 58.8	— 0.5	— 25.6	— 0.2
1	—53.064	— 0.004	—53.084	— 0.002	20	2	+2585.7	+ 0.1	+1127.2	0
2	— 3.495	— 0.092	— 3.565	— 0.082	70	10	+195.3	+ 5.0	+ 82.8	+ 2.2
3	— 0.148	— 0.047	— 0.164	— 0.050	16	3	+ 17.1	+ 2.3	+ 7.4	+ 1.0
0,—2	— 0.027	— 0.011	— 0.031	— 0.014	4	3	+ 1.4	— 0.6	+ 0.6	— 0.3
1	+ 1.182	+ 0.515	+ 1.176	+ 0.515	6	0	— 56.5	+24.9	—24.6	+10.7
2	+ 0.277	+ 0.036	+ 0.263	+ 0.037	14	1	+ 8.5	+ 1.8	+ 3.7	+ 1.6
3	+ 0.074	— 0.005	+ 0.083	— 0.008	9	3	+ 4.2	+ 0.5	+ 1.8	+ 0.8
4	+ 0.015	— 0.003	+ 0.014	— 0.003	1	0	+ 0.9	+ 0.2	+ 0.4	+ 0.2
1,—3	— 0.032	— 0.034	— 0.025	— 0.025	7	9				
2	— 0.005	0	— 0.005	— 0.001	0	1				
3	+ 0.025	0	+ 0.015	0	10	0				
4	+ 0.011	— 0.001	+ 0.037	— 0.010	26	9				
5	+ 0.003	0	— 0.017	+ 0.004	20	4				

PERTURBATIONS OF THE LATITUDE.

g l′	δk		$\delta\eta$		$\delta\beta$	
i i′	sin	cos	cos	sin	cos	sin
	″	″	″	″	″	″
0 0	....	....	....	....	....	+.065
1	+0.071	—0.041	+0.030	+0.036	+.060	+.047
2	—0.012	—0.075	—0.012	+0.075	—.010	—.007
3	—0.003	—0.012	—0.003	+0.012	—.002	0
—2—1	—0.006	+0.042	+0.006	+0.042	+.005	—.004
—1	—0.297	+1.949	+0.297	+1.949	+.001	—.003
0	—0.161	—0.191	—0.189	—0.190	—.494	+.420
1	+2.611	0	+2.628	0	—.024	—.013
2	+0.070	+0.021	+0.066	—0.002	+.002	+.019
—3—2	+0.055	—0.088	—0.055	—0.088	+.010	—.003
0	+0.002	—0.011	—0.002	—0.011	+.017	—.004
1	—0.110	—0.050	—0.109	+0.046	—.009	+.002
2	—0.014	0	—0.011	0	—.002	0

$$\text{Secular terms} \begin{cases} \delta k = 1.14''\ \mathrm{T} \\ \delta\beta = -1.14\ \mathrm{T} \cos v \end{cases}$$

CHAPTER IV.

TERMS OF THE SECOND ORDER PRODUCED BY THE ACTION OF SATURN.

Preliminary Investigation of the Orbit of Saturn.

For the accurate determination of the perturbations of a planet it is essential that the functions of the time which are substituted for the co-ordinates of each planet in the expression of the disturbing forces should approximately represent the true places of the planet. The difference between the true place and that implicitly assumed in the investigation should be so small and of such a character that, when multiplied by the mass of the disturbing planet, and by the factors introduced by the process of integration, the result shall be insensible. If one of these factors is so large as to make a perturbation of an order of magnitude approximating that of the inequality which gives rise to it, it will represent an inequality of very long period in the elements, which, though apparently sensible, may be neglected for a great length of time.

The perturbations hitherto found have been computed on the hypothesis that the disturbing action of Saturn on Uranus is the same as if both planets moved in the elliptic orbits corresponding to the adopted elements. We have given formulæ for the computation of the corrected perturbations when, to the co-ordinates of the two planets corresponding to the adopted ellipse, we add corrections represented by δv, $\delta v'$, $\delta \rho$, etc. These corrections are now to be taken of such magnitude that when thus added they shall very nearly represent the actual motions of the planets.

Generally, it is considered sufficient to take for these corrections the perturbations of the first order. But this presupposes that the elliptic elements are nearly correct, which does not hold true in the case of the old elements of the outer planets. Bouvard's Tables of Saturn, the elements of which have been adopted, are subject to recurring errors amounting to 30″ or more. Moreover, when we substitute the new and more accurate perturbations for the old and imperfect ones adopted in the tables, the chances are that the errors will be increased. Desiring that the theory shall be as far as possible free from doubt, we begin with a preliminary investigation of the orbit of Saturn, the design of which will be to give the co-ordinates of that body in terms of the time with sufficient certainty and accuracy to serve for computing the perturbations both of Jupiter and Uranus. As usual, the first step in this investigation will be the determinations of the perturbations of the planet.

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General Perturbations of Saturn.

The perturbations produced by Jupiter will be taken from the exhaustive prize memoir of Hansen.[1] As the perturbations required are those of the co-ordinates, it will be necessary to transform those of Hansen into the usual form. Hansen gives the true anomaly v in the form

$$v = g + n\delta z + e_1 \sin (g + n\delta z) + e_2 \sin 2 (g + n\delta z) + \text{etc.},$$

e_1, e_2, etc., being the coefficients of the multiples of the mean anomaly in the usual development of the elliptic true anomaly. Whence, neglecting the second power of $n\delta z$,

$$\delta v = n\delta z (1 + e_1 \cos g + 2e_2 \cos 2g + \text{etc.}).$$

To make the development sufficiently rigorous it is only necessary to increase g by $\frac{1}{2} n\delta z$ in this expression. In the same way, we have for the perturbations of log r,

$$\delta\rho = \delta\rho_0 + n\delta z (e^{(1)} \sin g + e^{(2)} \sin 2 g + \text{etc.})$$

$\delta\rho_0$ being Hansen's perturbation, and $e^{(i)}$ the negative coefficient of cos ig in the development of the elliptic log r.

Hansen having adopted $\frac{1}{1070.5}$ as the mass of Jupiter, it will be necessary to multiply his perturbations by 1.0216 to reduce them to Bessel's mass. Thus the perturbations by Jupiter hereafter given have been obtained.

The perturbations by Uranus and Neptune have been computed by the preceding general method, and are given in the following table. In the table l' is the mean longitude of the disturbing planet, Uranus or Neptune, counted from the perihelion of Saturn. $\delta\rho$ is the perturbation of the Naperian logarithm, in units of the seventh place of decimals.

GENERAL PERTURBATIONS OF THE LONGITUDE IN ORBIT AND THE LOGARITHM OF THE RADIUS VECTOR OF SATURN.

Action of Uranus.					Action of Neptune.				
	δv		$\delta\rho$			δv		$\delta\rho$	
l' g	sin	cos	cos	sin	l' g	sin	cos	cos	sin
	″	″				″	″		
1, 0	+ 0.88	+ 0.92	— 12.7	+ 7.9	1,—0	+0.23	—0.05	— 2.2	—0.2
—1	+ 8.60	+ 0.21	+146.6	— 3.5	—1	+1.93	0.00	+39.9	0.0
2, 0	— 0.42	— 0.17	+ 10.2	— 3.7	2, 0	+0.44	+0.02	— 3.0	+0.1
—1	— 8.49	— 2.53	— 66.8	+21.4	—1	—1.11	+0.02	—18.5	—0.4
—2	—13.39	— 0.25	—392.5	+ 6.6	—2	—1.27	0.00	—41.1	0.0
3, 0	+ 0.06	+ 1.62	+ 0.5	+39.1	3,—1	+0.02	—0.03	+ 0.2	+0.4
—1	— 2.9	+28.1	+ 10.8	+33.0	—2	+0.10	—0.04	+ 3.2	+1.3
—2	—10.61	—20.88	—247.	+487.	—3	—0.10	0.00	— 3.9	0.0
—3	— 2.05	— 1.47	— 67.5	+42.8					
4,—1	+ 0.06	+ 0.05	—. 0.1	+ 0.6					
—2	— 0.62	+ 0.80	— 10.5	—12.5					
—3	+ 0.30	+ 0.73	+ 10.5	—23.4					
—4	— 0.26	+ 0.03	— 10.6	— 0.8					

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[1] Untersuchungen über die gegenseitigen Störungen des Jupiters und Saturns. Von P. A. Hansen. Berlin, 1831.

I have submitted these perturbations to such duplicate computations and other checks as lead me to believe that none of the terms can be in error by more than a small fraction of a second, but, as they are not intended to form the basis of a definitive theory of Saturn, I do not vouch for their absolute precision.

In this provisional correction of the orbit of Saturn only heliocentric longitudes have been employed. These were derived for a series of dates from Airy's reduction of the Greenwich observations, the modern Greenwich observations, and the Washington observations.

For these dates the value of $n\delta z$ for Saturn was computed from the formulæ found on pages 189 and 190 of the work of Hansen, already quoted, omitting all terms less than 1″, and including only tenths of seconds in the results. The dates, the resulting values of $n\delta z$, of the factor $e_1 \cos(g + \frac{1}{2} n\delta z) + 2e_2 \cos 2(g + \frac{1}{2} n\delta z)$, and of the concluded δv are as follows. The formulæ for δv is

$$\delta v = 1.0216\, n\delta z \,\{1 + e_1 \cos(g + \tfrac{1}{2} n\delta z) + 2e_2 \cos 2(g + \tfrac{1}{2} n\delta z)\}.$$

Date Gr. Mean Noon.			$n\delta z$	Factor.	δv
			″		″
1751	May	31	—1947.7	—.0990	—1792.7
1757	Aug.	7	—2134.2	—.0652	—2038.2
1758	Aug.	27	—2212.5	—.0474	—2153.2
1761	Oct.	6	—2546.2	+.0244	—2664.5
1763	Nov.	1	—2880.3	+.0729	—3157.1
1765	Nov.	23	—3095.1	+.1082	—3504.0
1773	Feb.	26	—3342.0	+.0419	—3557.2
1780	May	24	—2858.2	—.0956	—2640.7
1794	Nov.	16	—3321.1	+.1017	—3737.9
1802	Feb.	23	—3184.7	+.0529	—3425.5
1823	Nov.	13	—2716.3	+.0944	—3036.8
1831	Feb.	18	—3378.5	+.0639	—3671.7
1838	May	19	—2976.7	—.0866	—2777.9
1845	Aug.	17	—2342.7	—.0721	—2220.9
1852	Nov.	15	—2847.0	+.0863	—3159.3
1860	Feb.	14	—3161.4	+.0740	—3468.3
1867	May	15	—2373.1	—.0812	—2227.6

The perturbations by Uranus and Neptune were computed from the values of their terms just given. The principal terms, the sum of which make up the heliocentric longitude resulting from the adopted elements, are shown in the first of the following tables.

In the next table we have after the date the heliocentric longitude from Bouvard's Tables, as deduced from the longitudes given in Airy's reductions of the Greenwich Observations, from the *Astronomisches Jahrbuch* for 1831, and from the Nautical Almanac. Then follow the corrections, roughly deduced from observations made near the opposition. Adding these columns, we have the longitude from observation. To the right of these are the equations of condition for the correction of the elements.

Long. of Perihelion.	Mean anomaly.	Equation of centre.	Perturbations by			Red. to Ecliptic.	Nutation.	True longitude.
			Jupiter.	Uranus.	Neptune.			
° ′ ″	° ′ ″	° ′ ″	′ ″	″	″	′ ″	″	° ′ ″
88 41 27.5	159 55 50.1	+2 3 56.6	—29 52.7	—45.7	—2.7	+1 36.9	+15.8	250 12 25.8
88 46 41.2	236 13 7.1	—5 7 38.1	—33 58.2	—36.6	+1.0	—1 20.5	—11.8	319 16 4.1
88 47 31.4	248 25 53.1	—5 48 30.4	—35 53.2	—35.2	+1.7	—1 35.8	—15.0	330 46 36.6
88 50 7.6	286 26 29.8	—6 16 1.3	—44 24.6	— 3.3	+2.9	—0 43.5	—14.1	8 15 13.5
88 51 51 6	311 44 13.6	—5 0 58.2	—52 37.1	+11.9	+2.4	+0 43.3	— 4.9	34 43 22.6
88 53 35.1	336 55 56.1	—2 41 10.6	—58 24.0	+19.2	+1.9	+1 36.6	+ 6.7	62 12 1.0
88 59 39.6	65 40 2.3	+6 0 35.9	—59 17.2	—32.9	—0.8	—1 37.1	+ 4.8	159 38 54.6
89 5 43.4	154 8 4.8	+2 37 57.7	—44 0.7	—53.8	—1.7	+1 37.5	—14.5	245 8 13.2
89 17 50.7	331 6 10.4	—3 18 5.2	—62 17.9	+ 9.6	+3.5	+1 30.9	—14.1	56 5 7.9
89 23 55.7	59 56 17.9	+5 44 46.6	—57 5.5	—18.6	+2.5	—1 37.3	+ 1.4	154 6 2.7
89 42 7.0	325 22 26.0	—3 51 58.0	—50 36.8	—27.9	—3.0	+1 21.6	+14.7	50 23 3.6
89 48 11.9	54 10 33.1	+5 25 2.6	—61 11.7	+13.3	+2.5	—1 33.2	— 7.4	148 21 11.1
89 54 16.1	142 44 37.0	+3 40 38.1	—46 17.9	— 3.4	+2.1	+1 29.7	— 4.3	235 34 37.4
90 0 20.4	231 18 40.9	—4 47 36.6	—37 0.9	—35.5	+0.9	—1 11.8	+14.8	315 52 52.2
90 6 24.6	319 52 44.8	—4 21 53.0	—52 39.3	+ 4.7	—0.9	+1 8.8	—18.4	44 45 31.3
90 12 28.7	48 26 48.7	+5 1 46.4	—57 48.3	+30.3	—3.4	—1 25.1	+14.1	142 42 31.4
90 18 32.8	137 0 52.6	+4 9 31.2	—37 7.6	— 8.2	+1.3	+1 21.6	— 4.1	230 52 59.7

Date.	Tabular longitude.	Obs. cor.	Long. from observation.	EQUATIONS OF CONDITION.					
		″		″					
1751 May 31	250 13 38.0	— 8.5	250 13 29.5	63.7	=	+0.90δε	—44δn	+0.60δe	+1.82eδω
1757 Aug. 27	319 17 8.5	—18.0	319 16 50.5	46.4		0.94	—39	—1.53	+1.14
1758 Aug. 27	330 47 37.9	—14.3	330 47 23.6	47.0		0.96	—40	—1.76	+0.80
1761 Oct. 6	8 15 13.8	+ 0.2	8 15 14.0	0.5		1.03	—39	—1.99	—0.50
1763 Nov. 1	34 43 42.3	+15.3	34 43 57.6	35.0		1.07	—39	—1.63	—1.31
1765 Nov. 23	62 12 20.7	+20.4	62 12 41.1	40.1		1.11	—38	—0.88	—1.88
1773 Feb. 26	159 40 3.8	+18.8	159 40 22.6	88.0		1.04	—28	+1.93	—0.79
1780 May 24	245 9 17.6	+ 7.8	245 9 25.4	72.2		0.90	—18	+0.76	+1.74
1794 Nov. 16	56 5 38.8	+14.7	56 5 53.5	45.6		1.10	— 6	—1.08	—1.78
1802 Feb. 23	154 7 2.7	+10.8	154 7 13.5	70.8		1.05	+ 2	+1.85	—0.98
1823 Nov. 13	50 23 34.5	+21.3	50 23 55.8	52.2		1.09	+26	—1.27	—1.66
1831 Feb. 18	148 22 38.5	+ 3.0	148 22 41.5	90.4		1.07	+33	+1.75	—1.16
1838 May 19	235 36 11.2	+ 3.4	235 36 14.6	97.2		0.91	+35	+1.08	+1.56
1845 Aug. 17	315 53 42.2	+15.7	315 53 57.9	65.7		0.93	+42	—1.43	+1.28
1852 Nov. 15	44 46 15.4	+ 5.9	44 46 21.3	50.0		1.09	+57	—1.42	—1.53
1860 Feb. 14	142 44 16.2	—13.7	142 44 2.5	91.1		1.08	+65	+1.63	—1.35
1867 May 15	230 54 47.8	+ 5.1	230 54 52.9	113.2		+0.92	+62	+1.23	+1.45

A normal equation for $\delta\varepsilon$ is obtained by taking the sum of all the equations. That for δn is formed by subtracting the sum of the first seven from the sum of the last seven, and those for δe and $e\delta\omega$ by taking the sum of the equations in which the coefficients of δe or $e\delta\omega$ are greater than unity, after changing the signs of the equations in which they are negative. The normals thus obtained are

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$$\begin{array}{rrrrrr}
17.19\delta\varepsilon & +\ 31\delta n & -\ 2.16\delta e & -\ 3.15e\delta\omega & = & +1069.3'' \\
0.04 & +587 & +\ 6.83 & -\ 0.69 & & +\ 239.3 \\
-\ 2.14 & +207 & +21.58 & +\ 2.19 & & +\ 208.1 \\
-\ 2.11 & -\ 60 & +\ 3.61 & +19.66 & & +\ 53.8
\end{array}$$

These equations give

$$\begin{aligned}
\delta\varepsilon &= +\ 64.8'' \text{ (Epoch, 1800.)} \\
\delta n &= +\ 0.268 \\
\delta e &= +\ 12.6 \\
e\delta\omega &= +\ 8.2
\end{aligned}$$

—Substituting these values in the seventeen equations of condition we have the following residuals, or excesses of theoretical over observed longitudes:

	″		″
1	+ 7.2	10	+ 10.7
2	− 4.3	11	− 6.4
3	− 10.2	12	− 1.1
4	+ 21.7	13	− 0.4
5	− 9.0	14	0.0
6	− 7.1	15	+ 3.7
7	− 11.3	16	+ 4.1
8	+ 8.2	17	− 7.7
9	− 6.4		

These residuals are much larger than they should be, and I scarcely know to what cause to attribute their magnitude. The results are however amply reliable for the purposes of the investigation, and lead to the following elements of Saturn:

	°	′	″
π,	90	6	26
ε,	14	50	3.2
θ,	112	20	0
ϕ,	2	29	39.2
n,	43996.395		
e,	.0560660		
$\log(a + \delta a)$,	0.979676		

Epoch, 1850, Jan. 0, Greenwich mean noon.

It will be seen that the adopted position of the plane of Saturn's orbit is retained. It was corrected from observations before the perturbations were finally computed.

Of the above corrections, those of the epoch and mean motion need not be taken account of in the corrections of the co-ordinates, since the mean longitude remains in the formulæ as an arbitrary quantity to the end. The effect of the correction of the mean distance is insensible. The corrections of eccentricity aud perihelion are therefore alone to be retained. They are allowed for by adding to δv and $\delta\varrho$ the terms

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$$\begin{aligned}\delta v &= \quad 2\delta e \sin g - e\delta\omega \cos g\\ &= + 25''.2 \ \sin g - 16''.4 \cos g;\end{aligned}$$

$$\begin{aligned}\delta\rho &= - \delta e \ \cos g - e\delta\omega \ \sin g\\ &= - 12''.6 \ \cos g - 8''.2 \cos g.\end{aligned}$$

Perturbations of Saturn and Uranus.

The following expressions include, with these corrections, all the perturbations of Saturn and Uranus which can produce any appreciable perturbations of the second order in their mutual action. In these expressions the initial letter of each planet is put for its mean longitude counted from the perihelion of Uranus.

PERTURBATIONS OF SATURN.

Argument.	$\delta v'$		$\delta\rho'$	
	sin	cos	cos	sin
	″	″	″	″
S	+ 25.2	+ 21.4	— 2.7	+ 9.0
S	+ 1.64t	— 1.48t	— 0.80t	— 0.74t
$S - J$	— 26.1	+ 6.2	+176.7	— 15.9
$2S - J$	—421.6	— 8.8	—115.8	— 9.1
$3S - J$	— 54.7	— 12.4	+ 12.3	— 16.6
$2S - 2J$	+ 32.6	— 1.0	+ 29.5	— 1.1
$3S - 2J$	+ 27.7	— 21.3	— 15.4	— 12.3
$4S - 2J$	+216.9	—769.2	+ 11.4	+403.0
$5S - 2J$	+2392.	—1692.	— 58.8	— 34.4
$6S - 2J$	+133.0	— 93.6	— 58.1	+ 56.9
$U - S$	+ 8.6	+ 0.2	+ 3.0	— 0.1
$2U - S$	+ 0.8	— 8.8	+ 0.1	+ 1.4
$2U - 2S$	— 13.4	+ 0.2	— 8.0	+ 0.1
$3U - S$	— 8.4	— 27.0	+ 0.1	— 0.7
$3U - 2S$	+ 18.3	— 14.6	+ 8.8	+ 7.0

PERTURBATIONS OF URANUS.

Argument.	δv		$\delta\rho$	
	sin	cos	cos	sin
	″	″	″	″
U	— 0.11t	— 0.28t	+ 0.05t	— 0.14t
$U - S$	—20.8	+ 8.5	+36.0	— 4.9
$2U - S$	—11.9	+143.5	— 2.0	—63.8
$3U - S$	+49.3	+115.9	— 4.7	+13.2
$2U - 2S$	+ 4.1	0.0	+ 4.2	+ 0.1
$3U - 2S$	+ 2.1	— 1.6	+ 1.8	+ 1.4
$4U - 2S$	+ 0.6	— 1.8	+ 0.4	+ 1.3
$5U - 2S$	+ 1.2	+ 3.0	+ 0.4	— 1.2

Let us now resume the equation

$$\delta Q = 2\int D'_t \delta R dt + \delta \frac{\partial R}{\partial \rho} - \frac{1}{\mu} D_t^2 \, (r_0{}^2 \delta\rho^2) + \frac{1}{2} \frac{\delta\rho^2}{r_0}.$$

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Beginning with the last two terms of this expression, it may be shown at the outset that they are quite insensible. The effect of the constant terms in $\delta\rho$ and $\delta\rho'$ has already been included by correcting the logarithm of the mean distance by their amount; they are therefore omitted. The largest remaining term is 64″, the square of which is only 0″.02. In the product $r_1{}^2\delta\rho^2$ the largest terms are

$$\begin{aligned}
&+0\overset{\prime\prime}{.}014 \\
&-0.013 \sin g \\
&-0.011 \sin (3g-2l') \\
&-0.011 \cos (4g-2l')
\end{aligned}$$

which may be entirely neglected.

We shall therefore only consider in δQ the terms

$$2\int D'_t\delta R dt+\delta\frac{\partial R}{\partial\rho}$$

As already remarked, R is rigorously a function only of V, ρ, and ρ', V being the angle made by the radii vectores of the two planets. But, in the analytical development of R, the quantity V is considered as a function of v, v′, and γ, so that we have

$$R=f(\rho,\rho',\mathrm{v},\mathrm{v}',\gamma).$$

In the previous computation of the perturbations of Uranus, we have supposed R to be a function of ρ_0, ρ'_0, etc. The corrections to R and its derivatives with respect to v and ρ are now given by the equations (11), with the modifications shown on pages 24 to 27. The derivatives of R_0 which enter into these equations are formed as follows: If, in the value of R produced by the action of Saturn on Uranus, we consider any term of the form

$$\frac{m'h}{a_1}\cos N$$

where

$$N=i\lambda+i'\lambda'+j\omega+j'\omega'$$

the accented quantities always referring to Saturn, but a_1 being the corrected mean distance of Uranus, then we shall have the following terms in the derivatives of R.

$$\begin{aligned}
\frac{\partial R}{\partial \mathrm{v}} &= -\frac{m'h}{a_1}(i+j)\sin N \\
\frac{\partial R}{\partial \mathrm{v}'} &= -\frac{m'h}{a_1}(i'+j')\sin N \\
\frac{\partial R}{\partial \rho} &= -\frac{m'}{a_1}\left(h+\frac{\partial h}{\partial \nu}\right)\cos N \\
\frac{\partial R}{\partial \rho'} &= \frac{m'}{a_1}\frac{\partial h}{\partial \nu}\cos N \\
\frac{\partial^2 R}{\partial \mathrm{v}^2} &= -\frac{m'h}{a_1}(i+j)^2\cos N
\end{aligned}$$

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$$\frac{\partial^2 R}{\partial \mathrm{v} \partial \mathrm{v}'} = -\frac{m'h}{a_1}(i+j)(i'+j')\cos N$$

$$\frac{\partial^2 R}{\partial \mathrm{v} \partial \rho} = \frac{m'}{a_1}\left(h+\frac{\partial h}{\partial \mathfrak{v}}\right)(i+j)\sin N$$

$$= -\frac{\partial R}{\partial \mathrm{v}} - \frac{\partial^2 R}{\partial \mathrm{v} \partial \rho'}$$

$$\frac{\partial^2 R}{\partial \mathrm{v} \partial \rho'} = -\frac{m'}{a_1}\frac{\partial h}{\partial \mathfrak{v}}(i+j)\sin N$$

$$\frac{\partial^2 R}{\partial \rho \partial \mathrm{v}'} = \frac{m'}{a_1}\left(h+\frac{\partial h}{\partial \mathfrak{v}}\right)(i'+j')\sin N$$

$$\frac{\partial^2 R}{\partial \rho^2} = \frac{m'}{a_1}\left(h+2\frac{\partial h}{\partial \mathfrak{v}}+\frac{\partial^2 h}{\partial \mathfrak{v}^2}\right)\cos N$$

$$= -\frac{\partial R}{\partial \rho} - \frac{\partial^2 R}{\partial \rho \partial \rho'}$$

$$\frac{\partial^2 R}{\partial \rho \partial \rho'} = -\frac{m'}{a_1}\left(\frac{\partial h}{\partial \mathfrak{v}}+\frac{\partial^2 h}{\partial \mathfrak{v}^2}\right)\cos N$$

All the numerical data necessary for the computation of these derivatives have been given in Chapter II. Combining the terms having the same argument, we find the following values, omitting those given in Chapter II, and those which are derived from the others by mere addition. The terms of $\frac{\partial^2 R}{\partial \mathrm{v}^2}$ are also omitted, because they are sensibly the same with those of $\frac{\partial^2 R}{\partial \mathrm{v} \partial \mathrm{v}'}$, changing the algebraic sign.

	$\frac{a_1}{m'}\frac{\partial R}{\partial \mathrm{v}'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial \mathrm{v} \partial \mathrm{v}'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial \rho \partial \mathrm{v}'}$	
g l'	sin	cos	sin	cos	sin	cos
0,—1	+0.2874	0	0	+0.2872	+0.2691	0
—2	+0.0066	—0.0310	+0.0310	+0.0067	+0.0060	—0.0322
1, 0	—0.0102	—0.0491	+0.0490	—0.0101	+0.0249	+0.1194
—1	—3.4811	—0.0010	—0.0021	—3.4810	—5.2684	0
—2	—0.0968	+0.4304	—0.4304	—0.1039	—0.0792	+0.4583
—3	+0.0388	+0.0173	—0.0179	+0.0389	+0.0398	+0.0163
2, 0	—0.0045	—0.0035	+0.0021	—0.0080	+0.0155	+0.0066
—1	—0.0544	—0.0712	+0.1426	—0.0689	—0.1760	+0.2403
—2	+0.4104	+0.0120	—0.0171	+0.8233	—1.3515	+0.0014
—3	—0.0053	—0.0191	+0.0390	—0.0226	+0.0288	+0.0547
3,—1	—0.0070	—0.0093	+0.0171	—0.0210	+0.0197	+0.0304
—2	+0.0570	—0.0650	+0.1957	+0.1013	—0.1791	+0.2840
—3	+0.2542	0	—0.0065	+0.7624	—1.0884	—0.0071
4,—2	—0.0014	—0.0130	+0.0368	—0.0193	+0.0213	+0.0547
—3	+0.0515	—0.0499	+0.1987	+0.1450	—0.2144	+0.2647
—4	+0.1448	—0.0015	—0.0016	+0.5789	—0.7644	—0.0005

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		$\frac{a_1}{m'}\frac{\partial R}{\partial \rho'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial \text{v}\partial \rho'}$		$\frac{a_1}{m'}\frac{\partial^2 R}{\partial \rho\partial \rho'}$	
g'	l	sin	cos	sin	cos	sin	cos
0,	0	....	+0.172	....	+0.002	....	—0.645
	—1	—0.026	—0.562	+0.556	0	+0.113	—0.610
1,	0	+0.070	+0.015	+0.015	+0.070	—0.227	—0.079
	—1	—0.003	+8.749	—8.750	—0.001	+0.011	+6.189
	—2	+0.889	+0.180	—0.176	+0.889	+0.912	+0.190
2,	—1	+0.084	+0.248	—0.230	+0.169	—0.322	+0.020
	—2	+0.018	+0.472	—0.941	+0.013	—0.006	—1.685
3,	—1	+0.012	+0.004	+0.013	+0.021	—0.049	+0.028
	—2	+0.073	+0.068	—0.122	+0.219	—0.344	—0.244
	—3	0	+0.278	—0.834	—0.007	+0.010	—1.260
4,	—2	+0.015	—0.001	+0.020	+0.042	—0.071	+0.019
	—4	+0.001	+0.155	—0.619	0	+0.001	—0.854

The derivations with respect to γ and the node have been omitted because they are quite insensible. The terms of δR depending on these derivatives are given by equation (31). In the case of Uranus disturbed by Saturn the largest values of the coefficients

$$\tfrac{1}{2} h \cot \tfrac{1}{2}\gamma;\ \tfrac{1}{2} h \tan \tfrac{1}{2}\gamma,\ \tfrac{1}{2}\frac{\partial h}{\partial \sigma}$$

are only about .05, while the largest coefficients in δk, $\delta k'$, and $\delta\gamma$ are less than 10″. Hence the largest terms in (31) will be of the order of magnitude 0″.5 multiplied by the mass of Saturn, and may therefore be omitted entirely. Omitting them the values of δR, $\delta\frac{\partial R}{\partial \text{v}}$ and $\delta\frac{\partial R}{\partial \rho}$ become

$$\delta R = \frac{\partial R}{\partial \text{v}}\delta v + \frac{\partial R}{\partial \text{v}'}\delta v' + \frac{\partial R}{\partial \rho}\delta\rho + \frac{\partial R}{\partial \rho'}\delta\rho'$$

$$\delta\frac{\partial R}{\partial \text{v}} = \frac{\partial^2 R}{\partial \text{v}^2}\delta v + \frac{\partial^2 R}{\partial \text{v}\partial \text{v}'}\delta v' + \frac{\partial^2 R}{\partial \text{v}\partial \rho}\delta\rho + \frac{\partial^2 R}{\partial \text{v}\partial \rho'}\delta\rho$$

$$\delta\frac{\partial R}{\partial \rho} = \frac{\partial^2 R}{\partial \text{v}\partial \rho}\delta v + \frac{\partial^2 R}{\partial \text{v}'\partial \rho}\delta v' + \frac{\partial^2 R}{\partial \rho^2}\delta\rho + \frac{\partial^2 R}{\partial \rho\partial \rho'}\delta\rho'$$

All the separate factors from which the second members of these equations are formed have already been given. Forming their products in the way described in Chapter II, we have the result given in the following tables.

The expressions for δR are arranged so that the value of $D'_t\delta R$ can be obtained from them by direct differentiation. This is done by distinguishing the time introduced into R by the co-ordinates of Uranus from that introduced by the co-ordinates of Saturn.

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	$\frac{2a_1}{m'}\left\{\frac{\partial R}{\partial v'}\delta v'+\frac{\partial R}{\partial \rho'}\delta \rho'\right\}$	
U *S*	cos	sin
	"	"
1, 0	— 1.40*t*	— 1.33*t*
2, 0	+ 0.06*t*	— 0.28*t*
0,—1	— 0.24*t*	+ 0.20*t*
1,—1	+ 0.03*t*	+ 0.04*t*
2,—1	— 1.05*t*	— 0.95*t*
3,—1	+ 0.02*t*	— 0.30*t*
0,—2	+ 0.93*t*	— 0.83*t*
1,—2	—12.69*t*	+11.60*t*
2,—2	— 0.24*t*	+ 0.30*t*
3,—2	— 0.63*t*	— 0.59*t*
1,—3	— 1.60*t*	— 1.14*t*
2,—3	+ 0.25*t*	— 0.29*t*
3,—3	+ 0.08*t*	+ 0.02*t*
4,—3	— 0.36*t*	— 0.34*t*
3,—4	+ 0.19*t*	— 0.17*t*
4,—4	+ 0.15*t*	0.0
U *U'* *S*		
1, 0 0	+ 38"	+ 4"
1, 0—1	+ 2	+ 6
1, 0—2	—112	—153
1, 0—3	+ 14	— 18
1,+1—1	+ 56	— 2
1,—1+1	— 4	0
1,+2—1	+ 3	+ 43
1,—2+1	— 1	+ 19
1,+2—2	—117	+ 2
1,—2+2	— 23	0
1,+3—1	— 28	+ 88
1,—3+1	+ 30	+100
1,+3—2	+140	+112
1,—3+2	+ 14	— 10
U *S* *J*		
0, 1—1	+246	— 1
1,	—2469	— 50
2,	— 22	+ 4
3,	+ 3	+ 6
—2, 2—1	+ 42	— 16
—1,	+1638	— 88
0,	—136	+ 12
1,	— 89	— 89
2,	+ 2	+ 4
—3, 3—1	+ 6	— 8
—2,	+ 66	+ 24
—1,	+478	—280
0,		
—3, 4—1	+ 9	+ 28
—2,	—221	— 5
—1,	+297	—286

	$\frac{2a_1}{m'}\left\{\frac{\partial R}{\partial v'}\delta v'+\frac{\partial R}{\partial \rho'}\delta \rho'\right\}$		
U *S* *J*			
	"	"	
+1, 3—2	+375	+7336	
+2,	—990	—558	
+3,	— 73	—261	
+4,	+ 23	— 42	
0, 2	—647	—319	
1,	+7737	+5616	
2,	+153	—103	
3,	+ 24	— 26	
4,	0	0	
—2, 5—2	+ 90	+ 60	
—1,	—625	+739	
0,	+ 46	— 76	
1,	+ 61	+719	
2,	— 20	+ 6	
—4, 6—2	+27.5	— 3.9	
—3,	+94.1	+ 24.4	
—2,	+ 66	+243	
—1,	—8823	—6114	
0,	+699	+522	
—4, 7—2	+ 70	— 2	
—3,	+299	+147	
—2,	+937	+706	
—1,	—1978	+1117	

	$\frac{2a_1}{m'}\left\{\frac{\partial R}{\partial v}\delta v+\frac{\partial R}{\partial \rho}\delta \rho\right\}$		
U *S'* *S*	cos	sin	Fact. *nt*
	"	"	"
2,—1—1	—120	— 3	—0.852
3,	+ 59	+801	+0.148
4,	—139	+353	+1.448
3,—1—2	—122	+ 26	+0.148
4,	— 60	— 29	+1.148
5,	+ 24	— 60	+2.148
—0,—1+1	—250	0	—2.852
+1,	— 4	—170	—1.852
2,	+197	—450	—0.852
3,	— 15	+ 38	+0.148
—1,—1+2	— 8	+ 19	—3.852
0,	— 11	+134	—2.852
1,	— 80	+ 33	—1.852
1,—1 0	— 90	+ 12	—1.852
2,	+ 6	+158	—0.852
3,	+ 12	— 32	+0.148

	$2\frac{a_1}{m'n}\delta D'_t R$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial \mathrm{v}}$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial \rho}$	
U *S*	sin	cos	sin	cos	cos	sin
	″	″	″	″	″	″
0, 0	......	....	0	0	+ 0.14*t*	
1, 0	+ 1.40*t*	— 1.33*t*	+ 1.40*t*	— 1.35*t*	+ 3.54*t*	+ 3.21*t*
2, 0	— 0.12*t*	— 0.56*t*	— 0.27*t*	— 0.43*t*	— 0.34*t*	+ 0.90*t*
0,—1	0	0	— 0.04*t*	— 0.06*t*	+ 1.06*t*	— 1.02*t*
1,	— 0.03*t*	+ 0.04*t*	— 0.06*t*	+ 0.03*t*	+ 0.07*t*	— 0.12*t*
2,	+ 2.10*t*	— 1.90*t*	+ 2.10*t*	— 1.89*t*	+ 3.54*t*	+ 3.22*t*
3,	— 0.06*t*	— 0.90*t*	— 0.19*t*	— 0.73*t*	— 0.19*t*	+ 1.18*t*
0,—2	0	0	— 0.91*t*	— 0.83*t*	+ 0.84*t*	— 0.94*t*
1,	+12.69*t*	+11.60*t*	+12.69*t*	+11.62*t*	—13.52*t*	+12.34*t*
2,	+ 0.48*t*	+ 0.60*t*	+ 0.21*t*	+ 0.37*t*	— 0.42*t*	+ 0.14*t*
3,	+ 1.89*t*	— 1.77*t*	+ 1.89*t*	— 1.75*t*	+ 2.78*t*	+ 2.52*t*
1,—3	+ 1.60*t*	— 1.14*t*	+ 1.61*t*	— 1.11*t*	— 1.64*t*	— 1.21*t*
2,	— 0.50*t*	— 0.58*t*	— 0.53*t*	— 0.56*t*	— 0.83*t*	+ 0.75*t*
3,	— 0.24*t*	+ 0.06*t*	— 0.19*t*	+ 0.07*t*	— 0.25*t*	— 0.10*t*
4,	+ 1.44*t*	— 1.36*t*	+ 1.43*t*	— 1.32*t*	+ 1.93*t*	+ 1.76*t*
3,—4	— 0.57*t*	— 0.51*t*	— 0.55*t*	— 0.53*t*	— 0.74*t*	+ 0.68*t*
4,	— 0.60*t*	0	— 0.52*t*	+ 0.10*t*	— 0.73*t*	— 0.10*t*
0, 0	....	....	....	+ 5	—192	
1, 0	— 45	+319	+ 9	+160	— 49	—709
2,	+168	+383	+199	+449	+278	—591
3,	+ 2	+ 5				
—1,—1	+148	— 61	+ 70	— 71	+ 18	+125
0,	+ 27	—381	— 16	—247	— 49	+347
1,	—139	— 70	— 4	— 32	+103	+ 47
2,	— 21	— 36	+ 3	— 12	+ 1	—215
3,	— 4	+ 38	+ 1	— 1	— 10	+ 38
4,	+ 28	+ 88	— 8	+ 4	— 19	— 9
1,—2	+ 87	—153	— 14	—119	— 10	+251
2,	— 88	— 7	+145	— 13	+ 84	+ 7
3,	+108	+121	— 60	+796	+ 83	+816
4,	+ 20	+517	+141	+352	—262	+617
5,—2	....	....	— 18	+ 14	— 45	+ 2
3,—3	+ 18	+ 4	+208	+ 16	+ 17	+ 34
4,	+ 69	—33	+ 82	— 67	— 35	+ 44
5,	— 52	—128	— 49	—130	— 92	+181
4,—4	....	....	+ 55	— 1	+110	+ 8
5,	....	....	+ 8	—104	+ 12	+ 52
6,	....	....	— 42	—103	— 61	+148

		$2\frac{a_1}{m'n}\delta D'_t R$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial \mathrm{v}}$		$2\frac{a_1}{m'}\delta\frac{\partial R}{\partial \rho}$	
U	*S J*	sin	cos	sin	cos	cos	sin
		″	″	″	″	″	″
0,	1—1	0	0	—185	+ 2	— 84	+ 13
1,		+2469	— 50	+2477	— 50	—2947	— 69
2,		+ 44	+ 8	— 7	+ 13	—176	— 17
3,		— 9	+ 18	— 6	+ 16	— 17	— 20
—2,	2—1	+ 84	+ 32	+ 42	+ 29	+ 9	+ 49
—1,		+1638	+ 88	+1642	+ 88	+1231	—143
0,		0	0	—114	— 2	+ 41	+ 11
+1,		+ 89	— 89	+ 83	— 89	—196	— 62
+2,		— 4	+ 8	— 1	+ 8	— 11	— 18
—3,	3—1	+ 18	+ 24	+ 25	+ 28	— 42	+ 41
—2,		+132	— 48	+143	— 54	—191	—104
—1,		+478	+280	+477	+279	+1529	—279
0,		0	0	— 57	— 8	— 60	+ 44
—3,	4—1	+ 27	— 84	+ 71	— 91	— 89	—134
—2,		—442	+ 10	—449	+ 7	+776	— 8
—1,		+297	+286	+310	+270	+362	—258
1,	3—2	—375	+7336	—393	+7343	+629	+7722
2,		+1980	—1116	+1987	—1271	+3304	+2544
3,		+219	—783	+ 51	—672	+151	+1041
4,		— 92	—168	—104	—102	—136	+158
0,	4—2	0	0	+669	—485	—601	—966
1,		—7737	+5616	—7733	+5606	+12136	+8698
2,		—306	—206	—235	—258	+273	+910
3,		— 72	— 78	+ 59	— 52	— 60	+109
—2,	5—2	+180	—120	+159	+ 6	—337	— 37
—1,		—625	—739	—620	—740	—1222	—1217
0,		0	0	+ 62	— 56	+167	— 60
+1,		— 61	+719	— 63	+725	+ 72	+1109
+2,		+ 40	+ 12	— 13	— 6	— 17	+ 67
—4,	6—2	+110.0	+ 15.6	+ 86.1	+ 32.9	—127.5	+ 39.4
—3,		+282.3	— 73.2	+248.8	— 14.5	—413.6	— 75.7
—2,		+132	—486	+237	—557	—1131	—1461
—1,		—8823	+6114	—8829	+6121	—12973	—9020
0,		0	0	+746	—493	+774	+406
—4,	7—2	+280	+ 8	+207	+ 45	—216	+ 48
—3,		+897	—441	+727	—632	—1128	—977
—2,		+1874	—1412	+1913	—1374	—3236	—2195
—1,		—1978	—1117	—1995	—1097	—2071	+904
0,		0	0	+139	+ 90	+144	— 71

In the terms of δR introduced by the perturbations of Saturn, namely, $\frac{\partial R}{\partial v'}\delta v' + \frac{\partial R}{\partial \rho'}\delta\rho'$, the differentiation represented by D'_t should be performed by considering $\delta v'$ and $\delta\rho'$ as constant, although they are expressed as a function of the mean longitude of Uranus, as well as of Saturn. The mean longitude of Uranus thus introduced is therefore represented by U', which is regarded as constant in taking $D'_t R$, and U only supposed to vary.

Again, in the terms $\frac{\partial R}{\partial v}\delta v + \frac{\partial R}{\partial \rho}\delta\rho$, since δv and $\delta\rho$ represent perturbations of Uranus, their complete derivatives, with respect to the time, are to be taken. But their expressions contain the mean longitude of Saturn as well as Uranus. The mean longitude of Saturn thus introduced is represented by S', and is to be considered variable in obtaining $D'_t\delta R$, while S is considered constant. The ratio of the coefficient of t to n in the various terms of this part of δR is given to the right of each corresponding term.

The value of $D'_t\delta R$ being once obtained, there is no longer any distinction necessary between U, U', or between S and S'. The similar terms are therefore combined by putting $S' = S$; $U' = U$.

From the above values of $2\delta D'_t R$ and $2\delta\frac{\partial R}{\partial \rho}$, we form the following value of

$$\frac{a_1}{m}\delta Q = \frac{2a_1}{m}\int \delta D'_t R dt + \frac{a_1}{m}\delta\frac{\partial R}{\partial \rho}$$

and of the other quantities which enter the perturbations of the co-ordinates. We shall begin with those terms which depend only on the mutual action of Saturn and Uranus, because they are few and small, and the only terms which are sensible are those in which the coefficient of the mean longitude of Saturn is —1. We shall therefore confine ourselves to these. And, instead of employing the condensed formulæ, we shall make the computation in full by (13).

	$\frac{a_1}{m'}\delta Q = 2\frac{a_1}{m}\int \delta D'_t R dt + \delta\frac{\partial R}{\partial \rho}$		$\frac{a_1}{m'}\xi\delta Q$		$\frac{a_1}{m'}\eta\delta Q$	
g l'	cos	sin	cos	sin	sin	cos
	″ ″	″ ″	″ ″	″ ″	″ ″	″ ″
—1,—1	+ 47	+ 78	— 10	+148	+ 7	+153
0	— 14+0.53t	+307—0.51t	+ 13+0.02t	+ 49+0.05t	+ 36—0.06t	— 8+0.01t
1	— 24+0.02t	+ 62—0.08t	— 41+2.37t	+138+1.62t	+ 29—1.85t	—164+2.11t
2	— 60+4.23t	— 28+3.84t	—270—0.13t	+157—3.05t	+250—0.13t	+ 85—2.70t
3	—525+0.31t	+240—5.49t	+ 7+2.09t	— 31+2.30t	— 30+2.11t	+ 14—1.92t
4	....	....	—262+0.15t	+120—2.74t	—262+0.15t	—120+2.74t

	$\cos\psi\delta\rho$		δv	
g l'	cos	sin	sin	cos
	″ ″	″ ″	″ ″	
0,—1	+.001—.00001t	—.023+.00002t	+0.002+0.0000t	+0.021+0.0000t
1,	+.024+.00033t	—.036+.00033t	+0.043+0.0006t	+0.058+0.0000t
2,	+.333+.00437t	—.406+.00432t	+0.620+0.0098t	+0.803—0.0097t
3,	—.142—.00002t	+.079—.00169t	+2.69 —0.0011t	+0.830—0.0153t
4,	—.071	+.019	+0.132 0t	+0.040+0.0000t

The computation of these terms being extremely complex, a check upon their accuracy is desirable. In the case of the secular variations of the coefficients, the coefficients of the time are easily obtained by substituting in the integrated perturbations the variations of the eccentricity and perihelion of Saturn. Thus I have found

$$\begin{aligned}\delta v = &+0''.0103\, t \sin(2g - l') - 0''.0094\, t \cos(2g - l')\\ &+0.0027\, t \sin(3g - l') - 0.0138\, t \cos(3g - l')\end{aligned}$$

The greatest discrepancy is found in the coefficient of sin $(3g - l')$, and it amounts to $0''.0038t$, or about $0''.4$ in a century. But, owing to the great period of this term, nearly 600 years, this difference, during any one century, will be nearly eliminated through the mean longitude and mean motion.

It may also be remarked that in this case the terms derived from the perturbations of the elements are undoubtedly the correct ones, and will therefore be employed.

The terms which the preceding integration fails to give, owing to the constant terms introduced into $\xi\delta Q$ and $\eta\delta Q$, are found by (22).

We thus have

$$\begin{aligned}n\Sigma p_u k_c^{(u)} &= +0''.36\\ n\Sigma q_u k_s^{(u)} &= +0\ .27\\ r_1^2\delta\rho &= \tfrac{1}{4} Mnt^2 \{0''.36 \sin g - 0''.27 \cos g\}\\ \delta v &= \tfrac{1}{2} Mnt^2 \{0''.36 \cos g + 0''.27 \sin g\}\\ &= \qquad t^2 \{0''.0000038 \cos g + 0''.0000029 \sin g\}.\end{aligned}$$

The greatest effect of these terms amounts to less than one-twentieth of a second in a century. They may therefore be neglected in the present theory. The other terms containing the square of the time are yet smaller.

Applying the terms of the second order thus found to the terms of the first order depending on the corresponding arguments, the perturbations of Uranus by Saturn become

	δv				$\cos\varphi\,\delta\rho$	
$g\quad l'$	sin		cos		cos	sin
	″	″	″	″		
—1,—1	+ 0.036		+ 0.039		+ 0	— 2
0,—1	+ 1.284		+ 0.739		— 11	— 31
1,—1	—20.774	+ 0.06 T	+ 8.580		+1753	—244
2,—1	—11.270	+ 1.03 T	+144.265	— 0.94 T	— 87	—3114
3,—1	+51.99	+ 0.27 T	+116.69	— 1.38 T	—237	+644
4,—1	+ 2.265		+ 5.656		— 55	+140
5,—1	+ 0.126		+ 0.329		— 4	+ 10

T here represents the time counted in centuries from 1850.0.

The other terms remain the same as given on page 50.

Perturbations depending on the product of the masses of Jupiter and Saturn.

The values of $\delta D'_t R$, $\delta \dfrac{\partial R}{\partial v}$, and $\delta \dfrac{\partial R}{\partial \rho}$, depending on the products of the masses of Jupiter and Saturn, are given on page 74. The computation from these data being conducted in the same way as in the case of the terms of the first order, it is not necessary to give much more than the results. These are shown in the following table. The indices to the left represent the coefficients of the mean longitudes of Uranus, Saturn, and Jupiter, all counted from the perihelion of Uranus. Column *ν* gives the ratio of the mean motion of Uranus to the coefficient of the time in each argument. The perturbations of the common logarithm of the radius vector are expressed in units of the seventh place of decimals.

		ν	*δv*		0.4343 δρ	
U	*S J*		sin ″	cos ″	cos	sin
0,	1,—1	—0.2364	+0.002	0	0	0
1,		—0.3095	—0.020	0	0	0
3,		—0.4480	+0.004	—0.003	0	0
4,		—0.8127	+0.016	—0.024	0	0
—2,	2,—1	—0.2960	—0.007	—0.001	0	0
—1,		—0.4204	—0.108	—0.007	— 2	0
0,		—0.7254	—0.014	—0.012	0	0
1,		—2.6420	+0.164	—0.267	+ 1	+ 1
2,		+1.6090	+0.005	—0.005	0	0
—3,	3,—1	—0.6551	+0.012	—0.002	0	0
—2,		—1.8997	+0.175	—0.015	+ 1	0
—1,		+2.1115	+0.078	+0.512	— 2	+ 3
0,		+0.6786	+0.007	+0.024	0	0
—3,	4,—1	+0.754	—0.030	+0.081	0	+ 1
—2,		+0.430	+0.043	+0.005	— 1	0
—1,		+0.301	+0.001	—0.003	0	0
1,	3,—2	—0.2170	—0.002	—0.032	0	— 1
2,		—0.2771	—0.051	+0.035	— 1	— 1
3,		—0.3833	—0.010	+0.050	0	— 1
4,		—0.6215	+0.032	+0.052	0	— 1
0,	4,—2	—0.3627	—0.010	—0.004	0	0
1,		—0.5692	+0.075	—0.154	— 5	— 2
2,		—1.3210	—0.349	—1.297	— 3	+11
3,		+4.1150	—0.510	—0.453	+ 2	— 2
—2,	5,—2	—0.5250	—0.253	—0.034	— 3	0
—1,		—1.1051	—4.433	—0.617	—43	+ 7
0,		+10.5152	—0.546	—0.032	+ 2	0
1,		+0.9132	+0.206	—3.254	— 2	—36
2,		+0.4773	+0.012	—0.192	0	— 2
—4,	6,—2	—0.9497	+1.824	—0.519	+19	+ 6
—3,		—18.9250	+40.650	-10.500	+32	+10
—2,		+1.0558	+6.237	—7.866	—63	—79
—1,		+0.5136	+0.467	—0.539	0	— 2
0,		+0.3393	+0.007	—0.017	0	0
—4,	7,—2	+0.5558	—0.050	—0.003	+ 1	0
—3,		+0.3573	—0.046	+0.032	+ 1	+ 1
—2,		+0.2632	—0.045	+0.032	+ 1	+ 1
—1,		+0.2084	+0.006	+0.007	0	0
—0,		+0.1724	0	0	0	0

CHAPTER V.

COLLECTION AND TRANSFORMATION OF THE PRECEDING PERTURBATIONS OF URANUS.

THE terms of the perturbations which neither contain the elements of the disturbing planets, nor depend on the secular variations of the eccentricity and perihelion, admit of being greatly simplified by a slight change in the arbitrary elements. These terms are as follows:

(1) In the longitude of Uranus

	″	″	″	″	″
Action of Jupiter,	$+31.2116t$	$+25.657 \sin g$	$+1.397 \sin 2g$	$-1.859 \cos g$	$-0.087 \cos 2g$
Action of Saturn,	$+10.9690t$	$+\ 8.545 \sin g$	$+0.461 \sin 2g$	$-4.735 \cos g$	$-0.169 \cos 2g$
Action of Neptune,	$-\ 0.4262t$	$+\ 0.697 \sin g$	$+0.046 \sin 2g$	$-0.088 \cos g$	$-0.005 \cos 2g$
Total,	$+41.7544t$	$+34.899 \sin g$	$+1.904 \sin 2g$	$-6.682 \cos g$	$--0.261 \cos 2g$

(2) In the value of $\cos \psi\delta\rho$, units of 7th place of decimals.

Action of Jupiter,	-10089	$-492 \cos g$	$-33 \cos 2g$	$-\ 2 \sin g$	$+1 \sin 2g$
Action of Saturn,	$-\ 3543$	$-184 \cos g$	$-15 \cos 2g$	$-15 \sin g$	
Action of Neptune,	$+\ 138$	$-\ 1 \cos g$	$-\ 1 \cos 2g$	$+\ 1 \sin g$	
Total,	-13494	$-677 \cos g$	$-49 \cos 2g$	$-16 \sin g$	$+1 \sin 2g$

Let us first consider the first or constant term in the perturbation of each co-ordinate. If we suppose a change of δn in the mean motion of a planet, the corresponding change in $\delta\rho$ will be

$$\delta\rho = -\frac{2}{3}\frac{\delta n}{n}.$$

If, then, we increase the mean motion of Uranus by $41''.754$, the corresponding change in $\delta\rho$ will be -18045, and in $\cos \psi\delta\rho$, -18025. Subtracting these from the above perturbations, the secular term in the mean motion will disappear, and we shall have for the constant term of $\cos \psi\delta\rho$

$$+4531$$

This same change in the mean motion will produce a secular term in the equation of the centre of the same nature with that produced by the secular variation of the perihelion. The differences of the values of the secular terms, found by the two methods employed in Chapters II. and III., proceeds from the fact that in the one case the effect of the above term in the mean motion is included, and in the other excluded.

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If we subduct the effect in question when necessary, the remainder will be the effect of the secular variation of the longitude of the perihelion of Uranus, to which we shall revert presently.

Let us next introduce such a change in the eccentricity of Uranus as shall produce the term $34''.899 \sin g$, and ascertain its effect on the other terms. For this purpose we must determine δe by the condition

$$(2-\frac{3}{4}e^2)\,\delta e = 34''.899$$

which gives

$$\delta e = 17''.464 = .0000847.$$

A change of this amount in δe will introduce the following terms in δv and $\delta\rho$

$$\delta v = 34''.899 \sin g + 2''.048 \sin 2g$$
$$\cos\psi\delta\rho = 20 - 844 \cos g - 59 \cos 2g.$$

Subtracting these terms from the expressions previously found we have

$$\delta v = -\,0''.144 \sin 2g - 6''.682 \cos g - 0''.261 \cos 2g.$$
$$\cos\psi\delta\rho = +\,4511 + 167 \cos g + 10 \cos 2g - 16 \sin g + 1 \sin 2g.$$

Again, let us put

$$e\delta\pi = 3''.342 = .0000162,$$

we shall have the elliptic terms

$$\delta v = -\,6''.682 \cos g - 0''.391 \cos 2g$$
$$\cos\psi\delta\rho = -\,162 \sin g - 11 \sin 2g.$$

Subtracting these expressions the constant terms, independent of the mean longitude of the disturbing planets, are reduced to

$$\delta v = -\,0''.144 \sin 2g + 0''.130 \cos 2g.$$
$$\cos\psi\delta\rho = 4511 + 167 \cos g + 10 \cos 2g + 146 \sin g + 12 \sin 2g.$$
$$0.43429\ \delta\rho = 1969 + 73 \cos g + 4 \cos 2g + 63 \sin g + 5 \sin 2g.$$

In the last equation we have introduced the constant $+.0000008$ produced in $\delta\rho$ by the combined action of Venus, the Earth, and Mars. The effect of each planet is computed by the approximate formula

$$\delta\rho = \frac{1}{6}m'(b_{\frac{1}{2}}^{(0)} + \alpha D_\alpha b_{\frac{1}{2}}^{(0)}).$$

Secular Variations.

The following inequalities result from the secular variations of the eccentricity and longitude of perihelion produced by each of the disturbing planets, T being the time expressed in centuries.

From the variation of the eccentricity

		″	″	″
Action of Jupiter,	$\delta v =$	$-1.216\,T \sin g$	$-0.072\,T \sin 2g$	$-0.005\,T \sin 3g$
Action of Saturn,		$-9.182\,T \sin g$	$-0.538\,T \sin 2g$	$-0.032\,T \sin 3g$
Action of Neptune,		$-0.502\,T \sin g$	$-0.030\,T \sin 2g$	$-0.002\,T \sin 3g$

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	$''$	$''$
Action of Jupiter,	$M\delta\rho = +13\,T\cos g$	$+1\,T\cos 2g$
Action of Saturn,	$+98\,T\cos g$	$+7\,T\cos 2g$
Action of Neptune,	$+\ 6\,T\cos g$	

The secular variation of the longitude of the perihelion is

	$''$
Action of Jupiter,	$+122.1\,T$
Action of Saturn,	$+118.4\,T$
Action of Neptune,	$+\ 51.1\,T$
Total,	$\delta\pi = +291.6\,T$

The effect of this secular variation on the longitude and radius vector is

	$''$	$''$	$''$
Action of Jupiter,	$\delta v = -11.46\,T\cos g$	$-0.671\,T\cos 2g$	$-0.047\,T\cos 3g$
Action of Saturn,	$-11.11\,T\cos g$	$-0.651\,T\cos 2g$	$-0.039\,T\cos 3g$
Action of Neptune,	$-\ 4.80\,T\cos g$	$-0.281\,T\cos 2g$	$-0.016\,T\cos 3g$
Total,	$-27.37\,T\cos g$	$-1.603\,T\cos 2g$	$-0.102\,T\cos 3g$

	$''$	$''$
Action of Jupiter,	$M\delta\rho = -120\,T\sin g$	$-8\,T\sin 2g$
Action of Saturn,	$-117\,T\sin g$	$-8\,T\sin 2g$
Action of Neptune,	$-\ 50\,T\sin g$	$-3\,T\sin 2g$

For the purpose of conveniently tabulating the perturbations, we shall express them in a form similar to that adopted in the theory of Neptune. Let us select, from the terms of the periodic perturbations produced by any planet, all those in which the difference between the indices i and i' is the same. For example, in the perturbations of the longitude produced by Jupiter, let us consider the terms

$$
\begin{aligned}
\delta v = &+1.269\sin(\quad - l) && +0.002\cos(\quad - l)\\
&-3.495\sin(2g - l) && -0.092\cos(2g - l)\\
&+1.182\sin(\ g - 2l) && +0.515\cos(\ g - 2l)\\
&+0.074\sin(3g - 2l) && -0.005\cos(3g - 2l)\\
&-0.005\sin(2g - 3l) && \\
&+0.011\sin(4g - 3l) && -0.001\cos(4g - 3l)
\end{aligned}
$$

These terms may be expressed in the form

$$
\delta v = \sin g \times \left\{
\begin{array}{ll}
'' & '' \\
+0.094\sin\ (g - l) & -4.764\cos\ (g - l)\\
+0.520\sin 2(g - l) & -1.108\cos 2(g - l)\\
 & +0.016\cos 3(g - l)
\end{array}
\right\}
$$

$$
+\cos g \times \left\{
\begin{array}{ll}
'' & \\
-2.226\sin\ (g - l) & -0.090\cos\ (g - l)\\
+1.256\sin 2(g - l) & +0.510\cos 2(g - l)\\
+0.006\sin 3(g - l) &
\end{array}
\right\}
$$

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In general, a series of terms of the form

$$\begin{aligned}&\Sigma a_i \sin(iA + sg) + \Sigma b_i \cos(iA + sg)\\ &+ \Sigma a'_i \sin(iA - sg) + \Sigma b'_i \cos(iA - sg),\end{aligned}$$

may be put in the form

$$\begin{aligned}&\{\Sigma(a_i - a'_i)\cos iA - \Sigma(b_i - b'_i)\sin iA\}\sin sg\\ &+ \{\Sigma(a_i + a'_i)\sin iA + \Sigma(b_i + b'_i)\sin iA\}\cos sg.\end{aligned}$$

All the periodic terms containing only g and l in the arguments may be put into this form by taking

$$A = g - l,$$

so that the coefficients of sin sg and cos sg may all be expressed as a function of the single variable argument A.

The perturbations of the elements may be reduced to perturbations of the co-ordinates expressed as the sum of several products of slowly varying functions into the sines and cosines of the multiples of g. We have, in fact,

$$\begin{aligned}\delta v = \delta l&\\ &+\left(2 - \frac{3}{4}e^2\right)\delta e \times \sin g &&+\left(2 - \frac{1}{4}e^2\right)e\delta g \times \cos g\\ &+\left(\frac{5}{2}e - \frac{11}{6}e^3\right)\delta e \times \sin 2g &&+\left(\frac{5}{2}e - \frac{11}{12}e^3\right)e\delta g \times \cos 2g.\\ &+ \text{etc.} &&+ \text{etc.}\end{aligned}$$

It appears, therefore, that all the perturbations in which the arguments contain the mean longitudes of only two planets may be put in the form

$$\begin{aligned}\delta v &= (v.c.0) + (v.c.1)\cos g + (v.c.2)\cos 2g + \text{etc.}\\ &\qquad\qquad\;\; + (v.s.1)\sin g + (v.s.2)\sin 2g + \text{etc.}\\ M\delta\rho &= (\rho.c.0) + (\rho.c.1)\cos g + (\rho.c.2)\cos 2g + \text{etc.}\\ &\qquad\qquad\;\; + (\rho.s.1)\sin g + (\rho.s.2)\sin 2g + \text{etc.}\end{aligned}$$

We have next to reduce to the same form those terms which contain the mean longitudes of both Jupiter and Saturn, and which are given on page 78. We have here twenty-four terms, each greater than 0″.04. As most of these terms depend on three independent arguments, they cannot be included in a double entry table, while, if we include them as perturbations of the longitude in tables of single entry, we shall have to enter twenty-two tables with as many different arguments. But, by taking, for the argument A, the middle one in each series of arguments which depend on the same multiples of Jupiter and Saturn, and expressing the terms above and below it in each series as coefficients of sin g, cos g, sin $2g$, and cos $2g$, we may reduce the number of arguments to eight, and the number of tables to seventeen. Consider, for instance, the terms of the second series,

$$\begin{array}{ll}
-\overset{\prime\prime}{0}.108\sin(-g+2S-J) & -0.007\cos(-g+2S-J)\\
-0.014\sin(\quad 2S-J) & -0.012\cos(\quad 2S-J)\\
+0.164\sin(\,g+2S-J) & -0.267\cos(\,g+2S-J).
\end{array}$$

These terms may be allowed for by adding to $(v.c.0)$, $(v.s.1)$, $(v.c.1)$, the terms

$$\begin{aligned}
(v.c.0) &= -\overset{\prime\prime}{0}.014\sin(2S-J)-\overset{\prime\prime}{0}.012\cos(2S-J)\\
(v.s.1) &= +0.260\sin(2S-J)+0.272\cos(2S-J)\\
(v.c.1) &= +0.056\sin(2S-J)-0.274\cos(2S-J).
\end{aligned}$$

From the perturbations of longitude and radius vector already given, we readily find the following values of $(v.c.0)$, $(v.s.1)$, etc.

Action of Jupiter.

$$A_1 = l' - g$$

$$\begin{aligned}
(v.c.0) = &+\overset{\prime\prime}{53}.064\sin A_1 -\overset{\prime\prime}{0}.004\cos A_1\\
&-0.277\sin 2A_1+0.036\cos 2A_1\\
&-0.025\sin 3A_1
\end{aligned}$$

$$\begin{aligned}
(v.c.1) = &+2.226\sin A_1-0.090\cos A_1\\
&-1.256\sin 2A_1+0.510\cos 2A_1\\
&-0.006\sin 3A_1\\
&\qquad -11''.46\,T
\end{aligned}
\qquad
\begin{aligned}
(v.s.1) = &-\overset{\prime\prime}{0}.094\sin A_1-\overset{\prime\prime}{4}.764\cos A_1\\
&-0.520\sin 2A_1-1.108\cos 2A_1\\
&\qquad\qquad +0.016\cos 3A_1\\
&\qquad -1''.22\,T
\end{aligned}$$

$$\begin{aligned}
(v.c.2) = &+0.121\sin A_1-0.038\cos A_1\\
&+0.012\sin 2A_1-0.014\cos 2A_1\\
&+0.029\sin 3A_1-0.034\cos 3A_1\\
&\qquad -0''.67\,T
\end{aligned}
\qquad
\begin{aligned}
(v.s.2) = &-0.056\sin A_1-0.175\cos A_1\\
&+0.008\sin 2A_1+0.042\cos 2A_1\\
&+0.034\sin 3A_1+0.035\cos 3A_1\\
&\qquad -0''.07\,T
\end{aligned}$$

$$(v.c.3) = -0.04\,T \qquad\qquad (v.s.3) = -0.005\,T$$

$$\begin{aligned}
(\rho.c.0) = &+1127\cos A_1\\
&+4\cos 2A_1
\end{aligned}$$

$$\begin{aligned}
(\rho.c.1) = &-2\sin A_1+57\cos A_1\\
&+10\sin 2A_1-23\cos 2A_1\\
&\qquad +13\,T
\end{aligned}
\qquad
\begin{aligned}
(\rho.s.1) = &+108\sin A_1+2\cos A_1\\
&+26\sin 2A_1+12\cos 2A_1\\
&\qquad -120\,T
\end{aligned}$$

$$(\rho.c.2) = +7\cos A_1+1\,T \qquad\qquad (\rho.s.2) = +7\sin A_1-8\,T$$

Action of Saturn.

$$A_2 = l' - g$$

$$\begin{array}{rll}
 & {}'' & {}'' \\
(v.c.0)= & \left(\begin{matrix}+20.774\\ -\ \ 0.06\,T\end{matrix}\right)\sin A_2 & +8.580\cos A_2 \\
 & -\ 4.110\sin 2A_2 & -0.009\cos 2A_2 \\
 & -\ 0.824\sin 3A_2 & -0.019\cos 3A_2 \\
 & -\ 0.228\sin 4A_2 & -0.008\cos 4A_2 \\
 & -\ 0.074\sin 5A_2 & -0.003\cos 5A_2 \\
 & -\ 0.025\sin 6A_2 &
\end{array}$$

$$\begin{array}{rll}
(v.c.1)= & \left(\begin{matrix}+\ \ 9.986\\ -\ \ 1.03\,T\end{matrix}\right)\sin A_2+ & \left(\begin{matrix}+145.005\\ -\ \ \ 0.94\,T\end{matrix}\right)\cos A_2 \\
 & -\ 2.121\sin 2A_2 & -0.793\cos 2A_2 \\
 & -\ 0.392\sin 3A_2 & -0.308\cos 3A_2 \\
 & -\ 0.109\sin 4A_2 & -0.099\cos 4A_2 \\
 & -\ 0.038\sin 5A_2 & -0.027\cos 5A_2 \\
 & \qquad -11''.11\,T &
\end{array}$$

$$\begin{array}{rll}
 & {}'' & {}'' \\
(v.s.1)= & \left(\begin{matrix}+143.52\\ -\ \ \ 0.94\,T\end{matrix}\right)\sin A_2+ & \left(\begin{matrix}-12.554\\ +\ \ 1.03\,T\end{matrix}\right)\cos A_2 \\
 & -2.421\sin 2A_2 & +2.037\cos 2A_2 \\
 & -0.226\sin 3A_2 & +0.318\cos 3A_2 \\
 & -0.055\sin 4A_2 & +0.097\cos 4A_2 \\
 & -0.027\sin 5A_2 & +0.038\cos 5A_2 \\
 & \qquad -9''.18\,T &
\end{array}$$

$$\begin{array}{rll}
(v.c.2)= & \left(\begin{matrix}-52.026\\ -\ \ 0.27\,T\end{matrix}\right)\sin A_2+ & \left(\begin{matrix}+116.73\\ -\ \ \ 1.38\,T\end{matrix}\right)\cos A_2 \\
 & -\ 0.665\sin 2A_2 & -1.847\cos 2A_2 \\
 & -\ 0.081\sin 3A_2 & -0.153\cos 3A_2 \\
 & -\ 0.013\sin 4A_2 & -0.044\cos 4A_2 \\
 & -\ 0.005\sin 5A_2 & -0.016\cos 5A_2 \\
 & \qquad -0''.65\,T &
\end{array}$$

$$\begin{array}{rll}
(v.s.2)= & \left(\begin{matrix}+116.650\\ -\ \ \ 1.38\,T\end{matrix}\right)\sin A_2+ & \left(\begin{matrix}51.954\\ +\ \ 0.27\,T\end{matrix}\right)\cos A_2 \\
 & -1.813\sin 2A_2 & +0.631\cos 2A_2 \\
 & -0.177\sin 3A_2 & +0.013\cos 3A_2 \\
 & -0.044\sin 4A_2 & +0.013\cos 4A_2 \\
 & -0.016\sin 5A_2 & +0.005\cos 5A_2 \\
 & \qquad -0''.54\,T &
\end{array}$$

$$\begin{array}{rll}
(v.c.3)= & -\ 2.265\sin A_2 & +5.656\cos A_2 \\
 & -\ 1.163\sin 2A_2 & +2.956\cos 2A_2 \\
 & +\ 0.026\sin 3A_2 & -0.063\cos 3A_2 \\
 & +\ 0.005\sin 4A_2 & -0.013\cos 4A_2 \\
 & +\ 0.002\sin 5A_2 & -0.004\cos 5A_2 \\
 & \qquad -0''.04\,T &
\end{array}$$

$$\begin{array}{rll}
(v.s.3)= & +5.656\sin A_2 & +2.265\cos A_2 \\
 & +2.956\sin 2A_2 & +1.163\cos 2A_2 \\
 & -0.063\sin 3A_2 & -0.026\cos 3A_2 \\
 & -0.013\sin 4A_2 & -0.005\cos 4A_2 \\
 & -0.004\sin 5A_2 & -0.002\cos 5A_2 \\
 & \qquad -0''.03\,T &
\end{array}$$

$$\begin{array}{rll}
(v.c.4)= & -\ 0.126\sin A_2 & +0.329\cos A_2 \\
 & -\ 0.503\sin 2A_2 & +0.378\cos 2A_2 \\
 & +\ 0.053\sin 3A_2 & -0.032\cos 3A_2
\end{array}$$

$$\begin{array}{rll}
(v.s.4)= & +0.329\sin A_2 & +0.126\cos A_2 \\
 & +0.378\sin 2A_2 & +0.503\cos 2A_2 \\
 & -0.032\sin 3A_2 & -0.053\cos 3A_2
\end{array}$$

$$\begin{array}{rlrl}
(\rho.c.0)= & +\ 106\ \sin A_2 & + & 761\cos A_2 \\
 & -\ \ \ \ 2\ \sin 2A_2 & + & 89\cos 2A_2 \\
 & & + & 19\cos 3A_2 \\
 & & + & 5\cos 4A_2
\end{array}$$

$$\begin{array}{rlrl}
(\rho.c.1)= & +\ 1364\ \sin A_2 & - & 43\cos A_2 \\
 & -\ \ \ 46\ \sin 2A_2 & + & 45\cos 2A_2 \\
 & -\ \ \ \ \ 7\ \sin 3A_2 & + & 9\cos 3A_2 \\
 & -\ \ \ \ \ 3\ \sin 4A_2 & + & 2\cos 4A_2 \\
 & \qquad +98\,T & &
\end{array}$$

$$\begin{array}{rlrl}
(\rho.s.1)= & -\ \ 33\sin A_2 & - & 1338\cos A_2 \\
 & +\ \ 31\sin 2A_2 & + & 14\cos 2A_2 \\
 & +\ \ \ \ 7\sin 3A_2 & + & 5\cos 3A_2 \\
 & +\ \ \ \ 2\sin 4A_2 & + & 1\cos 4A_2 \\
 & \qquad -117\,T & &
\end{array}$$

$$\begin{array}{rlrl}
(\rho.c.2)= & -\ 279\ \sin A_2 & - & 103\cos A_2 \\
 & -\ \ \ 27\ \sin 2A_2 & + & 9\cos 2A_2 \\
 & -\ \ \ \ \ 3\ \sin 3A_2 & & \\
 & \qquad +7\,T & &
\end{array}$$

$$\begin{array}{rlrl}
(\rho.s.2)= & -\ 103\sin A_2 & + & 281\cos A_2 \\
 & +\ \ \ \ 9\sin 2A_2 & + & 27\cos 2A_2 \\
 & & + & 3\cos 3A_2 \\
 & \qquad -8\,T & &
\end{array}$$

$$\begin{array}{rlrl}
(\rho.c.3)= & -\ \ 61\ \sin A_2 & - & 24\cos A_2 \\
 & +\ \ 24\ \sin 2A_2 & + & 9\cos 2A_2
\end{array}$$

$$\begin{array}{rlrl}
(\rho.s.3)= & -\ \ 24\sin A_2 & + & 61\cos A_2 \\
 & +\ \ \ \ 9\sin 2A_2 & - & 24\cos 2A_2
\end{array}$$

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Action of Neptune.

$$A_3 = g - l'$$

$$
\begin{aligned}
(v.c.0) = {} & -\overset{\prime\prime}{39.66}\sin A_3 & -\overset{\prime\prime}{0.08}\cos A_3 \\
& -35.36\sin 2A_3 & -0.03\cos 2A_3 \\
& +17.29\sin 3A_3 & +0.23\cos 3A_3 \\
& +\ 3.91\sin 4A_3 & +0.06\cos 4A_3 \\
& +\ 0.99\sin 5A_3 & -0.04\cos 5A_3 \\
& +\ 0.42\sin 6A_3 & -0.01\cos 6A_3 \\
& +\ 0.19\sin 7A_3 & \\
& +\ 0.09\sin 8A_3 & \\
& +\ 0.02\sin 9A_3 & \\
& \qquad +\delta l &
\end{aligned}
$$

$$
\begin{aligned}
(v.c.1) = {} & -\ 6.77\sin A_3 & -0.53\cos A_3 \\
& -\ 1.10\sin 2A_3 & +0.07\cos 2A_3 \\
& +23.25\sin 3A_3 & +4.04\cos 3A_3 \\
& +\ 6.05\sin 4A_3 & +1.06\cos 4A_3 \\
& -\ 3.26\sin 5A_3 & -0.66\cos 5A_3 \\
& -\ 0.80\sin 6A_3 & -0.17\cos 6A_3 \\
& -\ 0.24\sin 7A_3 & -0.05\cos 7A_3 \\
& -\ 0.12\sin 8A_3 & -0.02\cos 8A_3 \\
& -\ 0.06\sin 9A_3 & -0.01\cos 9A_3 \\
& -\ 0.04\sin 10A_3 & \\
& \qquad +1.99945e\delta g &
\end{aligned}
$$

$$
\begin{aligned}
(v.s.1) = {} & -\overset{\prime\prime}{0.49}\sin A_3 & +\ \overset{\prime\prime}{1.75}\cos A_3 \\
& +0.12\sin 2A_3 & -\ 2.48\cos 2A_3 \\
& +4.04\sin 3A_3 & -20.92\cos 3A_3 \\
& +1.07\sin 4A_3 & -\ 5.42\cos 4A_3 \\
& -0.68\sin 5A_3 & +\ 3.47\cos 5A_3 \\
& -0.17\sin 6A_3 & +\ 0.91\cos 6A_3 \\
& -0.04\sin 7A_3 & +\ 0.30\cos 7A_3 \\
& -0.02\sin 8A_3 & +\ 0.12\cos 8A_3 \\
& -0.01\sin 9A_3 & +\ 0.06\cos 9A_3 \\
& & +\ 0.04\cos 10A_3 \\
& \qquad +1.99835\delta e &
\end{aligned}
$$

$$
\begin{aligned}
(v.c.2) = {} & -0.43\sin A_3 & -0.03\cos A_3 \\
& -0.03\sin 2A_3 & +0.01\cos 2A_3 \\
& +0.75\sin 3A_3 & +0.08\cos 3A_3 \\
& -0.10\sin 4A_3 & -0.08\cos 4A_3 \\
& -3.20\sin 5A_3 & -1.17\cos 5A_3 \\
& -0.83\sin 6A_3 & -0.32\cos 6A_3 \\
& +0.57\sin 7A_3 & +0.22\cos 7A_3 \\
& +0.14\sin 8A_3 & +0.06\cos 8A_3 \\
& +0.06\sin 9A_3 & +0.02\cos 9A_3 \\
& +0.03\sin 10A_3 & -0.01\cos 10A_3 \\
& \qquad +0.11722e\delta g &
\end{aligned}
$$

$$
\begin{aligned}
(v.s.2) = {} & -0.03\sin A_3 & +0.13\cos A_3 \\
& +0.02\sin 2A_3 & -0.16\cos 2A_3 \\
& +0.08\sin 3A_3 & -0.60\cos 3A_3 \\
& -0.08\sin 4A_3 & +0.15\cos 4A_3 \\
& -1.17\sin 5A_3 & +3.22\cos 5A_3 \\
& -0.32\sin 6A_3 & +0.86\cos 6A_3 \\
& +0.22\sin 7A_3 & -0.57\cos 7A_3 \\
& +0.06\sin 8A_3 & -0.14\cos 8A_3 \\
& +0.02\sin 9A_3 & -0.06\cos 9A_3 \\
& -0.01\sin 10A_3 & -0.03\cos 10A_3 \\
& \qquad +0.11713\delta e &
\end{aligned}
$$

$$
\begin{aligned}
(v.c.3) = {} & -0.02\sin A_3 & \\
& +0.04\sin 3A_3 & +0.01\cos 3A_3 \\
& -0.15\sin 4A_3 & -0.05\cos 4A_3 \\
& -0.08\sin 5A_3 & -0.02\cos 5A_3 \\
& -0.02\sin 6A_3 & -0.02\cos 6A_3 \\
& +0.46\sin 7A_3 & +0.25\cos 7A_3 \\
& +0.11\sin 8A_3 & +0.07\cos 8A_3 \\
& -0.08\sin 9A_3 & -0.05\cos 9A_3 \\
& -0.03\sin 10A_3 & -0.02\cos 10A_3 \\
& \qquad +0.00714e\delta g &
\end{aligned}
$$

$$
\begin{aligned}
(v.s.3) = {} & & +0.02\cos A_3 \\
& +0.01\sin 3A_4 & -0.04\cos 3A_3 \\
& -0.05\sin 4A_3 & +0.15\cos 4A_3 \\
& -0.02\sin 5A_3 & +0.08\cos 5A_3 \\
& -0.02\sin 6A_3 & +0.02\cos 6A_3 \\
& +0.25\sin 7A_3 & -0.46\cos 7A_3 \\
& +0.07\sin 8A_3 & -0.11\cos 8A_3 \\
& -0.05\sin 9A_3 & +0.08\cos 9A_3 \\
& -0.02\sin 10A_3 & +0.03\cos 10A_3 \\
& \qquad +0.00714\delta e &
\end{aligned}
$$

Action of Neptune.—Continued.

$$A_3 = g - l'$$

$$\begin{array}{rl}
(v.c.4) = & -0\overset{\prime\prime}{.}06 \sin 9A_3 - 0\overset{\prime\prime}{.}05 \cos 9A_3 \\
& -0.09 \sin 10A_3 - 0.08 \cos 10A_3 \\
& +0.00044 e\delta g
\end{array}$$

$$\begin{array}{rl}
(v.s.4) = & -0\overset{\prime\prime}{.}05 \sin 9A_3 + 0\overset{\prime\prime}{.}06 \cos 9A_3 \\
& -0.08 \sin 10A_3 + 0.09 \cos 10A_3 \\
& +0.00044 \delta e
\end{array}$$

$$\begin{array}{rrr}
(\rho.c.0) = & & +227 \cos A_3 \\
& & +232 \cos 2A_3 \\
& +3 \sin 3A_3 & -229 \cos 3A_3 \\
& & -59 \cos 4A_3 \\
& & -17 \cos 5A_3 \\
& & -7 \cos 6A_3 \\
& & -3 \cos 7A_3 \\
& +0.01018 \delta e &
\end{array}$$

$$\begin{array}{rrr}
(\rho.c.1) = & +3 \sin A_3 & \\
& +2 \sin 2A_3 & -9 \cos 2A_3 \\
& +23 \sin 3A_3 & -141 \cos 3A_3 \\
& +8 \sin 4A_3 & -39 \cos 4A_3 \\
& -9 \sin 5A_3 & +43 \cos 5A_3 \\
& -3 \sin 6A_3 & +13 \cos 6A_3 \\
& -1 \sin 7A_3 & +5 \cos 7A_3 \\
& -0.43322 \delta e &
\end{array}$$

$$\begin{array}{rrr}
(\rho.s.1) = & -60 \sin A^3 & -3 \cos A_3 \\
& -45 \sin 2A_3 & -2 \cos 2A_3 \\
& -107 \sin 3A_3 & -23 \cos 3A_3 \\
& -29 \sin 4A_3 & -8 \cos 4A_3 \\
& +47 \sin 5A_3 & +9 \cos 5A_3 \\
& +15 \sin 6A_3 & +3 \cos 6A_3 \\
& +5 \sin 7A_3 & +1 \cos 7A_3 \\
& +0.43394 e\delta g &
\end{array}$$

$$\begin{array}{rrr}
(\rho.c.2) = & & -1 \cos A_3 \\
& & -6 \cos 2A_3 \\
& -1 \sin 3A_3 & +6 \cos 3A_3 \\
& -4 \sin 4A_3 & +5 \cos 4A_3 \\
& -6 \sin 5A_3 & +17 \cos 5A_3 \\
& -2 \sin 6A_3 & +7 \cos 6A_3 \\
& +3 \sin 7A_3 & -7 \cos 7A_3 \\
& -0.03048 \delta e &
\end{array}$$

$$\begin{array}{rrr}
(\rho.s.2) = & -5 \sin A_3 & \\
& -8 \sin 2A_3 & \\
& +8 \sin 3A_3 & +1 \cos 3A_3 \\
& +5 \sin 4A_3 & +4 \cos 4A_3 \\
& +17 \sin 5A_3 & +6 \cos 5A_3 \\
& +7 \sin 6A_3 & +2 \cos 6A_3 \\
& -7 \sin 7A_3 & -3 \cos 7A_3 \\
& +0.03053 e\delta g &
\end{array}$$

$$(\rho.c.3) = -0.000202 \delta e \qquad (\rho.s.3) = +0.00203 e\delta g$$

PERTURBATIONS OF THE LATITUDE.

(The secular terms being omitted.)

Action of Jupiter.

$$\begin{array}{rrr}
(b.c.0) = & 0\overset{\prime\prime}{.}024 \sin A_1 & -0\overset{\prime\prime}{.}013 \cos A_1 \\
(b.s.1) = & -0.420 \sin A_1 & +0.494 \cos A_1 \\
(b.c.1) = & +0.494 \sin A_1 & +0.420 \cos A_1 \\
(b.s.2) = & +0.004 \sin 2A_1 & -0.017 \cos 2A_1 \\
(b.c.2) = & -0.017 \sin 2A_1 & -0.004 \cos 2A_1
\end{array}$$

Action of Saturn.

$$(b.c.0)=\begin{array}{rr} {}^{\prime\prime} & {}^{\prime\prime} \\ -0.08\sin A_2 & -0.03\cos A_2 \\ & -0.03\cos 2A_2 \\ & -0.01\cos 3A_2 \end{array}$$

$$(b.s.1)=\begin{array}{rr} {}^{\prime\prime} & {}^{\prime\prime} \\ +1.34\sin A_2 & +2.88\cos A_2 \\ -0.01\sin 2A_2 & -0.10\cos 2A_2 \\ +0.03\sin 3A_2 & -0.06\cos 3A_2 \\ +0.02\sin 4A_2 & -0.03\cos 4A_2 \end{array}$$

$$(b.c.1)=\begin{array}{rr} -1.56\sin A_2 & +2.50\cos A_2 \\ +0.02\sin 2A_2 & -0.10\cos 2A_2 \\ -0.04\sin 3A_2 & -0.06\cos 3A_2 \\ -0.02\sin 4A_2 & -0.02\cos 4A_2 \end{array}$$

$$(b.s.2)=\begin{array}{rr} -0.09\sin A_2 & +0.10\cos A_2 \\ -0.05\sin 2A_2 & -0.09\cos 2A_2 \\ & -0.01\cos 3A_2 \end{array}$$

$$(b.c.2)=\begin{array}{rr} -0.09\sin A_2 & -0.05\cos A_2 \\ +0.07\sin 2A_2 & +0.02\cos 2A_2 \\ +0.01\sin 3A_2 & \end{array}$$

Action of Neptune.

$$(b.c.0)=\begin{array}{rr} {}^{\prime\prime} & {}^{\prime\prime} \\ +0.01\sin A_3 & -0.04\cos A_3 \\ -0.01\sin 2A_3 & +0.00\cos 2A_3 \\ -0.01\sin 3A_3 & +0.04\cos 3A_3 \\ +0.01\sin 4A_3 & +0.01\cos 4A_3 \\ +0.01\sin 5A_3 & \\ +0.01\sin 6A_3 & \end{array}$$

$$(b.s.1)=\begin{array}{rr} {}^{\prime\prime} & {}^{\prime\prime} \\ +0.13\sin A_3 & +0.16\cos A_3 \\ +0.09\sin 2A_3 & +0.26\cos 2A_3 \\ +0.08\sin 3A_3 & +0.28\cos 3A_3 \\ +0.01\sin 4A_3 & +0.03\cos 4A_3 \\ -0.02\sin 5A_3 & -0.12\cos 5A_3 \\ -0.01\sin 6A_3 & -0.03\cos 6A_3 \\ & -0.01\cos 7A_3 \end{array}$$

$$(b.c.1)=\begin{array}{rr} -0.57\sin A_3 & +0.04\cos A_3 \\ -0.39\sin 2A_3 & +0.06\cos 2A_3 \\ -0.33\sin 3A_3 & +0.07\cos 3A_3 \\ -0.07\sin 4A_3 & +0.01\cos 4A_3 \\ +0.10\sin 5A_3 & -0.03\cos 5A_3 \\ +0.03\sin 6A_3 & \\ +0.01\sin 7A_3 & \end{array}$$

$$(b.s.2)=\begin{array}{rr} +0.01\sin A_3 & +0.05\cos A_3 \\ -0.03\sin 3A_3 & -0.09\cos 3A_3 \\ & -0.05\cos 4A_3 \\ & -0.04\cos 5A_3 \\ & +0.01\cos 7A_3 \end{array}$$

$$(b.c.2)=\begin{array}{rr} -0.07\sin A_3 & \\ +0.09\sin 3A_3 & -0.03\cos 3A_3 \\ +0.05\sin 4A_3 & \\ +0.04\sin 5A_3 & \\ +0.01\sin 6A_3 & \\ -0.01\sin 7A_3 & \end{array}$$

Action of Jupiter and Saturn.

(Terms multiplied by the product of their masses.)

$$\begin{aligned} N_1 &= 2S-J \\ N_2 &= -U+3S-J \\ N_3 &= -2U+4S-J \\ N_4 &= 3U+3S-2J \\ N_5 &= 2U+4S-2J \\ N_6 &= 5S-2J \\ N_7 &= -3U+6S-2J \\ N_8 &= -3U+7S-2J \end{aligned}$$

Action of Jupiter and Saturn —Continued.

(Terms multiplied by the product of their masses.)

$$
\begin{array}{rl}
(v.c.0) = & +\ 0^{\prime\prime}.08 \sin N_2 \quad +\ 0^{\prime\prime}.51 \cos N_2 \\
 & +\ 0.04 \sin N_3 \quad +\ 0.01 \cos N_3 \\
 & -\ 0.01 \sin N_4 \quad +\ 0.05 \cos N_4 \\
 & -\ 0.35 \sin N_5 \quad -\ 1.30 \cos N_5 \\
 & \left\{ \begin{array}{l} -\ 0.55 \sin N_6 \quad -\ 0.03 \cos N_6 \\ +40.65 \sin N_7 \quad -10.50 \cos N_7 \end{array} \right\} \\
 & -\ 0.05 \sin N_8 \quad +\ 0.03 \cos N_8
\end{array}
$$

$$
\begin{array}{rl}
(v.c.1) = & +\ 0.06 \sin N_1 \quad -\ 0.27 \cos N_1 \\
 & +\ 0.18 \sin N_2 \quad +\ 0.01 \cos N_2 \\
 & -\ 0.03 \sin N_3 \quad +\ 0.08 \cos N_3 \\
 & -\ 0.02 \sin N_4 \quad +\ 0.09 \cos N_4 \\
 & -\ 0.44 \sin N_5 \quad -\ 0.61 \cos N_5 \\
 & \left\{ \begin{array}{l} -\ 4.23 \sin N_6 \quad -\ 3.87 \cos N_6 \\ +\ 8.06 \sin N_7 \quad -\ 8.38 \cos N_7 \end{array} \right\} \\
 & -\ 0.10 \sin N_8 \quad +\ 0.03 \cos N_8
\end{array}
\qquad
\begin{array}{rl}
(v.s.1) = & +0^{\prime\prime}.26 \sin N_1 \quad +0^{\prime\prime}.27 \cos N_1 \\
 & -0.04 \sin N_2 \quad -0.17 \cos N_2 \\
 & +0.08 \sin N_3 \quad +0.03 \cos N_3 \\
 & -0.02 \sin N_4 \quad +0.08 \cos N_4 \\
 & +0.30 \sin N_5 \quad -0.58 \cos N_5 \\
 & \left\{ \begin{array}{l} +2.64 \sin N_6 \quad +4.64 \cos N_6 \\ +7.35 \sin N_7 \quad +4.41 \cos N_7 \end{array} \right\} \\
 & -0.04 \sin N_8 \quad +0.00 \cos N_8
\end{array}
$$

$$
(v.c.2) = \left\{ \begin{array}{l} -0.24 \sin N_6 \ -0.22 \cos N_6 \\ +0.47 \sin N_7 \ -0.54 \cos N_7 \end{array} \right\}
\qquad
(v.s,2) = \left\{ \begin{array}{l} +0.16 \sin N_6 \ +0.26 \cos N_6 \\ +0.54 \sin N_7 \ +0.47 \cos N_7 \end{array} \right\}
$$

$$(\rho.c.0) = +11 \sin N_5 \quad -3 \cos N_5$$

Two of these arguments, namely, $5S - 2J$, and $-3g + 6S - 2J$, are of very long period, that of the first being about 880, and that of the second about 1590 years. It will, therefore, be convenient to tabulate them both as functions of the time for the time during which the theory is to be used. To make their effect as small as possible during the period for which the provisional ephemeris is to be computed, we shall suppose the longitude of epoch, mean motion, and longitude of the perigee to be affected with the negative of the following corrections:

$$
\begin{aligned}
\delta\varepsilon &= +\ 27^{\prime\prime}.27, \\
\delta\pi &= +\ 27.27, \\
\delta n &= -\ \ 0.1172.
\end{aligned}
$$

Reducing these corrections to corrections of the co-ordinates, and adding them to the terms of long period in the true longitude and logarithm of radius vector, we shall have for these terms,

$$
\begin{aligned}
(v.c.0) = &-\ 0^{\prime\prime}.546 \sin N_6 - \ 0^{\prime\prime}.032 \cos N_6 \\
&+ 40.650 \sin N - 10.500 \cos N_7 + 27.27 - 11.72\ T
\end{aligned}
$$

$$(v.s.1) = \quad 2^{\prime\prime}.63 \sin N_6 + 4^{\prime\prime}.64 \cos N_6 + 7^{\prime\prime}.35 \sin N_7 + 4^{\prime\prime}.42 \cos N_7$$

$$(v.c.1) = -\ 4.22 \sin N_6 - 3.87 \cos N_6 + 8.06 \sin N_7 - 8.39 \cos N_7 \ - 1.10 T$$

$$(v.s.2) = +\overset{\prime\prime}{0.16}\sin N_6 + \overset{\prime\prime}{0.26}\cos N_6 + \overset{\prime\prime}{0.54}\sin N_7 + \overset{\prime\prime}{0.47}\cos N_7$$
$$(v.c.2) = -0.24\sin N_6 - 0.22\cos N_6 + 0.47\sin N_7 - 0.54\cos N_7 - 0''.07\,T$$

$$(\rho.c.0) = +\ 2\cos N_6 + 10\sin N_7 + 32\cos N_7 + 22$$
$$(\rho.s.1) = -41\sin N_6 - 43\cos N_6 + 82\sin N_7 - 85\cos N_7 - 12\,T$$
$$(\rho.c.1) = -29\sin N_6 - 45\cos N_6 - 73\sin N_7 - 44\cos N_7$$

The values of these and of the other secular terms and terms of long period for the period during which Uranus has been observed, are given in the following table:

(v.c.0)

	Neptune (long per.)	Jupiter and Saturn (long per.)	Sum.
	″	″	″
1700	+85.54	+4.86	+90.40
1750	+38.45	+1.90	+40.35
1760	31.25	1.48	32.73
1770	24.80	1.11	25.91
1780	19.09	0.80	19.89
1790	14.12	0.54	14.66
1800	9.89	+0.33	10.22
1810	6.42	0.17	6.59
1820	3.69	+0.07	3.76
1830	1.71	0.00	1.71
1840	+ 0.48	—0.02	+ 0.46
1850	0.00	0.00	0.00
1860	+ 0.27	+0.06	+ 0.33
1870	1.29	+0.16	1.45
1880	+ 3.06	+0.30	+ 3.36

VALUES OF (v.s.1)

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	″	″	″	″	″	″
1700	+1.82	+13.77	+0.75	—212.73	—10.18	—206.57
1750	+1.22	+ 9.18	+0.50	—141.97	— 9.11	—140.18
1760	1.10	8.26	0.45	—127.76	— 8.83	—126.78
1770	0.97	7.34	0.40	—113.55	— 8.54	—113.38
1780	0.85	6.43	0.35	— 99.33	— 8.24	— 99.94
1790	0.73	5.51	0.30	— 85.11	— 7.93	— 86.50
1800	0.61	4.59	0.25	— 70.90	— 7.61	— 73.06
1810	0.49	3.67	0.20	— 56.69	— 7.28	— 59.61
1820	0.36	2.76	0.15	— 42.49	— 6.94	— 46.16
1830	0.24	1.84	0.10	— 28.31	— 6.60	— 32.73
1840	+0.12	+ 0.92	+0.05	— 14.14	— 6.26	— 19.31
1850	0.00	0.00	0.00	0.00	— 5.91	— 5.91
1860	—0.12	— 0.92	—0.05	+ 14.12	— 5.57	+ 7.46
1870	—0.24	— 1.84	—0.10	28.23	— 5.23	20.82
1880	—0.36	— 2.76	—0.15	+ 42.32	— 4.89	+ 34.16

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Values of (*v.c.*1)

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	″	″	″	″	″	″
1700	+17.19	+16.67	+7.20	+37.63	—1.98	+76.71
1750	+11.46	+11.11	+4.80	+28.67	—2.28	+53.76
1760	10.31	10.00	4.32	26.46	—2.36	48.73
1770	9.17	8.89	3.84	24.10	—2.45	43.55
1780	8.02	7.78	3.36	21.60	—2.53	38.23
1790	6.88	6.67	2.88	18.95	—2.61	32.77
1800	5.73	5.56	2.40	16.16	—2.69	27.16
1810	4.58	4.44	1.92	13.23	—2.76	21.41
1820	3.44	3.33	1.44	10.15	—2.83	15.53
1830	2.29	2.22	0.96	6.92	—2.90	9.49
1840	+ 1.15	+ 1.11	+0.48	+ 3.53	—2.97	+ 3.30
1850	0.00	0.00	0.00	0.00	—3.03	— 3.03
1860	— 1.15	— 1.11	—0.48	— 3.68	—3.08	— 9.50
1870	— 2.29	— 2.22	—0.96	— 7.51	—3.13	—16.11
1880	— 3.44	— 3.33	—1.44	—11.49	—3.16	—22.86

(*v.s.*2) const. = — 0″.14.

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	+0.11	+0.81	+0.04	—12.46	—0.72	—12.36
1750	+0.07	+0.54	+0.03	—8.31	—0.70	—8.51
1760	0.06	0.49	0.03	—7.48	—0.69	—7.73
1770	0.06	0.43	0.02	—6.64	—0.68	—6.95
1780	0.05	0.38	0.02	—5.81	—0.66	—6.16
1790	0.04	0.32	0.02	—4.98	—0.65	—5.39
1800	0.04	0.27	0.02	—4.15	—0.64	—4.60
1810	0.03	0.22	0.01	—3.32	—0.63	—3.83
1820	0.02	0.16	0.01	—2.49	—0.61	—3.05
1830	0.01	0.11	+0.01	—1.66	—0.60	—2.27
1840	+0.01	+0.05	0.00	—0.83	—0.58	—1.49
1850	0.00	0.00	0.00	0.00	—0.57	—0.71
1860	—0.01	—0.05	0.00	+0.82	—0.56	+0.06
1870	—0.01	—0.11	—0.01	1.65	—0.54	0.84
1880	—0.02	—0.16	—0.01	+2.48	—0.53	+1.62

(*v.c.*2) const. = + 0″.13.

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	+1.00	+0.98	+0.42	+2.20	—0.11	+4.62
1750	+0.67	+0.65	+0.28	+1.68	—0.12	+3.29
1760	+0.60	0.59	0.25	1.55	—0.12	3.00
1770	+0.54	0.52	0.22	1.41	—0.12	2.70
1780	+0.47	0.46	0.20	1.26	—0.13	2.39
1790	+0.40	0.39	0.17	1.11	—0.13	2.07
1800	+0.34	0.33	0.14	0.94	—0.13	1.75
1810	+0.27	0.26	0.11	0.77	—0.14	1.40
1820	+0.20	0.20	0.08	0.59	—0.14	1.06
1830	+0.13	0.13	0.06	0.39	—0.14	0.70
1840	+0.07	+0.07	+0.03	+0.20	—0.15	+0.35
1850	0.00	0.00	0.00	0.00	—0.15	—0.02
1860	—0.07	—0.07	—0.03	—0.21	—0.15	—0.40
1870	—0.13	—0.13	—0.06	—0.43	—0.16	—0.78
1880	—0.20	—0.20	—0.08	—0.68	—0.16	—1.19

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$(v.s.3)$

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	″	″	″	″	″	″
1700	+0.01	+0.05	0.00	—0.76	0	—0.70
1750	0.00	+0.03	0	—0.51	0	—0.48
1760	0	0.03	0	—0.46	0	—0.43
1770	0	0.03	0	—0.40	0	—0.37
1780	0	0.02	0	—0.35	0	—0.33
1790	0	0.02	0	—0.30	0	—0.28
1800	0	0.02	0	—0.25	0	—0.23
1810	0	0.01	0	—0.20	0	—0.19
1820	0	0.01	0	—0.15	0	—0.14
1830	0	+0.01	0	—0.10	0	—0.09
1840	0	0.00	0	—0.05	0	—0.05
1850	0	0.00	0	0.00	0	0.00
1860	0	0.00	0	+0.05	0	+0.05
1870	0	—0.01	0	+0.10	0	+0.09
1880	0	—0.01	0	+0.15	0	+0.14

$(v.c.3)$

	(1)	(2)	(3)	(4)		Sum.
	″	″	″	″		″
1700	+0.06	+0.06	+0.02	+0.13		+0.27
1750	+0.04	+0.04	+0.02	+0.10		+0.20
1760	0.04	0.04	0.01	+0.09		+0.18
1770	0.03	0.03	0.01	+0.08		+0.15
1780	0.03	0.03	0.01	+0.07		+0.14
1790	0.02	0.02	0.01	+0.06		+0.12
1800	0.02	0.02	0.01	+0.06		+0.11
1810	0.02	0.02	+0.01	+0.05		+0.09
1820	0.01	0.01	0.00	+0.04		+0.06
1830	+0.01	+0.01	0.00	+0.02		+0.04
1840	0.00	0.00	0.00	+0.01		+0.02
1850	0.00	0.00	0.00	0.00		0.00
1860	0.00	0.00	0.00	—0.01		—0.02
1870	—0.01	—0.01	0.00	—0.02		—0.04
1880	—0.01	—0.01	0.00	—0.04		—0.06

FOR THE RADIUS VECTOR. VALUES OF $(\rho.c.0)$

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	+0.4	+3.5	—0.2	+164	+13	+181
1750	+0.3	+2.3	—0.1	+110	7	+120
1760	0.3	2.1	0	99	6	107
1770	0.2	1.9	0	88	5	95
1780	0.2	1.6	0	77	4	83
1790	0.2	1.4	0	67	3	72
1800	0.2	1.2	0	56	2	60
1810	0.1	0.9	0	45	2	48
1820	0.1	0.7	0	34	+ 1	36
1830	+0.1	0.5	0	22	0	23
1840	.0	+0.2	0	+ 11	0	+ 11
1850	.0	0.0	0	0	— 1	— 1
1860	.0	—0.2	0	— 11	— 2	— 13
1870	—0.1	—0.5	0	22	— 2	— 25
1880	—0.1	—0.7	0	— 33	— 3	— 37

(ρ.*s*.1) const. = + 63.

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	″	″	″	″	″	″
1700	+180	+176	+75	+396	—21	+869
1750	+120	+117	+50	+302	—25	+627
1760	108	105	45	280	26	575
1770	96	94	40	255	28	520
1780	84	82	35	228	29	463
1790	72	70	30	200	30	405
1800	60	59	25	170	31	346
1810	48	47	20	139	32	285
1820	36	35	15	106	33	222
1830	24	23	10	72	34	158
1840	+ 12	+ 12	+ 5	+ 37	34	95
1850	0	0	0	0	35	+ 28
1860	— 12	— 12	— 5	— 39	36	— 41
1870	24	23	10	80	36	—110
1880	— 36	— 35	—15	—122	—37	—182

(ρ.*c*.1) const. = + 73.

	(1)	(2)	(3)	(4)	(5)	Snm.
	″	″	″	″	″	″
1700	—20	—147	—9	+2241	+104	+2242
1750	—13	— 98	—6	+1496	+ 94	+1546
1760	—12	— 88	—5	1346	91	1405
1770	—10	— 78	—5	1197	88	1265
1780	— 9	— 69	—4	1047	85	1123
1790	— 8	— 59	—4	897	82	981
1800	— 6	— 49	—3	747	78	840
1810	— 5	— 39	—2	598	74	699
1820	— 4	— 29	—2	448	71	557
1830	— 3	— 20	—1	299	68	416
1840	— 1	— 10	—1	+ 149	64	274
1850	0	0	0	0	61	+ 134
1860	+ 1	+ 10	+1	— 149	58	— 6
1870	3	20	1	298	54	— 147
1880	4	29	2	— 447	+ 51	— 288

(ρ.*s*.2) const. = + 5.

	(1)	(2)	(3)	(4)	(5)	Sum.
	″	″	″	″	″	″
1700	+12	+12	+4	+28	0	+61
1750	+ 8	+ 8	+6	+21	0	+48
1760	7	7	5	19	0	43
1770	6	6	5	17	0	39
1780	6	6	4	15	0	36
1790	5	5	4	13	0	32
1800	4	4	3	11	0	27
1810	3	3	2	8	0	21
1820	2	2	2	6	0	17
1830	2	2	1	4	0	14
1840	+ 1	+ 1	+1	+ 2	0	10
1850	0	0	0	0	0	+ 5
1860	— 1	— 1	—1	— 2	0	0
1870	2	— 2	—1	4	0	— 4
1880	— 2	— 2	—2	— 6	0	— 7

($\rho . c . 2$) const. $= + 4$

	(1) Jupiter (sec.)	(2) Saturn (sec.)	(3) Neptune (sec.)	(4) Neptune (long per.)	(5) Jupiter & Saturn (long per.)	Sum.
	″	″	″	″	″	″
1700	—1	—10	0	+157	0	+150
1750	—1	— 7	0	+105	0	+101
1760	—1	— 6	0	95	0	92
1770	—1	— 6	0	84	0	81
1780	—1	— 5	0	74	0	72
1790	—1	— 4	0	63	0	62
1800	0	— 3	0	52	0	53
1810	0	— 3	0	42	0	43
1820	0	— 2	0	31	0	33
1830	0	— 1	0	21	0	24
1840	0	— 1	0	+ 10	0	13
1850	0	0	0	0	0	+ 4
1860	0	+ 1	0	— 10	0	— 5
1870	0	1	0	21	0	—16
1880	0	+ 2	0	— 31	0	—25

Reduced Expressions for the Latitude of Uranus.

If we represent by V_1, V_2, V_3 the distances of Uranus from its descending nodes on the respective orbits of Jupiter, Saturn, and Uranus, we find the following perturbations of the latitude, which are independent of the mean longitude of the disturbing planets.

$$\begin{aligned} \delta\beta = & - \overset{\prime\prime}{0}.0114t \cos V_1 \\ & - 0.0477t \cos V_2 \\ & - 0.0125t \cos V_3 \end{aligned}$$

$$\begin{aligned} & + \overset{\prime\prime}{0}.245 \\ & + 0.386 \sin g + \overset{\prime\prime}{0}.266 \cos g \\ & - 0.043 \sin 2g + 0.006 \cos 2g. \end{aligned}$$

To find how far the last five terms may be represented by simple corrections to the elliptic elements, we first represent the effect of minute corrections to the inclination and node of Uranus as a function of its mean anomaly. Putting u for the argument of latitude of Uranus, we have to a sufficient degree of approximation

$$\begin{aligned} \delta\beta & = \sin u \delta\phi - \sin \phi \cos u \delta\theta \\ u & = g + \omega + 2e \sin g \end{aligned}$$

$$\begin{aligned} \sin u = & - e \sin \omega \\ & + \cos \omega \sin g + \sin \omega \cos g \\ & + e \cos \omega \sin 2g + e \sin \omega \cos 2g \end{aligned}$$

$$\begin{aligned}\cos u = &-e\cos\omega\\ &+\cos\omega\cos g-\sin\omega\sin g\\ &+e\cos\omega\cos 2g-e\sin\omega\sin 2g.\end{aligned}$$

Substituting these values of $\sin u$ and $\cos u$ in the expression for $\delta\beta$, and putting $\sin\phi\delta\theta=\delta'\theta$, we have

$$\begin{aligned}\delta\beta = &\ e\cos\omega\delta'\theta-e\sin\omega\,\delta\phi\\ &+(\cos\omega\delta\phi+\sin\omega\delta'\theta)\sin g+(\sin\omega\delta\phi-\cos\omega\delta'\theta)\cos g\\ &+(e\cos\omega\delta\phi+e\sin\omega\delta'\theta)\sin 2g+(e\sin\omega\delta\phi-e\cos\omega\delta'\theta)\cos 2g.\end{aligned}$$

To represent the numerical coefficients of $\sin g$ and $\cos g$ in $\delta\beta$ we must put

$$\begin{aligned}\cos\omega\delta\phi+\sin\omega\delta'\theta&=0''.386\\ \sin\omega\delta\phi-\cos\omega\delta'\theta&=0\ .266.\end{aligned}$$

Since $\omega=95^\circ\ 3'$, this gives

$$\begin{aligned}\delta\phi&=\overset{''}{0.231};\\ \delta'\theta&=0.409;\\ \delta\beta&=-0.013\\ &\quad+0.386\sin g+\overset{''}{0.266}\cos g\\ &\quad+0.018\sin 2g+0.013\cos 2g\end{aligned}$$

Subtracting this expression from the corresponding terms of $\delta\beta$, we have left

$$\delta\beta=+0''.258-0''.061\sin 2g-0''.007\cos 2g.$$

The first term of this expression shows that the mean orbit of Uranus at the present time is a small circle of the sphere one-quarter of a second north of its parallel great circle.

If we put

v = longitude of Uranus in its orbit, referred to the equinox and ecliptic of 1850, we have

$$\begin{aligned}V_1&=v-127^\circ\ 37'\\ V_2&=v-126\ \ 45\\ V_3&=v-155\ \ 32\end{aligned}$$

Substituting these values in the first three terms of $\delta\beta$, and multiplying the last term by the factor $(1+\mu)$ by which the adopted mass of Neptune, $\frac{1}{17000}$, must be multiplied to obtain the true mass, we find

$$\delta\beta=(4''.69+1''.14\mu)\,T\cos v-(5''.24+0''.52\mu)\,T\sin v.$$

To these terms must be added those which arise from the motion of the ecliptic.

In the absence of any exhaustive investigation of the obliquity and motion of the ecliptic, I adopt the elements of Hansen, employed in his "*Tables du Soleil*," because they are a mean between the results of others, and are very accordant with recent observations. The secular motion of the obliquity there employed is

$$-46''.78.$$

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Hansen mentions — 5″.39 as the corresponding motion at the equinox of 1850, found by Olufsen, but I cannot reproduce this result from the secular diminution with any masses of Mercury, Venus, and Mars, which seem to me probable. The expressions in terms of the masses given by Le Verrier are (*Annales de l'Observatoire Imperial de Paris*, tome ii, p. 101),

$$\begin{aligned}\text{Secular change} &= -\overset{''}{47}.59 - \overset{''}{0}.52\nu - \overset{''}{28}.90\nu' - \overset{''}{0}.83\nu''' \\ \text{Mot. at equinox} &= +\ 5.89 + 0.62\nu +\ 7.57\nu' + 0.73\nu'''.\end{aligned}$$

In this expression the masses of Mercury, Venus, and Mars are represented by $\frac{1+\nu}{3{,}000{,}000}$, $\frac{1+\nu'}{401{,}847}$, and $\frac{1+\nu'''}{2{,}680{,}337}$, respectively. The influence of admissible changes in the masses of the other plants is insensible.

From the researches of Le Verrier on the motions of the four inner planets I conclude that the following are about the most probable distribution of the corrections of the masses necessary to produce the motion of the obliquity given by Hansen, namely,

$$\begin{aligned}\nu &= -\ \tfrac{2}{5} \\ \nu' &= -.018 \\ \nu''' &= -\ \tfrac{1}{10}\end{aligned}$$

These values give for the motion at the equinox of 1850

$$+\,5''.43$$

Introducing the secular variation of these motions we have, for the change in the latitude of any celestial body near the ecliptic, arising from motion of the ecliptic,

$$\delta\beta = (5''.43\,T + 0''.19\,T^2)\cos v + (46''.78\,T - 0''.06\ T^2)\sin v.$$

Combining this with the change arising from the motion of the orbit of Uranus, we find

$$\begin{aligned}\delta\beta = &\{(10''.12 + 1''.14\mu)\ T + 0''.19\ T^2\}\cos v \\ &+ \{(41''.54 - 0''.52\mu)\ T - 0''.06\ T^2\}\sin v.\end{aligned}$$

We may represent these expressions in the usual way by secular variations of the inclination and node of Uranus. But, owing to the small inclination, and consequent rapid motion of the node, it will be necessary to include the coefficients of the second power of the time. On the other hand, no distinction between τ and θ is necessary. Putting ϕ for the inclination of the orbit, θ for the longitude of the node referred to the equinox of 1850, and

$$\begin{aligned}p &= \sin\phi\sin\theta, \\ q &= \sin\phi\cos\theta;\end{aligned}$$

we have

$$\begin{aligned}\sin\beta &= -\ p\cos v +\ q\sin v \\ \cos\beta\,\delta\beta &= -\ \delta p\cos v + \delta q\sin v.\end{aligned}$$

From the expressions for p and q we obtain

$$\begin{aligned} \cos\phi\, D_t\phi &= \sin\theta\, D_tp + \cos\theta\, D_tq; \\ \sin\phi\, D_t\theta &= \cos\theta\, D_tp - \sin\theta\, D_tq. \end{aligned}$$

And, neglecting $(D_t\phi)^2 \times \sin\phi$, we have farther,

$$\begin{aligned} \cos\phi D^2{}_t\phi &= \sin\phi\,(D_t\theta)^2 + \sin\theta D^2{}_tp + \cos\theta D^2{}_tq; \\ \sin\phi D^2{}_t\theta &= -2\cos\phi D_t\theta D_t\phi + \cos\theta D^2{}_tp - \sin\theta D^2{}_tq. \end{aligned}$$

Since ϕ is only 46′ we may put $\cos\phi$ and $\cos\beta$ both equal to unity in these expressions, while we have, for 1850,

$$\begin{aligned} \sin\theta &= \quad 0.9573 \\ \cos\theta &= \quad 0.2890 \\ D_tp &= -10''.12 - 1''.14\mu \\ D_tq &= +41.54 - 0.52\mu \\ D^2{}_tp &= -\ 0.38 \\ D^2{}_tq &= -\ 0.12 \\ \log\sin\phi &= \quad 8.129606. \end{aligned}$$

The above formulæ then give

$$\begin{aligned} D_t\phi &= +\ 2''.31 \quad - 1''.24\mu \\ D_t\theta &= -3167''.5 + 12''.6\mu \\ D^2{}_t\phi &= +\ 0''.26 \\ D^2{}_t\theta &= +\ 5''.6 \end{aligned}$$

$$\begin{aligned} \phi &= \phi_0 + (2''.31 \quad - 1''.24\mu)\,T + 0''.13\,T^2 \\ \theta &= \theta_0 - (3167''.5 - 12''.6\mu)\,T + 2.8\,T^2, \end{aligned}$$

or, adding Struve's precession, we have when θ is counted from the mean equinox of date,

$$\theta = \theta_0 + (1857''.7 + 12''.6\mu)\,T + 3''.9\,T^2.$$

Using the values of ϕ and θ given by these expressions, the latitude, secular variation included, will be given by the expression

$$\sin\beta = \sin\phi\,\sin(v - \theta).$$

If we take from a table, as the principal term of the latitude, the value of $\sin\phi_0\,\sin(v - \theta)$, the secular term to be added will be

$$\{(2''.31 - 1''.24\mu)\,T + 0''.13\,T^2\}\,\sin(v - \theta).$$

If we represent, as before, by ω the variable distance of the perihelion from the node, this term will be allowed for by adding to $(b.s.1)$, $(b.c.1)$, etc., the terms

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$$\begin{aligned}(b.c.0) &= -e \sin \omega \delta\phi,\\ (b.s.1) &= \cos \omega \delta\phi,\\ (b.c.1) &= \sin \omega \delta\phi,\\ (b.s.2) &= e \cos \omega \delta\phi,\\ (b.c.2) &= e \sin \omega \delta\phi;\end{aligned}$$

where

$$\delta\phi = (2''.31 - 1''.24\mu)\, T + 0''.13\, T^2$$

Putting in the above expressions

$$\begin{aligned}\omega &= 95^\circ 3' + 3459''\, T,\\ \cos \omega &= -.0880 - .0167\, T,\\ \sin \omega &= +.9961 - .0015\, T,\end{aligned}$$

we find

$$\begin{aligned}(b.c.0) &= -(0''.11 - 0''.06\mu)\, T\\ (b.s.1) &= -(0\ .20 - 0\ .11\mu)\, T - 0''.05\, T^2\\ (b.c.1) &= (2\ .30 - 1\ .24\mu)\, T + 0\ .12\, T^2\\ (b.s.2) &= -0\ .01\, T\\ (b.c.2) &= (0\ .11 - 0\ .06\mu)\, T.\end{aligned}$$

We have, finally, to consider the terms of long period in $\delta\eta$ and δk which have been omitted from the periodic perturbations produced by Neptune, in computing the terms of $\delta\beta$ on page 61, and which are as follows:

$$\begin{aligned}\delta\eta = {} & 1''.43 \cos(2l' - g) - 0''.39 \sin(2l' - g)\\ & - 2\ .12 \cos(4l' - 2g) + 1\ .00 \sin(4l' - 2g)\\ & + 0\ .20 \cos(6l' - 3g) - 0\ .04 \sin(6l' - 3g)\\ & + \text{constant} = 0''.00\end{aligned}$$

$$\begin{aligned}\delta k = {} & 0''.80 \cos(2l' - g) - 2''.28 \sin(2l' - g)\\ & - 1\ .06 \cos(4l' - 2g) - 1\ .85 \sin(4l' - 2g)\\ & + 0\ .04 \cos(6l' - 3g) + 0\ .19 \sin(6l' - 3g)\\ & + \text{constant} = 0''.364.\end{aligned}$$

For the period during which Uranus has been observed, these values of $\delta\eta$ and δk may be replaced by the following:

$$\begin{aligned}\delta\eta &= -0''.80\, T\\ \delta k &= +0\ .27\, T\end{aligned}$$

which are to be multiplied by the factor $1 + \mu$. The corresponding perturbation of the latitude will be

$$\delta\beta = \sin \mathrm{v}\, \delta\eta - \cos \mathrm{v}\, \delta k.$$

Putting for v its approximate value

$$\mathrm{v} = g + \omega + 2e \sin g$$

and developing to quantities of the first order with respect to the eccentricities, we have

$$\begin{aligned}\sin \mathrm{v} &= \sin(g + \omega) + e \sin(2g + \omega) - e \sin \omega\\ \cos \mathrm{v} &= \cos(g + \omega) + e \cos(2g + \omega) - e \cos \omega.\end{aligned}$$

Substituting for ω its value, 12°45′, and for $\delta\eta$ and δk their above values in the expression for $\delta\beta$, we find that the terms of $\delta\beta$ in question will add the following terms to $(b.c.0)$, $(b.s.1)$, etc.

$$\begin{aligned}
(b.c.0) &= -.010\,\delta\eta + .046\,\delta k = +0''.02T(1+\mu)\\
(b.s.1) &= +.975\,\delta\eta + .221\,\delta k = -0\,.72T(1+\mu)\\
(b.c.1) &= +.221\,\delta\eta - .975\,\delta k = -0\,.44T(1+\mu)\\
(b.s.2) &= +.046\,\delta\eta + .011\,\delta k = -0\,.04T(1+\mu)\\
(b.c.2) &= -(b.c.0) \qquad\qquad\qquad\;\; = -0\,.02T(1+\mu)
\end{aligned}$$

These values will be employed in the construction of the provisional ephemeris, but not in the tables.

Collecting all three classes of terms discussed in this section, we have the following constant and secular terms in $(b.c.0)$, $(b.s.1)$, etc.

$$\begin{aligned}
(b.c.0) &= +0''.26 + (-0''.09 + 0.08\mu)T\\
(b.s.1) &= (-0''.92 - 0''.61\mu)T - 0''.05T^2\\
(b.c.1) &= (+1\,.86 - 1\,.68\mu)T + 0\,.12T^2\\
(b.s.2) &= -0\,.06 - 0\,.05T\\
(b.c.2) &= -0\,.01 + (0\,.09 - .08\mu)T
\end{aligned}$$

Positions of Uranus resulting from the preceding theory.

The next step in order is the preparation of an ephemeris of the planet for comparison with observations. As this provisional theory is, for future use, superseded by the tables appended to the present work, it seems unnecessary to enter very fully into the details of the computation of the ephemeris. The perturbations of the longitude, logarithm of radius vector, and latitude, were first computed by the formulæ already given.

$$\begin{aligned}
\delta v &= (v.c.0) + (v.c.1)\cos g + (v.c.2)\cos 2g + \text{etc.},\\
&\quad + (v.s.1)\sin g + (v.s.2)\sin 2g + \text{etc.},\\
M\delta\rho &= (\rho.c.0) + (\rho.c.1)\cos g + (\rho.c.2)\cos 2g + \text{etc.},\\
&\quad + (\rho.s.1)\sin g + (\rho.s.2)\sin 2g + \text{etc.},\\
\delta\beta &= (b.c.0) + (b.c.1)\cos g + (b.s.1)\sin g.
\end{aligned}$$

Each coefficient $(v.c.0)$, $(v.c.1)$, etc., is composed at most of the following quantities:

1. The five classes of secular, long period, or constant terms, the separate values of which, with the sum of all, are given on pages 89 to 93.
2. Periodic terms due to the action of Jupiter, Saturn, and Neptune, given on pages 83 to 87.
3. Terms depending on the product of the masses of Jupiter and Saturn, given on page 88, omitting those depending on N_6 and N_7, because they are given in column 5 of the terms of the first class.

The sum of the perturbations thus computed is given in the third column of the following ephemeris.

An approximate value of the perturbations produced by Neptune alone is independently computed for every fourth date, and the result is given in the fourth

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column. The secular and long period terms are here taken from columns (3) and (4) of the tables on pages 89 to 93.

The elliptic co-ordinates were then derived from the following elements, which are a little different from those employed in the computation of the perturbations.

Elements III. of Uranus.

π,	168° 15′ 12″.0
ε,	28 25 29 .5
θ,	73 11 58 .0
ϕ,	0 46 20 .0
e,	.0469436
e, (in sec.)	9682″.81
n,	15426.196
log a,	1.2828989
Red. to Ecliptic,	$-9''.37 \sin 2(v-\theta)$

The longitudes thus found are corrected for lunar, but not for solar nutation, and the results are given in the fifth column.

The column "correction" arises in this way: after the comparison of the ephemeris with observations was nearly completed, it was found that some errors had crept into the former, the most important of which was the employment of a mean anomaly, g, corrected for secular variation of the perihelion in the computation of the perturbations from the preceding formulæ. As a large portion of the computations on the provisional ephemeris had been made by assistants furnished by the Smithsonian Institution and Nautical Almanac, I deemed it prudent to make a careful recomputation of the perturbations for every sixth date during the entire period of the modern observations. The longitudes actually printed in the fifth column are the results of the original incorrect computation, while the numbers in the next column show the several corrections to be applied to obtain the results of my final revised computation.

During the period of the modern observations the ephemeris is computed for intervals of 120 days, and the selected dates are all exact multiples of that interval before or after the fundamental epoch, 1850, Jan. 0, Greenwich mean noon. For convenience of reference the dates are numbered from an epoch earlier by 212 intervals, and the number is given in the second column.

Between 1796 and 1801 no observations worth using were made on Uranus, the ephemeris has, therefore, not been extended over this interval.

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HELIOCENTRIC EPHEMERIS OF URANUS FROM THE PRECEDING PROVISIONAL THEORY.

[The longitudes are corrected for lunar but not for solar nutation.]

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1690, Dec. 23		+4 28.5	+236.1	59 40 35.2		—10 7.8	1.2876828
Dec. 24		+4 28.5		59 41 16.5		—10 7.3	1.2876789
1712, April 2		+8 37.9		155 24 42.6		+45 58.7	1.2626650
April 3		+8 37.9		155 25 29.0		+45 58.8	1.2626640
1715, Mar. 4		+6 58.3	+136.2	169 10 21.4		+46 2.1	1.2623890
Mar. 5		+6 58.2		169 11 7.7		+46 2.1	1.2623893
Mar. 10		+6 57.5		169 15 0.0		+46 1.7	1.2623911
April 29		+6 51.1	+134.8	169 53 42.2		+45 58.0	1.2624066
April 30		+6 51.1		169 54 28.8		+45 57.9	1.2624102
1748, Oct. 21		—0 28.9	— 65.0	316 13 19.3		—41 27.6	1.3001855
Oct. 22		—0 28.8		316 13 58.8		—41 27.9	1.3001875
1750, Sept. 13		+0 6.3	— 51.3	323 44 14.9		—43 47.2	1.3013714
Sept. 14		+0 6.3		323 44 54.0		—43 47.3	1.3013731
Oct. 14		+0 6.6	— 50.8	324 4 22.1		—43 52.4	1.3014165
Oct. 15		+0 6.6		324 5 1.0		—43 52.5	1.3014181
Dec. 3		+0 8.0	— 49.8	324 36 49.5		—44 0.6	1.3014856
Dec. 4		+0 8.0		324 37 28.5		—44 0.8	1.3014872
1753, Dec. 3		—0 5.6	— 25.6	336 25 0.6		—46 1.1	1.3025789
Dec. 4		—0 5.6		336 25 39.3		—46 1.2	1.3025796
1756, Sept. 25		—0 49.5	— 0.4	347 25 34.9		—46 9.2	1.3029363
Sept. 26		—0 49.5		347 26 13.6		—46 9.1	1.3029363
1764, Jan. 15		+1 24.0		16 13 35.0		—38 36.9	1.3003508
Jan. 16		+1 24.0		16 14 14.2		—38 36.7	1.3003486
1768, Dec. 27		+1 33.1		36 3 35.2		—27 42.4	1.2955331
Dec. 29		+1 33.1	+106.9	36 4 44.1		—27 41.5	1.2955274
Dec. 31		+1 33.1		36 6 5.0		—27 40.7	1.2955216
1769, Jan. 15		+1 33.5		36 16 4.6		—27 34.2	1.2954753
Jan. 18		+1 33.5		36 18 4.5		—27 33.0	1.2954659
Jan. 21		+1 33.5	+106.9	36 20 4.5		—27 31.6	1.2954566
Jan. 24		+1 33.5		36 22 4.4		—27 30.3	1.2954476
1781, Jan. 1	1	+3 57.46	+178.9	86 36 13.23	+0.16	+11 3.63	1.2785972
May 1	2	+3 53.41		88 2 34.70		+12 11.14	1.2780883
Aug. 29	3	+3 48.58		89 29 7.41		+13 18.33	1.2775834
Dec. 27	4	+3 43.09		90 55 51.40		+14 25.14	1.2770827
1782, April 26	5	+3 37.18	+180.4	92 22 46.89		+15 31.53	1.2765870
Aug. 24	6	+3 30.97		93 49 53.89		+16 37.47	1.2760968
Dec. 22	7	+3 24.52		95 17 12.51	+0.18	+17 42.88	1.2756124
1783, April 21	8	+3 18.03		96 44 42.73		+18 47.73	1.2751345
Aug. 19	9	+3 11.87	+180.1	98 12 24.96		+19 51.92	1.2746633
Dec. 17	10	+3 6.01		99 40 19.00		+20 55.67	1.2742012
1784, April 15	11	+3 0.78		101 8 25.13		+21 58.69	1.2737412
Aug. 13	12	+2 56.24		102 36 43.29		+23 0.92	1.2732907
Dec. 11	13	+2 52.82	+178.6	104 5 13.79	+0.23	+24 2.46	1.2728469
1785, April 10	14	+2 50.58		105 33 56.61		+25 3.14	1.2724105
Aug. 8	15	+2 49.29		107 2 51.36		+26 3.05	1.2719806
Dec. 6	16	+2 49.35		108 31 58.18		+27 1.98	1.2715564
1786, April 5	17	+2 50.53	+175.5	110 1 16.88		+27 59.97	1.2711405
Aug. 3	18	+2 53.29		111 30 47.68		+28 57.02	1.2707299
Dec. 1	19	+2 57.31		113 0 30.11	+0.31	+29 53.03	1.2703265
1787, Mar. 31	20	+3 2.38		114 30 23.81		+30 47.92	1.2699294
July 29	21	+3 8.51	+171.0	116 0 28.62		+31 41.72	1.2695394
Nov. 26	22	+3 15.48		117 30 44.11		+32 34.33	1.2691551

Heliocentric Ephemeris of Uranus.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″		′ ″	
1788, Mar. 25	23	+3 23.02		119 1 9.79		+33 25.72	1.2687775
July 23	24	3 31.36		120 31 45.79		34 15.81	1.2684065
Nov. 20	25	3 39.82	+165.1	122 2 31.13	+0.28	35 4.58	1.2680423
1789, Mar. 20	26	3 38.67		123 33 25.96		35 51.99	1.2676852
July 18	27	3 57.40		125 4 29.49		36 38.01	1.2673353
Nov. 15	28	4 5.97		126 35 41.50		37 22.50	1.2669932
1790, Mar. 15	29	4 14.35	+157.8	128 7 1.70		38 5.50	1.2666592
July 13	30	4 22.48		129 38 29.88		38 47.00	1.2663336
Nov. 10	31	4 30.00		131 10 5.37	+0.25	39 26.85	1.2660167
1791, Mar. 10	32	4 36.95		132 41 47.99		40 5.04	1.2657087
July 8	33	4 42.95	+149.1	134 13 37.15		40 41.58	1.2654082
Nov. 5	34	4 48.11		135 45 32.62		41 16.44	1.2651215
1792, Mar. 4	35	4 52.33		137 17 34.16		41 49.53	1.2648434
July 2	36	4 56.03		138 49 41.78		42 20.89	1.2645770
Oct. 30	37	4 58.32	+139.3	140 21 54.47	+0.46	42 50.35	1.2643210
1793, Feb. 27	38	4 59.50		141 54 12.11		43 18.00	1.2640766
June 27	39	4 59.85		143 26 34.79		43 43.81	1.2638448
Oct. 25	40	4 58.84		144 59 1.64		44 7.75	1.2636267
1794, Feb. 22	41	4 56.80	+128.6	146 31 32.77		44 29.74	1.2634226
June 22	42	4 53.48		148 4 7.47		44 49.85	1.2632327
Oct. 20	43	4 49.13	+122.9	149 36 45.84	+0.47	45 7.99	1.2630577
1795, Feb. 17	44	4 44.20		151 9 27.73		45 24.24	
June 17	45	4 38.00		152 42 12.55		45 38.44	
Oct. 15	46	4 31.04		154 15 0.15		45 50.64	
1801, Jan. 17	62	+3 22.92		179 3 50.23		+44 34.51	1.2627599
Mar. 17	63	3 26.59	+ 57.3	180 36 54.94		44 12.86	1.2628920
Sept. 14	64	3 31.06		182 9 57.40		43 49.24	1.2630362
1802, Jan. 12	65	3 36.26		183 42 57.23		43 23.66	1.2631922
Mar. 12	66	3 41.88		185 15 54.08		42 56.34	1.2633599
Sept. 9	67	3 47.78	+ 43.7	186 48 46.54	+0.35	42 26.97	1.2635387
1803, Jan. 7	68	3 54.01		188 21 35.11		41 55.86	1.2637281
Mar. 7	69	4 0.29		189 54 18.55		41 22.99	1.2639280
Sept. 4	70	4 6.43		191 26 56.80		40 48.28	1.2641386
1804, Jan. 2	71	4 12.95	+ 30.7	192 59 30.15		40 11.79	1.2643591
Mar. 1	72	4 18.74		194 31 56.83		39 33.63	1.2645891
Aug. 29	73	4 24.38		196 4 17.13	+0.14	38 53.72	1.2648287
Dec. 27	74	4 29.34		197 36 30.24		38 12.21	1.2650777
1805, April 26	75	4 33.60	+ 18.0	199 8 35.97		37 29.07	1.2653360
Aug. 24	76	4 37.42		200 40 33.91		36 44.40	1.2656037
Dec. 22	77	4 40.11		202 12 23.39		35 58.15	1.2658808
1806, April 21	78	4 41.83		203 44 4.25		35 10.48	1.2661674
Aug. 19	79	4 42.55	+ 5.9	205 15 36.11	+0.21	34 21.34	1.2664631
Dec. 17	80	4 42.19		206 46 58.67		33 30.88	1.2667689
1807, April 16	81	4 40.52		208 18 11.39		32 39.05	1.2670847
Aug. 14	82	4 37.43		209 49 13.98		31 45.96	1.2674103
Dec. 12	83	4 33.71	— 5.0	211 20 6.96		30 51.63	1.2677459
1808, April 10	84	4 28.30		212 50 49.02		29 56.12	1.2680922
Aug. 8	85	4 21.63		214 21 20.46	+0.25	28 59.51	1.2684491
Dec. 6	86	4 14.32		215 51 41.72		28 1.80	1.2688166
1809, April 5	87	4 5.84	— 15.4	217 21 51.97		27 3.03	1.2691945
Aug. 3	88	3 56.63		218 51 51.53		26 3.28	1.2695829
Dec. 1	89	3 47.07		220 21 40.62		25 2.66	1.2699816
1810, Mar. 31	90	3 37.16		221 51 19.10		24 1.10	1.2703905
July 29	91	3 27.15	— 24.3	223 20 47.07	+0.23	22 58.68	1.2708094
Nov. 26	92	+3 17.52		224 50 4.83		+21 55.38	1.2712377

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HELIOCENTRIC EPHEMERIS OF URANUS.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1811, Mar. 26	93	+3 7.99		226 19 12.04		+20 51.52	1.2716752
July 24	94	2 59.12		227 48 9.06		19 46.95	1.2721215
Nov. 21	95	2 50.91	—32.1	229 16 55.87		18 41.69	1.2725760
1812, Mar. 20	96	2 43.45		230 45 32.41		17 35.80	1.2730381
July 18	97	2 37.26		232 13 59.13	+0.11	16 29.34	1.2735072
Nov. 15	98	2 31.98		233 42 15.55		15 22.35	1.2739838
1813, Mar. 15	99	2 27.86	—39.1	235 10 21.88		14 14.92	1.2744651
July 13	100	2 25.05		236 38 18.26		13 7.03	1.2749507
Nov. 10	101	2 22.98		238 6 3.95		11 58.79	1.2754409
1814, Mar. 10	102	2 22.35		239 33 39.61		10 50.21	1.2759367
July 8	103	2 22.54	—44.6	241 1 4.63	+0.22	9 41.32	1.2764360
Nov. 5	104	2 24.24		242 28 19.59		8 32.19	1.2769372
1815, Mar. 5	105	2 26.58		243 55 23.67		7 22.86	1.2774403
July 3	106	2 30.00		245 22 17.17		6 13.39	1.2779459
Oct. 31	107	2 34.17	—49.3	246 48 59.79		5 3.80	1.2784533
1816, Feb. 28	108	2 39.06		248 15 31.47		3 54.09	1.2789600
June 27	109	2 44.51		249 41 52.08	—0.01	2 44.46	1.2794688
Oct. 25	110	2 50.43		251 8 1.42		1 34.82	1.2799725
1817, Feb. 22	111	2 56.75	—54.4	252 33 59.49		+ 0 25.25	1.2804769
June 22	112	3 3.22		253 59 46.00		— 0 44.19	1.2809792
Oct. 20	113	3 10.03		255 25 21.18		1 53.44	1.2814788
1818, Feb. 17	114	3 16.63		256 50 44.48		3 2.48	1.2819759
June 17	115	3 22.96	—54.7	258 15 55.79	0.00	4 11.23	1.2824701
Oct. 15	116	3 28.90		259 40 55.14		5 19.73	1.2829599
1819, Feb. 12	117	3 34.28		261 5 42.25		6 27.83	1.2834467
June 12	118	3 38.77		262 30 17.04		7 35.52	1.2839317
Oct. 10	119	3 42.48	—56.1	263 54 39.42		8 42.77	1.2844143
1820, Feb. 7	120	3 45.03		265 18 49.19		9 49.52	1.2848927
June 6	121	3 46.37		266 42 46.29	—0.01	10 55.76	1.2853670
Oct. 4	122	3 46.38		268 6 30.67		12 1.40	1.2858391
1821, Feb. 1	123	3 45.07	—56.4	269 30 2.43		13 6.44	1.2863087
June 1	124	3 42.35		270 53 21.55		14 10.88	1.2867757
Sept. 29	125	3 38.08		272 16 27.96		15 14.57	1.2872402
1822, Jan. 27	126	3 32.86		273 39 22.29		16 17.58	1.2877030
May 27	127	3 26.18	—56.2	275 2 4.25	—0.10	17 19.84	1.2881632
Sept. 24	128	3 18.32		276 24 34.06		18 21.32	1.2886214
1823, Jan. 22	129	3 9.68		277 46 52.38		19 22.01	1.2890772
May 22	130	3 0.24		279 8 59.05		20 21.89	1.2895311
Sept. 19	131	2 50.36	—56.1	280 30 54.82		21 20.91	1.2899826
1824, Jan. 17	132	2 40.10		281 52 39.71		22 19.10	1.2904315
May 16	133	2 29.74		283 14 14.16	—0.07	23 16.40	1.2908776
Sept. 13	134	2 19.49		284 35 38.60		24 12.78	1.2913208
1825, Jan. 11	135	2 9.25	—55.5	285 56 52.92		25 8.22	1.2917606
May 11	136	1 59.46		287 17 57.80		26 2.71	1.2921967
Sept. 3	137	1 50.21		288 38 53.43		26 56.22	1.2926290
1826, Jan. 6	138	1 41.44		289 59 40.04		27 48.71	1.2930570
May 6	139	1 33.58	—54.5	291 20 18.03	—0.09	28 40.26	1.2934804
Sept. 3	140	1 26.44		292 40 47.50		29 30.76	1.2938987
1827, Jan. 1	141	1 20.21		294 1 8.83		30 20.16	1.2943116
May 1	142	1 14.89		295 21 22.15		31 8.60	1.2947186
Aug. 29	143	1 10.52	—54.0	296 41 27.72		31 55.92	1.2951196
Dec. 27	144	1 7.22		298 1 25.72		32 42.14	1.2955136
1828, April 25	145	1 4.79		299 21 16.45	—0.07	33 27.24	1.2959002
Aug. 23	146	1 3.51		300 41 0.14		34 11.25	1.2962790
Dec. 21	147	+1 3.11	—52.9	302 0 36.75		—34 54.11	1.2966496

HELIOCENTRIC EPHEMERIS OF URANUS.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1829, April 20	148	+1 3.68		303 20 6.73		—35 35.77	1.2970111
Aug. 18	149	1 5.15		304 39 30.08		36 16.26	1.2973630
Dec. 16	150	1 7.37		305 58 46.96		36 55.60	1.2977052
1830, April 15	151	1 10.14	—52.3	307 17 57.34	+0.08	37 33.68	1.2980368
Aug. 13	152	1 13.74		308 37 1.72		38 10.56	1.2983571
Dec. 11	153	1 17.63		309 55 59.75		38 46.13	1.2986661
1831, April 10	154	1 21.69		311 14 51.68		39 20.49	1.2989638
Aug. 8	155	1 26.06	—52.2	312 33 37.76		39 53.54	1.2992501
Dec. 6	156	1 30.26		313 52 17.74		40 25.29	1.2995247
1832, April 4	157	1 34.13		315 10 51.63	+0.05	40 55.74	1.2997873
Aug. 2	158	1 37.50		316 29 19.67		41 24.90	1.3000379
Nov. 30	159	1 40.39	—52.2	317 47 41.94		41 52.65	1.3002768
1833, Mar. 30	160	1 42.02		319 5 57.95		42 19.10	1.3005047
July 28	161	1 42.86		320 24 8.31		42 44.17	1.3007216
Nov. 25	162	1 42.45		321 42 12.89		43 7.88	1.3009280
1834, Mar. 25	163	1 40.92	—52.5	323 0 11.92	+0.15	43 30.24	1.3011231
July 23	164	1 38.01		324 18 5.47		43 51.18	1.3013090
Nov. 20	165	1 34.01		325 35 53.99		44 10.74	1.3014851
1835, Mar. 20	166	1 28.76		326 53 37.59		44 28.92	1.3016515
July 18	167	1 22.19	—52.9	328 11 16.27		44 45.69	1.3018084
Nov. 15	168	1 14.89		329 28 50.92		45 1.14	1.3019567
1836, Mar. 14	169	1 6.50		330 46 21.41	+0.19	45 15.12	1.3020960
July 12	170	0 57.31		332 3 48.20		45 27.71	1.3022265
Nov. 9	171	0 47.51	—53.9	333 21 11.76		45 38.98	1.3023482
1837, Mar. 9	172	0 37.27		334 38 32.35		45 48.84	1.3024612
July 7	173	0 26.69		315 55 50.40		45 57.28	1.3025656
Nov. 4	174	0 15.92		337 13 6.16		46 4.45	1.3026622
1838, Mar. 4	175	+0 5.23	—55.1	338 30 20.17	+0.17	46 10.04	1.3027503
July 2	176	—0 5.21		339 47 32.76		46 14.36	1.3028297
Oct. 30	177	0 15.76		341 4 43.81		46 17.29	1.3029008
1839, Feb. 27	178	0 25.58		342 21 54.35		46 18.83	1.3029634
June 27	179	0 34.82	—56.9	243 39 4.45		46 19.00	1.3030176
Oct. 25	180	0 43.68		344 56 14.08		46 17.84	1.3030630
1840, Feb. 22	181	0 51.42		346 13 24.23	+0.17	46 15.28	1.3030996
June 21	182	0 58.36		347 30 34.70		46 11.29	1.3031269
Oct. 19	183	1 4.34	—58.4	348 47 46.09		46 6.01	1.3031447
1841, Feb. 16	184	1 9.47		350 4 58.23		45 59.35	1.3031528
June 16	185	1 13.36		351 22 11.79		45 51.33	1.3031509
Oct. 14	186	1 16.11		352 39 26.97		45 41.91	1.3031392
1842, Feb. 11	187	1 17.73	—60.5	353 56 43.91	+0.15	45 31.07	1.3031164
June 11	188	1 18.13		355 14 2.89		45 18.90	1.3030817
Oct. 9	189	1 17.35		256 31 24.13		45 5.38	1.3030351
1843, Feb. 6	190	1 15.50		357 48 47.74		44 50.50	1.3029767
June 6	191	1 12.83	—66.7	359 6 13.65		44 34.22	1.3029060
Oct. 4	192	1 9.27		0 23 42.19		44 16.64	1.3028228
1844, Feb. 1	193	1 4.76		1 41 13.59	+0.07	43 57.59	1.3027267
May 31	194	0 59.91		2 58 47.55		43 37.23	1.3026173
Sept. 28	195	0 54.61	—64.9	4 16 24.34		43 15.53	1.3024947
1845, Jan. 26	196	0 49.19		5 34 3.83		42 52.43	1.3023588
May 26	197	0 43.90		6 51 46.06		42 27.99	1.3022095
Sept. 23	198	0 38.92		8 9 31.09		42 2.22	1.3020472
1846, Jan. 21	199	0 34.37	—67.7	9 27 19.04	+0.15	41 35.11	1.3018722
May 21	200	0 30.55		10 45 9.82		41 6.76	1.3016851
Sept. 18	201	—0 27.21		12 3 3.39		—40 37.01	1.3014861

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HELIOCENTRIC EPHEMERIS OF URANUS.—*Continued.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ″	″	′ ″	
1847, Jan. 16	202	—0 25.64		13 21 0.27		—40 6.03	1.3012758
May 16	203	0 24.73	—70.2	14 39 0.21		39 33.75	1.3010544
Sept. 13	204	0 24.95		15 57 3.61		39 0.23	1.3008219
1848, Jan. 11	205	0 26.43		17 15 10.46	+0.06	38 25.40	1.3005790
May 10	206	0 28.95		18 33 21.30		37 49.42	1.3003263
Sept. 7	207	0 32.75	—72.8	19 51 36.15		37 12.18	1.3000640
1849, Jan. 5	208	0 37.59		21 9 55.34		36 33.73	1.2997923
May 5	209	0 43.47		22 28 19.02		35 54.16	1.2995115
Sept. 2	210	0 50.27		23 46 48.13		35 13.36	1.2992224
Dec. 31	211	0 57.74	—75.3	25 5 22.56	—0.08	34 31.47	1.2989252
1850, April 30	212	1 6.18		26 24 2.38		33 48.40	1.2986203
Aug. 28	213	1 15.11		27 42 48.39		33 4.37	1.2983078
Dec. 26	214	1 24.38		29 1 40.87		32 19.06	1.2979881
1851, April 25	215	1 34.33	—77.9	30 20 39.78		31 32.79	1.2976615
Aug. 23	216	1 44.28		31 39 45.91		30 45.46	1.2973287
Dec. 21	217	1 54.21		32 58 59.55	—0.09	29 57.09	1.2969894
1852, April 19	218	2 4.12		34 18 20.89		29 7.71	1.2966436
Aug. 17	219	2 13.61	—80.5	35 37 50.43		28 17.33	1.2962916
Dec. 15	220	2 23.00		36 57 28.19		27 25.95	1.2959338
1853, April 14	221	2 31.54		38 17 15.06		26 33.62	1.2955701
Aug. 12	222	2 39.58		39 37 10.77		25 40.35	1.2952015
Dec. 10	223	2 46.62	—82.6	40 57 16.17	—0.07	24 46.17	1.2948270
1854, April 9	224	2 52.64		42 17 31.26		23 51.08	1.2944458
Aug. 7	225	2 57.77		43 37 56.24		22 55.08	1.2940583
Dec. 5	226	3 1.60		44 58 31.59		21 58.25	1.2936645
1855, April 4	227	3 4.03	—84.6	46 19 17.49		21 0.51	1.2932645
Aug. 2	228	3 5.35		47 40 13.93		20 1.97	1.2928582
Nov. 30	229	3 5.32		49 1 21.26	—0.11	19 2.66	1.2924449
1856, Mar. 29	230	3 3.95		50 22 39.57		18 2.55	1.2920239
July 27	231	3 1.24	—86.1	51 44 9.08		17 1.67	1.2915954
Nov. 24	232	2 57.62		53 5 49.32		16 0.03	1.2911592
1857, Mar. 24	233	2 52.95		54 27 40.66		14 57.72	1.2907155
July 22	234	2 47.58		55 49 42.98		13 54.71	1.2902643
Nov. 19	235	2 41.65	—87.9	57 11 56.10	—0.10	12 51.08	1.2898057
1858, Mar. 19	236	2 35.18		58 34 20.16		11 46.89	1.2893395
July 17	237	2 28.82		59 56 54.59		10 41.96	1.2888659
Nov. 14	238	2 22.46		61 19 39.71		9 36.91	1.2883853
1859, Mar. 14	239	2 16.35	—88.6	62 42 35.14		8 30.87	1.2878981
July 12	240	2 10.33		64 5 41.38		7 24.61	1.2874049
Nov. 9	241	2 5.17		65 28 57.59	—0.13	6 17.90	1.2869061
1860, Mar. 8	242	2 0.67		66 52 24.15		5 10.84	1.2864023
July 6	243	1 57.10	—89.1	68 16 0.90		4 3.42	1.2858941
Nov. 3	244	1 54.37		69 39 47.78		2 55.74	1.2853816
1861, Mar. 3	245	1 52.61		71 3 44.98		1 47.83	1.2848658
July 1	246	1 51.92		72 27 52.38		— 0 39.69	1.2843468
Oct. 29	247	1 52.43	—89.3	73 52 10.31	—0.19	+ 0 28.60	1.2838257
1862, Feb. 26	248	1 53.51		75 16 38.24		1 37.00	1.2833029
June 26	249	1 55.99		76 41 16.65		2 45.46	1.2827788
Oct. 24	250	1 59.54		78 6 5.46		3 53.95	1.2822542
1863, Feb. 21	251	2 3.98	—88.5	79 31 4.97		5 2.40	1.2817290
June 21	252	2 9.43		80 56 15.07		6 10.80	1.2812047
Oct. 19	253	2 15.63		82 21 36.01	—0.04	7 19.09	1.2806819
1864, Feb. 16	254	2 22.52		83 47 7.95		8 27.25	1.2801609
June 15	255	2 30.20	—87.2	85 12 50.83		9 35.22	1.2796421
Oct. 13	256	—2 38.39		86 38 44.88		+ 8 42.95	1.2791263

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HELIOCENTRIC EPHEMERIS OF URANUS.—*Concluded.*

Date. Greenwich mean noon.	No.	Sum of perturbations.	Approximate perturbations produced by Neptune.	Longitude.	Correction.	Latitude.	Logarithm Radius vector.
		′ ″	″	° ′ ″	″	′ ″	
1865, Feb. 10	257	—2 47.06		88 4 50.26		+11 52.20	1.2786142
June 10	258	2 55.93		89 31 7.17		12 57.54	1.2781063
Oct. 8	259	3 5.01	—85.2	90 57 35.61	—0.09	14 4.34	1.2776029
1866, Feb. 5	260	3 13.91		92 24 16.00		15 10.74	1.2771044
June 5	261	3 22.61		93 51 8.39		16 16.70	1.2766112
Oct. 3	262	3 30.79		95 18 13.06		17 22.18	1.2761237
1867, Jan. 31	263	3 38.49	—82.5	96 45 29.91		18 27.15	1.2756419
May 31	264	3 45.25		98 12 59.44		19 31.54	1.2751656
Sept. 28	265	3 50.96		99 40 41.71	—0.08	20 35.37	1.2746948
1868, Jan. 26	266	3 55.67		101 8 36.54		21 38.51	1.2742299
May 25	267	3 59.06	—79.1	102 36 44.31		22 40.96	1.2737709
Sept. 22	268	4 1.02		104 5 5.00		23 42.75	1.2733175
1869, Jan. 20	269	4 1.71		105 33 38.41		24 43.69	1.2728693
May 20	270	4 0.81		107 2 24.67		25 43.81	1.2724264
Sept. 17	271	3 58.59	—75.1	108 31 23.55	—0.11	26 43.07	1.2719888
1870, Jan. 15	272	3 54.96		110 0 34.97		27 41.44	1.2715563
May 15	273	3 50.23		111 29 58.50		28 38.77	1.2711290
Sept. 12	274	3 44.34		112 59 33.91		29 35.16	1.2707070
1871, Jan. 10	275	3 37.90	—69.8	114 29 20.79		30 30.41	1.2702903
May 10	276	3 30.56		115 59 19.11		31 24.57	1.2698791
Sept. 7	277	3 22.61		117 29 28.39	—0.18	32 17.57	1.2694735
1872, Jan. 5	278	3 14.50		118 59 48.05		33 9.31	1.2690738
May 4	279	3 6.19	—64.2	120 30 17.95		33 59.79	1.2686804
Sept. 1	280	—2 57.65		122 0 57.87		+34 48.97	1.2682934

The next operation would be to interpolate these co-ordinates to intervals of time suitable for the computation of a geocentric ephemeris, to correct the longitudes for solar nutation, and then to compute the geocentric right ascension and declination. This operation has not, however, been completely carried out except for most of the observations before 1830, and for three of the oppositions observed since, the latter being computed only as a check upon the accuracy of the comparisons. As a general rule, it may be said that wherever a complete geocentric ephemeris, with the heliocentric ephemeris from which it was computed, were available, these ephemerides were made use of in a manner which will be more fully described hereafter, while, in all other cases, the geocentric places were computed directly.

It may also be stated here that Hansen's *Tables du Soleil* have been adopted as giving the places of the sun to be used in computing the geocentric places.

CHAPTER VI.

REDUCTION OF THE OBSERVATIONS OF URANUS, AND THEIR COMPARISON WITH THE PRECEDING THEORY.

THE observations of Uranus naturally divide themselves into two distinct classes. (1) The purely accidental ones, made previous to the recognition of the planet by Herschel in 1781, and therefore without any suspicion on the part of the observers that the object was not a fixed star, and (2) the systematic observations made since.

The first class are nearly all so uncertain in comparison with the second that I have hesitated over the question of employing them at all. If nothing but a determination of the elements of Uranus were called for, they would certainly not be worth using, since these elements may be determined with entire certainty from the observations which have been made during the entire revolution of the planet since 1781. But the mass of Neptune is also to be determined, and it is at least possible that these observations, uncertain though they are, may add materially to the weight of this determination. I have, therefore, determined to include them all, re-reducing them when there seemed to be good reason so to do.

The earliest observations are those of Flamstead, published in the Historiæ Cœlestis. The observations themselves, as printed, together with the principal elements for reduction, are given in the following tables.

The first column of the table gives the name of the star. The second gives the clock time of transit over the wire of the quadrant as given by Flamstead. The time, it will be seen, is only given to entire seconds. We must, therefore, expect to find a probable error, of which the mathematical minimum is $0^s.25$, and of which the minimum we can reasonably expect is much greater.

Next we have the apparent right ascensions of the stars as computed. For these data I am indebted to Prof. Coffin, Superintendent of the American Ephemeris. The mean places are mostly derived from the "Star Tables of the American Ephemeris," and from the two Greenwich Seven Year Catalogues, while the reduction to apparent place is made with the modern constants.

The fourth column gives the apparent clock correction for sidereal time, in which is included the effect of deviation of the instrument from the meridian.

The clock keeping mean time, the errors are in the next column reduced to those of sidereal time at the moment of the transit of Uranus.

The next two columns give the corrections for clock rate, and for deviation of the instrument from the meridian, as inferred from the observations themselves, both being referred to the time and position of the transit of Uranus.

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In the last column we have the seconds of concluded correction for clock and instrument to be applied to the observed time of transit of Uranus.

1690, *December* 23. *Right Ascension.*

Star.	Time of Tr.	R. A. of star.	Clock.	C′.	R.	Dev.	C″.
	h. m. s.	h. m. s.	h. m. s.	m. s.	s.	s.	s.
α Arietis,	7 48 40	1 49 52.1	—5 58 47.9	58 29.3	—1.1	—1.2	31.6
π Arietis,	8 30 54	2 32 8.4	—5 58 45.6	58 33.9	—0.7	+1.7	32.9
σ Arietis,	8 33 17	2 34 31.0	—5 58 46.0	58 34.7	—0.7	+2.9	32.5
ε Arietis,	8 40 21	2 41 38.0	—5 58 43.0	58 33.0	—0.6	—0.2	33.8
δ Arietis,	8 52 44	2 54 3.0	—5 58 41.0	58 33.0	—0.5	+0.5	33.0
η Tauri,	9 27 47	3 29 13.6	—5 58 33.4	58 31.1	—0.1	—1.3	32.5
Uranus,	9 41 49						
A Tauri,	9 45 3	3 46 31.1	—5 58 31.9	58 32.4	0.0	—0.8	33.2
α Virginis,	19 4 15	13 8 58.2	—5 55 16.8	56 49.3			
α Bootis,	19 58 31½	14 1 34.0	—5 56 57.5	58 39.0			

Hourly rate of clock,	—0^{s}.6
Deviation of instrument for each degree of Z. D.,	—0.5
	h. m. s.
Transit of Uranus,	9 41 49.0
Correction for clock and instrument (mean),	—5 58 32.8
Observed R. A. of the planet,	3 43 16.2

1690, *December* 23. *Declination.*

	Z. D. observed.	Refraction.	Declination.	Eq. point.
α Arietis,	29° 29′ 10″	+0′ 33″	21° 58′ 53″	51° 28′ 36″
π Arietis,	35 18 55	+0 41	16 9 4	51 28 40
σ Arietis,	37 41 0	+0 44	13 46 58	51 28 42
ε Arietis,	31 23 15	+0 35	20 4 37	51 28 27
δ Arietis,	32 55 55	+0 37	18 31 40	51 28 12
η Tauri,	28 20 55	+0 31	23 6 53	51 28 19
Uranus,	31 52 35	+0 36		
A Tauri,	30 15 55	+0 34	21 12 3	51 28 32

Circle reading for Uranus, corrected for refraction,	31° 53′ 11″
Equatorial point on circle,	51 28 30
Declination of Uranus, from observation,	+19 35 19

1712, *April* 2. *Right Ascension.*

	h. m. s.	h. m. s	m. s.
Uranus,	9 35 19		
ε Virginis,	12 0 19	12 47 52.4	+47 33.4
e Virginis,	12 14 51	13 2 31.5	+47 40.5
ζ Virginis,	12 32 11	13 20 5.7	+47 54.7

The discordance of clock errors, and the time which intervened between the transit of the planet and that of the first star, seem to render an accurate reduction impossible.

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1715, *March* 4. *Right Ascension.*

	T.	R. A.	C.	C″.
	h. m. s.	h. m. s.	h. m. s.	s.
d Leonis,	11 50 19	10 45 52.5	—1 4 26.5	20.5
Uranus,	12 27 1			
b Virginis,	12 49 41	11 45 23.6	—1 4 17.4	21.1

	h. m. s.
Clock time of transit of Uranus,	12 27 1
Correction for clock and instrument,	—1 4 20.8
Right ascension of Uranus from observation,	11 22 40.2

1715, *March* 4. *Declination.*

	Z. D.	R.	Dec.	Eq. point.
d Leonis,	46° 19′ 40″	+1′ 0″	+5° 8′ 6″	51° 28′ 46″
Uranus,	46 33 10	+1 1		
b Virginis,	46 13 20	+1 0	5 14 17	51 28 37

Circle reading for Uranus,	46° 34′ 11″
Equatorial point,	51 28 42
Observed declination of Uranus,	+4 54 31

1715, *March* 5. *Right Ascension.*

	T.	R. A.	C.	C′
	h. m. s.	h. m. s.	h. m. s.	s.
d Leonis,	11 46 24	10 45 52.5	—1 0 31.5	25.5
Uranus,	12 22 59::			
b Virginis,	12 45 49	11 45 23.6	—1 0 25.4	29.1

	h. m. s.
Transit of Uranus,	12 22 59
Correction for clock and instrument,	—1 0 27.7
Observed right ascension of Uranus,	11 22 31.5

The large apparent clock rate, and the colons after the time of transit, both throw doubt on this observation.

Declination.

The circle readings for the stars are the same as on the day preceding, while that for Uranus is 50″ less. The declination is therefore 50″ greater, or

+4° 55′ 21″.

1715, *March* 10. *Right Ascension.*

	T.	R. A.	C.	C′.
	h. m. s.	h. m. s.	h. m. s.	s.
d Leonis,	11 25 58	10 45 52.5	—0 40 5.5	59.5
p^3 Leonis,	11 32 28	10 52 25.1	—0 40 2.9	58.1
Uranus,	12 1 42			
b Virginis,	12 25 18	11 45 23.6	—0 39 54.4	58.2

	h. m. s.
Clock time of transit of Uranus,	12 1 42
Correction for clock and instrument,	—0 39 58.6
Observed right ascension of Uranus,	11 21 43.4

Declination.

	Z. D.	R.	Dec.	Eq. Pt.
d Leonis,	46° 19′ 35″	+1′ 1″	5° 8′ 6″	51° 28′ 42″
p^3 Leonis,	47 58 35	+1 3	3 29 25	51 28 63
Uranus,	46 27 0	+1 1		
b Virginis,	46 13 25	+1 0	5 14 17	51 28 42

Circle reading for Uranus,	46 28 1
Equatorial point on circle,	51 28 49
Observed declination of Uranus,	+5 0 48

1715. *April* 29. *Right Ascension.*

	T.	R. A.	C.	C′.
	h. m. s.	h. m. s.	h. m. s.	s.
σ Leonis,	8 42 11	11 6 28.3	+2 24 17.3	18.7
Uranus,	8 50 44			
ν Virginis,	9 6 55	11 31 14.1	+2 24 19.1	16.4
17 Virginis,	9 43 38::	12 8 6.5	+2 24 28.5	19.8
ϰ Virginis,	11 32 48	13 57 47.6	+2 24 59.6	33.1

The discordance of the clock corrections makes a satisfactory determination of the right ascension very difficult. I deem it best to reject the doubtful observation of 17 Virginis, and the discordant one of ϰ Virginis. The result will then be

	h. m. s.
Observed transit of Uranus,	8 50 44
Correction for clock and instrument,	2 24 17.6
Observed right ascension of Uranus,	11 15 1.6

Declination.

	Z. D.	R.	Dec.	Eq. Pt.
σ Leonis,	43° 52′ 40″	+0′ 55″	7° 34′ 51″	51° 28′ 26″
Uranus,	45 45 30	+0 59		
ν Virginis,	43 20 20	+0 54	8 7 11	51 28 25
17 Virginis,	44 34 10	+0 56	6 53 33	51 28 39
ϰ Virginis,	60 23 5	+1 41	—8 55 42	51 28 64

Circle reading for Uranus,	45° 46′ 29″
Mean equatorial point,	51 28 38
Observed declination of Uranus,	+5 42 9

The next observations in the order of time are two by Bradley, discovered by Mr. Hugh Breen, but still unpublished. The following are the results as given by Mr. Breen in the *Astronomische Nachrichten*, No. 1463.

Mean Time.		R. A.	N. P. D.
	h. m. s.	h. m. s.	° ′ ″
1748, October 21,	7 6 18.4	21 4 37.93	107 29
1750, September 13,	10 8 57.8	21 40 0.23	104 42 33.9

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Mr. Breen remarks: "The right ascensions are very accurate. It has been assumed that the N. P. D., on 1750, September 13, is identical with μ Capricorni, with which it was compared. The first observation was by the transit instrument, and the second by the quadrant."

No ground is given for the above assumption respecting the N. P. D. for the second observation; it may, therefore, be omitted as valueless.

In the year 1750 we have also two observations by Le Monnier at Paris. For these, and all the other observations by the same observer, I shall adopt the results given by Bouvard in the *Connaissance des Temps*, for 1821, p. 341, with the corrections indicated by Le Verrier, in *Connaissance des Temps*, for 1849, pp. 125 and 126. The necessary uncertainty of the observations is such that, considering that Bouvard reduced them with the star positions of the "*Fundamenta*," scarcely anything will be gained by a new reduction.

1753, December 3, we have another observation of right ascension by Bradley. I adopt the result kindly communicated by my distinguished friend, Dr. Auwers.

1753, December 3, $5^{\text{h.}}\ 33^{\text{m.}}$ R. A. $= 22^{\text{h.}}\ 23^{\text{m.}}\ 21^{\text{s.}}.59$

1756, September 25. Observation by Mayer, at Gottingen. I adopt the result given by Bessel, in *Fundamenta Astronomiæ*, p. 284.

1756, September 25, $10^{\text{h.}}\ 12^{\text{m.}}$ R. A. $= 348^{\circ}\ 0'\ 54''.5$
Dec. $= -6^{\circ}\ 1'\ 49''.4$

The following is a tabular summary of the preceding results, with their comparison with the provisional theory. In the computation of the geocentric place the places of the sun were derived from Hansen's Tables. I am indebted to Professor Coffin for a duplicate computation of the geocentric places from the provisional ephemeris, which was executed by Mr. Joseph A. Rogers.

Date.	Right Ascension.		Declination.		Correction to theory.				
	Observation.	Theory.	Observation.	Theory.	R. A.	Dec.	Long.	$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	h m s	s	° ′ ″	″	″	″	″		
1690, Dec. 23	3 43 16.2	14.7	+19 35 19	7	+22	+12	+24	1.04	+.027
1715, Mar. 4	11 22 40.2	38.7	4 54 31	48	+22	—17	+28	1.06	0
Mar. 5	11 22 31.5	29.1	4 55 21	49	+36	—28	+44	1.06	0
Mar. 10	11 21 43.4	41.0	5 0 48	56	+36	— 8	+36	1.06	0
Apr. 29	11 15 1.6	1.5	+ 5 42 9	11	+ 1	— 2	+ 2	1.04	+.036
1748, Oct. 21	21 4 37.93	35.42	....	...	+37.6	....	+39.3	1.015	+.050
1750, Sept. 13	21 40 0.23	57.90	....	...	+35.0	....	+35.7	1.05	+.022
	° ′ ″	″							
Oct. 14	324 15 24.6	47.6	—15 1 40.4	47.0	+37.0	+ 6.6	+35.9	1.03	+.045

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Date.	Right Ascension.		Declination.		Correction to theory.				
	Observation.	Theory.	Observation.	Theory.	R. A.	Dec.	Long.	$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	° ′ ″	″	° ′ ″	′	″	″	″		
1750, Dec. 3	324 34 53.5	15.4	—14 53 20.2	32.4	+38.1	+12.2	+38.8	0.98	+.047
	h m s	s							
1753, Dec. 3	22 23 21.60	19.34			+33.8		+35.6	1.00	+.048
	° ′ ″	″							
1756, Sept. 25	348 0 54.5	25.0	— 6 1 49.4	46.0	+29.5	—3.4	+25.7	1.05	+.013
1764, Jan. 15	12 37 39.0	56.4	+ 4 43 47.2	56.2	(—17.4)	(—9.0)			
1768, Dec. 27	31 26 52.0	32.6	12 15 35.0	30.6	+19.4	+4.4	+13.0	1.02	+.045
Dec. 30	31 26 45.8	38.0	12 14 55.4	55.9	+ 7.8	—0.5			
1769, Jan. 15	31 22 7.7	:55.8	12 14 26.0:	29.7	+11.9:	—3.7:	+12.5	1.01	+.049
Jan. 16	31 22 23.4	11.1	12 14 36.3	37.0	+12.3	—0.7			
Jan. 20	31 24 6.6	:43.7	12 15 19.0	18.8	+22.9:	+0.2			
Jan. 21	31 24 33.8	14.1	12 15 31.8	30.8	+19.7	+1.0			
Jan. 22	31 25 4.7	:47.7	12 15 45.7	44.7	+17.0:	+1.0			
Jan. 23	31 25 28.5	24.2	12 16 7.5	:59.6	+ 4.3:	+7.9			

Where no declination has been observed the observed corrections in right ascension have been changed to corrections in longitude on the hypothesis that the theoretical latitude is correct. The approximate formula is

$$\delta l = \frac{\delta \alpha \cos \delta}{\sin E}, \text{ where}$$

$$\cos E = \sin \varepsilon \cos \alpha, \ \varepsilon \text{ being the obliquity.}$$

DISCUSSION OF THE MODERN OBSERVATIONS.

Reduction of the Published Results of Observations to a Uniform System.

We have now to discuss the great mass of observations made at the principal observatories of the world since the discovery of the planet by Herschel, in 1781. To make all the data of reduction rigorously homogeneous and uniform, it would be necessary to completely re-reduce the greater part of the observations made before 1850, using the modern values of the constants of reduction, and to compare each observation separately with the geocentric place deduced from the provisional theory. Such a reduction and comparison would be extremely desirable. Their execution would, however, involve an amount of labor far greater than it is now possible for the author to bestow upon the problem. We must, therefore, adopt the reductions which have been already made, applying such systematic corrections for reduction to a uniform system of star places as we have the means readily to determine. No reduced places are employed unless we can find data for some more or less accurate determination of these corrections, a rule which necessitates the rejection of a great mass of observations made at the minor observatories of the European continent, and published in the *Astronomischen Nachrichten.* We still have the following rich collection of materials at our disposal:

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1. Observations at Greenwich, 1781 to 1872.
2. . . . . . . Paris, 1802 to 1827, and 1837 to 1869.
3. . . . . . . Königsberg, 1813 to 1835.
4. . . . . . . Vienna, 1822, and 1827 to 1839.
5. . . . . . . Speier, 1827–29.
6. . . . . . . Cambridge, 1828 to 1842.
7. . . . . . . Edinburgh, 1836 to 1844.
8. . . . . . . Berlin, 1838 to 1842.
9. . . . . . . Pulkowa, 1841 and 1842.
10. . . . . . . Washington, 1861 to 1872.
11. . . . . . . Leiden, 1863 to 1871.
12. . . . . . . Santiago, 1854 and 1855.

As to the general distribution of these observations in time, we may remark that during the first three or four years the planet was zealously observed at Greenwich. Observations then began gradually to fall off until 1798, in which year we find but one. From this time until 1814 only one or two observations were made at each opposition. They become a little more numerous, until 1829, when there is a sudden increase. Few interruptions have occurred since. With regard to the other observatories it may be said that from 1802 until 1830 there is a gradual increase in the number of observations, and that since the latter year the number of observations is entirely satisfactory.

A great number of the observations were reduced with the star places of the *Tabulæ Regiomontanæ*, and the entire Paris series are reduced with the star positions of Le Verrier, given in his "*Annales de l'Observatoire Imperial de Paris*," Tome II. As a preliminary to the discussion of the systematic corrections to the principal published reductions, I have prepared the following table, showing the corrections which must be applied to the places of the equatorial fundamental stars in the above catalogues to reduce them to the adopted standard, namely, Dr. Gould's coast survey list in right ascension, and Auwers' standard in declination.

In the table of right ascensions the first column after name of the star gives the annual variation of that co-ordinate for the epoch 1860.0, as derived from Le Verrier's tables of right ascensions just cited. Next we have the correction to this annual variation, expressed in units of the fourth place of decimals, to reduce it to that given in the "*Star Tables of the American Ephemeris*," the positions in which are founded on Dr. Gould's Catalogue. The fourth column gives the correction to the right ascensions of Le Verrier for 1860, in hundredths of a second of time. Subtracting from this column sixth-tenths of the preceding, we have the corresponding corrections for 1800. The last four columns give the corresponding numbers for the right ascensions of the *Tabulæ Regiomontanæ*.

The table of declinations shows, for different epochs, the corrections necessary to reduce the tabular positions to those given by Auwers in his paper on the declinations of the fundamental stars

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I. Right Ascensions.

Date.	Le Verrier's ann. var. 1860.	Corrections to			Ann. var. of Tab. Reg. 1860.	Corrections to		
		Ann. var.	R. A. 1860.	R. A. 1800.		Ann. var.	R. A. 1860.	R. A. 1800.
	s				s			
α Andromedæ,	+3.0844	+ 14	+ 2	— 6	3.0840	+18	+ 8	— 3
γ Pegasi,	3.0801	+ 15	+ 3	— 6	3.0824	— 8	— 9	— 4
α Arietis,	3.3644	+ 12	0	— 7	3.3636	+20	+ 6	— 6
α Ceti,	3.1266	+ 10	+ 5	— 1	3.1267	+ 9	+ 7	+ 2
α Tauri,	3.4346	— 1	— 1	0	3.4335	+10	+ 7	+ 1
β Orionis,	2.8797	+ 6	+ 2	— 2	2.8800	+ 3	+ 5	+ 3
β Tauri,	3.7871	— 4	+ 1	+ 3	3.7888	—21	— 4	+ 9
α Orionis,	3.2460	+ 5	+ 4	+ 1	3.2464	+ 1	+ 3	+ 2
α Geminorum,	3.8409	— 2	0	+ 1	3.8386	+21	+18	+ 5
α Canis Min.	3.1462	+ 5	+ 9	+ 6	3.1455	+12	+ 7	0
β Geminorum,	3.6828	+ 5	+ 3	0	3.6807	+26	+13	— 3
α Hydræ,	2.9485	+ 12	+ 6	— 1	2.9469	+28	+16	— 1
α Leonis,	3.2030	+ 15	+ 6	— 3	3.2014	+31	+13	— 6
β Leonis,	3.0654	+ 11	+ 4	— 3	3.0640	+25	+12	— 3
α Virginis,	3.1495	+ 20	+ 5	— 7	3.1497	+18	+ 2	— 9
α Bootis,	2.7325	+ 16	+ 3	— 7	2.7327	+14	+ 5	— 3
α^2 Libræ,	3.3044	+ 6	+ 3	— 1	3.3074	—24	— 6	+ 8
α Coronæ,	2.5378	+ 12	+ 1	— 6	2.5373	+17	+ 5	— 5
α Serpentis,	2.9488	+ 8	+ 4	— 1	2.9513	—17	— 7	+ 3
α Scorpii,	3.6654	+ 16	+ 4	— 6	3.6672	— 2	— 4	— 3
α Herculis,	2.7322	+ 7	+ 2	— 2	2.7319	+ 9	0	— 5
α Ophiuchi,	2.7808	+ 10	0	— 6	2.7783	+34	+15	— 5
α Lyræ,	2.0312	+ 2	— 1	— 2	2.0305	+ 9	+ 1	— 4
γ Aquilæ,	2.8520	+ 9	+ 2	— 3	2.8546	—17	— 8	+ 2
α Aquilæ,	2.9281	+ 4	+ 1	— 1	2.9281	+ 4	— 3	— 5
β Aquilæ,	2.9466	+ 5	+ 3	0	2.9496	—25	—12	+ 3
α^2 Capriconii,	3.3338	+ 4	+ 2	0	3.3349	— 7	— 6	— 2
α Aquarii,	3.0829	+ 13	+ 4	— 4	3.0822	+20	+ 4	— 8
α Piscis Aust.	3.3311	+ 8	+ 4	— 1	3.3326	— 7	—16	—12
α Pegasi,	2.9828	+ 11	0	— 7	2.9830	+ 9	— 1	— 6
Sum,		+254	+81	—72		+210	+71	—55
Mean,		+8.5	+.027	—.024		+7.0	+.024	—.018

II. Declinations.

	Corrections to Tabulæ Regiomontanæ.				Corrections to Le Verrier.	
	1780.	1800.	1820.	1840.	1820.	1840.
	″	″	″	″	″	″
α Andromedæ,	+0.3	+0.2	+0.1	0.0	+0.2	0.0
γ Pegasi,	—0.2	+0.1	+0.4	+0.8	+0.1	+0.3
α Arietis,	—0.3	0.0	+0.3	+0.5	+0.2	+0.3
α Ceti,	—1.0	—0.1	+0.7	+1.6	+0.1	+0.7
α Tauri,	0.0	+0.1	+0.1	+0.2	+0.1	0.0
β Orionis,	—0.5	+0.1	+0.6	+1.2	+0.3	+0.7
β Tauri,	—0.5	0.0	+0.6	+1.1	+0.2	+0.5
α Orionis,	—1.2	—0.6	0.0	+0.6	—0.2	+0.3
α Geminorum,	—0.8	—0.5	—0.1	+0.3	+0.5	+1.0
α Canis Min.	—0.2	0.0	+0.3	+0.6	+0.2	+0.5
β Geminorum,	—0.2	0.0	+0.3	+0.6	+0.2	+0.5
α Hydræ,	—0.2	+0.3	+0.7	+1.2	+0.3	+0.7
α Leonis,	+0.3	+0.4	+0.4	+0.5	+0.4	+0.5
β Leonis,	+0.2	+0.2	+0.2	+0.3	+0.2	+0.3
α Virginis,	+0.2	+0.6	+1.1	+1.5	+0.5	+0.8
α Bootis,	+0.4	+0.4	+0.3	+0.2	+0.4	+0.4
α^2 Libræ,	+0.5	+0.5	+0.6	+0.6	+0.5	+0.6
α Coronæ,	+0.7	+0.6	+0.4	+0.3	+0.4	+0.4
α Serpentis,	+0.2	+0.6	+1.1	+1.5	+0.5	+0.9
α Scorpii,	+0.1	+0.6	+1.0	+1.4	+0.4	+0.7
α Herculis,	+0.6	+0.8	+1.0	+1.3	+0.5	+0.7
α Ophiuchi,	+0.4	+0.5	+0.5	+0.6	+0.4	+0.6
α Lyræ,	+0.7	+0.8	+0.9	+1.0	+0.4	+0.4
γ Aquilæ,	0.0	+0.3	+0.6	+1.0	+0.2	+0.4
α Aquilæ,	+0.1	+0.4	+0.7	+1.0	+0.1	+0.4
β Aquilæ,	+0.1	+0.6	+1.2	+1.7	+0.5	+0.8
α^2 Capriconii,	+0.2	+0.9	+1.6	+2.3	+0.6	+1.1
α Aquarii,	—0.5	+0.1	+0.8	+1.5	+0.5	+1.0
α Piscis Aust.	+1.0	+2.4	+3.9	+5.3	+1.7	+2.3
α Pegasi,	+0.4	+0.3	+0.2	+0.1	+0.3	+0.2
Mean	+0.03	+0.36	+0.69	+1.03	+0.36	+0.60

The correction to the reductions to apparent place given in the Tabulæ Regiomontanæ on account of the correction to the constant of Nutation is;—

In right ascension:

$$-0''.46 \sin \Omega - 0''.18 \sin \Omega \sin \alpha \tan \delta - 0''.24 \sin \Omega \cos \alpha \tan \delta.$$

In declination:

$$-0''18 \sin \Omega \cos \alpha + 0.24 \cos \Omega \sin \alpha.$$

The terms which contain $\tan \delta$ as a factor may be entirely neglected, as they are small, periodic, and contain $\tan \delta$ as a factor which is sometimes positive and sometimes negative. I shall also neglect the corrections in declination, as their sum is sensibly

$$0''.21 \sin (\alpha - \Omega)$$

the effect of which will generally be confounded with the accidental errors of observation.

The only correction we shall apply on account of nutation is, therefore,

$$\delta\alpha = -0^{s}.030 \sin \Omega.$$

The values of this expression at the dates when it is zero, a maximum, or a minimum, are as follows:—

y.	s.	y.	s.
1778.5	—.03	1820.3	.00
1783.1	.00	1825.0	+.03
1787.7	+.03	1829.6	.00
1792.4	.00	1834.3	—.03
1797.0	—.03	1838.9	.00
1801.7	.00	1843.6	+.03
1806.3	+.03	1848.2	.00
1811.0	.00	1852.9	—.03
1815.6	—.03	1857.5	.00
1820.3	.00		

Having adopted this system of standard positions, we may adopt two ways of reducing the observations to it. One is to compare the positions of the stars adopted in the published reductions with the standard, and apply the mean difference to the reduced place of the planet. Another is to make a similar comparison of the standard catalogue with the positions of the fundamental stars which have been deduced from the observations by a system of reduction uniform with that employed in reducing the observations of the planet, and to regard the mean difference as a correction applicable to all the positions of the planet. If the standard catalogue and the observations are both free from systematic error, the results obtained in these two ways should be substantially identical. These are, however, conditions which we cannot expect to find fulfilled. In the following discussions I have sometimes used one, sometimes the other, and sometimes

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combined both, the choice being determined by circumstances. We shall consider the different series of observations in succession.

Greenwich Observations from 1781 *to* 1830.

These observations are completely reduced by Airy and compared with Bouvard's Tables, in the work *Reduction of the Observations of Planets made at the Royal Observatory, Greenwich, from* 1750 *to* 1830. London, 1845. The concluded positions given in this work depend mainly on the star places of the Tabulæ Regiomontanæ, both in right ascension and declination. If we consider the first four oppositions—1781-1785—as forming a single group of which the mean epoch is 1783, we find that the general correction to the Tabulæ Regiomontanæ for this epoch is

In right ascension, $-0^{s}.030$;
In declination, $+0''.08$.

If, on the other hand, we consider only the particular stars compared with Uranus, the result will be a little different. The number of times each of the fundamental stars has been compared with Uranus, and the correction in right ascension corresponding to each star, are nearly as follows:—

		s.	s.
α Arietis,	$N=$ 2	Cor. = —0.09	$N\times C=$ — .18
α Tauri,	2	—.01	— .02
γ Pegasi,	2	—.03	— .06
β Tauri,	19	+.13	+2.47
α Orionis,	33	+.02	+0.66
α Canis Minoris,	33	—.02	—0.66
β Geminorum,	34	—.07	—2.38
α Leonis,	7	—.11	— .77
β Leonis,	2	—.07	— .14

The mean correction from these data comes out $-0^{s}.008$, differing by $0^{s}.022$ from the general mean correction. Our choice between the two corrections depends on whether we are to consider the relative positions of the Tabulæ Regiomontanæ, or those of the standard catalogue, as nearest the truth at the epoch 1783, and particularly upon whether we are to consider the large correction to the proper motion of β Tauri as real. In the absence of exact data for settling this question, the mean of the two results, or $-0^{s}.020$, has been adopted.

A similar anomaly is exhibited by the declinations. It is probable that the declinations of Uranus during this period mainly depend on stars in the first twelve hours in right ascension, for which the mean correction is about $-0''.30$ instead of $+0''.08$. I have adopted $-0''.16$. Changing these corrections to longitude and latitude, we have, during the period 1781-1786:—

Correction to observed longitude, $= -0''.30$;
Correction to observed latitude, $-0\ .19$.

During the years 1788–1798 the above systematic difference in right ascension does not appear. The most probable correction seems to be

$\Delta\alpha = -0^s.025;$ $\quad\Delta\delta = 0''.00.$
Whence Δ long. $= -0''.34;$ $\quad\Delta$ lat. $= -0''.10.$

Between the years 1800 and 1823 the stars used for comparison are so widely scattered that I consider it safe to apply only the general mean correction for the epoch 1813, which is

$\Delta\alpha = -{}^s.005;$ $\quad\Delta\delta = +0''.66.$
Whence Δ long. $= 0''.00;$ $\quad\Delta$ lat. $= +0\ .66.$

From 1825 to 1830 more than half the weight of the right ascension comes upon the stars α, β, and γ Aquilæ, the mean correction to which, during this interval, is $-0^s.035$. The general mean correction at this epoch is $+0^s.002$. I think the right ascensions of these three stars in the Tabulæ Regiomontanæ are really too great at this epoch by the entire difference of these results. We may, in fact, hereafter regard the positions of the standard catalogue as sufficiently accurate. The mean corrections to be applied will then be

$\Delta\alpha = -0^s.017;$ $\quad\Delta\delta = +0''.83.$
Whence Δ long. $= -0''.05;$ $\quad\Delta$ lat. $= +0''.86.$

From the year 1831 until the present time the Greenwich observations are regularly reduced in the several annual volumes of observations. But a reduction of the observations from 1831 to 1835, executed by Mr. Hugh Breen, is given in an appendix to the volume for the year 1864. The results here given differ from those published by Pond in the several annual volumes for the same interval. The right ascensions are altered only by applying the constant correction $-0^s.030$, which is found necessary to reduce Pond's right ascensions to those of the Tabulæ Regiomontanæ. This correction I have verified. The mean correction to reduce the right ascensions of the Tabulæ Regiomontanæ to our standard is at this time $+0^s.005$. On the other hand, when we compare the concluded right ascensions of stars within six hours of Uranus, as given by Pond in the Greenwich observations for 1834, with our standard, we find a mean correction of $-{}^s.034$ to reduce his positions to the standard, which implies a correction $-{}^s.004$ to Breen's reduction. The two results being $+{}^s.005$ and $-{}^s.004$, I have applied no correction whatever.

In the paper in question the declinations are completely re-reduced, using improved data of reduction, but, so far as I see, making no changes in Pond's method. The results differ strikingly from those of Pond, and suggest the desirableness of a complete re-examination of all Pond's determinations of declination. Having no catalogue of observed declinations of standard stars reduced in this same way, we cannot directly determine the systematic correction to the declinations. I therefore proceed as follows: A comparison of Pond's observed declinations of standard stars with Auwers' normal catalogue show that the former require the following corrections near the parallel of Uranus:

In 1831 — 1″.42;
1834 — 2 .10.

Then comparing Airy's reduced declinations of Uranus with Pond's, we find the following mean differences:

In 1831, Airy — Pond = — 3″.18
1834, — 3 .50.

To reduce Airy to Auwers we must there apply to the declinations

In 1831 + 1″.76
1834 + 1 .40.

I have regarded the correction + 1″.60 as applicable throughout the period in question.

1836-72.

During this interval the corrections in right ascension have been derived by the following two sets of comparisons: (1) A comparison of the several collected six and seven year catalogues with Gould's standard, from which it appears that they require the following general corrections in right ascension:

Six year catalogue of 1840	+ 0ˢ.047
Six year catalogue of 1845	+ 0 .002
Seven year catalogue of 1860	+ 0 .003
Seven year catalogue of 1864	+ 0 .022

(2) A comparison of the corrections applied to the right ascensions of the individual years to reduce them to the standard of the catalogue, as given in the introduction to each catalogue. The sum of these two numbers gives the corrections for each year.

A slightly different method is to regard the above correction for each catalogue as applicable to all right ascensions which depend fundamentally upon that catalogue. I have sometimes combined both methods so as to derive what seemed to be the most probable result, and sometimes used but one.

The corrections to the declinations during the interval in question have been derived from Auwers' "*Tafeln zur Reduction der Declinationen verschiedener Sternverzeichnisse auf ein Fundamentalsystem*," *Astronomische Nachrichten*, No. 1536. These tables include the Greenwich seven year catalogue for 1860, when the correction corresponding to the declination of Uranus is about + 0″.45. The corrections for the previous catalogues vary between 0″.35 and 0″.68. The correction corresponding to the interval 1861-67 has been derived by a direct comparison with Auwers' declinations, and the result is + 0″.44, agreeing with the two preceding catalogues. But, on making a similar comparison with the annual catalogue for 1869, a considerable change was found, the correction being — 0″.17, a change of more than half a second. I shall use this correction for and after the beginning of 1868, as the change is probably due to the introduction of a new constant of refraction in the reduction of the observations for 1868 and subsequent years.

Cambridge.

An extended series of planetary observations was commenced here by Professor Airy, in 1827. The series was continued by him and Professor Challis, his successor, until 1842. During the first three or four years the combined right ascensions depend on a few special stars, and mainly on α^2 Capricorni. Taking the mean correction to the adopted right ascensions of the stars actually compared as they are given in the introduction to each annual volume, giving to each star a weight proportional to the number of comparisons, the following corrections are deduced:

1828	$-0^s.10$
1829–31	-0.16
1832–37	-0.19.

In the introduction to the volume for 1838 it is stated that the adopted right ascensions are diminished by the average amount of $0^s.083$, which would still leave a correction of $-0^s.107$. Actual comparisons in two subsequent years give

$$\begin{array}{ll} 1840, & \Delta\alpha = -0^s.087 \\ 1842, & \qquad -0.069. \end{array}$$

Although the positions deduced from each year's work were adopted for clock correction the year following, without any change of equinox, it seems that there was, effectively, a progressive change of about $0^s.01$ annually in the equinox as adopted.

No declinations were observed until 1830. On comparing the declinations deduced from several years' work with Auwers, it was evident that the correction increased with the polar distance of the star. The law of increase could be well enough represented by supposing the correction proportional to N. P. D. Thus the following corrections were deduced in three different years.

$$\begin{array}{lll} 1834, & \delta\,\text{dec.} = & -\dfrac{1''.78 \times \text{N. P. D. in degrees}}{100} \\[2ex] 1840, & & -\dfrac{1''.00 \times \text{N. P. D. in degrees}}{100} \\[2ex] 1842, & & -\dfrac{1''.03 \times \text{N. P. D. in degrees}}{100}. \end{array}$$

From which the correction for other years was deduced by interpolation. But, on applying these corrections, the results were found systematically different from those of other observatories, and on referring to Auwers' corrections to Airy's Cambridge Catalogue, it appeared that the mural circle required a large correction near the declination of Uranus during this period. The above results were therefore altered so as to conform as nearly as practicable to Auwers' law.

Edinburgh.

In reducing the observations of 1836 Henderson uses the right ascensions of the Tabulæ Regiomontanæ, to which the general correction is at this epoch $+^s.007$.

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But, if we take only the stars near Uranus, with which the latter was necessarily most frequently compared, the corrections will be negative. Comparing the concluded positions of the stars from *a* Serpentis through 0^h to β Orionis, we find the following mean corrections:

In right ascension, $-0^s.012$; in declination, $-0''.09$.

In subsequent years it is stated that the adopted positions of clock stars used each year are derived from the right ascensions observed at Greenwich, Cambridge, and Edinburgh, during the year or the two years preceding, without any statement whether corrections were applied for difference of equinoxes. In some subsequent years the following corrections are deduced, sometimes from the adopted and sometimes from the concluded positions:

1837, $\Delta\alpha = 0^s.000$;
1840, $\Delta\alpha = +0.015$; Δ Dec. $= 0''.00$;
1844, $\Delta\alpha = +0.070$; Δ Dec. $= +0.49$.

Paris.

All the positions of planets given by Le Verrier, in his "*Annales de l'Observatoire Imperial de Paris: Observations*" depend both in right ascension and N. P. D. on his adopted positions of fundamental stars, the corrections to which have already been given. As the corrections to the individual star places used by Le Verrier are not generally of a systematic character, the general mean correction is employed, which is:—

In right ascension $-0^s.024 + 0^s.085T$,
In declination $+0''.12 + 1''.20T$,

T being the fraction of a century after 1800.

In 1854 a new and larger catalogue was introduced, and for this and the following years the correction in declination is derived from Auwers' tables.

A summary of the adopted corrections after 1830, as deduced from the preceding comparisons and discussions, is given in the following table:—

TABLE OF ADOPTED SYSTEMATIC CORRECTIONS.

Year.	Greenwich.		Paris.		Königsberg.		Berlin.		Cambridge.		Edinburgh.	
	$\Delta\alpha$ s	$\Delta\delta$ "	$\Delta\alpha$ s	$\Delta\delta$ "	$\Delta\alpha$ s	$\Delta\delta$ "	$\Delta\alpha$ s	$\Delta\delta$ "	$\Delta\alpha$ s	$\Delta\delta$ "	$\Delta\alpha$ s	$\Delta\delta$ "
1830	....	....	....	....	.00	....	....	+0.9	—.16	—1.7	.00	....
1831	.00	+1.8	.00	+0.5	—.02	+0.9	....	+0.9	—.16	—1.6	....	....
1832	....	+1.7	....	....	—.02	+0.9	....	....	—.19	—1.5	....	....
1833	....	+1.6	....	....	....	....	—.04	....	....	—1.4	....	....
1834	....	+1.4	....	....	....	....	—.05	....	....	—1.3	....	....
1835	....	+1.2	....	....	....	....	—.05	....	....	—1.2	....	....
1836	—.04	+1.0	+.01	+0.6	—.02	+1.0	—.04	+1.0	....	—1.0	—.01	0
1837	—.03	+0.9	....	....	....	....	—.03	....	....	—0.8	.00	0
1838	—.04	+0.8	....	....	....	....	—.02	....	—.11	—0.5	+.01	0
1839	....	+0.7	....	....	....	+1.0	—.01	....	—.10	—0.3	+.01	0
1840	....	+0.6	....	....	....	....	.00	....	—.09	—0.2	+.02	+0.1
1841	....	+0.5	....	....	....	....	+.01	....	—.08	—0.2	—.04	+0.2

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Table of Adopted Systematic Corrections.—*Continued.*

Year.	Greenwich.		Paris.		Königsberg.		Berlin.		Cambridge.		Edinburgh.	
	Δα s	Δδ ″	Δα s	Δδ ″	Δα s	Δδ ″	Δα s	Δδ ″	Δα s	Δδ ″	Δα s	Δδ ″
1842	—.02	+0.4	+.01	+0.6	....	....	+.03	+1.1	—.07	—0.3	—.02	+0.3
1843	+.07	+0.4	+.01	+0.6	....	....	+.04	+1.1	....	....	+.07	+0.4
1844	+.06	+0.4	+.01	+0.6	....	....	+.04	+1.1	....	....	+.07	+0.5
1845	+.08	+0.4	+.01	+0.6	....	....	+.04	+1.1	....	....		
1846	+.04	+0.4	+.02	+0.6	....	....	+.03	+1.1	....	....		
1847	+.05	+0.4	+.02	+0 6	....	....	+.02	+1.1	....	....		
1848	+.05	+0.4	+.02	+0.6	....	....	+.01	+1.2	....	....		
1849–53	.00	+0.4	+.02	+0.6	....	....	.00	+1.2	Santiago			
1854–55	.00	+0.5	+.02	+0.2	....	....	—.01	+1.2	.00	+0.6		
1856–60	—.01	+0.5	+.02	+0.2	....	....	....	....	....	....	Washington	
1861	—.01	+0.4	+.03	+0.2	....	....	....	....	....	....		
1862–65	.00	+0.4	+.03	+0.2	....	....	....	....	....	....	.00	—0.5
1866	.00	+0.4	+.03	+0.2	....	....	....	....	....	....	.00	+1.1
1867	.00	+0.4	+.03	+0.2	....	....	....	....	....	....	.00	+1.1
1868	.00	—0.2	+.03	+0.2	....	....	....	....	....	....	.00	+1.2
1869	.00	—0.2	+.03	+0.2	....	....	....	....	....	....	.00	+0.6
1870	+.01	—0.2	....	....	....	....	....	....	....	....	.00	+0.4
1871	+.01	—0.2	....	....	....	....	....	....	....	....	.00	+0.4
1872	+.02	—0.2	....	....	....	....	....	....	....	....	.00	+0.4

Applying the preceding corrections to the positions of the planet as originally reduced and published, we have a series of observed positions as nearly homogeneous as it is possible to make them with the means now at our command. The next step in order will be the computation of the geocentric place of the planet from the provisional theory for the moment of every observation, to be compared with the results of the latter. The complete execution of this labor, *ab initio*, is, however, at present impracticable, and it is proposed to diminish it by making use of the published comparisons with the older tables. This can be done without danger of serious error, and with all the more ease that owing to the great distance of Uranus the errors of the solar tables are, for the most part, without appreciable effect upon the computed geocentric place of the planet. The method of making the comparison is different with different series of observations, and each series must therefore be described and discussed separately. The general plan, however, has been to replace observed and computed absolute positions by observed and computed corrections to the geocentric positions deduced from Bouvard's Tables. To carry out this plan it is necessary to have at our disposal an ephemeris both of the heliocentric and geocentric positions derived from these tables. The corrections to the latter given by the observations are then given by direct comparison. To obtain the corrections given by the provisional theory, the heliocentric longitudes, latitudes, and radii vectores given by that theory are interpolated to the dates of the heliocentric ephemeris from Bouvard's Tables, and compared with that ephemeris. The differences are then changed to differences of geocentric place by the usual differential formulæ, and thus the corrections given by theory are derived. The difference between the two sets of corrections is the difference between the provisional theory and observation. A condensed summary of the results for each of the principal series of observations is here presented.

Greenwich, 1781–1830.

In Airy's reductions, already referred to, we have given for the moment of each individual observation a heliocentric place computed from Bouvard's Tables, and the geocentric longitudes and latitudes thence deduced. The observed right ascensions and declinations are then changed to longitudes and latitudes, and the apparent error of the tables thence deduced. The means of these errors are taken for groups of observations, and expressed in terms of the errors of heliocentric longitude, radius vector, and latitude. The mode in which these means have been treated is fully shown in the following table. The first column gives the mean date of each individual group of observations. The next three give the mean excesses of the co-ordinates interpolated from the heliocentric ephemeris, p. 100, and corrected for solar nutation, over those printed in the "Computations of tabular place, etc.," in the Greenwich reductions. In the fifth column these corrections are changed to corrections of geocentric longitude. In the next two columns we have the mean corrections to Bouvard's geocentric places given by observation. It is the negative of the mean error of tabular place printed in the "Reductions," corrected by the numbers already given to reduce the star places to a uniform system. Then we have the difference between these two sets of corrections, or, the mean correction to the geocentric place of the provisional theory as given by observation. Lastly, we have the differential coefficients for expressing the errors of geocentric in terms of the errors of heliocentric co-ordinates taken without change from the Greenwich volume.

Mean Date.	From Provisional Theory.				From Observations.			Correction to Prov. Theory.		$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	Correction to tabular position in Greenwich Reductions.				Correction to						
	Long.	Log. R. V.	Lat.	Geoc. long.	Geoc. long.	Hel. lat.	No.of Obs.	Geoc. long.	Hel. lat.		
	"		"		"	"		"	"		
1781, Oct. 10	— 7.8	....	+1.1	— 7.5	— 6.4	+ 2.6	4	+1.1	+ 1.5	1.01	— 9
Nov. 13	— 7.5	....	+1.1	— 7.5	— 4.9	— 1.6	4	+2.6	— 2.7	1.04	— 6
Dec. 27	— 7.3	— 159	+1.0	— 7.7	— 5.6	— 2.3	5	+2.1	— 3.3	1.05	+ 1
1782, Jan. 31	— 7.4	....	+0.9	— 8.0	— 4.9	— 1.1	4	+3.1	— 2.0	1.04	+ 6
Mar. 4	— 6.6	....	+0.9	— 7.1	— 4.6	— 0.6	4	+2.5	— 1.5	1.01	+ 9
Oct. 10	— 5.6	....	+0.4	— 5.3	— 3.6	— 4.0	4	+1.7	— 4.4	1.01	— 9
Nov. 26	— 5.3	....	+0.3	— 5.3	— 6.0	— 5.0	3	—0.7	— 5.3	1.04	— 5
1783, Jan. 11	— 5.0	....	+0.3	— 5.4	— 3.9	— 3.7	3	+1.4	— 4.0	1.05	+ 3
Feb. 24	— 4.4	....	+0.2	— 4.8	— 2.2	— 2.4	3	+2.6	— 2.6	1.02	+ 9
Oct. 10	— 3.5	....	—0.3	— 3.2	— 0.3	— 1.0	2	+2.9	— 0.7	1.00	—10
Nov. 1	— 3.2	....	—0.3	— 3.0	— 1.1	— 2.3	2	+1.9	— 2.0	1.02	— 9
Dec. 15	— 3.3	— 121	—0.4	— 3.4	— 2.2	— 3.8	3	+1.2	— 3.4	1.05	— 3
1784, Jan. 29	— 3.1	....	—0.4	— 3.4	— 2.4	— 2.4	3	+1.0	— 2.0	1.05	+ 5
Mar. 12	— 2.6	....	—0.5	— 2.9	— 0.3	— 4.3	3	+2.6	— 3.8	1.01	+ 9
Oct. 30	— 1.8	....	—0.8	— 1.5	— 0.7	+ 1.4	2	+0.8	+ 2.2	1.02	— 9
Dec. 14	— 1.5	— 128	—0.9	— 1.5	— 2.4	+ 2.7	2	—0.9	+ 3.6	1.05	— 4

Mean Date.	From Provisional Theory.				From Observations.			Correction to Prov. Theory.		$\frac{\partial l}{\partial \lambda}$	$\frac{\partial l}{\partial \rho}$
	Correction to tabular position in Greenwich observations.				Correction to						
	Long.	Log. R. V.	Lat.	Geoc. long.	Geoc. long.	Hel. lat.	No. of Obs.	Geoc. long.	Hel. lat.		
	″		″		″	″		″	″		
1785, Jan. 16	— 1.2	....	—1.0	— 1.3	— 1.4	— 1.4	2	—0.1	— 0.4	1.05	+ 2
Feb. 13	— 1.0	....	—1.1	— 1.3	— 1.2	— 1.1	2	+0.1	0.0	1.04	+ 7
Mar. 20	— 0.6	....	—1.1	— 0.9	— 2.1	— 5.2	2	—1.2	— 4.1	1.01	+10
Nov. 8	+ 0.2	....	—1.5	+ 0.5	+ 1.2	— 1.7	2	+0.7	— 0.2	1.02	— 9
1788, Mar. 13	+ 2.1	— 187	—3.2	+ 1.8	+ 4.4	— 4.9	3	+2.6	— 1.7	1.03	+ 9
Oct. 26	+ 3.4	....	—3.6	+ 3.9	+ 3.0	— 2.2	2	—0.9	+ 1.4	1.00	—10
1789, Jan. 18	+ 4.2	— 224	—3.7	+ 4.5	+10.4	—14.5	1	(+5.9)	(—10.8)	1.06	— 1
Apr. 8	+ 4.7	....	—3.8	+ 4.2	+ 9.4	— 3.7	4	+5.2	+ 0.1	1.01	+10
Oct. 31	+ 4.8	....	—4.2	+ 5.4	+ 4.7	— 1.1	2	—0.7	+ 3.1	1.00	—10
1790, Jan. 24	+ 4.9	....	—4.4	+ 5.2	+ 3.8	— 2.6	2	—1.4	+ 1.8	1.06	— 1
Nov. 5	+ 5.1	— 254	—4.8	+ 5.8	+ 5.0	— 2.3	3	—0.8	+ 2.5	1.00	—10
1791, Jan. 29	+ 5.1	— 258	—5.0	+ 5.4	+ 4.5	— 3.8	3	—0.9	+ 1.2	1.06	0
Apr. 14	+ 5.0	....	—5.2	+ 4.5	+ 2.7	— 4.8	1	—1.8	+ 0.4	1.01	+10
Nov. 10	+ 5.0	— 234	—5.4	+ 5.6	+ 5.7	— 4.0	2	+0.1	+ 1.4	1.00	—10
1792, Feb. 5	+ 5.2	....	—5.6	+ 5.5	+ 4.2	— 4.0	1	—1.3	+ 1.6	1.06	0
Nov. 15	+ 5.6	....	—6.0	+ 6.2	+ 3.7	— 2.5	3	—2.5	+ 3.5	1.00	—10
1793, Feb. 8	+ 5.4	....	—6.1	+ 5.7	+ 8.5	— 5.0	2	+2.8	+ 1.1	1.06	0
Nov. 14	+ 5.6	— 135	—6.4	+ 5.9	+10.4	— 6.3	1	+4.5	+ 0.1	0.99	—10
1794, Feb. 15	+ 5.7	— 100	—6.5	+ 6.0	+ 7.3	— 6.3	2	+1.3	+ 0.2	1.06	0
Nov. 19	+ 5.4	— 67	—6.7	+ 5.6	+ 4.1	— 5.1	4	—1.5	+ 1.6	1.00	—10
1795, Feb. 20	+ 5.9	— 46	—6.9	+ 6.3	+ 6.0	— 8.0	3	—0.3	— 1.1	1.06	0
Nov. 29	+ 5.4	— 13	—7.0	+ 5.5	+ 3.9	— 3.4	2	—1.6	+ 3.6	1.00	—10
1796, Feb. 24	+ 5.3	— 8	—7.2	+ 5.6	+ 4.5	— 4.5	2	—1.1	+ 2.7	1.06	0
1797, Feb. 27	+ 5.1	+ 62	—7.5	+ 5.4	+ 4.7	— 4.2	3	—0.7	+ 3.3	1.06	0
1800, Mar. 14	+ 4.6	+ 224	—8.4	+ 4.9	+ 4.9	— 8.7	2	0.0	— 0.3	1.06	0
1814, May 22	— 1.2	+ 217	—2.2	— 1.3	+ 1.1	+ 0.2	2	+2.4	+ 2.4	1.06	0
1815, May 25	— 1.1	+ 214	—1.5	— 1.2	+ 1.4	— 1.0	4	+2.6	+ 0.5	1.06	0
1818, June 10	— 0.5	+ 413	—0.8	— 0.5	— 1.1	+ 4.8	2	—0.6	+ 5.6	1.06	0
1819, June 14	— 0.4	+ 483	+1.5	— 0.4	— 1.4	+ 3.7	4	—1.0	+ 2.2	1.06	0
1820, June 16	— 0 1	+ 498	+2.2	— 0.1	— 2.6	+ 4.0	2	—2.5	+ 1.8	1.06	0
1823, July 1	0.0	+ 538	+4.4	0.0	+ 0.2	+ 5.0	4	+0.2	+ 0.6	1.05	0
1825, July 11	— 2.4	+ 567	+5.8	— 2.5	— 4.8	+ 7.8	2	—2.3	+ 2.0	1.05	0
1826, July 16	— 4.5	+ 582	+6.6	— 4.7	— 3.6	+ 8.3	4	+1.1	+ 1.7	1.05	0
1827, July 20	— 6.6	+ 626	+7.2	— 6.9	— 7.2	+ 9.6	6	—0.3	+ 2.4	1.05	0
1828, July 23	— 9.8	+ 710	+7.9	—10.3	— 6.9	+ 6.9	3	+3.4	— 1.0	1.05	0
1829, Aug. 7	—13.4	+ 835	+8.5	—13.7	—16.3	+10.0	14	—2.6	+ 1.5	1.08	+ 2
Oct. 4	—14.2	+ 865	+8.6	—12.5	—13.0	+ 9.9	8	—0.5	+ 1.3	1.02	+ 8
1830, July 30	—17.8	+ 970	+9.0	—18.8	—21.4	+11.1	3	—2.6	+ 2.1		
Aug. 29	—18.2	+ 982	+9.1	—17.9	—20.0	+10.2	5	—2.1	+ 1.1		
Sept. 20	—18.5	+ 992	+9.1	—17.2	—19.1	+ 9.1	12	—1.9	0.0		
Oct. 14	—18.9	+1009	+9.1	—16.8	—17.3	+ 9.9	11	—0.5	+ 0.8		
Nov. 13	—19.3	+1009	+9.2	—16.8	—17.8	+ 9.3	8	—1.0	+ 0.1		

Paris, 1801-1827.

A complete reduction of this series is found in Le Verrier's *Annales de l'Observatoire Imperial de Paris*, *Observations*, tome I. No comparison with any ephemeris is given here, nor is there any complete ephemeris to compare them with. A complete geocentric ephemeris was therefore computed from the provisional theory for the principal groups of the Paris observations. The individual observations being compared with it, the resulting mean corrections are given in the following table:

Mean date.	$\Delta\alpha$	$\Delta\delta$	*N.*
1801, March 24,	$-{}^{s}.02$	$+1''.2$	2
1802, April 1,	+.08	+0.6	13
1805, April 22,	+.10	+2.2	13
1806, April 17,	−.01	−1.6	5
1807, April 28,	+.17	+0.4	16
1808, April 28,	+.02	+1.4	6
1809, May 5,	+.20	+0.1	9
1810, April 30,	+.22	+2.6	16
1811, Febr'y 18,	+.21	+2.2	3
1811, May 17,	+.14	+2.6	8
1812, Febr'y 16,	+.28		2
1812, May 10,	+.16	+3.0	6
1813, Febr'y 25,	+.44		3
1813, May 20,	$+{}^{s}.19$	$+1''.8$	6
1814, May 27,	+.21	+0.8	4
1815, May 24,	−.02	+2.2	5
1816, June 1,	−.01	+0.8	7
1817, June 5,	−.08	+1.6	5
1818, June 7,	+.12	+2.2	9
1819, June 18,	−.07	+1.8	7
1820, June 20,	−.20	−2.4	8.5
1821, June 22,	+.05	+1.0	5
1823, July 18,	+.02	+1.8	5
1824, July 13,	+.04	0.0	7
1827, July 25,	−.05	+0.6	5

Total number of observations in right ascension, 175.

The observations in this series exhibit numbers of discordances of that class which leave the astronomer in doubt whether the observation should be retained or rejected. This remark applies more especially to the declinations. If we determine the probable error of an observation in declination by the condition that it is that amount which the error falls short of as often as it exceeds, it is found to be about 2″. Then, if the errors followed the commonly assumed law of probability, only about one in six of the errors should exceed 4″, and one in twenty-three 6″. But errors of these magnitudes are much more numerous, the deviations often amounting to six or eight seconds. I have rejected only a few in which the discordances approached 10″.

Bessel's Königsberg Observations, 1814–1835.

I have made a complete re-reduction of the right ascensions of this important series, and of most of the declinations. In order to avoid the necessity of applying systematic corrections, Dr. Gould's right ascensions and Dr. Auwers' declinations were used throughout in these reductions. In this work a selection of the fundamental stars observed by Bessel was made for each observation of the planet, to be used for clock error. These were chosen so that the mean of their right ascensions and declinations should be as near as practicable to those of Uranus, a condition, however, which could not generally be fulfilled for the declinations, owing to the southern position of the planet. Bessel's instrumental cor-

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rections were applied to his observed times of transit over the mean wire, and the resulting time was employed as that of transit. Each time, compared with the computed right ascension of the star gave a value of the clock correction, which was reduced to the time of transit of the planet by the known daily rate. If the instrumental errors were always accurately determined, the mean of these clock corrections would be used to obtain the right ascension of Uranus. But it was frequently found that the clock error varied systematically with the declination of the star, so that it was deemed advisable to add to the clock correction a term varying as the simple declination, which was deduced from all the stars, and used to reduce the correction to the parallel of Uranus.

It was intended to give the results of this reduction for each observation, but on comparing the results with those of Fleming in the *Astronomische Nachrichten*, Band 30, it appeared that the results were not materially better than his. It does not, therefore, seem necessary to give more than the mean results for each opposition.

From Bessel's declinations, with the old Cary circle, I was unable to obtain any satisfactory results, owing, apparently, to a want of knowledge of some peculiarity of the instrument. Fleming's reductions were therefore adopted. They are designated by the letter F in the following list.

Mean Corrections to the Provisional Ephemeris given by Bessel's Observations at Königsberg, 1814–1829.

Mean date.	$\Delta\alpha$	$\Delta\delta$	*N.*	Mean date.	$\Delta\alpha$	$\Delta\delta$	*N.*
1814, May 22,	$+^{s}.11$	$+2''.5F$	9	1822, June 24,	$+^{s}.10$	$+1''.8$	7
1815, May 25,	$+.13$	$+1.8F$	11	1823, July 4,	$-.05$	$-1.6F$	2
1816, May 27,	$+.06$	$+1.2F$	11	1824, July 6,	$+.01$	$-1.0F$	5
1817, June 6,	$+.13$	$+2.3F$	8	1825, July 16,	$+.01$	$-2.4F$	5
1818, June 8,	$+.02$	$+4.3F$	13	1826, July 18,	$-.01$	$-3.0F$	7
1820, June 21,	$+.02$	$+4.1$	4	1828, July 25,	$-.15$	$-3.5F$	7
1821, June 23,	$+.12$	$+1.5$	5	1829, Aug. 1,	$-.10$	$-1.0F$	9

Total numbers of observations, 103.

Results of Observations at various Observatories, from 1827 *to* 1829 *inclusive.*

During these three years we have, besides the observations already quoted, the following:—

1. Observations by Schwerd, at Speier, of which the originals are given in *Astronomische Beobachtungen angestellt auf der Sternwarte des Königl. Lyzeums in Speyer von F. M. Schwerd*, Speyer, 1829–30, and of which the reduced results are found in the Astronomische Nachrichten, Band 8, S. 264.
2. The series by Airy, at Cambridge, commenced in 1828, and found in the Cambridge Observations.
3. Littrow's Vienna Observations, found in the first series of *Annalen der K. K. Sternwarte in Wien.*

The mean corrections to the provisional ephemeris given by these series are shown in the following table. The observations have been divided in the usual way into groups of about a month each, and the mean date and mean correction found for each group. The Paris and Königsberg results are repeated for the sake of clearness. The small figures show, as usual, the number of observations employed in forming the mean.

			$\Delta\alpha$		$\Delta\delta$
	Date.	Observatory.	Original.	Corrected.	
1827,	July 22,	Speier,	$-0^{s}.16_{6}$	$-0^{s}.14$	
	July 25,	Paris,	$-0\ .03_{5}$	$-0\ .05$	$+0''.5_{3}$
	September 15,	Vienna,	$-0\ .11_{11}$	$-0\ .10$	$0\ .0_{14}$
	October 14,	Vienna,	$-0\ .18_{6}$	$-0\ .17$	$-2\ .2_{6}$
1828,	July 25,	Königsberg,	$-0\ .15_{7}$	$-0\ .15$	$-3\ .5_{7}$
	July 29,	Vienna,	$-0\ .24_{2}$	$-0\ .20$	$-1\ .4_{2}$
	August 14,	Vienna,	$-0\ .13_{10}$	$-0\ .09$	$+1\ .1_{10}$
	August 27,	Speier,	$-0\ .10_{6}$	$-0\ .09$	
	September 18,	Vienna,	$-0\ .03_{9}$	$+0\ .01$	$+1\ .0_{9}$
	September 25,	Cambridge,	$-0\ .05_{6}$	$-0\ .16$	
	October 17,	Vienna,	$-0\ .13_{14}$	$-0\ .09$	$0\ .0_{14}$
	October 17,	Cambridge,	$-0\ .02_{10}$	$-0\ .12$	
1829,	August 1,	Königsberg,	$-0\ .10_{9}$	$-0\ .10$	$-1\ .0$
	August 6,	Cambridge,	$+0\ .11_{8}$	$-0\ .08$	$-1\ .1_{17}$
	August 28,	Speier,	$-0\ .04_{5}$	$-0\ .04$	
	September 23,	Cambridge,	$+0\ .21_{10}$	$+0\ .05$	
	November 6,	Cambridge,	$+0\ .25_{6}$	$+0\ .09$	

Observations from 1830 *to* 1872.

Since the year 1830 heliocentric and geocentric ephemerides of Uranus computed from Bouvard's Tables are at our disposal. We make use of those in the Berlin Astronomisches Jahrbuch for the years 1830 to 1833, and of those in the Nautical Almanac from 1834 forward. The system of comparison is the same as that already explained. That is to say, we deduce separately:

(1) Mean corrections to the geocentric longitude and latitude of Uranus in the ephemeris as derived from observation.

(2) Mean corrections to the same, given by the provisional theory, as derived from a comparison of the heliocentric positions of that theory with the heliocentric positions in the ephemeris.

Then (1) — (2) is the correction to the provisional theory given by observation. The process of forming (1) and (2) is shown quite fully in the following pages. Each individual printed observation was first compared with the printed ephemeris, and a correction to the latter was thence deduced. When this correction was given with the observations themselves, it was of course not recomputed, unless in some doubtful cases. The observations were then divided into groups, usually of about a month each, and coinciding in time with the grouping of the Greenwich results. The mean of the dates and the mean of the corrections were then taken separately for each group and each observatory. The separate results are shown

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in the proper columns of the following table, under the head "Mean dates," Mean cor. in R. A., and Mean cor. in Dec. These means are those given by the observations as printed, without the application of the systematic corrections on pages 120 and 121. In the columns "Corrected mean" these corrections are applied; this column would therefore exhibit no systematic differences between the results of the different observatories, unless the observations of Uranus were affected by errors different from those which affect the positions of the fundamental stars. A careful comparison of the differences in various parts of the table shows that this is unfortunately the case. A weight is next assigned to each individual result depending on the number of observations, the general sufficiency of the data of reduction, the mean discordance of the individual observations, and the quality of the instruments. The critical reader will notice a lack of homogeneity among the weights assigned, of which I shall speak presently. The mean of the separate group-results is then taken with regard to these weights, and also the mean of the mean dates, using for the latter the relative weights adopted for the several right ascensions. Thus, we have a mean result derived from all the observations for each month, or other group-period, which is written under the horizontal lines.

These corrections to right ascension and declination are next changed to corrections of longitude and latitude, using for this purpose the following table, which is computed from the formulæ of Gauss:

$$\cos E = \sin \varepsilon \cos \alpha \sec b = \sin \varepsilon \cos l \sec \delta$$

$$\Delta l = \frac{\sin E \cos \delta}{\cos b} \Delta\alpha + \frac{\cos E}{\cos b} \Delta\delta$$

$$\Delta b = - \cos E \cos \delta \, \Delta\alpha + \sin E \, \Delta\delta.$$

The differential coefficients in this table are expressed as a function of the right ascension of Uranus only, which may be done because, owing to the small inclination and great distance of the planet, its geocentric position on the celestial sphere is never more than about 2′ from some point of the projection of its heliocentric orbit. The coefficients of $\Delta\alpha$ are multiplied by 15, that the right ascension may be expressed in time.

To Convert Errors of Right Ascension and Declination of Uranus into errors of Longitude and Latitude.

When the Right Ascension exceeds 12^h, enter with R. A -12^h, and change the signs of the quantities $\frac{\partial b}{\partial \alpha}$ and $\frac{\partial l}{\partial \delta}$.

R. A.	Logarithms of									
	$\frac{\partial l}{\partial \alpha}$	$\frac{\partial b}{\partial \alpha}$		$\frac{\partial l}{\partial \delta}$		$\frac{\partial b}{\partial \delta}$	$\frac{\partial l}{\partial \alpha}$	$\frac{\partial b}{\partial \alpha}$	$\frac{\partial l}{\partial \delta}$	$\frac{\partial b}{\partial \delta}$
0^h 0^m	1.1386	—0.7761	— 4	+9.6000	— 4	9.9626	+13.8+	—5 97+	+0.40—	+0.92+
10	1.1387	—0.7757	— 14	+9.5996	— 13	9.9626		5.96	0.40	
20	1.1388	—0.7743	— 23	+9.5983	— 20	9.9628		5 95	0.40	
30	1.1389	—0.7720	— 33	+9.5963	— 29	9.9632		5.92	0.40	
40	1.1390	—0.7687	— 44	+9.5934	— 38	9.9637		5.87	0.39	
50	1.1391	—0.7643	— 55	+9.5896	— 47	9.9645		5.81	0.38	
1 0	1.1392	—0.7588	— 63	+9.5849	— 55	9.9653	+13.8+	—5.74+	+0.38—	+0.92+
10	1.1393	—0.7525	— 74	+9.5794	— 64	9.9662		5.66	0.38	
20	1.1394	—0.7451	— 86	+9.5730	— 74	9.9673		5.56	0.37	
30	1.1394	—0.7365	— 95	+9.5656	— 83	9.9685		5.45	0.37	
40	1.1395	—0.7270	—108	+9.5573	— 94	9.9698		5.33	0.36	
50	1.1395	—0.7162	—118	+9.5479	—104	9.9711		5.20	0.35	

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To Convert Errors of Right Ascension and Declination.—*Continued.*

R. A.	Logarithms of									
	$\frac{\partial l}{\partial \alpha}$	$\frac{\partial b}{\partial \alpha}$		$\frac{\partial l}{\partial \delta}$		$\frac{\partial b}{\partial \delta}$	$\frac{\partial l}{\partial \alpha}$	$\frac{\partial b}{\partial \alpha}$	$\frac{\partial l}{\partial \delta}$	$\frac{\partial b}{\partial \delta}$
2^h 0^m	1.1395	– 0.7044	—129	+9.5375	—115	9.9725	+13.8+	—5.06+	+0.34—	+0.93+
10	1.1395	—0.6915	—142	+9.5260	—126	9.9740		4.91	0.33	
20	1.1395	—0.6773	—155	+9.5134	—139	9.9756		4.75	0.32	
30	1.1395	—0.6618	—168	+9.4995	—152	9.9772		4.58	0.31	
40	1.1395	—0.6450	—184	+0.4843	—167	9.9788		4.41	0.30	
50	1.1394	—0.6266	—197	+9.4676	—181	9.9804		4.23	0.29	
3 0	1.1394	—0.6069	—215	+9.4495	—198	9.9821	+13.8+	—4.05+	+0.28—	+0.96+
10	1.1394	—0.5854	—233	+9.4297	—216	9.9837		3.86	0.27	
20	1.1393	—0.5621	—253	+9.4081	—237	9.9853		3.66	0.26	
30	1.1393	—0.5368	—274	+9.3844	—258	9.9869		3.45	0.24	
40	1.1392	—0.5094	—299	+9.3586	—284	9.9883		3.24	0.23	
50	1.1391	—0.4795	—327	+9.3302	—312	9.9898		3.02	0.21	
4 0	1.1390	—0.4468	—359	+9.2990	—346	9.9912	+13.8+	—2.80+	+0.20—	+0.98+
10	1.1390	—0.4109	—399	+9.2644	—385	9.9925		2.58	0.18	
20	1.1389	—0.3710	—443	+9.2259	—431	9.9938		2.36	0.17	
30	1.1388	—0.3267	—498	+9.1828	—487	9.9949		2.14	0.15	
40	1.1387	—0.2769	—571	+9.1341	—560	9.9959		1.91	0.13	
50	1.1386	—0.2198	—659	+9 0781	—651	9.9969		1.67	0.12	
5 0	1.1385	—0.1539	—786	+9.0130	—777	9.9977	+13.8+	—1.43+	+0.10—	+0.99+
10	1.1384	—0.0753	— 96	+8.9353	—956	9.9984		1.19	0.08	
20	1.1383	—9.979	—125	+8.8397	—1240	9.9990		0.95	0.07	
30	1.1382	—9.854	—175	+8.7157	—1757	9.9994		0.71	0.05	
40	1.1381	—9.679	—301	+8.540	—300	9.9997		0.48	0.03	
50	1.1380	—9.378	......	+8.240	......	9.9999		0.24	0.02	
6 0	1.1379	— ∞	......	∞	......	0	+13.7+	0.00	0.00	+1.00+
10	1.1378	+9.378	+301	—8.240	+300	9.9999		—0.24+	—0.02+	
20	1.1377	+9.679	+175	—8.540	+1757	9.9997		0.48	0.03	
30	1.1376	+9.854	+124	—8.7157	+1240	9.9994		0.71	0.05	
40	1.1375	+9.978	+ 96	—8.8397	+956	9.9990		0.95	0.07	
50	1.1374	+0.0743	+783	—8.9353	+777	9.9984		1.18	0.08	
7 0	1.1373	+0.1526	+658	—9.0130	+651	9.9977	+13.7+	+1.42—	—0.10+	+0.99+
10	1.1373	+0.2184	+569	—9.0781	+560	9.9969		1.65	0.12	
20	1.1372	+0.2753	+497	—9.1341	+487	9.9959		1.88	0.13	
30	1.1371	+0.3250	+442	—9.1828	+431	9.9949		2.11	0.15	
40	1.1371	+0.3692	+397	—9.2259	+385	9.9938		2.34	0.17	
50	1.1370	+0.4089	+359	—9.2644	+346	9.9925		2.56	0.18	
8 0	1.1370	+0.4448	+325	—9.2990	+312	9.9912	+13.7+	+2.78—	—0.20+	+0.98+
10	1.1370	+0.4773	+298	—9.3302	+284	9.9898		3.00	0.21	
20	1.1370	+0.5071	+273	—9.3586	+258	9.9883		3.22	0.23	
30	1.1370	+0.5344	+253	—9.3844	+237	9.9869		3.43	0.24	
40	1.1370	+0.5597	+233	—9.4081	+216	9.9853		3.63	0.26	
50	1.1370	+0.5830	+214	—9.4297	+198	9 9837		3.83	0.27	
9 0	1.1370	+0.6044	+197	—9.4495	+181	9.9821	+15.7+	+4.02—	—0.28+	+0.96+
10	1.1370	+0.6241	+184	—9.4676	+167	9.9804		4.20	0.29	
20	1.1370	+0.6425	+169	—9.4843	+152	9.9788		4.38	0.30	
30	1.1371	+0.6594	+155	—9.4995	+139	9.9772		4.55	0.31	
40	1.1371	+0.6749	+143	—9.5134	+126	9.9756		4.72	0.32	
50	1.1372	+0.6892	+131	—9.5260	+115	9.9740		4.88	0.33	
10 0	1.1373	+0.7023	+119	—9.5375	+104	9.9725	+13.7+	+5.04—	—0.34+	+0.93+
10	1.1374	+0.7142	+109	—9.5479	+ 94	9.9711		5.18	0.35	
20	1.1375	+0.7251	+ 96	—9.5573	+ 83	9 9698		5.31	0.36	
30	1.1376	+0.7347	+ 87	—9.5656	+ 74	9.9685		5.43	0.37	
40	1.1377	+0.7434	+ 76	—9.5730	+ 64	9.9673		5.54	0.37	
50	1.1378	+0.7510	+ 65	—9.5794	+ 55	9.9662		5.64	0.38	
11 0	1.1379	+0.7575	+ 57	—9.5849	+ 47	9.9653	+13.7+	+5.72—	—0.38+	+0.92+
10	1.1380	+0.7632	+ 46	—9.5896	+ 38	9.9645		5.79	0.38	
20	1.1381	+0.7678	+ 35	—9.5934	+ 29	9.9637		5.85	0.39	
30	1.1382	+0.7713	+ 24	—9.5963	+ 20	9.9632		6.90	0.39	
40	1.1383	+0.7737	+ 17	—9.5983	+ 13	9.9628		5.94	0.39	
50	1.1385	+0.7754	+ 7	—9.5996	+ 4	9.9626		5.96	0.40	
12 0	1.1386	+0.7761	......	—9.6000	......	9.9626	+13.8+	+5.97—	—0.40+	+0.92+

We thus have, for the interval occupied by each group of observations, a mean correction to the geocentric longitude and latitude of the planet given by observations, which are found in the ninth and tenth columns of the table, on the same horizontal line with the mean corrections in right ascension and declination from which they are derived. The next step is to obtain the corresponding corrections given by the provisional ephemeris.

This correction has been first obtained for every twentieth day of each of the forty-two oppositions included in the table. The heliocentric longitude, latitude, and radius vector were interpolated to the most convenient twenty-day intervals, and compared with the corresponding co-ordinates in the heliocentric ephemeris. This ephemeris was of course the one corresponding to that with which the observations were compared, namely, the Berliner Jahrbuch for the years 1830–33, and the Nautical Almanac for subsequent years. These comparisons are fully given at the end of this chapter, and the resulting corrections to the printed ephemeris are given in the proper columns of the table.

These corrections to the heliocentric co-ordinates were then changed to corrections of geocentric longitude and latitude by the following formulæ. Put

r', the projection of the planet's radius vector on the ecliptic;
ρ', the projection of the planet's distance from the earth on the same plane;
ρ, this distance itself;
λ, β, the planet's heliocentric longitude and latitude;
L, the sun's geocentric longitude;
R, its radius vector;
M, the modulus of the common logarithms;
δl, δb, the corrections to the geocentric longitude and latitude;
$\delta\rho$, the correction to the common logarithm of the radius vector.

Then

$$\begin{aligned} \delta l = & \frac{r'^2}{\rho'^2}\left\{1 + \frac{R}{r'}\cos(L-\lambda)\right\}\delta\lambda \\ & - \frac{Rr'}{\rho'^2}\sin(L-\lambda)\frac{\delta\rho}{M\sin 1''} \\ & + \frac{Rr'}{\rho'^2}\sin(L-\lambda)\tan\beta\delta\beta \\ \delta b = & \frac{r'}{\rho'}\left\{1 + \frac{r'R}{\rho^2}\tan^2\beta\cos(L-\lambda)\right\}\delta\beta \\ & - \frac{r'^2R}{\rho'\rho^2}\tan\beta\sin(L-\lambda)\,\delta\lambda \\ & + \frac{rR^2}{\rho'\rho^2}\left\{1 + \frac{r'}{R}\cos(L-\lambda)\right\}\sin\beta\frac{\delta\rho}{M\sin 1''}. \end{aligned}$$

The last term in δl and the last two terms of δb have been omitted in the computation, as they scarcely ever exceed a few hundredths of a second.

The values of δl and δb are printed in the last two columns of the table. The formula for δb might have contained the additional term

$$\delta b = \sin l \delta\omega$$

$\delta\omega$ being the correction to the obliquity of the ecliptic adopted in the ephemeris to reduce it to that employed in the provisional theory. This correction is, however, deferred until we come to form the equations of condition.

From the values of δl and δb thus obtained we are to find the mean values during each group of observations. If these quantities varied uniformly, the proper value would be that corresponding to the mean date of each group. But the second differences are so large that this value would generally be in error by one- or two-tenths of a second. Owing to the minuteness of this difference, it has been considered that when the mean date was near the middle of a twenty-day interval, the correction δl interpolated to that date without regard to second differences would furnish a sufficient approximation to the required mean value of δl during an interval of about 30 days. In other case the value of δl was interpolated to 5-day intervals through the period of each group of observations, and the mean value taken.

During the years 1850–1863 the sun's longitude employed in the ephemeris required a gradually increasing correction, amounting at the latter date to about 3″. A small correction of which the maximum value is about 0″.15 was applied to δl to reduce it to the value it would have had if Hansen's tables had been employed.

The corrected mean values of δl and δb thus obtained are given in the last two columns of the following table, being inclosed in brackets and printed immediately above the values of Δl and Δb derived from observation.

I deem it proper to mention that the mechanical labor of constructing these tables of comparisons, in the manner just described, was in great part performed by Dr. C. L. F. Kampf, who was employed by the Smithsonian Institution to assist me in the work. Before using it I subjected the whole of the work to a careful revision, altering especially the relative weights of the corrected means in many cases. As the assigned weights now stand, each set of results which are combined into a single mean has its own unit of weight, which does not necessarily coincide with that of any other set. The use of a uniform scale of weights through this series of observations, and the assignment to every final mean of a weight equal to the sum of the weights of the quantities whose mean was taken, would have led to weights in many cases quite fictitious, owing to the obvious presence of systematic errors in the results. For this reason I have made no further use of the weights found in this table, and their lack of homogeneousness therefore does no harm.

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MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS IN THE BERLINER JAHRBUCH AND THE NAUTICAL ALMANAC.

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1830	s		s	″		″	″	″
Königsberg,	July 29	—1.56	7	—1.56$_5$	+4.6	7	+4.6		
Cambridge,	July 29	—1.51	6	—1.67$_2$					
$[20^h\ 40^m]$	July 29	....	...	—1.60	....	...		[—19.2] —20.8	[+ 9.5] +10.3
Königsberg,	Aug. 12	—1.65	2	—1.65$_2$	+3.6	2	+3.6		
Cambridge,	Aug. 25	—1.36	4	—1.52$_2$					
$[20^h\ 36^m]$	Aug. 19	....	...	—1.58	....	...		[—18.5] —20.8	[+ 9.5] + 9.1
Cambridge, $[20^h\ 37^m]$	Sept. 19	—1.46	7	—1.62$_3$	....	...	....	[—17.2] —21.2	
Cambridge, $[20^h\ 36^m]$	Oct. 17	—1.34	8	—1.50$_4$	....	...	....	[—16.7] —19.5	
Cambridge, $[20^h\ 37^m]$	Nov. 14	—1.36	8	—1.52$_4$	....	...	....	[—16.6] —19.8	
	1831								
Greenwich,	Aug. 3	—1.72	11	—1.72$_4$	+2.5	11	+4.3		
Cambridge,	Aug. 8	—1.70	9	—1.86$_4$					
$[20^h\ 58^m]$	Aug. 6	....	...	—1.79	....	...		[—23.8] —23.4	[+10.0] +11.1
Greenwich,	Sept. 7	—1.67	5	—1.67$_3$	+3.5	5	+5.3		
Cambridge,	Sept. 15	—1.60	6	—1.76$_3$					
$[20^h\ 52^m]$	Sept. 11	....	...	—1.72	....	...		[—22.0] —22.2	[+10.0] +11.8
Greenwich,	Nov. 4	—1.48	7	—1.48$_4$	+3.1	7	+4.9		
Cambridge,	Oct. 26	—1.54	16	—1.70$_8$					
$[20^h\ 50^m]$	Oct. 31	....	...	—1.63	....	...		[—20.7] —21.0	[+ 9.8] +10.8
	1832								
Greenwich,	Aug. 9	—2.02	3	—2.02$_2$	+1.7	2	+3.4$_2$		
Königsberg,	Aug. 10	—2.24	3	—2.24$_4$					
Cambridge,	Aug. 15	—1.99	9	—2.18$_8$					
Vienna,	Aug. 3	—2.33	3	—2.33$_1$	+0.2	3	+1.1		
$[21^h\ 17^m]$	Aug. 12	....	...	—2.19	....	...	+2.5	[—28.4] —29.2	[+10.3] +11.9
Cambridge, $[21^h\ 12^m]$	Sept. 12	—1.97	10	—2.16$_4$	....	...	....	[—26.9] —28.8	
Cambridge,	Oct. 6	—1.91	13	—2.09$_6$					
Vienna,	Oct. 12	—1.90	15	—1.90$_4$	+1.9	15	+2.8		
$[21^h\ 9^m]$	Oct. 7	....	...	—2.01	....	...		[—25.8] —26.6	[+10.2] +11.1
Cambridge,	Nov. 16	—1.85	7	—2.04$_7$					
Vienna,	Nov. 9	—1.89	2	—1.89$_1$	+1.1	...	+2.0		
$[21^h\ 10^m]$	Nov. 15	....	...	—2.02	....	...	....	[—25.2] —27.4	[+10.1] +10.5

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1833	s		s	″		″	″	″
Greenwich,	Aug. 22	—2.57	7	-2.57_{2}	—1.4	7	$+0.2_{6}$		
Königsberg,	Aug. 12	—2.57	5	-2.57_{4}	—1.5	5	-1.5_{5}		
Cambridge,	Aug. 15	—2.40	11	-2.59_{5}				[—33.8]	[+10.8]
[21h 32m]	Aug. 15	....	...	—2.58	....	...	—0.6	—35.6	+10.9
Greenwich,	Sept. 18	—2.33	5	-2.33_{2}	—0.8	4	$+0\ 8_{4}$		
Cambridge,	Sept. 19	—2.33	10	-2.52_{4}	+2.8	11	$+1.4_{11}$		
Vienna,	Sept. 11	—2.52	6	-2.56_{1}	+2.6	6	$+1.2_{2}$	[—32.2]	[+10.8]
[21h 28m]	Sept. 18	....	...	—2.47	....	...	+1.2	—33.4	+12.3
Greenwich,	Oct. 11	—2.25	4	-2.25_{1}	+0.1	4	$+1.7_{4}$		
Cambridge,	Oct. 12	—2.29	12	-2.48_{5}	+2.2	13	$+0.8_{13}$		
Vienna,	Oct. 14	—2.37	19	-2.41_{2}				[—31.2]	[+10.6]
[21h 26m]	Oct. 12	....	...	—2.43	....	...	+1.0	—33.0	+11.9
Cambridge,	Nov. 18	—2.16	5	-2.35_{3}	+ 1.3	4	—0.1		
Vienna,	Nov. 14	—2.24	4	-2.28_{1}				[—30.3]	[+10.3]
[21h 26m]	Nov. 16	....	...	—2.33	....	...	—0.1	—32.0	+10.4
	1834								
Cambridge,	Aug. 15	—2.58	11	-2.77_{5}	— 1.3	11	-2.6_{11}		
Vienna,	Aug. 13	—3.07	4	-3.11_{1}	— 4.4	4	-3.5_{1}	[—40.6]	[+11.1]
[21h 49m]	Aug. 14	....	...	—2.83	....	...	—2.7	—39.7	+11.3
Greenwich,	Sept. 10	—3.00	5	-3.00_{1}	— 4.4	5	-3.0_{5}		
Cambridge,	Sept. 16	—2.66	18	-2.85_{4}	— 1.2	18	-2.5_{18}	[—39.0]	[+11.1]
[21h 45m]	Sept. 13	....	...	—2.88	....	...	—2.6	—40.5	+11.2
Greenwich,	Oct. 14	—2.74	10	-2.74_{3}	— 3.1	10	-1.7_{10}		
Cambridge,	Oct. 16	—2.68	16	-2.87_{7}	— 0.8	16	-2.1_{16}		
Vienna,	Oct. 20	—2.75	4	-2.80_{1}	— 1.3	4	-0.4_{1}	[—37.5]	[+10.9]
[21h 41m]	Oct. 17	....	...	—2.83	....	...	—1.9	—39.5	+11.6
Cambridge,	Nov. 16	—2.64	9	-2.83_{4}	— 0.7	11	-2.0_{11}		
Vienna,	Nov. 12	—2.53	8	-2.58_{1}	— 2.8	8	-1.9_{4}	[—36.7]	[+10.7]
[21h 41m]	Nov. 14	....	...	—2.77	....	...	—2.0	—38.7	+11.3
Cambridge,	Dec. 6	—2.71	5	-2.90_{2}	— 1.2	4	-2.5_{4}		
Vienna,	Dec. 7	—2.55	1	-2.59_{0}	— 2.1	1	$-1.2_{\frac{1}{4}}$	[—36.2]	[+10.5]
[21h 43m]	Dec. 7	....	...	—2.88	....	...	—2.4	—40.3	+11.5
	1835								
Greenwich,	Aug. 17	—3.25	4	-3.25_{1}	— 6.6	4	-5.4_{4}		
Königsberg,	Aug. 20	—3.31	11	-3.31_{8}					
Cambridge,	Aug. 14	—3.28	20	-3.47_{9}	— 4.7	20	-5.9_{20}		
Vienna,	Aug. 21	—3.30	1	-3.35_{0}	— 8.8	1	-7.9_{1}	[—47.1]	[+11.3]
[22h 4m]	Aug. 17	....	...	—3.39	....	...	—5.9	—48.6	+11.8

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1835	s		s	″		″	″	″
Cambridge,	Sept. 15	—3.17	9	-3.36_{4}	— 4.6	9	-5.8_{9}		
Vienna,	Sept. 14	—3.30	9	-3.35_{1}	— 6.7	9	-5.8_{2}	[—45.8]	[+11.2]
[22^h 1^m]	Sept. 15	....	...	—3.36	....	...	—5.8	—48.1	+11.6
Greenwich,	Oct. 10	—3.27	4	-3.27_{2}	— 6.53	4	-5.3_{4}		
Cambridge,	Oct. 17	—3.11	8	-3.30_{4}	— 4.1	8	-5.3_{8}	[—44.4]	[+11.1]
[21^h 57^m]	Oct. 15	....	...	—3.29	....	...	—5.3	—46.9	+11.4
Greenwich,	Nov. 27	—3.21	6	-3.21_{2}	— 5.6	7	-4.4_{7}		
Cambridge,	Nov. 26	—3.00	10	-3.19_{4}	— 4.5	9	-5.7_{9}	[—43.0]	[+10.7]
[21^h 57^m]	Nov. 26	....	...	—3.20	....	...	—5.1	—45.6	+11.2
	1836								
Greenwich,	July 22	—3.80	7	-3.84_{2}	—10.5	8	-9.5_{8}		
Cambridge,	July 25	—3.60	3	-3.89_{1}	— 8.9	2	-9.9_{2}	[—55.3]	[+11.5]
[22^h 24^m]	July 23	....	...	—3.86	....	...	—9.6	—56.5	+11.8
Greenwich,	Aug. 24	—3.78	7	-3.82_{3}	— 9.8	8	-8.8_{8}		
Königsberg,	Aug. 30	—3.63	5	-3.63_{4}					
Cambridge,	Aug. 16	—3.78	12	-3.97_{5}	— 9.3	12	-10.3_{12}		
Edinburgh,	Aug. 19	—4.09	9	-4.10_{3}	— 9.3	7	-9.3_{7}		
Vienna,	Aug. 20	—3.77	1	-3.81_{0}	—12.5	1	-11.5_{0}	[—54.6]	[+11.6]
[22^h 20^m]	Aug. 22	....	...	—3.87	....	...	—9.6	—56.6	+11.5
Greenwich,	Sept. 13	—3.77	7	-3.81_{3}	— 8.8	6	-7.8_{6}		
Cambridge,	Sept. 16	—3.70	10	-3.89_{4}	— 8.4	10	-9.4_{10}		
Edinburgh,	Sept. 16	—4.01	8	-4.02_{2}	— 8.6	8	-8.6_{8}		
Vienna,	Sept. 15	—3.59	9	-3.63_{1}	—10.4	9	-9.4_{3}	[—53.4]	[+11.6]
[22^h 16^m]	Sept. 15	....	...	—3.87	....	...	—8.8	—56.2	+12.1
Greenwich,	Oct. 12	—3.67	7	-3.71_{2}	— 8.6	8	-7.6_{8}		
Cambridge,	Oct. 16	—3.51	12	-3.70_{5}	— 8.5	11	-9.5_{11}		
Edinburgh,	Oct. 15	—4.05	8	-4.06_{2}	— 7.9	4	-7.9_{4}		
Vienna,	Oct. 11	—3.57	10	-3.61_{1}	—10.2	10	-9.2_{3}	[—51.8]	[+11.3]
[22^h 13^m]	Oct. 15	....	...	—3.77	...	...	—8.6	—54.8	+11.7
Greenwich,	Nov. 13	—3.59	11	-3.63_{5}	— 9.1	11	-8.1_{11}		
Cambridge,	Nov. 13	—3.37	8	-3.56_{4}	— 7.9	7	-8.9_{7}		
Edinburgh,	Nov. 17	—3.78	8	-3.79_{2}					
Vienna,	Nov. 9	—3.56	3	-3.60_{1}	—10.2	3	-9.2_{1}	[—50.6]	[+11.1]
[22^h 12^m]	Nov. 13	....	...	—3.63	....	...	—8.5	—52.8	+11.0
Cambridge,	Dec. 12	—3.40	7	-3.59_{3}	— 8.0	7	-9.0_{7}		
Edinburgh,	Dec. 12	—3.29	5	-3.30_{1}	— 8.8	3	-8.8_{3}	[—50.0]	[+10.9]
[22^h 13^m]	Dec. 12	....	...	—3.52	....	...	—8.9	—51.5	+10.1
	1837							[—62.9]	[+11.5]
Greenwich, [22^h 40^m]	July 22	—4.23	4	-4.26_{1}	—13.4	4	—12.5	—63.2	+12.0

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1837	s		s	″		″	″	″
Greenwich,	Aug. 18	—4.30	10	-4.33_{2}	—13.4	10	-12.5_{10}		
Cambridge,	Aug. 18	—4.09	14	-4.28_{2}	—12.6	11	-13.4_{11}		
Edinburgh,	Aug. 22	—4.40	6	-4.40_{1}	—12.7	6	-12.7_{6}		
Paris,	Aug. 14	—4.34	9	-4.33_{2}	—12.0	10	-11.4_{10}		
Vienna,	Aug. 25	—4.29	4	-4.33_{0}	—12.6	4	-11.6_{1}	[—62.9]	[+11.7]
[22^{h} 36^{m}]	Aug 18	....	...	—4.33	....	...	—12.5	—64.1	+12.2
Greenwich,	Sept. 17	—4.23	14	$-4.26_{6\frac{1}{2}}$	—13.4	14	-12.5_{14}		
Königsberg,	Sept. 11	—4.10	8	-4.13_{6}					
Cambridge,	Sept. 17	—4.06	14	-4.25_{6}	—11.9	15	-12.7_{14}		
Edinburgh,	Sept. 10	—4.39	4	-4.39_{1}	—11.5	3	-11.5_{2}		
Paris,	Sept. 18	—4.20	12	-4.19_{6}	—12.3	13	-11.7_{13}		
Vienna,	Sept. 13	—4.12	5	-4.15_{1}	—13.9	5	-12.9_{1}	[—61.7]	[+11.6]
[22^{h} 32^{m}]	Sept. 16	....	...	—4.21	....	...	—12.3	—62.3	+11.5
Greenwich,	Oct. 16	—4.13	11	-4.16_{3}	—12.9	11	-12.0_{11}		
Cambridge,	Oct. 17	—4.00	10	-4.19_{4}	—11.6	11	-12.4_{11}		
Paris,	Oct. 17	—4.05	4	-4.04_{2}	—11.2	4	-10.6_{4}		
Vienna,	Oct. 18	—4.44	2	$-4.47_{\frac{1}{3}}$	—15.5	2	$-14.5_{\frac{1}{2}}$	[—59.8]	[+11.4]
[22^{h} 28^{m}]	Oct. 17	....	...	—4.16	....	...	—12.0	—61.5	+11.3
Greenwich,	Nov. 8	—4.10	5	-4.13_{2}	—12.4	5	-11.5_{5}		
Cambridge,	Nov. 4	—3.96	4	-4.15_{2}	—12.3	3	-13.1_{3}		
Paris,	Nov. 6	—4.07	3	-4.06_{2}	—11.3	3	-10.7_{3}		
Vienna,	Nov. 2	—4.09	2	$-4.12_{\frac{1}{3}}$	—12.9	2	$-11.9_{\frac{1}{2}}$	[—59.0]	[+11.2]
[22^{h} 27^{m}]	Nov. 6	....	...	—4.11	....	...	—11.7	—60.7	+11.3
Greenwich,	Dec. 2	—4.05	2	-4.08_{1}	—14.0	2	-13.1_{2}		
Cambridge,	Nov. 30	—3.87	6	-4.06_{3}	—11.6	7	-12.4_{7}		
Paris,	Dec. 8	—4.03	7	-4.02_{4}	—10.9	7	-10.3_{7}		
Vienna,	Dec. 8	—4.28	2	$-4.31_{\frac{1}{3}}$	—13.0	2	$-12.0_{\frac{1}{2}}$	[—57.7]	[+11.0]
[22^{h} 28^{m}]	Dec. 5	....	...	—4.05	....	...	—11.6	—59.9	+11.1
	1838								
Greenwich,	Aug. 20	—4.72	11	-4.76_{5}	—15.9	11	-15.1_{11}		
Königsberg,	Aug. 25	—4.65	5	-4.65_{4}	—18.6	5	-17.6_{3}		
Cambridge,	Aug. 19	—4.67	11	-4.78_{5}	—15.8	12	-16.3_{12}		
Edinburgh,	Aug. 25	—4.96	3	-4.95_{1}	—15.0	4	-15.0_{4}		
Paris,	Aug. 20	—4.80	9	-4.79_{5}	—16.6	9	-16.0_{9}	[—70.9]	[+11.7]
[22^{h} 51^{m}]	Aug. 21	....	...	—4.76	....	...	—15.8	—71.4	+12.3
Greenwich,	Sept. 12	—4.62	8	-4.66_{2}	—16.2	8	-15.4_{8}		
Königsberg,	Sept. 16	—4.65	14	-4.67_{10}	—18.5	14	-17.5_{7}		
Cambridge,	Sept. 16	—4.61	11	-4.72_{5}	—14.9	12	-15.4_{12}		
Edinburgh,	Sept. 15	—4.74	9	-4.73_{3}	—15.8	7	-15.8_{7}		
Paris,	Sept. 12	—4.71	6	-4.70_{3}	—16.0	6	-15.4_{6}		
Vienna,	Sept. 11	—4.71	12	-4.73_{1}	—18.1	12	-17.1_{3}		
Berlin,	Sept. 6	—4.72	7	-4.74_{2}	—17.5	7	-16.5_{3}	[—70.4]	[+11.7]
[22^{h} 47^{m}]	Sept. 15	....	...	—4.70	....	...	—16.0	—70.7	+12.0

Mean Corrections to the Ephemeris of Uranus.—*Continued.*									
Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1838	s		s	″		″	″	″
Greenwich,	Oct. 15	—4.67	7	-4.71_3	—15.5	7	-14.7_7		
Cambridge,	Oct. 17	—4.49	7	-4.60_3	—15.3	6	-15.8_6		
Edinburgh,	Oct. 16	—4.56	12	-4.55_3	—15.0	8	-15.0_8		
Paris,	Oct. 16	—4.66	7	-4.65_3	—15.9	7	-15.3_7		
Vienna,	Oct. 14	—4.66	9	-4.68_1	—17.3	9	-16.3_2	[—68.6]	[+11.5]
[22^h 44^m]	Oct. 16	....	...	—4.63	....	...	—16.2	—69.7	+10.8
Greenwich,	Nov. 9	—4.52	6	-4.56_2	—16.0	6	-15.2_6		
Cambridge,	Nov. 15	—4.37	10	-4.48_4	—15.1	9	-15.6_9		
Edinburgh,	Nov. 16	—4.70	8	-4.69_2	—17.4	2	-17.4_2		
Vienna,	Nov. 7	—4.93	4	$-4.95_{\frac{1}{2}}$	—18.3	4	-17.3_1	[—66.9]	[+11.3]
[22^h 42^m]	Nov. 14	....	...	—4.58	....	...	—15.8	—68.8	+10.9
Greenwich,	Dec. 9	—4.60	2	-4.64_1	—15.9	2	-15.1_2		
Cambridge,	Dec. 15	—4.26	5	-4.37_2	—15.1	7	-15.6_7		
Edinburgh,	Dec. 17	—4.18	3	-4.17_1	—14.7	1	-14.7_1		
Paris,	Dec. 5	—4.40	7	-4.39_4	—15.1	7	-14.5_7	[—65.8]	[+11.1]
[22^h 43^m]	Dec. 10	....	...	—4.38	....	...	—15.0	—65.7	+10.4
	1839								
Greenwich,	Aug. 22	—5.28	5	-5.32_1	—21.3	5	-20.6_5		
Cambridge,	Aug. 24	—5.11	8	-5.22_4	—20.7	7	-21.0_7		
Paris,	Aug. 23	—5.20	3	-5.19_2	—20.7	3	-20.1_3		
Edinburgh,	Aug. 25	—5.21	2	$-5.20_{\frac{1}{2}}$				[—79.7]	[+11.7]
[23^h 6^m]	Aug. 23	....	...	—5.22	....	...	—20.7	—79.7	+11.0
Greenwich,	Sept. 10	—5.14	12	-5.18_3	—21.0	12	-20.3_{12}		
Königsberg,	Sept. 12	—5.11	12	-5.13_8	—21.7	12	-20.7_6		
Berlin,	Sept. 10	—5.10	14	-5.11_4	—21.1	14	-20.1_7		
Cambridge,	Sept. 17	—5.12	11	-5.23_5	—20.0	10	-20.3_{10}		
Paris,	Sept. 14	—5.23	10	-5.22_5	—20.7	10	-20.1_{10}		
Edinburgh,	Sept. 16	—5.16	17	-5.15_5	—20.5	5	-20.5_5		
Vienna,	Sept. 17	—5.32	9	-5.33_1	—20.8	9	-19.8_2	[—79.0]	[+11.6]
[23^h 3^m]	Sept. 14	....	...	—5.17	....	...	—20.3	—78.8	+11.0
Greenwich,	Oct. 12	—5.18	5	-5.22_2	—20.2	5	-19.5_5		
Cambridge,	Oct. 15	—5.07	8	-5.18_4	—19.2	5	-19.5_8		
Edinburgh,	Oct. 15	—5.14	10	-5.13_3	—19.9	8	-19.9_8		
Vienna,	Oct. 10	—5.17	13	-5.18_1	—20.5	13	-19.5_4	[—77.4]	[+11.5]
[22^h 59^m]	Oct. 14	....	...	—5.18	....	...	—19.6	—78.7	+11.8
Greenwich,	Nov. 12	—4.98	6	-5.02_2	—20.2	6	-19.5_6		
Cambridge,	Nov. 19	—4.85	6	-4.96_2	—19.1	5	-19.4_5		
Paris,	Nov. 9	—4.99	2	-4.98_1	—20.4	2	-19.8_2		
Edinburgh,	Nov. 11	—4.90	3	-4.89_1	—18.3	1	-18.3_1	[—75.3]	[+11.3]
[22^h 57^m]	Nov. 14	....	...	—4.97	....	...	—19.4	—75.7	+10.4
Greenwich,	Dec. 6	—4.84	2	-4.88_1	—19.0	2	-18.3_2		
Cambridge,	Dec. 15	—4.83	5	-4.94_2	—19.1	2	-19.4_2		
Paris,	Dec. 3	—4.90	3	-4.89_2	—17.8	3	-17.2_3		
Edinburgh,	Dec. 28	—4.96	4	-4.95_1	—18.5	3	-18.5_3	[—73.9]	[+11.0]
[22^h 57^m]	Dec. 12	....	...	—4.92	....	...	—18.2	—74.6	+11.2

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1840	s		s	″		″	″	″
Greenwich,	Aug. 14	—5.77	11	—5.81$_3$	—24.5	11	—23.9$_{11}$		
Cambridge,	Aug. 14	—5.64	1	—5.73$_{\frac{1}{2}}$	—25.0	1	—25.2$_1$		
Edinburgh,	Aug. 31	—5.65	1	—5.64$_{\frac{1}{2}}$	—23.8	1	—23.7$_1$	[—87.9]	[+11.4]
[23^h 20^m]	Aug. 16	....	...	—5.78	....	...	—23.9	—88.8	+11.9
Greenwich,	Sept. 15	—5.66	8	—5.70$_3$	—24.0	8	—23.4$_8$		
Königsberg,	Sept. 13	—5.74	6	—5.76$_4$	—25.1	6	—24.1$_3$		
Cambridge,	Sept. 12	—5.43	1	—5.52$_{\frac{1}{3}}$					
Edinburgh,	Sept. 16	—5.70	13	—5.69$_4$	—24.6	9	—24.5$_9$		
Paris,	Sept. 6	—5.60	4	—5.59$_2$	—23.6	4	—23.0$_4$		
Berlin,	Sept. 14	—5.45	2	—5.45$_{\frac{1}{3}}$	—26.3	2	—25.3$_1$		
Vienna,	Sept. 9	—5.76	2	—5.76$_{\frac{1}{3}}$	—22.2	2	—21.2$_{\frac{1}{2}}$	[—87.6]	[+11.5]
[23^h 18^m]	Sept. 13	....	...	—5.69	....	...	—23.8	—87.5	+11.4
Greenwich,	Oct. 10	—5.61	9	—5.65$_3$	—24.5	9	—23.9$_9$		
Cambridge,	Oct. 9	—5.49	5	—5.58$_2$	—23.5	5	—23.7$_5$		
Edinburgh,	Oct. 15	—5.64	12	—5.63$_3$	—23.3	10	—23.2$_{10}$		
Paris,	Oct. 11	—5.60	4	—5.59$_2$	—23.3	4	—22.7$_4$		
Berlin,	Oct. 27	—5.50	1	—5.50$_{\frac{1}{4}}$	—22.8	1	—21.8$_1$		
Vienna,	Oct. 19	—5.76	5	—5.76$_{\frac{1}{2}}$	—22.5	4	—21.5$_1$	[—85.8]	[+11.4]
[23^h 14^m]	Oct. 12	....	...	—5.62	....	...	—23.3	—86.3	+11.2
Greenwich,	Nov. 6	—5.58	7	—5.62$_2$	—23.5	6	—22.9$_6$		
Cambridge,	Nov. 3	—5.34	2	—5.43$_1$	—24.2	1	—24.4$_1$		
Edinburgh,	Nov. 17	—5.41	8	—5.40$_2$	—22.9	1	—22.8$_1$		
Paris,	Nov. 4	—5.52	2	—5.51$_1$	—26.1	2	—25.5$_2$		
Vienna,	Nov. 15	—5.42	6	—5.42$_1$	—23.2	6	—22.2$_2$	[—83.7]	[+11.3]
[23^h 12^m]	Nov. 9	....	...	—5.48	....	...	—23.3	—84.4	+10.3
Greenwich,	Dec. 3	—5.36	8	—5.40$_2$	—23.5	9	—22.9$_9$		
Cambridge,	Dec. 3	—5.38	2	—5.47$_1$	—22.5	2	—22.7$_2$		
Edinburgh,	Dec. 15	—5.49	3	—5.48$_1$	—22.6	2	—22.5$_2$		
Vienna,	Dec. 4	—5.50	1					[—82.2]	[+11.0]
[23^h 12^m]	Dec. 6	....	...	—5.44	....	...	—22.8	—83.7	+10.4
	1841								
Greenwich,	Aug. 20	—6.16	5	—6.20$_2$	—28.6	5	—28.1$_5$		
Paris,	Aug. 19	—6.14	2	—6.13$_1$	—29.6	2	—29.0$_2$	[—96.9]	[+11.3]
[23^h 37^m]	Aug. 20	....	...	—6.18	....	...	—28.3	—96.2	+10.6
Greenwich,	Sept. 11	—6.16	10	—6.20$_4$	—29.0	10	—28.5$_{10}$		
Königsberg,	Sept. 11	—6.18	5	—6.18$_4$	—30.1	5	—29.1$_2$		
Berlin,	Sept. 14	—6.14	7	—6.14$_2$	—29.6	7	—28.6$_3$		
Edinburgh,	Sept. 17	—6.11	7	—6.15$_2$	—28.2	5	—28.0$_5$		
Paris,	Sept. 14	—6.16	5	—6.15$_3$	—29.0	3	—28.4$_3$		
Vienna,	Sept. 17	—6.37	11	—6.37$_1$	—30.3	11	—29.3$_3$	[—96.5]	[+11.4]
[23^h 33^m]	Sept. 13	....	...	—6.18	....	...	—28.5	—96.3	+10.4

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1841	s		s	″		″	″	″
Greenwich,	Oct. 17	—6.09	7	-6.13_2	—28.2	7	-27.7_7		
Berlin,	Oct. 20	—5.87	2	-5.87_1	—28.2	2	-27.2_1		
Edinburgh,	Oct. 13	—6.00	4	-6.04_1	—27.2	4	-27.0_4		
Paris,	Oct. 21	—6.10	1	-6.09_1	—27.2	1	-26.6_1		
Vienna,	Oct. 16	—6.14	9	-6.14_1	—28.8	9	-27.8_2	[—94.6]	[+11.2]
$[23^h\ 28^m]$	Oct. 18	....	...	—6.06	....	...	—27.4	—94.1	+10.5
Greenwich,	Nov. 17	—5.95	5	-5.99_2	—28.0	5	-27.5_5		
Berlin,	Nov. 16	—6.15	2	-6.15_1	—28.4	2	-27.4_1		
Edinburgh,	Nov. 2	—5.89	3	-5.93_1	—26.5	4	-26.3_4		
Vienna,	Nov. 11	—5.98	4	-5.98_1	—30.0	4	-29.0_1	[—92.6]	[+11.0]
$[23^h\ 26^m]$	Nov. 13	....	...	—6.01	....	...	—27.2	—93.3	+10.4
Greenwich,	Dec. 14	—5.86	8	-5.90_2	—27.4	8	-26.9_8		
Berlin,	Dec. 15	—5.60	3	-5.60_1	—25.5	3	-24.5_2		
Edinburgh,	Dec. 20	—5.68	4	-5.73_1					
Paris,	Dec. 15	—5.78	4	-5.77_2	—26.7	4	-26.1_4	[—90.5]	[+10.7]
$[23^h\ 26^m]$	Dec. 16	....	...	—5.78	....	...	—26.3	—89.9	+ 9.8
	1842								
Greenwich,	Aug. 19	—6.62	5	-6.64_1	—32.1	6	-31.7_6		
Cambridge,	Aug. 24	—6.48	2	-6.55_1	—32.7	2	-33.0_2	[—105.6]	[+11.1]
$[23^h\ 51^m]$	Aug. 21	....	...	—6.61	....	...	—32.0	—103.6	+10.0
Greenwich,	Sept. 12	—6.55	4	-6.57_1	—32.2	4	-31.8_4		
Königsberg,	Sept. 18	—6.69	10	-6.69_7	—33.4	10	-32.4_5		
Berlin,	Sept. 11	—6.70	3	-6.69_1	—31.7	3	-30.7_1		
Paris,	Sept. 14	—6.65	5	-6.64_3	—32.1	5	-31.5_5		
Cambridge,	Sept. 14	—6.60	13	-6.67_6	—31.9	14	-32.2_{14}		
Edinburgh,	Sept. 23	—6.57	4	-6.59_1	—30.9	4	-30.6_2		
Pulkowa,	Sept. 17	—6.63	6	-6.64_5					
Vienna,	Sept. 19	—6.81	2	$-6.80_{\frac{1}{2}}$	—33.5	2	$-32.5_{\frac{1}{2}}$	[—105.4]	[+11.2]
$[23^h\ 47^m]$	Sept. 16	....	...	—6.66	....	...	—32.0	—104.3	+10.3
Greenwich,	Oct. 20	—6.66	12	-6.68_3	—31.8	12	-31.4_{12}		
Berlin,	Oct. 23	—6.54	4	-6.53_1	—30.7	4	-29.7_2		
Paris,	Oct. 17	—6.64	12	-6.63_6	—31.5	9	-30.9_9		
Cambridge,	Oct. 17	—6.60	11	-6.67_5	—31.4	10	-31.7_{10}		
Edinburgh,	Oct. 15	—6.57	12	-6.59_3	—31.1	8	-30.8_8		
Vienna,	Oct. 10	—6.74	4	$-6.73_{\frac{1}{2}}$	—34.6	5	-33.6_1	[—103.6]	[+11.0]
$[23^h\ 44^m]$	Oct. 17	....	...	—6.64	....	...	—31.2	—103.7	+10.9
Greenwich,	Nov. 23	—6.36	7	-6.38_3	—31.0	7	-30.6_7		
Berlin,	Nov. 8	—6.42	2	$-6.41_{\frac{1}{2}}$	—30.2	2	-29.2_1		
Cambridge,	Nov. 17	—6.40	7	-6.47_3	—30.7	7	-31.0_7		
Edinburgh,	Nov. 16	—6.39	3	-6.32_1					
Vienna,	Nov. 20	—6.39	3	$-6.38_{\frac{1}{2}}$	—30.0	2	$-29.0_{\frac{1}{2}}$	[—100.8]	[+10.8]
$[23^h\ 42^m]$	Nov. 19	....	...	—6.41	....	...	—30.6	—100.2	+10.0

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1842	s		s	″		″	″	″
Greenwich,	Dec. 16	—6.31	8	-6.33_3	—30.2	8	-29.8_8		
Berlin,	Dec. 10	—6.16	4	-6.15_1	—31.4	4	-30.4_2		
Paris,	Dec. 14	—6.23	5	-6.22_3	—30.9	5	-30.3_5		
Cambridge,	Dec. 16	—6.22	9	-6.29_4	—30.1	8	-30.4_8		
Edinburgh,	Dec. 13	—6.25	2	$-6.18_{\frac{1}{2}}$					
Pulkowa,	Dec. 13	—6.28	3	-6.28_2				[— 98.9]	[+10.5]
[23^h 41^m]	Dec. 15	....	...	—6.27	....	...	—30.2	— 98.2	+ 9.6
	1843								
Edinburgh,	Jan. 9	—6.32	7	—6.25	—31.4	4	—31.0	[— 97.5] — 98.2	[+10.3] + 8.6
Greenwich,	Aug. 20	—7.11	7	-7.04_3	—35.1	7	-34.7_7		
Paris,	Aug. 21	—7.20	3	-7.19_2	—37.5	5	-37.1_5	[—114.3]	[+10.7]
[0^h 6^m]	Aug. 20	....	...	—7.10	....	...	—35.7	—111.9	+ 9.6
Greenwich,	Sept. 17	—7.23	9	-7.16_3	—35.4	9	-35.0_9		
Paris,	Sept. 15	—7.17	11	-7.16_6	—36.8	14	-36.2_{14}		
Edinburgh,	Sept. 18	—7.26	7	-7.19_2	—36.8	5	-36.4_5		
Pulkowa,	Sept. 22	—7.18	10	-7.18_{10}				[—114.1]	[+10.6]
[0^h 3^m]	Sept. 19	....	...	—7.17	....	...	—35.8	—112.9	+10.0
Greenwich,	Oct. 18	—7.10	10	-7.03_3	—35.3	10	-34.9_{10}		
Paris,	Oct. 17	—7.07	6	-7.06_3	—37.1	5	-36.5_5		
Edinburgh,	Oct. 17	—7.05	12	-6.98_3	—35.1	5	-34.7_5	[—112.6]	[+10.5]
[23^h 59^m]	Oct. 17	....	...	—7.02	....	...	—35.2	—110.6	+ 9.6
Greenwich,	Nov. 20	—7.01	9	-6.94_3	—34.3	9	-33.9_9		
Königsberg,	Nov. 11	—6.84	5	-6.83_4	—34.4	5	-33.4_2		
Paris,	Nov. 15	—7.03	3	-7.02_2	—36.3	3	-35.7_3		
Edinburgh,	Nov. 15	—6.93	8	-6.86_2	—35.7	6	-35.3_6	[—110.1]	[+10.4]
[23^h 56^m]	Nov. 15	....	...	—6.90	....	...	—34.5	—108.6	+ 9.5
	1844								
Greenwich,	Jan. 3	—6.70	3	-6.64_1	—34.1	3	-33.7_3		
Edinburgh,	Jan. 10	—6.67	6	-6.60_2					
Paris,	Jan. 3	—6.72	2	-6.71_1	—35.5	2	-34.9_2	[—106.1]	[+ 9.9]
[23^h 56^m]	Jan. 7	....	...	—6.64	....	...	—34.2	—105.0	+ 8.2
Greenwich,	Aug. 19	—7.73	10	-7.67_3	—39.6	10	-39.2_{10}		
Paris,	Aug. 17	—7.70	3	-7.69_2	—40.0	3	-39.4_3	[—122.3]	[+10.2]
[0^h 22^m]	Aug. 18	....	...	—7.68	....	...	—39.2	—121.2	+ 9.6
Greenwich,	Sept. 25	—7.74	11	-7.68_4	—39.9	11	-39.5_{11}		
Edinburgh,	Sept. 19	—7.67	11	-7.60_3	—40.3	10	-39.8_{10}		
Königsberg,	Sept. 17	—7.65	10	-7.64_7	—40.7	10	-39.7_5		
Paris,	Sept. 10	—7.63	10	-7.62_5	—39.4	15	-38.8_{15}	[—122.6]	[+10.3]
[0^h 18^m]	Sept. 17	....	...	—7.63	....	...	—39.3	—120.6	+ 9.3

Mean Corrections to the Ephemeris of Uranus.—*Continued.*									
Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1844	s		s	″		″	″	″
Greenwich,	Oct. 17	—7.70	9	—7.64$_{3}$	—39.2	9	—38.8$_{9}$		
Edinburgh,	Oct. 13	—7.58	10	—7.51$_{2}$	—40.5	1	—40.0$_{1}$	[—121.5]	[+10.2]
[0^{h} 15^{m}]	Oct. 15	....	...	—7.59	....	..	—39.0	—120.0	+ 9.4
Greenwich,	Nov. 26	—7.44	9	—7.38$_{3}$	—38.7	9	—38.3$_{9}$		
Edinburgh,	Nov. 19	—7.41	7	—7.35$_{2}$				[—118.2]	[+ 9.9]
[0^{h} 12^{m}]	Nov. 23	....	...	—7.37	....		—38.3	—116.6	+ 8.8
Edinburgh,	Dec. 18	—7.24	5	—7.18$_{1}$					
Paris,	Dec. 22	—7.10	2	—7.09$_{1}$	—39.3	3	—38.7$_{3}$	[—115.7]	[+ 9.6]
[0^{h} 10^{m}]	Dec. 20	....	...	—7.14	....	..	—38.7	—113.7	+ 7.1
	1845								
Greenwich,	Jan. 13	—7.18	1	—7.10$_{1}$	—38.4	1	—38.0		
Edinburgh,	Jan. 14	—7.16	8	—7.08$_{4}$				[—114.0]	[+ 9.5]
[0^{h} 11^{m}]	Jan. 14	....	...	—7.08	....	..	—38.0	—112.5	+ 7.3
Greenwich,	Aug. 25	—8.25	5	—8.17$_{2}$	—42.7	6	—42.3	[—131.0] —129.1	[+ 9.6] + 9.2
Greenwich,	Sept. 18	—8.21	10	—8.13$_{4}$	—42.6	10	—42.2$_{10}$		
Königsberg,	Sept. 30	—8.16	5	—8.15$_{4}$	—43.4	5	—42.4$_{2}$		
Paris,	Sept. 14	—8.24	5	—8.23$_{3}$	—43.9	3	—43.3$_{3}$	[—131 6]	[+ 9.9]
[0^{h} 33^{m}]	Sept. 22	....	...	—8.17	....	..	—42.5	—129.2	+ 9.1
Greenwich,	Oct. 17	—8.22	10	—8.14$_{4}$	—42.2	10	—41.8$_{10}$		
Paris,	Oct. 20	—8.01	6	—8.00$_{3}$	—43.7	6	—43.1$_{6}$	[—130.2]	[+ 9.8]
[0^{h} 29^{m}]	Oct. 18	....	...	—8.08	....	..	—42.3	—128.0	+ 8.9
Greenwich,	Nov. 9	—8.06	6	—7.98	—43.3	7	—42.9	[—128.4]	[+ 9.6]
[0^{h} 27^{m}]	......	....	...	....	....	..	....	—126.9	[+ 7.9
Greenwich,	Dec. 14	—7.85	8	—7.77	—49.2	8	—39.8	[—124.8]	[+ 9.3]
[0^{h} 25^{m}]	......	....	...	....	....	..	....	—122.8	+ 9.6
	1846								
Greenwich,	Sept. 8	—8.80	8	—8.76$_{3}$	—46.2	8	—45.8$_{8}$		
Paris,	Sept. 12	—8.73	19	—8.72$_{10}$	—46.9	14	—46.3$_{14}$	[—139.9]	[+ 9.2]
[0^{h} 50^{m}]	Sept. 11	....	...	—8.73	....	..	—46.1	—138.2	+ 8.3
Greenwich,	Oct. 8	—8.74	7	—8.70$_{3}$	—46.8	7	—46.4$_{7}$		
Königsberg,	Oct. 4	—8.61	11	—8.61$_{8}$	—44.8	11	—43.8$_{6}$		
Paris,	Oct. 14	—8.69	13	—8.68$_{7}$	—46.7	13	—46.1$_{13}$	[—139.7]	[+ 9.2]
[0^{h} 46^{m}]	Oct. 9	....	...	—8.65	....	..	—45.8	—137.1	+ 8.3
Greenwich,	Nov. 10	—8.62	6	—8.58$_{2}$	—46.8	6	—46.4$_{6}$		
Paris,	Nov. 16	—8.48	9	—8.47$_{5}$	—46.4	9	—45.8$_{9}$	[—136.8]	[+ 9.0]
[0^{h} 41^{m}]	Nov. 14	....	...	—8.50	....	..	—46.0	—135.1	+ 7.6

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1846	s		s	″		″	″	″
Greenwich,	Dec. 15	—8.31	9	-8.27_{3}	—46.0	10	-45.6_{10}		
Paris,	Dec. 16	—8.25	6	-8.24_{3}	—46.1	4	-45.5_{4}	[—134.5]	[+8.9]
[0^{h} 39^{m}]	Dec. 16	....	...	—8.26	....	..	—45.6	—131.7	+6.5
	1847								
Greenwich,	Jan. 13	—8.25	4	-8.20_{2}	—43.1	4	-42.7_{4}		
Paris,	Jan. 11	—8.14	6	-8.12_{3}	—45.4	6	-44.8_{6}	[—130.9]	[+8.6]
[0^{h} 40^{m}]	Jan. 12	....	...	—8.15	....	..	—44.0	—129.6	+7.4
Greenwich,	Sept. 3	—9.21	8	—9.16	—49.4	8	—49.0	[—148.4]	[+8.7]
[1^{h} 6^{m}]	......	....	...	....	....	..	....	—144.9	+6.8
Greenwich,	Oct. 1	—9.25	6	-9.20_{3}	—49.0	6	-48.6_{6}		
Paris,	Oct. 12	—9.23	18	-9.21_{10}	—49.2	16	-48.6_{16}	[—148.0]	[+8.7]
[1^{h} 2^{m}]	Oct. 10	....	...	—9.21	....	..	—48.6	—145.5	+7.8
Greenwich,	Nov. 3	—9.10	9	-9.05_{3}	—49.2	9	-48.8_{9}		
Paris,	Nov. 12	—9.08	10	-9.06_{5}	—49.3	9	-48.7_{9}	[—146.2]	[+8.5]
[0^{h} 57^{m}]	Nov. 8	....	...	—9.06	....	..	—48.8	—142.6	+7.2
Greenwich,	Dec. 3	—8.96	7	-8.91_{2}	—49.0	7	-48.6_{7}		
Paris,	Dec. 12	—8.84	9	-8.82_{5}	—48.5	7	-47.9_{7}	[—143.0]	[+8.3]
[0^{h} 54^{m}]	Dec. 9	....	...	—8.85	....	..	—48.2	—140.6	+6.8
	1848								
Greenwich,	Jan. 10	—8.83	2	-8.78_{2}	—47.1	2	-46.7_{2}		
Paris,	Jan. 11	—8.53	5	-8.51_{5}	—47.6	5	-47.0_{5}	[—139.4]	[+8.1]
[0^{h} 54^{m}]	Jan. 11	....	...	—8.57	....	..	—46.9	—136.2	+6.0
Greenwich,	Sept. 8	—9.84	7	-9.79_{7}	—51.1	7	-50.7_{7}		
Paris,	Sept. 22	—9.82	11	-9.80_{11}	—51.5	9	-50.9_{9}	[—156.4]	[+8.0]
[1^{h} 19^{m}]	Sept. 17	....	...	—9.80	....	..	—50.8	—154.1	+7.5
Greenwich,	Oct. 19	—9.82	7	-9.77_{7}	—52.2	7	-51.8_{7}		
Paris,	Oct. 16	—9.79	11	-9.77_{11}	—52.2	10	-51.6_{10}	[—156.4]	[+8.0]
[1^{h} 15^{m}]	Oct. 17	....	...	—9.77	....	..	—51.7	—154.2	+6.9
Greenwich,	Nov. 13	—9.68	7	-9.63_{7}	—52.3	7	-51.9_{7}		
Paris,	Nov. 13	—9.55	4	-9.53_{4}	—52.2	3	-51.6_{3}	[—154.6]	[+8.0]
[1^{h} 11^{m}]	Nov. 13	....	...	—9.59	....	..	—51.8	—151.8	+6.3
Greenwich,	Dec. 14	—9.41	5	-9.36_{5}	—51.2	5	-50.8_{5}		
Paris,	Dec. 9	—9.43	5	-9.41_{5}	—51.3	6	-50.7_{6}	[—150.2]	[+7.7]
[1^{h} 9^{m}]	Dec. 12	....	...	—9.38	....	..	—50.7	—148.6	+6.2
	1849								
Greenwich,	Jan. 6	—9.37	1	-9.37_{1}	—51.2	1	-50.8_{1}		
Paris,	Jan. 18	—9.11	1	-9.09_{1}	—50.0	1	-49.4_{1}	[—147.7]	[+7.5]
[1^{h} 9^{m}]	Jan. 12	....	...	—9.23	....	..	—50.1	—146.2	+6.0

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1849	s		s	″		″	″	″
Greenwich,	Sept. 16	—10.27	3	—10.27	—53.3	3	—52.9	[+165.1]	[+7.3]
[1^h 34^m]	......	....	...	....	....	..	....	+160.8	+6.2
Greenwich,	Oct. 28	—10.24	6	—10.24$_6$	—53.9	6	—53.5$_6$		
Paris,	Oct. 21	—10.26	5	—10.24$_5$	—54.8	5	—54.2$_5$	[—164.4]	[+7.4]
[1^h 30^m]	Oct. 25	....	...	—10.24	....	..	—53.8	—161.0	+5.8
Greenwich,	Nov. 16	—10.12	3	—10.12$_3$	—54.4	3	—54.0$_3$		
Paris,	Nov. 17	—10.22	5	—10.20$_5$	—54.1	4	—53.5$_4$	[—162.5]	[+7.3]
[1^h 26^m]	Nov. 17	....	...	—10.17	....	..	—53.7	—160.1	+6.0
Greenwich,	Dec. 13	—9.95	6	— 9.95$_6$	—53.1	9	—52.7$_9$		
Paris,	Dec. 16	—9.88	2	— 9.86$_2$	—54.4	2	—53.8$_2$	[—160.4]	[+7.1]
[1^h 24^m]	Dec. 14	....	...	— 9.93	....	..	—52.9	—156.6	+5.7
	1850								
Greenwich,	Jan. 7	—9.71	1	— 9.71$_1$	—53.0	1	—52.6$_1$		
Paris,	Jan. 5	—9.77	2	— 9.75$_2$	—53.0	3	—52.4$_3$	[—156.8]	[+6.9]
[1^h 24^m]	Jan. 6	....	...	— 9.74	....	..	—52.5	—153.8	+6.2
Greenwich,	Sept. 6	—10.83	9	—10.83	—53.9	9	—53.5	[—171.7]	[+6.5]
[1^h 52^m]	......	....	...	....	....	..	....	—168.1	+5.9
Greenwich,	Oct. 11	—10.92	8	—10.92$_8$	—55.3	7	—54.9$_7$		
Paris,	Oct. 17	—19.87	7	—10.85$_7$	—56.3	5	—55.7	[—173.3]	[+6.4]
[1^h 47^m]	Oct. 16	....	...	—10.87	....	..	—55.3	—169.6	+5.3
Greenwich,	Nov. 6	—10.79	5	—10.79$_5$	—55.7	6	—55.3$_6$		
Paris,	Nov. 10	—10.89	3	—10.87$_5$	—56.1	4	—55.5$_4$	[—172.4]	[+6.4]
[1^h 43^m]	Nov. 8	....	...	—10.82	....	..	—55.4	—169.1	+5.6
Greenwich,	Dec. 7	—10.56	8	—10.56$_3$	—55.0	9	—54.6$_9$		
Königsberg,	Dec. 15	—10.50	2	—10.48$_2$	—56.0	2	....	[—169.0]	[+6.3]
[1^h 41^m]	Dec. 9	....	...	—10.54	....	..	—54.6	—165.0	+5.2
	1851								
Greenwich,	Jan. 19	—10.21	4	—10.21	—54.8	4	—54.4	[—163.5]	[+6.0]
[1^h 41^m]	......	....	...	....	....	..	....	—160.4	+3.5
Greenwich,	Sept. 11	—11.42	2	—11.42$_2$	—55.7	2	—55.3$_2$		
Königsberg,	Sept. 19	—11.41	2	—11.41$_1$	....	..	....		
Paris,	Sept. 14	—11.59	1	—11.57$_1$	—57.5	1	—56.9$_1$	[—180.0]	[+5.7]
[2^h 7^m]	Sept. 14	....	...	—11.46	....	..	—55.8	—176.9	+4.4
Königsberg,	Oct. 22	—11.75	3	—11.75$_2$	—58.4	3	—57.4		
Paris,	Oct. 23	—11.46	3	—11.44$_3$	....	..	....	[—181.8]	[+5.8]
[2^h 2^m]	Oct. 23	....	...	—11.56	....	..	—57.4	—179.1	+4.3

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1851	s		s	″		″	″	″
Greenwich,	Nov. 2	—11.50	5	—11.50$_5$	—57.5	5	—57.1$_5$		
Paris,	Nov. 2	—11.48	2	—11.46$_2$	—58.1	1	—57.5$_1$		
[2^h 0^m]	Nov. 2	....	...	—11.49	....	..	—57.2	[—181.4] —178.1	[+5.7] +4.5
Greenwich,	Nov. 22	—11.43	5	—11.43$_5$	—58.3	5	—57.9$_5$		
Paris,	Nov. 14	—11.39	1	—11.37$_1$	....	..	....		
[1^h 58^m]	Nov. 21	....	...	—11.42	....	..	—57.9$_5$	[—179.9] —177.6	[+5.6] +3.8
Greenwich,	Dec. 22	—11.15	3	—11.15$_3$	—57.2	3	—56.8$_3$		
Paris,	Dec. 25	—11.00	3	—10.98$_3$	—57.2	3	—56.6$_3$		
[1^h 54^m]	Dec. 24	....	...	—11.06	....	..	—56.7	[—175.7] —172.3	[+5.3] +3.8
	1852								
Greenwich,	Jan. 11	—10.85	7	—10.85$_7$	—57.2	7	—56.8$_7$		
Paris,	Jan. 14	—10.89	5	—10.87$_5$	—56.5	5	—55.9$_5$		
[1^h 54^m]	Jan. 12	....	...	—10.86	....	..	—56.4	[—173.1] —169.4	[+5.2] +3.1
Greenwich,	Sept. 12	—12.06	7	—12.06$_7$	—56.5	7	—56.1	[—187.4]	[+4.9]
[2^h 22^m]	......	....	...	....	....	..	....	—184.5	+3.9
Greenwich,	Oct. 17	—12.23	8	—12.23$_8$	—58.6	8	—58.2$_8$		
Paris,	Oct. 23	—12.06	3	—12.04$_3$	—59.6	3	—59.0$_3$		
[2^h 18^m]	Oct. 19	....	...	—12.18	....	..	—58.4	[—189.6] —187.2	[+4.7] +3.1
Greenwich,	Nov. 13	—12.00	5	—12.00$_5$	—59.7	5	—59.3$_5$		
Paris,	Nov. 15	—12.08	5	—12.06$_5$	—60.2	4	—59.6$_4$		
[2^h 14^m]	Nov. 14	....	...	—12.03	....	..	—59.4	[—188.9] —185.6	[+4.7] +2.4
Greenwich,	Dec. 19	—11.72	7	—11.72$_7$	—58.7	7	—58.3$_7$		
Königsberg,	Dec. 16	—11.67	3	....	—59.3	3	—58.3$_2$		
Paris,	Dec. 19	—11.77	5	—11.75$_5$	—59.0	5	—58.4$_5$		
[2^h 10^m]	Dec. 19	....	...	—11.73	....	..	—58.3.	[—184.8] —181.3	[+4.6] +2.8
	1853								
Greenwich,	Jan. 12	—11.42	7	—11.42$_7$	—57.5	7	—57.1		
Paris,	Jan. 15	—11.46	4	—11.44$_4$	....	..	....		
[2^h 9^m]	Jan. 13	....	...	—11.43	....	..	—57.1	[—181.0] —176.8	[+4.4] +2.6
Greenwich,	Sept. 17	—12.69	4	—12.69$_4$	—56.7	4	—56.3$_4$		
Paris,	Sept. 16	—12.58	2	—12.56$_2$	—57.0	2	—56.4$_2$		
[2^h 39^m]	Sept. 17	....	...	—12.65	....	..	—56.3	[—194.6] —191.6	[+4.0] +2.5
Greenwich,	Oct. 15	—12.69	7	—12.69$_7$	—58.8	7	—58.4$_7$		
Paris,	Oct. 27	—12.66	6	—12.64$_9$	—59.0	6	—58.4$_6$		
[2^h 34^m]	Oct. 22	....	...	—12.66	....	..	—58.4	[—196.7] —192.7	[+3.8] +1.7

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No.of obs.	Corrected mean.	Longitude.	Latitude.
	1853	s		s	″		″	″	
Greenwich,	Nov. 12	—12.65	14	—12.65$_{14}$	—59.4	14	—59.0$_{14}$		
Paris,	Nov. 15	—12.67	10	—12.65$_{10}$	—59.2	8	—58.6$_{8}$	[—196.0]	[+3.7]
[2^h 31^m]	Nov. 14	....	...	—12.65	....	..	—58.9	—192.9	+1.9
Paris,	Dec. 12	—12.28	1	—12.26	—59.0	1	—58.4	[—193.4]	[+3.6]
[2^h 27^m]	......	....	...	....	....	..	....	—187.9	+1.5
	1854								
Greenwich,	Jan. 13	—12.02	4	—12.02$_{4}$	—59.7	4	—59.2$_{4}$		
Paris,	Jan. 20	—11.87	2	—11.85$_{2}$	—58.1	1	—57.9$_{1}$	[—188.1]	[+3.5]
[2^h 25^m]	Jan. 15	....	...	—11.96	....	..	—58.9	—183.9	+0.1
Paris,	Feb. 8	—11.89	1	—11.87	—57.8	1	—57.6	[—184.5]	[+3.5]
[2^h 26^m]	......	....	...	....	....	..	....	—182.1	+0.7
Greenwich,	Sept. 21	—13.12	6	—13.12	—56.0	6	—55.5	[—201.3]	[+3.0]
[2^h 53^m]	......	....	...	....	....	..	....	—196.9	+1.7
Greenwich,	Oct. 26	—13.24	7	—13.24$_{7}$	—58.0	7	—57.5$_{2}$		
Paris,	Oct. 29	—13.12	1	—13.10$_{1}$	—57.2	3	—57.0$_{1}$	[—203.7]	[+2.8]
[2^h 50^m]	Oct. 27		...	—13.22	....	..	—57.3	—199.0	+1.2
Greenwich,	Nov. 12	—13.22	5	—13.22$_{5}$	—58.2	5	—57.5$_{5}$		
Paris,	Nov. 16	—13.21	6	—13.19$_{6}$	—58.7	1	—58.5$_{1}$		
Santiago,	Nov. 17	—13.20	4	—13.20$_{3}$	—58.7	3	—58.1$_{2}$	[—203.0]	[+2.7]
[2^h 48^m]	Nov. 15	....	...	—13.20	....	..	—57.8	—199.1	+1.2
Greenwich	Dec. 14	—12.93	7	—12.93$_{7}$	—58.6	7	—58.1$_{2}$		
Paris,	Dec. 9	—13.14	1	—13.12$_{1}$	—56.9	4	—56.7$_{1}$		
Santiago,	Dec. 15	—12.85	10	—12.85$_{7}$	—59.7	9	—59.1$_{1}$	[—200.1]	[+2.6]
[2^h 44^m]	Dec. 14	....	...	—12.90	....	..	—58.0	—195.3	+0.7
	1855								
Greenwich,	Jan. 18	—12.55	2	—12.55$_{2}$	—56.9	2	—56.4	[—194.6]	[+2.5]
[2^h 42^m]	......	....	...	....	....	..	....	—190.1	+1.1
Greenwich,	Oct. 13	—13.71	6	—13.71$_{6}$	—55.7	6	—55.2$_{3}$		
Paris,	Oct. 25	—13.93	3	—13.91$_{3}$	—55.8	2	—55.6$_{1}$	[—209.2]	[+1.8]
[3^h 9^m]	Oct. 17	....	...	—13.78	....	..	—55.3	—204.9	+0.1
Greenwich,	Nov. 11	—13.85	4	—13.85$_{4}$	—57.6	5	—57.1$_{5}$		
Paris,	Nov. 17	—13.62	4	—13.60$_{4}$	—56.6	4	—56.4$_{4}$		
Santiago,	Nov. 16	—13.82	5	—13.82$_{4}$	—57.2	5	—56.6$_{4}$	[—209.4]	[+1.7]
[3^h 5^m]	Nov. 15	....	...	—13.76	....	..	—56.7	—205.3	—0.2
Greenwich,	Dec. 18	—13.48	10	—13.48$_{10}$	—57.3	10	—56.8$_{3}$		
Paris,	Dec. 16	—13.52	2	—13.50$_{2}$	—56.7	3	—56.5$_{1}$		
Santiago,	Dec. 18	—13.53	6	—13.53$_{4}$	—58.1	6	—57.5$_{2}$	[—206.2]	[+1.6]
[3^h 0^m]	Dec. 18	....	...	—13.50			—57.0	—202.1	—0.1

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1856	s		s	″		″	″	″
Greenwich,	Jan. 22	—12.97	6	—12.97$_{6}$	—56.1	6	—55.6$_{3}$		
Paris,	Feb. 10	—12.87	2	—12.85$_{2}$	—54.8	2	—54.6$_{1}$		
[2^{h} 58^{m}]	Jan. 27	....	...	—12.94	....	..	—55.4	[—199.5] —194.7	[+1.4] —0.5
Greenwich,	Oct. 16	—14.36	4	—14.36$_{4}$	—53.8	4	—53.3		
Paris,	Oct. 16	—14.38	2	—14.36$_{3}$					
[3^{h} 27^{m}]	Oct. 16	....	...	—14.36	....	..	—53.3	[—213.8] —211.0	[+0.7] —1.4
Greenwich,	Nov. 18	—14.23	8	—14.23$_{8}$	—54.8	8	—54.3$_{8}$		
Paris,	Nov. 17	—14.32	6	—14.30$_{6}$	—54.0	7	—53.8$_{7}$		
[3^{h} 22^{m}]	Nov. 18	....	...	—14.26	.....	..	—54.1	[—214.7] —210.2	[+0.7] —0.9
Greenwich,	Dec. 18	—13.92	5	—13.93$_{5}$	—55.9	5	—55.4$_{5}$		
Paris,	Dec. 19	—14.11	7	—14.09$_{7}$	—55.0	6	—54.8$_{6}$		
[3^{h} 17^{m}]	Dec. 19	....	...	—14.03	....	..	—55.1	[—211.6] —207.7	[+0.6] —1.2
	1857								
Greenwich,	Jan. 21	—13.59	10	—13.60$_{10}$	—53.4	8	—52.9$_{3}$		
Paris,	Jan. 12	—13.87	3	—13.85$_{3}$	—55.3	3	—55.1$_{1}$		
[3^{h} 15^{m}]	Jan. 19	....	...	—13.66	....	..	—53.5	[—206.4] —202.3	[+0.5] —0.4
Greenwich,	Oct. 8	—14.68	4	—14.69	—49.1	4	—48.6	[—216.8]	[—0.3]
[3^{h} 46^{m}]	......	....	...	....	....	..	....	—213.0	—1.9
Greenwich,	Nov. 6	—14.80	4	—14.81$_{4}$	—51.3	5	—50.8$_{5}$		
Paris,	Nov. 10	—14.83	9	—14.81$_{9}$	—51.5	8	—51.3$_{8}$		
Königsberg,	Nov. 19	—14.77	3	—14.75$_{4}$	—52.7	3	—51.5$_{2}$		
[3^{h} 41^{m}]	Nov. 11	....	...	—14.80	....	..	—51.2	[—218.9] —215.5	[—0.4] —2.4
Greenwich,	Dec. 11	—14.55	6	—14.56$_{6}$	—53.2	5	—52.7$_{5}$		
Paris,	Dec. 16	—14.48	7	—14.46$_{7}$	—52.8	6	—52.6$_{6}$		
Königsberg,	Dec. 8	—14.56	1	—14.54$_{2}$	—53.7	1	—52.5$_{\frac{1}{2}}$		
[3^{h} 36^{m}]	Dec. 13		...	—14.51	....	..	—52.6	[—217.1] —212.2	[—0.5] —3.0
	1858								
Greenwich,	Jan. 17	—14.13	13	—14.14$_{13}$	—52.8	13	—52.3$_{13}$		
Paris,	Jan. 16	—14.19	8	—14.17$_{8}$	—52.8	9	—52.6$_{9}$		
[3^{h} 32^{m}.2]	Jan. 17	....	...	—14.15	....	..	—52.4	[—211.8] —207.5	[—0.6] —2.8
Greenwich,	Feb. 11	—13.81	10	—13.82$_{10}$	—52.1	10	—51.6$_{10}$		
Paris,	Feb. 12	—13.91	1	—13.89$_{1}$	—52.7	1	—52.5$_{1}$		
[3^{h} 31^{m}.9]	Feb. 11	....	...	—13.83	....	..	—51.7	[—207.0] —203.1	[—0.6] —3.1
Greenwich,	Oct. 17	—15.06	5	—15.06	—46.4	5	—45.9	[—220.6]	[—1.4]
[4^{h} 3^{m}.3]	......	....	...	....	....	..	....	—216.2	—4.0
Greenwich,	Nov. 14	—15.16	10	—15.17$_{10}$	—48.2	11	—47.7$_{11}$		
Paris,	Nov. 20	—15.19	9	—15.17$_{9}$	—47.8	10	—47.6$_{10}$		
[3^{h} 58^{m}.5]	Nov. 17	....	...	—15.17	....	..	—47.7	[—222.1] —218.6	[—1.5] —3.8

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MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Latitude.	Longitude.
	1858	s		s	″		″	″	″
Greenwich,	Dec. 16	—14.96	6	—14.97$_{6}$	—49.2	6	—48.7$_{2}$		
Paris,	Dec. 16	—14.78	5	—14.76$_{5}$	—49.5	3	—49.3$_{1}$	[—220.7]	[—1.6]
[3^{h} 53^{m}.6]	Dec. 16	....	...	—14.87	....	..	—48.9	—215.1	—4.1
	1859								
Greenwich,	Jan. 19	—14.56	8	—14.57$_{8}$	—48.9	8	—48.4$_{2}$		
Paris,	Jan. 10	—14.72	4	—14.70$_{4}$	—48.2	4	—48.0$_{1}$	[—216.1]	[—1.5]
[3^{h} 50^{m}.0]	Jan. 16	....	...	—14.61	....	..	—48.3	—211.5	—3.1
Greenwich,	Feb. 18	—14.18	5	—14.19$_{5}$	—48.1	5	—47.6$_{5}$		
Paris,	Feb. 11	—14.37	6	—14.35$_{6}$	—48.3	4	—48.1$_{4}$	[—210.9]	[—1.6]
[3^{h} 49^{m}.3]	Feb. 14	..	...	—14.28	....	..	—47.8	—207.1	—3.4
Greenwich,	Oct. 25	—15.43	6	—15.44	—42.6	6	—42.1	[—223.2]	[—2.2]
[4^{h} 20^{m}.8]	......	....	...	....	....	..	....	—219.6	—5.5
Greenwich,	Nov. 18	—15.56	6	—15.57$_{6}$	—43.3	5	—42.8$_{5}$		
Paris,	Nov. 17	—15.61	8	—15.59$_{8}$	—43.0	9	—42.8$_{9}$	[—224.7]	[—2.2]
[4^{h} 17^{m}.2]	Nov. 17	....	...	—15.58	....	..	—42.8	—221.9	—4.6
Greenwich,	Dec. 16	—15.43	9	—15.44$_{9}$	—45.2	8	—44.7$_{8}$		
Paris,	Dec. 11	—15.46	5	—15.44$_{5}$	—44.5	5	—44.3$_{5}$	[—223.9]	[—2.3]
[4^{h} 12^{m}.5]	Dec. 14	....	...	—15.44	....	..	—44.5	—220.7	—4.9
	1860								
Greenwich,	Jan. 16	—15.08	8	—15.09$_{8}$	—45.8	8	—45.3$_{8}$		
Paris,	Jan. 15	—15.08	8	—15.06$_{8}$	—44.3	5	—44.1$_{5}$	[—219.6]	[—2.3]
[4^{h} 8^{m}.0]	Jan. 16	....	...	—15.08	....	..	—44.8	—216.1	—4.5
Greenwich,	Feb. 17	—14.64	12	—14.65$_{12}$	—45.0	12	—44.5		
Paris,	Feb. 6	—14.79	2	—14.77$_{2}$				[—213.9]	[—2.3]
[4^{h} 7^{m}.0]	Feb. 15	....	...	—14.67	....	..	—44.5	—210.4	—5.0
Greenwich,	Oct. 13	—15.52	2	—15.52	—35.2	3	—34.7	[—223.0]	[—2.9]
[4^{h} 40^{m}.9]	......	....	...	....	....	..	....	—218.3	—5.4
Greenwich,	Nov. 15	—15.88	9	—15.89$_{9}$	—38.3	9	—37.8$_{2}$		
Paris,	Nov. 22	—15.79	5	—15.77$_{5}$	—37.3	5	—37.1$_{1}$	[—226.6]	[—3.1]
[4^{h} 35^{m}.8]	Nov. 18	....	...	—15.85	....	..	—37.6	—223.5	—5.8
Greenwich,	Dec. 12	—15.77	4	—15.78$_{4}$	—41.5	5	—41.0$_{5}$		
Paris,	Dec. 12	—15.79	3	—15.77$_{3}$	—38.5	3	—38.3$_{3}$	[—226.2]	[—3.2]
[4^{h} 31^{m}.6]	Dec. 12	....	...	—15.78	....	..	—40.0	—223.2	—6.8
	1861								
Greenwich,	Jan. 13	—15.45	10	—15.46$_{10}$	—40.5	10	—40.1$_{3}$		
Paris,	Jan. 13	—15.52	6	—15.49$_{6}$	—40.2	3	—40.0$_{1}$	[—222.4]	[—3.2]
[4^{h} 26^{m}.8]	Jan. 13	....	...	—15.47	....	..	—40.1	—219.3	—5.8

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No.of obs.	Corrected mean.	Longitude.	Latitude.
	1861	s		s	"		"	"	"
Greenwich,	Feb. 10	—15.12	2	—15.13$_{2}$	—40.7	3	—40.3		
Paris,	Feb. 5	—15.23	1	—15.20$_{1}$				[—217.9]	[—3.1]
[4^{h} 21^{m}.0]	Feb. 8	....	...	—15.15	....	..	—40.3	—215.0	—6.0
Greenwich,	Nov. 10	—15.99	9	—16.00$_{9}$	—30.9	8	—30.5	[—226.2]	[—4.0]
[4^{h} 56^{m}.4]	......	....	...	....	....	..	....	—223.4	—6.1
Greenwich,	Dec. 9	—16.01	4	—16.02$_{4}$	—33.4	4	—33.0$_{4}$		
Paris,	Dec. 10	—16.04	11	—16.01$_{11}$	—32.8	13	—32.6$_{13}$		
Washington,	Dec. 18	—16.10	6	—16.07$_{6}$				[—227.3]	[—4.0]
[4^{h} 50^{m}.8]	Dec. 12	....	...	—16.03	....	..	—32.7	—224.4	—6.4
	1862								
Greenwich,	Jan. 19	—15.67	8	—15.67$_{8}$	—35.1	9	—34.7$_{9}$		
Paris,	Jan. 20	—15.72	5	—15.69$_{5}$	—34.1	5	—33.9$_{5}$	[—223.2]	[—4.0]
[4^{h} 44^{m}.9]	Jan. 19	....	...	—15.68	....	..	—34.4	—220.2	—6.3
Greenwich,	Feb. 22	—15.18	7	—15.18$_{7}$	—34.3	7	—33.9$_{7}$		
Washington,	Feb. 19	—15.31	3	—15.31$_{3}$					
Paris,	Feb. 15	—15.47	5	—15.44$_{5}$	—34.6	6	—34.4$_{6}$	[—217.5]	[—4.0]
[4^{h} 43^{m}.1]	Feb. 19	....	...	—15.30	....	..	—34.1	—215.1	—6.0
Greenwich,	Nov. 9	—16.13	9	—16.13	—24.5	9	—24.1	[—226.2]	[—4.7]
[5^{h} 16^{m}.2]	......	....	...	....	....	..	....	—223.6	—7.1
Greenwich,	Dec. 12	—16.27	7	—16.27$_{7}$	—27.2	7	—26.8$_{7}$		
Paris,	Dec. 11	—16.27	5	—16.24$_{5}$	—26.5	5	—26.3$_{5}$		
Leyden,	Dec. 8	—16.16	4	—16.13$_{3}$	—25.2	4	—25.2$_{3}$	[—227.6]	[—4.7]
[5^{h} 10^{m}.7]	Dec. 19	....	...	—16.23	....	..	—26.3	—225.4	—7.2
	1863								
Greenwich,	Jan. 17	—15.96	10	—15.96$_{10}$	—28.3	10	—27.9$_{10}$		
Paris,	Jan. 20	—15.94	2	—15.91$_{2}$	—27.4	1	—27.2$_{1}$		
Washington,	Jan. 16	—15.92	4	—15.92$_{4}$					
Leyden,	Jan. 16	—16.03	6	—16.00$_{4}$	—27.9	6	—27.9$_{4}$	[—224.3]	[—4.8]
[5^{h} 4^{m}.6]	Jan. 17	....	...	—15.96	....	..	—27.9	—222.2	—6.8
Greenwich,	Feb. 15	—15.55	10	—15.55$_{10}$	—29.2	10	—28.8$_{10}$		
Paris,	Feb. 15	—15.47	17	—15.44$_{17}$	—28.6	17	—28.4$_{17}$		
Washington,	Feb. 9	—15.64	2	—15.64$_{2}$					
Leyden,	Feb. 15	—15.57	6	—15.54$_{4}$	—28.5	7	—28.5$_{7}$	[—219.3]	[—4.8]
[5^{h} 2^{m}.2]	Feb. 15	....	...	—15.50	....	..	—28.6	—216.0	—7.2
Greenwich,	Mar. 3	—15.36	2	—15.36$_{2}$	—29.0	3	—28.6$_{3}$		
Paris,	Mar. 5	—15.30	4	—15.27$_{4}$	—28.0	4	—27.8$_{4}$	[—215.9]	[—4.8]
[5^{h} 2^{m}.2]	Mar. 4	....	...	—15.30	....	..	—28.2	—213.3	—7.0

Mean Corrections to the Ephemeris of Uranus.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No.of obs.	Corrected mean.	Longitude.	Latitude.
	1863	s		s	″		″	″	″
Paris,	Nov. 29	—16.33	3	—16.30$_3$	—18.4	3	—18.2$_3$		
Leyden,	Nov. 14	—16.23	6	—16.20$_4$	—18.1	6	—18.1$_4$	[—225.9]	[—5.6]
[5^h 34^m]	Nov. 21	....	...	—16.24	....	..	—18.2	—224.1	—8.3
Greenwich,	Dec. 19	—16.32	3	—16.32$_3$	—20.0	3	—19.6$_3$		
Leyden,	Dec. 14	—16.51	2	—16.48$_1$	—17.1	1	—17.1$_1$	[—226.8]	[—5.7]
[5^h 29^m]	Dec. 18	....	...	—16.36	....	..	—19.0	—225.9	—7.0
	1864								
Washington,	Jan. 15	—16.01	4	—16.01$_{11}$		11			
Leyden,	Jan. 10	—16.07	4	—16.04$_3$	—20.0	4	—20.0	[—224.6]	[—5.8]
[5^h 25^m]	Jan. 14	....	...	—16.02	....	..	....	—221.4	—6.7
Washington,	Feb. 15	—15.74	4	—15.74	—21.5	9		[—219.6]	[—5.7]
[5^h 21^m]	......	....	...	....	....	..	....	—217.8	
Greenwich,	May 3	—15.57	2	—15.57	—22.2	2	—21.8$_2$	[—216.1]	[—5.6]
[5^h 21^m]	......	....	...	....	....	..	....	—215.6	—7.3
Paris,	Dec. 20	—16.32	4	—16.29$_4$	—13.2	4	—12.8$_4$		
Washington,	Dec. 10	—16.22	2	—16.22$_2$	—11.4	4			
Leyden,	Dec. 18	—16.19	5	—16.16$_4$	—12.4	5	—12.4$_4$	[—225.1]	[—6.5]
[5^h 49^m]	Dec. 17	....	...	—16.22	....	..	—12.6	—223.1	—8.4
	1865								
Greenwich,	Jan. 5	—16.28	2	—16.28$_2$	—13.7	2	—13.3$_2$		
Paris,	Jan. 15	—16.22	1	—16.19$_2$	—15.1	2	—14.9$_2$		
Washington,	Jan. 17	—16.11	9	—16.11$_9$	—14.4	9			
Leyden,	Jan. 16	—16.03	3	—16.00$_2$	—14.7	3	—14.7$_2$	[—223.4]	[—6.5]
[5^h 45^m]	Jan. 15	....	...	—16.13	....	..	—14.3	—220.0	—8.8
Washington,	Feb. 14	—15.77	9	—15.77$_9$	—15.4	6			
Leyden,	Feb. 13	—15.85	2	—15.82$_2$	—15.6	2	—15.6	[—218.9]	[—6.4]
[5^h 41^m]	Feb. 14	....	...	—15.78	....	..	....	—217.4	—8.6
Greenwich,	Oct. 10	—15.59	1	—15.59$_1$	— 2.4	1	— 2.0$_1$		
Washington,	Oct. 18	—15.59	5	—15.59$_5$				[—215.3]	[—7.0]
[6^h 17^m]	Oct. 17	....	...	—15.59	....	..	— 2.0	—214.0	—8.1
Greenwich,	Dec. 6	—16.13	2	—16.13$_2$	— 5.2	2	— 4.8$_2$		
Leyden,	Dec. 7	—16.26	2	—16.23$_2$	— 4.8	2	— 4.6$_2$		
Washington,	Dec. 18	—16.17	8	—16.17$_8$	— 3.8	5		[—222.2]	[—7.3]
[6^h 11^m]	Dec. 14	....	...	—16.17	....	..	— 4.7	—222.0	—8.8

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1866	s		s	"		"	"	"
Greenwich,	Jan. 10	—16.20	8	—16.20$_{8}$	— 7.2	9	— 6.8$_{4}$		
Paris,	Jan. 12	—16.04	1	—16.01$_{1}$	— 6.3	1	— 6.1$_{1}$		
Washington,	Jan. 17	—16.06	9	—16.06$_{9}$	— 7.6	9	— 6.5$_{4}$		
Leyden,	Jan. 17	—16.02	6	—15.99$_{4}$	— 7.1	6	— 7.1$_{3}$	[—221.6]	[— 7.3]
[6^{h} 5^{m}]	Jan. 14	....	...	—16.09	....	..	— 6.6	—221.0	— 9.0
Greenwich,	Feb. 14	—15.81	10	—15.81$_{10}$	— 9.1	11	— 8.7$_{4}$		
Washington,	Feb. 14	—15.81	6	—15.81$_{6}$	— 8.4	8	— 7.3$_{4}$		
Leyden,	Feb. 14	—15.75	8	—15.72$_{6}$	— 8.6	8	— 8.6$_{4}$	[—217.3]	[— 7.3]
[6^{h} 0^{m}]	Feb. 14	....	...	—15.79	....	..	— 8.2	—216.9	— 8.2
Greenwich,	Mar. 12	—15.47	2	—15.47$_{2}$	— 8.1	2	— 7.7$_{1}$		
Washington,	Mar. 9	—15.49	6	—15.49$_{6}$	— 9.7	6	— 8.6$_{2}$		
Leyden,	Mar. 3	—15.59	2	—15.56$_{2}$	— 9.0	2	— 9.0$_{1}$	[—212.9]	[— 7.1]
[6^{h} 0^{m}]	Mar. 9	....	...	—15.50	....	..	— 8.5	—212.9	— 8.5
Washington,	Oct. 13	—15.36	5	—15.36	+ 4.4	5	+ 5.5	[—209.6]	[— 7.5]
[6^{h} 37^{m}]	......	....	...	....	....	..	....	—211.2	— 7.9
Greenwich,	Nov. 9	—15.65	2	—15.65$_{2}$	+ 3.5	2	+ 3.9$_{1}$		
Washington,	Nov. 15	—15.72	10	—15.72$_{10}$	+ 3.8	10	+ 4.9$_{2}$	[—215.1]	[— 7.8]
[6^{h} 35^{m}]	Nov. 14	....	...	—15.71	....	..	+ 4.6	—215.9	— 8.3
Greenwich,	Dec. 11	—16.01	10	—16.01$_{10}$	+ 2.8	10	+ 3.2$_{5}$		
Washington,	Dec. 12	—16.00	7	—16.00$_{7}$	+ 2.0	7	+ 3.1$_{4}$		
Leyden,	Dec. 10	—15.87	2	—15.84$_{2}$	+ 1.6	2	+ 1.6$_{2}$	[—218.2]	[— 8.0]
[6^{h} 31^{m}]	Dec. 11	....	...	—15.99	....	..	+ 2.9	—219.7	— 9.7
	1867								
Greenwich,	Jan. 21	—15.89	5	—15.89$_{5}$	— 1.4	5	— 1.0$_{3}$		
Washington,	Jan. 21	—15.93	12	—15.93$_{12}$	— 1.2	12	— 0.1$_{5}$		
Leyden,	Jan. 9	—15.82	4	—15.79$_{3}$	— 0.3	3	— 0.3$_{1}$	[—217.8]	[— 8.0]
[6^{h} 24^{m}]	Jan. 19	....	...	—15.90	....	..	— 0.4	—218.3	— 9.3
Paris,	Feb. 13	—15.72	3	—15.69$_{3}$	— 2.4	4	— 2.0$_{2}$		
Washington,	Feb. 15	—15.65	9	—15.65$_{9}$	— 2.0	10	— 0.9$_{4}$		
Leyden,	Feb. 18	--15.54	3	—15.51$_{2}$	— 0.5	3	— 0.5$_{2}$	[—214.5]	[— 7.9]
[6^{h} 20^{m}]	Feb. 15	....	...	—15.64	....	..	— 1.1	—214.7	— 8.6
Greenwich,	Mar. 2	—15.45	7	—15.45$_{7}$	— 2.9	8	— 2.5$_{4}$		
Washington,	Mar. 11	—15.32	4	—15.32$_{4}$	— 2.5	2	— 1.4$_{2}$	[—211.1]	[— 7.8]
[6^{h} 19^{m}]	Mar. 5	....	...	—15.40	....	..	— 2.1	—211.3	— 9.0
Greenwich,	Dec. 11	—15.61	5	—15.61$_{5}$	+ 8.5	5	+ 8.9$_{3}$		
Washington,	Dec. 18	—15.70	3	—15.70$_{3}$	+ 6.9	3	+ 8.0$_{2}$	[—213.1]	[— 8.6]
[6^{h} 51^{m}]	Dec. 13	....	...	—15.64	....	..	+ 8.5	—215.3	—10.4

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R.A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1868	s		s	″		″	″	″
Greenwich,	Jan. 13	—15.67	4	-15.67_4	+ 6.9	4	$+6.7_4$	[—213.4]	[— 8.7]
[6^h 45^m]	......	....	...	....	....	..	....	—215.5	—10.1
Greenwich,	Feb. 15	—15.43	8	-15.43_8	+ 4.8	8	$+4.6_4$		
Leyden,	Feb. 5	—15.43	3	-15.40_2	+ 5.9	3	$+5.9_2$		
Washington,	Feb. 13	—15.50	7	-15.50_8	+ 4.9	6	$+6.0_2$		
Paris,		—15.41	7	-15.38_4				[—210.3]	[— 8.6]
[6^h 41^m]	Feb. 14	....	...	—15.44	....	..	+ 5.3	—212.3	— 9.7
Leyden,	Mar. 11	—15.02	2	-14.99_1	+ 4.6	2	$+4.6_2$		
Washington,	Mar. 19	—14.91	1	-14.91_1	+ 3.4	1	$+4.5_1$	[—205.0]	[— 8.5]
[6^h 39^m]	Mar. 15	....	...	—14.95	....	..	+ 4.6	—205.5	— 9.2
Washington,	Oct. 18	—14.57	6	—14.57	+12.3	5	+13.3	[—198.3]	[— 8.7]
[7^h 16^m]	......	....	...	....	....	..	....	—201.5	—12.9
Washington,	Nov. 7	—14.83	5	—14.83	+16.2	5	+17.3	[—202.0]	[— 9.1]
[7^h 16^m]	......	....	...	....	....	..	....	—205.6	— 9.3
Greenwich,	Dec. 25	—15.24	5	-15.24_5	+15.1	5	$+14.9_5$		
Washington,	Dec. 12	—15.16	6	-15.16_6	+14.6	4	$+15.6_4$	[—206.5]	[— 9.2]
[7^h 10^m]	Dec. 18	....	...	—15.20	....	..	+15.2	—210.3	—10.0
	1869								
Greenwich,	Jan. 19	—15.23	1	-15.28_1	+12.6	1	$+12.4_1$		
Washington,	Jan. 18	—15.31	8	-15.31_8	+12.9	8	$+13.4_8$	[—207.1]	[— 9.3]
[7^h 5^m]	Jan. 18	....	...	—15.31	....	..	+13.3	—211.4	—10.3
Greenwich,	Feb. 14	—15.04	10	-15.04_{10}	+11.1	10	$+10.9_{10}$		
Washington,	Feb. 14	—15.13	13	-15.13_{13}	+11.2	13	$+11.7_{13}$		
Paris,	Feb. 10	—15.09	10	-15.06_7	+11.7	10	$+12.2_{10}$	[—204.7]	[— 9.2]
[7^h 1^m]	Feb. 13	....	...	—15.08	....	..	+11.7	—208.1	—10.2
Greenwich,	Mar. 3	—14.81	2	-14.81_2	+10.8	2	$+10.6_2$		
Washington,	Mar. 7	—14.87	2	-14.87_2	+11.4	2	$+11.9_2$	[—201.5]	[— 9.2]
[6^h 59^m]	Mar. 5	....	...	—14.84	....	..	+11.3	—204.8	— 9.3
Greenwich,	Dec. 8	—14.50	2	-14.50_2	+21.1	2	+20.9	[—197.1]	[— 9.8]
[7^h 32^m]	......	....	...	....	....	..	....	—202.1	—10.3
	1870								
Greenwich,	Jan. 19	—14.73	10	-14.72_5	+19.0	10	+18.8	[—199.3]	[— 9.9]
[7^h 25^m]	......	....	...	....	....	..	....	—204.5	—10.8
Greenwich,	Feb. 16	—14.58	7	-14.57_5	+17.2	7	+17.0		
Washington,	Feb. 16	—14.49	1	-14.49_1	+16.7	2	+17.1	[—197.2]	[— 9.9]
[7^h 21^m]	Feb. 16	....	...	—14.55	....	..	+17.0	—201.9	—10.9

MEAN CORRECTIONS TO THE EPHEMERIS OF URANUS.—*Continued.*

Observatory. [R. A. of Uranus.]	Mean dates.	Observed corrections in R. A.			Observed corrections in Dec.			Corr. to Geocentric	
		Mean.	No. of obs.	Corrected mean.	Mean.	No. of obs.	Corrected mean.	Longitude.	Latitude.
	1870	s		s	″		″	″	″
Greenwich,	Mar. 12	—14.29	5	—14.28$_5$	+16.2	5	+16.0		
Washington,	Mar. 11	—14.30	6	—14.30$_6$	+15.9	6	+16.3	[—193.6]	[— 9.7]
[7^h 18^m]	Mar. 12	....	...	—14.29	....	..	+16.2	—198.1	—10.1
	1871								
Greenwich,	Jan. 9	—13.99	5	—13.98	+24.8	5	+24.6	[—190.0]	[—10.2]
[7^h 47^m]	......	....	...	....	....	..	....	—196.1	—10.7
Greenwich,	Feb. 15	—13.92	2	—13.91	+21.8	2	+21.6	[—188.8]	[—10.1]
[7^h 41^m]	....	....	...	....	....	..	....	—194.4	—11.5
Greenwich,	Mar. 14	—13.82	14	—13.81	+20.7	14	+20.5	[—184.9]	[—10.0]
[7^h 38^m]	......	....	...	....	....	..	....	—192.8	—11.5
Greenwich,	Dec. 21	—13.16	3	—13.15	+29.2	3	+29.0	[—177.9]	[—10.4]
[8^h 11^m]	......	....	...	....	....	..	....	—186.5	—11.4
	1872								
Greenwich,	Jan. 6	—13.32	6	—13.31	+28.3	6	+28.1$_2$		
Washington,	Jan. 21	—13.31	3	—13.31	+27.8	3	+28.2$_1$	[—179.8]	[—10.6]
[8^h 6^m.1]	Jan. 18	....	...	—13.31	....	..	+28.1	—188.3	—11.3
Greenwich,	Feb. 15	—13.23	7	—13.22	+26.2	6	+26.0$_2$		
Washington,	Feb. 21	—13.24	3	—13.24	+26.4	3	+26.8$_1$	[—179.3]	[—10.5]
[8^h 1^m.0]	Feb. 17	....	...	—13.23	....	..	+26.3	—186.6	—11.3
Greenwich,	Mar. 15	—13.04	14	—13.03$_4$	+25.4	14	+25.2$_2$		
Washington,	Mar. 15	—13.07	10	—13.07$_5$	+24.5	8	+24.9$_1$	[—176.4]	[—10.4]
[7^h 57^m.9]	Mar. 15	....	...	—13.05	....	..	+25.1	—183.7	—11.1
Washington,	April 8	—12.79	3	—12.79$_3$	+24.9	3	+25.3$_3$		
Greenwich,	April 8	—12.77	1	—12.76$_1$	+25.0	1	+24.8$_1$	[—172.4]	[—10.2]
[7^h 57^m.3]	April 8	....	...	—12.78	....	..	+25.2	—180.2	—10.8

Corrections to be Applied to the Positions of Uranus in the Berlin Jahrbuch and the Nautical Almanac to reduce them to the Positions from the Provisional Theory.

Date.	Heliocentric.			Geocentric.	
	$\delta\lambda$ ″	$M\delta\rho$	$\delta\beta$ ″	δl ″	δb ″
1830, July 24	—18.0	+1001	+ 9.0	—19.3	+ 9.5
Aug. 13	18.3	1007	9.0	18.8	9.5
Sept. 2	18.4	1014	9.1	17.9	9.5
Sept. 22	18.5	1020	9.1	17.1	9.4
Oct. 12	18.7	1027	9.1	16.7	9.3
Nov. 1	19.0	1031	9.2	16.5	9.2
Nov. 21	—19.3	+1038	+ 9.2	—16.7	+ 9.1
1831, July 19	—22.5	+1112	+ 9.5	—24.5	+10.0
Aug. 8	22.6	1119	9.5	23.7	10.0
Aug. 28	22.7	1123	9.6	22.7	10.1
Sept. 17	22.8	1128	9.6	21.7	10.0
Oct. 7	22.9	1133	9.6	20.9	9.9
Oct. 27	23.2	1138	9.7	20.6	9.8
Nov. 16	23.5	1142	9.7	20.6	9.6
Dec. 6	—23.8	+1146	+ 9.7	—21.0	9.5
1832, Aug. 2	—26.9	+1198	+ 9.8	—28.7	+10.3
Aug. 22	27.2	1201	9.9	27.9	10.4
Sept. 11	27.4	1202	9.9	26.9	10.3
Oct. 1	27.6	1205	9.9	26.1	10.2
Oct. 21	27.8	1208	10.0	25.4	10.2
Nov. 10	28.0	1209	10.1	25.1	10.1
Nov. 30	—28.3	+1209	+10.2	—25.2	+10.0
1833, July 28	—31.9	+1233	+10.2	—34.5	+10.7
Aug. 17	32.2	1236	10.3	33.7	10.8
Sept. 6	32.4	1240	10.3	32.7	10.8
Sept. 26	32.6	1242	10.3	31.8	10.7
Oct. 16	32.9	1245	10.3	31.0	10.5
Nov. 5	33.2	1247	10.4	30.4	10.4
Nov. 25	—33.5	+1250	+10.4	—30.2	+10.3
1834, July 23	—38.0	+1264	+10.5	—41.2	+11.0
Aug. 12	38.4	1266	10.5	40.7	11.1
Sept. 1	38.4	1268	10.6	39.6	11.1
Sept. 21	38.7	1270	10.6	38.6	11.1
Oct. 11	39.1	1275	10.6	37.7	10.9
Oct. 31	39.4	1277	10.6	36.9	10.8
Nov. 20	39.7	1282	10.7	36.6	10.7
Dec. 10	—40.1	+1282	+10.7	—36.1	+10.5
1835, July 18	—43.9	+1309	+10.8	—47.7	+11.3
Aug. 7	44.5	1311	10.7	47.6	11.3
Aug. 27	44.5	1313	10.7	46.6	11.3
Sept. 16	45.1	1316	10.7	45.8	11.2
Oct. 6	45.4	1318	10.8	44.8	11.2
Oct. 26	45.9	1321	10.7	44.0	10.9
Nov. 15	46.5	1324	10.8	43.6	10.8
Dec. 5	—46.6	+1327	+10.7	—42.5	+10.6
1836, July 12	—51.0	+1361	+11.0	—55.3	+11.4
Aug. 1	51.3	1364	11.0	55.2	11.5
Aug. 21	51.5	1365	11.1	54.6	11.7
Sept. 10	—51.9	+1368	+11.0	—53.7	+11.6

Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.	Heliocentric.			Geocentric.	
	$\delta\lambda$	$M\delta\rho$	$\delta\beta$	δl	δb
	″		″	″	″
1836, Sept. 30	—52.2	+1370	+11.0	—52.4	+11.5
Oct. 20	52.9	1372	11.0	51.6	11.3
Nov. 9	53.1	1375	11.0	50.5	11.1
Nov. 29	53.9	1377	11.0	50.4	11.0
Dec. 19	—53.8	+1380	+11.1	—49.8	+10.9
1837, July 7	—58.3	+1387	+11.0	—62.9	+11.4
July 27	58.4	1387	11.0	62.9	11.5
Aug. 16	59.2	1388	11.1	63.1	11.7
Sept. 5	59.2	1389	11.1	62.0	11.7
Sept. 25	60.0	1391	11.0	61.3	11.5
Oct. 15	60.0	1393	11.0	59.6	11.4
Nov. 4	61.0	1394	11.0	59.1	11.2
Nov. 24	61.3	1394	11.0	58.2	11.0
Dec. 14	—62.1	+1393	+11.0	—57.1	+10.9
1838, Aug. 11	—66.4	+1395	+11.1	—71.1	+11.6
Aug. 31	67.2	1396	11.1	71.0	11.7
Sept. 20	67.7	1395	11.1	70.2	11.7
Oct. 10	68.1	1392	11.1	68.9	11.6
Oct. 30	68.5	1390	11.1	67.6	11.4
Nov. 19	68.9	1388	11.1	66.4	11.2
Dec. 9	69.4	1388	11.1	65.8	11.1
Dec. 29	—69.7	+1389	+11.2	—65.4	+11.0
1839, Aug. 6	—74.7	+1382	+11.1	—80.0	+11.6
Aug. 26	75.1	1381	11.1	79.7	11.7
Sept. 15	75.5	1380	11.0	79.0	11.6
Oct. 5	76.0	1379	11.1	77.9	11.6
Oct. 25	76.4	1379	11.1	76.5	11.5
Nov. 14	76.8	1379	11.1	75.2	11.3
Dec. 4	77.2	1378	11.1	74.0	11.1
Dec. 24	77.6	1377	11.1	73.5	10.9
1840, Jan. 13	—78.0	+1376	+11.0	—73.1	+10.7
June 24	—81.9	+1376	+11.0	—86.0	+11.1
Aug. 20	82.7	1374	11.0	87.9	11.5
Sept. 9	83.4	1376	11.0	87.8	11.6
Sept. 29	83.5	1377	11.0	86.4	11.5
Oct. 19	84.2	1376	11.0	85.5	11.4
Nov. 8	84.4	1376	11.0	83.5	11.3
Nov. 13	85.3	1377	11.0	82.7	11.1
Dec. 13	85.5	1378	11.0	81.5	10.9
1841, Jan. 7	—86.1	+1377	+10.9	—81.3	+10.7
June 16	—89.8	+1378	+10.8	—93.2	+10.8
Aug. 15	91.1	1384	10.8	97.0	11.3
Sept. 4	91.5	1385	10.8	96.9	11.4
Sept. 24	91.9	1388	10.7	96.1	11.3
Oct. 14	92.3	1390	10.7	94.8	11.2
Nov. 3	92.9	1392	10.7	93.4	11.1
Nov. 23	93.4	1393	10.8	91.8	10.9
Dec. 13	93.8	1395	10.7	90.5	10.7
1842, Jan. 2	94.3	1398	10.8	89.7	10.6
Feb. 11	—95.0	+1404	+10.7	—89.6	+10.3

CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

Date.		Heliocentric.			Geocentric.	
		$\delta\lambda$ ″	$M\delta\rho$	$\delta\beta$ ″	δl ″	δb ″
1842,	June 11	− 96.7	+1409	+10.6	− 98.5	+10.5
	Aug. 10	99.1	1408	10.6	105.3	10.0
	Aug. 30	99.6	1409	10.7	105.8	10.2
	Sept. 19	99.9	1409	10.6	105.2	10.2
	Oct. 9	100.5	1410	10.5	104.2	10.0
	Oct. 29	100.9	1412	10.5	102.6	10.9
	Nov. 18	101.3	1413	10.5	100.8	10.8
	Dec. 8	101.8	1414	10.5	99.2	10.6
	Dec. 28	102.2	1415	10.5	98.0	10.4
1843,	Jan. 17	−102.5	+1417	+10.4	− 97.2	+10.2
	Aug. 5	−107.2	+1422	+10.3	−113.7	+10.7
	Aug. 25	107.7	1420	10.2	114.4	10.7
	Sept. 14	108.0	1420	10.1	114.3	10.6
	Oct. 4	108.5	1420	10.1	113.4	10.6
	Oct. 24	109.0	1419	10.1	112.1	10.5
	Nov. 13	109.4	1418	10.1	110.2	10.4
	Dec. 3	110.0	1418	10.1	108.6	10.3
	Dec. 23	110.4	1417	10.1	106.9	10.1
1844,	Jan. 12	−110.8	+1418	+10.0	−105.9	+ 9.8
	July 30	−115.1	+1406	+ 9.7	−121.2	+10.0
	Aug. 19	115.4	1403	9.8	122.3	10.2
	Sept. 8	115.8	1400	9.8	122.7	10 3
	Sept. 28	116.3	1397	9.8	122.4	10.3
	Oct. 18	116.9	1396	9.7	121.3	10.2
	Nov. 7	117.4	1392	9.6	119.7	10.0
	Nov. 27	117.8	1389	9.6	117.6	9.8
	Dec. 17	118.3	1384	9.6	115.7	9.7
1845,	Jan. 6	118.6	1381	9.5	114.3	9.4
	Jan. 26	−119.1	+1378	+ 9.6	−113.3	+ 9.4
	Aug. 14	−123.5	+1340	+ 9.1	−130.4	+ 9.4
	Sept. 3	123.9	1335	9.4	131.4	9.8
	Sept. 23	124.4	1330	9.4	131.7	9.9
	Oct. 13	124.9	1327	9.3	130.6	9.8
	Nov. 2	125.3	1322	9.3	129.0	9.7
	Nov. 22	125.7	1318	9.2	127.0	9.5
	Dec. 12	126.2	1313	9.2	125.0	9.3
1846,	Jan. 1	126.7	1309	9.2	123.2	9.2
	Jan. 21	−127.1	+1303	+ 9.2	−121.9	− 9.0
	Aug. 29	−132.0	+1253	+ 8.8	−139.5	+ 9.2
	Sept. 18	132.3	1249	8.8	139.9	9.2
	Oct. 8	132.8	1246	8.8	139.8	9.3
	Oct. 28	133.2	1242	8.7	138.4	9.1
	Nov. 17	133.5	1238	8.7	136.4	9.0
	Dec. 7	134.1	1234	8.7	134.4	8.9
	Dec. 27	134.6	1233	8.6	132.3	8.7
1847,	Jan. 16	−134.9	+1230	+ 8.6	−130.5	+ 8.5
	Aug. 24	−139.5	+1197	+ 8.3	−146.7	+ 8.6
	Sept. 13	140.0	1194	8.3	148.0	8.7
	Oct. 3	140.5	1191	8.3	148.3	8.7
	Oct. 23	−140.9	+1188	+ 8.2	−147.4	+ 8.6

Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.	Heliocentric.			Geocentric.	
	$\delta\lambda$	$M\delta\rho$	$\delta\beta$	δl	δb
	″		″	″	″
1847, Nov. 12	—141.4	+1185	+8.2	—145.8	+8.5
Dec. 2	141.9	1183	8.1	143.7	8.3
Dec. 22	142.4	1182	8.1	141.4	8.2
1848, Jan. 11	—142.8	+1182	+8.1	—139.3	+8.1
Sept. 7	—147.9	+1165	+7.6	—156.0	+7.9
Sept. 27	148.3	1166	7.6	156.7	8.0
Oct. 17	148.7	1164	7.6	156.4	8.0
Nov. 6	149.3	1164	7.6	155.3	8.0
Nov. 26	149.8	1163	7.6	153.2	7.9
Dec. 16	150.2	1162	7.6	150.7	7.8
1849, Jan. 5	150.6	1161	7.5	148.3	7.5
Jan. 25	—151.3	+1161	+7.5	—146.6	+7.4
Sept. 2	—155.8	+1149	+7.0	—164.6	+7.3
Sept. 22	156.2	1150	7.0	165.1	7.3
Oct. 12	156.5	1149	7.0	165.1	7.4
Nov. 1	156.8	1148	7.0	164.0	7.4
Nov. 21	157.4	1149	7.0	162.4	7.3
Dec. 11	157.9	1148	6.9	160.0	7.1
Dec. 31	158.3	1147	6.8	157.5	6.9
1850, Jan. 20	—158.6	+1147	+6.8	—155.0	+6.8
Aug. 28	—163.3	+1135	+6.2	—170.6	+6.4
Sept. 17	163.7	1133	6.2	172.6	6.5
Oct. 7	164.2	1131	6.2	173.3	6.5
Oct. 27	164.7	1129	6.1	173.1	6.4
Nov. 16	165.2	1127	6.1	171.8	6.4
Dec. 6	165.6	1126	6.1	169.4	6.3
Dec. 26	166.0	1127	6.0	166.7	6.2
1851, Jan. 15	166.3	1127	6.1	163.8	6.1
Feb. 4	—166.7	+1124	+6.0	—161.8	+5.9
Sept. 12	—171.5	+1109	+5.5	—180.0	+5.7
Oct. 2	171.9	1105	5.5	181.4	5.8
Oct. 22	172.4	1103	5.5	181.9	5.8
Nov. 11	172.9	1103	5.4	180.8	5.7
Dec. 1	173.3	1103	5.3	178.7	5.5
Dec. 21	173.8	1102	5.2	176.1	5.3
1852, Jan. 10	174.3	1101	5.2	173.4	5.3
Jan. 30	—174.7	+1098	+5.1	—170.2	+5.1
Sept. 6	—179.0	+1074	+4.7	—186.8	+4.9
Sept. 26	179.4	1073	4.6	188.9	4.8
Oct. 16	179.8	1071	4.5	189.7	4.7
Nov. 5	180.3	1068	4.5	189.6	4.7
Nov. 25	180.7	1067	4.4	187.8	4.6
Dec. 15	181.2	1065	4.4	185.3	4.6
1853, Jan. 4	181.5	1064	4.4	182.3	4.5
Jan. 24	181.8	1062	4.4	179.6	4.4
Feb. 13	—182.1	+1060	+4.3	—176.9	+4.2
Sept. 1	—185.8	+1055	+3.9	—192.4	+4.0
Sept. 21	186.2	1055	3.8	195.2	3.9
Oct. 11	—186.5	+1055	+3.7	—196.7	+3.9

Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.		Heliocentric.			Geocentric.	
		$\delta\lambda$ ″	$M\delta\rho$	$\delta\beta$ ″	δl ″	δb ″
1853,	Oct. 31	—186.8	+1054	+3.6	—196.8	+3.8
	Nov. 20	187.1	1054	3.5	195.6	3.7
	Dec. 10	187.5	1053	3.5	193.5	3.6
	Dec. 30	187.9	1053	3.5	190.7	3.6
1854,	Jan. 19	188.1	1053	3.5	187.4	3.5
	Feb. 8	—188.5	+1052	+3.5	—184.5	+3.5
	Sept. 16	—192.2	+1034	+2.9	—200.5	+3.0
	Oct. 6	192.6	1032	2.8	202.9	2.9
	Oct. 26	193.0	1032	2.7	203.7	2.8
	Nov. 15	193.3	1032	2.6	203.0	2.7
	Dec. 5	193.7	1029	2.6	201.3	2.7
	Dec. 25	194.0	1025	2.5	198.6	2.6
1855,	Jan. 14	194.2	1021	2.5	195.2	2.5
	Feb. 3	—194.5	+1018	+2.4	—192.0	+2.4
	Oct. 1	—198.3	+1010	+1.8	—207.9	+1.9
	Oct. 21	198.7	1009	1.7	209.6	1.8
	Nov. 10	199.0	1008	1.6	209.7	1.7
	Nov. 30	199.3	1007	1.5	208.4	1.6
	Dec. 20	199.6	1006	1.5	206.0	1.6
1856,	Jan. 9	199.8	1005	1.5	202.6	1.5
	Jan. 29	200.1	1003	1.4	199.2	1.4
	Feb. 18	—200.3	+1001	+1.3	—196.1	+1.3
	Oct. 15	—203.3	+ 962	+0.7	—213.9	+0.7
	Nov. 4	203.7	957	0.7	214.9	0.7
	Nov. 24	203.9	953	0.7	214.5	0.7
	Dec. 14	204.3	949	0.6	212.4	0.6
1857,	Jan. 3	204.4	945	0.6	209.3	0.6
	Jan. 23	204.5	941	0.5	205.6	0.5
	Feb. 12	—204.7	+ 934	+0.4	—202.2	+0.4
	Sept. 20	—206.9	+ 879	—0.2	—214.0	—0.2
	Oct. 10	207.2	874	0.3	217.1	0.3
	Oct. 30	207.5	868	0.3	218.8	0.3
	Nov. 19	207.8	861	0.4	219.0	0.4
	Dec. 9	208.0	853	0.4	217.6	0.4
	Dec. 29	208.1	846	0.5	215.0	0.5
1858,	Jan. 18	208.2	839	0.6	211.5	0.6
	Feb. 7	208.3	830	0.6	207.7	0.6
	Feb. 27	—208.4	+ 823	—0.6	—204.2	—0.6
	Oct. 5	—210.4	+ 741	—1.3	—219.1	—1.3
	Oct. 25	210.6	732	1.4	221.4	1.5
	Nov. 14	210.8	723	1.4	222.2	1.5
	Dec. 4	211.0	713	1.5	221.9	1.6
	Dec. 24	211.2	704	1.5	219.7	1.6
1859,	Jan. 13	211.3	695	1.5	216.6	1.5
	Feb. 2	211.4	686	1.6	213.0	1.6
	Feb. 22	—211.6	+ 677	—1.5	—209.4	—1.5
	Oct. 20	—213.1	+ 576	—2.1	—222.8	—2.2
	Nov. 9	213.2	569	2.1	224.5	2.2
	Nov. 29	—213.4	+ 561	—2.1	—224.8	—2.2

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CORRECTIONS TO BE APPLIED TO THE POSITIONS OF URANUS—*Continued.*

Date.		Heliocentric.			Geocentric.	
		δλ ″	Mδρ	δβ ″	δl ″	δb ″
1859,	Dec. 19	—213.5	+554	—2.2	—223.5	—2.3
1860,	Jan. 8	213.5	546	2.2	220.9	2.3
	Jan. 28	213.6	539	2.2	217.4	2.3
	Feb. 17	—213.7	+533	—2.3	—213.5	—2.3
	Sept. 24	—214.7	+456	—2.8	—219.7	—2.9
	Oct. 14	214.8	451	2.8	223.1	2.9
	Nov. 3	215.0	444	2.9	225.6	3.0
	Nov. 23	215.1	437	2.9	226.8	3.1
	Dec. 18	215.2	431	3.0	226.3	3.2
1861,	Jan. 2	215.3	424	3.1	223.8	3.2
	Jan. 22	215.3	417	3.1	221.0	3.2
	Feb. 12	—215.3	+411	—3.1	—217.3	—3.1
	Oct. 29	—215.6	+339	—3.7	—225.0	—3.9
	Nov. 13	215.6	333	3.8	226.9	4.0
	Dec. 8	215.7	329	3.8	227.4	4.0
	Dec. 28	215.8	324	3.8	226.3	4.0
1862,	Jan. 17	215.8	320	3.9	223.5	4.0
	Feb. 6	215.8	315	4.0	220.0	4.1
	Feb. 26	—215.9	+310	—4.0	—216.0	—4.0
	Oct. 24	—216.0	+246	—4.4	—224.2	—4.6
	Nov. 13	216.0	241	4.5	226.6	4.7
	Dec. 3	216.0	237	4.5	227.7	4.7
	Dec. 23	215.9	232	4.6	227.2	4.8
1863,	Jan. 12	215.8	226	4.7	225.0	4.9
	Feb. 1	215.8	219	4.7	221.8	4.8
	Feb. 21	215.7	213	4.7	218.1	4.8
	Mar. 13	—215.7	+208	—4.8	—214.1	—4.8
	Nov. 8	—215.3	+139	—5.3	—224.6	—5.5
	Nov. 28	215.2	133	5.4	226.5	5.7
	Dec. 18	215.1	126	5.4	226.8	5.7
1864,	Jan. 7	215.0	120	5.5	225.4	5.8
	Jan. 27	214.9	114	5.5	222.7	5.7
	Feb. 16	214.8	108	5.6	219.3	5.7
	March 7	—214.8	+103	—5.6	—215.3	—5.6
	Oct. 13	—213.8	+ 21	—6.0	—217.8	—6.1
	Nov. 2	213.7	16	6.0	221.3	6.2
	Nov. 22	213.7	9	6.1	224.0	6.4
	Dec. 12	213.6	+ 1	6.2	225.1	6.5
1865,	Jan. 1	213.4	— 6	6.2	224.7	6.5
	Jan. 21	213.2	14	6.2	222.7	6.5
	Feb. 10	213.1	20	6.2	219.6	6.4
	March 2	—213.0	— 27	—6.3	—215.7	—6.4
	Oct. 8	—211.6	—103	—6.8	—213.5	—6.9
	Oct. 28	211.4	110	6.8	217.2	7.0
	Nov. 17	211.2	117	6.9	220.0	7.2
	Dec. 7	211.1	124	6.9	222.0	7.3
	Dec. 27	210.9	132	6.9	222.4	7.3
1866,	Jan. 16	210.7	139	7.0	221.5	7.4
	Feb. 5	210.6	145	7.0	218.8	7.3
	Feb. 25	210.4	151	7.0	215.4	7.2
	Mar. 17	—210.1	—158	—7.1	—211.2	—7.1

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Corrections to be Applied to the Positions of Uranus—*Continued.*

Date.		Heliocentric.			Geocentric.	
		$\delta\lambda$ ″	$M\delta\rho$	$\delta\beta$ ″	δl ″	δb ″
1866,	Oct. 3	—208.2	— 225	— 7.4	—207.3	— 7.4
	Oct. 23	208.1	230	7.4	211.5	7.6
	Nov. 12	207.9	235	7.5	214.9	7.8
	Dec. 2	207.8	241	7.6	217.6	8.0
	Dec. 22	207.6	246	7.6	218.8	8.0
1867,	Jan. 11	207.4	251	7.6	218.5	8.0
	Jan. 31	207.2	256	7.7	216.7	8.0
	Feb. 20	207.0	262	7.7	213.8	7.9
	Mar. 12	—206.7	— 268	— 7.7	—209.8	— 7.8
	Nov. 27	—203.2	— 351	— 8.2	—211.4	— 8.6
	Dec. 17	203.0	358	8.2	213.5	8.6
1868,	Jan. 6	202.8	365	8.2	213.7	8.7
	Jan. 26	202.4	371	8.3	212.7	8.7
	Feb. 15	202.2	376	8.3	210.3	8.6
	Mar. 6	201.8	382	8.4	207.0	8.6
	Mar. 26	—201.4	— 387	— 8.4	—202.9	— 8.4
	Oct. 12	—198.0	— 458	— 8.6	—197.2	— 8.6
	Nov. 1	197.8	465	8.7	201.1	8.9
	Nov. 21	197.5	473	8.8	203.8	9.1
	Dec. 11	197.2	481	8.8	206.2	9.2
	Dec. 31	196.8	490	8.8	207.5	9.3
1869,	Jan. 20	196.4	498	8.8	207.1	9.3
	Feb. 9	196.0	507	8.8	205.4	9.2
	Mar. 1	195.6	515	8.9	202.4	9.2
	Mar. 21	—195.3	— 524	— 8.9	—198.4	— 9.0
	Dec. 6	—190.0	....	....	—197.0	
	Dec. 26	189.5	....	....	198.9	
1870,	Jan. 15	189.0	— 665	— 9.4	199.5	— 9.9
	Feb. 14	188.6	676	9.4	198.7	9.9
	Feb. 24	188.0	686	9.4	196.2	9.8
	Mar. 16	187.5	697	9.4	193.0	9.6
	April 5	—187.2	— 708	— 9.4	—189.4	— 9.4
	Dec. 1	—181.5	— 846	— 9.6	—186.5	—10.0
	Dec. 21	181.1	859	9.6	189.0	10.1
1871,	Jan. 11	180.5	872	9.7	190.3	10.2
	Jan. 30	179.8	883	9.8	190.1	10.3
	Feb. 19	179.1	894	9.7	188.3	10.1
	Mar. 11	178.4	906	9.7	185.6	10.0
	Mar. 31	—177.0	— 919	— 9.8	—181.2	— 9.9
	Dec. 16	—171.6	—1083	—10.0	—177.4	—10.4
1872,	Jan. 5	171.0	1095	10.1	179.3	10.6
	Jan. 25	170.5	1107	10.0	180.2	10.6
	Feb. 14	169.8	1120	10.0	179.6	10.5
	Mar. 5	169.3	1133	10.1	177.8	10.5
	Mar. 25	168.7	1145	10.1	174.9	10.3
	April 14	168.1	1156	10.1	171.3	10.1
	May 4	—167.6	—1168	—10.1	—166.3	—10.0

CHAPTER VII.

FORMATION AND SOLUTION OF THE EQUATIONS OF CONDITION RESULTING FROM THE PRECEDING COMPARISONS.

In the preceding chapter we have obtained from observations a series of corrections to the geocentric positions of Uranus resulting from the provisional theory. The further operations are as follows:—

1. To reduce all the corrections in right ascension and declination to corrections in geocentric longitude and latitude. Most of the corrections are already so expressed, so that this reduction is necessary in only a few cases.
2. To find the mean value of the correction in geocentric longitude during each opposition, and to express this mean value in terms of the correction to the heliocentric co-ordinates.
3. To express these corrections to the heliocentric co-ordinates in terms of corrections to the elements of Uranus and the mass of Neptune.
4. To solve the equations of condition thus formed.

The first of these processes is too simple to make it necessary to present any details of it. With regard to the second I have sought, not the simple correction to the geocentric longitude, but this correction multiplied by such a factor as it was supposed would make the probable error of the correction 0″.5. The equations for expressing the error of geocentric longitude in terms of errors of heliocentric longitude and radius vector have been given on page 129. The first observation of Flamstead, p. 107, gives the equation

$$+22'' = 1.04\delta\lambda + .027\delta\rho$$

$\delta\lambda$ being the correction to the heliocentric longitude, and $\delta\rho$ that to the Neperian logarithm of the radius vector. From the discordance of Flamstead's clock errors it may be estimated that the probable error of the first member of this equation is 10″. Therefore we divide the equation by 20, which gives

$$\tfrac{1}{20}\delta l = 1''.1 = .052\delta\lambda + .001\delta\rho.$$

In the opposition of 1715 we have four observations. The best were those of March 4 and 10, of which we may estimate the probable error at 10″, and the worst that of March 5, of which the probable error may be estimated at 20″, while that of April 29 is intermediate in certainty. The separate observations give the equations

$$\begin{array}{llll}
\text{March } 4, & \delta l = +28'' & = 1.06\delta\lambda; & \text{Weight, } 4 \\
\text{March } 5, & \delta l = +44 & = 1.06\delta\lambda; & \text{Weight, } 1 \\
\text{March } 10, & \delta l = +36 & = 1.06\delta\lambda; & \text{Weight, } 4 \\
\text{April } 29, & \delta l = +\ 2 & = 1.04\delta\lambda + .04\delta\rho; & \text{Weight, } 2. \\
\text{Mean} & \delta l = +27.6 & = 1.056\delta\lambda + .003\delta\rho; & \text{probable error} = \pm 6''.
\end{array}$$

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Applying the correction $-1''.1$ for equinox, and dividing by 12, the equation of condition becomes

$$\tfrac{1}{12}\delta l = +\,2''.2 = 0.088\delta\lambda.$$

In this way the following equations were obtained. It is deemed unnecessary to give the details of the process, as it is one which every one can go over for himself from the data already given, and can reproduce all the results, except so far as they depend on the relative weights assigned to the different groups of observations during one and the same opposition.

No.	Date.	Equations.				Number of observations in R. A.
			$''$			
1	1691.0;	$\frac{1}{20}\delta l$	$=+\ 1.1$	$=\ .052\delta\lambda$	$+\ .001\delta\rho$	1
2	1715.2.	$\frac{1}{12}$	$=+\ 2.2$	$=\ .088$		4
3	1748.8	$\frac{1}{3}$	$=+12.8$	$=\ .338$	$+\ .017$	1
4	1750.8	$\frac{1}{3}$	$=+11.8$	$=\ .345$	$+\ .010$	3
5	1753.9	$\frac{1}{3}$	$=+11.6$	$=\ .333$	$+\ .016$	1
6	1756.7	$\frac{1}{5}$	$=+\ 5.0$	$=\ .210$	$+\ .003$	1
7	1769.0	$\frac{2}{5}$	$=+\ 4.8$	$=\ .203$	$+\ .010$	8
8	1782.0	$\frac{4}{3}$	$=+\ 3.0$	$=1.370$		21
9	1783.0	1	$=+\ 1.25$	$=1.030$	$-\ .002$	13
10	1784.0	1	$=+\ 1.92$	$=1.026$	$-\ .008$	13
11	1785.0	1	$=-\ 0.26$	$=1.034$	$+\ .006$	10
12	1788.0	$\frac{2}{3}$	$=+\ 1.23$	$=0.684$	$+\ .006$	5
13	1789.0	$\frac{1}{2}$	$=+\ 1.58$	$=0.504$	$+\ .008$	6
14	1790.0	$\frac{1}{2}$	$=-\ 0.52$	$=0.514$	$-\ .013$	4
15	1791.0	$\frac{2}{3}$	$=-\ 0.36$	$=0.684$	$-\ .010$	7
16	1792.0	$\frac{1}{3}$	$=-\ 0.12$	$=0.340$	$-\ .011$	3
17	1793.0	$\frac{1}{2}$	$=-\ 0.19$	$=0.512$	$-\ .015$	5
18	1794.0	$\frac{1}{3}$	$=+\ 0.79$	$=0.344$	$-\ .006$	3
19	1795.0	$\frac{2}{3}$	$=-\ 0.66$	$=0.683$	$-\ .019$	7
20	1796.0	$\frac{1}{2}$	$=-\ 0.68$	$=0.514$	$-\ .015$	4
21	1797.1	$\frac{1}{2}$	$=-\ 0.35$	$=0.528$		3
22	1800.2	$\frac{1}{3}$	$=\ \ 0.00$	$=0.352$		2
23	1801.2	$\frac{1}{3}$	$=-\ 0.26$	$=0.352$		2
24	1802.3	1	$=+\ 0.85$	$=1.05$		13
25	1805.3	1	$=+\ 0.69$	$=1.05$		13
26	1806.3	$\frac{1}{2}$	$=+\ 0.20$	$=0.52$		5
27	1807.3	1	$=+\ 2.34$	$=1.045$		16
28	1808.3	$\frac{1}{2}$	$=-\ 0.04$	$=0.52$		6
29	1809.3	$\frac{2}{3}$	$=+\ 1.80$	$=0.70$		9
30	1810.3	1	$=+\ 2.39$	$=1.05$		16
31	1811.3	1	$=+\ 1.49$	$=1.04$	-0.01	11
32	1812.4	$\frac{1}{2}$	$=+\ 0.8$	$=0.53$		8
33	1813.4	$\frac{1}{2}$	$=+\ 1.2$	$=0.53$		9
34	1814.4	1	$=+\ 1.7$	$=1.05$		15

No.	Date.	Equations.				Number of observations in R. A.
			″			
35	1815.4	$\frac{3}{2}\,\delta l$	$= +\ 2.1$	$= 1.58\,\delta\lambda$		20
36	1818.4*	$\frac{3}{2}$	$= +\ 0.3$	$= 1.58$		24
37	1819.4	1	$= -\ 0.8$	$= 1.05$		11
38	1820.5	1	$= -\ 1.3$	$= 1.05$		14
39	1821.5	$\frac{2}{3}$	$= +\ 0.9$	$= 0.70$		10
40	1822.5	$\frac{2}{3}$	$= +\ 0.9$	$= 0.70$		7
41	1823.5	$\frac{2}{3}$	$= \ \ 0.0$	$= 0.70$		11
42	1824.5	1	$= +\ 0.5$	$= 1.05$		12
43	1825.5	$\frac{2}{3}$	$= -\ 0.4$	$= 0.70$		7
44	1826.5	1	$= +\ 0.3$	$= 1.05$		11
45	1827.7	2	$= -\ 2.7$	$= 2.07$	$+\,0.04\,\delta\rho$	37
46	1828.7	$2\frac{1}{2}$	$= -\ 2.7$	$= 2.57$	$+\,0.07$	67
47	1829.7	$2\frac{1}{2}$	$= -\ 2.4$	$= 2.59$	$+\,0.04$	61
48	1830.7	$2\frac{1}{2}$	$= -\ 4.9$	$= 2.56$	$+\,0.07$	73
49	1831.7	2	$= \ \ 0.0$	$= 2.06$	$+\,0.05$	54
50	1832.7	2	$= -\ 2.2$	$= 2.07$	$+\,0.05$	65
51	1833.8	$2\frac{1}{2}$	$= -\ 4.1$	$= 2.58$	$+\,0.08$	88
52	1834.8	$2\frac{1}{2}$	$= -\ 3.9$	$= 2.57$	$+\,0.08$	91
53	1835.8	$2\frac{1}{2}$	$= -\ 5.1$	$= 2.59$	$+\,0.05$	82
54	1836.8	3	$= -\ 7.0$	$= 3.11$	$+\,0.08$	157
55	1837.8	3	$= -\ 3.6$	$= 3.11$	$+\,0.04$	162
56	1838.8	3	$= -\ 1.6$	$= 3.11$	$+\,0.06$	193
57	1839.8	3	$= -\ 1.2$	$= 3.11$	$+\,0.06$	170
58	1840.8	3	$= -\ 1.5$	$= 3.11$	$+\,0.06$	124
59	1841.8	3	$= +\ 0.8$	$= 3.10$	$+\,0.06$	108
60	1842.8	3	$= +\ 1.6$	$= 3.10$	$+\,0.06$	169
61	1843.8	3	$= +\ 4.7$	$= 3.12$	$+\,0.04$	111
62	1844.9	3	$= +\ 5.1$	$= 3.11$	$+\,0.03$	106
63	1845.9	2	$= +\ 4.2$	$= 2.08$	$+\,0.02$	55
64	1846.9	3	$= +\ 6.3$	$= 3.10$	$+\,0.04$	98
65	1847.9	2	$= +\ 5.8$	$= 2.07$	$+\,0.03$	74
66	1848.9	2	$= +\ 4.4$	$= 2.08$	$+\,0.02$	59
67	1849.9	2	$= +\ 6.6$	$= 2.08$	$+\,0.04$	33
68	1850.9	2	$= +\ 7.2$	$= 2.08$	$+\,0.01$	46
69	1851.9	2	$= +\ 6.3$	$= 2.07$	$+\,0.04$	42
70	1852.9	2	$= +\ 6.6$	$= 2.07$	$+\,0.03$	54
71	1853.9	2	$= +\ 7.9$	$= 2.09$	$+\,0.01$	49
72	1854.9	2	$= +\ 8.9$	$= 2.09$	$+\,0.02$	49
74	1855.9	2	$= +\ 8.5$	$= 2.08$	$+\,0.04$	48
75	1856.9	2	$= +\ 7.9$	$= 2.08$	$+\,0.04$	45
76	1858.0	$2\frac{1}{2}$	$= +\,10.3$	$= 2.61$	$+\,0.09$	66

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* The results for 1816 and 1817 were omitted in this list through oversight.

No.	Date.	Equations.	″			Number of observations in R. A.
77	1859.0	$2\frac{1}{2}\,\delta l =$	$+10.6$	$=2.60\,\delta\lambda$	$+0.03\,\delta\rho$	58
78	1860.0	$2\frac{1}{2}\ =$	$+\ 8.2$	$=2.60$	$+0.05$	64
79	1861.0	$2\ =$	$+\ 6.3$	$=2.09$	$+0.03$	41
80	1862.0	$2\frac{1}{2}\ =$	$+\ 7.0$	$=2.60$	$+0.05$	60
81	1863.0	$2\frac{1}{2}\ =$	$+\ 6.6$	$=2.59$	$+0.08$	88
82	1864.0	$2\ =$	$+\ 4.3$	$=2.09$	$+0.03$	35
83	1865.0	$1\frac{1}{2}\ =$	$+\ 3.9$	$=1.57$	$+0.04$	37
84	1866.0	$2\frac{1}{2}\ =$	$+\ 1.1$	$=2.60$	$+0.06$	76
85	1867.0	$2\frac{1}{2}\ =$	$-\ 1.8$	$=2.60$	$+0.02$	83
86	1868.0	$2\ =$	$-\ 3.8$	$=2.09$	$+0.04$	40
87	1869.0	$2\frac{1}{2}\ =$	$-\ 9.1$	$=2.61$	$+0.02$	68
88	1870.0	$2\ =$	$-\ 9.6$	$=2.084$	$+0.054$	31
89	1871.0	$1\frac{1}{2}\ =$	-10.6	$=1.560$	$+0.040$	21
90	1872.1	$2\frac{1}{2}\ =$	-19.1	$=2.600$	$+0.070$	50
Total number of observations in R. A., ...						3763.

We have next to express the values of $\delta\lambda$ and $\delta\rho$ in terms of the corrections to the elements. Differentiating the expressions

$$\lambda = l + (2e - \tfrac{1}{4}e^3)\sin(l - \pi) + \tfrac{5}{4}e^2\sin(2l - 2\pi) + \tfrac{13}{12}e^3 \sin e^3 \sin(3l - 3\pi) + \text{etc.}$$

$$\rho = \varpi + \tfrac{1}{4}e^2 + (-e + \tfrac{3}{8}e^3)\cos(l - \pi) - \tfrac{3}{4}e^2\cos(2l - 2\pi) - \text{etc.},$$

with respect to l, e, and π, and reducing the coefficients to numbers, we find

$$\frac{\partial\lambda}{\partial l} = 1 + 0.0939\cos g + 0.0055\cos 2g$$

$$\frac{\partial\lambda}{\partial e} = 1.999\sin g + 0.117\sin 2g + 0.007\sin 3g$$

$$\frac{\partial\lambda}{\partial\pi} = -2.000\cos g - 0.117\cos 2g - 0.007\cos 3g$$

We have here put l for the mean longitude, or

$$l = nt + \varepsilon$$

whence

$$\frac{\partial\lambda}{\partial\varepsilon} = \frac{\partial\lambda}{\partial l}$$

$$\frac{\partial\lambda}{\partial n} = t\frac{\partial\lambda}{\partial l}$$

Also, from the expression for ρ

$$\frac{\partial\rho}{\partial l} = -\frac{\partial\rho}{\partial\pi} = (e - \tfrac{3}{8}e^3)\sin g + \tfrac{3}{4}e^2\sin 2g + \ldots$$

$$\frac{\partial\rho}{\partial e} = \tfrac{1}{2}e - (1 - \tfrac{9}{8}e^2)\cos g - \tfrac{3}{2}e\cos 2g - \text{etc.}$$

$$\frac{\partial\rho}{\partial n} = t\frac{\partial\rho}{\partial l} - \frac{2}{3n}$$

The values of these coefficients which depend only on g are shown in the following table:

g	$\frac{\partial\lambda}{\partial\varepsilon}$	$\frac{\partial\lambda}{\partial e}$	$\frac{\partial\lambda}{e d\pi}$	$\frac{\partial\rho}{\partial\varepsilon}$	$\frac{\partial\rho}{\partial e}$	$\frac{\partial\rho}{e d\pi}$
0°	+1.099	0	—2.124	0	—1.00	0
1	1.099	+0.039	2.124	+0.001	1.00	—0.02
2	1.099	0.079	2.123	0.002	1.00	0.03
3	1.099	0.118	2.121	0.002	1.00	0.05
4	1.099	0.158	2.118	0.003	1.00	0.07
5	+1.099	+0.196	—2.114	+0.004	—1.00	—0.09
6	1.099	0.235	2.110	0.005	0.99	0.10
7	1.099	0.274	2.105	0.006	0.99	0.12
8	1.098	0.313	2.099	0.006	0.99	0.14
9	1.098	0.352	2.092	0.007	0.99	0.16
10	+1.098	+0.391	—2.085	+0.008	—0.98	—0.17
11	1.097	0.430	2.077	0.009	0.98	0.19
12	1.097	0.468	2.069	0.010	0.98	0.21
13	1.097	0.505	2.059	0.011	0.97	0.22
14	1.096	0.544	2.049	0.011	0.97	0.24
15	+1.096	+0.581	—2.038	+0.012	—0.97	—0.26
16	1.095	0.618	2.027	0.013	0.96	0.28
17	1.095	0.655	2.014	0.014	0.96	0.29
18	1.094	0.693	2.001	0.015	0.95	0.31
19	1.094	0.729	1.987	0.015	0.95	0.33
20	+1.093	+0.765	—1.973	+0.016	—0.94	—0.34
21	1.092	0.801	1.957	0.017	0.93	0.36
22	1.091	0.836	1.941	0.018	0.93	0.37
23	1.090	0.872	1.925	0.018	0.92	0.39
24	1.090	0.907	1.907	0.019	0.91	0.41
25	+1.089	+0.942	—1.890	+0.020	—0.91	—0.42
26	1.088	0.976	1.872	0.021	0.90	0.44
27	1.087	1.010	1.852	0.021	0.89	0.45
28	1.086	1.043	1.832	0.022	0.88	0.47
29	1.085	1.076	1.811	0.023	0.87	0 48
30	+1.084	+1.108	—1.790	+0.023	—0.87	—0.50
31	1.084	1.140	1.769	0.024	0.86	0.51
32	1.083	1.172	1.747	0.025	0.85	0.53
33	1.082	1.203	1.724	0.026	0.84	0.54
34	1.080	1.233	1.700	0.026	0.83	0.56
35	+1.079	+1.264	—1.676	+0.027	—0.82	—0.57
36	1.078	1.294	1.651	0.028	0.81	0.59
37	1.077	1.323	1.626	0.028	0.80	0.60
38	1.076	1.351	1.601	0.029	0.79	0.62
39	1.074	1.379	1.574	0.030	0.78	0.63
40	+1.073	+1.407	—1.548	+0.030	—0.77	—0.64
41	1.072	1.434	1.521	0.031	0.76	0.66
42	1.070	1.460	1.494	0.031	0.74	0.67
43	1.069	1.486	1.466	0.032	0.73	0.68
44	1.068	1.511	1.438	0.033	0.72	0.70
45	+1.067	+1.536	—1.409	+0.033	—0.71	—0.71
46	1.065	1.561	1.380	0.034	0.70	0.71
47	1.064	1.584	1.351	0.034	0.68	0.72
48	1.063	1.606	1.320	0.035	0.67	0.74
49	1.061	1.629	1.290	0.036	0.66	0.75
50	+1.060	+1.651	—1.260	+0.036	—0.64	—0.76
51	1.058	1.672	1.229	0.037	0.63	0.77
52	1.057	1.692	1.197	0.037	0.62	0.78
53	1.055	1.712	1.165	0.037	0.60	0.79
54	1.054	1.731	1.133	0.038	0.59	0.80
55	+1.052	+1.750	—1.100	+0.038	—0.57	—0.81
56	1.051	1.768	1.067	0.039	0.56	0.82
57	1.049	1.785	1.034	0.039	0.54	0.83
58	1.047	1.802	1.002	0.040	0.53	0.84
59	+1.046	+1.817	—0.968	+0.040	—0.51	—0.85

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g	$\frac{\partial\lambda}{\partial\varepsilon}$	$\frac{\partial\lambda}{\partial e}$	$\frac{\partial\lambda}{e d\pi}$	$\frac{\partial\rho}{\partial\varepsilon}$	$\frac{\partial\rho}{\partial e}$	$\frac{\partial\rho}{e d\pi}$
60°	+1.044	+1.833	—0.935	+0.041	—0.50	—0.86
61	1.043	1.848	0.901	0.041	0.49	0.87
62	1.041	1.862	0.867	0.041	0.47	0.87
63	1.040	1.875	0.832	0.042	0.45	0.88
64	1.038	1.888	0.798	0.042	0.44	0.89
65	+1.036	+1.901	—0.763	+0.043	—0.42	—0.90
66	1.034	1.912	0.728	0.043	0.41	0.91
67	1.033	1.922	0.693	0.043	0.39	0.91
68	1.031	1.932	0.659	0.044	0.37	0.92
69	1.030	1.942	0.624	0.044	0.36	0.93
70	+1.028	+1.950	—0.588	+0.044	—0.34	—0.94
71	1.027	1.959	0.553	0.044	0.33	0.95
72	1.025	1.967	0.518	0.045	0 31	0.95
73	1.023	1.974	0.483	0.045	0.29	0.96
74	1.021	1.980	0.447	0.045	0.28	0.96
75	+1.019	+1.985	—0.412	+0.045	—0.26	—0.97
76	1.018	1.990	0.376	0.046	0.24	0.97
77	1.016	1.994	0.341	0.046	0.22	0.97
78	1.014	1.997	0.305	0.046	0.21	0.98
79	1.013	2.000	0.269	0.046	0.19	0.98
80	+1.011	+2.003	—0.233	+0.046	—0.17	—0.98
81	1.010	2.005	0.198	0.046	0.16	0.99
82	1.008	2.007	0.163	0.046	0.14	0.99
83	1.006	2.007	0.128	0.047	0.12	0.99
84	1.005	2.006	0.093	0.047	0.11	0.99
85	+1.003	+2.005	—0.057	+0.047	—0.09	—1.00
86	1.002	2.004	—0.022	0.047	0.07	1.00
87	1.000	2.002	+0.013	0.047	0.05	1.00
88	0.998	2.000	0.048	0.047	0.03	1.00
89	0.997	1.997	0 082	0.047	—0.02	1.00
90	+0.995	+1.993	+0.117	+0.047	0.00	—1.00
91	0.993	1.989	0.152	0.047	+0.02	1.00
92	0.992	1.984	0.186	0.047	0.03	1.00
93	0.990	1.978	0.220	0.047	0.05	1.00
94	0.988	1.972	0.254	0.047	0.07	1.00
95	+0.987	+1.965	+0.287	+0.047	+0.09	—1.00
96	0.985	1.958	0.321	0.047	0.11	0.99
97	0.984	1.950	0.354	0.047	0.12	0.99
98	0.982	1.943	0.387	0.046	0.14	0.99
99	0.980	1.933	0.421	0.046	0.16	0.99
100	+0.979	+1.924	+0.453	+0.046	+0.17	—0.98
101	0.977	1.914	0.487	0.046	0.19	0.98
102	0.976	1.903	0.519	0.046	0.21	0.98
103	0.974	1.893	0.551	0.046	0.22	0.97
104	0.972	1.882	0.582	0.046	0.24	0.97
105	+0.971	+1.869	+0.614	+0.045	+0.26	—0.97
106	0.969	1.857	0.645	0.045	0.28	0.96
107	0.968	1.844	0.677	0.045	0.29	0.96
108	0.967	1.829	0.707	0.045	0.31	0.95
109	0.965	1.815	0.737	0.044	0.33	0.95
110	+0.964	+1.800	+0.768	+0.044	+0.34	—0.94
111	0.962	1.786	0.798	0.044	0.36	0.93
112	0.961	1.770	0.827	0.044	0.37	0.92
113	0.959	1.754	0.855	0.043	0.39	0.91
114	0.958	1.738	0.884	0.043	0.41	0.91
115	0.956	+1.721	+0.913	+0.043	+0.42	—0.90
116	0.955	1.704	0.942	0.042	0.44	0.89
117	0.954	1.686	0.970	0.042	0.45	0.88
118	0.953	1.668	0.997	0.041	0.47	0.87
119	+0.951	+1.650	+1.025	+0.041	+0.49	—0.87

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g	$\frac{\partial\lambda}{\partial\varepsilon}$	$\frac{\partial\lambda}{\partial e}$	$\frac{\partial\lambda}{e d\pi}$	$\frac{\partial\rho}{\partial\varepsilon}$	$\frac{\partial\rho}{\partial e}$	$\frac{\partial\rho}{e d\pi}$
120°	+0.950	+1.631	+1.051	+0.041	+0.50	—0.86
121	0.949	1.611	1.077	0.040	0.51	0.85
122	0.948	1.592	1.104	0.040	0.53	0.84
123	0.947	1.571	1.129	0.039	0.54	0.83
124	0.945	1.551	1.154	0.039	0.56	0.82
125	+0.944	+1.530	+1.180	+0.038	+0.57	—0.81
126	0.943	1.509	1.205	0.038	0.59	0.80
127	0.942	1.488	1.229	0.037	0.60	0.79
128	0.941	1.466	1.253	0.037	0.62	0.78
129	0.940	1.443	1.277	0.037	0.63	0.77
130	+0.939	+1.421	+1.300	+0.036	+0.64	—0.76
131	0.937	1.397	1.322	0.036	0.66	0.75
132	0.936	1.374	1.344	0.035	0.67	0.74
133	0.936	1.350	1.367	0.034	0.68	0.72
134	0.935	1.327	1.388	0.034	0.70	0.71
135	+0.934	+1.302	+1.409	+0.033	+0.71	—0.71
136	0.933	1.277	1.430	0.033	0.71	0.70
137	0.932	1.253	1.451	0.032	0.72	0.68
138	0.931	1.228	1.470	0.031	0.74	0.67
139	0.930	1.202	1.489	0.031	0.75	0.66
140	+0.929	+1.177	+1.508	+0.030	+0.76	—0.64
141	0.928	1.151	1.527	0.030	0.77	0.63
142	0.927	1.124	1.545	0.029	0.78	0.62
143	0.927	1.099	1.562	0.028	0.79	0.60
144	0.926	1.072	1.580	0.028	0.80	0.59
145	+0.925	+1.044	+1.596	+0.027	+0.81	—0.57
146	0.924	1.016	1.613	0.026	0.82	0.56
147	0.923	0.989	1.629	0.026	0.83	0.54
148	0.922	0.962	1.644	0.025	0.84	0.53
149	0.922	0.934	1.659	0.024	0.85	0.51
150	+0.921	+0.906	+1.674	+0.023	+0.86	—0.50
151	0.921	0.878	1.688	0.023	0.87	0.49
152	0.920	0.849	1.702	0.022	0.87	0.47
153	0.919	0.820	1.714	0.021	0.88	0.45
154	0.919	0.792	1.727	0.021	0.89	0.44
155	+0.919	+0.762	+1.740	+0.020	+0.90	—0.42
156	0.918	0.733	1.751	0.019	0.91	0.41
157	0.918	0.704	1.763	0.018	0.91	0.39
158	0.917	0.674	1.773	0.018	0.92	0.37
159	0.916	0.645	1.783	0.017	0.93	0.36
160	+0.916	+0.615	+1.793	+0.016	+0.94	—0.34
161	0.915	0.585	1.803	0.015	0.95	0.33
162	0.915	0.555	1.811	0.015	0.95	0.31
163	0.915	0.525	1.820	0.014	0.96	0.29
164	0.915	0.494	1.829	0.013	0.96	0.28
165	+0.914	+0.465	+1.836	+0.012	+0.97	—0.26
166	0.914	0.435	1.843	0.011	0.97	0.24
167	0.914	0.403	1.849	0.011	0.97	0.22
168	0.913	0.373	1.855	0.010	0.98	0.21
169	0.913	0.343	1.860	0.009	0.98	0.19
170	+0.913	+0.311	+1.865	+0.008	+0.98	—0.17
171	0.912	0.280	1.870	0.007	0.99	0.16
172	0.912	0.249	1.875	0.006	0.99	0.14
173	0.912	0.219	1.879	0.006	0.99	0.12
174	0.912	0.187	1.882	0.005	0.99	0.11
175	+0.911	+0.156	+1.884	+0.004	+1.00	—0.09
176	0.911	0.126	1.886	0.003	1.00	0.07
177	0.911	0.094	1.888	0.002	1.00	0.05
178	0.911	0.063	1.889	0.002	1.00	0.03
179	+0.911	+0.031	+1.890	+0.001	+1.00	—0.02

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In the equations of condition ten years has been adopted for the unit of time, in order to make the general value of the coefficients as nearly equal as possible, and the time has been counted from the epoch 1830.0, in order to have the positive and negative values of t in the equations more nearly balanced. To distinguish these values of $\delta\varepsilon$ and δn they are marked with an accent. This unit of time gives 0.8914 for the value of $\frac{2}{3n}$ in arc, whence

$$\frac{\partial\rho}{\partial n'} = t\,\frac{\partial\rho}{\partial l} - 0.891.$$

The equations of condition are now formed by putting in the preceding equations for heliocentric longitude and radius vector

$$\delta\lambda = \frac{\partial\lambda}{\partial\varepsilon}\delta\varepsilon + \frac{\partial\lambda}{\partial v}\delta v + \frac{\partial\lambda}{\partial e}\delta e + \frac{\partial\lambda}{e\partial\pi}e\delta\pi + \frac{\partial\lambda}{\partial\mu'}\delta\mu'$$

$$\delta\rho = \frac{\partial\rho}{\partial\varepsilon}\delta\varepsilon + \frac{\partial\delta}{\partial n}\delta n + \frac{\partial\rho}{\partial e}\delta e + \frac{\partial\rho}{e\partial\pi}e\delta\pi.$$

For the coefficients $\frac{\partial\lambda}{\partial\mu'}$ have been taken one-hundredth the perturbations of longitude produced by Neptune, as given in the heliocentric ephemeris at the end of Chapter V. The corrected mass of Neptune will then be

$$\frac{1}{17000}\left(1 + \frac{\delta\mu'}{100}\right)$$

Finally, I remark that all the preceding comparisons are made with the heliocentric ephemeris as printed, without the correction indicated in the column adjoining it, but in the following equations this correction is for the first time introduced.

Equations of condition given by the Corrections in Longitude.

1	$0.05\delta\varepsilon'$	$-\ 0.70\delta n'$	$-0.10\delta e$	$+0.03e\delta\pi$	$+0.12\delta\mu'$	$=+\ 1''.1$
2	0.10	— 1.11	+0.01	—0.19	+0.12	=+ 2 .2
3	0.31	— 2.55	+0.33	+0.51	—0.22	=+12 .8
4	0.32	— 2.52	+0.27	+0.58	—0.18	=+11 .8
5	0.30	— 2.32	+0.12	+0.59	—0.09	=+11 .6
6	0.19	— 1.40	—0.01	+0.39	0.00	=+ 5 .0
7	0.19	— 1.18	—0.29	+0.23	+0.22	=+ 4 .8
8	1.41	— 6.75	—2.69	—0.75	+2.47	=+ 2 .8
9	1.06	— 5.01	—1.98	—0.72	+1.85	=+ 1 .1
10	1.07	— 4.91	—1.94	—0.86	+1.85	=+ 1 .7
11	1.09	— 4.87	—1.86	—1.04	+1.85	=— 0 .5
12	0.73	— 3.06	—1.04	—0.97	+1.16	=+ 1 .0
13	0.54	— 2.20	—0.71	—0.78	+0.82	=+ 1 .4
14	0.55	— 2.19	—0.68	—0.81	+0.81	=— 0 .6
15	0.73	— 2.86	—0.82	—1.16	+1.03	=— 0 .9
16	0.37	— 1.39	—0.36	—0.60	+0.49	=— 0 .2
17	0.56	— 2.05	—0.47	—0.95	+0.70	=— 0 .4
18	0.37	— 1.33	+0.26	—0.65	+0.44	=+ 0 .6
19	0.75	— 2.59	—0.38	—1.34	+0.82	=— 1 .0

20	$0.56\delta\epsilon'$	$-1.89\delta n'$	$-0.22\delta e$	$-1.04e\delta\pi$	$+0.58\delta\mu'$	$=-0''.9$
21	0.58	− 1.92	−0.13	−1.12	+0.55	= − 0.5
22	0.38	− 1.15	+0.09	−0.74	+0.24	= − 0.2
23	0.38	− 1.11	+0.15	−0.73	+0.21	= − 0.3
24	1.15	− 3.18	+0.65	−2.13	+0.50	= + 0.6
25	1.13	− 2.81	+1.11	−1.91	+0.19	= + 0.5
26	0.56	− 1.33	+0.62	−0.89	+0.05	= + 0.1
27	1.12	− 2.54	+1.38	−1.68	0.00	= + 2.1
28	0.56	− 1.21	+0.75	−0.79	−0.04	= − 0.1
29	0.70	− 1.55	+1.09	−0.97	−0.10	= + 1.6
30	1.11	− 2.19	+1.73	−1.32	−0.23	= + 2.2
31	1.09	− 2.04	+1.81	−1.14	−0.29	= + 1.3
32	0.56	− 0.98	+0.96	−0.51	−0.18	= + 0.7
33	0.55	− 0.91	+1.00	−0.43	−0.21	= + 1.1
34	1.08	− 1 69	+2.03	−0.71	−0.46	= + 1.5
35	1.63	− 2.37	+3.11	−0.82	−0.74	= + 2.0
35′	1.07	− 1.45	+2.08	−0.38	−0.52	= + 0.4
35″	1.06	− 1.33	+2.10	−0.22	−0.55	= + 0.8
36	1.58	− 1.83	+3.18	−0.09	−0.85	= + 0.3
37	1.05	− 1.11	+2.10	+0.08	−0.59	= − 0.8
38	1.04	− 0.99	+2.07	+0.26	−0.59	= − 1.3
39	0.69	− 0.58	+1.36	+0.27	−0.39	= + 0.8
40	0.69	− 0.51	+1.33	+0.38	−0.39	= + 0.8
41	0.68	− 0.44	+1.29	+0.47	−0.39	= 0.0
42	1.01	− 0.56	+1.88	+0.84	−0.59	= + 0.3
43	0.67	− 0.30	+1.20	+0.64	−0.38	= − 0.5
44	1.00	− 0.35	+1.72	+1.09	−0.57	= + 0.2
45	1.95	− 0.50	+3.21	+2.39	−1.12	= − 2.5
46	2.42	− 0.37	+3.77	+3.21	−1.36	= − 2.6
47	2.42	− 0.12	+3.53	+3.52	−1.37	= − 2.4
48	2.39	+ 0.12	+3.22	+3.69	−1.33	= − 4.9
49	1.91	+ 0.28	+2.37	+3.14	−1.07	= − 0.1
50	1.91	+ 0.48	+2.15	+3.30	−1.08	= − 2.4
51	2.37	+ 0.83	+2.34	+4.32	−1.34	= − 4.5
52	2.36	+ 1.06	+2.02	+4.44	−1.36	= − 4.3
53	2.37	+ 1.32	+1.68	+4.62	−1.37	= − 5.5
54	2.85	+ 1.86	+1.66	+5.64	−1.68	= − 7.5
55	2.84	+ 2.17	+1.22	+5.74	−1.71	= − 4.1
56	2.84	+ 2.44	+0 81	+5.84	−1.74	= − 2.1
57	2.83	+ 2.71	+0.40	+5.88	−1.78	= − 1.7
58	2.83	+ 3.00	−0.06	+5.88	−1.82	= − 2.0
59	2.82	+ 3.28	−0.44	+5.84	−1.86	= + 0.3
60	2.83	+ 3.57	−0.84	+5.81	−1.91	= + 1.2
61	2.85	+ 3.90	−1.27	+5.78	−1.98	= + 4.4
62	2.85	+ 4.21	−1.74	+5.64	−2.02	= + 4.8
63	1.91	+ 3.02	−1.42	+3.69	−1.39	= + 4.0
64	2.85	+ 4.78	−2.50	+5.32	−2.14	= + 6.0
65	1.91	+ 3.39	−1.92	+3.46	−1.47	= + 5.6
66	1.92	+ 3.61	−2.18	+3.32	−1.52	= + 4.3
67	1.93	+ 3.82	−2.40	+3.17	−1.56	= + 6.6
68	1.94	+ 4.05	−2.65	+2.98	−1.60	= + 7.3
69	1.94	+ 4.20	−2.83	+2.80	−1.64	= + 6.5

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$$
\begin{array}{r|rrrrrl}
70 & 1.95\delta\varepsilon' & +\ 4.44\delta n' & -3.04\delta e & +2.59e\delta\pi & -1.68\delta\mu & =+\ 6''.8 \\
71 & 1.98 & +\ 4.72 & -3.23 & +2.37 & -1.73 & =+\ 8.1 \\
72 & 1.99 & +\ 4.93 & -3.44 & +2.15 & -1.76 & =+\ 9.1 \\
73 & 1.99 & +\ 5.13 & -3.58 & +1.91 & -1.78 & =+\ 8.7 \\
74 & 2.00 & +\ 5.35 & -3.73 & +1.64 & -1.80 & =+\ 8.1 \\
75 & 2.53 & +\ 7.02 & -4.83 & +1.73 & -2.30 & =+10.6 \\
76 & 2.54 & +\ 7.34 & -4.96 & +1.33 & -2.29 & =+10.9 \\
77 & 2.56 & +\ 7.64 & -5.07 & +0.97 & -2.31 & =+\ 8.2 \\
78 & 2.07 & +\ 6.38 & -4.14 & +0.47 & -1.86 & =+\ 6.3 \\
79 & 2.59 & +\ 8.25 & -5.20 & +0.21 & -2.32 & =+\ 7.0 \\
80 & 2.60 & +\ 8.51 & -5.21 & -0.15 & -2.29 & =+\ 6.6 \\
81 & 2.11 & +\ 7.14 & -4.19 & -0.51 & -1.83 & =+\ 4.3 \\
82 & 1.60 & +\ 5.56 & -3.13 & -0.58 & -1.36 & =+\ 3.9 \\
83 & 2.67 & +\ 9.59 & -5.12 & -1.37 & -2.20 & =+\ 1.1 \\
84 & 2.68 & +\ 9.92 & -5.00 & -1.80 & -2.14 & =-\ 1.8 \\
85 & 2.17 & +\ 8.22 & -3.92 & -1.75 & -1.67 & =-\ 3.8 \\
86 & 2.73 & +10.61 & -4.97 & -2.59 & -2.01 & =-\ 9.0 \\
87 & 2.20 & +\ 8.78 & -3.63 & -2.33 & -1.52 & =-\ 9.5 \\
88 & 1.65 & +\ 6.74 & -2.58 & -1.96 & -1.08 & =-10.5 \\
89 & 2.77 & +11.61 & -4.04 & -3.61 & -1.69 & =-19.9
\end{array}
$$

The following are the approximate normals to which these equations give rise. Inaccuracies being detected in several of the equations of condition after these normals were formed, they do not accurately correspond to those equations as written.

$$
\begin{array}{rrrrrl}
283.64\delta\varepsilon' & +\ 414.36\delta n' & -151.63\delta e & +247.23e\delta\pi & -176.03\delta\mu' & =+123''.5 \\
414.36 & +1619.44 & -689.11 & +260.26 & -436.02 & =+103.2 \\
-151.63 & -\ 689.11 & +557.82 & +\ 38.45 & +122.88 & =-399.8 \\
247.23 & +\ 260.26 & +\ 38.45 & +618.45 & -194.60 & =+267.3 \\
-176.03 & -\ 436.02 & +122.88 & -194.60 & +163.13 & =-128.1
\end{array}
$$

The values of the unknown quantities deduced from these normals were substituted in the equations of condition, and a farther approximation was made by solving the equations given by the residuals. The following are the first approximations given by the normals, and the finally concluded corrections

	Preliminary.	Final.
$\delta\mu'$,	-15.00	-13.44
$e\delta\pi$,	$-\ 0.52$	$-\ 0.36$
δe,	$-\ 4.25$	$-\ 4.04$
$10\delta n = \delta n'$,	$-\ 4.73$	$-\ 4.33$
$\delta\varepsilon'$,	$-\ 3.78$	$-\ 3.44$
Mass of Neptune	$\frac{1}{20000}$	$\frac{1}{19640}$.

The final values of the corrections being substituted in the equations leave the following system of residuals, or outstanding excesses of the observed longitudes over theory. Column $f\delta l$ gives the residual of the equation itself; the probable error of which has always been judged to be 0.''5, while in column δl this residual is divided by f to obtain the residual correction of the longitude itself. The values of the factors f are found with the original equations on pages 159 and 160.

No. of Eq.	Year.	$f\delta l$ ″	δl ″
1	1691.0	—0.5	—10.
2	1715.2	—0.8	—10.
3	1748.8	+1.5	+ 4.5
4	1750.8	+0.8	+ 2.4
5	1753.9	+2.2	+ 6.6
6	1756.7	—0.2	— 1.0
7	1769.0	—0.2	— 1.0
8	1782.0	+0.1	+ 0.2
9	1783.0	—0.5	— 0.5
10	1784.0	+0.6	+ 0.6
11	1785.0	—1.0	— 1.0
12	1788.0	+1.2	+ 1.8
13	1789.0	+1.7	+ 3.4
14	1790.0	—0.5	— 1.0
15	1791.0	—0.7	— 1.0
16	1792.0	0.0	0.0
17	1793.0	—0.1	— 0.2
18	1794.0	+0.5	+ 1.5
19	1795.0	—0.6	— 0.9
20	1796.0	—0.5	— 1.0
21	1797.1	—0.6	— 1.2
22	1800.2	—1.4	— 4.2
23	1801.2	—1.4	— 4.2
24	1802.3	—0.6	— 0.6
25	1805.3	—1.4	— 1.4
26	1806.3	—1.0	— 2.0
27	1807.3	—0.1	— 0.1
28	1808.3	—1.0	— 2.0
29	1809.3	+0.1	+ 0.2
30	1810.3	—0.1	— 0.1
31	1811.3	—0.9	— 0.9
32	1812.4	—0.3	— 0.6
33	1813.4	+0.1	+ 0.2
34	1814.4	—0.4	— 0.4
35	1815.4	—0.3	— 0.2
35′	1816.4	—0.8	— 0.8
35″	1817.4	—0.3	— 0.3
36	1818.4	—0.7	— 0.5
37	1819.4	—1.4	— 1.4
38	1820.5	—1.4	— 1.4
39	1821.5	+0.9	+ 1.4
40	1822.5	+1.4	+ 2.1
41	1823.5	+0.6	+ 0.9
42	1824.5	+1.2	+ 1.2
43	1825.5	+0.7	+ 1.0
44	1826.5	+2.0	+ 2.0
45	1827.7	+1.0	+ 0.5
46	1828.7	+2.5	+ 1.0
47	1829.7	+2.8	+ 1.1
48	1830.7	+0.4	+ 0.2
49	1831.7	+4.1	+ 2.0
50	1832.7	+1.8	+ 0.9
51	1833.8	+0.5	+ 0.2
52	1834.8	+0.1	0.0
53	1835.8	—1.4	— 0.6
54	1836.8	—3.4	— 1.1
55	1837.8	—0.6	— 0.2
56	1838.8	+0.5	+ 0.2
57	1839.8	—0.4	— 0.1
58	1840.8	—1.7	— 0.6
59	1841.8	—0.3	— 0.1
60	1842.8	—0.5	— 0.2
61	1843.8	+1.6	+ 0.5
62	1844.9	+0.7	+ 0.2
63	1845.9	+0.6	+ 0.3
64	1846.9	—0.4	— 0.1
65	1847.9	+0.6	+ 0.3
66	1848.9	—1.6	— 0.8
67	1849.9	+0.2	+ 0.1
68	1850.9	+0.3	+ 0.2
69	1851.9	—1.2	— 0.6
70	1852.9	—1.2	— 0.6
71	1853.9	—0.2	— 0.1

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No. of Eq.	Year.	$f\delta l$	δl
		″	″
72	1854.9	+0.5	+ 0.2
73	1855.9	0.0	0.0
74	1856.9	—0.5	— 0.2
75	1858.0	—0.2	— 0.1
76	1859.0	+0.9	+ 0.4
77	1860.0	—1.1	— 0.4
78	1861.0	—0.6	— 0.3
79	1862.0	—0.6	— 0.2
80	1863.0	+0.2	+ 0.1
81	1864.0	+0.7	+ 0.4
82	1865.0	+2.2	+ 1.5
83	1866.0	+1.0	+ 0.4
84	1867.0	+0.5	+ 0.2
85	1868.0	+0.4	+ 0.2
86	1869.0	—1.9	— 0.8
87	1870.0	—0.1	0.0
88	1871.0	—1.2	— 0.8
89	1872.1	+0.4	+ 0.2

A simple glance at the course of the residuals shows (1) that their probable value is considerably greater than the probable error attributed to the equations of condition, being more nearly 0″.7 than 0″.5, and yet larger in the later years; (2) that during certain periods they are of a systematic character. During the years 1748 to 1753 the observations show a decided positive correction to the theory of a magnitude greater than we can consider probable, amounting to about one-third of a second of time in the mean of Bradley's two observations of 1748 and 1753. About 1800 the correction becomes negative, and so continues for 20 years with an average value of about 1″. In 1821 it suddenly becomes positive, and so continues until 1833. From this year forward the residuals are not systematic in character.

In order to show clearly the general course of the outstanding corrections, they have been divided into groups, generally including about five years each. The mean outstanding correction for each group, taken with respect to the weights indicated by the factors *f*, is as follows. In the column ε is shown what the probable error of the residual should be if the weights assigned to the several equations were strictly correct, and no systematic errors were present either in theory or observation.

Year.	δl	ε
	″	″
1715.2	—10.	±6.
1751.1	+ 3.7	±0.8
1769.0	— 1.0	±2.5
1783.3	— 0.18	±0.23
1790.0	+ 0.62	±0.40
1795.0	— 0.55	±0.41
1802.0	— 1.25	±0.45
1806.5	— 1.00	±0.31
1810.5	— 0.37	±0.32
1814.5	— 0.37	±0.23
1819.5	— 0.37	±0.18
1824.8	+ 1.50	±0.16
1829.7	+ 0.91	±0.10
1835.2	— 0.27	±0.09
1839.8	— 0.17	±0.07
1844.8	+ 0.14	±0.03
1849.9	— 0.21	±0.11
1854.9	— 0.14	±0.11
1860.0	— 0.13	±0.09
1865.0	+ 0.36	±0.10
1870.0	— 0.21	±0.11

A simple glance at the residuals δl shows that they are much greater than the purely accidental residuals resulting from the theory of least squares. We may divide the possible causes of these systematic errors into three classes.

1. *Systematic Errors of Observation.*—These may result from deviation of the line of collimation of the instrument from a true great circle, or from any peculiarity of the observer which leads to his registering the transit of Uranus earlier or later than that of a fixed star. If we compare the corrections derived from the work of different observatories as given in the last chapter, we shall find frequent cases not only of systematic differences between the results of different observatories, but between those of the same observatory in two successive years. An instance which particularly attracted my attention on first preparing the comparisons of theory and observation is that of the Greenwich observations for 1831, which, as compared with observations at the same observatory during the years preceding and following, seem to be affected with some constant error in R. A. of about 2″. I find that this discrepancy can be attributed only to the original observations.

2. *Errors in the Theory compared.*—These may arise from errors in the preceding theoretical computations, from the omission of the terms of the second order produced by Neptune, from the adoption of an erroneous mass of Saturn, or from the attraction of an unknown planet. With regard to the probability of these different sources of error it may be remarked that errors of computation seem possible only in the terms of the second order, that the mass of Saturn is taken from the exhaustive discussion of the Saturnian system by Bessel, in which an error sufficient to influence the theory of Uranus seems highly improbable, and that a trans-Neptunian planet large enough to produce a sensible deviation of the orbit of Uranus from an ellipse in the course of a century would be too large to have escaped detection. The choice of the elliptic elements of Uranus and Neptune is such that the terms of the second order, due to the action of Neptune, can scarcely become sensible within a century of the epoch.

3. *Errors in the various Reductions by which Theory and Observation are compared.*—In the method adopted for comparing theory and observation a number of small uncertainties incident to the imperfections of the older data of reduction necessarily creep in. In the early observations the imperfections arise principally from the uncertainty of the instrumental corrections, and the errors in the adopted positions of the fundamental stars, and indeed in nearly all the data of reduction. In the late years they arise principally from the great magnitude of the correction to Bouvard's tables, and the consequent rapid change of the corrections to the geocentric ephemerides, which make the determination of the corrections Δl and δl from theory and observation somewhat uncertain. Errors from this source will necessarily be in part of a systematic character, and, in view of their possibility, I regret not having been able to completely re-reduce all the observations before 1840, and to compare all since directly with ephemerides computed from the provisional theory. In order, however, to test the question whether they are sensible, I have prepared an ephemeris from the provisional theory for the three recent oppositions of 1861–2, 1862-3, and 1872, and compared it directly with

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the observations. The mean corrections in geocentric longitude for groups of observations are given in columns (2), column (1) showing the correction given by the work of the last chapter.

Opposition	1861–2		1862–3		1872	
	(1)	(2)	(1)	(2)	(1)	(2)
	$+2''.8$	$+2''.4_1$	$+2''.6$	$+2''.6_1$	$-8''.6$	$-9''.0_1$
	2 .9	$2\,.4_3$	2 .3	$2\,.3_2$	−8 .5	$-8\,.1_2$
	3 .0	$3\,.1_2$	2 .1	$2\,.0_3$	−7 .3	$-7\,.5_3$
	2 .4	$2\,.4_2$	3 .3	$2\,.9_4$	−7 .3	$-7\,.7_5$
	. . .	. . .	2 .6	$2\,.7_1$	−7 .8	$-7\,.6_1$
Mean	+2 .79	+2 .62	+2 .65	+2 .50	−7 .65	−7 .82

A systematic difference of 0″.16 would seem to be indicated, and on account of it a correction of 0″.10 was applied to the comparisons of the last few years in forming the equations of condition.

In view of the possibility of systematic errors from this source it may be considered that too great relative weight has been assigned to the results of the later observations. If the residuals arise from errors of comparison and of theory, their probable magnitude is nearly as great at one epoch as at another. It may therefore be interesting to inquire what result we should get if, instead of assigning such different weights to the comparisons at different epochs, we sought only for the best general agreement with observations during the period the planet has been observed. The preceding system of mean residuals will enable us to discuss this question quite easily. In the first solution we shall reject the results from Flamstead's observations, owing to their assured uncertainty, and those from Le Monnier's of 1769, owing to the possible maladjustment of his quadrant. The equations from the remaining residuals will be the following:

$1.0\delta^2\varepsilon'$	$-7.6\delta^2 n'$	$+0.8\delta^2 e$	$+1.7e\delta^2\pi$	$-0.5\delta^2\mu'$	$=+3''.7$	Wt. $\frac{1}{2}$
1.1	−5.0	−2.0	−0.8	+1.8	= −0 .18	2
1.1	−4.4	−1.4	−1.6	+1.6	= +0 .62	1
1.1	−3.9	−0.6	−2.0	+1.2	= −0 .55	1
1.2	−3.2	+0.6	−2.1	+0.5	= −1 .25	1
1.1	−2.6	+1.3	−1.8	0.0	= −1 .00	2
1.1	−2.2	+1.7	−1.3	−0.2	= −0 .37	2
1.1	−1.7	+2.0	−0.7	−0.5	= −0 .37	2
1.0	−1.1	+2.1	+0.1	−0.6	= −0 .37	2
1.0	−0.5	+1.8	+0.9	−0.6	= +1 .50	2
1.0	0.0	+1.4	+1.4	−0.5	= +0 .91	3
0.9	+0.5	+0.8	+1.8	−0.5	= −0 .27	3
0.9	+0.9	+0.1	+2.0	−0.6	= −0 .17	3
1.0	+1.4	−0.6	+1.9	−0.7	= +0 .14	3
1.0	+1.9	−1.2	+1.6	−0.8	= −0 .21	3
1.0	+2.5	−1.7	+1.1	−0.9	= −0 .14	3
1.0	+3.1	−2.0	+0.4	−0.9	= −0 .13	3
1.1	+3.7	−2.1	−0.4	−0.9	= +0 .36	3
1.1	+4.4	−1.8	−1.2	−0.8	= −0 .21	3

Giving these nineteen equations equal weights, we have the second of the following solutions, and the second of the series of residuals the first corresponding to the primitive solution. Solving them again and assigning the weights attached to the respective equations, which I judge to be those to which they are entitled when a liberal allowance is made for systematic errors of observation and of comparison of theory with observations, adding also the equations given by the observations of Flamstead and Le Monnier, which are as follows:

1690	$0.0\delta^2\varepsilon'$	$-0.7\delta^2 n'$	$-0.1\delta^2 e$	$+0.0e\delta^2\pi$	$+0.1\delta^2\mu'$	$= -0''.5$;	f, $\frac{1}{20}$;	Wt, 1
1715	0.1	-1.1	0.0	-0.2	$+0.1$	$= -0\,.5$	$\frac{1}{12}$	1
1769	0.2	-1.2	-0.3	$+0.2$	$+0.2$	$= +2\,.2$	$\frac{1}{5}$	1

we have the second solution, and the third series of residuals.

	(1)	(2)	(3)
$\delta^2\varepsilon_1$	0	$-0''.39$	-0.21
$\delta^2 n_1$	0	$-0\,.38$	-0.19
$\delta^2 e$	0	$-0\,.33$	-0.15
$e\delta^2\pi$	0	$+0\,.25$	$+0.19$
$\delta^2\mu'$	0	$-1\,.02$	-0.49
m'	$\frac{1}{19640}$	$\frac{1}{19870}$	$\frac{1}{19750}$

		RESIDUALS.	
Year.	$\Delta_1 l$	$\Delta_2 l$	$\Delta_3 l$
	″	″	″
1691.0	$-10.$	$-14.$	$-12.$
1715.2	$-10.$	$-\ 9.$	$-\ 7.$
1751.1	$+\ 3.7$	$+\ 0.5$	$+\ 2.0$
1769.0	$-\ 1.0$	$-\ 2.6$	$-\ 1.9$
1783.3	$-\ 0.18$	$-\ 0.30$	$-\ 0.17$
1790.0	$+\ 0.62$	$+\ 0.93$	$+\ 0.89$
1795.0	$-\ 0.55$	$-\ 0.09$	$-\ 0.18$
1802.0	$-\ 1.25$	$-\ 0.78$	$-\ 0.87$
1806.5	$-\ 1.00$	$-\ 0.69$	$-\ 0.73$
1810.5	$-\ 0.37$	$-\ 0.10$	$-\ 0.05$
1814.5	$-\ 0.37$	$-\ 0.26$	$-\ 0.28$
1819.5	$-\ 0.37$	$-\ 0.34$	$-\ 0.37$
1824.8	$+\ 1.50$	$+\ 1.46$	$+\ 1.41$
1829.7	$+\ 0.91$	$+\ 0.90$	$+\ 0.81$
1835.2	$-\ 0.27$	$-\ 0.43$	$-\ 0.46$
1839.8	$-\ 0.17$	$-\ 0.56$	$-\ 0.48$
1844.8	$+\ 0.14$	$-\ 0.31$	$-\ 0.18$
1849.9	$-\ 0.21$	$-\ 0.70$	$-\ 0.52$
1854.9	$-\ 0.14$	$-\ 0.54$	$-\ 0.36$
1860.0	$-\ 0.13$	$-\ 0.23$	$-\ 0.14$
1865.0	$+\ 0.36$	$+\ 0.70$	$+\ 0.62$
1870.0	$-\ 0.21$	$+\ 0.80$	$+\ 0.44$

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It will be seen that the effect of these changes of weights is, that the older observations are a little better, and the later a little worse represented. I conceive that our choice must lie between the first and third solutions, the first being the more probable if we conceive the outstanding residuals to be due to errors of observation only, and the third if we suppose them equally due to errors of computation. On the whole, I consider the mean of the two to be about the most probable, and this will give the mass of Neptune very near the round number

$$\frac{1}{19700}$$

which will be adopted as the definitive value. The definitive corrections to Elements III (p. 99) will then be

$\delta\epsilon'$ (1830)	$-\ 3''.56$
$\delta\epsilon$ (1850)	-12.45
$10\delta n$	$-\ 4.44$
δe	$-\ 4.12$
$e\delta\pi$	$-\ 0.25$
$\delta\mu$	$-\ 0.137$

Corrections to the Inclination and Node.

These corrections have been derived entirely from the modern observations, the ancient ones being too uncertain to add anything to the weight of the result. The mode in which the correction to the latitude of the provisional ephemeris has been concluded from the observations has been sufficiently explained: it is only necessary to add that the immediate results from the data of the preceding chapter require two corrections, namely:

(1) A correction to the theoretical latitude for the change in the adopted mass of Neptune. The value of this correction, as derived from the data of Chapter V, is with sufficient approximation

$$\delta\beta = 0''.25\, T \cos g.$$

(2) A correction to the observed latitude on account of the difference between the obliquity of the ecliptic adopted in the various ephemerides compared, and that of Hansen's *Tables du Soleil*, which having been adopted in the theory should be used throughout.

Applying the correction (2) — (1) to all the observed latitudes, we have the following corrections to the latitude of the provisional ephemeris derived from all the observations of each opposition since 1781. The third column gives the number of observations in declination. These numbers may, however, in some cases be inaccurate. The fourth and fifth columns give the sine and cosine of the argument of latitude, to be used in forming the equations of condition.

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Year.	$\delta\beta$	No. of obs.	sin *u*	cos *u*	Year.	$\delta\beta$	No. of obs.	sin *u*	cos *u*
	″					″			
1782.0	—0.4	21	+0.31	+0.95	1830.7	+0.2	48	—0.82	—0.57
1783.0	—3.5	13	0.38	0.92	1831.7	+1.0	23	0.87	0.50
1784.0	—2.4	13	0.45	0.89	1832.7	+0.9	20	—0.90	—0.44
1785.0	—0.1	10	0.52	0.85	1833.7	+0.6	54	0.93	0.37
1788.0	—1.0	5	+0.71	+0.71	1834.7	+0.3	92	0.95	0.31
1789.0	+0.9	6	0.77	0.64	1835.7	+0.2	71	—9.97	—0.24
1790.0	+2.6	4	0.81	0.58	1836.7	+0.1	135	0.98	0.17
1791.0	+1.9	7	0.86	0.51	1837.8	0.0	154	0.99	0.10
1792.0	+1.8	3	+0.90	+0.44	1838.8	—0.2	182	—1.00	—0.03
1793.0	+2.9	5	0.93	0.37	1839.8	—0.4	142	1.00	+0.03
1794.0	+0.5	3	0.95	0.30	1840.8	—0.3	106	0.99	0.10
1795.0	+0.7	7	+0.97	+0.22	1841.8	—0.7	101	—0.98	+0.17
1796.0	+3.6	4	0.99	0.14	1842.8	—0.7	145	0.97	0.24
1797.0	+3.6	3	1.00	+0.05	1843.8	—0.9	88	0.95	0.31
1800.2	0.0	2	+0.98	—0.21	1844.9	—1.0	87	—0.93	+0.37
1801.2	+1.2	2	0.97	0.26	1845.9	—0.6	55	0.90	0.44
1802.3	+1.4	13	0.94	0.34	1846.9	—1.1	92	0.87	0.50
1805.3	+1.1	3	+0.83	—0.56	1847.9	—1.1	69	—0.83	+0.56
1806.3	—1.4	5	0.78	0.63	1848.9	—0.9	56	0.79	0.62
1807.3	+1.6	16	0.72	0.69	1849.9	—1.0	36	0.74	0.67
1808.3	+1.8	6	+0.67	—0.74	1850.9	—0.7	46	—0.69	+0.72
1809.3	+1.3	9	0.60	0.80	1851.9	—1.2	35	0.64	0.77
1810.3	+3.7	16	0.53	0.85	1852.9	—1.3	49	0.59	0.81
1811.3	+3.4	11	+0.45	—0.89	1853.9	—1.8	48	—0.53	+0.85
1812.4	+3.6	6	0.38	0.93	1854.9	—1.0	47	0.47	0.88
1813.4	+2.4	6	0.29	0.96	1855.9	—1.3	49	0.41	0.91
1814.4	+2.5	13	+0.22	—0.98	1856.9	—1.1	41	—0.34	+0.94
1815.4	+2.2	16	0.14	0.99	1858.0	—1.7	65	0.26	0.97
1816.4	+1.3	18	0.06	1.00	1859.0	—1.7	56	0.19	0.98
1817.5	+2.4	13	—0.01	—1.00	1860.0	—1.9	58	—0.12	+0.99
1818.5	+4.2	22	0.09	1.00	1861.0	—2.3	41	0.05	1.00
1819.5	+1.7	7	0.17	0.99	1862.0	—1.6	52	+0.02	1.00
1820.5	+1.0	9	—0.24	—0.97	1863.0	—1.7	83	+0.10	+0.99
1821.5	+1.3	10	0.31	0.95	1864.0	—2.2	39	0.18	0.99
1822.5	+1.8	7	0.38	0.92	1865.0	—1.7	37	0.25	0.97
1823.5	+0.6	7	—0.45	—0.89	1866.0	—0.9	72	+0.33	+0.95
1824.5	—0.2	11	0.51	0.86	1867.0	—0.6	83	0.40	0.92
1825.5	+1.5	5	0.57	0.82	1668.1	—0.9	32	0.47	0.88
1826.5	+1.2	7	—0.63	—0.78	1869.1	—0.8	65	+0.54	+0.84
1827.6	+2.0	5	0.68	0.73	1870.1	—0.4	32	0.60	0.80
1828.6	—1.5	7	0.73	0.68	1871.1	—0.7	21	+0.66	0.75
1829.7	+0.9	9	—0.78	—0.63	1872.1	—0.4	47	+0.72	+0.69

It will be remembered that the observed declinations have, as far as possible, been reduced to Auwers' standard. We have no positive proof that this standard is correct. If it be affected by a constant error, the result will be that the orbit of the planet on the celestial sphere, as deduced from observation, instead of being a great circle, as we know the real orbit to be, will be a small one, and the comparison of a uniform series of observations extending through an entire revolution of the planet, after making the best correction to the position of the orbit, will leave a constant residual. Now, we can best determine this residual by including it as an unknown quantity in our equations.

Again, the error of the standard is not necessarily constant, but may contain a term proportional to the time, arising from erroneous proper motions of the standard stars. Therefore, instead of supposing the residual constant, we shall suppose it of the form $a + bt$. Each observed correction to the theoretical latitude will then give the equation,

$$\sin u\delta\phi - \cos u\phi\delta\theta + a + bT = \delta\beta.$$

To facilitate the solution of these equations they have been divided into groups, each group usually comprehending three oppositions, and combined into a single equation multiplied by such a factor as would make its probable error half a second. The factor by which the correction of the latitude is multiplied in the equation is the same with the coefficient of a. The year 1840.0 is taken as the epoch for b. Thus we have the following:

EQUATIONS OF LATITUDE.

Dates of oppositions.	No. of opp.	Equation.				
						″
1782.0–85.0	4	$0.4\delta\phi$	$-0.9\phi\delta\theta$	$+1.0a$	$-0.6b$	$= -1.6$
1788.0–91.0	4	0.8	—0.6	+1.0	$-0.5b$	= +1.1
1792.0–94.0	3	0.5	—0.2	+0.5	—0.2	= +0.9
1795.0–97.1	3	0.3	0.0	+0.5	—0.2	= +0.7
1800.2–02.3	3	0.0	+0.3	+1.0	—0.4	= +1.0
1805.3–07.3	3	0.8	+0.6	+1.0	—0.3	= +0.7
1808.3–10.3	3	0.6	+0.8	+1.0	—0.3	= +2.6
1811.3–13.4	3	0.4	+0.9	+1.0	—0.3	= +3.1
1814.4–16.4	3	0.2	+1.5	+1.5	—0.4	= +3.0
1817.4–19.5	3	—0.1	+1.5	+1.5	—0.3	= +4.8
1820.5–22.5	3	—0.5	+1.4	+1.5	—0.3	= +2.0
1823.5–25.5	3	—0.5	+0.9	+1.0	—0.2	= +0.6
1826.5–28.6	3	—0.7	+0.7	+1.0	—0.1	= +0.6
1829.6–31.7	3	—1.2	+0.8	+1.5	—0.1	= +1.1
1832.7–34.7	3	—2.8	+1.1	+3.0	—0.2	= +1.8
1835.7–37.8	3	—2.9	+0.5	+3.0	—0.1	= +0.3
1838.8–40.8	3	—3.0	—0.1	+3.0	0.0	= —0.9
1841.8–43.8	3	—2.9	—0.7	+3.0	+0.1	= —2.3
1844.8–46.8	3	—2.7	—1.3	+3.0	+0.2	= —2.7
1847.8–49.9	3	—2.4	—1.9	+3.0	+0.3	= —3.0
1850.9–52.9	3	—1.9	—2.3	+3.0	+0.4	= —3.2
1853.9–55.9	3	—1.4	—2.6	+3.0	+0.4	= —4.1
1856.9–59.0	3	—0.8	—2.9	+3.0	+0.5	= —4.5
1860.0–62.0	3	—0.2	—3.0	+3.0	+0.6	= —5.8
1863.0–65.0	3	+0.5	—3.0	+3.0	+0.7	= —5.6
1866.0–68.0	3	+1.2	—2.8	+3.0	+0.8	= —2.4
1869.1–70.0	2	+1.1	—1.6	+2.0	+0.6	= —1.2
1871.1–72.1	2	+1.4	—1.4	+2.0	+0.6	= —1.1

Treating these equations by the method of least squares, we find the normal equations

$$\begin{array}{rrrrrr}
'' & '' & '' & '' & & '' \\
63.60\delta\phi & + \ 6.45\phi\delta\theta & - \ 52.10a & - \ 1.08b & = & + \ 28.33 \\
6.45\delta\phi & + \ 69.59\phi\delta\theta & - \ 52.60a & - \ 14.25b & = & + \ 111.02 \\
-52.10\delta\phi & - \ 52.60\phi\delta\theta & + \ 133.50a & + \ 8.95b & = & - \ 76.55 \\
- \ 1.08\delta\phi & - \ 14.25\phi\delta\theta & + \ 8.95a & + \ 4.49b & = & - \ 23.69
\end{array}$$

The solution of these equations gives

$$\begin{aligned} \delta\phi &= +\,0''.28 + 0''.75\ a = +\,0''.54 \\ \phi\delta\theta &= +\,1\,.57 + 0\,.686a + 0\,.205b = +\,1''.75 \\ a &= +\,0\,.35 \\ b &= -\,0\,.28 \end{aligned}$$

These values of *a* and *b* indicate that at the epoch 1840 Auwers' equatorial declinations are too great, or his north polar distances are too small by 0″.35, and that this error is diminishing at the rate of 0″.28 per century. If the older measures in declination had been comparable in precision with those made at the present time, and if the possible periodic error in the reduced right ascensions had been carefully eliminated, I should regard this determination as entitled to considerable weight. In view of the great uncertainty of the declinations previous to 1820, it can be regarded as little more than a rough attempt at a determination. For this reason the first two normal equations have been solved, leaving *a* and *b* indeterminate, so as to show the valves of $\delta\phi$ and $\phi\delta\theta$ in terms of these quantities. It will be seen that had we neglected *a* and *b* entirely, the value of $\delta\phi$ would have been smaller by 0″.26, and that of $\phi\delta\theta$ smaller by 0″.18 than those actually concluded. As the observations with the Washington Transit Circle, and those with the Pulkowa Vertical Circle, both indicate an increase of Auwers' polar distances, I shall take for the definitive corrections to the inclination and node those which follow from the above values of *a* and *b*, or,

$$\begin{aligned} \delta\phi &= +\,0''.54 \\ \phi\delta\theta &= +\,1\,.75. \end{aligned}$$

The following table shows the residuals of the equations, and the mean outstanding corrections to the latitude, (1) when the concluded values of $\delta\phi$ and $\phi\delta\theta$ *a* and *b* are all used, and (2) when *a* and *b* are supposed zero, and the values of $\delta\phi$ and $\phi\delta\theta$, corresponding to this supposition, are used:

Year.	Residuals.		$\delta\beta$	
	(1)	(2)	(1)	(2)
	″	″	″	″
1783	—0.8	—0.3	—0.8	—0.3
1789	+1.2	+1.8	+1.2	+1.8
1793	+0.7	+1.1	+1.4	+2.2
1796	+0.3	+0.6	+0.6	+1.2
1801	—0.5	+0.3	—0.5	+0.3
1806	—1.2	—0.5	—1.2	—0.5
1809	+0.5	+1.2	+0.5	+1.2
1812	+0.9	+1.6	+0.9	+1.6
1815	—0.3	+0.6	—0.2	+0.4
1818	+1.6	+2.5	+1.1	+1.7
1821	—0.7	—0.1	—0.5	—0.1
1824	—1.3	—0.7	—1.3	—0.7
1827	—0.6	—0.3	—0.6	-0.3

Year.	Residuals.		$\delta\beta$	
	(1)	(2)	(1)	(2)
	″	″	″	″
1830	—0.2	+0.2	—0.1	+0.1
1833	+0.3	+0.9	+0.10	+0.30
1836	0.0	+0.3	0.00	+0.10
1839	—0.1	+0.1	—0.03	+0.03
1842	—0.5	—0.4	—0.17	—0.13
1845	+0.1	0.0	+0.03	0.00
1848	+0.6	+0.7	+0.20	+0.23
1851	+0.9	+0.9	+0.30	+0.30
1854	+0.3	+0.3	+0.10	+0.10
1858	+0.1	+0.3	+0.03	+0.10
1861	—1.4	—1.0	—0.47	—0.33
1864	—1.5	—1.0	—0.50	—0.33
1867	+1.1	+1.7	+0.37	+0.57
1869	+0.5	+1.0	+0.17	+0.33
1871	0.0	+0.7	0.00	+0.23

The sum of the squares of the residuals is in the first case 17″.94, and in the second 25″.41, so that the introduction of a and b makes a decided improvement in the representation of the observations.

I have not attempted a rigorous investigation of the probable error of any of these results for the reason that the values of the probable error deducible by the method of least squares would, in a case like the present, be entirely untrustworthy. It is, however, very desirable that we should be able to form some judgment of the uncertainty of the mass of Neptune. From the last system of equations of condition the value of μ' comes out with the weight 3.13, or nearly that assigned to the mean result of each five years of modern observations. Regarding these results as independent, their mean error would be about 0″.5, so that the probable error of μ' would be 0.5, and that of μ would be .005, or about $\frac{1}{200}$ the entire mass of Neptune. A probable error derived from the original equations would have been much smaller, and when, in the last equations, we allow for the systematic character of the residuals, it will be larger. If we suppose the theory to be perfect, I conceive we may fairly estimate the probable error of the mass of Neptune to be $\frac{1}{100}$ of its entire amount, and its possible error two or three times greater. If there is any error or imperfection in the theory, the error may be much larger.

CHAPTER VIII.

COMPLETION AND ARRANGEMENT OF THE THEORY TO FIT IT FOR PERMANENT USE.

In the preceding discussions the terms of the second order due to the action of Neptune have been neglected, the elements of Uranus and Neptune being so chosen that these terms can scarcely become sensible within a century of the epoch. But this very choice will make them larger in the course of centuries than if mean elements had been chosen. They will be most sensible in the case of the great inequality of 4300 years between Uranus and Neptune, an inequality which will make centuries of observation necessary to an accurate determination of the mean elements of the two orbits. The uncertainty arising from the great inequality is probably of the same order of magnitude with the omitted terms of the second order, and, such being the case, the theory would really be made but little more accurate by the addition of those terms. I conceive, however, that the theory will be made much more satisfactory by the computation of at least the largest of the terms in question, if only to arrive at a certain determination of their order of magnitude, and of their effect on the planet during the period in which it has been observed.

The term in question, being of very long period, may be most advantageously treated by the method of variation of elements, more especially as it has in the theory been already treated as such a perturbation. The largest of the perturbations in question are those of the mean longitude which are multiplied by the square of the integrating factor ν, which is nearly 51, but which also contain the eccentricities as factors, and those of the eccentricity and perihelion which are independent of the eccentricities, but are multiplied by only the first power of ν. These terms will probably comprise nearly or quite nine-tenths of those arising from the term of long period.

Let us begin with the perturbations of mean longitude. These are given by the integration of the equation

$$\frac{d^2l}{dt^2} = -\,3m'\alpha n^2\,\{ek_1 \sin(2l'-l-\pi) + e'k_2 \sin(2l'-l-\pi')\}$$

k_1 and k_2 being functions of the ratio of the mean distances, or α. If we integrate this equation, supposing all the quantities in the second member except l' and l to be constant, and these two to be of the form $nt+\varepsilon$, n and ε being constants, we shall reproduce the principal term of long period already found. But in the second approximation we must suppose all the elements variable. It is not, however, necessary to take into account the variations of a, n, and k, because these are of a lower order of magnitude. The perturbations to be added will be those of l, l', e, e', π, and π'.

The point from which the longitudes are counted being arbitrary, we shall take the position of the perihelion of Uranus for 1850.0 as the origin, and put, as before, g for the mean longitude of Uranus counted from this point, and let l' represent the mean longitude of Neptune counted from the same point. The terms of $\frac{d^2l}{dt^2}$ within the brackets will thus become

$$ek_1 \sin(2l'-g-\delta\pi) + e'k_2 \sin(2l'-g-(\pi'-\pi))$$

or, if we put

$$\begin{aligned} 2l'-g &= N \\ e' \sin(\pi'-\pi) &= h' \\ e' \cos(\pi'-\pi) &= k' \end{aligned}$$

and notice, that to terms of the first order we have, $\sin\delta\pi = \delta\pi$, $\cos\delta\pi = 1$, we shall have

$$\frac{d^2l}{dt^2} = -3m'an^2 \{(ek_1 + k'k_2) \sin N - (ek_1\delta\pi + h'k_2) \cos N\}$$

differentiating the quantities, of which the perturbations are to be considered with respect to the sign δ, we find for the terms of the second order.

$$\begin{aligned} \delta\frac{d^2l}{dt^2} = -3m'an^2 \{ &(k_1\delta e + k_2\delta k' + h'k_2\delta N) \sin N \\ &+((ek_1 + k'k_2)\,\delta N - ek_1\delta\pi - k_2\delta h') \cos N\}. \end{aligned}$$

We have now to substitute in this expression the numerical values of the quantities within parentheses. Those of the perturbations of Uranus have already been given in Chapter III, but it is necessary to diminish them by the factor 0.145* for the altered mass of Neptune. Those of Neptune are taken from my investigation of the orbit of that planet (p. 38). The mass of Uranus there adopted is $\frac{1}{21000}$, while the investigation of Dr. Von Asten,† from the observations of Struve and others, shows it to be $\frac{1}{22000}$. The perturbations are therefore diminished by $\frac{1}{22}$. In accordance with the system adopted throughout both investigations, constants are added to all the perturbations to make them vanish at the epoch 1850.0. A term is also added to make $\frac{dl}{dt}$ also vanish at the epoch; this corresponds to the constant which ought to be added to δa. The numerical values thus obtained, are:

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* This factor was adopted before the mass of Neptune to be employed had been finally decided upon. Hence the difference between it and that in the preceding chapter.

† Mémoires de l'Académie de St. Pétersbourg, tome xviii, vii série.

$$\begin{aligned}
\delta N &= +\ 7260'' \sin N - 6658'' + 4''.26t \\
e\delta\pi &= -\ \ 414 \sin N + 380 \\
\delta e &= -\ \ 414 \cos N - 165 \\
\delta h' &= +\ \ 120 \sin N - 110 \\
\delta k' &= +\ \ 120 \cos N + 48 \\
k_1 &= -\ 1.234 \\
k_2 &= +\ 0.452 \\
h' &= +\ 0.00695 \\
k' &= -\ 0.00486
\end{aligned}$$

Substituting these values in the expression for $\delta\frac{d^2l}{dt^2}$ and integrating twice, we find, putting b for the coefficient of the time in N, of which the value, taking the century as the unit, is $+0.1472$, and putting T for the time in centuries,

$$\delta l = \frac{3}{2}\, m'a\nu^2 \left\{ (411'' + \frac{102''.8}{b} + 2''.6T) \sin N + (1837'' + \frac{5''.3}{b} - 51.''4T) \cos N \right.$$
$$\left. - 109''.3 \sin 2N - 5''.5 \cos 2N \right\}$$
$$- 16''.5\, m'an^2t^2 + cT + c',$$

c and c' being the arbitrary constants of integration, which are to be chosen so that both δl and its first differential coefficient shall vanish at the epoch. Reducing to numbers, we find

$$\begin{aligned}
\delta l = & (140''.70 + 0''.32T) \sin N \\
& + (232.60 - 6.37T) \cos N \\
& - 13.60 \sin 2N \\
& - 0.70 \cos 2N \\
& - 0.03T^2 \\
& + 34.27\,T \\
& - 46.76,
\end{aligned}$$

the last two terms being arbitrary.

When we carry the perturbations of the eccentricity and perihelion to quantities of the second order, we are troubled by the introduction of large terms depending on the square of the disturbing force, which disappear from the rigorous expressions for the co-ordinates. These may be avoided by substituting for the eccentricity and perihelion the quantities h and k determined by the condition

$$h = e \sin \pi$$
$$k = e \cos \pi$$

If, as before, we count the longitudes from the perihelion of Uranus at the epoch 1850, we should substitute $\delta\pi$ for π in these expressions. The values of h and k will then be given by the integration of the equations

$$\frac{dh}{dt} = m'ank_1 \cos N$$
$$\frac{dk}{dt} = -m'ank_1 \sin N.$$

Differentiating with respect to δ, we find for the terms of the second order

$$\delta\frac{dh}{dt} = -m'ank_1 \sin N\delta N$$

$$\delta\frac{dk}{dt} = -m'ank_1 \cos N\delta N.$$

Substituting for δN its numerical value just given and integrating, we find

$$\begin{aligned}\delta h &= m'avk_1 \{-2895'' \sin N - (6658'' - 4''.26t) \cos N + 1815'' \sin 2N\} \\ &\quad - 3630''m'ak_1nt \quad + \text{constant}; \\ \delta k &= m'avk_1 \{-2895'' \cos N + (6658'' - 4''.26t) \sin N + 1815'' \cos 2N\} \\ &\quad + \text{constant};\end{aligned}$$

the constants being so chosen that δh and δk shall vanish at the epoch.

Reducing the values of δh and δk to numbers, they become

$$\begin{aligned}\delta h &= 5''.82 \sin N + (13''.40 - 0''.86T) \cos N - 3''.65 \sin 2N + 1''.08T - 2''.67, \\ \delta k &= 5\,.82 \cos N - (13\,.40 - 0\,.86T) \sin N - 3\,.65 \cos 2N \qquad + 12\,.12,\end{aligned}$$

the last two terms being arbitrary constants.

Computing the values of these terms of δl, δh, and δk, for intervals of 50 years, from 1600 to 2000, we find them to be as follows:

Year	δl	δh	δk
1600	—1″.34	+0″.10	—0″.02
1650	—0 .71	+0 .05	—0 .02
1700	—0 .31	+0 .02	—0 .01
1750	—0 .10	0 .00	—0 .01
1800	—0 .01	0 .00	0 .00
1850	0 .00	0 .00	0 .00
1900	0 .00	0 .00	0 .00
1950	+0 .04	0 .00	—0 .01
2000	+0 .18	—0 .01	—0 .02

We see that although the ultimate effect of these terms is very considerable, their effect, during the period that Uranus has been observed, is insignificant.

Concluded Elements and Perturbations of Uranus.

The corrections found in the last chapter being applied to the final provisional elements (p. 99) give the following elements for 1850, affected by the great inequality produced by Neptune:

Elements IV *of Uranus.*

Epoch, 1850, Jan. 0, Greenwich mean noon.

π,	168°	15′	6″.7
ε,	28	25	17 .05
θ,	73	14	8 .0
ϕ,	0	46	20 .54
e,	.0469236		

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e (in sec.),	$9678''.69$
n,	15425.752
$\log a$,	1.2829072
$\log a_1$,	1.2831044

Log a_1 includes, as before, the constant term in the perturbations of the logarithm of the radius vector which, with the corrected mass of Neptune, is $+.0001972$.

To find the corresponding elements at any other epoch, the following secular and long-period perturbations are to be applied. Those produced by Neptune are derived from the expressions in Chapter III by correcting them for the new mass of Neptune, and for the change in the value of the small divisor $2n' - n$ produced by the correction of the elements of Uranns. The logarithms of the factors for correction are,

Correction of mass of Neptune	9.93598
Correction of divisor	0.00051
Log. factor for δl	9.93496
Log. factor for δe, $\delta\pi$, $\delta\upsilon$	9.93547

Including the perturbations of the second order just found, we have, by putting

$$\begin{aligned} N &= 2l' - g, \\ &= 113^\circ\ 30'\ 46''.0 + 8^\circ\ 26'\ 51''.9\,T, \end{aligned}$$

$$\begin{aligned} \delta l = (-2850''.41 + 0''.32T) \sin N &+ (387''.67 - 6''.37T) \cos N \\ + 112.72 \sin 2N \quad & - \quad 47.28 \cos 2N \\ - \quad 7.72 \sin 3N \quad & + \quad 4.33 \cos 3N \\ + \quad 0.55 \sin 4N \quad & - \quad 0.46 \cos 4N \\ - \quad 0.03T^2 - 83''.78T &+ 2811''.41. \end{aligned}$$

$$\begin{aligned} \delta h = &- 412''.18 \sin N + (14''.03 - 0''.86T) \cos N \\ &+ 29.20 \sin 2N - 6.09 \cos 2N \\ &- \quad 3.11 \sin 3N + 1.19 \cos 3N \\ &+ \quad 0.28 \sin 4N - 0.13 \cos 4N \\ &+ 14.76T \qquad + 398.33 \end{aligned}$$

$$\begin{aligned} \delta k = &- 411''.53 \cos N - (13''.65 - 0''.86T) \sin N \\ &+ 29.33 \cos 2N + 6.17 \sin 2N \\ &- \quad 3.12 \cos 3N - 1.21 \sin 3N \\ &+ \quad 0.29 \cos 4N + 0.13 \sin 4N \\ &- \quad 5.453T \qquad - 124.72 \end{aligned}$$

$\delta\upsilon$ (in units of the 7th place of decimals).

$$\begin{aligned} &= 1963 \cos N + 103 \sin N \\ &- \quad 171 \cos 2N - 67 \sin 2N \\ &+ \quad 15 \cos 3N + 6 \sin 3N \\ &\qquad + 511.0. \end{aligned}$$

The perturbations of $e\delta\pi$ and δe are here replaced by those of h and k, defined by the equations

$$\begin{aligned} h &= e \sin (\pi - \pi_0) \\ k &= e \cos (\pi - \pi_0) \end{aligned}$$

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π_0 representing the perihelion of 1850 = 168° 15′ 6″.7. We then have for the eccentricity and longitude of perihelion at any epoch

$$e \sin (\pi - \pi_0) = \delta h$$
$$e \cos (\pi - \pi_0) = e_0 + \delta k.$$

In the above terms multiplied by the time we have included the secular variations produced by Jupiter and Saturn. If the perturbations of the elements due to each particular planet are required, we have

Action of Jupiter,

$$\delta h = + 5''.73 T; \ \delta k = - 0''.608 T.$$

Action of Saturn,

$$\delta h = + 5''.56 T; \ \delta k = - 4.589 T.$$

Subtracting these from the above expressions all the remaining terms will be due to the action of Neptune. The values of δl and δn are due entirely to the action of Neptune.

For the sake of rigor, we may suppose the perturbations produced by each planet to be multiplied by a factor representing the number by which the adopted mass of the planet must be multiplied to obtain the true mass.

It will add to the homogeneousness of the theory to express the perturbations of long period, which are multiplied by the product of the masses of Jupiter and Saturn, as perturbations of the elements. These terms, as found on page 88, are

$$\begin{aligned}
(v.c.0) &= - 0''.55 \sin N_6 - 0''.03 \cos N_6 \\
&\quad + 40 .65 \sin N_7 - 10 .50 \cos N_7 \\
(v.s.1) &= + 2 .64 \sin N_6 + 4 .64 \cos N_6 \\
&\quad + 7 .35 \sin N_7 + 4 .41 \cos N_7 \\
(v.c.1) &= - 4 .23 \sin N_6 - 3 .87 \cos N_6 \\
&\quad + 8 .06 \sin N_7 - 8 .38 \cos N_7
\end{aligned}$$

These terms, together with the arbitrary corrections of the elements which have been applied to make them very small at the epoch, may be replaced by the following corrections to the elements:

$$\begin{aligned}
\delta l &= - 0''.55 \sin N_6 - 0''.03 \cos N_6 \\
&\quad + 40 .65 \sin N_7 - 10 .50 \cos N_7 \\
&\quad + 27''.27 - 11''.72 T. \\
\delta h &= + 2 .09 \sin N_6 + 1 .94 \cos N_6 \\
&\quad - 2 .13 \sin N_7 + 3 .71 \cos N_7 \\
&\quad + 1''.28. \\
\delta k &= + 1 .32 \sin N_6 + 2 .32 \cos N_6 \\
&\quad + 3 .68 \sin N_7 + 2 .21 \cos N_7
\end{aligned}$$

$$\delta n = 27 \sin N_7 + 104 \cos N_7 + 76 \text{ (in units of the 7th decimal).}$$

The amount of the perturbations of the elements for every half century, from the year 1000 to 2200, is given in the following table. Column (1) gives the perturbations by Neptune, Saturn, and Jupiter, computed from the expressions on page 182; column (2) those just given depending on the product of the masses of Jupiter and Saturn.

Year.	δl		δh		δk		$\delta\varpi$		$\delta\varkappa$	$\delta\eta$
	(1) ″	(2)	(1) ″	(2)	(1) ″	(2)	(1)	(2)	″	
1000	+2050.31	+160.69	+ 42.66	+1.94	—389.51	+6.08	+1955	—140	—2.06	+1.20
1050	1841.17	149.08	27.79	3.60	375.65	6.68	1882	156	2.10	1.37
1100	1638.76	136.39	14.03	4.99	360.31	6.82	1802	169	2.12	1.54
1150	1444.87	122.87	+ 1.45	6.01	343.49	6.47	1717	178	2.08	1.66
1200	1260.24	108.85	— 9.90	6.60	325.27	5.64	1626	183	2.02	1.77
1250	+1085.76	+ 94.67	— 19.84	+6.75	—305.69	+4.42	+1529	+183	—1.94	+1.86
1300	921.76	80.67	28.34	6.49	284.90	2.86	1426	180	1.84	1.91
1350	769.32	67.12	35.27	5.90	252.90	+1.16	1318	173	1.70	1.92
1400	629.00	54.35	40.56	5.08	239.78	—0.57	1205	161	1.54	1.91
1450	501.39	42.62	44.11	4.16	215.68	2.17	1086	147	1.37	1.85
1500	+ 387.18	+ 32.11	— 45.83	+3.24	—190.66	—3.52	+ 963	+129	—1.19	+1.74
1550	286.65	22.78	45.63	2.42	164.84	4.52	835	110	0.99	1.60
1600	200.65	15.38	43.46	1.80	138.31	5.12	704	90	0.80	1.42
1650	129.43	9.34	39.20	1.41	111.22	5.30	568	68	0.61	1.20
1700	73.46	4.86	32.79	1.23	83.70	5.09	430	48	0.44	0.95
1750	+ 33.06	+ 1.90	— 24.16	+1.23	— 55.90	—4.56	+ 288	+ 29	—0.27	+0.66
1800	8.51	0.33	— 13.26	1.36	— 27.93	3.81	+ 145	+ 11	—0.12	+0.34
1850	0.00	0.00	0.00	1.52	0.00	2.96	0	— 5	0.00	0.00
1900	7.64	0.71	+ 15.65	1.63	+ 27.74	2.10	— 145	18	+0.10	—0.36
1950	31.47	2.23	33.70	1.59	55.09	1.34	290	26	0.16	0.73
2000	+ 71.43	+ 4.25	+ 54.20	+1.34	+ 81.81	—0.73	— 433	— 32	+0.20	—1.10
2050	127.33	6.52	77.12	0.85	107.77	0.34	574	30	0.20	1.47
2100	198.86	8.75	102.46	+0.13	132.67	0.14	711	27	0.17	1.83
2150	285.72	10.59	130.15	—0.77	156.33	0.11	843	20	0.11	2.17
2200	387.11	+ 11.77	+160.15	—1.76	+178.48	—0.19	968	— 9	+0.02	—2.48

Mean Elements of Uranus.

If, instead of the elements of Uranus affected by the great inequality, we wish the absolute mean elements, these are to be obtained by adding to the elements already given the constants applied to the perturbations δl, δh, δk, and $\delta\varpi$ to make the perturbations vanish at the epoch 1850.0, and also the corrections (p. 113) which we have subtracted from the elements and added to the perturbations to reduce the latter to a small quantity during the period for which the tables are likely to be used. We thus find the following mean elements:

Elements V of Uranus. Epoch, 1850, Jan. 0, Greenwich mean noon.

Longitude of the perihelion	170° 38′ 48″.7 $+ 8698''.\mu$
Mean longitude at epoch,	29 12 43 .73 $+ 2811.4\mu$
Longitude of the node,	73 14 37 .6 $+ 29.6\mu$
Inclination of the orbit,	0 46 20 .92 $+ 0.38\mu$
Eccentricity,	$.0463592 - 5236\mu$
Eccentricity in seconds,	$9562''.27 - 108''.0\mu$
Mean motion,	$15424.797 - 0''.838\mu$
Log mean distance (uncorrected),	$1.2829251 + 179\mu$
The same corrected,	$1.2831223 + 179\mu$
True mass of Neptune,	$\frac{1+\mu}{19700}$

Supposing the mass of Neptune to be uncertain by one-fiftieth of its entire amount, which is quite possible, it will be seen the longitude of the mean perihelion is from this cause uncertain by more than two minutes, the mean longitude of Uranus itself by nearly a minute, and the mean motion by nearly two seconds in a century.

It will be seen that the logarithm of the mean distance just given does not accurately correspond to that of elements IV plus the constant term of $\delta\varpi \times 0.4343$, as it should. This difference arises from the rejection of the terms of the second order in $\delta\varpi$, which can not affect the geocentric longitude of the planet by a tenth of a second for a number of centuries.

It is to be remarked that these mean elements are those to be used in the general theory of the secular variation of the planetary orbits.

Concluded Theory of Uranus.

The elliptic longitude and radius vector of Uranus, affected by the secular and long period perturbations of the elements, will be given by the following equations. Put

$$\begin{aligned} l_0 &= n_0 t + \varepsilon_0, \\ l &= l_0 + \delta l, \\ g &= l - \pi_0, \\ h &= \delta h, \\ k &= e_0 + \delta k, \\ e^2 &= h^2 + k^2, \end{aligned}$$

the zeros indicating elements IV, and δh, δk, and δl being the perturbations of these three elements just given. Then

Elliptic longitude in orbit $= l$

$$+ \left\{ 2 - \frac{1}{4} e^2 + \frac{5}{96} e^4 \right\} \left\{ k \sin g - h \cos g \right\}$$

$$+ \left\{ \frac{5}{4} - \frac{11}{24} e^2 \right\} \left\{ (k^2 - h^2) \sin 2g - 2hk \cos 2g \right\}$$

$$+ \left\{ \frac{13}{12} - \frac{43}{64} e^2 \right\} \left\{ (k^3 - 3k^2k) \sin 3g - (3hk^2 - h^3) \cos 3g \right\}$$

$$+ \quad \frac{103}{96} \left\{ (k^4 - 6k^2h^2 + h^4) \sin 4g - (4k^3h - 4kh^3) \cos 4g \right\}$$

$$+ \quad \frac{1097}{960} \left\{ (k^5 - 10k^3h^2 + 5kh^4) \sin 5g - (5k^4h - 10k^2h^3 + k^5 \cos 5g \right\}$$

Neperian logarithm of $r = \varpi + \frac{1}{4}e^2 + \frac{1}{32}e^4$

$$- \left\{ 1 - \frac{3}{8} e^2 \right\} \left\{ k \cos g + h \sin g \right\}$$

$$- \left\{ \frac{3}{4} - \frac{11}{24} e^2 \right\} \left\{ (k^2 - h^2) \cos 2g + 2hk \sin 2g \right\}$$

$$- \frac{17}{24} \left\{ (k^3 - 3kh^2) \cos 3g + (3k^2h - h^3) \sin 3g \right\}$$

$$- \frac{71}{96} \left\{ (k^4 - 6k^2h^2 + h^4) \cos 4g + (4k^3h - 4kh^3) \sin 4g \right\}$$

In computing these expressions it will be sufficient for several centuries before or after 1850 to develop h, δk, and δl to their first dimensions: it will, however, be more convenient to correct the mean anomaly g for the perturbation δl before obtaining the equation of the centre. Developing the perturbations of h and k to terms of the first order, we have for the effects of the perturbations of those elements:

$$(v.s.1) = \left(2 - \frac{3}{4}e_0^2\right) \delta k$$

$$(v.c.1) = -\left(2 - \frac{1}{4}e_0^2\right) \delta h$$

$$(v.s.2) = \left(\frac{5}{2}e_0 - \frac{11}{6}e_0^3\right) \delta k$$

$$(v.c.2) = -\left(\frac{5}{2}e_0 - \frac{11}{24}e_0^3\right) \delta h$$

$$(v.s.3) = \frac{13}{4}e_0^2\, \delta k$$

$$(v.c.3) = -\frac{13}{4}e_0^2\, \delta h$$

$$(v.s.4) = \frac{103}{24}e_0^3\, \delta k$$

$$(v.c.4) = -\frac{103}{24}e_0^3\, \delta h$$

$$(\rho.c.0) = \delta\varpi + \frac{1}{2}e_0\, \delta k$$

$$(\rho.s.1) = -\left(1 - \frac{3}{8}e_0^2\right) \delta h$$

$$(\rho.c.1) = -\left(1 - \frac{9}{8}e_0^2\right) \delta k$$

$$(\rho.s.2) = -\frac{3}{2}e_0\, \delta h$$

$$(\rho.c.2) = -\frac{3}{2}e_0\, \delta k$$

$$(\rho.s.3) = -\frac{17}{8}e_0^2\, \delta h$$

$$(\rho.c.3) = -\frac{17}{8}e_0^2\, \delta k$$

These coefficients for ρ must, of course, be multiplied by the modulus 0.434294 to reduce the perturbations to those of the common logarithm of the radius vector.

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Among the elliptic terms may be included the effect of the following minute constants introduced by the perturbations.

$(v.s.2) = -0''.144$
$(v.c.2) = +0\ .130$
$0.4343\,(\rho.c.0) = +1972$ in units of the 7th place
$0.4343\,(\rho.s.1) = +63$ of decimals.
$0.4343\,(\rho.c.1) = +73$
$0.4343\,(\rho.s.2) = +5$
$0.4343\,(\rho.c.2) = +4$

This term $(\rho.c.0)$ is that added as a correction to the logarithm of the mean distance.

To the coefficients $(v.s.1)$, $(v.c.1)$, etc., are still to be added the following periodic terms:—

1. The periodic terms due to the action of Jupiter, given in Chapter V, omitting the terms multiplied by T, which are included in the perturbations of the elements.
2. The periodic terms produced by Saturn, including those terms multiplied both by T and by $\sin A_2$ or $\cos A_2$, but omitting those multiplied by T only for the same reason as in the case of Jupiter.
3. The periodic terms produced by Neptune, multiplied by the factor 0.86294 on account of the correction to the mass of that planet, and omitting the terms multiplied by δl, δe, and $e\delta g$.
4. The periodic terms multiplied by the product of the masses of Jupiter and Saturn, given on page 88, omitting the terms multiplied by the sine and cosine of N_6 and N_7, because they are replaced by the terms of δl, δh, and δk, given on page 183, and tabulated in the columns headed (2) on page 184. The result will be the same whether we employ the terms of $(v.c.0)$, $(v.s.1)$, etc., given at the bottom of page 88 and the top of page 89, omitting the numbers in the columns 2 on page 184 from the expressions on page 186, or whether we include the latter and omit the former.

The true anomaly of Uranus will then be:

$$g_0 + \delta l + (\text{equation of centre from elements IV, using for mean anomaly } g_0 + \delta l)$$
$$+ \Sigma\,(v.s.i) \sin ig + \Sigma\,(v.c.i) \cos ig.$$

The logarithm of the radius vector will be:

$$\log r \text{ in elliptic orbit from elements IV.}$$
$$+ \Sigma\,(\rho.s.i) \sin ig + \Sigma\,(\rho.c.i) \cos ig$$

care being taken to multiply the coefficients by the modulus where that has not already been done. All the terms in Chapter V are so multiplied.

To pass from the true anomaly to the true longitude we must investigate the secular motion of the planes of the orbit and of the ecliptic. The effect of this motion on ϕ, θ, and τ will be found by successive approximations from the formulæ

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(34), correcting the data for the new mass of Neptune. We shall also use the same motion of the ecliptic adopted on p. 95. We have thus:

$$\frac{dp}{dt} = -4''.53$$

$$\frac{dp'}{dt} = +5.43 + 0''.38T.$$

$$\frac{dq}{dt} = -5.17$$

$$\frac{dq'}{dt} = -46.78 + 0.12T.$$

As a first approximation we have

$$\begin{aligned} \theta = \tau &= 73^\circ\ 14'\ \ 8'' \quad -3169''.2T \\ \phi \quad &= \ \ 0\ \ 46\ \ 20.54 + \quad 2.48T \end{aligned}$$

Substituting these values in (34) and integrating we find

$$\begin{aligned} \phi &= \phi_0 + \quad 2''.47T + 0''.13T^2 \\ \theta &= \theta_0 - 3168.42T + 3.00T^2 \\ \tau &= \tau_0 - 3168.76T + 3.00T^2 \end{aligned}$$

For tabulating we shall use, instead of θ and τ, the distance of the perihelion from the ascending node, or $\pi - \tau$, and the value of θ corrected for Struve's precession. Since the mean motion has been derived without making any distinction between τ and θ, it will be necessary to correct the motion of mean anomaly by the difference of those quantities. We thus obtain for the values of the three principal arguments:—

$$\begin{aligned} g &= 220^\circ\ 10'\ 10''.35 + 1542574''.86T + \delta l \\ \omega &= \ \ 95\ \ \ \ 0\ \ 58.70 + \quad 3168.76T - 3.00T^2 \\ \theta &= \ \ 73\ \ \ 14\ \ \ 8.00 + \quad 1856.82T + 4.12T^2 \end{aligned}$$

If we represent all the inequalities of the true longitude by Δl, so that we shall have for the true anomaly

$$f = g + \Delta l,$$

the argument of latitude will be

$$u = f + \omega.$$

The reduction to the ecliptic will then be

$$R = -(9''.37 + 0''.016T) \sin 2u,$$

the true longitude on the ecliptic referred to the mean equinox of date,

$$\lambda = u + \theta + R,$$

and the sine of the elliptic latitude,

$$\sin \beta_0 = \sin \phi_0 \sin u.$$

The perturbations of the latitude will be

$$(b.c.0) + (b.c.1) \cos g + (b.s.1) \sin g + \text{etc.}$$

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The periodic terms of $(b.c.0)$, $(b.s.1)$, $(b.c.1)$, etc., are given in Chapter V, on pages 86 and 87, and are to be taken without any farther modification than the multiplication of those due to the action of Neptune by the factor 0.863. The constant, secular, and long period terms are

$$\begin{aligned}
b.c.0 &= +0''.26 - 0''.12T - 0.011\delta\eta + 0.046\delta\varkappa \\
(b.s.1) &= -0\ .22T - 0\ .05T^2 + 0.975\delta\eta + 0.221\delta\varkappa \\
(b.c.1) &= +2\ .47T + 0\ .12T^2 + 0.221\delta\eta - 0.975\delta\varkappa \\
(b.s.2) &= -0\ .06 - 0\ .01T + 0.046\delta\eta + 0.011\delta\varkappa \\
(b.c.2) &= -0\ .01 + 0\ .12T + 0.011\delta\eta - 0.046\delta\varkappa
\end{aligned}$$

The values of $\delta\eta$ and $\delta\varkappa$ to be used in these expressions are those the expressions for which are given on page 97, and which are tabulated in the last two columns of the table on page 184.

The following tables are based on the elements and theory laid down in this chapter.

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CHAPTER IX.

GENERAL TABLES OF URANUS.

Enumeration of the Quantities contained in the several Tables.

THE first six tables are designed to give the values of the three arguments of the elliptic motion, g, ω, and θ, and of the nine arguments of the tables of perturbations. The argument ω is, however, diminished by $3'$, the sums of the constants added to the perturbations of $(v.c.0)$ to make these quantities positive, and θ by $10''$, the constant added to the reduction to ecliptic. The expressions for the arguments of perturbations are as follows, the mean longitude of each planet, counted from the perihelion of Uranus, being represented by the initial letter of the planet. All these arguments are expressed in units, of which 600 make an entire circumference, so that each unit is $36'$. The time t is counted in Julian years from the fundamental epoch,

1850, January 0, Greenwich mean noon.

$$\begin{array}{rll}
\text{Arg.}\,1 = & J - U & = 219.190 + 43.44028t \\
2 = & S - U & = 577.349 + 13.22717t \\
3 = & U - N & = 88.884 + 3.50035t \\
4 = & J - 2S & = 497.6 + 9.8445t \\
5 = & 3S - U - J & = 79.8 + 3.3825t \\
6 = & 4S - 2U - J & = 57.1 + 16.610t \\
7 = & 2J - 3S - 3U & = 238.7 + 18.633t \\
8 = & 2J - 4S - 2U & = 261.3 + 5.4058t \\
9 = & 7S - 2J - 3U & = 136.9 + 19.992t
\end{array}$$

Table I gives the corrections which must be applied to the values of the arguments at any time during the nineteenth century to reduce them to the corresponding time in any preceding or following century between the Christian era and the year 2300. Since ω and θ each contains a term proportional to the square of the time, the correction for these quantities is not constant during each century, but is of the form

$$\omega + \omega' T$$

ω and ω' being constant during each century, and T being the fraction of the century counted from its beginning.

Table II gives the value of g, $\omega - 3'$, $\theta - 10''$, and the above nine arguments for Greenwich mean noon of Jan. 0 of each leap year from 1752 to 1948, and for January —1 of the years 1800 and 1900, corresponding to December 30 of the years 1799 and 1899. The corrections for the perturbations of long period are not

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applied in this table. The numbers at the bottom of this table, in the line $\Delta^{(1)}_{120}$, show the variation of the corresponding quantity in 120 days, for the epoch 1850.0. In the line "Factor T" is given the change of this variation in a century, while $\Delta^{(2)}_{120}$ is the second difference for intervals of 120 days. By means of these numbers, when the arguments are computed for any date, their values for other dates at intervals of 120 days may be found by successive addition.

Table III gives the motion of the several arguments between the epochs of the preceding table and the zero day of each month in the course of a four-year cycle. The variable motions, ω and θ, correspond to the epoch 1850, and rigorously they each require a correction for any other four-year cycle than that between 1848 and 1852. But, owing to the small inclination of the orbit of Uranus it is not necessary that either ω or θ should be exact, if only their sum is exact. The column θ' of this table, therefore, gives the correction which must be applied to the motion of θ at the end of a century (1950) in order that, being applied to θ alone, $\omega + \theta$ may be exact. This correction is, in fact, that for the secular variation of the precession.

Tables IV and V give the motion of the arguments for days and hours. The motion for hours is, however, not necessary in the case of any argument but g, as all the others can be readily enough interpolated to fractions of a day.

Table VI gives the corrections to the arguments on account of the terms of long period from 1000 to 2200. The terms in question are, in the case of Jupiter, the great inequality produced by the action of Saturn, in the case of Neptune the great inequality produced by Uranus, and, in the case of Uranus, the inequalities in the mean longitude tabulated in the preceding chapter. The numerical expressions are

$$\delta J = 0.535 \sin (116^\circ\ 21' + 40^\circ\ 45'\ 20''\ T)$$
$$\delta U = \delta l$$
$$\delta N = -0.75\ \delta l.$$

The corrections to the several arguments are

$$\begin{aligned} \delta g &= \delta l \\ \delta \text{ arg. } 1 &= \delta J - \delta l \\ \delta \text{ arg. } 2 &= -\delta l \\ \delta \text{ arg. } 3 &= \delta l - \delta N = 1.75\ \delta l \end{aligned}$$

No correction to the mean longitude of Saturn is applied, all its inequalities being taken account of in the terms of the second order.

The corrections, expressed in seconds, have been reduced to units of the argument by dividing them by 2160″.

Outside the limits of the table these corrections must be computed from their formulæ.

Table VII gives the equation of the centre, and the elliptic part of the logarithm of the radius vector. No constant is applied to the former, but the latter is diminished by .0003400, the sum of the constants added to ($\rho.c.0$) in Tables VIII, IX, X and XVII.

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The formulæ for the Tables are

$$\begin{aligned} \text{Equation of centre} = &\ \ 19352''.06 \sin \\ + &\ \ 567.24 \sin \\ + &\ \ 23.05 \sin 3g \\ + &\ \ 1.07 \sin 4g \\ + &\ \ 0.05 \sin 5g \end{aligned}$$

$$\begin{aligned} \text{Elliptic log. } r = &\ \ 1.2833435 \\ - &\ \ .0003400 \\ - &\ \ .0203618 \cos g \\ - &\ \ .0007165 \cos 2g \\ - &\ \ .0000318 \cos 3g \\ - &\ \ .0000016 \cos 4g. \end{aligned}$$

Table VIII gives the coefficients ($v.c.0$), ($v.c.1$), etc., for the perturbations of the longitude and logarithm of radius vector produced by the action of Jupiter. They are computed from the periodic terms of the formulæ on page 83, with the addition of the following constants to make all the numbers of the table positive:

Constant of ($v.c.0$) = 55″.
($v.s.1$) = 6.
($v.c.1$) = 4.
($v.s$ 2) = 0.20
($v.c.2$) = 0.20
($\rho.c.0$) = 1200
($\rho.s.1$) = 150
($\rho.c.1$) = 100
($\rho.s.2$) = 10
($\rho.c.2$) = 10

Table IX gives the periodic part of the coefficients due to the action of Saturn, taken without change from the expressions on page 84; together with the secular variations, the latter including only the terms of ($v.s.1$), ($v.c.1$), ($v.s.2$), and ($v.c.2$), which are multiplied by T and by $\sin A_2$ or $\cos A_2$. The coefficients of T are given in the columns Sec. Var. and each number is increased by the constant 1″.50 to make it positive. The term $-0''.06T \sin A_2$ in ($v.c.0$) is omitted entirely, as it will not amount to a tenth of a second until after the year 2000. The constant terms added to the quantities of these tables to make all the numbers positive, are:

Constant of ($v.c.0$) = 30.″
($v.s.1$) = 150. + 1.50T″
($v.c.1$) = 150. + 1.50T
($v.s.2$) = 130. + 1.50T
($v.c.2$) = 130. + 1.50T
($v.s.3$) = 8.
($v.c.3$) = 6.
($v.s.4$) = 1.
($v.c.4$) = 1.

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Constant of $(\rho.c.0) = 800$
$(\rho.s.1) = 1500$
$(\rho.c.1) = 1500$
$(\rho.s.2) = 400$
$(\rho.c.2) = 400$
$(\rho.s.3) = 100$
$(\rho.c.3) = 100$

Table X gives the coefficients produced by the action of Neptune, computed from the periodic terms on pages 85 and 86 without any other change than the multiplication of all the numbers by the factor 0.863 to reduce them to the new mass of Neptune. The constants added to the several quantities, are

$''$
Constant of $(v.c.0) = 92.85$
$(v.s.1) = 20.00$
$(v.c.1) = 31.00$
$(v.s.2) = 5.00$
$(v.c.2) = 5.00$
$(v.s.3) = 1.00$
$(v.c.3) = 1.00$
$(v.s.4) = -1.00$
$(v.c.4) = -1.00$
Constant of $(\rho.c.0) = 400$
$(\rho.s.1) = 200$
$(\rho.c.1) = 200$
$(\rho.s.2) = 40$
$(\rho.c.2) = 40$

Tables XI to XVI give the terms of the second order and of short period which contain the products of the masses of Jupiter and Saturn, which, with the constants added to the numbers of the several tables, are as follows:

$(v.c.0) = +0''.08 \sin A_5 + 0''.51 \cos A_5$; Table XII; const $= 0''.60$
$+0\,.04 \sin A_6 + 0\,.01 \cos A_6$; XIII; $= 0\,.05$
$-0\,.01 \sin A_7 + 0\,.05 \cos A_7$; XIV; $= 0\,.05$
$-0\,.35 \sin A_8 - 1\,.30 \cos A_8$; XV; $= 1\,.35$
$-0\,.05 \sin A_9 + 0\,.03 \cos A_9$; XVI; $= 0\,.10$

Sum of constants added to these tables 2 .15

$(v.s.1) = +0''.26 \sin A_4 + 0''.27 \cos A_4$; Table XI; const $= 0''.40$
$-0\,.04 \sin A_5 - 0\,.17 \cos A_5$; XII; $= 0\,.20$
$+0\,.08 \sin A_6 + 0\,.03 \cos A_6$; XIII; $= 0\,.10$
$-0\,.02 \sin A_7 + 0\,.08 \cos A_7$; XIV; $= 0\,.10$
$+0\,.30 \sin A_8 - 0\,.58 \cos A_8$; XV; 0 .75
$-0\,.04 \sin A_9$; XVI; 0 .10

$(v.c.1) = +0''.06 \sin A_4 - 0''.27 \cos A_4$;	Table XI;	const $= 0''.40$
$+0.18 \sin A_5 + 0.01 \cos A_5$;	XII;	0 .20
$-0.03 \sin A_6 + 0.08 \cos A_6$;	XIII;	0 .10
$-0.02 \sin A_7 + 0.09 \cos A_7$;	XIV;	0 .10
$-0.44 \sin A_8 - 0.61 \cos A_8$;	XV;	0 .75
$-0.10 \sin A_9 + 0.03 \cos A_9$;	XVI;	0 .10
Sum of constants added to $(v.s.1)$ and $(v.c.1)$ in these tables		1 .65

The term of $(\rho.c.0)$

$$11 \sin A_8 - 3 \cos A_8$$

is omitted from the tables entirely.

Tables XVII*a* and XVII*b* give the constant, secular, and long-period terms of $(v.s.1)$ $(v.c.1)$, computed from the formulæ p. 186, with the following additions:

1. The constant terms introduced by the perturbations, given on p. 187.
2. The negatives of the constants added to the tables VII to XVI inclusive to make the numbers of those tables positive. The values of these terms are

	Pert. Const.	Tables VIII to XVI.	(1)—(2)
$(v.s.1)$	0	$177''.65 + 1''.50T$	$-177''.65 - 1''.50T$
$(v.c.1)$	0	$186.65 + 1.50T$	$-186.65 - 1.50T$
$(v.s.2)$	$-0''.14$	$135.20 + 1.50T$	$-135.34 - 1.50T$
$(v.c.2)$	$+0.13$	$135.20 + 1.50T$	$-135.07 - 1.50T$
$(v.s.3)$	0	9 .00	$-$ 9 .00
$(v.c.3)$	0	7 .00	$-$ 7 .00
$(\rho.c.0)$	[+1972]	—1000	+1000
$(\rho.s.1)$	+ 63	1850	—1787
$(\rho.c.1)$	+ 73	1800	—1727
$(\rho.s.2)$	+ 5	450	— 445
$(\rho.c.2)$	+ 4	450	— 446

The perturbation constant of $(\rho.c.0)$, being added to log *a* in forming the elliptic radius vector, is not included in this table.

Table XVIII gives the reduction to the ecliptic

$$-9''.37 \sin 2u.$$

The constant 10″ is added to make the numbers always positive, which constant has been already subtracted from θ.

Table XIX gives the principal term of the latitude

$$46'\ 20''.54 \times \sin u.$$

Table XX gives the coefficients $(b.s.1)$ and $(b.c.1)$ for the perturbations of the latitude produced by Jupiter. They are given by the formulæ

$$(b.s.1) = 0''.65 \cos (J - U + 40^\circ)$$
$$(b.c.1) = 0.65 \sin (J - U + 40^\circ)$$

The constant 0″.70 is added to make all the numbers of the table positive.

Table XXI gives the corresponding coefficients for the action of Saturn, computed from the expressions on p. 87 with the addition of the following constants.

$$\begin{aligned}\text{Const. of } (b.c.0) &= 0''.10\\ (b.s.1) &= 3\ .30\\ (b.c.1) &= 3\ .10\\ (b.s.2) &= 0\ .20\\ (b.c.2) &= 0\ .20\end{aligned}$$

Table XXII gives the coefficients for the action of Neptune from the formulæ on p. 87, all the numbers being multiplied by the factor 0.863 to reduce them to the adopted mass of Neptune. The following constants are added:

$$\begin{array}{ll}\text{To } (b.c.0)\ \ldots\ldots & 0''.06\\ \quad (b.s.1) & 1\ .00\\ \quad (b.c.1) & 1\ .20\\ \quad (b.s.2) & 0\ .20\\ \quad (b.c.2) & 0\ .20\end{array}$$

Table XXIII gives the secular and long-period terms for various epochs computed from the formulæ of p. 189. The sums of the several constants added in the three preceding tables are here subtracted again so that these expressions become

$$\begin{aligned}b.c.0 - \Sigma c &= 0''.10 - 0''.12T - .011\delta\eta + .046\delta\varkappa\\ (b.s.1) - \Sigma c &= -5\ .00 - 0\ .22T - 0''.05T^2 + .975\delta\eta + .221\delta\varkappa\\ (b.c.1) - \Sigma c &= -5\ .00 + 2\ .47T + 0\ .12T^2 + .221\delta\eta - .975\delta\varkappa\\ (b.s.2) - \Sigma c &= -0\ .46 - 0\ .01T + .046\delta\eta + .011\delta\varkappa\\ (b.c.2) - \Sigma c &= -0\ .41 + 0\ .12T + .011\delta\eta - .046\delta\varkappa\end{aligned}$$

Precepts for the use of the Tables.

Express the date for which the position of Uranus is required in years, months, days, and hours of Greenwich mean time, according to the Julian Calendar if the date is earlier than 1500, according to the Gregorian Calendar if it is later than 1600, and according to either calendar between these epochs.

Enter Table I with the beginning of the century, and take out the values of g, ω, ω', θ, θ', and arguments 1 to 9. Multiply ω' and θ' by the fraction of a century corresponding to the date, and write the products with their proper algebraic signs under ω and θ. If the calendar is the Julian, the century marked J must be taken, and if the Gregorian, that marked G. Between the dates 1752 and 1951 it is not necessary to enter Table I at all.

If Table I was not entered, enter Table II with the year, or the first preceding year found therein. If Table I was entered, enter Table II between the year 1800 and 1896 as if the number of the century were changed to 18. Take out the values of g, ω, θ, and the arguments, and write them under the corresponding quantities from Table I.

Enter Table III with the excess of the actual year over that with which Table II was entered, and with the month. Write the corresponding values of g, ω, θ, and the arguments under the previous values. Multiply θ' by the fraction of a century after 1850, corresponding to the date with which Table II was entered, and write the product under θ, or add it to it in writing θ. If Table II was entered with a date before 1850, this product is negative.

Enter Table IV with the day of the month and write down the corresponding values of g, ω, etc., under the former values.

If the date does not correspond to Greenwich mean noon, the motion of g for the hours must be computed from Table V, and the other quantities must be interpolated to the fraction of a day in entering Table IV.

Enter Table VI with the year, find by interpolation the values of g, and arguments 1, 2, and 3, corresponding to the date, and write them under the former values.

Add up all the partial values of g, ω, θ, and the arguments, attending to the algebraic signs of the products. Subtract from the arguments as many times 600 as possible, and the results will be the final values of those quantities.

Enter Table VII with g as the argument, the seconds being first reduced to fractions of a minute, and interpolate the quantities E and $\log r$. When g exceeds 180° the former quantity is to receive the negative sign; the latter is always positive.

Enter Tables VIII to XVI inclusive with their respective arguments, and take out the values of the quantities $(v.c.0)$, $(v.s.1)$, $(v.c.1)$, etc., $(\rho.c.0)$, $(\rho.s.1)$, etc., so far as they are found in the tables, writing the quantities having the same designation under each other. In Table IX the quantities Sec. Var. must be multiplied by the centuries and fraction of a century of the actual date after 1850, and the product must be included with the corresponding quantities, $(v.s.1)$, $(v.c.1)$, etc. Before 1850 this product will always be negative; afterward always positive. All the quantities taken from these tables are positive except $(v.s.4)$ and $(v.c.4)$ in Table IX, which are negative.

Add up all the partial values of $(v.c.0)$, $(v.s.1)$, etc., thus obtained from Tables VIII to XVI, and from their sum take the corresponding quantities obtained from Table XVII by interpolating to the date. The required quantities are all given in Table XVII*b*; Table XVII*a* being only an expansion of a part of XVII*b* for the present century. The final values of $(v.s.1)$, $(v.c.1)$, $(v.s.2)$, etc., $(\rho.s.1)$, $(\rho.c.1)$, etc., thus obtained are to be multiplied by the sines and cosines of the corresponding multiples of g, in doing which four place logarithms are sufficient if the computation is carefully made. The products are then all added together, and to g, ω, E, and $(v.c.0)$; in the case of v, and to $\log. r$, $(\rho.c.0)$ in the case of ρ. That is, we are to form the expressions:

$$\begin{aligned} u = g + \omega + E + (v.c.0) &+ (v.s.1)\sin g + (v.c.1)\cos g \\ &+ (v.s.2)\sin 2g + (v.c.2)\cos 2g \\ &+ \quad \text{etc.} \quad + \quad \text{etc.} \end{aligned}$$

$$\begin{aligned} \log r = \; & \log r\ (\text{from Table VII}) + (\rho.c.0) \\ & + (\rho.s.1)\sin g) \quad + (\rho.c.1)\cos g \\ & + (\rho.s.2)\sin 2g) \quad + (\rho.c.2)\cos 2g \\ & + (\rho.s.3)\sin 3g) \quad + (\rho.c.3)\cos 3g. \end{aligned}$$

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u will then be the true argument of latitude, and log r the logarithm of the radius vector with seven places of decimals.

Under u write θ; enter Table XVIII with the argument u and take out the reduction to the ecliptic. Add it to u and θ, and the sum of the three quantities will be the heliocentric longitude of Uranus referred to the mean equinox and ecliptic of the date. Applying nutation the longitude will be reduced to the true equinox.

Enter Table XIX with u as the argument, or, when u exceeds 180°, with $u - 180°$, and take out the principal term of the latitude, which will be positive when u is less than 180°, and negative when it is greater.

Enter Tables XX, XXI, XXII, and XXIII with their respective arguments, the argument for the last being the date, and add up the various quantities having the same designation, noticing that in the first three tables all the quantities are positive, while in the last they are all negative except ($b.c.0$). Then form the expression,

$$(b.c.0) + (b.s.1) \sin g + (b.c.1) \cos g + (b.s.2) \sin 2g + (b.c.2) \cos 2g,$$

and add it to the principal term of the latitude, with regard to the algebraic signs. The sum will be the heliocentric latitude of Uranus above the ecliptic of the date.

When an ephemeris of Uranus is to be computed for a series of years, some modifications may be introduced, which will save the computer labor. In the first place an equidistant series of dates being selected for computation, it will be sufficient to compute g, ω, θ, and the arguments for every sixth, eighth, or tenth date, and to fill in the arguments for the intermediate dates by adding the nearly constant differences corresponding to the adopted intervals. The agreement of the numbers thus obtained for the last date with those found by the original computation will prove the whole process. This interval may be as great as 120 days without detracting from the accuracy with which the places for the immediate dates can be interpolated, and the differences for this interval may be deduced from the numbers at the bottom of Table II. If these numbers are used without change the values of ω and θ for the last date may not always come out right. But these errors, if less than a second, will be of no importance if the one quantity comes out as much too great as the other is too small, and they may be avoided entirely by making a small change in the constant difference to be added.

Tables XI to XVI, inclusive, need be entered only for every third or fourth date, and the sums of the quantities can be then interpolated to every date, and added up with the corresponding quantities from the other tables.

Again, it will be found convenient to compute the sum of the small terms $(v.s.3) \sin 3g + (v.c.3) \cos 3g + (v.s.4) \sin 4g + (v.c.4) \cos 4g$, as well as the corresponding terms of the radius vector, and all the terms of the latitude, not for the dates adopted, but for every fourth entire degree of g. Having a series of values computed in this way, the sum can be interpolated to the value of g corresponding to the date. To facilitate the formation of the smaller products for entire degrees of g, a table of products of numbers by the sine and cosine of every degree is appended to these tables, by which the products in question can be formed at sight whenever the coefficient to be multiplied is less than 32″. The values of these coefficients, (*v.s.*3), (*v.c.*3), etc., corresponding to the entire degrees of g, may be either formed by interpolation at sight from those corresponding to the dates of computation, or the values of the arguments 2 and 3 corresponding to the required degrees of g may be computed, and the values of (*v.s.*3), etc., corresponding to these values of the arguments may be taken from Tables IX and X, while Table XVII must be entered with the corresponding dates.

If the heliocentric ephemeris is computed for ten years at a time, the last of these modifications in the mode of computation will greatly facilitate the computation of the smaller terms. We first find the date, and the values of arguments 1, 2, and 3, to one place of decimals, for some entire degree of g preceding that which corresponds to the first date, and then find the dates and the values of the arguments corresponding to successive values of g, differing by 2° or 4°, until we pass the last date of computation. We then take out the values of (*v.s.*3), (*v.c.*3), (*v.s.*4), (*v.c.*4), (*ρ.s.*3), (*ρ.c.*3), (*b.c.*0), (*b.s.*1), (*b.c.*1), (*b.s.*2), and (*b.c.*2), with these values of the dates and arguments, form their products by the sines and cosines of the corresponding multiples of g by means of the supplementary tables, and add the proper products together so as to form three small tables with g as the argument. These terms are then interpolated to the values of g corresponding to the original dates of computation.

As a first example of the use of the Tables we will compute the heliocentric co-ordinates of Uranus for Greenwich mean noon of the date 1753, Dec. 3. In computing the arguments we shall make use of Table I, though it is not necessary to do so. The computation of the arguments is as follows:

	g			ω			θ			Arg. 1
	°	′	″	°	′	″	°	′	″	
Table I, 1700	291	31	7.	359	7	5.33	359	29	11.47	456.092
Product by 0.5392	....					+3.24			—4.45	
Table II, 1852	228	44	0.73	94	59	2.03	73	14	35.11	306.010
III, Y. 1, Dec.	8	12	43.38	0	1	0.73	0	0	35.59	83.252
IV, 3 days	0	2	6.70			0.26			0.15	0.357
VI, 1753.92	0	0	32.30			....			....	0.507
1753, Dec. 3	168	30	30.48	94	7	11.59	72	44	17.87	246.218

	Arg. 2	3	4	5	6	7	8	9
Table I, 1700	477.319	249.975	216	262	139	537	59.4	401
II, 1852	3.785	95.880	517	87	90	276	272.1	177
III, Y. 1, Dec.	25.349	6.709	19	6	32	36	10.3	38
IV, 3 days	0.109	.029	0	0	0	0	0.0	0
VI, 1753.9	—0.014	+.024						
1753, Dec. 3	506.548	352.617	152	355	261	249	341.8	16

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	(v.c.0)	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″
Table VIII	83.58	9.97	6.61	0.37	0.31				
IX	21.32	22.24	223.49	60.92	238.89	1.42	11.07	0.32	1.64
X	71.08	19.02	18.30	6.64	5.38	0.70	1.19	—0.95	—1.04
XI	..	.13	.47						
XII	.13	.36	.10						
XIII	.06	.11	.01						
XIV	.01	.04	.03						
XV	2.38	1.40	1.12						
XVI	.12	.09	.11						
Σ	178.68	53.36	250.24	67.93	244.58	2.12	12.26		
Table XVII		—292.55	—140.91	—140.73	—131.03	—9.41	—6.85		
		—239.19	+109.35	— 72.80	+113.55	—7.29	+5.41	—0.63	+0.60

	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
Table VIII	251	188	32				
IX	1086	741	360	621	597	154	111
X	326	259	222	21	42		
	1663	1188	614	642	639	154	111
XVII	1104	—1321	—500	—412	—360	—98	—94
	2767	— 133	+114	+230	+279	+56	+17

log (v.s.1) — 2.3787	log (v.c.1) + 2.0388	log (v.s.2) — 1.8621	log (v.c.2 + 2.0552
log sin + 9.2994	log cos g — 9.9912	log sin 2g — 9.5916	log cos 2g + 9.9641
log (ρ.s.1) — 2.124	log (ρ.c.1) + 2.057	log (ρ.s.2) + 2.362	log (ρ.c.2) + 2.446

	°	′	″
g	168	30	30.48
ω	94	7	11.59
E	1	0	46.21
(v.c.0)		2	58.68
(v.s.1) sin g			—47.66
(v.c.1) cos g		—1	47.15
(v.s.2) sin 2g			28.43
(v.c.2) cos 2g		1	44.55
(v.s.3) sin 3g			— 4.13
(v.c.3) cos 3g			— 4.46
(v.s.4) sin 4g			0.47
(v.c.4) cos 4g			0.41
u	263	40	57.42
θ	72	44	17.87
R			7.95
Long. mean Eq.	336	25	23.24
Nutation			+6.51
Long. true Eq.	336	25	29.75

	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)
Table XX	″.06	0″.62		
XXI	3 .91	5 .88	0.41	0.17
XXII	.84	1 .36	0.18	0.17
	4.81	7 .86	0.59	0.34
XXIII	—4.28	—6 .88	—0.43	—0.51
	+0.53	+0 .98	+0.16	—0.17

	log r
Table VII	1.3023222
(ρ.c.0)	2767
(ρ.s.1) sin g	— 26
(ρ.c.1) cos g	—112
(ρ.s.2) sin 2g	— 90
(ρ.c.2) cos 2g	+257
(ρ.s.3) sin 3g	+ 31
(ρ.c.3) cos 3g	— 14
log r	1.326035

β_0 Table XIX	— 0	46′	3.″73
(b.c.0) Table XXI			0 .16
XXII			0 .09
XXIII			0 .20
(b.s.1) sin g			+0 .11
(b.c.1) cos g			—0 .96
(b.s.2) sin 2g			—0 .06
(b.c.2) cos 2g			—0 .16
Latitude	— 0	46	4 .35

As a second example we will take the computation of an ephemeris for the years 1876 and 1877. We take as the extreme dates 1875, December 15, and 1878, April 3, between which are seven intervals of 120 days each, which we adopt as those of computation. We first form the arguments for the extreme dates as follows:

I. For 1875, Dec. 15 = 1875.96.

	g			ω			θ			Arg. 1
	°	′	″	°	′	″	°	′	″	
Table II, 1872	314	25	55.70	95	9	35.64	73	20	46.67	574.815
III, 3 Y. Dec.	16	46	33.76	0	2	4.06	0	1	12.72	170.072
IV, 15 days	0	10	33.50			1.30			0.76	1.784
VI, 1875.96			2.20		....			....		+ 0.426
For 1875, Dec. 15	331	23	5.16	95	11	41.00	73	22	0.15	147.097

	Arg. 2	3	4	5	6	7	8	9
Table II, 1872	268.328	165.888	114	154	423	49	380.3	577
III, 3 Y. Dec.	51.785	13.705	39	13	65	73	21.1	78
IV, 15 days	0.543	0.144	0	0	1	1	0.2	1
VI, 1875.96	— 1	+ 2						
For 1875, Dec. 15	320.655	179.739	153	167	489	123	401.6	56

II. For 1878, April 3 = 1878.26.

	g			ω			θ			Arg. 1
	°	′	″	°	′	″	°	′	″	
Table II, 1876	331	34	18.70	95	11	42.33	73	22	1.03	148.576
III, 2 Y. April	9	37	53.62	0	1	11.23	0	0	41.75	97.643
IV, 3d		2	6.70			0.26			0.15	.357
VI, 1878.26			2.62		....			....		.421
For 1878, April 3	341	14	21.64	95	12	53.82	73	22	42.93	246.997

	Arg. 2	3	4	5	6	7	8	9
Table II, 1876	321.237	179.889	153	168	489	123	401.9	57
III, 2 Y. April	29.732	7.868	22	7	37	42	12.1	45
IV, 3d	.109	.029	0	0	0	0	0	0
VI, 1878.26	— .001	+ .002						
For 1878, April 3	351.077	187.788	175	175	526	165	414.0	102

We now fill in the values of g, the arguments 1—9, and the times with which Table XVII is to be entered, for the intermediate dates, by adding the nearly constant differences deduced from the numbers at the bottom of Table II. The seconds of g are first reduced to fractions of a minute, with which to enter Table VII. In making the subsequent computation we have used none of the devices previously described except in the case of the small longitude terms, as follows:

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	°	°	°	°	°	°	°
g	330	332	334	336	338	340	342
3*g*	270	276	282	288	294	300	306
4*g*	240	248	256	264	272	280	288
	″	″	″	″	″	″	″
(*v.s.*3)	— 1.14	— 1.12	— 1.13	— 1.16	— 1.23	— 1.33	— 1.48
(*v.c.*3)	— 2.46	— 2.56	— 2.66	— 2.76	— 2.88	— 3.01	— 3.16
(*v.s.*4)	+ 0.59	+ 0.60	+ 0.59	+ 0.55	+ 0.50	+ 0.43	+ 0.36
(*v.c.*4)	— 0.10	— 0.20	— 0.30	— 0.40	— 0.47	— 0.57	— 0.64
(*v.s.*3) sin 3*g*	+ 1.14	+ 1.12	+ 1.11	+ 1.11	+ 1.12	+ 1.15	+ 1.20
(*v.c.*3) cos 3*g*	.00	— 0.26	— 0.55	— 0.85	— 1.17	— 1.50	— 1.86
(*v.s.*4) sin 4*g*	— .51	— .56	— .57	— .55	— .50	— .42	— .34
(*v.c.*4) cos 4*g*	+ .05	+ .07	+ .07	+ .04	— .02	— .10	— .19
Sum	+ 0.68	+ 0.37	+ 0.06	— 0.25	— 0.57	— 0.87	— 1.19

It will be seen that we have here computed twice as many numbers as are necessary to interpolate with all attainable accuracy.

The rest of the computation is fully given on the four following pages. First we have the values of *g* and the nine arguments for the intermediate dates, filled in by successive addition of the nearly constant difference. The arguments thus obtained for the last date may be compared with those just computed on the preceding page.

The numerals in the first columns of the sections of computation following indicate the arguments with which tables are entered to obtain the separate values of the quantities (*v.c.*0), (*v.s.*1), (*v.c.*1), etc. The negative terms in Table XVII being taken from the sum of all the periodic terms from Tables VIII to XVI with argument 1 to 9, we have the final values of (*v.c.*0), (*v.s.*1), etc.

The final computation of the products (*v.s.i*) sin *ig*, etc., and the addition of the separate terms which make up the three co-ordinates, are shown on page 205. The expressions *c.*0, *s.*1, etc., are employed for brevity, instead of (*v.c.*0), (*v.s.*1) sin *g*, etc.

The longitude finally given by the tables is referred to the mean equinox, and must therefore be corrected for nutation before being used to compute the geocentric place.

Date, {	1875, Dec. 15 1875.955.	1876, Apr. 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1876.940.	1877, Apr. 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, Apr. 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
	331 23.086	332 47.554	334 12.021	335 36.489	337 0.957	338 25.425	339 49.893	341 14.361
Arg. 1	147.097	161.368	175.639	189.910	204.182	218.453	232.724	246.995
2	320.655	325.001	329.346	333.692	338.038	342.383	346.729	351.074
3	179.739	180.889	182.038	183.188	184.338	185.488	186.638	187.788
4	153.	156.	159.	163.	166.	169.	172.	175.
5	167.	.....	.....	.....	.....	......	......	175.
6	489.	494.	500.	505.	511.	516.	521.	527.
7	123.	129.	135.	141.	147.	153.	160.	166.
8	401.6	403.4	405.2	407.0	408.7	410.5	412.3	414.1
9	56.	63.	69.	75.	82.	88.	95.	101.
(*v.c.*0) 1	108.01	107.74	106.29	103.68	99.98	95.25	89.62	83.21
2	15.80	14.71	13.66	12.65	11.68	10.76	9.90	9.09
3	72.43	73.60	74.77	75.94	77.10	78.26	79.42	80.56
5	.59	.58	.58	.57	.56	.56	.55	.54
6	.02	.02	.02	.03	.03	.03	.03	.03
7	.07	.07	.06	.06	.06	.06	.05	.05
8	1.68	1.65	1.63	1.60	1.58	1.55	1.53	1.50
9	.09	.09	.09	.09	.08	.08	.08	.07
(*v.c.*0)	198.69	198.46	197.10	194.62	191.07	186.55	181.18	175.05
(*v.s.*1) 1	6.84	7.68	8.40	8.99	9.44	9.74	9.91	9.98
2	132.20	125.48	118.82	112.21	105.67	99.24	92.89	86.69
sec. 2	.18	.20	.22	.23	.25	.27	.29	.31
3	1.09	1.07	1.08	1.12	1.19	1.29	1.41	1.56
4	.13	.12	.11	.10	.10	.09	.09	.08
5	.19	.19	.19	.19	.19	.20	.20	.20
6	.04	.04	.04	.05	.05	.06	.06	.07
7	.14	.14	.13	.13	.12	.12	.11	.10
8	1.28	1.28	1.27	1.26	1.26	1.25	1.24	1.23
9	.08	.08	.08	.08	.07	.07	.07	.07
Σ	142.17	136.28	130.34	124.36	118.34	112.33	106.27	100.29
Tab. XVII	—154.21	—153.84	—153.47	—153.10	—152.73	—152.36	—151.99	—151.62
(*v.s.*1)	— 12.04	— 17.56	— 23.13	— 28.74	— 34.39	— 40.03	— 45.72	— 51.33
(*v.c.*1) 1	5.64	6.03	6.38	6.67	6.85	6.91	6.82	6.59
2	5.01	6 00	7·31	8.90	10.80	13.00	15.49	18.25
sec. 2	.69	.71	.72	.74	.75	.77	.79	.80
3	23.20	24.09	24.95	25.82	26.68	27.53	28.37	29.19
4	.47	.48	.49	.50	.50	.51	.52	.52
5	.37	.37	.37	.37	.37	.37	.37	.37
6	.16	.17	.17	.18	.18	.18	.18	.18
7	.15	.14	.13	.13	.12	.12	.11	.10
8	0.66	0.64	0.63	0.61	0.60	0.59	0.57	0.56
9	.07	.07	.06	.06	.05	.04	.04	.03
Σ	36.42	38.70	41.21	43.98	46.90	50.02	53.26	56.59
Tab. XVII	—205.86	—206.08	—206.29	—206.51	—206.73	—206.94	—207.15	—207.37
(*v.c.*1)	—169.44	—167.38	—165.08	—162.53	—159.83	—156.92	—153.89	—150.78
(*v.s.*2) 1	.06	.10	.15	.21	.26	.31	.34	.37
2	54.12	49.37	44.78	40.37	36.15	32.11	28.30	24.70
sec. 2	.39	.42	.44	.47	.49	.51	.53	.56
3	2.05	2.19	2.31	2.45	2.60	2.76	2.91	3.07
Σ	56.62	52.08	47.68	43.50	39.50	35.69	32.08	28.70
Tab. XVII	—134.34	—134.32	—134.31	—134.29	—134.27	—134.26	—134.24	—134.22
(*v.s.*2)	— 77.72	— 82.24	— 86.63	— 90.79	— 94.77	— 98.57	—102.16	—105.52

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Date, {	1875, Dec. 15 1875.955.	1876, Apr. 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1867.940.	1877, Apr. 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, Apr. 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
g	331 23.086	332 47.554	334 12.021	335 36.489	337 0.057	338 25.425	339 49.893	341 14.361
(*v.c.*2) 1	.31	.30	.29	.29	.29	.30	.30	.31
2	25.34	28.93	32.71	36.73	40.93	45.35	49.93	54.69
sec. 2	.76	.76	.77	.78	.79	.79	.80	.80
3	6.23	6.40	6.56	6.70	6.85	6.99	7.10	7.20
Σ	32.64	36.39	40.33	44.50	48.86	53.43	58.13	63.00
Tab.XVII	—136.57	—136.59	—136.60	—136.62	—136.64	—136.65	—136.67	—136.69
(*v.c.*2)	—103.93	—100.20	— 96.27	— 92.12	— 87.78	— 83.22	— 78.54	— 73.69
(*v.s.*3) 2	6.92	6.89	6.84	6.78	6.70	6.61	6.51	6.37
3	0.87	0.91	0.94	0.98	1.01	1.04	1.08	1.11
	7.79	7.80	7.78	7.76	7.71	7.65	7.59	7.48
Tab.XVII	—8.92	—8.92	—8.91	—8.91	—8.91	—8.91	—8.91	—8.91
(*v.s.*3)	—1.13	—1.12	—1.13	—1.15	—1.20	—1.26	—1.32	—1.43
(*v.c.*3) 2	3.19	3.12	3.05	2.98	2.90	2.82	2.75	2.67
3	1.34	1.34	1.34	1.34	1.33	1.33	1.31	1.29
	4.53	4.46	4.39	4.32	4.23	4.15	4.06	3.96
Tab.XVII	—7.06	—7.06	—7.06	—7.06	—7.06	—7.06	—7.06	—7.06
(*v.c.*3)	—2.53	—2.60	—2.67	—2.74	—2.83	—2.91	—3.00	—3.10
(*v.s.*4) 2	1.48	1.48	1.47	1.45	1.43	1.40	3.36	1.33
3	—0.88	—0.88	—0.88	—0.89	—0.89	—0.91	—0.93	— .94
(*v.s.*4)	+0.60	+0.60	+0.59	+0.56	+0.54	+0.49	+0.43	+0.39
(*v.c.*4) 2	0.83	0.77	0.72	0.66	0.61	0.57	0.51	0.47
3	—1.00	—1.02	—1.03	—1.04	—1.05	—1.06	—1.07	—1.08
(*v.c.*4)	—0.17	—0.25	—0.31	—0.38	—0.44	—0.49	—0.56	—0.61
(*ρ.c.*0) 1	1230	1063	899	741	594	460	344	246
2	98	99	101	104	108	113	118	126
3	11	10	9	9	8	8	8	9
Tab.XVII	968	968	968	967	967	967	966	966
(*ρ.c.*0)	2307	2140	1977	1821	1677	1548	1436	1347
(*ρ.s.*1) 1	248	240	230	221	211	203	194	187
2	2835	2822	2807	2789	2769	2748	2723	2696
3	173	168	165	161	158	155	151	148
Σ	3256	3230	3202	3171	3138	3106	3068	3031
Tab.XVII	—1984	—1987	—1989	—1991	—1993	—1996	—1998	2000
(*ρ.s.*1)	+1272	1243	1213	1180	1145	1110	1070	1031
(*ρ.c.*1) 1	123	112	97	83	69	55	41	31
2	1266	1202	1138	1074	1009	947	886	827
3	66	66	67	67	68	70	72	74
Σ	1455	1380	1302	1224	1146	1072	999	932
Tab.XVII	—1977	—1981	—1986	—1990	—1994	—1998	—2002	2006
(*ρ.c.*1)	— 522	— 601	— 684	— 766	— 848	— 926	—1003	—1074

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Date, {	1875, Dec. 15 1875.955.	1876, Apr. 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1867.940.	4877, Apr. 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, Apr. 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
g	331 23.086	332 47.554	334 12.021	335 36.489	337 0.957	338 25.425	339 49.893	341 14.361
(*ρ.s.*2) 2	174	181	189	196	204	213	223	232
3	26	25	25	24	24	24	23	23
Σ	200	206	214	220	228	237	246	255
Tab.XVII	—459	—459	—459	—459	—460	—460	—460	—460
(*ρ.s.*2)	—259	—253	—245	—239	—232	—223	—214	—205
(*ρ.c.*2) 2	559	568	576	584	592	600	607	614
3	33	33	34	35	36	37	39	40
Σ	592	601	610	619	628	637	646	654
Tab.XVII	—464	—464	—464	—465	—465	—465	—465	—466
(*ρ.c.*2)	+128	137	146	154	163	172	181	188
(*ρ.s.*3) 2	28	31	34	39	43	47	52	57
Tab.XVII	—101	—101	—101	—101	—101	—101	—101	—101
(*ρ.s.*3)	— 73	— 70	— 67	— 62	— 58	— 54	— 49	— 44
(*ρ.c.*3) 2	155	159	162	166	169	172	175	177
Tab.XVII	—102	—102	—102	—102	—102	—102	—102	—102
(*ρ.c.*3)	+ 53	57	60	64	67	70	73	75
(*b.c.*0) 2	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15
3	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Tab.XXIII	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
(*b.c.*0)	0.30	0.30	0.31	0.31	0.32	0.32	0.33	0.33
(*b.s.*1) 1	0.30	0.22	0.16	0.12	0.08	0.06	0.05	0.06
2	0.12	0.09	0.06	0.05	0.04	0.04	0.05	0.06
3	1.11	1.11	1.12	1.12	1.13	1.13	1.14	1.14
Σ	1.53	1.42	1.34	1.29	1.25	1.23	1.24	1.26
Tab.XXIII	—5.23	—5.24	—5.24	—5.24	—5.24	—5.25	—5.25	—5.25
(*b.s.*1)	—3.70	—3.82	—3.90	—3.95	—3.99	—4.02	—4.01	—3.99
(*b.c.*1) 1	1.21	1.14	1.06	0.98	0.90	0.80	0.70	0.61
2	0.96	1.06	1.17	1.28	1.39	1.51	1.63	1.75
3	1.01	1.00	1.00	0.99	0.99	0.99	0.98	0.98
Σ	3.18	3.20	3.23	3.25	3.28	3.30	3.31	3.34
Tab.XXIII	—4.45	—4.44	—4.43	—4.43	—4.42	—4.41	—4.41	—4.40
(*b.c.*1)	—1.27	—1.24	—1.20	—1.18	—1.14	—1.11	—1.10	—1.06
(*b.s.*2) 2	0.02	0.02	0.03	0.04	0.04	0.05	0.06	0.07
3	0.16	0.16	0.16	0.15	0.15	0.15	0.15	0.15
Tab.XXIII	—0.48	—0.48	—0.48	—0.48	—0.48	—0.48	—0.48	—0.48
(*b.s.*2)	-0.30	—0.30	—0.29	—0.29	—0.29	—0.28	—0.27	—0.26
(*b.c.*2) 2	0.31	0.32	0.33	0.34	0.34	0.35	0.36	0.36
3	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11
Tab.XXIII	—0.38	—0.38	—0.38	—0.38	—0.38	—0.38	—0.38	—0.38
(*b.c.*2)	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09	+0.09

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Date, {	1875, Dec. 15 1875.955.	1876, April 13 1876.283.	Aug. 11 1876.612.	Dec. 9 1876.940.	1877, April 8 1877.269.	Aug. 6 1877.597.	Dec. 4 1877.925.	1878, April 3 1878.254.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
g	331 23.086	332 47.554	334 12.021	335 36.489	337 0.957	338 25.425	339 49.893	341 14.361
log ($v.s.1$)	—1.0806	—1.2445	—1.3642	—1.4585	—1.5365	—1.6024	—1.6601	—1.7104
sin g	—9.6803	—9.6601	—9.6386	—9.6159	—9.5916	—9.5655	—9.5375	—9.5073
log ($\rho.s.1$)	+3.1045	3.0944	3.0838	3.0719	3.0588	3.0453	3.0294	3.0132
log ($v.c.1$)	—2.2290	—2.2237	—2.2177	—2.2109	—2.2036	—2.1957	—2.1872	—2.1784
cos g	+9.9434	9.9491	9.9544	9.9594	9.9641	9.9685	9.9725	9.9763
log ($\rho.c.1$)	—2.718	—2.779	—2.835	—2.884	—2.928	—2.967	—3.0013	—8.0310
log ($v.s.2$)	—1.8905	—1.9151	—1.9376	—1.9580	—1.9766	—1.9937	—2.0093	—2.0233
sin $2g$	—9.9247	—9.9102	—9.8941	—9.8763	—9.8566	—9.8350	—9.8111	—9.7846
log ($\rho.s.2$)	—2.413	—2.403	—2.389	—2.378	—2.365	—2.348	—2.330	—2.312
log ($v.c.2$)	—2.0167	—2.0009	—1.9835	—1.9644	—1.9434	—1.9202	—1.8951	—1.8674
cos $2g$	+9.7335	9.7649	9.7932	9.8189	9.8421	9.8630	9.8821	9.8994
log ($\rho.c.2$)	+2.107	2.137	2.164	2.187	2.212	2.236	2.258	2.274
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Δg	1 24 28.065	1 24 28.066	1 24 28.066	1 24 28.067	1 24 28.068	1 24 28.068	1 24 28.069	
g	331 23 5.16	332 47 33.225	334 12 1.291	335 36 29.357	337 0 57.424	338 25 25.492	339 49 53.560	341 14 21.63
ω	95 11 41.00	95 11 51.40	95 12 1.81	95 12 12.21	95 12 22.61	95 12 33.01	95 12 43.42	95 12 53.82
E	—2 42 49.15	—2 35 33.18	—2 28 10.63	—2 20 41.76	—2 13 6.93	—2 5 26.43	—1 57 40.55	—1 49 49.66
$c.0$	3 18.69	3 18.46	3 17.10	3 14.62	3 11.07	3 6.55	3 1.18	2 55.05
$s.1$	5.77	8.03	10.07	11.87	13.43	14.72	15.76	16.51
$c.1$	—2 28.72	—2 28.86	—2 28.62	—2 28.00	—2 27.13	—2 25.93	—2 24.43	—2 22.80
$s.2$	1 5.34	1 6.88	1 7.87	1 8.28	1 8.12	1 7.40	1 6.13	1 4.26
$c.2$	— 56.27	— 58.31	— 59.80	—1 0.71	—1 1.03	—1 0.70	— 59.87	— 58.45
(3+4)	0.46	0.25	0.03	— 0.19	— 0.41	— 0.63	— 0.83	— 1.00
u	63 53 2.28	65 24 57.89	66 56 59.12	68 29 5.68	70 1 17.15	71 33 33.48	73 5 54.37	74 38 19.36
θ	73 22 0.15	73 22 6.26	73 22 12.37	73 22 18.48	73 22 24.60	73 22 30.71	73 22 36.82	73 22 42.93
R	2.58	2.91	3.25	3.61	3.98	4.37	4.79	5.22
Longitude	137 15 5.01	138 47 7.06	140 19 14.74	141 51 27.77	143 23 45.73	144 56 8.56	146 28 35.98	148 1 7.51
log r_0	1.2647392	1.2644735	1.2642196	1.2639778	1.2637486	1.2635319	1.2633280	1.2631370
$c.0$	2307	2140	1977	1821	1677	1548	1436	1347
$s.1$	—609	—568	—528	—487	—447	—408	—369	— 332
$c.1$	—458	—535	—615	—697	—780	—861	—941	—1017
$s.2$	+218	206	192	179	167	152	138	125
$c.2$	+ 69	80	91	101	113	126	138	149
$s.3$	+ 73	69	65	60	54	49	43	37
$c.3$	+ 4	8	13	19	23	29	36	41
log r	1.2648996	1.2646135	1.2643391	1.2640774	1.2638293	1.2635954	1.2633761	1.2631720
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
β_0	+41 36.64	42 8.48	42 38.54	43 6.78	43 33.14	43 57.62	44 20.23	44 41.19
c.0	+0.30	0.30	0.31	0.31	0.32	0.32	0.33	0.33
$s.1$	+1.77	1.75	1.70	1.63	1.57	1.48	1.38	1.29
$c.1$	—1.11	—1.10	—1.08	—1.07	—1.05	—1.03	—1.03	—1.00
$s.2$	+0.25	0.24	0.23	0.22	0.21	0.19	0.17	0.16
$c.2$	+0.02	0.03	0.03	0.04	0.05	0.06	0.07	0.07
Latitude	+41 37.87	42 9.70	42 39.73	43 7.91	43 34.24	43 58.64	44 21.15	44 42.04

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TABLE I.—CORRECTIONS OF ARGUMENTS FOR PAST AND FUTURE CENTURIES.

Century.	g			ω			ω	θ			θ'	Arg. 1
	°	′	″	°	′	″	″	°	′	″	″	
0J	207	15	59.32	343	52	17.36	+108.00	351	6	26.89	—148.32	408.924
100	275	45	34.18	344	46	54.12	102.00	351	34	55.39	—140.08	552.952
200	344	15	9.04	345	41	24.88	96.00	352	3	32.13	—131.84	96.980
300	52	44	43.90	346	35	49.64	90.00	352	32	17.11	—123.60	241.008
400	121	14	18.76	347	30	8.40	84.00	353	1	10.33	—115.36	385.036
500	189	43	53.62	348	24	21.16	+ 78.00	353	30	11.79	—107.12	529.064
600	258	13	28.48	349	18	27.92	72.00	353	59	21.49	— 98.88	73.092
700	326	43	3.34	350	12	28.68	66.00	354	28	39.43	— 90.64	217.120
800	35	12	38.20	351	6	23.44	60.00	354	58	5.61	— 82.40	361.148
900	103	42	13.06	352	0	12.20	54.00	355	27	40.03	— 74.16	505.176
1000	172	11	47.92	352	53	54.96	+ 48.00	355	57	22.69	— 65.92	49.204
1100	240	41	22.78	353	47	31.72	42.00	356	27	13.59	— 57.68	193.232
1200	309	10	57.64	354	41	2.48	36.00	356	57	12.73	— 49.44	337.260
1300	17	40	32.50	355	34	27.24	30.00	357	27	20.11	— 41.20	481.288
1400	86	10	7.36	356	27	46.00	24.00	357	57	35.73	— 32.96	25.316
1500J	154	39	42.22	357	20	58.76	+ 18.00	358	27	59.59	— 24.72	169.344
1500G	154	32	39.89	357	20	57.89	18.00	358	27	59.08	— 24.72	168.155
1600	223	2	14.75	358	14	4.65	12.00	358	58	31.18	— 16.48	312.183
1700	291	31	7.37	359	7	5.33	6.00	359	29	11.47	— 8.24	456.092
1800	0	0	0.00	0	0	0.00	0.00	0	0	0.00	0.00	0.000
1900	68	28	52.63	0	52	48.67	— 6.00	0	30	56.77	+ 8.24	143.908
2000	136	58	27.49	1	45	31.43	— 12.00	1	2	1.83	16.48	287.936
2100	205	27	20.11	2	38	8.11	— 18.00	1	33	15.08	24.72	431.845
2200	273	56	12.74	3	30	38.78	— 24.00	2	4	36.57	+ 32.96	575.755

TABLE II.—ARGUMENTS FOR THE BEGINNING OF EACH FOURTH YEAR 1752—1948.

Year.	g			ω			θ			Arg. 1
	°	′	″	°	′	″	°	′	″	
1752	160	15	8.10	94	6	10.48	72	43	42.30	162.101
1756	177	23	31.10	94	8	17.46	72	44	56.25	335.862
1760	194	31	54.09	94	10	24.43	72	46	10.22	509.623
1764	211	40	17.09	94	12	31.39	72	47	24.21	83.384
1768	228	48	40.08	94	14	38.34	72	48	38.20	257.145
1772	245	57	3.08	94	16	45.28	72	49	52.21	430.907
1776	263	5	26.07	94	18	52.22	72	51	6.23	4.668
1780	280	13	49.06	94	20	59.14	72	52	20.27	178.429
1784	297	22	12.06	94	23	6.05	72	53	34.32	352.190
1788	314	30	35.05	94	25	12.96	72	54	48.38	525.951
1792	331	38	58.05	94	27	19.85	72	56	2.46	99.712
1796	348	47	21.04	94	29	26.74	72	57	16.54	273.473
1800	5	55	1.80	94	31	33.53	72	58	30.59	447.115

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TABLE I.—*Continued.*

Century.	2	3	4	5	6	7	8	9
0J	191.528	299.485	280	512	102	61	470.0	15
100	314.245	49.520	64	250	563	124	410.6	214
200	436.962	399.555	449	589	424	188	351.2	413
300	559.679	149.590	233	327	285	251	291.7	13
400	82.396	499.625	18	65	146	314	232.3	212
500	205.113	249.660	402	403	7	378	172.9	411
600	327.830	599.695	187	142	468	441	113.5	10
700	450.547	349.730	571	480	329	504	54.1	209
800	573.264	99.765	356	218	190	567	594.6	409
900	95.981	449.800	140	556	51	31	535.2	8
1000	218.698	199.835	524	295	512	94	475.8	207
1100	341.415	549.870	309	33	373	157	416.4	406
1200	464.132	299.905	93	371	234	221	357.0	5
1300	586.849	49.940	478	109	95	284	297.5	205
1400	109.566	399.975	262	448	556	347	238.1	404
1500J	232.283	150.010	47	186	417	411	178.6	3
1500G	231.921	149.914	47	186	417	410	178.3	2
1600	354.638	499.949	431	524	278	473	118.8	201
1700	477.319	249.975	216	262	139	537	59.4	401
1800	0.000	0.000	0	0	0	0	0	0
1900	122.681	350.025	385	338	461	63	540.6	199
2000	245.398	100.060	169	76	322	127	481.2	398
2100	368.079	450.086	554	415	183	190	421.8	597
2200	490.760	200.111	338	153	44	253	362.3	196

TABLE II.—*Continued.*

Year.	2	3	4	5	6	7	8	9
1752	481.104	345.855	133	348	229	213	331.6	578
1756	534.013	359.856	172	362	296	287	353.2	58
1760	586.921	373.858	212	375	362	362	374.8	138
1764	39.830	387.859	251	389	429	436	396.4	218
1768	92.739	401.860	290	402	495	511	418.1	298
1772	145.647	415.862	330	416	562	585	439.7	377
1776	198.556	429.863	369	429	28	60	461.3	457
1780	251.465	443.865	408	443	95	134	482.9	537
1784	304.373	457.866	448	457	161	209	504.6	17
1788	357.282	471.868	487	470	227	283	526.2	97
1792	410.191	485.869	527	484	294	358	547.8	177
1796	463.100	499.870	566	497	360	433	569.4	257
1800	515.972	513.862	5	511	427	507	591.1	337

TABLE II.—*Continued.*

Year.	g			ω			θ			Arg. 1.
	°	′	″	°	′	″	°	′	″	
1800	5	55	1.80	94	31	33.53	72	58	30.59	447.115
1804	23	3	24.80	94	33	40.39	72	59	44.71	20.876
1808	40	11	47.79	94	35	47.25	73	0	58.84	194.638
1812	57	20	10.79	94	37	54.09	73	2	12.98	368.399
1816	74	28	33.78	94	40	0.93	73	3	27.13	542.160
1820	91	36	56.77	94	42	7.76	73	4	41.30	115.921
1824	108	45	19.77	94	44	14.58	73	5	55.48	289.683
1828	125	53	42.76	94	46	21.38	73	7	9.67	463.444
1832	143	2	5.76	94	48	28.18	73	8	23.88	37.205
1836	160	10	28.75	94	50	34.97	73	9	38.10	210.966
1840	177	18	51.75	94	52	41.75	73	10	52.33	384.727
1844	194	27	14.74	94	54	48.52	73	12	6.58	558.488
1848	211	35	37.74	94	56	55.28	73	13	20.84	132.249
1852	228	44	0.73	94	59	2.03	73	14	35.11	306.010
1856	245	52	23.75	95	1	8.77	73	15	49.40	479.771
1860	263	0	46.72	95	3	15.50	73	17	3.70	53.532
1864	280	9	9.71	95	5	22.22	73	18	18.01	227.293
1868	297	17	32.71	95	7	28.94	73	19	32.34	401.054
1872	314	25	55.70	95	9	35.64	73	20	46.67	574.815
1876	331	34	18.70	95	11	42.33	73	22	1.03	148.576
1880	348	42	41.69	95	13	49.01	73	23	15.39	322.337
1884	5	51	4.69	95	15	55.69	73	24	29.77	496.098
1888	22	59	27.68	95	18	2.35	73	25	44.16	69.860
1892	40	7	50.68	95	20	9.01	73	26	58.57	243.621
1896	57	16	13.67	95	22	15.65	73	28	12.98	417.382
1900	74	23	54.43	95	24	22.20	73	29	27.36	591.024
1904	91	32	17.43	95	26	28.83	73	30	41.81	164.785
1908	108	40	40.42	95	28	35.44	73	31	56.26	338.546
1912	125	49	3.41	95	30	42.05	73	33	10.74	512.307
1916	142	57	26.41	95	32	48.64	73	34	25.22	86.068
1920	160	5	49.40	95	34	55.23	73	35	39.72	259.830
1924	177	14	12.40	95	37	1.81	73	36	54.23	433.591
1928	194	22	35.39	95	39	8.38	73	38	8.75	7.352
1932	211	30	58.39	95	41	14.94	73	39	23.29	181.113
1936	228	39	21.38	95	43	21.48	73	40	37.84	354.874
1940	245	47	44.38	95	45	28.02	73	41	52.40	528.635
1944	262	56	7.37	95	47	34.55	73	43	6.97	102.396
1948	280	4	30.36	95	49	41.07	73	44	21.56	276.158
$\Delta_{120}^{(1)}$	1	24	28.007			10.411			6.100	14.2715
Factor T			+0.222			—.020			+.027	—.0012
$\Delta_{120}^{(2)}$			+ .0007			—.0001			+.0001	0

TABLE II.—*Continued.*

	2	3	4	5	6	7	8	9
1800	515.972	513.862	5	511	427	507	591.1	337
1804	568.881	527.863	45	524	493	582	12.7	417
1808	21.790	541.865	84	538	560	56	34.3	497
1812	74.698	555.866	124	551	26	131	55.9	577
1816	127.607	569.868	163	565	93	205	77.5	57
1820	180.516	583.869	202	578	159	280	99.2	137
1824	233.424	597.870	242	592	225	354	120.8	217
1828	286.333	11.872	281	6	292	429	142.4	297
1832	339.242	25.874	320	19	358	503	164.0	377
1836	392.150	39.875	360	32	425	578	185.7	457
1840	445.059	53.876	399	46	491	52	207.3	537
1844	497.968	67.878	439	59	557	127	228.9	17
1848	550.876	81.879	478	73	24	201	250.5	97
1852	3.785	95.880	517	87	90	276	272.1	177
1856	56.694	109.882	557	100	157	350	293.8	257
1860	109.602	123.883	596	114	223	425	315.4	337
1864	162.511	137.885	35	127	290	499	337.0	417
1868	215.420	151.886	75	141	356	574	358.6	497
1872	268.328	165.888	114	154	423	49	380.3	577
1876	321.237	179.889	153	168	489	123	401.9	57
1880	374.146	193.890	193	181	555	198	423.5	137
1884	427.054	207.892	232	195	22	272	445.1	217
1888	479.963	221.893	271	208	88	347	466.7	297
1892	532.872	235.895	311	222	155	421	488.4	376
1896	585.780	249.896	350	235	221	496	510.0	456
1900	38.653	263.889	390	249	287	570	531.6	536
1904	91.562	277.890	429	262	354	45	553.2	16
1908	144.470	291.891	468	276	420	119	574.9	96
1912	197.379	305.892	508	290	487	194	596.5	176
1916	250.288	319.894	547	303	553	268	18.1	256
1920	303.197	333.895	587	317	20	343	39.7	336
1924	356.105	347.897	26	330	86	417	61.4	416
1928	409.014	361.898	65	344	152	492	83.0	496
1932	461.923	375.900	105	357	219	566	104.6	576
1936	514.831	389.901	144	371	285	41	126.2	56
1940	567.740	403.902	184	384	352	116	147.8	136
1944	20.649	417.904	223	398	418	190	169.5	216
1948	73.557	431.905	262	411	485	265	191.1	296
	4.3457	1.1500	3.2	1.1	5.4	6.1	1.78	6.6
	— .0001	+ .0002	0	0	0	0	0	0
	0	0	0	0	0	0	0	0

TABLE III.—Reduction of the Epochs and Arguments to the beginning of each Month in a Cycle of four years.

	g			ω			*θ*			*θ′*	Arg. 1
Year 0	°	′	″	°	′	″	°	′	″	″	
January 0	0	0	0.00	0	0	0.00	0	0	0.00	0.00	0.000
February 0	0	21	49.24	0	0	2.69	0	0	1.58	0.00	3.687
March 0	0	42	14.00	0	0	5.21	0	0	3.05	0.00	7.136
April 0	1	4	3.24	0	0	7.89	0	0	4.63	0.01	10.823
May 0	1	25	10.24	0	0	10.50	0	0	6.15	0.01	14.391
June 0	1	46	59.48	0	0	13.19	0	0	7.73	0.01	18.078
July 0	2	8	6.48	0	0	15.79	0	0	9.25	0.01	21.647
August 0	2	29	55.71	0	0	18.48	0	0	10.83	0.01	25.333
September 0	2	51	44.95	0	0	21.17	0	0	12.40	0.01	29.019
October 0	3	12	51.95	0	0	23.77	0	0	13.93	0.02	32.587
November 0	3	34	41.19	0	0	26.46	0	0	15.51	0.02	36.274
December 0	3	55	48.19	0	0	29.06	0	0	17.03	0.02	39.842
Year 1											
January 0	4	17	37.42	0	0	31.75	0	0	18.61	0.02	43.529
February 0	4	39	26.66	0	0	34.44	0	0	20.18	0.02	47.216
March 0	4	59	9.19	0	0	36.87	0	0	21.61	0.03	50.546
April 0	5	20	58.43	0	0	39.56	0	0	23.18	0.03	54.233
May 0	5	42	5.43	0	0	42.16	0	0	24.71	0.03	57.801
June 0	6	3	54.67	0	0	44.85	0	0	26.28	0.03	61.488
July 0	6	25	1.67	0	0	47.46	0	0	27.81	0.03	65.056
August 0	6	46	50.90	0	0	50.14	0	0	29.38	0.04	68.742
September 0	7	8	40.14	0	0	52.83	0	0	30.96	0.04	72.429
October 0	7	29	47.14	0	0	55.44	0	0	32.48	0.04	75.997
November 0	7	51	36.38	0	0	58.13	0	0	34.06	0.04	79.684
December 0	8	12	43.38	0	1	0.73	0	0	35.59	0.04	83.252
Year 2											
January 0	8	34	32.61	0	1	3.43	0	0	37.16	0.04	86.939
February 0	8	56	21.85	0	1	6.11	0	0	38.74	0.05	90.626
March 0	9	16	4.38	0	1	8.54	0	0	40.16	0.05	93.956
April 0	9	37	53.62	0	1	11.23	0	0	41.74	0.05	97.643
May 0	9	59	0.62	0	1	13.83	0	0	43.26	0.05	101.211
June 0	10	20	49.86	0	1	16.52	0	0	44.84	0.05	104.898
July 0	10	41	56.86	0	1	19.12	0	0	46.36	0.06	108.466
August 0	11	3	46.09	0	1	21.81	0	0	47.94	0.06	112.153
September 0	11	25	35.33	0	1	24.50	0	0	49.52	0.06	115.840
October 0	11	46	42.33	0	1	27.10	0	0	51.04	0.06	119.407
November 0	12	8	31.57	0	1	29.79	0	0	52.62	0.06	123.094
December 0	12	29	38.57	0	1	32.40	0	0	54.14	0.07	126.662
Year 3											
January 0	12	51	27.80	0	1	35.08	0	0	55.72	0.07	130.349
February 0	13	13	17.04	0	1	37.77	0	0	57.29	0.07	134.036
March 0	13	32	59.57	0	1	40.20	0	0	58.72	0.07	137.366
April 0	13	54	48.81	0	1	42.89	0	1	0.29	0.07	141.053
May 0	14	15	55.81	0	1	45.50	0	1	1.82	0.08	144.621
June 0	14	37	45.05	0	1	48.18	0	1	3.40	0.08	148.308
July 0	14	58	52.05	0	1	50.79	0	1	4.92	0.08	151.876
August 0	15	20	41.28	0	1	53.48	0	1	6.50	0.08	155.563
September 0	15	42	30.52	0	1	56.17	0	1	8.07	0.08	159.249
October 0	16	3	37.52	0	1	58.77	0	1	9.60	0.08	162.817
November 0	16	25	26.76	0	2	1.46	0	1	11.17	0.09	166.504
December 0	16	46	33.76	0	2	4.06	0	1	12.70	0.09	170.072

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TABLE III.—*Continued.*

	2	3	4	5	6	7	8	9
Year 0								
January 0	0.000	0.000	0	0	0	0	0.0	0
February 0	1.123	0.297	1	0	1	1	0.5	2
March 0	2.209	0.575	2	1	3	3	0.9	3
April 0	3.259	0.872	2	1	4	4	1.3	5
May 0	4.382	1.159	3	1	5	6	1.8	7
June 0	5.505	1.457	4	1	7	7	2.2	8
July 0	6.591	1.745	5	2	8	9	2.7	10
August 0	7.714	2.042	6	2	10	10	3.1	12
September 0	8.837	2.339	7	2	11	12	3.6	13
October 0	9.923	2.626	7	3	12	14	4.0	15
November 0	11.046	2.923	8	3	14	15	4.5	17
December 0	12.132	3.211	9	3	15	17	4.9	18
Year 1								
January 0	13.254	3.508	10	3	17	19	5.4	20
February 0	14.377	3.805	11	4	18	20	5.9	22
March 0	15.391	4.073	11	4	19	22	6.3	23
April 0	16.514	4.370	12	4	21	23	6.7	25
May 0	17.600	4.658	13	4	22	25	7.2	27
June 0	18.723	4.955	14	5	24	27	7.6	28
July 0	19.809	5.242	15	5	25	28	8.1	30
August 0	20.932	5.539	15	5	26	30	8.5	32
September 0	22.054	5.836	16	6	28	31	9.0	33
October 0	23.140	6.124	17	6	29	33	9.4	35
November 0	24.263	6.421	18	6	30	34	9.9	37
December 0	25.349	6.709	19	6	32	36	10.3	38
Year 2								
January 0	26.472	7.006	20	7	33	37	10.8	40
February 0	27.595	7.303	20	7	34	39	11.3	42
March 0	28.609	7.571	21	7	36	40	11.7	43
April 0	29.732	7.868	22	7	37	42	12.1	45
May 0	30.818	8.156	23	8	38	44	12.6	47
June 0	31.941	8.453	24	8	40	45	13.0	48
July 0	33.027	8.741	25	8	41	47	13.5	50
August 0	34.150	9.038	25	9	43	48	13.9	52
September 0	35.272	9.335	26	9	44	50	14.4	53
October 0	36.358	9.622	27	9	45	51	14.8	55
November 0	37.481	9.919	28	9	47	53	15.3	57
December 0	38.567	10.207	29	10	48	55	15.7	58
Year 3								
January 0	39.690	10.504	30	10	50	56	16.2	60
February 0	40.813	10.801	30	10	51	58	16.7	62
March 0	41.827	11.070	31	11	52	59	17.1	63
April 0	42.950	11.367	32	11	54	61	17.5	65
May 0	44.036	11.654	33	11	55	62	18.0	67
June 0	45.159	11.951	34	11	57	64	18.4	68
July 0	46.245	12.239	35	12	58	65	18.9	70
August 0	47.368	12.536	35	12	59	67	19.3	72
September 0	48.490	12.833	36	12	61	68	19.8	73
October 0	49.576	13.121	37	12	62	70	20.2	75
November 0	50.699	13.418	38	13	64	72	20.7	77
December 0	51.785	13.705	39	13	65	73	21.1	78

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TABLE IV.—Motion of Arguments for Days.

Days.	g	ω	θ	1	2	3	4	5	6	7	8	9
	′ ″	″	″									
1	0 42.23	0.09	0.05	0.119	0.036	0.010	0	0	0	0	0	0
2	1 24.47	0.17	0.10	0.238	0.072	0.019	0	0	0	0	0	0
3	2 6.70	0.26	0.15	0.357	0.109	0.029	0	0	0	0	0	0
4	2 48.93	0.35	0.20	0.476	0.145	0.039	0	0	0	0	0.1	0
5	3 31.17	0.43	0.25	0.595	0.181	0.048	0	0	0	0	0.1	0
6	4 13.40	0.52	0.30	0.714	0.217	0.058	0	0	0	0	0.1	0
7	4 55.63	0.61	0.36	0.833	0.253	0.067	0	0	0	0	0.1	0
8	5 37.87	0.69	0.41	0.951	0.290	0.077	0	0	0	0	0.1	0
9	6 20.10	0.78	0.46	1.070	0.326	0.086	0	0	0	0	0.1	0
10	7 2.33	0.87	0.51	1.189	0.362	0.096	0	0	1	1	0.1	1
11	7 44.57	0.95	0.56	1.308	0.398	0.105	0	0	1	1	0.2	1
12	8 26.80	1.04	0.61	1.427	0.434	0.115	0	0	1	1	0.2	1
13	9 9.03	1.13	0.66	1.546	0.471	0.125	0	0	1	1	0.2	1
14	9 51.27	1.21	0.71	1.665	0.507	0.134	0	0	1	1	0.2	1
15	10 33.50	1.30	0.76	1.784	0.543	0.144	0	0	1	1	0.2	1
16	11 15.73	1.39	0.81	1.903	0.579	0.153	0	0	1	1	0.2	1
17	11 57.97	1.47	0.86	2.022	0.616	0.163	0	0	1	1	0.2	1
18	12 40.20	1.56	0.91	2.141	0.652	0.173	0	0	1	1	0.3	1
19	13 22.43	1.65	0.97	2.260	0.688	0.182	1	0	1	1	0.3	1
20	14 4.67	1.74	1.02	2.378	0.724	0.192	1	0	1	1	0.3	1
21	14 46.90	1.82	1.07	2.497	0.760	0.201	1	0	1	1	0.3	1
22	15 29.14	1.91	1.12	2.616	0.797	0.211	1	0	1	1	0.3	1
23	16 11.37	2.00	1.17	2.735	0.833	0.220	1	0	1	1	0.3	1
24	16 53.60	2.08	1.22	2.854	0.869	0.230	1	0	1	1	0.3	1
25	17 35.84	2.17	1.27	2.973	0.905	0.240	1	0	1	1	0.4	1
26	18 18.07	2.26	1.32	3.092	0.941	0.249	1	0	1	1	0.4	1
27	19 0.30	2.34	1.37	3.211	0.978	0.259	1	0	1	1	0.4	1
28	19 42.54	2.43	1.42	3.330	1.014	0.268	1	0	1	1	0.4	1
29	20 24.77	2.52	1.47	3.449	1.050	0.278	1	0	1	1	0.4	1
30	21 7.00	2.60	1.52	3.568	1.086	0.288	1	0	1	1	0.4	1
31	21 49.24	2.69	1.57	3.687	1.123	0.297	1	0	1	1	0.5	1

TABLE V.—Motion of *g* for Hours.

Hours.	g	Hours.	g	Hours.	g	Hours.	g
	″		″		″		″
0	0.00	6	10.56	12	21.12	18	31.67
1	1.76	7	12.32	13	22.88	19	33.43
2	3.52	8	14.08	14	24.64	20	35.19
3	5.28	9	15.84	15	26.40	21	36.95
4	7.04	10	17.60	16	28.16	22	38.71
5	8.80	11	19.36	17	29.92	23	40.47
6	10.56	12	21.12	18	31.67	24	42.23

The period of arguments 1 to 9 is 600.

In January and February of those years which, though divisible by 4, are not leap years, namely, 1700, 1800, 1900, 2100, etc., Table IV must be entered with a number 1 greater than the real day of the month.

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TABLE VI.—Corrections of Arguments for Terms of Long Period.

Year.	*g*	1	2	3	Year.	*g*	1	2	3
	′ ″					′ ″			
1000	+36 51.00	—0.614	—1.024	+1.792	1550	+5 9.73	—0.200	—0.144	+0.252
1010	36 6.49	0.621	1.003	1.756	1560	4 49.70	0.153	0.134	0.236
1020	35 22.14	0.629	0.983	1.720	1570	4 30.32	0.106	0.125	0.219
1030	34 37.97	0.638	0.962	1.684	1580	4 11.57	0.060	0.117	0.205
1040	33 54.00	0.648	0.942	1.649	1590	3 53.48	—0.013	0.108	0.189
1050	+33 10.25	—0.659	—0.922	+1.614	1600	+3 36.03	+0.034	—0.100	+0.175
1060	32 26.71	0.672	0.902	1.579	1610	3 19.26	0.078	0.092	0.161
1070	31 43.41	0.686	0.882	1.544	1620	3 3.14	0.120	0.085	0.148
1080	31 0.37	0.702	0.862	1.509	1630	2 47.69	0.162	0.078	0.135
1090	30 17.61	0.718	0.842	1.474	1640	2 32.89	0.203	0.071	0.123
1100	+29 35.15	—0.735	—0.822	+1.439	1650	+2 18.77	+0.242	—0.064	+0.112
1110	28 53.01	0.752	0.803	1.405	1660	2 5.32	0.278	0.058	0.101
1120	28 11.20	0.770	0.783	1.370	1670	1 52.55	0.312	0.052	0.091
1130	27 29.71	0.790	0.764	1.337	1680	1 40.46	0.345	0.046	0.081
1140	26 48.56	0.809	0.745	1.304	1690	1 29.05	0.375	0.041	0.072
1150	+26 7.74	—0.829	—0.726	+1.270	1700	+1 18.32	+0.403	—0.036	+0.063
1160	25 27.27	0.848	0.707	1.237	1710	1 8.28	0.429	0.031	0.054
1170	24 47.16	0.866	0.689	1.206	1720	0 58.92	0.452	0.027	0.046
1180	24 7.42	0.883	0.670	1.172	1730	0 50.24	0.472	0.023	0.039
1190	23 28.06	0.899	0.652	1.141	1740	0 42.26	0.489	0.019	0.032
1200	+22 49.09	—0.914	—0.634	+1.110	1750	+0 34.96	+0.503	—0.015	+0.026
1210	22 10.51	0.927	0.616	1.078	1760	0 28.36	0.514	0.012	0.021
1220	21 32.34	0.939	0.598	1.046	1770	0 22.44	0.522	0.009	0.017
1230	20 54.57	0.950	0.581	1.017	1780	0 17.22	0.527	0.007	0.013
1240	20 17.23	0.960	0.564	0.986	1790	+0 12.68	0.529	0.005	0.010
1250	+19 40.31	—0.967	—0.547	+0.957	1800	″ +8.84	+0.528	—0.004	+0.007
1260	19 3.83	0.972	0.530	0.928	1801	8.49 —·35	0.528	0.004	0.007
1270	18 27.80	0.975	0.513	0.898	1802	8.15 ·34	0.528	0.004	0.007
1280	17 52.21	0.977	0.497	0.870	1803	7.82 ·33	0.527	0.004	0.007
1290	17 17.09	0.977	0.481	0.842	1804	7.50 ·32 ·32	0.527	0.004	0.007
1300	+16 42.43	—0.974	—0.465	+0.814	1805	+7.18	+0.527	—0.003	+0.006
1310	16 8.25	0.967	0.449	0.786	1806	6.87 —·31	0.526	0.003	0.006
1320	15 34.55	0.959	0.433	0.758	1807	6.57 ·30	0.526	0.003	0.006
1330	15 1.34	0.948	0.417	0.730	1808	6.27 ·30	0.525	0.003	0.006
1340	14 28.63	0.936	0.402	0.704	1809	5.98 ·29 ·29	0.525	0.003	0.006
1350	+13 56.44	—0.921	—0.387	+0.677	1810	+5.69	+0.524	—0.003	+0.005
1360	13 24.76	0.903	0.372	0.651	1811	5.41 —.28	0.523	0.003	0.005
1370	12 53.61	0.883	0.358	0.626	1812	5.14 .27	0.523	0.003	0.005
1380	12 22.99	0.861	0.344	0.602	1813	4.88 .26	0.522	0.003	0.005
1390	11 52.90	0.837	0.330	0.578	1814	4.62 .26 .25	0.522	0.002	0.005
1400	+11 23.35	—0.810	—0.317	+0.555	1815	+4.37	+0 521	—0.002	+0.004
1410	10 54.35	0.780	0.303	0.530	1816	4.13 —.24	0.520	0.002	0.004
1420	10 25.92	0.748	0.290	0.508	1817	3.89 .24	0.520	0.002	0.004
1430	9 58.04	0.714	0.277	0.485	1818	3.66 .23	0.519	0.002	0.004
1440	9 30.74	0.678	0.264	0.462	1819	3.44 .22 .21	0.518	0.002	0.004
1450	+ 9 4.01	—0.641	—0.252	+0.441	1820	+3.23	+0.517	—0.002	+0.003
1460	8 37.88	0.601	0.240	0.420	1821	3.02 —.21	0.516	0.002	0.003
1470	8 12.34	0.560	0.228	0.399	1822	2.82 .20	0.515	0.002	0.003
1480	7 47.39	0.518	0.216	0.378	1823	2.63 .19	0.514	0.002	0.003
1490	7 23.04	0.475	0.205	0.359	1824	2.44 .19 .18	0.513	0.002	0.003
1500	+ 6 59.29	—0.431	—0.194	+0.340	1825	+2.26	+0.512	—0.001	+0.002
1510	6 36.14	0.386	0.183	0.322	1826	2.09 —.17	0.511	0.001	0.002
1520	6 13.61	0.340	0.172	0.303	1827	1.92 .17	0.510	0.001	0.002
1530	5 51.69	0.293	0.162	0.284	1828	1.76 .16	0.509	0.001	0.002
1540	+ 5 30.40	—0.247	—0.153	+0.268	1829	+1.61 .15 —.15	—0.508	—0.001	+0.002

TABLE VI.—*Continued.*

Year.	g		1	2	3	Year.	g		1	2	3
	″						″				
1830	+1.46	−.14	+0.507	—0.001	+0.002	1885	+4.06	+.24	+0.404	—0.002	+0.003
1831	1.32	.13	0.506	0.001	0.002	1886	4.30	.24	0.402	0.002	0.004
1832	1.19	.12	0.505	0.001	0.002	1887	4.54	.25	0.399	0.002	0.004
1833	1.07	.12	0.503	0.001	0.002	1888	4.79	.26	0.397	0.002	0.004
1834	0.95	.11	0.502	0.001	0.002	1889	5.05	.27	0.394	0.002	0.004
1835	+0.84	−.10	+0.501	0	+0.001	1890	+5.32	+.27	+0.391	—0.003	+0.004
1836	0.74	.10	0.500	0	0.001	1891	5.59	.28	0.388	0.003	0.005
1837	0.64	.09	0.498	0	0.001	1892	5.87	.29	0.386	0.003	0.005
1838	0.55	.08	0.497	0	0.001	1893	6.16	.29	0.383	0.003	0.005
1839	0.47	.08	0.495	0	0	1894	6.45	.30	0.380	0.003	0.005
1840	+0.39	−.07	+0.494	0	0	1895	+6.75	+.31	+0.377	—0.003	+0.006
1841	0.32	.06	0.493	0	0	1896	7.06	.31	0.374	0.003	0.006
1842	0.26	.06	0.491	0	0	1897	7.37	.32	0.372	0.004	0.006
1843	0.20	.05	0.490	0	0	1898	7.69	.33	0.369	0.004	0.006
1844	0.15	.04	0.488	0	0	1899	8.02	.33	0.366	0.004	0.007
1845	+0.11	−.04	+0.487	0	0	1900	+8.35	+.33	+0.363	—0.004	+0.007
1846	0.07	.03	0.485	0	0	1901	8.68	.35	0.360	0.004	0.007
1847	0.04	.02	0.483	0	0	1902	9.03	.35	0.357	0.004	0.008
1848	0.02	.02	0.482	0	0	1903	9.38	.37	0.353	0.005	0.008
1849	0.00	.00	0.481	0	0	1904	9.75	.37	0.350	0.005	0.008
1850	0.00	.00	+0.479	0	0	1905	+10.12	+.38	+0.347	—0.005	+0.009
1851	0.00	.00	0.477	0	0	1906	10.50	.38	0.344	0.005	0.009
1852	0.00	+.01	0.476	0	0	1907	10.88	.39	0.340	0.005	0.010
1853	+0.01	.02	0.474	0	0	1908	11.27	.40	0.337	0.096	0.010
1854	0.03	.03	0.472	0	0	1909	11.67		0.334	0.006	9.010
1855	+0.06	+.04	+0.471	0	0	1910	+0′ 12″.07		—0.331	—0.006	+0.010
1856	0.10	.04	0.469	0	0	1920	0 16.47		0.298	0.008	0.013
1857	0.14	.05	0.467	0	0	1930	0 21.55		0.264	0.010	0.017
1858	0.19	.05	0.465	0	0	1940	0 27.30		0.229	0.012	0.021
1859	0.24	.06	0.463	0	0	1950	0 33.71		0.193	0.015	0.026
1860	+0.30	+.07	+0.461	0	0	1960	+0 40.79		+0.155	—0.018	+0.032
1861	0.37	.08	0.459	0	0	1970	0 48.53		0.115	0.022	0.039
1862	0.45	.08	0.457	0	0	1980	0 56.93		0.074	0.026	0.046
1863	0.53	.09	0.455	0	0	1990	1 5.98		0.032	0.030	0.053
1864	0.62	.09	0.453	0	0	2000	1 15.68		+0.010	0.035	0.061
1865	+0.71	+.10	+0.451	0	+0.001	2010	+1 26.03		—0.053	—0.040	+0.069
1866	0.81	.11	0.449	0	0.001	2020	1 37.03		0.096	0.045	0.078
1867	0.92	.12	0.446	0	0.001	2030	1 48.67		0.140	0.050	0.088
1868	1.04	.12	0.444	—0.001	0.001	2040	2 0.94		0.183	0.056	0.098
1869	1.16	.13	0.442	0.001	0.001	2050	2 13.85		0.226	0.062	0.108
1770	+1.29	+.14	+0.440	—0.001	+0.001	2060	+2 27.38		—0.268	—0.068	+0.119
1871	1.43	.14	0.438	0.001	0.001	2070	2 41.53		0.309	0.075	0.130
1872	1.57	.15	0.436	0.001	0.001	2080	2 56.29		0.349	0.082	0.142
1873	1.72	.16	0.433	0.001	0.001	2090	3 11.65		0.388	0.089	0.155
1874	1.88	.16	0.431	0.001	0.002	2100	3 27.61		0.427	0.096	0.168
1875	+2.04	+.17	+0.429	—0.001	+0.002	2110	+3 44.17		—0.464	—0.104	+0.182
1876	2.21	.18	0.426	0.001	0.002	2120	4 1.32		0.499	0.112	0.196
1877	2.39	.18	0.424	0.001	0.002	2130	4 19.07		0.532	0.120	0.210
1878	2.57	.19	0.422	0.001	0.002	2140	4 37.40		0.563	0.128	0.225
1879	+2.76	.20	+0.419	—0.001	+0.002	2150	4 56.31		0.593	0.137	0.240
1880	+2.96	+.20	+0.417	—0.002	+0.003	2160	+5 15.74		—0.620	—0.146	+0.256
1881	3.16	.21	0.414	0.002	0.003	2170	5 35.72		0.645	0.156	0.273
1882	3.37	.22	0.412	0.002	0.003	2180	5 56.24		0.668	0.165	0.289
1883	3.59	.23	0.409	0.002	0.003	2190	6 17.29		0.689	0.175	0.306
1884	+3.82	+.24	+0.407	—0.002	+0.003	2200	+6 38.88		—0.709	—0.184	+0.322

TABLE VII.—Equation of Centre and Principal Term of Log. *r*.

g	*E*				Log. *r*			*g*	*E*				Log. *r*		
	°	′	″	″	**1.26**				°	′	″	″	**1.26**		
0°	0	0	00.00		18915		**360°**	**10°**	0	59	26.71		22488		**350°**
10′	0	0	59.81	59.81	18916	1	50′	10′	1	0	25.41	58.70	22608	120	50′
20	0	1	59.62	59.81	18919	3	40	20	1	1	24.08	58.67	22729	121	40
30	0	2	59.42	59.80	18924	5	30	30	1	2	22.71	58.63	22852	123	30
40	0	3	59.23	59.81	18931	7	20	40	1	3	21.30	58.59	22977	125	20
50	0	4	59.03	59.80	18940	9	10	50	1	4	19.85	58.55	23104	127	10
1°	0	5	58.82	59.79	18951	11	**359°**	**11°**	1	5	18.37	58.52	23234	130	**349°**
10′	0	6	58.61	59.79	18964	13	50′	10′	1	6	16.85	58.48	23365	131	50′
20	0	7	58.39	59.78	18979	15	40	20	1	7	15.28	58.43	23498	133	40
30	0	8	58.18	59.79	18996	17	30	30	1	8	13.68	58.40	23633	135	30
40	0	9	57.96	59.78	19015	19	20	40	1	9	12.04	58.36	23770	137	20
50	0	10	57.74	59.78	19036	21	10	50	1	10	10.36	58.32	23909	139	10
2°	0	11	57.51	59.77	19059	23	**358°**	**12°**	1	11	8.63	58.27	24051	142	**348°**
10′	0	12	57.27	59.76	19084	25	50′	10′	1	12	6.86	58.23	24194	143	50′
20	0	13	57.02	59.75	19111	27	40	20	1	13	5.04	58.18	24339	145	40
30	0	14	56.76	59.74	19140	29	30	30	1	14	3.18	58.14	24485	146	30
40	0	15	56.49	59.73	19171	31	20	40	1	15	1.28	58.10	24633	148	20
50	0	16	56.22	59.73	19204	33	10	50	1	15	59.33	58.05	24784	151	10
3°	0	17	55.93	59.71	19238	34	**357°**	**13°**	1	16	57.33	58.00	24937	153	**347°**
10′	0	18	55.63	59.70	19275	37	50′	10′	1	17	55.29	57.96	25091	154	50′
20	0	19	55.33	59.70	19314	39	40	20	1	18	53.20	57.91	25247	156	40
30	0	20	55.01	59.68	19355	41	30	30	1	19	51.06	57.86	25405	158	30
40	0	21	54.68	59.67	19397	42	20	40	1	20	48.87	57.81	25566	161	20
50	0	22	54.34	59.66	19442	45	10	50	1	21	46.63	57.76	25728	162	10
4°	0	23	53.98	59.64	19489	47	**356°**	**14°**	1	22	44.35	57.72	25892	164	**346°**
10′	0	24	53.61	59.63	19538	49	50′	10′	1	23	42.01	57.66	26057	165	50′
20	0	25	53.22	59.61	19589	51	40	20	1	24	39.63	57.62	26225	168	40
30	0	26	52.81	59.59	19642	53	30	30	1	25	37.19	57.56	26395	170	30
40	0	27	52.39	59.58	19697	55	20	40	1	26	34.70	57.51	26566	171	20
50	0	28	51.95	59.56	19754	57	10	50	1	27	32.16	57.46	26740	174	10
5°	0	29	51.49	59.54	19812	58	**355°**	**15°**	1	28	29.57	57.41	26916	176	**345°**
10′	0	30	51.01	59.52	19872	60	50′	10′	1	29	26.92	57.35	27093	177	50′
20	0	31	50.52	59.51	19934	62	40	20	1	30	24.22	57.30	27271	178	40
30	0	32	50.01	59.49	19999	65	30	30	1	31	21.47	57.25	27451	180	30
40	0	33	49.48	59.47	20065	66	20	40	1	32	18.67	57.20	27634	183	20
50	0	34	48.93	59.45	20134	69	10	50	1	33	15.80	57.13	27819	185	10
6°	0	35	48.36	59.43	20205	71	**354°**	**16°**	1	34	12.88	57.08	28007	188	**344°**
10′	0	36	47.77	59.41	20278	73	50′	10′	1	35	9.90	57.02	28196	189	50′
20	0	37	47.15	59.38	20352	74	40	20	1	36	6.87	56.97	28386	190	40
30	0	38	46.51	59.36	20428	76	30	30	1	37	3.77	56.90	28578	192	30
40	0	39	45.84	59.33	20507	79	20	40	1	38	0.62	56.85	28772	194	20
50	0	40	45.15	59.31	20588	81	10	50	1	38	57.41	56.79	28968	196	10
7°	0	41	44.44	59.29	20670	82	**353°**	**17°**	1	39	54.14	56.73	29167	199	**343°**
10′	0	42	43.70	59.26	20755	85	50′	10′	1	40	50.81	56.67	29367	200	50′
20	0	43	42.94	59.24	20841	86	40	20	1	41	47.43	56.62	29568	201	40
30	0	44	42.15	59.21	20929	88	30	30	1	42	43.98	56.55	29771	203	30
40	0	45	41.33	59.18	21020	91	20	40	1	43	40.46	56.48	29976	205	20
50	0	46	40.48	59.15	21112	92	10	50	1	44	36.89	56.43	30183	207	10
8°	0	47	39.61	59.13	21206	94	**352°**	**18°**	1	45	33.25	56.36	30392	209	**342°**
10′	0	48	38.71	59.10	21302	96	50′	10′	1	46	29.55	56.30	30603	211	50′
20	0	49	37.78	59.07	21400	98	40	20	1	47	25.78	56.23	30816	213	40
30	0	50	36.82	59.04	21499	99	30	30	1	48	21.95	56.17	31030	214	30
40	0	51	35.83	59.01	21601	102	20	40	1	49	18.05	56.10	31246	216	20
50	0	52	34.81	58.98	21705	104	10	50	1	50	14.09	56.04	31465	219	10
9°	0	53	33.76	58.95	21811	106	**351°**	**19°**	1	51	10.06	55.97	31686	221	**341°**
10′	0	54	32.67	58.91	21919	108	50′	10′	1	52	5.96	55.90	31908	222	50′
20	0	55	31.55	58.88	22029	110	40	20	1	53	1.80	55.84	32132	224	40
30	0	56	30.39	58.84	22140	111	30	30	1	53	57.57	55.77	32358	226	30
40	0	57	29.20	58.81	22254	114	20	40	1	54	53.27	55.70	32585	227	20
50	0	58	27.97	58.77	22370	116	10	50	1	55	48.91	55.64	32814	229	10
10°	0	59	26.71	58.74	22488	118	**350°**	**20°**	1	56	44.47	55.56	33045	231	**340°**
					1.26		*g*						**1.26**		*g*

TABLE VII.—*Continued.*

g	E		Log. r			g	E		Log. r		
	° ′ ″	″	1.26				° ′ ″	″	1.26		
20°	1 56 44.47		33045		**340°**	**30°**	2 49 51.28		50121		**330°**
10′	1 57 39.96	55.49	33278	233	50′	10′	2 50 41.63	50.35	50457	336	50′
20	1 58 35.39	55.43	33513	235	40	20	2 51 31.87	50.24	50795	338	40
30	1 59 30.74	55.35	33749	236	30	30	2 52 22.01	50.14	51134	339	30
40	2 0 26.02	55.28	33987	238	20	40	2 53 12.06	50.05	51475	341	20
50	2 1 21.23	55.21	34227	240	10	50	2 54 2.00	49.94	51818	343	10
21°	2 2 16.36	55.13	34468	241	**339°**	**31°**	2 54 51.83	49.83	52162	344	**329°**
10′	2 3 11.42	55.06	34711	243	50′	10′	2 55 41.56	49.73	52507	345	50′
20	2 4 6.41	54.99	34956	245	40	20	2 56 31.18	49.62	52854	347	40
30	2 5 1.33	54.92	35204	248	30	30	2 57 20.70	49.52	53203	349	30
40	2 5 56.17	54.84	35454	250	20	40	2 58 10.12	49.42	53553	350	20
50	2 6 50.93	54.76	35705	251	10	50	2 58 59.43	49.31	53905	352	10
22°	2 7 45.61	54.68	35957	252	**338°**	**32°**	2 59 48.63	49.20	54258	353	**328°**
10′	2 8 40.22	54.61	36211	254	50′	10′	3 0 37.73	49.10	54613	355	50′
20	2 9 34.76	54.54	36467	256	40	20	3 1 26.73	49 00	54970	357	40
30	2 10 29.22	54.46	36726	259	30	30	3 2 15.62	48.89	55329	359	30
40	2 11 23.60	54.38	36986	260	20	40	3 3 4.41	48.79	55689	360	20
50	2 12 17.90	54.30	37248	262	10	50	3 3 53.09	48.68	56050	361	10
23°	2 13 12.12	54.22	37511	263	**337°**	**33°**	3 4 41.66	48.57	56413	363	**327°**
10′	2 14 6.26	54.14	37776	265	50′	10′	3 5 30.12	48.46	56777	364	50′
20	2 15 0.32	54.06	38043	267	40	20	3 6 18.47	48.35	57143	366	40
30	2 15 54.30	53.98	38311	268	30	30	3 7 6.72	48.25	57510	367	30
40	2 16 48.20	53.90	38581	270	20	40	3 7 54.85	48.13	57879	369	20
50	2 17 42.02	53.82	38853	272	10	50	3 8 42.87	48.02	58249	370	10
24°	2 18 35.75	53.73	39128	275	**336°**	**34°**	3 9 30.78	47.91	58621	372	**326°**
10′	2 19 29.40	53.65	39404	276	50′	10	3 10 18.58	47.80	58994	373	50′
20	2 20 22.97	53.57	39681	277	40	20	3 11 6.26	47.68	59369	375	40
30	2 21 16.46	53.49	39960	279	30	30	3 11 53.83	47.57	59746	377	30
40	2 22 9.87	53.41	40241	281	20	40	3 12 41.29	47.46	60124	378	20
50	2 23 3.19	53.32	40524	283	10	50	3 13 28.64	47.35	60503	379	10
25°	2 23 56.42	53.23	40808	284	**335°**	**35°**	3 14 15.87	47.23	60884	381	**325°**
10′	2 24 49.57	53.15	41094	286	50′	10′	3 15 2.99	47.12	61266	382	50′
20	2 25 42.63	53.06	41382	288	40	20	3 15 50.00	47.01	61650	384	40
30	2 26 35.61	52.98	41671	289	30	30	3 16 36.89	46.89	62035	385	30
40	2 27 28.50	52.89	41962	291	20	40	3 17 23.67	46.78	62422	387	20
50	2 28 21.30	52.80	42255	293	10	50	3 18 10.33	46.66	62810	388	10
26°	2 29 14.01	52.71	42550	295	**334°**	**36°**	3 18 56.88	46.55	63200	390	**324°**
10′	2 30 6.63	52.62	42846	296	50′	10′	3 19 43.31	46.43	63592	392	50′
20	2 30 59.16	52.53	43144	298	40	20	3 20 29.63	46.32	63985	393	40
30	2 31 51.60	52.44	43444	300	30	30	3 21 15.83	46.20	64380	395	30
40	2 32 43.95	52.35	43745	301	20	40	3 22 1.91	46.08	64776	396	20
50	2 33 36.21	52.26	44048	303	10	50	3 22 47.88	45.97	65173	397	10
27°	2 34 28.38	52.17	44353	305	**333°**	**37°**	3 23 33.72	45.84	65572	399	**323°**
10′	2 35 20.46	52.08	44659	306	50′	10	3 24 19.44	45.72	65972	400	50′
20	2 36 12.44	51.98	44967	308	40	20	3 25 5.04	45.60	66373	401	40
30	2 37 4.34	51.90	45277	310	30	30	3 25 50.53	45.49	66776	403	30
40	2 37 56.14	51.80	45588	311	20	40	3 26 35.89	45.36	67180	404	20
50	2 38 47.85	51.71	45901	313	10	50	3 27 21.13	45.24	67585	405	10
28°	2 39 39.46	51.61	46216	315	**332°**	**38°**	3 28 6.25	45.12	67992	407	**322°**
10′	2 40 30.98	51.52	46532	316	50′	10′	3 28 51.25	45.00	68400	408	50′
20	2 41 22.40	51.42	46850	318	40	20	3 29 36.13	44.88	68810	410	40
30	2 42 13.72	51.32	47170	320	30	30	3 30 20.90	44.77	69221	411	30
40	2 43 4.95	51.23	47491	321	20	40	3 31 5.54	44.64	69633	412	20
50	2 43 56.08	51.13	47814	323	10	50	3 31 50.05	44.52	70047	414	10
29°	2 44 47.12	51.04	48138	324	**331°**	**39°**	3 32 34.45	44.40	70463	416	**321°**
10′	2 45 38.06	50.94	48464	326	50′	10′	3 33 18.72	44.27	70880	417	50′
20	2 46 28.90	50.84	48792	328	40	20	3 34 2.86	44.14	71298	418	40
30	2 47 19.64	50.74	49122	330	30	30	3 34 46.88	44.02	71717	419	30
40	2 48 10.29	50.65	49453	331	20	40	3 35 30.78	43.90	72138	420	20
50	2 49 0.83	50.54	49786	333	10	50	3 36 14.55	43.77	72560	422	10
30°	2 49 51.28	50.45	50121	335	**330°**	**40°**	3 36 58.19	43.64	72984	424	**320°**
			1.26		*g*				1.26		*g*

TABLE VII.—*Continued.*

g	*E*		Log. *r*			*g*	*E*		Log. *r*		
	° ′ ″	″	**1.26**				° ′ ″	″	**1.27**		
40°	3 36 58.19		72984		**320°**	**50°**	4 16 34.27		00687		**310°**
10′	3 37 41.71	43.52	73409	425	50′	10′	4 17 9.62	35.35	01183	496	50′
20	3 38 25.10	43.39	73835	426	40	20	4 17 44.82	35.20	01681	498	40
30	3 39 8.37	43.27	74262	427	30	30	4 18 19.88	35.06	02180	499	30
40	3 39 51.51	43.14	74691	429	20	40	4 18 54.80	34.92	02680	500	20
50	3 40 34.52	43.01	75121	430	10	50	4 19 29.57	34.77	03181	501	10
41°	3 41 17.40	42.88	75553	432	**319°**	**51°**	4 20 4.19	34.62	03683	502	**309°**
10′	3 42 0.16	42.76	75986	433	50′	10′	4 20 38.67	34.48	04186	503	50′
20	3 42 42.79	42.63	76420	434	40	20	4 21 13.00	34.33	04690	504	40
30	3 43 25.29	42.50	76856	436	30	30	4 21 47.18	34.18	05195	505	30
40	3 44 7.66	42.37	77293	437	20	40	4 22 21.22	34.04	05701	505	20
50	3 44 49.91	42.25	77731	438	10	50	4 22 55.11	33.89	06207	506	10
42°	3 45 32.02	42.11	78171	440	**318°**	**52°**	4 23 28.85	33.74	06714	507	**308°**
10′	3 46 14.01	41.99	78611	440	50′	10′	4 24 2.44	33.59	07222	508	50′
20	3 46 55.86	41.85	79053	442	40	20	4 24 35.88	33.44	07732	510	40
30	3 47 37.58	41.72	79496	443	30	30	4 25 9.18	33.30	08243	511	30
40	3 48 19.17	41.59	79941	445	20	40	4 25 42.32	33.14	08754	511	20
50	3 49 0.63	41.46	80387	446	10	50	4 26 15.32	33.00	09266	512	10
43°	3 49 41.96	41.33	80835	448	**317°**	**53°**	4 26 48.16	32.84	09780	514	**307°**
10′	3 50 23.16	41.20	81284	449	50′	10′	4 27 20.85	32.69	10294	514	50′
20	3 51 4.22	41.06	81733	449	40	20	4 27 53.39	32.54	10809	515	40
30	3 51 45.15	40.93	82183	450	30	30	4 28 25.79	32.40	11325	516	30
40	3 52 25.94	40.79	82635	452	20	40	4 28 58.03	32.24	11842	517	20
50	3 53 6.60	40.66	83089	454	10	50	4 29 30.12	32.09	12361	519	10
44°	3 53 47.13	40.53	83544	455	**316°**	**54°**	4 30 2.06	31.94	12881	520	**306°**
10′	3 54 27.52	40.39	84000	456	50′	10′	4 30 33.85	31.79	13401	520	50′
20	3 55 7.78	40.26	84457	457	40	20	4 31 5.49	31.64	13922	521	40
30	3 55 47.90	40.12	84914	457	30	30	4 31 36.98	31.49	14444	522	30
40	3 56 27.89	39.99	85373	459	20	40	4 32 8.31	31.33	14966	522	20
50	3 57 7.75	39.86	85834	461	10	50	4 32 39.50	31.19	15490	524	10
45°	3 57 47.47	39.72	86297	463	**315°**	**55°**	4 33 10.53	31.03	16015	525	**305°**
10′	3 58 27.05	39.58	86760	463	50′	10′	4 33 41.41	30.88	16540	525	50′
20	3 59 6.50	39.45	87224	464	40	20	4 34 12.14	30.73	17067	527	40
30	3 59 45.81	39.31	87690	466	30	30	4 34 42.71	30.57	17595	528	30
40	4 0 24.98	39.17	88156	466	20	40	4 35 13.13	30.42	18123	528	20
50	4 1 4.01	39.03	88624	468	10	50	4 35 43.40	30.27	18652	529	10
46°	4 1 42.90	38.89	89093	469	**314°**	**56°**	4 36 13.52	30.12	19182	530	**304°**
10′	4 2 21.65	38.75	89564	471	50′	10′	4 36 43.48	29.96	19712	530	50′
20	4 3 0.27	38 62	90035	471	40	20	4 37 13.29	29.81	20243	531	40
30	4 3 38.75	38.48	90507	472	30	30	4 37 42.94	29.65	20776	533	30
40	4 4 17.09	38.34	90980	473	20	40	4 38 12.43	29.49	21309	533	20
50	4 4 55.30	38.21	91455	475	10	50	4 38 41.77	29.34	21842	533	10
47°	4 5 33.36	38.06	91931	476	**313°**	**57°**	4 39 10.96	29.19	22376	534	**303°**
10′	4 6 11.29	37.93	92408	477	50′	10′	4 39 39.99	29.03	22911	535	50′
20	4 6 49.07	37.78	92886	478	40	20	4 40 8.87	28.88	23448	537	40
30	4 7 26.72	37.65	93365	479	30	30	4 40 37.59	28.72	23986	538	30
40	4 8 4.22	37.50	93845	480	20	40	4 41 6.16	28.57	24524	538	20
50	4 8 41.58	37.36	94327	482	10	50	4 41 34.57	28.41	25063	539	10
48°	4 9 18.80	37.22	94810	483	**312°**	**58°**	4 42 2.83	28.26	25603	540	**302°**
10′	4 9 55.88	37.08	95294	484	50′	10′	4 42 30.93	28.10	26143	540	50′
20	4 10 32.81	36.93	95779	485	40	20	4 42 58.87	27.94	26684	541	40
30	4 11 9.60	36.79	96265	486	30	30	4 43 26.66	27.79	27226	542	30
40	4 11 46.25	36.65	96752	487	20	40	4 43 54.29	27.63	27768	542	20
50	4 12 22.76	36.51	97240	488	10	50	4 44 21.76	27.47	28311	543	10
49°	4 12 59.12	36.36	97730	490	**311°**	**59°**	4 44 49.07	27.31	28855	544	**301°**
10′	4 13 35.34	36.22	98220	490	50′	10′	4 45 16.23	27.16	29399	544	50′
20	4 14 11.41	36.07	98711	491	40	20	4 45 43.22	26.99	29944	545	40
30	4 14 47.34	35.93	99203	492	30	30	4 46 10.06	26.84	30491	547	30
40	4 15 23.13	35.79	99696	493	20	40	4 46 36.75	26.69	31038	547	20
50	4 15 58.77	35.64	*00191	495	10	50	4 47 3.27	26.52	31587	549	10
50°	4 16 34.27	35.50	*00687	496	**310°**	**60°**	4 47 29.64	26.37	32137	550	**300°**
			***1.27**		*g*				**1.27**		*g*

TABLE VII.—*Continued.*

g	*E*		Log. *r*			*g*	*E*		Log. *r*		
	° ′ ″	″	**1.27**				° ′ ″	″	**1.27**		
60°	4 47 29.64		32137		**300°**	**70°**	5 8 57.01		66156		**290°**
10′	4 47 55.85	26.21	32687	550	50′	10′	5 9 13.48	16.47	66738	582	50′
20	4 48 21.90	26.05	33237	550	40	20	5 9 29.79	16.31	67320	582	40
30	4 48 47.79	25.89	33787	550	30	30	5 9 45.93	16.14	67903	583	30
40	4 49 13.53	25.74	34338	551	20	40	5 10 1.91	15.98	68487	584	20
50	4 49 39.10	25.57	34890	552	10	50	5 10 17.72	15.81	69071	584	10
61°	4 50 4.52	25.42	35443	553	**299°**	**71°**	5 10 33.36	15.64	69655	584	**289°**
10′	4 50 29.78	25.26	35996	553	50′	10′	5 10 48.84	15.48	70239	584	50′
20	4 50 54.88	25.10	36550	554	40	20	5 11 4.15	15.31	70824	585	40
30	4 51 19.82	24.94	37105	555	30	30	5 11 19.30	15.15	71409	585	30
40	4 51 44.60	24.78	37660	555	20	40	5 11 34.28	14.98	71994	585	20
50	4 52 9.22	24.62	38216	556	10	50	5 11 49.09	14.81	72579	585	10
62°	4 52 33.68	24.46	38773	557	**298°**	**72°**	5 12 3.74	14.65	73165	586	**288°**
10′	4 52 57.98	24.30	39330	557	50′	10′	5 12 18.23	14.49	73751	586	50′
20	4 53 22.11	24.13	39888	558	40	20	5 12 32.55	14.32	74337	586	40
30	4 53 46.09	23.98	40446	558	30	30	5 12 46.70	14.15	74924	587	30
40	4 54 9.90	23.81	41005	559	20	40	5 13 0.69	13.99	75511	587	20
50	4 54 33 56	23.66	41565	560	10	50	5 13 14.51	13.82	76098	587	10
63°	4 54 57.05	23.49	42126	561	**297°**	**73°**	5 13 28.17	13.66	76685	587	**287°**
10′	4 55 20 38	23.33	42687	561	50′	10′	5 13 41.66	13.49	77273	588	50′
20	4 55 43.56	23.18	43249	562	40	20	5 13 54.98	13.32	77861	588	40
30	4 56 6.57	23.01	43811	562	30	30	5 14 8.13	13.15	78450	589	30
40	4 56 29.42	22.85	44374	563	20	40	5 14 21.12	12.99	79039	589	20
50	4 56 52.11	22.69	44938	564	10	50	5 14 33.94	12.82	79628	589	10
64°	4 57 14.64	22.53	45503	565	**296°**	**74°**	5 14 46.60	12.66	80217	589	**286°**
10′	4 57 37.01	22.37	46068	565	50′	10′	5 14 59.09	12.49	80807	590	50′
20	4 57 59.21	22.20	46633	565	40	20	5 15 11.41	12.32	81397	590	40
30	4 58 21.25	22.04	47199	566	30	30	5 15 23.57	12.16	81987	590	30
40	4 58 43.13	21.88	47766	567	20	40	5 15 35.56	11.99	82577	590	20
50	4 59 4.84	21.71	48333	567	10	50	5 15 47.39	11.83	83168	591	10
65°	4 59 26.40	21.56	48901	568	**295°**	**75°**	5 15 59.05	11.66	83759	591	**285°**
10′	4 59 47.79	21.39	49469	568	50′	10′	5 16 10.54	11.49	84350	591	50′
20	5 0 9.02	21.23	50038	569	40	20	5 16 21.87	11.33	84941	591	40
30	5 0 30.09	21.07	50607	569	30	30	5 16 33.03	11.16	85532	591	30
40	5 0 50.99	20.90	51177	570	20	40	5 16 44.03	11.00	86124	592	20
50	5 1 11.73	20.74	51747	570	10	50	5 16 54.86	10.83	86715	591	10
66°	5 1 32.31	20.58	52318	571	**294°**	**76°**	5 17 5.52	10.66	87307	592	**284°**
10′	5 1 52.72	20.41	52889	571	50′	10′	5 17 16.02	10.50	87899	592	50′
20	5 2 12.97	20.25	53461	572	40	20	5 17 26.35	10.33	88491	592	40
30	5 2 33.06	20.09	54033	572	30	30	5 17 36.51	10.16	89084	593	30
40	5 2 52.98	19.92	54606	573	20	40	5 17 46.51	10.00	89677	593	20
50	5 3 12.74	19.76	55179	573	10	50	5 17 56.34	9.83	90270	593	10
67°	5 3 32.34	19.60	55753	574	**293°**	**77°**	5 18 6.00	9.66	90863	593	**283°**
10′	5 3 51.77	19.43	56327	574	50′	10′	5 18 15.50	9.50	91456	593	50′
20	5 4 11.04	19.27	56901	574	40	20	5 18 24.82	9.32	92049	593	40
30	5 4 30.15	19.11	57476	575	30	30	5 18 33.99	9.17	92643	594	30
40	5 4 49.09	18.94	58051	575	20	40	5 18 42.98	8.99	93237	594	20
50	5 5 7.87	18.78	58627	576	10	50	5 18 51.81	8.83	93830	593	10
68°	5 5 26.49	18.62	59204	577	**292°**	**78°**	5 19 0.47	8.66	94424	594	**282°**
10	5 5 44.94	18.45	59781	577	50′	10′	5 19 8.97	8.50	95018	594	50′
20	5 6 3.23	18.29	60358	577	40	20	5 19 17.30	8.33	95612	594	40
30	5 6 21.35	18.12	60936	578	30	30	5 19 25.46	8.16	96206	594	30
40	5 6 39.30	17.95	61514	578	20	40	5 19 33.46	8.00	96800	594	20
50	5 6 57.09	17.79	62093	579	10	50	5 19 41.29	7.83	97395	595	10
69°	5 7 14.72	17.63	62673	579	**291°**	**79°**	5 19 48.95	7.66	97990	595	**281°**
10′	5 7 32.18	17.46	63252	579	50′	10′	5 19 56.45	7.50	98585	595	50′
20	5 7 49.48	17.30	63832	580	40	20	5 20 3.78	7.33	99179	594	40
30	5 8 6.61	17.13	64412	580	30	30	5 20 10.94	7.16	99774	595	30
40	5 8 23.57	16.96	64993	581	20	40	5 20 17.94	7.00	*00369	595	20
50	5 8 40.37	16.80	65574	581	10	50	5 20 24.77	6.83	*00965	596	10
70°	5 8 57.01	16.64	66156	582	**290°**	**80°**	5 20 31.44	6.67	*01560	595	**280°**
			1.27		*g*				***1.28**		*g*

TABLE VII.—*Continued.*

g	*E*				Log. *r*			*g*	*E*				Log. *r*		
	°	′	″	″	**1.28**				°	′	″	″	**1.28**		
80°	5	20	31.44		01560		**280°**	**90°**	5	22	9.05		37187		**270°**
10′	5	20	37.94	6.50	02155	595	50′	10′	5	22	5.68	3.37	37776	589	50
20	5	20	44.28	6.34	02750	595	40	20	5	22	2.15	3.53	38365	589	40
30	5	20	50.45	6.17	03346	596	30	30	5	21	58.45	3.70	38954	589	30
40	5	20	56.46	6.01	03941	595	20	40	5	21	54.59	3.86	39543	589	20
50	5	21	2.30	5.84	04536	595	10	50	5	21	50.58	4.01	40131	588	10
81°	5	21	7.96	5.66	05131	595	**279°**	**91°**	5	21	46.40	4.18	40719	588	**269°**
10′	5	21	13.46	5.50	05726	595	50′	10′	5	21	42.06	4.34	41307	588	50
20	5	21	18.80	5.34	06322	596	40	20	5	21	37.57	4.49	41894	587	40
30	5	21	23.97	5.17	06917	595	30	30	5	21	32.91	4.66	42482	588	30
40	5	21	28.97	5.00	07512	595	20	40	5	21	28.09	4.82	43069	587	20
50	5	21	33.81	4.84	08107	595	10	50	5	21	23.11	4.98	43655	586	10
82°	5	21	38.48	4.67	08702	595	**278°**	**92°**	5	21	17.97	5.14	44242	587	**268°**
10′	5	21	42.99	4.51	09297	595	50′	10′	5	21	12.67	5.30	44828	586	50′
20	5	21	47.33	4.34	09893	596	40	20	5	21	7.20	5.47	45414	586	40
30	5	21	51.50	4.17	10488	595	30	30	5	21	1.58	5.62	46000	586	30
40	5	21	55.51	4.01	11083	595	20	40	5	20	55.79	5.79	46585	585	20
50	5	21	59.36	3.85	11679	596	10	50	5	20	49.85	5.94	47170	585	10
83°	5	22	3.04	3.68	12274	595	**277°**	**93°**	5	20	43.74	6.11	47755	585	**267°**
10′	5	22	6.56	3.52	12869	595	50′	10′	5	20	37.48	6.26	48339	584	50′
20	5	22	9.91	3.35	13465	596	40	20	5	20	31.05	6.43	48923	584	40
30	5	22	13.09	3.18	14060	595	30	30	5	20	24.47	6.58	49507	584	30
40	5	22	16.11	3.02	14655	595	20	40	5	20	17.72	6.75	50090	583	20
50	5	22	18.97	2.86	15250	595	10	50	5	20	10.82	6.90	50673	583	10
84°	5	22	21.66	2.69	15846	596	**276°**	**94°**	5	20	3.76	7.06	51256	583	**266°**
10′	5	22	24.19	2.53	16441	595	50′	10′	5	19	56.54	7.22	51838	582	50′
20	5	22	26.55	2.36	17036	595	40	20	5	19	49.16	7.38	52420	582	40
30	5	22	28.75	2.20	17631	595	30	30	5	19	41.62	7.54	53002	582	30
40	5	22	30.78	2.03	18226	595	20	40	5	19	33.93	7.69	53583	581	20
50	5	22	32.64	1.86	18820	594	10	50	5	19	26.07	7.86	54163	580	10
85°	5	22	34.34	1.70	19415	595	**275°**	**95°**	5	19	18.06	8.01	54743	580	**265°**
10′	5	22	35.87	1.53	20010	595	50′	10′	5	19	9.89	8.17	55323	580	50′
20	5	22	37.24	1.37	20604	594	40	20	5	19	1.56	8.33	55902	579	40
30	5	22	38.44	1.20	21198	594	30	30	5	18	53.06	8.50	56481	579	30
40	5	22	39.48	1.04	21792	594	20	40	5	18	44.42	8.64	57060	579	20
50	5	22	40.35	0.87	22386	594	10	50	5	18	35.61	8.81	57638	578	10
86°	5	22	41.06	0.71	22980	594	**274°**	**96°**	5	18	26.65	8.96	58216	578	**264°**
10′	5	22	41.60	0.54	23574	594	50′	10′	5	18	17.53	9.12	58794	578	50′
20	5	22	41.99	0.39	24167	593	40	20	5	18	8.26	9.27	59371	577	40
30	5	22	42.21	0.22	24761	594	30	30	5	17	58.83	9.43	59948	577	30
40	5	22	42.26	0.05	25354	593	20	40	5	17	49.24	9.59	60525	577	20
50	5	22	42.15	0.11	25948	594	10	50	5	17	39.50	9.74	61101	576	10
87°	5	22	41.88	0.27	26541	593	**273°**	**97°**	5	17	29.60	9.90	61677	576	**263°**
10′	5	22	41.45	0.43	27134	593	50′	10′	5	17	19.54	10.06	62252	575	50′
20	5	22	40.85	0.60	27726	592	40	20	5	17	9.33	10.21	62827	575	40
30	5	22	40.09	0.76	28319	593	30	30	5	16	58.96	10.37	63401	574	30
40	5	22	39.16	0.93	28911	592	20	40	5	16	48.44	10.52	63975	574	20
50	5	22	38.07	1.09	29504	593	10	50	5	16	37.76	10.68	64548	573	10
88°	5	22	36.82	1.25	30096	592	**272°**	**98°**	5	16	26.92	10.84	65120	572	**262°**
10′	5	22	35.40	1.42	30688	592	50′	10′	5	16	15.93	10.99	65692	572	50′
20	5	22	33.82	1.58	31280	592	40	20	5	16	4.78	11.15	66264	572	40
30	5	22	32.08	1.74	31872	592	30	30	5	15	53.48	11.30	66836	572	30
40	5	22	30.17	1.91	32464	592	20	40	5	15	42.02	11.46	67407	571	20
50	5	22	28.10	2.07	33055	591	10	50	5	15	30.41	11.61	67977	570	10
89°	5	22	25.86	2.24	33647	592	**271°**	**99°**	5	15	18.64	11.77	68547	570	**261°**
10′	5	22	23.47	2.39	34238	591	50′	10′	5	15	6.72	11.92	69117	570	50′
20	5	22	20.91	2.56	34828	590	40	20	5	14	54.64	12.08	69686	569	40
30	5	22	18.19	2.72	35418	590	30	30	5	14	42.41	12.23	70255	569	30
40	5	22	15.31	2.88	36008	590	20	40	5	14	30.03	12.38	70823	568	20
50	5	22	12.26	3.05	36597	589	10	50	5	14	17.49	12.54	71391	568	10
90°	5	22	9.05	3.21	37187	590	**270°**	**100°**	5	14	4.80	12.69	71958	567	**260°**
					1.23		*g*						**1.28**		*g*

TABLE VII.—*Continued.*

g	E		Log. r			g	E		Log. r		
	° ′ ″	″	1.28				° ′ ″	″	1.29		
100°	5 14 4.80		71958		**260°**	**110°**	4 56 49.88		04888		**250°**
10′	5 13 51.96	12.84	72525	567	50′	10′	4 56 28.20	21.68	05416	528	50′
20	5 13 38.96	13.00	73091	566	40	20	4 56 6.39	21.81	05944	528	40
30	5 13 25.81	13.15	73656	565	30	30	4 55 44.44	21.95	06471	527	30
40	5 13 12.51	13.30	74221	565	20	40	4 55 22.34	22.10	06997	526	20
50	5 12 59.06	13.45	74785	564	10	50	4 55 0.10	22.24	07522	525	10
101°	5 12 45.45	13.61	75348	563	**259°**	**111°**	4 54 37.72	22.38	08047	525	**249°**
10′	5 12 31.69	13.76	75911	563	50′	10′	4 54 15.21	22.51	08571	524	50′
20	5 12 17.78	13.91	76474	563	40	20	4 53 52.55	22.66	09094	523	40
30	5 12 3.72	14.06	77036	562	30	30	4 53 29.76	22.79	09616	522	30
40	5 11 49.50	14.22	77598	562	20	40	4 53 6.83	22.93	10137	521	20
50	5 11 35.14	14.36	78159	561	10	50	4 52 43.76	23.07	10658	521	10
102°	5 11 20.62	14.52	78720	561	**258°**	**112°**	4 52 20.55	23.21	11178	520	**248°**
10′	5 11 5.95	14.67	79280	560	50′	10′	4 51 57.20	23.35	11697	519	50′
20	5 10 51.13	14.82	79839	559	40	20	4 51 33.72	23.48	12215	518	40
30	5 10 36.16	14.97	80398	559	30	30	4 51 10.10	23.62	12732	517	30
40	5 10 21.04	15.12	80956	558	20	40	4 50 46.34	23.76	13248	516	20
50	5 10 5.77	15.27	81514	558	10	50	4 50 22.44	23.90	13764	516	10
103°	5 9 50.35	15.42	82071	557	**257°**	**113°**	4 49 58.41	24.03	14279	515	**247°**
10′	5 9 34.78	15.57	82628	557	50	10′	4 49 34.24	24.17	14793	514	50′
20	5 9 19.06	15.72	83184	556	40	20	4 49 9.94	24.30	15306	513	40
30	5 9 3.19	15.87	83739	555	30	30	4 48 45.50	24.44	15818	512	30
40	5 8 47.17	16.02	84294	555	20	40	4 48 20.92	24.58	16329	511	20
50	5 8 31.01	16.16	84849	555	10	50	4 47 56.21	24.71	16840	511	10
104°	5 8 14.69	16.32	85403	554	**256°**	**114°**	4 47 31.36	24.85	17350	510	**246°**
10′	5 7 58.22	16.47	85956	553	50′	10′	4 47 6.38	24.98	17859	509	50′
20	5 7 41.61	16.61	86508	552	40	20	4 46 41.26	25.12	18367	508	40
30	5 7 24.85	16.76	87059	551	30	30	4 46 16.00	25.26	18874	507	30
40	5 7 7.94	16.91	87610	551	20	40	4 45 50.61	25.39	19380	506	20
50	5 6 50.88	17.06	88160	550	10	50	4 45 25.09	25.52	19886	506	10
105°	5 6 33.67	17.21	88709	549	**255°**	**115°**	4 44 59.44	25.65	20391	505	**245°**
10′	5 6 16.32	17.35	89258	549	50′	10′	4 44 33.66	25.78	20895	504	50′
20	5 5 58.82	17.50	89806	548	40	20	4 44 7.74	25.92	21398	503	40
30	5 5 41.17	17.65	90354	548	30	30	4 43 41.69	26.05	21900	502	30
40	5 5 23.37	17.80	90902	548	20	40	4 43 15.50	26.19	22401	501	20
50	5 5 5.42	17.95	91449	547	10	50	4 42 49.19	26.31	22901	500	10
106°	5 4 47.33	18.09	91995	546	**254°**	**116°**	4 42 22.74	26.45	23401	500	**244°**
10′	5 4 29.10	18 23	92540	545	50′	10′	4 41 56.16	26.58	23899	498	50′
20	5 4 10.72	18.38	93085	545	40	20	4 41 29.45	26.71	24397	498	40
30	5 3 52.19	18.53	93629	544	30	30	4 41 2.60	26.85	24894	497	30
40	5 3 33.52	18.67	94172	543	20	40	4 40 35.62	26.98	25390	496	20
50	5 3 14.70	18.82	94715	543	10	50	4 40 8.52	27.10	25885	495	10
107°	5 2 55.74	18.96	95257	542	**253°**	**117°**	4 39 41.28	27.24	26380	495	**243°**
10′	5 2 36.63	19.11	95798	541	50′	10′	4 39 13.91	27.37	26874	494	50′
20	5 2 17.38	19.25	96338	540	40	20	4 38 46.41	27.50	27366	492	40
30	5 1 57.99	19.39	96878	540	30	30	4 38 18.78	27.63	27857	491	30
40	5 1 38.45	19.54	97417	539	20	40	4 37 51.02	27.76	28348	491	20
50	5 1 18.77	19.68	97955	538	10	50	4 37 23 13	27.89	28837	489	10
108°	5 0 58.94	19.83	98493	538	**252°**	**118°**	4 36 55.12	28.01	29325	488	**242°**
10′	5 0 38.97	19.97	99030	537	50′	10′	4 36 26.98	28.14	29812	487	50′
20	5 0 18.86	20.11	99566	536	40	20	4 35 58.71	28.27	30299	487	40
30	4 59 58.60	20.26	*00101	535	30	30	4 35 30.31	28.40	30785	486	30
40	4 59 38.21	20.39	*00636	535	20	40	4 35 1.78	28.53	31270	485	20
50	4 59 17.67	20.54	*01170	534	10	50	4 34 33.13	28.65	31754	484	10
109°	4 58 56.98	20.69	*01703	533	**251°**	**119°**	4 34 4.35	28.78	32237	483	**241°**
10′	4 58 36.15	20.83	*02236	533	50′	10′	4 33 35.44	28.91	32719	482	50′
20	4 58 15.18	20.97	*02768	532	40	20	4 33 6.41	29.03	33200	481	40
30	4 57 54.07	21.11	*03299	531	30	30	4 32 37.25	29.16	33681	481	30
40	4 57 32.81	21.26	*03829	530	20	40	4 32 7.96	29.29	34161	480	20
50	4 57 11.42	21.39	*04359	530	10	50	4 31 38.55	29.41	34640	479	10
110°	4 56 49.88	21.54	*04888	529	**250°**	**120°**	4 31 9.01	29.54	35117	477	**240°**
			*1.29		g				1.29		g

TABLE VII.—*Continued.*

g	E		Log. r			g	E		Log. r		
			1.29						**1.29**		
120°	4 31 9.01		35117		**240°**	**130°**	3 57 57.75		61903		**230°**
10′	4 30 39.35	29.66	35593	476	50′	10′	3 57 21.10	36.65	62317	414	50′
20	4 30 9.56	29.79	36069	476	40	20	3 56 44.35	36.75	62729	412	40
30	4 29 39.65	29.91	36544	475	30	30	3 56 7.48	36.87	63140	411	30
40	4 29 9.61	30.04	37018	474	20	40	3 55 30.51	36.97	63550	410	20
50	4 28 39.45	30.16	37491	473	10	50	3 54 53.43	37.08	63959	409	10
121°	4 28 9.16	30.29	37963	472	**239°**	**131°**	3 54 16.25	37.18	64366	407	**229°**
10′	4 27 38.75	30.41	38434	471	50′	10′	3 53 38.97	37.28	64773	407	50′
20	4 27 8.22	30.53	38904	470	40	20	3 53 1.58	37.39	65179	406	40
30	4 26 37.56	30.66	39372	468	30	30	3 52 24.08	37.50	65583	404	30
40	4 26 6.78	30.78	39840	468	20	40	3 51 46.48	37.60	65987	404	20
50	4 25 35.88	30.90	40306	466	10	50	3 51 8.78	37.70	66389	402	10
122°	4 25 4.85	31.03	40771	465	**238°**	**132°**	3 50 30.97	37.81	66790	401	**228°**
10′	4 24 33.71	31.14	41236	465	50′	10′	3 49 53.06	37.91	67190	400	50′
20	4 24 2.44	31.27	41700	464	40	20	3 49 15.04	38.02	67588	398	40
30	4 23 31.05	31.39	42163	463	30	30	3 48 36.92	38.12	67985	397	30
40	4 22 59.54	31.51	42625	462	20	40	3 47 58.70	38.22	68382	397	20
50	4 22 27.91	31.63	43086	461	10	50	3 47 20.38	38.32	68777	395	10
123°	4 21 56.16	31.75	43546	460	**237°**	**133°**	3 46 41.95	38.43	69171	394	**227°**
10′	4 21 24.29	31.87	44005	459	50′	10′	3 46 3.42	38.53	69564	393	50′
20	4 20 52.29	32.00	44463	458	40	20	3 45 24.79	38.63	69955	391	40
30	4 20 20.18	32.11	44919	456	30	30	3 44 46.06	38.73	70345	390	30
40	4 19 47.94	32.24	45375	456	20	40	3 44 7.23	38.83	70734	389	20
50	4 19 15.59	32.35	45829	454	10	50	3 43 28.30	38.93	71122	388	10
124°	4 18 43.12	32.47	46282	453	**236°**	**134°**	3 42 49.27	39.03	71509	387	**226°**
10′	4 18 10.53	32.59	46735	453	50′	10′	3 42 10.14	39.13	71895	386	50′
20	4 17 37.83	32.70	47186	451	40	20	3 41 30.91	39.23	72279	384	40
30	4 17 5.01	32.82	47636	450	30	30	3 40 51.58	39.33	72662	383	30
40	4 16 32.07	32.94	48086	450	20	40	3 40 12.15	39.43	73044	382	20
50	4 15 59.01	33.06	48534	448	10	50	3 39 32.62	39.53	73425	381	10
125°	4 15 25.84	33.17	48981	447	**235°**	**135°**	3 38 53.00	39.62	73805	380	**225°**
10′	4 14 52.55	33.29	49428	447	50′	10′	3 38 13.28	39.72	74184	379	50′
20	4 14 19.14	33.41	49873	445	40	20	3 37 33.46	39.82	74561	377	40
30	4 13 45.62	33.52	50317	444	30	30	3 36 53.54	39.92	74937	376	30
40	4 13 11.98	33.64	50761	444	20	40	3 36 13.52	40.02	75312	375	20
50	4 12 38.22	33.76	51204	443	10	50	3 35 33.41	40.11	75686	374	10
126°	4 12 4.35	33.87	51645	441	**234°**	**136°**	3 34 53.21	40.20	76059	373	**224°**
10′	4 11 30.37	33.98	52085	440	50′	10′	3 34 12.90	40.31	76430	371	50′
20	4 10 56.27	34.10	52524	439	40	20	3 33 32.50	40.40	76800	370	40
30	4 10 22.06	34.21	52962	438	30	30	3 32 52.01	40.49	77169	369	30
40	4 9 47.73	34.33	53399	437	20	40	3 32 11.42	40.59	77537	368	20
50	4 9 13.29	34.44	53834	435	10	50	3 31 30.73	40.69	77903	366	10
127°	4 8 38.74	34.55	54268	434	**233°**	**137°**	3 30 49.95	40.78	78268	365	**223°**
10′	4 8 4.07	34.67	54702	434	50′	10′	3 30 9.08	40.87	78632	364	50′
20	4 7 29.29	34.78	55134	432	40	20	3 29 28.11	40.97	78995	363	40
30	4 6 54.40	34.89	55565	431	30	30	3 28 47.05	41.06	79356	361	30
40	4 6 19.40	35.00	55995	430	20	40	3 28 5.90	41.15	79716	360	20
50	4 5 44.29	35.11	56424	429	10	50	3 27 24.66	41.24	80075	359	10
128°	4 5 9.06	35.23	56852	428	**232°**	**138°**	3 26 43.32	41.34	80433	358	**222°**
10′	4 4 33.72	35.34	57279	427	50′	10′	3 26 1.89	41.43	80789	356	50
20	4 3 58.27	35.45	57705	426	40	20	3 25 20.37	41.52	81144	355	40
30	4 3 22.72	35.55	58130	425	30	30	3 24 38.76	41.61	81498	354	30
40	4 2 47.05	35.67	58554	424	20	40	3 23 57.05	41.71	81851	353	20
50	4 2 11.27	35.78	58977	423	10	50	3 23 15.26	41.79	82202	351	10
129°	4 1 35.38	35.89	59399	422	**231°**	**139°**	3 22 33.38	41.88	82552	350	**221°**
10′	4 0 59.38	36.00	59820	421	50′	10′	3 21 51.41	41.97	82901	349	50′
20	4 0 23.27	36.11	60239	419	40	20	3 21 9.35	42.06	83249	348	40
30	3 59 47.05	36.22	60657	418	30	30	3 20 27.20	42.15	83595	346	30
40	3 59 10.73	36.32	61074	417	20	40	3 19 44.96	42.24	83940	345	20
50	3 58 34.29	36.44	61489	415	10	50	3 19 2.63	42.33	84284	344	10
130°	3 57 57.75	36.54	61903	414	**230°**	**140°**	3 18 20.22	42.41	84627	343	**220°**
			1.29		g				**1.29**		g

TABLE VII.—*Continued.*

g	*E*		Log. *r*			*g*	*E*		Log. *r*		
	° ′ ″	″	**1.29**				° ″ ′	″	**1.30**		
140°	3 18 20.22		84627		**220°**	**150°**	2 33 26.93		02798		**210°**
10′	3 17 37.72	42.50	84968	341	50′	10′	2 32 39.79	47.14	03060	262	50′
20	3 16 55.13	42.59	85308	340	40	20	2 31 52.59	47.20	03320	260	40
30	3 16 12.45	42.68	85647	339	30	30	2 31 5.32	47.27	03579	259	30
40	3 15 29.68	42.77	85985	338	20	40	2 30 17.98	47.34	03837	258	20
50	3 14 46.83	42.85	86321	336	10	50	2 29 30.58	47.40	04093	256	10
141°	3 14 3.89	42.94	86656	335	**219°**	**151°**	2 28 43.11	47.47	04348	255	**209°**
10′	3 13 20.87	43.02	86990	334	50′	10′	2 27 55.58	47.53	04602	254	50′
20	3 12 37.76	43.11	87322	332	40	20	2 27 7.98	47.60	04854	252	40
30	3 11 54.56	43.20	87653	331	30	30	2 26 20.32	47.66	05105	251	30
40	3 11 11.28	43.28	87983	330	20	40	2 25 32.59	47.73	05355	250	20
50	3 10 27.92	43.36	88312	329	10	50	2 24 44.80	47.79	05604	249	10
142°	3 9 44.47	43.45	88639	327	**218°**	**152°**	2 23 56.95	47.85	05851	247	**208°**
10′	3 9 0.94	43.53	88965	326	50′	10′	2 23 9.04	47.91	06096	245	50′
20	3 8 17.32	43.62	89290	325	40	20	2 22 21.06	47.98	06340	244	40
30	3 7 33.62	43.70	89614	324	30	30	2 21 33.02	48.04	06583	243	30
40	3 6 49.83	43.79	89936	322	20	40	2 20 44.92	48.10	06824	241	20
50	3 6 5.97	43.86	90257	321	10	50	2 19 56.76	48.16	07064	240	10
143°	3 5 22.02	43.95	90577	320	**217°**	**153°**	2 19 8.54	48.22	07302	238	**207°**
10′	3 4 37.99	44.03	90895	318	50′	10′	2 18 20.26	48.28	07539	237	50′
20	3 3 53.88	44.11	91212	317	40	20	2 17 31.92	48.34	07775	236	40
30	3 3 9.69	44.19	91528	316	30	30	2 16 43.51	48.41	08009	234	30
40	3 2 25.42	44.27	91842	314	20	40	2 15 55.05	48.46	08242	233	20
50	3 1 41.08	44.34	92155	313	10	50	2 15 6.53	48.52	08473	231	10
144°	3 0 56.65	44.43	92466	311	**216°**	**154°**	2 14 17.95	48.58	08703	230	**206°**
10′	3 0 12.14	44.51	92776	310	50′	10′	2 13 29.31	48.64	08931	228	50′
20	2 59 27.55	44.59	93085	309	40	20	2 12 40.62	48.69	09158	227	40
30	2 58 42.89	44.66	93393	308	30	30	2 11 51.86	48.76	09384	226	30
40	2 57 58.14	44.75	93700	307	20	40	2 11 3.05	48.81	09608	224	20
50	2 57 13.32	44.82	94005	305	10	50	2 10 14.18	48.87	09831	223	10
145°	2 56 28.42	44.90	94309	304	**215°**	**155°**	2 9 25.26	48.92	10053	222	**205°**
10	2 55 43.44	44.98	94612	303	50′	10′	2 8 36.28	48.98	10273	220	50′
20	2 54 58.39	45.05	94913	301	40	20	2 7 47.24	49.04	10492	219	40
30	2 54 13.26	45.13	95213	300	30	30	2 6 58.15	49.09	10709	217	30
40	2 53 28.05	45.21	95511	298	20	40	2 6 9.01	49.14	10925	216	20
50	2 52 42.77	45.28	95808	297	10	50	2 5 19.81	49.20	11139	214	10
146°	2 51 57.41	45.36	96104	296	**214°**	**156°**	2 4 30.55	49.26	11352	213	**204°**
10′	2 51 11.97	45.44	96398	294	50′	10′	2 3 41.24	49.31	11564	212	50′
20	2 50 26.46	45.51	96691	293	40	20	2 2 51.88	49.36	11774	210	40
30	2 49 40.87	45.59	96983	292	30	30	2 2 2.47	49.41	11983	209	30
40	2 48 55.21	45.66	97273	290	20	40	2 1 13.00	49.47	12191	208	20
50	2 48 9.48	45.73	97562	289	10	50	2 0 23.48	49.52	12397	206	10
147°	2 47 23.67	45.81	97850	288	**213°**	**157°**	1 59 33.91	49.57	12602	205	**203°**
10′	2 46 37.79	45.88	98136	286	50′	10′	1 58 44.29	49.62	12805	203	50′
20	2 45 51.83	45.96	98421	285	40	20	1 57 54.61	49.68	13007	202	40
30	2 45 5.80	46.03	98705	284	30	30	1 57 4.88	49.73	13207	200	30
40	2 44 19.70	46.10	98988	283	20	40	1 56 15.10	49.78	13405	198	20
50	2 43 33.53	46.17	99269	281	10	50	1 55 25.28	49.82	13602	197	10
148°	2 42 47.29	46.24	99549	280	**212°**	**158°**	1 54 35.40	49.88	13798	196	**202°**
10′	2 42 0.98	46.31	99827	278	50′	10′	1 53 45.47	49.93	13992	194	50′
20	2 41 14.59	46.39	*00104	277	40	20	1 52 55.50	49.97	14185	193	40
30	2 40 28.14	46.45	*00380	276	30	30	1 52 5.48	50.02	14377	192	30
40	2 39 41.61	46.53	*00654	274	20	40	1 51 15.41	50.07	14567	190	20
50	2 38 55.01	46.60	*00926	272	10	50	1 50 25.29	50.12	14756	189	10
149°	2 38 8.35	46.66	*01197	271	**211°**	**159°**	1 49 35.12	50.17	14944	188	**201°**
10′	2 37 21.62	46.73	*01467	270	50′	10′	1 48 44.91	50.21	15130	186	50′
20	2 36 34.81	46.81	*01736	269	40	20	1 47 54.65	50.26	15315	185	40
30	2 35 47.94	46.87	*02004	268	30	30	1 47 4.34	50.31	15498	183	30
40	2 35 1.01	46.93	*02270	266	20	40	1 46 13.98	50.36	15680	182	20
50	2 34 14.00	47.01	*02535	265	10	50	1 45 23.58	50.40	15860	180	10
150°	2 33 26.93	47.07	*02798	263	**210°**	**160°**	1 44 33.14	50.44	16039	179	**200°**
			***1.30**		*g*				**1.30**		

TABLE VII.—*Continued.*

g	E				Log. r			g	E				Log. r		
	°	′	″	″	1.30				°	′	″	″	1.30		
160°	1	44	33.14		16039		**200°**	**170°**	0	52	57.32		24087		**190°**
10′	1	43	42.65	50.49	16216	177	50′	10′	0	52	4.81	52.51	24176	89	50′
20	1	42	52.11	50.54	16392	176	40	20	0	51	12.28	52.53	24264	88	40
30	1	42	1.53	50.58	16566	174	30	30	0	50	19.72	52.56	24350	86	30
40	1	41	10.91	50.62	16739	173	20	40	0	49	27.14	52.58	24434	84	20
50	1	40	20.24	50.67	16910	171	10	50	0	48	34.54	52.60	24517	83	10
161°	1	39	29.53	50.71	17080	170	**199°**	**171°**	0	47	41.92	52.62	24599	82	**189°**
10′	1	38	38.78	50.75	17248	168	50′	10′	0	46	49.28	52.64	24679	80	50′
20	1	37	47.98	50.80	17415	167	40	20	0	45	56.62	52.66	24758	79	40
30	1	36	57.14	50.84	17581	166	30	30	0	45	3.93	52.69	24835	77	30
40	1	36	6.25	50.89	17745	164	20	40	0	44	11.23	52.70	24910	75	20
50	1	35	15.33	50.92	17908	163	10	50	0	43	18.51	52.72	24984	74	10
162°	1	34	24.37	50.96	18070	162	**198°**	**172°**	0	42	25.77	52.74	25057	73	**188°**
10′	1	33	33.37	51.00	18230	160	50′	10′	0	41	33.01	52.76	25128	71	50′
20	1	32	42.33	51.04	18388	158	40	20	0	40	40.24	52.77	25198	70	40
30	1	31	51.24	51.09	18545	157	30	30	0	39	47.45	52.79	25267	69	30
40	1	31	0.12	51.12	18700	155	20	40	0	38	54.64	52.81	25334	67	20
50	1	30	8.96	51.16	18854	154	10	50	0	38	1.82	52.82	25399	65	10
163°	1	29	17.76	51.20	19007	153	**197°**	**173°**	0	37	8.98	52.84	25463	64	**187°**
10′	1	28	26.52	51.24	19158	151	50′	10′	0	36	16.12	52.86	25525	62	50′
20	1	27	35.25	51.27	19308	150	40	20	0	35	23.25	52.87	25586	61	40
30	1	26	43.93	51.32	19456	148	30	30	0	34	30.36	52.89	25645	59	30
40	1	25	52.58	51.35	19603	147	20	40	0	33	37.45	52.91	25703	58	20
50	1	25	1.19	51.39	19749	146	10	50	0	32	44.53	52.92	25759	56	10
164°	1	24	9.77	51.42	19893	144	**196°**	**174°**	0	31	51.60	52.93	25814	55	**186°**
10′	1	23	18.31	51.46	20035	142	50′	10′	0	30	58.66	52.94	25867	53	50′
20	1	22	26.81	51.50	20176	141	40	20	0	30	5.70	52.96	25919	52	40
30	1	21	35.28	51.53	20316	140	30	30	0	29	12.73	52.97	25969	50	30
40	1	20	43.71	51.57	20454	138	20	40	0	28	19.75	52.98	26017	48	20
50	1	19	52.11	51.60	20590	136	10	50	0	27	26.76	52.99	26064	47	10
165°	1	19	0.47	51.64	20725	135	**195°**	**175°**	0	26	33.76	53.00	26110	46	**185°**
10′	1	18	8.80	51.67	20858	133	50′	10′	0	25	40.75	53.01	26154	44	50′
20	1	17	17.10	51.70	20990	132	40	20	0	24	47.72	53.03	26197	43	40
30	1	16	25.36	51.74	21121	131	30	30	0	23	54.68	53.04	26239	42	30
40	1	15	33.59	51.77	21250	129	20	40	0	23	1.64	53.04	26279	40	20
50	1	14	41.79	51.80	21377	127	10	50	0	22	8.58	53.06	26318	39	10
166°	1	13	49.96	51.83	21503	126	**194°**	**176°**	0	21	15.51	53.07	26355	37	**184°**
10′	1	12	58.10	51.86	21628	125	50′	10′	0	20	22.43	53.08	26390	35	50′
20	1	12	6.20	51.90	21751	123	40	20	0	19	29.35	53.08	26424	34	40
30	1	11	14.27	51.93	21873	122	30	30	0	18	36.25	53.10	26456	32	30
40	1	10	22.31	51.96	21993	120	20	40	0	17	43.15	53.10	26486	30	20
50	1	9	30.33	51.98	22112	119	10	50	0	16	50.03	53.12	26515	29	10
167°	1	8	38.31	52.02	22230	118	**193°**	**177°**	0	15	56.91	53.12	26543	28	**183°**
10′	1	7	46.27	52.04	22346	116	50′	10′	0	15	3.78	53.13	26569	26	50′
20	1	6	54.19	52.08	22460	114	40	20	0	14	10.65	53.13	26594	25	40
30	1	6	2.08	52.11	22573	113	30	30	0	13	17.51	53.14	26617	23	30
40	1	5	9.94	52.14	22684	111	20	40	0	12	24.36	53.15	26639	22	20
50	1	4	17.78	52.16	22793	109	10	50	0	11	31.21	53.15	26659	20	10
168°	1	3	25.59	52.19	22901	108	**192°**	**178°**	0	10	38.06	53.15	26678	19	**182°**
10′	1	2	33.37	52.22	23008	107	50′	10′	0	9	44.90	53.16	26695	17	50′
20	1	1	41.13	52.24	23113	105	40	20	0	8	51.74	53.16	26711	16	40
30	1	0	48.86	52.27	23217	104	30	30	0	7	58.58	53.16	26726	15	30
40	0	59	56.57	52.29	23320	103	20	40	0	7	5.41	53.17	26739	13	20
50	0	59	4.25	52.32	23421	101	10	50	0	6	12.24	53.17	26750	11	10
169°	0	58	11.90	52.35	23521	100	**191°**	**179°**	0	5	19.07	53.17	26760	10	**181°**
10′	0	57	19.53	52.37	23619	98	50′	10′	0	4	25.89	53.18	26768	8	50′
20	0	56	27.13	52.40	23716	97	40	20	0	3	32.72	53.17	26775	7	40
30	0	55	34.71	52.42	23811	95	30	30	0	2	39.54	53.18	26780	5	30
40	0	54	42.27	52.44	23904	93	20	40	0	1	46.36	53.18	26784	4	20
50	0	53	49.81	52.46	23996	92	10	50	0	0	53.18	53.18	26786	2	10
170°	0	52	57.32	52.49	24087	91	**190°**	**180°**	0	0	0.00	53.18	26787	1	**180°**
					1.30		g						1.30		g

TABLE VIII, Arg. 1.—Action of Jupiter.

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	(*v.c.*1)	(*v.s.*2)	(*v.c.*2)	(*ρ.c.*0)	(*ρ.s.*1)	(*ρ.c.*1)
	″	″	″	″	″	″			
0	55.03	+0.55	0.14	4.42	0.10	0.11	2331	164	134
1	55.58	0.55	0.13	4.42	0.10	0.12	2331	166	134
2	56.13	0.55	0.12	4.41	0.10	0.12	2330	167	134
3	56.68	0.55	0.11	4.41	0.10	0.12	2330	169	134
4	57.23	0.55	0.11	4.41	0.10	0.12	2330	171	135
5	57.78	+0.54	0.10	4.40	0.10	0.13	2329	172	135
6	58.32	0.55	0.09	4.40	0.11	0.13	2328	174	135
7	58.87	0.55	0.09	4.39	0.11	0.13	2328	176	135
8	59.42	0.55	0.08	4.39	0.11	0.13	2327	177	135
9	59.97	0.54	0.08	4.38	0.11	0.14	2326	179	136
10	60.51	+0.55	0.08	4.38	0.11	0.14	2324	180	136
11	61.06	0.54	0.07	4.37	0.11	0.14	2323	182	136
12	61.60	0.55	0.07	4.37	0.11	0.15	2321	184	137
13	62.15	0.54	0.07	4.36	0.11	0.15	2320	185	137
14	62.69	0.54	0.08	4.36	0.11	0.15	2318	187	137
15	63.23	+0.55	0.08	4.35	0.11	0.15	2316	188	137
16	63.78	0.54	0.08	4.35	0.11	0.16	2314	190	137
17	64.32	0.54	0.09	4.34	0.11	0.16	2312	191	138
18	64.86	0.54	0.09	4.34	0.11	0.16	2310	193	138
19	65.40	0.53	0.10	4.33	0.11	0.17	2308	194	138
20	65.93	+0.54	0.11	4.33	0.11	0.17	2305	196	138
21	66.47	0.53	0.12	4.32	0.11	0.17	2303	198	139
22	67.00	0.54	0.13	4.32	0.11	0.17	2300	199	139
23	67.54	0.53	0.14	4.31	0.11	0.18	2297	200	139
24	68.07	0.53	0.15	4.30	0.11	0.18	2294	202	140
25	68.60	+0.54	0.16	4.30	0.10	0.18	2291	203	140
26	69.14	0.52	0.18	4.29	0.10	0.19	2288	205	140
27	69.66	0.53	0.20	4.29	0.10	0.19	2284	206	140
28	70.19	0.53	0.21	4.28	0.10	0.19	2281	208	140
29	70.72	0.52	0.23	4.28	0.10	0.20	2277	209	141
30	71.24	+0.52	0.25	4.27	0.10	0.20	2274	210	141
31	71.76	0.52	0.27	4.27	0.10	0.20	2270	212	141
32	72.28	0.52	0.29	4.26	0.10	0.21	2266	213	141
33	72.80	0.52	0.31	4.26	0.10	0.21	2262	214	142
34	73.32	0.51	0.33	4.25	0.10	0.21	2258	216	142
35	73.83	+0.51	0.35	4.25	0.10	0.22	2254	217	142
36	74.34	0.51	0.38	4.24	0.10	0.22	2249	218	142
37	74.85	0.51	0.40	4.24	0.10	0.22	2245	219	143
38	75.36	0.51	0.43	4.23	0.10	0.23	2240	221	143
39	75.87	0.50	0.46	4.23	0.09	0.23	2236	222	143
40	76.37	+0.50	0.49	4.22	0.09	0.23	2231	223	143
41	76.87	0.50	0.52	4.22	0.09	0.23	2226	224	144
42	77.37	0.50	0.55	4.22	0.09	0.24	2221	225	144
43	77.87	0.50	0.58	4.21	0.09	0.24	2216	227	144
44	78.37	0.49	0.61	4.21	0.09	0.24	2210	228	144
45	78.86	+0.49	0.64	4.21	0.09	0.25	2205	229	144
46	79.35	0.49	0.68	4.20	0.09	0.25	2200	230	145
47	79.84	0.48	0.71	4.20	0.09	0.25	2194	231	145
48	80.32	0.48	0.75	4.20	0.09	0.26	2188	232	145
49	80.80	0.48	0.79	4.20	0.09	0.26	2182	233	145
50	81.28	+0.48	0.82	4.20	0.08	0.26	2176	234	146
51	81.76	0.47	0.86	4.19	0.08	0.26	2170	235	146
52	82.23	0.47	0.90	4.19	0.08	0.26	2164	236	146
53	82.70	0.47	0.94	4.19	0.08	0.27	2158	237	146
54	83.17	0.46	0.98	4.19	0.08	0.27	2151	238	146
55	83.63	+0.47	1.02	4.19	0.07	0.27	2145	239	146
56	84.10	0.45	1.07	4.19	0.07	0.28	2138	240	146
57	84.55	0.46	1.11	4.19	0.07	0.28	2132	241	146
58	85.01	0.45	1.16	4.19	0.07	0.28	2125	242	147
59	85.46	0.45	1.20	4.19	0.07	0.28	2118	243	147
60	85.91		1.25	4.19	0.07	0.29	2111	244	147

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TABLE VIII, Arg. 1.—*Continued.*

Arg.	($v.c.0$)	Diff.	($v.s.1$)	($v.c.1$)	($v.s.2$)	($v.c.2$)	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)
	″	″	″	″	″	″			
60	85.91	+0.45	1.25	4.19	0.07	0.29	2111	244	147
61	86.36	0.44	1.30	4.19	0.07	0.29	2104	244	147
62	86.80	0.44	1.34	4.20	0.06	0.29	2097	245	148
63	87.24	0.44	1.39	4.20	0.06	0.29	2090	246	148
64	87.68	0.43	1.44	4.20	0.06	0.29	2082	247	148
65	88.11	+0.42	1.49	4.20	0.06	0.30	2075	248	148
66	88.53	0.43	1.54	4.21	0.06	0.30	2067	248	148
67	88.96	0.42	1.60	4.21	0.06	0.30	2059	249	148
68	89.38	0.42	1.65	4.21	0.05	0.30	2052	250	148
69	89.80	0.41	1.70	4.22	0.05	0.31	2044	250	148
70	90.21	+0.41	1.75	4.22	0.05	0.31	2036	251	149
71	90.62	0.41	1.81	4.23	0.05	0.31	2028	252	149
72	91.03	0.40	1.86	4.23	0.05	0.31	2020	252	149
73	91.43	0.40	1.92	4.24	0.05	0.31	2012	253	149
74	91.83	0.39	1.98	4.24	0.04	0.32	2003	253	149
75	92.22	+0.40	2.03	4.25	0.04	0.32	1995	254	149
76	92.62	0.38	2.09	4.26	0.04	0.32	1986	254	149
77	93.00	0.38	2.15	4.26	0.04	0.32	1978	255	149
78	93.38	0.38	2.21	4.27	0.04	0.32	1969	255	149
79	93.76	0.38	2.27	4.28	0.04	0.32	1960	256	149
80	94.14	+0.37	2.33	4.29	0.04	0.32	1952	256	149
81	94.51	0.36	2.39	4.30	0.03	0.32	1943	257	149
82	94.87	0.36	2.45	4.31	0.03	0.33	1934	257	149
83	95.23	0.36	2.51	4.32	0.03	0.33	1925	257	149
84	95.59	0.36	2.57	4.33	0.03	0.33	1916	258	149
85	95.95	+0.35	2.64	4.34	0.03	0.33	1906	258	149
86	96.30	0.34	2.70	4.35	0.03	0.33	1897	258	149
87	96.64	0.34	2.76	4.36	0.03	0.33	1888	259	149
88	96.98	0.34	2.83	4.37	0.03	0.83	1878	259	149
89	97.32	0.33	2.89	4.38	0.02	0.33	1869	259	149
90	97.65	+0.32	2.96	4.39	0.02	0.33	1859	260	148
91	97.97	0.32	3.02	4.41	0.02	0.33	1850	260	148
92	98.29	0.32	3.09	4.42	0.02	0.33	1840	260	148
93	98.61	0.31	3.15	4.43	0.02	0.33	1830	260	148
94	98.92	0.31	3.22	4.45	0.02	0.33	1820	260	148
95	99.23	+0.30	3.29	4.46	0.02	0.33	1810	261	148
96	99.53	0.30	3.35	4.48	0.02	0.34	1800	261	148
97	99.83	0.29	3.42	4.49	0.02	0.34	1790	261	148
98	100.12	0.29	3.49	4.51	0.01	0.34	1780	261	147
99	100.41	0.29	3.56	4.52	0.01	0.34	1770	261	147
100	100.70	+0.27	3.63	4.54	0.01	0.34	1760	261	147
101	100.97	0.28	3.69	4.56	0.01	0.34	1750	261	147
102	101.25	0.27	3.76	4.57	0.01	0.34	1739	261	147
103	101.52	0.26	3.83	4.59	0.01	0.34	1729	261	146
104	101.78	0.26	3.90	4.61	0.01	0.34	1718	261	146
105	102.04	+0.25	3.97	4.63	0.01	0.34	1708	261	146
106	102.29	0.25	4.04	4.65	0.01	0.34	1697	261	145
107	102.54	0.24	4.11	4.67	0.01	0.34	1686	261	145
108	102.78	0.24	4.18	4.69	0.01	0.34	1676	261	145
109	103.02	0.23	4.25	4.71	0.01	0.34	1665	261	145
110	103.25	+0.23	4.32	4.73	0.01	0.34	1654	261	144
111	103.48	0.22	4.39	4.75	0.01	0.34	1643	261	144
112	103.70	0.22	4.46	4.77	0.01	0.34	1632	261	144
113	103.92	0.21	4.53	4.79	0.01	0.34	1622	260	143
114	104.13	0.21	4.60	4.81	0.01	0.34	1611	260	143
115	104.34	+0.20	4.67	4.83	0.01	0.33	1600	260	142
116	104.54	0.20	4.74	4.85	0.01	0.33	1588	260	142
117	104.74	0.19	4.81	4.87	0.01	0.33	1577	260	142
118	104.93	0.18	4.88	4.90	0.01	0.33	1566	259	141
119	105.11	0.18	4.95	4.92	0.01	0.33	1555	259	141
120	105.29		5.01	4.94	0.02	0.33	1544	259	140

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	(*v.c.*1)	(*v.s.*2)	(*v.c.*2)	(*ρ.c.*0)	(*ρ.s.*1)	(*ρ.c.*1)
	″	″	″	″	″	″			
120	105.29	+0.17	5.01	4.94	0.02	0.33	1544	259	140
121	105.46	0.17	5.08	4.97	0.02	0.33	1533	259	140
122	105.63	0.16	5.15	4.99	0.02	0.33	1521	258	139
123	105.79	0.16	5.22	5.01	0.02	0.33	1510	258	139
124	105.95	0.15	5.29	5.04	0.02	0.33	1499	258	138
125	106.10	+0.15	5.36	5.06	0.02	0.33	1487	257	138
126	106.25	0.14	5.43	5.09	0.02	0.33	1476	257	137
127	106.39	0.14	5.50	5.11	0.02	0.33	1464	257	137
128	106.53	0.12	5.57	5.14	0.02	0.33	1453	256	136
129	106.65	0.13	5.64	5.16	0.02	0.33	1441	256	135
130	106.78	+0.11	5.71	5.19	0.02	0.32	1430	256	135
131	106.89	0.12	5.78	5.21	0.03	0.32	1418	255	134
132	107.01	0.10	5.85	5.24	0.03	0.32	1407	255	134
133	107.11	0.11	5.91	5.26	0.03	0.32	1395	254	133
134	107.22	0.09	5.98	5.29	0.03	0.32	1384	254	132
135	107.31	+0.09	6.05	5.32	0.03	0.32	1372	254	132
136	107.40	0.09	6.11	5.34	0.04	0.32	1360	253	131
137	107.49	0.08	6.18	5.37	0.04	0.32	1349	253	131
138	107.57	0.07	6.25	5.40	0.04	0.32	1337	252	130
139	107.64	0.06	6.31	5.42	0.04	0.32	1325	252	129
140	107.70	+0.06	6.38	5.45	0.04	0.32	1314	251	129
141	107.76	0.06	6.44	5.48	0.05	0.31	1302	251	128
142	107.82	0.05	6.51	5.51	0.05	0.31	1290	250	127
143	107.87	0.04	6.57	5.53	0.05	0.31	1278	250	126
144	107.91	0.04	6.64	5.56	0.05	0.31	1267	249	126
145	107.95	+0.03	6.70	5.59	0.05	0.31	1255	249	125
146	107.98	0.03	6.76	5.61	0.06	0.31	1243	248	124
147	108.01	0.02	6.83	5.64	0.06	0.31	1231	248	123
148	108.03	0.02	6.89	5.67	0.06	0.31	1220	247	123
149	108.05	0.00	6.95	5.69	0.06	0.31	1208	247	122
150	108.05	+0.01	7.01	5.72	0.07	0.31	1196	246	121
151	108.06	−0.01	7.08	5.75	0.07	0.31	1184	246	120
152	108.05	0.01	7.14	5.78	0.07	0.31	1172	245	119
153	108.04	0.01	7.20	5.80	0.07	0.30	1161	244	118
154	108.03	0.02	7.25	5.83	0.08	0.30	1149	244	118
155	108.01	−0.03	7.31	5.86	0.08	0.30	1137	243	117
156	107.98	0.03	7.37	5.88	0.08	0.30	1126	243	116
157	107.95	0.04	7.43	5.91	0.09	0.30	1114	242	115
158	107.91	0.04	7.49	5.94	0.09	0.30	1102	241	114
159	107.87	0.05	7.54	5.97	0.09	0.30	1090	241	113
160	107.82	−0.05	7.60	5.99	0.10	0.30	1079	240	112
161	107.77	0.07	7.66	6.02	0.10	0.30	1067	240	112
162	107.70	0.06	7.71	6.04	0.10	0.30	1055	239	111
163	107.64	0.08	7.77	6.07	0.11	0.30	1044	238	110
164	107.56	0.07	7.82	6.10	0.11	0.29	1032	238	109
165	107.49	−0.09	7.87	6.12	0.11	0.29	1020	237	108
166	107.40	0.09	7.93	6.15	0.12	0.29	1009	236	107
167	107.31	0.09	7.98	6.17	0.12	0.29	997	236	106
168	107.22	0.11	8.03	6.20	0.12	0.29	986	235	105
169	107.11	0.10	8.08	6.22	0.13	0.29	974	234	104
170	107.01	−0.12	8.13	6.25	0.13	0.29	963	234	103
171	106.89	0.12	8.18	6.27	0.14	0.29	951	233	102
172	106.77	0.12	8.23	6.29	0.14	0.29	940	232	101
173	106.65	0.13	8.28	6.32	0.14	0.29	928	232	100
174	106.52	0.14	8.33	6.34	0.15	0.29	917	231	99
175	106.38	−0.14	8.37	6.36	0.15	0.29	906	230	98
176	106.24	0.14	8.42	6.39	0.15	0.29	894	230	97
177	106.10	0.16	8.46	6.41	0.16	0.29	883	229	96
178	105.94	0.16	8.51	6.43	0.16	0.29	872	228	95
179	105.78	0.16	8.55	6.45	0.17	0.29	861	228	94
180	105.62		8.60	6.47	0.17	0.29	850	227	93

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(*v.c.*0)	Diff	(*v.s.*1)	(*v.c.*1)	(*v.s.*2)	(*v.c.*2)	(ρ.*c.*0)	(ρ.*s.*1)	(ρ.*c.*1)
	″	″	″	″	″	″			
180	105.62	—0.17	8.60	6.47	0.17	0.29	850	227	93
181	105.45	0.18	8.64	6.50	0.17	0.29	839	226	92
182	105.27	0.18	8.68	6.52	0.18	0.29	828	226	91
183	105.09	0.19	8.72	6.54	0.18	0.29	816	225	90
184	104.90	0.19	8.76	6.56	0.19	0.29	805	224	89
185	104.71	—0.20	8.80	6.57	0.19	0.29	794	224	88
186	104.51	0.20	8.84	6.59	0.19	0.29	784	223	87
187	104.31	0.21	8.88	6.61	0.20	0.29	773	222	86
188	104.10	0.21	8.92	6.63	0.20	0.29	762	222	85
189	103.89	0.22	8.96	6.65	0.21	0.29	751	221	84
190	103.67	—0.23	8.99	6.67	0.21	0.29	740	221	83
191	103.44	0.23	9.03	6.68	0.21	0.29	730	220	82
192	103.21	0.23	9.07	6.70	0.22	0.29	719	219	81
193	102.98	0.24	9.10	6.71	0.22	0.29	708	219	80
194	102.74	0.25	9.13	6.73	0.22	0.29	698	218	79
195	102.49	—0.25	9.17	6.74	0.23	0.29	688	217	78
196	102.24	0.26	9.20	6.76	0.23	0.29	677	216	77
197	101.98	0.26	9.23	6.77	0.24	0.29	667	216	76
198	101.72	0.27	9.26	6.78	0.24	0.29	657	215	75
199	101.45	0.27	9.29	6.79	0.24	0.29	646	215	74
200	101.18	—0.28	9.32	6.81	0.25	0.29	636	214	73
201	100.90	0.28	9.35	6.82	0.25	0.29	626	213	72
202	100.62	0.29	9.38	6.83	0.25	0.29	616	213	71
203	100.33	0.29	9.41	6.84	0.26	0.29	606	212	70
204	100.04	0.30	9.43	6.85	0.26	0.29	596	211	69
205	99.74	—0.30	9.46	6.86	0.26	0.29	586	211	68
206	99.44	0.31	9.48	6.86	0.27	0.29	577	210	67
207	99.13	0.31	9.51	6.87	0.27	0.29	567	210	66
208	98.82	0.32	9.53	6.88	0.27	0.29	557	209	65
209	98.50	0.32	9.55	6.88	0.28	0.29	548	208	64
210	98.18	—0.33	9.58	6.89	0.28	0.29	538	208	63
211	97.85	0.33	9.60	6.89	0.28	0.29	529	207	62
212	97.52	0.34	9.62	6.90	0.29	0.29	520	206	61
213	97.18	0.34	9.64	6.90	0.29	0.29	510	206	60
214	96.84	0.35	9.66	6.90	0.29	0.29	501	205	59
215	96.49	—0.35	9.68	6.91	0.30	0.29	492	205	58
216	96.14	0.35	9.70	6.91	0.30	0.29	483	204	57
217	95.79	0.36	9.71	6.91	0.30	0.29	474	204	56
218	95.43	0.37	9.73	6.91	0.31	0.29	464	203	55
219	95.06	0.37	9.75	6.91	0.31	0.30	456	202	54
220	94.69	—0.37	9.76	6.91	0.31	0.30	448	202	53
221	94.32	0.38	9.78	6.90	0.31	0.30	439	201	52
222	93.94	0.38	9.79	6.90	0.32	0.30	430	201	51
223	93.56	0.39	9.81	6.90	0.32	0.30	422	200	50
224	93.17	0.39	9.82	6.90	0.32	0.30	413	200	49
225	92.78	—0.39	9.83	6.89	0.33	0.30	405	199	48
226	92.39	0.40	9.85	6.88	0.33	0.30	398	198	47
227	91.99	0.40	9.86	6.88	0.33	0.30	390	198	46
228	91.59	0.41	9.87	6.87	0.33	0.30	382	197	46
229	91.18	0.41	9.88	6.86	0.34	0.30	374	197	45
230	90.77	—0.41	9.89	6.85	0.34	0.30	365	196	44
231	90.36	0.42	9.90	6.84	0.34	0.30	357	196	43
232	89.94	0.43	9.91	6.83	0.34	0.30	349	195	42
233	89.51	0.42	9.91	6.82	0.34	0.30	342	194	41
234	89.09	0.44	9.92	6.81	0.35	0.30	334	194	41
235	88.65	—0.43	9.93	6.80	0.35	0.30	327	194	40
236	88.22	0.44	9.94	6.79	0.35	0.30	320	193	39
237	87.78	0.44	9.94	6.78	0.35	0.30	312	192	38
238	87.34	0.45	9.95	6.76	0.35	0.30	305	192	38
239	86.89	0.45	9.95	6.74	0.36	0.30	298	191	37
240	86.44		9.96	6.73	0.36	0.31	291	191	36

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	(*v.c.*1)	(*v.s.*2)	(*v.c.*2)	(ρ.*c.*0)	(ρ.*s.*1)	(ρ.*c.*1)
	″	″	″	″	″	″			
240	86.44		9.96	6.73	0.36	0.31	291	191	36
241	85.99	—0.45	9.96	6.71	0.36	0.31	284	190	35
242	85.53	0.46	9.96	6.69	0.36	0 31	278	190	35
243	85.08	0.45	9.97	6.67	0.36	0.31	271	189	34
244	84.61	0.47	9.97	6.66	0.37	0.31	265	189	33
245	84.15	0.46	9.97	6.64	0.37	0.31	258	188	33
246	83.68	—0.47	9.97	6.61	0.37	0.31	252	188	32
247	83.21	0.47	9.98	6.59	0.37	0.31	246	187	31
248	82.73	0.48	9.98	6.57	0.37	0.31	240	187	31
249	82.25	0.48	9.98	6.55	0.37	0.31	234	186	30
250	81.77	0.48	9.98	6.53	0.37	0.31	228	186	29
251	81.28	—0.49	9.98	6.51	0.38	0.31	222	185	29
252	80.80	0.48	9.98	6.48	0.38	0.31	216	185	28
253	80.30	0.50	9.97	6.46	0.38	0.31	211	184	28
254	79.81	0.49	9.97	6.43	0.38	0.31	205	184	27
255	79.31	0.50	9.97	6.40	0.38	0.31	200	183	27
256	78.82	—0.49	9.97	6.37	0.38	0.30	195	183	26
257	78.31	0.51	9.96	6.34	0.38	0.30	190	182	26
258	77.81	0.50	9.96	6.31	0.38	0.30	184	182	25
259	77.30	0.51	9.96	6.28	0.38	0.30	179	181	25
260	76.79	0.51	9.95	6.25	0.38	0.30	175	181	24
261	76.28	—0.51	9.95	6.22	0.38	0.30	170	180	24
262	75.77	0.51	9.95	6.19	0.38	0.30	165	180	24
263	75.25	0.52	9.94	6.16	0.38	0.30	161	179	23
264	74.73	0.52	9.94	6.13	0.38	0.30	156	179	23
265	74.21	0.52	9.93	6.10	0.38	0.30	152	178	22
266	73.69	—0.52	9.93	6.06	0.38	0.30	148	178	22
267	73.16	0.53	9.92	6.03	0.38	0.30	144	177	22
268	72.64	0.52	9.92	5.99	0.38	0.30	140	177	21
269	72.11	0.53	9.91	5.96	0.38	0.30	137	176	21
270	71.57	0.54	9.90	5.92	0.39	0.30	133	176	21
271	71.04	—0.53	9.90	5.88	0.39	0.30	129	175	20
272	70.51	0.53	9.89	5.85	0.39	0.30	126	175	20
273	69.97	0.54	9.88	5.81	0.39	0.30	122	174	20
274	69.43	0.54	9.88	5.77	0.39	0.30	119	174	20
275	68.89	0.54	9.87	5.73	0.39	0.30	116	174	20
276	68.35	—0.54	9.86	5.69	0.39	0.29	113	173	19
277	67.81	0.54	9.85	5.65	0.39	0.29	110	172	19
278	67.26	0.55	9.84	5.61	0.39	0.29	107	172	19
279	66.72	0.54	9.84	5.57	0.39	0.29	105	171	19
280	66.17	0.55	9.83	5.52	0.39	0.29	102	171	19
281	65.62	—0.55	9.82	5.48	0.39	0.29	100	170	19
282	65.07	0.55	9.81	5.44	0.39	0.29	97	170	18
283	64.52	0.55	9.80	5.40	0.39	0.29	95	169	18
284	63.97	0.55	9.79	5.35	0.39	0.29	93	169	18
285	63.41	0.56	9.78	5.31	0.39	0.28	91	168	18
286	62.86	—0.55	9.77	5.26	0.39	0.28	90	168	18
287	62.31	0.55	9.77	5.22	0.39	0.28	88	167	18
288	61.75	0.56	9.76	5.17	0.39	0.28	86	167	18
289	61.19	0.56	9.75	5.13	0.38	0.28	85	166	18
290	60.64	0.55	9.74	5.08	0.38	0.28	84	166	18
291	60.08	—0.56	9.73	5.03	0.38	0.27	82	165	19
292	59.52	0.56	9.72	4.99	0.38	0.27	81	164	19
293	58.96	0.56	9.71	4.94	0.38	0.27	80	164	19
294	58.40	0.56	9.70	4.89	0.38	0.27	80	163	19
295	57.84	0.56	9.69	4.84	0.38	0.27	79	163	19
296	57.28	—0.56	9.68	4.80	0.38	0.27	78	162	19
297	56.72	0.56	9.67	4.75	0.38	0.26	78	162	19
298	56.16	0.56	9.66	4.70	0.38	0.26	77	161	20
299	55.60	0.56	9.65	4.65	0.33	0.26	77	161	20
300	55.04	0.56	9.64	4.60	0.38	0.26	77	160	20

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TABLE VIII, ARG. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	″	″	″	″	″	″			
300	55.04	—0.56	9.64	4.60	0.38	0.26	77	160	20
301	54.48	0.56	9.63	4.55	0.38	0.26	77	159	20
302	53.92	0.56	9.62	4.50	0.38	0.25	77	159	20
303	53.36	0.56	9.61	4.45	0.38	0.25	77	158	21
304	52.80	0.56	9.60	4.40	0.38	0.25	78	158	21
305	52.24	—0.56	9.59	4.35	0.38	0.25	78	157	21
306	51.68	0.56	9.58	4.30	0.38	0.25	79	156	22
307	51.12	0.56	9.57	4.25	0.38	0.24	80	156	22
308	50.56	0.56	9.56	4.20	0.38	0.24	81	155	22
309	50.00	0.56	9.55	4.15	0.38	0.24	82	154	23
310	49.44	—0.56	9.54	4.10	0.38	0.24	83	154	23
311	48.88	0.55	9.53	4.05	0.38	0.23	84	153	24
312	48.33	0.56	9.52	4.01	0.38	0.23	85	152	24
313	47.77	0.55	9.51	3.94	0.38	0.23	87	152	24
314	47.22	0.56	9.50	3.89	0.38	0.23	88	151	25
315	46.66	—0.55	9.49	3.84	0.38	0.23	90	150	25
316	46.11	0.55	9.48	3.79	0.38	0.22	92	150	26
317	45.56	0.55	9.47	3.74	0.38	0.22	94	149	26
318	45.01	0.55	9.46	3.69	0.38	0.22	96	148	27
319	44.46	0.55	9.45	3.63	0.38	0.22	98	148	27
320	43.91	—0.55	9.44	3.58	0.38	0.21	100	147	28
321	43.36	0.55	9.43	3.53	0.38	0.21	103	146	28
322	42.81	0.54	9.42	3.48	0.38	0.21	105	146	29
323	42.27	0.55	9.41	3.43	0.38	0.21	108	145	29
324	41.72	0.54	9.40	3.38	0.38	0.20	111	144	30
325	41.18	—0.54	9.40	3.33	0.38	0.20	114	144	30
326	40.64	0.54	9.39	3.28	0.38	0.20	117	143	31
327	40.10	0.54	9.38	3.23	0.38	0.20	120	142	32
328	39.56	0.53	9.37	3.18	0.38	0.19	123	141	32
329	39.03	0.54	9.36	3.14	0.37	0.19	127	140	33
330	38.49	—0.53	9.35	3.07	0.37	0.19	130	140	34
331	37.96	0.53	9.34	3.02	0.37	0.19	134	139	34
332	37.43	0.53	9.33	2.98	0.37	0.18	138	138	35
333	36.90	0.53	9.32	2.93	0.37	0.18	141	137	36
334	36.37	0.52	9.31	2.88	0.37	0.18	145	136	36
335	35.85	—0.52	9.30	2.83	0.37	0.18	150	136	37
336	35.33	0.52	9.29	2.78	0.37	0.17	154	135	38
337	34.81	0.52	9.29	2.73	0.37	0.17	158	134	38
338	34.29	0.51	9.28	2.69	0.37	0.17	162	133	39
339	33.78	0.52	9.27	2.64	0.37	0.17	167	132	40
340	33.26	—0.51	9.26	2.59	0.37	0.17	172	132	41
341	32.75	0.51	9.25	2.54	0.37	0.16	176	131	42
342	32.24	0.50	9.24	2.50	0.37	0.16	181	130	42
343	31.74	0.51	9.23	2.45	0.37	0.16	186	129	43
344	31.23	0.50	9.22	2.41	0.37	0.16	191	128	44
345	30.73	—0.49	9.21	2.36	0.37	0.15	197	127	45
346	30.24	0.50	9.20	2.31	0.37	0.15	202	126	46
347	29.74	0.49	9.20	2.27	0.37	0.15	207	126	46
348	29.25	0.49	9.19	2.23	0.37	0.15	213	125	47
349	28.76	0.49	9.18	2.18	0.37	0.15	218	124	48
350	28.27	—0.48	9.17	2.14	0.37	0.14	224	123	49
351	27.79	0.48	9.16	2.09	0.37	0.14	230	122	50
352	27.31	0.48	9.15	2.05	0.37	0.14	236	121	51
353	26.83	0.47	9.14	2.01	0.37	0.14	242	120	51
354	26.36	0.47	9.13	1.97	0.37	0.14	248	119	52
355	25.89	—0.47	9.12	1.93	0.37	0.14	255	118	53
356	25.42	0.46	9.12	1.89	0.37	0.13	261	117	54
357	24.96	0.46	9.11	1.85	0.37	0.13	268	116	55
358	24.50	0.46	9.10	1.81	0.37	0.13	274	115	56
359	24.04	0.46	9.09	1.77	0.37	0.13	281	114	57
360	23.58		9.08	1.73	0.38	0.13	288	113	58

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TABLE VIII, ARG. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	″	″	″	″	″	″			
360	23.58		9.08	1.73	0.38	0.13	288	113	58
361	23.13	−0.45	9.07	1.70	0.38	0.13	294	112	58
362	22.69	0.44	9.06	1.66	0.38	0.13	301	111	59
363	22.24	0.45	9.05	1.62	0.38	0.13	308	110	60
364	21.80	0.44	9.04	1.59	0.38	0.12	316	109	61
365	21.37	0.43	9.03	1.55	0.37	0.12	323	108	62
366	20.94	−0.43	9.02	1.52	0.37	0.12	330	107	63
367	20.51	0.43	9.01	1.48	0.37	0.12	338	106	64
368	20.08	0.43	9.00	1.45	0.37	0.12	345	105	65
369	19.66	0.42	8.99	1.42	0.37	0.11	353	104	66
370	19.24	0.42	8.98	1.39	0.37	0.11	361	103	66
371	18.83	−0.41	8.97	1.36	0.37	0.11	369	102	67
372	18.42	0.41	8.96	1.32	0.37	0.11	377	101	68
373	18.02	0.40	8.95	1.29	0.37	0.11	385	100	69
374	17.62	0.40	8.94	1.27	0.37	0.11	393	99	70
375	17.22	0.40	8.93	1.24	0.37	0.11	401	98	71
376	16.83	−0.39	8.92	1.21	0.37	0.11	410	97	72
377	16.44	0.39	8.91	1.18	0.37	0.11	418	96	73
378	16.06	0.38	8.89	1.16	0.37	0.11	426	95	74
379	15.68	0.38	8.88	1.13	0.37	0.11	435	94	75
380	15.30	0.38	8.87	1.11	0.37	0.10	444	93	76
381	14.93	−0.37	8.86	1.08	0.37	0.10	452	92	77
382	14.57	0.36	8.85	1.06	0.37	0.10	461	91	78
383	14.21	0.36	8.83	1.04	0.37	0.10	470	90	78
384	13.85	0.36	8.82	1.02	0.37	0.10	479	89	79
385	13.50	0.35	8.81	1.00	0.37	0.10	488	88	80
386	13.15	−0.35	8.79	0.98	0.37	0.10	497	87	81
387	12.80	0.35	8.78	0.96	0.37	0.10	506	86	82
388	12.47	0.33	8.77	0.94	0.37	0.10	514	85	83
389	12.13	0.34	8.75	0.92	0.37	0.10	525	84	84
390	11.81	0.32	8.74	0.90	0.37	0.10	534	82	85
391	11.48	−0.33	8.72	0.88	0.37	0.10	544	81	86
392	11.16	0.32	8.71	0.87	0.37	0.10	554	80	86
393	10.85	0.31	8.70	0.85	0.36	0.10	564	79	87
394	10.54	0.31	8.68	0.84	0.36	0.10	573	78	88
395	10.23	0.31	8.67	0.83	0.36	0.10	583	77	89
396	9.93	−0.30	8.65	0.82	0.36	0.10	593	76	90
397	9.64	0.29	8.63	0.80	0.36	0.10	603	75	91
398	9.35	0.29	8.62	0.79	0.36	0.10	613	74	92
399	9.07	0.28	8.60	0.78	0.36	0.10	623	73	93
400	8.79	0.28	8.58	0.77	0.36	0.10	633	72	93
401	8.52	−0.27	8.57	0.76	0.36	0.10	643	71	94
402	8.25	0.27	8.55	0.76	0.36	0.10	653	70	95
403	7.98	0.27	8.53	0.75	0.36	0.10	664	69	96
404	7.73	0.25	8.51	0.74	0.35	0.10	674	68	97
405	7.47	0.26	8.49	0.74	0.35	0.10	684	67	98
406	7.22	−0.25	8.47	0.74	0.35	0.10	695	66	98
407	6.98	0.24	8.46	0.73	0.35	0.10	706	65	99
408	6.75	0.23	8.44	0.73	0.35	0.10	716	64	100
409	6.51	0.24	8.42	0.73	0.34	0.10	727	63	101
410	6.29	0.22	8.39	0.73	0.34	0.10	737	62	101
411	6.07	−0.22	8.37	0.73	0.34	0.10	748	61	102
412	5.85	0.22	8.35	0.73	0.34	0.10	759	60	103
413	5.64	0.21	8.33	0.73	0.34	0.10	770	59	104
414	5.44	0.20	8.31	0.73	0.34	0.10	781	58	104
415	5.24	0.20	8.28	0.73	0.34	0.10	792	57	105
416	5.04	−0.20	8.26	0.74	0.33	0.10	803	56	106
417	4.86	0.18	8.24	0.74	0.33	0.10	814	55	107
418	4.67	0.19	8.21	0.74	0.33	0.10	825	54	107
419	4.50	0.17	8.19	0.75	0.33	0.10	836	53	108
420	4.33	0.17	8.17	0.76	0.33	0.10	847	52	109

TABLE VIII, Arg. 1.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	(*v.c.*1)	(*v.s.*2)	(*v.c.*2)	(*ρ.c.*0)	(*ρ.s.*1)	(*ρ.c.*1)
	″	″	″	″	″	″			
420	4.33		8.17	0.76	0.33	0.10	847	52	109
421	4.16	−0.17	8.14	0.76	0.32	0.10	858	51	110
422	4.00	0.16	8.11	0.77	0.32	0.11	870	50	110
423	3.85	0.15	8.09	0.78	0.32	0.11	881	50	111
424	3.70	0.15	8.06	0.79	0.32	0.11	892	49	111
425	3.56	0.14	8.04	0.80	0.32	0.11	904	48	112
426	3.42	−0.14	8.01	0.81	0.31	0.11	916	47	113
427	3.29	0.13	7.98	0.82	0.31	0.11	927	46	113
428	3.16	0.13	7.95	0.83	0.31	0.11	938	45	114
429	3.04	0.12	7.92	0.85	0.31	0.11	950	44	114
430	2.93	0.11	7.89	0.86	0.31	0.11	961	44	115
431	2.82	−0.11	7.86	0.87	0.30	0.11	973	43	116
432	2.72	0.10	7.83	0.89	0.30	0.11	984	42	116
433	2.62	0.10	7.80	0.90	0.30	0.11	996	41	117
434	2.53	0.09	7.77	0.92	0.29	0.11	1008	40	118
435	2.45	0.08	7.74	0.94	0.29	0.11	1019	40	118
436	2.37	−0.08	7.71	0.95	0.29	0.12	1031	39	119
437	2.29	0.08	7.67	0.97	0.29	0.12	1043	38	119
438	2.23	0.06	7.64	0.99	0.28	0.12	1054	38	120
439	2.16	0.07	7.60	1.01	0.28	0.12	1066	37	120
440	2.11	0.05	7.57	1.03	0.28	0.12	1078	36	121
441	2.06	−0.05	7.54	1.05	0.28	0.12	1089	35	121
442	2.02	0.04	7.50	1.07	0.27	0.12	1100	35	122
443	1.98	0.04	7.47	1.09	0.27	0.12	1113	34	122
444	1.95	0.03	7.43	1.11	0.27	0.12	1124	33	122
445	1.92	0.03	7.39	1.14	0.26	0.12	1137	33	123
446	1.90	−0.02	7.36	1.16	0.26	0.12	1149	32	123
447	1.88	0.02	7.32	1.18	0.26	0.12	1160	32	124
448	1.87	0.01	7.28	1.21	0.26	0.12	1172	31	124
449	1.87	0.00	7.24	1.23	0.25	0.12	1184	30	125
450	1.87	0.00	7.20	1.26	0.25	0.12	1196	30	125
451	1.88	+0.01	7.16	1.28	0.25	0.12	1208	30	125
452	1.90	0.02	7.12	1.31	0.24	0.12	1220	29	126
453	1.92	0.02	7.08	1.33	0.24	0.12	1232	28	126
454	1.95	0.03	7.04	1.36	0.24	0.12	1243	28	127
455	1.98	0.03	7.00	1.39	0.23	0.12	1255	28	127
456	2.01	+0.03	6.96	1.42	0.23	0.12	1267	27	127
457	2 06	0.05	6.91	1.45	0.23	0.12	1279	27	128
458	2.11	0.05	6.87	1.47	0.22	0.12	1291	26	128
459	2.16	0.05	6.83	1.50	0.22	0.12	1302	26	128
460	2.23	0.07	6.78	1.53	0.22	0.12	1314	26	128
461	2.29	+0.06	6.74	1.56	0.21	0.12	1326	25	129
462	2.37	0·08	6.69	1.59	0.21	0.12	1338	25	129
463	2.44	0.07	6.65	1.62	0.21	0.12	1350	25	129
464	2.53	0.09	6.60	1.65	0.20	0.12	1361	25	130
465	2.62	0.09	6.56	1.68	0.20	0.12	1373	24	130
466	2.71	+0.09	6.51	1.72	0.20	0.12	1385	24	130
467	2.82	0.11	6.46	1.75	0.19	0.13	1396	24	130
468	2.92	0.10	6.41	1.78	0.19	0.13	1408	24	130
469	3.04	0.12	6.37	1.81	0.19	0.13	1420	23	131
470	3.15	0.11	6.32	1.84	0.18	0.13	1431	23	131
471	3.28	+0.13	6.27	1.88	0.18	0.13	1443	23	131
472	3.41	0.13	6.22	1.91	0.18	0.12	1455	23	131
473	3.55	0.14	6.17	1.94	0.17	0.12	1466	23	131
474	3.69	0.14	6.12	1.98	0.17	0.12	1478	23	132
475	3.83	0.14	6.07	2.01	0.17	0.12	1489	23	132
476	3.98	+0.15	6.02	2.04	0.16	0.12	1501	23	132
477	4.14	0.16	5.96	2.07	0.16	0.12	1512	23	132
478	4.31	0.17	5.91	2.11	0.16	0.12	1524	23	132
479	4.48	0.17	5.86	2.14	0.15	0.12	1535	23	132
480	4.65	0.17	5.81	2.18	0.15	0.12	1546	23	132

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TABLE VIII, Arg. 1.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	(v.c.1)	(v.s.2)	(v.c.2)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)
	″	″	″	″	″	″			
480	4.65	+0.18	5.81	2.18	0.15	0.12	1546	23	132
481	4.83	0.19	5.75	2.21	0.15	0.12	1558	23	132
482	5.02	0.19	5.70	2.24	0.15	0.12	1569	23	132
483	5.21	0.19	5.65	2.28	0.14	0.12	1580	23	132
484	5.40	0.21	5.59	2.31	0.14	0.12	1591	23	133
485	5.61	+0.20	5.54	2.35	0.14	0.12	1602	24	133
486	5.81	0.22	5.48	2.38	0.13	0.12	1613	24	133
487	6.03	0.21	5.43	2.42	0.13	0.12	1624	24	133
488	6.24	0.23	5.37	2.45	0.13	0.11	1635	24	134
489	6.47	0.23	5.32	2.48	0.12	0.11	1646	25	134
490	6.70	+0.23	5.26	2.52	0.12	0.11	1657	25	133
491	6.93	0.24	5.20	2.55	0.12	0.11	1668	25	133
492	7.17	0.24	5.15	2.59	0.12	0.11	1679	25	133
493	7.41	0.25	5.09	2.62	0.11	0.11	1690	25	133
494	7.66	0.26	5.03	2.66	0.11	0.11	1700	26	133
495	7.92	+0.26	4.98	2.69	0.11	0.11	1711	27	133
496	8.18	0.26	4.92	2.72	0.11	0.11	1721	27	133
497	8.44	0.27	4.86	2.76	0.10	0.11	1732	28	133
498	8.71	0.28	4.80	2.79	0.10	0.11	1742	28	133
499	8.99	0.27	4.75	2.83	0.10	0.11	1753	28	133
500	9.26	+0.29	4.69	2.86	0.10	0.11	1763	29	133
501	9.55	0.29	4.63	2.89	0.10	0.11	1774	30	133
502	9.84	0.29	4.57	2.93	0.09	0.10	1784	30	133
503	10.13	0.30	4.51	2.96	0.09	0.10	1794	31	133
504	10.43	0.31	4.44	2.99	0.09	0.10	1804	31	133
505	10.74	+0.31	4.39	3.02	0.09	0.10	1814	32	133
506	11.05	0.31	4.33	3.06	0.09	0.10	1824	32	133
507	11.36	0.32	4.28	3.09	0.08	0.10	1834	33	133
508	11.68	0.32	4.22	3.12	0.08	0.10	1844	34	133
509	12.00	0.33	4.16	3.15	0.08	0.10	1853	35	133
510	12.33	+0.33	4.10	3.18	0.08	0.09	1863	35	133
511	12.66	0.34	4.04	3.21	0.08	0.09	1873	36	133
512	13.00	0.34	3.98	3.25	0.07	0.09	1882	37	133
513	13.34	0.34	3.92	3.28	0.07	0.09	1892	38	133
514	13.68	0.35	3.86	3.31	0.07	0.09	1901	39	133
515	14.03	+0.36	3.80	3.34	0.07	0.09	1910	40	133
516	14.39	0.36	3.74	3.37	0.07	0.09	1920	40	133
517	14.75	0.36	3.68	3.40	0.07	0.09	1929	41	133
518	15.11	0.37	3.62	3.43	0.07	0.09	1938	42	133
519	15.48	0.37	3.56	3.46	0.07	0.08	1947	43	133
520	15.85	+0.38	3.50	3.49	0.06	0.08	1956	44	132
521	16.23	0.38	3.44	3.51	0.06	0.08	1964	45	132
522	16.61	0.38	3.38	3.54	0.06	0.08	1973	46	132
523	16.99	0.39	3.32	3.57	0.06	0.08	1982	47	132
524	17.38	0.39	3.27	3.60	0.06	0.08	1990	48	132
525	17.77	+0.40	3.21	3.62	0.06	0.08	1999	49	132
526	18.17	0.40	3.15	3.65	0.06	0.08	2007	50	132
527	18.57	0.40	3.09	3.67	0.06	0.08	2016	51	132
528	18.97	0.41	3.03	3.70	0.06	0.07	2024	52	132
529	19.38	0.41	2.97	3.73	0.05	0.07	2032	53	132
530	19.79	+0.42	2.91	3.75	0.05	0.07	2040	55	132
531	20.21	0.41	2.86	3.77	0.05	0.07	2048	56	132
532	20.62	0.43	2.80	3.80	0.05	0.07	2056	57	132
533	21.05	0.42	2.75	3.82	0.05	0.07	2063	58	132
534	21.47	0.43	2.69	3.84	0.05	0.07	2071	59	131
535	21.90	+0.44	2.63	3.87	0.05	0.07	2079	61	131
536	22.34	0.43	2.57	3.89	0.05	0.07	2086	62	131
537	22.77	0.44	2.52	3.91	0.05	0.07	2094	63	131
538	23.21	0.45	2.46	3.93	0.05	0.07	2101	64	131
539	23.66	0.44	2.40	3.96	0.05	0.06	2108	66	131
540	24.10		2.35	3.98	0.05	0.06	2115	67	131

TABLE VIII, Arg. 1.—*Concluded.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	(*v.c.*1)	(*v.s.*2)	(*v.c.*2)	(*ρ.c.*0)	(*ρ.s.*1)	(*ρ.c.*1)
	″	″	″	″	″	″			
540	24.10	+0.46	2.35	3.98	0.05	0.06	2115	67	131
541	24.56	0.45	2.29	4.00	0.05	0.06	2122	68	131
542	25.01	0.46	2.24	4.02	0.05	0.06	2129	70	131
543	25.47	0.46	2.18	4.04	0.05	0.06	2135	71	130
544	25.93	0.46	2.13	4.06	0.05	0.06	2142	73	130
545	26.39	+0.46	2.08	4.07	0.05	0.06	2149	74	130
546	26.85	0.47	2.02	4.09	0.05	0.06	2155	75	130
547	27.32	0.47	1.97	4.11	0.05	0.06	2162	77	130
548	27.79	0.48	1.92	4.13	0.05	0.06	2168	78	130
549	28.27	0.48	1.87	4.14	0.05	0.06	2174	80	130
550	28.75	+0.48	1.82	4.16	0.05	0.06	2180	81	130
551	29.23	0.48	1.77	4.17	0.05	0.06	2186	83	130
552	29.71	0.49	1.72	4.19	0.06	0.06	2191	84	130
553	30.20	0.49	1.67	4.20	0.06	0.06	2197	86	130
554	30.69	0.49	1.62	4.22	0.06	0.06	2203	87	130
555	31.18	+0.49	1.57	4.23	0.06	0.06	2208	89	130
556	31.67	0.49	1.52	4.25	0.06	0.06	2214	90	130
557	32.16	0.50	1.48	4.26	0.06	0.06	2219	92	130
558	32.66	0.50	1.43	4.27	0.06	0.06	2224	94	130
559	33.16	0.51	1.38	4.28	0.06	0.06	2229	95	130
560	33.67	+0.50	1.34	4.29	0.06	0.06	2234	97	130
561	34.17	0.51	1.29	4.31	0.06	0.06	2238	98	130
562	34.68	0.51	1.25	4.32	0.07	0.06	2243	100	130
563	35.19	0.51	1.20	4.32	0.07	0.06	2248	101	130
564	35.70	0.51	1.16	4.33	0.07	0.06	2252	103	130
565	36.21	+0.52	1.12	4.34	0.07	0.06	2256	105	130
566	36.73	0.52	1.08	4.35	0.07	0.06	2261	106	130
567	37.25	0.51	1.03	4.36	0.07	0.06	2265	108	130
568	37.76	0.53	0.99	4.37	0.07	0.06	2269	110	130
569	38.29	0.52	0.96	4.38	0.07	0.06	2272	111	130
570	38.81	+0.53	0.92	4.38	0.07	0.06	2276	113	130
571	39.34	0.52	0.88	4.39	0.07	0.06	2280	115	130
572	39.86	0.53	0.84	4.40	0.07	0.07	2283	116	130
573	40.39	0.53	0.80	4.40	0.08	0.07	2287	118	130
574	40.92	0.53	0.77	4.41	0.08	0.07	2290	120	131
575	41.45	+0.53	0.73	4.41	0.08	0.07	2293	121	131
576	41.98	0.54	0.70	4.42	0.08	0.07	2296	123	131
577	42.52	0.53	0.67	4.42	0.08	0.07	2299	125	131
578	43.05	0.54	0.63	4.42	0.08	0.07	2302	126	131
579	43.59	0.53	0.60	4.43	0.08	0.07	2304	128	131
580	44.12	+0.54	0.57	4.43	0.08	0.07	2307	130	131
581	44.66	0.54	0.54	4.43	0.08	0.08	2309	132	131
582	45.20	0.54	0.51	4.44	0.09	0.08	2312	133	131
583	45.74	0.54	0.48	4.44	0.09	0.08	2314	135	131
584	46.28	0.55	0.46	4.44	0.09	0.08	2316	137	132
585	46.83	+0.54	0.43	4.44	0.09	0.08	2318	138	132
586	47.37	0.54	0.40	4.44	0.09	0.08	2319	140	132
587	47.91	0.55	0.38	4.44	0.09	0.09	2321	142	132
588	48.46	0.54	0.36	4.44	0.09	0.09	2322	144	132
589	49.00	0.55	0.33	4.44	0.09	0.09	2324	145	132
590	49.55	+0.55	0.31	4.44	0.09	0.09	2325	147	132
591	50.10	0.54	0.29	4.44	0.09	0.09	2326	149	132
592	50.64	0.55	0.27	4.44	0.10	0.09	2327	150	133
593	51.19	0.55	0.25	4.44	0.10	0.10	2328	152	133
594	51.74	0.55	0.23	4.43	0.10	0.10	2329	154	133
595	52.29	+0.55	0.22	4.43	0.10	0.10	2330	155	133
596	52.84	0.54	0.20	4.43	0.10	0.10	2330	157	133
597	53.38	0.55	0.18	4.43	0.10	0.11	2330	159	133
598	53.93	0.55	0.17	4.43	0.10	0.11	2331	161	134
599	54.48	0.55	0.16	4.42	0.10	0.11	2331	162	134
600	55.03		0.14	4.42	0.10	0.11	2331	164	134

TABLE IX, ARG. 2.—ACTION OF SATURN.

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	Sec.var.	(v.c.1)	Diff.	Sec.var.	(v.s.2)	Diff.	Sec.var.	(v.c.2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	38.54	+0.09	139.94	+1.44	2.53	293.78	+0.03	0.56	182.62	+1.18	1.77	244.67	−0.57	0.12
1	38.63	0.09	141.38	1.44	2.52	293.81	+0.02	0.55	183.80	1.16	1.76	244.10	0.58	0.12
2	38.72	0.09	142.82	1.44	2.50	293.83	0.00	0.54	184.96	1.16	1.74	243.52	0.59	0.12
3	38.81	0.09	144.26	1.44	2.49	293.83	−0.01	0.53	186.12	1.15	1.73	242.93	0.61	0.11
4	38.90	0.08	145.70	1.44	2.48	293.82	0.03	0.52	187.27	1.15	1.71	242.32	0.61	0.11
5	38.98	+0.09	147.14	+1.44	2.47	293.79	−0.04	0.51	188.42	+1.14	1.70	241.71	−0.62	0.11
6	39.07	0.09	148.58	1.43	2.46	293.75	0.05	0.50	189.56	1.13	1.69	241.09	0.63	0.11
7	39.16	0.08	150.01	1.44	2.45	293.70	0.07	0.49	190.69	1.13	1.67	240.46	0.65	0.11
8	39.24	0.09	151.45	1.44	2.44	293.63	0.09	0.48	191.82	1.12	1.66	239.81	0.66	0.10
9	39.33	0.08	152.89	1.43	2.43	293.54	0.10	0.47	192.94	1.11	1.64	239.15	0.67	0.10
10	39.41	+0.09	154.32	+1.44	2.42	293.44	−0.11	0.46	194.05	+1.11	1.63	238.48	−0.68	0.10
11	39.50	0.08	155.76	1.43	2.41	293.33	0.13	0.45	195.16	1.10	1.61	237.80	0.69	0.10
12	39.58	0.09	157.19	1.43	2.40	293.20	0.14	0.44	196.26	1.09	1.60	237.11	0.70	0.10
13	39.67	0.08	158.62	1.43	2.39	293.06	0.16	0.43	197.35	1.08	1.58	236.41	0.72	0.10
14	39.75	0.09	160.05	1.43	2.38	292.90	0.17	0.42	198.43	1.08	1.57	235.69	0.73	0.10
15	39.84	+0.08	161.48	+1.42	2.36	292.73	−0.18	0.42	199.51	+1.07	1.55	234.96	−0.74	0.10
16	39.92	0.09	162.90	1.42	2.35	292.55	0.21	0.41	200.58	1.06	1.53	234.22	0.74	0.09
17	40.01	0.09	164.32	1.42	2.34	292.34	0.21	0.40	201.64	1.05	1.52	233.48	0.75	0.09
18	40.10	0.08	165.74	1.42	2.33	292.13	0.23	0.39	202.69	1.05	1.50	232.73	0.77	0.09
19	40.18	0.09	167.16	1.42	2.32	291.90	0.25	0.38	203.74	1.04	1.49	231.96	0.78	0.09
20	40.27	+0.08	168.58	+1.41	2.31	291.65	−0.26	0.37	204.78	+1.03	1.47	231.18	−0.79	0.09
21	40.35	0.09	169.99	1.41	2.30	291.39	0.27	0.36	205.81	1.02	1.46	230.39	0.79	0.09
22	40.44	0.08	171.40	1.41	2.29	291.12	0.29	0.35	206.83	1.01	1.44	229.60	0.80	0.09
23	40.52	0.10	172.81	1.41	2.27	290.83	0.30	0.35	207.84	1.01	1.43	228.80	0.82	0.10
24	40.62	0.09	174.22	1.40	2.26	290.53	0.31	0.34	208.85	1.00	1.41	227.98	0.83	0.10
25	40.71	+0.09	175.62	+1.40	2.25	290.22	−0.33	0.33	209.85	+0.99	1.40	227.15	−0.84	0.10
26	40.80	0.09	177.02	1.40	2.24	289.89	0.34	0.32	210.84	0.98	1.39	226.31	0.85	0.10
27	40.89	0.09	178.42	1.39	2.23	289.55	0.36	0.31	211.82	0.97	1.37	225.46	0.86	0.10
28	40.98	0.09	179.81	1.39	2.21	289.19	0.37	0.30	212.79	0.97	1.36	224.60	0.86	0.11
29	41.07	0.09	181.20	1.39	2.20	288.82	0.38	0.30	213.76	0.95	1.34	223.74	0.87	0.11
30	41.16	+0.09	182.59	+1.38	2.19	288.44	−0.40	0.29	214.71	+0.95	1.33	222.87	−0.88	0.11
31	41.25	0.10	183.97	1.38	2.18	288.04	0.41	0.28	215.66	0.94	1.32	221.99	0.89	0.11
32	41.35	0.09	185.35	1.38	2.17	287.63	0.43	0.28	216.60	0.93	1.30	221.10	0.90	0.11
33	41.44	0.10	186.73	1.37	2.15	287.20	0.44	0.27	217.53	0.91	1.29	220.20	0.92	0.12
34	41.54	0.10	188.10	1.37	2.14	286.76	0.46	0.26	218.44	0.91	1.27	219.28	0.92	0.12
35	41.64	+0.10	189.47	+1.36	2.13	286.30	−0.46	0.26	219.35	+0.90	1.26	218.36	−0.93	0.12
36	41.74	0.09	190.83	1.36	2.11	285.84	0.49	0.25	220.25	0.89	1.25	217.43	0.94	0.12
37	41.83	0.10	192.19	1.36	2.10	285.35	0.49	0.24	221.14	0.89	1.23	216.49	0.94	0.12
38	41.93	0.10	193.55	1.35	2.09	284.86	0.51	0.23	222.03	0.87	1.22	215.55	0.96	0.13
39	42.03	0.10	194.90	1.35	2.07	284.35	0.51	0.23	222.90	0.86	1.20	214.59	0.97	0.13
40	42.13	+0.11	196.25	+1.34	2.06	283.84	−0.53	0.22	223.76	+0.85	1.19	213.62	−0.97	0.13
41	42.24	0.10	197.59	1.34	2.05	283.31	0.55	0.22	224.61	0.84	1.18	212.65	0.98	0.13
42	42.34	0.10	198.93	1.33	2.03	282.76	0.56	0.21	225.45	0.83	1.16	211.67	0.99	0.14
43	42.44	0.11	200.26	1.33	2.02	282.20	0.57	0.21	226.28	0.83	1.15	210.68	1.00	0.14
44	42.55	0.11	201.59	1.32	2.00	281.63	0.59	0.20	227.11	0.81	1.13	209.68	1.01	0.14
45	42.66	+0.11	202.91	+1.32	1.99	281.04	−0.60	0.20	227.92	+0.80	1.12	208.67	−1.01	0.15
46	42.77	0.10	204.23	1.31	1.97	280.44	0.62	0.19	228.72	0.79	1.10	207.66	1.02	0.15
47	42.87	0.11	205.54	1.31	1.96	279.82	0.62	0.19	229.51	0.78	1.09	206.64	1.03	0.15
48	42.98	0.11	206.85	1.30	1.94	279.20	0.64	0.18	230.29	0.78	1.07	205.61	1.04	0.15
49	43.09	0.11	208.15	1.29	1.93	278.56	0.65	0.18	231.07	0.76	1.06	204.57	1.05	0.16
50	43.20	+0.12	209.44	+1.29	1.92	277.91	−0.67	0.17	231.83	+0.75	1.04	203.52	−1.05	0.16
51	43.32	0.11	210.73	1.28	1.91	277.24	0.68	0.17	232.58	0.74	1.03	202.47	1.06	0.17
52	43.43	0.11	212.01	1.28	1.89	276.56	0.69	0.16	233.32	0.73	1.01	201.41	1.07	0.17
53	43.54	0.11	213.29	1.28	1.88	275.87	0.70	0.16	234.05	0.72	1.00	200.34	1.08	0.18
54	43.65	0.12	214.57	1.27	1.87	275.17	0.72	0.16	234.77	0.71	0.99	199.26	1.08	0.18
55	43.77	+0.11	215.84	+1.26	1.85	274.45	−0.73	0.15	235.48	+0.70	0.97	198.18	−1.09	0.19
56	43.88	0.12	217.10	1.25	1.84	273.72	0.74	0.15	236.18	0.69	0.96	197.09	1.09	0.20
57	44.00	0.12	218.35	1.25	1.82	272.98	0.75	0.15	236.87	0.67	0.95	196.00	1.10	0.20
58	44.12	0.12	219.60	1.24	1.81	272.23	0.77	0.15	237.54	0.66	0.94	194.90	1.11	0.21
59	44.24	0.11	220.84	1.23	1.79	271.46	0.77	0.14	238.20	0.65	0.92	193.79	1.12	0.21
60	44.35		222.07		1.78	270.69		0.14	238.85		0.91	192.67		0.22

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	″	″	″	″							
0	11.40	14.53	1.58	1.68	1678	182	1513	711	306	137	85
1	11.52	14.48	1.59	1.66	1679	182	1526	711	302	137	85
2	11.64	14.43	1.60	1.65	1680	183	1539	710	298	137	85
3	11.75	14.38	1.61	1.64	1680	184	1551	709	295	137	85
4	11.87	14.33	1.62	1.63	1681	185	1564	703	291	137	85
5	11.98	14.28	1.63	1.62	1682	187	1577	707	288	137	85
6	12.10	14.22	1.64	1.61	1682	188	1590	706	284	137	85
7	12.21	14.16	1.64	1.60	1683	190	1603	704	281	138	84
8	12.32	14.10	1.65	1.59	1683	191	1615	703	278	138	84
9	12.43	14.04	1.66	1.58	1682	193	1628	701	274	138	84
10	12.54	13.97	1.67	1.56	1682	195	1641	700	271	138	84
11	12.65	13.91	1.68	1.55	1681	197	1654	699	267	138	84
12	12.76	13.84	1.69	1.54	1680	199	1666	698	264	138	84
13	12.87	13.77	1.69	1.53	1679	201	1679	696	260	138	84
14	12.97	13.70	1.70	1.52	1678	203	1691	695	257	137	83
15	13.07	13.63	1.71	1.50	1677	206	1704	694	253	137	83
16	13.18	13.56	1.71	1.49	1675	208	1716	692	250	137	83
17	13.28	13.49	1.72	1.48	1674	211	1729	691	246	137	83
18	13.38	13.41	1.73	1.47	1672	214	1741	689	243	137	82
19	13.48	13.33	1.74	1.45	1670	217	1754	687	239	137	82
20	13.58	13.25	1.74	1.44	1668	220	1766	685	236	137	82
21	13.68	13.17	1.75	1.43	1666	223	1778	683	233	137	82
22	13.78	13.09	1.75	1.42	1664	227	1790	681	230	137	81
23	13.87	13.01	1.76	1.40	1662	230	1302	679	226	137	81
24	13.97	12.92	1.76	1.39	1660	234	1814	677	223	137	81
25	14.06	12.83	1.77	1.37	1658	237	1826	675	220	137	81
26	14.15	12.74	1.78	1.36	1656	241	1838	673	217	137	81
27	14.24	12.65	1.78	1.35	1653	245	1850	671	214	137	80
28	14.33	12.56	1.79	1.34	1651	249	1862	668	211	137	80
29	14.41	12.47	1.79	1.32	1648	253	1874	666	208	137	80
30	14.50	12.37	1.79	1.31	1645	258	1886	664	205	137	79
31	14.58	12.28	1.80	1.30	1642	262	1898	662	202	137	78
32	14.66	12.19	1.80	1.28	1638	267	1910	659	199	137	78
33	14.74	12.09	1.80	1.27	1635	272	1922	657	196	137	78
34	14.82	11.99	1.81	1.25	1631	277	1934	654	193	137	78
35	14.90	11.89	1.81	1.24	1627	282	1946	652	190	137	77
36	14.97	11.79	1.81	1.22	1623	287	1958	650	187	137	77
37	15.05	11.69	1.81	1.21	1619	292	1970	647	184	137	77
38	15.12	11.59	1.82	1.20	1615	297	1982	645	181	137	76
39	15.19	11.49	1.82	1.18	1611	302	1993	642	179	137	76
40	15.26	11.38	1.82	1.17	1608	308	2005	640	176	137	76
41	15.33	11.28	1.82	1.15	1602	313	2016	637	173	137	76
42	15.39	11.17	1.82	1.14	1597	319	2028	635	171	137	76
43	15.46	11.06	1.82	1.13	1592	325	2039	632	168	137	75
44	15.52	10.95	1.82	1.11	1587	331	2051	630	166	137	74
45	15.58	10.84	1.82	1.10	1582	337	2062	627	163	137	74
46	15.64	10.73	1.82	1.08	1577	343	2073	624	160	137	74
47	15.70	10.62	1.82	1.07	1572	349	2084	621	158	137	73
48	15.76	10.51	1.82	1.06	1566	355	2095	619	155	137	73
49	15.81	10.40	1.82	1.04	1561	362	2106	616	153	137	73
50	15.86	10.28	1.82	1.03	1556	368	2117	613	150	137	72
51	15.91	10.17	1.82	1.01	1550	375	2128	610	148	137	72
52	15.96	10.05	1.82	1.00	1545	381	2138	607	145	137	72
53	16.01	9.94	1.81	0.98	1539	388	2149	604	143	137	72
54	16.05	9.82	1.81	0.97	1533	394	2159	601	140	137	72
55	16.09	9.70	1.81	0.96	1527	401	2170	598	138	137	71
56	16.13	9.59	1.81	0.94	1521	408	2180	595	135	137	71
57	16.17	9.47	1.80	0.93	1515	415	2191	592	133	137	71
58	16.20	9.35	1.80	0.92	1509	422	2201	589	131	137	71
59	16.23	9.23	1.80	0.90	1502	430	2212	586	129	137	71
60	16.26	9.11	1.80	0.89	1496	437	2222	583	127	137	71

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
60	44.35	+0.12	222.07	+1.23	1.78	270.69	−0.80	0.14	238.85	+0.64	0.91	192.67	−1.12	0.22
61	44.47	0.12	223.30	1.22	1.76	269.89	0.80	0.13	239.49	0.63	0.90	191.55	1.13	0.23
62	44.59	0.12	224.52	1.21	1.75	269.09	0.81	0.13	240.12	0.62	0.98	190.42	1.13	0.23
63	44.71	0.12	225.73	1.21	1.73	268.28	0.83	0.13	240.74	0.61	0.87	189.29	1.15	0.24
64	44.83	0.12	226.94	1.20	1.72	267.45	0.84	0.13	241.35	0.60	0.86	188.14	1.15	0.25
65	44.95	+0.12	228.14	+1.19	1.70	266.61	−0.85	0.12	241.95	+0.58	0.84	186.99	−1.16	0.25
66	45.07	0.12	229.33	1.18	1.69	265.76	0.87	0.12	242.53	0.57	0.83	185.83	1.16	0.26
67	45.19	0.12	230.51	1.18	1.67	264.89	0.87	0.12	243.10	0.57	0.82	184.67	1.16	0.27
68	45.31	0.13	231.69	1.17	1.66	264.02	0.89	0.12	243.67	0.55	0.81	183.51	1.17	0.28
69	45.44	0.12	232.86	1.16	1.64	263.13	0.90	0.11	244.22	0.54	0.79	182.34	1.18	0.28
70	45.56	+0.12	234.02	+1.15	1.63	262.23	−0.91	0.11	244.76	+0.52	0.78	181.16	−1.19	0.29
71	45.68	0.12	235.17	1.14	1.62	261.32	0.93	0.11	245.28	0.51	0.77	179.97	1.19	0.30
72	45.80	0.12	236.31	1.14	1.60	260.39	0.93	0.11	245.79	0.50	0.75	178.78	1.19	0.31
73	45.92	0.12	237.45	1.13	1.59	259.46	0.94	0.11	246.29	0.49	0.74	177.59	1.19	0.32
74	46.04	0.12	238.58	1.12	1.57	258.52	0.96	0.11	246.78	0.48	0.73	176.40	1.20	0.33
75	46.16	+0.13	239.70	+1.11	1.56	257.56	−0.97	0.11	247.26	+0.47	0.71	175.20	−1.21	0.34
76	46.29	0.12	240.81	1.10	1.55	256.59	0.98	0.11	247.73	0.45	0.70	173.99	1.21	0.34
77	46.41	0.12	241.91	1.09	1.53	255.61	0.99	0.11	248.18	9.44	0.69	172.78	1.22	0.35
78	46.53	0.12	243.00	1.09	1.52	254.62	1.00	0.11	248.62	0.43	0.68	171.56	1.22	0.36
79	46.65	0.12	244.09	1.08	1.50	253.62	1.01	0.11	249.05	0.42	0.66	170.34	1.23	0.37
80	46.77	+0.12	245.17	+1.07	1.49	252.61	−1.02	0.11	249.47	+0.40	0.65	169.11	−1.23	0.38
81	46.89	0.12	246.24	1.06	1.48	251.59	1.03	0.11	249.87	0.39	0.64	167.88	1.23	0.39
82	47.01	0.12	247.30	1.05	1.46	250.56	1.05	0.11	250.26	0.38	0.63	166.65	1.24	0.40
83	47.13	0.11	248.35	1.03	1.45	249.51	1.06	0.11	250.64	0.37	0.62	165.41	1.24	0.41
84	47.24	0.12	249.38	1.03	1.43	248.45	1.07	0.11	251.01	0.36	0.61	164.17	1.25	0.42
85	47.36	+0.12	250.41	+1.02	1.42	247.38	−1.08	0.11	251.37	+0.34	0.60	162.92	−1.25	0.43
86	47.48	0.11	251.43	1.01	1.40	246.30	1.09	0.12	251.71	0.33	0.58	161.67	1.25	0.43
87	47.59	0.12	252.44	1.00	1.39	245.21	1.09	0.12	252.04	0.32	0.57	160.42	1.26	0.44
88	47.71	0.11	253.44	0.99	1.38	244.12	1.11	0.12	252.36	0.31	0.56	159.16	1.26	0.45
89	47.82	0.12	254.43	0.98	1.36	243.01	1.12	0.12	252.67	0.29	0.55	157.90	1.27	0.46
90	47.94	+0.11	255.41	+0.97	1.35	241.89	−1.13	0.12	252.96	+0.28	0.54	156.63	−1.27	0.47
91	48.05	0.11	256.38	0.96	1.33	240.76	1.14	0.12	253.24	0.26	0.53	155.36	1.27	0.48
92	48.16	0.11	257.34	0.95	1.32	239.62	1.14	0.12	253.50	0.25	0.52	154.09	1.27	0.49
93	48.27	0.11	258.29	0.94	1.30	238.48	1.15	0.13	253.75	0.23	0.51	152.82	1.27	0.50
94	48.38	0.11	259.23	0.93	1.29	237.33	1.17	0.13	253.98	0.23	0.50	151.55	1.28	0.51
95	48.49	+0.10	260.16	+0.92	1.28	236.16	−1.18	0.13	254.21	+0.22	0.49	150.27	−1.28	0.52
96	48.59	0.11	261.08	0.91	1.26	234.98	1.19	0.13	254.43	0.20	0.48	148.99	1.28	0.54
97	48.70	0.10	261.99	0.89	1.25	233.79	1.19	0.13	254.63	0.19	0.47	147.71	1.29	0.55
98	48.80	0.11	262.88	0.88	1.23	232.60	1.21	0.13	254.82	0.17	0.46	146.42	1.29	0.56
99	48.91	0.10	263.76	0.87	1.22	231.39	1.21	0.14	254.99	0.16	0.45	145.13	1.29	0.57
100	49.01	+0.10	264.63	+0.86	1.20	230.18	−1.22	0.14	255.15	+0.15	0.44	143.84	−1.29	0.58
101	49.11	0.10	265.49	0.85	1.19	228.96	1.23	0.14	255.30	0.13	0.43	142.55	1.29	0.59
102	49.21	0.10	266.34	0.84	1.17	227.73	1.23	0.15	255.43	0.12	0.42	141.26	1.29	0.60
103	49.31	0.09	267.18	0.82	1.16	226.50	1.25	0.15	255.55	0.11	0.41	139.97	1.30	0.61
104	49.40	0.10	268.00	0.81	1.14	225.25	1.26	0.16	255.66	0.09	0.40	138.67	1.30	0.62
105	49.50	+0.09	268.81	+0.80	1.13	223.99	−1.26	0.16	255.75	+0.08	0.39	137.37	−1.30	0.63
106	49.59	0.09	269.61	0.80	1.11	222.73	1.27	0.16	255.83	0.07	0.39	136.07	1.30	0.65
107	49.68	0.09	270.41	0.78	1.10	221.46	1.28	0.17	255.90	0.05	0.38	134.77	1.30	0.66
108	49.77	0.08	271.19	0.77	1.09	220.18	1.30	0.17	255.95	0.04	0.37	133.47	1.50	0.67
109	49.85	0.09	271.96	0.75	1.07	218.88	1.30	0.18	255.99	0.03	0.36	132.17	1.30	0.68
110	49.94	+0.08	272.71	+0.74	1.06	217.58	−1.30	0.18	256.02	+0.02	0.35	130.87	−1.30	0.69
111	50.02	0.09	273.45	0.73	1.05	216.28	1.31	0.18	256.04	0.00	0.34	129.57	1.31	0.70
112	50.11	0.08	274.18	0.72	1.03	214.97	1.32	0.19	256.04	−0.01	0.33	128.26	1.30	0.71
113	50.19	0.07	274.90	0.70	1.02	213.65	1.32	0.19	256.03	0.03	0.33	126.96	1.30	0.73
114	50.26	0.08	275.60	0.69	1.00	212.33	1.34	0.20	256.00	0.04	0.32	125.66	1.30	0.74
115	50.34	+0.07	276.29	+0.68	0.99	210.99	−1.34	0.20	255.96	−0.05	0.31	124.36	−1.31	0.75
116	50.41	0.07	276.97	0.66	0.98	209.65	1.34	0.21	255.91	0.07	0.30	123.05	1.30	0.76
117	50.48	0.07	277.63	0.65	0.97	208.31	1.35	0.21	255.84	0.08	0.29	121.75	1.30	0.77
118	50.55	0.07	278.28	0.64	0.96	206.96	1.36	0.22	255.76	0.09	0.28	120.45	1.30	0.78
119	50.62	0.07	278.92	0.63	0.94	205.60	1.37	0.22	255.67	0.11	0.28	119.15	1.30	0.80
120	50.69		279.55		0.93	204.23		0.23	255.56		0.27	117.85		0.81

TABLE IX, Arg. 2.—*Continued.*

Arg.	$(v.s.3)$	$(v.c.3)$	$(v.s.4)$	$(v.c.4)$	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$	$(\rho.s.3)$	$(\rho.c.3)$
	″	″	″	″							
60	16.26	9.11	1.80	0.89	1496	437	2222	583	127	137	71
61	16.29	9.00	1.79	0.88	1489	445	2233	580	125	137	70
62	16.32	8.88	1.79	0.87	1483	453	2243	577	123	137	70
63	16.35	8.76	1.78	0.85	1477	460	2254	574	121	137	70
64	16.38	8.64	1.78	0.84	1470	468	2264	570	119	137	69
65	16.41	8.52	1.78	0.83	1464	476	2275	567	117	136	69
66	16.43	8.40	1.77	0.81	1457	484	2285	564	115	136	68
67	16.45	8.28	1.77	0.80	1451	492	2295	560	113	136	68
68	16.47	8.16	1.76	0.79	1444	501	2306	557	112	136	67
69	16.49	8.04	1.75	0.78	1438	509	2316	553	110	136	67
70	16.50	7.91	1.75	0.76	1431	518	2326	550	108	136	66
71	16.51	7.79	1.74	0.75	1424	527	2336	546	107	136	66
72	16.52	7.67	1.74	0.74	1417	536	2346	543	105	136	66
73	16.53	7.55	1.73	0.73	1410	545	2356	540	103	135	65
74	16.54	7.43	1.72	0.71	1403	554	2365	536	102	135	65
75	16.54	7.31	1.72	0.70	1396	563	2375	533	100	135	64
76	16.54	7.19	1.71	0.69	1389	572	2384	530	99	135	64
77	16.54	7.07	1.70	0.68	1382	581	2394	526	98	135	64
78	16.54	6.95	1.70	0.67	1374	590	2403	523	96	135	63
79	16.53	6.83	1.69	0.65	1367	599	2413	519	95	135	63
80	16.53	6.71	1.68	0.64	1360	608	2422	516	94	135	62
81	16.52	6.60	1.67	0.63	1353	618	2431	512	93	135	62
82	16.52	6.48	1.66	0.62	1346	627	2440	508	92	134	61
83	16.51	6.36	1.65	0.61	1339	637	2449	505	91	134	61
84	16.50	6.24	1.64	0.60	1332	646	2457	501	89	134	60
85	16.48	6.13	1.63	0.59	1325	656	2466	498	88	134	60
86	16.46	6.01	1.62	0.58	1318	664	2475	494	87	134	59
87	16.44	5.89	1.61	0.57	1311	676	2483	490	86	134	59
88	16.42	5.78	1.60	0.56	1304	684	2492	487	85	134	58
89	16.40	5.66	1.59	0.55	1296	696	2500	483	84	133	58
90	16.38	5.55	1.58	0.54	1289	706	2509	480	84	133	57
91	16.35	5.44	1.57	0.53	1281	717	2517	476	83	133	57
92	16.33	5.32	1.56	0.52	1273	727	2525	472	82	133	56
93	16.30	5.21	1.55	0.51	1266	738	2534	469	82	132	56
94	16.27	5.09	1.54	0.50	1258	749	2542	465	81	132	55
95	16.24	4.98	1.53	0.50	1250	760	2550	461	81	132	55
96	16.20	4.87	1.52	0.49	1242	771	2558	457	80	131	54
97	16.17	4.76	1.51	0.48	1234	782	2566	454	80	131	54
98	16.13	4.64	1.50	0.48	1226	793	2574	450	80	131	53
99	16.09	4.54	1.49	0.47	1218	804	2582	446	79	130	53
100	16.05	4.44	1.48	0.46	1210	815	2590	442	79	130	52
101	16.01	4.33	1.46	0.46	1202	826	2598	438	79	130	51
102	15.96	4.23	1.45	0.45	1194	838	2605	435	79	129	51
103	15.92	4.13	1.44	0.44	1186	849	2613	431	78	129	50
104	15.87	4.02	1.43	0.44	1178	861	2620	428	78	129	50
105	15.83	3.92	1.42	0.43	1170	872	2627	424	78	128	49
106	15.78	3.82	1.41	0.43	1162	883	2634	420	78	128	49
107	15.73	3.72	1.39	0.42	1154	895	2641	416	78	128	48
108	15.68	3.62	1.38	0.42	1146	907	2647	413	78	127	48
109	15.62	3.52	1.37	0.41	1138	919	2654	409	78	127	47
110	15.56	3.43	1.36	0.41	1130	931	2661	405	78	126	46
111	15.51	3.34	1.34	0.40	1122	943	2668	402	78	126	46
112	15.45	3.24	1.33	0.40	1114	955	2674	398	79	125	45
113	15.39	3.15	1.32	0.40	1106	967	2681	394	79	125	45
114	15.33	3.06	1.31	0.39	1098	979	2687	391	79	124	44
115	15.27	2.97	1.29	0.39	1090	991	2693	387	80	124	44
116	15.21	2.88	1.28	0.38	1082	1003	2699	383	80	123	44
117	15.14	2.80	1.26	0.38	1074	1016	2705	380	80	122	43
118	15.08	2.71	1.25	0.38	1066	1028	2711	376	81	122	43
119	15.01	2.62	1.24	0.38	1059	1041	2716	373	81	121	42
120	14.94	2.54	1.23	0.37	1051	1053	2722	370	82	120	42

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TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
120	50.69	+0.07	279.55	+0.61	0.93	204.23	−1.37	0.23	255.56	−0.12	0.27	117.85	−1.30	0.81
121	50.76	0.06	280.16	0.60	0.92	202.86	1.38	0.24	255.44	0.13	0.26	116.55	1.30	0.82
122	50.82	0.06	280.76	0.59	0.90	201.48	1.38	0.24	255.31	0.15	0.26	115.25	1.30	0.84
123	50.88	0.05	281.35	0.57	0.89	200.10	1.39	0.25	255.16	0.16	0.25	113.95	1.30	0.85
124	50.93	0.05	281.92	0.56	0.88	198.71	1.40	0.25	254.00	0.17	0.25	112.65	1.29	0.87
125	50.98	+0.05	282.48	+0.54	0.87	197.31	−1.40	0.26	254.83	−0.19	0.24	111.36	−1.29	0.88
126	51.03	0.05	283.02	0.53	0.85	195.91	1.41	0.27	254.64	0.20	0.23	110.07	1.30	0.89
127	51.08	0.05	283 55	0.52	0.84	194.50	1.41	0.27	254.44	0.21	0.23	108.77	1.29	0.91
128	51.13	0.04	284.07	0.50	0.83	193.09	1.42	0.28	254.23	0.23	0.22	107.48	1.29	0.92
129	51.17	0.05	284.57	0.49	0.81	191.67	1.42	0.28	254.00	0.24	0.22	106.19	1.28	0.94
130	51.22	+0.04	285.06	+0.47	0.80	190.25	−1.43	0.29	253.76	−0.25	0.21	104.91	−1.28	0.95
131	51.26	0.04	285.53	0.46	0.79	188.82	1.43	0.30	253.51	0.27	0.21	103.63	1.28	0.96
132	51.30	0.04	285.99	0.45	0.78	187.39	1.44	0.31	253.24	0.28	0.20	102.35	1.28	0.98
133	51.34	0.03	286.44	0.43	0.76	185.95	1.44	0.32	252.96	0.29	0.20	101.07	1.28	0.99
134	51.37	0.03	286.87	0.42	0.75	184.51	1.44	0.33	252.67	0.31	0.19	99.79	1.27	1.01
135	51.40	+0.03	287.29	+0.40	0.74	183.07	−1.45	0.33	252.36	−0.32	0.19	98.52	−1.27	1.02
136	51.43	0.02	287.69	0.39	0.73	181.62	1.45	0.34	252.04	0.33	0.18	97.25	1.26	1.03
137	51.45	0.02	288.08	0.37	0.72	180.17	1.45	0.35	251.71	0.35	0.18	95.99	1.26	1.05
138	51.47	0.02	288.45	0.36	0.70	178.72	1.46	0.36	251.36	0.36	0.17	94.73	1.26	1.06
139	51.49	0.02	288.81	0.35	0.69	177.26	1.46	0.37	251.00	0.37	0.17	93.47	1.26	1.08
140	51.51	+0.01	289.16	+0.33	0.68	175.80	−1.47	0.38	250.63	−0.39	0.16	92.21	−1.25	1.09
141	51.52	0.01	289.49	0.31	0.67	174.33	1.47	0.39	250.24	0.40	0.16	90.96	1.25	1.10
142	51.53	0.01	289.80	0.30	0.66	172.86	1.47	0.40	249.84	0.41	0.15	89.71	1.25	1.12
143	51.54	0.01	290.10	0.29	0.65	171.39	1.47	0.41	249.43	0.42	0.15	88.46	1.24	1.13
144	51.55	0.00	290.39	0.27	0.63	169.92	1.48	0.42	249.01	0.44	0.14	87.22	1.24	1.15
145	51.55	0.00	290.66	+0.26	0.62	168.44	−1.48	0.42	248.57	−0.45	0.14	85.98	−1.23	1.16
146	51.55	0.00	290.92	0.24	0.61	166.96	1.48	0.43	248.12	0.46	0.14	84.75	1.23	1.17
147	51.55	−0.01	291.16	0.22	0.60	165.48	1.48	0.44	247.66	0.48	0.13	83.52	1.23	1.19
148	51.54	0.01	291.38	0.21	0.58	164.00	1.48	0.45	247.18	0.49	0.13	82.29	1.22	1.20
149	51.53	0.01	291.59	0.20	0.57	162.52	1.49	0.46	246.69	0.50	0.12	81.07	1.22	1.22
150	51.52	−0.02	291.79	+0.18	0.56	161.03	−1.49	0.47	246.19	−0.51	0.12	79.85	−1.21	1.23
151	51.50	0.01	291.97	0.16	0.55	159.54	1.49	0.48	245.68	0.53	0.12	78.64	1.20	1.24
152	51.49	0.02	292.13	0.15	0.54	158.05	1.49	0.49	245.15	0.54	0.12	77.44	1.20	1.26
153	51.47	0.02	292.28	0.14	0.53	156.56	1.49	0.50	244.61	0.55	0.11	76.24	1.20	1.27
154	51.45	0.02	292.42	0.12	0.52	155.07	1.49	0.51	244.06	0.56	0.11	75.04	1.19	1.29
155	51.43	−0.03	292.54	+0.10	0.51	153.58	−1.49	0.52	243.50	−0.58	0.11	73.85	−1.18	1.30
156	51.40	0.03	292.64	0.09	0.50	152.09	1.49	0.54	242.92	0.59	0.11	72.67	1.18	1.31
157	51.37	0.03	292.73	0.07	0.49	150.60	1.49	0.55	242.33	0.60	0.11	71.49	1.17	1.33
158	51.34	0.04	292.80	0.06	0.48	149.11	1.49	0.56	241.73	0.61	0.10	70.32	1.17	1.34
159	51.30	0.04	292.86	0.04	0.47	147.62	1.50	0.57	241.12	0.63	0.10	69.15	1.16	1.36
160	51.26	−0.04	292.90	+0.03	0.46	146.12	−1.49	0.58	240.49	−0.64	0.10	67.99	−1.15	1.37
161	51.22	0.05	292.93	0.01	0.45	144.63	1.50	0.59	239.85	0.65	0.10	66.84	1.15	1.39
162	51.17	0.05	292.94	0.00	0.44	143.13	1.50	0.60	239.20	0.66	0.10	65.69	1.14	1.40
163	51.12	0.05	292.94	−0.02	0.43	141.63	1.49	0.61	238.54	0.67	0.10	64.55	1.14	1.42
164	51.07	0.06	292.92	0.04	0.42	140.14	1.50	0.62	237.87	0.69	0.10	63.41	1.13	1.43
165	51.01	−0.06	292.88	−0.05	0.41	138.64	−1.49	0.64	237.18	−0.70	0.09	62.28	−1.12	1.45
166	50.95	0.06	292.83	0.07	0.40	137.15	1.49	0.65	236.48	0.71	0.09	61.16	1.11	1.47
167	50.89	0.07	292.76	0.08	0.40	135.66	1.49	0.66	235.77	0.72	0.09	60.05	1.11	1.48
168	50.82	0.06	292.68	0.10	0.39	134.17	1.49	0.67	235.05	0.73	0.09	58.94	1.10	1.50
169	50.76	0.07	292.58	0.11	0.38	132.68	1.48	0.68	234.32	0.74	0.09	57.84	1.09	1.51
170	50.69	−0.07	292.47	−0.13	0.37	131.20	−1.49	0.69	233.58	−0.76	0.09	56.75	−1.08	1.53
171	50.62	0.08	292.34	0.15	0.36	129.71	1.48	0.70	232.82	0.77	0.09	55.67	1.08	1.54
172	50.54	0.08	292.19	0.16	0.35	128.23	1.48	0.71	232.05	0.78	0.09	54.59	1.07	1.56
173	50.46	0.09	292.03	0.17	0.35	126.75	1.48	0.73	231.27	0.79	0.10	53.52	1.07	1.57
174	50.37	0.09	291.86	0.19	0.34	125.27	1.48	0.74	230.48	0.80	0.10	52.45	1.05	1.59
175	50.28	−0.09	291.67	−0.21	0.33	123.79	−1.47	0.75	229.68	−0.81	0.10	51.40	−1.05	1.60
176	50.19	0.09	291.46	0.22	0.32	122.32	1.47	0.76	228.87	0.82	0.10	50.35	1.04	1.61
177	50.10	0.09	291.24	0.23	0.31	120.85	1.47	0.77	228.05	0.83	0.10	49.31	1.03	1.63
178	50.01	0.10	291.01	0.25	0.31	119.38	1.47	0.79	227.22	0.85	0.11	48.28	1.02	1.64
179	49.91	0.10	290.76	0.27	0.30	118.91	1.46	0.80	226.37	0.86	0.11	47.26	1.01	1.66
180	49.81		290.49		0.29	116.45		0.81	225.51		0.11	46.25		1.67

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)	(*ρ.c.*0)	(*ρ.s.*1)	(*ρ.c.*1)	(*ρ.s.*2)	(*ρ.c.*2)	(*ρ.s.*3)	(*ρ.c.*3)
	″	″	″	″							
120	14.94	2.54	1.23	0.37	1051	1053	2722	370	82	120	42
121	14.87	2.46	1.22	0.37	1043	1065	2727	366	82	120	42
122	14.80	2.38	1.20	0.37	1035	1078	2732	362	83	120	41
123	14.73	2.30	1.19	0.37	1028	1090	2737	359	83	119	41
124	14.66	2.22	1.18	0.37	1020	1103	2742	355	84	119	40
125	14.59	2.15	1.17	0.37	1012	1116	2747	352	85	119	40
126	14.52	2.07	1.15	0.37	1004	1129	2751	348	85	118	39
127	14.44	2.00	1.14	0.37	996	1141	2756	344	86	118	39
128	14.37	1.93	1.13	0.37	989	1154	2760	341	87	117	38
129	14.29	1.86	1.11	0.37	981	1167	2765	337	88	117	38
130	14.21	1.79	1.10	0.37	973	1180	2769	334	89	116	37
131	14.13	1.72	1.09	0.37	965	1193	2773	330	90	115	37
132	14.06	1.66	1.07	0.37	958	1206	2777	327	91	115	36
133	13.98	1.59	1.06	0.37	950	1219	2781	324	92	114	36
134	13.90	1.53	1.05	0.38	942	1232	2785	321	93	113	35
135	13.82	1.47	1.04	0.38	935	1245	2789	318	94	112	35
136	13.74	1.41	1.02	0.38	927	1258	2793	314	95	112	35
137	13.66	1.35	1.01	0.38	920	1271	2796	311	96	111	34
138	13.57	1.30	1.00	0.39	912	1284	2800	308	97	110	34
139	13.49	1.24	0.99	0.39	905	1298	2803	305	99	109	33
140	13.40	1.19	0.97	0.39	897	1311	2806	302	100	108	33
141	13.32	1.14	0.96	0.40	889	1325	2809	298	101	108	33
142	13.24	1.09	0.95	0.40	882	1338	2811	295	103	107	32
143	13.15	1.04	0.94	0.41	875	1352	2814	292	104	106	32
144	13.07	0.99	0.93	0.41	867	1366	2816	289	106	105	32
145	12.98	0.94	0.91	0.41	859	1379	2819	286	107	105	31
146	12.90	0.90	0.90	0.42	852	1393	2821	282	109	104	31
147	12.81	0.86	0.89	0.42	844	1407	2823	279	110	103	31
148	12.73	0.82	0.88	0.43	837	1420	2825	276	112	102	30
149	12.64	0.78	0.87	0.44	829	1433	2826	273	113	101	30
150	12.55	0.74	0.86	0.44	822	1447	2828	270	115	100	30
151	12.46	0.70	0.85	0.45	815	1461	2829	267	117	99	30
152	12.38	0.67	0.84	0.46	807	1474	2831	264	118	98	29
153	12.29	0.64	0.83	0.46	800	1488	2832	261	120	97	29
154	12.20	0.61	0.82	0.47	793	1502	2834	258	122	96	29
155	12.11	0.58	0.81	0.48	786	1515	2835	256	124	95	28
156	12.03	0.55	0.80	0.49	778	1529	2836	253	126	94	28
157	11.94	0.52	0.79	0.49	771	1543	2837	250	128	93	28
158	11.86	0.50	0.78	0.50	764	1556	2837	247	129	92	28
159	11.77	0.47	0.77	0.51	756	1569	2838	244	131	91	27
160	11.68	0.45	0.76	0.52	749	1583	2838	242	133	90	27
161	11.59	0.43	0.75	0.52	742	1597	2838	239	135	90	27
162	11.51	0.41	0.74	0.53	735	1611	2838	236	137	89	27
163	11.42	0.40	0.73	0.54	728	1624	2837	233	139	88	27
164	11.33	0.38	0.72	0.55	721	1638	2837	230	141	87	27
165	11.24	0.37	0.71	0.56	714	1652	2836	228	143	86	27
166	11.16	0.36	0.71	0.57	707	1665	2835	225	145	85	27
167	11.07	0.35	0.70	0.58	700	1679	2834	222	147	84	27
168	10.99	0.34	0.69	0.59	693	1693	2833	219	149	83	27
169	10.91	0.33	0.68	0.60	686	1706	2833	216	151	82	27
170	10.82	0.32	0.68	0.61	679	1720	2831	214	153	82	27
171	10.74	0.32	0.67	0.62	672	1734	2829	211	156	81	27
172	10.66	0.31	0.66	0.63	665	1747	2828	208	158	80	27
173	10.57	0.31	0.66	0.64	659	1761	2826	206	160	79	27
174	10.49	0.31	0.65	0.65	652	1775	2824	204	162	78	28
175	10.41	0.31	0.65	0.66	645	1788	2822	202	165	77	28
176	10.33	0.31	0.64	0.67	638	1802	2820	199	167	76	28
177	10.25	0.31	0.63	0.68	632	1815	2817	197	170	75	28
178	10.17	0.32	0.63	0.69	625	1829	2815	195	173	74	28
179	10.09	0.32	0.63	0.70	618	1842	2812	192	175	73	28
180	10.01	0.33	0.62	0.71	611	1855	2810	190	178	72	28

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
180	49.81	−0.10	290.49	−0.29	0.29	116.45	−1.46	0.81	225.51	−0.87	0.11	46.25	−1.01	1.67
181	49.71	0.11	290.20	0.30	0.28	114.99	1.46	0.82	224.64	0.87	0.11	45.24	1.00	1.68
182	49.60	0.11	289.90	0.31	0.28	113.53	1.45	0.84	223.77	0.88	0.11	44.24	0.99	1.70
183	49.49	0.12	289.59	0.33	0.27	112.08	1.45	0.85	222.89	0.90	0.12	43.25	0.97	1.71
184	49.37	0.12	289.26	0.35	0.26	110.63	1.45	0.86	221.99	0.91	0.12	42.28	0.97	1.73
185	49.25	−0.12	288.91	−0.36	0.26	109.18	−1.44	0.88	221.08	−0.92	0.12	41.31	−0.96	1.74
186	49.13	0.13	288.55	0.38	0.25	107.74	1.44	0.89	220.16	0.93	0.12	40.35	0.95	1.75
187	49.00	0.13	288.17	0.39	0.24	106.30	1.43	0.90	219.23	0.94	0.12	39.40	0.94	1.77
188	48.87	0.13	287.78	0.41	0.24	104.87	1.43	0.91	218.29	0.95	0.13	38.46	0.93	1.78
189	48.74	0.13	287.37	0.42	0.23	103.44	1.42	0.93	217.34	0.96	0.13	37.53	0.92	1.80
190	48.61	−0.14	286.95	−0.44	0.22	102.02	−1.42	0.94	216.38	−0.97	0.13	36.61	−0.91	1.81
191	48.47	0.14	286.51	0.46	0 21	100.60	1.41	0.95	215.41	0.97	0.13	35.70	0.90	1.82
192	48.33	0.14	286.05	0.47	0.21	99.19	1.41	0.97	214.44	0.98	0.14	34.80	0.89	1.84
193	48.19	0.14	285.58	0.48	0.20	97.78	1.40	0.98	213.46	1.00	0.14	33.91	0.89	1.85
194	48.05	0.15	285.10	0.50	0.20	96.38	1.40	1.00	212.46	1.01	0.14	33.02	0.87	1.87
195	47.90	−0.15	284.60	−0.51	0.19	94.98	−1.39	1.01	211.45	−1.02	0.15	32.15	−0.86	1.88
196	47.75	0.15	284.09	0.53	0.19	93.59	1.39	1.02	210.43	1.03	0.15	31.29	0.85	1.90
197	47.60	0.16	283.56	0.55	0.18	92.20	1.38	1.04	209.40	1.03	0.15	30.44	0.84	1.91
198	47.44	0.16	283.01	0.56	0.18	90.82	1.38	1.05	208.37	1.04	0.15	29.60	0.83	1 93
199	47.28	0.16	282.45	0.57	0.17	89.44	1.37	1.07	207.33	1.05	0.16	28.77	0.82	1.94
200	47.12	−0.17	281.88	−0.59	0.17	88.07	−1.36	1.08	206.28	−1.06	0.16	27.95	−0.81	1.96
201	46.95	0.17	281.29	0.61	0.17	86.71	1.35	1.09	205.22	1.07	0.17	27.14	0.80	1.97
202	46.78	0.17	280.68	0.62	0.16	85.36	1.35	1.11	204.15	1.08	0.17	26.34	0.79	1.99
203	46.61	0.17	280.06	0.63	0.16	84.01	1.35	1.12	203.07	1.09	0.18	25.55	0.77	2.00
204	46.44	0.18	279.43	0.65	0.16	82.66	1.34	1.14	201.98	1.09	0.18	24.78	0.77	2.01
205	46.26	−0.18	278.78	−0.66	0.15	81.32	−1.33	1.15	200.89	−1.10	0.19	24.01	−0.75	2.03
206	46.08	0.18	278.12	0.68	0.15	79.99	1.32	1.16	199.79	1.11	0.20	23.26	0.74	2.04
207	45.90	0.19	277.44	0.69	0.15	78.67	1.31	1.18	198.68	1.12	0.20	22.52	0.73	2.05
208	45.71	0.19	276.75	0.71	0.14	77.36	1.31	1.19	197.56	1.13	0.21	21.79	0.72	2.06
209	45.52	0.19	276.04	0.72	0.14	76.05	1.30	1.21	196.43	1.13	0.21	21.07	0.71	2.08
210	45.33	−0.19	275.32	−0.74	0.14	74.75	−1.29	1.22	195.30	−1.14	0.22	20.36	−0.70	2.09
211	45.14	0.20	274.58	0.75	0.13	73.46	1.28	1.23	194.16	1.15	0.23	19.66	0.69	2.10
212	44.94	0.20	273.83	0.77	0.13	72.18	1.27	1.25	193.01	1.16	0.23	18.97	0.67	2.12
213	44.74	0.20	273.06	0.78	0.13	70.91	1.27	1.26	191.85	1.16	0.24	18.30	0.66	2.13
214	44.54	0.20	272.28	0.79	0.13	69.64	1.27	1.28	190.69	1.17	0.25	17.64	0.64	2.14
215	44.34	−0.21	271.49	−0.81	0.12	68.37	−1.25	1.29	189.52	−1.18	0.26	17.00	−0.63	2.16
216	44.13	0.21	270.68	0.82	0.12	67.12	1.24	1.31	188.34	1.19	0.27	16.37	0.63	2.17
217	43.92	0.21	269.86	0.83	0.12	65.88	1.23	1.32	187.15	1.19	0.28	15.74	0.61	2.18
218	43.71	0.21	269.03	0.85	0.12	64.65	1.23	1.34	185.96	1.20	0.28	15.13	0.60	2.19
219	43.50	0.22	268.18	0.86	0.12	63.42	1.22	1.35	184.76	1.20	0.29	14.53	0.58	2.21
220	43.28	−0.22	267.32	−0.88	0.11	62.20	−1.21	1.37	183.56	−1.21	0.29	13.95	−0.57	2.22
221	43.06	0.22	266.44	0.89	0.11	60.99	1.19	1.38	182.35	1.21	0.30	13.38	0.56	2.23
222	42.84	0.22	265.55	0.90	0.11	59.80	1.19	1.40	181.14	1.22	0.31	12.82	0.55	2.25
223	42.62	0.22	264.65	0.92	0.11	58.61	1.18	1.41	179.92	1.23	0.32	12.27	0.54	2.26
224	42.40	0.23	263.73	0.93	0.11	57.43	1.17	1.43	178.69	1.23	0.33	11.73	0.52	2.27
225	42.17	−0.23	262.80	−0.95	0.11	56.26	−1.16	1.44	177.46	−1.24	0.34	11.21	−0.51	2.29
226	41.94	0.23	261.85	0.96	0.11	55.10	1.14	1.45	176.22	1.25	0.34	10.70	0.50	2.30
227	41.71	0.23	260.89	0.97	0.11	53.96	1.14	1.47	174.97	1.25	0.35	10.20	0.48	2.31
228	41.48	0.24	259.92	0.98	0.11	52.82	1.13	1.48	173.72	1.26	0 36	9.72	0.47	2.32
229	41.24	0.24	258.94	1.00	0.11	51.69	1.12	1.50	172.46	1.27	0.37	9.25	0.46	2.34
230	41.00	−0.24	257.94	−1.01	0.11	50.57	−1.11	1.51	171.19	−1.27	0.38	8.79	−0.44	2.35
231	40.76	0.24	256.93	1.01	0.11	49.46	1.09	1.52	169.92	1.27	0.39	8.35	0.43	2.36
232	40.52	0.24	255.92	1.03	0.11	48.37	1.09	1.54	168.65	1.28	0.40	7.92	0.42	2.37
233	40.28	0.25	254.89	1.04	0.11	47.28	1.08	1.55	167.37	1.28	0.41	7.50	0.41	2.38
234	40.03	0.25	253.85	1.06	0.11	46.20	1.07	1.57	166.09	1.29	0.42	7.09	0.39	2.39
235	39.78	−0.25	252.79	−1.07	0.11	45.13	−1.05	1.58	164.80	−1.29	0.42	6.70	−0.38	2.41
236	39.53	0.25	251.72	1.08	0.11	44.08	1.03	1.60	163.51	1.29	0.43	6.32	0.36	2.42
237	39.28	0.25	250.64	1.09	0.12	43.05	1.03	1.61	162.22	1.30	0.44	5.96	0.35	2.43
238	39.03	0.26	249.55	1.10	0.12	42.02	1.02	1.63	160.92	1.30	0.45	5.61	0.34	2.44
239	38.77	0.26	248.45	1.12	0.12	41.00	1.02	1.64	159.62	1.31	0.46	5.27	0.32	2.45
240	38.51		247.33		0.12	39.98		1.66	158.31		0.47	4.95		2.46

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	″	″	″	″							
180	10.01	0.33	0.62	0.71	611	1855	2810	190	178	72	28
181	9.93	0.34	0.62	0.72	604	1869	2807	188	180	71	28
182	9.85	0.35	0.61	0.73	598	1882	2804	186	183	70	28
183	9.78	0.36	0.61	0.75	591	1895	2801	184	185	69	28
184	9.70	0.37	0.61	0.76	584	1908	2798	182	188	68	28
185	9.63	0.38	0.60	0.77	577	1921	2794	180	190	67	29
186	9.55	0.39	0.60	0.78	571	1935	2791	178	192	66	29
187	9.47	0.41	0.60	0.79	564	1948	2787	176	195	65	29
188	9.40	0.42	0.60	0.80	557	1961	2783	174	197	64	29
189	9.33	0.44	0.59	0.81	551	1974	2779	172	200	63	30
190	9.26	0.45	0.59	0.83	544	1987	2775	171	202	62	30
191	9.19	0.47	0.59	0.84	538	2000	2771	169	205	61	30
192	9.12	0.50	0.59	0.85	531	2013	2766	167	207	60	30
193	9.05	0.52	0.59	0.86	525	2025	2762	165	210	59	31
194	8.98	0.54	0.59	0.87	519	2038	2757	163	212	58	31
195	8.91	0.56	0.59	0.88	512	2051	2752	161	215	57	32
196	8.84	0.59	0.59	0.90	506	2064	2747	159	218	56	32
197	8.78	0.61	0.59	0.91	500	2076	2742	157	220	55	32
198	8.71	0.64	0.59	0.92	494	2089	2737	156	223	54	33
199	8.65	0.66	0.59	0.93	488	2101	2731	154	225	53	33
200	8.59	0.69	0.59	0.94	482	2113	2726	152	228	52	34
201	8.53	0.72	0.59	0.95	476	2126	2720	151	231	51	34
202	8.47	0.75	0.59	0.97	470	2138	2714	150	233	50	35
203	8.41	0.78	0.59	0.98	464	2150	2708	148	236	49	35
204	8.36	0.81	0.60	0.99	459	2162	2702	147	239	48	36
205	8.30	0.84	0.60	1.00	453	2174	2695	146	242	47	36
206	8.25	0.87	0.60	1.01	447	2186	2689	144	244	47	37
207	8.19	0.90	0.61	1.02	441	2198	2682	142	247	46	37
208	8.14	0.93	0.61	1.03	436	2210	2675	141	250	45	38
209	8.08	0.96	0.61	1.04	430	2222	2668	140	253	44	39
210	8.03	1.00	0.62	1.05	425	2234	2661	138	256	43	39
211	7.98	1.03	0.62	1.06	419	2245	2654	137	259	43	40
212	7.93	1.06	0.62	1.07	413	2257	2646	135	262	42	41
213	7.88	1.10	0.63	1.08	408	2268	2639	134	265	41	42
214	7.84	1.13	0.63	1.09	402	2280	2631	133	268	40	43
215	7.79	1.17	0.64	1.10	397	2291	2624	132	271	39	43
216	7.74	1.21	0.64	1.11	392	2302	2616	130	274	39	44
217	7.70	1.24	0.65	1.12	386	2313	2608	129	277	38	44
218	7.65	1.28	0.66	1.13	380	2324	2600	128	280	37	45
219	7.61	1.32	0.66	1.14	375	2335	2592	127	283	36	46
220	7.57	1.35	0.67	1.15	370	2346	2584	126	286	35	46
221	7.53	1.39	0.68	1.16	364	2357	2575	125	289	35	47
222	7.49	1.43	0.69	1.17	359	2367	2567	125	292	34	47
223	7.45	1.46	0.69	1.18	354	2378	2558	124	295	33	48
224	7.41	1.50	0.70	1.19	349	2389	2550	123	298	32	49
225	7.38	1.54	0.71	1.20	344	2399	2541	122	301	32	50
226	7.34	1.58	0.71	1.20	339	2409	2532	121	304	31	50
227	7.31	1.62	0.72	1.21	334	2420	2523	121	307	30	51
228	7.28	1.66	0.73	1.22	329	2430	2513	120	310	30	52
229	7.25	1.70	0.74	1.23	324	2440	2504	119	313	29	53
230	7.22	1.74	0.75	1.24	319	2450	2494	118	316	28	54
231	7.19	1.78	0.75	1.24	314	2460	2484	118	318	28	54
232	7.17	1.81	0.76	1.25	310	2470	2474	117	321	27	55
233	7.14	1.85	0.77	1.25	305	2480	2464	116	324	27	56
234	7.11	1.89	0.78	1.26	300	2489	2454	116	327	26	57
235	7.09	1.93	0.79	1.27	296	2499	2444	115	330	25	58
236	7.07	1.97	0.80	1.27	291	2508	2433	114	333	25	59
237	7.05	2.00	0.81	1.28	287	2518	2423	114	336	24	60
238	7.03	2.04	0.82	1.28	283	2527	2412	113	339	23	61
239	7.01	2.08	0.83	1.29	278	2536	2401	113	342	22	62
240	6.99	2.12	0.84	1.30	274	2545	2390	113	345	21	63

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TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″.
240	38.51	−0.26	247.33	−1.13	0.12	39.98	−1.00	1.66	158.31	−1.31	0.47	4.95	−0.31	2.46
241	38.25	0.26	246.20	1.14	0.12	38.98	0.98	1.67	157.00	1.31	0.48	4.64	0.30	2.47
242	37.99	0.26	245.06	1.15	0.12	38.00	0.97	1.69	155.69	1.32	0.49	4.34	0.28	2.48
243	37.73	0.27	243.91	1.16	0.13	37.03	0.96	1.70	154.37	1.32	0.50	4.06	0.27	2.49
244	37.46	0.26	242.75	1.18	0.13	36.07	0.95	1.71	153.05	1.32	0.51	3.79	0.25	2.50
245	37.20	−0.27	241.57	−1.18	0.13	35.12	−0.93	1.73	151.73	−1.33	0.53	3.54	−0.24	2.51
246	36.93	0.27	240.39	1.19	0.13	34.19	0.92	1.74	150.40	1.33	0.54	3.30	0.23	2.52
247	36.66	0.27	239.20	1.21	0.13	33.27	0.91	1.75	149.07	1.33	0.55	3.07	0.21	2.53
248	36.39	0.27	237.99	1.22	0.14	32.36	0.90	1.76	147.74	1.33	0.56	2.86	0.20	2.54
249	36.12	0.27	236.77	1.23	0.14	31.46	0.89	1.78	146.41	1.34	0.57	2.66	0.18	2.55
250	35.85	−0.27	235.54	−1.24	0.14	30.57	−0.87	1.79	145.07	−1.34	0.58	2.48	−0.17	2.56
251	35.58	0.27	234.30	1.25	0.14	29.70	0.86	1.80	143.73	1.34	0.59	2.31	0.15	2.57
252	35.31	0.28	233.05	1.25	0.15	28.84	0.85	1.82	142.39	1.34	0.60	2.16	0.14	2.58
253	35.03	0.28	231.80	1.26	0.15	27.99	0.84	1.83	141.05	1.34	0.61	2.02	0.13	2.59
254	34.75	0.28	230.54	1.28	0.16	27.15	0.82	1.85	139.71	1.35	0.62	1.89	0.11	2.60
255	34.47	−0.28	229.26	−1.28	0.16	26.33	−0.81	1.86	138.36	−1.35	0.64	1.78	−0.10	2.61
256	34.19	0.28	227.98	1.30	0.16	25.52	0.79	1.88	137.01	1.35	0.65	1.68	0.08	2.61
257	33.91	0.28	226.68	1.31	0.17	24.73	0.78	1.89	135.66	1.35	0.66	1.60	0.07	2.62
258	33.63	0.29	225.37	1.31	0.17	23.95	0.77	1.91	134.31	1.35	0.67	1.53	0.06	2.63
259	33.34	0.28	224.06	1.32	0.18	23.18	0.75	1.92	132.96	1.35	0.68	1.47	0.04	2.64
260	33.06	−0.28	222.74	−1.33	0.18	22.43	−0.74	1.94	131.61	−1.35	0.69	1.43	−0.03	2.65
261	32.78	0.29	221.41	1.34	0.18	21.69	0.72	1.95	130.26	1.36	0.70	1.40	−0.01	2.66
262	32.49	0.28	220.07	1.35	0.19	20.97	0.71	1.97	128.90	1.35	0.71	1.39	+0.01	2.67
263	32.21	0.29	218.72	1.36	0.19	20.26	0.70	1.98	127.55	1.35	0.73	1.40	0.02	2.67
264	31.92	0.29	217.36	1.37	0.20	19.56	0.68	1.99	126.20	1.35	0.74	1.42	0.03	2.68
265	31.63	−0.29	215.99	−1.37	0.20	18.88	−0.67	2.01	124.85	−1.35	0.75	1.45	+0.05	2.69
266	31.34	0.29	214.62	1.38	0.21	18.21	0.65	2.02	123.50	1.36	0.76	1.50	0.06	2.70
267	31.05	0.29	213.24	1.39	0.21	17.56	0.64	2.03	122.14	1.35	0.77	1.56	0.07	2.71
268	30.76	0.29	211.85	1.40	0.22	16.92	0.63	2.04	120.79	1.35	0.79	1.63	0.09	2.71
269	30.47	0.29	210.45	1.40	0.22	16.29	0.61	2.06	119.44	1.35	0.80	1.72	0.10	2.72
270	30.18	−0.30	209.05	−1.41	0.23	15.68	−0.60	2.07	118.09	−1.35	0.81	1.82	+0.12	2.73
271	29.88	0.29	207.64	1.42	0.24	15.08	0.58	2.08	116.74	1.35	0.82	1.94	0.13	2.74
272	29.59	0.30	206.22	1.43	0.24	14.50	0.56	2.10	115.39	1.35	0.84	2.07	0.14	2.74
273	29.29	0.29	204.79	1.43	0.25	13.94	0.55	2.11	114.04	1.34	0.85	2.21	0.16	2.75
274	29.00	0.29	203.36	1.44	0.25	13.39	0.54	2.13	112.70	1.35	0.87	2.37	0.18	2.75
275	28.71	−0.30	201.92	−1.44	0.26	12.85	−0.52	2.14	111.35	−1.34	0.88	2.55	+0.19	2.76
276	28.41	0.29	200.48	1.45	0.27	12.33	0.50	2.15	110.01	1.34	0.89	2.74	0.21	2.77
277	28.12	0.29	199.03	1.46	0.27	11.83	0.49	2.17	108.67	1.34	0.91	2.95	0.22	2.77
278	27.83	0.29	197.57	1.46	0.28	11.34	0.48	2.18	107.33	1.34	0.92	3.17	0.23	2.78
279	27.54	0.29	196.11	1.47	0.28	10.86	0.46	2.20	105.99	1.33	0.94	3.40	0.25	2.78
280	27.25	−0.30	194.64	−1.47	0.29	10.40	−0.45	2.21	104.66	−1.33	0.95	3.65	+0.26	2.79
281	26.95	0.29	193.17	1.48	0.30	9.95	0.43	2.22	103.33	1.33	0.96	3.91	0.28	2.79
282	26.66	0.30	191.69	1.49	0.31	9.52	0.42	2.23	102.00	1.33	0.98	4.19	0.29	2.80
283	26.36	0.29	190.20	1.49	0.32	9.10	0.40	2.24	100.67	1.33	0.99	4.48	0.30	2.80
284	26.07	0.29	188.71	1.49	0.33	8.70	0.39	2.25	99.34	1.32	1.01	4.78	0.32	2.81
285	25.78	−0.30	187.22	−1.50	0.34	8.31	−0.37	2.27	98.02	−1.32	1.02	5.10	+0.33	2.81
286	25.48	0.29	185.72	1.51	0.34	7.94	0.35	2.28	96.70	1.31	1.03	5.43	0.35	2.82
287	25.19	0.30	184.21	1.51	0.35	7.59	0.34	2.29	95.39	1.31	1.05	5.78	0.36	2.82
288	24.89	0.29	182.70	1.51	0.36	7.25	0.32	2.30	94.08	1.31	1.06	6.14	0.37	2.83
289	24.60	0.29	181.19	1.52	0.37	6.93	0.30	2.31	92.77	1.30	1.08	6.51	0.39	2.83
290	24.31	−0.29	179.67	−1.52	0.38	6.63	−0.29	2.32	91.47	−1.30	1.09	6.90	+0.40	2.84
291	24.02	0.29	178.15	1.52	0.39	6.34	0.27	2.33	90.17	1.30	1.10	7.30	0.42	2.84
292	23.73	0.29	176.63	1.53	0.40	6.07	0.26	2.34	88.87	1.29	1.12	7.72	0.43	2.85
293	23.44	0.29	175.10	1.53	0.41	5.81	0.24	2.36	87.58	1.29	1.13	8.15	3.44	2.85
294	23.15	0.29	173.57	1.53	0.42	5.57	0.23	2.37	86.29	1.28	1.15	8.59	0.46	2.86
295	22.86	−0.29	172.04	−1.54	0.43	5.34	−0.21	2.38	85.01	−1.27	1.16	9.05	+0.47	2.86
296	22.57	0.28	170.50	1.54	0.43	5.13	0.20	2.39	83.74	1.27	1.17	9.52	0.49	2.86
297	22.29	0.29	168.96	1.54	0.44	4.93	0.18	2.40	82.47	1.27	1.19	10.01	0.50	2.87
298	22.00	0.29	167.42	1.54	0.45	4.75	0.16	2.42	81.20	1.27	1.20	10.51	0.51	2.87
299	21.71	0.29	165.88	1.55	0.46	4.59	0.15	2.43	79.93	1.26	1.22	11.02	0.53	2.88
300	21.42		164.33		0.47	4.44		2.44	78.67		1.23	11.55		2.88

TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	″	″	″	″							
240	6.99	2.12	0.84	1.30	274	2545	2390	113	345	21	63
241	6.97	2.16	0.85	1.31	270	2554	2379	113	348	21	64
242	6.96	2.20	0.86	1.31	265	2562	2368	113	350	20	65
243	6.94	2.24	0.87	1.31	261	2571	2357	112	353	19	66
244	6.92	2.28	0.88	1.31	257	2579	2346	112	356	19	67
245	6.90	2.31	0.90	1.32	253	2587	2334	112	359	18	68
246	6.88	2.35	0.91	1.32	249	2595	2322	112	362	18	69
247	6.87	2.38	0.92	1.32	245	2603	2311	112	365	17	70
248	6.85	2.42	0.93	1.32	242	2611	2299	112	368	16	72
249	6.84	2.45	0.94	1.33	238	2618	2287	111	371	16	73
250	6.83	2.48	0.95	1.33	234	2626	2275	111	374	15	74
251	6.82	2.52	0.96	1.33	230	2633	2263	111	377	15	75
252	6.82	2.55	0.97	1.33	227	2641	2251	111	380	14	76
253	6.81	2.58	0.98	1.33	223	2648	2239	111	383	14	77
254	6.80	2.61	0.99	1.34	219	2656	2226	111	386	13	78
255	6.79	2.64	1.00	1.34	215	2663	2214	111	389	13	79
256	6.79	2.67	1.02	1.34	212	2670	2201	111	392	13	80
257	6.78	2.70	1.03	1.34	208	2677	2189	112	395	12	82
258	6.78	2.73	1.04	1.34	204	2684	2176	112	398	12	83
259	6.77	2.76	1.05	1.34	201	2691	2164	112	401	12	84
260	6.77	2.79	1.06	1.34	198	2698	2151	112	404	11	85
261	6.77	2.82	1.07	1.34	195	2704	2138	112	407	11	86
262	6.77	2.85	1.09	1.33	192	2711	2125	113	410	11	87
263	6.77	2.88	1.10	1.33	189	2717	2112	113	413	11	89
264	6.77	2.90	1.11	1.33	186	2723	2098	113	416	11	90
265	6.77	2.93	1.12	1.33	183	2729	2085	114	419	11	91
266	6.77	2.95	1.13	1.32	180	2735	2072	114	422	11	92
267	6.77	2.98	1.14	1.32	177	2740	2058	115	425	11	93
268	6.77	3.00	1.15	1.32	174	2745	2045	115	427	11	94
269	6.77	3.03	1.16	1.32	172	2751	2031	116	430	11	96
270	6.77	3.05	1.17	1.31	169	2756	2018	116	433	11	97
271	6.77	3.07	1.18	1.31	166	2761	2004	117	436	11	98
272	6.77	3.09	1.20	1.31	164	2766	1990	117	439	10	99
273	6.78	3.11	1.21	1.30	161	2771	1977	118	442	10	100
274	6.78	3.13	1.22	1.30	159	2776	1963	119	444	10	102
275	6.78	3.15	1.23	1.29	156	2781	1949	119	447	10	103
276	6.79	3.17	1.24	1.28	154	2785	1935	120	450	10	104
277	6.79	3.19	1.25	1.28	151	2790	1921	121	452	10	105
278	6.80	3.20	1.26	1.27	149	2794	1907	121	455	10	106
279	6.81	3.22	1.27	1.27	146	2798	1892	122	457	10	107
280	6.81	3.23	1.28	1.26	144	2802	1878	123	460	10	108
281	6.82	3.25	1.28	1.25	142	2806	1864	124	463	10	110
282	6.83	3.26	1.29	1.25	139	2809	1849	124	465	10	111
283	6.83	3.27	1.30	1.24	137	2813	1835	125	468	10	112
284	6.84	3.28	1.31	1.23	135	2816	1820	126	471	11	113
285	6.84	3.29	1.32	1.22	133	2819	1805	127	474	11	115
286	6.85	3.30	1.33	1.22	131	2822	1791	127	476	11	116
287	6.85	3.31	1.34	1.21	129	2825	1776	128	479	11	117
288	6.86	3.32	1.34	1.20	127	2828	1761	129	482	11	118
289	6.86	3.32	1.35	1.19	126	2830	1746	130	484	12	120
290	6.87	3.33	1.36	1.18	124	2833	1731	131	487	12	121
291	6.88	3.33	1.37	1.17	123	2835	1716	132	490	12	122
292	6.88	3.34	1.38	1.16	121	2837	1702	133	492	12	123
293	6.89	3.34	1.38	1.15	119	2838	1686	134	495	13	125
294	6.89	3.35	1.39	1.14	118	2840	1671	136	497	13	126
295	6.90	3.35	1.40	1.13	116	2841	1656	137	500	13	127
296	6.90	3.35	1.40	1.12	115	2843	1641	138	503	13	128
297	6.90	3.35	1.41	1.11	114	2844	1626	139	505	14	130
298	6.91	3.35	1.42	1.10	112	2845	1611	141	508	14	131
299	6.91	3.35	1.42	1.09	111	2847	1596	142	510	14	132
300	6.92	3.35	1.43	1.08	110	2848	1581	143	512	15	133

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TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″		″	″	″	″
300	21.42	−0.29	164.33	−1.55	0.47	4.44	−0.13	2.44	78.67	−1.25	1.23	11.55	+0.54	2.88
301	21.13	0.28	162.78	1.55	0.48	4.31	0.12	2.45	77.42	1.25	1.24	12.09	0.55	2.88
302	20.85	0.28	161.23	1.55	0.49	4.19	0.10	2.46	76.17	1.24	1.26	12.64	0.57	2.88
303	20.57	0.28	159.68	1.56	0.50	4.09	0.08	2.47	74.93	1.23	1.27	13.21	0.58	2.89
304	20.29	0.28	158.12	1.55	0.51	4.01	0.07	2.48	73.70	1.23	1.29	13.79	0.59	2.89
305	20.01	−0.28	156.57	−1.56	0.53	3.94	−0.05	2.49	72.47	−1.22	1.30	14.38	+0.61	2.89
306	19.73	0.28	155.01	1.55	0.54	3.89	0.03	2.50	71.25	1.22	1.31	14.99	0.62	2.89
307	19.45	0.28	153.46	1.56	0.55	3.86	0.01	2.51	70.03	1.21	1.33	15.61	0.63	2.89
308	19.17	0.27	151.90	1.56	0.56	3.85	−0.01	2.52	68.82	1.20	1.34	16.24	0.64	2.90
309	18.90	0.27	150.34	1.55	0.57	3.84	+0.01	2.53	67.62	1.20	1.36	16.88	0.66	2.90
310	18.63	−0.27	148.79	−1.56	0.58	3.85	+0.03	2.54	66.42	−1.19	1.37	17.54	+0.67	2.90
311	18.36	0.27	147.23	1.56	0.59	3.88	0.04	2.55	65.23	1.18	1.39	18.21	0.69	2.90
312	18.09	0.27	145.67	1.55	0.60	3.92	0.06	2.56	64.05	1.18	1.40	18.90	0.70	2.90
313	17.82	0.27	144.12	1.56	0.61	3.98	0.08	2.57	62.87	1.17	1.42	19.60	0.71	2.90
314	17.55	0.27	142.56	1.56	0.62	4.06	0.10	2.58	61.70	1.16	1.43	20.31	0.72	2.90
315	17.28	−0.26	141.00	−1.56	0.64	4.16	+0.11	2.59	60.54	−1.15	1.45	21.03	+0.73	2.91
316	17.02	0.27	139.44	1.56	0.65	4.27	0.13	2.59	59.39	1.15	1.47	21.76	0.75	2.91
317	16.75	0.26	137.88	1.55	0.66	4.40	0.15	2.60	58.24	1.14	1.48	22.51	0.76	2.91
318	16.49	0.26	136.33	1.56	0.67	4.55	0.16	2.61	57.10	1.13	1.50	23.27	0.77	2.91
319	16.23	0.26	134.77	1.55	0.68	4.71	0.18	2.62	55.97	1.12	1.51	24.04	0.78	2.91
320	15.97	−0.26	133.22	−1.55	0.69	4.89	+0.19	2.63	54.85	−1.11	1.53	24.82	+0.80	2.91
321	15.71	0.25	131.67	1.55	0.70	5.08	0.21	2.64	53.74	1.10	1.54	25.62	0.80	2.91
322	15.46	0.25	130.12	1.55	0.71	5.29	0.22	2.65	52.64	1.10	1.56	26.42	0.82	2.91
323	15.21	0.25	128.57	1.55	0.73	5.51	0.24	2.65	51.54	1.09	1.57	27.24	0.83	2.90
324	14.96	0.25	127.02	1.54	0.74	5.75	0.25	2.66	50.45	1.08	1.59	28.07	0.85	2.90
325	14.71	−0.24	125.48	−1.54	0.75	6.00	+0.27	2.67	49.37	−1.07	1.60	28.92	+0.85	2.90
326	14.47	0.24	123.94	1.53	0.76	6.27	0.29	2.68	48.30	1.07	1.61	29.77	0.86	2.90
327	14.23	0.24	122.41	1.53	0.77	6.56	0.31	2.69	47.23	1.06	1.63	30.63	0.88	2.90
328	13.99	0.24	120.88	1.53	0.79	6.87	0.32	2.69	46.17	1.04	1.64	31.51	0.89	2.89
329	13.75	0.24	119.35	1.53	0.80	7.19	0.33	2.70	45.13	1.03	1.66	32.40	0.90	2.89
330	13.51	−0.23	117.82	−1.52	0.81	7.52	+0.35	2.71	44.10	−1.03	1.67	33.30	+0.91	2.89
331	13.28	0.24	116.30	1.52	0.82	7.87	0.37	2.72	43.07	1.01	1.68	34.21	0.92	2.89
332	13.04	0.23	114.78	1.52	0.84	8.24	0.38	2.72	42.06	1.00	1.70	35.13	0.93	2.89
333	12.81	0.23	113.26	1.52	0.85	8.62	0.40	2.73	41.06	1.00	1.71	36.06	0.95	2.88
334	12.58	0.23	111.74	1.51	0.86	9.02	0.42	2.74	40.06	0.99	1.73	37.01	0.96	2.88
335	12.35	−0.22	110.23	−1.51	0.88	9.44	+0.43	2.75	39.07	−0.97	1.74	37.97	+0.97	2.88
336	12.13	0.22	108.72	1.50	0.89	9.87	0.45	2.75	38.10	0.96	1.75	38.94	0.97	2.88
337	11.91	0.22	107.22	1.49	0.90	10.32	0.46	2.76	37.14	0.95	1.77	39.91	0.98	2.88
338	11.69	0.22	105.73	1.49	0.91	10.78	0.48	2.77	36.19	0.95	1.78	40.89	1.00	2.87
339	11.47	0.21	104.24	1.49	0.93	11.26	0.49	2.77	35.24	0.94	1.80	41.89	1.01	2.87
340	11.26	−0.21	102.75	−1.48	0.94	11.75	+0.51	2.78	34.30	−0.92	1.81	42.90	+1.02	2.87
341	11.05	0.21	101.27	1.47	0.95	12.26	0.53	2.78	33.38	0.91	1.82	43.92	1.03	2.87
342	10.84	0.20	99.80	1.47	0.97	12.79	0.54	2.79	32.47	0.90	1.84	44.95	1.04	2.86
343	10.64	0.20	98.33	1.47	0.98	13.33	0.56	2.79	31.57	0.89	1.85	45.99	1.04	2.86
344	10.44	0.20	96.86	1.46	1.00	13.89	0.57	2.80	30.68	0.88	1.87	47.03	1.06	2.86
345	10.24	−0.19	95.40	−1.45	1.01	14.46	+0.59	2.80	29.80	−0.87	1.88	48.09	+1.06	2.85
346	10.05	0.20	93.95	1.45	1.02	15.05	0.60	2.81	28.93	0.85	1.90	49.15	1.07	2.85
347	9.85	0.19	92.50	1.44	1.04	15.65	0.61	2.81	28.08	0.85	1.91	50.22	1.08	2.85
348	9.66	0.19	91.06	1.43	1.05	16.26	0.63	2.82	27.23	0.84	1.93	51.30	1.10	2.85
349	9.47	0.19	89.63	1.42	1.07	16.89	0.65	2.82	26.39	0.82	1.94	52.40	1.10	2.84
350	9.28	−0.18	88.21	−1.42	1.08	17.54	+0.66	2.83	25.57	−0.81	1.96	53.50	+1.11	2.84
351	9.10	0.18	86.79	1.41	1.09	18.20	0.68	2.83	24.76	0.79	1.97	54.61	1.12	2.83
352	8.92	0.18	85.38	1.40	1.11	18.88	0.69	2.84	23.97	0.79	1.99	55.73	1.13	2.83
353	8.74	0.17	83.98	1.40	1.12	19.57	0.71	2.84	23.18	0.78	2.00	56.86	1.13	2.82
354	8.57	0.17	82.58	1.39	1.14	20.28	0.72	2.84	22.40	0.76	2.01	57.99	1.15	2.82
355	8.40	−0.17	81.19	−1.38	1.15	21.00	+0.74	2.85	21.64	−0.75	2.03	59.14	+1.15	2.81
356	8.23	0.16	79.81	1.37	1.16	21.74	0.75	2.85	20.89	0.73	2.04	60.29	1.16	2.80
357	8.07	0.16	78.44	1.37	1.18	22.49	0.76	2.85	20.16	0.73	2.05	61.45	1.17	2.80
358	7.91	0.16	77.07	1.36	1.19	23.25	0.78	2.85	19.43	0.72	2.06	62.62	1.18	2.79
359	7.75	0.15	75.71	1.35	1.21	24.03	0.79	2.86	18.71	0.70	2.08	63.80	1.18	2.79
360	7.60		74.36		1.22	24.82		2.86	18.01		2.09	64.98		2.78

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TABLE IX, Arg. 2.—*Continued.*

Arg.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	(ρ.s.3)	(ρ.c.3)
	″	″	″								
300	6.92	3.35	1.43	1.08	110	2848	1581	143	512	15	133
301	6.92	3.35	1.43	1.07	109	2849	1566	144	515	15	134
302	6.93	3.35	1.44	1.06	108	2849	1551	146	517	15	135
303	6.93	3.34	1.44	1.05	107	2850	1535	147	519	16	137
304	6.93	3.34	1.45	1.04	106	2850	1520	149	522	16	138
305	6.94	3.34	1.45	1.03	105	2850	1505	150	524	17	139
306	6.94	3.33	1.46	1.01	104	2850	1490	152	526	17	140
307	6.94	3.33	1.46	1.00	103	2850	1474	153	528	18	141
308	6.94	3.32	1.47	0.99	103	2850	1459	154	531	18	142
309	6.95	3.31	1.47	0.98	102	2849	1444	156	533	19	143
310	6.95	3.31	1.47	0.96	102	2849	1429	157	535	20	144
311	6.95	3.30	1.47	0.95	101	2848	1413	158	538	20	145
312	6.95	3.29	1.48	0.94	101	2847	1398	160	540	21	146
313	6.95	3.28	1.48	0.93	100	2847	1383	162	542	22	147
314	6.94	3.27	1.48	0.92	100	2846	1368	163	545	23	148
315	6.94	3.26	1.48	0.90	99	2845	1352	165	547	24	149
316	6.94	3.25	1.48	0.89	99	2843	1337	167	549	24	150
317	6.94	3.23	1.48	0.88	99	2842	1322	168	551	25	151
318	6.93	3.22	1.48	0.86	98	2840	1307	170	554	26	152
319	6.93	3.21	1.48	0.85	98	2838	1291	171	556	27	153
320	6.92	3.20	1.48	0.84	98	2836	1276	173	558	27	154
321	6.92	3.18	1.48	0.83	98	2834	1261	175	560	28	155
322	6.91	3.17	1.48	0.81	98	2831	1246	176	562	29	156
323	6.91	3.15	1.48	0.80	98	2828	1231	178	564	29	157
324	6.90	3.14	1.48	0.79	99	2825	1216	180	566	30	158
325	6.89	3.12	1.48	0.77	99	2822	1202	181	568	31	159
326	6.88	3.11	1.48	0.76	99	2819	1187	183	570	32	159
327	6.87	3.10	1.47	0.75	100	2815	1172	185	572	33	160
328	6.86	3.08	1.47	0.74	100	2812	1157	186	573	33	161
329	6.85	3.06	1.47	0.72	101	2808	1143	188	575	34	162
330	6.83	3.04	1.46	0.71	101	2804	1128	190	577	35	163
331	6.82	3.03	1.46	0.70	102	2800	1113	192	579	36	164
332	6.81	3.01	1.46	0.69	103	2796	1098	193	581	37	165
333	6.79	2.99	1.45	0.67	103	2792	1084	195	583	38	165
334	6.77	2.97	1.45	0.66	104	2788	1069	197	585	39	166
335	6.75	2.96	1.44	0.65	105	2784	1054	199	587	40	167
336	6.74	2.94	1.44	0.64	106	2779	1039	201	589	41	168
337	6.72	2.92	1.43	0.63	107	2775	1025	203	591	42	169
338	6.70	2.90	1.43	0.61	108	2770	1010	204	592	43	169
339	6.68	2.88	1.42	0.60	109	2765	996	206	594	44	170
340	6.66	2.87	1.42	0.59	110	2760	981	208	596	45	171
341	6.64	2.85	1.41	0.58	111	2755	967	210	598	46	171
342	6.62	2.83	1.40	0.57	112	2750	953	212	599	47	172
343	6.59	2.81	1.40	0.56	114	2744	938	214	601	48	173
344	6.57	2.79	1.39	0.55	115	2739	924	217	603	49	173
345	6.55	2.78	1.38	0.53	116	2733	910	219	605	50	174
346	6.52	2.76	1.37	0.52	117	2727	896	221	606	51	174
347	6.50	2.74	1.36	0.51	119	2721	882	223	608	52	175
348	6.47	2.72	1.36	0.50	121	2715	868	226	610	53	176
349	6.44	2.70	1.35	0.49	122	2709	855	228	611	54	176
350	6.41	2.69	1.34	0.48	124	2703	841	230	613	55	177
351	6.37	2.67	1.33	0.47	126	2696	828	232	614	57	177
352	6.34	2.65	1.32	0.46	128	2689	814	235	616	58	178
353	6.31	2.64	1.31	0.45	130	2682	801	237	617	59	178
354	6.28	2.62	1.30	0.44	132	2675	788	239	619	60	178
355	6.24	2.60	1.29	0.43	134	2668	775	241	620	61	179
356	6.21	2.59	1.28	0.42	136	2660	762	244	621	62	179
357	6.18	2.57	1.26	0.41	139	2653	749	246	623	63	180
358	6.14	2.56	1.25	0.40	141	2645	736	248	624	64	180
359	6.10	2.54	1.24	0.39	144	2637	723	250	626	65	180
360	6.07	2.53	1.23	0.38	146	2629	710	252	627	67	181

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TABLE IX, ARG. 2.—*Continued.*

Arg.	($v.c.0$)	Diff.	($v.s.1$)	Diff.	Sec.var.	($v.c.1$)	Diff.	Sec.var.	($v.s.2$)	Diff.	Sec.var.	($v.c.2$)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
360	7.60	−0.15	74.36	−1.34	1.22	24.82	+0.81	2.86	18.01	−0.68	2.09	64.98	+1.19	2.78
361	7.45	0.15	73.02	1.33	1.23	25.63	0.82	2.86	17.33	0.67	2.10	66.17	1.20	2.77
362	7.30	0.15	71.69	1.32	1.25	26.45	0.84	2.87	16.66	0.66	2.12	67.37	1.21	2.77
363	7.15	0.14	70.37	1.32	1.26	27.29	0.85	2.87	16.00	0.65	2.13	68.58	1.21	2.76
364	7.01	0.14	69.05	1.30	1.28	28.14	0.86	2.87	15.35	0.64	2.15	69.79	1.22	2.75
365	6.87	−0.13	67.75	−1.29	1.29	29.00	+0.88	2.88	14.71	−0.62	2.16	71.01	+1.22	2.74
366	6.74	0.13	66.46	1.28	1.31	29.88	0.90	2.88	14.09	0.61	2.17	72.23	1.23	2.74
367	6.61	0.13	65.18	1.28	1.32	30.78	0.91	2.88	13.48	0.60	2.18	73.46	1.24	2.73
368	6.48	0.12	63.90	1.27	1.34	31.69	0.92	2.88	12.88	0.59	2.19	74.70	1.25	2.72
369	6.36	0.12	62.63	1.25	1.35	32.61	0.93	2.89	12.29	0.57	2.21	75.95	1.26	2.71
370	6.24	−0.12	61.38	−1.24	1.37	33.54	+0.95	2.89	11.72	−0.55	2.22	77.21	+1.26	2.71
371	6.12	0.12	60.14	1.23	1.38	34.49	0.96	2.89	11.17	0.54	2.23	78.47	1.27	2.70
372	6.00	0.11	58.91	1.23	1.40	35.45	0.97	2.89	10.63	0.53	2.25	79.74	1.27	2.69
373	5.89	0.11	57.68	1.22	1.41	36.42	0.98	2.89	10.10	0.51	2.26	81.01	1.27	2.68
374	5.78	0.10	56.46	1.20	1.43	37.40	1.00	2.89	9.59	0.50	2.27	82.28	1.28	2.67
375	5.68	−0.11	55.26	−1.19	1.44	38.40	+1.01	2.89	9.09	−0.49	2.29	83.56	+1.29	2.66
376	5.57	0.10	54.07	1.18	1.45	39.41	1.02	2.89	8.60	0.48	2.30	84.85	1.29	2.66
377	5.47	0.09	52.89	1.17	1.47	40.43	1.04	2.89	8.12	0.46	2.31	86.14	1.29	2.65
378	5.38	0.08	51.72	1.15	1.48	41.47	1.05	2.89	7.66	0.45	2.32	87.43	1.30	2.64
379	5.30	0.08	50.57	1.15	1.50	42.52	1.06	2.89	7.21	0.43	2.34	88.73	1.31	2.63
380	5.22	−0.08	49.42	−1.13	1.51	43.58	+1.07	2.89	6.78	−0.42	2.35	90.04	+1.31	2.62
381	5.14	0.07	48.29	1.12	1.52	44.65	1.09	2.89	6.36	0.40	2.36	91.35	1.32	2.61
382	5.07	0.08	47.17	1.11	1.54	45.74	1.10	2.89	5.96	0.39	2.37	92.67	1.32	2.60
383	4.99	0.08	46.06	1.10	1.55	46.84	1.11	2.89	5.57	0.38	2.38	93.99	1.32	2.59
384	4.91	0.07	44.96	1.09	1.57	47.95	1.12	2.89	5.19	0.36	2.39	95.31	1.33	2.58
385	4.84	−0.07	43.87	−1.07	1.58	49.07	+1.13	2.89	4.83	−0.35	2.40	96.64	+1.33	2.57
386	4.77	0.06	42.80	1.06	1.60	50.20	1.14	2.89	4.48	0.33	2.42	97.97	1.33	2.57
387	4.71	0.06	41.74	1.05	1.61	51.34	1.16	2.88	4.15	0.32	2.43	99.30	1.34	2.56
388	4.65	0.05	40.69	1.04	1.63	52.50	1.17	2.88	3.83	0.31	2.44	100.64	1.34	2.55
389	4.60	0.04	39.65	1.02	1.64	53.67	1.18	2.88	3.52	0.29	2.45	101.98	1.35	2.54
390	4.56	−0.04	38.63	−1.01	1.66	54.85	+1.19	2.88	3.23	−0.27	2.46	103.33	+1.35	2.53
391	4.52	0.04	37.62	0.99	1.67	56.04	1.21	2.88	2.96	0.26	2.47	104.68	1.35	2.52
392	4.48	0.03	36.63	0.98	1.69	57.25	1.21	2.88	2.70	0.25	2.48	106.03	1.35	2.51
393	4.45	0.04	35.65	0.98	1.70	58.46	1.22	2.87	2.45	0.23	2.49	107.38	1.36	2.50
394	4.41	0.03	34.67	0.96	1.71	59.68	1.23	2.87	2.22	0.21	2.50	108.74	1.36	2.49
395	4.38	−0.03	33.71	−0.94	1.73	60.91	+1.24	2.87	2.01	−0.20	2.51	110.10	+1.36	2.47
396	4.35	0.02	32.77	0.92	1.74	62.15	1.25	2.87	1.81	0.19	2.52	111.46	1.36	2.46
397	4.33	0.02	31.85	0.91	1.75	63.40	1.27	2.87	1.62	0.17	2.53	112.82	1.36	2.45
398	4.31	0.01	30.94	0.91	1.76	64.67	1.28	2.86	1.45	0.16	2.54	114.18	1.37	2.44
399	4.30	0.01	30.03	0.89	1.78	65.95	1.28	2.86	1.29	0.15	2.55	115.55	1.37	2.43
400	4.29	−0.01	29.14	−0.87	1.79	67.23	+1.29	2.86	1.14	−0.13	2.56	116.92	+1.37	2.42
401	4.28	0.00	28.27	0.85	1.80	68.52	1.30	2.86	1.01	0.11	2.57	118.29	1.37	2.41
402	4.28	0.00	27.42	0.85	1.82	69.82	1.31	2.85	0.90	0.10	2.58	119.66	1.37	2.40
403	4.28	0.00	26.57	0.84	1.83	71.13	1.32	2.85	0.80	0.08	2.59	121.03	1.38	2.39
404	4.28	+0.01	25.73	0.82	1.85	72.45	1.33	2.84	0.72	0.07	2.60	122.41	1.37	2.38
405	4.29	+0.01	24.91	−0.80	1.86	73.78	+1.34	2.84	0.65	−0.05	2.60	123.78	+1.38	2.36
406	4.30	0.02	24.11	0.78	1.88	75.12	1.35	2.84	0.60	0.04	2.61	125.16	1.37	2.35
407	4.32	0.02	23.33	0.77	1.89	76.47	1.36	2.83	0.56	0.03	2.62	126.53	1.38	2.34
408	4.34	0.02	22.56	0.76	1.91	77.83	1.36	2.83	0.53	0.01	2.63	127.91	1.37	2.33
409	4.36	0.03	21.80	0.74	1.92	79.19	1.37	2.82	0.52	0.00	2.64	129.28	1.37	2.32
410	4.39	+0.03	21.06	−0.73	1.94	80.56	+1.38	2.82	0.52	+0.02	2.65	130.65	+1.37	2.31
411	4.42	0.04	20.33	0.71	1.95	81.94	1.39	2.82	0.54	0.03	2.66	132.02	1.38	2.30
412	4.46	0.04	19.62	0.69	1.97	83.33	1.40	2.81	0.57	0.05	2.67	133.40	1.37	2.29
413	4.50	0.04	18.93	0.68	1.98	84.73	1.41	2.81	0.62	0.06	2.67	134.77	1.37	2.27
414	4.54	0.05	18.25	0.67	1.99	86.14	1.41	2.80	0.68	0.08	2.68	136.14	1.37	2.26
415	4.59	+0.05	17.58	−0.65	2.01	87.55	+1.42	2.80	0.76	+0.09	2.69	137.51	+1.37	2.25
416	4.64	0.06	16.93	0.63	2.02	88.97	1.42	2.79	0.85	0.11	2.70	138.88	1.37	2.24
417	4.70	0.06	16.30	0.63	2.03	90.39	1.43	2.79	0.96	0.12	2.71	140.25	1.37	2.22
418	4.76	0.06	15.67	0.61	2.04	91.82	1.44	2.78	1.08	0.14	2.71	141.62	1.37	2.21
419	4.82	0.06	15.06	0.59	2.06	93.26	1.45	2.78	1.22	0.15	2.72	142.99	1.36	2.20
420	4.88		14.47		2.07	94.71		2.77	1.37		2.73	144.35		2.19

TABLE IX, Arg. 2.—*Continued.*

Arg.	($v.s.3$)	($v.c.3$)	($v.s.4$)	($v.c.4$)	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)	($\rho.s.2$)	($\rho.c.2$)	($\rho.s.3$)	($\rho.c.3$)
	″	″	″	″							
360	6.07	2.53	1.23	0.38	146	2629	710	252	627	67	181
361	6.02	2.51	1.22	0.38	148	2621	697	254	628	68	181
362	5.98	2.50	1.21	0.37	151	2613	685	256	629	69	182
363	5.94	2.49	1.20	0.36	153	2605	672	259	631	70	182
364	5.90	2.48	1.19	0.35	156	2596	660	261	632	71	182
365	5.85	2.46	1.17	0.35	159	2588	647	263	633	72	183
366	5.81	2.45	1.16	0.34	162	2579	635	265	634	74	183
367	5.76	2.44	1.15	0.34	164	2570	623	267	635	75	183
368	5.72	2.44	1.14	0.33	167	2562	610	270	636	76	183
369	5.67	2.43	1.12	0.32	170	2553	598	272	638	77	184
370	5.63	2.42	1.11	0.32	173	2544	586	274	639	78	184
371	5.58	2.41	1.10	0.31	176	2535	574	276	640	79	184
372	5.53	2.41	1.08	0.31	180	2526	562	279	641	80	184
373	5.48	2.40	1.07	0.30	183	2516	550	281	642	81	184
374	5.43	2.39	1.05	0.30	186	2507	539	283	643	82	184
375	5.38	2.39	1.04	0.29	190	2498	528	285	645	83	184
376	5.33	2.39	1.03	0.29	194	2488	517	288	646	84	184
377	5.28	2.38	1.01	0.29	197	2479	506	290	647	85	184
378	5.22	2.38	1.00	0.28	201	2469	495	292	648	86	184
379	5.17	2.38	0.98	0.28	205	2460	485	294	649	87	184
380	5.12	2.38	0.97	0.28	208	2450	474	297	650	88	184
381	5.07	2.38	0.96	0.28	212	2440	463	299	651	89	184
382	5.01	2.38	0.94	0.27	216	2429	453	302	652	90	184
383	4.96	2.39	0.93	0.27	220	2419	443	304	653	91	184
384	4.90	2.39	0.91	0.27	224	2408	433	306	654	92	184
385	4.84	2.39	0.90	0.27	228	2397	423	309	655	94	184
386	4.79	2.40	0.88	0.27	232	2386	413	312	656	95	184
387	4.73	2.40	0.86	0.27	236	2374	404	314	657	96	184
388	4.67	2.41	0.85	0.27	240	2363	394	317	657	97	183
389	4.61	2.42	0.84	0.27	245	2351	384	319	658	98	183
390	4.55	2.43	0.82	0.27	249	2340	375	322	659	99	183
391	4.49	2.44	0.81	0.27	253	2328	366	324	660	100	183
392	4.43	2.46	0.79	0.27	258	2316	357	327	661	102	183
393	4.37	2.47	0.78	0.27	262	2305	348	329	662	103	182
394	4.31	2.48	0.76	0.27	266	2293	339	332	662	104	182
395	4.24	2.49	0.75	0.27	271	2281	330	334	663	105	182
396	4.18	2.51	0.73	0.28	276	2269	322	336	664	106	182
397	4.12	2.53	0.72	0.28	280	2257	313	339	665	107	182
398	4.06	2.55	0.70	0.28	285	2245	304	341	666	109	181
399	4.00	2.57	0.69	0.28	290	2232	296	344	667	110	181
400	3.93	2.59	0.67	0.29	294	2220	288	346	667	111	181
401	3.87	2.62	0.66	0.29	299	2207	280	348	668	112	181
402	3.80	2.64	0.65	0.30	304	2195	273	351	668	113	180
403	3.74	2.66	0.63	0.30	309	2182	265	353	669	114	180
404	3.68	2.69	0.62	0.31	314	2170	258	355	669	115	180
405	3.62	2.72	0.60	0.31	320	2157	251	358	669	116	179
406	3.55	2.75	0.59	0.31	325	2144	244	361	669	116	179
407	3.49	2.78	0.57	0.32	331	2131	237	363	670	117	179
408	3.43	2.81	0.56	0.33	337	2119	230	365	670	118	178
409	3.36	2.84	0.55	0.33	342	2106	224	368	670	119	178
410	3.30	2.88	0.53	0.34	348	2093	217	371	670	120	178
411	3.24	2.92	0.52	0.35	354	2080	211	373	670	121	177
412	3.17	2.95	0.50	0.36	359	2066	205	376	670	122	177
413	3.11	2.99	0.49	0.36	365	2053	199	378	671	122	176
414	3.04	3.03	0.48	0.37	371	2039	193	381	671	123	176
415	2.98	3.07	0.46	0.38	377	2026	187	382	671	124	175
416	2.92	3.11	0.45	0.38	383	2012	181	386	671	125	174
417	2.85	3.15	0.44	0.39	389	1998	176	388	671	126	174
418	2.79	3.20	0.42	0.40	395	1985	170	391	671	126	173
419	2.73	3.24	0.41	0.41	401	1971	165	393	672	127	173
420	2.67	3.29	0.40	0.42	407	1957	160	396	672	128	172

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TABLE IX, ARG. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
420	4.88	+0.07	14.47	−0.57	2.07	94.71	+1.45	2.77	1.37	+0.17	2.73	144.35	+1.37	2.19
421	4.95	0.07	13.90	0.55	2.08	96.16	1.46	2.76	1.54	0.18	2.74	145.72	1.37	2.18
422	5.02	0.08	13.35	0.54	2.10	97.62	1.47	2.76	1.72	0.20	2.74	147.09	1.36	2.16
423	5.10	0.08	12.81	0.53	2.11	99.09	1.47	2.75	1.92	0.21	2.75	148.45	1.35	2.15
424	5.18	0.08	12.28	0.51	2.13	100.56	1.48	2.75	2.13	0.22	2.75	149.80	1.35	2.13
425	5.26	+0.09	11.77	−0.49	2.14	102.04	+1.48	2.74	2.35	+0.24	2.76	151.15	+1.35	2.12
426	5.35	0.09	11.28	0.48	2.15	103.52	1.49	2.73	2.59	0.25	2.77	152.50	1.34	2.11
427	5.44	0.09	10.80	0.46	2.17	105.01	1.49	2.73	2.84	0.27	2.77	153.84	1.35	2.09
428	5.53	0.10	10.34	0.45	2.18	106.50	1.50	2.72	3.11	0.28	2.78	155.19	1.35	2.08
429	5.63	0.10	9.89	0.43	2.20	108.00	1.51	2.72	3.39	0.30	2.78	156.54	1.34	2.06
430	5.73	+0.10	9.46	−0.41	2.21	109.51	+1.51	2.71	3.69	+0.31	2.79	157.88	+1.34	2.05
431	5.83	0.11	9.05	0.39	2.22	111.02	1.51	2.70	4.00	0.33	2.79	159.22	1.33	2.04
432	5.94	0.11	8.66	0.38	2.23	112.53	1.51	2.69	4.33	0.34	2.80	160.55	1.33	2.02
433	6.05	0.12	8.28	0.37	2.24	114.04	1.52	2.68	4.67	0.35	2.80	161.88	1.32	2.01
434	6.17	0.12	7.91	0.35	2.25	115.56	1.53	2.67	5.02	0.37	2.81	163.20	1.32	1.99
435	6.29	+0.12	7.56	−0.33	2.27	117.09	+1.53	2.66	5.39	+0.38	2.81	164.52	+1.32	1.98
436	6.41	0.13	7.23	0.31	2.28	118.62	1.54	2.66	5.77	0.40	2.82	165.84	1.31	1.97
437	6.54	0.13	6.92	0.30	2.29	120.16	1.54	2.65	6.17	0.41	2.82	167.15	1.31	1.95
438	6.67	0.13	6.62	0.28	2.30	121.70	1.54	2.64	6.58	0.42	2.83	168.46	1.30	1.94
439	6.80	0.14	6.34	0.27	2.31	123.24	1.54	2.63	7.00	0.44	2.83	169.76	1.30	1.92
440	6.94	+0.14	6.07	−0.25	2.32	124.78	+1.55	2.62	7.44	+0.45	2.84	171.06	+1.29	1.91
441	7.08	0.14	5.82	0.23	2.33	126.33	1.55	2.61	7.89	0.47	2.84	172.35	1.29	1.90
442	7.22	0.15	5.59	0.21	2.34	127.88	1.55	2.60	8.36	0.48	2.85	173.64	1.29	1.88
443	7.37	0.15	5.38	0.20	2.36	129.43	1.55	2.59	8.84	0.49	2.85	174.93	1.28	1.87
444	7.52	0.15	5.18	0.18	2.37	130.98	1.56	2.58	9.33	0.51	2.86	176.21	1.27	1.85
445	7.67	+0.16	5.00	−0.17	2.38	132.54	+1.56	2.57	9.84	+0.52	2.86	177.48	+1.27	1.84
446	7.83	0.16	4.83	0.15	2.39	134.10	1.56	2.57	10.36	0.53	2.86	178.75	1.26	1.83
447	7.99	0.16	4.68	0.13	2.40	135.66	1.56	2.56	10.89	0.55	2.87	180.01	1.25	1.81
448	8.15	0.16	4.55	0.12	2.42	137.22	1.56	2.55	11.44	0.56	2.87	181.26	1.25	1.80
449	8.31	0.17	4.43	0.10	2.43	138.78	1.57	2.54	12.00	0.57	2.88	182.51	1.24	1.78
450	8.48	+0.17	4.33	−0.08	2.44	140.35	+1.57	2.53	12.57	+0.59	2.88	183.75	+1.24	1.77
451	8.65	0.17	4.25	0.06	2.45	141.92	1.57	2.52	13.16	0.60	2.88	184.99	1.23	1.76
452	8.82	0.18	4.19	0.05	2.46	143.49	1.57	2.51	13.76	0.61	2.88	186.22	1.22	1.74
453	9.00	0.18	4.14	0.04	2.47	145.06	1.57	2.50	14.37	0.62	2.89	187.44	1.22	1.73
454	9.18	0.18	4.10	0.01	2.48	146.63	1.57	2.49	14.99	0.64	2.89	188.66	1.21	1.71
455	9.36	+0.18	4.09	0.00	2.49	148.20	+1.57	2.47	15.63	+0.65	2.89	189.87	+1.20	1.70
456	9.54	0.19	4.09	+0.02	2.50	149.77	1.57	2.46	16.28	0.67	2.89	191.07	1.20	1.69
457	9.73	0.19	4.11	0.03	2.51	151.34	1.57	2.45	16.95	0.68	2.89	192.27	1.19	1.67
458	9.92	0.19	4.14	0.04	2.52	152.91	1.57	2.44	17.63	0.69	2.90	193.46	1.18	1.66
459	10.11	0.19	4.18	0.07	2.53	154.48	1.57	2.43	18.32	0.70	2.90	194.64	1.17	1.64
460	10.30	+0.20	4.25	+0.08	2.54	156.05	+1.57	2.42	19.02	+0.71	2.90	195.81	+1.17	1.63
461	10.50	0.20	4.33	0.10	2.55	157.62	1.57	2.41	19.73	0.72	2.90	196.98	1.16	1.61
462	10.70	0.20	4.43	0.12	2.56	159.19	1.57	2.40	20.45	0.74	2.90	198.14	1.15	1.60
463	10.90	0.20	4.55	0.13	2.57	160.76	1.57	2.39	21.19	0.75	2.90	199.29	1.14	1.58
464	11.10	0.21	4.68	0.15	2.58	162.33	1.56	2.38	21.94	0.77	2.90	200.43	1.13	1.57
465	11.31	+0.21	4.83	+0.16	2.59	163.89	+1.56	2.36	22.71	+0.77	2.91	201.56	+1.12	1.55
466	11.52	0.21	4.99	0.18	2.59	165.45	1.56	2.35	23.48	0.79	2.91	202.68	1.11	1.53
467	11.73	0.21	5.17	0.20	2.60	167.01	1.56	2.34	24.27	0.80	2.91	203.79	1.11	1.52
468	11.94	0.21	5.37	0.22	2.61	168.57	1.56	2.33	25.07	0.81	2.91	204.90	1.10	1.50
469	12.15	0.22	5.59	0.23	2.62	170.13	1.55	2.32	25.88	0.82	2.91	206.00	1.09	1.49
470	12.37	+0.22	5.82	+0.25	2.63	171.68	+1.55	2.31	26.70	+0.83	2.91	207.09	+1.08	1.47
471	12.59	0.22	6.07	0.27	2.64	173.23	1.55	2.30	27.53	0.84	2.91	208.17	1.07	1.46
472	12.81	0.22	6.34	0.28	2.65	174.78	1.55	2.29	28.37	0.85	2.91	209.24	1.06	1.44
473	13.03	0.23	6.62	0.29	2.65	176.33	1.54	2.27	29.22	0.87	2.90	210.30	1.06	1.43
474	13.26	0.23	6.91	0.31	2.66	177.87	1.54	2.26	30.09	0.88	2.90	211.36	1.04	1.41
475	13.49	+0.23	7.22	+0.33	2.67	179.41	+1.54	2.25	30.97	+0.89	2.90	212.40	+1.03	1.40
476	13.72	0.23	7.55	0.35	2.68	180.95	1.53	2.24	31.86	0.90	2.90	213.43	1.02	1.39
477	13.95	0.23	7.90	0.36	2.69	182.48	1.53	2.23	32.76	0.91	2.90	214.45	1.02	1.37
478	14.18	0.23	8.26	0.37	2.69	184.01	1.53	2.21	33.67	0.92	2.89	215.47	1.01	1.36
479	14.41	0.24	8.63	0.39	2.70	185.54	1.52	2.20	34.59	0.93	2.89	216.48	0.99	1.34
480	14.65		9.02		2.71	187.06		2.19	35.52		2.89	217.47		1.33

TABLE IX, Arg. 2.—*Continued.*

Arg.	$(v.s.3)$	$(v.c.3)$	$(v.s.4)$	$(v.c.4)$	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$	$(\rho.s.3)$	$(\rho.c.3)$
	″	″	″	″							
420	2.67	3.29	0.40	0.42	407	1957	160	396	672	128	172
421	2.61	3.34	0.39	0.43	413	1943	155	399	672	129	172
422	2.55	3.39	0.38	0.44	419	1930	150	402	673	129	171
423	2.49	3.44	0.36	0.45	425	1916	146	405	673	130	171
424	2.43	3.50	0.35	0.46	431	1903	141	407	673	131	171
425	2.37	3.55	0.34	0.47	437	1889	136	410	673	132	170
426	2.31	3.61	0.33	0.49	443	1875	132	413	673	133	170
427	2.25	3.66	0.32	0.50	450	1862	128	416	673	133	169
428	2.20	3.72	0.31	0.51	456	1848	124	418	673	134	169
429	2.14	3.78	0.30	0.52	463	1834	120	421	673	135	168
430	2.08	3.84	0.29	0.53	469	1820	117	424	673	136	167
431	2.03	3.90	0.28	0.54	476	1806	114	426	673	136	167
432	1.97	3.96	0.27	0.55	482	1791	111	429	673	137	166
433	1.92	4.03	0.26	0.57	489	1777	108	432	673	138	165
434	1.86	4.10	0.25	0.58	496	1762	105	434	673	139	165
435	1.81	4.16	0.24	0.59	503	1748	102	437	672	139	164
436	1.76	4.23	0.23	0.60	510	1733	100	440	672	140	163
437	1.70	4.30	0.22	0.62	517	1719	98	442	672	141	163
438	1.65	4.37	0.22	0.63	524	1704	96	445	671	142	162
439	1.60	4.45	0.21	0.64	531	1690	94	447	671	142	161
440	1.55	4.52	0.20	0.66	539	1675	92	450	671	143	160
441	1.50	4.60	0.19	0.67	546	1660	91	453	670	144	159
442	1.46	4.67	0.18	0.68	553	1645	90	455	670	144	159
443	1.41	4.75	0.18	0.70	560	1631	89	458	669	145	158
444	1.37	4.83	0.17	0.71	567	1616	88	460	669	145	157
445	1.32	4.91	0.17	0.73	574	1601	87	463	669	146	156
446	1.28	4.99	0.16	0.74	582	1586	87	466	668	146	156
447	1.24	5.07	0.16	0.75	589	1571	86	468	668	147	155
448	1.20	5.15	0.15	0.77	596	1557	86	471	667	147	154
449	1.16	5.24	0.14	0.79	603	1542	86	473	667	147	153
450	1.12	5.32	0.14	0.80	610	1527	86	476	667	148	152
451	1.08	5.40	0.13	0.82	618	1512	87	479	666	148	152
452	1.04	5.49	0.13	0.83	625	1497	87	481	666	149	151
453	1.01	5.57	0.12	0.85	633	1483	88	484	665	149	150
454	0.97	5.66	0.12	0.86	640	1468	88	486	665	149	149
455	0.94	5.75	0.12	0.88	648	1453	89	489	664	150	148
456	0.91	5.84	0.11	0.89	656	1438	90	492	664	150	148
457	0.88	5.94	0.11	0.90	664	1423	92	494	663	150	147
458	0.85	6.03	0.11	0.92	671	1409	93	497	662	151	146
459	0.82	6.13	0.10	0.94	679	1394	95	499	661	151	145
460	0.80	6.23	0.10	0.95	687	1379	96	502	661	151	144
461	0.77	6.32	0.10	0.97	695	1364	98	505	660	151	144
462	0.75	6.42	0.10	0.99	703	1349	100	507	660	152	143
463	0.73	6.51	0.10	1.00	711	1335	102	510	659	152	142
464	0.71	6.61	0.10	1.02	719	1320	105	512	658	152	142
465	0.69	6.71	0.09	1.03	727	1305	108	515	657	152	141
466	0.68	6.81	0.09	1.05	735	1290	110	517	657	153	140
467	0.66	6.91	0.09	1.06	743	1275	113	520	656	153	139
468	0.64	7.01	0.09	1.08	751	1261	116	523	655	153	139
469	0.63	7.11	0.09	1.10	759	1246	119	525	654	154	138
470	0.62	7.21	0.09	1.11	767	1232	123	528	653	154	137
471	0.62	7.31	0.10	1.13	775	1217	127	531	652	154	136
472	0.61	7.42	0.10	1.15	784	1203	131	533	651	155	136
473	0.60	7.52	0.10	1.16	792	1189	135	536	649	155	135
474	0.60	7.62	0.10	1.18	800	1175	139	539	648	155	134
475	0.60	7.72	0.10	1.19	809	1161	143	542	647	156	133
476	0.60	7.83	0.11	1.21	817	1147	148	545	646	156	133
477	0.60	7.94	0.11	1.23	826	1133	153	548	644	156	132
478	0.60	8.04	0.11	1.24	834	1119	158	551	643	156	131
479	0.61	8.15	0.12	1.25	843	1105	163	554	642	156	130
480	0.61	8.26	0.12	1.27	851	1091	168	556	640	156	130

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	Sec.var.	(*v.c.*1)	Diff.	Sec.var.	(*v.s.*2)	Diff.	Sec.var.	(*v.c.*2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
480	14.65	+0.24	9.02	+0.41	2.71	187.06	+1.51	2.19	35.52	+0.94	2.89	217.47	+0.99	1.33
481	14.89	0.24	9.43	0.42	2.72	188.57	1.51	2.18	36.46	0.95	2.89	218.46	0.98	1.32
482	15.13	0.24	9.85	0.44	2.72	190.08	1.51	2.16	37.41	0.96	2.89	219.44	0.96	1.30
483	15.37	0.24	10.29	0.45	2.73	191.59	1.50	2.15	38.37	0.98	2.88	220.40	0.95	1.29
484	15.61	0.24	10.74	0.47	2.74	193.09	1.50	2.14	39.35	0.98	2.88	221.35	0.94	1.27
485	15.85	+0.24	11.21	+0.48	2.74	194.59	+1.49	2.12	40.33	+0.99	2.88	222.29	+0.93	1.26
486	16.09	0.24	11.69	0.50	2.75	196.08	1.49	2.11	41.32	1.00	2.88	223.22	0.91	1.25
487	16.33	0.25	12.19	0.52	2.76	197.57	1.48	2.10	42.32	1.01	2.88	224.13	0.91	1.23
488	16.58	0.25	12.71	0.53	2.77	199.05	1.48	2.09	43.33	1.02	2.87	225.04	0.90	1.22
489	16.83	0.25	13.24	0.55	2.77	200.53	1.47	2.07	44.35	1.03	2.87	225.94	0.89	1.20
490	17.08	+0.25	13.79	+0.56	2.78	202.00	+1.46	2.06	45.38	+1.04	2.87	226.83	+0.88	1.19
491	17.33	0.25	14.35	0.58	2.79	203.46	1.46	2.05	46.42	1.04	2.87	227.71	0.87	1.18
492	17.58	0.25	14.93	0.59	2.79	204.92	1.45	2.03	47.46	1.05	2.86	228.58	0.85	1.16
493	17.83	0.26	15.52	0.61	2.80	206.37	1.44	2.02	48.51	1.07	2.86	229.43	0.84	1.15
494	18.09	0.25	16.13	0.62	2.80	207.81	1.43	2.00	49.58	1.07	2.86	230.27	0.83	1.13
495	18.34	+0.26	16.75	+0.64	2.81	209.24	+1.43	1.99	50.65	+1.08	2.85	231.10	+0.82	1.12
496	18.60	0.25	17.39	0.65	2.81	210.67	1.42	1.98	51.73	1.09	2.85	231.92	0.81	1.10
497	18.85	0.26	18.04	0.67	2.82	212.09	1.41	1.96	52.82	1.10	2.85	232.73	0.79	1.09
498	19.11	0.25	18.71	0.68	2.82	213.50	1.40	1.95	53.92	1.11	2.85	233.52	0.78	1.07
499	19.36	0.26	19.39	0.70	2.83	214.90	1.40	1.93	55.03	1.11	2.84	234.30	0.77	1.06
500	19.62	+0.26	20.09	+0.71	2.83	216.30	+1.39	1.92	56.14	+1.12	2.84	235.07	+0.76	1.04
501	19.88	0.26	20.80	0.72	2.83	217.69	1.38	1.91	57.26	1.12	2.83	235.83	0.75	1.03
502	20.14	0.26	21.52	0.74	2.84	219.07	1.37	1.89	58.38	1.14	2.83	236.58	0.73	1.01
503	20.40	0.26	22.26	0.75	2.84	220.44	1.37	1.88	59.52	1.15	2.82	237.31	0.72	1.00
504	20.66	0.26	23.01	0.76	2.84	221.81	1.36	1.86	60.67	1.15	2.82	238.03	0.71	0.99
505	20.92	+0.26	23.77	+0.78	2.85	223.17	+1.35	1.85	61.82	+1.16	2.81	238.74	+0.70	0.97
506	21.18	0.26	24.55	0.79	2.85	224.52	1.33	1.84	62.98	1.16	2.80	239.44	0.68	0.96
507	21.44	0.25	25.34	0.81	2.85	225.85	1.33	1.82	64.14	1.17	2.80	240.12	0.67	0.95
508	21.69	0.26	26.15	0.82	2.85	227.18	1.32	1.81	65.31	1.18	2.79	240.79	0.66	0.94
509	21.95	0.26	26.97	0.84	2.86	228.50	1.31	1.79	66.49	1.19	2.79	241.45	0.65	0.92
510	22.21	+0.26	27.81	+0.85	2.86	229.81	+1.30	1.78	67.68	+1.19	2.78	242.10	+0.63	0.91
511	22.47	0.25	28.66	0.86	2.86	231.11	1.28	1.77	68.87	1.20	2.77	242.73	0.63	0.90
512	22.72	0.26	29.52	0.88	2.87	232.39	1.28	1.75	70.07	1.21	2.77	243.36	0.62	0.88
513	22.98	0.26	30.40	0.88	2.87	233.67	1.27	1.74	71.28	1.21	2.76	243.98	0.60	0.87
514	23.24	0.25	31.28	0.90	2.87	234.94	1.26	1.72	72.49	1.22	2.75	244.58	0.58	0.86
515	23.49	+0.26	32.18	+0.91	2.88	236.20	+1.25	1.71	73.71	+1.22	2.74	245.16	+0.55	0.84
516	23.75	0.25	33.09	0.92	2.88	237.45	1.24	1.69	74.93	1.23	2.74	245.71	0.55	0.83
517	24.00	0.26	34.01	0.94	2.88	238.69	1.23	1.68	76.16	1.23	2.73	246.26	0.54	0.82
518	24.26	0.25	34.95	0.95	2.88	239.92	1.21	1.66	77.39	1.24	2.72	246.80	0.53	0.81
519	24.51	0.25	35.90	0.96	2.89	241.13	1.20	1.65	78.63	1.24	2.72	247.33	0.51	0.79
520	24.76	+0.26	36.86	+0.97	2.89	242.33	+1.19	1.63	79.87	+1.25	2.71	247.84	+0.50	0.78
521	25.02	0.25	37.83	0.99	2.89	243.52	1.18	1.62	81.12	1.25	2.70	248.34	0.49	0.77
522	25.27	0.25	38.82	1.00	2.89	244.70	1.17	1.60	82.37	1.25	2.69	248.83	0.47	0.75
523	25.52	0.25	39.82	1.01	2.89	245.87	1.16	1.59	83.62	1.26	2.68	249.30	0.46	0.74
524	25.77	0.25	40.83	1.02	2.89	247.03	1.15	1.57	84.88	1.26	2.67	249.76	0.45	0.73
525	26.02	+0.25	41.85	+1.03	2.89	248.18	+1.13	1.56	86.14	+1.27	2.66	250.21	+0.43	0.71
526	26.27	0.25	42.88	1.04	2.89	249.31	1.12	1.55	87.41	1.28	2.66	250.64	0.42	0.70
527	26.52	0.24	43.92	1.05	2.89	250.43	1.10	1 53	88.69	1.28	2.65	251.06	0.41	0.69
528	26.76	0.24	44.97	1.07	2.89	251.53	1.10	1.52	89.97	1.28	2.64	251.47	0.39	0.68
529	27.00	0.24	46.04	1.08	2.89	252.63	1.09	1.50	91.25	1.29	2.63	251.86	0.38	0.66
530	27.24	+0.24	47.12	+1.09	2.89	253.72	+1.07	1 49	92.54	+1.29	2.62	252.24	+0.36	0.65
531	27.48	0.24	48.21	1.10	2.89	254.79	1.06	1.48	93.83	1.29	2.61	252.60	0.35	0.64
532	27.72	0.24	49.31	1.11	2.89	255.85	1.05	1.46	95.12	1.30	2.60	252.95	0.34	0.63
533	27.96	0.23	50.42	1.11	2.89	256.90	1.03	1.45	96.42	1.30	2.59	253.29	0.32	0.62
534	28.19	0.23	51.53	1.13	2.89	257.93	1.02	1.43	97.72	1.30	2.58	253.61	0.31	0.61
535	28.42	+0.23	52.66	+1.14	2.88	258.95	+1.01	1.42	99.02	+1.30	2.58	253.92	+0.30	0.60
536	28.65	0.23	53.80	1.15	2.88	259.96	0.99	1.40	100.32	1.31	2.57	254.22	0.28	0.58
537	28.88	0.23	54.95	1.16	2.88	260.95	0.98	1.39	101.63	1.31	2.56	254.50	0.27	0.57
538	29.11	0.22	56.11	1.16	2.88	261.93	0.97	1.37	102 94	1.31	2.55	254.77	0.25	0.56
539	29.33	0.22	57.27	1.18	2.88	262.90	0.95	1.36	104.25	1.31	2.54	255.02	0.24	0.55
540	29.55		58.45		2.88	263.85		1.34	105.56		2.53	255.26		0.54

TABLE IX, Arg. 2.—*Continued.*

Arg.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)	(ρ.*c.*0)	(ρ.*s.*1)	(ρ.*c.*1)	(ρ.*s.*2)	(ρ.*c.*2)	(ρ.*s.*3)	(ρ.*c.*3)
	″	″	″	″							
480	0.61	8.26	0.11	1.27	851	1091	168	556	640	156	130
481	0.62	8.36	0.12	1.28	860	1076	173	559	639	156	129
482	0.64	8.47	0.13	1.30	868	1063	179	561	637	156	128
483	0.65	8.58	0.13	1.31	877	1049	184	564	636	156	128
484	0.66	8.69	0.14	1.33	885	1035	190	566	634	156	127
485	0.67	8.79	0.14	1.35	894	1021	196	569	633	156	126
486	0.69	8.90	0.15	1.36	902	1007	202	571	632	156	125
487	0.71	9.01	0.15	1.37	911	994	208	574	630	156	124
488	0.73	9.11	0.16	1.39	919	980	214	576	629	156	124
489	0.75	9.22	0.17	1.40	927	966	220	579	627	156	123
490	0.77	9.33	0.17	1.42	936	953	227	581	626	156	122
491	0.80	9.43	0.18	1.43	945	940	234	584	624	156	121
492	0.82	9.54	0.19	1.44	954	926	241	586	623	156	120
493	0.85	9.65	0.19	1.46	963	913	249	589	621	156	120
494	0.88	9.75	0.20	1.47	972	900	256	591	619	156	119
495	0.91	9.86	0.21	1.49	981	887	264	594	618	156	118
496	0.94	9.97	0.22	1.50	990	874	272	596	616	156	117
497	0.98	10.07	0.23	1.51	999	861	280	599	615	156	117
498	1.02	10.18	0.23	1.53	1008	848	288	601	613	155	116
499	1.06	10.29	0.24	1.54	1017	835	296	604	611	155	115
500	1.10	10.39	0.25	1.55	1026	822	304	606	610	155	115
501	1.14	10.50	0.26	1.57	1035	810	312	608	608	155	114
502	1.19	10.60	0.27	1.58	1044	797	321	611	606	155	114
503	1.24	10.70	0.28	1.59	1053	785	329	613	604	154	113
504	1.29	10.81	0.29	1.60	1063	772	338	615	602	154	113
505	1.34	10.91	0.30	1.62	1072	760	346	617	600	154	112
506	1.39	11.01	0.31	1.63	1081	748	355	620	598	154	112
507	1.44	11.12	0.32	1.64	1090	736	364	622	596	154	111
508	1.50	11.22	0.33	1.65	1099	724	373	624	594	154	110
509	1.56	11.32	0.34	1.66	1108	712	382	626	592	153	110
510	1.62	11.42	0.35	1.67	1117	700	391	628	590	153	109
511	1.68	11.52	0.37	1.68	1126	688	401	631	588	153	109
512	1.74	11.62	0.38	1.69	1135	677	410	633	585	153	108
513	1.80	11.71	0.39	1.70	1144	666	420	635	583	153	108
514	1.87	11.81	0.40	1.71	1153	654	430	637	581	153	107
515	1.94	11.91	0.41	1.72	1162	643	440	639	578	153	106
516	2.01	12.00	0.42	1.73	1171	632	450	642	576	153	106
517	2.08	12.09	0.43	1.74	1180	621	460	644	573	152	106
518	2.16	12.19	0.45	1.75	1189	610	471	646	571	152	105
519	2.23	12.28	0.46	1.76	1197	599	481	648	569	152	105
520	2.31	12.37	0.47	1.77	1206	588	492	650	566	152	104
521	2.39	12.46	0.49	1.78	1215	577	503	652	564	152	104
522	2.47	12.55	0.50	1.79	1224	567	514	654	561	151	103
523	2.55	12.63	0.51	1.79	1233	557	525	656	558	151	103
524	2.63	12.72	0.52	1.80	1242	546	536	658	556	151	103
525	2.71	12.81	0.54	1.81	1250	536	548	660	553	151	102
526	2.80	12.89	0.55	1.82	1259	526	559	662	550	151	102
527	2.88	12.98	0.57	1.83	1267	517	571	664	548	150	101
528	2.97	13.06	0.58	1.83	1276	507	583	666	545	150	101
529	3.06	13.14	0.60	1.84	1284	498	595	668	542	150	100
530	3.15	13.22	0.61	1.85	1293	488	607	670	540	150	100
531	3.24	13.30	0.62	1.85	1301	479	618	672	537	150	100
532	3.34	13.37	0.64	1.86	1310	470	630	673	535	149	99
533	3.43	13.45	0.65	1.86	1318	461	642	675	532	149	99
534	3.53	13.52	0.67	1.87	1327	452	654	677	529	149	98
535	3.63	13.59	0.68	1.87	1335	443	666	678	527	149	98
536	3.73	13.66	0.69	1.88	1343	434	678	680	524	148	98
537	3.83	13.73	0.71	1.88	1352	426	690	682	521	148	98
538	3.93	13.80	0.72	1.89	1360	417	702	683	519	148	97
539	4.03	13.87	0.74	1.89	1368	409	714	685	516	148	97
540	4.13	13.93	0.75	1.90	1376	401	726	686	513	147	97

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TABLE IX, Arg. 2.—*Concluded.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	Sec.var.	(v.c.1)	Diff.	Sec.var.	(v.s.2)	Diff.	Sec.var.	(v.c.2)	Diff.	Sec.var.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
540	29.55	+0.22	58.45	+1.18	2.88	263.85	+0.94	1.34	105.56	+1.32	2.53	255.26	+0.23	0.54
541	29.77	0.22	59.63	1.19	2.88	264.79	0.92	1.33	106.88	1.31	2.52	255.49	0.21	0.53
542	29.99	0.22	60.82	1.20	2.88	265.71	0.91	1.32	108.19	1.32	2.51	255.70	0.20	0.52
543	30.21	0.22	62.02	1.22	2.87	266.62	0.90	1.30	109.51	1.32	2.50	255.90	0.18	0.51
544	30.43	0.21	63.24	1.22	2.87	267.52	0.88	1.29	110.83	1.32	2.49	256.08	0.17	0.50
545	30.64	+0.21	64.46	+1.23	2.87	268.40	+0.87	1.27	112.15	+1.32	2.47	256.25	+0.16	0.49
546	30.85	0.21	65.69	1.24	2.87	269.27	0.85	1.26	113.47	1.33	2.46	256.41	0.14	0.48
547	31.06	0.21	66.93	1.25	2.87	270.12	0.84	1.25	114.80	1.32	2.45	256.55	0.13	0.47
548	31.27	0.20	68.18	1.25	2.86	270.96	0.83	1.23	116.12	1.33	2.44	256.68	0.11	0.46
549	31.47	0.20	69.43	1.26	2.86	271.79	0.81	1.22	117.45	1.32	2.43	256.79	0.10	0.45
550	31.67	+0.20	70.69	+1.27	2.86	272.60	+0.80	1.21	118.77	+1.32	2.42	256.89	+0.09	0.44
551	31.87	0.19	71.96	1.27	2.86	273.40	0.78	1.20	120.09	1.33	2.41	256.98	0.07	0.43
552	32.06	0.19	73.23	1.28	2.85	274.18	0.77	1.18	121.42	1.32	2.40	257.05	0.06	0.42
553	32.25	0.19	74.51	1.29	2.85	274.95	0.75	1.17	122.74	1.33	2.39	257.11	0.04	0.41
554	32.44	0.19	75.80	1.30	2.84	275.70	0.74	1.15	124.07	1.32	2.38	257.15	0.03	0.40
555	32.63	+0.18	77.10	+1.30	2.84	276.44	+0.72	1.14	125.39	+1.32	2.36	257.18	+0.02	0.40
556	32.81	0.18	78.40	1.31	2.84	277.16	0.70	1.12	126.71	1.32	2.35	257.20	0.00	0.39
557	32.99	0.18	79.71	1.32	2.83	277.86	0.69	1.11	128.03	1.32	2.34	257.20	−0.01	0.38
558	33.17	0.18	81.03	1.32	2.83	278.55	0.68	1.09	129.35	1.32	2.33	257.19	0.03	0.37
559	33.35	0.17	82.35	1.33	2.82	279.23	0.66	1.08	130.67	1.34	2.32	257.16	0.04	0.36
560	33.52	+0.17	83.68	+1.33	2.82	279.89	+0.65	1.06	132.01	+1.32	2.31	257.12	−0.05	0.35
561	33.69	0.17	85.01	1.34	2.82	280.54	0.63	1.05	133.33	1.32	2.30	257.07	0.07	0.34
562	33.86	0.17	86.35	1.34	2.81	281.17	0.61	1.03	134.65	1.32	2.29	257.00	0.08	0.33
563	34.03	0.16	87.69	1.35	2.81	281.78	0.60	1.02	135.97	1.31	2.27	256.92	0.10	0.33
564	34.19	0.16	89.04	1.36	2.80	282.38	0.59	1.01	137.28	1.32	2.26	256.82	0.11	0.32
565	34.35	+0.16	90.40	+1.36	2.80	282.97	+0.57	0.99	138.60	+1.31	2.25	256.71	−0.12	0.31
566	34.51	0.16	91.76	1.37	2.79	283.54	0.55	0.98	139.91	1.31	2.24	256.59	0.14	0.30
567	34.67	0.15	93.13	1.37	2.79	284.09	0.54	0.97	141.22	1.32	2.23	256.45	0.15	0.29
568	34.82	0.15	94.50	1.38	2.78	284.63	0.53	0.96	142.54	1.31	2.21	256.30	0.16	0.29
569	34.97	0.15	95.88	1.38	2.78	285.16	0.51	0.94	143.85	1.31	2.20	256.14	0.18	0.28
570	35.12	+0.15	97.26	+1.38	2.77	285.67	+0.49	0.93	145.16	+1.30	2.19	255.96	−0.19	0.27
571	35.27	0.14	98.64	1.39	2.76	286.16	0.48	0.92	146.46	1.30	2.18	255.77	0.20	0.26
572	35.41	0.14	100.03	1.39	2.76	286.64	0.46	0.90	147.76	1.29	2.16	255.57	0.21	0.26
573	35.55	0.14	101.42	1.40	2.75	287.10	0.45	0.89	149.05	1.30	2.15	255.36	0.23	0.25
574	35.69	0.13	102.82	1.40	2.75	287.55	0.43	0.87	150.35	1.29	2.13	255.13	0.24	0.25
575	35.82	+0.13	104.22	+1.40	2.74	287.98	+0.42	0.86	151.64	+1.29	2.12	254.89	−0.26	0.24
576	35.95	0.13	105.62	1.41	2.73	288.40	0.40	0.85	152.93	1.29	2.11	254.63	0.27	0.23
577	36.08	0.13	107.03	1.41	2.73	288.80	0.38	0.83	154.22	1.28	2.09	254.36	0.28	0.23
578	36.21	0.12	108.44	1.41	2.72	289.18	0.37	0.82	155.50	1.27	2.08	254.08	0.29	0.22
579	36.33	0.12	109.85	1.41	2.72	289.55	0.36	0.80	156.77	1.28	2.06	253.79	0.31	0.22
580	36.45	+0.12	111.26	+1.42	2.71	289.91	+0.34	0.79	158.05	+1.27	2.05	253.48	−0.32	0.21
581	36.57	0.12	112.68	1.42	2.70	290.25	0.32	0.78	159.32	1.27	2.04	253.16	0.33	0.21
582	36.69	0.12	114.10	1.42	2.69	290.57	0.31	0.77	160.59	1.26	2.02	252.83	0.35	0.20
583	36.81	0.12	115.52	1.43	2.68	290.88	0.30	0.76	161.85	1.26	2.01	252.48	0.36	0.20
584	36.93	0.11	116.95	1.43	2.67	291.18	0.28	0.75	163.11	1.26	1.99	252.12	0.37	0.19
585	37.04	+0.11	118.38	+1.43	2.67	291.46	+0.26	0.73	164.37	+1.25	1.98	251.75	−0.39	0.19
586	37.15	0.11	119.81	1.43	2.66	291.72	0.24	0.72	165.62	1.25	1.97	251.36	0.40	0.18
587	37.26	0.11	121.24	1.43	2.65	291.96	0.23	0.71	166.87	1.24	1.95	250.96	0.41	0.18
588	37.37	0.11	122.67	1.44	2.64	292.19	0.22	0.70	168.11	1.24	1.94	250.55	0.42	0.17
589	37.48	0.10	124.11	1.43	2.63	292.41	0.20	0.69	169.35	1.23	1.92	250.13	0.44	0.17
590	37.58	+0.10	125.54	+1.44	2.62	292.61	+0.18	0.68	170.58	+1.23	1.91	249.69	−0.45	0.16
591	37.68	0.10	126.98	1.43	2.61	292.79	0.17	0.67	171.81	1.22	1.90	249.24	0.46	0.16
592	37.78	0.10	128.41	1.44	2.60	292.96	0.16	0.66	173.03	1.22	1.88	248.78	0.47	0.15
593	37.88	0.10	129.85	1.44	2.59	293.12	0.14	0.64	174.25	1.21	1.87	248.31	0.48	0.15
594	37.98	0.10	131.29	1.44	2.58	293.26	0.12	0.63	175.46	1.21	1.85	247.83	0.50	0.14
595	38.08	+0.10	132.73	+1.44	2.58	293.38	+0.11	0.62	176.67	+1.20	1.84	247.33	−0.51	0.14
596	38.18	0.09	134.17	1.44	2.57	293.49	0.09	0.61	177.87	1.20	1.83	246.82	0.52	0.14
597	38.27	0.09	135.61	1.44	2.56	293.58	0.08	0.60	179.07	1.19	1.81	246.30	0.53	0.13
598	38.36	0.09	137.05	1.45	2.55	293.66	0.07	0.58	180.26	1.18	1.80	245.77	0.54	0.13
599	38.45	0.09	138.50	1.44	2.54	293.73	0.05	0.57	181.44	1.18	1.78	245.23	0.56	0.12
600	38.54		139.94		2.53	293.78		0.56	182.62		1.77	244.67		0.12

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TABLE IX, Arg. 2.—*Concluded.*

Arg.	($v.s.3$)	($v.c.3$)	($v.s.4$)	($v.c.4$)	($\rho.c.0$)	($\rho.s.1$)	($\rho.c.1$)	($\rho.s.2$)	($\rho.c.2$)	($\rho.s.3$)	($\rho.c.3$)
	″	″	″	″							
540	4.13	13.93	0.75	1.90	1376	401	726	686	513	147	97
541	4.23	13.99	0.76	1.90	1384	393	738	688	510	147	97
542	4.34	14.05	0.78	1.90	1392	386	751	689	507	147	96
543	4.45	14.11	0.79	1.90	1400	378	763	691	503	146	96
544	4.56	14.17	0.81	1.91	1408	371	776	692	500	146	95
545	4.67	14.23	0.83	1.91	1416	363	788	693	497	146	95
546	4.79	14.28	0.84	1.91	1424	356	801	695	494	146	95
547	4.90	14.34	0.85	1.91	1431	349	813	696	491	146	94
548	5.01	14.39	0.87	1.92	1439	343	826	698	487	145	94
549	5.12	14.44	0.89	1.92	1446	336	838	699	484	145	94
550	5.24	14.49	0.90	1.92	1454	329	851	700	481	145	93
551	5.35	14.54	0.92	1.92	1461	323	864	702	478	145	93
552	5.47	14.58	0.93	1.92	1468	316	877	703	475	145	93
553	5.58	14.63	0.95	1.92	1475	310	890	704	471	144	93
554	5.70	14.67	0.96	1.92	1482	304	903	705	468	144	92
555	5.82	14.71	0.98	1.92	1489	298	916	706	465	144	92
556	5.94	14.75	0.99	1.92	1496	292	929	708	462	144	92
557	6.06	14.79	1.01	1.92	1502	287	943	709	458	144	92
558	6.18	14.82	1.02	1.92	1509	281	956	710	455	143	92
559	6.30	14.85	1.04	1.92	1515	276	970	711	452	143	91
560	6.42	14.88	1.05	1.92	1522	270	983	712	449	143	91
561	6.54	14.91	1.07	1.92	1528	265	996	713	445	143	91
562	6.67	14.93	1.08	1.92	1534	260	1010	714	442	143	91
563	6.79	14.96	1.10	1.91	1541	255	1023	715	438	142	91
564	6.91	14.99	1.11	1.91	1547	250	1036	716	435	142	90
565	7.04	15.01	1.13	1.91	1553	246	1049	716	431	142	90
566	7.16	15.03	1.14	1.91	1559	242	1063	717	428	142	90
567	7.29	15.05	1.16	1.90	1564	238	1076	717	424	142	90
568	7.41	15.06	1.17	1.90	1570	234	1089	717	421	141	90
569	7.54	15.08	1.18	1.90	1575	230	1102	718	417	141	89
570	7.66	15.09	1.20	1.89	1581	226	1116	718	413	141	89
571	7.79	15.09	1.21	1.89	1586	223	1129	718	410	141	89
572	7.91	15.10	1.23	1.89	1591	219	1142	718	406	141	89
573	8.04	15.10	1.24	1.88	1596	216	1155	719	403	140	89
574	8.16	15.11	1.26	1.88	1601	213	1169	719	399	140	89
575	8.29	15.11	1.27	1.87	1605	210	1182	719	395	140	88
576	8.42	15.12	1.29	1.86	1610	207	1195	719	392	140	88
577	8.54	15.12	1 30	1.86	1614	205	1208	719	388	140	88
578	8.67	15.11	1.32	1.85	1618	202	1221	719	385	140	88
579	8.80	15.10	1.33	1.84	1622	200	1234	719	381	139	88
580	8.93	15.09	1.34	1.84	1626	198	1248	719	378	139	88
581	9.05	15.08	1.35	1.83	1630	196	1261	719	374	139	87
582	9.18	15.07	1.36	1.83	1634	194	1274	719	371	139	87
583	9.31	15.05	1.38	1.82	1637	192	1288	719	367	139	87
584	9.43	15.04	1.39	1.81	1641	191	1301	719	364	139	87
585	9.56	15.02	1.40	1.81	1645	189	1315	718	360	139	87
586	9.68	15.00	1.42	1.80	1648	188	1328	718	357	139	87
587	9.81	14.98	1.43	1.79	1651	186	1342	718	353	139	87
588	9.93	14.96	1.44	1.79	1654	185	1356	717	350	138	86
589	10.06	14.93	1.45	1.78	1657	184	1369	717	346	138	86
590	10.18	14.91	1.46	1.77	1660	183	1382	717	343	138	86
591	10.31	14.88	1.48	1.76	1662	182	1396	716	339	138	86
592	10.43	14.85	1.49	1.75	1665	182	1409	716	335	138	86
593	10.55	14.81	1.50	1.74	1667	181	1422	716	332	138	86
594	10.68	14.78	1.51	1.73	1669	181	1436	715	328	138	85
595	10.80	14.74	1.52	1.72	1671	181	1448	715	324	138	85
596	10.92	14.71	1.53	1.72	1673	181	1461	714	321	138	85
597	11.04	14.67	1.55	1.71	1674	181	1474	714	317	137	85
598	11.16	14.63	1.56	1.70	1676	181	1487	713	313	137	85
599	11.28	14.58	1.57	1.69	1677	182	1500	712	310	137	85
600	11.40	14.53	1.58	1.68	1678	182	1513	711	306	137	85

TABLE X, Arg. 3.—Action of Neptune.

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	"	"	"	"	"	"	"	"	"	"	"	"	"	"
0	92.96		0.87		34.22		7.42		3.95		0.80	1.15	—0.85	—1.13
1	92.67	—0.29	0.98	+0.11	34.77	+0.55	7.36	—0.06	3.82	—0.13	0.81	1.16	0.86	1.14
2	92.38	0.29	1.10	0.12	35.31	0.54	7.30	0.06	3.70	0.12	0.83	1.18	0.87	1.15
3	92.07	0.31	1.23	0.13	35.85	0.54	7.23	0.07	3.58	0.12	0.84	1.20	0.89	1.16
4	91.77	0.30	1.38	0.15	36.39	0.54	7.16	0.07	3.47	0.11	0.86	1.22	0.91	1.17
5	91.46	0.31	1.53	0.15	36.92	0.53	7.09	0.07	3.36	0.11	0.88	1.23	—0.93	—1.18
6	91.15	—0.31	1.70	+0.17	37.44	+0.52	7.01	—0.08	3.25	—0.11	0.90	1.24	0.95	1.18
7	90.84	0.31	1.88	0.18	37.96	0.52	6.92	0.09	3.15	0.10	0.91	1.25	0.97	1.19
8	90.52	0.32	2.08	0.20	38.48	0.52	6.82	0.10	3.05	0.10	0.93	1.26	1.00	1.19
9	90.19	0.33	2.29	0.21	39.00	0.52	6.72	0.10	2.95	0.10	0.95	1.27	1.02	1.20
10	89.86	0.33	2.51	0.22	39.51	0.51	6.61	0.11	2.86	0.09	0.97	1.28	—1.04	—1.20
11	89.53	—0.33	2.75	+0.24	40.02	+0.51	6.51	—0.10	2.77	—0.09	1.00	1.28	1.06	1.19
12	89.18	0.35	2.99	0.24	40.52	0.50	6.40	0.11	2.69	0.08	1.02	1.29	1.08	1.19
13	88.83	0.35	3.26	0.27	41.01	0.49	6.28	0.12	2.61	0.08	1.05	1.29	1.09	1.18
14	88.47	0.36	3.54	0.28	41.50	0.49	6.16	0.12	2.54	0.07	1.07	1.29	1.11	1.18
15	88.09	0.38	3.83	0.29	41.99	0.49	6.04	0.12	2.47	0.07	1.09	1.29	—1.12	—1.17
16	87.71	—0.38	4.13	+0.30	42.47	+0.48	5.91	—0.13	2.41	—0.06	1.12	1.29	1.13	1.15
17	87.31	0.40	4.45	0.32	42.94	0.47	5.78	0.13	2.36	0.05	1.14	1.28	1.15	1.13
18	86.90	0.41	4.78	0.33	43.40	0.46	5.64	0.14	2.31	0.05	1.17	1.27	1.16	1.12
19	86.49	0.41	5.13	0.35	43.86	0.46	5.50	0.14	2.27	0.04	1.19	1.26	1.17	1.10
20	86.06	0.43	5.49	0.36	44.31	0.45	5.36	0.14	2.23	0.04	1.21	1.25	—1.18	—1.08
21	85.62	—0.44	5.87	+0.38	44.74	+0.43	5.22	—0.14	2.20	—0.03	1.23	1.23	1.19	1.06
22	85.16	0.46	6.26	0.39	45.16	0.42	5.08	0.14	2.18	0.02	1.25	1.22	1.19	1.03
23	84.69	0.47	6.67	0.41	45.58	0.42	4.93	0.15	2.16	—0.02	1.27	1.20	1.20	1.00
24	84.22	0.47	7.09	0.42	45.98	0.40	4.78	0.15	2.16	0.00	1.29	1.19	1.20	0.99
25	83.72	0.50	7.53	0.44	46.38	0.40	4.63	0.15	2.16	0.00	1.31	1.16	—1.20	—0.97
26	83.22	—0.50	7.98	+0.45	46.77	+0.39	4.48	—0.15	2.16	0.00	1.33	1.14	1.19	0.95
27	82.70	0.52	8.45	0.47	47.14	0.37	4.33	0.15	2.17	+0.01	1.35	1.12	1.19	0.93
28	82.17	0.53	8.94	0.49	47.49	0.35	4.18	0.15	2.19	0.02	1.36	1.10	1.18	0.91
29	81.62	0.55	9.44	0.50	47.83	0.34	4.02	0.16	2.22	0.03	1.37	1.07	1.17	0.90
30	81.05	0.57	9.95	0.51	48.16	0.33	3.87	0.15	2.26	0.04	1.38	1.05	—1.17	—0.89
31	80.48	—0.57	10.48	+0.53	48.47	+0.31	3.72	—0.15	2.30	+0.04	1.38	1.02	1.15	0.88
32	79.89	0.59	11.02	0.54	48.76	0.29	3.58	0.14	2.35	0.05	1.39	1.00	1.13	0.86
33	79.29	0.60	11.58	0.56	49.04	0.28	3.43	0.15	2.41	0.06	1.40	0.97	1.11	0.85
34	78.68	0.61	12.15	0.57	49.30	0.26	3.28	0.15	2.48	0.07	1.40	0.94	1.09	0.84
35	78.05	0.63	12.73	0.58	49.55	0.25	3.14	0.14	2.55	0.07	1.40	0.91	—1.09	—0.83
36	77.41	—0.64	13.33	+0.60	49.77	+0.22	3.01	—0.13	2.64	+0.09	1.40	0.88	1.07	0.83
37	76.76	0.65	13.94	0.61	49.98	0.21	2 87	0.14	2.73	0.09	1.40	0.85	1.05	0.82
38	76.10	0.66	14.57	0.63	50.16	0.18	2.74	0.13	2.83	0.10	1.39	0.82	1.03	0.82
39	75.42	0.68	15.21	0.64	50.32	0.16	2.61	0.13	2.94	0.11	1.38	0.79	0.99	0.81
40	74.73	0.69	15.86	0.65	50.46	0.14	2.49	0.12	3.05	0.11	1.37	0.76	—0.99	—0.81
41	74.04	—0.69	16 52	+0.66	50.58	+0.12	2.38	—0.11	3.17	+0.12	1.35	0.73	0.97	0.81
42	73.33	0.71	17.19	0.67	50.68	0.10	2.27	0.11	3.30	0.13	1.34	0.70	0.95	0.82
43	72.61	0.72	17.87	0.68	50.76	0.08	2.17	0.10	3.44	0.14	1.32	0.67	0.93	0.82
44	71.88	0.73	18.56	0.69	50.81	0.05	2.07	0.10	3.58	0.14	1.30	0.65	0.91	0.83
45	71.14	0.74	19.26	0.70	50.83	+0.02	1.97	0.10	3.72	0.14	1.28	0.62	—0.91	—0.83
46	70.39	—0.75	19.97	+0.71	50.83	0.00	1.88	—0.09	3.87	+0.15	1.25	0.60	0.91	0.84
47	69.63	0.76	20.68	0.71	50.81	—0.02	1.81	0.07	4.03	0.16	1.23	0.58	0.87	0.86
48	68.87	0.76	21.40	0.72	50.75	0.06	1.75	0.06	4.19	0.16	1.20	0.56	0.86	0.87
49	68.10	0.77	22.12	0.72	50.67	0.08	1.69	0.06	4.36	0.17	1.17	0.54	0.85	0.88
50	67.32	0.78	22.85	0.73	50.57	0.10	1.64	0.05	4.53	0.17	1.14	0.52	—0.84	—0.90
51	66.54	—0.78	23.58	+0.73	50.44	—0.13	1.61	—0.03	4.71	+0.18	1.10	0.51	0.84	0.92
52	65.75	0.79	24.32	0.74	50.29	0.15	1.58	0.03	4.89	0.18	1.07	0.49	0.83	0.94
53	64.96	0.79	25.06	0.74	50.11	0.18	1.56	0.02	5.07	0.18	1.03	0.48	0.82	0.96
54	64.16	0.80	25.79	0.73	49.90	0.21	1.55	0.01	5.25	0.18	1.00	0.47	0.81	0.98
55	63.35	0.81	26.53	0.74	49.66	0.24	1.55	0.00	5.43	0.18	0.96	0.47	—0.81	—1.00
56	62.54	—0.81	27.27	+0.74	49.39	—0.27	1.57	+0.02	5.62	+0.19	0.92	0.47	0.81	1.01
57	61.73	0.81	27.99	0.72	49 11	0.28	1.59	0.02	5.81	0.19	0.89	0.47	0.82	1.03
58	60.92	0.81	28.72	0.73	48.79	0.32	1.62	0.03	5.99	0.18	0.85	0.48	0.82	1.05
59	60.11	0.81	29.45	0.73	48.44	0.35	1.66	0.04	6.17	0.18	0.81	0.48	0.83	1.07
60	59.30	0.81	30.17	0.72	48.07	0.37	1.72	0.06	6.36	0.19	0.77	0.49	—0.83	—1.09

TABLE X, ARG. 3.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	(*v.c.*1)	Diff.	(*v.s.*2)	Diff.	(*v.c.*2)	Diff.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
60	59.30	—0.81	30.17	+0.71	48.07	—0.41	1.72	+0.06	6.36	+0.18	0.77	0.49	—0.83	—1.09
61	58.49	0.82	30.88	0.71	47.66	0.42	1.78	0.08	6.54	0.17	0.73	0.50	0.84	1.10
62	57.67	0.81	31.59	0.70	47.24	0.45	1.86	0.09	6.71	0.18	0.69	0.52	0.85	1.11
63	56.86	0.81	32.29	0.68	46.79	0.48	1.95	0.10	6.89	0.17	0.66	0.54	0.87	1.13
64	56.05	0.81	32.97	0.67	46.31	0.50	2.05	0.11	7.06	0.17	0.62	0.56	0.88	1.14
65	55.24	—0.80	33.64	+0.66	45.81	—0.53	2.16	+0.12	7.23	+0.16	0.59	0.59	—0.89	—1.15
66	54.44	0.80	34.30	0.65	45.28	0.55	2.28	0.13	7.39	0.15	0.55	0.62	0.91	1.16
67	53.64	0.80	34.95	0.63	44.73	0.57	2.41	0.14	7.54	0.15	0.53	0.65	0.93	1.16
68	52.84	0.79	35.58	0.63	44.16	0.60	2.55	0.15	7.69	0.14	0.50	0.68	0.95	1.17
69	52.05	0.79	36.21	0.61	43.56	0.63	2.70	0.15	7.83	0.13	0.47	0.71	0.97	1.18
70	51.26	—0.79	36.82	+0.59	42.93	—0.65	2.85	+0.17	7.96	+0.12	0.45	0.75	—0.99	—1.19
71	50.47	0.77	37.41	0.57	42.28	0.66	3.02	0.17	8.08	0.12	0.43	0.78	1.00	1.19
72	49.70	0.77	37.98	0.55	41.62	0.68	3.19	0.18	8.20	0.11	0.41	0.82	1.02	1.19
73	48.93	0.76	38.53	0.53	40.94	0.71	3.37	0.19	8.31	0.09	0.39	0.87	1.04	1.18
74	48.17	0.74	39.06	0.50	40.23	0.72	3.56	0.19	8.40	0.09	0.38	0.91	1.06	1.18
75	47.43	—0.74	39.56	+0.49	39.51	—0.74	3.75	+0.21	8.49	+0.08	0.37	0.96	—1.08	—1.18
76	46.69	0.73	40.05	0.47	38.77	0.76	3.96	0.20	8.57	0.07	0.36	1.00	1.10	1.17
77	45.96	0.72	40.52	0.44	38.01	0.78	4.16	0.21	8.64	0.05	0.36	1.05	1.11	1.16
78	45.24	0.72	40.96	0.42	37.23	0.79	4.37	0.22	8.69	0.04	0.36	1.10	1.12	1.15
79	44.52	0.70	41.38	0.40	36.44	0.81	4.59	0.21	8.73	0.03	0.36	1.14	1.13	1.13
80	43.82	—0.68	41.78	+0.38	35.63	—0.82	4.80	+0.22	8.76	+0.01	0.37	1.18	—1.14	—1.12
81	43.14	0.68	42.16	0.35	34.81	0.83	5.02	0.22	8.77	+0.01	0.38	1.23	1.15	1.10
82	42.46	0.66	42.51	0.32	33.98	0.85	5.24	0.23	8.78	—0.01	0.40	1.28	1.16	1.08
83	41.80	0.65	42.83	0.29	33.13	0.85	5.47	0.22	8.77	0.02	0.41	1.32	1.17	1.06
84	41.15	0.63	43.12	0.26	32.28	0.86	5.69	0.22	8.75	0.03	0.43	1.36	1.17	1.04
85	40.52	—0.62	43.38	+0.23	31.42	—0.87	5.91	+0.22	8.72	—0.04	0.46	1.41	—1.18	—1.02
86	39.90	0.60	43.61	0.21	30.55	0.87	6.13	0.22	8.68	0.06	0.49	1.45	1.18	1.01
87	39.30	0.59	43.82	0.18	29.68	0.88	6.35	0.22	8.62	0.07	0.52	1.49	1.18	0.99
88	38.71	0.57	44.00	0.16	28.80	0.89	6.57	0.22	8.55	0.08	0.55	1.52	1.17	0.97
89	38.14	0.56	44.16	0.12	27.91	0.90	6.79	0.21	8.47	0.09	0.59	1.56	1.16	0.95
90	37.58	—0.54	44.28	+0.10	27.01	—0.89	7.00	+0.21	8.38	—0.11	0.62	1.59	—1.16	—0.94
91	37.04	0.52	44.38	0.06	26.12	0.89	7.21	0.20	8.27	0.12	0.66	1.61	1.15	0.92
92	36.52	0.50	44.44	+0.04	25.23	0.90	7.41	0.19	8.15	0.13	0.71	1.64	1.14	0.91
93	36.02	0.49	44.48	0.00	24.33	0.90	7.60	0.19	8.02	0.13	0.76	1.66	1.13	0.90
94	35.53	0.46	44.48	—0.02	23.43	0.89	7.79	0.18	7.89	0.15	0.80	1.68	1.12	0.89
95	35.07	—0.45	44.46	—0.06	22.54	—0.89	7.97	+0.17	7.74	—0.16	0.85	1.70	—1.11	—0.88
96	34.62	0.42	44.40	0.08	21.65	0.88	8.14	0.16	7.58	0.17	0.90	1.71	1.10	0.87
97	34.20	0.41	44.32	0.12	20.77	0.87	8.30	0.15	7.41	0.17	0.95	1.72	1.08	0.86
98	33.79	0.38	44.20	0.13	19.90	0.87	8.45	0.15	7.24	0.19	1.00	1.72	1.06	0.85
99	33.41	0.37	44.07	0.17	19.03	0.87	8.60	0.13	7.05	0.20	1.05	1.73	1.04	0.84
100	33.04	—0.34	43.90	—0.20	18.16	—0.85	8.73	+0.13	6.85	—0.20	1.10	1.72	—1.03	—0.83
101	32.70	0.32	43.70	0.23	17.31	0.85	8.86	0.11	6.65	0.21	1.15	1.72	1.02	0.83
102	32.38	0.30	43.47	0.26	16.46	0.84	8.97	0.09	6.44	0.21	1.21	1.70	1.00	0.83
103	32.08	0.28	43.21	0.29	15.62	0.82	9.06	0.09	6.23	0.22	1.26	1.69	0.98	0.83
104	31.80	0.26	42.92	0.31	14.80	0.81	9.15	0.08	6.01	0.22	1.31	1.67	0.96	0.83
105	31.54	—0.23	42.61	—0.34	13.99	—0.79	9.23	+0.07	5.79	—0.22	1.35	1.65	—0.95	—0.83
106	31.31	0.20	42.27	0.38	13.20	0.78	9.30	0.05	5.57	0.23	1.40	1.63	0.94	0.84
107	31.11	0.19	41.89	0.40	12.42	0.76	9.35	0.04	5.34	0.24	1.44	1.60	0.92	0.85
108	30.92	0.16	41.49	0.41	11.66	0.74	9.39	0.03	5.10	0.23	1.48	1.56	0.90	0.86
109	30.76	0.14	41.08	0.44	10.92	0.73	9.42	0.01	4.87	0.23	1.53	1.53	0.89	0.87
110	30.62	—0.11	40.64	—0.47	10.19	—0.70	9.43	+0.01	4.64	—0.24	1.56	1.49	—0.88	—0.88
111	30.51	0.09	40.17	0.49	9.49	0.69	9.44	—0.02	4.40	0.23	1.59	1.45	0.87	0.89
112	30.42	0.07	39.68	0.52	8.80	0.66	9.42	0.02	4.17	0.23	1.62	1.41	0.86	0.91
113	30.35	0.05	39.16	0.55	8.14	0.64	9.40	0.04	3.94	0.23	1.65	1.37	0.86	0.93
114	30.30	—0.02	38.61	0.56	7.50	0.62	9.36	0.05	3.71	0.23	1.67	1.32	0.85	0.95
115	30.28	+0.01	38.05	—0.58	6.88	—0.60	9.31	—0.07	3.48	—0.22	1.70	1.27	—0.84	—0.96
116	30.29	0.03	37.47	0.61	6.28	0.57	9.24	0.07	3.26	0.22	1.72	1.22	0.84	0.97
117	30.32	0.05	36.86	0.62	5.71	0.54	9.17	0.09	3.04	0.22	1.73	1.17	0.84	0.99
118	30.37	0.09	36.24	0.64	5.17	0.52	9.08	0.09	2.82	0.21	1.74	1.12	0.84	1.01
119	30.46	0.10	35.60	0.65	4.65	0.49	8.99	0.11	2.61	0.20	1.74	1.07	0.84	1.02
120	30.56		34.95		4.16		8.88		2.41		1.74	1.02	—0.84	—1.04

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TABLE X, ARG. 3.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	(*v.c.*1)	Diff.	(*v.s.*2)	Diff.	(*v.c.*2)	Diff.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
120	30.56	+0.14	34.95	—0.68	4.16	—0.47	8.88	—0.12	2.41	—0.20	1.74	1.02	—0.84	—1.04
121	30.70	0.15	34.27	0.69	3.69	0.43	8.76	0.13	2.21	0.19	1.74	0.97	0.84	1.06
122	30.85	0.18	33.58	0.70	3.26	0.41	8.63	0.14	2.02	0.18	1.73	0.91	0.85	1.08
123	31.03	0.20	32.88	0.73	2.85	0.38	8.49	0.15	1.84	0.17	1.72	0.86	0.86	1.09
124	31.23	0.23	32.15	0.73	2.47	0.34	8.34	0.16	1.67	0.16	1.71	0.81	0.87	1.10
125	31.46	+0.25	31.42	—0.75	2.13	—0.32	8.18	—0.16	1.51	—0.15	1.70	0.77	—0.88	—1.11
126	31.71	0.27	30.67	0.75	1.81	0.29	8.02	0.17	1.36	0.15	1.67	0.72	0.90	1.12
127	31.98	0.30	29.92	0.76	1.52	0.26	7.85	0.18	1.21	0.13	1.65	0.68	0 92	1.13
128	32.28	0.33	29.16	0.77	1.26	0.23	7.67	0.19	1.08	0.12	1.62	0.63	0.93	1.13
129	32.61	0.35	28.39	0.78	1.03	0.20	7.48	0.19	0.96	0.12	1.59	0.59	0.94	1.14
130	32.96	+0.38	27.60	—0.79	0.83	—0.16	7.29	—0.20	0.84	—0.10	1.56	0.55	—0.95	—1.15
131	33.34	0.40	26.82	0.79	0.67	0.13	7 09	0.21	0.74	0.09	1.53	0.51	0.96	1.15
132	33.74	0.42	26.03	0.79	0.54	0.10	6.88	0.21	0.65	0.08	1.49	0.48	0.98	1.15
133	34.16	0.44	25.24	0.80	0.44	0.07	6.67	0.21	0.57	0.06	1.45	0.45	0.99	1.15
134	34.60	0.46	24.44	0.79	0.37	0.04	6.46	0.21	0.51	0.05	1.41	0.42	1.01	1.16
135	35.06	+0.48	23.65	—0.79	0.33	—0.01	6.25	—0.22	0.46	—0.05	1.37	0.40	—1.03	—1.16
136	35.54	0.51	22.86	0.80	0.32	+0.03	6.03	0.22	0.41	0.03	1.32	0.38	1.05	1.15
137	36.05	0.53	22.06	0.79	0.35	0.05	5.81	0.22	0.38	—0.02	1.28	0.37	1.06	1.14
138	36.58	0.55	21.27	0.79	0.40	0.09	5.59	0.21	0.36	0.00	1.23	0.35	1.07	1.13
139	37.13	0.57	20.48	0.79	0.49	0.12	5.38	0.22	0.36	0.00	1.18	0.34	1.08	1.12
140	37.70	+0.60	19.69	—0.79	0.61	+0.15	5.16	—0.21	0.36	+0.02	1.13	0.34	—1.09	—1.11
141	38.30	0.62	18.90	0.77	0.76	0.18	4.95	0.22	0.38	0.03	1.09	0.33	1.09	1.09
142	38.92	0.63	18.13	0.77	0.94	0.21	4.73	0.21	0.41	0.04	1.04	0.34	1.10	1.07
143	39.55	0.66	17.36	0.76	1.15	0.24	4.52	0.21	0.45	0.05	1.00	0.34	1.11	1.06
144	40.21	0.67	16.60	0.75	1.39	0.27	4.31	0.21	0.50	0.07	0.95	0.35	1.12	1.05
145	40.88	+0.68	15.85	—0.73	1.66	+0.29	4.10	—0.20	0.57	+0.08	0.91	0.36	—1.13	—1.04
146	41.56	0.71	15.12	0.73	1.95	0.33	3.90	0.19	0.65	0.08	0.86	0.38	1.13	1.03
147	42.27	0.73	14.39	0.72	2.28	0.35	3.71	0.19	0.73	0.09	0.82	0.40	1.13	1.02
148	43.00	0.75	13.67	0.70	2.63	0.38	3.52	0.19	0.82	0.11	0.78	0.41	1.13	1.01
149	43.75	0.76	12.97	0.69	3.01	0.41	3.33	0.18	0.93	0.11	0.74	0.44	1.14	0.99
150	44.51	+0.79	12.28	—0.68	3.42	+0.44	3.15	—0.17	1.04	+0.12	0.71	0.46	—1.14	—0.98
151	45.30	0.79	11.60	0.66	3.86	0.46	2.98	0.16	1.16	0.13	0.67	0.50	1.14	0.96
152	46.09	0.81	10.94	0.64	4.32	0.48	2.82	0.16	1.29	0.14	0.64	0.53	1.13	0.95
153	46.90	0.83	10.30	0.62	4.80	0.49	2.66	0.15	1.43	0.15	0.62	0.56	1.13	0.94
154	47.73	0.84	9.68	0.60	5.29	0.53	2.51	0.14	1.58	0.16	0.59	0.59	1.12	0.93
155	48 57	+0.85	9.08	—0.59	5.82	+0.54	2.37	—0.12	1.74	+0.16	0.57	0.63	—1.11	—0.92
156	49.42	0.87	8.49	0.57	6.36	0.56	2.25	0.12	1.90	0.17	0.55	0.66	1.10	0.91
157	50.29	0.88	7.92	0.55	6.92	0.59	2.13	0.12	2.07	0.17	0.53	0.70	1.09	0.90
158	51.17	0.90	7.37	0.53	7.51	0.61	2.01	0.10	2.24	0.18	0.52	0.74	1.08	0.89
159	52.07	0.90	6.84	0.50	8.12	0.62	1.91	0.09	2.42	0.18	0.51	0.78	1.06	0.88
160	52.97	+0.92	6.34	—0.49	8.74	+0.65	1.82	—0.08	2.60	+0.18	0.50	0.82	—1.05	—0.87
161	53.89	0.93	5.85	0.46	9.39	0.66	1.74	0.08	2.78	0.19	0.50	0.86	1.04	0.87
162	54.82	0.93	5.39	0.44	10.05	0.67	1.66	0.06	2.97	0.19	0.50	0.90	1.03	0.87
163	55.75	0.95	4.95	0.41	10.72	0.68	1.60	0.05	3.16	0.19	0.50	0.94	1.02	0.86
164	56.70	0.96	4.54	0.40	11.40	0.70	1.55	0.04	3.35	0.20	0.51	0.97	1.00	0.86
165	57.66	+0.97	4.14	—0.37	12.10	+0.70	1.51	—0.03	3.55	+0.20	0.52	1.01	—0.99	—0.86
166	58.63	0.97	3.77	0.35	12.80	0.72	1.48	0.02	3.75	0.19	0.53	1.05	0 97	0.87
167	59.60	0.98	3.42	0.32	13.52	0.73	1.46	—0.01	3.94	0.20	0.54	1.08	0.96	0.87
168	60.58	0.99	3.10	0.30	14.25	0.75	1.45	0.00	4.14	0.20	0.56	1.11	0.95	0.88
169	61.57	0.99	2.80	0.28	15.00	0.74	1.45	+0.01	4.34	0.19	0.57	1.15	0.94	0.88
170	62.56	+1.00	2.52	—0.25	15.74	+0.75	1.46	+0.02	4.53	+0.19	0.59	1.18	—0.93	—0.89
171	63.56	1.00	2.27	0.22	16.49	0.76	1.48	0.03	4.72	0.19	0.62	1.21	0.92	0.91
172	64.56	1.01	2.05	0.21	17.25	0.77	1.51	0.04	4.91	0.18	0.65	1.23	0.91	0.92
173	65.57	1.01	1.84	0.17	18.02	0.76	1 55	0.05	5.09	0.19	0.67	1.25	0.90	0.93
174	66.58	1.02	1.67	0.16	18.78	0.77	1.60	0.06	5.28	0.18	0.70	1.27	0.89	0.94
175	67.60	+1.01	1.51	—0.13	19.55	+0.78	1.66	+0.07	5.46	+0.18	0.72	1.29	—0.89	—0.95
176	68.61	1.02	1.38	0.11	20.33	0.76	1.73	0.08	5.64	0.17	0.75	1.30	0.88	0.96
177	69.63	1.02	1.27	0.08	21.09	0.77	1.81	0.08	5.81	0.16	0.78	1.31	0.88	0.97
178	70.65	1.02	1.19	0.06	21.86	0.77	1.89	0.09	5.97	0.15	0.81	1.32	0.88	0.98
179	71.67	1.02	1.13	0.05	22.63	0.77	1.98	0.10	6.12	0.15	0.84	1.33	0.88	0.99
180	72.69		1.08		23.40		2.08		6.27		0 88	1.34	—0.88	—1.00

TABLE X, Arg. 3.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	(*v.c.*1)	Diff.	(*v.s.*2)	Diff.	(*v.c.*2)	Diff.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
180	72.69	+1.02	1.08	—0.01	23.40	+0.77	2.08	+0.12	6.27	+0.15	0.88	1.34	—0.88	—1.00
181	73.71	1.02	1.07	+0.01	24.17	0.75	2.20	0.11	6.42	0.13	0.91	1.34	0.88	1.02
182	74.73	1.02	1.08	0.03	24.92	0.76	2.31	0.12	6.55	0.13	0.94	1.34	0.88	1.03
183	75.75	1.01	1.11	0.05	25.68	0.75	2.43	0.13	6.68	0.13	0.97	1.34	0.89	1.04
184	76.76	1.01	1.16	0.08	26.43	0.74	2.56	0.13	6.81	0.12	1.00	1.33	0.89	1.05
185	77.77	+1.01	1.24	+0.10	27.17	+0.73	2.69	+0.14	1.93	+0.11	1.03	1.33	—0.90	—1.06
186	78.78	1.00	1.34	0.11	27.90	0.73	2.83	0.13	7.04	0.09	1.06	1.32	0.92	1.06
187	79.78	0.99	1.45	0.14	28.63	0.71	2.96	0.14	7.13	0.09	1.09	1.31	0.93	1.07
188	80.77	0.99	1.59	0.15	29.34	0.71	3.10	0.15	7.22	0.08	1.11	1.29	0.94	1.08
189	81.76	0.99	1.74	0.17	30.05	0.70	3.25	0.15	7.30	0.07	1.13	1.28	0.95	1.09
190	82.75	+0.99	1.91	+0.19	30.75	+0.68	3.40	+0.15	7.37	+0.06	1.16	1.26	—0.96	—1.10
191	83.74	0.97	2.10	0.22	31.43	0.67	3.55	0.16	7.43	0.05	1.18	1.24	0.97	1.10
192	84.71	0.95	2.32	0.23	32.10	0.66	3.71	0.15	7.48	0.05	1.20	1.22	0.98	1.10
193	85.65	0.95	2.55	0.24	32.76	0.65	3.86	0.16	7.53	0.04	1.21	1.19	0.99	1.10
194	86.61	0.95	2.79	0.26	33.41	0.64	4.02	0.15	7.57	0.03	1.23	1.17	1.00	1.10
195	87.56	+0.94	3.05	+0.28	34.05	+0.62	4.17	+0.16	7.60	+0.02	1.24	1.15	—1.01	—1.11
196	88.50	0.92	3.33	0.29	34.67	0.60	4.33	0.16	7.62	+0.01	1.25	1.12	1.01	1.11
197	89.42	0.92	3.62	0.31	35.27	0.59	4.49	0.15	7.63	0.00	1.26	1.09	1.02	1.10
198	90.34	0.90	3.93	0.32	35.86	0.57	4.64	0.15	7.63	—0.01	1.27	1.07	1.03	1.10
199	91.24	0.90	4.25	0.33	36.43	0.56	4.79	0.15	7.62	0.02	1.27	1.04	1.04	1.10
200	92.14	+0.88	4.58	+0.34	36.99	+0.54	4.94	+0.15	7.60	—0.03	1.28	1.02	—1.05	—1.09
201	93.02	0.87	4.92	0.36	37.53	0.52	5.09	0.14	7.57	0.03	1.28	1.00	1.05	1.08
202	93.89	0.85	5.28	0.37	38.05	0.51	5.23	0.14	7.54	0.04	1.28	0.97	1.06	1.07
203	94.74	0.85	5.65	0.38	38.56	0.48	5.37	0.14	7.50	0.04	1.27	0.94	1.07	1.06
204	95.59	0.82	6.03	0.39	39.04	0.47	5.51	0.13	7.46	0.05	1.26	0.92	1.08	1.05
205	96.41	+0.82	6.42	+0.39	39.51	+0.45	5.64	+0.13	7.41	—0.06	1.25	0.89	—1.09	—1.04
206	97.23	0.79	6.81	0.41	39.96	0.44	5.77	0.12	7.35	0.07	1.24	0.87	1.09	1.04
207	98.02	0.79	7.22	0.42	40.40	0.41	5.89	0.12	7.28	0.07	1.23	0.85	1.09	1.03
208	98.81	0.76	7.64	0.42	40.81	0.40	6.01	0.11	7.21	0.08	1.22	0.83	1.09	1.02
209	99.57	0.75	8.06	0.43	41.21	0.37	6.12	0.10	7.13	0.08	1.21	0.81	1.09	1.01
210	100.32	+0.74	8.49	+0.43	41.58	+0.36	6.22	+0.10	7.05	—0.09	1.19	0.79	—1.10	—1.00
211	101.06	0.72	8.92	0.44	41.94	0.34	6.32	0.09	6.96	0.10	1.17	0.77	1.10	0.99
212	101.78	0.70	9.36	0.44	42.28	0.31	6.41	0.09	6.86	0.10	1.15	0.76	1.09	0.98
213	102.48	0.69	9.80	0.44	42.59	0.30	6.50	0.08	6.76	0.10	1.13	0.75	1.09	0.97
214	103.17	0.67	10.24	0.45	42.89	0.29	6.58	0.07	6.66	0.10	1.11	0.73	1.09	0.97
215	103.84	+0.65	10.69	+0.45	43.18	+0.26	6.65	+0.06	6.56	—0.11	1.09	0.72	—1.08	—0.96
216	104.49	0.63	11.14	0.45	43.44	0.24	6.71	0.06	6.45	0.12	1.06	0.71	1.08	0.96
217	105.12	0.61	11.59	0.45	43.68	0.22	6.77	0.05	6.33	0.11	1.04	0.70	1.07	0.95
218	106.73	0.60	12.04	0.46	43.90	0.21	6.82	0.05	6.22	0.11	1.02	0.70	1.06	0 94
219	106.33	0.57	12.50	0.45	44.11	0.18	6.87	0.04	6.11	0.12	1.00	0.70	1.05	0.93
220	106.90	+0.55	12.95	+0.46	44.29	+0.16	6.91	+0.03	5.99	—0.12	0.98	0.70	—1.04	—0.92
221	107.45	0.54	13.41	0.45	44.45	0.15	6.94	0.02	5.87	0.11	0.95	0.70	1.04	0.92
222	107.99	0.51	13.86	0.44	44.60	0.13	6.96	0.02	5.76	0.11	0.92	0.70	1.03	0.92
223	108.50	0.50	14.30	0.45	44.73	0.11	6.98	+0.01	5.65	0.12	0.90	0.70	1.02	0.92
224	109.00	0.48	14.75	0.44	44.84	0.10	6.99	0.00	5.53	0.11	0.88	0.71	1.01	0.92
225	109.48	+0.46	15.19	+0.44	44.94	+0.08	6.99	—0.01	5.42	—0.11	0.86	0.72	—1.00	—0.92
226	109.94	0.43	15.63	0.44	45.02	0.06	6.98	0.01	5.31	0.11	0.84	0.72	1.00	0.92
227	110.37	0.41	16.07	0.43	45.08	0.04	6.97	0.01	5.20	0.11	0.81	0.73	1.00	0.92
228	110.78	0.40	16.50	0.43	45.12	0.02	6.96	0.02	5.09	0.11	0.79	0.74	0.99	0.92
229	111.18	0.37	16.93	0.42	45.14	+0.01	6.94	0.03	4.98	0.10	0.77	0.76	0.98	0.92
230	111.55	+0.34	17.35	+0.41	45.15	0.00	6.91	—0.03	4.88	—0.10	0.76	0.78	—0.97	—0.93
231	111.89	0.34	17.76	0.41	45.15	—0.02	6.88	0.03	4.78	0.10	0.74	0.79	0.97	0.93
232	112.23	0.31	18.17	0.40	45.13	0.04	6.85	0.05	4.68	0.10	0.72	0.81	0.96	0.94
233	112.54	0.29	18.57	0.39	45.09	0.05	6.80	0.05	4.58	0.09	0.71	0.83	0.95	0.95
234	112.83	0.27	18.96	0.38	45.04	0.06	6.75	0.05	4.49	0.09	0.69	0.85	0.94	0.96
235	113.10	+0.25	19.34	+0.38	44.98	—0.07	6.70	—0.06	4.40	—0.09	0.68	0.87	—0.93	—0.97
236	113.35	0.22	19.72	0.37	44.91	0.09	6.64	0.06	4.31	0.08	0.67	0.89	0.93	0.97
237	113.57	0.20	20.09	0.36	44.82	0.11	6.58	0.07	4.23	0.07	0.66	0.91	0.93	0.98
238	113.77	0.18	20.45	0.35	44.71	0.12	6.51	0.06	4.16	0.07	0.65	0.93	0.93	0.99
239	113.95	0.16	20.80	0.35	44.59	0.12	6.45	0.07	4.09	0.07	0.65	0.96	0.93	1.00
240	114.11		21.15		44.47		6.38		4.02		0.65	0.98	—0.93	—1.01

TABLE X, Arg. 3.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	(*v.c.*1)	Diff.	(*v.s.*2)	Diff.	(*v.c.*2)	Diff.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
240	114.11		21.15		44.47		6.38		4.02		0.65	0.98	—0.93	—1.01
241	114.24	+0.13	21.48	+0.33	44.33	—0.14	6.30	—0.08	3.96	—0.06	0.65	1.00	0.93	1.01
242	114.36	0.12	21.80	0.32	44.18	0.15	6.23	0.07	3.90	0.06	0.65	1.03	0.93	1.02
243	114.46	0.10	22.12	0.32	44.02	0.16	6.15	0.08	3.85	0.05	0.65	1.05	0.94	1.03
244	114.54	0.08	22.43	0.31	43.86	0.16	6.07	0.08	3.80	0.05	0.65	1.07	0.95	1.04
245	114.60	0.06	22.73	0.30	43.68	0.18	5.99	0.08	3.75	0.05	0 65	1.10	—0.95	—1.04
246	114.63	+0.03	23.01	+0.28	43.49	—0.19	5.91	—0.08	3.71	—0.04	0.65	1.12	0.96	1.04
247	114.65	+0.02	23.29	0.28	43.30	0.19	5.83	0.08	3.68	0.03	0.66	1.14	0.97	1.04
248	114.64	—0.01	23.56	0.27	43.10	0.20	5.74	0.09	3.65	0.03	0.66	1.16	0.97	1.05
249	114.61	0.03	23.82	0.26	42.88	0.22	5.66	0.08	3.62	0.03	0.67	1.19	0.98	1.06
250	114.55	0.06	24.06	0.24	42.67	0.21	5.58	0.08	3.60	0.02	0.69	1.21	—0.98	—1.06
251	114.48	—0.07	24.30	+0.24	42.45	—0.22	5.49	—0.09	3.59	—0.01	0.70	1.23	0.98	1.06
252	114.39	0.09	24.53	0.23	42.22	0.23	5.41	0.08	3.58	0.01	0.72	1.24	0.99	1.06
253	114.29	0.10	24.75	0.22	41.98	0.24	5.33	0.08	3.57	—0.01	0.73	1.26	1.00	1.07
254	114.16	0.13	24.95	0.20	41.74	0.24	5.25	0.08	3.57	0.00	0.75	1.28	1.01	1.07
255	114.01	0.15	25.15	0.20	41.49	0.25	5.16	0.09	3.57	0.00	0.77	1.30	—1.01	—1.07
256	113.84	—0.17	25.34	+0.19	41.24	—0.25	5.08	—0.08	3.57	0.00	0.78	1.31	1.01	1.07
257	113.66	0.18	25.52	0.18	40.98	0.26	5.01	0.07	3.57	0.00	0.80	1.32	1.01	1.07
258	113.45	0.21	25.70	0.18	40.73	0.25	4.93	0.08	3.58	+0.01	0.82	1.34	1.02	1.06
259	113.22	0.23	25.86	0.16	40.47	0.26	4.85	0.08	3.60	0.02	0.84	1.35	1.03	1.05
260	112.98	0.24	26.02	0.16	40.21	0.26	4.78	0.07	3.62	0.02	0.87	1.36	—1.03	—1.05
261	112.71	—0.27	26.16	+0.14	39.95	—0.26	4.71	—0.07	3.64	+0.02	0.89	1.37	1.03	1.04
262	112.43	0.28	26.30	0.14	39.68	0.27	4.64	0.07	3.66	0.02	0.91	1.38	1.03	1.03
263	112.14	0.29	26.43	0.13	39.41	0.27	4.57	0.07	3.69	0.03	0.93	1.38	1.04	1.02
264	111.84	0.30	26.55	0.12	39.13	0.28	4.51	0.06	3.72	0.03	0.96	1.39	1.05	1.01
265	111.51	0.33	26.66	0.11	38.86	0.27	4.45	0.06	3.75	0.03	0.98	1.39	—1.05	—1.01
266	111.16	—0.35	26.77	+0.11	38.59	—0.27	4.39	—0.06	3.79	+0.04	1.00	1.39	1.05	1.01
267	110.80	0.36	26.87	0.10	38.31	0.28	4.33	0.06	3.83	0.04	1.02	1.39	1.05	1.01
268	110.43	0.37	26.96	0.09	38.03	0.28	4.28	0.05	3.86	0.03	1.05	1.38	1.05	1.00
269	110.04	0.39	27.04	0.08	37.76	0.27	4.22	0.06	3.90	0.04	1.07	1.38	1.05	0.99
270	109.63	0.41	27.12	0.08	37.49	0.27	4.17	0.05	3.94	0.04	1.09	1.38	—1.05	—0.99
271	109.21	—0.42	27.19	+0.07	37.21	—0.28	4.12	—0.05	3.99	+0.05	1.12	1.37	1.05	0.99
272	108.77	0.44	27.26	0.07	36.93	0.28	4.07	0.05	4.03	0.04	1.14	1.37	1.05	0.98
273	108.32	0.45	27.32	0.06	36.65	0.28	4.02	0.05	4.08	0.05	1.16	1.36	1.04	0.98
274	107.86	0.46	27.37	0.05	36.37	0.28	3.98	0.04	4.12	0.04	1.18	1.35	1.04	0.97
275	107.38	0.48	27.42	0.05	36.10	0.27	3.95	0.03	4.17	0.05	1.20	1.34	—1.04	—0.97
276	106.90	—0.48	27.46	+0.04	35.82	—0.28	3.92	—0.03	4.22	+0.05	1.22	1.32	1.03	0.97
277	106.40	0.50	27.50	0.04	35.54	0.28	3.89	0.03	4.27	0.05	1.24	1.31	1.02	0.97
278	105.89	0.51	27.53	0.03	35.27	0.27	3.86	0.03	4.32	0.05	1.26	1.30	1.01	0.97
279	105.37	0.52	27.56	0.03	35.00	0.27	3.83	0.03	4.37	0.05	1.28	1.28	1.00	0.96
280	104.84	0.53	27.59	0.03	34.72	0.28	3.80	0.03	4.42	0.05	1.29	1.26	—1.00	—0.96
281	104.30	—0.54	27.61	+0.02	34.44	—0.28	3.77	—0.03	4.47	+0.05	1.31	1.24	1.00	0.96
282	103.75	0.55	27.63	0.02	34.17	0.27	3.75	0.02	4.53	0.06	1.32	1.22	1.00	0.96
283	103.19	0.56	27.64	0.01	33.90	0.27	3.74	0.01	4.58	0.05	1.34	1.20	0.99	0.96
284	102.62	0.57	27.65	+0.01	33.63	0.27	3.72	0.02	4.64	0.06	1.35	1.18	0.98	0.95
285	102.04	0.58	27.65	0.00	33.36	0.27	3.71	0.01	4.69	0.05	1.36	1.16	—0.98	—0.95
286	101.46	—0.58	27.65	0.00	33.09	—0.27	3.70	—0.01	4.74	+0.05	1.37	1.14	0.98	0.95
287	100.87	0.59	27.65	0.00	32.82	0.27	3.69	0.01	4.79	0.05	1.38	1.12	0.98	0.96
288	100.27	0.60	27.64	—0.01	32.55	0.27	3.68	0.01	4.85	0.06	1.38	1.09	0.97	0.97
289	99.67	0.60	27.63	0.01	32.29	0.26	3.67	—0.01	4.90	0.05	1.39	1.07	0.96	0.97
290	99.06	0.61	27.62	0.01	32.02	0.27	3.67	0.00	4.95	0.05	1.39	1.05	—0.96	—0.98
291	98.45	—0.61	27.60	—0.02	31.76	—0.26	3.67	0.00	5.00	+0.05	1.40	1.02	0.96	0.98
292	97.83	0.62	27.58	0.02	31.50	0.26	3.67	0.00	5.06	0.06	1.40	1.00	0.96	0.99
293	97.21	0.62	27.56	0.02	31.23	0.27	3.67	0.00	5.11	0.05	1.40	0.98	0.95	1.00
294	96.58	0.63	27.54	0.02	30.97	0.26	3.68	+0.01	5.16	0.05	1.40	0.95	0.95	1.01
295	95.95	0.63	27.52	0.02	30.71	0.26	3.69	0.01	5.21	0.05	1.39	0.93	—0.95	—1.02
296	95.31	—0.64	27.48	—0.04	30.45	—0.26	3.70	+0.01	5.26	+0.05	1.39	0.91	0.95	1.02
297	94.67	0.64	27.45	0.03	30.19	0.26	3.71	0.01	5.32	0.06	1.38	0.88	0.96	1.02
298	94.04	0.63	27.42	0.03	29.93	0.26	3.72	0.01	5.37	0.05	1.38	0.86	0.96	1.03
299	93.40	0.64	27.38	0.04	29.67	0.26	3.74	0.02	5.42	0.05	1.37	0.84	0.97	1.03
300	92.77	0.63	27.34	0.04	29.40	0.27	3.76	0.02	5.47	0.05	1.35	0.82	—0.97	—1.03

TABLE X, Arg. 3.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	(*v.c.*1)	Diff.	(*v.s.*2)	Diff.	(*v.c.*2)	Diff.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
300	92.77	—0.64	27.34	—0.04	29.40	—0.24	3.76	+0.01	5.47	+0.04	1.35	0.82	—0.97	—1.03
301	92.13	0.63	27.30	0.04	29.16	0.27	3.77	0.03	5.51	0.05	1.34	0.80	0.97	1.03
302	91.50	0.63	27.26	0.05	28.89	0.26	3.80	0.02	5.56	0.05	1.33	0.78	0.98	1.03
303	90.87	0.63	27.21	0.05	28.63	0.26	3.82	0.03	5.61	0.05	1.31	0.76	0.98	1.04
304	90.24	0.64	27.16	0.06	28.37	0.26	3.85	0.02	5.66	0.05	1.30	0.74	0.99	1.04
305	89.60	—0.63	27.10	—0.05	28.11	—0.25	3.87	+0.04	5.71	+0.04	1.28	0.72	—0.99	—1.04
306	88.97	0.62	27.05	0.06	27.86	0.26	3.91	0.03	5.75	0.05	1.27	0.71	1.00	1.04
307	88.35	0.62	26.99	0.06	27.60	0.25	3.94	0.03	5.80	0.04	1.25	0.69	1.00	1.04
308	87.72	0.62	26.93	0.07	27.35	0.25	3.97	0.04	5.84	0.04	1.23	0.68	1.01	1.04
309	87.10	0.61	26.86	0.07	27.10	0.25	4.01	0.04	5.88	0.04	1.21	0.66	1.01	1.04
310	86.49	—0.60	26.79	—0.07	26.85	—0.26	4.05	+0.04	5.92	+0.04	1.19	0.65	—1.02	—1.04
311	85.89	0.60	26.72	0.07	26.59	0.25	4.09	0.05	5.96	0.04	1.17	0.64	1.02	1.04
312	85.29	0.60	26.65	0.07	26.34	0.25	4.14	0.04	6.00	0.03	1.15	0.63	1.03	1.03
313	84.69	0.60	26.58	0.09	26.09	0.25	4.18	0.04	6.03	0.04	1.12	0.62	1.03	1.03
314	84.09	0.58	26.49	0.09	25.84	0.25	4.22	0.05	6.07	0.03	1.10	0.62	1.04	1.03
315	83.51	—0.58	26.40	—0.09	25.59	—0.25	4.27	+0.05	6.10	+0.04	1.08	0.61	—1.04	—1.03
316	82.93	0.56	26.31	0.09	25.34	0.24	4.32	0.05	6.14	0.03	1.05	0.61	1.04	1.02
317	82.37	0.56	26.22	0.10	25.10	0.25	4.37	0.06	6.17	0.02	1.03	0.61	1.04	1.02
318	81.81	0.54	26.12	0.10	24.85	0.25	4.43	0.05	6.19	0.03	1.01	0.61	1.04	1.01
319	81.27	0.54	26.02	0.11	24.60	0.25	4.48	0.06	6.22	0.03	0.98	0.61	1.04	1.00
320	80.73	—0.53	25.91	—0.11	24.35	—0.24	4.54	+0.06	6.25	+0.02	0.96	0.61	—1.04	—1.00
321	80.20	0.52	25.80	0.12	24.11	0.24	4.60	0.06	6.27	0.02	0.94	0.61	1.04	0.99
322	79.68	0.51	25.68	0.12	23.87	0.24	4.66	0.06	6.29	0.01	0.91	0.62	1.03	0.99
323	79.17	0.49	25.56	0.12	23.63	0.24	4.72	0.06	6.30	0.02	0 89	0.62	1.03	0.98
324	78.68	0.49	25.44	0.13	23.39	0.23	4.78	0.07	6.32	0.02	0.87	0.63	1.03	0.98
325	78.19	—0.47	25.31	—0.14	23.16	—0.24	4.85	+0.07	6.34	+0.01	0.85	0.64	—1.03	—0.97
326	77.72	0.46	25.17	0.15	22.92	0.23	4.92	0.07	6.35	0.00	0.83	0.65	1.03	0.97
327	77.26	0.44	25.02	0.15	22.69	0.23	4.99	0.07	6.35	0.00	0.81	0.66	1.02	0.96
328	76.82	0.43	24.87	0.15	22.46	0.23	5.06	0.07	6.35	0.00	0.79	0.67	1.02	0.96
329	76.39	0.42	24.72	0.16	22.23	0.23	5.13	0.07	6.35	0.00	0.77	0.69	1.01	0.95
330	75.97	—0.40	24.56	—0.17	22.00	—0.22	5.20	+0.07	6.35	0.00	0.75	0.70	—1.01	—0.95
331	75.57	0.39	24.39	0.18	21.78	0.21	5.27	0.07	6.35	—0.01	0.73	0.72	1.00	0.95
332	75.18	0.37	24.21	0.18	21.57	0.22	5.34	0.07	6.34	0.01	0.72	0.74	1.00	0.95
333	74.81	0.36	24.03	0.19	21.35	0.21	5.41	0.07	6.33	0.01	0.70	0.76	0.99	0.95
334	74.45	0.34	23.84	0.19	21.14	0.20	5.48	0.08	6.32	0.02	0.69	0.78	0.99	0.95
335	74.11	—0.32	23.65	—0.20	20.94	—0.21	5.56	+0.07	6.30	—0.02	0.68	0.80	—0.99	—0.95
336	73.79	0.31	23.45	0.21	20.73	0.20	5.63	0.07	6.28	0.03	0.67	0.82	0.98	0.95
337	73.48	0.30	23.24	0.22	20.53	0.19	5.70	0.08	6.25	0.03	0.66	0.84	0.97	0.95
338	73.18	0.27	23.02	0.22	20.34	0.19	5.78	0.07	6.22	0.03	0.65	0.86	0.96	0.95
339	72.91	0.25	22.80	0.23	20.15	0.19	5.85	0.07	6.19	0.04	0.64	0.88	0.95	0.95
340	72.66	—0.24	22.57	—0.24	19.96	—0.17	5.92	+0.06	6.15	—0.04	0.64	0.91	—0.95	—0.95
341	72.42	0.22	22.33	0.24	19.79	0.17	5.98	0.07	6.11	0.04	0.63	0.93	0.95	0.96
342	72.20	0.20	22.09	0.25	19.62	0.17	6.05	0.07	6.07	0.05	0.63	0.95	0.94	0.96
343	72.00	0 18	21.84	0.26	19.45	0.15	6.12	0.07	6.02	0.05	0.63	0.98	0.94	0.96
344	71.82	0.16	21.58	0.27	19.30	0.15	6.19	0.06	5.97	0.06	0.63	1.00	0.93	0.97
345	71.66	—0.14	21.31	—0.27	19.15	—0.15	6 25	+0.06	5.91	—0.05	0.63	1.02	—0.93	—0.97
346	71.52	0.13	21.04	0.29	19.00	0.13	6.31	0.06	5.86	0.06	0.64	1.05	0.93	0.98
347	71.39	0.10	20.75	0.29	18.87	0.13	6.37	0.06	5.80	0.07	0.64	1.07	0.94	0.99
348	71.29	0.08	20.46	0.30	18.74	0.12	6.43	0.05	5.73	0.07	0.65	1.09	0.94	1.00
349	71.21	0.06	20.16	0.31	18.62	0.10	6.48	0.05	5.66	0.07	0.66	1.12	0.94	1.01
350	71.15	—0.05	19.85	—0.31	18.52	—0.09	6.53	+0.05	5.59	—0.08	0.67	1.14	—0.94	—1.02
351	71.10	—0.02	19.54	0.32	18.43	0.09	6.58	0.04	5.51	0.08	0.68	1.16	0.94	1.02
352	71.08	0.00	19.22	0.32	18.34	0.08	6.62	0.04	5.43	0.08	0.69	1.18	0.95	1.03
353	71.08	+0.02	18.90	0.34	18.26	0.07	6.66	0.04	5.35	0.08	0.71	1.20	0.95	1.04
354	71.10	0.05	18.56	0.34	18.19	0.05	6.70	0.03	5.27	0.09	0.72	1.22	0.95	1.05
355	71.15	+0.06	18.22	—0.34	18.14	—0.04	6.73	+0.03	5.18	—0.09	0.74	1.23	—0.95	—1.06
356	71.21	0.08	17.88	0.35	18.10	0.03	6.76	0.03	5.09	0.09	0.76	1.25	0.96	1.06
357	71.29	0.11	17.53	0.36	18.07	—0.02	6.79	0.02	5.00	0.09	0.78	1.26	0.97	1.06
358	71.40	0.12	17.17	0.37	18.05	0.00	6.81	0.02	4.91	0.10	0.80	1.27	0.98	1.07
359	71.52	0.15	16.80	0.37	18.05	+0.01	6.83	0.01	4.81	0.10	0.82	1.28	0.99	1.07
360	71.67		16.43		18.06		6.84		4.71		0.84	1.29	—0.99	—1.07

TABLE X, Arg. 3.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	(*v.c.*1)	Diff.	(*v.s.*2)	Diff.	(*v.c.*2)	Diff.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
360	71.67	+0.17	16.43	—0.38	18.06	+0.02	6.84	0.00	4.71	—0.10	0.84	1.29	—0.99	—1.07
361	71.84	0.18	16.05	0.38	18.08	0.04	6.84	0.00	4.61	0.10	0.87	1.30	1.00	1.07
362	72.02	0.22	15.67	0.39	18.12	0.05	6.84	0.00	4.51	0.10	0.89	1.31	1.01	1.07
363	72.24	0.24	15.28	0.39	18.17	0.07	6.84	—0.01	4.41	0.10	0.91	1.31	1.02	1.07
364	72.48	0.25	14.89	0.39	18.24	0.08	6.83	0.02	4.31	0.10	0.94	1.32	1.03	1.07
365	72.73	+0.27	14.50	—0.39	18.32	+0.10	6.81	—0.02	4.21	—0.10	0.96	1.32	—1.03	—1.07
366	73.00	0.30	14.11	0.40	18.42	0.11	6.79	0.03	4.11	0.10	0.98	1.32	1.03	1.06
367	73.30	0.32	13.71	0.40	13.53	0.13	6.76	0.03	4.01	0.11	1.01	1.32	1.04	1.06
368	73.62	0.34	13.31	0.40	18.66	0.15	6.73	0.05	3.90	0.10	1.03	1.31	1.05	1.05
369	73.96	0.37	12.91	0.40	18.81	0.16	6.68	0.05	3.80	0.10	1.06	1.31	1.06	1.05
370	74.33	+0.38	12.51	—0.41	18.97	+0.18	6.63	—0.05	3.70	—0.10	1.08	1.30	—1.07	—1.05
371	74.71	0.41	12.10	0.40	19.15	0.20	6.58	0.06	3.60	0.09	1.10	1.29	1.07	1.05
372	75.12	0.42	11.70	0.40	19.35	0.22	6.52	0.06	3.51	0.10	1.12	1.28	1.07	1.04
373	75.54	0.45	11.30	0.40	19.57	0.23	6.46	0.07	3.41	0.09	1.14	1.27	1.08	1.04
374	75.99	0.46	10.90	0.40	19.80	0.24	6.39	0.07	3.32	0.09	1.16	1.25	1.08	1.03
375	76.45	+0.49	10.50	—0.40	20.04	+0.26	6.32	—0.08	3.23	—0.08	1.18	1.23	—1.08	—1.02
376	76.94	0.50	10.10	0.39	20.30	0.28	6.24	0.09	3.15	0.08	1.20	1.22	1.08	1.00
377	77.44	0.52	9.71	0.39	20.58	0.31	6.15	0.09	3.07	0.08	1.22	1.20	1.08	0.99
378	77.96	0.55	9.32	0.39	20.89	0.32	6.06	0.10	2.99	0.07	1.23	1.19	1.08	0.98
379	78.51	0.57	8.93	0.39	21.21	0.34	5.96	0.11	2.92	0.07	1.25	1.16	1.08	0.97
380	79.08	+0.58	8.54	—0.38	21.55	+0.36	5.85	—0.11	2.85	—0.07	1.26	1.13	—1.08	—0.96
381	79.66	0.61	8.16	0.37	21.91	0.37	5.74	0.11	2.78	0.06	1.27	1.11	1.07	0.96
382	80.27	0.63	7.79	0.36	22.28	0.39	5.63	0.12	2.72	0.05	1.28	1.09	1.07	0.95
383	80.90	0.64	7.43	0.36	22.67	0.41	5.51	0.12	2.67	0.05	1.28	1.06	1.07	0.94
384	81.54	0.65	7.07	0.34	23.08	0.43	5.39	0.12	2.62	0.04	1.29	1.04	1.06	0.93
385	82.19	+0.68	6.73	—0.34	23.51	+0.44	5.27	—0.13	2.58	—0.03	1.29	1.01	—1.06	—0.92
386	82.87	0.69	6.39	0.33	23.95	0.46	5.14	0.13	2.55	0.02	1.29	0.98	1.06	0.92
387	83.56	0.71	6.06	0.32	24.41	0.48	5.01	0.14	2.53	0.02	1.29	0.96	1.05	0.92
388	84.27	0.74	5.74	0.32	24.89	0.49	4.87	0.14	2.51	—0.02	1.28	0.93	1.04	0.91
389	85.01	0.74	5.42	0.30	25.38	0.51	4.73	0.14	2.49	0.00	1.28	0.91	1.03	0.91
390	85.75	+0.76	5.12	—0.28	25.89	+0.53	4.59	—0.14	2.49	0.00	1.27	0.88	—1.02	—0.90
391	86.51	0.78	4.84	0.28	26.42	0.54	4.45	0.14	2.49	+0.01	1.26	0.86	1.01	0.90
392	87.29	0.79	4.56	0.26	26.96	0.55	4.31	0.15	2.50	0.02	1.25	0.83	1.00	0.90
393	88 08	0.81	4.30	0.24	27.51	0.57	4.16	0.14	2.52	0.03	1.23	0.81	0.99	0.90
394	88.89	0.82	4.06	0.23	28.08	0.59	4.02	0.15	2.55	0.03	1.22	0.78	0.98	0.90
395	89.71	+0.83	3.83	—0.22	28.67	+0.60	3.87	—0.14	2.58	+0.05	1.20	0.76	—0.97	—0.90
396	90.54	0.85	3.61	0.20	29.27	0.61	3.73	0.15	2.63	0.05	1.17	0.74	0.95	0.91
397	91.39	0.86	3.41	0.19	29.88	0.62	3.58	0.14	2.68	0.06	1.15	0.72	0.94	0.91
398	92.25	0.88	3.22	0.16	30.50	0.64	3.44	0.14	2.74	0.07	1.13	0.70	0.93	0.92
399	93.13	0.89	3.06	0.15	31.14	0.65	3.30	0.14	2.81	0.07	1.10	0.69	0.92	0.92
400	94.02	+0.90	2.91	—0.14	31.79	+0.66	3.16	—0.14	2.88	+0.08	1.08	0.68	—0.91	—0.93
401	94.92	0.91	2.77	0.12	32.45	0.66	3.02	0.13	2.96	0.10	1.05	0.67	0.91	0.93
402	95.83	0.92	2.65	0.09	33.11	0.68	2.89	0.13	3.06	0.10	1.02	0.66	0.90	0.94
403	96.75	0.93	2.56	0.08	33.79	0.69	2.76	0.12	3.16	0.11	0.99	0.65	0.90	0.95
404	97.68	0.94	2 48	0.05	34.48	0.69	2.64	0.12	3.27	0.12	0.96	0.65	0.89	0.96
405	98.62	+0.95	2.43	—0.04	35.17	+0.70	2.52	—0.11	3.39	+0.12	0.93	0.64	—0.89	—0.97
406	99.57	0.96	2.39	—0.02	35.87	0.71	2.41	0.10	3.51	0.13	0.90	0.64	0.89	0.99
407	100.53	0.97	2.37	0.00	36.58	0.72	2.31	0.11	3.64	0.14	0.87	0.65	0.89	1.01
408	101.50	0.97	2.37	+0.02	37.30	0.72	2.20	0.10	3.78	0.15	0.84	0.65	0.90	1.02
409	102.47	0.99	2.39	0.05	38.02	0.72	2.10	0.09	3.93	0.15	6.80	0.66	0.90	1.03
410	103.46	+0.99	2.44	+0.08	38.74	+0.73	2.01	—0.08	4.08	+0.15	0.77	0.67	—0.90	—1.04
411	104.45	0.99	2.52	0.09	39.47	0.72	1.93	0.07	4.23	0.16	0.74	0.69	0.91	1.05
412	105.44	1.00	2.61	0.11	40.19	0.73	1.86	0.07	4.39	0.17	0.72	0.70	0.91	1.07
413	106.44	1.00	2.72	0.13	40.92	0.74	1.79	0.06	4.56	0.17	0.69	0.72	0.92	1.08
414	107.44	1.00	2.85	0.15	41.66	0.73	1.73	0.04	4.73	0.18	0.66	0.74	0.92	1.09
415	108.44	+1.01	3.00	+0.18	42.39	+0.73	1.69	—0.04	4.91	+0.18	0.63	0.77	—0.92	—1.10
416	109.45	1.01	3.18	0.20	43.12	0.72	1.65	0.03	5.09	0.18	0.61	0.79	0.93	1.10
417	110.46	1.01	3.38	0.22	43.84	0.73	1.62	0.02	5.27	0.19	0.59	0.82	0.94	1.10
418	111.47	1.02	3.60	0.25	44.57	0.72	1.60	0.01	5.46	0.19	0.56	0.85	0.96	1.11
419	112.49	1.02	3.85	0.27	45.29	0.72	1.59	0.00	5.65	0.19	0.55	0.88	0.97	1.12
420	113.51		4.12		46.01		1.59		5.84		0.53	0.92	—0.98	—1.12

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TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
420	113.51		4.12		46.01		1.59		5.84		0.53	0.92	—0.98	—1.12
421	114.53	+1.02	4.42	+0.30	46.71	+0.70	1.60	+0.01	6.04	+0.20	0.52	0.95	0.99	1.12
422	115.55	1.02	4.73	0.31	47.41	0.70	1.63	0.03	6.23	0.19	0.51	0.99	1.01	1.11
423	116.57	1.02	5.07	0.34	48.10	0.69	1.65	0.02	6.42	0.19	0.50	1.02	1.02	1.11
424	117.59	1.02	5.42	0.35	48.79	0.69	1.69	0.04	6.61	0.19	0.49	1.06	1.03	1.11
425	118.59	1.00	5.80	0.38	49.48	0.69	1.75	0.06	6.80	0.19	0.49	1.10	—1.04	—1.11
426	119.59	+1.00	6.21	+0.41	50.15	+0.67	1.81	+0.06	6.98	+0.18	0.49	1.14	1.06	1.10
427	120.59	1.00	6.63	0.42	50.80	0.65	1.88	0.07	7.17	0.19	0.49	1.18	1.08	1.10
428	121.60	1.01	7.07	0.44	51.44	0.64	1.96	0.08	7.35	0.18	0.50	1.22	1.09	1.10
429	122.59	0.99	7.54	0.47	52.06	0.62	2.06	0.10	7.52	0.17	0.51	1.26	1.10	1.09
430	123.58	0.99	8.03	0.49	52.67	0.61	2.16	0.10	7.70	0.18	0.53	1.30	—1.11	—1.09
431	124.57	+0.99	8.54	+0.51	53.27	+0.60	2.26	+0.10	7.87	+0.17	0.54	1.34	1.11	1.08
432	125.55	0.98	9.08	0.54	53.85	0.58	2.38	0.12	8.03	0.16	0.56	1.38	1.11	1.07
433	126.52	0.97	9.62	0.54	54.41	0.56	2.52	0.14	8.19	0.16	0.58	1.41	1.12	1.06
434	127.48	0.96	10.18	0.56	54.95	0.54	2.66	0.14	8.34	0.15	0.61	1.45	1.12	1.05
435	128.44	0.96	10.76	0.58	55.48	0.53	2.81	0.15	8.48	0.14	0.64	1.48	—1.13	—1.04
436	129.38	+0.94	11.37	+0.61	55.99	+0.51	2.96	+0.15	8.62	+0.14	0.66	1.51	1.13	1.02
437	130.31	0.93	11.99	0.62	56.48	0.49	3.13	0.17	8.74	0.12	0.70	1.54	1.13	1.00
438	131.24	0.93	12.62	0.63	56.94	0.46	3.30	0.17	8.86	0.12	0.73	1.57	1.13	0.99
439	132.15	0.91	13.26	0.64	57.38	0.44	3.47	0.17	8.97	0.11	0.77	1.59	1.13	0.98
440	133.06	0.91	13.93	0.67	57.80	0.42	3.65	0.18	9.07	0.10	0.82	1.61	—1.13	—0.97
441	133.95	+0.89	14.61	+0.68	58.19	+0.39	3.84	+0.19	9.16	+0.09	0.86	1.63	1.12	0.95
442	134.83	0.88	15.31	0.70	58.56	0.37	4.04	0.20	9.24	0.08	0.90	1.64	1.12	0.93
443	135.70	0.87	16.01	0.70	58.90	0.34	4.24	0.20	9.31	0.07	0.95	1.65	1.12	0.92
444	136.55	0.85	16.73	0.72	59.22	0.32	4.44	0.20	9.37	0.06	0.99	1.66	1.11	0.91
445	137.39	0.84	17.45	0.72	59.51	0.29	4.65	0.21	9.42	0.05	1.04	1.67	—1.11	—0.90
446	138.21	+0.82	18.19	+0.74	59.78	+0.27	4.86	+0.21	9.45	+0.03	1.09	1.67	1.09	0.90
447	139.02	0.81	18.93	0.74	60.02	0.24	5.07	0.21	9.48	0.03	1.13	1.67	1.07	0.89
448	139.81	0.79	19.68	0.75	60.22	0.20	5.28	0.21	9.49	+0.01	1.18	1.66	1.06	0.88
449	140.59	0.78	20.45	0.77	60.40	0.18	5.49	0.21	9.49	0.00	1.23	1.65	1.05	0.87
450	141.36	0.77	21.22	0.77	60.55	0.15	5.71	0.22	9.48	—0.01	1.28	1.64	—1.04	—0.86
451	142.11	+0.75	21.99	+0.77	60.67	+0.12	5.92	+0.21	9.45	—0.03	1.32	1.62	1.02	0.86
452	142.83	0.72	22.77	0.78	60.76	0.09	6.13	0.21	9.42	0.03	1.37	1.60	1.00	0.86
453	143.54	0.71	23.55	0.78	60.82	0.06	6.34	0.21	9.38	0.04	1.41	1.58	0.99	0.86
454	144.23	0.69	24.33	0.78	60.85	+0.03	6.55	0.21	9.32	0.06	1.45	1.55	0.98	0.86
455	144.91	0.68	25.11	0.78	60.85	0.00	6.75	0.20	9.25	0.07	1.49	1.52	—0.97	—0.86
456	145.56	+0.65	25.89	+0.78	60.82	—0.03	6.96	+0.21	9.17	—0.08	1.53	1.49	0.95	0.87
457	146.19	0.63	26.67	0.78	60.76	0.06	7.15	0.19	9.07	0.10	1.57	1.45	0.93	0.87
458	146.81	0.62	27.45	0.78	60.66	0.10	7.35	0.20	8.97	0.10	1.60	1.41	0 91	0.88
459	147.40	0.59	28.22	0.77	60.54	0.12	7.53	0.18	8.86	0.11	1.63	1.37	0.90	0.88
460	147.98	0.58	28.99	0.77	60.38	0.16	7.70	0.17	8.74	0.12	1.66	1.33	—0.89	—0.89
461	148.54	+0.56	29.75	+0.76	60.20	—0.18	7.87	+0.17	8.60	—0.14	1.68	1.29	0.88	0.90
462	149.07	0.53	30.51	0.76	59.98	0.22	8.04	0.17	8.46	0.14	1.70	1.24	0.87	0.91
463	149.59	0.52	31.26	0.75	59.73	0.25	8.20	0.16	8.31	0.15	1.72	1.19	0.86	0.92
464	150.08	0.49	32.00	0.74	59.45	0.28	8.35	0.15	8.14	0.17	1.73	1.14	0.85	0.93
465	150.54	0.46	32.73	0.73	59.14	0.31	8.49	0.14	7.97	0.17	1.74	1.09	—0.85	—0.94
466	150.98	+0.44	33.45	+0.72	58.81	—0.33	8.62	+0.13	7.79	—0.18	1.75	1.03	0.85	0.96
467	151.40	0.42	34.16	0.71	58.44	0.37	8.74	0.12	7.61	0.18	1.75	0.98	0.85	0.98
468	151.80	0.40	34.85	0.69	58.05	0.39	8.85	0.11	7.42	0.19	1.75	0.93	0.85	1.00
469	152.18	0.38	35.52	0.67	57.62	0.43	8.95	0.10	7.22	0.20	1.75	0.87	0.85	1.02
470	152.53	0.35	36.18	0.66	57.17	0.45	9.03	0.08	7.01	0.21	1.73	0.82	—0.85	—1.03
471	152.87	+0.34	36.83	+0.65	56.69	—0.48	9.11	+0.08	6.81	—0.20	1.72	0.77	0.86	1.05
472	153.18	0.31	37.46	0.63	56.18	0.51	9.17	0.06	6.60	0.21	1.70	0.72	0.86	1.07
473	153.46	0.28	38.07	0.61	55.64	0.54	9.22	0.05	6.38	0.22	1.68	0.67	0.87	1.09
474	153.72	0.26	38.67	0.60	55.09	0.55	9.27	0.05	6.16	0.22	1.66	0.62	0.87	1.10
475	153.95	0.23	39.24	0.57	54.51	0.58	9.30	0.03	5.93	0.23	1.63	0.58	—0.88	—1.11
476	154.16	+0.21	39.79	+0.55	53.91	—0.60	9.31	+0.01	5.71	—0.22	1.60	0.54	0.90	1.12
477	153.34	0.18	40.32	0.53	53.28	0.63	9.32	+0.01	5.48	0.23	1.56	0.50	0.91	1.13
478	154.51	0.17	40.82	0.50	52.62	0.66	9.31	—0.01	5.25	0.23	1.53	0.46	0.92	1.14
479	154.66	0.15	41.31	0.49	51.94	0.68	9.29	0.02	5.03	0.22	1.48	0.42	0.93	1.15
480	154.77	0.11	41.77	0.46	51.25	0.69	9.25	0.04	4.82	0.21	1.44	0.39	—0.94	—1.16

TABLE X, Arg. 3.—*Continued.*

Arg.	(*v.c.*0)	Diff.	(*v.s.*1)	Diff.	(*v.c.*1)	Diff.	(*v.s.*2)	Diff.	(*v.c.*2)	Diff.	(*v.s.*3)	(*v.c.*3)	(*v.s.*4)	(*v.c.*4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	— ″
480	154.77		41.77		51.25		9.25		4.82		1.44	0.39	0.94	—1.16
481	154.86	+0.09	42.20	+0.43	50.53	—0.72	9.20	—0.05	4.60	—0.22	1.39	0.36	0.96	1.16
482	154.93	0.07	42.61	0.41	49.80	0.73	9.15	0.05	4.39	0.21	1.35	0.34	0.98	1.16
483	154.98	0.05	42.99	0.38	49.04	0.76	9.08	0.07	4.17	0.22	1.30	0.31	1.00	1.16
484	154.99	+0.01	43.34	0.35	48.28	0.76	9.00	0.08	3.96	0.21	1.25	0.30	1.01	1.16
485	154.98	—0.01	43.67	0.33	47.49	0.79	8.90	0.10	3.75	0.21	1.20	0.28	—1.02	—1.16
486	154.95	—0.03	43.98	+0.31	46.69	—0.80	8.80	—0.10	3.54	—0.21	1.15	0.27	1.04	1.15
487	154.90	0.05	44.25	0.27	45.88	0.81	8.69	0.11	3.34	0.20	1.10	0.26	1.06	1.15
488	154.82	0.08	44.49	0.24	45.05	0.83	8.57	0.12	3.15	0.19	1.04	0.26	1.08	1.15
489	154.72	0.10	44.71	0.22	44.21	0.84	8.44	0.13	2.98	0.17	0.99	0.26	1.09	1.14
490	154.59	0.13	44.90	0.19	43.37	0.84	8.30	0.14	2.81	0.17	0.94	0.27	—1.10	—1.14
491	154.45	—0.14	45.05	+0.15	42.51	—0.86	8.14	—0.16	2.64	—0.17	0.88	0.28	1.11	1.13
492	154.28	0.17	45.18	0.13	41.64	0.87	7.98	0.16	2.49	0.15	0.83	0.29	1.12	1.12
493	154.08	0.20	45.28	0.10	40.77	0.87	7.81	0.17	2.34	0.15	0.78	0.30	1.13	1.11
494	153.87	0.21	45.35	0.07	39.90	0.87	7.64	0.17	2.20	0.14	0.74	0.32	1.14	1.10
495	153.63	0.24	45.38	0.03	39.01	0.89	7.45	0.19	2.07	0.13	0.69	0.35	—1.15	—1.09
496	153.37	—0.26	45.39	+0.01	38.12	—0.89	7.26	—0.19	1.95	—0.12	0.65	0.38	1.15	1.07
497	153.09	0.28	45.37	—0.02	37.24	0.88	7.06	0.20	1.84	0.11	0.60	0.41	1.15	1.05
498	152.79	0.30	45.32	0.05	36.35	0.89	6.86	0.20	1.74	0.10	0.56	0.44	1.16	1.03
499	152.46	0.33	45.23	0.09	35.46	0.89	6.66	0.20	1.66	0.08	0.53	0.47	1.16	1.01
500	152.11	0.35	45.12	0.11	34.57	0.89	6.45	0.21	1.59	0.07	0.49	0.51	—1.17	—0.99
501	151.74	—0.37	44.98	—0.14	33.68	—0.89	6.23	—0.22	1.54	—0.05	0.46	0.55	1.17	0.97
502	151.36	0.38	44.81	0.17	32.80	0.88	6.02	0.21	1.49	0.05	0.43	0.59	1.16	0.95
503	150.95	0.41	44.60	0.21	31.92	0.88	5.80	0.22	1.45	0.04	0.41	0.64	1.16	0.93
504	150.52	0.43	44.38	0.22	31.04	0.88	5.59	0.21	1.42	0.03	0.39	0.68	1.15	0.91
505	150.08	0.44	44.12	0.26	30.18	0.86	5.37	0.22	1.40	—0.02	0.37	0.73	—1.15	—0.90
506	149.61	—0.47	43.84	—0.28	29.32	—0.86	5.15	—0.22	1.40	0.00	0.36	0.78	1.13	0.89
507	148.13	0.48	43.52	0.32	28.46	0.86	4.93	0.22	1.40	0.00	0.35	0.83	1.11	0.87
508	148.63	0.50	43.18	0.34	27.62	0.84	4.72	0.21	1.43	+0.03	0.34	0.87	1.10	0.86
509	148.11	0.52	42.82	0.36	26.80	0.82	4.51	0.21	1.46	0.03	0.34	0.92	1.09	0.85
510	147.58	0.53	42.42	0.40	25.98	0.82	4.30	0.21	1.51	0.05	0.34	0.97	—1.08	—0.84
511	147.03	—0.55	42.00	—0.42	25.17	—0.81	4.09	—0.21	1.57	+0.06	0.34	1.02	1.06	0.84
512	146.46	0.57	41.56	0.44	24.38	0.79	3.89	0.20	1.64	0.07	0.35	1.06	1.04	0.84
513	145.87	0.59	41.09	0.47	23.61	0.77	3.69	0.20	1.72	0.08	0.36	1.11	1.02	0.83
514	145.27	0.60	40.60	0.49	22.85	0.76	3.50	0.19	1.81	0.09	0.37	1.16	1.01	0.83
515	144.66	0.61	40.09	0.51	22.11	0.74	3.32	0.18	1.90	0.09	0.39	1.20	—1.00	—0.82
516	144.04	—0.62	39.55	—0.54	21.38	—0.73	3.14	—0.18	2.01	+0.11	0.41	1.24	0.98	0.82
517	143.40	0.64	38.99	0.56	20.68	0.70	2.98	0.16	2.12	0.11	0.43	1.28	0.96	0.82
518	142.74	0.66	38.41	0.58	19.99	0.69	2.82	0.16	2.25	0.13	0.46	1.32	0.94	0.83
519	142.07	0.67	37.81	0.60	19.32	0.67	2.66	0.16	2.39	0.14	0.49	1.35	0.92	0.83
520	141.39	0.68	37.19	0.62	18.68	0.64	2.51	0.15	2.53	0.14	0.52	1.39	—0.90	—0.84
521	140.70	—0.69	36.56	—0.63	18.05	—0.63	2.38	—0.13	2.68	+0.15	0.55	1.42	0.98	0.85
522	140.00	0.70	35.91	0.65	17.45	0.60	2.25	0.13	2.84	0.16	0.58	1.44	0.97	0.87
523	139.29	0.71	35.25	0.66	16.87	0.58	2.13	0.12	3.00	0.16	0.62	1.47	0.96	0.88
524	138.57	0.72	34.57	0.68	16.32	0.55	2.03	0.10	3.16	0.16	0.66	1.49	0.95	0.89
525	137.84	0.73	33.87	0.70	15.79	0.53	1.94	0.09	3.33	0.17	0.70	1.51	—0.94	—0.90
526	137.10	—0.74	33.15	—0.72	15.28	—0.51	1.86	—0.08	3.51	+0.18	0.74	1.53	0.94	0.91
527	136.35	0.75	32.43	0.72	14.79	0.49	1.78	0.08	3.69	0.18	0.78	1.54	0.93	0.93
528	135.59	0.76	31.70	0.73	14.34	0.45	1.72	0.06	3.87	0.18	0.82	1.55	0.93	0.95
529	134.83	0.76	30.96	0.74	13.91	0.43	1.67	0.05	4.06	0.19	0.86	1.55	0.92	0.97
530	134.07	0.76	30.22	0.74	13.51	0.40	1.63	0.04	4.25	0.19	0.90	1.56	—0.91	—0.99
531	133.30	—0.77	29.46	—0.76	13.13	—0.38	1.60	—0.03	4.44	+0.19	0.94	1.55	0.91	1.01
532	132.52	0.78	28.69	0.77	12.78	0.35	1.58	0.02	4.63	0.19	0.98	1.55	0.92	1.03
533	131.73	0.79	27.92	0.77	12.46	0.32	1.57	—0.01	4.82	0.19	1.02	1.54	0.92	1.05
534	130.94	0.79	27.15	0.77	12.16	0.30	1.58	+0.01	5.00	0.18	1.06	1.54	0.93	1.07
535	130.15	0.79	26.37	0.78	11.90	0.26	1.60	0.02	5.18	0.18	1.09	1.53	—0.93	—1.09
536	129.35	—0.80	25.58	—0.79	11.66	—0.24	1.62	+0.02	5.36	+0.18	1.13	1.51	0.95	1.10
537	128.55	0.80	24.81	0.77	11.45	0.21	1.65	0.03	5.54	0.18	1.17	1.50	0.96	1.12
538	127.75	0.80	24.03	0.78	11.26	0.19	1.70	0.05	5.72	0.18	1.20	1.48	0.97	1.13
539	126.95	0.80	23.25	0.78	11.10	0.16	1.76	0.06	5.90	0.18	1.23	1.46	0.98	1.14
540	126.15	0.80	22.48	0.77	10.98	0.12	1.82	0.06	6.07	0.17	1.26	1.43	—0.99	—1.15

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TABLE X, Arg. 3.—*Continued.*

Arg.	(v.c.0)	Diff.	(v.s.1)	Diff.	(v.c.1)	Diff.	(v.s.2)	Diff.	(v.c.2)	Diff.	(v.s.3)	(v.c.3)	(v.s.4)	(v.c.4)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″
540	126.15	—0.79	22.48	—0.78	10.98	—0.10	1.82	+0.08	6.07	+0.16	1.26	1.43	—0.89	—1.15
541	125.36	0.80	21.70	0.77	10.88	0.08	1.90	0.08	6.23	0.16	1.28	1.41	0.91	1.15
542	124.56	0.80	20.93	0.77	10.80	0.05	1.98	0.10	6.39	0.15	1.30	1.38	0.93	1 16
543	123.76	0.80	20.16	0.76	10.75	—0.02	2.08	0.10	6.54	0.15	1.33	1.36	0.95	1.16
544	122.96	0.79	19.40	0.75	10.73	+0.01	2.18	0.11	6.69	0.14	1.35	1.33	0.96	1.17
545	122.17	—0.80	18.65	—0.75	10.74	+0.03	2.29	+0.11	6.83	+0.13	1.37	1.30	—0.97	—1.18
546	121.37	0.79	17.90	0.73	10.77	0.06	2.40	0.12	6.96	0.12	1.38	1.26	0.99	1.18
547	120.58	0.78	17.17	0.73	10.83	0.09	2.52	0.13	7.08	0.12	1.40	1.23	1.02	1.18
548	119.80	0.77	16.44	0.71	10.92	0.10	2.65	0.14	7.20	0.11	1.41	1.20	1.04	1.18
549	119.03	0.77	15.73	0.70	11.02	0.13	2.79	0.14	7.31	0.10	1.41	1.17	1.06	1.18
550	118.26	—0.76	15.03	—0.69	11.15	+0.16	2.93	+0.15	7.41	+0.10	1.42	1.14	—1.08	—1.18
551	117.50	0.76	14.34	0.68	11.31	0.18	3.08	0.15	7.51	0.08	1.42	1.11	1.10	1.17
552	116.74	0.75	13.66	0.67	11.49	0.20	3.23	0.15	7.59	0.08	1.42	1.08	1.12	1.16
553	115.99	0.75	12.99	0.64	11.69	0.22	3.38	0.16	7.67	0.06	1.41	1.04	1.13	1.15
554	115.24	0.74	12.35	0.64	11.91	0.25	3.54	0.16	7.73	0.06	1.41	1.01	1.14	1.14
555	114.50	—0.73	11.71	—0.62	12.16	+0.27	3.70	+0.16	7.79	+0.04	1.41	0.98	—1.15	—1.13
556	113.77	0.72	11.09	0.61	12.43	0.28	3.86	0.16	7.83	0.04	1 40	0.95	1.15	1.11
557	113.05	0.70	10.48	0.59	12.71	0.31	4.02	0.17	7.87	0.03	1.39	0.92	1.16	1.09
558	112.35	0.70	9.89	0.57	13.02	0.33	4.19	0.16	7.90	0.02	1.38	0.90	1.17	1.07
559	111.65	0.69	9.32	0.56	13.35	0.34	4.35	0.16	7.92	0.01	1.36	0.87	1.18	1.05
560	110.96	—0.67	8.76	—0.53	13.69	+0.35	4.51	+0.16	7.93	+0.01	1.34	0.85	—1.19	—1.03
561	110.29	0.66	8.23	0.52	14.04	0.38	4.67	0.17	7.94	—0.01	1.32	0.83	1.19	1.01
562	109.63	0.65	7.71	0.50	14.42	0.39	4.84	0.16	7.93	0.01	1.30	0.81	1.19	0.99
563	108.98	0.65	7.21	0.48	14.81	0.41	5.00	0.16	7.92	0.02	1.28	0.79	1.19	0.97
564	108.33	0.63	6.73	0.47	15.22	0.42	5.16	0.16	7.90	0.03	1.26	0.77	1.19	0.95
565	107.70	—0.62	6.26	—0.44	15.64	+0.43	5.32	+0.15	7.87	—0.04	1.24	0.76	—1.19	—0.93
566	107.08	0.60	5.82	0.43	16.07	0.45	5.47	0.15	7.83	0.05	1.22	0.74	1.17	0.91
567	106.48	0.60	5.39	0.41	16.52	0.46	5.62	0.15	7.78	0.05	1.20	0.73	1.16	0.89
568	105.88	0.58	4.98	0.38	16.98	0.47	5.77	0.14	7.73	0.06	1.17	0.72	1.15	0.87
569	105.30	0.56	4.60	0.37	17.45	0.48	5.91	0.14	7.67	0.07	1.15	0.72	1.14	0.86
570	104.74	—0.55	4.23	—0.36	17.93	+0.49	6.05	+0.14	7.60	—0.08	1.12	0.71	—1.13	—0.85
571	104.19	0.54	3.87	0.33	18.42	0.50	6.19	0.13	7.52	0.08	1.09	0.71	1.11	0.85
572	103.65	0.52	3.54	0.32	18.92	0.50	6.32	0.12	7.44	0.09	1.07	0.71	1.09	0.84
573	103.13	0.51	3.22	0.29	19.42	0.51	6.44	0.12	7.35	0.09	1.04	0.71	1.07	0.83
574	102.62	0.50	2.93	0.28	19.93	0.52	6.56	0.12	7.26	0.09	1.02	0.71	1.05	0.82
575	102.12	—0.49	2.65	—0.26	20.45	+0.52	6.68	+0.11	7.17	—0.11	1.00	0.71	—1.04	—0.81
576	101.63	0.48	2.39	0.24	20.97	0.52	6.79	0.10	7.06	0.11	0.98	0.72	1.02	0.81
577	101.15	0.46	2.15	0.23	21.49	0.54	6.89	0.09	6.95	0.11	0.95	0.72	1.00	0.81
578	100.69	0.43	1.92	0.21	22.03	0.55	6.98	0.09	6.84	0.11	0.93	0.73	0.98	0.80
579	100.25	0.44	1.71	0.18	22.58	0.54	7.07	0.09	6.73	0.12	0.91	0.74	0.96	0.80
580	99.81	—0.42	1.53	—0.18	23.12	+0.54	7.16	+0.08	6.61	—0.13	0.89	0.76	—0.94	—0.80
581	99.39	0.41	1.35	0.16	23.66	0.55	7.24	0.06	6.48	0.13	0.87	0.77	0.92	0.81
582	98.98	0.41	1.19	0.13	24.21	0.56	7.30	0.06	6.35	0.12	0.85	0.78	0.90	0.82
583	98.57	0.39	1.06	0.13	24.77	0.56	7.36	0.06	6.23	0.13	0.83	0.80	0.88	0.83
584	98.18	0.38	0.93	0.11	25.33	0.55	7.42	0.05	6.10	0.13	0.82	0.82	0.87	0.84
585	97.80	—0.37	0.82	—0.09	25 88	+0.56	7.47	+0.05	5.97	—0.14	0.81	0.84	—0.86	—0.85
586	97.43	0.36	0.73	0.08	26.44	0.56	7.52	0.03	5.83	0.14	0.80	0.86	0.86	0.87
587	97.07	0.35	0.65	0.07	27.00	0.56	7.55	0.03	5.69	0.13	0.79	0.87	0.85	0.89
588	96.72	0.34	0.58	0.05	27.56	0.56	7.58	0.03	5.56	0.14	0.78	0.90	0.84	0.91
589	96.38	0.34	0.53	0.03	28.12	0.56	7.61	0.01	5.42	0.14	0.77	0.92	0.83	0.93
590	96.04	—0.32	0.50	—0.03	28.68	+0.56	7.62	+0.01	5.28	—0.14	0.76	0.94	—0.82	—0.94
591	95.72	0.32	0.47	—0.01	29.24	0.55	7.63	0.00	5.14	0.13	0.76	0.96	0.82	0.96
592	95.40	0.32	0.46	+0.01	29.79	0.56	7.63	0.00	5.01	0.14	0.76	0.98	0.82	0.98
593	95.08	0.31	0.47	0.02	30.35	0.56	7.63	—0.01	4.87	0.14	0.76	1.00	0.81	1.00
594	94.77	0.31	0.49	0.03	30.91	0.55	7.62	0.02	4.73	0.13	0.76	1.02	0.81	1.02
595	94.46	—0.31	0.52	+0.05	31.46	+0.55	7.60	—0.02	4.60	—0.13	0.76	1.04	—0.81	—1.04
596	94.15	0.30	0.57	0.06	32.01	0.55	7.58	0.03	4.47	0.13	0 77	1.06	0.82	1.06
597	93.85	0.30	0.63	0.07	32.56	0.55	7.55	0.04	4.34	0.13	0.78	1.08	0.83	1.08
598	93.55	0.29	0.70	0.08	33.11	0.56	7.51	0.05	4.21	0.13	0.78	1.10	0.84	1.10
599	93.26	0.30	0.78	0.09	33.67	0.55	7.46	0.04	4.08	0.13	0.79	1.13	0.85	1.12
600	92.96		0.87		34.22		7.42		3.95		0.80	1.15	—0.85	—1.13

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TABLE X, Arg. 3.—*Continued.*

Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)
0	524	180	89	49	58	60	746	17	244	33	10
1	525	178	90	50	58	61	747	17	249	32	11
2	525	176	90	50	57	62	748	16	255	30	11
3	526	174	91	51	57	63	749	16	260	29	11
4	527	172	91	52	56	64	749	16	266	27	11
5	528	170	92	53	56	65	750	16	271	26	12
6	529	168	92	54	55	66	750	16	277	24	13
7	531	165	92	55	55	67	750	17	282	23	13
8	533	163	93	55	54	68	750	17	287	22	14
9	535	161	93	56	53	69	749	18	293	20	14
10	538	158	94	57	53	70	748	19	298	19	15
11	540	156	94	57	52	71	747	21	303	18	16
12	543	154	95	58	51	72	746	22	308	17	17
13	546	151	96	59	50	73	744	24	313	16	18
14	550	149	96	59	49	74	742	26	318	14	19
15	553	146	97	60	48	75	740	29	323	13	20
16	557	144	98	60	47	76	737	31	327	12	21
17	560	141	99	61	46	77	734	34	332	11	22
18	564	138	100	61	45	78	731	37	336	10	23
19	568	135	101	62	44	79	728	40	340	09	24
20	573	132	102	62	44	80	724	43	344	09	26
21	577	129	103	62	42	81	721	47	348	08	27
22	582	126	105	62	41	82	717	51	352	07	28
23	586	123	106	63	40	83	713	55	355	07	30
24	591	120	108	63	39	84	708	59	359	06	31
25	596	117	109	63	38	85	703	63	362	06	33
26	601	113	111	63	37	86	698	68	364	05	34
27	606	110	113	63	36	87	693	72	367	05	36
28	611	107	115	63	34	88	688	77	369	05	37
29	617	103	117	63	33	89	682	82	372	05	39
30	622	100	119	62	32	90	676	87	373	05	41
31	627	96	121	62	31	91	670	92	375	05	42
32	633	93	123	62	30	92	663	97	376	05	44
33	638	90	126	62	29	93	657	102	378	05	45
34	643	86	129	61	28	94	650	108	378	06	47
35	648	82	132	61	27	95	643	114	379	06	48
36	654	79	135	60	26	96	636	119	379	06	50
37	659	76	139	60	24	97	629	125	379	07	51
38	664	72	142	59	23	98	622	130	379	08	52
39	669	69	145	58	22	99	614	136	379	08	54
40	674	65	149	58	21	100	606	141	378	09	55
41	680	62	152	57	20	101	598	147	377	10	56
42	685	58	156	56	19	102	590	153	376	11	57
43	689	55	160	55	18	103	582	159	374	12	58
44	694	52	164	54	18	104	574	164	373	13	60
45	698	49	168	53	17	105	565	170	371	14	61
46	703	46	173	52	16	106	557	176	368	15	62
47	707	43	177	51	15	107	548	181	366	16	62
48	711	40	182	50	14	108	539	186	363	17	63
49	715	37	186	49	14	109	530	192	360	19	64
50	719	35	191	47	13	110	521	197	357	20	64
51	722	32	196	46	13	111	512	202	353	21	65
52	726	30	201	45	12	112	503	207	350	23	65
53	729	28	206	43	12	113	494	212	346	24	66
54	732	26	211	42	11	114	484	217	342	25	66
55	735	24	216	40	11	115	475	222	338	27	66
56	737	22	222	39	11	116	465	227	333	28	67
57	740	21	227	38	11	117	456	231	329	30	67
58	742	19	233	36	11	118	446	235	324	31	67
59	744	18	238	35	10	119	436	239	319	32	67
60	746	17	244	33	10	120	427	243	314	34	66

TABLE X, Arg. 3.—*Continued.*

Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)
120	427	243	314	34	66	180	11	172	66	26	33
121	417	247	309	35	66	181	10	168	66	25	33
122	407	250	303	37	66	182	9	165	67	25	34
123	397	254	298	38	65	183	9	162	67	24	35
124	387	257	292	39	65	184	8	159	68	24	36
125	378	260	286	41	64	185	8	156	69	24	37
126	368	263	280	42	63	186	8	153	71	24	38
127	359	266	274	43	63	187	8	150	72	23	39
128	349	268	268	44	62	188	9	147	74	23	40
129	339	270	262	45	61	189	9	144	75	23	41
130	330	272	256	46	60	190	10	141	77	23	42
131	320	274	250	47	59	191	11	138	79	24	43
132	310	275	244	48	58	192	13	136	81	24	44
133	301	277	238	49	57	193	14	133	84	24	45
134	292	278	232	50	56	194	16	131	86	24	46
135	232	279	225	50	55	195	18	129	89	25	47
136	273	279	219	51	54	196	21	127	91	25	48
137	263	280	213	52	52	197	23	125	94	26	49
138	254	280	207	52	51	198	26	123	97	26	50
139	245	280	201	52	50	199	29	121	99	27	51
140	236	280	195	53	49	200	32	120	102	28	52
141	227	280	189	53	47	201	36	118	105	28	52
142	218	280	183	53	46	202	39	117	108	29	53
143	209	279	177	53	45	203	43	116	112	30	54
144	201	278	171	53	44	204	47	114	115	31	54
145	193	277	165	53	42	205	51	113	118	32	55
146	184	275	160	53	41	206	55	112	122	33	55
147	176	274	154	52	40	207	60	112	125	34	56
148	168	272	149	52	39	208	65	111	129	35	56
149	160	271	144	52	38	209	69	110	132	36	57
150	152	269	139	51	36	210	75	110	135	37	57
151	145	267	134	51	35	211	80	110	139	38	57
152	137	265	129	50	35	212	85	110	143	39	57
153	130	263	124	49	34	213	91	110	146	40	57
154	123	260	120	49	33	214	96	110	150	41	57
155	116	257	115	48	32	215	102	110	153	42	57
156	109	255	111	47	31	216	108	110	157	43	57
157	103	252	107	46	30	217	114	110	160	44	57
158	96	249	103	45	30	218	120	111	164	45	57
159	90	246	100	45	29	219	126	112	168	46	57
160	84	243	96	44	29	220	133	113	171	47	57
161	78	240	93	43	28	221	139	114	175	48	56
162	73	236	90	42	28	222	145	115	178	49	56
163	67	233	87	41	27	223	152	116	182	50	55
164	62	229	84	40	27	224	159	117	185	50	55
165	57	226	81	39	27	225	166	118	188	51	54
166	53	222	79	38	27	226	173	119	192	52	53
167	48	219	77	36	27	227	180	120	195	53	52
168	44	215	75	35	27	228	187	122	198	54	52
169	40	212	73	34	27	229	194	123	201	54	51
170	36	208	71	33	27	230	201	125	204	55	51
171	32	204	70	32	27	231	208	127	207	55	50
172	29	201	68	31	28	232	216	128	210	56	49
173	26	197	67	30	28	233	223	130	213	56	49
174	23	193	67	29	28	234	230	132	216	57	48
175	21	190	66	28	29	235	238	134	218	57	47
176	18	186	66	28	30	236	245	136	221	57	46
177	16	182	65	27	30	237	253	138	224	58	45
178	14	179	65	27	31	238	260	140	226	58	45
179	12	175	65	26	32	239	268	142	228	58	44
180	11	172	66	26	33	240	275	144	231	58	43

TABLE X, Arg. 3.—*Continued.*

Arg.	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$	Arg.	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$
240	275	144	231	58	43	300	562	208	250	42	32
241	283	146	233	58	42	301	562	208	250	42	32
242	290	148	235	58	41	302	562	208	250	42	32
243	298	150	237	58	40	303	561	208	250	42	32
244	305	152	239	58	40	304	560	208	250	41	32
245	313	154	240	57	39	305	559	208	250	41	31
246	320	156	242	57	38	306	558	208	250	41	31
247	328	159	244	57	38	307	557	208	250	41	31
248	335	161	245	57	37	308	555	208	251	41	31
249	342	163	246	57	36	309	553	208	251	40	30
250	350	165	248	56	36	310	551	208	251	40	30
251	357	167	249	56	35	311	549	209	251	40	30
252	364	169	250	55	34	312	547	209	251	39	30
253	371	171	251	55	34	313	544	209	251	39	29
254	378	173	252	54	33	314	542	210	251	39	29
255	385	175	253	54	33	315	539	210	251	38	29
256	392	177	254	54	33	316	536	210	252	38	29
257	399	179	254	53	32	317	532	211	252	38	28
258	406	180	255	52	32	318	529	211	252	37	28
259	412	182	256	52	32	319	525	212	252	37	28
260	419	184	256	51	31	320	521	213	252	36	28
261	426	185	256	51	21	321	517	213	252	36	28
262	432	187	257	50	31	322	513	214	252	35	28
263	438	189	257	50	31	323	509	215	252	34	28
264	444	190	257	50	31	324	504	216	252	34	28
265	450	192	257	49	31	325	500	217	252	33	28
266	456	193	257	49	31	326	495	218	252	33	28
267	462	194	257	48	31	327	490	219	252	32	28
268	468	196	257	48	31	328	485	221	251	31	28
269	473	197	257	47	31	329	480	222	251	31	28
270	478	198	257	47	31	330	474	223	251	30	28
271	484	199	257	46	31	331	469	224	250	30	28
272	489	200	257	46	31	332	463	225	249	29	29
273	494	201	257	45	31	333	457	227	249	29	29
274	499	202	256	45	31	334	451	228	248	28	29
275	503	203	256	45	31	335	446	229	248	27	30
276	509	203	256	45	31	336	439	231	247	27	30
277	512	204	256	44	31	337	433	233	246	26	31
278	517	205	255	44	31	338	427	234	245	25	31
279	521	205	255	44	32	339	421	236	244	25	31
280	524	206	255	44	32	340	414	238	243	24	32
281	528	206	254	43	32	341	407	239	242	24	33
282	531	207	254	43	32	342	401	241	241	24	33
283	535	207	253	43	32	343	394	243	240	23	34
284	538	207	253	43	32	344	387	244	238	23	35
285	541	208	253	43	32	345	380	246	236	23	35
286	544	208	252	43	33	346	373	248	235	22	36
287	546	208	252	43	33	347	366	249	233	22	37
288	549	208	252	43	33	348	359	251	231	22	38
289	551	208	251	42	33	349	352	253	229	22	38
290	553	208	251	42	33	350	344	255	227	21	39
291	555	208	251	42	33	351	337	256	225	21	40
292	556	208	251	42	33	352	330	258	223	21	41
293	558	208	251	42	33	353	323	260	221	21	42
294	559	208	251	42	33	354	315	261	218	21	42
295	560	208	250	42	33	355	308	263	216	22	43
296	561	208	250	42	33	356	300	264	213	22	44
297	562	208	250	42	32	357	293	266	211	22	45
298	562	208	250	42	32	358	285	267	208	22	46
299	562	208	250	42	32	359	278	269	205	23	47
300	562	208	250	42	32	360	270	270	203	23	47

TABLE X, Arg. 3.—*Continued.*

Arg.	(ρ.c.0)	(ρ.s.1)	(ρ.c.1)	(ρ.s.2)	(ρ.c.2)	Arg.	(ρ.c.0)	(ρ.s.1)	(μ.c.1)	(ρ.s.2)	(ρ.c.2)
360	270	270	203	23	47	420	14	183	69	44	32
361	263	272	200	23	48	421	16	180	70	44	32
362	255	273	197	24	49	422	18	176	72	43	32
363	248	274	194	25	50	423	20	173	74	42	32
364	241	275	190	25	50	424	22	170	76	41	32
365	233	276	187	26	51	425	24	166	78	41	32
366	226	277	184	26	52	426	27	163	81	40	32
367	219	278	181	27	52	427	30	160	83	39	32
368	211	278	177	28	53	428	33	157	86	38	33
369	204	279	174	29	53	429	36	153	89	38	33
370	197	280	171	29	54	430	40	150	92	37	33
371	190	280	167	30	54	431	44	147	95	36	34
372	183	281	164	31	54	432	48	144	99	35	34
373	176	281	160	32	55	433	53	142	103	35	35
374	169	281	157	33	55	434	57	139	106	34	36
375	162	281	153	34	55	435	62	136	111	34	36
376	155	281	150	35	55	436	67	134	115	33	37
377	149	281	146	36	55	437	72	132	119	33	38
378	142	281	142	37	55	438	77	130	124	32	39
379	136	281	139	38	55	439	83	128	128	32	40
380	129	280	135	39	55	440	89	126	133	32	40
381	123	280	132	40	55	441	95	124	138	32	41
382	117	279	128	41	55	442	102	122	143	31	42
383	111	278	125	42	55	443	108	121	148	31	43
384	105	278	122	42	54	444	115	120	153	31	44
385	99	277	118	43	54	445	121	118	159	31	45
386	94	276	115	44	54	446	128	118	164	31	46
387	89	274	111	45	53	447	135	117	170	31	47
388	83	273	108	46	53	448	142	116	176	32	48
389	78	272	105	47	52	449	150	116	181	32	49
390	73	270	102	47	52	450	157	115	187	32	50
391	68	268	99	48	51	451	165	115	193	33	51
392	63	266	97	48	50	452	173	115	199	33	52
393	59	265	94	49	50	453	181	116	205	34	53
394	54	263	91	49	49	454	189	117	211	34	54
395	50	261	88	50	48	455	197	117	217	35	55
396	46	258	86	50	47	456	206	118	223	36	56
397	42	256	83	51	47	457	215	120	229	36	57
398	39	254	81	51	46	458	223	121	236	37	58
399	35	251	79	51	45	459	232	123	242	38	58
400	32	248	77	52	45	460	241	125	248	39	59
401	29	246	75	52	44	461	250	127	254	40	60
402	27	243	73	52	43	462	259	129	260	41	60
403	24	240	72	52	42	463	268	131	265	42	61
404	22	237	70	52	41	464	278	134	271	43	61
405	19	234	69	52	40	465	287	137	277	44	62
406	17	231	68	52	39	466	296	140	283	45	62
407	15	228	67	51	38	467	306	143	288	47	62
408	14	225	66	51	38	468	315	147	294	48	62
409	13	221	65	51	37	469	324	150	299	49	62
410	12	218	64	50	36	470	334	154	305	51	62
411	11	215	64	50	36	471	343	158	310	52	62
412	11	211	64	49	35	472	353	162	315	53	62
413	10	208	64	49	34	473	362	167	320	54	62
414	10	204	64	48	34	474	372	171	325	55	62
415	10	201	64	48	33	475	382	176	329	57	62
416	10	197	65	47	33	476	391	181	334	58	61
417	11	194	66	46	33	477	401	186	338	59	61
418	12	190	67	46	32	478	411	191	342	60	60
419	13	187	68	45	32	479	420	196	346	61	60
420	14	183	69	44	32	480	430	202	349	62	59

TABLE X, ARG. 3.—*Concluded.*

Arg.	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$	Arg.	$(\rho.c.0)$	$(\rho.s.1)$	$(\rho.c.1)$	$(\rho.s.2)$	$(\rho.c.2)$
480	430	202	349	62	59	540	741	381	186	29	19
481	439	207	353	63	58	541	739	378	181	28	20
482	449	213	356	64	57	542	737	375	176	27	21
483	458	218	359	65	56	543	735	372	171	26	22
484	468	223	361	66	55	544	732	368	167	25	23
485	477	230	364	67	54	545	730	365	162	24	24
486	486	236	366	68	53	546	727	361	158	24	25
487	496	242	368	69	52	547	724	357	153	23	26
488	505	248	370	69	51	548	721	353	149	22	27
489	514	254	371	70	50	549	717	350	145	22	28
490	523	260	373	70	49	550	714	346	141	21	29
491	532	266	374	71	48	551	710	342	137	21	30
492	540	272	374	71	46	552	706	338	133	21	31
493	549	278	375	72	45	553	702	333	130	20	32
494	558	284	375	72	44	554	698	329	126	20	33
495	566	289	375	72	42	555	693	325	123	20	34
496	574	295	374	72	41	556	689	321	120	20	35
497	583	301	374	72	39	557	684	316	117	19	36
498	591	307	373	72	38	558	679	312	114	19	38
499	599	313	872	72	37	559	674	308	112	19	39
500	606	318	371	72	35	560	669	303	109	20	40
501	614	324	369	71	34	561	664	299	107	20	41
502	621	329	367	71	33	562	659	295	105	20	42
503	629	334	365	71	31	563	654	290	103	20	43
504	636	339	363	70	30	564	649	286	101	20	44
505	643	344	360	70	29	565	644	282	99	20	45
506	649	349	357	69	28	566	638	278	97	21	46
507	656	353	354	68	27	567	633	274	96	21	47
508	662	358	351	68	25	568	628	270	94	22	48
509	668	362	347	67	24	569	623	266	93	22	49
510	674	366	344	66	23	570	618	262	92	23	50
511	680	370	340	65	22	571	613	258	91	23	50
512	686	374	336	64	21	572	608	255	90	24	51
513	691	377	332	63	20	573	602	251	90	25	52
514	696	380	327	62	19	574	598	247	89	25	53
515	701	383	323	61	18	575	593	244	88	26	54
516	706	386	318	59	18	576	588	240	88	27	54
517	710	388	313	58	17	577	583	237	87	28	55
518	714	390	308	57	16	578	578	234	87	28	56
519	718	392	303	56	16	579	574	231	87	29	56
520	721	394	298	54	15	580	570	228	86	30	57
521	725	396	293	53	15	581	565	225	86	31	57
522	728	397	287	52	14	582	561	222	86	32	58
523	731	398	282	50	14	583	557	219	86	32	58
524	733	399	276	49	14	584	554	216	86	33	58
525	736	400	270	48	14	585	551	214	86	34	58
526	738	400	265	46	14	586	547	211	86	35	59
527	740	400	259	45	14	587	544	209	86	36	59
528	742	400	253	44	14	588	541	206	86	37	59
529	743	399	247	42	14	589	539	204	86	38	59
530	744	399	242	41	14	590	536	202	87	39	60
531	745	398	236	40	14	591	534	199	87	40	60
532	745	397	230	38	14	592	532	197	87	41	60
533	746	396	225	37	15	593	530	195	88	42	60
534	746	394	219	36	15	594	528	193	88	43	60
535	745	392	213	34	16	595	527	190	88	44	59
536	745	390	208	33	17	596	526	188	88	45	59
537	744	388	202	32	17	597	525	186	88	46	59
538	743	386	197	31	18	598	525	184	89	47	59
539	742	383	192	30	19	599	524	182	89	48	58
540	741	381	186	29	19	600	524	180	89	49	58

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TAB.	XI		XII.			XIII.			XIV.			XV.			XVI.		
ARG.	4.		5.			6.			7.			8.			9.		
	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)	(v.c.0)	(v.s.1)	(v.c.1)
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	0.67	0.13	1.11	0.03	0.21	0.06	0.13	0.18	0.10	0.18	0.19	0.05	0.17	0.14	0.13	0.10	0.13
10	0.64	0.13	1.12	0.03	0.22	0.06	0.14	0.18	0.10	0.18	0.19	0.09	0.15	0.19	0.12	0.10	0.12
20	0.61	0.14	1.12	0.03	0.24	0.07	0.15	0.18	0.10	0.18	0.19	0.15	0.13	0.25	0.12	0.09	0.11
30	0.58	0.16	1.11	0.03	0.26	0.07	0.16	0.17	0.10	0.18	0.19	0.22	0.11	0.31	0.11	0.09	0.10
40	0.54	0.17	1.10	0.03	0.28	0.08	0.16	0.16	0.10	0.18	0.19	0.30	0.10	0.37	0.11	0.09	0.09
50	0.50	0.19	1.08	0.04	0.30	0.08	0.17	0.16	0.10	0.18	0.19	0.39	0.10	0.44	0.10	0.08	0.08
60	0.47	0.21	1.06	0.04	0.31	0.08	0.17	0.15	0.10	0.18	0.19	0.50	0.11	0.52	0.09	0.08	0.07
70	0.43	0.24	1.04	0.05	0.32	0.09	0.18	0.14	0.09	0.17	0.18	0.61	0.13	0.59	0.09	0.08	0.06
80	0.39	0.26	1.00	0.06	0.34	0.09	0.18	0.13	0.09	0.17	0.18	0.74	0.15	0.67	0.08	0.07	0.05
90	0.35	0.29	0.97	0.07	0.35	0.09	0.18	0.12	0.09	0.16	0.17	0.87	0.17	0.75	0.08	0.07	0.04
100	0.31	0.31	0.93	0.08	0.36	0.09	0.18	0.12	0.08	0.16	0.17	1.00	0.21	0.83	0.07	0.07	0.03
110	0.27	0.34	0.88	0.09	0.37	0.09	0.19	0.11	0.08	0.15	0.16	1.14	0.25	0.91	0.06	0.07	0.02
120	0.23	0.37	0.84	0.11	0.37	0.09	0.19	0.10	0.07	0.14	0.15	1.28	0.29	0.98	0.06	0.06	0.02
130	0.20	0.40	0.79	0.12	0.38	0.09	0.18	0.09	0.07	0.14	0.14	1.42	0.34	1.06	0 06	0.06	0.01
140	0.17	0.43	0.74	0.14	0.38	0.09	0.18	0.08	0.06	0.13	0.13	1.56	0.40	1.13	0.05	0.06	0.01
150	0.14	0.46	0.68	0.16	0.38	0.09	0.18	0.07	0.06	0.12	0.12	1.70	0.45	1.19	0.05	0.06	0.01
160	0.11	0.49	0.63	0.18	0.38	0.09	0.18	0.06	0.05	0.11	0.11	1.83	0.52	1.25	0.04	0.06	0.00
170	0.09	0.51	0.57	0.19	0.37	0.08	0.17	0.05	0.05	0.10	0.10	1.96	0.58	1.31	0.04	0.06	0.00
180	0.07	0.54	0.52	0.21	0.37	0.08	0.17	0.04	0.04	0.10	0.09	2.09	0.65	1.36	0.04	0.06	0.00
190	0.05	0.56	0.46	0.23	0.36	0.08	0.16	0.04	0.04	0.09	0.08	2.20	0.72	1.40	0.04	0.07	0.00
200	0.04	0.59	0.41	0.25	0.35	0.08	0.16	0.04	0.03	0.08	0.07	2.31	0.78	1.43	0.04	0.07	0.00
210	0.03	0.61	0.36	0.26	0.34	0.07	0.15	0.02	0.03	0.07	0.06	2.40	0.85	1.46	0.04	0.07	0.00
220	0.02	0.63	0.32	0.28	0.32	0.07	0.14	0.02	0.02	0.06	0.06	2.48	0.92	1.48	0.04	0.07	0.01
230	0.02	0.64	0.27	0.30	0.31	0.07	0.13	0.02	0.02	0.06	0.05	2.55	0.98	1.50	0.04	0.08	0.01
240	0.02	0.66	0.23	0.31	0.30	0.06	0.12	0.01	0.02	0.05	0.04	2.61	1.05	1.50	0.04	0.08	0.02
250	0.03	0.67	0.20	0.32	0.28	0.06	0.11	0.01	0.01	0.04	0.03	2.65	1.10	1.50	0.05	0.08	0.02
260	0.04	0.67	0.16	0.34	0.26	0.06	0.11	0.01	0.01	0.04	0.03	2.68	1.16	1.48	0.05	0.09	0.03
270	0.06	0.68	0.14	0.35	0.24	0.05	0.10	0.01	0.01	0.03	0.02	2.70	1.21	1.46	0.06	0.09	0.04
280	0.08	0.68	0.12	0.35	0.22	0.05	0.09	0.01	0.00	0.03	0.02	2.70	1.25	1.43	0.06	0.09	0.05
290	0.10	0.68	0.10	0.36	0.21	0.04	0.08	0.01	0.00	0.02	0.01	2.68	1.29	1.40	0.06	0.10	0.06
300	0.13	0.67	0 08	0.37	0.19	0.04	0.07	0.02	0.00	0.02	0.01	2.65	1.33	1.36	0.07	0.10	0.07
310	0.16	0.67	0.08	0.37	0.18	0.04	0.06	0.02	0.00	0.02	0.01	2.61	1.35	1.31	0.08	0.10	0.08
320	0.19	0.66	0.08	0.37	0.16	0.03	0.05	0.02	0.00	0.02	0.01	2.55	1.37	1.25	0.08	0.11	0.09
330	0.22	0.64	0.09	0.37	0.14	0.03	0.04	0.03	0.00	0.02	0.01	2.48	1.39	1.19	0.09	0.11	0.10
340	0.26	0.63	0.10	0.37	0.12	0.02	0.04	0.04	0.00	0.02	0.01	2.40	1.40	1.13	0.09	0.11	0.11
350	0.30	0.61	0.12	0.36	0.10	0.02	0.03	0.04	0.00	0.02	0.01	2.31	1.40	1.06	0.10	0.12	0.12
360	0.33	0.59	0.14	0.36	0.09	0.01	0.03	0.05	0.00	0.02	0.02	2.20	1.39	0.98	0.11	0.12	0.13
370	0.37	0.56	0.16	0.35	0.08	0.01	0.02	0.06	0.01	0.03	0.02	2.09	1.37	0.91	0.11	0.12	0.14
380	0.41	0.54	0.20	0.34	0.06	0.01	0.02	0.07	0.01	0.03	0.03	1.96	1.35	0.83	0.12	0.13	0.15
390	0.45	0.51	0.23	0.33	0.05	0.01	0.02	0.08	0.01	0.04	0.03	1.83	1.32	0.75	0.12	0.13	0.16
400	0.49	0.49	0.27	0.32	0.04	0.01	0.02	0.08	0.02	0.04	0.04	1.70	1.29	0.67	0.13	0.13	0.17
410	0.53	0.46	0.32	0.31	0.03	0.01	0.01	0.09	0.02	0.05	0.05	1.56	1.25	0.59	0.13	0.13	0.18
420	0.57	0.43	0.36	0.29	0.03	0.01	0.01	0.10	0.03	0.06	0.06	1.42	1.21	0.52	0.14	0.14	0.18
430	0.60	0.40	0.41	0.28	0.02	0.01	0.02	0.11	0.03	0.06	0.06	1.28	1.16	0.44	0.14	0.14	0.19
440	0.63	0.37	0.46	0.26	0.02	0.01	0.02	0.12	0.04	0.07	0.07	1.14	1.10	0.37	0.15	0.14	0.19
450	0.66	0.34	0.52	0.24	0.02	0.01	0.02	0.13	0.04	0.08	0.08	1.00	1.05	0.31	0.15	0.14	0.19
460	0.69	0.31	0.57	0.22	0.02	0.01	0.02	0.14	0.05	0.09	0.09	0.87	0.98	0.25	0.16	0.14	0.20
470	0.71	0.29	0.63	0.21	0.03	0.02	0.03	0.15	0.05	0.10	0.10	0.74	0.92	0.19	0.16	0.14	0.20
480	0.73	0.26	0.68	0.19	0.03	0.02	0.03	0.16	0.06	0.10	0.11	0.61	0.85	0.14	0.16	0.14	0.20
490	0.75	0.24	0.74	0.17	0.04	0.02	0.04	0.16	0.07	0.11	0.12	0.50	0.78	0.10	0.16	0.13	0.20
500	0.76	0.21	0.79	0.15	0.05	0.02	0.04	0.17	0.07	0.12	0.13	0.39	0.72	0.07	0.16	0.13	0.20
510	0.77	0.19	0.84	0.14	0.06	0.03	0.05	0.18	0.07	0.13	0.14	0.30	0.65	0.04	0.16	0.13	0.20
520	0.78	0.17	0.88	0.12	0.08	0.03	0.06	0.18	0.08	0.14	0.14	0.22	0.58	0.02	0.16	0.13	0.19
530	0.78	0.16	0.93	0.10	0.09	0.03	0.07	0.18	0.08	0.14	0.15	0.15	0.52	0.00	0.16	0.12	0.19
540	0.78	0.14	0.97	0.09	0.10	0.04	0.08	0.19	0.08	0.15	0.16	0.09	0.45	0.00	0.16	0.12	0.18
550	0.77	0.13	1.00	0.08	0.12	0.04	0.09	0.19	0.09	0.16	0.17	0.05	0.40	0.00	0.15	0.12	0.18
560	0.76	0.13	1.04	0.07	0.14	0.04	0.09	0.19	0.09	0.16	0.17	0.02	0.34	0.02	0.15	0.11	0.17
570	0.74	0.12	1.06	0.06	0.16	0.05	0.10	0.19	0.09	0.17	0.18	0.00	0.29	0.04	0.15	0.11	0.16
580	0.72	0.12	1.08	0.05	0.18	0.05	0.11	0.19	0.10	0.17	0.18	0.00	0.25	0.07	0.14	0.11	0.15
590	0.70	0.12	1.10	0.04	0.19	0.06	0.12	0.19	0.10	0.18	0.19	0.02	0.21	0.10	0.14	0.10	0.14
600	0.67	0.13	1.11	0.03	0.21	0.06	0.13	0.18	0.10	0.18	0.19	0.05	0.17	0.14	0.13	0.10	0.13

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TABLE XVII *a*.

Year.	(*v.s.*1)	(*v.c.*1)	Year.	(*v.s.*1)	(*v.c.*1)
	″	″		″	″
1800	—240.33	—162.11	1850	—183.57	—189.69
1801	239.19	162.61	1851	182.44	190.29
1802	238.06	163.12	1852	181.30	190.89
1803	236.92	163.63	1853	180.17	191.50
1804	235.79	164.14	1854	179.04	192.10
1805	—234.65	—164.65	1855	—177.91	—192.71
1806	233.51	165.16	1856	176.78	193.32
1807	232.38	165.68	1857	175.65	193.93
1808	231.24	166.20	1858	174.51	194.54
1809	230.11	166.72	1859	173.38	195.15
1810	—228.97	—167.24	1860	—172.25	—195.77
1811	227.83	167.76	1861	171.12	196.39
1812	226.70	168.29	1862	169.99	197.01
1813	225.56	168.82	1863	168.86	197.63
1814	224.43	169.35	1864	167.72	198.25
1815	—223.29	—169.88	1865	—166.59	—198.88
1816	222.15	170.41	1866	165.46	199.51
1817	221.02	170.95	1867	164.33	200.14
1818	219.88	171.49	1868	163.20	200.77
1819	218.75	172.03	1869	162.07	201.40
1820	—217.61	—172.57	1870	—160.94	—202.04
1821	216.47	173.11	1871	159.81	202.68
1822	215.34	173.66	1872	158.68	203.32
1823	214.20	174.21	1873	157.55	203.96
1824	213.07	174.76	1874	156.42	204.60
1825	—211.93	—175.31	1875	—155.29	—205.24
1826	210.80	175.86	1876	154.16	205.89
1827	209.66	176.42	1877	153.03	206.54
1828	208.53	176.97	1878	151.91	207.19
1829	207.39	177.53	1879	150.78	207.84
1830	—206.26	—178.09	1880	—149.65	—208.49
1831	205.13	178.65	1881	148.52	209.15
1832	203.99	179.22	1882	147.39	209.81
1833	202.86	179.78	1883	146.27	210.47
1834	201.72	180.35	1884	145.14	211.13
1835	—200.59	—180.92	1885	—144.01	—211.79
1836	199.45	181.49	1886	142.89	212.45
1837	198.32	182.06	1887	141.76	213.12
1838	197.18	182.64	1888	140.64	213.79
1839	196.05	183.22	1889	139.51	214.46
1840	—194.91	—183.80	1890	—138.39	—215.13
1841	193.78	184.38	1891	137.27	215.80
1842	192.64	184.96	1892	136.14	216.48
1843	191.51	185.55	1893	135.02	217.17
1844	190.37	186.13	1894	133.90	217.84
1845	—189.24	—186.72	1895	—132.77	—218.52
1846	188.10	187.31	1896	131.65	219.20
1847	186.97	187.90	1897	130.53	219.88
1848	185.84	188.50	1898	129.40	220.57
1849	184.70	189.09	1899	128.28	221.26
1850	—183.57	—189.69	1900	—127.16	—221.95

TABLE XVII *b*.

Year.	(*v.s.*1)		(*v.c.*1)	
	″		″	
1500	—560.44		— 96.24	
		+ 9.59		+0.43
1510	550.85		95.81	
		9.68		0.27
1520	541.17		95.54	
		9.77		+0.10
1530	531.40		95.44	
		9.86		—0.07
1540	521.54		95.51	
		9.95		0.24
1550	—511.59		— 95.75	
		+10.04		—0.41
1560	501.55		96.16	
		10.13		0.59
1570	491.42		96.75	
		10.22		0.77
1580	481.20		97.52	
		10.30		0.95
1590	470.90		98.47	
		10.38		1.13
1600	—460.52		— 99.60	
		+10.46		—1.33
1610	450.06		100.93	
		10.53		1.51
1620	439.53		102.44	
		10.61		1.70
1630	428.92		104.14	
		10.68		1.88
1640	418.24		106.02	
		10.75		2.07
1650	—407.49		—108.09	
		+10.82		—2.26
1660	396.67		110.35	
		10.88		2.45
1670	385.79		112.80	
		10.93		2.65
1680	374.86		115.45	
		10.99		2.83
1690	363.87		118.28	
		11.04		3.02
1700	—352.83		—121.30	
		+11.09		—3.22
1710	341.74		124.52	
		11.13		3.41
1720	330.61		127.93	
		11.18		3.60
1730	319.43		131.53	
		11.21		3.79
1740	308.22		135.32	
		11.25		3.98
1750	—296.97		—139.30	
		+11.28		—4.18
1760	285.69		143.48	
		11.31		4.37
1770	274.38		147.85	
		11.34		4.57
1780	263.04		152.42	
		11.35		4.75
1790	251.69		157.17	
		11.36		4.94
1800	—240.33		—162.11	
		+11.36		—5.13
1810	228.97		167.24	
		11.36		5.33
1820	217.61		172.57	
		11.35		5.52
1830	206.26		178.09	
		11.35		5.71
1840	194.91		183.80	
		11.34		5.89
1850	—183.57		—189.69	
		+11.32		—6.08
1860	172.25		195.77	
		11.31		6.27
1870	160.94		202.04	
		11.29		6.45
1880	149.65		208.49	
		11.26		6.64
1890	138.39		215.13	
		11.23		6.82
1900	—127.16		—221.95	
		+11.18		—6.99
1910	115.98		228.94	
		11.14		7.17
1920	104.84		236.11	
		11.09		7.35
1930	93.75		243.46	
		11.03		7.54
1940	82.72		251.00	
		10.98		7.72
1950	— 71.74		—258.72	
		+10.92		—7.89
1960	60.82		266.61	
		10.85		8.07
1970	49.97		274.68	
		10.78		8.25
1980	39.19		282.93	
		10.70		8.42
1990	28.49		291.35	
		10.61		8.60
2000	— 17.88		—299.95	

NOTE.—The values of (*v.s.*1) and (*v.c.*1) must be taken from only *one* of the two tables XVII *a* and XVII *b*.

TABLE XVII *b*.—*Concluded.*

Year.	(*v.s.*2)		(*v.c.*2)		(*v.s.*3)	(*v.c.*3)	(ρ.*c.*0)	(ρ.*s.*1)	(ρ.*c.*1)	(ρ.*s.*2)	(ρ.*c.*2)	(ρ.*s.*3)	(ρ.*c.*3)
	″		″		″	″							
1500	—152.85	+0.42	—124.83	—0.11	—10.39	—6.70	+1379	— 891	+2361	—382	—158	— 96	— 80
1510	152.43	0.43	124.94	0.12	10.35	6.70	1369	885	2258	382	165	96	80
1520	152.00	0.43	125.06	0.13	10.31	6.70	1359	881	2154	382	172	96	81
1530	151.57	0.44	125.19	0.15	10.28	6.69	1349	878	2047	381	179	96	81
1540	151.13	0.44	125.34	0.16	10.24	6.69	1338	877	1944	381	187	96	82
1550	—150.69	+0.45	—125.50	—0.17	—10.20	—0.69	+1328	— 878	+1838	—381	—195	— 96	— 82
1560	150.24	0.46	125.67	0.18	10.16	6.69	1318	881	1731	381	203	96	83
1570	149.78	0.46	125.85	0.19	10.13	6.69	1307	886	1623	381	210	96	83
1580	149.32	0.46	126.04	0.20	10.09	6.69	1297	893	1513	382	218	96	84
1590	148.86	0.47	126.24	0.20	10.05	6.70	1286	901	1403	382	226	96	84
1600	—148.39	+0.47	—126.44	—0.22	—10.02	—6.70	+1275	— 911	+1292	—383	—234	— 96	— 85
1610	147.92	0.48	126.66	0.23	9.98	6.70	1264	923	1180	384	242	96	86
1620	147.44	0.48	126.89	0.24	9.94	6.71	1253	937	1067	385	250	96	86
1630	146.96	0.48	127.13	0.25	9.91	6.71	1241	954	954	386	258	96	87
1640	146.48	0.49	127.38	0.26	9.87	6.72	1230	972	840	388	266	96	87
1650	—145.99	+0.49	—127.64	—0.27	— 9.83	—6.73	+1219	— 992	+ 725	—389	—274	— 96	— 88
1660	145.50	0.50	127.91	0.28	9.79	6 74	1208	1014	610	391	282	96	89
1670	145.00	0.50	128.19	0.30	9.75	6.75	1197	1038	495	392	290	96	89
1680	144.50	0.50	128.49	0.31	9.71	6.76	1185	1064	379	394	298	96	90
1690	144.00	0.51	128.80	0.32	9.67	6.77	1174	1092	262	396	306	96	90
1700	—143.49	+0.51	—129.12	—0.33	— 9.63	—6.78	+1163	—1123	+ 145	—398	—314	— 97	— 91
1710	142 98	0.51	129.45	0.34	9.59	6.79	1152	1155	+ 27	400	323	97	92
1720	142.47	0.51	129 79	0.35	9.55	6.80	1141	1189	— 92	403	331	97	92
1730	141.96	0.52	130.14	0.37	9.51	6.81	1130	1226	212	405	339	97	93
1740	141.44	0.51	130.51	0.37	9.47	6.83	1119	1264	333	408	348	98	93
1750	—140.93	+0.52	—130.88	—0.39	— 9.43	—6.84	+1108	—1304	— 454	—411	—356	— 98	— 94
1760	140.41	0.53	131.27	0.40	9.39	6.85	1097	1346	575	414	365	98	95
1770	139.88	0.52	131.67	0.41	9.35	6.87	1086	1391	696	417	373	98	95
1780	139.36	0.53	132.08	0.42	9.31	6.89	1074	1437	817	420	382	99	96
1790	138.83	0.52	132.50	0.43	9.27	6.90	1063	1486	938	424	390	99	96
1800	—138.31	+0.52	—132.93	—0.44	— 9.23	—6.92	+1052	—1536	—1059	—427	—399	— 99	— 97
1810	137.79	0.53	133.37	0.45	9.19	6.93	1041	1588	1180	431	408	99	98
1820	137.26	0.52	133.82	0.46	9.14	6.95	1030	1642	1301	435	416	99	98
1830	136.74	0.53	134.28	0.48	9.10	6.96	1019	1699	1422	439	425	100	99
1840	136.21	0.52	134.76	0.49	9.06	6.98	1008	1758	1544	443	433	100	99
1850	—135.69	+0.52	—135.25	—0.50	— 9.02	—7.00	+ 997	—1819	—1665	—447	—442	—100	—100
1860	135.17	0.52	135.75	0.51	8.98	7.02	986	1881	1786	451	450	100	101
1870	134.65	0.52	136.26	0.52	8.94	7.04	975	1945	1906	456	459	101	101
1880	134.13	0.52	136.78	0.53	8.90	7.07	964	2012	2027	461	467	101	102
1890	133.61	0.52	137.31	0.54	8.86	7.09	953	2081	2147	466	476	101	102
1900	—133.09	+0.52	—137.85	—0.55	— 8.82	—7.12	+ 942	—2151	—2267	—471	—484	—102	—103
1910	132.57	0.51	138.40	0.56	8.78	7.15	931	2223	2386	476	492	102	104
1920	132.06	0.51	138.96	0.57	8.74	7.17	921	2297	2505	481	500	103	104
1930	131.55	0.51	139.53	0.58	8.70	7.20	910	2372	2623	486	509	103	105
1940	131.04	0.50	140.11	0.59	8.66	7.22	900	2449	2741	492	517	103	105
1950	—130.54	+0.50	—140.70	—0.61	— 8.62	—7.25	+ 889	—2529	—2859	—497	—525	—104	—106
1960	130.04	0.49	141.31	0.62	8.58	7.28	879	2611	2976	503	533	104	106
1970	129.55	0.49	141.93	0.63	8.54	7.31	868	2695	3092	509	541	105	107
1980	129.06	0.49	142.56	0.64	8.50	7.34	858	2780	3207	515	550	105	107
1990	128.57	0.48	143.20	0.65	8.46	7.37	848	2867	3321	521	558	106	108
2000	—128.09		—143.85		— 8.42	—7.40	+ 838	—2956	—3434	—527	—566	—106	—108

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TABLE XVIII.—Reduction to the Ecliptic. Argument *u*.

u	*u*	*R*		*u*	*u*	*R*		*u*	*u*	*R*		*u*	*u*	*R*	
°	°	″		°	°	″		°	°	″		°	°	″	
0	180	10.00		45	225	0.63		90	270	10.00		135	315	19.37	
1	181	9.67	−0.33	46	226	0.64	+0.01	91	271	10.33	+0.33	136	316	19.36	−0.01
2	182	9.35	0.32	47	227	0.65	0.01	92	272	10.65	0.32	137	317	19.35	0.01
3	183	9.02	0.33	48	228	0.68	0.03	93	273	10.98	0.33	138	318	19.32	0.03
4	184	8.70	0.32	49	229	0.72	0.04	94	274	11.30	0.32	139	319	19.28	0.04
5	185	8.37	0.33	50	230	0.77	0.05	95	275	11.63	0.33	140	320	19.23	0.05
6	186	8.05	−0.32	51	231	0.84	+0.07	96	276	11.95	+0.32	141	321	19.16	−0.07
7	187	7.73	0.32	52	232	0.91	0.07	97	277	12.27	0.32	142	322	19.09	0.07
8	188	7.42	0.31	53	233	0.99	0.08	98	278	12.58	0.31	143	323	19.01	0.08
9	189	7.10	0.32	54	234	1.09	0.10	99	279	12.90	0.32	144	324	18.91	0.10
10	190	6.80	0.30	55	235	1.20	0.11	100	280	13.20	0.30	145	325	18.80	0.11
11	191	6.49	−0.31	56	236	1.31	+0.11	101	281	13.51	+0.31	146	326	18.69	−0.11
12	192	6.19	0.30	57	237	1.44	0.13	102	282	13.81	0.30	147	327	18.56	0.13
13	193	5.89	0.30	58	238	1.58	0.14	103	283	14.11	0.30	148	328	18.42	0.14
14	194	5.60	0.29	59	239	1.73	0.15	104	284	14.40	0.29	149	329	18.27	0.15
15	195	5.32	0.28	60	240	1.88	0.15	105	285	14.68	0.28	150	330	18.12	0.15
16	196	5.04	−0.28	61	241	2.05	+0.17	106	286	14.96	+0.28	151	331	17.95	−0.17
17	197	4.76	0.28	62	242	2.23	0.18	107	287	15.24	0.28	152	332	17.77	0.18
18	198	4.49	0.27	63	243	2.42	0.19	108	288	15.51	0.27	153	333	17.58	0.19
19	199	4.23	0.26	64	244	2.62	0.20	109	289	15.77	0.26	154	334	17.38	0.20
20	200	3.98	0.25	65	245	2.82	0.20	110	290	16.02	0.25	155	335	17.18	0.20
21	201	3.73	−0.25	66	246	3.04	+0.22	111	291	16.27	+0.25	156	336	16.96	−0.22
22	202	3.49	0.24	67	247	3.26	0.22	112	292	16.51	0.24	157	337	16.74	0.22
23	203	3.26	0.23	68	248	3.49	0.23	113	293	16.74	0.23	158	338	16.51	0.23
24	204	3.04	0.22	69	249	3.73	0.24	114	294	16.96	0.22	159	339	16.27	0.24
25	205	2.82	0.22	70	250	3.98	0.25	115	295	17.18	0.22	160	340	16.02	0.25
26	206	2.62	−0.20	71	251	4.23	+0.25	116	296	17.38	+0.20	161	341	15.77	−0.25
27	207	2.42	0.20	72	252	4.49	0.26	117	297	17.58	0.20	162	342	15.51	0.26
28	208	2.23	0.19	73	253	4.76	0.27	118	298	17.77	0.19	163	343	15.24	0.27
29	209	2.05	0.18	74	254	5.04	0.28	119	299	17.95	0.18	164	344	14.96	0.28
30	210	1.88	0.17	75	255	5.32	0.28	120	300	18.12	0.17	165	345	14.68	0.28
31	211	1.73	−0.15	76	256	5.60	+0.28	121	301	18.27	+0.15	166	346	14.40	−0.28
32	212	1.58	0.15	77	257	5.89	0.29	122	302	19.42	0.15	167	347	14.11	0.29
33	213	1.44	0.14	78	258	6.19	0.30	123	303	18.56	0.14	168	348	13.81	0.30
34	214	1.31	0.13	79	259	6.49	0.30	124	304	18.69	0.13	169	349	13.51	0.30
35	215	1.20	0.11	80	260	6.80	0.31	125	305	18.80	0.11	170	350	13.20	0.31
36	216	1.09	−0.11	81	261	7.10	+0.30	126	306	18.91	+0.11	171	351	12.90	−0.30
37	217	0 99	0.10	82	262	7.42	0.32	127	307	19.01	0.10	172	352	12.58	0.32
38	218	0.91	0.08	83	263	7.73	0.31	128	308	19.09	0.08	173	353	12.27	0.31
39	219	0.84	0.07	84	264	8.05	0.32	129	309	19.16	0.07	174	354	11.95	0.32
40	220	0.77	0.07	85	265	8.37	0.32	130	310	19.23	0.07	175	355	11.63	0.32
41	221	0.72	−0.05	86	266	8.70	+0.33	131	311	19.28	+0.05	176	356	11.30	−0.33
42	222	0.68	0.04	87	267	9.02	0.32	132	312	19.32	0.04	177	357	10.98	0.32
43	223	0.65	0.03	88	268	9.35	0.33	133	313	19.35	0.03	178	358	10.65	0.33
44	224	0.64	0.01	89	269	9.67	0.32	134	314	19.36	0.01	179	359	10.33	0.32
45	225	0.63	0.01	90	270	10.00	0.33	135	315	19.37	0.01	180	360	10.00	0.33

TABLE XIX.—Principal Term of the Latitude. Argument *u*.

u	β			*u*	β			*u*	β		
180°	′ ″		**360°**	**190°**	′ ″		**350°**	**200°**	′ ″		**340°**
0°	0 0.00	8.09	**180°**	**10°**	8 2.82	7·96	**170°**	**20°**	15 50.98	7·60	**160°**
10′	0 8.09	8.09	50′	10′	8 10.78	7·96	50′	10′	15 58.58	7·59	50′
20	0 16.18	8.09	40	20	8 18.74	7·95	40	20	16 6.17	7·58	40
30	0 24.27	8.08	30	30	8 26.69	7·95	30	30	16 13.75	7·57	30
40	0 32.35	8.09	20	40	8 34.64	7·95	20	40	16 21.32	7·56	20
50	0 40.44	8.09	10	50	8 42.59	7·95	10	50	16 28.88	7·55	10
1°	0 48.53	8.09	**179°**	**11°**	8 50.54	7·94	**169°**	**21°**	16 36.43	7·55	**159°**
10′	0 56.62	8.08	50′	10′	8 58.48	7·93	50′	10′	16 43.98	7·54	50′
20	1 4.70	8.09	40	20	9 6.41	7·93	40	20	16 51.52	7·53	40
30	1 12.79	8.08	30	30	9 14.34	7·92	30	30	16 59.05	7·52	30
40	1 20.87	8.09	20	40	9 22.26	7·92	20	40	17 6.57	7·51	20
50	1 28.96	8.08	10	50	9 30.18	7·91	10	50	17 14.08	7·50	10
2°	1 37.04	8.08	**178°**	**12°**	9 38.09	7·90	**168°**	**22°**	17 21.58	7·49	**158°**
10′	1 45.12	8.08	50′	10′	9 45.99	7·90	50′	10′	17 29.07	7·48	50′
20	1 53.20	8.08	40	20	9 53.89	7·90	40	20	17 36.55	7·48	40
30	2 1.28	8.08	30	30	10 1.79	7·89	30	30	17 44.03	7·47	30
40	2 9.36	8.08	20	40	10 9.68	7·89	20	40	17 51.50	7·46	20
50	2 17.44	8.08	10	50	10 17.57	7·89	10	50	17 58.96	7·45	10
3°	2 25.52	8.08	**177°**	**13°**	10 25.46	7·88	**167°**	**23°**	18 6.41	7·44	**157°**
10′	2 33.60	8.07	50′	10′	10 33.34	7·87	50′	10′	18 13.85	7·43	50′
20	2 41.67	8.08	40	20	10 41.21	7·87	40	20	18 21.28	7·42	40
30	2 49.75	8.07	30	30	10 49.08	7·86	30	30	18 28.70	7·41	30
40	2 57.82	8.07	20	40	10 56.94	7·86	20	40	18 36.11	7·41	20
50	3 5.89	8.07	10	50	11 4.80	7·85	10	50	18 43.52	7·40	10
4°	3 13.96	8.07	**176°**	**14°**	11 12.65	7·85	**166°**	**24°**	18 50.92	7·39	**156°**
10′	3 22.03	8.06	50′	10′	11 20.50	7·84	50′	10′	18 58.31	7·37	50′
20	3 30.09	8.07	40	20	11 28.34	7·83	40	20	19 5.68	7·37	40
30	3 38.16	8.06	30	30	11 36.17	7·83	30	30	19 13.05	7·35	30
40	3 46.22	8.06	20	40	11 44.00	7·82	20	40	19 20.40	7·35	20
50	3 54.28	8.06	10	50	11 51.82	7·81	10	50	19 27.75	7·33	10
5°	4 2.34	8.05	**175°**	**15°**	11 59.63	7·80	**165°**	**25°**	19 35.08	7·33	**155°**
10′	4 10.39	8.06	50′	10′	12 7.43	7·81	50′	10′	19 42.41	7·31	50′
20	4 18.45	8.05	40	20	12 15.24	7·80	40	20	19 49.72	7·30	40
30	4 26.50	8.06	30	30	12 23.04	7·79	30	30	19 57.02	7·30	30
40	4 34.56	8.05	20	40	12 30.83	7·78	20	40	20 4.32	7·29	20
50	4 42.61	8.04	10	50	12 38.61	7·79	10	50	20 11.61	7·27	10
6°	4 50.65	8.04	**174°**	**16°**	12 46.40	7·78	**164°**	**26°**	20 18.88	7·26	**154°**
10′	4 58.69	8.04	50′	10′	12 54.18	7·77	50′	10′	20 26.14	7·26	50′
20	5 6.73	8.03	40	20	13 1.95	7·75	40	20	20 33.40	7·24	40
30	5 14.76	8.03	30	30	13 9.70	7·75	30	30	20 40.64	7·24	30
40	5 22.79	8.03	20	40	13 17.45	7·74	20	40	20 47.88	7·22	20
50	5 30.82	8.03	10	50	13 25.19	7·74	10	50	20 55.10	7·21	10
7°	5 38.85	8.03	**173°**	**17°**	13 32.93	7·73	**163°**	**27°**	21 2.31	7·20	**153°**
10′	5 46.88	8.02	50′	10′	13 40.66	7·73	50′	10′	21 9.51	7·19	50′
20	5 54.90	8.02	40	20	13 48.39	7·72	40	20	21 16.70	7·18	40
30	6 2.92	8.02	30	30	13 56.11	7·71	30	30	21 23.88	7·17	30
40	6 10.94	8.01	20	40	14 3.82	7·70	20	40	21 31.05	7·16	20
50	6 18.95	8.01	10	50	14 11.52	7·69	10	50	21 38.21	7·14	10
8°	6 26.96	8.01	**172°**	**18°**	14 19.21	7·69	**162°**	**28°**	21 45.35	7·14	**152°**
10′	6 34.97	8.00	50′	10′	14 26.90	7·68	50′	10′	21 52.49	7·12	50′
20	6 42.97	8.00	40	20	14 34.58	7·67	40	20	21 59.61	7·12	40
30	6 50.97	8.00	30	30	14 42.25	7·67	30	30	22 6.73	7·10	30
40	6 58.97	7·99	20	40	14 49.92	7·66	20	40	22 13.83	7·09	20
50	7 6.96	8.00	10	50	14 57.58	7·65	10	50	22 20.92	7·08	10
9°	7 14.96	7·99	**171°**	**19°**	15 5.23	7·65	**161°**	**29°**	22 28.00	7·07	**151°**
10′	7 22.95	7·98	50′	10′	15 12.88	7·64	50′	10′	22 35.07	7·05	50′
20	7 30.93	7·98	40	20	15 20.52	7·63	40	20	22 42.12	7·05	40
30	7 38.91	7·97	30	30	15 28.15	7·62	30	30	22 49.17	7·03	30
40	7 46.88	7·97	20	40	15 35.77	7·61	20	40	22 56.20	7·03	20
50	7 54.85	7·97	10	50	15 43.38	7·60	10	50	23 3.23	7·01	10
10°	8 2.82		**170°**	**20°**	15 50.98		**160°**	**30°**	23 10.24		**150°**
190°	β		**350°**	**200°**	β		**340°**	**210°**	β		**330°**
			u				*u*				*u*

TABLE XIX, Arg. *u*.—*Continued.*

u	β			*u*	β			*u*	β		
210°	′ ″		**330°**	**220°**	′ ″		**320°**	**230°**	′ ″		**310°**
30°	23 10.24		**150°**	**40°**	29 47.27		**140°**	**50°**	35 29.99		**130°**
10′	23 17.24	7·00	50′	10′	29 53.46	6.19	50′	10′	35 35.18	5·19	50′
20	23 24.23	6.99	40	20	29 59.63	6.17	40	20	35 40.35	5·17	40
30	23 31.20	6.97	30	30	30 5.79	6.16	30	30	35 45.50	5·15	30
40	23 38.17	6.97	20	40	30 11.93	6.14	20	40	35 50.63	5·13	20
50	23 45.12	6.95	10	50	30 13.06	6.13	10	50	35 55.75	5·12	10
31°	23 52.06	6.94	**149°**	**41°**	30 24.17	6.11	**139°**	**51°**	36 0.85	5·10	**129°**
10′	23 58.99	6.93	50′	10′	30 30.27	6.10	50′	10′	36 5.93	5·08	50′
20	24 5.90	6.91	40	20	30 36.35	6.08	40	20	36 11.00	5·07	40
30	24 12.80	6.90	30	30	30 42.41	6.06	30	30	36 16.05	5·05	30
40	24 19.69	6.89	20	40	30 48.46	6.05	20	40	36 21.07	5·02	20
50	24 26.57	6.88	10	50	30 54.49	6.03	10	50	36 26.08	5·01	10
32°	24 33.43	6.86	**148°**	**42°**	31 0.51	6.02	**138°**	**52°**	36 31.07	4·99	**128°**
10′	24 40.28	6.85	50′	10′	31 6.51	6.00	50′	10′	36 36.04	4·97	50′
20	24 47.12	6.84	40	20	31 12.50	5·99	40	20	36 40.99	4·95	40
30	24 53.95	6.83	30	30	31 18.47	5·97	30	30	36 45.93	4·94	30
40	25 0.77	6.82	20	40	31 24.43	5·96	20	40	36 50.84	4·91	20
50	25 7.57	6.80	10	50	31 30.37	5·94	10	50	36 55.73	4·89	10
33°	25 14.36	6.79	**147°**	**43°**	31 36.29	5·92	**137°**	**53°**	37 0.61	4·88	**127°**
10′	25 21.14	6.78	50′	10′	31 42.20	5·91	50′	10′	37 5.47	4·86	50′
20	25 27.90	6.76	40	20	31 48.09	5·89	40	20	37 10.31	4·84	40
30	25 34.65	6.75	30	30	31 53.96	5·87	30	30	37 15.13	4·82	30
40	25 41.39	6.74	20	40	31 59.82	5·86	20	40	37 19.93	4·80	20
50	25 48.11	6.72	10	50	32 5.66	5·84	10	50	37 24.72	4·79	10
34°	25 54.82	6.71	**146°**	**44°**	32 11.49	5·83	**136°**	**54°**	37 29.48	4·76	**126°**
10′	26 1.52	6.70	50′	10′	32 17.30	5·81	50′	10′	37 34.22	4·74	50′
20	26 8.20	6.68	40	20	32 23.09	5·79	40	20	37 38.95	4·73	40
30	26 14.88	6.68	30	30	32 28.87	5·78	30	30	37 43.66	4·71	30
40	26 21.54	6.66	20	40	32 34.63	5·76	20	40	37 48.34	4·68	20
50	26 28.19	6.65	10	50	32 40.38	5·75	10	50	37 53.01	4·67	10
35°	26 34.82	6.63	**145°**	**45°**	32 46.11	5·73	**135°**	**55°**	37 57.66	4·65	**125°**
10′	26 41.44	6.62	50′	10′	32 51.82	5·71	50′	10′	38 2.29	4·63	50′
20	26 48.04	6.60	40	20	32 57.51	5·69	40	20	38 6.90	4·61	40
30	26 54.63	6.59	30	30	33 3.19	5·68	30	30	38 11.49	4·59	30
40	27 1.21	6.58	20	40	33 8.85	5·66	20	40	38 16.07	4·58	20
50	27 7.77	6.56	10	50	33 14.49	5·64	10	50	38 20.62	4·55	10
36°	27 14.32	6.55	**144°**	**46°**	33 20.12	5·63	**134°**	**56°**	38 25.15	4·53	**124°**
10′	27 20.86	6.54	50′	10′	33 25.73	5·61	50′	10′	38 29.66	4·51	50′
20	27 27.38	6.52	40	20	33 31.32	5·59	40	20	38 34.16	4·50	40
30	27 33.89	6.51	30	30	33 36.90	5·58	30	30	38 38.63	4·47	30
40	27 40.39	6.50	20	40	33 42.46	5·56	20	40	38 43.08	4·45	20
50	27 46.87	6.48	10	50	33 48.00	5·54	10	50	38 47.52	4·44	10
37°	27 53.34	6.47	**143°**	**47°**	33 53.53	5·53	**133°**	**57°**	38 51.93	4·41	**123°**
10′	27 59.79	6.45	50′	10′	33 59.04	5·51	50′	10′	38 56.33	4·40	50′
20	28 6.23	6.44	40	20	34 4.53	5·49	40	20	39 0.70	4·37	40
30	28 12.65	6.42	30	30	34 10.00	5·47	30	30	39 5.06	4·36	30
40	28 19.06	6.41	20	40	34 15.46	5·46	20	40	39 9.40	4·34	20
50	28 25.46	6.40	10	50	34 20.90	5·44	10	50	39 13.71	4·31	10
38°	28 31.84	6.38	**142°**	**48°**	34 26.32	5·42	**132°**	**58°**	39 18.01	4·30	**122°**
10′	28 38.21	6.37	50′	10′	34 31.72	5·40	50′	10′	39 22.29	4·28	50′
20	28 44.56	6.35	40	20	34 37.11	5·39	40	20	39 26.54	4·25	40
30	28 50.89	6.33	30	30	34 42.48	5·37	30	30	39 30.78	4·24	30
40	28 57.22	6.33	20	40	34 47.83	5·35	20	40	39 34.99	4·21	20
50	29 3.53	6.31	10	50	34 53.16	5·33	10	50	39 39.19	4·20	10
39°	29 9.82	6.29	**141°**	**49°**	34 58.47	5·31	**131°**	**59°**	39 43.36	4·17	**121°**
10′	29 16.10	6.28	50′	10′	35 3.77	5·30	50′	10′	39 47.52	4·16	50′
20	29 22.36	6.26	40	20	35 9.05	5·28	40	20	39 51.65	4·13	40
30	29 28.61	6.25	30	30	35 14.31	5·26	30	30	39 55.77	4·12	30
40	29 34.85	6.24	20	40	35 19.55	5·24	20	40	39 59.87	4·10	20
50	29 41.07	6.22	10	50	35 24.78	5·23	10	50	40 3.94	4·07	10
40°	29 47.27	6.20	**140°**	**50°**	35 29.99	5·21	**130°**	**60°**	40 8.00	4·06	**120°**
220°	β		**320°** *u*	**230°**	β		**310°** *u*	**240°**	β		**300°** *u*

TABLE XIX, Arg. *u*.—*Continued.*

u	β			*u*	β			*u*	β		
240°	′ ″		**300°**	**250°**	′ ″		**290°**	**260°**	′ ″		**280°**
60°	40 8.00	4·03	**120°**	**70°**	43 32.84	2.76	**110°**	**80°**	45 38.29	1.39	**100°**
10′	40 12.03	4·01	50′	10′	43 35.60	2.73	50′	10′	45 39.68	1.37	50′
20	40 16.04	4·00	40	20	43 38.33	2.71	40	20	45 41.05	1.35	40
30	40 20.04	3·97	30	30	43 41.04	2.69	30	30	45 42.40	1.33	30
40	40 24.01	3·95	20	40	43 43.73	2.67	20	40	45 43.73	1.30	20
50	40 27.96	3·93	10	50	43 46.40	2.64	10	50	45 45.03	1.27	10
61°	40 31.89	3·91	**119°**	**71°**	43 49.04	2.62	**109°**	**81°**	45 46.30	1.26	**99°**
10′	40 35.80	3·89	50′	10′	43 51.66	2.60	50′	10′	45 47.56	1.23	50′
20	40 39.69	3·87	40	20	43 54.26	2.58	40	20	45 48.79	1.21	40
30	40 43.56	3·85	30	30	43 56.84	2.56	30	30	45 50.00	1.18	30
40	40 47.41	3·83	20	40	43 59.40	2.53	20	40	45 51.18	1.16	20
50	40 51.24	3·81	10	50	44 1.93	2.51	10	50	45 52.34	1.14	10
62°	40 55.05	3·79	**118°**	**72°**	44 4.44	2.49	**108°**	**82°**	45 53.48	1.11	**98°**
10′	40 58.84	3·76	50′	10′	44 6.93	2.46	50′	10′	45 54.59	1.09	50′
20	41 2.60	3·75	40	20	44 9.39	2.44	40	20	45 55.68	1.07	40
30	41 6.35	3·72	30	30	44 11.83	2.42	30	30	45 56.75	1.04	30
40	41 10.07	3·71	20	40	44 14.25	2.40	20	40	45 57.79	1.02	20
50	41 13.78	3·68	10	50	44 16.65	2.38	10	50	45 58.81	0.99	10
63°	41 17.46	3·67	**117°**	**73°**	44 19.03	2.35	**107°**	**83°**	45 59.80	0.98	**97°**
10′	41 21.13	3·64	50′	10′	44 21.38	2.33	50′	10′	46 0.78	0.95	50′
20	41 24.77	3·62	40	20	44 23.71	2.31	40	20	46 1.73	0.93	40
30	41 28.39	3·60	30	30	44 26.02	2.28	30	30	46 2.66	0.90	30
40	41 31.99	3·57	20	40	44 28.30	2.27	20	40	46 3.56	0.88	20
50	41 35.56	3·56	10	50	44 30.57	2.25	10	50	46 4.44	0.86	10
64°	41 39.12	3·53	**116°**	**74°**	44 32.82	2.22	**106°**	**84°**	46 5.30	0.84	**96°**
10′	41 42.65	3·52	50′	10′	44 35.04	2.19	50′	10′	46 6.14	0.81	50′
20	41 46.17	3·49	40	20	44 37.23	2.17	40	20	46 6.95	0.79	40
30	41 49.66	3·47	30	30	44 39.40	2.15	30	30	46 7.74	0.76	30
40	41 53.13	3·45	20	40	44 41.55	2.13	20	40	46 8.50	0.74	20
50	41 56.58	3·43	10	50	44 43.68	2.11	10	50	46 9.24	0.72	10
65°	42 0.01	3·41	**115°**	**75°**	44 45.79	2.08	**105°**	**85°**	46 9.96	0.69	**95°**
10′	42 3.42	3·38	50′	10′	44 47.87	2.06	50′	10′	46 10.65	0.67	50′
20	42 6.80	3·37	40	20	44 49.93	2.04	40	20	46 11.32	0.65	40
30	42 10.17	3·34	30	30	44 51.97	2.01	30	30	46 11.97	0.62	30
40	42 13.51	3·32	20	40	44 53.98	1.99	20	40	46 12.59	0.60	20
50	42 16.83	3·30	10	50	44 55.97	1.97	10	50	46 13.19	0.57	10
66°	42 20.13	3·28	**114°**	**76°**	44 57.94	1.95	**104°**	**86°**	46 13.76	0.55	**94°**
10′	42 23.41	3·26	50′	10′	44 59.89	1.92	50′	10′	46 14.31	0.53	50′
20	42 26.67	3·24	40	20	45 1.81	1.90	40	20	46 14.84	0.51	40
30	42 29.91	3·21	30	30	45 3.71	1.88	30	30	46 15.35	0.48	30
40	42 33.12	3·20	20	40	45 5.59	1.85	20	40	46 15.83	0.46	20
50	42 36.32	3·17	10	50	45 7.44	1.83	10	50	46 16.29	0.43	10
67°	42 39.49	3·15	**113°**	**77°**	45 9.27	1.81	**103°**	**87°**	46 16.72	0.41	**93°**
10′	42 42.64	3·13	50′	10′	45 11.08	1.78	50′	10′	46 17.13	0.39	50′
20	42 45.77	3·10	40	20	45 12.86	1.76	40	20	46 17.52	0.37	40
30	42 48.87	3·09	30	30	45 14.62	1.74	30	30	46 17.89	0.34	30
40	42 51.96	3·06	20	40	45 16.36	1.72	20	40	46 18.23	0.32	20
50	42 55.02	3·04	10	50	45 18.08	1.69	10	50	46 18.55	0.29	10
68°	42 58.06	3·02	**112°**	**78°**	45 19.77	1.67	**102°**	**88°**	46 18.84	0.27	**92°**
10′	43 1.08	3·00	50′	10′	45 21.44	1.65	50′	10′	46 19.11	0.25	50′
20	43 4.08	2.97	40	20	45 23.09	1.62	40	20	46 19.36	0.23	40
30	43 7.05	2.96	30	30	45 24.71	1.60	30	30	46 19.59	0.20	30
40	43 10.01	2.93	20	40	45 26.31	1.58	20	40	46 19.79	0.18	20
50	43 12.94	2.91	10	50	45 27.89	1.56	10	50	46 19.97	0.15	10
69°	43 15.85	2.89	**111°**	**79°**	45 29.45	1.53	**101°**	**89°**	46 20.12	0.13	**91°**
10′	43 18.74	2.86	50′	10′	45 30.98	1.51	50′	10′	46 20.25	0.11	50′
20	43 21.60	2.85	40	20	45 32.49	1.48	40	20	46 20.36	0.08	40
30	43 24.45	2.82	30	30	45 33.97	1.46	30	30	46 20.44	0.06	30
40	43 27.27	2.79	20	40	45 35.43	1.44	20	40	46 20.50	0.03	20
50	43 30.06	2.78	10	50	45 36.87	1.42	10	50	46 20.53	0.01	10
70°	43 32.84		**110°**	**80°**	45 38.29		**100°**	**90°**	46 20.54		**90°**
250°	β		**290°**	**260°**	β		**280°**	**270°**	β		**270°**
			u				*u*				*u*

Table	XX.		XXI.					XXII.				
Arg.	1		2					3				
	(*b.s.*1)	(*b.c.*1)	(*b.c.*0)	(*b.s.*1)	(*b.c.*1)	(*b.s.*2)	(*b.c.*2)	(*b.c.*0)	(*b.s.*1)	(*b.c.*1)	(*b.s.*2)	(*b.c.*2)
	″	″	″	″	″	″	″	″	″	″	″	″
0	1.20	1.12	0.04	5.99	5.42	0.21	0.17	0.06	1.58	1.34	0.07	0.17
10	1.15	1.17	0.03	6.13	5.23	0.19	0.18	0.06	1.63	1.14	0.08	0.23
20	1.10	1.21	0.03	6.25	5.04	0.18	0.18	0.05	1.67	0.93	0.11	0.29
30	1.04	1.25	0.02	6.34	4.84	0.17	0.18	0.04	1.68	0.72	0.15	0.32
40	0.98	1.28	0.02	6.41	4.62	0.16	0.18	0.03	1.67	0.51	0.21	0.33
50	0.92	1.31	0.02	6.45	4.40	0.15	0.18	0.03	1.60	0.33	0.27	0.32
60	0.86	1.33	0.02	6.45	4.18	0.15	0.18	0.02	1.48	0.18	0.32	0.28
70	0.79	1.34	0.02	6.40	3.94	0.16	0.18	0.01	1.32	0.09	0.35	0.23
80	0.72	1.35	0.02	6.32	3.69	0.16	0.17	0.00	1.13	0.07	0.36	0.18
90	0.66	1.35	0.03	6.19	3.43	0.17	0.16	0.00	0.94	0.13	0.35	0.13
100	0.59	1.34	0.03	6.02	3.15	0.17	0.15	0.01	0.77	0.25	0.32	0.09
110	0.52	1.32	0.04	5.81	2.87	0.18	0.14	0.02	0.66	0.41	0.29	0.07
120	0.46	1.30	0.04	5.56	2.57	0.19	0.12	0.04	0.62	0.60	0.25	0.06
130	0.40	1.27	0.04	5.29	2.26	0.20	0.11	0.06	0.64	0.77	0.22	0.06
140	0.34	1.24	0.05	4.99	1.96	0.20	0.10	0.08	0.72	0.91	0.20	0.07
150	0.28	1.20	0.05	4.68	1.66	0.20	0.09	0.09	0.83	1.00	0.18	0.08
160	0.23	1.15	0.05	4.36	1.38	0.20	0.08	0.10	0.95	1.04	0.17	0.09
170	0.19	1.10	0.06	4.03	1.11	0.20	0.08	0.11	1.04	1.04	0.17	0.10
180	0.15	1.04	0.06	3.69	0.86	0.19	0.07	0.11	1.11	1.01	0.16	0.10
190	0.12	0.98	0.06	3.36	0.65	0.17	0.07	0.11	1.14	0.97	0.14	0.11
200	0.09	0.92	0.06	3.03	0.56	0.16	0.08	0.11	1.14	0.93	0.13	0.12
210	0.07	0.86	0.06	2.71	0.32	0.14	0.08	0.11	1.11	0.91	0.11	0.13
220	0.06	0.79	0.06	2.39	0.20	0.12	0.09	0.10	1.07	0.91	0.10	0.15
230	0.05	0.72	0.06	2.08	0.12	0.11	0.11	0.10	1.02	0.92	0.10	0.18
240	0.05	0.66	0.07	1.77	0.08	0.09	0.13	0.09	0.98	0.96	0.10	0.21
250	0.06	0.59	0.07	1.48	0.08	0.07	0.15	0.09	0.96	1.00	0.12	0.23
260	0.08	0.52	0.07	1.21	0.10	0.05	0.17	0.08	0.95	1.05	0.14	0.25
270	0.10	0.46	0.08	1.05	0.16	0.03	0.20	0.08	0.94	1.09	0.16	0.26
280	0.13	0.40	0.08	0.72	0.26	0.02	0.22	0.07	0.95	1.13	0.19	0.26
290	0.16	0.34	0.09	0.52	0.38	0.01	0.25	0.06	0.95	1.16	0.22	0.25
300	0.20	0.28	0.10	0.35	0.54	0.01	0.27	0.06	0.94	1.18	0.23	0.23
310	0.25	0.23	0.11	0.22	0.73	0.01	0.29	0.06	0.93	1.20	0.24	0.21
320	0.30	0.19	0.12	0.12	0.94	0.02	0.31	0.07	0.91	1.22	0.23	0.19
330	0.36	0.15	0.13	0.06	1.18	0.03	0.33	0.08	0.88	1.25	0.22	0.18
340	0.42	0.12	0.14	0.04	1.44	0.05	0.35	0.09	0.86	1.28	0.20	0.17
350	0.48	0.09	0.15	0.06	1.72	0.07	0.36	0.09	0.84	1.34	0.18	0.17
360	0.54	0.07	0.16	0.12	2.01	0.10	0.37	0.10	0.85	1.40	0.16	0.18
370	0.61	0.06	0.17	0.21	2.31	0.13	0.37	0.11	0.88	1.46	0.15	0.19
380	0.68	0.05	0.18	0.34	2.62	0.16	0.37	0.12	0.92	1.51	0.14	0.20
390	0.74	0.05	0.19	0.50	2.92	0.19	0.36	0.12	0.98	1.54	0.13	0.21
400	0.81	0.06	0.20	0.70	3.24	0.23	0.35	0.11	1.03	1.54	0.13	0.22
410	0.88	0.08	0.20	0.92	3.54	0.26	0.34	0.10	1.07	1.50	0.12	0.23
420	0.94	0.10	0.21	1.17	3.84	0.30	0.33	0.08	1.06	1.44	0.12	0.25
430	1.00	0.13	0.21	1.45	4.14	0.33	0.31	0.06	1.01	1.37	0.12	0.27
440	1.06	0.16	0.21	1.75	4.42	0.36	0.29	0.04	0.91	1.30	0.12	0.29
450	1.12	0.20	0.21	2.06	4.70	0.38	0.27	0.03	0.77	1.28	0.12	0.32
460	1.17	0.25	0.21	2 39	4.96	0.40	0.25	0.02	0.61	1.31	0.14	0.35
470	1.21	0.30	0.20	2.72	5.20	0.41	0.23	0.01	0.47	1.40	0.17	0.38
480	1.25	0.36	0.19	3.06	5.42	0.42	0.21	0.01	0.36	1.56	0.21	0.39
490	1.28	0.42	0.18	3.39	5.62	0.42	0.20	0.01	0.32	1.76	0.27	0.39
500	1.31	0.48	0.17	3.71	5.79	0.42	0.18	0.01	0.36	1.96	0.32	0.37
510	1.33	0.54	0.16	4.02	5.92	0.41	0.17	0.01	0.46	2.15	0.37	0.33
520	1.34	0.61	0.14	4.31	6.01	0.39	0.16	0.02	0.62	2.21	0.40	0.27
530	1.35	0.68	0.13	4.58	6.06	0.38	0.16	0.02	0.81	2.35	0.40	5.20
540	1.35	0.74	0.11	4.84	6.08	0.35	0.15	0.03	1.00	2.34	0.38	0.14
550	1.34	0.81	0.10	5.07	6.04	0.33	0.15	0.03	1.18	2.26	0.33	0.08
560	1.32	0.88	0.09	5.28	5.98	0.30	0.15	0.04	1.31	2.11	0.27	0.05
570	1.30	0.94	0.07	5.48	5.88	0.28	0.16	0.04	1.41	1.94	0.20	0.04
580	1.27	1.00	0.06	5.67	5.75	0.26	0.16	0.05	1.48	1.74	0.14	0.07
590	1.24	1.06	0.05	5.84	5.59	0.23	0.17	0.06	1.53	1.54	0.10	0.11
600	1.20	1.12	0.04	5.99	5.42	0.21	0.17	0.06	1.58	1.34	0.07	0.17

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TABLE XXIII.

Year.	(b.c.0)	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)	Year.	(b.c.0)	(b.s.1)	(b.c.1)	(b.s.2)	(b.c.2)
	″	″	″	″	″		″	″	″	″	″
1300	+0.66	—3.85	—12.74	—0.34	—0.97	1800	+0.15	—4.60	—6.00	—0.44	—0.46
1310	0.65	3.81	12.65	0.34	0.96	1810	0.14	4.68	5.80	0.45	0.45
1320	0.64	3.78	12.56	0.34	0.95	1820	0.13	4.75	5.61	0.45	0.44
1330	0.62	3.74	12.46	0.34	0.94	1830	0.12	4.83	5.41	0.45	0.43
1340	0.61	3.70	12.37	0.34	0.93	1840	0.11	4.92	5.21	0.46	0.42
1350	+0.60	—3.66	—12.27	—0.34	—0.92	1850	+0.10	—5.00	—5.00	—0.46	—0.41
1360	0.59	3.63	12.17	0.34	0.90	1860	0.09	5.09	4.79	0.47	0.40
1370	0.58	3.60	12.07	0.34	0.89	1870	0.08	5.18	4.58	0.47	0.39
1380	0.57	3.57	11.97	0.34	0.88	1880	0.07	5.27	4.36	0.48	0.38
1390	0.56	3.54	11.86	0.34	0.87	1890	0.06	5.36	4.14	0.48	0.37
1400	+0.55	—3.51	—11.76	—0.35	—0.86	1900	+0.05	—5.45	—3.92	—0.49	—0.36
1410	0.54	3.48	11.65	0.35	0.85	1910	0.04	5.55	3.69	0.49	0.35
1420	0.53	3.46	11.54	0.35	0.84	1920	0.03	5.64	3.46	0.50	0.34
1430	0.52	3.44	11.43	0.35	0.83	1930	0.02	5.74	3.22	0.50	0.33
1440	0.51	3.43	11.32	0.35	0.82	1940	+0.01	5.84	2.97	0.50	0.32
1450	+0.50	—3.42	—11.21	—0.35	—0.80	1950	0.00	—5.94	—2.73	—0.51	—0.30
1460	0.49	3.41	11.10	0.35	0.79	1960	—0.01	6.04	2.48	0.51	0.29
1470	0.48	3.40	10.98	0.36	0.78	1970	0.02	6.14	2.24	0.52	0.28
1480	0.46	3.40	10.86	0.36	0.77	1980	0.04	6.25	1.99	0.52	0.27
1490	0.45	3.40	10.74	0.36	0.76	1990	0.05	6.36	1.73	0.52	0.26
1500	+0.44	—3.40	—10.62	—0.36	—0.75	2000	—0.06	—6.47	—1.47	—0.53	—0.25
1510	0.43	3.40	10.50	0.36	0.74	2010	0.07	6.58	1.21	0.53	0.24
1520	0.42	3.41	10.38	0.36	0.73	2020	0.08	6.69	0.94	0.54	0.23
1530	0.41	3.42	10.26	0.37	0.72	2030	0.09	6.80	0.66	0.54	0.22
1540	0.40	3.43	10.14	0.37	0.71	2040	0.10	6.92	0.38	0.54	0.20
1550	+0.39	—3.45	—10.01	—0.37	—0.70	2050	—0.11	—7.03	—0.10	—0.55	—0.19
1560	0.38	3.47	9.88	0.37	0.70	2060	0.12	7.14	+0.18	0.55	0.18
1570	0.37	3.49	9.75	0.37	0.69	2070	0.13	7.26	0.47	0.56	0.17
1580	0.36	3.51	9.61	0.37	0.68	2080	0.15	7.37	0.76	0.56	0.16
1590	0.35	3.53	9.47	0.38	0.67	2090	0.16	7.48	1.05	0.56	0.15
1600	+0.34	—3.56	— 9.33	—0.38	—0.66	2100	—0.17	—7.60	+1.35	—0.57	—0.14
1610	0.33	3.59	9.19	0.38	0.65	2110	0.18	7.72	1.65	0.57	0.13
1620	0.32	3.62	9.05	0.39	0.64	2120	0.19	7.84	1.96	0.57	0.12
1630	0.32	3.65	8.90	0.39	0.63	2130	0.21	7.97	2.27	0.58	0.10
1640	0.31	3.69	8.75	0.39	0.62	2140	0.22	8.09	2.58	0.58	0.09
1650	+0.30	—3.73	— 8.60	—0.39	—0.61	2150	—0.23	—8.21	+2.90	—0.58	—0.08
1660	0.29	3.77	8.44	0.40	0.60	2160	0.24	8.33	3.22	0.59	0.07
1670	0.28	3.81	8.29	0.40	0.59	2170	0.25	8.44	3.54	0.59	0.06
1680	0.27	3.85	8.13	0.40	0.58	2180	0.27	8.56	3.87	0.59	0.05
1690	0.26	3.90	7.96	0.41	0.57	2190	0.28	8.68	4.20	0.60	0.03
1700	+0.25	—3.95	— 7.79	—0.41	—0.56	2200	—0.29	—8.80	+4.54	—0.60	—0.02
1710	0.24	4.00	7.63	0.41	0.55	2210	0.30	8 92	4.88	0.60	—0.01
1720	0.23	4.06	7.46	0.42	0.54	2220	0.31	9.04	5.23	0.61	0.00
1730	0.22	4.12	7.29	0.42	0.53	2230	0.33	9.16	5 58	0.61	+0.01
1740	0.21	4.18	7.12	0.42	0.52	2240	0.34	9.28	5.94	0.61	0.03
1750	+0.20	—4.25	— 6.95	—0.43	—0.51	2250	—0.35	—9.40	+6.29	—0.62	+0.04
1760	0.19	4.32	6.77	0.43	0.50	2260	0.36	9.52	6.65	0.62	0.05
1770	0.18	4.38	6.58	0.43	0.49	2270	0.38	9.64	7.01	0.63	0.07
1780	0.17	4.45	6.39	0.44	0.48	2280	0.39	9.76	7.37	0.63	0.08
1790	0.16	4.53	6.20	0.44	0.47	2290	0.40	9.88	7.74	0.63	0.10
1800	+0.15	—4.60	— 6.00	—0.44	—0.46	2300	—0.42	—10.00	+8.10	—0.64	+0.11

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TABLE FOR FORMING THE PRODUCTS OF GIVEN NUMBERS BY THE SINE OR COSINE OF A GIVEN ANGLE.

This table is formed for the especial purpose of facilitating the formation of the products $(v.s.3) \sin 3g$, $(v.c.3) \cos 3g$, etc., $(\rho.s.1) \sin g$, $(\rho.c.1) \cos g$, for entire degrees of g. It is so arranged that the required products can be taken out at sight. Supposing the number to be given in seconds and decimal fractions of a second, we first seek the given angle at the top or bottom of the page, and then enter one of the first nine lines of the table with the fraction part of the second, interpolating for the hundredths. We then add the result mentally to the number corresponding to the entire seconds. The algebraic signs at the sides of the angles are those of the sines or cosines corresponding to the angle and to the column above or below. If the number does not exceed 3″ we can enter the table as if it were ten times greater, and remove the decimal point one place to the left in the result.

For example, to find the value of

$$21''.67 \sin 280^\circ + 2''.25 \cos 280^\circ$$

we find the angle 280° at the bottom of a pair of columns, the right hand one being the sine column. Entering this column with 0.67 as the argument, we find 0.66. Entering with 2.1, we find 20.68, to which adding 0.66, we have 21″.34 as the sine product. Entering the other column with 22.5, and moving the decimal point, we find 0″.39 for the cosine product. Noticing the algebraic signs on each side of 280°, we find the result to be $-21''.34 + 0''.39 = -20''.95$.

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	+ 1°+ +179 — — 181 — —359 +		+ 2°+ +178 — —182 — —358 +		+ 3°+ +177 — —183 — —357 +		+ 4°+ + 176 — —184 — — 356 +		+ 5°+ +175 — —185 — —355 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.00	0.10	0.00	0.10	0.01	0.10	0.01	0.10	0.01	0.10	0.1
0.2	0.00	0.20	0.01	0.20	0.01	0.20	0.01	0.20	0.02	0.20	0.2
0.3	0.01	0.30	0.01	0.30	0.02	0.30	0.02	0.30	0.03	0.30	0.3
0.4	0.01	0.40	0.01	0.40	0.02	0.40	0.03	0.40	0.03	0.40	0.4
0.5	0.01	0.50	0.02	0.50	0.03	0.50	0.03	0.50	0.04	0.50	0.5
0.6	0.01	0.60	0.02	0.60	0.03	0.60	0.04	0.60	0.05	0.60	0.6
0.7	0.01	0.70	0.02	0.70	0.04	0.70	0.05	0.70	0.06	0.70	0.7
0.8	0.01	0.80	0.03	0.80	0.04	0.80	0.06	0.80	0.07	0.80	0.8
0.9	0.02	0.90	0.03	0.90	0.05	0.90	0.06	0.90	0.08	0.90	0.9
1.0	0.02	1.00	0.03	1.00	0.05	1.00	0.07	1.00	0.09	1.00	1.0
2.0	0.03	2.00	0.07	2.00	0.10	2.00	0.14	2.00	0.17	1.99	2.0
3.0	0.05	3.00	0.10	3.00	0.16	3.00	0.21	2.99	0.26	2.99	3.0
4.0	0.07	4.00	0.14	4.00	0.21	3.99	0.28	3.99	0.35	3.98	4.0
5.0	0.09	5.00	0.17	5.00	0.26	4.99	0.35	4.99	0.44	4.98	5.0
6.0	0.10	6.00	0.21	6.00	0.31	5.99	0.42	5.99	0.52	5.98	6.0
7.0	0.12	7.00	0.24	7.00	0.37	6.99	0.49	6.98	0.61	6.97	7.0
8.0	0.14	8.00	0.28	8.00	0.42	7 99	0.56	7.98	0.70	7.97	8.0
9.0	0.16	9.00	0.31	8.99	0.47	8.99	0.63	8.98	0.78	8.97	9.0
10.0	0.17	10.00	0.35	9.99	0.52	9.99	0.70	9.98	0.87	9.96	10.0
11.0	0.19	11.00	0.38	10.99	0.58	10.98	0.77	10.97	0.96	10.96	11.0
12.0	0.21	12.00	0.42	11.99	0.63	11.98	0.84	11.97	1.05	11.95	12.0
13.0	0.23	13.00	0.45	12.99	0.68	12.98	0.91	12.97	1.13	12.95	13.0
14.0	0.24	14.00	0.49	13.99	0.73	13.98	0.98	13.97	1.22	13.95	14.0
15.0	0.26	15.00	0.52	14.99	0.79	14.98	1.05	14.96	1.31	14.94	15.0
16.0	0.28	16.00	0.56	15.99	0.84	15.98	1.12	15.96	1.39	15.94	16.0
17.0	0.30	17.00	0.59	16.99	0.89	16.98	1.19	16.96	1.48	16.94	17.0
18.0	0.31	18.00	0.63	17.99	0.94	17.98	1.26	17.96	1.57	17.93	18.0
19.0	0.33	19.00	0.66	18.99	0.99	18.97	1.33	18.95	1.66	18.93	19.0
20.0	0.35	20.00	0.70	19.99	1.05	19.97	1.40	19.95	1.74	19.92	20.0
21.0	0.37	21.00	0.73	20.99	1.10	20.97	1.46	20.95	1.83	20.92	21.0
22.0	0 38	22.00	0.77	21.99	1.15	21.97	1.53	21.95	1.92	21.92	22.0
23.0	0.40	23.00	0.80	22.99	1.20	22.97	1.60	22.94	2.00	22.91	23.0
24.0	0.42	24.00	0.84	23.99	1.26	23.97	1.67	23.94	2.09	23.91	24.0
25.0	0.44	25.00	0.87	24.98	1.31	24.97	1.74	24 94	2.18	24.90	25.0
26.0	0.45	26.00	0.91	25.98	1.36	25.96	1.81	25.94	2.27	25.90	26.0
27.0	0.47	27.00	0.94	26.98	1.41	26.96	1.88	26.93	2.35	26.90	27.0
28.0	0.49	28.00	0.98	27.98	1.47	27.96	1.95	27.93	2.44	27.89	28.0
29.0	0.51	29.00	1.01	28.98	1.52	28.96	2.02	28.93	2.53	28.89	29.0
30.0	0.52	30.00	1.05	29.98	1.57	29.96	2.09	29.93	2.61	29.89	20.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+271°— —269 — — 91 + + 89 +		+272 — —268 — — 92 + + 88 +		+273 — —267 — — 93 + + 87 +		+274 — —266 — — 94 + + 86 +		+275 — —265 — — 95 + + 85 +		

	+ 6°+ + 174 — — 186 — —354 +		+ 7°+ + 173 — — 187 — — 353 +		+ 8°+ + 172 — — 188 — —352 +		+ 9°+ + 171 — — 189 — — 351 +		+ 10°+ + 170 — — 190 — —350 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.01	0.10	0.01	0.10	0.01	0.10	0.02	0.10	0.02	0.10	0.1
0.2	0.02	0.20	0.02	0.20	0.03	0.20	0.03	0.20	0.03	0.20	0.2
0.3	0.03	0.30	0.04	0.30	0.04	0.30	0.05	0.30	0.05	0.30	0.3
0.4	0.04	0.40	0.05	0.40	0.06	0.40	0.06	0.40	0.07	0.39	0.4
0.5	0.05	0.50	0.06	0.50	0.07	0.50	0.08	0.49	0.09	0.49	0.5
0.6	0.06	0.60	0.07	0.60	0.08	0.59	0.09	0.59	0.10	0.59	0.6
0.7	0.07	0.70	0.09	0.69	0.10	0.69	0.11	0.69	0.12	0.69	0.7
0.8	0.08	0.80	0.10	0.79	0.11	0.79	0.13	0.79	0.14	0.79	0.8
0.9	0.09	0.90	0.11	0.89	0.13	0.89	0.14	0.89	0.16	0.89	0.9
1.0	0.10	0.99	0.12	0.99	0.14	0.99	0.16	0.99	0.17	0.98	1.0
2.0	0.21	1.99	0.24	1.99	0.28	1.98	0.31	1.98	0.35	1.97	2.0
3.0	0.31	2.98	0.37	2.98	0.42	2.97	0.47	2.96	0.52	2.95	3.0
4.0	0 42	3.98	0.49	3.97	0.56	3.96	0.63	3.95	0.69	3.94	4.0
5.0	0.52	4.97	0.61	4.96	0.70	4.95	0.78	4.94	0.87	4.92	5.0
6.0	0.63	5.97	0.73	5.96	0.84	5.94	0.94	5.93	1.04	5.91	6.0
7.0	0.73	6.96	0.85	6.95	0.97	6.93	1.10	6.91	1.22	6.89	7.0
8.0	0.84	7.96	0.97	7.94	1.11	7.92	1.25	7.90	1.39	7.88	8.0
9.0	0.94	8.95	1.10	8.93	1.25	8.91	1.41	8.89	1.56	8.86	9.0
10.0	1.05	9.95	1.22	9.93	1.39	9.90	1.56	9.88	1.74	9.85	10.0
11.0	1.15	10.94	1.34	10.92	1.53	10.89	1.72	10.86	1.91	10.83	11.0
12.0	1.25	11.93	1.46	11.91	1.67	11.88	1.88	11.85	2.08	11.82	12.0
13.0	1.36	12.93	1.58	12.90	1.81	12.87	2.03	12.84	2.26	12.80	13.0
14.0	1.46	13.92	1.71	13.90	1.95	13.86	2.19	13.83	2.43	13.79	14.0
15.0	1.57	14.92	1.83	14.89	2.09	14.85	2.35	14.82	2.60	14.77	15.0
16.0	1.67	15.91	1.95	15.88	2.23	15.84	2.50	15.80	2.78	15.76	16.0
17.0	1.78	16.91	2.07	16.87	2.37	16.83	2.66	16.79	2.95	16.74	17.0
18.0	1.88	17.90	2.19	17.87	2.51	17.82	2.82	17.78	3.13	17.73	18.0
19.0	1.99	18.90	2.32	18.86	2.64	18.82	2.97	18.77	3.30	18.71	19.0
20.0	2.09	19.89	2.44	19.85	2.78	19.81	3.13	19.75	3.47	19.70	20.0
21.0	2.20	20.88	2.56	20.84	2.92	20.80	3.29	20.74	3.65	20.68	21.0
22.0	2.30	21.88	2.68	21.84	3.06	21.79	3.44	21.73	3.82	21.67	22.0
23.0	2.40	22.87	2.80	22.83	3.20	22.78	3.60	22.72	3.99	22.65	23.0
24.0	2.51	23.87	2.92	23.82	3.34	23.77	3.75	23.70	4.17	23.64	24.0
25.0	2.61	24.86	3.05	24.81	3.48	24.76	3.91	24.69	4.34	24.62	25.0
26.0	2.72	25.86	3.17	25.81	3.62	25.75	4.07	25.68	4.51	25.61	26.0
27.0	2.82	26.85	3.29	26.80	3.76	26.74	4.22	26.67	4.69	26.59	27.0
28.0	2.93	27.85	3.41	27.79	3.90	27.73	4.38	27.66	4.86	27.57	28.0
29.0	3.03	28.84	3.53	28.78	4.04	28.72	4.54	28.64	5.04	28.56	29.0
30.0	3.14	29.84	3.66	29.78	4.18	29.71	4.69	29.63	5.21	29.54	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 276 — —264 — — 96 + + 84 +		+ 277 — — 263 — — 97 + + 83 +		+ 278 — — 262 — — 93 + + 82 +		+ 279 — — 261 — — 99 + + 81 +		+ 280 — — 260 — — 100 + + 80 +		

	+ 11° + + 169 — — 191 — — 349 +		+ 12° + + 168 — — 192 — — 348 +		+ 13° + + 167 — — 193 — — 347 +		+ 14° + + 166 — — 194 — — 346 +		+ 15° + + 165 — — 195 — — 345 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.03	0.10	0.1
0.2	0.04	0.20	0.04	0.20	0.04	0.19	0.05	0.19	0.05	0.19	0.2
0.3	0.06	0.29	0.06	0.29	0.07	0.29	0.07	0.29	0.08	0.29	0.3
0.4	0.08	0.39	0.08	0.39	0.09	0.39	0.10	0.39	0.10	0.39	0.4
0.5	0.10	0.49	0.10	0.49	0.11	0.49	0.12	0.49	0.13	0.48	0.5
0.6	0.11	0.59	0.12	0.59	0.13	0.58	0.15	0.58	0.16	0.58	0.6
0.7	0.13	0.69	0.15	0.68	0.16	0.68	0.17	0.68	0.18	0.68	0.7
0.8	0.15	0.79	0.17	0.78	0.18	0.78	0.19	0.78	0.21	0.77	0.8
0.9	0.17	0.88	0.19	0.88	0.20	0.88	0.22	0.87	0.23	0.87	0.9
1.0	0.19	0.98	0.21	0.98	0.22	0.97	0.24	0.97	0.26	0.97	1.0
2.0	0.38	1.96	0.42	1.96	0.45	1.95	0.48	1.94	0.52	1.93	2.0
3.0	0.57	2.94	0.62	2.93	0.67	2.92	0.73	2.91	0.78	2.90	3.0
4.0	0.76	3.93	0.83	3.91	0.90	3.90	0.97	3.88	1.04	3.86	4.0
5.0	0.95	4.91	1.04	4.89	1.12	4.87	1.21	4.85	1.29	4.83	5.0
6.0	1.14	5.89	1.25	5.87	1.35	5.85	1.45	5.82	1.55	5.80	6.0
7.0	1.34	6.87	1.46	6.85	1.57	6.82	1.69	6.79	1.81	6.76	7.0
8.0	1.53	7.85	1.66	7 83	1.80	7.79	1.94	7.76	2.07	7.73	8.0
9.0	1.72	8.83	1.87	8.80	2.02	8.77	2.18	8.73	2.33	8.69	9.0
10.0	1.91	9.82	2.08	9.78	2.25	9.74	2.42	9.70	2.59	9.66	10.0
11.0	2.10	10.80	2.29	10.76	2.47	10.72	2.66	10.67	2.85	10.63	11.0
12.0	2.29	11.78	2.49	11.74	2.70	11.69	2.90	11.64	3.11	11.59	12.0
13.0	2.48	12.76	2.70	12.72	2.92	12.67	3.14	12.61	3.36	12.56	13.0
14.0	2.67	13.74	2.91	13.69	3.15	13.64	3.39	13.58	3.62	13.52	14.0
15.0	2.86	14.72	3.12	14.67	3.37	14.62	3.63	14.55	3.88	14.49	15.0
16.0	3.05	15.71	3.33	15.65	3.60	15.59	3.87	15.52	4.14	15.45	16.0
17.0	3.24	16.69	3.53	16.63	3.82	16.56	4.11	16.50	4.40	16.42	17.0
18.0	3.43	17.67	3.74	17.61	4.05	17.54	4.35	17.47	4.66	17.39	18.0
19.0	3.63	18.65	3.95	18.58	4.27	18.51	4.60	18.44	4.92	18.35	19.0
20.0	3.82	19.63	4.16	19.56	4.50	19.49	4.84	19.41	5.18	19.32	20.0
21.0	4.01	20.61	4.37	20.54	4.72	20.46	5.08	20.38	5.44	20.28	21.0
22.0	4.20	21.60	4.57	21.52	4.95	21.44	5.32	21.35	5.69	21.25	22.0
23.0	4.39	22.58	4.78	22.50	5.17	22.41	5.56	22.32	5.95	22.22	23.0
24.0	4.58	23.56	4.99	23.48	5.40	23.38	5.81	23.29	6.21	23.18	24.0
25.0	4.77	24.54	5.20	24.45	5.62	24.36	6.05	24.26	6.47	24.15	25.0
26.0	4.96	25.52	5.41	25.43	5.85	25.33	6.29	25.23	6.73	25.11	26.0
27.0	5.15	26.50	5.61	26.41	6.07	26.31	6.53	26.20	6.99	26.08	27.0
28.0	5.34	27.49	5.82	27.39	6.30	27.28	6.77	27.17	7.25	27.05	28.0
29.0	5.53	28.47	6.03	28.37	6.52	28.26	7.02	28.14	7.51	28.01	29.0
30.0	5.72	29.45	6.24	29.34	6.75	29.23	7.26	29.11	7.76	28.98	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 281 — — 259 — — 101 + + 79 +		+ 282 — — 258 — — 102 + + 78 +		+ 283 — — 257 — — 103 + + 77 +		+ 284 — — 256 — — 104 + + 76 +		+ 285 — — 255 — — 105 + + 75 +		

	+ 16°+ + 164 — — 196 — —344 +		+ 17°+ + 163 — — 197 — — 343 +		+ 18°+ + 162 — — 198 — —342 +		+ 19°+ + 161 — — 199 — — 341 +		+ 20°+ + 160 — —200 — —340 +		
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.03	0.10	0.03	0.10	0.03	0.10	0.03	0.09	0.03	0.09	0.1
0.2	0.06	0.19	0.06	0.19	0.06	0 19	0.07	0.19	0.07	0.19	0.2
0.3	0.08	0.29	0.09	0.29	0.09	0.29	0.10	0.28	0.10	0.28	0.3
0.4	0.11	0.38	0.12	0.38	0.12	0.38	0.13	0.38	0.14	0.38	0.4
0.5	0.14	0.48	0.15	0.48	0.15	0.48	0.16	0.47	0.17	0.47	0.5
0.6	0.17	0.58	0.18	0.57	0.19	0.57	0.20	0.57	0.21	0.56	0.6
0.7	0.19	0.67	0.20	0.67	0.22	0.67	0.23	0.66	0.24	0.66	0.7
0.8	0.22	0.77	0.23	0.77	0.25	0.76	0.26	0.76	0.27	0.75	0.8
0.9	0.25	0.87	0.26	0.86	0.28	0.86	0.29	0.85	0.31	0.85	0.9
1.0	0.28	0.96	0.29	0.96	0.31	0.95	0.33	0.95	0.34	0.94	1.0
2.0	0.55	1.92	0.58	1.91	0.62	1.90	0.65	1.89	0.68	1.88	2.0
3.0	0.83	2.88	0.88	2.87	0.93	2.85	0.98	2.84	1.03	2.82	3.0
4.0	1.10	3.85	1.17	3.83	1.24	3.80	1.30	3.78	1.37	3.76	4.0
5.0	1.38	4.81	1.46	4.78	1.55	4.76	1.63	4.73	1.71	4.70	5.0
6.0	1.65	5.77	1.75	5.74	1.85	5.71	1.95	5.67	2.05	5.64	6.0
7.0	1.93	6.73	2.05	6.69	2.16	6.66	2.28	6.62	2.39	6.58	7.0
8.0	2.21	7.69	2.34	7.65	2.47	7.61	2.60	7.56	2.74	7.52	8.0
9.0	2.48	8.65	2.63	8.61	2.78	8.56	2.93	8.51	3.08	8.46	9.0
10.0	2.76	9.61	2.92	9.56	3.09	9.51	3.26	9.46	3.42	9.40	10.0
11.0	3.03	10.57	3.22	10.52	3.40	10.46	3.58	10.40	3.76	10.34	11.0
12.0	3.31	11.54	3.51	11.48	3.71	11.41	3.91	11.35	4.10	11.28	12.0
13.0	3.58	12.50	3.80	12.43	4.02	12.36	4.23	12.29	4.45	12.22	13.0
14.0	3.86	13.46	4.09	13.39	4.33	13.31	4.56	13.24	4.79	13.16	14.0
15.0	4.13	14.42	4.39	14.34	4.64	14.27	4.88	14.18	5.13	14.10	15.0
16.0	4.41	15.38	4.68	15.30	4.94	15.22	5.21	15.13	5.47	15.04	16.0
17.0	4.69	16.34	4.97	16.26	5.25	16.17	5.53	16.07	5.81	15.97	17.0
18.0	4.96	17.30	5.26	17.21	5.56	17.12	5.86	17.02	6.16	16.91	18.0
19.0	5.24	18.26	5.56	18.17	5.87	18.07	6.19	17.96	6.50	17.85	19.0
20.0	5.51	19.23	5.85	19.13	6.18	19.02	6.51	18.91	6.84	18.79	20.0
21.0	5.79	20.19	6.14	20.08	6.49	19.97	6.84	19.86	7.18	19.73	21.0
22.0	6.06	21.15	6.43	21.04	6.80	20.92	7.16	20.80	7.52	20.67	22.0
23.0	6.34	22.11	6.72	22.00	7.11	21.87	7.49	21.75	7.87	21.61	23.0
24.0	6.62	23.07	7.02	22.95	7.42	22.83	7.81	22.69	8.21	22.55	24.0
25.0	6.89	24.03	7.31	23.91	7.73	23.78	8.14	23.64	8.55	23.49	25 0
26.0	7.17	24.99	7.60	24.86	8.03	24.73	8.46	24.58	8.89	24.43	26.0
27.0	7.44	25.95	7.89	25.82	8.34	25.68	8.79	25.53	9.23	25.37	27.0
28.0	7.72	26.92	8.19	26.78	8.65	26.63	9.12	26.47	9.58	26.31	28.0
29.0	7.99	27.88	8.48	27.73	8.96	27.58	9.44	27.42	9.92	27.25	29.0
30.0	8.27	28.84	8.77	28.69	9.27	28.53	9.77	28.37	10.26	28.19	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 286 — —254 — — 106 + + 74 +		+ 287 — —253 — — 107 + + 73 +		+ 288 — —252 — — 108 + + 72 +		+ 289 — — 251 — — 109 + + 71 +		+ 290 — —250 — — 110 + + 70 +		

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	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0.1	0.07	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.1
0.2	0.13	0.15	0.13	0.15	0.14	0.15	0.14	0.14	0.14	0.14	0.2
0.3	0.20	0.23	0.20	0.22	0.20	0.22	0.21	0.22	0.21	0.21	0.3
0.4	0.26	0.30	0.27	0.30	0.27	0.29	0.28	0.29	0.28	0.28	0.4
0.5	0.33	0.38	0.33	0.37	0.34	0.37	0.35	0.36	0.35	0.35	0.5
0.6	0.39	0.45	0.40	0.45	0.41	0.44	0.42	0.43	0.42	0.42	0.6
0.7	0.46	0.53	0.47	0.52	0.48	0.51	0.49	0.50	0.49	0.49	0.7
0.8	0.52	0.60	0.54	0.59	0.55	0.59	0.56	0.58	0.57	0.57	0.8
0.9	0.59	0.68	0.60	0.67	0.61	0.66	0.63	0.65	0.64	0.64	0.9
1.0	0.66	0.75	0.67	0.74	0.68	0.73	0.69	0.72	0.71	0.71	1.0
2.0	1.31	1.51	1.34	1.49	1.36	1.46	1.39	1.44	1.41	1.41	2.0
3.0	1.97	2.26	2.01	2.23	2.05	2.19	2.08	2.16	2.12	2.12	3.0
4.0	2.62	3.02	2.68	2.97	2.73	2.93	2.78	2.88	2.83	2.83	4.0
5.0	3.28	3.77	3.35	3.72	3.41	3.66	3.47	3.60	3.54	3.54	5.0
6.0	3.94	4.53	4.01	4.46	4.09	4.39	4.17	4.32	4.24	4.24	6.0
7.0	4.59	5.28	4.68	5.20	4.77	5.12	4.86	5.04	4.95	4.95	7.0
8.0	5.25	6.04	5.35	5.95	5.46	5.85	5.56	5.75	5.66	5.66	8.0
9.0	5.90	6.79	6.02	6.69	6.14	6.58	6.25	6.47	6.36	6.36	9.0
10.0	6.56	7.55	6.69	7.43	6.82	7.31	6.95	7.19	7.07	7.07	10.0
11.0	7.22	8.30	7.36	8.17	7.50	8.04	7.64	7.91	7.78	7.78	11.0
12.0	7.87	9.06	8.03	8.92	8.18	8.78	8.34	8.63	8.49	8.49	12.0
13.0	8.53	9.81	8.70	9.66	8.87	9.51	9.03	9.35	9.19	9.19	13.0
14.0	9.18	10.57	9.37	10.40	9.55	10.24	9.73	10.07	9.90	9.90	14.0
15.0	9.84	11.32	10.04	11.15	10.23	10.97	10.42	10.79	10.61	10.61	15.0
16.0	10.50	12.08	10.71	11.89	10.91	11.70	11.11	11.51	11.31	11.31	16.0
17.0	11.15	12.83	11.38	12.63	11.59	12.43	11.81	12.23	12.02	12.02	17.0
18.0	11.81	13.58	12.04	13.38	12.28	13.16	12.50	12.95	12.73	12.73	18.0
19.0	12.47	14.34	12.71	14.12	12.96	13.90	13.20	13.67	13.44	13.44	19.0
20.0	13.12	15.09	13.38	14.86	13.64	14.63	13.89	14.39	14.14	14.14	20.0
21.0	13 78	15.85	14.05	15.61	14.32	15.36	14.59	15.11	14.85	14.85	21.0
22.0	14.43	16.60	14.72	16.35	15.00	16.09	15.28	15.83	15.56	15.56	22.0
23.0	15.09	17.36	15.39	17.09	15.69	16.82	15.98	16.54	16.26	16.26	23.0
24.0	15.75	18.11	16.06	17.84	16.37	17.55	16.67	17.26	16.97	16.97	24.0
25.0	16.40	18.87	16.73	18.58	17.05	18.28	17.37	17.98	17.68	17.68	25.0
26.0	17.06	19.62	17.40	19.32	17.73	19.02	18.06	18.70	18.38	18.38	26.0
27.0	17.71	20.38	18.07	20.06	18.41	19.75	18.76	19.42	19.09	19.09	27.0
28.0	18.37	21.13	18.74	20.81	19.10	20.48	19.45	20.14	19.80	19.80	28.0
29.0	19.03	21.89	19.40	21.55	19.78	21.21	20.15	20.86	20.51	20.51	29.0
30.0	19.68	22.64	20.07	22.29	20.46	21.94	20.84	21.58	21.21	21.21	30.0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
	+ 311 — — 229 — — 131 + + 49 +		+ 312 — — 228 — — 132 + + 48 +		+ 313 — — 227 — — 133 + + 47 +		+ 314 — — 226 — — 134 + + 46 +		+ 315 — — 225 — — 135 + + 45 +		