

More in general, it must be admitted that all of the approaches or methods examined so far have not succeeded in ultimately verifying the Riemann Hypothesis (no matter how theoretically remarkable and mathematically true these results may be), because they still need to be completed, systematised, and properly understood in universal terms (even Complex Analysis has so far achieved either approximate or, at most, limited results).

Herewith attached, You can find a PDF file copy for the **submission of Dr. Federico Tambara's article containing his own problem-solving Essay on the Primes (vs Composite) Millennium Problem** (very recently published in the **Oriental Journal of Physical Sciences** and providing a comparatively simple but efficient arithmetical method for universally identifying both prime and composite numbers of any size, with **ORCID CODE : 0000-0002-9256-123X** , in relation to the Open Problem concerning **B. Riemann's Hypothesis**), which is susceptible of innumerable potential applications. This is the latest of Dr. Prof. Tambara Federico's **16 publications since August 2017**, starting from a worldwide known Essay published in the "**Journal of Pure and Applied Physics & Engineering**", followed up by **10 publications of e-books in "youcanprint.it" between March 2018 and March 2019** (in parallel with **other four publications of e-books in "Omniscriptum" (Lambert Publishing Company)** , two of which were also translated into eight different languages in March 2020 by the same Publishing Company.

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DR. PROF. TAMBARA FEDERICO'S FUNDAMENTAL MATHEMATICAL DISCOVERY ABOUT PRIMES MILLENNIUM PROBLEM AND ITS SOLUTION BY DR. PROF. TAMBARA FEDERICO.

The mathematical real value  $1/2$  corresponding to (real) SIGMA CONSTANT in the Euler-Riemann Zeta Function  $\zeta(s)$  , related to the Riemann Hypothesis:

$$(\zeta(s) = \zeta(1/2 + it)) ,$$

actually coincides with the constant relation between each power of 2 in the geometrical progression of Niccolò Tartaglia's / Blaise Pascal's Triangle: notice that this number is the very first and just the only **even prime**.

This is most of all verified by the following evidences:

- The exponents of 2-based powers are to correspond to **PRIMES** in so that they uninterruptedly give **integer quotos** (that is to say, **they give 0 as their decimal reports in relation to each of both "trivial" and "non-trivial" zeros lying along the so-called CRITICAL LINE of the Riemann Zeta Function**),

whenever their respective dividends are represented by all binomial coefficients in each number line limited by the two infinite symmetrical border diagonals of "One Numbers" (however there is no need to divide all binomial coefficients (to be found along a given number line) by their respective 2-based exponent as the second coefficients, it being sufficient to calculate quotos just up to the **FOURTH BINOMIAL COEFFICIENTS**, as will be explained in detail below). From this viewpoint, much more as a "SYMMETRICAL COMPLEMENT" (in the range of all possible types of quotos between binomial coefficients) than an actual "exception" must be considered the fact that **5-BASED POWERS ( CORRESPONDING TO EXPONENTS OF 2-BASED POWERS FOR EACH LINE OF PASCAL'S / TARTAGLIA'S TRIANGLE ) GIVE SYMMETRICALLY ORDERED DECIMAL QUOTOS ( = n.2 ) , STARTING FROM THEIR SIXTH BINOMIAL COEFFICIENTS .**

- The exponents of 2-based powers (that is to say, all **SECOND BINOMIAL COEFFICIENTS**) are to correspond to (composite) **EVEN NUMBERS** (multiples of 2), in so that they give **decimal quotos at symmetrical intervals**, in each number line of **Tartaglia's / Pascal's Triangle**, starting as early as from the **THIRD BINOMIAL COEFFICIENTS** (the first and the last symmetrical quotos being: **n.5**).
- The exponents of 2-based powers are to correspond to **COMPOSITE ODD NUMBERS** in so that they give decimal quotos at symmetrical intervals, in each number line (of the Triangle), starting as early as from the **FOURTH BINOMIAL COEFFICIENTS** (the first and the last symmetrical ordered decimal quotos being: **n.3 (periodical) ...**), **EXCEPT FOR 5-BASED POWERS**, as will be examined below. From this point of view, much more as a "**SYMMETRICAL COMPLEMENT**" (in the range of all possible types of quotos between binomial coefficients) than as an actual "exception" must be considered the fact that **5-BASED POWERS ( which in their turn correspond to EXPONENTS OF 2-BASED POWERS FOR EACH LINE IN PASCAL'S / TARTAGLIA'S TRIANGLE ) GIVE SYMMETRICALLY ORDERED DECIMAL QUOTOS , STARTING FROM THEIR SIXTH BINOMIAL COEFFICIENTS ( = n.2 ) ;** but as a matter of fact, this mathematical property cannot create any difficulty at all, in that **5-BASED POWERS ( WHICH EACH TIME END BY THEIR OWN BASIC FIGURE 5 , IN PARALLEL WITH WHAT OCCURS IN THE CASE OF 6-BASED POWERS ) ARE TO BE IMMEDIATELY VISUALIZED AS ODD COMPOSITES, WITHOUT NEEDING ANY PREVIOUS FACTORIZATION .**

In the case of **EXPONENT 1** (once again in relation to: **2<sup>1</sup>**), instead, the value of respectively the **FIRST / SECOND ( OR PENULTIMATE / LAST ) BINOMIAL COEFFICIENTS** are to coincide with 1 itself.

Finally, as far as all exponents from 2 upwards are concerned, it must be noticed that both their first and last symmetrical binomial coefficients ( = 1 ) in each number line are to represent any given exponent divided by itself, while both their second and penultimate binomial coefficients are to represent, viceversa, any given exponent divided by 1.

The fact that each coefficient in the Triangle is to represent the sum of two coefficients in the number line above (one on the right and the other on the left sides) can be schematised through the **C n, k** combinatory numbers relating to **Michael Stifel's formula** as follows:

$$C_{n, k} = C_{n-1, k-1} + C_{n-1, k} = (D_{n-1, k-1} + D_{n-1, k}) : P k .$$

And by combining this with **Isaac Newton's binomial formula** :  $(A + B)^n$  ( starting from  $A = 1$  and  $B = 1$  , so that:  $C_{n,0} + C_{n,1} + \dots + C_{n,n}$  ), we have:

$$[C_{n-1,k-1} + C_{n-1,k}] \times [A^{n-k} \times B^k] : P_k = (2^n) \times [C_{n-1,k-1} + C_{n-1,k}],$$

where the sum of terms is given by values of  $k$  ranging from  $0$  up to  $n$  , and where:

$$(A + B)^n = 2^n = C_{n,0} + C_{n,1} + \dots + C_{n,n} .$$

In this case, it is sufficient to calculate :

$$(C_{n-1,k-1} + C_{n-1,k}) \times (C_{n,0} + C_{n,1} + C_{n,2} + C_{n,3}) .$$

No doubt such mathematical guidelines can be used to instruct computer software in order to determine an unlimitedly wide progression of values relative to any **FOURTH BINOMIAL COEFFICIENTS in the Triangle**; but it is only through **dividing these values by their corresponding 2-based exponents** that a **systematical as well as unmistakeable identification of PRIMES vs ODD COMPOSITES** will be made possible (no matter how huge dividends and their respective divisors may be) , as herewith pointed out and verified by Dr. Prof. Tambara Federico himself. In this way the "hard core" of the Primes Millennium Problem has been effectively solved.

**DR. PROF. TAMBARA FEDERICO'S COMPREHENSIVE THEORETICAL DIGRESSION CONCERNING THE PRIMES MILLENNIUM PROBLEM .**

After verifying the biunivocal correspondence between the **Millennium mathematical aenigma concerning PRIME NUMBERS** and the **RIEMANN ZETA FUNCTION** , it is now necessary to outline a detailed **HISTORICAL-MATHEMATICAL AS WELL AS THEORETICAL BACKGROUND** , in order to provide sufficient academic documentation about.

The **Riemann Zeta Function ( or Euler-Riemann Zeta Function )  $\zeta(s)$**  , which plays a pivotal role in Analytic Number Theory and has applications in Physics, Probability Theory, and Applied Statistics, is a **function of a complex variable "s"** conventionally defined as:

$$s = \sigma + it,$$

where, according to the **Riemann Hypothesis**:

$$\zeta(s) = \zeta(1/2 + it).$$

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}},$$

This function, which is defined for  $\sigma > 1$  as a **sum of mathematical functions**, analytically continues the **sum of the Dirichlet Series** (many generalizations of the Riemann Zeta Function, such as **Dirichlet Series, Dirichlet L-Functions, and general L-Functions**, are known) as follows:

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots \quad \sigma = \text{Re}(s) > 1.$$

**This sum of mathematical functions converges when the real part of "s" is greater than "1" and, more in particular,  $\zeta(s)$  always converges according to the following infinite series :**

$$\zeta(s) = \frac{1}{1 - 2^{1-s}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s},$$

Last but not least, the above-reported equation relates **values of the Riemann Zeta Function at the points "s" and "1 - s", in particular relating even positive integers with odd negative integers**. Owing to the **zeros of the sine function**, the functional equation implies that  **$\zeta(s)$  has simple zeros, known as trivial zeros of  $\zeta(s)$ , at any even negative integers  $s = -2n$** .

On the other hand, **when "s" is an even positive integer, the product  $[\sin(\pi s / 2) \times \Gamma(1 - s)]$  on the right is non-zero, because  $\Gamma(1 - s)$  has a simple pole that cancels the simple zero of the sine factor**: incidentally, this relation gives an **equation for calculating  $\zeta(s)$  in the region  $0 < \text{Re}(s) < 1$** .

The Riemann Zeta Function can therefore be defined as the **"analytic continuation of the function defined for  $\sigma > 1$  by the sum of the preceding series"**.

In 1740 **Leonhard Euler** considered the above series for positive integer values of  $s$ , and later Chebyshev extended the definition to  **$\text{Re}(s) > 1$** , which can in its turn be defined as a **prototypical Dirichlet Series that converges absolutely to an analytic function for  $s$  with  $\sigma > 1$  and diverges for all other values of "s"**.

After that, in 1749, Euler also managed to conjecture an equivalent relationship for the Dirichlet Eta Function ( an alternating Zeta Function where the “ $\eta$ -series” is convergent , albeit “non- absolutely”, in the larger half-plane  $s > 0$  ) , first introducing and studying it as a function of a real variable .

Among other things, the values of the Euler-Riemann Zeta Function at even positive integers , that is the Euler Zeta Function  $\zeta(2)$  allowed the solution of the so-called “Basel Problem”.

Later, in 1859, Bernhard Riemann's paper “On the Number of Primes Less Than a Given Magnitude” extended the Euler definition to a COMPLEX VARIABLE , not only proving its meromorphic continuation and functional equation, but also establishing a close relationship between its zeros and the distribution of prime numbers , so that the Zeta Function can also be expressed in the terms of the following INTEGRAL :

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} \frac{x^{s-1}}{e^x - 1} dx$$

where:

$$\Gamma(s) = \int_0^{\infty} x^{s-1} e^{-x} dx$$

(with  $\Gamma(s)$  as the “Gamma Function”) .

This is an equality of meromorphic functions valid on the whole complex “s-plane”, which is holomorphic everywhere except for a simple pole at  $s = 1$  , whose residue is 1 and all positive even integers  $2n$  can be expressed by :

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = (1 - 2^{1-s}) \zeta(s).$$

More in general, for all non-positive integers we have:

$$\zeta(s) = \frac{1}{1 - 2^{1-s}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s},$$

for  $n \geq 0$  (when using the NIST convention according to which  $B_1 = -1/2$ ) .

However, for all odd positive integers no such simple expression is known, although these values are thought to be related to the algebraic K-theory of the integers in relation to special values of L-Functions .

In particular, “ $\zeta$ ” vanishes at the negative even integers where  $B_m = 0$  for all odd “ $m$ ” numbers other than 1 (so as to generate the Zeta Function's “trivial zeros”). Thus, via analytic continuation , we also have:

$$\xi(s) = \frac{1}{2} \pi^{-\frac{s}{2}} s(s-1) \Gamma\left(\frac{s}{2}\right) \zeta(s),$$

what gives a way to assign a finite result to the divergent series "1 + 2 + 3 + 4 + ...", which has been used in certain contexts such as the **String Theory**.

And similarly to the above, this also assigns a finite result to the series "1 + 1 + 1 + 1 + ...", which is employed, for example, in calculating **KINETIC LAYER PROBLEMS OF LINEAR KINETIC EQUATIONS**.

More in particular, Riemann showed that the function defined by the series on the half-plane of convergence can be continued analytically to **all complex values "s" ≠ 1**; while for **"s" = 1** the above-reported series is to correspond to a harmonic series that diverges to +∞, where:

$$\lim_{s \rightarrow 1} (s-1)\zeta(s) = 1.$$

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = (1 - 2^{1-s}) \zeta(s).$$

Just on the basis of all these preliminary universal statements, Dr. Prof. Tambara Federico could notice that the **ELEMENTARY, PIVOTAL AND SYMMETRICALLY DOUBLE "MIRROR" SEQUENCE IN THE FORM OF:**

**[ P(RIME) NUMBERS U "H(YPER)C(OMPOSITE )" NUMBERS (= 0 AND 1, IN SO THAT THEY BOTH CONTAIN INFINITE ROOTS AND POWERS OF THEMSELVES) ] ∩**

**∩ [ P(RIME) NUMBERS U C(OMPOSITE) NUMBERS ]**, that is :

**[ ±2 (= P) U ±1 (= HC) U 0**

**(= HC) U ±1 (= HC) U ±2**

**(= P) ] ∩ [ ±3 (= P) U ±4**

**(= C) U ±5 (= P) U ±6**

**(= C) ] ∩ [ ±7 (= P) U ±8**

**(= C) U ±9 (= C) U ±10**

**(= C) U ±11 (= P) ]**,

also corresponds, both in algebraic and in analytical terms, to **THREE SYMMETRICAL "MIRROR" COMBINATIONS OF REAL NUMBERS**, respectively:

$$\begin{aligned}
& \underline{[P(=-11) / C(=-10) / C(=-9) / C} \\
& \underline{(=-8) / P(=-7) / C(=-6) / P(=-5) / C(=-4) / P(=-3)]} \leftrightarrow [P(=-2) / \\
& \underline{(H)C(=-1) / (H)C(=0) / (H)C} \\
& \underline{(=+1) / P(=+2)]} \leftrightarrow \leftrightarrow [P(=+3) / C(=+4) / P(=+5) / C(=+6) / P \\
& \underline{(=+7) / C(=+8) / C(=+9) / C} \\
& \underline{(=+10) / P(=+11)]},
\end{aligned}$$

WHERE 5 PRIMES OUT OF 23 REAL NUMBERS AS A TOTAL ARE TO BE FOUND, ALL OF THEM BEING IN THEIR TURN CENTRED ON AND INTERCONNECTED BY TWO FURTHER (SYMMETRICALLY REVERSE) "MIRROR" PROGRESSIONS OF REAL NUMBERS , respectively:

$$\begin{aligned}
& \underline{[C(=-6) / P(=-5) / C(=-4) / P(=-3)]} \text{ VS } \underline{[P(=+3) / C(=+4) / P(=+5) / C} \\
& \underline{(=+6)]},
\end{aligned}$$

WHERE 2 PRIMES VS 2 COMPOSITES ( AS WELL AS 2 PRIMES OUT OF 8 REAL NUMBERS AS A TOTAL ) ARE TO BE FOUND , IN PARALLEL WITH THE FOUR FOUNDING NUMBER FIGURES: 0 AND 1 ( THE ONLY TWO "HYPERCOMPOSITES" ) FOLLOWED UP BY 2 AND 3 ( THE TWO VERY FIRST AS WELL AS ONLY CONSECUTIVE PRIMES , WHICH PRECEDE ANY COMPOSITES ALONG WITH ALL GREATER PRIMES ) .

It is therefore self-evident that ALL OTHER INFINITE SEQUENCES OF EITHER SMALLER ( NEGATIVE ) REAL NUMBERS ( LYING "ON THE LEFT SIDE" ) OR GREATER ( POSITIVE ) ONES ( LYING "ON THE RIGHT SIDE" ) ARE TO NECESSARILY BE INDEXED TO THE AFORESAID FUNDAMENTAL , DOUBLE "MIRROR" SEQUENCE :

$$\begin{aligned}
& \underline{[\pm 2(=P) \cup \pm 1(=HC) \cup 0} \\
& \underline{(=HC) \cup \pm 1(=HC) \cup \pm 2} \\
& \underline{(=P)] \cap [\pm 3(=P) \pm 4} \\
& \underline{(=C) \cup \pm 5(=P) \cup \pm 6} \\
& \underline{(=C)] \cap [\pm 7(=P) \cup \pm 8} \\
& \underline{(=C) \cup \pm 9(=C) \cup \pm 10} \\
& \underline{(=C) \cup \pm 11(=P)]},
\end{aligned}$$

WITHOUT ANY MORE POSSIBILITY TO REPRODUCE IT EVER AGAIN , IN THAT THE TWO "HYPERCOMPOSITES" 0 AND 1 , ALONG WITH THE PRIME 5 , WILL NEVER RETURN ONCE MORE .



And this is just the reason for an **INFINITE ASYMPTOTIC DISTRIBUTION OF ALL PRIME NUMBERS LYING OUTSIDE THE MATHEMATICAL VALUES OF THE SEQUENCE REPORTED ABOVE** .

More in particular, as regards the **PIVOTAL SEQUENCE** :

$$\begin{aligned}
 & \underline{[\pm 2 (= P) \cup \pm 1 (= HC) \cup 0} \\
 & \underline{= HC) \cup \pm 1 (= HC) \cup \pm 2} \\
 & \underline{= P)] \cap [\pm 3 (= P) \cup \pm 4} \\
 & \underline{= C) \cup \pm 5 (= P) \cup \pm 6} \\
 & \underline{= C)] \cap [\pm 7 (= P) \cup \pm 8} \\
 & \underline{= C) \cup \pm 9 (= C) \cup \pm 10 (= C) \cup \pm 11 (= P)]} ,
 \end{aligned}$$

it must be pointed out that **11** represents its **UPPERMOST LIMIT** .

In addition to all these considerations, however, Dr. Prof. Tambara Federico has thought it necessary to hint at further mathematical observations and results achieved over time up to our days, as well as to attach **FOUR EXEMPLIFYING DIAGRAMS** , with an aim at getting more effective visualization and analytical documentation of the mathematical concepts outlined above.

As a matter of fact, on the basis of these achievements other **four relevant mathematical standpoints** have been developed and proved in the following terms:

1. The **values at negative integer points (computed by Leonhard Euler in the first half of the eighteenth century without using Complex Analysis, which was not available at the time) are rational numbers and play an important role in the theory of modular forms.**
2. The functional equation shows that **the Riemann Zeta Function has zeros at  $-2, -4, \dots$  and so on, called trivial zeros** (in the sense that their existence is relatively easy to prove: for example, from  **$\sin \pi s/2$  being 0** in the functional equation).
3. The **distribution of non-trivial zeros** yields impressive results concerning prime numbers and related objects in number theory, as it is known that **each non-trivial zero lies in the open strip  $\{s \in \mathbb{C} : 0 < \text{Re}(s) < 1\}$ , or "critical strip"** : from this point of view, the **Riemann Hypothesis** (currently considered as one of the greatest unsolved problems in Mathematics) asserts that **any non-trivial "s" zero has** :

$$\text{Re}(s) = 1/2 ,$$

where **the set  $\{s \in \mathbb{C} : \text{Re}(s) = 1/2\}$  is called the "critical line"** .

4. **The  $\zeta(3)$  Zeta Function can only generate irrational numbers**, as proved by Roger Apéry in 1979.

As specifically concerns the Prime Number Millennium Problem as dealt about and formulated by researchers from the beginning of the twentieth century onwards, it must be first of all remembered that, in 1914, **Godfrey Harold Hardy** proved the fact that  **$\zeta(1/2 + it)$  has infinitely many real zeros** .

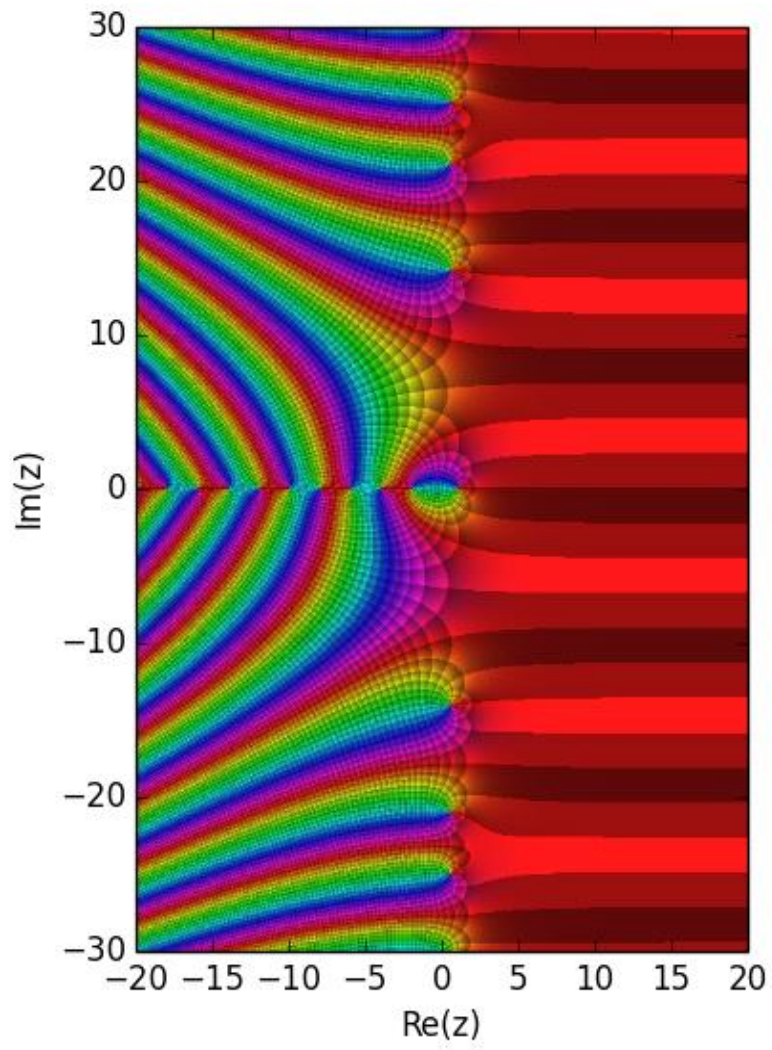
Just starting from this statement, both **G. H. Hardy** and **John Edensor Littlewood** were then to formulate **two conjectures on the density and distance between the zeros of " $\zeta(1/2 + it)$ " on intervals of large positive real numbers** , thus opening up new directions in the investigation of the Riemann Zeta Function.

In particular, by considering  **$N(T)$  as the total number of real zeros and  $N_0(T)$  as the total number of zeros of odd order belonging to the function  $\zeta(1/2 + it)$  and lying in the interval :**

**$[0, T]$**  , they could verify that:

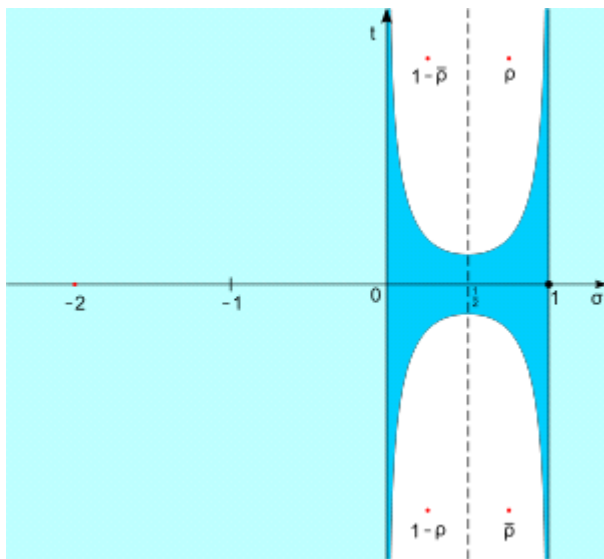
1. **for any  $\epsilon > 0$ , there exists a  $T_0(\epsilon) > 0$  such that when the interval  $[T, T + H]$  contains a zero of odd order ;**
2. **for any  $\epsilon > 0$ , there exists a  $T_0(\epsilon) > 0$  and  $c\epsilon > 0$  such that the inequality holds when the function " $\zeta(1/2 + it)$  lies in the interval  $[0, T]$  .**

As mentioned above, in order to effectively visualize these mathematical concepts, **FOUR DIAGRAMS** , herewith reproduced in colored pictures as well as related to **ZEROS, "CRITICAL LINE", AND RIEMANN HYPOTHESIS** , have been reported below as follows:



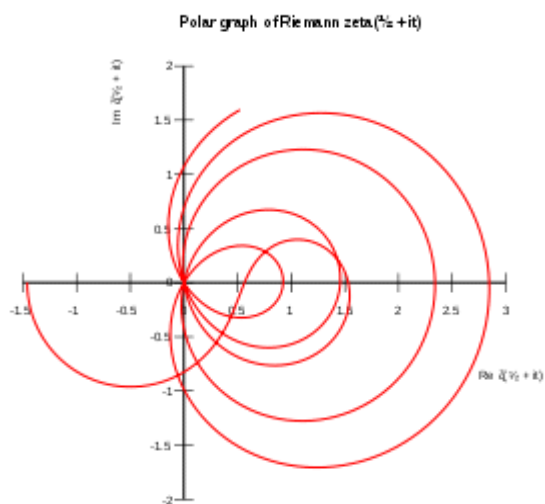
i.

ii.



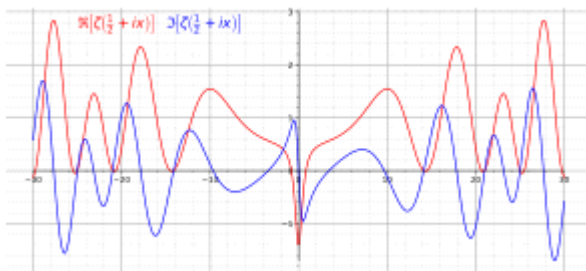
As regards these two diagrams, **apart from the trivial zeros , the Riemann Zeta Function has no zeros to the right of  $\sigma = 1$  and to the left of  $\sigma = 0$  (neither can zeros lie too close to those lines)** . Furthermore, **the non-trivial zeros are symmetric about the real axis and the line  $\sigma = 1/2$  , so that according to the Riemann Hypothesis, these zeros should all lie on the line  $\sigma = 1/2$  )** .

III.



The third diagram shows a plot of **the Riemann Zeta Function along the critical line for real values of “t” running from 0 to 34** . The first five zeros in the critical strip are clearly visible as the place where the spirals pass through the origin:

#### IV.



In this fourth diagram **the real part (in red) and imaginary part ( in blue ) of the Riemann Zeta Function along the critical line  $\text{Re}(s) = 1/2$**  , so that the first non-trivial zeros can, for example, be seen at:  **$\text{Im}(s) = \pm 14.135$  ,  $\pm 21.022$  , and  $\pm 25.011$**  .

As a matter of fact, if we approach from **numbers greater than 1** , this is to coincide with the **harmonic series where the Cauchy principal value is the Euler–Mascheroni Constant  $\gamma = 0.5772\dots$**  , which is employed in calculating the **critical temperature for a Bose–Einstein condensate** in a box with periodic boundary conditions, as well in calculating **kinetic boundary layer problems of linear kinetic equations and spin wave physics in magnetic systems** .

The demonstration of this equality is known as the so-called “**Basel Problem**”. The reciprocal of this sum answers the question: “*What is the probability that two numbers selected at random are **relatively prime**?*” Among other things, this number is called **Apéry's Constant**, which clearly appears when integrating the **Planck Law** to derive the **Stefan–Boltzmann Law** in Physics.

The connection between the **Zeta Function** and prime numbers was discovered by Euler, who employed his own **product formula** in order to prove the identity:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}},$$

where, by definition, the left hand side is  $\zeta(s)$  and the infinite product on the right hand side extends over **all prime numbers "p"** (such expressions are called "**Euler's products**") :

$$\prod_{p \text{ prime}} \frac{1}{1-p^{-s}} = \frac{1}{1-2^{-s}} \cdot \frac{1}{1-3^{-s}} \cdot \frac{1}{1-5^{-s}} \cdot \frac{1}{1-7^{-s}} \cdot \frac{1}{1-11^{-s}} \cdots \frac{1}{1-p^{-s}} \cdots$$

**Both sides of the Euler product formula converge for  $\text{Re}(s) > 1$**  , but the proof of Euler's identity uses only the formula for the geometric series and the fundamental Theorem of Arithmetics: **since the harmonic series obtained when "s" = 1 diverges , Euler's formula :  $\prod_p (p/p - 1)$  , implies that there are infinitely many primes .**

The Euler product formula can be used to calculate the **asymptotic probability that "s" randomly selected integers are set-wise coprimes**. Intuitively, the probability that any single number is divisible by a prime (or any integer) "p" is:  $1/p$ . Hence **the probability that "s" numbers are all divisible by this prime is  $1/p^s$  , and the probability that at least one of them is not is  $1 - p^{-s}$**  .

Now, for distinct primes, these divisibility events are mutually independent because the candidate divisors are coprime: consequently, **a number is divisible by coprime divisors "n" and "m" if and only if it is divisible by "nm", an event which occurs with probability  $1/nm$**  . Thus **the asymptotic probability that "s" numbers are coprime is given by a product over all primes .**

The **location of the Riemann Zeta Function's zeros** is of great importance in the Theory of Numbers: the **Prime Number Theorem** is equivalent to the fact that **there are no zeros of the Zeta Function on the "Re(s) = 1 line"** . And an even better result that follows from an effective form of **Vinogradov's Mean-Value Theorem** is that  **$\zeta(\sigma + it) \neq 0$  whenever  $|t| \geq 3$**  : the existence of this "**zero-free region**" represented a further step towards a mathematical proof of the truth of the Riemann Hypothesis. And as specifically regards the **function corresponding to the ARGUMENT OF THE RIEMANN ZETA FUNCTION , that is :  $\arg \zeta(1/2 + it)$**  , this function can be considered as the **INCREMENT OF AN ARBITRARY CONTINUOUS BRANCH OF  $\arg \zeta(s)$  ALONG THE BROKEN LINE RESPECTIVELY JOINING THE POINTS :**

$$[(2, 2) + it] ; [1/2 + it] .$$

Last but not least, the reciprocal of the Zeta Function may be expressed as a Dirichlet Series over the Möbius Function  $\mu(n)$ , for every complex number “ $s$ ” whose real part is greater than 1 .

And there are a number of similar relations involving various well-known multiplicative functions. From this point of view, the Riemann Hypothesis is equivalent to the claim that this expression is valid when the real part of “ $s$ ” is greater than  $1/2$  .

It is also known the existence of infinitely many zeros on the critical line: the “Critical Line Theorem” asserts that a positive proportion of nontrivial zeros lies on the critical line (from this point of view, the Riemann Hypothesis would imply that this proportion is equal to 1) , while in the critical strip, the zero with smallest non-negative imaginary part is  $1/2 + 14.1347251...i$  .

On the other hand, whenever all complex “ $s$ ”  $\neq 1$  , in such cases the Riemann Zeta Function's zeros are symmetric about the real axis. And by combining this symmetry with the Functional Equation, the non-trivial zeros are also symmetric about the critical line  $\text{Re}(s) = 1/2$  .

Among other properties relative to the **Riemann Zeta Function**, the **critical strip** has the remarkable property of **UNIVERSALITY** . This “Zeta-Function universality” states that there exists some location on the critical strip that approximates any holomorphic function arbitrarily well. Since holomorphic functions are very general, this property is quite remarkable. The first proof of universality was provided by Sergei Mikhailovitch Voronin in 1975, whose theorem was later extended to Dirichlet L-functions : their respective original series, if properly rearranged, allow for an extension of the area of convergence.

Consequently, for example, the series converging for  $\text{Re}(s) > 0$  and the series converging for  $\text{Re}(s) > -1$  can be extended to  $\text{Re}(s) > -k$  for any negative integer  $-k$  .

However, another series development concerning the rising factorial valid for the entire complex plane can be recursively used to extend the Dirichlet Series definition to all complex numbers.

And just on the basis of Weierstrass's Factorization Theorem , Hadamard managed to give an infinite product expansion where the product is over the non-trivial zeros  $\rho$  of  $\zeta$  and the letter  $\gamma$  again denotes the Euler–Mascheroni constant , by clearly displaying the simple pole at  $s = 1$  and the trivial zeros at  $-2n$  , what is due to both the gamma function term in the denominator and the non-trivial zeros at  $s = \rho$  .

But in order to ensure **CONVERGENCE** , the product should be taken over “matching pairs” of zeros , so that the factors for a pair of zeros of the form  $\rho$  and  $1 - \rho$  should be combined .

A globally convergent series for the Zeta Function, valid for all complex numbers  $s$  ( except when “ $s$ ” =  $1 + (2\pi)i / (\ln 2)n$  for  $n$  integers ) , was conjectured by Konrad Knopp and proven by Helmut Hasse in 1930 (the series only appeared in an appendix to Hasse's paper, and did not become generally known until it was discussed by Jonathan Sondow in 1994) .

Hasse also proved a globally converging series in the same publication, but research by Iaroslav Blagouchine has found that this latter series was actually first published by Joseph Ser in 1926.

New proofs for both of these results were offered by Demetrios Kanoussis in 2017. Other similar globally convergent series include harmonic numbers,  $H_n$ , along with the “Stirling numbers” of the first kind, the Pochhammer symbol:  $G_n$ , the Gregory coefficients:  $G(k)$  and  $C_n$ ; the Cauchy numbers of the second kind:  $C_n$  ( $C_1 = 1/2, C_2 = 5/12, C_3 = 3/8, \dots$ ), and the Bernoulli polynomials of the second kind:  $\psi_n(a)$ .

Moreover, Peter Borwein has developed an algorithm that applies the Chebyshev polynomials to the Dirichlet Eta Function to produce a very rapidly convergent series suitable for high-precision numerical calculating machines.

The Riemann Zeta Function can also appear in a form similar to the Mellin Transform in an integral over the Gauss–Kuzmin–Wirsing operator acting on  $xs - 1$  (whose context gives rise to a series expansion in terms not of the rising, but of the falling factorial).

There are also a number of related Zeta Functions that can be considered to be generalizations of the Riemann Zeta Function. These include the Hurwitz Zeta Function, whose convergent series representation was given by Helmut Hasse in 1930 and which coincides with the Riemann Zeta Function when  $q = 1$  ( but the lower limit of summation in the Hurwitz Zeta Function is 0, not 1 ), as well as the Dirichlet L-functions and the Dedekind Zeta Function; while the POLYLOGARITHM coincides with the Riemann Zeta Function when “ $z$ ” = 1.

Another series development using the rising factorial has proved itself valid for the entire complex plane, and this series can be used recursively to extend the Dirichlet series definition to all complex numbers.

The Riemann Zeta Function also appears in a form similar to the Mellin transform in an integral over the Gauss–Kuzmin–Wirsing operator acting on  $xs - 1$ , whose context gives rise to a series expansion in terms of the falling factorial. The Lerch transcendent coincides with the Riemann Zeta Function when “ $z$ ” = 1 and “ $q$ ” = 1 ( but the lower limit of summation in the Lerch transcendent is “0”, not “1” ).

The Clausen Function  $C_i(\theta)$  can also be chosen as the real or imaginary part of  $\text{Li}_s(e^{i\theta})$ , while all multiple Zeta Functions can be analytically continued to a  $n$ -dimensional complex space. The special values taken by these functions at positive integer arguments are called “multiple zeta values” by number theorists and have been connected to many different branches in Mathematics and Physics.

In addition, an important role in the field of Mathematical Sciences is nowadays played by some theorems on properties of the Function  $S(t)$ , among which the Mean Value Theorems for  $S(t)$  and its first integral on intervals of the real line.



To conclude this historical-mathematical and theoretical outline relating to Complex Analysis, it must also be remembered that the **applications of Zeta Function** occur in Applied Statistics, like in the cases, respectively, of the **Zipf Law** and the **Zipf–Mandelbrot Law** ; while the **method of Zeta Function regularization** is used as one possible means of regularization of divergent series and divergent integrals in **Quantum Field Theory**. In one notable example, the Riemann Zeta Function shows it up explicitly in one method of calculating the so-called **“Casimir Effect”**. The Zeta Function is also useful for the analysis of dynamical systems. In addition, the Zeta Function evaluated at equidistant positive integers appears in **infinite serial representations of a number of constants** .

Leaving now Complex Analysis out of consideration, a merely empirical, arithmetical-algorithmical method for individuating an infinite progression of primes "by exclusion" (so as to reproduce **Eratosthenes' Siver** in more synthetic as well as functional terms) consists in building up numberless **CONVERGING ARITHMETICAL-ALGORITHMIC PROGRESSIONS ( OR “GRIDS”) OF NUMBERS, THE VERY FIRST OF WHICH ( BASED ON THE ONLY EVEN PRIME , THAT IS 2 ) IS TO INDIVIDUATE ALL POSSIBLE EVEN INTEGERS, WHILE ALL OTHER “GRIDS” ARE TO INDIVIDUATE ANY POSSIBLE COMPOSITE ODD INTEGERS ONE AFTER ANOTHER , IN LINE WITH A “COMPOSITES VS PRIMES” BINARY SYSTEM .**

Herewith the first twenty-six examples of such numerical progressions (ranging from **2** up to **101** as their respective basic constant factors) have been reported, the products of **101** showing extremely interesting **chiastic combinations of number figures** as will be examined below, according to the following list:

**FIRST INFINITE ARITHMETICAL-ALGORITHMIC “GRID” FOR ALL EVEN INTEGERS , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :**

$\sum [ 1/2 + (1/2 + 2/2 n) + (1/2 + 2/2 n) + 1/2 ] , \underline{\text{AND WHERE IF } n = 0 :$

$(1/2 + 2/2 + 1/2) = 2 , \text{ VS ALL ODD NUMBERS THAT ARE EXCLUDED BY } \underline{\sum (2 + 2 + \dots 2n) .}$

**SECOND INFINITE ARITHMETICAL-ALGORITHMIC “GRID” FOR ALL ODD MULTIPLES OF 3 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :**

$\sum [ 1/2 + (1/2 + 6/2 n) + (1/2 + 6/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$

$(1/2 + 2/2 + 3/2) = 3 , \text{ VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY: } \underline{\sum (3 + 6 + \dots 6n) .}$

THIRD INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 5 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (3/2 + 10/2 n) + (3/2 + 10/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 6/2 + 3/2) = 5$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\underline{\sum (5 + 10 + \dots 10n)}$  .

FOURTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 7 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (5/2 + 14/2 n) + (5/2 + 14/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 10/2 + 3/2) = 7$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\underline{\sum (7 + 14 + \dots 14n)}$  .

FIFTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 11 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (9/2 + 22/2 n) + (9/2 + 22/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 18/2 + 3/2) = 11$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\underline{\sum (11 + 22 + \dots 22n)}$  .

SIXTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 13 AS A CONSTANT FACTOR , ALONG WITH EACH OTHER POSSIBLE FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (11/2 + 26/2 n) + (11/2 + 26/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 22/2 + 3/2) = 13$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (13 + 26 + \dots 26n)$  .

SEVENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD INTEGERS MULTIPLE OF 17 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\Sigma [ 1/2 + (15/2 + 34/2 n) + (15/2 + 34/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$  :

$(1/2 + 30/2 + 3/2) = 17$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (17 + 34 + \dots 34n)$  .

EIGHTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD INTEGERS MULTIPLE OF 19 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\Sigma [ 1/2 + (17/2 + 38/2 n) + (17/2 + 38/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$  :

$(1/2 + 34/2 + 3/2) = 19$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (19 + 38 + \dots 38n)$  .

NINTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 23 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\Sigma [ 1/2 + (21/2 + 46/2 n) + (21/2 + 46/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$  :

$(1/2 + 42/2 + 3/2) = 23$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (23 + 46 + \dots 46n)$  .

TENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 29 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (27/2 + 58/2 n) + (27/2 + 58/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$  :

$(1/2 + 54/2 + 3/2) = 29$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (29 + 58 + \dots 58n)$  .

ELEVENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 31 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (29/2 + 62/2 n) + (29/2 + 62/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$  :

$(1/2 + 58/2 + 3/2) = 31$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (31 + 62 + \dots 62n)$  .

TWELFTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 37 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (35/2 + 74/2 n) + (35/2 + 74/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$  :

$(1/2 + 70/2 + 3/2) = 37$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (37 + 74 + \dots 74n)$  .

THIRTEENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 41 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (39/2 + 82/2 n) + (39/2 + 82/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$  :

$(1/2 + 78/2 + 3/2) = 41$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (41 + 82 + \dots 82n)$  .

FOURTEENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 43 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (41/2 + 86/2 n) + (41/2 + 86/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 82/2 + 3/2) = 43$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\underline{\sum (43 + 86 + \dots 86n)}$  .

FIFTEENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 47 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (45/2 + 94/2 n) + (45/2 + 94/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 90/2 + 3/2) = 47$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\underline{\sum (47 + 94 + \dots 94n)}$  .

SIXTEENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 53 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (51/2 + 106/2 n) + (51/2 + 106/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 102/2 + 3/2) = 53$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\underline{\sum (53 + 106 + \dots 106n)}$  .

SEVENTEENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 59 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$$\sum [ 1/2 + (57/2 + 118/2 n) + (57/2 + 118/2 n) + 3/2 ] , \underline{\text{AND WHERE IF } n = 0 :$$

$(1/2 + 114/2 + 3/2) = 59$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (59 + 118 + \dots 118n)$  .

EIGHTEENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 61 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\Sigma [ 1/2 + (59/2 + 122/2 n) + (59/2 + 122/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 118/2 + 3/2) = 61$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (61 + 122 + \dots 122n)$  .

NINETEENTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 67 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\Sigma [ 1/2 + (65/2 + 134/2 n) + (65/2 + 134/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 130/2 + 3/2) = 67$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (67 + 134 + \dots 134n)$  .

TWENTIETH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 71 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\Sigma [ 1/2 + (69/2 + 142/2 n) + (69/2 + 142/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 138/2 + 3/2) = 71$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\Sigma (71 + 142 + \dots 142n)$  .

TWENTY-FIRST INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 73 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (71/2 + 146/2 n) + (71/2 + 146/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :  $(1/2 + 142/2 + 3/2) = 73$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (73 + 146 + \dots 146n)$  .

TWENTY-SECOND INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 79 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (77/2 + 158/2 n) + (77/2 + 158/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 154/2 + 3/2) = 79$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (79 + 158 + \dots 158n)$  .

TWENTY-THIRD INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 83 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (81/2 + 166/2 n) + (81/2 + 166/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 162/2 + 3/2) = 83$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (83 + 166 + \dots 166n)$  .

TWENTY-FOURTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 89 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (87/2 + 178/2 n) + (87/2 + 178/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 174/2 + 3/2) = 89$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (89 + 178 + \dots 178n)$  .

TWENTY-FIFTH INFINITE ARITHMETICAL-ALGORITHMIC "GRID" FOR ALL ODD MULTIPLES OF 97 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (95/2 + 194/2 n) + (95/2 + 194/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 190/2 + 3/2) = 97$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (97 + 194 + \dots 194n)$  .

TWENTY-SIXTH INFINITE ARITHMETICAL-ALGORITHMICAL "GRID" FOR ALL ODD MULTIPLES OF 101 AS A CONSTANT FACTOR , IN COMBINATION WITH EACH OTHER POSSIBLE FACTOR , WHERE PRIME NUMBERS DIVERGE FROM THE SUMMATION :

$\sum [ 1/2 + (99/2 + 198/2 n) + (99/2 + 198/2 n) + 3/2 ]$  , AND WHERE IF  $n = 0$ :

$(1/2 + 198/2 + 3/2) = 101$  , VS ALL THOSE PRIME NUMBERS THAT ARE EXCLUDED BY:  $\sum (101 + 202 + \dots 202n)$  .

**NOTICE :** This "GRID", which individuates ALL PRIMES THAT ARE EXCLUDED BY  $\sum (101 + 202 + \dots 202n)$  while generating ALL POSSIBLE ( BOTH EVEN AND ODD ) COMPOSITE INTEGERS , RESPECTIVELY FROM 202 ( WHOSE PRIME FACTORS ARE : 2 AND 101 ) AND 303 ( WHOSE PRIME FACTORS ARE : 3 AND 101 ) UPWARDS , ALONG WITH A SEQUENCE OF NUMBER FIGURES WHERE THESE ARE DOUBLED ONE AFTER ANOTHER , ACCORDING TO A CHIASTIC PROGRESSION , TO SUCH AN EXTENT AS TO GENERATE NO LESS THAN SEQUENCES OF "PERIODICAL INTEGERS" , like, for example: 202 , 303 , 404 , 505 ... 1,010 , 1,111 , 1,212 ... 2,121 , 2,222 , 2,323 ; ... 8,989 , 9,090 , 9,191 ; ... 10(periodical)... , 11(periodical)... , 12(periodical)... , 21(periodical)... , 22(periodical)... , 23(periodical)... ; ... 89(periodical)... , 90(periodical)... , 91(periodical)... , and so on up to : [ 99(periodical)... ]<sup>99</sup>(periodical)... , which symbolically expresses the greatest possible integer of all .

It must however be pointed out the fact that this traditionally empirical, arithmetical-algorithmical method will each time give only limited results , although it is fundamentally correct and also suitable for assessing ½ as the mathematical value of the real SIGMA CONSTANT (in correlation with the Riemann Hypothesis)

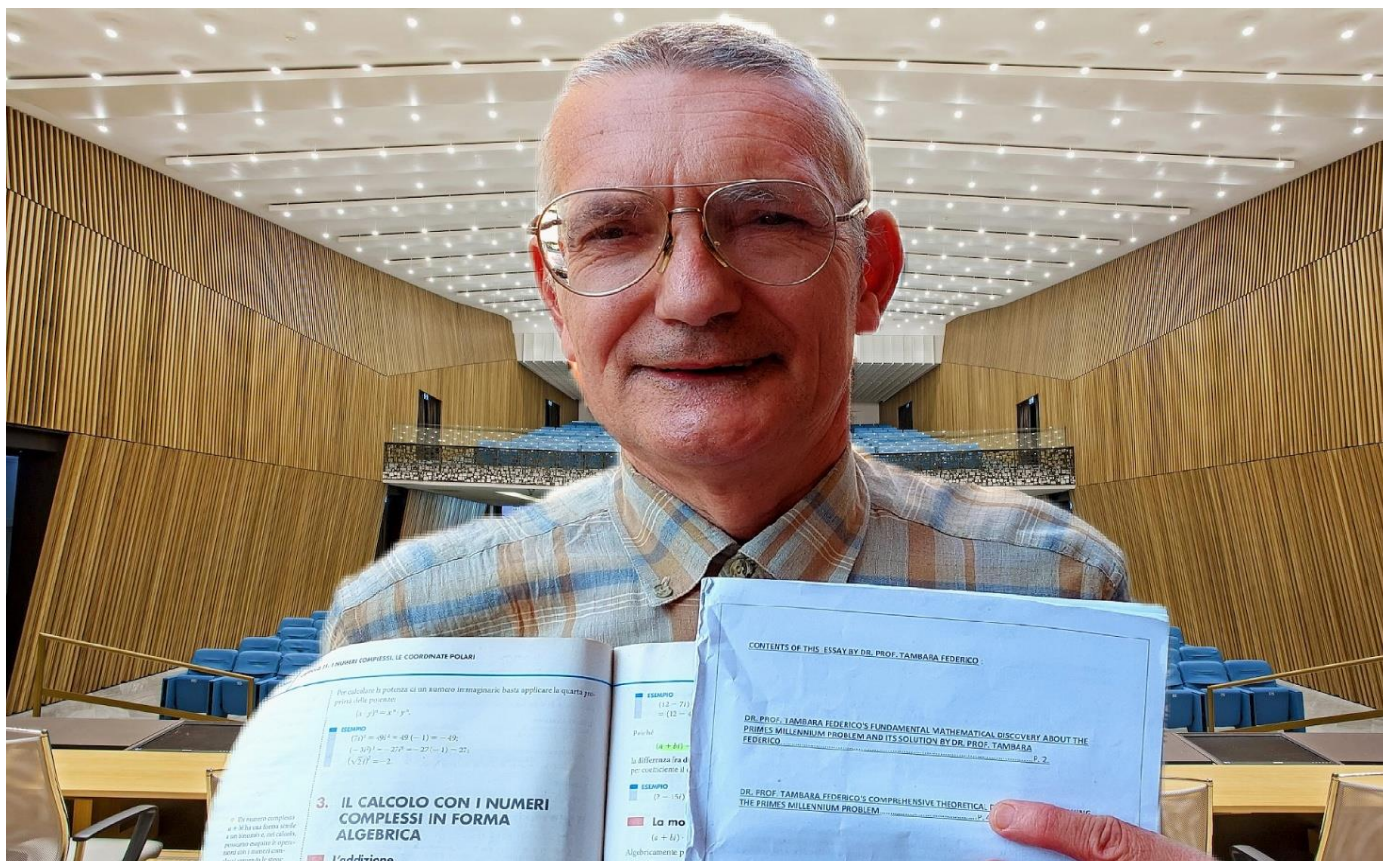


More in general, it must be admitted that all of the approaches or methods examined so far have not succeeded in ultimately verifying the Riemann Hypothesis (**no matter how theoretically remarkable and mathematically true these results may be**), because they still need to be completed, systematised, and properly understood in universal terms (even Complex Analysis has so far achieved either approximate or, at most, limited results).

In spite of all these difficulties, **DR. PROF. TAMBARA FEDERICO'S COMPARATIVELY SIMPLE ARITHMETICAL "DECRYPTION" OF TARTAGLIA'S / PASCAL'S TRIANGLE** can finally provide a comprehensive solution to the Millennium Problem examined above, just in compliance with **DR. PROF. TAMBARA FEDERICO'S MATHEMATICAL "MOTTO"** :

**" ALL COMPOSITES ARE THE 'CHILDREN' OF TWO OR MORE PRIMES ( OR THEIR PRODUCTS ) , BUT THEY ARE ALSO THE 'GRANDCHILDREN' OF 1 , SO THAT PRIMES ARE THE 'CHILDREN' OF 1 "** .

**(END OF THIS MATHEMATICAL ESSAY BY DR. PROF. TAMBARA FEDERICO).**





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Title of paper	DR. PROF. TAMBARA FEDERICO'S SOLUTION TO THE PRIMES MILLENNIUM PROBLEM.
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