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THESIS

A KALMAN FILTER APPLICATION TO THE ADVANCED TACTICAL INERTIAL GUIDANCE SYSTEM OF THE AIR-LAUNCHED LOW VOLUME RAMJET CRUISE MISSILE

by

John Archie Van Devender

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Thesis Advisor: H. A. Titus

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A Montecarlo simulation is conducted to ascertain performance of the ATIGS system in a proposed air-launched cruise missile configuration. The simulation is conducted within a local-level inertial frame consisting of down-range, cross-range and up as primary reference vectors. Efforts are made to measure the relative effects associated with the intended pure position reset provided by a MICRAD sensor as compared with those effects

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by

John A. Van Devender
Lieutenant, United States Navy
B.S.,University of Southern Mississippi,1968

Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

A Montecarlo simulation is conducted to ascertain performance of the ATIGS system in a proposed air-launched cruise missile configuration. The simulation is conducted within a local-level inertial frame consisting of down-range, cross-range and up as primary reference vectors. Efforts are made to measure the relative effects associated with the intended pure position reset provided by a micrad sensor as compared with those effects which could be expected from a linear suboptimal Kalman filtering scheme used in conjunction with the MICRAD sensor

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I. INTRODUCTION

The purpose of the Advanced Tactical Inertial Guidance System(ATIGS) program is to demonstrate the feasibility of a low cost inertial system to be used in the Air Launched Low Volume Ramjet (ALVRJ) cruise missile for mid course guidance. Within the framework of this stated purpose lies the intent to furnish moderate accuracy in a strapdown inertial navigator with high reliability of operation.

The strapdown inertial system requires a computer to provide inertial reference , hence the possibility of extending the computer's capability by installation of Kalman filtering algorithms is seen as an area for investigation. Previous work (ref. 1,2) in this field indicates that the computational burden associated with the Kalman filter limits its usefulness when position updating systems in the missile give highly accurate measurements of actual position. Most of the aforementioned computational burden resulted from the on-line gain generation required by a non-linear model within the Kalman filter. Hence if a linear model with sufficient performance were to be incorporated and the Kalman gains generated off-line and stored, then possibly the velocity estimation errors which are largely unaffected by the position updates could be reduced.

The purpose of this study was then threefold

1) Test a linear model of missile dynamics for use as a simulaticn tool.

2) Determine the inertial navigator accuracy within the six degree of freedom simulation when a pure position reset device is installed which provides position updates at two points along the flight path.

3) Determine improvements in missile performance if a Kalman filtering scheme were installed to estimate missile states between position updates.

The means by which accomplishment of the desired purposes was obtained were various Montecarlo simulations utilizing existing data on the proposed inertial guidance system. Extensive work,both in testing of physical equipment and in simulation, had previously been accomplished by various departments of the Naval Weapons Center, China Lake,California. Hence accurate data as to component performance were available. These data were utilized to construct models of the components for computer simulation.

The simulation of a strapdown inertial guidance system requires the nonlinear computations relating observed accelerations to inertial frame coordinates. This is normally accomplished within the guidance-navigator algorithm by a suitably chosen set of state variables and their related non-linear dynamics. The essential idea behind the linearization technique used in this report is that the non-linear calculations relating accelerations and angular rates to velocity changes within the inertial frame could be accomplished upon observation of the said accelerations and rates and utilized as forcing functions for a linear model of system dynamics. This corresponds to a free inertial system with observations physically aligned in the inertial frame of reference. Thus velocity changes in the inertial frame of the strapdown guidance system would be the non-linear combination of accelerations,angles and

angle rates which are treated as inputs

The model dynamics then are simple linear equations for which Kalman filtering gains can be calculated and stored.

The proof of the above linearization technique would be in comparison of the existing empirical performance of the ATIGS system with the observed corresponding simulation of the system without Kalman filtering installed. Ref.(3) provides ample data of the drift of the ATIGS system as a function of time under actual flight conditions. The data are for a pod mounted version of ATIGS installed on an A-7 aircraft. Information from this report indicated that ground test drift of the system was on the order of 1 nautical mile (nm) per hour under controlled temperature conditions. Under free flight test conditions without temperature control, performance was degraded to 4 nm per hour. The temperature instability was not incorporated into the simulation due to current effort to provide corrective measures within ATIGS. Hence verification of the model was assumed if simulation indicated drifts of 1 to 2 nm per hour.

Once verification of inertial-physical model was assumed, the next phase of observation of effect of pure position update was commenced. Ref.2 in an unclassified portion, contends that the optimal weighting of filtered position estimates and highly accurate measurement of position is such that the filtered estimates are ignored. This being the case, the computational burden imposed by a time varying Kalman filter may be unwarranted. The implicit assumption here is that the velocity errors incurred in an unfiltered system are not significantly decreased by the filter. Therefore the position reset feature would be sufficient to provide required accuracy at mid-course termination. This conclusion was tested by simulation of

missile flight under conditions of increased noise levels within the ATIGS system and comparison with "normal" performance obtained,which allowed visualization of the magnitude of end point error incurred under conditions of pure position reset and different noise level sensors.

The final phase of study was the filtering of the sensor outputs to provide more accurate estimates of velocity throughout the flight. No attempt was made to filter the position update measurements due to their reported accuracy (sigma=50 ft). Thus any gains in performance would have to be from the filtered estimates of position after position update and continuous filtered estimates of velocity. The primary indicator of accuracy in line with ref.2 was taken to be cross range error and cross range error covariance as a function of time.

II. BASIC DESCRIPTION OF THE ATIGS EQUIPMENT UTILIZATION

The approach selected for implementation of ATIGS within an actual cruise missile, was to employ a high-speed processor to handle transformation updating, earth rate torquing and other minor tasks in order to save on time requirements on the more complex navigation and guidance computer. This peripheral processor would then supply the central processing unit(CPU) with the necessary information that it required to compute position within the inertial guidance frame.

Thus the basic ATIGS unit involves ring laser gyros and accelerometers providing information to the peripheral processor wherein after suitable transformation, inertially referenced changes in state variables are supplied to the main navigation-guidance computer. The main computer (CPU) then has an auxiliary input from an external position measuring device, the Microwave Area Correlation fixer(MICRAD). The MICRAD system is designed to provide highly accurate position measurements at two or three preselected checkpoints along the route of flight. These position updates would then be utilized within the CPU to reset the inertial guidance estimate of position.

At present the only filtering system installed is an application to the initial alignment scheme wherein a two stage initialization process is used to align the missile inertial frame with the parent aircraft inertial frame. Filtering is not used presently during midcourse guidance due to the position checkpoint feature and the short time of flight.

A block diagram indicating proposed ATIGS utilization within the ALVRJ is shown in fig.1.

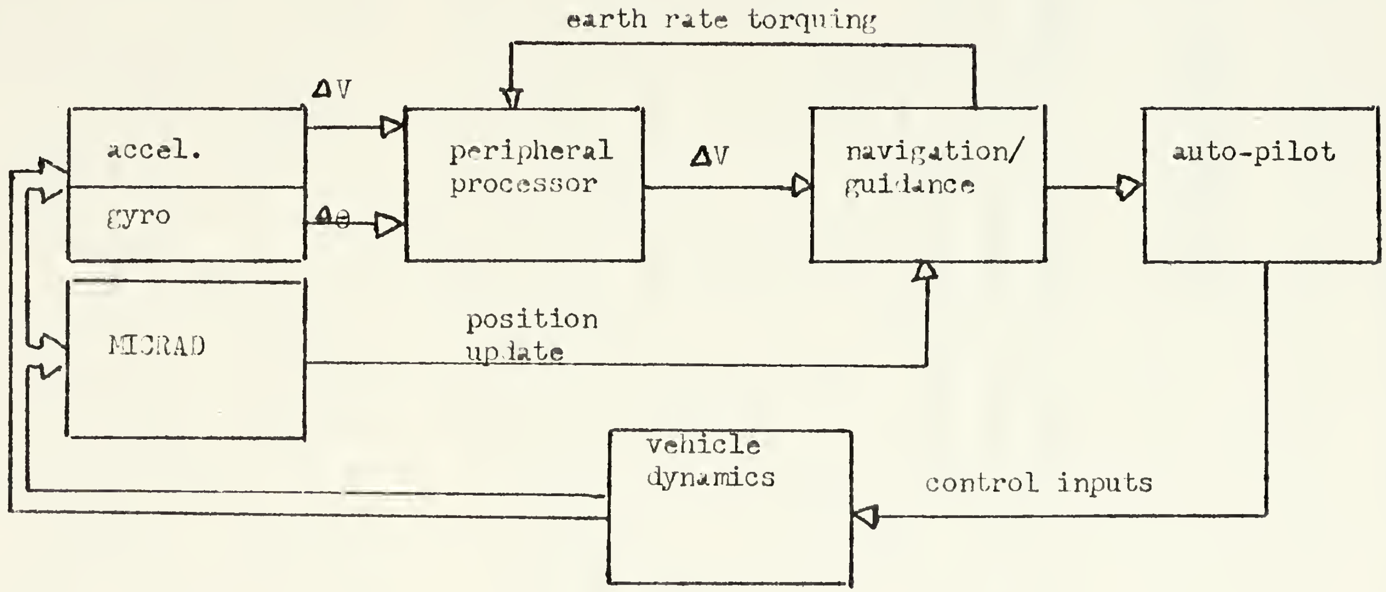

Figure 1 - FUNCTIONAL BLOCK DIAGRAM OF THE ATIGS INSTALLATION WITHIN THE ALVRJ CRUISE MISSILE

III. INERTIAL SYSTEM SIMULATION

A. SYSTEM CONSIDERATIONS

The inertial navigation system incorporated in ATIGS consists of the Honeywell GG-1300 Ring Laser gyro (RLG) and the Sunstrand Q-flex accelerometer acting as sensors. Empirical performance data for these sensors are found in table (1). Both sensors are considered to be of the integrating type in that the output of the RLG is in the form of total angle change per pulse and the output of the accelerometers are total velocity change per pulse. The measurement is accomplished within the RLG by means of a counter system which totals the number of fringe pattern passages during each pulse period and similarly within tne accelerometers, the total velocity change is proportional to the magnitude of the output pulse.

In view of the above characteristics it was felt that the inertial system as diagrammed in fig.4 could be modeled simply and linearly by using the outputs of the sensors as forcing functions vice part of the state vectors. The similarity between fig.1 and fig.4 should be noted. This would result in a net reduction in number of state variables by allowing the missile dynamics to consist of second order equations of displacement and first order equations for angular motion.

B. INERTIAL FRAME OF REFERENCE

1. ALVRJ Implementation

The proposed ATIGS application to the ALVRJ utilises a local-level co-ordinate frame for navigation to the target. The local-level frame is characterized by North,East and up as the respective axis of calculations. The non-spherical nature of the earth introduces an angle calculation which relates the local gravity vector to the position vector of the origin from the earth's center. In the ATIGS unit, the gravity vector calculation is accomplished by an inverse square gravitation model

$$G = -(KM/R^3)\ R \qquad (1)$$

where

G	gravity vector
K	earth's gravitation constant
M	mass of the earth
R	position vector from earth center to vehicle

which approximates the local gravity vector to the desired degree of accuracy.

The vector output of an orthogonal set of accelerometers is the geometric sum of all forces which act upon the vehicle and of course gravity is included. Since the above calculation is dependent on position, then the gravity vector is not constant during the time of flight. Thus to distinguish between the effect of external forces applied to the missile and the change in the gravity vector an equation such as

$$F_a = C_a^i \ddot{R}_i - G \qquad (2)$$

F_a force exerted on instruments

C^i_a coordinate transformation relating inertial axis(i) to accelerometer axis(a)

R_i inertially referenced acceleration

G gravity vector

resolves the time varying accelerometer outputs.

ATIGS accomplishes the above procedure within the missile and calculates the proper direction and range for a direct steer to the target.

2. System Simulation

The simulation of the ATIGS mission began by approximating the local-level inertial frame defined in 1. above as a dcwn-range,cross-range and up frame of reference. Due to the limited range of the missile the gravity vector was considered constant and known, hence the simulation simplified to a simple cartesian co-ordinate space wherein the navigator assumes knowledge of initial position, target position and range to target. The correct heading to the target was assumed to be the positive x-direction with the right-hand system defining positive cross-range accordingly.

The initial position of the inertial frame of reference was taken to be the origin and instantaneous headings were taken to be the difference between the longitudenal body axis and the positive x direction.

The ALVRJ maintains a constant wings level flight and this restriction was also placed upon the simulation.

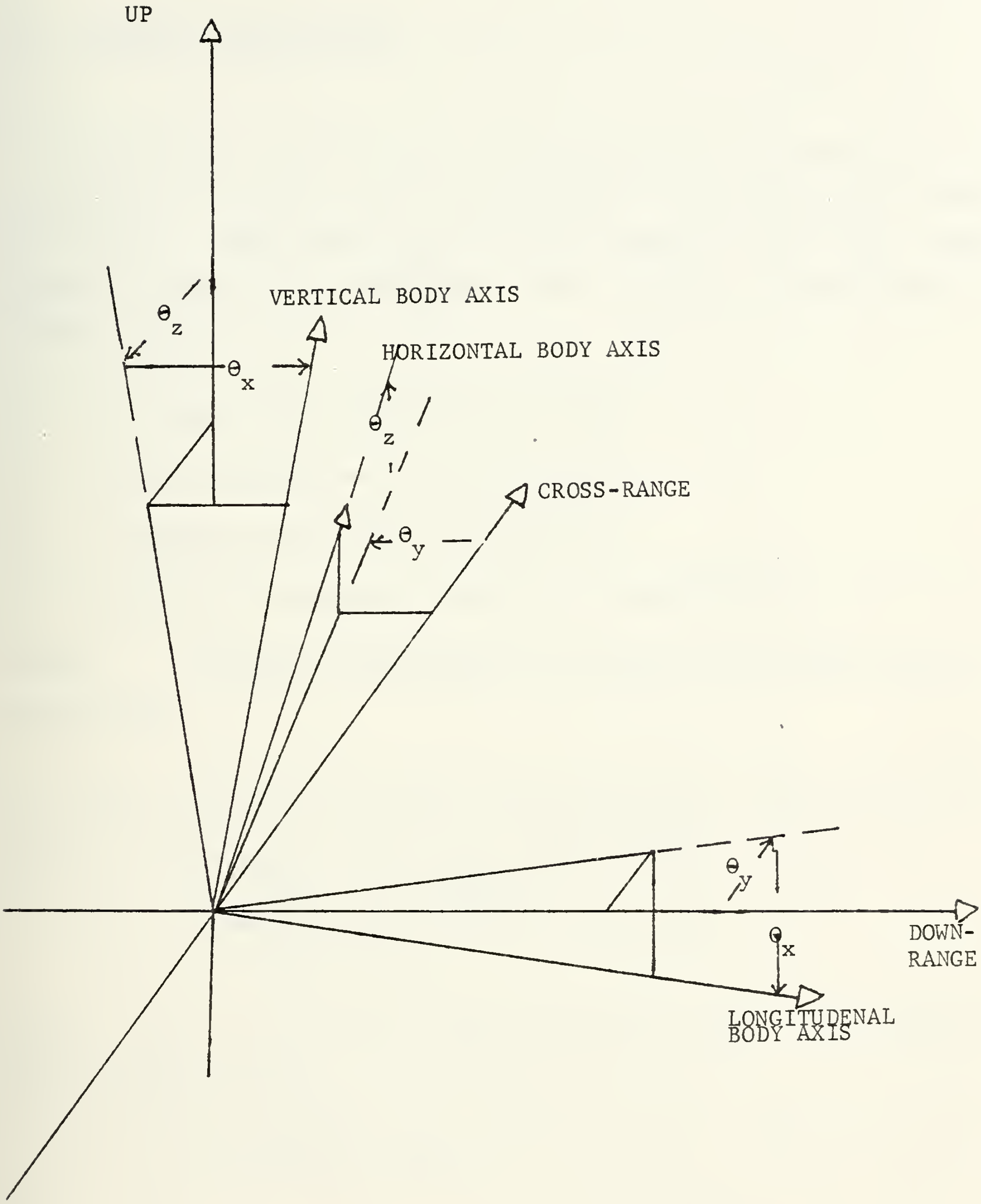

Figure 2 - SIMULATION FRAME OF REFERENCE

C. GENERAL DEVELOPMENT

Defining a set of state variables for an inertial system undergoing arbitrary two dimensional translation and making the careful restriction that small angles and very small angular rates are involved, one obtains (neglecting all noise inputs)

x Distance from origin down-range

$\dot{X}$ Velocity in down-range direction

Y Distance from track centerline

$\dot{Y}$ Velocity component vertical to centerline

Theta Angular displacement of body longitudinal axis to centerline

Due to the small angles and negligible effect of angular rates one can approximate the accelerometer outputs as

$$Z_1 = \text{accel. in longitudenal body axis} = A_{1b} = \Delta V_{1b} \qquad (3)$$
$$Z_2 = \text{accel. in lateral body axis} = A_{2b} = \Delta V_{2b}$$

where the B subscript indicates body axis. The output of the single gyro is

$$Z_3 = \dot{\theta}_1 * \Delta T = \Delta \theta_1 \qquad (4)$$

Now

$$\Delta \dot{X} = \Delta V_{1b} \cos \theta_1 - \Delta V_{2b} \sin \theta_1 + V_{1b} \Delta \cos \theta_1 - V_{2b} \Delta \sin \theta_1$$
$$\qquad (5)$$
$$\Delta \dot{Y} = \Delta V_{1b} \sin \theta_1 + \Delta V_{2b} \cos \theta_1 + V_{1b} \Delta \sin \theta_1 + V_{2b} \Delta \cos \theta_1$$

Where

$$\Delta\dot{X}=Z_1\cos\theta_1-Z_2\sin\theta_1+V_{1b}\Delta\cos\theta_1-V_{2b}\Delta\sin\theta_1 \tag{6}$$

$$\Delta\dot{Y}=Z_1\sin\theta_1+Z_2\cos\theta_1+V_{1b}\Delta\sin\theta_1+V_{2b}\Delta\cos\theta_1$$

Which can be further approximated by

$$\begin{aligned}\Delta\dot{X}&\cong Z_1-Z_2\theta_1-V_{1b}\theta_1\Delta\theta_1-V_{2b}\Delta\theta_1\\ &\cong Z_1-Z_2\theta_1-V_{2b}Z_3\\ \Delta\dot{Y}&\cong Z_1\theta_1+Z_2+V_{1b}\Delta\theta_1-V_{2b}\theta_1\Delta\theta_1\\ &\cong Z_1\theta_1+Z_2+V_{1b}Z_3\end{aligned} \tag{7}$$

Further utilization of small angle approximation yields

$$\begin{aligned}V_{1b}&=\dot{X}\\ V_{2b}&=\dot{Y}\end{aligned} \tag{8}$$

Hence

$$\begin{aligned}\Delta\dot{X}&=Z_1-Z_2\theta_1-\dot{Y}Z_3\\ \Delta\dot{Y}&=Z_1\theta_1+Z_2+\dot{X}Z_3\end{aligned} \tag{8a}$$

and for unit time intervals the discrete state equations are

$$\begin{aligned}X(k+1)&=X(k)+\dot{X}(k)+.5*\Delta\dot{X}(k)\\ \dot{X}(k+1)&=\dot{X}(k)+\Delta\dot{X}(k)\\ Y(k+1)&=Y(k)+\dot{Y}(k)+.5*\Delta\dot{Y}(k)\\ \dot{Y}(k+1)&=\dot{Y}(k)+\Delta\dot{Y}(k)\\ \theta_1(k+1)&=\theta_1(k)+\Delta\theta_1(k)\end{aligned} \tag{9}$$

Thus the observations can be treated as inputs to the system after appropriate substitution. The above model can be expanded to three dimensions and six degrees of freedom by the addition of one cartesian and two angular coordinates which would then consist of

$$Z(k), \dot{Z}(k), \theta_2, \theta_3$$

for a total of 9 states.

This development was accomplished without noise considerations. In the physical system noise would exist in the form of measurement noise in both the accelerometers and the gyros. Hence

$$
\begin{aligned}
Z_1 &= \Delta V_{1b} + \gamma_1 \\
Z_2 &= \Delta V_{2b} + \gamma_2 \qquad (10) \\
Z_3 &= \Delta \theta_1 + \varphi_1
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta\dot{X} &= Z_1 - Z_2\theta_1 - \dot{Y}Z_3 - \gamma_1 - \gamma_2\theta_1 - \dot{Y}\varphi_1 \\
\Delta\dot{Y} &= Z_1\theta_1 + Z_2 + \dot{X}Z_3 - \gamma_1\theta_1 + \gamma_2 + \dot{X}\varphi_1
\end{aligned} \qquad (11)
$$

The purpose of this approach was to utilize the outputs of the sensors as forcing functions for the linear model. Hence in the preprocessor the non-linear calculations involving observations and states can easily be accomplished such that one then obtains

$$
\begin{aligned}
U_1' &= Z_1 - Z_2\theta_1 - \dot{Y}Z_3 \\
U_2' &= Z_1\theta_1 + Z_2 + \dot{X}Z_3 \qquad (12) \\
U_3' &= Z_3
\end{aligned}
$$

as hypothetical and known forcing functions. Thus

substitution into the model of the system of $U_1'(k)$ for $\Delta\dot{x}(k)$ and $U_2'(k)$ for $\Delta\dot{y}(k)$ would result in

$$
\begin{aligned}
X(k+1) &= X(k) + \dot{X}(k) + .5*(\Delta\dot{X}(k) + \gamma_1 \\
&\quad + \gamma_2\theta_1 + \dot{Y}(k)\varphi_1) \\
\dot{X}(k+1) &= \dot{X}(k) + \Delta\dot{X}(k) + \gamma_1 + \gamma_2\theta_1(k) \\
&\quad + \dot{Y}(k)\varphi_1 \qquad (13) \\
Y(k+1) &= Y(k) + \dot{Y}(k) + .5*(\Delta\dot{Y}(k) + \gamma_1\theta_1(k) \\
&\quad + \gamma_2 + \dot{X}(k)\varphi_1) \\
\dot{Y}(k+1) &= \dot{Y}(k) + \Delta\dot{Y}(k) + \gamma_1\theta_1(k) + \gamma_2 \\
&\quad + \dot{X}(k)\varphi \\
\theta_1(k+1) &= \theta_1(k) + \Delta\theta_1(k) + \varphi_1
\end{aligned}
$$

Hence the net result is the addition of a process noise term to the model. Analysis of this noise term proceeds with the systematic elimination of the non-linear term involving γ and θ . This is easily justified due to the small value of γ and θ . Thus one is left with down-range process noise involving γ_1 and $\dot{y}\varphi_1$ and cross-range noise terms involving γ_2 and $\dot{x}\varphi_1$. Clearly the non-linear terms will dominate. The conclusion that logically follows is that the linearization technique utilized above will obviously be accurate for small angles in a manner proportional to the magnitudes of the quantities $\dot{y}\varphi_1$ and $\dot{x}\varphi_1$. Furthermore it indicates that Kalman filtering will be most effective in the estimation of angle information and of much smaller benefit in the filtering of accelerometer noise.

PARAMETER	UNIT	PERFORMANCE	UNITS
RANDOM WALK (^{o}/hr)	GG-1300-RLG	.0075	1 1hr.
BIAS STABILITY (^{o}/hr.)	SAME	.009	1
BIAS SENSITIVITY (^{o}/hr. ^{o}f)	SAME	.00037	
SCALE FACTOR (%)	SAME	.016	
BIAS UNCERTAINTY (ug)	Q-FLEX	79.0	1
SCALE FACTOR (ug/g)	SAME	63	1

TABLE 1-PUBLISHED UNCERTAINTIES OF THE ATIGS COMPONENTS

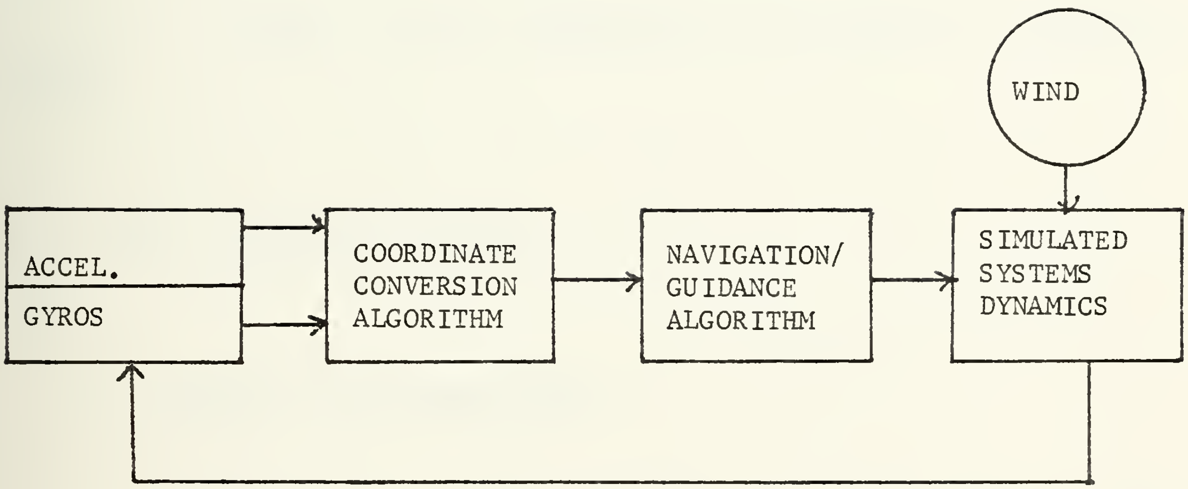

Figure 4 - BLOCK DIAGRAM FOR SIMULATION OF ATIGS INSTALLATION

Thus the total state vector would consist of 9 states as opposed to the 15 state vector considered essential in reference 2. The above technique was inspired by Kortum in his development (ref. 6) on Kalman filter applications. The inertial computation scheme is based on several assumptions, all of which are results of the short time of flight-short range requirements of the ALVRJ application. These assumptions are

1. Constant gravity vector over a 40 nm. flight path

2. Earth torquing not required

3. Small angle assumptions for largest portions of flight

D. BASIC SIMULATION DESIGN

1. General Considerations

In order to verify that the nine state system would be adequate for modeling purposes, a simulation program to test performance was conceived. The actual missile flight profile includes accelerations after launch

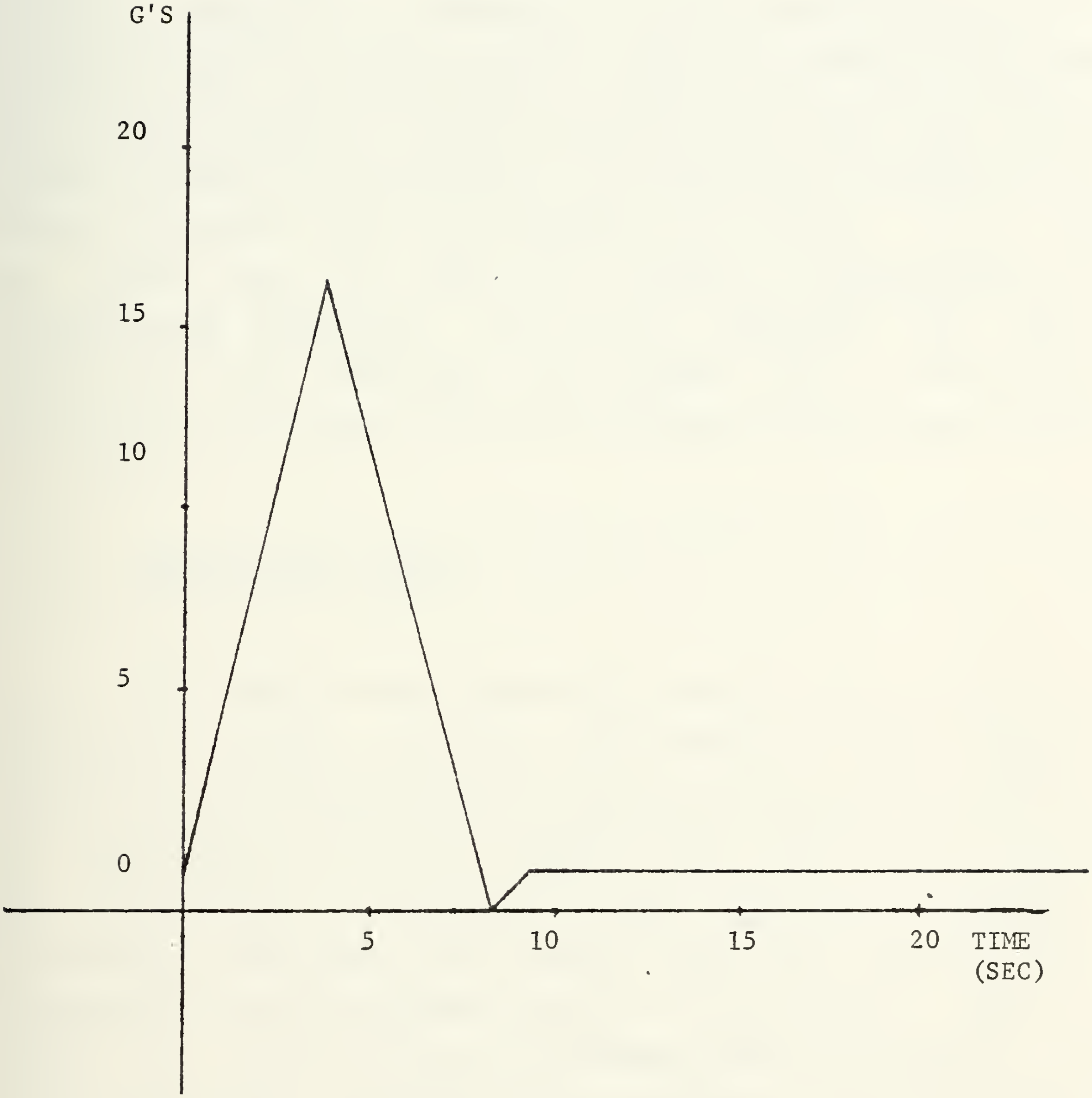

Figure 5 - ACCELERATION PROFILE

up to a maximum of 16 g's(fig.5)from the initial conditions given in fig.6. In addition, it is to be noted that for simplicity a homing type guidance command is given to the missile dynamics. This is recognised as ineffecient in practice but is simple in implementation and provides a basis for comparison purposes. Since missile flight controls respond to airspeed and not inertial speed, the velocities used in the missile dynamics portion of the algorithm are true airspeeds. However the inertial system must always compute in inertial velocities and hence must either adjust its calculations to include this difference or accept any error that this difference may entail. It is to be noted that no means of velocity measurement(i.e. doppler,mach gauge,etc.) is to be provided. In this simulation the difference is ignored due to the large magnitude of inertial velocities obtained and the short time of flight.

2. Noise Input Design

The randcm number generators used in this simulation were of two types; Gaussian and uniform. The Gaussian generators provided the noise inputs to the various sensors and the uniform generators provided the initial conditions and wind effects. The wind effects were such that constant bias directions of positive cross range and negative down-range conditions were imposed. The mean value of wind components in each direction was 30 ft per sec. with a range of + /-8 ft per sec. maximum change per second. No attempt was made to ascertain the relevance of the chosen wind model, its purpose was purely to introduce a bias into the system equations in order that the scale factor noise term of the gyros could be exercised. The scale factor term was finally dropped from the model of the gyros but the wind bias was retained.

The Gaussian generators provided noise inputs to each of the six installed sensors. The chosen model of the accelerometer noise term was

$$\gamma_{1,2,3} = EOG_{1,2,3} + WG_{1,2,3} * A_{1,2,3} \qquad (14)$$

where

EOG random bias term held constant over the entire flight but varied prior to each sample in the montecarlo

WG-scale factor term which varies through the flight

The chosen model for the gyro noise term was more complex consisting of bias terms and a random walk term. A random walk generatior is described in general terms as

$$\dot{E}_r = u_r \qquad (15)$$

where E_r is the error at a given instant and U_r is a white noise term with a standard deviation of σ_{U_r}. The variance of E_r grows linearly with time according to the relation

$$\sigma^2_{E_r} = t\,\sigma^2_{u_r} \qquad (16)$$

with σ_{E_r} given empirically in table 1 as random walk in °/hr with an uncertainty of $1\sigma 1\,hr = .0075$. For a sample generator to be used every second

$$\sigma_{u_r} = \sigma_{E_r} / 60 \qquad (17)$$

An error from each gyro is introduced into the inertial computation which is treated as a change in Θ.

$$\varphi(t) = EO + G(t) \qquad (18)$$

PHI error of each gyro per interval of time

EO constant bias term per flight

G result of random walk

3. Changes in Heading Resulting in Velocity Changes

The computation of velocity in the inertial frame consists of terms which relate the change in heading to changes in inertial velocities. For small angular rates the changes in velocity in the inertial frames show as inputs i.e. AWXX,AWXY, AWYY,AWYZ,AWZZ,AWZX.

AWXY is the change in velocity in the X(down-range) direction due to a change in direction Θ_Y . This is expressed in small angle approximations as

$$\begin{vmatrix} AWX \\ AWY \\ AWZ \end{vmatrix} = \begin{vmatrix} 1 & -\Delta\theta_z & \Delta\theta_y \\ \Delta\theta_z & 1 & \Delta\theta_x \\ -\Delta\theta_y & -\Delta\theta_x & 1 \end{vmatrix} \begin{vmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{vmatrix} \qquad (19)$$

AWX is the total change in velocity in the x direction due to small angle change. This calculation is performed in the appropriate missile dynamics portion of the simulation where the $\Delta\Theta$'s are the results of commands from the guidance system of the inertial system. In the inertial system these quantities are treated as $\Delta\Theta M$'s or measured changes and all velocity changes are computed based on gyro outputs.

4. Guidance System Design

The specific algorithm for generation of guidance commands was simple due to the homing type control employed. Inherent in the cross-range,down-range reference frame is the knowledge of distance remaining or "time-to-go" for termination of midcourse guidance and initiation of terminal guidance procedures. A parameter that continues to be significant within this project is the small angle, small rate assumption. In the guidance algorithm the one second time intervals chosen for use would require an inordinately long sequencing operation if commands were given in terms of rates. To clarify this statement, rate systems require an initiation and termination command, which for one second intervals would require a two second execution time. Therefore the guidance system employed within this project determines total angle change necessary and then commands an automatic pilot to accomplish this change. Thus the forcing function to the inertial navigator equations is not an input to the rate variables but rather to the angular displacement variables. No process noise was assumed for small angles hence the actual angle change was set equal to the commanded angle change. Notice that this is basically an open-loop process wherein the inertial navigator does not predict the next state based on the commanded heading change but rather on the noisy observed heading change.

The logical question then arises as to the effect of system drift during guidance. Normal procedure would be for a heading change command system in which an error signal generated by the navigator would be driven to null by the rotation of the vehicle. System drift during the heading change operation would result in process noise inputs to the navigator equations. The above was felt to be undesireable due to Kalman filter operation characteristics wherein steady state gains are non-zero for a linear system under process noise. The method of circumventing this discrepancy was to use as the forcing function the observed heading

change for the update of the navigator. Thus in the absence of process noise an accurate indicator of measurement noise would be the difference between observed heading change and commanded heading change. This concept was to be used during the Kalman filter application.

The algorithm for command guidance is given in fig.5. The "time-to-go" concept allows for a continual estimate of distance remaining, and a simple heading calculation computes the heading change necessary for homing.

$$\theta(k)_{required} = \frac{\text{cross-range position}}{\text{final position - present position}} = \frac{Y(k)}{240000-X(k)} \quad (20)$$

It should be noted that the flight profile simulation was terminated at a point where the inertially computed down-range position was greated than/or equal to the final position. This would correspond to the switchover point for terminal guidance.

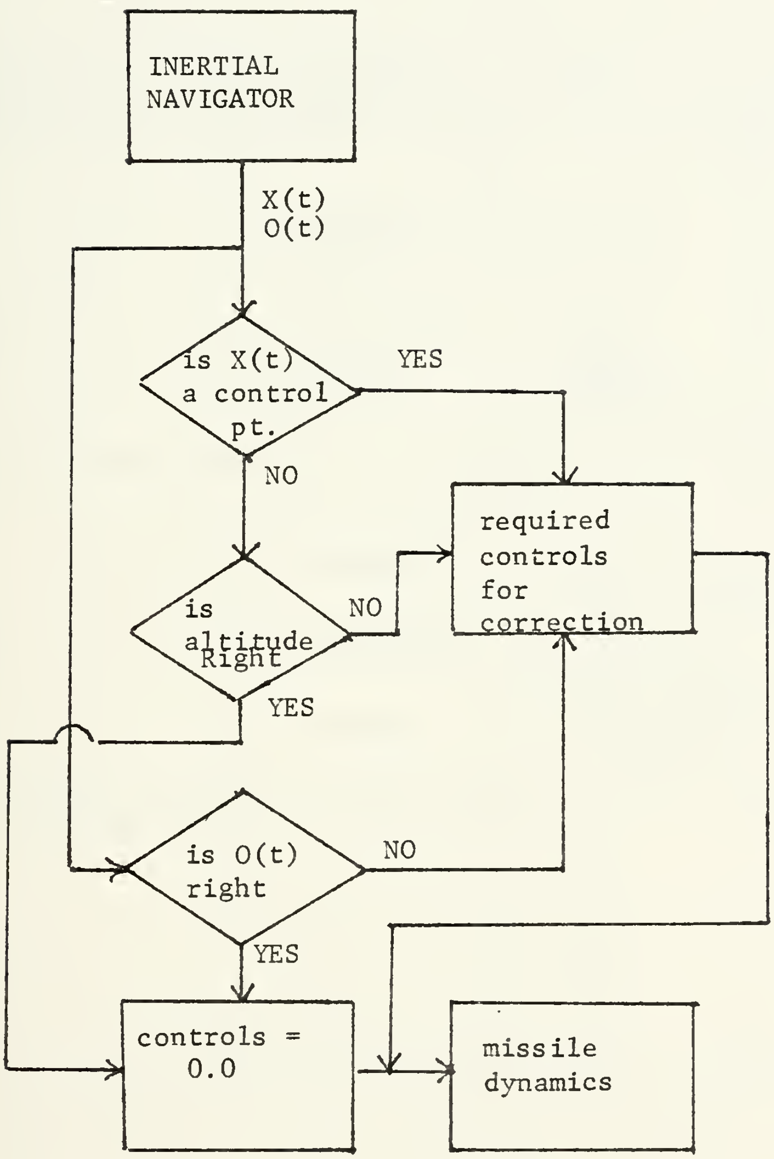

Figure 6 - GUIDANCE COMMANDS ALGORITHM

STATE VARIABLE	COORDINATE	MEAN	STD. DEVIATION
POSITION	downrange	0.0 ft (0.0 m)	1 = 387.0 ft (118 m)
	crossrange	0.0 ft (0.0 m)	1 = 387.0 ft (118 m)
	altitude	35000 ft (10668 m)	1 = 0.0 ft (0.0 m)
VELOCITY	downrange	670 ft/sec (204.2 m/sec)	1 =6.0 ft/s (1.83 m/
	crossrange	670 ft/sec (204.2 m/sec)	1 =6.0 ft/s (1.83 m/s
	vertical	0.0 ft /sec (0.0 m/sec)	1 =0.0 ft/s (0.0 m/s
ALIGNMENT	O_1	0.0°	1 = 2 min
	O_2	0.0°	1 = 2 min
	O_3	0.0°	1 = 2 min

Figure 7 - INITIAL CONDITIONS AT LAUNCH

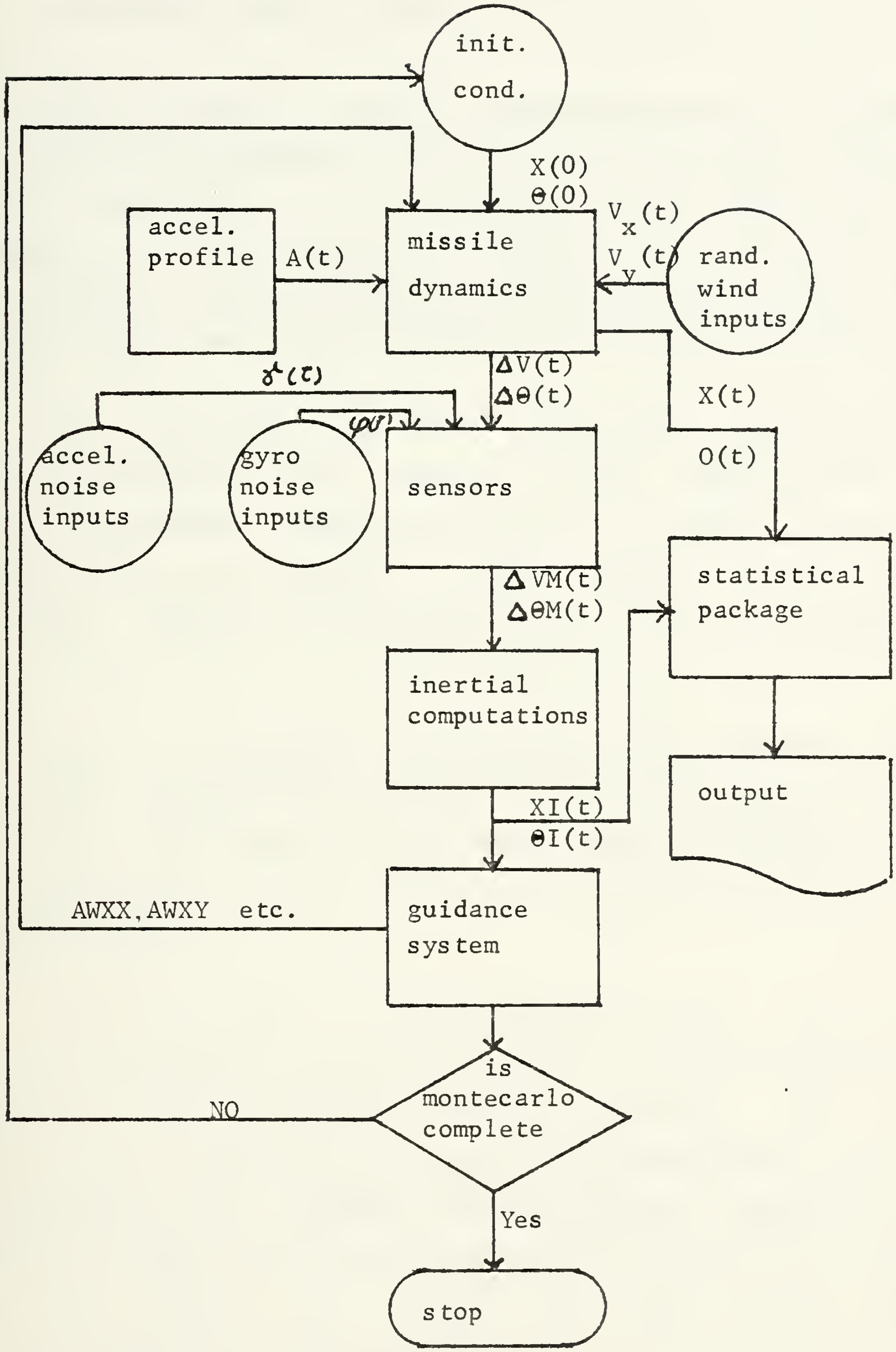

Figure 8 - ALGORITHM FOR MODEL OF MISSILE DYNAMICS WITH PURE INERTIAL COMPUTATIONS

5. Position Update System

The position update system(MICRAD) is designed to fix the missile position at various check points along the flight. Currently it is intended for the missile to navigate to each checkpoint inertially and upon arrival fix its position. The missile would then compute the course to the next check point and proceed to navigate to that point.

For purposes of this simulation and in order to reduce complexity and computer time requirements, the missile estimated position was set equal to its actual position at two discrete time steps. The inertial navigation system was thus reset at time=15 sec and time=80 sec.

The projected accuracy of the MICRAD position measurement system is $\sigma = 50.0$ ft at low altitudes. Thus a noise term was added to the measurement of position within the simulation in an effort to retain agreement with empirical data.

The magnitude of the deviation of the position fix is the fundamental argument in ref.2 for the elimination of the Kalman filter from the inertial system. The rational behind this assertion is that if two estimates of position are available (i.e. Kalman filter position estimate and a MICRAD estimate) then the weighting placed on each estimate would be heavily in favor of the more accurate estimate,logically the MICRAD fix. The optimal mix of the two estimates would then be

$$X = M X_n + (I-M) X_m \qquad (21)$$

where

X_n = navigation estimate = X + p

p = navigation estimate error

X_m = measurement estimate = X + r

r = measurement error

M = r / (r + p)

thus

$$X = (r/(r+p))X_n + (p/(r+p))X_m \quad (22)$$

since logically

$$r << p$$

$$X = X_m$$

and the Kalman filter estimates are ignored.

From the above, it can be seen that filtering for positon is required only in the intervals between observations and that the optimal mix of filtered position and observed position reduces to the observed position for highly accurate measurements.

6. <u>Statistical Formulations</u>

The primary measures of system performance for inertial navigation systems are mean of estimation error (i.e. the mean of actual position minus inertially calculated position) and variance of estimation error. The mean of estimation error reflects the result of bias within the inertial system and the variance of estimation error is an indicatior of system reliability. The simulation program adopted in this study evaluated the mean of position and

velocity as well as their variance at each one second interval along the flight path in addition to the estimation error mean and variance. The final position states were also computed and the mean and variance presented seperately. It was felt that this information could give a qualitative comparison of the missile performance with and without the Kalman filter installed.

The mean and variance equations for each time interval was computed using standard summation and averaging techniques. Thus

$$\overline{X} = 1/n \sum_{i=1}^{N} x_i \qquad (23)$$

with the inherent assumption that the relative frequency of occurence is analagous to the probability of occurence. The variance was computed similarly with

$$S^2 = (1/(n-1)) \sum_{i=1}^{N} (x_i - \overline{X})^2 \qquad (24)$$

IV. KALMAN FILTER

A. INTRODUCTORY REMARKS

Ref.12 discusses various aspects of the philosophy of Kalman filter applications in a very concise manner. Within this discussion the practical limitations of implementation are specified; the foremost limitation being that of a prerequisite knowledge of the exact statistical description for each random signal within the system. This a priori information determines the degree of optimality of the filter.

The filter is said to be optimum in the sense that it generates an unbiased, minimum variance estimate of the states of a linear system from some noisy measurement of a subset of those states. The requirements imposed upon the designer are : exact knowledge of the system dynamics, covariances of initial conditions, and noise inputs. Departure from optimality arises when either estimates of the above quantities are used or approximations to the state equations with lesser state variables comprise the model of system dynamics.

Previous studies indicate that best results are obtained with pessimistic estimations of design parameters and linearization of non-linear systems if possible. The pessimistic estimate of design parameters reduce the sensitivity of the design to deviations within the system while linearization results in off-line gain calculations

which significantly reduce the computational requirements.

The linearization technique analysis of this study indicates that the model of the system error terms are most critical in the estimation of theta. The cross-range and down-range position error being most dependent on these terms. Since digital filtering computational requirements increase roughly as the square of state variables, effeciency dictates that the number of filtered variables be minimized. Therefore it was felt that the best usage of Kalman filter techniques would be in the estimation of theta.

This evaluation is supported by various references(6,8) most notably ref.6.

B. GENERAL THEORY

Given a plant characterized by the linear discrete equations:

$$\underset{\sim}{X}(k+1) = \underset{\sim}{\phi}(k+1,k)\underset{\sim}{X}(k) + \underset{\sim}{\Delta}(k+1,k)\underset{\sim}{U}(k) + \underset{\sim}{W}(k)$$
$$\underset{\sim}{Z}(k) = \underset{\sim}{C}(k)\underset{\sim}{X}(k) + \underset{\sim}{V}(k) \qquad (25)$$

where

$\underset{\sim}{X}(K)$	Column matrix of states
$\underset{\sim}{\phi}(k+1,k)$	state transition matrix for time k to time k+1
$\underset{\sim}{\Delta}(k+1,K)$	Forcing transfer function
$\underset{\sim}{W}(K)$	Prccess noise term
$\underset{\sim}{Z}(K)$	Matrix of observations
$\underset{\sim}{V}(K)$	Measurement noise

With the appropriate assumptions concerning zero mean noise terms and knowledge of the covariance of initial conditions, the optimal estimate of the state vectors at time k can be arrived at through suitable use of a Kalman filter. This estimate will be characterized by a minimum variance of estimation error as its criteria for optimality. The Kalman filter equations are:

$$
\begin{aligned}
\underset{\sim}{G}(k) &= \underset{\sim}{P}(k/k-1)\underset{\sim}{C}(k)^{t}\left[\underset{\sim}{C}(k)\underset{\sim}{P}(k/k-1)\underset{\sim}{C}(k) + \underset{\sim}{R}(k)\right]^{-1} \\
\underset{\sim}{P}(k/k) &= \left[\underset{\sim}{I}-\underset{\sim}{G}(k)\underset{\sim}{C}(k)\right]\underset{\sim}{P}(k/k-1) \\
\underset{\sim}{P}(k+1/k) &= \underset{\sim}{\emptyset}(k+1,k)\underset{\sim}{P}(k/k)\underset{\sim}{\emptyset}^{t}(k+1,k) + \underset{\sim}{Q}(k) \qquad (26) \\
\underset{\sim}{X}(k/k) &= \underset{\sim}{X}(k/k-1) + \underset{\sim}{G}(k)\left[\underset{\sim}{Z}(k) - \underset{\sim}{C}(k)\underset{\sim}{X}(k/k-1)\right] \\
\underset{\sim}{X}(k+1/k) &= \underset{\sim}{\emptyset}(k+1,k)\underset{\sim}{X}(k/k) + \underset{\sim}{\Delta}(k+1,k)\underset{\sim}{U}(k)
\end{aligned}
$$

where the notation (k+1/k) implies the estimate at time k+1 given time k.

C. SPECIFIC THEORY

The current method of filtering inertial navigation systems is by building the filter specifically around a given error model of the gyro. The most commonly used error model is a Gauss-Markov drift model described by both correlated noise and bias terms.

$$\dot{D} = -(1/\Upsilon)D + r \qquad (27)$$

where D is the instantaneous value of error, Υ is the correlation time and r is a bias term. Statistically the time function has been found to be roughly equivalent to a random walk model such that

$$\dot{D} = w + r \qquad (28)$$

where W is a white noise term. Since the above drift is colored noise it has been handled previously by incorporating it in the state matrix such that it is part of the estimation process. The state equations would then become

$$\begin{aligned} \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= 0 \\ \dot{D} &= w + r \\ Z &= \theta_2 + \dot{D} \end{aligned} \qquad (29)$$

Thus the Kalman filter would be designed for a 3 state vector vice the needed two states. Also the presence of process noise requires a non zero steady state value for the gains of all equations in the update portion of the Kalman filter. This means that improper choice of parameters will bias the estimatior for all time

The overriding consideration of this study being the desire to maintain as few state variables as possible prompted a closer look at the above method of analysis.

The inertial navigator had been modeled as a first order system earlier(sect.I)

$$\theta I(k+1) = \theta I(k) + \theta M(k) \qquad (30)$$

however the Kalman filter requires an observation matrix and the above model uses the observation matrix $\Delta\theta M(k)$ as a forcing function therefore for filtering purposes the rate variables had to be redefined and a new state added.

$$\begin{aligned} \theta I_1(k+1) &= \theta I_1(k) + \theta I_2(k) + \Delta\theta M \\ \theta I_2(k+1) &= \theta I_2(k) \end{aligned} \qquad (31)$$

The new state variable θI_2 is to represent a small

angular rate which should be zero under normal conditions. Notice that the forcing function is solely upon the angular position and not upon the rate variable. Now the observation is the output of the gyro which is

$$\Delta\theta M(k) = \Delta\theta(k) + \theta I_2(k) * \Delta t + \dot{\varphi}(k) * \Delta t \qquad (32)$$

where $\dot{\varphi}$ is the noise term for the gyro given by

$$\dot{\varphi}(k) = EO + G(k) \qquad (33)$$

EO is a constant bias and g(k) is a white noise term. The noise equation then for unit time intervals are

$$\Delta\varphi(k)/\Delta t = EO + G(k) = \Delta\varphi(k) \qquad (34)$$

and the observation equation is

$$\begin{aligned} \Delta\theta M(k) &= \Delta\theta(k) + \theta I_2(k) + \Delta\varphi \\ &= \theta(k) + \theta I_2(k) + EO + G(k) \end{aligned} \qquad (35)$$

Now $\Delta\theta$ is a known forcing function generated by the inertial navigator therefore a new observation can be defined as

$$\begin{aligned} Z^*(k) &= \Delta\theta M(k) - \Delta\theta(k) \\ &= \theta I_2(k) + EO + G(k) \end{aligned} \qquad (36)$$

thus the discrete state equations are

$$\begin{aligned} \theta I_1(k+1) &= \theta I_1(k) + \theta I_2(k) + \Delta\theta(k) \\ \theta I_2(k+1) &= \theta I_2(k) \\ Z^*(k) &= \theta I_2(k) + EO + G(k) \end{aligned} \qquad (37)$$

Now if the constant bias term EO is added to the $\theta I_2(k)$ and redefined as $\theta I_2'(k) = \theta I_2(k) + EO$ (38)

Then it can be seen that the $\theta I_2(k)$ term takes on the significance of an estimated drift and the state equations then assume the form

$$\theta I_1(k+1) = \theta I_1(k) + \theta I_2'(k) + \Delta\theta(k)$$
$$\theta I_2(k+1) = \theta I_2'(k) \qquad (39)$$
$$Z^*(k) = \theta I_2'(k) + G(k)$$

Where g(k) is white noise and the state equations are now in standard form for Kalman filtering.

It was necessary to add one more state per gyro simulated but this brings the total state vector to only 12 which is still a net savings in state variables.

A block diagram of the algorithm is given in fig.09.

D. KALMAN FILTER RESULTS

The general Kalman equations of part B above were applied to the angular state variables in an off-line calculation by applying pessimistic estimates of initial condition variance such that

$$\sigma^2_{\theta_1(0)} = 1.00 \times 10^{-03}$$

$$(40)$$

$$\sigma^2_{\theta_2(0)} = 1.000 \times 10^{-03}$$

and the variance on the measurement noise to be

$$\sigma^2(\varphi) = 1.000 \times 10^{-03}$$

The resulting gains were found to be simple time

functions such that at time $T = K \Delta t$

$$G_1(k) = 1-(2/k+1)$$

$$G_2(k) = 1/k+1 \qquad (41)$$

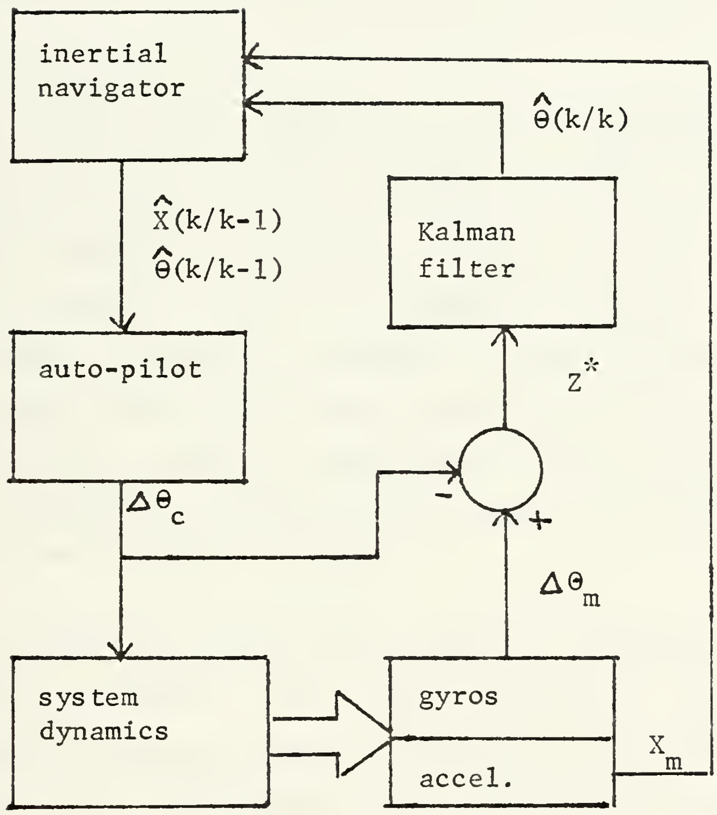

Figure 9 - ALGORITHM FOR PROPOSED KALMAN FILTER IMPLEMENTATION

V. ANALYSIS OF RESULTS

A. VERIFICATION OF RESULTS

Due to the high degree of simplification of plant dynamics, the basic plant model is felt to be weak. The linear dynamics applied to the given g profile produced a trajectory similar to the expected flight profile of the ALVRJ but (due to its simplified nature) with several defects. The descent produced a net increase in forward velocity after level off which would not be the case for a true non-linear model. This defect can be accommodated by the inertial system and hence was not felt to be detrimental to the purposes of this study.

Further work in this area would be recommended in order that basic defects in the missile such as thrust mis-alignment, process noise in control actuators and sensor misalignment be introduced into the simulation . A better tracking scheme could easily be instituted such that the guidance algorithm could institute a more efficient trajectory.

It was felt that for purposes of this study the plant model provided a reasonable approach to ALVRJ simulation. The time of flight and velocity profile are within the same general order of magnitude as the more complex simulations provided by ref.1 and correspond with physical intuition as to proper missile performance.

B. VERIFICATION OF ATIGS SIMULATION

It was felt that the simulation of the ATIGS would be accurate for purposes of this study if the error growth rate and the standard deviation growth rate incurred by the model were close to the observed quantities set forth in ref.1 and ref.3. Reference 3 stated that observed drift in the ATIGS test unit was approximately 1 nm /hr in ground test and 4 nm /hr. in airborne test. The simulation results show a numerical average drift of 3.35 nm /hr. which was felt to be in the range of the actual system. Ref.1 indicates that the simulation reported therein had a cross-range standard deviation growth rate of 1400 ft./min. between the first and second reset positions. The simulation within this study had a cross-range standard deviation growth rate of 1459.4 ft./min.

From the above correlation in performance the study proceeded under the assumption that the simulated model of the ATIGS would provide a reasonable background for analysis of position reset and Kalman filter performance in an actual installation.

C. EFFECT OF POSITION RESET ON SIMULATED PERFORMANCE

The unfiltered position reset feature of this simulation had the expected result of drastically decreasing the variance at mid-course termination. The pure inertial navigator must contend with both initial condition errors and integrated boost phase errors which combine to produce large scale variance at mid-course termination. The initial position reset occurs after boost is complete and thus virtually eliminates the initial condition and boost effect

on position. However, as expected, the velocity and angular errors of boost and initial condition still have effects on the final value of position at mid-course termination. The basic ATIGS navigator without position reset was found to have a radial uncertainty of 1608.6 ft. at mid-course termination (see table3). The addition of position reset to the basic model was found to reduce this uncertainty to 466.9 ft. However these figures reflect the inherent accuracy of the inertial navigator with the very low error terms in the sensors. If the noise terms in the inertial navigator are allowed to have their standard deviations increased by a factor of 5, thus simulating a very noisy inertial navigator, the basic ATIGS model without position reset demonstrates an uncertainty of 8646.5 ft., and after the addition of position reset,4128.0 ft. Thus it can be seen that,even with a position reset very close to mid-course termination, large uncertainty of position can accumulate due to the magnitude of the velocity errors which accrue throughout the flight.

D. EFFECT OF ADDITION OF LINEAR SUBOPTIMAL KALMAN FILTER

The Kalman filter proposed in this report had the effect of decreasing the variance of simulation behavior along the flight path for all tested situations (see table 2) . If cross-range standard deviations is taken as the criterion for performance quality, it can be seen that the filtered performance is readily superior at all noise levels. The "normal" noise level exhibits a radial uncertainty of 248.03 ft. at mid-course termination and the X5 noise level exhibits a radial uncertainty of 1083.1 ft. The fundamental reason for this increase in accuracy is shown in table 4, where the velocity errors at final update position are compared. The filtered velocity estimates, even at the

higher noise levels, are such that mid-course termination position is within much more reasonable bounds.

The overall effect of the Kalman filter proposed in this study then is to reduce the end point variance of missile position at mid-course termination in a significant manner. The unfiltered updates of position are seen to be valuable when used in conjunction with the Kalman filter but do not insure adequate missile performance if high noise levels are encountered throughout the flight.

RUN TYPE	NOISE	D-RANGE ERROR GROWTH	C-RANGE ERROR GROWTH	1-sigma ERROR GROWTH D-RANGE	1-sigma ERROR GROWTH C-RANGE
ATIGS	NORMAL	2.73 nm/hr	1.449 nm/hr	4016 ft/min	1459.4 ft/min
ATIGS	X5	68.64 nm/hr	107.9 nm/hr	14727 ft/min	10728 ft/min
ATIGS & MICRAD	NORMAL	1.849 nm/hr	1.52 nm/hr	4439.8 ft/min	1995.96 ft/min
ATIGS & MICRAD	X5	82.634 nm/hr	83.6 nm/hr	17986 ft/min	10240 ft/min
ATIGS MICRAD & KALMAN	NORMAL	.439 nm/hr	.87 nm/hr	3736 ft/min	664.59 ft/min
ATIGS MICRAD & KALMAN	X5	21.56 nm/hr	14.373 nm/hr	18836 ft/min	3053 ft/min

TABLE 2-SIMULATION PERFORMANCE WITH TYPE OF INSTALLATION

RUN TYPE	NOISE	MEAN FINAL D-RANGE POSITION 1-sigma	MEAN FINAL C-RANGE POSITION 1-sigma	MEAN FINAL D-RANGE ERROR 1-sigma	MEAN FINAL C-RANGE ERROR 1-sigma
ATIGS	NORMAL	240880 ft 1536.7 ft	217.72 ft 475.4 ft	6029.4 ft 3801.2 ft	816.9 ft 2964 ft
ATIGS	X5	241930 ft 8633.9 ft	413.39 ft 466 ft	29370 ft 18514 ft	1532.1 ft 12891 ft
ATIGS & MICRAD	NORMAL	239,610 ft 407.7 ft	74.17 ft 227.7 ft	307.4 ft 246.8 ft	97.5 ft 222.4 ft
ATIGS & MICRAD	X5	238180 ft 3564.7 ft	269.72 ft 2081.7 ft	2716 ft 1590.8 ft	293 ft 1107.5 ft
ATIGS, MICRAD & KALMAN	NORMAL	240000 ft 171.5 ft	63.0 ft 194 ft	239.77 ft 238.5 ft	34.036 ft 68.1 ft
ATIGS, MICRAD & KALMAN	X5	240540 ft 671.2 ft	61.62 ft 1018.9 ft	1900.7 ft 950.7 ft	59.37 ft 518.9 ft

Figure 11 - TABLE 3-FINAL POINT PERFORMANCE WITH TYPE OF INSTALLATION

type run	noise level	velocity error std. deviation
ATIGS	NORMAL	110.4 ft/sec
ATIGS	X5	562.47 ft/sec
ATIGS & MICRAD	NORMAL	104.65 ft/sec
ATIGS & MICRAD	X5	642 ft/sec
ATIGS, MICRAD & KALMAN	NORMAL	62.8 ft/sec
ATIGS, MICRAD & KALMAN	X5	262.47 ft/sec

Figure 12 - TABLE 4-VELOCITY ERROR OF TYPE OF INSTALLATION AT FINAL CHECKPOINT

APPENDIX A

RESULTS OF ATIGS SIMULATION WITHOUT FILTERING OR POSITION UPDATE

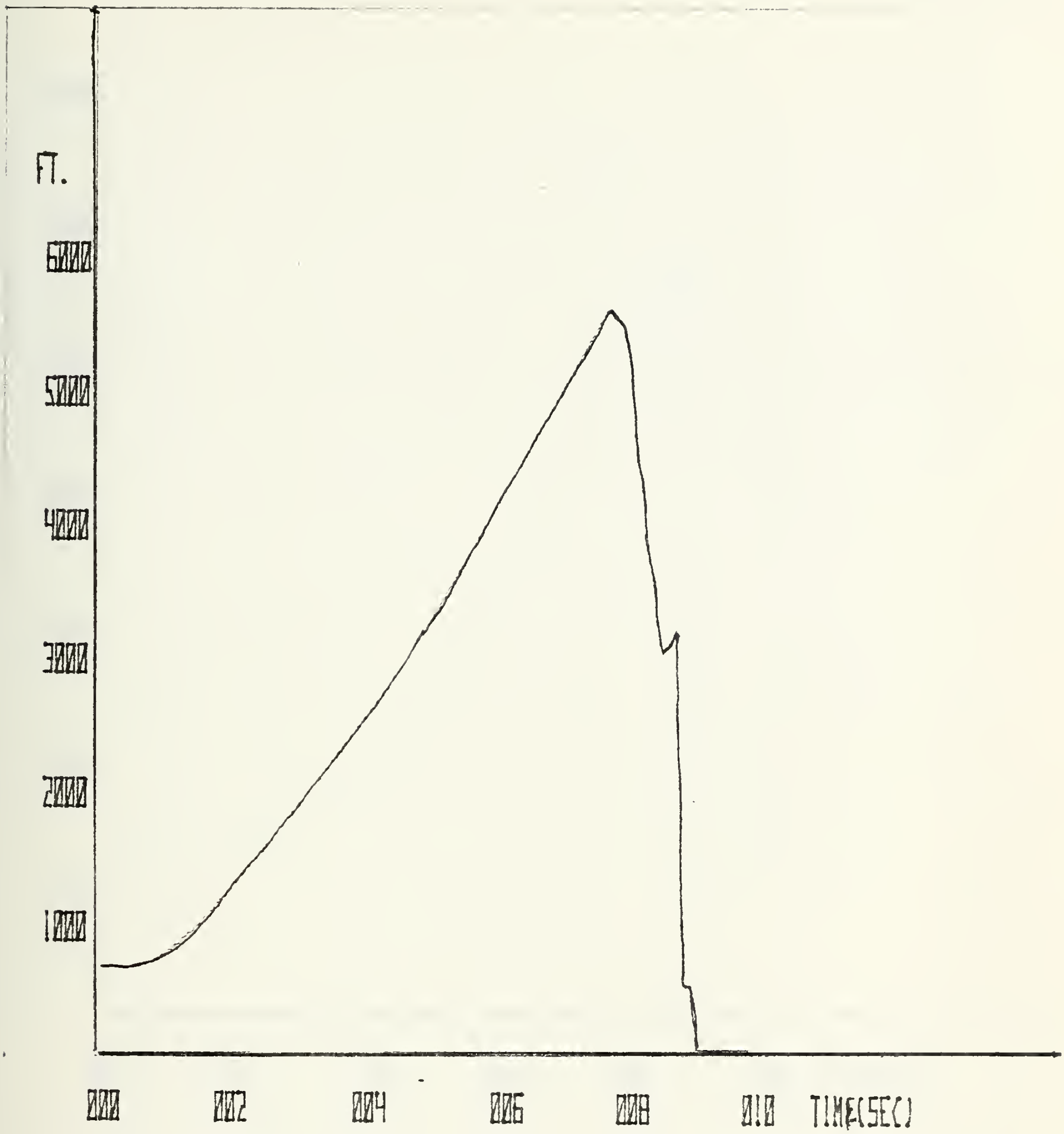

Figure 13 - SQRT OF DOWN RANGE VARIANCE (ATIGS ONLY)

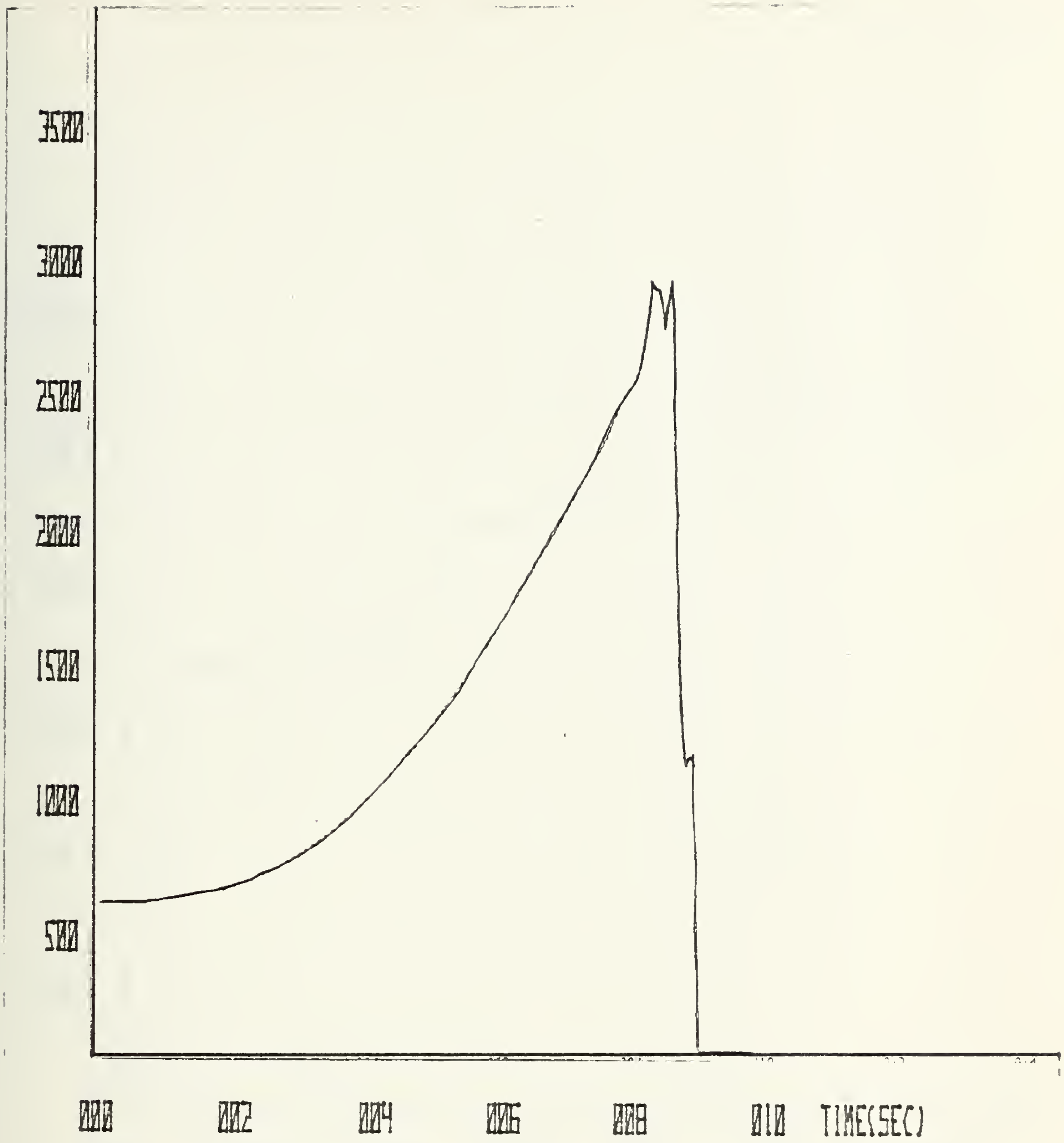

Figure 14 - SQRT OF CROSS RANGE VARIANCE (ATIGS ONLY)

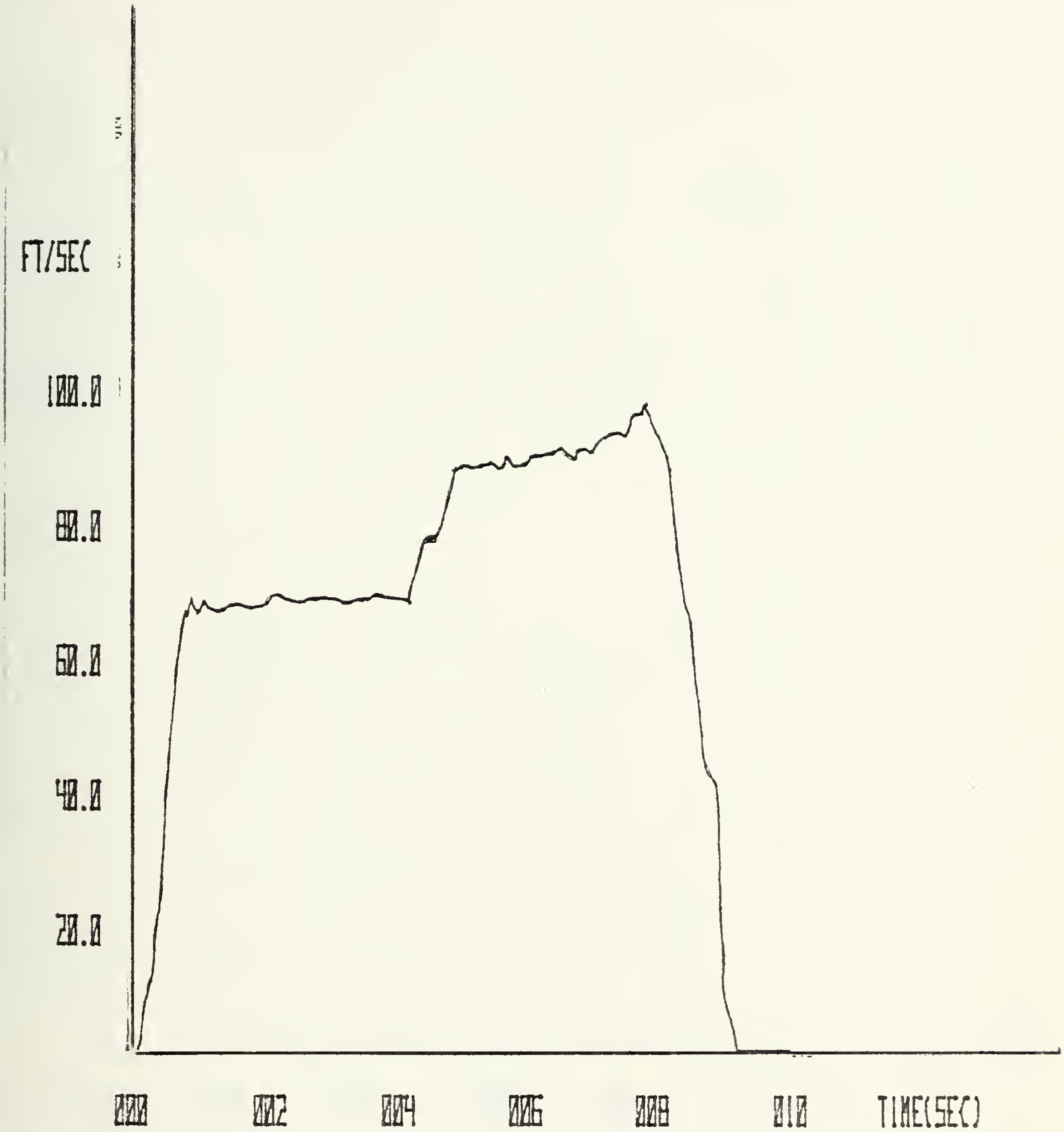

Figure 15 - SQRT• OF DOWN RANGE VELOCITY VARIANCE (ATIGS ONLY)

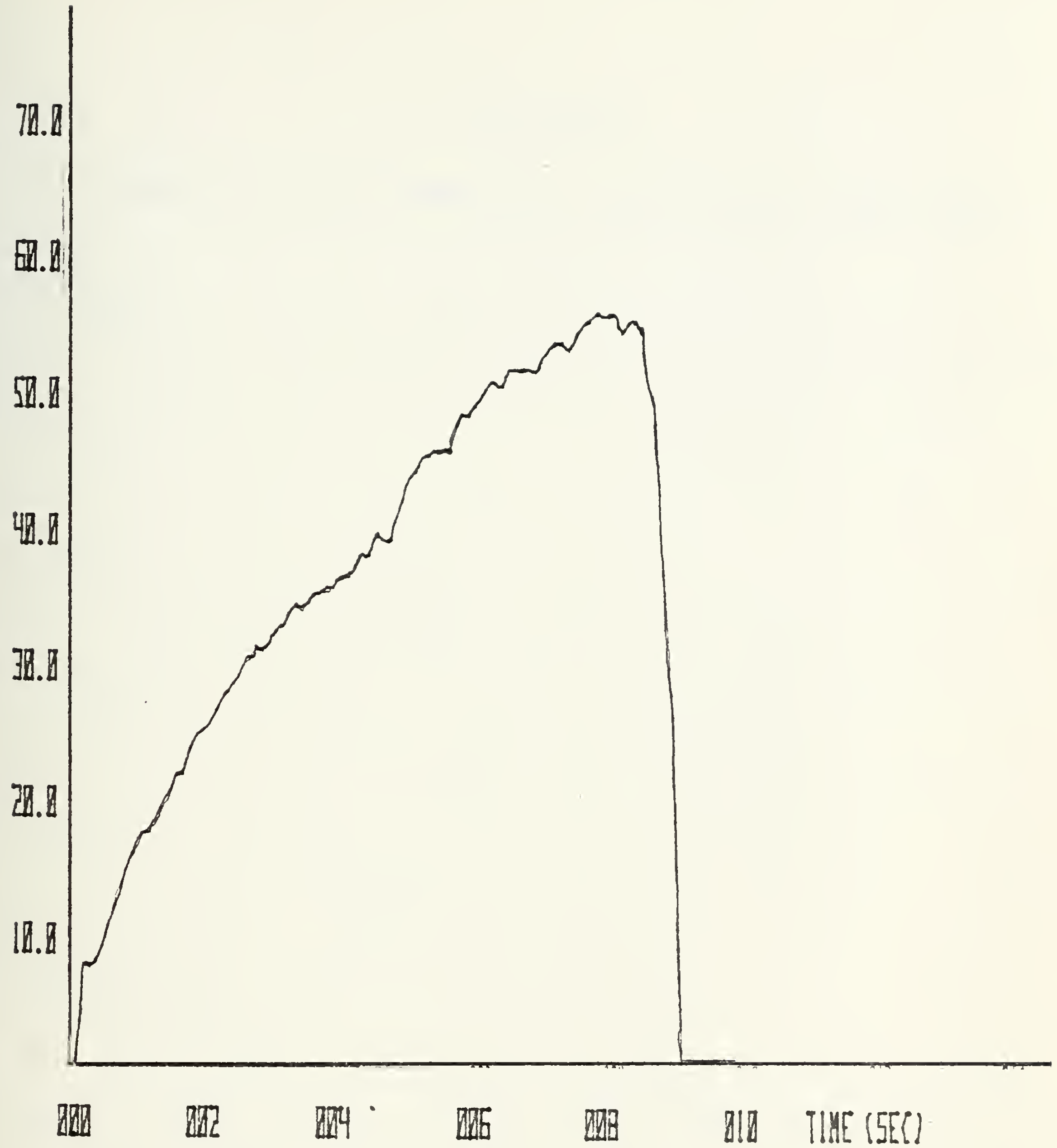

Figure 16 - SQRT OF CROSSRANGE VELOCITY VARIANCE (ATIGS ONLY)

APPENDIX B

RESULT OF ATIGS SIMULATION WITH POSITION RESET ONLY

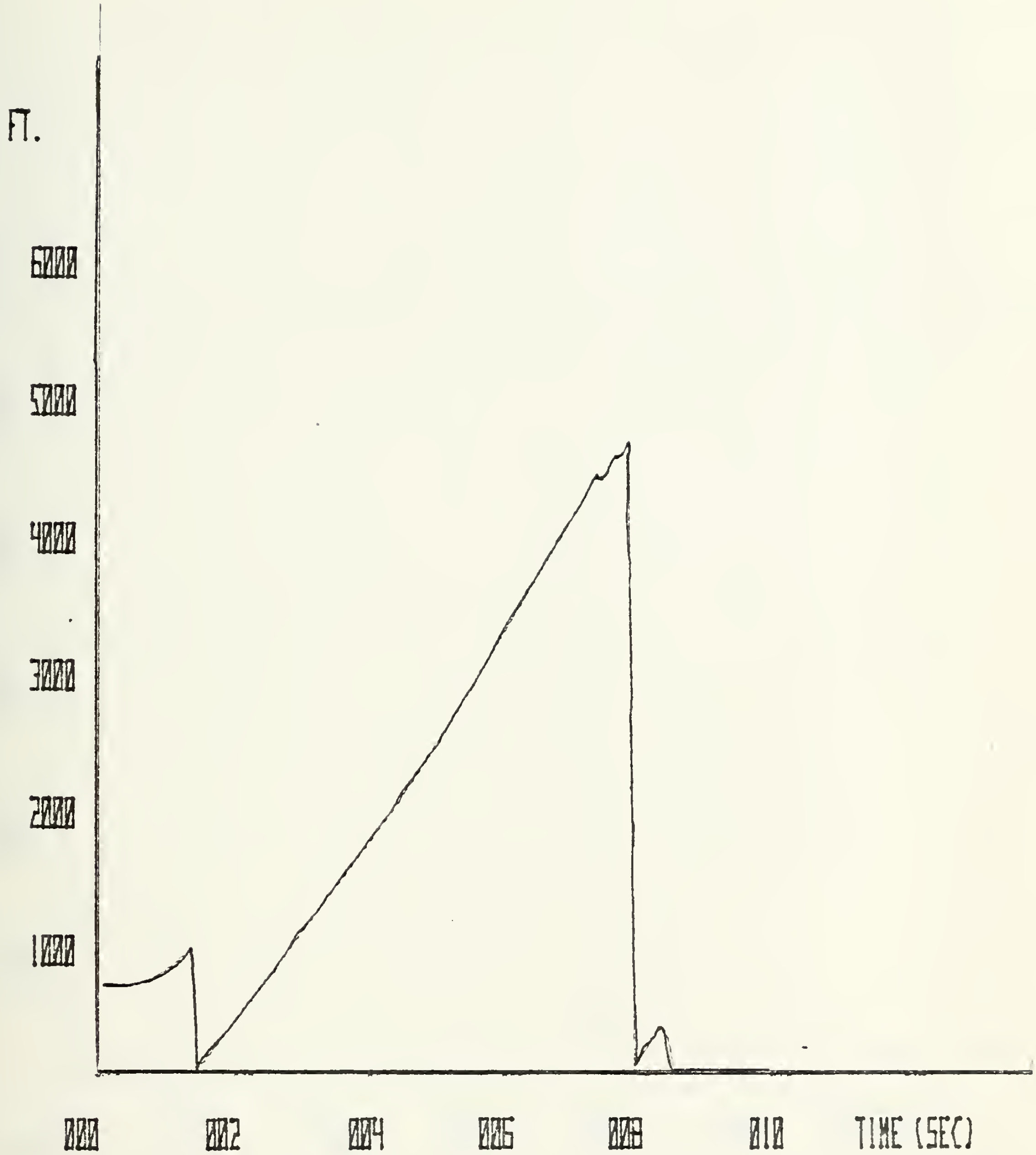

Figure 17 - SQRT OF DOWN RANGE VARIANCE (ATIGS WITH POSIT RESET)

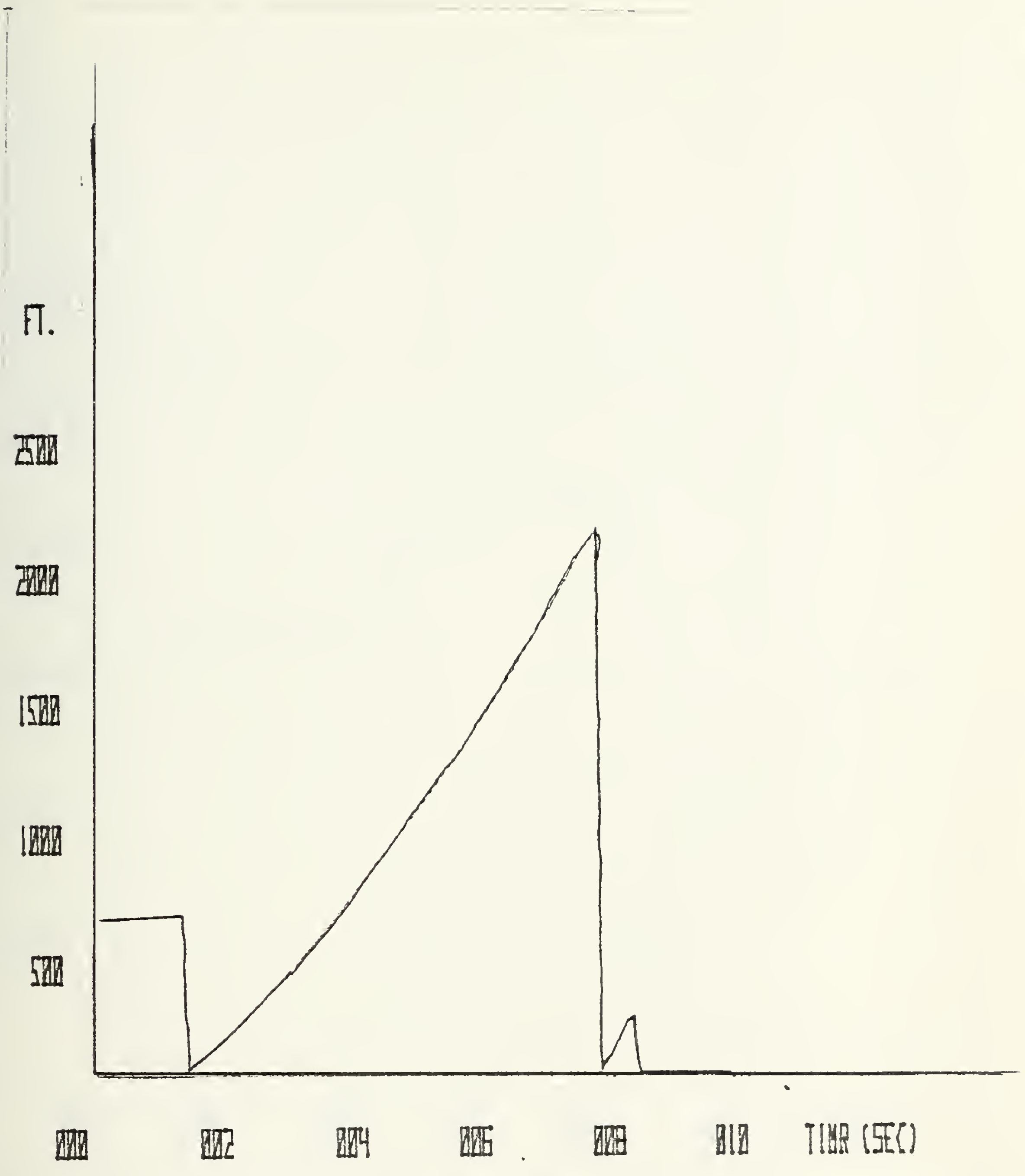

Figure 18 - SQRT OF CROSS RANGE VARIANCE (ATIGS WITH POSIT RESET)

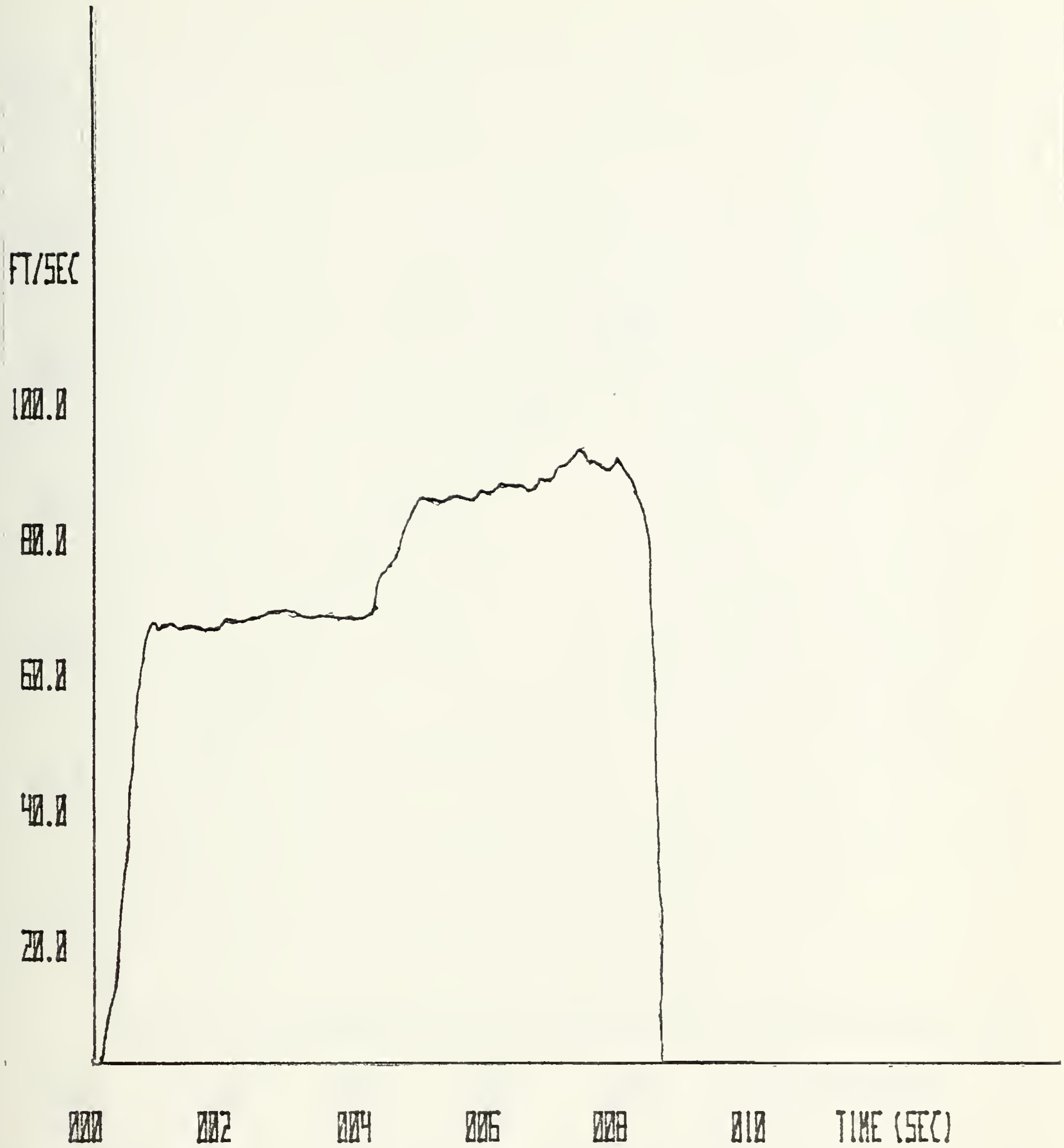

Figure 19 - SQRT OF DOWNRANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET)

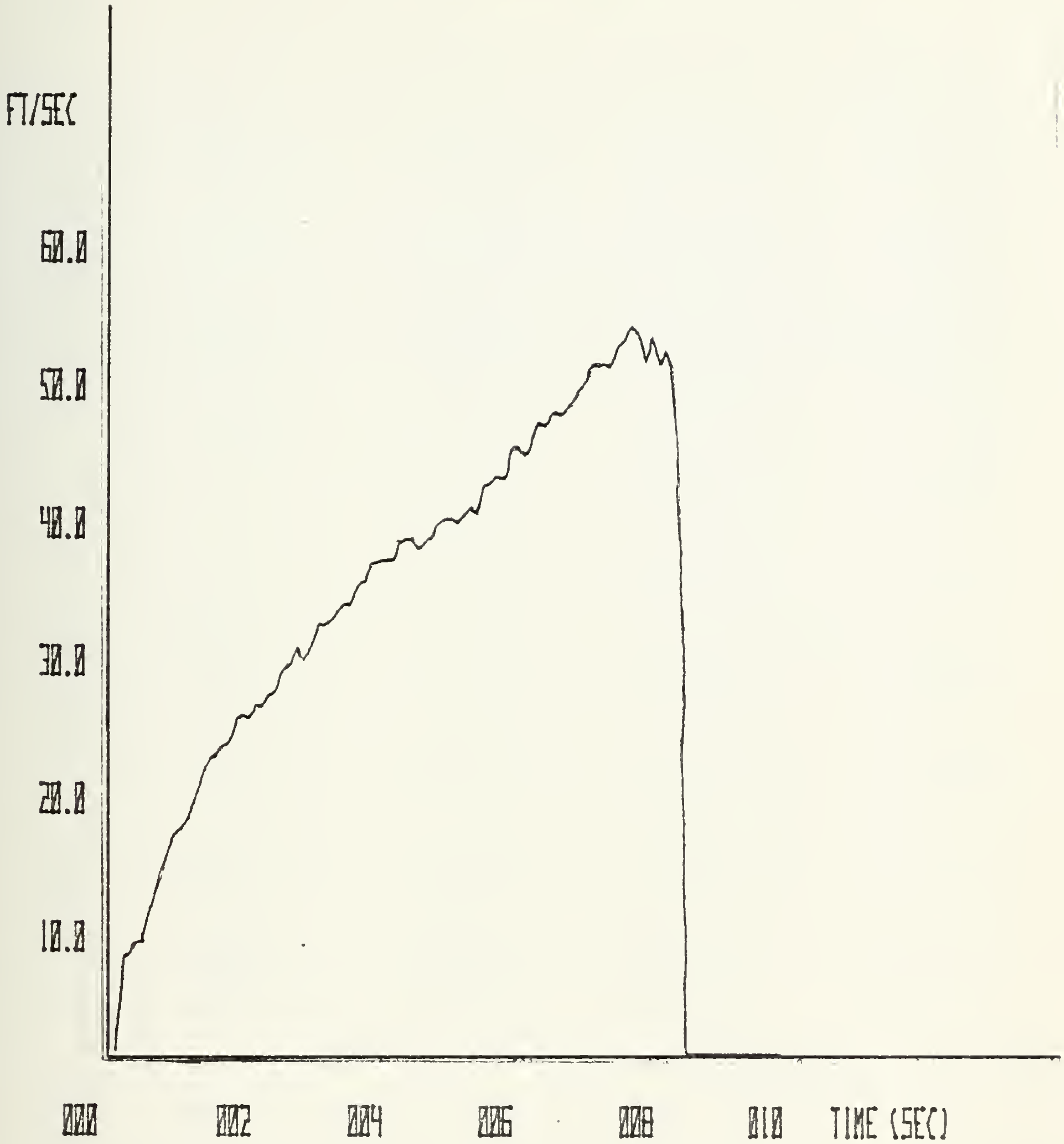

Figure 20 - SQRT OF CROSS RANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET)

```
THE NO. OF GYROS SIMULATED        3     THE NO.OF        ACCEL.SIMULATED        3     SIZE OF THE ENSEMBLE          200
SIGEO =0.330000E-04   SIGW =0.200000E-02   SIGK =0.250000E-01   SIGEG =0.644000E-03   SIGKG =0.330000E-01,  SIGT =0.582000E-03
THE UNCERTAINTY IN THE POSITION MEASUREMENT IS0.700000E 01FEET-RADIAL
```

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
1	X(1)	-0.63847D C2	0.40515D 06	-0.63847D 02	0.405150 06
	X(2)	0.70000C 03	0.10222D 00	0.300000 02	0.18775D-03
	X(3)	-0.23204D C3	0.36450D 06	-0.23204D 03	0.36450D 06
	X(4)	0.0	0.0	-0.30000D 02	0.187750-03
	X(5)	0.35000D 05	0.255550 03	0.0	0.0
	X(6)	0.0	0.0	0.0	0.0
2	X(1)	0.63827C C3	0.405380 06	-0.64029D 02	0.406060 06
	X(2)	0.76440D C3	0.12190D 00	0.297980 02	0.470810 02
	X(3)	-0.20192D C3	0.364180 06	-0.231610 03	0.364260 06
	X(4)	0.333980-C2	0.151320-02	-0.293720 02	0.50174D 02
	X(5)	0.35000D C5	0.25555D 03	0.72914D-02	0.44524D 00
	X(6)	0.0	0.0	0.14583D-01	0.178100 01
3	X(1)	0.146930 C4	0.405800 06	-0.644710 02	0.408310 06
	X(2)	0.957 6C0 03	0.19130D 00	0.293560 02	0.120970 03
	X(3)	-0.17226C 03	0.36408D 06	-0.23149D 03	0.36455D 06
	X(4)	0.133590-C1	0.242110-01	-0.29684D 02	0.57358D 02
	X(5)	0.350000 C5	0.25555D 03	-0.407340-01	0.486460 01
	X(6)	0.0	0.0	-0.11063D 00	0.411220 01
4	X(1)	0.255800 C4	0.405340 06	-0.65269D 02	0.410530 06
	X(2)	0.12796D 04	0.34158D C0	0.28715D 02	0.42481D 03
	X(3)	-0.14214D C3	0.364100 06	-0.230790 03	0.365370 06
	X(4)	0.30058D-C1	0.122570 00	-0.291330 02	0.67495D 02
	X(5)	0.35000D C5	0.25555D 03	-0.272120 00	0.184670 02
	X(6)	0.0	0.0	-0.35214D 00	0.82983D 01
5	X(1)	0.403350 C4	0.404510 06	-0.664220 02	0.41437D 06
	X(2)	0.173040 04	0.62466D 00	0.27919D 02	0.11703D 04
	X(3)	-0.1116CD 03	0.364660 06	-0.22917D 03	0.36715D 06
	X(4)	0.534360-C1	0.38738D 00	-0.28606D 02	0.744430 02
	X(5)	0.35000D 05	0.255550 03	-0.58918D 00	0.50201D 02
	X(6)	0.0	0.0	-0.281970 00	0.14592D 02
6	X(1)	0.596000 04	0.404770 06	-0.684140 02	0.424770 06
	X(2)	0.218120 C4	0.992520 00	0.26845D 02	0.23409D 04
	X(3)	-0.821350 C2	0.365230 06	-0.228580 03	0.36980D 06
	X(4)	-0.25677C-01	0.55220D 01	-0.29116D 02	0.11398D 03
	X(5)	0.350000 05	0.25555D 03	-0.841830 00	0.11109D 03
	X(6)	0.0	0.0	-0.22334D 00	0.22939D 02
7	X(1)	0.82720D C4	0.404430 06	-0.72535D 02	0.442920 06
	X(2)	0.250320 04	0.13072D 01	0.252270 02	0.351820 04
	X(3)	-0.52602C 02	0.36353D 06	-0.22800D 03	0.37134D 06
	X(4)	0.105350 C0	0.44585D 02	-0.287220 02	0.14657D 03
	X(5)	0.35000D C5	0.25555D 03	-0.11579D 01	0.21994D 03
	X(6)	0.0	0.0	-0.40887D 00	0.39375D 02
8	X(1)	0.108420 05	0.404020 06	-0.772950 02	0.470640 06
	X(2)	0.26964D C4	0.15170D 01	0.25005D 02	0.42745D 04
	X(3)	-0.22330C 02	0.35978D 06	-0.22662D 03	0.373120 06
	X(4)	0.163120 00	0.10741D 03	-0.287830 02	0.184000 03
	X(5)	0.35000D 05	0.255550 03	-0.164620 01	0.41529D 03
	X(6)	0.0	0.0	-0.56772D 00	0.68327D 02
9	X(1)	0.135400 C5	0.403300 06	-0.828710 02	0.507980 06
	X(2)	0.27608D 04	0.15908D 01	0.24414D 02	0.45143D 04
	X(3)	0.75175D 01	0.354050 06	-0.22546D 03	0.37446D 06
	X(4)	-0.386370 0C	0.15144D 03	-0.288320 02	0.242590 03
	X(5)	C.35000 05	0.255550 03	-0.211050 01	0.750980 03
	X(6)	0.0	0.0	-0.36082D 00	0.10054D 03

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
10	X(1)	0.16255D C5	0.40273D 06	-0.88287D 02	0.55422D 06
	X(2)	0.27286D C4	0.15549D 01	0.24761D 02	0.43559D 04
	X(3)	0.37339D C2	0.34769D 06	-0.22377D 03	0.37595D 06
	X(4)	-0.57036D C0	0.20025D 03	-0.28381D 02	0.27795D 03
	X(5)	0.3500CD 05	0.25555D 03	-0.24120D 01	0.12896D 04
	X(6)	0.0	0.0	-0.24220D 00	0.13010D 03
11	X(1)	0.18954D C5	0.40238D 06	-0.93398D 02	0.60975D 06
	X(2)	0.27286D C4	0.15557D 01	0.24839D 02	0.44280D 04
	X(3)	0.66259D C2	0.34143D 06	-0.22288D 03	0.37840D 06
	X(4)	-0.59907D 00	0.22079D 03	-0.28929D 02	0.29569D 03
	X(5)	0.3500CD 05	0.25555D 03	-0.24770D 01	0.21086D 04
	X(6)	0.0	0.0	0.11215D 00	0.16078D 03
12	X(1)	0.21652D C5	0.40333D 06	-0.98778D 02	0.67697D 06
	X(2)	0.27286D C4	0.15564D 01	0.24625D 02	0.44871D 04
	X(3)	0.96231D 02	0.33527D 06	-0.22083D 03	0.38130D 06
	X(4)	-0.52242D C0	0.26478D 03	-0.27956D 02	0.31910D 03
	X(5)	0.35000D C5	0.25555D 03	-0.23709D 01	0.32617D 04
	X(6)	0.0	0.0	0.10012D 00	0.19268D 03
13	X(1)	0.24351D C5	0.40289D 06	-0.10347D 03	0.75031D 06
	X(2)	0.27286D C4	0.15576D 01	0.25148D 02	0.43426D 04
	X(3)	0.12556D C3	0.32962D 06	-0.21922D 03	0.38499D 06
	X(4)	-0.81584D C0	0.28813D 03	-0.28812D 02	0.37022D 03
	X(5)	0.35000D 05	0.25555D 03	-0.21895D 01	0.48136D 04
	X(6)	0.0	0.0	0.26266D 00	0.22723D 03
14	X(1)	0.27050D C5	0.40346D 06	-0.10878D 03	0.83267D 06
	X(2)	0.27286D 04	0.15584D 01	0.24554D C2	0.43597D C4
	X(3)	0.15399D 03	0.32539D 06	-0.21888D 03	0.39014D 06
	X(4)	-0.15110D 01	0.31917D 03	-0.29694D 02	0.43777D 03
	X(5)	C.3500CD C5	0.25555D 03	-0.19450D 01	0.67919D 04
	X(6)	0.0	0.0	0.22629D 00	0.23846D 03
15	X(1)	0.29748D 05	0.40370D 06	-0.10584D 00	0.47197D 02
	X(2)	0.27286D C4	0.15620D 01	0.24358D 02	0.44022D 04
	X(3)	0.18158D C3	0.32108D 06	0.10418D 01	0.51009D 02
	X(4)	-0.24488D C1	0.36792D 03	-0.30089D 02	0.48802D 03
	X(5)	0.35000D C5	0.25555D 03	-0.16853D 01	0.92840D 04
	X(6)	0.0	0.0	0.29315D 00	0.29309D 03
16	X(1)	0.32446D C5	0.40375D 06	-0.55211D 01	0.45045D 04
	X(2)	0.27286D C4	0.15647D 01	0.24719D 02	0.43463D 04
	X(3)	0.20882D 03	0.31710D 06	0.12935D 01	0.57639D 03
	X(4)	-0.31367D C1	0.39820D 03	-0.29487D 02	0.50413D 03
	X(5)	0.35000D C5	0.25555D 03	-0.15030D 01	0.12386D 05
	X(6)	0.0	0.0	0.71515D-01	0.314C5D 03
17	X(1)	0.35146D 05	0.40331D 06	-0.99761D 01	0.17518D 05
	X(2)	0.27286D C4	0.15664D 01	0.25232D 02	0.43394D 04
	X(3)	0.23625D C3	0.31383D 06	0.20564D 01	0.21169D 04
	X(4)	-0.23098D C1	0.48143D 03	-0.29276D 02	0.53183D 03
	X(5)	0.35000C 05	0.25555D 03	-0.10199D 01	0.16113D 05
	X(6)	0.0	0.0	0.89464D 00	0.34845D 03
18	X(1)	0.37844D C5	0.40364D 06	-0.14984D 02	0.39046D 05
	X(2)	0.27286D C4	0.15666D 01	0.24689D 02	0.43399D 04
	X(3)	0.26302D C3	0.31054D 06	0.15189D 01	0.46599D 04
	X(4)	-0.25728D C1	0.48264D 03	-0.30230D 02	0.55583D 03
	X(5)	0.35000D C5	0.25555D 03	-0.23695D 00	0.20531D 05
	X(6)	0.0	0.0	0.67128D 00	0.38051D 03
19	X(1)	0.40543D C5	0.40440D 06	-0.20005D 02	0.69720D 05
	X(2)	0.27286D C4	0.15673D 01	0.24858D 02	0.44458D 04
	X(3)	0.29039D C3	0.30873D 06	0.19535D 01	0.83571D 04
	X(4)	-0.30210D 01	0.47773D 03	-0.29237D 02	0.62626D 03
	X(5)	0.35000D C5	0.25555D 03	0.32974D 00	0.25716D 05
	X(6)	0.0	0.0	0.46210D 00	0.41637D 03

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
20	X(1)	0.43241D 05	0.40499D 06	-0.26624D 02	0.10949D 06
	X(2)	0.27286D 04	0.15694D 01	0.23721D 02	0.45421D 04
	X(3)	0.31718D 03	0.30749D 06	0.29127D 01	0.13383D 05
	X(4)	-0.33963D 01	0.51794D 03	-0.28845D 02	0.64793D 03
	X(5)	0.34526D 05	0.69145D 05	-0.19963D 01	0.32443D 05
	X(6)	-0.94716D 03	0.27558D 06	-0.51141D 01	0.20169D 04
21	X(1)	0.45939D 05	0.40459D 06	-0.33648D 02	0.15762D 06
	X(2)	0.27286D 04	0.15717D 01	0.23832D 02	0.44817D 04
	X(3)	0.34354D 03	0.30729D 06	0.38721D 01	0.19430D 05
	X(4)	-0.37482D 01	0.55947D 03	-0.29093D 02	0.64361D 03
	X(5)	0.33434D 05	0.27588D 06	-0.99940D 01	0.42992D 05
	X(6)	-0.12380D 04	0.25304D 01	-0.10881D 02	0.23754D 04
22	X(1)	0.48638D 05	0.40457D 06	-0.38345D 02	0.21437D 06
	X(2)	0.27286D 04	0.15762D 01	0.25331D 02	0.45276D 04
	X(3)	0.36965D 03	0.30802D 06	0.47745D 01	0.26795D 05
	X(4)	-0.41901D 01	0.57442D 03	-0.29267D 02	0.69080D 03
	X(5)	0.32196D 05	0.27600D 06	-0.20902D 02	0.57958D 05
	X(6)	-0.12380D 04	0.25304D 01	-0.10934D 02	0.23739D 04
23	X(1)	0.51337D 05	0.40462D 06	-0.42598D 02	0.28056D 06
	X(2)	0.27286D 04	0.15802D 01	0.25102D 02	0.45480D 04
	X(3)	0.39479D 03	0.30878D 06	0.48613D 01	0.35182D 05
	X(4)	-0.47721D 01	0.59242D 03	-0.29808D 02	0.69604D 03
	X(5)	0.30958D 05	0.27613D 06	-0.31858D 02	0.77565D 05
	X(6)	-0.12380D 04	0.25304D 01	-0.10978D 02	0.24469D 04
24	X(1)	0.54036D 05	0.40495D 06	-0.47599D 02	0.35556D 06
	X(2)	0.27286D 04	0.15785D 01	0.24609D 02	0.45630D 04
	X(3)	0.42016D 03	0.31030D 06	0.57502D 01	0.44724D 05
	X(4)	-0.53818D 01	0.58950D 03	-0.29296D 02	0.72370D 03
	X(5)	0.29720D 05	0.27626D 06	-0.42952D 02	0.10216D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.11209D 02	0.24577D 04
25	X(1)	0.56734D 05	0.40496D 06	-0.53040D 02	0.43983D 06
	X(2)	0.27286D 04	0.15801D 01	0.24447D 02	0.45927D 04
	X(3)	0.44468D 03	0.31360D 06	0.61789D 01	0.55928D 05
	X(4)	-0.58850D 01	0.61897D 03	-0.30154D 02	0.75261D 03
	X(5)	0.28482D 05	0.27639D 06	-0.54204D 02	0.13176D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.11296D 02	0.24802D 04
26	X(1)	0.59433D 05	0.40504D 06	-0.58917D 02	0.53406D 06
	X(2)	0.27286D 04	0.15806D 01	0.24126D 02	0.46614D 04
	X(3)	0.46862D 03	0.31734D 06	0.59149D 01	0.68963D 05
	X(4)	-0.65244D 01	0.63776D 03	-0.30663D 02	0.83156D 03
	X(5)	0.27244D 05	0.27653D 06	-0.65640D 02	0.16625D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.11575D 02	0.25044D 04
27	X(1)	0.62131D 05	0.40549D 06	-0.64790D 02	0.63830D 06
	X(2)	0.27286D 04	0.15845D 01	0.24099D 02	0.46800D 04
	X(3)	0.49158D 03	0.32226D 06	0.49148D 01	0.83759D 05
	X(4)	-0.71409D 01	0.66638D 03	-0.30932D 02	0.85693D 03
	X(5)	0.26006D 05	0.27667D 06	-0.77429D 02	0.20597D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12005D 02	0.25942D 04
28	X(1)	0.64829D 05	0.40553D 06	-0.71550D 02	0.75141D 06
	X(2)	0.27286D 04	0.15812D 01	0.23543D 02	0.46838D 04
	X(3)	0.51467D 03	0.32812D 06	0.45660D 01	0.10052D 06
	X(4)	-0.74280D 01	0.66618D 03	-0.30507D 02	0.94083D 03
	X(5)	0.24767D 05	0.27682D 06	-0.89394D 02	0.25109D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.11925D 02	0.26205D 04
29	X(1)	0.67528D 05	0.40560D 06	-0.77897D 02	0.87475D 06
	X(2)	0.27286D 04	0.15840D 01	0.23949D 02	0.47487D 04
	X(3)	0.53721D 03	0.33450D 06	0.41682D 01	0.11820D 06
	X(4)	-0.80603D 01	0.69771D 03	-0.30850D 02	0.87746D 03
	X(5)	0.23529D 05	0.27698D 06	-0.10138D 03	0.30157D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12050D 02	0.26542D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
30	X(1)	0.70226D 05	0.40646D 06	-0.84474D 02	0.10062D 07
	X(2)	0.27286D 04	0.15864D 01	0.23534D 02	0.46217D 04
	X(3)	0.55852D 03	0.34315D 06	0.30723D 01	0.13754D 06
	X(4)	-0.87594D 01	0.69596D 03	-0.30799D 02	0.93359D 03
	X(5)	0.22291D 05	0.27714D 06	-0.11362D 03	0.35761D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12437D 02	0.27282D 04
31	X(1)	0.72924D 05	0.40699D 06	-0.92006D 02	0.11461D 07
	X(2)	0.27286D 04	0.15866D 01	0.22850D 02	0.46269D 04
	X(3)	0.57940D 03	0.35351D 06	0.17802D 01	0.16001D 06
	X(4)	-0.90410D 01	0.71383D 03	-0.31328D 02	0.10419D 04
	X(5)	0.21053D 05	0.27730D 06	-0.12637D 03	0.41925D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.13060D 02	0.27831D 04
32	X(1)	0.75622D 05	0.40842D 06	-0.99297D 02	0.12953D 07
	X(2)	0.27286D 04	0.15924D 01	0.23274D 02	0.46125D 04
	X(3)	0.60028D 03	0.36526D 06	0.11045D 01	0.18425D 06
	X(4)	-0.99273D 01	0.74667D 03	-0.30754D 02	0.10384D 04
	X(5)	0.19815D 05	0.27747D 06	-0.13935D 03	0.48655D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12888D 02	0.28301D 04
33	X(1)	0.78321D 05	0.40943D 06	-0.10607D 03	0.14531D 07
	X(2)	0.27286D 04	0.15985D 01	0.23543D 02	0.45861D 04
	X(3)	0.62007D 03	0.37742D 06	-0.24065D-01	0.20958D 06
	X(4)	-0.10874D 02	0.75458D 03	-0.31892D 02	0.10644D 04
	X(5)	0.18577D 05	0.27765D 06	-0.15210D 03	0.55969D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12627D 02	0.28818D 04
34	X(1)	0.81019D 05	0.40982D 06	-0.11255D 03	0.16195D 07
	X(2)	0.27286D 04	0.16027D 01	0.23607D 02	0.45987D 04
	X(3)	0.63863D 03	0.39184D 06	-0.18543D 01	0.23747D 06
	X(4)	-0.12310D 02	0.76089D 03	-0.32080D 02	0.11005D 04
	X(5)	0.17339D 05	0.27783D 06	-0.16478D 03	0.63834D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12723D 02	0.28691D 04
35	X(1)	0.83718D 05	0.40970D 06	-0.11880D 03	0.17959D 07
	X(2)	0.27286D 04	0.16078D 01	0.23826D 02	0.46319D 04
	X(3)	0.65603D 03	0.40723D 06	-0.42013D 01	0.26742D 06
	X(4)	-0.12754D 02	0.77755D 03	-0.32473D 02	0.11389D 04
	X(5)	0.16101D 05	0.27801D 06	-0.17735D 03	0.72261D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12424D 02	0.29374D 04
36	X(1)	0.86417D 05	0.41020D 06	-0.12440D 03	0.19840D 07
	X(2)	0.27286D 04	0.16075D 01	0.24171D 02	0.47154D 04
	X(3)	0.67270D 03	0.42449D 06	-0.65876D 01	0.29913D 06
	X(4)	-0.13713D 02	0.80484D 03	-0.32111D 02	0.11436D 04
	X(5)	0.14863D 05	0.27820D 06	-0.18976D 03	0.81272D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12397D 02	0.29523D 04
37	X(1)	0.89116D 05	0.41027D 06	-0.12951D 03	0.21768D 07
	X(2)	0.27286D 04	0.16070D 01	0.24394D 02	0.45297D 04
	X(3)	0.68948D 03	0.44299D 06	-0.82471D 01	0.33419D 06
	X(4)	-0.13636D 02	0.80144D 03	-0.32115D 02	0.12344D 04
	X(5)	0.13625D 05	0.27840D 06	-0.20216D 03	0.90876D 06
	X(6)	-0.12380D 04	0.25304D 01	-0.12394D 02	0.30127D 04
38	X(1)	0.91814D 05	0.41024D 06	-0.13629D 03	0.23794D 07
	X(2)	0.27286D 04	0.16090D 01	0.23139D 02	0.46065D 04
	X(3)	0.70556D 03	0.46244D 06	-0.10809D 02	0.37182D 06
	X(4)	-0.14419D 02	0.79043D 03	-0.33212D 02	0.12605D 04
	X(5)	0.12387D 05	0.27860D 06	-0.21466D 03	0.10116D 07
	X(6)	-0.12380D 04	0.25304D 01	-0.12603D 02	0.30846D 04
39	X(1)	0.94513D 05	0.41085D 06	-0.14310D 03	0.25933D 07
	X(2)	0.27286D 04	0.16105D 01	0.23318D 02	0.46340D 04
	X(3)	0.72041D 03	0.48306D 06	-0.14669D 02	0.41286D 06
	X(4)	-0.15422D 02	0.80932D 03	-0.34059D 02	0.13575D 04
	X(5)	0.11149D 05	0.27880D 06	-0.22761D 03	0.11209D 07
	X(6)	-0.12380D 04	0.25304D 01	-0.13307D 02	0.30845D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
40	X(1)	0.97211D 05	0.41127D 06	-0.15028D 03	0.28143D 07
	X(2)	0.27286D 04	0.16120D 01	0.22858D 02	0.45871D 04
	X(3)	0.73495D 03	0.50509D 06	-0.18116D 02	0.45661D 06
	X(4)	-0.15956D 02	0.82313D 03	-0.33278D 02	0.13670D 04
	X(5)	0.99109D 04	0.27901D 06	-0.24115D 03	0.12357D 07
	X(6)	-0.12380D 04	0.25304D 01	-0.13760D 02	0.30665D 04
41	X(1)	0.99909D 05	0.41131D 06	-0.15779D 03	0.30444D 07
	X(2)	0.27286D 04	0.16103D 01	0.22788D 02	0.46010D 04
	X(3)	0.74912D 03	0.52813D 06	-0.21344D 02	0.50247D 06
	X(4)	-0.16182D 02	0.81756D 03	-0.33673D 02	0.13763D 04
	X(5)	0.86728D 04	0.27923D 06	-0.25490D 03	0.13565D 07
	X(6)	-0.12380D 04	0.25304D 01	-0.13742D 02	0.31074D 04
42	X(1)	0.10261D 06	0.40940D 06	-0.16481D 03	0.32824D 07
	X(2)	0.27314D 04	0.15739D 04	0.22731D 02	0.46601D 04
	X(3)	0.76270D 03	0.55171D 06	-0.24727D 02	0.54986D 06
	X(4)	-0.17117D 02	0.81380D 03	-0.33545D 02	0.13726D 04
	X(5)	0.74379D 04	0.27939D 06	-0.26913D 03	0.14819D 07
	X(6)	-0.12315D 04	0.76148D 04	-0.14729D 02	0.30646D 04
43	X(1)	0.10532D 06	0.41166D 06	-0.17344D 03	0.35421D 07
	X(2)	0.27455D 04	0.91957D 04	0.20290D 02	0.53991D 04
	X(3)	0.77540D 03	0.57899D 06	-0.28350D 02	0.60207D 06
	X(4)	-0.17167D 02	0.82363D 03	-0.33386D 02	0.14900D 04
	X(5)	0.62215D 04	0.29073D 06	-0.28522D 03	0.16045D 07
	X(6)	-0.12009D 04	0.44586D 05	-0.17448D 02	0.27094D 04
44	X(1)	0.10807D 06	0.44512D 06	-0.18462D 03	0.38183D 07
	X(2)	0.28157D 04	0.41348D 05	0.16932D 02	0.55784D 04
	X(3)	0.78720D 03	0.60649D 06	-0.32879D 02	0.65745D 06
	X(4)	-0.17435D 02	0.81133D 03	-0.33875D 02	0.14879D 04
	X(5)	0.50980D 04	0.36006D 06	-0.30419D 03	0.17206D 07
	X(6)	-0.10461D 04	0.20080D 06	-0.20487D 02	0.24576D 04
45	X(1)	0.11094D 06	0.55489D 06	-0.19973D 03	0.41087D 07
	X(2)	0.29786D 04	0.77956D 05	0.13062D 02	0.57403D 04
	X(3)	0.79860D 03	0.63403D 06	-0.37769D 02	0.71425D 06
	X(4)	-0.18224D 02	0.78322D 03	-0.34359D 02	0.15012D 04
	X(5)	0.42314D 04	0.67289D 06	-0.32544D 03	0.18332D 07
	X(6)	-0.68703D 03	0.37068D 06	-0.22017D 02	0.24249D 04
46	X(1)	0.11396D 06	0.76542D 06	-0.21436D 03	0.44188D 07
	X(2)	0.31303D 04	0.64320D 05	0.17444D 02	0.61478D 04
	X(3)	0.80899D 03	0.66009D 06	-0.42548D 02	0.77232D 06
	X(4)	-0.19890D 02	0.74229D 03	-0.34105D 02	0.14436D 04
	X(5)	0.37116D 04	0.12635D 07	-0.34467D 03	0.19434D 07
	X(6)	-0.35271D 03	0.31244D 06	-0.16448D 02	0.25865D 04
47	X(1)	0.11712D 06	0.96728D 06	-0.22276D 03	0.47411D 07
	X(2)	0.32455D 04	0.23238D 05	0.24739D 02	0.65841D 04
	X(3)	0.81904D 03	0.68668D 06	-0.46402D 02	0.83244D 06
	X(4)	-0.20405D 02	0.69501D 03	-0.33986D 02	0.14727D 04
	X(5)	0.34858D 04	0.17774D 07	-0.35735D 03	0.20440D 07
	X(6)	-0.98861D 02	0.11275D 06	-0.89017D 01	0.22691D 04
48	X(1)	0.12035D 06	0.10572D 07	-0.22746D 03	0.50862D 07
	X(2)	0.32820D 04	0.46651D 04	0.27228D 02	0.70685D 04
	X(3)	0.82785D 03	0.71165D 06	-0.50587D 02	0.89446D 06
	X(4)	-0.21847D 02	0.66774D 03	-0.34266D 02	0.15191D 04
	X(5)	0.34272D 04	0.20032D 07	-0.36415D 03	0.21292D 07
	X(6)	-0.18376D 02	0.22632D 05	-0.47046D 01	0.18137D 04
49	X(1)	0.12361D 06	0.10742D 07	-0.22935D 03	0.54546D 07
	X(2)	0.32905D 04	0.27311D 01	0.29007D 02	0.73469D 04
	X(3)	0.83589D 03	0.73751D 06	-0.54187D 02	0.96119D 06
	X(4)	-0.22614D 02	0.69555D 03	-0.33471D 02	0.15839D 04
	X(5)	0.34181D 04	0.20545D 07	-0.36808D 03	0.22064D 07
	X(6)	0.19860D 00	0.22563D 01	-0.31582D 01	0.16551D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
50	X(1)	0.12687D 06	0.10747D 07	-0.23073D 03	0.58426D 07
	X(2)	0.32905D 04	0.27401D 01	0.28741D 02	0.73622D 04
	X(3)	0.84295D 03	0.76231D 06	-0.58063D 02	0.10300D 07
	X(4)	-0.22924D 02	0.69858D 03	-0.33944D 02	0.16089D 04
	X(5)	0.34183D 04	0.20546D 07	-0.37114D 03	0.22845D 07
	X(6)	0.19860D 00	0.22563D 01	-0.29503D 01	0.16854D 04
51	X(1)	0.13012D 06	0.10752D 07	-0.23243D 03	0.62434D 07
	X(2)	0.32905D 04	0.27416D 01	0.28627D 02	0.73184D 04
	X(3)	0.84922D 03	0.78865D 06	-0.62642D 02	0.11017D 07
	X(4)	-0.23765D 02	0.70348D 03	-0.34437D 02	0.16306D 04
	X(5)	0.34185D 04	0.20546D 07	-0.37428D 03	0.23674D 07
	X(6)	0.19860D 00	0.22563D 01	-0.33341D 01	0.18237D 04
52	X(1)	0.13339D 06	0.10763D 07	-0.23360D 03	0.66543D 07
	X(2)	0.32905D 04	0.28031D 01	0.29009D 02	0.72086D 04
	X(3)	0.85457D 03	0.81139D 06	-0.67227D 02	0.11737D 07
	X(4)	-0.25027D 02	0.62781D 03	-0.34225D 02	0.16060D 04
	X(5)	0.34187D 04	0.20547D 07	-0.37726D 03	0.24537D 07
	X(6)	0.19860D 00	0.22563D 01	-0.26176D 01	0.18139D 04
53	X(1)	0.13665D 06	0.10753D 07	-0.23471D 03	0.70830D 07
	X(2)	0.32905D 04	0.28070D 01	0.28904D 02	0.73190D 04
	X(3)	0.85951D 03	0.83177D 06	-0.70609D 02	0.12486D 07
	X(4)	-0.25766D 02	0.61804D 03	-0.33212D 02	0.16369D 04
	X(5)	0.34189D 04	0.20548D 07	-0.37955D 03	0.25421D 07
	X(6)	0.19860D 00	0.22563D 01	-0.19622D 01	0.18209D 04
54	X(1)	0.13991D 06	0.10760D 07	-0.23480D 03	0.75271D 07
	X(2)	0.32905D 04	0.28022D 01	0.29497D 02	0.73756D 04
	X(3)	0.86375D 03	0.85204D 06	-0.73531D 02	0.13285D 07
	X(4)	-0.26175D 02	0.61939D 03	-0.33050D 02	0.16913D 04
	X(5)	0.34153D 04	0.20432D 07	-0.38158D 03	0.26338D 07
	X(6)	-0.72660D 01	0.11063D 05	-0.21051D 01	0.18620D 04
55	X(1)	0.14317D 06	0.10773D 07	-0.23563D 03	0.79904D 07
	X(2)	0.32939D 04	0.22815D 04	0.28780D 02	0.74642D 04
	X(3)	0.86738D 03	0.86972D 06	-0.76610D 02	0.14082D 07
	X(4)	-0.26638D 02	0.58362D 03	-0.33196D 02	0.16468D 04
	X(5)	0.34118D 04	0.20371D 07	-0.38413D 03	0.27286D 07
	X(6)	0.20069D 00	0.22650D 01	-0.29958D 01	0.19335D 04
56	X(1)	0.14644D 06	0.10831D 07	-0.23673D 03	0.84633D 07
	X(2)	0.32939D 04	0.22815D 04	0.28710D 02	0.73682D 04
	X(3)	0.87001D 03	0.88941D 06	-0.80684D 02	0.14935D 07
	X(4)	-0.27380D 02	0.58621D 03	-0.34225D 02	0.18312D 04
	X(5)	0.34083D 04	0.20443D 07	-0.38696D 03	0.28245D 07
	X(6)	-0.72651D 01	0.11084D 05	-0.26645D 01	0.18966D 04
57	X(1)	0.14970D 06	0.10890D 07	-0.23774D 03	0.89476D 07
	X(2)	0.32972D 04	0.45423D 04	0.28554D 02	0.73378D 04
	X(3)	0.87188D 03	0.90795D 06	-0.84415D 02	0.15836D 07
	X(4)	-0.28984D 02	0.59572D 03	-0.33329D 02	0.18466D 04
	X(5)	0.33973D 04	0.20722D 07	-0.38985D 03	0.29235D 07
	X(6)	-0.14735D 02	0.22133D 05	-0.31169D 01	0.19376D 04
58	X(1)	0.15297D 06	0.10989D 07	-0.24004D 03	0.94567D 07
	X(2)	0.33041D 04	0.90115D 04	0.27902D 02	0.75426D 04
	X(3)	0.87225D 03	0.92589D 06	-0.88494D 02	0.16759D 07
	X(4)	-0.29503D 02	0.56270D 03	-0.34061D 02	0.18883D 04
	X(5)	0.33900D 04	0.20986D 07	-0.39329D 03	0.30277D 07
	X(6)	0.19900D 00	0.23083D 01	-0.37634D 01	0.20350D 04
59	X(1)	0.15625D 06	0.11255D 07	-0.24182D 03	0.99816D 07
	X(2)	0.33041D 04	0.90117D 04	0.28134D 02	0.75575D 04
	X(3)	0.87193D 03	0.94227D 06	-0.92497D 02	0.17699D 07
	X(4)	-0.30801D 02	0.56147D 03	-0.33615D 02	0.18619D 04
	X(5)	0.33790D 04	0.21197D 07	-0.39661D 03	0.31357D 07
	X(6)	-0.22197D 02	0.32936D 05	-0.28686D 01	0.20889D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
60	X(1)	0.15953D 06	0.11701D 07	-0.24397D 03	0.10521D 08
	X(2)	0.331420 04	0.35511D 05	0.27976D 02	0.747950 04
	X(3)	0.87068D 03	0.956800 06	-0.955720 02	0.187310 07
	X(4)	-0.32227D 02	0.54893D 03	-0.33061D 02	0.20473D 04
	X(5)	0.336050 04	0.219030 07	-0.39987D 03	0.32497D 07
	X(6)	-0.14735D 02	0.22120D 05	-0.36511D 01	0.225240 04
61	X(1)	0.16281D 06	0.12411D 07	-0.24593D 03	0.11079D 08
	X(2)	0.33210D 04	0.19744D 05	0.285620 02	0.77134D 04
	X(3)	0.86757D 03	0.96637D 06	-0.99645D 02	0.19808D 07
	X(4)	-0.33098D 02	0.56530D 03	-0.34180D 02	0.20743D 04
	X(5)	0.33533D 04	0.22345D 07	-0.40328D 03	0.33675D 07
	X(6)	0.197840 00	0.23273D 01	-0.317130 01	0.214940 04
62	X(1)	0.16611D 06	0.13475D 07	-0.24664D 03	0.11652D 08
	X(2)	0.33210D 04	0.19744D 05	0.28922D 02	0.76911D 04
	X(3)	0.86403D 03	0.972280 06	-0.10311D 03	0.20871D 07
	X(4)	-0.34492D 02	0.56126D 03	-0.33272D 02	0.20161D 04
	X(5)	0.33460D 04	0.22571D 07	-0.406780 03	0.34890D 07
	X(6)	-0.147330 02	0.22052D 05	-0.38269D 01	0.230750 04
63	X(1)	0.16940D 06	0.14812D 07	-0.24766D 03	0.12242D 08
	X(2)	0.33278D 04	0.23877D 05	0.289090 02	0.77265D 04
	X(3)	0.85919D 03	0.97667D 06	-0.10624D 03	0.21991D 07
	X(4)	-0.35665D 02	0.56565D 03	-0.33462D 02	0.21036D 04
	X(5)	0.33313D 04	0.23141D 07	-0.41055D 03	0.36145D 07
	X(6)	-0.14734D 02	0.22118D 05	-0.37280D 01	0.227960 04
64	X(1)	0.17270D 06	0.16408D 07	-0.24837D 03	0.12841D 08
	X(2)	0.33345D 04	0.27923D 05	0.29193D 02	0.76757D 04
	X(3)	0.85260D 03	0.98007D 06	-0.11046D 03	0.23195D 07
	X(4)	-0.369120 02	0.58490D 03	-0.34375D 02	0.22464D 04
	X(5)	0.33165D 04	0.23449D 07	-0.41478D 03	0.37427D 07
	X(6)	-0.14740D 02	0.22161D 05	-0.473170 01	0.23533D 04
65	X(1)	0.17601D 06	0.18511D 07	-0.25010D 03	0.13459D 08
	X(2)	0.33413D 04	0.31889D 05	0.28293D 02	0.77395D 04
	X(3)	0.84522D 03	0.980480 06	-0.114510 03	0.244410 07
	X(4)	-0.384110 02	0.59749D 03	-0.34294D 02	0.22229D 04
	X(5)	0.32980D 04	0.237970 07	-0.41982D 03	0.387790 07
	X(6)	-0.22202D 02	0.328840 05	-0.53383D 01	0.25318D 04
66	X(1)	0.179320 06	0.21268D 07	-0.25143D 03	0.14079D 08
	X(2)	0.33514D 04	0.37626D 05	0.290220 02	0.76263D 04
	X(3)	0.83616D 03	0.97955D 06	-0.11838D 03	0.257390 07
	X(4)	-0.40562D 02	0.600750 03	-0.34303D 02	0.23232D 04
	X(5)	0.32758D 04	0.24510D 07	-0.42435D 03	0.40182D 07
	X(6)	-0.221980 02	0.32904D 05	-0.37221D 01	0.242530 04
67	X(1)	0.18265D 06	0.245780 07	-0.25172D 03	0.14722D 08
	X(2)	0.336160 04	0.431670 05	0.29508D 02	0.77986D 04
	X(3)	0.82439D 03	0.97438D 06	-0.12341D 03	0.27070D 07
	X(4)	-0.42205D 02	0.65422D 03	-0.349680 02	0.231350 04
	X(5)	0.32574D 04	0.251600 07	-0.42886D 03	0.41657D 07
	X(6)	-0.147290 02	0.22055D 05	-0.53466D 01	0.27566D 04
68	X(1)	0.18595D 06	0.28508D 07	-0.25192D 03	0.15386D 08
	X(2)	0.336840 04	0.46743D 05	0.298120 02	0.79400D 04
	X(3)	0.81129D 03	0.96663D 06	-0.12881D 03	0.28448D 07
	X(4)	-0.43311D 02	0.68225D 03	-0.351680 02	0.23242D 04
	X(5)	0.32464D 04	0.25534D 07	-0.43374D 03	0.43204D 07
	X(6)	-0.72629D 01	0.11048D 05	-0.43575D 01	0.264270 04
69	X(1)	0.18933D 06	0.332580 07	-0.25273D 03	0.16061D 08
	X(2)	0.33717D 04	0.48479D 05	0.28824D 02	0.78388D 04
	X(3)	0.797040 03	0.955050 06	-0.13352D 03	0.29872D 07
	X(4)	-0.453280 02	0.71495D 03	-0.34381D 02	0.237230 04
	X(5)	0.32272D 04	0.262460 07	-0.438250 03	0.44808D 07
	X(6)	-0.31202D 02	0.48743D 05	-0.46729D 01	0.29113D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
70	X(1)	0.19268D 06	0.39124D 07	-0.25353D 03	0.16762D 08
	X(2)	0.33859D 04	0.61769D 05	0.29463D 02	0.92198D 04
	X(3)	0.78034D 03	0.93831D 06	-0.13876D 03	0.31360D 07
	X(4)	-0.46956D 02	0.80620D 03	-0.35018D 02	0.24563D 04
	X(5)	0.31957D 04	0.27275D 07	-0.44295D 03	0.46489D 07
	X(6)	-0.23729D 02	0.37862D 05	-0.47166D 01	0.28388D 04
71	X(1)	0.19604D 06	0.46816D 07	-0.25414D 03	0.17485D 08
	X(2)	0.33968D 04	0.73028D 05	0.29344D 02	0.82616D 04
	X(3)	0.76152D 03	0.91718D 06	-0.14436D 03	0.32923D 07
	X(4)	-0.49414D 02	0.91931D 03	-0.34963D 02	0.25596D 04
	X(5)	0.31722D 04	0.28009D 07	-0.44842D 03	0.48239D 07
	X(6)	-0.31202D 02	0.48729D 05	-0.62388D 01	0.30807D 04
72	X(1)	0.19941D 06	0.56681D 07	-0.25527D 03	0.18224D 08
	X(2)	0.34110D 04	0.85566D 05	0.29259D 02	0.84165D 04
	X(3)	0.74092D 03	0.89129D 06	-0.14921D 03	0.34553D 07
	X(4)	-0.51226D 02	0.10717D 04	-0.34181D 02	0.26899D 04
	X(5)	0.31381D 04	0.29165D 07	-0.45507D 03	0.50113D 07
	X(6)	-0.37127D 02	0.54370D 05	-0.70607D 01	0.33077D 04
73	X(1)	0.20280D 06	0.68308D 07	-0.25528D 03	0.18984D 08
	X(2)	0.34279D 04	0.92671D 05	0.30895D 02	0.86818D 04
	X(3)	0.71777D 03	0.85765D 06	-0.15302D 03	0.36218D 07
	X(4)	-0.54962D 02	0.12942D 04	-0.33313D 02	0.26972D 04
	X(5)	0.31121D 04	0.30079D 07	-0.46095D 03	0.52061D 07
	X(6)	-0.14727D 02	0.22045D 05	-0.46902D 01	0.31195D 04
74	X(1)	0.20620D 06	0.81435D 07	-0.25497D 03	0.19768D 08
	X(2)	0.34346D 04	0.95347D 05	0.30149D 02	0.88255D 04
	X(3)	0.69195D 03	0.81562D 06	-0.15565D 03	0.37922D 07
	X(4)	-0.57599D 02	0.16496D 04	-0.32934D 02	0.26758D 04
	X(5)	0.30847D 04	0.30849D 07	-0.46483D 03	0.54037D 07
	X(6)	-0.40208D 02	0.64321D 05	-0.30780D 01	0.31255D 04
75	X(1)	0.20953D 06	0.83896D 07	-0.18589D 03	0.19712D 08
	X(2)	0.34461D 04	0.10599D 06	0.31924D 02	0.83129D 04
	X(3)	0.66569D 03	0.76987D 06	-0.15335D 03	0.39839D 07
	X(4)	-0.60841D 02	0.21368D 04	-0.32101D 02	0.26980D 04
	X(5)	0.30649D 04	0.32165D 07	-0.46204D 03	0.56289D 07
	X(6)	-0.88640D 01	0.16235D 05	-0.32560D 01	0.31932D 04
76	X(1)	0.21295D 06	0.10032D 08	-0.18376D 03	0.20491D 08
	X(2)	0.34501D 04	0.11357D 06	0.32707D 02	0.84054D 04
	X(3)	0.63329D 03	0.71338D 06	-0.15601D 03	0.41645D 07
	X(4)	-0.63526D 02	0.28317D 04	-0.32794D 02	0.28358D 04
	X(5)	0.30561D 04	0.32756D 07	-0.46487D 03	0.58407D 07
	X(6)	-0.88620D 01	0.16187D 05	-0.23966D 01	0.31407D 04
77	X(1)	0.21637D 06	0.11978D 08	-0.18055D 03	0.21283D 08
	X(2)	0.34540D 04	0.12104D 06	0.33235D 02	0.83097D 04
	X(3)	0.59732D 03	0.65229D 06	-0.15850D 03	0.43523D 07
	X(4)	-0.67983D 02	0.38071D 04	-0.31774D 02	0.28990D 04
	X(5)	0.30480D 04	0.33080D 07	-0.46726D 03	0.60589D 07
	X(6)	-0.73176D 01	0.11158D 05	-0.23946D 01	0.32032D 04
78	X(1)	0.21959D 06	0.11287D 08	-0.62979D 02	0.21427D 08
	X(2)	0.34370D 04	0.96726D 05	0.34421D 02	0.81284D 04
	X(3)	0.56556D 03	0.59255D 06	-0.14647D 03	0.44917D 07
	X(4)	-0.71833D 02	0.45884D 04	-0.31407D 02	0.29946D 04
	X(5)	0.30659D 04	0.33131D 07	-0.41775D 03	0.61981D 07
	X(6)	-0.50240D 02	0.81184D 05	-0.12707D 01	0.33552D 04
79	X(1)	0.22282D 06	0.10085D 08	-0.51828D 02	0.22348D 08
	X(2)	0.34458D 04	0.10661D 06	0.34455D 02	0.85058D 04
	X(3)	0.52412D 03	0.52995D 06	-0.14347D 03	0.46802D 07
	X(4)	-0.78346D 02	0.64994D 04	-0.31222D 02	0.29084D 04
	X(5)	0.30632D 04	0.34064D 07	-0.36954D 03	0.62382D 07
	X(6)	-0.43045D 02	0.71282D 05	-0.23221D 01	0.39080D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
80	X(1)	0.22591D C6	0.82715D 07	0.28912D-01	0.61467D 02
	X(2)	0.34414D C4	0.11271D 06	0.40662D 02	0.82602D 04
	X(3)	0.48996D 03	0.46320D 06	0.29756D 00	0.54087D 02
	X(4)	-0.86664D 02	0.63343D 04	-0.29738D 02	0.26923D 04
	X(5)	0.30754D C4	0.34654D 07	-0.31137D 03	0.63490D 07
	X(6)	-0.77566D 01	0.11833D 05	0.11118D 01	0.33950D 04
81	X(1)	0.22921D 06	0.87188D 07	0.10557D 02	0.81386D 04
	X(2)	0.34298D C4	0.95646D 05	0.41065D 02	0.80296D 04
	X(3)	0.43101D C3	0.40237D 06	0.92754D 00	0.29162D 04
	X(4)	-0.98865D 02	0.10200D 05	-0.28777D 02	0.29127D 04
	X(5)	0.31115D 04	0.33785D 07	-0.26300D 03	0.64424D 07
	X(6)	-0.95122D C1	0.17427D 05	0.22369D 01	0.35261D 04
82	X(1)	0.23204D C6	0.48597D 07	0.14671D 02	0.30051D 05
	X(2)	0.33856D C4	0.56073D 05	0.37629D 02	0.75902D 04
	X(3)	0.37410D 03	0.32269D 06	0.47895D 01	0.10990D 05
	X(4)	-0.11366D 03	0.13514D 05	-0.28052D 02	0.26934D 04
	X(5)	0.32682D C4	0.28708D 07	-0.40190D 02	0.59667D 07
	X(6)	-0.16843D C2	0.25138D 05	0.77227D 01	0.28868D 04
83	X(1)	0.23481D C6	0.17651D 07	0.12414D 02	0.63094D 05
	X(2)	0.33425D C4	0.34023D 05	0.35201D 02	0.70793D 04
	X(3)	0.30275D C3	0.21608D 06	0.76325D 01	0.25196D 05
	X(4)	-0.12374D 03	0.20127D 05	-0.27957D 02	0.28062D 04
	X(5)	0.34690D C4	0.25382D 07	0.27938D 03	0.53713D 07
	X(6)	-0.90774D C1	0.13757D 05	0.13244D 02	0.27200D 04
84	X(1)	0.23785D C6	0.94441D 06	0.21892D 02	0.10031D 06
	X(2)	0.33124D 04	0.17654D 05	0.36085D 02	0.64457D 04
	X(3)	0.20925D 03	0.10570D 06	0.10279D 02	0.43654D 05
	X(4)	-0.14800D C3	0.33150D 05	-0.28284D 02	0.26671D 04
	X(5)	0.35312D C4	0.21847D 07	0.40533D 03	0.49566D 07
	X(6)	-0.19801D 02	0.29427D 05	0.16368D 02	0.25942D 04
85	X(1)	0.23961D C6	0.16626D 06	0.30737D 03	0.60902D 05
	X(2)	0.32805D C4	0.95547D 03	0.92218D 02	0.23982D 04
	X(3)	0.74175D C2	0.51840D 05	-0.97558D 02	0.49468D 05
	X(4)	-0.16827D C3	0.67391D 05	-0.52289D 02	0.18715D 04
	X(5)	0.23782D 04	0.11258D 07	-0.11654D 04	0.28286D 07
	X(6)	0.47992D 00	0.24002D 01	0.11322D 02	0.13981D 04

APPENDIX C

RESULTS OF ATIGS SIMULATION WITH POSITION RESET AND KALMAN FILTERING

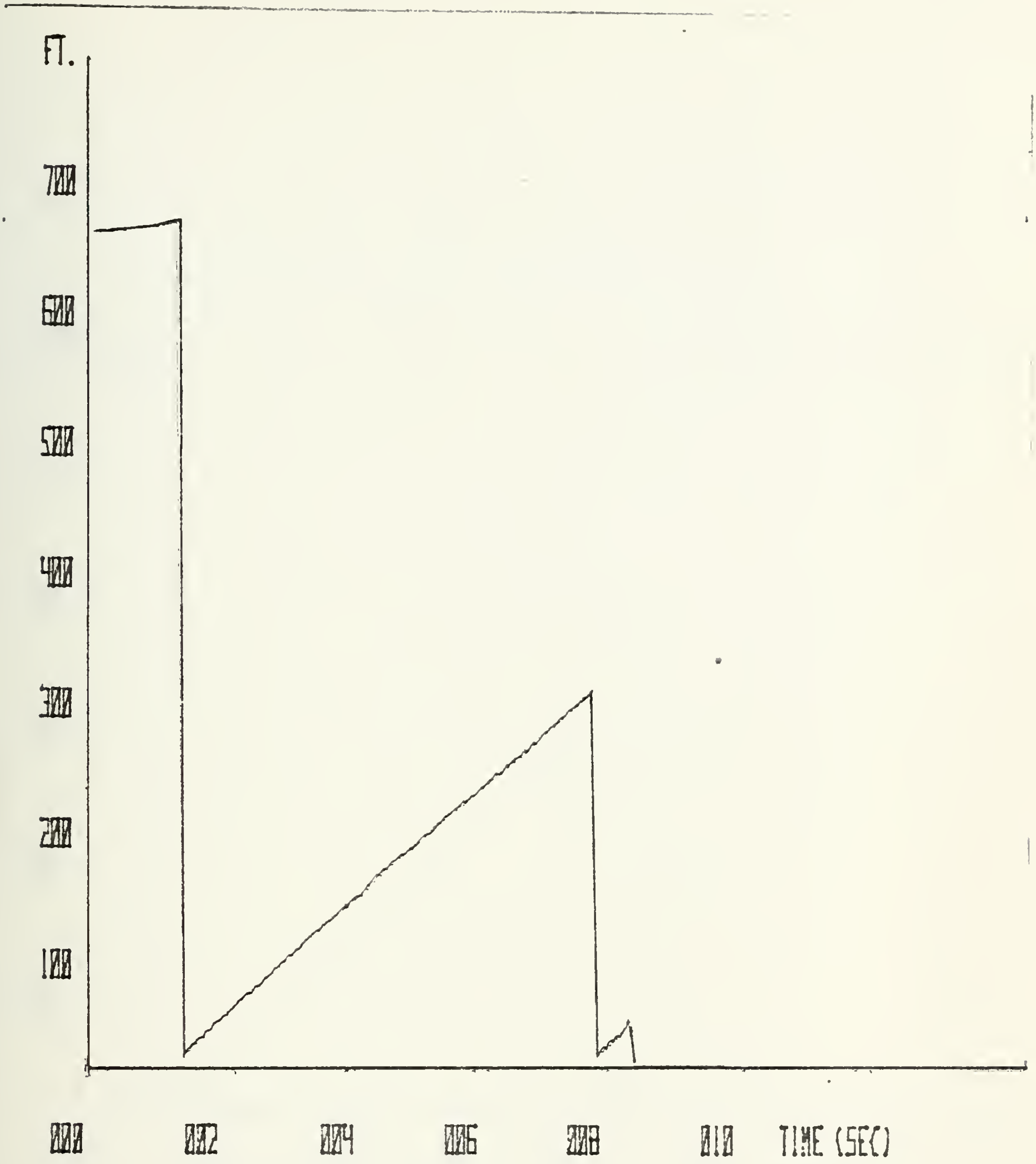

Figure 21 - SQRT OF DOWN RANGE VARIANCE (ATIGS WITH POSITION RESET AND KALMAN FILTERING)

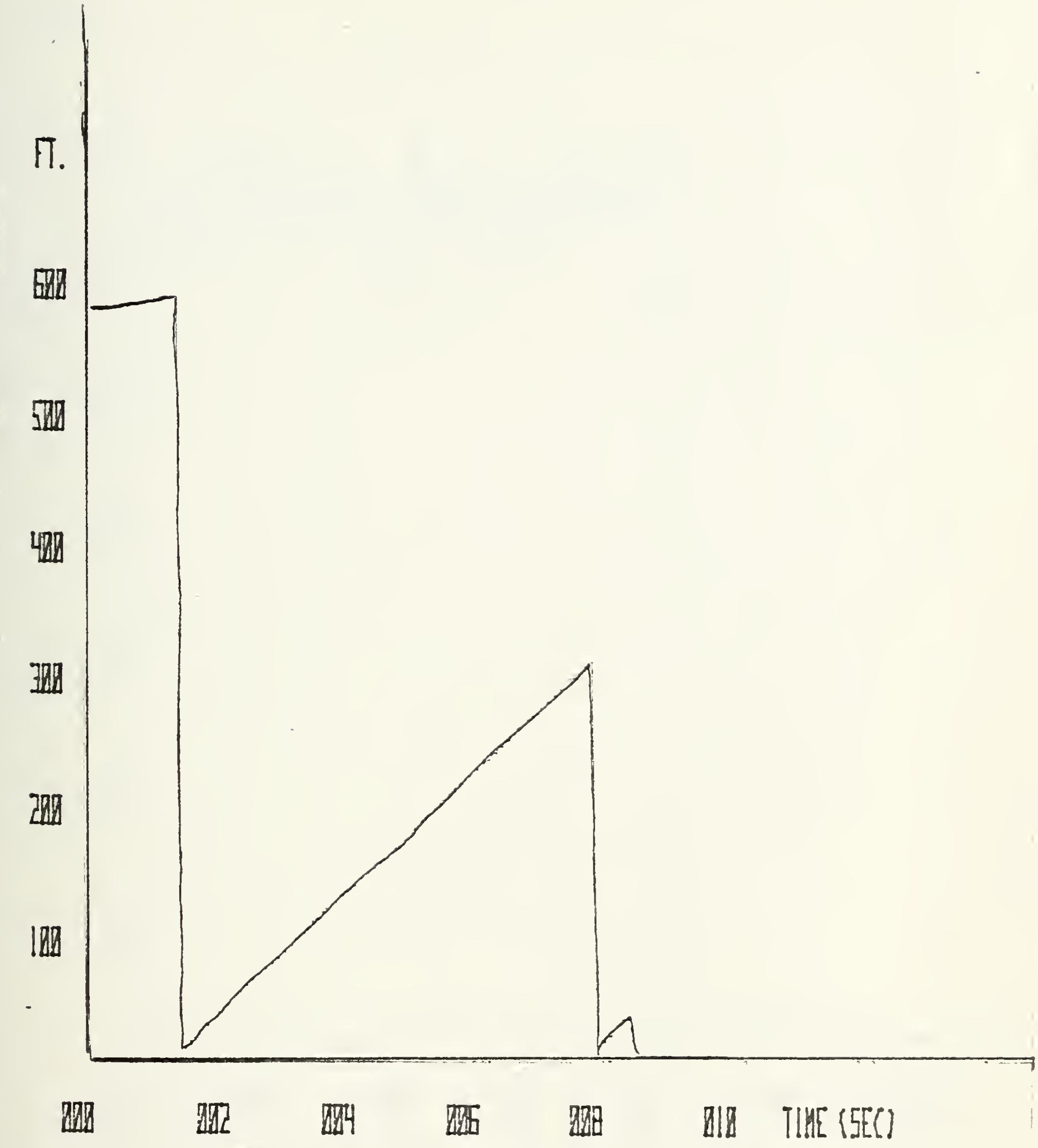

Figure 22 - SQRT OF CROSS RANGE VARIANCE (AFIGS WITH POSITION RESET AND KALMAN FILTERING)

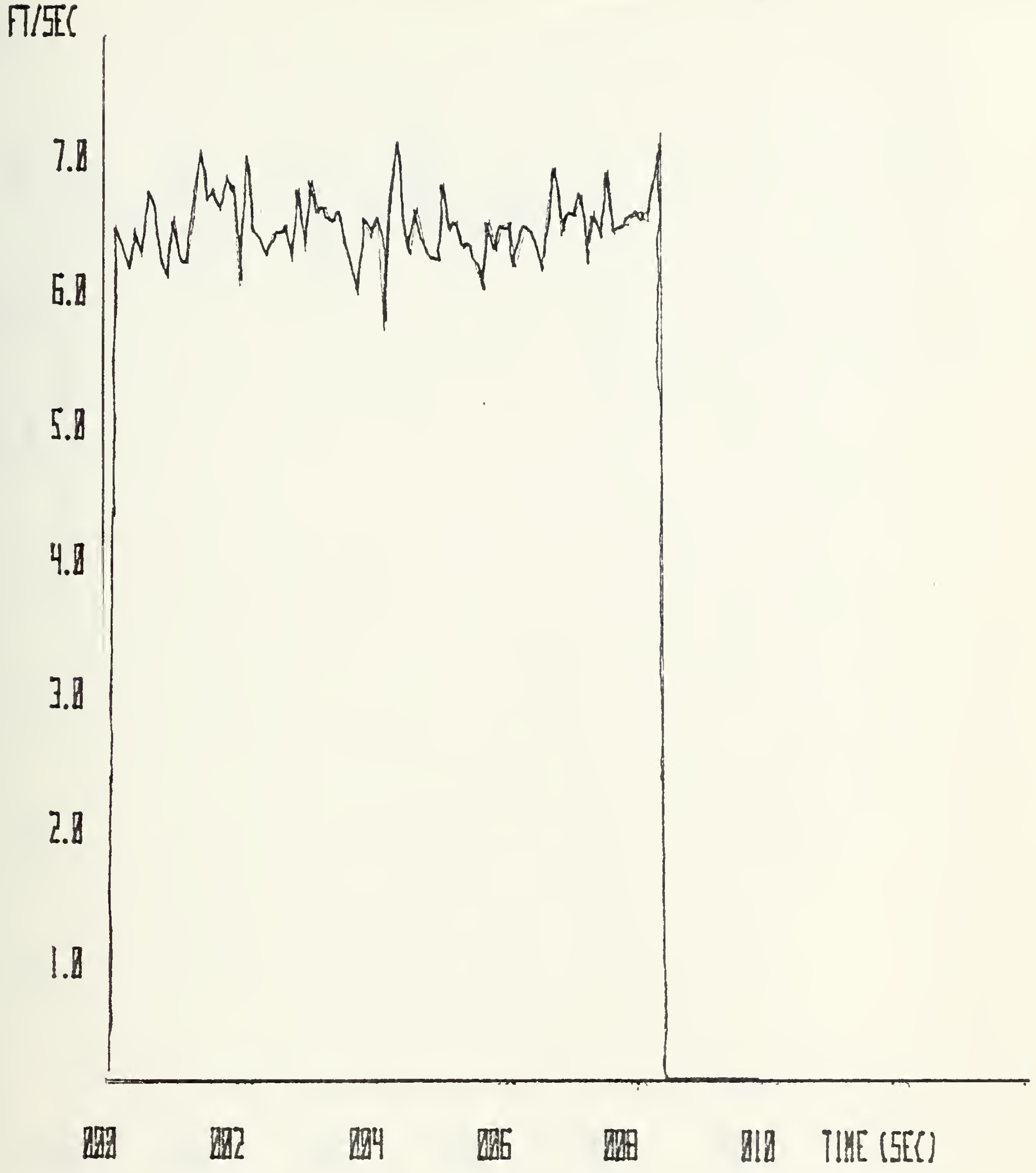

Figure 23 - SQRT• OF DOWN RANGE VELOCITY VARIANCE (ATIGS WITH POSITION RESET AND KALMAN FILTERING)

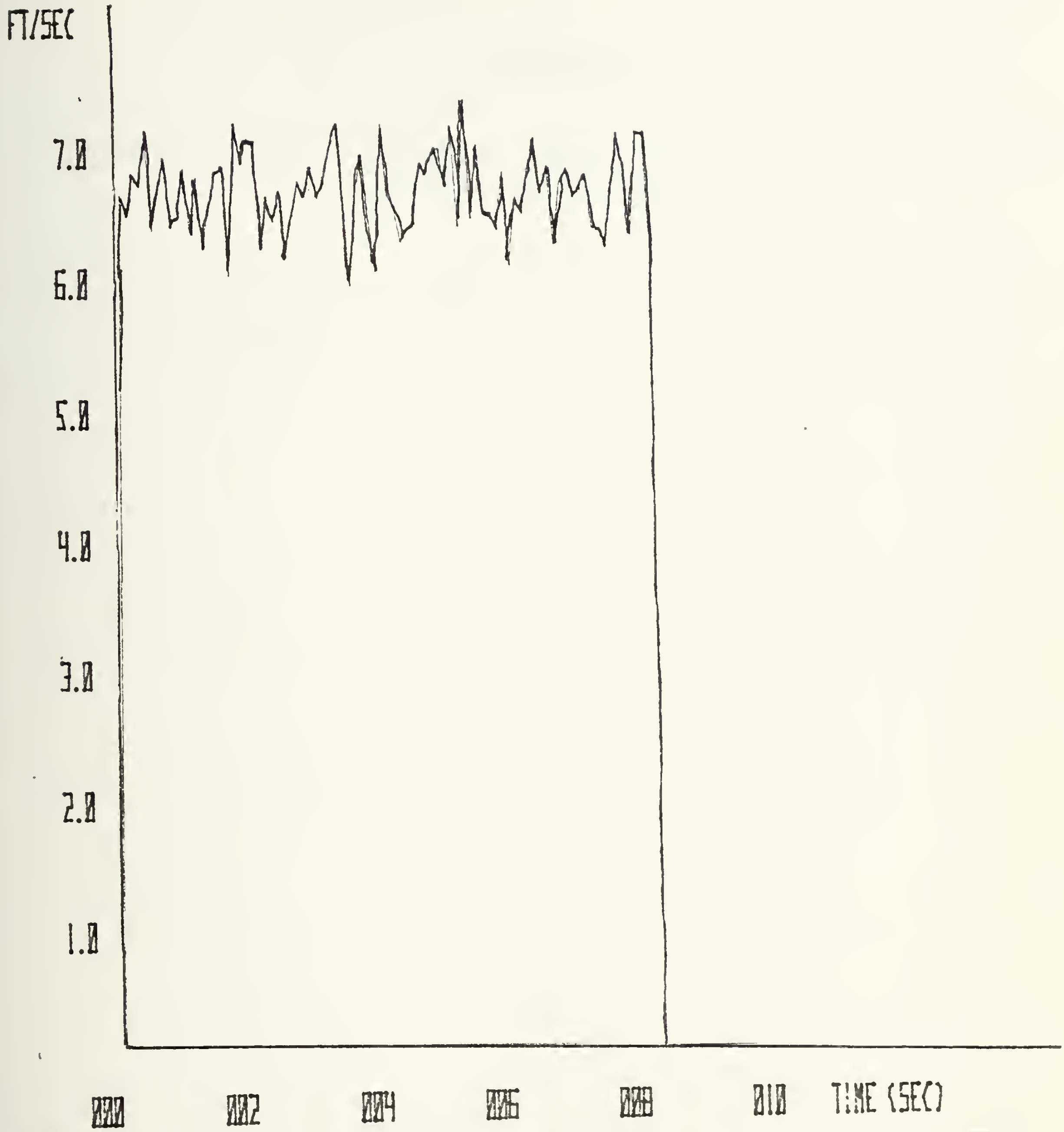

Figure 24 - SQRT. OF CROSS-RANGE VELOCITY VARIANCE (ATIGS WITH POSTION RESET AND KALMAN FILTERING)

APPENDIX D

RESULTS OF ATIGS SIMULATION WITH POSITION RESET AND KALMAN FILTERING AT X5 NOISE LEVEL

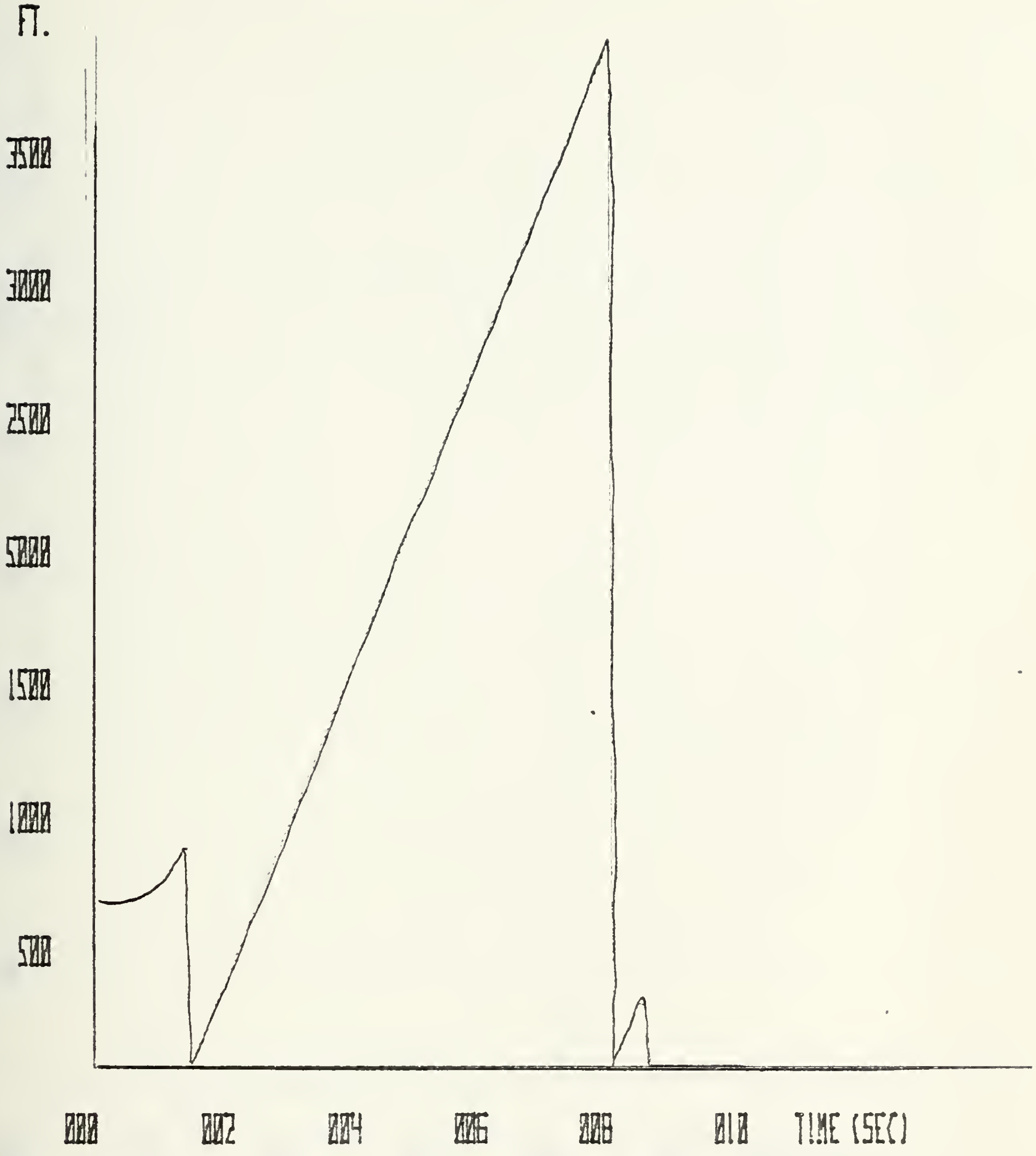

Figure 25 - SQRT. OF DOWN-RANGE VARIANCE (ATIGS WITH POSIT RESET AND KALMAN FILTERING AT X5 NOISE LEVEL)

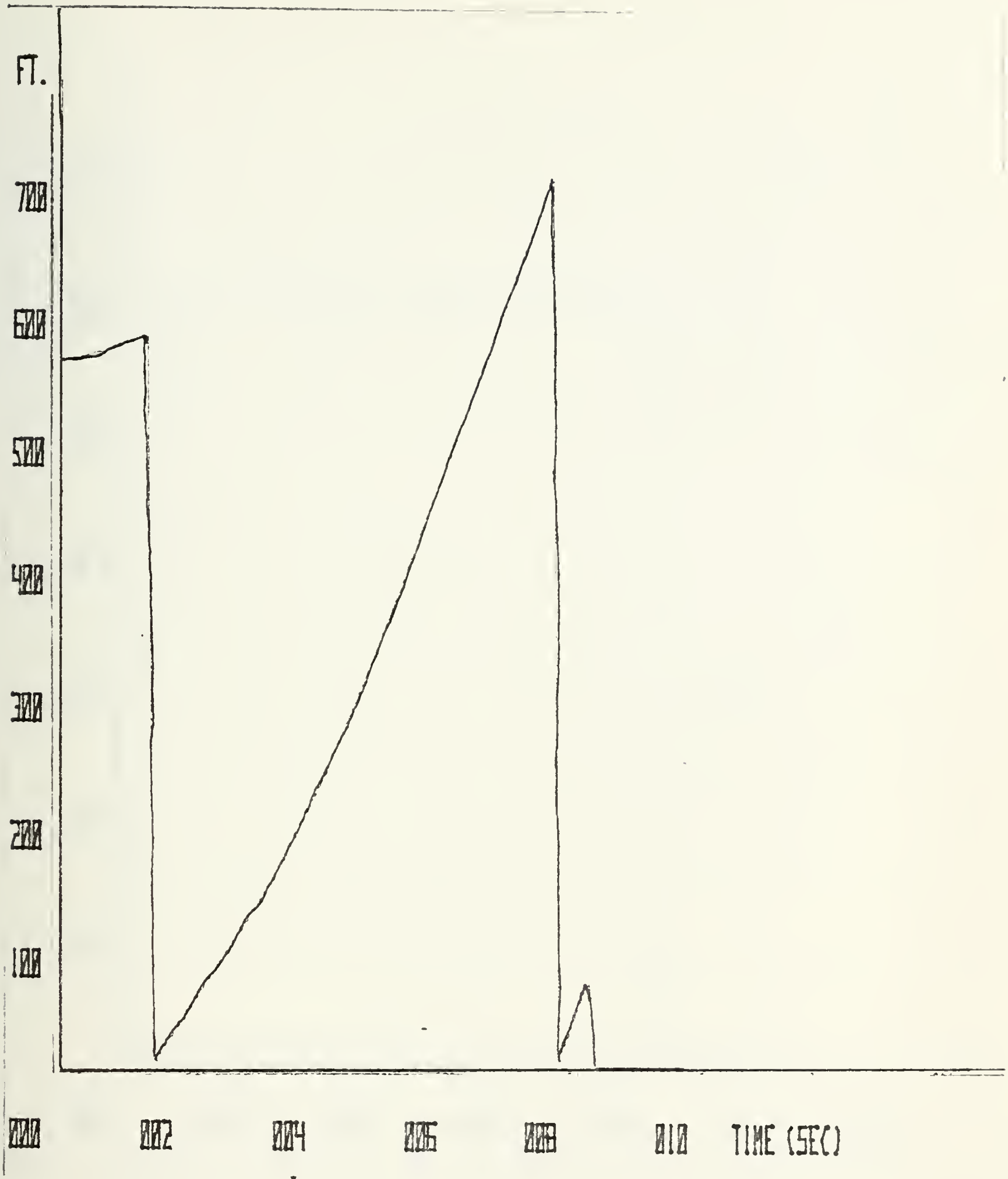

Figure 26 - SQRT• OF CROSS-RANGE VARIANCE (ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL)

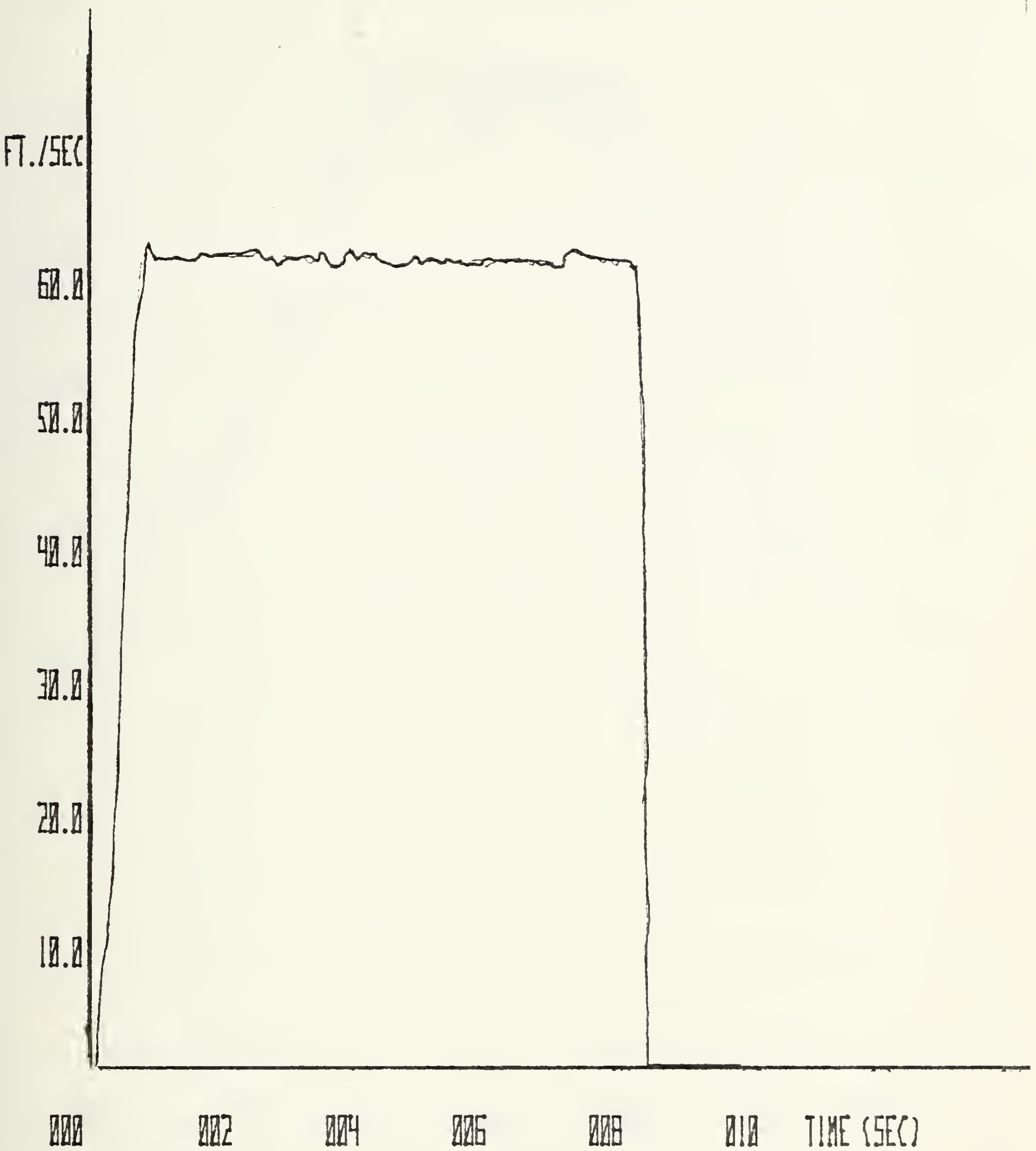

Figure 27 - SQRT• OF DOWN-RANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL)

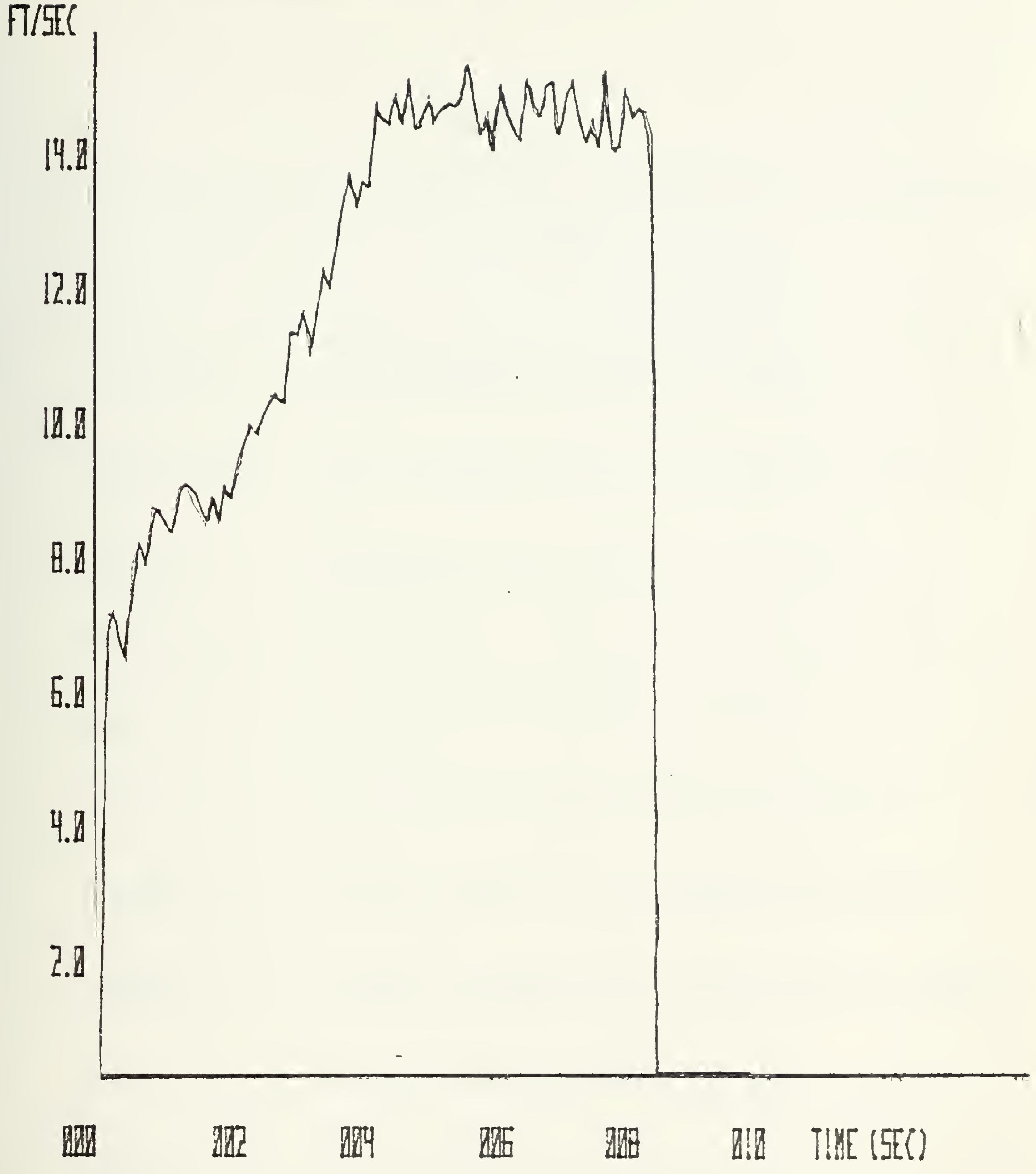

Figure 28 - SQRT. OF CROSS-RANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL)

APPENDIX E

PARTIAL LISTING OF SYMBOLS AND NOMENCLATURE OF SIMULATION PROGRAM

X(i,j)	i-th missile state at time j
THETA(i,j)	i-th angular state variable at time j
XI(i,j)	estimated i-th state at time j
THETA(i,j)	estimated i-th angular state at time j
XBAR(i,j)	mean of i-th state at time j
XBVAR(i,j)	variance of i-th state at time j
XMEAN(i,j)	error mean of i-th estimated state
XVAR(i,j)	error variance of i-th estimated state
A(j)	thrust acceleration at time j

AWXY
AWYY
AWYZ
AWZZ

AWXX velocity changes due to angular rotation
DELVX changes in wind components
DELVY

BETA1 BETA2 BETA3	body referenced accelerations
YIX YIY YIZ	inertial referenced accelerations
XPOS YPOS	micrad sensed positions
G1 G2	Kalman gains for filter
OMEGAX OMEGAY	extra states for Kalman filter
XINT	dummy variable for output
PSI	change in drift angle
XFIN(i)	final value of i-th state per track
NTERM	maximum time steps allowed
VXO	velocity of wind down-range
VYO	velocity of wind cross-range
IX	seed number for random number generators
DTHTAX	change in thetax
dthtxm	measured change in thetax
ZNI(j)	total tracks through time j

XIFIN(i) final value of estimated i-th state per track

XBFIN(i) mean of XFIN(i)

XIBFN(i) mean of XIFIN(i)

XBFV(i) variance of XFIN(i)

N number of gyros simulated

NNA number of accelerometers simulated

IENSB size of ensemble

SIGEO std.deviation of gyro bias

SIGW std.deviation of gyro random walk

SIGK std.deviation of gyro scale factor

SIGEG std.deviation of accelerometer bias

SIGKG std.deviation of accelerometer scale factor

SIGT std.deviation of initial condition on theta

APPENDIX F

SIMULATION PROGRAM

```
// EXEC FORTCLGP,REGION=180K
//FORT.SYSIN DD *
C
C      X(1,J)          THE ACTUAL POSITION ALONG THE DOWNRANGE
C                      AXIS
C      X(2,J)          THE AIRSPEED IN THE DOWNRANGE DIRECTION
C                      NOTE IT IS NOT THE ACTUAL VELOCITY
C      X(3,J)          CROSSRANGE POSITION
C      X(4,J)          CROSS RANGE AIRSPEED
C      XI(K,J)         THE INERTIALLY COMPUTED STATES ALONG THE
C                      DOWN-RANGE/CROSS-RANGE AXIS
C      XBAR            THE MATRIX OF MEAN VALUES OF THE MONTE-
C                      CARLO GENERATED TRACKS OF THE X MATRIX
C      XMEAN           THE MATRIX OF MONTECARLO GENERATED
C                      MEANS OF THE XI MATRIX
C
       DIMENSION NA(3),EOG(3),WG(3),PSI(150),GAM(3),ZNI(150),
       DIMENSION THETA(3,150),THETAI(3,150),G(3),EO(3),ZK(3),
       DIMENSION A(150),XFIN(6),XIFIN(6),YF(150),XINT(150),XP
       REAL*8 XBFIN(6),XIBFN(6),XBFV(6),XIBFV(6),XM1,XM3
       REAL*8 XBAR(6,155),XBVAR(6,155),X(6,155),XI(6,155)
       REAL*8 XMEAN(6,150),XVAR(6,150)
       DIMENSION ERR(6)
C
C      THIS SECTION READS IN THE VARIOUS NECESSARY SPECIFICAT
C        "N "  THE NUMBER OF GYROS INVOLVED PER SIMULATION
C        "NNA" THE NUMBER OF ACCELEROMETERS PER SIMULATION
C        "IENSB"  THE NUMBER OF THE ENSEMBLE
C
       READ(5,300)N,NNA,IENSB
C        "SIGEO" IS THE DIVIATION OF THE GYRO BIAS
C        "SIGW"  IS THE DEVIATION OF THE RANDOM WALK CONSTANT
C                FOR THE GYROS
C        "SIGK"  IS THE DEVIATION OF THE SCALE FACTOR (GYRO)
C        "SIGEG" IS THE DEVIATION OF THE ACCEL. BIAS
C        "SIGKG" IS THE DEVIATION OF THE ACCEL SCALE FACTOR
C        "SIGT"  IS THE DEVIATION OF THE INITIAL CONDITION
C                ON THETA
C
       READ(5,310)SIGEO,SIGW,SIGK,SIGEG,SIGKG,SIGT
C
C        "SIGMIC" IS THE DEVIATION OF THE POSITION MEASUREMEN
C
       READ(5,310)SIGMIC
       WRITE(6,420)N,NNA,IENSB
       WRITE(6,440)SIGEO,SIGW,SIGK,SIGEG,SIGKG,SIGT
       WRITE(6,500)SIGMIC
       CALL OVFLOW
       INDX1=1
       INDX2=1
       INDX3=1
       NDEBG=1
       YP=0.0
       NTERM=100
       IA=1
       IB=3
       IX=11111
       VXO=30.0
       VYO=30.0
       DO 02 J=1,NTERM
       DO 01 K=1,6
       XMEAN(K,J)=0.0
       XVAR(K,J)=0.0
       XBAR(K,J)=0.0
       XBVAR(K,J)=0.0
    01 CONTINUE
    02 CONTINUE
C
C      GENERATE THE THRUST ACCELERATION PROFILE
C
       A(1)=2.0
```

```
      A(2)=6.0
      A(3)=10.0
      A(4)=14.0
      A(5)=14.0
      A(6)=10.0
      A(7)=6.0
      A(8)=2.0
      A(9)=-1.0
      DO 03 I=10,150
   03 A(I)=0.0
      DO 04 J=1,6
      XBFIN(J)=0.0
      XIBFN(J)=0.0
      XBFV(J)=0.0
   04 XIBFV(J)=0.0
      DO 05 I=1,NTERM
      ZNI(I)=0.0
   05 A(I)=A(I)*32.2
C
C     START THE MONTECARLO SIMULATION
C
      DO 200 NI=1,IENSB
      TIMEX=1.0
      OMEGAY=0.0
      OMEGAX=0.0
C
C     THE PURPOSE OF THIS SECTION IS TO COMPUTE THE INITIAL
C     CONDITION FOR THE ACTUAL TRACK OF THE MISSILE.
C
      CALL RANDU(IX,IY,XFL)
      IX=IY
      X(1,1)=XFL*2240.0-1120.0
   10 CALL RANDU(IX,IY,YFL)
      IX=IY
      IF((XFL**2+YFL**2).GT.1.0)GO TO 10
      X(2,1)=700.0
      X(3,1)=YFL*2240.0-1120.0
      X(4,1)=0.0
      X(5,1)=35000.
      X(6,1)=0.0
C     GENERATE THE INITIAL CONDITIONS ON THETA
C
      CALL SNORM(IX,T,N)
      DO 11 J=1,N
      THETAI(J,1)=0.0
   11 THETA(J,1)=T(J)*SIGT
C
C     THE INITIAL SETTING OF THE INERTIAL NAVIGATOR IS
C     ZERO POSITION IN DOWN-RANGE AND CROSS-RANGE AND THE
C     ACTUAL VELOCITY AND ALTITUDE
C
      XI(1,1)=0.0
      XI(2,1)=670.0
      XI(3,1)=0.0
      XI(4,1)=30.0
      XI(5,1)=35000.0
      XI(6,1)=0.0
      DTHTAX=0.0
      DTHTAY=0.0
      DTHTAZ=0.0
C
C     THE PURPOSE OF THIS SECTION IS TO PRODUCE THE ACTUAL
C     TRACK OF THE SIMULATED MISSILE FOR COMPARISON WITH
C     OTHER ESTIMATES OF POSITION
C     WIND EFFECTS ARE COMPUTED FIRST
C
C     THE INITIAL VALUE OF C IS ALWAYS ZERO
      C=0.0
C
C     THE VARIOUS RANDOM INPUTS FOR MEASUREMENT DEVICES ARE
C     GENERATED
C
```

```
C       GENERATE GYRO BIAS  "EO"
        CALL SNORM(IX,EO,N)
        DO 18 KN=1,N
   18   EO(KN)=SIGEO*EO(KN)
C
C
C       GENERATE THE RANDOM SCALE FACTOR "ZK"
        CALL SNORM(IX,ZK,N)
        DO 19 KN=1,N
   19   ZK(KN)=SIGK*ZK(KN)
C
C       GENERATE RANDOM INPUTS TO ACCELEROMETER PACKAGE
C
C       GENERATE ACCELEROMETER BIAS "EOG"
        CALL SNORM(IX,EOG,NNA)
        DO 20 KN=1,NNA
   20   EOG(KN)=SIGEG*EOG(KN)
C
C       GENERATE ACCELEROMETER SCALE FACTOR "KG"
        CALL SNORM(IX,WG,NNA)
        DO 30 KN=1,NNA
   30   WG(KN)=SIGKG*WG(KN)
        DO 100 J=1,NTERM
        ZNI(J)=ZNI(J)+1.0
        JP1=J+1
        CALL RANDU(IX,IY,VY)
        IX=IY
        CALL RANDU(IX,IY,VX)
        VY=30+(VY*16.67-8.33)
        VX=30+(VX*16.67-8.33)
        IX=IY
        AWXX=X(6,J)*DTHTAX
        AWXY=-X(4,J)*DTHTAY
        AWYY=X(2,J)*DTHTAY
        AWYZ=X(6,J)*DTHTAZ
        AWZZ=-X(4,J)*DTHTAZ
        AWZX=-X(2,J)*DTHTAX
        AY=A(J)*THETA(2,J)
        THETA(1,JP1)=THETA(1,J)+DTHTAX
        THETA(2,JP1)=THETA(2,J)+DTHTAY
        THETA(3,JP1)=THETA(3,J)+DTHTAZ
        X(1,JP1)=X(1,J)+X(2,J)-VX+.5*(A(J)+AWXX+AWXY)
        X(2,JP1)=X(2,J)+A(J)+AWXX+AWXY
        X(3,JP1)=X(3,J)+X(4,J)+.5*(AY+AWYZ+AWYY)+VY
        X(4,JP1)=X(4,J)+AY+AWYZ+AWYY
        X(5,JP1)=X(5,J)+X(6,J)+.5*(AWZX+AWZZ)
        X(6,JP1)=X(6,J)+AWZX+AWZZ
C
C       GAM IS THE NOISE INPUT FOR EACH ACCELEROMETER
C
        GAM(1)=EOG(1)+WG(1)*A(J)
        GAM(2)=EOG(2)+WG(2)*C
        GAM(3)=EOG(3)
C
C       GENERATE THE BIAS TERM DUE TO RANDOM WALK
C
        CALL SNORM(IX,G,N)
        DO 50 JI=1,N
   50   G(JI)=SIGW*G(JI)
C
C       PSI IS THE CHANGE IN THE ANGLE BETWEEN THE COMPUTED
C       COORDINATE PLANE AND THE ACTUAL COORDINATE PLANE
C
        DO 51 JI=1,N
   51   PSI(JI)=EO(JI)+G(JI)
        DELVX=VXO-VX
        DELVY=VYO-VY
C
C       COMPUTE INERTIAL POSITION
C
        DTHTXM=DTHTAX+PSI(1)+DTHTAX*ZK(1)
        DTHTYM=DTHTAY+PSI(2)+DTHTAY*ZK(2)
```

```
      DTHTZM=DTHTAZ+PSI(3)+DTHTAZ*ZK(3)
C
C     THE GAINS G1 AND G2 ARE THE KALMAN GAINS GENERATED AS
C     A FUNCTION OF TIME
C
      G1=1.0-2.0/(TIMEX+1.0)
      G2=1.0/(TIMEX+1.0)
C
C     THE COMMANDED HEADING CHANGE IS SUBTRACTED FROM THE
C     OBSERVED HEADING CHANGE
C
      DLYJ=DTHTYM-DTHTAY
      DLXJ=DTHTXM-DTHTAX
C
C     THE FILTER UPDATE EQUATIONS FOLLOW
C
      THETAI(1,J)=THETAI(1,J)+G1*(DLXJ-OMEGAX)
      OMEGAX=OMEGAX+G2*(DLXJ-OMEGAX)
      THETAI(2,J)=THETAI(2,J)+G1*(DLYJ-OMEGAY)
      OMEGAY=OMEGAY+G2*(DLYJ-OMEGAY)
C
C     THE FILTERED UPDATES ARE USED TO PREDICT THE NEXT
C     STATE IN THE NAVIGATOR
C
      THETAI(1,JP1)=THETAI(1,J)+DTHTAX+OMEGAX
      THETAI(2,JP1)=THETAI(2,J)+DTHTAY+OMEGAY
      THETAI(3,JP1)=THETAI(3,J)+DTHTZM
      TIMEX=TIMEX+1.0
C
C     SENSED ACCELEROMETER INPUTS IN THE BODY AXIS FRAME
C
C     IN THE X(BODY FRAME) DIRECTION
C
      BETA1=A(J)-DELVX+DELVY*THETA(2,J)+GAM(1)
C
C     IN THE Y(BODY FRAME) DIRECTION
C
      BETA2=DELVX*THETA(2,J)+DELVY+GAM(2)
C
C     IN THE VERTICAL(BODY FRAME) DIRECTION
C
      BETA3=0.0
C
C
C     THE PURPOSE OF THIS SECTION IS TO GENERATE THE
C     INERTIAL ESTIMATES OF POSITION BASED ON A PURE
C     INERTIAL COMPUTATION
C
C     AWXXI IS THE ACCELERATIONS DUE TO HEADING CHANGE
C     AFFECTING THE X DIRECTION FROM THE ANGLE CHANGE THETA
C     X.  SIMILARLY AWXYI IS THE ACCELERATIONS  AFFECTING
C     THE X DIRECTION DUE TO THETAY
      AWXXI=XI(6,J)*DTHTAX
      AWXYI=-XI(4,J)*DTHTAY
      AWYYI=XI(2,J)*DTHTAY
      AWYZI=XI(6,J)*DTHTZM
      AWZZI=-XI(4,J)*DTHTZM
      AWZXI=-XI(2,J)*DTHTAX
      DTHTXM=0.0
      DTHTYM=0.0
      DTHTZM=0.0
      DTHTAX=0.0
      DTHTAY=0.0
      DTHTAZ=0.0
C
C     SENSED ACCELERATIONS IN THE BODY FRAME ARE CONVERTED
C     TO THE INERTIAL FRAME.
C
      YIX=BETA1-BETA2*THETAI(2,J)
      YIY=BETA1*THETAI(2,J)+BETA2
      IF(J.EQ.15)GO TO 48
      IF (J.NE.80)GO TO 49
```

```
C
C     GENERATE THE RANDOM ERROR IN THE POSITION MEASUREMENT
C
   48 CALL SNORM(IX,XPOS,1)
      XPOS=XPOS*SIGMIC
      CALL SNORM(IX,YPOS,1)
      YPOS=YPOS*SIGMIC
      XI(1,J)=XPOS+X(1,J)
      XI(3,J)=YPOS+X(3,J)
   49 CONTINUE
      XI(6,J)=X(6,J)
C
C     COMPUTE THE INERTIAL ESTIMATES OF POSITION
C     AND VELOCITY
C
      XI(1,JP1)=XI(1,J)+XI(2,J)+.5*(YIX+AWXXI+AWXYI)
      XI(2,JP1)=XI(2,J)+YIX+AWXXI+AWXYI
      XI(3,JP1)=XI(3,J)+XI(4,J)+.5*(YIY+AWYYI+AWYZI)
      XI(4,JP1)=XI(4,J)+YIY+AWYYI+AWYZI
      XI(5,JP1)=XI(5,J)+XI(6,J)+.5*(AWZZI+AWZXI)
      XI(6,JP1)=XI(6,J)+AWZZI+AWZXI
      VXO=VX
      VYO=VY
      IF (XI(1,JP1).GE.40000.)DTHTAX=.4538-THETA(1,JP1)
      IF (XI(5,JP1).LE.5000.)DTHTAX=-THETA(1,JP1)
      IF(DABS(XI(3,JP1)).LE.150.)GO TO 88
      REM=240000.-XI(1,J)
      IF(REM.LE..001)GO TO 88
      CONTRL=X(3,J)/REM
      DTHTAY=-CONTRL-THETAI(2,JP1)
   88 CONTINUE
C
C     THIS SECTION GENERATES THE REQUIRED STATISTICS
C
      DO 89 K=1,6
      ERR(K)=X(K,J)-XI(K,J)
      XMEAN(K,J)=XMEAN(K,J)+ERR(K)
      XBAR(K,J)=XBAR(K,J)+X(K,J)
      XBVAR(K,J)=XBVAR(K,J)+X(K,J)**2
      IF(ABS(ERR(K)).LE.0.001)GO TO 89
      XVAR(K,J)=XVAR(K,J)+ERR(K)**2
   89 CONTINUE
      IF(XI(1,J+1).GT.240000.) GO TO 90
      GO TO 100
   90 DO 91 I=1,6
   91 XFIN(I)=X(I,J)
      XIFIN(1)=XI(1,J)
      XIFIN(3)=XI(3,J)
      XIFIN(5)=XI(5,J)
      GO TO 110
  100 CONTINUE
      DO 101 I=1,6
  101 XFIN(I)=X(I,150)
      XIFIN(1)=XI(1,150)
      XIFIN(3)=XI(3,150)
      XIFIN(5)=XI(5,150)
C
C     THIS SECTION COMPUTES THE FINAL VALUE STATISTICS
C
  110 DO 120 J=1,6
  120 XBFIN(J)=XBFIN(J)+XFIN(J)
      XIBFN(1)=XIBFN(1)+XIFIN(1)
      XIBFN(3)=XIBFN(3)+XIFIN(3)
      XIBFN(5)=XIBFN(5)+XIFIN(5)
      XERR1=XFIN(1)-XIFIN(1)
      XERR3=XFIN(3)-XIFIN(3)
      IF(XERR1.LE..001)GO TO 121
      XBFV(1)=XBFV(1)+XERR1**2
  121 IF(XERR3.LE..001)GO TO 200
      XBFV(3)=XBFV(3)+XERR3**2
  200 CONTINUE
C
```

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