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Advances in nuclear and diesel-electric submarine technology have reduced the effectiveness of passive means of detection. The United States is faced with a multipolar threat in part due to the proliferation to Third World nations of advanced diesel-electric submarines. The use of active sonar must be explored to gain back the detection advantage the United States submarine force has enjoyed in the past. The use of bistatic sonar reduces the counter-detection threat resulting from active sonar.

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A BARRIER SEARCH MODEL USING ACTIVE BISTATIC SONAR TO PROTECT A CHANNEL

by

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ABSTRACT

Advances in nuclear and diesel-electric submarine technology have reduced the effectiveness of passive means of detection. The United States is faced with a multipolar threat in part due to the proliferation to Third World nations of advanced diesel-electric submarines. The use of active sonar must be explored to gain back the detection advantage the United States submarine force has enjoyed in the past. The use of bistatic sonar reduces the counter-detection threat resulting from active sonar.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. PURPOSE

The purpose of this thesis is to study an application of active bistatic sonar to anti-submarine warfare (ASW). In an active monostatic sonar system the source platform acts both as a source and receiver. A bistatic sonar system incorporates a separate source and receiver platform. Specifically, this research will concentrate on the use of an active bistatic sonar system to protect a channel from a transiting enemy submarine. A mathematical model can be developed to calculate signal excess as the enemy submarine transits down the channel for a given location of the source and receiver. This signal excess can be represented as a detection probability, which will be used as the payoff in a two-person zero-sum game.

B. BACKGROUND

Recent political changes in Russia and other former communist countries have resulted in a significant reduction of the perceived Eastern Bloc threat to the United States. The immediate cost of the Cold War victory is decreased world stability. Diminished Russian global influence has caused the instability throughout the Third World [Ref. 1:p. 146]. Reduced world stability has forced the United States to face a new multipolar threat. Consequently, U.S. military strategists now place a much higher priority on regional conflict planning. For the ASW mission, the United States must now also contend with diesel-electric submarines operating on their batteries, quieter nuclear submarines, and a shift in policy from preparing for global warfare to littoral warfare. The Russian navy, especially their submarine force, remains a viable fighting force.

The ASW advantage enjoyed by the United States and its allies has steadily decreased. Over the last ten years Russian submarines have undergone significant reductions of their radiated noise signatures [Ref. 1:p. 146]. During this same period improvements of passive detection by the United States advanced at a slower rate. The proliferation of advanced diesel-electric submarines to Third World nations adds a new dimension to the ASW problem for the United States. The new diesel-electric submarines are capable of extended patrols without having to snorkel as frequently as in the past. New battery technology provides a cheap alternative to nuclear submarines for Third World countries. To maintain its advantage over other countries in ASW, the United States must revolutionize detection technology.

1. Historical Perspective

The early 1960s saw the beginning of the tactical and strategic development of nuclear submarines. The United States development was significantly different from the Soviet

Union's. The United States invested in maintaining vital sea lanes of communications and assuring freedom of navigation in forward areas. To support this global ASW mission, the United States concentrated on the development of highly capable passive sonar systems, gaining a significant ASW advantage. For three decades after World War II the United States submarine force enjoyed a tactical and strategic advantage over the Soviet Union's submarine and ASW forces.

Since the late 1970s the United States ability to detect enemy submarines by passive means has significantly decreased. During the period 1975 to 1988, the radiated noise signal levels of Soviet submarines dropped by 30 dB or by a factor of 1000 [Ref. 1:p. 146]. The increased use of diesel-electric submarines by Third World countries compounds the passive detection problem.

Active sonar does not rely on target generated noise as with passive detection. The greatest concern for active sonar is counter-detection. It is very difficult for a submarine to remain undetected when using active sonar. However, accurate bearing and range information can be gathered by using active sonar. With passive systems only bearings can be found, thus requiring target motion analysis to find the range to the target.

2. Bistatic versus Monostatic Active Sonar

The most significant problem of active sonar is the loss of stealth of the searching platform. An active sonar can typically be detected by another platform at twice the detection range of the searching platform. The sound must travel both directions for the searcher but only one direction for the potential target platform.

With the use of a bistatic active sonar, where the transmitter (source) and receiver are two different platforms, the searching ship can remain undetected but also take advantage of the active sonar. It is, however, important to ensure the receiver is not counter-detected by the target as active sonar does not discriminate between searcher and target.

C. DESCRIPTION OF BISTATIC SEARCH SCENARIO

A source and receiver platform are searching out a rectangular barrier area attempting to detect and locate a transiting enemy submarine. The target submarine's orientation is known by the receiver and source platform. The receiver is placed within the barrier for two reasons. First, to constrain the problem to help make it solvable, and second, to keep the receiver within range to prosecute the target if necessary. The source platform can be located inside or outside the barrier area. The target enters the barrier along one side and transits through. Figure 1 shows the target entry into the barrier.

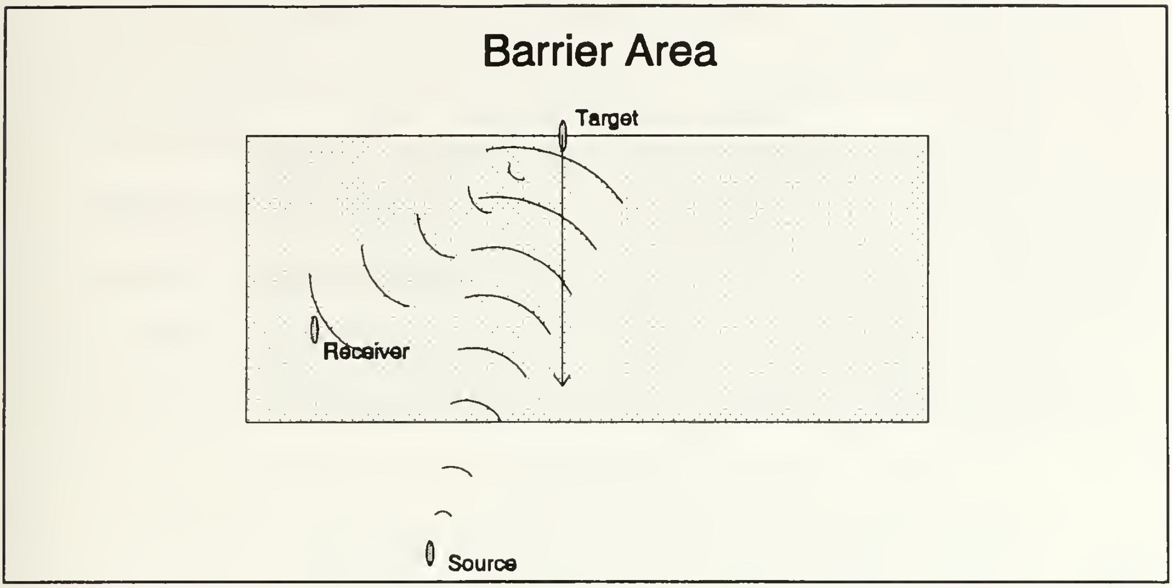

Figure 1. Target entry point into the barrier

The detection probability can be calculated, for a fixed source location, for each given case of target entry and receiver location. These probabilities represent a payoff matrix. On one axis of the matrix are the receiver locations and the other axis are the target entry points. To determine the result for a given combination of receiver location and target entry point, the detection probability for the receiver is the intersection point within the payoff matrix. The receiver does not know where the target will enter the barrier. The target also does not know where the receiver is located within the barrier. The receiver desires to maximize this value of detection, where the target wants to minimize the value. This method of one side maximizing and the other side minimizing is a two-person zero-sum game [Ref. 2:pp. 10-11].

There are two possible outputs to a two-person zero-sum game, a pure strategy and a mixed strategy. The pure strategy means that the element within the matrix represents both the largest column and the smallest row value [Ref. 2:p. 11] or in other words, the matrix contains a saddle point. The mixed strategy occurs when no saddle point exists, thus there is a probability distribution of pure strategies [Ref. 2:pp. 12-13].

In the evaluation of the payoff matrix formed by the model, the underlying desire is for a single pure strategy for any given case of a fixed source location. If there is a saddle point, the model results provide the receiver an optimal pure strategy to detect a transiting submarine through the barrier. If however, the results yield no saddle point, a mixed strategy is required and thus the receiver and the source must make tactical decisions regarding the choice of strategy.

II. MATHEMATICAL MODEL

A. MODEL ASSUMPTIONS

The mathematical model incorporates the bistatic active sonar equation to determine the optimal strategy for a receiver and a source to detect a transiting enemy submarine. The bistatic active sonar equation is

$$\overline{SE} = SL - TL_{ST} - TL_{TR} - NL - DT + TS + AG. \tag{1}$$

Where $\overline{SE}$ is the mean signal excess, SL is the source level of the active transmission from the source platform, TL_{ST} is the transmission loss from the source to the target, TL_{TR} is the transmission loss from the target to the receiver, NL is the ambient noise level of the surrounding sea, DT is the detection threshold of the receiver sonar system, TS is the bistatic target strength, and AG is the array gain of the receiver sonar. All parameters are measured in decibels (dB) and referenced to 1 micropascal.

Signal excess (SE) is assumed to be a normal random variable with mean given in equation 1 and standard deviation σ. Detection occurs whenever $SE>0$. The probability of detection is

$$P(SE>0) = \Phi\left(\frac{\overline{SE}}{\sigma}\right), \tag{2}$$

where Φ is the cumulative normal distribution and σ is the standard deviation [Ref. 3:p. 3-2]. Typical values for σ are between 3 and 9 dB [Ref. 3:p. 3-2]. The value of σ for the model is set at 8 dB. For computational purposes, a polynomial approximation to Φ from the Handbook of Mathematical Functions [Ref. 4:p. 932] was used.

The fictional active low frequency source is assumed to have a **source level (SL)** of **220 dB** re 1 micropascal. This source level provides a theoretical detection range in excess of 300 nautical miles (nm) [Ref. 1:p. 152].

Transmission loss (TL) results from attenuation and spreading. The model assumes the spreading loss is due to spherical spreading with no transition to cylindrical spreading. This assumption yields reasonable fits to actual measured data under various conditions, Urick [Ref. 5:p. 110]. Transmission loss can be expressed as

$$TL = 20\log R + \alpha R \times 10^{-3}, \tag{3}$$

from Urick [Ref. 5:pp. 110-111], where R is the range in yards from either the source to the target or the target to the receiver and α is the absorption coefficient in dB per kiloyard (kyd). For the model α = 0.0041 dB/kyd and is calculated from

$$\alpha = \frac{0.1 f^2}{1+f^2} + \frac{40 f^2}{4,100+f^2} + 2.75\times 10^{-4} f^2 + 0.003, \tag{4}$$

where f is the search frequency expressed in kHz [Ref. 5:p. 108]. For the model, f is assumed to be 0.1 kHz.

The background noise is assumed to be ambient noise limited, vice reverberation limited. The average **noise level (NL)** is **63 dB** re 1 micropascal for the search frequency of 100 Hz found on the Wentz-Knudsen curves [Ref. 5:p. 210]. Figure 2 shows the Wentz-Knudsen curves.

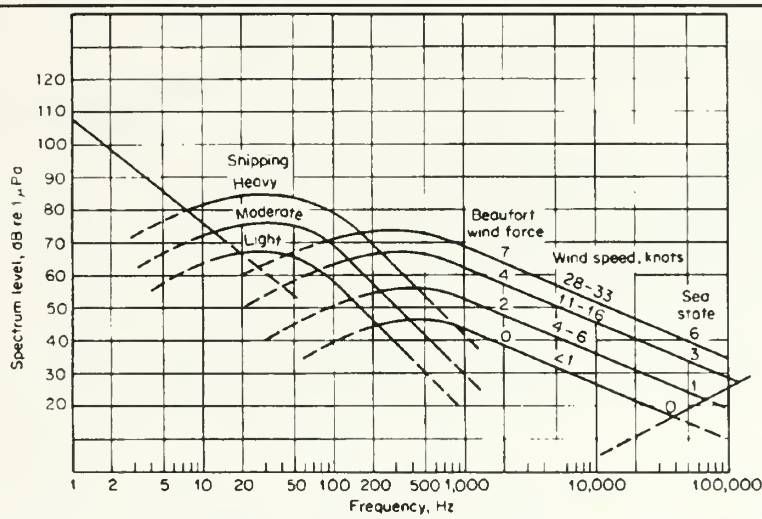

Figure 2. Average deep-water ambient-noise spectra.

The **detection threshold (DT)** for the receiver sonar is **12 dB** re 1 micropascal. Detection threshold can be calculated from one of two different cases [Ref 5:p. 394]. **Case I** assumes a known signal in Gaussian noise and a fixed point target at a known range. **Case II** assumes an unknown signal and a target of unknown range and range rate. **Case II** yields a higher value and thus is a more conservative estimate of detection threshold. **Case II** detection threshold is

$$DT = 5\log\frac{dw}{t}, \tag{5}$$

where ***d***, the detection index, is determined by the specified probability of detection and probability of false alarm. Selecting values for probability of detection (Pd) equal to 90% and probability of false alarm (Pfa) equal to 0.01%, yields $\boldsymbol{d} = 25$. $\boldsymbol{W}$ is the frequency bandwidth (100 Hz) and $\boldsymbol{t}$ is the pulse duration (10 seconds). [Ref. 1:p. 150].

The bistatic **target strength (TS)** is a function of the minimum ($\boldsymbol{TS_{min}}$) and maximum monostatic target strength ($\boldsymbol{TS_{max}}$) and the bistatic angle (θ_B) formed by the relative angles, (θ_S) and (θ_R), between the source and the target, and the receiver and the target, respectfully. Figure 3 contains a visual representation of the bistatic angle.

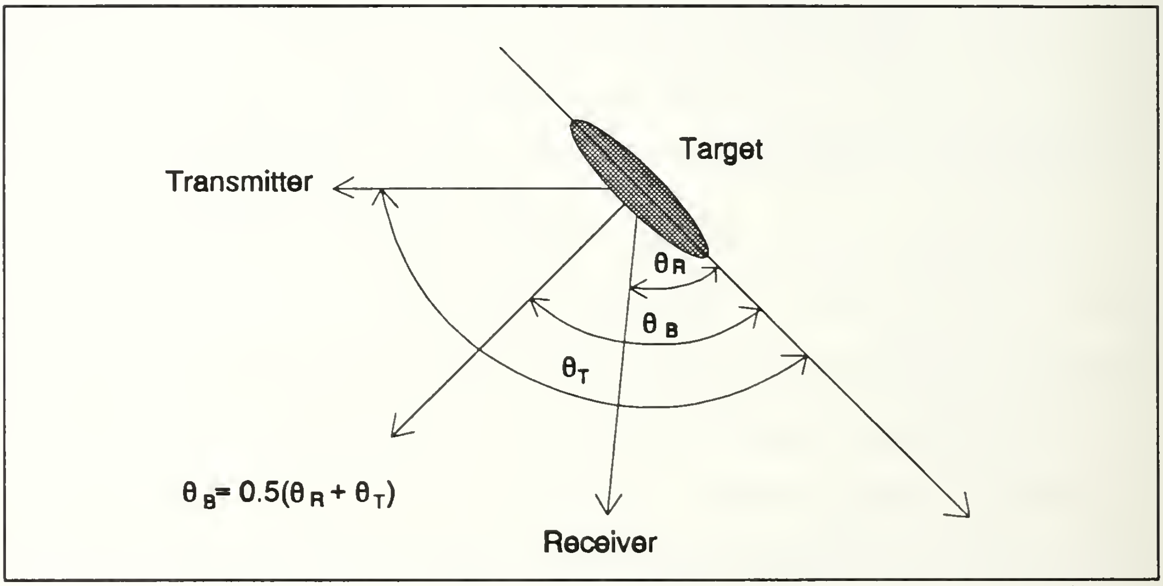

Figure 3. Bistatic angle for target strength calculation.

The equation for target strength is given by

$$TS = (TS_{max} - TS_{min}) + TS_{min}\sin^2(\theta_B)\,, \tag{6}$$

where, for a typical submarine, $\mathbf{TS_{max}}$**=25 dB** and $\mathbf{TS_{min}}$**=12 dB** re 1 micropascal. [Ref. 6:p. 2-3].

The final term for the active bistatic sonar equation is the **array gain (AG)**. Array gain is a measure of the sonar's ability to increase the ratio of the submarine signature to the background noise. Array gain of **15 dB** re 1 micropascal is consistent with the conclusions of Stefanick's work [Ref. 7:p. 257].

B. COMPUTER MODEL

The code for the computer model is written in Turbo Pascal 6.0 for DOS based computers. The computer program calculates the detection probability of the target as it transits through the barrier for a fixed entry point for the target and a fixed receiver and source position.

1. Inputs to Model

There are two input files for the main program. The first file contains specific information about the barrier, receiver, target, and source. The dimensions (width and depth) of the barrier are provided by the input file. Inputs for the receiver consist of the step size for receiver location in both the X and Y axis, and the detection threshold and array gain of the receiver sonar system. For the target, the step size for the target's barrier entry, speed of the target during transit and its minimum and maximum monostatic target strength are provided. The source inputs include active sonar pulse frequency, pulse length, ping interval, source level of active sonar and ambient background noise level. The second input file contains all source locations to evaluated.

2. Main Program

The program calculates for each given receiver position and target entry point, with a fixed source position, the detection probability for target transit through the barrier. The target entry points are based on the user input of the width and depth of the barrier, and the step size for target entry. For example, if the barrier is 200 kiloyards (kyds) in width, 40 kyds in depth and the target entry step size is 10 kyds then the target would enter the barrier at X position equal to (0,10,20,30,...,200) kyds with Y position equal 40 kyds. Based on the same width and depth of the barrier as above and with user input of step size of 5 kyds for both the X and Y position for the receiver, the receiver would be located at all possible combinations of X positions of (0,5,10,...,200) kyds and Y positions of (0,5,10,...,40) kyds.

To determine the overall detection probability for the target transit, the signal excess is calculated using the active bistatic sonar equation for the receiver at fixed points for the target on the transit. The signal excess can be represented as a detection probability as previously discussed in the Model Assumptions section. The probability of no detection is found by taking one minus the probability of detection. By taking one minus the product of all the probabilities of no detection the overall detection probability is determined as shown by

$$P_d = 1 - \prod (1 - p_d^i), \tag{7}$$

where P_d is the overall probability of detection of the transit and p_d^i is the probability of detection of the target on the i^{th} ping. Each P_d^i is assumed to be independent as the time between pings is typically greater than 5 minutes. The fixed points are based on the speed of the target, ping interval of the source active sonar and the length of the barrier. Each ping of the active sonar represents a fixed point for the target. Between pings the target advances through the barrier based on its speed.

Each possible entry point for the target is evaluated for detection probability (A_{xyz}) for a fixed receiver and source position. A_{xyz} is the probability of detection for a given (x,y) coordinate position of the receiver and entry point z for the target. This procedure is repeated for all possible receiver position combinations. The results for each possible combination of target entry and receiver position is written to an output file. The program is rerun for each possible source location. The code is shown in Appendix A.

3. GAMS Interface

The output from the program forms a probability matrix with one axis receiver position and the other target entry point. This probability matrix is for a given source position. Each different source position forms another separate probability matrix. Using Game Theory, the probability matrix can be evaluated to find an optimal strategy for both the receiver and the target. To find the optimal strategy numerous simultaneous equations must be solved. This problem can be formulated into a linear program as shown below:

$$
\begin{array}{lll}
\textit{maximize} & V & \\
\textit{subject to} & \sum_{x \in m} \sum_{y \in n} A_{xyz} P_{xy} - V \geq 0; & z \in q, \\
 & \sum_{x \in m} \sum_{y \in n} P_{xy} = 1, & \\
\textit{and} & P_{11}, \ldots, P_{mn} \geq 0, &
\end{array}
\tag{8}
$$

where m and n represents the number of different X and Y positions for the receiver respectfully, and q is the number of different entry points into the barrier for the target. V represents the overall probability of detection of the target for the optimal strategy (value of the game) [Ref. 8:pp. 31-32]. A_{xyz} is defined in the previous section. The z value for the target represents the X coordinate position as the Y coordinate position is at the top of the barrier. P_{xy} represents the percentage the receiver should spend in a given (x,y) coordinate position. The typical problem size, used specifically for this model, had 22 constraining equations and 369 variables.

The General Algebraic Modeling System (GAMS) MINOS5 linear program solver, version 2.25, was used to solve the linear program. Appendix B contains the model file for the solver. DOS batch files were used to integrate the Pascal code and the GAMS solver.

4. Counter-detection Determination

Three methods were considered for calculating the counter-detection threat. These methods correspond to three assumptions about what happens after a counter-detection:

1) nothing,
2) target evades, and
3) target attacks receiver.

The target's detection threshold is higher than the receiver's as the target does not have the same sonar system or the characteristics of the active pulse from the source platform as the receiver. For simplicity, the target's sonar system is assumed conservatively identical to the receiver's for counter-detection calculations. The following notation is used to help define the various counter-detection calculations:

$\boldsymbol{I}$ = ping on which receiver detects target for 1st time,

$\boldsymbol{J}$ = ping on which target detects receiver for 1st time, and

$\boldsymbol{I}$ or $\boldsymbol{J} = \infty$, if no detection during a given time period.

With this notation the $\boldsymbol{P_d}$ in equation 7 is the equivalent of

$$P(I \leq m), \tag{9}$$

where $\boldsymbol{m}$ is the time required for the target to transit through the barrier.

The first method determines counter-detection threat by calculating a counter-detection probability and comparing the result to the detection probability. Counter-detection probability is based on the optimal strategies determined by the linear program solver. The program, based on the optimal receiver location and target entry point, determines the counter-detection probability of the receiver. The counter-detection probability is multiplied by the probability the receiver will be located in that position and the probability the target will enter the barrier at that point. All possible combinations of optimal positions for both the target and receiver are summed up as shown by

$$P_{cd}=\sum\left[1-\prod\left(1-P_{cd}^{i}\right)\right]P_{xy}P_{z}, \tag{10}$$

where P_{cd} represents the overall counter-detection probability of the receiver for a given source location. P_{cd}^{i} is the counter-detection probability of the receiver on the i^{th} ping. P_{xy} is the mixed strategy probability for the receiver and P_{z} is the mixed strategy probability for the target. The mixed strategy probability for the receiver comes from the solution of the linear program in equation 8. The mixed strategy probability for the target is the dual solution of the linear program in equation 8.

The second method determines the counter-detection threat calculating the probability of the receiver detecting the target before the target detects the receiver and before time elapses, shown by

$$P(I<J \text{ and } I<\infty) . \qquad (11)$$

This takes place of the detection probability of the target,

$$P(I<\infty)=P(\text{receiver detects the target}), \qquad (12)$$

in the payoff matrix.

The third method finds the probability of the receiver detecting the target first and subtracts the probability of the target detecting the receiver first,

$$P(I<J)-P(J<I), \qquad (13)$$

and the result is stored in the payoff matrix. This method will show if the target will have a significant chance of detecting the receiver before the receiver detects the target and thus be able to attack first.

The payoff matrix for the last two methods, when evaluated using the linear program solver, yielded a strategy for the receiver very similar to the original method one payoff matrix. For model run-time considerations, only one method of counter-detection threat assessment is used. The first method is incorporated into the model as the strategy results are similar to the other methods and a specific value for counter-detection is found.

5. Output

The output from the GAMS linear program solver is written to an ASCII file and a Pascal program converts the information to the final output file. Appendix B contains the code for the conversion and output to the final output file. This allows the output file to be appended as multiple independent runs must be done to include data from all possible source locations. The final output file contains the optimal strategy for the receiver and the target, the overall detection probability (or value of the game), and a counter-detection probability of the receiver for each given source location.

III. RESULTS

A. INTERPRETATIONS OF RESULTS

The model uses the sonar equation assumptions discussed previously. The values for the barrier dimensions, receiver and source locations, and target entry points are also required for the model. The barrier dimensions are set at 200 kiloyards (kyds) in width and 40 kyds in depth. The target enters the top of the barrier at 10 kyds intervals and proceeds through. The possible receiver locations, within the barrier, are at intervals of 5 kyds in both the X and Y axis. Figure 4 graphically shows the barrier, target entry points and possible receiver locations.

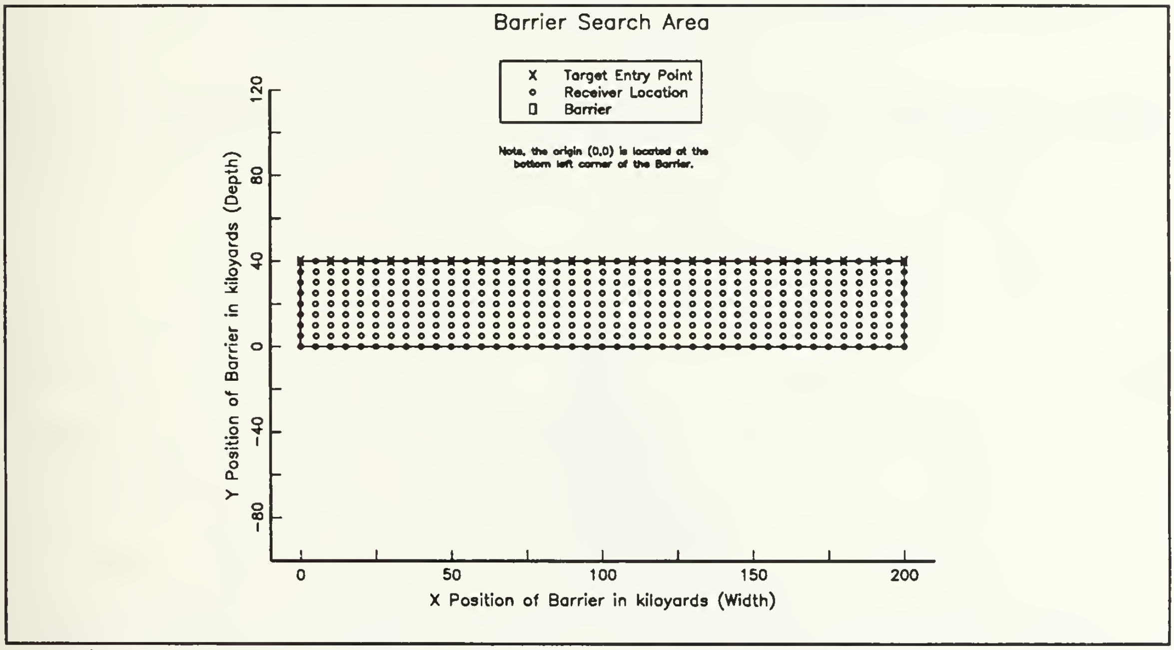

Figure 4. Target entry points and receiver locations.

Figure 5 shows the barrier imposed on all 4,469 possible source locations.

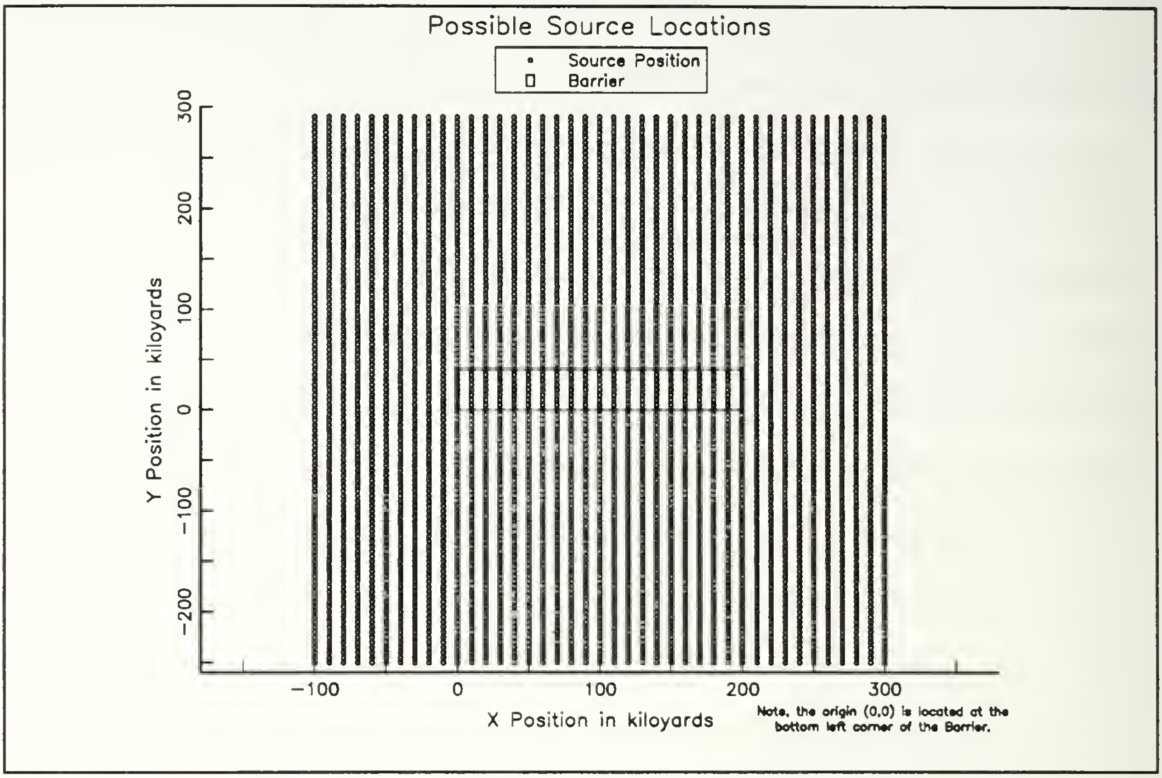

Figure 5. Possible source locations.

Note, in Figure 5 what appears to be straight vertical lines are actually many small circles, representing source locations, very close together.

At the completion of a model run for a given source location, an optimal strategy and the value of the game (overall detection probability) for the receiver is found. The dual solution of the results provides the optimal barrier entry point(s) for the target. In virtually every case the optimum strategy is mixed for both the receiver and the target platform. For example, with source located at the center of the barrier (100,20), the optimal pure strategies for the receiver were as follows:

(X,Y)	Probability
(45,10)	0.234
(55,20)	0.202
(85,20)	0.059
(115,20)	0.068
(145,20)	0.224
(155,10)	0.213,

and for the target the optimal pure strategies for entry into the barrier were:

(Z)	Probability
(0)	0.437
(90)	0.038
(100)	0.050
(110)	0.038
(200)	0.437.

In almost all other cases the optimum pure strategies were more numerous than for the example above.

Figure 6 shows the overall detection probability (value of the game) for various source positions.

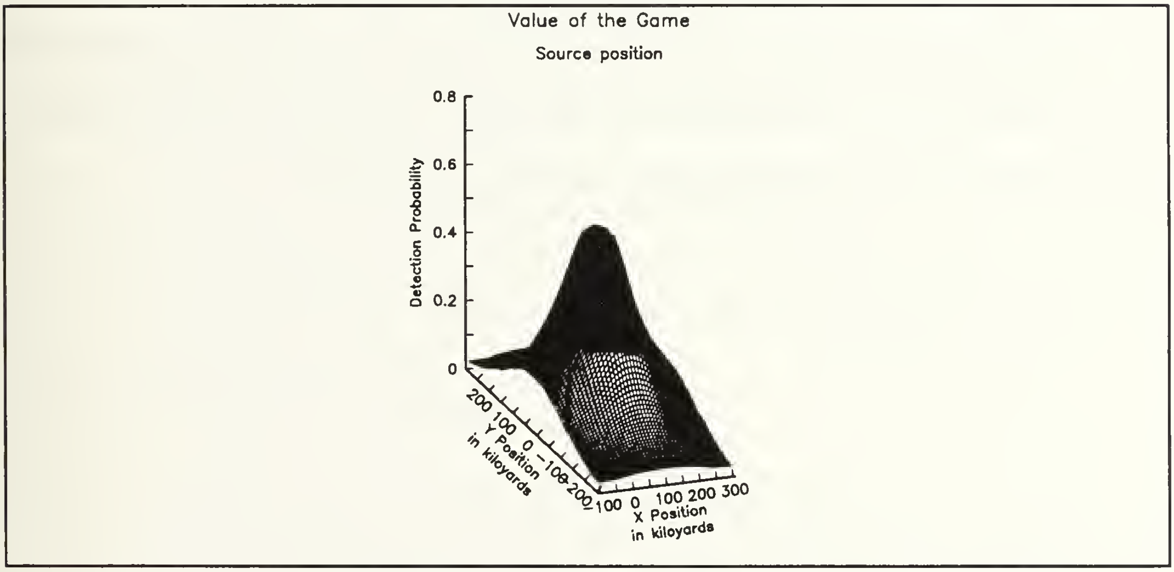

Figure 6. Value of the Game for various source positions.

Each of the 4,469 points in Figure 5 required separate solving on a computer linear program solver. The highest overall detection probability in Figure 6 occurs in two places, the source located just off the center of the barrier at X position 100 kyds and Y position 10 or 30 kyds. At this location the receiver has a overall detection probability of 0.57. This high value for detection demonstrates the usefulness of bistatic sonar. The detection probabilities are symmetric about the center of the barrier on both the X and Y axis. Values of detection probability around the center are fairly close to the highest value. For this reason the surface in Figure 6 has a well-rounded top. The receiver is located at least 20 kyds away from the source, based on the optimum mixed strategy. In most cases the receiver is located in excess of 60 kyds from the source.

The most significant problem of placing the source near the center of the barrier is the counter-detection threat to the receiver. Figure 7 shows the counter-detection probability for all possible source locations.

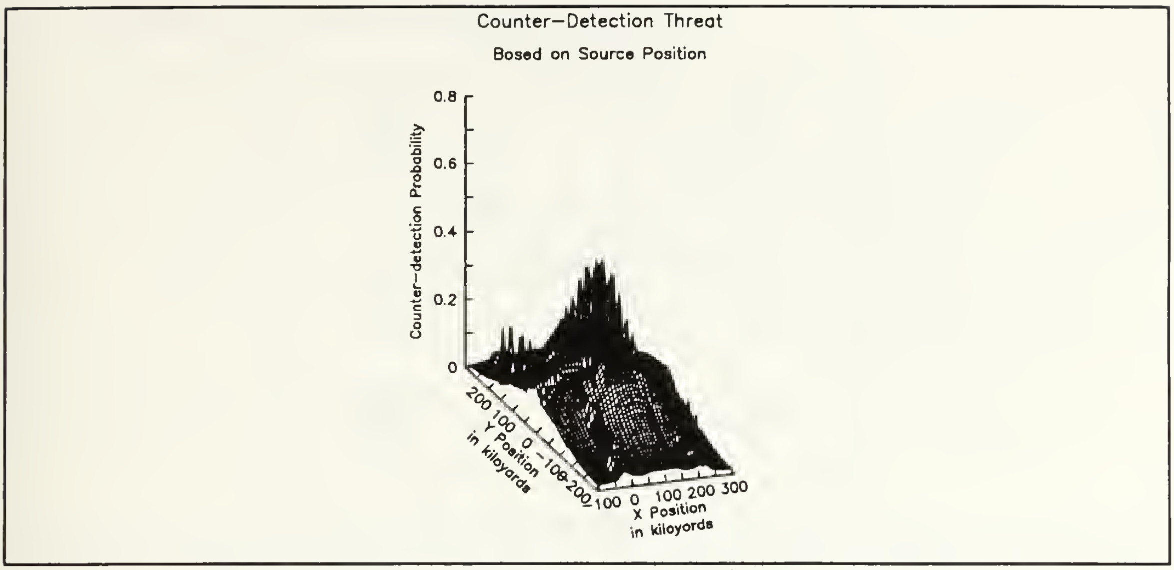

Figure 7. Counter-detection threat.

The apparent noise of the counter-detection threat plot results from the calculation method used. Counter-detection probability is based on the optimal solution for the receiver to detect the target. The receiver avoiding detection by the target is not optimized.

Counter-detection probability is reduced significantly by moving the source either up or down or left and right of the center of the barrier. With the source located off the center of the barrier, the detection probability is reduced only slightly. Figure 8 shows the contour graph of the counter-detection surface and better illustrates the sharp decline of counter-detection probability.

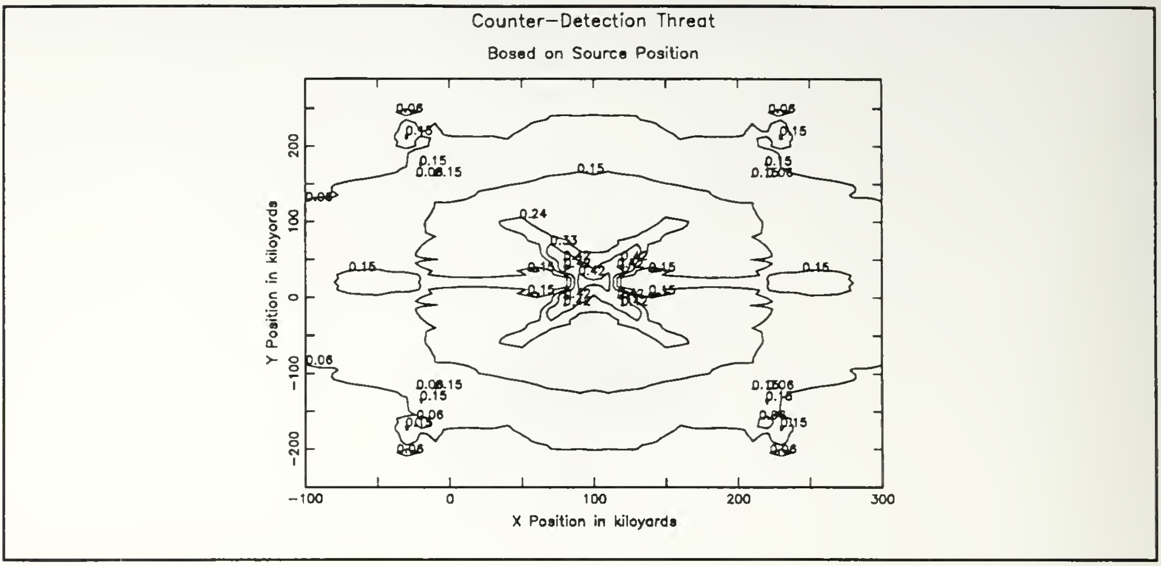

Figure 8. Contour graph for the counter-detection threat.

At the conclusion of a model run data provided includes overall detection probability, mixed strategy for the receiver, mixed strategy for the target and counter-detection threat to the receiver for a given source location. Multiple runs of the model with varied source position allows for the determination of the best strategy for both the receiver and the source. These results contribute a tool for decision making regarding the best receiver and source placement to detect an enemy submarine.

B. UTILITY OF APPLICATION

The model provides a decision making tool regarding submarine detection. Many different possible source and receiver position scenarios can be evaluated with the output data. Emphasis on detection of the target or counter-detection of the receiver can easily be incorporated into the decision making process.

The model provides a theoretical prediction of detection probability of an enemy submarine transiting through a barrier. The model results allow a user to evaluate the best strategy given any number of constraints. These constraints include source and receiver locations, barrier size, and target entry point. Limitations on these factors could be a result of geographical considerations or some other physical constraints.

Importance can be placed on either detection or the counter-detection probabilities by review of the output from the model. If the user is not concerned with being counter-detected more emphasis can be placed on detection probability. If however, the enemy submarine is very capable then the counter-detection threat can be considered with greater importance. These tools provided by the model allow the user to make logical decisions based on the current situation. Counter-detection cannot be minimized, however.

IV. PROGRAM

A. SYSTEM REQUIREMENTS TO RUN MODEL

The code portion of the model, written in Pascal computer language, can be used on any IBM compatible personal computer. The GAMS program requires at least a 386SX IBM compatible personal computer to execute the linear program solver for the model. A math coprocessor is not required, however it speeds up both the Pascal code execution and the GAMS linear program solver. Only 640 kilobytes of random access memory (RAM) is needed for execution of the model, but more RAM speeds up the GAMS linear program solver and allows for a virtual drive to be created. The virtual drive is used to speed up the storing and retrieving of temporary data during model runs.

B. LIMITATIONS OF THE CODE

The most significant problem with the model is execution time for the program. The model ran on an IBM compatible 486DX-25 MHz machine with 8 megabytes of RAM. Each run for a given source location took two hours to complete. When the model was run on a 386SX-20 MHz machine with math-coprocessor and 5 megabytes of RAM, execution time was six hours for each run. The model run time factor limits the number of source locations to be evaluated. The model runs takes advantage of symmetry about the center point of the barrier as discussed in the Results section. These long run-times are in spite of considerable effort spent in writing code efficiently. For example, the equation to determine target strength contains a sine squared ($\sin^2(\theta_B)$) term. With the use of some trigonometric functions, $\sin^2(\theta_B)$ could be represented by

$$\frac{1}{R_{ST}R_{TR}}\{h_{ST}h_{TR}+\frac{1}{4}\,[R_{SR}^2-(R_{ST}-R_{TR})^2]\}, \tag{14}$$

[Ref. 9], where $\boldsymbol{R_{ST}}$ is the distance between the source and target, $\boldsymbol{R_{TR}}$ is the distance between the target and the receiver, and $\boldsymbol{R_{ST}}$ is the distance between the source and the receiver. The other two terms ($\boldsymbol{h_{ST}}$ and $\boldsymbol{h_{TR}}$) are the horizontal distances between the source and the target, and the target and the receiver respectfully. The horizontal distances were the only new calculation required, as the distances between platforms are already calculated (each requires a square root) for the transmission loss terms.

V. CONCLUSIONS

A. SUMMARY

The use of active bistatic sonar provides a new method of detecting very quiet nuclear and diesel-electric submarines. Bistatic sonar takes advantage of the bearing and range information from active sonar and the stealth of passive sonar by using a separate source and receiver. The development of a model incorporating bistatic sonar allows for the determination of the optimal strategies for both the receiver and source platforms.

Multiple model runs are required to find the optimal strategy for the receiver and the source. In each case evaluated, for a given source location, the program yielded a mixed strategy for the receiver. Comparing the overall detection probability (value of the game) for each model run, for a given source location, provides a method for optimum source placement. There are multiple source locations yielding similar detection probabilities.

The mixed strategy for the receiver, with a given source location, places the receiver at least 20 kiloyards (kyds) from the source and in most cases outside 60 kyds. If the optimum receiver strategy placed it very close to the source, the use of bistatic sonar would be the same as a monostatic sonar.

The counter-detection threat to the receiver is calculated based on the optimal strategy of detection for the receiver. The highest threat occurs within the middle of the barrier. By placing the source away from the center of the barrier counter-detection threat drops significantly while the detection probability remains fairly constant.

All the previous results provide tools for the employment of a receiver and a source platform to best detect a transiting enemy submarine. With the counter-detection threat and mixed strategy results for the receiver, many different combinations of receiver and source locations can be considered.

B. POSSIBLE FOLLOW-ON RESEARCH

Two possible avenues for further research include: use of actual target strength, sonar system parameters and environmental data; and simulation of mixed strategies for the receiver, target and source. The use of real data for target strength, sonar system parameters and environmental data, would yield more accurate results. It also allows for evaluation of an actual search scenario with a known searcher and target platform. This approach would make the model more of a tactical decision aid.

Developing a simulation program would help confirm the results yielded from the model. It would allow for the comparison of the model to data generated by repeated simulations. AT&T Bell laboratories have developed a simulation program, Submarine Surveillance Simulator (S^3), that could be used in this capacity.

APPENDIX A

A. MAIN PROGRAM

```
Program Thesis;
{
LT Aasgeir Gangsaas USN
Naval Postgraduate School
Thesis work on Bistatic Sonar
Turbo Pascal 6.0

    The main program reads in the user inputs about the
receiver, target and the source.  Also the user inputs about
the environment.  With this information the program
calculates the signal excess as the target transits through
the barrier.  This signal excess is converted into a
probability.  The program contains three subroutines,
ProperSourceData, ReadIn and DetermineProb that are
explained below.}

Uses DOS, Normal, Signal4, Gamsform;

Const Sigma = 8;

Var Sx, Sy, WBarrier, DBarrier      : Real;
    Dz, Dx, Dy, Freq, PingIn, Num   : Integer;
    PulseL, TSpeed, Rx, Ry, Tx, Ty  : Real;
    TSmax, TSmin, NL, SL, AG, DT    : Real;
    Done                            : Boolean;

{***********************************************************}

Procedure ProperSourceData;

{   This procedure reads in the source location data for
multiple source locations.  It ensures the proper source
location sequence for the multiple model runs.}

Var I, J, N              : Integer;
    X, Y                 : Real;
    Temp, DataInSource : Text;

BEGIN
  Assign(Temp,'D:\TP\WORK\Num.txt');
  Assign(DataInSource,'D:\TP\WORK\THESSRCE.IN');
  Reset(DataInSource);
  Reset(Temp);
```

```
  Readln(Temp,J);
  Readln(DataInSource);
  FOR I := 1 to J DO
    Readln(DataInSource,N,X,Y);
  IF not eof (DataInSource) THEN
    Done := False
  ELSE
    Done := True;
  Sx := X;
  Sy := Y;
  Num := N;
  Rewrite(Temp);
  Writeln(Temp,(Num + 1));
  Close(Temp);
  Close(DataInSource);
END;

{***********************************************************}

Procedure ReadIn;

{   This procedure reads in from a file (Thes.in)
information about the receiver and the target needed for the
program.}

Var DataIn : Text;

BEGIN
  Assign(DataIn,'D:\TP\WORK\THES.IN');
  Reset(DataIn);
  ProperSourceData;
  Readln(DataIn);
  Readln(DataIn,PulseL);
  Readln(DataIn);
  Readln(DataIn, Freq);
  Readln(DataIn);
  Readln(DataIn,PingIn);
  Readln(DataIn);
  Readln(DataIn,WBarrier);
  Readln(DataIn);
  Readln(DataIn,DBarrier);
  Readln(DataIn);
  Readln(DataIn,Dz);
  Readln(DataIn);
  Readln(DataIn,TSpeed);
  Readln(DataIn);
  Readln(DataIn,Dx);
  Readln(DataIn);
  Readln(DataIn,Dy);
  Readln(DataIn);
  Readln(DataIn,TSmax);
```

```
  Readln(DataIn);
  Readln(DataIn,TSmin);
  Readln(DataIn);
  Readln(DataIn,NL);
  Readln(DataIn);
  Readln(DataIn,SL);
  Readln(DataIn);
  Readln(DataIn,AG);
  Readln(DataIn);
  Readln(DataIn,DT);
END; { Procedure ReadIn}

{************************************************************}

Procedure DetermineProb;

{   This procedure calculates the detection probability
matrix for the receiver for each receiver location and
target entry point.}

Var TotalDz, TotalDx, TotalDy    : Integer;
    C1, C2, C3, TotalTime,T1,T2 : Integer;
    Temp, SE, FreqSq, a, tq     : Real;
    DataOut                     : Text;

BEGIN
  ReadIn;
  Assign(DataOut,'F:\THES.OUT');
  Rewrite(DataOut);
  Tx := 0.0;
  Ty := DBarrier;
  Rx := 0.0;
  Ry := 0.0;
  TotalTime := Trunc((DBarrier/(TSpeed*2))*60);
  C1 := 0;
  C2 := 0;
  C3 := 0;
  FreqSq := SQR(Freq/1000);
  a := (0.1*FreqSq/(1+FreqSq))+(40*FreqSq/(4100+FreqSq))
                                    +((2.75E-04)*FreqSq);
  WHILE (C3 <= WBarrier) DO
    BEGIN
      WHILE (C2 <= DBarrier) DO
        BEGIN
          WHILE (C1 <= WBarrier) DO
            BEGIN
              T1 := 0;
              WHILE (T1 <= TotalTime) DO
                BEGIN
                  SE := SignalExcess(Sx, Sy, Tx, Ty, Rx, Ry,
                                     PulseL, a, TSmax, TSmin,
```

```
                                                  NL, SL, AG, DT);
                  IF (T1 = 0) THEN
                    Temp := 1 - NormalValue(SE/Sigma)
                  ELSE
                    Temp := Temp*(1-NormalValue(SE/Sigma));
                  Ty := Ty - (TSpeed/(3*10))*PingIn;
                  T1 := T1 + PingIn;
                END;
              Ty := DBarrier;
              T2 := 0;
              Write(Dataout,(1 - Temp):6:3);
              C1 := C1 + Dz;
              Tx := Tx + Dz;
            END;
          Writeln(DataOut);
          C2 := C2 + Dy;
          Tx := 0;
          Ry := C2;
          C1 := 0;
        END
      C3 := C3 + Dx;
      Rx := C3;
      C2 := 0;
      Ry := C2;
    END;
  Close(DataOut);
END;

{*********************  Main Program  *******************}

BEGIN
  DetermineProb;
  FormatG(Trunc(WBarrier),Trunc(DBarrier), Dx, Dy, Dz);
  IF (Done) THEN
     Exec('c:\command.com','/c copy done.bat answer.bat');
END.
```

B. SUBROUTINES

1. SignalExcess

```
Unit Signal4;

{       Determines signal excess for the receiver and the
target based on the bistatic active sonar equation.}

Interface

  Const  C = 8.685889638;

  Function SignalExcess(Sx, Sy, Tx, Ty, Rx, Ry, PL, a,
                 TSmax, TSmin, NL, SL, AG, DT : Real): Real;

Implementation

  Function SignalExcess(Sx, Sy, Tx, Ty, Rx, Ry, PL, a,
                 TSmax, TSmin, NL, SL, AG, DT : Real): Real;

  Var ThR, ThS, Rst, Rtr, Rsr, an, TS : Real;
      h1, h2, TLst, TLtr, Ftemp       : Real;

  BEGIN
    Rst := SQRT(SQR(Tx - Sx) + SQR(Ty - Sy));
    Rtr := SQRT(SQR(Tx - Rx) + SQR(Ty - Ry));
    Rsr := SQRT(SQR(Rx - Sx) + SQR(Ry - Sy));
    h1 := Tx - Sx;
    h2 := Tx - Rx;
    IF ((Rst + Rtr) < (Rsr + 4800*PL/3000)) THEN
      SignalExcess := -100.00
    ELSE
      BEGIN

      { Target Strength part, TS}
        an := (1/(Rst*Rtr))*(h1*h2 + 0.25*(SQR(Rsr) -
                                      SQR(Rst - Rtr)));
        TS := (TSmax - TSmin) + (TSmin*an);

      { Transmission Loss part for both paths, TLst, TLtr}
        TLst := (C*LN(Rst*1000)) + (a*Rst);
        TLtr := (C*LN(Rtr*1000)) + (a*Rtr);

        SignalExcess := SL-TLst-TLtr-NL-DT+TS+AG;
      END;
    END;
END.
```

2. NormalValue

```
Unit Normal;

{       This function determines the cumulative normal
distribution for a given value by using a polynomial
approximation.}

Interface

  Function NormalValue(x : Real): Real;

Implementation

  Const d1 = 4.9867347E-02;
        d2 = 2.11410061E-02;
        d3 = 3.2776263E-03;
        d4 = 3.80036E-05;
        d5 = 4.88906E-05;
        d6 = 5.383E-06;
        EPS = 0.00001;

  Function NormalValue(x : Real): Real;

  Var px, x1, x2, xt : Real;

  BEGIN
    xt := SQRT(SQR(x));
    IF (xt < EPS) THEN
      x2 := 0.5
    ELSE
      BEGIN
        x1 := 1 + d1*xt + d2*SQR(xt) + d3*EXP(3*ln(xt)) +
                    d4*SQR(SQR(xt)) + d5*EXP(5*ln(xt)) +
                                      d6*EXP(6*ln(xt));
        x2 := 0.5*EXP(-1*16*ln(x1));
      END;
    IF (x > 0.0) THEN
      px := 1 - x2
    ELSE
      px := x2;
    NormalValue := px;
  END;
END.
```

3. FormatG

```
Unit Gamsform;

{       This procedure converts the output from the main
program into a form which the GAMS linear program solver can
read in.}

Interface

  Procedure FormatG(WBarrier, DBarrier, Dx, Dy, Dz :
                                                    Integer);

Implementation

  Procedure AdvanceData(Var C1 : Integer; Dz, C : Integer;
                                        Var DataOutOLD : Text);

  Var I, J : Integer;
      T    : Real;

  BEGIN
    J := (C - 1)*7;
    FOR I := 1 to J DO
      Read(DataOutOLD,T);
    C1 := J*Dz;
  END;


  Procedure FormatG(WBarrier, DBarrier, Dx, Dy, Dz :
                                                    Integer);

  Var DataOutOLD, DataOutNEW     : Text;
      TC1, CT, LZ, C1, C2, C3    : Integer;
      TCA, TCB, C, Z             : Integer;
      Temp                       : Real;
      Txp, Typ                   : String[3];

  BEGIN
    Assign(DataOutOLD,'F:\THES.OUT');
    Assign(DataOutNEW,'F:\THES.DAT');
    Rewrite(DataOutNEW);
    Write(DataOUTNEW,'SET       X /');
    C1 := 0;
    C2 := 0;
    C3 := 0;
    Z := 1;
    WHILE (C3 <= WBarrier) DO
      BEGIN
```

```
    IF (C3 = 0) THEN
      Write(DataOutNEW,'00',C3)
    ELSE
      BEGIN
        IF (C3 < 10) THEN
          Txp := '00'
        ELSE
          BEGIN
            IF (C3 < 100) THEN
              Txp := '0'
            ELSE
              Txp := ''
          END;
        Write(DataOutNEW,',',Txp,C3);
        Z := Z + 1;
        IF (Z > 13) THEN
          BEGIN
            Writeln(DataOutNEW);
            Write(DataOUTNEW,'                    ');
            Z := 1;
          END;
      END;
    C3 := C3 + Dx;
  END;
Write(DataOutNEW,'/');
Writeln(DataOutNEW);
Z := 1;
Write(DataOutNEW,'          Y /');
WHILE (C2 <= DBarrier) DO
  BEGIN
    IF (C2 = 0) THEN
      Write(DataOutNEW,'00',C2)
    ELSE
      BEGIN
        IF (C2 < 10) THEN
          Typ := '00'
        ELSE
          BEGIN
            IF (C2 < 100) THEN
              Typ := '0'
            ELSE
              Typ := ''
          END;
        Write(DataOutNEW,',',Typ,C2);
        Z := Z + 1;
        IF (Z > 13) THEN
          BEGIN
            Writeln(DataOutNEW);
            Write(DataOUTNEW,'                    ');
            Z := 1;
          END;
```

```
      END;
    C2 := C2 + Dy;
  END;
Write(DataOutNEW,'/');
Writeln(DataOutNEW);
Z := 1;
Write(DataOutNEW,'            P /');
WHILE (C1 <= WBarrier) DO
  BEGIN
    IF (C1 = 0) THEN
      Write(DataOutNEW,C1)
    ELSE
      BEGIN
        Write(DataOutNEW,',',C1);
        Z := Z + 1;
      END;
    IF (Z > 13) THEN
       BEGIN
         Writeln(DataOutNEW);
         Write(DataOUTNEW,'                ');
         Z := 1;
       END;
    C1 := C1 + Dz;
  END;
Write(DataOutNEW,'/;');
Writeln(DataOutNEW);
Writeln(DataOutNEW);
Writeln(DataOutNEW,'TABLE     PAYOFF(X,Y,P)');
Writeln(DataOutNEW);
LZ := Trunc(WBarrier/Dz);
TC1 := 0;
CT := 1;
TCA := 1;
TCB := 1;
C := 0;
Write(DataOutNEW,'        ');
REPEAT
  C1 := 0;
  C2 := 0;
  C3 := 0;
  WHILE (((TCA MOD 8) <> 0) AND (C <= LZ)) DO
    BEGIN
      Write(DataOutNEW,TC1:7);
      TCA := TCA + 1;
      C := C + 1;
      TC1 := TC1 + Dz;
    END;
  Writeln(DataOutNEW);
  Reset(DataOutOLD);
  WHILE (C3 <= WBarrier) DO
    BEGIN
```

```
        WHILE (C2 <= DBarrier) DO
          BEGIN
            IF (C3 < 10) THEN
              Txp := '00'
            ELSE
              BEGIN
                IF (C3 < 100) THEN
                  Txp := '0'
                ELSE
                  Txp := ''
              END;
            IF (C2 < 10) THEN
              Typ := '00'
            ELSE
              BEGIN
                IF (C2 < 100) THEN
                  Typ := '0'
                ELSE
                  Typ := ''
              END;
            Write(DataOutNEW,Txp,C3,'.',Typ,C2,'  ');
            IF (CT > 1) THEN
              AdvanceData(C1, Dz, CT, DataOutOLD);
            TCB := 1;
            WHILE ((C1 <= WBarrier) AND ((TCB MOD 8) <>
                                                     0)) DO
              BEGIN
                Read(DataOutOLD,Temp);
                Write(DataOutNEW,Temp:7:3);
                C1 := C1 + Dz;
                TCB := TCB + 1;
              END;
            Readln(DataOutOLD);
            Writeln(DataOutNEW);
            C1 := 0;
            C2 := C2 + Dy;
          END;
        C2 := 0;
        C3 := C3 + Dx;
      END;
    Writeln(DataOutNEW);
    Writeln(DataOutNEW);
    TCA := 1;
    IF (C <= LZ) THEN
      Write(DataOutNEW,'    +   ');
    CT := CT + 1;
  UNTIL (C > LZ);
  Write(DataOutNew,';');
  Close(DataOutNEW);
END;
END.
```

APPENDIX B

A. GAMS FORMULATION PROGRAM

```
$TITLE          Bistatic Sonar Problem for Thesis
$STITLE         By LT Aasgeir Gangsaas USN

*--------------GAMS AND DOLLAR CONTROL OPTIONS--------------

$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS
  LIMCOL=0   , LIMROW =0     , SOLPRINT=OFF , DECIMALS=3
  RESLIM=100, ITERLIM=10000, OPTCR   =0.0 , SEED    =3141;

*---------------Definitions and Data------------------------

$INCLUDE F:\THES.DAT

*-------------------------Model-----------------------------

POSITIVE VARIABLE
  PR(X,Y)        distribution of effort;

VARIABLE
  V              value of the game;

EQUATIONS
  COL(P)
  PROB;

* maximize

* subject to
  COL(P)..       SUM((X,Y),PR(X,Y)*PAYOFF(X,Y,P)) =G= V;
  PROB..         SUM((X,Y),PR(X,Y)) =E= 1;

  MODEL ZEROSUM /ALL/;
  SOLVE ZEROSUM USING LP MAXIMIZING V;

*----------------------Reports-----------------------------

  COL.M(P)$(COL.M(P) EQ EPS) = 0.0000;
  DISPLAY V.L, PR.L, COL.M;
  file output1 /D:\TP\WORK\GAMOUT.DAT/;
  put output1;
  put V.L:6:3 /;
```

```
  LOOP((X,Y),
        put x.TL:7, y.TL:7, PR.L(X,Y):8:4 /;
      );
  file output2 /D:\TP\WORK\GAMDOUT.DAT/;
  put output2;
  LOOP(P,
       put p.TL:7, COL.M(P):8:4 /;
     );
```

B. FINAL OUTPUT CONVERSION

```
Program CumResults;

{   This procedure takes the output from the GAMS program
and outputs the results to files to cumulate results.}

Uses DOS, Normal, Signal4;

Const EPS = 0.0001;
      Sigma = 8;

Var INDATA1, INDATA2, INDATA3      : Text;
    INDATA4, OUTDATA, OUTDATATEMP : Text;
    OUTDATA1                      : Text;
    Sx, Sy, I, P1, P2, FreqSq     : Real;
    SumTemp, a, Tx, Ty, Rx, Ry    : Real;
    Temp, SE, TSmax, TSmin, NL    : Real;
    SL, AG, DT                    : Real;
    N, C, No, P, T1, T2           : Integer;
    PulseL, TSpeed,        Freq   : Integer;
    DBarrier, PingIn, TotalTime   : Integer;

{*****************************************************************}

Procedure ReadIn;

{   This procedure reads in from a file (Thes.in) which
contains information about the receiver, target and source
needed for the program.}

Var DataIn : Text;

BEGIN
  Assign(DataIn,'D:\TP\WORK\THES.IN');
  Reset(DataIn);
  Readln(DataIn);
  Readln(DataIn,PulseL);
  Readln(DataIn);
  Readln(DataIn, Freq);
  Readln(DataIn);
```

```
  Readln(DataIn, PingIn);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn, DBarrier);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn,TSpeed);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn);
  Readln(DataIn,TSmax);
  Readln(DataIn);
  Readln(DataIn,TSmin);
  Readln(DataIn);
  Readln(DataIn,NL);
  Readln(DataIn);
  Readln(DataIn,SL);
  Readln(DataIn);
  Readln(DataIn,AG);
  Readln(DataIn);
  Readln(DataIn,DT);
END; { Procedure ReadIn}


{******************** Main Program ************************}

BEGIN
  ReadIn;
  SumTemp := 0.0;
  Assign(INDATA1,'D:\TP\WORK\GAMOUT.DAT');
  Assign(INDATA2,'D:\TP\WORK\GAMDOUT.DAT');
  Assign(INDATA3,'D:\TP\WORK\THESSRCE.IN');
  Assign(INDATA4,'D:\TP\WORK\NUM.TXT');
  Assign(OUTDATA,'D:\TP\WORK\OUTPUT.DAT');
  Assign(OUTDATA1,'D:\TP\WORK\OUTPUT1.DAT');
  Assign(OUTDATATEMP,'D:\TP\WORK\TEMP.OUT');
  Reset(INDATA1);
  Reset(INDATA2);
  Reset(INDATA3);
  Reset(INDATA4);
  Rewrite(OUTDATATEMP);
  FreqSq := SQR(Freq/1000);
  a := (0.1*FreqSq/(1 + FreqSq)) + (40*FreqSq/(4100 +
                      FreqSq)) + ((2.75E-04)*FreqSq);
  TotalTime := Trunc((DBarrier/(TSpeed*2))*60);
  Readln(INDATA1,I);
```

```
Readln(INDATA4,N);
Readln(INDATA3);
FOR C := 1 to (N - 1) DO
  Readln(INDATA3,No,Sx,Sy);
IF (N = 2) THEN
  BEGIN
    Rewrite(OUTDATA);
    Rewrite(OUTDATA1)
  END
ELSE
  BEGIN
    Append(OUTDATA);
    Append(OUTDATA1);
  END;
Writeln(OUTDATA,' Source Position (in thousands of
                                         yards):');
Writeln(OUTDATA,'   X: ',Sx:5:1,' ','Y: ',Sy:5:1);
Writeln(OUTDATA);
Writeln(OUTDATA,' Value of the Game: ',I:5:3);
Writeln(OUTDATA);
Writeln(OUTDATA,' Optimal mixed strategy (for the
                                      Receiver)');
Writeln(OUTDATA,'   X     Y       probability');
WHILE not eof (INDATA1) DO
  BEGIN
    Readln(INDATA1,Rx,Ry,P1);
    IF (P1 > EPS) THEN
      BEGIN
        Writeln(OUTDATA,Rx:6:1,Ry:6:1,P1:13:3);
        Writeln(OUTDATATEMP,Rx,Ry,P1)
      END;
  END;
Close(OUTDATATEMP);
Writeln(OUTDATA);
Writeln(OUTDATA,' Dual Solution (Optimal Strategy for the
                                                   Target)');
Writeln(OUTDATA,'   X     probability');
WHILE not eof (INDATA2) DO
  BEGIN
    Readln(INDATA2,P,P2);
    Reset(OUTDATATEMP);
    P2 := -1 * P2;
    IF (P2 > EPS) THEN
      BEGIN
        Writeln(OUTDATA,P:5,P2:12:3);
        WHILE not eof (OUTDATATEMP) DO
          BEGIN
            Tx := P;
            Ty := DBarrier;
            Readln(OUTDATATEMP,Rx,Ry,P1);
            T1 := 0;
```

```
              WHILE (T1 <= TotalTime) DO
                BEGIN
                  SE := SignalExcess(Sx, Sy, Rx, Ry, Tx, Ty,
                                    PulseL, a, TSmax, TSmin,
                                            NL, SL, AG, DT);
                  IF (T1 = 0) THEN
                    Temp := 1 - NormalValue(SE/Sigma)
                  ELSE
                    Temp := Temp*(1-NormalValue(SE/Sigma));
                  Ty := Ty - (TSpeed/(3*10))*PingIn;
                  T1 := T1 + PingIn;
                END;
              SumTemp := SumTemp + (1 - Temp)*P1*P2;
            END;
        END;
    END;
  Writeln(OUTDATA);
  Writeln(OUTDATA,'Counterdetection probability:'
                                        ,SumTemp:5:3);
  Writeln(OUTDATA);
  Writeln(OUTDATA,'**************************************');
  Writeln(OUTDATA1,(N - 1):4,I:7:3,SumTemp:7:3);
  Writeln(N - 1);
  Close(OUTDATA1);
  Close(OUTDATA);
END.
```

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