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HELIUM RESEARCH CENTER

INTERNAL REPORT

CALIBRATION OF A PISTON GAGE BY MEANS OF A MERCURY COLUMN LESS THAN ONE METER HIGH. SIGNIFICANCE OF PISTON CONSTANTS AND THEIR APPLICATIONS

BY

Earle S. Burnett

Paul V. Mullins

BRANCH Fundamental Research

PROJECT NO. 4330

DATE June 1964

AMARILLO, TEXAS

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FORWORD

The authors of this report are commended for their ability to report in such detail, and with great clarity, the results of experimentation they conducted more than 30 years ago.

Mr. Burnett and Mr. Mullins reveal an intimate familiarity with the details and procedures required for the accurate calibration of a rotating piston gage and the determination of the piston constant for their gage. Their use of a short mercury column, less than one meter in height, is unusual. Nevertheless, the results of their measurements show that their decision was correct to perform the calibration by this simple procedure.

The authors present detailed corrections necessary for calibrating a piston gage, and the present report should serve as a stimulus to all who are seriously concerned with the most accurate calibration of a rotating piston gage.

L W Brandt

L. W. Brandt
Research Director
Helium Research Center

CONTENTS

Page

Foreword. 2

Abstract. 5

Introduction. 5

Preliminary considerations. 7

Experimental procedure. 10

The gage and testing arrangements. 10

Estimation of piston constant from experimental values 13

Piston constants at Amarillo 16

Comparison with 1924 calibration at MIT 19

Specific effect of part of piston in oil. 21

Graphical relations of piston constants 25

Composite buoyancy (deduction) estimation 28

Final remarks on piston constant. 31

Discussion of this and a later calibration 31

Appendix: 38

ILLUSTRATIONS

Fig.

1. Schematic of pressure measuring arrangement 11

2. Graphics of piston constants. 26

TABLES

Page

1. Sample data and calculations 17

2. Values calculated for graphical illustration (fig. 2; C_N = 0.814). 27

3. Composite buoyancy and deduction estimate. 29

4. Assembly of all experimental values and their indications. 35

CALIBRATION OF A PISTON GAGE BY MEANS OF A MERCURY COLUMN LESS THAN ONE METER HIGH. SIGNIFICANCE OF PISTON CONSTANTS AND THEIR APPLICATIONS

By

Earle S. Burnett[1/] and Paul V. Mullins[2/]

ABSTRACT

Rotating-piston gages have been used for many years for measurement of pressure to several hundred atmospheres. They have been calibrated by comparison of fluid pressures produced by various loads bearing on the piston bases, as measured by corresponding heights of balancing columns of mercury. When expressed in appropriate units, these ratios of column heights to piston loads are called piston constants. Experimental arrangements and procedures for their determination are presented in this paper, followed by a discussion of their significance and of their subsequent applications.

INTRODUCTION

In general, a piston gage is used as a secondary standard calibrated by comparing the pressure its loads produce with those

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produced by balancing columns of mercury measured by single column or multiple column manometers; or more recently (since 1927), by comparison with the vapor pressure of pure carbon dioxide vapor at the ice point. The mercury column is a highly satisfactory standard for pressure measurement but the high columns and multiple manometers that have generally been used for piston gage calibration are not convenient for most laboratories, and, furthermore, they require considerable experimental technique. This paper describes the calibration of a piston gage by comparison with a single short mercury column less than one meter in height. The simplicity of setting up and making measurements with this short column recommend it for use where conditions make it inconvenient to use one of the other methods.

Keyes and Dewey[3/] and Bridgeman[4/] have described the single

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3/ Keyes, F.G., and J. Dewey. An Experimental Study of the Piston Pressure Gage to Six Hundred Atmospheres. J. Opt. Soc. Am. and Rev. Sci. Instr., v. 14, No. 6, June 1927, pp. 491-504.

4/ Bridgeman, O.C. A Fixed Point for the Calibration of Pressure Gages. The Vapor Pressure of Liquid Carbon Dioxide at 0^{o} C. Jour. Am. Chem. Soc., v. 49, No. 5, May 1927, pp. 1174-83.

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column for 12 atmospheres pressure which was available for use at MIT for calibration of piston gages. Meyers and

Jessup[5] have described the multiple manometer system of the

5/ Meyers, C. H., and R. S. Jessup. A Multiple Manometer and Piston Gage for Precision Measurements. Bur. of Std. Jour. Res., v. 6, June 1931, pp. 1061-1102.

Bureau of Standards that was used for the same purpose. Others have used similar methods.[6]

6/ Roebuck, J. R., and H. W. Ibser. A Precision Multiple-Mercury-Column Manometer. Rev. Sci. Instr. v. 25, No. 1, 1954, pp. 46-51.

Calibration of a piston gage yields data which determine a factor, called the "piston constant", by which the net load in grams mass, including that of the piston, is multiplied to express the pressure thereby produced at the piston base. This pressure is expressed in standard units, usually in millimeters of mercury at 0° C and standard gravity, adjusted for air buoyancy.

PRELIMINARY CONSIDERATIONS

Before presenting the procedure account of this differential calibration, the following remarks are apropos.

In discussions of a piston constant, the loads on the piston, including the piston itself, are almost universally referred to as "weights", which is an ambiguous expression; it implies forces due to those loads which forces vary with location.

In this discussion it will be assumed that all such loads are known masses expressed in grams mass. Any such load of M grams mass is acted on by the pull of the local gravitation, g, resulting in a downward force Mg, which is by definition expressed

in dynes. Dividing these dynes by 980.665, which is the number of dynes per gram force, Mg/980.665 then is the number of grams force due to the gross load of M grams mass at the locality where gravity is "g".

Obviously this force is proportional to g so that when the above action occurs at a locality where $g = g_s$ the product $(Mg/980.665)(g_s/g) = M$, numerically, indicates that the number of grams mass expressing the loads also expresses the number of grams force which they produce at a locality where $g = g_s$.

It is to be noted here that in the equations developed in the following section (and in the appendix), each and every mass involved, and all balancing columns of air, oil, and mercury, each multiplied by its respective density and expressed as an equivalent mass of a column of mercury, plus the masses of the barometric columns of mercury, are multiplied by the prevailing factor g to express their effects as forces in dynes that measure the pressure on the piston base. Cancellation of that factor throughout permits the remaining expressions to represent masses in grams mass or forces in grams force that those masses would produce where standard gravity g_s prevails.

This means that for any piston load the corresponding balancing column values obtained at any and all localities are identical and that the forces expressed in grams, where $g = g_s$, are numerically the same as the gram masses involved; the only difference due to location is that the pressures produced by the

above masses are proportional to values of gravity obtaining. A further consequence of the above relations is that the ratio of H_{net} to M_{net}, i.e. the piston constant C_N, is independent of gravity, as appears also in the procedure account below.

The assumption that capillary effects on differential pressure measurements "balance out" is probably more nearly true than would be the results of any attempts to evaluate these effects for inclusion in the comparison. These effects include those of capillary attraction of the oil on the piston at its emergence into the atmosphere, which is a very slight depressing action. Countering this effect, more or less, is a very slight upward force on the piston due to leakage flow of oil past it, which must be a function of oil viscosity and of the difference between the oil pressure around the piston and that of the atmosphere above it. The piston itself is under compression due to its loads and to the pressure of the oil in which part of the piston is immersed, which immersion requires consideration, and to the pressure gradient accompanying the leakage flow. The containing cylinder is subject to expansion because of these pressures or to compression if it is under external pressure, in all of which action Poisson's ratio is a factor. These are minor matters as related to the differential low pressure calibration herein reported, and are of negligible importance.

Of like consideration are the very small differences of pressure due to variations of densities of air, oil, and mercury above masses are proportional to values of gravity obtaining. A further consequence of the above relations is that the ratio of R_{net} to M_{net}, i.e. the piston constant C_N, is independent of gravity, as appears also in the procedure account below.

The assumption that capillary effects on differential pressure measurements "balance out" is probably more nearly true than would be the results of any attempts to evaluate these effects for inclusion in the computation. These effects include those of capillary attraction of the oil on the piston at its emergence into the atmosphere, which is a very slight depressing action. Countering this effect, more or less, is a very slight upward force on the piston due to leakage flow of oil past it, which must be a function of oil viscosity and of the difference between the oil pressure around the piston and that of the atmosphere above it. The piston itself is under compression due to its loads and to the pressure of the oil in which part of the piston is immersed, which immersion requires consideration, and to the pressure gradient accompanying the leakage flow. The containing cylinder is subject to expansion because of these pressures or to compression if it is under external pressure, in all of which action Poisson's ratio is a factor. These are minor matters as related to the differential low pressure calibration herein reported, and are of negligible importance.

Of like consideration are the very small differences of pressure due to variations of densities of air, oil, and mercury that may exist due to differences of their temperatures and pressures at the several times when readings for the short and long columns were obtained. The assumption that the differences of pressure corresponding to these two mercury columns described below is equivalent to that produced by the change of load on the piston is justified. In any event, these very small differences tend to cancel, as is apparent from their comparisons in the appendix.

EXPERIMENTAL PROCEDURE

The Gage and Testing Arrangements

The gage calibrated is of the type described and illustrated by Keyes[7] except that the oscillating motion of the piston was

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7/ Keyes, Frederick G., High Pressure Technic. Ind. and Eng. Chem., v. 23, No. 12, December 1931, pp. 1375-1379.

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replaced by a continuous rotation and a small table was attached to the top of the piston to measure low pressures, thus eliminating the tare of the loading yoke and pan for heavy loads. The effective piston diameter at 20° C is 0.986164 cm, approximately 1 cm; its effective area A_s at 20° C is 0.736188 cm^2; both values are calculated from the piston constant $C_N = 0.999140 = 1/A_s\rho_{sm}$.[8]

The gage was connected to an open-end mercury manometer as shown schematically in figure 1. Air was used to transmit the pressure from the oil beneath the piston to the mercury in the manometer because direct contact of oil and mercury caused fouling of the

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8/ ρ_{sm} is standard density of mercury in (grams per cc)/10 = 1.35951 because H_{net} is expressed in mm of mercury column.

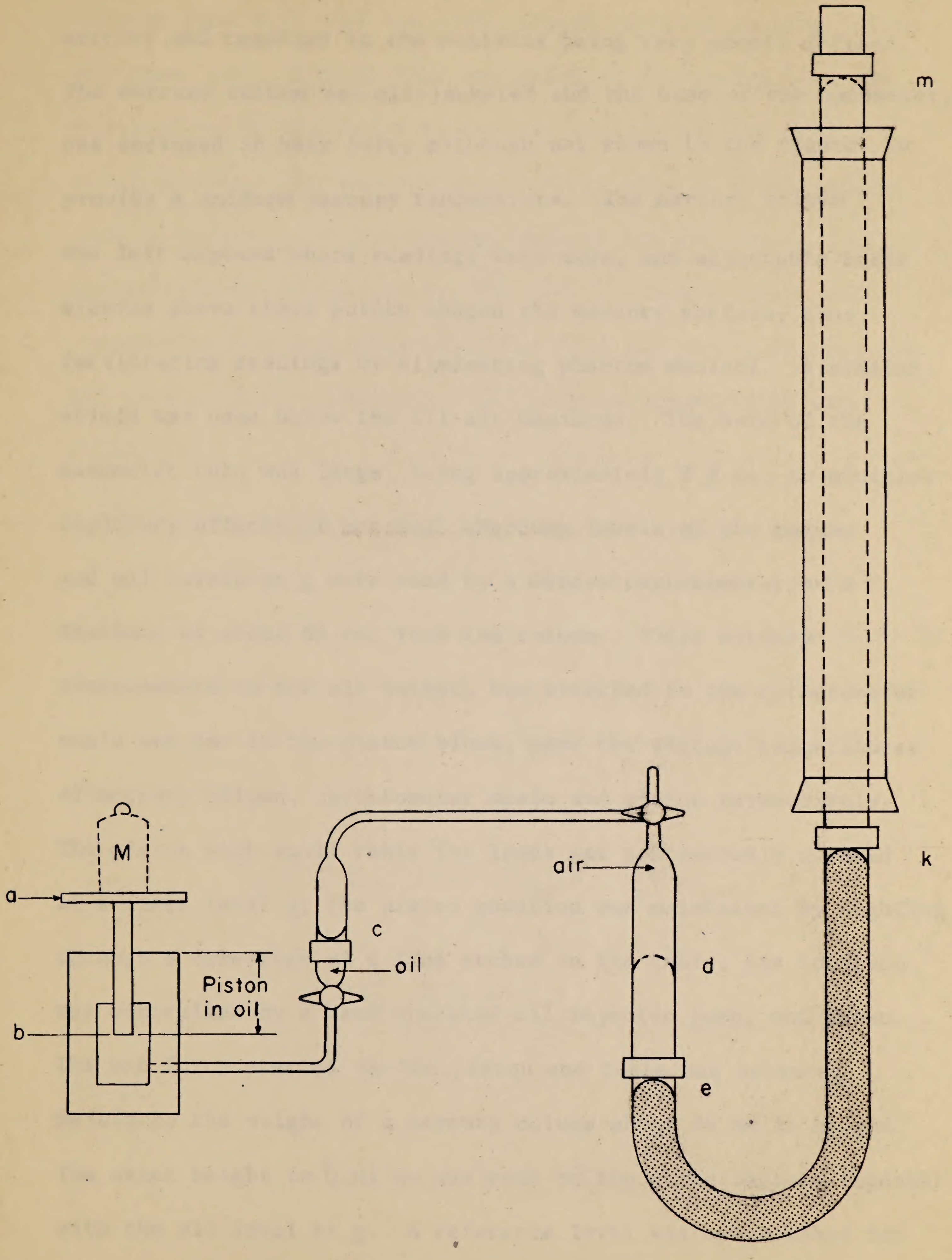

FIGURE 1. – Schematic of Pressure Measuring Arrangement

mercury and resulted in the meniscus being very poorly defined. The mercury column was air-jacketed and the base of the manometer, was enclosed in hair felt, although not shown in the figure, to provide a uniform mercury temperature. The mercury column was left exposed where readings were made, and adjustable brass sleeves above these points shaded the mercury surface, thus facilitating readings by eliminating phantom menisci. A similar shield was used below the oil-air meniscus. The bore of the manometer tube was large, being approximately 2.8 cm, to minimize capillary effects of menisci. Mercury levels of the column and oil levels at c were read by a Geneva cathetometer at a distance of about 65 cm. from the column. Three mercury thermometers in the air jacket, two attached to the cathetometer scale and one in the piston block, gave the average temperatures of mercury column, cathetometer scale and piston respectively. The piston with small table for loads was continuously rotated at a fixed level a; the piston position was maintained by sighting through a telescope at a line etched on the table; its location was controlled by a hand operated oil injector pump, not shown. The net force exerted by the piston and table was balanced mainly by the weight of a mercury column about 94 mm in height. The exact height to 0.01 mm was read on the cathetometer, together with the oil level at c. A reference level was established for the oil, and each mercury column reading was corrected for oil head caused by displacement from this reference level. All mercury and resulted in the meniscus being very poorly defined. The mercury column was air-jacketed and the base of the manometer, was enclosed in hair felt, although not shown in the figure, to provide a uniform mercury temperature. The mercury column was left exposed where readings were made, and adjustable brass sleeves above these points shaded the mercury surface, thus facilitating readings by eliminating phantom menisci. A similar shield was used below the oil-air meniscus. The bore of the manometer tube was large, being approximately 2.8 cm, to minimize capillary effects of menisci. Mercury levels of the column and oil levels at c were read by a Geneva cathetometer at a distance of about 95 cm. from the column. Three mercury thermometers in the air jacket, two attached to the cathetometer scale and one in the piston block, gave the average temperatures of mercury column, cathetometer scale and piston respectively. The piston with small table for loads was continuously rotated at a fixed level d; the piston position was maintained by sighting through a telescope at a line etched on the table; its location was controlled by a hand operated oil injector pump, not shown. The net force exerted by the piston and table was balanced mainly by the weight of a mercury column about 94 mm in height. The exact height to 0.01 mm was read on the cathetometer, together with the oil level at c. A reference level was established for the oil, and each mercury column reading was corrected for oil head caused by displacement from this reference level. All thermometers were read for each mercury meniscus reading. Then 850 grams mass of brass load was added to the piston load and balanced on the manometer by forcing mercury into it from a leveling bulb. The mercury in the short side of the manometer rose to compress the air between mercury and oil until equilibrium was established. Oil level, mercury levels and thermometer readings were taken for this new piston loading and the change in air density due to the above compression was considered.

Estimation of Piston Constant from Experimental Values

In accordance with the conclusion expressed in the paragraph preceding the above section, evidence for which is given in the appendix, we are justified in assuming that our determining equation is based only upon the change of gross piston load. That equation is

$$M_{gr}(1 - \rho_a/\rho_M)/A_s = H\rho_{sm}(1 - \rho_a/10\rho_{sm})$$

or

$$M_{net}/A_s = H_{net} \cdot \rho_{sm}$$

whence

$$H_{net}/M_{net} = C_N = 1/A_s\rho_{sm}$$

in which H_{net} is the net change in height in international mm of a column of mercury that exactly balances a change in net load,

M_{net} grams, on a piston base of effective area A_s sq cm at 20°C. Densities are represented by ρ with subscripts of obvious significance.

The piston constant C_N as above determined has dimensions of length per unit mass and obviously is inversely proportional to the product of the effective area of the piston and the standard density of mercury.

For convenience, a piston constant C_M may be defined as the ratio of the change of net mercury column, H_n, to change of total load, M_{gross}

$$H_{net}/M_{gross} = C_M = C_N \cdot C_B$$

in which $C_B = (1 - \rho_a/\rho_M)$ is a buoyancy factor that is ordinarily evaluated in terms of average densities of air and of loads at the locality of its application. (~~Usually~~ Usually and properly included with the loads in air is that part of the piston in oil.)[9/]

In either case the products

$$C_N \cdot M_{net} = H_{net} \text{ and } C_M \cdot M_{gross} = H_{net}$$

determine the heights of standardized mercury columns that define the changes of pressure at the base of a piston, due to those changes of loads, that obtain only at a location where gravity is g_s, standard, and when the deductions for buoyancy are always proportional to ΔM_{gross}.

9/ Helium Research Center Memorandum Report No. 53, "Buoyancy Effect of Air and Oil on Rotating Piston Gage Loads," by E. S. Burnett, in process.

In both cases the gravity factor, $C_g = g/g_s$, must be applied so that the resulting H, equal to $C_g \cdot H_{net}$ then is a measure of the actual change of pressure at the piston base at the locality of its use where the value of gravity is g. Therefore, we have, for use at that locality the piston constant

$$C_P = C_M \cdot C_g = C_N \cdot C_B \cdot C_g$$

The importance of accuracy in the evaluation of the buoyancy factor C_B is here specially emphasized. In the past, apparently, the total load, including the piston, has usually been assumed to be of the same material of density ρ_M and the buoyancy deduction has been based on the assumption that air of average density has been the buoyant medium.

Strictly the ratio ρ_a/ρ_M should be a composite ratio reflecting the buoyancy of surrounding air on each different material of which the total piston loads may consist, including that part of the piston which is in oil, which environment may require a further deduction.

Historically, this last mentioned probable deduction appears to have been ignored or completely overlooked, although it usually is of major relative importance as is obvious in the example given later. (But note also that this "deduction may be negative instead of positive and should be handled accordingly.)

Piston Constants at Amarillo

From the final values of the sample calculations in Table 1 for this differential Keyes gage calibration, we obtain

$$C_N = 849.166/849.897 = 0.9991399$$

$$C_M = 849.166/850.000 = 0.9990188$$

$$C_B = 1 - 0.001045/8.63 = 0.9998789 = \Delta M_n/\Delta M_{gr}$$
(assuming air only is the buoyant fluid)

$$C_g = 979.402/980.665 = 0.9987120$$

$$C_M = C_N \cdot C_B = (0.9991399)(0.9998789) = 0.9990188$$

$$C_P = C_M \cdot C_g = (0.9990188)(0.9987120) = 0.9977308$$
$$= C_N \cdot C_B \cdot C_g = (0.9991399)(0.9998789)(0.9987120)$$

Application: $C_P \cdot \Delta M_{gross} = 0.9977308 \times 850. = 848.071$ mm change of standardized mercury column that measures the change of pressure at the piston base, produced by a change in gross piston load of 850 grams of brass mass, at Amarillo.

The above value for the piston constant, C_P, applies only to the change in M, when of brass, at this particular locality where gravity has the value given as calculated for the latitude and elevation assumed for our laboratory from formula given in ICT, v. 1, pp. 401.

Latitude assumed: 35° 17' 30"

Elevation assumed: 3740 ft. = 1141.50 meters

TABLE 1. - Sample Data and Calculations

Mercury Column

Temperatures °C			
Cathetometer	25.7°, 25.7°	25.7°, 25.7°	Av. 25.7°
Mercury	24.5°, 25.1°, 24.8°	24.5°, 25.1°, 24.8°	Av. 24.8°
Piston block	31.3°	31.3°	Av. 31.3°
Cathetometer Reading		Upper Hg level	963.11 mm
Cathetometer Reading		Lower Hg level	15.90 mm
Difference			948.21 mm

Reference Oil Level 32.68 mm

Cathetometer reading of oil level 10.53 mm

Difference from reference -22.15 mm

Millimeters of mercury equivalent to -22.15 mm oil	-1.42 mm
Cathetometer corrected for oil level	946.79 mm

Cathetometer temperature correction		$25.7 \times 0.0_4 183$	= +0.000496
Mercury	" "	$24.8 \times 0.0_3 1818$	= -0.004509
Piston	" "	$(31.3-20) \times 0.0_4 23$	= +0.000260
			-0.003753

Cathetometer reading corrected for oil head	946.79 mm
Temperature correction -0.003753 x 946.79 mm	-3.55 mm
Cathetometer reading corrected for temperature	943.24 mm

TABLE 1. - Sample Data and Calculations (Con.)

Mercury Column

Average of 74 high columns	943.336 mm
Average of 74 short columns	94.104 mm
Difference	849.232 mm
Air buoyancy correction $\frac{H_a \rho_a}{10\rho_{sm}}$ with	
ρ_a = 0.00105 gm/cc at 25° C and 670 mm, $\frac{849.232 \times 0.00105}{13.5951}$	- 0.066 mm
Fully corrected mercury column difference, H_n	849.166 mm

Load on piston

Change of load, M gms, on piston	850.000 gm
Buoyancy correction, B, at 25° C; = volume of brass load x ρ_a	
$= \frac{M_{gr}}{\rho_b} \times \rho_a = \frac{850 \times 0.00105}{8.63}$	-0.103 gm
Differential piston load, M_n	849.897 gm

Piston constant, C_N, for mm of mercury at 0° C.

per gm net piston load, piston at 20° C, is 849.166/849.897 = 0.999140

Note: Density of oil at t°, $(\rho_o)_t$ = 0.887 (1 - 0.0007t) grams per cc.

Linear temperature coefficient of expansion of cathetometer scale, $+0.0_4183$.

α, areal temperature coefficient of expansion of steel piston, $+0.0_4230$.

β, density temperature coefficient of expansion of mercury -0.0_31818.

For any and all cases the height of the barometric pressure column of mercury, H_b, measured at the piston base level, must be added to the net mercury column, H_n, to estimate the total pressure at the piston base. Since H_b represents just another mass of mercury in a closed-end tube, it must therefore be multiplied by $C_g = g/g_s$ for the estimate indicated. It may be noted in passing that H_{net} plus H_b is exactly equivalent to adding H_T to H_{gross}, where H_T is the barometric column at the top level of H_{gross} which itself is the height of the balancing mercury column before its final adjustment for air buoyancy.

COMPARISON WITH 1924 CALIBRATION AT MIT

Piston constants have been variously defined and used. The original calibration of this gage was made at MIT in 1924 by comparison with a mercury column that was varied between 4 meters and 9 meters in height. The low column due to piston and tare appears from our report of the tests to have been maintained at a fixed position. The changes in heights of the columns due to changes in applied piston loads from approximately 4,000 grams to 9,000 grams were recorded for 11 such changes. These equivalent mercury columns after correction for oil levels, measuring tape inaccuracies, air buoyancy, and mercury density, all at 22° C. were standardized for density of mercury at 0° C; they were then multiplied by the ratio of g at MIT to g at 45° latitude and sea level, then taken as 980.370/980.616 = 0.999749 = C_g.

The changes of mercury columns as above adjusted in mm were then divided by the corresponding changes of gross piston loads in grams, to yield eleven values averaging 0.99880 ± 0.00010 as a piston constant, $C_g \cdot C_M$, which is $C_g \cdot (\Delta H_{net})/(\Delta M)_{gross}$. Later data indicate that C_g should be 980.398/980.665 = 0.999727 .. which, if applied, would reduce 0.998800 to 0.998778.

Assuming a Boston air density of 0.001200 gms/cc at the temperature of the tests (22^o C) and normal barometric pressure (760 mm Hg) and a steel load density of 7.84 gms/cc, there results an air buoyancy correction factor of 0.999847 which applies to the change of the steel loads on the piston.

Using total values of the changes in loads and in mercury columns indicated as proportional to their respective averages, the net standardized changes of Hg column totaled 82,235.9 mm. The total changes of applied steel loads was 82,315.0 gm which, corrected for air buoyancy by the factor 0.999847, becomes 82,302.4 gm. The ratio 82,235.9/82,302.4 = 0.999192 is C_{net}, or C_N a piston constant. The product C_P = 0.999192 x 0.999847 x 0.999727 = 0.998766 is for use at Boston. The gross change of steel loads in grams on the piston base multiplied by this constant yields the height of a standardized mercury column which determines the corresponding change in absolute pressure at the level of the piston base; effects of air buoyancy on the steel load change, and of gravity, are included in that constant.

The changes of mercury columns as above adjusted in mm were then divided by the corresponding changes of gross piston loads in grams, to yield eleven values averaging 0.99880 ± 0.00010 as a piston constant, $C_g \cdot C_M$, which is $C_g \cdot (\Delta H_{net})/(\Delta M)_{gross}$. Later data indicate that C_g should be 980.398/980.665 = 0.999727 which, if applied, would reduce 0.998800 to 0.998728.

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C_{net} determined at Amarillo in 1932, 0.999140, is to be compared with the above 1924 value, 0.999192. Each agrees with their average to ±1/38,500, which may indicate a slight enlargement of effective piston area over eight years' use.

Whatever the combination of ratios that produces a piston constant for use at a given location, it still has to be multiplied by the factor $1 - \alpha_p (t - t_s)$ as previously defined, table 1, to account for variation of the piston temperature. We believe as before stated that the fundamental ratio $H_n/M_n = C_N$ where properly determined should define the piston constant. Adjustments for buoyancy on the loads and for gravitation can be added for local use, but they vary with each location and with load material distribution; also the temperature of the piston varies during use, as does the density of the oil column contributing to H_n.

SPECIFIC EFFECT OF PART OF PISTON IN OIL

In the original 1932 internal report of this calibration mention was made of the necessity of including this effect when measuring absolute pressures with a piston gage. Consideration thereof has been shown not to be necessary in a differential calibration. The magnitude and significance of that effect can readily be demonstrated, however, by using values from table 1.

Two mercury columns are listed which, corrected for air buoyancy, are 943.263 and 94.097 mm. These differ by 849.166 mm which corresponds to a change of net piston load of 849.897 grams mass. How

much more change of net load, z grams, must be made to reduce the net mercury column, 94.097 mm to zero?

By simple proportion z = 94.097(849.897/849.166) = 94.178 net grams. Our records show that the piston and table plus rotating arm amounted to 99.60 gms gross. Hence the difference, M_d, 5.422 grams, is the net upward force due to upward flow of oil past the piston, downward capillary pull of oil at piston emergence into the air, and to possible errors of estimating the height, H_n, of the equivalent balancing column of mercury.

While 5.422 grams net uplift may seem excessive, it nevertheless appears to have been operative in this calibration. (Graphically it is the intercept, M_d, on the M_{gross} axis of a straight line through the coordinated points representing the long and short columns.) Hence the equation for excess pressure above atmosphere at the piston base level due to the total loads on the piston base is in this case:

$$P_B - P_b = \left[M_{gross} \left(1 - \rho_a/\rho_M\right) - M_d \right] /A_s = M_n/A_s = H_n\rho_{sm}$$

The total effective deduction is then $M_{gr}(\rho_a/\rho_M) + M_d$ which leads to the deduction factor $C_B = 1 - \frac{\text{total deduction}}{\text{gross load}} = M_n/M_{gr}$

If now we assume that $(C_M)_{apparent}$ was taken to be the ratio of $(H_n)_{true}$ to $(M_{gr})_{observed}$ at some random calibration value of $(M_{gr})_{obs}$ (without knowledge or consideration of M_d) and values of $(H_n)_{app}$ were computed therefrom, the question arises what is the true relation between $(\triangle P)_{true}$ and $(\triangle P)_{app}$ in which $\triangle P = P_B - P_b$?

That relation is:

$$(\Delta P)_{true}/(\Delta P)_{app} = (C_B)_{at\ (P_B)_{true}} \div (C_B)\ \text{at calibration point.}$$

For the same H_n as at the calibration point,

$$C_M = H_n/(M_{gross})_{observed}; \quad C_N = H_n/(M_n)_{correct}$$

whence

$$\frac{(\Delta P)_{true} \sim (H_n)_{true} = C_N M_n = M_n(H_n/M_n)_{at\ calib}}{(\Delta P)_{app} \sim (H_n)_{app} = C_M M_{gr} = M_{gr}(H_n/M_{gr})_{at\ calib}} =$$

$$\frac{M_n/M_{gr}}{(M_n/M_{gr})_{at\ calib}} = \frac{C_B}{(C_B)_{at\ calib}} = \frac{1 - \rho_a/\rho_M - M_d/M_{gr}}{(1 - \rho_a/\rho_M - M_d/Mgr)_{at\ calib}}$$

Obviously this ratio cannot be evaluated without knowledge of the value of M_d. However this value may be approximately estimated by assuming that it is equivalent to a pseudo buoyancy of the part M_o of the piston that is in oil. On that assumption, $M_d = M_o\rho_o/\rho_M$ and $M_o = M_d\rho_M/\rho_o$. In this calibration $M_o = 5.422(7.84/0.870) = 48.85$ gms.

The equivalent length in oil of this piston of area 0.736 cm^2 is 48.85/(7.84 x 0.736) = 8.5 cm = 3.34". This means that our steel piston if immersed in the cylinder oil to a depth of 8.5 cm below the oil-atmosphere surface would be buoyed upward by a force M_d of 5.422 gms. These values are mutually consistent with actual dimensions of the piston. Subtracting 48.85 from 99.6 leaves in air 50.75 gms of the short column load on which the air buoyancy is about 0.007 gms; this is about 1/800 of 5.422 gms which emphasizes the necessity of taking M_d into account.

The advantage of this assumption is that for some other piston gage calibration for which its M_d has not been reported, but for which its $(C_M)_{apparent}$ appears to have been based upon its $(M_{gr})_{observed}$ it may be possible to estimate a probable M_d for it based upon its cross sectional area and its depth of immersion in oil. Thus assume area $A = 1\ cm^2$, $L = 7$ cm, density of steel piston $\rho_M = 7.84$ gms per cm^3; density of piston oil at calibration pressure $\rho_o = 0.900$ gms per cm^3. Then

$$(M_d)_{estimated} = M_o\rho_o/\rho_M = A \cdot L \cdot \rho_M\rho_o/\rho_M = AL\rho_o = 1 \cdot 7 \cdot 0.9 = 6.3 \text{ grams}$$

This estimated value of M_d permits evaluation of the above ratio when the load at calibration is known; that load usually is given in the report of the piston calibration; M_d is seldom if ever mentioned.

When the piston constant appears to be based upon comparison of observed load for one value only of H_n (usually about 9 m of H_g), then M_d can be estimated approximately only by the above method.

Several relevant values applying to the two calibration points of table 1 are listed below.

	H_n mm	M_{gr} gms	Total Deduc. gms	M_n gms	$C_M = H_n/M_{gr}$	$C_N = H_n/M_n$	$C_B = M_n/M_{gr}$
Long Col.	943.263	949.6	5.525	944.075	0.993327	0.999140	0.994182
Short Col.	94.097	99.6	5.422	94.078	0.944749	0.999140	0.945562
			Net Deduc.		$\Delta H_n/M_{gr}$	$\Delta H_n/\Delta M_n$	$\Delta M_n/\Delta M_{gr}$
Differences	849.166	850.0	0.103	849.897	0.999019	0.999140	0.999878

This comparison again emphasizes the necessity of accurate estimate of the total deduction for each loading when total loads are involved to determine the net load M_n.

For the densities of this calibration and a calibration load of about 9000 gms of steel as at MIT with M_d taken to be 5.4 grams, the correction ratio $C_B/(C_B)_{at\ calib}$ evaluates to approximately $1.000600 - 5.40/M_{gr}$ for which the following values apply.

M_{gr}, gms:	100	200	600	1,000	2,000	6,000
Ratios:	0.9466	0.9736	0.9916	0.9942	0.9979	0.9997
M_{gr}, gms:	10,000	20,000	60,000	100,000	200,000	600,000
Ratios:	1.00006	1.00033	1.00051	1.000546	1.000573	1.000591

GRAPHICAL RELATIONS OF PISTON CONSTANTS

Figure 2 is drawn to an exaggerated scale with H_n = 70 mm at the upper right corner over M_{gross} equal to 100 gms so that the slope of the diagonal evaluates C_M as 0.700 for that calibration point only. The buoyancy deduction of M_{gr} is arbitrarily taken as $M_{gr}/10$; the constant deduction M_d is taken to be 4 gms.

Subject to these arbitrary assumptions, the total deduction prevailing at the upper right corner of fig. 2 is (100/10) + 4 = 14; M_n is then 100 - 14 = 86; C_N is 70/86 = 0.814; $C_B = C_M/C_N$ = 0.700/0.814 = 0.860 = 1 - (14/100) = M_n/M_{gr} = 86/100.

Now take M_{gross} = 80; the total deduction is (80/10) + 4 = 12; M_n = 80 - 12 = 68; a line starting from 12 on the H_n = 0 axis rising with a slope C_N = 0.814 over the distance M_n = 68 meets the vertical M_{gr} = 80 at H_n = 55.35. C_M for this H_n is the slope of a line from there to the origin or $(C_M)_{80}$ = 55.35/80 = 0.692. $C_B = C_M/C_N$ = 0.692/0.814 = 0.850 = 1 - (12/80) = 1 - 0.15 = M_n/M_{gr} = 68/80.

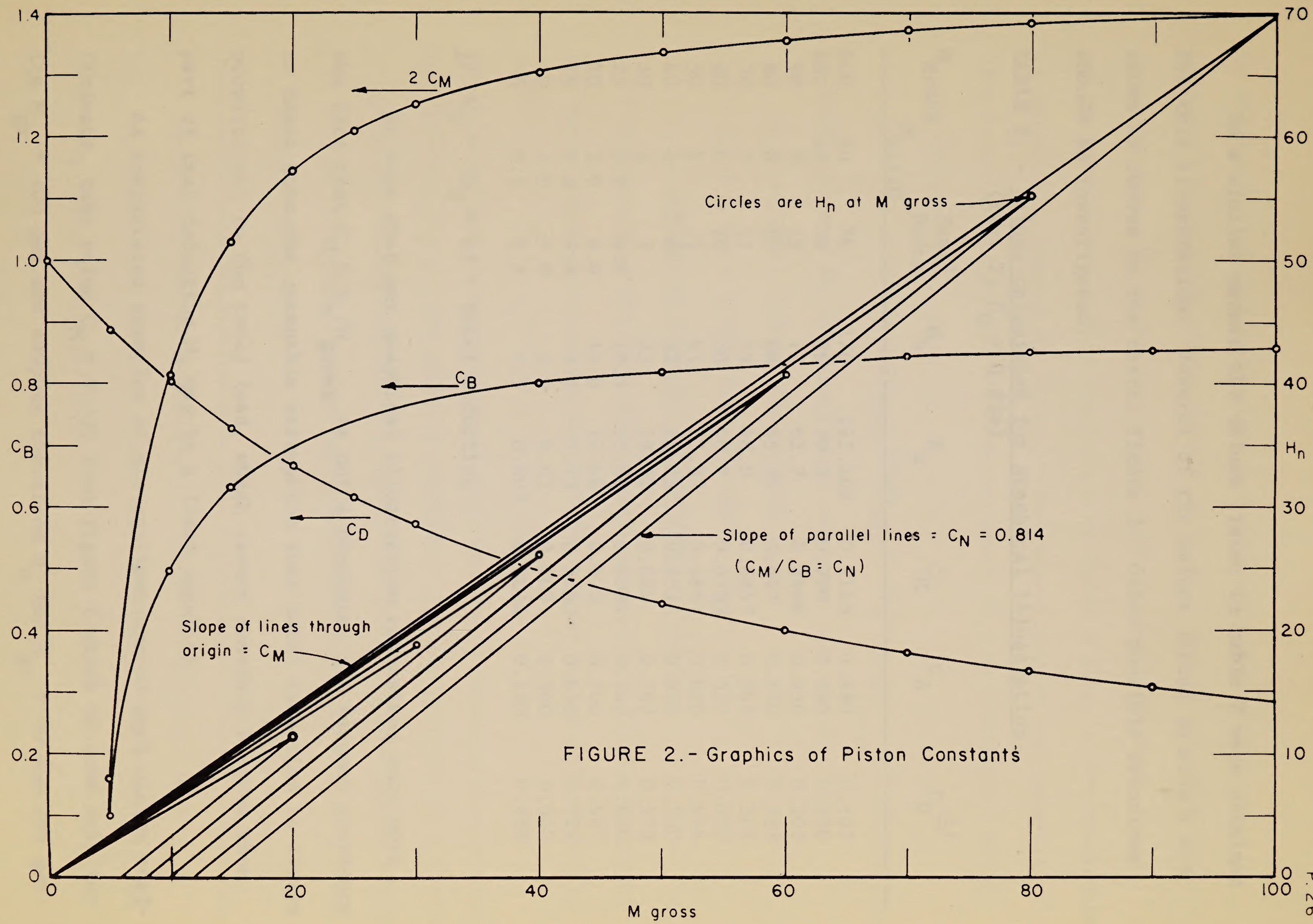

FIGURE 2.- Graphics of Piston Constants

In a similar manner the values listed in table 2 were obtained for this illustration. Several of the values listed in table 2 are shown as curves on the chart, figure 2. Other possible relations should be investigated.

TABLE 2. - Values calculated for graphical illustration (fig. 2; $C_N = 0.814$)

M_{gross}	$M_{gr/10}$	Total Deduc.	M_n	H_n	C_M	C_B	C_D 1/
200	20	24	176	143.264	0.7163	0.880	0.167
100	10	14	86	70.0	0.700	0.860	0.296
90	9	13	77	62.7	0.6966	0.856	0.308
80	8	12	68	55.35	0.692	0.850	0.333
70	7	11	59	48.0	0.6857	0.843	0.363
60	6	10	50	40.7	0.6783	0.837	0.400
50	5	9	41	33.37	0.6667	0.820	0.444
40	4	8	32	26.05	0.651	0.800	0.500
30	3	7	23	18.72	0.624	0.767	0.573
25	2.5	6.5	18.5	15.06	0.6024	0.740	0.616
20	2.0	6.0	14.0	11.40	0.570	0.700	0.667
15	1.5	5.5	9.5	7.73	0.5153	0.633	0.729
10	1.0	5.0	5.0	4.07	0.407	0.500	0.800
5	0.5	4.5	0.5	0.407	0.0814	0.100	0.889

1/ $C_D = (M_d = 4) \div$ Total deduction

We hope that our graphical illustration emphasizes once more the fact that $C_M = H_n/M_{gross}$ is not a constant; its assumed constancy is based upon the untenable assumption that total deduction is always proportional to the total load, which cannot possibly be true when part of that deduction M_d may be a fixed quantity.

As exaggerated examples of mis-applications of such piston coefficients, take value $C_M = 0.700$ from figure 2 which is good only for its $M_{gr} = 100$ gms and compute therefrom H_n' for $M_{gr} = 50$ and 200 gms.

We obtain H'_n = 35 and 140 mm respectively compared to the true values 33.37 and 143.264 mm from $H_n = C_N \cdot M_n$. The H'_n values are too large by about 5% in the first case, and too small by about 2 1/4% in the second case. This comparison provides a clue to the correction of values for changes of pressure thus miscalculated and suggests skepticism regarding the absolute accuracy of some pressures listed in the literature.

The true relation $(\Delta P)_{true}/(\Delta P)_{app} = (C_B)_{at (P)_{true}}/(C_B)_{at\ calibr}$ is substantiated by the exaggerated examples just given, for

143.264/140 = 1.0233 = 0.88/0.86 and

33.37/ 35 = 0.9534 = 0.82/0.86

COMPOSITE BUOYANCY (DEDUCTION) ESTIMATE

The data are from Mullins' observations on the determination of the vapor pressure of carbon dioxide at 0 C, as recently observed by us.[10/]

10/ Mullins, P. V., and Earle S. Burnett. The Calibration of a Piston Gage by Comparison with the Vapor Pressure of Liquid Carbon Dioxide at the Ice Point. Helium Research Center Internal Report No. 45, January 1964, 16 pp.

As Mullins' piston loads were of several materials, a part in oil, but all buoyed up by air, the several effects are listed in the following table 3 in grams mass.

TABLE 3. - Composite buoyancy and deduction estimate

Gross Mass Loads, grams		x Buoyancy Factors	=	Deductions	Net Loads, M_n
Cast iron	24,042.14	x 0.00105/7.08	=	3.57	24,038.57
Steel	1,109.85	x 0.00105/7.84	=	0.15	1,109.70
Brass	352.20	x 0.00105/8.63	=	0.04	352.16
Total load	25,504.19	x $(0.00105/7.48)_{av}$	=	3.76	25,500.43
Constant deduction due to piston in oil environment				5.42	
Total deduction				9.18	25,495.01

Composite total deduction factor = 9.18/25,504.2 = 0.000360

Hence M_n/M_{gr} = 0.999640 = C_B = 25,495.0/25,504.2 compared to 0.999852 that Mullins used.

Mullins' net load on the piston base for one determination was 25,495.0 grams mass. Multiplying this number by our piston constant $C_N \cdot C_g$ = 0.999140 x 0.998712 = 0.997853 yields 25,440.3; the temperature of the piston in this case was 7° above the 20° C normal, enlarging its effective area by 1/6200 which is equivalent to subtracting 4.2 mm, yielding 25,436.1. Adding further 1.20 mm for diaphragm correction and 1.67 mm for head of CO_2 vapor yields 25,439.0; and lastly adding the height of the barometric column of mercury, 689.64, corrected for air buoyancy, (by -0.05) and gravity, (by -0.89) to 688.7 mm yields a final figure of 26,127.7[11/] as the

11/ This equals 34.378 + atmospheres to compare with 34.379 given in ICT, v. VIII, p. 235, by an eight-constant equation.

height of the balancing column in international mm of mercury that defines the vapor pressure of carbon dioxide at 0 C.

The corrected value published by Roebuck & Cram[12/] for their

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12/ Roebuck, J. R., and W. Cram. Multiple Column Mercury Manometer for Presures to 200 Atmospheres. Rev. Sci. Inst. v. 8, No. 6, pp. 215-220 (1937)

---

average of twelve determinations is 26,129.8 mm which is 1/13,400 more than this single one of Mullins.

The deduction M_d = 5.42 gms due to the piston environment in oil was overlooked by Mullins, Burnett, and others in 1937; and the piston constant then used was

$$C'_P = C'_N \cdot C'_g \cdot C'_B = (0.999142)(0.998758)(0.999852)_{c.i.} = 0.997752$$

According to our analysis herein presented, it should have been

$$C_P = C_N \cdot C_g \cdot C_B = (0.999140)(0.9998712)(0.999640)_{comp} = 0.997492$$

The difference, $C_P - C'_P$ = -0.000260, which multiplied by our earlier (1937) average value, 26,136.4 mm yields a deduction of 6.8 mm reducing it to 26,129.6 mm. This value almost exactly agrees (to -1/130,000) with our revised value of 26,129.8 mm from the direct multiple mercury manometer determination announced[12/] by Roebuck and Cram in 1937. Their average of 12 determinations previous to gravity adjustment, 26,136.0 mm multiplied by 980.365/980.665 (ICT values for Madison, Wisconsin) yields 26,128.0 mm; compressibility of mercury adjustment adds 1.8 mm and brings it to 26,129.8 mm.

FINAL REMARKS ON PISTON CONSTANT

We say that if a "piston constant" is to be assigned to a given rotating piston assembly, it must have the value $C_N = H_{net}/M_{net}$ to be applicable to all conditions defined by that ratio and should not be otherwise used nor should any other ratio be called a "piston constant." This requirement involves the determination of $M_{net} = M_{gross} \times \frac{1 - \text{total deduction}}{\text{total load}}$ for each and every load, which is no more of a chore than are the determinations of H_{net}. Involved in the total deduction is that part of it, M_d, due to the piston in oil environment which must not be overlooked.

The piston constant C_N as above defined has been shown to be equivalent to $1/A_s\rho_{sm}$; A_s is the effective area of the piston at some standard temperature t_s, and ρ_{sm} is the standard density of mercury in grams mass per cu cm divided by 10 for H_n expressed in mm; therefore any method of calibration which determines the effective piston area at a standard reference temperature combined with standard mercury density serves to evaluate that piston constant.

<u>Discussion of this and a later calibration</u>

In all, 74 high columns and 74 low columns of mercury were measured; table 4 contains a non-chronological arrangement of the data.

The first 39 pairs of these observations were made in the laboratory building under somewhat adverse circumstances. That location was not far from a group of gas compressors, the ground vibration from which was quite disturbing, particularly when their speeds happened

nearly to coincide to produce a "beat" effect. We would get about set to read a mercury column level through the cathetometer telescope when the mercury meniscus would start quivering, thus delaying the observation and causing some uncertainty of its value.

The last 35 pairs of observations were made after removing our apparatus to a small building remoter from the powerhouse. The vibration interference was not eliminated but its disturbance was ameliorated. The observations of this group were more consistent than were those of the former group, but both indicated essentially the same average value, to about 1/100,000.

When the duly corrected values of the heights of the 74 short columns are placed opposite those of the 74 long columns as in table 4, both in descending order of magnitude, the opposing differences are also generally, but not everywhere consecutively, in descending order of magnitude. The extreme individual differences vary from the average value, 849.232 mm from +0.088 to -0.079, or from +1/9640 to -1/10750; that is roughly by $\pm$1/10,000. The arithmetic average of all differences from the mean value regardless of their signs, is ±0.035 or 1/24,300. By least square formula,

$$R = 2\sqrt{\frac{\Sigma(v^2)}{n(n-1)}} = 2\sqrt{\frac{0.142198}{74.73}} = \pm 0.010_{26}$$

in which each v is one of the 74 differences. This indicates that R, the probable uncertainty of the mean is not more than $\pm$0.0103 mm Hg on a twenty to one basis; that the most probable value of the mean difference is about 849.232 (1 $\pm$ 1/82,700); the equation says that

it is about 20 times as likely that the true value lies within this range as it does without it.

Because each individual determination of either the long or short column is independent of all others, it appears permissible to combine them in all possible ways and thus obtain $(74)^2 = 5476$ virtually independent differences. This was done and an extensive analysis was made of those results utilizing least squares methods applied to various groupings of the two sets of data. The results obtained were interesting and informative but hardly more satisfying than those presented above as used in this report.

During the summer of 1938 when the helium plant was shut down for a brief period and disturbing vibration was not harrassing us, another short mercury column calibration of the same Keyes gage was made. This time the mercury column arrangements were mounted directly on the Geneva cathetometer base. Its telescope was removed and replaced by a viewing tube provided with narrow parallel slits for proper sighting on mercury levels, which were read directly on the cathetometer scale. Without reporting details, let it be said that individual 80 cm column differences were so mutually agreeable that only 17 were determined.

In the meantime, necessary data had been obtained concerning the original calibration against a nine-meter mercury column at MIT in which steel "weights" had been used. Some of the data were obtained through personal correspondence with Dr. F. G. Keyes who with A. G. Loomis and C. W. Seibel of the Bureau, participated in that 1924 event.

Suffice it to say here that our 1932 calibration "piston constant" exceeded the 1924 value by about 1/20,000 and was less than our 1938 value by about the same amount, as estimated by the same methods.

From these comparisons at least two conclusions are obvious: a short column of mercury appreciably less than one (1) meter high can serve to calibrate a piston gage satisfactorily; and our Keyes gage did not appear to have changed in size significantly during fourteen years of use, although a possible very slight enlargement is suggested of the order of 1/10,000 of its effective area.

TABLE 4 - Assembly of all Experimental Values and Their Indications 1/

	Mercury Columns			Departures from Averages			$10^6 V_D^2$
	Long	Short	Diff.	Diff.	Long	Short	
No.	943.+	94.+	849.+	849.232	943.336	94.104	
				V_D	V_L	V_s	
1.	.474	.177	.297	+.065	+.138	+.077	4225
2.	.473	.158	.315	.083	.137	.054	6889
3.	.471	.158	.313	.081	.135	.054	6561
4.	.471	.155	.316	.084	.135	.051	7056
5.	.469	.149	.320	.088	.133	.045	7744
6.	.466	.149	.317	.085	.130	.045	7225
7.	.455	.146	.309	.076	.119	.042	5776
8.	.447	.144	.303	.071	.111	.040	5041
9.	.438	.144	.294	.062	.102	.040	3844
10.	.436	.139	.297	.065	.100	.035	4225
11.	.433	.139	.294	.062	.097	.035	3844
12.	.427	.138	.289	.057	.091	.034	3249
13.	.418	.138	.280	.048	.082	.034	2304
14.	.404	.135	.269	.035	.078	.031	1225
15.	.402	.132	.270	.038	.066	.028	1444
16.	.401	.130	.271	.039	.065	.026	1521
17.	.399	.127	.272	.040	.063	.023	1600
18.	.394	.126	.268	.036	.058	.022	1296
19.	.391	.126	.265	.033	.055	.022	1089
20.	.391	.126	.265	.033	.055	.022	1089
21.	.388	.125	.263	.031	.052	.021	961
22.	.377	.125	.252	.020	.041	.021	400
23.	.366	.125	.241	.009	.030	.021	81
24.	.365	.124	.241	.009	.029	.020	81
25.	.362	.124	.238	.006	.026	.020	36
26.	.359	.122	.237	.005	.023	.018	25
27.	.352	.119	.233	.001	.016	.015	1
28.	.352	.118	.234	.002	.016	.014	4
29.	.352	.118	.234	.002	.016	.014	4
30.	.349	.112	.237	.005	.013	.008	25
31.	.348	.111	.237	.005	.012	.007	25
32.	.348	.111	.237	.005	.012	.007	25
33.	.347	.111	.236	.004	.011	.007	16
34.	.345	.110	.235	.003	.009	.006	9
35.	.344	.109	.235	.003	.008	.005	9
36.	.344	.108	.236	+.004	.008	.004	16
37.	.339	.107	.232	-.010	+.003	.003	100
38.	.336	.106	.230	-.002	----	.002	4
39.	.328	.106	.222	-.010	-.008	.002	100
40.	.327	.105	.222	-.010	-.009	+.001	100
41.	.326	.104	.222	.010	.010	----	100

TABLE 4 - Assembly of all Experimental Values and Their Indications (Con.) [1/]

No.	Mercury Columns Long	Short	Diff.	Departures from Averages Diff.	Long	Short	$10^6 V_D^2$
No.	943.+	94.+	849.+	849.232	943.336	94.104	
42.	.322	.102	.220	-.012	-.014	-.002	144
43.	.317	.102	.215	.017	.019	.002	289
44.	.317	.097	.220	.012	.019	.007	144
45.	.315	.094	.221	.011	.021	.010	121
46.	.315	.090	.225	.012	.021	.014	144
47.	.313	.090	.223	.014	.023	.014	196
48.	.309	.088	.221	.011	.027	.016	121
49.	.308	.088	.220	.012	.028	.016	144
50.	.306	.088	.218	.014	.030	.016	196
51.	.298	.087	.211	.021	.038	.017	441
52.	.298	.086	.212	.020	.038	.018	400
53.	.292	.085	.207	.025	.044	.019	625
54.	.291	.084	.207	.025	.045	.020	625
55.	.280	.084	.196	.036	.056	.020	1296
56.	.279	.081	.198	.034	.057	.023	1156
57.	.274	.081	.193	.039	.062	.023	1521
58.	.274	.079	.195	.037	.062	.025	1369
59.	.274	.078	.196	.036	.062	.026	1296
60.	.273	.074	.199	.033	.063	.030	1089
61.	.269	.073	.196	.036	.067	.031	1296
62.	.268	.071	.197	.035	.068	.033	1225
63.	.265	.070	.195	.037	.071	.034	1369
64.	.264	.069	.195	.037	.072	.035	1369
65.	.250	.068	.182	.050	.086	.036	2500
66.	.238	.060	.178	.054	.098	.044	2916
67.	.222	.059	.163	.069	.114	.045	4761
68.	.217	.059	.158	.074	.119	.045	5476
69.	.211	.050	.161	.071	.125	.054	5041
70.	.210	.050	.160	.072	.126	.054	5184
71.	.201	.042	.159	.073	.135	.062	5329
72.	.195	.042	.153	.079	.141	.062	6249
73.	.192	.038	.154	.078	.144	.066	6084
74.	.186	.015	.171	.061	.150	.089	3721
Totals				+ 1.295	+ 2.275	+ .976	
				- 1.289	- 2.272	-1.006	
Totals	24.855	7.662	17.206	2.584	4.547	1.982	142198

TABLE 4 - Assembly of all Experimental Values and Their Indications (Con.) 1/

	Mercury Columns			Departures from Averages			$10^6 V_D^2$
	Long	Short	Diff.	Diff.	Long	Short	
No.	943.+	94.+	849.+	849.232	943.336	94.104	
Grand Ave.	943.336 -	94.104 =	849.232 849.232	Average Arithmetic Differences 0.0356	0.0614	0.0267	
Proportions of averages				1/23,600	1/15,350	1/3,525	

$$\frac{\text{Average column difference}}{\text{Average column - length}} = \frac{0.044}{518.220} = 0.000085 = 1/11,800$$

1/ Data of 1932, 85 cm mercury column calibration of a Keyes-type piston gage. "Corrected" long and short columns in mm. of Hg. as recorded for determination of their differences due to the addition to or removal from 850 grams mass load in brass on the piston tare mass, arranged in decreasing order of magnitude, and other relations. The numbers are mm of Hg.

By least square calculations the probability

$$R = 2\sqrt{\frac{\Sigma V_D^2}{n(n-1)}} = 2\sqrt{\frac{0.142198}{74 \times 73}} = \pm 0.010_{26}$$

indicates that the sought for magnitude is

$$849.232 \pm 0.010_{26} \quad \text{or} \quad 849.232 \ (1 \pm 1/82700)$$

that is the chances are about 20 to 1 that the true value lies between those limits as outside of them.

APPENDIX

The insignificance of residual differences of pressure, $P_2 - P_1$, when expressed as due to differences of piston loads, and as differences of equivalent balancing columns of mercury, air and oil (pp. 10-13 mms) is shown. [1/] (Refer to figure 1).

1/ For the short column of mercury the density of air confined between c and e of Figure 1 is its normal density ρ_a (at 670 mm H_g pressure and at room temperature at Amarillo) multiplied by the ratio (670 + 94.1)/670 = 1.14; for the long column ρ_a is multiplied by the ratio (670 + 943.3)/670 = 2.41; and 2.41/1.14 = 2.11.

In this demonstration M_T represents the tare load including the part of the piston that is in oil - about 100 grams, M_L the change (in brass) of the load on the piston - 850 grams.

For pressures expressed in equivalent columns of oil, air and mercury, the following relations obtain:

$$P_1 = \left[h_{bc}\rho_o - 1.14h_{ce}\rho_a + h_{ek}\rho_m + h_{bar}\rho_m - h_{bk}\rho_a\right]_1 g + \text{capillarity}_1$$

$$P_2 = \left[h_{bc}\rho_o - 2.41\,h_{cd}\rho_a + h_{dm}\rho_m + h_{bar}\rho_m - h_{bm}\rho_a\right]_2 g + \text{capillarity}_2$$

$$P_2 - P_1 = \Big[h_{bc}(\rho_{o_2} - \rho_{o_1}) - (2.41h_{cd} - 1.14h_{ce})\rho_a - (h_{bm}\rho_{a_2} - h_{bk}\rho_{a_1}) + (h_{dm}\rho_{m_2} - h_{ek}\rho_{m_1})$$
$$+ (h_{bar}\rho_{m_2} - h_{bar}\rho_{m_1})\Big]\, g + \left[(\text{cap}_2 - \text{cap}_1) = 0\right].$$

In the equation for $P_2 - P_1$, $(h_{ce} - h_{cd})\rho_a \equiv h_{de}\rho_a)$ is the air buoyancy change accompanying the change in total length of the shorter mercury column. The third parenthesis of this equation is the air buoyancy change accompanying the change in total length of the longer mercury column. The difference of these changes is the net change of air buoyancy $H\rho_a$ accompanying the net change of mercury column $H\rho_m$ indicated in the fourth ().

Hence, for the second and third () we may write

$$(0.14\, h_{ce} - 1.41\, h_{cd})\, \rho_a - (h_{bm}\rho_{a_2} - h_{bk}\rho_{a_1} - h_{de}\, \rho_a)$$

whence

$$P_2 - P_1 = \Big[h_{bc}(\rho_{o_2} - \rho_{o_1}) + (0.14\, h_{ce} - 1.41\, h_{cd})\rho_a + H(\rho_m - \rho_a) + (h_{bar}\,\rho_{m_2} - h_{bar}\,\rho_{m_1}) \Big]\, g$$

For pressures expressed in net loads on the piston base, the expressions below obtain:

$$P_1 = M_T\, \frac{g}{A_s} \left[\frac{1 - \rho_a/\rho_M}{1 + \alpha_p\, \Delta t} \right]_1 + (h_{bar}\rho_m g)_1 + (\text{capillarity, etc.})_1$$

$$P_2 = M_T\, \frac{g}{A_s} \left[\frac{1 - \rho_a/\rho_M}{1 + \alpha_p\, \Delta t} \right]_2 + (h_{bar}\rho_M g)_2 + (\text{capillarity, etc.})_2$$

$$+ M_L\, \frac{g}{A_s} \left[\frac{1 - \rho_a/\rho_M}{1 + \alpha_p\, \Delta t} \right]_2$$

$$P_2 - P_1 = M_T g/A_s \left[(1 - \rho_a/\rho_M - \alpha_p \Delta t)_2 - (1 - \rho_a/\rho_M - \alpha_p \Delta t)_1\right]$$

$$+ M_L \frac{g}{A_s} \left[\frac{1 - \rho_a/\rho_M}{1 + \alpha_p \Delta t}\right]_2 + \left[(h_{bar}\rho_m)_2 - (h_{bar}\rho_m)_1\right] g$$

$$+ \left[cap_2 - cap_1 = 0\right]$$

in which $\Delta t = t - 20°$; t_1 and t_2 refer both to piston and oil within the cylinder: α_p is the areal coefficient of expansion of steel, 0.000023. If these two final expressions for $P_2 - P_1$ are equated, the barometric terms and factor g cancel out exactly, leaving

$$h_{bc} (\rho_{o_2} - \rho_{o_1}) + (0.14 h_{ce} - 1.41\, h_{cd}) \rho_a + H\rho_m (1 - \rho_a/\rho_m)$$

$$= \frac{M_T}{A_s} \left[\frac{\rho_{a_1} - \rho_{a_2}}{\rho_M} - \alpha_p (t_2 - t_1)\right] + \frac{M_L}{A_s} \left[\frac{1 - \rho_a/\rho_M}{1 + \alpha_p \Delta t}\right]_2 .$$

It is to be noted here that h_{bc} and h_{cd}, both small in any case, can each be made equal to zero so that values in the involved products are negligible. Because of the inability to separate $(\rho_a, \rho_m)_2$ from $(\rho_a, \rho_m)_1$ in the H term, average values must be used; however the product $(H\rho_m)_{av}$ tends to stay constant. The unlikely maximum variation of the value of the M_T term on the right side of the equation is of the order of $M_L/40{,}000$. Hence our previous conclusion that ΔH_n is proportional to ΔM_n expressed on page 13 is justified.