

Report No. \_\_\_\_46

# UNITED STATES DEPARTMENT OF THE INTERIOR BUREAU OF MINES HELIUM ACTIVITY HELIUM RESEARCH CENTER

# INTERNAL REPORT

CALIBRATION OF A PISTON GAGE BY MEANS OF A MERCURY COLUMN LESS THAN ONE

METER HIGH. SIGNIFICANCE OF PISTON CONSTANTS AND THEIR APPLICATIONS

# BY

Earle S. Burnett

Paul V. Mullins

 BRANCH
 Fundamental Research

 PROJECT NO.
 4330

 DATE
 June 1964

HD 9660 .H43 M56 no.46

1

AMARILLO, TEXAS





#914877999

1288069869

HD 9660 . H43 M56 No.46

Report No. 46

JUREAU OF LAND MANAGEMENT LIBRARY BLDG: 50, DENVER FEDERAL CENTER DENVER, COLORADO 80225 DENVER, COLORADO 80225

HELIUM RESEARCH CENTER INTERNAL REPORT

CALIBRATION OF A PISTON GAGE BY MEANS OF A MERCURY COLUMN LESS THAN ONE METER HIGH. SIGNIFICANCE OF PISTON CONSTANTS AND THEIR APPLICATIONS

> Earle S. Burnett Paul V. Mullins

#### Fundamental Research Branch

Project 4330

June 1964



#### FORWORD

The authors of this report are commended for their ability to report in such detail, and with great clarity, the results of experimentation they conducted more than 30 years ago.

Mr. Burnett and Mr. Mullins reveal an intimate familiarity with the details and procedures required for the accurate calibration of a rotating piston gage and the determination of the piston constant for their gage. Their use of a short mercury column, less than one meter in height, is unusual. Nevertheless, the results of their measurements show that their decision was correct to perform the calibration by this simple procedure.

The authors present detailed corrections necessary for calibrating a piston gage, and the present report should serve as a stimulus to all who are seriously concerned with the most accurate calibration of a rotating piston gage.

Brandt

L. W. Brandt Research Director Helium Research Center

The anthors of this capact are commanded for their sollies to report in and detail, and with creat clarity, the reality of errecteducation they conducted note than he years are

in Airpait and Mr. Multim roomal ar inclasse familiants, with the details and procedures required for the address calibracion of a receing ristom ange and the determination of the phatom constant for their segn. Their use of a short mercan roturn, have then are meter in height, is unusual herertheight. the regules of their resourcemute even that the fest detision was correct to useform the calibration by this state of a state of the

The Authors present detailed corrections cacessary for callbratics a plater rege, and the present repart should serve as a stimulus to all won are sectorally converged with the must securate calification of a recating platon gage.

L. N. STANGE Recentric Ultractor Balting Research Division

#### 123031299

### CONTENTS

Foreword	2
Abstract	5
Introduction	5
Preliminary considerations	7
Experimental procedure	10
The gage and testing arrangements	10
Estimation of piston constant from experimental values	13
Piston constants at Amarillo	16
Comparison with 1924 calibration at MIT	19
Specific effect of part of piston in oil	21
Graphical relations of piston constants	<b>2</b> 5
Composite buoyancy (deduction) estimation	28
Final remarks on piston constant	31
Discussion of this and a later calibration	31
Appendix:	38

## ILLUSTRATIONS

# Fig.

1.	Schematic of pressure measuring arrangement	•	• '	•	• •	11
2.	Graphics of piston constants	•			•, •	26

Page

....

3														, _		
												/				
E1																

#### ILLUSTWATIONS

#### 112.

#### TABLES

1.	Sample data and calculations	17
2.	Values calculated for graphical illustration (fig. 2; C <sub>N</sub> = 0.814)	27
3.	Composite buoyancy and deduction estimate	29
4.	Assembly of all experimental values and their indications	35

measurement of pressure to several hundred streepheres. They have been calibrated by comparison of fluid pressures produced by wartons loady bearing on the platon bases, as measured by correaponding heights al, balancing columns of mercury. When represent in appropriate suits, these ratios of column heights to platon loads are called plator constants. Experimental arrangements and procedures for their decormination are presented in this paper, collowed by a discussion of their significance and of their sub-

#### SVERGIDITCT LOR

In general, a platon gage is used as a secondary standard malibrated by comparing the pressure its loads broduce with those

١

Page

•

<sup>1/</sup> Machanical Research Hogineer (General), Hellum Research Center, Bureau of Minney, Amerillo, Texas

<sup>2/</sup> Densial Manager, Hellus Operations, Bursen of Minse, Amerilio, Texas

#### TABLES.

															ί, αι		

#### CALIBRATION OF A PISTON GAGE BY MEANS OF A MERCURY COLUMN LESS THAN ONE METER HIGH. SIGNIFICANCE OF PISTON CONSTANTS AND THEIR APPLICATIONS

By

Earle S. Burnett<sup>1/</sup> and Paul V. Mullins<sup>2/</sup>

#### ABSTRACT

Rotating-piston gages have been used for many years for measurement of pressure to several hundred atmospheres. They have been calibrated by comparison of fluid pressures produced by various loads bearing on the piston bases, as measured by corresponding heights of balancing columns of mercury. When expressed in appropriate units, these ratios of column heights to piston loads are called piston constants. Experimental arrangements and procedures for their determination are presented in this paper, followed by a discussion of their significance and of their subsequent applications.

#### INTRODUCTION

In general, a piston gage is used as a secondary standard calibrated by comparing the pressure its loads produce with those

- <u>1</u>/ Mechanical Research Engineer (General), Helium Research Center, Bureau of Mines, Amarillo, Texas
- <u>2</u>/ General Manager, Helium Operations, Bureau of Mines, Amarillo, Texas

Work on manuscript completed May 1964

CALIBRATION OF A PISTON CAGE BY MEANS OF A MERCURY COLUMN LESS MALL ONE METER HIGH. SIGNIFICANCE OF PISTON CONSTANTS AND THEIR APPLICATIONS

By.

Warle S. Burnett and Paul V. Mulilne-

#### ABSTRACT

Rotating-pieton gages have been used for many years for measurement of pressure to several hundred atmospheres. They have been calibrated by comparison of fluid pressures produced by various loads bearing on the piston bases, as measured by corresponding heights of balancing columns of mercury. When expressed in appropriate units, these ratios of column heights to piscon loads are called piston constants. Experimental arrangements and procedures for their determination are presented in this paper, followed by a discussion of their significance and of their sub-

#### MOLLOUDOLLON

lo general, a piston gage is used as a secondary standard calibrated by comparing the pressure its toads produce with those

- Mechanical Research Engineer (Ceneral), Malium Research Center, Burcau of Mines, Amarillo, Texas
- 2/ General Manager, Helium Operations, Mureau of Mines, Amerilio, Texes

Work on manuscript completed May 1964

produced by balancing columns of mercury measured by single column or multiple column manometers; or more recently (since 1927), by comparison with the vapor pressure of pure carbon dioxide vapor at the ice point. The mercury column is a highly satisfactory standard for pressure measurement but the high columns and multiple manometers that have generally been used for piston gage calibration are not convenient for most laboratories, and, furthermore, they require considerable experimental technique. This paper describes the calibration of a piston gage by comparison with a single short mercury column less than one meter in height. The simplicity of setting up and making measurements with this short column recommend it for use where conditions make it inconvenient to use one of the other methods.

Keyes and Dewey<sup> $\frac{3}{}$ </sup> and Bridgeman<sup> $\frac{4}{}$ </sup> have described the single

- 3/ Keyes, F.G., and J. Dewey. An Experimental Study of the Piston Pressure Gage to Six Hundred Atmospheres. J. Opt. Soc. Am. and Rev. Sci. Instr., v. 14, No. 6, June 1927, pp. 491-504.
- <u>4</u>/ Bridgeman, O.C. A Fixed Point for the Calibration of Pressure Gages. The Vapor Pressure of Liquid Carbon Dioxide at 0<sup>°</sup> C. Jour. Am. Chem. Soc., v. 49, No. 5, May 1927, pp. 1174-83.

column for 12 atmospheres pressure which was available for use at MIT for calibration of piston gages. Meyers and

6

produced by balancing columns of metcury measures by single column of multiple column manameters; or wore recently tained 1927), by semparison with the vapor pressure of pure carbon dioxide vapor at the ice bolds. The mercury column is a highly satisfactory standard for pressure measurement but the high columns and multiple manometers that have generally been used for platan gage calibration are and convenience for event laboraturies, and, furthermore, they require sumsiderable experimental technique. This paper describes the calibration of a staten gage by comparison with a singlicity of setting up and making measurements with this abort column recommend it for use where conditions make 16

· Kayes and Devey? and Bridgeman? have described the bingle

- 3/ Keyes, F.G. and J. Beasy. An Experimental Study of the Flaton Freesure Gage to Six Hundred Armospheres. J. Opt. Soc. An and Rev. Sci. Instr., v. 14, No. 6, Nume 1917, no. 191-504.
- Stillgeman, D.C. A Fixed Point for the Calibration of Pressure Gages. The Vapor Pressure of Light Garbon Dioxide at 0<sup>0</sup> G. Jour. Am. Chem. Soc., v. 49, No. 5, Kay 1927, pp. 1174-83.

column for 12 atmospheres pressure which was available for use at MIT for calibration of piston pages. Mevers and Jessup<sup>5/</sup> have described the multiple manometer system of the

5/ Meyers, C. H., and R. S. Jessup. A Multiple Manometer and Piston Gage for Precision Measurements. Bur. of Std. Jour. Res., v. 6, June 1931, pp. 1061-1102.

Bureau of Standards that was used for the same purpose. Others have used similar methods. $\frac{6}{}$ 

<u>6</u>/ Roebuck, J. R., and H. W. Ibser. A Precision Multiple-Mercury-Column Manometer. Rev. Sci. Instr. v. 25, No. 1, 1954, pp. 46-51.

Calibration of a piston gage yields data which determine a factor, called the "piston constant", by which the net load in grams mass, including that of the piston, is multiplied to express the pressure thereby produced at the piston base. This pressure is expressed in standard units, usually in millimeters of mercury at 0° C and standard gravity, adjusted for air buoyancy.

#### PRELIMINARY CONSIDERATIONS

Before presenting the procedure account of this differential calibration, the following remarks are apropos.

In discussions of a piston constant, the loads on the piston, including the piston itself, are almost universally referred to as "weights", which is an ambiguous expression; it implies forces due to those loads which forces vary with location.

In this discussion it will be assumed that all such loads are known masses expressed in grams mass. Any such load of M grams mass is acted on by the pull of the local gravitation, g, resulting in a downward force Mg, which is by definition expressed

7

foraup" have described the multiple menometer evetem of the

5/ Mayers, G. H., and R. S. Jessuy, A Multiple Manometer and Piston Gage for Precision Measurements, Bur. of Std. Jaur. Nes., v. 6, June 1931, pp. 1061-1102.

Bureau of Scandards that was used for the same purpose. Others have

Manometer, Nev. Sci. Instr. V. 25, No. 1. 1954, pp. 45-51.

Calibration of a pieron gaps yields data which detaining a factor, called the "pieron constant", by which the net load in grant mass, including that of the pieron, is multiplied to express the pressure thereby produced at the pieron base. This pressure is expressed in standard units, usually in a limeters of persons at 0.00 and standard gravity, adjusted for sit buoyancy.

#### PREDIMINARY CONSIDERATIONS

Before presenting the procedure account of this attratential calibration, the following remarks are appopts in discussions of a piston constant, the lusds on the pieros, including the piston tracht, are almost universally referred to as "weights", which is an ambiguous expression; if implies forces due to those lusds which forces viry with location.

In this distingtion in will be assumed that all such loads are known masses expressed in grams mass. Any such load of M grams mass is acted on by the pull of the local gravitation, a. resulting to a downward force MG, which is by definition expressed in dynes. Dividing these dynes by 980.665, which is the number of dynes per gram force, Mg/980.665 then is the number of grams force due to the gross load of M grams mass at the locality where gravity is "g".

Obviously this force is proportional to g so that when the above action occurs at a locality where  $g = g_s$  the product (Mg/980.665)( $g_s/g$ ) = M, numerically, indicates that the number of grams mass expressing the loads also expresses the number of grams force which they produce at a locality where  $g = g_s$ .

It is to be noted here that in the equations developed in the following section (and in the appendix), each and every mass involved, and all balancing columns of air, oil, and mercury, each multiplied by its respective density and expressed as an equivalent mass of a column of mercury, plus the masses of the barometric columns of mercury, are multiplied by the prevailing factor g to express their effects as forces in dynes that measure the pressure on the piston base. Cancellation of that factor throughout permits the remaining expressions to represent masses in grams mass or forces in grams force that those masses would produce where standard gravity g<sub>s</sub> prevails.

This means that for any piston load the corresponding balancing column values obtained at any and all localities are identical and that the forces expressed in grams, where  $g = g_s$ , are numerically the same as the gram masses involved; the only difference due to location is that the <u>pressures</u> produced by the

8

in dynes Dividing Lasse dynam by 980 665, which is the number of dynes per gram force. Hg/930 665 then is the number of grams force due to the grass lass at M grams make at the Locality where gravity is "g".

OBVIOUSIY this force is proportional to g so that when the above action become at a incality where g = e the bacduct (Mg/360.465)(g /g) H, merically, incleates the header of grame mass exercisating the fourty mist extremes the moder of grame force which they produce at a incalier where g = g

It is to be noted have that in the equations developed in the following section (and in the momential, each and over, asso involved, and all balancing columns of sir, cil, and accoupt, acch multiplied by its respective density and expressed as an equivalent mass of a column of marchy, also the masses of the harcometric columns of mercety, are multiplied by the prevating the pressure on the platen base. Compliation of that factor the operators that remaining expressions to the the operating the pressure on the platen base. Compliation of that factor to grass one of forces in grand force that there remains to grass ones of lowering expressions to represent coups produce where standard gravity 6, prevently.

This means that for any platon load the corresponding balancing column volues obtained at my and all invaligher are identical and that the forces expressed in grame, under g = g, are numerically the same as the gram masses involved, the only difference due to location is that the pressures or durated by the above masses are proportional to values of gravity obtaining. A further consequence of the above relations is that the ratio of  $H_{net}$  to  $M_{net}$ , i.e. the piston constant  $C_N$ , is independent of gravity, as appears also in the procedure account below.

The assumption that capillary effects on differential pressure measurements "balance out" is probably more nearly true than would be the results of any attempts to evaluate these effects for inclusion in the comparison. These effects include those of capillary attraction of the oil on the piston at its emergence into the atmosphere, which is a very slight depressing action. Countering this effect, more or less, is a very slight upward force on the piston due to leakage flow of oil past it, which must be a function of oil viscosity and of the difference between the oil pressure around the piston and that of the atmosphere above it. The piston itself is under compression due to its loads and to the pressure of the oil in which part of the piston is immersed, which immersion requires consideration, and to the pressure gradient accompanying the leakage flow. The containing cylinder is subject to expansion because of these pressures or to compression if it is under external pressure, in all of which action Poisson's ratio is a factor. These are minor matters as related to the differential low pressure calibration herein reported, and are of negligible importance.

Of like consideration are the very small differences of pressure due to variations of densities of air, oil, and mercury above masses are proportional to values of gravity childred. A forther consequence of the above relations is that the ratio of  $R_{mat}$  to  $M_{opt}$ , i.e. the placen constant  $\Omega_{N}$  is independent of gravity, as appears also in the procedure account become

Of like consideration are the vary small differences of pressure due to variations of densities of sir, oil, and mercury

that may exist due to differences of their temperatures and pressures at the several times when readings for the short and long columns were obtained. The assumption that the differences of pressure corresponding to these two mercury columns described below is equivalent to that produced by the change of load on the piston is justified. In any event, these very small differences tend to cancel, as is apparent from their comparisons in the appendix.

#### EXPERIMENTAL PROCEDURE

#### The Gage and Testing Arrangements

The gage calibrated is of the type described and illustrated by Keyes  $\frac{7}{}$  except that the oscillating motion of the piston was

<u>7</u>/ Keyes, Frederick G., High Pressure Technic. Ind. and Eng. Chem.,
 v. 23, No. 12, December 1931, pp. 1375-1379.

replaced by a continuous rotation and a small table was attached to the top of the piston to measure low pressures, thus eliminating the tare of the loading yoke and pan for heavy loads. The effective piston diameter at 20° C is 0.986164 cm, approximately 1 cm; its effective area  $A_s$  at 20° C is 0.736188 cm<sup>2</sup>; both values are calculated from the piston constant  $C_N = 0.999140 = 1/A_s \rho_{sm} \frac{8}{2}$ 

The gage was connected to an open-end mercury manometer as shown schematically in figure 1. Air was used to transmit the pressure from the oil beneath the piston to the mercury in the manometer because direct contact of oil and mercury caused fouling of the  $\underline{8}/ \rho_{sm}$  is standard density of mercury in (grams per cc)/10 = 1.35951 because H is expressed in mm of mercury column. that may exist due to differences of their temperatures and measures at the several times when readings for the abort and long columns were obtained. The assumption that the differences of measure corresponding to these two mercury actions described below is equivalent to that produced by the charge of load on the piston is justified. In any event, these very small differences rand to cancel, as is apparent from their compations in the appacits.

#### EXPERIMENTAL PROCEDERS

#### The Gane and Testing Arrangements

The gage calibrated is of the type described and bluetrated by Keyes I except that the veciliating motion of the piston was

2/ Mayres, Frederick G., High Frassure Technic. Ind. and Eng. Gham., v. 23. No. 12, December 1931, pp. 1375-1379.

replaced by a continuous rotarion and a small table was attached to the top of the piston to measure low pressures, thus eliminating the tare of the loading yele and pan for heavy loads. The effective piston dismeter at 20° C is 0.988164 cm, approximately I cm; its effective area  $A_{\mu}$  at 20° C is 0.736188 cm<sup>2</sup>; both values are calculated from the piston constant  $C_{\mu} = 0.999140 = 1/A_{\mu} C_{\mu}$ 

The gage was connected to an open-and service manometer as about schematically in figure 1. Air was used to transmit the prosents from the oil beneath the piston to the marcury in the manometer because direct contact of oil and marcury coused fouling of the  $\frac{3}{2}$ ,  $\rho_{\rm gm}$  is standard density of marcury to grams per covid - 1.35951 because H = 1a expressed in mo of marcury column.

0.5

-







mercury and resulted in the meniscus being very poorly defined. The mercury column was air-jacketed and the base of the manometer, was enclosed in hair felt, although not shown in the figure, to provide a uniform mercury temperature. The mercury column was left exposed where readings were made, and adjustable brass sleeves above these points shaded the mercury surface, thus facilitating readings by eliminating phantom menisci. A similar shield was used below the oil-air meniscus. The bore of the manometer tube was large, being approximately 2.8 cm, to minimize capillary effects of menisci. Mercury levels of the column and oil levels at c were read by a Geneva cathetometer at a distance of about 65 cm. from the column. Three mercury thermometers in the air jacket, two attached to the cathetometer scale and one in the piston block, gave the average temperatures of mercury column, cathetometer scale and piston respectively. The piston with small table for loads was continuously rotated at a fixed level a; the piston position was maintained by sighting through a telescope at a line etched on the table; its location was controlled by a hand operated oil injector pump, not shown. The net force exerted by the piston and table was balanced mainly by the weight of a mercury column about 94 mm in height. The exact height to 0.01 mm was read on the cathetometer, together with the oil level at c. A reference level was established for the oil, and each mercury column reading was corrected for oil head caused by displacement from this reference level. All

thermometers were read for each mercury meniscus reading. Then 850 grams mass of brass load was added to the piston load and balanced on the manometer by forcing mercury into it from a leveling bulb. The mercury in the short side of the manometer rose to compress the air between mercury and oil until equilibrium was established. Oil level, mercury levels and thermometer readings were taken for this new piston loading and the change in air density due to the above compression was considered.

#### Estimation of Piston Constant from Experimental Values

In accordance with the conclusion expressed in the paragraph preceding the above section, evidence for which is given in the appendix, we are justified in assuming that our determining equation is based only upon the change of gross piston load. That equation is

$$M_{gr} (1 - \rho_a / \rho_M) / A_s = H \rho_{sm} (1 - \rho_a / 10 \rho_{sm})$$

or

$$M_{net}/A_s = H_{net} \rho_{sm}$$

whence

$$H_{net}/M_{net} = C_N = 1/A_s \rho_{sm}$$

in which H is the net change in height in international mm of a column of mercury that exactly balances a change in net load,

13

chermonaters ware read for each mercury maniscus reading. Then 850 grams mass of brass load was added to the piston hold and balanced on the maxameter by forcing mercury into it from a leveling bulb. The mercury in the short side of the memonter cose to compress the air barwean mercury and oil udtil equilibrium was established. Gil level, mercury invels and theremoneter readings were taken for this new piston levels and the change in air density due to the above compression was comsidered.

#### Sectantion of Platon Constant from Experimental Values

In adcordance with the conclusion expressed in the paragraph proceding the above section, evidence for which is given in the appendix, we are justified in assuming that out detarmining equation is based only upon the clange of gross piston load. That equation is

20

spanity

Hat Mar = CN = 1/Angun

in which H net is the net change in height in international an of a column of mercury that exactly balances a charge in net load.

1.3

 $M_{net}$  grams, on a piston base of effective area A sq cm at 20 $^{\circ}$ C. Densities are represented by  $\rho$  with subscripts of obvious significance.

The piston constant  $C_N$  as above determined has dimensions of length per unit mass and obviously is inversely proportional to the product of the effective area of the piston and the standard density of mercury.

For convenience, a piston constant  $C_M$  may be defined as the ratio of the change of net mercury column,  $H_n$ , to change of total load,  $M_{gross}$ 

$$H_{net}/M_{gross} = C_M = C_N \cdot C_B$$

in which  $C_B = (1 - \rho_a / \rho_M)$  is a buoyancy factor that is ordinarily evaluated in terms of average densities of air and of loads at the locality of its application. (Usually and properly included with the loads in air is that part of the piston in oil.)  $\frac{9}{2}$ 

In either case the products

3

$$C_N \cdot M_{net} = H_{net}$$
 and  $C_M \cdot M_{gross} = H_{net}$ 

determine the heights of standardized mercury columns that define the changes of pressure at the base of a piston, due to those changes of loads, that obtain only at a location where gravity is  $g_s$ , standard, and when the deductions for buoyancy are always proportional to  $\Delta M_{gross}$ .

9/ Helium Research Center Memorandum Report No. 53, "Buoyancy Effect of Air and Oil on Rotating Piston Gage Loads," by E. S. Burnett, in process. M<sub>aet</sub> grams, on a piston base of effective ares A sq in at 20°C. Densities are represented by a with subscripts of ubvious significance.

The piston constant C<sub>N</sub> as showe deterpised has dimensions of length per unit case and obviously is inversely proporcional to the product of the sitective area of the piston and the standard density of carcury

Nor convenience, a platon constant G<sub>q</sub> may be defined as the ratio of the change of nor marcary column, B<sub>n</sub>, to change of total load, M<sub>gross</sub>

net "groas " M " SH' CE

in which  $E_B = (1 - \rho_d/\rho_H)$  is a bunyancy instar that is ordinarily evaluated in terms of average departure of air and of heads at the locasity of its application. (Newally and properly included with the loads in air is that part of the sister in oil )  $2^j$ 

In strings pass the products

determine the heights of standardized mercury columns that define the shanges of pressure at the base of a piston, due to these obacges of loads, their obtain only at a location where gravity is by standard, and when the seductions for hunyandy are always prepartional to b K

3/ Hellum Research Canter Menorandum Report So. 53, "Budyancy Effect of Air and Oil on Rotating Fiston Gage Louds," by E. S. Burnett, in process. In both cases the gravity factor,  $C_g = g/g_s$ , must be applied so that the resulting H, equal to  $C_g \cdot H_{net}$  then is a measure of the actual change of pressure at the piston base at the locality of its use where the value of gravity is g. Therefore, we have, for use at that locality the piston constant

 $C_p = C_M \cdot C_g = C_N \cdot C_B \cdot C_g$ 

The importance of accuracy in the evaluation of the buoyancy factor  $C_B$  is here specially emphasized. In the past, apparently, the total load, including the piston, has usually been assumed to be of the same material of density  $\rho_M$  and the buoyancy deduction has been based on the assumption that air of average density has been the buoyant medium.

Strictly the ratio  $\rho_a/\rho_M$  should be a composite ratio reflecting the buoyancy of surrounding air on each different material of which the total piston loads may consist, including that part of the piston which is in oil, which environment may require a further deduction.

Historically, this last mentioned probable deduction appears to have been ignored or completely overlooked, although it usually is of major relative importance as is obvious in the example given later. (But note also that this "deduction may be negative instead of positive and should be handled accordingly.) In both cases the gravity factor,  $C_g = g/g_g$ , must be applied so that the resulting H, equal to  $C_g$  H and then is a measure of the actual change of pressure at the piston base at the locality of its use where the value of gravity is g Therefore, we have, for use at that locality the piston constant

The importance of accuracy in the evaluation of the huoyancy factor C<sub>B</sub> is here specially emphasized in the past, apparently, the total leaf, including the platue, has usually been assumed to be of the same material of density o<sub>M</sub> and the huoyancy deduction has been based on the assumption that air of average density has here the boyant medium.

Strictly the ratio  $\mu_{A}/\mu_{A}$  should be a composite ratio refrecting the buoyanty of surrounding sir on each different material of which the total platon loads may constat, including that part of the staton which is in oil, which environment may require a further deduction

Bistorically, this last mentioned probable deduction appears to have been ignored or completely overlooked, although it usually is of major relative importance as is obvious in the example given later. (But note also that this "deduction may be negative 'material of positive and should be handled accordingly.)

51

#### Piston Constants at Amarillo

From the final values of the sample calculations in Table 1 for this differential Keyes gage calibration, we obtain

$$C_{N} = 849.166/849.897 = 0.9991399$$

$$C_{M} = 849.166/850.000 = 0.9990188$$

$$C_{B} = 1 - 0.001045/8.63 = 0.9998789 = \Delta M / \Delta M (assuming air only is the buoyant fluid)$$

$$C_{g} = 979.402/980.665 = 0.9987120$$

$$C_{M} = C_{N} \cdot C_{B} = (0.9991399)(0.9998789) = 0.9990188$$

 $C_{P} = C_{M} \cdot C_{g} = (0.9990188) (0.9987120) = 0.9977308$ =  $C_{N} \cdot C_{B} \cdot C_{g} = (0.9991399) (0.9998789) (0.9987120)$ 

Application:  $C_p \cdot \Delta M_{gross} = 0.9977308 \times 850. = 848.071 \text{ mm}$  change of standardized mercury column that measures the change of pressure at the piston base, produced by a change in gross piston load of 850 grams of brass mass, at Amarillo.

The above value for the piston constant,  $C_p$ , applies only to the <u>change</u> in M, when of <u>brass</u>, at this particular locality where gravity has the value given as calculated for the latitude and elevation assumed for our laboratory from formula given in ICT, v. 1, pp. 401.

> Latitude assumed: 35° 17' 30" Elevation assumed: 3740 ft. = 1141.50 meters

#### Ploton Constants at Amarillo

From the final values of the apple calculations to Libic i

 $C_{19} = .849.156/849.897 = 0.9991399$   $C_{16} = .849.166/850.000 = 0.9970189$   $C_{16} = .649.166/850.000 = 0.9970189$  $C_{16} = .1 - 0.001045/8.63 = 0.9958789 = .00 /034$ 

C - 979\_402/980 665 - 0 9987120

 $C_{M} = C_{N} - C_{B} = (0.9991339) (0.9998789) = 0.9990128$  $C_{F} = C_{H} - C_{B} = (0.9990188) (0.9987120) = 0.9977308$  $= C_{H} - C_{B} - C_{C} = (0.9991399) (0.9983789) (0.9983789) (0.9987139)$ 

Application:  $C_{\rm F} \cdot \frac{2M}{3}$  gross = 0.9377308 x 850. = 848.071 ms change of standardized mercury column that measures the change of pressure at the piston base, produced by a change in gross piston field of 850 grams of brees mass, at faurillo.

The shows value for the present constant, C<sub>p</sub> spalles only to the <u>channo</u> in M. when of <u>brass</u> at this particular locality where gravity has the value given as releviated for the latitude and elevation associes for our laboratory from formula given in 107. v. 1. pp. 401.

Lattitude resument 35° 17' 30"

Elevation sammed: 1740 fr. = 1141.50 meter

d [

### TABLE 1. - Sample Data and Calculations

Mercury Column

Temperatures	°c	nury Colomn	
Cathetometer	25.,7°, 25.7°	25.7°, 25.7°	Av. 25.7 <sup>0</sup>
Mercury	24.5°, 25.1°, 24.8°	$24.5^{\circ}, 25.1^{\circ}, 24.8^{\circ}$	Av. 24.8 <sup>0</sup>
Piston block	31.3°	31.3°	Av. 31.3 <sup>0</sup>
Cathetometer	Reading	Upper Hg level	963.11 mm
Cathetometer	Reading	Lower Hg level	15,90 mm
Difference			948.21 mm
Reference Oil	Level	32.68 mm	849.166
Cathetometer	reading of oil leve	el <u>10.53 mm</u>	850.000
Difference fr	com reference	-22.15 mm	
Millimeters o	of mercury equivale	nt to -22.15 mm oil	-1.42 mm
Cathetometer	corrected for oil	level	946.79 mm
Cathetometer	temperature correct	tion 25.7 x 0.0 <sub>4</sub> 183 =	+0.000496
Mercury	H H	$24.8 \times 0.0_{3}1818 =$	-0.004509
Piston	1	$(31.3-20) \times 0.0_4 23 =$	+0.000260
			-0.003753
Cathetometer	reading corrected :	for oil head	946.79 mm
Temperature o	correction -0.00375	3 x 946.79 mm	-3.55 mm
Cathetometer	reading corrected	for temperature	943.24 mm

i.

ż

#### TABLE 1. - Sample Date and Calculations

Marcury Column

 Temperatures "C

 Cathecometer 25.7°, 25.7°
 25.7°, 25.7°

 Marcury
 24.5°, 25.1°, 25.1°, 24.8°
 Av. 24.8°

 Marcury
 24.5°, 25.1°, 25.1°, 24.8°
 Av. 24.8°

 Platon block 31.3°
 36.5°, 25.1°, 24.8°
 Av. 21.3°

 Cathetometer Reading
 Upper Re level
 963.11 mm

 Cathetometer Kending
 Lover Ne level
 963.11 mm

 Differance
 Sading
 Lover Ne level
 963.11 mm

Reference 011 Level

------

Difference from reference -22.15 mm oll -1.42 cm Millimeters of mercury equivalent (o -22.15 mm oll -1.42 cm Cathetometer corrected for oil level 946.79 mm

Cathertoneter temperature correction 25.7 x 0.0<sub>0</sub>183 =  $\pm 0.00000$ Hercury " 20.8 x 0.0<sub>0</sub>1818 = -0.000503Platon " (31.3-20) x 0.0<sub>0</sub>33 =  $\pm 0.000280$ 

Catherometer reading corrected for oil head 948.79 mm Temperature correction -0.003753 x 946.79 mm -3.55 mm Catherometer reading corrected for temperature 943.24 mm
# TABLE 1. - Sample Data and Calculations (Con.)

## Mercury Column

Average of 74 high columns	943.336 mm
Average of 74 short columns	94.104 mm
Difference	849.232 mm
Air buoyancy correction $\frac{\frac{1}{a}a}{10\rho_{sm}}$ with	

 $\rho_a = 0.00105 \text{ gm/cc} \text{ at } 25^\circ \text{ C} \text{ and } 670 \text{ mm}, \frac{849.232 \times 0.00105}{13.5951} - 0.066 \text{ mm}$ 

Fully corrected mercury column difference, H

# Load on piston

Change of load, M gms, on piston

Buoyancy correction, B, at  $25^{\circ}$  C; = volume of brass load x  $\rho_{a}$ 

$$= \frac{M_{gr}}{\rho_{h}} \times \rho_{a} = \frac{850 \times 0.00105}{8.63} -0.103 \text{ gm}$$

849.897 gm

Piston constant,  $C_N$ , for mm of mercury at 0° C.

per gm net piston load, piston at 20° C, is 849.166/849.897 = 0.999140 Note: Density of oil at t°,  $(\rho_0)_t = 0.887 (1 - 0.0007t)$  grams per cc. Linear temperature coefficient of expansion of cathetometer scale,  $\pm 0.0_{\hat{L}}$ 183.  $\alpha$ , areal temperature coefficient of expansion of steel piston,  $\pm 0.0_{\hat{L}}$ 230.  $\beta$ , density temperature coefficient of expansion of mercury  $\pm 0.0_{3}$ 1818.

- D9004

850.000 gm

849.166 mm

### TABLE 1. - Sample Data and Culculations (Con.)

#### Marchry Column

Average of 74 high columns Average of 74 chort columns Difference Alt buoyancy correction  $\frac{M_{e^{2}}}{M_{e^{2}}}$  with alt buoyancy correction  $\frac{M_{e^{2}}}{M_{e^{2}}}$  with

Part 0.00105 ga/cc at 25° C and 670 mm. B94.222 × 0.00105 13.5931 Fully corrected mercury column difference, H

#### Load on piscon

Change of load, N gma, on placon . 850.000 gm

Sucyancy correction, 8, at 25" C: - volume of brass load x p

Differential piston load, M

Fiston constant, C., for am of serviry at 0° C.

per ga net piston load, piston at 10" C. is 849.166/849.897 - 0.999140

bore: Density of oil at t', (0) = 0.887 (1 - 0.0007t) grams per co. Linear temperature coefficient of expandin of cathetoerier scale. +0.0,183. G. archi temperature coefficient of expansion of steel piscos, +0.0,230. B. density temperature coefficient of expansion of mercury +0.0,1818.

For any and all cases the height of the barometric pressure column of mercury,  $H_b$ , measured at the piston base level, must be added to the net mercury column,  $H_n$ , to estimate the total pressure at the piston base. Since  $H_b$  represents just another mass of mercury in a closed-end tube, it must therefore be multiplied by  $C_g = g/g_s$  for the estimate indicated. It may be noted in passing that  $H_{net}$  plus  $H_b$  is exactly equivalent to adding  $H_T$  to  $H_{gross}$ , where  $H_T$  is the barometric column at the top level of  $H_{gross}$  which itself is the height of the balancing mercury column before its final adjustment for air buoyancy.

## COMPARISON WITH 1924 CALIBRATION AT MIT

Piston constants have been variously defined and used. The original calibration of this gage was made at MIT in 1924 by comparison with a mercury column that was varied between 4 meters and 9 meters in height. The low column due to piston and tare appears from our report of the tests to have been maintained at a fixed position. The changes in heights of the columns due to changes in applied piston loads from approximately 4,000 grams to 9,000 grams were recorded for 11 such changes. These equivalent mercury columns after correction for oil levels, measuring tape inaccuracies, air buoyancy, and mercury density, all at 22° C. were standardized for density of mercury at 0° C; they were then multiplied by the ratio of g at MIT to g at 45° latitude and sea level, then taken as 980.370/980.616 = 0.999749 = C<sub>g</sub>.

"A Barr.

For any and all cases the intight of the barometric pressure column of marcury,  $H_{\rm b}$ , assaured at the piston base level, must be added to the net mercury column,  $H_{\rm n}$ , to estimate the rotal pressure at the piston base. Since  $H_{\rm b}$  represents just annihier mass of mercury in a closed-end tube, it must therefore in multiplied by  $G_{\rm g} = s/g_{\rm g}$  for the estimate indicated. It may be noted in passing that  $H_{\rm het}$  plus  $H_{\rm b}$  is exactly equivalent to adding  $H_{\rm c}$  to  $H_{\rm gross}$ , where  $H_{\rm c}$  is the barometric column at the top level of  $H_{\rm gross}$  which itself is the height of the balancing acting to the start start adjustment for sit buyency.

#### COMPARISON WITH 1924 CALIBRATION AT MIT

Pletron constants have been variously defined and used. The original calification of this gags was made at NIT in 1924 by comparison with a mercury column that was varied between a meters and 9, meters in height. The low column due to platon and tare appears from our report of the tests to have head maintained at a fixed position. The changes in heights of the columns due to changes in applied platon loads from approximately 4,000 grams are 9,000 grams were recorded for 11 such changes. These equivalent inaccuracies, air buoyancy, and mercury density, all at  $22^{\circ}$  C. wore starderdized for file out of 0 2; they were then multiplied by the ratio of x at NTT to g at  $45^{\circ}$  facitude then multiplied by the ratio of x at NTT to g at  $45^{\circ}$  facitude and sea level, them taken as 980.370/980.616 = 0.393749 = C.

The changes of mercury columns as above adjusted in mm were then divided by the corresponding changes of gross piston loads in grams, to yield eleven values averaging 0.99880  $\pm$  0.00010 as a piston constant,  $C_g \cdot C_M$ , which is  $C_g \cdot (\Delta H_{net}) / (\Delta M)_{gross}$ . Later data indicate that  $C_g$  should be 980.398/980.665 = 0.999727 ... which, if applied, would reduce 0.998800 to 0.998778.

Assuming a Boston air density of 0.001200 gms/cc at the temperature of the tests (22<sup>0</sup> C) and normal barometric pressure (760 mm Hg) and a steel load density of 7.84 gms/cc, there results an air buoyancy correction factor of 0.999847 which applies to the change of the steel loads on the piston.

Using total values of the changes in loads and in mercury columns indicated as proportional to their respective averages, the net standardized changes of Hg column totaled 82,235.9 mm. The total changes of applied steel loads was 82,315.0 gm which, corrected for air buoyancy by the factor 0.999847, becomes 82,302.4 gm. The ratio 82,235.9/82,302.4 = 0.999192 is  $C_{net}$ , or  $C_N$  a piston constant. The product  $C_P$  = 0.999192 x 0.999847 x 0.999727 = 0.998766 is for use at Boston. The gross change of steel loads in grams on the piston base multiplied by this constant yields the height of a standardized mercury column which determines the corresponding change in absolute pressure at the level of the piston base; effects of air buoyancy on the steel load change, and of gravity, are included in that constant. The changes of mercury columns as above adjusted in universion of the changes of mercury columns as above adjusted in university of the corresponding changes of groat platen loads in grams, to yield eleven values averaging 0.99880  $\pm$  0.00010 as a platen constant,  $C_g C_N$ , which is  $C_g (M_{out})/(M_{groat})$  and the correspondence that  $C_g$  should be 980.398/980.605 = 0.998717 which, if applied, would reduce 0.998800 to 0.998781.8

.

Assuming a Boston are denaity of 0.001200 gma/co at the temperature of the tests, (22°C) and normal barometric pressure (760 mm Hg) and a steel load density of 7.84 gma/ct, there results an air buoyancy correction factor of 0.799847 which applies to the change of the steel loads on the piston.

Using total values of the changes in loads and in mercury columns indicated as proportional to their respective averages. the set standardized changes of hg column totaled 82,235.9 mm. The total changes of applied steel loads was 82,315.0 gm which, corrected for air buoyaney by the factor 0.95986.7, beccase 32,302.4 gm. The ratio 82,235.9/82,302.4 = 0.999192 is  $C_{\rm nef}$ , or  $C_{\rm g}$  a platom constant. The product  $C_{\rm p}$  = 0.999192 s 0.999867 z 0.999727 = 0.998766 is for use at boaton. The gross change of steel loads in grams on the platom base multiplied by this which determines the corresponding change in absolute pressure at the level of the pistom base; offects of air buoyancy us the steel load the pistom base; offects of air buoyancy us the steel load the pistom base; offects of air buoyancy us the steel load the pistom base; offects of air buoyancy us the steel load the pistom base; offects of air buoyancy us the steel load the pistom base; offects of air buoyancy us the steel load the pistom base; offects of air buoyancy us the steel load the pistom base; offects of air buoyancy us the steel load change, and of gravity, are included in that constant;

 $C_{net}$  determined at Amarillo in 1932, 0.999140, is to be compared with the above 1924 value, 0.999192. Each agrees with their average to  $\pm 1/38,500$ , which may indicate a slight enlargement of effective piston area over eight years' use.

Whatever the combination of ratios that produces a piston constant for use at a given location, it still has to be multiplied by the factor  $1 - \alpha_p$  (t - t<sub>s</sub>) as previously defined, table 1, to account for variation of the piston temperature. We believe as before stated that the fundamental ratio  $H_n/M_n = C_N$  where properly determined should define the piston constant. Adjustments for buoyancy on the loads and for gravitation can be added for local use, but they vary with each location and with load material distribution; also the temperature of the piston varies during use, as does the density of the oil column contributing to  $H_n$ .

# SPECIFIC EFFECT OF PART OF PISTON IN OIL

In the original 1932 internal report of this calibration mention was made of the necessity of including this effect when measuring absolute pressures with a piston gage. Consideration thereof has been shown not to be necessary in a differential calibration. The magnitude and significance of that effect can readily be demonstrated, however, by using values from table 1.

Two mercury columns are listed which, corrected for air buoyancy, are 943.263 and 94.097 mm. These differ by 849.166 mm which corresponds to a change of net piston load of 849.897 grams mass. How

Cnet determined at Amarillo in 1932, 0.909140, is to be compared with the above 1924 value, 0.999192. Each agrees with their average to ±1/38,500, which may indicate a slight sulargoment of effective niston area over cight years' use.

Whatever the combination of ratios that produces a practon constant for use at a given incation, it still has to be unitiplied by the factor  $1 - \alpha_p (t - t_g)$  as previously defined, table 1, to account for variation of the elstan temperature. We believe as before atsted that the fundamental ratio  $K_{\mu}^{\mu} = t_{\mu}$  where properly determined should define the platen constant. Adjustments for total use, but they vary with each location and with load material distribution; also the respectature of the platen varies doring use, as does the density of the oil column contribution to  $R_{\mu}$ .

## STECIFIC EVENCT OF PART OF PIETON IN GIL

In the original 1932 internal report of this calibration mention was made of the decreasity of including this effect when measuring absolute pressures with a platon gage. Consideration therwol has been shown not to be necessary in a differential calibration. The magnitude and significance of that effect can teadily be demonstrated, however, by using values from table 1.

Two marcury columns are Hared which, corrected for air buoyancy, are 941,263 and 94.097 mm. These differ by 849.166 mm which correaponda to a change of net piston load of 849.637 grams mass. How

2.2

much more change of net load, z grams, must be made to reduce the net mercury column, 94.097 mm to zero?

By simple proportion z = 94.097(849.897/849.166) = 94.178 net grams. Our records show that the piston and table plus rotating arm amounted to 99.60 gms gross. Hence the difference,  $M_d$ , 5.422 grams, is the net upward force due to upward flow of oil past the piston, downward capillary pull of oil at piston emergence into the air, and to possible errors of estimating the height,  $H_n$ , of the equivalent balancing column of mercury.

While 5.422 grams net uplift may seem excessive, it nevertheless appears to have been operative in this calibration. (Graphically it is the intercept, M<sub>d</sub>, on the M<sub>gross</sub> axis of a straight line through the coordinated points representing the long and short columns.) Hence the equation for excess pressure above atmosphere at the piston base level due to the total loads on the piston base is in this case:

$$P_B - P_b = \left[ M_{gross} (1 - \rho_a / \rho_M) - M_d \right] / A_s = M_n / A_s = H_n \rho_{sm}$$
  
total effective deduction is then  $M_a(\rho_b / \rho_M) + M_1$  which lead

to the deduction factor  $C_B = 1 - \frac{\text{total deduction}}{\text{gross load}} = M_n/M_{gr}$ 

The

If now we assume that  $(C_M)_{apparent}$  was taken to be the ratio of  $(H_n)_{true}$  to  $(M_{gr})_{observed}$  at some random calibration value of  $(M_{gr})_{obs}$  (without knowledge or consideration of  $M_d$ ) and values of  $(H_n)_{app}$  were computed therefrom, the question arises what is the true relation between  $(\Delta P)_{true}$  and  $(\Delta P)_{app}$  in which  $\Delta P = P_B - P_b$ ?

22

S

much more change of not load, a grume, must be made to reduce the

By simple proportion a = 94.097(849.897/349.100) = 96.178 met grams. Our records show that the miston and table plus rotation arm amounted to 99.60 gms gross. Hence the difference, N<sub>0</sub>, 5.422 grams, is the net upward force due to opward flow of oil past the miston, dosnward capiliary pull of oil at pluros emergence into the air, and to possible arrors of estimating the height, B<sub>0</sub>, of the squivalent balancing column of mercury.

While 5.422 grams nat uplift may seem excersive, it newether less appears to have been orwrative in this selibration. (Graphscally it is the intercent, "g, on the N gross axis of a straight line through the coordinated points representing the long and short colomns.) Hence the equation for axcess pressure above atmosphere at the piston have level due to the total loads on the piston have is in this case:

The total effective deduction is then  $h_{gf}(a_A/a_B) + M_{d}$  which hads to the deduction factor  $C_B = 1 - \frac{\text{Dotal deduction}}{\text{prove load}} = M_A/M_{gf}$ 

If now we assume that  $(C_{pl})_{apparant}$  was taken to be the ratio of  $(H_{p})_{true}$  to  $(O_{pr})_{observed}$  at some random calibration value of  $(M_{pr})_{obs}$  (without knowledge or consideration of  $M_{d}$ ) and values of  $(G_{n})_{app}$  were computed thereform, the question arises what is the true relation between  $(OP)_{rous}$  and  $(OP)_{app}$  in which  $OP = P_{B} - P_{D}^{2}$ 

That relation is:

$$(\triangle P)_{true}/(\triangle P)_{app} = (C_B)_{at} (P_B)_{true} \div (C_B)_{at calibration point.}$$

For the same  $H_n$  as at the calibration point,

 $C_{M} = H_{n} / (M_{gross})_{observed};$   $C_{N} = H_{n} / (M_{n})_{correct}$ 

whence

$$\frac{(\triangle P)_{true} \sim (H_n)_{true} = C_N M_n = M_n (H_n/M_n)_{at calib}}{(\triangle P)_{app} \sim (H_n)_{app} = C_M M_g = M_g (H_n/M_g)_{at calib}}$$

$$\frac{\frac{M_n / M_{gr}}{(M_n / M_{gr})_{at calib}}}{(M_n / M_{gr})_{at calib}} = \frac{C_B}{(C_B)_{at calib}} = \frac{1 - \rho_a / \rho_M - M_d / M_{gr}}{(1 - \rho_a / \rho_M - M_d / M_{gr})_{at calib}}$$

Obviously this ratio cannot be evaluated without knowledge of the value of  $M_d$ . However this value may be approximately estimated by assuming that it is equivalent to a pseudo buoyancy of the part  $M_o$  of the piston that is in oil. On that assumption,  $M_d = M_o \rho_o / \rho_M$  and  $M_o = M_d \rho_M / \rho_o$ . In this calibration  $M_o = 5.422 (7.84/0.870) = 48.85$  gms.

The equivalent length in oil of this piston of area 0.736 cm<sup>2</sup> is  $48.85/(7.84 \ge 0.736) = 8.5 \ cm = 3.34''$ . This means that our steel piston if immersed in the cylinder oil to a depth of 8.5 cm below the oil-atmosphere surface would be buoyed upward by a force  $M_d$  of 5.422 gms. These values are mutually consistent with actual dimensions of the piston. Subtracting 48.85 from 99.6 leaves in air 50.75 gms of the short column load on which the air buoyancy is about 0.007 gms; this is about 1/800 of 5.422 gms which emphasizes the necessity of taking  $M_d$  into account.

That relation is:

$$(\Delta P)_{true}/(\Delta P)_{app} = (C_B)_{at} (P_B)_{true} + (C_B)_{at, calibration point.}$$

For the same II as at the calibration point,

whence

$$\frac{M_{n}/M_{n}}{m_{n}} = \frac{G_{M}}{G_{n}} = \frac{1 - p_{n}/p_{M} - M_{0}/M_{nT}}{(1 - p_{n}/p_{M} - M_{0}/M_{nT})}$$

Obviously this table cannot be evaluated without knowledge of the value of  $M_d$ . However this value may be approximately estimated by assuming that it is equivalent to a pseudo buoyancy of the part  $M_d$  of the piston that is in oil. On that assumption,  $M_d = M_0 o_0 \rho_{\rm H}$  and  $M_s = M_0 J_0 o_0 \rho_{\rm H}$  and

The equivalent length in oil of this platon of area 0.720 cm to 48.65/(7.24 x 0.736) = 8.5 cm = 3.34". This means that our steel platon 11 immersed in the cylinder oil to a depth of 8.5 cm below the oil-armosphere surface could be buoyed upward by a force M<sub>d</sub> of 5.422 gas. These values are mutually consistent with actual dimenstons of the piston. Subtracting 48.85 from 99.6 leaves in air 50.75 gms of the short column load on which the sit buoyancy is about 0.007 gms; this is about 1/800 of 5.422 gms which emphasizes the necessity of taking M, into account The advantage of this assumption is that for some other piston gage calibration for which its  $M_d$  has not been reported, but for which its  $(C_M)_{apparent}$  appears to have been based upon its  $(M_g)_{gr}_{observed}$ it may be possible to estimate a probable  $M_d$  for it based upon its cross sectional area and its depth of immersion in oil. Thus assume area  $A = 1 \text{ cm}^2$ , L = 7 cm, density of steel piston  $\rho_M = 7.84 \text{ gms per}$ cm<sup>3</sup>; density of piston oil at calibration pressure  $\rho_o = 0.900 \text{ gms}$ per cm<sup>3</sup>. Then

 $(M_d)_{estimated} = M_o \rho_o / \rho_M = A \cdot L \cdot \rho_M \rho_o / \rho_M = A L \rho_o = 1 \cdot 7 \cdot 0.9 = 6.3 \text{ grams}$ This estimated value of  $M_d$  permits evaluation of the above ratio when the load at calibration is known; that load usually is given in the report of the piston calibration;  $M_d$  is seldom if ever mentioned.

When the piston constant appears to be based upon comparison of observed load for one value only of  $H_n$  (usually about 9 m of  $H_g$ ), then  $M_d$  can be estimated approximately only by the above method.

Several relevant values applying to the two calibration points of table 1 are listed below.

	H n mm	M gr gms	Total Deduc. gms	M n gms	C <sub>M</sub> = H <sub>n</sub> /M <sub>gr</sub>	$C_{N} = H_{M} / M_{n}$	$C_{B}^{=}$ M_n/M_{gr}
Long Col.	943.263	949.6	5.525	944.075	0.993327	0.999140	0.994182
Short Col.	94.097	99.6	5.422	94.078	0.944749	0,999140	0.945562
			Net Deduc,		∆H <sub>n</sub> /M <sub>gr</sub>	$\Delta H_n / \Delta M_n$	∆M <sub>n</sub> /∆M <sub>gr</sub>

Differences 849.166 850.0 0.103 849.897 0.999019 0.999140 0.999878

This comparison again emphasizes the necessity of accurate estimate of the total deduction for each loading when total loads are involved to determine the net load  $M_n$ .

The advantage of this assumption is that for some other pieton gage calibration for which its  $M_d$  has not been reported, but for which its  $(C_M)_{apparent}$  appears to have been based upon its  $(M_{gr})_{observed}$ it may be possible to estimate a probable  $M_d$  for it based upon its cross sentional area and its depth of numerator in oil. Thus assume area  $A = 1 \text{ cm}^2$ , L = 7 cm, density of store piston  $p_M = 7.84$  gms per cm<sup>3</sup>; density of piston oil at calibration pressure  $p_0 = 0.900$  gms cer cm<sup>3</sup>. Then

 $(M_d)$  estimated =  $M_0 \rho_0 / \rho_M = A \cdot L \cdot \rho_M \rho_0 / \rho_M = A L \rho_0 = 1.7.0.9 = 6.3 grams$ This astimated value of  $M_d$  permits evaluation of the above ratio when the load at calibration is known; that load usually is given in the report of the platon calibration;  $M_d$  is acidom if ever mentioned.

When the piston constant appears to be based upon comparison of observed load for one value only of H<sub>11</sub> (usually about 9 m of H<sub>2</sub>), then M<sub>2</sub> can be estimated approximately only by the above method. Several relevant values applying to the two calibration points

of table | are listed below.

This comparison again empinaizes the necessity of accurate estimate of the total deduction for each loading when total loads are involved to determine the net load M.

For the densities of this calibration and a calibration load of about 9000 gms of steel as at MIT with M<sub>d</sub> taken to be 5.4 grams, the correction ratio  $C_B/(C_B)_{Bat}$  calib evaluates to approximately  $1.000600 - 5.40/M_{gr}$  for which the following values apply. M, gms: 100 200 600 1,000 2,000 6,000 0.9466 0.9736 0.9942 Ratios: 0.9916 0.9979 0.9997 M , gms: 10,000 100,000 20,000 60,000 200,000 600,000 1.00006 1.00033 1.00051 Ratios: 1.000546 1.000573 1.000591

## GRAPHICAL RELATIONS OF PISTON CONSTANTS

Figure 2 is drawn to an exaggerated scale with  $H_n = 70$  mm at the upper right corner over  $M_{gross}$  equal to 100 gms so that the slope of the diagonal evaluates  $C_M$  as 0.700 for that calibration point only. The buoyancy deduction of  $M_{gr}$  is arbitrarily taken as  $M_{gr}/10$ ; the constant deduction  $M_d$  is taken to be 4 gms.

Subject to these arbitrary assumptions, the total deduction prevailing at the upper right corner of fig. 2 is (100/10) + 4 = 14; M<sub>n</sub> is then 100 - 14 = 86; C<sub>N</sub> is 70/86 = 0.814; C<sub>B</sub> = C<sub>M</sub>/C<sub>N</sub> = 0.700/0.814 = 0.860 = 1 - (14/100) = M<sub>n</sub>/M<sub>gr</sub> = 86/100.

Now take  $M_{gross} = 80$ ; the total deduction is (80/10) + 4 = 12;  $M_n = 80 - 12 = 68$ ; a line starting from 12 on the  $H_n = 0$  axis rising with a slope  $C_N = 0.814$  over the distance  $M_n = 68$  meets the vertical  $M_{gr} = 80$  at  $H_n = 55.35$ .  $C_M$  for this  $H_n$  is the slope of a line from there to the origin or  $(C_M)_{80} = 55.35/80 = 0.692$ .  $C_B = C_M/C_N =$  $0.692/0.814 = 0.850 = 1 - (12/80) = 1 - 0.15 = M_n/M_{gr} = 68/80$ . For the densities of this calibration and a calibration load of about 9000 gas of steel as at MIT with  $H_d$  taken to be 5.4 grams, the correction ratio  $G_g^{/(C_B)}a_1$  sullated to approximately 1.000600 - 5.40/M for which the following values apply. M , gms: 100 200 600 1.000 2.000 6.000 Ratios: 0.9466 0.9736 0.9916 0.9942 0.9978 0.9997 M , gms: 10,000 20.000 00,000 100,000 200,000 Ratios: 1.00006 1.00033 1.00031 1.000348 1.000573 1.00039

#### GRAPHICAL RELATIONS OF FISTON CONSTANTS

Figure 2 is drawn to an exaggerated soals with  $H_{ij} = 10$  mm at the upper right corner over  $M_{ij}$  equal to 100 gms so that the slope of the diagonal evaluates  $C_{ij}$  as 0.700 for that calibration point unly. The buoyancy deduction of  $M_{ij}$  is arbitrarily taken as  $M_{ij}/10$ ; the constant deduction  $M_{ij}$  is taken to be 4 gms.

Subject to those arbitrary assumptions, the total dedaction prevaliting at the upper right corner of fig. 2 is (100/10) + 4 = 14;  $M_{a}$ is then 100 - 14 = 86;  $G_{N}$  is 70/85 = 0.814;  $G_{a} = G_{M}/G_{m} = 0.700/0.814 =$ 0.850 = 1 - (14/100) = 8 / M = 35/100.

Now take M  $_{gross}$  - 80; the total deduction is (80/10) + 4 = 12; M = 80 - 12 = 58; a line starting from 12 on the H = 0 sais rising with a slope  $C_{\rm H}$  = 0.816 over the distance M = 58 heats the vertical M = 80 at H = 55.35.  $C_{\rm H}$  for this H is the slope of a line from there to the origin or  $(C_{\rm H})_{\rm SO}$  = 55.35/80 = 0.692.  $C_{\rm H}$  =  $C_{\rm H}/C_{\rm H}$  = 0.992/0.814 = 0.830 = 1 - (12/80) = 1 - 0.15 = M/M = 58/80;





In a similar manner the values listed in table 2 were obtained for this illustration. Several of the values listed intable 2 are shown as curves on the chart, figure 2. Other possible relations should be investigated.

M gros	ss Mgr/10	Total Deduc.	Mn	Hn	с <sub>м</sub>	CB	$c_{D}^{1/2}$
				175742	abb ge	22 (2) -	Bat CAL
200	20	24	176	143.264	0.7163	0.880	0.167
100	10	14	86	70.0	0.700	0.860	0.296
90	9	13	77	62.7	0.6966	0.856	0.308
80	8	12	68	55.35	0.692	0.850	0.333
70	7	11	59	48.0	0.6857	0.843	0.363
60	6	10	50	40.7	0.6783	0.837	0.400
50	5	9	41.	33.37	0.6667	0.820	0.444
40	4	8	32	26.05	0.651	0.800	0.500
30	3	7	23	18.72	0.624	0.767	0.573
25	2.5	6.5	18.5	15.06	0.6024	0.740	0,616
20	2.0	6.0	14.0	11.40	0.570	0.700	0.667
15	1.5	5.5	9.5	7.73	0.5153	0.633	0.729
10	1.0	5.0	5.0	4.07	0.407	0.500	0.800
5	0.5	4.5	0.5	0.407	0.0814	0.100	0.889

TABLE 2. - <u>Values calculated for graphical illustration</u> (fig. 2; C<sub>N</sub> = 0.814)

 $\underline{1}/C_{D} = (M_{d} = 4) \div \text{Total deduction}$ 

We hope that our graphical illustration emphasizes once more the fact that  $C_M = H/M_{model}$  is not a constant; its assumed constancy is based upon the untenable assumption that total deduction is always proportional to the total load, which cannot possibly be true when part of that deduction  $M_d$  may be a fixed quantity.

As exaggerated examples of mis-applications of such piston coefficients, take value  $C_{M} = 0.700$  from figure 2 which is good only for its  $M_{gr} = 100$  gms and compute therefrom  $H'_{n}$  for  $M_{gr} = 50$  and 200 gms. In a similar manner the values listed in table 2 were obtained for this illustration. Several of the values listed intable 2 are shown as curves on the thert, figure 2. Other possible relations

I C = (M = 4) + Total deduction

We hope that our graphical illustration emphasizes once more the fact that  $C_M = \frac{H}{n} \frac{M}{n}$  is not a constant; its assumed constancy is based upon the unterable assumption that total deduction is always proportional to the rotal load, which cannot possibly be true when

As exaggerated examples of mis-applications of such pieton coefficients, take value  $C_{\rm M}=0.700$  from figure 2 which is good only for its M = 100 gas and compute therefrom H for M = 50 and 200 gas.

We obtain  $H'_n = 35$  and 140 mm respectively compared to the true values 33.37 and 143.264 mm from  $H_n = C_N \cdot M_n$ . The  $H'_n$  values are too large by about 5% in the first case, and too small by about 2 1/4% in the second case. This comparison provides a clue to the correction of values for changes of pressure thus miscalculated and suggests skepticism regarding the absolute accuracy of some pressures listed in the literature.

The true relation  $(\triangle P)_{true}/(\triangle P)_{app} = (C_B)_{at(P)_{true}}/(C_B)_{at calibr}$ is substantiated by the exaggerated examples just given, for

143.264/140 = 1.0233 = 0.88/0.86 and

33.37/35 = 0.9534 = 0.82/0.86

## COMPOSITE BUOYANCY (DEDUCTION) ESTIMATE

The data are from Mullins' observations on the determination of the vapor pressure of carbon dioxide at 0 C, as recently observed by  $\frac{10}{us}$ .

10/ Mullins, P. V., and Earle S. Burnett. The Calibration of a Piston Gage by Comparison with the Vapor Pressure of Liquid Carbon Dioxide at the Ice Point. Helium Research Center Internal Report No. 45, January 1964, 16 pp.

As Mullins' piston loads were of several materials, a part in oil, but all buoyed up by air, the several effects are listed in the following table 3 in grams mass. We obtain  $H'_{0} = 15$  and 140 am respectively compared to the tran values 33.37 and 143.264 mm form  $H'_{0} = C_{H}^{-1} H'_{0}$ . The  $H'_{0}$  values are too large by about 5% in the first case, and too small by about 3 1/4% in the second case. This comparison provides a clue to the carraction of values for changes of pressure thus miscalculated and suggests skepticism regarding the shaplule accuracy of some pressures listed in the literature.

The true relation  $(2^{\circ})_{\text{true}} (2^{\circ})_{\text{spp}} = (0^{\circ})_{\text{true}} (0^{\circ})_{$ 

TRUTTEL (NOLLOGEN) ANVIOUR STISDEROO

The data are from Mullins' observations on the determination of the vapor pressure of carbon dioxide at 0 C, as repently observed by  $u = \frac{10}{2}$ 

10/ Multime, F. V., and Marie S. Burnett The Calibration of a Staton Gage by Comparison with the Varor Pressure of Linuid Sarbon Dioxide at the Lee Point. Helium Sassarch Center Incornel Report No. 45, January 1964, 16 pp.

As Mullins' pizzon loads ware of several materials, a part in oil, but all buoyed up by air, the several effects are listed in the following table 3 in grang mass.

-85

Gross Mass Loa	ads, grams x	Buoyancy	Factors =	Deductions	Net Loads, M		
				1000 C			
Cast iron	24,042.14 x	0.00105/7	.08 =	3.57	24,038.57		
Steel	1,109.85 x	0.00105/7	7.84 =	0.15	1,109.70		
Brass	352.20 x	0.00105/8	3.63 =	0.04	352.16		
Total load	25,504.19 x	(0.00105/7	.48) =	3.76	25,500.43		
Constant deduc	ction due to	piston ir	av				
oil environmer	nt			5.42			
Total deductio	on			9.18	25,495.01		
					1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		
Composite total deduction factor = $9.18/25,504.2 = 0.000360$							

### TABLE 3. - Composite buoyancy and deduction estimate

Hence  $M_n/M_{gr} = 0.999640 = C_B = 25,495.0/25,504.2$  compared to 0.999852 that Mullins used.

Mullins' net load on the piston base for one determination was 25,495.0 grams mass. Multiplying this number by our piston constant  $C_N \cdot C_g = 0.999140 \ge 0.998712 = 0.997853$  yields 25,440.3; the temperature of the piston in this case was 7° above the 20° C normal, enlarging its effective area by 1/6200 which is equivalent to subtracting 4.2 mm, yielding 25,436.1. Adding further 1.20 mm for diaphragm correction and 1.67 mm for head of  $CO_2$  vapor yields 25,439.0; and lastly adding the height of the barometric column of mercury, 689.64, corrected for air buoyancy, (by -0.05) and gravity, (by -0.89) to 688.7 mm yields a final figure of 26,127.7<sup>11</sup>/as the

11/ This equals 34.378 + atmospheres to compare with 34.379 given in ICT, v. VIII, p. 235, by an eight-constant equation.

	Breas Breas Total load 25,504.19 x 0.00105/8.93.
	Constant deduction due to platon in -
9,18	

Composite tetal deduction (actor = 9.18/25.504.2 = 0.000360 Hence  $M_0/R_{gr} = 0.999640 = 0_0 = 25.495.0/25.504.2 compared to$ 0.000361 the mailtine mank

Multics' net load on the piston have for one determinetion was 21,405.0 grame wass. Multiplying this number by our piston constant  $C_N \cdot C_g = 0.923140 \times 0.923512 = 0.397853 yields 25,440.3; the term$ parature of the piston in this case was 7° above the 20° C normal,constructing its attactive area by 1/5200 which is evolvatent to subtracting 4.2 ms, yielding 25,435.1. Adding further 1.20 mm for $displayers correction and 1.67 mm for hand of <math>CO_2$  vapor yields 25,43% 0; and larriy adding the height of the baromeette column of matcury, 638.04, corrected for all baroyanoy, (by -0.05) and gravity, (by -0.88) to 358.7 mm yields a final figure of 26,127.11 as the

11/ This equals 34.378 \* sequepheres to compare with 34.379 given in ICT, c. VIII; p. 235, by an eight-constant equation.

height of the balancing column in international mm of mercury that defines the vapor pressure of carbon dioxide at 0 C.

The corrected value published by Roebuck &  $Cram^{12}$  for their

<u>12</u>/ Roebuck, J. R., and W. Cram. Multiple Column Mercury Manometer for Presures to 200 Atmospheres. Rev. Sci. Inst. v. 8, No. 6, pp. 215-220 (1937)

average of twelve determinations is 26,129.8 mm which is 1/13,400 more than this single one of Mullins.

The deduction  $M_d = 5.42$  gms due to the piston environment in oil was overlooked by Mullins, Burnett, and others in 1937; and the piston constant then used was

 $C_{p} = C'_{N} \cdot C'_{g} \cdot C_{B} = (0.999142)(0.998758)(0.999852)_{c.i.} = 0.997752$ According to our analysis herein presented, it should have been  $C_{p} = C_{N} \cdot C_{g} \cdot C_{B} = (0.999140)(0.9998712)(0.999640)_{comp} = 0.997492$ The difference,  $C_{p} - C'_{p} = -0.000260$ , which multiplied by our earlier (1937) average value, 26,136.4 mm yields a deduction of 6.8 mm reducing it to 26,129.6 mm. This value almost exactly agrees (to -1/130,000) with our revised value of 26,129.8 mm from the direct multiple mercury manometer determination announced  $\frac{12}{}$  by Roebuck and Cram in 1937. Their average of 12 determinations previous to gravity adjustment, 26,136.0 mm multiplied by 980.365/980.665 (ICT values for Madison, Wisconsin) yields 26,128.0 mm; compressibility of mercury adjustment adds 1.8 mm and brings it to 26,129.8 mm. height at the balancing column in international an of mercury that defines the vapar preasure of carbon dioxide at 0 C.

12/ Manhuch, J. N., and M. Gran, Multiple Column Mercury Manometer for Presentes to 200 Atmospheres Rev. Sci. Inst. v. B. Sci. 6. on. 215-220 (1937)

average of twelve determinations is 25,129.8 mm which is 1/13,400

The associan M<sub>d</sub> = 5.41 ges due to the posing environment in oil was overlooked by italiins; Burnett, and others in 1933; and the pisten constant then used was

 $C_p = C_n^2 \cdot C_n^2 \cdot C_n^2 \cdot C_n^2 \cdot (0.999192)(0.9993238)(0.999323)_{0.1}^2 - 0.997732$ Ancoraing to juur unalysis barava presented. it should have been  $C_p = E_n^2 \cdot C_n^2 \cdot C_n^2 - (0.999196)(0.9993212)(0.999660)_{comp} - 0.997492$ The difference,  $C_p - C_p^2 - 0.000260$ , which multiplied by our setter (1937) answare value, 35,136.4 mm vision a desertion of 5.8 mm viducing it to 25,129.6 mm. This waite elevation of 5.8 mm vi--1/130,000) with our revised value of 26,129.8 on from the direct multiple versury sememer between erastly agrees (to and Gran in 1937. Their eversely of 12 determinations provide to gravity adjustments. 26,135.0 mm values of 12 determinations provide to values for Multiple Values of 13 determinations provide to of escentry adjustments adde 1.8 mm and bridge it to 26,129.6 mm.

3.0

### FINAL REMARKS ON PISTON CONSTANT

We say that if a "piston constant" is to be assigned to a given rotating piston assembly, it must have the value  $C_N = H_n/M_{net}$  to be applicable to all conditions defined by that ratio and should not be otherwise used nor should any other ratio be called a "piston constant." This requirement involves the determination of  $M_{net} = M_{met} \times \frac{1 - \text{total deduction}}{\text{total load}}$  for each and every load, which is no more of a chore than are the determinations of  $H_{net}$ . Involved in the total deduction is that part of it,  $M_d$ , due to the piston in oil environment which must not be overlooked.

The piston constant  $C_N$  as above defined has been shown to be equivalent to  $1/A_s \rho_{sm}$ ;  $A_s$  is the effective area of the piston at some standard temperature  $t_s$ , and  $\rho_{sm}$  is the standard density of mercury in grams mass per cu cm divided by 10 for  $H_n$  expressed in mm; therefore any method of calibration which determines the effective piston area at a standard reference temperature combined with standard mercury density serves to evaluate that piston constant.

# Discussion of this and a later calibration

In all, 74 high columns and 74 low columns of mercury were measured; table 4 contains a non-chronological arrangement of the data.

The first 39 pairs of these observations were made in the laboratory building under somewhat adverse circumstances. That location was not far from a group of gas compressors, the ground vibration from which was quite disturbing, particularly when their speeds happened

### FINAL MERAND S ON FIELDS CONSTANT

We say that it a "states constant" is to be assigned to a given rotating piston essenbly. It was have the value  $C_0 = \frac{1}{2} \int_{0}^{\infty} \int_{0}^{\infty} dt$  for be similarly to all conditions defined by that ratio and should not constant." This requirement involves the deter varies of  $M_{\rm est}$  "  $\frac{1-creel deduction}{cotal loud}$  for each and every lood, which is no access a choice that are the determinations of  $M_{\rm est}$ . The total loud the total deduction is chair part of the determinations of  $M_{\rm est}$  involved in the total deduction is chair part of the determinations of  $M_{\rm est}$ . Involved in the total deduction is chair part of the  $M_{\rm est}$  involved in the total deduction is chair part of the  $M_{\rm est}$  and the ratio of the determinations of  $M_{\rm est}$ .

The piston constant C<sub>1</sub> as above defined has been shown to has aquivalent to 1/A<sub>2</sub> and A<sub>2</sub> is the effective area of the piston at some standard temperature t<sub>1</sub> and 2<sub>20</sub> is the areadard density of mercury in grave mass per core divided of 10 for H<sub>2</sub> expressed in mit there fore any method of calibration vatch dersemines the effective piston area at a standard reference temperature combined with areadard

### Discounting of the and a later calibration

In all, 75 high column and 76 how column of mercury were means ored; table 4 contains a mon-chromological arrangement of the data. The first 30 pairs of these observations were rade in the laboracosy building under someoner edverse circumstances. That location was not far from a group of gas compressors, the ground vibration from

nearly to coincide to produce a "beat" effect. We would get about set to read a mercury column level through the cathetometer telescope when the mercury meniscus would start quivering, thus delaying the observation and causing some uncertainty of its value.

The last 35 pairs of observations were made after removing our apparatus to a small building remoter from the powerhouse. The vibration interference was not eliminated but its disturbance was ameliorated. The observations of this group were more consistent than were those of the former group, but both indicated essentially the same average value, to about 1/100,000.

When the duly corrected values of the heights of the 74 short columns are placed opposite those of the 74 long columns as in table 4, both in descending order of magnitude, the opposing differences are also generally, but not everywhere consecutively, in descending order of magnitude. The extreme individual differences vary from the average value, 849.232 mm from  $\pm 0.088$  to -0.079, or from  $\pm 1/9640$  to -1/10750; that is roughly by  $\pm 1/10,000$ . The arithmetic average of all differences from the mean value regardless of their signs, is  $\pm 0.035$  or 1/24,300. By least square formula,

$$R = 2 \sqrt{\frac{\sum (v^2)}{n(n-1)}} = 2 \sqrt{\frac{0.142198}{74.73}} = \pm 0.010_{26}$$

in which each v is one of the 74 differences. This indicates that R, the probable uncertainty of the mean is not more than  $\pm 0.0103$  mm Hg on a twenty to one basis; that the most probable value of the mean difference is about 849.232 (1  $\pm$  1/82,700); the equation says that meanly to coinclute to produce a "beat" allect. We would get about set to read a mercury solures level through the rethetometer talkacope when the mercury montacus would start quivering, inte dataving the absorption and causing some uncartainty of the value.

the last 35 pairs of chatervations were made after read ing our apparates to a small huridide resuler from the powerhouse the vibrorion interference was not eliminated out its distarchance was ameliarated. The observations of this group wave core constance was than were three of the former group, but both indicated casedially the same storage value. to stout 1/100,000.

When the doly correcter values of the beights of the 24 short colucus are placed opposite those of the 74 long colucus as in rabie 4, both in descending order of megatrade, the opposing differences are also generally, but not everywhere concernitively, in descending order of megatrade. The extreme individual differences very from the average value, 843,132 am from +0,088 to -0.076, or from +1/9643 co -1/10756; one is coughly by ±1/10,000. The actednativ average of all differences from the mean value regardless of their stars (a 50,035 or 1/24,300, 39 least square formula.

$$R = 2 \int \frac{2(\sqrt{2})}{n(m-1)} = 2 \int \frac{\pi}{76.73} = \pm 0.01026$$

In which each with one of the 74 differences. This indicates that R, the probable uncertainty of the mean is not more than  $\pm 0.0103$  and Rg on a twenty to one basis; that the most probable value of the man difference is shown 649.232 (1 + 1/82.700); the equation save that

it is about 20 times as likely that the true value lies within this range as it does without it.

Because each individual determination of either the long or short column is independent of all others, it appears permissible to combine them in all possible ways and thus obtain  $(74)^2 = 5476$  virtually independent differences. This was done and an extensive analysis was made of those results utilizing least squares methods applied to various groupings of the two sets of data. The results obtained were interesting and informative but hardly more satisfying than those presented above as used in this report.

During the summer of 1938 when the helium plant was shut down for a brief period and disturbing vibration was not harrassing us, another short mercury column calibration of the same Keyes gage was made. This time the mercury column arrangements were mounted directly on the Geneva cathetometer base. Its telescope was removed and replaced by a viewing tube provided with narrow parallel slits for proper sighting on mercury levels, which were read directly on the cathetometer scale. Without reporting details, let it be said that individual 80 cm column differences were so mutually agreeable that only 17 were determined.

In the meantime, necessary data had been obtained concerning the original calibration against a nine-meter mercury column at MIT in which steel "weights" had been used. Some of the data were obtained through personal correspondence with Dr. F. G. Keyes who with A. G. Loomis and C. W. Seibel of the Bureau, participated in that 1924 event. it is about 20 tires as likely that the true value lies within this range as it does without it.

because each individual determination of effer the lang at short column is independent of all others, it appears perviselble to conbine them in all possible ways and thus obtain (14)<sup>2</sup> - 5476 virtually independent differencet. This was done and an exceptive analysis was made at phose results utilizing least squares periods applied to various groupings of the two sets of data. The results chained wave interesting and informative but bardly more estimativing them

brring the summer of 1938 when the Anlian plant was abut down for a brief period and disturbing vibration was not herefering us, another short mercury column calibration of the same layers rage was made. This time the mercury column arrangements were emuted directly on the Geneva cathetowarar base. Its telescope was removed and replaced by a viewing tube provided with mercew parallel allia for proper sighting on moreary layels, which were read directly on the cathetomater scale. Without reporting details, let it be said that individual 80 cm column differences were an mutually agreeable that only 17 were determined.

In the meantime, necessary data had been obtained conterning the original calibration against a nine-meter marcury column at MII in which steel "weights" had been usud. Some of the data were obtained through personal correspondence with Dr. F. C. Keyes who with A. C. Loomis and C. W. Seibel of the Sureau, participated in that 1924 event

"EK

Suffice it to say here that our 1932 calibration "piston constant" exceeded the 1924 value by about 1/20,000 and was less than our 1938 value by about the same amount, as estimated by the same methods.

From these comparisons at least two conclusions are obvious: a short column of mercury appreciably less than one (1) meter high can serve to calibrate a piston gage satisfactorily; and our Keyes gage did not appear to have changed in size significantly during fourteen years of use, although a possible very slight enlargement is suggested of the order of 1/10,000 of its effective area. Suffice it to any have that our 1932 calibration "platen constant" exceeded the 1924 value by about 1/20.000 and was less than our 1938 value by about the same amount, as estimated by the same methods.

iton these comparisons at least two conclusions are obvious: a short column of mercury appreciably less than one (i) mater high can rerve to calibrate a piston rago satisfactorily; and our Keres gage did not appear to have changed in size significantly during fourtaen years of use, although a possible very slight enlargement is suggested of the order of 1/10,000 of its offective area

	Me	rcury Colu	mns	Departu	$10^{6} v_{\rm D}^{2}$		
	Long	Short	Diff.	Diff.	Long	Short	
No.	943.+	94.+	849.+	849.232 V <sub>D</sub>	943.336 V <sub>T</sub>	94.104 V <sub>S</sub>	
1.	.474	.177	. 297	+.065	+ 138	+ 077	4225
2.	.473	.158	.315	.083	.137	.054	6889
3.	.471	.158	.313	.081	.135	. 0.54	6561
4.	.471	.155	.316	.084	.135	.051	7056
5.	.469	.149	. 320	.088	.133	.045	7744
6.	.466	.149	.317	.085	.130	.045	7225
7.	.455	.146	. 309	.076	.119	.042	5776
8.	.447	. 144	. 303	.071	. 111	.040	5041
9.	.438	. 144	.294	.062	. 102	.040	3844
10.	.436	.139	.297	.065	. 100	.035	4225
11.	.433	.139	. 294	.062	.097	.035	3844
12.	.427	.138	. 289	.057	.091	.034	3249
13.	.418	.138	. 280	.048	.082	.034	2304
14.	.404	. 135	.269	.035	.078	.031	1225
15.	.402	.132	.270	.038	.066	.028	1444
16.	.401	. 1.30	.271	039	.065	.026	1521
17.	.399	. 1.27	.272	.040	.063	.023	1600
18.	. 394	. 126	. 268	.036	.058	.022	1296
19.	.391	.126	.265	.033	.055	.022	1089
20.	. 391	. 126	.265	.033	.055	.022	1089
21.	. 388	. 125	.263	.031	.052	.021	961
22.	.311	. 125	. 454	.020	.041	.021	400
23.	. 366	. 120	· 44 1	.009	.030	. UZI	01
24.	. 305	. 1.24	. 24.1	.009	.029	.020	26
22.	. 302	. 124	. 200	.006	.020	.020	20
20.	. 339	. 122	. 6.21	.005	.025	.010	20
21.		.119	. 233	.001	.010	01/	1.
20.	352	. 110	. 2.34	002	.016	.014	4
29.	3/10	. 1.10	234	.002	013	008	25
31	3/18	·	237	.005	012	007	25
32	3/18	111	230	005	012	007	25
32.	347	111	236	004	011	007	16
34	345	110	235	003	.009	.006	9
35	344	109	235	.003	.008	.005	9
36	344	108	236	+ 004	.008	.004	16
37.	. 339	107	.232	-,010	+.003	.003	100
38.	.336	.106	230	-,002		.002	4
39.	. 328	.106	.222	-,010	<b>₽.008</b>	.002	100
40.	. 327	.105	.222	010	009	+.001	100
41.	. 326	.104	.222	.010	.010		100

TABLE 4 - Assembly of all Experimental Values and Their Indications 1/

		SEC .WAR		, +.14	+.242	
					B10	
					·	
01 1						

TABLE 4 - Astrophy of all Proprimental Values and These

QL.
	Mercury Columns			Depart	Departures from Averages		
	Long	Short	Diff.	Diff.	Long	Short	
No.	943.+	94.+	849.+	849.232	943.336	94.104	
42.	. 322	. 102	.220	012	- 014	002	144
43.	.317	. 102	.215	.017	.019	.002	289
44.	.317	.097	.220	.012	.019	.007	144
45.	. 315	.094	.221	.011	.021	.010	121
46.	. 315	.090	. 225	.012	.021	.014	144
47.	.313	.090	.223	.014	.023	.0.14	196
48.	. 309	.088	.221	.011	.027	.016	121
49	.308	.088	.220	.012	.028	.016	144
50.	. 306	.088	.218	.014	.030	.016	196
51.	. 298	.087	.211	. 021	.038	.017	441
52.	. 298	.086	.212	.020	.038	.018	400
53.	. 292	.085	.207	.025	.044	.019	625
54.	.291	.084	. 207	.025	.045	.020	625
55.	. 280	.084	. 196	.036	.056	.020	1296
56.	.279	.081	. 198	.034	.057	.023	1156
57.	.274	.081	. 193	.039	.062	.023	1521
58.	.274	.079	. 195	.037	.062	.025	1369
59.	.274	.078	. 1.96	.036	.062	.026	1296
60 .	.273	.074	. 199	.033	.063	.030	1089
61.	.269	.073	.196	.036	.067	.031	1296
62.	.268	.071	. 197	.035	.068	.033	1225
63.	.265	.070	.195	.037	.071	.034	1369
64.	. 264	.069	. 195	.037	.072	.035	1369
65.	.250	.068	. 182	. 050	.086	.036	2500
66.	.238	.060	. 1/8	.054	.098	.044	2916
67.	.222	.059	. 16.3	.069	.114	.045	4/61
68.	.21/	.059	. 158	.014	. 119	.045	54/6
69.	. 211	.050	. 161	.071	. 125	.054	5041
/0.	. 210	.050	. 160	.012	. 126	.054	5184
11.	.201	.042	. 109	.073	. 135	.062	5329
12.	. 195	.042	. 103	.079	. 1.4.1.	.062	6249
13.	. 192	.038	. 1.74	.078	. 144	.000	0084
/4.	. 186	.015	· 1 / 1	.061	. 190	.089	3/21
Tota	ls			+ 1.295	+ 2.275	+ .976	
				- 1.289	- 2.272	-1.006	
Tota	<b>1s</b> 24.855	7.662	17.206	2.584	4.547	1.982	142198

TABLE 4 - Assembly of all Experimental Values and Their Indications (Con.)  $\frac{1}{}$ 

				,#. AP	, +.CA		
	* 87.F.M						

TABLE 4 - Ansertaly of 11 Fagor mental Value and Thair

32

TABLE 4 - Assembly of all Experimental Values and Their Indications (Con.)  $\frac{1}{}$ 

							( )
	Mercury Columns		Departures from Averages				$10^{6} V_{D}^{2}$
	Long	Short	Diff.	Diff.	Long	Short	
No.	943.+	94.+	849.+	849.232	943.336	94.104	
Grand Ave.	943.336 -	94.104 =	849.232 849.232	Average Ax 0.0350	cithmetic D: 0.0614	ifference 0.0267	S
Averag	<u>e column di</u>	fference =	$\frac{0.044}{510.220} =$	0.000085 =	1/11,800	., , , , , , , , , , , , , , , , , , ,	

1/ Data of 1932, 85 cm mercury column calibration of a Keyes-type piston gage. "Corrected" long and short columns in mm. of Hg. as recorded for determination of their differences due to the addition to or removal from 850 grams mass load in brass on the piston tare mass, arranged in decreasing order of magnitude, and other relations. The numbers are mm of Hg.

By least square calculations the probability

518.220

$$R = 2 \sqrt{\frac{\Sigma V_D^2}{n(n-1)}} = 2 \sqrt{\frac{0.142198}{74 \times 73}} = \pm 0.010_{26}$$

indicates that the sought for magnitude is

Average column - length

 $849.232 \pm 0.010_{26}$  or  $849.232 (1 \pm 1/82700)$ 

that is the chances are about 20 to 1 that the true value lies between those limits as outside of them.

	Departures from Anoungus 10						
						, gool	
					. 19:20		

.

$$x = x \int \frac{z v_0^2}{n(n-1)} = z \int \frac{0.142198}{46 \times 73} = \frac{2}{2} 0.010_{20}$$

.

## APPENDIX

The insignificance of residual differences of pressure,  $P_2 - P_1$ , when expressed as due to differences of piston loads, and as differences of equivalent balancing columns of mercury, air and oil (pp. 10-13 mms) is shown.  $\frac{1}{2}$  (Refer to figure 1).

<u>1</u>/ For the short column of mercury the density of air confined between c and e of Figure 1 is its normal density  $\rho_a$  (at 670 mm H pressure and at room temperature at Amarillo) multiplied by the ratio (670 + 94.1)/670 = 1.14; for the long column  $\rho_a$  is multiplied by the ratio (670 + 943.3)/670 = 2.41; and 2.41/1.14 = 2.11.

In this demonstration  $M_T$  represents the tare load including the part of the piston that is in oil - about 100 grams,  $M_L$  the change (in brass) of the load on the piston - 850 grams.

For pressures expressed in equivalent columns of oil, air and mercury, the following relations obtain:

$$P_{1} = \left[h_{bc}\rho_{o} - 1.14h_{ce}\rho_{a} + h_{ek}\rho_{m} + h_{bar}\rho_{m} - h_{bk}\rho_{a}\right]_{1}^{g} + capillarity_{1}$$

$$P_{2} = \left[h_{bc}\rho_{o} - 2.41h_{cd}\rho_{a} + h_{dm}\rho_{m} + h_{bar}\rho_{m} - h_{bm}\rho_{a}\right]_{2}g + capillarity_{2}$$

$$P_{2} - P_{1} = \left[h_{bc}(\rho_{o_{2}}-\rho_{o_{1}}) - (2.41h_{cd}-1.14h_{ce})\rho_{a} - (h_{bm}\rho_{a_{2}}-h_{bk}\rho_{a_{1}}) + (h_{dm}\rho_{m_{2}}-h_{ek}\rho_{m_{1}}) + (h_{bar}\rho_{m_{2}} - h_{bar}\rho_{m_{1}})\right]g + \left[(cap_{2} - cap_{1}) = 0\right].$$

The insignificance of residual differences of pressure,  $P_2 - P_1$ , when expressed as due to differences of platon leads, and as differences of squivalent balancing columns of merciry, air and oil (pp. 17-13 mas) is shown. M (Refer to figure 1).

If For the short column of servery the density of als confined benews to and e of Figure 1 is its roughly behalty of at 670 cm  $E_{\rm B}$  presents and at room temperature at Amarillo) multiplied by the ratio (670 + 94.1)/670 = 1.14, for the long column of 10 multiplied by the ratio (670 + 943.3)/670 = 2.41, and

In this demonstration My represents the tare load including the part of the piston that is in all - shout 100 grams, My the change (in brass) of the load on the piston - 550 grams

For pressures expressed in equivalent columns of oil, sir and . mercury, the following relations obtains

$$P_{1} = \left[h_{0}e^{\phi_{0}} - 1.14h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}}\right]_{2} + caottiarter;$$

$$P_{2} = \left[h_{0}e^{\phi_{0}} - 2.41h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}}\right]_{2} + caottiarter;$$

$$P_{3} = \left[h_{0}e^{\phi_{0}} - 2.41h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}}\right]_{2} + caottiarter;$$

$$P_{3} = P_{1} - \left[h_{0}e^{\phi_{0}} - 2.41h_{0}e^{\phi_{0}} + h_{0}e^{\phi_{0}} + h_{0}e^{\phi} + h$$

In the equation for  $P_2 - P_1$ ,  $(h_{ce} - h_{cd}) \rho_a \equiv h_d \rho_a)$  is the air buoyancy change accompanying the change in total length of the shorter mercury column. The third parenthesis of this equation is the air buoyancy change accompanying the change in total length of the longer mercury column. The difference of these changes is the net change of air buoyancy Hp<sub>a</sub> accompanying the net change of mercury column Hp<sub>m</sub> indicated in the fourth ().

Hence, for the second and third () we may write

$$(0.14 h_{ce} - 1.41 h_{cd}) \rho_a - (h_{bm} \rho_{a2} - h_{bk} \rho_{a_1} - h_{de} \rho_a)$$

whence

$$P_{2} - P_{1} = \left[h_{bc}(\rho_{0} - \rho_{0}) + (0.14 h_{ce} - 1.41 h_{cd})\rho_{a} + H(\rho_{m} - \rho_{a})\right]$$

+ 
$$(h_{bar} p_{m_2} - h_{bar} p_{m_1})$$
 g

For pressures expressed in net loads on the piston base, the expressions below obtain:

$$P_{1} = M_{T} \frac{g}{A_{s}} \left[ \frac{1 - \rho_{a}/\rho_{M}}{1 + \alpha_{p} \Delta t} \right]_{1}^{+} (h_{bar}^{\rho} \rho_{m}^{g})_{1} + (capillarity, etc.)_{1}$$

$$P_{2} = M_{T} \frac{g}{A_{s}} \left[ \frac{1 - \rho_{a}/\rho_{M}}{1 + \alpha_{p} \Delta t} \right]_{2}^{+} (h_{bar}^{\rho} \rho_{M}^{g})_{2}^{+} (capillarity, etc.)_{2}$$

$$+ M_{L} \frac{g}{A_{s}} \left[ \frac{1 - \rho_{a}/\rho_{M}}{1 + \alpha_{p} \Delta t} \right]_{2}^{-}$$

In the equation for  $P_2 = P_1$ ,  $(h_{co} - h_{cd})P_a = h_{c}P_a$  is the air buoyancy change accompanying the change in total length of the aborter mercury column. The third paranthesis of this squation is the str buoyancy change accompanying the change in total length of the longer mercury column. The difference of these changes is the net change of sit buoyancy  $R_{c}$  accompanying the out change of mercury column  $R_{c}$  is the net change of sit buoyancy  $R_{c}$  accompanying the out change of mercury column  $R_{c}$  is buoyancy  $R_{c}$  accompanying the out change of mercury column  $R_{c}$  is buoyance  $R_{c}$  accompanying the out change of mercury column  $R_{c}$  is

Hence, for the second and coird () we may write

on shu

$$P_2 = P_1 = \left[ \frac{n}{bc} \left( e_{\alpha_2} - e_{\alpha_1} \right) + \left( 0.14 \ b_{co} - 1.41 \ b_{cd} \right) e_{\alpha} + H(e_{\alpha} - e_{\alpha}) \right]$$

For pressures expressed in met lauds on the piston base, the

$$I = M_{T} \frac{c}{\Lambda_{B}} \left[ \frac{1 - \rho_{B} / \rho_{M}}{1 + \sigma_{B} \Delta t} \right] + (b_{Bar} \rho_{B} s)_{1} + (capillarticy, etc.)_{1}$$

$$2 = M_{T} \frac{c}{\Lambda_{B}} \left[ \frac{1 - \rho_{A} / \rho_{M}}{1 + \sigma_{B} \Delta t} \right] + (h_{Bar} \rho_{M} s)_{2} + (capillarticy, etc.)_{2}$$

$$+ M_{T} \frac{c}{\Lambda_{B}} \left[ \frac{1 - \rho_{A} / \rho_{M}}{1 + \sigma_{B} \Delta t} \right] + (h_{Bar} \rho_{M} s)_{2} + (capillarticy, etc.)_{2}$$

$$P_{2} - P_{1} = M_{T}g/A_{s} \left[ (1 - \rho_{a}/\rho_{M} - \alpha_{p}\Delta t)_{2} - (1 - \rho_{a}/\rho_{M} - \alpha_{p}\Delta t)_{1} \right] + M_{L} \frac{g}{A_{s}} \left[ \frac{1 - \rho_{a}/\rho_{M}}{1 + \alpha_{p}\Delta t} \right]_{2} + \left[ (h_{bar}\rho_{m})_{2} - (h_{bar}\rho_{m})_{1} \right] g + \left[ cap_{2} - cap_{1} = 0 \right]$$

in which  $\Delta t = t - 20^{\circ}$ ;  $t_1$  and  $t_2$  refer both to piston and oil within the cylinder:  $\alpha_p$  is the areal coefficient of expansion of steel, 0.000023. If these two final expressions for  $P_2 - P_1$  are equated, the barometric terms and factor g cancel out exactly, leaving

$${}^{h}_{bc} (\rho_{o_{2}} - \rho_{o_{1}}) + (0.14h_{ce} - 1.41h_{cd}) \rho_{a} + H\rho_{m} (1 - \rho_{a}/\rho_{m})$$

$$= \frac{M_{T}}{A_{s}} \left[ \frac{\rho_{a_{1}} - \rho_{a_{2}}}{\rho_{M}} - \alpha_{p}(t_{2} - t_{1}) \right] + \frac{M_{L}}{A_{s}} \left[ \frac{1 - \rho_{a}/\rho_{M}}{1 + \alpha_{p}\Delta t} \right]_{2}$$

It is to be noted here that  $h_{bc}$  and  $h_{cd}$ , both small in any case, can each be made equal to zero so that values in the involved products are negligible. Because of the inability to separate  $(\rho_a, \rho_m)_2$ from  $(\rho_a, \rho_m)_1$  in the H term, average values must be used; however the product  $(H\rho_m)_{av}$  tends to stay constant. The unlikely <u>maximum</u> <u>variation</u> of the value of the  $M_T$  term on the right side of the equation is of the order of  $M_L/40,000$ . Hence our previous conclusion that  $\Delta H_n$  is proportional to  $\Delta M_n$  expressed on page 13 is justified.

$$\frac{e_{2} \cdot e_{1} - u_{1}e^{A}}{e} \left[ (1 - e_{0})u_{1} - u_{0}e^{A} + (1 - e_{0})e^{A} + (1 - e_{0})u_{1} \right] \\ + u_{1}e^{A} \left[ \frac{1 - e_{0}}{e^{A}} \frac{h_{1}}{h} \right] + \left[ (u_{2}e^{A})e^{A} + (u_{2}e^{A}) + (u_{2}e^{A})u_{1} \right] \\ + \left[ eae_{2} \cdot eae_{1} - e_{0} \right]$$

in which  $\Delta t = t - 20^{\circ}$ ;  $t_1$  and  $t_2$  refer both to platon and oil within the cylinder:  $J_p$  is the areal coefficient of expansion of steel, 0.000023. If there two final expressions for  $r_2 - r_1$  are equated, the barometric terms and factor g cancel out exactly, lawing

$$\frac{M_{\rm pc}}{M_{\rm p}} \left[ \frac{\rho_{a_1} - \rho_{a_2}}{\rho_{a_1}} + \frac{(0.14h_{\rm cc} - 1.41h_{\rm cd})_{a_1} + H_{\rm b}}{\rho_{a_1}} \frac{(1 - \rho_{a_1}/\rho_{a_1})_{a_1}}{\rho_{a_1}} \right] + \frac{M_{\rm cc}}{M_{\rm s}} \left[ \frac{1 - \rho_{a_1}/\rho_{a_1}}{\rho_{a_1}} \right] + \frac{M_{\rm cc}}{M_{\rm s}} \left[ \frac{1 - \rho_{a_1}/\rho_{a_1}}{\rho_{a_1}} \right] + \frac{M_{\rm cc}}{M_{\rm s}} \left[ \frac{1 - \rho_{a_1}/\rho_{a_1}}{\rho_{a_1}} \right] + \frac{M_{\rm cc}}{\rho_{a_1}} \left[ \frac{1 - \rho_{a_1}/\rho_{a_1}}{\rho_{a_1}}$$

It is to be noted here that  $h_{pr}$  and  $h_{ed}$ , both small in any case, can each be made equal to zero so that values in the involved products are negligible. Because of the inshillty to expansive  $(D_{a}, D_{a})_{2}$ irow  $(D_{a}, D_{a})_{1}$  in the H term, average values must be used, however the product  $(H_{0})_{av}$  tends to stay constant. The unlikely maximum variation is of the value of the  $H_{1}$  term on the right aide of the aquation is of the order of  $M_{1}/40,000$ . Hence our previous con-



