WORKING DRAWINGS

HOW TO MAKE AND USE THEM

BY

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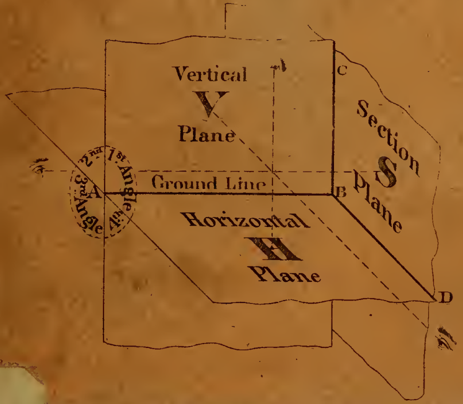
THE PLANES



PHILADELPHIA.

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1881.

WORKING DRAWINGS

AND

HOW TO MAKE AND USE THEM.

DESIGNED FOR

INDUSTRIAL, TECHNICAL, NORMAL, AND THE HIGHER GRADE GRAMMAR SCHOOL; ACADEMIES AND NIGHT SCHOOLS; AND ARTISANS DESIRING A KNOWLEDGE OF THE PRINCIPLES OF PATTERN AND TEMPLATE MAKING

BY LEWIS M. HAUPT,

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PREFACE.

THE large number of pupils of the public schools who are removed, by necessity, from their studies at the end of the grammar school course, and who are sent into mills, factories and shops as apprentices, seldom become more than a part of the machine which they are required to attend and operate, from lack of instruction in the one element which forms the basis of all constructive development, namely, the *science* of drawing.

It is true that, of late years, the *art* of drawing has been taught in the schools; that is, the manner of handling and using the necessary instruments to enable the student to copy from the flat or from models; but all of this may be done mechanically, and is almost entirely destitute of intellectual culture.

It is believed that there is but little in the present course of instruction that tends to develop the imagination to such an extent as to enable a pupil to form a mental conception of an object from a mere verbal description, and yet every new invention is but the fruit of such a process. It must first be clearly conceived in the mind, thence transferred to paper in the language of the artisan, and finally reproduced in substance out of appropriate materials. The inventor cannot convey, nor the workman interpret his idea, unless both are familiar with the conventional language which must be employed to represent objects, not as they appear to, but as they do actually, exist, according to their true dimensions, relations and proportions.

The present system of teaching drawing is useful only in so far as it cultivates the faculties of observing the form and position of objects, and the manual skill of representing them, requiring the exercise of the judgment and memory; but is comparatively worthless for all practical purposes in the trades, except, perhaps, for the designer of free-hand patterns for tapestry, carvings and similar applications. The mere copying of pictures or models, or the construction of perspectives by rules-of-thumb, whose principles are not understood, is no more able to produce a draughtsman or artisan than would the copying of any number of sheets of music be able to make a musician, or the reproduction of hieroglyphics, a linguist. Any one who can handle a pencil may soon be taught to make a copy, although the characters so duplicated may be unintelligible.

To represent any object so that it may be constructed from the drawing, requires that it should be dissected and its several parts so projected on the plane of the paper that the artisan shall know just where to find them and what they represent; in short, a knowledge of *projections*, *scales*, and the *conventions* used in working drawings must be understood.

This is the missing link between theory and practice, which it is the effort of the author to introduce. Without it all attempts to coördinate the industrial school features with our common school system must fail. The principles employed are as simple and readily understood as those of elementary Geometry, upon which they are based; and it is believed there is nothing in this Elementary Treatise on *Working Drawings, and How to Make and Use Them* that is beyond the comprehension of the average intellectual capacity found in the higher grades of our grammar schools.

Whoever can make from his own working drawings a model of an object, should also, with appropriate tools and materials, be able to construct the object itself, or in other words, become a practical artisan so far as the general principles of framing and construction are concerned.

In other countries numerous texts upon the subject are in use, but in America very few, and those only in our higher institutions of learning; hence, one reason why many of our draughtsmen, designers and most successful artisans are foreigners.

The plan of this work is to state the general principles involved in any theorem or problem, giving in the same connection its analysis, construction, and one or more of its numerous applications when practicable, thus fixing the principles much more efficiently than can be done by the ordinary methods of proceeding

By this means it is hoped that the mental training resulting from a study of this subject, will greatly assist in qualifying pupils for the more useful occupations in life, and reduce the number of graduates who are incompetent to perform any other than clerical service, and who spend a large portion of their lives in seeking office.

The application which may be made of such information is very extended. As a disciplinary study it is one of the first order, developing the conceptive faculties and enabling one to grasp an idea readily. It has its application in nearly all manufactured articles and in all constructions and designs, in wood, iron, stone or other materials. It is used constantly by the engineer, architect, builder, pattern-maker, iron or sheet metal-worker, stair-builder, stone-cutter, designer and many others. It is the basis of all perspective drawings, which are generally made by rule and without reason, and is essential to a correct interpretation of all suggestions relating to constructions of any kind. It is used to explain and reinforce verbal language, and should be so used whenever possible.

One of its most important applications must not be overlooked. To the statistician as well as the merchant it is valuable as furnishing at a glance information which, if expressed in a mass of figures, would be unintelligible. It cannot be surpassed as a method of exhibiting rapidly the distribution of population, of products, of poverty or wealth, of crime or morality, of vital, or in fact any statistics which may be expressed numerically. To the physicist it is also particularly useful in investigations into the properties of molecular or mass physics, and enables him to discover almost immediately many of the laws governing the motions of matter.

Fluctuations of prices, in the market values of daily commodities, may be more intelligently expressed by this means than any other, and can be compared at a glance. In short, the number of intelligent and eminently practical applications that may be made of projections is almost limitless.

Its introduction into the grammar, normal and other schools would supplant a certain amount of mnemonical by rational and manual development, and would thus be a relief to a system already overtaxed with memorizing.

In this brief treatise the consideration of the subject is limited to straight lines and planes; but enough is given to enable the teacher to measure the capacity of the student, and to determine whether he has the elements necessary to continue the subject with profit, in its application to curved lines, curved and warped surfaces, and solids.

Those students who may be incapable of developing the imagination to such an extent as readily to understand this part of the subject, should be allowed to stop here; but all who desire to become successful engineers or artisans must pursue the course still further, and take up the various subjects of intersections and developments of surfaces and solids, as applied to pattern-making, machine-drawing, stone-cutting, and many other of the useful arts. These the author hopes to provide in subsequent numbers. This number is only intended to serve as a test or gauge by which the teacher may determine which of his pupils may reasonably expect success in the many occupations in which these principles are employed, and also to serve as a basis for their further application to the trades and professions.

The necessity for such instruction is becoming every year more urgent, in consequence of the abolition of the system of apprenticeships. The author knows of no other means that will so rapidly and cheaply meet this difficulty than a thorough ground-work in the principles of projections as applied to making and reading working drawings.

L. M. H.

SUGGESTIONS TO TEACHERS.

As this subject may be new to many teachers, the author has given, in addition to the ordinary projections, as in the figures lettered (*a*), a perspective view of the objects in space, to assist the imagination. The paper planes will also be found a great help. The drawings of the problems should be made upon them unfolded; they may then be turned up so as to form a solid angle, and the position of the object (point, line or plane,) be indicated by holding a pencil, pointer, or card, in the proper position.

Frequent practice of this kind will render the student very expert at reading the drawings.

The teacher will find a hinged blackboard of two leaves very useful, and a card or thin board with a hole in it, through which a needle or wire may be placed, will enable him to illustrate most of the problems by a model.

Students should also be required to prepare drawings of the problems given in the text, assuming the parts in different positions, and to apply the principles to other objects than those suggested.

It will be found beneficial to require the student to give all the principles involved in the solution of any problem, and to state where they are applied.

By attention to these few hints, the author hopes that the teacher who may take up this subject for the first time will find it simple, pleasant and instructive.

L. M. H.

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INTRODUCTORY.

DRAWING DEFINED.

POINTS AND LINES CLASSIFIED AND DEFINED.

1. Drawing is the art of representing objects or ideals (*a*) as they exist or (*b*) appear to exist.

(*a*) The representation of objects as they exist, showing their forms, sizes and positions, is included in those subdivisions of the art known as Orthographic and Isometrical Projections.

(*b*) The representations of objects as they appear to exist is known as Scenographic Projections, or Linear Perspective.

To draw correctly, therefore, a knowledge of projections must first be acquired as a foundation for subsequent work.

2. All objects possess two general characteristics, *form* and *color*, which impress themselves upon the minds of all rational, seeing beings. The color, which gives expression to objects, is limited by and contained within the form. The student's first care should therefore be to represent the form correctly; and since the outlines of all objects are composed of lines, either straight or curved, or both combined, he must neces-

sarily devote some time to the consideration of these elements in their order, before taking up the subject of projections.

POINTS, LINES AND ANGLES.

POINTS.

3. The origin of all lines is THE POINT, which, mathematically considered, is a mere ideal having no material existence, and consequently "*neither length, breadth nor thickness, but position only.*" The points used in drawings have, however, a visible existence, and are represented by mere dots, so small indeed, that their actual dimensions are supposed to be incapable of being measured. They are used simply to indicate positions.

A POINT IS GIVEN OF DETERMINED when its position with reference to other points or lines is known.

The intersection of two lines is always a point, whose position is determined when that of the lines is known.

LINES.

If a point be put in motion it will generate a line called the PATH or LOCUS of the point.

4. If the motion of the point be always in the *same* direction, the path will be a RIGHT OF STRAIGHT LINE; if it continually change its direction, in accordance with some law, it will be a CURVED LINE. A curved line may also be defined as one in which no three consecutive points lie in the same direction. Thus A LINE *is a geometrical magnitude composed of a succession of points lying in the same or different directions.*

A BROKEN LINE *is one composed of portions of straight lines.*

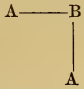
ANGLES.

5. AN ANGLE *is the portion of space included between two lines or planes which intersect.* If the lines are straight, the included space is a *plane angle;* if arcs of spheres, a *spherical angle.*

6. Two right lines are PERPENDICULAR or "SQUARE" to each other when the angles around their point of intersection are all equal.* The angles are then RIGHT ANGLES. An angle *less* than any one of these is said to be ACUTE; one *greater*, OBTUSE.

DIRECTIONS AND RELATIVE POSITIONS OF LINES.

7. Every straight line has two directions, as that from left to right, or from bottom to top, as from A to B, or the reverse, as from B to A.



8. Two right lines are PARALLEL, when they both lie in the same direction. The angle between them is therefore zero.† Two concentric circumferences, or those described from the same centre, are parallel, as well as *right* lines lying in the same direction; for their tangents (12) at corresponding points will be parallel.

*It is not necessary that two lines should lie in the same plane nor intersect to be PERPENDICULAR, *for they are so if their directions in space are at right angles to each other.*

† This fact is important, as it is the basis of the method of finding the vanishing point of any system of parallel lines in perspective.

9. An OBLIQUE line is one which makes any angle with another line different from zero or a right angle. It may have any position, therefore, between the parallel and the perpendicular. These positions may be easily illustrated by revolving a line in a plane about one of its extremities, and noting the angles which it makes with its first position.

10. A VERTICAL line is one which is perpendicular to the surface of water at rest, or which is parallel to a plumb line.

11. A HORIZONTAL line is one which is at right angles or perpendicular to a vertical line. A horizontal line is therefore parallel to the surface of water at rest, or to the earth's sensible horizon.

12. A TANGENT is a line which simply touches another at any point, but does not cut it.

13. A SECANT is a line which cuts another in one or more points.

14. A DIAGONAL is a line joining non-adjacent angles in any figure.

15. The SEGMENTS of a line are the portions into which it is divided by any cutting line or point.

A GIVEN LINE, OR SURFACE, is one whose position is known or assumed.

PLANES.

16. A PLANE is a surface generated by a straight line, moving so as to touch two parallel straight lines, or so as to revolve in a direction perpendicular to a given straight line. Hence, if any two points of a plane be

joined by a right line, that line will be wholly within the plane.

17. A plane is *determined*—that is, fixed in position—by the conditions, that it shall pass through two straight lines which may intersect, or be parallel; or through a right line and a point; or through three points not in the same straight line, or through a given point and be perpendicular to a given line; or through a given line and be perpendicular to a given plane, and by many other reasonable conditions.

RIGHT LINE AND PLANE.

18. The relative positions of a right line and a plane are similar to those already given for two right lines.

19. A line is said to *pierce* a plane, and a plane to *cut* a line. When the line passes through or pierces the plane, the intersection will be a point; but when the *plane passes through the line*, the line lies wholly within the plane

TWO PLANES.

20. The *intersection* of two planes is always a right line.

21. The *relative positions* of two planes are similar to those of two right lines—that is, they may be perpendicular, parallel, or oblique to each other; secant, diagonal, horizontal or vertical. (See Page 5.)

22. The angle formed by two planes which intersect, is called a DIEDRAL ANGLE; the planes are the FACES, and the line of intersection the EDGE, or ARRIS.

The diedral angle is MEASURED BY, or equal to, the plane angle formed by intersecting the faces by a plane *perpendicular* to the edge at any point.

CURVED LINES.

23. *Curved lines* may be divided into two general classes, as those of SINGLE and those of DOUBLE CURVATURE, *according as the generating point moves in a plane, or in space.*

PLANE CURVES.

24. A PLANE CURVE, or CURVE OF SINGLE CURVATURE, is one, every point of which lies in the same plane. The principal curves of single curvature are:

25. The CIRCUMFERENCE, which is generated by a point moving in a plane, so as to remain at the same distance from a fixed point called the centre.

26. An ARC is any portion of a circumference. (Care must be taken to distinguish between a circle, which is an area, having two dimensions, and its circumference, or bounding line, having but one.)

27. THE PARABOLA, every point of which is equally distant from an assumed point, *or focus,* and a straight line not passing through the point.

28. THE ELLIPSE, every point of which is so situated that the *sum* of its distances from two fixed points, *or foci,* is equal to a given straight line.

29. THE HYPERBOLA, every point of which is so situated, that the *difference* of its distances from the two foci is equal to a given straight line.

[NOTE.—These four curves constitute the conic sections, since they may all be cut from the surface of a cone by a plane. They may be constructed from the definitions above given, as will be shown hereafter in the application of these principles.]

30. The OVAL, a plane curve composed of arcs of circles, which

are tangent to each other, two and two, and closely resembling the ellipse in appearance.

CYCLOIDS.

31. The Cycloid is the path described by any point in the plane of a circle, which is rolled along its tangent. The term is generally limited to the curve generated by a point *on* the circumference. If the point is *within* the circumference, the curve is a *prolate cycloid*; if *without*, a *curtate* or *contracted cycloid.*

32. The Epicycloid is the curve generated by a point on the circumference of a circle which rolls around another. If the generating circumference moves upon the outside of the *fundamental* circle, the epicycloid is *external;* if on the inside, it is *internal,* or more generally an HYPOCYCLOID.

(These curves are important as forming the outlines of toothed wheels, and of rack and pinion gearings.)

33. The Involute is a plane curve generated by the end of a flexible line which is unwound from the circumference of a circle or other curve.

34. The Evolute is the curve about which the line is wound.

35. If more than one turn be made, the line generated will be a Spiral. It may be constructed from the above definition of an involute, by drawing tangents to an evolute of any assumed form (but generally a circumference is taken), and laying off on these tangents the lengths of the evolute, from the origin of the spiral to the respective tangent points. This construction may be readily illustrated practically by wrapping a string around a circular block or wheel, and placing a pencil or piece of chalk at the loose end of the string, the other being fastened to the block. If now the pencil point be moved away from the block or evolute, as the string is unwound keeping it stretched tightly, the curve described will be a spiral. The term spiral is often incorrectly used for a curve of *double* curvature, which it is not, as of "spiral" stairs or a "spiral" screw thread, etc. These curves are helices, as will appear presently. (40.) A steel watch-spring is a familiar instance of a *spiral.*

36. The Catenary is a plane curve formed by suspending a flexible chord or chain, uniformly loaded, from two fixed points of support.

37. A REVERSE curve is one composed of two simple curves, which lie on opposite sides of a common tangent, thus ; in the trades it is called an ogee, or "O. G."

38. A COMPOUND curve is one composed of two simple curves of different radii lying on the same side of a common tangent, as in an oval.

39. There are many other plane curves having definite names and properties, as the *Ogees*, or cyma recta and cyma reversa, *Ovolos*, *Cusps*, *Sinoids*, etc., which it is unnecessary to consider in this connection.

CURVES OF DOUBLE CURVATURE.

40. The principal curve of this class is THE HELIX, which is generated by a point moving uniformly in the direction of a straight line, while at the same time it revolves so as always to remain at a constant distance from it. (The curve can be illustrated by wrapping a right-angled triangle cut from a piece of paper around a cylinder. It is the same as the edge of a screw-thread or hand-rail to a winding stair-case.)

41. The distance through which the point moves in making one revolution, measured parallel to the straight line or axis, is called the PITCH.

(The number of curves of this class is practically infinite, but this is the only one which it is necessary to study in this connection.)

CHAPTER I.

WORKING DRAWINGS.

42. WORKING DRAWINGS are projections (47) of the objects to be represented, or made, drawn to a scale.

SCALES.

43. A SCALE is an instrument for determining the ratio of the object to the drawing. Thus, if the drawing is made so small that one inch on it represents one foot of object, the scale is then *one foot* (*or twelve inches*) *to one inch*, or $\frac{1}{12}$.

As the *object* to be drawn has generally a definite size, whilst the *drawing* may be made of any convenient magnitude, it is better to take the former as the divisor or denominator of the fraction expressing the ratio, and the corresponding dimension of the drawing as the numerator.

44. Thus we have the ratio of the object to its drawing, $\frac{\text{Drawing}}{\text{Object}}$ or $= \frac{\text{D.}}{\text{O.}}$

The scale of a drawing or map is then obtained by dividing the *second* quantity or drawing, by the *first* or object. Thus, a map drawn to a scale of one mile to an inch, evidently means one mile of ground to one inch of paper, or 63360 inches of ground to onc inch of the drawing—that is, $\frac{1}{63360}$—so four feet to one inch means that 48 inches of the object or model are represented by one inch of the drawing; hence, the scale is $\frac{1}{48}$. This scale is frequently erroneously called "$\frac{1}{4}$ inch to 1 foot," which is just the reverse of that intended.

45. The quantities used in expressing the ratio must *always* be reduced to the same denomination.

46. Instead of expressing the scale fractionally, it may be drawn out on the paper, and marked so that its divisions shall represent the proper size of the object. Thus, a scale of 1 foot to 1 inch or $\frac{1}{12}$ may be represented by a bar or line divided into inches, but marked feet; thus

The left hand inch may be subdivided into tenths, twelfths, sixteenths, or any other suitable fraction.*

PROJECTIONS.

47. The word project means, literally, to throw down or upon; hence the PROJECTION of an object is the representation of it made by throwing it vertically down or upon the plane on which the drawing is to be made.

48. Three things are essential in all projections: they are, the position of the point from which the object is seen, called the *point of sight; the position of the object in space;* and the *position of the planes upon which the drawing is to be made.*

*Mechanics generally use the duodecimal scales; engineers, the decimal.

THE PLANES OF PROJECTION.

49. To represent the three dimensions of any solid body there must be at least two planes, intersecting at right angles, upon one of which the length and breadth can be measured off, and upon the other the height; but since these two planes do not show a section of the body, it is necessary to use a third, at right angles to the intersection of the others, as in the three adjacent faces forming the solid angle of a cube. See Fig. 1. These are called the PLANES OF PROJECTION, and are designated as the horizontal, ground or (H) plane; the vertical, wall, or (V) plane, and the side, section, or (S) plane. They are also called *coördinate planes.*

50. The line of intersection A B, of H and V, is called the GROUND LINE.

51. The diedral angles formed by these H and V planes produced are designated as indicated on the figure, that above H and in front of V being the FIRST ANGLE, and so on around as in Fig. 1.

These coördinate planes are supposed to be of indefinite extent, so that any object in space will be found either in some one of the four angles or on the planes which separate them; and as the size of the object is not affected by its distance from the planes, it will be found more convenient to imagine them placed so close to the object that it may rest upon them.

POINTS OF SIGHT.

52. In all working drawings the point of sight is situated in lines perpendicular to the planes of projec-

tion, and at an infinite distance from them; and since there are three such planes, there must be three different positions for the points of sight, one for horizontal projections, another for vertical, and the third for side-views.

53. From the positions and distance of the point of sight it follows that all visual rays drawn through points of the object to either plane will be parallel to each other and *perpendicular to that plane*. Such rays are called the PROJECTING LINES or PROJECTANTS of the object, and the points in which they pierce the planes of projection are the PROJECTIONS of the points through which they pass.

NOMENCLATURE OF A POINT.

54. To distinguish the vertical from the horizontal projections, it is necessary to designate them by some characteristic signs; hence for a point in space the capital letters are used, as P; while for its *horizontal* projection the corresponding small letter is taken, as p; and for its *vertical* projection the same small letter accented, as p′ (called p prime). The revolved position is generally designated by the same small letter with the "second" mark, as p″.

THE PROJECTIONS OF A POINT IN SPACE.

55. PROBLEM.—*To project any given point* we have only to draw through it its *projectants*, and find where they pierce the planes of projection. These points will be the *projections* of the given point, the one in H being

the *horizontal* projection, and the one in V the *vertical*. Thus if a point (P), Fig 2, be situated in the first angle, at a distance of three-fourths of an inch above H, three-fourths of an inch in front of V, and half an inch to the left of S, we have only to lay off on BA one-half inch from B; on c p three-fourths of an inch, and on p P three-fourths of an inch, and the given position of the point will be represented.

NOTE—Pupils should draw the positions of a sufficient number of assumed points to enable them to conceive readily of their positions in space, and should hold the end of a pencil or pointer in the position of the imaginary point in space to indicate its place.

EXERCISES.

56. In the following exercises the coördinates of several points are assumed for practice. The distances given are the actual distances measured perpendicular to the planes of projection, as indicated under the letters S, V and H, respectively. In the following assumed positions, all distances measured to the right from S, to the front from V, and upward from H, are plus (+), or positive; all distances measured to the left from S, to the rear from V, or downward from H, are minus (—), or negative; hence the coördinates of points in the first angle and to the right of S will all be positive; for those to the left, the S distances will be negative; for those in the second angle, distances from V will be negative; for those in the third, both the V and H distances will be negative; and for those in fourth angle,

distances from V will be positive, and those from H negative.

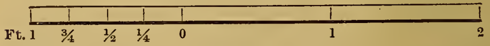
CO-ORDINATES OF POINTS TO BE PLATTED (NATURAL SCALE).

Name of Point.	Distance from S.	Distance from V.	Distance from H.	In what angle ?
Fig. 2. P......	— $\frac{1}{2}$ inch.	$\frac{3}{4}$ inch.	$\frac{3}{4}$ inch.	First, etc., to be filled out by pupil.
Q......	—1 "	1 "	$\frac{1}{4}$ "	
R......	—1 "	— $\frac{1}{2}$ "	$\frac{1}{2}$ "	
S......	—1$\frac{1}{2}$ "	—1$\frac{1}{8}$ "	—1$\frac{1}{2}$ "	
T......	—2 "	$\frac{3}{4}$ "	—1 "	
U......	— $\frac{3}{4}$ "	1$\frac{1}{2}$ "	2 "	
Fig. 2. { p......	— $\frac{1}{2}$ "	$\frac{3}{4}$ "	0 "	On H.
p′......	— $\frac{1}{2}$ "	0 "	$\frac{1}{2}$ "	On V.
c......	— $\frac{1}{2}$ "	0 "	0 "	On Ground Line.

57. It will be seen from the last three cases that, if the points be in the planes of projection or at their intersections, one or both of their projections will be on the ground line, and their positions will be determined in the same manner as when in space.

58. If the above co-ordinates be multiplied by twelve, the number of inches in one foot, the numbers in the table will represent the distances of the points from the planes in feet, instead of inches ; but the same drawing may still be used by changing the scale, or assuming it to be $\frac{1}{12}$ the full size ; this is done by marking the inch divisions on the scale to represent feet, thus :

Scale 1 ft. to 1 in., or $\frac{1}{12}$.



In the same manner any other scale may be applied, as 2 feet to 1″ or $\frac{1}{24}$; three feet, or one yard, to one inch, or $\frac{1}{36}$, etc.

FIG. 2.

59. *If the point be in the second angle*, as at P_2, its projections will be p_2 and p'_2.

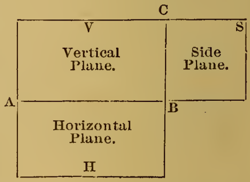
60. *If the point be in the third angle*, its vertical projection p'_3 will be on that part of V below the ground line, and its horizontal projection p_3 on that part of H behind AB.

61. *In the fourth angle*, the vertical projection p'_4 is below the ground line, and the horizontal projection p_4 is in front.

REVOLVED POSITIONS.

REVOLUTION OF THE PLANES OF PROJECTION.

62. Since it is practically impossible to make drawings upon three sheets of paper placed at right angles to each other, it becomes necessary to turn the vertical planes down so as to coincide with the horizontal. For this purpose the side plane is revolved 90° to the right, about the line BC (Fig. 2) as an axis, until it lies in the vertical plane produced. The vertical plane is then turned down *backward* through the second angle until it coincides with the horizontal plane. Thus the three planes of projection are represented on the same sheet of paper. The ground line AB separates the horizontal or lower portion from the vertical or upper portion, whilst the side plane will lie to the right of the vertical line BC, as above.



63. All the points projected on the vertical planes will remain in them, and be found in the same relative positions with reference to the lines AB and BC, as before the revolution. Hence, it follows that *all horizontal projections of objects in the first angle will be* BELOW *the ground line, all vertical* ABOVE, *and all side projections, looking from the left, will be* ABOVE *AB and to the right of BC.*

REVOLUTION OF POINTS ABOUT AN AXIS.

64. A point is said to REVOLVE about a right line as

an axis, when it moves in a plane which is perpendicular to the line, and remains at a constant distance from it. The path of the point will then be a circumference, of which the *radius* is the distance of the point from the axis. The axis may be in any position—as horizontal, vertical, or oblique.

65. Problem.—*To find the revolved position of a point about a given axis.*

Note.—The position of a given point with reference to a given line is determined when we know both its *distance* and *direction* from some point of the line.

66. Analysis.—The *distance* of a point from the axis is always measured by the perpendicular from the point to the axis; this distance is the radius of the circle of revolution; and since this radius must remain perpendicular to the axis, the *direction* to any new position of the generating point must always be *perpendicular* to the axis. Hence to find the revolved position of a point, through the centre, draw a line perpendicular to the axis, and make it equal to the radius of the circle described by the generating point.

67. *To determine the length of this radius and the position of the centre when the point is given by its projections.*

The distance of the point, from an axis in the horizontal plane, is the hypothenuse of a right angled triangle, the height of which is the distance of the point above the horizontal plane, or the distance of its vertical projection above the ground line, and the base is the distance from the horizontal projection of the point to the axis.

68. Construction.—Thus, in Fig. 3 and (*b*), let P be a point in the first angle; let M N be an axis lying in the horizontal plane, about which it is proposed to revolve the point P until it falls upon said plane. The horizontal projection of P is p, and the vertical projection p′. The radius of the path Pp″ is c P, which is evidently the hypothenuse of the right angled triangle cPp, whose plane is perpendicular to MN; and therefore cP must be perpendicular to MN.* Hence *to find c, the centre*, drawn through p, which is always given, a line perpendicular to the axis; the point of intersection c will be the centre; and to find the radius draw anywhere two straight lines at right angles to each other, as cp and pP, Fig. 3 (*a*). and lay off on them the lengths of the base and altitude of the triangle cpP, as given by the lines c p and P p or its equal p′d. Fig. 3. The line joining the points thus determined will be the hypothenuse or radius required.

69. Rule.—Hence we have this GENERAL RULE FOR FINDING THE REVOLVED POSITION OF A POINT ABOUT AN AXIS IN H. *Through the horizontal projection of the point draw a straight line perpendicular to the axis; lay off on it, from the axis, a distance equal to the hypothenuse of a right angled triangle, the base of which is equal to the distance of the horizontal projection of the*

* For if a *plane* is perpendicular to a line, *any line in that plane* will be perpendicular to the line.

point from the axis, and the altitude, the distance of the vertical projection of the point from the ground line.

If the axis be in V, the same rule will apply by interchanging the words *vertical* and *horizontal.*

70. To find the position of the projections of points after the revolution of V into H. (See 63.)

To apply this rule to the revolution of V into H, Fig. 2, we see that the axis AB must pass *through the horizontal projection* of all points in V, and hence the base of the triangle vanishes or becomes zero (0), whilst the hypothenuse and altitude coincide or are equal. The radius is therefore the distance from the point in V to the ground line, and the revolved position of p′ will be found at p″, in the perpendicular to AB at c, and at a distance from it, p″c equal to p′c.

71. Hence, if *the point be in the first angle*, *as P*, *its vertical projection must be above the ground line*, *and its horizontal projection below.*

72. If in the *second angle*, as P_2 (Figs. 2 and 4), since all vertical projections above the horizontal plane are found above the ground line, and since after revolution that part of V coincides with that part of H which is behind the ground line, *both projections will be on the same side or* ABOVE *the ground line*, and can only be distinguished by their letters (see 54).

73. If in the *third angle*, as P_3 (Figs. 2 and 4), the rear portion of H after revolution will be above AB, and the lower portion of V below; hence, the *horizontal projection will be above, and the vertical below the ground line* —just the reverse of the point in the first angle.

74. If in the *fourth angle*, as P_4, it will be the reverse of that in the second, or *both projections will fall* BELOW *the ground line.* (See Figs. 2 and 4.)

NOTE.—Since the side plane is only useful when sections are desired, it may be omitted for the present, and the attention be confined only to the planes V and H, which are now supposed to lie in the same plane, but to be separated by the ground line as represented in Fig. 4.

75. THEOREM.—*The two projections of a point are in the same straight line perpendicular to the ground line.*

ANALYSIS.—If a plane be passed through any point (P) in space, perpendicular to the ground line, it will contain the projecting lines of the point, and will cut the planes of projection in two lines, which are perpendicular to the ground line at the same point. Now, when V is revolved into H, these last two lines will lie in the same plane and be perpendicular to the ground line at the same point; hence, they must coincide or form one and the same straight line.

CONSTRUCTION.—Fig. 2. Let P be the point. The plane formed by the lines Pp and Pp′ is perpendicular to AB; hence, AB is perpendicular to the lines cp and cp′, which lie in the plane Pc. But when cp′ is revolved it will coincide with cp″, which is also perpendicular to AB, and therefore pc and cp″ form one straight line upon which the projections p and p′ or p″ are found; hence, both projections of the same point must always lie in the same straight line perpendicular to the ground line.

Principles of Geometry involved. Fig. 2.

76. 1. Two straight lines intersecting at a point P, determine a plane p′Pp.

77. 2. If a straight line (Pp or Pp′) is perpendicular to a plane (H or V), any plane passed through that line must be perpendicular to that plane; hence, p′Ppc is perpendicular to both H and V at the same time.

78. 3. If a plane (p′Ppc) is perpendicular to two others (H and V) which intersect, it must be perpendicular to their intersection (AB).

79. 4. If a plane (p′Ppc) is perpendicular to a line (AB), any lines (cp′ or cp) lying in that plane, will be perpendicular to the line (AB).

80. 5. If a point p′ revolve about a line (AB) as an axis, it must move in a plane perpendicular to the axis.

THE POINT IN SPACE.

81. Theorem.—*The distance of a point from H is measured by the distance of its vertical projection from the ground line, and the distance of a point from V is measured by the distance of its horizontal projection from the same line.*

Since the figure cp′Pp (Fig. 2) is a rectangle, whose opposite sides are equal and parallel, we have cp=p′P, or the distance of the horizontal projection from the ground line, is equal to the distance of P in front of V, and since Pp=p′c, the distance of the vertical projection of P above the ground line is equal to the height of the point P above H.

Thus, in Fig 4, the position of P will be fully represented by the projections pp′ on a perpendicular to the ground line (AB) drawn through the point c at any given distance from B.

82. Corollary.—*If either projection of a point be given, the other projection must lie on the line drawn perpendicular to the ground line and passing through the given projection.*

83. Problem.—*To determine the point in space when its projections are given.* Figs 2 or 4.

If through the given projections p and p′ perpendiculars be erected, they will lie in the same plane and intersect each other at the point P, thus fixing its position.

In figure 4, the perpendicular through p and p′, after revolution, would be parallel, and therefore not intersect; but it must be remembered that V is only *supposed to be revolved* into H for convenience in representing the parts of the drawing, whereas it is in reality vertical, and *must always be imagined to be so when reading a drawing.* The pupil should not neglect to practice indicating the position of the points in space, by holding a pencil point in the proper place, always remembering that V is supposed to stand vertical. The position of P will then be indicated by holding a point three-fourths of an inch above p, which will be where the perpendiculars through p and p′ would intersect in space.

EXERCISES.

84. The planes are supposed to be revolved as in Fig. 4. The points on AB may be assumed at pleasure.

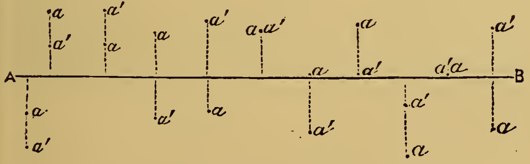
The + and — signs are used as before (56) to indicate directions.

SCALE—FULL SIZE.

Points.	Distances.		In what angle ?
	From V.	From H.	
D	$+\frac{5}{12}''$	$-\frac{8}{12}''$	
E	$-\frac{6}{12}''$	$-1''$	
F	$-\frac{10}{12}''$	$+\frac{14}{12}$	
G	$-\frac{4}{12}$	$-\frac{4}{12}$	
H	$+\frac{5}{12}$	± 0	
I	0	0	

This practice should be extended until the student becomes entirely familiar with the positions of points, for this is the key to the successful interpretation of drawings, and too much stress cannot be laid upon it. The position of points once thoroughly understood, all the rest follows very readily, since lines and surfaces are but collections of points, and the same principles are applicable to them as to points.

85. Let the angle or plane in which the following points are situated be indicated by the student :



CONVENTIONALITIES.

86. In making the drawings, to avoid confusion it is necessary to adopt certain conventional methods of expressing the parts to be represented. Thus, the line joining the projections p and p′ of a point should

always be dotted, as .... ; the projectants of a point, as Pp or Pp′ (Fig. 3), should be broken: — — — — — — — ; all given lines or the parts of such lines which are visible in space should be drawn full and heavy, thus: ━━━━━; the projections of such lines should be full, but light, as ———— ; all invisible or auxiliary lines should be drawn broken: — — — — — — — ; all visible intersections of planes are drawn full: ———— ; and all such intersections which are invisible should be broken and dotted, as — • • — • • — • • —.

87. All sections of solids should be shaded with section lines or cross-hatchings. See Fig. 19 (*a*).

88. All shade lines should be drawn heavy. The position of these lines will depend upon the direction of the light, the form and position of the object, and the position of the point of sight; but since parallel rays of light are generally assumed to enter from the upper left hand corner at an angle of 45° to the plane of the drawing, it has become conventional to make *the right hand and bottom lines of all projecting surfaces, and the left hand and upper lines of all recesses, shade or heavy lines.*

CHAPTER II.

THE RIGHT LINE.

89. PRINCIPLES TO BE REMEMBERED.—1. *Between two points only one straight line can be drawn; hence, the line is given when the position of the points is known.*

90. 2. *The intersection of two planes is a straight line.*

91. 3. *Two parallel lines "determine" a plane.*

92. 4. If a plane pass through two points it must contain the straight line joining them.

93. If two lines be perpendicular to the same plane, *they must be parallel.*

94. PROBLEM.— *To represent a given straight line by its projections.* Fig. 5.

ANALYSIS.—We have already seen that the projections of a point are found by passing lines through it perpen dicular to the planes H and V. If now this operation be applied to all the points in a given straight line, and the points at the feet of the perpendiculars be joined by lines, they will be the projections of the given line.

The horizontal projecting perpendiculars, or projectants, being perpendicular to the same plane (H) will be parallel (93), and since they all pass through the given line they must lie in the same plane (91); hence, their intersections with H will be on a *straight* line (90). As two parallel lines determine a plane, *any two* points may be taken on the given line, and their projections being joined by a right line, will give the desired projections (89 and 92).

Construction.—Thus, in Fig. 5, let P and Q be two points whose co-ordinates are given, and let PQ be the given right line joining them. Through P let fall the projectant p′p upon H, and through Q the projectants Qq and Qq′ upon H and V. Then pq will be the horizontal, and p′q′ the vertical projection of the given line.

95. *Remark.*—If a line be produced so as to pierce either plane of projection, the point in which it pierces the plane will be one point of its projection on that plane. For P and p′ are the same point. Also, *if a point be on a right line, its projections must be on the projections of the line.*

96. Problem.—*To assume a point on a line whose projections are given.*

Assume a point on either projection of the line, and through it draw a line perpendicular to the ground line, until it intersects the other projection of the given line. This will be the other projection of the point on the line. Thus, in Fig. 5 (a), assume any point, as q, on the horizontal projection of the line, and draw q q′ perpendicular to the ground line until it intersects p′r′ at q′. These will be the projections of Q on P R.

The line may be situated in any one of the four angles, but we will discuss here only its possible positions in the *first.*

POSITIONS OF RIGHT LINES IN SPACE.

97. 1. A line in the *first angle* may be oblique to the ground line—that is, to both H and V.

2. It may be oblique to H and parallel to V.

3. It may be parallel to H and oblique to V.

4. It may be parallel to both H and V—that is, to the ground line.

5. It may be perpendicular to H.

6. It may be perpendicular to V.

7. It may be perpendicular to their intersection.

8. It may lie on the horizontal plane, and be either parallel, perpendicular, or oblique to the ground line; or,

9. It may have similar positions in the vertical plane.

NOTE.—Let the pupil hold a pointer or straight edge in their several positions, to aid his imagination in conceiving of their actual places in space. The wall and floor of a room may be taken as the planes V and H, or the leaves of a book turned at right angles. For such practice, model planes will be found very useful. These various positions of a line in space are represented in Figs. 5 to 12 inclusive. There are two diagrams given for each case; the first being the isometric projection of the line as seen in space; the second, or figure marked (a), being the ordinary orthographic projection after the revolution of V.

98. OBSERVATIONS.—It will be seen from the FIRST CASE, that the projections p′r′—p r, Fig. 5, are inclined to the ground line, and in revolving V about AB the point p′ simply describes a quarter of a circumference

about the axis, remaining at the same distance from it, so that the angle which p′r′ makes with the ground line is not changed; and the position of p r in H is not affected by revolving V; hence, in (a) the projections p′r′—p r will represent the same line PR as in Fig. 5. We conclude, then, that *when a line is oblique to both planes of projection, its projections will both be oblique to the ground line;* and since p and p′, r and r′ must be in the perpendiculars to the ground line, *the two projections of the same line must be found immediately above and below each other.*

99. In CASE 2, Figs. 6 and (a), it should be observed that since the line is *parallel to* V, *its vertical projection* p′r′ *must be parallel to the line itself, and its horizontal projection parallel to the ground line.* The angle which the line PR makes with H is equal to that which p′r′ makes with the ground line, for "*two plane angles are equal when their sides are parallel and lie in the same direction.*"

100. In CASE 3, Figs. 7 and (a), the conditions are similar to the above, only interchanging V and H. In general, therefore, *if a line be* PARALLEL *to either plane of projection, its projection on that plane will be parallel to the line itself,* and *its other projection will be parallel to the ground line.* Also, *the angle which the line makes with the plane to which it is not parallel, will equal that made by its projection on the other plane with the ground line.*

101. In CASE 4, Fig. 8, both projections must be par-

allel to the ground line; for, *a line which is parallel to two planes at the same time is parallel to their intersection.* Also, *if through a line which is parallel to a given plane any planes be passed, the intersections of these planes with the given plane will be straight lines parallel to the given line.*

102. CASE 5, Fig. 9. If a line be perpendicular to H, it is evident that all of its points will have the same horizontal projection p, and that the lower extremity of the line, or Q, will coincide with p, or the three points, Q, p, and q, all form one point. The vertical projection p′q′ will be parallel and equal to the given line, for the reason given in case 4, and also because PQp′q′ is a rectangle.

103. CASE 6, Fig. 10, corresponds in principles to the one just given; hence, we see that *if a line be* PERPENDICULAR *to either* H *or* V, *its projection on* H *or* V *will be a point, and on* V *or* H, *a right line perpendicular to the ground line, and equal in length to the given line.*

104. CASE 7, Fig. 11. When the line is perpendicular to the ground line, it must lie in a plane perpendicular to the ground line. This figure shows two positions of such a line, one of which intersects the ground line, and the other piercing H and V. In Fig. (a) the horizontal and vertical projections form one line, and hence in this case the position of the line in space is only determined when the projections of two of its points are known. Or the projection may be made upon the side plane to which the line is parallel, when it becomes the same as Case 2, by substituting S for V. Hence,

When a line is perpendicular to the ground line, both of its projections must be perpendicular to that line at the same point.

105. CASE 8, Fig. 12. *If a line lie in either plane of projection in any position, it is its own projection on that plane, and its projection on the other plane must be in the ground line, between the perpendiculars to it through the extremities of the given line.*

CASE 9 is similar to the above.

106. PROBLEM.—*To determine the position of the line in space, having given its projections.*

ANALYSIS.—Since the projectants of any two points of the given line are parallel, they form a plane which is perpendicular to the plane of projection and passes through the given line. This plane, as P r r′, Fig. 5, is the VERTICAL PROJECTING PLANE of the line, and its intersection p′r′ with V is evidently the vertical projection of the given line. The other plane, Ppr, is called the HORIZONTAL PROJECTING PLANE,* and its intersection with H is the horizontal projection of the given line; *hence, if we pass planes through the given projections of a line, perpendicular to the planes* H *and* V, *their intersections will be the required line* P R *in space.*

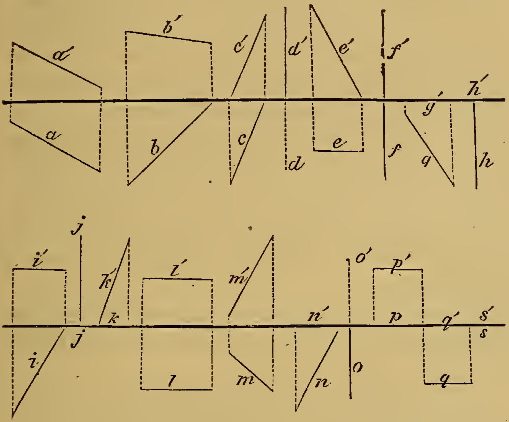
EXERCISES.

107. The following lines are given by their projections after V has been revolved into H, and the student is requested to indicate the position of the line in

* Note the difference between the *horizontal projecting plane*, and the *horizontal plane of projection* (H).

space as well as to describe it, assuming V to stand at right angles to H.

108. These lines are all given in the first angle, and may be designated as the lines A, B, etc.



109. Problem.—*To find the point in which a given right line pierces either plane of projection.* Fig. 5.

Principles.—It has already been shown (95) that the point in which a line pierces either plane of projection is a point of its projection on that plane; that if a point is in either H or V, its projection on V or H must be on the ground line (57); that if a point is on a line, its projection must be on the projections of the line (95); and finally, a point can only be on two lines at the same time when it is at their intersection.

2*

Problem.— *To find the point in which a line pierces* H. The required point is in H, and also on the line; hence, its vertical projection must be on the ground line as well as on the vertical projection of the line, or *at their intersection* (r′) Fig. 5, and the horizontal projection of the same point must be on the perpendicular to the ground line through this vertical projection, and also on the horizontal projection of the line, or where they intersect at r. We have then the following:

110. Rule.— *To find the point in which a straight line pierces* H, *produce its* VERTICAL *projection until it intersects the ground line; at this point erect a perpendicular until it cuts the horizontal projection of the line, produced if necessary.* This last intersection will be the required point.

This same rule will apply to a point in V, by transposing the words vertical and horizontal, and is true for any angle.

111. If the line is parallel to either plane, it is evident it cannot pierce it, for its projection on the other will be parallel to the ground line, and hence will not intersect it.

Construction.—Let P Q, Fig. 5 (a), be the given line. To find where it pierces H, produce the *vertical* projection p′q′ until it cuts the ground line at r′; erect the perpendicular r′r until it intersects the horizontal projection of the line p q produced at r. Hence, r is the desired point.

EXERCISE.

Let the student find the points in which the lines

ABC, given in (108), pierce H and V, and also assume other lines for this purpose.

TWO LINES WHICH INTERSECT.

112. THEOREM.—*If two lines intersect at a point, their projections must pass through the projections of the point.* See Figs. 13 and (a).

ANALYSIS.—Two intersecting lines may be either oblique or at right angles to each other; in either case they must have one common point through which, if the projectants be drawn, they will lie in the projecting planes of the two lines, and hence be at their intersection. But these projecting planes intersect H and V in the projections of the lines, which must therefore pass through the projections of the common point.

CONSTRUCTION.—Let PQ and RS be the two given lines intersecting at O (not shown in (a)). If now the horizontal projecting planes p′pq and s′sr be passed through them, they will intersect each other in the vertical line Oo, which is the projectant of the point of intersection, and which pierces H at o, the intersection of the horizontal projections of the lines p q and r s. In the same way it will appear that Oo′, the vertical projectant, pierces V at o′, where the vertical projections of the given lines intersect; but these points oo′ are the projections of the same point O, in which the given lines intersect. The converse of this is also true, viz.:

113. *If the projections of two lines pass through the projections of the same point, the lines must pass through the point in space.*

114. Corollary.—It also follows that *to draw a line through a given point on another line*, it is only necessary to assume a point on that line (96), and draw the projections of the second line through that point; or,

115. *To draw two or more lines through a given point*, draw their projections at pleasure through the projections of the point. See the figures.

The student should also show that p′ and s′ are the points in which the given lines pierce V, and q and r, the points in which they pierce H, by applying the rule given in 110.

EXERCISES (After the revolution of V into H).

116. Let the co-ordinates of a point be S= —½″, V= 1″, and H= 1″, natural scale.

1. Draw through the above point a line parallel to the ground line also two others, one perpendicular to H, the other to V.

2. Draw a line through the same point perpendicular to the ground line.

3. Draw a line parallel to V, and making an angle of 45° with H (there may be two such lines, one on either side).

4. Assume a point on the line parallel to the ground line, and drop perpendiculars from it to V and H, thus completing the parallelopipedon.

5. Assume any point in space, and draw any two oblique lines through it.

6. Find the points in which these lines pierce the planes of projection.

CHAPTER III.

PLANES.

117. A plane is determined when it passes through three given points; through a right line and a point; through two right lines which intersect; through two parallel lines; through a point and perpendicular to a given line; through a point and parallel to a plane, and by many other conditions. Since two intersecting straight lines determine a plane, if the points be found in which they pierce H and V, these points will lie in the plane of the two lines, and the straight lines joining them (one of the lines being in H, the other in V), will also lie in the plane of the given lines; hence, *they are the intersections of the plane of the given lines with* H *and* V.

118. *These intersections are called the* TRACES *of the plane on* H *and* V.

119. THEOREM.—*The traces of an oblique plane must intersect each other*, *if at all*, *on the ground line.* For the horizontal trace is a line of H, and it is not parallel to the ground line, which is also a line in H. Since then these two lines lie in the same plane, and are not parallel, they must intersect at some point. For the same reason the vertical trace and gruond line must

intersect at some point. But the two given traces also lie in the same oblique plane, and are not parallel; hence, they must intersect each other as well as the ground line; therefore, these three lines must have one point in common, which is where the traces meet the ground line. There can be but one such point, since a straight line (AB) can pierce a plane but once.

The two traces and the ground line form the three edges of a triangle or pyramid whose vertex is their intersection.

120. Problem.—*To represent a plane in any position by its traces.*

A plane, like a line, may have a great variety of positions in space. The following are some of those in the first angle:

1. It may be oblique to both H and V or to the ground line, as p′Tr, Fig. 13.

2. It may be perpendicular to H but inclined to V, as s′sr, Figs. 13 and (*a*), in which case the *vertical* trace is perpendicular to the ground line, and the *horizontal* trace makes an angle Tsr with the ground line equal to that between the planes—p′pq is another instance of a similar position.

3. It may be perpendicular to both H and V at any point of AB, in which case both traces will be perpendicular to AB at the same point, and will form one straight line. As in Figs. 14 and (*a*).

4. It may be parallel to H, in which case it can only intersect V in a line parallel to the ground line. It will

then have but *one* trace parallel to AB, and at a distance from it equal to that between the planes as shown in Figs. 15 and (*a*).

Or it may be parallel to V, when its trace will be on H and parallel to AB. Figs. 16 and (*a*).

These traces are only distinguishable from the projections of lines parallel to AB by the letters T′, T, t′, t, used to designate them.

5. And finally the plane may be parallel to the ground line and yet intersect both H and V, as in Figs. 17 and (*a*).

In this case the intersection of the given with the side plane will determine the angle of inclination.

The student should indicate these positions by placing a card in the angle of the model planes of projection.

APPLICATIONS.

121. PROBLEM.—*To represent a simple geometrical solid, as a rectangular block of wood, stone, or brick, by its projections.*

To represent a brick, the dimensions of which are $8\frac{1}{2}$x 4x$2\frac{1}{2}$ inches, by a drawing made to a scale of $\frac{1}{8}$ full size, the dimensions of the drawing must be one-eighth of the above figures, or $1\frac{1}{16}$x$\frac{1}{2}$x$\frac{5}{16}$ inches; hence, if we assume the brick to lie with its edges parallel to H and V, and touching them, the length may be laid off on the ground line; and from its ends lay off the height above and the breadth below it. See Fig. 18, (*a*). From this drawing a model $\frac{1}{8}$ the full size may readily be made. By changing the scale, the model may be made of any desired size.

122. The portion of a drawing made on the vertical plane or wall is called the ELEVATION; that on the horizontal plane or ground, the GROUND PLAN, or simply the PLAN, and those on the side planes, may be either END ELEVATIONS or VERTICAL SECTIONS.

The great simplicity of Fig. 18 (*a*), over the oblique perspective of the same solid, as given in Fig. 18, is at once apparent, and yet the latter gives all the information contained in the former; for it will be seen at once that the points P and Q are in planes perpendicular to the ground line, to which the edge PQ is parallel; that the plane of the upper face is parallel to H, and hence has but one trace, p′q′, whilst the front face is parallel to V and intersects H in the line pq, etc.

123. PROBLEM.— *To represent a box of the same dimensions as above.*

If we wish to construct a box, as in Fig. 19, it would be better to show the cross section, although the size of the interior might be expressed in Fig. 18 (*a*) by drawing dotted lines at the proper distance from the projections of the outer edges, as in 19 (*a*), in which the cross section is also shown.

In this case the thickness of the sides of the box is assumed to be $\frac{1}{2}$ of an inch, which will be represented in the drawing by $\frac{1}{16}''$, the scale being as before, $\frac{1}{8}$ full size.

The inner and outer surfaces of the box are in reality two rectangular prisms, but the edges of the inner one, being concealed by the outer prism, are drawn dotted.

Sections.—The plane cutting out a right section may be taken perpendicular to the edge at any point. In the drawing it is represented at t′Tt, and its intersection with the solid material of the box is shaded with section lines. This section is projected upon the side plane, which is then revolved to the right about BC, until it coincides with V. This brings the section on t′t on the right side of Fig. 19 (*a*) and above the ground line.

124. Thus the true sizes of the pattern are obtained from this drawing for all the various parts of the box. It is simply the unfolding of three adjacent faces, and is evidently one-half of the whole surface. The amount of material required can therefore be readily estimated.

The student should compute the required amount.

125. The dimensions are marked on the drawing for full size, and the scale should be marked to correspond. The advantage of numbering the subdivisions of the first unit to the left will become apparent in using the dividers to measure distances.

This is the simplest position in which to represent a rectangular figure, but there are many others which may be necessary. These will be explained subsequently.

126. PROBLEM.—To represent *an assemblage of rectangular blocks of the same or different sizes.* Figs. 20 and (a).

The same principles are applied in projecting a number of parts as in representing any unit; thus in Fig. 20, the bricks or blocks in a wall are shown in precisely the same manner as the single block in Fig. 18, but on a smaller scale. Here the scale is $\frac{1}{24}$ full size, or 24 inches to 1 inch; that is, two feet to one inch, or one foot to one-half an inch. In drawing the scale, therefore, one-half an inch should be marked to represent one foot.

127. PROBLEM.—*To represent a square mortise and tenon joint.* Fig. 21.

This joint consists of a rectangular projection called a tenon, cut upon the end of a piece of timber, which is made to fit a corresponding recess, called a mortise, cut on the face of another piece, which is to be framed or joined to the first.

Thus, in Fig. 21, is shown the tenon T, and under it the mortise M. The same parts are shown in Fig. 21 (a) in plan, side and end elevation. In this case, the lower block, called a "*sill*," does not rest against V, but stands out from it a distance equal to m′n. If then m′C be the axis of revolution for the side plane, the revolved position of n will be at n″, and of m at m″. These points revolve in H, which is perpendicular to the axis, and remaining at distances from it equal to m m′ and n m′, the radii of their respective circumferences. So of any other point, as R, which revolves about the point r'_1, in the axis. Its projection upon the side plane is at R_1, and this point when revolved will be found at r''_s, at a distance r'_1 r''_s from the axis equal to the radius $m'r'_1$. The plane in which the point moves being perpendicular to the axis at the centre r'_1, will have but one trace, which will pass through that point and be parallel to the ground line, and it is in this trace that r''_s is found. Hence this

128. RULE.—*To find the revolved position of the projections of a point on a side plane. Through the vertical projection of the point draw a right line perpendicular to the vertical trace of* S; *and lay off on this line from the trace a distance equal to the distance of the horizontal projection of the point from the ground line.*

Such are a few of the large number of applications that may be made of the principles already given. The pupil can readily extend them to a variety of other cases.

CHAPTER IV.

PROBLEMS RELATING TO LINES AND PLANES.

129. Problem.—*To find the true length of an oblique line joining two given points in space.* Case I.

It will be seen from the preceding problems that the projections of a line are only equal to the line itself when the line is *parallel* to the planes of projection. In order, then, to find the true length of any line given by its projections, it will be necessary to revolve the line either into or parallel to one of the planes of projection. Its projection on that plane will then be equal to the line itself.

Analysis.—If the line does not pierce either H or V within the limits of the drawing, imagine either of its projecting planes to pass through it, and revolve this plane, containing the given line, about its trace as an axis, until it coincides with the plane of projection. The revolved position of the line will then give its true length.

Construction.—Let PQ, Figs. 22 and (a), be any oblique line. Through it pass its horizontal projecting plane Pq. It cuts H in pq, which is the projection of the line. Now revolve the plane Pq about pq and find the revolved position of P and Q in H, by the rule already given (64). The line p″q″, will be the true length of PQ. The perspective Fig. 22 does not give the true length.

A similar construction, using p′q′ as an axis, would have given the length upon V.

130. *Another method.*—Or, the plane Pq may be revolved about the perpendicular Pp as an axis until parallel to V, when PQ will be projected upon V in its true size.

This position is not shown in Fig. 22, to avoid confusion, but is given in (a), where o′ is the centre of the circumference described by Q moving in a plane parallel to H, and whose vertical trace is, therefore, the line o′q′. The radius being pq, when the plane Pq is revolved until parallel to V, the point q will be found at q_1q'', and p′q″ will be the required length.

Note.—It will be seen that the length of an oblique line is always greater than that of its projections.

131. Case II.—*When the line pierces either* H *or* V.

Analysis.—The point in which it pierces the plane is a point of its projection on that plane, and hence is on the axis. It is only necessary, therefore, to revolve the other end of the line into the plane of projection, and join this point with that in the axis for the true length.

Let the pupil assume such a line, and determine its length.

132. Problem.—*To assume a line in any oblique plane.*

Case 1. *When the line is oblique to both planes of projection.*

Analysis.—Since the straight line joining any two points of a plane must be a line of that plane, and since the traces of the given plane are also lines of the plane, if any point in one of these traces be joined to any point in the other by a right line, that line must lie in the plane. The line is represented by its projections, as in Figs. 13 and (a). The converse of this proposition is also true.

Construction.—(Same Fig.) Let p be any point in the vertical trace of the plane p′Tr, and q be any point in the horizontal trace of the same plane, the line P Q joining them must lie in the plane. Its vertical projection is p′q′, and its horizontal pq.

133. Case 2. *When the line is parallel to either plane of projection.*

Analysis.—If a line be drawn through a point of a plane parallel to a line of that plane, it must lie in the plane; hence, if a line be drawn through any point of one trace parallel to the other, it will lie in the given plane, and be parallel to H or V.

Construction.—Figs. 23 and (a). Let t′Tt be the given plane. Through any assumed point, as n, in the horizontal trace tT, draw a line, as NM parallel to Tt′. This line being parallel to Tt′, will be parallel to the plane in which it lies, and therefore its projection on V will be parallel to the line itself, NM, or to its parallel line Tt′. Its horizontal projection will be parallel to the ground line (99).

134. Case 3. *If the line be perpendicular to the ground line.*

In this case its projections must be perpendicular to the ground line (104). In Figs. 23 and (a) the line is PQ. It intersects MN, which is also a line of tTt′ in O, as will be seen from its projections. From this case it also appears that if any point of either trace be joined with the points of any line of the given plane by straight lines, these lines will lie in the plane.

135. Problem.—*To pass a plane through three given points.*

Analysis.—If through any two of the points right lines be drawn intersecting at the third, these lines will line in the plane of the points. The traces of this plane will be found by producing the lines until they pierce H and V, and joining the points thus found. Should the lines not pierce the planes of projection within the limits of the drawing, assume any other points (96) on the two lines joining the given points, and join them by a line which will give points of the required traces.

Application.—Figs. 13 and (*a*). Let R O N be the three given points; join R and O by a straight line, and find the points r and s′ in which it pierces H and V. These will be points in the required traces. Join O and N, and find the points p′ and q as before. The lines r q and p′ s′ will be the horizontal and vertical traces of the plane of the points. If the construction be correct, these traces will intersect each other, if at all, on the ground line at T.

136. Problem.—*To pass a plane through two lines which intersect or are parallel.*

The general solution of this problem is the same as that of the preceding after the points have been joined by right lines. See figs. 13 and (*a*).

In case the parallel lines are parallel to the ground line, or to either plane of projection, it is only necessary to assume points on the given lines, and join them by other lines which will pierce the planes H or V.

137. Theorem.—*If two or more lines are parallel, their projections must be parallel;* for the horizontal projections are determined by passing planes through the lines and perpendicular to H. These projecting planes must therefore be parallel, and hence their intersections with H, or the horizontal projections of the lines, are parallel to each other.* For the same reason the vertical projections are parallel to each other.

138. Problem.—*To pass a plane through one given line parallel to another.*

Analysis.—If a line be drawn through any point of the given line (114) parallel to the other (137), these intersecting lines will determine the required plane. The traces are found as in (135) or (109). Let the pupil construct the problem.

139. Problem.—*To pass a plane through a right line and a point.*

Analysis—Let the point be joined to any assumed

* "If two parallel planes be intersected by a third plane, the lines of intersection will be parallel."

point on the given line, and the plane of the intersecting lines will be the one required. The traces are found as in (136).

TWO PLANES.

140. PROBLEM. *To find the intersection of two oblique planes whose traces intersect.*

ANALYSIS.—The required intersection must be a straight line lying in both planes, but we have seen (132) that if a line lies in a plane it must pierce the planes of projection, if at all, in some point of the traces of the planes. As the line of intersection is in *both* planes, it can only pierce the planes H and V in the points where the traces of the given planes intersect each other; hence if a line be drawn, by its projections, between these points, it will be the required intersection.

CONSTRUCTION.—Let sSs′ and tTt′, Figs. 24 and (*a*), represent the two given oblique planes. The vertical traces intersect at p′, and the horizontal at r, the line P R, joining them, is the desired intersection; its projections are *pr* and *p′r′*.

141. PROBLEM.—*To find the angle which the line of intersection* P R *makes with either plane of projection, as* H.

ANALYSIS.—Revolve the line about its projection on H as an axis, into the horizontal plane, when the angle between the revolved position and the axis will be the one required.

CONSTRUCTION.—Since the point r, Fig. 24 (*a*), is on the axis rp, it remains fixed. The revolved position of p′ will be found, as in (64), at p″. Hence p″r is the revolved position of the line, and p″rp is the angle.

142. Remark.—The angle which a line makes with any plane is the same as that which it makes with its projection on that plane.

143. Special Cases.—If the *intersecting planes are perpendicular to* H only, the vertical traces must be perpendicular to the ground line (120, case 2), but the horizontal traces will be oblique; and since the planes intersect, their traces must meet in some point, through which the line of intersection must pass. As the planes are perpendicular to H, this line must also be perpendicular to H or parallel to the vertical traces, hence its position is fully determined, as it passes through a given point and is parallel to a given line.

Construction.—In Fig 25 and (*a*) the planes sSs′ and tTt′ are perpendicular to H, their vertical traces, perpendicular to the ground line, the line of intersection PR passes through the point r, in which the horizontal traces intersect, and is parallel to V, or to the vertical traces. The pupil should construct the figure for the case in which the planes are perpendicular to V, which is just the reverse of this.

144. Problem.—*When the intersecting planes are parallel to the ground line.*

Analysis.—In this case the intersection will be parallel to the ground line.* Its position is best de-

* A line can only be parallel to two planes at the same time when it is parallel to their intersection.

termined by cutting the two given planes by a side or section plane, finding their traces upon this plane, and through their point of intersection, drawing a line parallel to the ground line.

CONSTRUCTION.—In figures 26 and (*a*), the planes tt′P and ss′P are cut by the side plane in the lines tt′ and ss′, which intersect at R, the line through which, parallel to the ground line, is the required line. In Fig. (*a*) the side plane is revolved about its vertical trace, and the vertical projection r″ is found in its revolved position; its true projections r and r′ are easily determined, and the required line P R is then known.

145. PROBLEM.—*To find the intersection of two oblique planes when the traces do not intersect within the limits of the drawing.* In this case let us assume that the horizontal traces do not intersect. If we suppose a third plane to be passed parallel to the vertical plane, it will cut the given oblique planes in two lines which will be parallel to the vertical traces of those planes[1] (99), and these lines must cut each other at some point of the required line of intersection.[2] A second point of the line may be found in the same manner, and thus the line will be determined by points.

CONSTRUCTION.—Let r′Tt and r′Ss be the given oblique planes whose horizontal traces do not intersect within

[1] If an oblique plane cut two parallel planes, the lines of interection will be parallel ; and the projections of parallel lines are parallel.

[2] If two lines lie in the same planc and are not parallel, they must intersect.

the drawing. Pass a plane whose H trace is *n m*, parallel to V. It will cut the given planes in the lines PM and PN, the vertical projections of which are p′m′ and p′n′ parallel to the vertical traces and intersecting at p′, the vertical projection of P, one of the required points. Since P is in a vertical plane, its H projection must be found in the horizontal trace *mn* of that plane, and also in the perpendicular to the ground line through *p′*, hence at p. In like manner *q′* and *q* are found, and the line joining Q and P to them should pass through the point in which the vertical traces intersect. If the vertical traces do not intersect, the construction is the same.

146. Problem.—*To find the point in which a given right line pierces any oblique plane.*

Analysis.—If through the given line any auxiliary plane be passed, it will cut the given plane, if at all, in a line which must meet the given line at the required point.* Hence if a plane be passed through the given line so as to cut the given plane, the line of intersection of these planes will meet the given line in the required point, which will be found where the projections of the lines intersect (113).

Construction.—Let tTt′, Figs. 28 and (*a*), be the given plane, and P Q the line. For simplicity, pass through P Q its horizontal projecting plane pSt′. Its

*For every right line of a plane must pierce any other plane to which the line is not parallel in the common intersection of the two planes.

traces intersect tTt′ in the points m and t′; hence the line mt′ is the intersection of the two planes. This line intersects P Q in O, which is therefore the desired point.

147. PROBLEM.—*To find a point in a plane when either of its projections is given.*

ANALYSIS.—Through the given projection of the point erect a perpendicular to the plane of projection. This must pass through the required point. Hence if the point in which the perpendicular pierces the given plane be found, as in the last problem, it will be the one required.

CONSTRUCTION.—Let *o*, Figs. 28 and (*a*), be the given projection of the point, and Oo the perpendicular through it. Pass any plane, as pSt′, through the perpendicular. It intersects the given plane tTt′ in the line t′m, which cuts the perpendicular at O or o′ in Fig. (*a*), which is therefore the required point.

PRACTICAL APPLICATIONS.

148. PROBLEM.—*To construct a simple dove-cote, dog-kennel or similar structure out of boards, without a frame.*

REMARKS.—The first step in this problem is to decide upon the form and dimensions. This requires an exercise of the imagination, and knowledge of the habits and size of the proposed occupants of the house. The particular style is a matter of taste, and may be varied to suit the idea of the designer. In this case the building is intended for a medium-sized dog. Its dimensions will be assumed, therefore, at 24 inches in length, 18 in breadth, and 34 in height, measured outside. The boards are to be one inch thick. The next step will be to select a convenient scale, so as to represent all the necessary parts upon a page or paper of limited dimensions. In this case it is one-sixteenth the full size, which will just enable the three projections to be placed upon the Plate (viii), Fig. 29.

DESCRIPTION OF PARTS.—The structure consists of 7 parts, viz.: 2 ends or gables, 2 sides, 2 roof boards, and 1 base plate. This latter may be allowed to project one or more inches all around. The doors and windows should be cut out before the parts are put together. The dormer windows in the roof are merely imitations, made of wedge-shaped blocks covered with half-inch boards. Before beginning any work, the amount of material required should always be determined, and a bill made out in the following form:

BILL OF MATERIAL.

1 Base-board,	28×22= 616	square inches.
2 Sides,	22×18= 792	"
2 Gables,	32×18=1152	"
2 Roof-boards,	30×23=1380	"
	144)3940	
	27+	square feet.

As the material cut out for doors and gables will furnish blocks for dormers, etc., it will not be necessary to make a large allowance for waste; so that thirty feet of lumber will be sufficient.

The *cost of materials* will then be, for

30 feet 1-inch boards @ $0.05=	$1.50
2 pounds assorted nails @ $.05=	.10
	$1.60

CONSTRUCTION.—Having the materials and tools, the pupil can proceed to erect the structure, without detailed instruction from a text-book, as to the manner of using them. A few failures and a little patience will prove to be the best instructors. The angles of the bevels on the edges of the roof and sides are given by the drawing, Fig. 29, in the end elevation. The eaves of the roof may be left square.

In the same manner any similar structure composed of planes and straight lines may be erected.

FIGS. 1–2.

THE PLANES

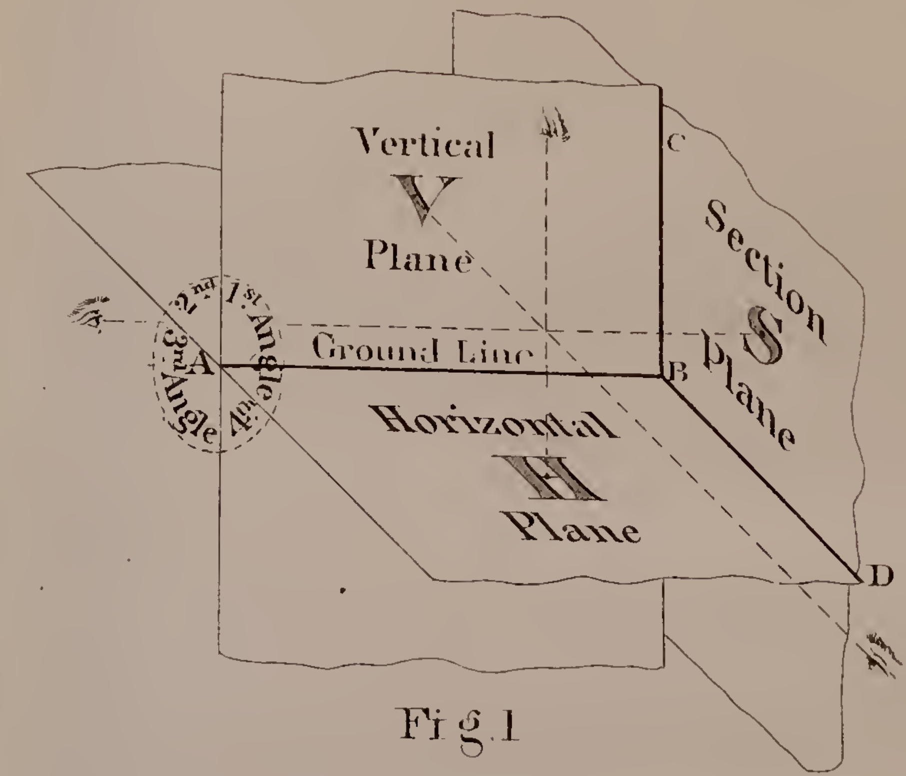


Fig. 1

POINTS

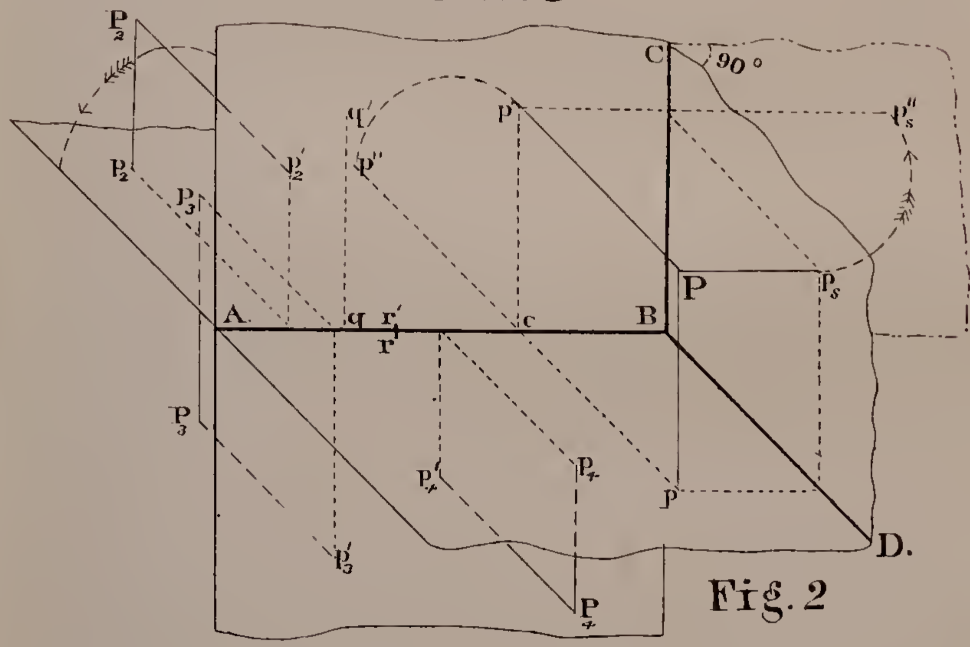
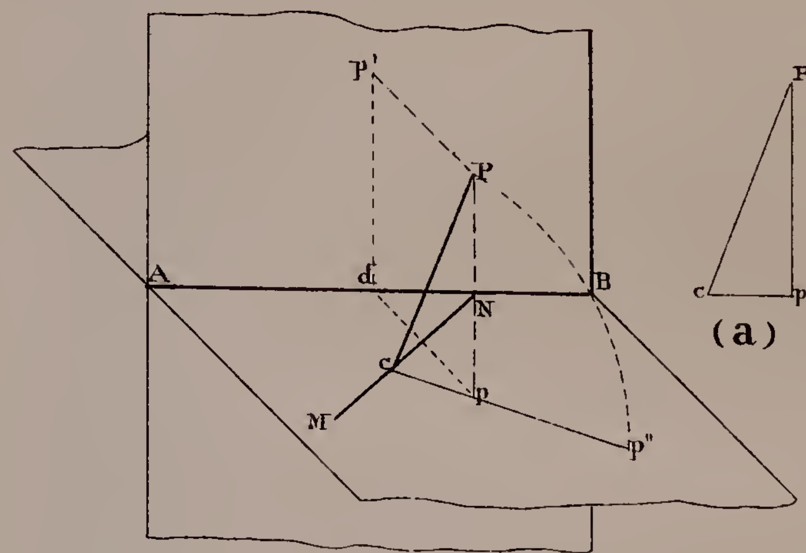


Fig. 2

Scale in inches. Full Size.

FIGS. 3–4.

REVOLUTION OF POINTS AND PLANES.



(a)

Fig 3.

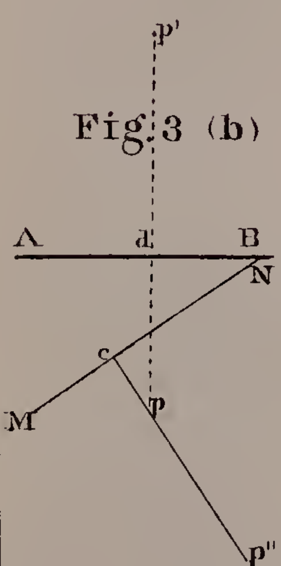


Fig. 3 (b)

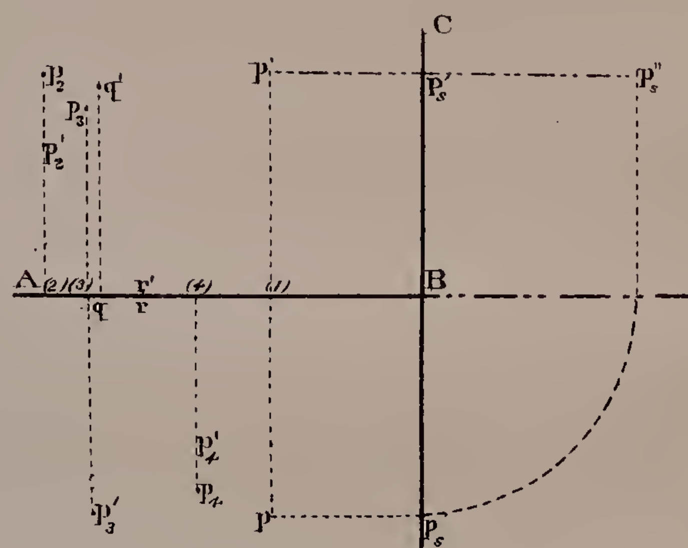
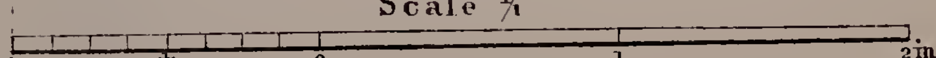


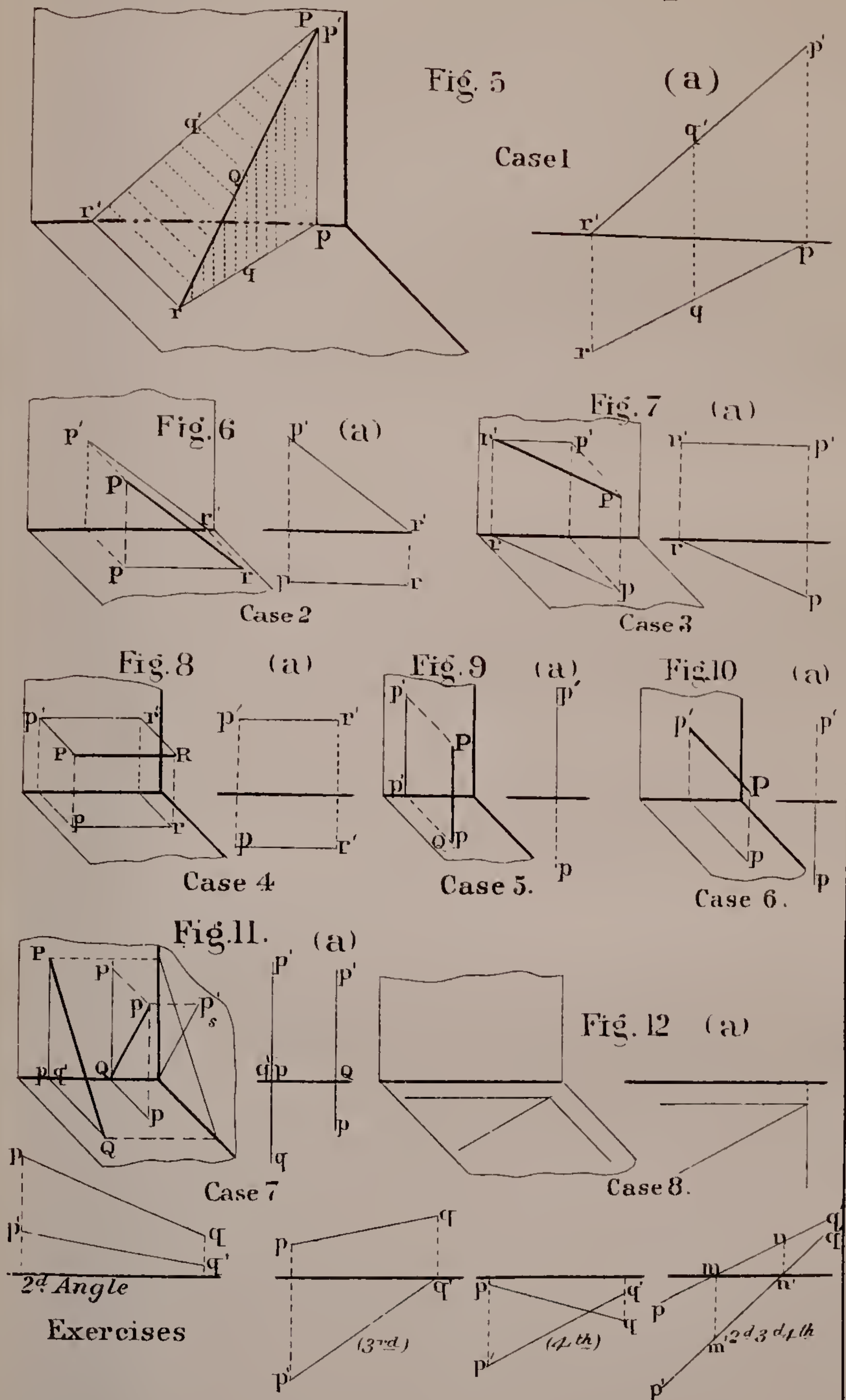
Fig. 4

Scale 1/1



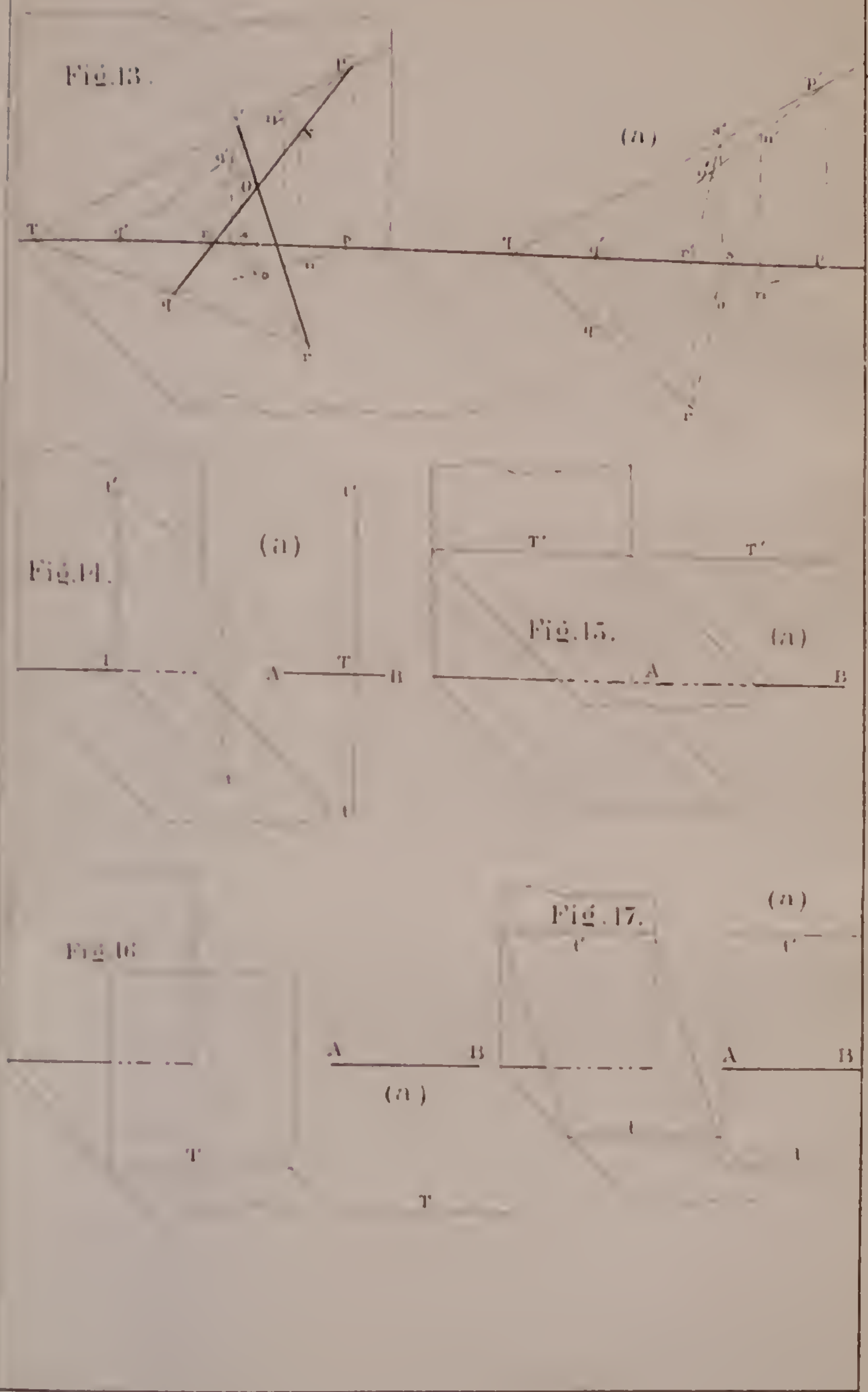
FIGS. 5–12.

PROJECTIONS OF A RIGHT LINE



FIGS. 13–17. 1. 4

PLANES.



FIGS. 18–21.

APPLICATIONS

Fig.18.

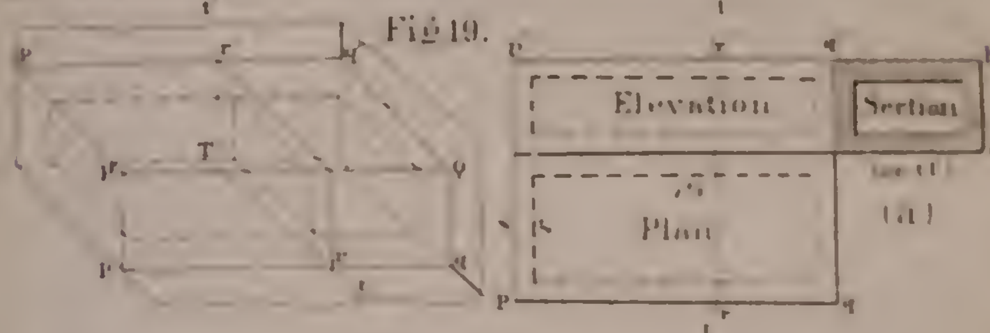


Fig.19.

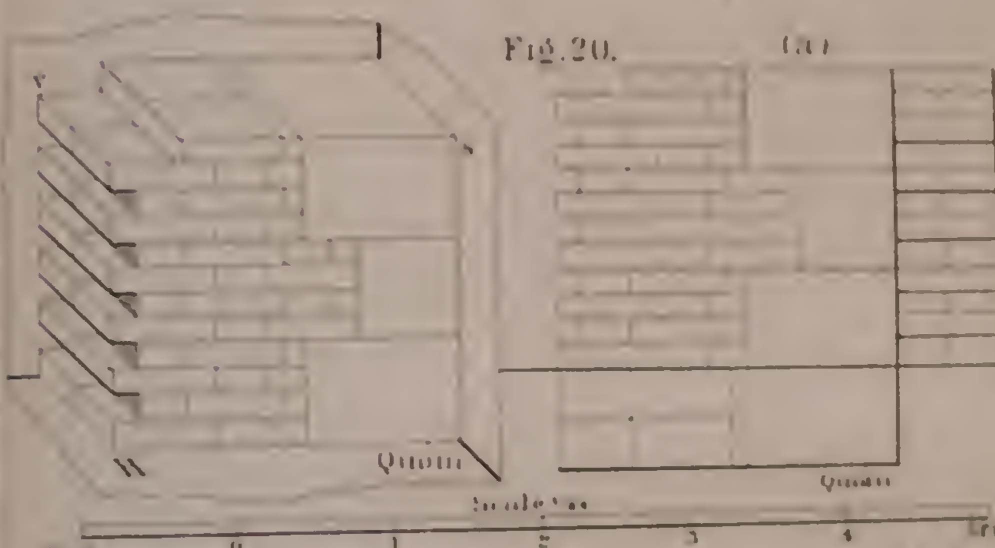


Fig.20.

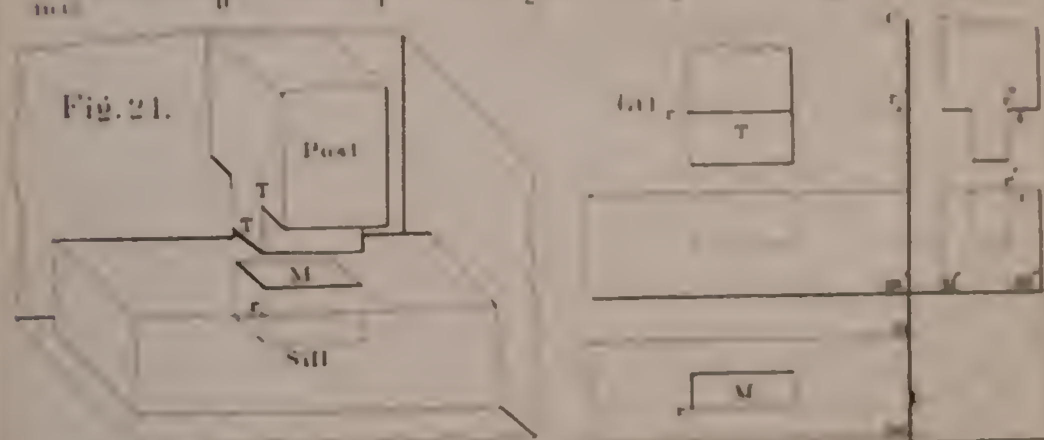
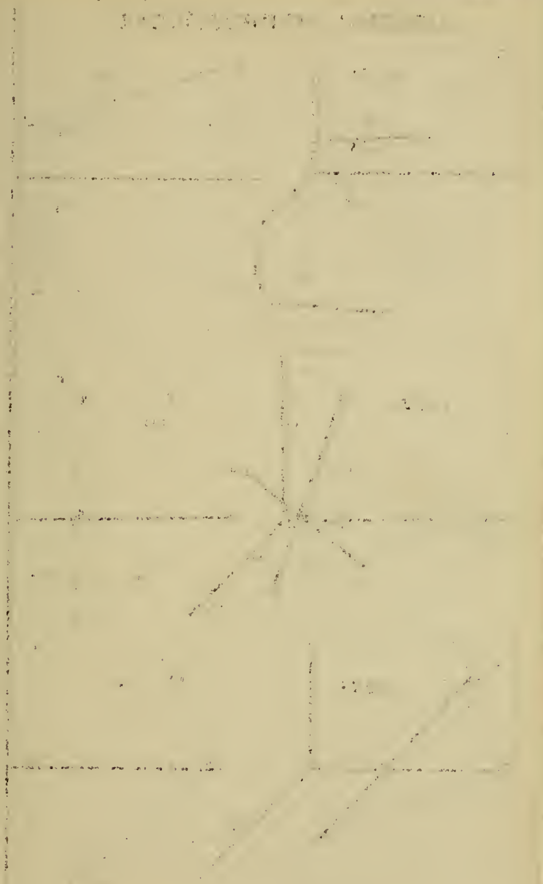
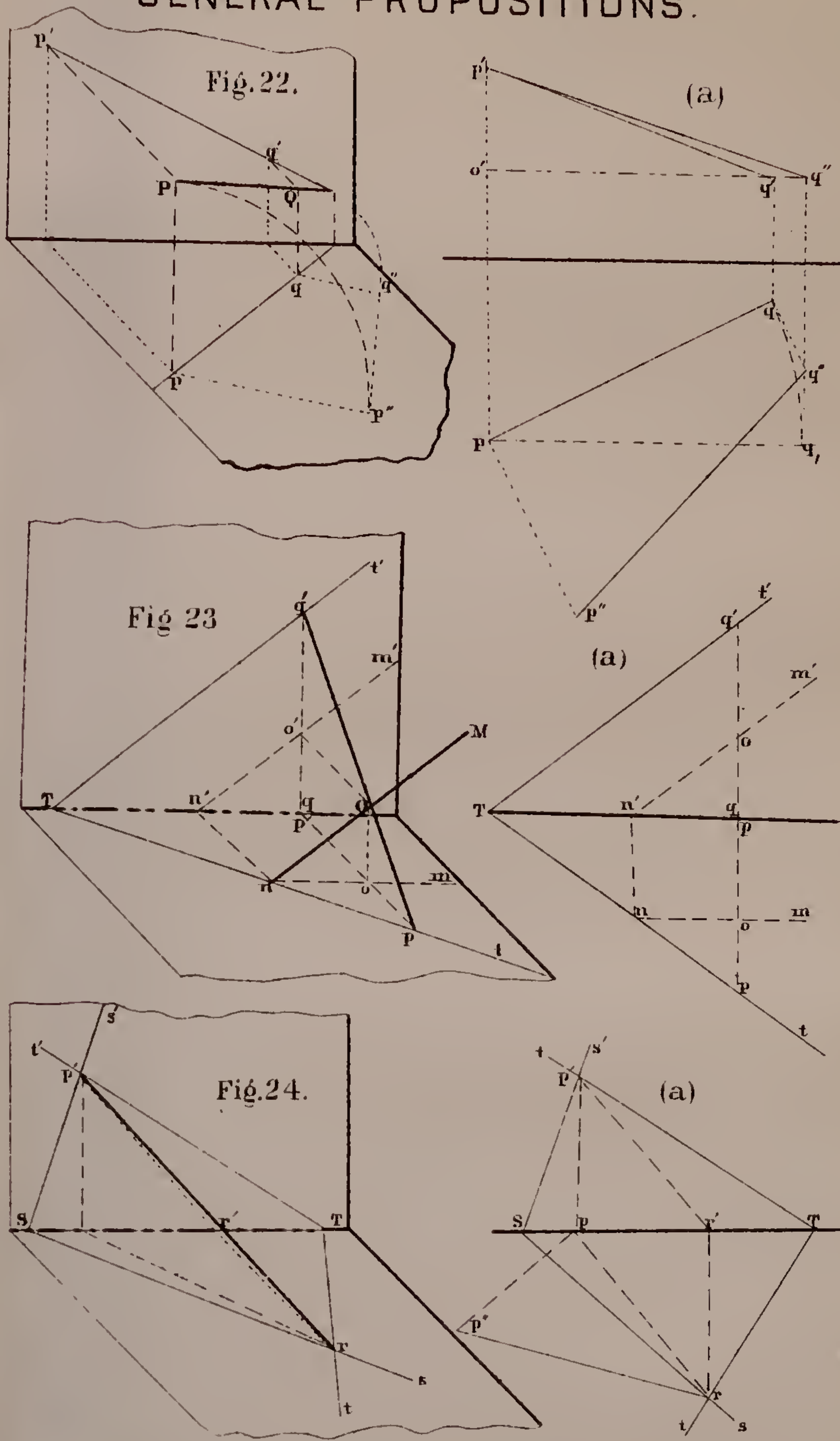


Fig.21.

FIGS. 22–24



GENERAL PROPOSITIONS.



FIGS. 25–27

INTERSECTIONS OF PLANES

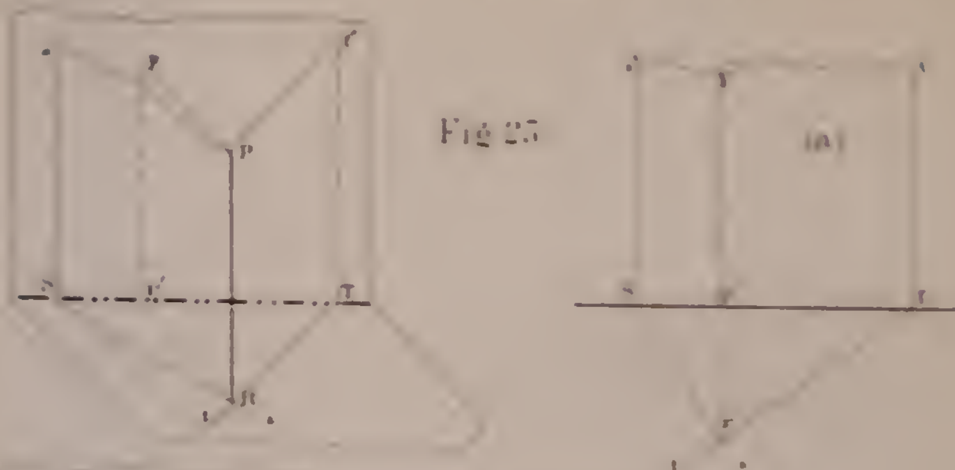


Fig 25.

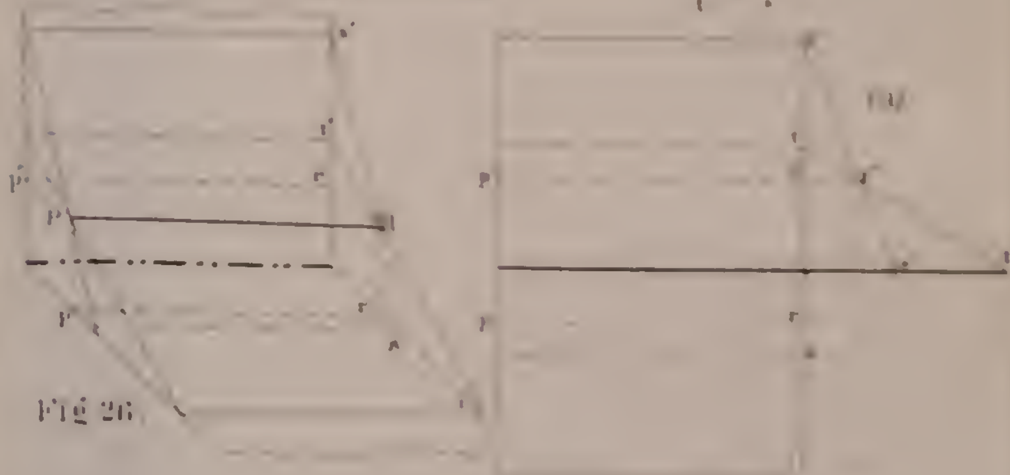


Fig 26.

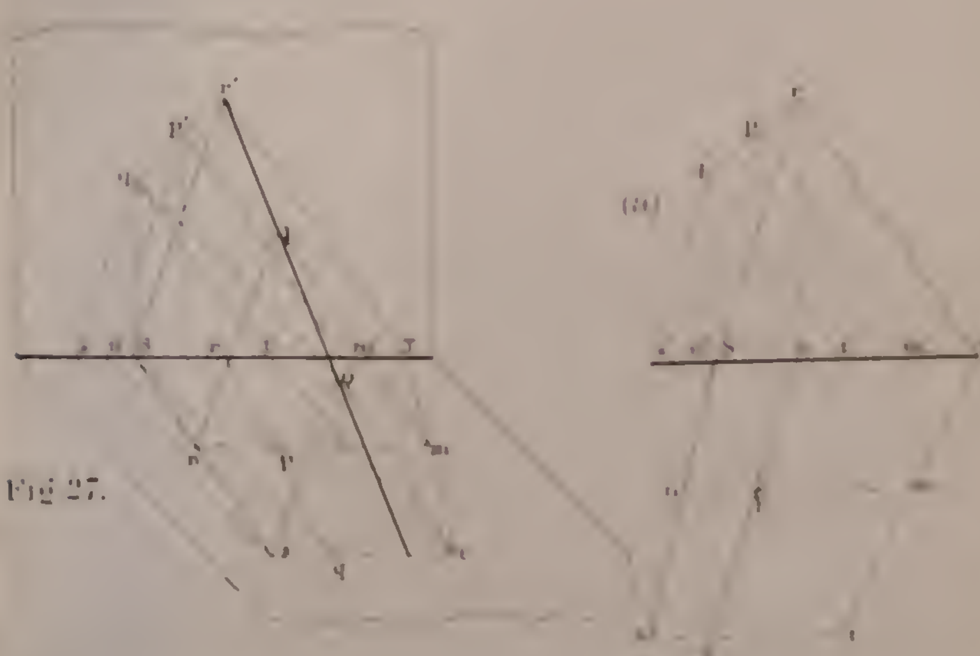


Fig 27.

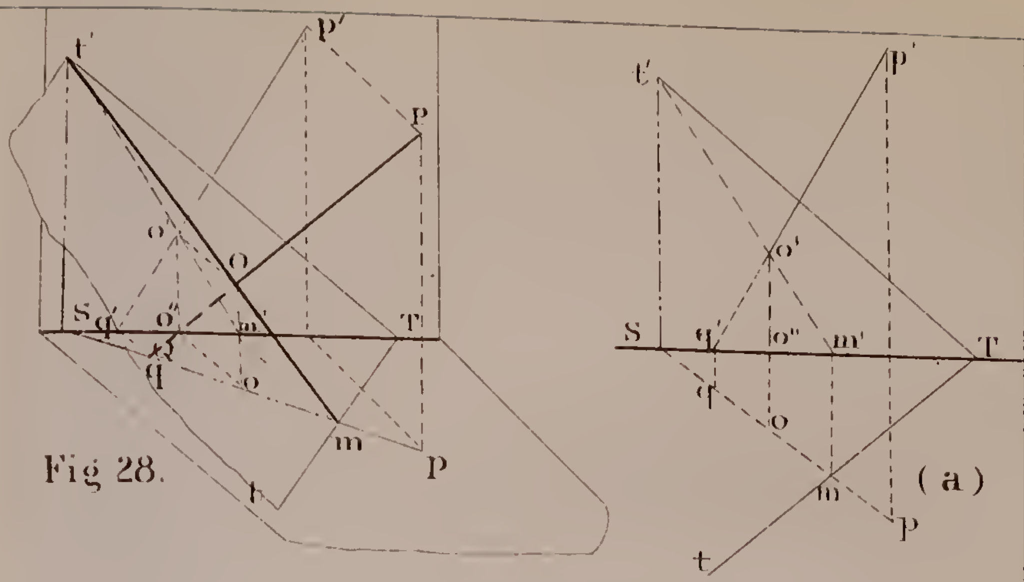


Fig 28.

(a)

APPLICATIONS

Fig 29.

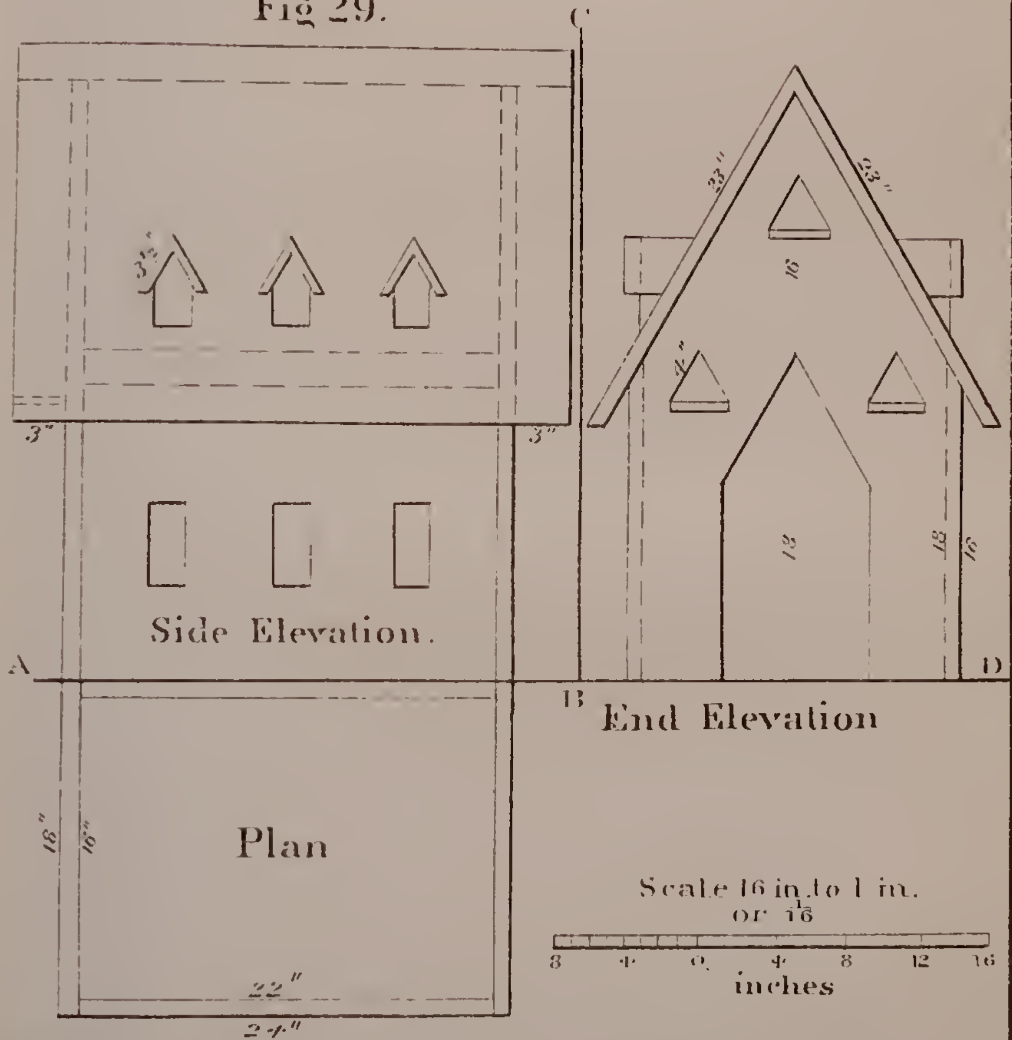


FIG. 30.

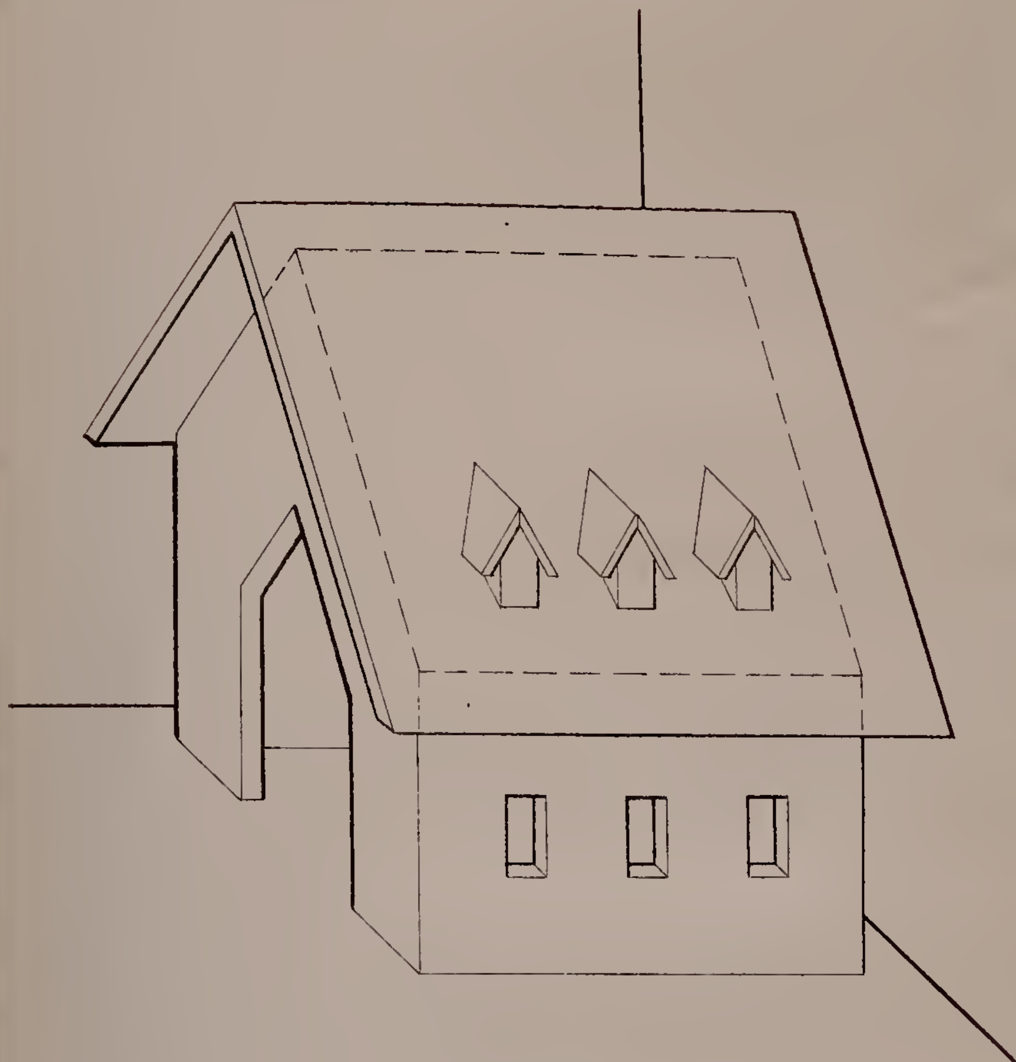
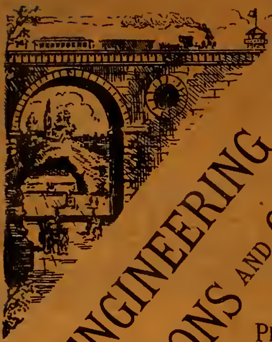


Fig 30



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