intellus LEARNING OPEN COURSES

Motion Along a Straight Line

Module Overview

Acknowledgments

This presentation is based on and includes content derived from the following OER resource:

University Physics Volume 1

An OpenStax book used for this course may be downloaded for free at: https://openstax.org/details/books/university-physics-volume-1

Position and Displacement

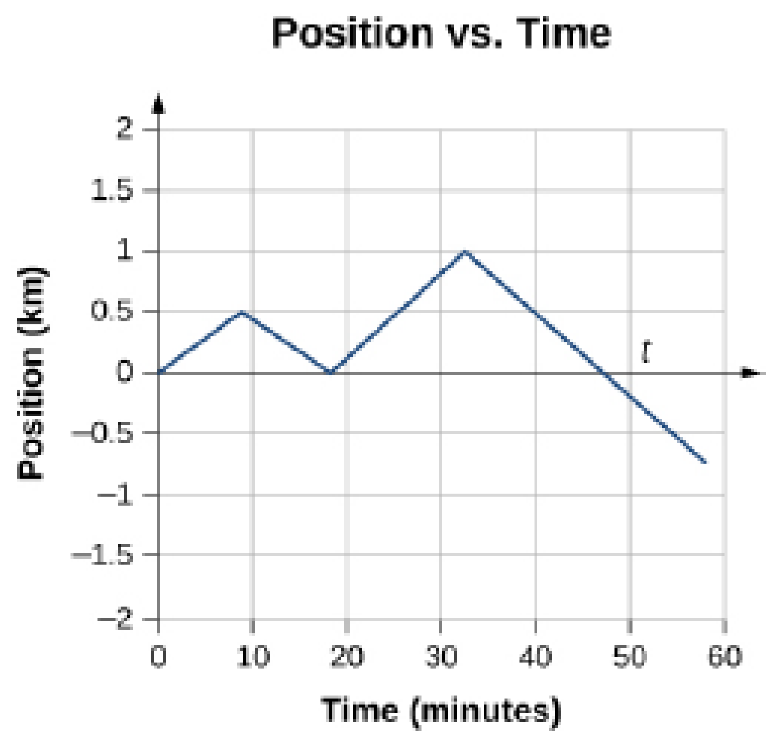
The **position** of an object is its location at any particular time. An object's **displacement,** Δx, is a vector that describes a change in its position. It is defined by the equation, $\Delta x = x_{\mathrm{f}} - x_0$, where x_0 and x_{f} are the initial and final positions, respectively.

If an object undergoes a series of displacements, its **total displacement** is given by the sum of each individual displacement, $\Delta x_{\mathrm{Total}} = \sum \Delta x_{\mathrm{i}}$, where the Δx_{i} are the individual displacements. **Distance traveled** is the sum of the magnitudes of individual displacements, $x_{\mathrm{Total}} = \sum |\Delta x_{\mathrm{i}}|$. Distance traveled is always greater than or equal to total displacement.

Average Velocity

The **average velocity** of an object, $\bar{v}$, is the vector quantity given by the displacement between two points divided by the time taken to travel between them. The **elapsed time**, Δt, is the time taken to travel between two points.

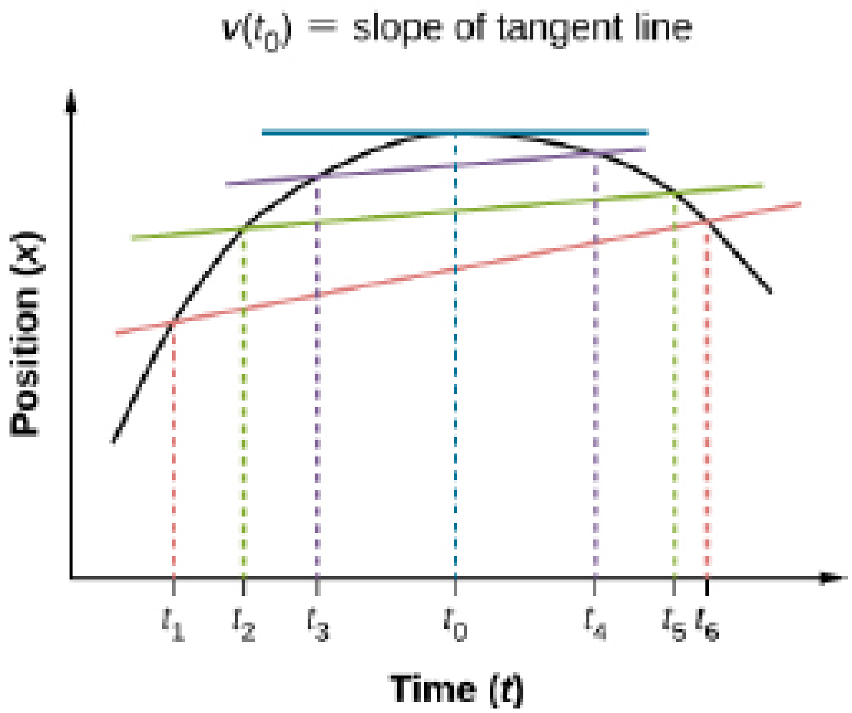
Mathematically, the average velocity is written as $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$.



(University Physics Volume 1. OpenStax. Fig. 3.5)

Instantaneous Velocity

The quantity that tells us how fast an object is moving anywhere along its path is the **instantaneous velocity**, or just velocity. Instantaneous velocity is defined as the time derivative of position, $v(t) = \frac{d}{dx}x(t)$.



(University Physics Volume 1. OpenStax. Fig. 3.6)

Speed

In physics, speed and velocity are not the same because speed is a scalar with no direction, unlike velocity.

- **Average speed**, $\bar{s}$, is total distance traveled divided by elapsed time.
- **Instantaneous speed** is the magnitude of instantaneous velocity.

$$\text{Average speed} = \bar{s} = \frac{\text{Total distance}}{\text{Elapsed time}}$$

$$\text{Instantaneous speed} = |v(t)|$$

Calculating Instantaneous Velocity

To calculate instantaneous velocity, we need the explicit form of the position, $x(t)$.

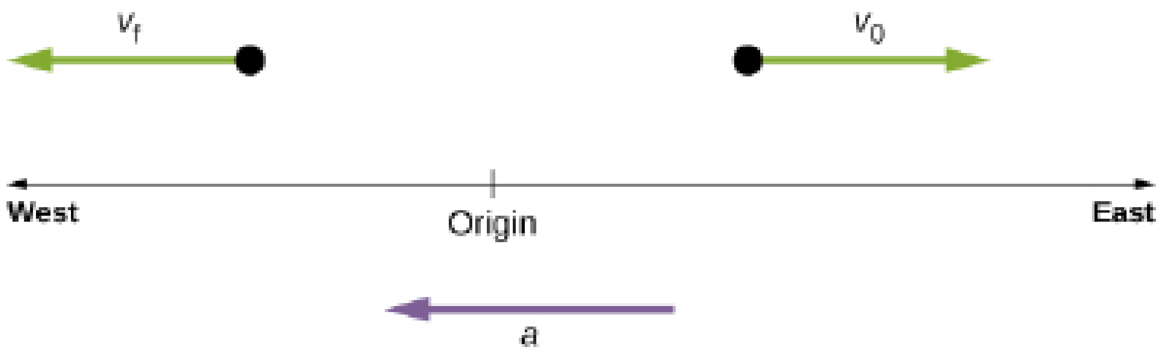
Let $x(t) = At^n$. Then the velocity is easily found by differentiating.

$$x(t) = At^n$$

$$v(t) = \frac{d}{dx}x(t) = nAt^{n-1}$$

Average Acceleration

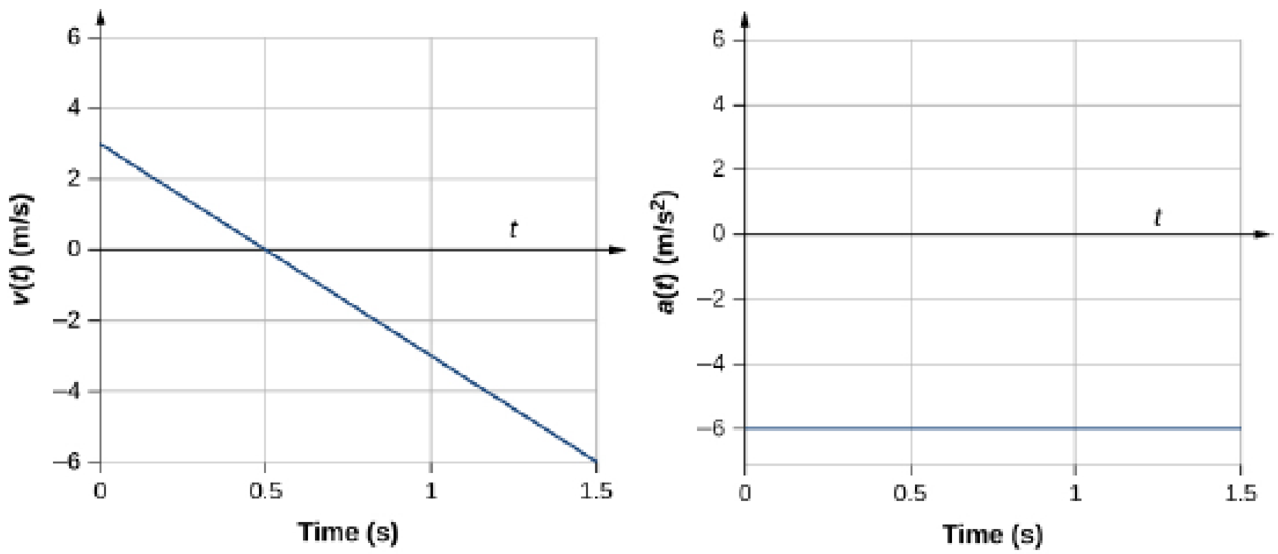
Average acceleration, $\bar{a}$**,** is the rate at which an object's velocity changes, $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\mathrm{f}} - v_0}{t_{\mathrm{f}} - t_0}$. Acceleration is a vector that points in the direction of velocity change. Since velocity is a vector, acceleration is a change in speed, direction, or both.



(University Physics Volume 1. OpenStax. Fig. 3.11)

Instantaneous Acceleration

Instantaneous acceleration, a, is the acceleration of an object at a specific instant in time. It is expressed mathematically as the derivative of velocity, $a(t) = \frac{d}{dt}v(t)$. As Δt approaches 0, $\bar{a}$ approaches a.



(a) Velocity (b) Acceleration

(University Physics Volume 1. OpenStax. Fig. 3.15)

Getting a Feel for Acceleration

acceleration event	acceleration (m/s^2)
high-speed train	0.25
object in freefall at sea level	9.8
baseball struck by a bat	30,000
proton in the Large Hadron Collider	1.9×10^{19}

Motion with Constant Acceleration

The first special case we consider is motion with constant acceleration. The average and instantaneous accelerations are equal, $\bar{a} = a = \text{constant}$. Constant acceleration occurs in many scenarios. Even in scenarios where acceleration is not constant, we can often approximate the motion by assuming constant acceleration equal to the average acceleration. When acceleration does differ drastically, the motion can often be broken into separate parts with approximately constant acceleration for each part.

Displacement and Position from Velocity

Starting with the definition of average velocity, substitute in expressions for Δx and Δt. Solving for x gives an expression for position as a function of time.

Under constant acceleration, the average velocity is simply the average of the initial and final velocities.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$$

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

Solving for Final Velocity from Acceleration and Time

Starting with the definition of average acceleration, substitute in expressions for Δv and Δt. Solving for v gives an expression for velocity as a function of time.

Final velocity depends on how large the acceleration is and how long it lasts.

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$$

$$v = v_0 + at$$

Solving for Final Position with Constant Acceleration

Starting with the equation for velocity in terms of acceleration and time, add v_0 to both sides and divide by 2. The left side is equal to average velocity for constant acceleration.

Substitute into the equation for x to get a third kinematic equation.

Displacement depends on the square of the time elapsed unless $a = 0$.

$$v = v_0 + at$$

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at = \bar{v}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Solving for Final Velocity from Distance and Acceleration

Start by solving the equation for final velocity, $v = v_0 + at$, for t.

Substitute the result and the equation for average velocity into the equation for final displacement.

Final velocity depends on how large the acceleration is and the distance over which it acts.

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a},\ \bar{v} = \frac{v_0 + v}{2}, \text{ and } x = x_0 + \bar{v}t$$

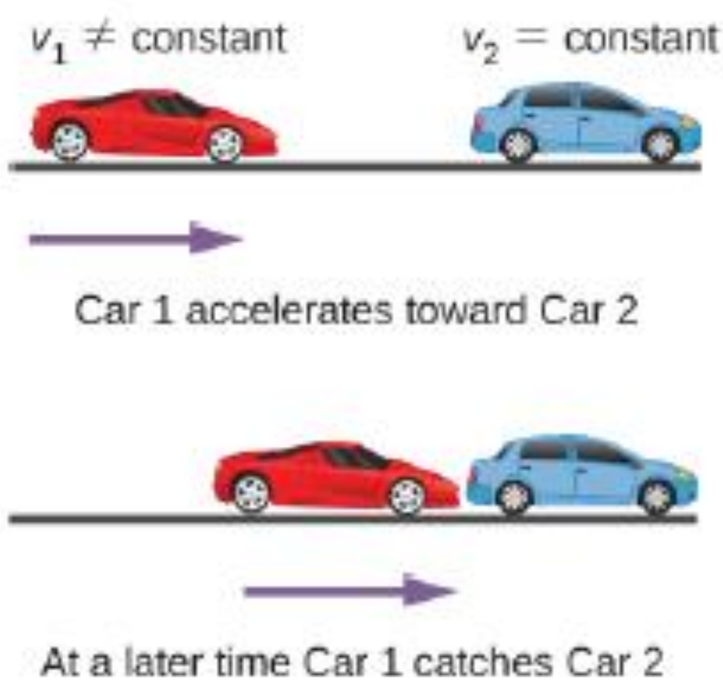
$$v^2 = v_0^2 + 2a(x - x_0)$$

Putting Equations Together

kinematic equations for constant acceleration
$x = x_0 + \bar{v}t$
$\bar{v} = \dfrac{v_0 + v}{2}$
$v = v_0 + at$
$x = x_0 + v_0 t + \dfrac{1}{2}at^2$
$v^2 = v_0^2 + 2a(x - x_0)$

Two-Body Pursuit Problems

In **two-body pursuit problems**, the motion of two objects is coupled. In order to solve for unknowns, the equations of motion must be written down for both objects and solved simultaneously.



(University Physics Volume 1. OpenStax. Fig. 3.25)

Gravity

If air resistance and friction are negligible, all objects at a given location fall with the same constant acceleration, independent of their mass. This ideal situation is called **free fall**.

The **acceleration due to gravity**, g, has the average value $g = 9.81\ \mathrm{m}/s^2$. The direction of the acceleration due to gravity is what defines our use of the term vertical.

One-Dimensional Motion Involving Gravity

We consider the special case of vertical motion in a straight line involving gravity. In this case, we use the position coordinate y, and the acceleration is $-g$, since the acceleration due to gravity is downward.

kinematic equations for constant acceleration
$v = v_0 - gt$
$y = y_0 + v_0 t - \frac{1}{2} g t^2$
$v^2 = v_0^2 - 2g(y - y_0)$

Kinematic Equations from Integral Calculus, Part 1

By integrating the definitions of acceleration and velocity, we can derive the kinematic equations. For constant acceleration, we can evaluate the integrals explicitly. The integral of velocity gives the previously derived expression for final velocity.

$$\frac{d}{dt}v(t) = a(t)$$

$$v(t) = \int a(t)dt + C_1$$

$$v(t) = at + C_1$$
$$v(0) = C_1 = v_0$$
$$v(t) = v_0 + at$$

Kinematic Equations from Integral Calculus, Part 2

Substituting the expression for velocity into its definition in terms of the time derivative of position allows the integral to be directly evaluated. The integral of position gives the previously derived expression for final position.

$$\frac{d}{dt}x(t) = v(t)$$

$$\mathrm{x}(t) = \int (v_0 + at)dt + C_2$$

$$x(t) = v_0 t + \frac{1}{2}at^2 + C_2$$

$$x(0) = C_2 = x_0$$

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

intellus LEARNING OPEN COURSES

How to Study this Module

- Read the syllabus or schedule of assignments regularly.
- Understand key terms; look up and define all unfamiliar words and terms.
- Take notes on your readings, assigned media, and lectures.
- As appropriate, work all questions and/or problems assigned and as many additional questions and/or problems as possible.
- Discuss topics with classmates.
- Frequently review your notes. Make flow charts and outlines from your notes to help you study for assessments.
- Complete all course assessments.

intellus
LEARNING
OPEN COURSES

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.