ACCURACY ANALYSIS FOR A LOWER CONFIDENCE LIMIT PROCEDURE FOR SYSTEM RELIABILITY

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THESIS

ACCURACY ANALYSIS FOR
A LOWER CONFIDENCE LIMIT PROCEDURE
FOR SYSTEM RELIABILITY

by

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September 1976

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by

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ABSTRACT

This thesis examines a proposed empirical method for determining the the $100(1-\alpha)\%$ lower confidence limit for the reliability of a system composed of a mixture of series and parallel connected components for which only component level test data is available. The method is an extension of the Log-gamma procedure originally proposed for series connected systems only. The accuracy of the method is assessed for some representative system reliability constructs using a computer simulation procedure. The simulation results are examined with a view toward identification of accuracy indication parameters which may be estimated prior to the component tests.

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I. INTRODUCTION

Often, in the design and production of complex systems, an important question to be answered early in the process concerns the demonstrable system reliability based upon observed mission test results. Usually only components and minor subassemblies can be tested in any but the very latest stages of production. For reasons of prototype cost, the lack of complete systems for early tests, the expense involved in total system operational tests, and the need to make production decisions as early as feasible, reliance must be shifted to the component test results to provide at least an approximate answer to this question. Component data must be used in spite of the moderately strong assumptions needed to transform such test results into an estimate of overall system reliability.

Since estimates are, by their nature, inexact, it would be reasonable to expect that the most credible estimate would be one which was both conservative and carried with it an indicator of the degree of conservatism associated with its computation. This, of course, is a description fitting a lower confidence limit when referring to estimates of system reliability. Thus, when one speaks of a "$100(1-\alpha)$% lower confidence limit" and obtains a value for it, the assertion is that the obtained lower confidence limit value is less than the actual system reliability with probability $1-\alpha$.

In 1968, J. R. Borsting and W. M. Woods developed what is known as the Log-Gamma method for computing lower confidence limits for system reliability based upon

component mission test results only [Ref. 1]. As originally developed, the Log-Gamma method was restricted to systems composed of all series connected components. It did, however, offer the advantage of allowing for unequal sample sizes, so the number of component tests could vary arbitrarily from component to component without degrading the procedure accuracy. In addition, no assumption is needed concerning the failure distribution of any component.

Woods has recently proposed a modification to the procedure which, it was hoped, would extend its application to a mixed system of series and parallel connected components. The purpose of this study is to assess the accuracy of the revised procedure when applied to some typical component failure data from several representative series and series-parallel systems. An attempt is also made to discern parameters (constructed from or inherent to the sample data, component reliability goals, testing plan, and system design) which could provide useful indicators of the expected accuracy of the procedure.

II. THE PROCEDURAL MODEL

It should be noted that two rather strong assumptions need to be made with respect to system and component reliabilities. First, it must be assumed that component tests were conducted in a test environment not unlike that encountered in the system operational mission environment profile. Second, no component reliability degradation will result from interactions with other components when assembled into the system. The degree to which these two assumptions are met will have an unknown but potentially great effect upon the validity of the model and indeed upon the whole concept of system reliability prediction from component attributes data.

A. DEVELOPMENT FOR SERIES SYSTEMS

This section gives a modified and elaborated development of the basic reliability model and estimation procedure based upon that presented by Woods and Borsting [Ref. 1].

For a system of k components in logical series, the system reliability, Rs, can be expressed in terms of the individual component reliabilities as the product

$$Rs = \prod_{i=1}^{k} Pi \qquad (2.1)$$

where Pi is the true reliability of the i-th component. From Pi the component unreliability, Qi, is defined as

$$Qi = 1-Pi.$$

Taking the natural logarithm in equation (2.1) and defining the new variable, S:

$$S = -\ln Rs = -\sum_{i=1}^{k} \ln(1-Qi) \qquad (2.2)$$

The natural logarithm can be expanded by the infinite series

$$\ln(1-X) = -\sum_{j=1}^{\infty} (X)^j/j \qquad \text{for } 0 \le X \le 1$$

$$S = \sum_{i=1}^{k} \sum_{j=1}^{\infty} (Qi)^j/j \qquad (2.3)$$

$$S = \sum_{i=1}^{k} [\,Qi + Qi^2/2 + Qi^3/3 + \ldots\,]$$

For Qi small, say 0.1 or less, the above series may be approximated by the first two terms with a maximum error of 0.34% of the true value. Thus,

$$S \doteq \sum_{i=1}^{k} [\,Qi + Qi^2/2\,] = \sum_{i=1}^{k} Ti \qquad (2.4)$$

where $Ti = Qi + Qi^2/2$. For the remainder of the derivation this approximation is treated as an equality.

Appendix A derives an unbiased estimator, $\hat{T}i$, for Ti:

$$\hat{T}i = Ai\hat{Q}i + Bi[\hat{Q}i^2]/2 \qquad (2.5)$$

where $Ai = [2Ni-3]/[2Ni-2]$

$Bi = Ni/[Ni-1]$

$\hat{Q}i = Fi/Ni$

and, Fi is the number of failures resulting from Ni mission tests conducted on the i-th component.

An unbiased estimator of S will be obtained from

$$\hat{S} = \sum_{i=1}^{k} \hat{T}i \tag{2.6}$$

$\hat{S}$ will be an unbiased estimator of S, since it is the sum of the unbiased estimators, $\hat{T}i$, of Ti.

Appendix B shows that, for the reasonably expected range of values for Qi and Ni, the variance of $\hat{S}$ may be approximated

$$Var[\hat{S}] = \sum_{i=1}^{k} Var[\hat{T}i] \doteq \sum_{i=1}^{k} [Ti/Ni] \tag{2.7}$$

By intuitively assuming the probability distribution of $\hat{S}$ is of the gamma type with parameters r and θ, an expression can be derived for a confidence interval for Rs.

The gamma probability density may be written as

$$f_{\hat{S}}(x,r,\theta) = \begin{cases} \dfrac{1}{\Gamma(r) \cdot \theta^{r}} x^{r-1} e^{-x/\theta} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{2.8}$$

with mean = rθ and variance = $r\theta^2$.

To determine the values of r and θ, the following relationships are established:

$E[\hat{S}] = r\theta$ from Eq. (2.8)

$E[\hat{S}] = \sum_{i=1}^{k} Ti$ from Eq. (2.4)

$Var[\hat{S}] = r\theta^2$ from Eq. (2.8)

$Var[\hat{S}] = \sum_{i=1}^{k} \frac{Ti}{Ni}$

These equations may be solved simultaneously to yield the

following expressions for r and θ:

$$r = \frac{\left[\sum_{i=1}^{k} Ti\right]^2}{\sum_{i=1}^{k} \frac{Ti}{Ni}} \quad \text{and} \quad \theta = \frac{\sum_{i=1}^{k} \frac{Ti}{Ni}}{\sum_{i=1}^{k} Ti}$$

and an estimator, $\hat{r}$, for r would be

$$\hat{r} = \frac{\left[\sum_{i=1}^{k} \hat{T}i\right]^2}{\sum_{i=1}^{k} \frac{\hat{T}i}{Ni}} \qquad (2.9)$$

Note that for $\hat{S}$ distributed as gamma(r,θ), $2\hat{S}/\theta$ will be distributed as chi-square with 2r degrees of freedom [Ref. 2, p. 181]. The following probability statements can then be made:

$$1-\alpha = P\left[\frac{2\hat{S}}{\theta} \geq \chi^2_{2r,1-\alpha}\right] \qquad (2.10)$$

$$1-\alpha = P\left[\theta \leq \frac{2\hat{S}}{\chi^2_{2r,1-\alpha}}\right]$$

Since $\theta = E[\hat{S}]/r$ from Eq. (2.8) and $E[\hat{S}] = -\ln Rs$ from Eq. (2.2),

$$1-\alpha = P\left[\frac{-\ln Rs}{r} \leq \frac{2\hat{S}}{\chi^2_{2r,1-\alpha}}\right]$$

$$1-\alpha = P\left[\ln Rs \leq \frac{-2r\hat{S}}{\chi^2_{2r,1-\alpha}}\right]$$

Thus the 100(1-α)% lower confidence limit, $Rs^*(\alpha)$, for Rs is

$$Rs^*(\alpha) = \exp\left[\frac{-2r\hat{S}}{\chi^2_{2r,1-\alpha}}\right] \qquad (2.11)$$

A 100(1-α)% lower confidence limit estimate, $\hat{R}s^*(\alpha)$, for Rs would be

$$\hat{R}s^*(\alpha) = \exp\left[\frac{-2\hat{r}\hat{s}}{\chi^2_{2\hat{r},1-\alpha}}\right] \qquad (2.12)$$

The primary difficulty this formulation presents lies in the non-integer nature of values for $2\hat{r}$ as defined. Although some computer routines are available to approximate chi-square values with non-integer degrees of freedom, most users would find such a requirement difficult to fulfill at least. Therefore $2\hat{r}$ is replaced with the expression

$$[2\hat{r}] = \text{smallest integer} \geq 2\hat{r}$$

This is justified by noting that the ratio of the chi-square value divided by its number of degrees of freedom varies slowly with changes in the degrees of freedom for values of degrees of freedom greater than about five. Further, since $2\hat{r}$ may be changed moderately, by up to 1.0, without great change in the ratio value, it is reasonable to expect that the probability distribution of $[2\hat{r}]/\chi^2_{[2\hat{r}],1-\alpha}$ has small variance. This expression can be subsittuted into equation (2.12) to yield

$$Rs^*(\alpha) = \exp\left[\frac{-[2\hat{r}]\hat{s}}{\chi^2_{[2\hat{r}],1-\alpha}}\right] \qquad (2.13)$$

Since the computer routines available for this research included the capability of computation of chi-square values with non-integer degrees of freedom, some simulation runs were performed using both equations (2.12) and (2.13) for comparison. The results indicated that the effects were limited to the third decimal place at worst and thus had little significance with respect to the procedural accuracy.

B. REVISION FOR SERIES-PARALLEL SYSTEM

Proceding from the strictly series system procedure developed above, the principal revision for the mixed series-parallel system procedure was to redefine $\hat{T}i$ and $\hat{r}$ as follows:

$$\hat{T}i = \begin{cases} Ai\dfrac{Fi}{Ni} + Bi\dfrac{Fi^2}{2Ni^2} & i=1,2,\ldots,K1 \\ (1-\hat{R}i) + (1-\hat{R}i)^2/2 & i=K1+1,\ldots,K2 \end{cases} \quad (2.14)$$

$$\hat{r} = \max\left[1.0, \frac{\left[\sum_{i=1}^{K1+K2} \hat{T}i\right]^2}{\sum_{i=1}^{K1+K2} \dfrac{\hat{T}i}{Ni}}\right] \quad (2.15)$$

$$\hat{S} = \sum_{i=1}^{K1+K2} \hat{T}i \quad (2.16$$

where Ai, Bi, Fi, and Ni are defined as before

K1 is the number of series components

K2 is the number of series-parallel subassemblies interconnected in series

$\hat{R}i$ is the reliability point estimate for the i-th subassembly determined in the usual manner from the individual point estimates for the subassembly components as derived from the raw failure data.

Although some comparison runs were made using Ai and Bi coefficients for the subassembly contribution to $\hat{S}$, no significant improvement in accuracy was noted and the use of these coefficients in this manner was abandoned due to the difficulty of computation with varying sample sizes within a subassembly.

C. SPECIAL CIRCUMSTANCES

In the event of no failures in any components or a lack of sufficient failures in subassembly components such that the basic procedure yields an $\hat{S}$ value of zero and, thus, a system reliability lower confidence limit estimate of one, an alternate approach must be employed.

Statistically, the optimum solution might be to continue testing until sufficient component failures have been observed such that the above conditions no longer exist. However, such a solution might also not be economically feasible. Therefore, the simulation included provision for an alternate procedure when $\hat{S}$ equals zero at the end of the programmed number of tests on all components. In addition, several cases were included where such conditions would be expected to exist in a significant number of the simulation runs, in order to observe the efficacy of the alternate procedure.

The derivation of this alternate procedure is given in Appendix C. The procedure may be summarized as follows:

(i) Find the series component or subassembly with the smallest actual or equivalent number of trials, N or N', respectively. (See section IV.B.1)

(ii) Change the number of component or subassembly failures to F' by the following scheme:

α	F'
0.2	0.37
0.1	0.25
0.05	0.16

NOTE: For best results, N or N' should be at least 10.

If a subassembly was selected, treat the subassembly as a component with F' failures out of N' mission trials for the remainder of this procedure.

(iii) Recompute the lower confidence limit estimate using the basic log-gamma procedure with the revised failure data. This estimate is approximately equivalent to α raised to the 1/N power as explained in Appendix C.

III. THE SIMULATION

A. PROCEDURE

The computer simulation program is listed in Appendix F. The following procedure was incorporated into the program:

Step One: Initialize and read in values for the total number of components in series, K1, the number of subassemblies in series, K2, the number of components within each subassembly, the number of tests performed upon each component, Ni, the true reliability of each component, Ri, and the seed value for the random number generator.

Step Two: For each component in turn, generate the observed number of failures as a binomially distributed random variable, Fi, with parameters Ni and (1-Ri).

Step Three: Compute $\hat{T}i$ for each series component and subassembly, $\hat{S}$ from the sum of the $\hat{T}i$, and $\hat{r}$ according to equations (2.14), (2.15), and (2.16) respectively.

Step four: Compute the resulting estimate of the $100(1-\alpha)\%$ lower confidence limit, $\hat{R}^{*(\alpha)}$, according to equation (2.13) for representative values of alpha and store each estimate in a vector array of estimates. Alpha values used in this study were 0.20, 0.10, and 0.05.

Step Five: Repeat Steps Two through Four until the three vector arrays of estimates corresponding to the three

different alpha values are 1000 values in length.

Step Six: Order the elements of each vector array from low to high and determine the sample mean and standard deviation.

Step Seven: Compute the true system reliability, Rs, from the input component reliabilities.

Step Eight: Extract the $100(1-\alpha)$ percentile value from the corresponding array and compare with the true system reliability. If the estimation procedure is accurate these two values should agree. If agreement does not occur, the differences should be small, preferably conservative, and, hopefully, can be made to converge toward zero.

Step Nine: Output all data which was input in Step One in order to insure against input error. Output starting and ending random number generator seed values. Output true system reliability, $100(1-\alpha)$ percentile values, differences from true reliability, sample means and standard deviations, and various percentile values of interest. Provision is made in the program for output of all of the sample values both before and after ordering, if the user desires.

Step Four was performed in two different ways for four of the cases where there was a high frequency of runs without failures, called 'null runs' in the output. The normal way of handling this problem was to discard that run data and repeat it from step one. Thus the complete arrays contained estimates from only those runs which had sufficient failures for the procedure to be valid without the special circumstances modification discussed in Section II.C. In order to observe the effect of utilizing the modification, however, cases 18 through 21 were repeated as cases 30 through 33 with the alternate procedure invoked

whenever $\hat{S}$ equalled zero at the end of step three.

Three proprietary routines were utilized in the program and are therefore not reproduced in the program listing. These routines were used to obtain the binomially distributed random variables, the chi-square function values, and to sort the estimate arrays [Ref. 4].

B. DESCRIPTION OF THE CASES

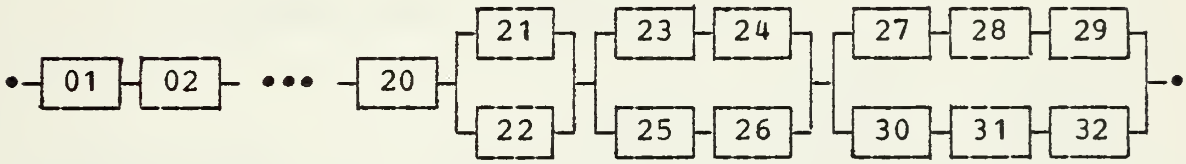
1. General

The cases chosen for simulation represent three different configurations; a series of components connected in series with a series of simple series-parallel subassemblies of components, a strictly series-connected system of components, and a system which incorporates two somewhat unusual but representative "real-world" subassembly reliability relationships. The first configuration was chosen to demonstrate the general performance of the revised estimation procedure with a moderately complex, series-parallel system. The second configuration consisted primarily of some cases which had previously been investigated for the original Log-gamma procedure [Ref. 1]. Since the revised procedure varied somewhat from the originally developed version in the treatment of the wholly series system, a comparison was desirable in order to ascertain whether significant effects had occurred in the procedure accuracy. The third configuration was chosen to observe the robustness of the procedure when applied to a much more complex system which is not easily reducable to a simpler series-parallel reliability representation.

In general, the simulated number of tests was the same for each component within each case. However, this number was varied among cases with the same true system reliability in order to identify its effect upon the procedure accuracy. The two exceptions were case 13 wherein the number of tests varied from component to component and case 14 for which only one value of simulated component tests was used without any corresponding cases with different numbers of tests.

2. Cases One Through Twelve

a. Reliability Block Diagram



b. Test Parameters

(1) Number of Component Tests, Ni.

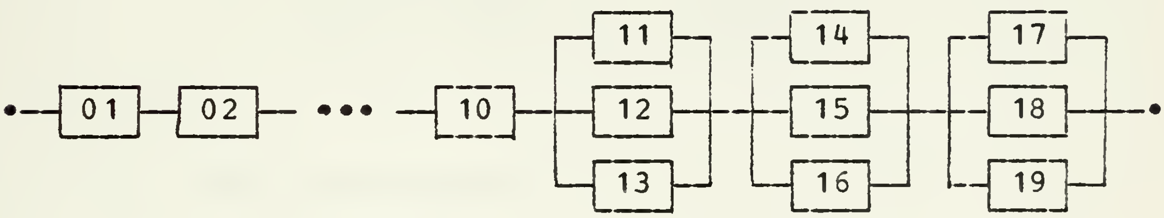
Cases	Ni	
01, 04, 07, 10	20	i=1,2,...,32
02, 05, 08, 11	50	i=1,2,...,32
03, 06, 09, 12	100	i=1,2,...,32

(2) True System and Component Reliabilities, Rs and Ri.

Cases	Rs	Ri	
01, 02, 03	.87559	.995	i=1,2,...,20
		.950	i=21,...,32
04, 05, 06	.79984	.995	i=1,2,...,20
		.900	i=21,...,32
07, 08, 09	.79167	.990	i=1,2,...,20
		.950	i=21,...,32
10, 11, 12	.72317	.990	i=1,2,...,20
		.900	i=21,...,32

3. Case Thirteen

a. Reliability Block Diagram



b. Test Parameters

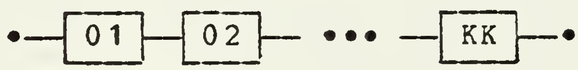
i	Ni	Ri	i	Ni	Ri	i	Ni	Ri
1	150	.995	7	28	.967	13	59	.682
2	90	.985	8	125	.995	14	5	.729
3	75	.979	9	63	.970	15	5	.729
4	100	.988	10	125	.995	16	5	.729
5	125	.982	11	59	.682	17	19	.536
6	18	.980	12	59	.682	18	19	.536
Rs: 0.72329						19	19	.536

c. Discussion

This case corresponds to Case No. 1 of reference 1. The only change was to replace the last three components each with three components in parallel with a subsystem reliability equivalent to the reliability of the replaced component.

4. Cases Fourteen Through Twenty-one and Thirty Through Thirty-three

a. Reliability Block Diagram



b. Test Parameters

(1) Number of Component Tests, Ni.

Cases	Ni	
14	20	i=1,2,...,15
15,18,30	25	i=1,2,...,25
16,17,31	50	i=1,2,...,25
17,20,32	100	i=1,2,...,25
21,33	200	i=1,2,...,25

(2) True System and Component Reliabilities, Rs and Ri.

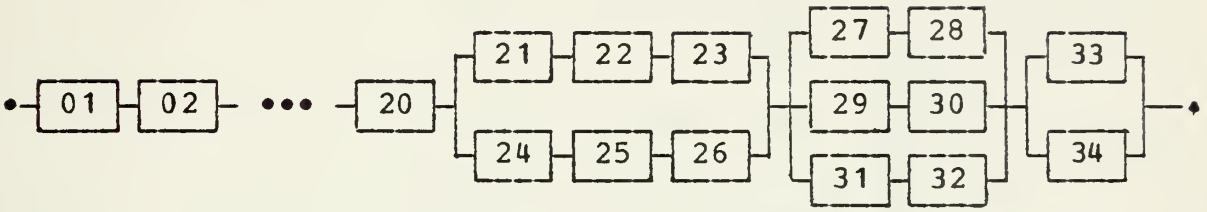
Cases	Rs	Ri	
14	.79239	.995	i=1,2,...,14
		.850	i=15
15,16,17	.88222	.995	i=1,2,...,25
18,19,20,21 30,31,32,33	.97530	.999	i=1,2,...,25

c. Discussion

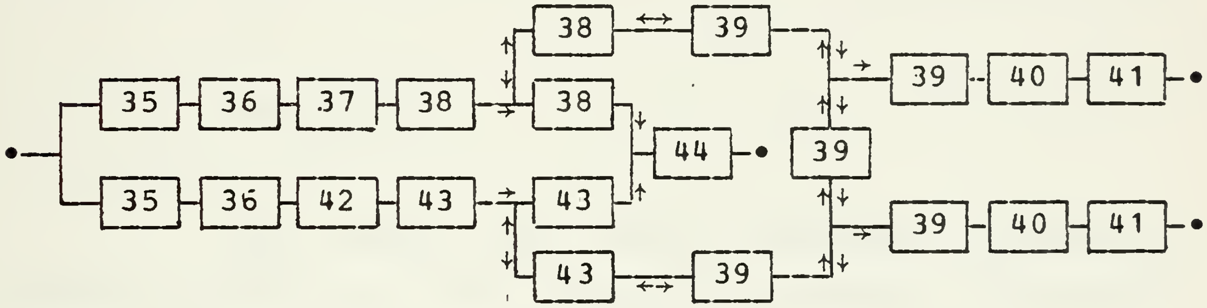
These cases correspond to several cases from Ref. 1 with the exception of cases 21 and 33 of the present study. These were added when it was noted that a large number of runs with zero failures were being observed with as many as 100 tests on each component. Cases 30 through 33 are identical with cases 18 through 21. They are identified as separate cases, since a slightly different procedure, as described in section III.A, was followed in their simulation.

5. <u>Cases Twenty-two Through Twenty-nine</u>

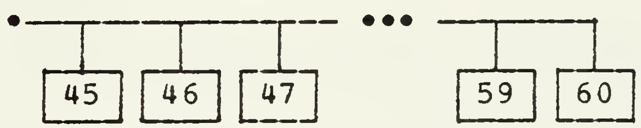
a. Reliability Block Diagram



In logical series with



In logical series with



b. Test Parameters

(1) <u>Number of Component Tests, Ni.</u>

Cases	Ni	
22,26	20	i=1,2,...,60
23,27	25	i=1,2,...,60
24,28	50	i=1,2,...,60
25,29	100	i=1,2,...,60

(2) <u>True System and Component Reliabilities, Rs and Ri.</u>

Cases	Rs	Ri	
22 through 29	(See below)	.999	i=1,2,...,26
		.886	i=27,28,...,32
		.900	i=33,34
		.999	i=35,36,...,41
		.990	i=42,43,44
22 to 25	.87306	$(.995)^j$	j=i-44, i=45,46,...,60
26 to 29	.83293	$(.990)^j$	j=i-44, i=45,46,...,60

c. Discussion

The second and third subassembly reliability diagrams above were drawn from the Lockheed Reliability Evaluation Plan for the Trident Missile [Ref. 3].

In the second diagram, component 44 and both components 41 must operate successfully for system success. An additional feature of this design is the capability of obtaining successful functioning of component 44 by following a logical path through component 39 and continuing in a reverse direction to reach component 44 from the opposite side of the diagram, if necessary. This capability is illustrated with double arrows to indicate that the logical success path may be traced in either direction for that portion of the reliability diagram. Note also that identically numbered components are, in fact, separate components but identical in operation. Therefore, component test results for a particular component type are assumed to be equally applicable to all components of that type, regardless of the specific application. The reliability

equation for this portion of the diagram, as adapted from reference 3, is as follows:

$$R_{SS} = M_4 \Big[T_1T_2T_3L_3M_3 \,[1 - (1-M_1)(1-L_2M_2)]$$
$$+ (1-T_1)L_1L_2L_3T_3M_3 \,[1 - (1-M_2)(1-T_2M_1)]\Big]$$
$$+ (1-M_4) \Big[M_1M_2M_3T_2T_3L_2L_3 \,[1 - (1-T_1)(1-L_1)]$$
$$+ (1-M_1)\, M_2M_3T_1T_2T_3L_1L_2L_3$$
$$+ (1-M_2)\, M_1M_3T_1T_2T_3L_1L_2L_3\Big] \qquad (3.1)$$

where

$T_1 = R_{35}R_{36}R_{37}R_{38}$ $\qquad M_1 = R_{38}$

$T_2 = R_{38}R_{39}$ $\qquad M_2 = R_{43}$

$T_3 = R_{39}R_{40}R_{41}$ $\qquad M_3 = R_{44}$

$L_1 = R_{35}R_{36}R_{42}R_{43}$ $\qquad M_4 = R_{39}$

$L_2 = R_{43}R_{39}$ $\qquad L_3 = T_3$

The third subassembly diagram illustrates another type of reliability problem in that the subassembly reliability is defined in Ref. 3 as follows:

$$R_{SS} = \frac{\sum_{i=45}^{60} R_i}{16}$$

Although these diagrams represent current systems, the reliability values chosen bear no intentional relation to actual values other than that acquired when choosing them on a criteria of reasonableness. Therefore the values used are not classified.

IV. RESULTS AND CONCLUSIONS

A. SIMULATION RESULTS

The simulation output is listed in detail in Appendix D for each run. Each case was simulated three times with different random number generator seed values to insure against the occurrance of non-representative singularities in the observed data. Each simulation is distinguished by a letter suffix, a, b, or c, to the case number in Appendix D. The starting seed used each time was the ending seed value from the previous run. This insured that each run would be independent [Ref. 5].

The demonstrated accuracy of the estimation procedure can be measured in terms of the difference between the $100(1-\alpha)$ percentile of the simulation results, $\hat{R}^{*o}$, and the true system reliability, Rs. This difference, labeled D in the output tables, is defined as follows:

$$D = \hat{R}^{*o} - Rs \qquad (4.1)$$

As mentioned in Section III.A, the absolute value of D should be small and reflect a conservative error. This desired conservativism is satisfied so long as the sign of D is negative. Otherwise the lower confidence limit estimation procedure would yield overly-optomistic estimates and could not be a valid vehicle for the description of system reliability. The one instance where this condition was not satisfied occurred in Case 13c. However, the value of D was extremely small, represented less than one-half of

one percent error, and differed from the conservative results of the other runs by less than one percent.

It should also be noted that the distributions of the estimates $\hat{T}i$, $\hat{S}$, and $\hat{r}$ are discrete in nature and the values are determined by the number and arrangement of components and the number of component tests. Estimates based upon few failures will exhibit the most highly discrete sample distribution scheme. The effect of this discreteness will generally be to increase the observed error magnitude unless some form of continuity correction can be developed.

B. ANALYSIS OF RESULTS

1. General Characteristics

The simulation output for Cases One through Twelve was expanded to include the complete ordered arrays of estimated confidence limit values in order to establish the nature of the sample. Some properties were evident from inspection. The samples were unimodal and discrete, as expected. Discreteness was most pronounced in the higher valued estimates. The sample distribution appeared nearly symmetric and this supposition was checked by comparing the sample mean and median values and also by plotting complementary quantiles. The first test showed very close agreement throughout. The second test indicated that only minor left skewness exists in most cases. Skewness seemed more pronounced for higher values of system reliability, as might be expected might be suspected, since the distribution is bounded by an upper estimate value of 1.0.

The shape of the distribution was further described

by examining the ratio of the interquartile range divided by the interdecile range and comparing it with the same ratio from the Normal distribution which is about 0.526. The observed values were generally on the order of 0.50 which is in good agreement considering the discrete nature of the sample distribution. Systems designed with high reliability coupled with insufficient testing, such as Cases 18 and 19, exhibited great variations in the ratio, from 0.2 to 1.0, due to the sparcity of different sample values occuring in the pronouncedly discrete sample distribution.

Another useful measure of shape is the sample standard deviation. It is particularly helpful in quantifying the spread of the data in terms of the units of measure. The preceeding ratio, on the other hand, was independent of the units. The sample standard deviation, however, gives no indication of skewness nor the discreteness of the underlying distribution. The standard deviation is most useful when these higher order effects are small.

The ability to predict the standard deviation from the test plan and system design parameters would provide an additional measure of the system reliability. This concept evolves from a consideration of the 100(1-α) percent lower confidence limit. Of itself, it tells us only that there is a (1-α) probability that the true system reliability is greater than a particular value. The standard deviation of the estimate of this confidence limit can provide a feel for how much the true system reliability might vary from the estimate. For example, given an eighty percent lower confidence limit of 0.78 and a predicted standard deviation of the estimate of 0.005, we can feel fairly certain that the true system reliability does not exceed 0.82. However, had the standard deviation been 0.01 or 0.02, there would be increasing uncertainty about the range of likely values of

true reliability. Therefore, the sample standard deviation has been included in the regression section later in this chapter in an effort to not only predict procedure accuracy, but also estimate spread.

2. Derivation and Discussion of TT

Previous tests of the original Log-Gamma procedure have indicated that the quantity

$$TT = \sum_{i=1}^{k} (Ni)(Qi) \qquad (4.2)$$

could be used as an indicator of relative accuracy of the procedure, where Ni is the number of mission tests performed upon the i-th component and Qi is the design or expected unreliability goal for the i-th component [Ref. 1]. The quantity determined in this manner was the total amount of testing relative to the component unreliabilities, or, approximately, the expected number of test failures summed over all components and tests.

A similiar measure was sought for the revised procedure examined in this paper. However, a simple expression, such as equation (4.2), could not be directly applied to the mixed series-parallel system. If the equation were used, it could lead to extremely large values of TT, since the parallel components would tend to dominate the sum. This is reasonable to expect, since the components connected in parallel, taken individually, would tend to have greater unreliability than the series components.

The first revised equation proposed for TT can be used whenever all components within each parallel subsystem are given identical numbers of mission tests.

$$TT = \sum_{i=1}^{K1} (Ni)(Qi) + \sum_{i=K1+1}^{K1+K2} (Ni')(Qi') \qquad (4.3)$$

where K1 is the number of series components in the system

K2 is the number of series-parallel subassemblies connected in logical series in the system

Ni, Qi are defined as above.

Ni' is the common number of tests given each component in the i-th subassembly.

Qi' is the unreliability of the i-th subassembly computed from its component unreliability goals using standard series-parallel reliability computation techniques.

Equation (4.3), in effect, reduces each subassembly to an equivalent 'pseudo-component' connected in series in the system. The quantity TT is now approximately equal to the expected number of system-affecting failures summed over all components, subassemblies, and tests. However, the drawback of this equation is the necessity of testing all components within a subassembly an identical number of times. It would seem unnecessarily restrictive to conduct the same number of tests on a medium reliability component and on a high reliability component just to satisfy the equation.

Therefore an alternate formulation for the term (Ni')(Qi') was sought which would provide a meaningful measure, and yet would allow unequal amounts of testing within subassemblies. The following ad hoc formulation is proposed:

Each wholly series-connected branch within a subsystem will first be reduced to an equivalent single component by the following formula

$$Nb = \frac{\sum_{i \in B} (Ni)(1-Ri)}{1 - Rb} \qquad (4.4)$$

where Rb is the classically determined branch reliability goal $Rb = \prod_{i \in B} Ri$ for series-connected components.

Ri is the reliability goal of the i-th component.

B is the set of all components in the branch.

Nb represents an "equivalent" number of branch tests (need not be integer).

The series-connected branch is replaced in the remaining computations by a single equivalent component with reliability goal Ri = Rb and number of mission tests Ni = Nb.

After all wholly series-connected branches have been reduced in this manner, each wholly parallel-connected branch in the revised subassembly is reduced to an equivalent component in the following manner:

Compute the equivalent reliability goal

$$Rb = 1 - \prod_{i \in B} (1-Ri)$$

in the usual manner.

The number of possible combinations of individual test results for the branch components is $\prod_{i \in B} Ni$, which is set equal to the same number of possible combinations obtained if each component had been tested Nb times. Thus the equivalent number of mission tests, Nb, is computed as the geometric mean of the component tests

$$Nb = \left[\prod_{i \in B} Ni \right]^{1/m}$$

The above procedure is repeated until the entire subassembly is reduced to a single equivalent component, i',

with its equivalent number of mission trials, Ni', and equivalent unreliability goal, Qi' = 1-Ri'. TT may then be computed as before:

$$TT = \sum_{i=1}^{K1} (Ni)(Qi) + \sum_{i=K1+1}^{K1+K2} (Ni')(Qi') \qquad (4.3)$$

3. Accuracy Prediction

Appendix E contains the equations and methodology used to attempt to relate the accuracy measure, D, and the estimate standard deviation to the quantity TT. The principal conclusion to be drawn from the results is that accuracy is a predictable quantity only with respect to particular system configurations for which models have been generated and tested as in this paper. Accuracy seems an unstable variable from configuration to configuration, although it appears to be fairly well behaved within any one configuration.

With respect to the estimate standard deviation, the value appears to be related more to the number of component tests than to the expected number of system affecting failures. Reduction in the magnitude of the standard deviation by about 0.01 appears to require alomst double the number of component tests. However, the magnitudes are generally less than 0.07 for component test sample sizes of 20 or more. The standard deviation values also appear to be fairly insensitive to changes in system configuration.

4. Comparison with Earlier Results.

Cases 13 through 20 were taken from Ref. 1 in order to provide a basis for checking relative accuracy when

extending the Log-gamma procedure beyond simple series systems. Unfortunately, an apparent shortcoming of the original simulation makes the 1968 data suspect.

The original results tended to be too optimistic for those cases, in particular, where the number of component tests was moderately low and the true component reliabilities were high. Significantly high estimates at the $100(1-\alpha)$ percentile were observed in the cases corresponding to Cases 13, 15, 18, 19, and 20 of the present study. Although, on this basis, the comparison is very favorable to the present procedure, it is thought that the original simulation results are in error with respect to the procedure. Reference 1 does not specifically identify the random number generator used, however some uniform random variate generators do not perform well for very small or very large numbers near the boundaries, such as would be encountered in the simulation procedure used in the original study. The present simulation binomial random variate generator was tested prior to use to insure proper performance throughout the range of simulation values used.

Of the remaining cases (14, 16, and 17) the original results were much closer to the true system reliability in the first two cases and comparable in the last case. However, these results cannot be considered meaningful in light of the suspected problems with the random number generator.

In addition, many instances of cases found in Ref. 1 had prima-facia unreasonable results for the $100(1-\alpha)$ percentile values, in that they were considerably higher than the upper bound imposed by the zero-failure, binomial system test treatment discussed in Appendix C.

Another difference between the original study and

the present one is the use of an arbitrary continuity correction in the earlier procedure. This could also have been a contributor to the excess optimism in the reported simulation results.

In light of the foregoing paragraphs, no comparison with the results of this study was deemed worthwhile.

C. CONCLUSIONS

The primary conclusion to be drawn from this study is that the Log-gamma lower confidence level procedure for series-parallel systems can be accurate. How accurate it will be in any specific application is a function of the magnitude of the component testing program, the individual component reliabilities, and the complexity of the system. In addition, of course, the validity of the assumptions concerning component interaction and mission environment will also bear directly upon the accuracy of any procedure which purports to estimate system mission reliability from component test results. The effects of these assumptions, however will be controlled by the proper design of the component test procedures and are not addressed in this study.

For the cases considered the Log-gamma procedure tends to be conservative, which is necessary if one takes the position that pleasant surprises with better than expected performance cost less than unpleasant surprises in the form of degraded demonstrated reliability. Of course, too much underestimation of system reliability could drive up the development costs in an unnecessary search for a much more reliable design.

The accuracy indicator TT developed in Section IV.B.2 can be used as a rough gauge of expected procedure accuracy. The following scheme is suggested for minimum values of TT for fairly good expected accuracy:

Approximate Order of Accuracy	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$
0.02	6.1	12.8	19.2
0.01	11.7	23.6	32.6

However, since procedure accuracy is so demonstrably sensitive to variation in the system configuration, the best approach prior to use of the Log-gamma procedure is to perform a simulation as described in Section III.A for the particular system design to be exercised. Such a simulation could be used to evaluate many different combinations of component test plans and also support the claimed accuracy of the lower confidence limit derived from the adopted test plan results. The investment in such a simulation effort should not be excessive from the standpoint of computer time, since the most complex cases examined in this study required less than five minutes of Central Processor Unit (CPU) time from start to end. Without the support of such a simulation specific to the system, the accuracy of the procedure can be predicted only very approximately.

For extremely reliable systems, for which few failures are expected with large amounts of testing, the binomial lower confidence limit procedure developed in Appendix C might provide the desired magnitude of the lower confidence limit without the extra testing required to achieve the desired accuracy of the Log-gamma procedure. In fact, for cases of one failure or less, the binomial procedure is probably preferable on the basis of both theoretical rigor and ease of computation.

Finally, the matter of a continuity correction should be investigated. An appropriately determined continuity correction procedure could conceivably reduce the component testing requirements for the same expected procedure accuracy.

APPENDIX A

DERIVATION OF UNBIASED ESTIMATE FOR Ti

For

$$Ti = Qi + Qi^2/2$$

an unbiased estimator is desired, based upon the component failure data. Let Fi be the number of failures observed in Ni mission tests on the i-th component. The expected value of Fi is NiQi and an unbiased estimator, $\hat{Q}i$, for Qi is Fi/Ni, where Fi is binomially distributed with parameters Qi and Ni; Qi = 1-Ri; and Ri is the true reliability of the i-th component.

An estimator, $\hat{T}i$, for Ti will be chosen of the form

$$\hat{T}i = Ai[\hat{Q}i] + Bi[\hat{Q}i^2]/2 \quad (A.1)$$

and the constants Ai and Bi chosen so as to make the estimator unbiased, that is the expected value of Ti is equal to Ti,

$$E[\hat{T}i] = Ti = Qi + Qi^2/2 \quad (A.2)$$

$$= AiE[\hat{Q}i] + (Bi/2)E[\hat{Q}i^2]$$

$$= (Ai/Ni)E[Fi] + (B/2Ni^2)E[Fi^2]$$

$$= (Ai/Ni)[NiQi] + (Bi/2Ni^2)[NiQi-NiQi^2+Ni^2Qi^2]$$

$$= AiQi + BiQi/2Ni - BiQi^2/2Ni + BiQi^2/2$$

$$= [Ai + Bi/2Ni]Qi + [Bi/2 - Bi/2Ni]Qi^2 \quad (A.3)$$

Equating equation (A.3) term by term with equation (A.2) yields the following two equations

$$Ai + Bi/2Ni = 1$$

and

$$Bi - Bi/Ni = 1$$

which reduce to expressions for the unbiased estimator constants:

$$Ai = (2Ni-3)/(2Ni-2) \qquad (A.4)$$

and

$$Bi = Ni/(Ni-1) \qquad (A.5)$$

APPENDIX B

APPROXIMATION OF $Var[\hat{T}i]$ BY Ti/Ni

A. DERIVATION OF $Var[\hat{T}i]$

The variance of $\hat{T}i$ is derived as follows:

From appendix A,

$$\hat{T}i = Ai\hat{Q}i + (Bi\hat{Q}i^2)/2 \tag{B.1}$$

thus,

$$\hat{T}i^2 = Ai^2\hat{Q}i^2 + AiBi\hat{Q}i^3 + (Bi^2\hat{Q}i^4)/4 \tag{B.2}$$

$$Var[\hat{T}i] = E[\hat{T}i^2] - E^2[\hat{T}i] \tag{B.3}$$

$$= E[\hat{T}i^2] - Ti^2$$

$$\begin{aligned} &= (Ai^2/Ni^2)E[Fi^2] + (AiBi/Ni^3)E[Fi^3] \\ &\quad + (Bi^2/4Ni^4)E[Fi^4] \\ &\quad - [Qi^2+Qi^3+(Qi^4/4)] \end{aligned} \tag{B.4}$$

Using the moment generating functions for the binomially distributed variable Fi, with parameters Qi and Ni, yields

$$E[Fi] = NiQi \tag{B.5}$$

$$E[Fi^2] = NiQi + NiQi^2(Ni-1) \tag{B.6}$$

$$E[Fi^3] = NiQi + NiQi^2(Ni-1)[3+Qi(Ni-2)] \tag{B.7}$$

$$\begin{aligned} E[Fi^4] &= NiQi^4(Ni-1)(Ni-2)(Ni-3) + 7NiQi^2(Ni-1) \\ &\quad + 6NiQi^3(Ni-1)(Ni-2) + NiQi \end{aligned} \tag{B.8}$$

Substituting the moment equations (B.5) through (B.8) into equation (B.4) and simplifying the resulting expression yields the following equation for the variance of $\hat{T}i$:

$$Var[\hat{T}i] = \frac{4Ni^2-3Ni+1}{4Ni(Ni-1)^2} Qi + \frac{3Ni-4}{2Ni(Ni-1)} Qi^2 - \frac{1}{Ni-1} Qi^3 - \frac{2Ni-3}{2Ni(Ni-1)} Qi^4 \quad (B.9)$$

or, in terms of Ai, Bi, Ni, and Qi:

$$Var[\hat{T}i] = \frac{Bi^2+2Ni^2Bi(Ai+Bi)}{4Ni^3} Qi + \frac{2Ai+1}{2Ni} Qi^2 - \frac{Bi}{Ni} Qi^3 - \frac{Ai}{Ni} Qi^4 \quad (B.10)$$

To test the approximation of $Var[\hat{T}i]$ by Ti/Ni, equation (B.9) was solved with representative values of Qi and Ni and compared with $E[\hat{T}i]/Ni$. The percent error of the approximation was computed for each value and is given in table B-1. As expected, increasing Ni or decreasing Qi improves the accuracy of the approximation.

By way of comparison, table B-2 shows the relative accuracy of this same approximation when Ti is defined as the first three, vice two, terms of the logarithmic series expansion of S [Eq. (2.3)]. In this case the unbiased estimator for Ti will be

$$\hat{T}i = Ai'\hat{Q}i + Bi'\hat{Q}i^2/2 + Ci'\hat{Q}i^3/3 \quad (B.11)$$

where

$$Ai' = \frac{6(Ni-1)(Ni-2)-3(Ni-4)-2}{6(Ni-1)(Ni-2)}$$

$$Bi' = \frac{Ni(Ni-4)}{(Ni-1)(Ni-2)}$$

$$Ci' = \frac{Ni^2}{(Ni-1)(Ni-2)}$$

Qi	Ni = 10	Ni = 50	Ni = 100
0.001	-5.08%	-1.10%	-0.60%
0.003	-5.25%	-1.29%	-0.79%
0.006	-5.52%	-1.58%	-1.09%
0.01	-5.87%	-1.96%	-1.47%
0.03	-7.50%	-3.73%	-3.26%
0.06	-9.64%	-6.07%	-5.62%
0.10	-12.0%	-8.65%	-8.23%
0.20	-15.8%	-12.9%	-12.5%
0.30	-17.0%	-14.4%	-14.1%
0.40	-15.8%	-13.4%	-13.1%
0.50	-12.0%	-9.67%	-9.38%
0.60	-4.77%	-2.40%	-2.11%
0.70	+7.70%	+10.3%	+10.6%

Table B-1. Error Percentage of approximation for Var[Ti] Using Two-term Form of Ti.

Qi	Ni = 10	Ni = 50	Ni = 100
0.001	-0.06%	-0.05%	-0.05%
0.003	-0.17%	-0.15%	-0.15%
0.006	-0.33%	-0.31%	-0.30%
0.01	-0.56%	-0.51%	-0.51%
0.03	-1.68%	-1.54%	-1.53%
0.06	-3.36%	-3.09%	-3.06%
0.10	-5.57%	-5.11%	-5.06%

Table B-2. Error Percentage of approximation for Var[Ti] Using Three-term Form of Ti.

Due to time constraints and the general acceptability of the values noted in table B-1, the use of the more complex definition of Ti was left for future study. However, figure B-1 plots the accuracy as a function of Qi, Ni, and form of Ti.

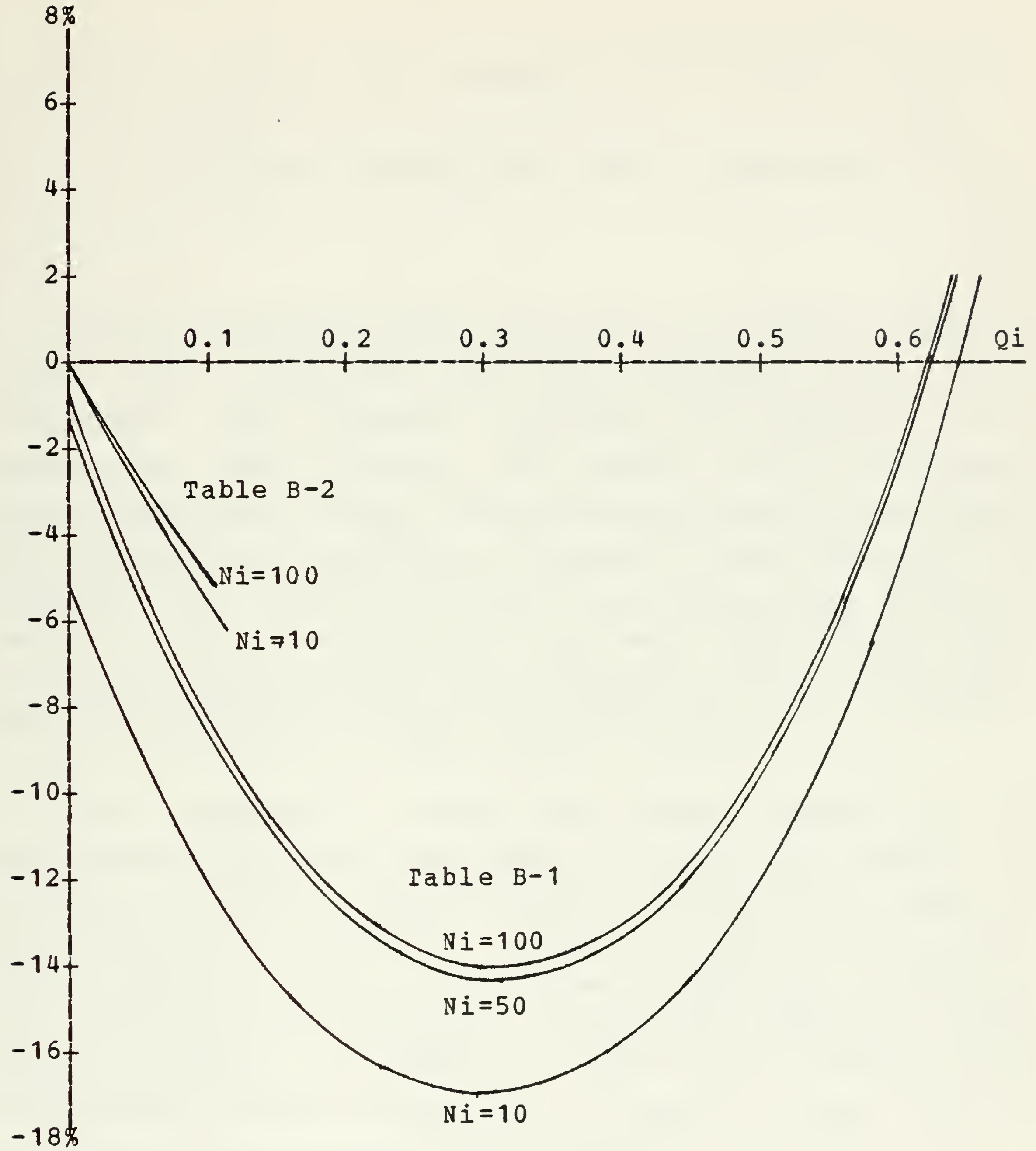


Figure B-1. PERCENTAGE ERROR OF APPROXIMATION FOR VAR[$\hat{T}i$] VS. COMPONENT UNRELIABILITY.

APPENDIX C

SPECIAL CONSIDERATIONS WHEN $\hat{S}$ EQUALS ZERO

When all components have completed the course of mission tests and no failures have been observed along some reliability path through the system, the estimator $\hat{S}$ will equal zero. This, in turn, will result in the log-gamma lower confidence limit estimate being equal to one unless the procedure is modified to include some "virtual" or "pseudo" test failure value for the computations when this occurs. The alternative is to seek some other lower confidence limit procedure which applies with zero failures. Such an alternative is also suggested below.

For simplicity, assume all series components and subassemblies have been subjected to an equal number of mission trials, N, without failure. This may be thought of as equivalent to conducting N mission trials on the system without failure, subject to the usual assumptions concerning test environment and component interaction effects. When unequal numbers of component trials are involved, the conservative course of action is to choose N equal to the smallest of the component trial values.

Now the number of system mission trial successes, W, can be related to system mission reliability, Rs, by the binomial distribution formula

$$\sum_{j=W}^{N} \binom{N}{j} (Rs)^{j} (1 - Rs)^{N-j} = \text{Prob}(\# \text{ successes} \geq W) \quad (C.1)$$

where N = number of mission trials

The 100(1−α)% Lower Confidence Limit Estimate for system reliability would then be the solution for R* in the following formula:

$$\sum_{j=W}^{N} \binom{N}{j} (R^*)^{j} (1 - R^*)^{N-j} = \alpha \qquad \text{(C.2)}$$

which, when W = N, reduces to

$$(R^*)^{N} = \alpha \quad \text{or} \quad R^* = (\alpha)^{1/N}$$

This result may then be substituted into the Log-gamma procedure formula for $\hat{R}^*(\alpha)$ and solved for the number of equivalent component failures which would yield the same estimate of system reliability.

$$\hat{R}^*(\alpha) = \exp\left[\frac{-[2\hat{r}]\,\hat{S}}{\chi^2_{[2\hat{r}],1-\alpha}}\right] = (\alpha)^{1/N} \qquad \text{(C.3)}$$

where

$$\hat{S} = \sum_{i=1}^{K} \hat{T}i \quad , \quad \hat{r} = \max\left[1.0,\ \frac{\left[\sum_{i=1}^{K} \hat{T}i\right]^2}{\sum_{i=1}^{K} \frac{\hat{T}i}{Ni}}\right]$$

which reduce to $\hat{S} = \hat{T}i$ and $\hat{r} = \max[1, (Ni)(\hat{T}i)]$, since only one component or subassembly will make a non-zero contribution to the sums.

Now, if $\hat{r} = 1.0$, then $(Ni)(\hat{T}i) \le 1.0$ and $\hat{T}i \le 1/Ni$. Also recall that

$$\hat{T}i = (Ai)(\hat{Q}i) + (Bi/2)(\hat{Q}i^2) \qquad \text{(2.5)}$$

where

$$Ai = \frac{2Ni - 3}{2(Ni-1)}, \quad Bi = \frac{Ni}{Ni - 1}, \quad \hat{Q}i = \frac{Fi}{Ni}$$

Or, equivalently,

$$(Bi/2Ni^2)(Fi^2) + (Ai/Ni)(Fi) - \hat{T}i = 0 \qquad (C.4)$$

Substituting for Ai, Bi and the earlier inequality for $\hat{T}i$,

$$\left[\frac{Ni}{2(Ni-1)(Ni^2)}\right](Fi^2) + \left[\frac{2Ni-3}{2(Ni-1)(Ni)}\right](Fi) - \frac{1}{Ni} \leq 0$$

which reduces to

$$Fi^2 + (2Ni-3)Fi - 2(Ni-1) \leq 0$$

which, in turn, can be factored as

$$(Fi - 1)[Fi + 2(Ni-1)] \leq 0 \qquad (C.5)$$

The only positive values for Fi which can satisfy this last equation are Fi less than or equal to one. This means that for any size Ni, the value of $[2\hat{r}]$ will be two. We can now solve the reliability estimate equation for Fi*, the number of equivalent failures for the Log-gamma method.

$$\exp\left[\frac{-[2\hat{r}]\hat{S}}{\chi^2_{[2\hat{r}],1-\alpha}}\right] = (\alpha)^{1/N} \qquad (C.6)$$

$$\hat{S} = \left[\frac{\chi^2_{2,1-\alpha}}{2N}\right](\ln \alpha) = \frac{Ai(Fi^*)}{N} + \frac{Bi(Fi^*)^2}{2N^2} \qquad (C.7)$$

which reduces to

$$Fi^{*2} + (2N-3)Fi^* + (N-1)(\ln \alpha)\left[\chi^2_{2,1-\alpha}\right] = 0 \qquad (C.8)$$

This quadratic may be solved for Fi* as a function of N and α. The results are listed in table C-1. From the table, choosing Fi* at the N = 10 level will be conservative for all N greater than 10, yet will yield relatively accurate results for all larger N due to the slow rate of change of Fi* with respect to N.

N	α=0.2	α=0.1	α=.05
5	0.3888	0.2670	0.1713
10	0.3721	0.2531	0.1610
20	0.3652	0.2475	0.1570
100	0.3603	0.2435	0.1542
1000	0.3593	0.2427	0.1536

Table C-1. Solution for Fi* with Various N and α Values.

APPENDIX D

TABULATED SIMULATION RESULTS

The following pages contain two tables listing the principal simulation output data. The tables are presented concurrently for each case, one case per page. The cases are listed in alpha-numeric order. The following list of terms applies:

Case: The case number as explained in Section III.B. The letter suffix distinguishes between different runs of the same case using different seeds for the binomial random variate generating function.

KK: The total number of components in the system.

TT: The accuracy indicator as discussed in seciton IV.B.

TM: The modified accuracy indicator described in Appendix E.

Rs: True system reliability.

CL: Confidence Level, $CL = 100(1-\alpha)$.

$\hat{R}*^{o}$: The $100(1-\alpha)$ percentile of the simulation estimates of the $100(1-\alpha)\%$ lower confidence limit for system reliability.

D: The primary accuracy measure. As discussed in Section IV.A, $D = \hat{R}*^{o} - Rs$.

Mean: The sample mean of the estimates.

S.D.: The sample standard deviation of the estimates.

Null Runs: The number of runs which occured during the simulation for which $\hat{S}$ equalled zero. Refer to sections II.C and III.A and Appendix C for further elaboration.

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
01a	32	2.67	1.73	0.8756	80.0	0.8206	-0.0550	0.7728	0.0668
					90.0	0.7452	-0.1304	0.6896	0.0608
					95.0	0.6457	-0.2299	0.5897	0.0586
01b	32	2.67	1.73	0.8756	80.0	0.8193	-0.0563	0.7727	0.0646
					90.0	0.7464	-0.1292	0.6892	0.0584
					95.0	0.6461	-0.2295	0.5894	0.0560
01c	32	2.67	1.73	0.8756	80.0	0.8235	-0.0521	0.7721	0.0706
					90.0	0.7500	-0.1256	0.6893	0.0652
					95.0	0.6551	-0.2205	0.5898	0.0624

Table D-1. Simulation Results.

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
01a	0.87559	3	.80	.53944	.69032	.73043	.77577	.81214	.84858	.98884
			.90	.48173	.62211	.65534	.68976	.71470	.74522	.97652
			.95	.37693	.53409	.56606	.59030	.61304	.63225	.95232
01b	0.87559	2	.80	.58990	.69065	.73087	.77493	.81276	.84287	.98884
			.90	.52912	.62342	.65790	.68654	.71696	.74642	.97652
			.95	.37693	.53492	.56598	.58906	.61356	.63413	.95232
01c	0.87559	4	.80	.54338	.68722	.72971	.77453	.81283	.84552	.97779
			.90	.48590	.61906	.65396	.68826	.71614	.75004	.95352
			.95	.37693	.53220	.56440	.58946	.61321	.63796	.90680

Table D-2. Additional Simulation Output

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
02a	32	6.68	4.33	0.8756	80.0	0.8609	-0.0147	0.8232	0.0442
					90.0	0.8416	-0.0340	0.7883	0.0434
					95.0	0.8127	-0.0629	0.7525	0.0407
02b	32	6.68	4.33	0.8756	80.0	0.8593	-0.0163	0.8220	0.0440
					90.0	0.8411	-0.0345	0.7871	0.0431
					95.0	0.8108	-0.0648	0.7512	0.0401
02c	32	6.68	4.33	0.8756	80.0	0.8581	-0.0175	0.8219	0.0419
					90.0	0.8382	-0.0374	0.7871	0.0413
					95.0	0.8079	-0.0677	0.7515	0.0388

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
02a	0.87559	0	.80	.65891	.76871	.79558	.82406	.85334	.87724	.93761
			.90	.62550	.73444	.76068	.78970	.81891	.84163	.89509
			.95	.59525	.70164	.72700	.75476	.78227	.80256	.83969
02b	0.87559	0	.80	.67925	.76328	.79249	.82399	.85253	.87684	.95942
			.90	.64552	.72870	.75819	.78935	.81748	.84113	.91599
			.95	.61468	.69577	.72474	.75520	.78037	.80123	.83924
02c	0.87559	0	.80	.69563	.76882	.79494	.82324	.85206	.87362	.92467
			.90	.66188	.73484	.76030	.78868	.81707	.83819	.88568
			.95	.63083	.70172	.72656	.75330	.78017	.79999	.83678

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
03a	32	13.4	8.67	0.8756	80.0	0.8691	-0.0065	0.8425	0.0325
					90.0	0.8628	-0.0128	0.8220	0.0330
					95.0	0.8535	-0.0221	0.8025	0.0329
03b	32	13.4	8.67	0.8756	80.0	0.8655	-0.0101	0.8392	0.0323
					90.0	0.8573	-0.0183	0.8187	0.0328
					95.0	0.8476	-0.0280	0.7991	0.0327
03c	32	13.4	8.67	0.8756	80.0	0.8678	-0.0078	0.8421	0.0314
					90.0	0.8603	-0.0153	0.8216	0.0319
					95.0	0.8520	-0.0236	0.8021	0.0319

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values Minimum	10 %	25 %	Median	75 %	90 %	Maximum
03a	0.87559	0	.80	.75991	.80069	.82015	.84283	.86422	.88257	.93326
			.90	.73837	.77952	.79940	.82246	.84404	.86278	.91309
			.95	.71882	.75992	.77990	.80288	.82450	.84349	.89037
03b	0.87559	0	.80	.73571	.79738	.81835	.83998	.86190	.87707	.93497
			.90	.71376	.77624	.79741	.81930	.84186	.85726	.91529
			.95	.69399	.75672	.77787	.79968	.82239	.83799	.89310
03c	0.87559	0	.80	.72448	.80323	.82190	.84334	.86270	.87996	.94266
			.90	.70250	.78193	.80095	.82283	.84268	.86025	.92343
			.95	.68279	.76221	.78131	.80313	.82319	.84080	.90110

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
04a	32	4.59	2.97	0.7998	80.0	0.7555	-0.0443	0.6962	0.0680
					90.0	0.6988	-0.1010	0.6250	0.0596
					95.0	0.6196	-0.1802	0.5506	0.0488
04b	32	4.59	2.97	0.7998	80.0	0.7524	-0.0474	0.6948	0.0673
					90.0	0.6975	-0.1023	0.6247	0.0605
					95.0	0.6200	-0.1798	0.5525	0.0501
04c	32	4.59	2.97	0.7998	80.0	0.7544	-0.0454	0.6953	0.0664
					90.0	0.6956	-0.1042	0.6247	0.0588
					95.0	0.6161	-0.1837	0.5515	0.0471

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
04a	0.79984	0	.80	.48972	.61041	.65058	.69590	.74771	.78197	.87023
			.90	.43625	.54882	.58596	.62685	.67005	.69881	.77064
			.95	.38537	.48419	.52212	.55678	.58542	.60847	.65042
04b	0.79984	0	.80	.44002	.60848	.65274	.69752	.74341	.77654	.87521
			.90	.39024	.54672	.58755	.62925	.66842	.69749	.75399
			.95	.34712	.48617	.52372	.55975	.58732	.60935	.65509
04c	0.79984	0	.80	.47640	.60950	.65157	.69646	.74662	.77755	.88866
			.90	.42249	.54783	.58604	.62869	.66911	.69563	.77875
			.95	.37508	.48818	.52285	.55888	.58553	.60642	.65455

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
05a	32	11.5	7.43	0.7998	80.0	0.7839	-0.0159	0.7451	0.0450
					90.0	0.7689	-0.0309	0.7108	0.0448
					95.0	0.7458	-0.0541	0.6783	0.0437
05b	32	11.5	7.43	0.7998	80.0	0.7816	-0.0183	0.7449	0.0429
					90.0	0.7631	-0.0367	0.7106	0.0428
					95.0	0.7452	-0.0547	0.6781	0.0416
05c	32	11.5	7.43	0.7998	80.0	0.7832	-0.0167	0.7454	0.0464
					90.0	0.7696	-0.0302	0.7111	0.0461
					95.0	0.7460	-0.0538	0.6786	0.0450

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
05a	0.79984	0	.80	.59499	.68927	.71713	.74444	.77731	.80302	.86177
			.90	.56231	.65536	.68309	.71020	.74243	.76893	.82783
			.95	.53328	.62407	.65145	.67778	.70872	.73432	.79186
05b	0.79984	0	.80	.60950	.68998	.71483	.74415	.77564	.79795	.88186
			.90	.57666	.65578	.68059	.71000	.74129	.76313	.84399
			.95	.54698	.62435	.64876	.67767	.70816	.72939	.80112
05c	0.79984	0	.80	.54462	.68834	.71679	.74954	.77669	.80420	.85368
			.90	.51291	.65416	.68272	.71535	.74256	.76962	.82016
			.95	.48516	.62324	.65105	.68300	.70906	.73594	.78529

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
06a	32	22.9	14.9	0.7998	80.0	0.7891	-0.0107	0.7625	0.0314
					90.0	0.7810	-0.0188	0.7409	0.0318
					95.0	0.7716	-0.0282	0.7213	0.0317
06b	32	22.9	14.9	0.7998	80.0	0.7886	-0.0113	0.7619	0.0320
					90.0	0.7828	-0.0170	0.7403	0.0322
					95.0	0.7748	-0.0250	0.7207	0.0322
06c	32	22.9	14.9	0.7998	80.0	0.7879	-0.0120	0.7613	0.0311
					90.0	0.7774	-0.0225	0.7397	0.0312
					95.0	0.7702	-0.0297	0.7200	0.0312

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
06a	0.79984	0	.80	.65710	.72329	.74237	.76278	.78275	.80208	.86406
			.90	.63521	.70147	.72061	.74120	.76129	.78104	.84358
			.95	.61595	.68192	.70099	.72156	.74159	.76154	.82429
06b	0.79984	0	.80	.64967	.72275	.74021	.76058	.78276	.80405	.84749
			.90	.62777	.70090	.71848	.73890	.76130	.78282	.82723
			.95	.60855	.68131	.69886	.71927	.74161	.76317	.80793
06c	0.79984	0	.80	.65784	.72172	.74177	.76064	.78345	.79871	.85272
			.90	.63582	.69981	.71998	.73895	.76205	.77736	.83260
			.95	.61643	.68017	.70032	.71931	.74241	.75778	.81336

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
07a	32	4.67	3.03	0.7917	80.0	0.7557	-0.0359	0.6850	0.0807
					90.0	0.7012	-0.0904	0.6144	0.0715
					95.0	0.6202	-0.1715	0.5406	0.0578
07b	32	4.67	3.03	0.7917	80.0	0.7627	-0.0290	0.6883	0.0811
					90.0	0.7034	-0.0883	0.6176	0.0716
					95.0	0.6254	-0.1663	0.5436	0.0572
07c	32	4.67	3.03	0.7917	80.0	0.7503	-0.0414	0.6844	0.0773
					90.0	0.6996	-0.0921	0.6144	0.0687
					95.0	0.6202	-0.1715	0.5419	0.0566

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
07a	0.79167	0	.80	.44802	.58087	.63122	.68742	.73980	.79090	.95813
			.90	.39723	.52024	.56678	.61866	.66591	.70123	.91337
			.95	.35308	.46297	.50207	.54601	.58169	.60634	.83005
07b	0.79167	0	.80	.45284	.57832	.63451	.69260	.74744	.79082	.94593
			.90	.40095	.51803	.57087	.62358	.67360	.70336	.88892
			.95	.35576	.46283	.50859	.55297	.58580	.61029	.78501
07c	0.79167	0	.80	.38869	.58239	.63401	.68801	.73831	.78287	.97779
			.90	.34237	.52236	.56991	.61936	.66531	.69960	.95352
			.95	.30297	.46551	.50816	.54743	.58297	.60634	.90680

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
08a	32	11.7	7.57	0.7917	80.0	0.7788	-0.0129	0.7368	0.0520
					90.0	0.7689	-0.0228	0.7026	0.0518
					95.0	0.7528	-0.0389	0.6702	0.0504
08b	32	11.7	7.57	0.7917	80.0	0.7777	-0.0140	0.7331	0.0541
					90.0	0.7694	-0.0222	0.6989	0.0537
					95.0	0.7551	-0.0365	0.6666	0.0524
08c	32	11.7	7.57	0.7917	80.0	0.7767	-0.0150	0.7327	0.0530
					90.0	0.7632	-0.0285	0.6986	0.0526
					95.0	0.7487	-0.0429	0.6663	0.0512

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
08a	0.79167	0	.80	.58162	.67194	.69966	.73759	.76997	.80417	.90349
			.90	.54914	.63828	.66556	.70304	.73573	.76888	.86814
			.95	.52041	.60763	.63410	.67082	.70297	.73495	.82644
08b	0.79167	0	.80	.57794	.66379	.69643	.73256	.76951	.80434	.89135
			.90	.54570	.63015	.66267	.69836	.73545	.76942	.85627
			.95	.51722	.59962	.63157	.66632	.70212	.73532	.81640
08c	0.79167	0	.80	.58949	.66094	.69694	.73291	.76725	.79764	.91141
			.90	.55696	.62707	.66308	.69875	.73315	.76332	.86608
			.95	.52813	.59640	.63194	.66673	.70036	.72952	.81783

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
09a	32	23.4	15.1	0.7917	80.0	0.7871	-0.0046	0.7551	0.0383
					90.0	0.7817	-0.0100	0.7335	0.0386
					95.0	0.7764	-0.0152	0.7139	0.0385
09b	32	23.4	15.1	0.7917	80.0	0.7857	-0.0060	0.7542	0.0385
					90.0	0.7813	-0.0104	0.7326	0.0388
					95.0	0.7742	-0.0175	0.7130	0.0388
09c	32	23.4	15.1	0.7917	80.0	0.7839	-0.0078	0.7538	0.0383
					90.0	0.7819	-0.0098	0.7322	0.0385
					95.0	0.7755	-0.0162	0.7125	0.0385

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
09a	0.79167	0	.80	.63478	.70642	.72995	.75499	.78073	.80272	.86981
			.90	.61288	.68446	.70810	.73337	.75940	.78166	.84982
			.95	.59372	.66487	.68846	.71376	.73958	.76216	.83037
09b	0.79167	0	.80	.62609	.70625	.72913	.75489	.77990	.80230	.88091
			.90	.60430	.68427	.70722	.73326	.75852	.78128	.86134
			.95	.58526	.66467	.68754	.71365	.73892	.76175	.84203
09c	0.79167	0	.80	.61157	.70578	.72972	.75441	.77767	.80320	.86563
			.90	.58980	.68377	.70785	.73274	.75610	.78190	.84564
			.95	.57085	.66415	.68820	.71310	.73634	.76221	.82627

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
10a	32	6.59	4.27	0.7232	80.0	0.6889	-0.0343	0.6233	0.0758
					90.0	0.6471	-0.0761	0.5597	0.0697
					95.0	0.5932	-0.1300	0.4983	0.0606
10b	32	6.59	4.27	0.7232	80.0	0.6897	-0.0335	0.6227	0.0800
					90.0	0.6544	-0.0688	0.5592	0.0736
					95.0	0.5985	-0.1247	0.4976	0.0636
10c	32	6.59	4.27	0.7232	80.0	0.6906	-0.0326	0.6340	0.0780
					90.0	0.6520	-0.0712	0.5602	0.0717
					95.0	0.5932	-0.1299	0.4984	0.0625

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
10a	0.72317	0	.80	.38169	.52320	.56765	.62362	.67774	.71491	.82429
			.90	.33622	.46646	.50850	.56045	.61015	.64708	.74116
			.95	.29763	.41592	.45525	.49977	.54475	.57482	.63872
10b	0.72317	0	.80	.40848	.51967	.56637	.61984	.67426	.72627	.88751
			.90	.36039	.46321	.50735	.55732	.60872	.65438	.77661
			.95	.31920	.41270	.45360	.49868	.54328	.57726	.65117
10c	0.72317	0	.80	.41254	.51980	.56890	.62469	.68110	.72407	.84018
			.90	.36448	.46295	.50943	.56162	.61399	.65197	.74113
			.95	.32326	.41345	.45571	.50127	.54580	.57756	.64522

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*^o$	D	Mean	S.D.
11a	32	16.5	10.7	0.7232	80.0	0.7099	-0.0132	0.6647	0.0520
					90.0	0.6953	-0.0279	0.6311	0.0514
					95.0	0.6779	-0.0453	0.6004	0.0502
11b	32	16.5	10.7	0.7232	80.0	0.7060	-0.0171	0.6636	0.0510
					90.0	0.6938	-0.0294	0.6300	0.0505
					95.0	0.6817	-0.0415	0.5994	0.0493
11c	32	16.5	10.7	0.7232	80.0	0.7115	-0.0116	0.6672	0.0525
					90.0	0.7011	-0.0220	0.6335	0.0520
					95.0	0.6843	-0.0389	0.6028	0.0508

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
11a	0.72317	0	.80	.47279	.59689	.62798	.66587	.70222	.72980	.82596
			.90	.44301	.56430	.59476	.63212	.66834	.69533	.79100
			.95	.41741	.53534	.56494	.60167	.63701	.66304	.75559
11b	0.72317	0	.80	.48277	.59915	.63033	.66350	.69693	.72775	.84057
			.90	.45267	.56623	.59708	.62984	.66292	.69383	.80645
			.95	.42673	.53695	.56710	.59929	.63184	.66190	.77143
11c	0.72317	0	.80	.52162	.59916	.63076	.66657	.70357	.73559	.82379
			.90	.49048	.56624	.59752	.63266	.66908	.70114	.78845
			.95	.46339	.53696	.56769	.60180	.63763	.66848	.75267

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
12a	32	32.9	21.4	0.7232	80.0	0.7151	-0.0081	0.6849	0.0358
					90.0	0.7096	-0.0136	0.6630	0.0358
					95.0	0.7019	-0.0213	0.6435	0.0356
12b	32	32.9	21.4	0.7232	80.0	0.7147	-0.0085	0.6845	0.0368
					90.0	0.7083	-0.0149	0.6627	0.0367
					95.0	0.7003	-0.0229	0.6433	0.0366
12c	32	32.9	21.4	0.7232	80.0	0.7165	-0.0067	0.6868	0.0362
					90.0	0.7120	-0.0112	0.6649	0.0363
					95.0	0.7045	-0.0187	0.6455	0.0360

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values: Minimum	10 %	25 %	Median	75 %	90 %	Maximum
12a	0.72317	0	.80	.59098	.63998	.66109	.68369	.70760	.73135	.80584
			.90	.56944	.61814	.63921	.66167	.68570	.70958	.78478
			.95	.55076	.59901	.61990	.64216	.66617	.69002	.76528
12b	0.72317	0	.80	.55970	.63851	.66109	.68655	.71031	.73015	.78661
			.90	.53843	.61662	.63915	.66439	.68836	.70831	.76518
			.95	.52009	.59744	.61975	.64485	.66876	.68868	.74550
12c	0.72317	0	.80	.58726	.64023	.66333	.68675	.70933	.73387	.79268
			.90	.56573	.61830	.64138	.66470	.68732	.71203	.77146
			.95	.54707	.59904	.62205	.64515	.66768	.69238	.75189

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
13a	19	15.4	10.7	0.7233	80.0	0.7195	-0.0038	0.6658	0.0647
					90.0	0.7176	-0.0057	0.6335	0.0662
					95.0	0.7119	-0.0114	0.6042	0.0672
13b	19	15.4	10.7	0.7233	80.0	0.7198	-0.0035	0.6676	0.0622
					90.0	0.7143	-0.0090	0.6354	0.0637
					95.0	0.7123	-0.0110	0.6060	0.0649
13c	19	15.4	10.7	0.7233	80.0	0.7263	+0.0030	0.6696	0.0680
					90.0	0.7264	+0.0031	0.6373	0.0696
					95.0	0.7257	+0.0024	0.6080	0.0707

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
13a	0.72329	0	.80	.48075	.57589	.62474	.66831	.70837	.74695	.85199
			.90	.44732	.54331	.59298	.63561	.67681	.71758	.82642
			.95	.41620	.51302	.56258	.60410	.64881	.69143	.80113
13b	0.72329	0	.80	.46207	.58839	.62658	.67098	.71049	.74397	.85327
			.90	.42661	.55217	.59377	.63913	.67878	.71427	.82692
			.95	.39601	.52107	.56404	.60892	.65022	.68551	.80068
13c	0.72329	0	.80	.42256	.58572	.62423	.67044	.71730	.75410	.85472
			.90	.38883	.55042	.58986	.63877	.68509	.72639	.82958
			.95	.36002	.51810	.55940	.60920	.65589	.70087	.80469

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
14a	15	4.40	3.14	0.7924	80.0	0.7561	-0.0363	0.6903	0.0758
					90.0	0.6865	-0.1059	0.6163	0.0647
					95.0	0.6388	-0.1536	0.5375	0.0652
14b	15	4.40	3.14	0.7924	80.0	0.7561	-0.0363	0.6898	0.0776
					90.0	0.6865	-0.1059	0.6162	0.0669
					95.0	0.6388	-0.1536	0.5379	0.0665
14c	15	4.40	3.14	0.7924	80.0	0.7561	-0.0363	0.6890	0.0744
					90.0	0.6865	-0.1059	0.6162	0.0642
					95.0	0.6388	-0.1536	0.5394	0.0633

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
14a	0.79239	5	.80	.43820	.58497	.62901	.69696	.75814	.79927	.80327
			.90	.38840	.52379	.56639	.62751	.67696	.68654	.72709
			.95	.34527	.44252	.49880	.55675	.57773	.61078	.63884
14b	0.79239	9	.80	.40335	.58721	.62901	.70586	.75814	.79927	.80327
			.90	.35524	.52621	.56639	.62751	.67696	.68654	.72709
			.95	.31410	.44252	.49880	.55675	.57773	.61078	.63884
14c	0.79239	10	.80	.42400	.58497	.62901	.69696	.74888	.79927	.80327
			.90	.37505	.52379	.56639	.62751	.66470	.68654	.72709
			.95	.33289	.45711	.50391	.55675	.57659	.61078	.63884

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
15a	25	3.12	2.09	0.8822	80.0	0.8236	-0.0586	0.7790	0.0535
					90.0	0.7402	-0.1420	0.6969	0.0392
					95.0	0.6455	-0.2367	0.5962	0.0696
15b	25	3.12	2.09	0.8822	80.0	0.8236	-0.0586	0.7795	0.0520
					90.0	0.7402	-0.1420	0.6997	0.0394
					95.0	0.6455	-0.2367	0.6030	0.0653
15c	25	3.12	2.09	0.8822	80.0	0.8236	-0.0586	0.7789	0.0536
					90.0	0.7402	-0.1420	0.6991	0.0407
					95.0	0.6455	-0.2367	0.6025	0.0651

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values Minimum	10 %	25 %	Median	75 %	90 %	Maximum
15a	0.88222	40	.80	.57786	.69497	.75679	.79095	.82358	.83590	.84003
			.90	.52740	.64006	.68406	.69292	.72127	.74017	.77600
			.95	.45815	.45815	.57625	.62593	.64372	.64372	.70008
15b	0.88222	53	.80	.55247	.69497	.75679	.79095	.82358	.83590	.84003
			.90	.50185	.64006	.68406	.69292	.74017	.74017	.77600
			.95	.45625	.45815	.60180	.62593	.64372	.64372	.70008
15c	0.88222	45	.80	.57525	.69497	.75679	.79095	.82358	.83590	.84003
			.90	.52461	.64006	.68406	.70531	.74017	.74017	.77600
			.95	.45815	.45815	.60180	.63745	.64372	.64372	.70008

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
16a	25	6.25	4.18	0.8822	80.0	0.8699	-0.0123	0.8285	0.0457
					90.0	0.8493	-0.0329	0.7923	0.0432
					95.0	0.8023	-0.0799	0.7540	0.0388
16b	25	6.25	4.18	0.8822	80.0	0.8699	-0.0123	0.8277	0.0461
					90.0	0.8493	-0.0329	0.7916	0.0437
					95.0	0.8023	-0.0799	0.7534	0.0392
16c	25	6.25	4.18	0.8822	80.0	0.8699	-0.0123	0.8278	0.0452
					90.0	0.8493	-0.0329	0.7914	0.0425
					95.0	0.8023	-0.0799	0.7527	0.0386

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
16a	0.88222	0	.80	.66422	.77665	.79675	.83156	.86994	.88935	.91737
			.90	.63061	.74214	.76304	.79577	.82708	.84928	.88207
			.95	.60010	.69549	.72955	.75911	.78548	.80232	.83826
16b	0.88222	2	.80	.66560	.77722	.79675	.83156	.85381	.88935	.91737
			.90	.63208	.74214	.76304	.79577	.82032	.84928	.88207
			.95	.60165	.69549	.72955	.75911	.78548	.80232	.83826
16c	0.88222	4	.80	.69530	.76147	.79629	.83156	.85381	.88935	.91428
			.90	.66153	.72767	.76252	.79577	.82032	.84928	.86134
			.95	.63046	.69498	.72897	.75911	.78548	.79841	.82273

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
17a	25	12.5	8.36	0.8822	80.0	0.8816	-0.0006	0.8493	0.0341
					90.0	0.8716	-0.0107	0.8291	0.0346
					95.0	0.8614	-0.0208	0.8098	0.0343
17b	25	12.5	8.36	0.8822	80.0	0.8728	-0.0095	0.8479	0.0338
					90.0	0.8716	-0.0107	0.8277	0.0344
					95.0	0.8650	-0.0172	0.8084	0.0341
17c	25	12.5	8.36	0.8822	80.0	0.8816	-0.0006	0.8508	0.0334
					90.0	0.8735	-0.0087	0.8308	0.0340
					95.0	0.8614	-0.0208	0.8113	0.0336

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values Minimum	10 %	25 %	Median	75 %	90 %	Maximum
17a	0.88222	0	.80	.72335	.80692	.82532	.85305	.87263	.89156	.94305
			.90	.70154	.78597	.80472	.83297	.85304	.87156	.92156
			.95	.68198	.76656	.78540	.81378	.83397	.85146	.89572
17c	0.88222	0	.80	.72300	.80702	.82511	.84394	.87263	.89156	.93270
			.90	.70116	.78608	.80448	.82371	.85304	.87156	.91237
			.95	.68159	.76668	.78514	.80451	.83397	.85146	.88947
17c	0.88222	0	.80	.73138	.80702	.82532	.85328	.87263	.89261	.93270
			.90	.70962	.78609	.80472	.83323	.85304	.87353	.91237
			.95	.69006	.76668	.78540	.81406	.83397	.85448	.88947

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
18a	25	0.62	0.42	0.9753	80.0	0.8359	-0.1394	0.8305	0.0195
					90.0	0.7402	-0.2351	0.6993	0.0277
					95.0	0.6437	-0.3316	0.5104	0.0836
18b	25	0.62	0.42	0.9753	80.0	0.8359	-0.1394	0.8306	0.0200
					90.0	0.7402	-0.2351	0.6972	0.0263
					95.0	0.6437	-0.3316	0.5044	0.0804
18c	25	0.62	0.42	0.9753	80.0	0.8359	-0.1394	0.8304	0.0198
					90.0	0.7402	-0.2351	0.6981	0.0267
					95.0	0.6437	-0.3316	0.5073	0.0817

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
18a	0.97530	1184	.80	.72345	.82358	.82358	.83590	.83590	.83590	.84003
			.90	.66288	.68406	.68406	.68406	.72127	.74017	.77600
			.95	.45815	.45815	.45815	.45815	.63745	.63745	.70008
18b	0.97530	1164	.80	.72802	.82358	.83590	.83590	.83590	.83590	.84003
			.90	.67179	.68406	.68406	.68406	.69292	.74017	.77600
			.95	.45815	.45815	.45815	.45815	.62593	.63745	.70008
18c	0.97530	1251	.80	.75679	.82358	.82358	.83590	.83590	.83590	.84003
			.90	.68406	.68406	.68406	.68406	.72127	.74017	.77600
			.95	.45815	.45815	.45815	.45815	.63745	.63745	.70008

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
19a	25	1.25	0.84	0.9753	80.0	0.9143	-0.0610	0.9069	0.0156
					90.0	0.8603	-0.1150	0.8409	0.0194
					95.0	0.8023	-0.1730	0.7372	0.0622
19b	25	1.25	0.84	0.9753	80.0	0.9143	-0.0610	0.9067	0.0163
					90.0	0.8603	-0.1150	0.8402	0.0193
					95.0	0.8023	-0.1730	0.7352	0.0618
19c	25	1.25	0.84	0.9753	80.0	0.9143	-0.0610	0.9064	0.0167
					90.0	0.8603	-0.1150	0.8396	0.0191
					95.0	0.8023	-0.1730	0.7342	0.0615

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
19a	0.97530	427	.80	.83156	.88935	.90751	.91428	.91428	.91428	.91737
			.90	.79577	.82708	.82708	.82708	.86033	.86033	.88207
			.95	.67687	.67687	.67687	.67687	.79841	.80232	.83826
19b	0.97530	424	.80	.81518	.88935	.90751	.91428	.91428	.91428	.91737
			.90	.78157	.82708	.82708	.82708	.86033	.86033	.88207
			.95	.67687	.67687	.67687	.67687	.79841	.80232	.83826
19c	0.97530	438	.80	.81299	.88935	.90751	.91428	.91428	.91428	.91737
			.90	.77752	.82708	.82708	.82708	.86033	.86033	.88207
			.95	.67687	.67687	.67687	.67687	.79841	.80232	.83826

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
20a	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9439	0.0181
					90.0	0.9275	-0.0478	0.9168	0.0102
					95.0	0.8971	-0.0782	0.8770	0.0322
20b	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9438	0.0182
					90.0	0.9275	-0.0478	0.9165	0.0098
					95.0	0.8971	-0.0782	0.8764	0.0324
20c	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9436	0.0184
					90.0	0.9275	-0.0478	0.9164	0.0099
					95.0	0.8971	-0.0782	0.8763	0.0321

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
20a	0.97530	89	.80	.88240	.92226	.93511	.94636	.95263	.95618	.95800
			.90	.86306	.90232	.90944	.92156	.92754	.92754	.93949
			.95	.82272	.82272	.87538	.89354	.89572	.89572	.91598
20b	0.97530	96	.80	.88265	.92402	.93270	.94636	.95263	.95618	.95800
			.90	.86334	.90572	.90944	.91705	.92754	.92754	.93949
			.95	.82272	.82272	.87127	.89354	.89572	.89572	.91598
20c	0.97530	102	.80	.88265	.92373	.93511	.94636	.95263	.95618	.95800
			.90	.86334	.90563	.90944	.91705	.92754	.92754	.93949
			.95	.82272	.82272	.87127	.89354	.89572	.89572	.91598

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
21a	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9603	0.0158
					90.0	0.9631	-0.0122	0.9493	0.0137
					95.0	0.9472	-0.0281	0.9363	0.0126
21b	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9607	0.0157
					90.0	0.9600	-0.0153	0.9496	0.0136
					95.0	0.9472	-0.0281	0.9362	0.0128
21c	25	5.00	3.34	0.9753	80.0	0.9711	-0.0122	0.9492	0.0141
					90.0	0.9631	-0.0122	0.9492	0.0141
					95.0	0.9472	-0.0281	0.9360	0.0125

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values Minimum	10 %	25 %	Median	75 %	90 %	Maximum
21a	0.97530	8	.80	.91857	.94488	.95494	.96034	.97111	.97603	.97883
			.90	.90748	.93475	.94449	.94990	.95768	.96309	.96935
			.95	.89683	.91889	.93019	.93849	.94527	.94643	.95717
21b	0.97530	5	.80	.91860	.94488	.95494	.96138	.97111	.97603	.97883
			.90	.90752	.93475	.94449	.95183	.95768	.95998	.96935
			.95	.89687	.91889	.93019	.93849	.94527	.94643	.95717
21c	0.97530	3	.80	.91349	.94485	.95030	.96034	.97111	.97603	.97883
			.90	.90223	.93471	.94037	.94990	.95768	.96309	.96935
			.95	.89149	.91889	.93019	.93849	.94527	.94643	.95717

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
22a	60	2.70	1.62	0.8731	80.0	0.8262	-0.0469	0.7790	0.0679
					90.0	0.7512	-0.1219	0.6943	0.0634
					95.0	0.6521	-0.2209	0.5911	0.0660
22b	60	2.70	1.62	0.8731	80.0	0.8288	-0.0443	0.7811	0.0680
					90.0	0.7525	-0.1206	0.6953	0.0625
					95.0	0.6530	-0.2201	0.5899	0.0642
22c	60	2.70	1.62	0.8731	80.0	0.8246	-0.0484	0.7761	0.0676
					90.0	0.7478	-0.1253	0.6909	0.0616
					95.0	0.6495	-0.2236	0.5873	0.0636

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
22a	0.87306	0	.80	.52180	.69050	.73853	.78167	.81674	.85286	.99093
			.90	.46500	.62229	.66195	.69235	.72130	.75116	.98088
			.95	.37713	.52792	.56676	.59199	.61670	.63962	.96110
22b	0.87306	0	.80	.55336	.69440	.73977	.78298	.82177	.85451	.99474
			.90	.49445	.62650	.66268	.69292	.72486	.75247	.98890
			.95	.37971	.52699	.56710	.59095	.61793	.63880	.97731
22c	0.87306	0	.80	.52401	.69267	.73345	.78012	.81573	.84941	.99585
			.90	.46732	.62483	.65758	.69134	.72028	.74775	.99124
			.95	.37810	.52751	.56343	.58907	.61514	.63736	.98207

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
23a	60	3.37	2.02	0.8731	80.0	0.8410	-0.0321	0.7942	0.0570
					90.0	0.7785	-0.0945	0.7255	0.0499
					95.0	0.7003	-0.1728	0.6442	0.0464
23b	60	3.37	2.02	0.8731	80.0	0.8419	-0.0312	0.7949	0.0616
					90.0	0.7825	-0.0906	0.7270	0.0560
					95.0	0.7029	-0.1701	0.6467	0.0514
23c	60	3.37	2.02	0.8731	80.0	0.8391	-0.0340	0.7926	0.0593
					90.0	0.7767	-0.0964	0.7242	0.0523
					95.0	0.7001	-0.1729	0.6434	0.0471

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
23a	0.87306	0	.80	.59988	.72351	.75778	.79763	.83538	.85795	.97649
			.90	.54795	.66462	.69580	.72965	.75616	.77854	.95084
			.95	.45881	.59143	.62638	.65049	.67132	.69025	.90157
23b	0.87306	0	.80	.59403	.72129	.75771	.79692	.83623	.86128	.98131
			.90	.54167	.66402	.69603	.73001	.75780	.78248	.96081
			.95	.46142	.59407	.62577	.65024	.67127	.69244	.92110
23c	0.87306	0	.80	.62630	.71817	.75536	.79716	.83404	.85771	.96837
			.90	.57296	.66041	.69307	.73122	.75588	.77669	.93418
			.95	.46050	.58928	.62165	.64834	.67042	.68987	.86939

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
24a	60	6.75	4.04	0.8731	80.0	0.8640	-0.0091	0.8260	0.0442
					90.0	0.8445	-0.0286	0.7910	0.0433
					95.0	0.8118	-0.0613	0.7548	0.0402
24b	60	6.75	4.04	0.8731	80.0	0.8619	-0.0112	0.8239	0.0437
					90.0	0.8425	-0.0305	0.7890	0.0429
					95.0	0.8122	-0.0608	0.7531	0.0402
24c	60	6.75	4.04	0.8731	80.0	0.8573	-0.0158	0.8194	0.0445
					90.0	0.8408	-0.0323	0.7846	0.0439
					95.0	0.8090	-0.0641	0.7491	0.0413

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
24a	0.87306	0	.80	.66551	.77050	.79687	.82778	.85871	.88109	.93288
			.90	.63138	.73626	.76276	.79314	.82338	.84450	.88911
			.95	.60036	.70318	.72856	.75803	.78587	.80365	.89870
24b	0.87306	0	.80	.64083	.76983	.79571	.82485	.85311	.87970	.93627
			.90	.60736	.73544	.76107	.79011	.81862	.84252	.89290
			.95	.57719	.70207	.72747	.75513	.78299	.80259	.84319
24c	0.87306	0	.80	.66632	.76170	.79211	.82243	.84971	.87603	.93976
			.90	.63226	.72751	.75776	.78764	.81534	.84081	.89862
			.95	.60128	.69529	.72426	.75260	.77863	.80057	.83745

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
25a	60	13.5	8.09	0.8731	80.0	0.8679	-0.0052	0.8411	0.0324
					90.0	0.8609	-0.0122	0.8206	0.0328
					95.0	0.8519	-0.0212	0.8011	0.0328
25b	60	13.5	8.09	0.8731	80.0	0.8693	-0.0037	0.8426	0.0317
					90.0	0.8622	-0.0109	0.8221	0.0322
					95.0	0.8524	-0.0206	0.8026	0.0321
25c	60	13.5	8.09	0.8731	80.0	0.8678	-0.0053	0.8411	0.0331
					90.0	0.8616	-0.0115	0.8206	0.0335
					95.0	0.8529	-0.0202	0.8011	0.0333

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
25a	0.87306	0	.80	.73071	.79953	.82120	.84188	.86174	.88054	.92451
			.90	.70890	.77826	.80022	.82141	.84177	.86092	.90512
			.95	.68930	.75857	.78082	.80200	.82235	.84155	.88415
25b	0.87306	0	.80	.71861	.80235	.82333	.84342	.86329	.88198	.92267
			.90	.69672	.78122	.80252	.82297	.84318	.86215	.90405
			.95	.67713	.76145	.78301	.80349	.82390	.84271	.88429
25c	0.87306	0	.80	.73682	.79911	.81992	.84246	.86295	.88116	.92694
			.90	.71493	.77780	.79914	.82207	.84275	.86162	.90814
			.95	.69522	.75818	.77971	.80271	.82339	.84207	.88779

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
26a	60	3.58	2.14	0.8329	80.0	0.7999	-0.0330	0.7462	0.0671
					90.0	0.7262	-0.1067	0.6670	0.0580
					95.0	0.6341	-0.1988	0.5770	0.0510
26b	60	3.58	2.14	0.8329	80.0	0.8013	-0.0317	0.7480	0.0657
					90.0	0.7259	-0.1070	0.6687	0.0579
					95.0	0.6388	-0.1941	0.5786	0.0542
26c	60	3.58	2.14	0.8329	80.0	0.8016	-0.0313	0.7479	0.0683
					90.0	0.7291	-0.1038	0.6689	0.0606
					95.0	0.6389	-0.1941	0.5790	0.0550

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
26a	0.83293	0	.80	.50494	.65423	.70476	.75277	.79207	.82132	.99003
			.90	.44907	.58884	.63339	.67574	.70396	.72624	.97899
			.95	.38278	.51215	.55374	.58451	.60620	.62661	.95730
26b	0.83293	0	.80	.44914	.66803	.70691	.75246	.79448	.81974	.99182
			.90	.39837	.60232	.63512	.67374	.70225	.72594	.98276
			.95	.35423	.52336	.55745	.58320	.60475	.62804	.96488
26c	0.83293	0	.80	.49078	.65833	.70811	.75293	.79397	.82203	.99675
			.90	.43589	.59225	.63602	.67552	.70268	.72909	.99313
			.95	.38318	.52008	.55696	.58450	.60797	.62949	.98594

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
27a	60	4.47	2.68	0.8329	80.0	0.8104	-0.0226	0.7572	0.0615
					90.0	0.7577	-0.0752	0.6938	0.0546
					95.0	0.6870	-0.1459	0.6246	0.0459
27b	60	4.47	2.68	0.8329	80.0	0.8106	-0.0223	0.7589	0.0606
					90.0	0.7543	-0.0786	0.6954	0.0535
					95.0	0.6848	-0.1482	0.6257	0.0452
27c	60	4.47	2.68	0.8329	80.0	0.8126	-0.0203	0.7591	0.0624
					90.0	0.7601	-0.0729	0.6962	0.0565
					95.0	0.6901	-0.1428	0.6278	0.0478

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
27a	0.83293	0	.80	.50452	.67785	.71738	.75978	.79907	.83384	.94963
			.90	.45751	.62261	.65951	.69857	.73381	.75769	.89629
			.95	.41601	.56529	.59974	.63164	.65610	.67397	.79846
27b	0.83293	0	.80	.51810	.67842	.72067	.76371	.79991	.83328	.94105
			.90	.46947	.62258	.66315	.70358	.73528	.75428	.87922
			.95	.42629	.56412	.60144	.63689	.65631	.67342	.76751
27c	0.83293	0	.80	.54827	.67671	.72177	.76238	.80267	.83756	.97651
			.90	.49872	.61950	.66378	.70030	.73596	.76007	.95089
			.95	.45421	.56337	.60268	.63476	.65904	.67770	.90167

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
28a	60	8.94	5.36	0.8329	80.0	0.8221	-0.0108	0.7846	0.0459
					90.0	0.8066	-0.0263	0.7502	0.0456
					95.0	0.7856	-0.0473	0.7165	0.0439
28b	60	8.94	5.36	0.8329	80.0	0.8249	-0.0080	0.7854	0.0478
					90.0	0.8092	-0.0237	0.7510	0.0477
					95.0	0.7911	-0.0419	0.7172	0.0459
28c	60	8.94	5.36	0.8329	80.0	0.8251	-0.0078	0.7832	0.0473
					90.0	0.8080	-0.0249	0.7487	0.0470
					95.0	0.7845	-0.0484	0.7150	0.0453

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
28a	0.83293	0	.80	.63064	.72377	.75268	.78790	.81578	.84218	.94019
			.90	.59758	.68954	.71818	.75363	.78143	.80659	.89934
			.95	.56788	.65760	.68544	.72030	.74695	.77134	.83842
28b	0.83293	0	.80	.62536	.71988	.75456	.78770	.81720	.84323	.91227
			.90	.59199	.68591	.71996	.75361	.78241	.80921	.87495
			.95	.56204	.65447	.68714	.72032	.74817	.77337	.82952
28c	0.83293	0	.80	.62536	.72284	.75191	.78245	.81647	.84227	.91056
			.90	.59200	.68852	.71714	.74805	.78204	.80799	.87266
			.95	.56205	.65650	.68452	.71466	.74790	.77207	.82558

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
29a	60	17.9	10.7	0.8329	80.0	0.8269	-0.0060	0.8002	0.0326
					90.0	0.8198	-0.0131	0.7791	0.0329
					95.0	0.8111	-0.0219	0.7595	0.0329
29b	60	17.9	10.7	0.8329	80.0	0.8288	-0.0041	0.8002	0.0339
					90.0	0.8230	-0.0099	0.7791	0.0324
					95.0	0.8137	-0.0192	0.7595	0.0343
29c	60	17.9	10.7	0.8329	80.0	0.8299	-0.0030	0.8016	0.0344
					90.0	0.8227	-0.0103	0.7805	0.0348
					95.0	0.8156	-0.0173	0.7609	0.0348

Table D-1. Simulation Results (continued).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values Minimum	10 %	25 %	Median	75 %	90 %	Maximum
29a	0.83293	0	.80	.67933	.75979	.78117	.80152	.82098	.84042	.90200
			.90	.65728	.73825	.75985	.78054	.80016	.81979	.88303
			.95	.63776	.71851	.74026	.76102	.78049	.80022	.86379
29b	0.83293	0	.80	.67260	.75726	.77803	.80023	.82430	.84348	.88634
			.90	.65057	.73553	.75649	.77902	.80359	.82301	.86695
			.95	.63112	.71581	.73675	.75938	.78410	.80374	.84770
29c	0.83293	0	.80	.70176	.75812	.77888	.80255	.82497	.84331	.89342
			.90	.67973	.73646	.75741	.78138	.80425	.82268	.87374
			.95	.66021	.71679	.73774	.76162	.78484	.80304	.85396

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
30a	25	0.62	0.42	0.9753	80.0	0.9377	-0.0376	0.8894	0.0554
					90.0	0.9120	-0.0633	0.8156	0.1078
					95.0	0.8871	-0.0882	0.7159	0.1969
30b	25	0.62	0.42	0.9753	80.0	0.9377	-0.0376	0.8895	0.0554
					90.0	0.9120	-0.0633	0.8152	0.1083
					95.0	0.8871	-0.0882	0.7144	0.1983
30c	25	0.62	0.42	0.9753	80.0	0.9377	-0.0376	0.8917	0.0551
					90.0	0.9120	-0.0633	0.8203	0.1071
					95.0	0.8871	-0.0882	0.7243	0.1954

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
30a	0.97530	549	.80	.72345	.82358	.83590	.93765	.93765	.93765	.93765
			.90	.66288	.68406	.68406	.91201	.91201	.91201	.91201
			.95	.45815	.45815	.45815	.88707	.88707	.88707	.88707
30b	0.97530	550	.80	.75679	.82358	.83590	.93765	.93765	.93765	.93765
			.90	.68406	.68406	.68406	.91201	.91201	.91201	.91201
			.95	.45815	.45815	.45815	.88707	.88707	.88707	.88707
30c	0.97530	571	.80	.75679	.82358	.83590	.93765	.93765	.93765	.93765
			.90	.68406	.68406	.68406	.91201	.91201	.91201	.91201
			.95	.45815	.45815	.45815	.88707	.88707	.88707	.88707

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}*o$	D	Mean	S.D.
31a	25	1.25	0.84	0.9753	80.0	0.9683	-0.0070	0.9257	0.0313
					90.0	0.9550	-0.0203	0.8764	0.0554
					95.0	0.9418	-0.0335	0.8018	0.1076
31b	25	1.25	0.84	0.9753	80.0	0.9683	-0.0070	0.9247	0.0306
					90.0	0.9550	-0.0203	0.8732	0.0548
					95.0	0.9418	-0.0335	0.7943	0.1076
31c	25	1.25	0.84	0.9753	80.0	0.9683	-0.0070	0.9256	0.0317
					90.0	0.9550	-0.0203	0.8753	0.0560
					95.0	0.9418	-0.0335	0.7984	0.1091

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
31a	0.97530	311	.80	.83156	.88935	.90751	.91428	.96832	.96832	.96832
			.90	.79577	.82708	.82708	.86033	.95499	.95499	.95499
			.95	.67687	.67687	.67687	.79841	.94184	.94184	.94184
31b	0.97530	288	.80	.81518	.88935	.90751	.91428	.96832	.96832	.96832
			.90	.78157	.82708	.82708	.86033	.95499	.95499	.95499
			.95	.67687	.67687	.67687	.79841	.94184	.94184	.94184
31c	0.97530	309	.80	.81299	.88935	.90751	.91428	.96832	.96832	.96832
			.90	.77752	.82708	.82708	.86033	.95499	.95499	.95499
			.95	.67687	.67687	.67687	.79841	.94184	.94184	.94184

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
32a	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9470	0.0209
					90.0	0.9283	-0.0470	0.9214	0.0195
					95.0	0.9705	-0.0048	0.8841	0.0398
32b	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9473	0.0211
					90.0	0.9283	-0.0470	0.9218	0.0199
					95.0	0.9705	-0.0048	0.8847	0.0405
32c	25	2.50	1.67	0.9753	80.0	0.9562	-0.0191	0.9472	0.0217
					90.0	0.9395	-0.0358	0.9220	0.0208
					95.0	0.9705	-0.0048	0.8854	0.0408

Table D-1. Simulation Results (continued).

				Lower Confidence Limit Estimate Percentile Values						
Case	Rs	Null Runs	CL	Minimum	10 %	25 %	Median	75 %	90 %	Maximum
32a	0.97530	79	.80	.88240	.92417	.94305	.95263	.95618	.95618	.98403
			.90	.86306	.90590	.90944	.92156	.92754	.92832	.97724
			.95	.82272	.82272	.87577	.89354	.89572	.90735	.97049
32b	0.97530	86	.80	.88265	.92417	.93511	.95263	.95618	.95618	.98403
			.90	.86334	.90590	.90944	.92156	.92754	.92832	.97724
			.95	.82272	.82272	.88077	.89354	.89572	.90735	.97049
32c	0.97530	94	.80	.88265	.92402	.93511	.95263	.95618	.95800	.98403
			.90	.86334	.90572	.90944	.92156	.92754	.93949	.97724
			.95	.82272	.82272	.87577	.89354	.89572	.91598	.97049

Table D-2. Additional Simulation Output (continued)

Case	KK	TT	TM	Rs	CL	$\hat{R}^{*o}$	D	Mean	S.D.
33a	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9605	0.0160
					90.0	0.9631	-0.0122	0.9496	0.0140
					95.0	0.9472	-0.0281	0.9366	0.0133
33b	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9609	0.0158
					90.0	0.9631	-0.0122	0.9498	0.0138
					95.0	0.9472	-0.0281	0.9365	0.0132
33c	25	5.00	3.34	0.9753	80.0	0.9711	-0.0042	0.9604	0.0160
					90.0	0.9631	-0.0122	0.9494	0.0142
					95.0	0.9472	-0.0281	0.9362	0.0128

Table D-1. Simulation Results (concluded).

Case	Rs	Null Runs	CL	Lower Confidence Limit Estimate Percentile Values Minimum	10 %	25 %	Median	75 %	90 %	Maximum
33a	0.97530	8	.80	.91857	.94488	.95494	.96034	.97111	.97603	.99199
			.90	.90748	.93475	.94449	.94990	.95768	.96309	.98855
			.95	.89683	.91889	.93019	.93849	.94527	.94643	.98513
33b	0.97530	5	.80	.91860	.94488	.95494	.96138	.97111	.97603	.99199
			.90	.90752	.93475	.94449	.95183	.95768	.96309	.98855
			.95	.89687	.91889	.93019	.93849	.94527	.94643	.98513
33c	0.97530	3	.80	.91349	.94485	.95030	.96034	.97111	.97603	.99199
			.90	.90223	.93471	.94037	.94990	.95768	.96309	.98855
			.95	.89149	.91889	.93019	.93849	.94527	.94643	.98513

Table D-2. Additional Simulation Output (concluded)

APPENDIX E

REGRESSION OF OBSERVED ACCURACY

Examination of the tabulated data in Appendix D suggests that a linear relationship is not directly applicable to regression of the accuracy measure, d, on any of the other variables. The terms TT, developed in section IV.B.2, and KK, the number of components, appear to be the most likely candidate independent variables for the model. Plots of these terms versus D suggest that an exponential decreasing or inverse power curve could be used to model procedure accuracy on TT, i.e.

$$D = a \bullet \exp(b \bullet TT) \qquad (E.1)$$

$$D = a \bullet TT^{b} \qquad (E.2)$$

where a and b are appropriate constants from the underlying response function. Each of these models can be easily transformed into a linear model by taking the natural logarithm and redefining variables and coefficients where convenient.

The plots indicate, however, that the system configuration has a strong effect. Therefore, curve fitting will be initially confined to similar configurations in the following groups: Cases 01 to 12, Cases 14 to 17, and Cases 22 to 29. Cases 18 to 21 will be considered only with reservations due to the artificial bias introduced by the particular procedure used which ignored failureless runs.

Cases 30 to 33 will not be fitted since the procedure employed caused the sample to be drawn from two quite unlike distributions and the plots demonstrated an apparent discontinuity between the results of the two methods (see Section II.C and Appendix C). Finally, if the group fits are good, a composite fit will be attempted for Cases 01 to 17 and 22 to 29. This fit would not be expected to be good due to the observed inter-group plot variation.

A. EXPONENTIAL CURVE FIT

Consider the linearized exponential model

$$D' = A + b \bullet TT \tag{E.3}$$

where $D' = \ln(-D)$ and $A = \ln(-a)$. From the Method of Least Squares, expressions for A and b can be derived directly.

$$b = \frac{\sum_{i=1}^{n} TT_i D'_i - \frac{1}{n}\left[\sum_{i=1}^{n} TT_i\right]\left[\sum_{i=1}^{n} D_i\right]}{\sum_{i=1}^{n} TT_i^2 - \frac{1}{n}\left[\sum_{i=1}^{n} TT_i\right]^2}$$

$$A = \frac{1}{n}\left[\sum_{i=1}^{n} D'_i\right] - \frac{b}{n}\left[\sum_{i=1}^{n} TT_i\right]$$

and the coefficient of determination [Ref. 6] is

$$r^2 = \frac{\left[\sum_{i=1}^{n} TT_i D'_i - \frac{1}{n}\left[\sum_{i=1}^{n} D'_i\right]\left[\sum_{i=1}^{n} TT_i\right]\right]^2}{\left[\sum_{i=1}^{n} TT_i^2 - \frac{1}{n}\left[\sum_{i=1}^{n} TT_i\right]^2\right]\left[\sum_{i=1}^{n} D'^2_i - \frac{1}{n}\left[\sum_{i=1}^{n} D'_i\right]^2\right]}$$

The results of the exponential curve fit were as follows:

Cases	α	a	b	r^2
01-12	.20	-0.040	-0.065	0.690
	.10	-0.094	-0.076	0.724
	.05	-0.073	-0.048	0.738
14-17	.20	-0.118	-0.290	0.942
	.10	-0.301	-0.282	0.938
	.05	-0.480	-0.260	0.988
22-29	.20	-0.049	-0.156	0.896
	.10	-0.153	-0.170	0.899
	.05	-0.346	-0.216	0.917

B. INVERSE POWER CURVE FIT

The power curve may be linearized to the form

$$D' = A + b \bullet TT' \qquad (E.3)$$

where A and D' are defined as before and TT' = ln(TT). The coefficients, A and b, and the coefficient of determination may be found as described in the preceding section with the substitution of TT' for TT where occuring.

The results of the power curve fit were as follows:

Cases	α	a	b	r^2
01-12	.20	-0.130	-0.868	0.844
	.10	-0.368	-1.016	0.883
	.05	-0.780	-1.124	0.893
14-17	.20	-0.664	-2.085	0.986
	.10	-1.589	-2.016	0.972
	.05	-2.110	-1.828	0.992
22-29	.20	-0.159	-1.305	0.989
	.10	-0.543	-1.412	0.977
	.05	-1.041	-1.430	0.980
01-17	.20	-0.136	-1.060	0.648
22-29	.10	-0.459	-1.228	0.793
	.05	-0.936	-1.303	0.845

Comparing these results with those of the exponential curve fit indicates that the power curve provides a better fit for the observed simulation data. However, the variation in the coefficients between different system

configurations implies that, while the fit may be good within like groups, the fit is much more approximate across the groups. This is borne out by the composite fit listed last in the preceding table.

C. REGRESSION OF D ON TT AND KK

For this regression the model chosen was a combined power curve

$$D = a \bullet TT^{b} KK^{c} \qquad (E.5)$$

which linearizes to

$$D' = A + b \bullet TT' + c \bullet KK' \qquad (E.6)$$

where D', A, and TT' are defined as before and KK' = ln(KK).

Solving for the coefficient values by the Method of Least Squares yields the following:

$$c = \frac{\left[\frac{S1 \bullet S2}{n} - S6\right]\left[\frac{S1 \bullet S3}{n} - S8\right] - \left[\frac{S2 \bullet S3}{n} - S7\right]\left[\frac{S1^2}{n} - S4\right]}{\left[\frac{S1^2}{n} - S4\right]\left[\frac{S3^2}{n} - S5\right] - \left[\frac{S1 \bullet S3}{n} - S8\right]^2}$$

$$b = \frac{\left[\frac{S1 \bullet S2}{n} - S6\right] - c\left[\frac{S1 \bullet S3}{n} - S8\right]}{\frac{S1^2}{n} - S4}$$

$$a = \exp\left[\frac{1}{n}S2 - \frac{b}{n}S1 - \frac{c}{n}S3\right]$$

where

$$S1 = \sum_{i=1}^{n} TT'_i \qquad S2 = \sum_{i=1}^{n} D'_i \qquad S3 = \sum_{i=1}^{n} KK'_i$$

$$S4 = \sum_{i=1}^{n} TT'^2_i \qquad S5 = \sum_{i=1}^{n} KK'^2_i \qquad S6 = \sum_{i=1}^{n} \left[TT'_i \bullet D'_i\right]$$

$$S7 = \sum_{i=1}^{n} \left[D'_i \bullet KK'_i \right] \qquad \text{and} \qquad S8 = \sum_{i=1}^{n} \left[TT'_i \bullet KK'_i \right]$$

From the data for cases 01 through 17 and 22 through 29, the following values for a, b, and c were obtained:

α = 0.20	a = -0.080	b = -1.028	c = 0.129
α = 0.10	a = -0.386	b = -1.200	c = 0.033
α = 0.05	a = -1.038	b = -1.279	c = -0.043

An approximate coefficient of determination was calculated by defining

$$TM = TT \bullet KK^{c/b}$$

so that

$$D = a \bullet TM^{b}$$

which is the inverse power curve model and the two variable procedure of Section B can be used. This yielded an r^2 value of 0.626 for α = 0.2, which shows no improvement of fit for the three variable approach.

Appendix D lists TM values for α = 0.2 for information.

D. REGRESSION OF STANDARD DEVIATION

Examination of the sample standard deviation values listed in Appendix D also suggests that the power curve form would be appropriate to model the standard deviation on TT.

$$S.D. = a \bullet TT^{b}$$

Proceeding as in Section B, the following values are obtained:

$\alpha = 0.20$	$a = 0.094$	$b = -0.300$	$r^2 = 0.523$
$\alpha = 0.10$	$a = 0.075$	$b = -0.219$	$r^2 = 0.374$
$\alpha = 0.05$	$a = 0.072$	$b = -0.215$	$r^2 = 0.408$

which indicates a poor fit at best.

However, the data suggests that the sample standard deviation may be better predicted as a function of the number of component tests, i.e.

$$S.D. = a \bullet N^{b}$$

where N represents the "equivalent" number of system tests as derived in Section IV.B.2 for use in Equation (4.3).

The regression yields the following values:

$\alpha = 0.20$	$a = 0.213$	$b = -0.391$	$r^2 = 0.853$
$\alpha = 0.10$	$a = 0.186$	$b = -0.365$	$r^2 = 0.825$
$\alpha = 0.05$	$a = 0.164$	$b = -0.339$	$r^2 = 0.799$

This fit is better and implies that the primary mechanism to reduce the sample standard deviation is to increase the number of component tests. As the data base is limited, little more than that may be averred at this level of analysis.

APPENDIX F

COMPUTER PROGRAM

The following pages contain a representative listing of the main computer program utilized in the simulation. Three proprietary routines were used for binomial random variate generation (GGBIN), inverse Chi-Square value determination (MDCHI), and ordering of the estimate arrays (VSORTA). Reference 4 provides information concerning the contents and use of these routines. Equivalent routines could be used without penalty to either accyracy or consistence of the simulation results.

The program was written in the FORTRAN IV language for use on the IBM-360 computer system installed at the Naval Postgraduate School.

Portions of the program must be changed as the subassembly configurations are varied in order to tailor the computations to the specific system being modeled.

```
      DIMENSION EREL(1000,5),ZZ(99),KAPN(20),REL(100),FAIL(100),         SREL001
     1TRIAL(100),RHAT(100),PROB(5),XMEAN(5),XVAR(5),XSD(5)               SREL002
      DATA PROB/0.2,0.1,.05,0.9,0.9/                                     SREL003
      READ (5,5) JSD                                                     SREL004
    5 FORMAT(I10)                                                        SREL005
      JSED = JSD                                                         SREL006
C                                                                        SREL
C     K1 = NUMBER OF SINGLE COMPONENTS IN SERIES
C     K2 = NUMBER OF SUBASSEMBLIES IN SERIES
C                                                                        SREL
      READ (5,10) K1,K2                                                  SREL007
   10 FORMAT (2I5)                                                       SREL008
      NTOT = K1                                                          SREL009
      IF (K2.LT.1) GO TO 21                                              SREL010
C                                                                        SREL
C     KAPN(I) = NUMBER OF COMPONENTS IN THE I-TH SUBASSEMBLY
C                                                                        SREL
      READ (5,12) (KAPN(I), I=1,K2)                                      SREL011
   12 FORMAT (16I5)                                                      SREL012
      DO 20 I=1,K2                                                       SREL013
      NTOT = NTOT+KAPN(I)                                                SREL014
   20 CONTINUE                                                           SREL015
   21 CONTINUE                                                           SREL016
C                                                                        SREL
C     READ IN COMPONENT  RELIABILITIES
C                                                                        SREL
      K = K1+1                                                           SREL017
      READ (5,30) (REL(I), I=1,K1)                                       SREL018
      IF (K2.LT.1) GO TO 25                                              SREL019
      READ (5,30) (REL(I), I=K,NTOT)                                     SREL020
   25 CONTINUE                                                           SREL021
C                                                                        SREL
C    READ IN NUMBER OF TRIALS PERFORMED UPON EACH COMPONENT
C                                                                        SREL
      READ (5,30) (TRIAL(I),I=1,NTOT)                                    SREL022
   30 FORMAT (8F10.4)                                                    SREL023
      KNOT = 0                                                           SREL034
      T1 = 0.0                                                           SREL024
      T2 = 0.0                                                           SREL025
C                                                                        SREL
C     COMPUTE T VALUES FOR SERIES COMPONENTS
C                                                                        SREL
      DO 38  I=1,K1                                                      SREL025
      T1 = T1+TRIAL(I)*(1.0-REL(I))                                      SREL026
   38 CONTINUE                                                           SREL027
C                                                                        SREL
C     PERFORM 1000 SIMULATIONS OF COMPONENT TEST DATA AND ESTIMATE SYSTEM REL.
C                                                                        SREL
```

```
      DO 120 K=1,1000                                                   SREL035
   40 ZK1S = 0.0                                                        SREL036
      ZK2S = 0.0                                                        SREL037
      DO 45 N=1,K1                                                      SREL038
C                                                                       SREL
C     DETERMINE NUMBER OF FAILURES FOR THIS COMPONENT  (BINOMIALLY DISTRIBUTED)
C                                                                       SREL
      Q = 1.0-REL(N)                                                    SREL039
      NTEST = TRIAL(N)                                                  SREL040
      FAIL(N) = GGBIN(JSD,NTEST,Q)                                      SREL041
      FRAT = FAIL(N)/TRIAL(N)                                           SREL044
      AA = (2.0*TRIAL(N)-3.0)/(2.0*TRIAL(N)-2.0)                        SREL042
      BB = TRIAL(N)/(TRIAL(N)-1.0)                                      SREL043
      ZZ(N) = (AA+BB*FRAT/2.0)*FRAT                                     SREL045
      ZK1S = ZK1S+ZZ(N)/TRIAL(N)                                        SREL046
      ZK2S = ZK2S+ZZ(N)                                                 SREL047
   45 CONTINUE                                                          SREL048
      IF (K2.LT.1) GO TO 1101                                           SREL049
C                                                                       SREL
C     COMPUTE SUBASSEMBLY RESULTS
C                                                                       SREL
      KSUM = K1+K2                                                      SREL050
      NUM = K1+1                                                        SREL051
      KNUM = NUM                                                        SREL052
      DO 100 N=NUM,KSUM                                                 SREL053
      KK = N-K1                                                         SREL054
      JNUM = KNUM + KAPN(KK)-1                                          SREL055
      DO 55 NN=KNUM,JNUM                                                SREL056
      Q = 1.0-REL(NN)                                                   SREL057
      NTEST = TRIAL(NN)                                                 SREL058
      FAIL(NN) = GGBIN(JSD,NTEST,Q)                                     SREL059
      RHAT(NN) = 1.0 - FAIL(NN)/TRIAL(NN)                               SREL060
   55 CONTINUE                                                          SREL061
C
C     COMPUTE SUBASSEMBLY RELIABILITY ESTIMATES
C
      IF (KK.GT.1) GO TO 56                                             CASE22
      ARHAT = 1.0-(1.0-RHAT(KNUM)  *RHAT(KNUM+1)*RHAT (KNUM+2))*(1.0-   CASE22
     1RHAT(KNUM+3)*RHAT(KNUM+4)*RHAT(KNUM+5))                           CASE22
      GO TO 90                                                          CASE22
   56 IF (KK.GT.2) GO TO 57                                             CASE22
      ARHAT = 1.0-(1.0-RHAT(KNUM)*RHAT(KNUM+1))*(1.0-RHAT(KNUM+2)*      CASE22
     1RHAT(KNUM+3))*(1.0-RHAT(KNUM+4)*RHAT(KNUM+5))                     CASE22
      GO TO 90                                                          CASE22
   57 IF (KK.GT.3) GO TO 58                                             CASE22
      ARHAT = 1.0-(1.0-RHAT(KNUM)) * (1.0-RHAT(KNUM+1))                 CASE22
      GO TO 90                                                          CASE22
   58 IF (KK.GT.4) GO TO 59                                             CASE22
```

```
      A1 = RHAT(KNUM)*RHAT(KNUM+1)*RHAT(KNUM+2)*RHAT(KNUM+3)              CASE22
      A2 = RHAT(KNUM+3)*RHAT(KNUM+4)                                      CASE22
      A3 = (RHAT(KNUM+4)*RHAT(KNUM+5)*RHAT(KNUM+6))**2                    CASE22
      A4 = RHAT(KNUM)*RHAT(KNUM+1)*RHAT(KNUM+7)*RHAT(KNUM+8)              CASE22
      A5 = RHAT(KNUM+4)*RHAT(KNUM+8)                                      CASE22
      A6 = RHAT(KNUM+3)                                                   CASE22
      A7 = RHAT(KNUM+8)                                                   CASE22
      A8 = RHAT(KNUM+9)                                                   CASE22
      A9 = RHAT(KNUM+4)                                                   CASE22
      ARHAT = A1*A2*A3*A8*(1.0-(1.0-A6)*(1.0-A5*A7))                      CASE22
      ARHAT = (ARHAT+(1.0-A1)*A4*A5*A3*A8*(1.0-(1.0-A7)*(1.0-A2*A6)))*A9CASE22
      BRHAT = (1.0-A6)*A7+A6*(1.0-A7)                                     CASE22
      BRHAT = BRHAT*A1*A2*A3*A4*A5*A8                                     CASE22
      BRHAT = BRHAT+A6*A7*A8*(1.0-(1.0-A1)*(1.0-A4))*A2*A3*A5             CASE22
      ARHAT = ARHAT+(1.0-A9)*BRHAT                                        CASE22
      GO TO 90                                                            CASE22
   59 ARHAT = 0.0                                                         CASE22
      NNUM = KNUM-2                                                       CASE22
      DO 60 I=2,17                                                        CASE22
      ARHAT = ARHAT+RHAT(NNUM+I)**I                                       CASE22
   60 CONTINUE                                                            CASE22
      ARHAT = ARHAT/16.0                                                  CASE22
   90 ZZ(N) = 1.5+ARHAT*(ARHAT-4.0)/2.0                                   SREL077
      KNUM = KNUM+KAPN(KK)                                                SREL078
      ZK1S = ZK1S+ZZ(N)/TRIAL(N)                                          SREL079
      ZK2S = ZK2S+ZZ(N)                                                   SREL080
  100 CONTINUE                                                            SREL081
 1101 CONTINUE                                                            SREL082
      IF(ZK1S.GE.1.E-10) GO TO 105
C
C     PROCEDURE FOR S-HAT EQUAL ZERO
C
      KNOT = KNOT+1                                                       SCHG
C
C    FOLLOWING CARD FOR REITERATIVE CASE ONLY
C
//FORT.SYSPRINT  DD  DUMMY
 2101 N = 1                                                               SCHG
      XTRL = TRIAL(1)                                                     SCHG
      IF (K1.LT.2) GO TO 102                                              SCHG
      DO 101 I=2,K1                                                       SCHG
      IF (XTRL.GE.TRIAL(I)) GO TO 101                                     SCHG
      XTRL = TRIAL(I)                                                     SCHG
      N = I                                                               SCHG
  101 CONTINUE                                                            SCHG
  102 CONTINUE                                                            SCHG E
      DO 104 I=1,3                                                        SCHG A
      EREL(K,I) = PROB(I)**(1.0/TRIAL(N))                                 SCHG B
```

```
  104 CONTINUE                                                          SCHG C
      GO TO 112                                                         SCHG D
  105 ARG = ZK2S**2/ZK1S                                                SCHG
C
C     DETERMINE THE DEGREES OF FREEDOM FOR THE CHI-SQUARE VALUE
C
      DF = AMAX1(1.0,ARG)*2.0                                           SREL085
      ACF = DF                                                          SREL086
      DF = AINT(DF)                                                     SREL087
      ADF = ADF - DF                                                    SREL088
      IF (ADF.GT.0.0) DF=DF+1.0                                         SREL089
  108 DO 110 I=1,3                                                      SREL093
C
C     GET CHI-SQUARE VALUE
C
      CALL MDCHI(PROB(I),DF,CHISQ,IER)                                  SREL094
      EREL(K,I) = EXP(-DF*ZK2S/CHISQ)                                   SREL095
  110 CONTINUE                                                          SREL096
  112 CCNTINUE                                                          SREL096A
      EREL(K,4) = ZK1S                                                  SREL097
      EREL(K,5) = ZK2S                                                  SREL098
  120 CONTINUE                                                          SREL099
C     WRITE (6,114)                                                     SREL100
  114 FCRMAT(' UNORDERED RESULTS')                                      SREL101
C     WRITE (6,115)    (K,(EREL(K,I),I=1,5),K=1,1000)                   SREL102
  115 FORMAT (2(4X,I4,1X,5(G10.4,1X)))                                  SREL103
      LA = 1000                                                         SREL104
      DO 130  I=1,3                                                     SREL105
C
C     ORDER THE ESTIMATES
C
      CALL VSORTA(EREL(1,I),LA)                                         SREL106
  130 CCNTINUE                                                          SREL107
C
C     CCMPUTE TRUE SYSTEM RELIABILITY
C
      SREL = 1.0                                                        SREL108
      DO 135 N=K1,NTOT                                                  CASE22
      RHAT(N) = REL(N)                                                  CASE22
  135 CONTINUE                                                          CASE22
      DO 140 N=1,K1                                                     SREL109
      SREL = SREL*REL(N)                                                SREL110
  140 CONTINUE                                                          SREL111
      IF (K2.LT.1) GO TO 147                                            SREL112
      KNUM = NUM                                                        SREL113
      DO 146 N=NUM,KSUM                                                 SREL114
      KK = N-K1                                                         SREL115
      IF (KK.GT.1) GO TO 141                                            CASE22
```

```
      ARHAT = 1.0-(1.0-RHAT(KNUM)  *RHAT(KNUM+1)*RHAT  (KNUM+2))*(1.0-    CASE22
     1RHAT(KNUM+3)*RHAT(KNUM+4)*RHAT(KNUM+5))                            CASE22
      SREL = SREL*ARHAT                                                   CASE22
      T2 = TRIAL(K1+1)*(1.0-REL(K1+1)) + TRIAL(K1+2)*(1.0-REL(K1+2))
     1   + TRIAL(K1+3)*(1.0-REL(K1+3))
      T2 = T2/(1.0-REL(K1+1)*REL(K1+2)*REL(K1+3))
      T3 = TRIAL(K1+4)*(1.0-REL(K1+4)) + TRIAL(K1+5)*(1.0-REL(K1+5))
     1   + TRIAL(K1+6)*(1.0-REL(K1+6))
      T3 = T3/(1.0-REL(K1+4)*REL(K1+5)*REL(K1+6))
      T2 = SQRT(T2*T3)*(1.0-ARHAT)
      GO TO 145                                                           CASE22
  141 IF (KK.GT.2) GO TO 142                                              CASE22
      ARHAT = 1.0-(1.0-RHAT(KNUM)*RHAT(KNUM+1))*(1.0-RHAT(KNUM+2)*       CASE22
     1RHAT(KNUM+3))*(1.0-RHAT(KNUM+4)*RHAT(KNUM+5))                      CASE22
      SREL = SREL*ARHAT                                                   CASE22
      T3 = TRIAL(K1+7)*(1.0-REL(K1+7)) + TRIAL(K1+8)*(1.0-REL(K1+8))
      T3 = T3/(1.0-REL(K1+7)*REL(K1+8))
      T4 = TRIAL(K1+9)*(1.0-REL(K1+9)) + TRIAL(K1+10)*(1.0-REL(K1+10))
      T4 = T4/(1.0-REL(K1+9)*REL(K1+10))
      T5 = TRIAL(K1+11)*(1.0-REL(K1+11))+TRIAL(K1+12)*(1.0-REL(K1+12))
      T5 = T5/(1.0-REL(K1+11)*REL(K1+12))
      T2 = T2 + EXP(ALOG(T3*T4*T5)/3.0)*(1.0-ARHAT)
      GO TO 145                                                           CASE22
  142 IF (KK.GT.3) GO TO 143                                              CASE22
      ARHAT = 1.0-(1.0-RHAT(KNUM)) * (1.0-RHAT(KNUM+1))                   CASE22
      SREL = SREL*ARHAT                                                   CASE22
      T2 = T2 + SQRT(TRIAL(K1+13)*TRIAL(K1+14))*(1.0-ARHAT)
      GO TO 145                                                           CASE22
  143 IF (KK.GT.4) GO TO 144                                              CASE22
      A1 = RHAT(KNUM)*RHAT(KNUM+1)*RHAT(KNUM+2)*RHAT(KNUM+3)              CASE22
      A2 = RHAT(KNUM+3)*RHAT(KNUM+4)                                      CASE22
      A3 = (RHAT(KNUM+4)*RHAT(KNUM+5)*RHAT(KNUM+6))**2                    CASE22
      A4 = RHAT(KNUM)*RHAT(KNUM+1)*RHAT(KNUM+7)*RHAT(KNUM+8)              CASE22
      A5 = RHAT(KNUM+4)*RHAT(KNUM+8)                                      CASE22
      A6 = RHAT(KNUM+3)                                                   CASE22
      A7 = RHAT(KNUM+8)                                                   CASE22
      A8 = RHAT(KNUM+9)                                                   CASE22
      A9 = RHAT(KNUM+4)                                                   CASE22
      ARHAT = A1*A2*A3*A8*(1.0-(1.0-A6)*(1.0-A5*A7))                      CASE22
      ARHAT = (ARHAT+(1.0-A1)*A4*A5*A3*A8*(1.0-(1.0-A7)*(1.0-A2*A6)))*A9CASE22
      BRHAT = (1.0-A6)*A7+A6*(1.0-A7)                                     CASE22
      BRHAT = BRHAT*A1*A2*A3*A4*A5*A8                                     CASE22
      BRHAT = BRHAT+A6*A7*A8*(1.0-(1.0-A1)*(1.0-A4))*A2*A3*A5             CASE22
      ARHAT = ARHAT+(1.0-A9)*BRHAT                                        CASE22
      SREL = SREL*ARHAT                                                   CASE22
      T2 = T2 + TRIAL(K1+15)*(1.0-ARHAT)
      GO TO 145                                                           CASE22
  144 ARHAT = 0.0                                                         CASE22
```

```
      NNUM = KNUM-2                                                     CASE22
      DO 1144 I=2,17                                                    CASE22
      ARHAT = ARHAT+RHAT(NNUM+I)**I                                     CASE22
 1144 CONTINUE                                                          CASE22
      T2 = T2 + TRIAL(K1+25)*(1.0-ARHAT/16.0)
      SREL = SREL*ARHAT/16.0                                            CASE22
  145 KNUM = KNUM+KAPN(KK)                                              SREL132
  146 CONTINUE                                                          SREL133
  147 CONTINUE                                                          SREL134
C                                                                       SREL
C     COMPUTE TOTAL TT VALUE FOR SYSTEM
C                                                                       SREL
      TT = T1+T2                                                        SREL033
      DO 149 J=1,3                                                      SREL135
      XMEAN(J) = 0.0                                                    SREL136
      XVAR(J) = 0.0                                                     SREL137
  149 CONTINUE                                                          SREL138
C
C     COMPUTE SAMPLE MEAN AND VARIANCE OF THE ESTIMATES
C
      DO 160 I=1,1000                                                   SREL139
      DO 150 J=1,3                                                      SREL140
      XMEAN(J) = XMEAN(J)+EREL(I,J)                                     SREL141
      XVAR(J) = XVAR(J)+EREL(I,J)**2                                    SREL142
  150 CONTINUE                                                          SREL143
  160 CONTINUE                                                          SREL144
      DO 170 J=1,3                                                      SREL145
      XVAR(J) = (XVAR(J)-XMEAN(J)**2/1000.0)/999.0                      SREL146
      XMEAN(J) = XMEAN(J)/1000.0                                        SREL147
      XSD(J) = SQRT(XVAR(J))                                            SREL148
  170 CONTINUE                                                          SREL149
      WRITE (6,1006) JSED                                               SREL150
 1006 FORMAT ('1STARTING SEED:',I12//)                                  SREL151
      WRITE (6,194) JSD                                                 SREL187
  194 FORMAT(' FINAL SEED: ',I12)                                       SREL188
      WRITE (6,1035)                                                    SREL152
 1035 FORMAT ('      COMPNT   RELIABILITY   TRIALS      COMPNT   RELIABISREL153
     1LITY   TRIALS       COMPNT   RELIABILITY   TRIALS')               SREL154
      WRITE (6,1036) (I,REL(I),TRIAL(I), I=1,NTOT)                      SREL155
 1036 FORMAT(3(8X,I4,7X,F7.3,5X,F5.1))                                  SREL156
      WRITE (6,178) SREL                                                SREL157
  178 FORMAT(1H0,20X,'COMPUTATION/SIMULATION RESULTS'///' TRUE SYSTEM RESREL158
     1LIABILITY:',F10.5/)                                               SREL159
      WRITE (6,179) T1,T2,TT                                            SREL160
  179 FORMAT (' T1:',F10.3,5X,'T2:',F10.3,5X,'TT:',F10.3/)              SREL161
      DO 1179 I=1,3                                                     SREL162
      PROB(I) = 1.0-PROB(I)                                             SREL163
 1179 CONTINUE                                                          SREL164
```

```
      WRITE (6,180)(PROB(I),I=1,3)                                      SREL165
  180 FORMAT (' CONFIDENCE LEVEL',16X,':',3X,3(F8.2,4X)/)               SREL166
      WRITE (6,182) EREL(800,1),EREL(900,2),EREL(950,3)                 SREL167
  182 FORMAT (' CONF. LEVEL PERCENTILE'/' ESTIMATED RELIABILITY',11X,':'SREL168
     1,2X,3F12.5/)                                                      SREL169
      WRITE (6,184) (XMEAN(I), I=1,3)                                   SREL170
  184 FORMAT (' ATTRIBUTES OF',8X,'SAMPLE MEAN:   ',5F12.5)             SREL171
      WRITE (6,186) (XVAR(I), I=1,3)                                    SREL172
  186 FORMAT (' THE ESTIMATED    SAMPLE VARIANCE:  ',5F12.5)            SREL173
      WRITE (6,188) (XSD(I), I=1,3)                                     SREL174
  188 FORMAT (' RELIABILITIES     SAMPLE STD DEV:  ',3F12.5/)           SREL175
      WRITE (6,190) (EREL(1,I),I=1,3),(EREL(250,I),I=1,3),(EREL(500,I), SREL176
     1I=1,3),(EREL(750,I),I=1,3),(EREL(1000,I),I=1,3)                   SREL177
  190 FORMAT(' OTHER',20X,'MINIMUM:   ',3F12.5/' OBSERVED',9X,'25TH PERCESREL178
     1NTILE:  ',3F12.5/' SAMPLE',20X,                                   SREL179
     $                   'MEDIAN: ',3F12.5/' POINT',12X,'75TH',   '     SREL180
     2PERCENTILE:  ',3F12.5/' VALUES',19X,'MAXIMUM:  ',3F12.5/)         SREL181
      XSD(1) = EREL(800,1) -SREL                                        SREL182
      XSD(2) = EREL(900,2) -SREL                                        SREL183
      XSD(3) = EREL(950,3) -SREL                                        SREL184
      WRITE(6,192)(XSD(I),I=1,3)                                        SREL185
  192 FORMAT(' DIFFERENCE OF ESTIMATE FROM TRUE:  ',3F12.5/)            SREL186
      WRITE (6,196) KNOT                                                SREL189
  196 FORMAT(' NUMBER OF RUNS WITHOUT FAILURES:',I6)                    SREL190
      WRITE (6,191) (EREL(100,I),I=1,3),(EREL(900,I),I=1,3)             SREL190A
  191 FORMAT (1H0,17X,'10TH PERCENTILE:  ',3F12.5/1H ,17X,'90TH PERCENTISREL190B
     1LE:  ',3F12.5/)                                                   SREL190C
C     WRITE (6,198)                                                     SREL191
  198 FORMAT ('1ORDERED RESULTS:'//' %-TILE    80% CL    90% CL    95% CSREL192
     1L',9X,'%-TILE    80% CL    90% CL    95% CL')                     SREL193
C     WRITE (6,200) (I,(EREL(I,K),K=1,3),I=1,1000)                      SREL194
  200 FORMAT (1H ,2(I6,3F10.5,9X))                                      SREL195
      STOP                                                              SREL196
      END                                                               SREL197
```

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